



Flow in a channel with two facing cavities

Florian Tuerke

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NNT : 2017SACLS078



Comprendre le monde,
construire l'avenir

THÈSE DE DOCTORAT
DE
L'UNIVERSIDAD DE BUENOS AIRES
ET DE
L'UNIVERSITÉ PARIS-SACLAY
PRÉPARÉE À L'UNIVERSITÉ PARIS-SUD

Ecole doctorale n°579

Sciences mécaniques et énergétiques, matériaux et géosciences

Spécialité de doctorat: Mécanique des Fluides

par

M. FLORIAN ALEXANDER TUERKE

Interaction entre deux cavités opposées dans un canal
hydrodynamique

sous la direction de

Prof. Dr. Guillermo Artana, Dr. François Lusseyran et Dra. Denisse Sciamarella

Thèse présentée et soutenue à Buenos Aires, le 7 avril 2017.

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Titre : Interaction entre deux cavités opposées dans un canal hydrodynamique

Mots clefs : cavité, double cavité, instabilité linéaire

Résumé : Dans ce travail, nous étudions l'écoulement au sein d'un canal symétrique avec une expansion et une contraction soudaines. Cette configuration peut-être considérée comme constituée de deux cavités face à face, deux cavités en miroir, que nous dénommons "double cavité". Le sujet est traité expérimentalement, numériquement et analytiquement, en faisant varier la vitesse d'entrée et de la distance entre cavités, mais en restant à des nombres de Reynolds modérés. L'accent est mis sur l'interaction entre les deux couches de cisaillement et sur le mécanisme de rétroaction intracavitaire dans la limite des écoulements incompressibles. Expérimentalement, on mesure la vitesse par Vélocimétrie par Images de Particules non résolue en temps (PIV 2D2C) dans un plan longitudinal permettent de quantifier le champ de vitesse en moyenne temporelle. Par ailleurs, des mesures par Vélocimétrie Laser à effet Doppler (LDV) et des mesures résolues en temps par PIV 2D2C permettent d'accéder à la composition spectrale des fluctuations de vitesse dans la direction de l'écoulement. L'écoulement est caractérisé à partir des séries temporelles, enregistrées dans les couches de cisaillement d'une des deux cavités, pour une large gamme de vitesses d'entrée et des distances entre cavités. Des simulations numériques directes 2D et 3D, per-

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Title : Flow in a channel with two facing cavities

Keywords : cavity flow, double cavity flow, linear instability

Abstract : This work investigates the flow in a symmetric channel with a sudden expansion and contraction, creating two facing cavities, a so called double cavity. Double cavity flow at moderate Reynolds numbers is studied experimentally, numerically and analytically, as the inflow velocity and the distance between the cavities are varied. The focus is put on the interaction of the two shear layers and the intracavitary hydrodynamic feedback in the incompressible limit. Experimentally, standard 2D2C particle image velocimetry (PIV) measurements in a given spanwise plane provide information on the instantaneous and mean velocity flow fields. Laser Doppler velocimetry and time resolved 2D2C PIV measurements reveal the richness of the streamwise fluctuating velocity spectra. The flow is characterized based on times series, recorded in one of the cavity's shear layers, for a wide range of inflow velocities and cavity distances. Two dimensional and three dimensional direct numerical simulations, which give easy access to the entire flow field, are used to study the intracavitary hydrodynamic feedback mechanism. Vorticity fields,

obtained from 2D numerical simulations, show the importance of the recirculating intracavitary back flow. Vorticity packages, injected by the oscillating and impinging shear layer at the downstream cavity edge, are advected upstream in the recirculation region, creating a “carousel-like” pattern. The interaction of this vortex carousel with the oscillating shear layer is found to be responsible for the self-sustained oscillations observed experimentally in single and double cavity flow. The quantification of three characteristic time scales of the rotation allows to identify in which regime the flow resides. Temporal and spatio-temporal inviscid linear stability analyses are applied to a one dimensional base flow of single and double cavity flows. To account for the finite extent of the system, the spatio-temporal linear stability analysis is conditioned by a so called Kulikowskii condition, which allows the reflection of hydrodynamic instability waves within the cavity domain. This feedback mechanism yields a set of discrete, non-harmonic frequencies, some of which compare well with experimental results.

The Defense

The defense took place in Buenos Aires, Argentina
on April 7th, 2017

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Preface

This work has been carried out in the framework of a joint affiliation (“cotutela” in Spanish or “cotutelle” in French), involving both Universidad de Buenos Aires in Argentina and Université Paris-Saclay (Paris-Sud XI) in Orsay in France. The research was conducted over a period of five years, of which four years were spent at the Laboratorio de Fluidodinámica de la Facultad de Ingeniería, and one year at the Laboratoire d’Informatique pour la Mécanique et les Sciences de l’Ingénieur (LIMSI).

Through such a collaboration, different aspects of the subject were considered, using different approaches and techniques, depending on the expertise of each laboratory. Hence, the thesis includes different aspects of double cavity flow, which combine to yield a global picture of the flow physics encountered. The research was realized under the co-supervision of Prof. Dr. Guillermo Artana (LFD-FIUBA), Dra. Denisse Sciamarella (LIMSI / LFD) and Dr. François Lusseyran (LIMSI). Frequent consultations were held with Dr. Luc Pastur (LIMSI).

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Declaration of Authorship

I, Florian Tuerke, declare that this thesis titled, “Flow in a channel with two facing cavities” and the work presented in it are my own. I confirm that:

- This work was done wholly while in candidature for a research degree at the participating Universities.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Florian Tuerke

December 2016

“No te salves ahora. Ni nunca.”

Mario Benedetti

UNIVERSIDAD DE BUENOS AIRES &
UNIVERSITÉ PARIS-SACLAY
(PARIS-SUD XI)

Abstract

Laboratorio de Fluidodinámica de la Facultad de Ingeniería
Laboratoire d'Informatique pour la Mécanique et les Sciences de l'Ingénieur

PhD Thesis

Flow in a channel with two facing cavities

by Florian Tuerke

This work investigates the flow in a symmetric channel with a sudden expansion and contraction, creating two facing cavities, a so called double cavity. Double cavity flow at moderate Reynolds numbers is studied experimentally, numerically and analytically, as the inflow velocity and the distance between the cavities are varied. The focus is put on the interaction of the two shear layers and the intracavitary hydrodynamic feedback in the incompressible limit. Experimentally, standard 2D2C particle image velocimetry (PIV) measurements in a given spanwise plane provide information on the instantaneous and mean velocity flow fields. Laser Doppler velocimetry and time resolved 2D2C PIV measurements reveal the richness of the streamwise fluctuating velocity spectra. The flow is characterized based on times series, recorded in one of the cavity's shear layers, for a wide range of inflow velocities and cavity distances. Two dimensional and three dimensional direct numerical simulations, which give easy access to the entire flow field, are used to study the intracavitary hydrodynamic feedback mechanism. Vorticity fields, obtained from 2D numerical simulations, show the importance of the recirculating intracavitary back flow. Vorticity packages, injected by the oscillating and impinging shear layer at the downstream cavity edge, are advected upstream in the recirculation region, creating a “carousel-like” pattern. The interaction of this vortex carousel with the oscillating shear layer is found to be responsible for the self-sustained oscillations observed experimentally in single and double cavity flow. The quantification of three characteristic time scales of the rotation allows to identify in which regime the flow resides. Temporal and spatio-temporal inviscid linear stability analyses are applied to a one dimensional base flow of single and double cavity flows. To account for the finite extent of the system, the spatio-temporal linear stability analysis is conditioned by a so called Kulikowskii condition, which allows the reflection of hydrodynamic instability waves within the cavity domain. This feedback mechanism yields a set of discrete, non-harmonic frequencies, some of which compare well with experimental results.

Dans ce travail, nous étudions l'écoulement au sein d'un canal symétrique avec une expansion et une contraction soudaines. Cette configuration peut-être considérée comme constituée de deux cavités face à face, deux cavités en miroir, que nous dénommons "double cavité". Le sujet est traité expérimentalement, numériquement et analytiquement, en faisant varier la vitesse d'entrée et de la distance entre cavités, mais en restant à des nombres de Reynolds modérés. L'accent est mis sur l'interaction entre les deux couches de cisaillement et sur le mécanisme de rétroaction intracavitaire dans la limite des écoulements incompressibles. Expérimentalement, on mesure la vitesse par Vélocimétrie par Images de Particules non résolue en temps (PIV 2D2C) dans un plan longitudinal permettant de quantifier le champ de vitesse en moyenne temporelle. Par ailleurs, des mesures par Vélocimétrie Laser à effet Doppler (LDV) et des mesures résolues en temps par PIV 2D2C permettent d'accéder à la composition spectrale des fluctuations de vitesse dans la direction de l'écoulement. L'écoulement est caractérisé à partir des séries temporelles, enregistrées dans les couches de cisaillement d'une des deux cavités, pour une large gamme de vitesses d'entrée et des distances entre cavités. Des simulations numériques directes 2D et 3D, permettent d'étudier le mécanisme hydrodynamique de rétroaction intracavitaire, à partir des champs de vitesse complet. Le champ de vorticité issu des simulations numériques 2D montre l'importance de la rotation d'ensemble au sein de la cavité qui transporte les injections de vorticité induites par les oscillations de la couche de mélange conduisant à une structure de type "carrousel" elle même à l'origine du mécanisme de rétroaction responsable des oscillations auto-entretenues de la couche de cisaillement. La quantification des temps caractéristiques de cette rotation permet d'identifier le régime dans lequel se trouve l'écoulement. Une analyse de stabilité en temps seul, ainsi qu'en temps et espace est réalisée pour des écoulements non visqueux, en prenant un écoulement de base unidimensionnel pour chacun des cas: cavité simple ou double. Pour prendre en compte l'extension finie du système, dans le cas de l'analyse de stabilité linéaire spatio-temporel, on ajoute la condition dite de Kulikowskii, qui permet de prendre en compte la réflexion des ondes d'instabilité hydrodynamique aux bornes du domaine de la cavité. Ce mécanisme de rétroaction produit un ensemble discret de fréquences non-harmoniques, dont certaines correspondent effectivement aux données expérimentales.

Für
meine Eltern.

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I feel honored to having gathered such an outstanding academic jury for my thesis defense. I would like to thank Dr. José Eduardo Wesfreid, Dr. Jorge Silvestrini, Dr. Norberto Nigro, Dr. Luc Pastur, Dr. Pablo Cobelli and Dr. Alejandro Otero for having accepted to evaluate my work. I am looking forward to their review and comments.

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Contents

The Defense	iv
Preface	v
Declaration of Authorship	vi
Abstract	x
Acknowledgements	xiv
Contents	xvii
List of Figures	xxi
List of Tables	xxiv
Abbreviations	xxv
1 Introduction	2
1.1 Background	2
1.2 The double cavity	4
1.3 Objectives	8
1.4 Organization of the thesis	9
I Methodologies and Results	11
2 Experimental Setup	12
2.1 Introduction and background	12
2.2 Wind tunnel facilities	15
2.3 PIV and LDV measurements	17
2.3.1 PIV Paris-campaign	17
2.3.2 PIV BA-campaign	18
2.3.3 LDV	20
2.4 Uncertainty analysis	21
2.4.1 LDV	21
2.4.2 PIV	23

3	Experimental Results	25
3.1	Flow description	25
3.1.1	Instantaneous flow fields	25
3.1.2	Symmetry properties	29
3.1.3	Mean flow fields	31
3.1.4	Vorticity flow field	32
3.2	Velocity profiles	33
3.2.1	Blasius fit	39
3.3	Spectral Results	40
3.3.1	Comparison: Paris vs. Buenos Aires	40
3.3.2	Global spectra	41
3.3.3	Local spectra	44
3.3.4	Detailed spectral composition	48
3.3.5	Y-offset graph	50
3.4	Concluding remarks	50
4	Numerical Setup	53
4.1	Numerical simulation code	54
4.2	Setup 2D simulations	56
4.3	Setup 3D Simulations	59
5	Numerical Results	63
5.1	Introduction	63
5.2	2D results: A nonlinear dynamical system analysis	63
5.2.1	Time series	63
5.2.2	Phase space analysis	65
5.2.3	Minimal phase space dimension	67
5.2.4	Evidence of chaotic behavior	68
5.2.5	Parameter space	71
5.2.6	Spectral results	71
5.2.7	Intermittency	73
5.2.8	Velocity and vorticity flow fields	74
5.2.9	Concluding remarks	75
5.3	3D results	76
5.3.1	Spectral results	76
5.3.2	Single vs. double cavity	78
5.3.3	Velocity and Vorticity flow fields	79
5.3.4	Chaos and turbulence	79
5.3.5	Concluding remarks	82
5.4	Comparison: Simulations and experiments	83
5.4.1	Comparison of the inflow conditions	83
5.4.2	Velocity flow fields	83
5.4.3	Fluctuating velocity intensity fields	89
5.4.4	Spectra	89
5.4.5	Concluding remarks	91

II	Analysis and Discussion	93
6	Linear stability analysis	94
6.1	Introduction and background	94
6.2	The linear model	96
6.2.1	The computational domains	98
6.2.2	The base flows	99
6.2.3	The effective momentum thickness	100
6.2.4	The dispersion relation	102
6.3	Temporal linear stability analysis	102
6.3.1	Results and Discussion	104
6.3.2	Concluding remarks: temporal analysis	110
6.4	The spatio-temporal analysis	110
6.4.1	Amplification and non-transparency	110
6.4.2	Asymptotic behavior of the response of the system	111
6.5	The Finite Domain	113
6.5.1	Reflection coefficients	115
6.5.2	Interpretation of the Kulikowskii Condition	116
6.5.3	The infinite limit	117
6.5.4	Evaluation of the Kulikowskii Condition	118
6.6	Results for single cavity flow	119
6.6.1	Evaluation of the Kulikowskii Condition	119
6.6.2	Evolution of Kulikowskii Modes in space and time	123
6.6.3	Eigenfunctions of Kulikowskii Modes	123
6.6.4	Phase speed of Kulikowskii Modes	124
6.6.5	The total growth rate in the finite domain	125
6.6.6	Discussion	128
6.7	Results for double cavity flow	130
6.7.1	Evaluation of the Kulikowskii Condition	131
6.7.2	Results	133
6.8	Effect of slowly varying base flow	135
6.8.1	The slowly varying base flow	136
6.8.2	The Kulikowskii condition for a slowly varying base flow	139
6.8.3	Results	140
6.9	Conclusions	144
7	The Vortex Carousel	147
7.1	Introduction	147
7.2	The vorticity flow field	148
7.3	Characteristic time scales	150
7.4	Flow field dynamics of the periodic regime	150
7.5	Flow field dynamics of the intermediate regime	151
7.6	Coupling Strength	154
7.7	The outflow pattern	156
7.8	Experimental example	157
7.9	Bio-inspired example	159

7.9.1	Background	159
7.9.2	Setup	159
7.9.3	Results	161
7.10	Conclusions	163
8	Conclusions and Outlook	165
9	Summaries	171
9.1	Resumen en Español	171
9.2	Résumé en Français	173
	Bibliography	176
A	Linear stability code	183
B	Blasius boundary layer fit	188

List of Figures

1.1	Sketch of a double cavity	5
1.2	Flow pattern of steady flow	6
1.3	Flow pattern of jet-like case	6
1.4	Flow pattern double cavity flow	7
2.1	Sketch of experimental setup	14
2.2	Wind tunnel setup LIMSI	14
2.3	Wind tunnel setup LFD	15
2.4	Picture of the double cavity	15
2.5	Schematic top view of double cavity	16
2.6	Variances of the mean and the rms	21
2.7	Probability density functions	23
3.1	Inst. flow fields from Paris-campaign	26
3.2	Inst. flow fields from BA-campaign	27
3.3	QR-code for double cavity flow	28
3.4	Experimental correlation plots	30
3.5	Mean velocity field from BA-campaign	31
3.6	Vorticity contour plot from experimental data	32
3.7	Experimental and Blasius BL profiles	34
3.8	Momentum thickness of Paris-campaign	34
3.9	Momentum thickness of BA-campaign	35
3.10	Fit of sinh-profile to experimental data	37
3.11	Normalized vorticity thickness $\delta_\omega/\delta_{\omega 0}$	38
3.12	Blasius BL profile fitting	39
3.13	Comparison Paris-BA	41
3.14	Power spectra at multiple locations	42
3.15	Mean flow field of $D/L = 0.4$ at $Re_L = 8900$	43
3.16	Power spectra for all cases from Paris-campaign	45
3.17	Detailed shear layer spectra from Paris campaign	47
3.18	Y offset graph of power spectral density	49
4.1	Staggered 2D grid	54
4.2	Computational domain 2D simulations	57
4.3	Inflow velocity profiles for 2D simulations	57
4.4	3D double cavity domain	59

4.5	3D MPI partitioning	59
4.6	Mesh for 3D simulations	60
4.7	Probe point location for 3D simulations	62
5.1	2D spectra from various probe points	64
5.2	Phase portraits from 2D simulation data	65
5.3	Phase space dimensions	68
5.4	Auto-correlation plots	69
5.5	2D chaotic time series and spectra	70
5.6	Parameter space plot 2D simulations	70
5.7	Bifurcation diagrams 2D simulations	72
5.8	Examples of intermittency	73
5.9	Typical inst. velocity fields from 2D simulation	74
5.10	Power spectra of 3D cases	77
5.11	Shear layer spectra: single vs double cavity	78
5.12	Vorticity flow field of 3D numerical simulation	80
5.13	Vorticity field of $L/H = 1$ case	80
5.14	Power Spectra for 3D case with $L/H = 1$	81
5.15	Comparison of momentum thickness	84
5.16	Flow field comparison simulations and experiments	86
5.17	RMS field comparison simulations and experiments	88
5.18	Comparison 2D and 3D spectra	90
5.19	Comparison of experimental and numerical spectra	91
6.1	Schematic view of cavity geometry for linear stability analysis	98
6.2	Hyperbolic tangent base profile	99
6.3	Hyperbolic sine base profile	99
6.4	Choice of momentum thickness	101
6.5	Effective momentum thickness of temporal linear stability analysis	103
6.6	Eigenmodes of case $D/L = 0.2$	105
6.7	Linear stability curves of DC temporal analysis	105
6.8	Results from linear stability analysis for DC	108
6.9	Experimental temporal growth rates	109
6.10	Schematic view of wave reflection scenario	113
6.11	Reflection coefficients not equal unity	116
6.12	Movement of integration contours	118
6.13	Illustration of Briggs method	120
6.14	Branches in complex k -plane	121
6.15	Branch in complex ω -plane	121
6.16	Kulikowskii stability curve	121
6.17	Comparison of results in Y offset graph	122
6.18	Values of k_i	124
6.19	Values of ω_i	124
6.20	Eigenmodes of Kulikowskii modes	125
6.21	Phase speed of Kulikowskii modes k_+	126
6.22	Phase speed of Kulikowskii modes k_-	126

6.23	Fitting procedure total growth rate	127
6.24	Total growth rate over Strouhal	128
6.25	Comparison of results	130
6.26	Velocity profiles for DC lin. stab. analysis	131
6.27	Effective momentum thickness over distance for DC cases	132
6.28	Pinch Point DC	132
6.29	Kulikowskii curves DC	134
6.30	Summary results LSAFD for DC	135
6.31	Schematic view of computational domain	136
6.32	Fitting of velocity profiles	138
6.33	Velocity base profiles	139
6.34	Pinch point locations	141
6.35	Graphical evaluation of Kulikowskii condition	144
7.1	Flow patterns in SC flow	148
7.2	Flow patterns in DC flow	149
7.3	Flow patterns for variable distance	153
7.4	Carousel speed	154
7.5	QR-code for 2D visualization	156
7.6	Vorticity flow pattern from experiments and 3D simulations	157
7.7	Spatial Chronos-Koopman modes of 2D double cavity	159
7.8	Larynx geometry and computational domain	160
7.9	Larynx power spectra	160
7.10	Larynx vorticity field	162

List of Tables

2.1	Summary of experimental cases of Paris-campaign	13
2.2	Summary of experimental cases of BA-campaign	13
2.3	Data for PIV uncertainty computation	24
3.1	Comparison with Maurel et al.	28
4.1	Summary of 2D numerical cases	57
4.2	Summary of 3D numerical cases	61
4.3	Summary of computational 3D setup	62
5.1	Summary of cases for flow field comparison	85
6.1	Summary parameters of linear stability analysis	103
6.2	Summary of $L/H = 2$ cavity cases	119
6.3	Summary of all double cavity cases for linear stability analysis.	131
6.4	Summary of cases for analysis with slowly varying base flow	138

Abbreviations

2D	Two dimensional
3D	Three dimensional
BL	Boundary layer
BP	Branch Point
CNRS	Centre national de la recherche scientifique (France)
CONICET	Consejo Nacional De Investigaciones Científicas Y Técnicas (Argentina)
dB	Decibel
DMD	Dynamical Mode Decomposition
DC	Double Cavity
EVP	Eigen value problem
FIUBA	Facultad de Ingeniería de la Universidad de Buenos Aires
KH	Kelvin-Helmholtz
LDV	Laser Doppler Velocimetry
LFD	Laboratorio de Fluidodinámica
LIMSI	Laboratoire d’Informatique pour la Mécanique et les Sciences de l’Ingénieur
LSAFD	Linear Stability Analysis in a Finite Domain
MPI	Message Passing Interface
NS	Navier-Stokes
NTR	Non time resolved
OpenMP	Open Multi-Processing
POD	Proper orthogonal decomposition
PIV	Particle Image Velocimetry
PP	Pinch Point
RMS	Root Mean Square
SC	Single Cavity
SNR	Signal-to-Noise-Ratio
SVD	Singular value decomposition
TR	Time resolved

Nomenclature

β	Parameter for spatial development of base profile
Δf	Frequency resolution
Δk	Interaction wave length
Δk_r	Wave number range
ΔSt_L	Strouhal number range
Δt	Time difference
ΔU	Mean velocity profile difference (shear strength)
Δu	Uncertainty of velocity field
Δx	Displacement in streamwise direction
Δy	Displacement in vertical direction
Δ	Dispersion relation
δ_ω	Vorticity thickness or shear layer thickness
$\delta_{\omega,0}$	Vorticity thickness at leading edge
Γ	Cavity aspect ratio
γ	Lag time in Rossiter formula
$\hat{u}$	Eigenvector of normal mode solution for streamwise component
$\hat{v}$	Eigenvector of normal mode solution for vertical component
$\hat{w}$	Eigenvector of normal mode solution for spanwise component
κ	Speed ratio in Rossiter formula
λ	Wave length
$\mathbf{A}$	Left singular matrix
$\mathbf{B}$	Right singular matrix
$\mathbf{M}$	Matrix of velocity field snapshots
$\mathcal{H}$	Heaviside step function
ν	Kinematic viscosity
ω	Angular frequency
ω_0	Frequency of pinch point
ω_T	Angular velocity
$\omega_{i,max}$	Maximum temporal amplification rate
ω_i	Imaginary part of angular frequency or temporal amplification rate
ω_r	Real part of angular frequency

Ψ	Wave function
Σ	Matrix containing the singular values
σ	Total growth rate
σ_U	Standard deviation
τ	Time delay of embedding
τ_c	Convective time unit
Θ	Momentum thickness
Θ_0	Momentum thickness at inflow
Θ_N	Normalized momentum thickness at inflow
Θ_{eff}	Effective momentum thickness
$\tilde{U}$	Average streamwise velocity component for linear stability analysis
$\vec{e}_x$	Streamwise cartesian coordinate
$\vec{e}_y$	Vertical cartesian coordinate
$\vec{e}_z$	Spanwise cartesian coordinate
$\vec{U}$	Mean Velocity vector
$\vec{u}$	Velocity vector
ζ	Pixel size
A	Wave amplitude
a	Parameter of sinh base profile
a_∞	Speed of sound
b	Parameter of sinh base profile
$c_\pm$	Phase speed of upstream and downstream traveling waves
D	Intercavitary distance
f	Frequency
F_k	Integration contour in the complex wave number plane
H	Cavity depth
H^*	Normalized cavity depth
k	Wavenumber
k_+	Downstream traveling perturbation wave
k_-	Upstream traveling perturbation wave
k_i	Imaginary part of wave number or spatial amplification rate
$k_{r,max}$	Real part of wavenumber for which amplification is maximal
k_r	Real part of wavenumber
L	Cavity length
L^*	Normalized cavity length
L_1	Inflow length
L_2	Outflow length
L_ω	Integration contour in the complex frequency number plane
M	Mach number

m	Integer number
n	Integer number
N_x	Grid points in x
N_y	Grid points in y
N_z	Grid points in z
P	Probe point
p	Total pressure
P_{LDV}	Main LDV probe point
R	Auto-correlation function
R_1	Downstream reflection coefficient
R_2	Upstream reflection coefficient
Re_L	Reynolds number based on the cavity length
S	Spanwise cavity extension
$s(t)$	Time series
St_L	Strouhal number based on the cavity length
$St_{L,min}$	Minimal selectable Strouhal number
t	Temporal variable
T_{lt}	Life time scale
T_{shl}	Shear layer time scale
T_{to}	Turn over time scale
U	Mean streamwise velocity component
u	Streamwise velocity component
u'	Streamwise fluctuating velocity component
U_∞	Maximum centerline velocity or free stream velocity
U_N	Nominal velocity inflow profile
U_{base}	Analytical base velocity profile
U_{exp}	Experimental mean velocity profile
u_{rms}	Root mean square of streamwise fluctuating velocity component
V	Mean vertical velocity component
v	Vertical velocity component
v'	Vertical fluctuating velocity component
V_{CR}	Carousel speed
W	Mean spanwise velocity component
w	Spanwise velocity component
w'	Spanwise fluctuating velocity component
X	First time delayed vector of phase space
x	Streamwise spatial variable
x_0	Inflow length of Blasius profile
x_{min}	Streamwise position of best fit

Y	Second time delayed vector of phase space
y	Vertical spatial variable
y_0	Zero wall position of Blasius profile
Z	Third time delayed vector of phase space
z	Spanwise spatial variable

Chapter 1

Introduction

1.1 Background

Self excited resonances, appearing in spatially developing flows, explain the sharp selection of definite wavenumber and frequencies observed in experiments. A paradigmatic example of hydrodynamic resonance is the wake flow behind bluff bodies at low Reynolds numbers, in which oscillations occur with a distinct frequency and wavenumber ([Koch \(1985\)](#); [Pierrehumbert \(1984\)](#)). The nature of this resonance is related to the stability characteristics of wake flows that present the possibility of amplifying perturbations in the region upstream of the source. In shear flows like mixing layers or boundary layers, perturbations are swept away from the source, and unless a coherent perturbation is imposed, the measured spectra remain broadband. However, when a feedback mechanism is possible, these flows may exhibit discrete frequency spectra. The feedback usually takes place as a consequence of downstream reflection, and upstream propagation, of some of the waves of the perturbing wavepacket, which then reach again the region where the perturbation originated. The reflection may occur either in a turning point ([Kulikowskii & Shikina \(1996\)](#)), or in a fixed boundary.

The latter case results in impinging flows on solid boundaries, extensively studied as reported in [Rockwell & Naudascher \(1978\)](#); [Knisely & Rockwell \(1982\)](#); [Ziada & Rockwell \(1982\)](#). As mentioned in [Rockwell & Naudascher \(1978\)](#), the highly organized oscillations of an impinging flow are sustained through a series of interacting events:

- (a) Feedback, or upstream propagation, of disturbances from the impingement region to the sensitive area of the free shear layer near separation,
- (b) Inducement of localized vorticity fluctuations in this region by the arriving perturbations,

- (c) Amplification of these vorticity fluctuations in the shear layer between separation and impingement
- (d) Production of organized disturbances at impingement

Event (a), disturbance feedback, is an essential feature of this sequence. It provides direct communication between processes near the impingement surface and the separation region, thereby ensuring that the shear-layer oscillation is a globally organized phenomenon. Event (c) is the energy supply to the self-sustained oscillations, while events (b) and (d) are the description of a feedback mechanism, which can take different forms.

In particular, in open cavity flows, the impingement of the unstable shear layer onto the downstream edge of the cavity produces this global organization of the flow, leading to a global feedback, responsible for self-sustained oscillations of the flow beyond a critical value of both Reynolds number and cavity length. Those self-sustained oscillations are known to be the source of noise in many applications (*e.g.* fast train, aircraft land gear bay) or harmonious sound in wind instruments (*e.g.* the Saxophone). At high speeds (compressible flows), where the acoustic wavelength is of the same order as the impingement length, the consideration of the acoustic speed becomes important in evaluating the delay time between the impingement surface and the location of shear-layer separation. [Rossiter \(1964\)](#) first formulated a semi-empirical model, based on this acoustic feedback mechanism, known until then by the name of edge tones ([Powell \(1953\)](#)). In the Rossiter mechanism, small disturbances in the free shear layer, spanning the cavity, are amplified via Kelvin-Helmholtz instabilities. Their impingement onto the trailing cavity edge(s) gives rise to pressure perturbations that travel upstream, where they excite further disturbances to the free shear layer, especially near the cavity leading edge. Due to the time needed for the disturbance to travel upstream, there is an acoustic delay. Resonance occurs at frequencies where the phase change of the disturbance leads to constructive reinforcement. The Rossiter mechanism is widely accepted to govern the feedback in compressible open cavity flow ([Wilson *et al.* \(1971\)](#); [Morel \(1979\)](#); [Knisely & Rockwell \(1982\)](#); [Delprat \(2006\)](#); [Bres & Colonius \(2008\)](#)), though hydrodynamics feedback is not excluded, as shows experimental work by [Gharib & Roshko \(1987\)](#) and numerical work by [Rowley *et al.* \(2002\)](#), who studied the so-called *wake-mode* in open cavity flows.

Incompressible open cavity flows still remain an active field of research ([Podvin *et al.* \(2006\)](#); [Lusseyran *et al.* \(2008\)](#); [Pastur *et al.* \(2008a\)](#); [Basley *et al.* \(2011\)](#); [Douay *et al.* \(2015\)](#)) to cite only a few of them, since the feedback mechanism through far-field acoustic waves traveling upstream loses importance. The acoustic wavelength at such low speeds is much greater than the impingement length and it may be suggested that other, purely hydrodynamic, mechanisms gain influence. Experiments of cavity flow in water

by Burroughs & Stinebring (1994) show the same results as in air at low Mach numbers (Lusseyran *et al.* (2008); Basley *et al.* (2011) among others), confirming the hypothesis that other mechanisms must be at play. The study of two such alternative purely hydrodynamic mechanisms in the incompressible limit forms part of the present work. In chapter 6 the reflection of hydrodynamic instability waves at the cavity walls is considered. This novel approach (Tuerke *et al.* (2015)) is able to describe the origin of the non-harmonic quantization of frequencies reported in different experiments. The analysis, however, is unable to distinguish, among the set of possible frequencies, which ones are selected by the flow. This limitation indicates that yet other mechanisms have to be considered additionally.

In chapter 7 the role of the recirculating zone of (double) cavity flow is studied with respect to the complex intracavity vorticity patterns formed by vortex structures of different sizes that are expected to participate in the feedback mechanism, as noted by Burroughs & Stinebring (1994) and Basley *et al.* (2013) and confirmed in the framework of the present work (Tuerke *et al.* (2016)). The interaction of these structures with the onset of the shear instability is nevertheless delayed by the time lag necessary for their transport from the impinging region to the upstream flow. This non-linear delayed action on the upstream shear layer instability has been modeled in confined jets with a formalism referred to as the Non-Linear Delayed Saturation Model (Villermaux & Hopfinger (1994)). Inspired by this work, the dynamics of each cavity can be conceived in terms of non-linear oscillators. The small structures, present in the recirculation region, produce discontinuous and delayed feedback, giving rise to non-linear dynamical features, such as quasi-periodic oscillations, phase-locked oscillations, or even chaotic dynamics (Villermaux & Hopfinger (1994)).

1.2 The double cavity

When two identical self-oscillating systems are brought together, the dynamics is expected to get enriched with additional couplings and interactions. Some of these examples are side-by-side cylinder flows (Zhou *et al.* (2002); Kang (2003); Landel *et al.* (2012)) or the two mirrored backward facing steps (Durst *et al.* (1974); Fearn *et al.* (1990)). At very close distances the two systems lose identity and behave as a new system.

When two open cavities face each other, a so-called double cavity configuration is created, and one may presume the appearance of dynamical couplings of the two shear layers originating from each wall of the inlet channel. Research related to different industrial and bio-mechanical applications is concerned with these flows. The sudden expansion and contraction in pipe flows (Drikakis (1997)), the exit of confined jets (Maurel *et al.* (1996); Righolt *et al.* (2015)) and the laryngeal ventricle or Morgagni's sinus in the human

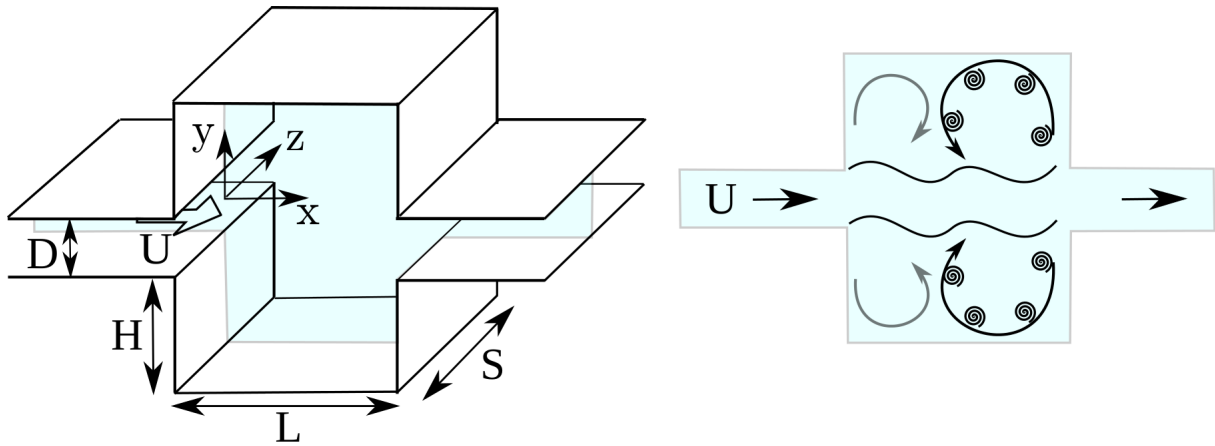


FIGURE 1.1: Sketch of a double cavity geometry with intercavity distance D , cavity length L , cavity depth H and spanwise cavity extension S as well as inflow velocity U_∞ .

phonatory system (Agarwal *et al.* (2003); Chisari *et al.* (2011)) are just a few examples among many, in engineering applications and nature, in which the understanding of the flow physics of double cavity dynamics is of importance.

Maurel *et al.* (1996) considered a planar jet, issuing into a large cavity, a geometry that is similar to the double cavity. The experiments were performed with a ratio between the inlet channel height D and the cavity depth H equal to $1/12$, and the varied geometrical parameter was the length L of the cavity. The authors found a stable flow regime with no oscillations, a regime with self-sustained oscillations and a regime with turbulent free jet characteristics. In the self-sustained oscillation regime, the authors observed that the flow was mainly two dimensional, and proposed a subdivision of this regime into a sinuous mode with large scale wavelengths and a varicose mode with much shorter wavelengths. The case with constant L/H and varying D was not investigated by Maurel *et al.* (1996). By sufficiently increasing the ratio D/H , the two shear layers extend but do not merge along the cavity length L . The confined jet behavior vanishes and double cavity (DC) flow arises. For sufficiently large D the single cavity flow behavior is expected to be retrieved. Mizushima & Shiotani (2001), Mullin *et al.* (2003) investigated the symmetry, stability and bifurcation properties of double cavity flow as the Reynolds number is varied. For some cases the Reynolds numbers is comparable to the present work, however L/H is much greater, leading to asymmetric flow configurations.

This work addresses the fluid flow through a symmetric channel with a sudden expansion and contraction, creating two facing cavities, a so called double cavity. A sketch of the double cavity configuration is shown in figure 1.1. The flow enters from the left and leaves to the right. Both cavities are rectangular and confined by walls in spanwise direction. The double cavity is symmetric with respect to the horizontal mid-plane $y = 0$. As

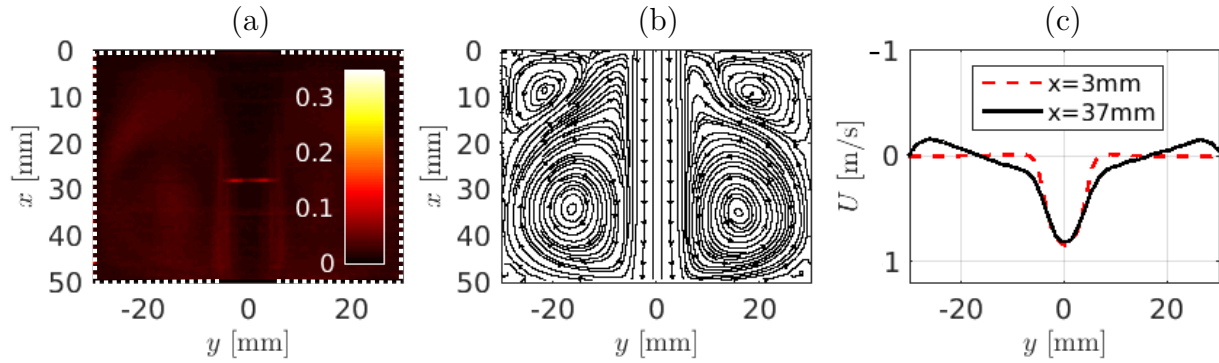


FIGURE 1.2: Flow pattern for steady case. Experimental results for $D/L = 0.2$ at $Re_L = 2800$ (a) rms of the streamwise velocity component (the dotted lines indicate the cavities' boundaries). (b) Streamlines over the time averaged field. (c) Profiles of mean streamwise velocity at indicated x-locations. Flow from top to bottom. Dashed white lines in (a) indicate cavity walls.

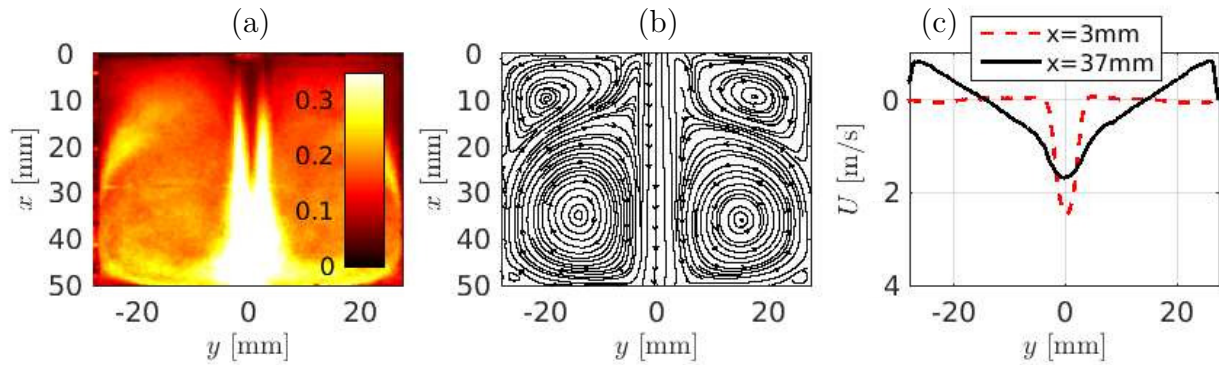


FIGURE 1.3: Flow pattern for jet-like case. Experimental results for $D/L = 0.1$ at $Re_L = 8600$ (a) rms of the streamwise velocity component. (b) Streamlines over the time averaged field. (c) Profiles of mean streamwise velocity at indicated x-locations. Flow from top to bottom.

primary parameters, the cavity distance D and the inflow velocity U_∞ , expressed in term of its Reynolds number, are varied. The well known and extensively studied single open cavity flow can be retrieved as a double cavity in the limit of $D \rightarrow \infty$. The Reynolds number in this work is defined in term of the maximum velocity of the flow in the inflow channel U_∞ , the cavity length L and the dynamic viscosity ν , according to

$$Re_L = \frac{U_\infty L}{\nu} \quad (1.1)$$

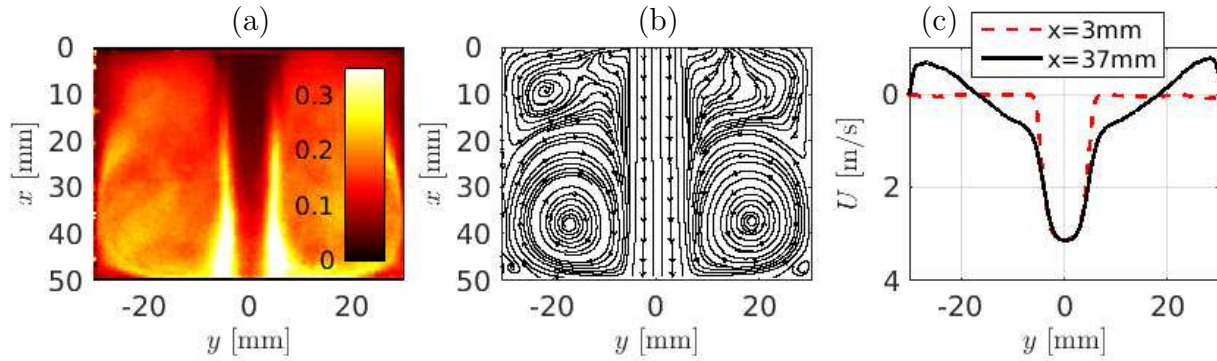


FIGURE 1.4: Flow pattern for double cavity flow. Experimental results for $D/L = 0.2$ at $Re_L = 11\,200$ (a) rms of the streamwise velocity component. (b) Streamlines over the time averaged field. (c) Profiles of mean streamwise velocity at indicated x -locations. Flow from top to bottom.

For a fixed intercavity distance D , downstream of the leading edge of the double cavity, different flow regimes can be observed as the Reynolds number Re_L is increased.

The flow is steady at low Reynolds numbers. The fluctuations of the velocity components are almost negligible compared to the mean velocity, as illustrated with the field of the root-mean-square (rms) of the streamwise velocity components of the flow shown in figure 1.2(a). In figure 1.2(b) time-averaged streamlines for the steady flow state are shown. In figure 1.2(c), the mean velocity profiles at two fixed streamwise stations show that the velocities in the intracavity region are relatively low. The flow becomes time-periodic above a critical value of Re_L , which depends on D (figures 1.3 and 1.4). The rms field now shows important values compared to the mean velocity, particularly in the shear layer region. The streamlines are comparable to the steady regime, though the velocities in the intracavity regions are enhanced. In this regime we can find cases in which the flow in between the cavities has the characteristics of a jet (figure 1.3). In these situations, as can be seen from the rms field of the velocity components, the shear layers merge at some distance from the leading edge. For such small distances the velocity profile at the cavities' leading edge becomes in our case a parabolic (Poiseuille-like) channel profile, and the resulting jet impinges alternately on the downstream sides of the cavities. The jet-like case, studied in detail in [Maurel *et al.* \(1996\)](#), yields significantly different flow physics, and is out of scope of the current study.

For larger separations the two shear layers are separate all along the cavity length as can be seen from the rms field in figure 1.4(a). At this, or larger distances a different flow type appears, hereafter named double cavity flow (Figure 1.4). In double cavity flow the two shear layers can be easily identified in the rms flow fields all along the cavity length. In the present work we will focus on this kind of flow. The velocity profiles at the leading edge of the cavities correspond to a double Blasius boundary layer profile.

1.3 Objectives

The objective of the present work is the investigation of the flow characteristics of the flow in a channel with two facing cavities, a so called double cavity, in the incompressible limit. As mentioned in the introduction, single open cavity flow has been extensively studied over the past decades. The question this work addresses is, how the flow characteristics change when a second, identical cavity is facing the single cavity. In the limit of very large intercavity distances the single cavity will be retrieved, while in the other limit of very small intercavity distances a confined jet flow, as described in the introduction, is encountered. The present work fills the missing gap between both extremes and aims at contributing to the general understanding of double cavity flow, and where applicable also to its sibling, single cavity flow. New insights could be used to design simple but effective models to control and study the phenomenon in question and thus broaden the fluid dynamical understanding of the geometry. In order to accomplish these goals, the most influential factors and parameters must first be revealed and characterized.

To address these questions, double cavity flow is analyzed by means of theoretical, numerical and experimental methods, as two main parameters, the inflow velocity U_∞ and the intercavity distance D , are varied. The focus is put on the description of the rich composition of the shear layer spectra, which are determined by both the impinging shear layers and the intracavity recirculation regions.

During the course of the project it became apparent that even for single open cavity flow in the incompressible limit, the complex composition of the shear layer spectrum still lacks a comprehensive explanation. For that matter, the single cavity, which can be regarded as a double cavity with infinite intercavity distance, was included in the current study. This allowed us first to study the impinging shear layer in its simpler form (single cavity), before advancing to the more complex double cavity geometry and the interaction of two shear layers.

An experimental study aims at obtaining a general picture of the phenomena of single and double cavity flow. By means of particle image velocimetry (PIV) the structure of instantaneous and mean velocity fields is studied. Laser Doppler velocimetry (LDV) is used to analyze the spectral response of the double cavity.

A flow field analysis is conducted based on 2D and 3D numerical simulations, which allow access to highly resolved data in space and time. The question of the role of intracavity coherent structures (vortices), originating from the shear layers, is addressed. The shear layer spectra, which are a result of both the impinging shear layers and the intracavity recirculation regions, are investigated as the inflow velocity and the distance between the cavities are varied.

The comparison of results from 2D/3D numerical simulations and wind tunnel experiments will give indications to which extent both methods are compatible.

The theoretical work concerns the implementation and development of a linear stability analysis in a finite domain, given by the confined geometry of the single or double cavity. Using this purely theoretical approach a new perspective on the experimentally recorded shear layer spectra is developed.

1.4 Organization of the thesis

A variety of aspects of double cavity flow are addressed in the present work, structured into two main parts. In part I, named “Methodologies and Results”, the data acquisition techniques and the results are presented. To begin with, in chapter 2 the experimental setup of the wind tunnel measurements and the data acquisition techniques are presented, followed by the presentation of the results, in chapter 3. In chapter 4 the numerical simulations are introduced. The numerical procedure to solve the incompressible Navier-Stokes equations is shortly outlined and the computational domains for the 2D and 3D direct numerical simulations are presented. The results of both the 2D and 3D simulations are presented in chapter 5. Results from the 2D simulations are analyzed using tools from the nonlinear dynamical system theory. At the end of this chapter experimental and numerical results are compared for some specific cases.

In part II, named “Analysis and Discussion” results from part I are used to conduct two different types of analysis, both related to the hydrodynamic feedback mechanisms, mentioned in the introduction. In chapter 6, a novel approach to a local spatio-temporal linear stability in a finite domain is considered, using a wave reflection scenario, first proposed by [Kulikowskii \(1966\)](#). The theoretical background of the linear stability analysis in a finite domain as well as the application to single and double cavity flow is addressed. Also, a temporal linear stability analysis is performed, to explain certain frequency results, observed experimentally in double cavity flow. In chapter 7, based on the results from 2D numerical simulations, an intracavity hydrodynamic feedback mechanism is revealed in which upstream advected vorticity structures are responsible for the rich composition of shear layer spectra and the transition from periodic flow conditions to chaotic flow conditions. The same vorticity structures can also be observed in real 3D flow. At the end of chapter 7 examples from 3D numerical simulations and experiments are shown together with an example from a bio-inspired double cavity flow.

A summary, concluding remarks and perspectives for future work are presented in chapter 8.

Part I

Methodologies and Results

Wind Tunnel Experiments and Numerical Simulations

Chapter 2

Experimental Setup

2.1 Introduction and background

In the present work double cavity flow is studied experimentally by means of particle image velocimetry (PIV) and laser Doppler velocimetry (LDV) for a wide range of cases in the $Re_L - D$ parameter space. The setup of the experiments are presented in this chapter. In the course of the project three major experimental campaigns were performed. Two at the Laboratorio de Fluidodinámica (LFD-FIUBA) of the Engineering Faculty of the University of Buenos Aires in Argentina, and one at LIMSI-CNRS in Orsay, France. They differ from one another in the experimental data acquisition techniques used and are in that sense complementary. Their results are compared whenever possible, thus providing a first validation of the acquired data. At LFD-FIUBA time resolved Particle Image Velocimetry (TR-PIV) and non-time resolved Particle Image Velocimetry (NTR-PIV) measurements were carried out, while at LIMSI-CNRS, non-time resolved PIV (NTR-PIV) and Laser-Doppler-Velocimetry (LDV) measurements were carried out. The two experimental campaigns conducted in Buenos-Aires (BA) will be merged and referred to as the *BA-campaign*, while the experimental campaign, carried out at LIMSI-CNRS near Paris will be referred to as the *Paris-campaign*. Both campaigns are summarized in tables 2.1 and 2.2, respectively.

While the inflow dimensions, the cavity aspect ratio $\Gamma = L/H$, as well as the outflow dimensions of the wind tunnel geometry were identical on either side of the Atlantic ocean, two mayor differences between the experimental setups in Buenos Aires and Paris have to be mentioned:

- Due to the given geometry of the test section of the wind tunnels, the spanwise extension S (*cf.* figure 2.1) is different in both cases. While for the LIMSI-campaign the spanwise extension is $S = 75$ mm, for the BA-campaign the spanwise extension

TABLE 2.1: Summary of experimental cases of LIMSI-campaign

Cases	D [mm]	D/L [-]	Symbol	TR-PIV	NTR-PIV	LDV	Re_L range
DC05	5	0.1	+	X	✓	✓	3300 - 12500
DC10	10	0.2	▼	X	✓	✓	3200 - 12000
DC15	15	0.3	○	X	✓	✓	3800 - 12900
DC20	20	0.4	△	X	✓	✓	3500 - 12500
DC25	25	0.5	×	X	✓	✓	3700 - 13300
DC30	30	0.6	◁	X	✓	✓	3800 - 13700
DC40	40	0.8	◇	X	✓	✓	3800 - 13800
DC50	50	1.0	►	X	✓	✓	3900 - 13700
SC50	50	1.0	□	X	✓	✓	3700 - 13700

TABLE 2.2: Summary of experimental cases of the BA-campaign.

Cases	D [mm]	D/L [-]	Symbol	TR-PIV	NTR-PIV	LDV	Re_L range
DC05	5	0.1	+	X	✓	X	1780 - 10400
DC10	10	0.2	▼	✓	✓	X	1810 - 11400
DC15	15	0.3	○	X	✓	X	2300 - 12100
DC20	20	0.4	△	✓	✓	X	2300 - 12000
DC30	30	0.6	◁	✓	✓	X	2150 - 12630

is with $S = 180$ mm more than twice as large. This is an important difference, that will be discussed in chapter 5.

- The wind tunnel at LIMSI-CNRS, schematically depicted in figure 2.2, is open, while the wind tunnel at LFD-FIUBA, shown schematically in figure 2.3, is closed. This difference should, if at all, only play a minor role.

Note, that the NTR-PIV and LDV measurements of the Paris-campaign were carried out simultaneously and in the same spanwise plane. Hence, the LDV laser interfered with the laser plane for the PIV measurements, which lead to poor PIV results in the area close to the rear cavity edge.

The chapter is organized as follows: First both wind tunnel setups are presented, followed by details on the measurement techniques of both campaigns. Finally, an uncertainty analysis for the LDV and PIV measurements is presented. Results are discussed in the following chapter 3.

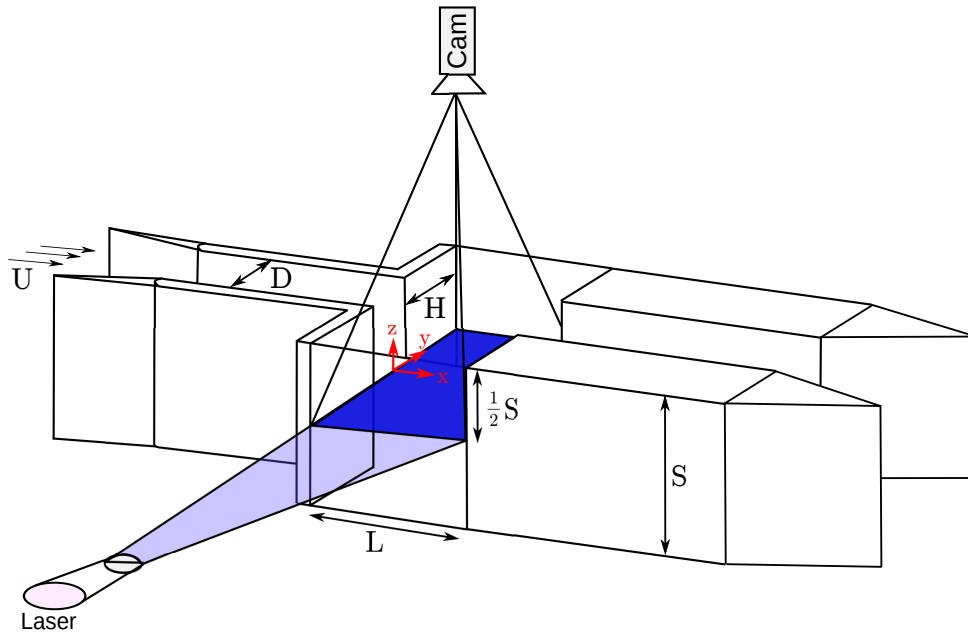


FIGURE 2.1: Sketch of general experimental setup: Three dimensional double cavity geometry of distance D , length L , depth H , spanwise extension S and inflow velocity U_∞ with horizontal laser sheet and camera position.

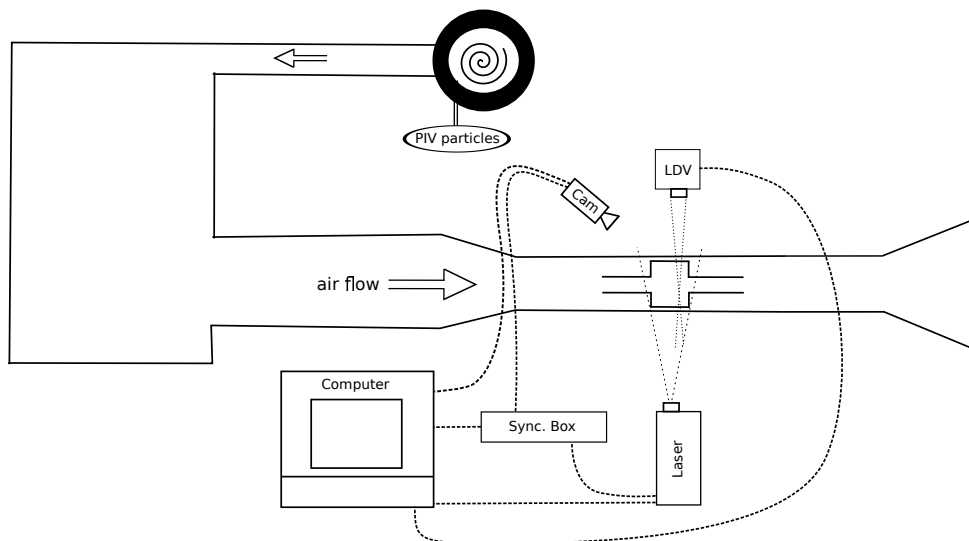


FIGURE 2.2: The subsonic open wind tunnel and experimental setup at LIMSI-CNRS used for the Paris-campaign.

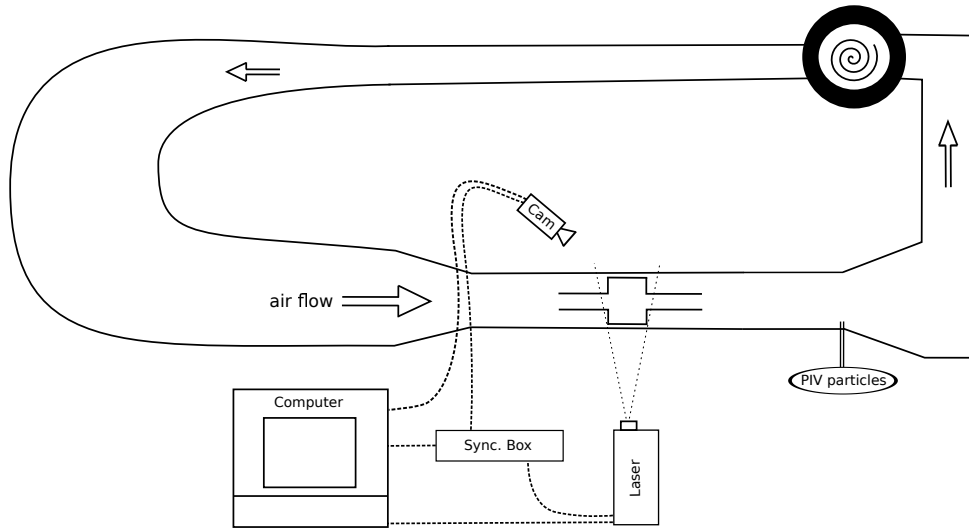


FIGURE 2.3: The subsonic closed wind tunnel and experimental setup at LFD-FIUBA used for the BA-campaign.

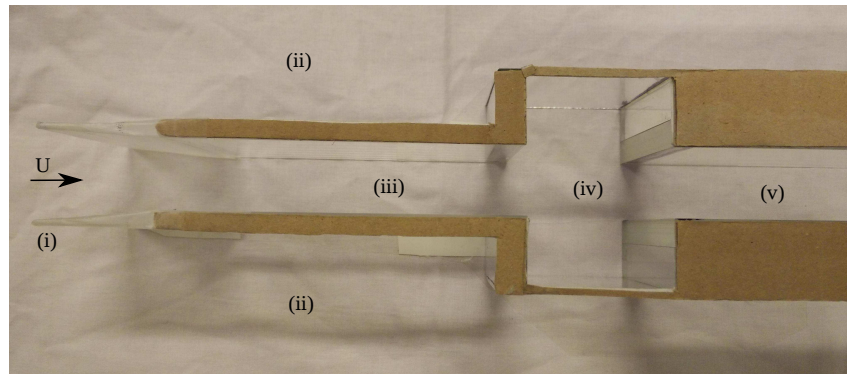


FIGURE 2.4: Picture of the double cavity geometry that was placed in the wind tunnel. Flow from left to right as indicated by the arrow. See text for indications.

2.2 Wind tunnel facilities

The wind tunnel at LIMSI-CNRS, used for the Paris-campaign and schematically depicted in figure 2.2, is comprised of a centrifugal fan, providing stationary volume flow, a settling chamber that contributes to equalize the flow and a honey-comb panel, placed at the inlet of the contraction, which contributes to laminarize the flow.

The wind tunnel at LFD-FIUBA, used for the BA-campaign and schematically depicted in figure 2.3, is a closed facility. It is driven by and an axial fan that provides a stationary volume flow. The divergent-convergent form contributes to equalize the flow and provides a uniform laminar flow in the test section.

In each test section the double cavity geometry is placed, schematically depicted in perspective view in figure 2.1, and in top view in figure 2.5. A photograph of the double cavity (iv), the inflow channel (iii) and part of the outflow channel (v) is shown in figure 2.4. The profiled and by approximately 4 degrees inclined leading edges (i) separate

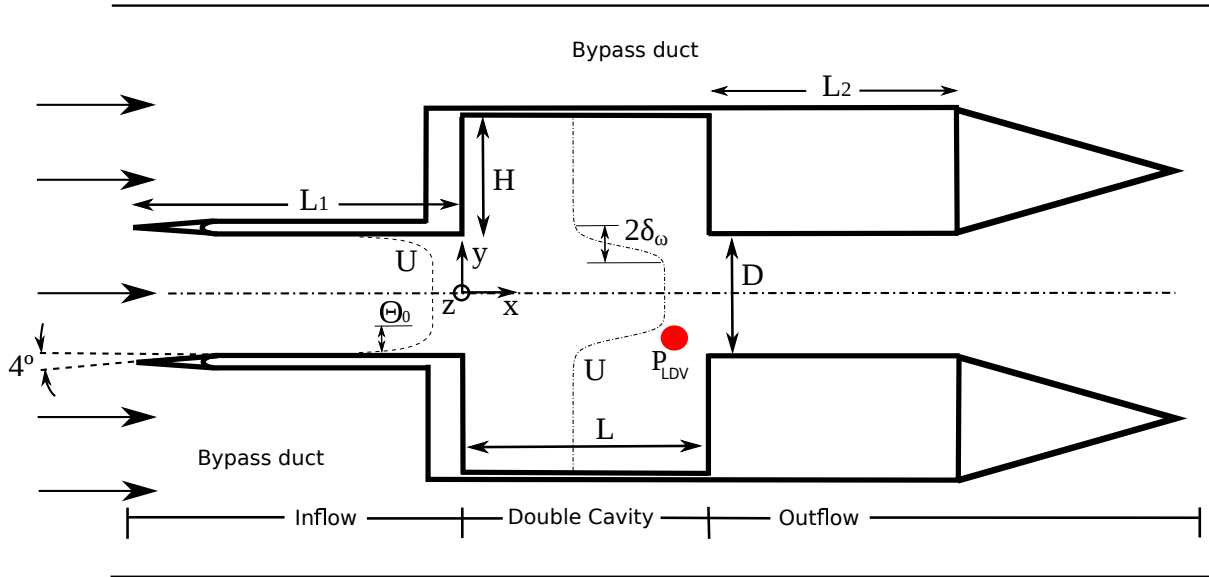


FIGURE 2.5: Schematic top view of figure 2.1. Double cavity geometry with variable distance $0.1 \leq D/L \leq 1$, constant length $L = 50$ mm, constant depth $H = 0.5L$, constant spanwise extension $S = 1.5$ (Paris-campaign) and $S = 3.6$ (BA-campaign), respectively, constant inflow and outflow channel length $L_1 = 3.4$ mm and $L_2 = 2.6$ mm, respectively, and variable inflow velocity U_∞ . The red filled circle indicates the main LDV probe point at $P_{LDV} = [45\text{mm}, 5\text{mm}]$. The inflow velocity profile (--) with the center velocity U_∞ and the momentum thickness Θ_0 and the double cavity profile (-.-) with the shear layer thickness δ_ω are depicted.

the flow into two bypass streams (ii) at either side of the geometry, and a center flow (iii), which enters the inflow channel leading to the double cavity test section (iv). Consequently, two, with respect to the double cavity center line symmetrical, laminar boundary layers are created, that develop in the inflow channel of length $L_1 = 3.4L$, before entering the double cavity. The wind tunnel walls, as well as the double cavity side walls, are made of 2 mm thick reflection treated glass, allowing for complete optical access. The rest of the double cavity geometry is made of acrylic glass. Adaptable shields are mounted at the test section outlet, in order to minimize perturbations of the inflow. The open wind tunnel facility at LIMSI-CNRS is additionally equipped with shields at the wind tunnel outflow to minimize perturbations traveling upstream.

The cavity ratio $\Gamma = L/H = 2$ is kept constant throughout the entire study. The inter-cavity distance D is varied from $D/L = 0.1$ to $D/L = 1$. The cavity length $L = 50$ mm, as well as the inflow length and the outflow length, are kept constant at $L_1 = 3.4L$ and $L_2 = 2.6L$, respectively. The spanwise extension is kept constant at $S = 1.5L$ for the Paris-campaign and at $S = 3.6L$ for the BA-campaign, respectively, both times limited by the height of the wind tunnel test section. As a reference, a single cavity case is added to the double cavity cases in the Paris-campaign, by closing one of the two facing cavities at a distance of $D/L = 1$. A Cartesian coordinate system is $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$ is set mid-span

at the cavity leading corner, as shown in figures 2.1 and figures 2.5. The study concerns Reynolds numbers of $Re_L \approx 1800$ to $Re_L \approx 13700$, where the Reynolds number, based on the cavity length and the velocity in the inflow channel at $x = 0$, is defined as

$$Re_L = \frac{U_\infty L}{\nu} \quad (2.1)$$

with $\nu = 15 \cdot 10^{-6} [m^2/s]$ being the dynamic viscosity of air at room temperature (24 degrees Celsius). The inflow velocity profile U_∞ is taken slightly upstream of the entry to the double cavity at $x = -30$ mm. The momentum thickness Θ_0 is obtained at the same x -position. The total turbulent intensity on the inflow channel center line was measured to less than 0.4% in all cases.

2.3 PIV and LDV measurements

2.3.1 PIV Paris-campaign

In the Paris-campaign standard two component (2D2C) particle image velocimetry (PIV) and laser Doppler velocimetry (LDV) were carried out simultaneously. The PIV system consists of a high repetition CCD camera, a laser system, a synchronization box and a personal computer (PC), as shown in figure 2.2. The DALSA Genie Camera is equipped with a FUJINON 1 : 1.8/50 mm lens and records images with a resolution of 1368×1024 pixels, which are encoded in 8 bits, saved and post-processed on a PC. The light source, which is synchronized by a synchronization box with the camera, is a Quantel Laser from Big Sky Laser Tech., Inc. Model PIV190 PS2, that sends two laser pulses with a time difference of Δt . The wavelength of the lasers is $\lambda = 532$ nm, providing a maximum energy output of 2×200 mJ in the green part of the visible light spectrum. The lasers are set to pulsate at its maximum rate of 15 Hz, with each pulse lasting roughly 10 ns. The light pulses are projected into a horizontal plane by a concave lens, creating a laser sheet of about 1 mm width at $z/S = 0$ as shown in figure 2.1. In this plane the displacement vector $\Delta \vec{x} = [\Delta x, \Delta y]^T$, together with the known time difference between the two laser pulses, Δt , yield the velocity vector $\vec{u} = \Delta \vec{x} / \Delta t = [u, v]^T$. The time difference Δt varies between $275 \mu s$ and $700 \mu s$, depending on the inflow velocity. Seeding particles are liquid droplets of mineral oil DEHS – di(2-ethylhexyl)sebacate –, sprayed at the fan entrance. DEHS density is 0.9 and droplet diameter are of the order of $1 \mu m$. Measurements start only after the seeding particle distribution is uniform inside the double cavity. The field of vision is adapted, such that for all intercavity distances the complete double cavity area is covered. The PIV measurement plane is kept constant in the symmetry plane at $z/S = 0$. After image pre-processing, including mean field subtraction as well as intensity corrections, displacement fields are computed using an FFT based cross-correlation algorithm of two corresponding interrogation windows, that are consecutively decreased

from 64×64 pixels down to 8×8 pixels in size, using a 50% overlap. For each case 400 images, resulting in 200 instantaneous vector fields, are recorded. The average was found to be well converged after around 100-150 fields, depending on the Reynolds number.

2.3.2 PIV BA-campaign

In the BA-campaign both time resolved (TR) and non-time resolved (NTR) standard two component (2D2C) particle image velocimetry (PIV) were carried out. The reason for that is, that the TR PIV was not able to resolve the fast moving channel center due to light source limitations of the continuous laser, used to visualize the flow. In order to obtain information about the mean flow in the channel center NTR PIV was carried out additionally.

Non-time resolved PIV

The NTR PIV system consists of a high repetition CCD camera, a laser system, a synchronization box and a personal computer (PC), as shown in figure 2.3. The ImagerPro CCD Camera is equipped with a Nikon 1 : 1.8/50 mm lens and records images with a resolution of 1600×1400 pixels, which are encoded in 8 bits, saved and post-processed on a PC. The light source, which is synchronized by a synchronization box with the camera, is a Litron NanoL SN LM0764 laser, that sends two laser pulses with a time difference of Δt . The wavelength of the lasers is $\lambda = 532$ nm, providing a maximum energy output of 2×200 mJ in the green part of the visible light spectrum. The lasers are set to pulsate at its maximum rate of 14 Hz, with each pulse lasting roughly 10 ns. The light pulses are projected into a horizontal plane by a concave lens, creating a laser sheet of about 1 mm width at $z/S = 0$ as shown in figure 2.1. In this plane the displacement vector $\Delta \vec{x} = [\Delta x, \Delta y]^T$, together with the known time difference between the two laser pulses, Δt , yield the velocity vector $\vec{u} = \Delta \vec{x} / \Delta t = [u, v]^T$. The time difference Δt varies between $150 \mu s$ and $500 \mu s$, depending on the inflow velocity, such that a mean displacement Δx of roughly 15 pixels is reached in the channel center.

A difficulty inherent to the double cavity geometry is the difference, of roughly one order of magnitude, between the velocities in the channel center and the much slower intracavity recirculation region. The time difference Δt was hence chosen such that it accounts for both regions, *i.e.* $\Delta \vec{x}$ is slightly above the optimum value in the channel center and slightly below the optimum value in the intracavity region.

Seeding particles are produced by the LaVision V-Z droplet seeder, with a mean droplet diameter of $1 \mu m$ and are sprayed upstream of the fan and downstream of the test section as indicated in figure 2.3. Measurements start only after the seeding particle distribution is uniform inside the double cavity. The field of vision is adapted, such that for all intercavity

distances the complete double cavity area is covered. As for the Paris-campaign, the PIV measurement plane is kept constant in the symmetry plane at $z/S = 0$. After image pre-processing, including mean field subtraction as well as intensity corrections, displacement fields are computed using LaVision's software package DaVis. The program applies an FFT based cross-correlation algorithm of two corresponding interrogation windows, that are consecutively decreased from 64×64 pixels down to 8×8 pixels in size, using a 50% overlap. For each case 340 images, resulting in 170 instantaneous vector fields, are recorded. As in the Paris-campaign the average was found to be well converged after around 100-140 fields, depending on the Reynolds number.

Time resolved PIV

The TR PIV system consists of a high repetition CCD camera, a continuous laser, and a personal computer (PC). It is the same setup as shown in figure 2.3 except for the synchronization box, which is not needed since a continuous laser is used. The SpeedCam MiniVis CCD Camera is equipped with a Nikon 1 : 1.8/50 mm lens and records images with a resolution of 800×800 pixels, which are encoded in 8 bits, saved and post-processed on a PC. The continuous light source is a Tolket laser model TKL-445-2W with a Nichia blue laser diode model NDB7875. The wavelength of the lasers is $\lambda = 445$ nm, providing a maximum energy output of effectively 1.6 W in the blue part of the visible light spectrum. All the remaining setup is equal to the setup of the NTR PIV, described in the previous paragraph. Time series are recorded over 8-10 seconds, depending on the case. The camera frame rate is set to $500 - 1000 \mu\text{s}$, depending on the inflow velocity. The aperture opening time is kept constant at $\Delta t = 400 \mu\text{s}$. Lower values of Δt are not possible due to maximum power limitations of the continuous laser. The power spectral density (PSD) of a time series is obtained by the help of the [Welch \(1967\)](#) algorithm, using an overlapped segment averaging estimator with a Hamming window size equal to 4.1 seconds and a segment overlap of 98%. The resulting frequency resolution is $\Delta f = 0.24$ Hz. One TR PIV recording hence consists of roughly 100 windows for averaging.

Note that, as mentioned above, TR PIV was only able to correctly resolve the slow moving intracavity flow regions. The maximum power of the continuous light source limits the camera's minimum aperture opening time, which is the limiting factor in order to freeze the particles in the taken image. Hence, due to a relatively long minimum aperture opening time, the fast moving flow in the channel center results in smeared fluid particles, which cannot be used for velocity computations. In the intracavity region the slower moving particles do not smear and the flow field can be computed.

2.3.3 LDV

Laser Doppler velocimetry (LDV) is a nonintrusive technique, used in the present work to measure time resolved time series data of the streamwise velocity component. A detailed description of the technique is given by [Adrian \(1996\)](#); [Iyer & Woodmansee \(2005\)](#). In short, a pair of monochromatic laser beams from an 30 mW argon-ion laser with a wavelength of 660 nm is focused to a point in the flow. The two laser beams are crossed, which due to interference of the light waves, creates a fringe pattern at their intersection. One of the laser beams is modulated at 80 Hz using a Bragg cell, which makes the fringes move and hence allows to measure zero velocities. Small seed particles, obtained from vaporization mineral oil, are introduced into the flow. The particles pass through the fringe pattern and scatter the light via so-called “Mie scattering”. Two filtered photo detectors and associated spherical lenses collect a portion of the scattered light for processing. The frequency of the scattered light, often referred to as the Doppler frequency (hence the name of the technique), is dependent upon the spacing of the fringes in the probe volume as well as the velocity of the particles as they cross the fringes maxima and minima. Since the fringe-spacing is known from the lasers wavelength, the velocity of the particle can be calculated.

In the experimental setup of the Paris-campaign, LDV measurements, using a Dantec BSA-system, are performed simultaneously with the PIV measurements. In the main probe point P_{LDV} , located 5 mm upstream and 5 mm above the trailing edge, as indicated in figure 2.5, the streamwise velocity component u is recorded. As also found by [Basley *et al.* \(2011\)](#) for the case of the single cavity, in the vicinity of P_{LDV} , shear layer oscillations are of highest amplitude, while low frequency content of the recirculation region is still present, but not dominant. LDV series are recorded over 30 seconds for each case, at an LDV count rate of the order of 3 kHz. Time series are resampled at the mean particle sampling frequency by linear interpolation before performing Fourier analysis. The power spectral density (PSD) of a time series is obtained by the help of the [Welch \(1967\)](#) algorithm, using an overlapped segment averaging estimator with a Hamming window size equal to 5.2 seconds and a segment overlap of 98%. The resulting frequency resolution is $\Delta f = 0.18$ Hz. One LDV recording hence consists of roughly 260 windows for averaging. All frequency data is represented in terms of the non-dimensional frequency (Strouhal number)

$$St_L = \frac{fL}{U_\infty}, \quad (2.2)$$

where f is the measured frequency, L , the cavity length and U_∞ the maximum center velocity of the incoming channel flow. Whenever the power spectral densities is given in dB units, it refers to

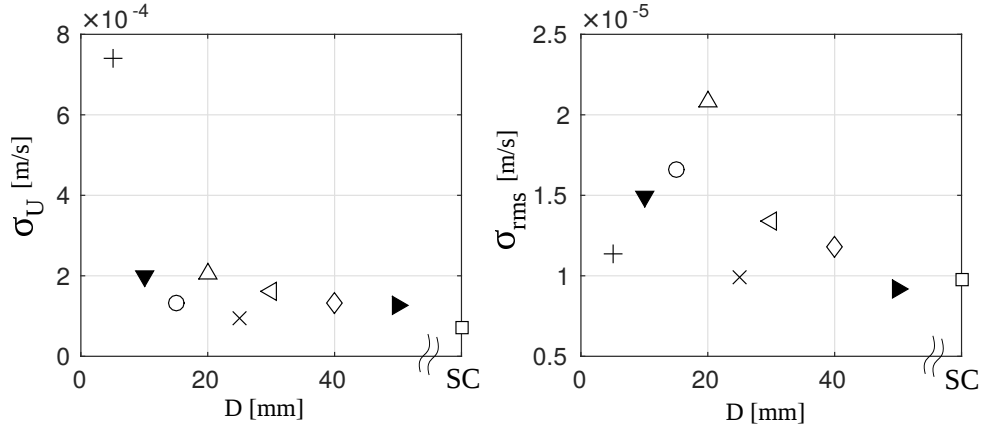


FIGURE 2.6: Variances of the mean (left) and the rms (right) as a function of the intercavity distance at Reynolds numbers in the range $10\,500 \lesssim Re_L \lesssim 11\,100$.

$$\text{PSD}(f) = \ln(2\text{PSD}(f)\Delta f), \quad (2.3)$$

where $\ln$ denotes the natural logarithm, Δf is the frequency resolution and $\text{PSD}(f)$ is the power spectral density. Note that this definition does not strictly coincide with the common definition of Decibel but varies by a constant factor.

2.4 Uncertainty analysis

2.4.1 LDV

Following [Iyer & Woodmansee \(2005\)](#), the uncertainties of the frequency measurements, using laser Doppler velocimetry (LDV), are estimated. The variance of the mean velocity U is calculated according to:

$$\sigma_U = \sqrt{\frac{\sigma^2}{n}}, \quad (2.4)$$

where σ is the standard deviation (normalized with its mean) of the independent measurement and n is the number of velocity measurements (*i.e.* the number of data points of the time series). According to equation 2.4, the uncertainty in the mean velocity diminishes as the sample size n increases. Therefore, the influence of stochastic noise introduced by the LDV system can be minimized by increasing the times series length and hence n . In the present case the time series lengths are of the order of $n \approx 100\,000$ velocity measurements. The resulting variance of the mean velocity is summarized in figure 2.6(a) for different intercavity distances at Reynolds numbers in the range $10\,500 \lesssim Re_L \lesssim 11\,100$. For the closest cavity distance the highest value is obtained ($\sigma_U \approx 8 \cdot 10^{-4}$). All other cases yield similar values close to $\sigma_U \approx 2 \cdot 10^{-4}$. The smallest variance of the mean is obtained for

the single cavity (SC) case ($\sigma_U \approx 1 \cdot 10^{-4}$).

Again, following [Iyer & Woodmansee \(2005\)](#), the variance of the fluctuating velocity (rms velocity) is calculated by

$$\sigma_{rms} = \sqrt{\frac{\sigma}{n-1} \left[1 + \left(\frac{U}{u_{rms}} \right)^2 \right]}, \quad (2.5)$$

where σ is the standard deviation (normalized with its mean) of the independent measurement and n is the number of velocity measurements, U is the mean velocity and $u_{rms} = \sqrt{\overline{u^2}}$ is the root-mean-square (rms) of the streamwise velocity component (the only velocity component recorded with the LDV technique). The results are summarized in figure 2.6(a) for different intercavity distances and the same Reynolds number range as before. All values of σ_{rms} are found in the range between $1 \cdot 10^{-5} \lesssim \sigma_U \lesssim 2 \cdot 10^{-5}$. The highest value is obtained for $D/L = 0.4$, the lowest for $D/L = 1.0$.

Both the variance of the mean and the variance of the velocity fluctuations are sufficiently low to neglect any uncertainties from the LDV measurements.

The 95% confidence interval is calculated to quantify the uncertainty interval of the power spectra obtained from LDV measurements. The 95% confidence interval relates to the reliability of the estimation procedure, *i.e.* how much the average value is likely to fluctuate. For the LDV measurements in the present work the 95% confidence interval is determined directly from the [Welch \(1967\)](#) algorithm and is found to be maximal $[\pm \simeq 0.15 \text{ dB}]$. The Welch algorithm divides a time series of length N into n segments of length N_s (with $N_s < N$), over which it averages. The procedure reduces the confidence interval by a factor of $1/\sqrt{n}$. However, care has to be taken, since shorter segments reduce the precision in frequency, since the frequency resolution is given by

$$\Delta f = \frac{1}{N_s \Delta t}, \quad (2.6)$$

where $\Delta t = t_i - t_{i-1}$ is the time step of the time series. Also, it must be ensured, that the window size contains the longest characteristic time scales t_c , *i.e.* $N_s \Delta t \gg t_c$.

The 95% confidence interval of the frequency measurements by means of time resolved particle image velocimetry (TR PIV) is on average equal to $[\pm \simeq 0.30 \text{ dB}]$. The higher uncertainty is due to shorter sample sizes, which translates into lesser windows for averaging, and in general noisier data from PIV measurements, compared to data from LDV measurements.

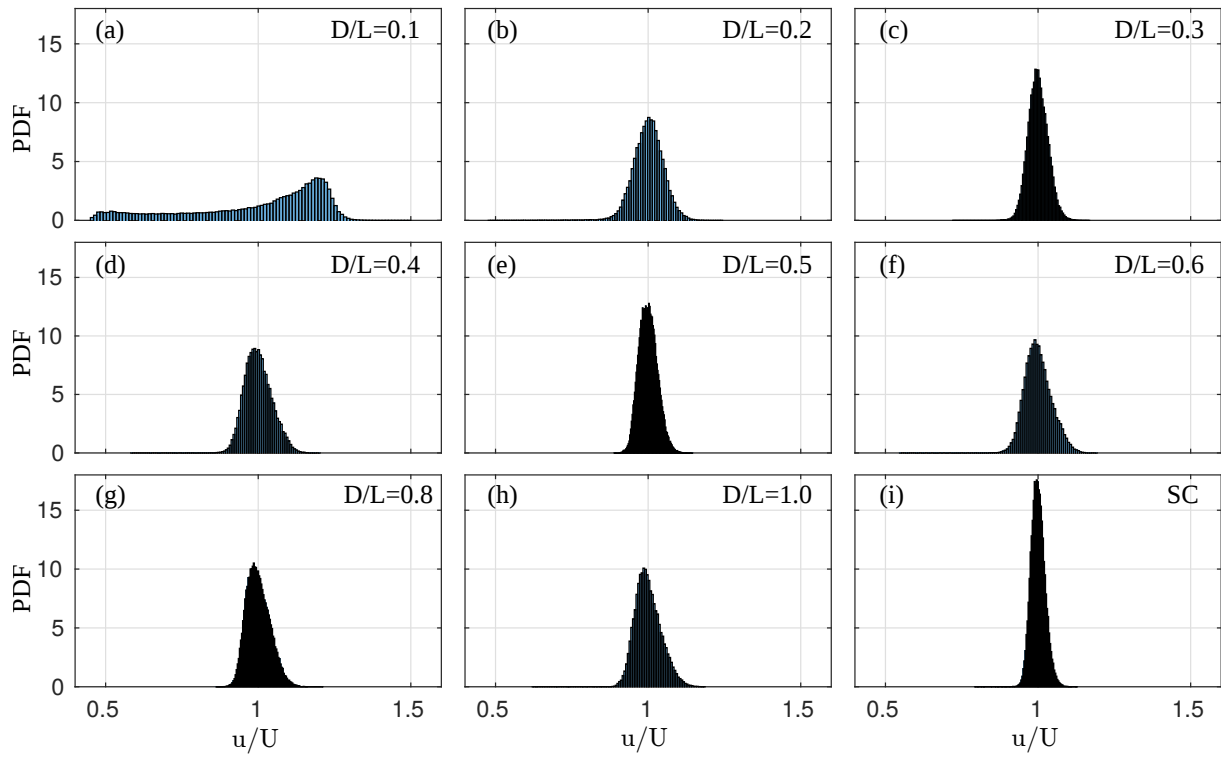


FIGURE 2.7: Probability density functions (PDF) of the normalized streamwise velocity times series u/U for all intercavity distances at Reynolds numbers $10\,500 \lesssim Re_L \lesssim 11\,100$.

Figure 2.7 shows the probability density functions (PDFs), computed from time series of the normalized streamwise velocity component (u/U) for all intercavity distances in the Reynolds number range $10\,500 \lesssim Re_L \lesssim 11\,100$. The PDFs of double cavity cases with $D/L \geq 0.2$ look all very similar. The velocities are distributed around the mean in what looks like a normal (or *Gaussian*) distribution. The single cavity case exhibits less fluctuations around the mean, which results in a slimmer PDF (smaller σ). At the closest intercavity distance ($D/L = 0.1$) the PDF shows a very different distribution. Greater fluctuations around the mean are accompanied by a strong skewness. This negative skewness indicates that at such a close distance different flow physics are encountered. We will come back to this case later in the work.

2.4.2 PIV

Concerning PIV uncertainties, a conservative value can be calculated following [Hart \(2000\)](#). The system resolution for distances can be estimated as the product of the pixel resolution (1/4 pix) and the pixel size for a study on PIV uncertainties). Thus, the resolution of the velocity field can be estimated dividing the latter value by the time between a pair of laser pulses. For the selected case to be controlled, this time value varies between $\Delta t = 150\,\mu\text{s}$ and $500\,\mu\text{s}$. The pixel size ζ is given in table 2.3 for the three

TABLE 2.3: Data for PIV uncertainty computation

Technique	Campaign	Δt [μ s]	ζ [mm/pix]	Δu [m/s]
TR PIV	BA	400	0.16129	0.1008
NTR PIV	BA	150 - 500	0.05469	0.0911 - 0.0273
NTR PIV	Paris	275 - 700	0.09473	0.0861 - 0.0338

PIV campaigns described above. This leads to a conservative value of uncertainty for the velocity field of

$$\Delta u = \frac{1}{4} \text{ pixel } \zeta \frac{\text{mm}}{\text{pixel}} \frac{1}{\Delta t} \quad (2.7)$$

The resulting uncertainties for the velocity calculations of the respective experimental campaigns are summarized in table 2.3. Maximum uncertainties of $\Delta u = 0.108 \text{ m/s}$ were obtained for the TR PIV measurements. This results in a relative error of $\Delta u/U_\infty$ between 2% for the highest inflow velocity and 16% for the lowest inflow velocity.

Chapter 3

Experimental Results

The results of the two experimental campaigns, introduced in the previous chapter, are presented in this chapter. Tables 2.1 and 2.2 summarize all studied cases. In the Paris-campaign, eight double cavity (DC) cases with different intercavity distances D are complemented by one single cavity case (SC) as a reference in the limit $D \rightarrow \infty$. In the BA-campaign, five double cavity cases with intercavity distances varying between $D/L = 0.1$ and $D/L = 0.6$ are presented.

3.1 Flow description

3.1.1 Instantaneous flow fields

Figures 3.1(a)-(h) show selected instantaneous flow fields of the Paris-campaign for the indicated intercavity distances ($0.1 \leq D/L \leq 1.0$) at intermediate Reynolds numbers ($5000 \lesssim Re_L \lesssim 6000$). Figures 3.2(a)-(j) show selected instantaneous flow fields of the BA-campaign for the indicated intercavity distances ($0.1 \leq D/L \leq 0.6$) for a low Reynolds number case (figures 3.2(a)-(e)) and a high Reynolds number case (figures 3.2(f)-(j)), respectively, thus completing the picture obtained from the results of the Paris-campaign. The flow fields from both campaigns indicate similar results, that will be discussed in the following paragraph. Note, that the small irregularities, in the downstream part of the flow fields of the Paris campaign (figure 3.1) are traces of the laser from the simultaneously conducted LDV measurements. Flow visualizations, using smoke, can also be found online scanning the QR code in figure 3.3 with a suitable device.

The flow in all sub-figures of figures 3.1 and 3.2 is from top to bottom. For $D/L \geq 0.2$, for any Reynolds number, the center flow separates the two cavities, in such a way, that no fluid is exchanged. Indeed, there is at least one streamline in the channel center that runs parallel to the streamwise direction. At a smaller distance (*i.e.* $D/L = 0.1$), the center flow starts to oscillate, as can be seen from figures 3.1(b) and 3.2(a)-(b), respectively. The

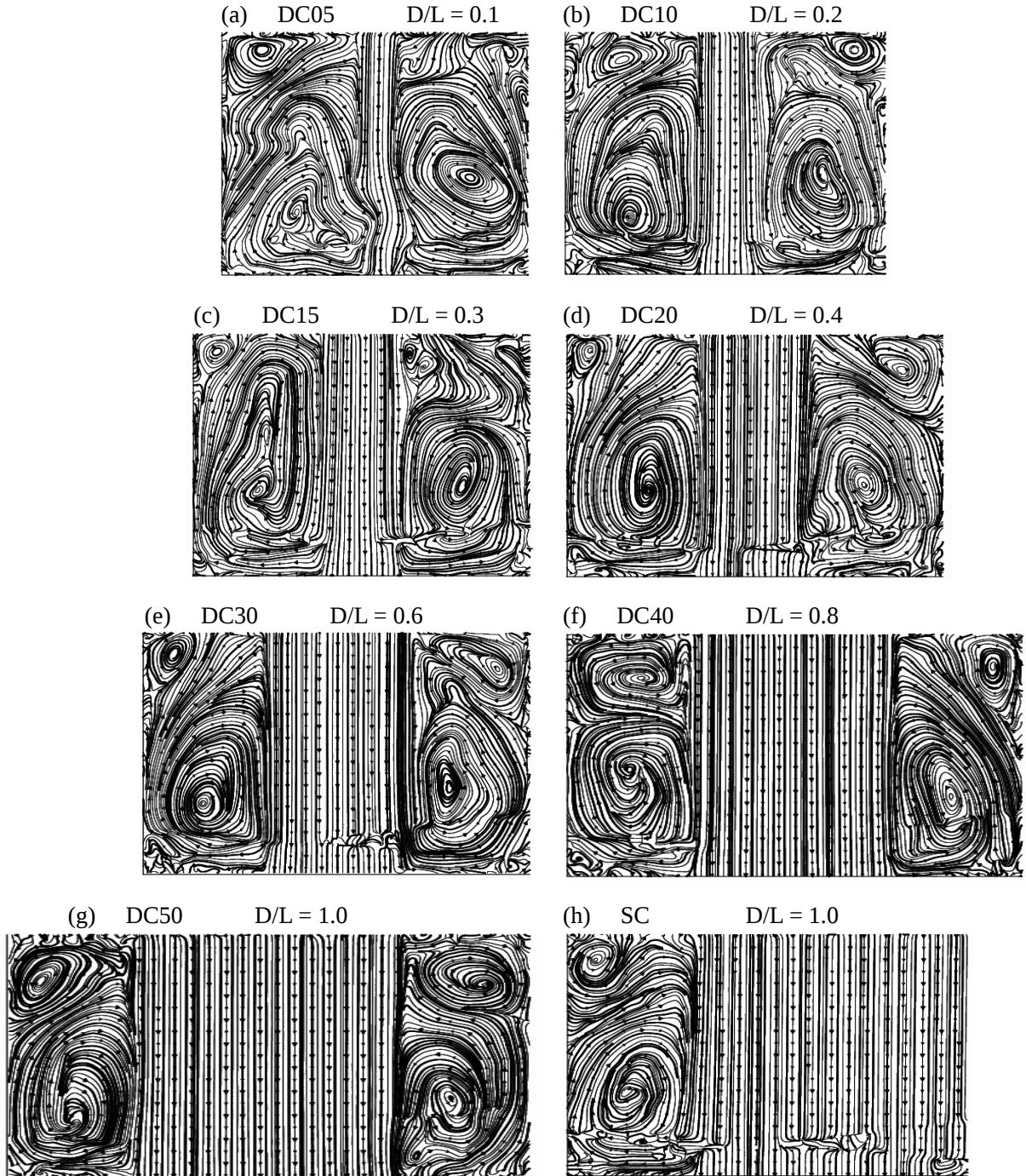


FIGURE 3.1: Streamlines of typical instantaneous fields at $5000 \lesssim Re_L \lesssim 6000$. Flow from top to bottom. Note, the irregularities in the flow field in the down stream part of the double cavities are due to the perturbation of the LDV laser beam.

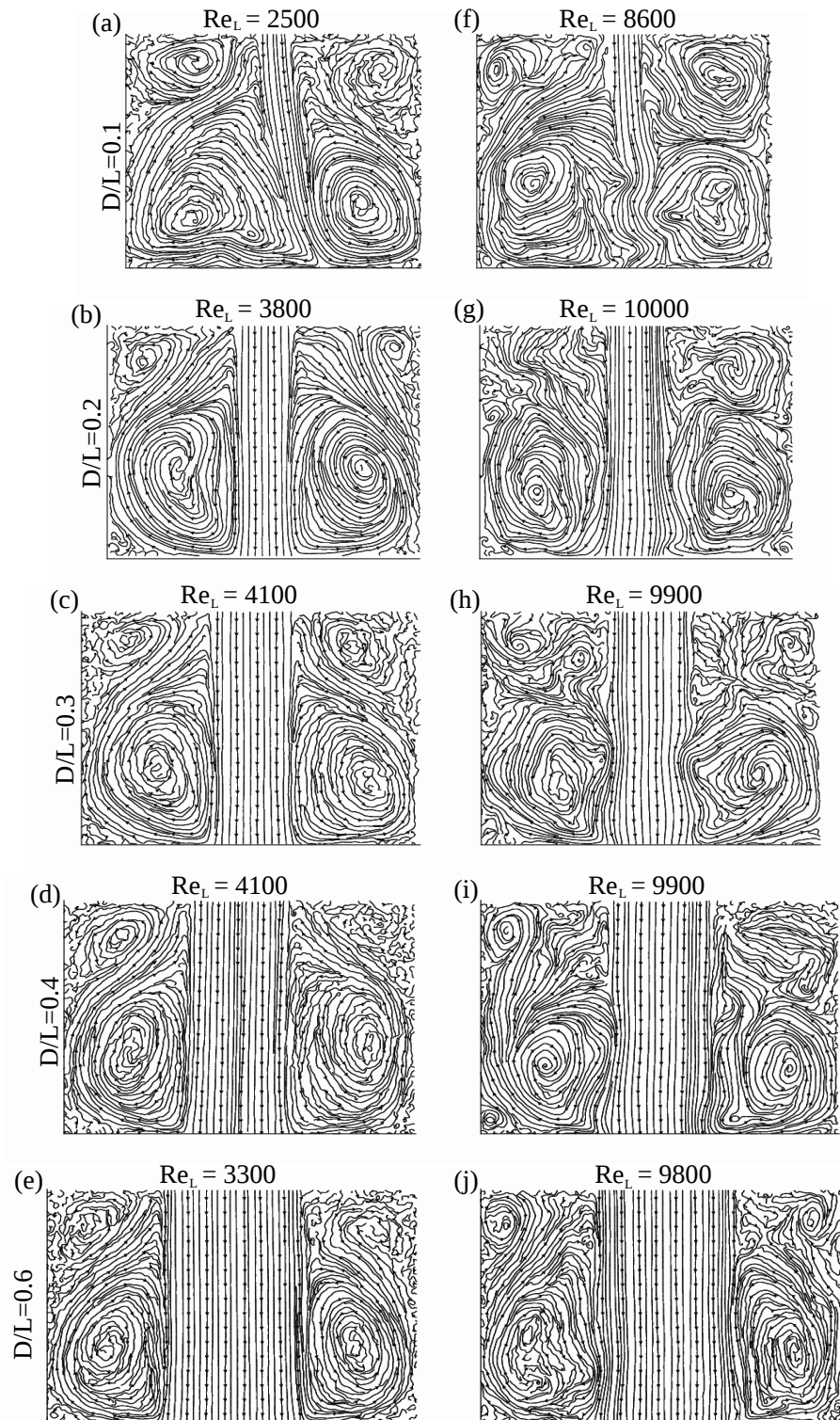


FIGURE 3.2: Instantaneous flow fields from BA-campaign at indicated Reynolds numbers and geometry ratio D/L . Flow from top to bottom.



FIGURE 3.3: QR-code for wind tunnel visualization of double cavity flow.

TABLE 3.1: Comparison for case $D/L = 0.1$ from present work with case $L/D = 10$ from [Maurel et al. \(1996\)](#). Data of the present work is calculated with $L = 50$ mm, $D = 5$ mm y $\epsilon = 1/4$.

current work					Maurel et al.	
Re_L	Re_D	λ	λ/D		λ/D^*	Mode N
2500	250	$\approx 4L$	≈ 40		40	0
5800	580	$\approx 1/2L$	≈ 5		4.4	2
8600	860	$\approx 1/3L$	≈ 3.3		3.01	3

* computed from equation 3.1.

streamlines of the center channel bend and momentum is alternately spilled into either cavity. The extra momentum spilled in a given cavity enforces the respective backflow in the intracavity recirculation region, resulting in a destabilizing effect on the incoming center flow through hydrodynamic feedback. As mentioned in the introduction, at such small distances the flow corresponds to a confined planar jet flow, similar to the one discussed in [Maurel et al. \(1996\)](#)). For low Reynolds numbers (figure 3.2(a)), the jet oscillates between the two cavities with a wave length $\lambda \approx 4L$. For increasing Reynolds numbers (figures 3.2(f) and 3.1(a)) the wave length of the jet decreases to $\lambda \approx 1/2L$ and $\lambda \approx 1/3L$, respectively. This behavior corresponds to the “cavity oscillations” regime defined by [Maurel et al. \(1996\)](#). These authors suggest the following relation to compute the selected wavelength:

$$\lambda = \frac{L}{N + \epsilon}, \quad (3.1)$$

where L is the impingement length (cavity length), N is the mode number and $\epsilon = 1/4$ is an end correction of around a quarter of a wavelength.

The closest intercavity distance ($D/L = 0.1$) of the present work coincides with the largest intercavity distances investigated by [Maurel et al. \(1996\)](#) ($L/D = 10$). As stated in their

article on page 4, in a given geometry, an increase of the Reynolds number can induce a jump to a higher mode. Hence, in general mode II is found at a higher Reynolds number than mode I for the given geometry ($D/L = 0.1$). The exact Reynolds numbers could not be determined from the provided data. However, the maximum Reynolds number studied in [Maurel *et al.* \(1996\)](#) is $Re_D \approx 370$, which is at the lower end of the Reynolds number range of the present work ($250 \lesssim Re_D \lesssim 13000$).

In the current work, for the lowest Reynolds number (figure 3.2(a)), we obtain $\lambda \approx 4L$ (figure 3.2(a)) which translates into $\lambda/D \approx 40$. Such a high value for λ/D is not mentioned in [Maurel *et al.* \(1996\)](#). However, we can apply equation 3.1 with $N = 0$. This yields the exact same value $\lambda/D = 40$. For the next higher Reynolds number we obtain $\lambda/D \approx 5.0$ (figure 3.1(a)). [Maurel *et al.* \(1996\)](#) obtain a value of $\lambda/D = 4.4$ for $N = 2$. For the highest Reynolds number we find $\lambda/D \approx 3.3$ (figure 3.2(f)), while from equation 3.1 we compute $\lambda/D = 3.01$ for $N = 3$. All three cases are in good agreement. Table 3.1 provides a summary of the results. Note, however, the values of λ of the present work are only rough estimates, obtained from streamline graphs of the instantaneous flow fields in figures 3.2(a),(f) and 3.1(a). Hence, uncertainties are inevitable.

For all cases with $D/L \geq 0.2$ the two cavities are separated by the center channel flow and the flow fields look very similar, independent of D/L and the Reynolds number. The streamlines in the channel center are aligned with the streamwise direction and no mass is exchanged between the cavities. The intracavity flow is structured into a main recirculation region, occupying most of the cavity, and a smaller recirculation region, located in the upstream cavity corner. However, the sizes of the two recirculation bubbles are not constant. The larger bubble, located further downstream, is fed by the oscillating shear layer, while the smaller bubble, located further upstream, is fed by the larger bubble. The interaction cycle happens as follows: As the shear layer injects mass and momentum into the larger downstream bubble, it extends into the upstream area. It moves mass and momentum into the smaller bubble, which grows and pushes the larger bubble back further downstream. As the larger bubble becomes smaller, it stops injecting mass and momentum into the upstream region and makes the upstream bubble diminish again, which gives way for the larger bubble to grow again and hence to start the next cycle.

3.1.2 Symmetry properties

The instantaneous flow fields also provide information on the spatial symmetry of the velocity field. Considering a spatial reflection symmetry (with respect to the mid plane $y = 0$) the following two flow configurations can be distinguished using the streamwise velocity fluctuations $u(x, y)$:

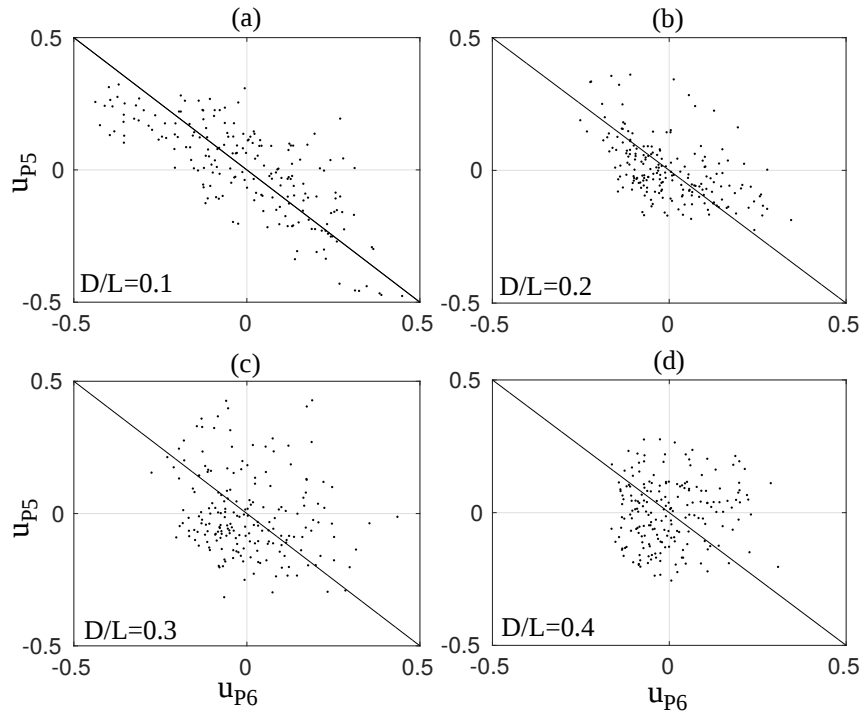


FIGURE 3.4: Fluctuating streamwise velocity correlations of shear layer probe points (cf. figure 3.14) at a streamwise position $x_{P5} = x_{P6} = 0.6L$ and $y_{P5,P6} = \pm D/2$ for (a) $D/L = 0.1$, (b) $D/L = 0.2$, (c) $D/L = 0.3$ and (d) $D/L = 0.4$. The Reynolds number range is $5\,000 \lesssim Re_L \lesssim 6\,000$.

$$u(x, y) = u(x, -y) \Rightarrow \text{spatially symmetric} \quad (3.2)$$

$$u(x, y) \neq u(x, -y) \Rightarrow \text{spatially asymmetric} \quad (3.3)$$

Following the wording of linear stability analysis, the spatially symmetric type is commonly called a *varicose* type flow configuration, while the spatially asymmetric type is usually called a *sinuous* type flow.

While the flow field for $D/L = 0.1$ is clearly of sinuous type (figure 3.2(f)), the flow field for greater distances show both varicose (figure 3.2(h)) and sinuous (figure 3.2(i)) behavior. In fact it was found that at such high Reynolds numbers, for which the study was conducted, varicose and sinuous type of flows may coexist. For a given geometry at a fixed Reynolds number the flow might switch between the varicose and the sinuous type. The reason for this behavior will be addressed and explained by means of a temporal linear stability analysis in chapter 6.

Figure 3.4 shows the spatial correlation graphs of the streamwise fluctuating velocity component between two shear layer points $P_5 = [x_{P5}, y_{P5}]$ and $P_6 = [x_{P6}, y_{P6}]$ (see caption

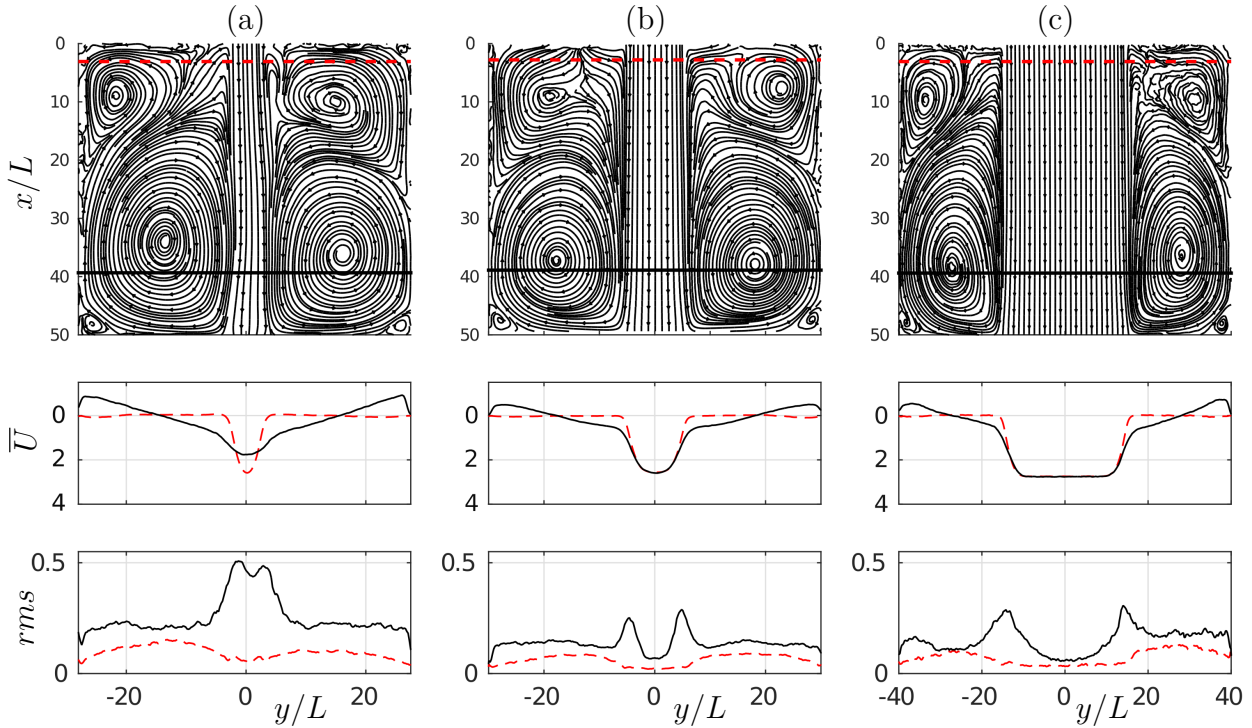


FIGURE 3.5: Top row: Streamlines over the time averaged field for double cavity cases (a) $D/L = 0.1$ (b) $D/L = 0.2$ and (c) $D/L = 0.6$ at $Re_L \approx 8700$. Flow from top to bottom. Middle row: Velocity profiles at $x/L = 0.1$ (red $--$) and at $x/L = 0.8$ (black $—$). Lower row: rms profiles at $x/L = 0.1$ (red $--$) and at $x/L = 0.8$ (black $—$). Flow from top to bottom.

of figure 3.4 for details), for the four closest intercavity distances $0.1 \leq D/L \leq 0.4$. At $D/L = 0.1$ the flow is spatially asymmetric (sinuous type), as can be seen from the elongated cloud of points, spread along the anti-diagonal in figure 3.4(a). This corresponds to a sinuous configuration. At $D/L = 0.2$ (figure 3.4(b)) the flow is still mainly asymmetric, however, with increasing distance (figures 3.4(c)-(d)), the cloud of points transforms from the elongated form to a circular form. The distribution of the points becomes aleatory, indicating that the correlation and hence the symmetry is lost. In agreement with the visual analysis of the flow field in the previous paragraph both the varicose and the sinuous modes are then equally probable.

3.1.3 Mean flow fields

The streamlines of the mean velocity fields for three cases at approximately the same Reynolds number are depicted in figure 3.5 (top row). In agreement with what was found for the instantaneous velocity fields, the intracavity flow of the mean velocity fields is structured into a main recirculation region, occupying most of the respective cavity, and a second smaller one, located further upstream. The mean fields additionally reveal in both downstream cavity corners a very small recirculation region, which could not easily be observed in the instantaneous fields. Also depicted in figure 3.5, are the velocity

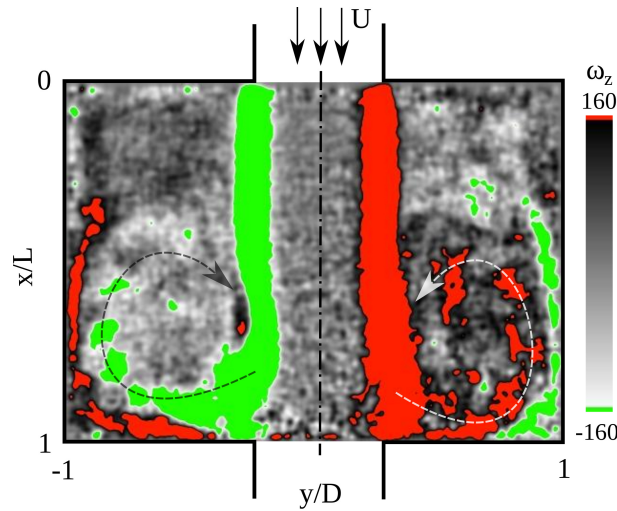


FIGURE 3.6: (Color in PDF) Contour plot of spanwise instantaneous vorticity $\omega_z = \partial_x v - \partial_y u$ of experimental double cavity configuration with $D/L = 0.4$ at $Re_L \approx 8600$ from the BA-campaign. The vorticity field was computed from the 30 most energetic POD modes of the velocity field. Flow from top to bottom as indicated by the arrows.

profiles, together with the root mean square (rms) profiles at the indicated streamwise positions.

For the $D/L = 0.1$ case (figure 3.5(a)), the velocity profile further downstream at $x/L \approx 0.8$ shows a clear deficit in the maximum center line velocity with respect to the profile at the double cavity entrance $x/L \approx 0.1$. At the same time the intracavity velocity profiles show higher values than for cases with greater intercavity distances. The two shear layers, formed by the separating boundary layers, join in the channel center, resulting in a double peak of the rms value at the downstream location $x/L \approx 0.8$. This flow configuration corresponds to a confined jet case, as mentioned above. For larger intercavity distances (figure 3.5(b)-(c)), the two shear layers stay well separated all along the cavity length as can be seen from the rms profiles at the position $x/L \approx 0.8$. It can also be noted that there is no center line velocity deficit for cavity distances $D/L \geq 0.2$ as can be seen in figures 3.5(b)-(c). As mentioned in the introduction, the forthcoming study will focus mainly on cases where the two shear layers can be distinguished all along the cavity length. The slightly asymmetric and wrinkly rms profiles suggest that more velocity fields are needed to obtain fully converged data.

3.1.4 Vorticity flow field

Figure 3.6 shows a typical instantaneous field of the spanwise vorticity component $\omega_z = \partial_x v - \partial_y u$ of a $D/L = 0.6$ case at $Re_L \approx 8000$ computed from the 30 most energetic proper orthogonal modes of the velocity field. Proper orthogonal decomposition (POD)

allows to extract the most energetic features of a flow field. To that end, a series of N two dimensional velocity field snapshots is assembled in a $m \times n$ matrix

$$M = [\vec{u}(\vec{x}, t_0), \vec{u}(\vec{x}, t_1), \dots, \vec{u}(\vec{x}, t_N)] \quad (3.4)$$

which is then decomposed, using a singular value decomposition (SVD) according to

$$M = A \Sigma B^T \quad (3.5)$$

where Σ is an $m \times n$ diagonal matrix containing the singular values on its diagonal. Matrices A and B^T are $m \times m$ and $n \times n$ unitary matrices, respectively. The columns of B^T form a set of orthonormal vectors onto which the velocity field is then projected. The SVD decomposition ranks the mode according to the energy. Here, the 30 most energetic modes are used, which account for about 75% of the total energy. The advantage of applying the POD technique to the velocity field before computing the vorticity field is that only the dynamically most energetic modes are retained while low-energy structures (such as noise) are discarded.

Both shear layers, originating from the separated boundary layers of the inflow channel, can clearly be distinguished. When the shear layers impinge on the cavity rear edge, vorticity is regularly injected into the respective cavities. The injected vorticity forms small vortices at the frequency of the shear-layer oscillations. These regularly spaced vortices circumvent in the recirculation region, limited by a vortex sheet of opposite sign. The injected vortices create a circular “carousel” like formation. This phenomenon will be discussed in detail in chapter 7. It can also be observed in flow fields of the single cavity, shown in [Basley *et al.* \(2011\)](#).

3.2 Velocity profiles

Inflow profiles

Due to restrictions of translucency of the geometry at $x = 0$, the incoming boundary layer is obtained at position $x_0 = -30$ mm. Depending on the inflow velocity and the intercavity distance the streamwise inflow profile $U(y)$ either corresponds to a parabolic profile of the form

$$U_p(y) = U_\infty \left(1 - \frac{y^2}{D^2} \right) \quad (3.6)$$

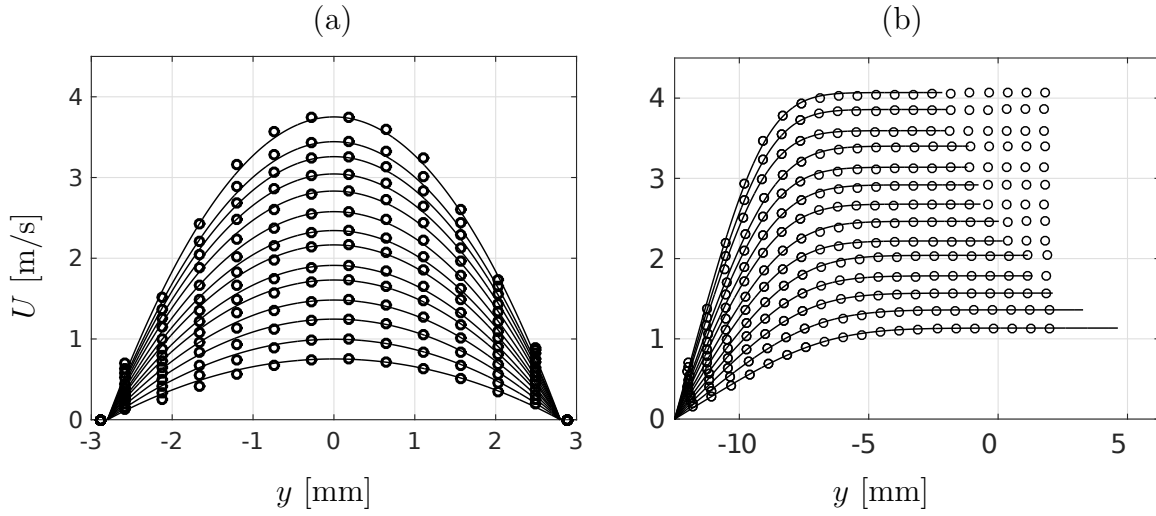


FIGURE 3.7: Experimental and analytical velocity profiles at inflow position $x = -30$ mm for cases at (a) $D/L = 0.1$ (parabolic fit) and at (b) $D/L = 0.5$ (Blasius boundary layer fit).

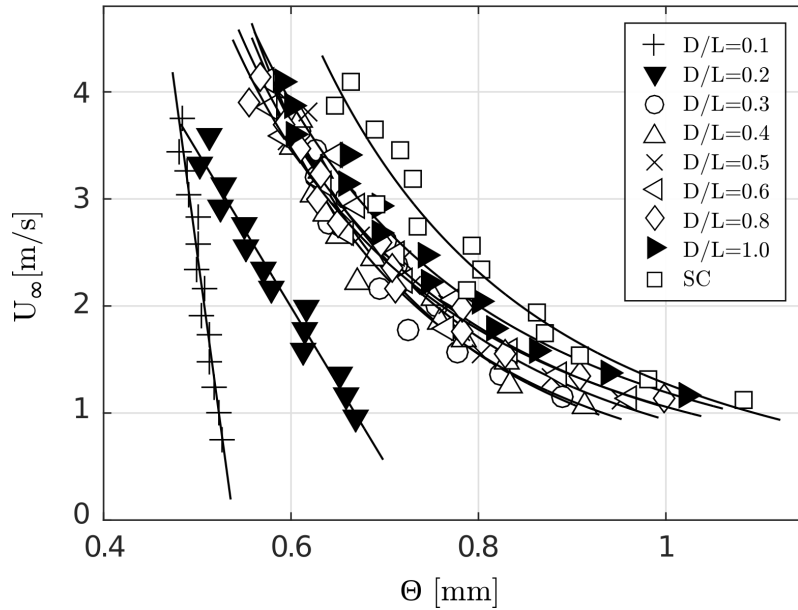


FIGURE 3.8: Momentum thickness of incoming boundary layer for all cases of the Paris-campaign summarized in table 2.1. Symbols, as in table 2.1, show experimental data. Solid lines (—) indicate least square fits.

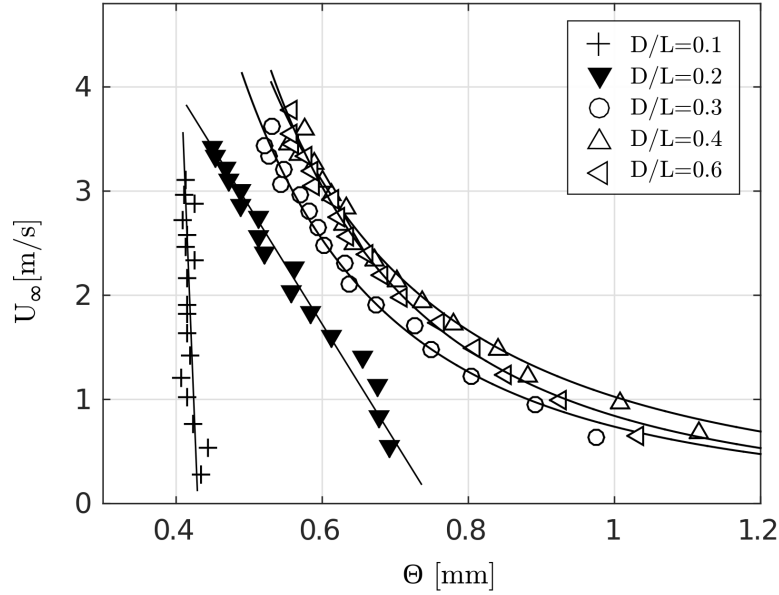


FIGURE 3.9: Momentum thickness of incoming boundary layer for all cases of the BA-campaign summarized in table 2.2. Symbols, as in table 2.2, show experimental data. Solid lines (—) indicate least square fits.

or to a symmetric (double) Blasius boundary layer profile. The parabolic profile is obtained for a fully developed channel flow, *i.e.* when the two opposing boundary layers join in the channel center. In the present setup this happens for all Reynolds numbers at the distance $D/L = 0.1$ and for low Reynolds number cases at the distance $D/L = 0.2$. The parabolic fit, together with experimentally measured profiles for all Reynolds numbers at $D/L = 0.1$, are shown in figure 3.7(a). A good agreement between the analytic profiles and the experimental profiles is observed. For all other distances, the Blasius boundary layer profile (solution of the Blasius equation) is fitted to the experimental data. The fits are performed by minimizing the $\mathcal{L}2$ -norm difference of experimental streamwise velocity profile and the analytical profile. The fitting algorithm is outlined in detail in section 3.2.1 and the Matlab code is documented in Appendix B. The results for the distance $D/L = 0.5$ are exemplified in figure 3.7(b). A good agreement between experiments and analytical profile was found for all inflow velocities. The shape factor of the profiles vary between 2.62 and 2.72, strongly suggesting a laminar inflow. The shear layers, that form upon separation at the double cavity's leading edge, depend on the momentum thickness Θ_0 of the incoming velocity profile. The momentum thickness is calculated according to:

$$\Theta_0 = \int_0^\infty \frac{U_{x0}(y)}{U_\infty} \left(1 - \frac{U_{x0}(y)}{U_\infty} \right) dy \quad (3.7)$$

Figure 3.8 shows the momentum thickness at inflow Θ_0 , calculated from experimental data

of the Paris-campaign, for all cases of table 2.1, as a function of the center line inflow velocity U_∞ . Figure 3.9 shows the momentum thickness at inflow Θ_0 , calculated from experimental data of the BA-campaign, for all cases of table 2.2, as a function of the center line inflow velocity U_∞ . Both campaigns show similar results. For the parabolic profiles, the momentum thickness is in theory independent of the center line velocity U_∞ and depends only on the intercavity distance. In accordance with the theory, at $D/L = 0.1$, in both campaigns, Θ_0 is found almost independent of U_∞ , varying linearly, as indicated by the linear fit to the experimental data. At $D/L = 0.2$, for which the velocity profiles start to depart from the parabolic form, a somewhat stronger dependency on U_∞ is found, though Θ_0 still varies linearly with U_∞ . All other cases ($D/L \geq 0.3$) were fitted with a power fit function of the form $y = cx^d$, by minimizing the $\mathcal{L}_2$ -norm difference of the analytical function and the experimental data. For $D/L \geq 0.3$, the momentum thickness is largely independent of the intercavity distance. Interestingly, for the single cavity case, slightly larger values of Θ_0 are obtained than for the closely related $D/L = 1.0$ case, suggesting, that even though the cavities are far apart, the incoming flow is still affected by the presence of the second cavity.

Intracavity profiles

The streamwise velocity profiles inside the double cavity domain are fitted to a hyperbolic sine (sinh) profile, borrowed from the analysis of a cylinder wake [Huerre & Rossi \(1998\)](#). The analytical form of the non-dimensional hyperbolic sine profile reads:

$$U_{sinh}(y, N, a) = [1 + \sinh^{2N} [y \sinh^{-1}(a)]]^{-1}, \quad (3.8)$$

where $N \in \mathbb{N}$ and $a \in \mathbb{R}$ are the free parameters to be fitted. For the single cavity case the classical hyperbolic tangent (tanh) profile

$$U_{tanh}(y) = \frac{1}{2} + \frac{1}{2} \tanh\left(\frac{y}{2}\right) \quad (3.9)$$

in its non-dimensional form is proposed. Figure 3.10(a) shows the results of the fit for all cases in table 2.2, measured at $x = x_{min}$ for a Reynolds number of roughly $Re_L \approx 9000$. Again, the fits are performed by minimizing the $\mathcal{L}_2$ -norm difference of the analytical profile and the experimental mean velocity profile, varying both N and a . The streamwise position x_{min} denotes the location where the best fit (in the least square sense) is obtained. More details on x_{min} can be found in section 6.2.3. In the channel center and the shear layer region good agreement between the analytic and experimental profiles is observed

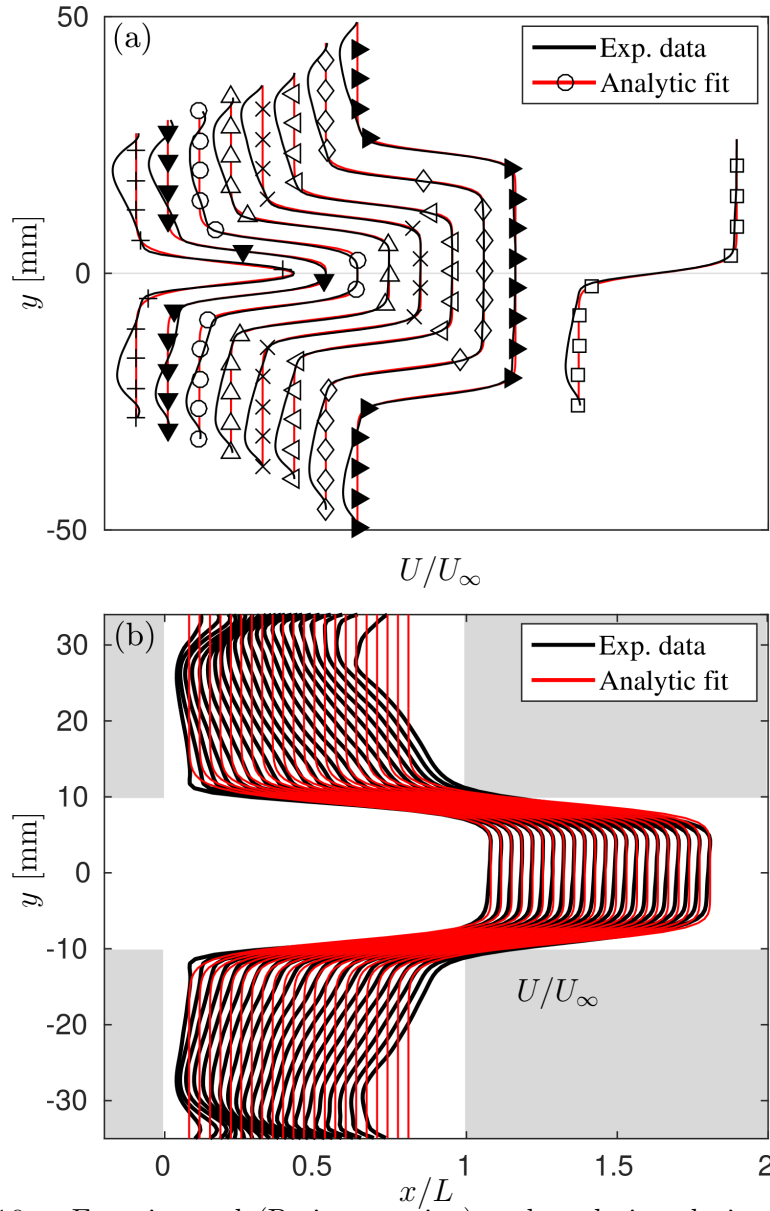


FIGURE 3.10: Experimental (Paris-campaign) and analytic velocity profiles (a) at streamwise position $x = x_{min}$ for all cases in table 2.1 at $Re_L \approx 9000$ and (b) along a double cavity with $D/L = 0.4$ at $Re_L = 8800$. The analytic profile is given in equation 3.8. Symbols as in table 2.1.

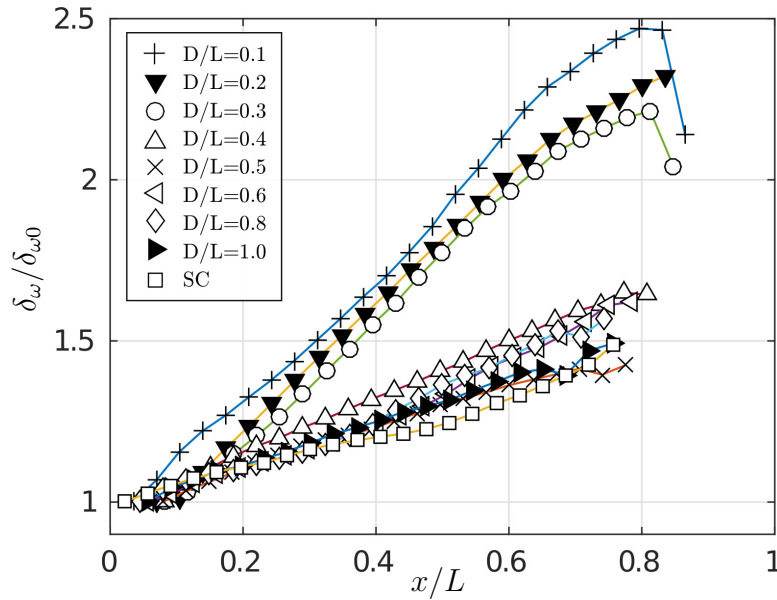


FIGURE 3.11: Normalized vorticity thickness $\delta_\omega/\delta_{\omega 0}$ for all cases in table 2.1 at $Re_L \approx 9000$ computed from experimental Paris-campaign data. Symbols as in table 2.1.

for all intercavity distances. The recirculation regions inside both cavities is not captured by the sinh-profile.

Figure 3.10(b) exemplifies for a case with $D/L = 0.4$ at $Re_L = 8800$ the development of the streamwise velocity profile along the double cavity, together with the fitted sinh-profile.

In the channel center the sinh profile follows closely the experimental data at all streamwise positions. Very close to the leading edge ($x/L < 0.15$) as well as close to the trailing edge ($x/L > 0.6$) both profiles show differences in the shear region, while close to the streamwise position x_{min} good agreement is obtained over the entire shear region. As mentioned before, inside the cavities we observe a variation between the experimental and the analytical profiles due to the recirculation region which is not modeled by the sinh-profile.

The vorticity thickness

The vorticity thickness, defined as

$$\delta_\omega = \frac{\Delta U}{\left(\frac{\partial U}{\partial y}\right)_{\max}}, \quad (3.10)$$

is calculated along the cavity length from experimental velocity profiles for all cases in table 2.1 at $Re_L \approx 9000$. The inflexion point is determined such that the gradient $(\partial U/\partial y)$ is maximal. The shear-strength is given as $\Delta U = U_{\max} - U_{\min}$. The vorticity thickness δ_ω ,

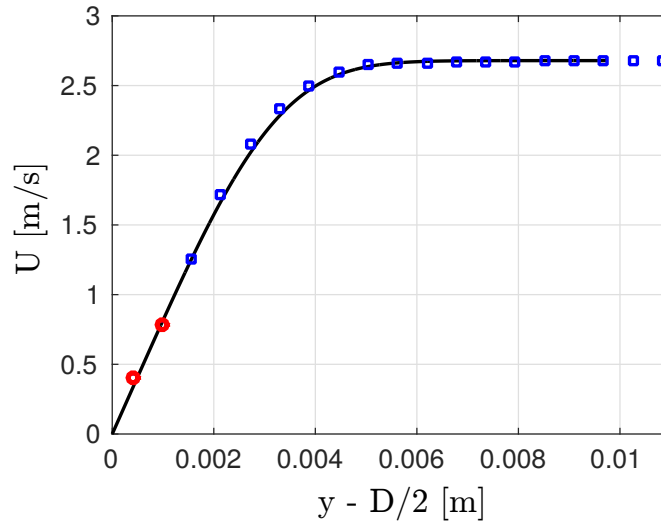


FIGURE 3.12: (Color in PDF) Exemplary Blasius boundary layer fit for a case with $D/L = 0.4$ and $Re_L = 8800$ (Paris-campaign). Red circles indicate data points omitted for fitting. Blue squares indicate data points used for fitting. Black solid line shows the Blasius boundary layer fit.

normalized with the vorticity thickness at the cavity's leading edge ($\delta_{\omega 0} = \delta_{\omega}(x = 0)$), is depicted in figure 3.11. The vorticity thickness at the cavity's leading edge $\delta_{\omega 0}$ is essentially determined by the momentum thickness Θ_0 of the incoming flow. The development along the double cavity domain was found to be largely independent of the Reynolds number, but strongly dependent on the intercavity distance. For intercavity distances $D/L \leq 0.3$ the vorticity thickness δ_{ω} increases much stronger compared to larger distances and the value of δ_{ω} at $x/L = 0.8$ is roughly 50% higher. The trend is close to linear up to $x/L \approx 0.8$, where it flattens out and even drops due to the effect of the rear cavity edge. For intermediate distances $0.4 \leq D/L \leq 0.8$ the vorticity thickness grows less strongly and for $D/L \geq 0.5$ it starts showing a plateau-like zone between $0.2 \leq x/L \leq 0.5$. All cases at these distances show similar values. The plateau, which was also observed by [Basley *et al.* \(2011\)](#) for the single cavity case, flattens out for increasing distances.

For the cases where the inflow profile corresponds to a Blasius boundary layer profile ($D/L \geq 0.3$), the value of $\delta_{\omega,0}$ at the cavities leading edge is $2\delta_{\omega,0} \approx 4.5\Theta_0$, which reasonably satisfies the theoretical properties of Blasius profile mixing layers, as defined by Huerre and Monkevitz [Huerre & Monkevitz \(1985\)](#). A similar value ($2\delta_{\omega 0} \approx 4.6\Theta_0$) was found by [Basley *et al.* \(2013\)](#) for the single cavity.

3.2.1 Blasius fit

The Blasius boundary layer profile, which is derived on the bases of a similarity solution of the Navier-Stokes equations, was fitted to the experimental data using the nonlinear fitting

algorithm *fminsearch* in Matlab. The corresponding Matlab code is given in Appendix B. The Blasius boundary layer describes a steady two-dimensional laminar boundary layer that forms on a semi-infinite plate which is held parallel to a constant unidirectional flow. Due to the relatively coarse resolution of the PIV results, data points very close to the wall in general do not coincide with the wall limits. For that reason the fitting algorithm was adapted so it varied not only the inflow length x_0 but also the zero position y_0 of the wall. The procedure is illustrated in figure 3.12. To increase the fitting quality, only the blue data points (squares) are used for the fit, while the red data points (circles) are excluded. The black solid line in figure 3.12 shows the fitted Blasius boundary layer profile. It coincides well with the experimental data.

The fitting procedure consists of the following three steps:

- A non-dimensional Blasius profile on a very fine grid is computed (roughly 9000 data points)
- The data points of the non-dimensionalized experimental profile closest to the non-dimensional Blasius profile are computed
- The non-linear fitting function *fminsearch* finds the best parameter set for the zero position y_0 of the Blasius profile and the distance from the origin of the boundary layer x_0 assuming a constant kinematic viscosity of $\nu = 1.5 \cdot 10^{-5} \frac{m^2}{s}$

3.3 Spectral Results

3.3.1 Comparison: Paris vs. Buenos Aires

Figure 3.13 compares the power spectra in the impingement region of the Paris-campaign and the BA-campaign for a low Reynolds number case (figure 3.13(a)) and a high Reynolds number case (figure 3.13(b)). In general, the power spectra obtained by means of LDV (Paris-campaign) are smoother and the peak-to-noise-ratio is lower. This is due to the fact that the time series of the Paris-campaign are roughly three times longer than the time series of the BA-campaign.

Nonetheless, both spectra show the same qualitative spectral features: at the low, as well as the high Reynolds number, both campaigns capture the typical shear layer peak at $St_L \approx 1$ and the low frequency peak close to $St_L \approx 0$. The low frequency peak of the BA-campaign is more amplified, most likely due to the fact that the spectrum is obtained closer to the recirculation region (*cf.* zone 3 in figure 3.15 and LDV probe point P_{LDV} in figure 2.5). At low Reynolds numbers, the spectrum of the BA-campaign contains also the first harmonic of $St_L = 2St_{L,1} \approx 2$, which is not observed in the spectrum of

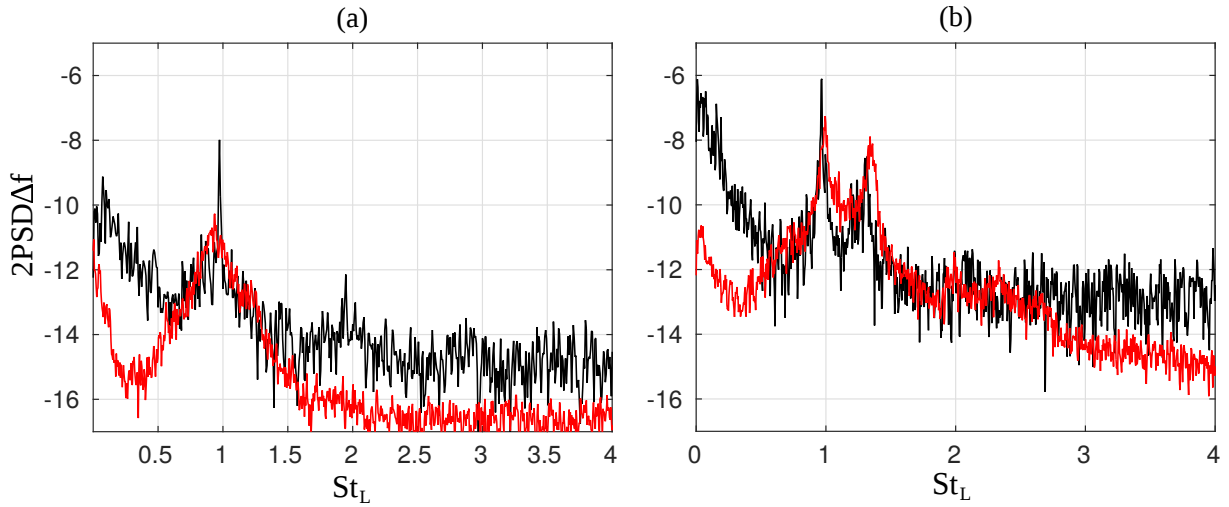


FIGURE 3.13: (Color in PDF) Comparison of power spectra from Paris-campaign (red) in probe point P_{LDV} and BA-campaign (black) spatially averaged over zone 3 (see figure 3.15) for cases with $D/L = 0.4$ (a) at $Re_L \approx 6100$ and (b) at $Re_L \approx 8800$.

the Paris-campaign. For increased Reynolds numbers (figure 3.13(b)) a second peak at $St_{L,2} \approx 1.4$ appears in both cases, which is incommensurate with respect to the first peak at $St_{L,1} \approx 1$. The spectrum of the Paris-campaign also contains the first harmonics of both peaks, $2St_{L,1}$ and $2St_{L,2}$.

In general, both spectra indicate the same physical behavior and the fact that they were recorded in different facilities, located in different hemispheres, underlines the universality of the forthcoming results.

3.3.2 Global spectra

Paris-campaign

Figure 3.14 shows spectra recorded with Laser Doppler Velocimetry (LDV) at nine different probe locations inside the double cavity domain, for a case with $D/L = 0.4$ at $Re_L = 10\,500$. As expected, the spectrum at the inflow position (P_7) shows no significant spectral content. Only a very low frequency peak at $St_L \approx 0$ is present, which is, as mentioned by [de Vicente *et al.* \(2014\)](#) for the case of the single cavity, most likely due to quasi-steady features of the flow. The six probe points located on the channel center-line show the same spectral structure: two dominant non-harmonic frequency peaks at Strouhal numbers $St_{L,1} \approx 1$ and $St_{L,2} \approx 1.4$, together with their first harmonics and linear combinations, which are roughly two orders of magnitude smaller in amplitude. A rather broadband peak, one order of magnitude smaller in amplitude than the peak at $St_{L,1}$, is found at around $St_{L,3} \approx 0.6$. The low frequency content at $St_{L,0} \approx 0$ is transmitted from the inflow. Probe points P_8 and P_9 are located inside the respective cavities. Spectra from time series recorded at these probe points are different from spectra recorded at

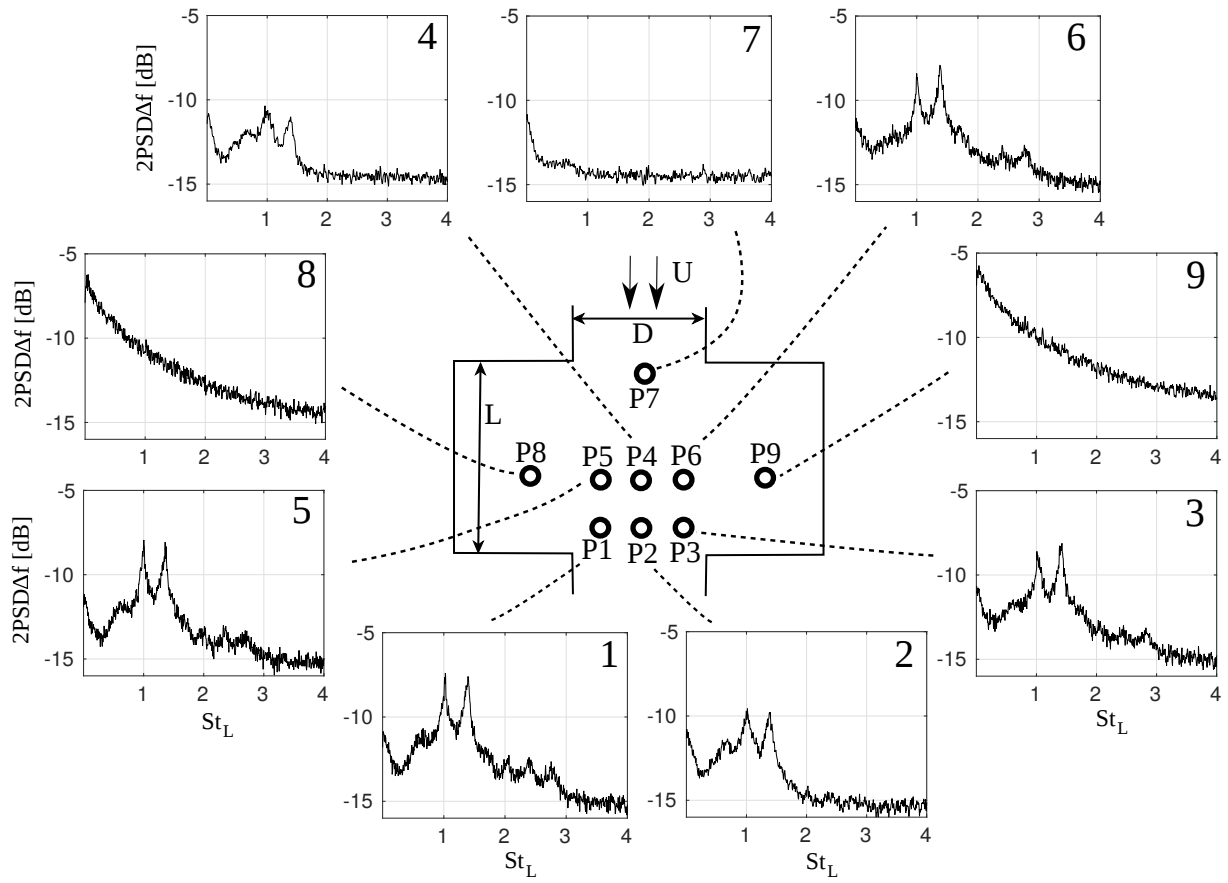


FIGURE 3.14: Power spectra at multiple probe locations inside the double cavity, measured with LDV (Paris-campaign). Double cavity case with $D/L = 0.5$ and $Re_L = 9\,600$.

probe points, located in the channel center or in one of the shear layers. Only a strongly amplified broadband low frequency peak at $St \approx 0$ can be found in the power spectra of the intracavity probe points. The typical shear layer peaks can not be observed. This is in agreement with [Basley *et al.* \(2011\)](#), who found the same spectral structure inside the cavity of single open cavity flow. In this work we are mainly interested in the interaction of the two shear layers. Since probe points P_1 to P_6 give very similar results we will mostly focus on time series recorded at probe point P_1 (which is coincident with the position of P_{LDV} in figure 2.5).

BA-campaign

In general the spectra of the TR PIV measurements are noisier than the spectra obtained by means of LDV. This is due to shorter time series length and the uncertainties transmitted from the PIV measurements. However, the great advantage of TR PIV is, that it allows to evaluate the spectra at any point in either of the two cavities. As mentioned above, in the channel center no TR PIV data is available due to light source restrictions of the continuous laser. Hence only the intracavity regions will be evaluated. Figure 3.15

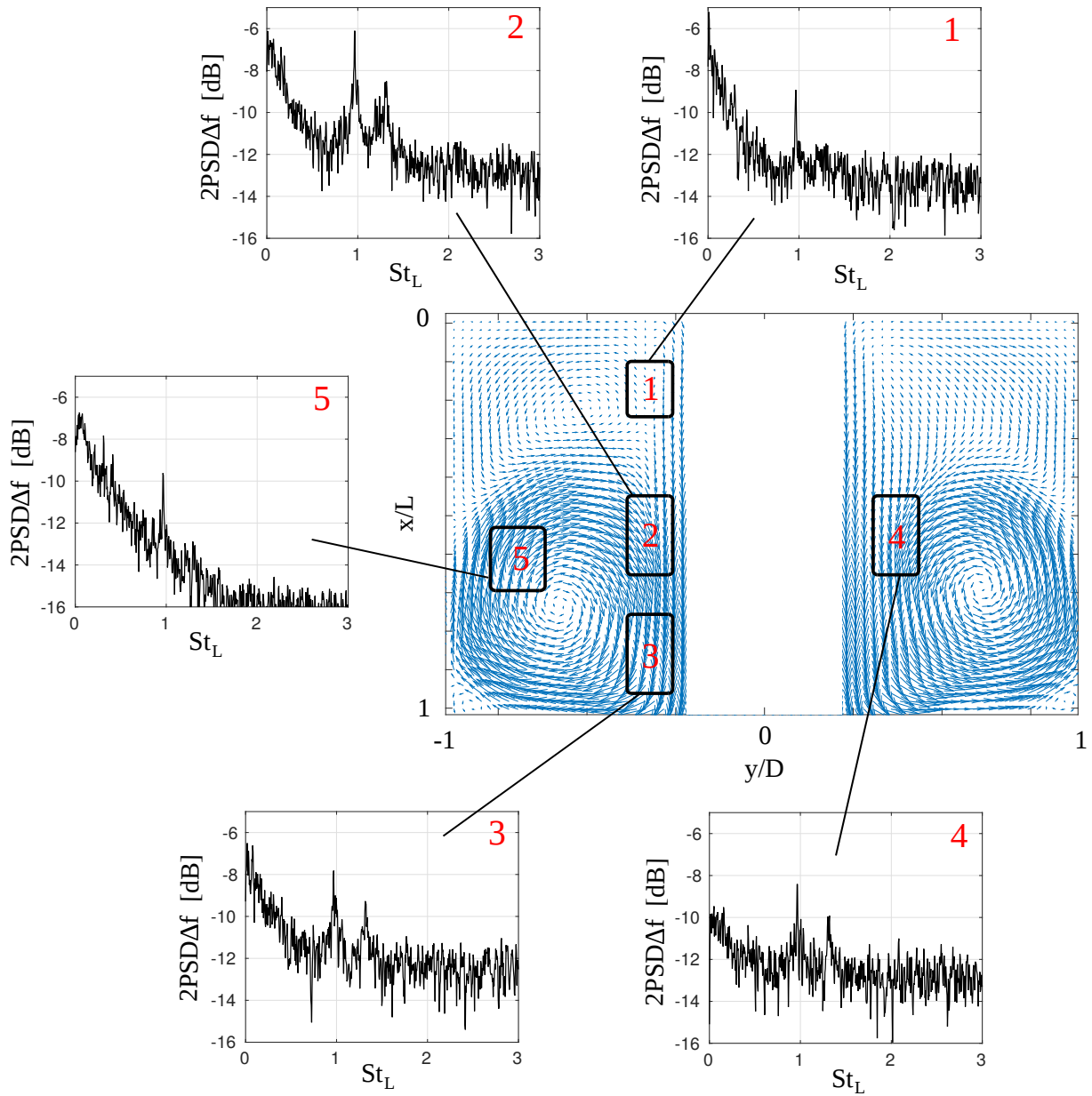


FIGURE 3.15: Mean flow field from TR PIV of $D/L = 0.4$ at $Re_L = 8900$. Zones 1-5 indicate spatial averaging windows for spectral computation as explained in the text.

compares spectra from different locations in the flow field for a typical double cavity configuration with $D/L = 0.4$ at $Re_L = 8900$. In order to decrease the spectral background noise of the signals, the time series from which the spectra are calculated, are averaged over small zones (1-5) in the flow field.

Common to all spectra is a broadband low frequency peak close to $St_{L,0} \approx 0$ and a narrow band frequency peak at $St_{L,1} \approx 1$. While the low frequency peak is commonly associated with three dimensional intracavity instabilities and modulations ([Basley et al. \(2013\)](#), [Douay et al. \(2015\)](#)), the peak at $St_{L,1} \approx 1$ can be linked to the Kelvin-Helmholtz shear layer instabilities. Close to the inflow (zone 1) the peak at $St_{L,1} \approx 1$ dominates the

shear layer spectrum. Moving further downstream a second peak, incommensurate with respect to the first peak, appears at $St_{L,2} \approx 1.4$ (zone 2 and 4, respectively). Close to the impinging edge (zone 3) the spectral content does not change much with respect to zone 2. Besides the low frequency content, the two non-harmonic peaks dominate the shear layer spectrum.

In the intracavity regions mostly flat spectra are observed, however, zones can be found for which the spectrum contains a peak at $St_{L,1} \approx 1$. The second peak at $St_{L,2} \approx 1.4$ could not be observed anywhere in the intracavity region.

3.3.3 Local spectra

By means of laser Doppler velocimetry (LDV), time series for a wide range of Reynolds numbers and intercavity distances were recorded at point P_{LDV} . Figure 3.16 summarizes the power spectra of these time series in terms of the Strouhal number St_L , for the non-dimensional intercavity distance D/L and Reynolds number Re_L . The light gray checkered areas indicate the four cases, which were selected for a detailed spectral analysis in section 3.3.4. Different spectral compositions are observed, depending on the intercavity distance and the Reynolds number. For sufficiently low Reynolds numbers all cases are in a steady regime, where no significant shear layer oscillations are present. By increasing the Reynolds number, the typical shear layer peak at $St_{L,1} \approx 1$ arises in all cases, except the very close distance case $D/L = 0.1$ (figure 3.16(i)). The lowest threshold to shear layer oscillations is found at the intermediate distance $D/L = 0.5$. For both, close distances ($D/L = 0.1$) and very far distances the shear layer oscillations arise at a roughly 25% higher Reynolds number than the one corresponding to this intermediate distance. In the next sections the most significant features of the power spectra in figure 3.16 are described as a function of the intercavity distance.

Single Cavity Spectra

Figure 3.16(a) shows the spectra of a single cavity case as a reference to the eight double cavity cases depicted in figures 3.16(b)-(i). The results of the single cavity agree well with other works in literature, such as [Basley *et al.* \(2013\)](#); [Lusseyran *et al.* \(2008\)](#). The peak associated with single peak shear layer oscillations at $St_{L,1} \approx 1$ is observed when a threshold of the Reynolds number is passed. For high enough Reynolds numbers a quasi-periodic flow regime is reached, where two non-harmonic frequencies coexist. The coexistence and nonlinear mode competition in the flow over an open cavity was studied in detail by [Pastur *et al.* \(2008a\)](#). While the first peak and its harmonics are related to the impinging shear layer, the second peak is connected to a fluid dynamic feedback process ([Tuerke *et al.* \(2016\)](#)), as will be discussed in detail in chapter 7. From the literature ([Basley \(2012\)](#) and [Basley *et al.* \(2013\)](#)) we know, that for increasing L/Θ_0 , the

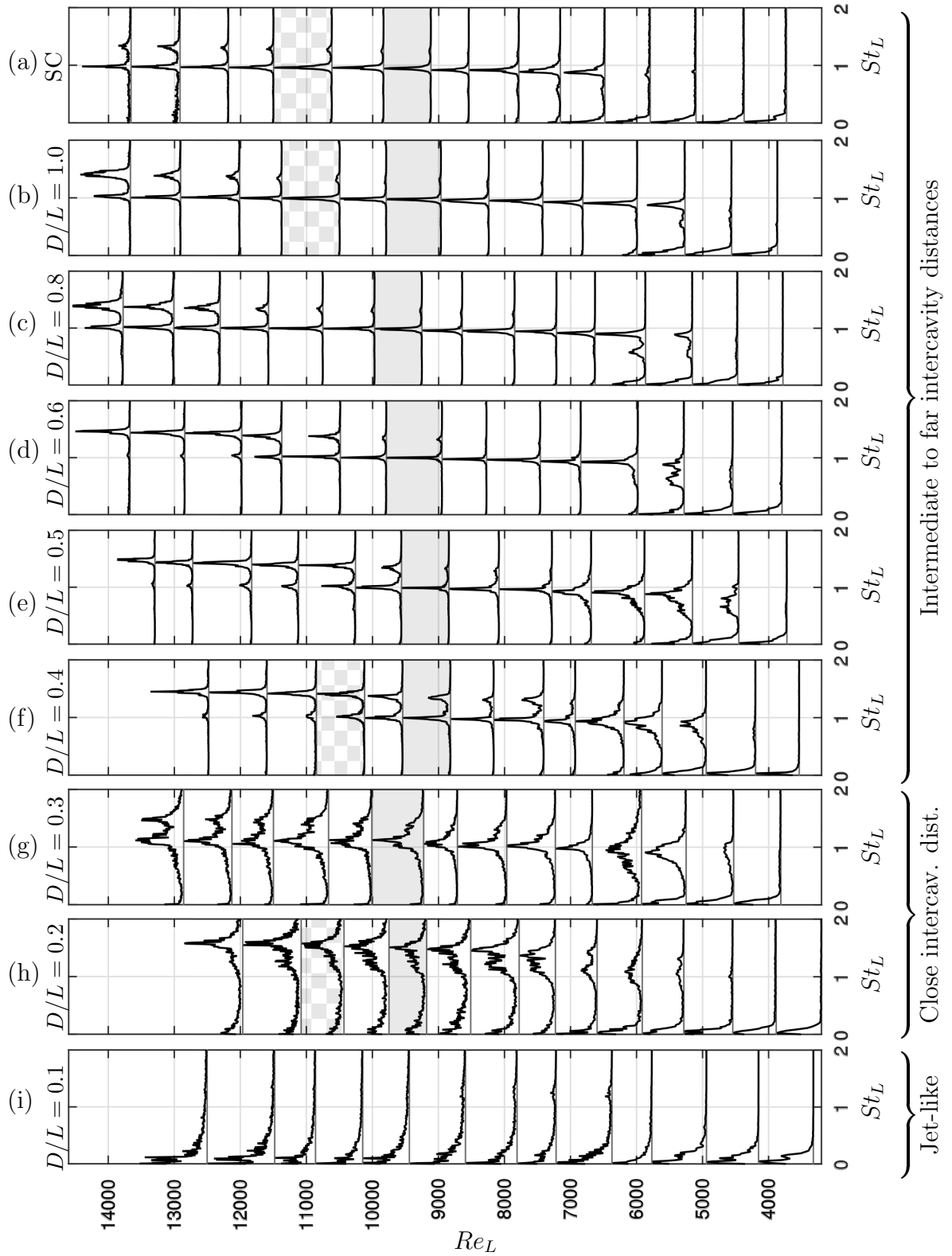


FIGURE 3.16: Normalized power spectra $PSD/\max(PSD)$. The cases indicated by the checkered area are discussed in detail in section 3.3.4. The light gray shaded areas indicate cases for which a local temporal linear stability was carried out in chapter 6.

second peak at $St_{L,2}$ can even overtake the first $St_{L,1}$ peak in amplitude. As shown in [Basley \(2012\)](#), this cascade process can reach up to $3St_{L,1}$ and $2St_{L,2}$, respectively, and is predominantly influenced by the term L/Θ_0 . In the present study, due to a relatively low value of L/Θ_0 , the takeover of the second peak is not observed for the single cavity but, as will be shown below, this phenomenon appears when both cavities are approached.

Spectra of intermediate to far intercavity distances

Let us consider the range of intermediate to far intercavity distances $0.4 \leq D/L \leq 1$, depicted in figures 3.16(b)-(f). At these distances, the spectral composition of the double cavity cases is similar to the one of the single cavity (figure 3.16(a)). The single shear layer peak at $St_{L,1} \approx 1$ is observed over a wide range of Reynolds numbers, that increases with D/L . For increasing Reynolds numbers the quasi-periodic regime appears and the second non-harmonic frequency peak at $St_{L,2} \approx 1.4$ grows with increasing Re_L . The onset of the second peak continually increases with D/L . At $D/L = 0.4$ the Reynolds number of the onset is about 50% lower than for the single cavity. The peak of $St_{L,2}$ is less sharp than the one associated with $St_{L,1}$. The takeover of the second peak indicates that the proximity of the second cavity advances the cascade process, described above. For cases at $0.4 \leq D/L \leq 0.6$, this even leads to the disappearance of the first peak $St_{L,1}$ at the highest Reynolds numbers investigated in the present work. A reason for the advancement of the cascade process may be the decreasing value of Θ_0 with decreasing D/L , as shown in figures 3.8). A decreasing value of Θ_0 increases the term L/Θ_0 , which, as mentioned before, advances the cascade process (*cf.* [Basley et al. \(2013\)](#)).

Spectra of close intercavity distances

For intercavity distances $D/L = 0.2$ and $D/L = 0.3$ the spectral composition changes strongly with respect to the single cavity. The steady regime is still followed by a periodic regime with a frequency peak at $St_{L,1} \approx 1$, however, the peaks are much broader.

At a distance of $D/L = 0.3$ the second peak ($St_{L,2}$) eventually starts growing, though at a Reynolds number roughly 30% higher than for the neighboring case at $D/L = 0.4$. Both peaks show a similar increase in bandwidth. For the distance $D/L = 0.2$ the peaks broaden further and the second peak ($St_{L,2}$) overtakes the first peak ($St_{L,1}$) at a much lower Reynolds number, compared to the other cases.

Furthermore it can be observed that with decreasing intercavity distance, the most amplified frequencies of both peaks increase continuously.

Jet-like behavior

As mentioned above, at the closest intercavity distance $D/L = 0.1$, a significantly different spectral composition is observed. The regime is called *jet-like*, as it is similar to the “cavity

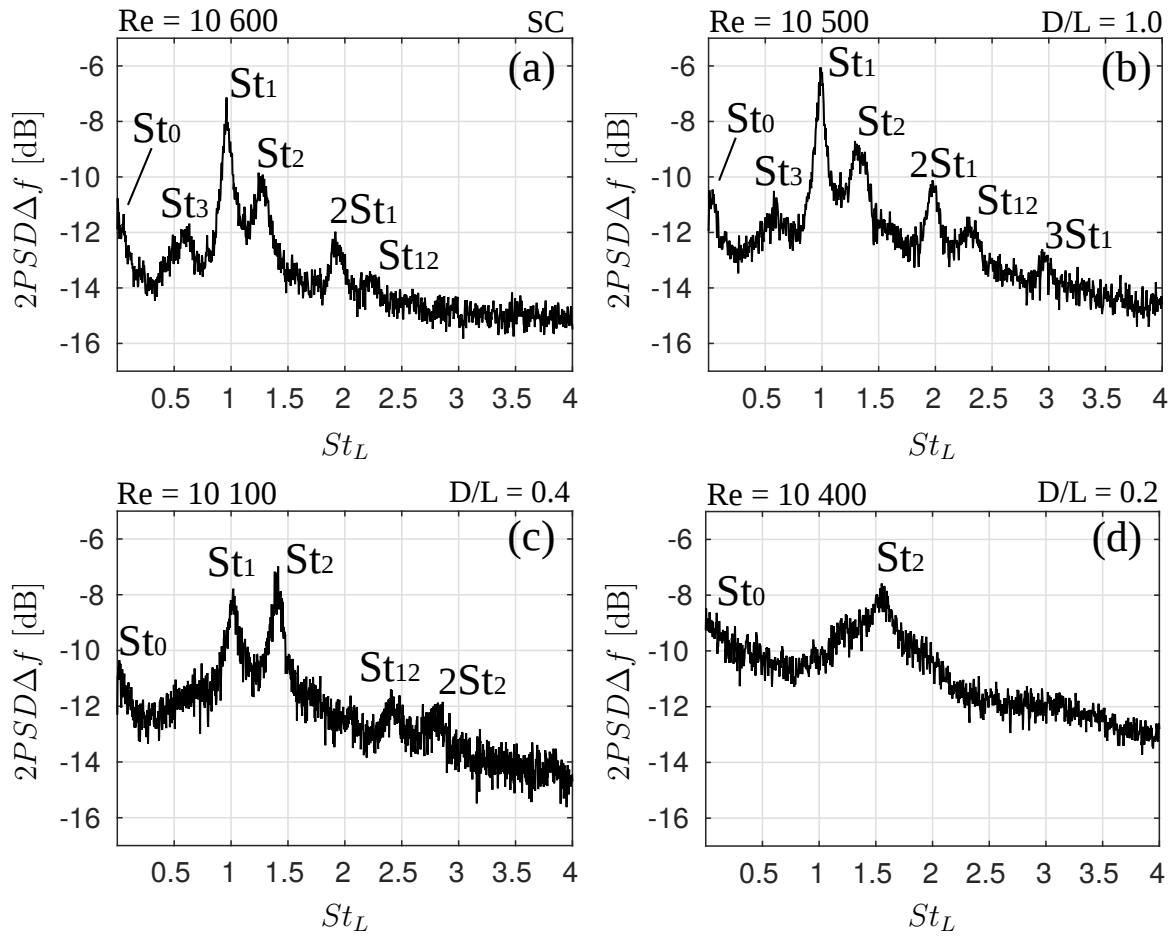


FIGURE 3.17: Shear layer spectra recorded with LDV in probe point P_{LDV} for cases at (a) $D/L = 0.1$, (b) $D/L = 0.4$, (c) $D/L = 1.0$ and (d) SC of Paris-campaign at indicated Reynolds numbers.

jet oscillation regime” observed in the experiments reported by [Maurel *et al.* \(1996\)](#). In this regime for all Reynolds numbers, low frequency content dominates the spectra. The low frequency content results from the proximity of the probe point to the recirculation region and from the change in the flow physics (jet flow vs. double cavity flow). As shown in section 3.1, due to the close intercavity distance, the double Blasius profile turns into a parabolic inflow profile. The resulting jet becomes unstable and, under the influence of the recirculation region, starts to flap irregularly. These unstable jet fluctuations can also be noted in the streamline plot of the instantaneous field in figures 3.1(a), 3.2(a) and 3.2(f), respectively. The spectral trace of these jet fluctuations can be observed in figure 3.16 for Reynolds numbers between $6400 \lesssim Re_L \lesssim 8600$ for which very subtle peaks at around $St_L \approx 1.25$ appear. These peaks compare well with mode 3 found by [Maurel *et al.* \(1996\)](#) for a similar geometry with $D/L = 0.08$, though for lower Reynolds numbers ($2000 \lesssim Re_L \lesssim 3500$).

3.3.4 Detailed spectral composition

In this section the composition of three typical double cavity spectra at different distances and one typical single cavity spectrum are analyzed in detail. Figure 3.17 shows the power spectra for four cases with comparable Reynolds numbers ($10100 \leq D/L \leq 10600$) but different intercavity distances. As we see the logarithmic scaling reveals a rich composition of the spectrum.

The spectrum of the single cavity, shown in figure 3.17(a), exhibits a main frequency peak very close to the typical shear layer frequency at $St_1 = 0.96$. A second, about one order of magnitude less energetic and non-harmonic frequency peak can be found at $St_2 = 1.28$. The first harmonic of St_1 can be found at $2St_1 = 1.92$. St_{12} and St_3 are linear combinations of the two non-harmonic peaks and its harmonics: $St_{12} = St_1 + St_2 = 2.22$ and $St_3 = 2St_1 - St_2 \approx 0.6$, respectively. A low frequency peak at $St_0 \approx 0$ is common to all four cases.

When a second cavity faces the first cavity, the structure of the spectrum stays largely the same. The spectrum of a case with $D/L = 1$ is shown in figure 3.17(b). The second cavity acts like an amplifier, leading to a similar but more amplified spectrum, when compared to the single cavity case. The main peak at $St_1 = 0.99$ increases slightly and moves closer to unity. The second non-harmonic peak also increases slightly to $St_2 = 1.32$. All other peaks are again harmonics or linear combinations of these two peaks: $St_3 = 2St_1 - St_2 = 0.58$ and $St_{12} = St_1 + St_2 = 2.30$. Due to the increased amplification, even the third harmonic $3St_1 = 2.94$ can now be observed.

For even closer distances, at $D/L = 0.4$ shown in figure 3.17(c), we note that the two non-harmonic peaks at $St_1 = 1.02$ and $St_2 = 1.41$ further increase in Strouhal number. The amplitudes of both peaks are now within the same order of magnitude. In fact, the second peak even overtakes the first peak, showing the advancement in the cascade process. As mentioned earlier in this section, this is most likely related to the decrease of Θ_0 or respectively the increase of L/Θ_0 as D/L decreases (*cf.* figures 3.8 and 3.9). As before, linear combinations of the two non-harmonic peaks are present: we find the second harmonic of the second peak at $2St_2 = 2.83$ and the sum of the two non-harmonic peaks at $St_{12} = St_1 + St_2 = 2.43$. We note that the peak at St_3 disappears and the peak-to-noise-ratio decreases, while both peaks broaden slightly.

For yet closer distances, the spectrum changes strongly. The only prevailing two peaks are the low frequency peak at $St_0 \approx 0$ and the second shear layer peak at $St_2 = 1.54$. The latter broadens strongly, extending over roughly $\Delta St \approx 1$. Side band peaks cannot be clearly identified due to the broadening of the peaks and a further decrease of the peak-to-noise-ratio.

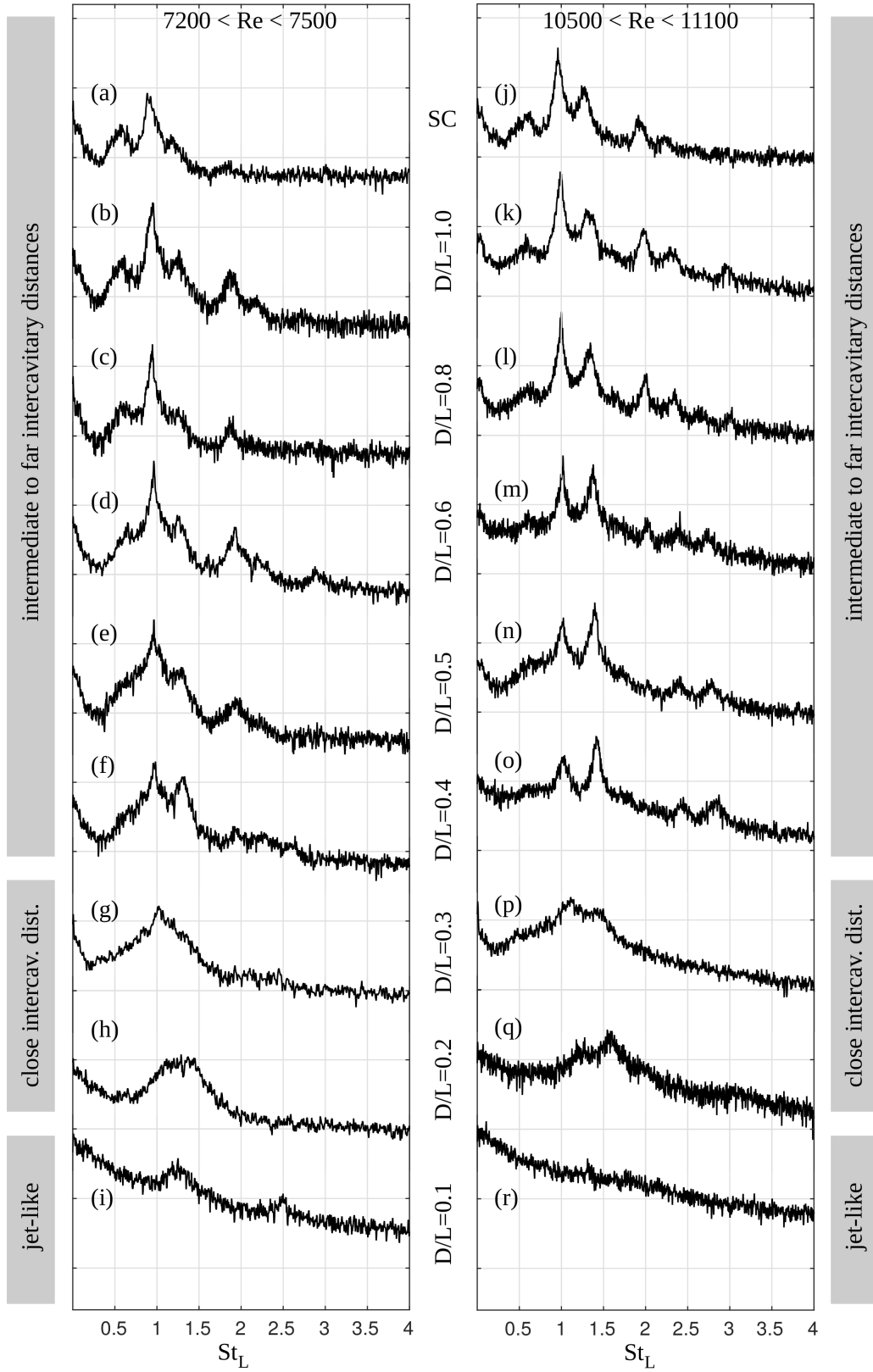


FIGURE 3.18: Y-offset graph of power spectral density ($2PSD\Delta f$ in [dB]) over Strouhal number for all intercavity distances at low Reynolds numbers (a)-(i) and high Reynolds numbers (j)-(r).

3.3.5 Y-offset graph

Figure 3.18 exemplifies once more the spectral composition of double cavity flow as the intercavity distance varies. The Y-offset graph shows the power spectral density in logarithmic scaling as a function of the Strouhal number for different intercavity distances (D/L) at low (figure 3.18(a)-(i)) and high Reynolds numbers (figure 3.18(j)-(r)). The logarithmic scaling of the ordinate reveals a more detailed picture than the linear scaling used in figure 3.16.

At intermediate to far intercavity distances the peak at St_1 is present in all spectra. At the higher Reynolds number the non-harmonic peak at St_2 is equal in amplitude and depending on the distance may even dominate. Decreasing D/L increases the amplitude of the peak at St_2 . A strong change in the power spectra is observed for close intercavity distances: the peaks broaden and hence grow together. Also, the remaining peak moves to slightly higher Strouhal numbers. The peak-to-noise ratio decreases which makes all other peaks (harmonic and linear combinations of the two non-harmonic peaks) disappear from the spectrum. For the smallest intercavity distance ($D/L = 0.1$), at the low Reynolds number (figure 3.18(i)), the logarithmic scaling of the ordinate reveals a peak at $St_1 = 1.25$ together with its first harmonic. As mentioned above, this peak compares well with mode 3 found by [Maurel *et al.* \(1996\)](#) for a similar cavity aspect ratio. At the same intercavity distance but higher Reynolds numbers the spectrum is found to be flat with the low frequency peak at $St_0 \approx 0$ being the only remaining maximum in the power spectrum. At such small distances the flow physics changes and the double cavity flow turns into a confined jet flow.

3.4 Concluding remarks

The flow through two facing cavities was studied experimentally by means of PIV (particle image velocimetry) and LDV (laser Doppler velocimetry) measurements, when the Reynolds number and the distance between the cavities were varied. A wide range of Reynolds number ($3\,000 \lesssim Re_L \lesssim 14\,000$) and intercavity distances ($0.1 \lesssim D/L \lesssim 1$) were investigated. A single cavity case was added as a case of reference. The velocity profiles in the inflow channel, as well as in the double cavity were characterized. The momentum thickness of the incoming velocity and the vorticity thickness inside the double cavity were computed for all cases. The streamwise velocity profiles inside the double cavity were found to fit well a hyperbolic sine (sinh) profile for all intercavity distances. The inflow profiles were found to depend strongly on D for small intercavity distances. At $D/L \lesssim 0.2$, and low Reynolds numbers, a parabolic profile was obtained, while for larger distances ($D/L \gtrsim 0.2$), independent of the Reynolds numbers, a symmetric Blasius profile

was measured. For $D/L \geq 0.2$, the streamlines of the instantaneous intracavity velocity fields did not show a strong dependence on the intercavity distance. For $D/L = 0.1$ the picture changed, and the instantaneous fields showed a fluctuating jet, flapping between the two cavities. This went along with a change in the inflow velocity profile from a double Blasius profile (at $D/L \geq 0.2$) to a parabolic profile (at $D/L = 0.1$). The mean RMS field showed that for $D/L \geq 0.2$ the two shear layers stayed separate all along the cavity length, while for $D/L = 0.1$ the two shear layers grew together. This close distance case ($D/L = 0.1$) was found to compare well to the work by [Maurel *et al.* \(1996\)](#) who investigated a confined jet flow.

In general, the flow in each cavity was found to be structured around two recirculation regions. A smaller one, located in the upstream corner, fed by a larger one, located further downstream. The larger, downstream recirculation bubble receives mass and momentum, directly injected from the channel center, while the smaller, upstream bubble, is fed by the larger, downstream bubble. A back and forth movement of the two intracavity bubbles was observed in all cases. By means of spatial correlation plots of the streamwise velocity fluctuations, it was found that for close distances the flow is asymmetric (sinuous), while for larger distances no symmetry could be observed.

Spectra, recorded at various probe locations inside the double cavity domain, revealed its global nature. Spectra at a single probe point in the impingement zone of one of the shear layers were recorded for a wide range of distances and Reynolds numbers in order to characterize the spectral signature as a function of the Reynolds number and the intercavity distance (*i.e.* the coupling strength). For sufficiently low Reynolds numbers, all cases start out in the steady regime, followed by a flow regime with periodic oscillations, first reached for intermediate intercavity distances. For $D/L \geq 0.3$ a quasi-periodic regime, where a second, non-harmonic frequency arises in the spectrum, is found at the highest Reynolds numbers. This regime is reached at lower Reynolds numbers the smaller D/L .

The coupling between the cavities evolves with the intercavity distance. At very close distances ($D/L = 0.1$) the two systems lose identity and behave as a new system. A different flow structure, observed in the instantaneous fields, was confirmed by the absence of the dominant shear layer frequency peaks. All other cases show strong spectral activity at the typical shear layer frequency $St_L \approx 1$. For intercavity distances $0.2 \leq D/L \leq 0.3$ the strong coupling between the cavities broadens the frequency peaks and moves them to higher Reynolds numbers. At distances $D/L \geq 0.4$, the coupling continuously ceases and the spectral signature looks increasingly similar to the single cavity. The only noticeable effect of the facing cavity at these distances is the Reynolds number threshold of the quasi-periodic regime. A detailed spectral analysis of some selected cases, at approximately

the same Reynolds number, elucidated the influence of the intercavity distance on the frequency peaks. A complementary Y-offset graph summarizes the spectral composition of double cavity flow for a low and a high Reynolds number range.

Chapter 4

Numerical Setup

In this chapter the setup of the numerical simulations of the double cavity geometry is described. Numerical simulations allow easy access to all flow variables at great resolution in space and time, which makes it a suitable complement to the experimental wind tunnel study, presented in the previous chapters 2 and 3, respectively. Additionally, simulations are usually free of external noise and its accuracy is restricted only by the precision of the numerical schemes and the computational grid. Once validated by experiments, numerical simulations present a powerful tool to study a phenomenon.

The double cavity geometry is studied numerically in both a domain with two spatial dimensions (2D) and a domain with three spatial dimensions (3D). The computationally relatively cheap 2D simulations ($\approx 100\,000$ grid points) allow the study of the double cavity in a well refined $Re_L - D$ parameter space. More than 270 individual 2D simulations were carried out for the present study, an impossible number for the computationally much more expensive 3D simulations with close to $\approx 12\,600\,000$ grid points. The results of the 2D simulations are studied in chapter 5, using tools borrowed from nonlinear dynamical system theory (or chaos theory). Based on these results and the analysis of the vorticity flow field, the study of the hydrodynamic feedback mechanism (Tuerke *et al.* (2016)), presented in chapter 7, is realized. 3D simulations of specific, well chosen configurations were conducted in order to validate experimental and 2D numerical results as well as to answer questions about the influence of the spanwise extension, the “carousel mechanism” (*cf.* chapter 7) and the existence of a chaotic flow regime, encountered at sufficiently high Reynolds numbers in 2D numerical simulations.

Numerical simulations, especially in 2D but also in 3D are idealized setups, that have to be regarded in close relation to physical wind tunnel experiments in order to confirm its validity and to not loose sight of the “real” flow. For that purpose at the end of chapter 5 a comparison and validation of the numerical results with experimental results from

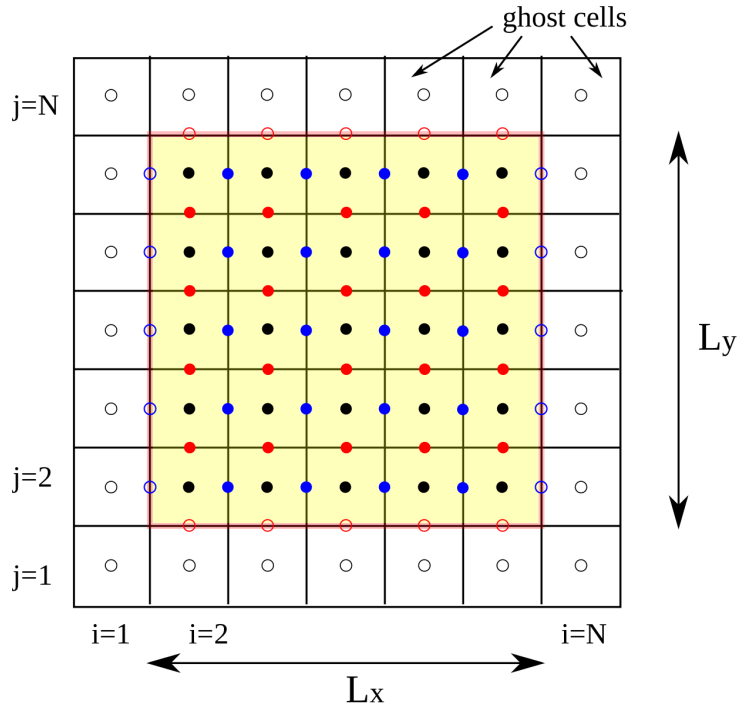


FIGURE 4.1: (Color in PDF) Schematic staggered grid in 2D. Pressure (black) is defined in the cell centers, while the velocities u (blue) and v (red) are defined on the cell boundaries. Shaded (yellow) zone indicates interior of computational domain. White cells indicates ghost cells. Nodes inside the domain are filled, outside the domain and on the domain boundary they are circled.

chapter 3 is presented. Agreements and difference of the two methods are presented and analyzed with respect to the results discussed in the present work.

The present chapter is organized as follows: First, a short outline of the numerical method, used to solve the incompressible Navier-Stokes equations is presented. Secondly, the description of the numerical domain for the 2D computations is given. In a last section, the 3D computational domain is presented. Results from both 2D and 3D simulations, as well as the comparison with experiments are found in the chapter 5.

4.1 Numerical simulation code

The numerical method is briefly outlined. More details and applications of this numerical methods can be found in [Gadoin *et al.* \(2001\)](#), [Podvin *et al.* \(2006\)](#), [Pastur *et al.* \(2008b\)](#) and [Rizi *et al.* \(2015\)](#). The numerical code, used for both 2D and 3D simulations is called *Sunfluidh* and is developed at LIMSI-CNRS since 2011 by Yann Fraigneau. *Sunfluidh* can treat different types of natural and forced convective flows and allows to define complex geometries using immersed bodies on a structured grid.

The equations of motion describe the incompressible and isothermal flow, given by the non-dimensional Navier-Stokes equations:

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{U} = -\nabla P + \frac{1}{Re_L} \Delta \mathbf{U} \quad (4.1)$$

$$\nabla \cdot \mathbf{U} = 0 \quad (4.2)$$

where $\mathbf{U}$ is the non-dimensional velocity, P the non-dimensional pressure and $Re_L = U_\infty L / \nu$ the Reynolds number based on the cavity length L , in which U_∞ is the maximum of the inflow velocity and ν is the kinematic viscosity coefficient.

The numerical method used to solve equations 4.1 and 4.2 is based on the prediction-projection method, reported by [Guermond *et al.* \(2006\)](#), with the incremental approach proposed by [Goda \(1979\)](#). The Navier-Stokes equations are discretized, following a finite volume approach on the staggered structured grid, schematically depicted in figure 4.1, with a second order approximation in time and space. Every quantity (pressure and velocities in the present case) is defined on its own mesh. The meshes are shifted by half a mesh size in a particular direction. In other words, the pressure is defined in the cell centers of the control volume, while the velocities are defined at the cell faces. This is different from a collocated grid arrangement, where all variables are stored in the same positions. The staggered formulation has a great advantage when it comes to the conformity of gradient operators and second order divergence and Laplacian operators. Using a staggered grid is a simple way to avoid odd-even decoupling between the pressure and velocity. Odd-even decoupling is a discretization error that can occur on collocated grids and which leads to checkerboard patterns in the solutions. The staggered formulation also results intrinsically in a better numerical stability of the prediction-projection method. The cells surrounding the computational domain are called “ghost cells” (*cf.* figure 4.1) and are used to impose the boundary conditions.

Advection fluxes and viscous terms are calculated with a second order centered-scheme. The time derivatives are approximated by a second order differentiation formulation. An implicit discretization scheme is carried out on the viscous terms in order to increase the numerical stability; δt being the time step for integration, to estimate the advection flux at time $(n+1)\delta t$, the code uses an Adams-Bashford’s extrapolation from the results at time $n\delta t$ and $(n-1)\delta t$. Note, that pressure gradients are explicitly defined, as suggested in the projection method.

The semi-implicit discretization scheme of the velocity equation leads to a Helmholtz-like equation for each velocity component, of the form:

$$\left(I - \frac{2\delta t}{3Re_L} \nabla^2 \right) U_i^* = S^{n,n-1} \quad (4.3)$$

where U_i^* is the field of the i^{th} velocity component, estimated at time $(n + 1)\delta t$. The velocity field ($\mathbf{U}^*$) does not satisfy the divergence-free condition yet. $S^{n,n-1}$ contains all explicit terms defined at time $n\delta t$ and $(n - 1)\delta t$. For each time step, these equations are solved by means of an alternating direction implicit method (see [Hirsch \(1987\)](#)). The divergence-free condition on the velocity field and the pressure field are updated at time $(n + 1)\delta t$ by solving Poisson's Equation

$$\nabla^2 \Phi = \frac{\nabla \cdot \mathbf{U}^*}{\delta t}, \quad (4.4)$$

where

$$\Phi = P^{n+1} - P^n - \frac{1}{Re_L} \nabla \cdot \mathbf{U}^*. \quad (4.5)$$

The solution Φ is calculated by means of the successive over-relaxation method coupled with a geometric multi-grid method in order to improve the convergence efficiency ([Wesseling \(1992\)](#)). The pressure field is directly updated at time $(n + 1)\delta t$ from the previous relation and the velocity field $\mathbf{U}^*$ is corrected such that the divergence-free condition is satisfied:

$$\mathbf{U}^{n+1} = \mathbf{U}^* - \frac{2}{3} \delta t \nabla \Phi. \quad (4.6)$$

A convergence study revealed that the grid convergence of the numerical scheme is of second order. A grid convergence study was carried out by successively refining the mesh until the residual of the $\mathcal{L}_2$ -norm fell below the tolerance of 10^{-5} .

For the two dimensional (2D) simulations OpenMP (Open Multi Processing) parallelization is used. For the three dimensional (3D) simulations both OpenMP and MPI (Message Passing Interface) parallelization are used in order to reduce computation time. With these parallelization techniques a 2D case finishes in approximately 20 hours using a single processor, while a 3D case needs on average 14 days on an IBM cluster using 192 Intel ES 4650 processors to finish.

4.2 Setup 2D simulations

The two dimensional (2D) simulations were computed in the computational domain, depicted in figure 4.2. It has a cavity length L , with a length over depth ratio $\Gamma = 2$, kept constant throughout the entire study. The inflow length is $L_1 = 0.6L$ and the outflow length is $L_2 = 1.2L$. Longer outflow lengths did not produce significant changes on the

TABLE 4.1: Summary of 2D numerical cases

Config.	D/L	N_x	N_y	Re_L
SC	1.40	512	256	3833 – 8330
DC	0.90	512	512	3666 – 7333
DC	0.60	512	512	3666 – 7333
DC	0.50	512	256	3666 – 7333
DC	0.40	512	256	3666 – 7333
DC	0.35	512	256	3666 – 7333
DC	0.30	512	256	3666 – 7333
DC	0.25	512	256	3666 – 7333
DC	0.20	512	256	3666 – 7333

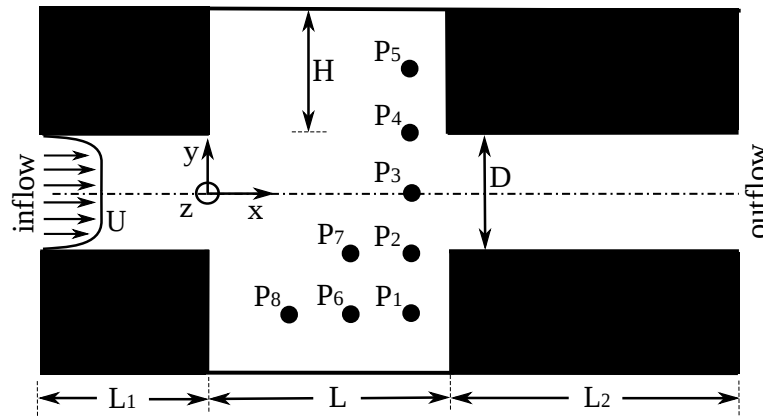


FIGURE 4.2: Computational domain of double cavity geometry with variable distance D , inflow length $L_1 = 0.6L$, cavity length L , cavity aspect ratio $L/H = 2$, outflow length $L_2 = 1.2L$ and probe points P_1 to P_8 . Arrows indicate flow direction.

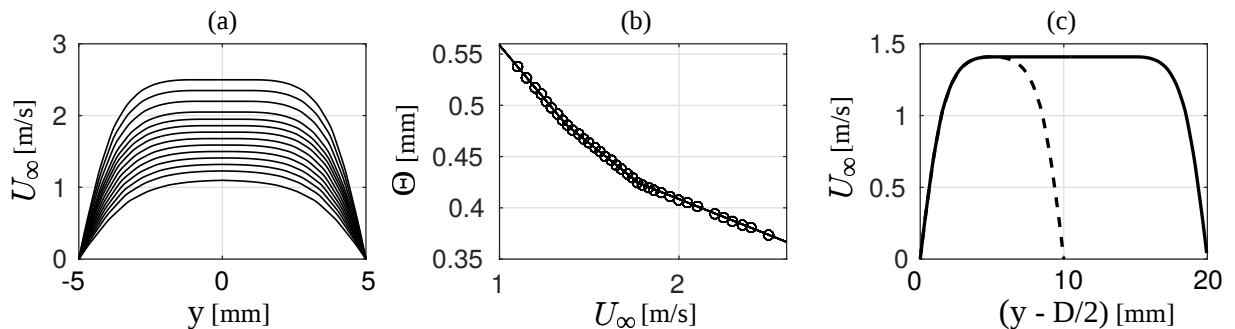


FIGURE 4.3: (a) Inflow velocity profiles for case with $D/L = 0.2$ for different inflow velocities (not all velocities depicted). (b) Momentum thickness Θ as a function of the maximum inflow velocity U_∞ . (c) Extension of a $D/L = 0.2$ profile (---) to a $D/L = 0.4$ profile (—)

flow inside the double cavity. Usual Neumann boundary conditions are applied for solving the pressure Poisson's equation. For the velocity, the inflow boundary conditions are of Dirichlet type, with an imposed velocity profile in order to control the flow rate. In a test case noise was applied to the inflow velocity profile, however, resulting in no noticeable change in the velocity fluctuations inside the double cavity domain. The outlet boundary conditions are defined by solving a simple 1D advection equation along the normal direction to estimate the velocity profile in the outlet plane. The normal velocity component is then corrected such that the inlet flow rate is preserved. This simple approach ensures mass conservation and limits spurious effects on the solution at the vicinity of the outlet. The usual no-slip and impermeability conditions are applied at the walls. The laminar inflow profiles, imposed at the inlet boundary of the computational domain, were obtained considering an additional length of the upstream channels of $2.8L$ in which a laminar boundary layer develops on either side of the channel. A set of approximately 30 velocity profiles was created for each distance D . The profiles for case $D/L = 0.2$ and the variation of the momentum thickness Θ with U_∞ are depicted in figure 4.3(a) and (b), respectively. Inflow profiles for larger cavity distances are obtained by extending the $D/L = 0.2$ profiles in the y -direction, *i.e.* filling the resulting gap with the value of the respective maximum velocity. It is thus ensured that while D is varied, the maximum velocity U_∞ and the momentum thickness Θ of the profiles are kept constant. Figure 4.3 (c) shows an example of the extension of a $D/L = 0.2$ to a $D/L = 0.4$ case.

The domain is covered with 512 cells in the x -direction, and 256 or 512, depending on D , in the y -direction. The mesh is refined close to the walls of the inflow and outflow channel as well as in the shear layers region and close to the impinging edge, in order to resolve the strong velocity gradients in these areas. Each simulation is carried out over a time duration of approximately 400 to 600 convective time units $\tau_c = L/U_\infty$, depending on the case. The code automatically adapts the time step size to ensure numerical stability, respecting $CFL = 0.25$. As a reference for the double cavity (DC), a single cavity (SC) case is added, by closing one of the cavities at a distance of $D = 1.4L$. A total of $2 \cdot 10^6$ integration time steps were computed for each case in order to obtain time series of a length close to 20 seconds.

A total of 276 simulations were conducted, summarized in table 4.1. At each distance the i th simulation at the Reynolds number $Re_L(i)$ is initialized with a converged instantaneous flow field of the previous simulation at a slightly lower Reynolds number $Re_L(i-1)$, while the inflow profile is updated such that it corresponds to $Re_L(i)$. This sweeping technique minimizes the perturbation of the flow at the start of a simulation and mimics the stepwise increase of the wind tunnel speed in laboratory experiments.

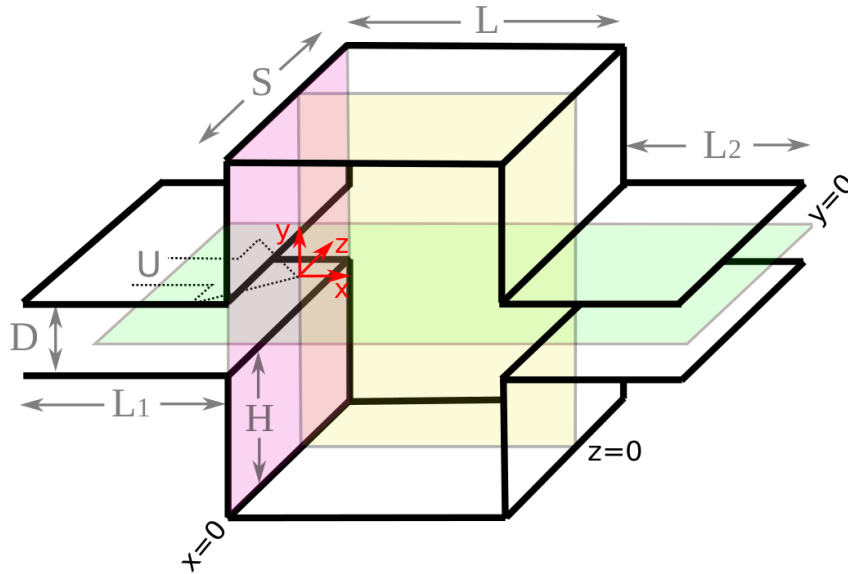


FIGURE 4.4: Schematic view of the 3D computational double cavity domain with cavity length L , cavity depth H , intercavity distance D , spanwise extension S , inflow length L_1 and outflow length L_2 . The indicated planes are the three zero planes: $x = 0$, $y = 0$ and $z = 0$, respectively.

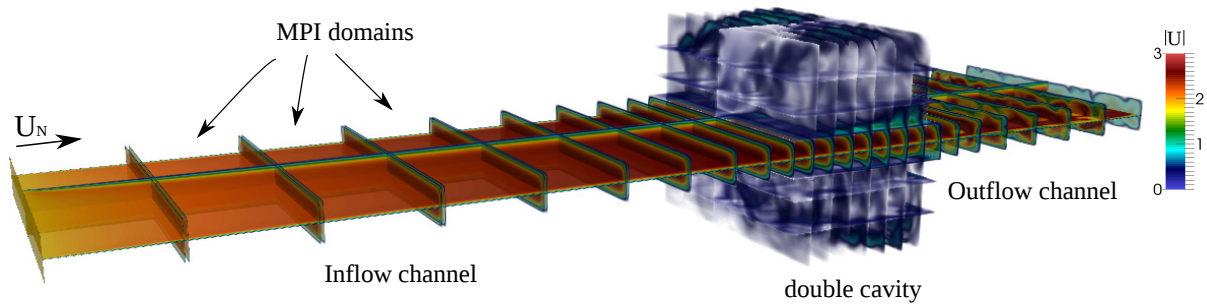


FIGURE 4.5: 3D double cavity with $D/L = 0.2$ and constant inflow velocity U_N . Color indicates absolute velocity. Sections indicate domain partitioning for MPI computation.

4.3 Setup 3D Simulations

In this section the setup of the three dimensional (3D) numerical simulations is presented. Simulations in three spatial dimensions are computed in the computational domain, depicted in figures 4.4 and 4.5, using the numerical code presented in section 4.1. The cavity depth H is kept constant, while the cavity length varies. Cases with $L/H = 2$ and $L/H = 1$ are investigated. The inflow length is $L_1 = 3.6L$ and the outflow length is $L_2 = 1.6L$. Spanwise extensions of $S = 1.5L$ and $S = 3.6L$ are used, resembling the the spanwise extensions of the experimental setups of the Paris- and the BA-campaign, respectively. Usual Neumann boundary conditions are applied for solving the pressure

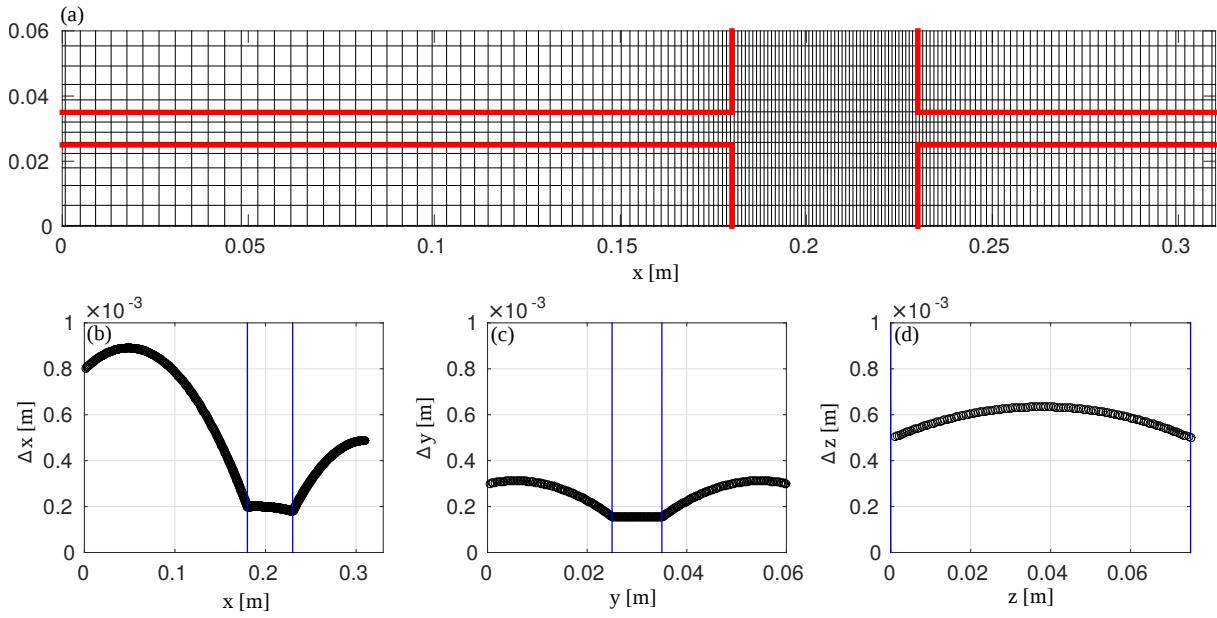


FIGURE 4.6: (Color in PDF) (a) 2D plane of 3D computational mesh of a DC10 case ($D/L = 0.2$). Line spacing is reduced 20 fold in each direction. Red solid lines indicate double cavity domain. (b) Mesh spacing in streamwise direction. (c) Mesh spacing in vertical direction. (d) Mesh spacing in spanwise direction. Blue vertical lines in (b)-(d) indicate double cavity domain.

Poisson's equation. For the velocity, the inflow boundary conditions are of Dirichlet type, with an imposed constant velocity profile of value U_N in order to control the flow rate. The inflow length L_1 was designed such that the momentum thickness obtained at the leading edge of the double cavity is approximately equal to the momentum thickness measured experimentally. The outlet boundary conditions are defined by solving a simple 1D advection equation along the normal direction to estimate the velocity profile in the outlet plane. The normal velocity component is then corrected such that the inlet flow rate is preserved. This simple approach ensures mass conservation and limits spurious effects on the solution at the vicinity of the outlet. The usual no-slip and impermeability conditions are applied at the walls. MPI parallelization is used to reduce the computational time. An exemplary computational domain, divided into several MPI sub-domains is depicted in figure 4.5. A typical computational grid for a 3D computation is shown in figure 4.6(a). In figure 4.6(b)-(d) the corresponding distribution of the grid spacing in the streamwise, vertical and spanwise direction, respectively, are depicted. As in the 2D cases, the mesh is refined close to the walls of the inflow and outflow channel as well as in the shear layers region and close to the impinging edge, in order to resolve the strong velocity gradients in these areas. The grid spacing is designed such that the maximum to minimum cell aspect ratio is kept below 5 ($\Delta x_{max}/\Delta y_{min} = 4.75$) everywhere in the domain.

TABLE 4.2: Summary of 3D numerical cases. U_N and U_∞ are given in [m/s], while S and Θ_0 are given in [mm].

No	Config	Symbol	U_N	U_∞	Re_L	D/L	L/H	S	Θ_0	L/Θ_0	Regime
1	DC10*	▼	—	1.35	4 500	0.2	2	75	0.4839	103.3	periodic
2	DC10*	▼	—	1.71	5 700	0.2	2	75	0.4332	115.4	periodic
3	DC10	▼	1.35	1.94	6 500	0.2	2	75	0.5845	85.5	periodic
4	DC10	▼	1.60	2.50	8 300	0.2	2	75	0.5423	92.2	periodic
5	DC15	○	1.35	1.77	5 900	0.3	2	75	0.6542	76.4	steady
6	DC15	○	1.80	2.29	7 600	0.3	2	75	0.5888	84.9	periodic
7	DC20	△	1.35	1.69	5 600	0.4	2	75	0.6800	73.5	periodic
8	DC20	△	1.80	2.19	7 300	0.4	2	75	0.6027	82.9	periodic
9	DC20	×	1.80	2.15	7 200	0.4	2	180	0.6167	81.1	periodic
10	SC	□	1.90	2.13	7 100	—	2	75	0.6369	78.5	periodic
11	DC10	★	1.80	1.89	3 200	0.2	1	75	0.3535	70.7	chaotic
12	DC10	★	1.40	1.49	2 500	0.2	1	75	0.3911	63.9	chaotic

* different inflow conditions. See text for details.

The numerical simulations are carried out over a time duration of approximately 200 to 400 convective time units $\tau_c = L/U_\infty$, depending on the case. The code automatically adapts the time step size to ensure numerical stability, respecting $CFL = 0.25$. Approximately 800 000 integration time steps were computed for each case. As summarized in table 4.2 eleven double cavity cases were complemented by one single cavity (SC) case, by closing one of the cavities at a distance of $D = 1.4L$. Table 4.3 summarizes the numerical setup of all 3D cases. The total mesh size $N_x \times N_y \times N_z$, the mesh size in the double cavity domain $N_{x,DC} \times N_{y,DC} \times N_{z,DC}$, the mesh block size of each MPI subdomain and the total number of processor are listed. Note, that cases 1 and 2, marked by an asterisk (*) were computed with only OpenMP parallelization (no MPI) and with a slightly different inflow condition. As in the 2D cases, a developed laminar profile was imposed at $x = -0.03$, in order to shorten the inflow length.

Probe points are located in multiple positions in four different $z = \text{constant}$ planes:

- $z = -0.0325 \text{ mm}$ or $z/S = 0.4333$,
- $z = -0.0175 \text{ mm}$ or $z/S = -0.2333$,
- $z = 0$ and
- $z = 0.0175 \text{ mm}$ or $z/S = 0.2333$

TABLE 4.3: Summary of computational setup for all 3D cases

No	Config	$N_x \times N_y \times N_z$	$N_{x,DC} \times N_{y,DC} \times N_{z,DC}$	Block size	No. of procs
1	DC10*	$256 \times 128 \times 128$	$128 \times 128 \times 128$	$256 \times 128 \times 128$	1
2	DC10*	$256 \times 128 \times 128$	$128 \times 128 \times 128$	$256 \times 128 \times 128$	1
3	DC10	$768 \times 256 \times 128$	$256 \times 256 \times 128$	$32 \times 32 \times 64$	192
4	DC10	$768 \times 256 \times 128$	$256 \times 256 \times 128$	$32 \times 32 \times 64$	192
5	DC15	$768 \times 256 \times 128$	$256 \times 256 \times 128$	$32 \times 32 \times 64$	192
6	DC15	$768 \times 256 \times 128$	$256 \times 256 \times 128$	$32 \times 32 \times 64$	192
7	DC20	$832 \times 288 \times 128$	$256 \times 288 \times 128$	$32 \times 32 \times 64$	252
8	DC20	$832 \times 288 \times 128$	$256 \times 288 \times 128$	$32 \times 32 \times 64$	252
9	DC20	$832 \times 288 \times 128$	$256 \times 288 \times 128$	$32 \times 32 \times 64$	630
10	SC	$832 \times 288 \times 128$	$256 \times 288 \times 128$	$32 \times 32 \times 64$	308
11	DC10	$544 \times 256 \times 128$	$160 \times 256 \times 128$	$32 \times 32 \times 32$	256
12	DC10	$544 \times 256 \times 128$	$160 \times 256 \times 128$	$32 \times 32 \times 32$	256

* different inflow conditions. See text for details.

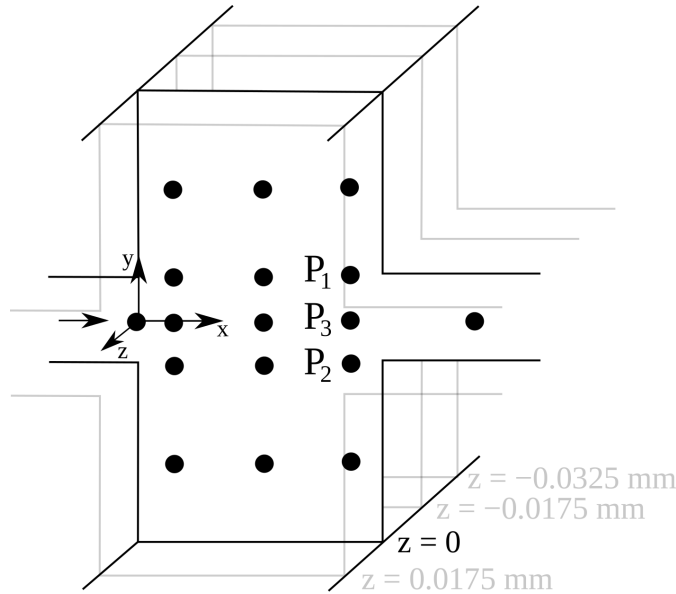


FIGURE 4.7: Probe point locations $P_1 - P_3$ in the $z = 0$ plane used for the analysis. Flow from left to right as indicated by the arrow. Black dots show probe point locations.

Figure 4.7 shows exemplary the probe point location in the plane $z = 0$. Time series are recorded at these probe points with a sampling frequency of 3 kHz. The times series are resampled and interpolated onto an equidistant mesh before post-processing.

Chapter 5

Numerical Results

5.1 Introduction

In this chapter the results from the numerical simulations, described in the previous chapter 4, are presented. First, in section 5.2, the 2D simulations are analyzed, using tools borrowed from nonlinear dynamical system analysis. Based on these results and the analysis of the vorticity flow field, the study of the hydrodynamic feedback mechanism (Tuerke *et al.* (2016)), mentioned in the introduction and presented in chapter 7, is realized. Results from 3D simulations are presented in section 5.3 to answer some specific questions raised by the results of the 2D simulations and also to bridge the gap between the (naturally 3D) experimental results from chapter 3 and idealized 2D numerical results. Finally in section 5.4 numerical and experimental results are compared.

5.2 2D results: A nonlinear dynamical system analysis

In this section the results from the 2D numerical simulations are presented and a detailed nonlinear dynamical system analysis is carried out based on the time series obtained from discrete probe points in the computational domain.

5.2.1 Time series

Probe points are located in various positions inside the domain, as indicated in figures 4.2 and 5.1, respectively. Time series are recorded at these probe points with a sampling frequency of roughly 5 kHz. The Strouhal number is defined as $St_L = fL/U_\infty$, in which f is the measured frequency. The initial 30% of each time series is discarded in order to avoid transient effects. The Welch algorithm (Welch (1967)) with a window size of 5.1 s and an overlap of 98% is used to compute the power spectral density (PSD), from the

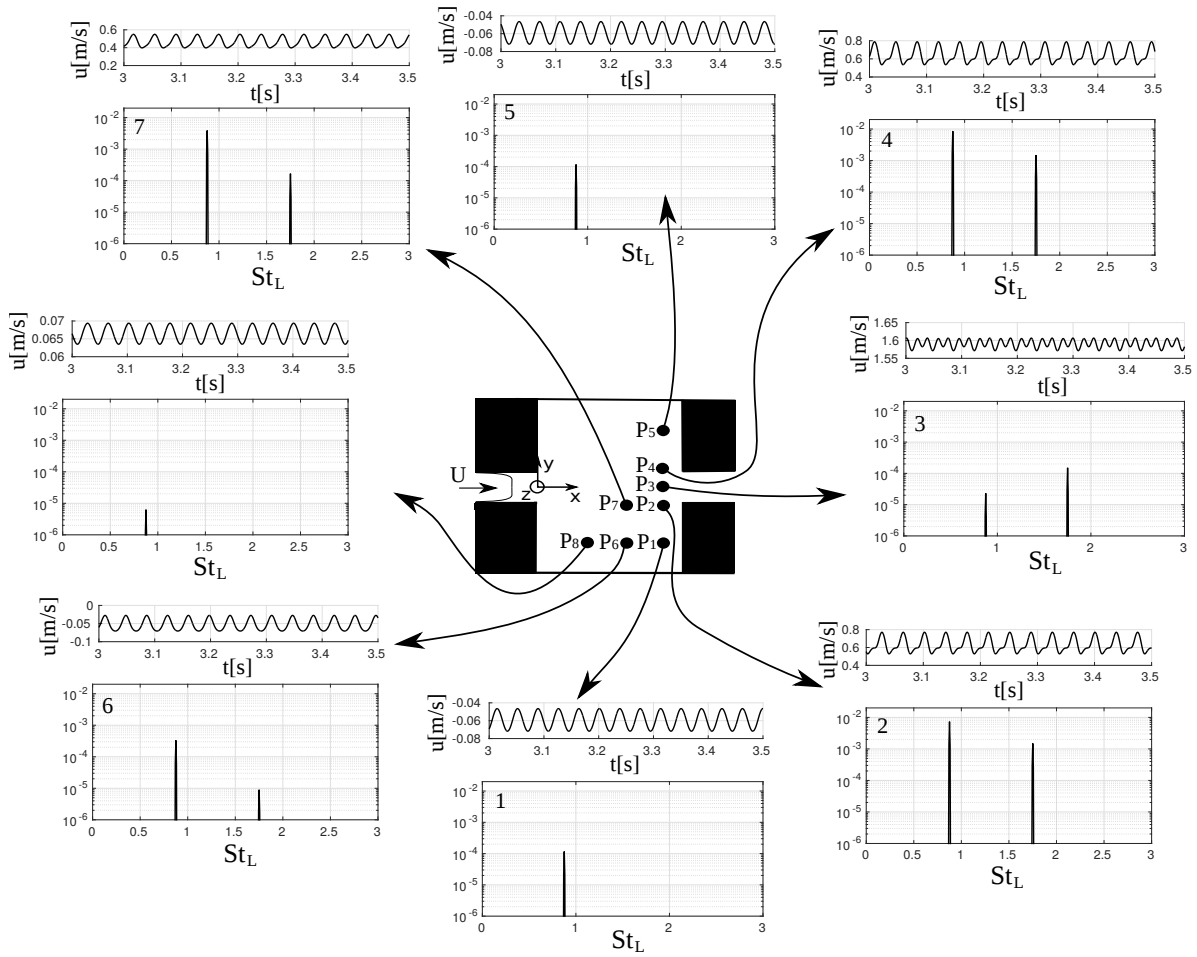


FIGURE 5.1: Time series of streamwise velocity component u and its power spectral density from probe locations $P_1 - P_8$ of case $D/L = 0.2$ at $Re_L = 5100$

streamwise component of the velocity fluctuations. This results in a spectral resolution of $\Delta St \approx 0.01$. All power spectra depicted in this work are normalized by pre-multiplying with two times the frequency resolution: $2\text{PSD}\Delta f$. Time series from eight probe points together with their power spectra are shown in figure 5.1 for a typical case. The same frequency ($St \approx 1$) is amplified in the entire domain, which suggests a global nature of the spectrum. Hence, the spectral analysis of the different points can be reduced to a single point. Probe points P_2 and P_4 were chosen for most of the forthcoming spectral analyses, since they carry information from both the respective shear layers as well as the recirculation region. The reason why in point P_3 the second harmonic rises above its fundamental frequency will be explained in chapter 7. Inside the cavities, the second harmonic has smaller amplitudes than in the shear layer.

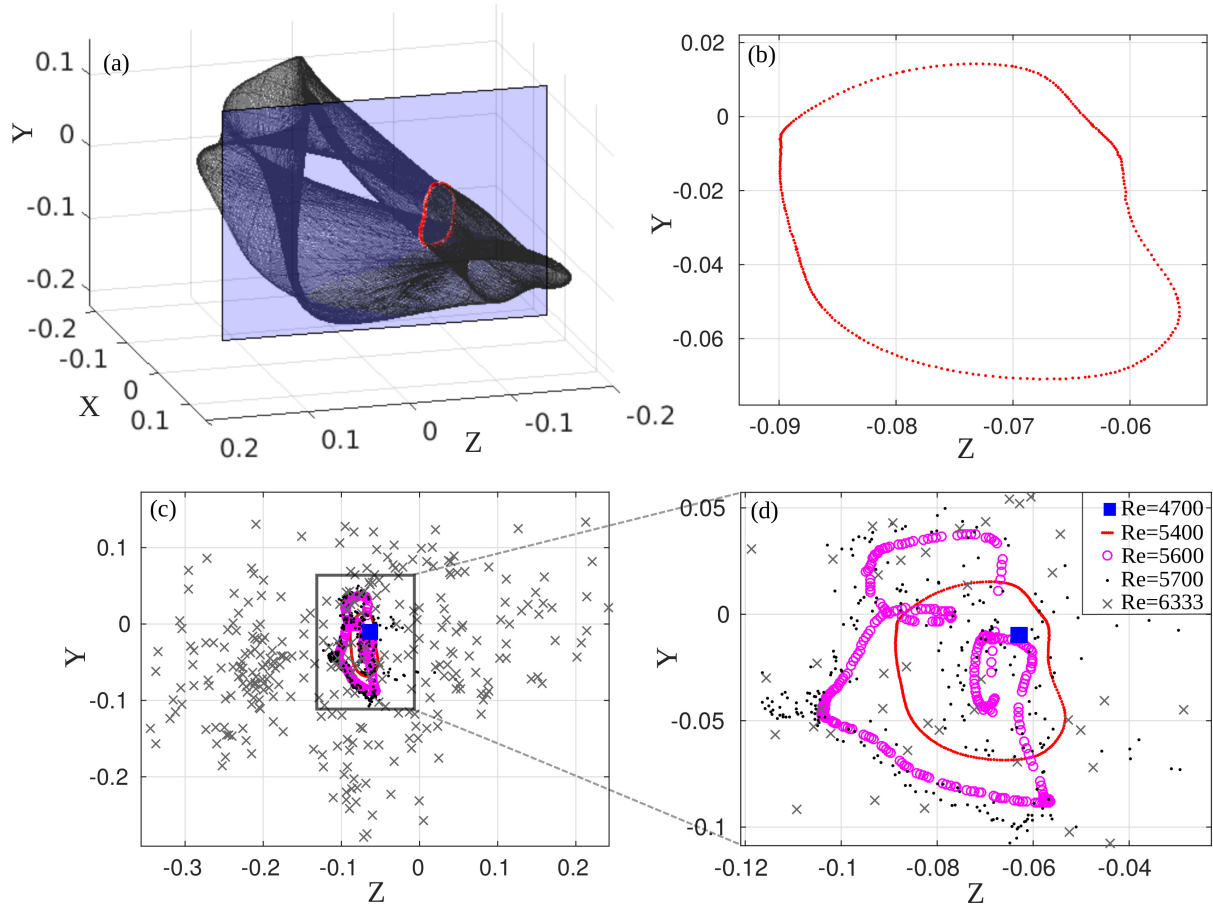


FIGURE 5.2: (Color in PDF) (a) Phase Portrait: 3D projection of the reconstructed flow from time series of $D/L = 0.2$ case at $Re_L = 5400$ using time delay embedding ($\tau = 0.015$ s). Vertical plane (blue) at $X = 0.0055$ indicates position of Poincaré section. Intersections of phase portrait with plane is indicated by red symbols. (b) Poincaré section: Intersection of phase portrait in (a) with plane $X = 0.0055$. (c) Poincaré sections ($X = 0.0055$) $Re_L = 4700$ (blue filled square, periodic regime), $Re_L = 5400$ (red solid line, intermediate regime (quasi-periodic case)), $Re_L = 5600$ (magenta $\circ$, folded torus), $Re_L = 5700$ (black dots, low order chaotic regime). $Re_L = 6333$ (gray crosses, high order chaotic regime). (d) zoom-in view of (c).

5.2.2 Phase space analysis

Different regimes are encountered when the Reynolds number $Re_L = U_\infty L / \nu$ is varied. We choose phase space representations to characterize these regimes. Phase space is reconstructed using time delay embedding of time series $s(t)$ (in our case the streamwise component of the fluctuating velocity) recorded at probe point P_2 . The 3D projection of the reconstructed phase space, spanned by the time delayed vectors

$$[X, Y, Z]^T = [s(t), s(t - \tau), s(t - 2\tau)]^T,$$

where τ is the time delay, is shown in figure 5.2(a) for an illustrative case with $D/L = 0.2$. The choice of τ is detailed in section 5.2.3.

Figure 5.2(b) depicts the Poincaré section, a 2D subspace of the 3D phase space in figure 5.2(a). It is obtained as an intersection of the plane at $X = 0.0055$ with the phase portrait in figure 5.2(a). The Poincaré section was chosen transversal to the flow in agreement with the three conditions for a global Poincaré section (see Solari *et al.* (1996)). Figure 5.2(c) exemplifies the Poincaré sections of the $D/L = 0.2$ geometry for different Reynolds numbers. Figure 5.2(d) shows a zoomed-in view of figure 5.2(c). Based on these Poincaré sections, the flow regimes are defined.

Once the flow evolves from the *steady regime* to the *periodic regime*, a period-1 limit cycle is reached in phase space and a single point can be observed in the Poincaré section ($Re_L \approx 4700$). When the value of the Reynolds number is further increased ($Re_L \approx 5400$), a so-called *intermediate regime* is reached. This regime includes quasi-periodicity where a second non-harmonic frequency appears in the spectrum. This induces the creation of a torus in phase space, which is characterized by a closed curve in the Poincaré section. Frequency locking may occur on the torus, resulting in periodic windows. At $Re_L \approx 5600$ the torus surface is folded by wrinkles. The line associated with the torus, self-intersects in the Poincaré section, which is only allowed in a state space with dimension of at least 4. The Cao (1997) algorithm, applied to the time-signal, in this flow regime, gives an actual dimension of 5 as can be seen from figure 5.4 which is discussed in section 5.2.4. This relatively small (local) dimension does not preclude potentially higher, though finite, dimension for the full state space. For $Re_L \approx 5700$, more complex dynamics are reached, though still structured around the torus. Finally, for Reynolds number $Re_L = 6333$, the Poincaré map yields a cloud of points (see figure 5.2(c)) without any identifiable structure. The last two regimes are reminiscent of chaotic behavior and hence are labeled *chaotic regime*. Yet note, that the dispersion of points in the Poincaré map alone is not a quantitative proof, though a strong indication of the chaoticity of the system. The described route to chaos is common to small channel heights ($D/L < 0.30$) and is reminiscent of a Curry & Yorke (1978) scenario: steady (fixed) point $\rightarrow$ period-1 limit cycle $\rightarrow$ quasi-periodicity $\rightarrow$ torus breakdown leading to toroidal chaos, as also recently observed for the van der Pol system by Letellier *et al.* (2008). For larger channel heights ($D/L > 0.35$), however, the route to chaos looks slightly different, while configurations with $D/L = 0.35$ and $D/L = 0.30$ have their own distinctive details, as we shall see in the following sections. Henceforth, our so-called intermediate regime is not exclusive to quasi-periodic behavior, as it may also include periodic windows, that occur due to frequency locking on the torus.

5.2.3 Minimal phase space dimension

Due to the discretized problem the dimension of phase space is finite, though large ($\approx 100\,000$ degrees of freedom (dof)). Yet, the strongly organized flow, observed in this study, suggest that the dynamics actually evolves on a manifold of much smaller (finite) dimension. The method proposed by [Cao \(1997\)](#) is used to compute the optimal embedding dimension and the corresponding time delay for the time delay embedding. The advantage of this method is, that it is not subjective and computationally not very expensive. The minimum embedding dimension is computed using an idea similar to the one employed in the false neighbor method, which states that any two points which stay close in the d -dimensional reconstructed space will be still close in the $(d+1)$ -dimensional reconstructed space. Such a pair of points is called true neighbors, otherwise, they are called false neighbors since they became neighbors only through folding of the subspace due to an insufficient number of embedding dimensions. Folding makes a curve in phase space intersect, which is non-physical since the system is deterministic and hence the system's future must be uniquely defined.

Following the method of [Cao \(1997\)](#) the time series is embedded in higher and higher dimensions until no false neighbors can be found. In the program this means that the quantity E_1 , which is defined as the relative difference between the distance of points in successive embedding dimensions, approaches unity. Additionally, a second quantity E_2 is defined which is useful to distinguish deterministic signals from stochastic signals. For random data signals E_2 is equal to unity for all embedding dimensions. For deterministic data the value cannot be equal to unity for all embedding dimensions. Exact definition and more detailed information can be found in [Cao \(1997\)](#). The computation is carried out for a time delay of $\tau = 100$ data points and a time series length of $N_{max} = 20\,000 - 70\,000$ data points, depending on the case. It was made sure that the embedding dimension is saturated with respect to N_{max} and τ . As a matter of fact, the time delay τ was chosen so that the minimum embedding dimension does not depend on τ , using the method of [Cao \(1997\)](#). Other types of embedding techniques, such as single value decomposition (SVD) embedding, were tested in order to confirm an adequate choice of the time delay τ .

Figure 5.3 shows the time series and the results from the computation of the minimal embedding dimension for an exemplary case with $D/L = 0.2$ at different Reynolds numbers, using the described method. $E_1 \approx 1$ gives the minimal embedding dimension. At low Reynolds numbers the flow resides in the periodic regime and a very low embedding dimension $N = 2$ is obtained. In the quasi-periodic regime a minimum dimension of $N = 4$ is obtained as also follows from the the intersection of the Poincaré section in figure 5.2(d) at $Re_L = 5600$. As suggested by the clouds of points in the Poincaré section for higher Reynolds numbers, the minimal embedding dimensions for the chaotic cases are much

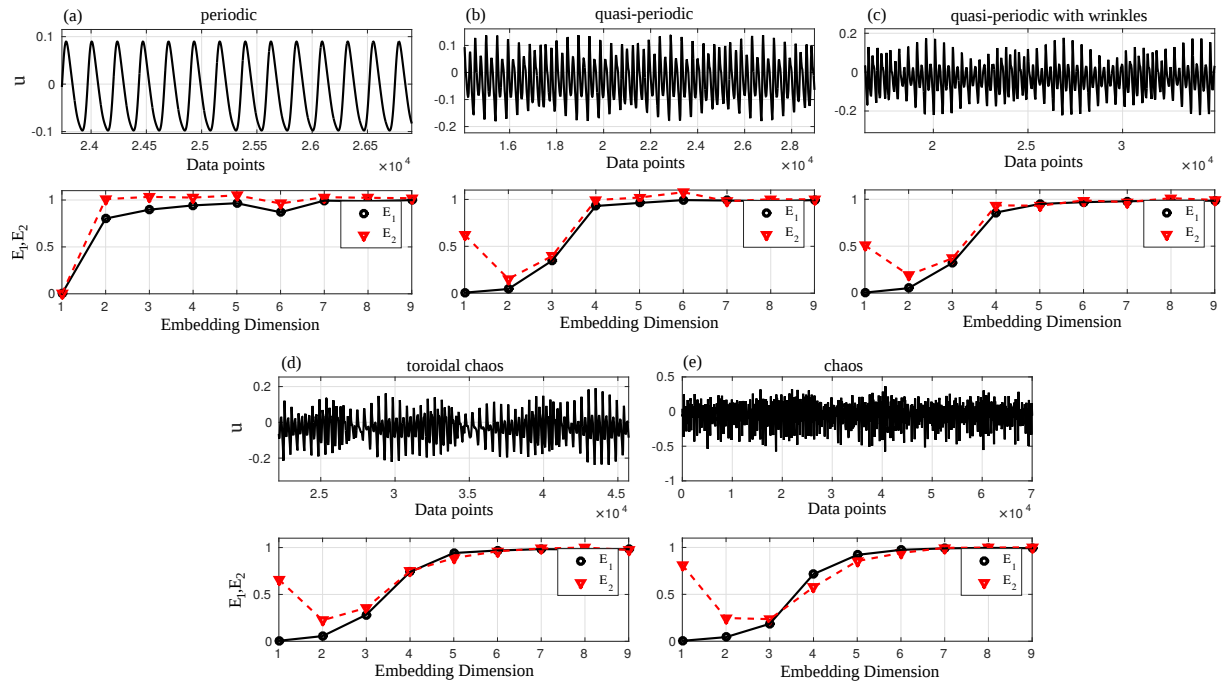


FIGURE 5.3: (Color in PDF) Minimal embedding dimension for a case with $D/L = 0.2$ at (a) $Re_L = 4700$, (b) $Re_L = 5400$, (c) $Re_L = 5600$, (d) $Re_L = 5700$, (e) $Re_L = 6333$, using the method of [Cao \(1997\)](#).

greater than 3. In fact for cases $Re_L = 5700$ and $Re_L = 6333$ embedding dimensions of $N = 5$ and $N = 6$, respectively, were computed, as can be seen from figures 5.3(d)-(e).

5.2.4 Evidence of chaotic behavior

As mentioned previously in section 5.2.2, for $Re_L \approx 5700$, toroidal chaos is reached and for even higher Reynolds numbers ($Re_L = 6333$), the Poincaré map yields a cloud of points without any identifiable structure (see figure 5.2(c)). These two regimes are strongly reminiscent of chaotic behavior. However, the Lyapunov exponents were not computed in order to provide conclusive evidence of chaos. In this section, results helping to conclude on the nature of the flow regime are presented. First, we take a look at the auto-correlation functions, computed from the respective time series. In figure 5.4 the auto-correlation functions are shown as a function of the convective time unit $\tau_c = tU_\infty/L$ for the same Reynolds numbers as in figure 5.2. The auto-correlation functions are defined for a zero-mean time series $s(t)$ as:

$$R(t, \tau) = \frac{E[s(t)s(t + \tau)]}{\sigma_s^2}, \quad (5.1)$$

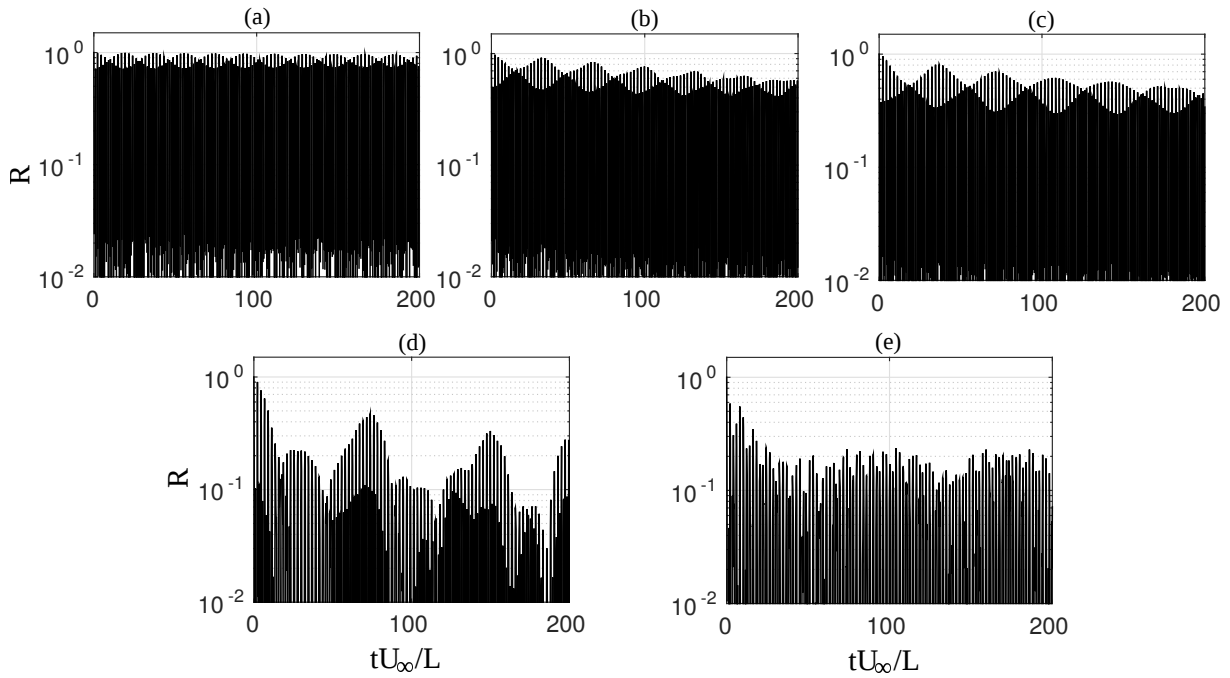


FIGURE 5.4: Auto-correlations R as a function of the convective time unit $\tau_c = tU_\infty/L$ for time series of $D/L = 0.2$ at (a) $Re_L = 4700$ (periodic) (b) $Re_L = 5400$ (quasi-periodic) (c) $Re_L = 5600$ (advanced quasi-periodic) (d) $Re_L = 5700$ (toroidal chaos) (e) $Re_L = 6333$ (chaos).

where $E[\cdot]$ is the expected value operator and σ_s is the variance. For $Re_L = 4700$ the periodic time series yields an auto-correlation function R which cyclically comes back to values equal to unity. For $Re_L = 5400$ and $Re_L = 5600$, the quasi-periodic behavior yields an auto-correlation function that cyclically returns to values close to unity but with increasing time the correlation drops. For $Re_L = 5700$ and 6333 instead, $R(t, \tau)$ first decrease exponentially, before eventually oscillating up to values far from unity. Together with the broadening of power spectra and disordering of the Poincaré section, these are strong indications, that the dynamics at the highest Reynolds numbers is indeed chaotic (Ketzmerick *et al.* (1992)).

Secondly, a closer look at the chaotic spectra is taken. Figure 5.5 shows the time series and spectrum of a typical chaotic case with $D/L = 0.2$ at $Re_L = 7000$. The spectrum peaks close to $St_L \approx 0$ and then drops exponentially. Between $0.2 \lesssim St_L \lesssim 3$ the slope is close to the for turbulence typical value of $-5/3$. For Strouhal numbers $St_L \gtrsim 5$, the slope decreases more rapidly than $-5/3$. Hence, figure 5.5 provides another indication that the dynamics of 2D (double) cavity flow at high Reynolds number is chaotic. In section 5.3.4 the chaotic 2D spectrum will be compared to the corresponding 3D spectrum.

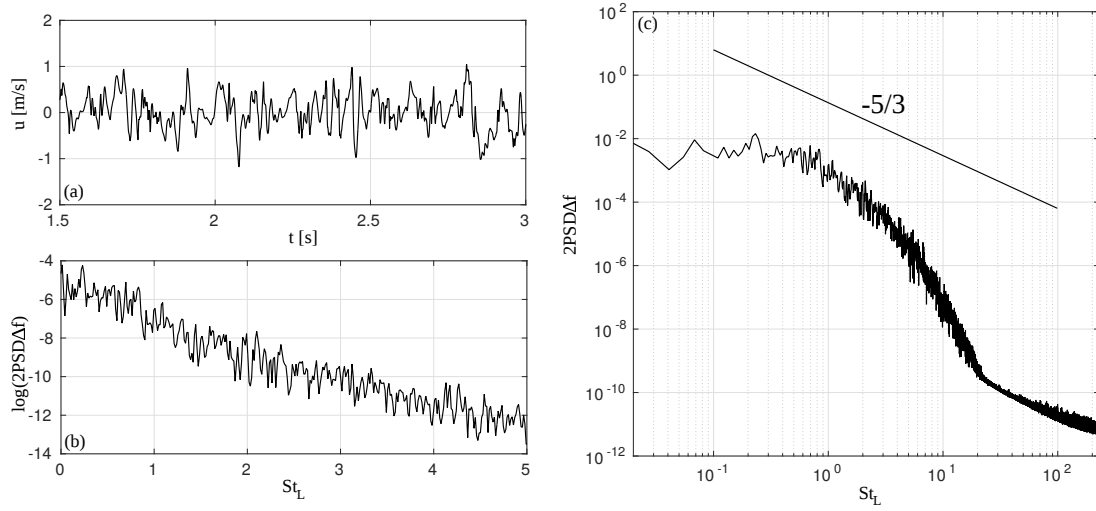


FIGURE 5.5: Time series (a) and spectra (b,c) of vertical fluctuating velocity component of a typical chaotic case with $D/L = 0.2$ at $Re_L = 7000$. (b) and (c) only differ in the axis scaling. Straight line with slope $-5/3$ in figure (c).

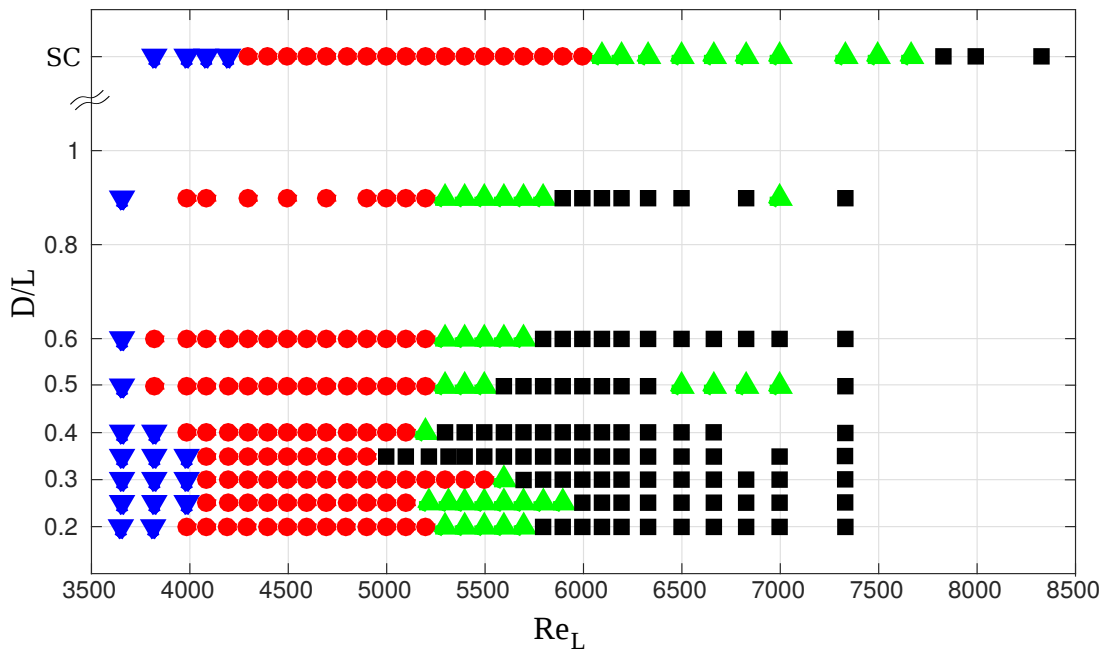


FIGURE 5.6: (Color in PDF) Parameter space plot. Blue ▼: steady regime. Red ●: Periodic regime. Green ▲: intermediate regime. Black ■: chaotic regime. Note, the top line corresponds to a single cavity (SC) channel.

5.2.5 Parameter space

Based on the characterizations from section 5.2.2, a parameter space plot, depicted in figure 5.6, is built. It summarizes the regimes, found when the Reynolds number and the cavity distance are varied. Double cavity flow is equivalent to the canonical single cavity (SC) flow when the distance D is very large. In general, the respective regimes are found at higher Reynolds numbers for single cavity flow when compared to double cavity flow. The steady regime is the first regime observed. It ends with the development of instabilities, that produce self-sustained oscillations of the flow. The associated limit cycle is observed in our study for Reynolds numbers lying in the range between $Re_L \approx 3700$ and $Re_L \approx 4300$ depending on the value of D . The periodic regime is prone to develop at lower Re_L for intermediate cavity distances ($D/L = 0.5$ and $D/L = 0.6$) and at higher Re_L for small ($D/L \leq 0.4$) as well as for very large distances ($D/L \geq 0.9$). The case $D/L = 0.3$ is an exception to the other close distance cases: its periodic regime is observed for much larger Reynolds numbers, compared to its neighbors. This finding will be discussed in greater detail in chapter 7 where the hydrodynamic feedback mechanism and the symmetry properties of the flow are addressed. Depending on the values of D/L , the intermediate regime begins around $Re_L \approx 5500$ for the double cavity, while for the single cavity it starts at $Re_L \approx 6000$. The Reynolds number range of the intermediate regime depends on the cavity distance. The longest range is found for the single cavity, while the shortest range is found for distances close to $D/L = 0.35$. At $D/L = 0.35$ the intermediate regime disappears all together in what looks like a hard (in amplitude) transition to chaos (Lopez-Rebollal & Sanmartin (1995)) and the chaotic regime is encountered in direct continuation to the periodic regime. However, it cannot be fully excluded that a quasi-periodic regime be recovered with a smaller step size in the value of the Reynolds number.

5.2.6 Spectral results

In this section we characterize the spectra associated with each regime. Figures 5.7(a)-(i) show spectral bifurcation diagrams for all cases in table 4.1 when the Reynolds number is varied. In the steady regime no oscillations and hence no frequency peak is present. In the periodic regime, a salient peak at $St_L \approx 1$ prevails over a wide range of the Reynolds number. As expected, harmonics of this dominant mode are also present in the spectrum. The bifurcation diagrams in these regions show strong dark lines equispaced in the vertical coordinate, reminiscent of the teeth of a “comb”. For $D/L \geq 0.4$, towards the end of the periodic regime, the dominant Strouhal number $St_L \approx 1$ is replaced by its first harmonic $St_L \approx 2$. In the Poincaré section this is accompanied by an additional intersection. The underlying physics of this phenomenon will be addressed in chapter 7. In the intermediate regime, the peak at $St_L \approx 1$, and its harmonics, are still present, but

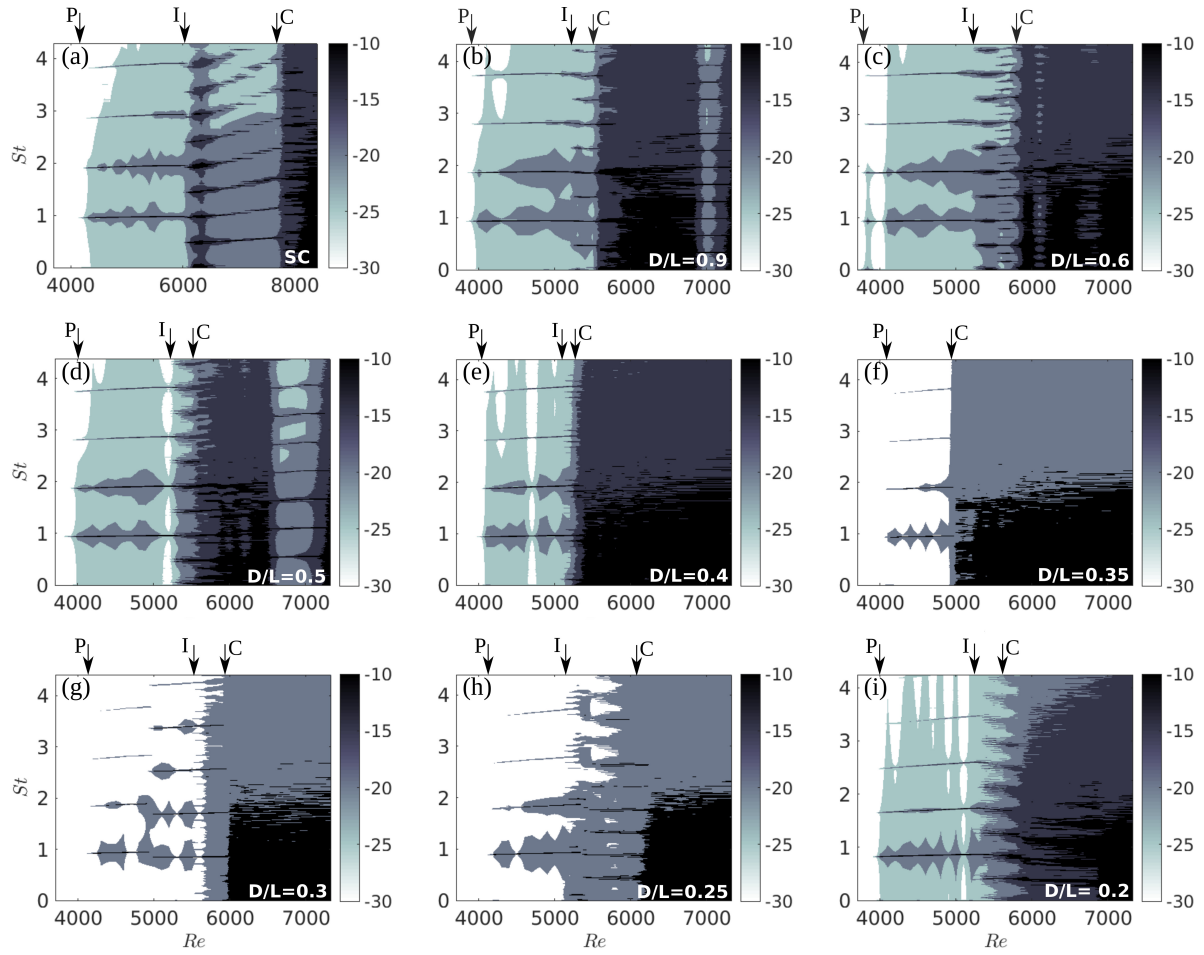


FIGURE 5.7: Spectral bifurcation diagrams: Power spectra vs. increasing ramps of the Reynolds number for (a) single cavity (SC), (b) $D/L = 0.9$, (c) $D/L = 0.6$, (d) $D/L = 0.5$, (e) $D/L = 0.4$, (f) $D/L = 0.35$, (g) $D/L = 0.3$, (h) $D/L = 0.25$, (i) $D/L = 0.2$. Color code depicts the normalized power spectra in log scale: $\log(2\text{PSD}\Delta f)$. Arrows indicate where the periodic (P), the intermediate (I) and the chaotic (C) regime commence in each case.

new peaks of incommensurate frequencies arise in the spectra, a phenomenon that is in correspondence with the creation of a torus in phase space mentioned in section 5.2.2. Peaks of much lower values of non-dimensional frequencies than the previous dominant Strouhal number indicate that a slow dynamics now leaves the signature in the spectrum. As a consequence of the rise of incommensurate frequencies, its harmonics and linear interactions, a tighter comb, with a reduced teeth separation is observed in the bifurcation diagram. Some exceptions may occur, as illustrated in figures 5.7(g)-(h). We may notice in these graphs a discontinuity in the evolution of the dominant frequency at $Re_L \approx 5000$ and $Re_L \approx 5300$, respectively. While for $D/L = 0.25$ (figure 5.7(h)) this discontinuity may be associated with some frequency-locking phenomenon, as commonly observed in quasi-periodic dynamics, for $D/L = 0.30$ (figure 5.7(g)) the discontinuity occurs in the periodic regime and will be discussed in section 7. In the intermediate regime of the SC,

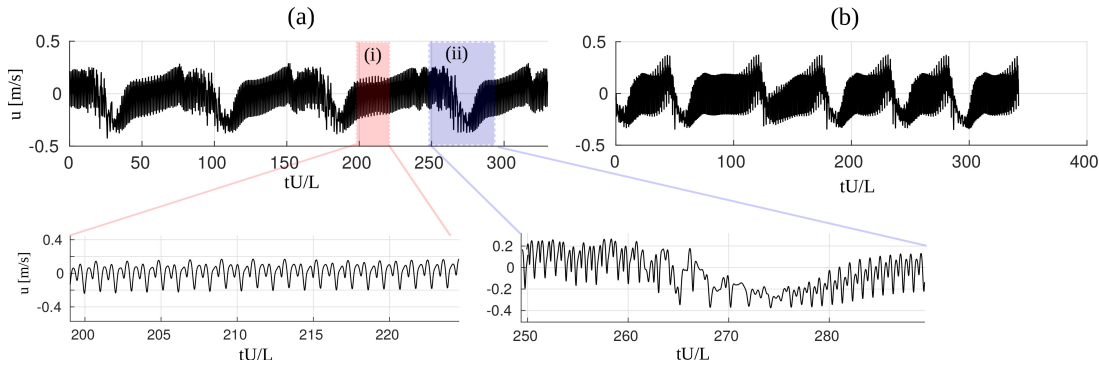


FIGURE 5.8: Example of intermittent cases (a) double cavity with $D/L = 0.9$ at $Re_L = 5600$ and (b) single cavity with $D/L = 0.9$ at $Re_L = 6300$. Zoomed-in view (i): periodic behavior; (ii): chaotic burst.

the Strouhal number increases monotonically with the Reynolds number and the teeth are inclined. The trajectory in phase space is locked on the torus during most of the intermediate regime of the SC, resulting in a periodic window with $St_L \approx 0.5$ and its harmonics over a wide range of Reynolds numbers. In the spectra corresponding to the intermediate regime of the double cavity, the frequencies of the third and fourth peaks are in general in a ratio $\simeq 3/4$. Furthermore, the frequencies of the first and third peaks keep a ratio of $\simeq 1/3$. We will discuss the reason of this behavior in chapter 7. With the exception of $D/L = 0.35$, all transitions to the chaotic regime take the quasi-periodic route. As the Reynolds number is further increased, the spectrum becomes richer, filling in with linear combinations of both incommensurate frequencies. The dynamics enters the chaotic regime. At even higher Reynolds numbers, the spectrum eventually becomes much flatter, peaking at $St_L \approx 0.4$. Notice that the chaotic regime is not necessarily definite as Re_L is increased, and periodic windows may appear. For instance, in figure 5.7(d) for the case $D/L = 0.5$, such a window is observed between $Re_L = 6500$ and $Re_L = 7000$, where the spectrum simplifies and the flow returns from chaotic to the intermediate regime with $St_L = 0.5$. For $D/L = 0.9$ a stability window occurs at $Re_L = 7000$. Also, windows of simplified spectra may appear even inside the periodic regime, as can be observed for instance for $D/L = 0.4$, at $Re_L \approx 4700$, in figure 5.7(e).

5.2.7 Intermittency

At large intercavity distances ($D/L = 0.9$ and SC) there exist Reynolds number ranges, for which intermittent flow behavior is observed. Typical time series of two cases are depicted in figure 5.8. The signal, recorded at probe point P_2 exhibits irregular alternations between periodic dynamics (i) and chaotic bursts (ii). A typical cycle can be described as follows: The system, exhibiting periodic dynamics, slowly drifts away from the unstable periodic orbit. Low frequencies slowly modulate and dephase the signal. The system gets affected by the chaotic dynamics that surrounds the periodic orbit in phase space.

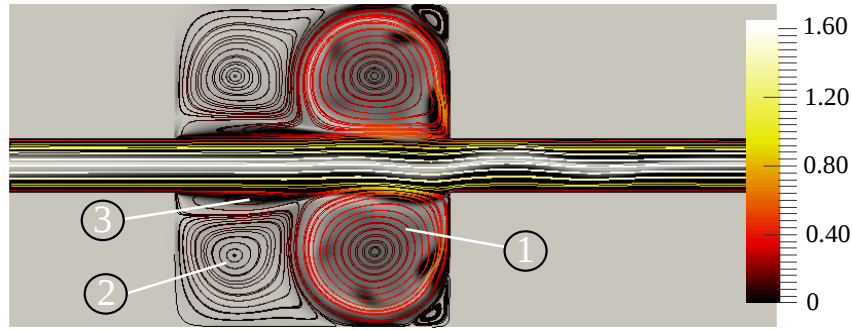


FIGURE 5.9: (Color in PDF) Instantaneous velocity flow field with streamlines of case $D/L = 0.2$ at $Re_L = 4900$ (periodic regime). Color code of the streamlines correspond to the velocity magnitude. Background corresponds to vorticity magnitude. Numbers 1-3 indicate intracavity flow recirculation regions.

The system becomes chaotic and moves back towards the periodic orbit. When it reaches the vicinity of the initial periodic orbit the amplitude of the signal is reduced and the periodic dynamics are recovered. This is a typical textbook scenario for intermittency ([Ruelle \(1995\)](#)). The underlying physical processes will be discussed in chapter 7.

5.2.8 Velocity and vorticity flow fields

As can be seen in figure 5.9, in each cavity, the inner-flow is structured into two large main recirculation regions and a third smaller one. The main recirculation region, located in the downstream half of the cavity, remains almost circular up to the end of the intermediate regime. The secondary recirculation, located in the upstream half of the cavity, is also closely circular. The line connecting the centers of recirculation regions 1 and 2 (cf. figure 5.9) is parallel to the free-stream direction. The dynamics of both recirculation regions are however quite different, as the magnitude of the velocity field in the secondary recirculation region is one order of magnitude smaller than in the main recirculation region. The third, smaller, recirculation bubble is located on top of the secondary recirculation region, below the shear-layer close to the leading edge. These observations are in accordance with [Mizushima & Shiotani \(2001\)](#), who found essentially the same streamline pattern for a geometry with a slightly greater cavity aspect ratio $\Gamma = L/H = 2.6$. The vorticity field, depicted in the background of figure figure 5.9, shows small vortex structures, which are injected by the oscillating shear layers into the respective cavities near their trailing edges. The small vortices travel in a circular trajectory inside the recirculation region and interact with the shear layer, spanning the respective cavity, once they return to its origin. A detailed analysis of this feedback mechanism is found in chapter 7.

5.2.9 Concluding remarks

A nonlinear dynamical system analysis of the double and single cavity based on 2D numerical simulations, introduced in chapter 4, was conducted. It was found that the signal of a single probe was in general rich enough to capture the salient features of the global behavior. On this basis, a study was performed using time series from a single probe. As Re_L is increased, for a given ratio D/L , we found the following successive regimes: steady $\rightarrow$ periodic $\rightarrow$ intermediate $\rightarrow$ chaotic. A map was constructed which enables to identify these regimes for different Reynolds numbers and different separation distances between the cavities. In phase space, the periodic regime describes a limit cycle with a spectrum, that always exhibits a peak at $St_L \approx 1$. It is observed for $Re_L \gtrsim 4000$, though this value depends on the cavity distances. For $D/L = 0.5$ and $D/L = 0.6$ it is reached at somewhat lower Reynolds numbers. As the Reynolds number is increased, the limit cycle disappears and the trajectories in phase space lie on a torus. The spectra become more complex, exhibiting combinations of the two basic incommensurate frequencies. Frequency lockings and un-lockings may occur on the torus, before it breaks down and bifurcates towards a chaotic regime. In the chaotic regime any phase coherence between the two shear layers is lost and oscillation amplitudes are one order of magnitude above the amplitudes of the periodic regime. We could verify that the transitions to the chaotic regime in almost all cases take the quasi-periodic route, in which a frequency smaller than the dominant Strouhal number appears. This route to chaos is reminiscent of a [Curry & Yorke \(1978\)](#) scenario.

The study raises questions, some of which are addressed in the next chapters, others call for further research:

- When the cavities are wide apart the system behaves as two independent single cavities. In such situations ($D/L = 0.9$ and SC) an intermittent regime may take place as indicated by the simulations. However no experiments have reported this regime so far. The frozen degrees of freedom in the span, appear important to relax some constraint on the shear-layer self-sustained oscillations, but the real incidence of these constraints are far from being clearly understood.
- When the inflow is laminar, there are no experimental reports of the chaotic regime for single cavity flows. However, this can not be excluded as this requires specially designed experiments with well-controlled inflow conditions. Also, a reduction of the lifetime of the small vortices related to a non-zero spanwise velocity component, seems another plausible reason for the absence of the chaotic regime.

5.3 3D results

In this section the results of the three dimensional (3D) numerical simulations, introduced in chapter 4, are presented. 3D simulations were performed in order to address four major questions, which could not be answered satisfactorily with the results from the 2D simulations and the wind tunnel experiments of chapter 3. The four questions are:

- How does the spectrum and the flow field change from 2D simulations to 3D simulations and how do both compare to the experimental results presented in chapter 3?
- The only difference between the experimental wind tunnel geometry of the BA-campaign and the Paris-campaign is the spanwise extension. Therefore a natural question is, what is the effect of the spanwise extension on the shear layer spectra?
- How do the flow velocity and vorticity field differ between 2D numerical simulations, 3D numerical simulations and wind tunnel experiments?
- The chaotic flow regime, as observed in the 2D numerical simulations, was not observed in the experiments. A natural question is if it can be observed in 3D numerical simulations or if it is a 2D phenomenon?

To address these questions, ten numerical simulations of the double cavity geometry with three spatial dimension (3D) at three different intercavity distances and for two different cavity aspect ratios $\Gamma = L/H$ are performed using MPI (Message Passing Interface) parallelization. The numerical setup is described in section 4.3. The set of ten double cavity simulations is complemented by one single open cavity flow simulation as a case of reference. All cases are summarized in tables 4.2 and 4.3, respectively.

5.3.1 Spectral results

Time series are recorded with a sampling frequency of roughly 3 kHz at different probe point locations in the computational domain (see figure 4.7 for probe point location). As before, the Strouhal number is defined as $St_L = fL/U_\infty$, in which f is the measured frequency. The initial 20-30% of each time series is discarded to ensure the absence of transient effects. The [Welch \(1967\)](#) algorithm with a window size of 2.3 s and an overlap of 98% is used to compute the power spectral density (PSD), from the streamwise component of the velocity fluctuations. This results in a spectral resolution of $\Delta St \approx 0.012$. All power spectra depicted in this work are normalized by pre-multiplying with two times the frequency resolution: $2PSD\Delta f$.

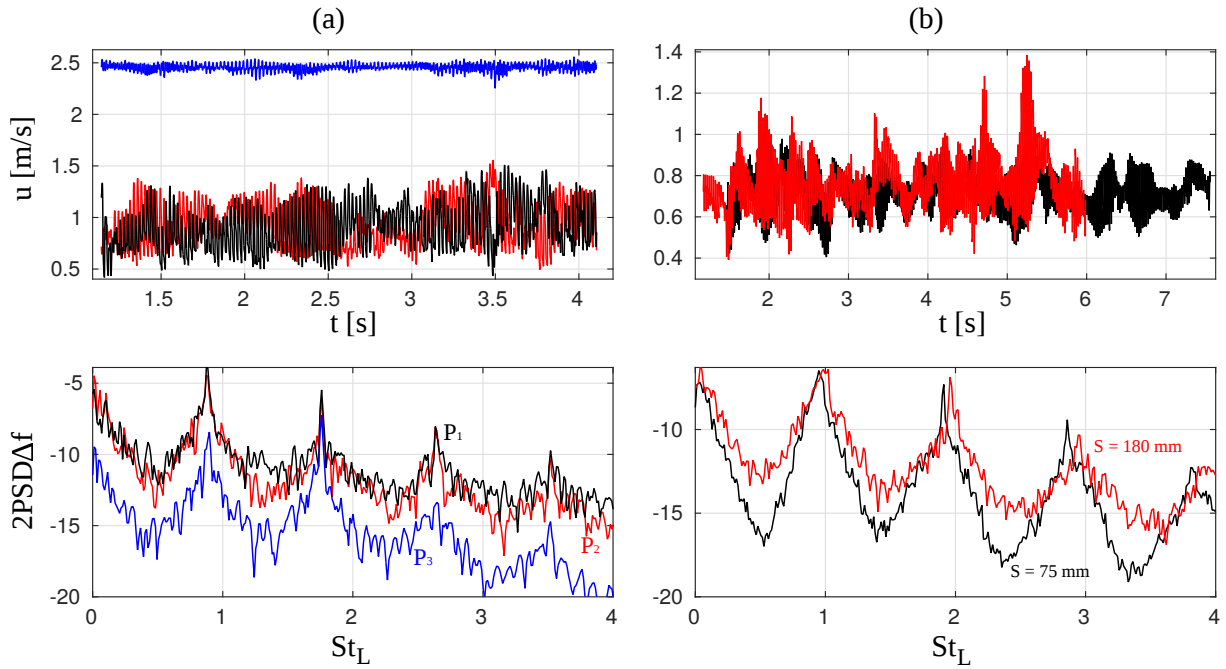


FIGURE 5.10: (Color in PDF) (a) Time series and power spectra of 3D case $D/L = 0.2$ with $S = 75$ mm and $Re_L = 8300$ in probe points $P_1 = [0.04, 0.01, 0]$ (red), $P_2 = [0.04, -0.01, 0]$ (black) and $P_3 = [0.04, 0, 0]$ (blue). (b) Comparison of time series and spectra of 3D cases with $D/L = 0.4$ and $S = 180$ mm (red) and $S = 75$ mm (black) at $Re_L = 7200$ and $Re_L = 7300$, respectively. Data in both cases recorded at probe point $P_1 = [0.04, 0.01, 0]$.

It was found that the spectra in all investigated spanwise planes ($z = -0.0325$ mm, $z = -0.0175$ mm, $z = 0$ and $z = 0.0175$ mm) are similar. Hence, only the spanwise plane $z = 0$ is considered from now on. Figure 5.10(a) compares the time series and the power spectra from probe points P_1 , P_2 and P_3 in plane $z = 0$ (cf. figure 4.7). The two shear layer spectra (P_1 and P_2) are close to identical and exhibit a dominant peak at $St_1 = 0.89$ (and its harmonics) and a low frequency peak at $St_0 \approx 0$. The time series recorded in probe point P_3 , located in the channel center, is less in amplitude but shows a similar spectral composition: the low frequency peak at $St_0 \approx 0$ and a shear layer peak at $St_1 = 0.89$. However, the most amplified peak is not the peak at $St_1 = 0.89$, but its first harmonic at $St = 2St_1$. This is in agreement with what was found for the 2D simulations in section 4.2. Due to the asymmetric (sinuous) flow type, probe point P_3 records the effect of both cavities, which doubles the dominant frequency. For larger distances ($D/L \geq 0.3$) this effect ceases.

The spanwise extension

Figure 5.10(b) compares the shear layer spectra (P_1) of two cases with $D/L = 0.4$ and $Re_L = 7200$ for two different spanwise extensions $S = 75$ mm, as used in the experimental

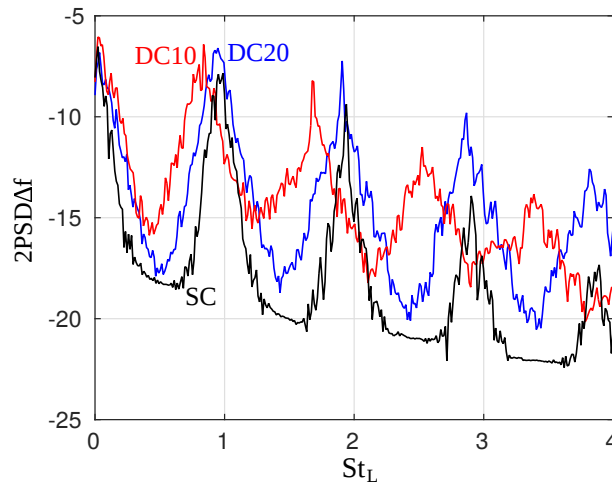


FIGURE 5.11: (Color in PDF) Comparison of shear layer spectra recorded in probe point $P_2 = [0.04, -0.01, 0]$ of 3D double cavity cases 4 ($D/L = 0.2$, $Re_L = 8\,300$), 3D double cavity case 8 ($D/L = 0.4$, $Re_L = 7\,300$) and 3D single cavity (SC) case 10 ($Re_L = 7\,100$) all detailed in table 4.2.

Paris-campaign and $S = 180$ mm, as used in the experimental BA-campaign (*cf.* cases 8 and 9 from table 4.2). At first sight the spectra look very similar. Both spectra contain a low frequency peak at $St_L \approx 0$ and the typical shear layer peak at $St_L \approx 1$ together with its less amplified harmonics. A closer look reveals three main differences:

1. The peaks are broader and the signal-to-noise-ratio (SNR) is smaller for the larger spanwise extensions
2. The larger spanwise extension leads to oscillations at slightly higher frequencies: $St_{S180} > St_{S75}$.
3. The low frequency peak St_0 is more amplified for the larger spanwise extension

As found by Basley *et al.* (2014) and Douay *et al.* (2015), three-dimensional intracavity structures are responsible for the side band and the low frequency peaks in open cavity flow. Their findings are confirmed by the observations made in double cavity flow: the case with a larger spanwise extension naturally allows for stronger three-dimensional structures which then leave their imprint on the shear layer spectra. Hence, the spectra of the double cavity with the larger spanwise extension shows stronger side band peaks and a stronger low frequency peak.

5.3.2 Single vs. double cavity

In this section, results from the 3D numerical simulation of the single cavity (case 10 in table 4.2) are compared to two double cavity cases with different intercavity distances

(cases 4 and 8 from table 4.2). Figure 5.11 shows shear layer spectra for the double cavity cases (4 red, 8 blue) and the single cavity reference case (10 black). All three spectra show a strong low frequency peak and the typical shear layer peak at $St_{L,1} \approx 1$. Their harmonics are also present. The single cavity shear layer peak is more narrow band and found at slightly higher frequencies, closer to $St_{L,1} = 1$, when compared to the double cavity shear layer peaks. The following inequality holds for the shear layer peaks at $St_{L,1} \approx 1$:

$$St_{L,1}(DC10) \lesssim St_{L,1}(DC20) \lesssim St_{L,1}(SC)$$

This is in accordance with the results from the 2D numerical simulations, where the same trend of the most amplified frequencies was observed when the intercavity distance was varied (*cf.* figure 5.7).

5.3.3 Velocity and Vorticity flow fields

Figure 5.12 depicts a typical vorticity and velocity field of a 3D numerical simulation with $D/L = 0.2$ at $Re_L = 8333$. In the z-plane, depicted in figure 5.12(a), it can be observed how the fluctuating shear layers, spanning each cavity, regularly inject small packages of vorticity into the respective cavity. The vorticity packages in each cavity circulate in the backflow of the recirculation region and return to the shear layer close to its origin, near $x/L \approx 0.1$. From the x-plane, depicted in figure 5.12(b), it becomes apparent that the vorticity is distributed in discrete vortex packages not only in the x-y-plane but also along the spanwise direction (z-y-plane). Figure 5.12(c) shows the instantaneous velocity field of the same instant as in figure 5.12(a)-(b). The velocity field clearly shows the recirculation region in which the vortex structures are advected upstream. In the spanwise direction the fluctuating center flow shows a wavy form, indicating that there are spanwise dependencies of the instantaneous flow field. The analysis of the (certainly important) spanwise dynamics is however out of scope of the present work.

5.3.4 Chaos and turbulence

In this section, the question will be addressed whether or not a chaotic flow regime, encountered in 2D numerical simulations of open (double) cavity flows, can also be found in 3D flows and if so, under which conditions this regime occurs and whether or not it is turbulent.

For the highest Reynolds number cases of the 3D simulations with $L/H = 2$ clear frequency peaks at $St_L = 1$ and harmonics are observed. A chaotic regime as in the 2D simulations can therefore not be confirmed for cases with aspect ratio $L/H = 2$. However, for the cases with $L/H = 1$ the picture changes. The cavity length L is shortened, such

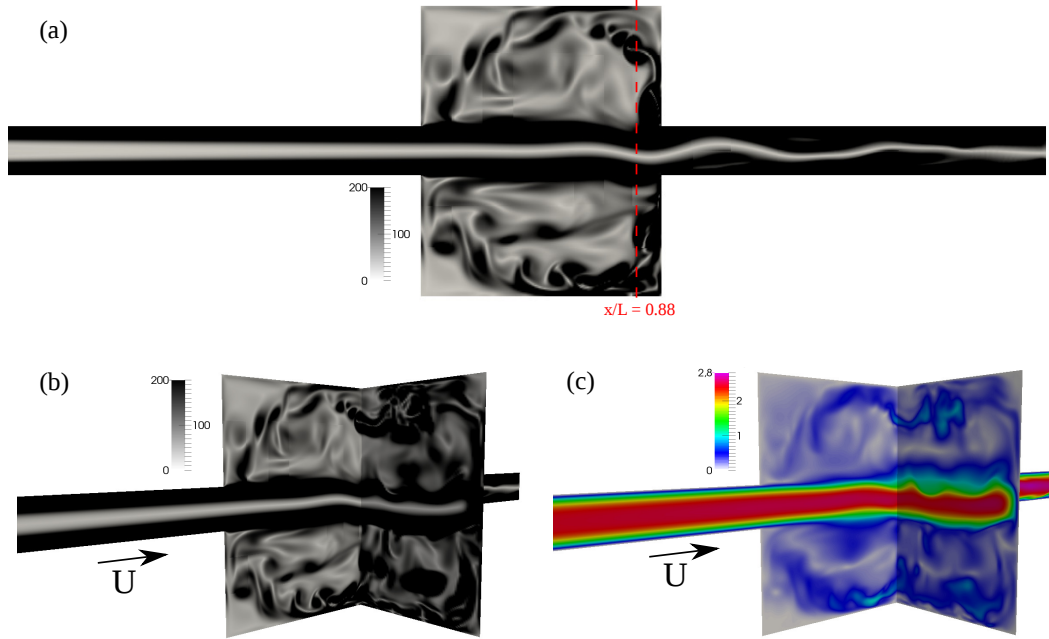


FIGURE 5.12: (a) Vorticity magnitude flow field of 3D numerical simulation at $Re = 8333$ with $D/L = 0.2$ in plane $z/L = 0$. (b) Same as (a) but for both planes $z/L = 0$ and $x/L = 0.96$. Red dashed line in (a) indicates position $x/L = 0.96$. (c) Velocity magnitude field of same instant and same planes as in (b).

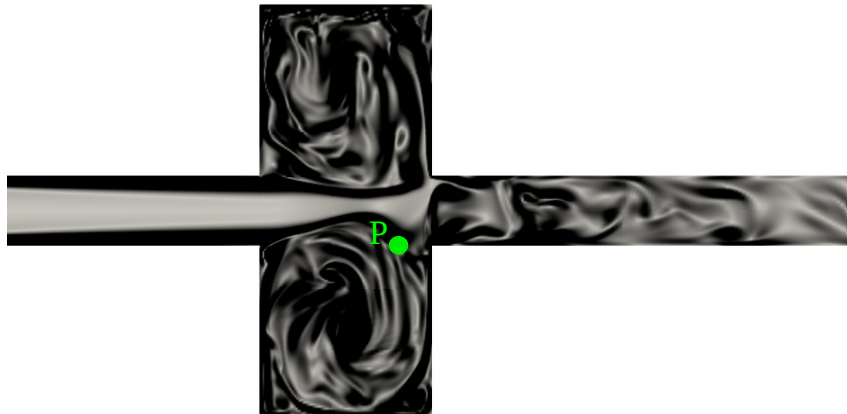


FIGURE 5.13: Vorticity magnitude field of case 12 in table 4.2 ($L/H = 1$, $D/L = 0.2$, $Re_L = 2500$) with probe point location P .

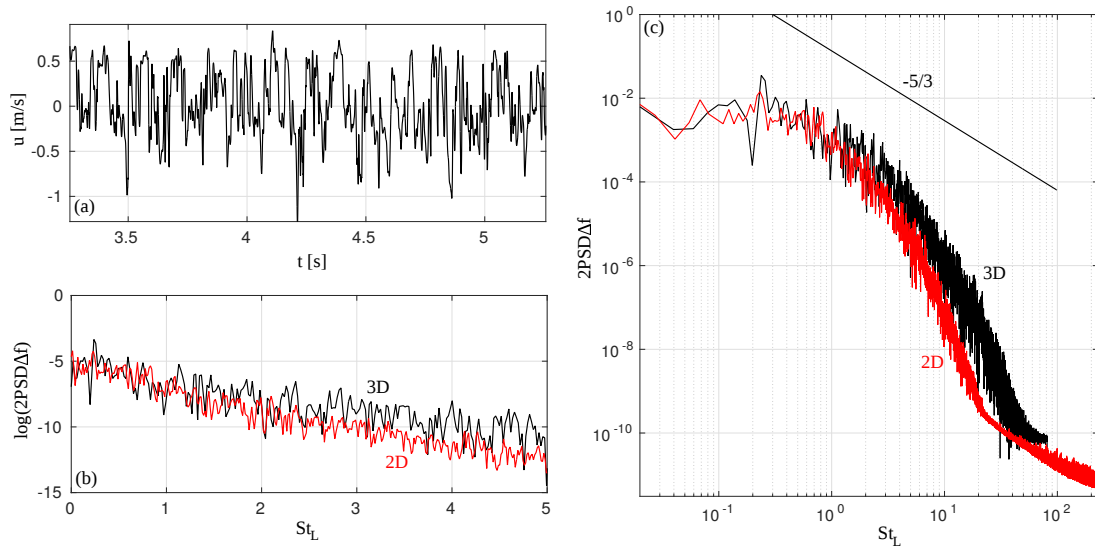


FIGURE 5.14: (Color in PDF) (a) Time series and (b) power spectra from probe point $P = [0.02, 0.01, 0]$ (*cf.* figure 5.13) of case 12 in table 4.2 ($L/H = 1$, $D/L = 0.2$, $Re_L = 2500$). (c) Comparison of 3D case (black) as in (b) and chaotic 2D case (red) with $D/L = 0.2$ at $Re_L = 7000$. The straight line in subfigure (c) indicates a slope of $-5/3$.

that $L/H = 1$, and the inflow length $L_1 = 2.2L$ is shortened, in order to decrease Θ_0 . This is necessary to keep the term L/Θ_0 constant, which according to [Basley *et al.* \(2011\)](#) characterizes the stability properties of the impinging shear layer. As can be seen from table 4.2, the terms L/Θ_0 are similar for both the $L/H = 2$ and the $L/H = 1$ simulations.

Interestingly, chaotic flow conditions were encountered for both cases with $L/H = 1$. As in the 2D case the spectra are flat and no dominant frequency peak at the typical shear layer frequency $St_L \approx 1$ is found. Figure 5.14(a) shows a typical times series recorded in the shear layer probe point P (*cf.* figure 5.13) and figures 5.14(b)-(c) show the associated power spectrum in different axis scaling. The straight line in figure 5.14(c) corresponds to a slope of $-5/3$. It fits well the 3D spectrum in a range $0.2 \lesssim St_L \lesssim 8$, suggesting that the spectrum belongs to a fully turbulent process. The 2D spectrum, shown in red color in figures 5.14(b)-(c), in general agrees well with the 3D spectrum. At low frequencies ($St_L \lesssim 3$ both spectra are close to identical. For higher frequencies the 2D spectrum drops faster than the 3D spectrum.

A possible reason why in double cavity flow a chaotic (turbulent) regime is encountered for cases with $L/H = 1$, but not for cases with $L/H = 2$, is discussed in chapter 7 with respect to the hydrodynamic feedback mechanism.

5.3.5 Concluding remarks

Results from three dimensional (3D) numerical simulations of double and single cavity flow were presented. As in the 2D case, the analysis was restricted to cases in which the inflow profile is laminar, in order to exclude possible excitations associated with the intrinsic fluctuations of turbulent flows. The correct inflow profile was obtained by letting a constant inflow develop in an inflow channel, that was designed such that approximately the same inflow conditions as in the wind tunnel experiments from chapter 2 were obtained. Due to the increased computational cost of 3D simulations, only a small number of cases in the $Re_L - D/L$ parameter space were investigated. The main objective was to address questions, left unanswered from section 5.2 and to compare the numerical results to the (naturally 3D) experimental results from chapter 3.

Spectra were recorded in various probe points in different spanwise planes $z = \text{const.}$ It was found that the spectra in all $z = \text{const.}$ were identical and that considering the spanwise plane ($z = 0$) is sufficient. In a given plane the same peaks are amplified in all probe points, though the dominance may change. As in the experimental study two different spanwise extensions had been used, one of the questions set out to answer with the help of 3D numerical simulations was in which way and to which extend the spanwise extension influences the spectra recorded in the midplane ($z = 0$). The comparison of the spectra from time series recorded in the midplane ($z = 0$) of two cases at the same Reynolds number but different spanwise extensions ($S = 75 \text{ mm}$ and $S = 180 \text{ mm}$) showed that the signal-to-noise ratio (SNR) is smaller for the larger spanwise extensions and the low frequency peak $St_{L,0} \approx 0$ is more amplified for the larger spanwise extension. Also, the larger spanwise extension lead to oscillations at slightly higher frequencies. The analysis of the vorticity field in spanwise and streamwise planes revealed small intracavity packages of predominantly spanwise vorticity, injected by the fluctuating shear layer upon impingement, an observation that had also been made for the 2D simulations. The comparison of single and double cavity flow cases showed, that both configurations amplify similar frequency peaks, though in the case of the single cavity the frequency peaks are more narrow band and are found at slightly higher frequencies compared to the double cavity cases. Chaotic flow behavior, as found in 2D simulations, was not observed in 3D simulations for cases with aspect ratios $L/H = 2$. However, when the cavity aspect ratio was reduced to $L/H = 1$, while L/Θ_0 was kept approximately constant, chaotic flow behavior was observed at relatively low Reynolds numbers $Re_L = 2500$. The spectrum looks very similar to the chaotic spectra observed in 2D simulations.

5.4 Comparison: Simulations and experiments

In this section the results from the 2D and 3D numerical simulations, presented above, and the wind tunnel experiments, presented in chapter 3, are compared. To this end, the inflow conditions, the mean flow fields, the fluctuating velocity intensity fields and the shear layer power spectra, all measured in the plane $z = 0$, of typical cases are considered.

5.4.1 Comparison of the inflow conditions

Figure 5.15 summarizes and compares the characteristic inflow velocity U_∞ vs. the non-dimensional cavity length L/Θ_0 for the numerical simulations (2D, 3D) and the experimental campaigns (Paris-campaign, BA-campaign). The term L/Θ_0 , which is the cavity length L , normalized with the momentum thickness at the leading edge of the (double) cavity Θ_0 , characterizes the inflow conditions of the (double) cavity and strongly influences the stability properties of the shear layers as noted by Basley (2012) and confirmed in the present work. A higher value of L/Θ_0 yields a more unstable shear layer and is therefore denoted a more unstable inflow condition.

As mentioned in section 5.2, the inflow velocity profiles for the 2D numerical simulations are designed such that they are independent of the intercavity distance D . Hence, all 2D cases lie on the curve formed by the blue circles in figure 5.15. Due to the fixed inflow length of the experimental setup and the setup of the 3D numerical simulations, L/Θ_0 of the Paris-campaign (black), the BA-campaign (red) and the 3D numerical simulations (green) depend on D . As mentioned above, the inflow of 3D cases 1 and 2 (*cf.* table 4.3) were obtained by imposing a 2D profile, uniform in the spanwise direction. This simplified method is the reason why two of the 3D cases (green upside down triangles) coincides with the corresponding 2D cases (blue circles). 2D cases are in general found at higher values of L/Θ_0 than their experimental and 3D numerical counterparts. Both experimental campaigns agree rather well. By design the values for L/Θ_0 of the 3D numerical simulation cases are found within the range of values of the two experimental campaigns.

5.4.2 Velocity flow fields

In this section the flow fields and velocity profiles of simulations and experiments are compared. Figure 5.16 shows the streamlines plots and the intracavity velocity profiles at the indicated streamwise position for (I) numerical data from 3D simulation, (II) numerical data from 2D simulation, (III) experimental data from the Paris-campaign and (IV) experimental data from the BA-campaign. All four cases are summarized in table 5.1. Besides the streamlines, mean velocity profiles at five streamwise positions, indicated by

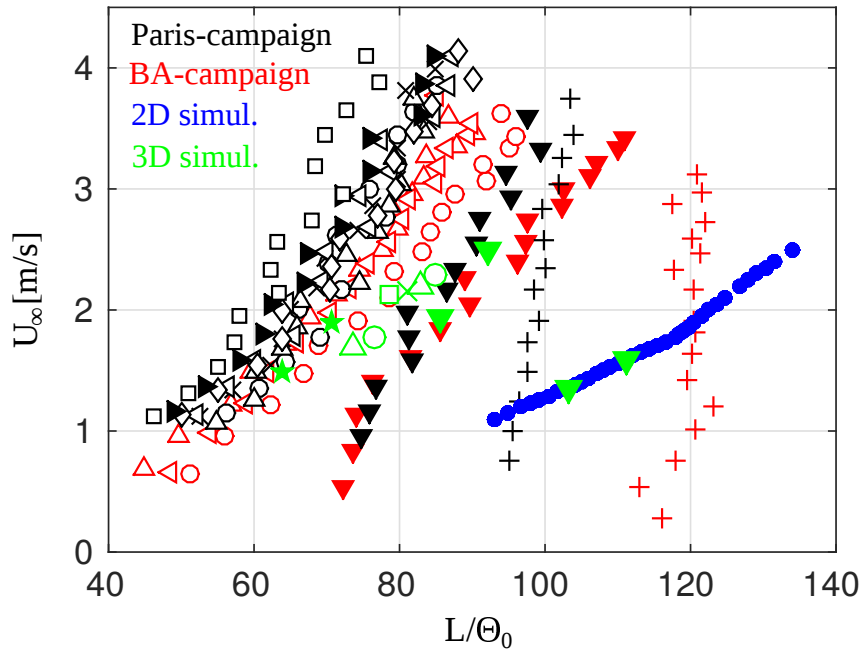


FIGURE 5.15: (Color in PDF) Comparison of term L/Θ_0 of both experimental campaigns and both 2D and 3D numerical simulations. Symbols for Paris-campaign as in table 2.1, for the BA-campaign as in table 2.2 and for the 3D numerical simulations as in table 4.2. 2D numerical simulation data is independent of the intercavity distance.

the solid horizontal lines in figure 5.16(a)-(d), are compared. The normalized streamwise and vertical velocity profiles at these positions 1-5 are shown in figure 5.16(e)-(n).

The flow field of the double cavity can in general be divided in three main regions: the center flow ($-5 \leq y \leq 5$), which separates the two intracavity zones ($-30 \leq y \leq -5$) and ($5 \leq y \leq 30$), respectively. All mean flow fields are symmetric with respect to the channel center line at $y = 0$ and show a main recirculation region, located in the downstream part of each cavity and a secondary recirculation region, located further upstream. Experiments and 3D numerical simulations show much larger main recirculation regions at the expense of the smaller secondary recirculation regions. In the case of the 2D simulation, the secondary recirculation region is almost of the same size as the main recirculation region and the center of the two recirculation regions in each cavity are aligned with the streamwise direction. This is not true for the 3D simulation and the experiments, where the center of the second recirculation region is found further outwards (higher absolute value of y) than the center of the primary recirculation region. Also the streamwise position of the recirculation center differ. For the 3D simulation the two center are found furthest upstream at $x/L \approx 0.1$ and $x/L \approx 0.4$, respectively. The 2D simulation exhibits the largest secondary recirculation region and their center are found at $x/L \approx 0.25$ and

TABLE 5.1: Summary of cases for flow field comparison

Case	Method	D/L	L/H	U_∞ [m/s]	Re_L	Θ_0 [mm]	L/Θ_0
I	3D simulation	0.2	2	1.74	5 800	0.532	94
II	2D simulation	0.2	2	1.74	5 800	0.498	100
III	Exp. Paris	0.2	2	1.65	5 500	0.612	82
IV	Exp. BA	0.2	2	1.62	5 400	0.610	82

$x/L \approx 0.7$, respectively. The streamwise position of the recirculation center of the experimental Paris-campaign is found at $x/L \approx 0.1$ and $x/L \approx 0.55$, respectively, close to the values obtained from the BA-campaign $x/L \approx 0.15$ and $x/L \approx 0.65$.

The absolute velocity values, encountered in the secondary recirculation region, are usually about one order of magnitude smaller than the absolute velocity values in the main recirculation region. The 2D case even produces a third smaller recirculation region close to the inflow channel. This third recirculation region is neither confirmed by the 3D simulation nor by the experiments. A more detailed investigation (especially of the vorticity) flow fields will be postponed until chapter 7, where a detailed analysis of the vorticity flow field reveals a feedback mechanism, that provides an explanation for the spectral composition of double cavity flow. Double cavity flow was investigated before by [Mizushima & Shiotani \(2001\)](#) and [Mullin *et al.* \(2003\)](#), though for lower Reynolds numbers, fixed inter-cavity distance and larger cavity aspect ratios. Their setup results in both symmetric and asymmetric steady flow conditions. The velocity flow fields of the symmetric steady flow conditions compare well with the flow fields shown in 5.16(a)-(d).

Figures 5.16(e)-(i) compare the streamwise mean velocity profiles $U(y)$ of the four cases (simulations and experiments) at five streamwise positions in the double cavity domain, as indicated by the solid horizontal lines in figures 5.16(a)-(d). As suggested by the streamline plots, all velocity profiles are symmetric with respect to $y = 0$.

In general it can be observed, that the velocity profiles of all four cases agree well in the channel center, while in the intracavity regions, certain differences are noticeable. The 2D simulation strongly over predicts intracavity velocity minima and maxima. This is due to the fact, that in the 2D numerical simulations all energy is kept in the z -plane, since it is not redistributed into the third dimension. At the inflow, position (1) (figures 5.16(e)), the 3D simulation and both experimental campaigns are in good agreement, while the 2D simulation shows much higher velocities in the intracavity zones, due to the more pronounced second recirculation region. Moving further downstream at position (2) (figures 5.16(f)), the 3D simulation slightly underestimates the velocities in the intracavity flow zones when compared to the experimental data. The 2D simulation data shows much higher intracavity velocities of opposite sign. Again, this is due to the different flow field

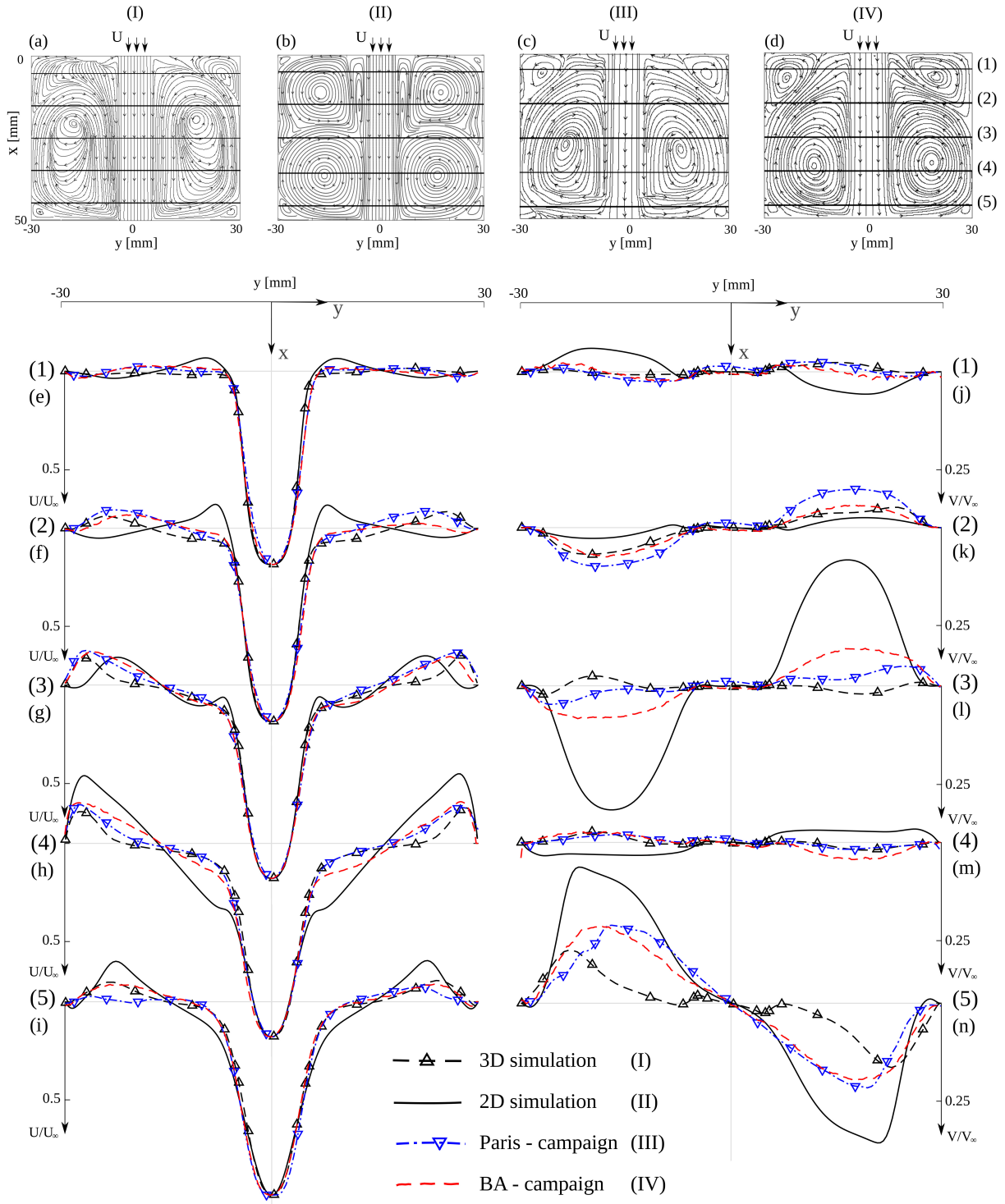


FIGURE 5.16: Mean velocity stream lines and mean velocity profiles of $D/L = 0.2$ double cavities. (a) Numerical data from 3D simulation at $Re \approx 5700$. (b) Numerical data from 2D simulation at $Re \approx 5800$. (c) Experimental data from Paris-campaign at $Re \approx 5500$. (d) Experimental data from BA-campaign $Re \approx 5400$. Subfigures (e)-(i) show with maximum center line velocity U_∞ normalized streamwise mean velocity profiles $U(y)$ at the indicated streamwise locations (1)-(5). Subfigures (j)-(n) show with maximum center line velocity U_∞ normalized vertical mean velocity profiles $V(y)$ at the indicated streamwise locations $x/L \approx 0.1$ (1), $x/L \approx 0.3$ (2), $x/L \approx 0.5$ (3), $x/L \approx 0.7$ (4) and $x/L \approx 0.9$ (5).

structure of the 2D simulation with respect to the other cases. The larger secondary recirculation region induces these velocities. At position (3) (figures 5.16(g)), all four methods are in better agreement when it comes to the velocity minima in the intracavity zones, however, the y -location of the minimum velocity is closer to the center channel for the 2D simulation when compared to the other three cases. Towards the rear end of the double cavity at position (4) and (5) (figures 5.16(h)-(i)), the cases I, II, and IV show similar intracavity profiles. The maxima and minima of these cases coincide rather well. Case II (2D simulation) again over predicts the velocity minimum in the recirculation zone and due to the stronger primary recirculation region it shows increased velocities at position (4) even near the channel center.

Figures 5.16(j)-(n) compare the vertical mean velocity profiles $V(y)$ of the four cases (simulations and experiments) at the same five streamwise positions as before. Note that the axis scaling of the vertical velocity profiles is different from the scaling of the streamwise velocity profiles by a factor of 2. As in the case of the streamwise mean velocity profiles, the vertical mean velocity profiles from the 2D numerical simulations show significantly higher velocities at almost all positions. As expected, all profiles are symmetric with respect to the origin. At the inflow, position (1) (figure 5.16(j)), the vertical mean velocities are relatively low. The 3D simulation and both experimental campaigns are in good agreement. The 2D simulation, however, shows higher velocities in the intracavity zones, due to the more pronounced second recirculation region. At position (2) (figure 5.16(k)) the recirculating flow of cases I, II and IV return towards the channel center, which increases the vertical mean velocities. Case II (2D simulation) shows lower velocities at this position which is in agreement with the more pronounced second recirculation region. The main recirculation region of case II returns towards the channel center at position (3) (figure 5.16(l)), which yields much higher vertical mean velocities than for the cases I, II and IV. Due to the different sizes of the recirculation region in all four cases, the vertical velocities in position (3) do not agree well. At position (4) (figure 5.16(m)) in all cases very low vertical velocity components are measured, since in all four cases a strong streamwise backflow dominates. Position (5) (figures 5.16(n)) is located closest to the rear edge of the double cavity. All four cases show strong vertical velocity components. As before, the vertical velocity of the 2D simulation is roughly twice as high as in the other cases. The two experimental campaigns agree quite well on the maximum velocity and also the vertical location of the maximum. The 3D simulation shows lower vertical velocities and the location of the maximum is moved somewhat further outwards with respect to the other three cases.

In summary, the comparison of the streamwise and vertical velocity fields show that the velocity profiles of the simulations and experiments agree well in the channel center,

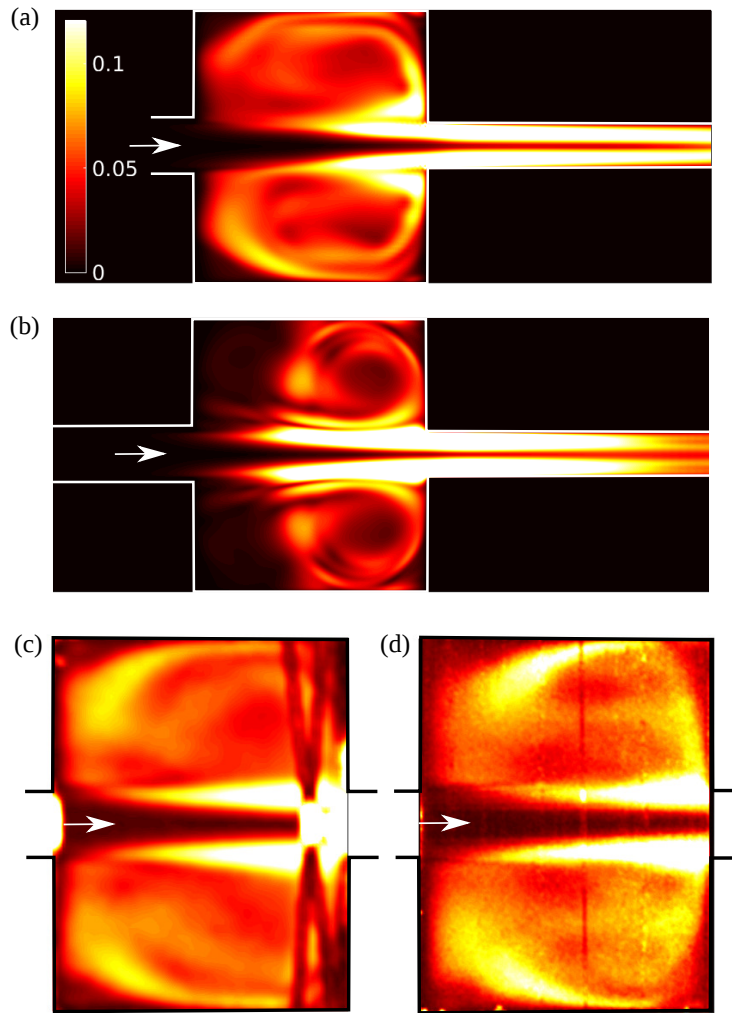


FIGURE 5.17: (Color in PDF) Root-mean-square (rms) fields of the streamwise fluctuating velocity component u_{rms} of $D/L = 0.2$ double cavities. (a) Numerical data from 3D simulation at $Re \approx 5700$. (b) Numerical data from 2D simulation at $Re \approx 5800$. (c) Experimental data from Paris-campaign at $Re \approx 5500$. (d) Experimental data from BA-campaign $Re \approx 5400$. Colorbar in (a) valid for all subfigures. Flow in the direction indicated by the arrows. Note, the irregularities close to the downstream cavity edge in (c) is due to the interference of the LDV laser.

while the difference in the structure of the recirculation regions yields different intracavity velocity profiles. The two experimental campaigns and the 3D simulation agree reasonably well. The 2D simulation results in a recirculation flow which is about twice as strong as in the other cases.

5.4.3 Fluctuating velocity intensity fields

The root-mean-square (rms) fields of the streamwise fluctuating velocity component u' of the 2D and 3D numerical simulations and the two experimental campaigns are compared in figure 5.17. The rms of streamwise fluctuating velocity component u' in each point in the flow field is defined as

$$u_{\text{rms}} = \sqrt{u'^2} \quad (5.2)$$

The rms fields confirm the reflection symmetry (with respect to the cavity center line $y = 0$), found for the mean velocity streamlines. Also, the form of the intracavity rms flow field reflects the structure observed in the mean velocity streamline plots. The results from the 3D simulations and the two experimental campaigns agree well. The main recirculation region of the 2D simulation is smaller, occupying only about half of each cavity. The shear layers can clearly be distinguished all along the cavity length in all four cases. In contrary to a jet flow where the shear regions merge, the double cavity geometry can be considered as two separate but coupled systems. The onset of shear layer fluctuations occur after the geometrical separation of the flow at the double cavity's leading edge. While in the case of the 2D numerical simulation strong shear layer fluctuations are observed starting at around $x/L \approx 0.3$, the onset of shear layer fluctuations in the case of the 3D numerical simulations is found further downstream at $x/L \approx 0.5$. Both experimental campaigns show similar onsets of the shear layer oscillations at around $x/L \approx 0.3$.

5.4.4 Spectra

Figure 5.18 compares the power spectra from 2D and 3D numerical simulations at two different Reynolds numbers for a case with $D/L = 0.2$ and $L/H = 2$.

At the lower Reynolds number (figure 5.18(a)) both simulations agree on the most dominant frequency peak, which is found at $St_L = 0.87$ and $St_L = 0.88$, respectively. The respective harmonics are present in both cases. Two major differences can be observed:

1. The 3D spectrum contains a strong low frequency peak at $St_L \approx 0$, whereas the 2D spectrum does not exhibit any low frequency content. This observation is in agreement with [Basley et al. \(2013\)](#) and [Douay et al. \(2015\)](#), who found that spanwise dynamics modulates the intracavity flow in single open cavity flow and hence is responsible for the low frequency peak in the shear layer spectrum.

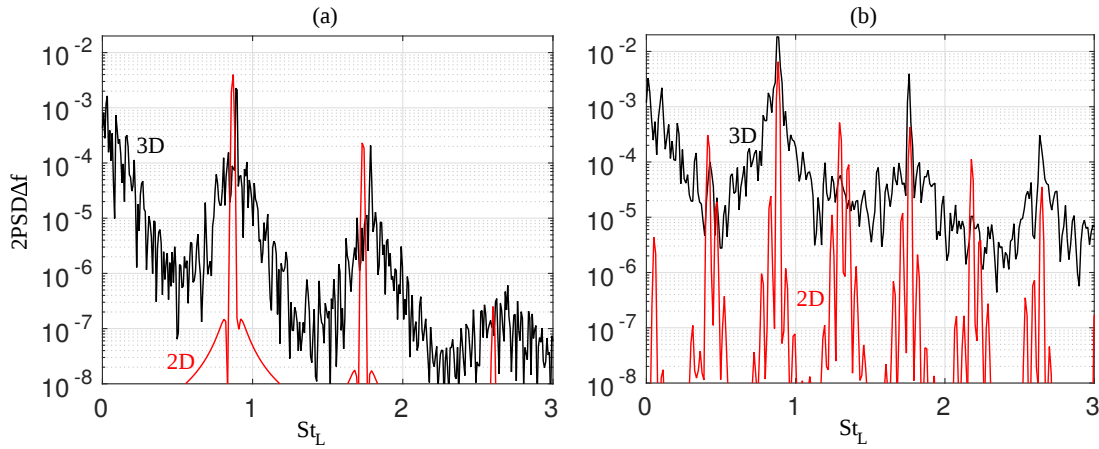


FIGURE 5.18: (Color in PDF) Comparison of power spectra from 2D (red) and 3D (black) simulations for a double cavity with $D/L = 0.2$ and $L/H = 2$. (a) in the periodic regime (at $Re_L = 4500$) and (b) 2D in the quasi-periodic regime (at $Re_L = 5400$) and 3D in the periodic regime (at $Re_L = 8300$).

2. The 3D spectrum shows symmetric (with respect to the most amplified frequency) side band peaks, whereas the 2D case does not show any side band peaks. Again, this is in agreement with the literature ([Basley *et al.* \(2013\)](#) and [Douay *et al.* \(2015\)](#)), who related the side band peaks to the modulation induced by spanwise dynamics in single open cavity flow.

Note that the peak of the 3D simulation at a first glance seems very narrow band compared to other 3D simulations. This is due to the fact that this simulation is very close to the threshold of the onset of the periodic regime, resulting in less modulation and hence less pronounced side band peaks.

At a higher Reynolds number (figure 5.18(b)) results from 2D and 3D numerical simulations do not agree entirely. The typical shear layer peak at $St_L \approx 1$ is still the most amplified peak in both cases, though the peak-to-noise ratio in the 3D spectrum decreases and in the 2D spectrum a second non-harmonic peak at $St_{L,2} \approx 1.3$ appears. Combinations of the two non-harmonic frequencies account for the remaining peaks. For even higher Reynolds numbers, as mentioned before, the 2D simulations transition towards a chaotic regime, which was not observed in the 3D simulations with cavity aspect ratios $\Gamma = L/H = 2$, but only for cavity aspect ratios $\Gamma = L/H = 1$.

Figure 5.19 compares spectra from the experimental Paris-campaign with spectra from numerical 3D simulations for (a) a low Reynolds number case ($Re_L \approx 5\,600$) and (b) a high Reynolds number case ($Re_L \approx 7\,300$), both with $D/L = 0.4$. As was shown in chapter 3, for low Reynolds numbers and $D/L \geq 0.2$, the double cavity spectrum shows a single peak typically found at around $St_{L,1} \approx 1$. Numerical simulations in 3D (and as was shown in

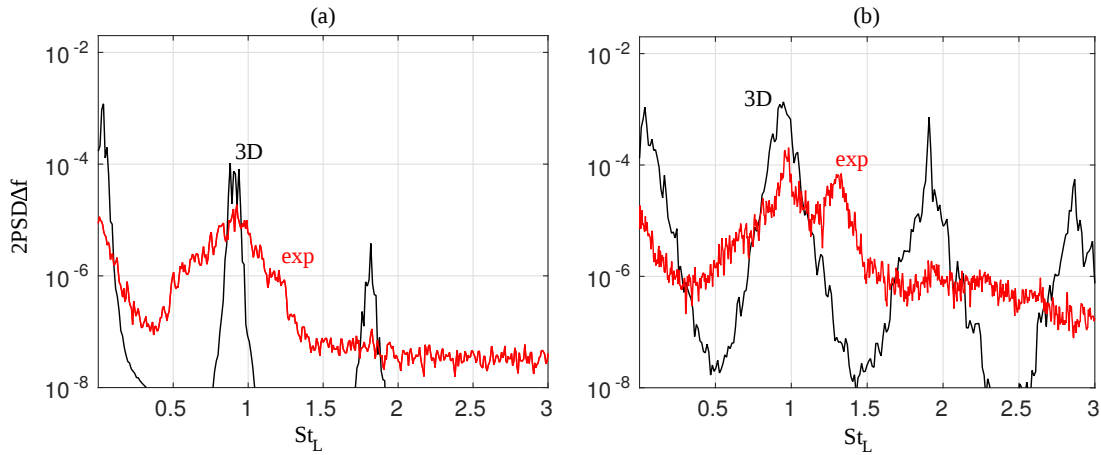


FIGURE 5.19: (Color in PDF) Comparison of power spectra from experimental (Paris-campaign, red) and numerical 3D simulation data (black) (a) at $Re_L \approx 5600$ and (b) at $Re_L \approx 7300$ for cases with $D/L = 0.4$ and $L/H = 2$.

the previous paragraph also in 2D) are able to reproduce this peak rather accurately as can be seen from figure 5.19(a). It can be observed that the spectrum of the 3D simulation is less broadband than the experimental spectrum. The first harmonic, that appears in the numerical spectrum is not observed in the experimental spectrum. In figure 5.19(b) an experimental spectrum of a $D/L = 0.4$ cavity at $Re_L = 7300$ is compared to a spectrum of a 3D numerical simulation with $D/L = 0.4$ at the same Reynolds number. The numerical 3D simulation spectrum shows a single peak at $St_{L,1} \approx 1$ and its harmonics, while the experimental spectrum shows two non-harmonic peaks. One possibility for the discrepancy is that the second peak will appear eventually at higher Reynolds numbers in numerical simulations. The proof of this speculation is currently addressed in numerical simulations at higher Reynolds numbers (higher values of L/Θ_0). Interestingly, the 2D numerical simulations, presented in the previous paragraph were able to reproduce the second non-harmonic frequency peak $St_{L,2}$, as shown in figure 5.18(b). Another possible explanation for the absence of this peak will be discussed in chapter 7 with respect to the hydrodynamic feedback mechanism.

5.4.5 Concluding remarks

Results from numerical simulations and wind tunnel experiments were compared. The inflow conditions (U_∞ and L/Θ_0) of both experiments and simulations are found at similar values. By design the inflow conditions of the 3D simulations are identical to the experiments. The velocity profiles imposed on the 2D simulations are found at slightly more unstable conditions.

The mean velocity flow fields of both experimental campaigns were compared to 2D and 3D numerical simulations. In all four cases, the flow field is composed of a center flow that separates the two facing cavities. The streamline plots of the intracavity mean velocity field show two main recirculation regions, though their relative sizes differ. The streamwise velocity profiles at different streamwise positions show good agreement in the channel center. In the recirculation region the velocity profiles of 2D/3D simulations and experiments differ as also suggest the different structures of the streamline plots. For the 2D simulations substantially higher intracavity velocities are observed. Both experimental campaigns and 3D simulations agree reasonably well.

Considering the rms fields of the streamwise fluctuating velocity component, both experimental campaigns show very similar results. The 2D numerical simulations compare better to the experiments when it comes to the onset of the development of the shear layer fluctuations, while the 3D numerical simulations compare better to the experiments, when the structure of the intracavity flow regions is considered.

The comparison of spectra from 3D simulations and 2D simulations resulted in good agreement for low Reynolds numbers. In both cases the typical shear layer peak ($St_{L,1} \approx 1$) was observed. The strong low frequency peak, observed in 3D simulations, is absent in 2D simulations, in agreement with the literature. At low Reynolds good agreement between experimental results and 3D simulations is observed. At higher Reynolds numbers the 2D simulations enter a so-called intermediate regime (*cf.* section 5.2), where a second non-harmonic frequency peak appears. While in the 3D simulations this peak was not observed, the experimental results from chapter 3 confirm the non-harmonic peak.

Part II

Analysis and Discussion

Towards the Understanding of Feedback & Coupling

Chapter 6

Linear stability analysis

6.1 Introduction and background

As shown in chapter 3, the shear layer spectra of single and double cavity flows exhibit non-harmonic frequency peaks that still lack a comprehensive explanation and which are not observed in shear layer spectra of backward facing steps or expanding channels. As stated in the introduction it can be safely reasoned that the existence of these peaks is due to some kind of feedback mechanism induced by the downstream cavity wall. In this chapter, the problem is approached from the theoretical point of view, by means of a spatio-temporal linear stability analysis, which is conditioned to be applicable in a finite domain by taking into account the upstream and downstream cavity edges.

As mentioned in the introduction of this work, in the compressible subsonic regime, the well known semi-empirical [Rossiter \(1964\)](#) formula, which was derived based on the assumption of acoustic feedback, describes the resonance frequencies observed in such flows reasonably well ([Delprat \(2006\)](#)). Interestingly, non-harmonic modes were found experimentally also in the incompressible case, as confirmed by [Lusseyran *et al.* \(2008\)](#); [Basley \(2012\)](#) and also by the experimental data presented in chapter 3. [Burroughs & Stinebring \(1994\)](#) confirmed the same shear layer modes for cavity flow in water, thus confirming their existence in an essentially incompressible fluid. [Yamouni *et al.* \(2013\)](#) recently tried to link the compressible to the incompressible dynamics by means of a global linear stability analysis. His results, however, do not compare well with experimental data in the incompressible limit by [Lusseyran *et al.* \(2008\)](#); [Basley \(2012\)](#).

In the same decade as [Rossiter \(1964\)](#) presented his semi-empirical formula, [Kulikowskii \(1966\)](#) derived a condition for a linear stability analysis in a finite domain (LSAFD) that takes into account an amplified downstream traveling perturbation wave k_+ , as well as an

evanescent upstream traveling perturbation wave k_- . The waves are reflected at the downstream and upstream boundaries, which are characterized by the reflection coefficients R_1 and R_2 , respectively. In order for the perturbation to be self-sustaining, the amplitude of a perturbation wave at a given point in space must, after successive reflections, coincide with the amplitude of the original perturbation. This notion yields a condition that selects certain modes from the spectrum of the linear stability analysis in an infinite domain. Doare & de Langre (2006) retrieved the same results as Kulikowskii, considering the complex Ginzburg-Landau amplitude equation in a finite domain together with a phase and energy closure principle first proposed by Gallaire & Chomaz (2003). Kulikowskii (2006) himself continued research on this subject, considering a two dimensional domain, where waves can propagate and be reflected.

Under the incompressible assumption, single and double cavity flows are commonly described in terms of two physically different flow phenomena: the recirculating flow inside the cavity and the shear layer flow above. The emitted frequencies observed experimentally are most likely the result of both the intracavitary recirculation region and the shear layer instabilities, as suggested by Bres & Colonius (2008). None of the previous studies, however, considers the possibility (and consequences) of including the reflection of instability waves. This will be the subject of the present chapter.

In a first analysis, a local temporal linear stability analysis is carried out for a set of double cavity flows at roughly constant inflow conditions and variable intercavity distance. This analysis seeks to explain the experimentally observed amplification of higher Strouhal numbers and the broadening of the frequency peaks as the two cavities are brought closer together. The relatively simple local linear stability analysis is, however, not able to capture the two non-harmonic frequency peaks commonly found in shear layer spectra of single and double cavity flows.

In order to gain further insight into the nature of these non-harmonic frequency peaks, the phenomenon is analyzed by means of a linear stability analysis in a finite domain. The wave reflection scenario, first proposed by Kulikowskii (1966), leads to a so called Kulikowskii Condition, which characterizes the finiteness of the system. In this chapter it will be applied within the time asymptotic theory of spatio-temporal linear stability analysis, described in detail in the textbook by Landau & Lifschitz (1985) for plasma physics. The work by the author and collaborators (Tuerke *et al.* (2015)) is the first time that solutions of the Kulikowskii Condition are combined with a spatio-temporal linear stability analysis of open cavity flow, thus obtaining a theoretical account for the non-harmonic mode coexistence, observed experimentally in open cavity flows. The results will be compared with six experiments of the $L/H = 2$ single open cavity geometry, published by Lusseyran *et al.* (2008) and Basley (2012), as well as to the experimental results of double cavity flow from chapter 3.

It must be mentioned that a stability analysis and experimental measurements do not strictly highlight the same state of the system. Linear stability analysis is concerned with the onset and the nature of the flow instabilities whereas the experiments deal with the final state of nonlinearly saturated dynamics. Linear stability analysis involves an ideal noiseless one-dimensional basic flow (often, as in the case of the present work, the mean flow is used), as opposed to the real conditions. By contrast experimental measurements only give access to partial imperfect information with uncertainties.

The chapter is organized as follows: The linear stability model is presented in section 6.2. A temporal linear stability analysis is conducted in section 6.3, followed by an overview of the conventional spatio-temporal linear stability analysis in section 6.2. The physical argument of the Kulikowskii Condition and the method developed to find solutions is presented in section 6.5. In sections 6.6 and 6.7 the results of the linear stability analysis in a finite domain are compared to experimental data from the literature and from chapter 3. In section 6.8 the consequences of a slowly varying base flow are addressed. Final conclusions are drawn in section 6.9.

6.2 The linear model

The flow is assumed to be steady and if not noted otherwise parallel and unidirectional. The streamwise development of the base flow is addressed in section 6.8. The analysis of flow stability considers the two-dimensional, inviscid and incompressible Euler equations given by

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p \quad (6.1)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (6.2)$$

where $\mathbf{u} = (u, v)^T$. As in Huerre and Monkewitz [Huerre & Monkewitz \(1985\)](#) the flow variables $\mathbf{q}(x, y, t) = (u, v, p)^T$ and coordinates x, y are non-dimensionalized using alternatively the maximum center line velocity U_∞ or the mean velocity $\tilde{U} = 0.5U_\infty$, depending on the case, and an effective momentum thickness Θ_{eff} , introduced in section 6.2.3. The flow is decomposed into a steady laminar base flow

$$\overline{\mathbf{Q}}(y) = (\overline{U}, \overline{V}, \overline{P})^T \quad (6.3)$$

upon which small amplitude perturbations

$$\mathbf{q}'(\mathbf{x}, t) = (u', v', p')^T \quad (6.4)$$

are permitted ($\mathbf{x} = (x, y)^T$). The customary parallel flow assumptions $\bar{V} = 0$, $\bar{P} = \text{const.}$ and $\partial_x \bar{Q}$ are applied. The decomposed flow field is subsequently linearized by neglecting higher order perturbation terms. Equations 6.1 and 6.2 together with equations 6.3 and 6.4 yield the linearized equations of motion for a 2D fluid flow:

$$(\partial_t + \bar{U}\partial_x) u' + \bar{U}_y v' = -\partial_x p' \quad (6.5)$$

$$(\partial_t + \bar{U}\partial_x) v' = -\partial_y p' \quad (6.6)$$

$$\partial_x u' + \partial_y v' = 0 \quad (6.7)$$

Normal mode solutions

$$\mathbf{q}'(\mathbf{x}, t) = \hat{\mathbf{q}}(y) e^{i(kx - \omega t)} \quad (6.8)$$

with $\hat{\mathbf{q}}(y) = (\hat{u}, \hat{v}, \hat{p})^T$ are then considered to model the spatial and temporal development of the perturbations. Note that k and ω are also non-dimensionalized using the average velocity $\tilde{U}$ and the effective momentum thickness Θ_{eff} , defined and discussed in detail in section 6.2.3. Throughout this work subscripts r and i stand for real and imaginary part, respectively. Considering equation 6.8 with equations 6.5 - 6.7 (equivalent to replacing $\partial_x \Rightarrow ik$ and $\partial_t \Rightarrow -i\omega$) yields

$$(-i\omega + ik\bar{U}) \hat{u} + \hat{v}\bar{U}_y + ik\hat{p} = 0 \quad (6.9)$$

$$(-i\omega + \bar{U}ik) \hat{v} + \partial_y \hat{p} = 0 \quad (6.10)$$

$$ik\hat{u} + \partial_y \hat{v} = 0 \quad (6.11)$$

Equations 6.9 - 6.11 can be rewritten in matrix formulation, which yields the dispersion relation of the spatio-temporal linear stability analysis

$$\Delta(\omega, k) = \left[\underbrace{\omega \begin{pmatrix} -i & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\mathbf{B}} + \underbrace{\begin{pmatrix} 0 & \bar{U}_y & 0 \\ 0 & 0 & \partial_y \\ 0 & \partial_y & 0 \end{pmatrix}}_{\mathbf{C}} + k \underbrace{\begin{pmatrix} i\bar{U} & 0 & i \\ 0 & i\bar{U} & 0 \\ i & 0 & 0 \end{pmatrix}}_{\mathbf{D}} \right] \underbrace{\begin{pmatrix} \hat{u} \\ \hat{v} \\ \hat{p} \end{pmatrix}}_{\hat{\mathbf{q}}} = 0 \quad (6.12)$$

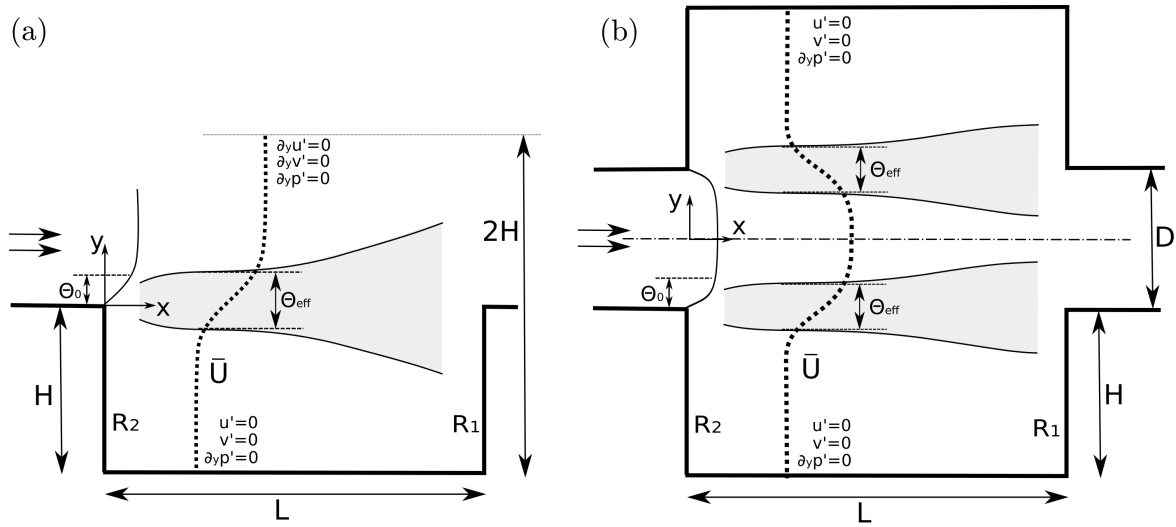


FIGURE 6.1: Schematic view of (a) single cavity computational domain and (b) double cavity computational domain, each with base flow $\bar{U}$, reflection coefficients R_1 and R_2 , boundary conditions, inflow momentum thickness Θ_0 and effective momentum thickness Θ_{eff} used in the linear stability analysis and defined in section 6.6. Flow from left to right as indicated by the arrows. The gray shaded areas indicate the shear layer regions.

This is a linear system that can be expressed as a general eigenvalue problem (EVP), either for a complex frequency $\omega = \omega_r + i\omega_i$ and a real wave number k (temporal analysis), which yields

$$\omega \mathbf{B} \hat{\mathbf{q}} = -\mathbf{A}_2 \hat{\mathbf{q}}, \quad (6.13)$$

where $\mathbf{A}_2 = \mathbf{C} + k \cdot \mathbf{D}$, or for a complex wave number $k = k_r + ik_i$ and a real frequency ω (spatial analysis or signaling problem), which results in

$$k \mathbf{D} \hat{\mathbf{q}} = -\mathbf{A}_1 \hat{\mathbf{q}} \quad (6.14)$$

For a given base flow $U = f(y)$, both eigenvalue formulations can be solved in *e.g.* Matlab, using the functions $\text{eig}(\mathbf{B}, -\mathbf{A}_2)$ or $\text{eig}(\mathbf{D}, -\mathbf{A}_1)$, respectively. The numerical code for the linear stability analysis was written and adapted to the present problem based on a code published by Hoepffner (2004). It is documented in Appendix A.

6.2.1 The computational domains

The linear stability problem of single and double cavity flows are considered in the computational domains, depicted in figures 6.1(a) and 6.1(b), respectively. To start with, in section 6.3, a temporal analysis is carried out in the infinite domain, *i.e.* not taking

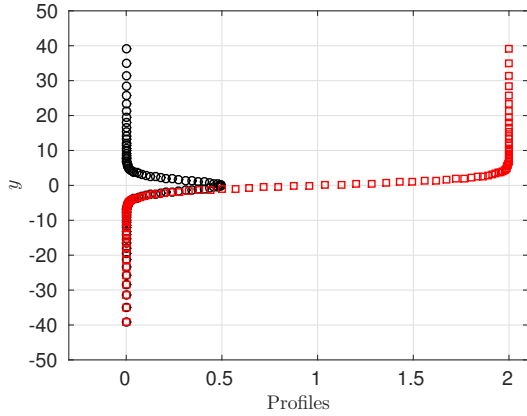


FIGURE 6.2: Hyperbolic tangent velocity base profile from equation 6.16 (red squares) and its shear strength $\partial_y \bar{U}$ (black circles) used for the analysis of the single cavity.

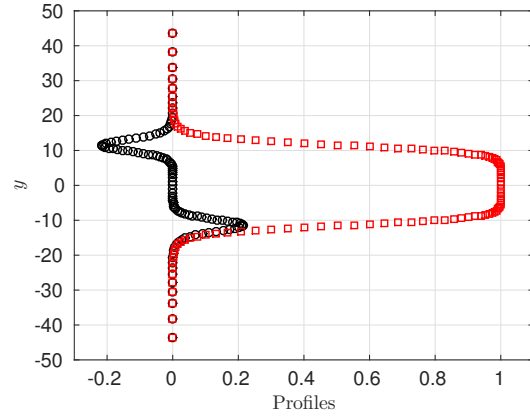


FIGURE 6.3: Hyperbolic sine velocity base profile from equation 6.18 (red squares) and its shear strength $\partial_y \bar{U}$ (black circles) used for the analysis of the double cavity.

into account the downstream cavity edge. Thereafter, in section 6.5, the consequences of a finite computational domain, taking into account the downstream cavity edge, are elucidated.

For the single cavity case the free stream boundary conditions ($y = H$) of the EVP are of Neumann type for all three variables. In the cavity floor ($y = -H$) the boundary conditions for the velocities are of Dirichlet type due to the no-slip condition at the solid wall. The boundary condition for the pressure is of Neumann type, as follows from the momentum equation. For the double cavity all velocity boundary conditions at both ends ($y = \pm H$) are of Dirichlet type, while the boundary condition for the pressure is still of Neumann type.

6.2.2 The base flows

In the case of the single cavity, following [Huerre & Monkewitz \(1985\)](#), the hyperbolic tangent ($\tanh$) profile is used to model the one dimensional base state. In its dimensional form it reads

$$\tilde{U}(y) = \tilde{U} + \frac{\Delta U}{2} \tanh\left(\frac{2y^*}{\delta_\omega}\right) \quad (6.15)$$

where the velocity difference is $\Delta U = U_{max} - U_{min} = 2\tilde{U} = U_\infty$ and y^* is the dimensional vertical coordinate. The vorticity thickness δ_ω is commonly approximated by $\delta_\omega = 4\Theta_{eff}$ ([Huerre & Monkewitz \(1985\)](#)), where Θ_{eff} is the effective momentum thickness, discussed in section 6.2.3. Normalized with the average velocity $\tilde{U}$ and Θ_{eff} the velocity profile reads

$$\bar{U}(y) = 1 + \tanh\left(\frac{1}{2}y\right) \quad (6.16)$$

Alternatively, normalized with the free stream velocity U_∞ , we get

$$\bar{U}(y) = \frac{1}{2} + \frac{1}{2} \tanh\left(\frac{1}{2}y\right) \quad (6.17)$$

Both forms are used in the present work. The profiles are fitted to experimental data from chapter 3 and from [Lusseyran *et al.* \(2008\)](#); [Basley \(2012\)](#), respectively, by searching for the best fit in the least square sense, varying the vorticity thickness $\delta_\omega \approx 4\Theta_0$, as explained in section 6.2.3.

For the double cavity a hyperbolic sine (sinh) profile is used to approximate the one dimensional base state. It is borrowed from [Huerre & Rossi \(1998\)](#), who used it to model the wake flow behind a cylinder. Non-dimensionalized with the maximum center line velocity U_∞ and the effective momentum thickness Θ_{eff} (*cf.* section 6.2.3), it reads

$$\bar{U}(y) = 1 + \left(\sinh\left(y (\sinh(a))^{-1}\right)^{2b} \right)^{-1} \quad (6.18)$$

where $a \in \mathbb{R}$ and $b \in \mathbb{N}$ are fitting parameter, which are fitted to appropriately normalized experimental data from chapter 3, using a non-linear fitting algorithm described in detail in Appendix B. Examples of both the tanh and the sinh base profiles and their first derivatives are depicted in figures 6.2 and 6.3, respectively.

6.2.3 The effective momentum thickness

The momentum thickness Θ is the only variable parameter in the analysis. As noted by [Ho & Huerre \(1984\)](#) (and confirmed in the present work), the analysis is quite sensitive to Θ . It is therefore crucial to choose the momentum thickness with care. The base profile is assumed to follow the shape of a hyperbolic tangent function in the case of the single cavity and the hyperbolic sine function in the case of the double cavity. Hence, the incoming Blasius boundary layer needs a certain time (and space) to relax, in order to fit the hyperbolic tangent/sine assumption.

The idea is to use the momentum thickness of the streamwise location where the analytic profile best fits the experimental data. The procedure is exemplified in this section for the case of a hyperbolic tangent profile, but can be easily adapted to the hyperbolic sine

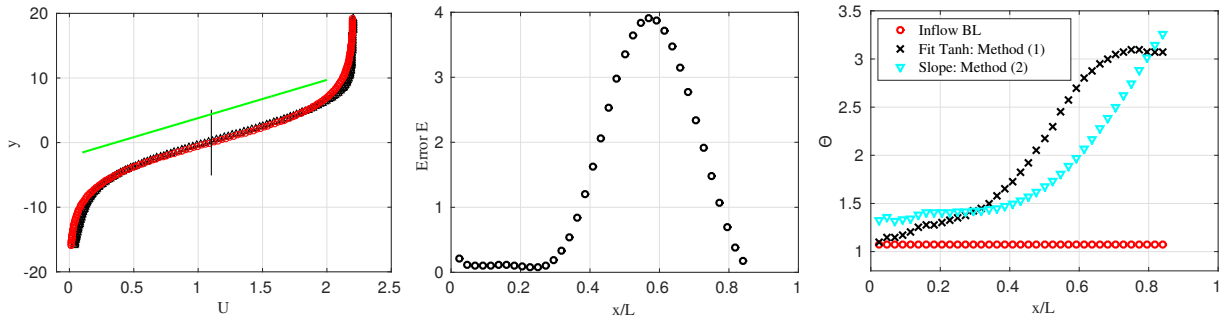


FIGURE 6.4: (a) Fit of tanh velocity base profile (red) to experimental data (black). Green straight line indicates fit to maximum slope of base profile. (b) Relative error E between fit and experimental data in (a). (c) Momentum thickness over cavity length for three methods discussed in the text.

profile. The momentum thickness Θ along the cavity length is computed according to two different methods:

1. As shown in figure 6.4(a), the analytic tanh profile (red) is fitted to the experimental profile (black) along the cavity domain by minimizing the error

$$E = \sum (U_{base} - U_{exp})^2$$

between the experimental and the analytical tanh profile. The error as a function of x/L is shown in figure 6.4(b). The momentum thickness is obtained from the best fit at each streamwise location (Tam & Block (1987)).

2. The momentum thickness is computed from the vorticity thickness, according to

$$\Theta = \frac{1}{4} \delta_\omega = \frac{1}{4} \frac{\Delta U}{\left(\frac{\partial U}{\partial y} \right)_{\max}},$$

where the maximum slope $(\partial_y U)_{\max}$ of the velocity profile is obtained by fitting the straight solid (green) line to the experimental velocity profile at $y = 0$, as shown in figure 6.4(a).

Both methods are compared in figure 6.4(c) together with the momentum thickness of the incoming boundary layer Θ_0 , which is also sometimes used as length scale. It becomes clear, that simply using Θ_0 underestimates the true value of Θ at the streamwise location, where the velocity profile corresponds to the analytic tanh profile. Method (2) yields a well pronounced plateau where $\frac{\partial \Theta}{\partial x} \approx 0$. Hammond & Redekopp (1997) propose to choose Θ in the vicinity of this plateau, where the momentum thickness varies only marginally, *i.e.* $\frac{\partial \Theta}{\partial x} \approx 0$. Since the plateau in our case is found over a range of streamwise positions ($0.1 < x/L < 0.4$), method (1) is used and Θ is taken at the streamwise position x_{min} ,

for which the least-square error $E = \sum (U_{base} - U_{exp})^2$, shown in figure 6.4(b), exhibits a minimum. In all analyzed cases x_{min} was found in the range of $0.20 L \lesssim x_{min} \lesssim 0.25 L$, which is in good agreement with [Ho & Huerre \(1984\)](#), who suggest to use the value of Θ one instability wave length downstream of the leading edge and before any significant nonlinear interactions occur. Thus, instead of the inflow momentum thickness Θ_0 , an effective momentum thickness $\Theta_{eff} = \Theta(x_{min})$ is used as length scale.

6.2.4 The dispersion relation

The eigenvalue problem (EVP) together with the base flow assumptions yield the dispersion relation of the system, given by the complex equation

$$\Delta(\omega_r, \omega_i, k_r, k_i) = 0 \quad (6.19)$$

Its roots $\omega(k)$ provide the eigen-frequencies and growth rates. In general, both the wave number k and the frequency ω are considered complex. For $k_i = 0$ the problem is reduced to the temporal stability problem discussed in section 6.3. The dispersion relation is solved numerically using a Matlab code, given in Appendix A, which is based on a Chebyshev collocation method with $N = 100 - 120$ collocation points in the wall normal direction y , depending on the case and the base profile. Differentiation is carried out using a second order non-equidistant finite difference method. According to the Rayleigh criterion, the inflexion point of the base profile is a necessary (yet not sufficient) condition for the base-flow to be unstable to small perturbations. Whether instabilities occur and if so, whether they are amplified or evanescent in space and time has to be determined by criteria described in [Landau & Lifschitz \(1985\)](#). It builds the basis for the linear stability analysis in a finite domain (LSAFD) and will be shortly outlined in section 6.4. First, however, in section 6.3 a classical temporal linear stability analysis in an semi-infinite domain will be considered.

6.3 Temporal linear stability analysis

In this section, the results of a temporal local linear stability analysis of single and double cavity flow in an infinite domain are presented. The effect and consequences of the downstream cavity edges are not taken into account in this section. The computational domains of the single and double cavity are depicted in figure 6.1(a) and 6.1(b), respectively. Eight cases, summarized in table 6.1, at variable intercavity distances and approximately the same inflow conditions ($U_\infty \approx 2.7$), are considered.

The goal is to elucidate the coupling of the two cavities in terms of the linear stability properties of the two shear layers. As found in chapter 3, when the two cavities are

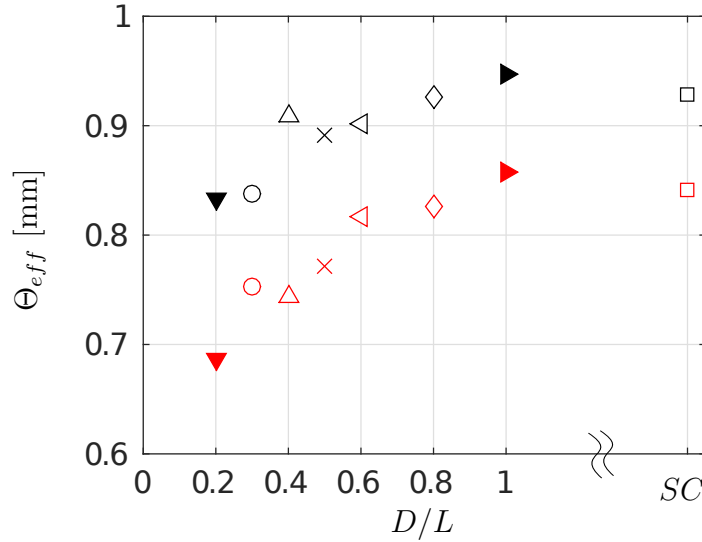


FIGURE 6.5: Effective momentum thickness Θ_{eff} over distance D for all cases in table 6.1 computed from analytic profiles (black) and from experimental profiles (red). Symbols as in table 6.1.

TABLE 6.1: Summary parameters of linear stability analysis

Cases	D/L	a	Re_L	N	Symbol
DC10	0.2	2.28	2	9 200	▼
DC15	0.3	2.68	3	9 300	○
DC20	0.4	3.04	4	8 800	△
DC25	0.5	3.23	5	8 900	×
DC30	0.6	3.43	6	9 000	◁
DC40	0.8	3.74	8	9 300	◇
DC50	1.0	3.98	10	9 000	►
SC50	1.0	—	—	9 100	□

brought closer together, the shear layer spectra change. By means of a relatively simple temporal local linear stability analysis, the effect of the intercavitary distance D/L on the stability characteristics is addressed.

The set of equation and assumptions from section 6.2 are applied. The linear system 6.12 is solved for a real wave number $k = k_r$, resulting generally in a complex frequency $\omega = \omega_r + i\omega_i$. This is commonly referred to as a temporal analysis in the literature. The one dimensional base states of the velocity field $\bar{U} = f(y)$, described in section 6.2.2, are imposed to solve the dispersion relation. They are non-dimensionalized with the freestream velocity U_∞ and the effective momentum thickness Θ_{eff} , which is obtained from the experimental velocity profiles as described in section 6.2.3 and depicted in figure 6.5 for the eight cases listed in table 6.1. As can be seen from figure 6.5, Θ_{eff} increases

monotonically with D/L . For the double cavity cases the sinh velocity profile (equation 6.18) is fitted to the experimental data and then used as base profile in the stability calculation. For the single cavity case the tanh profile (6.17) is fitted to the experimental profile. The fitting parameters N and a of the sinh profile (6.18) are summarized in table 6.1 for the eight cases under study. The velocity profiles together with the fitted analytic profiles at $x = x_{min}$ are depicted in figure 3.10 in chapter 3. The momentum thickness of the fitted analytic velocity profiles is compared to the experimental values in figure 6.5 for all analyzed cases. While the trend is well captured by the fitted profiles, the absolute values lie slightly above the experimental values of Θ_{eff} .

The EVP of the temporal linear stability analysis, given in equation 6.13, together with the base flow assumptions yield the dispersion relation of the system, given by the complex equation

$$\Delta(\omega_r, \omega_i, k_r) = 0 \quad (6.20)$$

For a real wavenumber k_r , the roots $\omega(k)$ of this equation provide the eigen-frequencies and growth rates, respectively. The dispersion relation is solved numerically as described in section 6.2.4.

The results are expressed in terms of the Strouhal number St_L , which is obtained by

$$St_L = \frac{fL}{U_\infty} = \omega_r \frac{U_\infty}{\Theta_{eff}} \frac{1}{2\pi} \frac{L}{U_\infty} = \omega_r \frac{L}{2\pi\Theta_{eff}} \quad (6.21)$$

where ω_r is the non-dimensional frequency resulting from the linear stability analysis. The Strouhal number is independent of the inflow velocity and only depends on the non-dimensional cavity length. The light gray shaded areas in figure 3.16 indicate the cases, for which the linear stability analysis is carried out.

6.3.1 Results and Discussion

As found by [Huerre & Rossi \(1998\)](#), the temporal linear stability analysis of the sinh profile results in two unstable branches: a symmetric (varicose) and a asymmetric (sinuous) branch. Figure 6.6 exemplifies both eigenmodes for case DC10 ($D/L = 0.2$). The tanh profile results in a single (sinuous) branch, which is in accordance with [Michalke \(1964\)](#).

Figure 6.7(a) shows the temporal amplification rate ω_i over the Strouhal number St_L , exemplary for two cases: a close distance case at $D/L = 0.2$ and a far distance case at

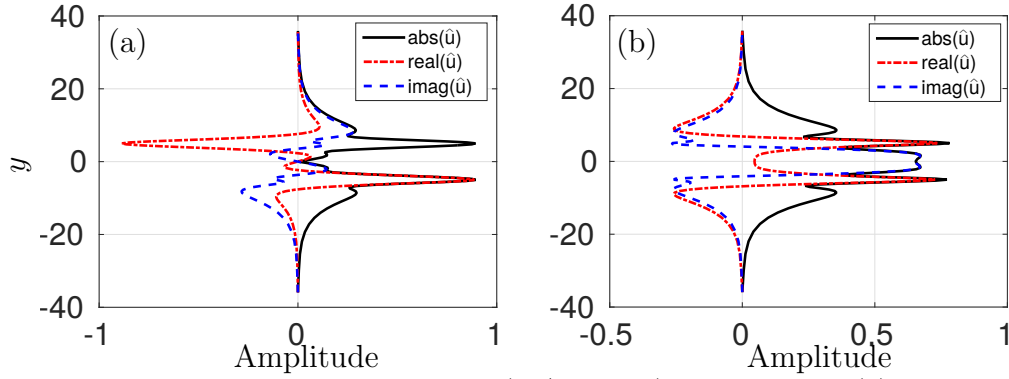


FIGURE 6.6: Eigenmodes of case DC10 ($D/L = 0.2$) at $\omega_{i,max}$ for (a) sinuous mode (more amplified) (b) varicose mode (less amplified).

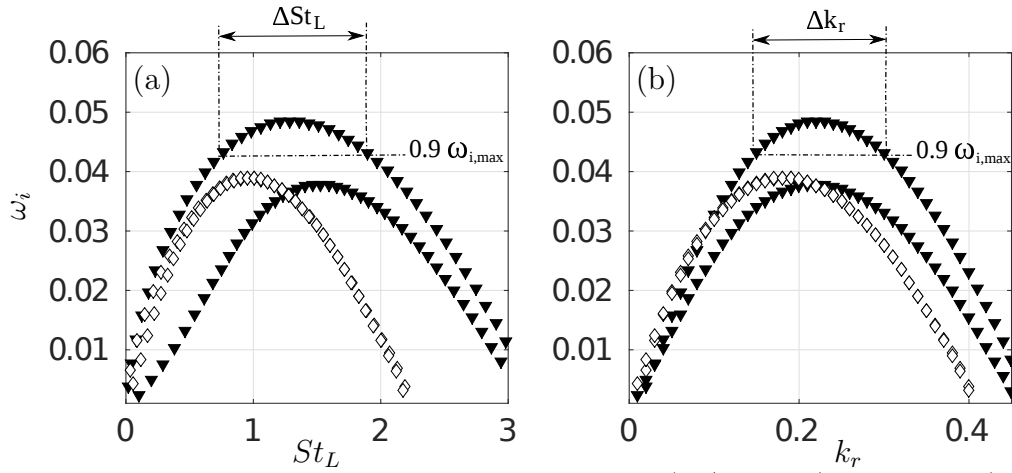


FIGURE 6.7: Linear stability curves for cases DC10 ($D/L = 0.2$) and DC40 ($D/L = 0.8$). Symbols as in table 6.1. For the definition of the Strouhal number range ΔSt_L and the wave number range Δk_r see text.

$D/L = 0.6$. At $D/L = 0.2$ the two branches are well separated. The sinuous branch yields higher values of ω_i and hence is more unstable. The difference between the branches decreases with distance, such that for distances $D/L \geq 0.4$, the difference between the two branches becomes negligible and both modes exhibit a similar temporal amplification rate ($\Delta\omega_i < 3\%$), *i.e.* both modes are equally prone to amplification. These theoretical predictions are in agreement with the experimental results from the shear layer correlations in chapter 3 (figure 3.4), where it was found, that for small intercavity distances the flow is asymmetric (sinuous type), while for larger intercavity distances ($D/L \geq 0.4$) the symmetry is lost, most likely due to the fact, that both sinuous and varicose modes are equally susceptible to amplification.

The Strouhal number St_L , at which the growth rate ω_i is maximal, is defined as

$$St_{L,max} = St_L(\omega_{i,max}) \quad (6.22)$$

and in the same manner the wave number k_r , at which the growth rate ω_i is maximal, is defined as

$$k_{r,max} = k_r(\omega_{i,max}) \quad (6.23)$$

In figure 6.7, the temporal growth rate is plotted as a function of the Strouhal number and the wave number. When the two branches are distinguishable, $St_{L,max}$ of the varicose branch is found at higher Strouhal numbers than $St_{L,max}$ of the sinuous mode. The wave number of the maximal growth rate $k_{r,max}$, however, is found to be equal for the two branches, as can be seen from figure 6.7(b).

Figure 6.8 summarizes the results from the linear stability analysis for all eight cases in table 6.1 and compares them to the experimental data from the Paris-campaign discussed in detail in section 3.3.3. The Strouhal number of the maximum growth rate $St_{L,max}$, the Strouhal number range ΔSt_L , the wave number range Δk_r and the maximum growth rate $\omega_{i,max}$ are plotted over the non-dimensional intercavity distance D/L .

In figure 6.8(a), at $D/L = 0.2$, the value of $St_{L,max} = 1.37$, obtained by the linear stability analysis, is slightly below the experimental value of $St_{L,max} \approx 1.5$. At the distance $D/L = 0.3$, the linear stability analysis predicts $St_{L,max} = 1.26$, while experimentally we obtain a somewhat lower value of $St_{L,max} \approx 1.12$. Experimentally, for $D/L \geq 0.4$, the frequency peaks become more narrow band and start to be distinguishable. The experimental data for $D/L \geq 0.4$ shows the amplification of two incommensurate peaks, $St_{L,1}$ (red) and $St_{L,2}$ (blue). For cases with $0.3 \leq D/L \leq 0.6$, the first mode $St_{L,1}$ lies below the value obtained from the linear stability analysis, while the second mode $St_{L,2}$ is found at a higher Strouhal number. The far distance cases $0.8 \leq D/L \leq 1.0$ coincide well with the first experimental mode $St_{L,1}$, found close to unity. For the single cavity (SC), the predicted value $St_{L,max}$ of the linear stability analysis increases slightly to $St_{L,max} = 1.1$ and lies exactly in between the values obtained experimentally ($St_{L,1} = 0.96$ and $St_{L,2} = 1.24$, respectively). This is in accordance with [Tuerke et al. \(2015\)](#), who found that, in order to predict both incommensurate peaks, the finiteness of the system has to be taken into account (*cf.* section 6.7).

In figure 6.8(b)-(c), the Strouhal number range ΔSt_L and the wave number range Δk_r are depicted. They are respectively defined as

$$\Delta St_L = \{St_L \in \mathbb{R} \mid 0.9 \omega_{i,max} \leq \omega_i(St_L) \leq \omega_{i,max}\} \quad (6.24)$$

and

$$\Delta k_r = \{k_r \in \mathbb{R} \mid 0.9 \omega_{i,max} \leq \omega_i(k_r) \leq \omega_{i,max}\} \quad (6.25)$$

Figure 6.7 shows both definitions applied to $D/L = 0.2$ case. The experimental values of the Strouhal number range and the wave number range were obtained, considering an intensity cut-off threshold, according to

$$\Delta St_L = \{St_L \in \mathbb{R} \mid P(St_L) > -10\text{dB}\} \quad (6.26)$$

and

$$\Delta k_r = \{k_r \in \mathbb{R} \mid P(St_L) > -10\text{dB}\} \quad (6.27)$$

where $P = 2PSD\Delta f$ is the normalized power spectral density. As found in section 3.3, for close intercavity distances, $0.2 \leq D/L \leq 0.3$, the frequency peaks broaden, *i.e.* a greater range of frequencies is amplified. This is reflected in the increased value of ΔSt_L . The results from the linear stability analysis compare well to the experimental data. For intermediate distances $0.4 \leq D/L \leq 0.6$ both the linear stability prediction and the experimental data show a drop in ΔSt_L , however, the amplified frequency range of the experimental data drops stronger than the linear stability analysis predicts. The arbitrarily chosen frequency range selection criterion ($0.9 \omega_{i,max}$) could be adapted to increase the accuracy. The rebound of ΔSt_L for large intercavity distances $D/L \geq 0.8$ and the single cavity, found experimentally, is also predicted by the linear stability analysis. The wave number range Δk_r , depicted in figure 6.8(c), shows a nearly linear and monotonic decrease with D/L . This suggests, that a closer intercavity distance is favorable to a greater set of wave numbers being amplified. In figure 6.8(d), the maximum temporal growth rate $\omega_{i,max}$, obtained from the linear stability analysis, also decreases with D/L , however, only up to a distance of $D/L = 0.8$. For $D/L > 0.8$ it increases again and finally settles for the single cavity at almost the same value as at close intercavity distances.

In order to compare the temporal growth rates of the linear stability analysis to the experimental results from chapter 3, the experimental temporal growth rates are calculated for all cases in table 6.1. In figure 6.9 the rms (root-mean-square) peak values $A(x)$ of the streamwise fluctuating velocity component are plotted over the cavity length in a range

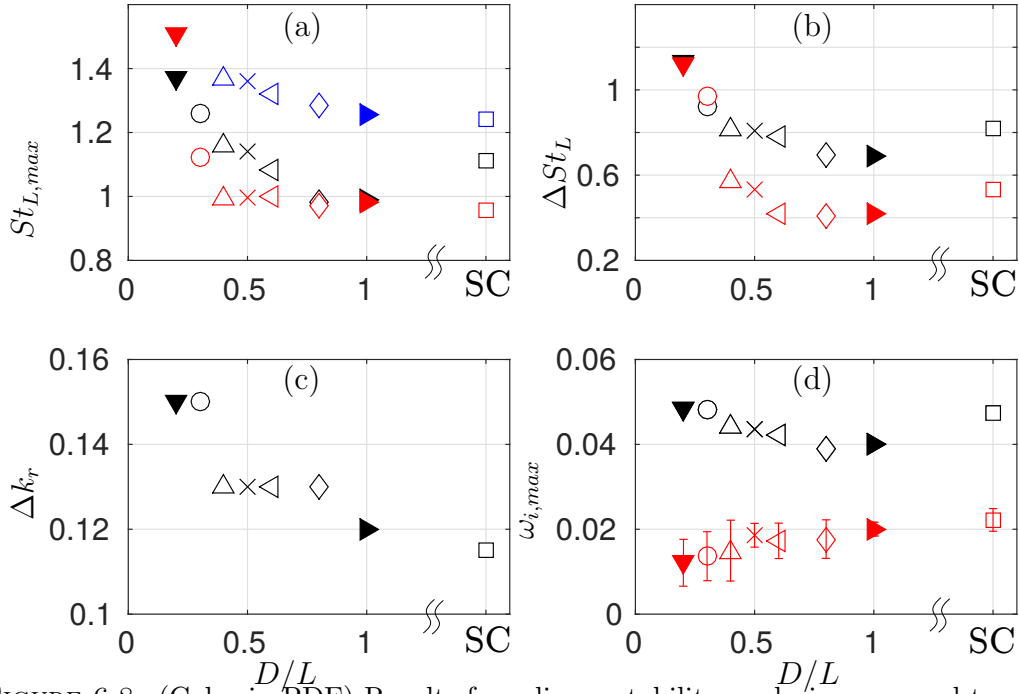


FIGURE 6.8: (Color in PDF) Results from linear stability analysis compared to experimental results. Data given as a function of the non-dimensional intercavity distance D/L . (a) Strouhal number of the first experimental mode $St_{L,1}$ (red), the second experimental mode $St_{L,2}$ (blue) and linear stability analysis (black). (b) Strouhal number range ΔSt_L , (c) wave number range Δk_r and (d) maximum growth rate $\omega_{i,max}$ from experimental results (red) and from linear stability analysis (black). Error bars show 95% confidence interval of fits in figure 6.9. Symbols in all figures as in table 6.1.

of $0.2 \lesssim x/L \lesssim 0.6$, exemplary for three cases. The maximum value at each streamwise position is normalized with $A(x_0)$, where $x_0 \approx 0.2L$. For $x < x_0$ the velocity profile is stable and no significant shear layer fluctuations are observed. For $x/L \gtrsim 0.6$ nonlinearities saturate the fluctuations. In order to obtain the experimental temporal growth rate, an exponential function of the form

$$A(x) = \gamma e^{\kappa x} \quad (6.28)$$

is fitted to the experimental data by minimizing the $\mathcal{L}_2$ -norm difference, varying the fitting parameters γ and κ . After normalization with the maximum center line velocity U_∞ and the effective momentum thickness Θ_{eff} , we obtain the temporal growth rate according to

$$\omega_{i,exp} = \kappa c_g \frac{\Theta_{\text{eff}}}{U_\infty} = \kappa \frac{\Theta_{\text{eff}}}{2} \quad (6.29)$$

where $c_g = \frac{\partial \omega}{\partial k}$ is the group velocity of the perturbing wave package, which was found to

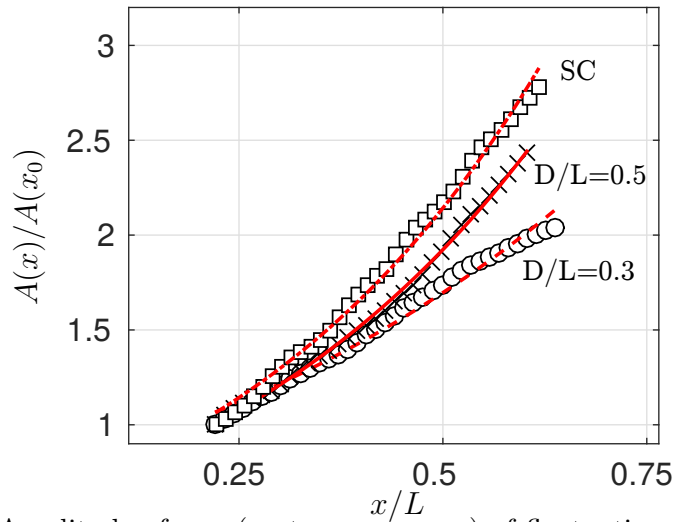


FIGURE 6.9: Amplitude of rms (root-mean-square) of fluctuating streamwise velocity component over cavity length exemplary for cases $D/L = 0.3$ ($\circ$ and $---$), $D/L = 0.5$ ($\times$ and $---$) and single cavity SC ($\square$ and $---$). Symbols show experimental data, curves indicate exponential fits.

be $c_g \approx 0.5 U_\infty$, independent of the intercavity distance. Figure 6.9 exemplifies the fitting procedure for three cases. Interestingly, at close distances ($D/L \leq 0.4$) the amplitude of the streamwise velocity fluctuations increases nearly linearly along the cavity length. For larger distances ($D/L \geq 0.5$) the typical exponential growth is observed and equation 6.28 fits well the experimental data. The proximity of the facing cavity seems to inhibit the exponential growth of fluctuations. In figure 6.8(d) the experimental growth rates $\omega_{i,exp}(x)$, calculated according to equation 6.29 for all cases in table 6.1 are compared to the maximum temporal growth rates $\omega_{i,max}$, obtained from the linear stability analysis. As expected from an inviscid analysis, the experimental growth rates are found at somewhat lower values than the theoretical predictions but within the same order of magnitude. For close intercavity distances the trend of the predictions does not resemble the experimental data. This is most likely due to the linear course of $A(x)$ for $D/L \leq 0.4$, which reduces $\omega_{i,exp}$. For distances $D/L \gtrsim 0.5$, independent of the intercavity distance, similar growth rates of approximately $\omega_{i,exp} \approx 0.02$ are obtained.

The temporal linear stability analysis only predicts a single frequency peak. In section 6.5, a mechanism of quantization of the frequency spectrum is elucidated, by taking into account the finite extent of the system. As will be seen, this so called linear stability analysis in a finite domain (LSAFD), allows to explain the existence of non-harmonic frequency peaks. LSAFD is applied to the single cavity in 6.6 and to the double cavity in section 6.7.

6.3.2 Concluding remarks: temporal analysis

A temporal local linear stability analysis was carried out, based on the experimental sinh velocity profiles for the double cavity and the tanh profile for the single cavity. By means of this simple model we were able to explain why the peaks for close intercavity distances broaden (stronger side bands), and why for decreasing intercavity distance the frequency peaks move to higher Strouhal numbers. The most amplified Strouhal number shows the same trend as the experimental data, when D/L is varied. However, without taking into account the finiteness of the system, only a single peak can be predicted. The range of amplified frequencies, and the collapse of the varicose and the sinuous branches also coincide with experimental results from chapter 3.

6.4 The spatio-temporal analysis

In this section a spatio-temporal linear stability analysis is considered. It lays the foundation for a linear stability analysis in a finite domain, which takes into account the finiteness of the streamwise variable (in this work the cavity length L). First, the theoretical background required for the spatio-temporal analysis is given, before the so called Kulikowskii Condition is presented, which characterizes the finite system. The spatio-temporal linear stability analysis together with the Kulikowskii condition forms the theoretical argument that leads to a linear stability analysis in a finite domain (LSAFD). The novel theory is then applied to single as well as double cavity flow and discussed with respect to other methods and results found in the literature.

6.4.1 Amplification and non-transparency

In order to present the argument that leads to the linear stability analysis in a finite domain (LSAFD), a brief outline of the steps needed to determine whether a medium, governed by its dispersion relation, amplifies or damps an infinitesimal perturbation, is given. This theoretical consideration for the infinite domain is on one hand used to distinguish between upstream and downstream traveling waves (k_- and k_+ , respectively), and on the other hand, to study the asymptotic behavior of these waves. In the present work, and in this chapter in particular, plane waves are considered. As explained in [Lighthill \(1990\)](#), plane waves are waves with a single frequency that propagate in space. The parallel planes are wave fronts which are theoretically infinite in extend. In reality a domain is never infinite and hence a plane wave has to be regarded as locally plane, which is a good approximation. Note, since a plane wave is rather difficult to visualize, they are commonly visualized in its sine-form.

Normal mode properties, sustained by a given base state, are encapsulated in its dispersion relation (6.19). As mentioned before, in general, both the wave number k and the frequency ω are considered complex. Solutions of the dispersion relation for given complex wave numbers are called temporal branches, whereas solutions of the dispersion relation for given complex frequencies are called spatial branches. The response of the system (the perturbation wave) to a source

$$g(x, t) = C \cdot \mathcal{H}(t) \delta(x) e^{-i\omega_0 t} \quad (6.30)$$

of a given frequency ω_0 is given by

$$\Psi(x, t) = \frac{C}{2\pi} \int_{L_\omega} \underbrace{\int_{F_k} \frac{e^{ikx}}{\Delta(\omega, k)} dk}_{\phi(\omega, x)} \frac{e^{-i\omega t}}{i(\omega - \omega_0)} d\omega \quad (6.31)$$

where $\Delta(\omega, k)$ denotes the dispersion relation, $\mathcal{H}(t)$ the Heaviside step function in time, $\delta(x)$ the Dirac Delta in space and C a constant. The integration contours in the complex ω -plane and the complex k -plane, respectively, are defined according to $L_\omega = (-\infty + i\omega_i, +\infty + i\omega_i)$ where $\omega_i > 0$ and $F_k = (-\infty, +\infty)$. The integration contour L_ω is located above all the poles in the complex ω -plane, as required by the “causality principle”, which states that “effect cannot precede cause”, meaning, the perturbation occurs only after the source has been turned on at $t = 0$.

In the next paragraph we discuss the asymptotic behavior of the response of the system perturbed with upstream and downstream traveling waves. To this end we consider the asymptotic behavior of expression 6.31.

6.4.2 Asymptotic behavior of the response of the system

In space, a perturbation can travel to $x < 0$ and to $x > 0$, whereas time is unidirectional and thus only $t > 0$ makes sense. Therefore, the sign of k cannot serve as an indicator of stability or instability, as in the temporal case. The physical nature of the behavior of the waves is embedded in its dispersion relation, which depends on the medium. In a transparent medium, the wave only possesses real parts of the frequency and the wave number. Thus, it passes without being affected by the medium. On the other hand, if the medium affects the wave it acquires an imaginary part, which accordingly yields an amplified or an evanescent wave. If the perturbation tends to zero as $x \rightarrow \pm\infty$ the wave is evanescent, while if the perturbation increases as $x \rightarrow \pm\infty$ the wave is amplified.

Following [Landau & Lifschitz \(1985\)](#), we now look for the asymptotic form of the response of the system $\Psi(x, t)$, far from the source, *i.e.* for $|x| \rightarrow \infty$, and long after the time origin ($t \rightarrow \infty$). The asymptotic form $t \rightarrow \infty$ has to be taken before $|x| \rightarrow \infty$, as a perturbation

cannot propagate to infinity in a finite amount of time. Applying Briggs (1964) method, we consider equation 6.31 and move L_ω downwards (for a given $\omega_r = \omega_0$) in order to get the asymptotic expression in time. The highest located singularity in the complex ω -plane is ω_0 . Once this pole is reached, the asymptotic form of equation 6.31 reads

$$\Psi(x, t) \propto e^{-i\omega_0 t} \phi(\omega_0, x) \quad (6.32)$$

where only the second term $\phi(\omega_0, x)$ is of interest, since the first term simply oscillates. The poles of $\phi(\omega_0, x)$ are the zeros of the dispersion relation $\Delta(\omega_0, k)$ of the system. By definition, let $k_+(\omega)$ denote the poles located in the positive k -half-plane and $k_-(\omega)$ the poles located in the negative k -half-plane for $\omega_i \rightarrow \infty$. When the L_ω contour is lowered, the poles in the complex k -plane move. They might stay in their original half-plane or might cross into the other half-plane. The poles of interest are the ones that are closest to the real k axis if they have not crossed, or the ones that are farthest from the real axis in the case they have crossed the k_r axis. With these values of k_+ and k_- the asymptotic behavior of the response of the system can be evaluated, considering

$$\begin{aligned} \Psi(x, t) &\propto e^{i[k_+(\omega_0)x - \omega_0 t]} \\ &= e^{i[k_{r+}(\omega_0)x - \omega_{r0} t]} e^{-k_{i+}(\omega_0)x + \omega_{0i} t} \end{aligned} \quad (6.33)$$

for $x > 0$ and

$$\begin{aligned} \Psi(x, t) &\propto e^{i[k_-(\omega_0)x - \omega_0 t]} \\ &= e^{i[k_{r-}(\omega_0)x - \omega_{r0} t]} e^{-k_{i-}(\omega_0)x + \omega_{0i} t} \end{aligned} \quad (6.34)$$

for $x < 0$. In the case studied in the present work, a k_+ branch crosses the k_r axis, while all k_- branches stay in the lower half-plane. Thus, $k_{i+} < 0$ and $k_{i-} < 0$, which means that downstream traveling waves are amplified (equation (6.33)), while upstream traveling waves are evanescent (equation (6.34)). This is commonly known as convective instability.

Once the L_ω contour reaches the most unstable pole, a so-called branch point (BP) is formed. According to the textbook by Schmid & Henningson (2000), it can be shown that when a Branch Point occurs in the complex ω -plane, a so called Pinch Point (PP) occurs simultaneously in the complex k -plane. Invoking the arguments introduced above, k_+ and k_- waves can be distinguished, by considering their behavior for $\omega_i \rightarrow \infty$. A pole that moves to the positive half plane when $\omega \rightarrow \infty$ forms part of the k_+ family while a pole that moves to (or stays in) the negative half plane when $\omega \rightarrow \infty$ forms part of the k_- family.

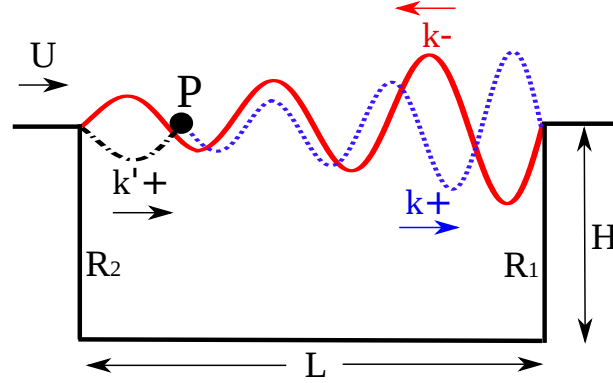


FIGURE 6.10: Schematic view of wave reflection scenario in a finite domain exemplary for open cavity flow with $L/H = 2$. Instability wave originates in point P . Downstream traveling k_+ waves (blue dashed and black dot dashed waves, respectively), upstream traveling k_- wave (red solid wave), reflection coefficients R_1 and R_2 . Flow from left to right.

6.5 The Finite Domain

The finite extent of the domain becomes relevant when the times taken into consideration are larger than the time needed by the perturbation to travel along the cavity length. Let us consider the geometry depicted in figure 6.10, which is characterized by its normalized length $L^* = L/\Theta_{eff}$, the cavity height $H^* = H/\Theta_{eff}$ and the two reflection coefficients R_2 and R_1 at the upstream and downstream boundaries, respectively. Θ_{eff} is the effective momentum thickness that is discussed in detail in section 6.2.3. The cavity length L is of the same order as the wavelength of the perturbation and therefore cannot be neglected.

The Kulikowskii Condition is obtained, following [Landau & Lifschitz \(1985\)](#) and [Kulikowskii \(1966\)](#). The argument is schematically outlined in figure 6.10. Let P be an arbitrary point inside the domain emitting two counter-traveling perturbation waves $k_+(\omega)$ and $k_-(\omega)$. Branches associated with $k_+(\omega)$ and $k_-(\omega)$ are found as solutions of the dispersion relation $\Delta(\omega, k)$ for complex wave numbers and complex frequencies in an infinite domain. As stated in [Landau & Lifschitz \(1985\)](#), the characteristic oscillations of a finite system may be regarded as the result of the superposition of traveling waves reflected by the two boundaries, represented by R_1 and $R_2 \in \mathbb{R}$. The reflections are accompanied by a mutual transformation of waves belonging to different branches of the spectrum. Formally the Kulikowskii Condition is obtained by considering the perturbation wave

$$\Psi(x, t) = A_0 e^{i[k_+(\omega)x - \omega t]} \quad (6.35)$$

which is emitted at P ($0 < P < L^*$). A_0 is the initially infinitesimal amplitude. When it reaches the downstream boundary, Ψ is reflected according to reflection coefficient R_1 , and is sent back upstream. The wave on its way upstream is described by

$$\Psi(x, t) = R_1 A_0 e^{ik_+(\omega)L^*} e^{i[k_-(\omega)(x-L^*)-\omega t]} \quad (6.36)$$

It reaches the upstream boundary, where it is reflected again, according to the reflection coefficient R_2 . When it travels downstream again the wave is described by

$$\Psi(x, t) = R_1 R_2 A_0 e^{i[k_+(\omega)-k_-(\omega)]L^*} e^{i[k_+(\omega)x-\omega t]} \quad (6.37)$$

Due to the requirement that $\Psi(x, t)$ must be single valued, equations (6.35) and (6.37) must coincide, leading to the Kulikowskii Condition

$$R_1 R_2 e^{i(k_+-k_-)L^*} = 1 \quad (6.38)$$

or in a reorganized form

$$i(k_+ - k_-) = \frac{1}{L^*} \ln \left[\frac{1}{R_1 R_2} \right] \quad (6.39)$$

Note that the frequencies for the downstream traveling waves (6.35) and the upstream traveling wave (6.36) and (6.37) do not change. Hence the frequencies of the $k_+(\omega)$ and $k_-(\omega)$ waves are the same. Splitting the Kulikowskii Condition into a real and an imaginary part yields

$$\Delta k_r = k_{r+}(\omega) - k_{r-}(\omega) = \frac{\pi m}{L^*}, \quad (6.40)$$

$$\Delta k_i = k_{i-}(\omega) - k_{i+}(\omega) = \frac{1}{L^*} \ln \left[\frac{1}{R_1 R_2} \right] \approx 0, \quad (6.41)$$

where $m \in \mathbb{N}_0$. Poles of the linear stability analysis which also solve equations (6.40) and (6.41) will hereafter be called *Kulikowskii Points*. Note, that the RHS of equation (6.40) is $\pi m/L^*$ since

$$e^{i(k_+-k_-)L^*} = \frac{1}{R_1 R_2} \quad (6.42)$$

$$e^{i\Delta k_r L^*} e^{\Delta k_i L^*} = \frac{1}{R_1 R_2} \quad (6.43)$$

$$e^{\Delta k_i L^*} [\cos(\Delta k_r L^*) + i \cdot \sin(\Delta k_r L^*)] = \frac{1}{R_1 R_2} \quad (6.44)$$

Splitting equation (6.44) into a real and an imaginary parts as well as considering the wave envelope (*i.e.* the modulus) yields

$$\left| e^{\Delta k_i L^*} \cos(\Delta k_r L^*) \right| = \left| \frac{1}{R_1 R_2} \right| \quad (6.45)$$

$$\sin(\Delta k_r L^*) = 0 \quad (6.46)$$

The sine function equals zero at πm with $m \in \mathbb{N}_0$ and hence the RHS of equation (6.40).

6.5.1 Reflection coefficients

As we consider rigid cavity walls, the reflection coefficients are assumed to be real with a phase shift of π . Following [Doare & de Langre \(2006\)](#) we impose a zero total deformation boundary condition at the upstream and downstream corner of the cavity, which is expressed as

$$\Psi(x = 0, t) = 0 \quad (6.47)$$

$$\Psi(x = L, t) = 0 \quad (6.48)$$

Let us denote A_1^+ the amplitude of the wave at the downstream boundary (1) before it was reflected and let us denote A_1^- the amplitude of the wave at the downstream boundary (1) after it was reflected. Then we can write the deformation $\Psi(x = 0, t)$ at the location where the boundary condition is imposed as the sum of the two waves k_+ and k_- , that have to add up to zero. This yields (for all times t)

$$\Psi(x = 0, t) = [A_2^+ e^{ik_+ x} + A_2^- R_1 e^{ik_- x}] e^{-i\omega t} = 0 \quad (6.49)$$

$$\Psi(x = 0, t) = A_2^+ + A_2^- R_1 = 0 \quad (6.50)$$

$$R_1 = -\frac{A_2^+}{A_2^-} = -1 \quad (6.51)$$

$$(6.52)$$

if we assume that no forcing is induced by the boundaries and therefore the amplitudes before and after the reflection are the same. The same reasoning holds for the upstream boundary (2) from which follows $R_2 = -1$.

Physically, it is plausible to expect the product of the reflection coefficients to be smaller than unity since a part of the perturbation wave could be lost and travel off to infinity. A rough computation following [Lighthill \(1990\)](#) gave a value of $R_1 R_2 \approx 0.8$ which translates into $\Delta k_i = 0.0029$ for case LH21 (see table 6.2) when applied to equation (6.41). The effect of this positive difference of the spatial amplification rates is depicted in figure 6.11.

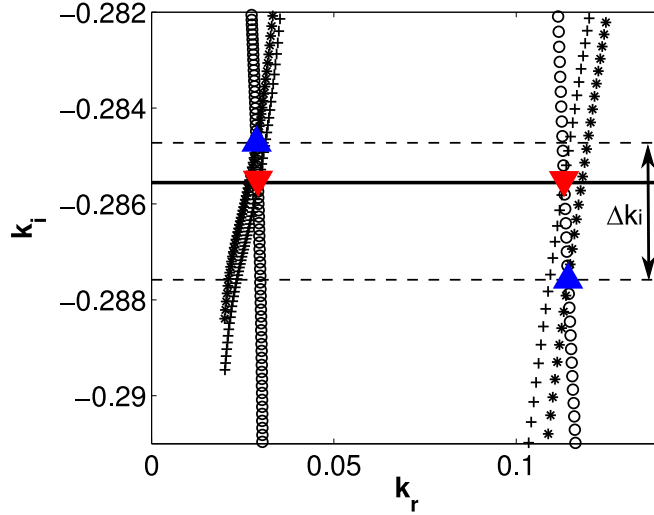


FIGURE 6.11: LSAFD with $\Delta k_i = 0.0029$, which translates into $R_1 R_2 = 0.80$. Case LH21 in table 6.2. Branches (+) and branches (•) correspond to different $\omega_i = \text{const}$; Branches (o) correspond to $\omega_r = \text{const}$. Horizontal solid line indicates $\Delta k_i = 0$ ($R_1 R_2 = 1$) for Kulikowskii points (red ∇). Horizontal dashed lines separate Kulikowskii points (blue $\triangle$) by $\Delta k_i = 0.0029$ ($R_1 R_2 = 0.8$).

Since the real part of the frequency does not change upon reflection, the Kulikowskii Points for $\Delta k_i = 0$ (red ∇) must move on branches $\omega_r = \text{const}$ (indicated by black circles in figure 6.11) until the corresponding Δk_i is reached. The resulting difference in the real part of the wave number Δk_r is approximately equal to Δk_r for $R_1 R_2 = 1$. This is true even for values $R_1 R_2 < 0.8$ since the branches $\omega_r = \text{const}$ are close to vertical and therefore Δk_r changes only little when Δk_i is increased.

Based on the considerations outlined above we propose a first order approximation, assuming the reflection coefficients to be $R_1 = R_2 = -1$ for all the cases considered in this work.

6.5.2 Interpretation of the Kulikowskii Condition

Physically, equation (6.40) states that the finite domain can only contain an integer number of waves, which is in agreement with the wavelength selection criterion found by Maurel *et al.* (1996). This condition yields a discretization of a continuous spectrum of solutions and thus will lead to a quantization mechanism of the associated frequency. Note that in the infinite domain ($L \rightarrow \infty$) such a discretization does not occur since $\Delta k_r = 0 \ \forall m \in \mathbb{N}$.

The second equation (6.41) states the coincidence condition in terms of the spatial growth rates. The difference of the spatial growth rates for the upstream and downstream traveling waves has to be compensated by the reflection coefficients R_1 and R_2 , to prevent

the system from getting out of balance. Hence, equation (6.41) expresses the self-limited nature of the system. For the case of total reflection, the RHS of equation (6.41) is zero, which means that if the amplitude of the perturbation wave is not diminished by reflection, the spatial amplification rates of the upstream and downstream traveling waves must be equal in absolute terms. The more general case of $R_1 R_2 \neq 1$ yields a strictly positive RHS of equation (6.41) and the amplification rates adapt accordingly in order to maintain valid equation (6.41). However, since as mentioned above the product of the reflection coefficients is close to unity, the numerical value of Δk_i is small and does not affect the results significantly.

In short, the solutions to the Kulikowskii Condition consist of two different values of k_r , which are associated with one single set of values of k_i , ω_r and ω_i , *i.e.*:

$$\omega_{i+} = \omega_{i-} \quad (6.53)$$

$$\omega_{r+} = \omega_{r-} \quad (6.54)$$

$$k_{i+} = k_{i-} \quad (6.55)$$

$$k_{r+} = k_{r-} + \frac{\pi m}{L^*} \quad m \in \mathbb{N}_0 \quad (6.56)$$

Poles of the linear stability analysis in the infinite domain that also solve the finite domain constraint, namely the Kulikowskii Condition, are denoted *Kulikowskii Points*. Kulikowskii Points therefore characterize the instability behavior of the finite system.

As stated in [Landau & Lifschitz \(1985\)](#), in general we can assume that the spectrum of characteristic oscillations will be governed by the boundary condition and the finite length of the system. Thus the characteristic oscillations of a finite system are a superposition of waves reflected at its boundaries, accompanied by mutual transformation of waves belonging to different branches of the spectrum.

6.5.3 The infinite limit

In reference to the question posed by Prof. Dr. Patrick Huerre after the presentation of the present work at the *Bifurcation and Instability in Fluid Dynamics Conference (BIFD)* in Paris in July 2015, about “what happens for $L \rightarrow \infty$ ”, it can be said that the system returns to the properties of the infinite system. For $L \rightarrow \infty$ the right-hand side of both Kulikowskii equations becomes zero. This means that the only pole (mode) that solves the Kulikowskii equation is the Pinch Point itself, just as in the infinite system. Kulikowskii modes do not appear for $L \rightarrow \infty$ as one would expect when no reflection takes place. By

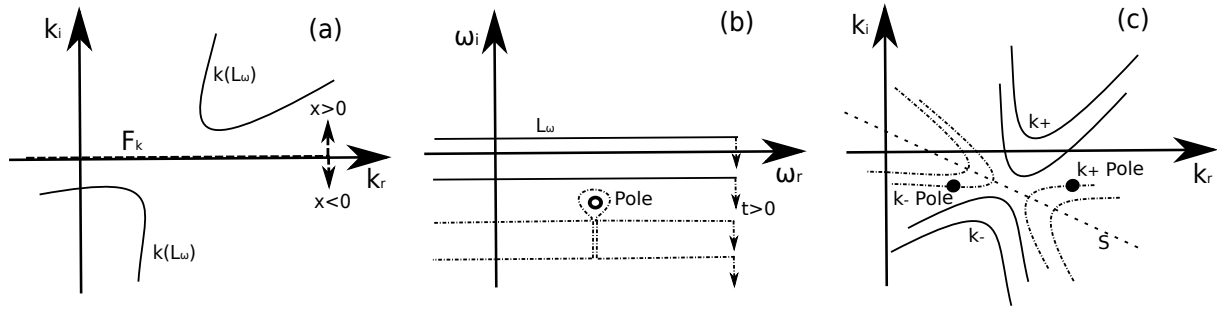


FIGURE 6.12: (a): Complex k plane with k branches for $\omega_i \rightarrow \infty$. (b): Complex ω plane with integration contours before (—) and after (— · —) the branch point is crossed. (c): k branches before (—) and after (— · —) the branch point is crossed, together with two exemplary poles (•) that solve the Kulikowskii Condition. The straight dashed line S (---), separates k_+ poles from k_- poles. The dashed arrows indicate movement of respective poles when $\omega_i \rightarrow \infty$.

reaching that limit the right hand side of the first Kulikowskii Condition $\pi m/L^*$ becomes smaller and smaller and the discretization of the frequency axis disappears successively.

6.5.4 Evaluation of the Kulikowskii Condition

As stated above, the RHS of equation (6.41) is always positive or zero, since $0 \leq R_1 R_2 \leq 1$. For the conventional case (*i.e.* $\omega_i > \omega_{iBP}$), however, the respective locations of the k_+ branch and the k_- branch (see figure 6.12(a)) do not allow for such a solution, since the k_+ branch is located above the k_- branch, which will always yield $k_-(\omega) - k_+(\omega) \leq 0$. The only point where the equation holds is the PP itself. However, if the BP is crossed (*i.e.* $\omega_i < \omega_{iBP}$) by means of analytic continuation of the L_ω contour, as depicted in figure 6.12(b), the branches change their location so that non-zero solutions for equation (6.41) become possible. As stated in section 6.4.1, once the integration contour L_ω crosses the least stable pole in the ω -plane, causality is violated. This holds for the infinite domain in which the reflection of the perturbation wave is not taken into account. In the case of a finite domain, the concepts of “before” and “after” become inadequate, since due to reflection, a wave has to be considered as cause and effect at the same time. In order to know whether a pole is located on a k_+ branch or a k_- branch, the same criterion as before is applied. A pole that moves to the upper half plane as $\omega_i \rightarrow \infty$ is located on a k_+ branch, while a pole that stays in the lower half plane when $\omega_i \rightarrow \infty$, is located on a k_- branch. The straight dashed line (S) in figure 6.12(c) separates the two types of branches according to this criterion.

TABLE 6.2: Summary of $L/H = 2$ cavity cases. Free stream velocity, U_∞ . Momentum thickness of boundary layer at $x = 0$, Θ_0 . Cavity length, L . Location of best fit, x_{min} . Effective momentum thickness, Θ_{eff} . Normalized cavity height, $H_{eff}^* = \frac{H}{\Theta_{eff}}$. Selected Kulikowskii modes, m_1 and m_2 .

Case	Symbols	$U_\infty(\frac{m}{s})$	$\Theta_0(mm)$	$L(mm)$	$x_{min}/L(-)$	$\Theta_{eff}(mm)$	$H_{eff}^*(-)$	m_1	m_2
LH20	●	2.250	—	100	—	1.296 ¹	38.6	3	8
LH21	○	2.180	1.072	100	0.227	1.325	37.7	4	8
LH22	△	1.898	1.294	100	0.227	1.425	35.1	4	8
LH23	▽	1.715	1.260	100	0.250	1.525	32.8	5	9
LH24	□	1.379	1.310	100	0.227	1.675	29.9	5	—
LH25	▲	0.992	1.420	100	0.250	1.975	25.3	5	—

¹ Momentum Thickness was not obtained experimentally but extrapolated from cases LH21 to LH25.

6.6 Results for single cavity flow

The linear stability analysis in a finite domain (LSAFD), as explained in the previous sections, is now applied to single open cavity flow and compared to experimental results by [Lusseyran *et al.* \(2008\)](#) and [Basley \(2012\)](#). Single open cavity flow presents a canonical flow geometry of practical as well as academic interest in fluid mechanics. Note, results by [Lusseyran *et al.* \(2008\)](#) and [Basley \(2012\)](#) are in excellent agreement with the results of the single open cavity flow, presented in the present work in chapter 3. However, at the time of the development of the LSAFD, the experimental data presented in chapter 3, was not yet available and therefore data from the literature was chosen to test the novel theory.

The computational domain and the boundary conditions are depicted in figure 6.1(a). Table 6.2 summarizes the six cases analyzed in the present work. The cavity aspect ratio L/H is kept constant and only the free stream velocity U_∞ is varied. With U_∞ changes Θ_0 and therefore Θ_{eff} , which likewise affects the normalized cavity height and length, H^* and L^* , respectively.

6.6.1 Evaluation of the Kulikowskii Condition

Using [Briggs \(1964\)](#) method, schematically outlined in figure 6.13, equispaced grids are mapped through the dispersion relation of the linear stability analysis (6.19) from the complex frequency plane to the complex wave number plane or vice versa. Figure 6.14 shows four branches, that result from mapping $\omega_i = \text{const}$ through the dispersion relation into the complex k -plane, before (circles) and after (crosses) the branch point is crossed. Comparison between the (○)-branch and the (+)-branch shows that, as stated above, solutions of the Kulikowskii Condition (equations (6.40) and (6.41)) are possible only after the BP is reached. Solutions to the Kulikowskii Condition are called Kulikowskii

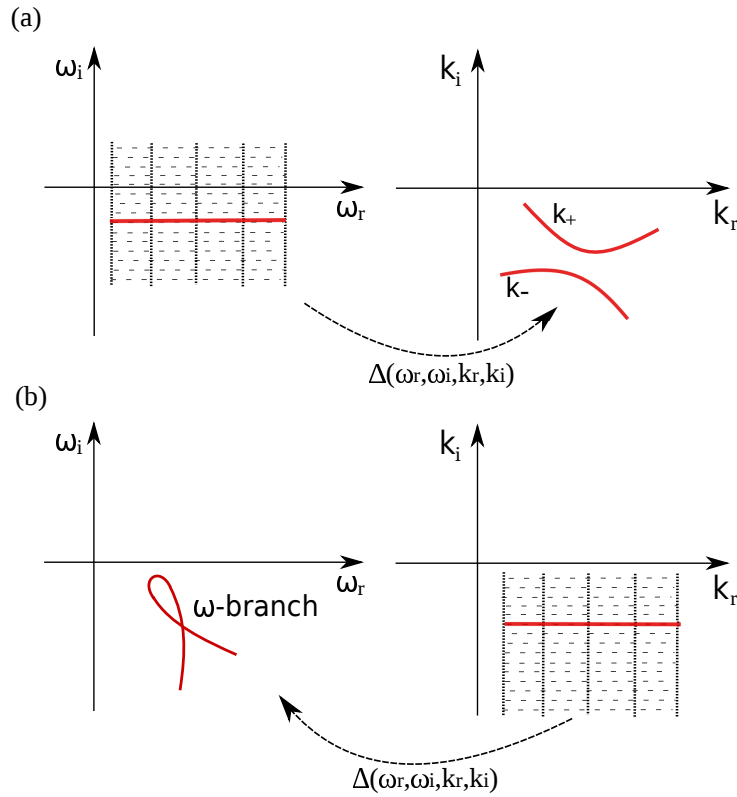


FIGURE 6.13: Schematic illustration of the Briggs (1964) method. Through the complex dispersion relation 6.19 a grid is mapped (a) from the complex frequency plane to the complex wave number plane or (b) vice versa.

Points. The image of a Kulikowskii Point, mapped from the k -plane into the ω -plane, is depicted in figure 6.15. By means of a spatial analysis, using Briggs (1964) method (figure 6.13(b)), a line corresponding to $k_i = \text{const}$ is mapped through the dispersion relation into the complex ω -plane, resulting in a self-intersecting branch. The intersection yields a Kulikowskii Point for which, as mentioned before, the following conditions are met:

$$\omega_{i+} = \omega_{i-} \quad (6.57)$$

$$\omega_{r+} = \omega_{r-} \quad (6.58)$$

$$k_{i+} = k_{i-} \quad (6.59)$$

$$k_{r+} = k_{r-} + \frac{\pi m}{L^*} \quad m \in \mathbb{N}_0 \quad (6.60)$$

A special solution to the Kulikowskii Condition is obtained for $m = 0$. This double root of the dispersion relation is commonly called *pinch point* (PP) in the literature. Evaluating the second Kulikowskii Condition (equation (6.41)) results in the continuous red graph in figure 6.16. For various $k_i = \text{const}$, $\Delta k_r(\omega_r) = k_{r+}(\omega_r) - k_{r-}(\omega_r)$ is plotted over the associated frequency ω_r . The quadratic form confirms the validity of the Taylor series expansion of the dispersion relation around the pinch point, as proposed by Landau &

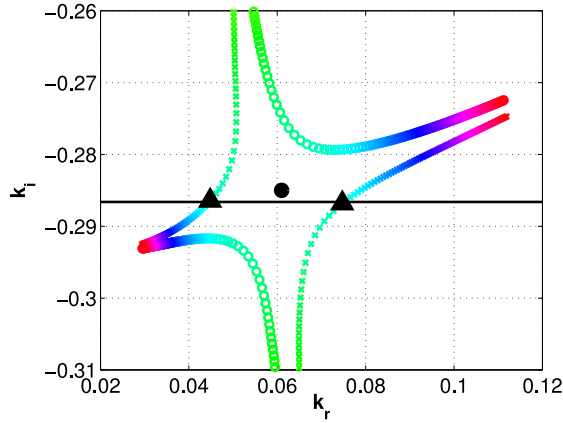


FIGURE 6.14: (Color in PDF) k branches in complex k -plane for $\omega_i = -0.0632$ (o) and $\omega_i = -0.0636$ (x) for case LH21. Color scale indicates value of increasing ω_r from light green ($\omega_r = 0.1676$) to red ($\omega_r = 0.1728$). Pinch Point marked by $\bullet$. Kulikowskii Points marked by $\blacktriangle$. Horizontal solid line (—) indicates $k_i = \text{const.}$

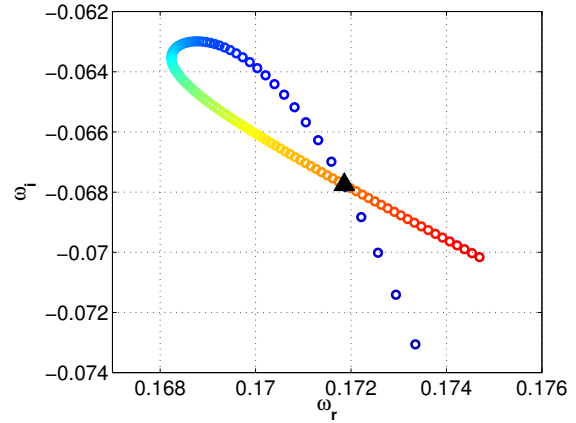


FIGURE 6.15: (Color in PDF) ω branch for $k_i = -0.3015$. Intersection ($\blacktriangle$) indicates Kulikowskii point. Color scale indicates value of increasing k_r from blue ($k_r = 0.0220$) to red ($k_r = 0.1400$).

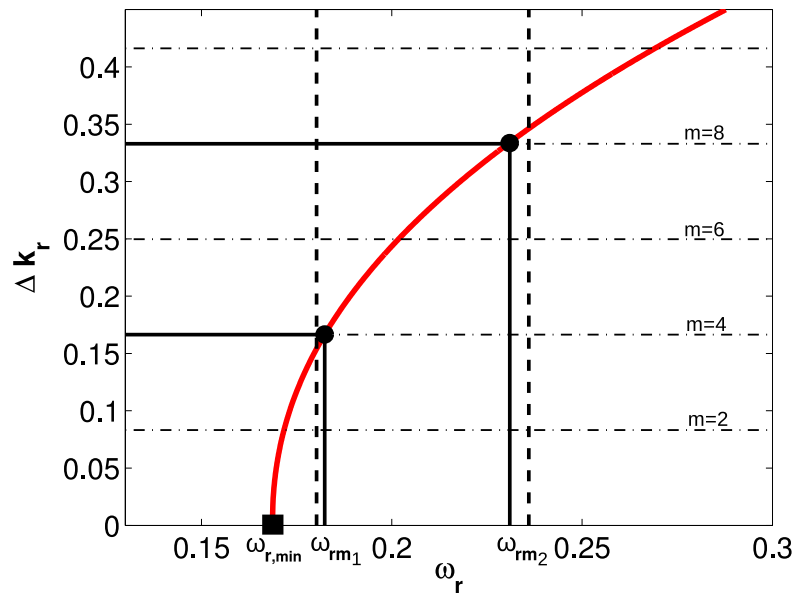


FIGURE 6.16: (Color in PDF) Red solid graph (—) shows continuous set of solutions to equation (6.41). Horizontal lines (— · —) depict solutions to equation (6.40) for $m = 2, \dots, 10$. Vertical solid lines (—) indicate by LSAFD selected frequencies ω_{rm1} and ω_{rm2} . Dashed vertical lines (— · —) indicate frequencies obtained experimentally (Lusseyran *et al.* (2008); Basley (2012)). The pinch point is marked by $\blacksquare$ and indicates the minimum frequency $\omega_{r,min}$ selected by the LSAFD. Kulikowskii Points are marked by $\bullet$. Only case LH21 is depicted for the sake of clarity.

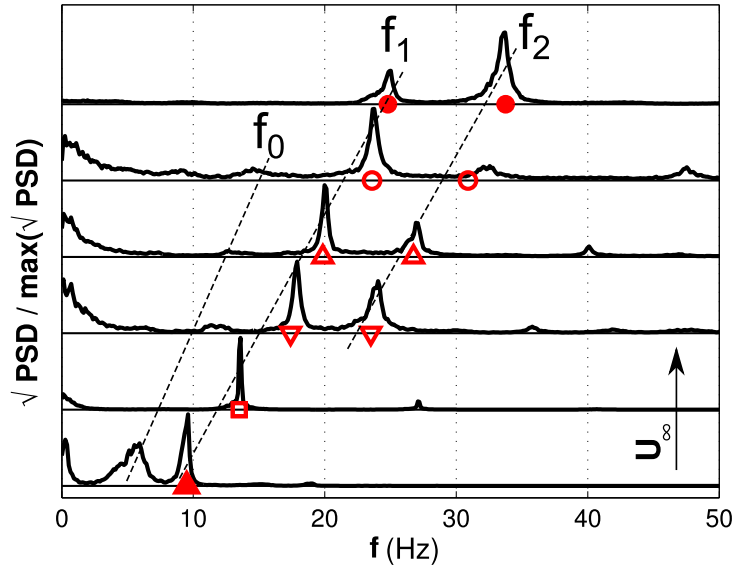


FIGURE 6.17: (Color in PDF) Y offset graph comparing the results from LSAFD (red symbols as in table 6.2) with the square root of the normalized power spectral density (PSD) of experimental results from Lusseyran *et al.* (2008); Basley (2012) (black graphs), measured using a LDV technique. The streamwise velocity component for the six cases in table 6.2 is depicted. Dashed lines indicate frequency branches f_0 , f_1 and f_2 . The arrow indicates the direction of increasing U_∞ .

Lifschitz (1985), which yields $k - k_0 \sim \pm(\omega - \omega_0)^{\frac{1}{2}}$. The continuous spectrum obtained by the second Kulikowskii Condition is discretized by the RHS of the first Kulikowskii Condition (equation (6.40)), which adds the horizontal lines to figure 6.16 and thus selects a set of discrete frequencies ω_{rm} , $m \in \mathbb{N}_0$. Certain frequencies, labeled ω_{rm_1} and ω_{rm_2} , compare well with experimental results from Lusseyran *et al.* (2008); Basley (2012).

Figure 6.17 compares experimental results for the open cavity flow from Lusseyran *et al.* (2008); Basley (2012) with the results of the LSAFD. The square root of the power spectral density (PSD), normalized by the respective maximum value, is depicted as a function of the freestream velocity U_∞ and the frequency f . The cases investigated in the present work are represented by the thick lines and U_∞ varies in the vertical axis as indicated by the arrow. Results of the LSAFD are indicated by the red symbols. Three incommensurable frequency branches (denoted f_0, f_1, f_2 in the graph) are amplified depending on U_∞ . At low velocities f_0 and f_1 coexist, though f_0 is dominant. While f_0 disappears for velocities larger than $U_\infty \approx 1.4$, f_1 keeps growing until it dies out for velocities $U_\infty > 2.5$. f_2 starts to develop around $U_\infty \approx 1.6$ and keeps growing from there on. f_0 lies outside of the frequency range selected by the Kulikowskii condition and is therefore assumed to be due to other effects, mentioned in the introduction (Bres & Colonius (2008); Villermaux & Hopfinger (1994)). The two high frequency branches f_1 and f_2 enter the regime predicted

by the LSAFD for all velocities.

Frequencies selected by the Kulikowskii Condition for $m = 4, 5$ and $m = 8, 9$, respectively, are in good agreement with experimental data from [Lusseyran *et al.* \(2008\)](#); [Basley \(2012\)](#). However, the Kulikowskii Condition selects a larger number of discrete frequencies than experimentally observed.

6.6.2 Evolution of Kulikowskii Modes in space and time

The values of k_i and ω_i along the Kulikowskii Points are depicted in figure 6.18 and 6.19, respectively. In accordance with equation (6.33) and (6.34), a negative value of k_i amplifies downstream traveling waves and attenuates upstream traveling waves as required by the Kulikowskii condition. Close to the cut-off frequency $\omega_{r,min}$ (which is nearly equal for all six cases), the value of k_i changes rapidly, whereas the increase or decrease depends on the local topology of the PP. Further away from $\omega_{r,min}$ the value of k_i levels out and tends to a constant, hence being approximately equal for both amplified frequencies ω_{rm_1} and ω_{rm_2} .

The value of ω_i is negative throughout the entire frequency band. This is in accordance with the convective nature of the instabilities in the present flow and the idea of spatially amplified waves. If ω_i were positive the perturbations would grow exponentially in time in every point in space and thus contaminate the entire flow (absolute instability). The value of ω_i as a function of ω_r drops linearly (with slope ≈ -1) moving away from the cut-off frequency. This means that mode m_2 is temporally more damped than mode m_1 . A mechanism which eludes the predominance of either one of the modes, as discussed in [Pastur *et al.* \(2008a\)](#), could not be identified by the present theory. However, it should be noticed that the temporal growth rate has only a weak meaning in the present work since we do not consider an infinitely extended shear layer, but a system confined by boundaries within which the question of self-sustaining modes induced by the constructive interference of reflected waves is addressed.

6.6.3 Eigenfunctions of Kulikowskii Modes

The location of Kulikowskii Points in their respective complex planes yields information on the stability behavior as well as on the wave number and frequency of the perturbation waves. The corresponding eigenfunctions provide further information on the structure of the respective waves in their three components u , v and p . Graphs (a)-(c) in figure 6.20 show the eigenmodes for the streamwise velocity perturbation u , wall normal velocity perturbation v and the pressure perturbation p of a single set of Kulikowskii Points (k_+ and k_-) at approximately the frequency corresponding to mode m_1 ($\omega_r = 0.1848$) of case LH21. For the same frequency, graphs 6.20(d)-(f) show the u , v and p eigenmodes corresponding

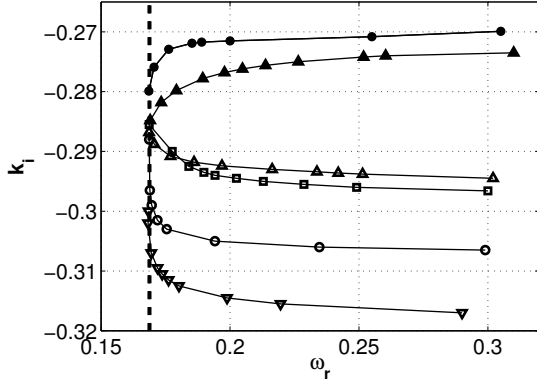


FIGURE 6.18: Values of k_i for different cases.

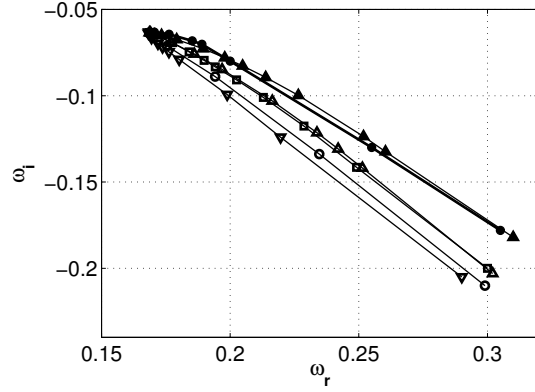


FIGURE 6.19: Values of ω_i for different cases. Symbols as in table 6.2.

to the pole on Kelvin-Helmholtz (KH) branch. Comparing the k_+ perturbations structure with the KH perturbation structure it becomes clear that the amplifying k_+ wave is a KH-like perturbation. This is in accordance with the general picture of the cavity mechanism. The components of the evanescent k_- wave (dashed lines in figure 6.20(a)-(c)) do not resemble the KH modes. In all three components their amplitudes show a more dispersed behavior. The velocity components of the k_- modes are most active inside the cavity ($y < 0$), while the pressure component shows a maximum above the cavity ($y > 0$). Which one of the components is responsible for the reflection mechanism cannot be clearly identified, however it can be stated that while the downstream traveling k_+ waves are active in a rather narrow region close to the center line ($y = 0$), the upstream traveling k_- waves are active over the entire domain ($-H < y < H$). It is worthwhile noting that the choice of the KH-like k_+ branch is not arbitrary, but follows directly from the spatio-temporal stability theory described in section 6.4.1, which states that the k_+ branch to be chosen is the one with the greatest negative value of k_{i+} (Landau & Lifschitz (1985)). The choice of the k_- branch is dictated by the Kulikowskii Condition.

6.6.4 Phase speed of Kulikowskii Modes

Figures 6.21 and 6.22 show the phase speed of the downstream traveling k_+ waves and the upstream traveling k_- waves, respectively, as a function of the frequency for the six cases summarized in table 6.2. The dimensionless phase speed of the downstream and upstream traveling waves, respectively, is defined as

$$c_{\pm} = \frac{\omega}{k_{\pm}} \quad (6.61)$$

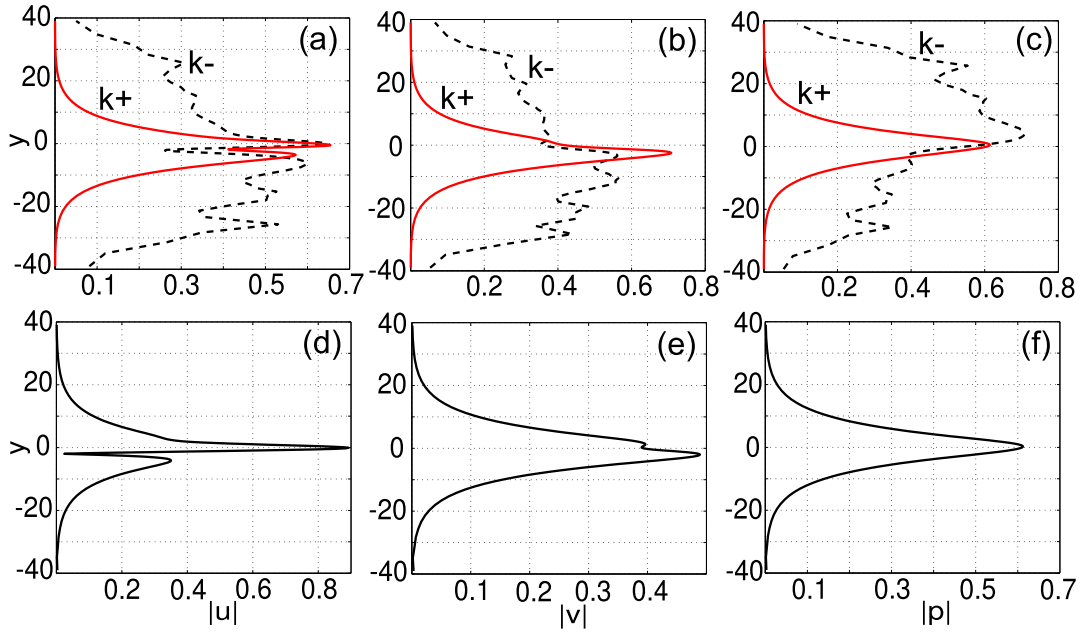


FIGURE 6.20: (a)-(c) Eigenmodes of amplifying k_+ waves (solid lines) and evanescent k_- waves (dashed lines) of $m = m_1$ of case LH21 ($\omega_r = 0.1848$, $\omega_i = -0.0801$). (d)-(f) Kelvin-Helmholtz modes of same frequency $\omega_r = 0.1848$, but $\omega_i = 0$.

As shown in figure 6.21, c_+ increases near the pinch point singularity but tends to a constant value of approximately $c_+ \approx 0.6$ for larger frequencies. All six cases collapse in a single line. Figure 6.22 depicts the phase speed of the upstream traveling waves for the different cases together with the non-dimensional speed of sound in air

$$a = \frac{a_\infty}{U_\infty} \quad \text{with} \quad a_\infty = 340 \text{ m/s} \quad (6.62)$$

For the feedback mechanism to be acoustic, waves must travel at the speed of sound. As shows figure 6.22 the upstream traveling waves travel in fact substantially faster than the downstream traveling waves, but do still not reach the speed of sound in the frequency band where the amplified frequencies are observed experimentally ($0.18 < \omega_r < 0.27$). These results suggest that the feedback mechanism in the incompressible limit is rather due to slower traveling instability waves than to acoustic pressure waves.

6.6.5 The total growth rate in the finite domain

As mentioned in chapter 3, for an increasing value of L/Θ_0 the incommensurate frequency peaks in single and double cavity flow move successively to higher harmonics in a so called cascade process, as also mentioned by Basley (2012). This cascade process still lacks a comprehensive explanation. Note, since for the cases in this section L is kept constant and Θ_0 decreases strictly monotonously with increasing U_∞ , the Reynolds number $Re_L =$

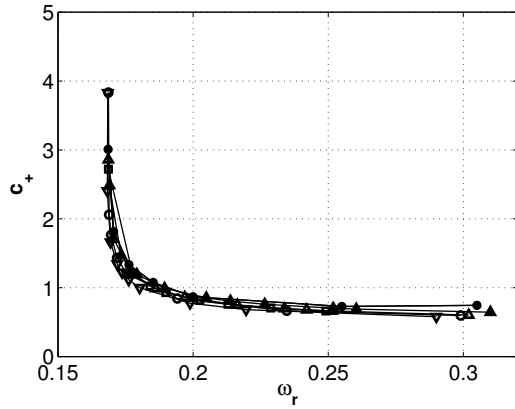


FIGURE 6.21: Phase speeds of k_+ waves for all cases.

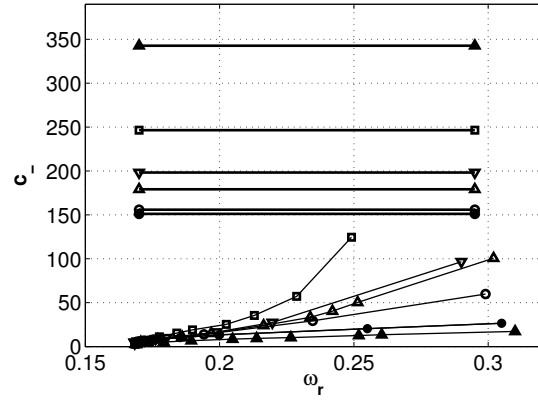


FIGURE 6.22: Phase speeds of k_- waves. Symbols as in table 6.2. Dashed horizontal lines indicate non-dimensionalized speed of sound in air $a^* = \frac{a_\infty}{U_\infty}$ of respective case.

$U_\infty L/\nu$ is used instead of L/Θ_0 to characterize the cascade process. To shed some light on this phenomenon, using the results obtained from the linear stability analysis in a finite domain, the total growth rate based on the interaction wave length Δk_r according to

$$\sigma = \omega_i - k_i \frac{\partial \Delta k_r}{\partial \omega_r} \quad (6.63)$$

is considered. Instead of computing the total growth rates for the downstream and the upstream traveling waves separately, as commonly done in infinite domains, the total growth rate for the finite domain is computed from the interaction wave length

$$\Delta k_r = k_{r+} - k_{r-}, \quad (6.64)$$

which corresponds to the difference between the real parts of the downstream traveling k_+ and the upstream traveling k_- wave, as follows directly from first Kulikowskii Condition (equation 6.40). The oscillations are self-sustained, which means that the total growth rate must be zero or at least close to zero. Otherwise, oscillations would either amplify everywhere in the domain and hence lead to an absolutely unstable flow, or die out everywhere and no oscillations would be observed. In the case of total reflection at the boundaries of the domain, the quantities ω_r , ω_i and k_i are equal for downstream and upstream traveling waves as follows directly from the Kulikowskii condition (equations 6.40 and 6.41).

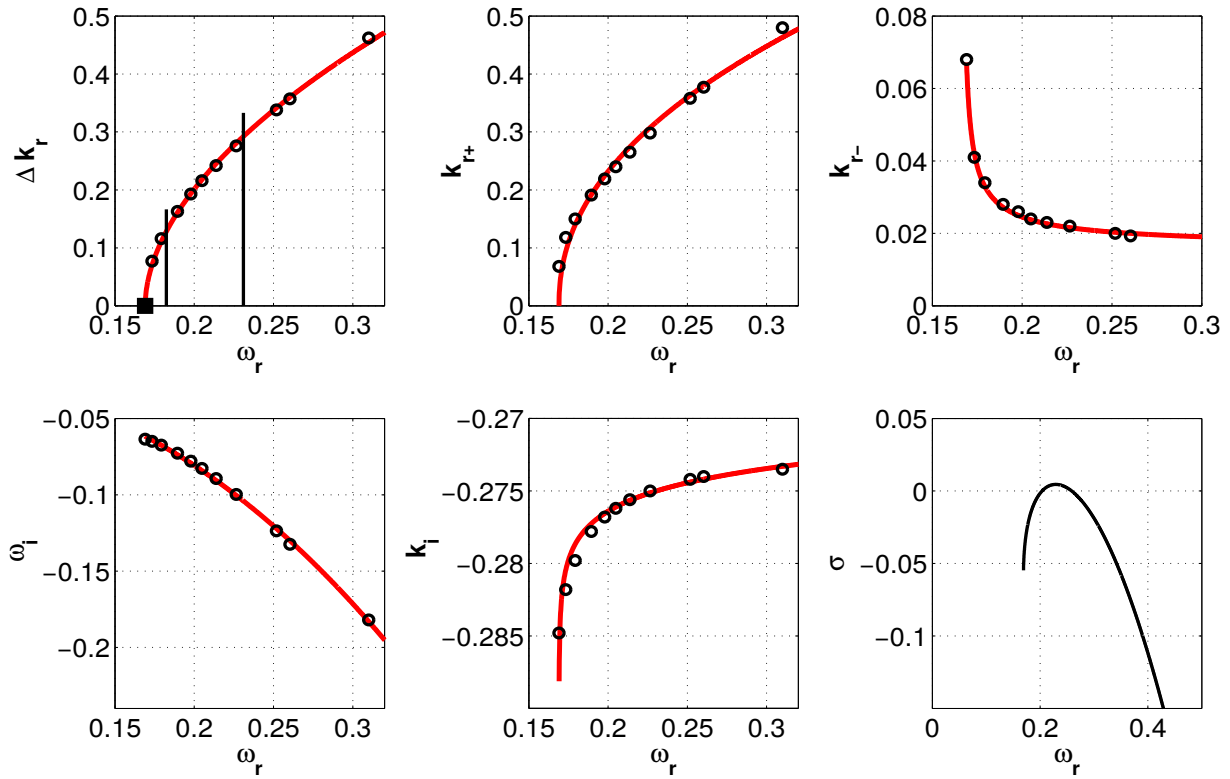


FIGURE 6.23: Example of fitting procedure in order to compute total growth rate σ from equation 6.63. Case LH25 in table 6.2.

In order to compute smooth derivatives in equation 6.63, the set of continuous solutions, obtained from the second Kulikowskii condition (equation 6.41) is fitted using a least square algorithm. Figure 6.23 shows an example of the fitting procedure for case LH25 in table 6.2 and the resulting curve for the total growth rate $\sigma(\omega_r)$. The black dots indicate random points at which the second Kulikowskii condition (equation 6.41) has been evaluated. The red solid curves show the least-square fits, which are then used to calculate the total growth rate σ from equation 6.63.

With this procedure the growth rate $\sigma(\omega_r)$ is calculated for each case in table 6.2. In figure 6.24 the total growth rate σ is shown as a function of the Strouhal number $St_L = \omega L^*/4\pi$. As expected, σ is close to zero in the frequency band where self-sustained oscillations are observed. It can be observed that for a low Reynolds number (black triangles) the total growth rate becomes zero around $St_L \approx 1$. When the Reynolds number is increased, the zone where $\sigma \approx 0$ broadens and at the same time moves to higher Strouhal numbers. This is in agreement with the cascade phenomenon described above and observed experimentally in chapter 3 and by Basley (2012).

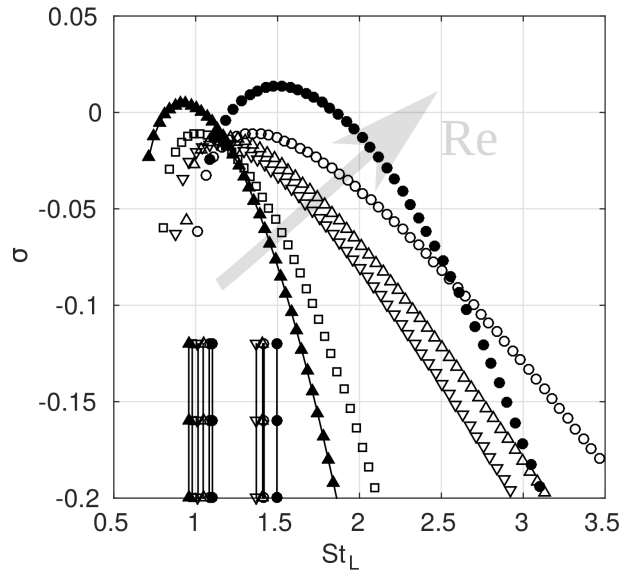


FIGURE 6.24: Total growth rate σ as a function of the Strouhal number St_L for all six cases from table 6.2. Symbols as in table 6.2. Arrow in (b) indicates direction of increasing Reynolds number Re_L (corresponding to increasing L/Θ_{eff}).

These results indicate, that the total growth rate based on the interaction wave length Δk , may be responsible for the fact that the amplified frequency peaks switch to higher harmonics when the Reynolds number (or L/Θ_0 for that matter) is increased.

6.6.6 Discussion

Figure 6.25 compares the results obtained by the LSAFD with data from several other works in literature. Results from the LSAFD ($St_{L,FD} = \omega L^*/4\pi$, red symbols) compare well with experimental results from Basley (2012) ($St_{L,exp} = f_{exp}L/U_\infty$, black symbols). The blue diamond $\diamond$ (or star $\star$) symbols in figure 6.25 show the Strouhal number $St_{L,LS} = \omega_{LS}L^*/4\pi$ of a conventional local temporal (or spatial) linear stability analysis in an infinite domain (*i.e.* no Kulikowskii Condition applied), where $\omega_{LS}(k_r)$ is the frequency for which $\omega_i(k_r)$ (or $k_i(\omega_r)$) is maximal (or minimal). The conventional (temporal or spatial) linear stability analysis can however predict only one single amplified frequency which tends to fall in between the two non-harmonic peaks observed experimentally and predicts neither of them with clarity.

As mentioned in the introduction, high subsonic compressible open cavity flow is commonly related to the acoustic feedback mechanism (*Rossiter mechanism*) expressed in terms of the Strouhal number $St_{L,R}$ by the semi-empirical Rossiter Rossiter (1964) formula

$$St_{L,R} = \frac{f_R L}{U_\infty} = \frac{n - \gamma}{M + \frac{1}{\kappa}}, \quad (6.65)$$

which was derived under the assumption that the feedback waves travel upstream at the speed $c = \frac{\omega}{k}$ where c is the speed of sound c . In the incompressible limit the Mach number is zero and the cavity length L is given in table 6.2. In equation (6.65) $n = 1, 2, 3 \dots$ is the mode number, κ is the ratio between the convection speed of the vortices and the free stream velocity and γ is the lag time between the impact of a vortex on the cavity edge and the emission of an acoustic wave. The values $\kappa = 0.66$ and $\gamma = 0.25$ were used (Delprat (2006)). As mentioned by Rowley *et al.* (2002) for the $L/H = 2$ cavity the first two Rossiter modes are prevalent in the compressible case. Yamouni *et al.* (2013) tried to link the Rossiter mechanism to the results of a global instability analysis when approaching the incompressible limit. However, a good agreement could be found only for mode $n = 2$. In figure 6.25 Rossiter's results for $M = 0$ (vertical lines) are compared with the global linear stability results (Yamouni *et al.* (2013)), with the results from the LSAFD (red symbols), the results from a spatial linear stability analysis (blue stars), the results from a temporal linear stability analysis (blue diamonds) and the experimental results from Lusseyran *et al.* (2008); Basley (2012) (black symbols) for the six cases evaluated in the present work (see table 6.2).

Rossiter Mode 1 does not enter the frequency range selected by the LSAFD (gray shaded area in figure 6.25), given by the cut-off Strouhal number St_{min} which is defined by the pinch point frequency $\omega_{r,min}$ in figure 6.16. This is in agreement with Yamouni *et al.* (2013) who found Rossiter's mode 1 to be absent when approaching $M = 0$. Frequencies smaller than St_{min} are outside of the selectable frequency range, while the frequency band above $St_{L,min}$ is discretized by the finite extent of the domain. Experimentally obtained frequencies (Lusseyran *et al.* (2008); Basley (2012)) can be found at $St > St_{min}$. Rossiter modes for $n = 2$ and $n = 3$ are also found within the discretized frequency band, however they do not compare well with experimental results by Lusseyran *et al.* (2008); Basley (2012). Rossiter's mode 2 is somewhat close to experimental mode 1, but Rossiter's mode 3 is found at substantially higher frequencies than the experimental mode 2. Global modes (Yamouni *et al.* (2013)) compare reasonably well to Rossiter's modes but neither to the experimental results nor to the results obtained by the LSAFD.

These results indicate that in the incompressible limit the mechanisms responsible for the existence of non-harmonic modes are at least partly due to the reflection and the linear interaction of instabilities in the shear layer. This mechanism is not included in the acoustic feedback mechanism of Rossiter's formula.

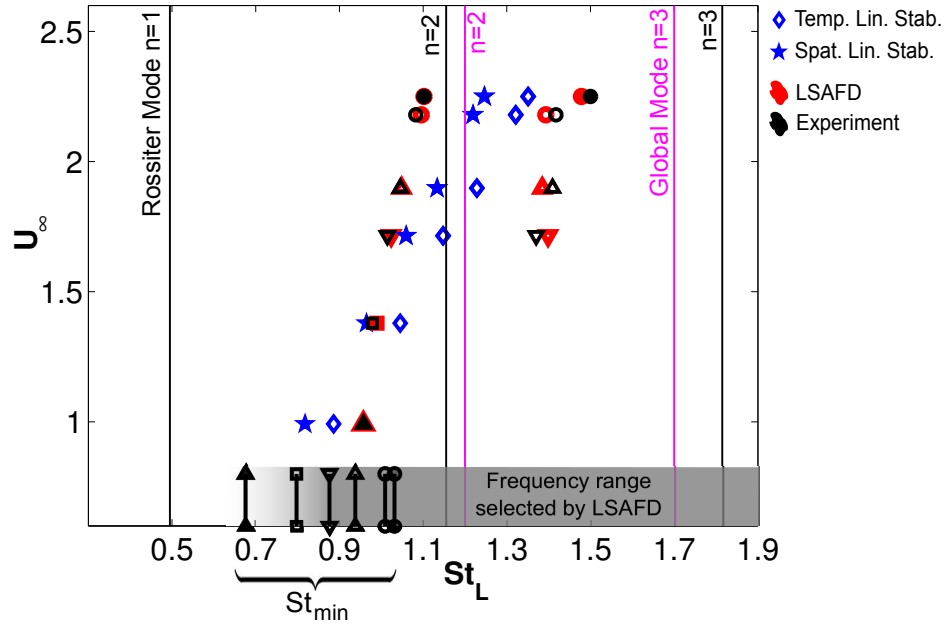


FIGURE 6.25: (Color in PDF) Comparison of experimental results from Lusseyran *et al.* (2008); Basley (2012) (black) with LSAFD (red) for all six cases as well as with a temporal (spatial) linear stability in infinite domain marked by blue $\diamond$ ($\star$). Long vertical black lines show Rossiter modes $n = 1, 2, 3$ from equation (6.65). Long vertical magenta lines show Global Modes $n = 2, 3$ from Yamouni *et al.* (2013). Short vertical lines show minimum Strouhal number $St_{min} = \omega_{r,min} L^* / 4\pi$, defined by pinch point frequency of the respective cases (see figure 6.16). Grey shaded area shows frequency range selected by LSAFD ($St > St_{min}$). Symbols as in table 6.2.

6.7 Results for double cavity flow

In this section, results obtained from the application of the linear stability analysis in a finite domain (LSAFD) to the double cavity geometry are presented and compared to experimental results from chapter 3. The computational domain and the boundary conditions are depicted in figure 6.1(b). The sinh profile from equation 6.18 is used for the analysis and, as described in section 6.2.2, it is fitted to the experimental data from chapter 3. The results are exemplified in figure 6.26, where the experimental velocity profile, the fitted analytic sinh profile and the shear strength, computed from the derivation of the sinh profile, are depicted for two different intercavity distances. The grid spacing is refined in the zone of high shear by combining two Chebyshev grids and the respective transformation. The velocity profile approximates very well the experimental profile in the channel center. However, the back flow of the recirculation region is not captured by the sinh profile.

Table 6.3 summarizes the eight double cavity cases analyzed in this section. The cavity aspect ratio L/H and the Reynolds number Re_L are kept constant, while the intercavity

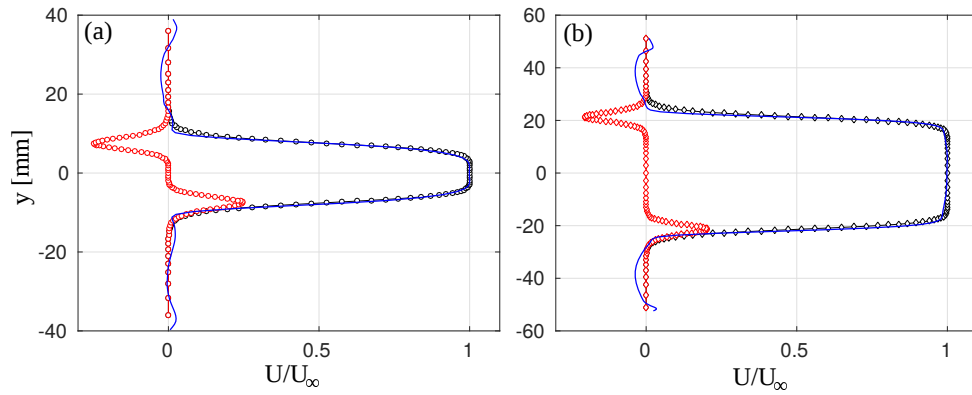


FIGURE 6.26: Exemplary velocity base profiles at x_{min} for linear stability analysis of double cavity with (a) $D/L = 0.3$ and (b) $D/L = 0.8$. Velocity profile in (black symbols), the shear strength (red symbols) and the experimental profile (blue solid line).

TABLE 6.3: Summary of all double cavity cases for linear stability analysis.

Case	D/L	D [mm]	Re_L	L [mm]	x_{min}/L	Θ_{eff} [mm]	H_{eff}^*	Mode	m_1	m_2
DC05	0.1	5	10200	50	0.22	0.559	44.6	var	—	—
DC10	0.2	10	10400	50	0.20	0.648	38.5	var	—	6
DC15	0.3	15	10700	50	0.20	0.715	34.9	var	3	6
DC20	0.4	20	10100	50	0.24	0.698	35.7	sin	3	6
DC25	0.5	25	10200	50	0.22	0.706	35.3	sin	3	6
DC30	0.6	30	10500	50	0.22	0.748	33.4	var	4	7
DC40	0.8	40	10800	50	0.22	0.766	32.6	var	4	7
DC50	1.0	50	10500	50	0.24	0.774	32.2	sin	4	7

distance is varied from $D/L = 0.1$ to $D/L = 1.0$. The effective momentum thickness Θ_{eff} , which is used as the length scale, varies with D/L as shown in figure 6.27 and hence affects the normalized cavity height and length, H^* and L^* , respectively. The analysis in this section is carried out closely following the analysis presented in the previous section 6.6, where the LSAFD was applied to single cavity flow.

6.7.1 Evaluation of the Kulikowskii Condition

As in section 6.6.1, the Briggs (1964) method, schematically outlined in figure 6.13, is used to map an equispaced grid through the dispersion relation of the linear stability analysis (equation 6.19) from the complex frequency plane to the complex wave number plane and vice versa. Figure 6.28(a) shows the k branches obtained from the spatio-temporal analysis before the pinch point is crossed, while figure 6.28(b) shows the branches after the pinch point has been crossed. According to the theory by Landau & Lifschitz (1985), described in section 6.4, the most unstable condition, *i.e.* the branch point in the complex

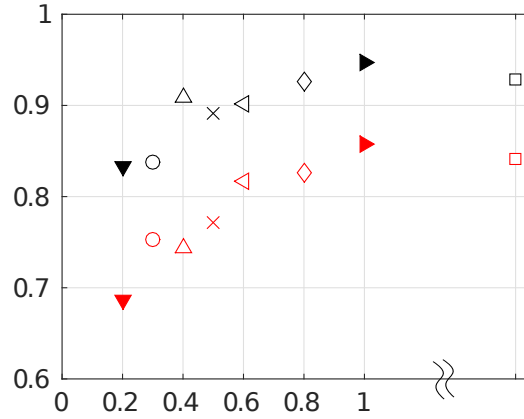


FIGURE 6.27: Effective momentum thickness Θ_{eff} over intercavity distance for cases in table 6.3, computed from analytic profiles (black) and from experimental profiles (red). Error bars indicate sum of squared errors between both profiles in the shear region. Symbols as in table 6.3.

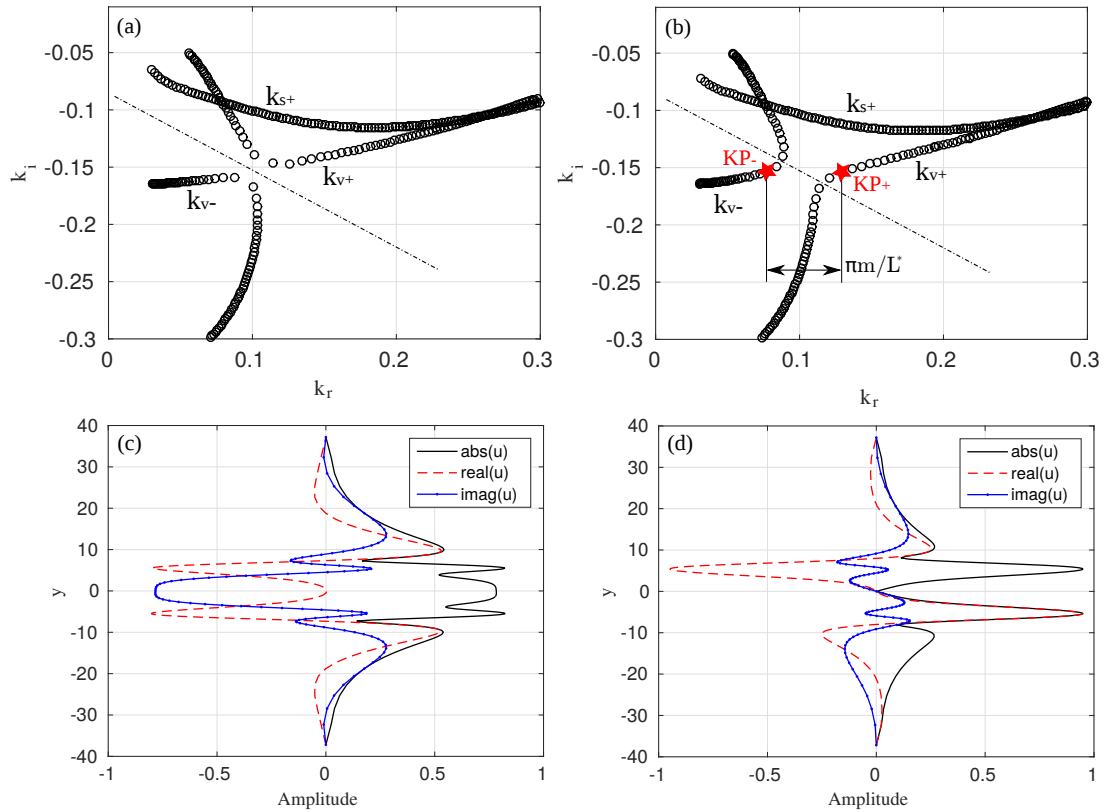


FIGURE 6.28: Spatio temporal linear stability analysis of DC10 ($D/L = 0.2$) double cavity case. (a) k branches just before pinch point and (b) just after pinch point of the varicose branch. (c) Exemplary varicose mode and (d) exemplary sinuous mode splitted up in real, imaginary (imag) and absolute (abs) parts, respectively. Red stars in (a) and (b) indicate possible set of Kulikowskii points (*cf.* equations 6.66 - 6.69). Diagonal dash dotted lines in (a) and (b) separate plus and minus modes.

k -plane that lies closest to the real k -axis, characterizes the stability properties of the flow. The tanh profile (single cavity) results in a single k_+ branch pinched that pinches with a k_- branch. The sinh profile, however, results in two k_+ branches (a varicose and a sinuous branch, *cf.* section 6.3). It was found, that the sinuous branch yields a higher temporal amplification rate and hence is temporally more unstable. Now, the obvious question that arises is what happens in the spatio-temporal case, *i.e.* which branch pinches first once the L_ω contour is lowered? Interestingly this question cannot be answered in general. For close cavity distances $0.1 \leq D/L \leq 0.3$ very clearly the varicose branch pinches first. In other words, the value for ω_i at which the varicose branch pinches is much higher than the value for ω_i at which the sinuous branch pinches. For larger distances both branches pinch at nearly the same value of ω_i and the branch that pinches first may even change. In fact for distances $0.4 \leq D/L \leq 0.5$ the varicose branch pinches first. For distances $0.6 \leq D/L \leq 0.8$ the sinuous branch pinches first, while for $D/L = 1.0$ the sinuous comes first again. As in the temporal analysis where for $D/L \geq 0.4$ the two branches nearly coincide, in the spatio-temporal analysis for $D/L \geq 0.4$ it seems to be rather arbitrary which branch pinches first. In table 6.3 the mode of the branch that first pinches is listed for each case (“var” stands for varicose and “sin” for sinuous).

Once the first branch pinches the analysis is straight forward as outlined in detail in section 6.6 for the single cavity. Assuming again total reflection at the boundaries yields $R_1 R_2 = 1$. Modes, that comply with the Kulikowskii condition (equations (6.40) and (6.41)):

$$\omega_{i+} = \omega_{i-} \quad (6.66)$$

$$\omega_{r+} = \omega_{r-} \quad (6.67)$$

$$k_{i+} = k_{i-} \quad (6.68)$$

$$k_{r+} = k_{r-} + \frac{\pi m}{L^*} \quad m \in \mathbb{N}_0, \quad (6.69)$$

are then searched for in the spectrum of the local linear stability analysis in an infinite domain. These modes are the Kulikowskii modes that characterize the finite system.

6.7.2 Results

The pinch point location in the complex wave number plane changes with the intercavity distance (D/L). The topology surrounding the pinch point of each case was confirmed to agree well with the quadratic form of the Taylor series expansion of the dispersion relation as proposed by Landau & Lifschitz (1985), which yields $k - k_0 \sim \pm(\omega - \omega_0)^{\frac{1}{2}}$. Figure 6.29 shows the results from the linear stability analysis in a finite domain for

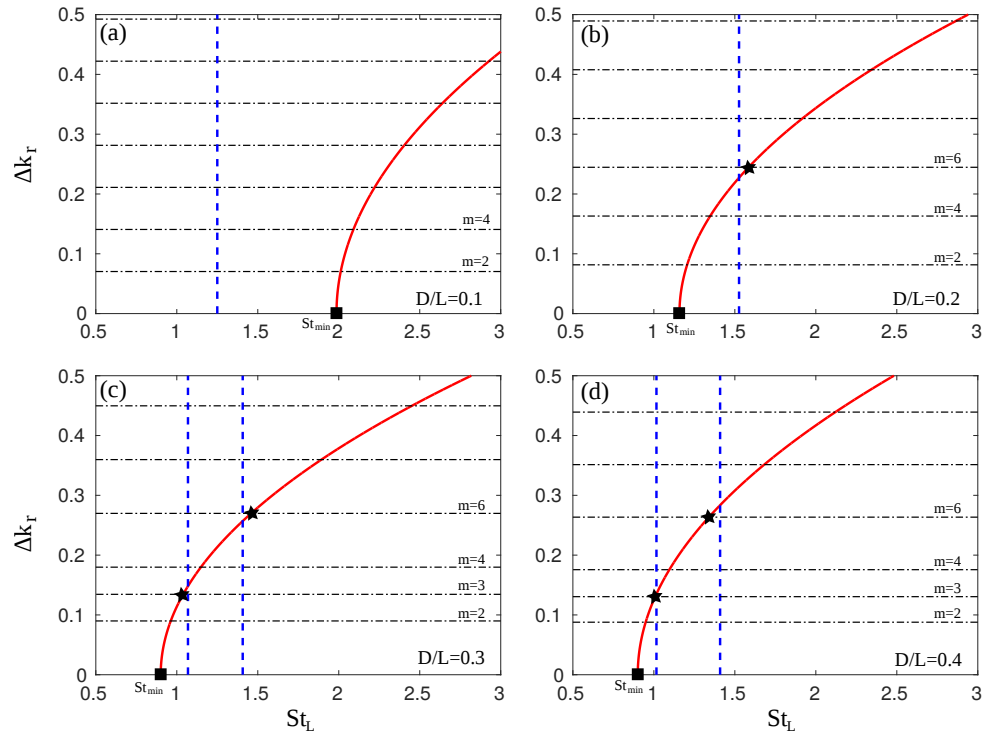


FIGURE 6.29: (Color in PDF) Kulikowskii curves for (a) $D/L = 0.1$, (b) $D/L = 0.2$, (c) $D/L = 0.3$, (d) $D/L = 0.4$ (*cf.* table 6.3). St_{min} indicates minimum selectable frequency. Black stars ★ indicate solutions to the Kulikowskii condition (*cf.* equations 6.66 - 6.69). The horizontal dashed (blue) lines indicate the experimentally measured frequencies from chapter 3. Horizontal dash dotted lines correspond to $\Delta k_r = \pi m/L^*$ (*cf.* first Kulikowskii condition, equation 6.40). The pinch point in each subfigure is marked by ■ and indicates the minimum frequency St_{min} .

four typical double cavity cases with $0.1 \leq D/L \leq 0.4$. The real part of the frequency of the pinch point ω_r defines the minimum selectable frequency St_{min} , marked in each subfigure. Only frequencies $St_L > St_{min}$ can be selected. The horizontal dashed (blue) lines indicate the experimentally measured frequency peaks presented in chapter 3. The horizontal dash dotted lines correspond to $\Delta k_r = \pi m/L^*$ (*cf.* first Kulikowskii condition, equation 6.40), while the red solid line follows a quadratic form and corresponds to the continuous sets of solutions to the second Kulikowskii condition (equation 6.41). The intersections of the quadratic curve with the lines $\Delta k_r = \pi m/L^*$ solve both the first and the second Kulikowskii equation and hence indicate the Kulikowskii points. The black stars indicate solutions to the Kulikowskii condition (*cf.* equations 6.66 - 6.69), that are closest to the experimentally measured frequency peaks. The value of the integer number m that corresponds to this solution is listed in table 6.3 for each case. As mentioned in section 6.6, a mechanism that indicates the correct value of m could not be found. The Kulikowskii mechanism quantizes the frequency spectrum, but the selection is most likely up to another mechanism. A possible candidate is presented in chapter 7.

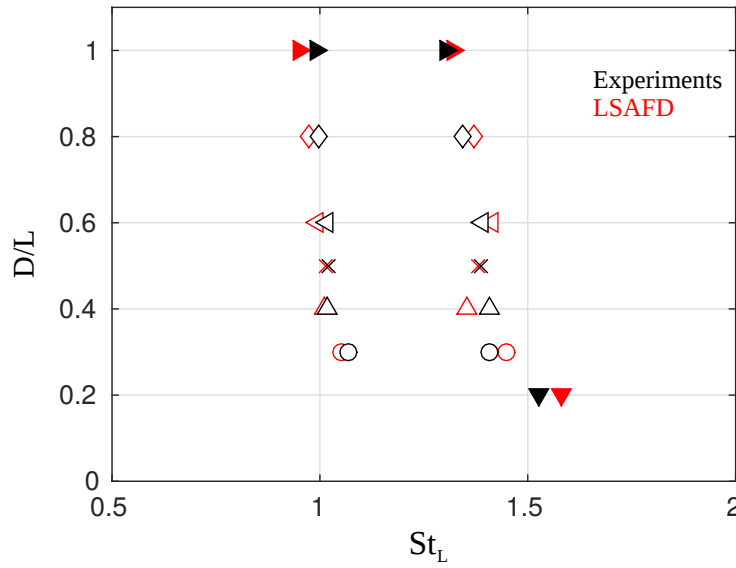


FIGURE 6.30: (Color in PDF) Comparison of experimental results from chapter 3 (black) with LSAFD (red) for the seven experimental Paris-campaign cases from table 6.3 with $D/L \geq 0.2$.

Figure 6.30 compares the results of all double cavity cases with $D/L \leq 0.2$ to the experimental results from chapter 3. For small intercavity distances $D/L \leq 0.5$ modes $m = 3$ and $m = 6$ agree well with experimental results, while for larger distances $D/L \geq 0.6$ modes $m = 4$ and $m = 7$ agree best with experiments. Case DC05 ($D/L = 0.1$) yields a minimum Strouhal number of $St_{min} \approx 2$. Hence, the LSAFD does not select any frequencies within the typical shear layer frequency bandwidth of around $0.9 \lesssim St_L \lesssim 1.5$. This is in agreement with the experimental observations of chapter 3, where no dominant shear layer peaks were observed for case DC05 ($D/L = 0.1$). Case DC10 ($D/L = 0.2$) experimentally yields a single frequency peak at around $St_{L,2} \approx 1.5$ and no peak at the typical shear layer frequency $St_{L,1} \approx 1$. The minimum Strouhal number of $St_{min} \approx 1.2$, obtained from the LSAFD correctly predicts the experimental observations. Experimental results of the double cavity with $D/L \geq 0.3$ show two non-harmonic shear layer peaks, one close to unity and the other one at around $St_{L,2} \approx 1.4$. For these distances the LSAFD yields a minimum Strouhal number of $St_{min} < 1$, which makes it possible to select both peaks. The LSAFD therefore again correctly captures the experimental observations.

6.8 Effect of slowly varying base flow

The linear stability analysis described in the previous sections considers a constant velocity profile along the cavity length. This parallel flow assumption is a valid first approximation, though an immediate question that arises is what may be the effect of a varying base flow. The implications and consequences of a slowly varying base flow will be discussed in this section. The Kulikowskii condition, derived above under a parallel flow assumption, is

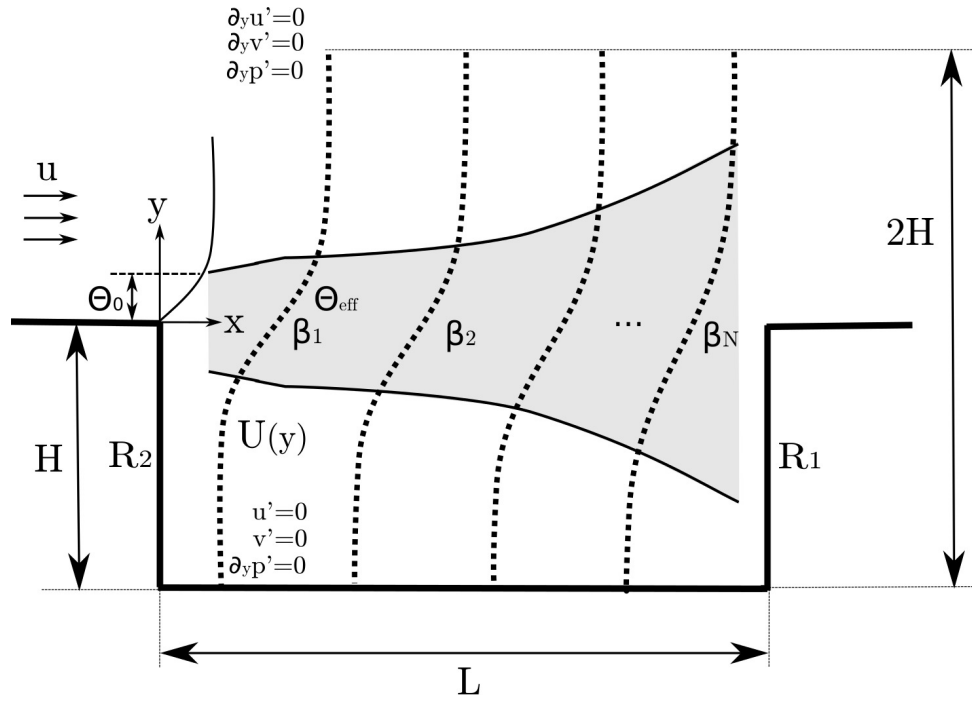


FIGURE 6.31: Schematic view of computational domain: single open cavity flow with slowly varying base flow $\bar{U}$, reflection coefficients R_1 and R_2 , boundary conditions, inflow momentum thickness Θ and effective momentum thickness Θ_{eff} .

extended to account for a slowly varying base flow and is then applied to the same open cavity flow as in section 6.6.

Figure 6.31 shows schematically the domain of interest with the slowly varying base flow $\bar{U}(x, y)$. Experimental data by [Basley \(2012\)](#) is used to fit the streamwise velocity profiles to the analytical hyperbolic tangent profile at each streamwise position. To account for the inhomogeneous background, the linear stability analysis of the cavity flow is decomposed into several equivalent parallel flow problems, resulting in a so-called “strip-theory” approach ([Triantafyllou et al. \(1986\)](#)). Hence, the same dispersion relation as presented in section 6.2 can be used and only the correct base flow has to be updated at each streamwise position. Results are compared to experimental data from [Basley \(2012\)](#) and to the theoretical predictions of the LSAFD under the parallel flow assumption, *i.e.* without a varying base flow.

6.8.1 The slowly varying base flow

The computational domain is schematically depicted in Fig. 6.31. Along the single cavity the hyperbolic tangent profile is assumed to hold. In its dimensional form it reads:

$$U(y, x) = \tilde{U} + \frac{\Delta U}{2} \tanh\left(\frac{2\beta y}{\delta_\omega}\right), \quad (6.70)$$

where $\tilde{U}$ is the mean velocity, $\Delta U = U_{max} - U_{min} = 2\tilde{U} = 0.5U_\infty$ is the velocity difference and $\beta = f(x)$ characterizes the spatial development of the flow. Normalized with the maximum (free stream) velocity U_∞ and the momentum thickness $\Theta_{eff} = \delta_\omega/4$, we obtain the following non-dimensionalized velocity profile

$$\bar{U}(y, x) = \frac{1}{2} [1 + \tanh(\beta y)], \quad (6.71)$$

from which the base profile at any given position can be computed and then be imposed on the linear stability analysis. The variation in streamwise direction x manifests itself in the increase of the vorticity thickness δ as shown in figure 6.32, which translates directly into the parameter β . The vorticity thickness is computed from the following formula

$$\delta_\omega = \frac{\Delta U}{\left. \frac{\partial U}{\partial y} \right|_{max}} \quad (6.72)$$

where the maximum gradient $\left. \frac{\partial U}{\partial y} \right|_{max}$ is obtained from fitting a straight line to the inflection point area of the experimental velocity profile as shown in figure 6.32 (blue dashed line). The momentum thickness of the local profile is then obtained by the relation $\Theta(x) = \delta_\omega(x)/4$. The normalized momentum thickness, which we defined as

$$\Theta_N(x) = \frac{\Theta(x)}{\Theta_{eff}}, \quad (6.73)$$

changes from $\Theta_N = 1$ at the position where Θ_{eff} is obtained, to $\Theta_N = 2.30$ close to the trailing edge of the cavity, as shown in figure 6.32(c). The effective momentum thickness Θ_{eff} is taken for the profile at $x/L = 0.22$ according to the method described in section 6.2.3. The parameter β , shown in figure 6.32(d), can be calculated directly from the normalized momentum thickness. The relation between β and Θ_N is given by

$$\beta(x) = \frac{1}{2\Theta_N(x)}. \quad (6.74)$$

Six profiles, summarized in table 6.4 and depicted in figure 6.33, were used in the present study to characterize the slowly varying base flow in the linear stability analysis in a finite domain.

Details on the linear stability analysis can be found in section 6.2. The different base profiles are consecutively imposed on the dispersion relation 6.12, resulting in several equivalent parallel flow problems as proposed by the “strip-theory” approach ([Triantafyllou et al. \(1986\)](#)).

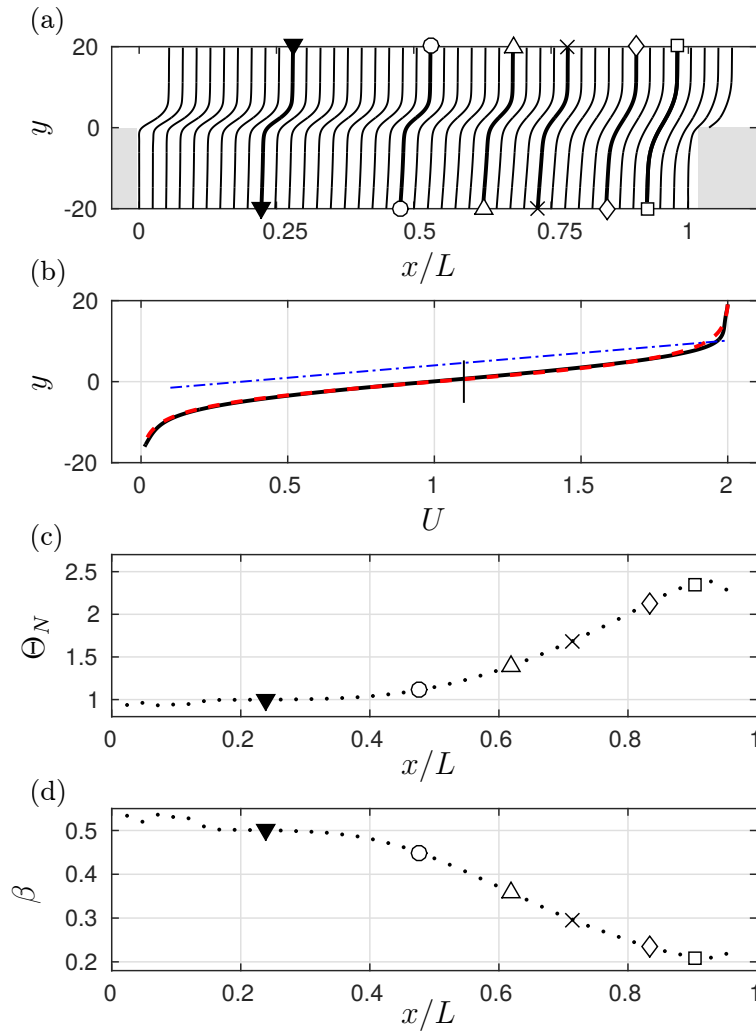


FIGURE 6.32: (Color in PDF) Fitting of analytical velocity profile from equation (6.71) to experimental data from Basley et al. (2010). (a) Cross-stream velocity profiles inside the cavity. In bold with symbols the selected cases for the linear stability analysis. (b) Exemplary fitting of the analytical profile (red $--$) to the experimental profile (black $—$) at $x/L = 0.93$. The fitted maximum velocity gradient is depicted in blue ($-\cdot-$). (c) Normalized momentum thickness Θ_N and (d) parameter β as a function of the cavity length. Symbols as in table 6.4.

TABLE 6.4: Summary of cases for analysis with slowly varying base flow

Name	x/L	Θ_N	β	Symbol
C01	0.23	1.00	0.50	$\blacktriangledown$
C02	0.48	1.11	0.45	$\circ$
C03	0.63	1.40	0.35	$\triangle$
C04	0.73	1.70	0.29	$\times$
C05	0.85	2.08	0.24	$\diamond$
C06	0.93	2.30	0.21	$\square$

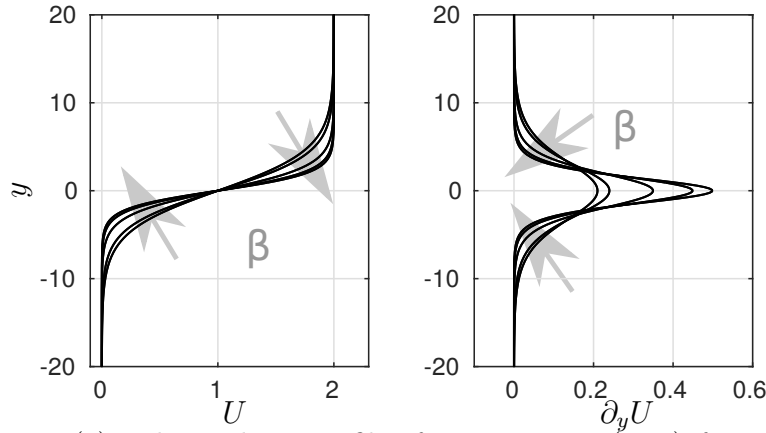


FIGURE 6.33: (a) Velocity base profiles from equation (6.71) for case LH21 from chapter 3 for $0.21 < \beta < 0.5$. (b) Shear strength base profiles. Arrows indicate how profiles change for increasing β .

6.8.2 The Kulikowskii condition for a slowly varying base flow

In this section, the Kulikowskii condition, derived in section 6.5, is extended to a slowly spatially developing base flow. The same argument as in section 6.5 is invoked. The sole difference is that the wave number k is now a function of the streamwise variable x and hence its integral along the streamwise direction is considered. As before, total reflection at the upstream and downstream cavity edges is assumed. The initial infinitesimal amplitude is called A_0 . Upstream traveling waves are denoted k_+ , while downstream traveling waves are denoted k_- . A perturbation wave

$$\Psi(x, t) = A_0 e^{i \int_0^x k_+(\omega, x') dx' - i\omega t} \quad (6.75)$$

is considered, which is emitted at P ($0 < P < L$), where L is the normalized cavity length. When it reaches the downstream boundary, Ψ is reflected according to reflection coefficient R_1 , and sent back upstream. The wave on its way upstream is described by

$$\Psi(x, t) = R_1 A_0 e^{i \int_0^L k_+(\omega, x) dx} e^{i \int_0^x k_-(\omega, x') d(x'-L) - i\omega t} \quad (6.76)$$

It reaches the upstream boundary, where it is reflected again, according to the reflection coefficient R_2 . When it travels downstream again the wave is described by

$$\Psi(x, t) = R_1 R_2 A_0 e^{i \int_0^L [k_+(\omega, x) - k_-(\omega, x)] dx} e^{i \int_0^x k_+(\omega, x') dx' - i\omega t} \quad (6.77)$$

Due to the requirement that $\Psi(x, t)$ must be single valued, equations (6.75) and (6.77) must coincide, which leads to the Kulikowskii Condition for a varying base flow:

$$R_1 R_2 e^{i \int_0^L (k_+(\omega, x) - k_-(\omega, x)) dx} = 1 \quad (6.78)$$

Note that the frequencies for the downstream traveling $k_+(\omega)$ wave and the upstream traveling $k_-(\omega)$ wave do not change. Hence, the frequencies of $k_+(\omega)$ and $k_-(\omega)$ are the same. The Kulikowskii condition is a complex equation which can be split up into a real and an imaginary part. This yields

$$\int_0^L \Delta k_r dx = \int_0^L k_{r+}(\omega, x) - k_{r-}(\omega, x) dx = \pi m \quad (6.79)$$

$$\int_0^L \Delta k_i dx = \int_0^L k_{i-}(\omega, x) - k_{i+}(\omega, x) dx = \ln \left[\frac{1}{R_1 R_2} \right] \approx 0, \quad (6.80)$$

where $m \in N_0$. As above, poles of the linear stability analysis which also solve equations (6.83) and (6.84) will hereafter be called *Kulikowskii Points*. For a finite number of streamwise evaluation points M the integrals are converted into sums and we obtain

$$\sum_{j=1}^M [k_{r+}(\omega, x_j) - k_{r-}(\omega, x_j)] \frac{\Delta x_j}{M} = \pi m \quad (6.81)$$

$$\sum_{j=1}^M [k_{i-}(\omega, x_j) - k_{i+}(\omega, x_j)] \frac{\Delta x_j}{M} = \ln \left[\frac{1}{R_1 R_2} \right] \approx 0, \quad (6.82)$$

where Δx_j is the fraction of the cavity length over which a local profile is considered constant. Note that $\sum_{j=1}^M \Delta x_j = L$ and hence for an equidistant distribution of the velocity profiles the equations (6.81) and (6.82), respectively, reduce to

$$\overline{\Delta k_r} = \frac{1}{M} \sum_{j=1}^M k_{r+}(\omega, x_j) - k_{r-}(\omega, x_j) = \frac{1}{M} \sum_{j=1}^M \Delta k_r(\omega, x_j) = \frac{\pi m}{L} \quad (6.83)$$

$$\overline{\Delta k_i} = \frac{1}{M} \sum_{j=1}^M k_{i-}(\omega, x_j) - k_{i+}(\omega, x_j) = \frac{1}{M} \sum_{j=1}^M \Delta k_i(\omega, x_j) = \frac{1}{L} \ln \left[\frac{1}{R_1 R_2} \right] \approx 0, \quad (6.84)$$

which can be understood as a spatial average of Δk over all local conditions, assuming local parallelism. All graphs $\Delta k_r(\omega, x_j)$ are summed over j and then divided by its total number M . This yields an average curve $\overline{\Delta k_r} - \omega_r$ which according these equations characterizes the weakly non-parallel flow condition.

6.8.3 Results

The results of the linear stability analysis in a finite domain for a slowly varying base flow for case LH21 from section 6.6 are presented. The first question that is addressed is how the pinch point (PP) in the complex wave number plane and respectively the branch point (BP) in the complex frequency plane changes, when different profiles along the streamwise

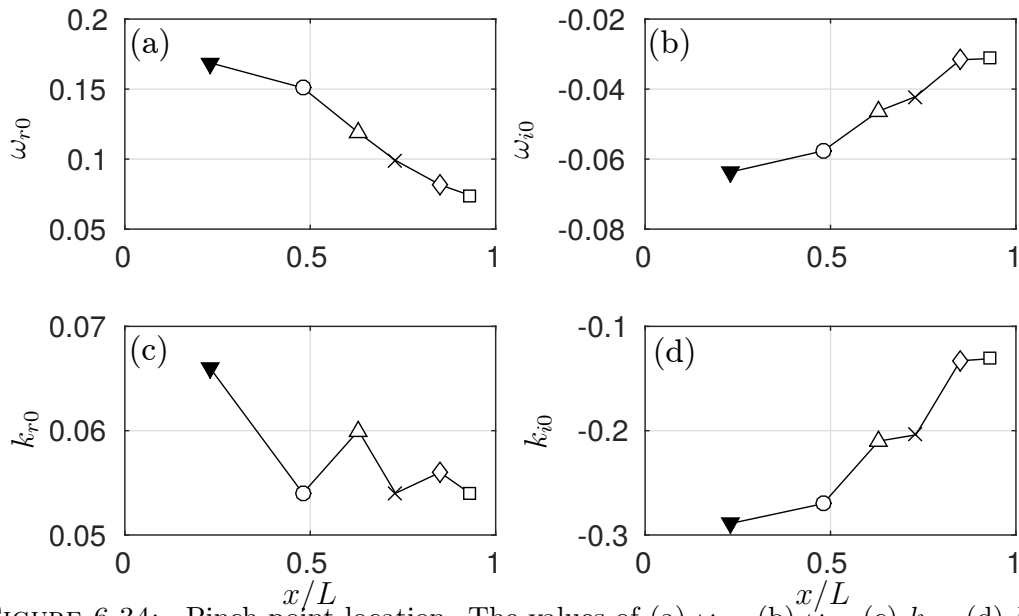


FIGURE 6.34: Pinch point location. The values of (a) ω_{r0} , (b) ω_{i0} , (c) k_{r0} , (d) k_{i0} as a function of the cavity length. Symbols as in table 6.4.

direction of the cavity are considered. The location of these singularities characterize the stability of the system (Landau & Lifschitz (1985); Huerre & Monkewitz (1985)). In the present analysis we obtain a PP and a BP for each local profile by carrying out a local linear stability analysis in an infinite domain, not considering the streamwise finiteness of the system. In order to obtain characteristic modes of the finite system the Kulikowskii condition (equations (6.83) and (6.84)) is applied. As shown in section 6.6 solutions to the Kulikowskii condition are found in the vicinity of the pinch point and branch point singularities. Finally, it will be shown how the different locally parallel approaches are assembled to a global solution for the inhomogeneous background.

Movement of the pinch point

Figure 6.34 shows the evolution of the complex frequency and the complex wave number of the saddle point (pinch point) along the cavity length (increasing β or in equal measure increasing x). The frequency ω_{r0} , which defines the selectable frequency range, decreases linearly along the cavity length (figure 6.34(a)). If ω_{r0} decreases along the cavity length, the selectable frequency range increases and subsequently lower frequencies can be selected from the frequency spectrum.

The imaginary part of the frequency ω_{i0} is an important quantity. As outlined in detail in Landau & Lifschitz (1985) and Huerre & Monkewitz (1985), it indicates whether the perturbation is absolutely or convectively unstable and the frequency selection criterion by Chomaz *et al.* (1991) is based on the local maximum of ω_{i0} . The value must be negative for the system to be convectively unstable. A positive value of ω_{i0} leads to an

absolute instability. As can be seen from figure 6.34(b), ω_{i0} increases monotonically along the cavity length, which means that the system becomes more unstable and closer to an absolute instability. However it does not reach the real ω axis and hence the flow stays convectively unstable in accordance with all known experiments. Note that ω_{i0} exhibits no maximum along the cavity length and hence Chomaz's selection criterion would simply select the most downstream position.

The real part of the wave number k_{r0} , depicted in figure 6.34(c), has little meaning for the present analysis since it is rather the difference of the real part of the wave numbers $\Delta k_r = k_{r+} - k_{r-}$ of the downstream and upstream traveling wave (Δk_r) that is of importance. In the pinch point by definition $\Delta k_r = 0$ since $k_{r+} = k_{r-}$. Δk_r will be addressed in the following section.

The imaginary part of the wave number k_{i0} determines whether the pinch point is located in the upper or the lower k half plane and is hence responsible for the spatial amplification. Figure 6.34(d) shows how the flow becomes more stable with increasing x . It does not cross the real axis and hence the findings from section 6.6 stay valid: the downstream traveling wave is amplified (positive x and negative k_{i0}), while the upstream traveling wave is evanescent (negative x and negative k_{i0}).

In all cases the quadratic form of the pinch point $k - k_0 \sim \pm(\omega - \omega_0)^{\frac{1}{2}}$ is confirmed as postulated by Landau and Lifschitz (1985). In short, the flow without boundaries becomes temporally more unstable with increasing x and allows for a lower frequencies to be selected.

Evaluation of the global Kulikowskii Condition

The global Kulikowskii condition was evaluated for the six local profiles along the streamwise direction. The results are depicted in figure 6.35(a). The black solid lines indicate the $\Delta k_r(x)$ curves, obtained by the evaluation of the second Kulikowskii condition (equation 6.84) at different streamwise positions. The discretization of the continuous solution of equation 6.84 is obtained by applying the first Kulikowskii condition (equation 6.83), which results in the dashed dotted horizontal lines $\Delta k_r = \text{const}$. As above, the intersection of the horizontal lines with the black solid curves yield the Kulikowskii points, which define a set of frequencies that characterizes the finite system.

So far we used a single length scale (Θ_{eff}) to describe the problem. With this single length scale at each streamwise position a different set of Kulikowskii points is obtained as follows from the different curves in figure 6.35(a). The global Kulikowskii equations (6.83 and 6.84) then require to form a mean quantization curve from these local curves. However, when instead the quantization curves at each position are normalized with their local length scale, *i.e.* when each local quantization curve is multiplied with the respective normalized momentum thickness $\Theta_N(x)$ according to

$$\Delta k_r = \Delta k_r(x)\Theta_N(x) \quad \text{and} \quad (6.85)$$

$$\omega_r = \omega_r(x)\Theta_N(x), \quad (6.86)$$

all $\Delta k_r - \omega_r$ curves collapse onto a single curve as can be seen in figure 6.35(b). As follows from the definition of Θ_N , the collapsed curves are identical to the curve that results from the streamwise position at which Θ_{eff} is obtained (red dashed curve in figure 6.35(b)). This is the same quantization curve as obtained from the analysis assuming a parallel base flow (*cf.* section 6.6). The collapsed profiles makes it unnecessary to take the mean.

Each of the black curves in figure 6.35(a) is the result of a linear stability analysis in a finite domain, performed with the local streamwise velocity profile $U(y, x)$ plugged into the dispersion relation. At all streamwise positions the equations of motion were normalized with the same effective momentum thickness Θ_{eff} , obtained at the position $x/L = 0.22$ in agreement with [Tuerke *et al.* \(2015\)](#). The linear stability analysis at each streamwise position results in a pinch point (PP) around which Kulikowskii points are located. Depending on the form of the local velocity profile the PP moves within the complex wave number plane (*cf.* figure 6.34), which results in different values of $\omega_{r,0}$. The topology around the PP determines the form of the Kulikowskii curve $\Delta k_r - \omega_r$. Since all profile share the same maximum (free stream) velocity U_∞ , the only parameter left is the momentum thickness. When multiplied with the normalized momentum thickness $\Theta_N(x) = \Theta(x)/\Theta_{eff}$, which characterizes the deviation of the velocity profile from the position at which Θ_{eff} is obtained, all curves collapse onto a single curve. The collapsed curve is identical to the curve which was obtained for Θ_{eff} under the parallel flow assumption.

This result indicates, that the parallel flow assumption, proposed in section 6.2, is in fact a valid assumption even for a weakly non-parallel flow, since at each streamwise position the flow sees the local conditions, which when used to normalize the results, collapse the frequency quantization mechanism onto a single curve.

The immediate question that arises is concerned with the meaning of the two non-dimensional numbers $\Delta k_r \Theta_N$ and $\omega_r \Theta_N$, respectively. The term $\Delta k_r \Theta_N$ refers to a ratio of characteristic length scales, while the term $\omega_r \Theta_N$ refers to a ratio of characteristic velocities. Let us picture an instability wave traveling inside the finite domain. We can imagine that the two characteristic length scales are the vertical length scale, which is characterized by the effective momentum thickness Θ_{eff} , and the horizontal length scale, characterized by the interaction wave length $\Delta k_r = k_{r+} - k_{r-}$. The two velocity scales are

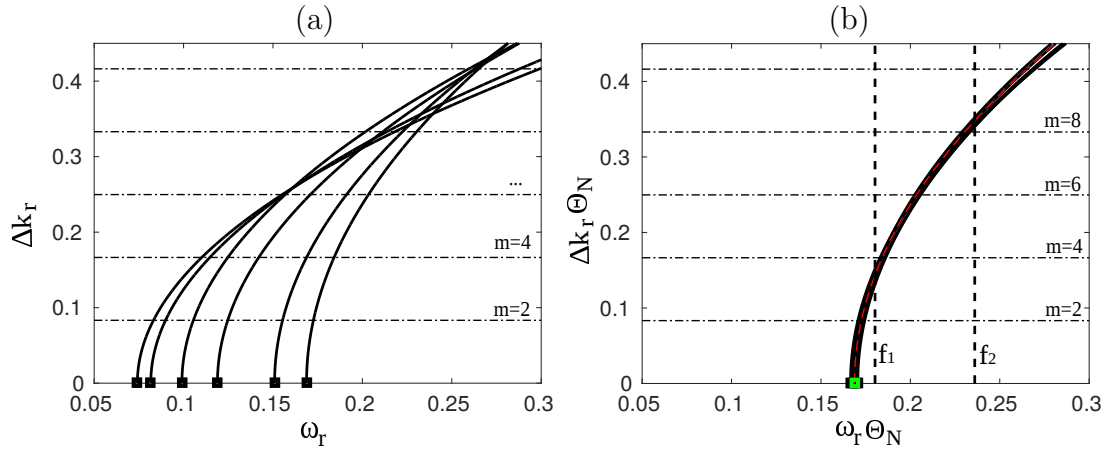


FIGURE 6.35: (a) Graphical evaluation of Kulikowskii condition (equations 6.83 and 6.84) for the local approaches at different streamwise positions. Black squares indicate ω_{r0} for the local cases. (b) Collapse of all curves from figure (a) after multiplying with Θ_N according to equation 6.85. Red dashed curve and green square indicate results obtained under the parallel flow assumption with $\Theta = \Theta_{eff}$ (*cf.* section 6.6). Dashed vertical lines indicate experimentally measured frequency peaks f_1 and f_2 from [Basley \(2012\)](#).

the vertical velocity of the wave versus the horizontal propagation velocity (*i.e.* the group velocity) of the wave. Interestingly, the relation between both non-dimensional numbers is constant along the cavity which yields a self-similarity of the instability properties in the finite domain.

These results open new perspectives on the global nature of the linear stability analysis in a finite domain and are subject of on-going research.

6.9 Conclusions

The complex composition of shear layer spectra of single and double cavity flow, observed experimentally, was studied with the tools of linear stability theory.

First, a temporal local linear stability analysis in an semi-infinite domain was carried out, based on the experimental sinh velocity profiles for the double cavity and the tanh profile for the single cavity. The analysis allowed to explain experimental findings from chapter 3, in which it was observed that the peaks for close intercavity distances broaden (stronger side bands) and move to higher Strouhal numbers. The range of amplified frequencies, and the collapse of the symmetric and asymmetric branch, also coincide with experimental results presented in chapter 3. However, without taking into account the finiteness of the system, only a single peak can be predicted.

The problem of the non-harmonic frequency peaks, commonly found in single and double cavity flow, was addressed in a second analysis, which takes into account the finiteness of the system in streamwise direction. A theoretical framework for a linear stability analysis in a finite streamwise direction was developed. A wave reflection scenario, first introduced by Kulikowskii (1966) and later revisited by Landau & Lifschitz (1985), was applied in which downstream traveling k_+ waves are reflected into upstream traveling k_- waves and vice versa at the respective boundary. Total reflection was assumed, though the implications and consequences of non-total reflection were outlined. As a result the wave reflection condition, the so-called *Kulikowskii Condition*, discretizes the frequency band in which the amplified non-harmonic frequencies are observed experimentally. It was found that the downstream traveling k_+ waves are spatially amplified and temporally damped, while the upstream traveling k_- waves are spatially and temporally attenuated. This results in a convective type of instability, a necessary condition for Kulikowskii's theory to be applicable. Solutions to the Kulikowskii Condition become possible only after the pinch point singularity is crossed, which yields a minimum frequency $\omega_{r,min}$ (or St_{min} , respectively) that can be selected by the LSAFD (Linear Stability Analysis in a Finite Domain). A criterion for choosing the momentum thickness of the base profile, used as the length scale in the linear stability analysis, was presented in agreement with previous works in the literature (Ho & Huerre (1984); Hammond & Redekopp (1997); Tam & Block (1987)). Kulikowskii's Condition was evaluated for single and double cavity flow, both times, yielding discrete frequencies ω_{rm_1} and ω_{rm_2} , respectively, that compare well with experiments from chapter 3 and from the literature Lusseyran *et al.* (2008); Basley (2012). It was found that the finite extent of the geometry is a necessary condition for the discretization of the frequency band.

The single cavity was investigated at six different Reynolds numbers. The low frequency content of the spectrum of the experimental data of Lusseyran *et al.* (2008); Basley (2012) f_0 (figure 6.17) was found to lie outside of the predicted regime ($\omega > \omega_{r,min}$). It is concluded that another mechanism (Bres & Colonius (2008); Villermaux & Hopfinger (1994)) must be at play than for the higher frequencies peaks f_1 and f_2 which enter the predicted regime and compare well with experimental results (Lusseyran *et al.* (2008); Basley (2012)). The frequency of mode 1 predicted by Rossiter's semi-empirical formula (equation (6.65)), which is based on an compressible assumption, is also found outside of the regime ($St < St_{min}$). Frequencies of Rossiter's mode 2 and 3 enter the predicted regime, which is in agreement with Yamouni *et al.* (2013). However, the results of Yamouni *et al.* (2013) do not compare well with experimental data by Lusseyran *et al.* (2008); Basley (2012). Results indicate that the total growth rate, computed from the interaction wave length Δk_r , is responsible for the amplification of higher harmonics with increasing L/Θ_0 , as observed experimentally in chapter 3 and also reported in the literature by Basley

(2012).

The double cavity was investigated at a fixed Reynolds number and variable intercavity distance D/L . It was found that for case DC05 ($D/L = 0.1$) the minimum selectable frequency St_{min} is higher than the typical shear layer frequencies, which is in agreement with experimental findings, that did not show any significant frequency peaks in this Strouhal number range. For $D/L = 0.2$ only the second peak $St_{L,2} \approx 1.4$ enters the regime selected by the LSAFD, which again is in agreement with experimental results from chapter 3, where the peak at the typical cavity shear layer frequency $St_{L,1} \approx 1$ was found to be absent. For $D/L > 0.3$ the experimental results from chapter 3 show two incommensurate frequency peaks which compare well to certain modes m_1 and m_2 , selected by the LSAFD.

In a final section it was shown how the theory for a linear stability analysis in a finite domain under a parallel flow assumption, can be extended to a weakly non-uniform base flow. The equations for the wave reflection scenario were derived and the analysis was carried out exemplary for one case of single open cavity flow. The quantization mechanism, found for the analysis under parallel flow assumptions, was confirmed. When the results at each streamwise position were multiplied by the local normalized momentum thickness Θ_N , all Kulikowskii curves collapsed on a single curve, identical to the curve found under the parallel flow assumption. The results suggest, that the parallel flow assumption is in fact a valid assumption even for a weakly non-parallel flow, since at each streamwise position the flow sees the local conditions, which when used to normalize the results, collapse the frequency quantization mechanism onto a single curve.

The results presented in this chapter indicate that the rather simple linear wave interaction model which is based on a local linear stability analysis and Kulikowskii's Condition, can explain the existence of non-harmonic modes, observed experimentally. Thus in the incompressible limit the mechanisms responsible for the existence of non-harmonic modes are at least partly due to the reflection and the linear interaction of instabilities in the shear layer. This mechanism is not included in the acoustic feedback mechanism of Rossiter's formula. In our model the upstream traveling k_- waves are found to travel substantially faster than the downstream traveling k_+ waves, though they do not reach the speed of sound.

Chapter 7

The Vortex Carousel

7.1 Introduction

In this chapter the recirculation region of open single cavity (SC) and double cavity (DC) flow is examined with respect to the role it plays in a hydrodynamic feedback mechanism. [Burroughs & Stinebring \(1994\)](#) conducted experiments of open cavity flow in water and noted that the cavity tones are generated by vortices, shed at the upstream edge of the cavity and injected into the cavity at the downstream edge. The tone is produced when vortices, which are transported within the cavity back to the upstream edge, interact with the new vortices shed at the upstream edge to amplify the instabilities in the separated flow at the upstream edge of the cavity. Based on data from the 2D numerical simulations presented in chapters 4 and 5, the described mechanism will be illuminated in greater detail in this section.

To this end, results from the dynamical system analysis, carried out in section 5.2, are linked to the flow dynamics, especially the vorticity flow field in the intracavitary region of SC and DC flow. This reveals a feedback mechanism, which enhances organized oscillations. As mentioned by [Rockwell & Naudascher \(1978\)](#) and also [de Vicente *et al.* \(2014\)](#), shear-layer modes are primarily two-dimensional so that they remain confined in the (x,y)-plane, generating predominantly spanwise vorticity. The present work therefore focuses mostly on data from 2D numerical simulations, though data from 3D direct numerical simulations as well as experiments is discussed and compared at the end of the chapter in order to underline the validity of this approximation.

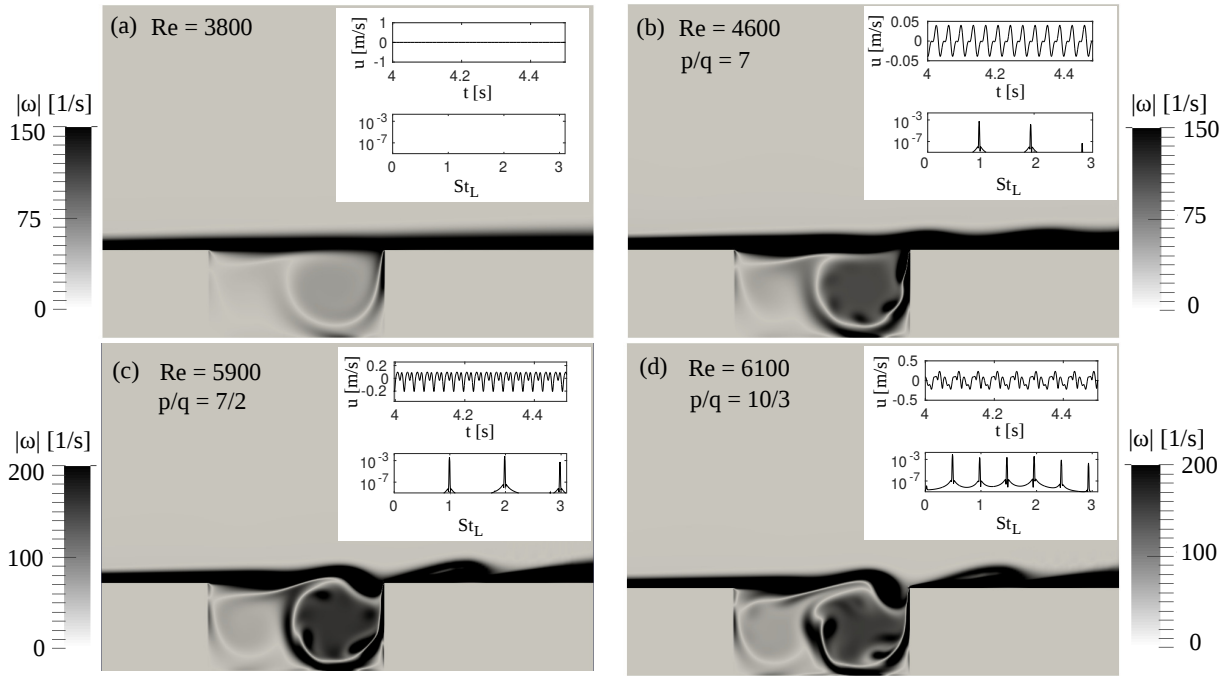


FIGURE 7.1: Flow patterns for the single cavity in the (a) steady regime, (b) periodic regime with $St_L \approx 1$ most amplified, (c) periodic regime with $St_L \approx 2$ most amplified, (d) intermediate regime (QP locked-on torus), together with respective time series recorded in probe point P_2 and its power spectra. Color code indicates vorticity magnitude. For details on the ratio p/q see text.

7.2 The vorticity flow field

The vorticity magnitude field is plotted in figure 7.1 for SC flow, and in figure 7.2 for DC flow ($D/L = 0.2$) at different Reynolds numbers. Also, the time series of the streamwise velocity signal, recorded in probe point P_2 and the associated power spectra are depicted. For low Reynolds numbers, $Re \lesssim 4000$, the flow is steady. A circular dipolar vortex sheet limits the main recirculation region, as can be seen for instance in figures 7.1(a) and 7.2(a). In the periodic regime, the shear layer exhibits oscillations of such magnitude that vorticity of the shear layer is regularly injected at the rear edge of the cavity into the main recirculation region. The injected vorticity forms small vortices at the frequency of the shear-layer oscillations. These regularly spaced vortices circumvent along the dipolar vortex sheet, creating a “carousel” type formation. The number of small vortices in the carousel depends on the value of D and on the velocity of the incoming flow, which defines the frequency at which the shear layer oscillates, and hence the rate at which small vortex structures are injected into the cavities. The vorticity strength of these vortices decreases along their circular motion in the carousel. The existence of these small vortices is readily seen in the spectrum. They leave a signature on the probe signal as a consequence of their cyclic passage.

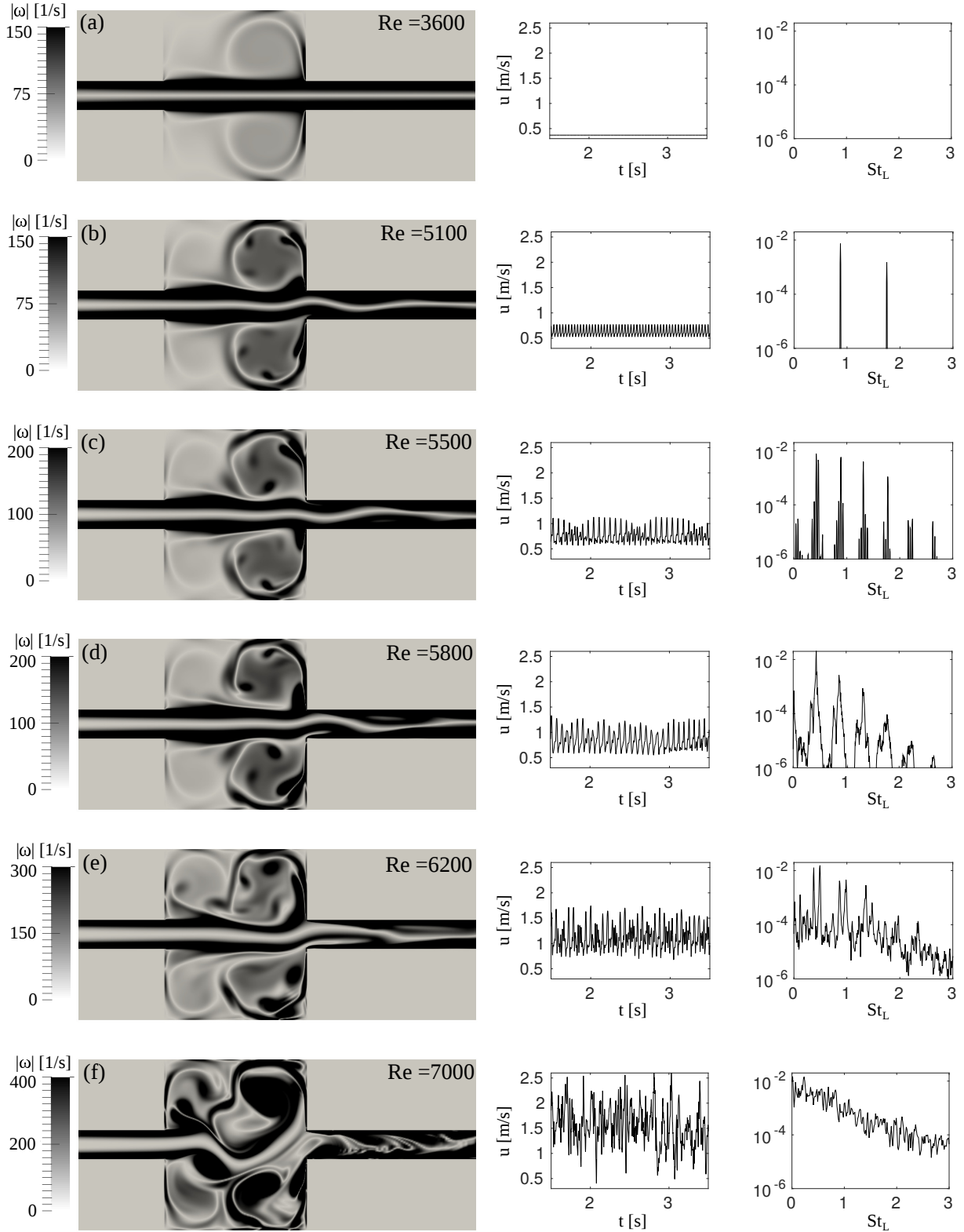


FIGURE 7.2: Flow patterns for case $D/L = 0.2$ in (a) steady regime, (b) periodic regime, (c) intermediate regime (torus), (d) torus folding, (e) toroidal chaos, (f) fully developed chaos, together with time series recorded in probe point P_2 and respective power spectra. Color code indicates vorticity magnitude.

Henceforth, the analysis will focus on the interaction of the shear-layer with the main recirculation region.

7.3 Characteristic time scales

Different time scales of interest appear in the carousel phenomenon. We define T_{shl} as the characteristic time of the prevailing oscillations of the shear layer, T_{to} the turn over time of the main recirculation region, and T_{lt} the characteristic lifetime of the small vortex structures. The advective time $\tau_c = L/U_\infty$ is the natural reference time. It turns out that $T_{shl}/\tau_c \simeq 1$ in all cases. If the main recirculation region was considered as being driven only by the shear-layer in solid rotation, T_{to} should scale with U_∞ and H . In this respect, one could therefore expect that $T_{to}\Gamma/\tau_c$ (where $\Gamma = L/H$) also be little sensitive to the value of Re_L . However, this is not the case. Instead, we observe a dependence of both $T_{to}\Gamma/\tau_c$ and T_{to}/T_{shl} with Re_L . The ratio T_{to}/T_{shl} depends additionally on the momentum thickness Θ of the incoming boundary layer, which defines the shear layer thickness, among other parameters that will be further clarified. The lifetime T_{lt} is not easily determined as it is a function of the viscosity of the fluid, of the strength of the small vortices and of their interaction with the neighboring vorticity. A reference time can be considered assuming $T_{lt} \sim \eta^2/\nu$ where η is the radius of the small vortices, which depends on the amplitude of the shear layer oscillations and therefore on Re_L . The ratio T_{lt}/T_{to} determines, whether in a single snapshot of the vorticity field, the small vortices in the image were all created in the same round trip (figure 7.1(a)), or vortices from two or more consecutive round trips coexist (figures 7.1(c) and (d)).

7.4 Flow field dynamics of the periodic regime

For the periodic regime, the simplest configuration is obtained when $T_{lt} < T_{to}$, *i.e.* the small vortices dissipate before they complete one round trip in the carousel. Figure 7.1(b) illustrates this configuration, for the SC flow, where seven small vortices are formed during one turn over cycle ($T_{to}/T_{shl} \simeq 7$). These vortices have such a small lifetime, that during one cycle they do not appreciably reach the region of vorticity injection. The only frequency measured in probe point 2 is therefore the frequency of the shear layer oscillations.

For $2T_{to} \gtrsim T_{lt} \gtrsim T_{to}$, the vortex completes one round trip and returns to the injection position, exiting the shear layer in a feedback process. A periodic regime may still be encountered in this case over a wide range of the Re number. In such a situation, both the shear layer oscillations and the carousel turn over are synchronized, which is something usually expected from two coupled oscillators. Periodic behavior implies $T_{to}/T_{shl} = p/q$, with $p, q \in \mathbb{N}$ and $p > q$. Many different possible scenarios are found in SC and DC flows,

depending on the values of Re_L , ratio $T_{to}/T_{shl} = p/q$, and D . We shall illustrate some of them below.

In figure 7.2(b) we observe, for a DC flow case, a situation in which $T_{to}/T_{shl} = 4$. The vorticity plots show four small equispaced vortices per cavity in the main recirculation region. The structures of the carousels of each cavity are in opposite phase. Once a small vortex returns to the area of injection at the rear edge of the respective cavity, the shear layer injects a new vortex in its position. The power spectrum exhibits in this case the main peak at $St_L \simeq 1$, as the cyclic passage of the different vortices at probe position occurs at the frequency of the shear layer oscillations. The amplitude of the first harmonic is enhanced by the opposed carousel. The set of two carousels, exhibiting phase opposition, produces a signal that doubles the frequency of the shear layer. This is clearly illustrated in the spectrum of figure 5.1 in section 5.2.1 at probe point P_3 . There, the peak at $St \simeq 2$ is of larger amplitude than the peak at $St \simeq 1$.

Figure 7.1(c) illustrates the case of an extremely large value of D/L or equivalently a SC flow. For this case the ratio is $T_{to}/T_{shl} = 7/2$ and the dynamics is periodic. The power spectrum exhibits a dominant peak at $St_L \simeq 2$ in probe point P_2 , while a peak at $St_L \simeq 1$ is still present. The reason for this spectral distribution, is that $T_{lt}/T_{to} \simeq 2$. Consequently, the carousel is made of two intertwined patterns of small vortices, which doubles the frequency in P_2 . The first pattern is weakened since it undertakes its second turn in the recirculation region, while the second pattern carries a stronger vorticity as it comes along for the carousel ride in its first round trip (figure 7.1(c)).

The ratio T_{to}/T_{shl} , however, evolves as the Re number is increased. By further increasing Re_L , the single cavity reaches a periodic regime with $St_L \simeq 0.5$ in which $T_{to}/T_{shl} = 10/3$. A snapshot of this flow regime is shown in figure 7.1(d). Due to $T_{lt}/T_{to} \simeq 2$ the carousel pattern, though still periodic, becomes more complex.

To summarize, a periodic regime is obtained for either i) $T_{lt} < T_{to}$ or ii) $T_{lt} \geq T_{to}$ when at the same time $T_{to}/T_{shl} = p/q$, with $p, q \in \mathbb{N}$. In the first case the rhythm is set by the shear layer, since the small vortices dissipate before completing one round trip in the carousel. In the second case, the shear layer and the carousel are locked in by $T_{to}/T_{shl} = p/q$, with $p, q \in \mathbb{N}$. We did not observe cases with $T_{lt} \gtrsim 2T_{to}$ in the periodic regime.

7.5 Flow field dynamics of the intermediate regime

In the intermediate regime, incommensurate frequencies appear in the spectrum and the trajectory in phase space lies on a torus. We can expect that this new behavior manifests

itself in the dynamics of the carousel. We have shown in section 5.2.6, that many different situations are encountered in the intermediate regime. Let us illustrate some examples.

For $D/L = 0.2$ at $Re = 5500$ (figure 7.2(c)), the lifetime of the small vortices lasts approximately two turnover times $T_{lt} \simeq 2T_{to}$ before they dissipate. The carousel pattern exhibits recurrent states with $n = 3$ or $n = 4$ vortices. Expressed in terms of time scales we get: $3 < T_{to}/T_{shl} < 4$. The small vortices are not equispaced as a consequence of the continuous transitions between both states. The spectrum displays the frequency associated with the shear layer instability at $St_L \approx 1$ and a second incommensurate frequency $St_L \approx 0.45$ associated with the carousel's patterns.

Continuing with figure 7.2(d) at $Re_L = 5800$, the three-vortex pattern per cavity prevails. The spectrum shows both the frequency associated with the shear layer oscillations ($St_L \approx 1$) and with the passage of the three small vortices. The frequency ratio between the values of the frequencies of the third and fourth modes, which is $\simeq 3/4$, can be associated with the recurrence in each cavity of a three vortex pattern instead of a four vortices pattern, synchronized with the shear layer instability. The ratio of the new peaks, at $St_L \simeq 0.45$ and $St_L \simeq 1.35$, reaches a value close to $1/3$ that can be associated with the cyclic behavior of the three-vortex pattern in the intracavitary structures.

At Reynolds number close to 6200 (figure 7.2(e)), the intermediate regime loses coherence and the small vortices are not regularly spaced any more. Note, however, that the vortex carousel persists.

For the SC at $Re = 6200$ and the DC with $D/L = 0.9$ at $Re_L = 5600$, a small phase mismatch between the period of the shear-layer oscillations and the inner-flow recirculation time makes the time series at probe P_2 evolve slowly, in a seemingly periodic way, with a fundamental frequency $St \simeq 0.02$, generating a time-series that exhibits features of intermittency as discussed in section 5.2.7. This suggests that the two oscillators provided by the shear-layer oscillations, on the one hand, and the main recirculation region, on the other hand, may intermittently break phase-locking.

Finally, at Reynolds numbers close to 7000 (figure 7.2(f)), the recirculation regions deform strongly. The recurrent patterns of vortices in the carousel become largely disordered and are not easily determined. In fact, the vortices injected into the cavities by the shear layers are of such intensity, that after one turnover, they largely disturb the incoming shear layer and make the periodic feedback mechanism impossible. The counterpart in phase space, to this last dynamics, is the torus breakdown (cf. figure 5.2).

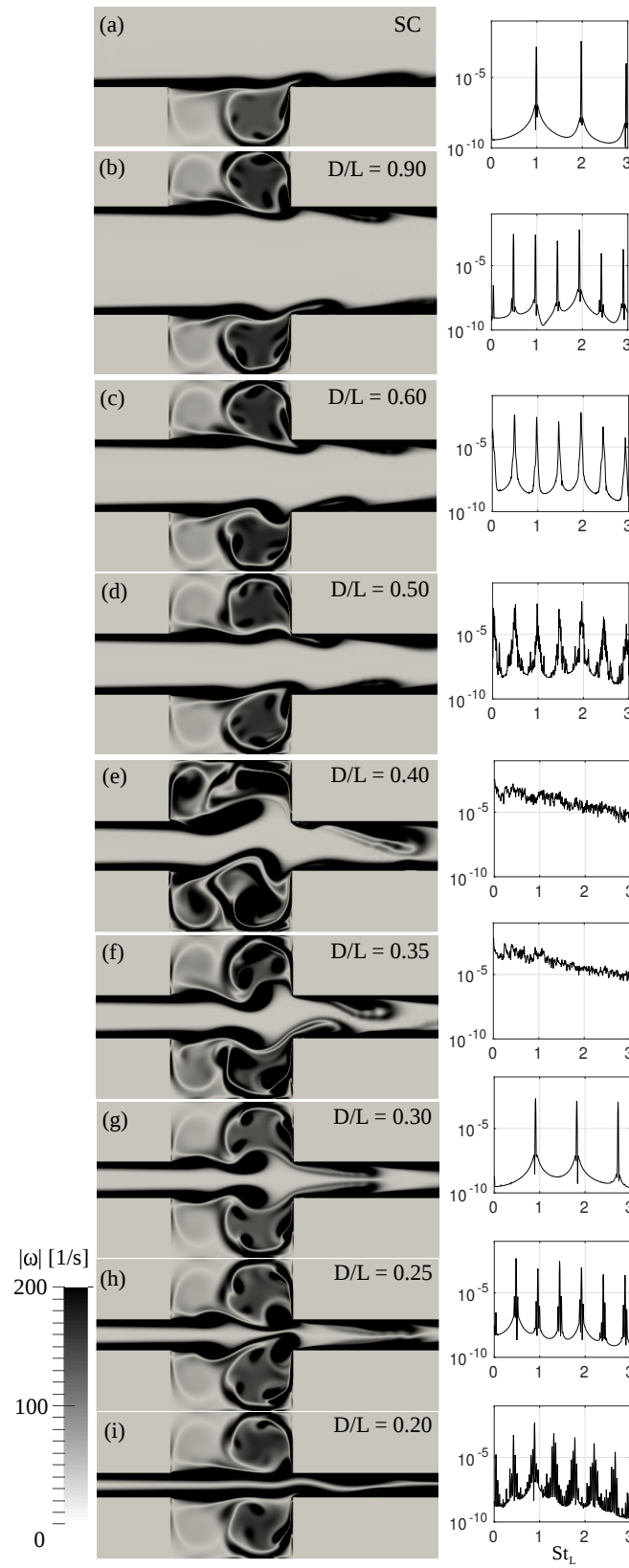


FIGURE 7.3: Flow patterns for different cavity distances D/L at $Re = 5500$ and the respective power spectra from time series recorded in probe point P_2 . Color code indicates vorticity magnitude. (a) periodic regime, (b) intermediate regime, (c) transitional regime, (d) transitional regime, (e) chaotic regime, (f) chaotic regime, (g) periodic regime, (h) intermediate regime, (i) intermediate regime.

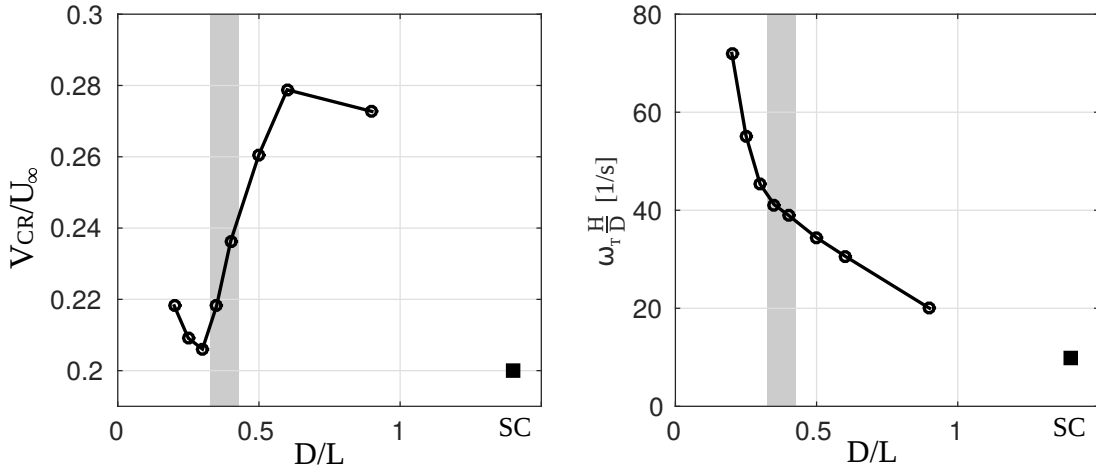


FIGURE 7.4: (a) Normalized carousel speed V_{CR} and (b) angular velocity $\omega_T = \frac{V_{CR}}{H/2}$ multiplied by the depth over distance ratio H/D as a function of the normalized cavity distance D/L for all cases in figure 7.3 ($Re = 5500$). The SC case is indicated by ■. The shaded areas indicate the distances for which chaotic behavior occurs (see figure 7.3).

7.6 Coupling Strength

Let us consider the spatial symmetry of the 2D vorticity magnitude field $|\omega(x, y)|$ in figure 7.3. Considering a spatial reflection symmetry (with respect to the center line $y = 0$), we distinguish between the following two cases:

$$\begin{cases} |\omega(x, y)| = |\omega(x, -y)| \Rightarrow \text{spatially symmetric (or varicose)} \\ |\omega(x, y)| \neq |\omega(x, -y)| \Rightarrow \text{spatially asymmetric} \end{cases}$$

Note, in the periodic regime the spatially asymmetric cases correspond to a sinuous flow configuration. Both symmetric and asymmetric cases are observed, though in most cases the flow is of the asymmetric type. For a given Reynolds number, as D/L is varied and the coupling between the cavities evolves, the flow symmetry may change. This is illustrated in figure 7.3 for $Re_L = 5500$. For close facing cavities, when $D/L < 0.25$, the flow configuration is spatially asymmetric. The symmetry changes as D/L increases and gets closer to $D/L = 0.3$, a value for which the flow adopts a spatially symmetric (varicose) configuration. Increasing D/L again, the flow returns to the a spatially asymmetric configuration. The emergence of spatial symmetry explains the discontinuity in the Strouhal number as a function of the Reynolds number for $D/L = 0.25$ at $Re_L = 5400$ and $D/L = 0.3$ at $Re_L = 5000$, respectively (*cf.* figure 5.7(g)-(h)). At these Reynolds numbers the flow adopts the varicose flow configuration. For the case $D/L = 0.3$ the change in symmetry occurs in the periodic regime without hysteresis, while for the case

$D/L = 0.25$ the change in symmetry is located in the intermediate regime and a hysteresis is observed. The change in symmetry before the transition to unsteady or turbulent flow conditions is also observed for the flow past a sphere by [Johnson & Patel \(1999\)](#) and [Tomboulides & Orszag \(2000\)](#).

Let us analyze the interaction of the feedback mechanisms of the carousel as we modify the ratio D/L for a fixed value of Reynolds number. For $Re_L = 5\,500$, the flow passes through all regimes as D/L is varied. As exemplified in figure 7.3(a), the number of small vortices in each recirculation region is four when the cavities are infinitely far from each other (SC). This corresponds to a case in which $T_{to}/T_{shl} = 4$ and $T_{lt} \simeq T_{to}$ and the recorded value compares favorably with the average recirculation velocity reported by other researchers ([Villiermaux & Hopfinger \(1994\)](#); [Back & Roschke \(1972\)](#)). A noticeable change in the carousel is observed, when a second cavity is present at proximity (figure 7.3(b)-(d)). We observe, that the frequency is almost constant but the number of small vortices in the carousel diminishes, which means that the carousel must turn faster, since the shear layer instabilities can not be strongly modified, as neither the Reynolds number nor the momentum thickness have changed. Figure 7.4(a) shows the carousel speed, normalized with the center line velocity U_∞ , as a function of the cavity distance. We observe, that the second cavity has a strong effect. The carousel speed increases by 43%, when D/L is varied from 1.4 (SC) to 0.9 (DC). This increase in angular velocity has in turn a critical effect on the feedback mechanism between the carousel and the shear-layer. Although the carousel is turning faster, the lifetime of the circumventing vortices has not been significantly modified. As a consequence, the injected vortices remain strong when they come back to the shear-layer region, which they now excite. As a result, the amplitude of the shear-layer oscillations is reinforced by this excitation, and the vorticity injected into the carousel at the impingement, becomes stronger. Mutual induction between the stronger and (relatively) long-living circumventing vortices can further increase their speed in the carousel. Indeed, the angular velocity of the carousel increases as D/L is reduced and reaches a maximal value for $D/L \approx 0.6$. For more contiguous cavities, the angular velocity of the carousel decreases again. This may be understood as a “blocking” effect due to the proximity of the shear-layer of the facing cavities. The shaded zones in figure 7.4 indicate the D/L -range for which chaotic behavior is observed. The speed drops to a minimum when $D/L = 0.3$, where the flow exhibits a varicose symmetry, as can be seen from figure 7.3(g). The carousel speed slightly increases for the closest distances (cases $D/L = 0.25$ and $D/L = 0.2$). In figure 7.4(b) the angular velocity, multiplied by the depth to distance ratio, is plotted against the normalized cavity distance D/L . Interestingly, the resulting curve decreases monotonously, while the trend for V_{CR} , as a function of D/L , is quite different, as shown in figure 7.4(a). Note however, that ω_T and V_{CR} are connected



FIGURE 7.5: QR-code for 2D visualization of the carousel pattern for a case with $D/L = 0.3$ at $Re_L = 5000 - 5100$ when symmetry switching occurs.

through the relation:

$$\omega_T \frac{H}{D} = \frac{V_{CR}}{H/2} \frac{H}{D}$$

The change in the carousel speed is even more striking when D/L is fixed and the Reynolds number is varied. In the single cavity, for instance, the number of circumventing vortices in the carousel changes from $n = 7$ to about $n = 3$ when the Reynolds number passes from 4 600 to 5 100, as can be seen in figure 7.1. The relative change in the in-coming flow velocity is about 10% while the turnover time increases by 50%. This non-linear behavior is common to all cases. The significance of the carousel mechanism to understand the coupling between shear layer and intracavitary flow and also to understand the mutual interactions between facing cavities, encourages future work on the subject.

Figure 7.5 depicts a QR-code, to be scanned with an appropriate device, in order to see a 2D visualization of the carousel pattern for a case with $D/L = 0.3$ at $Re_L = 5000 - 5100$. At this Reynolds number symmetry switching occurs.

7.7 The outflow pattern

It is also interesting to observe how the flow is modified at the exit channel as the different regimes take place (*cf.* figure 7.2). The outflow in the steady regime and in the periodic regime is well organized and the presence of the double cavity does not introduce significant modifications between the inflow and the outflow. As the Reynolds number is increased and the intermediate regime begins, the outflow becomes more complex and once the chaotic regime is reached, strong mixing is produced. As a potential application, DC flow could hence be designed as a laminar chaotic mixing device. In Figure 5.6, we can observe that the chaotic regime for DC flows is reached at much lower Reynolds numbers for closer cavity distances. This reflects the consequence of the strong coupling of the two

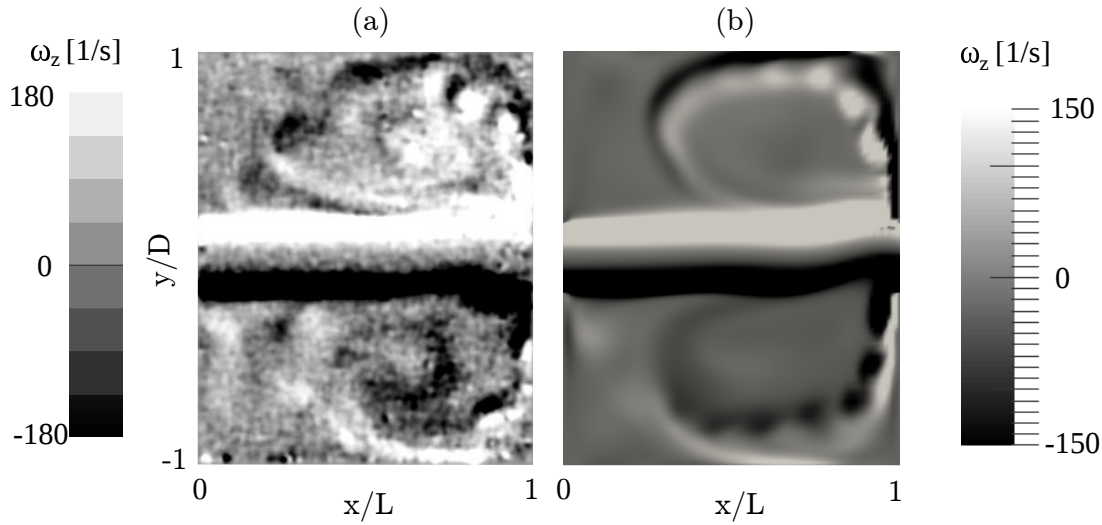


FIGURE 7.6: (a) Experimental instantaneous vorticity flow pattern from PIV measurements (BA-campaign) in the double cavity mid-plane for case $D/L = 0.2$ at $Re \approx 7500$. (b) Instantaneous vorticity field in spanwise direction from three-dimensional simulation at $Re \approx 5600$. The variable represented in the color maps of both figures is the vorticity in z .

individual cavities, and the advantage of a DC configuration when mixing enhancement is pursued.

7.8 Experimental example

In this section, the link between the dynamics of a flow confined to a two-dimensional space with the dynamics found in fully three-dimensional flows is illustrated. First the two-dimensional flow pattern is compared to the instantaneous field obtained by particle image velocimetry for a double cavity flow, see figure 7.6(a) and also figure 3.6 in section 3.1.4. In these experiments, the three dimensional flow also exhibits a carousel-like structure, although the recirculation region is less circular and undergoes low frequency oscillations in the streamwise direction, absent from their two-dimensional counterpart. In the illustrative example of figure 7.6(a), T_{lt} is small compared to T_{to} . The small vortices do not survive a complete loop and hence do not return to the downstream edge of the cavity, even though $Re_L \approx 7500$ is already quite large. [Basley *et al.* \(2011\)](#) and [Basley \(2012\)](#) also reported smoke visualizations and instantaneous PIV fields in which the “carousel” structure can clearly be identified - see for instance figure 2 and 3 in [Basley *et al.* \(2011\)](#). The same carousel structure is observed in three-dimensional numerical simulations of the DC flow, as shown in the snapshot of the vorticity field of figure 7.6(b), for $Re_L = 5600$ and figure 5.12 in chapter 5 at $Re_L = 8333$.

As already mentioned, the flow patterns resulting from these small vortices are strongly dependent on the relative values of the three time scales T_{shl} , T_{to} and T_{lt} . One may expect that the lifetime of small vortices T_{lt} be quite different in three-dimensional than in two-dimensional flows, because of the effect of the spanwise velocity on the time evolution of vorticity. In fact this hypothesis is backed by intracavitary velocity profiles shown in figure 5.16 in section 5.4.2. The intracavitary velocity profiles of both experimental campaigns and 3D numerical simulations are roughly half the value of the 2D numerical simulations. This reduces the carousel speed, which translates into a reduced T_{to} . The carousel does not complete a full round trip and hence does not reach and interact with the shear layer spanning the cavity. This offers a plausible explanation why in the 3D simulations the second non-harmonic peak is not observed (*cf.* chapter 5). Also the vorticity strength is less strong in the 3D simulations when compared to the 2D simulations (*cf.* scales in figure 7.6(b) and *e.g.* figure 7.2). The perfect cyclic behavior, observed in 2D simulations (figure 7.2 for $Re_L = 5100$), is therefore difficult to “tune” in a three-dimensional experiment. One possibility is to force the carousel into a circular trajectory by decreasing the cavity length, while maintaining the same cavity depth. This results in a cavity aspect ratio $\Gamma = L/H = 1$. As presented in chapter 4, two 3D simulation with $\Gamma = 1$ were performed to test this hypothesis. The results were presented in chapter 5. Interestingly, chaotic flow behavior was obtained in both cases, even though the maximum inflow velocity U_∞ and the normalized cavity length L/Θ_0 were kept constant with respect to the simulations with $\Gamma = 2$. Further research is needed, though these results strongly suggest that the carousel, once forced into a circular trajectory, disturbs the shear layer strongly enough to disequilibrate the whole system. Just as in the 2D case chaotic flow behavior follows.

These results are encouraging but indicate that careful experimental and numerical efforts are still necessary to conclude about the possible scenarios to be found in three-dimensional configurations of DC flows.

Additional evidence of the existence of the carousel can be found in Schmid (2010), who conducted a dynamical mode decomposition (DMD) analysis of a numerical (linearized) single square cavity and found a circular pattern of amplified modes. By means of a Chronos-Koopman analysis of our (non-linear) 2D numerical simulation data, closely following Cammilleri *et al.* (2013), we can confirm the circular pattern of the spatial Chronos-Koopman modes associated with the most amplified frequency peaks. Figure 7.7 exemplifies the spatial structure of a Chronos-Koopman mode of the absolute velocity at a frequency $St_L = 1.77$. A double cavity case with $D/L = 0.2$ in the quasi-periodic regime at $Re_L = 5500$ is used. The circular pattern of the spatial KM at $St_L = 1.77$ confirms the feedback mechanism.

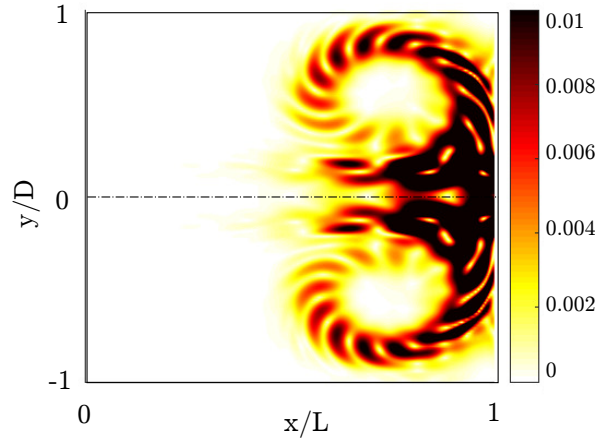


FIGURE 7.7: (Color in PDF) Spatial Chronos-Koopman mode at frequency $St_L = 1.77$. Color code shows absolute velocity. Double cavity case $D/L = 0.2$ with $Re_L = 5\,500$ in quasi-periodic regime.

7.9 Bio-inspired example

7.9.1 Background

The laryngeal ventricle, also called laryngeal sinus or Morgagni's ventricle, is a fusiform fossa situated between the vocal folds and the ventricular folds, also called true and false vocal folds, respectively. The geometry of the laryngeal ventricle defines a cavity on either side (left and right), whose function has attracted the attention in laryngology for well over a century ([Hodgkinson \(1896\)](#)). Recent studies have focused on scenarios with ventricular fold motion, which is atypical in normal voice ([Bailly *et al.* \(2014\)](#)). Yet, no studies have previously addressed the question of the double cavity fluid flow features of the Morgagni's ventricular geometry. In line with the research subject considered in this thesis, we examine the DC problem in a realistic laryngeal geometry, obtained from a CT scan of a non-pathological patient who was asked to sustain vowel /e/ in Spanish, as the scan was performed. This measurement was practiced at CINEOT-CONICET in the HIBA, an institute that participates in the MoSiME Project of CAFCI (CNRS-CONICET).

7.9.2 Setup

The scan was not time-resolved, and therefore the obtained geometry is a time-averaged picture of the larynx during normal phonation. As a result, the vibrating focal folds are slightly blurred in the image. The studied case makes use of the mid-coronal section of the CT scan, depicted in figure 7.8(a), which shows the same section that is outlined in figure 7.8(b). In order to obtain a mirrored double cavity geometry (symmetrical with

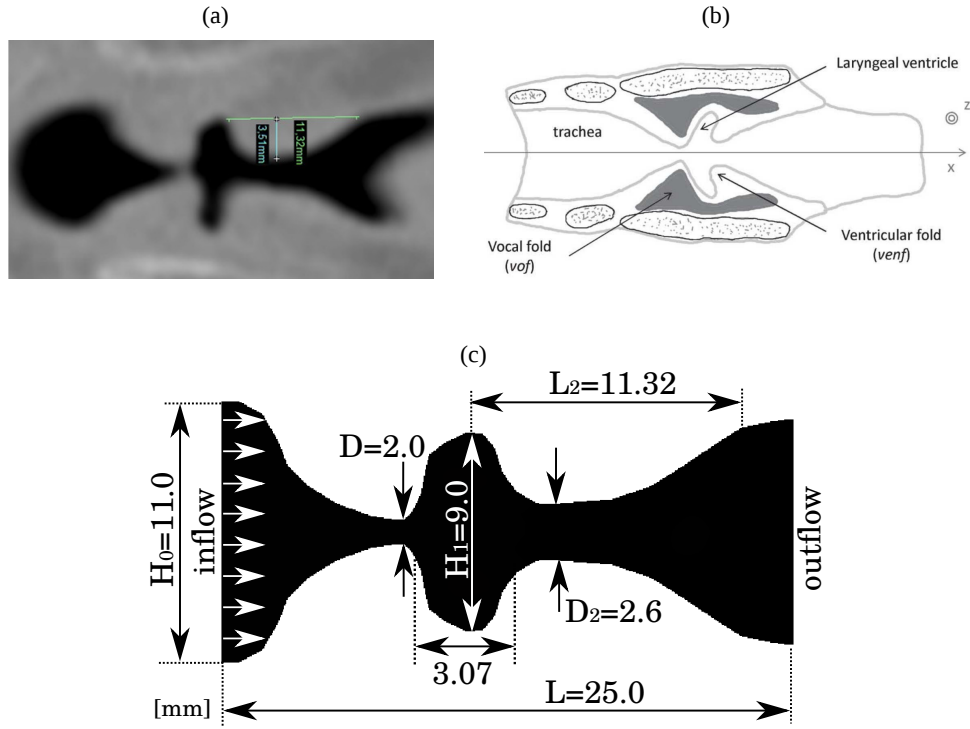


FIGURE 7.8: (a) Mid-coronal section of CT scan of larynx geometry performed at CINEOT-CAFCI. (b) Sketch of the coronal section of larynx, from [Bailly *et al.* \(2014\)](#). (c) Computational domain. Black background indicates the interior of the larynx geometry.

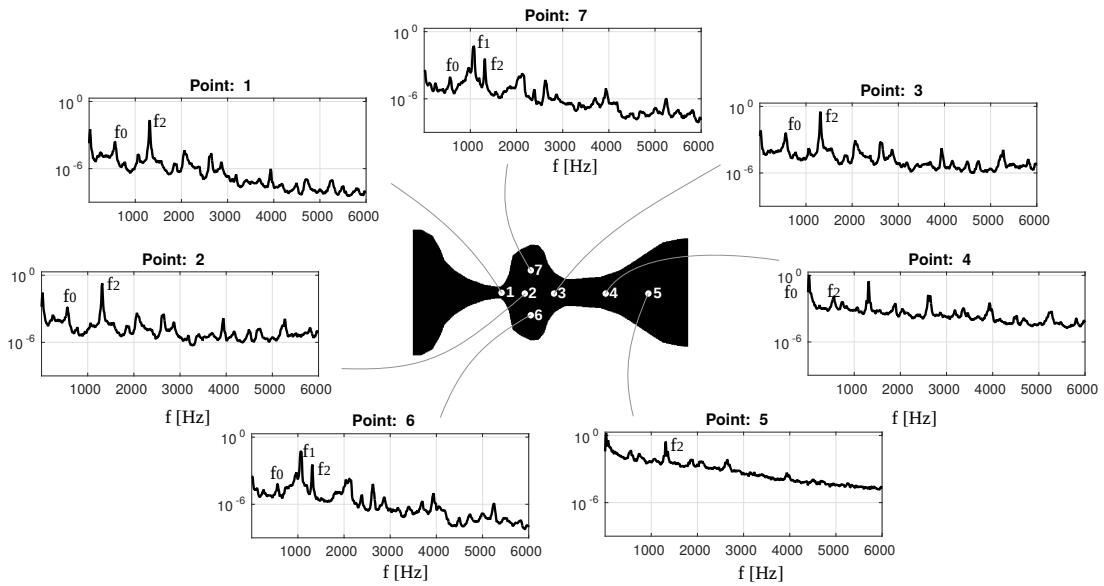


FIGURE 7.9: Power spectra, recorded at respective probe point locations in computational domain. Main frequency peaks f_0 , f_1 and f_2 are indicated.

respect to x axis), only the left side of the scan is used: the other side is obtained by reflection symmetry. The resulting computational domain is depicted in figure 7.8(c). The question addressed in this section is, whether or not the carousel phenomenon is present in this bio-inspired configuration, with the parameters that are typical of voice production.

The CT scan data was imported into the open source flow solver “Gerris”, a general-purpose fluid mechanics code, which solves the incompressible Navier-Stokes equations on a structured Cartesian adaptive grid. The numerical code *Sunfluidh*, presented in chapter 4, could not be used since it does not allow for arbitrary immersed geometries. Figure 7.8(c) shows the computational domain. The flow is from left to right, as indicated by the arrows. The cavity aspect ratio $\Gamma = L/H$ is close to unity. The boundary conditions are of Dirichlet type at the inflow and of Neumann type at the outflow. The inflow velocity profile is kept constant in time. The Reynolds number based on domain height is $Re_{H_0} = 2000$ as in [Sciamarella & Qu  r   \(2008\)](#).

7.9.3 Results

Figure 7.9 shows the spectra recorded at the indicated probe points in the computational domain for a case with a constant inflow velocity. The spectra show a rich composition with three main frequency peaks that dominate the spectra. The peaks are respectively found at

- $f_0 = 565 \text{ Hz}$
- $f_1 = 1076 \text{ Hz}$
- $f_2 = 1315 \text{ Hz}$

and as can easily be seen are incommensurate with respect to each other. The peaks are not present in all probe point signals, but vary with respect to the location. The three probe point on the center line of the domain 1, 2 and 3, respectively, show as expected very similar spectra due to its strong correlation in streamwise direction. In all three spectra, peak f_2 is most amplified while f_1 is absent. Moving further downstream towards the outflow (probe points 4 and 5) the peak-to-noise ratio decreases, though peaks f_0 and f_2 are still recognizable. Peak f_1 is not observed. Spectra of the intracavitary probe points 6 and 7 are very close to identical, suggesting that both cavities work in the same manner. In both spectra peak f_1 is most amplified, though now all three peaks are visible and f_2 is the second most amplified, suggesting that an intracavitary mechanism may be responsible for its presence.

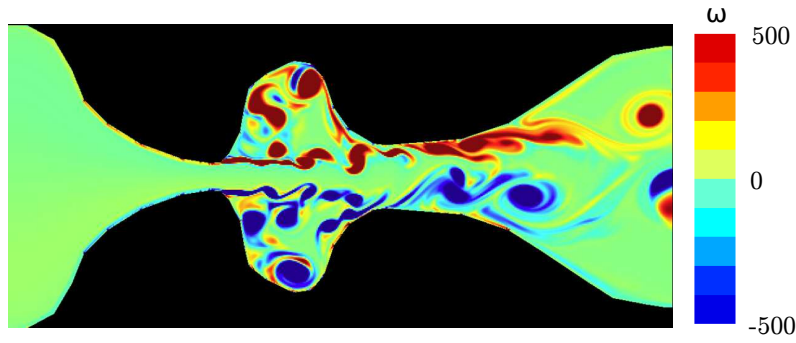


FIGURE 7.10: (Color in PDF) (a) Instantaneous vorticity field of the larynx model.

The observation of the vorticity field (figure 7.10) reveals small intracavitary vortex structures, that are injected by the fluctuating shear layers, issuing from the inflow channel. The small vortex structures circle inside the cavities in the same carousel-like manner as described above. At this specific Reynolds number, the small vortex structures are strong enough to come back to the shear layer with which they interact. As was shown in section 7.5, this is the reason why incommensurate frequencies appear in the spectra and hence the flow through the ventricle of Morgagni confirms the carousel mechanism for the first time in a bio-inspired geometry.

It is important to remark that the frequencies related to the carousel phenomenon are within the range of the distinctive frequency components of the acoustic signal produced by speech and singing (formants). The fact that the carousel phenomenon is encountered in a scenario that mimics the main aspects of voice production introduces a new perspective in terms of vocal-ventricular aerodynamic interactions that will be subject of future research.

On-going research includes imposing a pulsating inflow. This is a commonplace method to model glottal flow without considering vocal-fold motion (Chisari *et al.* (2011)). Using the same model as described above, with an inflow, pulsed at $f_G = 100$ Hz, it was found, that the times scales, defining the carousel mechanism, are within the same order of magnitude as the vocal fold pulsation. In fact, the life time T_{lt} of the intracavitary vortices was observed to be roughly three times greater than the period $T_f = 1/f_G$ of the pulsation. Therefore the intracavitary vortices of the carousel are active even when no mass flow passes the vocal chords. A thorough analysis of the three characteristic time scales, described above, *i.e.*, the turnover time of the main recirculation region, the lifetime of the small intracavitary vortices and the period of the shear layer oscillation with respect to the period of the pulsation T_f is subject of ongoing research.

7.10 Conclusions

An analysis of the vorticity field of 2D numerical simulations was carried out, in order to link the phase space dynamics from section 5 to the dynamics of the physical space. The visualization of patterns of small intracavitary structures serves to understand the underlying mechanisms of the feedback process and helps to identify the flow regime. The small vortices are created as a consequence of the shear layer oscillations, which regularly inject vorticity at the rear edge of the cavities. They describe a quasi-circular motion inside the main intracavitary recirculation region, forming a “carousel” like structure in each cavity. These vortices, when returning to the region of the instability onset, excite the shear layers. The relative phase synchronization of the carousels indicates the degree of coupling between the two cavities. The angular velocity of the carousel and the number of vortices present in the carousel depend, non trivially, on the Reynolds number and the distance between the cavities. For a given Reynolds number, as D/L is varied and the coupling between the cavities evolves, the flow symmetry and the flow regime may change. Also, the feedback mechanism was found to depend on the coupling strength.

The interplay of three characteristic time scales determines the properties of the observed phenomena: the turnover time of the main recirculation region, the lifetime of the small intracavitary vortices and the period of the shear layer oscillations. We illustrated the importance of these three time scales on the dynamics of the physical space and the associated dynamics of the phase space. The simplest scenarios correspond to lifetimes so short, that the vortices cannot accomplish one complete round trip in the carousel. In this situation, time series indicate a periodic regime with $St_L \approx 1$, associated with the shear layer oscillations. When the lifetime is long enough, the small circumventing vortices keep an intensity capable to excite the shear layers, and therefore to produce a significant feedback process. Depending on the separation D/L , the small vortices may even leave their signature in the signal of probes placed in the facing cavity. A periodic regime may be found for lifetimes larger than the turnover time, when the carousel synchronizes in such a way that any new vortex superposes with a dissipating one. When new vortices are injected in the space between two circumventing vortices, which have already completed one round trip, the spectrum reflects the frequency of the injection ($St_L \approx 1$) and of the surviving carousel structures. Lifetimes up to two round trips were observed, which explains the complex spectral composition found for the intermediate regime. The chaotic regime occurs when the feedback process becomes too strong: the vortices of the carousel do not decay sufficiently and do not synchronize with the natural oscillations of the shear layer.

Experimental evidence of the vortex carousel mechanism in double cavity flow was presented and can also be found in literature for open single cavity flows. Applying the Chronos-Koopman analysis, the spatial Chronos-Koopman modes confirmed the circular intracavity carousel structure. Numerical simulations of the flow in Morgagni's ventricle, an bio-inspired example of the double cavity, confirmed the existence of the carousel in a 2D approximation.

Chapter 8

Conclusions and Outlook

Context

The flow in a channel with two facing, identical and symmetric cavities, forming a so-called double cavity, was studied experimentally, numerically and analytically. A double cavity can be thought of as two identical coupled self-oscillating systems, subject to feedback due to the streamwise confinement. Previous works in the literature focused mostly on the two extremes of double cavity flow, *i.e.* confined jet-like flow, that is obtained when the cavities are very close (strongly coupled), and single open cavity flow, that is encountered when the intercavity distance becomes very large ($D \rightarrow \infty$ or uncoupled). In this work, intermediate cavity distances ($0.1 \leq D/L \leq 1$) at moderate Reynolds numbers ($3\,000 \lesssim Re_L \lesssim 14\,000$) were considered, in which the two shear layers, issuing from the laminar boundary layers of the incoming channel flow, did not merge along the cavity length.

The streamwise confinement of the geometry results in a feedback, which leads to information of the downstream impinging flow traveling upstream. Such a feedback organizes the flow globally and leads to well defined, sometimes non-harmonic, amplified frequency peaks in the shear layer spectrum. Over the past five decades most works in the literature have focused on the semi-empirical flow acoustic feedback mechanism named after [Rossiter \(1964\)](#), to explain the non-harmonic amplified frequencies observed experimentally. In the present work, the feedback in incompressible (double) cavity flow has been investigated from a purely hydrodynamical point of view. Results from a linear stability analysis, conditioned to a finite domain, show that the finiteness of the domain is responsible for the non-harmonic quantization of the frequency spectrum. The investigation of the intracavity vorticity field of two-dimensional numerical simulations revealed an hydrodynamic feedback mechanism, in which upstream traveling vorticity structures close the feedback loop and, depending on specific time scales, amplify harmonic and

non-harmonic frequencies.

Results

Data for the analysis has been acquired by mean of wind tunnel experiments and direct numerical simulations.

Two experimental campaigns with different measurement techniques were aimed at understanding the flow behavior of double cavity flow in the $D/L - Re_L$ parameter space. Time resolved and non-time resolved PIV (particle image velocimetry) as well as LDV (laser Doppler velocimetry) measurements were conducted for a wide range of Reynolds number ($3\,000 \lesssim Re_L \lesssim 14\,000$) and intercavity distances ($0.1 \lesssim D/L \lesssim 1$). Results from both campaigns were found in good agreement. Based on the recorded experimental data base, instantaneous and mean velocity flow fields, were investigated. Velocity inflow profiles were fitted to adequate analytical profiles from the literature. The intracavity double cavity profiles were found to fit well a hyperbolic sine profile. The momentum thickness of the incoming channel flow was documented for a great number of cases in the $D/L - Re_L$ parameter space and a method for the adequate choice of the length scale was proposed. Spectra, recorded at various probe locations inside the double cavity domain, revealed its global nature. Spectra at a single probe point in the impingement zone of one of the shear layers were recorded for a wide range of distances and Reynolds numbers in order to characterize the spectral signature of double cavity flow in the $Re_L - D/L$ parameter space. The downstream confinement organizes the flow globally, resulting in well defined shear layer peaks. It was shown how the shear layer spectra change as a result of the coupling between the two facing cavities.

Two dimensional (2D) and three dimensional (3D) high fidelity numerical simulations of the double cavity geometry have been performed using a numerical code based on the finite volume method. We restricted our analysis to cases in which the inflow profile was laminar, in order to exclude possible excitations associated with the intrinsic fluctuations of turbulent flows. Inflow conditions, mean velocity fields, rms fields and shear layer spectra of the streamwise fluctuating velocity were compared to the experimental results. With respect to the velocity flow field, 3D numerical simulations agreed better with the experiments than 2D numerical simulations. Intracavity velocities were overestimated by 2D numerical simulations. The spectral results from 3D numerical simulations compared well to experimental data in the periodic regime where a single frequency peak and its harmonics dominate the spectrum. However, the quasi-periodic regime, where two non-harmonic frequency peaks co-exist, was not observed in 3D numerical simulations.

In contrast, in 2D numerical simulations a quasi-periodic regime was observed. A chaotic regime was observed in all 2D simulations and in 3D simulations when the cavity aspect ratio was reduced. Experimentally the chaotic regime was not observed, however, it can not be excluded that it may exist at higher Reynolds numbers. An inherent difficulty of such high Reynolds number experiments is to keep the inflow laminar.

Based on time series obtained by 2D numerical simulations, a detailed nonlinear dynamical system analysis has been carried out, identifying different flow regime and a characteristic route to chaotic flow behavior for increasing Reynolds number.

The complex composition of shear layer spectra of single and double cavity flow, observed experimentally, has been studied with the tools of linear stability analysis. By means of a local temporal linear stability analysis in an semi-infinite domain, the coupling between the two facing cavities was studied. The results explained the broadening of frequency peaks and its shift to higher Strouhal numbers for close cavity distances (strong coupling). To account for the streamwise confinement of the flow, a novel theory for a linear stability analysis in a finite domain has been developed, based on a local spatio-temporal linear stability analysis, combined with a wave reflection scenario, first mentioned by [Kulikowskii \(1966\)](#). Kulikowskii's condition takes into account the reflection and interaction of downstream and upstream traveling instability waves. For both single and double cavity flows the novel analysis resulted in a discretized frequency spectrum, where some of the modes compared well to experimental data. It was found that the finite extent of the geometry is a necessary condition for the quantization of the frequency band. The presented results have indicated that the rather simple linear wave interaction model, which is based on a local linear stability analysis and Kulikowskii's Condition, may explain the non-harmonic modes observed experimentally. Thus in the incompressible limit the mechanisms responsible for the existence of non-harmonic modes have to be considered at least partly due to the reflection and the linear interaction of instabilities in the shear layer. This mechanism is not included in the acoustic feedback mechanism of Rossiter's formula. The upstream traveling waves from the Kulikowskii model have been found to travel substantially faster than the downstream traveling waves, though they do not reach the speed of sound.

A condition which selects a definite mode could however not be identified by the wave reflection model. It was concluded, that other additional mechanisms must play a role in the selection of the frequency modes. A detailed investigation of the back flow in the recirculation region of 2D numerical simulations revealed small vortex structures that are being advected upstream in "carousel" like manner, thus creating a hydrodynamic feedback. The small vortices are created as a consequence of the shear layer oscillations, which regularly inject vorticity at the rear edge of the cavities. They describe a quasi-circular motion inside the main intracavitary recirculation region, forming a carousel-like structure

in each cavity. When returning to the region of the instability onset, the small vortices interact with the shear layers, spanning the cavities and thus closing the feedback loop. The interplay of three characteristic time scales was found to determine the properties of the observed phenomena: the turnover time of the main recirculation region, the lifetime of the small intracavitary vortices and the period of the shear layer oscillations. For a given Reynolds number, as D/L is varied and the coupling between the cavities evolves, the flow symmetry and the flow regime may change. It was found that the carousel rotation velocity and hence also the feedback mechanism depend on the coupling strength.

Evidence of the vortex carousel was confirmed by the results of the 3D numerical simulations and the experimental results as well as by results from numerical simulations of the flow in a bio-inspired geometry. It can also be observed in the literature for open single cavity flow by various authors.

In summary, the presented work characterizes double cavity flow using experimental, numerical and analytical methods. Spectral analysis reveals the effect of coupling between the facing cavities. By means of a linear stability analysis conditioned to a finite domain and the analysis of intracavitary coherent structures (vortices) we present two novel, purely hydrodynamic feedback mechanisms and apply them to both double cavity flow and open single cavity flow. Hydrodynamic feedback is found to be capable of explaining experimentally observed non-harmonic amplified frequency peaks in the incompressible limit, a phenomenon common to many self-sustained impinging flows. Over the past decades many works in the literature have focused mainly on the compressible flow acoustic feedback mechanism, while leaving unnoticed the great importance and potential of hydrodynamics feedback.

Perspectives

The presented work answered some questions and raised many more. Future research is needed to address the following questions:

1. Elaboration of a model for (double) cavity flow, based on the feedback mechanism, revealed in the present work. A possible candidate for a model is the nonlinear delayed action model proposed by [Villermaux & Hopfinger \(1994\)](#).
2. Combining the quantization mechanism from the linear stability analysis conditioned to a finite domain with the hydrodynamic carousel feedback in order to obtain a coherent global model.
3. Application of the feedback mechanisms, revealed in the present work, to other impinging flows in the incompressible limit.

4. Design the double cavity as a laminar mixing device for industrial applications.
5. Continue and expand the started analysis of the vortex carousel in the realistic vocal fold flow geometry. 3D simulations and elastic vocal folds will contribute to more realistic conditions.
6. 2D numerical simulations revealed an intermittent regime in single and double cavity flow that has so far not been reported in the literature. Specially designed experiments and/or numerical simulations (3D numerical simulations) are needed to conclude on this subject.
7. When the inflow is laminar, there are no experimental reports of the chaotic regime for single cavity flows. Experiments with well-controlled inflow conditions are needed to answer this question.
8. Add additional complexities to the double cavity geometry, such as *e.g.* a pulsating flow and/or elastic walls.
9. Works by [Basley \(2012\)](#) and [Douay *et al.* \(2015\)](#) are concerned with the 3D features in single cavity flow. An interesting question to ask would be what are the 3D features in double cavity flow, how do they change with the intercavitary distance and what is their role in the feedback mechanism?
10. The current work is concerned with incompressible flow. An interesting question is how the presented results change for compressible flow conditions.
11. The collapse of the instability curves in the linear stability analysis in a finite domain with a varying base flow presents an interesting result that we will pursue in future work.

Chapter 9

Summaries

9.1 Resumen en Español

Esta tesis aborda el problema del flujo en un canal simétrico, con una expansión y contracción rápidas que configuran dos cavidades dispuestas frente a frente geometría que en adelante denominaremos doble cavidad. Se estudia el flujo de doble cavidad a números de Reynolds moderados con técnicas experimentales, numéricas y analíticas, haciendo variar la velocidad de entrada y la distancia entre cavidades. El estudio se focaliza sobre la interacción entre las dos capas de corte y la retroacción hidrodinámica intracavitaria en el límite de incompresibilidad. Con el objeto de obtener información sobre el comportamiento complejo del escurrimiento de doble cavidad, adoptamos distintas técnicas experimentales. Se utiliza la velocimetría por imágenes de partículas no resuelta en el tiempo (PIV 2D2C) para cuantificar el campo de velocidades en un plano elegido en el sentido de la envergadura. Los campos y los perfiles de velocidad en el canal de entrada y el dominio de la doble cavidad son característicos de una amplia gama de velocidades de entrada y de distancias entre cavidades. Para distancias intercavitarias intermedias o grandes, las velocidades de entrada pueden ser aproximadas correctamente por un perfil simétrico de capa límite tipo Blasius, y por un perfil de canal parabólico para distancias más pequeñas. En el interior de la cavidad, las velocidades en la dirección del escurrimiento se pueden aproximar de manera satisfactoria utilizando un perfil de seno hiperbólico. La PIV resuelta en el tiempo para las regiones intracavitarias permite estudiar en detalle la composición espectral del flujo intracavitario para ciertas configuraciones. Se utiliza la velocimetría Laser Doppler (LDV) para recuperar series temporales en sondas situadas en la región de interés. La técnica LDV se aplica a la caracterización del escurrimiento de doble cavidad para una vasta gama de velocidades de entrada y de distancias intercavitarias. A Reynolds bajo, el flujo se encuentra en un régimen estacionario, independiente de la distancia entre cavidades. A medida que se incrementa el valor del número de Reynolds, las capas de corte comienzan a oscilar a la frecuencia característica. Para

distancias intermedias, los espectros son similares a los espectros de la cavidad simple. Para distancias más pequeñas, los picos característicos de la capa de corte se ensanchan y se desplazan hacia valores ligeramente más altos del número de Strouhal. Cuando la distancia es mínima las dos capas límites evolucionan juntas, formando un jet oscilante y confinado.

Se realizan también simulaciones numéricas directas de la doble cavidad en dos o tres dimensiones en el espacio de parámetros $Re_L - D$ con Sunfluidh, un código basado en el método de volúmenes finitos. Las simulaciones bidimensionales son utilizadas para caracterizar el escurrimiento de doble cavidad en base a los datos registrados por una sonda puntual, utilizando herramientas de la teoría de sistemas dinámicos (teoría del caos). Se realizan simulaciones numéricas 3D para ciertas condiciones elegidas especialmente con el objeto de colmar la brecha entre las simulaciones bidimensionales y los resultados experimentales, necesariamente tridimensionales. Se presentan las semejanzas y discrepancias entre simulaciones numéricas 2D/3D y los resultados de los experimentos.

Se emplean con herramientas de la teoría de inestabilidad lineal para estudiar la composición compleja de los espectros de la capa de corte en la cavidad simple y en la cavidad doble. Se desarrolla un marco teórico para el análisis de un dominio finito en la dirección del flujo a partir de un análisis de inestabilidad lineal, local e incompresible en un dominio finito junto con un escenario de reflexión de ondas (retroacción), conocido como condición de Kulikowskii. La consideración de un dominio finito y la aplicación de la condición de Kulikowskii comporta una discretización de la banda de frecuencias: experimentalmente se observan frecuencias no armónicas que se amplifican. Este análisis novedoso describe exitosamente tanto el caso de la simple como el de la doble cavidad. Los resultados presentados indican que un modelo sencillo de interacción de ondas (retroacción) permite dar cuenta de la existencia de los modos no armónicos observados experimentalmente.

Un análisis del campo de vorticidad a partir de las simulaciones 2D conduce a (la identificación de) un segundo mecanismo de retroacción, puramente hidrodinámico, en la simple y en la doble cavidad. Este análisis permite vincular el análisis de la dinámica no lineal con el espacio físico. La visualización de patrones de pequeñas estructuras intracavitarias revela el mecanismo que subyace el proceso de retroacción y permite identificar el régimen del escurrimiento. Los pequeños vórtices, creados por la inyección de vorticidad en la cavidad a través de la capa de corte, describen un movimiento cuasi-circular en el interior de la zona de recirculación intracavitaria, formando una estructura de tipo “calesita” en cada cavidad. Los vórtices en la calesita realizan un recorrido que regresa a la región en la que se desencadena la inestabilidad, excitando la capa de corte y creando una retroacción determinada por la interacción de tres tiempos característicos: el tiempo de revolución en la zona principal de recirculación, el tiempo de vida de los vórtices intracavitarios y el período de las oscilaciones de la capa de corte. Se presenta evidencia experimental del mecanismo tipo calesita, que puede encontrarse también en la literatura para los

escurrimientos de cavidad simple.

En síntesis, esta tesis estudia escurrimientos de doble cavidad con técnicas experimentales, numéricas y analíticas en el límite de la incompresibilidad. Se considera una amplia gama de velocidades de entrada y distancias intercavitarias. Sobre la base de un análisis de estabilidad lineal en tiempo y espacio, condicionado por la finitud del dominio a través de la reflexión de ondas, conseguimos explicar la existencia de picos de frecuencias no armónicas para las capas de corte. El análisis del campo de vorticidad intracavitario obtenido mediante simulaciones numéricas 2D, permite poner en evidencia un mecanismo de retroacción hidrodinámica, que da cuenta de la composición espectral de las capas de corte en el escurrimiento de doble cavidad.

9.2 Résumé en Français

Cette thèse adresse le problème d'un écoulement dans un canal symétrique avec une expansion et une contraction rapides qui créent deux cavités face à face - une géométrie dorénavant dénommée "double cavité". Nous étudions l'écoulement de double cavité à des Reynolds modérés par des techniques expérimentales, numériques et analytiques, en faisant varier la vitesse d'entrée et la distance entre cavités. L'accent est mis sur l'interaction entre les deux couches de cisaillement et la rétroaction hydrodynamique intracavitaine dans la limite incompressible.

Afin d'obtenir information sur le comportement complexe de l'écoulement de double cavité, nous adoptons des techniques expérimentales différentes. La vélocimétrie par image de particules (PIV) 2D2C non résolu en temps est utilisée pour quantifier le champ de vitesse de l'écoulement dans un plan choisi dans le sens de l'envergure. Les champs et les profils de vitesse dans le canal d'entrée et le domaine de la double cavité sont caractérisés pour une large plage de vitesses d'entrée et des distances entre cavités. Les vitesses d'entrée peuvent être correctement approximées par un profil symétrique de couche limite de Blasius pour des distances intercavitaires intermédiaires ou larges, et un profil de canal parabolique pour des distances plus réduites. A l'intérieur du domaine de la cavité, les vitesses dans la direction de l'écoulement sont bien approximées par un profil de sinus hyperbolique.

La PIV résolu en temps pour les régions intracavitaires dans certaines configurations nous permet d'étudier en détail la composition spectrale de l'écoulement intracavitaine. La vélocimétrie Doppler laser (LDV) est employée pour situer des sondes permettant de récupérer des séries temporelles. La technique est aussi appliquée pour caractériser l'écoulement de double cavité pour une vaste gamme de vitesse d'entrée et des distances entre cavités. A bas Reynolds, l'écoulement est dans un régime stationnaire, indépendamment de la distance entre cavités. Au fur et à mesure que l'on monte en

Reynolds, les couches de cisaillement commencent à osciller à leur fréquence caractéristique. A des distances intermédiaires, les spectres sont similaires aux spectres de la cavité simple. A des distances plus petites, les pics caractéristiques de la couche de cisaillement s'élargissent et se déplacent vers des nombres de Strouhal légèrement plus élevés. Lorsque la distance est minimale, les deux couches limites progressent ensemble, formant un jet oscillant et confiné.

Des simulations numérique directes de la double cavité en deux et trois dimensions d'espace sont réalisées dans l'espace de paramètres $Re_L - D$ avec Sunfluidh, un code basé sur la méthode de volumes finis. Les simulations 2D sont utilisées pour caractériser l'écoulement de double cavité sur la base des données enregistrées dans une sonde ponctuelle, utilisant des outils de la théorie de systèmes dynamiques (théorie du chaos). Des simulations numériques 3D sont réalisées pour certaines configurations choisies à fin de combler l'écart entre les simulations bidimensionnelles et les résultats expérimentaux, forcément tridimensionnels. Les affinités et dissemblances entre les simulations numériques 2D/3D et les résultats expérimentaux sont présentées.

La composition complexe des spectres de la couche de cisaillement pour la cavité simple et double est étudiée avec des outils de la théorie d'instabilités linéaires. Sur la base d'une analyse d'instabilité linéaire, locale et incompressible, dans un domaine fini, conjointement avec un scénario de réflexion des ondes (rétroaction) - la condition dite de Kulikowskii, nous développons un cadre théorique pour l'analyse d'un domaine fini dans la direction de l'écoulement. La considération d'un domaine fini et l'application de la condition de Kulikowskii entraîne une discrétisation de la bande de fréquences: les fréquences non-harmoniques qui sont amplifiées sont observées expérimentalement. Cette analyse nouvelle est appliquée avec succès au cas de la simple et de la double cavité. Les résultats présentés indiquent qu'un modèle simple d'interaction d'ondes (rétroaction) peut rendre compte de l'existence des modes non-harmoniques qui sont observés expérimentalement.

Une analyse du champ de vorticit   issu des simulations num  riques 2D conduit    un deuxi  me m  canisme de r  troaction, purement hydrodynamique, dans la cavit   simple et double. Cette analyse associe l'analyse de la dynamique non lin  aire, r  alis  e avec les r  sultats de simulations 2D, avec l'espace physique. La visualisation des patrons des petites structures intracavitaires r  v  le le m  canisme sous-jacent le proc  s de r  troaction et permet d'identifier le r  gime de l'  coulement. Les petits vortex, cr   s par l'injection de vorticit   dans la cavit   par les couches de cisaillement, d  crivent un mouvement quasi-circulaire    l'int  rieur de la zone de recirculation intracavitaire, formant une structure de type "carrousel" dans chaque cavit  . Les tourbillons dans le carrousel font un parcours qui retourne    la r  gion o   l'instabilit   se d  clenche, excitant la couche de cisaillement et

créant une rétroaction qui est déterminée par l'interaction de trois temps caractéristiques: le temps de révolution dans la zone principale de recirculation, le temps de vie des vortex intracavitaires et la période des oscillations de la couche de cisaillement. Nous présentons évidence expérimentale du mécanisme vortical du carrousel, qui peut être également rencontré dans la littérature pour les écoulements de cavité simple.

En somme, nous étudions l'écoulement de double cavité avec des techniques expérimentales, numériques et analytiques dans la limite incompressible. Nous considérons une gamme vaste de vitesses d'entrée et de distances intercavitaires. Sur la base d'une analyse de stabilité linéaire en temps et espace, conditionnée par la finitude du domaine à travers la réflexion d'ondes, nous arrivons à expliquer l'existence des pics de fréquences non harmoniques pour les couches de cisaillement. L'analyse du champ de vortacité intracavitaire obtenu par simulations numériques 2D, met en évidence un mécanisme de rétroaction hydrodynamique, permettant d'expliquer la composition spectrale des couches de cisaillement dans l'écoulement de double cavité.

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Appendix A

Linear stability code

Matlab code for the computation of the local spatio-temporal linear stability analysis is given. The code was adapted from [Hoepffner \(2004\)](#).

```
format compact; clear all
%%% type of analysis: 2 spatial stability, 1 temporal stability
spatemp=1;
N=100;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% finite y:
ex=48
[y1,D0] = meshSL(N+ex,10);
y=y1(ex/2+1:end-ex/2);
D1 = fddifflo(y,N);
%%% base flow
u=1+tanh(0.5*y);
up=D1*u;
%figure(12);h=plot(up,y,'ko',u,y,'ro');set(h,'LineWidth',2);
hold on; xlabel('profile'); ylabel('y');
grid on;xlim([-0.1,2.1]); hold on;
%%% coordinate selectors
uu=1:N; vv=uu+N; pp=vv+N;
bou=[1,N]; % boundaries indices
%%% dynamic matrices:
Z=zeros(N,N);I=eye(N);
B=blkdiag(-1i*I,-1i*I,Z); % temporal operator
C=[Z, diag(up), Z; Z, Z, D1; Z, D1, Z];
D=[1i*diag(u), Z, 1i*I; Z, 1i*diag(u), Z; 1i*I, Z, Z];
DD=blkdiag(D1,D1,D1);II=eye(3*N);
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
iii=1
```

```

for kr=0:0.05:0.5
    ii=1;
for ki=0
    k=kr+i*ki
switch spatemp
    case 1 % temporal stability analysis
AA0=-(C+k*D); % =A2
AA1=B;
    case 2 % spatial stability analysis
AA0=-(om*B+C); % =A1
AA1=D;
end
%%% impose boundary conditions
II=eye(3*N);DD=blkdiag(D1,D1,D1);
bbouD=[1;N;N+1;2*N]; % Dirichlet, u,v top y bottom
bbouN=[2*N+1;3*N]; % Neumann p top y bottom
AA0(bbouD,:)=0;
AA0(bbouN,:)=0;
AA1(bbouD,:)=II(bbouD,:); % Dirichlet
AA1(bbouN,:)=DD(bbouN,:); % Neumann
%%% solve for eigenvalues
disp('computing eigenvalues')
tic; [UU,S]=eig(AA0,AA1); toc
S=diag(S);
kmt{iii}{ii}=k;
SSk{iii}{ii}=S;
UUk{iii}{ii}=UU;
ii=ii+1
iii
end
iii=iii+1
end
savefile0 = 'kgrid.mat';
save(savefile0,'kmt','SSk','UUk','ii','iii','N','y','u')
display('Run successful!!')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Plot map
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all;
color={'k','r','c','m','g','b','k','r','c','m','g','b','r'}
symb={'o','x','+','v','*','^','o','o','x','+','v','*','^'}
load('kgrid.mat');

```

```

L=50;
r=iii-1; i=ii-1;
uu=[1:N]; vv=[N+1:2*N]; pp=[2*N+1:3*N];
h=figure(55); set(gcf,'position',[50 50 1200 600])
for ii=1%:1:i
    for ri=1:1:r
        for ind=1:10
h1=plot(real(SSk{ri}{ii}(ind)),imag(SSk{ri}{ii}(ind)),symb{ri});
set(h1,'Color',color{ii},'LineWidth',2); hold on
tt=['figure(5);aa=' num2str(ind) ';ra=' num2str(ri) ';ia=' num2str(ii) ';
h2=plot(
abs(UUk{ra}{ia}(uu,aa)),y,'r-',
abs(UUk{ra}{ia}(vv,aa)),y,'b-',
abs(UUk{ra}{ia}(pp,aa)),y,'k-'
);
legend('abs(u)','abs(v)','abs(p)');
set(h2,'LineWidth',2);xlabel('Amplitude');ylabel('y'); ylim([-L,L]);
hold on; grid on; figure(4);
h1=plot(real(SSk{ra}{ia}(aa)),imag(SSk{ra}{ia}(aa)),'ro');
set(h1,'Color','r','LineWidth',2);
xlim([0,0.5]); ylim([-0.3, 0.2]);hold off; grid on'];
        set(h1,'buttondownfcn',tt);
        end
    end
end
xlabel('\omega_r'); ylabel('\omega_i'); h3 = get(gca, 'ylabel'); h4 = get(gca,
    'xlabel');
set(h3,'FontName','Arial','FontSize',16,'FontWeight','bold');
    set(h4,'FontName','Arial','FontSize',16,'FontWeight','bold');
set(gca,'fontsize',16)
xlim([0,0.5]); ylim([0,0.15]);
grid on; hold on
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
return
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Subroutine 1:
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function D1 = fddifflo(y,N)
D=zeros(N,N);

```

```

for k=2:N-1
    hw(k)=y(k)-y(k-1);
    he(k)=y(k+1)-y(k);

    a(k)=hw(k)/(he(k)*(he(k)+hw(k)));
    b(k)=(he(k)-hw(k))/(he(k)*hw(k));
    c(k)=-he(k)/(hw(k)*(he(k)+hw(k)));
end
a(1)=0;a(N)=0;
b(1)=0;b(N)=0;
c(1)=0;c(N)=0;
D=diag(c(2:N),-1)+diag(b)+diag(a(1:N-1),1);
% Boundaries: (first order approx)
D(N,N-1)=-1/(y(N)-y(N-1));
D(N,N)=1/(y(N)-y(N-1));

D(1,1)=-1/(y(1)-y(2));
D(1,2)=1/(y(1)-y(2));
D1=D;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Subroutine 2:
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [y,D] = meshSL(n,ly)
% define Gauss-Lobatto points xi,
% eta is the equidistant vector
eta = (0:n-1)'*pi/(n-1);
xi = cos(eta);
% build derivative matrix D
% such that F'(i) = D(i,:)*F(:)
% with F(i) = F(y(i)) in y(i) as defined above.
A=zeros(n);
C = ones(1,n); C(1) = 2; C(n) = 2;
for i=1:n
    for j=1:n
        if (i~=j)
            A(i,j) = C(i)*(-1)^(i+j) / (C(j)*(xi(i)-xi(j)));
        elseif (i==1)
            A(i,i) = (2*(n-1)^2 + 1)/6;
        elseif (i==n)
            A(i,i) = -(2*(n-1)^2 + 1)/6;

```


Appendix B

Blasius boundary layer fit

Matlab code for Blasius fit.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% fminsearch for Blasius
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
yx=(yx-yx(1))*1e-3;
Uexp=UN(anf:end-ende); % Define velocity profile, used for fitting: anf:ende
yexp=yx(anf:end-ende);
% Definition of Uinf
Uinf=max(UN); % oder: Uinf=mean(UN(end-5:end));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% initialisation Blasius
eta=[0:0.9e-2:9]; % very fine (almost continuous)
[U2, V] = blasius(eta,1,1,1); % one time: computation of non-dimensional
    Blasius profile (very fine)
for i2=1:length(Uexp)
    [tmp,iu(i2)]=min(abs(U2-Uexp(i2)./Uinf));
end
G0=[4e-4,0.21]; %valeur de depart (units physiques) pour la recherche du
    minimum
[G,fval,exitflag,output]=fminsearch('fitblasius2',G0);
nu=15e-6;
% Blasius Plot
figure(99)
plot(eta.*sqrt(nu*G(2)/Uinf),U2.*Uinf,'k','LineWidth',2);grid on; hold on
plot(yx+G(1),UN,'bs','LineWidth',2); grid on; hold on
plot(yexp+G(1),Uexp,'ro','LineWidth',3); grid on; hold on
xlabel('$y$ [m]','interpreter','latex');
ylabel('$U$ [m/s]','interpreter','latex');
set(gca,'FontSize',16)
```

```
h3 = get(gca, 'ylabel'); h4 = get(gca, 'xlabel');  
set(h3,'FontName','Arial','FontSize',16);  
set(h4,'FontName','Arial','FontSize',16);  
%xlim([-1e-3, 10e-3]); ylim([-0.1 1.1]);  
hold on
```

Frequency-selection mechanism in incompressible open-cavity flows via reflected instability waves

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We present an alternative perspective on nonharmonic mode coexistence, commonly found in the shear layer spectrum of open-cavity flows. Modes obtained by a local linear stability analysis of perturbations to a two-dimensional, incompressible, and inviscid sheared flow over a cavity of finite length and depth were conditioned by a so-called coincidence condition first proposed by Kulikowskii [J. Appl. Math. Mech. **30**, 180 (1966)] which takes into account instability wave reflection within the cavity. The analysis yields a set of discrete, nonharmonic frequencies, which compare well with experimental results [Phys. Fluids **20**, 114101 (2008); Exp. Fluids **50**, 905 (2010)].

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I. INTRODUCTION

Open-cavity flow, a canonical flow geometry of practical as well as academic interest in fluid mechanics, exhibits shear layer instabilities that have been subject to extensive, theoretical [1,2], and experimental [3,4,11,12] as well as numerical [5,6] studies over the course of the past decades. Resonant frequencies, commonly observed in open-cavity flows, were explained in terms of flow-acoustic feedback mechanisms [7] coupled with secondary hydrodynamic instabilities in the recirculation region of the cavity as well as three-dimensional instabilities [6]. Yet another mechanism was proposed by Villermaux and Hopfinger [8] to explain the low frequency content in the shear layer spectrum as a result of the recirculation region.

In the compressible subsonic regime, the well-known semiempirical Rossiter [7] formula, which was derived based on the assumption of acoustic feedback, describes the resonance frequencies observed in such flows reasonably well [9]. Interestingly, the nonharmonic modes were found experimentally also in the incompressible case by different groups [11,12]. Yamouni *et al.* [10] recently tried to link the compressible to the incompressible dynamics by means of a global linear stability analysis. His results, however, do not compare well with experimental data [11,12]. Hence the phenomenon still lacks a comprehensive explanation.

In the same decade as Rossiter presented his formula, Kulikowskii [13] derived a condition for linear stability analysis in a finite domain (LSAFD) that takes into account an amplified downstream traveling perturbation wave k_+ as well as an evanescent upstream traveling perturbation wave k_- . The waves are reflected at the downstream and upstream boundaries characterized by the reflection coefficients R_1 and R_2 , respectively. In order for the perturbation to be self-sustaining, the amplitude of a perturbation wave at a given point in space must, after successive reflections, coincide with the amplitude of the original perturbation. Doaré and Langre [14] retrieved the same results as Kulikowskii, considering

the complex Ginzburg-Landau amplitude equation in a finite domain together with a phase and energy closure principle first proposed by Gallaire and Chomaz [15]. Lindzen and Rosenthal [16] showed that Kelvin-Helmholtz instabilities result from wave overreflection in vertical direction with their energy extracted from the mean flow. Kulikowskii [17] himself continued research on this subject considering a two-dimensional domain, where waves can propagate and be reflected.

Open-cavity flow under the incompressible assumption is commonly described in terms of two physically different flow types: the recirculating flow inside the cavity and the shear layer flow above. The emitted frequencies observed experimentally are most likely the result of both the three-dimensional instabilities in the recirculation region and the shear layer instabilities [6]. None of the previous studies, however, consider the possibility (and consequences) of including the reflection of instability waves. In order to gain further insight into the nature of the mode coexistence in open-cavity flows, the present work analyzes the phenomenon through a local linear stability analysis (in the sense of Ref. [18]) together with the reflection of the instability waves from both cavity edges. The Kulikowskii condition is applied within the time asymptotic theory of spatiotemporal linear stability analysis, described in detail in Ref. [1] for plasma physics. The present work presents solutions of the Kulikowskii condition combined with a spatiotemporal linear stability analysis, thus obtaining a theoretical account for the nonharmonic mode coexistence commonly found in open-cavity flows. The results will be compared with six experiments of the $L/H = 2$ cavity geometry, published in Refs. [11] and [12].

The article is organized as follows. A short overview of the conventional spatiotemporal linear stability analysis is given in Sec. II. The physical argument of the Kulikowskii condition and the method developed to find solutions is presented in Sec. III. In Sec. IV the experiments with which the theory is compared are introduced, and thereafter the results of the LSAFD are presented. Conclusions are drawn in Sec. V.

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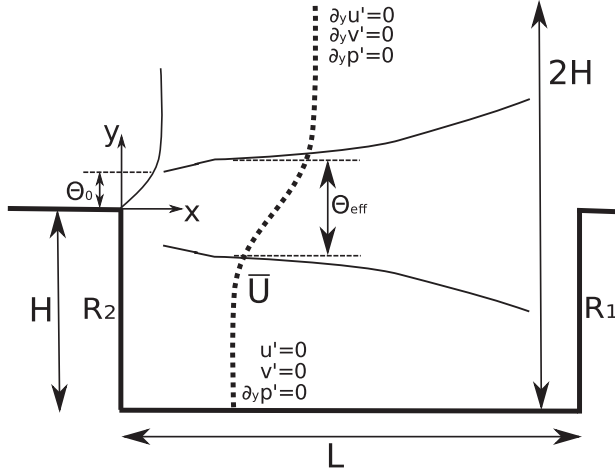


FIG. 1. Schematic view of cavity geometry with base flow $\bar{U}$, reflection coefficients R_1 and R_2 , boundary conditions, inflow momentum thickness Θ_0 , and effective momentum thickness Θ_{eff} used in the linear stability analysis and defined in Sec. IV A.

II. LINEAR STABILITY ANALYSIS

The flow is assumed to be steady, parallel, and unidirectional. The streamwise development of the base flow is therefore entirely neglected. The analysis of flow stability considers the two-dimensional, inviscid, and incompressible Euler equations given by

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p, \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (2)$$

where $\mathbf{u} = (u, v)^T$. As in Huerre and Monkewitz [2] the flow variables $\mathbf{q}(x, y, t) = (u, v, p)^T$ and coordinates x, y are nondimensionalized using the average velocity $\bar{U}$ and an effective momentum thickness Θ_{eff} , introduced in Sec. IV. The flow is decomposed into a steady laminar base flow $\bar{\mathbf{Q}}(y) = (\bar{U}, \bar{V}, \bar{P})^T$ upon which small amplitude perturbations $\mathbf{q}'(\mathbf{x}, t) = (u', v', p')^T$ are permitted ($\mathbf{x} = (x, y)^T$). Normal mode solutions $\mathbf{q}'(\mathbf{x}, t) = \hat{\mathbf{q}}(y)e^{i(kx - \omega t)}$ with $\hat{\mathbf{q}}(y) = (\hat{u}, \hat{v}, \hat{p})^T$ are then considered to model the spatial and temporal development of the perturbations. Note that k and ω are also nondimensionalized using the average velocity $\bar{U}$ and the effective momentum thickness Θ_{eff} , defined and discussed in detail in Sec. IV A. The decomposed flow field is subsequently linearized by neglecting higher order perturbation terms. This yields a system of linear equations, which is solved as an eigenvalue problem (EVP) for the wave number $k = k_r + ik_i$ or for the frequency $\omega = \omega_r + i\omega_i$. Subscripts r and i stand for real and imaginary part, respectively.

The linear stability problem is considered in a finite domain in the streamwise direction, represented by the Kulikowski condition, and in the wall-normal direction, characterized by the boundary conditions as depicted in Fig. 1. The free stream boundary conditions ($y = H$) of the EVP are of Neumann type for all three variables. In the cavity floor ($y = -H$) the boundary conditions for the velocities are of Dirichlet type

due to the no-slip condition at the solid wall. The boundary condition for the pressure is of Neumann type, as follows from the momentum equation. The customary parallel flow assumptions $\bar{V} = 0$, $\bar{P} = \text{const}$ and $\partial_x \bar{Q} = 0$ are applied. The one-dimensional base state of the velocity field $\bar{U} = f(y)$ was nondimensionalized with the average velocity $\bar{U}$ and the momentum thickness Θ_{eff} . It reads

$$\bar{U}(y) = 1 + \tanh\left(\frac{1}{2}y\right). \quad (3)$$

The EVP together with the base-flow assumptions yield the dispersion relation of the system, given by the complex equation

$$\Delta(\omega_r, \omega_i, k_r, k_i) = 0. \quad (4)$$

Its roots $\omega(k)$ provide the eigenfrequencies and growth rates. The dispersion relation is solved numerically using a Matlab code based on a Chebyshev collocation method with $N = 100$ collocation points in the wall-normal direction y . Differentiation is carried out using a second order nonequidistant finite difference method. According to the Rayleigh criterion, the inflexion point of the base profile is a necessary (yet not sufficient) condition for the base flow to be unstable to small perturbations. Whether instabilities occur and if so, whether they are amplified or evanescent in space and time has to be determined by criteria described in Ref. [1]. It builds the basis for the LSAFD and will be shortly outlined in the next section.

A. Amplification and nontransparency

In order to present the argument that leads to the LSAFD, a brief outline of the steps needed to determine whether a medium, governed by its dispersion relation, amplifies or damps an infinitesimal perturbation is given. This theoretical consideration for the infinite domain is on one hand used to distinguish between upstream and downstream traveling waves (k_- and k_+ , respectively) and, on the other hand, to study the asymptotic behavior of these waves. Normal mode properties, sustained by a given base state, are encapsulated in its dispersion relation (4). In general, both the wave number k and the frequency ω are considered complex. Solutions of the dispersion relation for given complex wave numbers are called temporal branches, whereas solutions of the dispersion relation for given complex frequencies are called spatial branches. The response of the system (the perturbation wave) to a source

$$g(x, t) = C \cdot \mathcal{H}(t)\delta(x)e^{-i\omega_0 t} \quad (5)$$

of a given frequency ω_0 is given by

$$\Psi(x, t) = \frac{C}{2\pi} \int_{L_\omega} \underbrace{\int_{F_k} \frac{e^{ikx}}{\Delta(\omega, k)} dk}_{\phi(\omega, x)} \frac{e^{-i\omega t}}{i(\omega - \omega_0)} d\omega, \quad (6)$$

where $\Delta(\omega, k)$ denotes the dispersion relation, $\mathcal{H}(t)$ the Heaviside step function in time, $\delta(x)$ the Dirac delta in space, and C a constant. The integration contours in the complex ω plane and the complex k plane, respectively, are defined according to $L_\omega = (-\infty + i\omega_i, +\infty + i\omega_i)$ where $\omega_i > 0$ and $F_k = (-\infty, +\infty)$. The integration contour L_ω is located above all the poles in the complex ω plane, as required by the

“causality principle,” which states that “effect cannot precede cause,” meaning that the perturbation occurs only after the source has been turned on at $t = 0$.

Below we discuss the asymptotic behavior of the response of the system perturbed with upstream and downstream traveling waves. To this end we consider the asymptotic behavior of expression (6).

B. Asymptotic behavior of the response of the system

In space, a perturbation can travel to $x < 0$ and to $x > 0$, whereas time is unidirectional and thus only $t > 0$ makes sense. Therefore, the sign of k cannot serve as an indicator of stability or instability, as in the temporal case. The physical nature of the behavior of the waves is embedded in its dispersion relation, which depends on the medium. In a transparent medium, the wave only possesses real parts of the frequency and the wave number. Thus, it passes without being affected by the medium. On the other hand, if the medium affects the wave it acquires an imaginary part, which accordingly yields an amplified or an evanescent wave. If the perturbation tends to zero as $x \rightarrow \pm\infty$ the wave is evanescent, while if the perturbation increases as $x \rightarrow \pm\infty$ the wave is amplified.

Following Ref. [1], we now look for the asymptotic form of the response of the system $\Psi(x, t)$, far from the source, i.e., for $|x| \rightarrow \infty$, and long after the time origin ($t \rightarrow \infty$). The asymptotic form $t \rightarrow \infty$ has to be taken before $|x| \rightarrow \infty$, as a perturbation cannot propagate to infinity in a finite amount of time. Applying Briggs’ method [19] we consider Eq. (6) and move L_ω downwards (for a given $\omega_r = \omega_0$) in order to get the asymptotic expression in time. The highest located singularity in the complex ω plane is ω_0 . Once this pole is reached, the asymptotic form of Eq. (6) reads

$$\Psi(x, t) \propto e^{-i\omega_0 t} \phi(\omega_0, x), \quad (7)$$

where only the second term $\phi(\omega_0, x)$ is of interest, since the first term simply oscillates. The poles of $\phi(\omega_0, x)$ are the zeros of the dispersion relation $\Delta(\omega_0, k)$ of the system. By definition, let $k_+(\omega)$ denote the poles located in the positive k half plane and $k_-(\omega)$ the poles located in the negative k half plane for $\omega_i \rightarrow \infty$. When the L_ω contour is lowered, the poles in the complex k plane move. They might stay in their original half plane or might cross into the other half plane. The poles of interest are the ones that are closest to the real k axis if they have not crossed, or the ones that are farthest from the real axis in the case they have crossed the k_r axis. With these values of k_+ and k_- the asymptotic behavior of the response of the system can be evaluated, considering

$$\begin{aligned} \Psi(x, t) &\propto e^{i[k_+(\omega_0)x - \omega_0 t]} \\ &= e^{i[k_{r+}(\omega_0)x - \omega_{r0} t]} e^{-k_{i+}(\omega_0)x + \omega_{0i} t} \end{aligned} \quad (8)$$

for $x > 0$ and

$$\begin{aligned} \Psi(x, t) &\propto e^{i[k_-(\omega_0)x - \omega_0 t]} \\ &= e^{i[k_{r-}(\omega_0)x - \omega_{r0} t]} e^{-k_{i-}(\omega_0)x + \omega_{0i} t} \end{aligned} \quad (9)$$

for $x < 0$. In the case studied in the present work, a k_+ branch crosses the k_r axis, while all k_- branches stay in the lower half plane. Thus, $k_{i+} < 0$ and $k_{i-} < 0$, which means that

downstream traveling waves are amplified [Eq. (8)], while upstream traveling waves are evanescent [Eq. (9)]. This is commonly known as convective instability. Once the L_ω contour reaches the most unstable pole, a so-called branch point (BP) is formed. According to Ref. [20], it can be shown that when a BP occurs in the complex ω plane, a so-called pinch point (PP) occurs simultaneously in the complex k plane. Invoking the arguments introduced above, k_+ and k_- waves can be distinguished by considering their behavior for $\omega_i \rightarrow \infty$. A pole that moves to the positive half plane when $\omega \rightarrow \infty$ forms part of the k_+ family while a pole that moves to (or stays in) the negative half plane when $\omega \rightarrow \infty$ forms part of the k_- family.

III. THE FINITE DOMAIN

The finite extent of the domain becomes relevant when the times taken into consideration are larger than the time needed by the perturbation to travel along the cavity length. The geometry under consideration is characterized by its normalized length $L^* = \frac{L}{\Theta_{\text{eff}}}$, the cavity height $H^* = \frac{H}{\Theta_{\text{eff}}}$, and the two reflection coefficients R_2 and R_1 at the upstream and downstream boundaries, respectively. Θ_{eff} is the effective momentum thickness that is discussed in detail in Sec. IV A. L is of the same order as the wavelength of the perturbation and therefore cannot be neglected. The Kulikowskii condition is obtained, following Refs. [1] and [13]. Let P be an arbitrary point inside the domain emitting two countertraveling perturbation waves $k_+(\omega)$ and $k_-(\omega)$. Branches associated with $k_+(\omega)$ and $k_-(\omega)$ are found as solutions of the dispersion relation $\Delta(\omega, k)$ for complex wave numbers and complex frequencies in an infinite domain. As stated in Ref. [1], the characteristic oscillations of a finite system may be regarded as the result of the superposition of traveling waves reflected by the two boundaries, represented by R_1 and $R_2 \in \mathbb{R}$. The reflections are accompanied by a mutual transformation of waves belonging to different branches of the spectrum. Formally the Kulikowskii condition is obtained by considering the perturbation wave

$$\Psi(x, t) = A_0 e^{i[k_+(\omega)x - \omega t]}, \quad (10)$$

which is emitted at P ($0 < P < L^*$). A_0 is the initially infinitesimal amplitude. When it reaches the downstream boundary, Ψ is reflected according to reflection coefficient R_1 , and sent back upstream. The wave on its way upstream is described by

$$\Psi(x, t) = R_1 A_0 e^{i k_+(\omega) L^*} e^{i[k_-(\omega)(x - L^*) - \omega t]}. \quad (11)$$

It reaches the upstream boundary, where it is reflected again, according to the reflection coefficient R_2 . When it travels downstream again the wave is described by

$$\Psi(x, t) = R_1 R_2 A_0 e^{i[k_+(\omega) - k_-(\omega)] L^*} e^{i[k_+(\omega)x - \omega t]}. \quad (12)$$

Due to the requirement that $\Psi(x, t)$ must be single valued, Eqs. (10) and (12) must coincide, leading to the Kulikowskii condition

$$R_1 R_2 e^{i(k_+ - k_-) L^*} = 1. \quad (13)$$

Note that the frequencies for the downstream traveling waves (10) and the upstream traveling wave (11) and (12) do not change. Hence the frequencies of the $k_+(\omega)$ and $k_-(\omega)$

TABLE I. Summary of $L/H = 2$ cavity cases. Free stream velocity, U_∞ . Momentum thickness of boundary layer at $x = 0$, Θ_0 . Cavity length, L . Location of best fit, $x_{\min}$. Effective momentum thickness, Θ_{eff} . Normalized cavity height, $H_{\text{eff}}^* = \frac{H}{\Theta_{\text{eff}}}$. Selected Kulikowskii modes, m_1 and m_2 .

Case	Symbols	$U_\infty (\frac{\text{m}}{\text{s}})$	Θ_0 (mm)	L (mm)	$x_{\min}/L (-)$	Θ_{eff} (mm)	$H_{\text{eff}}^* (-)$	m_1	m_2
LH20	●	2.250	—	100	—	1.296 ^a	38.6	3	8
LH21	○	2.180	1.072	100	0.227	1.325	37.7	4	8
LH22	△	1.898	1.294	100	0.227	1.425	35.1	4	8
LH23	▽	1.715	1.260	100	0.250	1.525	32.8	5	9
LH24	□	1.379	1.310	100	0.227	1.675	29.9	5	—
LH25	▲	0.992	1.420	100	0.250	1.975	25.3	5	—

^aMomentum thickness was not obtained experimentally but extrapolated from cases LH21 to LH25.

waves are the same. The Kulikowskii condition is a complex equation which can be split up into a real and an imaginary part. This yields

$$\Delta k_r = k_{r+}(\omega) - k_{r-}(\omega) = \frac{\pi m}{L^*}, \quad (14)$$

$$\Delta k_i = k_{i-}(\omega) - k_{i+}(\omega) = \frac{1}{L^*} \ln \left[\frac{1}{R_1 R_2} \right] \approx 0, \quad (15)$$

where $m \in \mathbb{N}_0$. Poles of the linear stability analysis which also solve Eqs. (14) and (15) will hereafter be called *Kulikowskii points*.

A. Reflection coefficients

As we consider a rigid cavity walls, the reflection coefficients are assumed to be real with a phase shift of π . Following Ref. [14] we impose a zero total deformation boundary condition at the upstream and downstream corner of the cavity, which is expressed as

$$\Psi(x = 0, t) = 0, \quad (16)$$

$$\Psi(x = L, t) = 0. \quad (17)$$

Let us denote A_1^+ the amplitude of the wave at the downstream boundary (1) before it was reflected and let us denote A_1^- the amplitude of the wave at the downstream boundary (1) after it was reflected. Then we can write the deformation $\Psi(x = 0, t)$ at the location where the boundary condition is imposed as the sum of the two waves k_+ and k_- , which have to add up to zero. This yields (for all times t)

$$\Psi(x = 0, t) = [A_2^+ e^{ik_+ x} + A_2^- R_1 e^{ik_- x}] e^{-i\omega t} = 0, \quad (18)$$

$$\Psi(x = 0, t) = A_2^+ + A_2^- R_1 = 0, \quad (19)$$

$$R_1 = -\frac{A_2^+}{A_2^-} = -1 \quad (20)$$

if we assume that no forcing is induced by the boundaries and therefore the amplitudes before and after the reflection are the same. The same reasoning holds for the upstream boundary (2) from which follows $R_2 = -1$.

Physically it is plausible to expect the product of the reflection coefficients to be smaller than unity since a part of the perturbation wave could be lost and travel off to infinity. A rough computation following Lighthill [21] gave a value

of $R_1 R_2 \approx 0.8$ which translates into $\Delta k_i = 0.0029$ for case LH21 (see Table I) when applied to Eq. (15). The effect of this positive difference of the spatial amplification rates is depicted in Fig. 2. Since the real part of the frequency does not change upon reflection, the Kulikowskii points for $\Delta k_i = 0$ (red ▽) must move on branches $\omega_r = \text{const}$ (indicated by black circles in Fig. 2) until the corresponding Δk_i is reached. The resulting difference in the real part of the wave number Δk_r is approximately equal to Δk_r for $R_1 R_2 = 1$. This is true even for values $R_1 R_2 < 0.8$ since the branches $\omega_r = \text{const}$ are predominantly vertical and therefore Δk_r changes only little when Δk_i is increased.

Based on the considerations outlined above we propose a first order approximation, assuming the reflection coefficients to be $R_1 = R_2 = -1$.

B. Interpretation of the Kulikowskii condition

Physically, Eq. (14) states that the finite domain can contain only an integer number of waves, which is in agreement with the wavelength selection criterion found in Ref. [22]. This

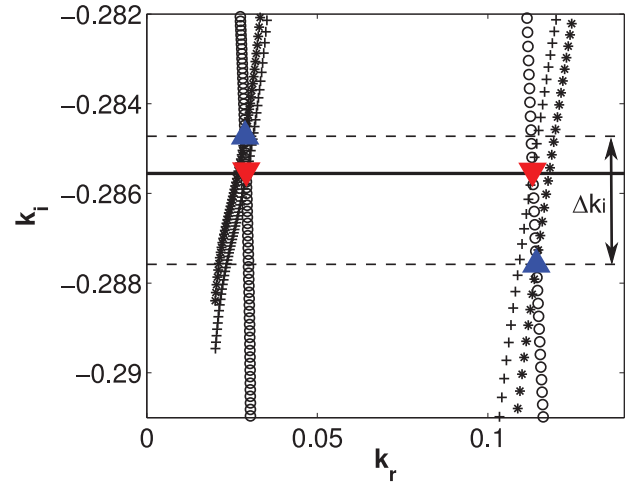


FIG. 2. (Color online) LSAFD with $\Delta k_i = 0.0029$, which translates into $R_1 R_2 = 0.80$. Case LH21: Branches (+) and branches (●) correspond to different $\omega_i = \text{const}$; Branches (○) correspond to $\omega_r = \text{const}$. Horizontal solid line indicates $\Delta k_i = 0$ ($R_1 R_2 = 1$) for Kulikowskii points (red ▽). Horizontal dashed lines separate Kulikowskii points (blue △) by $\Delta k_i = 0.0029$ ($R_1 R_2 = 0.8$).

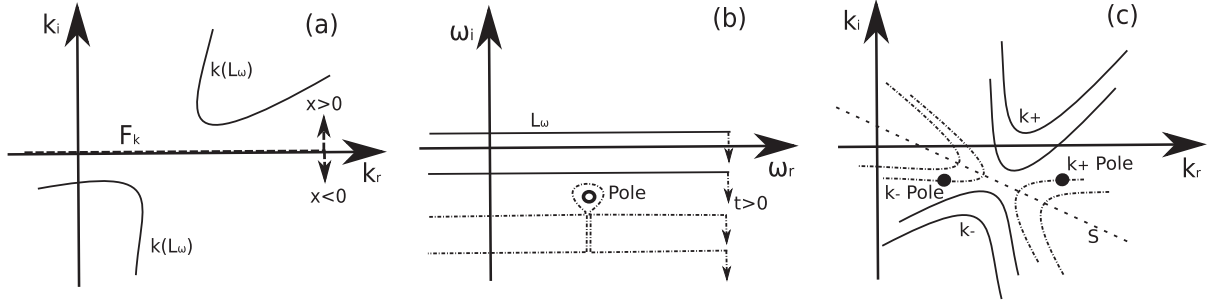


FIG. 3. (a) Complex k plane with k branches for $\omega_i \rightarrow \infty$. (b) Complex ω plane with integration contours before (—) and after (---) the branch point is crossed. (c) k branches before (—) and after (---) the branch point is crossed, together with two exemplary poles (●) that solve the Kulikowskii Condition. The straight dashed line S (---), separates k_+ poles from k_- poles. The dashed arrows indicate movement of respective poles when $\omega_i \rightarrow \infty$.

yields a discretization of a continuous spectrum of solutions and thus will lead to a selection mechanism of the associated frequency. Note that in the infinite domain ($L \rightarrow \infty$) such a discretization does not occur since $\Delta k_r = 0 \quad \forall m \in \mathbb{N}$. The second equation (15) states the coincidence condition in terms of the spatial growth rates. The difference of the spatial growth rates for the upstream and downstream traveling waves has to be compensated by the reflection coefficients R_1 and R_2 , to prevent the system from getting out of balance. Hence, Eq. (15) expresses the self-limited nature of the system. For the case of total reflection, the right-hand side of Eq. (15) is zero, which means that if the amplitude of the perturbation wave is not diminished by reflection, the spatial amplification rates of the upstream and downstream traveling waves must be equal in absolute terms. The more general case of $R_1 R_2 \neq 1$ yields a strictly positive right-hand side of Eq. (15) and the amplification rates adapt accordingly in order to maintain valid Eq. (15). However, since as mentioned above the product of the reflection coefficients is close to unity, the numerical value of Δk_i is small and does not affect the results significantly.

In short, the solutions to the Kulikowskii condition consist of two different values of k_r , which are associated with one single set of values of k_i , ω_r , and ω_i . Poles of the linear stability analysis in the infinite domain that also solve the finite domain constraint, namely, the Kulikowskii condition, are denoted Kulikowskii points. Kulikowskii points therefore characterize the instability behavior of the finite system.

C. Evaluation of the Kulikowskii condition

As stated above, the right-hand side of Eq. (15) is always positive or zero, since $0 \leq R_1 R_2 \leq 1$. For the conventional case (i.e., $\omega_i > \omega_{iBP}$), however, the respective locations of the k_+ branch and the k_- branch [see Fig. 3(a)] do not allow for such a solution, since the k_+ branch is located above the k_- branch, which will always yield $k_-(\omega) - k_+(\omega) \leq 0$. The only point where the equation holds is the PP itself. However, if the BP is crossed (i.e., $\omega_i < \omega_{iBP}$) by means of analytic continuation of the L_ω contour, as depicted in Fig. 3(b), the branches change their location so that nonzero solutions for Eq. (15) become possible. As stated in Sec. II A, once the integration contour L_ω crosses the least stable pole in the ω plane, causality is violated. This holds for the infinite domain

in which the reflection of the perturbation wave is not taken into account. In the case of a finite domain, the concepts of “before” and “after” become inadequate, since due to reflection, a wave has to be considered as cause and effect at the same time. In order to know whether a pole is located on a k_+ branch or a k_- branch, the same criterion as before is applied. A pole that moves to the upper half plane as $\omega_i \rightarrow \infty$ is located on a k_+ branch, while a pole that stays in the lower half plane when $\omega_i \rightarrow \infty$, is located on a k_- branch. The straight dashed line in Fig. 3(c) separates the two types of branches according to this criterion.

IV. RESULTS

A. Experiments and momentum thickness

The results of the LSAFD are now compared to experimental results obtained by Refs. [11] and [12]. The momentum thickness Θ is the only variable parameter in the analysis. As noted in Ref. [23] (and confirmed in the present work), the analysis is quite sensitive to Θ . It is therefore crucial to choose the momentum thickness with care. The base profile is assumed to follow the shape of a hyperbolic tangent function. Hence, the incoming Blasius boundary layer needs a certain time (and space) to relax, in order to fit the hyperbolic tangent assumption. The authors of Ref. [24] propose to choose Θ in the vicinity of the plateau where the momentum thickness varies only marginally, i.e., $\frac{\partial \Theta}{\partial x} \approx 0$. Since the plateau in our case is found at approximately $0.1 < x/L < 0.45$, the base profile [Eq. (3)] was fitted in this region to the experimental velocity profiles [12] along the cavity length with a least-square fit method as done by Ref. [25] and Θ was taken at the streamwise position $x_{\min}$, for which the least-square error $(U_{\tan} - U_{\exp})^2$ exhibits a minimum. $x_{\min}$ was found to be at approximately one quarter of the cavity length. This is in agreement with Ref. [23], which suggests using the value of Θ one instability wave length downstream of the trailing edge and before any significant nonlinear interactions occur. Thus instead of the inflow momentum thickness Θ_0 , an effective momentum thickness $\Theta_{\text{eff}} = \Theta(x_{\min})$ is used as length scale.

Table I summarizes the six cases analyzed in the present work. The cavity geometry (L/H) is kept constant and only the free stream velocity U_∞ is varied. With U_∞ changes Θ_0 and

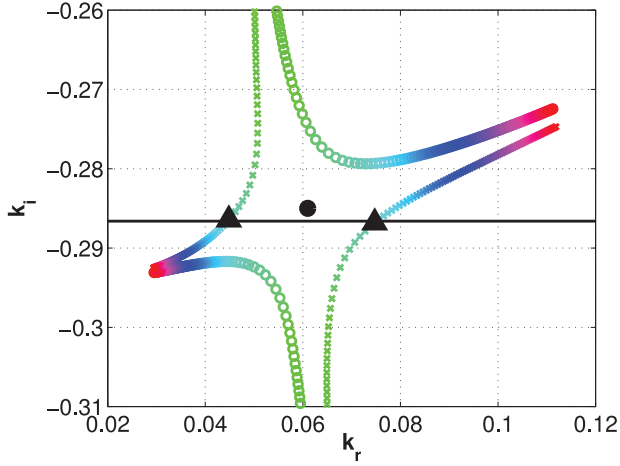


FIG. 4. (Color online) k branches in complex k plane for $\omega_i = -0.0632$ ($\circ$) and $\omega_i = -0.0636$ ($\times$) for case LH21. Color scale indicates value of increasing ω_r from light green ($\omega_r = 0.1676$) to red ($\omega_r = 0.1728$). Pinch point marked by $\bullet$. Kulikowskii points marked by $\blacktriangle$. Horizontal solid line (—) indicates $k_i = \text{const}$.

therefore Θ_{eff} , which likewise affects the normalized cavity height and length, H^* and L^* , respectively.

B. Evaluation of the Kulikowskii condition

Figure 4 shows four branches, that result from mapping $\omega_i = \text{const}$ through the dispersion relation into the complex k plane, before (circles) and after (crosses) the branch point is crossed. Comparison between the ($\circ$) branch and the ($\times$) branch shows that, as stated above, solutions of the Kulikowskii condition [Eqs. (14) and (15)] are possible only after the BP is reached. Solutions to the Kulikowskii condition are called Kulikowskii points. The mapping of a Kulikowskii point from the k plane into the ω plane is depicted in Fig. 5. By means of a spatial analysis $k_i = \text{const}$ is mapped through the

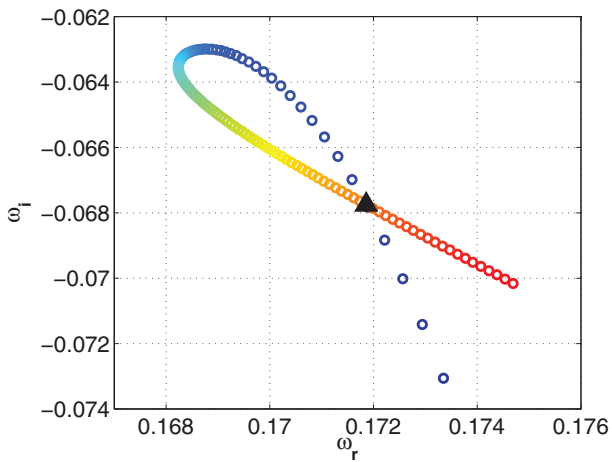


FIG. 5. (Color online) ω branch for $k_i = -0.3015$. Intersection ($\blacktriangle$) indicates Kulikowskii point. Color scale indicates value of increasing k_r from blue ($k_r = 0.0220$) to red ($k_r = 0.1400$).

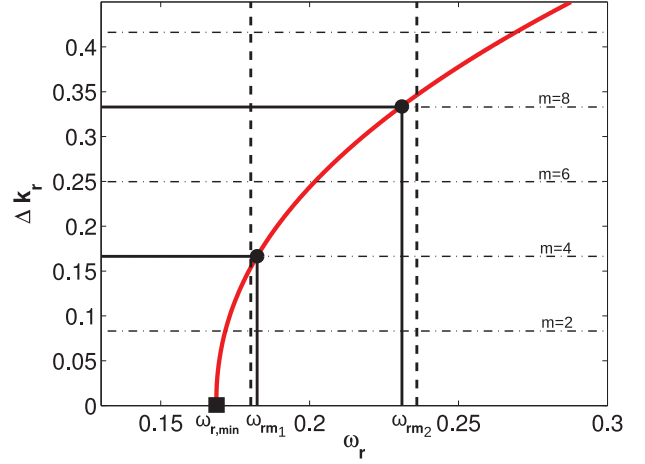


FIG. 6. (Color online) Red solid graph (—) shows continuous set of solutions to Eq. (15). Horizontal lines (---) depict solutions to Eq. (14) for $m = 2, \dots, 10$. Vertical solid lines (—) indicate by the LSAFD selected frequencies ω_{rm1} and ω_{rm2} . Dashed vertical lines (---) indicate frequencies obtained experimentally [11,12]. The pinch point is marked by $\blacksquare$ and indicates the minimum frequency $\omega_{r,\text{min}}$ selected by the LSAFD. Kulikowskii points are marked by $\bullet$. Only case LH21 is depicted for the sake of clarity.

dispersion relation into the complex ω plane and results in a self-intersecting branch. The intersection yields a Kulikowskii point as the following conditions are met:

$$\omega_{i+} = \omega_{i-}, \quad (21)$$

$$\omega_{r+} = \omega_{r-}, \quad (22)$$

$$k_{i+} = k_{i-}, \quad (23)$$

$$k_{r+} = k_{r-} + \frac{\pi m}{L^*} \quad m \in \mathbb{N}_0. \quad (24)$$

A special solution to the Kulkowskii condition is obtained for $m = 0$. This double root of the dispersion relation is commonly called a *pinch point* in the literature. Evaluating the second Kulikowskii condition [Eq. (15)] results in the continuous red graph in Fig. 6. For various $k_i = \text{const}$, $\Delta k_r(\omega_r) = k_{r+}(\omega_r) - k_{r-}(\omega_r)$ is plotted over the associated frequency ω_r . The quadratic form confirms the validity of the Taylor series expansion of the dispersion relation around the pinch point [1], which yields $k - k_0 \sim \pm(\omega - \omega_0)^{1/2}$. The continuous spectrum obtained by the second Kulikowskii condition is discretized by the right-hand side of the first Kulikowskii condition [Eq. (14)], which adds the horizontal lines to Fig. 6 and thus selects a set of discrete frequencies ω_{rm} , $m \in \mathbb{N}_0$. Certain frequencies, labeled ω_{rm1} and ω_{rm2} , compare well with experimental results [11,12] shown in Fig. 7.

Figure 7 compares experimental results for the open-cavity flow [11,12] with the results of the LSAFD. The square root of the power spectral density (PSD), normalized by the respective maximum value, is depicted as a function of the free stream velocity U_∞ and the frequency f . The cases investigated

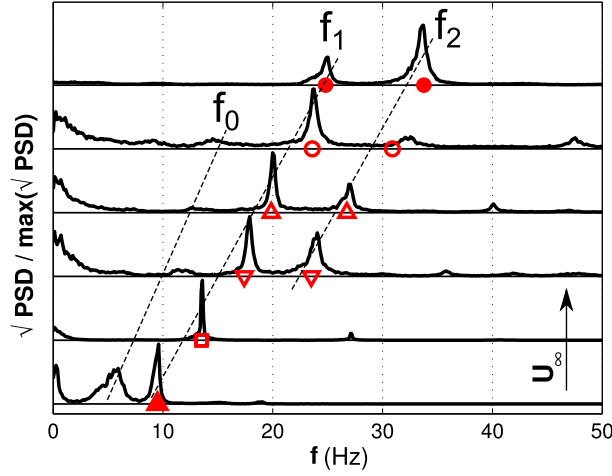


FIG. 7. (Color online) Y offset graph comparing the results from LSAFD (red symbols as in Table I) with the square root of the normalized power spectral density (PSD) of experimental results [11,12] (black graphs), measured using an LDV technique. The streamwise velocity component for the six cases in Table I is depicted. Dashed lines indicate frequency branches f_0 , f_1 , and f_2 . The arrow indicates the direction of increasing U_∞ .

in the present work are represented by the thick lines and U_∞ varies in the vertical axis as indicated by the arrow. Results of the LSAFD are indicated by the red symbols. Three incommensurable frequency branches (denoted f_0 , f_1 , f_2 in the graph) are amplified depending on U_∞ . At low velocities f_0 and f_1 coexist, though f_0 is dominant. While f_0 disappears for velocities larger than $U_\infty \approx 1.4$, f_1 keeps growing until it dies out for velocities $U_\infty > 2.5$. f_2 starts to develop around $U_\infty \approx 1.6$ and keeps growing from there on. f_0 lies outside of the frequency range selected by the Kulikowskii condition and is therefore assumed to be due to other effects, mentioned in the introduction [6,8]. The two high-frequency branches f_1 and f_2 enter the regime predicted by the LSAFD for all velocities.

Frequencies selected by the Kulikowskii condition for $m = 4, 5$ and $m = 8, 9$, respectively, are in good agreement with experimental data [11,12]. However, the Kulikowskii condition selects a larger number of discrete frequencies than experimentally observed.

C. Evolution of Kulikowskii modes in space and time

The values of k_i and ω_i along the Kulikowskii points are depicted in Figs. 8 and 9, respectively. In accordance with Eqs. (8) and (9), a negative value of k_i amplifies downstream traveling waves and attenuates upstream traveling waves as required by the Kulikowskii condition. Close to the cutoff frequency $\omega_{r,\min}$ (which is nearly equal for all six cases), the value of k_i changes rapidly, whereas the increase or decrease depends on the local topology of the PP. Farther away from $\omega_{r,\min}$ the value of k_i levels out and tends to a constant, hence being approximately equal for both amplified frequencies ω_{rm_1} and ω_{rm_2} .

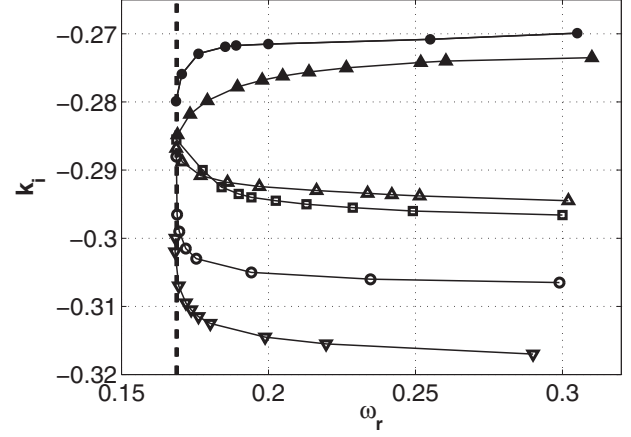


FIG. 8. Values of k_i for different cases.

The value of ω_i is negative throughout the entire frequency band. This is in accordance with the convective nature of the instabilities in the present flow and the idea of spatially amplified waves. If ω_i were positive, the perturbations would grow exponentially in time in every point in space and thus contaminate the entire flow (absolute instability). The value of ω_i as a function of ω_r drops linearly (with slope ≈ -1) moving away from the cutoff frequency. This means that mode m_2 is temporally more damped than mode m_1 . A mechanism which eludes the predominance of either one of the modes, as discussed in Ref. [26], could not be identified by the present theory. However, it should be noticed that the temporal growth rate has only a weak meaning in the present work since we do not consider an infinitely extended shear layer, but a system confined by boundaries within which the question of self-sustaining modes induced by the constructive interference of reflected waves is addressed.

D. Eigenfunctions of Kulikowskii modes

The location of Kulikowskii points in their respective complex planes yields information on the stability behavior as

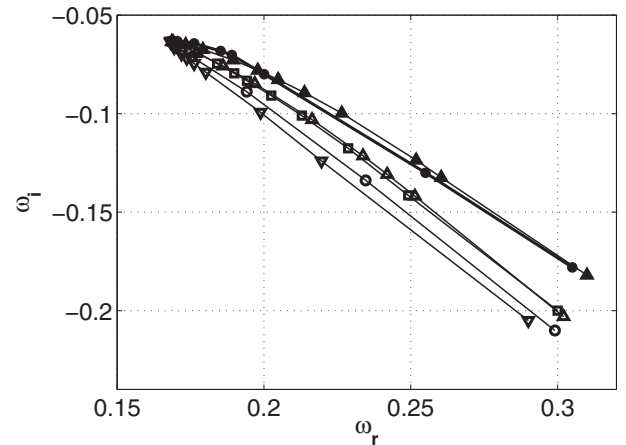


FIG. 9. Values of ω_i for different cases. Symbols as in Table I.

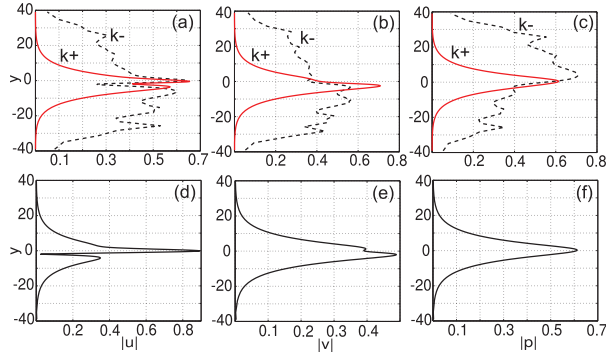


FIG. 10. (Color online) (a–c) Eigenmodes of amplifying k_+ waves (solid lines) and evanescent k_- waves (dashed lines) of $m = m_1$ of case LH21 ($\omega_r = 0.1848$, $\omega_i = -0.0801$). (d–f) Kelvin-Helmholtz modes of same frequency $\omega_r = 0.1848$, but $\omega_i = 0$.

well as on the wave number and frequency of the perturbation waves. The corresponding eigenfunctions provide further information on the structure of the respective waves in their three components u , v , and p . Figures 10(a)–10(c) shows the eigenmodes for the streamwise velocity perturbation u , wall-normal velocity perturbation v , and pressure perturbation p of a single set of Kulikowskii points (k_+ and k_-) at approximately the frequency corresponding to mode m_1 ($\omega_r = 0.1848$) of case LH21. For the same frequency, Figs. 10(d)–10(f) shows the u , v , and p eigenmodes corresponding to the pole on the Kelvin-Helmholtz (KH) branch. Comparing the k_+ perturbations structure with the KH perturbation structure it becomes clear that the amplifying k_+ wave is a KH-like perturbation. This is in accordance with the general picture of the cavity mechanism. The components of the evanescent k_- wave [dashed lines in Figs. 10(a)–10(c)] do not resemble the KH modes. In all three components their amplitudes show a more dispersed behavior. The velocity components of the k_- modes are most active inside the cavity ($y < 0$), while the pressure component shows a maximum above the cavity ($y > 0$). Which one of the components is responsible for the reflection mechanism cannot be clearly identified; however, it can be stated that while the downstream traveling k_+ waves are active in a rather narrow region close to the center line ($y = 0$), the upstream traveling k_- waves are active over the entire domain ($-H < y < H$). It is worthwhile noting that the choice of the KH like k_+ branch is not arbitrary but follows directly from the spatiotemporal stability theory described in Sec. II, which states that the k_+ branch to be chosen is the one with the greatest negative value of k_{i+} [1]. The choice of the k_- branch is dictated by the Kulikowskii condition.

E. Phase speed of Kulikowskii modes

Figures 11 and 12 show the phase speed of the downstream traveling k_+ waves and the upstream traveling k_- waves, respectively, as a function of the frequency for the six cases summarized in Table I. The dimensionless phase speed of the downstream and upstream traveling waves, respectively, is

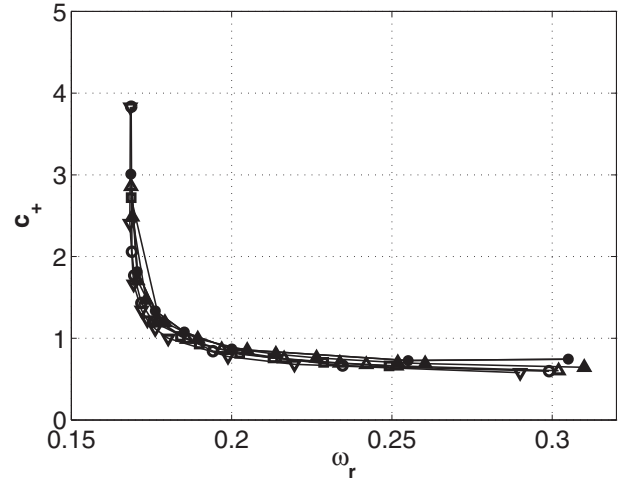


FIG. 11. Phase speeds of k_+ waves for all cases.

defined as

$$c_{\pm} = \frac{\omega}{k_{\pm}}. \quad (25)$$

As shown in Fig. 11, c_+ increases near the pinch point singularity but tends to a constant value of approximately $c_+ \approx 0.6$ for larger frequencies. All six cases collapse in a single line. Figure 12 depicts the phase speed of the upstream traveling waves for the different cases together with the nondimensional speed of sound in air

$$a = \frac{a_{\infty}}{U_{\infty}} \quad \text{with} \quad a_{\infty} = 340 \text{ m/s}. \quad (26)$$

For the feedback mechanism to be acoustic, waves must travel at the speed of sound. As shown in Fig. 12 the upstream traveling waves travel in fact substantially faster than the downstream traveling waves, but do still not reach the speed of sound in the frequency band where the amplified

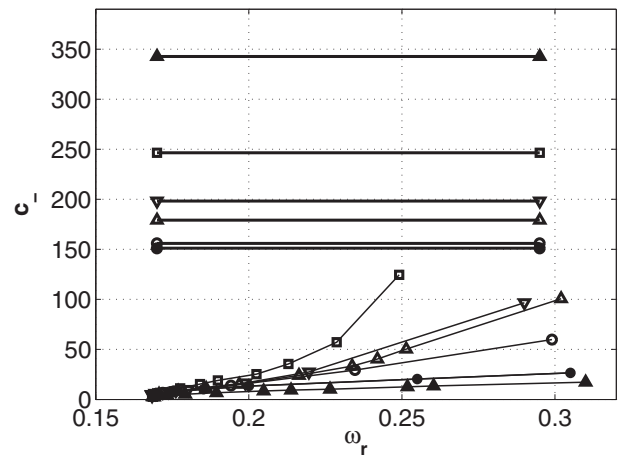


FIG. 12. Phase speeds of k_- waves. Symbols as in Table I. Dashed horizontal lines indicate nondimensionalized speed of sound in air $a^* = \frac{a_{\infty}}{U_{\infty}}$ of the respective case.

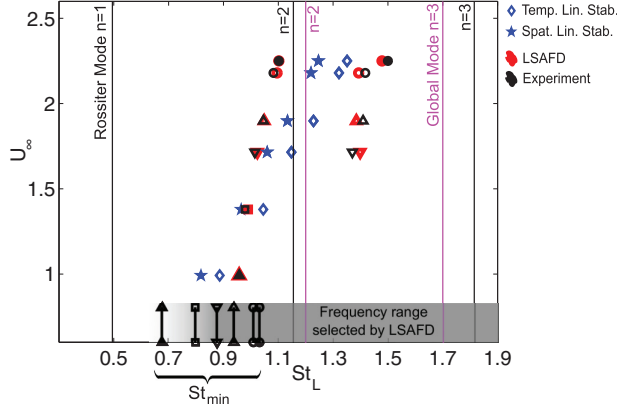


FIG. 13. (Color online) Comparison of experimental results [11,12] (black) with LSAFD (red) for all six cases as well as with a temporal (spatial) linear stability in infinite domain marked by blue $\diamond$ ($\star$). Long vertical black lines show Rossiter modes $n = 1, 2, 3$ from Eq. (27). Long vertical magenta lines show global modes $n = 2, 3$ from Ref. [10]. Short vertical lines show minimum Strouhal number $St_{\min} = \frac{\omega_{r,\min} L^*}{4\pi}$, defined by pinch point frequency of the respective cases (see Fig. 6). Gray shaded area shows frequency range selected by LSAFD ($St > St_{\min}$). Symbols as in Table I.

frequencies are observed experimentally ($0.18 < \omega_r < 0.27$). These results suggest that the feedback mechanism in the incompressible limit is rather due to slower traveling instability waves than to acoustic pressure waves.

E. Discussion

Figure 13 compares the results obtained by the LSAFD with data from several other works in the literature. Results from the LSAFD ($St_{L,FD} = \frac{\omega_{L,FD}^*}{4\pi}$, red symbols) compare well with experimental results [12] ($St_{L,exp} = \frac{f_{exp} L}{U_\infty}$, black symbols). The blue diamond $\diamond$ (or star $\star$) symbols in Fig. 13 show the Strouhal number $St_{L,LS} = \frac{\omega_{LS} L^*}{4\pi}$ of a conventional local temporal (or spatial) linear stability analysis in an infinite domain (i.e., no Kulikowskii condition applied), where $\omega_{LS}(k_r)$ is the frequency for which $\omega_i(k_r)$ [or $k_i(\omega_r)$] is maximal (or minimal). The conventional (temporal or spatial) linear stability analysis can, however, predict only one single amplified frequency which tends to fall between the two nonharmonic peaks observed experimentally and does not predict either one of them with clarity.

As mentioned in the introduction, high subsonic compressible open-cavity flow is commonly related to the acoustic feedback mechanism (*Rossiter mechanism*) expressed in terms of the Strouhal number $St_{L,R}$ by the semiempirical Rossiter [7] formula

$$St_{L,R} = \frac{f_R L}{U_\infty} = \frac{n - \gamma}{M + \frac{1}{\kappa}}, \quad (27)$$

which was derived under the assumption that the feedback waves travel upstream at the speed $c = \frac{\omega}{k}$ where c is the speed of sound c . In the incompressible limit the Mach number is zero and the cavity length L is given in Table I. In Eq. (27) $n = 1, 2, 3 \dots$ is the mode number, κ is the ratio between the

convection speed of the vortices and the free stream velocity, and γ is the lag time between the impact of a vortex on the cavity edge and the emission of an acoustic wave. The values $\kappa = 0.66$ and $\gamma = 0.25$ were used [9]. As mentioned by Ref. [5] for the $L/H = 2$ cavity the first two Rossiter modes are prevalent in the compressible case. Reference [10] tried to link the Rossiter mechanism to the results of a global instability analysis when approaching the incompressible limit. However, only for mode $n = 2$ could a good agreement be found. In Fig. 13 Rossiter's results for $M = 0$ (vertical lines) are compared with the global linear stability results [10], with the results from the LSAFD (red symbols), the results from a spatial linear stability analysis (blue stars), the results from a temporal linear stability analysis (blue diamonds), and the experimental results [11,12] (black symbols) for the six cases evaluated in the present work (see Table I).

Rossiter mode 1 does not enter the frequency range selected by the LSAFD (gray shaded area in Fig. 13), given by the cutoff Strouhal number $St_{\min}$, which is defined by the pinch point frequency $\omega_{r,\min}$ in Fig. 6. This is in agreement with Ref. [10], which found Rossiter's mode 1 to be absent when approaching $M = 0$. Frequencies smaller than $St_{\min}$ are outside of the selectable frequency range, while the frequency band above $St_{L,\min}$ is discretized by the finite extent of the domain. Experimentally obtained frequencies [11,12] can be found at $St > St_{\min}$. Rossiter modes for $n = 2$ and $n = 3$ are also found within the discretized frequency band; however, they do not compare well with experimental results by Refs. [11] and [12]. Rossiter's mode 2 is somewhat close to the experimental mode 1, but Rossiter's mode 3 is found at substantially higher frequencies than the experimental mode 2. Global modes [10] compare reasonably well to Rossiter's modes but neither to the experimental results nor to the results obtained by the LSAFD.

These results indicate that in the incompressible limit the mechanisms responsible for the existence of nonharmonic modes are at least partly due to the reflection and the linear interaction of instabilities in the shear layer. This mechanism is not included in the acoustic feedback mechanism of Rossiter's formula.

V. CONCLUSIONS

This work reports results obtained by combining a local, incompressible linear stability analysis in the infinite domain with the so-called Kulikowskii condition, first introduced by Kulikowskii [13] and later revisited by Landau and Lifshitz [1], which limits the streamwise coordinate to L and takes into account the reflection of the perturbation waves. A theoretical framework for a linear stability analysis in a finite streamwise direction was developed. A wave reflection scenario was introduced in which downstream traveling k_+ waves are reflected into upstream traveling k_- waves and vice versa at the respective boundary. Total reflection was assumed, though the implications and consequences of nontotal reflection were outlined. As a result the Kulikowskii condition discretizes the frequency band in which the amplified nonharmonic frequencies are observed experimentally. It was found that the downstream traveling k_+ waves are spatially amplified and temporally damped, while the upstream traveling k_- waves are spatially and temporally attenuated. This results in a convective

type of instability, a necessary condition for Kulikowskii's theory to be applicable. Solutions to the Kulikowskii condition become possible only after the pinch point singularity is crossed, which yields a minimum frequency $\omega_{r,\min}$ (or $St_{\Theta,\min}$, respectively) that can be selected by the LSADF.

The momentum thickness of the base profile used in the local linear stability analysis was chosen by a least square fit to experimental data [11,12] in accordance with previous works [23–25]. Kulikowskii's condition was evaluated for six different free stream velocities in a $L/H = 2$ cavity, yielding discrete frequencies ω_{rm1} and ω_{rm2} , respectively, that compare well with experiments [11]. It was found that the finite extent of the geometry is a necessary condition for the discretization of the frequency band. The low frequency content of the spectrum of the experimental data [11,12] f_0 (Fig. 7) was found to lie outside of the predicted regime ($\omega > \omega_{r,\min}$). It is concluded that another mechanism [6,8] must be at play than for the higher frequencies peaks f_1 and f_2 which enter the predicted regime and compare well with experimental results [11,12]. The frequency of mode 1 predicted by Rossiter's semiempirical formula [Eq. (27)], which is based on an compressible assumption, is also found outside of the regime ($St < St_{\Theta,\min}$). Frequencies of Rossiter's mode 2 and 3 enter the predicted regime, which is in agreement with Ref. [10].

However, the results of Ref. [10] do not compare well with experimental data in Refs. [11] and [12].

Our results indicate that the rather simple linear wave interaction model which is based on a local linear stability analysis and Kulikowskii's condition, describes well the nonharmonic modes observed experimentally. Thus in the incompressible limit the mechanisms responsible for the existence of nonharmonic modes are at least partly due to the reflection and the linear interaction of instabilities in the shear layer. This mechanism is not included in the acoustic feedback mechanism of Rossiter's formula. In our model the upstream traveling k_- waves are found to travel substantially faster than the downstream traveling k_+ waves, though do not reach the speed of sound.

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Nonlinear dynamics and hydrodynamic feedback in two-dimensional double cavity flow

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This paper reports results obtained with two-dimensional numerical simulations of viscous incompressible flow in a symmetric channel with a sudden expansion and contraction, creating two facing cavities; a so-called double cavity. Based on time series recorded at discrete probe points inside the double cavity, different flow regimes are identified when the Reynolds number and the intercavity distance are varied. The transition from steady to chaotic flow behaviour can in general be summarized as follows: steady (fixed) point, period-1 limit cycle, intermediate regime (including quasi-periodicity) and torus breakdown leading to toroidal chaos. The analysis of the intracavity vorticity reveals a ‘carousel’ pattern, creating a feedback mechanism, that influences the shear-layer oscillations and makes it possible to identify in which regime the flow resides. A relation was found between the ratio of the shear-layer frequency peaks and the number of small intracavity structures observed in the flow field of a given regime. The properties of each regime are determined by the interplay of three characteristic time scales: the turnover time of the large intracavity vortex, the lifetime of the small intracavity vortex structures and the period of the dominant shear-layer oscillations.

Key words: aerodynamics, nonlinear dynamical systems

1. Introduction

The self-excited resonances appearing in spatially developing flows enable us to explain the sharp selection of definite wavenumbers and frequencies observed in experiments. A paradigmatic example of hydrodynamic resonance is the wake flow behind bluff bodies at low Reynolds number, in which oscillations occur with a distinct frequency and wavenumber (Pierrehumbert 1984; Koch 1985). The nature of this resonance is related to the stability characteristics of wake flows, which presents the possibility of amplifying perturbations in the region upstream of the

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source. In shear flows like mixing layers or boundary layers, perturbations are swept away from the source, and unless a coherent perturbation is imposed, the measured spectra remain broadband. However, when a feedback mechanism is possible, these flows may also exhibit discrete frequency spectra. The feedback may take place as a consequence of the downstream reflection, and upstream propagation, of some of the waves of the perturbing wavepacket, which then reach again the region where the perturbation originated. The reflection may occur either in a turning point (Kulikowskii & Shikina 1996), or in a fixed boundary. The latter case results in impinging flows on solid boundaries, extensively studied as reported in Rockwell & Naudascher (1978), Knisely & Rockwell (1982), Ziada & Rockwell (1982). In particular, in open cavity flows, the impingement of the unstable shear layer onto the downstream edge of the cavity produces a global feedback, responsible for self-sustained oscillations of the flow beyond a critical value of both Reynolds number and cavity length – see for instance Rockwell & Naudascher (1978) for an early review on the topic. Those self-sustained oscillations are known to be the source of noise in many applications (e.g. fast train, aircraft land gear bay) or harmonious sound in wind instruments (e.g. in a saxophone). In compressible flows, this phenomenon is commonly linked to the acoustic feedback mechanism first introduced by Rossiter (1964). Incompressible open cavity flows still remain an active field of research including Rowley, Colonius & Basuz (2002), Lusseyran, Pastur & Letellier (2008), Basley *et al.* (2011), to cite only a few of them, as the feedback mechanism through far-field acoustic waves travelling upstream loses importance due to other pure hydrodynamics mechanisms. A recent linear instability analysis by Tuerke *et al.* (2015), considering reflection of hydrodynamic waves at the wall of the cavity, was able to describe the origin of the non-harmonic quantization of frequencies reported in different experiments. This analysis, however, is unable to distinguish, among the set of possible frequencies, which ones are selected by the flow. This limitation indicates that other hydrodynamic mechanisms, associated with the confinement of the flow, have to be taken into account. In an open cavity, a recirculating zone is established with complex intracavity vorticity patterns formed by vortex structures of different sizes that are expected to participate in the feedback mechanism (Basley *et al.* 2013). The interaction of these structures with the onset of the shear instability is nevertheless delayed by the time lag necessary for their transport from the impinging region to the upstream flow. This nonlinear delayed action on the upstream shear-layer instability has been modelled in confined jets with a formalism referred to as the nonlinear delayed saturation model (Villermaux & Hopfinger 1994). Inspired by this work, the dynamics of each cavity can be conceived in terms of nonlinear oscillators. The small structures, present in the recirculation region, produce discontinuous and delayed feedback, giving rise to nonlinear dynamical features, such as quasi-periodic oscillations, phase-locked oscillations or even chaotic dynamics (Villermaux & Hopfinger 1994).

When two identical self-oscillating systems are brought together, the dynamics can be expected to be enriched with additional couplings and interactions. Some of these examples are side-by-side cylinder flows (Zhou, Zhang & Yiu 2002; Kang 2003; Landel, Caulfield & Woods 2012) or two mirrored backward facing steps (Durst, Melling & Whitelaw 1974; Fearn, Mullin & Cliffe 1990). At very close distances the two systems lose identity and behave as a new system. When two open cavities face each other, a so-called double cavity configuration is created, and one may presume the appearance of dynamical couplings of the two shear layers originating from each wall of the inlet channel. Research related to different industrial and bio-mechanical applications is concerned with these flows: the sudden expansion and contraction in

pipe and channel flows (Drikakis 1997; Mizushima & Shiotani 2001; Mullin, Shipton & Tavener 2003), the exit of confined jets (Maurel *et al.* 1996; Righolt *et al.* 2015) and the laryngeal ventricle or Morgagni's sinus in the human phonatory system (Agarwal, Scherer & Hollien 2003; Chisari, Artana & Sciamarella 2011) are just a few examples, among others, in engineering applications and nature, in which the understanding of the flow physics of double cavity dynamics is of importance.

Maurel *et al.* (1996) considered a confined planar jet issuing into a large cavity, a geometry that is similar to the double cavity. The experiments were performed with a ratio between the inlet channel height (D) and the cavity depth (H) equal to $1/12$, and the varied geometrical parameter was the length L of the cavity. The authors found a stable flow regime with no oscillations, a regime with self-sustained oscillations and a regime with turbulent-free jet characteristics. In the self-sustained oscillation regime, the authors observed that the flow was mainly two-dimensional, and proposed a subdivision of this regime into a sinuous mode with large scale wavelengths and a varicose mode with much shorter wavelengths. The case with constant L/H and varying D was not investigated by Maurel *et al.* (1996). By sufficiently increasing the ratio D/H , the two shear layers extend but do not merge along the cavity length L . The confined jet behaviour vanishes and double cavity flow arises. For sufficiently large D the single cavity flow behaviour is expected to be retrieved. Mizushima & Shiotani (2001), Mullin *et al.* (2003) investigated the symmetry, stability and bifurcation properties of double cavity flow as the Reynolds number is varied. For some cases the Reynolds numbers is comparable to the present work, however L/H is much greater, leading to asymmetric flow configurations.

The aim of this paper is to understand the nonlinear frequency selection mechanisms in double cavity flow, focusing on the role of the backflow structures of the recirculation region, observed experimentally for single cavity flow by Basley *et al.* (2011). We address the question of how this feedback mechanism develops as the Reynolds number is increased and whether or not a general route to chaos can be observed. Conceptually, this approach is similar to Johnson & Patel (1999) and Tomboulides & Orszag (2000), who characterized the flow regimes around a sphere in the transition to turbulence.

The present analysis is based on results issued from two-dimensional numerical simulations. The numerical study we propose has the advantage of allowing a detailed parametric study, in a reasonable simulation time, with a greater control of variables than in physical experiments. In the laboratory, for instance, the momentum thickness of the velocity profile, which largely determines the instabilities that develop in the shear layer, is difficult to control without strongly modifying other variables of interest. The computational domain is composed of an upstream inlet channel, the two facing cavities and a downstream exit channel of the same height as the inflow channel. For a given flow rate, the inlet channel characteristics imposes the velocity profile at the leading edges of the two facing cavities. We shall restrict our analysis to cases in which this incoming flow is laminar. By doing this, extrinsic excitations of the flow from turbulent fluctuations are excluded. Furthermore, in order to simplify the study and easily identify the contribution to the dynamics of the shear-layer oscillations, we limit the analysis to cases in which the flow in the inlet channel is not fully developed, i.e. an irrotational core remains in between the two shear layers when reaching the leading edge of the cavities.

The paper is organized as follows. In §2, a detailed description of the numerical simulations is given. In §3 we illustrate the characteristics of the nonlinear system, analysing time series of a single probe and propose a parameter space plot in terms of Reynolds number and the ratio D/L . In §4 we link the dynamical system analysis

to the flow field dynamics of the recirculation region and discuss different scenarios of the delayed feedback mechanism. A summary of our results is given in § 5.

2. Aspects of the numerical study

2.1. Numerical simulation code

We briefly outline the numerical method used. More details and applications of this numerical methods can be found in Gadoin, Quéré & Daube (2001), Podvin *et al.* (2006), Pastur *et al.* (2008) and Rizi *et al.* (2015). The equations of motion describe the incompressible and isothermal flow, given by the non-dimensional Navier–Stokes equations:

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{U} = -\nabla P + \frac{1}{Re_L} \Delta \mathbf{U} \quad (2.1)$$

$$\nabla \cdot \mathbf{U} = 0, \quad (2.2)$$

where $\mathbf{U}$ is the non-dimensional velocity, P the non-dimensional pressure and $Re_L = U_\infty L / \nu$ the Reynolds number based on the cavity length L , in which U_∞ is the maximum of the inflow velocity and ν is the kinematic viscosity coefficient.

The numerical method used to solve (2.1) and (2.2) is based on the projection method, reported by Guermond, Mineev & Shen (2006), with the incremental approach proposed by Goda (1979). The Navier–Stokes equations are discretized, following a finite volume approach on a staggered structured grid with a second-order approximation in time and space. Pressure is defined at the cell centre, whereas the velocity components are defined at the centre of cell faces. Advection fluxes and viscous terms are calculated with a second-order centred scheme. The time derivatives are approximated by a second-order differentiation formulation. An implicit discretization scheme is carried out on the viscous terms in order to increase the numerical stability; δt being the time step for integration, to estimate the advection flux at time $(n+1)\delta t$, the code uses an Adams–Bashford extrapolation from the results at time $n\delta t$ and $(n-1)\delta t$. The semi-implicit discretization scheme of the velocity equation leads to a Helmholtz-like equation for each velocity component, of the form:

$$\left(I - \frac{2\delta t}{3Re_L} \nabla^2 \right) U_i^* = S^{n,n-1}, \quad (2.3)$$

where U_i^* is the field of the i th velocity component, estimated at time $(n+1)\delta t$. The velocity field ($\mathbf{U}^*$) does not satisfy the divergence-free condition yet. $S^{n,n-1}$ contains all explicit terms defined at time $n\delta t$ and $(n-1)\delta t$. For each time step, these equations are solved by means of an alternating direction implicit method (see Hirsch 1987). The divergence-free condition on the velocity field and the pressure field are updated at time $(n+1)\delta t$ by solving Poisson's equation

$$\nabla^2 \Phi = \frac{\nabla \cdot \mathbf{U}^*}{\delta t}, \quad (2.4)$$

where

$$\Phi = P^{n+1} - P^n - \frac{1}{Re_L} \nabla \cdot \mathbf{U}^*. \quad (2.5)$$

The solution Φ is calculated by means of the successive over-relaxation method coupled with a geometric multi-grid method in order to improve the convergence efficiency (Wesseling 1992). The pressure field is directly updated at time $(n+1)\delta t$

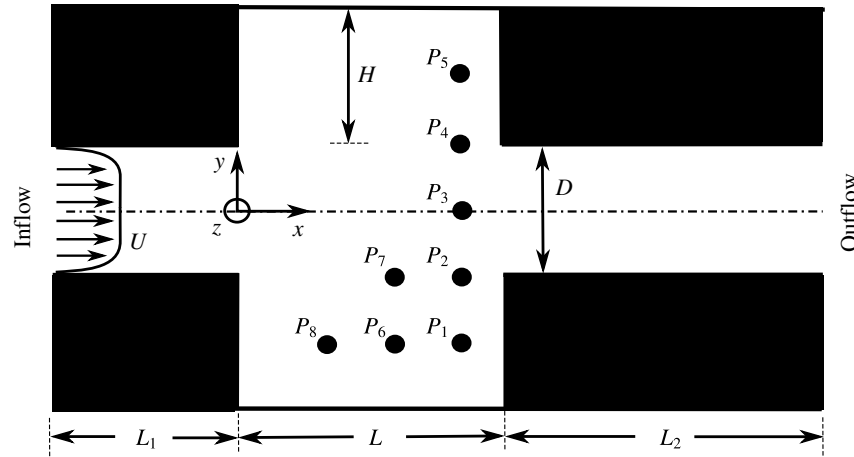


FIGURE 1. Computational domain of double cavity geometry with variable distance D , inflow length $L_1 = 0.6L$, cavity length L , cavity aspect ratio $L/H = 2$, outflow length $L_2 = 1.2L$ and probe points P_1 – P_8 . Arrows indicate flow direction.

from the previous relation and the velocity field U^* is corrected such that the divergence-free condition is satisfied:

$$U^{n+1} = U^* - \frac{2}{3} \delta t \nabla \Phi. \quad (2.6)$$

2.2. Computational domain and boundary conditions

The computational domain, depicted in figure 1, has a cavity length L , with a length over depth ratio $\Gamma = 2$, kept constant throughout the entire study. The inflow length is $L_1 = 0.6L$ and the outflow length is $L_2 = 1.2L$. Longer outflow lengths did not produce significant changes on the flow inside the double cavity. Usual Neumann boundary conditions are applied for solving the pressure Poisson equation. For the velocity, the inflow boundary conditions are of Dirichlet type, with an imposed velocity profile in order to control the flow rate. The outlet boundary conditions are defined by solving a simple one-dimensional advection equation along the normal direction to estimate the velocity profile in the outlet plane. The normal velocity component is then corrected such that the inlet flow rate is preserved. This simple approach ensures mass conservation and limits spurious effects on the solution at the vicinity of the outlet. The usual no-slip and impermeability conditions are applied at the walls. The laminar inflow profiles, imposed at the inlet boundary of the computational domain, were obtained considering an additional length of the upstream channels of $2.8L$ in which a laminar boundary layer develops on either side of the channel. A set of approximately 30 velocity profiles was created for each distance D . The profiles for case $D/L = 0.2$ and the variation of the momentum thickness Θ with U_∞ are depicted in figure 2(a,b), respectively. Inflow profiles for larger cavity distances are obtained by extending the $D/L = 0.2$ profiles in the y -direction, i.e. filling the resulting gap with the value of the respective maximum velocity. It is thus ensured that while D is varied, the maximum velocity U_∞ and the momentum thickness Θ of the profiles are kept constant. Figure 2(c) shows an example of the extension of a $D/L = 0.2$ to a $D/L = 0.4$ case.

The domain is covered with 512 cells in the x -direction, and 256 or 512, depending on D , in the y -direction. The grid convergence of the numerical scheme is of second

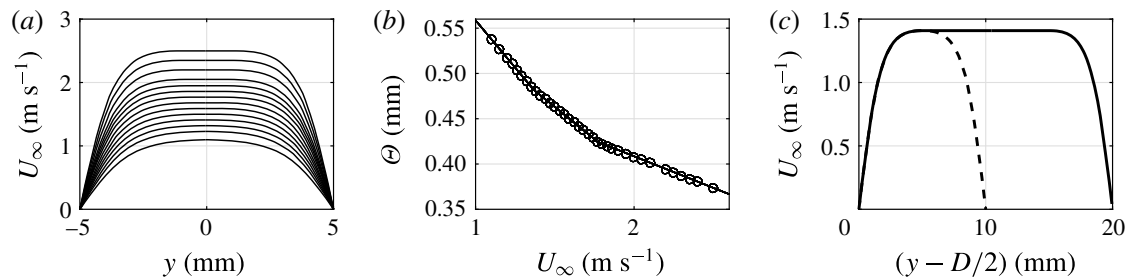


FIGURE 2. (a) Inflow velocity profiles for case with $D/L = 0.2$ for different inflow velocities (for clarity not all velocity profiles are depicted). (b) Momentum thickness Θ as a function of the maximum inflow velocity U_∞ . (c) Extension of a $D/L = 0.2$ profile (---) to a $D/L = 0.4$ profile (—).

Config.	D/L	N_x	N_y	Re_L
SC	1.40	512	256	3833–8330
DC	0.90	512	512	3666–7333
DC	0.60	512	512	3666–7333
DC	0.50	512	256	3666–7333
DC	0.40	512	256	3666–7333
DC	0.35	512	256	3666–7333
DC	0.30	512	256	3666–7333
DC	0.25	512	256	3666–7333
DC	0.20	512	256	3666–7333

TABLE 1. Summary of cases.

order. The mesh is refined close to the walls of the inflow and outflow channel as well as in shear-layer regions and close to the impinging edge, in order to resolve the strong velocity gradients in these areas. The numerical simulations are carried out over a time duration of approximately 400–600 convective time units $\tau_c = L/U_\infty$, depending on the case. The code automatically adapts the time step size to ensure numerical stability, respecting $CFL = 0.25$, where CFL is the Courant number. As a reference for the double cavity (DC), a single cavity (SC) case is added, by closing one of the cavities at a distance of $D = 1.4L$. All simulated cases are summarized in table 1.

3. Time series

Probe points are located in various positions inside the domain, as indicated in figures 1 and 3. Time series are recorded at these probe points with a sampling frequency of approximately 5 kHz. The Strouhal number is defined as $St_L = fL/U_\infty$, in which f is the measured frequency. The initial 30% of each time series is discarded in order to avoid transient effects. The Welch algorithm (Welch 1967) with a window size of 5.1 s and an overlap of 98% is used to compute the power spectral density (PSD) from the streamwise component of the velocity fluctuations. The spectral resolution is thus $\Delta St \approx 0.01$. All power spectra depicted in this work are normalized by pre-multiplying with two times the frequency resolution: $2PSD\Delta f$. Time series from eight probe points together with their power spectra are shown in figure 3 for a typical case. The same frequency ($St \approx 1$) is amplified in the entire

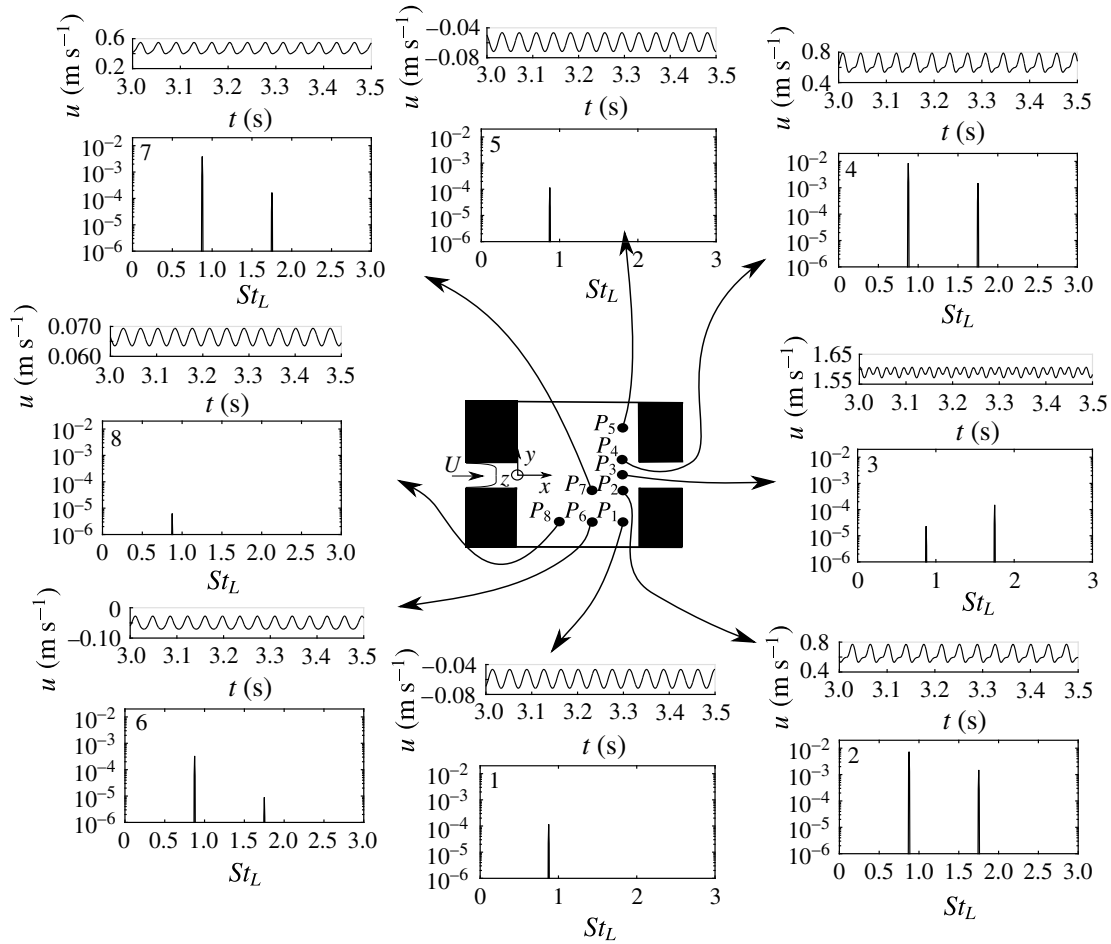


FIGURE 3. Time series of streamwise velocity component u and its power spectral density from probe locations P_1 – P_8 of case $D/L = 0.2$ at $Re_L = 5100$.

domain, which suggests that the spectrum is representative of the global dynamics. Hence, the spectral analysis of the different points can be reduced to a single point. Probe points P_2 and P_4 were chosen for most of the forthcoming spectral analyses, since they carry information from both the respective shear layers as well as the recirculation region. The reason why in point P_3 the second harmonic rises above its fundamental frequency will be explained later in the article. Inside the cavities, the second harmonic has smaller amplitudes than in the shear layer.

3.1. Phase space analysis

Different regimes are encountered when the Reynolds number Re_L is varied. We choose phase space representations to characterize these regimes. Phase space is reconstructed using time delay embedding of time series $s(t)$ (in our case the streamwise component of the fluctuating velocity) recorded at probe point P_2 . The three-dimensional projection of the reconstructed phase space, spanned by the time delayed vectors

$$[X, Y, Z]^T = [s(t), s(t - \tau), s(t - 2\tau)]^T, \quad (3.1)$$

where τ is the time delay, is shown in figure 4(a) for an illustrative case with $D/L = 0.2$. The time delay τ was chosen such that the embedding dimension does not depend on τ , using the method of Cao (1997). Other types of embedding

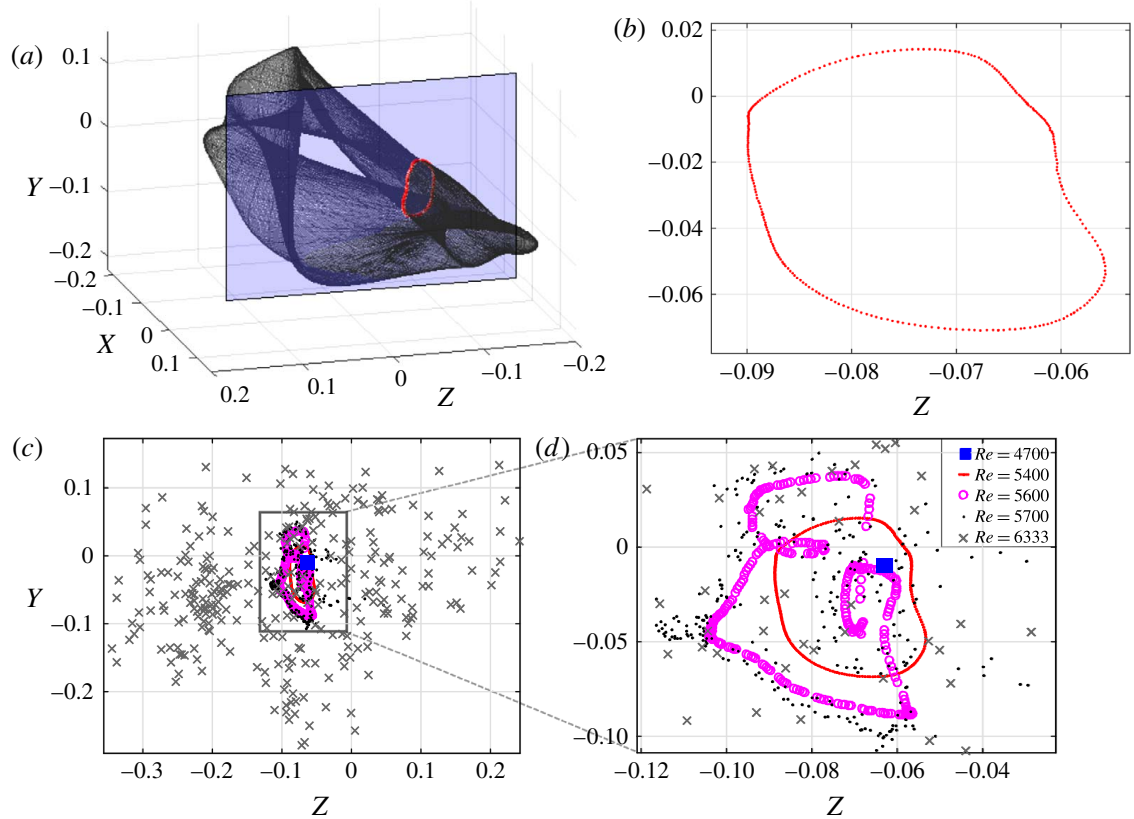


FIGURE 4. (Colour online) (a) Phase portrait: three-dimensional projection of the reconstructed flow from time series of $D/L = 0.2$ case at $Re_L = 5400$ using time delay embedding ($\tau = 0.015$ s). Vertical plane (blue) at $X = 0.0055$ indicates position of the Poincaré section. Intersections of phase portrait with plane is indicated by red symbols. (b) Poincaré section: intersection of phase portrait in (a) with plane $X = 0.0055$. (c) Poincaré sections ($X = 0.0055$) at $Re_L = 4700$ (blue filled square, periodic regime), $Re_L = 5400$ (red solid line, intermediate regime (quasi-periodic case)), $Re_L = 5600$ (magenta $\circ$, folded torus), $Re_L = 5700$ (black dots, low-order chaotic regime). $Re_L = 6333$ (grey crosses, high-order chaotic regime). (d) Zoom-in view of (c).

techniques, such as single value decomposition (SVD) embedding, were tested, and the conclusions remain the same. Figure 4(b) depicts the Poincaré section, a two-dimensional subspace of the three-dimensional phase space in figure 4(a). It is obtained as an intersection of the plane at $X = 0.0055$ with the phase portrait in figure 4(a). The Poincaré section was chosen transverse to the flow in the best agreement with the three conditions for a global Poincaré section (see Solari, Natiello & Mindlin 1996). Figure 4(c) exemplifies the Poincaré sections of the $D/L = 0.2$ geometry for different Reynolds numbers. Figure 4(d) shows a zoomed-in view of figure 4(c). Based on these Poincaré sections, we can now define the flow regimes.

Once the flow evolves from the steady regime to the periodic regime, a period-1 limit cycle is reached in phase space and a single point can be observed in the Poincaré section ($Re_L \approx 4700$). When the value of the Reynolds number is further increased ($Re_L \approx 5400$), a so-called intermediate regime is reached. This regime includes quasi-periodicity, i.e. a second frequency appears in the spectrum, incommensurate with the dominant frequency. This induces the creation of a torus

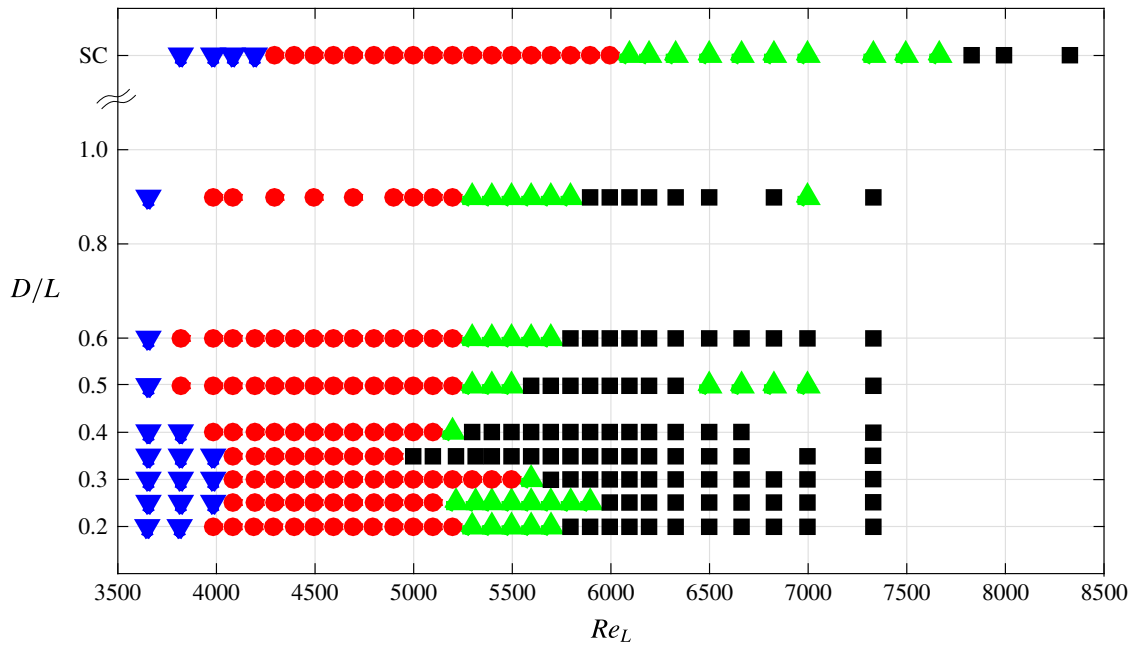


FIGURE 5. (Colour online) Parameter space plot. Blue ∇ : steady regime. Red $\bullet$: Periodic regime. Green $\blacktriangle$: intermediate regime. Black $\blacksquare$: chaotic regime. Note, the top line corresponds to a SC channel.

in phase space, which is characterized by a closed curve in the Poincaré section. Frequency locking may occur on the torus, resulting in periodic windows. At $Re_L \approx 5600$ the torus surface is folded by wrinkles. The Poincaré section of the torus presents self-intersections, indicating that the state space has at least a dimension equal to 4. The Cao (1997) algorithm, applied to the time signal in this flow regime gives an actual dimension of 5. This relatively small (local) dimension does not preclude potentially higher, though finite, dimensions for the full state space. For $Re_L \approx 5700$, more complex dynamics is reached, though still structured around the torus. Finally, for Reynolds number $Re_L = 6333$, the Poincaré map yields a cloud of points (see figure 4c) without any identifiable structure. The last two regimes are reminiscent of chaotic behaviour and hence are labelled chaotic regime. Yet note, that the dispersion of points in the Poincaré map alone is not a quantitative proof, though a strong indication of the chaotic nature of the system. The described route to chaos is common to small channel heights ($D/L < 0.30$) and is similar to the Curry & Yorke (1978) scenario: steady (fixed) point $\rightarrow$ period-1 limit cycle $\rightarrow$ quasi-periodicity $\rightarrow$ torus breakdown leading to toroidal chaos, as also recently observed for the van der Pol system by Letellier, Messenger & Gilmore (2008). For larger channel heights ($D/L > 0.35$), however, the route to chaos looks slightly different, while configurations with $D/L = 0.35$ and $D/L = 0.30$ have their own distinctive details, as we shall see in the following sections. Henceforth, our so-called intermediate regime is not exclusive to quasi-periodic behaviour, as it may also include periodic windows, that occur due to frequency locking on the torus.

3.2. Parameter space

Based on the characterizations from § 3.1, a parameter space plot, depicted in figure 5, is constructed. It summarizes the regimes, found when the Reynolds number and the

cavity distance are varied. Double cavity flow is equivalent to the canonical SC flow when the distance D is very large. In general, the respective regimes are found at higher Reynolds numbers for SC flow when compared to DC flow. The steady regime is the first regime observed. It ends with the development of instabilities that produce self-sustained oscillations of the flow. The associated limit cycle is observed in our study for Re_L lying in the range between ≈ 3700 and ≈ 4300 depending on the value of D . The periodic regime is prone to develop at lower Re_L for intermediate cavity distances ($D/L = 0.5$ and $D/L = 0.6$) and at higher Re_L for small ($D/L \leq 0.4$) as well as for very large distances ($D/L \geq 0.9$). The case $D/L = 0.3$ is an exception to the other close distance cases: its periodic regime is observed for much larger Reynolds numbers, compared to its neighbours. Depending on the values of D/L , the intermediate regime begins around $Re_L \approx 5500$ for the double cavity, while for the single cavity it starts at $Re_L \approx 6000$. The Reynolds number range of the intermediate regime depends on the cavity distance. The longest range is found for the single cavity, while the shortest range is found for distances close to $D/L = 0.35$. At $D/L = 0.35$ the intermediate regime disappears all together in what looks like a sudden (hard in amplitude) transition to chaos (Lopez-Rebollal & Sanmartin 1995) and the chaotic regime is encountered in direct continuation to the periodic regime. However, it cannot be fully excluded that a quasi-periodic regime be recovered with a smaller step size in the value of the Reynolds number.

3.3. Spectral analysis

In this section we characterize the spectra associated with each regime. Figure 6(a–i) show the power spectra for all cases in table 1 when the Reynolds number is varied. In the steady regime no oscillations and hence no frequency peak is present. In the periodic regime, a salient peak at $St_L \approx 1$ prevails over a wide range of the Reynolds number. As expected, harmonics of this dominant mode are also present in the spectrum. The power spectra in these regions show strong dark lines equispaced in the vertical coordinate, reminiscent of the teeth of a ‘comb’. For $D/L \geq 0.4$, when approaching the intermediate regime, the dominant Strouhal number $St_L \approx 1$ is gradually replaced by its first harmonic $St_L \approx 2$. In the Poincaré section this is accompanied by an additional intersection. The underlying physics of this phenomenon will be addressed in §4.2. In the intermediate regime, the peak at $St_L \approx 1$, and its harmonics, are still present, but new peaks of incommensurate frequencies arise in the spectra, a phenomenon that is in correspondence with the creation of a torus in phase space mentioned in §3.1. Peaks of much lower values of non-dimensional frequencies than the previous dominant Strouhal number indicate that a slow dynamics now leaves the signature in the spectrum. As a consequence of the rise of incommensurate frequencies, its harmonics and linear combinations, a tighter comb with a reduced teeth separation is observed in the power spectra of figure 6. Some exceptions may occur, as illustrated in figure 6(g,h). We may notice in these graphs a discontinuity in the evolution of the dominant frequency at $Re_L \approx 5000$ and $Re_L \approx 5300$, respectively. The underlying physics of these discontinuities will be discussed in §4.4. In the intermediate regime of the SC, the Strouhal number increases monotonically with the Reynolds number and the teeth are inclined. The trajectory in phase space is locked on the torus during most of the intermediate regime of the SC, resulting in a periodic window with $St_L \approx 0.5$ and its harmonics over a wide range of Reynolds numbers. In the spectra corresponding to the intermediate regime of the DC, the third and fourth peaks are in general in the ratio $\simeq 3/4$. Furthermore, the first and third peaks

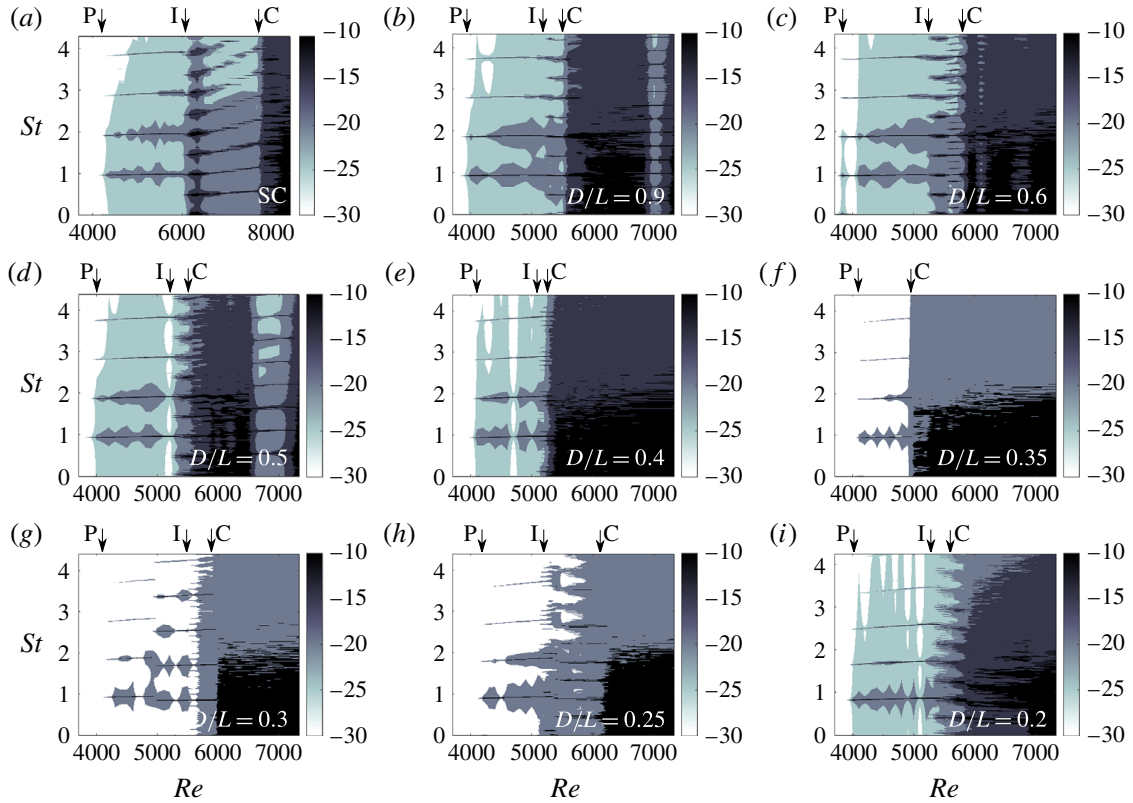


FIGURE 6. (Colour online) Power spectra versus increasing ramps of the Reynolds number for (a) SC, (b) $D/L = 0.9$, (c) $D/L = 0.6$, (d) $D/L = 0.5$, (e) $D/L = 0.4$, (f) $D/L = 0.35$, (g) $D/L = 0.3$, (h) $D/L = 0.25$, (i) $D/L = 0.2$. Colour code depicts the normalized power spectra in log scale: $\log(2\text{PSD}\Delta f)$. Arrows indicate where the periodic (P), the intermediate (I) and the chaotic (C) regime commence in each case.

keep a ratio of $\simeq 1/3$. We will discuss the reason of this behaviour in § 4. With the exception of $D/L = 0.35$, all transitions to the chaotic regime take the quasi-periodic route. As the Reynolds number is further increased, the spectrum becomes richer, filling in with linear combinations of both incommensurate frequencies. Nonlinear effects promote the dynamics to enter the chaotic regime. At even higher Reynolds numbers, the spectrum eventually becomes much flatter, peaking at $St_L \approx 0.4$. The decay of the time-correlation functions strongly indicates that the dynamics is chaotic in this regime. As expected when Re_L is increased in the chaotic regime, chaos resorption may occur over finite windows of Re_L . For instance, in figure 6(d) for the case $D/L = 0.5$, such a window is observed between $Re_L = 6500$ and $Re_L = 7000$, where the spectrum simplifies and the flow returns from chaotic to the intermediate regime with $St_L = 0.5$. For $D/L = 0.9$ a stability window occurs at $Re_L = 7000$. Also, windows of simplified spectra may appear even inside the periodic regime, as can be observed for instance for $D/L = 0.4$, at $Re_L \approx 4700$, in figure 6(e).

4. Flow field analysis

In this section, the dynamical system analysis is linked to the flow dynamics. We first discuss the salient characteristics of the flow structure. As we see in figure 7, in each cavity, the inner flow is structured in two large main recirculation regions and a third smaller one. The main recirculation region, located in the downstream

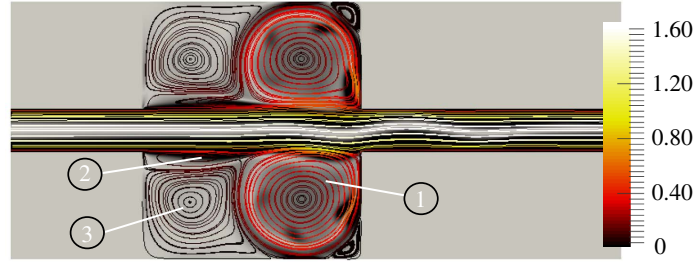


FIGURE 7. (Colour online) Instantaneous velocity flow field with streamlines of case $D/L = 0.2$ at $Re_L = 4900$ (periodic regime). Colour code of the streamlines correspond to the velocity magnitude. Background corresponds to vorticity magnitude. Numbers 1–3 indicate intracavity flow recirculation regions.

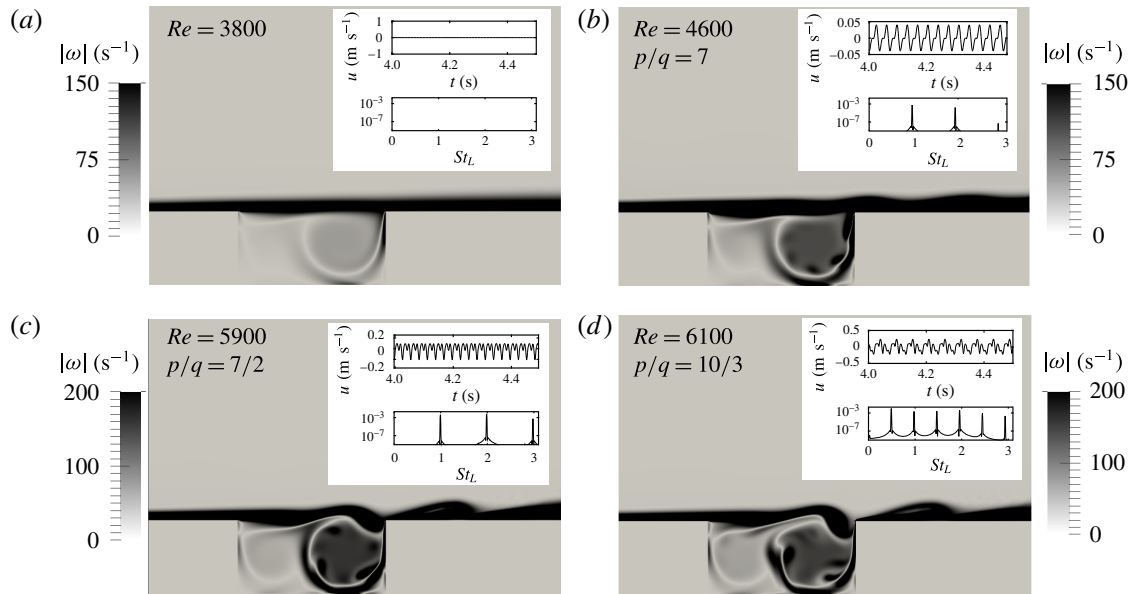


FIGURE 8. (Colour online) Flow patterns for the single cavity in the (a) steady regime, (b) periodic regime with $St_L \approx 1$ most amplified, (c) periodic regime with $St_L \approx 2$ most amplified, and (d) intermediate regime (quasi-periodic locked-on torus), together with respective time series recorded in probe point P_2 and its power spectra. Colour code indicates vorticity magnitude. For details on the ratio p/q see text.

half of the cavity, remains almost circular up to the end of the intermediate regime. The secondary recirculation, located in the upstream half of the cavity, is also closely circular. The line connecting the centres of recirculation regions 1 and 2 (cf. figure 7) is parallel to the free-stream direction. The dynamics of both recirculation regions is however quite different, as the magnitude of the velocity field in the secondary recirculation region is one order of magnitude smaller than in the main recirculation region. The third, smaller, recirculation bubble is located on top of the secondary recirculation region, below the shear-layer close to the leading edge. These observations are in agreement with Mizushima & Shiotani (2001), who found essentially the same flow pattern for a geometry with a slightly greater cavity aspect ratio $\Gamma = L/H = 2.6$.

The vorticity magnitude field is plotted in figures 8 for the single cavity flow, and in figure 9, for the $D/L = 0.2$ DC flow, for different Reynolds numbers, with time series

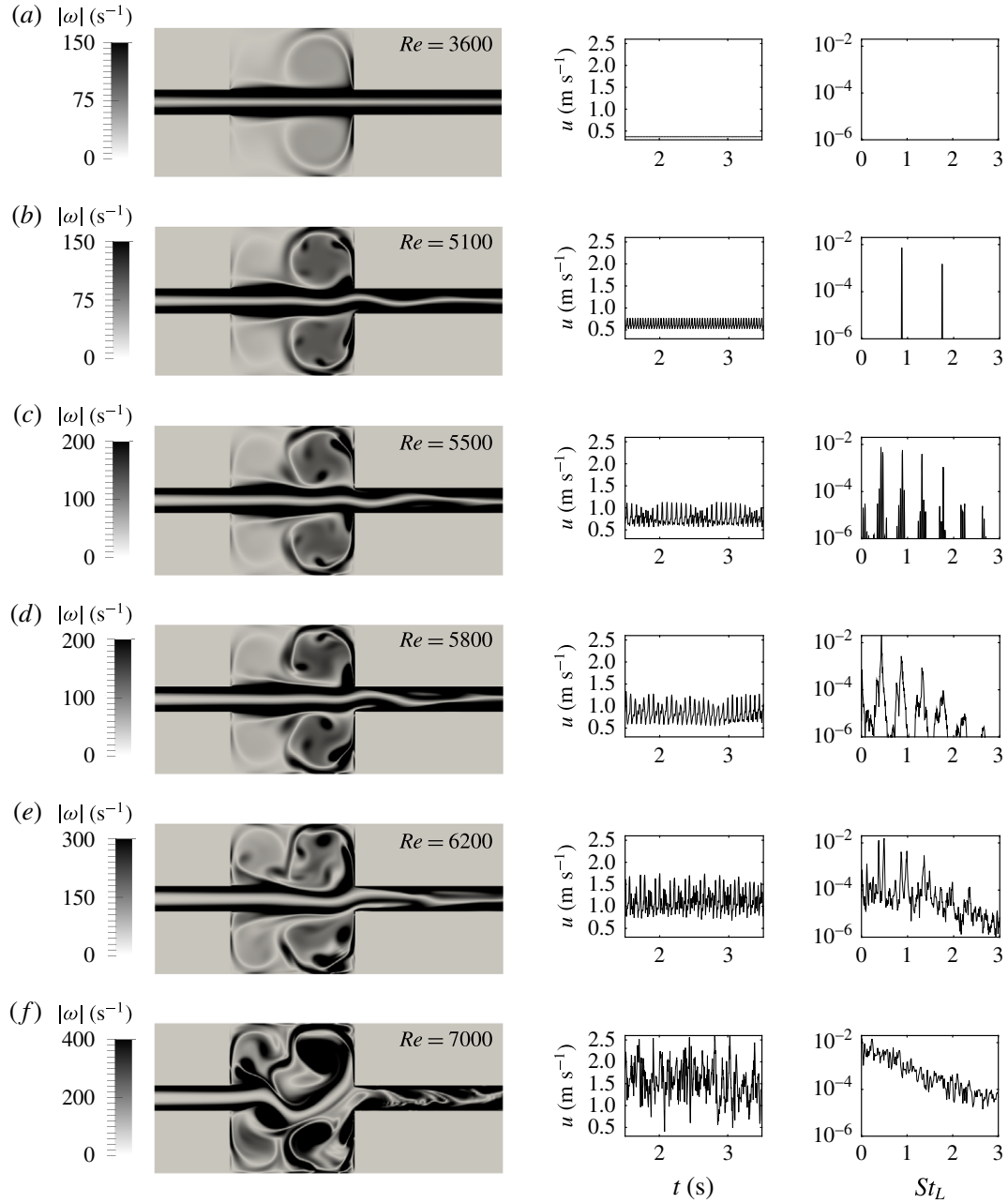


FIGURE 9. (Colour online) Flow patterns for case $D/L = 0.2$ in (a) steady regime, (b) periodic regime (limit cycle), (c) intermediate regime (torus), (d) intermediate regime (torus folding), (e) chaotic regime (toroidal chaos), and (f) chaotic regime (exploded torus) together with time series recorded in probe point P_2 and respective power spectra. Colour code indicates vorticity magnitude.

of the signal, recorded in probe point P_2 and the associated power spectra. For low Reynolds numbers, $Re \lesssim 4000$, the flow is steady. A circular dipolar vortex sheet limits the main recirculation region, as can be seen for instance in figures 8(a) and 9(a). In the periodic regime, the shear layer exhibits oscillations of such magnitude that vorticity of the shear layer is regularly injected at the rear edge of the cavity into the main recirculation region. The injected vorticity forms small vortices at the

frequency of the shear-layer oscillations. These regularly spaced vortices circumvent along the dipolar vortex sheet, creating a ‘carousel’ type formation. The number of small vortices in the carousel depends on the value of D and on the velocity of the incoming flow, which defines the frequency at which the shear layer oscillates, and hence the rate at which small vortex structures are injected into the cavities. The vorticity strength of these vortices decreases along their circular motion in the carousel. The existence of these small vortices is readily seen in the spectrum. They leave a signature on the probe signal as a consequence of their cyclic passage.

Henceforth, the analysis will focus on the interaction of the shear layer with the main recirculation region.

4.1. Characteristic time scales

Different time scales of interest appear in this phenomenon. We define T_{shl} as the characteristic time of the prevailing oscillations of the shear layer, T_{to} the turnover time of the main recirculation region, and T_{lt} the characteristic lifetime of the small vortex structures. The advective time $\tau_c = L/U_\infty$ is the natural reference time. It turns out that $T_{shl}/\tau_c \simeq 1$ in all cases. If the main recirculation region was considered as being driven only by the shear layer in solid rotation, T_{to} should scale with U_∞ and H . In this respect, one could therefore expect that $T_{to}\Gamma/\tau_c$ also be little sensitive to the value of Re_L . However, this is not the case. Instead, we observe a dependence of both $T_{to}\Gamma/\tau_c$ and T_{to}/T_{shl} with Re_L . The ratio T_{to}/T_{shl} depends additionally on the momentum thickness Θ of the incoming boundary layer, which defines the shear-layer thickness, among other parameters that will be further clarified. The lifetime T_{lt} is not easily determined as it is a function of the viscosity of the fluid, of the strength of the small vortices and of their interaction with the neighbouring vorticity. A reference time can be considered assuming $T_{lt} \sim \eta^2/\nu$ where η is the radius of the small vortices, which depends on the amplitude of the shear-layer oscillations and therefore on Re_L . The ratio T_{lt}/T_{to} determines, whether in a single snapshot of the vorticity field, the small vortices in the image were all created in the same round trip (figure 8a), or vortices from two or more consecutive round trips coexist (figure 8c,d).

4.2. Flow field dynamics of the periodic regime

For the periodic regime, the simplest configuration is obtained when $T_{lt} < T_{to}$, i.e. the small vortices dissipate before they complete one round trip in the carousel. Figure 8(b) illustrates this configuration, for the SC flow, where seven small vortices are formed during one turnover cycle ($T_{to}/T_{shl} \simeq 7$). These vortices have such a small lifetime, that during one cycle they do not appreciably reach the region of vorticity injection. The only frequency measured in probe point 2 is therefore the frequency of the shear-layer oscillations.

For $2T_{to} \gtrsim T_{lt} \gtrsim T_{to}$, the vortex completes one round trip and returns to the injection position, exiting the shear layer in a feedback process. A periodic regime may still be encountered in this case over a wide range of the Re number. In such a situation, both the shear-layer oscillations and the carousel turnover are synchronized, which is something usually expected from two coupled oscillators. Periodic behaviour implies $T_{to}/T_{shl} = p/q$, with $p, q \in \mathbb{N}$ and $p > q$. Many different possible scenarios are found in SC and DC flows, depending on the values of Re_L , ratio $T_{to}/T_{shl} = p/q$, and D . We shall illustrate some of them below.

In figure 9(b) we observe, for a DC flow case, a situation in which $T_{to}/T_{shl} = 4$. The vorticity plots show four small equispaced vortices per cavity in the main recirculation

region. The structures of the carousels of each cavity are in opposite phase. Once a small vortex returns to the area of injection at the rear edge of the respective cavity, the shear layer injects a new vortex in its position. The power spectrum exhibits in this case the main peak at $St_L \simeq 1$, as the cyclic passage of the different vortices at probe position occurs at the frequency of the shear-layer oscillations. The amplitude of the first harmonic is enhanced by the opposed carousel. The set of two carousels, exhibiting phase opposition, produces a signal that doubles the frequency of the shear layer. This is clearly illustrated in the spectrum of figure 3 at probe point P_3 . There, the peak at $St \simeq 2$ is of larger amplitude than the peak at $St \simeq 1$.

Figure 8(c) illustrates the case of an extremely large value of D/L or equivalently a single cavity flow. For this case the ratio is $T_{to}/T_{shl} = 7/2$ and the dynamics is periodic. The power spectrum exhibits a dominant peak at $St_L \simeq 2$ in probe point P_2 , while a peak at $St_L \simeq 1$ is still present. The reason for this spectral distribution, is that $T_{lt}/T_{to} \simeq 2$. Consequently, the carousel is made of two intertwined patterns of small vortices, which doubles the frequency in P_2 . The first pattern is weakened since it enters its second turn in the recirculation region, while the second pattern carries a stronger vorticity as it enters for the first time (figure 8c).

The ratio T_{to}/T_{shl} , however, evolves as the Re number is increased. By further increasing Re_L , the single cavity reaches a periodic regime with $St_L \simeq 0.5$ in which $T_{to}/T_{shl} = 10/3$. A snapshot of this flow regime is shown in figure 8(d). Due to $T_{lt}/T_{to} \simeq 2$ the carousel pattern, though still periodic, becomes more complex.

To summarize, a periodic regime is obtained for either (i) $T_{lt} < T_{to}$ or (ii) $T_{lt} \geq T_{to}$ when at the same time $T_{to}/T_{shl} = p/q$, with $p, q \in \mathbb{N}$. In the first case the rhythm is set by the shear layer, since the small vortices dissipate before completing one round trip in the carousel. In the second case, the shear layer and the carousel are locked in by $T_{to}/T_{shl} = p/q$, with $p, q \in \mathbb{N}$. We did not observe cases with $T_{lt} \gtrsim 2T_{to}$ in the periodic regime.

4.3. Flow field dynamics of the intermediate regime

In the intermediate regime, incommensurate frequencies appear in the spectrum and the trajectory in phase space lies on a torus. We can expect that this new behaviour manifests itself in the dynamics of the carousel. We have shown in § 3.3, that many different situations are encountered in the intermediate regime. Rather than describing all the possible scenarios, let us instead illustrate some examples.

For $D/L = 0.2$ at $Re = 5500$ (figure 9c), the lifetime of the small vortices lasts approximatively two turnover times $T_{lt} \simeq 2T_{to}$ before they dissipate. The carousel pattern exhibits recurrent states with $n = 3$ or $n = 4$ vortices. Expressed in terms of time scales we get: $3 < T_{to}/T_{shl} < 4$. The small vortices are not equispaced as a consequence of the continuous transitions between both states. The spectrum displays the frequency associated with the shear-layer instability at $St_L \approx 1$ and a second incommensurate frequency $St_L \approx 0.45$ associated with the carousel's patterns.

Continuing with figure 9(d) at $Re_L = 5800$, the three vortex pattern per cavity prevails. The spectrum shows both the frequency associated with the shear-layer oscillations ($St_L \approx 1$) and with the passage of the three small vortices. The frequency ratio between the values of the frequencies of the third and fourth modes, which is $\simeq 3/4$, can be associated with the recurrence in each cavity of a three vortex pattern instead of a four vortices pattern, synchronized with the shear-layer instability. The ratio of the new peaks, at $St_L \simeq 0.45$ and $St_L \simeq 1.35$, reaches a value close to $1/3$ that can be associated with the cyclic behaviour of the three vortex pattern in the intracavity structures.

At Reynolds number close to 6200 (figure 9e), the intermediate regime loses coherence and the small vortices are not regularly spaced anymore. Note however, that the vortex carousel persists. Finally, at Reynolds numbers close to 7000 (figure 9f), the recirculation regions deform strongly. The recurrent patterns of vortices in the carousel become largely disordered and are not easily determined. In fact, the vortices injected into the cavities by the shear layers are of such intensity, that after one turnover, they largely disturb the incoming shear layer and make the periodic feedback mechanism impossible. The counterpart in phase space, to this last dynamics, is the torus breakdown (cf. figure 4).

4.4. Coupling strength

Let us consider the spatial symmetry of the two-dimensional vorticity magnitude field $|\omega(x, y)|$ in figure 10. Considering a spatial reflection symmetry (with respect to the centreline $y=0$), we distinguish between the following two cases:

$$\left. \begin{aligned} |\omega(x, y)| &= |\omega(x, -y)| \Rightarrow \text{spatially symmetric (or varicose)} \\ |\omega(x, y)| &\neq |\omega(x, -y)| \Rightarrow \text{spatially asymmetric.} \end{aligned} \right\} \quad (4.1)$$

Note, in the periodic regime the spatially asymmetric cases correspond to a sinuous flow configuration. Both symmetric and asymmetric cases are observed, though in most cases the flow is of the asymmetric type. For a given Reynolds number, as D/L is varied and the coupling between the cavities evolves, the flow symmetry may change. This is illustrated in figure 10 for $Re_L = 5500$. For close facing cavities, when $D/L < 0.25$, the flow configuration is spatially asymmetric. The symmetry changes as D/L increases and gets closer to $D/L = 0.3$, a value for which the flow adopts a spatially symmetric (varicose) configuration. Increasing D/L again, the flow returns to a spatially asymmetric configuration. The emergence of spatial symmetry explains the discontinuity in the Strouhal number as a function of the Reynolds number for $D/L = 0.25$ at $Re_L = 5400$ and $D/L = 0.3$ at $Re_L = 5000$, respectively (cf. figure 6g,h). At these Reynolds numbers the flow adopts the varicose flow configuration. For the case $D/L = 0.3$ the change in symmetry occurs in the periodic regime without hysteresis, while for the case $D/L = 0.25$ the change in symmetry is located in the intermediate regime and a hysteresis was observed. The change in symmetry before the transition to unsteady or turbulent flow conditions was also observed for the flow past a sphere by Johnson & Patel (1999) and Tomboulides & Orszag (2000).

Let us analyse the interaction of the feedback mechanisms of the carousel as we modify the ratio D/L for a fixed value of Reynolds number. For $Re_L = 5500$, the flow passes through all regimes as D/L is varied. As exemplified in figure 10(a), the number of small vortices in each recirculation region is four when the cavities are infinitely far from each other (SC). This corresponds to a case in which $T_{to}/T_{shl} = 4$ and $T_{lt} \simeq T_{to}$ and the recorded value compares favourably with the average recirculation velocity reported by other researchers (Villermaux & Hopfinger 1994; Back & Roschke 1972). A noticeable change in the carousel is observed, when a second cavity is present at proximity (figure 10b–d). We observe, that the frequency is almost constant but the number of small vortices in the carousel diminishes, which means that the carousel must turn faster, since the shear-layer instabilities can not be strongly modified, as neither the Reynolds number nor the momentum thickness have changed. Figure 11(a) shows the carousel speed, normalized with the centreline velocity U_∞ , as a function of the cavity distance. We observe, that the second cavity has a strong effect. The carousel speed increases by 43 %, when D/L is varied from

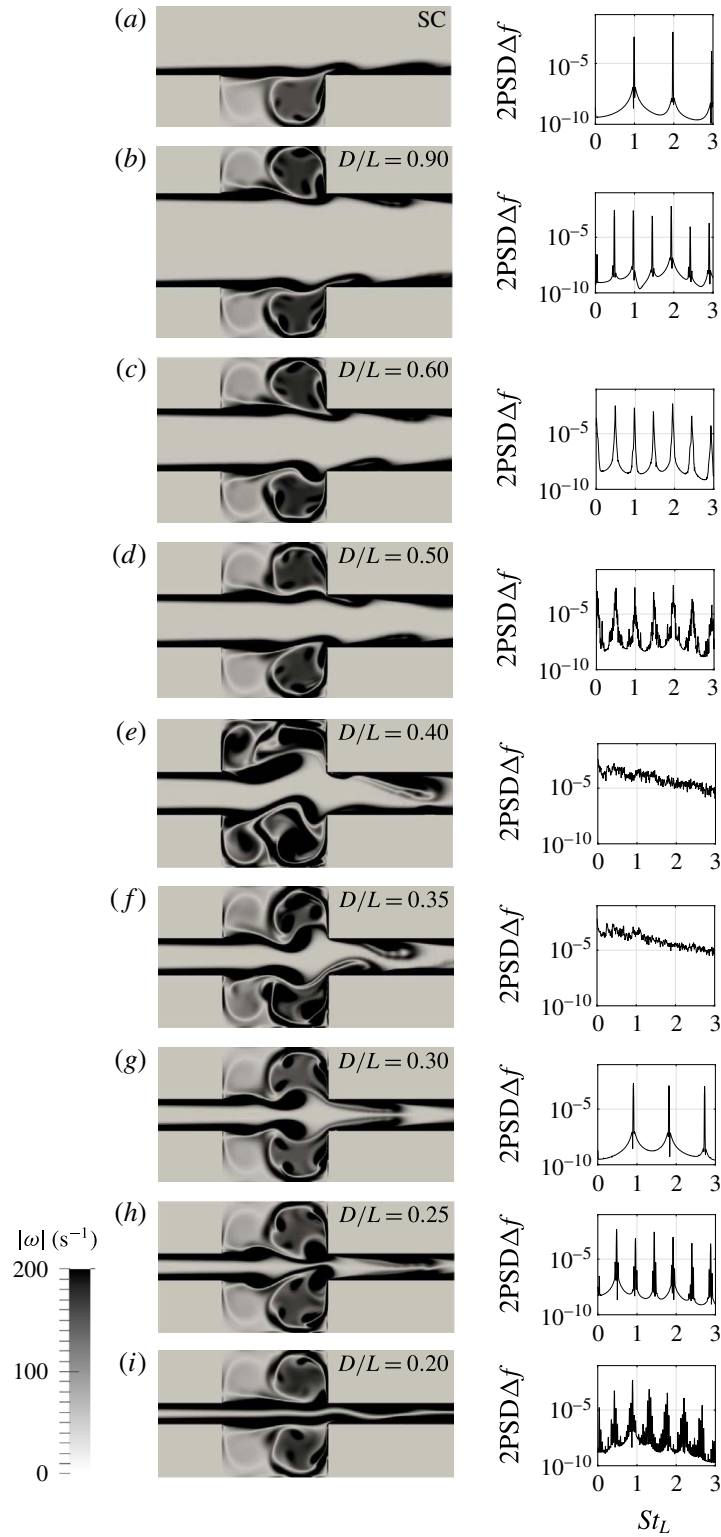


FIGURE 10. (Colour online) Flow patterns for different cavity distances D/L at $Re = 5500$ and the respective power spectra from time series recorded in probe point P_2 . Colour code indicates vorticity magnitude. (a) Periodic regime, (b) intermediate regime, (c) intermediate regime, (d) intermediate regime, (e) chaotic regime, (f) chaotic regime, (g) periodic regime, (h) intermediate regime, and (i) intermediate regime.

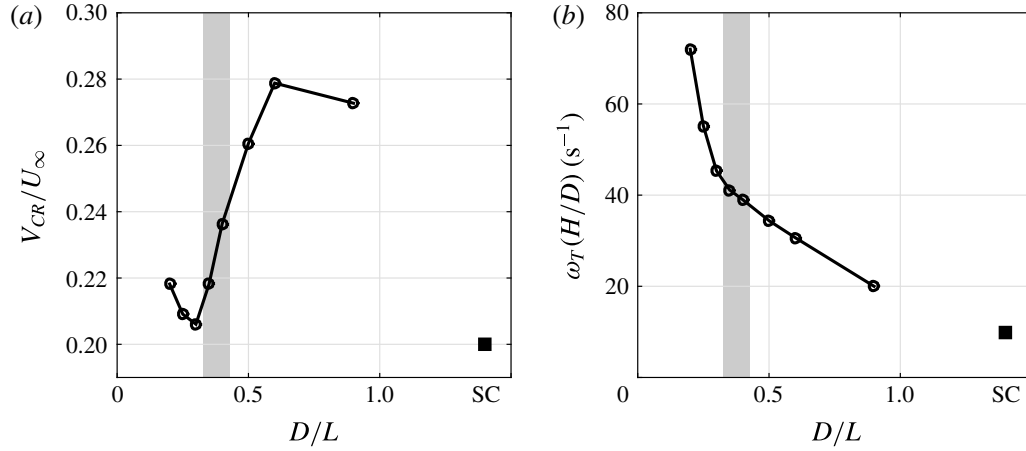


FIGURE 11. (a) Normalized carousel speed V_{CR} and (b) angular velocity $\omega_T = V_{CR}/(H/2)$ multiplied by the depth over distance ratio H/D as a function of the normalized cavity distance D/L for all cases in figure 10 ($Re = 5500$). The SC case is indicated by ■. The shaded areas indicate the distances for which chaotic behaviour occurs (see figure 10).

1.4 (SC) to 0.9 (DC). This increase in angular velocity has in turn a critical effect on the feedback mechanism between the carousel and the shear layer. Although the carousel is turning faster, the lifetime of the circumventing vortices has not been significantly modified. As a consequence, the injected vortices remain strong when they come back to the shear-layer region, which they now excite. As a result, the amplitude of the shear-layer oscillations is reinforced by this excitation, and the vorticity injected into the carousel at the impingement, becomes stronger. Mutual induction between the stronger and (relatively) long living circumventing vortices can further increase their speed in the carousel. Indeed, the angular velocity of the carousel increases as D/L is reduced and reaches a maximal value for $D/L \approx 0.6$. For more contiguous cavities, the angular velocity of the carousel decreases again. This may be understood as a ‘blocking’ effect due to the proximity of the shear layer of the facing cavities. The shaded zones in figure 11 indicate the D/L -range for which chaotic behaviour is observed. The speed drops to a minimum when $D/L = 0.3$, the distance at which the flow becomes symmetric, as can be seen from figure 10(g). The carousel speed slightly increases for the closest distances (cases $D/L = 0.25$ and $D/L = 0.2$). In figure 11(b) the angular velocity, multiplied by the depth to distance ratio, is plotted against the normalized cavity distance D/L . Interestingly, the resulting curve decreases monotonously, while the trend for V_{CR} , as a function of D/L , is quite different, as shown in figure 11(a). Note however, that ω_T and V_{CR} are connected through the relation:

$$\omega_T \frac{H}{D} = \frac{V_{CR}}{H/2} \frac{H}{D}. \quad (4.2)$$

The change in the carousel speed is even more striking when D/L is fixed and the Reynolds number is varied. In the SC, for instance, the number of circumventing vortices in the carousel changes from $n = 7$ to about $n = 3$ when the Reynolds number passes from 4600 to 5100, as can be seen in figure 8. The relative change in the incoming flow velocity is approximately 10% while the turnover time increases by 50%. This behaviour is common to all cases. The significance of the carousel mechanism to understand the coupling between shear-layer and intracavity flow and

also to understand the mutual interactions between facing cavities, encourages future work on the subject.

Experimental evidence of the vortex carousel can be found in the literature for open cavity flow by e.g. Basley *et al.* (2011) and Basley *et al.* (2013). In these works, smoke visualizations and particle image velocimetry measurements for a $\Gamma = L/H = 1.5$ cavity show the mechanism. The length to depth ratio Γ most likely does become important for $\Gamma \gg 2$, as in Mizushima & Shiotani (2001) and Mullin *et al.* (2003), who showed that the flow field can become heavily asymmetric due to large cavity lengths L . Such an asymmetry will certainly alter the carousel mechanism.

4.5. The outflow pattern

It is also interesting to observe how the flow is modified at the exit channel as the different regimes take place (cf. figure 9). The outflow in the steady regime and in the periodic regime is well organized and the presence of the double cavity does not introduce significant modifications between the inflow and the outflow. As the Reynolds number is increased and the intermediate regime begins, the outflow becomes more complex and once the chaotic regime is reached, strong mixing is produced. As a potential application, DC flow could hence be designed as a laminar chaotic mixing device. In figure 5, we can observe that the chaotic regime for DC flows is reached at much lower Reynolds numbers for closer cavity distances. This reflects the consequence of the strong coupling of the two individual cavities, and the advantage of a DC configuration when mixing enhancement is pursued.

5. Conclusions

We studied a system, composed of two facing cavities, driven by a flow entering from a channel that determines the characteristics of the velocity profile at the leading edge of the cavities. We restricted our analysis to cases in which the inflow profile was laminar, in order to exclude possible excitations associated with the intrinsic fluctuations of turbulent flows. A first analysis, considering time series of the velocity signal from two-dimensional numerical simulations, was carried out. We found that the signal of a single probe was in general rich enough to capture the salient features of the global behaviour. On this basis, a study was performed using time series from a single probe. As Re_L is increased, for a given ratio D/L , we found the following successive regimes: steady $\rightarrow$ periodic $\rightarrow$ intermediate $\rightarrow$ chaotic. A map was constructed which enables to identify these regimes for different Reynolds numbers and different separation distances between the cavities. In phase space, the periodic regime describes a limit cycle with a spectrum, that always exhibits a peak at $St_L \approx 1$. In most cases, the oscillations of the shear layer is of the asymmetric type. The periodic regime is observed for $Re_L \gtrsim 4000$, though this value depends on the cavity distances. For $D/L = 0.5$ and $D/L = 0.6$ it is reached at somewhat lower Reynolds numbers. As the Reynolds number is increased, the limit cycle disappears and the trajectories in phase space lie on a torus. The spectra become more complex, exhibiting combinations of the two basic incommensurate frequencies. Frequency lockings and un-lockings may occur on the torus, before it breaks down and bifurcates towards a chaotic regime. In this last regime, any phase coherence between the two shear layers is lost and oscillation amplitudes are one order of magnitude above the amplitudes of the periodic regime. We could verify that the transitions to the chaotic regime in almost all cases take the quasi-periodic route, in

which a frequency smaller than the dominant Strouhal number appears. This route to chaos is reminiscent of a Curry & Yorke (1978) scenario.

A second analysis was carried out, focusing on the vorticity fields, in order to link the phase space dynamics to the dynamics of the physical space. The visualization of patterns of small intracavity structures serves to understand the underlying mechanisms of the feedback process and helps to identify the flow regime. The small vortices are created as a consequence of the shear-layer oscillations, which regularly inject vorticity at the rear edge of the cavities. They describe a quasi-circular motion inside the main intracavity recirculation region, forming a 'carousel-like' structure in each cavity. These vortices, when returning to the region of the instability onset, excite the shear layers. The relative phase synchronization of the carousels indicates the degree of coupling between the two cavities. The angular velocity of the carousel and the number of vortices present in the carousel depend, non-trivially, on the Reynolds number and the distance between the cavities.

The interplay of three characteristic time scales determines the properties of the observed phenomena: the turnover time of the main recirculation region, the lifetime of the small intracavity vortices and the period of the shear-layer oscillations. We illustrated the importance of these three time scales on the dynamics of the physical space and the associated dynamics of the phase space. The simplest scenarios correspond to lifetimes so short, that the vortices cannot accomplish one complete round trip in the carousel. In this situation, time series indicate a periodic regime with $St_L \approx 1$, associated with the shear-layer oscillations. When the lifetime is long enough, the small circumventing vortices keep an intensity capable to excite the shear layers, and therefore to produce a significant feedback process. Depending on the separation D/L , the small vortices may even leave their signature in the signal of probes placed in the facing cavity. A periodic regime may be found for lifetimes larger than the turnover time, when the carousel synchronizes in such a way that any new vortex superposes with a dissipating one. When new vortices are injected in the space between two circumventing vortices, which have already completed one round trip, the spectrum reflects the frequency of the injection ($St_L \approx 1$) and of the surviving carousel structures. Lifetimes up to two round trips were observed, which explains the complex spectral composition found for the intermediate regime. The chaotic regime occurs when the feedback process becomes too strong: the vortices of the carousel do not decay sufficiently and do not synchronize with the natural oscillations of the shear layer. Experimental evidence of the vortex carousel mechanism can be found in literature for open single cavity flows.

Our study raises questions that call for further research:

- (i) Two-dimensional simulations indicate that there is a narrow range in D/L where it is possible to find a symmetric (varicose) flow configurations. This state inhibits the intermediate regime and the flow stays in the periodic regime up to higher Reynolds numbers. It would be of interest to understand why the selection of this state occurs in such a narrow range in a two-dimensional context and if this state effectively occurs in three-dimensional flows.
- (ii) When the inflow is laminar, there are no experimental reports of the chaotic regime for single cavity flows. However, this cannot be excluded as this requires specially designed experiments with well-controlled inflow conditions. Yet, a reduction of the lifetime of the small vortices related to a non-zero spanwise velocity component, seems another plausible reason for the absence of the chaotic regime.

Specifically dedicated experimental and/or numerical efforts seem necessary to clarify these points.

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