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Heterogeneity, financialization and price formation in commodity derivative markets

Edouard Jaeck

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THÈSE DE DOCTORAT

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Hétérogénéité, financiarisation et formation des prix dans
les marchés dérivés de matières premières

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**Heterogeneity, financialization and price
formation in commodity derivative markets**

DOCTORAL DISSERTATION

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February 23, 2017

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L'université Paris-Dauphine n'entend donner aucune approbation, ni improbation aux opinions émises dans cette thèse; elles doivent être considérées comme propres à leur auteur.

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Introduction générale

Les marchés dérivés de matières premières existent depuis des siècles. Dans leur version de gré à gré, ils ont probablement toujours existé sans en porter le nom. Dans leur version standardisée, il est possible de retracer leur existence depuis la fin du 19^e siècle aux Etats-Unis. Un tel historique pourrait laisser penser que tout a été dit sur ces marchés, que nous n'avons que des certitudes et plus de questions. Néanmoins, en raison de leur évolution permanente en nouveaux systèmes économiques, rien n'est moins vrai.

En 1864, les premiers contrats *futures*¹ à être développés au sein du Chicago Board of Trade (CBOT) portent sur des matières premières agricoles, et donc stockables avec des contraintes physiques importantes (contrainte de localisation, contrainte de non négativité des stocks...). Les produits dérivés qui s'échangent sur ces marchés sont complexes et sophistiqués, leur trading est donc réservé à des spécialistes (agents commerciaux ayant une activité industrielle ou traders spécialisés sur les matières premières). Cette spécialisation des agents induite par les coûts d'entrée élevés (coût informel d'acquisition d'information ou coût formel de trading) explique la segmentation assez importante qui existait entre les marchés dérivés de matières premières et les marchés financiers des autres classes d'actifs. Cette situation historique correspond à un premier système économique, celui d'un marché dérivé segmenté des autres marchés, portant sur une matière première stockable.

Ce n'est que bien plus tard, à la fin du 20^e siècle et au début du 21^e siècle, qu'ont eu lieu en parallèle deux évolutions distinctes et notables : i) le développement des marchés dérivés portant sur une matière première non-stockable : l'électricité ; et ii) la *financiarisation* des marchés dérivés de matières premières.

Les marchés dérivés sur l'électricité ont commencé à se développer à travers le monde au rythme des libéralisations et des privatisations. C'est la première fois que se développent des marchés de produits dérivés sur une matière première non-stockable. Ces marchés sont encore des marchés principalement réservés à des agents spécialisés. Jusque-là, tous les modèles d'évaluation et de gestion des risques avaient été pensés et développés pour des ma-

1. Il est possible en français de parler de contrats à terme. Néanmoins, cette terminologie est générique et s'applique aussi bien à des contrats standardisés (*futures* en anglais) qu'à des contrats de gré à gré (*forward* en anglais). Comme cette thèse s'intéresse particulièrement à des contrats standardisés, la terminologie futures anglo-saxonne est retenue.

tières premières stockables, et très rapidement la question de l'utilisation de ces modèles pour l'électricité s'est posée. Cette situation correspond à un second système économique, celui d'un marché dérivé de matières premières segmenté portant sur un actif sous-jacent non-stockable.

En parallèle, et de façon complètement distincte, la financiarisation a eu lieu. Ce terme décrit le phénomène de modification du fonctionnement des marchés dérivés de matières premières. Un des symptômes de cette financiarisation est la modification de la participation sur les marchés dérivés de matières premières. Ces marchés ne sont alors plus réservés à quelques agents spécialisés et voient arriver des flux d'investissements en provenance d'investisseurs non spécialisés. Cette situation correspond à un troisième système économique, celui d'un marché dérivé de matières premières qui est alors considéré comme un actif financier quelconque.

L'objectif de cette thèse est de mieux appréhender le fonctionnement des marchés de produits dérivés de matières premières. Ceci, en tenant compte de cette constante évolution et de la grande hétérogénéité de ces marchés afin d'offrir un panorama, le plus large et complet possible de ce qui est souvent considéré, à tort, comme une classe d'actifs homogènes. Concrètement, cela passe par la réalisation de trois études portant sur les marchés futures de matières premières. Bien que chaque étude se place dans un des systèmes économiques présentés précédemment, les deux premières partagent la même question de recherche : Comment le comportement dynamique des prix (et de la structure par terme des prix) sur un marché futures de matières premières segmenté est-il influencé par ses caractéristiques physiques? La question de recherche de la dernière étude est la suivante : Comment la fonction de partage des risques d'un marché futures de matières premières évolue avec la financiarisation? Ainsi, les deux premiers chapitres de cette thèse mettent l'accent sur ce qui fait qu'il existe des divergences entre les matières premières et les autres classes d'actifs, tandis que le troisième chapitre montre dans quelle mesure la financiarisation peut être un vecteur d'homogénéisation de ces différentes classes d'actifs.

La suite de cette introduction générale a pour objectif de présenter les notions fondamentales des marchés dérivés de matières premières sur lesquelles cette thèse repose, ainsi que la littérature existante et l'apport de mon travail de recherche.

Notions fondamentales des marchés dérivés de matières premières

L'objectif de cette section est de fournir quelques notions fondamentales des marchés dérivés de matières premières. Ces rappels permettent de clarifier certains concepts qui se-

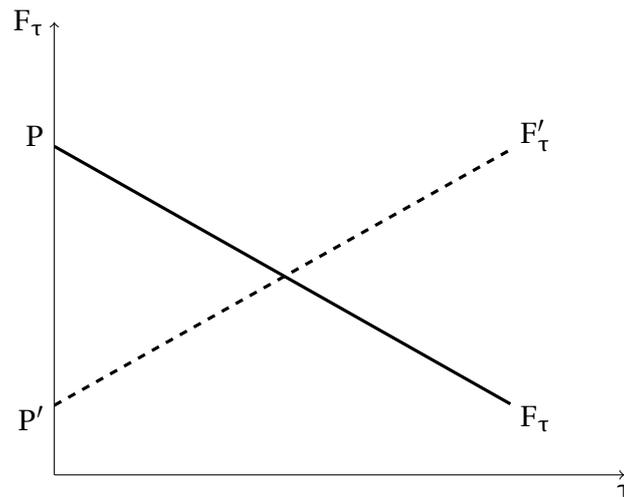
ront réutilisés tout au long de cette thèse.

Pour commencer, il est important de faire la distinction entre les marchés physiques de matières premières, et les marchés dérivés. Nous appelons marché physique d'une matière première le lieu où les transactions engendrent un échange réel de matières premières. Pour chaque matière première, il existe une multitude de marchés physiques à travers le monde, sans qu'aucune forme de standardisation n'existe. Le prix observé sur ces marchés est appelé *prix spot*. Du fait de la multiplicité de ces marchés et des qualités pour une matière première, ses prix spots peuvent diverger.

Le marché dérivé fait lui référence à un marché le plus souvent unique et standardisé où s'échangent des produits financiers dont le sous-jacent est une matière première. Les principaux produits financiers échangés sur ces marchés sont les contrats futures et les options. Cette thèse porte intégralement sur les contrats futures. Ce sont des contrats via lesquels l'acheteur (ou le vendeur) acquière l'obligation d'acheter (de vendre) à terme une certaine quantité de matières premières à un prix pré-déterminé. Les options ne fournissent qu'un droit et non une obligation d'achat ou de vente. Les produits financiers échangés dans les marchés dérivés donnent généralement lieu à une compensation financière (*cash settlement* en anglais) entre les contreparties. La livraison physique (*physical delivery* en anglais) n'est qu'une possibilité offerte au dénouement du contrat. Néanmoins, l'existence de cette possibilité est très importante car elle rend possible les opérations d'arbitrage de type *cash-and-carry*. Une telle opération d'arbitrage est composée de la prise de deux positions en parallèle : l'achat de la matière première sur le marché physique et son stockage d'un côté, et la vente d'un contrat dérivé permettant la livraison dans le futur de la matière première stockée. Ce faisant, il existe une relation entre les prix spot et futures sur les marchés de matières premières.

Concernant les contrats futures, il est important d'avoir en tête certaines notions. Pour commencer, comme le contrat futures permet d'acheter ou de vendre à terme, la notion de maturité est importante. Les contrats futures pour une matière première n'ont alors pas une seule échéance possible, mais une multitude d'échéances. La courbe qui donne le prix d'un contrat futures en fonction de sa maturité est la *structure par terme des prix*. Comme explicité dans la figure 1, la structure par terme peut avoir différentes formes. Les deux principales sont le *déport* (ligne pleine) et le *report* (ligne pointillée). Le déport ou *backwardation* en anglais (report ou *contango* en anglais) correspond à une situation où la structure par terme des prix d'une matière première est décroissante (croissante).

L'existence de cette structure par terme des prix permet au marché futures de matières premières de remplir deux fonctions importantes pour l'économie réelle : le partage des risques et la découverte d'information. La fonction de partage des risques est celle qui a justifié la création et le développement des marchés dérivés de matières premières. En prenant



Cette figure montre les deux principales formes des structures par terme des prix futures. τ représente la maturité, F_τ et F'_τ représentent des prix futures d'échéance τ et P et P' représentent des prix spots. La ligne pleine montre une structure par terme en déport et la ligne pointillée montre une structure par terme en report.

FIGURE 1 – Principales formes des structures par terme des prix futures

des positions sur des contrats futures des agents (*hedgers* en anglais) se couvrent contre une évolution défavorable des prix pour leur profit. Le fonctionnement de ces marchés est basé sur le fait que tous les agents n'ont pas le même risque de prix. Ainsi des agents cherchant à se prémunir contre une hausse des prix échangent des contrats futures avec des agents cherchant à se prémunir contre une baisse des prix. Cependant, comme rien n'assure que les demandes de couverture soient équilibrées la présence d'agents purement spéculatifs est nécessaire pour absorber le niveau de risque supplémentaire, en l'échange d'une rémunération. La fonction de découverte d'information, quant à elle, joue un rôle important dans le processus de décision d'investissement dans des projets dont les revenus futurs dépendent des prix d'une matière première. En effet, plutôt que de formuler des hypothèses ou des scénarios d'évolution des prix, l'évaluation de ces revenus futurs peut être faite sur la base de la structure par terme des prix.

De plus, à la notion de structure par terme des prix sont associées les notions de *base* et indirectement de *prime de risque*. La base est la différence entre le prix spot et le prix futures pour une certaine maturité. La prime de risque est la différence entre l'espérance du prix spot et le prix futures. C'est une notion plus abstraite que la base car elle fait référence à une quantité subjective : l'espérance du prix spot. Le signe et la valeur de ces quantités donnent des informations sur un marché de matières premières. Les informations qu'elles contiennent ont été mises en avant par la *théorie du stockage* pour la base, et la *théorie du déport normal* de Keynes [1930] pour la prime de risque. Selon ces théories, développées plus amplement dans la prochaine section, la base dépend du niveau des stocks de la matière première et des coûts de portage associés, tandis que la prime de risque dépend du déséqui-

libre des besoins de couverture des agents cherchant à se prémunir contre une hausse et contre une baisse des prix.

Revue de la littérature et apport de mon travail de recherche

La littérature sur les marchés dérivés de matières premières s'est naturellement développée en parallèle de l'évolution de ces marchés. Ainsi, depuis le début du 20^e siècle, cette littérature a tenté d'appréhender les modifications du fonctionnement des marchés dérivés de matières premières, tout en ayant recourt à des outils de modélisation de plus en plus sophistiqués. L'objet de cette section est de présenter le niveau de connaissance qui a servi de base à la rédaction de cette thèse, ainsi que les apports de celle-ci.

Le comportement dynamique des prix des matières premières dans un marché segmenté

Pour commencer, nous nous intéressons à la littérature concernant la première question de recherche, à savoir celle sur le comportement dynamique des prix des matières premières dans un marché segmenté. Cette littérature, qui est la plus importante et la plus ancienne des marchés dérivés de matières premières, cherche à comprendre les déterminants importants du processus de formation des prix et donc des primes de risque.

Conceptuellement, cette littérature est dominée par deux grandes théories datant de la première moitié du 20^e siècle : la théorie du déport normal de Keynes [1930] et la théorie du stockage. Ces deux théories, bien que souvent mises en concurrence, ne cherchent pas à expliquer le même phénomène.

Avec la théorie du déport normal, Keynes [1930] cherche à expliquer l'existence d'une prime de risque sur les marchés dérivés de matières premières. Cette prime de risque, définie comme la différence entre le prix futures et l'espérance du prix spot est le plus souvent négative. Autrement dit, l'espérance du prix spot est plus importante que le prix futures, on parle alors de déport². Selon Keynes [1930] ce fait empirique s'explique par la structure de la participation des agents sur les marchés dérivés de matières premières. Empiriquement, il y a plus d'agents cherchant à se couvrir contre une baisse des prix (qui vendent le contrat futures) que contre une hausse des prix. Cette propension plus importante à la couverture de certains agents par rapport aux autres semble avant tout être un fait empirique robuste. Il a été mis en avant que les producteurs de matières premières qui cherchent à vendre à terme leur production ont la capacité de se couvrir, mais que les consommateurs qui pour-

2. La théorie de la pression à la couverture (*hedging pressure theory* en anglais) de De Roon, Nijman, and Veld [2000] élargit le spectre d'analyse de la théorie du déport normal aux situations de report.

raient chercher à acheter à terme ne le font pas par manque de sophistication financière. La validation empirique de cette théorie n'a jamais été concluante. Ceci s'explique en partie par le fait qu'elle repose sur une quantité subjective difficilement estimable (l'espérance du prix spot).

En parallèle, les travaux de Kaldor [1939], Working [1949] et Brennan [1958] ont permis le développement de la théorie du stockage. Cette théorie, contrairement à la théorie du déport normal, ne s'intéresse pas à la prime de risque sur les marchés dérivés de matières premières, mais à la base (la différence entre le prix futures et le prix spot). Cette théorie met en avant le lien qui existe entre le prix spot et le prix futures d'une matière première via le comportement d'arbitrage des agents sur le marché. Ainsi, les opérations de cash-and-carry justifient la relation suivante : $F_{t,T} = P_t e^{s_t(T-t)}$ avec $F_{t,T}$ le prix futures en t de maturité T , P_t le prix spot en t et $s_t = r_t - c_t$ le coût net de portage de la matière première. Cette relation fonctionne très bien lorsque le prix futures est supérieur au prix spot (structure par terme en report) mais n'explique pas l'existence de stocks en déport. C'est pourquoi Kaldor [1939] a introduit la notion de *convenience yield* dans l'analyse. Ce *convenience yield* correspond à la compensation dont bénéficie le détenteur d'un stock de matière première. Cette compensation est une fonction décroissante des stocks. Cette notion, très utile pour l'analyse, a longtemps fait l'objet d'intenses débats.

Empiriquement, un certain nombre de faits stylisés caractéristiques des marchés dérivés de matières premières ont été mis en avant. Dans cette thèse, nous nous intéressons principalement à deux de ces faits : le comportement asymétrique de la base et l'effet Samuelson.

Concernant la base, en conséquence directe de la théorie du stockage, son comportement est souvent décrit comme asymétrique. C'est-à-dire qu'en report, la base est stable et limitée alors qu'en déport, elle est instable et théoriquement illimitée. Cette différence en situation de déport et de report de la structure par terme provient du stockage et de l'impossibilité de vendre à découvert une matière première. En effet, en report, il existe des stocks de matières premières qui permettent la mise en place des opérations d'arbitrage cash-and-carry. Ces opérations d'arbitrage sont telles que le prix futures ne peut pas excéder le prix spot d'un montant supérieur aux coûts de portage de la matière première sur la période considérée. En revanche, en déport, aucune opération d'arbitrage ne peut être mise en place. Etant donnée l'absence d'arbitrage, les prix spot et futures ne sont plus liés et peuvent évoluer de façon indépendante. Le lien entre le stockage et la variabilité de la base a été étudié empiriquement par Fama and French [1987]. Ils ont mis en avant l'impact des coûts de stockage sur le comportement de la base en montrant que l'écart-type de la base des métaux précieux est plus faible que celui de la base des produits agricoles qui est plus faible que celui de la base des produits animaliers. Bien que Kang, Rouwenhorst, and Tang [2014] remettent en avant cette relation, elle n'a jamais été étudiée théoriquement.

L'effet Samuelson (Samuelson [1965]) est le nom communément donné à un important

fait stylisé des marchés dérivés de matières premières. Il décrit la décroissance des volatilités le long de la structure par terme des prix. Autrement dit, la volatilité du prix d'un contrat futures de matières premières augmente à mesure que le contrat se rapproche de sa date d'échéance. La littérature empirique testant cet effet sur des marchés de matières premières stockables ou sur des sous-jacents de nature financière est large³. Un consensus existe, et l'existence d'un tel effet est reconnu pour les matières premières énergétiques et agricoles. Cette littérature montre aussi que cet effet est peu présent sur les métaux et n'existe pas pour les produits dérivés ayant un actif sous-jacent financier. En revanche, la question de l'effet Samuelson sur un marché de matière première non-stockable n'a que très peu été étudiée. Il existe à notre connaissance un article de Walls [1999] utilisant une base de données peu importante et un working paper non publié de Allen and Cruickshank [2002]. C'est un manque dans la littérature d'autant plus important que d'un point de vue théorique un débat existe sur les causes de cet effet. D'un côté, Bessembinder, Coughenour, Seguin, and Smoller [1996] mettent en avant le caractère stockable d'une matière première comme condition nécessaire à l'effet Samuelson. En effet, pour ces auteurs c'est le comportement de retour à la moyenne des prix des matières premières qui conduit à l'existence de l'effet Samuelson. Or, dans leur cadre d'analyse ce comportement de retour à la moyenne provient uniquement des actions de (de)stockage des agents. D'un autre côté, Anderson and Danthine [1983b] basent leur analyse sur le rôle des flux d'informations et sur le rythme avec lequel l'incertitude est résolue. Pour ces auteurs, les prix futures sont volatiles quand beaucoup d'informations sont révélées au marché. Ainsi, l'effet Samuelson existerait uniquement dans le cas particulier où la résolution de l'incertitude dans un marché dérivé de matières premières augmente à mesure que le contrat futures se rapproche de son échéance.

Dans le chapitre 1 de cette thèse nous cherchons à comprendre les spécificités en terme de comportement des prix que la non-stockabilité engendre. Pour cela, nous étudions empiriquement l'existence de l'effet Samuelson sur les marchés dérivés d'électricité. Notre étude empirique à grande échelle porte sur quatre marchés dérivés d'électricité dans le monde (le marché Allemand, le NordPool, le marché Australien et le PJM aux Etats-Unis) pour une période allant de 2008 à 2014. Afin de comparer et de vérifier notre procédure empirique nous avons incorporé dans l'analyse le pétrole Américain (WTI). Cette étude contient deux étapes.

Tout d'abord, nous testons statistiquement l'existence de l'effet Samuelson sur les marchés dérivés d'électricité considérés. Pour ce faire, nous nous reposons sur trois implications empiriques de l'effet Samuelson. Deux de ses implications empiriques ont déjà été utilisées pour des tests sur d'autres matières premières. Nous proposons et testons la troisième implication empirique : l'effet Samuelson devrait se traduire par des spillovers de volatilité de la partie court terme de la structure par terme vers la partie long terme. De plus, l'intensité des

3. Se reporter à l'annexe 1.D pour plus de détails.

spillovers de volatilité devrait décroître à mesure que l'on se déplace vers la partie long terme de la structure par terme des prix. Après avoir montré l'existence de l'effet Samuelson sur les marchés dérivés d'électricité, nous cherchons à répondre à la question de l'importance du stockage dans l'existence de l'effet Samuelson en regardant l'importance du stockage dans l'explication du comportement des prix de l'électricité. Pour cela, nous nous basons sur le récent concept de la stockabilité indirecte de l'électricité (Aïd, Campi, Huu, and Touzi [2009] et Aïd, Campi, and Langrené [2013]) : une certaine forme de stockage existe pour l'électricité sous la forme des matières premières utilisées pour sa production. Nous cherchons à tester l'existence d'une telle capacité de stockage sur les marchés d'électricité considérés. En effet, cela pourrait réconcilier l'analyse de Bessembinder et al. [1996] avec l'existence de l'effet Samuelson sur l'électricité.

Ce chapitre montre que l'effet Samuelson existe sur les principaux marchés dérivés d'électricité dans le monde et qu'il ne semble pas y avoir de lien direct et unidirectionnel entre la volatilité du prix de l'électricité et les volatilités des prix des matières premières nécessaires à sa production. Cela a plusieurs conséquences : i) d'un point de vue théorique, le stockage ne semble pas être une condition nécessaire à l'existence de cet effet ; et ii) d'un point de vue pratique, comme c'est le cas pour les matières premières stockables, cet effet devrait être pris en compte dans l'évaluation de produits dérivés d'électricité, dans les pratiques de couverture des risques ou encore dans les procédures de gestion des risques financiers par les régulateurs.

D'un point de vue méthodologique, les théories du stockage et du déport normal ont fourni un cadre théorique général qui a permis l'émergence de modèles cherchant à étudier le comportement dynamique des prix ou des primes de risque en général, et les faits stylisés décrits précédemment en particulier. L'évolution de ces modèles s'est faite au fil de l'amélioration des techniques de modélisation.

Tout d'abord, des modèles statiques ont été développés par Anderson and Danthine [1983a] et Hirshleifer [1988, 1989a]. L'objectif principal de ces modèles était de comprendre les déterminants de la prime de risque dans les marchés dérivés de matières premières. Ils ont été les premiers à mettre en avant formellement le rôle des besoins de couverture et du déséquilibre des agents sur les marchés dérivés de matières dans l'explication de la prime de risque. En ce sens, ces modèles s'intègrent dans le cadre théorique de la théorie du déport normal. Cette modélisation statique a été remise au goût du jour par Acharya, Lochstoer, and Ramadorai [2013] et Ekeland, Lautier, and Villeneuve [2016]. Les problématiques posées par ces modèles récents sont plus larges que la compréhension des déterminants de la prime de risque. Le modèle développé par Acharya et al. [2013] cherche à comprendre l'effet des contraintes financières des spéculateurs sur les marchés dérivés de matières premières, tandis que le modèle développé par Ekeland et al. [2016] cherche à comprendre l'effet de la spéculation et les conséquences que son développement très important peut avoir sur les

marchés de matières premières.

Des modèles dynamiques ont eux aussi été développés. Parmi ces modèles, le modèle de stockage compétitif avec des anticipations rationnelles par Williams and Wright [1991] a vu le jour et est rapidement devenu la référence pour étudier le comportement dynamique des prix des matières premières. Ce type de modèle, en mettant l'accent sur le rôle du stockage dans les marchés dérivés de matières premières, s'intègre tout naturellement dans le cadre théorique de la théorie du stockage. Ce modèle, avec un horizon infini et des agents (stockeurs) compétitifs neutres au risque a l'avantage de la simplicité. En revanche, il ne s'intéresse qu'au marché physique des matières premières et ne permet pas d'avoir une réelle analyse des marchés dérivés. Ce modèle a été utilisé sur des données par Deaton and Laroque [1992] qui ont mis en avant le rôle crucial du stockage dans le comportement des prix des matières premières. Le premier modèle à étudier à la fois le marché physique et dérivé d'une matière première dans ce type de modélisation dynamique est celui de Routledge, Seppi, and Spatt [2000]. Néanmoins, l'analyse se fait toujours dans le cadre d'une industrie du stockage compétitive et neutre au risque. En conséquence, le marché futures alors modélisé ne peut induire de prime de risque.

Afin d'avoir une analyse dynamique des marchés physique et futures d'une matière première, les premiers modèles comprenant des agents hétérogènes⁴ ont été développés. Ces modèles sont les premiers à tenter d'étendre la théorie du déport normal au cadre dynamique. Du fait de l'hypothèse d'aversion pour le risque des agents, ils permettent d'analyser réellement le fonctionnement et l'effet d'un marché dérivé de matières premières. Hirshleifer [1989b] est le premier à modéliser le comportement des différents agents hétérogènes sur un marché dérivé de matières premières. Ce faisant, il permet une première étude dynamique précise des déterminants de la prime de risque. Plus récemment, en parallèle de la réalisation de cette thèse, les articles de Vercaemmen and Doroudian [2014] et Baker [2016] ont eux aussi réussi à faire cohabiter un modèle à horizon infini permettant une analyse dynamique, avec des agents hétérogènes et averses au risque dans un marché futures de matières premières.

Le chapitre 2 de cette thèse s'intègre dans cette dernière branche de la littérature. Son objectif est double. Tout d'abord, nous souhaitons expliquer par les caractéristiques physiques d'un marché dérivé de matières premières l'existence ou non de certains faits stylisés (effet Samuelson et comportement asymétrique de la base). Ensuite, nous souhaitons analyser le rôle de la spéculation dans un marché dérivé de matières premières.

Afin de réaliser ces objectifs, nous développons un modèle stationnaire à horizon infini des prix spots et futures d'une matière première. Dans ce modèle, les différents agents (transformateurs, stockeurs et spéculateurs) averses au risque forment des anticipations ra-

4. L'hétérogénéité ici décrite concerne le rôle économique des différents agents. Ceci se traduit par des fonctions objectives différentes lors de la modélisation.

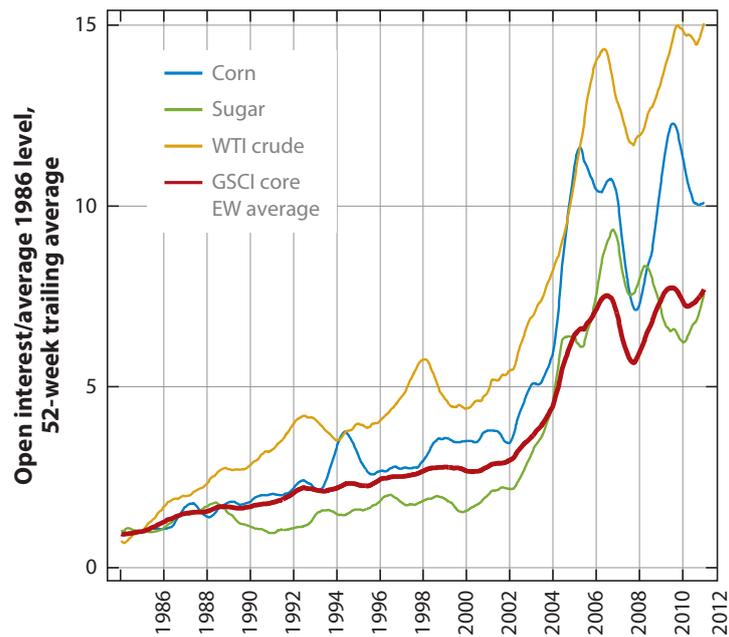
tionnelles. Ces agents peuvent prendre des positions à la fois sur le marché physique et le marché dérivé de la matière première considérée. L'équilibre se forme alors simultanément sur ces deux marchés. Nous incorporons dans ce modèle l'impossibilité de vendre à découvert la matière première physique. Ceci est une spécificité très importante des marchés de matières premières.

La résolution numérique de ce modèle et les simulations qui s'en suivent nous permettent d'aboutir aux résultats suivants. Tout d'abord, nous complétons la littérature théorique existante concernant les deux faits stylisés décrits précédemment : le comportement asymétrique de la base (différence entre le prix futures et le prix spot) en déport et en report, et la diminution de la volatilité le long de la structure par terme des prix (l'effet Samuelson) pour les matières premières agricoles et énergétiques. Nous montrons que : i) le coût de stockage impacte le comportement asymétrique de la base. En effet, en report, la volatilité de la base est une fonction croissante des coûts de stockage, alors qu'elle en est indépendante en déport; ii) l'effet Samuelson est plus important dans les marchés de matières premières ayant des contraintes de stockage importantes (peu de flexibilité à court terme); et iii) la pente de la structure par terme des volatilités dépend du niveau observé des stocks. Cette structure par terme peut être plate (et non décroissante) dans certains cas de sur-accumulation des stocks. Ensuite, nous étudions le rôle de la spéculation dans les marchés dérivés de matières premières en augmentant le poids de l'industrie spéculative. Nous montrons que l'augmentation de la spéculation modifie le fonctionnement des marchés dérivés de matières premières. De façon générale, cela permet une réduction des primes de risque (coût de couverture) et une augmentation de la pression à la couverture. Les différents prix sont impactés de façon hétérogène en fonction de l'agent dominant sur le marché. Ainsi, l'augmentation de ce type de spéculation est globalement favorable pour l'économie, mais cela se fait au bénéfice de certains agents et au détriment des autres.

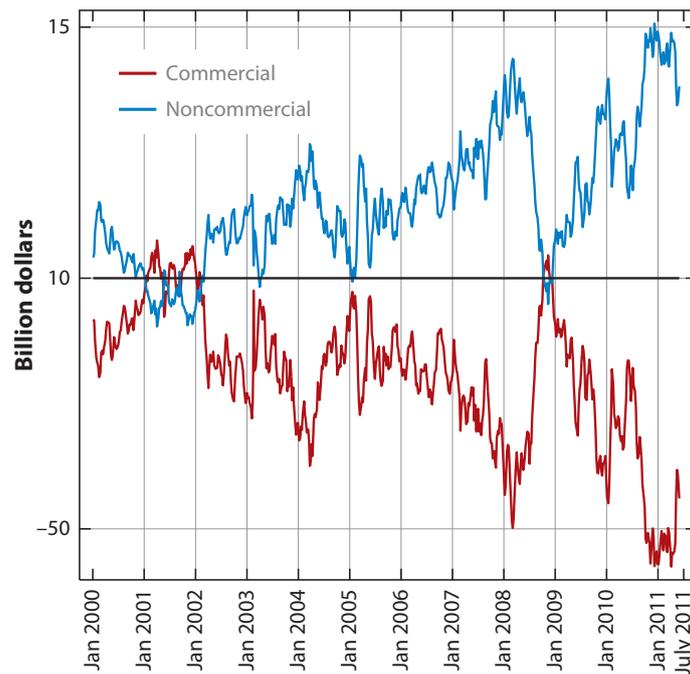
La financiarisation des marchés dérivés de matières premières

Nous nous intéressons maintenant à la littérature liée à notre deuxième question de recherche, celle cherchant à étudier ce qui a probablement été le fait le plus marquant des marchés dérivés de matières premières depuis le début des années 2000 : la financiarisation. Tout comme ce phénomène, cette littérature est récente.

Le concept de financiarisation des marchés dérivés de matières premières décrit la profonde modification du fonctionnement de ces marchés depuis le début des années 2000. Comme explicité par Cheng and Xiong [2014] dans leur revue de la littérature, cette financiarisation se traduit principalement par deux faits marquants : un sur les participants et un sur les prix.



(a) Open interest



(b) Commitment of traders

Cette figure montre l'open interest normalisé à sa moyenne de 1986 pour différentes matières premières entre 1986 et 2012 dans la sous-figure a) et la valeur notionnelle nette aggregée des "commitments" par groupe de traders entre 2000 et 2011 dans la sous-figure b). La catégorisation retenue est celle de la CFTC qui classe les traders ayant une activité de couverture en "commercial" et les autres en "noncommercial". Source : Cheng and Xiong [2014].

FIGURE 2 – Financiarisation : participation sur les marchés futures des matières premières

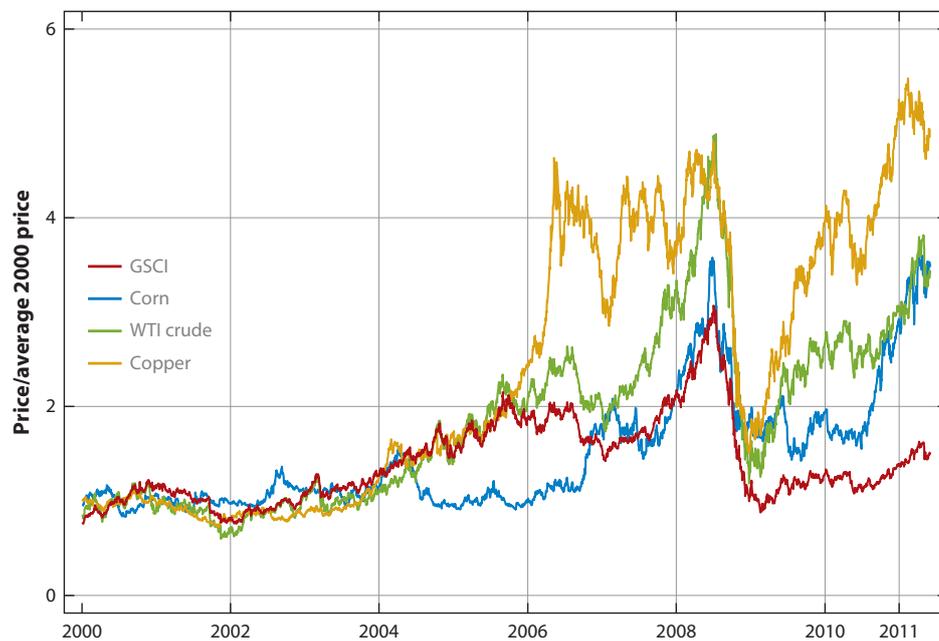
Tout d'abord, comme montré dans la figure 2 extraite de l'article de Cheng and Xiong [2014], la participation dans les marchés futures de matières premières a été bouleversée. Pour commencer, le volume des positions brutes a explosé entre 2004 et 2006. Cela s'est traduit par une forte progression des positions ouvertes (*open interest* en anglais), c'est-à-dire du nombre de contrats futures non compensés en circulations. De plus, les positions nettes des agents financiers (noncommercial traders⁵ selon la Commodity Futures Trading Commission (CFTC), l'organisme de régulation des marchés dérivés de matières premières aux USA) spéculant sur des indices ont engendré un développement très important du côté acheteur sur les marchés de matières premières agricoles. Cette augmentation est due au souhait de diversification des investisseurs financiers. Cette diversification a été rendue possible par le développement de nouveaux véhicules d'investissement à bas coûts et à la faible corrélation des matières premières avec la plupart des autres classes d'actifs. En conséquence de ce développement, les producteurs (commercial traders selon la CFTC) ont eux augmenté leurs positions vendeuses.

De façon concomitante, le comportement des prix des futures de matières premières après 2004 a complètement changé. La figure 3 montre que la plupart des matières premières ont connu des épisodes de hausses de prix importantes, suivi par des krachs (boom/bust cycle). Plus précisément, entre 2000 et 2004, les prix restent relativement stables. En revanche, dès 2004 ils explosent jusqu'à atteindre leur plus haut historique en 2008 (quatre fois leur niveau de 2000), avant de rechuter en quelques mois à leur niveau de 2000. En parallèle de la modification du niveaux des prix, les corrélations entre matières premières, et avec les autres classes d'actifs ont elles aussi évolué. Ces corrélations ont fortement augmenté avant et pendant le krach de 2008.

L'arrivée massive de flux d'investissements sur les marchés dérivés de matières premières a très vite été décriée et mise en avant afin de justifier ces mouvements importants de prix entre 2004 et 2008. Par exemple, en 2008 lors de son témoignage devant le Sénat Américain, Michael Master a explicitement fait le lien entre les flux d'investissements et l'explosion puis le krach des prix des matières premières. Ce débat public a motivé un nombre important d'études empiriques cherchant à analyser ce lien (voir par exemple Brunetti and Buyuksahin [2009], Buyuksahin and Robe [2011], Singleton [2013] et Hamilton and Wu [2015]). Ces études ont eu des résultats mitigés et ont avant tout montré la difficulté économétrique d'étudier ce genre de lien de cause à effet.

Face à ce constat, la littérature cherche maintenant à comprendre comment la financiarisation modifie la capacité d'un marché dérivé de matières premières à faire face à ses deux

5. Les notions de commercial et noncommercial traders de la CFTC sont relativement larges. Seront catégorisées comme commercial traders les entités pouvant justifier de l'utilisation de contrats futures à des fins de couverture. Les autres seront catégorisées comme noncommercial.



Cette figure montre les prix futures pour le GSCI total return index et trois matières premières entre 2000 et 2011. Ces prix sont normalisés à leur niveau moyen de 2000. Le GSCI total return index est un indice investissable de matières premières, actuellement émis par S&P Dow Jones Indices. Les pondérations se font en fonction de la production de chaque matière première. Source : Cheng and Xiong [2014].

FIGURE 3 – Financiarisation : prix futures des matières premières

fonctions essentielles : la fonction de partage des risques et de découverte des prix. Concernant l'effet sur la fonction de partage de risques, certains résultats ont déjà été montrés. Tout d'abord, Hamilton and Wu [2014] et Baker [2016] ont montré que la financiarisation a permis de réduire la prime de risque sur le marché du pétrole brut. Hamilton and Wu [2014] justifie empiriquement ce résultat en mettant en avant que les positions prises par les investisseurs sont opposées aux positions prises par les agents traditionnels cherchant à se couvrir. Baker [2016] justifie ce résultat avec un modèle à participation limitée des ménages calibré sur les données du pétrole brut. Dans leur étude empirique, Brunetti and Reiffen [2014] sont arrivés à la même conclusion pour les marchés agricoles. Ekeland et al. [2016] montrent théoriquement que la prime de risque sur un marché dérivé de matières premières devrait décroître quand la capacité d'absorption des risques des spéculateurs sur un marché augmente.

Ensuite, Tang and Xiong [2012] ont montré que les marchés de matières premières sont plus intégrés entre eux à cause de l'investissement sur indice de matières premières. Silvennoinen and Thorp [2013], Buyuksahin and Robe [2014] et Boons, De Roon, and Szymanowska [2014] ont montré qu'avec la financiarisation, les marchés de matières premières sont devenus plus intégrés avec les autres classes d'actifs. Ces résultats empiriques concernant les corrélations ont été confirmés théoriquement par Basak and Pavlova [2016] dans un modèle où la financiarisation est décrite comme le trading dans les marchés dérivés de matières premières d'investisseurs institutionnels dont les préférences dépendent d'un benchmark.

Le chapitre 3 de cette thèse s'inscrit particulièrement dans cette littérature qui cherche à comprendre le lien entre la financiarisation et la fonction de partage des risques d'un marché dérivé de matières premières. Ainsi, il ne décrit pas un marché dérivé de matières premières cloisonné du reste de l'économie, mais un marché sur lequel les traders peuvent aussi bien être des agents spécialisés dans le trading de matières premières que des investisseurs détenant des positions sur plusieurs classes d'actifs. De plus, l'analyse est axée sur l'étude de la structure par terme des primes de risque.

Afin d'étudier cela, j'ai développé un modèle d'équilibre d'un marché dérivé de matières premières avec une structure par terme et une participation limitée dans l'esprit de Hirshleifer [1988] et Boons et al. [2014]. Dans ce modèle, des agents traditionnels (producteurs et spéculateurs) font face à des investisseurs constituants des portefeuilles en utilisant plusieurs classes d'actifs. Comme dans le chapitre 2, tous les agents sont averses au risque. En revanche, dans ce chapitre les agents prennent uniquement des positions sur les marchés dérivés de matières premières, l'analyse des positions physiques étant laissée de côté. L'analyse de la financiarisation se fait en plusieurs étapes.

Tout d'abord, le modèle est résolu pour une économie pré-financiarisation dans laquelle uniquement les agents traditionnels sont actifs. Cette première étape permet entre autres

d'étendre certains résultats du chapitre 2 lorsqu'il existe une structure par terme. Ensuite, en ajoutant des investisseurs contraints et non-contraints, le modèle est résolu pour deux économies post-financiarisation. Dans la première économie, les investisseurs sont contraints d'investir uniquement dans le contrat futures d'échéance la plus proche, tandis que dans la seconde économie les investisseurs peuvent investir dans tous les contrats de la structure par terme. Ces choix de modélisation ont été faits afin de décrire au mieux l'évolution des marchés dérivés de matières premières. Dans un premier temps, cette évolution est caractérisée par d'importants flux d'investissement via les Commodity Index Traders prenant des positions importantes uniquement sur les contrats de faibles maturités. Ensuite, par l'évolution des stratégies visant à se positionner sur des maturités plus importantes afin de minimiser les pertes liées au "roll" des contrats futures. Ce modèle, quelle que soit l'économie considérée, est résolu analytiquement. De plus, il a été calibré afin d'illustrer et de quantifier les résultats.

Ce chapitre montre que la financiarisation modifie profondément la nature des marchés dérivés de matières premières, au moins en changeant leur fonction de partage des risques. Tout d'abord, du fait de l'arrivée d'investisseurs non spécialisés, ces marchés deviennent moins segmentés par rapport aux autres classes d'actifs, et notamment par rapport aux marchés actions. De plus, la demande des investisseurs sur les marchés dérivés de matières premières modifie les déterminants des primes de risque sur ces marchés. La rémunération du risque n'est plus liée uniquement aux besoins de couverture des producteurs mais aussi à la demande d'investissement des investisseurs. Enfin, quelle que soit la stratégie d'investissement considérée (avec ou sans contraintes), les primes de risque pour toutes les maturités sont impactées par la financiarisation. C'est-à-dire que quelle que soit la maturité sur la structure par terme, une partie de la rémunération du risque provient de facteurs de risques liés au marché action et non à la matière première. Ceci traduit une propagation le long de la structure par terme de l'effet de la financiarisation. Il est important de noter que qualitativement ces impacts existent quelle que soit la matière première considérée. En revanche, quantitativement ils dépendent du sous-jacent du marché dérivé considéré. En effet, les caractéristiques physiques de la matière première et du marché déterminent les prix, les primes de risque et ainsi, leur comportement dynamique.

Ainsi, les trois essais de cette thèse étudient théoriquement et empiriquement les marchés futures de matières premières dans différentes conditions de fonctionnement. Le premier essai est une étude empirique qui montre l'existence de l'effet Samuelson sur les marchés futures d'électricité. Le second essai est un modèle qui montre comment le comportement dynamique des prix d'une matière première stockable sur un marché futures segmenté du reste de l'économie est impacté par ses caractéristiques physiques, et notamment par le coût de stockage. Enfin, le troisième essai est un modèle qui montre que la financiarisation modifie la fonction de partage des risques des marchés futures de matières premières.

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Part I

Le comportement dynamique des prix sur des marchés segmentés

Chapter 1

Volatility in electricity derivative markets: the Samuelson effect revisited ¹

Abstract

This article proposes an empirical study of the Samuelson effect in electricity markets. Our motivations are twofold. First, although the literature largely assesses the decreasing pattern in the volatilities along the price curve in commodity markets, it has not extensively tested the presence of such a dynamic feature in electricity prices. Second, the analysis of a non-storable commodity enriches the literature on the behavior of commodity prices. Indeed, it has been sometimes asserted that the Samuelson effect results from the presence of inventories. We examine the four most important electricity futures markets worldwide for the period from 2008 to 2014: the German, Nordic, Australian, and US markets. We also use the American crude oil market as a benchmark for a *storable* commodity negotiated on a *mature* futures market. Our analysis has two steps: i) in addition to the traditional tests, we propose and test a new empirical implication of the Samuelson effect: price shocks should spread *from* the physical market *to* the paper market, and not the reverse; ii) based on the concept of "indirect storability", we investigate the link between the Samuelson effect and the storability of the commodity. We find evidence of a Samuelson effect in all of the electricity markets and show that storage is not a necessary condition for such an effect to appear. These results should be taken into account for the understanding of the dynamic behavior of commodity prices, for the valuation of electricity assets, and for hedging operations.

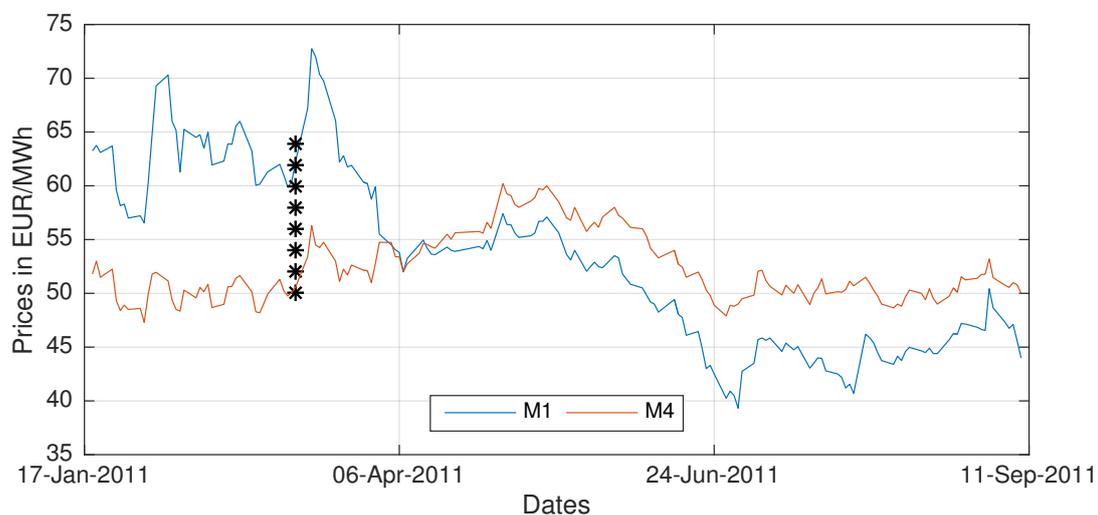
1. This chapter is based on an article written with my PhD Supervisor Delphine Lautier. It has been published in the Volume 59 of Energy Economics in September 2016. It has been presented during three international conferences (the 31st AFFI Conference, the 14th IAEE European Energy Conference and the 3rd ISEFI Symposium) and a research seminar at the Université Paris-Dauphine.

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1.1 Introduction

The most important dynamic feature of commodity futures prices is probably the difference between the behavior of the prices of the first-nearby and deferred contracts. The movements of the former are large and erratic, while the latter are relatively stable. This difference results in a decreasing pattern in the volatilities along the price curve. The same is true for the correlations between the nearest futures price and subsequent prices, which decline with the maturity. This phenomenon is usually called the Samuelson effect or the maturity effect. The reasoning behind this phenomenon is that a shock that affects the short-term price has an effect on the succeeding prices that decreases as the maturity increases (Samuelson [1965]). Indeed, when a futures contract reaches its expiration date, it reacts more strongly to information shocks because of the ultimate convergence of the futures to the spot prices at maturity. The demand and supply shocks borne in the physical market are responsible for this price disturbance that mostly influences the short-term part of the curve. Figure 1.1 gives an example of such an effect. It represents the prices of electricity on a European futures market (the Nasdaq OMX commodities market, also called the NordPool) around the Fukushima nuclear disaster of March 2011. The jump recorded in the prices just after the plant failure is far more important for the short- rather than the long-term prices. This higher volatility clearly continues in the weeks following the disaster.



This figure displays the prices of electricity futures for two maturities on the NordPool market around the Fukushima disaster of March 2011. "M1" and "M4" respectively stand for the one- and four-month futures contracts. The vertical line of stars identifies the day of the accident.

Figure 1.1 – Electricity futures prices on the NordPool market around the Fukushima catastrophe

In this article, we offer a large scale study of the Samuelson effect on different electricity markets worldwide. We enlarge the spectrum of the empirical studies on the maturity effect

and we address the question of non-storability.

The literature devoted to empirical tests of the Samuelson effect on storable commodities and financial assets is quite large². This literature has reached an overall consensus about the Samuelson effect, and finds a strong effect on energy products and agricultural commodities (grains, soft commodities, meats). The picture is more nuanced for metals, with weak or null evidence for precious metals. Further, there is no Samuelson effect on financial assets. As far as electricity is concerned, the only available studies, to the best of our knowledge, are the article by Walls [1999] (14 futures contracts traded on the Nymex between March and November 1996 for two US markets) and the working paper of Allen and Cruickshank [2002] (42 futures contracts for two Australian electricity markets, from 1997 to 1999).

Regarding the economic explanation of the Samuelson effect, the literature is mixed about its origins. Bessembinder, Coughenour, Seguin, and Smoller [1996] establish a relation between the Samuelson effect and the mean reversion in the dynamics of commodity prices. For them, these dynamics are the direct consequence of storability and reflect the behavior of the operators in the physical market. Other studies use this kind of analysis, such as Schwartz [1997]. In the same line of reasoning, a temporary excess in inventories in the physical market might act as a cushion and decrease the volatility of short-term prices. Fama and French [1988], among others, find that violations of the Samuelson effect might occur at short-term horizons when inventories are high. They show that for industrial metals, when the inventory is high, the spot and futures prices have the same variability. But in the case of scarcity, there is a decreasing pattern in the volatilities. This finding is consistent with the storage theory (Working [1949], Brennan [1958]). In this framework, the marginal convenience yield is a nonmonotonic and decreasing function of the inventory level. Routledge, Seppi, and Spatt [2000] reiterate the proposition of Fama and French with an equilibrium model of the term structure of forward prices for storable commodities.

In contrast Anderson and Danthine [1983] propose a theoretical framework that allows for an analysis of the relation between the Samuelson effect and the resolution of uncertainty over time. In this setting, storage does not remain the most important explanatory factor for the behavior of volatility. What matters more is production uncertainty and the way this uncertainty diffuses into the market. According to the authors, "futures prices are volatile in times when much uncertainty is resolved and are stable when little uncertainty is

2. See among others; Anderson [1985] for a study on nine markets for agricultural products and metals; Milonas [1986] who examines 11 markets for agricultural products, financial assets, and metals; Duong and Kalev [2008]: 20 futures markets for agricultural products, financial assets, and metals (these authors are the only ones who examine high frequency data. However, such a study is out of reach in the case of electricity, due to a lack of data), from 1996 to 2003; Lautier and Raynaud [2011]: 13 commodity and financial futures markets from 1998 to 2010; and Brooks [2012]: 50 futures markets for commodities and financial assets, from 1993 to 2012. Appendix 1.D gives more details.

resolved. Whether most uncertainty is resolved near the delivery date of a futures contract is an empirical matter".

This debate naturally raises questions about the dynamic behavior of the futures prices of a non-storable commodity like electricity. We thus examine this behavior in the four most important electricity futures markets worldwide for the period from 2008 to 2014: the German market, NordPool (representative of European Nordic countries), Australian market, and the PJM Western Hub in the United States. We also rely on the American crude oil market as a benchmark for a storable commodity and as an example of a mature futures contract. We adopt a two-step process.

The first step is to test the Samuelson effect on electricity markets, which requires a thoughtful analysis of its empirical implications. To the best of our knowledge, the research has only tested two implications of this dynamic behavior up to now. The first is the closest to the idea developed by Samuelson: if price shocks arising from the physical market influence the futures contracts particularly when these contracts are close to their expiration date, then the volatility must be a decreasing function of the remaining days before maturity (Anderson [1985], Milonas [1986], Walls [1999], Bessembinder et al. [1996]). The second implication is that if there is a decreasing relation between the volatility and the time-to-maturity, then the volatility of the one-month contract should be higher than that of a two-month contract, which in turn should be higher than that of a three-month contract, and so on. In other words, there should be an ordering in the time series of the volatilities across maturities that results in a decreasing pattern (Duong and Kalev [2008], Lautier and Raynaud [2011]). In this article, we propose and test a third empirical implication: the shocks emerging in the physical market should spillover *in the direction* of the futures market with a decreasing intensity when the contract's maturity rises. Thus, not only should the volatilities be ordered according to the maturity; there should also be volatility spillovers from the physical market to the paper market and not the reverse. Such reasoning is consistent with what is expected from a derivative market as regards to the risk management function it performs. In order to test this assumption, we rely on the method developed by Diebold and Yilmaz [2012] for the analysis of volatility spillovers between the markets for US stocks, bonds, foreign exchanges, and commodities.

The second step of our analysis aims to give insights into the debate about the role of inventories in the existence of the Samuelson effect. We rely on the third empirical implication of the Samuelson effect, and on the recent concept of "indirect storability": there could be some storability in electricity due to its inputs³. In this context, if there is an indirect

3. As early as in 2001, Routledge, Seppi, and Spatt [2001] underline that the potential storability in the form of fuels motivates the exploration of the relation between electricity and fuel prices. This idea was later reformulated under the idea of "indirect storability". Going further, Aid, Campi, and Langrené [2013] propose considering electricity as a portfolio of futures contracts on its inputs and show that this is the case in the French market. For a short review on this concept, see e.g., Huisman and Kilic [2012].

storability effect in the market of electricity, then one possible explanation for the presence of a Samuelson effect is that price shocks borne in the markets for the inputs spread to the electricity market. Should that be the case, then electricity should not be considered as very different from any other storable commodity.

The large-scale investigation of the Samuelson effect on electricity markets performed in this article is important for at least two reasons.

First, although most developed countries have considered electricity as a public good over time, it is now regarded as a tradable commodity. Since they were launched 20 years ago, electricity derivative markets have had sustained increases in their transaction volumes. Even if these markets are still young, which raises empirical issues such as the lack of historical data or of long-dated contracts, there is now enough information to understand precisely how they function and to compare them with other markets for traditional commodities.

A second and more general reason is that industrial and financial agents as well as regulatory authorities need a deeper knowledge of the Samuelson effect. The traditional hedgers on commodity markets are producers, industrial processors, and trading companies. They use the futures markets to hedge their physical exposure, and they are rationally induced to minimize their hedging costs. The existence of the Samuelson effect could affect the choice of their hedging horizon and/or their hedging ratio. Moreover, volatility is one of the most important parameters in the pricing of options. Whenever the framework of a constant volatility (as in the Black and Scholes [1973] model) is relaxed (see, for e.g., the Heston [1993] model), the Samuelson effect must be taken into account. Further, the maturity effect concerns clearing houses and regulatory authorities when setting margin requirements and managing risk exposures.

The remainder of the article is organized as follows. In Section 1.2 we describe the data. Section 1.3 has an explanation of how we test the three empirical implications of the Samuelson effect and displays our results. In Section 1.4 we further examine the maturity effect by introducing the concept of "indirect storability" to the analysis. Section 1.5 is the conclusion.

1.2 Data and descriptive statistics

Our database comprises the daily settlement prices of monthly futures contracts⁴ that we extract from Datastream. These data cover the four following electricity futures markets: the German, NordPool, Australian, and the American PJM. These markets are characterized, worldwide, by the most important trading volumes on electricity. In addition, we collect data for the Light Sweet Crude Oil contract (also known as the West Texas Intermediate, hereafter WTI). This market is used as a benchmark in this study for three reasons: i) in the period

4. The Australian market, with quarterly expiration dates, is the exception.

under examination, it is the largest commodity market in regard to transaction volumes; ii) it is storable; and iii) the mean reversion in the behavior of the futures prices is established in the literature (see among others, Gibson and Schwartz [1990], Schwartz [1997], Routledge et al. [2000]). Further, we collect price data for the analysis of the indirect storability of electricity performed in Section 1.4. These are nearby futures prices for the main inputs used in the production of electricity in the American PJM market (heating oil, natural gas, and coal) and in the German market (natural gas and coal). Table 1.1 summarizes the most important characteristics of our data set.

As specified in Table 1.1, our study covers more than five years. It starts at different dates in 2008 (August for crude oil, December for the German market, October for the PJM, July for the Australian market) and ends in August 2014. Due to a lack of data for some expiration dates, we reduce the time period for the NordPool contract, which starts in January 2011. This change leaves a total of 319 futures contracts used for the study of the Samuelson effect and 375 futures contracts for the study of the indirect storability.

Most of our empirical tests rely on the continuous time series of the futures prices with constant maturities. Thus while keeping the raw data, we use them to reconstitute the daily term structures of the futures prices. Because our data set contains futures contracts that mature periodically and because on the same observation date there are quotes for contracts with different maturities, we create the continuous time series by using a rollover technique. In our case, the rollover takes place at each expiration date; for example, the first time series contains futures prices for the nearest contract, and the second futures prices for the second closest-to-maturity contract.

Further, the length of the term structure is different for each market: we have maturities up to six months for the PJM contract, five months for the German market, four months for the NordPool market, and up to six quarters for the Australian market. As far as crude oil is concerned, even if the traded maturities reach several years (nine) in the American market, we retain only the first six months. Because inputs futures prices enter the analysis only in Section 1.4 which does not rely on their term structure, we construct only one continuous fixed maturity time series for each of these markets.

Figure 1.2 presents these continuous time series of futures prices for the WTI and German electricity markets for two different maturities. Initially, there are no common trends nor similar behaviors in these prices. Crude oil is clearly less volatile than electricity, and the distance between the two maturities is lower for the first of these two markets. The same is true when we compare different electricity markets against each other (the figures for all other contracts are available in Appendix 1.A.1).

Table 1.2 gives more insight into the electricity and crude oil markets: it displays the average number of contracts traded each day over the study period, both for all of the maturities

	Futures contract	Exchange (Country)	Frequency of expiration	# of contracts	Start date	# of dates	# of fixed mat. time series
Crude Oil	WTI	NYMEX (United States)	monthly	79	08/21/08	1 518	6
	Phelix	EEX (Germany)	monthly	78	12/01/08	1 457	5
Electricity	PJM	NYMEX (United States)	monthly	79	10/01/08	1 490	6
	NEC	Nasdaq OMX Com (Norway)	monthly	47	01/19/11	906	4
	NSW	ASX (Australia)	quarterly	36	07/01/08	1 563	6
Inputs (PJM market)	Heating Oil	NYMEX (United States)	monthly	75	10/01/08	1 490	1
	Natural Gas	NYMEX (United States)	monthly	75	10/01/08	1 490	1
	Coal	NYMEX (United States)	monthly	75	10/01/08	1 490	1
Inputs (Phelix market)	TTF Natural Gas	ICE (United Kingdom)	monthly	75	12/01/08	1 457	1
	Rotterdam Coal	ICE (United Kingdom)	monthly	75	12/01/08	1 457	1

This table sums up the features of the data contained in our data set: for the analysis of the Samuelson effect, there are 319 futures contracts including 240 on electricity from different start dates in 2008 to 28 August 2014; for the analysis of the indirect storability of electricity, there are 375 futures contracts for the inputs of the PJM and Phelix markets. The names of the futures contracts are the following: WTI stands for West Texas Intermediate, Phelix for Physical Electricity Index, NEC for Nordic Electricity Contract, NSW for New South Wales, and TTF for Title Transfer Facility. The acronym PJM Western Hub is based on the corresponding regional transmission organization in the United States. As far as the exchanges are concerned, NYMEX stands for the New York Mercantile Exchange, EEX for the European Energy Exchange, Nasdaq OMX Com for the Nasdaq Options Market Exchange Commodities, ASX for the Australian Securities Exchange, and ICE for the Inter Continental Exchange. The "# of contracts" column gives the information about the raw data we start with. The "# of dates" column shows the number of daily observation dates. The "# of fixed mat. time series" stands for the maturity up to which we create fixed maturity time series.

Table 1.1 – Futures price data

and maturity by maturity⁵. Even if there are important differences between these contracts because of their underlying assets (crude oil *vs* electricity) and because of the contract specifications for the electricity markets (MW per contract, delivery hours...), the transaction volumes show that electricity futures markets, with daily average volumes ranging from 22.9 to 112.6 contracts, are much smaller than the crude oil market that is characterized by 86,411.4 contracts per day on average for all of its maturities. Further, as far as the electricity markets are concerned, the NordPool and the German markets have higher volumes. Finally, for all of the markets, the trading volume is concentrated at the first maturity and decreases regularly with the time to expiration. This feature is typical of derivative markets.

Maturity	WTI		Phelix		PJM		NEC		NSW	
All	86,411.4		95.5		47		112.6		All	22.9
M1	264,302.1	51%	287.4	60.2%	84.7	30%	305.1	67.7%	Q1	12.9 9.4%
M2	133,455.7	25.7%	129.1	27.1%	42.7	15.1%	90.1	20%	Q2	29.4 21.5%
M3	54,494.8	10.5%	38.1	8%	37	13.1%	33.9	7.5%	Q3	25.1 18.3%
M4	30,888.6	6%	15.2	3.2%	40.2	14.3%	21.4	4.8%	Q4	25.8 18.8%
M5	20,301.3	3.9%	7.3	1.5%	38.6	13.7%			Q5	25.3 18.5%
M6	15,025.8	2.9%			39	13.8%			Q6	18.6 13.5%

This table shows the transactions recorded between 2008 and 2014 on each electricity market (German, American, Australian and Nordic) and on the American crude oil market. The first line displays the average daily volume for all maturities, the others give details for each monthly or quarterly maturity. The percentages represent the share of each maturity in the total volume traded. In the electricity markets, the volumes are in MW and the nominal is one contract per MW, except for the PJM contract, where it is 1 for 2.5. In the crude oil market, the volume is in contracts; one contract represents 1,000 barrels.

Table 1.2 – Transaction volumes, crude oil and electricity markets, 2008-2014

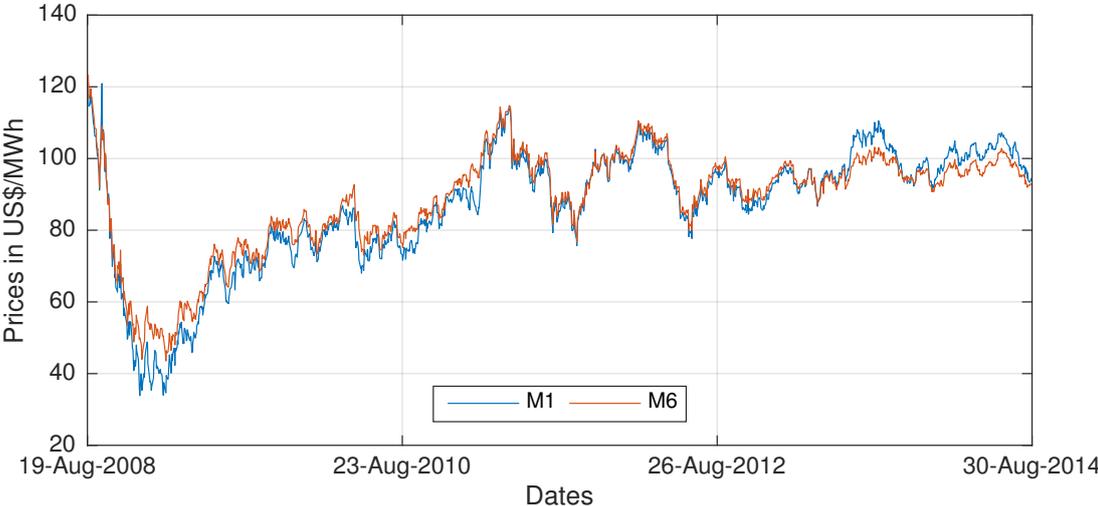
Table 1.3 provides another comparison between the markets under consideration. For each market, it displays some descriptive statistics about the volatility of the nearby futures prices. Specifically, at t we measure the daily realized volatility σ_t^k of a futures contract with fixed maturity k . To do so we retain, as in Bessembinder et al. [1996], the absolute value of the daily futures price returns:

$$\sigma_t^k = \left| \ln \left(\frac{F_t^k}{F_{t-1}^k} \right) \right| * 100 \quad (1.1)$$

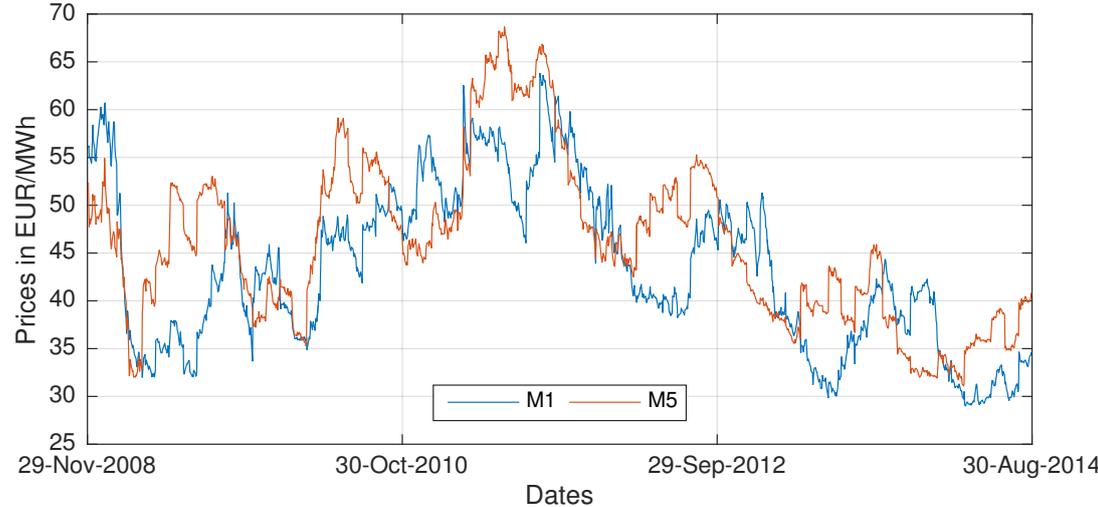
where F_t^k and F_{t-1}^k are the settlement prices of the futures contract with fixed maturity k at dates t and $t-1$.

The use of the other measures of daily volatility, such as the High-Low volatility measure of Parkinson [1980] and Garman and Klass [1980], is not possible because of the lack of data

5. Note that the PJM contract size corresponds to 2.5 MW, whereas it amounts to 1 MW for all of the other electricity contracts.



(a) WTI contract



(b) Phelix contract

This figure shows the continuous time series of the prices for two maturities in two markets. That is the one- and six-month futures contracts for the crude oil market and the one- and five-month futures contracts for the German electricity market.

Figure 1.2 – Time series of prices of fixed maturity contracts, WTI and Phelix markets, 2008-2014

on high and low prices in certain markets and/or periods. And we do not use the usual measure of volatility, which is the standard-error of the daily returns on a rolling window, because we want to have a daily measure and to avoid some overlapping issues.

	WTI	Phelix	PJM	NEC	NSW
# of observations	1 518	1 457	1 490	906	1 563
Mean	1.597	1.133	1.676	1.998	1.052
Median	1.055	0.793	1.178	1.451	0.272
Standard-deviation	1.774	1.139	2.415	1.902	2.438
Skewness	2.72	3.40	10.01	2.23	7.73
Kurtosis	13.21	30.08	164.62	10.96	96.71
ADF	-19.22*	-19.60*	-17.93*	-14.52*	-26.10*
LB	2 314*	234*	1 508*	367*	432*
Jarque-Bera	8 452*	47 315*	1 646 644*	3 139*	587 510*

This table sums up the descriptive statistics of the daily volatilities $\sigma_t^k = \left| \ln \left(\frac{F_t^k}{F_{t-1}^k} \right) \right| * 100$ recorded on the closest-to-maturity contracts for each electricity market (German, American, Australian, and Nordic) and for the American crude oil market from 2008 to 2014. The "ADF", "LB", and "Jarque-Bera" respectively stand for the test statistics of the Augmented Dickey-Fuller test for unit roots without a lag, the Ljung-Box test for autocorrelation with 15 lags, and the Jarque-Bera test for normality. The associated null hypothesis H_0 are the presence of a unit root for the ADF test, that the data are independently distributed for the LB test, and that the data follow a normal law for the JB test. The star (*) means that we reject the assumption H_0 at the 1% level of confidence.

Table 1.3 – Descriptive statistics of the daily volatilities, crude oil and electricity markets, 2008-2014

Table 1.3 displays the mean, median, standard-deviation, and the skewness and kurtosis for the daily volatilities from 2008 to 2014. The charts of the daily volatilities are available in Appendix 1.A.2. We also conduct some statistical tests for the autocorrelation (Ljung-Box test⁶) and the normality (Jarque-Bera test⁷) of the series, as well as for the presence of unit roots (ADF test⁸).

The table shows the following: first, the NordPool appears to be the most volatile market, according to both the mean and the median. The PJM market comes second. Then the crude oil market, followed by the two other electricity markets. The volatility of the crude oil market is rather surprising. Because it is the only storable commodity in the sample, it should be the less volatile. Second, there are some doubts about the normality of our time series of volatilities: all of the markets have a non-normal skewness with coefficients ranging from 2.23 to 10.01, which is well above zero. In the same way, with values between 10.96 and 164.62, all of the markets have a non-normal kurtosis. Third, the results are homogeneous as

6. H_0 : The data are independently distributed.

7. H_0 : Normality.

8. H_0 : Presence of a unit root.

regards to the statistical tests: i) no series contains unit roots which allows us to study them without pretreatment; ii) the results of the Ljung-Box test show the presence of autocorrelations in the time series of volatilities; and iii) the Jarque-Bera test confirms that the series do not follow a normal distribution. These results justify the use of nonparametric tests⁹ to study the maturity effect on the electricity derivative markets.

1.3 Does the Samuelson effect hold for electricity markets?

This section examines whether or not the Samuelson effect is a common feature in electricity markets, as is the case for other energy and agricultural commodities. We define and test three different empirical implications of the Samuelson effect on the electricity and crude oil markets.

1.3.1 Is volatility a function of the Time-To-Maturity (TTM) of the contracts?

We test the first implication of the Samuelson effect by using a linear regression between the volatility of the futures prices and the time-to-maturity of the contracts. Theoretically, the volatility should increase when the maturity of the futures contract comes near.

There are several methods to perform such a regression. The first one relies on raw data: it consists of extracting the prices of a futures contract during its whole life and running the analysis on these prices (see Walls [1999]). In our case, with 319 futures contracts, this method means running and interpreting 319 regressions. We thus use the continuous times series of the futures prices presented in section 1.2, as done in Anderson [1985], Milonas [1986] and Bessembinder et al. [1996]. In this case, each time series corresponds to a "fixed maturity". For example, the time-to-maturity of the first month contract ranges from 1 to 20 trading days and that of the second contract ranges from 21 to 40 days. If we take the example of crude oil, there are 61 months between August 2008 and August 2014. Consequently, there are 61 TTM of one day, 61 of two days... up to 61 TTM of 120 days because the longest maturity retained for this market is six months.

With the continuous time series, the regression between the volatility and the time-to-maturity can be expressed as follows:

$$\sigma_i^k = \alpha + \beta \text{TTM}_i^k + \varepsilon_i, \forall k \quad (1.2)$$

where σ_i^k is the volatility of the futures prices, the superscript k stands for the maturity of the futures contracts (in month or in quarter), and the index i corresponds to the Time-To-Maturity in days. The α is a constant, the TTM_i^k is the number of days until the expiration

9. Or, at least, methods that are compatible with non-normal data.

of the contract k . The ε_i stands for noise. Because our volatility measure is by definition positive, the same must be true for α . Moreover, if the volatility increases when the contract reaches maturity, we expect β to be negative.

Table 1.4 has the values of the coefficients obtained for each market. The results are homogeneous; for all four electricity markets and for the WTI we obtain positive constants and negative betas. Moreover, all of these coefficients (both α and β) are statistically significant at the 1% level. This significance is consistent with the Samuelson effect. Nevertheless, our coefficients of determination are low. There are at least two reasons for this result. First, the time-to-maturity explains only a small part of the volatility. If our objective were to *explain* volatility, we should add other explanatory variables to the regression. The second reason is that our data violate some assumptions¹⁰ of the linear regression. We thus consider these results as a first step in the validation, which must be confirmed with nonparametric tests.

	WTI	Phelix	PJM	NEC	NSW
α	1.5960	1.0781	1.6596	1.9624	0.8104
(p-value)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
β	-0.0024	-0.0037	-0.0063	-0.0097	-0.0010
(p-value)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
R^2	0.0031	0.0152	0.0295	0.0266	0.0086
# of observations	9 108	7 285	8 940	3 624	9 378

This table provides the results obtained when testing the first implication of the Samuelson effect on the four electricity derivative markets (German, American, Australian, and Nordic) and on the American crude oil market. The table shows the coefficients of a linear regression between the daily volatilities σ_i^k of the futures prices with maturity k and the Time-To-Maturity TMM_i^k , that is, the number of days between i and the maturity k of the contract: $\sigma_i^k = \alpha + \beta \text{TMM}_i^k + \varepsilon_i, \forall k$. This regression uses all of the available maturities. The daily volatility is computed as follows: $\sigma_i^k = \left| \ln \left(\frac{F_i^k}{F_{i-1}^k} \right) \right| * 100$. The p-values of the coefficients (in parentheses) and the coefficients of determination R^2 are also displayed.

Table 1.4 – Relation between the daily volatility and the Time-To-Maturity, crude oil and electricity markets, 2008-2014

1.3.2 Are the time series of volatilities ordered?

The second implication of the Samuelson effect is a consequence of the first one. If there is a decreasing relation between volatility and the time-to-maturity, then the volatility of the one-month contract should be higher than that of the two-month, and so on.

10. Principally homoscedasticity, no-autocorrelation, and normality.

In order to examine this second implication, we use two different methods. The first one consists of computing the daily volatilities according to the maturity of the futures contracts and to compare them (see Lautier and Raynaud [2011]). The second, which is more formal, is a nonparametric test that checks whether or not the volatilities significantly decrease in order by maturity (see Duong and Kalev [2008]).

Table 1.5 contains the results obtained with the first method. It reproduces the median volatilities for each maturity and for each market under consideration. The results support the Samuelson effect for three contracts: the entire term structure of the volatilities is downward sloping for the WTI, the PJM, and the NEC. However, for the Phelix contract, the last maturity, which is also the least liquid maturity (see Table 1.2), is higher than expected. Further, the volatility curve is S-shaped in the Australian market (NSW futures contract). But the maturities for this market range from 3 to 18 months.

Medians	WTI	Phelix	PJM	NEC	NSW
$\bar{\sigma}_1$	1.055	0.793	1.178	1.451	0.272
$\bar{\sigma}_2$	0.999	0.654	0.909	1.208	0.365
$\bar{\sigma}_3$	0.962	0.578	0.830	1.069	0.281
$\bar{\sigma}_4$	0.941	0.576	0.777	1.020	0.269
$\bar{\sigma}_5$	0.904	0.585	0.758		0.289
$\bar{\sigma}_6$	0.880		0.742		0.279

This table shows the median $\bar{\sigma}_k$ of the daily volatilities $\sigma_t^k = \left| \ln \left(\frac{F_t^k}{F_{t-1}^k} \right) \right| * 100$ for each maturity k of each electricity market (German, American, Australian, and Nordic) and for the American crude oil market.

Table 1.5 – Median volatilities, crude oil and electricity markets, 2008-2014

At first glance, if the results on the term structure of the volatilities are globally consistent with the Samuelson effect, this is more evident for the short- rather than for the long-term maturities.

To gain more insight into the phenomenon, following Duong and Kalev [2008], we complement this analysis with a nonparametric test. The latter is especially suited in our case because the time series are non-normal. More precisely, we use the Jonckheere-Terpstra (hereafter JT) test developed by Jonckheere [1954] and Terpstra [1952] that shows if the medians of the time series of volatilities significantly decrease in order of maturity. The following are the null and the alternative hypotheses (respectively H_0 and H_1) of the JT test:

$$\left\{ \begin{array}{l} H_0 : \bar{\sigma}_k = \bar{\sigma}_{k-1} = \dots = \bar{\sigma}_1 \\ H_1 : \bar{\sigma}_k \leq \bar{\sigma}_{k-1} \leq \dots \leq \bar{\sigma}_1 \end{array} \right. \quad (1.3)$$

where $\bar{\sigma}_k$ is the median volatility of the k th maturity. We accept the existence of a maturity effect when the null hypothesis of the JT test is rejected.

To perform this test we have to compute two statistics, Z and J. In order to obtain J, we compare the observations of each time series of volatilities to those in the successive time series. In other words, we pair each volatility recorded in the first maturity time series with each one recorded in the second maturity, in the third, and so on. For each comparison, we attribute a value of one (zero) if the first component of the pair is bigger (smaller) than the second one. A value of 0.5 is recorded in the case of a tie. Finally, we sum up all of these values to get the test statistic J. The statistic Z is then computed as follows:

$$Z = \frac{J - [(N^2 - \sum_{i=1}^k n_i^2)/4]}{\sqrt{[N^2(2N+3) - \sum_{i=1}^k n_i^2(2n_i+3)]/72}} \quad (1.4)$$

where N is the total number of observations, and n_i is the number of observations in the time series of volatility with maturity i . For large sample sizes like ours, the JT test statistic, Z, is approximately normally distributed with a zero mean and a variance equal to one.

Table 1.6 reports the results of the JT test. It shows that we can reject the null hypothesis at a 1% level for all of the markets; the Samuelson effect holds for the WTI market and for all four of the electricity futures markets.

	WTI	Phelix	PJM	NEC	NSW
Z-statistic	5.11	9.51	14.40	8.41	2.38
(p-value)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
# of observations by maturity	1 518	1 457	1 490	906	1 563
# of maturities	6	5	6	4	6

This table reports the results (Z-statistics, associated p-values) obtained with the Jonckheere-Terpstra test for each electricity market (German, American, Australian, and Nordic) and for the American crude oil market. This nonparametric test examines the null hypothesis of equal volatilities against the alternative hypothesis of ordered volatilities. The rejection of H_0 means the acceptance of the Samuelson effect. The test statistic Z is computed as follows: $Z = \frac{J - [(N^2 - \sum_{i=1}^k n_i^2)/4]}{\sqrt{[N^2(2N+3) - \sum_{i=1}^k n_i^2(2n_i+3)]/72}}$.

Table 1.6 – Ordering of volatilities across maturities, crude oil and electricity markets, 2008-2014

Taking into account the non-normality of the time series gives results that are now fully consistent with the second empirical implication of the Samuelson effect. This is especially remarkable for the Australian market (NSW futures contract) and can probably be explained by the lack of liquidity in certain maturities (see Table 1.2) because it is the market with the lowest transaction volumes.

1.3.3 Do the price shocks spread from the physical market to the paper market?

In what follows, we propose and test a third empirical implication to the Samuelson effect. Following Lautier, Raynaud, and Robe [2014], we assume that when a derivative market performs its hedging function correctly, the price shocks in the paper market should be the result of the price shocks emerging from the physical market, and not the reverse. Because we measure the price shocks with volatility, we should thus observe a volatility that transmits *from* the physical market *to* the paper market with a decreasing intensity when the contracts' maturity rises. In other words, not only should the volatilities be ordered according to the maturity; there should also be a direction to the propagation.

A preliminary answer to this question could be tackled with a Granger analysis (Granger [1969]). However, with this method, we would only have information about the direction of the price shock. To make sure that the volatility transmitted from the physical market to the paper market diminishes with the maturities, we also need information about quantities. The method developed by Diebold and Yilmaz [2012], hereafter DY, gives such indications. Whereas those authors apply it to different assets, we use it for different maturities of the same futures contract. Moreover, we use the first nearby futures price as a proxy for the spot price.

Volatility spillover measures: the method

The DY method (2012) is an extension of the index they developed three years before (Diebold and Yilmaz [2009]). This method improves the previous index in two ways. First, the 2009 index provides aggregated information about the *total* spillover of volatilities; it tells how much volatility spreads across all of the markets and gives a measure of the markets' integration. By comparison, the method developed in 2012 provides disaggregated information about how much volatility spreads *from* one market *to* one or to all of the others; it gives information about the direction of the spillover. Second, the previous method is based on a vector autoregressive (VAR) framework for which the results can be order-dependent due to the Cholesky factor orthogonalization: to make sure that a shock impacts one variable at a time, there is an ordering of the variables impacted by the shock. This choice can influence the results. By comparison, the measures of 2012 are based on a generalized VAR framework in which the forecast-error variance decompositions are invariant to the variable ordering.

In what follows, we explain how we apply this method. We retain for each market only three maturities: the nearest, the longest, and the one situated in the middle of the curve. For example, in the case of the PJM contract, we use the first month M1, six months M6, and three months M3. The first step of the method consists of setting the generalized VAR framework. The second leads to the total spillover index. The third indicates the directionality.

Generalized vector autoregressive framework We first build a generalized VAR framework that states the dependencies between the three series of volatilities. We consider the following covariance stationary¹¹ N-variable VAR(p), where N = 3:

$$x_t = \sum_{i=1}^p \phi_i x_{t-i} + \varepsilon_t \quad (1.5)$$

where:

- x_t is a (3x1) vector gathering the values of daily volatilities at date t,
- p is the number of lags in days,
- ϕ_i is the (3x3) coefficient matrix at lag i ,
- $\varepsilon \sim (0, V)$ is a (3x1) vector of independently and identically distributed disturbances.

Using the Wold's representation theorem (Wold [1938]), we can write the moving average representation of x_t as follows:

$$x_t = \sum_{i=0}^{\infty} A_i \varepsilon_{t-i} \quad (1.6)$$

where the $N \times N$ coefficient matrices A_i obey the recursion:

$$A_i = \phi_1 A_{i-1} + \phi_2 A_{i-2} + \dots + \phi_p A_{i-p} \quad (1.7)$$

with A_0 being an $N \times N$ identity matrix and $A_i = 0$ for $i < 0$. Once the moving average coefficients are determined, they can be used to understand the dynamics of the system with variance decompositions.

These decompositions allow the assessment of the fraction of the H-step-ahead error variance in forecasting x_i that is due to shocks to x_j , $\forall j \neq i$, for each i where H is the horizon of forecasting and x_i is the volatility that corresponds to maturity i . In order to make sure that forecast-error variance decompositions are invariant to the variable ordering (i.e., to avoid the use of the Cholesky factorization), DY rely on the generalized VAR framework of Koop, Pesaran, and Potter [1996] and Pesaran and Shin [1998] - hereafter KPPS. The KPPS H-step-ahead forecast error variance decompositions $\theta_{ij}(H)$, for $H = 1, 2, \dots$, are:

$$\theta_{ij}(H) = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} (e_i' A_h V e_j)^2}{\sum_{h=0}^{H-1} (e_i' A_h V A_h' e_i)} \quad (1.8)$$

where σ_{jj} is the standard deviation of the error term for the j th maturity, e_i is the (3x1) selection vector with one as the i th element and zeros otherwise, and V is the (3x3) variance matrix of the error vector ε . The terms with an apostrophe are transposes of the original matrices.

11. As stated in Table 1.3, our time series of volatilities are stationary.

Further, each entry of the variance decomposition matrix $\theta_{ij}(H)$ is normalized by the row sum that gives the spillover measures from maturity i to maturity j at horizon H , $\tilde{\theta}_{ij}(H)$:

$$\tilde{\theta}_{ij}(H) = \frac{\theta_{ij}(H)}{\sum_{j=1}^N \theta_{ij}(H)} \quad (1.9)$$

Using the KPPS variance decomposition, the authors propose one index of total spillover, and two measures of directional spillovers and net pairwise spillovers.

Total spillover index This quantity measures the contribution of spillovers of volatility shocks across maturities to the total forecast error variance. In other words, this index gives information about the degree of integration of the market under study. In percentage, it is constructed as follows:

$$S(H) = \frac{\sum_{i,j=1, i \neq j}^N \tilde{\theta}_{ij}(H)}{\sum_{i,j=1}^N \tilde{\theta}_{ij}(H)} \cdot 100 = \frac{\sum_{i,j=1, j \neq i}^N \tilde{\theta}_{ij}(H)}{N} \cdot 100 \quad (1.10)$$

Directional spillovers The total spillover index can be decomposed into directional spillovers, also expressed in percentage, that give information about the direction of the volatility spillovers across maturities. The volatility spillover received by maturity i from all others is:

$$S_{.i}(H) = \frac{\sum_{j=1, j \neq i}^N \tilde{\theta}_{ij}(H)}{\sum_{i,j=1}^N \tilde{\theta}_{ij}(H)} \cdot 100 = \frac{\sum_{j=1, j \neq i}^N \tilde{\theta}_{ij}(H)}{N} \cdot 100 \quad (1.11)$$

In a similar way, the volatility spillover transmitted by the maturity i to all others is written as:

$$S_{.i}(H) = \frac{\sum_{j=1, j \neq i}^N \tilde{\theta}_{ji}(H)}{\sum_{i,j=1}^N \tilde{\theta}_{ji}(H)} \cdot 100 = \frac{\sum_{j=1, j \neq i}^N \tilde{\theta}_{ji}(H)}{N} \cdot 100 \quad (1.12)$$

The net pairwise volatility spillover gives information about how much maturity i contributes to the volatility of maturity j :

$$S_{ij}(H) = \left(\frac{\tilde{\theta}_{ji}(H)}{\sum_{i,k=1}^N \tilde{\theta}_{ik}(H)} - \frac{\tilde{\theta}_{ij}(H)}{\sum_{j,k=1}^N \tilde{\theta}_{jk}(H)} \right) \cdot 100 = \left(\frac{\tilde{\theta}_{ji}(H) - \tilde{\theta}_{ij}(H)}{N} \right) \cdot 100 \quad (1.13)$$

For the estimation, as in DY we use the following parameters: $p = 4$ lags for the VAR and $H = 10$ for the forecast error variance decompositions. We also perform estimations with other values for the parameters (the range chosen for p is from 2 to 6, and the one for H from 6 to 9) without any change in our results (this sensitivity analysis is available on request).

Static analysis of volatility spillovers between maturities

We first measure the total spillover index and then the directional and net pairwise spillovers between the futures prices for different maturities in each market for the sample period. To

	# of observations	Total spillover index (%)	Directional spillovers			Net pairwise spillovers		
			To all	From all	Net			
WTI	1 518	64.76	M1	23.05	20.085	2.965	M1-M3	0.989
			M3	22.042	21.986	0.055	M1-M6	1.976
			M6	19.673	22.693	-3.020	M3-M6	1.044
Phelix	1 457	51.42	M1	21.462	13.439	8.023	M1-M3	3.889
			M3	15.784	19.178	-3.394	M1-M5	4.135
			M5	14.176	18.806	-4.630	M3-M5	0.495
PJM	1 490	46.75	M1	18.734	7.130	11.605	M1-M3	6.43
			M3	15.984	20.038	-4.054	M1-M6	5.174
			M6	12.036	19.587	-7.551	M3-M6	2.376
NEC	906	57.34	M1	21.642	14.786	6.856	M1-M3	3.501
			M3	18.418	21.456	-3.037	M1-M4	3.355
			M4	17.279	21.098	-3.819	M3-M4	0.464
NSW	1 563	24.08	Q1	12.826	1.705	11.120	Q1-Q3	6.778
			Q3	5.442	13.117	-7.676	Q1-Q6	4.342
			Q6	5.808	9.253	-3.444	Q3-Q6	-0.898

This table shows the total spillover indexes and the directional volatility spillover measures between maturities for each electricity market (German, American, Australian, and Nordic) and for the American crude oil market. The total spillover index measures the contributions of the spillovers of volatility shocks across maturities to the total forecast error variance (see equation (1.10)). The directional spillovers "To all (from i)" and "From all (to i)" respectively give information about the volatility spillovers transmitted by the maturity i to all others (see equation (1.12)) and the volatility spillover received by maturity i from all others (see equation (1.11)). The net directional spillover is the difference between the two and shows if a maturity contributes more than it receives. The net pairwise volatility spillover gives information about how much the maturity i contributes to the volatility of maturity j (see equation (1.13)). In the column representing the directional spillovers, "Net" stands for "To all - From all". In the column headed "Net pairwise spillovers", "Mi-Mj is the measure "From Mi to Mj - To Mi from Mj".

Table 1.7 – Static analysis: Volatility spillovers between maturities, crude oil and electricity markets, 2008-2014

simplify the interpretation of the results, we focus on three of the available maturities: the nearest contract (Q1 for the NSW, M1 for all others), the intermediate maturity (M3 or Q3), and the longest maturity. Except for the NSW contract, the results of the static analysis are quite homogeneous, as Table 1.7 shows.

The second column reproduces the total spillover index of each market. It shows that crude oil is the most integrated market: in this case, almost 65% of the volatility of the prices are due to co-movements. This ranking of the WTI contract is reasonable because crude oil is the only storable commodity in our study. Moreover, oil is characterized by very high transaction volumes. Maturities are thus linked by arbitrage strategies. With total spillover indexes of 57.34%, 51.42%, and 46.75%, respectively, the different maturities of the NEC, Phe-lix, and PJM futures contracts remain quite heavily integrated. The lowest value is obtained for the NSW contract with a total spillover index of only 24.08%. This is probably due to the fact that the maturities under consideration are quarters and not months.

The third column is devoted to the decomposition of the total spillover index into direc-tional spillovers. It is separated into four sub-columns. The first indicates the maturity, the second provides the directional spillovers from one maturity i to all of the others; the third, from all of the others to the maturity i ; and the fourth gives the difference between the sec-ond and third sub-columns. First, for each futures contract, the volatility sent by the nearest maturity to all of the others is always higher than the volatility received. Because the nearest maturity is the closest to the physical market, this finding is in line with what is expected: the shocks arising in the physical market are higher than those coming from the paper market. Second, the longer maturities are characterized by negative net spillovers. This reinforces the previous observation. Third, if we compare the volatility sent by the nearest maturity with those sent by more deferred contracts, a decreasing pattern emerges (except for the NSW). This is consistent with the third empirical implication of the Samuelson effect.

These findings are corroborated by the figures for the net pairwise spillovers shown in the last column. The latter is divided into two sub-columns. The first sub-column indicates the pair of maturities taken into account, that is, (M1 - M3), (M1 - M6) and (M3 - M6); the second gives the difference between the two directional spillovers. First, we always obtain a positive measure of the net pairwise spillover when the pair includes the first maturity. Second, the net spillover between the two extreme maturities (M1 - M6) is always higher than the one linking the intermediate and last maturities (M3 - M6).

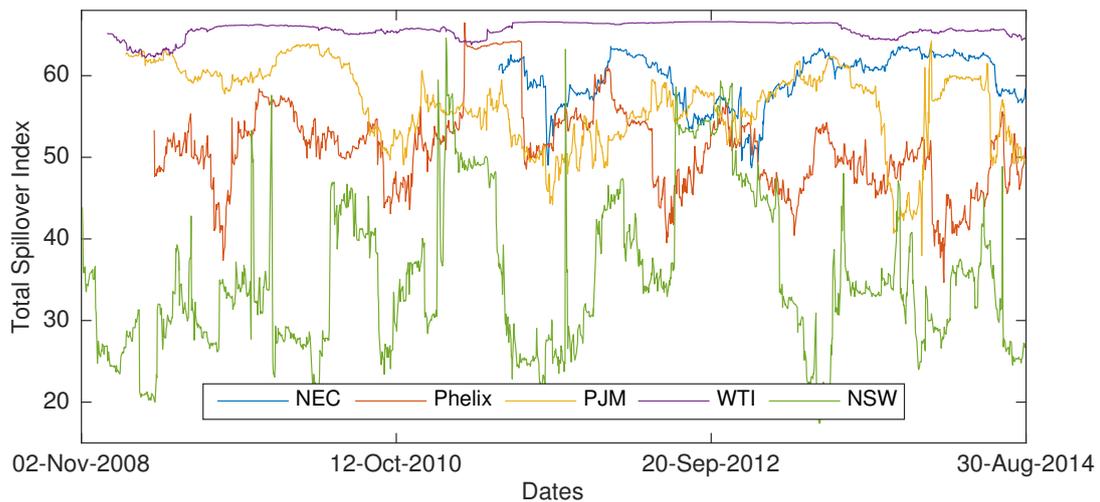
The dynamic analysis gives more insights about the evolution of the volatility spillover through time and reinforces the conclusions made in the static case.

Dynamic analysis of volatility spillovers between maturities

This analysis relies on a rolling window of 90 days (increasing or decreasing the length of the rolling window only smooths or un-smooths the results). The dynamic analysis globally

reinforces the conclusions made in the static case.

Figure 1.3 depicts the evolution of the total spillover indexes during the period. It shows that the stability of integration changes dramatically with the futures contract. The most integrated market, crude oil, is also the most stable by far. As far as electricity contracts are concerned, the most stable is the PJM, the least is the NSW.



This figure exhibits the total spillover index measures in a dynamic framework for each electricity market (German, American, Australian, and Nordic) and for the American crude oil market on the basis of a 90-day rolling window. The total spillover index measures the contribution of spillovers of volatility shocks across maturities to the total forecast error variance (see equation (1.10)).

Figure 1.3 – Dynamic analysis: total spillover index between maturities, crude oil and electricity markets, 2008-2014

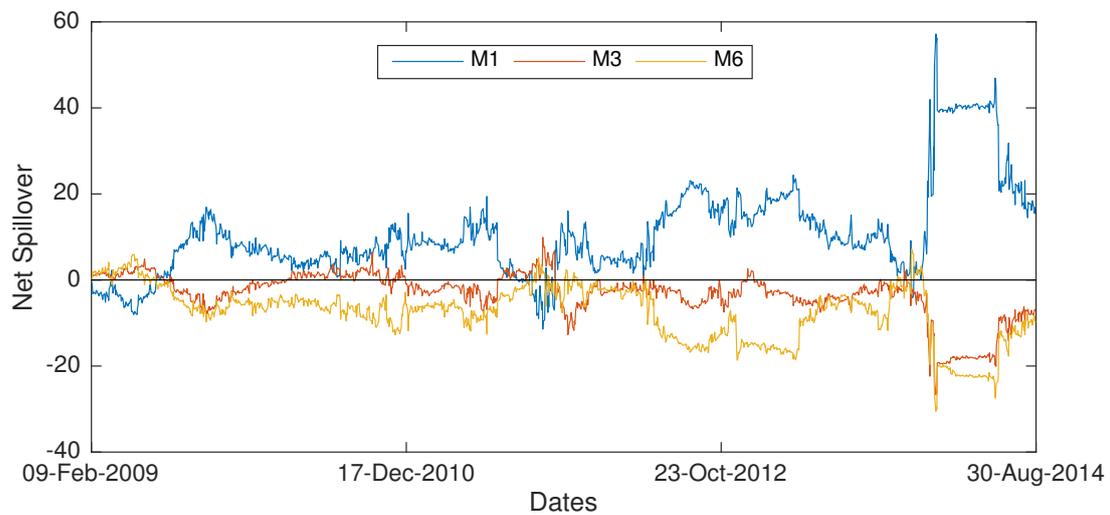
Table 1.8 contains the descriptive statistics on the time series of spillover measures in a dynamic framework for each market. It provides information on the total spillover indexes and on the net spillover measures. First, the frequency of the positive values for the net directional spillover when considering the volatility sent by the first maturity (M_1) is very high: over 89% for all of the markets (around 98% for the WTI and NEC contracts). In contrast, the frequency of the positive values for the volatility sent by the last maturity (M_L) is very low, except for the NSW where the contracts send volatility to the other maturities in 21.24% of the cases. The standard deviation of the net spillover is also the lowest for the WTI and NEC contracts. We observe the lowest extreme values (minimum and maximum) recorded for the net spillover from M_1 to all of the other maturities for the WTI. A more precise examination, through Figure 1.8 in Appendix 1.B, shows that for this market, the number of high values is concentrated in very short periods. These findings are consistent with the high level of integration previously observed for the WTI market.

Figure 1.4 gives a good illustration of the results provided by Table 1.8. It reproduces the example of the PJM contract and is representative of what can be seen in electricity markets

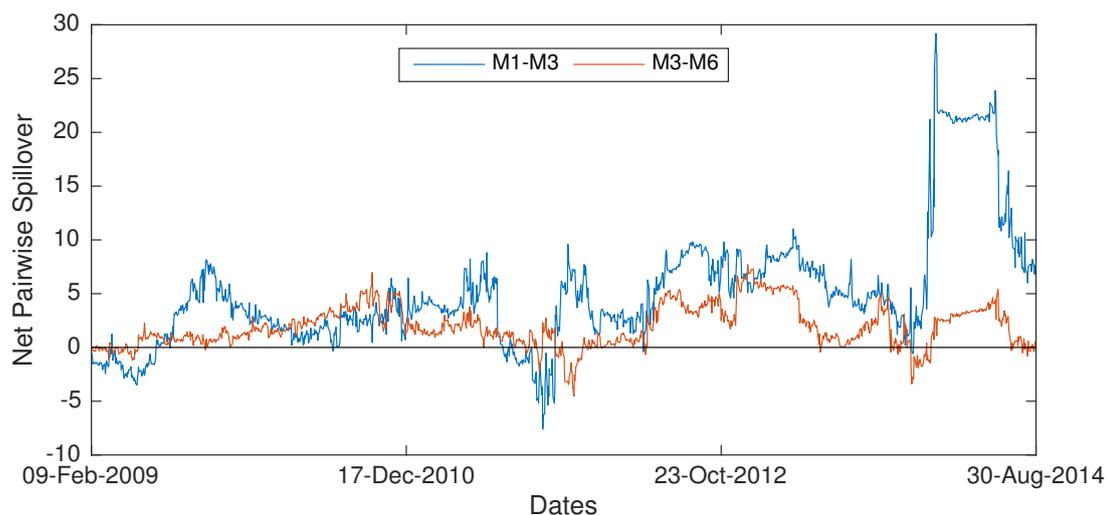
	# of observations		Total Spillover	Net Directional		Net Pairwise		
			Index	For M1	For M _L	M1M3	M3M _L	M1M3-M3M _L
WTI	1 518	Min	62.19	-2.77	-8.67			
		Max	66.62	11.27	0.38			
		StDev	0.98	1.98	1.79			
		\oplus (%)		97.97	0.07	93.98	100	
		$>$ (%)						15.20
Phelix	1 457	Min	34.69	-10.75	-19.31			
		Max	66.46	28.88	13.89			
		StDev	5.35	7.12	5.36			
		\oplus (%)		91.44	15.22	92.17	55.45	
		$>$ (%)						82.44
PJM	1 490	Min	37.95	-11.43	-30.52			
		Max	64.26	57.19	7.16			
		StDev	4.82	10.78	6.44			
		\oplus (%)		89.71	9.79	87.50	87.86	
		$>$ (%)						73.71
NEC	906	Min	48.71	-1.46	-11.63			
		Max	63.63	29.55	1.66			
		StDev	3.18	4.59	2.4			
		\oplus (%)		98.16	3.68	96.32	65.93	
		$>$ (%)						87.13
NSW	1 563	Min	17.41	-23.97	-30.99			
		Max	64.59	63.79	55.02			
		StDev	9.32	11.75	9.06			
		\oplus (%)		92.60	21.24	93.08	42.50	
		$>$ (%)						91.38

This table shows the summary statistics of the total spillover indexes and the directional volatility spillover measures between maturities, in a dynamic framework, for each electricity market (German, American, Australian, and Nordic) and for the American crude oil market on the basis of a 90-day rolling window. The total spillover index measures the contribution of the spillovers of volatility shocks across maturities to the total forecast error variance (see equation (1.10)). The directional spillovers "To all (from i)" and "From all (to i)" respectively give information about the volatility spillovers transmitted by maturity i to all others (see equation (1.12)) and the volatility spillover received by maturity i from all others (see equation (1.11)). The net directional spillover is the difference between the two and shows if a maturity contributes more than it receives. The net pairwise volatility spillover gives information about how much maturity i contributes to the volatility of maturity j (see equation (1.13)). The M_L is the longest maturity available for each contract. The \oplus (%) represents the percentage of positive values recorded over the period. The $>$ (%) is used for net pairwise measures only. The M1M3-M3M_L represents the percentage of measures of net pairwise volatility between the maturities M1 and M3, which are higher than those between M3 and M_L.

Table 1.8 – Dynamic analysis: volatility spillovers between maturities, crude oil and electricity markets, 2008-2014



(a) Net Spillovers between maturities, PJM market



(b) Net Pairwise Spillovers, between maturities, PJM market

This figure displays the directional volatility spillover measures between maturities for the PJM electricity contract traded in the United States in a dynamic framework. A 90-day rolling window is used. The first chart represents for each maturity the net directional spillovers. The second chart represents the net pairwise spillovers between two maturities. The directional spillovers "To all (from i)" and "From all (to i)" respectively give information about the volatility spillovers transmitted by the maturity i to all others (see equation (1.12)) and the volatility spillover received by maturity i from all others (see equation (1.11)). The net directional spillover is the difference between the two and shows if a maturity contributes more than it receives. The net pairwise volatility spillover gives information about how much maturity i contributes to the volatility of maturity j (see equation (1.13)).

Figure 1.4 – Dynamic analysis: directional spillovers between maturities, PJM market, 2008-2014

(the figures for all of the other markets are in Appendix 1.B). The first part of the figure is devoted to net directional spillovers, the second to the net pairwise spillovers. The first part shows that the M1 contract has a positive volatility spillover and transmits shocks. The periods in which the nearest contract receives the volatility are exceptional. Another striking feature is that the value of the net directional spillovers emanating from the longest maturity (M_L) apparently mirror those obtained for the M1 (a more precise analysis, relying on Table 1.8 gives a different picture, especially for the Phelix and even more so for the NSW contracts). As far as the second part of the Figure 1.4 is concerned, it shows that the "M1-M3" line that represents what is sent between the first pair of maturities (M1 and M3) is above the "M3-M6" line, which represents what is sent between the last pair of maturities (M3 and M_L).

For the electricity markets other than the Australian as well as for crude oil, there is a transmission of the price shocks from the physical market to the paper market with a decreasing intensity. This transmission confirms the previous results of this article that are based on the two first empirical implications of the Samuelson effect. Further, for very short periods of time, the direction of the transmission changes significantly, and the Samuelson effect is less important for the crude oil market. This difference between electricity and crude oil might be explained by the storability of the latter, which allows for a better transmission of the shocks.

1.4 Going deeper in the analysis of the maturity effect: the link with the indirect storability

Up to now we have shown, via three different empirical implications, that the Samuelson effect is an important feature of electricity markets. This confirms and extends the results of Walls [1999] and Allen and Cruickshank [2002] with a large database. Moreover, as is the case for a vast majority of studies in this literature (see Bessembinder et al. [1996], Brooks [2012], Lautier and Raynaud [2011], Daal, Farhat, and Wei [2006] and Galloway and Kolb [1996]), we also find a Samuelson effect in the crude oil market¹².

Finding a strong decreasing pattern in the volatilities for electricity seems to go against the conclusions of Bessembinder et al. [1996], who emphasize the importance of storability. Nevertheless, at this point, one could imagine that the existence of the Samuelson effect in electricity markets is due to its potential storability in the form of its inputs, also called indirect storability. This concept, recently proposed for electricity markets (see Routledge et al. [2001], Aïd, Campi, Huu, and Touzi [2009] or Aïd et al. [2013]), could indeed reconcile

12. This result for the crude oil market goes against the finding of Duong and Kalev [2008]. This divergence could be due to the difference in the frequency of the data used (daily versus intraday). High frequency data indeed give insights into microstructure effects that can not be seen with daily data.

the existence of the Samuelson effect in electricity markets and the storability as a necessary condition for it. In order to explore this assumption we study the relation between the prices of electricity and those of its inputs, and we examine whether or not the price shocks borne in the input markets are transmitted to the electricity markets.

In what follows we investigate the effect of indirect storability in the American and German electricity markets. Compared with the others, these two markets are characterized by the fact that a significant part of their inputs are tradable commodities: the electricity traded under the PJM contract is mainly produced on the basis of coal, natural gas, and petroleum products. According to the PJM Regional Transmission Organization (RTO), in 2014 these inputs accounted for 76% (40%, 28%, and 8% respectively) of the installed capacity in the geographical area under consideration. As far as the Phelix contract is concerned, according to the Fraunhofer Institute, coal and natural gas accounted for 43.8% (27.8% and 16.11% respectively) of the net installed generation capacity in 2014.

We retain the same time period and prices for the electricity contracts (see Table 1.1). For the inputs of the PJM market, the heating oil, natural gas, and coal prices correspond to the futures contracts negotiated on the NYMEX. For the inputs of the Phelix market, we use the prices of the Rotterdam coal futures and those of the TTF natural gas futures. Both contracts are traded on the ICE. Tables 1.11 and 1.12 in Appendix 1.C display the descriptive statistics of the time series of volatilities of the input prices. They show that the series do not contain unit roots, are autocorrelated, and do not follow a normal distribution.

To study the potential effect of indirect storability, we rely on the Diebold and Yilmaz [2012] method. For all of the markets, we retain the one-month continuous time series (M1). As before, we use $p = 4$ lags for the VAR and $H = 10$ for the forecast error variance decompositions¹³. We first perform a static, and then a dynamic, analysis on the two electricity markets. Because the results are similar for the two contracts, we only give those for the PJM market. The study of the Phelix market is in Appendix 1.C.2.

Table 1.9 displays the results of the static analysis. First, the total spillover index is 26.25%, which means that the volatility in the PJM market is mainly explained by its own shocks rather than by links with its inputs. Further, this figure is much lower than those found for the different maturities of each contract, with the exception of the NSW market (see Table 1.7). Moreover the heating oil and coal markets receive the most important amount of volatility from the three others. The heating oil and coal are characterized by net directional spillovers of -3.66 and -4.03 respectively. Moreover, with positive net pairwise spillovers the PJM market delivers volatility to the three input markets.

The dynamic analysis, performed on the basis of 90-day rolling windows and illustrated by Figure 1.5 and Table 1.10 gives further insights. In net terms, the PJM and natural gas markets respectively send volatility 58.62% and 92.35% of the time, while the heating oil and

13. These choices do not affect our results; we have computed the volatility spillover measures with other parameters and find no significant changes. The results are available on request.

# of observations		Total spillover index (%)	Directional to all others	Directional from all others	Net	Net Pairwise against PJM
1 490	PJM		6.48	4.23	2.25	
	Heating oil	26.25	4.47	8.13	-3.66	0.81
	Natural gas		11.51	6.07	5.44	0.67
	Coal		3.79	7.82	-4.03	0.78

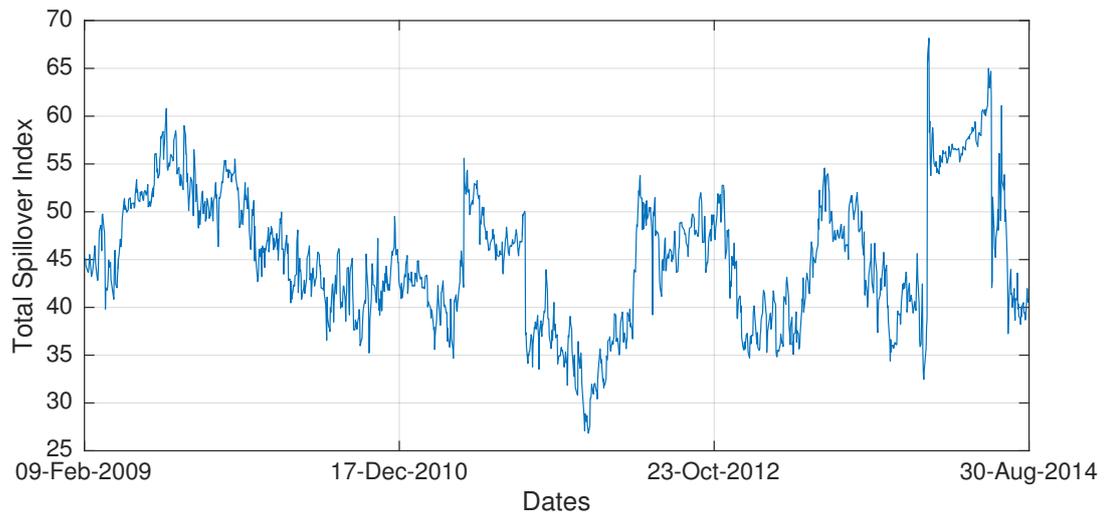
This table shows the total spillover index and the directional volatility spillover measures in a static framework between the PJM prices and three inputs: heating oil, natural gas, and coal. The total spillover index measures the contribution of the spillovers of volatility shocks across markets to the total forecast error variance (see equation (1.10)). The directional spillovers "To all (from i)" and "From all (to i)" respectively give information about the volatility spillovers transmitted by the market i to all others (see equation (1.12)) and the volatility spillover received by market i from all others (see equation (1.11)). The net directional spillover is the difference between the two and shows if a market contributes more than it receives. The net pairwise volatility spillover gives information about how much market i contributes to the volatility of market j (see equation (1.13)).

Table 1.9 – Static analysis: volatility spillovers between electricity and its inputs, PJM and input markets, 2008-2014

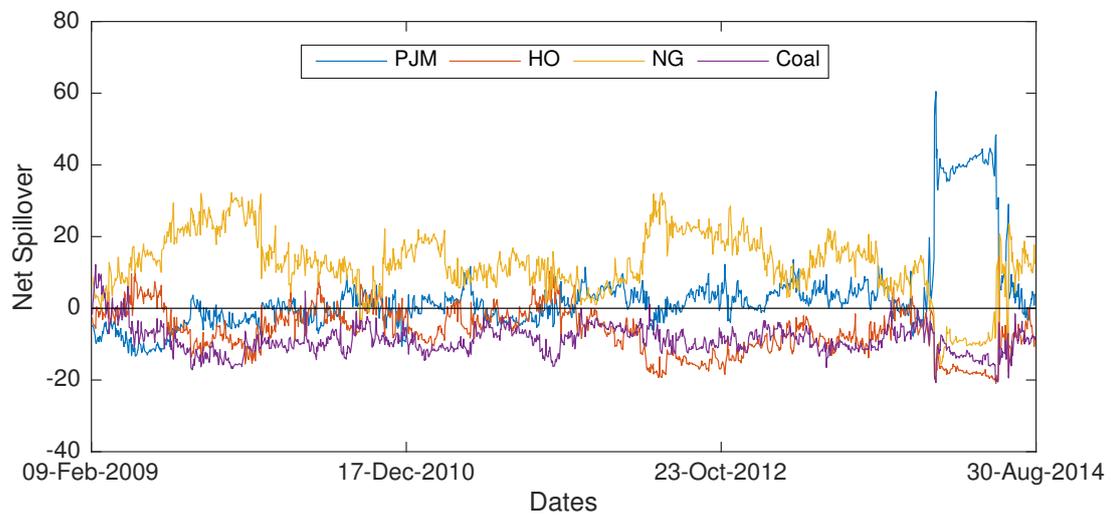
coal markets respectively receive it 83.92% and 97.43% of the time. At the end of our period, during the winter of 2014, there is a sudden change in the behavior of the prices and the PJM market turns into a volatility emitter. This exceptional period coincides with a higher integration of our markets (high total spillover index) than usual and leads to an overestimation in the static case of the net volatility spillover coming from the PJM market. However, as is the case in the static framework, the dynamic pairwise analysis shows that most of the time, the PJM market sends volatility to the heating oil and coal markets and receives volatility from the natural gas market.

All of these findings are in contradiction with what we expected: under the influence of an indirect storability effect, the PJM prices should *receive* the volatility from the input markets. We reach the same conclusion for the German market (see Appendix 1.C.2). So even if these two electricity markets interact with their input markets, only a small part of the behavior of the electricity prices can be explained by that of its inputs.

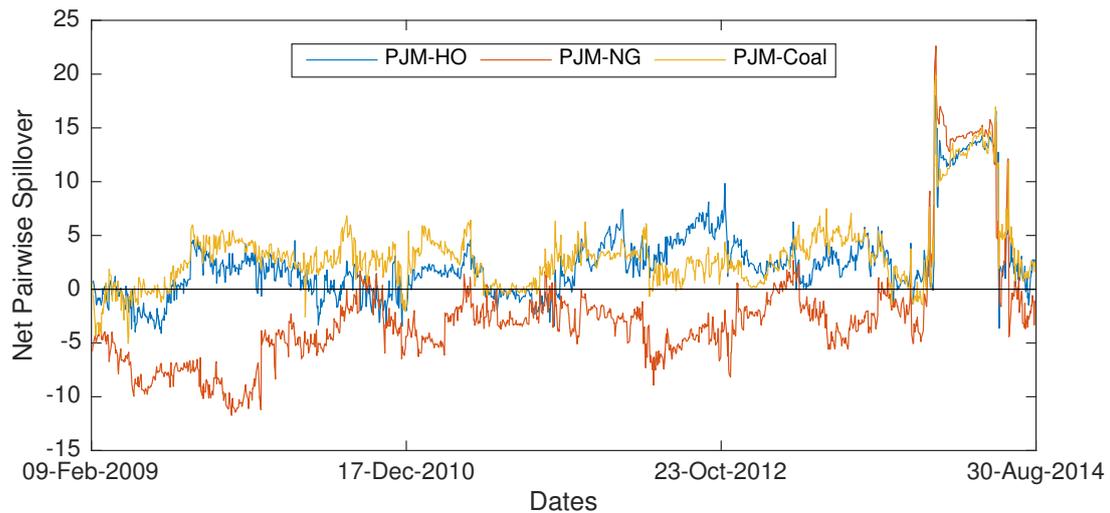
In the presence of a Samuelson effect without storage or indirect storage, where do we stand in the debate on the economic explanation of the Samuelson effect? One would be tempted, at first, to conclude that our findings are consistent with the theoretical framework of Anderson and Danthine [1983], where what matters is the resolution of production uncertainty over time, and inconsistent with that of Bessembinder et al. [1996] who focuses on inventories. However, the answer is more nuanced. Bessembinder et al. [1996] rely primarily on the negative covariance between net carrying costs and spot prices to explain the Samuelson effect. Then they assume that such a negative covariance is due to the presence



(a) Indirect Storability in the American market: Total Spillover Index



(b) Indirect Storability in the American market: Net Directional Spillover



(c) Indirect Storability in the American market: Net Pairwise Spillover

This figure shows the total spillover index and directional volatility spillover measures between electricity prices and input prices in the PJM market and in a dynamic framework with a 90-day rolling window. The first chart represents the total spillover index. The second chart represents for each market the net directional spillovers. The third chart represents the net pairwise spillovers against the PJM market. The total spillover index measures the contribution of the spillovers of volatility shocks across markets to the total forecast error variance (see equation (1.10)). The directional spillovers "To all (from i)" and "From all (to i)" respectively give information about the volatility spillovers transmitted by market i to all others (see equation (1.12)) and the volatility spillover received by market i from all others (see equation (1.11)). The net directional spillover is the difference between the two and shows if a market contributes more than it receives. The net pairwise volatility spillover gives information about how much market i contributes to the volatility of market j (see equation (1.13)).

Figure 1.5 – Dynamic analysis: volatility spillovers between electricity and its inputs, PJM and input markets, 2008-2014

# of observations	Total Spillover Index	Net Directional				Net Pairwise		
		PJM	HO	NG	Coal	PJM-HO	PJM-NG	PJM-C
1 490	Min	26.84	-13.37	-20.94	-19.63	-20.75		
	Max	68.16	60.48	14.44	32.31	12.18		
	StDev	6.83	11.26	6.37	8.95	3.99		
	⊕(%)		58.62	16.08	92.35	2.57	77.34	14.65

This table shows the summary statistics for the total spillover index and the directional volatility spillover measures between the PJM market and its inputs, in a dynamic framework, on the basis of a 90-day rolling window. In this table, HO stands for Heating Oil, NG for Natural Gas, and C for Coal. The $\oplus(\%)$ represents the percentage of positive values recorded over the period. The total spillover index measures the contribution of the spillovers of volatility shocks across markets to the total forecast error variance (see equation (1.10)). The directional spillovers "To all (from i)" and "From all (to i)" respectively give information about the volatility spillovers transmitted by market i to all others (see equation (1.12)) and the volatility spillover received by market i from all others (see equation (1.11)). The net directional spillover is the difference between the two and shows if a market contributes more than it receives. The net pairwise volatility spillover gives information about how much market i contributes to the volatility of market j (see equation (1.13)).

Table 1.10 – Dynamic analysis: volatility spillovers between electricity and its inputs, PJM and input markets, 2008-2014

of inventories and convenience yields¹⁴. Further, in the case of electricity, we also find a negative covariance between the net carrying costs and the spot prices (see Appendix 1.E for the tests of the *negative covariance hypothesis* of Bessembinder et al. [1996] and of the *state variable hypothesis* of Anderson and Danthine [1983]). Yet there is no stock, nor indirect storability, at play. Why? Because even in the absence of a buffering effect on prices due to inventories, there is still some flexibility in the electricity markets. This flexibility is associated with flexible production capacities, like thermal units.

Thus, as in Bessembinder et al. [1996], Daal et al. [2006], and Duong and Kalev [2008], our results on electricity do not definitively reject or validate one of the two theoretical frameworks used to explain the Samuelson effect; both frameworks are helpful. However, the analysis proposed by Bessembinder et al. [1996] should not be restricted to storable commodities.

14. Bessembinder et al. [1996]: " (...) we argue that the most plausible reason for substantial time variation in inventory carrying costs derives from the variation of real service flows or 'convenience yields'. In particular, a positive covariation between convenience yields and spot prices leads to mean-reverting spot prices in equilibrium, and is sufficient to support the Samuelson hypothesis. Since financial assets do not provide service flows, we predict that the Samuelson hypothesis will not hold for financial futures".

1.5 Conclusion

This article provides insights for the literature on commodity derivative markets in several directions. First, it proposes a new empirical implication of the Samuelson effect and offers a method to test it. Second, it enhances the knowledge about the dynamics of the futures prices in the four most important electricity markets in the world, and we find evidence of a Samuelson effect for all of the markets under consideration. Even if electricity is non-storable, the comparison with crude oil does not give evidence of a specific behavior. Third, contrary to what was proposed by Bessembinder et al. [1996], this empirical study shows that storage is not a necessary condition for a Samuelson effect to appear. This is interesting, as most of the models of the term structure of commodity prices rely on the storage theory (see, e.g., Brennan [1958], Brennan and Schwartz [1985], and Cortazar and Schwartz [2003]). This result is reinforced by our finding that there is no evidence of an "indirect storability" effect in the markets under examination. The results do not show the presence of persistent directionality effects from the inputs to the electricity prices.

This evidence of an existing time-to-maturity effect for electricity markets calls for improvements in the valuation of electricity derivative assets. This need is all the more true because maturity and volatility are essential components of the asset's value. For example, this is the case, for term structure models of futures prices and for options. This improvement in the valuation should be followed by an enhancement of risk management procedures: it is necessary to take the Samuelson effect into account for hedging operations in electricity markets and for the design of markets protection tools by clearing houses and regulatory authorities.

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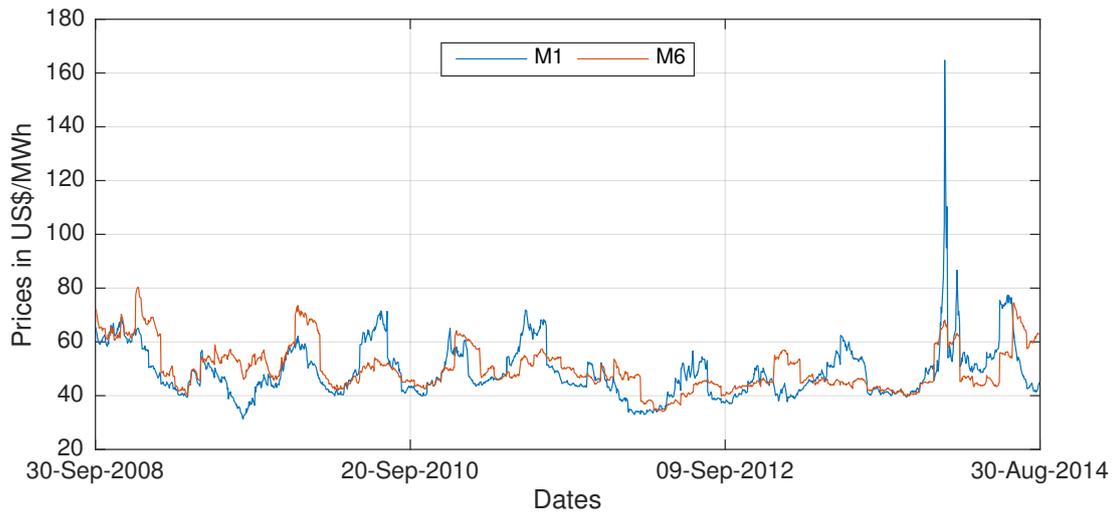
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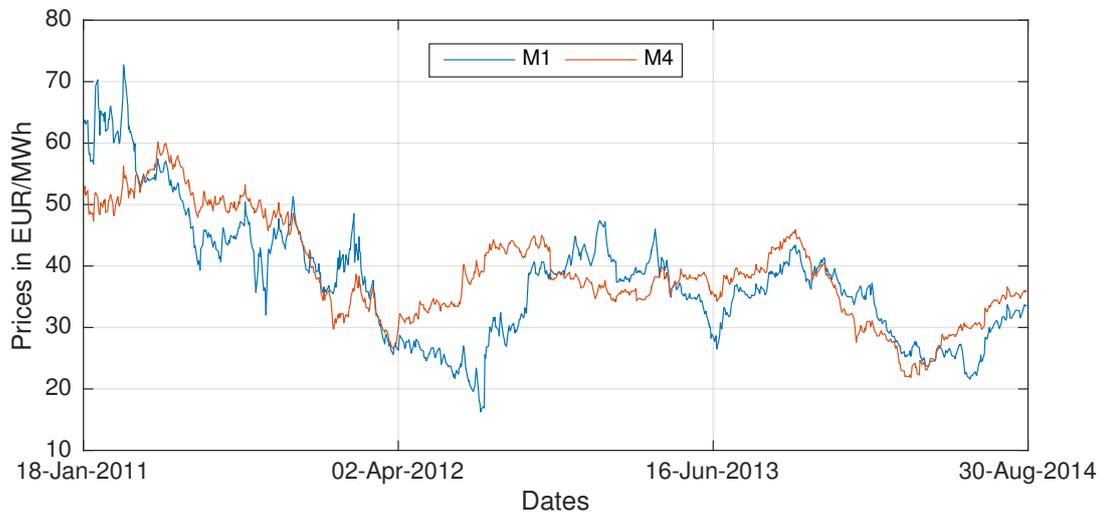
1.A Appendix: Prices and volatilities

1.A.1 Continuous time series of prices for different maturities

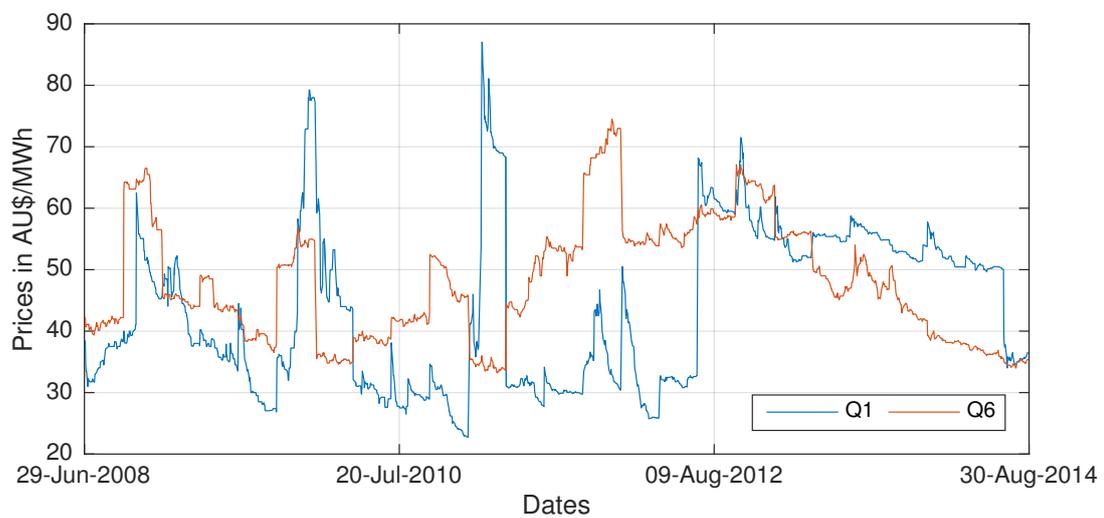
This figure displays the continuous time series of prices for two different constant maturities in three electricity futures markets: the PJM, the NEC, and the NSW.



(a) PJM contract



(b) NEC contract



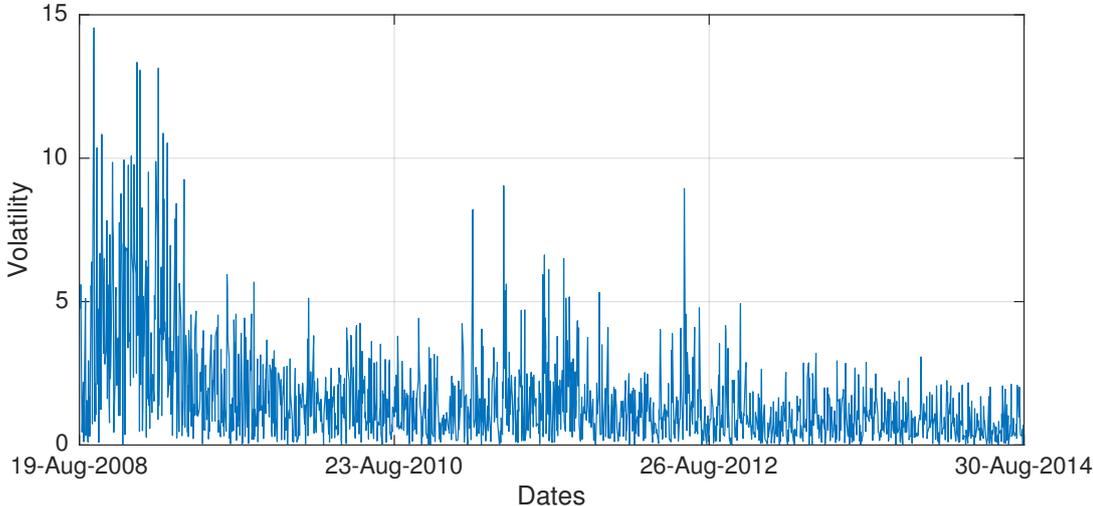
(c) NSW contract

This figure shows the continuous time series of the prices for two maturities in three electricity markets. That is the one- and six-month futures contracts for the PJM market, the one- and four-month futures contracts for the Nordic market and the one- and six-quarter futures contracts for the Australian market.

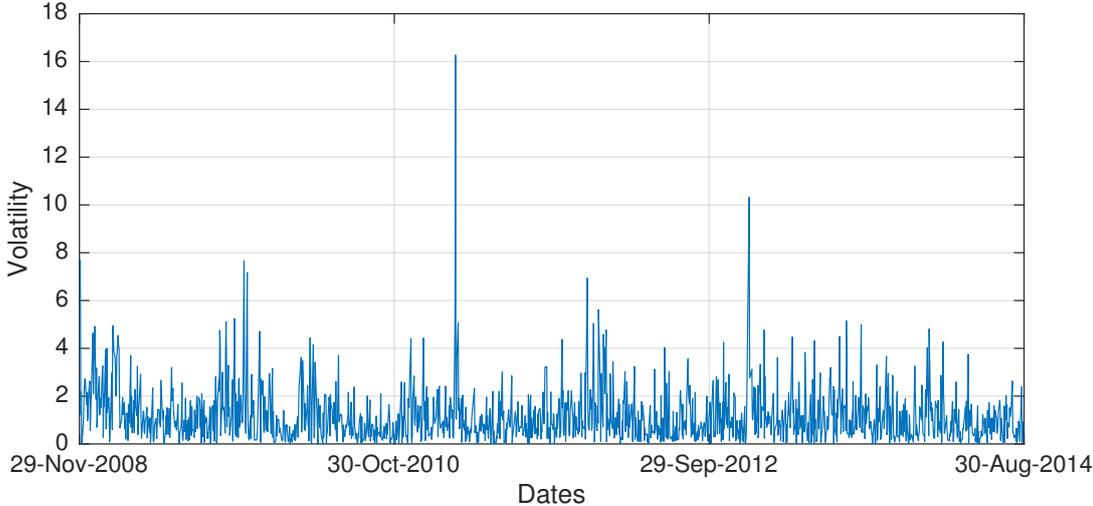
Figure 1.6 – Time series of prices of fixed maturity contracts, PJM, NEC and NSW markets, 2008-2014

1.A.2 Daily volatilities of the closest-to-maturity time series

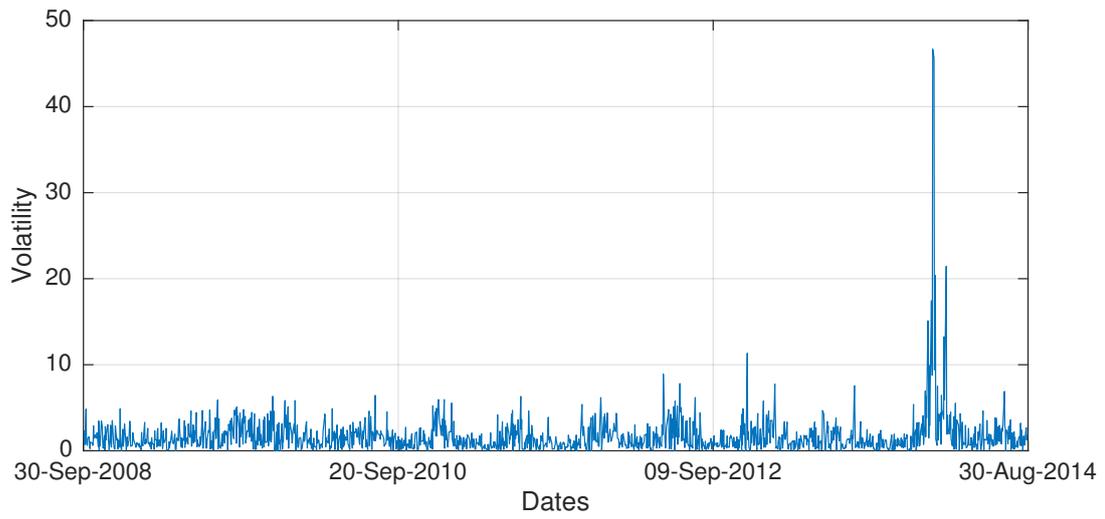
This appendix contains a chart for each market, displaying the volatility of the closest-to-maturity time series.



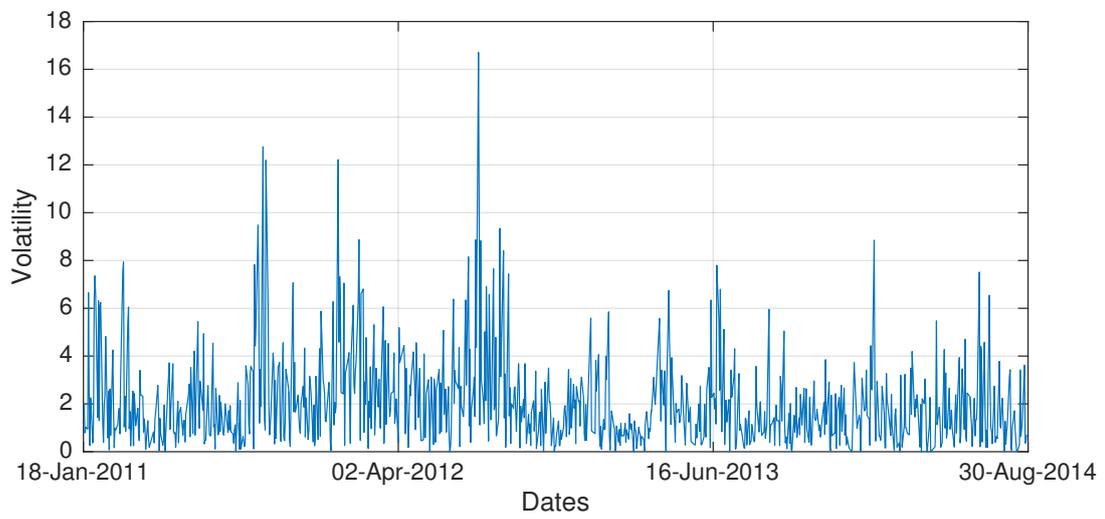
(a) WTI contract



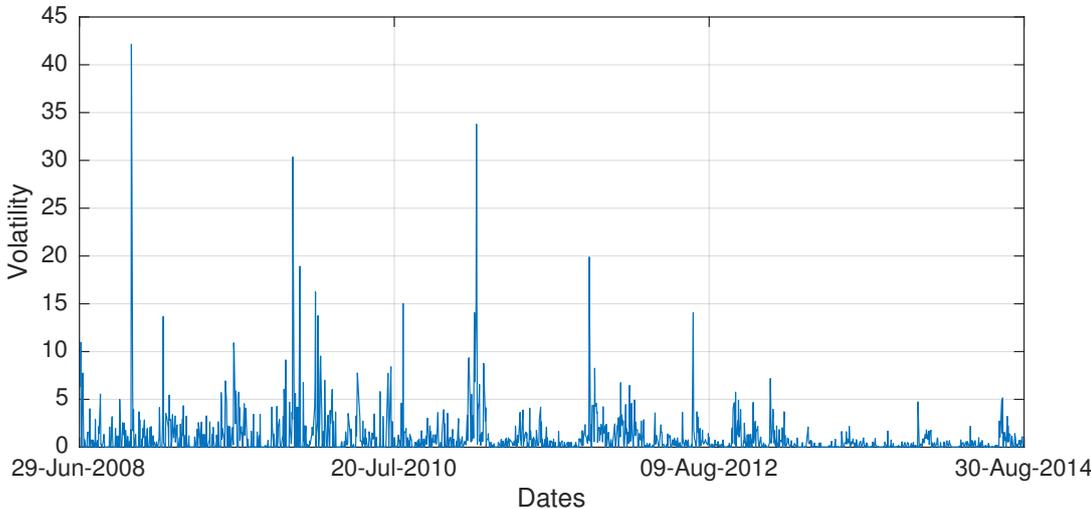
(b) Phelix contract



(c) PJM contract



(d) NEC contract



(e) NSW contract

This figure shows for each market, the time series of the daily volatilities $\sigma_t^k = \left| \ln \left(\frac{F_t^k}{F_{t-1}^k} \right) \right| * 100$ for the closest-to-maturity contract.

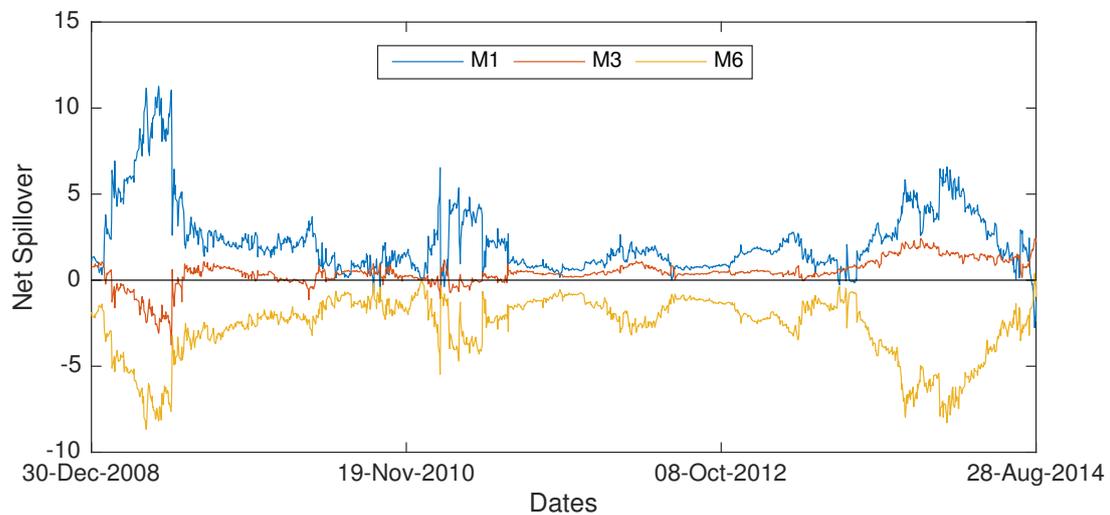
Figure 1.7 – Daily Volatilities of the closest-to-maturity contract, crude oil and electricity markets, 2008-2014

1.B Appendix: Dynamic analysis of spillovers from the physical market to the paper market

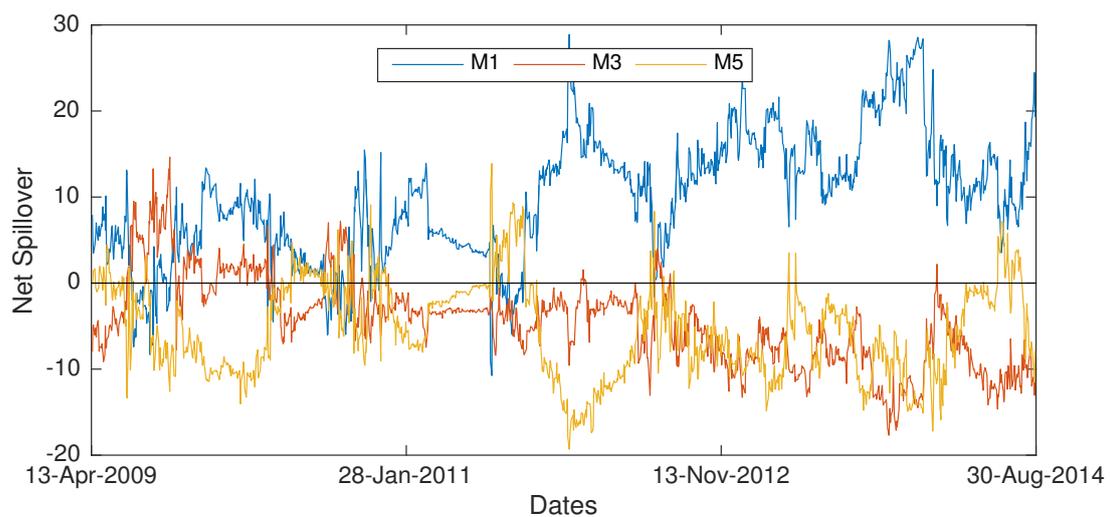
This appendix contains complementary charts for the spillover measures in a dynamic framework. That is, two charts by market not presented in the main sections of the article (WTI, Phelix, NEC, NSW).

1.B.1 Dynamic net spillovers from the physical to the paper markets

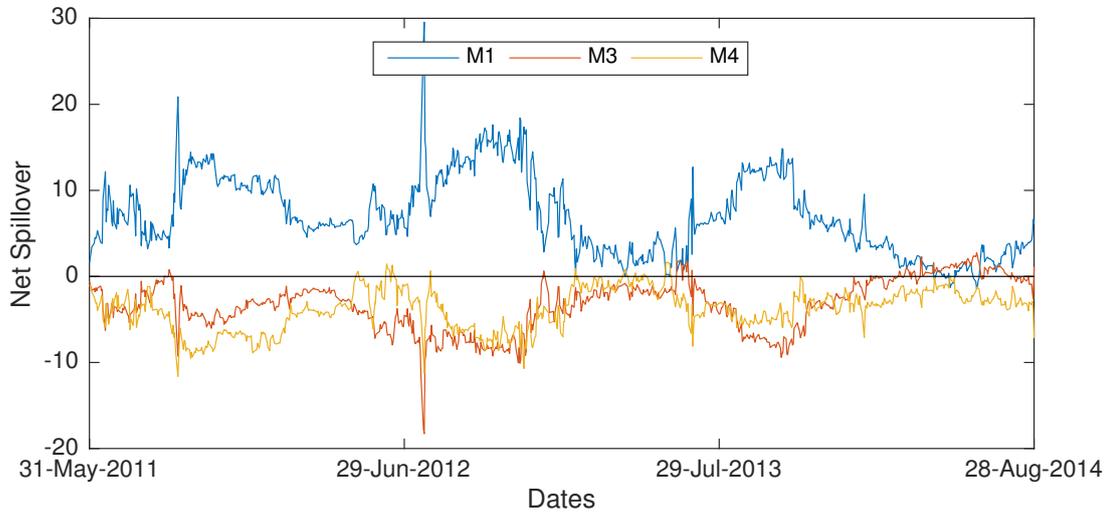
These charts display the dynamic net spillovers from the physical to the paper markets for each contract.



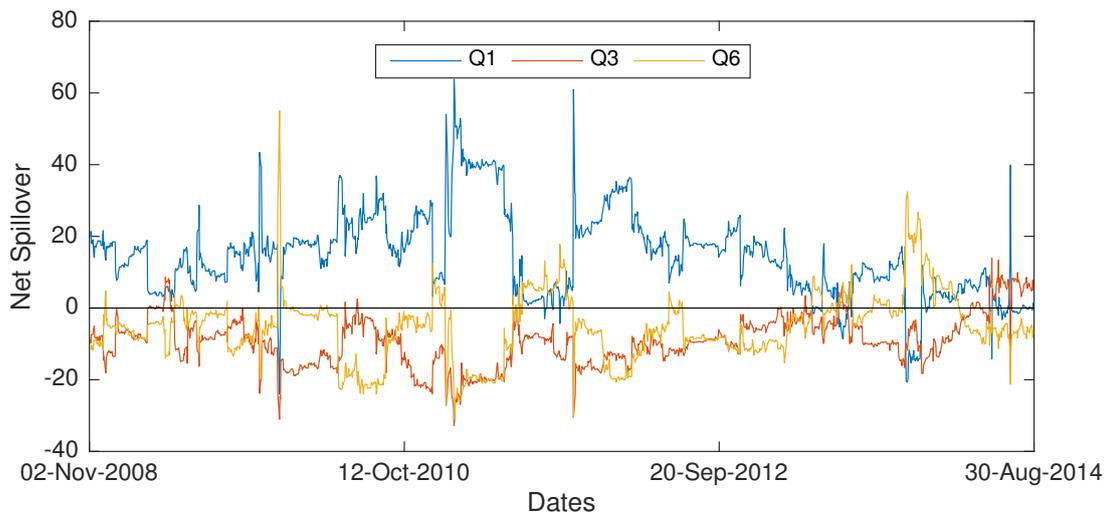
(a) WTI contract



(b) Phelix contract



(c) NEC contract



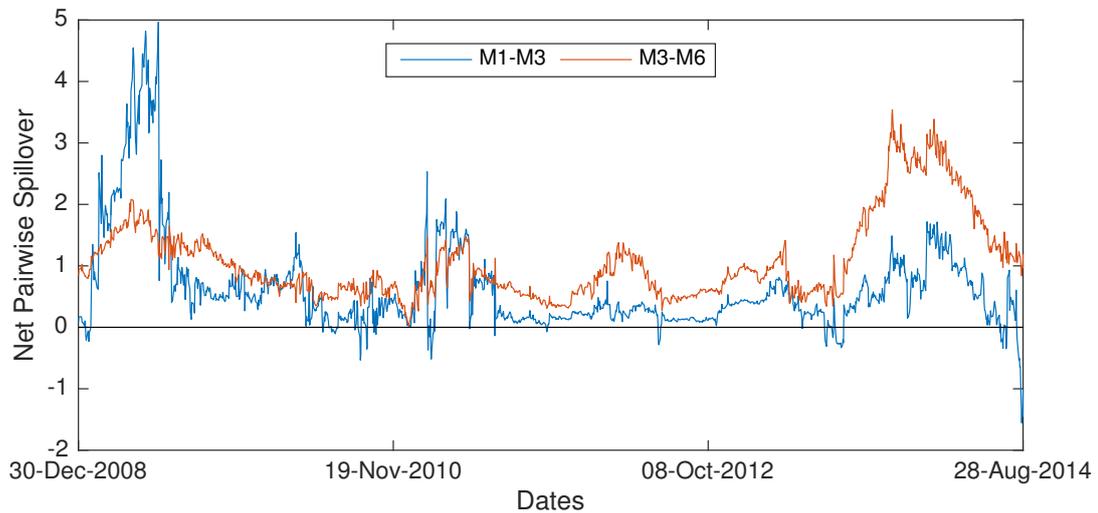
(d) NSW contract

This figure shows the net directional volatility spillover measures between maturities for each market in a dynamic framework using a rolling window of 90 days. The directional spillovers "To all (from i)" and "From all (to i)" respectively give information about the volatility spillovers transmitted by maturity i to all others (see equation (1.12)) and the volatility spillover received by maturity i from all others (see equation (1.11)). The net directional spillover is the difference between the two and shows if a maturity contributes more than it receives.

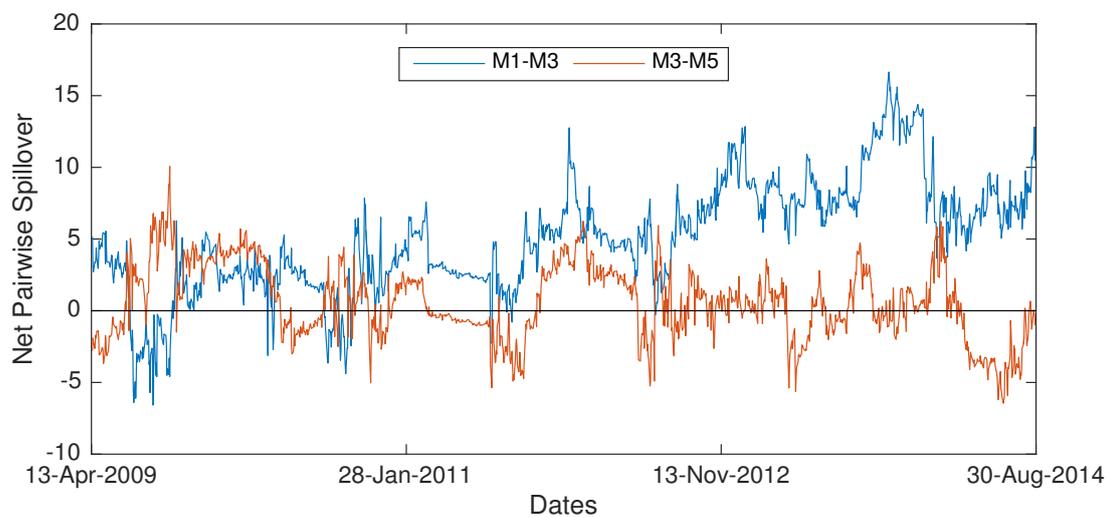
Figure 1.8 – Dynamic analysis: net spillovers between maturities, crude oil and electricity markets, 2008-2014

1.B.2 Dynamic net pairwise spillovers from the physical to the paper markets

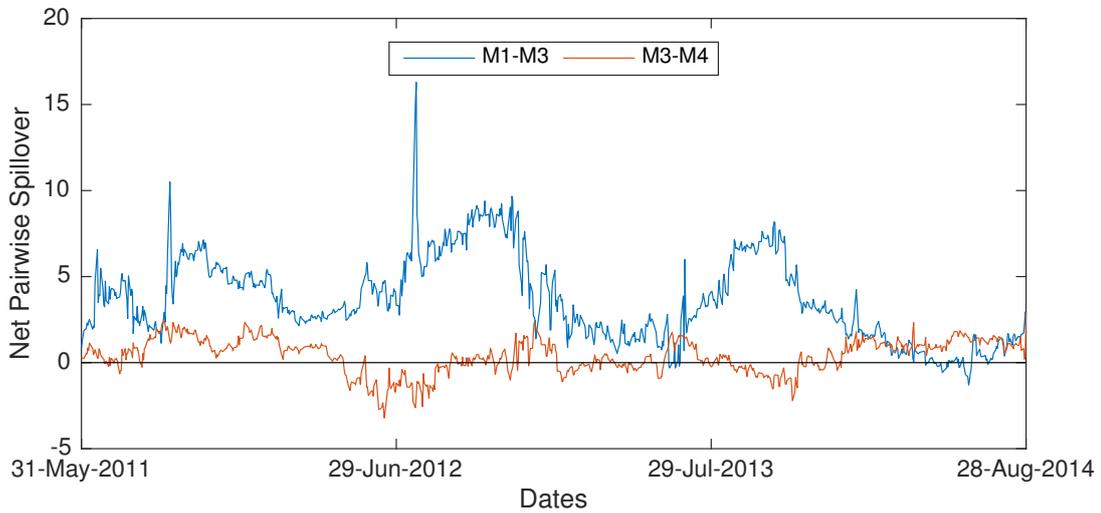
These charts display the dynamic net pairwise spillover from the physical to the paper market for each market.



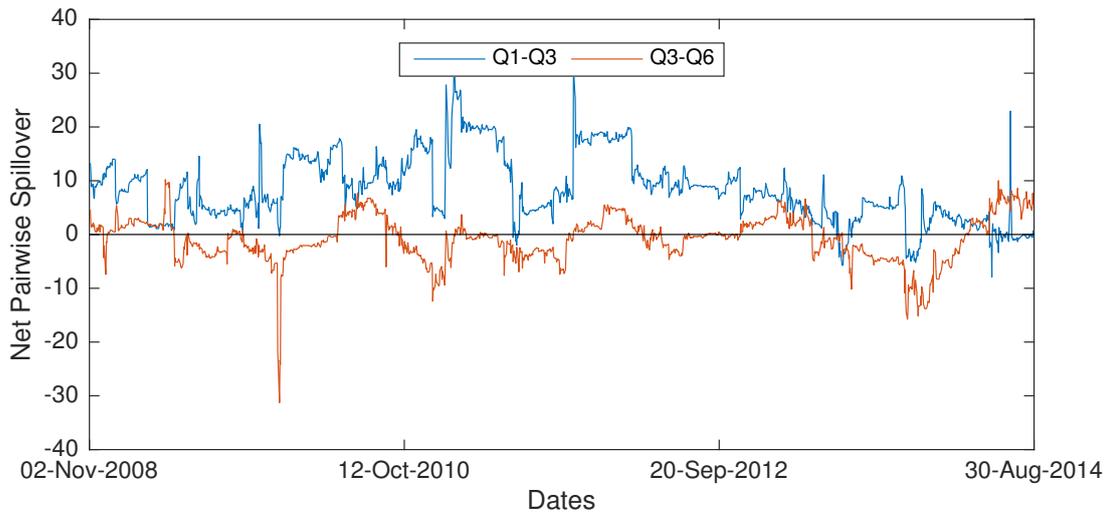
(a) WTI contract



(b) Phelix contract



(c) NEC contract



(d) NSW contract

This figure shows the net pairwise directional volatility spillover measures between two consecutive maturities for each market in a dynamic framework using a rolling window of 90 days. The net pairwise volatility spillover gives information about how much maturity i contributes to the volatility of maturity j (see equation (1.13)).

Figure 1.9 – Dynamic analysis: net pairwise spillovers between maturities, crude oil and electricity markets, 2008-2014

1.C Appendix: Analysis of the indirect storability on the American and the German markets

1.C.1 Complements of the analysis on the PJM market

Table 1.11 sums up the descriptive statistics of the daily volatilities recorded on the closest-to-maturity contracts for each market from 2008 to 2014.

	PJM	Heating Oil	Natural Gas	Coal
# of observations	1 490	1 490	1 490	1 490
Mean	1.676	1.344	2.309	0.96
Median	1.178	0.950	1.759	0.651
Standard-deviation	2.415	1.366	2.18	1.09
Skewness	10.01	2.361	2.76	3.04
Kurtosis	164.62	10.76	18.886	17.69
ADF	-17.93*	-19.05*	-20.06*	-18.82*
LB	1 508*	1 118*	471*	1 364*
Jarque-Bera	1 646 644*	5 123*	17 559*	15 688*

This table sums up the descriptive statistics of the daily volatilities $\sigma_t^k = \left| \ln \left(\frac{F_t^k}{F_{t-1}^k} \right) \right| * 100$ recorded on the closest-to-maturity contracts for the PJM market and its inputs (heating oil, natural gas, and coal) from 2008 to 2014. The "ADF", "LB", and "Jarque-Bera" respectively stand for the test statistics of the Augmented Dickey-Fuller test for unit roots without a lag, the Ljung-Box test for autocorrelation with 15 lags, and the Jarque-Bera test for normality. The associated null hypothesis H_0 is the presence of a unit root for the ADF test, that the data are independently distributed for the LB test, and that the data follow a normal law for the JB test. The star (*) means that we reject the assumption H_0 at the 1% level of confidence.

Table 1.11 – Descriptive statistics of the daily volatilities, PJM and input markets, 2008-2014

1.C.2 Analysis of the Phelix market

In this paragraph we reproduce for the Phelix market the same analysis as for the PJM market regarding the concept of indirect storability. In 2014, on the Phelix market, the coal and the natural gas accounted respectively for 27.8% and 16.11% of the net installed generation capacity according to the Fraunhofer Institute. The analysis uses prices for the one-month Phelix futures contract, the one-month Rotterdam coal futures, and the one-month TTF natural gas futures traded on the ICE. Table 1.12 sums up the descriptive statistics of the daily volatilities recorded on the closest-to-maturity contracts from 2008 to 2014 for each market. The results obtained are the same as before, that is, our series do not contain unit roots, are autocorrelated, and do not follow a normal distribution.

	Phelix	Natural Gas	Coal
# of observations	1 457	1 457	1 457
Mean	1.133	1.598	0.613
Median	0.793	1.051	0.341
Standard-deviation	1.139	1.72	0.816
Skewness	3.40	2.176	3.116
Kurtosis	30.08	9.12	15.88
ADF	-19.60*	-18.11*	-19.64*
LB	234*	1 563*	1 033
Jarque-Bera	47 315*	3 423*	12 430*

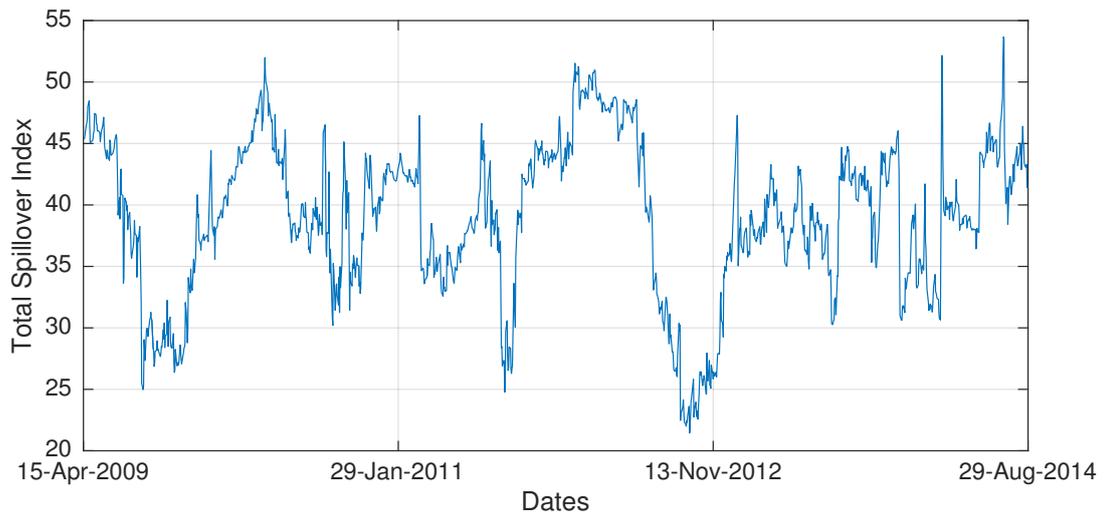
This table sums up the descriptive statistics of the daily volatilities $\sigma_t^k = \left| \ln \left(\frac{F_t^k}{F_{t-1}^k} \right) \right| * 100$ recorded on the closest-to-maturity contracts for the Phelix market and its inputs (natural gas and coal) from 2008 to 2014. The "ADF", "LB" and "Jarque-Bera" respectively stand for the test statistics of the Augmented Dickey-Fuller test for unit roots without a lag, the Ljung-Box test for autocorrelation with 15 lags, and the Jarque-Bera test for normality. The associated null hypothesis H_0 is the presence of a unit root for the ADF test, that the data are independently distributed for the LB test, and that the data follow a normal law for the JB test. The star (*) means that we reject the assumption H_0 at the 1% level of confidence.

Table 1.12 – Descriptive statistics of the daily volatilities, Phelix and input markets, 2008-2014

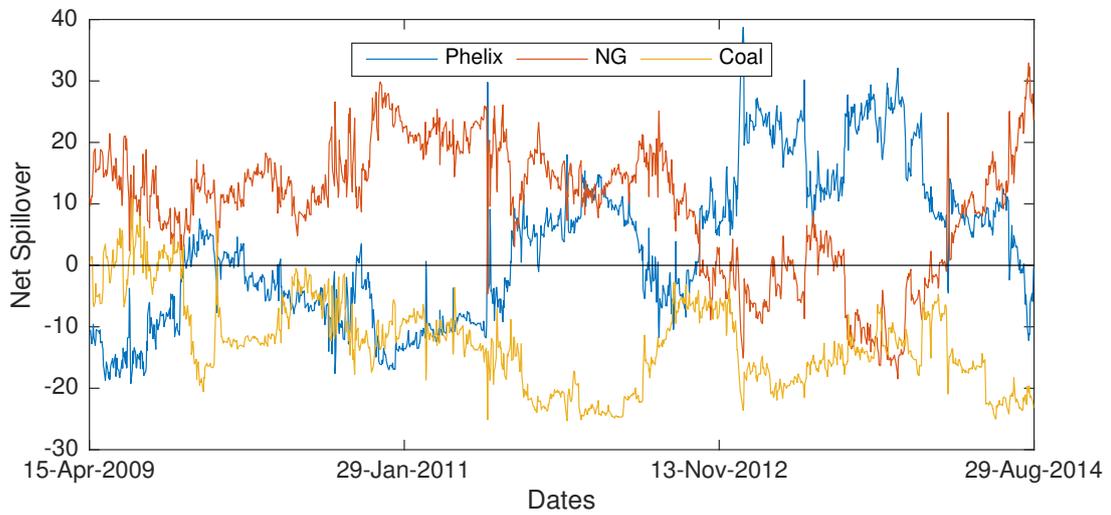
Table 1.13 displays the results of the static analysis on the entire sample. First, the total spillover index is 28.58%, near the one recorded in the analysis for the PJM. This value shows that the volatility on these markets is mainly explained by their own shocks and not by the links between markets. Moreover, the results show that the Phelix and the coal markets receive most of their volatility from natural gas. The Phelix and the coal markets are indeed characterized by a net directional spillover of -1.24 and -8.957 respectively. With a positive net pairwise spillover of 10.197, the natural gas market *delivers* volatility to the two other markets.

A more dynamic analysis performed on the basis of 90-day rolling windows and illustrated by Figure 1.10 and Table 1.14 gives more insights. In net terms, the Phelix and the natural gas markets send volatility respectively 54.61% and 80.75% of the time, while the coal market receives it 93.92% of the time.

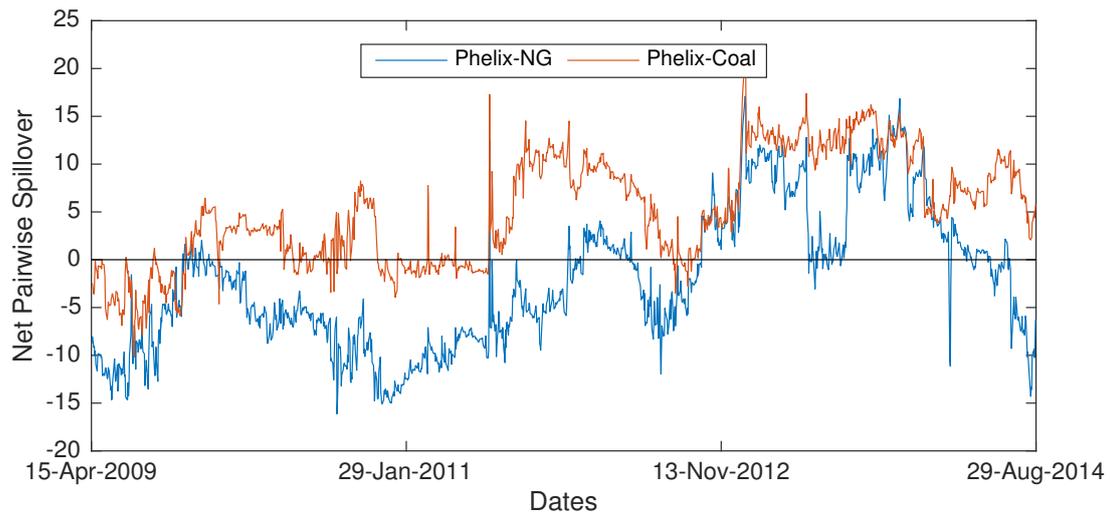
However, as was the case in the static framework, the dynamic pairwise analysis shows that most of the time, the Phelix market sends volatility to the coal markets and receives volatility from the natural gas market.



(a) Indirect Storability in the German market: Total Spillover Index



(b) Indirect Storability in the German market: Net Directional Spillover



(c) Indirect Storability in the German market: Net Pairwise Spillover

This figure displays the total spillover index and directional volatility spillover measures between electricity prices and input prices in the Phelix market and in a dynamic framework with a 90-day rolling window. The first chart represents the total spillover index. The second chart represents for each market the net directional spillovers. The third chart represents the net pairwise spillovers against the Phelix market. The total spillover index measures the contribution of the spillovers of volatility shocks across markets to the total forecast error variance (see equation (1.10)). The directional spillovers "To all (from i)" and "From all (to i)" respectively give information about the volatility spillovers transmitted by market i to all others (see equation (1.12)) and the volatility spillover received by market i from all others (see equation (1.11)). The net directional spillover is the difference between the two and shows if a market contributes more than it receives. The net pairwise volatility spillover gives information about how much market i contributes to the volatility of market j (see equation (1.13)).

Figure 1.10 – Dynamic analysis: volatility spillovers between electricity and its inputs, Phelix and input markets, 2008-2014

# of observations		Total spillover index (%)	Directional to all others	Directional from all others	Net	Net Pairwise against Phelix
	Phelix		7.76	8.999	-1.24	
1 457	Natural gas	28.58	16.45	6.254	10.197	-3.693
	Coal		4.367	13.324	-8.957	2.454

This table shows the total spillover index and the directional volatility spillover measures between electricity prices and natural gas and coal in a static framework. The total spillover index measures the contribution of the spillovers of volatility shocks across markets to the total forecast error variance (see equation (1.10)). The directional spillovers "To all (from i)" and "From all (to i)" respectively give information about the volatility spillovers transmitted by market i to all others (see equation (1.12)) and the volatility spillover received by market i from all others (see equation (1.11)). The net directional spillover is the difference between the two and shows if a market contributes more than it receives. The net pairwise volatility spillover gives information about how much market i contributes to the volatility of market j (see equation (1.13)).

Table 1.13 – Static analysis: volatility spillovers between electricity and its inputs, Phelix and input markets, 2008-2014

# of observations		Total Spillover	Net Directional			Net Pairwise	
		Index	Phelix	NG	Coal	Phelix-NG	Phelix-C
1 457	Min	21.44	-19.22	-18.47	-25.31		
	Max	53.66	38.7	32.93	11.09		
	StDev	6.32	12.08	10.21	7.15		
	⊕(%)		54.61	80.75	6.08	35.72	75.84

This table shows the summary statistics of the total spillover index and the directional volatility spillover measures between the Phelix market and its inputs, in a dynamic framework, on the basis of a 90-day rolling window. In this table, NG represents natural gas and C represents coal. The $\oplus(\%)$ represents the percentage of positive values recorded over the period. The total spillover index measures the contribution of the spillovers of volatility shocks across markets to the total forecast error variance (see equation (1.10)). The directional spillovers "To all (from i)" and "From all (to i)" respectively give information about the volatility spillovers transmitted by market i to all others (see equation (1.12)) and the volatility spillover received by market i from all others (see equation (1.11)). The net directional spillover is the difference between the two and shows if a market contributes more than it receives. The net pairwise volatility spillover gives information about how much market i contributes to the volatility of market j (see equation (1.13)).

Table 1.14 – Dynamic analysis: volatility spillovers between electricity and its inputs, Phelix and input markets, 2008-2014

1.D Appendix: Empirical literature on the Samuelson effect

This appendix contains a table that sums up the most important empirical literature on the Samuelson effect. This table focuses on large scale studies and puts emphasis on studies on energy products. Once we were sure that this would not change the overall picture, in order to enhance the readability of the table, we have voluntarily omitted certain articles. The work of Khoury and Yourougou [1993] for example, is not mentioned; it is indeed a study devoted to agricultural markets; more importantly, the results are in line with the other studies on agricultural products displayed in the table. The same is true for the article by Grammatikos and Saunders [1986] on exchange rates. We have also withdrawn the articles that rely on the existence of a Samuelson effect, but are not devoted to empirical tests on this effect. Among these articles are very famous works on term structure models, like Schwartz [1997], Routledge et al. [2000] and Kogan, Livdan, and Yaron [2009]. All were published in the *Journal of Finance*. Their empirical results are in line with those exhibited in this table.

The table is divided into four subtables: energy products, metals, agricultural products and financial assets. It provides, for each underlying asset: the names of the authors, the year of publication, the length of the study in years, the frequency of the data, the category of tests used to examine the possible existence of a Samuelson effect, the conclusion regarding this existence and, when available, the economic explanation of the Samuelson effect tested.

	Authors [Year]	Length (years)	Freq.	Samuelson Effect		
				Tests	Existence	Origin
Am. Crude Oil	Duong and Kalev [2008]	3	Intraday	OLS, NPT	No	B*
	Bessembinder et al. [1996]	8	Daily	OLS	Yes	B*
	Brooks [2012]	19	Daily	OLS	Yes	
	Lautier and Raynaud [2011]	11	Daily	NPT	Yes	
	Daal et al. [2006]	17	Daily	OLS	Yes	AD*
	Galloway and Kolb [1996]	9	Daily	OLS, PT	Yes	
Brent crude	Brooks [2012]	11	Daily	OLS	Yes	
	Lautier and Raynaud [2011]	9	Daily	NPT	Yes	
Gasoil	Lautier and Raynaud [2011]	9	Daily	NPT	Yes	
Heating oil	Brooks [2012]	19	Daily	OLS	Yes	
	Lautier and Raynaud [2011]	11	Daily	NPT	Yes	
	Daal et al. [2006]	21	Daily	OLS	Yes	AD*
	Galloway and Kolb [1996]	23	Daily	OLS, PT	Yes	
Nat gas (us)	Brooks [2012]	19	Daily	OLS	Yes	
	Lautier and Raynaud [2011]	11	Daily	NPT	Yes	
Nat gas (eu)	Lautier and Raynaud [2011]	12	Daily	NPT	Yes	
Electricity	Walls [1999]	0.5	Daily	OLS	Yes	
	Allen and Cruickshank [2002]	2	Daily	OLS, GAR	Yes	AD

This subtable sums up, for energy products, the main existing empirical literature on the Samuelson effect. It provides: the names of the authors, the year of publication of the article, the length of the study period (in years), the frequency of the data in the column "Freq", the type of tests retained to examine the presence of the Samuelson effect, the result of these tests and, when available, the economic explanation of the Samuelson effect tested. In the column "Test", "OLS" stands for Ordinary Least Squares and refers to a linear regression, "PT" stands for Parametric Test, "NPT" stands for Non-Parametric Test and "GAR" for GARCH modeling. The column "Origin" stands for the main economic explanation given as the source of the Samuelson effect, when available. It displays "B" if the theory of [Bessembinder et al., 1996] is found as the main explanation of the Samuelson effect, or "AD" if it is the theory of information flows by Anderson and Danthine [1983]. A star (★) next to the "B" or "AD" means that both theories have been tested by the authors, but that the one indicated has been found to be more important.

(a) Energy products

	Authors [Year]	Length (years)	Freq.	Samuelson effect		
				Test	Existence	Origin
Wheat (CBOT)	Anderson [1985]	14	Daily	OLS, NPT	Yes (weak)	
	Milonas [1986]	11	Daily	OLS	Yes	
	Bessembinder et al. [1996]	9	Daily	OLS	Yes	B*
	Brooks [2012]	19	Daily	OLS	Yes	
	Lautier and Raynaud [2011]	11	Daily	NPT	Yes	
	Daal et al. [2006]	40	Daily	OLS	Yes (weak)	AD*
	Galloway and Kolb [1996]	23	Daily	OLS, PT	Yes	
Oats	Anderson [1985]	14	Daily	OLS, NPT	Yes	
	Brooks [2012]	19	Daily	OLS	Yes	
	Daal et al. [2006]	40	Daily	OLS	Yes (weak)	AD*
	Galloway and Kolb [1996]	18	Daily	OLS, PT	No	
Corn	Anderson [1985]	14	Daily	OLS, NPT	No	
	Milonas [1986]	11	Daily	OLS	No	
	Duong and Kalev [2008]	7	Intraday	OLS, NPT	Yes	B*
	Brooks [2012]	19	Daily	OLS	Yes	
	Daal et al. [2006]	40	Daily	OLS	Yes (weak)	AD*
	Galloway and Kolb [1996]	23	Daily	OLS, PT	No	
Soybean (all types)	Anderson [1985]	14	Daily	OLS, NPT	Yes	
	Milonas [1986]	11	Daily	OLS	Yes	
	Duong and Kalev [2008]	7	Intraday	OLS, NPT	Yes	B*
	Brooks [2012]	19	Daily	OLS	Yes	
	Lautier and Raynaud [2011]	11	Daily	NPT	Yes	
	Daal et al. [2006]	40	Daily	OLS	Yes (weak)	AD*
	Galloway and Kolb [1996]	23	Daily	OLS, PT	Yes	
	Rutledge [1976]		Daily	OLS, PT	No	
World Sugar	Bessembinder et al. [1996]	9	Daily	OLS	Yes	B*
	Brooks [2012]	19	Daily	OLS	Yes	
Orange juice	Bessembinder et al. [1996]	9	Daily	OLS	Yes	B*
	Brooks [2012]	19	Daily	OLS	Yes	
	Daal et al. [2006]	33	Daily	OLS	Yes (weak)	AD*
	Galloway and Kolb [1996]	19	Daily	OLS, PT	Yes	
Cocoa	Anderson [1985]	14	Daily	OLS, NPT	Yes	
	Brooks [2012]	19	Daily	OLS	Yes	
	Daal et al. [2006]	40	Daily	OLS	Yes (weak)	AD*
	Galloway and Kolb [1996]	12	Daily	OLS, PT	Yes	
	Rutledge [1976]		Daily	OLS, PT	Yes	
Leans hogs	Duong and Kalev [2008]	3	Intraday	OLS, NPT	Yes	B*
	Brooks [2012]	19	Daily	OLS	Yes	
	Daal et al. [2006]	30	Daily	OLS	Yes (weak)	AD*
	Galloway and Kolb [1996]	22	Daily	OLS, PT	Yes	
Live cattle	Anderson [1985]	14	Daily	OLS, NPT	Yes	
	Duong and Kalev [2008]	7	Intraday	OLS, NPT	Yes	B*
	Bessembinder et al. [1996]	9	Daily	OLS	Yes	B*
	Brooks [2012]	19	Daily	OLS	Yes	
	Daal et al. [2006]	35	Daily	OLS	Yes (weak)	AD*
	Galloway and Kolb [1996]	16	Daily	OLS, PT	Yes	

This subtable sums up, for agricultural products, the main existing empirical literature on the Samuelson effect. It provides: the names of the authors, the year of publication of the article, the length of the study period (in years), the frequency of the data in the column "Freq", the type of tests retained to examine the presence of the Samuelson effect, the result of these tests and, when available, the economic explanation of the Samuelson effect tested. In the column "Test", "OLS" stands for Ordinary Least Squares and refers to a linear regression, "PT" stands for Parametric Test, "NPT" stands for Non-Parametric Test. The column "Origin", when available, stands for the main economic explanation given as the source of the Samuelson effect. It displays "B" if the theory of [Bessembinder et al., 1996] is found as the main explanation of the Samuelson effect, or "AD" if it is the theory of information flows by Anderson and Danthine [1983]. A star (★) next to the "B" or "AD" means that both theories have been tested by the authors, but that the one indicated has been found to be more important.

	Authors [Year]	Length (years)	Freq.	Samuelson effect		
				Test	Existence	Origin
Gold	Milonas [1986]	8	Daily	OLS	Yes (weak)	
	Duong and Kalev [2008]	4	Intraday	OLS, NPT	No	B*
	Bessembinder et al. [1996]	9	Daily	OLS	Yes (weak)	B*
	Brooks [2012]	19	Daily	OLS	Yes (weak)	
	Lautier and Raynaud [2011]	11	Daily	NPT	Yes (weak)	
	Daal et al. [2006]	25	Daily	OLS	No	AD*
	Galloway and Kolb [1996]	17	Daily	OLS, PT	No	
Silver	Anderson [1985]	14	Daily	OLS, NPT	No	
	Milonas [1986]	8	Daily	OLS	Yes (weak)	
	Bessembinder et al. [1996]	9	Daily	OLS	Yes (weak)	B*
	Brooks [2012]	19	Daily	OLS	Yes (weak)	
	Daal et al. [2006]	36	Daily	OLS	No	AD*
	Galloway and Kolb [1996]	19	Daily	OLS, PT	No	
	Rutledge [1976]		Daily	OLS, PT	Yes	
Platinum	Bessembinder et al. [1996]	9	Daily	OLS	Yes (weak)	B*
	Brooks [2012]	19	Daily	OLS	Yes (weak)	
	Galloway and Kolb [1996]	19	Daily	OLS, PT	No	
Copper	Milonas [1986]	8	Daily	OLS	Yes	
	Brooks [2012]	19	Daily	OLS	Yes (weak)	
	Daal et al. [2006]	40	Daily	OLS	Yes (weak)	AD*
	Galloway and Kolb [1996]	20	Daily	OLS, PT	Yes	
Palladium	Brooks [2012]	19	Daily	OLS	Yes (weak)	
	Daal et al. [2006]	23	Daily	OLS	No	AD*
	Galloway and Kolb [1996]	9	Daily	OLS, PT	No	

This subtable sums up, for metals, the main existing empirical literature on the Samuelson effect. It provides: the names of the authors, the year of publication of the article, the length of the study period (in years), the frequency of the data in the column "Freq", the type of tests retained to examine the presence of the Samuelson effect, the result of these tests and, when available, the economic explanation of the Samuelson effect tested. In the column "Test", "OLS" stands for Ordinary Least Squares and refers to a linear regression, "PT" stands for Parametric Test, "NPT" stands for Non-Parametric Test. The column "Origin" stands for the main economic explanation given as the source of the Samuelson effect, when available. It displays "B" if the theory of [Bessembinder et al., 1996] is found as the main explanation of the Samuelson effect, or "AD" if it is the theory of information flows by Anderson and Danthine [1983]. A star (★) next to the "B" or "AD" means that both theories have been tested by the authors, but that the one indicated has been found to be more important.

(c) Metals

	Authors [Year]	Length (years)	Freq.	Samuelson effect		
				Test	Existence	Origin
S&P 500	Duong and Kalev [2008]	7	Intraday	OLS, NPT	No	B*
	Bessembinder et al. [1996]	9	Daily	OLS	No	B*
	Brooks [2012]	19	Daily	OLS	No	
	Daal et al. [2006]	18	Daily	OLS	No	AD*
	Galloway and Kolb [1996]	20	Daily	OLS, PT	No	
Nikkei 225	Brooks [2012]	19	Daily	OLS	No	
	Daal et al. [2006]	10	Daily	OLS	No	AD*
10 year T-Notes	Duong and Kalev [2008]	2	Intraday	OLS, NPT	No	B*
	Brooks [2012]	19	Daily	OLS	No	
	Daal et al. [2006]	18	Daily	OLS	No	AD*
Eurodollar	Duong and Kalev [2008]	7	Intraday	OLS, NPT	No	B*
	Brooks [2012]	19	Daily	OLS	No	
	Lautier and Raynaud [2011]	12	Daily	NPT	No	
	Daal et al. [2006]	18	Daily	OLS	No	AD*
	Galloway and Kolb [1996]	20	Daily	OLS, PT	No	

This subtable sums up, for financial assets, the main existing empirical literature on the Samuelson effect. It provides: the names of the authors, the year of publication of the article, the length of the study period (in years), the frequency of the data in the column "Freq", the type of tests retained to examine the presence of the Samuelson effect, the result of these tests and, when available, the economic explanation of the Samuelson effect tested. In the column "Test", "OLS" stands for Ordinary Least Squares and refers to a linear regression, "PT" stands for Parametric Test, "NPT" stands for Non-Parametric Test. The column "Origin", when available, stands for the main economic explanation given as the source of the Samuelson effect. It displays "B" if the theory of [Bessembinder et al., 1996] is found as the main explanation of the Samuelson effect, or "AD" if it is the theory of information flows by Anderson and Danthine [1983]. A star (★) next to the "B" or "AD" means that both theories have been tested by the authors, but that the one indicated that the one indicated has been found to be more important.

(d) Financial assets

Table 1.15 – Empirical literature on the Samuelson effect: a synthesis

1.E Appendix: *Negative covariance hypothesis versus state variable hypothesis*

This appendix exhibits the results of the tests related to the *negative covariance hypothesis* proposed by Bessembinder et al. [1996] and to the *state variable hypothesis* proposed by Anderson and Danthine [1983]. We performed these tests on our data.

Table 1.16 reproduces the results of a regression that examines the *negative covariance hypothesis* on the four electricity derivative markets (German, American, Australian and Nordic) and on the American crude oil market. The regression is the following:

$$\Delta c_t = \alpha + \beta \Delta S_t + \varepsilon_t \quad (1.14)$$

with $c_t = \frac{\ln(F_t^m/F_t^n)}{m-n}$ the temporal basis between two maturities and $F_t^n = S_t$ the proxy for the spot price. We retained the nearby maturity in the regression. Overall, Table 1.16 shows that the *negative covariance hypothesis* holds (the β coefficients are significantly negative) on all markets and for all maturities.

Nearby		WTI	Phelix	PJM	NEC	NSW
2	β	-161.53	-696.36	-439.98	-476.93	-342.25
	(p-value)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
3	β	-49.49	-771.07	-585.76	-403.55	-533.57
	(p-value)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
4	β	-37.71	-913.53	-574.21	-392.79	-536.71
	(p-value)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
5	β	-33.26	-974.15	-581.7		-577.08
	(p-value)	(0.00)	(0.00)	(0.00)		(0.00)
6	β	-28.58		-688.84		-538.6
	(p-value)	(0.00)		(0.00)		(0.00)
	# of obs	1 517	1 456	1 489	905	1 562

This table provides the results (coefficients and p-values) obtained when testing the *negative covariance hypothesis* on the four electricity derivative markets (German, American, Australian and Nordic) and on the American crude oil market with the following regression: $\Delta c_t = \alpha + \beta \Delta S_t + \varepsilon_t$, where c_t is the temporal basis between two maturities. "Nearby 2" corresponds to the regression between the two first nearest maturities, "Nearby 3" to the regression between the second and the third nearest maturities, etc. For exhibition reasons, prices have been divided by their own mean and the dependent variable has been multiplied by 25,000.

Table 1.16 – Test of the *negative covariance hypothesis*, crude oil and electricity markets, 2008-2014

Table 1.17 reproduces the results of two regressions that test the relative importance of uncertainty in explaining the Samuelson effect on the four electricity derivative markets (German, American, Australian and Nordic) and on the American crude oil market. The regressions, as in Bessembinder et al. [1996], are the following. The first one tests for the

existence of a Samuelson effect:

$$\sigma_i^k = \alpha + \beta_1 \text{TTM}_i^k + \varepsilon_i \quad (1.15)$$

$\forall k > 1$. The second controls for the effects of uncertainty, measured by the volatility of the spot price:

$$\sigma_i^k = \alpha + \beta_1 \text{TTM}_i^k + \beta_2 \sigma_i^1 + \varepsilon_i \quad (1.16)$$

$\forall k > 1$ with $\sigma_i^k = \left| \ln \left(\frac{F_i^k}{F_{i-1}^k} \right) \right| * 100$ the daily volatility of the futures prices with maturity k , σ_i^1 the daily volatility of the first nearby futures price and TTM_i^k the number of days until the expiration of the futures contract.

Except for the Australian market, short-term uncertainty matters for the Samuelson effect (statistically significant positive β_2 coefficients and better R^2). However the time-to-maturity effect remains statistically significant (statistically significant negative β_1 coefficients).

	WTI		Phelix		PJM		NEC		NSW	
	(i)	(ii)	(i)	(ii)	(i)	(ii)	(i)	(ii)	(i)	(ii)
α	1.564	0.253	0.908	0.346	1.313	0.968	1.647	0.739	0.527	0.466
(p-value)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
β_1	-0.002	-0.0017	-0.0014	-0.0006	-0.003	-0.0027	-0.0044	-0.0045	0	0
(p-value)	(0.00)	(0.00)	(0.00)	(0.10)	(0.00)	(0.00)	(0.00)	(0.00)	(0.97)	(0.4)
β_2		0.803		0.452		0.186		0.457		0.073
(p-value)		(0.00)		(0.00)		(0.00)		(0.00)		(0.00)
R^2	0.002	0.867	0.002	0.372	0.01	0.238	0.005	0.513	0	0.054
# of obs	7 590		5 828		7 450		2 718		7 815	

This table provides the results obtained when testing the impact of the short-term uncertainty on the Samuelson effect on the four electricity derivative markets (German, American, Australian and Nordic) and on the American crude oil market. The model (i) exhibits the coefficients of a linear regression between the daily volatilities σ_i^k of the futures prices with maturity k and the time-to-maturity TTM_i^k , ie the number of days between i and the maturity k of the contract: $\sigma_i^k = \alpha + \beta_1 \text{TTM}_i^k + \varepsilon_i, \forall k > 1$. The model (ii) exhibits the coefficients of a linear regression between the daily volatilities of the futures prices with maturity k , the number of days TTM_i^k between i and the maturity k of the contract and the volatility of the spot price: $\sigma_i^k = \alpha + \beta_1 \text{TTM}_i^k + \beta_2 \sigma_i^1 + \varepsilon_i, \forall k > 1$. These regressions use all the available maturities as dependent variables, except the first one. The volatility of the front month contract serves as a proxy for the volatility of the spot price. The p-values of the coefficients (in brackets) and the coefficients of determination R^2 are also displayed.

Table 1.17 – Test of the *state variable hypothesis*, crude oil and electricity markets, 2008-2014

Chapter 2

Equilibrium relations between the spot and futures markets for commodities: an infinite horizon model ¹

Abstract

We give new insights into the theory of the dynamic behavior of commodity prices with an infinite horizon rational expectations equilibrium model for spot and futures commodity prices. Numerical simulations of the model emphasize the heterogeneity that exists in the behavior of commodity prices by showing the link between the physical characteristics of a market and some stylized facts of commodity futures prices. They show the impact of storage costs on both the variability of the basis and on the Samuelson effect. Finally, the simulations of the model show that an increase in the speculative activity on commodity futures markets has an overall positive effect on risk premia. However, not all of the agents benefit from it.

1. This chapter is based on an article written with Ivar Ekeland, Delphine Lautier and Bertrand Villeneuve. It has been presented during two international conferences (the 33rd AFFI Conference and the 39th IAEE International Conference) and multiple research seminars or workshops.

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2.1 Introduction

We develop a general stationary model for spot and futures commodity prices. This infinite horizon rational expectations equilibrium model features: i) heterogeneous agents (processors, storers and speculators) with relevant economic functions and more importantly, naturally opposite positions in the futures market; ii) simultaneous equilibrium on the spot and the futures markets. Because we explicitly model the behavior of the agents on both markets, the prices and the risk premium are endogenous. They come from the clearing of the markets at the equilibrium; iii) non-negativity constraint on inventories. This is important since it creates a nonlinearity in the model and the prices; iv) random production at each period. Compared to Scheinkman and Schechtman [1983], we do not allow supply response in our model. Therefore, the available quantity on the spot market at time t is \bar{z}_t . It is the sum of the random production \tilde{w}_t and the quantity left after the settlement of the physical contracts from the preceding period. This quantity creates the recursive structure of the model. Our model is general: it is not specified to match one particular commodity but can describe a wide range of commodities depending on the parameters we use. Moreover, this model encompasses the risk-neutral case and the case where there exists no futures market.

We numerically solve the model and perform simulations to study different issues. First, we extend the existing theoretical and empirical literature regarding two stylized facts of commodity markets: the asymmetric behavior of the basis (the difference between contemporaneous spot and futures prices) in backwardation and in contango, and the decreasing pattern of volatilities along the term structure of futures prices (the Samuelson effect) for energy and agricultural commodities. We show that: i) the asymmetric behavior of the basis is impacted by the level of storage costs because the volatility of the basis in contango is a positive function of the storage costs, whereas in backwardation it is independent. Then, for some level of storage costs, the variability of the basis can be the same in the two situations; ii) the Samuelson effect is stronger in markets for commodities with storage constraints (high cost of storage or few storage capacities); and iii) the actual level of inventories predicts the steepness of the term structure of volatilities which can be flat for some important level of inventories. In other words, violations of the Samuelson effect can exist.

Then, we emphasize the existing heterogeneity between different commodity markets depending on their own physical characteristics. We use the general structure of the model to simulate specific markets by looking at the characteristics of the gold market (high level of inventories) and the electricity market (non-storability).

Finally, we perform an analysis of the impact of the speculative activity for different situations of the market depending on the current scarcity and show that the impact of the speculation is not always the same. First, regardless of the current scarcity of the market an increase in the speculative activity decreases the risk premium and increases the hedg-

ing pressure. On the contrary, the spot price, the expected spot price and the futures price react differently to an increase in the speculative activity according to the level of current scarcity. These different effects can be explained by the fact that in each market situation, the marginal agent in the market, or the agent who influences the most the price formation process is different. In other words, an increase in the speculative activity always benefits to the economy, but not always to the same agent.

These results are important for a variety of agents, from traditional participants in commodity markets to regulators. First, even though some general stylized facts exist, our model emphasizes that the physical characteristics of each commodity market imply different dynamic behavior of prices, and therefore that there exist not one big commodity market but different heterogeneous commodity markets. This conclusion is very important for: i) industrial agents (e.g. producers, storers, processors), because they use these markets to hedge their physical exposure to commodity prices and want to minimize their hedging cost. To do so, they need to have a perfect knowledge of the economic mechanism that drives the prices and the risk premium in their specific market; ii) specialized speculators or market makers because they take position in commodity markets in order to earn a risk premium. As for the industrial agents, they must have a perfect knowledge of each specific commodity market in which they intend to participate in order to be able to maximize their profit from the trading activity. This is even more important, because as emphasized before, it seems that overall, such speculators benefit to the economy; and iii) regulators because they need to be able to understand each commodity market independently in order to adopt the best specific regulation market by market, or at least to understand how a new regulation will impact the different participants commodity market by commodity market.

Second, our model shows precisely the impact of having more specialized speculators. This conclusion has mixed implications for the hedgers depending on their side of the market. Indeed, there exists a risk premium in commodity markets due to the imbalance in the hedging needs of industrial agents. This risk premium is marginally determined by the most important category of agents who will have to pay this hedging cost, whereas the other group of hedgers, the smaller one, benefits from the risk premium. Then, an increase in the speculative activity means a decrease in the hedging cost for the former group of hedgers and an increase in the hedging cost for the latter. Therefore, both groups have opposite interests. For the speculators, an increase in the speculative activity means an increase in the competition and then a decrease in the individual profit of each speculator. Then, these agents may be reluctant to the entry of new competitors. Finally, the implication for the regulators is that having more dedicated speculators does not seem to destabilize commodity markets and therefore the first goal of the regulation should not be to limit their activity. Be careful that this conclusion is valid in this context for speculators who do not engage in cross-asset trading and do not have time varying risk-aversion.

The article is organized as follows: Section 2.2 reviews the relevant literature. Section 2.3 describes the economic framework of the model. The optimality conditions and the equilibrium analysis are developed in Section 2.4. Section 2.5 is devoted to the study of the stylized facts, Section 2.6 to the analysis of speculation.

2.2 Literature review

Our article is linked to different strands of the economic literature regarding commodity markets. First, in term of modeling, our model is at the cross-road of two literatures: one regarding traditional competitive storage models (Williams and Wright [1991], Deaton and Laroque [1992], Routledge, Seppi, and Spatt [2000]), and one based on models with heterogeneous agents (Hirshleifer [1989], Vercaemmen and Doroudian [2014] and Baker [2016]).

The standard rational expectations competitive storage model described by Williams and Wright [1991] is the reference framework for the study of the dynamic behavior of commodity prices. It is an infinite horizon model, where homogeneous risk-neutral competitive storers optimize their expected utility by choosing an optimal nonlinear storage behavior (this nonlinearity in the storage behavior is a feature of our model as well). The only asset is the physical commodity (storable) traded on a spot market. The futures market is not modeled and futures contracts are not traded. In term of methodology, the authors emphasize that because of the non-negativity constraint on the inventories and the dynamic rational expectations equilibrium the stochastic dynamic programming principle, or other recursive methods must be used. Moreover, as it is the case in our paper, they show that most of the time analytical solutions do not exist and numerical solutions are needed. Deaton and Laroque [1992] apply the traditional framework of Williams and Wright [1991] to real spot data and emphasize that the storage behavior is a critical determinant of the price formation process. Therefore, any commodity model should take care of it. The model by Routledge et al. [2000] is the first on this tradition of rational expectations competitive storage model to include a futures market in the analysis. Nevertheless, because the model features only homogeneous risk-neutral agents, the futures market remains an adhoc construction and does not allow to study futures prices bias for example.

Hirshleifer [1989], in his finite horizon model of storage (dynamic programming principle up to maturity T), is the first to model an active futures market. Compared to the previous models, he proposes an explicit modeling of the behavior of the heterogeneous risk-averse traders on this market. By solving for the joint equilibrium in the spot and the futures markets his model allows to study precisely the determinants of the risk premium. This model is fundamental for our paper because it emphasizes that storage and hedging decisions need to be study in a dynamic framework, and because we adopt a trading structure between dif-

ferent heterogeneous agents on the futures market which is close to the one used by the author. The papers by Vercammen and Doroudian [2014] and Baker [2016] are close to our. With different modelings they succeed to gather infinite horizon storage model with heterogeneous risk-averse agents and an active futures market. First, Vercammen and Doroudian [2014] have extended the paper of Routledge et al. [2000] by adding cross-asset risk-averse speculators in the analysis. Even if the modeling is not the same, this model could be the closest to our model in term of reasoning with simulations based on calibrated parameters. Then, as far as the modeling is concerned we are closer to Baker [2016], but we do not devote our paper to the study of the financialization.

So far we have focused on dynamic models, but since our model is the dynamic and stationary extension of the paper by Ekeland, Lautier, and Villeneuve [2016], it is indirectly linked to most of the literature cited in this paper about the original static framework. This literature remains important since it initiated the theoretical work on commodity markets.

Second, in term of analysis, our paper is linked to the following literatures regarding commodity markets. First, because we give results regarding important stylized facts of commodity markets, we contribute to the empirical literature on the behavior of commodity prices. We first assess the question of the Samuelson effect on commodity markets. That is the fact that there exists a decreasing pattern of volatilities along the term structure of prices. While, initially the storability of the commodity was described as a necessary condition (Bessembinder, Coughenour, Seguin, and Smoller [1996]) for this feature to hold², the debate is now more concentrated on whether an excess of storage can induce a violation of the maturity impact as stated by Fama and French [1988] and Routledge et al. [2000]. Then, regarding the volatility of the basis, it is often assumed that due to the presence of inventories the basis in contango is stable and bounded, whereas in backwardation it is volatile and theoretically unbounded. Nevertheless, as already emphasized by Fama and French [1987] in an empirical study on a wide range of commodities, there exists a more complicated link between the volatility of the basis and the storage cost. Our model allows us discuss these issues. Moreover, we contribute to the literature regarding the classification of commodity markets which most of the time have been tackle through large scale empirical studies, except for Ekeland et al. [2016]. For instance, the descriptive statistics in the paper of Kang, Rouwenhorst, and Tang [2014] which is focused on the liquidity provision on futures markets give a good overview of the basis and the risk premia for 26 commodities.

Second, this article also contributes to the growing literature on the financialization of commodity markets. There is an important imbalance between the empirical and the theoretical literature on the financialization. On the empirical side, there is a significant and still growing literature on different issues link to the financialization for a large variety of com-

2. This view has been challenged by Jaeck and Lautier [2016] with an empirical study of the Samuelson effect on electricity markets.

modities. Brunetti and Buyuksahin [2009], Buyuksahin and Harris [2011], Singleton [2013], Hamilton and Wu [2015] are some references with mixed results for the direct link between Commodity Index Traders (CITs) and prices. Tang and Xiong [2012] show that the correlations between different commodities have increased after 2004. Buyuksahin and Robe [2011, 2014] show that the cross-asset correlation has increased after 2008 and link this to the trading of hedge funds. Hamilton and Wu [2014] have shown that the risk premium in oil futures markets has significantly decreased due to the potential hedging pressure from financial investors. On the other hand, the theoretical side of the literature is still scarce. One paper is the one of Basak and Pavlova [2016] to assess the impact of institutional investors (and a benchmarked investment) on commodity markets. Baker [2016] tries to assess the impact of the entry of households on the futures markets by calibrating his model to the crude oil market to assess the "true" parameters to use. Finally Boons, De Roon, and Szymanowska [2014] study the impact of hedging by investors of their commodity risk, on commodity returns. While those three papers study the impact of the introduction of a new agent on the commodity markets, in our model, we answer a different question which is, what if the existing agent (speculator) becomes more and more important. For a more detailed review see Cheng and Xiong [2014].

2.3 Economic setting

The time and the markets: It is an infinite horizon model. The interest rate r and the associated discounting factor Δ (with $\Delta = (1 + r)^{-1}$) are assumed to be positive and constant. At each period $t \geq 0$ two markets are open: i) the spot market for the trading of the physical commodity with immediate delivery. As it is usual for spot markets, the clearing implies the equality between the total supply (production and storage) and the total demand (for storage and consumption) of the commodity. The demand for consumption is assumed to be linear. The spot prices on this market are assumed to be bounded: there is a price level p_{min} below which the remaining quantity of the commodity is destroyed. This could figure the case where the limits of the storage capacities are reached. There is also a price level p_{max} above which a substitute of the commodity is used; ii) the futures market for the trading of derivatives contracts written on the physical commodity.

The heterogeneous agents: There are four categories of agents. The first one, the consumer, is not explicitly modeled and is represented by a linear demand function. The others are three types of risk-averse agents (storer, processor and speculator), each one representing a competitive industry. We associate to each one of those agents a quantity $n_i \forall i = I, P, S$ which can be interpreted either as the number of agents, or as the elasticity of this type of agent. Those three agents want to maximize their expected utility in a mean-variance frame-

work, but they have different goals (objective functions) and can have different sets of positions (spot and futures markets or only futures market). Their names represent our vision of the market and can be changed. What matters the most is their economic functions and the associated futures demands for hedging and speculation. The characteristics of these agents are: i) The *storer* maximizes his profit from carrying over the commodity from periods $t - 1$ to t , with a quadratic storage cost. He has to choose both his optimal level x^* of inventories and his optimal position q_f^* on the futures contract. He is not allowed to carry over a negative amount of the commodity. He has access to both the spot market, where he buys at $t - 1$ and sells at t , and the futures market, where his natural hedging position is a short one, and where he initiates a position at $t - 1$ and compensates it at t ; ii) The *processor* maximizes his profit from transforming the commodity into a final good (sold at a fixed price), using a costly production process. He has to choose at $t - 1$ the optimal level of his input y^* for a processing activity that will be performed at t . This input will be bought on the spot market at t . He also has to choose his optimal position q_p^* on the futures contract, where he has a natural long hedging position and where he initiates the position at $t - 1$ and compensates it at t ; iii) The *speculator* maximizes his profit from trading on the futures market only. He has to choose the optimal position q_s^* that he will hold between $t - 1$ and t . He does not have access to any other physical or financial market.

The information structure and the uncertainty: At each period t the agents know the quantity \tilde{z}_t available on the physical market. This quantity depends first on past choices of the agents: the storage and processing decisions. Because it creates the recursive structure of the model, this component of the available quantity is a fundamental notion, which can also be interpreted as the hedging pressure on the futures market. Naturally, \tilde{z}_t also depends on the random production or harvest $\tilde{\omega}_t$ of the period. The $\tilde{\omega}_t$ are assumed to be independent, with a known distribution (we assume that it is Gaussian). Based on this information and relying on the expectation and variance of future spot prices, the agents make new choices on the physical market.

2.4 Equilibrium analysis

In this section we show how to derive the equilibrium equations of the model. Unfortunately, there exists no closed form solutions for the equilibrium, so we will prove both its existence and its uniqueness (under progress) and then we will rely on numerical procedures to find it.

2.4.1 Optimal positions

The first step to derive the equilibrium equations in our model is to find the optimal positions in the spot and the futures markets of each one of the agent. Because the agents are assumed to be risk-averse and because we use a Gaussian random variable, we retain a mean-variance framework. We thus need to maximize for any agent i :

$$E_t[\tilde{\pi}_i] - \frac{\alpha_i}{2} \text{Var}_t[\tilde{\pi}_i]$$

where $\tilde{\pi}_i$ is the profit and α_i the risk aversion of agent i . $E_t[\cdot]$ and $\text{Var}_t[\cdot]$ are respectively the expectation and the variance at t .

The speculator: his profit $\tilde{\pi}_S$ can be written

$$\tilde{\pi}_S(q_{S,t}) = q_{S,t} \Delta (\tilde{p}_{t+1} - f_t)$$

where $q_{S,t}$ is the number of futures contracts bought ($q_{S,t} > 0$) or sold ($q_{S,t} < 0$) at date t , p_t is the spot price at date t and f_t is the futures price at t . Maximizing the expected utility according to the quantities hold on the futures market $q_{S,t}$ gives:

$$q_{S,t}^* = \frac{1}{\Delta} \frac{E_t[\tilde{p}_{t+1}] - f_t}{\alpha_S \text{Var}_t[\tilde{p}_{t+1}]} \quad (2.1)$$

The storer: we assume that the storage activity is characterized by quadratic costs ($\frac{\gamma}{2} x^2$), so that the profit of the storer is written:

$$\tilde{\pi}_I(q_{I,t}, x_t) = q_{I,t} \Delta (\tilde{p}_{t+1} - f_t) + x_t (\Delta \tilde{p}_{t+1} - p_t) - \frac{\gamma}{2} x_t^2$$

Maximizing the expected utility according to the quantities hold on the futures market $q_{I,t}$ and on the physical market x_t gives:

$$q_{I,t}^* = \frac{1}{\Delta} \frac{E_t[\tilde{p}_{t+1}] - f_t}{\alpha_I \text{Var}_t[\tilde{p}_{t+1}]} - x_t^* \quad (2.2)$$

$$x_t^* = \frac{1}{\gamma} \max\{\Delta f_t - p_t, 0\} \quad (2.3)$$

The processor: we denote by Q the fixed price of the output, we retain the idea of quadratic transforming costs ($\frac{\beta}{2} y^2$). We also assume that the processing activity is instantaneous. The profit of the processor becomes:

$$\tilde{\pi}_P(q_{P,t}, y_t) = q_{P,t} \Delta (\tilde{p}_{t+1} - f_t) + y_t \Delta (Q - \tilde{p}_{t+1}) - \frac{\beta}{2} y_t^2$$

The optimal positions are:

$$q_{P,t}^* = \frac{1}{\Delta} \frac{E_t[\tilde{p}_{t+1}] - f_t}{\alpha_P \text{Var}_t[\tilde{p}_{t+1}]} + y_t^* \quad (2.4)$$

$$y_t^* = \frac{1}{\beta Q} \max\{\Delta(Q - f_t), 0\} \quad (2.5)$$

As in Anderson and Danthine [1983a,b], Boons et al. [2014] and Ekeland et al. [2016] the storer and the processor have positions on the futures market that can be decomposed into two parts: a hedging one (short for the storer, long for the processor) and a speculative one. Moreover, the latter is alike the position of the speculator. Note also that the equations (2.1) to (2.5) express the quantities $(q_{S,t}^*, q_{I,t}^*, q_{P,t}^*, x_t^*, y_t^*)$ as functions of p_t , f_t and p_{t+1} . Thus prices determine the positions of the operators on all markets.

2.4.2 Clearing of the markets

Using the previous results on the optimal positions at each period t of the different agents, we can clear both the spot and the futures markets. For the sake of simplicity, and without loss of generality, we set $\gamma = 1$ and $\beta Q = 1$, as done in the model by Ekeland et al. [2016]. We also introduce a minimum spot price p_{\min} and a maximum spot price p_{\max} on the physical market. If the spot price falls below p_{\min} any quantity of the commodity in excess will be destroyed. We assume that the destruction cost is negligible. If the price goes beyond p_{\max} , a substitute replaces the commodity.

The clearing of the spot market: At t , once the transactions initiated during the previous period by the n_I inventory holders and the n_P processors are settled, the quantity \tilde{z}_t is available on the physical market. This quantity also depends on the harvest $\tilde{\omega}_t$ and is equal to:

$$\tilde{z}_t = n_I x_{t-1} - n_P y_{t-1} + \tilde{\omega}_t \quad (2.6)$$

The amount of physical products traded during the previous period is given by:

$$h_t := n_I \max\{\Delta f_t - p_t, 0\} - n_P \max\{\Delta(Q - f_t), 0\} \quad (2.7)$$

Note that h_t also corresponds to the net hedging demand on the futures market, i.e. the hedging pressure. The equation (2.6) can thus be rewritten:

$$\tilde{z}_t = \tilde{\omega}_t + h_{t-1} \quad (2.8)$$

This quantity is fundamental in the model. It creates its recursive structure and represents one of the links between the physical and the futures markets.

Axiom 2.1 We will assume that z_t , p_t and f_t are \mathcal{F}_t -measurable for all t : they are stochastic processes adapted to the filtration \mathcal{F}_t . In other words for a given t , z_t , p_t and f_t depend only on the ω_s , for $s \leq t$.

Definition 2.2 We will say that (z_t, p_t, f_t) is an equilibrium process if all markets are cleared at any time.

If we now introduce the minimum and maximum prices in the demand function of the spot traders, $D(p)$, and if we set $p_{\min} = 0$ and $D(p_{\max}) = 0$ we have:

$$D(p) = \begin{cases} [M, \infty) & \text{if } p = 0 \\ M - mp & \text{if } 0 \leq p \leq Mm^{-1} \\ (-\infty, 0] & \text{if } p = Mm^{-1} \end{cases}$$

where m stands for the price elasticity, and M for the maximal demand. The clearing of the spot market at date t is thus:

$$z_t \in (n_I x_t^* + D(p_t)) \quad (2.9)$$

Note that it is possible to have $z_t \leq 0$. In such case, $D(p_t) < 0$, which means that $p = Mm^{-1}$ and that the spot traders act as suppliers. They provide the quantity $(n_I x_t^* - z_t)$ to the physical market. Taking into account the different values of the demand function gives:

$$z_t = n_I x_t^* + M - mp_t = n_I \max\{\Delta f_t - p_t, 0\} + M - mp_t \text{ if } 0 \leq p_t \leq Mm^{-1} \quad (2.10a)$$

$$z_t \geq n_I x_t^* + M = n_I \max\{\Delta f_t - p_t, 0\} + M \text{ if } p_t = 0 \quad (2.10b)$$

$$z_t \leq n_I x_t^* = n_I \max\{\Delta f_t - p_t, 0\} \text{ if } p_t = Mm^{-1} \quad (2.10c)$$

The clearing of the futures market: At t , the clearing implies zero net supply. That is:

$$n_S q_{S,t}^* + n_P q_{P,t}^* + n_I q_{I,t}^* = 0$$

which gives

$$E_t[\tilde{p}_{t+1}] - f_t = \alpha \Delta h_t \text{Var}_t[\tilde{p}_{t+1}] \quad (2.11)$$

where the constant α represents the risk aversion at the market level:

$$\alpha := \frac{1}{\frac{n_P}{\alpha_P} + \frac{n_I}{\alpha_I} + \frac{n_S}{\alpha_S}} \quad (2.12)$$

Equation (2.11) is known as the risk premium and provides the relation between the expected spot price $E_t[\tilde{p}_{t+1}]$ in t for $t+1$ and the futures price f_t in t . As implied by the theory of normal backwardation of Keynes [1930] and shown in Ekeland et al. [2016] a risk premium exists only if: i) agents are risk-averse ($\alpha \neq 0$); ii) a risk exists ($\text{Var}_t[\tilde{p}_{t+1}] \neq 0$); and iii) physical hedgers want to hedge and there is an imbalance in the market ($h_t \neq 0$).

Definition 2.3 A \mathcal{F}_t -adapted process is an equilibrium price process if there are some \mathcal{F}_t -adapted processes z_t and f_t that verify the equilibrium equations (2.8), (2.10a), (2.10b), (2.10c) and (2.11).

2.4.3 Equilibrium equations

For the equilibrium analysis, we assume that the spot price \tilde{p}_t depends only on the quantity available on the physical market, \tilde{z}_t . In such case, the expectations $E_t[\tilde{p}_{t+1}]$ and $\text{Var}_t[\tilde{p}_{t+1}]$ also depend only on \tilde{z}_t .

Definition 2.4 For $(z, e, v) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}_+$ we will denote by $P(z, e, v)$, $F(z, e, v)$ and $H(z, e, v)$ the solutions of the following equations:

$$z = M - mP + n_I \max\{\Delta F - P, 0\} \quad \text{if } 0 \leq P \leq Mm^{-1} \quad (2.13)$$

$$z \geq M + n_I \max\{\Delta F - P, 0\} \quad \text{if } P = 0 \quad (2.14)$$

$$z \leq n_I \max\{\Delta F - Mm^{-1}, 0\} \quad \text{if } P = Mm^{-1} \quad (2.15)$$

$$F = e - \alpha \Delta v H \quad (2.16)$$

$$H = n_I \max\{\Delta F - P, 0\} - n_P \max\{\Delta(Q - F), 0\} \quad (2.17)$$

In what follows we determine explicitly the functions $P(z, e, v)$, $F(z, e, v)$ and $H(z, e, v)$. Once these functions are known, the equilibrium equations can be simply written:

$$\tilde{p}_t = P(z_t, E_t[\tilde{p}_{t+1}], \text{Var}_t[\tilde{p}_{t+1}])$$

Definition 2.5 We will say that $E(z)$ and $V(z)$, with $V \geq 0$, are rational expectations if, for almost all z , we have:

$$E(z) = E[P(H(z) + \omega), E(H(z) + \omega), V(H(z) + \omega))]$$

$$V(z) = \text{Var}[P(H(z) + \omega), E(H(z) + \omega), V(H(z) + \omega))]$$

Theorem 2.6 If (E, V) are rational expectations, the process:

$$\tilde{p}_t = P(\tilde{z}_t, E(\tilde{z}_t), V(\tilde{z}_t))$$

is an equilibrium price process. It is Markovian, and the corresponding processes z_t and f_t are given by:

$$\tilde{z}_{t+1} = H(t, E(\tilde{z}_t), V(\tilde{z}_t)) + \tilde{\omega}_{t+1}$$

$$f_t = E_t[\tilde{p}_{t+1}] - \alpha \Delta \text{Var}_t[\tilde{p}_{t+1}] H(t, E(\tilde{z}_t), V(\tilde{z}_t))$$

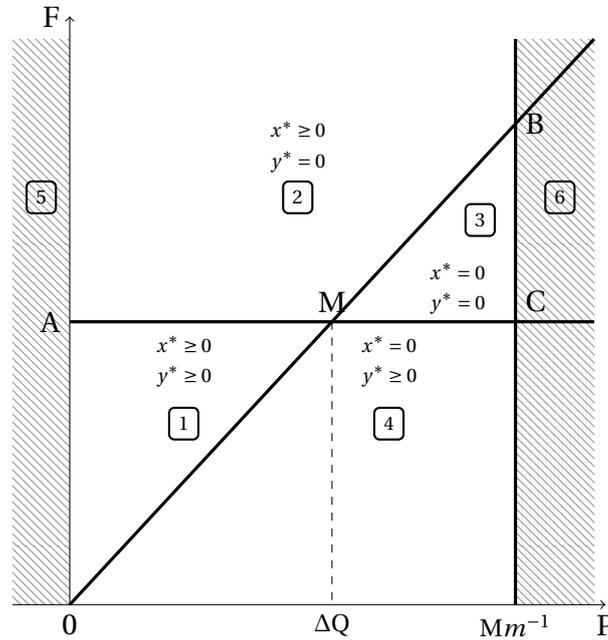
2.4.4 The different regions at the equilibrium

In order to analyze what happens at the equilibrium, we compute $P(z, e, v)$, $F(z, e, v)$ and $H(z, e, v)$. As done in Ekeland et al. [2016], we start from the space (F, P) . In this space, for a given v , and because of the nonlinearity of the equilibrium equations, we consider 6 regions:

- **Region 1:** $0 \leq P \leq Mm^{-1}$, $\Delta F > P$ and $Q > F$. In this Region, all agents are active.

- **Region 2:** $0 \leq P \leq Mm^{-1}$, $\Delta F > P$ and $Q < F$. In this Region the processors are not active.
- **Region 3:** $0 \leq P \leq Mm^{-1}$, $\Delta F < P$ and $Q < F$. In this Region, there is no activity on the physical and the futures markets.
- **Region 4:** $0 \leq P \leq Mm^{-1}$, $\Delta F < P$ and $Q > F$. In this Region the storers do not have an incentive to operate.
- **Region 5:** $P \leq p_{min} = 0$
- **Region 6:** $P \geq p_{max} = Mm^{-1}$

These regions in the space (F, P) are depicted by Figure 2.1. The point situated at the intersection of the four first regions is $M(\Delta Q, Q)$.



This figure shows the different regions in the space (P, F) considered in order to solve the nonlinear system. Each region corresponds to a particular market situation.

Figure 2.1 – Regions in the space (P, F)

Relying on Definition 2.4, we can compute the images of these six regions by the application Φ :

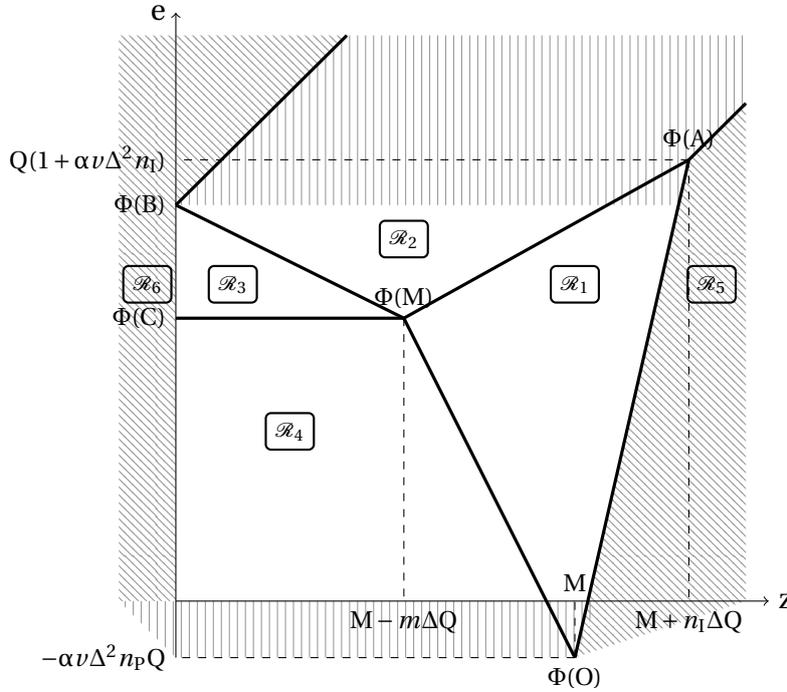
$$\Phi : \begin{pmatrix} P \\ F \end{pmatrix} \rightarrow \begin{pmatrix} M - mP + n_I \max\{\Delta F - P, 0\} \\ F + \Delta\alpha v [n_I \max\{\Delta F - P, 0\} - n_P \max\{\Delta(Q - F), 0\}] \end{pmatrix} = \begin{pmatrix} z \\ e \end{pmatrix} \quad (2.18)$$

We note \mathcal{R}_i , with $i = 1$ to 6 , the images of the regions in the space (e, z) . The image of M by Φ is the point $\Phi(M)$, with $(M - m\Delta Q, Q)$ as coordinates. Moreover, we assume that:

$$p_{min} = 0 \leq Q \leq p_{max} = Mm^{-1}$$

$$M_{min} = 0 \leq M - m\Delta Q \leq M_{max} = M$$

The regions in (e, z) are depicted by Figure 2.2. The details of the computations are given in the Appendix 2.A.1. Note that they are obtained for a given v . More precisely, the regions \mathcal{R}_1 to \mathcal{R}_4 are delimited by four half-lines emanating from the point $\Phi(M)$.



This figure shows the images of the regions by Φ in the space (e, z) . Each region corresponds to a particular market situation.

Figure 2.2 – Images of the regions by Φ in the space (e, z)

In each of the regions in the space (e, z) , it is possible to compute the values of $P(z, e, v)$, $F(z, e, v)$ and $H(z, e, v)$. For example, in region 1 there are storage as well as processing activities. Thus, the equilibrium equations can be written:

$$\begin{aligned} z &= n_1 (\Delta F - P) + M - mP \\ e &= F + \alpha \Delta v [n_1 (\Delta F - P) - \Delta n_p (Q - F)] \end{aligned}$$

This gives, for a given v :

$$F(z, e, v) = \frac{(m + n_1) [e + \alpha \Delta^2 v n_p Q] + \alpha \Delta v n_1 (M - z)}{m + n_1 + \alpha \Delta^2 v (n_p m + n_p n_1 + n_1 m)} \quad (2.19)$$

$$P(z, e, v) = \frac{n_1 \Delta [e + \alpha \Delta^2 v n_p Q] + (M - z) [1 + \alpha \Delta^2 v (n_1 + n_p)]}{m + n_1 + \alpha \Delta^2 v (n_p m + n_p n_1 + n_1 m)} \quad (2.20)$$

Remind that:

$$H(z, e, v) = n_1 (\Delta F(z, e, v) - P(z, e, v)) - \Delta n_p (Q - F(z, e, v))$$

Thus, for a given v , we have:

$$H(z, e, v) = \frac{e\Delta [n_P m + n_P n_I + n_I m] - n_P \Delta Q (m + n_I) - n_I (M - z)}{m + n_I + \alpha \Delta^2 v (n_P m + n_P n_I + n_I m)} \quad (2.21)$$

Region 1 being the most complicated one, the values of $P(z, e, v)$, $F(z, e, v)$ and $H(z, e, v)$ in the other regions can be found easily. The details can be found in the Appendix 2.A.2.

2.4.5 Parameters for the numerical simulations

In order to numerically solve for the equilibrium and to perform a simulation analysis, we must identify the twelve parameters of the model: the interest rate r , the risk aversion of each category of agents (α_i), the number of agents (n_i) also interpreted as elasticities, the two parameters describing the demand function (M and m), the price Q of the final good and the two parameters μ and σ , that describe the first moments of the distribution function of the production $\tilde{\omega}$.

	Domain	Interpretation	Reference set of parameters	Changes for		
				Gold	Electricity	Speculation
r	\mathbb{R}_+	Interest rate	1%			
α_i	\mathbb{R}_+	Risk aversion of i	2			
$n_{i=I,P}$	\mathbb{R}_+	Elasticity of $i = I, P$	$n_I = n_P = 1$	$n_P = 0.1$	$n_I = 0.1$	
n_S	\mathbb{R}_+	Elasticity of S	2			{2, 10}
M	\mathbb{R}_+	Maximum demand	75			
m	[0, 1]	Sensibility of the demand	0.5			
Q	\mathbb{R}_+	Price of the final good	60	80		
μ	\mathbb{R}_+	Mean of the Normal law	50			
σ	\mathbb{R}_+	Standard-deviation of the Normal law	7			

This table gathers the information about the parameters of the model. It displays the domain, the economic interpretation and the values of the parameters. The "Reference set of parameters" is for the values used in the main simulations, and the column "Changes for" shows, when it is needed, the new value used in a specific part of the article.

Table 2.1 – Parameters of the model

Most of these parameters can not be directly calibrated on empirical data, but as done in Vercaemmen and Doroudian [2014] we set realistic values based on economic reasoning. The latter are reported in Table 2.1. The "reference" set of parameters is designed in order to represent a market for a storable commodity. The underlying asset could be the crude oil, for example, and the futures the Light Sweet Crude Oil contract. Let us comment the most important choices done for the parameters' values in this reference set.

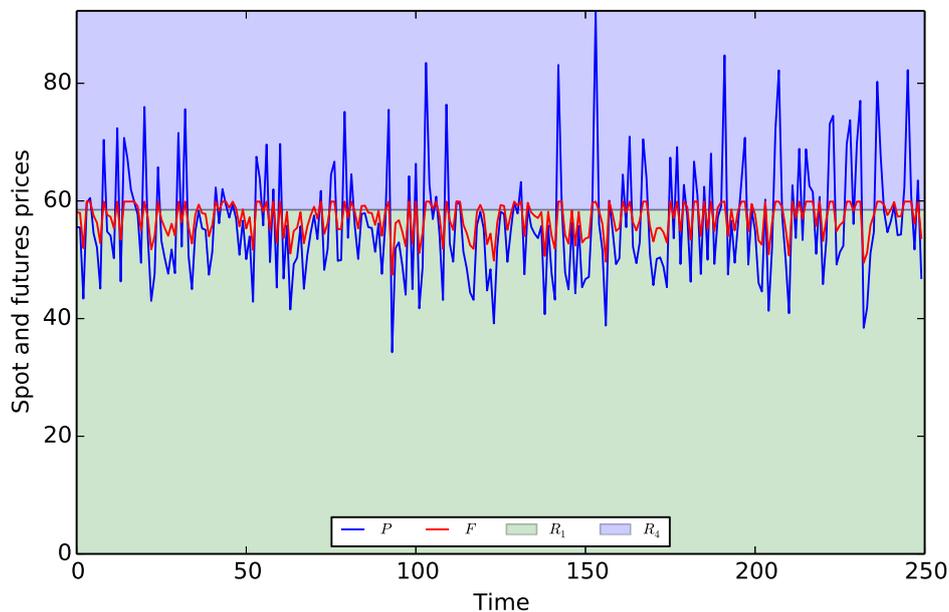
- $\alpha_i = 2, \forall i = P, I, S$ which is a common value of risk aversion.
- $n_i = 1, \forall i = P, I$. This parameter represents the number of agents belonging to the category i . We interpret this parameter as the price elasticity characterizing each of the two industries under scrutiny. An increase in n_i can indeed be seen either as an increase in the number of agents, or as a decrease in their storage or processing costs, because such a decrease would lead the industrial operators to a more intense activity. Even if the existence of a disparity between the agents belonging to the same category is theoretically possible, the simulations rely on an assumption of homogeneity. Finally, in the paragraph 2.5.3 devoted to the gold market we retain a very low value for the number of processors ($n_p = 0.1$): gold is more a reserve of value than the input of a production process. Thus, this market is characterized by a huge amount of inventories. On the contrary, the analysis of the electricity market performed in the paragraph 2.5.3 assumes that $n_l = 0.1$ because electricity is non-storable. Moreover, Appendix 2.B.2 shows a sensitivity analysis of our model to these two parameters.
- $n_s = 2$. The interpretation of this parameter is the same than before but applied to a financial agent. It is reasonable to retain a greater price elasticity for the speculators: their "production costs" are lower than those of the industrial agents. In Section 2.6 devoted to the analysis of the speculation we retain for this parameter a range of values between 2 and 10.
- $m = 0.5$. This parameter is also the price elasticity of the demand of the consumers. We set this low value in order to stress the fact that the consumption and the supply of a commodity are naturally inelastic.
- $M = 75$ because with $m = 0.5$ then $P_{max} = Mm^{-1} = 150$. We then have $P \in [0, 150]$ which seems realistic for a commodity like crude oil.
- $Q = 60$ is maybe the most arbitrary choice we made. It corresponds to a market where the price of the final good is not really expensive. In Section 2.5.3, we use $Q = 80$ because the gold is a commodity characterized by an important intrinsic value. We look in the Appendix 2.B.1 at how the resolution of the model is impacted by the change in Q .
- $N(\mu, \sigma) = N(50, 7)$ in order to have a physical market which is neither undersupplied nor oversupplied.

In what follows, we change one or two parameters in the reference set in order to capture particularities of specific markets or to analyze the impact of the speculation.

2.5 Analysis of the dynamic behavior of commodity prices

In this section we discuss some common stylized facts of commodity markets and we emphasize their heterogeneity by simulating extreme markets like the gold and electricity markets.

First, Figure 2.3 gives a representative overview of the simulations we run. It shows one simulated path of prices for 250 periods for the reference set of parameters. One important feature that comes out of our simulations is the stationary behavior of prices generated by the model. This is a direct consequence of the choice of independent and stationary production shocks.



This figure exhibits one simulated path on 250 periods of time, for the spot and futures prices. We use the reference set of parameters, i.e.: $r = 1\%$, $\alpha_p = \alpha_f = \alpha_s = 2$, $n_p = n_f = 1$, $n_s = 2$, $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$. The area at the top of the figure corresponds to the region \mathcal{R}_4 and the one at the bottom corresponds to \mathcal{R}_1 .

Figure 2.3 – Simulated spot and futures prices

Second, regarding the common stylized facts, we know from previous empirical analysis of a large number of commodity markets that: i) the Samuelson effect holds most of the time (for energy and agricultural futures markets); and ii) the behavior of the basis is different for commodities in backwardation and for those in contango. We will first see if our model allows us to replicate these stylized facts in general, and then we will study more carefully their determinants. More precisely, we shed light on their link with the level of storage costs.

Finally, in order to emphasize the heterogeneity of commodity markets, we use our model to simulate more specific markets. The ones retained are the electricity market characterized by its non-storability and the gold market which behaves almost like a financial asset.

In what follows, for exhibition reasons, we display only one simulated path by figure but it is representative of our simulations.

2.5.1 The Samuelson effect

An important well known feature of commodity markets is the Samuelson effect, or in other words, the decreasing pattern of volatilities along the term structure. The empirical literature has reached an overall consensus, and finds a strong effect on energy products and agricultural commodities (grains, soft commodities, meats). The picture is more nuanced for metals, with weak or null evidence for precious metals. However, the theoretical literature is mixed about its origins, and there exist two dominant views. Bessembinder et al. [1996] establish a relation between the Samuelson effect and the mean reversion in the dynamics of commodity prices. For them, this mean reversion is the direct consequence of storability and reflects the behavior of the operators in the physical market. In contrast, Anderson and Danthine [1983b] propose a theoretical framework that allows for an analysis of the relation between the Samuelson effect and the resolution of uncertainty over time. In this setting, storage does not remain the most important explanatory factor for the behavior of volatility. What matters more is production uncertainty and the way this uncertainty diffuses into the market.

As in Fama and French [1988], we study the Samuelson effect by comparing the volatility of the spot price and the one of the futures price. Because we do not have analytical formulas for the volatility of the spot and futures prices, our analysis is based on simulations in which we compute the one-period volatility in t as the absolute value of the return times 100 ($\sigma_t^p = 100 * |\ln(\frac{P_{t+1}}{P_t})|$). The use of a one-period volatility measure allows to see the impact of the storage on the volatility, and eventually to see violations of the Samuelson effect. Table 2.2 shows the descriptive statistics of the ratio of the volatility of the futures price to the volatility of the spot price ($\frac{\sigma_f}{\sigma_p}$). This ratio is less than one if the Samuelson effect holds and close to one if not.

Table 2.2 shows that with the reference set of parameters, the mean ratio over the paths is 0.3419, with a maximum value below 0.5. In other words, the generality is that the volatility of the spot price is higher than the one of the futures price and the Samuelson effect holds in our simulations. This is what was expected as our reference set of parameters is designed to be representative of a commodity market such as the crude oil market. In some extreme cases, the ratio is equal to 0. In these cases which correspond to situations of backwardation, there is no inventory and the spot price becomes highly volatile while the futures price

	Reference	"Gold"	"Electricity"
Mean	0.3419	0.8479	0.0575
Std	0.1602	0.0616	0.0232
Min	0	0.2596	0
Max	0.4904	0.8935	0.0883

This table shows the descriptive statistics of the ratio of volatilities of simulated spot and futures prices for three different situations: the reference set of parameters, the "gold" market and the "electricity" market. Values are the mean over multiple simulated paths.

Table 2.2 – Descriptive statistics of the ratio of volatilities

reaches its maximum value and then becomes constant.

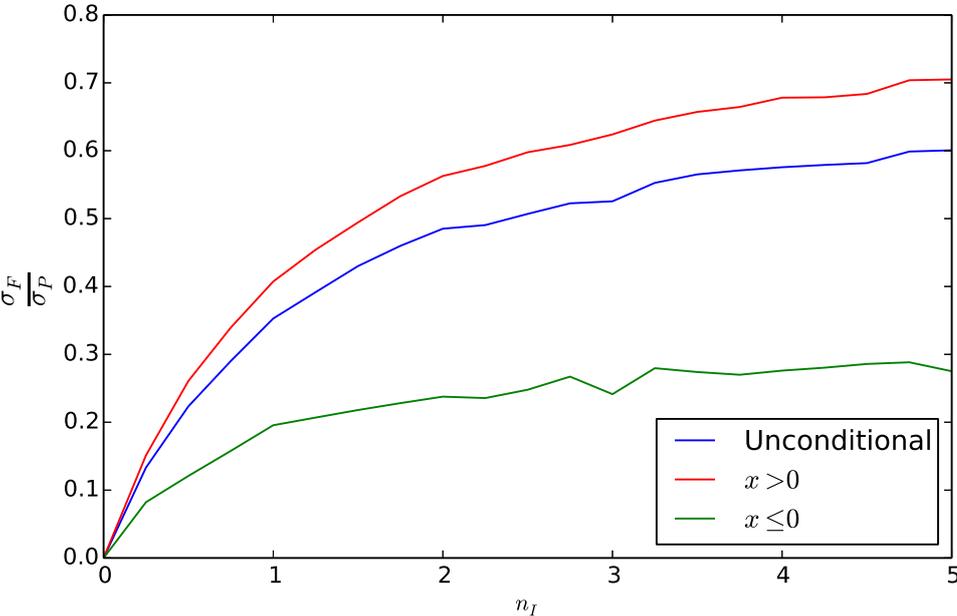
We investigate the role of inventories on the Samuelson effect by running simulations of our model for different values of n_1 ³. Figure 2.4 shows the (un)conditional mean ratios of volatilities as functions of n_1 . We compute the mean ratio of volatilities conditionally on the sign of $x = \Delta F - P$: $E_{sup} = E[\frac{\sigma_F}{\sigma_P} | x > 0]$ and $E_{less} = E[\frac{\sigma_F}{\sigma_P} | x < 0]$.

First, as the number of storers increases, the unconditional ratio of volatilities increases. That is the Samuelson effect is less important as storage capacities increase. Then, a market characterized by important storage facilities exhibits a flatter term structure of volatilities (see the example of the gold market in Section 2.5.3). This suggests, contrary to the theoretical framework of Bessembinder et al. [1996], that the storability of the commodity is not a necessary condition for the Samuelson effect to exist. Indeed, when n_1 is very close to 0, which is a proxy for a non-storable commodity, the Samuelson effect is very strong in our simulations (ratio of volatilities very close to 0). This has been empirically illustrated for the electricity market by Jaeck and Lautier [2016].

Then, according to Fama and French [1988] and Routledge et al. [2000], some violations of the Samuelson effect are possible when there is an excess of storage. That is, sometimes, the volatility of the futures price becomes roughly the same as the one of the spot price (ratio close to one in our simulations). To see if the actual level of storage can predict the steepness of the term structure of volatilities we rely on the conditional mean ratios of volatilities. To confirm the existence of violations of the Samuelson effect due to the excess of storage we expect to have $E_{sup} > E_{less}$ and $E_{sup} \approx 1$. Figure 2.4 shows that E_{sup} is always higher than the unconditional ratio and is increasing in n_1 , whereas E_{less} is always lower than the unconditional ratio, and is increasing at first, but then seems to be constant⁴. These results

3. An increase in n_1 can be interpreted either as a decrease in the storage costs or an increase in the number of storers.

4. In Figure 2.4, E_{sup} is closer to the unconditional mean ratio of volatilities than E_{less} because the frequency of periods of contango is much more important than the one of periods of backwardation.



This figure shows the (un)conditional mean ratios of volatilities for different level of n_I . The basic set of parameters is the following: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = 1$, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$. We use $n_I \in [0, 5]$ as a proxy of the level of storage costs. That is when n_I increases, the level of storage costs decreases.

Figure 2.4 – Mean ratio of volatilities and storage costs

first show that the actual level of inventories provides information on the term structure of volatilities. Moreover, they show that the Samuelson effect can be less important from time to time, because with a high level of inventories, the volatilities of the spot price and of the futures price tend to be the same.

2.5.2 The dynamic behavior of the basis

Compared to the theoretical literature, the empirical literature focusing on the basis ($B = F - P$) on commodity derivative markets is important. This literature gives its empirical features.

First, according to the physical characteristics of a market, the basis can be both positive (term structure in contango) or negative (term structure in backwardation). Bhardwaj, Gorton, and Rouwenhorst [2015] show that commodity markets are most of the time in contango. The simulated path of the basis plotted in Figure 2.5, and Table 2.3 show that in our model with the reference set of parameters it is also the case.

	Reference	"Gold"	"Electricity"
Mean	1.038	2.601	7.125
% of contango ($B > 0$)	74.28	96.88	75.04
% of backwardation ($B < 0$)	25.72	3.12	24.96
$\sigma(B)_{B>0}$	2.5725	1.0068	8.0254
$\sigma(B)_{B<0}$	6.5966	4.6135	6.9145

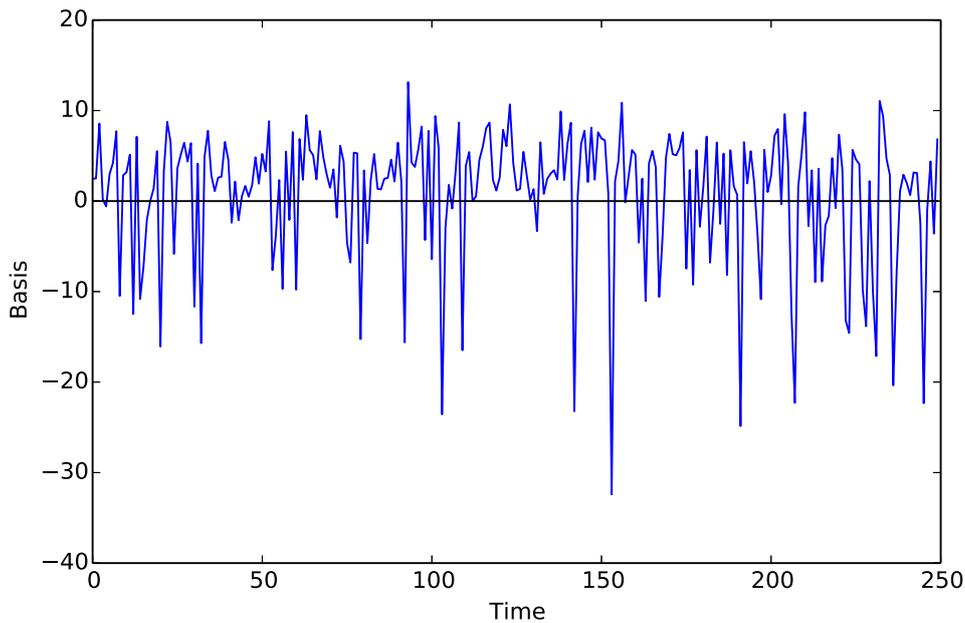
This table shows the descriptive statistics of the simulated basis (futures price minus spot price) for three different situations: the reference set of parameters, the "gold" market and the "electricity" market. Values are the mean over multiple simulated paths.

Table 2.3 – Descriptive statistics of the simulated basis

Then, the basis is known to have an asymmetric behavior. That is, in contango the basis is stable and capped whereas in backwardation it is unstable and theoretically unlimited. This asymmetric behavior is the consequence of the non-negativity constraint on inventories. Indeed, in contango inventories make possible cash-and-carry arbitrages. As stated in the theory of storage initiated with Kaldor [1939], these arbitrages create a link between the spot price and the futures price. On the opposite, in backwardation cash-and-carry arbitrages are not economically profitable and reverse cash-and-carry arbitrages are impossible because of the lack of inventories. Then, the link between the spot and futures prices is broken and they become independent.

As shown by the simulated path of the basis plotted in Figure 2.5, our model reproduces this asymmetric behavior. When the basis is positive it seems concentrated, while when it

is negative, we can observe important negative jumps. Moreover, the maximum absolute value reached by the basis in backwardation is far more important than the one reached by the basis in contango. Table 2.3, which shows the standard deviation of the basis in contango ($\sigma(B)_{B>0}$) and in backwardation ($\sigma(B)_{B<0}$) confirms these observations. Indeed, for the reference set of parameters, the former at 2.5725 on average is lower than the latter at 6.5966.

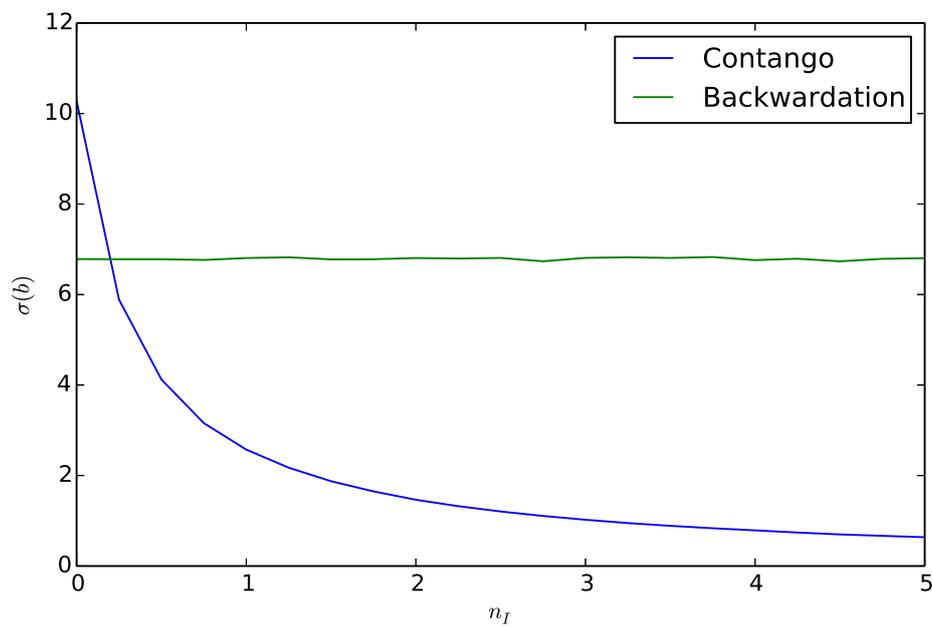


This figure shows one simulated path on 250 periods of time, for the basis (futures price minus spot price). We use the reference set of parameters, i.e.: $r = 1\%$, $\alpha_p = \alpha_I = \alpha_S = 2$, $n_p = n_I = 1$, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$. A positive basis represents a situation of a contango, and a negative basis a situation of backwardation.

Figure 2.5 – Basis of simulated spot and futures prices

Finally, some empirical evidence show that the level of the storage cost is an important determinant of the behavior of the basis. Fama and French [1987] find that the standard deviation of the basis for precious metals is lower than the one for agricultural products, which is lower than the one for animal products. Kang et al. [2014] also find this result. However, this has never been demonstrated theoretically and we intend to fill this gap. We base our analysis on time series simulated from the model, as the one in Figure 2.5 for the reference set of parameters. Figure 2.6 gives information regarding the stability of the basis as a function of the storage costs contained in n_I . As in the previous section, we use the number of storers as a proxy for the level of storage costs. That is, we increase the number of storers, to decrease the level of storage costs.

We find that there exists an asymmetric link between storage costs and the variability of



This figure shows the conditional (on the sign of the basis) volatility (standard deviation) of the basis for different level of n_I . The basic set of parameters is the following: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = 1$, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$. We use $n_I \in [0, 5]$ as a proxy of the level of storage costs. That is when n_I increases, the level of storage costs decreases.

Figure 2.6 – Stability of the basis and storage costs

the basis. That is, in contango the variability of the basis is a positive function of storage costs, whereas in backwardation storage costs do not impact the variability of the basis. In other words, when storage costs decrease, the volatility of the basis in contango (backwardation) decreases (does not change). It is noteworthy that, for some high level of storage costs, the basis in contango is more volatile than the basis in backwardation. The reasoning behind this particular link is the following: in backwardation, without any convenience yield, there is no storage for any storage cost. However, in contango the arbitrage opportunity is directly linked to the cost of making such an arbitrage. This cost comes almost entirely from the cost of storage associated with the holding of the commodity. Then, a higher cost of storage is associated with less arbitrage opportunities between the spot and futures prices and a weaker link between the two.

2.5.3 The heterogeneity of commodity markets

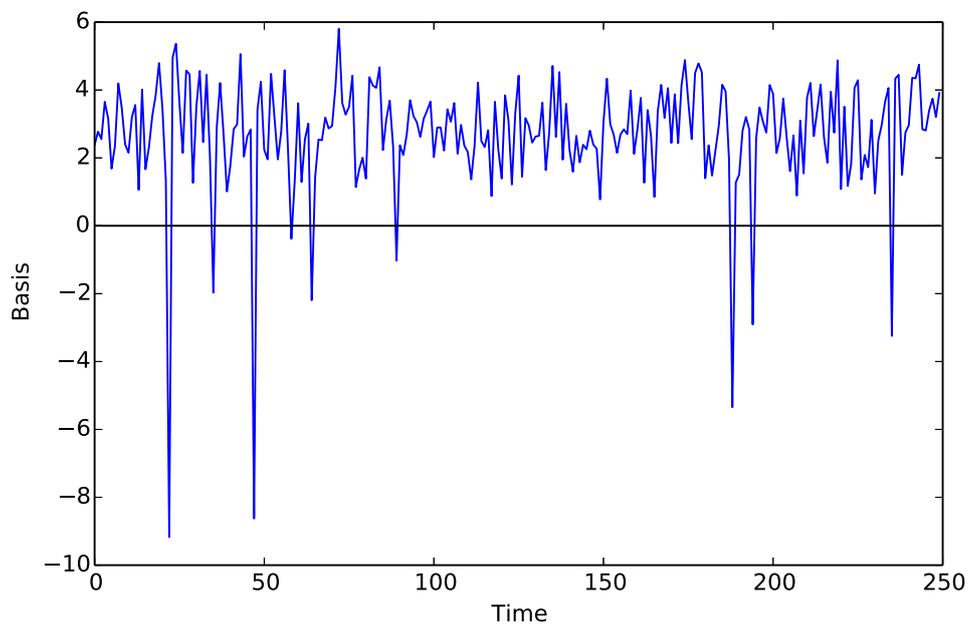
The previous sections were dedicated to the discussion of the economic mechanisms behind some stylized facts that are common to all major commodity markets. In this section, we want to emphasize the important heterogeneity that exists between commodity markets by studying more in details some markets which are somehow extreme cases: the gold market and the electricity market.

The gold market

The gold market is particular in the commodity world since gold is mainly used as a financial asset and not in production processes as other commodities. As a consequence, in this market, most of the participants are storers and a few are processors. As described by Fama and French [1988], this structure of the market induces some specific stylized facts for gold prices. First, regarding the basis. The term structure of gold prices is most of the time in contango ($B > 0$). As a consequence, the basis is very stable. Then, regarding the term structure of volatilities. The Samuelson effect does not hold for the short-term part of the curve (see Lautier and Raynaud [2011]). Indeed, the volatilities of the spot price and of the futures price are the same.

We can use our model to simulate the gold market using a setting with a very low value of n_p and a higher value of Q . Tables 2.2 and 2.3 and Figures 2.7 and 2.8 show the results obtained by setting $n_p = 0.1$ and $Q = 80$.

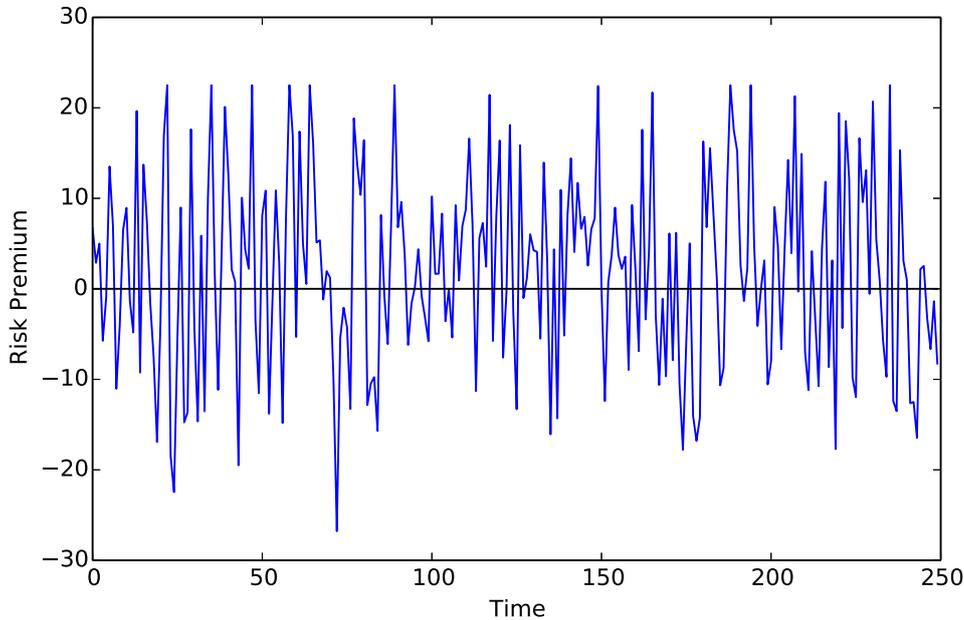
First, the Figure 2.7 shows one path of simulated basis and the Table 2.3 some descriptive statistics of the simulated basis. As for the gold market, our simulations exhibit most of the time a market in contango, with a stable basis compared to other simulations presented before.



This figure shows one simulated path on 250 periods of time, for the basis (futures price minus spot price). We use the set of parameters for "gold", i.e.: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = 0.1$, $n_I = 1$, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 80$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.7 – Simulated basis for "gold"

Second, Table 2.2 shows the descriptive statistics of the ratio of the volatility of the futures price to the volatility of the spot price ($\frac{\sigma_F}{\sigma_P}$). In the case of gold, this ratio is very close to 1 and relatively stable, so the Samuelson effect does not seem to hold in this case.



This figure shows one simulated path on 250 periods of time, for the risk premium (futures price minus expected spot price). We use the set of parameters for "gold", i.e.: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = 0.1$, $n_I = 1$, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 80$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.8 – Risk premium for "gold"

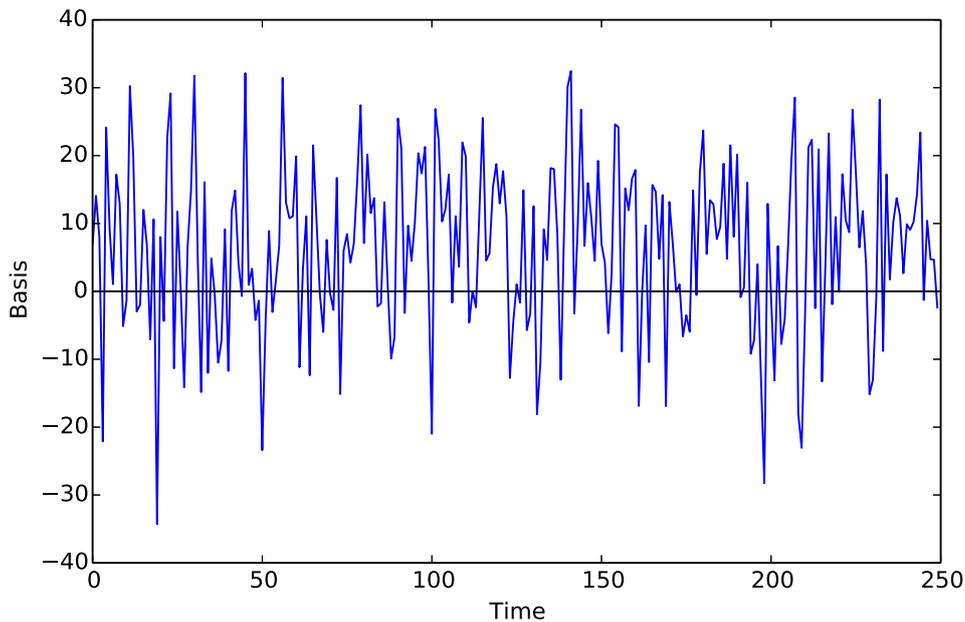
Finally, an interesting thing that comes out of the simulations is that the simulated risk premia in Figure 2.8 are very unstable. It can be both negative and positive and goes from -30 to 30 . This might be explained by the arbitrage behavior of storers which makes the futures prices almost perfectly following the spot price while the expected spot price is more or less always the same.

The electricity market

Electricity is different from other commodities, even from other energies because of its non-storability. This non-storability makes intertemporal arbitrage impossible and the pricing of derivative contracts using the storage theory irrelevant. As a consequence, electricity prices exhibit specific stylized facts. First, they are mean-reverting with an important seasonality. Moreover, prices can be negative for some specific situations. Then, regarding the basis, the term structure of electricity prices can exhibit a lot of shapes (backwardation, contango, U-shaped, bell-shaped...). Therefore, the basis is very unstable. Finally, regard-

ing the term structure of volatilities. The volatility of the spot price is higher than that of a storable commodity, and the Samuelson effect holds (see Jaeck and Lautier [2016]).

Because our model is general, we are not able to reproduce stylized facts that are too specific as the seasonality or the negative prices, but we can study the electricity market by setting a very low value of n_I . Tables 2.2 and 2.3 and Figures 2.9 and 2.10 show the results obtained by setting $n_I = 0.1$ in our simulations.



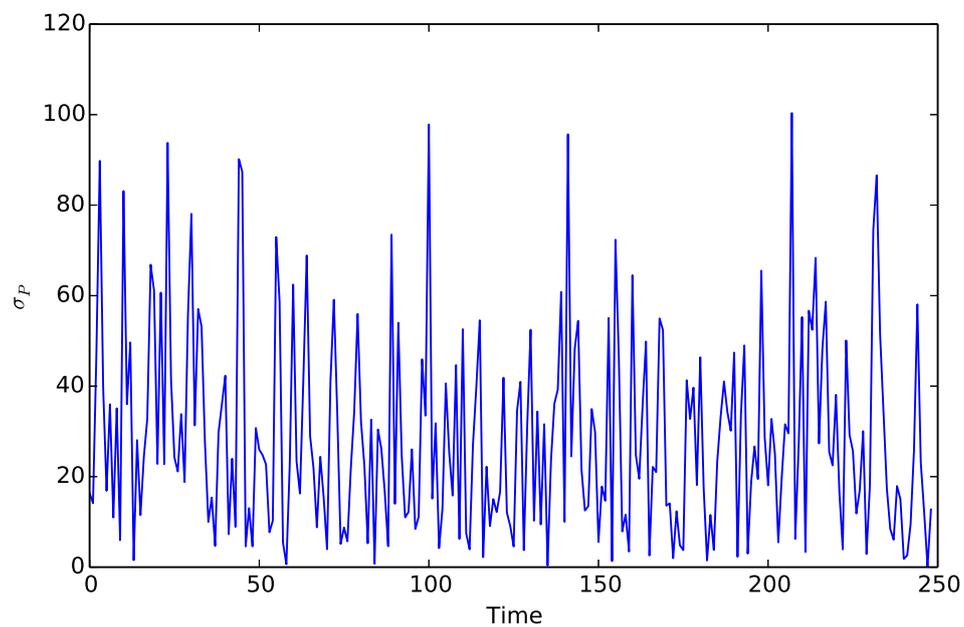
This figure shows one simulated path on 250 periods of time, for the basis (futures price minus spot price). We use the set of parameters for "electricity", i.e.: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = 1$, $n_I = 0.1$, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.9 – Simulated basis for "electricity"

First, Table 2.3 and Figure 2.9 show that, as for electricity markets, the basis is very unstable and that independently of the shape of the term structure (contango or backwardation). Indeed, most of the time the basis is positive, that is the market is in contango, but the standard deviation of the basis in these cases is as important as the one in backwardation.

Second, Figure 2.10 shows the volatility of the simulated spot price. As for the electricity market, we can see that the values are more important than the one we obtained for other specifications with storage. In other words, the market we have simulated as the electricity market is without any doubt more volatile than other markets, and this because of the very low level of storage allowed.

Finally, the comparison of the same ratio of volatilities as before in Table 2.2 gives op-



This figure shows one simulated path on 250 periods of time, for the volatility of the spot price. We use the set of parameters for "electricity", i.e.: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = 1$, $n_I = 0.1$, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.10 – Volatility of the spot price for "electricity"

posite results as for the "gold" market because values are very low and close to 0. In other words, there is a strong Samuelson effect in our simulations of the electricity market.

2.6 Analysis of the speculative activity

In what follows, relying on the sets of parameters described by Table 2.1, we numerically examine how the functioning of the market and the resulting quantities are impacted by an increase in the number of speculators⁵.

We look at the effect of the speculative activity on the equilibrium functions. As before, at the equilibrium we consider a market which is expected to be neither under nor over-supplied. Nevertheless, in a dynamic setting, the present state of the economy inherits from past decisions. Then, all situations between the two extreme cases of complete shortage ($z = 0$) and complete abundance (important z) of the commodity are possible. Then, we study the optimal quantities we obtain at the equilibrium as functions of z , the level of supply today. Table 2.4 summarizes these results and shows that some quantities are impacted in the same way in situations of current scarcity or abundance, but that the reaction of others is situation dependent.

	Current scarcity (Low z)	Current abundance (High z)
Regions		{4, 1, 5}
Impact on		
P_t	↔	↗
F_t	↘	↗
$E_t[\tilde{P}_{t+1}]$	↗	↘
$\text{Var}_t[\tilde{P}_{t+1}]$	↗	↘
$ \text{RP}_t = F_t - E_t[P_{t+1}] $	↘	↘
$ H_t $	↗	↗
$ B_t = F_t - P_t $	↗	↗

This table summarizes the results of the analysis of the speculation activity. P_t is the spot price, $E_t[\tilde{P}_{t+1}]$ is the expected spot price, F_t is the futures price, $|\cdot|$ stands for absolute values, $\text{Var}_t[\cdot]$ is the variance and H_t is the Hedging pressure.

Table 2.4 – Summary of the impacts of the speculation

5. We increase the number of speculators n_s , but we could also have decreased the risk aversion α_s of the speculators because both lead to a decrease in the parameter α of the global risk aversion of the market.

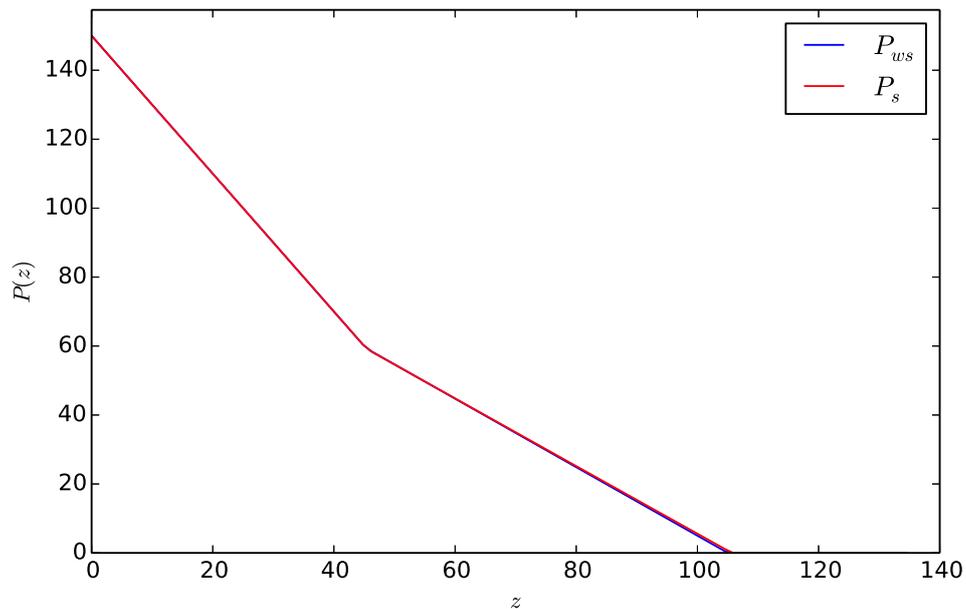
2.6.1 Quantities heterogeneously affected by the speculation

In this section we focus on the analysis of the quantities that are ambiguously impacted by an increase in the level of the speculative activity, because their reactions will depend on the level of supply today. These quantities, depicted in Figures 2.11, 2.12, 2.13 and 2.14 are the spot price, the expected spot price, the variance of the spot price and the futures price.

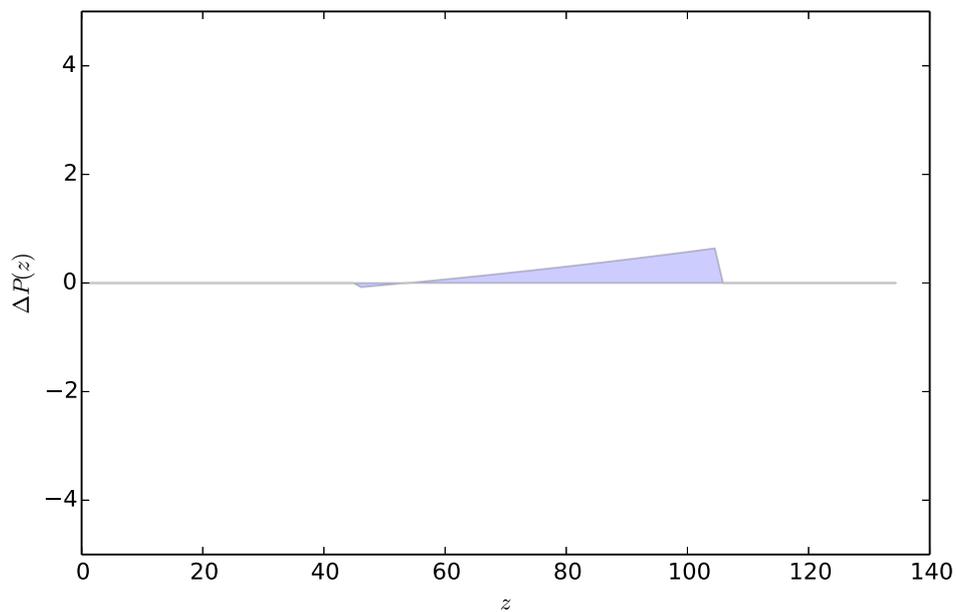
First, regarding the spot price, Figure 2.11 shows that the introduction of more speculative activity with low values of current supply z leads to unchanged spot prices and with high values of current supply leads to higher spot prices. This is the case because the speculative activity motivates the building of inventories by the storers, but only when the current level of supply is important enough. Then, in a situation of scarcity, storers can not buy more commodity to store it and the spot price is unchanged, whereas when the supply is important enough, the increase in the speculative activity leads to an accumulation of inventories by the storers and then an upward pressure on the spot price.

Second, Figures 2.12 and 2.13 show that the reaction of the level of the expected spot price and of the variance of the spot price are the same: they increase for low level of supply z and decrease in situations of abundance. The economic reasoning behind this result is similar for these two quantities. For low z , the increase in the number of speculators induces an increase in the hedging demand from processors and an increase in the physical demand by processors in $t + 1$ on the spot market. This increased demand, by stressing the spot market in $t + 1$ will have two consequences: i) a higher spot price; and ii) a more volatile spot price. The reverse happens for high z , the increase in the speculation induces an increase in the hedging demand from storers (more important than the one from processors) and, due to inventory accumulation, an increase in the physical supply in $t + 1$. This extra supply, by relaxing the spot market will lead to a lower and less volatile spot price.

Finally, Figure 2.14 shows that the fluctuation interval of futures prices is smaller with speculation. That is, futures prices decrease for low levels of current supply and increase for high level of current supply. This is because for low z the hedging pressure is negative, which means that processors, who want to buy futures contracts are more important on the market. Then, the new speculators will optimally sell futures contracts leading to a decrease in the futures price. For high z , the economic reasoning is exactly the same but the new speculators buy futures contracts from the storers in order to earn the risk premium. By doing this, the speculators smooth the variations of the futures price.

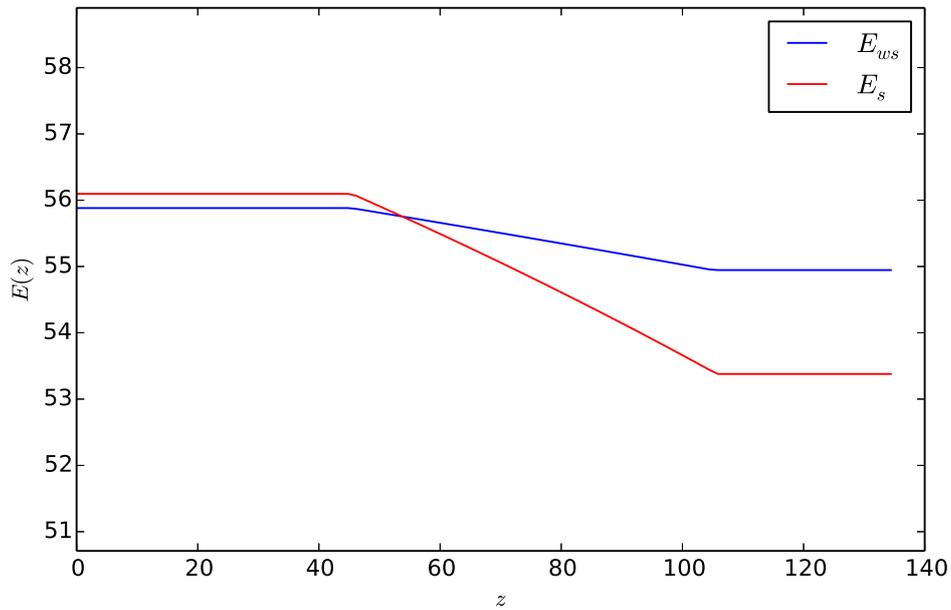


(a) Spot price

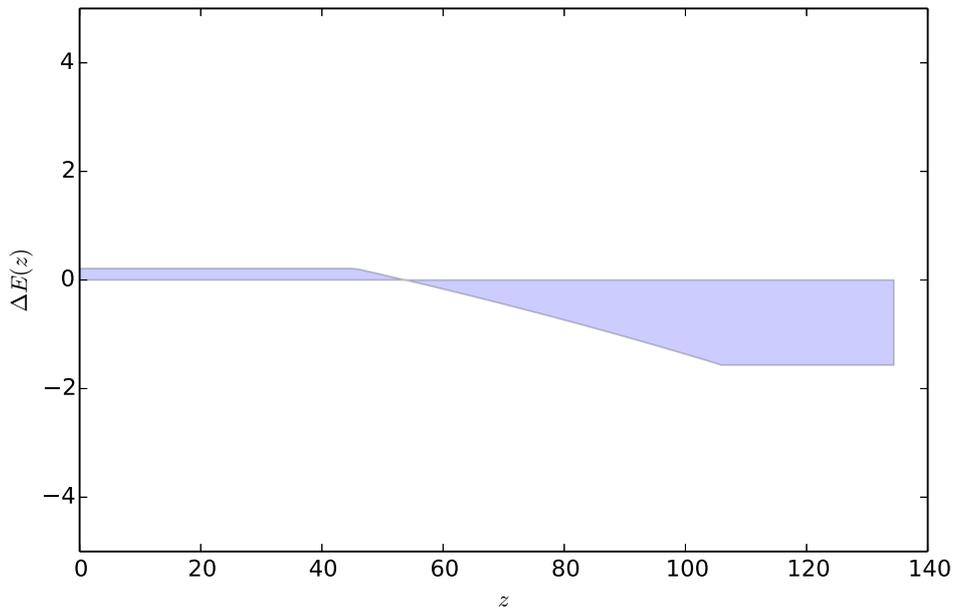
(b) Δ Spot price

This figure shows the impact of a change in the level of the speculative activity n_s on the spot price. The chart on the sub-figure a) represents the equilibrium functions. The blue line is for the function at the equilibrium without speculation (ws), the red line is for the function at the equilibrium with speculation (s). The chart on the sub-figure b) is for the variation with the increase in the speculative activity. The parameters used to solve our model are the following: $r = 1\%$, $\alpha_p = \alpha_I = \alpha_S = 2$, $n_p = n_I = 1$, $n_S = 1$ in ws , $n_S = 10$ in s , $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.11 – Impact of the speculative activity on the spot price



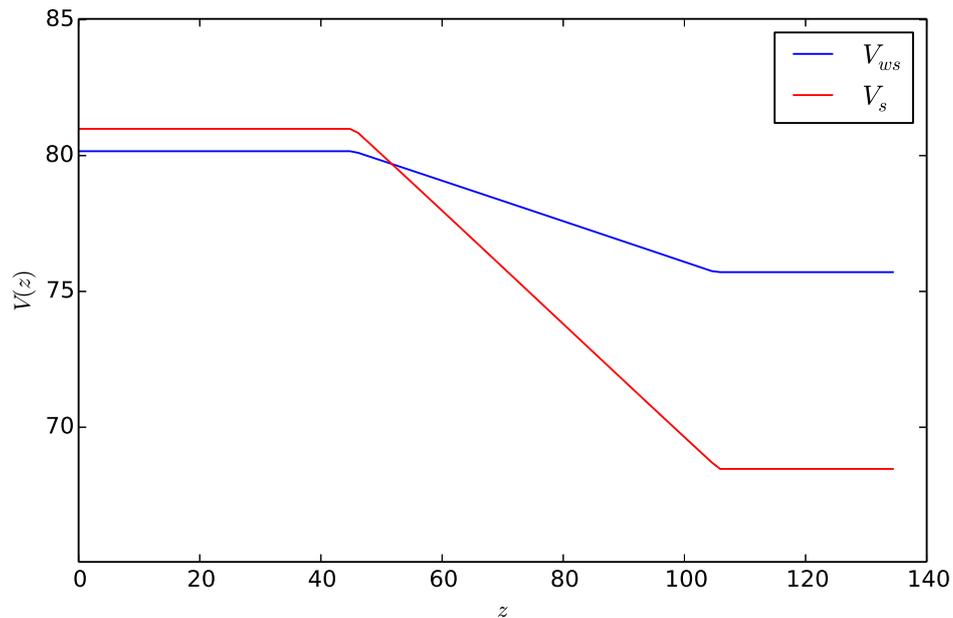
(a) Expected spot price



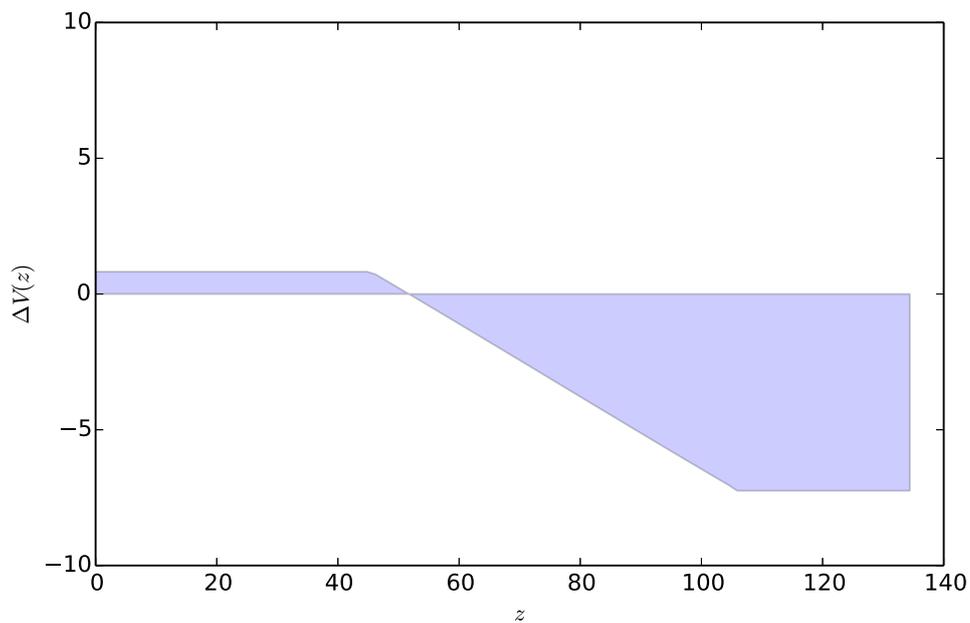
(b) Δ Expected spot price

This figure shows the impact of a change in the level of the speculative activity n_s on the expected spot price. The chart on the sub-figure a) represents the equilibrium functions. The blue line is for the function at the equilibrium without speculation (ws), the red line is for the function at the equilibrium with speculation (s). The chart on the sub-figure b) is for the variation with the increase in the speculative activity. The parameters used to solve our model are the following: $r = 1\%$, $\alpha_p = \alpha_l = \alpha_s = 2$, $n_p = n_l = 1$, $n_s = 1$ in ws , $n_s = 10$ in s , $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.12 – Impact of the speculative activity on the expected spot price

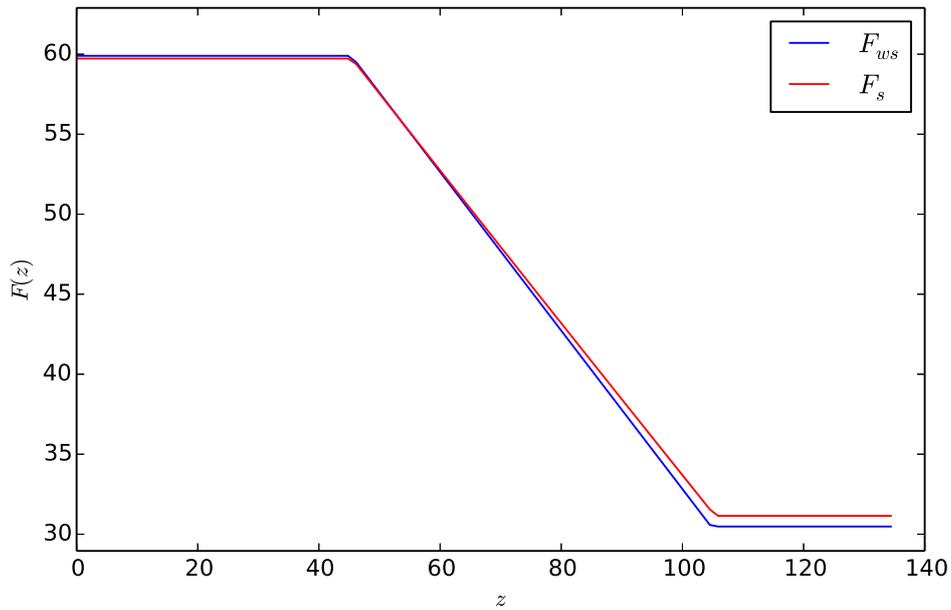


(a) Variance of the spot price

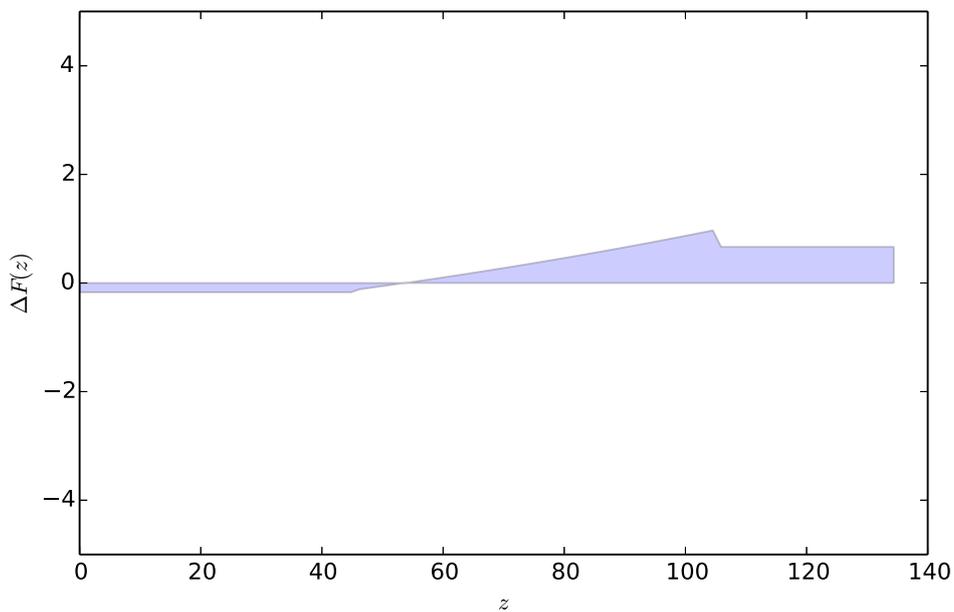
(b) Δ Variance of the spot price

This figure shows the impact of a change in the level of the speculative activity n_S on the variance of the spot price. The chart on the sub-figure a) represents the equilibrium functions. The blue line is for the function at the equilibrium without speculation (ws), the red line is for the function at the equilibrium with speculation (s). The chart on the sub-figure b) is for the variation with the increase in the speculative activity. The parameters used to solve our model are the following: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = n_I = 1$, $n_S = 1$ in ws , $n_S = 10$ in s , $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.13 – Impact of the speculative activity on the variance of the spot price



(a) Futures price



(b) Δ Futures price

This figure shows the impact of a change in the level of the speculative activity n_s on the futures price. The chart on the sub-figure a) represents the equilibrium functions. The blue line is for the function at the equilibrium without speculation (ws), the red line is for the function at the equilibrium with speculation (s). The chart on the sub-figure b) is for the variation with the increase in the speculative activity. The parameters used to solve our model are the following: $r = 1\%$, $\alpha_p = \alpha_I = \alpha_S = 2$, $n_p = n_I = 1$, $n_S = 1$ in ws , $n_S = 10$ in s , $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.14 – Impact of the speculative activity on the futures price

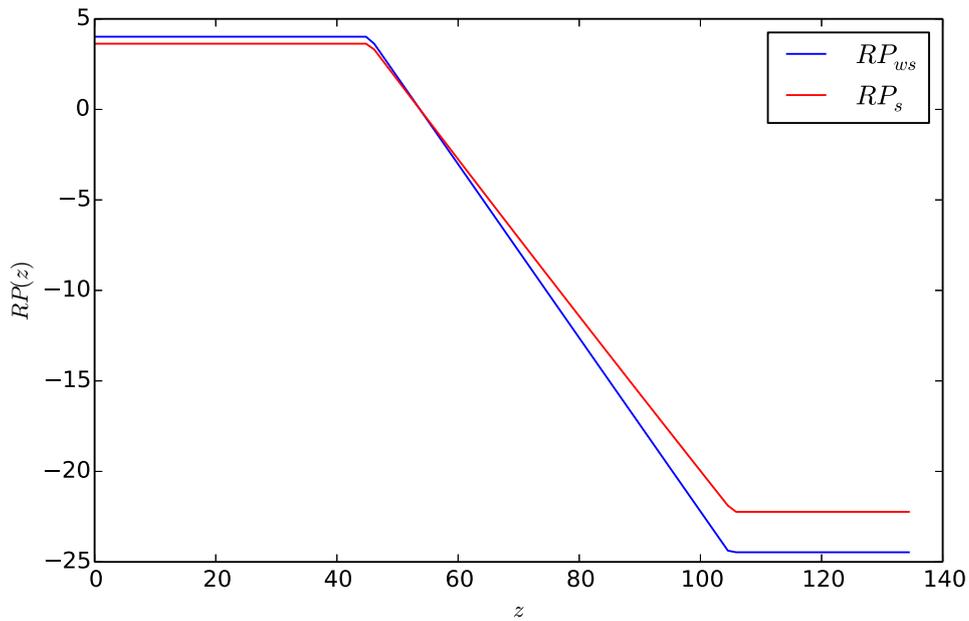
2.6.2 Quantities homogeneously affected by the speculation

In this section we focus on the analysis of the quantities that are always impacted in the same way by an increase in the level of the speculative activity. These quantities depicted in Figures 2.15, 2.16 and 2.17 are the risk premium, the hedging pressure and the basis.

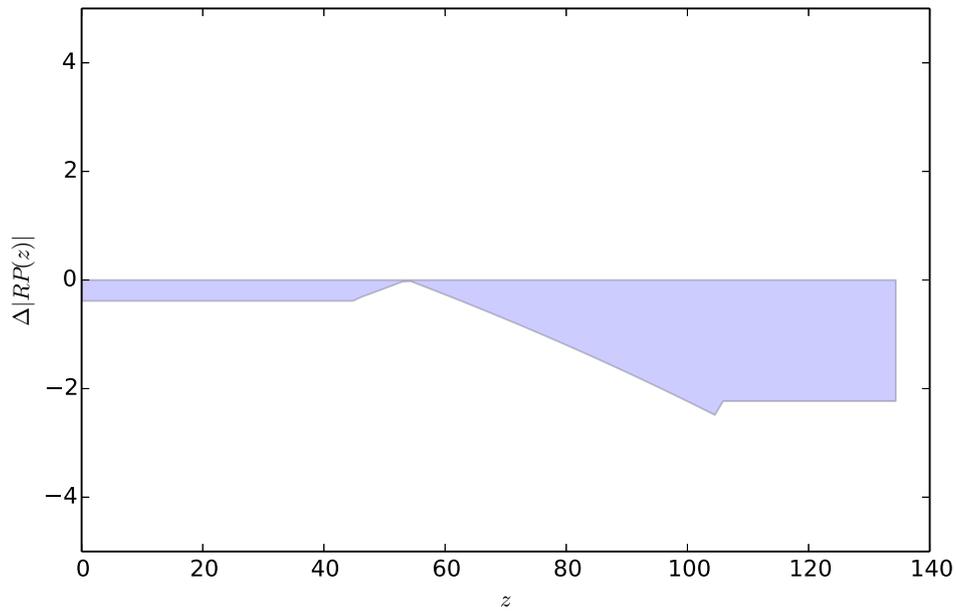
First, Figure 2.15 shows that an increase in the speculative activity always leads to an increase in the overall capacity to absorb risk, which makes the hedging cheaper. In other words, the absolute value of the risk premium decreases with the number of speculators. This result is intuitive if we think of the risk premium as the price of risk, and of the increase in the speculative activity as an increase in the competition between speculators to bear the risk. Then, as it is usually the case, the increased competition in the risk bearing market leads to a decrease in the price of risk. At the individual level, this result has mixed implications because a decrease in the risk premium is beneficial for some agents but harmful for others. Indeed, only the main agent on the market, the one who marginally determines the risk premium (the processors in scarcity and the speculators in abundance) pays the risk premium and then benefits from its decrease. The other agents earn the risk premium by speculating or hedging on the opposite side of the main agent and will therefore dislike an increase in the speculative activity because it decreases their profits.

Second, as a direct consequence of the decrease in the risk premium, Figure 2.16 shows that the hedging pressure is amplified in absolute value and that sometimes it is beneficial for the processors and sometimes for the storers. Indeed, when it is negative for low z , it becomes even more negative meaning that processors engage into more processing. On the contrary, when the hedging pressure is positive (situation of abundance), it becomes even more important meaning that storers accumulate inventories.

Finally, Figure 2.17 shows that the speculation changes the shape of the term structure of commodity prices. Indeed, we can see that with more speculation, the absolute value of the basis is most of the time more important than without speculation. This is a consequence of the change in the behavior of spot and futures prices described previously. Indeed, the market is in backwardation ($B < 0$) for low z when an increase in the speculative activity does not impact the spot price but decreases the futures, which leads to a more important backwardation. In a situation of contango for important values of z , the futures price increases more rapidly with the speculation than the spot price and then the contango becomes more important. In the framework of the storage theory of Working [1949] and Brennan [1958] with constant interest rates this can be directly interpreted as an increase in the marginal storage cost or a decrease in the convenience yield due to the inventory accumulation implied by the extra speculation.



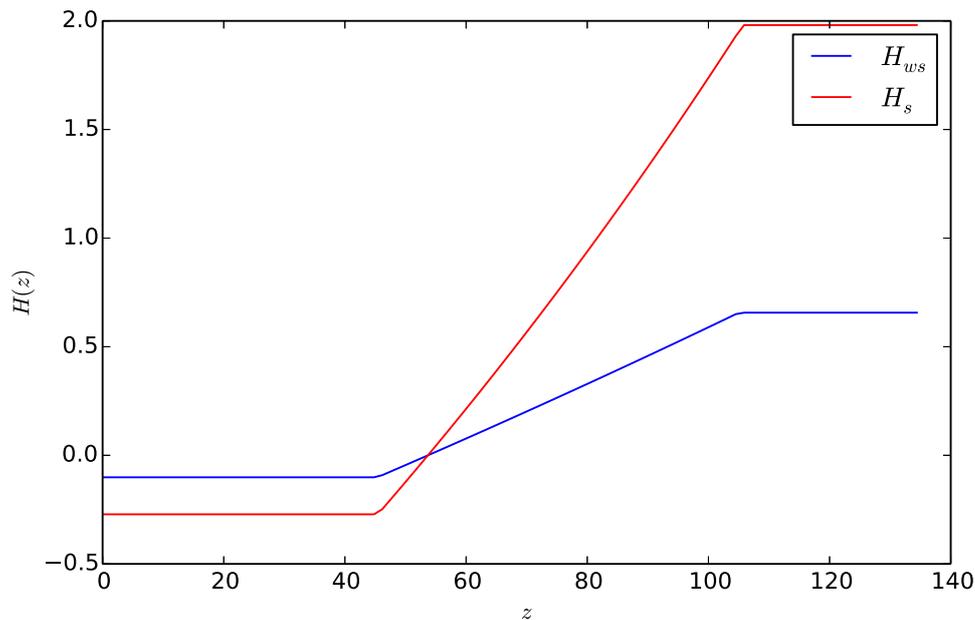
(a) Risk premium



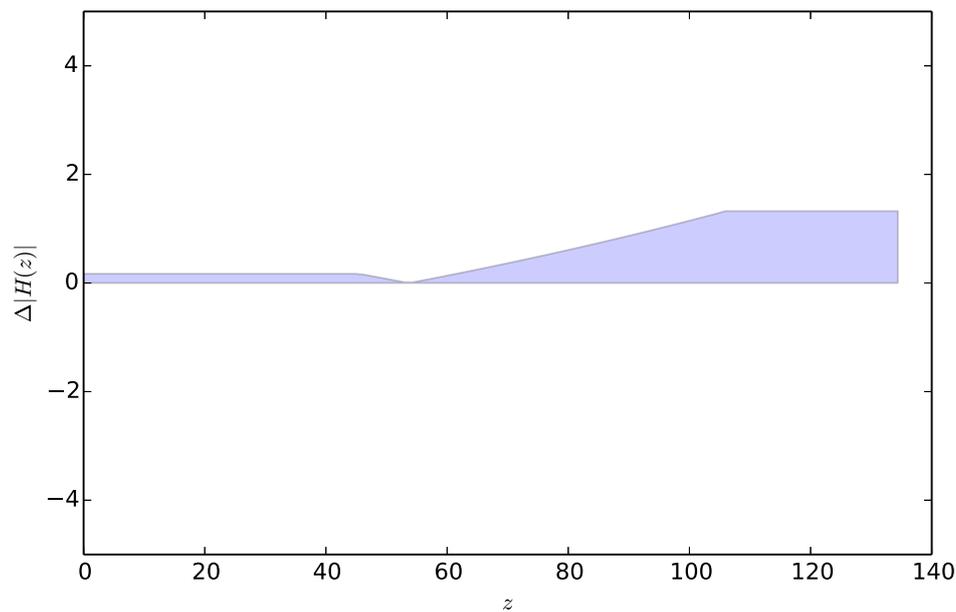
(b) Δ Risk premium (absolute value)

This figure shows the impact of a change in the level of the speculative activity n_S on the risk premium defined as $RP(z) = F(z) - E(z)$. The chart on the sub-figure a) represents the equilibrium functions. The blue line is for the function at the equilibrium without speculation (ws), the red line is for the function at the equilibrium with speculation (s). The chart on the sub-figure b) is for the variation with the increase in the speculative activity. The parameters used to solve our model are the following: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = n_I = 1$, $n_S = 1$ in ws , $n_S = 10$ in s , $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.15 – Impact of the speculative activity on the risk premium

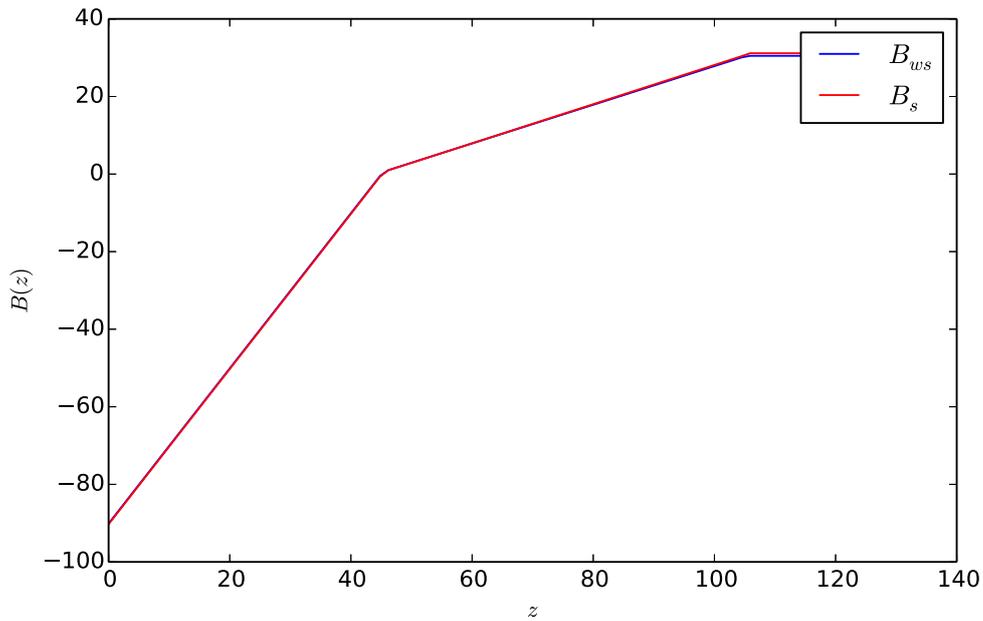


(a) Hedging pressure

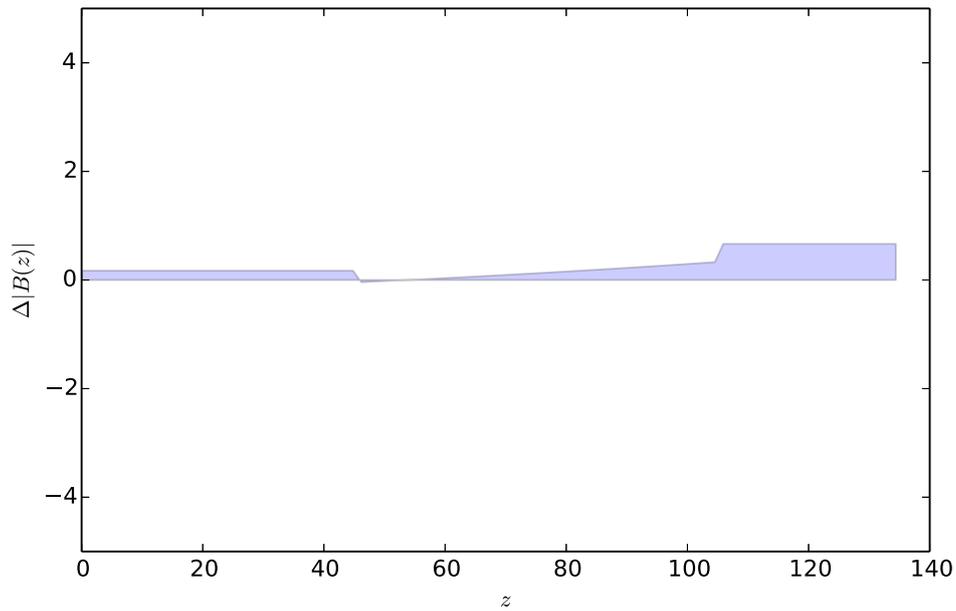
(b) Δ Hedging pressure (absolute value)

This figure shows the impact of a change in the level of the speculative activity n_S on the hedging pressure. The chart on the sub-figure a) represents the equilibrium functions. The blue line is for the function at the equilibrium without speculation (ws), the red line is for the function at the equilibrium with speculation (s). The chart on the sub-figure b) is for the variation with the increase in the speculative activity. The parameters used to solve our model are the following: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = n_I = 1$, $n_S = 1$ in ws , $n_S = 10$ in s , $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.16 – Impact of the speculative activity on the hedging pressure



(a) Basis



(b) Δ Basis (absolute value)

This figure shows the impact of a change in the level of the speculative activity n_s on the basis defined as $B(z) = F(z) - P(z)$. The chart on the sub-figure a) represents the equilibrium functions. The blue line is for the function at the equilibrium without speculation (ws), the red line is for the function at the equilibrium with speculation (s). The chart on the sub-figure b) is for the variation with the increase in the speculative activity. The parameters used to solve our model are the following: $r = 1\%$, $\alpha_p = \alpha_l = \alpha_s = 2$, $n_p = n_l = 1$, $n_s = 1$ in ws , $n_s = 10$ in s , $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.17 – Impact of the speculative activity on the basis

2.7 Conclusion

Our infinite horizon rational expectations equilibrium model shows the interaction, in a dynamic setting, between spot and futures markets for commodities. It exhibits a surprising variety of behaviors. In equilibrium, there might be a contango or a backwardation, the futures prices might be higher or lower than the expected spot price, inventories might be held or not, the commodity might be processed or not, and adding speculators might increase or decrease the hedging benefits. This variety of situations is found in real commodity markets. Moreover, this model is able to reproduce the dynamic behavior of spot and futures prices for a wide range of commodities including non-storable ones like electricity. It also gives insights about some important stylized facts for which the theoretical literature is scarce (Samuelson effect, link between the volatility of the basis and the storage costs). Our model explains why speculation can stabilize or destabilize markets, by moving expected prices or volatilities. We show that the functioning of a commodity market depends on the fundamentals of the physical market, the realization of shocks, and on past decisions. The predictions on the effect of speculation and financialization can be much more precise once these elements are considered.

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2.A Appendix: Equilibrium

2.A.1 The images of the regions at the equilibrium

In the space (F, P) , for a given v , we consider 6 regions:

- **Region 1:** $0 \leq P \leq Mm^{-1}$, $\Delta F > P$ and $Q > F$. In this Region, all agents are present.
- **Region 2:** $0 \leq P \leq Mm^{-1}$, $\Delta F > P$ and $Q < F$. In this Region there are no processors.
- **Region 3:** $0 \leq P \leq Mm^{-1}$, $\Delta F < P$ and $Q < F$. In this Region, there is no activity on the physical and the futures markets.
- **Region 4:** $0 \leq P \leq Mm^{-1}$, $\Delta F < P$ and $Q > F$. In this Region there are no storers.
- **Region 5:** $P \leq p_{min} = 0$
- **Region 6:** $P \geq p_{max} = Mm^{-1}$

We assume that:

$$p_{min} = 0 \leq Q \leq p_{max} = Mm^{-1}$$

$$M_{min} = 0 \leq M - m\Delta Q \leq M_{max} = M$$

Let us compute the image of these six regions:

$$\Phi: \begin{pmatrix} P \\ F \end{pmatrix} \rightarrow \begin{pmatrix} M - mP + n_I \max\{\Delta F - P, 0\} \\ F + \Delta\alpha v (n_I \max\{\Delta F - P, 0\} - n_P \max\{\Delta(Q - F), 0\}) \end{pmatrix} = \begin{pmatrix} z \\ e \end{pmatrix}$$

The point situated at the intersection of the four first regions is $M(\Delta Q, Q)$. Its image by Φ is the point $\Phi(M)(M - m\Delta Q, Q)$. We note \mathcal{R}_i , for $i = 1$ to 6 the images of the regions by Φ . Let us first consider the four half-lines emanating from $\Phi(M)$:

- \mathcal{R}_1 and \mathcal{R}_2 are separated by \mathcal{D}_{12} , which is the image of the half-line $Q = F > 0$, with $\Delta F > P$:

$$\mathcal{D}_{12} = \left\{ \begin{pmatrix} M - mt + n_I(\Delta Q - t) \\ Q + \Delta\alpha v n_I(\Delta Q - t) \end{pmatrix} \mid t < \Delta Q \right\}$$

It is the northeastward half-line emanating from $\Phi(M)$, with the slope $\frac{\Delta\alpha v n_I}{m + n_I}$. The equation of \mathcal{D}_{12} is :

$$e = \frac{\alpha v \Delta n_I}{m + n_I} (z - M + m\Delta Q) + Q$$

- \mathcal{R}_2 and \mathcal{R}_3 are separated by \mathcal{D}_{23} , which is the image of the half-line $\Delta F = P > 0$, with $Q < F$:

$$\mathcal{D}_{23} = \left\{ \begin{pmatrix} M - mt \\ \frac{t}{\Delta} \end{pmatrix} \mid t > \Delta Q \right\}$$

It is the northwestward half-line emanating from $\Phi(M)$, with the slope $-\frac{1}{\Delta m}$. The equation of \mathcal{D}_{23} is:

$$e = \frac{(M - z)}{m\Delta}$$

- \mathcal{R}_3 and \mathcal{R}_4 are separated by \mathcal{D}_{34} , which is the image of the half-line $Q = F > 0$, with $\Delta F < P$:

$$\mathcal{D}_{34} = \left\{ \left(\begin{array}{c} M - mt \\ Q \end{array} \right) \mid t > \Delta Q \right\}$$

It is the horizontal half-line emanating from $\Phi(M)$, directed toward the west.

- \mathcal{R}_4 and \mathcal{R}_1 are separated by \mathcal{D}_{41} , which is the image of the half-line $P = \Delta F > 0$, with $Q > F$:

$$\mathcal{D}_{41} = \left\{ \left(\begin{array}{c} M - mt \\ \frac{t}{\Delta} - \Delta \alpha \nu n_p (\Delta Q - t) \end{array} \right) \mid t < \Delta Q \right\}$$

It is the half-line with the slope $-\frac{1}{\Delta m} (1 + \Delta^2 \alpha \nu n_p)$ emanating from $\Phi(M)$ and directed toward the south-east:

$$e = \frac{(1 + \alpha \nu \Delta^2 n_p)}{m \Delta} (M - z) - \alpha \nu \Delta^2 n_p Q$$

- \mathcal{R}_4 and \mathcal{R}_5 are separated by \mathcal{D}_{45} which is the image of the half-line $P = 0$, $\Delta F \leq 0$:

$$\mathcal{D}_{45} = \left\{ \left(\begin{array}{c} M \\ (t - \alpha \nu \Delta^2 n_p (Q - t)) \end{array} \right) \mid t \leq 0 \right\}$$

It is the vertical half-line emanating from the point $(M, -\alpha \nu \Delta^2 n_p Q)$ and directed toward the south.

- \mathcal{R}_1 and \mathcal{R}_5 are separated by \mathcal{D}_{15} which is the image of $P = 0$, $0 \leq F \leq Q$:

$$\mathcal{D}_{15} = \left\{ \left(\begin{array}{c} M + n_1 \Delta t \\ t + \alpha \nu \Delta^2 [n_1 t - n_p (Q - t)] \end{array} \right) \mid 0 \leq t \leq Q \right\}$$

It is the segment that associates the point: $(M + n_1 \Delta Q, Q(1 + \alpha \nu \Delta^2 n_1))$ to the preceding point (on \mathcal{D}_{45}).

- \mathcal{R}_2 and \mathcal{R}_5 are separated by \mathcal{D}_{25} that is the image of $P = 0$, $F \geq Q$. This gives, in parametric:

$$\mathcal{D}_{25} = \left\{ \left(\begin{array}{c} M + n_1 \Delta t \\ t(1 + \Delta^2 n_1 \alpha \nu) \end{array} \right) \mid t \geq Q \right\}$$

It is the half line with the slope $\left(\frac{1 + \Delta^2 n_1 \alpha \nu}{\Delta n_1}\right)$ emanating from the preceding point and directed toward North-East.

- \mathcal{R}_4 and \mathcal{R}_6 are separated by \mathcal{D}_{46} that is the image of the half-line $P = M m^{-1}$, $F \leq Q$. This gives, in parametric:

$$\mathcal{D}_{46} = \left\{ \left(\begin{array}{c} 0 \\ t - \alpha \nu \Delta^2 n_p (Q - t) \end{array} \right) \mid t \leq Q \right\}$$

It is the vertical half-line emanating from $(0, Q)$ and directed toward the south.

- \mathcal{R}_3 and \mathcal{R}_6 are separated by \mathcal{D}_{36} that is the image of the segment $P = Mm^{-1}$, $Q \leq F \leq Mm^{-1}$. In parametric, we have:

$$\mathcal{D}_{36} = \left\{ \left(\begin{array}{c} 0 \\ t \end{array} \right) \mid Q \leq t \leq M(m\Delta)^{-1} \right\}$$

It is the vertical segment that make the junction between the preceding point and the point: $(0, M(m\Delta)^{-1})$.

- \mathcal{R}_2 and \mathcal{R}_6 are separated by \mathcal{D}_{26} that is the image of the segment $P = Mm^{-1}$, $F \geq Mm^{-1}$. In parametric we have:

$$\mathcal{D}_{26} = \left\{ \left(\begin{array}{c} n_1(\Delta t - Mm^{-1}) \\ t + \Delta\alpha v n_1(\Delta t - Mm^{-1}) \end{array} \right) \mid t \geq Mm^{-1} \right\}$$

It is the half-line with the same slope than \mathcal{D}_{25} .

The frontiers between these regions are the following:

	Slope	Direction
\mathcal{D}_{12}	$(\alpha v \Delta n_1)(m + n_1)^{-1}$	N-E
\mathcal{D}_{23}	$-(m\Delta)^{-1}$	N-W
\mathcal{D}_{34}	0	W
\mathcal{D}_{41}	$-((1 + \alpha v \Delta^2 n_p))(m\Delta)^{-1}$	S-E

Only the frontiers of \mathcal{R}_1 , i.e. the two half-lines \mathcal{D}_{12} and \mathcal{D}_{41} depend on v . When v increases from 0 to $+\infty$, \mathcal{R}_1 becomes bigger. Indeed, \mathcal{D}_{12} , that was initially an horizontal half-line toward the East, becomes vertical and North oriented. Similarly, \mathcal{D}_{41} , initially in an horizontal position toward the West, becomes a South oriented vertical half-line.

2.A.2 The values of $P(z, e, v)$, $F(z, e, v)$ and $H(z, e, v)$ in the Regions 2 to 6

Region 2 If $(z, e) \in \mathcal{R}_2$, the processors are inactive. The equilibrium system (2.18) becomes:

$$\begin{aligned} z &= n_1(\Delta F - P) + M - mP \\ e &= F + \alpha\Delta v n_1(\Delta F - P) \end{aligned}$$

Then, the equilibrium functions are the same as in the Region 1, with $n_p = 0$. We find:

$$\begin{aligned} F(z, e, v) &= \frac{e(m + n_1) + \alpha\Delta v n_1(M - z)}{m + n_1 + \alpha\Delta^2 v n_1 m} \\ P(z, e, v) &= \frac{e\Delta n_1 + (M - z)[1 + \alpha\Delta^2 v n_1]}{m + n_1 + \alpha\Delta^2 v n_1 m} \\ H(z, e, v) &= \frac{e\Delta n_1 m - n_1(M - z)}{m + n_1 + \alpha\Delta^2 v n_1 m} \end{aligned}$$

Region 3 If $(z, e) \in \mathcal{R}_3$, nobody operates. The equilibrium equations can be written:

$$\begin{aligned} z &= M - mP \\ e &= F \end{aligned}$$

which gives:

$$\begin{aligned} F(z, e, v) &= e \\ P(z, e, v) &= \frac{M - z}{m} \\ H(z, e, v) &= 0 \end{aligned}$$

Region 4 If $(z, e) \in \mathcal{R}_4$, the storers do not operate on the physical market. With $n_I = 0$ the equilibrium equations become:

$$\begin{aligned} z &= M - mP \\ e &= F - \alpha\Delta^2 v n_P (Q - F) \end{aligned}$$

Thus:

$$\begin{aligned} F(z, e, v) &= \frac{e + \alpha\Delta^2 v n_P Q}{1 + \alpha\Delta^2 v n_P} \\ P(z, e, v) &= \frac{M - z}{m} \\ H(z, e, v) &= \frac{\Delta n_P (e - Q)}{1 + \alpha\Delta^2 v n_P} \end{aligned}$$

Region 5 In this case $(z, e) \in \mathcal{R}_5$, and:

$$P(z, e, v) = 0$$

We will note:

$$H(z, e, v) = H(\pi_5(e), e, v)$$

Region 6 If $(z, e) \in \mathcal{R}_6$ we have:

$$P(z, e, v) = P_{\max} = Mm^{-1}$$

We will note:

$$H(z, e, v) = H(\pi_6(e), e, v)$$

2.A.3 Analytical expressions of the basis

Based on the explicit formulas from Sections 2.4 and 2.A.2 we can compute the explicit basis $B_i = F_i - P_i$ in each region i :

$$B_1 = \frac{[n_1(1 - \Delta) + m][e + \alpha v \Delta^2 n_p Q] + (M - z)[n_1 \alpha v \Delta(1 - \Delta) - 1 - \alpha v \Delta^2 n_p]}{n_1 + m + \alpha v \Delta^2 [n_1 m + n_p(n_1 + m)]}$$

$$B_2 = \frac{[n_1(1 - \Delta) + m]e + (M - z)[n_1 \alpha v \Delta(1 - \Delta) - 1]}{n_1 + m + \alpha v \Delta^2 n_1 m}$$

$$B_3 = \frac{me - (M - z)}{m}$$

$$B_4 = \frac{m[e + \alpha v \Delta^2 n_p Q] - (M - z)[1 + \alpha v \Delta^2 n_p]}{m + \alpha v \Delta^2 n_p m}$$

2.B Appendix: Sensitivity to parameters

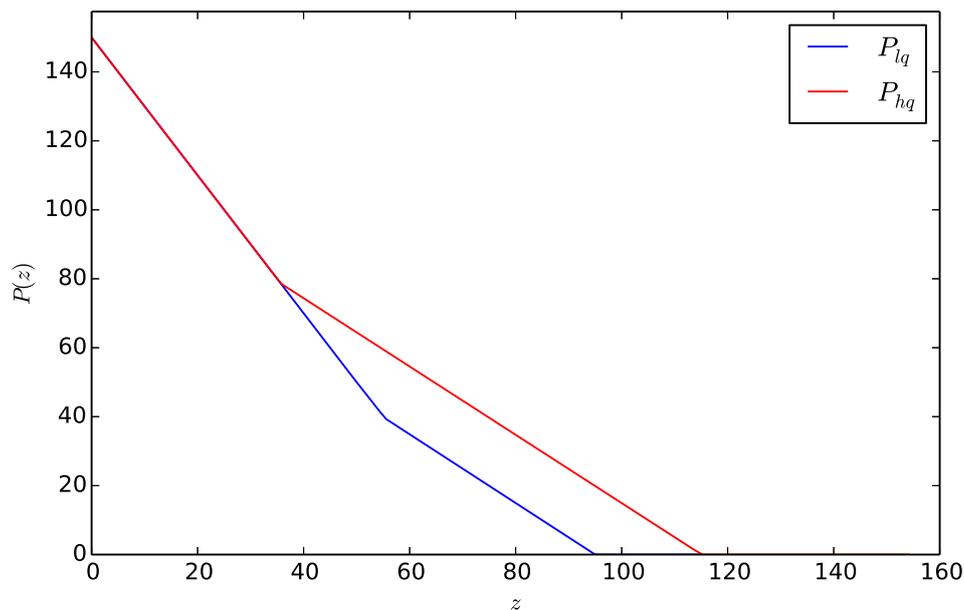
2.B.1 Sensitivity to the price level Q of the final product

In this appendix we study the sensitivity of our model to the price level of the final good (Q). We rely on the reference set of parameters, changing only the level of Q , for $Q = 40$ or $Q = 80$. A general result is that the areas at the equilibrium change with the level of Q . That is the situation of the market for a given level of supply changes with the price level of the final good.

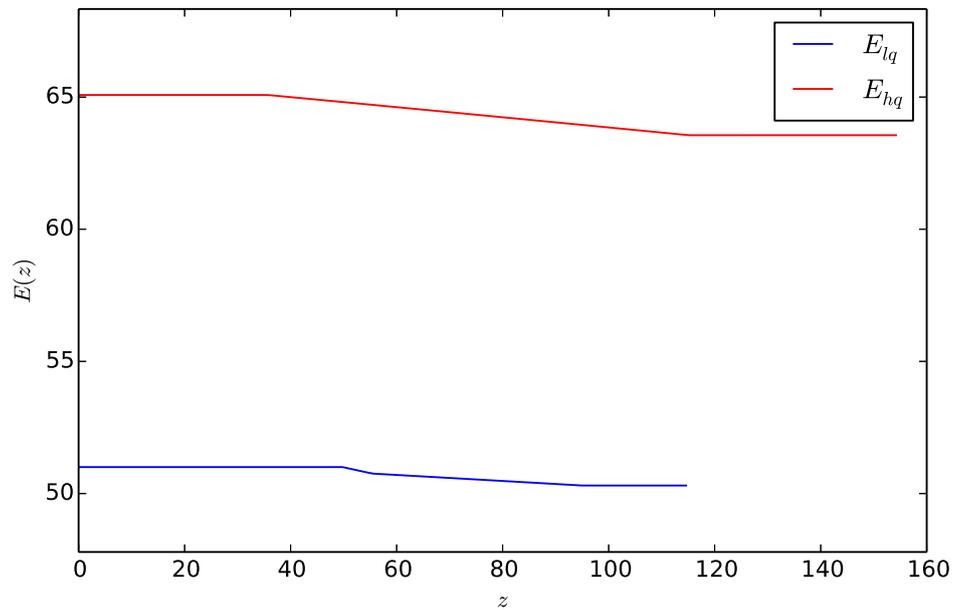
The impact on the level of prices

First, we look at the impact on the level of spot, expected spot and futures prices with Figure 2.18. The main result, which is the same for all prices, is that reducing Q leads to a decrease in the prices. This is the case because a lower price of the final good means a lower profit for the processor.

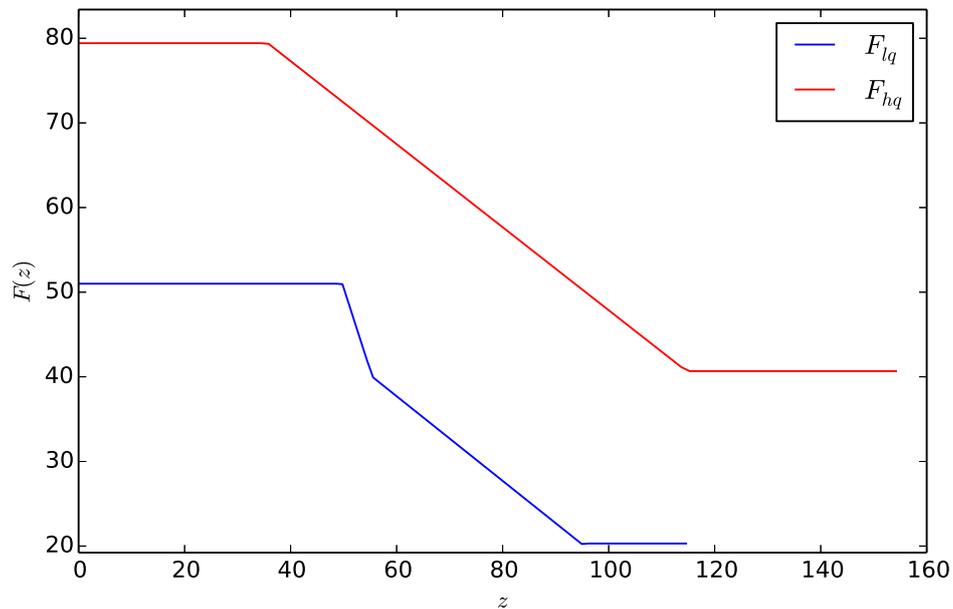
More precisely, the processing activity becomes less profitable, therefore processors buy less commodity on the spot market and hedge less on the futures market. As a consequence on the spot market, due to the lower demand both the spot price and the expected spot price decrease. On the futures market, the processors buy less futures contracts and the futures price decreases.



(a) Spot price



(b) Expected spot price



(c) Futures price

This figure shows the impact of a change in the level of the fixed price of the final good Q on spot, expected spot and futures prices. For each quantity, the blue line is for the function at the equilibrium with a low Q , the red line is for the function at the equilibrium with a high Q . The parameters used to solve our model are the following: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = n_I = 1$, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 40$ in low Q and $Q = 80$ in high Q , $N(\mu, \sigma) = N(50, 7)$.

Figure 2.18 – Impact of the price level of the final product on prices

The impact on the risk premium

Figure 2.19 shows the effects of a change in the price level Q of the final good on the risk premium (the futures price minus the expected spot price). Because at the equilibrium the areas are different in the two situations (low and high Q), the shapes of the risk premium according to the level of supply are very different.

For a low level of Q and for a market in a situation of scarcity (low z) nobody is active on the market and the risk premium is null. This is the case because storers are never active in tight markets, and because due to the low profitability of the processing activity processors are not active. However, when the supply on the market increases (higher z) storers begin to trade (buy the commodity and sell futures contracts). Their trading decreases the futures price and then creates a negative risk premium. At some point, the futures price becomes so low that the processing activity becomes profitable and processors enter the market.

For a high level of Q , the risk premium can be both negative and positive. In tight markets, processors buy the futures contract to hedge their activity (because the processing activity is economically profitable due to the high price level of the final good) and storers can not trade. Therefore, the risk premium is positive. When the supply on the market increases, storers trade more and more to sell the futures contracts, decreasing the risk premium (down to negative values).

The impact on the basis

Figure 2.20 shows the effects of a change in the price level Q of the final good on the basis (the futures price minus the spot price). In the two situations (low and high Q), the term structure is in backwardation when the market is in scarcity (low z) and in contango when there is more supply.

For a low price level of the final product, the backwardation (contango) is more (less) important. This is the case because the buying pressure on the futures contract from the processors is less important due to the non-profitability of the processing activity.

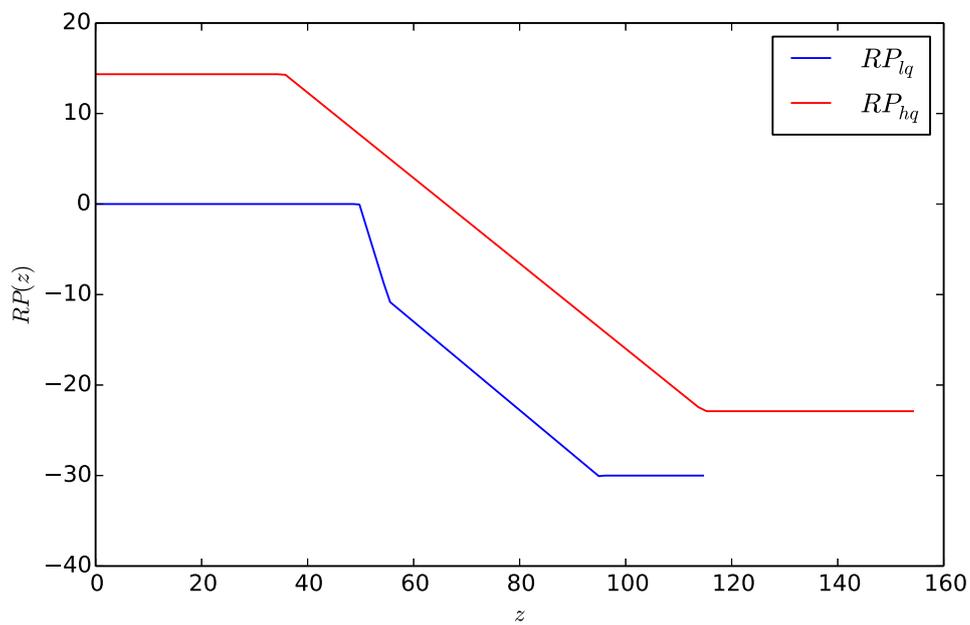
The impact on the variance of prices

We turn to the analysis of the impact of a decrease in Q on the variance of prices with the Figure 2.21. First, in both situations (low and high Q) the volatility of the spot price is a negative function of the current supply of the market (z).

Then, the volatility of the spot price decreases with the price level of the final good.

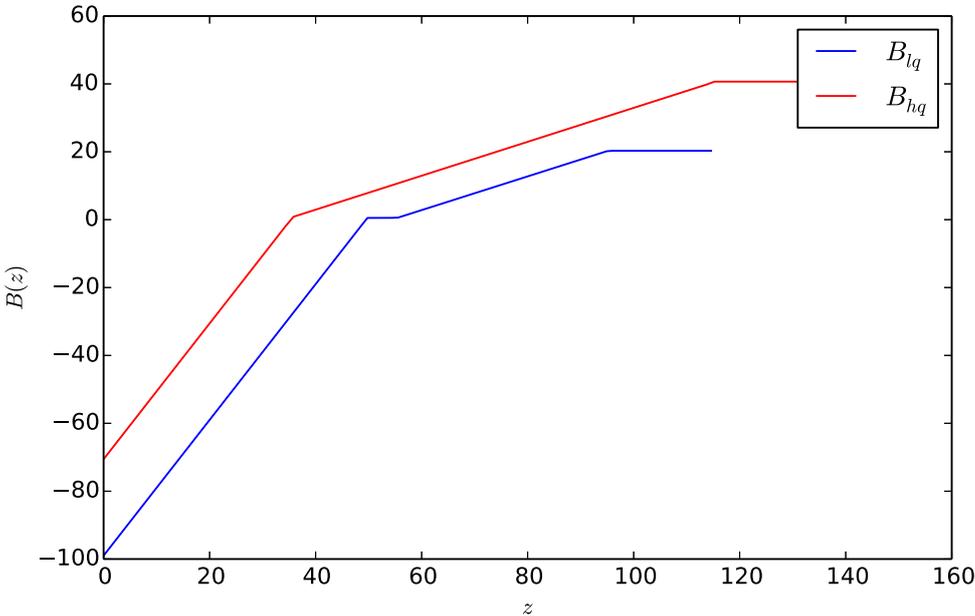
The impact on quantities

Figure 2.22 shows the effects of a change in the price level Q of the final good on quantities (or the hedging pressure). Because at the equilibrium the areas are different in the two



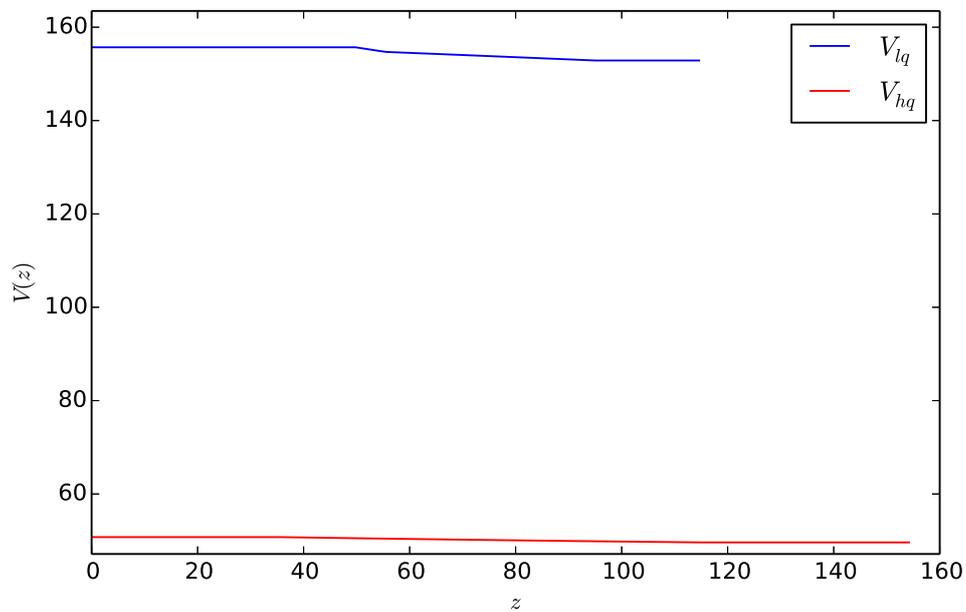
This figure shows the impact of a change in the level of the fixed price of the final good Q on the risk premium. For each quantity, the blue line is for the function at the equilibrium with a low Q , the red line is for the function at the equilibrium with a high Q . The parameters used to solve our model are the following: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = n_I = 1$, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 40$ in low Q and $Q = 80$ in high Q , $N(\mu, \sigma) = N(50, 7)$.

Figure 2.19 – Impact of the price level of the final product on the risk premium



This figure shows the impact of a change in the level of the fixed price of the final good Q on the basis. For each quantity, the blue line is for the function at the equilibrium with a low Q, the red line is for the function at the equilibrium with a high Q. The parameters used to solve our model are the following: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = n_I = 1$, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 40$ in low Q and $Q = 80$ in high Q, $N(\mu, \sigma) = N(50, 7)$.

Figure 2.20 – Impact of the price level of the final product on the basis



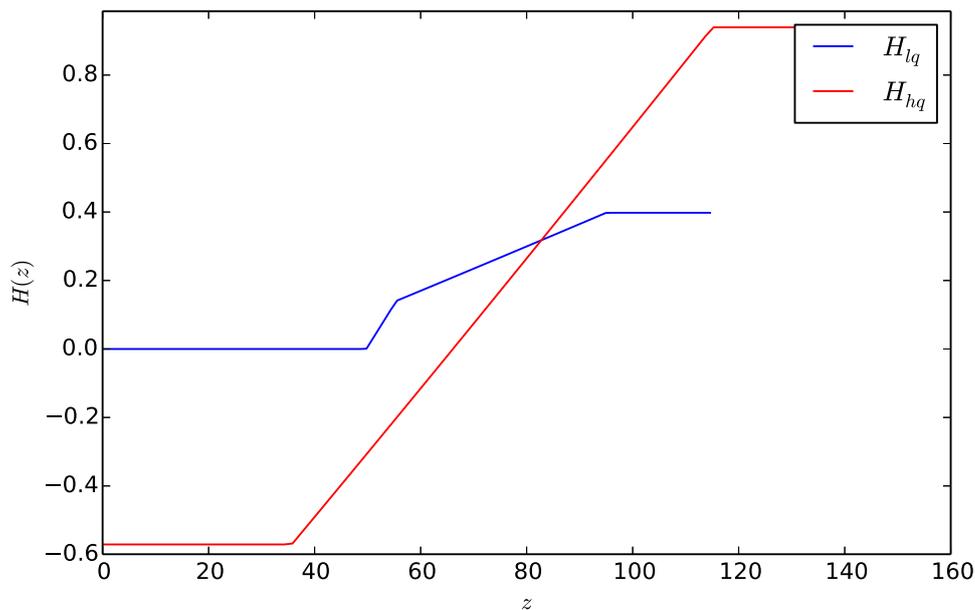
This figure shows the impact of a change in the level of the fixed price of the final good Q on the variance of the spot price. For each quantity, the blue line is for the function at the equilibrium with a low Q , the red line is for the function at the equilibrium with a high Q . The parameters used to solve our model are the following: $r = 1\%$, $\alpha_p = \alpha_l = \alpha_s = 2$, $n_p = n_l = 1$, $n_s = 2$, $m = 0.5$, $M = 75$, $Q = 40$ in low Q and $Q = 80$ in high Q , $N(\mu, \sigma) = N(50, 7)$.

Figure 2.21 – Impact of the price level of the final product on the variance

situations (low and high Q), the shapes of the hedging pressure h according to the level of supply are very different.

For a low level of Q and for a market in a situation of scarcity (low z) nobody is active on the market and the hedging pressure is null. This is the case because storers are never active in tight markets, and because due to the low profitability of the processing activity processors are not active. However, when the supply on the market increases (higher z) storers begin to trade (buy the commodity and sell futures contracts). Therefore the hedging pressure becomes positive.

For a high level of Q , the hedging pressure can be both negative and positive. In tight markets, processors buy the futures contract to hedge their activity (because the processing activity is economically profitable due to the high price level of the final good) and storers can not trade. Therefore, the hedging pressure is negative. When the supply on the market increases, storers trade more and more to sell the futures contracts, increasing the hedging pressure (up to positive values).



This figure shows the impact of a change in the level of the fixed price of the final good Q on quantities. For each quantity, the blue line is for the function at the equilibrium with a low Q , the red line is for the function at the equilibrium with a high Q . The parameters used to solve our model are the following: $r = 1\%$, $\alpha_p = \alpha_I = \alpha_S = 2$, $n_p = n_I = 1$, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 40$ in low Q and $Q = 80$ in high Q , $N(\mu, \sigma) = N(50, 7)$.

Figure 2.22 – Impact of the price level of the final product on quantities

2.B.2 Sensitivity to the structure of the market

In this appendix we look at the impact of the structure of the market (in term of number of agents) on prices and quantities. To do this we solve our model for three cases: a benchmark case identical to our reference set of parameters with $n_p = n_l = 1$ and $n_s = 2$, a case with an excess of processors ($n_p = 5$) and a case with an excess of storers ($n_l = 5$).

The impact on the level of prices

Figure 2.23 shows the effects of a change in the structure of the market on spot, expected spot and futures prices. The impact on each price is different.

First, there is no ambiguity on the spot price. When processors or storers are in excess compared to the benchmark case, the spot price stays the same for low values of z (situation of scarcity) and increases for high values of z (situation of abundance). This increase is due to a more important storage activity.

Then, the impacts on the expected spot price and on the futures price depends on the dominant agent. In a market with an excess of producers, both the expected spot price and the futures price increase. This is because the surplus of processors leads to a more important processing activity. This activity leads to an increase in the physical commodity bought on the spot market and in the futures contract bought for hedging purposes. As a consequence, both prices increase.

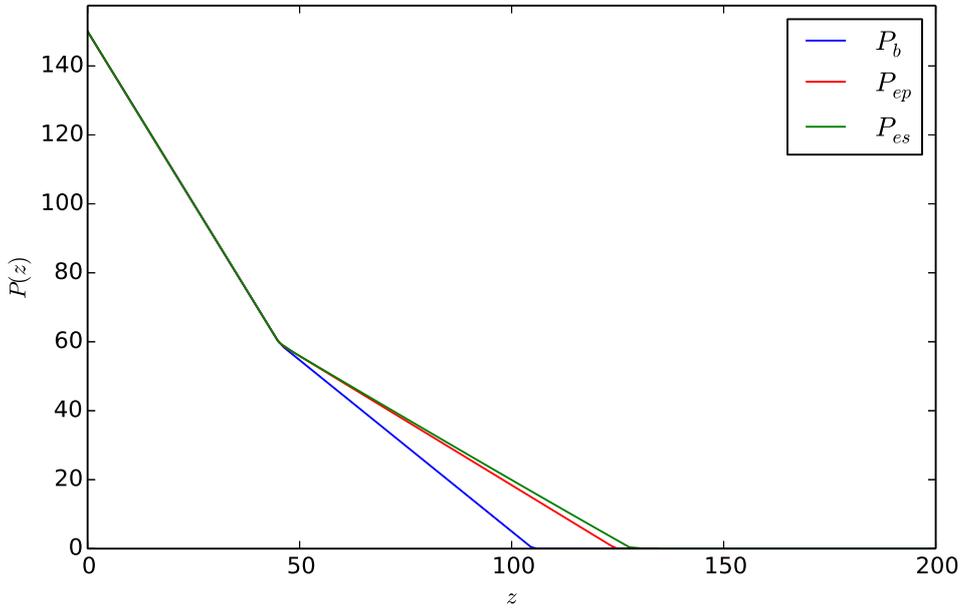
On the opposite, in a market with an excess of storers the expected spot price increases in periods of scarcity (low z) and decreases in periods of abundance (high z). This lower expected spot price in period of abundance can be explained by the more important storage activity, and then the more important expected release of inventories. In this situation, the futures price is also lower than in the benchmark case due to the selling pressure from the storers who hedge their activity.

The impact on the risk premium

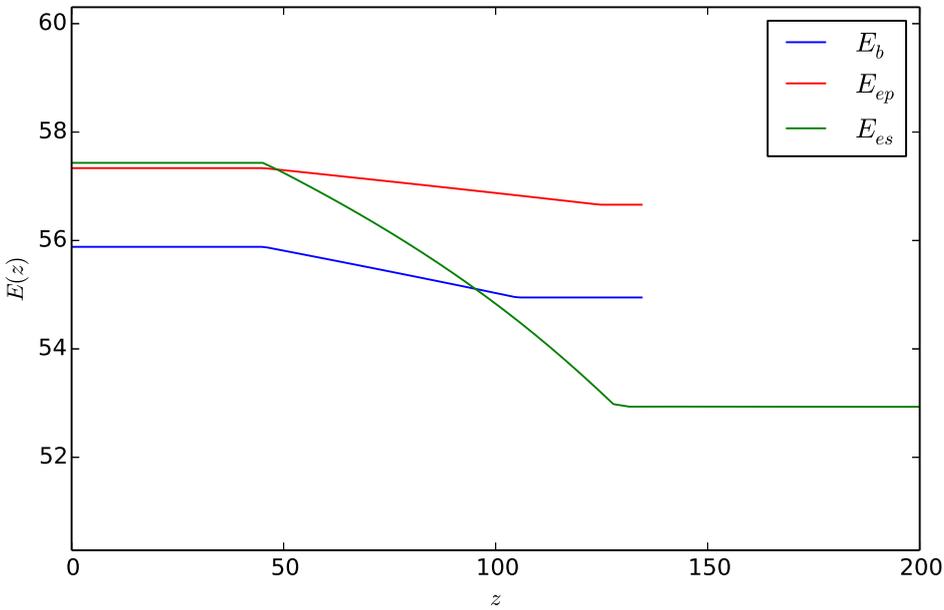
Figure 2.24 shows the impact of the structure of the market on the risk premium. In each case the risk premium is positive for low values of z , and negative for high values of z , but the impact is different depending of which type of agent is in excess.

In a market dominated by processors, the absolute value of the risk premium always decreases (regardless of the supply of the market).

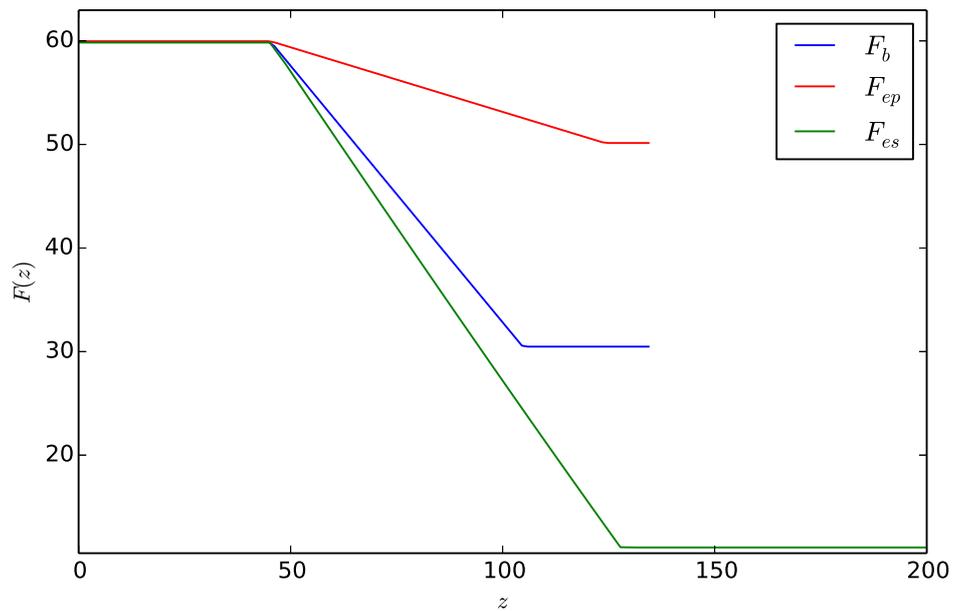
In a market dominated by storers, the risk premium decreases in situations of scarcity



(a) Spot price



(b) Expected spot price

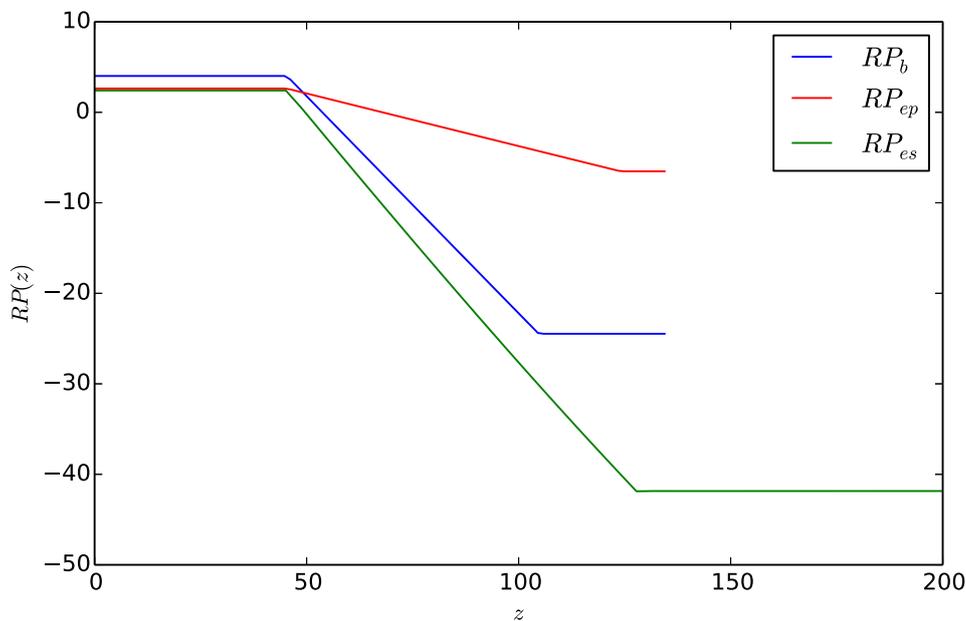


(c) Futures price

This figure shows the impact of a change in the structure of the market (number of the different agents) on spot, expected spot and futures prices. For each quantity, the blue line is for the function at the equilibrium in the benchmark case, the red line is for the function at the equilibrium with an excess of processors and the green line for the function at the equilibrium with an excess of storers. The parameters used to solve our model are the following: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = n_I = 1$ in the benchmark case, $n_P = 5$ in the excess of processors case, $n_I = 5$ in the excess of storers case, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.23 – Impact of the structure of the market on prices

but increases in absolute value in situations of abundance. This is because in abundance, storers are dominant on the market and when their number increases the market becomes even more unbalanced.

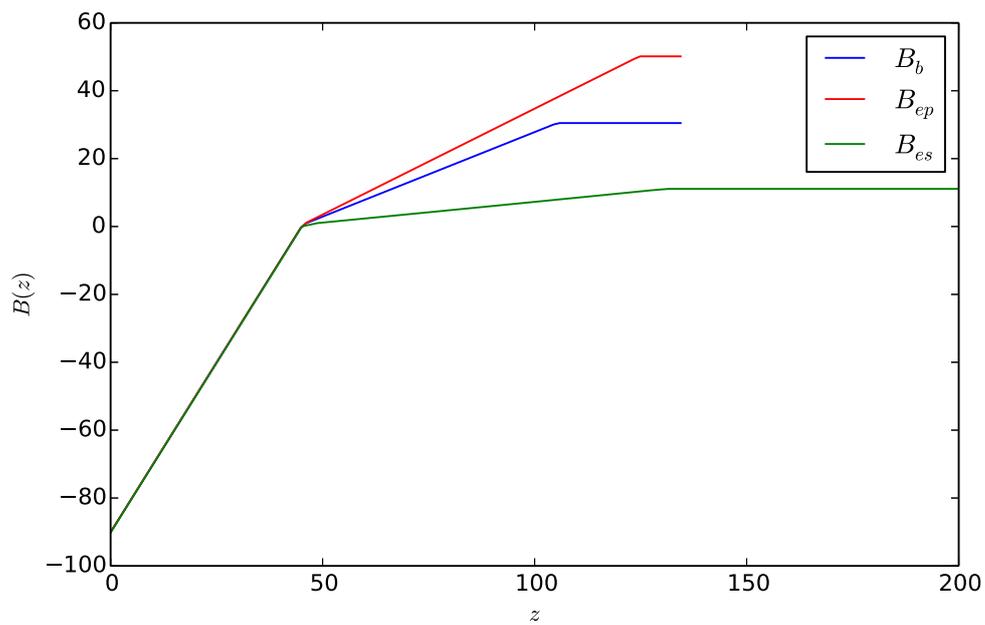


This figure shows the impact of a change in the structure of the market (number of the different agents) on the risk premium (futures price minus expected spot price). The blue line is for the function at the equilibrium in the benchmark case, the red line is for the function at the equilibrium with an excess of processors and the green line for the function at the equilibrium with an excess of storers. The parameters used to solve our model are the following: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = n_I = 1$ in the benchmark case, $n_P = 5$ in the excess of processors case, $n_I = 5$ in the excess of storers case, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.24 – Impact of the structure of the market on the risk premium

The impact on the basis

Figure 2.25 shows the impact of the structure of the market on the basis. It shows that in backwardation, for low levels of z , the structure of the market has no effect. However, in contango, the slope of the term structure of prices is impacted by the structure of the market. Indeed, in a market dominated by processors, the slope increases and the term structure of futures prices becomes steeper. On the contrary, in a market dominated by storers, the slope decreases and the term structure of futures prices becomes flatter. This is because, by their activity, storers arbitrage the term structure of futures prices (buy the commodity, sell the futures contract), whereas processors have an impact on the futures price but not on the spot price.

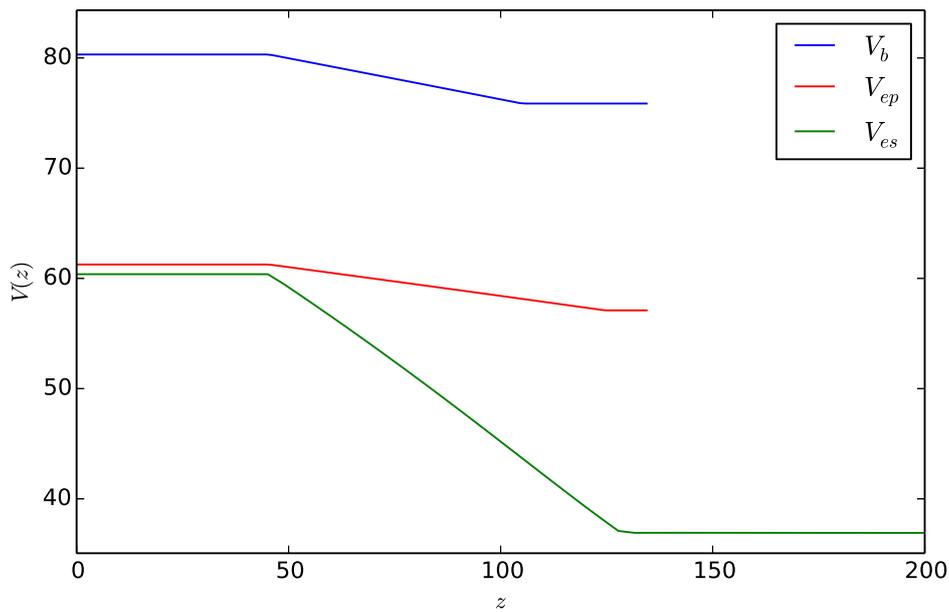


This figure shows the impact of a change in the structure of the market (number of the different agents) on the basis (futures price minus spot price). The blue line is for the function at the equilibrium in the benchmark case, the red line is for the function at the equilibrium with an excess of processors and the green line for the function at the equilibrium with an excess of storers. The parameters used to solve our model are the following: $r = 1\%$, $\alpha_p = \alpha_I = \alpha_S = 2$, $n_p = n_I = 1$ in the benchmark case, $n_p = 5$ in the excess of processors case, $n_I = 5$ in the excess of storers case, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.25 – Impact of the structure of the market on the basis

The impact on the variance of prices

Figure 2.26 shows the impact of the structure of the market on the variance of the spot price. Overall, it shows that the variance of prices decreases with the number of agents, but this is even more the case when the number of storers increases. This is because storers can move the commodity between different dates in order to smooth the variation of the spot price.



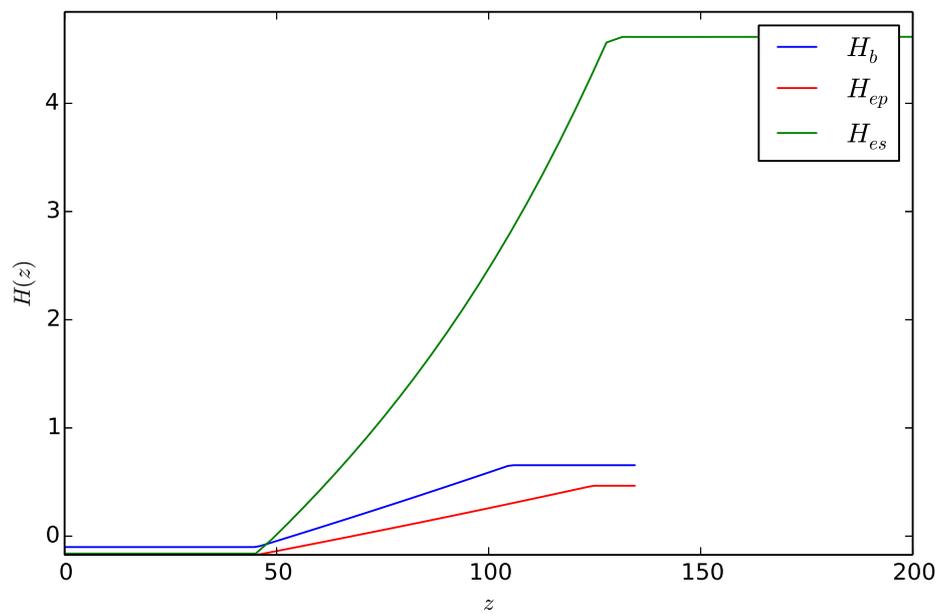
This figure shows the impact of a change in the structure of the market (number of the different agents) on the variance of the spot price. The blue line is for the function at the equilibrium in the benchmark case, the red line is for the function at the equilibrium with an excess of processors and the green line for the function at the equilibrium with an excess of storers. The parameters used to solve our model are the following: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = n_I = 1$ in the benchmark case, $n_P = 5$ in the excess of processors case, $n_I = 5$ in the excess of storers case, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.26 – Impact of the structure of the market on the variance

The impact on quantities

Figure 2.27 shows the impact of the structure of the market on the quantities (or the hedging pressure). First, it shows that when the number of processors increases the hedging pressure decreases in absolute value.

Then, it shows that when the number of storers increases, the hedging pressure decreases in situations of scarcity but increases in situation of abundance. This is because storers are not really active in tight markets but are very important during periods of abundance.



This figure shows the impact of a change in the structure of the market (number of the different agents) on the quantities. The blue line is for the function at the equilibrium in the benchmark case, the red line is for the function at the equilibrium with an excess of processors and the green line for the function at the equilibrium with an excess of storers. The parameters used to solve our model are the following: $r = 1\%$, $\alpha_P = \alpha_I = \alpha_S = 2$, $n_P = n_I = 1$ in the benchmark case, $n_P = 5$ in the excess of processors case, $n_I = 5$ in the excess of storers case, $n_S = 2$, $m = 0.5$, $M = 75$, $Q = 60$ and $N(\mu, \sigma) = N(50, 7)$.

Figure 2.27 – Impact of the structure of the market on quantities

Part II

L'évolution de la fonction de partage des risques avec la financiarisation

Chapter 3

The financialization of the term structure of risk premia in commodity markets¹

Abstract

In this paper, I examine how financialization affects the term structure of risk premia by using an equilibrium model for commodity futures markets. I define financialization as the entry of cross-asset investors, who are exposed to a commodity risk, into a commodity market. Qualitatively, the model shows that the financialization decreases the segmentation between commodity markets and the stock market. It also shows that speculators and investors both provide and consume liquidity and that the *investment pressure* from investors creates new risk premia. Further the model shows that financialization affects the entire term structure of risk premia. Quantitatively, these effects depend on the physical characteristics of the commodity market under study.

1. This chapter is based on an article available on SSRN.

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3.1 Introduction

Since 2000, commodity futures markets have experienced an in-depth modification in their trading participation, also known as financialization. This evolution is characterized by the entry of new investors and is symptomatic of the recent prevailing view of commodities as a financial asset class. As estimated by the CFTC [2008], these new investors have led to the transfer of \$200 billions in investment flows from traditional asset classes to various commodity futures markets between 2000 and 2008. The reasoning behind this appetite for commodity markets is "low-cost" diversification. The opportunities for diversification in these markets come from the development of new investment vehicles for index investing (Commodity Index Traders or CITs, Exchange Traded Funds or ETFs...) and from the historical segmentation of commodity markets with traditional asset classes.

The focus of this paper is to theoretically examine the consequences of this financialization process for commodity futures markets and their participants. The focus is on the risk sharing function of these markets. This is important because financial investors see commodity markets through the lens of index investments. Therefore, they may have an aggregated view of commodities as another financial asset class. However, commodity markets are complex economic systems which fulfill a role in the global economy through their risk sharing and price discovery functions. Moreover, they have some particularities. First, no single unique commodity market exists, instead there are multiple heterogeneous markets with their own specific physical characteristics. These characteristics are the most important determinants of the prices and risk premia. As a consequence, the dynamic behaviors of commodity prices have different patterns across markets (e.g., very volatile electricity prices versus very stable gold prices). Second, for each commodity there are multiple futures contracts with different maturities trading simultaneously. This is known as the term structure of the futures prices. This structure allows commodity markets to perform their fundamental economic functions (risk sharing and price discovery). This maturity component is well-known in the economics literature dedicated to commodity markets but has largely been ignored in the literature regarding financialization. This gap exists because the investment strategies for commodity indices rely on taking important positions on short-term futures contracts and rolling them.

Because of these specific features of commodity markets, financialization naturally raises questions: What is the effect on the risk sharing function of commodity markets? Do the determinants of the risk premia evolve with the trading of cross-asset investors? Do these new investors act as the suppliers or consumers of liquidity? Does the heterogeneity of commodity markets make the effects of the financialization market specific? Another natural question is whether financialization changes the level of segmentation both between commodity markets and between commodity markets and other asset classes? Further, it is important

to know if the financialization changes the shape of the term structure of the risk premia. In other words, are the risk premia for different maturities affected equally by the specific investment behaviors of outside investors?

The answers to these questions are of great interest for a variety of agents that range from traditional market participants to regulators. First, industrial agents use commodity futures markets daily to hedge their physical exposure to commodity prices. The existence and the good functioning of these markets allow them to secure their activity, by accepting to pay a cost of hedging: the risk premium. As a consequence, any structural modification on commodity futures markets due to the financialization process, good or bad, affects them. Therefore, they may have to adjust their behavior, for instance by using more short- or long-term futures contracts, if the modification affects the shape of the term structure of the risk premia. Thus, any change in the cost of hedging to industrial agents is likely to spillover to consumers. Second, these questions are useful to understand the evolution of the market making industry in commodity markets. This industry is composed of specialized speculators providing liquidity to hedgers in exchange for the risk premia. Therefore, the study of the role of cross-asset investors helps in understanding whether financialization increases the competition among traditional speculators (market makers) or whether it creates new profit opportunities for them. Further, by being in charge of the well-being of the economy, regulators need to have a good understanding of the implications of the financialization process in commodity markets. For instance, they have to make decisions relative to the regulation of investments or the optimal design of clearing houses. They may face a trade-off between having more or less segmented markets. On the one hand, less segmentation can cause more efficient risk sharing but on the other hand it can reinforce spillover effects between markets and increase the systemic risk of the global system.

To answer these questions, I develop a three-date equilibrium model of commodity futures markets with limited participation. This model follows Hirshleifer [1988] and Boons, De Roon, and Szymanowska [2014] in which traditional agents (producers and speculators) can face new cross-asset investors who hold a commodity risk. All agents in the economy are risk averse and try to maximize their expected utility under the mean-variance framework by choosing their optimal positions on the futures markets. The specialized speculators and cross-asset investors have short-term horizons. The producers, who have random production, have a long-term horizon and a preferred habitat. There are three risky assets: two futures contracts on the same commodity with different maturities (the term structure) and one stock market index. Because the focus of the paper is on the study of the behavior of commodity futures markets, the model of the stock market is basic.

To study the effect of financialization on the term structure of risk premia in commodity markets, I adopt a multistage process. First, the model is solved for a pre-financialization economy in which only the producers and the speculators can trade in the futures markets.

This economy provides a good comparison to the main model and also identifies known and new results regarding the determinants of the risk premia in commodity markets when a term structure exists. Then, the model is solved for two post-financialization economies by adding (un)constrained cross-asset investors to the analysis. In the first post-financialization economy, the investors are constrained to a position on the front-month contract only. In the second one, the investors are unconstrained and can trade all of the futures contracts available (both the front-month and the deferred contracts). The modeling of constrained investors is motivated by the fact that index investors hold most of the time positions in short-term futures contracts that they roll from month to month. In other words, the cross-asset investors use the majority of the investment flows to trade short-term futures contracts and ignore the other maturities. The modeling of the unconstrained investors accounts for the new, more sophisticated investment strategies that use contracts with longer maturities. These strategies have been developed in response to the recent durable situations of contango in the crude oil market. Indeed, in such a market configuration, a strategy based on rolling over short-term futures contracts leads to important roll losses. Buyuksahin, Haigh, Harris, Overdahl, and Robe [2009] describe this evolution in the WTI crude oil futures market between 2000 and 2008 as "open interest at maturities greater than one year grew nearly twice as fast as open interest at shorter maturities".

The model is solved analytically, but because of the complexity of the equilibrium equations, most of the analysis is done through visual representations (figures) of the risk premia for a specific set of parameters. These parameters are based on assumptions and on an empirical calibration of the crude oil market for the commodity futures market and on the S&P 500 for the stock market.

In the pre-financialization economy, I find the following: i) commodity markets are segmented from the stock market, that is, the performance of the stock market does not affect the risk premia in commodity futures markets; ii) the existence of a *hedging pressure* from producers is a necessary condition for the existence of risk premia that lead specialized speculators to enter the commodity markets only to allow the producers to share their risk; iii) speculators link the futures contracts for different maturities, particularly when producers have a preferred habitat and trade only some maturities; and iv) speculators act both as providers and consumers of liquidity because they have optimal diversified portfolios with futures contracts of different maturities.

In the post-financialization economies, I find the following: i) commodity markets become less segmented from the stock market; ii) regardless of the maturity, the *investment pressure* from investors creates a risk premium, even without hedging pressure from the producers. Therefore, the commodity markets do not exist only for the benefit of producers but also for the cross-asset investors who can hedge their commodity risk; iii) the financialization affects all of the risk premia along the term structure even if investors are constrained

to trade only the front-month contract. In other words, the optimal trading behaviors of the other agents in general and of the speculators in particular lead to a spillover of the stock market risk along the term structure. The intensity of the spillover depends on the level of integration of the prices for different maturities; and iv) investors act both as providers and consumers of liquidity.

More generally, my model emphasizes that the effect of financialization on the term structure of risk premia is market specific. That is, each commodity market according to its physical characteristics reacts differently to the financialization. As a consequence, some may profit from it and some may suffer.

The article is organized as follows: Section 3.2 has a review of the relevant literature. In Section 3.3 I describe the economic setting of the model. The optimality conditions and the equilibrium analysis are developed in Sections 3.4 and 3.5. Sections 3.6 and 3.7 contain the numerical analysis of the model and Section 3.8 concludes.

3.2 Related literature

This paper contributes to different strands of the literature on commodity futures markets. The first one is the important literature on the behavior of commodity prices. Theoretical papers by Anderson and Danthine [1983a,b], Hirshleifer [1988, 1989a,b], Acharya, Lochstoer, and Ramadorai [2013], Ekeland, Lautier, and Villeneuve [2016b] and Ekeland, Jaeck, Lautier, and Villeneuve [2016a] study the economic mechanisms underlying the joint process of spot and futures prices in static and dynamic frameworks. These models focus on the existence and the determinants of the risk premia for a commodity market without a term structure. They aim to replicate the empirical stylized facts of commodity markets described in important empirical studies like Fama and French [1987], Deaton and Laroque [1992] and Bhardwaj, Gorton, and Rouwenhorst [2015] for commodity prices and De Roon, Nijman, and Veld [2000] and Szymanowska, De Roon, Nijman, and Goorbergh [2014] for risk premia.

I contribute to this literature by extending the analysis of the determinants of the risk premia in a commodity market with a term structure. Indeed, for tractability reasons most of the papers that model both active spot and futures markets with heterogeneous risk-averse agents focus their analysis on one maturity. An equilibrium analysis with a term structure can be found in Routledge, Seppi, and Spatt [2000], but the risk neutral framework, by construction, makes any study of the risk premium impossible.

Second, this paper contributes to the literature dedicated to the *Preferred Habitat Theory*. This theory was introduced by Modigliani and Sutch [1966] for the term structure of

interest rates as an extension of the *Market Segmentation Hypothesis* emphasized by Culbertson [1957]. According to these authors, in the *Market Segmentation Hypothesis* the hedgers have different time preferences and only trade futures contracts of a specific maturity. As a consequence, the futures prices (and risk premia) for different maturities are determined in separate markets by their own supply and demand schedules. They are independent. In the *Preferred Habitat Theory*, hedgers can when it is economically profitable, trade futures contracts with a different maturity than the one they prefer. Therefore, the arbitrage behavior of the hedgers links the risk premia for different maturities together. These theories have been adapted to commodity markets by Gabillon [1995], Lautier [2005], and Buyuksahin et al. [2009]. They have shown that segmentation exists in at least two parts of the term structure of crude oil futures prices.

My paper offers an alternative extension to the *Market Segmentation Hypothesis*. It shows that, even if producers do not exit their preferred habitat, the risk premia are not independent. This is because of the arbitrage behavior of the speculators.

Further, my paper contributes to the emerging literature regarding the so-called financialization of commodity markets. This literature has emerged as a consequence of the boom and bust cycles around 2008 in many commodities (crude oil and agricultural products) in parallel with an in-depth modification of the structure and the functioning of commodity markets (electronization, entry of new agents in the markets, development of new investment vehicles). Michael Master established the link between investment flows via the Commodity Index Traders (CITs) and the boom/bust cycle in his 2008 testimony to the US Senate. This link has motivated a lot of empirical studies that look for direct evidence of price distortion due to investment flows into commodity markets (see, e.g., Brunetti and Buyuksahin [2009], Buyuksahin and Harris [2011], Singleton [2013], Hamilton and Wu [2015] and Cheng and Xiong [2014] for a complete literature review). These studies have mixed results mainly because of the difficulty in finding a proper econometric procedure to tackle this kind of issue. As a consequence, this literature focuses now on how the financialization of commodity markets can modify their two fundamental economic functions: risk sharing and information discovery.

My paper sheds light on the link between the financialization and the modification of the risk sharing function in commodity markets. Historically, commodity markets have always been seen as segmented markets with little co-movements both between commodities² (Erb and Harvey [2006]) and between commodities and other asset classes (Gorton and Rouwenhorst [2006]). As a consequence, they are characterized by inefficient risk sharing. Indeed, this inefficiency in the risk sharing function of futures markets is one way to interpret the *theory of normal backwardation* of Keynes [1930]. It states that, because of unbalanced hedging

2. However, Pindyck and Rotemberg [1990] identify periods of excess co-movement between commodity prices.

needs, hedgers have to pay a premium to give outside speculators an incentive to enter the market and bear the risk. Hirshleifer [1988] formalizes this inefficient view in a model based on limited participation by outside speculators. Based on that, the question is whether the financialization helps to reduce the inefficiencies and the segmentation or whether it reinforces them. My paper, by studying the effect of the entry of investors bearing a commodity risk into futures markets on the term structure of risk premia, reinforces and complements the following results in the literature: i) Regarding the inefficiencies of the futures markets, both Hamilton and Wu [2014] and Baker [2016] show that financialization has helped to reduce the risk premia in the crude oil market. Hamilton and Wu [2014] justify this result by showing that the important investment flows from the commodity index funds take the opposite side of the hedgers. Baker [2016], on a model in the spirit of Hirshleifer [1988], focuses on the entry of households into commodity futures markets. Moreover, based on the trading of the commodity index traders, Brunetti and Reiffen [2014] reach the same conclusion for agricultural markets. Finally, the static model of Ekeland et al. [2016b] and its dynamic counterpart by Ekeland et al. [2016a] show that, even without the entry of a new agent, the risk premium decreases if the risk bearing capacity of the existing speculator increases. ii) Regarding the integration of commodity markets, the empirical studies show that after financialization, the commodity markets are more integrated because of the investment in commodity indices (see Tang and Xiong [2012]) and with other asset classes (see Silvennoinen and Thorp [2013], Buyuksahin and Robe [2014] and Boons et al. [2014]). Moreover, Basak and Pavlova [2016] provide a theoretical framework which confirms the previous empirical evidence and details clear economic mechanisms through which correlations can increase between equity and a commodity and among commodities. In this model, the financialization is described as the trading in commodity futures markets by institutional investors with index dependent preferences (the utility depends on the performance of a benchmark).

3.3 Economic setting

The time, the assets, and the markets: There are three dates in the model ($t = 0, 1, 2$), and agents have to make decisions during the two first. There are three different assets: The first is a risk free asset with a null risk free rate. The second is an index representing the stock market. As in Hirshleifer [1988, 1989a], I model the financial market through a representative index to focus on the results that link to the commodity markets. This index is traded by the investors at $t = 0, 1$, but the clearings of these markets are not under the scope of this paper. The return, expected return, and the variance of the stock index between $t - 1$ and t are respectively $R_{r,t}$, $\mu_{r,t}$, and $\sigma_{r,t}^2$. The third is a term structure of futures contracts. At $t = 0$, two futures contracts written on the same commodity are traded, one with maturity $t = 1$ (the front-month contract) and one with maturity $t = 2$ (the deferred contract). At $t = 1$,

only one contract stays alive because the first futures contract matures. Then, the second contract that matures at $t = 2$ becomes the front-month contract. These contracts lead to the clearing of three different markets: two markets at $t = 0$ for the front-month contract and the deferred contract and one market at $t = 1$ for the contract with maturity $t = 2$. The return, expected return, and the variance of a futures contract with maturity T between $t - 1$ and t are respectively $R_{F,t,T}$, $\mu_{F,t,T}$, and $\sigma_{t,T}^2$.

This modeling with three dates and two futures contracts of different maturities is the simplest way to accommodate both the existence of a term structure of prices and the tractability of the model.

The risk-averse agents: There are three different risk-averse agents with different time horizons and available sets of investments. The assumption of risk-averse agents has been extensively used in the literature (see, e.g., Hirshleifer [1988, 1989a,b], Boons et al. [2014], Ekeland et al. [2016a,b], and Baker [2016]). As emphasized by Bessembinder and Lemmon [2002], the corporate risk management literature motivates its use for the producers. As emphasized by Acharya et al. [2013], the limits to arbitrage literature motivates its use for the speculators. This assumption enables the solving of the model under the mean-variance framework.

The risk-averse agents are the following: First are the N_p *producers with a preferred habitat*³. There are two types of producers: one is short term and exists between $t = 0$ and $t = 1$, has random production at $t = 1$, and trades only the front-month contract at $t = 0$. The other is long term and exists at all three dates, has random production only at $t = 2$, and can trade only the futures contract maturing at $t = 2$ (i.e., the deferred contract at $t = 0$ and the front-month contract at $t = 1$). These producers are identical in terms of number and risk aversion and trade on the futures market for hedging purposes. At $t = 0, 1$, they choose their positions $f_{t+1,T}^p$ in the futures contract with maturity T to hold until $t + 1$. This idea of the existence of producers with heterogeneous time preferences dates back to the *Preferred Habitat Theory* of Modigliani and Sutch [1966] on the term structure of interest rates.

Second are the N_s *specialized speculators*. There are two successive generations of short-term speculators that trade only on the futures markets. They do not have physical exposure to the commodity. At $t = 0, 1$, they choose position $f_{t+1,T}^s$ in the futures contract with maturity T to hold until $t + 1$. The first generation exists between $t = 0$ and $t = 1$ and has access at $t = 0$ to both the front-month and the deferred contracts. All positions that are initiated in the deferred futures contract which does not mature at $t = 1$ have to be canceled on the markets at $t = 1$. The second generation exists between $t = 1$ and $t = 2$ and has access at $t = 1$ to the remaining front-month contract.

3. Another version of the model is developed in Appendix 3.C. In this case there is only one long-term producer who exists at all three dates, has random productions at $t = 1$ and $t = 2$, and can trade all futures contracts without liquidity or regulatory issues. The results are qualitatively the same.

The third are the N_{in} *cross-asset investors*. There are two successive generations of short-term investors. Initially, they only trade on the stock market but begin to trade on the futures markets with the financialization. They hold a commodity risk and then have an incentive to trade in the futures markets for hedging purposes. At $t = 0, 1$, they choose position $f_{t+1,T}^w$ in the futures contract with maturity T to hold until $t + 1$, and they choose position w_t on the stock market index. I successively model two versions of investors in this paper: i) The constrained investors who only trade the front-month contracts, that is, at $t = 0$ ($t = 1$) they trade the contract maturing at $t = 1$ ($t = 2$). This case is motivated by the predominance of CITs in the futures markets during a long time period. ii) The unconstrained investors who can trade all of the futures contracts, that is, the first generation of investors at $t = 0$ can trade both the front-month and the deferred contracts. Furthermore, at $t = 1$, there is no difference between these cases because only one futures contract can be traded.

The modeling of hedgers in the commodity futures markets as producers is made for tractability reasons but is consistent with the literature. Indeed, the articles by Hirshleifer [1988], Acharya et al. [2013], and Boons et al. [2014] are based on the same assumption. Moreover, in the spirit of the *theory of normal backwardation* of Keynes [1930], the empirical studies (see Cheng and Xiong [2014]) show that aggregated hedgers on the futures markets more often short a commodity than go long. This fact can be explained by the predominance of producers on the futures market who have a natural long exposure to the commodity but a short position on the futures markets.

Finally, the modeling of speculators and investors with a short-term horizon is motivated by Kang, Rouwenhorst, and Tang [2014]. They show that the trading behavior of speculators and hedgers is different and more precisely that the former trade more impatiently.

The randomness and the physical market: The productions \tilde{q}_t of the producers at time $t = 1, 2$ are random. The modeling of physical decisions (production or storage) is not under the scope of this article. The \tilde{q}_1 and \tilde{q}_2 are assumed to be independent and normally distributed. At the market level, the aggregated production at time t is $\tilde{Q}_t = N_p * \tilde{q}_t$. On the spot market, this supply faces the linear demand of consumers characterized by the inverse demand function $Q_t^D = g(S_t)$ for the commodity. Therefore, the spot price S_t depends on the available quantity on the spot market and is such that $\tilde{Q}_t = Q_t^D$. The return and variance of the spot price between $t - 1$ and t are respectively $R_{s,t}$ and $\sigma_{s,t}^2$.

For convenience, Appendix 3.A synthesizes the parameters and their notations.

3.4 Optimal behavior of the agents

The first step of the analysis is to find the optimal positions of the different risk-averse agents in a mean-variance framework. That is, agent i with risk aversion γ_i at t maximizes his expected utility by solving the following problem:

$$\max_{f_{t+1,T}^i} E_t[\pi_{t+1}] - \frac{\gamma_i}{2} \text{Var}_t[\pi_{t+1}] \quad (3.1)$$

3.4.1 Short-term specialized speculators

As mentioned in Section 3.3, there are two successive generations of short-term speculators trading only on the futures markets without constraints on the traded contracts. They choose their position $f_{t,T}^s$ in the futures contract with maturity T to hold until t . Because the set of investments available for the first generation is bigger than those for the second generation, the profits at $t = 1$ and $t = 2$ of speculators from the first and the second generation are respectively $\pi_1 = R_{F_{1,1}} f_{1,1}^s + R_{F_{1,2}} f_{1,2}^s$ and $\pi_2 = R_{F_{2,2}} f_{2,2}^s$.

Without loss of generality, the two generations of speculators are assumed to be identical in terms of number and risk aversion, that is, $N_s^1 = N_s^2 = N_s$ and $\gamma_s^1 = \gamma_s^2 = \gamma_s$.

At $t = 1$, solving the problem (3.1) for a speculator of the second generation gives the following optimal position:

$$f_{2,2}^{s*} = \frac{\mu_{F_{2,2}}}{\gamma_s \sigma_{2,2}^2} \quad (3.2)$$

This position is a pure speculative position, which is well-known in the literature on commodity derivative markets with one period and one commodity (see, e.g., Anderson and Danthine [1983a], Ekeland et al. [2016a,b], and Boons et al. [2014]). The speculator takes a long (short) position whenever the risk premium is positive (negative), that is, whenever he thinks that the expected spot price is higher (lower) than the futures price. This speculative position is adjusted by the risk aversion of the speculator and by the risk of the futures contract.

At $t = 0$, solving the problem (3.1) for a speculator of the first generation gives rise to a more general result:

$$f_{1,1}^{s*} = \frac{\mu_{F_{1,1}} \sigma_{1,2}^2 - \mu_{F_{1,2}} \sigma_{[11,12]}}{\gamma_s (\sigma_{1,1}^2 \sigma_{1,2}^2 - \sigma_{[11,12]}^2)} \quad (3.3)$$

$$f_{1,2}^{s*} = \frac{\mu_{F_{1,2}} \sigma_{1,1}^2 - \mu_{F_{1,1}} \sigma_{[11,12]}}{\gamma_s (\sigma_{1,1}^2 \sigma_{1,2}^2 - \sigma_{[11,12]}^2)} \quad (3.4)$$

where $\sigma_{[t_1 T_1, t_2 T_2]}$ is the covariance between the returns $R_{F_{t_1, T_1}}$ and $R_{F_{t_2, T_2}}$.

The optimal position in each futures contract has two components: i) a traditional speculative component which depends on the risk premium and the riskiness of the contract; and ii) a diversification component which depends on the risk premium attached to the other contract, on the covariance with the other contract, and on its riskiness. In other words, because of the existence of a term structure, the speculator does not allocate all of his wealth to one contract but creates an optimal portfolio of futures contracts.

3.4.2 Producers with a preferred habitat

As the speculators, the producers trade only on the futures market, but they hold a physical exposure to the commodity through their random production \tilde{q}_t . As a consequence, they choose their position $f_{t,T}^p$ in the futures contract with maturity T to hold until t primarily for hedging purposes.

My modeling is based primarily on the *Market Segmentation Hypothesis* because the producers can never trade in the other futures contract, that is, there are two producers: one short-term and one long-term. They are assumed to be identical in terms of number and risk aversion, that is $N_p^{sh} = N_p^l = N_p$ and $\gamma_p^{sh} = \gamma_p^l = \gamma_p$.

Optimal position of the short-term producer: At $t = 0$, solving the problem (3.1) with the producer's profit $\pi_1 = \tilde{q}_1 R_{s,1} + R_{F_{1,1}} f_{1,1}^p$ leads to the following result:

$$f_{1,1}^{p*} = \frac{\mu_{F_{1,1}}}{\gamma_p \sigma_{1,1}^2} - \frac{\rho_{[1,11]}}{\sigma_{1,1}^2} \quad (3.5)$$

where $\rho_{[t,t_1 T_1]}$ is the covariance between the physical revenue between $t - 1$ and t and the return $R_{F_{t_1, T_1}}$.

This position has two components and is well-known in the literature on commodity derivative markets (see, e.g., Anderson and Danthine [1983a], Ekeland et al. [2016a,b], and Boons et al. [2014]). The first one is a speculative component and is exactly the position of the speculator in the same period. The second component is the hedging component. It depends on both the riskiness of the futures contract and on the physical exposure of the producer to the commodity. This exposure is characterized by the covariance $\rho_{[1,11]}$ of its physical revenue with the futures price. If this covariance is positive, then the futures price increases when the producer's revenue increases. Therefore, in order to be hedged the producer must short the futures contract. Then, if the futures price decreases, his gains on buying back the futures contract will compensate for the losses on the spot market. Because a producer typically has a long exposure to the commodity market, this covariance is assumed to be positive. As a consequence, the producer is naturally short on the futures market.

Optimal positions of the long-term producer: This producer maximizes the expected utility at the final date $t = 2$ which comes from his profit $\pi_2 = \pi_1 + \tilde{q}_2 R_{s,2} + R_{F_{2,2}} f_{2,2}^p$ with $\pi_1 = R_{F_{1,2}} f_{1,2}^p$.

At $t = 1$, solving the problem (3.1) with the producer's profit $\pi_2 = \pi_1 + \tilde{q}_2 R_{s,2} + R_{F_{2,2}} f_{2,2}^p$ leads to the following optimal position:

$$f_{2,2}^{p*} = \frac{\mu_{F_{2,2}}}{\gamma_p \sigma_{2,2}^2} - \frac{\rho_{[2,22]}}{\sigma_{2,2}^2} \quad (3.6)$$

This is the traditional speculative/hedging position when there is only one futures contract available in a one period setting.

At $t = 0$, the long-term producer solves the following problem:

$$\max_{f_{1,2}^p} E_0[\pi_2] - \frac{\gamma_p}{2} \text{Var}_0[\pi_2] \quad (3.7)$$

Which leads to the following optimal position $f_{1,2}^{p*}$:

$$f_{1,2}^{p*} = \frac{\mu_{F_{1,2}}}{\gamma_p \sigma_{1,2}^2} - \frac{\mu_{F_{2,2}} \sigma_{[12,22]}}{\gamma_p \sigma_{1,2}^2 \sigma_{2,2}^2} + \frac{\rho_{[2,22]} \sigma_{[12,22]}}{\sigma_{1,2}^2 \sigma_{2,2}^2} - \frac{\rho_{[2,12]}}{\sigma_{1,2}^2} \quad (3.8)$$

This futures position has three components: i) the speculative component; ii) an intertemporal diversification component which depends on the expected return of the same futures contract one period ahead and on the auto-correlation of the futures contract; and iii) an intertemporal hedging component which depends on the covariance of the futures contract with the physical revenue at the next period.

This hedging strategy by a producer implies an evolution of the hedging position over time.

3.4.3 Short-term cross-asset investors

There are two successive generations of short-term investors who hold a commodity risk (φ). As pointed out by Boons et al. [2014], the existence of this commodity risk can be motivated by at least two arguments: i) the inflation risk of the investor because commodity prices are an important and volatile component of inflation; and ii) the importance in terms of the investment-consumption decisions of the commodity prices. Following the literature (see Driesprong, Jacobsen, and Maat [2008]) and the idea that inflation decreases the real returns, investors are assumed to have a negative exposure to the commodity ($\varphi < 0$).

Initially, investors only trade on the stock market via a representative index⁴. Two costs can explain this non-diversification across assets (see Hirshleifer [1988] and Boons et al.

4. The use of an index does not qualitatively change the results because the focus of this article is on understanding the effect on commodity markets and not on the cross-section of the stock market.

[2014]). It can be an explicit cost of entry on alternative investments based on physical assets or an implied cost of becoming informed. Then, for exogenous reasons (for instance the decrease in the explicit entry cost on futures market thanks to the development of ETFs or CITS), investors start to trade on the futures markets. This entry of investors describes the financialization of commodity markets.

Optimal positions pre-financialization

This subsection first considers the optimal positions in the stock market index of the investors who do not participate in the futures market. The two successive generations of investors adopt the same behavior because they have the same set of investments (the stock market index) and the same kind of risks (commodity risk).

An investor who lives between $t - 1$ and t solves the problem (3.1) over his position w_t in the stock market index, with his profit $\pi_t = w_t R_{r,t} + \varphi_t R_{s,t}$ and optimally chooses the following position in the stock market index:

$$w_t^* = \frac{\mu_{r,t}}{\gamma_{in} \sigma_{r,t}^2} - \frac{\varphi_t \sigma_{[r,t,s_t]}}{\sigma_{r,t}^2}, \forall t = 1, 2 \quad (3.9)$$

where $\sigma_{[r,t,s_t]}$ is the covariance between the returns of the stock index and of the spot price between $t - 1$ and t .

This optimal position, due to the use of a stock market index, is a simplified version of the one in Boons et al. [2014]. The investor first invests in the stock market for speculative reasons. Then, because he is prevented from investing in the futures market, the investor hedges his commodity exposure by adjusting his position in the stock market index. The investor adjusts his position according to the risk φ and the covariance between the index and the spot price of the commodity.

Optimal positions of constrained investors

This subsection now considers the first type of financialization. In this case, investors trade only on the short-term part of the term structure of commodity futures prices (front-month contract). As before, the two successive generations of investors have the same behavior because they have the same set of investments (the stock market index and the front-month futures contract) and the same kind of risks (commodity risk).

The investors who live between $t - 1$ and t solve the problem (3.1) over their positions w_t in the stock market index and $f_{t,t}^w$ in the futures contract to hold until its maturity in t . With

profit $\pi_t = w_t R_{r,t} + \varphi_t R_{s,t} + f_{t,t}^w R_{F,t}$ they optimally choose the following positions:

$$w_t^* = \frac{\mu_{r,t} \sigma_{t,t}^2 - \mu_{F,t} \sigma_{[r_t, F_{t,t}]}}{\gamma_{in} (\sigma_{t,t}^2 \sigma_{r,t}^2 - \sigma_{[r_t, F_{t,t}]}^2)} + \frac{\varphi_t \{ \sigma_{[r_t, F_{t,t}]} \sigma_{[s_t, F_{t,t}]} - \sigma_{t,t}^2 \sigma_{[r_t, s_t]} \}}{(\sigma_{t,t}^2 \sigma_{r,t}^2 - \sigma_{[r_t, F_{t,t}]}^2)}, \forall t = 1, 2 \quad (3.10)$$

$$f_{t,t}^{w*} = \frac{\mu_{F,t} \sigma_{r,t}^2 - \mu_{r,t} \sigma_{[r_t, F_{t,t}]}}{\gamma_{in} (\sigma_{t,t}^2 \sigma_{r,t}^2 - \sigma_{[r_t, F_{t,t}]}^2)} + \frac{\varphi_t \{ \sigma_{[r_t, F_{t,t}]} \sigma_{[r_t, s_t]} - \sigma_{r,t}^2 \sigma_{[s_t, F_{t,t}]} \}}{(\sigma_{t,t}^2 \sigma_{r,t}^2 - \sigma_{[r_t, F_{t,t}]}^2)}, \forall t = 1, 2 \quad (3.11)$$

where $\sigma_{[s_t, F_{t_1, T_1}]}$ is the covariance between the return of the spot price between $t - 1$ and t and the return $R_{F_{t_1, T_1}}$; and $\sigma_{[r_t, F_{t_1, T_1}]}$ is the covariance between the return of the stock index between $t - 1$ and t and the return $R_{F_{t_1, T_1}}$.

The investors create an optimal portfolio with the stock index and the futures contract. Each position includes: i) A speculative part in which their position on the stock index (futures contract) is first dedicated to speculation on the stock (futures). ii) A diversification part in which they adjust their position on the stock index (futures contract) according to their position on the other contract and its covariance. iii) A hedging part in which unlike in the article by Boons et al. [2014], the investors do not hedge their commodity risk entirely by using the futures contract but use the two assets according to their covariance.

For instance, if the covariance between the stock market index and the spot price of the commodity is null ($\sigma_{[r_t, s_t]} = 0$), then the investors hedge their commodity risk with the futures contract ($-\varphi_t \sigma_{r,t}^2 \sigma_{[s_t, F_{t,t}]}$) and hedge the mismatch of their position on the futures contract with the stock market index ($\varphi_t \sigma_{[r_t, F_{t,t}]} \sigma_{[s_t, F_{t,t}]}$).

Optimal positions of unconstrained investors

This subsection addresses the second type of financialization. In this case, cross-asset investors trade on the futures markets without a constraint on the traded contracts. Empirically, Buyuksahin et al. [2009] justify this evolution after 2004 by the switch from backwardation to contango on the crude oil market⁵. This situation created important roll losses for the investors. Therefore, the set of investments available for the first generation is bigger than the one for the second generation. The profits at $t = 1$ and $t = 2$ for investors from the first and second generations are respectively $\pi_1 = w_1 R_{r_1} + \varphi_1 R_{s,1} + f_{1,1}^w R_{F_{1,1}} + f_{1,2}^w R_{F_{1,2}}$ and $\pi_2 = w_2 R_{r_2} + \varphi_2 R_{s,2} + f_{2,2}^w R_{F_{2,2}}$.

At $t = 1$, solving the problem (3.1) with profit $\pi_2 = w_2 R_{r_2} + \varphi_2 R_{s,2} + f_{2,2}^w R_{F_{2,2}}$ leads to the positions (3.10) and (3.11) at $t = 2$. These positions result because there is only one contract to trade at $t = 1$, and the investor lives for only one period.

5. Buyuksahin et al. [2009]: "(...) the growth of swap dealers' backdated positions accelerated in the second half of 2004 (at the time when the WTI futures market contangoed, after a long period of backwardation)."

At $t = 0$, solving the problem (3.1) with profit $\pi_1 = w_1 R_{r_1} + \varphi_1 R_{s,1} + f_{1,1}^w R_{F_{1,1}} + f_{1,2}^w R_{F_{1,2}}$ leads to optimal positions that are more complex versions of equations (3.10) and (3.11). However, these positions, which are displayed in Appendix 3.B, contain the same elements: i) A speculative part where the position of the investor on the stock index (futures contracts) is first dedicated to speculation on the stock (futures). ii) A diversification part where the investor adjusts his position on the stock index (futures contracts) according to his position on the other contracts and their covariances. iii) A hedging part where the investor hedges his commodity risk using not only the futures contracts but the three available assets according to their covariances.

3.5 Pre- and post-financialization equilibria

This section contains the equilibrium analysis of the model before and after the financialization. The model is first solved for the pre-financialization economy, that is, without investors. Then, it is solved for the two post-financialization economies with constrained and unconstrained investors. This order provides the possibility to draw general results on the functioning of commodity markets when there is a term structure. Then, it shows exactly how these markets are affected by the entry of new investors. To solve the model, regardless of the economy, three futures markets at two dates have to be cleared, that is, at $t = 0$ the market for the contract maturing at $t = 1$ (front-month contract) and for the contract maturing at $t = 2$ (deferred contract), and at $t = 1$ the market for the contract maturing at $t = 2$ (the new front-month contract).

3.5.1 Equilibrium pre-financialization

In pre-financialization, three types of market participants are trading on the futures markets: the N_s short-term speculators and the N_p short- and N_p long-term producers who are identical. The clearing equations are the following:

$$\begin{aligned} t=0, \text{ maturing in } 1: N_s f_{1,1}^{s*} + N_p f_{1,1}^{p*} &= 0 \\ t=0, \text{ maturing in } 2: N_s f_{1,2}^{s*} + N_p f_{1,2}^{p*} &= 0 \\ t=1, \text{ maturing in } 2: N_s (f_{2,2}^{s*} - f_{1,2}^{s*}) + N_p (f_{2,2}^{p*} - f_{1,2}^{p*}) &= 0 \end{aligned}$$

Using the optimal positions of the agents from equations (3.2), (3.3), (3.4), (3.5), (3.6), and

(3.8); the equilibrium expected returns or risk premia are:

$$\mu_{F_{1,1}}^* = \frac{\lambda_p \gamma_p \left\{ (\lambda_p + \lambda_s) \sigma_{2,2}^2 \left[\rho_{[1,11]} \left((\lambda_p + \lambda_s) \sigma_{1,1}^2 \sigma_{1,2}^2 - \lambda_p \sigma_{[11,12]}^2 \right) + \lambda_s \sigma_{1,1}^2 \rho_{[2,12]} \sigma_{[11,12]} \right] - \lambda_s^2 \sigma_{1,1}^2 \sigma_{[11,12]} \sigma_{[12,22]} \rho_{[2,22]} \right\}}{(\lambda_p + \lambda_s) \sigma_{2,2}^2 \left[(\lambda_p + \lambda_s)^2 \sigma_{1,1}^2 \sigma_{1,2}^2 - \lambda_p^2 \sigma_{[11,12]}^2 \right]} \quad (3.12a)$$

$$\mu_{F_{1,2}}^* = \frac{\lambda_p \gamma_p \left\{ (\lambda_p + \lambda_s) \sigma_{2,2}^2 \left[\rho_{[2,12]} \left((\lambda_p + \lambda_s) \sigma_{1,1}^2 \sigma_{1,2}^2 - \lambda_p \sigma_{[11,12]}^2 \right) + \lambda_s \sigma_{1,2}^2 \rho_{[1,11]} \sigma_{[11,12]} \right] - \lambda_s \rho_{[2,22]} \sigma_{[12,22]} \left((\lambda_p + \lambda_s) \sigma_{1,1}^2 \sigma_{1,2}^2 - \lambda_p \sigma_{[11,12]}^2 \right) \right\}}{(\lambda_p + \lambda_s) \sigma_{2,2}^2 \left[(\lambda_p + \lambda_s)^2 \sigma_{1,1}^2 \sigma_{1,2}^2 - \lambda_p^2 \sigma_{[11,12]}^2 \right]} \quad (3.12b)$$

$$\mu_{F_{2,2}}^* = \frac{\lambda_p \gamma_p \rho_{[2,22]}}{(\lambda_p + \lambda_s)} \quad (3.12c)$$

where $\lambda_i = \frac{N_i}{Y_i}$ is the elasticity of agent i .

Result 3.1 synthesizes the main results from equations (3.12a), (3.12b), and (3.12c). Its two first points and the fourth one can be developed.

Result 3.1 (Pre-financialization risk premia)

Without investors, the risk premia: i) exist only if there are risk-averse producers ($\lambda_p \neq 0$ and $\gamma_p \neq 0$) in the markets with associated hedging pressures (covariances between the physical revenues and the returns of the futures prices); ii) can decrease or increase with the speculators (number and risk aversion) because of their diversification behavior; iii) depend on the riskiness of the futures contracts (variances) and on the link between the contracts (covariances); and iv) the front-month futures contract maturing at $t = 1$ is affected by the long-term variables.

First, the result i) is well-known in the literature on commodity markets and justifies the existence of a risk premium because of the hedging needs of the physical hedgers. This view of the risk premia on commodity futures markets has been initiated by the *theory of normal backwardation* (Keynes [1930]) and has been reinforced more recently by De Roon et al. [2000], Bessembinder and Lemmon [2002], and Ekeland et al. [2016a,b].

Second, the result ii) goes against most of the theoretical papers on the determinants of the risk premia using a mono-commodity framework (Anderson and Danthine [1983a] and Ekeland et al. [2016a,b]) which conclude that speculators by being a counterpart to the physical hedgers decrease the risk premia. This is not the case in my model because of the existence of a term structure of futures contracts. This term structure gives the opportunity to the speculator to trade both for speculative (as in the literature) and diversification reasons.

Therefore, when the speculators trade to diversify their portfolios they might become consumers of liquidity and not providers. This result has been empirically illustrated by Kang et al. [2014].

Third, the result iv) goes against an important result of the *Market Segmentation Hypothesis* and complements the *Preferred Habitat Theory* of Modigliani and Sutch [1966]. Indeed, according to the former, because some agents naturally hedge in the short term and others in the long term, the two parts of the term structure should be governed by different state variables. According to the latter, the two parts of the curve should be integrated because the agents can move out of their preferred habitat if extreme differences exist. However, my result shows that even if producers stay in their preferred habitat, the risk premia are affected by the same state variables because of the speculators. Indeed, they play an important role in integrating the different parts of the term structure.

3.5.2 Equilibria post-financialization

In post-financialization, four types of market participants are trading on the futures markets: the N_s short-term speculators, the N_p short- and N_p long-term producers and the N_{in} (un)constrained investors. The clearing equations in the two types of financialization are:

With constrained investors:

$$\begin{aligned} t=0, \text{ maturing in 1: } N_s f_{1,1}^{s*} + N_p f_{1,1}^{p*} + N_{in} f_{1,1}^{w*} &= 0 \\ t=0, \text{ maturing in 2: } N_s f_{1,2}^{s*} + N_p f_{1,2}^{p*} &= 0 \\ t=1, \text{ maturing in 2: } N_s (f_{2,2}^{s*} - f_{1,2}^{s*}) + N_p (f_{2,2}^{p*} - f_{1,2}^{p*}) + N_{in} f_{2,2}^{w*} &= 0 \end{aligned}$$

With unconstrained investors:

$$\begin{aligned} t=0, \text{ maturing in 1: } N_s f_{1,1}^{s*} + N_p f_{1,1}^{p*} + N_{in} f_{1,1}^{w*} &= 0 \\ t=0, \text{ maturing in 2: } N_s f_{1,2}^{s*} + N_p f_{1,2}^{p*} + N_{in} f_{1,2}^{w*} &= 0 \\ t=1, \text{ maturing in 2: } N_s (f_{2,2}^{s*} - f_{1,2}^{s*}) + N_p (f_{2,2}^{p*} - f_{1,2}^{p*}) + N_{in} (f_{2,2}^{w*} - f_{1,2}^{w*}) &= 0 \end{aligned}$$

Results 3.2 and 3.3 synthesize the results obtained from the clearing of the markets with constrained and unconstrained investors respectively, using the optimal positions of the agents from equations (3.2), (3.3), (3.4), (3.5), (3.6), (3.8), (3.11), (3.16), and (3.17).

Result 3.2 (Risk premia with constrained investors)

With constrained investors, the risk premia for front-month contracts: i) are affected by the same factors as without investors (hedging pressure, number and risk aversion of the traditional agents...); ii) can exist even without producers because of the hedging/speculative demand from investors; iii) can decrease or increase with the investors because of their hedging,

speculative, and diversification demands; and iv) become dependent on financial variables (expected return, variance, and covariance with the stock index).

With constrained investors, the risk premium for the deferred contract: i) is affected by the same factors as without investors (hedging pressure, number and risk aversion of the traditional agents...); ii) can exist even without producers because of the hedging/speculative demand from investors; iii) can decrease or increase with the investors because of their hedging, speculative, and diversification demands; and iv) becomes dependent on financial variables (expected return, variance, and covariance with the stock index).

Equations (3.13a), (3.13b), and (3.13c) show the equilibrium expected returns with constrained investors and $\lambda_p = 0$. They are obtained from the clearing of the markets with constrained investors by using the optimal positions of the agents from equations (3.2), (3.3), (3.4), (3.5), (3.6), (3.8), and (3.11).

$$\mu_{F_{1,1}}^{\star, \lambda_p=0} = \frac{\lambda_{in} \sigma_{1,1}^2 \left\{ \mu_{r_1} \sigma_{[r_1, F_{1,1}]} + \Upsilon_{in} \varphi_1 \left(\sigma_{r_1}^2 \sigma_{[s_1, F_{1,1}]} - \sigma_{[r_1, F_{1,1}]} \sigma_{[r_1, s_1]} \right) \right\}}{(\lambda_{in} + \lambda_s) \sigma_{1,1}^2 \sigma_{r_1}^2 - \lambda_s \sigma_{[r_1, F_{1,1}]}^2} \quad (3.13a)$$

$$\mu_{F_{1,2}}^{\star, \lambda_p=0} = \frac{\lambda_{in} \sigma_{[11, 12]} \left\{ \mu_{r_1} \sigma_{[r_1, F_{1,1}]} + \Upsilon_{in} \varphi_1 \left(\sigma_{r_1}^2 \sigma_{[s_1, F_{1,1}]} - \sigma_{[r_1, F_{1,1}]} \sigma_{[r_1, s_1]} \right) \right\}}{(\lambda_{in} + \lambda_s) \sigma_{1,1}^2 \sigma_{r_1}^2 - \lambda_s \sigma_{[r_1, F_{1,1}]}^2} \quad (3.13b)$$

$$\mu_{F_{2,2}}^{\star, \lambda_p=0} = \frac{\lambda_{in} \sigma_{2,2}^2 \left\{ \mu_{r_2} \sigma_{[r_2, F_{2,2}]} + \Upsilon_{in} \varphi_2 \left(\sigma_{r_2}^2 \sigma_{[s_2, F_{2,2}]} - \sigma_{[r_2, F_{2,2}]} \sigma_{[r_2, s_2]} \right) \right\}}{(\lambda_{in} + \lambda_s) \sigma_{2,2}^2 \sigma_{r_2}^2 - \lambda_s \sigma_{[r_2, F_{2,2}]}^2} \quad (3.13c)$$

The first part of Result 3.2 and Result 3.3 provide important results. That is, the determinants of the risk premia of the futures contracts that are traded by investors are different than those that are not. The points i) and iv) state that the risk premia are still subject to variables linked to the physical market and become subject to variables linked to the stock market. The most interesting result is that an investment pressure is associated with the trading of the investors. As shown by equations (3.13a) and (3.13c), as stated by the point ii) and as in Boons et al. [2014], this investment pressure leads to the existence of a risk premium even without risk-averse producers (and an associated hedging pressure). Moreover, these equations and the point iii) show that this investment pressure can either compensate (as in Hamilton and Wu [2014]) or reinforce the hedging pressure of the producers and then decrease or increase the risk premia. It is noteworthy that this investment pressure exists even without commodity risk ($\varphi_t = 0$) for the investors because of their diversification behavior.

Result 3.3 (Risk premia with unconstrained investors)

With unconstrained investors, the risk premia: i) are affected by the same factors as without investors (hedging pressure, number and risk aversion of the traditional agents...); ii) can exist even without producers, because of the hedging/speculative demand from investors; iii)

can decrease or increase with the investors because of their hedging, speculative, and diversification demands; and iv) become dependent on financial variables (expected return, variance, and covariance with the stock index).

The second part of Result 3.2, illustrated by equation (3.13b), emphasizes the propagation effect of the financialization. It shows that, even without being traded by the investors, the deferred futures contract is affected by financial variables. This propagation is the consequence of the trading behavior of the speculators and the producers. In other words, the trading on the front-month contract by financial investors propagates to the entire term structure by the trading of the other agents. This idea of propagation to other contracts is the subject of an example in Hamilton and Wu [2014]. An interesting point in my model is that the intensity of the propagation depends on the level of integration of the term structure of prices. That is, a market like the crude oil market for which futures prices for different maturities tend to move together (important covariances) will see the effect of the investors propagate more rapidly than a market like the electricity market where futures prices for different maturities are more independent (small covariances) (see Section 3.7.1).

3.6 Numerical analysis of a representative market

This section, which is based on a calibrated version of the model, shows a quantification of the effect of financialization on the risk premia. This calibration is a static analysis to study the effect of the entry of new investors on a given specification of the market (*ceteris paribus*) and not a complete and dynamic analysis to study the effect of the entry of new investors on the specification of the market.

3.6.1 Calibration

Regarding the calibration, some parameters can be estimated using empirical data (e.g., variances and covariances). Some do not have an empirical counterpart and are chosen by assumption.

First, the parameters empirically estimated are gathered in Sub-table 3.1a. The estimation uses S&P500 prices as stock market parameters, prices of the WTI front-month contract as spot market parameters, prices of the WTI second nearby maturity as the parameters relative to the front-month contracts, and the prices of the WTI sixth maturity as the parameters relative to the deferred contract. The data are extracted from Datastream for the time period of January 2013 to February 2014. Moreover, the estimation relies on some assumptions: i) the descriptive statistics of the front-month contracts are stationary ($\sigma_{1,1}^2 = \sigma_{2,2}^2$); ii) the descriptive statistics of the stock market are stationary ($\mu_{r_1} = \mu_{r_2}$ and $\sigma_{r,1}^2 = \sigma_{r,2}^2$); and iii) the

links between the different assets are stationary ($\sigma_{[r_1, F_{1,1}]} = \sigma_{[r_2, F_{2,2}]}$, $\sigma_{[s_1, F_{1,1}]} = \sigma_{[s_2, F_{2,2}]}$ and $\sigma_{[r_1, s_1]} = \sigma_{[r_2, s_2]}$).

More important than these values is that the parameters disclose some stylized facts about commodity futures markets. First, they show that the variance in the front-month contract is higher than that in the deferred contract ($\sigma_{1,1}^2 = \sigma_{2,2}^2 > \sigma_{1,2}^2$), which is known as the Samuelson effect (Samuelson [1965]). Second, they show that commodity markets are more volatile than the stock market ($\sigma_{1,1}^2 = \sigma_{2,2}^2 > \sigma_{r,1}^2 = \sigma_{r,2}^2$). Third, these parameters show that the co-movement between the stock market and the commodity market is low compared to the co-movement between different maturities of the same commodity ($\sigma_{[s_1, F_{1,1}]} = \sigma_{[s_2, F_{2,2}]} > \sigma_{[r_1, s_1]} = \sigma_{[r_2, s_2]}$).

Second, the values of the other parameters presented in Sub-table 3.1b are based on assumptions. The principal assumptions are: i) the covariances between non-contemporaneous futures contracts are null ($\sigma_{[11,22]} = \sigma_{[12,22]} = 0$); ii) the covariances between the physical revenues and non-contemporaneous futures contracts are null ($\rho_{[2,11]} = \rho_{[2,12]} = 0$); iii) the covariance between the physical revenue and the front-month contract is stationary ($\rho_{[1,11]} = \rho_{[2,22]}$); and iv) the commodity risk of the investors is stationary ($\varphi_1 = \varphi_2$).

The values of the remaining parameters are based on arbitrary choices but their signs are important. The parameters $\rho_{[1,11]}$, $\rho_{[1,12]}$, and $\rho_{[2,22]}$ determine the hedging pressure on the model and are assumed to be positive in accordance with the *theory of normal backwardation*. Thus, the covariances with the front-month contract are set to one and the covariance with the deferred contract is assumed to be lower than the two others at 0.7. The parameters φ_1 and φ_2 that determine the commodity exposure of the investors are assumed to be negative as in Boons et al. [2014] because of the existence of an inflation risk for the investors.

3.6.2 Risk premia contract by contract

Using the parameters in Section 3.6.1, I look at the expected returns as functions of the number of investors for different numbers of producers.

Figure 3.1 shows for the front-month and deferred contracts at $t = 0$ the associated risk premia pre- and post-financialization (with constrained and unconstrained investors). This figure illustrates Results 3.1, 3.2, and 3.3 and shows the following general results: i) In pre-financialization without producers (red solid lines), there is no risk premium. However, when there are producers (blue and green solid lines), then the risk premia are positive, increase with the elasticity of the producers, and are independent of the elasticity of the investors (λ_{in}). ii) In post-financialization (dashed and dotted lines), if there are no investors ($\lambda_{in} = 0$), then the risk premia are the same as in pre-financialization. iii) In post-financialization, without producers and with investors (dashed and dotted red lines), there are negative risk

Parameters	Description	Value
$\sigma_{1,1}^2, \sigma_{2,2}^2$	Variance of the front-month futures contract at each period	1.25
$\sigma_{1,2}^2$	Variance of the deferred futures contract in the first period	0.98
$\sigma_{[11,12]}$	Covariance between the front-month and the deferred futures contracts in the first period	1.07
μ_{r_1}, μ_{r_2}	Expected return of the stock market index at each period	0.08
$\sigma_{r_1}^2, \sigma_{r_2}^2$	Variance of the return of the stock market index at each period	0.5
$\sigma_{[r_1, F_{1,1}]}, \sigma_{[r_2, F_{2,2}]}$	Covariance between the stock market index and the front-month contract at each period	.31
$\sigma_{[r_1, F_{1,2}]}$	Covariance between the stock market index and the deferred contract in the first period	0.29
$\sigma_{[s_1, F_{1,1}]}, \sigma_{[s_2, F_{2,2}]}$	Covariance between the spot price and the front-month contract at each period	1.26
$\sigma_{[s_1, F_{1,2}]}$	Covariance between the spot price and the deferred contract in the first period	1.06
$\sigma_{[r_1, s_1]}, \sigma_{[r_2, s_2]}$	Covariance between the stock market index and the spot price at each period	0.3

(a) Empirically estimated parameters

Parameters	Description	Value
$\sigma_{[11,22]}, \sigma_{[12,22]}$	Covariances between non-contemporaneous futures contracts in the first period	0
$\rho_{[1,11]}, \rho_{[2,22]}$	Covariance between the physical revenue and the front-month contract at each period	1
$\rho_{[1,12]}$	Covariance between the physical revenue and the deferred contract in the first period	.7
$\rho_{[2,11]}, \rho_{[2,12]}$	Covariances between the physical revenue and non-contemporaneous futures contracts in the first period	0
φ_1, φ_2	Commodity risk of the investors at each period	-2
$\gamma_i, \gamma_p, \gamma_s$	Risk aversion of the agents	1
λ_s	Elasticity of the speculators	2

(b) Other parameters

This table provides the description for each parameter and its value in the numerical analysis. Sub-table (a) contains the empirically estimated parameters, and Sub-table (b) contains the parameters whose values are based on assumptions. The estimation uses S&P500 prices as the stock market parameters, prices of the WTI front-month contract as the spot market parameters, prices of the WTI second nearby maturity as the parameters relative to the front-month contracts, and the prices of the WTI sixth maturity as the parameters relative to the deferred contract for the time period of January 2013 to February 2014.

Table 3.1 – Parameters of the numerical analysis

premia because of the choice of the commodity exposure. These risk premia increase in absolute value with the elasticity of the investors (λ_{in}). iv) In post-financialization, the combined effect of the producers' hedging pressure and the investors' investment pressure leads to a decrease in the absolute value of the risk premia when the elasticity of both increases. The reasoning behind this finding is because each has an opposite exposure to the commodity. Section 3.7.2 shows that this is also the case when producers and investors have the same exposure to the commodity.

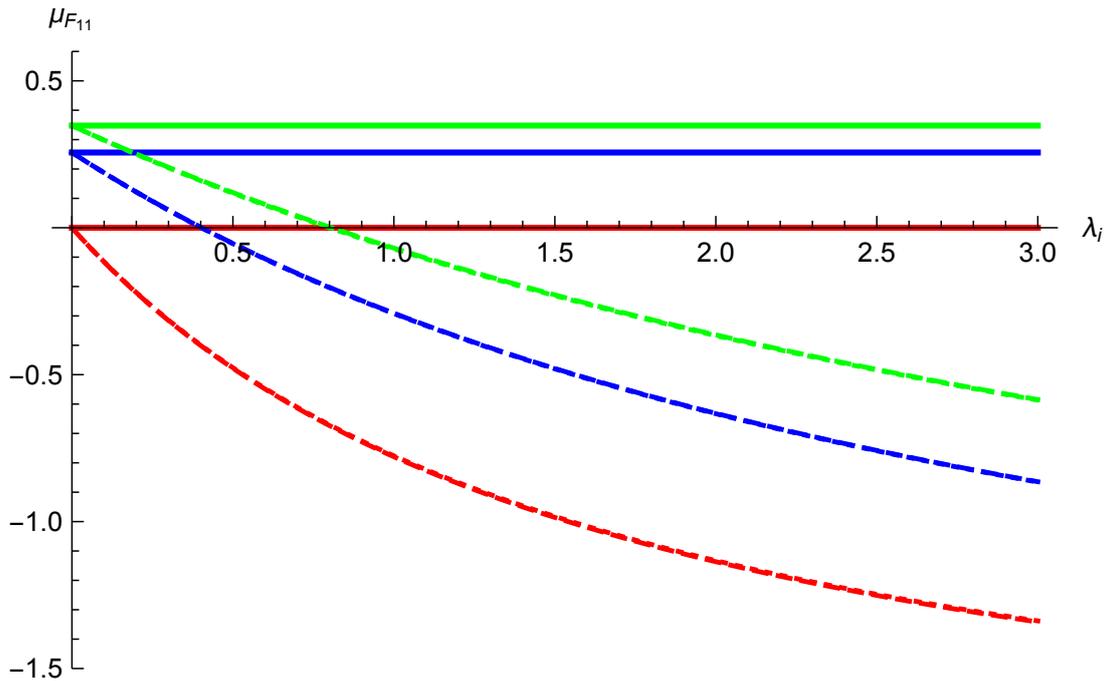
By describing lower and most of the time negative post-financialization risk premia, Figure 3.1 confirms the empirical results of Hamilton and Wu [2014] regarding the level of the risk premia pre- and post-financialization. According to these authors, a positive risk premium is associated to the holding of the front-month futures contract before 2005. However, after 2005, the risk premium is lower and even sometimes negative. Moreover, my results show that when the market is characterized by a bigger investment industry (λ_{in}) than a producing industry (λ_p), then the negative risk premia reach more important levels in absolute values. In other words, the investment pressure does more than compensate for the hedging pressure.

Finally, the effects of the financialization on the risk premia with constrained and unconstrained investors are very close (dashed versus dotted lines). There is only a small difference for the deferred contract. This result is driven by the important integration of the different maturities along the term structure of the prices (important covariance). This integration implies that the trading of some futures contracts of the term structure by financial investors propagates to all of the other existing contracts.

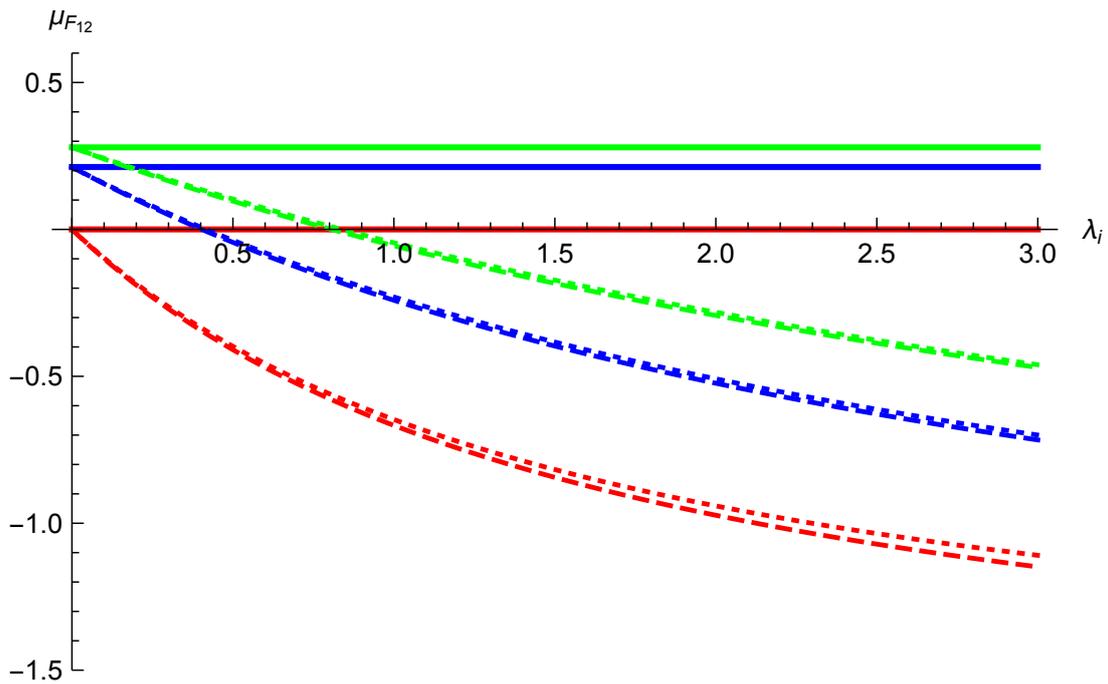
3.6.3 The term structure of risk premia

This section focuses on the study of the effect of financialization on the term structure of the risk premia. Figure 3.2 shows the term structure of risk premia before and after financialization. In pre-financialization (blue line), it shows the following: i) The term structure of the risk premia is downward sloping (backwardation), that is, at $t = 0$, the risk premium of the front-month contract is higher than that of the deferred contract. ii) The steepness of the term structure of the risk premia is independent of the investors. In post-financialization, it shows that: i) When the number of investors is smaller relative to the number of producers (black lines), then the term structure of the risk premia flattens. ii) When the number of investors is larger relative to the number of producers (orange lines), then the term structure of the risk premia is in contango (upward sloping term structure with negative risk premia) and steeper. iii) Because of the high level of integration of the market under consideration, the steepness of the term structure of the risk premia is roughly the same with constrained (dashed lines) and unconstrained investors (dotted lines).

In other words, before financialization, the term structure of the risk premia is downward



(a) Risk premium of the front-month contract

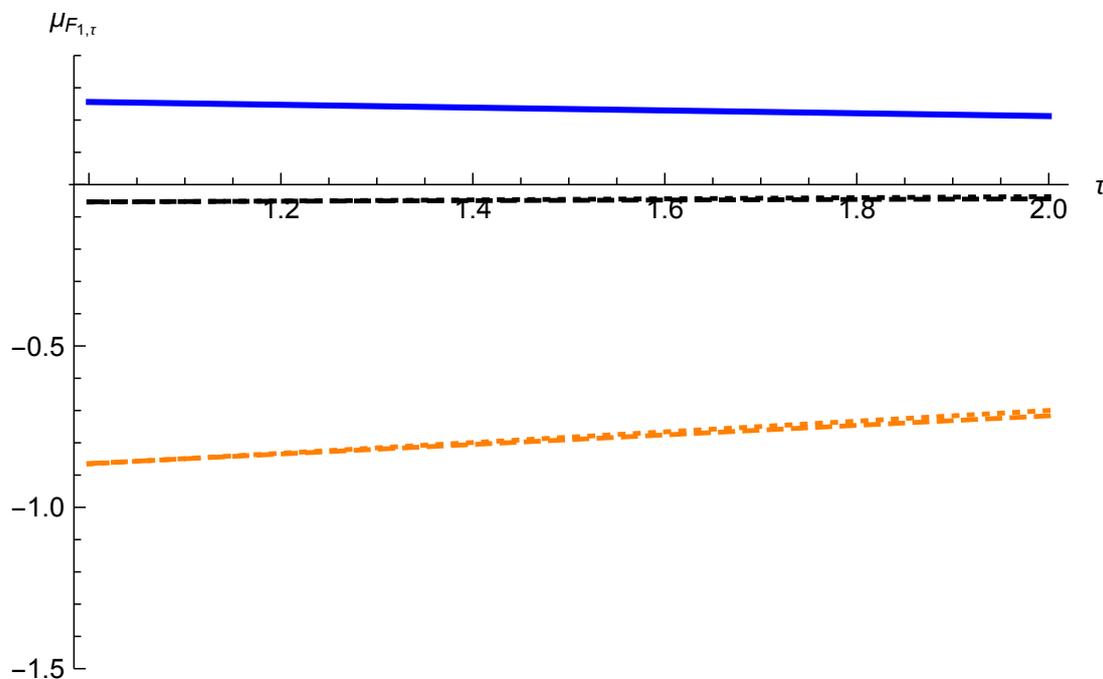


(b) Risk premium of the deferred contract

This figure shows the risk premia of the front-month ($\mu_{F_{1,1}}$) and deferred ($\mu_{F_{1,2}}$) contracts at $t = 0$, before and after the financialization, as functions of the number of investors (λ_i) for three different numbers of producers. The red lines are for $\lambda_p = 0$, the blue lines are for $\lambda_p = 1$, and the green lines are for $\lambda_p = 2$. The thick lines are for the pre-financialization, the dashed lines are for the financialization with constrained investors, and the dotted lines are for the financialization with unconstrained investors. The charts are obtained using the estimated parameters described in Section 3.6.1.

Figure 3.1 – Risk premia of the front-month and deferred contracts at $t = 0$

sloping because the front-month contract is the one most used for hedging by producers. Then, after financialization, as long as the investors remain a small group of participants in the market they help to flatten the term structure by absorbing the producers' hedging pressure. In contrast, as soon as they become more important, their investment pressure overcompensates for the producers' hedging pressure on the front month-contract. As a consequence, the term structure of the risk premia goes from backwardation to contango and becomes steeper. This result is driven by the fact that the front month futures contract is the most correlated with the physical revenue and then the one mainly used for hedging and trading.



This figure shows the term structure of risk premia before and after financialization for a given number of producers ($\lambda_p = 1$). The blue line is for the pre-financialization economy ($\lambda_{in} = 0$), the black lines are for the post-financialization economies with $\lambda_{in} = 0.5$, and the orange lines are for the post-financialization economies with $\lambda_{in} = 3$. The dashed lines are for the financialization with constrained investors, and the dotted lines are for the financialization with unconstrained investors. τ stands for the date of the maturity and $\mu_{F_{1,\tau}}$ for the risk premium at $t = 0$ of the futures contract with maturity τ . The charts are obtained using the estimated parameters described in Section 3.6.1.

Figure 3.2 – Term structure of risk premia at $t = 0$

The slope of the term structure of risk premia can be interpreted in terms of prices: $\mu_{F_{1,1}} =$

$E_0[S_1] - F_{0,1}$ and $\mu_{F_{1,2}} = E_0[S_2] - F_{0,2}$, then:

$$\begin{aligned}\mu_{F_{1,1}} - \mu_{F_{1,2}} &= E_0[S_1] - F_{0,1} - E_0[S_2] + F_{0,2} \\ &= F_{0,2} - F_{0,1} + E_0[S_1 - S_2] \\ &= \text{Basis} - E_0[\Delta S]\end{aligned}\tag{3.14}$$

As explained in Section 3.3, the spot price S_t depends on the available quantity on the spot market which comes entirely from random production \tilde{q}_t . There are no inventories at play. Next, assuming that the random productions \tilde{q}_t at $t = 1, 2$ have the same Gaussian distribution, then $E_0[S_1] = E_0[S_2]$ and $E_0[\Delta S] = 0$. Therefore, the term structure of the risk premia gives information on the term structure of the prices.

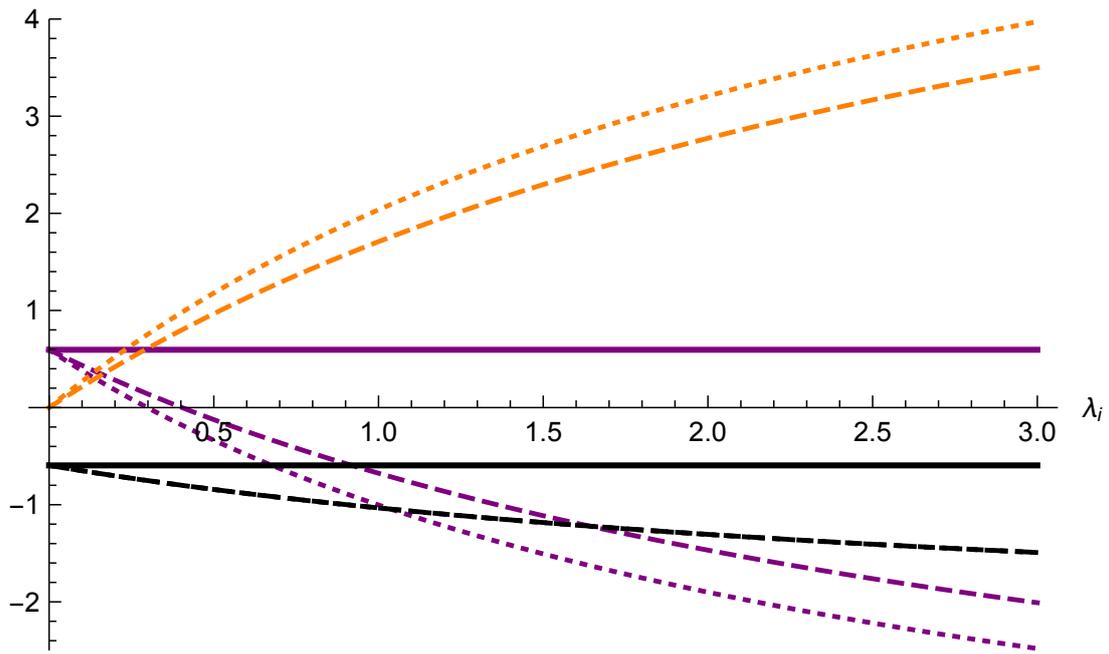
Under the assumptions of this section, the term structure of prices is in contango before the financialization. Then, according to the relative importance of the cross-asset investors, the term structure of prices can stay in contango and flatten or switch to backwardation with the financialization. This is because the front-month contract is the most relevant to use for trading. As a consequence, before financialization the producers sell more front-month contracts than deferred contracts and their price becomes lower. After the financialization, the cross-asset investors buy more front-month than deferred contracts and they become more expensive. This result describes only a partial effect and goes against the conclusion of Baker [2016] who states that backwardation should decrease with financialization. This contrast is because my model focuses on the effect of the hedging pressure on futures prices without taking into account any dynamic storage behavior.

Overall, my model features term structures of prices and risk premia with opposite shapes. That is, in pre-financialization, a term structure of prices in contango is associated with a term structure of risk premia in backwardation. The reverse is true post-financialization. Moreover, the model emphasizes that the financialization does affect the shape of the term structure of risk premia (and prices).

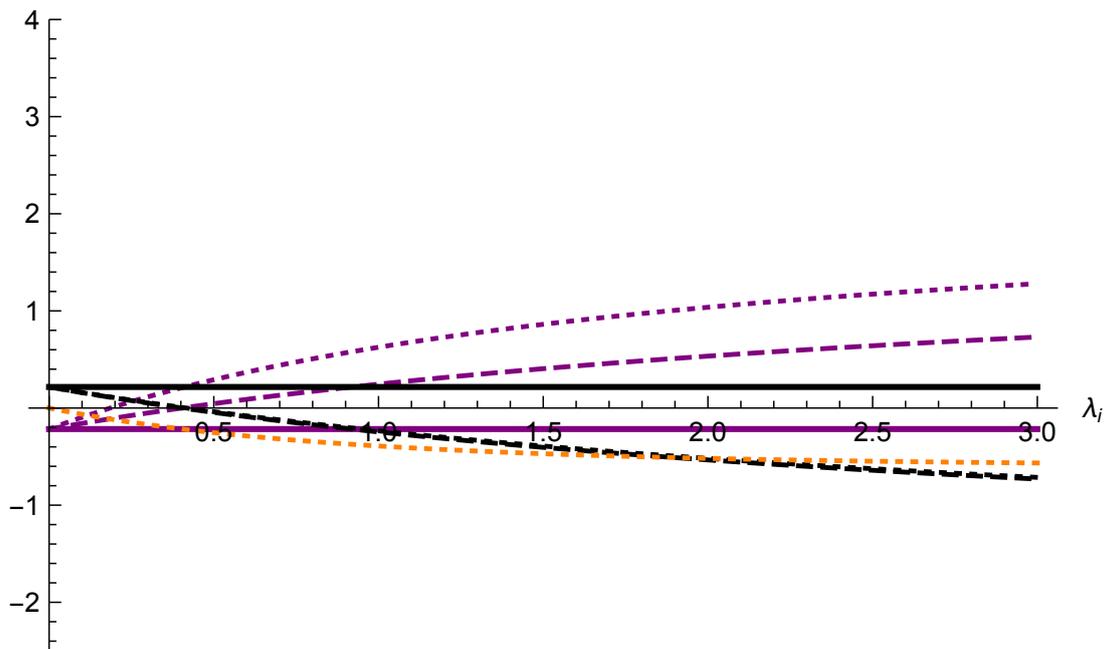
3.6.4 Liquidity provision by the agents

This subsection aims to study the role of each type of agent as a provider or consumer of liquidity in the futures market. The traditional view of commodity markets is that speculators provide liquidity to hedgers in exchange for a risk premium. As a consequence, as in Ekeland et al. [2016a,b] when the competition between speculators increases, then the risk premium decreases. However, financialization poses an empirical challenge to this view.

Figure 3.3 shows the aggregated optimal positions of the agents pre- and post-financialization for the front-month and deferred contracts at $t = 0$. Generally, trading volumes are more important for the front-month contract than for the deferred contract. This is a well-known feature of the term structure of commodity prices.



(a) Aggregated positions in the front-month contract



(b) Aggregated positions in the deferred contract

This figure shows the associated aggregated optimal positions of the agents before and after the financialization for each futures contract of the first period (Sub-figures) as functions of the number of investors (λ_i). The purple lines are for speculators, the black lines for producers, and the orange lines for investors. The thick lines are for the pre-financialization, the dashed lines are for the financialization with constrained investors, and the dotted lines for the financialization with unconstrained investors. The charts are obtained using the estimated parameters described in Section 3.6.1 and $\lambda_p = 1$.

Figure 3.3 – Aggregated positions by agent in the front-month and deferred contracts at $t = 0$

First, in the pre-financialization economy (solid lines), producers always sell front-month futures contracts (black solid line in Figure 3.3a). This is in agreement with their natural long exposure to the commodity. Therefore, speculators act as liquidity providers by buying futures contracts (purple solid line). In contrast, hedgers buy deferred futures contracts (black solid line in Figure 3.3b). By taking this position that is opposite to their hedging needs, they provide liquidity to speculators. These speculators ask for liquidity in the futures contracts in order to create optimal well-diversified portfolios. This result, in contradiction with the traditional view, has been empirically illustrated by Kang et al. [2014]. They show that hedgers provide short-term liquidity to the futures market and then to speculators.

Then, with financialization (dashed and dotted lines), investors massively buy the front-month futures contracts (orange lines in Figure 3.3a). As a consequence, when the number of investors increases, producers sell more and more futures contracts (dashed and dotted black lines), and speculators stop buying futures contracts to sell them (dashed and dotted purple lines). This feature has been empirically illustrated by Cheng and Xiong [2014]. They show that the entry of CITs into agricultural futures markets has resulted in an important expansion in the long side of the market. And that, as a consequence, producers have expanded their short positions.

Finally, with financialization, speculators buy more and more deferred contracts (dashed and dotted purple lines in Figure 3.3b). Their strategy is to hedge their position in the front-month contract with the deferred contract and therefore ask for liquidity on this contract to hedgers and investors.

These results show that, both pre- and post-financialization, the traditional view of commodity futures markets as places where hedgers find liquidity is incomplete. Under some circumstances, hedgers may have to provide liquidity to speculators and investors. This is the case because they have other trading motives than providing liquidity to hedgers.

3.7 Heterogeneity of commodity markets and the financialization

This section aims to emphasize that the quantitative effects of financialization are market specific. To do so, I study two specific cases by changing some of the parameters. Changing these parameters is equivalent to studying a market with different physical characteristics.

3.7.1 Non-integrated markets

Up to now, most of the results are identical (or at least close) when I consider constrained and unconstrained investors. This is the case because the model is calibrated with

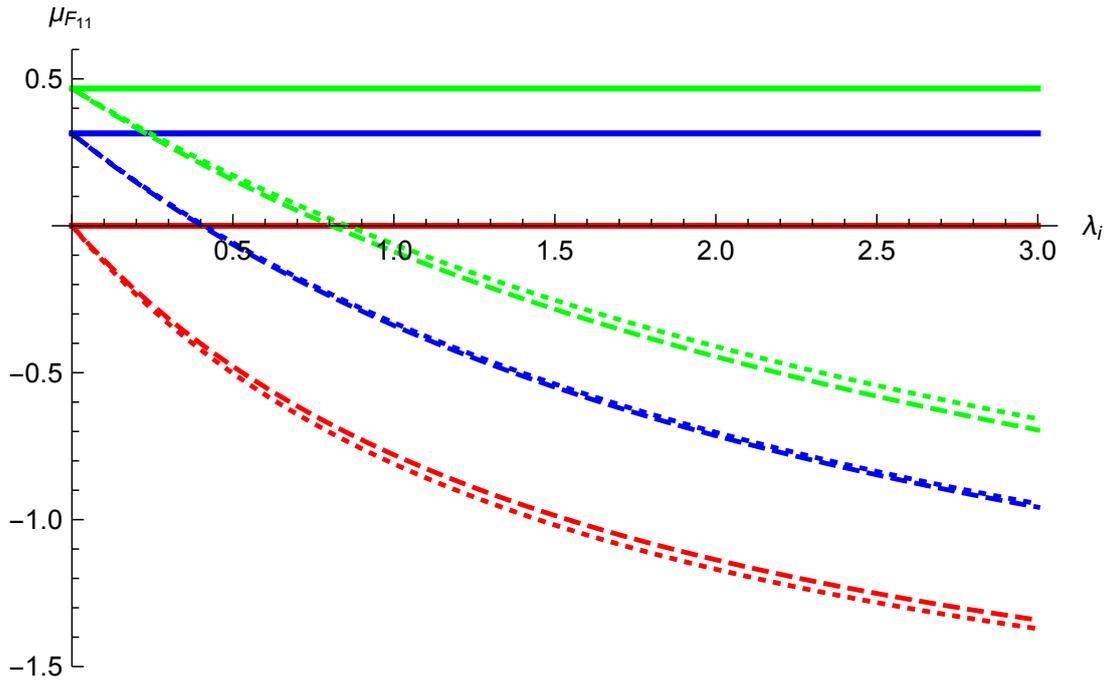
futures prices for the crude oil futures market. This market is characterized by a high level of temporal integration (an important covariance between the futures prices of different maturities). Some markets, like the electricity market tend to be less integrated (a small covariance between the futures prices of different maturities)⁶. This subsection shows that on these markets, the effect of financialization on non-traded futures contracts is different.

In the basic set of estimated parameters, the correlation between the front-month and deferred futures contracts ($\frac{\sigma_{[11,12]}}{\sqrt{\sigma_{1,1}^2}\sqrt{\sigma_{1,2}^2}}$) is approximately 0.96. For electricity futures markets, the correlation is around 0.5. Then, in order to see the effect of financialization in such a market, I set $\sigma_{[11,12]} = 0.55$. Without changing the level of variability in the market ($\sigma_{1,1}^2$ and $\sigma_{1,2}^2$), this covariance leads to a correlation equal to 0.5.

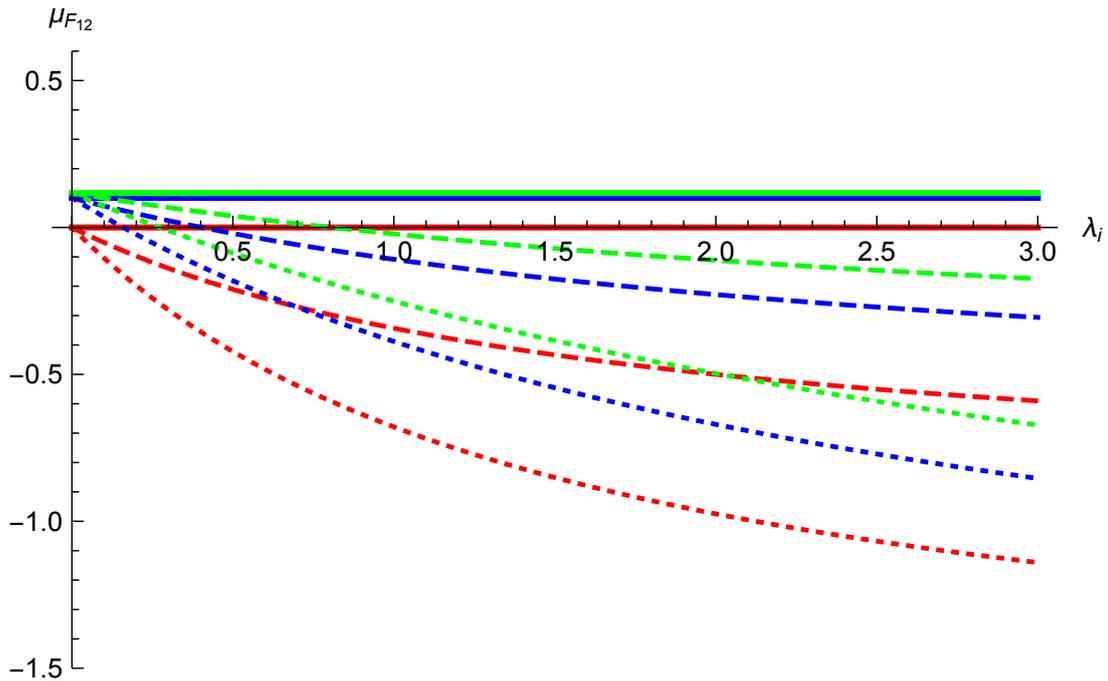
Figure 3.4 shows the expected returns for the front-month and deferred contracts at $t = 0$. It gives the following results: i) As for the integrated market, there is no difference in the effect of financialization on the front-month contract with constrained and unconstrained investors (dashed and dotted lines in Figure 3.4a). This effect is the same as the one described in the previous sections. ii) Contrary to what has been said for the integrated market, the effect on the risk premium of the deferred contract ($\mu_{F_{1,2}}$) is situation dependent. That is, the magnitude of the effect is less important with constrained (dashed lines in Figure 3.4b) than with unconstrained investors (dotted lines). More precisely, the risk premium in the first situation goes in the same direction (decreases and becomes negative) as in the second situation but it always stays smaller in absolute value. In general, the magnitude of the effect has two sources: an indirect propagation effect and a direct investment pressure effect. In this case the propagation effect is low because of the low integration of the different maturities, and the direct effect does not exist for the deferred contract with constrained investors.

Figure 3.5 represents the term structure of risk premia before (blue line) and after financialization (black and orange lines). It shows that the evolution of the shape of the term structure with unconstrained investors (dotted lines) stays roughly the same as described in Section 3.6.3 for the integrated market. In contrast, it changes dramatically with constrained investors (dashed lines). In the latter situation, the steepness of the term structure of the risk premia is much more important than with unconstrained investors. Indeed, the slope of the term structure can be up to three or four times in absolute value above the level with unconstrained investors. This result can be explained by the important direct investment pressure from constrained investors on the front-month contract which decreases its risk premium ($\mu_{F_{1,1}}$). Whereas the risk premium ($\mu_{F_{1,2}}$) of the deferred contract changes only through the diversification behavior of the agents. But because the market is not very integrated, the

6. See Jaeck and Lautier [2016] for an illustration of such differences in the level of integration of the term structures of futures prices between the crude oil market and electricity markets.



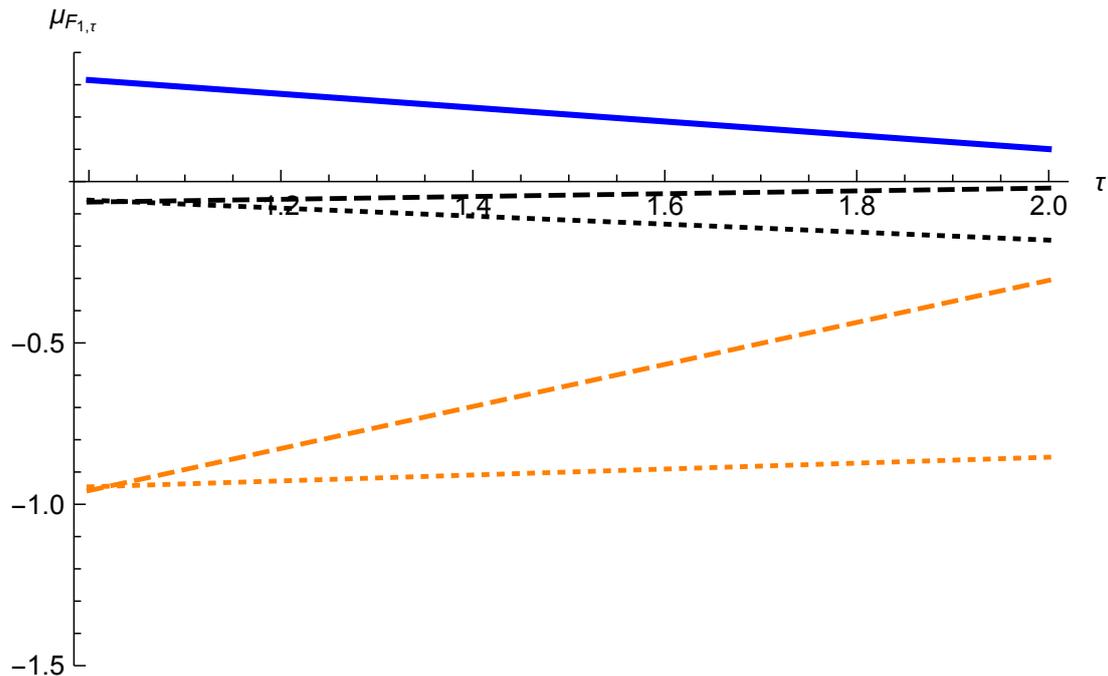
(a) Risk premium of the front-month contract



(b) Risk premium of the deferred contract

This figure shows the risk premia of the front-month ($\mu_{F_{1,1}}$) and deferred ($\mu_{F_{1,2}}$) contracts at $t = 0$, before and after the financialization, as functions of the number of investors (λ_i) for three different numbers of producers. The red lines are for $\lambda_p = 0$, the blue lines are for $\lambda_p = 1$, and the green lines are for $\lambda_p = 2$. The thick lines are for the pre-financialization, the dashed lines are for the financialization with constrained investors, and the dotted lines are for the financialization with unconstrained investors. The charts are obtained by using the estimated parameters described in Section 3.6.1 except for the covariance between the two contemporaneous futures contracts which becomes $\sigma_{[11,12]} = 0.55$.

Figure 3.4 – Risk premia of the front-month and deferred contracts at $t = 0$ (non-integrated market)



This figure shows the term structure of risk premia before and after financialization for a given number of producers ($\lambda_p = 1$). The blue line is for the pre-financialization economy ($\lambda_{in} = 0$), the black lines are for the post-financialization economies with $\lambda_{in} = 0.5$, and the orange lines are for the post-financialization economies with $\lambda_{in} = 3$. The dashed lines are for the financialization with constrained investors, and the dotted lines are for the financialization with unconstrained investors. τ stands for the date of the maturity and $\mu_{F_{1,\tau}}$ for the risk premium at $t = 0$ of the futures contract with maturity τ . The charts are obtained by using the estimated parameters described in Section 3.6.1 except for the covariance between the two contemporaneous futures contracts which becomes $\sigma_{[11,12]} = 0.55$.

Figure 3.5 – Term structure of risk premia at $t = 0$ (non-integrated market)

transmission of the shock (the entry of new investors) is slow.

Overall, it seems that financialization reinforces the non-integration of the market when only constrained investors enter the market and does not change it when unconstrained investors enter the market.

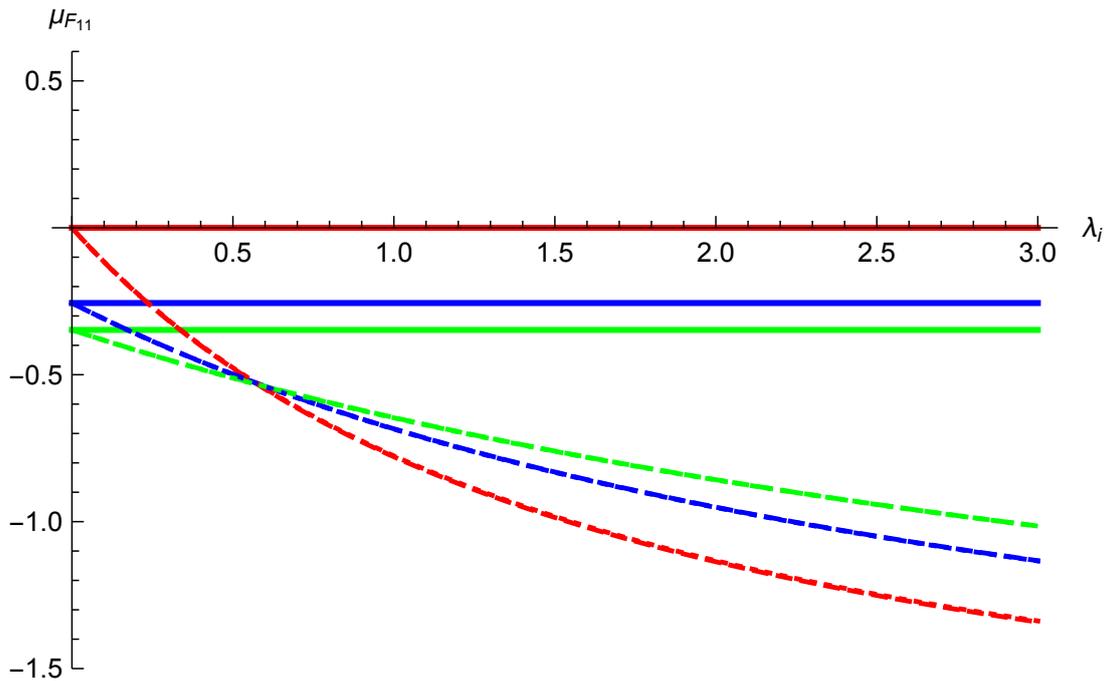
3.7.2 Markets dominated by long hedgers

So far, the results are based on the assumption that commodity markets are mainly used by short hedgers as described by the *theory of normal backwardation* (Keynes [1930]). Nevertheless, as emphasized by De Roon et al. [2000], the *hedging pressure theory* states that there are substantial variations inside each commodity market and from market to market in the level and the sign of the hedging pressure. Assuming a negative covariance between the physical revenue of the producers and the futures prices ($\rho_{[1,11]} < 0$, $\rho_{[1,12]} < 0$ and $\rho_{[2,22]} < 0$), this subsection shows how a commodity market dominated by long hedgers reacts to the introduction of new (un)constrained investors.

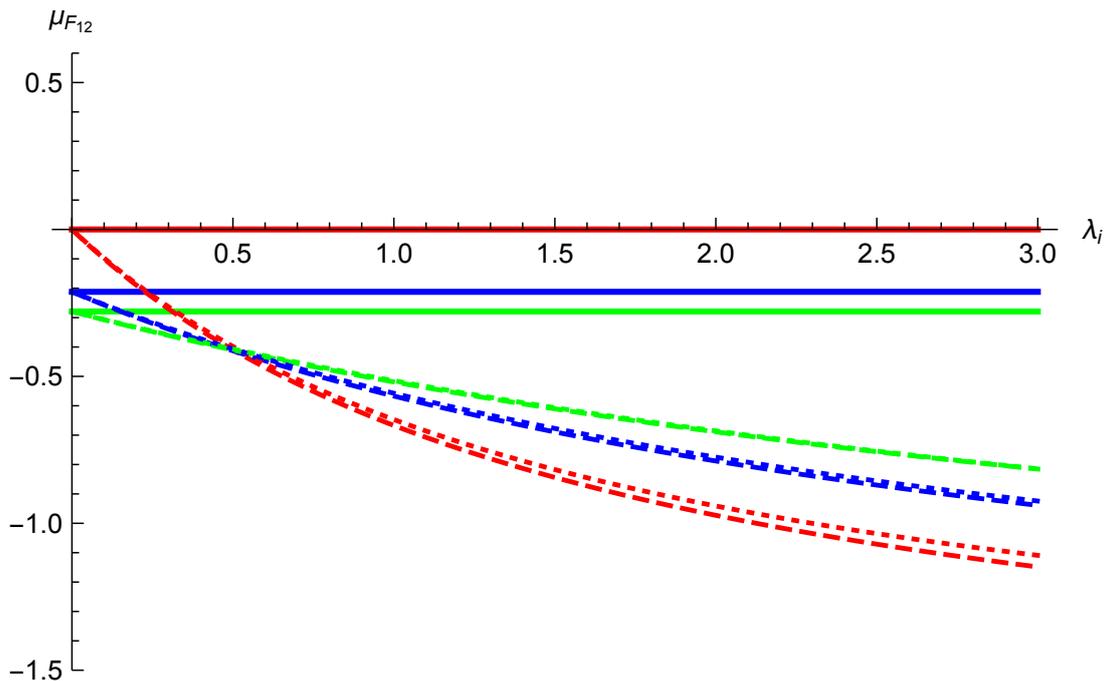
Compared to the basic set of estimated parameters, I change the sign of the relevant parameters without changing their absolute value. Therefore, this subsection uses $\rho_{[1,11]} = -1$, $\rho_{[1,12]} = -0.7$, and $\rho_{[2,22]} = -1$.

Figure 3.6 describes the risk premia for the front-month and deferred contracts at $t = 0$ when the futures market is dominated by long hedgers. It shows that: i) In pre-financialization (solid lines), risk premia are negative and increase in absolute value as the number of producers λ_p increases. Producers in this situation have a negative exposure to the commodity (they lose money when the price increases) and then buy the futures contract. Because of that, the futures price increases and becomes bigger than the expected spot price. ii) In post-financialization (dashed and dotted lines), the risk premia decrease in absolute value when both the number of producers (λ_p) and the number of investors (λ_{in}) increase. This combined effect of the hedging and investment pressures is more unexpected. Indeed, because the risk premia increase in absolute value in the pre-financialization with the number of producers and that without producers the risk premia increase in absolute value with the number of investors, maybe the hedging pressure and the investment pressure should have reinforced each other.

Figure 3.7 gives more insight into what happens. It shows that the unexpected combined effect of the hedging pressure and of the investment pressure is due to the hedgers. As in Kang et al. [2014], they act as liquidity providers on the futures markets. Indeed, the aggregated optimal position of the producers (dashed lines) decreases when the aggregated position of the investors (dotted lines) increases. The adjustment of the futures position of the producers is even more important when there are more producers. When the number of investors is bigger than the number of producers, the latter hold a short futures position.



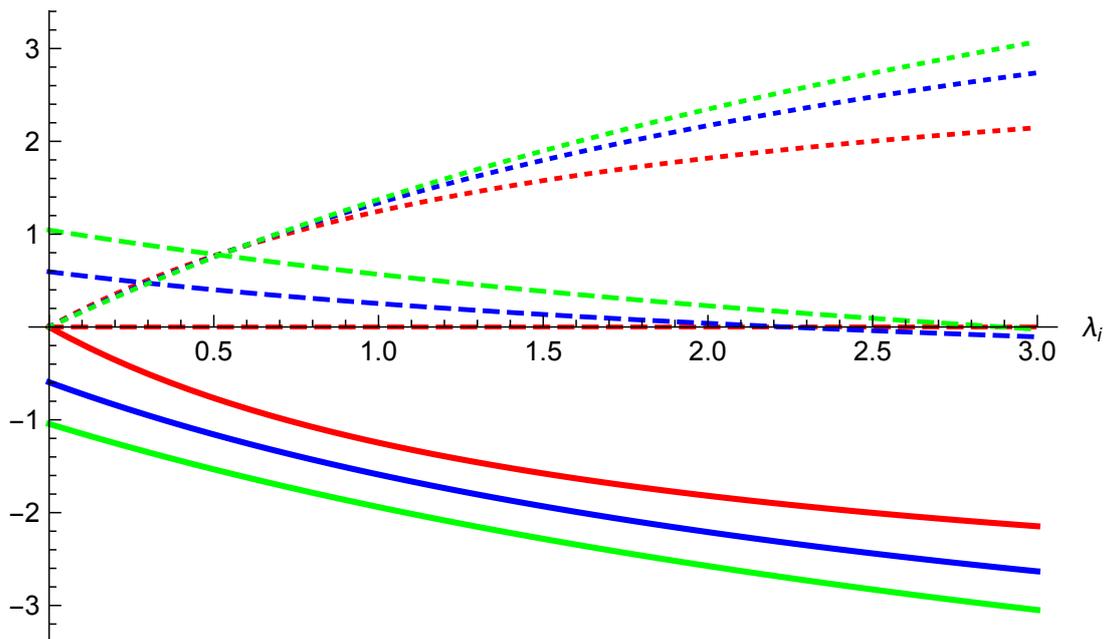
(a) Risk premium of the front-month contract



(b) Risk premium of the deferred contract

This figure shows the risk premia of the front-month ($\mu_{F_{1,1}}$) and deferred ($\mu_{F_{1,2}}$) contracts at $t = 0$, before and after the financialization, as functions of the number of investors (λ_i) for three different numbers of producers. The red lines are for $\lambda_p = 0$, the blue lines are for $\lambda_p = 1$, and the green lines are for $\lambda_p = 2$. The thick lines are for the pre-financialization, the dashed lines are for the financialization with constrained investors, and the dotted lines are for the financialization with unconstrained investors. The charts are obtained by using the estimated parameters described in Section 3.6.1 except for the covariances between the physical revenue of the producers and the futures prices which become $\rho_{[1,11]} = -1$, $\rho_{[1,12]} = -0.7$, and $\rho_{[2,22]} = -1$.

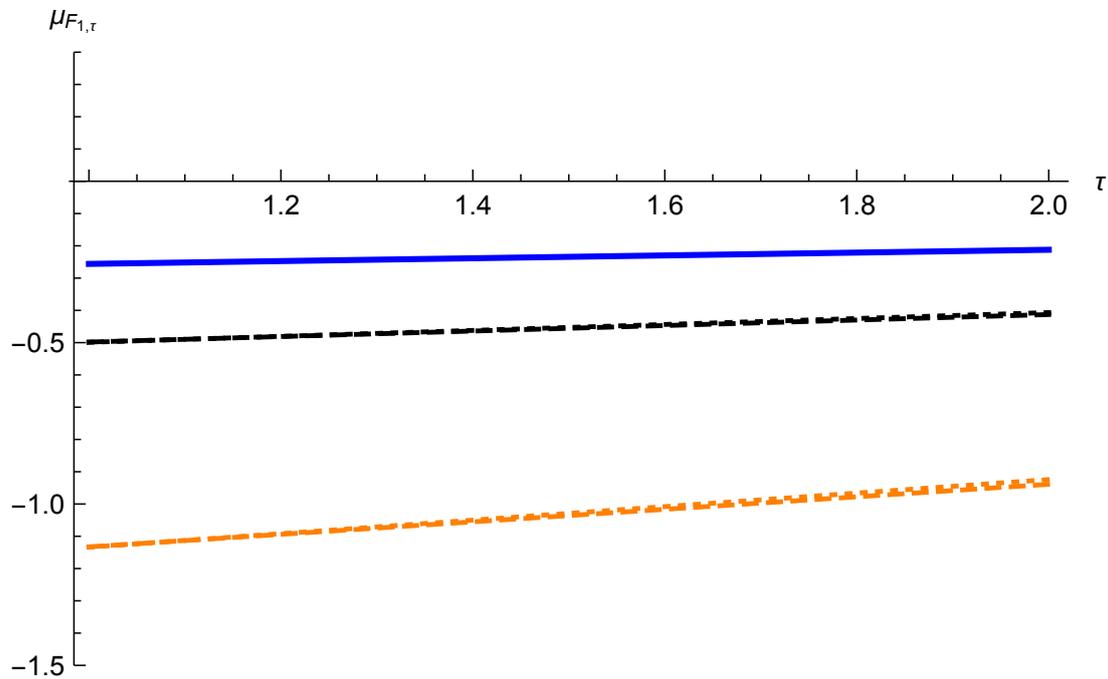
Figure 3.6 – Risk premia of the front-month and deferred contracts at $t = 0$ (long hedgers)



This figure shows the aggregated optimal positions of the agents for the front-month futures contract at $t = 0$ with constrained investors as a function of the number of investors (λ_i) for three different numbers of producers. The red lines are for $\lambda_p = 0$, the blue lines are for $\lambda_p = 1$, and the green lines are for $\lambda_p = 2$. The solid lines are for the speculators, the dashed line are for the producers, and the dotted lines are for the constrained investors. The charts are obtained by using the estimated parameters described in Section 3.6.1 except for the covariances between the physical revenue of the producers and the futures prices which become $\rho_{[1,11]} = -1$, $\rho_{[1,12]} = -0.7$, and $\rho_{[2,22]} = -1$.

Figure 3.7 – Positions in the front-month contract at $t = 0$ with constrained investors (long hedgers)

This is the opposite of their hedging needs.



This figure shows the term structure of risk premia before and after financialization for a given number of producers ($\lambda_p = 1$). The blue line is for the pre-financialization economy ($\lambda_{in} = 0$), the black lines are for the post-financialization economies with $\lambda_{in} = 0.5$, and the orange lines are for the post-financialization economies with $\lambda_{in} = 3$. The dashed lines are for the financialization with constrained investors, and the dotted lines are for the financialization with unconstrained investors. τ stands for the date of the maturity and $\mu_{F_{1,\tau}}$ for the risk premium at $t = 0$ of the futures contract with maturity τ . The charts are obtained by using the estimated parameters described in Section 3.6.1 except for the covariances between the physical revenue of the producers and the futures prices which become $\rho_{[1,11]} = -1$, $\rho_{[1,12]} = -0.7$, and $\rho_{[2,22]} = -1$.

Figure 3.8 – Term structure of risk premia at $t = 0$ (long hedgers)

Figure 3.8 represents the term structure of risk premia before (blue line) and after financialization (black and orange lines). It shows that the term structure of the risk premia is always upward sloping (contango). Further, in post-financialization, the term structure of the risk premia is always steeper than in pre-financialization. This was not the case previously and is a consequence of the hedging and investment pressures being on the same side of the market.

In term of prices, following the same reasoning as in Section 3.6.3, Figure 3.8 shows that when the futures market is dominated by long hedgers, the term structure of prices is always in backwardation. Moreover, the financialization increases the steepness of the term structure of the prices.

3.8 Conclusion

I develop an equilibrium model of commodity futures markets in which traditional risk-averse agents (producers and speculators) face new cross-asset investors. Because it features a term structure of futures prices, I first extend to this framework the results regarding the functioning of commodity markets before financialization. For instance, I emphasize the role of speculators as both providers and consumers of liquidity and their role in the integration of the risk premia along the term structure. Then, I show that the financialization changes the nature of commodity markets, at least by changing their risk sharing function. Indeed, they become less segmented from the stock market, and the investment pressure from cross-asset investors becomes an important determinant of the risk premia. Moreover, my analysis shows that all of the existing maturities on a futures market are affected by the financialization, even in a context of a short-term constrained investment. This propagation effect depends on the market under consideration because it depends on the integration of the futures prices for different maturities.

The economic implications of the financialization are: that the cost of hedging of traditional hedgers is greatly affected, that the shape of the term structure of risk premia (and of prices) changes, that speculators can face more competition from investors to earn the risk premium from the hedging pressure of the hedgers but can also have new profit opportunities when the investment pressure from investors is important, and that there is more efficient risk sharing because of the decreased fragmentation of the markets, but this may create stronger spillover effects. Regulators need to take this into account when monitoring the systemic risk of the system.

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3.A Appendix: Summary of the notations

Regarding the futures market:

- Position of the agent i between $t - 1$ and t in a futures contract with maturity T : $f_{t,T}^i$
- Return between $t - 1$ and t of a futures contract with maturity T : $R_{F_{t,T}}$
- Expected return between $t - 1$ and t of a futures contract with maturity T : $\mu_{F_{t,T}}$
- Variance between $t - 1$ and t of the return a futures contract with maturity T : $\sigma_{t,T}^2$
- Covariance between the returns of the two futures contracts $R_{F_{t_1,T_1}}$ and $R_{F_{t_2,T_2}}$: $\sigma_{[t_1T_1, t_2T_2]}$
- Elasticity of the agent i : $\lambda_i = \frac{N_i}{Y_i}$

Regarding the spot market:

- Return between $t - 1$ and t of the spot commodity: $R_{s,t}$
- Variance between $t - 1$ and t of the spot commodity: $\sigma_{s,t}^2$
- Commodity risk in t of an investor: φ_t

Regarding the stock market:

- Return between $t - 1$ and t of a stock: R_{r_t}
- Expected return between $t - 1$ and t of a stock: μ_{r_t}
- Variance between $t - 1$ and t of the return a stock: $\sigma_{r,t}^2$

Regarding the link between assets:

- Covariance between the spot return between $t - 1$ and t and the return of the futures contract $R_{F_{t_1,T_1}}$: $\sigma_{[s_t, F_{t_1,T_1}]}$
- Covariance between the physical revenue between $t - 1$ and t and the return of the futures contract $R_{F_{t_1,T_1}}$: $\rho_{[t, t_1T_1]}$
- Covariance between the return of the stock and the return of the futures contract $R_{F_{t_1,T_1}}$: $\sigma_{[r_t, F_{t_1,T_1}]}$
- Covariance between the return of the stock and of the spot commodity between $t - 1$ and t : $\sigma_{[r_t, s_t]}$

3.B Appendix: Optimal positions of unconstrained investors

This appendix shows the optimal positions at $t = 0$ in each asset of an unconstrained investor. They are obtained by solving the problem (3.1) with profit $\pi_1 = w_1 R_{r_1} + \varphi_1 R_{s,1} + f_{1,1}^w R_{F_{1,1}} + f_{1,2}^w R_{F_{1,2}}$.

$$w_1^* = \frac{\mu_{r_1} (\sigma_{1,1}^2 \sigma_{1,2}^2 - \sigma_{[11,12]}^2) + \mu_{F_{1,1}} (\sigma_{[11,12]} \sigma_{[r_1, F_{1,2}]} - \sigma_{1,2}^2 \sigma_{[r_1, F_{1,1}]}) + \mu_{F_{1,2}} (\sigma_{[11,12]} \sigma_{[r_1, F_{1,1}]} - \sigma_{1,1}^2 \sigma_{[r_1, F_{1,2}]})}{\text{Yin} \left(2\sigma_{[11,12]} \sigma_{[r_1, F_{1,1}]} \sigma_{[r_1, F_{1,2}]} + \sigma_{1,1}^2 (\sigma_{1,2}^2 \sigma_{r,1}^2 - \sigma_{[r_1, F_{1,2}]}^2) - \sigma_{[11,12]}^2 \sigma_{r,1}^2 - \sigma_{1,2}^2 \sigma_{[r_1, F_{1,1}]}^2 \right)} + \frac{\varphi_1 \left\{ \sigma_{1,1}^2 \sigma_{[r_1, F_{1,2}]} \sigma_{[s_1, F_{1,2}]} - \sigma_{[11,12]} (\sigma_{[r_1, F_{1,2}]} \sigma_{[s_1, F_{1,1}]} + \sigma_{[r_1, F_{1,1}]} \sigma_{[s_1, F_{1,2}]} \right\} + \sigma_{[11,12]}^2 \sigma_{[r_1, s_1]}}{\left(2\sigma_{[11,12]} \sigma_{[r_1, F_{1,1}]} \sigma_{[r_1, F_{1,2}]} + \sigma_{1,1}^2 (\sigma_{1,2}^2 \sigma_{r,1}^2 - \sigma_{[r_1, F_{1,2}]}^2) - \sigma_{[11,12]}^2 \sigma_{r,1}^2 - \sigma_{1,2}^2 \sigma_{[r_1, F_{1,1}]}^2 \right) + \sigma_{1,2}^2 (\sigma_{[r_1, F_{1,1}]} \sigma_{[s_1, F_{1,1}]} - \sigma_{1,1}^2 \sigma_{[r_1, s_1]})} \quad (3.15)$$

$$f_{1,1}^{w*} = \frac{\mu_{F_{1,1}} (\sigma_{1,2}^2 \sigma_{r,1}^2 - \sigma_{[r_1, F_{1,2}]}^2) + \mu_{r_1} (\sigma_{[11,12]} \sigma_{[r_1, F_{1,2}]} - \sigma_{1,2}^2 \sigma_{[r_1, F_{1,1}]}) + \mu_{F_{1,2}} (\sigma_{[r_1, F_{1,1}]} \sigma_{[r_1, F_{1,2}]} - \sigma_{[11,12]} \sigma_{r,1}^2)}{\text{Yin} \left(2\sigma_{[11,12]} \sigma_{[r_1, F_{1,1}]} \sigma_{[r_1, F_{1,2}]} + \sigma_{1,1}^2 (\sigma_{1,2}^2 \sigma_{r,1}^2 - \sigma_{[r_1, F_{1,2}]}^2) - \sigma_{[11,12]}^2 \sigma_{r,1}^2 - \sigma_{1,2}^2 \sigma_{[r_1, F_{1,1}]}^2 \right)} + \frac{\varphi_1 \left\{ \sigma_{r,1}^2 \sigma_{[11,12]} \sigma_{[s_1, F_{1,2}]} - \sigma_{[r_1, F_{1,2}]} (\sigma_{[r_1, F_{1,1}]} \sigma_{[s_1, F_{1,2}]} + \sigma_{[11,12]} \sigma_{[r_1, s_1]}) \right\} + \sigma_{[r_1, F_{1,2}]}^2 \sigma_{[s_1, F_{1,1}]}}{\left(2\sigma_{[11,12]} \sigma_{[r_1, F_{1,1}]} \sigma_{[r_1, F_{1,2}]} + \sigma_{1,1}^2 (\sigma_{1,2}^2 \sigma_{r,1}^2 - \sigma_{[r_1, F_{1,2}]}^2) - \sigma_{[11,12]}^2 \sigma_{r,1}^2 - \sigma_{1,2}^2 \sigma_{[r_1, F_{1,1}]}^2 \right) + \sigma_{1,2}^2 (\sigma_{[r_1, F_{1,1}]} \sigma_{[r_1, s_1]} - \sigma_{r,1}^2 \sigma_{[s_1, F_{1,1}]})} \quad (3.16)$$

$$f_{1,2}^{w*} = \frac{\mu_{F_{1,2}} (\sigma_{1,1}^2 \sigma_{r,1}^2 - \sigma_{[r_1, F_{1,1}]}^2) + \mu_{r_1} (\sigma_{[11,12]} \sigma_{[r_1, F_{1,1}]} - \sigma_{1,1}^2 \sigma_{[r_1, F_{1,2}]}) + \mu_{F_{1,1}} (\sigma_{[r_1, F_{1,1}]} \sigma_{[r_1, F_{1,2}]} - \sigma_{[11,12]} \sigma_{r,1}^2)}{\text{Yin} \left(2\sigma_{[11,12]} \sigma_{[r_1, F_{1,1}]} \sigma_{[r_1, F_{1,2}]} + \sigma_{1,1}^2 (\sigma_{1,2}^2 \sigma_{r,1}^2 - \sigma_{[r_1, F_{1,2}]}^2) - \sigma_{[11,12]}^2 \sigma_{r,1}^2 - \sigma_{1,2}^2 \sigma_{[r_1, F_{1,1}]}^2 \right)} + \frac{\varphi_1 \left\{ \sigma_{r,1}^2 \sigma_{[11,12]} \sigma_{[s_1, F_{1,1}]} - \sigma_{[r_1, F_{1,1}]} (\sigma_{[r_1, F_{1,2}]} \sigma_{[s_1, F_{1,1}]} + \sigma_{[11,12]} \sigma_{[r_1, s_1]}) \right\} + \sigma_{[r_1, F_{1,2}]}^2 \sigma_{[s_1, F_{1,2}]}}{\left(2\sigma_{[11,12]} \sigma_{[r_1, F_{1,1}]} \sigma_{[r_1, F_{1,2}]} + \sigma_{1,1}^2 (\sigma_{1,2}^2 \sigma_{r,1}^2 - \sigma_{[r_1, F_{1,2}]}^2) - \sigma_{[11,12]}^2 \sigma_{r,1}^2 - \sigma_{1,2}^2 \sigma_{[r_1, F_{1,1}]}^2 \right) + \sigma_{1,1}^2 (\sigma_{[r_1, F_{1,2}]} \sigma_{[r_1, s_1]} - \sigma_{r,1}^2 \sigma_{[s_1, F_{1,2}]})} \quad (3.17)$$

3.C Appendix: A model with frictionless producers

In this appendix I solve a different version of the model. In this version there is one population of long-term producers who live between $t = 0$ and $t = 2$ and can use the entire term structure to hedge their physical exposure to the commodity. This case is more general, but it supposes that producers have no trading frictions. Indeed, it induces important intertemporal diversification and hedging positions. Qualitatively, most of the results are the same as in the version of the model presented in Sections 3.4, 3.5, and 3.6. However, quantitatively they can differ.

In what follows I derive the optimal positions of the long-term producers. The optimal positions of the other agents are the same as in the previous version of the model. Then, I solve the model for the pre-financialization economy that is without investors. After that, I solve the model for the two types of investors (constrained and unconstrained).

3.C.1 Optimal position of the long-term frictionless producer

This version of the model is characterized by the existence of a long-term producer who lives at three dates and gets random productions \tilde{q}_1 and \tilde{q}_2 at $t = 1$ and $t = 2$. The producer can trade all futures contracts without constraints. The implied assumptions are that the liquidity is good enough for all the term structure or even that the producer does not face regulatory issues limiting his investment opportunities.

Because of the long-term horizon, the producer maximizes his expected utility at the final date $t = 2$ coming from the profit $\pi_2 = \pi_1 + \tilde{q}_2 R_{s,2} + R_{F_{2,2}} f_{2,2}^p$ with $\pi_1 = \tilde{q}_1 R_{s,1} + R_{F_{1,1}} f_{1,1}^p + R_{F_{1,2}} f_{1,2}^p$. I adopt a two-step backward reasoning by first solving the problem (3.1) for the producer at $t = 1$ over π_2 in order to find $f_{2,2}^{p*}$ and then solve the problem (3.19) for the producer at $t = 0$ over π_2 to find $f_{1,1}^{p*}$ and $f_{1,2}^{p*}$ for the given optimal $f_{2,2}^{p*}$.

At $t = 1$, the first step of the reasoning gives the following optimal position for the long-term producer:

$$f_{2,2}^{p*} = \frac{\mu_{F_{2,2}}}{\gamma_p \sigma_{2,2}^2} - \frac{\rho_{[2,22]}}{\sigma_{2,2}^2} \quad (3.18)$$

This position is the same as the one at $t = 1$ of the long-term producer with a preferred habitat in Section 3.4.

At $t = 0$, the long-term agent solves the following program:

$$\max_{f_{1,1}^p, f_{1,2}^p} E_0[\pi_2] - \frac{\gamma_p}{2} \text{Var}_0[\pi_2] \quad (3.19)$$

Which leads to the following optimal positions:

$$f_{1,1}^{p\star} = \frac{\mu_{F_{1,1}}\sigma_{1,2}^2 - \mu_{F_{1,2}}\sigma_{[11,12]}}{\gamma_p(\sigma_{1,1}^2\sigma_{1,2}^2 - \sigma_{[11,12]}^2)} + \frac{\sigma_{[11,12]}\{\rho_{[2,12]} + \rho_{[1,12]}\} - \sigma_{1,2}^2\{\rho_{[2,11]} + \rho_{[1,11]}\}}{(\sigma_{1,1}^2\sigma_{1,2}^2 - \sigma_{[11,12]}^2)} + \frac{\mu_{F_{2,2}}(\sigma_{[12,22]}\sigma_{[11,12]} - \sigma_{[11,22]}\sigma_{1,2}^2) + \gamma_p\rho_{[2,22]}(\sigma_{[11,22]}\sigma_{1,2}^2 - \sigma_{[12,22]}\sigma_{[11,12]})}{\gamma_p\sigma_{2,2}^2(\sigma_{1,1}^2\sigma_{1,2}^2 - \sigma_{[11,12]}^2)} \quad (3.20a)$$

$$f_{1,2}^{p\star} = \frac{\mu_{F_{1,2}}\sigma_{1,1}^2 - \mu_{F_{1,1}}\sigma_{[11,12]}}{\gamma_p(\sigma_{1,1}^2\sigma_{1,2}^2 - \sigma_{[11,12]}^2)} + \frac{\sigma_{[11,12]}\{\rho_{[2,11]} + \rho_{[1,11]}\} - \sigma_{1,1}^2\{\rho_{[2,12]} + \rho_{[1,12]}\}}{(\sigma_{1,1}^2\sigma_{1,2}^2 - \sigma_{[11,12]}^2)} + \frac{\mu_{F_{2,2}}(\sigma_{[11,22]}\sigma_{[11,12]} - \sigma_{[12,22]}\sigma_{1,1}^2) + \gamma_p\rho_{[2,22]}(\sigma_{[12,22]}\sigma_{1,1}^2 - \sigma_{[11,22]}\sigma_{[11,12]})}{\gamma_p\sigma_{2,2}^2(\sigma_{1,1}^2\sigma_{1,2}^2 - \sigma_{[11,12]}^2)} \quad (3.20b)$$

These optimal positions are different than the ones at $t = 0$ of the producer with a preferred habitat in Section 3.4. They describe a sophisticated hedging behavior with strong speculative and diversification parts. The important result is that without any constraint (linked for instance to liquidity issues or to the regulatory framework), the producer should create an optimal portfolio of futures contracts for hedging purposes which embeds speculative and diversification positions. More precisely, there are two kinds of diversification and hedging in these positions: at the same period and intertemporal between $t = 1$ and $t = 2$.

3.C.2 Pre- and post-financialization equilibria

Pre-financialization: clearing of the futures markets without investors

The clearing of the futures markets in the pre-financialization is the simplest case with only two types of market participants: the N_s short-term speculators and the N_p long-term producers. The clearing equations are the following:

$$\begin{aligned} t=0, \text{ maturing in 1: } & N_s f_{1,1}^{s\star} + N_p f_{1,1}^{p\star} = 0 \\ t=0, \text{ maturing in 2: } & N_s f_{1,2}^{s\star} + N_p f_{1,2}^{p\star} = 0 \\ t=1, \text{ maturing in 2: } & N_s (f_{2,2}^{s\star} - f_{1,2}^{s\star}) + N_p (f_{2,2}^{p\star} - f_{1,2}^{p\star}) = 0 \end{aligned}$$

Using the optimal positions of the agents from equations (3.2), (3.3), (3.4), (3.18), (3.20a),

and (3.20b), the equilibrium expected returns or risk premia are:

$$\mu_{F_{1,1}}^* = \frac{\lambda_p \gamma_p \left\{ (\lambda_p + \lambda_s) \sigma_{2,2}^2 (\rho_{[1,11]} + \rho_{[2,11]}) - \lambda_s \sigma_{[11,22]} \rho_{[2,22]} \right\}}{(\lambda_p + \lambda_s)^2 \sigma_{2,2}^2} \quad (3.21a)$$

$$\mu_{F_{1,1}}^* = \frac{\lambda_p \gamma_p \left\{ (\lambda_p + \lambda_s) \sigma_{2,2}^2 (\rho_{[1,12]} + \rho_{[2,12]}) - \lambda_s \sigma_{[12,22]} \rho_{[2,22]} \right\}}{(\lambda_p + \lambda_s)^2 \sigma_{2,2}^2} \quad (3.21b)$$

$$\mu_{F_{2,2}}^* = \frac{\lambda_p \gamma_p \rho_{[2,22]}}{(\lambda_p + \lambda_s)} \quad (3.21c)$$

The main results from equations (3.21a), (3.21b), and (3.21c) are the same as in Result 3.1 of the previous version of the model described in Section 3.5.

Financialization: clearing of the futures markets with investors

The clearing of the futures markets in the financialization era, with three types of market participants (the N_s short-term speculators, the N_p long-term producers, and the N_{in} investors) are given by the following equations:

With constrained investors:

$$t=0, \text{ maturing in 1: } N_s f_{1,1}^{s*} + N_p f_{1,1}^{p*} + N_{in} f_{1,1}^{w*} = 0$$

$$t=0, \text{ maturing in 2: } N_s f_{1,2}^{s*} + N_p f_{1,2}^{p*} = 0$$

$$t=1, \text{ maturing in 2: } N_s (f_{2,2}^{s*} - f_{1,2}^{s*}) + N_p (f_{2,2}^{p*} - f_{1,2}^{p*}) + N_{in} f_{2,2}^{w*} = 0$$

With unconstrained investors:

$$t=0, \text{ maturing in 1: } N_s f_{1,1}^{s*} + N_p f_{1,1}^{p*} + N_{in} f_{1,1}^{w*} = 0$$

$$t=0, \text{ maturing in 2: } N_s f_{1,2}^{s*} + N_p f_{1,2}^{p*} + N_{in} f_{1,2}^{w*} = 0$$

$$t=1, \text{ maturing in 2: } N_s (f_{2,2}^{s*} - f_{1,2}^{s*}) + N_p (f_{2,2}^{p*} - f_{1,2}^{p*}) + N_{in} (f_{2,2}^{w*} - f_{1,2}^{w*}) = 0$$

The results in the post-financialization economies obtained by using the optimal positions of the agents from equations (3.2), (3.3), (3.4), (3.10), (3.16), (3.17), (3.18), (3.20a), and (3.20b) are the same as in Results 3.2 and 3.3 of the previous version of the model described in Section 3.5.

Conclusion générale

Cette thèse étudie les marchés futures de matières premières dans différentes conditions de fonctionnement. Ce faisant, elle met en avant la diversité et la complexité de ces marchés tout en tenant compte de la financiarisation. L'hétérogénéité des marchés futures de matières premières provient de l'hétérogénéité des matières premières sous-jacentes à ces marchés. Il existe un spectre très large de matières premières allant des produits agricoles servant à l'alimentation aux produits énergétiques en passant par les métaux (précieux ou industriels). Cette variété explique en partie leur complexité. Historiquement, les caractéristiques physiques (type et utilisation finale de la matière première, structure des agents cherchant à se couvrir sur le marché...) ont été les premiers éléments à jouer un rôle dans le processus de formation des prix des matières premières. Néanmoins, avec la financiarisation observée ces dernières années, des éléments de nature financière se sont ajoutés à cela. Ces éléments ont modifié le processus de formation des prix futures de matières premières, entraînant une intégration plus importante de ces marchés avec les marchés financiers.

Les chapitres 1 et 2 de cette thèse mettent en avant l'hétérogénéité des marchés futures de matières premières. Ils étudient le processus dynamique de formation des prix dans des marchés futures de matières premières segmentés.

L'analyse empirique du chapitre 1 se sert de l'étude d'un fait stylisé reconnu pour les matières premières stockables afin d'analyser le processus de formation des prix d'un marché futures portant sur une matière première non-stockable. Parce que nous validons empiriquement l'existence de l'effet Samuelson sur plusieurs marchés dérivés d'électricité dans le monde, les implications de ce chapitre sont de deux natures. Premièrement, théoriquement parlant, la justification de l'effet Samuelson ne semble pas être liée au caractère stockable d'une matière première, mais à sa rigidité à court terme. Ceci rend nécessaire le développement d'un cadre théorique permettant de mieux comprendre le lien entre les caractéristiques physiques d'un marchés futures de matières premières et le processus de formation des prix. Ensuite, même si l'électricité est non-stockable, nous retrouvons certaines similitudes dans le comportement dynamique de ses prix avec d'autres matières premières stockables. Ce constat permet de parfaire les modèles de structure par terme d'électricité utilisés à des fins de trading, d'évaluation d'actifs dérivés et de gestion des risques.

Le modèle théorique du chapitre 2 montre le lien qui existe entre le comportement dynamique des prix (spot et futures) d'une matière première stockable et ses caractéristiques physiques. Ce modèle affirme la grande hétérogénéité des marchés futures de matières premières. Ainsi, il met en avant le paradoxe qui consiste à parler des matières premières comme d'une classe d'actifs homogènes. Par exemple, l'or et le pétrole sont deux marchés de matières premières avec des caractéristiques physiques très différentes, et donc des comportements de prix très différents. Dès lors, cette hétérogénéité doit être prise en compte dans différentes circonstances. Premièrement, lors de la réalisation de modèles d'évaluation d'actifs dérivés. Ces modèles devraient prendre en compte les particularités de chaque marché afin d'avoir une librairie de modèles spécifiques pour chaque matière première, et non un modèle général applicable à toutes. De plus, cette hétérogénéité doit aussi être prise en compte lors de la régulation des marchés. En effet, la spéculation sur les marchés futures de matières premières semble favoriser les agents dominants et défavoriser les agents dominés sur un marché futures de matières premières. La mise en place de régulation visant à modifier le niveau de spéculation ne va donc pas avoir le même effet sur les différents marchés de matières premières.

Le chapitre 3 s'intéresse à la modification du processus de formation des prix induite par la financiarisation. Ce modèle théorique montre que les marchés futures de matières premières, au travers de leur fonction de partage des risques, évoluent avec l'arrivée d'investisseurs prenant des positions sur plusieurs classes d'actifs. Les implications économiques de ce modèle sont importantes. Premièrement, la financiarisation modifie le coût de couverture payé par les agents cherchant à se couvrir sur les marchés futures de matières premières. Selon les marchés, cette modification se fait au bénéfice de certains agents et au détriment d'autres. Ensuite, la forme de la structure par terme des primes de risques (et des prix) évolue. Enfin, la défragmentation des marchés de matières premières avec les marchés d'autres classes d'actifs améliore le partage des risques individuels mais augmente le risque systémique en favorisant la propagation des chocs.

Cette thèse s'intéresse à des problématiques d'économie financière importantes et d'actualité. Ainsi, sur la base de ces travaux, d'autres axes de recherche sont envisageables.

Pour commencer, la compréhension de l'impact de la financiarisation sur les marchés dérivés de matières premières n'est pas parfaite. Un axe envisageable est celui de l'analyse de la modification de la fonction de découverte des prix de ces marchés. Est-ce que le contenu informationnel de la structure par terme se modifie avec le trading d'investisseurs multi classes d'actifs? Un autre axe est celui de l'analyse des conséquences de la vitesse de trading sur les marchés futures de matières premières. Avec l'automatisation de ces marchés, une nouvelle catégorie d'agents a émergé, celle des traders haute-fréquence. La littérature s'intéressant à ces agents est importante pour les autres classes d'actifs mais relativement

rare pour les matières premières.

Ensuite, concernant le développement des marchés dérivés de matières premières non-stockables. Un axe envisageable est celui de l'étude approfondie des faits stylisés propres à ces marchés. Par exemple, des prix négatifs sont régulièrement observés sur les marchés futures d'électricité. Le fait de devoir payer pour vendre une matière première peut paraître contre-intuitif et pose des questions très importantes en terme d'évaluation des produits dérivés et de régulation.

Le développement de ces axes de recherche est en dehors de la portée de cette thèse, mais peut-être pourront-ils motiver de nouvelles recherches dans les années à venir?

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Résumé

Les marchés futures de matières premières existent depuis des siècles. Néanmoins, depuis le début du 21^e siècle, le développement en parallèle de la financiarisation et de marchés futures sur une matière première non-stockable (l'électricité) a bouleversé leur fonctionnement.

Les trois essais de cette thèse étudient théoriquement et empiriquement les marchés futures de matières premières dans différentes conditions de fonctionnement.

Le premier essai est une étude empirique qui montre l'existence de l'effet Samuelson sur les marchés futures d'électricité. Ce faisant, il montre que le stockage n'est pas une condition nécessaire à l'existence d'un tel effet.

Le second essai est un modèle qui montre comment le comportement dynamique des prix d'une matière première stockable sur un marché futures segmenté du reste de l'économie est impacté par ses caractéristiques physiques, et notamment par le coût de stockage.

Enfin, le troisième essai est un modèle qui montre que la financiarisation modifie la fonction de partage des risques des marchés futures de matières premières, et ce, quelle que soit la maturité concernée.

Mots Clés

Matières premières ; Marchés dérivés ; Structure par terme ; Prime de risque ; Financiarisation ; Effet Samuelson.

Abstract

Commodity futures markets have a long history. However, since the beginning of the 21st century, both the financialization process and the development of futures markets on a non-storable commodity (the electricity) have shake up their functioning.

The three essays of this thesis study theoretically and empirically commodity futures markets in different situations of functioning.

The first essay is an empirical study that shows that the Samuelson effect exists on electricity derivative markets. As a consequence, it shows that storage is not a necessary condition for such an effect.

The second essay is a model that shows how the dynamic behavior of storable commodity prices on a segmented futures market is affected by its physical characteristics, and more precisely by the cost of storage.

Further, the third essay is a model that shows that financialization changes the risk sharing function of commodity futures markets, whatever the concerned maturity.

Keywords

Commodities ; Derivative markets ; Term structure ; Risk premium ; Financialization ; Samuelson effect.