

Experimental study and application of homogenization based on metamaterials

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THÈSE DE DOCTORAT DE l'UNIVERSITÉ PIERRE ET MARIE CURIE

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Étude expérimentale de structures basées sur les metamatériaux; application de l'homogénéisation à ces structures

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Abstract

In order to retrieve effective parameter of the periodic structure, people have made effort to develop the theory of homogenization, which regards the periodic structure as a homogeneous medium. This thesis mainly focuses on the proposition and validation of classical and second order homogenization. A kind of method for homogenization is applied to obtain effective parameters of multilayer structures. Moreover, classical homogenization is adopted to calculate transmission properties of dielectric multilayer structures and the dispersion relation of metallic cylinder array mounted on a smooth metallic ground plane. From the dispersion relation, its band gap property at a certain frequency band is revealed and is applied to reshape the radiation pattern of traditional patch antenna by suppressing the surface wave propagating on the ground plane. We have published his part of the work in [22].

We experimentally demonstrate one important limitation of classical homogenization when calculating the transmission property of an ultra-thin metallic grating. The measured transmission coefficient is much smaller than that calculated by classical homogenization, although the interferences may be caused by the imperfect experiment facility are eliminated. Thus we propose a new second order homogenization, which is able to get the transmission coefficient correspondent to numerical results. Furthermore, second order homogenization has been experimentally validated by several metallic grating with the same dimension except for the thickness. This part of work will be included in a chapter of a book will be published by Intech Publisher.

Chapter 1

Introduction

This thesis focuses on homogenization of metamaterials, which are periodic structures exhibiting extraordinary properties that do not exist in nature. The structuration of metamaterial being at the subwavelength scale, they can be replaced by equivalent, homogenized material.

1.1 Background

Since 1960s Veselago proposed the conception of negative reflective index metamaterials that exhibit negative permittivity ε and permeability μ , lots of scientists have made effort in this domain. After approximate 30 years, thin wires and split-ring array structures, which have negative ε and μ respectively, are invented. In order to determine the effective constitute parameters, several methods have been proposed.

The first one is to adopt the numerical tools to simulate the effective parameters of metamaterials, since the properties of effective metamaterials can be achieved from simulation by inferring refraction n and relative impedance Z, which can be derived from transmission and reflection coefficients [54]. Finite Element Method (FTDT), Method of Moments (MOM) and Finite Integration Technology (FIT) are applied in the numer-

ical tools such as High-Frequency Electromagnetic Field Simulation (HFSS) and CST Microwave Studio (CST). The second method uses the scattering parameters to retrieve the constitute parameters, which is proposed by Smith et all in 2002 [62]. Actually, retrieve method is mainly used to dealing with isotropic medium [35, 63]. The third one is so-called homogenization, which allows us to replace the subwavelength periodic structures that have complicated boundary condition with an equivalent and effective medium. [8, 2, 57, 26, 45, 27, 47]

1.2 Aim of this thesis

In this thesis, in order to deal with the metamaterials with one or two-dimensional periodic structures, an approximate analytical model is adopted to homogenize the structure to obtain its transmission properties and dispersion relation in a simple and fast way. The metallic material is taken into consideration in the far infrared, thus in a frequency range where metal behaves as an opaque medium. Such metallic arrays are used in the designation of metallo-dielectric structures, as the Artificial Magnetic Conductors used in the design of the antenna, and there is currently renewed interest in such array since they are the key piece of so-called metasurfaces (with a typical resonant behavior as in the mushroom structure of Sievenpiper [60]).

In addition to being able to produce unexpected scattering of electromagnetic waves, these structures can support guided waves, and it is for this property that they have been proposed by Pendry and coworkers [53]; in this context, these guided waves have been called 'spoof plasmons' since they mimic, in the far infrared regime, the behavior of plasmons observed in the visible range (plasmons are the wave guided at the flat interface between air and metal, and this requires a negative permittivity). In this thesis, this kind of metasurface is used to reshape the radiation pattern of traditional patch antenna by suppressing the guided waves on the metal ground in a certain range of frequency. Furthermore, the drawbacks of classical homogenization for the ultra-thin structures are reviewed. A new method named the second order of homogenization is investigated.

1.3 Outline of thesis

The overview of this thesis is as follows:

In chapter 2, starting from the basic phenomenon in daily life, the conceptions of wave propagation and metamaterials are introduced. Moreover, a brief history of the development of metamaterials is reviewed. Several landmark discoveries after the publication of Maxwell Equations are presented, such as the proposition of the idea of metamaterials and the experimental validation of cloaking. In addition, several important applications of metamaterials are demonstrated, such as Electric Band Gap (EBG), High Impedance Surface (HIS) and Left Hand Material (LHM) that is able to realize negative-index. Finally, a short introduction and history of homogenization are shown to provide a general view of the background.

In Chapter 3, classical homogenization is investigated and validated to predict the transmission properties of a multilayer structure. Start from the simplest multilayer structure that consists of two kinds of dielectric layered medium, numerical results is provided to validate the analytical result of multilayer slab and corrugated surface. In order to solve the same problem for a metallic corrugated surface, continuous field condition is taken into consideration to calculate its transmission property. And also, a thin structure can support surface plasmons is experimental validated.

Furthermore, a meta-surface made of a finite array of grounded metallic cylinders is proposed to realize a transmission bandgap is demonstrated. Such a property has been shown to reshape the radiation field of a printed patch antenna. Also, the resonances among the metallic cylinders are analyzed, and the field distributions are shown at different frequencies.

Chapter 4 starts from a phenomenon of disagreement caused by the method of classical homogenization in calculating the transmission property of an ultra-thin periodic structure. The classical homogenization of second order is thus introduced. Six different prototypes are manufactured to validate the second order homogenization, which proves that classical homogenization can be used only when the thickness of periodic structures is larger than 0.01λ , otherwise, classical homogenization of second order should be employed.

Moreover, investigations of Artificial Magnetic Conductor (AMC) that consists of ultra-thin structures by using interface homogenization are shown. The "jump" boundary condition was employed instead of bulk region boundary condition to determine the phase shift property of AMC. Experimental validations are given to show that exactly phase shift due to the different thickness of metal layer.

Chapter 2

State of art

The periodic phenomenon is the most important basic elements to constitute complicated movements of the world, even the Galaxy. In our daily life, the water wave that we see in the lake or the sea results from oscillation of the surface of water; the sound we hear results from the vibration of the sound source induce the oscillation of the air molecule from distance to excite our eardrum; the wireless video we can see on the screen is because the video information is modulated on the carrier wave that results from the oscillation of the electrons in the crystal oscillator. And, this kind of oscillation of lots of elements causes "wave". For example, more than century ago a Germany scientist Hertz experimentally proved the existence of electromagnetic wave. Until now people have found lots of properties and applications such as telecommunication, power transmission, detecting and so on.

Trying to control and tail transmission of the electromagnetic wave in the different medium is always an important and meaningful subject. In the near past several decades, scientists found that not only the properties of the material that we used in the experiment is able to affect the wave property. Moreover, the arrangement of the material has advanced impact when tailing the wave propagation, entitling traditional material with extraordinary properties that do not exist in nature. Normally such arrangement refers to periodic, for which the structure is so-called 'metamaterial'. Moreover, in order to study inner physical mechanism, lots of methods are proposed such as Numerical calculation, Retrieval method, Field averaging method. These methods are briefly introduced in this chapter.

2.1 Metamaterials

2.1.1 Introduction

'Metamaterial' is a material composed of a periodic arrangement of metallo-dielectric unit cells at a scale smaller the wavelength of the propagating wave. The 'meta' comes from Greek prefix ' $\mu\epsilon\tau\alpha$ -', which means beyond. In another word, metamaterials describe those materials can provide original properties that could not be found in nature. For the nature materials, their resonance frequencies mainly rely on the polarization of their molecule, whose dimension is extremely smaller than the wavelength of the incident wave. Although they can resonant in the same mechanism as normal materials, the dimension of the array and the unit cells is also much smaller than the wavelength of the incident wave. Consequently, designation of the array can affect the resonance property of this structure, which gives us a new method to manipulate the incident wave.

Actually, people's understanding of the electromagnetism can be dated back to the 19th century when James Clerk Maxwell developed the formulation of Maxwell Equation. After that, scientists have made enormous progress in the telecommunication system. More importantly, people have an advanced view towards the interaction of the electromagnetic wave and different materials, including metamaterials.

Those extraordinary characteristics of metamaterials do not only rely on the properties of the traditional materials, but also their peculiar designation including geometry, size, and shape, which give them special capable for blocking, absorbing or selecting waves. Well-designed metamaterials can affect the radiation or propagation of electromagnetic, acoustic and water waves. As early in the 1960s, a Russia physicist Veselago first proposed the existence of metamaterials [66] Although the mechanism between the electromagnetic wave and metamaterial is not very clear, he predicted that such kind of medium has new capability such as negative refractive index.

In 1990s, engineers experimentally demonstrated thin metal wires array and split ring array, shown in Fig.2.1 (a), which has negative permittivity and permeability respectively. Electromagnetic wave can excite special resonances in these kinds of artificial structure, which is different from those properties in conventional materials.



Figure 2.1: (a) Split-ring and wires array. (b) Cloaking

In the past decades, one of the most famous conceptions related to metamaterial is cloaking. Lots of scientists and engineers have made effort in such filed. In 2006, Prof. Pendry and his team realized a cloaking can work in microwave frequency [58]. This cloak is made of metamaterials that consist of periodically arranged elements, shown in Fig.2.1 (b), which is able to change the transmission properties of the electromagnetic wave that propagate through the cloak. And consequently, make the stuff placed in the cloaking invisible in microwave frequency band.

Generally speaking, well-designed electromagnetic metamaterials have several extraordinary properties that could not be found in nature, shown as following.

1) **Single Negative Metamaterials**: Single negative metamaterials (SNG) have negative permittivity or negative permeability. In [73], the authors realize DNG metamaterials by combining two SNG structures. To conduct wave reflection experiments, the slab of negative mu materials and negative epsilon materials have been joined. Like DNG metamaterials, SNGs change their parameters such as refraction index n.

2) **Double Negative Metamaterials**: Double negative metamaterials (DNG) are the metamaterials that have both negative permittivity and permeability. These are also known as negative index metamaterials (NIM) [15]. Another name for DNGs is left-handed media, because in this kind of medium the electric vector, magnetic vector, and wave propagating vector satisfy the left-hand rule [78].

3) Electromagnetic Band Gap (EBG) Metamaterials: Electromagnetic band gap metamaterials can control wave propagation by tuning the parameter to form a bandgap at certain frequency band. It is achieved either by photonic crystals (PC), or left-handed materials (LHM).

After decades of investigation, there are tremendous kinds of metamaterials are proposed, such as:

1, Textured Surface

Textured surface has extraordinary properties in comparison with smooth metallic or dielectric surfaces, such as high impedance, electric bandgap or frequency selective. For example, High Impedance Surface (HIS) is one of the important applications of metamaterials. By coupling a textured surface on the normal smooth metal surface, we can manipulate its radio-frequency electromagnetic property [60]. The typical smooth conducting sheet has small surface impedance, however, such kind of textured surface has relatively high impedance. A well-known HIS is a mushroom-like structure, shown in Fig.2.2, designed by Sievenpiper [60]. The structure consists of an array of square metal patches, connected to a metal ground by vertical conducting vias. When the unit cells are much smaller than the operating wavelength, its behavior is similar to resonant LC



Figure 2.2: HIS (a) Side view (b) Top view

circuits, which perform as an electric filter to suppress the current flow along the sheet. Consequently, the surface impedance is very high, and the tangential magnetic field is very small. Such structure can be described as artificial magnetic conductor (AMC).

Figure 2.3: EBG Structure

2, Photonic Metamaterials

Photonic metamaterials are the type of electromagnetic metamaterials that designed to work in optical frequencies. Photonic metamaterials radiate the electronic waves at optical wavelengths. Furthermore, the sub-wavelength period differentiates the photonic metamaterials from photonic band gap structure. This is because the optical properties do

not come from the photonic band gaps, but from a sub-wavelength interaction with the light spectrum. The metamaterials with the capability of zero index of refraction (ZIMs) and negative index of refraction (NIMs) is the active area of research in optical materials.

3, Chiral Metamaterials

A chiral medium is composed of particles that cannot be superimposed on their mirror images [39]. A chiral medium has different responses for a left circularly polarized (LCP) wave and a right circularly polarized (RCP) wave due to the intrinsic chiral asymmetry of the medium [67]. Besides, cross-coupling between the electric field and magnetic field going through a chiral medium is also existent.



Figure 2.4: Different kinds of Chiral metamaterials

2.1.2 Application of Metamaterials

Lots of sophisticated applications of metamaterials including optical fiber, aerospace device, detect sensor and super lens have been proposed and realized in the past decades. Metamaterials are cross subjects involving electrical engineering, electromagnetic, optical antenna engineering, and material science. In the following, several important properties and applications of metamaterials in electromagnetic domain are shown.

1, Left Hand Materials

One of the most typical applications of metamaterials realized by scientists is lefthand materials (LHM). Russia physicist Mandelshtan first theoretically predicted this kind of material in 1940. Almost 30 years later, from Maxwell equation, another physicist Vesolago found that in this special materials, electric vector, magnetic vector, and pointing vector do not satisfy the Right-Hand Rules, but Left Hand Rules. However, no more big progress is made in the following another 30 years. Until 1996, an English physicist Pendry realized a metal wires array, which has negative permittivity. In 1999, he achieved negative permeability by arranging the metal split-ring (SRRs). In 2000, an American physicist D. R. Smith and his team combined those two structures in a proper array and firstly achieved double negative materials (DNM), whose permittivity and permeability are negative.

Actually, all the materials can be separated into four categories by using the positive or negative value of permittivity ε and permeability μ , shown in Fig.2.5. In the first



Figure 2.5: Categories of materials based on the value of ε and μ

quadrant, where $\varepsilon > 0$ and $\mu > 0$, it represents the normal right-hand material, whose electric vector *E*, magnetic vector *H* and the wave propagate vector *k* satisfy the right-

hand material. In the second and fourth quadrant, only one of ε and μ is positive, while the other is negative. Those kinds of material only can support evanescent decaying wave, in another word, waves could not propagate in such medium.

The most important and interesting materials locate at the third quadrant, whose ε and μ are all negative, thus so-called double negative material. It follows the left-hand rules because its wave propagation vector reverses in comparison with normal materials.

This double negative material has many special properties, such as negative refraction index, reversal Doppler Effect. From Snell's law, when the lights, or electromagnetic waves, propagate against the interface between air and DNM, the reflect energy do not flow on the original direction, however, reverse to the opposite one, shown in Fig.2.6 (a).



Figure 2.6: Application of left hand materials

Due to its negative refraction index, when the wave propagation traveling through an LHM slab, the wave would focus on it and then create an image when leaving the slab, which has been realized in the experiment, shown in Fig.2.6 (b).

2, Filter

Because of the bandgap and high impedance property of metamaterials, they can be applied to design microwave devices, such as filter. In 2003, Martín proposed a left hand medium consists of an array of split-ring resonator (SRR) over a coplanar waveguide (CPW) to control the wave propagation along the CPW [41], see Fig.2.7 (a). The SRR



provide a bandgap for the transmission and can be tuned by varying their dimension.

Figure 2.7: (a) Split-ring resonator perform as filter to provide band gap for a coplanar waveguide [41];(b) Square C slot act as low pass filter on a traditional transmission line [1].

In 2007, Ali et al presented a low pass filter which provides flat pass band response but slow attenuation transmission [1], by etching a double square C slots on the metallic ground of a transmission line, see Fig.2.7 (b). This kind of structure improves the filter selective, besides it is easy to be manufactured.

3, Antenna

The antenna is one of the most important elements in the telecommunication system, such as radio, broadcast television, radar or wireless computer network. Since the very beginning at the 1880s when Hertz invented dipole antenna, tremendous kinds of antennas are designed for different applications. Due to various requirements, lots of parameters, such as radiation pattern, bandwidth, gain, return loss or even size, are taken into first consideration by engineers and scientists.

However, some defect of antenna comes from the intrinsic properties. For example, the radiation of traditional printed antenna is due to resonance of electromagnetic wave in

the dielectric between radiating patch and the ground plane. Consequently, the size of the patch antenna is hard to minimize. Meanwhile, although horn antenna is able to radiate high power electromagnetic wave, its radiation pattern is not easy to control. Thanks to the realization of metamaterial, engineers have found that by employing metamaterial, many kinds of properties of the different antenna can be enhanced.

1, Radiated Power Enhancement

DNG metamaterials can be used to increase the radiated power of small antenna [77]. A small dipole antenna with radome made of DNG metamaterials has the larger radiated power than the conventional antenna.

2, Directivity Enhancement

Metamaterials have extraordinary property to controls the electromagnetic radiation, thus to confine the originating energy in a small angular [76]. A DNG material can be used to enhance the directivity of the antenna. For instance, the metamaterials antennas decrease the beam width and side lobe ratio [16] and thus enhance the directivity and reduce the return loss of the antenna.

3, Bandwidth Enhancement

Antenna compensated with Metamaterials is able to achieve wider bandwidth as compared to the conventional patch antenna [40]. This is achieved by use of superstrate of metamaterials over a conventional antenna or by employing of LHM.

2.2 Homogenization

In the early 1960s, the Geometrical Theory of Diffraction (GTD) was proposed and applied to solve the problem of rays that hit edges, corners or vertical of boundary surface [32]. As the development of electromagnetic theory, engineers met more and more complicated problems in reality. Lots of Numerical methods were proposed, such as Finite-difference time-domain method (FTDT), Transmission line method (TLM), Finite element method (FEM), Method of moments (MOM) and so on. These numerical tools are good at solving the problem of big volume, high frequency, and complicated boundary conditions. However, these kinds of method require high-speed modern computers and relatively long time to solve the problem.

The transmission properties of metamaterials can be determined by another simplified way, because of its periodic arrangement structure. This method is so called 'homogenization', which allows us to replace the subwavelength periodic structures who have complicated boundary condition with an equivalent and effective medium. Motivated by



Figure 2.8: Homogenizaed materials

the development of theoretical physics, the problem related to the periodic small structures was investigated in a perspective of macroscopic view of physical phenomena. Actually, any well-mixture materials, whose basic elements are smaller than a certain scale, can be regard as homogenous materials, shown in Fig.2.8.

2.2.1 Field averaging method

Another way to derive the effective parameter of periodic structures is so called "field averaging" method, a straightforward way of homogenization presented by Pendry et al. in 2006 [64]. It aims to obtain the average value of the field in the unit cell instead of the field that varies throughout the unit cell. In this method, averaged electric and magnetic

field \overline{E} and \overline{H} can be described as

$$\begin{cases} \overline{H}_{i} = d^{-1} \int \mathbf{H} \cdot dx_{i} \\ \overline{E}_{i} = d^{-1} \int \mathbf{E} \cdot dx_{i} \end{cases}$$
(2.1)

where *d* is the length of the side of unit cell. In another words, \overline{E} and \overline{H} can be calculated from line integral of local field in the area of unit cell. Consequently, it resembles a finite differencing of Maxwell equation [64].



Figure 2.9: Unit cell for field averaging

Fig.2.9 shows the definition of averaging field in unit cell of periodic structure. The general idea is that the averaging magnetic field of top surface H_z is parallel to Z axis and coms from curl of electric field E_x and E_y which lie on the edges of the four cubic lattices. Besides, electric field E_y can be derived from the curl of magnetic field H_x and H_z on the edges of the adjacent cubics. In order to develop a general method, Maxwell equation subjected to the periodic boundary condition should be solved to find each mode of local fields. The curl equation of Maxwell is

$$\begin{cases} \nabla \times \mathbf{E} = i\omega \mathbf{B} \\ \nabla \times \mathbf{H} = -i\omega \mathbf{D} \end{cases}$$
(2.2)

Normally, the unit cell is inhomogenous, thus for a periodic medium, the solution of Eq.2.1 has the form of

$$g(x) = h(x)exp(iqx)$$
(2.3)

where q(x) is any of the components in Eq.2.1 h(x+d) = h(x) is the periodic function. Thus, in the case of $d \ll \lambda$, where λ is the operating wavelength in the effective medium, the averaging field can describe relative accurate field for us to determine its effective electromagnetic parameter.

2.2.2 Retrieval method

The general idea of retrieval method is to obtain the material parameter from its scattering properties such as transmission or reflection coefficient. Here we take 1-D problem as an example, shown in Fig.2.11 the transmission property of slab can be expressed by transmission matrix



Figure 2.10: Scattering parameter on a homogenous 1-D slab

$$W_t = T \cdot W_i \tag{2.4}$$

where $W_i = [E, H]$ is the incident wave, *E* and *H* are the complex electric and magnetic wave amplitude on the boundary between the slab and the air, W_t is the transmission wave.

According to [61], the transfer matrix of such a homogenous slab is

$$T = \begin{pmatrix} \cos(nkd) & -\frac{z}{k}\sin(nkd) \\ \frac{z}{k}\sin(nkd) & \cos(nkd) \end{pmatrix}$$
(2.5)

where *n* is refractive index and *Z* is wave impedance of the slab. *n*, *Z* can be expressed by its permittivity ε and permeability μ :

$$\varepsilon = n/Z, \quad \mu = nZ.$$

Thus, the scattering matrix related to the transmission coefficient can be obtain by using T matrix [61]:

$$S_{21} = \frac{2}{T_{11} + T_{22} + (ikT_{12} + \frac{T_{21}}{ik})} \quad S_{11} = \frac{T_{11} - T_{22} + (ikT_{12} - \frac{T_{21}}{ik})}{T_{11} + T_{22} + (ikT_{12} + \frac{T_{21}}{ik})}$$

$$(2.6)$$

$$S_{22} = \frac{T_{22} - T_{11} + (ikT_{12} - \frac{T_{21}}{ik})}{T_{11} + T_{22} + (ikT_{12} + \frac{T_{21}}{ik})} \quad S_{12} = \frac{2\det(T)}{T_{11} + T_{22} + (ikT_{12} + \frac{T_{21}}{ik})}$$

for the homogenous material, we have $T_{11} = T_{22} = T_s$, det(T) = 1 and S is symmetric, thus

$$S_{21} = S_{12} = \frac{1}{T_s + \frac{1}{2}(ikT_{12} + \frac{T_{21}}{ik})}$$

$$S_{11} = S_{22} = \frac{\frac{1}{2}(ikT_{21} - \frac{T_{12}}{ik})}{T_s + \frac{1}{2}(ikT_{12} + \frac{T_{21}}{ik})}.$$
(2.7)
Consequently, *n* and *z* can be achieved by inverting Eq.2.7

$$n = \frac{1}{kd} \cos^{-1} \left[\frac{1}{2S_{21}} (1 - S_{11}^2 + S_{21}^2) \right]$$

$$Z = \sqrt{\left(\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2} \right)}.$$
(2.8)

This is the typical retrieval method to describe a homogenous slab. Generally speaking, retrieval methods is widely used because its easy way to retrieve the effective parameters of the medium in a macroscopical view such as permittivity, permeability and magneto-electric coupling coefficient of the bianisotropic metamaterial [37].

2.2.3 Near Resonance method

Actually, there are several kinds of methods to homogenize the periodic structures, for instance, the retrieval method, are well established. However, their solution based on a two-dimensional method that ignoring the case of a linear polarized magnetic field. Thus confine their application to infinite cylindrical obstacles. GUY and his team proposed a dielectric structure performing a full three-dimensional asymptotic analysis [8].

In their method, they focus on the problem of complex medium in 3-D geometric domain, denote by Ω . The periodicity between the unit cell that cause diffraction by fast oscillating behavior when the wave propagate in it is infinitesimal, denotes by *h*, shown in Fig. As demonstrated in [7], they proved that the structure behaves as local material is



Figure 2.11: Diffracting structures

described by a frequency-dependent tensor, when periodicity approaching to zero.

Chapter 3

Classical homogenization for metamaterials structures at the subwavelength scale

Metamaterials have promoted many research interests in the last decades because of their extraordinary properties that do not exist in nature to tail wave propagation in several domains such as electromagnetic, acoustic or water waves [9]. The most common metamaterials are made of periodic structures. When the spatial period of the structure is of wavelength size, the structures are referred as photonic or phononic crystals. When the scale of the periodicity is much smaller than the operating wavelength, the structures are considered as metamaterials. Homogenization approaches are preferred for the modeling of these metamaterials. In this case, an equivalent homogeneous (and possibly anisotropic) medium is taken into consideration. The classical homogenization is applied in the low-frequency regime assuming that all wavelengths involved in the structure are larger than the periodicity [3, 46, 47]. Extensions of this classical homogenization have been proposed, known as high frequency homogenization [12, 27] or high contrast homogenization [17]. Alternatively to these homogenization methods, retrieval methods[20] or theories of effective medium [53, 31] are also commonly used.

The electromagnetic homogenization for natural and artificial materials has a long history, and lots of theories are proposed to design effective constitutive parameters of the periodic structure. By averaging the microscopic field at the atomic or molecular scale, we can get the definition of permittivity and permeability; with the same idea, by using a proper method to average the field in the artificial periodic structure with a relatively large scale to avoid solving the fields of each unit cells, such method is known as 'homogenization'[47]. Here, the relatively large scale actually is a limitation for which the wavelength of the wave should be always much larger than the periodicity of the structure.

With such limitation, this method can accurately solve the field of artificial structures with less CPU time than the traditional numerical ways. One of the simple homogenization techniques is so called retrieving method, which is able to get the effective parameters from the scattering parameters of a metamaterial sample. For example, the Nicholson-Ross-Weir (NRW) retrieval method assumes the equivalence between a complex periodic structure and a simple slab of the same thickness, usually limited to permittivity and permeability. [17, 11].

3.1 Homogenization for Layered Medium

In this section, the capability of classical homogenization to predict the transmission property of layered structure at the sub-wavelength scale is demonstrated. The periodic interfaces in the real problem are replaced by a slab of a homogeneous medium, with an effective mass density tensor and an effective bulk modulus. Thus, explicit dispersion relation can be derived, corresponding to guided waves in the homogenized problem [47]. Homogenization approaches provide effective parameters by considering the permittivity ε , permeability μ of the materials and the geometry of the microstructure, which is dif-

ferent from the retrieval methods. As we mentioned before, a periodic structure can be regarded as effective medium, as long as the periodicity of the structure is much smaller than the wavelength of the electromagnetic wave. Classical homogenization related to periodic structures in this thesis should satisfy $kh \ll 1$, where k is the wavenumber of EM wave and h is the periodicity of the structure. Next, the simplest one dimension periodic multilayer structures are first taken into consideration. Secondly, analytical expression and numerical validation are given to describe the transmission property of EM wave in such medium.

Multilayer medium

In the following, we calculate the field in a layered structure that consists of two kinds of penetrable dielectric for electromagnetic waves, shown in Figure.3.1(a). Transmission property of such structure described by an analytical expression derived from Helmholtz Equation is given. Furthermore, the result reveals that the transmission property can be strongly affected by not only the permittivity of the materials but also the filling fraction of the two dielectrics.

Consider a plane wave propagating in the free space, the magnetic field H can be obtained from the Helmholtz Equation

$$\nabla \cdot [a\nabla H(x,y)] + k^2 b H(x,y) = 0, \qquad (3.1)$$

where $a = 1/\varepsilon$ and $b = \mu$, ε and μ are permittivity and permeability of the medium respectively. Magnetic field can be described as:

$$H(\vec{r},t) = H_0 \cdot e^{j(k \cdot \vec{r} - \omega t)}$$
(3.2)

where ω is the angular frequency of the plane wave, H_0 is the amplitude of vector of $H(\vec{r},t)$, and \vec{r} is the position vector of $H(\vec{r},t)$, \vec{k} is the wave vector of the propagating wave. From the theory of electromagnetic fields, we know that in the free space the

electronic fields $E(\vec{r},t)$ and magnetic fields $H(\vec{r},t)$ are mutually perpendicular, and also $E(\vec{r}), H(\vec{r})$ and \vec{k} satisfy the right handed screw rule as $E(\vec{r}) \times H(\vec{r}) = \vec{k}$, consequently, the electric field $E(\vec{r},t)$ can be expressed as $E(\vec{r},t) = E_0 \cdot e^{j(\vec{k}\cdot\vec{r}-\omega t)}$.

Before the study of the metamaterials, it is necessary to make it clear for the different plane waves. Mechanical waves are distinguished by the different oscillation direction of the particles. For example, acoustic waves are defined as "longitudinal" wave since those molecules in the air vibrate along the direction of sound propagation. On the contrary, water waves are defined as "transverse" wave because the surface of water vibrate up and down, which is vertical to the wave propagation direction. From such definition, electromagnetic wave is transverse wave.

Circular and linear polarizations are two kinds of transverse plane wave. Electric or magnetic field vectors are often used to describe the variation of the fields. Once the start point of the electric vector is fixed at a specific point, the track of the end of the vector travels along a circular path when the circular polarization wave propagates; otherwise, the track of the end of the vector travels along straight line and go back and forth, the wave is linear polarized. Since the circularly polarized wave can be composed of two linear polarized waves [34], in this thesis only the linear polarized transverse waves are taken into consideration for simplicity.

For electromagnetic waves, the electric and magnetic field, which are always perpendicular to each other in free space, are transverse to the propagation direction. In the transverse electric (TE) mode, the electric field is transverse to the direction of propagation while the magnetic field is normal to the direction of propagation. In our calculation, we take transverse magnetic (TM) mode into consideration, whose magnetic field is transverse to the direction of propagation.

Figure 3.1 (a) shows the side view of a grating that we considered. An alternating layers, whose thickness are w_1 and w_2 , of two non-magnetic dielectric material (with relative permeability almost equal to 1) with relative permittivities ε_1 and ε_2 are proposed.



Figure 3.1: (a) Side view of multilayer medium and (b)Three dimensional slab and the wavevector

To begin with, we take TM incident wave into consideration. Thus the magnetic field can be written as $\mathbf{H} = H_0 e^{j(\vec{k}\cdot\vec{r})}$ with $\vec{r} = (x, y)$, where the time-dependence $e^{-j\omega t}$ was omitted because we do not take the variation of the field correspondent to time, then **H** should satisfies Helmholtz Equation shown in Eq.3.1.

We know that for this multilayer structure, in the principal directions of anisotropy (x,y), shown in Figure.3.1, *a* is a diagonal tensor and denoted [18, 43] :

$$a = \begin{pmatrix} 1/\varepsilon_{\parallel} & 0\\ 0 & 1/\varepsilon_{\perp} \end{pmatrix}$$
(3.3)

where

$$\frac{1}{\varepsilon_{||}} = \frac{w_1}{w_1 + w_2} \frac{1}{\varepsilon_1} + \frac{w_2}{w_1 + w_2} \frac{1}{\varepsilon_2}$$

and

$$\varepsilon_{\perp} = \frac{w_1}{w_1 + w_2} \varepsilon_1 + \frac{w_2}{w_1 + w_2} \varepsilon_2$$

b = 1 inside dielectric medium, including the air. It means that such sub-wavelength layered grating can be described as an anisotropic homogeneous medium by using the homogenization theory of multilayers structure, and the effective permittivity along the two axis can be affected by the geometry parameters.

In the following part, to begin with, several different periodic layered structures are demonstrated and analyzed. Moreover, experimental results are provided to validate these theoretical predictions as well.

3.1.1 Multilayer Slab

In order to calculate the transmission property of such multilayer structure, and make it possible to realize in the experiment, a slab of grating that consists of two kinds of medium layered along the Y axis is mounted in region 2, shown in Figure.3.1 (b). Region 1 and 3 are free space respectively, where $\varepsilon_r = 1$ and $\mu_r = 1$. To determine its transmission property such as reflection or transmission coefficient when it was exposed in TM incident wave, from Eq.3.1 and Eq.3.3, Helmholtz Equation can be written as:

$$\nabla \cdot \left[\left(\begin{array}{cc} 1/\varepsilon_{||} & 0\\ 0 & 1/\varepsilon_{\perp} \end{array} \right) \nabla H \right] + k^2 H = 0$$
(3.4)

where $H = H_0 \cdot exp(-jk_xx - jk_yy)$ in Cartesian coordinate system, k_x and k_y are the x and y components of wavevector along X and Y axis respectively. The multilayer slab in region 2, shown in Figure.3.1(b) can be homogenized into an anisotropic medium. And the effect permittivity along X and Y axis have be given as $\varepsilon_{||}$ and ε_{\perp} respectively[44]. We assume that in both region 1 and 2 of Figure.3.1 (b), the total magnetic field H_1 and H_2 consist of the incident wave and reflected wave. From the Law of Reflection, we know that wave vectors of incident and reflected wave have the same X and Y component in both region 1 and 2. In the other hand, the magnetic field in region 3 only has the transmission wave. Consequently, we can have the expression of magnetic field in each region as following:

$$\begin{cases}
H_1 = H_0 \cdot e^{-jk_{1x}x - jk_{1y}y} + R \cdot H_0 \cdot e^{jk_{1x} - jk_{1y}} \\
H_2 = A \cdot H_0 \cdot e^{-jk_{2x}x - jk_{2y}y} + B \cdot H_0 \cdot e^{jk_{2x}x - jk_{2y}y} \\
H_3 = H_0 \cdot T \cdot e^{-jk_{3x}(x-l) - jk_{3y}y}
\end{cases} (3.5)$$

where k_{1x}, k_{2x}, k_{3x} are the X component of wave vector, k_{1y}, k_{2y}, k_{3y} are the Y component of wave vector. So, $e^{-jk_{1x}x}$ and $e^{jk_{1x}x}$ are the X component of those waves propagate to two opposite directions. A and T are the transmission coefficient in region 2 and 3, R and B are the reflection coefficient in region 2 and 3 respectively.

From the continuity condition of the field, we know that the filed on the boundary of two medium should be continuous. Besides, the first-order derivative of field along the normal vector of the boundary should also be continuous. Together with Eq.3.5, we can have two set of combined equations:

$$\begin{cases} H_1(0^-, y) = H_2(0^+, y) \\ H_2(l^-, y) = H_3(l^+, y) \end{cases}$$
(3.6)

and

$$\begin{cases} \partial_x H_1(0^-, y) = \frac{1}{\varepsilon_{||}} \partial_x H_2(0^+, y) \\ \frac{1}{\varepsilon_{||}} \partial_x H_2(l^-, y) = \partial_x H_3(l^+, y) \end{cases}$$
(3.7)

we can get

$$\begin{cases} k_{2y} = k_{3y} = k_{1y} = k_{1} \sin \theta \\ k_{3x} = k_{1} \sin \theta \\ k_{2x} = t_{0}k_{1}, \qquad \text{where } t_{0} = \sqrt{(1 - 1/\varepsilon_{\perp} \sin^{2} \theta)\varepsilon_{\parallel}} \end{cases}$$
(3.8)

Furthermore, transmission and reflection coefficient are achieved as :

$$\begin{cases} T = \frac{4Z_r e^{jk_1 \cos \theta L} M}{(1 + Z_r + M - MZ_r)(1 + Z_r - M + MZ_r)} \\ R = \frac{(Z_r^2 - 1)(M^2 - 1)}{(1 + Z_r + M - MZ_r)(1 + Z_r - M + MZ_r)} \end{cases}$$
(3.9)

where $M = e^{-jk_1t_0l}$ and $Z_r = \varepsilon_{||} \cos \theta / t_0$ is the relative impedance of the homogenized grating with respect to the air.

In the case of TE waves ($\mathbf{E} = \mathbf{E}(\mathbf{r})\mathbf{\hat{z}}$), we have

$$\nabla \cdot \nabla E(\mathbf{r}) + \mathbf{k}^2 \varepsilon(\mathbf{r}) \mathbf{E}(\mathbf{r}) = \mathbf{0}$$
(3.10)

from which the homogenization gives an equivalent isotropic medium

$$\nabla \cdot \nabla E + k^2 \varepsilon_{eff} E = 0, \quad with \quad \varepsilon_{eff} = \varepsilon_{\perp}$$
(3.11)

So with the same method, we can get the dispersion relation as $k_{2x} = \sqrt{1/\varepsilon_{||} - \sin^2 \theta}$ and $Z_r = \cos \theta / t_0$, and the transmission and reflection coefficient can be expressed as the same formation with Eq.3.9.

3.1.2 Corrugated Surface

Compared to the multilayer structures that we demonstrated before, corrugated surfaces have more sophisticated configuration and provide interesting applications. In this part, we are going to show our homogenization approach can be applied to corrugated surfaces. We consider the corrugation surface shown in Figure.3.2. With the same previous method, region 2 of Figure.3.2 (a) can be treated as a multilayer slab and modeled as we did in last section.



Figure 3.2: (a)Corrugated surface (b) Homogenized corrugated surface

The expressions of the fields and boundary conditions have been proposed in the last section. The corrugated surface model can be achieved by replace the medium in region 3 with layer 1 in Figure 3.1 (a), whose permittivity is ε_1 . So it is not difficult to get the

reflection property of such structure from Eq. 3.5 and Eq.3.6. For the TM incident wave, we can get the expression of transmission and reflection coefficient

$$\begin{cases} T = \frac{4exp^{j\sqrt{\varepsilon_{3}\mu_{3} - \sin^{2}\theta}k_{1}L}Z_{1}M}{(Z_{1}Z_{2} + 1)(1 - M^{2}) + (Z_{1} + Z_{2})(1 + M^{2})} \\ R = \frac{(Z_{1}Z_{2} - 1)(1 - M^{2}) + (Z_{1} - Z_{2})(1 + M^{2})}{(Z_{1}Z_{2} + 1)(1 - M^{2}) + (Z_{1} + Z_{2})(1 + M^{2})} \end{cases}$$
(3.12)

where

$$\begin{cases} M = e^{-jt_0k_1L} \\ Z_1 = \varepsilon_{||}\cos\theta/t_0, & \& Z_2 = \frac{\varepsilon_{||}\sqrt{\varepsilon_3\mu_3 - \sin^2\theta}}{\varepsilon_3t_0} \end{cases}$$

3.1.3 Slanted Slab

In the past two sections, we have studied the transmission properties of grating and corrugated surface, whose slabs or teeth are parallel to the horizontal axis. In the following, more practical structures are proposed. The orientation of the slab is not along the x axis whereas with slanted angles α , as is shown in Figure.3.3



Figure 3.3: Slanted Multilayers slab and Corrugated surface

Here, the thickness of the dielectric slab and the periodicity are exactly as the multilayered slab that we mentioned in the past section, and when the slanted angle $\alpha = 0$, this problem is simplified as the situation that we have solve in section 3.1.1 and 3.1.2. From what have been done in last section, the field in the slab can be written as the solution of Helmholtz Equation, shown in Eq. 3.1. On the contrary, the tensor a is not diagonal matrix shown the Eq.3.3, but a symmetrical diagonal matrix. From the work in [47], under this condition the tensor can be written as

$$a = \begin{pmatrix} a_x & a_{xy} \\ a_{xy} & a_y \end{pmatrix}$$
(3.13)

where

$$\begin{cases} a_x \equiv a_o \sin^2 \alpha + a_e \cos^2 \alpha \\ a_y \equiv a_o \cos^2 \alpha + a_e \sin^2 \alpha , \\ a_{xy} \equiv (a_e - a_o) \cos \alpha \sin \alpha \end{cases}$$
(3.14)

and

$$\begin{cases} \frac{1}{a_o} = \frac{w_1}{w_1 + w_2} \frac{1}{\varepsilon_1} + \frac{w_2}{w_1 + w_2} \frac{1}{\varepsilon_2} \\ \\ a_e = \frac{w_1}{w_1 + w_2} \varepsilon_1 + \frac{w_2}{w_1 + w_2} \varepsilon_2 \end{cases}$$
(3.15)

Thus the Helmholtz Equation 3.1 can be written as

$$\nabla \cdot \left[\left(\begin{array}{cc} a_x & a_{xy} \\ a_{xy} & a_y \end{array} \right) \nabla H \right] + k^2 H = 0$$
(3.16)

In the last section, the multilayer slab in the region 2 is symmetric, so we assume that in this region the transmit and reflect wave share the same absolute wavenumber value along x axis, but with the different sign of them because of the reversed propagation direction. Besides, such assumption is proved in the result of dispersion relation. However, in slanted situation with slanted angle of α , this assumption should not be made arbitrarily like this, on the other hand we use k_2^+ and k_2^- as the wavenumber along axis for the two

waves of different propagating direction as follows:

$$\begin{cases}
H_1 = H_0 \cdot e^{-jk_{1x}x - jk_{1y}y} + R \cdot H_0 \cdot e^{jk_{1x} - jk_{1y}} \\
H_2 = A \cdot H_0 \cdot e^{jk_{2x}^+ x - jk_{2y}y} + B \cdot H_0 \cdot e^{jk_{2x}^- x - jk_{2y}y} \\
H_3 = H_0 \cdot T \cdot e^{-jk_{3x}(x-l) - jk_{3y}y}
\end{cases} (3.17)$$

With the same method to calculate the field in the multilayer slab in Eq.3.5, we can get the scattering properties. The continuity relation at the interfaces y = 0 and y = l are imposed by the structure of Eq. 3.1, namely the continuity of the field *H* and the continuity of the normal component of the vector $[a \cdot \nabla H]$, which lead to

$$\begin{cases} H_1(0^-, y) = H_2(0^+, y) & a_x \partial_y H_1(0^-, y) = a_x \partial_x H_2(0^+, y) + a_{xy} \partial_y H_2(0^+, y) \\ H_2(l^-, y) = H_3(l^+, y) & a_x \partial_x H_2(l^-, y) + a_{xy} \partial_y H_2(l^-, y) = 1/\varepsilon_3 \partial_x H_3(l^+, y) \end{cases}$$
(3.18)

Therefore we can get the dispersion relation and the each wavenumber along different axis for this slanted multilayered slab:

$$k_{3y} = k_{2y} = k_{1y} = k_{1} \sin \theta , \ k_{3x} = t_{03}k_{1}$$

$$t_{03} = \sqrt{\varepsilon_{3}\mu_{3} - \sin^{2}\theta}$$

$$\begin{cases} k_{2x}^{+} = k_{1}[-\frac{a_{xy}}{a_{x}}\sin\theta + \frac{1}{a_{x}}\sqrt{(a_{xy}\sin\theta)^{2} - a_{x}a_{y}\sin^{2}\theta + a_{x}\mu_{3}}] \\ k_{2x}^{-} = k_{1}[-\frac{a_{xy}}{a_{x}}\sin\theta - \frac{1}{a_{x}}\sqrt{(a_{xy}\sin\theta)^{2} - a_{x}a_{y}\sin^{2}\theta + a_{x}\mu_{3}}] \end{cases} (3.19)$$

where μ_3, ε_3 are the permeability and permittivity of the medium in region 3. From Eq.3.5, Eq.3.18 and Eq.3.19, the transmission properties of the plane wave propagating in this slanted multilayer can be found as:

$$\begin{cases} T = \frac{2M(Y_1 + Y_2)}{(1 - X_1 + Y_2 + X_1Y_2) - (1 - X_2 - Y_1 + X_2Y_1)M^2} \\ R = \frac{(1 - X_1 + Y_2 - X_1Y_2) - (1 + X_2 - Y_1 - X_2Y_1)M^2}{(1 - X_1 + Y_2 + X_1Y_2) - (1 - X_2 - Y_1 + X_2Y_1)M^2} \end{cases}$$
(3.20)

where

$$\begin{cases} M = e^{-jLt_2k_1} \\ X_1 = (a_x t_0 + a_{xy}\sin\theta)/\cos\theta \quad Y_1 = \varepsilon_3(a_x t_0 + a_{xy}\sin\theta)/t_{03} \\ X_2 = (a_x t_0 - a_{xy}\sin\theta)/\cos\theta \quad Y_1 = \varepsilon_3(a_x t_0 - a_{xy}\sin\theta)/t_{03} \end{cases}$$

The calculation of different kinds dielectric layered structures is shown in the past sections. In the following part, metallic structures are investigated. Besides, spoof plasmons phenomenon, which can be supported on the textured metallic surface is briefly introduced.

3.2 Spoof plasmons

Surface plasmons (SPs) are electromagnetic surface waves propagating along a metaldielectric interface that can be observed in the visible wavelength regime. SPs have been proposed for tailoring the light propagation at sub-wavelength scale thanks to the strong confinement of light at the metal-dielectric interface [14]. In 2004, Spoof Surface plasmons (SSP) were proposed to engineer SPs at THz and microwave frequencies [53]. This result has been obtained by cutting grooves on a scale much smaller than the wavelength to increase the penetration of the fields into the metal.

Corrugated surfaces are able to support the propagation of surface waves in the microwave regime. The existence of spoof plasmons has been proved experimentally[28]. The periodic subwavelength structure that confined electromagnetic field is investigated in microwave and THz waveguide[68]. The properties of these surface waves mainly depend on the geometry of the corrugations. Such spoof plasmons solutions can be obtained by the previous homogenization approach.
3.2.1 Corrugated Metallic Surface

In the following part, the simplest 1-D textured metallic surface is studied, furthermore, transmission property of such structure will be derived from the method that we used for the dielectric configurations. Figure.3.4 shows the side view of the 1-D corrugated metallic surface whose periodicity is h = 3mm with metal fraction $\varphi = 2/3$. However, the method that we deal with metallic layers is different from of dielectric materials, because of its impenetrable property for eletromagnetic waves.



Figure 3.4: Metallic Corrugated Surface

From the electromagnetic theory, we know that magnetic field H satisfy $\Delta H + k^2 H = 0$ in the air, and in the single substance material, no matter metal or dielectric, magnetic field H satisfy $a\nabla H + k^2 bH = 0$. For the dielectric medium, $a = 1/\varepsilon$ and $b = \mu(\mu = 1)$ for non-magnetic material, but for the metal, there is not a clear value for permittivity, since from the definition of the permittivity of metal it is infinite. Here, we have to find the value of *a* and *b* for metal before the next step.

Figure 3.5 is the sketch map for the interface between smooth metal (region 2) and air (region 1). The two kinds of boundary conditions on the interface can be written as

$$\begin{cases} H|_A = H|_B \\ \partial_n H|_A = a\partial_n H|_B \end{cases}$$
(3.21)

Since there is no explicit way to get the *a* and *b*, if we can give a specific value for both of them which can satisfy the phenomenon that we observed, then we can safely regard



Figure 3.5: Interface of smooth metal and air

such value as *a* and *b*. Here, we assume that a = b = 0. We shall lose the information of *H* inside the region B. Fortunately, there is not necessary and impossible for us to determine the field inside the metallic material. However, for the Second boundary condition, we can get $\partial_n H|_A = 0$, which tell us that the partial differential of H along the normal vector of the interface is ZERO, which can be proved in the experiment. So, it is safe for us to apply a = b = 0 for the calculation related to metal. Next, the multilayer metallic slab is studied by using the classical homogenization.

Obviously, the magnetic field is zero within the metal, and on the interface, the fields meet Boundary Condition.

Thus the main propagation equation in this anisotropic medium can be derived from

$$\nabla \cdot \left[\left(\begin{array}{cc} 0 & 0 \\ 0 & \phi \end{array} \right) \nabla H \right] + \phi k^2 H = 0$$
(3.22)

and then, in the air the propagation equation can be written as

$$\Delta H + k^2 H = 0 \tag{3.23}$$

we look for the solution of the evanescent wave in the air is derived from the former two

equations, and find that

$$\begin{cases} H = e^{-\alpha(y+L) + i\beta x} & \text{for } y > L \\ H = A\cos(Ky)e^{i\beta x} & \text{for } 0 < y < L \end{cases}$$
(3.24)

which can satisfy the boundary condition. By applying the continuity for the perpendicular component of *H* on the interface $\phi \partial_1 H(x, L^+) = \partial_1 H(x, L^-)$, the dispersion relation is obtained :

$$\beta = k\sqrt{1 + (1 - \phi)^2 \cos^2 \frac{kL}{\cos \alpha}}, \quad \text{where} \quad tan(kL) < 0. \tag{3.25}$$

Fig.3.6 shows the dispersion relation of metal corrugated surface, where β is the x com-



Figure 3.6: Dispersion Relation of metallic corrugated surface, whose periodic is h = 21mm, filling fraction is phi = 1/7, and length of the cylinder is L = 27mm.

ponent of the wave vector in the corrugation, f is the frequency. The blue line describes the dispersion relation of corrugated metallic surface. Meanwhile, the dot line is the light line in free space. The curve of dispersion relation of corrugated surface is lower than the light line, which means that for the same frequency, the wavenumber in the corrugated surface is larger than that in the free space. In another word, we can tail the wavelength by using this kind of structure on the metallic surface.

3.2.2 Experimental Validation of Metallic Thin structure

Vertical thin cylinder array

Surface plasmons cannot be supported by the smooth electric conductor at frequency band lower than the optical spectrum. From the investigation in the last section, the corrugated metal surface is able to sustain "spoof surface plasmons" at microwave frequency band, which has similar properties with the real surface plasmons that mainly due to its geometry, but also was restrained to be applied to some portable or miniaturized situation. Actually, there are several kinds of structures are able to support spoof plasmons.

Recently, a split-ring resonator designed to sustain spoof plasmons was proposed in [50]. A planar waveguide based on the element of the split-ring resonator is numerically analyzed to show high out- and in-plane confinement. Moreover, in [59], the concept of conformal surface plasmons (CSPs), surface plasmon waves that can propagate on ultrathin and flexible films too long distances in a wide broadband range from microwave to mid-infrared frequencies.

The aim of this experiment is to demonstrate the capability of the printed array to support spoof plasmons on the surface of a metallic ground plane. The rods of the proposed structure are made here by bolting cylindrical screws in a metallic plane. The schematic view and a photography of the designed structure are shown in Fig.3.7. The length of the screws is L = 27 mm and their diameter is 3 mm. They are disposed of in a one-dimensional array using a lattice constant of a=21 mm.

The array is mounted on a metallic ground plane that is placed next to a waveguide which is used to produce surface wave on the metallic ground, see Fig.3.8 (a). The detector is fixed on a motor that can move along the horizontal bar, which is mounted to another motor. The second motor is able to move along a fixed vertical, shown in Figure.3.8 (b). The field distribution near the array can be measured when the two motors moving correspondent to each other.



Figure 3.7: The schematic view of the three dimensional structure composed of an array of rods disposed on a metallic plane (L = 27 mm and h = 21 mm).



Figure 3.8: (a) A photography of the realized structure and the setup used for the experimental characterization. (b)Experimental facility to measure the field distribution near the cylinder array.

The dispersion diagram of this structure is presented in Fig.3.9. Branches that support very large wavenumbers (β) compared to free air wavenumber (β_0) can be obtained. These solutions correspond to the sub-wavelength waves that can propagate on the structure usually called spoof-plasmons.

Fig.3.9 shows that spoof-plasmons providing a sub-wavelength propagation can be obtained with the proposed structure. The wavelength of these guided waves is comparable to the one in free space at low frequencies. This wavelength becomes smaller and smaller when increasing the frequency. The smallest values that can be observed for the



Figure 3.9: The dispersion diagram of the considered structure.

first branch are near 1.9 GHz. One can also notice that an electromagnetic band gap is imposed by the structure between 2 and 5 GHz.

In order to characterize the proposed structure and provide a validation of the theoretical prediction, the experimental setup presented in Fig.3.8 (b) is used. We use a microwave waveguide to excite the array by near-field coupling. A small loop antenna is then used to measure the magnetic near field in a plane parallel to the rods and disposed at a distance of 5 mm.

Fig.3.10 (a) shows the measured transmission magnitude, between 1.7 and 2.7 GHz,

through the proposed structure using two waveguides. A transmission band can be observed above 2 GHz, while an electromagnetic band gap is obtained above this frequency. This result is consistent with the dispersion relation, calculated by the homogenization approach presented previously, given in Fig.3.10 (b).



Figure 3.10: The dispersion diagram of the considered structure.

The measured magnetic fields at different frequencies are presented in Fig.3.11. We can see that when increasing the frequency, smaller wavelengths can be obtained. This result is consistent with the theoretical prediction for the first branch. Moreover, the electromagnetic band gap can be observed above 2 GHz. This is shown in the last presented field, measured at 2.1 GHz.

Another simple validation of these experimental results compared with the theoretical calculations can be made by evaluating the wavelength of the spoof-plasmon at each frequency. The obtained results are summarized in the table below. These results show the good agreement between the theoretical predictions and the experiments.



Figure 3.11: The magnetic field distribution measured at different frequencies [1.5 (a), 1.7 (b), 1.9 (c) and 2 GHz (d)] using a small loop probe at a distance of 5 mm from the structure.

Frequency (GHz)	Calculated (Dispersion	Maggurad wavalangth (m)	
	relation) wavelength (m)	Measured wavelength (III)	
1.5	0.18	0.19	
1.7	0.11	0.125	
1.9	0.05	0.07	

Table 3.1: Comparison between the calculated and the measured wavelengths.

Slanted thin cylinder array

The aim of this experiment is to demonstrate the different band gap of the cylinder array with different slanted angles, shown in Fig.3.12 (a). In order to show the different transmission property of one dimensional thin cylinder array, the prototypes are mounted on a metallic ground plane that is placed in the middle of two waveguide, shown in 3.12 (b).





(a) Vertical and Slanted Thin Cylinder Array



Figure 3.12: (a) Both of the two kinds of cylinder array with the same periodicity of h = 6mm, diameter of l = 1mm and the same value of length L = 17.5mm, besides, the second one with slanted angle of $\alpha = 50^{\circ}$. (b) Experimental facility to measure the transmission between the two waveguide.

results

By employing Eq.3.25, we established in section 2, dispersion relation of these two kinds of cylinder array can be derived, shown in Fig.3.13 (a). From dispersion relation curve of vertical array, it is clear that there is band gap introduced by this structure above 4GHz, represented as the gray area named 'Band gap 1'. Beside, the slanted array with 50 degree angle has 'Band gap 2' that is above 2.6GHz, shown in Fig.3.13 (a).

The measurement results are depict in Figure.3.13. The black curve is the transmission of the vertical array, which goes up as the frequency increases from 2.2GHz to 2.9GHz, and remains high transmission until 3.6GHz. At around 4GHz, a band gap is appeared, whose transmission is lower than 0.4. The red curve stands for the transmission of slanted array. However, clear band gap at 2.75GHz is observed, which is at lower frequency



Figure 3.13: Dispersion relation and transmission property of the two kinds of one-dimensional array

than that of vertical array. Such phenomenon is mainly because the longer length of the cylinders. Although the hight of he arrays are equal, the effective length for the slanted array is larger and induces a lower resonance frequency. This phenomenon can be used

for the miniaturization of microwave and optical devices based on spoof plasmon.

3.3 Application to patch antennas

In the former sections, analytical predictions, numerical and experimental validation for multilayer and thin structure are provided. At the end of the last section, the property of the metal cylinders to support or suppress EM wave is roughly demonstrated. Next, we are going to investigate into details of such ability, which can be applied to a radiating source to enhance the performance of antennas.

*The aim of the work in this part is to apply the metasurface that we have studied to enhance the directivity of the traditional antenna. Besides, it has been published in [22]. We insert the paper here just after page 45 for the reader who wants get more details, or the reader can skip this part and go back to the manuscript.

3.3.1 Background

Electromagnetic radiating sources involve planar structures susceptible to support surface waves. These waves are bounded to the interface, they diffract in free space when propagating to the edges of the ground plane. This mechanism is used as the main radiating mechanism in the traveling-wave antennas (or leaky-wave antennas) based on the design of a guiding structure enable to efficiently couple the surface waves to free space. It is in contrast to resonant antennas, which employ resonances of radiating elements (as monopoles or dipoles). However, even in this case, surface waves are involved but in an undesirable way. Indeed, such antennas are mounted over a larger plane surface (the ground plane), which is susceptible to support surface waves, as surface plasmons for a metallic ground plane in the optical frequencies. The antennas are designed by considering an infinite ground plane that is ignoring surface wave radiation. Because of the finite size of the ground plate in the practice, the surface waves are scattered, and thus, interact with the field of the radiating elements. It results in the appearance of undesired ripples and high-level side lobes in the radiation pattern, responsible for a decrease in the overall performances of the antenna. To avoid such perturbations, one has to prevent the wave propagation on the ground plane. This means that the surface waves have to be associated with a dispersion relation with frequency ranges where the propagation is prevented, namely with band gaps. Lots of work have been done to show the ability of the textured surface to suppress surface wave on the interface between the electric conductor and another medium. In 1999, two designs of improved antennas were proposed [24, 60], where the planar interface of the ground plane is replaced by a periodically structured interface. In the former reference [24], the structure consists in piercing a square array of air columns in a dielectric substrate. The array being at wavelength scale realizes a photonic crystal type substrate, which presents a forbidden band gap at the Bragg frequency. In the latter reference [60], the forbidden frequency band is realized using a mushroom-like

array designed at a subwavelength scale.

The concept of surface impedances is used to characterize the response of the structure, resulting in the so-called high impedance surfaces, associated with forbidden frequencies for the surface waves. The effect of the excitation of surface waves on radiating elements has been studied in many contexts [33, 72]. Recently, a microstrip patch antenna array and a beam-scanning circularly polarized antenna based on the spoof plasmonic waveguide of a blind-hole array, which can efficiently convert spoof plasmon waves into free-space emissions, have been realized [71, 4]. A review on the application of structured surfaces to control spoof plasmon waves can be found in [70].

3.3.2 Dispersion Relation

1-D structure

In this section, we follow the idea of using the dispersion relation of surface waves to enhance the directivity of printed antennas. Here, we consider the surface waves, usually called spoof plasmons that we have introduced in section 2, guided at the corrugated interface of a perfect conductor, which consists of rectangular grooves with height *L* and volume fraction of air φ , shown in Figure.3.14. In this case, the dispersion relation of the spoof plasmons, derived in [31] for acoustic waves and in [53, 23] for electromagnetic waves with transverse magnetic polarization (the unit vector of H(x, z) along *y* axis, shown in 3.14) gives

$$k_{\rm SP} = k \sqrt{1 + \varphi^2 \tan^2 kL}, \quad \text{for } \tan kL > 0,$$
 (3.26)

where $k_{\rm SP}$ is the wavevector of the spoof plasmon along *x* and *k* the wavenumber in air (or say, the frequency). Note that the restriction to $\tan kL > 0$ is not mentioned in [31, 53, 23] although it is essential to define the band gaps, and it can be obtained by using classical homogenization technique [44, 47]. The wavefield in air is $H(x,z) = H_0 e^{ik_{\rm SP}x - \alpha z}$, with $k^2 = k_{\rm SP}^2 - \alpha^2$. The forbidden frequency ranges correspond to negative values of $\tan kL$,

leading to the usual first total band gap is for $\pi/2L \le k \le \pi/L$ [47]. In the following, we refer to the forbidden frequency range, the corresponding range of frequency f such that $c/4L \le f \le c/2L$, with $c = 3.10^8$ m.s⁻¹. In the context of improved antenna, the exis-



Figure 3.14: Schematic view and dispersion diagram of a corrugated surface (factor $\varphi = 0.46$ and a = 13 mm) for two different groove lengths.

tence of stop bands is interpreted in terms of high impedance surface (HIS) [60], with the surface impedance being $Z_s = E_x/H$ with our notations. Being the ratio of two conserved quantities, Z_s is notably continuous at the interface z = 0 between the corrugated layer and air. Thus, if $Z_s(0^-)$ is known, one can solve the problem only in the air ($z \ge 0$) using the boundary condition at the equivalent surface z = 0 (obviously, the difficulty is precisely to find Z_s , which requires to solve the problem for z < 0). In our two dimensional configurations, Z_s imposes a boundary condition of Robin type $\partial_z H(x = 0, z) = i\omega Z_s H(x = 0, z)$ (since $\partial_z H = i\omega E_x$ when considering a time dependence $e^{-i\omega t}$). When looking for a surface waves, $H(x, z > 0) = H_0 e^{ik_{SP}x - \alpha z}$, we simply get $\alpha = -i\omega Z_s$. This means that surface waves (with α real positive) will be supported for Z_s having a positive imaginary part (inductive impedance). Next, if Z_s tends to infinity, α and $k_{SP} = k^2 + \alpha^2$ also tends to infinity, which indeed coincides with the limits of the stop bands in our case, Eq. 3.26, at

 $\tan kL \to \pm \infty$.

In the following, we use the concept of plasmon band gap (given by the condition $\tan kL < 0$ in Eq. 3.26) to design our metamaterial substrate and we demonstrate experimentally and numerically its capability to improve the performances of the patch antenna.

2-D structure

Start by considering the simplest structure (one dimensional case) with rectangular grooves and parameters $\varphi a = 6$ mm and a = 13 mm. Two different groove lengths, L = 6 and L = 15, mm are considered. The corresponding dispersion relations given by Eq.3.26 is shown in Fig.3.14. The light line that corresponds the dispersion relation in the vacuum is also given for comparison. It is clear that the maximum value of the black continuous line, which stands for depth of the grooves L = 6mm, is 8.4 GHz, in another word, such dimension of grooves is able to provide a band gap above 8.4 GHz. Meanwhile, the lower red dash lines are the first mode of L = 15mm, and the other one is the first mode. So, the bandgap that L = 15mm can provide is from 3.8 to 9.5 GHz. Spoof plasmon modes are clearly observed over a wide band at low frequencies for both structures, and also, the band gap is obtained in each geometry and can be easily tuned by the length of the groove.

Brillouin Zone

It has been shown recently that the dispersion relation of surface waves in ultrathin and flexible films is practically identical to the relation provided by corrugated surfaces [59, 42]. Here, we propose to take advantage of this phenomenon to go toward a twodimensional structure. For practical consideration, we use the two-dimensional configuration illustrated in Fig. 3.16(a). Instead of massive rectangular grooves invariant along y, an array of cylindrical rods periodically located along x and y with the same parameters,



Figure 3.15: (a) Square array (b) Unit cell of square array

lattice a = 13mm and $2R = \varphi a$.

In the last section, dispersion relation of the corrugated surface was given by analytical approach, and then to predict the property of 2-D cylinder array. Here, the commercial electromagnetic simulation software CST MWS that we introduced in the last section was adopted to determine the dispersion relation for the 2-D structure. Periodic structure can be calculated in many ways, one of the most efficient and widely used is to reconstruct the whole field by the solution of single unit cell.

Brillouin Zone of the considered structure is illustrated in Fig 3.16(a). Two different cylinder lengths, L = 6 and L = 15, mm are considered. The obtained results are shown in Fig. 3.16(b).

As expected, along $\Gamma - X$, the obtained dispersion relation resembles the dispersion relation given by Eq. 3.26 presented in Fig.3.14. A full transmission band gap for surface waves is clearly observed for frequency in the range 3.8 - 9.5 GHz for L = 15 mm. The band gap for L = 6 mm starts at f=8.5 GHz. The proposed two dimensional metamaterial supports the propagation of spoof plasmon modes ruled by a dispersion relation that can be tuned by the length of the rods as predicted by Eq. 3.26.



Figure 3.16: (a) Schematic view of the metamaterial made of an array of grounded rods. (b) Dispersion diagram of the proposed metamaterial calculated for two different cylinder lengths. The light line is plotted as well (dotted line).

3.3.3 Reshaping the radiation pattern of antenna

Antenna with metasurface

We now inspect the ability of our metamaterial substrate to avoid such unpleasant effects by preventing surface wave, or spoof plasmon, to propagate. We realize a patch antenna resonating around 10 GHz. The radiating patch fed by a coax cable via a hole near the center of the ground plane is $6 \times 5.8 mm^2$, shown in Figure.3.17 (a). The antenna element is then placed in the center of a 6 by 6 cylinder array with L = 6 mm, as shown in the Fig. 3.17(a), and from Fig. 3.16(b), we know that this structure provides a band gap that surrounds the resonant frequency of the antenna even after adding the thin layer of the epoxy substrate.

The reflection coefficient of the patch antenna alone and antenna that is surrounded by the cylinder array, shown in Fig.3.17(b), is given by the Network Analyser comes from





Figure 3.17: (a) Schematic view and a photography of the printed antenna surrounded by the metamaterial structure. (b) Reflection coefficients calculated for the antenna with and without the metamaterial ($f_1 = 7.75$ GHz, $f_2 = 7.87$ GHz and $f_3 = 8.34$ GHz).

Agilent Company that we mentioned before. In both cases, a minimum of the reflection is obtained around 10 GHz, corresponding to the operating frequency of the antenna. Although a slight red shift, the presence of the array does not significantly affect the operating frequency. This shift is due to the modification of the effective permittivity caused by the use of the metamaterial. What is more, it is also depicted in Fig. 3.17(b) that there are three resonances appeared at frequencies, $f_1 = 7.75$ GHz, $f_2 = 7.87$ GHz and $f_3 = 8.34$ GHz, although very weak in the present case; additional results on these low-frequency resonances are reported at the end of this section, where we show that the

structure can be optimized to work efficiently at these frequencies [36, 52, 29].

Radiation Pattern of antenna (working in the band gap)



Figure 3.18: Experiment set up

The radiating patterns have been measured in a semi-anechoic chamber using a Network Analyzer. Figure.3.18 demonstrates the facility of the experiment. The patch antenna is perpendicularly mounted on the center of a rotatable platform drove by a motor which is controlled by a computer and is excited by port 1 of Network Analyzer via coaxial cable. Meanwhile, a horn antenna who works from 8 GHz to 12.2 GHz to detect the radiated energy of patch antenna is connected to Port 2 of the analyzer. The Network Analyzer can record the transmission from patch antenna to horn antenna step by step as the platform rotates from 0 to 360 degrees. Here, both the patch and horn antenna are linear polarized, so the most important in this experiment is to make sure the two antennas are placed to the same polarization. For example, the E-plane of both patch and horn antenna ought to parallel to XOY plane when measuring the radiation pattern of E-plane, since platform rotates in the same plane, as is shown in Figure.3.18.

The measurements have been performed at the main resonance, f = 10.6 GHz, in the



Figure 3.19: (a) Measured and simulated radiation patterns in the E plane for the antenna alone and the antenna surrounded by the metamaterial surface.(b) Same representation as in (a) for the measurements in the H plane.

E and H planes for the realized antenna. The results are presented in Figs.3.19. A significant enhancement of the directivity of the antenna is achieved due to the metamaterial substrate. A reduction of the backward radiation and the side lobe levels, in particular for angles higher than 90 degree, are also obtained on the two planes. An average value of reduction of 10 dB is observed. Moreover, the gain of the antenna is increased of about 2 dB at the operating frequency. This improvement is higher (of about 1 dB) than the one obtained with photonic crystals (PCs) and high impedance surfaces (HISs) when considering similar lateral dimensions.

Radiation Pattern of antenna (working out of the band gap)

In order to further investigate the role of the band structure on the radiation patterns, we consider a second metamaterial for which the spoof plasmon propagation is possible at the operating frequency ($f_0 \simeq 10$ GHz) of the printed antenna, shown in Figure.3.16(b).

This is done by considering a metamaterial composed of higher cylinders (L = 15 mm). The reflection coefficient of the antenna in this configuration is presented in Fig. 3.20(a). As previously, the operating frequency is not significantly affected by the presence of



Figure 3.20: (a) Reflection coefficients of the antenna in the absence and in the presence of the metamaterial surface. (b) and (c) Radiation patterns of the printed antenna surrounded by the metamaterial measured respectively in the E plane and in the H plane.

the array. A slight red shift occurs because of the modification of the effective medium around the patch antenna by using the metamaterial. The main resonance is obtained at f = 10.45 GHz. Figs. 3.20(b) and (c) show the radiation patterns measured at the main resonance for the last configuration.

The appearance of high secondary lobes are noticeable in the E plane at ± 45 degrees, and this effect is accompanied by a decrease in the main lobe gain, which is found to be lower of about 2 dB when compared to the gain of the antenna alone. Clearly, the spoof plasmons mode has enabled edge effects and the directivity of the antenna has not been improved in this configuration. This confirms that the efficiency of the antenna previously observed is due to the band structure of the meta-substrate.

3.3.4 Electromagnetic radiation from the finite size metamaterial

In the previous configurations, we have investigated the effect of the array on the radiation pattern at the operating frequency of the antenna. We actually studied two configurations where this frequency is considered inside and outside the spoof plasmons band gap. Nevertheless, we have not yet discussed all the perceptible effects on the reflection spectrum of the antenna observed in the first configuration. While we studied the behavior of the structure at the main resonance, we have not investigated the collection of resonant peaks observed at the lower frequencies of the reflection spectrum presented in Figure.3.17(b) . We propose now to focus on this phenomenon. It appears that such reflection coefficient response was recently observed in a finite size structure called metalens [36, 52, 29]. This structure consists of a finite size periodic array of sub-wavelength resonators. The formalisms used to study this type of resonant media have shown that the coupling splits the single resonance of the atom, considered in each unit cell, to different modes that show at different frequencies. This hybridization process produces a number of modes that equals at best the number of atoms. However, due to mode overlapping and symmetries, it is not always possible to individually excite each of these modes.

While designed to mimic corrugated surfaces in order to support spoof SPs in a twodimensional configuration, our metamaterial can be considered as a cluster of a resonant grounded rods disposed of periodically according to a two-dimensional square lattice. This overview regards this structure as a metalens and predicts the appearance of the collection of resonant modes presented in Figure.3.17(b).



Figure 3.21: Electric field distribution (left) and radiation patterns (center for 3D and right for 2D patterns) calculated at the resonant mode frequencies : $f_1 = 7.75 \text{ GHz}(a), f_2 = 7.87 \text{ GHz}(b) \text{ and } f_3 = 8.34 \text{ GHz}(c)$. The frequencies are defined in Figure.3.17(b). Measurements are also given for the in-plane 2D patterns.

In our configuration, we observe that the first branch of the dispersion relation, calculated for the infinite array of the grounded rods, covers the collection of resonant frequencies. At this frequency band, the printed antenna alone is not resonating. The obtained resonances are obtained at low frequencies by coupling power from the near evanescent field of the (non-radiating) antenna to the finite size metamaterial. In order to take a closer look in the near field variations at these resonant frequencies, we have performed numerical simulations. Fig.3.21 shows the cartography of the electric field variation above the structure at the three main resonant frequencies. The spatial fluctuations on the near field patterns relate the Fabry-Perot resonant behavior of the structure. These modes can be expressed in terms of the in-plane wavenumber $k_{sp} = \frac{\pi}{a}(m\vec{x} + n\vec{y})$, where *m* and *n* are integers inferior or equal receptively to the number of the columns and lines of the considered array and where *a* is the lattice constant of the array.

These results match the metalens behavior presented above [29]. Thereby, we expect a far field radiation in the plane of the structure with a specific pattern at each resonant frequency. We have performed calculations and measurements in order to characterize the radiation patterns. Fig.3.21 shows the results at the three main considered frequencies, 7.75 GHz, 7.87 GHz and 8.34 GHz respectively. We can notice that each pattern expresses the fluctuation observed in the corresponding near-field resonant mode. The



Figure 3.22: Radiation Pattern and Magnetic Field Distribution on plane

resonant modes obtained here by hybridization do not present resonances with high magnitude because the coupling of the structure to the main source (patch antenna) has not been optimized here to that aim. In Fig.3.22, we present the spectrum obtained with a

configuration where such coupling is amplified. In such configuration, efficient radiation at each resonance and a wide diversity of radiation patterns can be obtained. This structure can be used as an original and efficient solution to reshape the radiation pattern for specific configurations.

Chapter 4

Classical homogenization of Second Order

In the last chapter, classical homogenization theory is introduced and verified to show its ability to predict the transmission properties of periodic structures. One important application is to enhance the directivity of conventional patch antenna by designing a metasurface to block the surface wave on the ground. However, classical homogenization results cannot provide the desired results in some unique condition, such as the ultra-thin structures.

*This part of work mainly focuses on the problem of the disagreement between the theoretical and measured results when we deal with the ultra-thin structure by using the first order homogenization. It is included in a chapter of a book that will be published by Intech Publisher. In order to provide the reader more details, we insert the chapter after this page.



Figure 4.1: (a) Top view of the set up to measure the transmission property of periodic structure. (b) The result of measured transmission coefficient of classical homogenization.

Figure.4.1 (a) shows the top view of the set up to measure such periodic structure that consists of thin metal bars with thickness of e, width of l and was periodically arranged along x axis. The theoretical and measured transmission coefficients are shown in Figure.4.1 (b). The blue dash line is the transmission coefficient derived from classical homogenization theory, and the red circles depict the measured results of such structure with different thickness e from 0.03mm to 20mm respectively. It is clearly that when e is larger than 4mm, where $k \cdot e \approx 0.84$ and k is wave vector of the incident wave, the result of classical homogenization correspond to the measured data. However, in the other case, a significant disagreement between them is observed when e < 4mm.

One of the possible reasons is the loss. In the experiment, which is demonstrated in the later section, metal bars are made of aluminum that is not a perfect conductor. Thus a certain value of attenuation would exist when electromagnetic waves transit through the grating, therefore decrease the transmission property. But in the theoretical calculation, we only add a small value of the loss, which might not be efficient. In order to find the ideal value of attenuation in the theoretical prediction, we increase the loss of metal gradually, shown in Figure.4.2, to see how the attenuation affects the transmission.



Figure 4.2: Transmission coefficient of different lossy

Although a slightly decrease of transmission property is achieved from e = 0.1mmto e = 1mm, there is still large disagreement between the measurement results and theoretical prediction. Moreover, in the measured results, a pulse is observed at e = 10mm. However, such high transmission value vanishes in the theoretical results when the attenuation increases. In another word, the attenuation is not the only cause of the disagreement in the left part of those curve.

Another explanation for this disagreement is the applicability of classical homogenization when the thickness is smaller than a certain value, here we see that is e = 4mm. A kind of modification of the classical homogenization is necessary, which is proposed by other authors and named Second Order homogenization.



Figure 4.3: In the figure (a), the problem of the multilayered medium is the usual continuity conditions on the boundaries of the layers. In figure (b), an equivalent slab made of an effective homogeneous and anisotropic medium and jump conditions was applied at the boundaries of the slab.

4.1 Theory

In this section, the second order homogenization is briefly explained. Generally speaking, second order homogenization is used to solve the problem of structure that the thickness *e* is small value. The word "homogenization" is attempted to replace the complex structure consists of periodic layers with an effective medium, shown in Figure.4.3. Wave in the bulk region can be described as the solution of Helmholtz Equation as follows:

$$div\Sigma + k^2\varphi H = 0 \tag{4.1}$$

where $\Sigma = \begin{pmatrix} 0 & 0 \\ 0 & \varphi \end{pmatrix} \nabla \cdot H$ in the bulk medium, $\Sigma = \nabla \cdot H$ in the air, and $\varphi = l/h$ is the filling factor of the metal and the air in the grating. For the classical homogenization *e* is relatively large, for instance $k \cdot e > 1$. On the boundary of the bulk region, jump condition is $[\![H]\!] = 0$ and $[\![\Sigma \cdot \mathbf{n}]\!] = \mathbf{0}$, where $[\![H]\!] \equiv H^+ + H^-$, which means that on the boundary of the bulk region, the electromagnetic wave is continuous. However, when *e* is too small that almost can be neglect, for example $k \cdot e \ll 1$. We need to use the second order homogenization that employ the same equation of first order homogenization in the bulk, but with the different boundary condition $\llbracket H \rrbracket \neq 0$ and $\llbracket \Sigma \cdot \mathbf{n} \rrbracket \neq \mathbf{0}$, because the field of two side of the bulk is no longer continuous. Here $\llbracket H \rrbracket$ and $\llbracket \Sigma \cdot \mathbf{n} \rrbracket$ can be expressed as

$$\begin{cases} \llbracket H \rrbracket = \frac{h\mathscr{B}}{2} (\Sigma^{-} + \Sigma^{+}) \cdot \mathbf{n} \\ \\ \\ \llbracket \Sigma \rrbracket \cdot \mathbf{n} = -\frac{h\mathscr{C}}{2} (\frac{\partial^{2} H^{-}}{\partial x^{2}} + \frac{\partial^{2} H^{+}}{\partial x^{2}}), \end{cases}$$
(4.2)

where $\mathscr{B} = -\frac{1}{\pi} \log \sin \frac{\pi \varphi}{2}$ and $\mathscr{C} \approx \frac{pi}{16} \varphi^2$. Then with the same method that is employed in Chapter 2, the transmission properties of second order homogenization can be shown as follows:

$$\begin{cases} R = -\frac{z_1^* z_2^* e^{ike} - z_1 z_2 e^{-ike}}{z_1^{*2} e^{ike} - z_2^2 e^{-ike}} \\ T = \frac{|z_1|^2 - |z_2|^2}{z_1^{*2} e^{ike} - z_2^2 e^{-ike}}, \end{cases}$$
(4.3)

where

$$\begin{cases} z_1 \equiv (1 - \frac{\cos \theta}{\varphi}) + ikh(\mathscr{B}\cos \theta + \mathscr{C}\frac{\sin^2 \theta}{\phi}) - (kh)^2 \sin^2 \theta \frac{\mathscr{B}\mathscr{C}}{4}(1 + \frac{\cos \theta}{\varphi}) \\ z_1 \equiv (1 + \frac{\cos \theta}{\varphi}) - ikh(\mathscr{B}\cos \theta - \mathscr{C}\frac{\sin^2 \theta}{\phi}) + (kh)^2 \sin^2 \theta \frac{\mathscr{B}\mathscr{C}}{4}(1 + \frac{\cos \theta}{\varphi}). \end{cases}$$

With this expression, the transmission coefficient of the structure at 10 GHz can be derived, and added into Figure.4.1 (b). these curves can almost correspond to measured results, shown as follows:

It depicts that second order homogenization successfully predict the transmission of the metal grating when e is very small. Furthermore, the results of second order homogenization correspond to numerical result at each value of thickness e. In the following section, each prototype with different thickness of e is shown to validate the second order homogenization.



Figure 4.4: Results of the transmission coefficient of the structure in Figure 4.4: Results of the transmission coefficient of the structure in Figure.4.1 (a), where width of the bars is l = 3.5mm and periodicity is h = 4.5mm, blue dash line is the results of classical homogenization, black curve and red curve are the analytical and numerical results of second order homogenization respectively, red circles are the measured results of thin structure.

4.2 Experimental Validation for Structured Metallic Layers

In this section, numerical and experimental validation is provided to verify the second order homogenization technique. It is employed to calculate the transmission property of a one dimensional grating consists of slim bars, whose thickness *e* can be tuned, shown in Figure.4.5. Six prototypes are manufactured and tested in this experiment, which have the same width l = 3.5mm and periodicity h = 4.5mm. Figure.4.5 shows the schematic view of the prototype. From the definition of these gratings, they are supposed to be infinite along Z axis. Therefore the boundary condition on XOY plane at the two ends of the bars should be periodic in the numerical calculation.



Figure 4.5: Schematic of Structured metallic layers

The parameters of the each prototype are shown in Table.4.1. Moreover, the overview of different materials that are used to support the prototypes is mentioned. All the elements of the prototypes are made of metal or other materials covered with aluminum paper, whose thickness is larger than microwave skin depth of aluminum to exhibit as metal bars in the radiation of electromagnetic waves. All the information is provided in Table.4.1. The glue and scotch are used to sustain the structures.

	P 1	P 2	P 3	P 4	P 5	P 6
Thickness e	0.03mm	0.24mm	1mm	4mm	14mm	20mm
k · e	0.0063	0.05	0.21	0.84	2.93	4.19
Length of prototype	405mm	150mm	150mm	150mm	150mm	400mm
Bars	aluminum paper	aluminum paper	enclothed epoxy	en- clothed	en- clothed	Metal / en- clothed
Sustainer	Y	Y	Y	N	N	N
Glue	Y	Y	Y	N	N	N
Scotch	N	N	N	Y	Y	Y

 Table 4.1: Different value of e for each prototype

4.2.1 Experiment Set up

In order to measure the transmission property of the gratings, two X-band frequency horn antennas are placed at the two ends of a electromagnetic chamber respectively, shown in Figure.4.5. They are connected to port 1 and 2 of the network analyzer. The grating is



Figure 4.6: Experiment setup to measure transmission property of metallic grating.

vertically amounted on a motor that is located at the centre of the chamber. The motor is connected to a computer and can be rotated from 0 to 180 with 2 degree steps, as we used in the experiment of Chapter 2 to measure the radiation pattern of antenna. Here, in order to use TM incident wave, the magnetic vector of wave should be always parallel to the metal bars and vertical to the ground of the chamber. Moreover, the magnetic field vector of the horn antennas should be vertical to the ground either.

4.2.2 Results

In order to verify the second order homogenization theory, six prototypes with different value of thickness e at 10 GHz are manufactured and tested. Furthermore, angular sweep measurements of some prototypes are shown as a complementary. In the following part, measurement of all the prototypes is shown.

Prototype 1 with e=0.03mm ($ke \approx 0.0063$)

Prototype 1 has the smallest thickness and the largest disagreement between the experimental result and theoretical prediction of classical homogenization. It is made of 90 alumina paper taps with thickness e = 0.03mm and width l = 3.5mm. Because the aluminum taps are too thin to keep standing on the ground, they are smeared by glue on one side and parallel stuck on a hard plane paper with periodicity of h = 4.5mm, shown in Figure.4.7 (a).

Although the permittivity of the hard paper is approximately equal to 1, it still can slightly affect the transmission property of the structure. In order to eliminate the undesirable affection caused by the hard paper, another hard paper without the aluminum taps is necessary to be a reference. Thus, after the calibration of the network analyzer, the measured data of transmission between the two horn antenna throughout the empty hard paper is record as $s21_r$. Next, it is replaced with prototype 1 and with the same method to get its transmission value s21. From the mathematical calculation, the transmission coefficient of prototype 1 can be deduced as $s21/s21_r$.



Figure 4.7: (a) Photograph of prototype 1 (b) Transmission properties of measured results and Numerical, classical and second order homogenization of prototype 1.

Figure.4.7 (b) depicts the simulated and measured results of prototype 1 with the incident angle of 0 degree. It is clear that the measurement result can meet the second order homogenized prediction. Moreover, this theory is almost perfectly correspondent to numerical calculation results, which means that the second order homogenization is able to give more precise result of the tiny thickness structure than the classic method.



Figure 4.8: Numerical (a) and measured (b) angular sweep results for prototype 1

Figure.4.8 shows the numerical and measured results of transmission coefficient of angular sweep from 0 to 70 degrees for prototype 1 respectively. They can roughly match each other, except the ripples in the lower incident angles, which is partly because the standing waves between the antennas and the structure. At lower incident angle, more energy would be reflected directly back to antenna. Standing wave would be formed at different frequencies. Since at only several frequencies the standing wave can reach the maximum amplitude, consequently induce the variations on the amplitude of the received wave at low incident angle.
Prototype 2 with e=0.24mm ($ke \approx 0.05$)

The total thickness of Prototype 2 is 0.24mm, which consists of three layers of copper tap with width of 5mm and thickness of 0.08mm. As in prototype 1, these taps are smeared by glue and stuck on a piece of paper with periodicity of h = 4.5mm to make them align on a plane surface and also vertical to the ground. Compared to Figure 4.9 (b), which shows the transmission property, increasing the value of thickness *e* causes the decrease of the transmission.



Figure 4.9: (a) Photograph of prototype 2 (b) Transmission properties of measured results and Numerical, classical and second order homogenization of prototype 2.

In this experiment, the measured results of transmission coefficient are slightly smaller than that of the second order homogenization and numerical results. It can be explained by the tiny thickness of glue between the layers, which would enlarge the effective thickness and consequently making the transmission coefficient lower than the numerical results. Nevertheless, the measured results can still meet the theoretical prediction.

Prototype 3 with e=1mm ($ke \approx 0.21$)

Prototype 3 is made of epoxy plate whose thickness is 1mm and width is 3.5mm respectively. Each bars is covered by the aluminum paper, shown in Figure 4.10 (a). Meanwhile, the bars are smeared of glue on one side and stuck on a piece of thin paper since the affection caused by the tiny thickness paper can be neglect.



Figure 4.10: (a) Photograph of prototype 3 (b) Transmission properties of measured results and Numerical, classical and second order homogenization of prototype 3. (c) Numerical angular sweep results for prototype 3. (d) Measured angular sweep results for prototype 3.

The results are shown in Figure.4.10 (b), and the measured results correspond to the Chapter 4

second order homogenization theory. However, the classical homogenization method still provide nearly 0.2 larger than the second order homogenization result. Furthermore, by comparing the results of prototype 2 and 3 shown in Figure.4.9 (b) and Figure.4.10 (b), when the thickness *e* was enlarged, the value of transmission coefficient drop from 0.78 to 0.58. Figure.4.10 (c) and (d) show the transmission when we varying the incident angle of numerical and measured setup respectively. They correspond to each other fairly well when the incident angle is small, for example from 0 to 50 degrees. However, for the large incident angle the measured results are lower than the numerical data. It is because the apertures that we employ to depress the undesired diffraction from the edge of the prototype are made of absorber with 5*mm* thickness. Consequently, when rotating the motor to realize large incident angle, the aperture would has smaller projection on the original plane. Thus, less energy pass through the aperture to cause inaccuracy in measured results. In the future work, in order to get more precise measure data, to manufacture larger prototype and larger aperture is necessary.

Prototype 4 with e=4mm ($ke \approx 0.84$)

When the thickness of prototype is larger than that of prototype 3, which is e = 1mm, multilayer method is not efficient. Because the more layers are used, the more glue or other materials are employed to assemble the layers, consequently affect the measured results. Prototype 4 consists of 25 plastic bars that are covered by aluminum paper tightly and the thickness and width of the bars are e = 4mm and l = 3.5mm respectively. These bars are aligned on a plane table and parallel to each other. Meanwhile, several small pieces of epoxy plate of dimension $4 \times 5mm^2$ are inserted between every two bars, shown in Figure 4.11 (a).

The function of such small pieces epoxy is to make space between the bars. Furthermore, the bars are long enough to hide these sustainers behind the absorb to eliminate the affection. By doing this, the grating is already firm, thus the paper or hard paper is not



Figure 4.11: (a) Photograph of prototype 4 (b) Transmission properties of measured results and Numerical, classical and second order homogenization of prototype 4. Numerical (c) and measured (d) angular sweep results for prototype 4.

necessary. Figure.4.11 (b) shows the transmission coefficient of prototype 4. We can see that its transmission is kept on decreasing the thickness e reaches 4mm. Moreover, the two kinds of homogenization are already close to each other, measured results are more close to classical one.

Prototype 5 with e=14mm ($ke \approx 2.93$)



Figure 4.12: (a) Photograph of prototype 5 (b) Transmission properties of measured results and Numerical, classical and second order homogenization of prototype 5.

Prototype 5 is made of 25 plastic bars, whose dimension is $14 \times 5 \times 200 mm^2$. With the same method that of prototype 4, it was enclosed by transparent scotch to make it stable, shown in Figure.4.12 (a). Figure.4.12 (b) depicts that for this dimension there is a high transmission, which has been predicted by the both kinds of homogenization. The only one difference is that such peak happens at different frequencies. And also the measured data corresponds to second order homogenization as well.



Figure 4.13: (a) Photograph of prototype 6 (b) Transmission properties of measured results and Numerical, classical and second order homogenization of prototype 6. Numerical (c) and measured (d) angular sweep results for prototype 6.

Prototype 6 with e=20mm ($ke \approx 4.19$)

Prototype 6, which has the largest thickness e, is made of two kinds of bars. Parts of them are metal bars, whose parameters of cross sections are $20 \times 5 mm^2$. In order to get a better result for the variety of incident angle, extra bars made of plastic enclothed by the aluminum paper are used, shown in Figure.4.13 (a). As in prototype 5, the purpose of the small pieces between the bars is to separate the bars with an exact distance. Transmission property is shown in Figure.4.13 (b), the measured results agree with second order homogenization results, especially at the low incident angle. Classical homogenization

other frequency validation

The picture that was shown at the beginning of this chapter is aimed to show the disagreement between the measured results and classical homogenization, besides the different transmission of different thickness of the metal layer. Therefore, several prototypes are investigated to verify the second order homogenization, aimed at 10GHz, and the 'ke' values are presented as well.

*The work in this part has been adopted into the first part of the paper that shown in the end of this thesis.

Figure.4.14 clearly shows that in each frequency, the difference between classical and second order homogenization is acceptable when the thickness of this structure e is larger than 4mm, which can be validated by the experiment results. Also, it proved that the classical homogenization can be applied into relatively large value of e. Furthermore, second order homogenization is much precise in calculate the transmission property, especially at 9.5GHz it correspondent to measurement data pretty well.

What is more, the transmission calculated from classical homogenization remains close to 1 when the thicknesses e < 0.2 mm. However, transmission that from second order homogenization decreases when the frequency reach 12 GHz, which correspond to



Figure 4.14: Variation of transmission correspond to thickness of the metal layer at different frequencies: (a) 8 GHz, (b) 9.5 GHz, (c) 10 GHz, (d) 12 GHz

the measured results.

4.3 Application to the tuning of the reflection phase of AMC

In the last section, a second order homogenization theory is provided to calculate the transmission property of the thin structures. Actually, such tiny thickness structures have been widely used in the past decades. In the modern electronic system, printed circuit board (PCB) was widely used because of its multiple advantages such as low profile, low cost, and easy manufacture. One important application is that it can be used for designation of printed microstrip antenna. However, low gain, narrow bandwidth, low directivity and other undesired properties limit its application in many research fields, such as the radar system and onboard equipment. In order to overcome those disadvantages and apply patch antenna into many other fields, lots of artificial materials for instance electromagnetic band gap structures (EBG), high impedance surface (HIS), photonic band gap structure (PBG) and artificial magnetic conductor (AMC) have been proposed for antenna designation and have improved antenna performance.

Artificial materials were first introduced in the 1960s, and lots of terminologies have been proposed for negative permittivity and permeability by different researchers. Several techniques based on metamaterials have been proposed to enhance the performance of patch antennas. One of the approaches is using multiple antennas to reduce surface wave that deteriorates radiation pattern. Frequency selective surface (FSS) and EBG structures can be used to enhance bandwidth and gain. Another method used to enhance gain and bandwidth is the use of multilayer patches, but the total volume of the antenna is also increased simultaneously [20,21].

AMC was proposed to solve the thickness problem at the beginning, then researchers

find that AMC has many other properties. The first importance application of AMC is that those periodic structures can be engineered to achieve electromagnetic band gap. This band gap is able to limit surface wave to propagate on the ground plane, which enables AMC structure suitable for antenna applications [38]. For example, an AMC surface, proposed by Sievenpiper [60] possesses such band gap. Another valuable feature of AMC is its capability to reflect electromagnetic waves without phase shift. The traditional ground plane of the patch antenna is made of a good electric conductor, which is impenetrable for electromagnetic waves thus act as a reflector to prevent electromagnetic waves radiate to backside [60]. However, introducing an image of a radiating source with a negative phase in comparison of the radiation can deteriorate the radiation pattern. In order to cancel such undesirable out phase, researchers usually employ $\lambda/4$ thickness between the radiated patch and the ground plane at the beginning, which would induce relative large thickness, shown in Figure.4.15, especially at lower frequency band. By using AMC structure instead of traditional metal ground, the volume of the antenna can be dramatically reduced.



Figure 4.15: schematic map for the function of relector

The third important property of AMC, together with HIS, to improve the impedance characteristic [60]. Actually, for the conventional printed antenna, image current introduced by ground plane is not the only reason which is responsible for the bad performance [69]–input impedance matching is also counted for this. A spiral antenna demonstrates wide impedance frequency band when adopted high impedance type artificial surface [49]. An improvement of an invert F antenna is also published in [48]. Moreover, AMCs and HISs have been used as a ground plane of printed microstrip antenna to enhance both impedance bandwidth and gain [55, 21, 5]. In the last section, the interface homogenization theory was introduced and validated. Actually, interface homogenization has another application such as AMC. In this chapter, we mainly focus on the validation of the analytical calculation for the transmission property of AMC structure.

4.3.1 Theory

The conventional metallic ground plane is able to reflect electromagnetic waves, however, the phase of the reflect wave would be shifted out of π compare to the incident wave. Thus, at the beginning for the designation of traditional patch antenna, shown in Figure.4.15, in order to depress the interference between the radiated wave r1 and the reflected wave r2, normally the distance *d* between the patch antenna and the metallic reflector should be $\lambda/4$, where λ is the effective wavelength of the radiated wave in the substrate.

One of the most valuable properties of HIS structure proposed in [60] is that such structure can be tailored to reflect wave without phase shift. Consequently, the total thickness of patch antenna that is employed in such mechanism would decrease dramatically.

The accurate method to calculate the transmission property of thin structures has been proposed in the last section. Although the detailed procedures are not demonstrated in this thesis, the idea can be explained in the following way. When the thickness of the grating, or other periodic structures, is diminishing to a tiny scale, show in Figure.4.16 (a), the two boundaries would be very close to each other. In another word, the bulk region is small enough to be regard as an "interface" of the other two regions, shown in Figure.4.16 (a). For the bulk region with big thickness situation, the component of the magnetic vector along the normal direction of the boundary is continuous on the both



(a) From bulk region to interface

(b) From classical homogenization to interface homogenization

Figure 4.16: Bulk boundary to interface boundary

interfaces. However, when the bulk becomes such interface, the component of magnetic is no longer continuous, so it is called "jump boundary condition". The main idea in this part is to employ the same idea to treat the AMC structure, shown in Figure.4.16 (b), to induce its reflection property.

4.3.2 Validation

The method to measure the phase shift property of AMC is presented in this part. Actually, no matter what kind of measurement, a "standard" or "reference" is essential to get the phase shift correctly. To find the standard or reference when we use the Network analyzer is another story, so-called Calibration that we have mentioned in chapter 2. For AMC structure, we aim to calculate and measure its ability of phase shifting. Here, the phase shifting is compared to the simple metal surface. From the wave equation, is it clear that each point on the propagated direction corresponds to a certain phase. In another word, the different position has a different phase at a specific time. So, in order to get the exact phase shift, remaining a distance from the antenna to the reflector is very crucial. Next,

the experiment set up and the data processing method would be shown in the following part.

Numerical Validation

Set up of the experiment

As we mentioned before, in order to measure the phase shift, the distance "Dis" between the antenna aperture and the reflector should remain the same value during the measurement, shown in Figure.4.17. The emitting antenna and receiving antennas are placed at the same end of the electromagnetic chamber and with the same polarization to keep **H** vector vertical to the ground. The reflector consists of three layers, metallic grating, substrate, which is made of epoxy, and the ground plane.

In this measurement, electromagnetic waves start from the emitting antenna to the smooth metallic reflector and then go back to receiving antenna. Thus, the transmission between the two antennas should be

$$S_{21} = \frac{E}{E_0} \quad , \tag{4.4}$$

where E_0 and E is the emitting and receiving electric field respectively. Afterward, this metallic ground plane is replaced by AMC structure, transmission can be written as

$$S_{21}^* = \frac{E^*}{E_0} , \qquad (4.5)$$

where E_0^* is the receiving signal in the new configuration. From Eq.4.4 and Eq.4.5, we can get

$$\frac{E^*}{E} = \frac{S_{21}^*}{S_{21}} , \qquad (4.6)$$

then the phase of the two sides of Eq.4.6 also should be equal. Besides, the traditional metallic plane reflector would induce π phase shift, thus the phase shift caused by the AMC structure should be

$$\phi_{shift} = \arctan\{\frac{S_{21}^*}{S_{21}} * e^{j\pi}\} \quad . \tag{4.7}$$



(b) Set up of experiment

Figure 4.17: (a) 3-D view of the AMC structure that consists of three layers: 1, the unit cell of metal bars whose width are l, thickness are e and periodicity is h; 2, the substrate layer; 3, the metallic ground plane. T is the total thickness of the three layers. (b) Experiment set up to measure the phase shift. The two antennas are connected to the two ports of the analyzer to measure the emitting and receiving signal, "Dis" is the distance between the antenna aperture and the surface of the metal grating. The ground is made of absorber and the whole set up is put in a microwave anechoic chamber to remove the interference from the environment.

In the following, several groups of AMC structures are simulated and get the phase shift by using such a method to find the ideal dimension to work at around 10 GHz.

Numerical results

In order to validate the theory experimentally in X-band ($8GHz \sim 12GHz$), an ideal parameter should be chosen firstly, because the reflection property can be affected by the dimension of the structure and correspondent to the frequency band. In this part, several groups of parameters are employed to find the relation between the parameter and the phase shift property of AMC structure.

1, Fixed width of the bars

First, all the dimensions are fixed, for example the width of the bars is set as l = 1 mm, the thickness of the bars is e = 1.5 mm and the total thickness of AMC are T = 2.5mm, except the periodicity of the array *h*, which is varied as h = 5, 7, 9, 12mm respectively. The



Figure 4.18: Study for the relation between the periodicity and the phase shift while other dimensions are fixed as e = 1.5 mm and T = 2.5 mm. (a) l = 1 mm, (b) l = 3 mm.

result is shown in Figure.4.18 (a). It is clear that when the thickness of the bars is small,

no matter the value of periodicity is, the phase shifting property of the AMC does not change obviously. When a larger value of width is employed, the tendency of the curve begins to appear, shown in Figure.4.18 (b). The phase shift result from AMC becomes larger when the periodicity is increased.

2, Fixed periodicity of the array



Figure 4.19: Study for the relation between the width of the bars and the phase shift while other dimensions are fixed as h = 9 mm, e = 1.5 mm and T = 2.5 mm

The second group simulation that we have done is fixing the periodicity as h = 9 mm, and thickness of metal grating as e = 1.5 mm, then change the width of bars from 1 mm to 5 mm. From the numerical results, shown in Figure.4.19, larger width of the bars introduce smaller phase shift, which has the opposite effect as the periodicity.

3, Vary the periodicity of the array

Here, the different thickness of substrate layer is studied to see if the dielectric between the periodicity metallic grating and the round affect the phase shift property. From the result we can see that as the decrease of the thickness of the substrate, phase shift



Figure 4.20: Study for the relation between the total thickness of substrate for AMC structure and the phase shift while other dimensions are fixed as h = 3 mm, l = 8 mm and e = 2.5 mm

increases.

4, Final dimension



Figure 4.21: Phase shift property of final dimension of AMC structure, whose periodicity h = 4.5mm, width l = 3.5mm and total thickness of the structure T = 2.5mm, whereas the different thickness of metallic layer e = 0.03mm, 1.5mm and 2mm respectively.

From the calculation that we have done above, we know that in order to get larger value of phase shift, smaller thickness of substrate, smaller width of bars or larger periodicity should be employed. Take consideration of all the factors, such parameter are finally chosen, h = 4.5 mm, l = 3.5 mm, T = 2.5 mm. The thickness of the metallic grating is chosen as e = 0.03 mm, 1.5 mm and 2 mm. All the interface homogenization and numerical results are shown in Figure.4.21. As the value of e is changed, the phase shift property is also varies, which is useful to design the exactly value of phase shift by just changing the thickness of metal layer of AMC structure.

Experimental validation

From the discussion of the parameter for the three prototypes, three group of parameter are determined, shown in Table 4.2. The results of each prototypes are demonstrated one by one in the following.

	P 1	P 2	Р3
Thickness e	0.03mm	1.5 mm	2mm
ke	0.0063	0.31	0.42
type of array	PCBs	aluminum Bars	aluminum Bars

Table 4.2: Different parameters for three AMC prototypes

e=0.03mm (*ke* ≈ 0.0063)

The first component prototype 1 is made of Printed Circuit Boards (PCBs) with the printed copper layer on one side, and the thickness of the substrate is 0.5mm. The copper layer is etched into several parallel wide wires with periodicity of h = 4.5mm and width of l = 3.5mm, shown in Figure.4.22. The thickness of copper layer is 0.03mm, so the value of $ke \approx 0.0063$ is much smaller than 1. The other components of prototype 1 is two epoxy substrates with thickness of 1mm, and one of them has copper layer which act as a reflector on one side, then the total thickness of prototype 1 is 2.5mm. Such three components are layered together as shown in Figure.4.17 and 4.22. Then this AMC structure is placed in the microwave anechoic chamber facing to the two antennas instead of the plane metallic surface to get its reflection property. The result would be demonstrated in the after.



Figure 4.22: Prototype 1 of AMC structure consists of three layers. The top one consists of a one-side PCBs, which was etched into 1-D array with periodicity of h = 4.5mm. The middle layer is an epoxy substrate and the bottom layer is an epoxy substrate with one side copper layer as the reflector, both of them are 1mm thick.

e=1.5mm (*ke* ≈ 0.31)

The thickness of the grating of prototype 2 is much bigger than that of prototype 1, so it cannot be manufactured by etching the PCBs. Figure.4.23 (a) shows several single elements of the grating, which is cut into 3.5mm width from a 1.5mm thick aluminum board, then the bars have thickness of e = 1.5mm and l = 3.5mm. Those aluminum bars are stuck on one side of an epoxy substrate with periodicity of h = 4.5mm, shown in Figure.4.23 (b). The substrate is 1mm thick, and one side is covered by copper layer.

e=2mm (*ke* ≈ 0.42)

Prototype 3, which is similar with prototype 2 shown in Figure.4.23, also consists of aluminum bars and a substrate. The difference is that here the thickness of the aluminum bars and the substrate are 2mm and 0.5mm respectively, then the total thickness of this structure remains T = 2.5mm. Until now, all the three prototypes are introduced, in the following part, and the measurement result will be demonstrated.

Etude expérimentale de structures basées sur les metamatériaux



Figure 4.23: (a) Single metallic bars of prototype 2 and 3 (b) Prototype 2 consists of metallic bars whose thickness is e = 1.5mm.

results

Figure.4.24 demonstrate all the data measured for the prototypes with different thickness of metallic grating, and the same total thickness of the AMC structures. From the figure



Figure 4.24: Measurement and numerical data for phase shift of the prototypes in X-band.

we can see that as we increase the thickness of the metal layer, the phase shift of the

reflected wave is also increase. In another word, the phase shift can be tunning from $-100 \deg \sim 130 \deg$ at 10 GHz, and also, can be tunning from $0 \deg \sim 150 \deg$ at 8 GHz and from $-120 \deg \sim 50 \deg$ at 12 GHz. With such ability, the unit cells of planar reflect array antennas can be designed to have the same size with different thickness, which is huge advantage for the designation of the array. With such ability, the unit cell of planar reflect array antennas can be designed have the same size with different thickness, which is huge advantage for the designation of the array.

Chapter 5

Conclusion

In this chapter, we present the main achievements and contributions made in this dissertation to homogenize the periodic structure in different situations. Effective parameters of the one and two-dimensional periodic structures are derived by an approximate analytical model and applied to enhance the performance of the traditional antenna. The second order homogenization is proposed to overcome the inaccuracy of the classical homogenization when dealing with the ultra-thin structures.

In Chapter 3, a brief method is proposed to determine the transmission properties of the dielectric multilayer structures and corrugated surface based on the Helmholtz equation and the equivalence expression of tensor a [11,32]. Besides, this method is also applied to dealing with metallic multilayer structures, which can support 'spoof plasmons'. In the experiment, we use a line metallic cylinder array, consists of cylinders with the height of 27mm, the diameter of 3mm and periodicity of 21mm, to approximately replace the multilayer structure to control the wavelength in the array. In the air, the wavelength is 0.2m, 0.17m and 0.15m at 1.5GHz, 1.7GHz and 1.9GHz respectively. Within the structure, we find that the operating wavelength is 0.9m, 0.125m and 0.07m respective, which are approximately correspondent to the analytical prediction.

Furthermore, by using this method, we design a two-dimensional cylinder array mounted

on a metallic ground that is able to provide a band gap between 8 to 12 GHz, in other words, it can suppress the surface wave propagating on the plane. Thus, we apply this structure into the designation of traditional patch antenna to reduce the side and back radiation caused by the surface wave. Consequently, 10 dB of the side radiation is achieved, which is meaningful to reshape the radiation pattern for antennas.

In addition, in order to find out the inner physical mechanism in the cylinder array, further study has been done. We notice that there are weak resonances when we tune the band gap of the array. Actually, resonances are correspondent to each mode of transmission pattern, which have been measured at the different frequency. In other words, the near field excited by the radiated patch between the cylinders can be mutual canceled or lapped because of the different frequency. As a consequence, different near-field distribution and band gap properties are formed.

In chapter 4, we find the disagreements between the classical homogenization prediction and the experimental results when we study the transmission property of the ultrathin metallic grating. Besides such disagreements always remain although we eliminate the interferences caused by the experimental facility. Thus, we propose a new method to homogenize the thin structures, so-called second order homogenization. In this method, jump boundary condition is proposed to replace the original bulk boundary condition. Six different prototypes with different thickness are manufactured to experimentally validate the second order homogenization. The measured transmission coefficients at different incident angle reveal that seconder order homogenization can determine the precise outcome of the transmission property for the thin metamaterial structures.

Moreover, investigations of thin Artificial Magnetic Conductor (AMC) structures have been conducted by using interface homogenization. The 'jump' boundary condition was employed instead of bulk region boundary condition to determine the phase shift property of AMC. Experimental validations are given to show that phase shift due to the different thickness of the metal layer.

Appendix

1, Numerical tools

In the following, numerical results calculated by electromagnetic simulation software are given to valid the analytical solution for multilayered structures. Actually, there are numerous methods adopted by the different commercial company to design their own software to get the numerical solution for the electromagnetic problem. For example, Advanced Design System (ADS) produced by Agilent Technologies based on the Method of Moments (MOM) is designed for RF, microwave, and high-speed digital applications. ADS provides a vast array of simulation modes and models. Another software worth to be mentioned is High-Frequency Structural Simulator (HFSS), a commercial finite element method (FEM) solver for electromagnetic structures from Ansys company. It is one of the several commercial tools used for the designation of antennas, complex RF electronic circuit elements including filters and transmission lines.

The last one CST MICROWAVE STUDIO (CST MWS) is designed for high-frequency devices. It enables the fast and accurate analysis of antennas, filters, couplers, planar and multi-layer structures and SI and EMC effects etc. In this thesis, most of the numerical calculation were carried by CST, because of its multiple solvers, for instance, Time Domain Solver, Frequency Domain Solver and Eigenmode Solver which can provide dispersion relation for the periodic structures.

2, Patch antenna

At microwave frequency (300 MHz \sim 300 GHz), because the requirement of low profile and portable size for the modern communication system, many kinds of printed patch antenna were widely used in many types of equipment due to its easy for manufacture and designable size. The original type of patch antenna was proposed by Howell at 1972 [6], shown in Figure. 5.1(a). After several decades' development, more and more types of printed patch antenna were invented to meet the different requirement in reality.



(a) typical printed patch antenna (b) printed vivaldi antenna (c) printed Yagi antenna

Figure 5.1: schematic view of several printed antenna

Vivaldi antenna, shown in Figure. 5.1(b), is a co-planar broadband antenna, which is printed on a dielectric plate with metal layers on both sides. The feeding line excites a circular space via a microstrip line, terminated with a sector-shaped area. From the circular resonant area, the energy reaches an exponential pattern via a symmetrical slot line[13, 75]. Yagi-Uda antenna, commonly known as a Yagi antenna, is a directional antenna consisting of multiple parallel elements in a line[25], usually half-wave dipoles

made of metal rods. Because of its excellent performance in high directivity, Yagi antenna was transformed to printed antenna [74, 30], shown in Figure 5.1(c), and also have higher directivity than other printed antennas.

However, as we mentioned before, the patch antenna is placed on a dielectric substrate over a metallic ground plane which can support surface wave and couple power into the substrate. Figure.5.2 (a) demonstrates the top view of the patch antenna which is located on the dielectric layers above the metallic ground plane, shown in Figure.5.2 (b).



Figure 5.2: (a) Top view of printed patch antenna. (b) Side view of the patch antenna.

The diffraction of these waves at the edges of the substrate can radiate into free space and couples with the main radiation of patch antenna, then affects the antenna efficiency and causes ripples and high-level side lobes in the radiation pattern. In the following part, this kind of patch antenna is used as the radiation source to test the ability of the metasurface to confine the surface wave on the metal-dielectric surface around the patch antenna.

3, Numerical Method Solving Electromagnetic Problem

As the development of both theory and experimental tools of the wireless system, higher frequency band was in use and a more complicated system was proposed. More and more numerical method and tools are invented to meet the high-speed calculation requirement.

Finite-Difference Time-Domain method

Finite-difference time-domain is a numerical analysis technique used for modeling computational electrodynamics. As a time-domain method, FDTD solutions can cover a wide frequency range, and treat nonlinear material properties in a natural way.

The FDTD method is a general grid-based differential numerical modeling methods. The time-dependent Maxwell's equations (in partial differential form) are discretized using central-difference approximations to space and time partial derivatives. The resulting finite-difference equations are solved in either software or hardware in a leapfrog manner: the electric field vector components in a volume of space are solved at a given instant in time; then the magnetic field vector components in the same spatial volume are solved at the next instant in time; and the process is repeated over and over again until the desired transient or steady-state electromagnetic field behavior is fully evolved.

Finite Element Method

The finite element method (FEM) is a numerical technique for solutions of boundary value problems for partial differential equations. It is also referred to as finite element analysis (FEA). FEM subdivides a large problem into smaller, simpler, parts, which is so called finite elements. The simple equations that model these finite elements are then assembled into a larger system of equations that models the entire problem. FEM then uses variational methods from the calculus of variations to approximate a solution by

minimizing an associated error function.

Method of Moments

In statistics, the method of moments is a method of estimation of population parameters. One starts with deriving equations that relate the population moments to the parameters of interest. Then a sample is drawn and the population moments are estimated from the sample. The equations are then solved for the parameters of interest, using the sample moments in place of the (unknown) population moments. This results in estimates of those parameters. The method of moments was introduced by Karl Pearson in 1894.

With the help of the numerical tools that employ the method above, the telecommunication industry develops very fast and minimize the development cycle of the equipment in a large scale.

4, Maxwell Equation

Maxwell's equations are a set of four partial differential equations, it comes from the four laws that form the foundation of classical electrodynamics, classical optics, and electric circuits. Maxwell's equations mainly describe how electric and magnetic fields are interacted and altered by each other and by other charges and currents. They are named after physicist and mathematician James Clerk Maxwell, who published those equations between 1861 and 1862. All the four elements would be introduced as follows.

Gauss's law

In physics, Gauss's law is the principle of the of the distribution of the electric field result from electric charges. The integral form of Gauss's law can be expressed as



Figure 5.3: Electric force line between two positive charges

where E is the electric field, S is the surface of the volume V that enclose the charges with total charge of q. From the divergence theory, Gauss's law can be alternatively expressed as differential form

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0},$$

which tells us that the divergency of electric charges has a certain value. And also, the electric flux, which is defined as $\Phi_E = \oiint_S \mathbf{E} \cdot d\mathbf{S}$ and indicates the flow of the electric field through a given area, of positive charge is a positive value, in another words, the electric field line is alway out of positive charges, shown in the picture.

Gauss's law for magnetism

The integral form of Gauss's law for magnetism, $\nabla \cdot \mathbf{B} = 0$, reveals that the magnetic field **B** has divergence equal to zero[10], which means that magnetic field is a solenoidal vector

field. In another words, magnetic monopoles do not exist. The law in this form tells us that for each volume in space, no "magnetic charge" can build up in any point in space. For example, the magnet has exactly same strong south pole and north pole, and single south poles without accompanying north poles (magnetic monopoles) are not exist. In contrast, this is not true for other fields such as electric fields or gravitational fields, where total electric charge or mass can found in a volume of space.

Faraday's law of induction

Faraday's law of induction, $\oint_{\partial \Sigma} \mathbf{E} \cdot dl = -\frac{d}{dt} \iint_{\Sigma} \mathbf{B} d\mathbf{S}$, is a primary law of electromagnetism to describe how a magnetic field could interact with an electric field. It is the fundamental operating principle of transformers, inductors, and many types of electrical motors, generators and solenoids[56].

Also, from its differential form

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

it indicating that constantly changing magnetic field is able to generate electric field, which is an essential fundamental mechanism in electromagnetic theory.

Ampère's circuital law

Ampère's law proposed by André-Marie Ampère in 1823[51], determines the magnetic field generated by a given current and a changing electric field, and describes the relation between the integrated magnetic field around a loop to the electric current passing through the loop. In its original form, Ampère's circuital law relates a magnetic field to its electric current source. The law are related by the Kelvin-Stokes theorem. It can also be written in terms of either the B or H magnetic fields. Then Maxwell derived it again using hydrodynamics in his paper "On Physical Lines of Force" and it is now one of the

Maxwell equations, which form the basis of classical electromagnetism:

$$\oint_{\partial \Sigma} \mathbf{B} \cdot dl = \mu_0 \iint_{\Sigma} \mathbf{J} \, d \, \mathbf{S} + \mu_0 \boldsymbol{\varepsilon}_0 \frac{d}{dt} \iint_{\Sigma} \mathbf{E} \, d \, \mathbf{S}.$$

Where J is the enclosed conduct current or free current density, and dS is the surface element where the current flows by and the electric field passes through.

Table 5.1: Maxwell equation

Name	Integral Form	Differential Form
Gauss's Law	$ \oiint_{\mathbf{S}} \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} \iiint_{\Omega} \rho d\mathbf{V} $	$ abla \cdot {f E} = { ho \over {m arepsilon_0}}$
Gauss's Law for magnetism	$ \oint\!$	$\nabla \cdot \mathbf{B} = 0$
Faraday's Law induction	$\oint_C \mathbf{E} \cdot dl = -\frac{d}{dt} \iint_{\mathbf{S}} \mathbf{B} d \mathbf{S}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
Ampère circuital Law	$\oint_C \mathbf{B} \cdot dl = \iint_{\mathbf{S}} (\mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}) d\mathbf{S}$	$\nabla \times \mathbf{B} = \mu_0 (\mathbf{J} + \boldsymbol{\varepsilon}_0 \frac{\partial \mathbf{E}}{\partial t})$

The full set of Maxwell's equations, both integral and differential form describe the electromagnetic field in any kind of medium, are shown in Table.5.1. In vacuum or air, there is no conductive current **J** and the charges, so by employing J = 0 and $\rho = 0$, the Maxwell equation in vacuum is given as:

$$\nabla \cdot \mathbf{E} = 0 \qquad (1)$$

$$\nabla \cdot \mathbf{B} = 0 \qquad (2)$$

$$\begin{cases} \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (3) \\ \nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \qquad (4) \end{cases}$$

The equation here is symmetric for **E** and **B** except for the factor of $\mu_0 \varepsilon_0$ and a minus sign in Eq.5 (3). If we take the curl of **E** in Eq.5 (3), and insert it into Eq.5 (4), then we can obtain

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

From the vector identical equation [19], $A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$, when *A* and *B* are replaced by differential operator ∇ , we can have $\nabla \times (\nabla \times E) = \nabla (\nabla \cdot E) - (\nabla \cdot \nabla) \mathbf{E}$. From Maxwell equation Eq. (1) $\nabla \cdot \mathbf{E} = 0$, then becomes

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \nabla^2 \mathbf{E}.$$
(5.2)

With the same method we can derive the same expression of magnetic field as

$$\frac{\partial^2 \mathbf{B}}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \nabla^2 \mathbf{B}.$$
(5.3)

It is clear that electromagnetic wave have the same form wave equation as that of mechanical wave of Eq.5.2. Although the mechanical wave and electromagnetic wave has the totally different physical mechanism, from the basic principle of oscillation and electromagnetic rules, the same form of wave equation is derived, which demonstrate the unity and beauty of mathematic and physics.

The expression of the electromagnetic wave in free space is the solution of Helmholtz Equation, thus from Eq.5.3, the expression of magnetic filed can be written as

$$H(\vec{r},t) = H_0 \cdot e^{j(\vec{k}\cdot\vec{r}-\omega t)} .$$
(5.4)

In order to simplify the expression, here we use magnetic field intensity *H* in stead of Magnetic induction intensity *B*. Where H_0 is the amplitude of *H*, \vec{k} is the wave vector and $|\vec{k}| = \frac{1}{\mu_0 \varepsilon_0}$ in free space. Hertz proved the theory by engineering instruments to transmit and receive radio pulses using experimental procedures that ruled out all other known wireless phenomena. The unit of frequency was named as "hertz" in his honor [65]. From then on, investigation for the electromagnetic waves from theory to the application.

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