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Evolution of urban systems : a physical approach

Giulia Carra

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Evolution of urban systems: a physical approach

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ABSTRACT

Despite the availability of always more precise data the quantitative understanding of the structure and growth of urban systems, remains very partial. In this thesis, we aim to contribute to identify the hierarchy of the processes governing the city evolution, using tools and approaches coming from statistical physics.

The manuscript is organized in two main parts. The first part focuses on the physical structure of the city that is investigated at two different spatial resolutions: at the scale of the building lot in the first section and at a more coarse-grained scale in the second one.

In the first section we study the phenomenon of urbanization beginning with an empirical analysis of geolocalized historical data, at the spatial scale of the building. We discuss how the number of buildings evolves with population and we show on different datasets that this "fundamental diagram" evolves in a possibly universal way, independent from historical and geographical features. We propose then a stochastic model based on simple mechanisms to contribute to the understanding of the empirical observations.

In the second section we aim to propose a continuous description of urban sprawl. We study a dispersion model that represents a good candidate for describing the growth of an urban area based on a double process, the growth of surface area and the absorption of neighbouring towns. We are interested in understanding and exploring the different behaviors that the model can produce in order to get a better insight on the variety of behaviors empirically observed.

In the second part of the manuscript we focus on socio-economic aspects. We study the commuting patterns and its relation to income for Denmark, US and UK, highlighting the empirical regularities. We consider the important economic job search model, the McCall model that is based on an optimal strategy. We study its implications for the spatial distribution of distances between residences and jobs as a function of the income and we show that they are not supported by empirical evidences. In a last part we propose an alternative model based on the closest opportunity that meets the expectation of each individual, and that is able to predict correctly the empirically behaviors observed. More generally, we proposed here an alternative framework to study human or animal behavior, in which actions are taken not on the basis of an optimal strategy but on the first opportunity that is good enough.

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Part I

INTRODUCTION

Ten thousand years ago humans lived in small and nomadic groups eating wild plants and animals. It is during the neolithic period that the first stone tools were shaped and plants and animals were domesticated. This agricultural revolution and the technological innovations brought to a greater sedentism and to the emergence of the first villages [1, 2]. These have been the prerequisites for the urban revolution of 4000 b.c., as defined by Childe [3]. During this period the first cities arose in the fertile areas of Mesopotamia and Egypt, characterized by settlements greater than any known neolithic village and by a complexity of the social organization.

The first big change in the organization of the city is related to the industrial revolution of the first half of the 18th century, and another important factor that has shaped and is still shaping cities are transportation technologies and in particular cars [4].

The urbanization process measured by the fraction of individuals living in urban areas describes a continuous process that gradually increased in many countries with a quick growth since the middle of the 19th century until reaching values around 80% in most European countries [5]. The majority of individuals in the world now live in urban areas [6] and the proportion is expected to increase in the next decades.

Cities are the key of cultural, social and technological innovations and of job creation [7]. Yet a large number of challenges are related to them that we are still not able to control: the emergence of inequalities, pollution, transportation and energy issues.

Understanding what governs the evolution of urban systems has thus become of paramount importance. The nowadays availability of large scale data allows a glimpse into the dawn of a new science of cities, interdisciplinary and based on data [8, 9].

A LITTLE BIT OF HISTORY

Cities have been for a long time the subject of numerous studies in a large number of fields and different approaches have been used and developed. We choose here to focus on the quantitative approaches developed in the fields of spatial economics and quantitative geography. The purpose is not to provide a complete survey of the history of these two fields but to review a little bit their origins revealing some of the main features characterizing their approach to the study of urban systems.

1.1 SPATIAL ECONOMY

The 19th century has been characterized by a scientific spirit that has strongly influenced the social sciences and the economy. The purpose of these disciplines was no longer just to describe reality, but to seek fundamental laws governing the reality [10].

In this intellectual atmosphere, spatial economy finds its origin as a sub-field of economy. It first developed in Germany as an essentially theoretical and deductive discipline.

The history of geographical economics can be summarized through the three most important models of location of economies activities: the Von Thunen and the Weber models and the Christaller theory. These models have been elaborated between the first half of the 19th century and the beginning of the 20th, and represent the bedrock of economic geography.

The Von Thunen model [12] is the first model of spatial economy and was proposed in an historical period characterized essentially by agricultural activity.

According to this model the agriculture land use around the city is determined by the distance from the center and the related transport costs. The various agricultural productions would distribute in circular bands around the urban market. In the nearer bands we find located the type of crops that can provide a higher rent by reducing the distance to the market and thus the transport costs. In the outward bands, are located crops that need to pay lower incomes, i.e. those for which the savings on transport costs determined by the reduction of the distance from the market are progressively less significant, see Fig. 1.

Different assumptions are made in this model, among them:

- the city as monocentric, homogeneous and isotropic

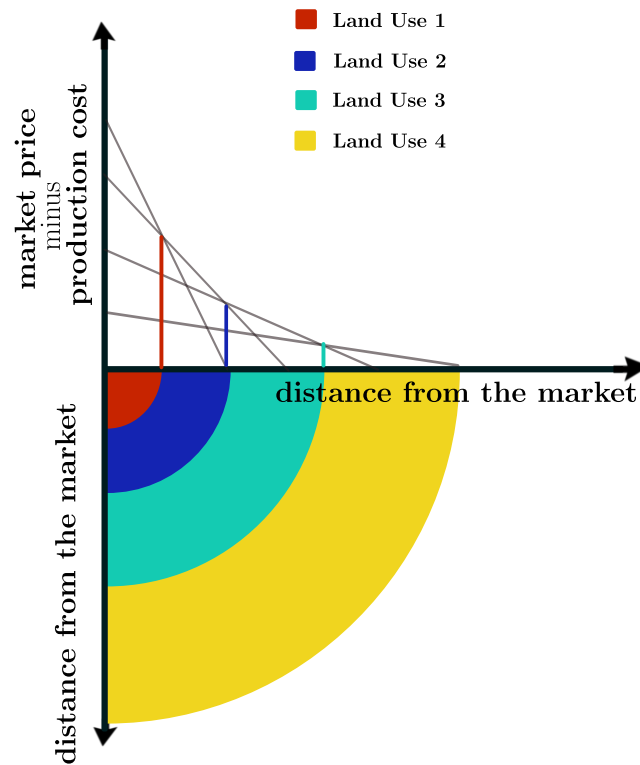


Figure 1: An illustration of the Von Thunen model (adapted from [11])

- the farmers are characterized by a rational behavior with the aim to maximize their utility (their profit) subject to budget constraint.

We note that these assumptions are still widely used in economic models.

Moreover, in the Von Thünen approach we find the ideas characterizing the bid-rent theory that will be applied in the context of urban analysis by William Alonso [13] to explain urban segregation and that represent one of the pillar of classical economic theory.

The Von Thünen model was considered revolutionary for his time because it shows that it is possible to describe an organized space neglecting physical and morphological considerations.

While the world of Von Thünen, in the first half of the 19th century, was still a world based on agriculture, the one of Alfred Weber, between the late 19th and early of 20th, is instead a world that has already undergone numerous transformations such as industrialization and the introduction of railways.

The location model of Weber [14] aims to provide a model for any kind of industry analysing the problem in respect of a single production unit.

As in the Von Thünen model, the firm chooses its location through a rational behavior, in order to maximize the profit and having a

perfect knowledge of the other activity locations. Nevertheless, it is important to remark that Weber introduces a new ingredient that influences the production costs: the agglomeration economies. These consist in a reduction in the costs of production caused by the fact that the production activity takes place in the same place.

Weber considered the problem of the location of individual firms: the optimal location for any firm will be the one that maximizes its profit. Now the problem was no longer to find the optimal location of the firm, but the one of maximizing the efficiency of the economic system overall.

A fundamental work is the one of the German geographer Walter Christaller [15]. Christaller was looking for some fundamental laws to explain the size, the number and the spatial distribution of urban settlements. The interesting idea of his theory is to consider human settlement as "central places" providing services to the population in the surrounding areas. He studied the settlement patterns in southern Germany and he found possible to model the pattern of settlement locations using geometric shapes. According to his theory there is a hierarchy of central places that are not organized at random and that has the following properties [16] :

- the shape of each dominant region is a regular hexagon.
- the dominant regions at the same level have the same area.
- there is a hierarchy of centers and at each level i of the hierarchy there are n_i corresponding local centers. The ratio

$$K_i = \frac{n_i}{n_{i-1}} \quad (1)$$

is constant.

This central place theory is a good example to highlight a first difference between economists and geographers. Indeed, as mentioned by Krugman [17] this is not an economic model as it does not explain the emergent macroscopic behavior as a result of the microscopic socio-economic processes.

Nevertheless, it has been the starting point of the German economist August Lösch [18], that tried to resume the logic of Christaller's model, extending it into a real spatial economic theory based on the hypothesis of general equilibrium.

We note indeed, that the theoretical and philosophical context of the economy at the beginning of the 20th century postulates that economic mechanisms and individual choices are able to self-regulate exchanges, and therefore they tend to a condition of equilibrium. This idea of equilibrium is still at the basis of most economic theories.

To summarize these three models presented above allow us to highlight and to present some assumptions and methodology that are still widely used in economic models:

- monocentric city
- equilibrium assumption
- rational behavior
- agglomeration economies

and the necessity to explain the emergent macroscopic behavior as a result of the microscopic ones. For the interested reader, in [19] Krugman present a more complete but concise summary of the intellectual traditions that strongly influence the new economic geography.

1.2 QUANTITATIVE GEOGRAPHY

The mathematical models of Von Thünen, Weber and Christaller highlighted the possibility to study urban systems by building abstract and deductive models based on postulates and axioms.

Nevertheless, these works were not known to most geographers, and only at a later stage, after the World War II, they began to be part of the historical baggage of quantitative geography.

This period was characterized, especially in the United States and in England, by a real epistemological change, a *Quantitative Revolution* whose manifesto is the work of William Bunge: "Theoretical Geography" [20]. The revolution brought to a "New Geography" resolutely deductive with the aim to highlight statistical regularities and identify laws. The movement reached France only twenty years later. Its history and its specific features are described by Denis Pumain and Marie-Claire Robic, in [21].

In [22] the geographer Lena Sander highlights the new methodological approaches developed during the quantitative revolution that have still a big influence in geography.

The first concerns the study of spatial differentiations. Most of the tools adopted at the time came from statistics [23, 24], methods of data analysis as factor analysis and classifications have been very successful in geography [25, 26]. Matrices of geographic information were built crossing a set of places (cities, districts, etc. . .) with socio-economic indicators (population, income, etc. . .) [27].

These methods made it possible to analyze empirical observations, to highlight relations between different variables characterizing the places, to classify locations depending on their similarities and dissimilarities or to reveal the existence of fundamental patterns.

The second methodology was about models and laws. An example is what Tobler in 1970 defined as *the first law of geography* related to the role of the distance [28]:

Everything is related to everything else, but near things are more related than distant things.

The most used and known model based on this concept is the gravitational law, firstly introduced by William Reilly [29] to analyse retail trade. Reilly's law is based explicitly on Newton's universal gravitation law, through a parallelism between physics and the behavior of the spatial markets.

This law establishes that the flow of goods, services and people between two locations grows proportionally to the product of their masses (values of goods, or number of inhabitants etc...), while it decreases with the increasing of the reciprocal distance.

1.3 DISCUSSION

In the previous sections we discussed very briefly the origins of spatial economics and quantitative geography. Between these two disciplines that frequently focus on the same subject the interaction is often difficult. In [30] the authors show through a bibliographic study that the two disciplines are not in conflict but rather simply ignore each other. Indeed, the study shows that only 1.9% of the citations in the leading journal of economic geography are from economic literature and the 2.8% of the citations in geographical economics come from geography.

Moreover they remark that geographers cite papers from a wide variety of other disciplines such as political science, sociology, anthropology, while economists seem to be only concerned with the work of other economists. This observation is related to a second difference (the first one has been showed in the discussion about Christaller theory) between economists and geographers that can be summarized by the metaphor of the lions and butterfly [30]

The lions/economists appear to labour the same core questions over and over again (why does economic activity agglomerate, what are the drivers of urban growth, etc), whereas the butterflies/geographers seem to enjoy exploring a much broader variety of directions and hop from question to question

Lena Sanders in [22] uses this citation to insist on the fact that while economy are characterized by a deductive approach with the systematic use of mathematical formulation, geography is a wide and varied field of research that is not characterised by methodological convergences, and that is open and influenced by other fields.

Many of the subjects the two communities study are similar and a number of general conclusions overlap (see for example [31]). Nevertheless the questions they ask and the use they do of models are quite different.

Some of these differences can be discussed through the models and

methodologies presented in the previous sections.

We remarked in the previous section that the Christaller theory was not "enough" for economists because the resulting geometrical paths of localities were not derived starting from the microscopic behaviors of agents. This was the reason why Lösch [18] tried to formalize it within a classical economy theory of general equilibrium.

The same happens for the gravitational law: "how to relate the flux to the optimizing behavior of agents?" asks Thisse in [31]. Economists reproach this law not to explain the theoretical foundations relating to the behaviors of the elementary entities, and they proposed different theoretical frameworks compatible with the gravitational law [32]. Interesting instead is the different approach of geographers, that given the law expressing a universal empirical fact, this is used as a "filter" [22, 33]. That is, they use the law to identify and interpret the residuals, i.e. the discrepancies between observed fluxes and fluxes estimated by the model, a positive residue revealing a preferential flow and a negative residue revealing a barrier effect.

For the geographer the priority is indeed to give an explanation of the specific and detailed reality observed.

Hence, geographers mainly focus on semi-aggregated approaches that provide a good fit of empirical data and then they look at details and "exceptions" not following the law. The main issue of economists instead, is to understand the microscopic mechanisms from which the law emerges, building a model, rigorous from the mathematical point of view, and compatible with pre-existing models often based on the assumptions presented in section 1.1. This discussion is widely developed by Thisse [31].

To conclude, we can remark that the empirical facts have not the same place in the two approaches, and they are quite marginal in economy, where often models come before empirical evidences.

In the next section we discuss when and why physicists began to be interested in cities, and in section 4 we present the approach to the understanding of urban systems used in this thesis, based on tools and methods of statistical physics.

CITIES AS COMPLEX SYSTEMS

2.1 COMPLEX SYSTEMS SCIENCE

Complexity and complex systems science have become widely used words in the last years and pervaded very different fields from physics to social sciences. We talk of *physics of complex systems* and the term complexity began to be used in geography [34] and economics [19] as well. This brought some of the geographers and economists communities to a change in the way they study cities by anyway keeping some of their main approaches. The development of the physics of complex systems brought physicists to be interested in problems belonging to other fields, such as biology, social sciences, economy and geography.

The success of the science of complexity and the nowadays availability of always more data open the possibility of a new interdisciplinary science of cities [8]. In this section we are going to present what a complex system is, how this concept represents a change of paradigm in the approach used to understand phenomena, and how in physics it has naturally arisen from the evolution of statistical physics.

A complex system is a system made up of a large number of interacting parts that give rise to non trivial emergent behaviors. These parts can be atoms, molecules but also people, firms, etc. . . It is characterized by emergent behaviors and fundamental laws that can be understood by focusing on the structure of interactions, rather than on the individual elements. Indeed, in complex systems the behavior of the whole system cannot be predicted by the knowledge of the single element composing it. An example is the brain, the knowledge of a neuron does not allow to describe the electrical activity produced by the brain, [35]. Other classical examples are the flocks of birds or the ant colonies. These represent the emergent behavior of a large number of interacting birds or ants that cannot be anticipated from the properties of the individual bird or ant.

2.1.1 *Statistical physics and complex systems*

As discussed by Giorgio Parisi [36] there have been three revolutions in physics, and each of this revolution had a big influence on the meaning of the world prediction and on the question we can ask about a system:

- (1) the introduction of statistical mechanics with the first probabilistic reasoning

- (2) the discovery of quantum mechanics
- (3) statistical physics of critical phenomena and complex systems

Among these, two of them, the (1) and the (3) concern the relation between a microscopic description of a system and a macroscopic one, that is the relation between the system studied at two different scales. At the time of Boltzmann and Gibbs, the main motivation for leaving the classical physics point of view was that it was often useless [37]. Consider for example the physical system constituted by a given gas, when we study this system from a microscopic point of view we face a big problem. In the deterministic approach, the experiment should allow us to measure the positions and the velocity of a huge number of atoms. Nevertheless, this task is not only impossible, but also useless. Indeed, the knowledge of its pressure and density are enough to determine all other physical quantities as temperature, viscosity etc. In other words the values of the pressure and the density are enough to determine the macroscopic state of the system, even if they are obviously not sufficient to determine its microscopic structure, which can be instead described using a probabilistic approach. We note moreover that to a macroscopic state correspond a big number of possible microscopic states. Summarizing what we can retain from this first change of paradigm is that:

- Large systems can be characterized by a small number of macroscopic parameters that are independent from all microscopic variables
- Some physical quantities do not take always the same value. In this case it is necessary to use a probabilistic approach.

This shows that in physics the description of a system usually depends on the scale at which the phenomenon is considered and also on the particular aspect we want to describe. The change of paradigm (3) happened inside statistical physics itself when physicists were facing the difficult problem of understanding phase transitions and critical phenomena. A classical example is the one of a magnetic system that above a critical temperature T_c is in a disorder state with zero magnetization, while below this temperature they acquire a (partial) order with an average magnetization different from zero. Near the critical temperature large fluctuations at all scales are observed. The system cannot be described anymore through simple equations for average quantities. Through these studies physicists understood that interactions play an essential role: without interactions there is usually no emergent behavior, since the new properties that appear at large scales, result from the interaction between constituents. Moreover, at a critical point, the system is typically characterized by power law distributions whose behavior tends to be universal. This means

that they do not depend on the details of the interactions but only on a small number of parameters and different kind of systems can be described through the same power laws. Since then, many new concepts have been developed in the attempts to understand these new problems: scaling laws, renormalization group, fractal geometry [38]. To summarize the importance of scales in the description of a system, the link between microscopic interaction and emergent collective behavior and the universal behavior that can be observed in very different systems, are important reasons that pushed physicists to think that some of these tools and concepts could actually be used to study also complex socioeconomic systems, for example cities. These ideas will guide us in the study of urban phenomena that we will carry out throughout this thesis. The approach used will be present in more details in section 4.

2.2 WHAT IS A CITY?

Cities are the place where we go to work, where we have a house, meet friends or strangers, go to the swimming pool and do many other activities. We live everyday in the city, but what is a city? *Cities are about connecting people*, according to Batty [8]. Thanks also to the recent success and development of the complex systems science, most scientists now agree on the concept of cities as complex systems. They are the results of a huge number of individual actions and interactions and only occasionally they can be structurally modified by top-down actions as it happened for example in Paris during the Haussmann period [39]. We can expect, as previously discussed, that the large number of interacting individuals lead to collective behaviors characterized by universal features. And indeed, even if there is much variety among cities in terms of morphology, population, density distribution, and also functions, despite these differences, we observe statistical regularities for some socio-economic indicators. Some examples are the gravitation law discussed above, the Zipf law [40], some recent empirical observations highlighting scaling relations with population for various socio-economical indicators [7, 41] and other two illustrations of this "universality" will be presented in chapter 6 and chapter 8.

Many questions can be asked about a city and many different phenomena can be studied: urban sprawl, effects of congestion, the spatial distribution of activities and residences, or the effect of new transportation infrastructures for example. Each of these phenomena is characterized by its own spatial and time scale and by some main ingredients or mechanisms that govern its behavior.

2.2.1 Different Scales

Most complex structures are hierarchical [42–44] and cities are such an example. They are indeed characterized by different scales, each with a particular fundamental element, particular questions associated to it and few dominant mechanisms associated to each phenomenon.

The microscopic scale of the city is represented by individuals, at this level we can ask questions about human behavior such as daily mobility, choice of the job location or of mode of transport etc.

A large number of individuals interacting with each others and characterized by spatial proximity, give rise to the next scale that is the city (or also a district). At this scale, we can ask questions about aggregated quantities such as population, segregation, pollution or about the structure of the city: density distribution, infrastructures, roads, amenities.

Finally, especially in this historical period characterized by globalization, a city is not isolated but rather is part of a system of cities, at the regional, national or global level. We thus have a higher scale constituted by system of cities.

Each aspect of the city is not only characterized by a spatial but also by a temporal scale over which the phenomenon occur (from an hour to a century).

The smallest time scale is the day and is related to the daily mobility: commuting trips, stop to the supermarket, going to the cinema at night or vacation trip as well. At the time scale of the order of years we can observe socio-economics changes and at the order of century one has changes in infrastructures.

In Fig. 2 is shown a summary of the processes occurring in urban systems according to these different scales, as presented in [9].

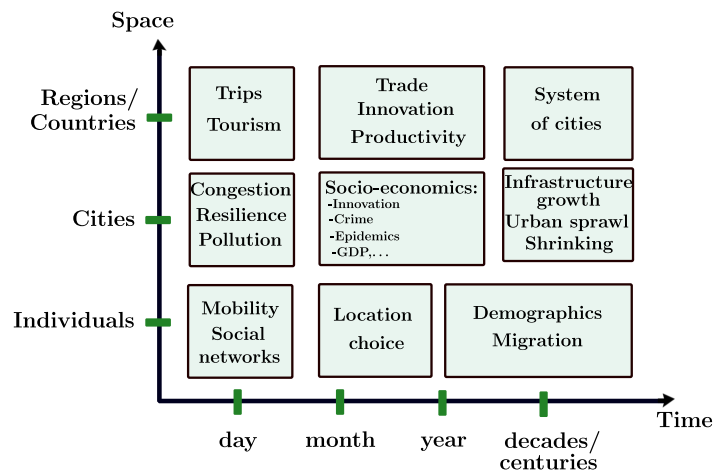


Figure 2: A summary of the processes occurring in urban systems according to their spatial and time scale, Figure from [9].

In addition to the development of the science of complexity, another important factor that brought physicists to be interested in cities is the recent availability of new data. Data play a dominant role in understanding urban systems and in building a new and interdisciplinary science of cities [8, 45]. Indeed, they allow us to gain a first insight into the phenomenon studied and to test predictions of theoretical models.

Agencies, state-owned and private enterprises began to release a various amount of data and to create open-data websites. The sources of these different data are usually related to new technologies and social networks such as smartphones and GPS devices or twitter.

We have data about almost all aspects of urban life, at different time scales and at different spatial resolutions and we are beginning to learn how to extract useful information from these new kind of data. Concerning the physical structure of the city the collaborative project OpenStreetMaps aim to build an editable map of the world that allow free access to street and road networks but also to buildings properties such as height or lot area; transportation networks data are also often available. Moreover the digitalisations of old documents and maps allow to investigate the historical evolution of the city structure [46]. Geolocalized historical data are also available at the level of the building lots. An example is the PLUTO dataset (short for Property Land Use Tax lot Output) released by the city of New York (US), where tax lot records contain very useful information about the urbanization process. In addition to the location, property value, square footage etc, this dataset gives access to the construction date for each building and thus allow in particular to produce "age maps" where the construction date of buildings is displayed on a map. Many age building maps are now available: Chicago [47]. New York City (US) [48], Ljubljana (Slovenia) [49], Reykjavik (Iceland) [50], etc. This type of geolocalized data at a very small spatial scale allows to monitor the urbanization process in time and at a very good spatial resolution (see section 6).

Going now to a smaller time scale we have mobility data. The classic source of mobility data are registers and surveys, but in the last years we witnessed the attempt to use new sources of data to track human mobility: movements of dollar bills [51], credit card transitions [52], GPS data [53], public transportation RFIDs [54], and cell phone calls [55–58]. These latter have been widely exploited for different issues [59–61]: every year an important conference, "NETMOB"

is organized on the scientific analysis of mobile phone datasets and the telecommunications company Orange organizes a "Data for development" (D4D) challenge, with the release of different mobile phone datasets with the purpose to contribute to societal development [62]. An example of what we can do using these data is shown in Fig. 3.

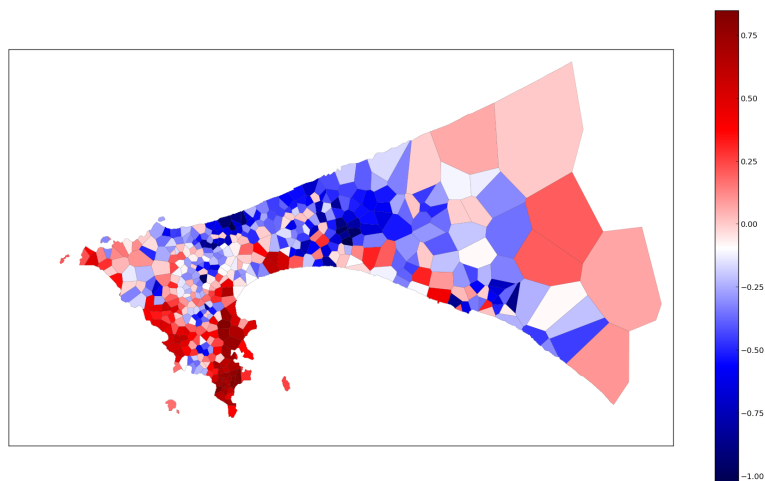


Figure 3: Relative difference of antenna mass between night and day in Dakar. Each Voronoi cell is colored with respect to the value $\frac{M_{\text{day}}(i) - M_{\text{night}}(i)}{M_{\text{day}}}$

In the report of the D4D Senegal challenge [63] we compute the mass of each antenna i during the day M_{day} and during the night M_{night} defines as the activity of the antenna i (i.e. the total number of calls and text messages sent and received from antenna i) during the day and night respectively, renormalized by the daily and nightly activity of the entire city. This quantity gives an idea of the changes in the spatial location of population over time.

An important issue concerning these new datasets is the accuracy of the information we extract from them. This is still not always well understood [58] and in a previous phase new methodologies to extract data should be compared with the more traditional ones. This for example has been done by the authors in [64] who investigated this issue for the origin-destination matrix comparing the results obtained with twitter, cell phone data and surveys finding a good agreement.

George Box claimed [65] "Essentially, all models are wrong, but some are useful". Helbing [66] added "Several models are right". It is moreover interesting to note that a model can have different interpretations. This has been discussed several times by Thisse in [31] and Helbing in [67]. An example is the multinomial logit model [68, 69] that can be derived in a utility-maximizing framework assuming rational individuals, but it can be also related to distributions of statistical physics and be compatible with a framework where individuals act without a strategy [70].

I am really tempted and interested in starting a philosophical discussion about modeling but it would go far beyond the goal of this thesis. I refer the interested reader to the introduction of the Helbing book [66] that is focused on the modeling of social systems.

In this section, I will quickly summarize the modeling approaches used in economics, geography and computer science and then I will focus on the physicist's one.

In the paradigm of complexity, geographers and computer scientists began to widely use and develop agent-based models. These consist in really complex numerical simulations that grasp as many details as possible. These kind of models however are often characterized by over-fitting and fine-tuning problems.

"With four parameters I can fit an elephant and with five I can make it wiggle his trunk" (Attributed to von Neumann by Enrico Fermi, as quoted by [71]).

The big number of parameters and variables taken into account in these models, make it difficult to specify their inter-dependencies, to understand which of the parameters and mechanisms introduced are relevant for the phenomenon studied and which can be neglected. In order to understand the phenomenon is essential to identify the variables and interactions that play a crucial role.

On the other side, most of economics model are very simplified and abstract, based on very strong assumptions such as the rational choice and the general equilibrium (see section 1.1). This strict methodology can be questioned [72].

The equilibrium assumption for example: the city is an out-of-equilibrium system [73] characterised by different time and space scales over which the variables evolve. The Equilibrium hypothesis can be done depending on the particular question asked after a discussion on the spatial and temporal scale that are involved (see [74] for a wider discussion on this).

Concerning instead the rationality of individuals, Paul Ormerod argues [75]:

"In many social and economic contexts, self-awareness of agents is of little consequence... No matter how advanced the cognitive abilities of agents in abstract intellectual terms, it is as if they operate with relatively low cognitive ability within the system... The more useful 'null model' in social science agent modeling is one close to zero intelligent. It is only when this fails that more advances cognition of agent should be considered. "

However, beyond these particular and strict assumptions, writing complicated equations that are essentially impossible to solve does not contribute much to our understanding. Indeed hypothesis and approximations are necessary in order to obtain quantitative and empirically testable predictions. Economic models are thus interesting to get an insight on the different urban processes and on the possible socio-economic variables needed when studying urban systems, such as the spatial structure of rent and income, the job choice process, the impact of amenities and transportation. Nevertheless, these model have often a big drawback that is the lack of the relation between models and empirical evidences [72] as also the economist Krugman highlights at the end of its review [19]. The scientific validity of these models is thus questionable.

Empirical analysis is fundamental to understand urban systems behaviors, but is also not enough. Data can indeed be misleading without a theoretical framework for interpreting them (see for example [45, 76]) and understanding which are the interactions and processes underpinning the observations is important to inform and help urban policies makers. We thus need models, and empirical observations to put constraints on them and help to identify universal regularities. Both data and models and the feedback loop between them are necessary in order to make progresses.

For a physicist the purpose of a model is to explain empirical observations and to predict. But what does it mean to explain? to predict? to understand? As we discussed talking about the revolution in physics, physicists began to learn and they keep learning to change the meaning of the word prediction and the way to tackle problems depending on the studied system, its typical scales and its characteristics. Sometimes if we want to solve a problem we are just obliged to view it from another point of view.

"Models may be compared with city maps. It is clear that maps simplify facts, otherwise they would be quite confusing. We do not want to see any single detail (e.g. each tree) in them. Rather we expect a map to show the facts we are interested in, and depending on the respective purpose, there are

quite different maps (showing streets, points of interest, topography, supply networks, industrial production, mining of natural resources, etc.)." [66]

Our aim is not to construct a mathematical model that reflect the full complexity of social interactions. We try instead to ask simple questions about urban systems and to propose simple models using ideas and tools from statistical physics, quantitative geography and spatial economics. These models will be characterized by the minimum number of variables needed to reproduce a certain effect, phenomenon or system behavior and they will give mathematical relations that can be tested against data. The important and difficult issue is to identify the main ingredients and mechanisms involved in the particular phenomenon studied.

We do not want to describe all the details of the system, we aim instead to reach a better understanding of the so-called "stylized facts" and universal properties that we observe in data and that go beyond the details characterizing each city.

Roughly speaking we can summarize the physical approach through the following steps [77]

1. Ask simple questions
2. Look at data
3. Make a guess about the dominant mechanisms and build a minimal model
4. Compute the consequences the model implies
5. Compare with data

and if there is not agreement with data, the model is wrong and you have to make another guess. In applying this methodology to urban systems, our choice of the ingredients and mechanisms to introduce in the model will be guided by geography and economic studies.

ABOUT THIS THESIS

Given the huge variety of problems related to cities, it is impossible to discuss all aspects of cities at once.

The manuscript is organized in two parts. The first part [ii](#) focuses on the physical structure of the city that is investigated at two different spatial resolutions: at the scale of the building lot in the first section [6](#), and at a more coarse-grained scale in the second section [7](#). In the first section we tackle the phenomenon of urbanization beginning with an empirical analysis of geolocalized historical data, at the spatial scale of the building. This allow to monitor the urbanization process in time and at a very good spatial resolution.

In particular, we discuss how the number of buildings evolves with population and we show on different datasets that this "fundamental diagram" evolves in a possibly universal way with three distinct phases. Once the universal pattern has been determined, we propose a stochastic model based on simple mechanisms to contribute to the understanding of the empirical observations. These results bring evidences for the possibility of constructing a minimal model that could serve as a tool for understanding quantitatively urbanization and the future evolution of cities.

In the second section we propose a continuous description of urban sprawl at a coarse-grained level with respect to the spatial unit of the building lots analysed in the first section.

If we begin by looking at data we observe that cities are characterized by a large number of different behaviors and do not display a simple unique pattern.

We then propose a slightly different approach focused on the study of a dispersion model that has been used extensively in the investigation of dispersions of animals in theoretical ecology, and also as a simplified model for the growth of cancerous tumours. This model represents a good candidate to describe the growth of an urban area based on a double process, the growth of surface area and the absorption of neighbouring towns. We are interested in understanding and exploring the different behaviors the model can produce and this can help us to get a better insight on the variety of behaviors observed in empirical data, that can be reinterpreted and analyzed in light of the new acquired knowledge. In the second part of the thesis we introduce in our study socio-economic aspects. Indeed, we focus on commuting patterns and its relation to income. Once again we start from data, and we study the commmting patterns of Denmark, US and England highlighting some regularities we observe. In a second step, we

consider the important economic job search model, the McCall model [78] that is based on an optimal strategy. We study the implications of the McCall model for the spatial distribution of distances between residences and jobs depending on the income and we show that they are not supported by empirical evidences. In a last part we propose a model based on the closest opportunity that meets the expectation of each individual that is able to predict correctly the behavior of the average commuting distance with income in terms of the density of jobs offers. More importantly, this model is able to correctly predict the form of the commuting distance distribution, its broad tail, and the data collapse predicted by its form. More generally, we propose here an alternative framework to study human or animal behavior, in which actions are taken not on the basis of an optimal strategy but on the first opportunity that is good enough. This framework would potentially find some applications in our understanding of foraging for example and other applications in ecology or finance where optimal control might be an incorrect assumption.

Part II

THE PHYSICAL STRUCTURE OF CITIES

This part focuses on the physical structure of the city with the aim to get a better quantitative insight in the process of urbanization and its consequences. In section 6 we analyze urban changes looking at the evolution in time of two urban indicators: the population and the number of buildings. In section 7 instead, we use a coarse-grained scale with the aim to study one of the consequences of the urbanization process that is the urban sprawl. We present a dispersal model as a candidate for describing the growth of an urban area based on a double process, the growth of surface area and the absorption of neighboring towns.

A FUNDAMENTAL DIAGRAM OF URBANIZATION

6.1 INTRODUCTION

Understanding urbanization and the evolution of urban system is a long-standing problem tackled by geographers, historians, and economists and has been abundantly discussed in the literature but still represents a widely debated problem (see [5]).

6.1.1 *What do we mean by urbanization?*

The first question we should ask when we aim to get a better insight into the urbanization process is "what do we mean by urbanization"? A well defined subject is an essential step to get a better understanding of it. The term urbanization has been indeed used in the literature with various definitions, and depending on the particular choice it can be considered as a continuous or an intermittent process [5].

Most of the time urbanization is measured by the fraction of individuals living in urban areas. With this definition, urbanization represents a continuous process that gradually increased in many countries with a quick growth since the middle of the 19th century until reaching values around 80% in most european countries, [79]. In Fig. 4 we show this trend for more and less developed regions and for the entire world. Although this characterization is widely used, it does not allow to get a deep understanding of the urban changes occurring in cities. Does the fraction of people living in urban areas increases because fixed and stable defined urban systems become denser or does it increases because rural areas become urban and thus the number of urban centers increases or even because of urban sprawl?

Another definition has been introduced by [75] in its empirical analysis of french cities and presented by [80] as a *theory of differential urbanization* in which urban changes are analysed looking at the changes in the population distribution in a system of cities characterized by different sizes. The theory assumes that in general we observe the three regimes of urbanization, polarization reversal and counter-urbanization, and that are characterized by a net migration which favors the primate, intermediate, and small-sized cities, respectively (see Fig. 5). It thus describes a process of population redistribution down the urban hierarchy, in which the term urbanization is associated to the regime where the growth of the primate city is the faster one.

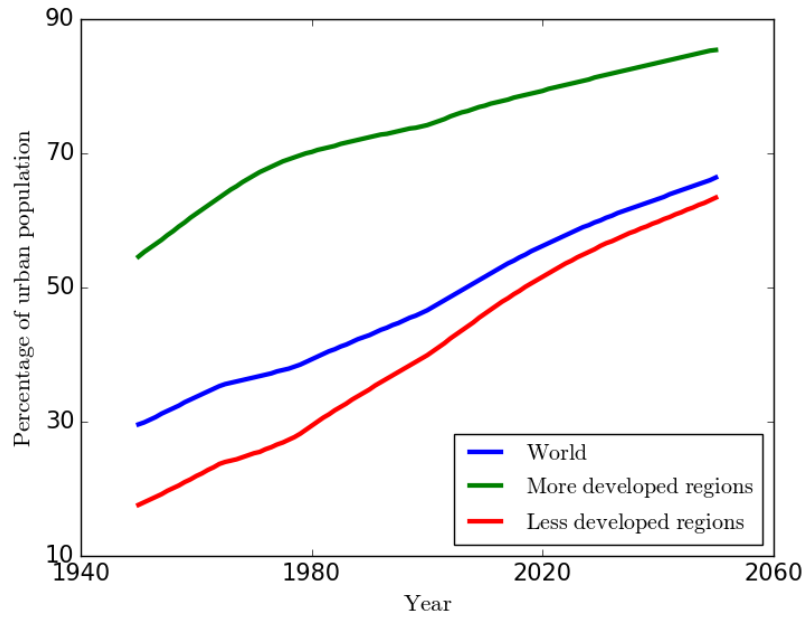


Figure 4: Percentage of urban population. Data from [6].

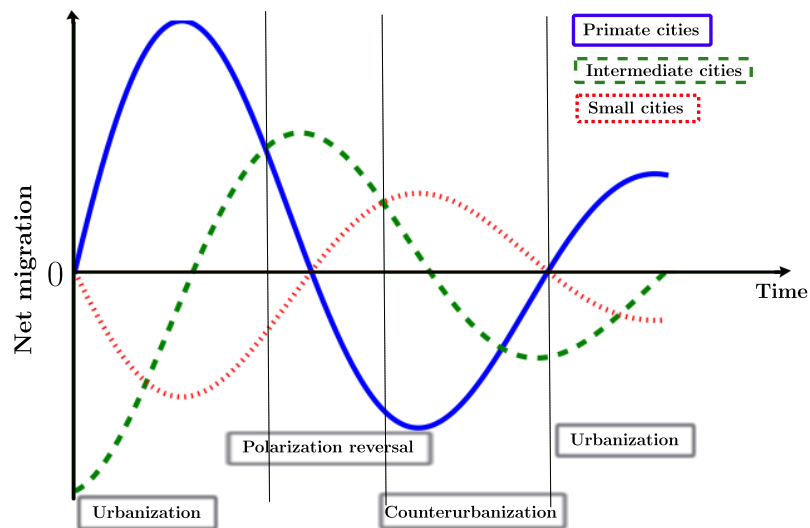


Figure 5: The figure represents the phases of urban development of a system of cities, according to the *theory of differetial urbanization*. Figure adapted from [80].

Yet another approach in the study of urban changes is presented in the *stages of urban development* proposed by [81], (see Fig. 6) where the phases of development are analyzed for a single urban agglomeration distinguishing the behavior of its inner part, the 'core' from the one of the surrounding area, the 'ring'.

According to this model, the city has a life cycle going from an early growing phase to an older phase of stability or decline, and four

main intermediate phases of development are identified. The first one called *urbanization* consists of a concentration of the population in the city core by migration of the people from outer rings. The second phase of *suburbanization* is characterized by a population growth of the urban agglomeration as a whole but with a population loss of the inner city and an increase in urban rings. During the third phase of (*counterurbanization* or *disurbanization*) the urban population decreases both in the core and the ring. Finally, the last phase of *reurbanization* displays a re-increase of the urban population. Within this framework, we observe that for most post-second war western countries *urbanization* was dominating in the 1950s followed by a *suburbanization* in the 1960s during which the population moved from the city core to the suburbs. The standard theory of suburbanization suggests that this is driven by a combination of technological progress (leading to transport infrastructure development) and rising incomes [4, 79, 82]. In the 1970s we observe in many urbanized areas a regime of *counterurbanization* where the population decreases. The significance of this regime and of the re-urbanization period for the 1980s and beyond, and more generally the possibility of a cyclic development are controversial topics (see for example [5]).

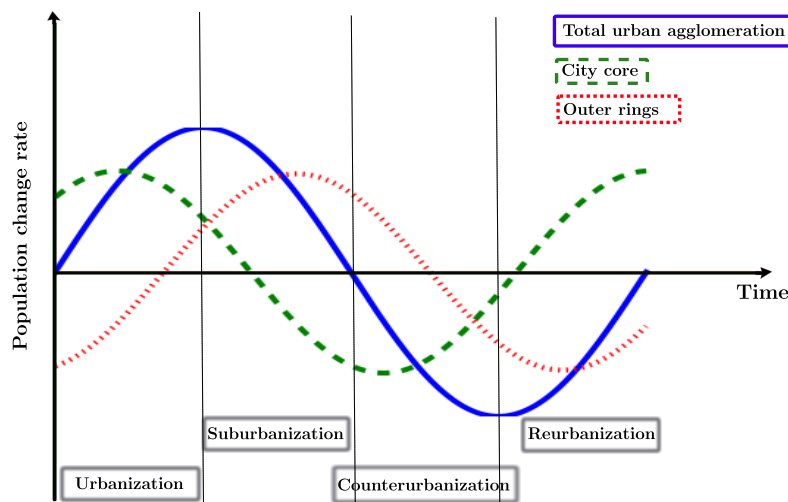


Figure 6: The figure represents the stages of urban development. Figure adapted from [79].

6.1.1.1 Data and definitions issues

In the previous section we discussed urban changes and the different meanings attributed to the word *urbanization*. It can concern the study of aggregate quantities as in the first definition, or a system of cities hierarchically organized in the second one, or a single urban agglomeration, analysing the relation between the core and its surrounding rings. When from a qualitative discussion we move to the

empirical analysis to better understand the phenomenon and test theories, we have to face the following questions: how do we establish what is urban and what is rural? Spatially, how do we determine the boundaries of the urban system? What is its core? And what its ring? Long time ago this task was easier, the urban center was enclosed by walls and beyond it was the countryside. From the 18th century on, gradually, city walls began to be broken down and the urban agglomerations started to spread out, the surrounding countryside began to be affected by an 'urbanization process' as well. Establishing the delimitation between urban and rural became a challenging task and the difficulty mainly arise from the fact that the concept of urban from which city boundaries are determined, is an abstraction that involves different factors, such as population density, labour supply and demand, and administration [83].

There are different definitions of cities depending on the historical period and on the country considered, which makes it difficult to conduct comparative and historical studies. Cities were defined in terms of population densities, or in terms of administrative boundaries which tell nothing about urban sprawl. A more recent approach, is to study cities in terms of functional boundaries as the MSAs in US, the FUAs for the OECD and the LUZs in Europe, in which urban center are defined according to a population density threshold ρ^* and are connected to other areas for which the commuting flow to the urban center is higher than a threshold τ^* . However, these functional definitions are not consistent with each other. Recently, a non-ambiguous way to measure the extent of human agglomerations have been proposed. It is based on clustering techniques and called City Clustering Algorithm (CCA) [84, 85]. We remark moreover that all these definitions depend on the choice of threshold parameters. A variant has been proposed in [86], where different definitions and different choices of the thresholds are discussed, analyzing how they modify power-law behaviors observed in data.

6.1.2 *Quantitative understanding*

Even though urban development and the spatial distribution of residences in urban areas are long-standing problems and were indeed discussed in many fields such as geography, history and economics; few of these approaches tackled this problem from a quantitative point of view ([28, 87–94]). Anas [95] presented an economic model for the dynamics of urban residential growth where different zones of a region exchange goods, capital, etc. according to some optimization rule. In the same framework the authors of [96] proposed a dynamical central place model highlighting the importance of both determinism and fluctuations in the evolution of urban systems. For a review on different approaches, one can consult [97], where different

studies of population dynamics modeling are presented. In particular, the author discusses the ecological approach, where ideas from mathematical ecology models are introduced for modeling urban systems. The work in [98] is such an example. In it, the authors show how the phase portraits of differential equations can bring qualitative insights on urban systems behaviors. Other important theoretical approaches comprise the classical Alonso-Muth-Mills model ([82]) developed in urban economics, and also numerical simulations based on cellular automata ([99]). More recently, the fractal nature of city structures (see the review [100]) served as a guide for the development of models ([94, 101, 102]). In particular, in [102], the authors proposed a variant of percolation models for describing the evolution of the morphological structure of urban areas. This coarse-grained approach however neglects all economical ingredients and suggests that an intermediate way between these purely morphological approaches and economical models should be found.

For most of these quantitative studies however, numerical models usually require a large number of parameters that makes it difficult to test their validity and to identify the main mechanisms governing the urbanization process. On the other hand, theoretical approaches propose in general a large set of coupled equations that are difficult to handle and amenable to quantitative predictions that can be tested against data. In addition, even if a qualitative understanding is brought by these theoretical models, empirical tests are often lacking.

6.1.3 *New data*

The recent availability of geolocalized, historical data (such as in [46] for example) from world cities [103], has the potential to change our quantitative understanding of urban areas and allows us to revisit with a fresh eye long-standing problems. Many cities created open-data websites [104] and the city of New York (US) played an important role with the release of the PLUTO dataset (short for Property Land Use Tax lot Output), where tax lot records contain very useful information about the urbanization process. For example, in addition to the location, property value, square footage etc, this dataset gives access to the construction date for each building. This type of geolocalized data at a very small spatial scale allows to monitor the urbanization process in time and at a very good spatial resolution.

These datasets allow in particular to produce ‘age maps’ where the construction date of buildings is displayed on a map (see Figure 7 for the example of the Bronx borough in New York City). Many age building maps are now available: Chicago [47], New York City US [48], Ljubljana (Slovenia) [49], Reykjavik (Iceland) [50], etc. In addition to be visually attractive (see for example [105, 106]), these maps together with new mapping tools (such as the urban layers

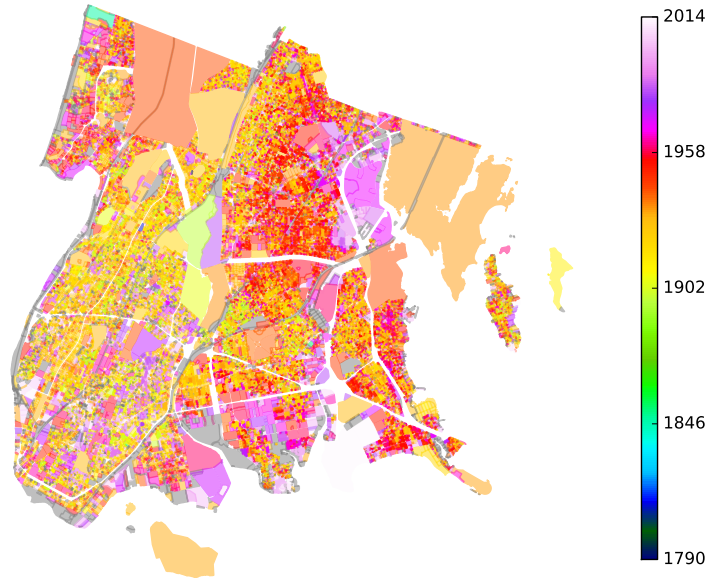


Figure 7: Map of buildings construction date for the case of the Bronx (New York City, US). Most of the buildings were constructed during the beginning of the 20th century, followed by the construction in some localized areas of buildings in the second half of the 20th century. (See section 6.2.1 for details on the dataset).

proposed in [106]) provide qualitative insights into the history of specific buildings and also into the evolution of entire neighborhoods. [107] studied the evolution of the city of Portland (Oregon, US) from 1851 and observed that only 942 buildings are still left from the end of the 19th century, while 75,434 buildings were built at the end of the 20th century and are still standing, followed by a steady decline of new buildings construction since 2005. Inspired by Palmer’s map, [108] constructed a map of building ages in his home town of Ljubljana, Slovenia, and proposed a video showing the growth of this city from 1500 until now ([109]). Plahuta observed that the number of new buildings constructed each year displays huge spikes that signalled important events: an important spike occurred when people were able to rebuild a few years after a major earthquake hit the area in 1899, and other periods of rebuilding occurred after the two world wars. In the case of Los Angeles (USA), the ‘Built:LA project’ shows the ages of almost every building in the city and allows to reveal the city growth over time ([110]).

Thanks to these new datasets, it is possible to monitor urban processes at a very small spatial resolution. In particular, we will focus on a given district or zone, without considering for the moment their position and their role in the whole urban agglomeration they belong to. We will ask quantitative questions about the evolution over time of the population and of the number of buildings, in order to understand if different districts of different cities can be compared with

each other. Surprisingly enough, such a dual information is difficult to find and – up to our knowledge – was not thoroughly studied at the quantitative level (except at a morphological level with fractal studies, [100]).

In this chapter we will show that the number of buildings versus the population follows the same unique pattern for all the cities studied here. Despite the small number of cities analyzed, the strong similarities observed suggest the possibility of a universal behavior that can be tested quantitatively. In order to go further in our understanding of this unique pattern, we then propose a theoretical model and empirical evidences supporting it.

We investigate the urban growth of four different cities: Chicago (US), London (UK), New York (US), and Paris (France). The urbanization process can be described by many different aspects and we will concentrate on two main indicators. First, we consider the evolution of the population of urban areas and second, the evolution of the number of buildings. These aspects concern both an individual-related aspect (the population) and an important physical aspect of cities, the buildings. Analyzing how the number of buildings varies with the population, we will try to understand the relation between these two elements.

In most datasets, we essentially have access to buildings that were built and survived until now. In this respect we do not take into account the destruction, replacement or modifications of buildings. Although replacement or modifications do not alter our discussion, replacement with buildings of another land-use certainly has an impact on the evolution of the population and could potentially lead to a major impact on the evolution of cities. As we will see in our model this can be in a way encoded in the ‘conversion’ process where a residential building is converted into a non-residential one. The important point is to describe the temporal evolution of buildings and their function, and we encode all these aspects in the simpler quantity that is the number of buildings. Further studies are however certainly needed in order to clarify the impact of these points on our results.

Finally, before proceeding to the description of the data and the empirical results, we want to remark that although these cities are among the most urbanized ones, they are characterized by quite different historical paths, with US cities being usually ‘younger’ compared to the European ones. Chicago for example is a young city founded at the beginning of the 19th century, and Paris instead has an history of about two thousands years.

6.2.1 *Data description*

Data will be studied at the spatial scale of the district and we will discuss this choice in the next section. This means that an important limitation that guided us for choosing these cities is the simultaneous availability of building age and historical data for district population. In particular, this second kind of data is difficult to find, mainly due to the change in time of administrative borders. In the following we describe the datasets, the historical range they cover and the way we exploited it.

6.2.1.1 *Chicago data*

We used the Building Footprints dataset (deprecated August 2015) provided by the Data portal of the City of Chicago [111]. For each building we have the information on the built year, the position and the geometrical shape from which we compute the building surface. By using the shapefiles of the 77 Chicago communities [112], we can deduce the community (and thus the side) where the building is located in. For each side we compute the average building surface a_1 by averaging the building lot area over all the buildings with known built year, situated in the side. For Chicago the percentage of buildings with known built year is 54%. Population data from each community area come from [113] and they cover the period from 1930 to 2010.

6.2.1.2 *London data*

We used the dataset ‘Dwelling Age Group Counts (LSOA)’ [114], which contain the residential dwelling ages, grouped into approximately 10-year age bins from pre-1900 to 2015 (the bin 1940 – 1944 is missing). The number of properties is given for each LSOA area and each age bin. From these data we deduce the number of buildings for each London district as function of the year. Data for the historical population of the London boroughs were obtained from ‘A Vision of Britain through time’ [115]. Finally we used OSOpenMapLocal [116] containing the geometrical shape of London buildings to compute the average footprint surface for each district. We note that in this last dataset some buildings are aggregated and rendered as homogenized zones. For this reason we computed the average building surface of each district by averaging over all the buildings belonging to the district having a footprint surface smaller than 700m^2 . In order to locate the district to which a building belongs to, we used the shapefile of London districts boundaries [117].

6.2.1.3 *New York data*

We used data from the Primary Land Use Tax Lot Output (PLUTO) data file, developed by the New York City Department of City Planning’s Information Technology Division (ITD)/Database and Application Development Section [118]. It contains extensive land use and geographic data at the tax lot level. PLUTO data files contain three basic types of data: tax lot characteristics, building characteristics and geographic/political/administrative districts. In particular for each building of the city we are interested in the building’s borough, the building age and the surface of the lot. For each borough we compute the average building surface a_1 (assumed to be given by the average building lot surface) over all the buildings in the borough and with known age (for New York city we have this information for 94% of

buildings). New York data cover the period from 1790 to 2013. For the historical population data, we used different sources [119–122]

6.2.1.4 *Paris data*

We used the dataset ‘Emprise Batié Paris’ provided by the open data initiative of the ‘Atelier Parisien d’urbanisme (APUR)’ [123]. For each building we have the information on the geometrical shape, from which we compute the building surface, the year built and the arrondissement the building is situated in. For each arrondissement we compute the average building surface a_1 averaging this quantity over all the buildings with known built year (i.e. the 57% of the buildings), situated in the arrondissement. Population data comes from [124] and since the actual arrondissements were defined in 1859, population data at the level of the arrondissements covers the period from 1861 to 2011.

6.2.2 *Choice of the areal unit*

An important discussion concerns the choice of the scale at which we study the urbanization process. This is related to the Modifiable Areal Unit Problem (MAUP) introduced by Oppenshaw [125] to highlight that the results of an empirical spatial analysis depend on the space zoning used (district, city, region, country). However, as argued in [22] this is not necessarily an issue, indeed different results at different scales can be considered as a source of knowledge on the studied phenomenon. Moreover, as discussed in the introduction (see section 2.2.1), depending on the question asked there will be a more appropriate spatial scale to choose. Here, we aim to analyse urban changes at a spatial scale that is large enough in order to obtain statistical regularities, but not too large as different zones may evolve differently. Indeed geographers observed that the population density is not homogeneous and decreases in general with the distance to the center ([126, 127]). Also, most cities during their evolution tend to spread out, with the density decreasing in central districts and increasing in the outer ones ([128]); for this reason in the literature the core of the city is often analyzed in relation to its suburbs. In this study we aim to simplify the analysis and we focus on a fixed area without considering its role in the whole urban agglomeration; nevertheless we would like this area to be mostly homogeneous and not mixing zones behaving in different ways.

We choose to focus here on the evolution of administrative districts of each city. At this level, data is available and we can hope to exclude longer term processes. We will show in the following that districts in the different cities considered here, display homogeneous growth. More precisely, we consider the 5 boroughs of New York, the 9 sides of Chicago, the 20 arrondissements of Paris and the 33 Lon-

don districts. Also, in this way we do not have to tackle the difficult problem of city definition and its impact on various measures (see section 6.1.1.1) and focus on the urban changes of a given zone with fixed surface area.

The cities studied here display very different scales, ranging from Paris with 20 districts for 2 – 3 millions inhabitants and an average of 5km² per district, to New York City with 5 boroughs of very diverse area (from 60km² for Manhattan to 183km² and 283km² for Brooklyn and Queens, respectively). The most important assumption that we will use here is that the development in each of these districts is relatively homogeneous. We test this assumption on Chicago, New York City and Paris for which we have the exact localization of new buildings (which we don't have for London). For each district and at each point in time we compute the average distance \bar{d} (normalized by the maximum distance in the district d_{\max}) between new buildings in this district. We also compute the same quantity for a 'null' model for which the new buildings are distributed uniformly. The results are shown (Fig 8) for a selection of districts of the different cities. In this figure, we observe that despite the very diverse sizes of these districts, in all cities studied here, the development of new buildings is consistent with a uniform distribution, within these districts. This is a rather unexpected result as for example in Paris we have relatively homogeneous districts while in New York City, the boroughs are much larger and aggregate together a variety of urban spaces. These results therefore show that despite the variety of cases, this choice of aerial unit provides a reasonable partition of space where the growth is homogeneous. In particular, it implies that a smaller area is not necessarily a good choice for studying the evolution of the number of buildings as it would suffer from strong sampling effects.

6.2.3 Population density growth

In order to provide an historical context, we first measure the evolution of the population density and then analyse the evolution of the number of buildings in a given district as a function of its population. In Fig. 9 we show the average population density for the four cities studied here. This plot reveals that these different cities follow similar dynamics, at least at a coarse-grained level. After a positive growth and a population increase that accelerates around 1900, we observe a density peak. After this peak, the density decreases (even sharply in the case of NYC) or stays roughly constant. This decreasing regime is associated to the post World War years, defined by geographers as the suburbanization/counter-urbanization period. In the last years, New York City, Paris and London display a re-densification period. The possibility of this latter period has been proposed in some cyclic model as the *stages of urban development* one (see section 6.1.1). Nev-

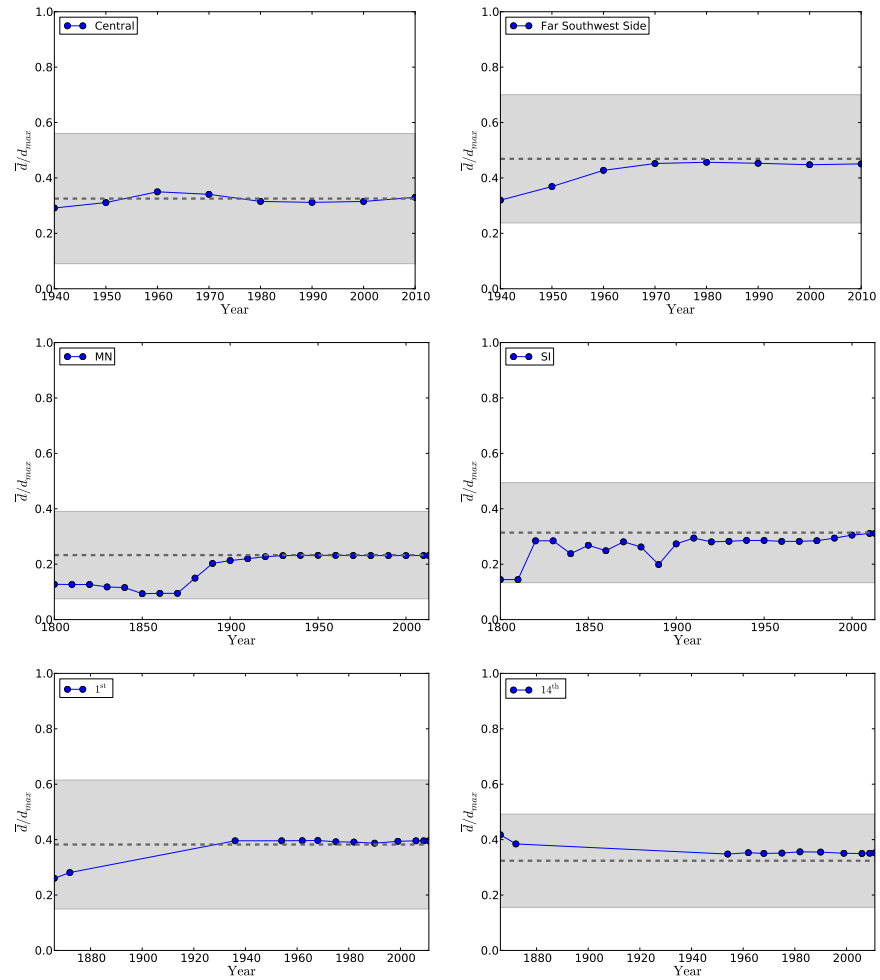


Figure 8: **Homogeneity of growth in districts.** Average distance between buildings at a given time (this distance is normalized by the maximum distance found for each district). Top: Chicago (central and far southwest sides). Middle: New York City (Manhattan and Staten Islands). Bottom: Paris (1st and 14th arrondissements). The dotted line represents the average value computed for a random uniform distribution and the grey zone the dispersion computed with this null model.

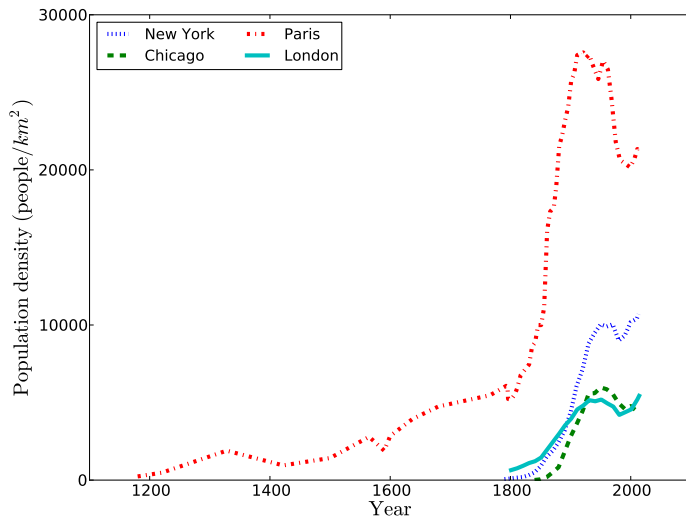


Figure 9: **Population density versus time.** The average population density versus time for the four cities studied in the paper. All these cities display a density peak in the first half of the 20th century.

ertheless, evidences or interpretations about this phase are still an highly discussed topic. At least, this first figure highlights the existence of a seemingly ‘universal’ pattern governing the urban change process, probably driven by technological changes.

However, at the smaller scale of districts, these large cities display different behaviors shown in Fig. 10 where we plot the time evolution of some district densities. In the case of London (Fig. 10, top panels), we note that the district City of London reached a density peak before 1800 while other districts (for example Lewisham, Brent and Newham) display all the different phases of urbanization described above. For Chicago (not plotted for the sake of clarity) and Paris (Fig. 10, bottom panels), the different districts are not all synchronized and display simultaneously different urbanization phases. The central districts of Paris (the 1st and the 4th for example) typically reached their density peak before 1860, while less central districts (11th to 20th) reached their density peak in the first half of the 20th century, consistently with the idea of a centrifugal urbanization process.

For the five boroughs of New York (not plotted for the sake of clarity), we observe that Manhattan (MN), the Bronx (BX) and Brooklyn (BK) already passed through the different phases of urbanization, and are now in a re-densification period. In contrast, Staten Island (SI) and Queens (QN) are still in the urbanization period characterized by a positive population growth rate and didn’t reach yet a density peak.

These preliminary results highlight the importance of spatial delimitations when studying a city. The dynamics of different districts might be the same as also suggested by qualitative models presented in the introduction, but are not necessary simultaneous mainly be-

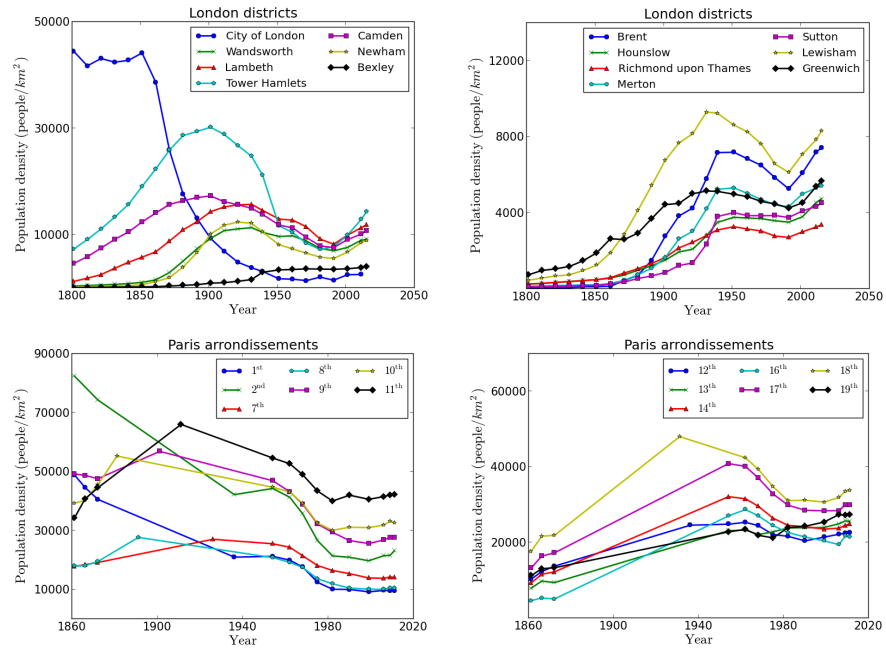


Figure 10: **Population density versus time.** Local population densities for a selection of London districts (top), and a selection of Paris arrondissements (bottom). For the sake of clarity we did not plot all the districts studied.

cause of the difference between districts belonging to the core of the city and districts belonging to the ring, and further is the distance from the core of the city, later the district will reach the second phase. For this reason, we will not consider in the following cities as a whole, but rather follow the evolution of various quantities for each district which display a better level of homogeneity.

We note here that a large number of empirical studies have already been performed where the densification and the disurbanization phase were observed [75, 129–133]. In most of these studies, the analysis was performed focusing on the dependence between the behavior of the core and of the ring districts or on the size of the urban agglomeration.

6.2.4 Number of building vs. population

We now turn to the characterization of the urbanization from the point of view of both the physical aspect via the number of buildings, and the individual aspect described here by the population.

For each district we study the relation between the number of buildings N_b and the population P , and plot N_b versus P (Fig. 11). We thus connect an element of the infrastructure - the building - to the population which allows us to get rid of exogenous effects that govern the time evolution of population for example. This plot encodes these

two basic fundamental aspects of the urbanization process and we refer to this representation as ‘the fundamental diagram’.

In Fig. 11, we observe an apparent diversity of behaviors but, as we will see in the following, they can all be interpreted and compared in the framework of a simple quantitative model. In Fig. 11 (a) we show the results for the nine sides of Chicago, and we observe a clear growth phase followed by a ‘saturation’ (corresponding to the density peak) for the Far North, Northwest, Southwest, Far Southeast and Far Southwest sides (plotted in continuous line). In contrast, the other sides (Central, North, West and South), in dotted line, seem to have reached a saturation before 1930. Indeed, the dotted lines (that have to be read chronologically from the right to the left) do not display the growth regime, suggesting that it stopped before 1930, year of the earliest available data. In Fig. 11 (b) for London districts, we observe that all districts displayed here reached a saturation, but that the district Tower Hamlets (dotted line) reached it before 1900, year of the first available data. In Fig. 11 (c) we plot the five boroughs of New York City. We observe that Staten Island and Queens (dashed lines) are in a growing phase characterized by a positive value of dN_b/dP , while Manhattan, Brooklyn and Bronx (plotted in continuous line) reached other dynamical regimes. In Fig. 11 (d), we represent the evolution for some Paris arrondissements. We observe the growth regime followed by a saturation for the 10th, 12th, 16th and 18th arrondissement (in continuous line), the 13th seem not having reached a saturation yet, while the others have reached saturation before 1861.

These various plots show that for different districts we have essentially the same trajectory in the plane (P, N_b) at different stages of their evolution. In Fig. 11 (e)-(f), we show illustrative examples for various cities that reached the second regime after a saturation point (while other districts are still in the first regime). The evolution of these ‘mature’ districts can thus be represented by a typical path shown in Fig. 12. This path is characterized by a first phase of rapid growth of the number of buildings versus population. In a second regime, the population decreases while the number of buildings stays roughly constant. In a last – and more recent – phase, both the number of buildings and population grow again. The behavior of the urban changes emerging by studying the relation between population and number of buildings in a fixed area is thus analogous to the one described in the *stages of urban development* model of [81], in which a qualitative understanding of the first two phases is widely recognized, while the last one remain widely discussed. We remark that the year at which the second or the third phase begins is not necessary the same for all districts and depends mainly on the role and function of the district in the whole urban agglomeration.

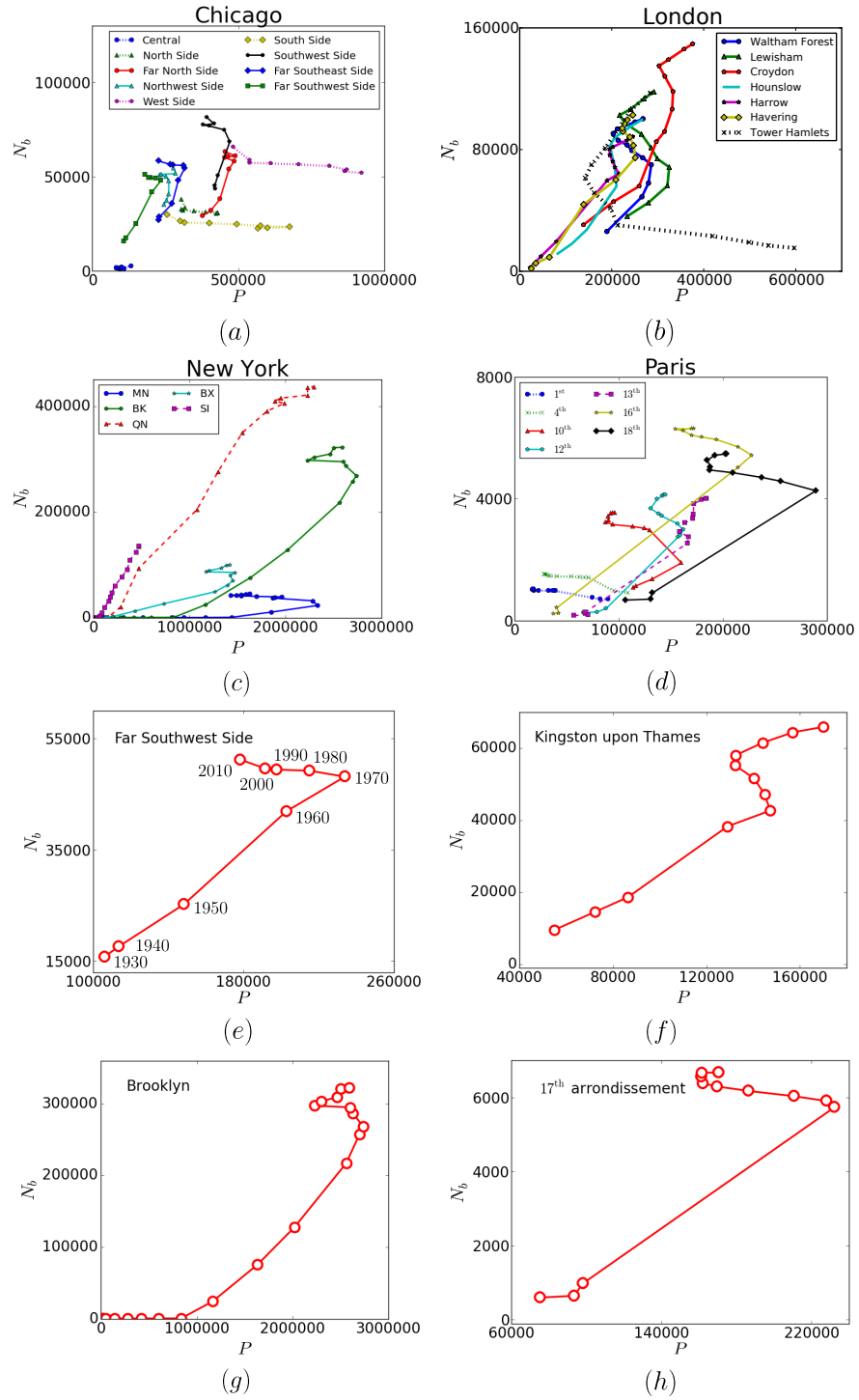


Figure 11: **Number of buildings versus population.** We represent with continuous lines the districts that have reached their density peak, with dashed lines for districts that are still in the growing phase. We use dotted line for the districts that reached the density peak before the first year available in the dataset. (Figures (a)-(d)) Results for districts in the cities studied here. (Figures (e)-(h)) We show examples illustrating the ‘universal’ diagram for districts in different cities that display all the regimes described in the text. In Figure (e) we added to the (P, N_b) points the corresponding year.

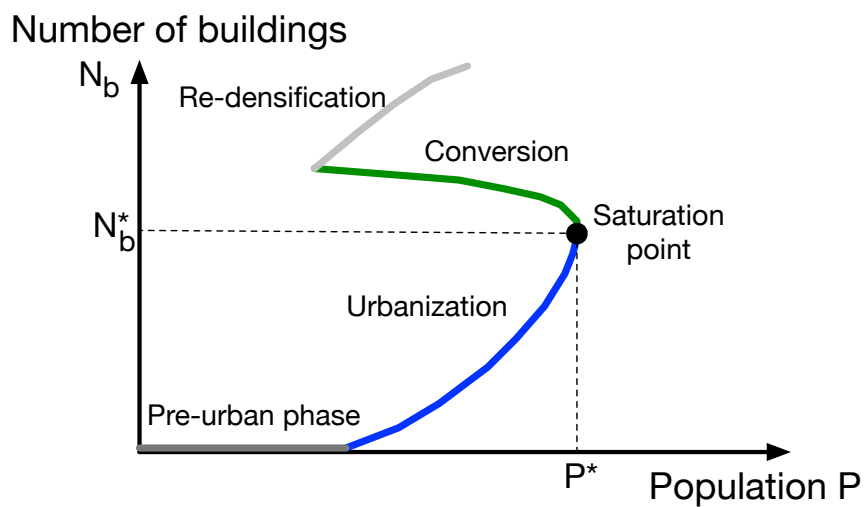


Figure 12: **Schematic representation of the fundamental curve.** We represent here the typical district growth curve characterized by three main phases: after a pre-urbanization period, there is first an urbanization phase with a positive growth rate dN_b/dP that stops at the 'saturation point' (P^*, N_b^*). A second 'conversion' phase follows, during which the population decreases. Finally, we observe a last re-densification phase where both the population and the number of buildings increase.

6.3 THEORETICAL MODELING

The data studied in the previous section display a pattern that seems to encompass specific features of the different cities and we propose a simple theoretical model based on the following interpretation for the different regimes observed in data. The first regime corresponds to the urbanization where buildings are constructed on empty lots until the ‘saturation point’ (P^*, N_b^*), which signals the beginning of the second regime (we note that not all districts reached this saturation point and can still be in the first growing phase). In this second regime, land-use is modified (for example from residential to stores or offices) and the population naturally decreases while the number of buildings stays approximately constant. We emphasize here that this ‘conversion’ is meant as a generic term that describes the process in which a part or the whole of a building changes from a residential use to a non-residential one. In the last regime, both the number of buildings and the population grow again, corresponding to the ‘re-densification’ of cities. This last phase which occurs mostly at the same period for the different cities seems to be triggered by external factors such as governance. This is why we will focus on the first two regimes and to understand the reasons and control parameters of the saturation point. In order to provide quantitative evidences for these first two phases, we propose a simple model based on the simple interpretation described above and – very importantly – that allows us to make predictions that we can test against data.

We model the evolution of a given zone of building land area A by a two-dimensional square grid where each cell of surface a_ℓ represents an empty, constructible lot. The maximum number of lots is then given by $N_{\max} = A/a_\ell$. Each cell can be empty or occupied (a building has already been built) and each building on a lot i is characterized by its number of residential floors $h_r(i)$ and commercial floors $h_c(i)$ (the total number of floors is $h(i) = h_r(i) + h_c(i)$); see Fig. 13 for an illustration.

At each time step $t \rightarrow t + \Delta t$ (in the following we count the time t in units of Δt), we pick at random a cell i and if it is empty we build a residential floor and we update the variables in the following way

$$\begin{cases} P \rightarrow P + \Delta P , \\ N_b \rightarrow N_b + 1 , \\ h(i) = h_r(i) = 1 , \\ h_c(i) = 0 , \end{cases} \quad (2)$$

where N_b is the total number of buildings in the area, ΔP is the number of people per residential floor (we assume here that the number of person per floor does not change too much in time which is certainly true in terms of order of magnitude). If a building is already present

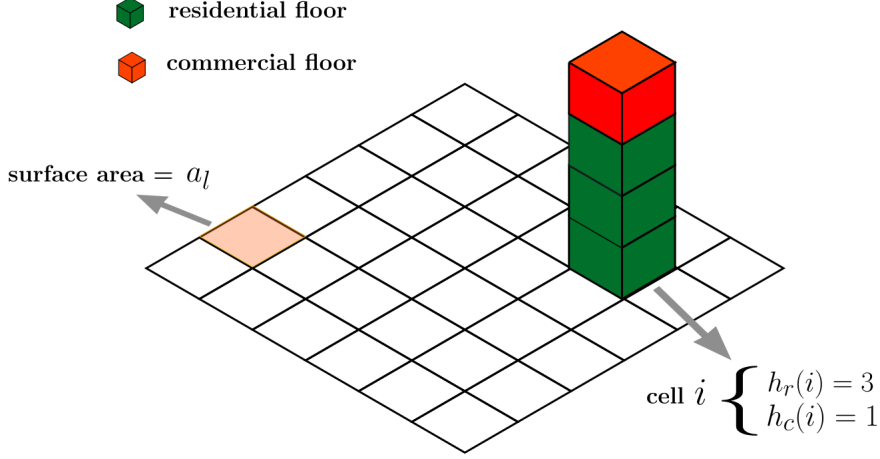


Figure 13: Illustration of the representation of a city district.

on the chosen cell, we add an extra residential floor with probability p_h or convert a residential floor into a non-residential one (such as offices or stores) with probability p_c :

$$\left\{ \begin{array}{l} h_r(i) \rightarrow h_r(i) + 1 \\ h_c(i) \rightarrow h_c(i) \\ P \rightarrow P + \Delta P \end{array} \right. \quad \text{with prob. } p_h \quad (3)$$

$$\left\{ \begin{array}{l} h_r(i) \rightarrow h_r(i) - 1 \\ h_c(i) \rightarrow h_c(i) + 1 \\ P \rightarrow P - \Delta P \end{array} \right. \quad \text{with prob. } p_c \quad (4)$$

Finally, nothing happens with probability $1 - p_h - p_c$.

We remark that we describe the building growing process as an average increase of the number of floors, using a simplifying assumption for a process which in reality is discontinuous and not necessarily homogeneous across different districts.

According to this simple model, each district is characterized by the parameters ΔP , p_c and p_h , that are assumed to be constant over time (even if it is not necessary true in the reality).

The mean-field equations describing the evolution of $H_r = \sum_i h_r(i)$ (the total number of residential floors in the district), the total number of buildings N_b and the total population P in the district are

$$\frac{dH_r}{dt} = \frac{N_b}{N_{max}}(p_h - p_c) + \left(1 - \frac{N_b}{N_{max}}\right), \quad (5)$$

$$\frac{dN_b}{dt} = 1 - \frac{N_b}{N_{max}}, \quad (6)$$

$$\frac{dP}{dt} = \Delta P \frac{dH_r}{dt}. \quad (7)$$

Solving Eq. (6) and Eq. (7) leads to

$$N_b(t) = N_{\max} \left(1 - e^{-t/N_{\max}} \right), \quad (8)$$

$$P(t) = \Delta P[(p_h - p_c)t + N_{\max}(1 + p_c - p_h)(1 - e^{-t/N_{\max}})]. \quad (9)$$

Eqs. (5),(6),(7) imply that the population is an increasing function of the number of building up to a saturation value N_b^* corresponding to the population P^* , after which the population decreases (ie. above which dP/dt becomes negative). Indeed, if we solve:

$$\frac{dP}{dt} \geq 0$$

$$\Delta P \frac{N_b}{N_{\max}} (p_h - p_c) + \Delta P \left(1 - \frac{N_b}{N_{\max}} \right) \geq 0,$$

we obtain the following condition

$$N_b \leq \frac{N_{\max}}{1 + p_c - p_h} \quad (10)$$

which means that the population grows if $N_b < N_b^*$ and decreases otherwise, with

$$N_b^* = \frac{N_{\max}}{1 + p_c - p_h}. \quad (11)$$

We can now compute the time t^* at which the saturation happens. From Eq. (8) we can write

$$N_b^* = N_{\max} \left(1 - e^{-t^*/N_{\max}} \right), \quad (12)$$

from which we obtain

$$t^* = N_{\max} \log \left(\frac{1 + p_c - p_h}{p_c - p_h} \right). \quad (13)$$

Finally, Eq. (9) allows to determine the saturation value of the population P^* , that is given by

$$\frac{P^*}{\Delta P N_{\max}} = (p_h - p_c) \log \left(\frac{1 + p_c - p_h}{p_c - p_h} \right) + 1. \quad (14)$$

We remark that the saturation happens only if $N_b^* < N_{\max}$ and thus if $p_c > p_h$ which expresses the fact that the conversion rate should be large enough in order to observe a saturation point (if the conversion rate is too small, the first phase of growth will continue indefinitely). Defining the normalized variables $N_b^{*'} = N_b^*/N_{\max}$ and $P^{*'} = P^*/N_{\max}$, we can rewrite the above equation as

$$P^{*'} = \Delta P \left[1 + (1/N_b^{*'} - 1) \log(1 - N_b^{*'}) \right]. \quad (15)$$

This relation allows us to determine the average number of people per building floor ΔP for each district. It is important to note that the

theoretical results given by Eq. (8) and Eq. (9) imply a scaling that can be checked empirically. Indeed, if we make the following change of variables

$$\begin{aligned} X(t) &= \frac{N_b(t)}{N_{\max}}, \\ Z(t) &= \frac{\frac{P(t)}{\Delta P N_{\max}} - \frac{N_b(t)}{N_b^*}}{\frac{1}{N_b^{*'}} - 1}, \end{aligned} \quad (16)$$

then the curves for the different districts at different times should all collapse on the same curve given by

$$Z = \log(1 - X), \quad (17)$$

that is independent on the specific parameters characterizing each district. In order to test this model, we focus on all districts that have already reached saturation (the others are still in the first growth phase) and we exclude the re-densification regime that is not reproduced by the model. From the data we know the area A of each district and the average building footprint surface a_l of each district. This allows us to compute the maximum number of buildings $N_{\max} = A/a_l$ of the district. Moreover, the empirical curves allow us to determine the saturation values (P^*, N_b^*) , corresponding to the value of the population and the number of buildings after which the density growth rate becomes negative (and we can then compute $(P^{*'}, N_b^{*'})$ and ΔP through Eq. (15)). At this point, we thus have estimated from empirical data all the parameters that characterize a district, without performing any fit. We can now test the scaling Eq. (17) predicted by the model. As explained above, the curves obtained for different districts should all collapse on the theoretical one. In Fig. 14 we plot the theoretical prediction (red line) and the values of the rescaled variables for the different districts (represented by different symbols and different colors for the different districts). An excellent collapse is observed, supporting the validity of the model.

This collapse is a validation of the model: it shows that the non-trivial relation between variables (Eq. 17) predicted by the model is in agreement with the data.

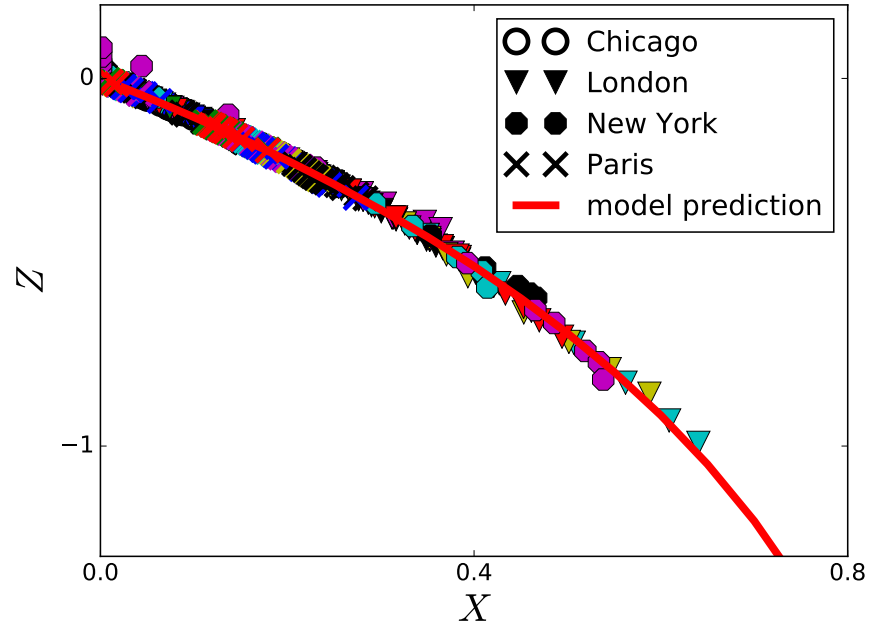


Figure 14: **Collapse for the rescaled variable Z and X .** We plot the rescaled variables Z versus X (Eqs. (16)) for all the 47 saturated districts of all cities excluding the points corresponding to the re-densification period. Each city is characterized by a different symbol and each district by a different color. The continuous red line is the theoretical prediction given by Eq. (17). All the cities considered in this study are present and we kept the districts that have saturated and for which we can compute (P^*, N_b^*) .

6.4 DISCUSSION

As discussed in the introduction (see section 4), theoretical urban models can be roughly divided in two categories. On one hand there are economics models characterized by complex mathematical equations rarely amenable to quantitative predictions that can be tested against data. On the other hand, there are computer simulations (such as agent-based models or cellular automata) that are characterized by a large number of parameters, preventing to understand the hierarchy of processes governing the phenomenon. In the approach used here we propose a simple model with the smallest number of parameters that is able to describe quantitatively the evolution of various macroscopic quantities such as the number of buildings and the population for a given district. The model is based on simplifying assumptions, as for example to suppose the probability p_c and p_h constant over time, which is not necessary true in the reality. Despite this simplifying hypothesis, the model is able to reproduce from a macroscopic point of view the relation between the number of buildings and the population we observed in data. It thus highlights some mechanisms that are important in the process of urbanization.

The agreement with data is tested with a data collapse which does not rely on a parameter fit. However, this agreement is not a definitive proof that the model described here is the fundamental one. Ideally one should compare with other existing models but in this case our proposal seems to be the first attempt to describe quantitatively the evolution of fundamental quantities with the help of simple fundamental mechanisms.

Further quantitative studies are however needed and are of two types. First other datasets for other cities are needed in order to test for the validity of the quantitative behavior observed here and a data analysis on the change in time of the building land-use would be necessary to support the idea of the relation between land-use 'conversion' and population decays. Also, the comparison with other competing theoretical models could be very fruitful.

To summarize, our empirical analysis confirms that there are essentially three different phases of the urbanization process: a growth phase where we observe an increase of both the number of buildings and the population; a second regime where the population decreases while the number of buildings stays roughly constant, and a last phase where both population and the number of buildings are increasing. The first two phases are well described by the simple model proposed here and which integrate the crucial ingredient of converting residential space into commercial activities. We observe empirically the existence of a 're-densification' phase where both population and the number of buildings increase after the conversion phase. This phase seems to happen simultaneously for the different districts in a

city which suggests that it is an effect due to planning decisions and not resulting from self-organization. Modeling the appearance of this regime is thus at this point a challenge for future studies.

Moreover it is important to note another issue of this model. At this stage we are not able to make prediction. Indeed the value of the parameters $p_c - p_h$ and ΔP characterizing the district are obtained thanks to the knowledge of the saturation point, without which is not possible to determine if or when the district will saturate.

This means that it would be interesting to estimate and better understand the values $p_c - p_h$, and ΔP knowing which we could try to predict the saturation point for the districts that did not reach it, yet. Nevertheless, to do it we would need an higher precision in data. In order to discuss this issue, we provide a rough estimation of the error we have on our evaluation of ΔP . From Eq. 15 we can write

$$\Delta P = g(P^*, N_b^*, N_{max}) = \frac{P^*}{N_{max}} f(N_b, N_{max}) \quad (18)$$

ΔP is thus a function of P^* , N_b^* and N_{max} and its error depend on the error of its three variables ΔP^* , ΔN_b^* and ΔN_{max} . One has

$$\Delta(\Delta P) = \left(\left(\frac{\partial g}{\partial P^*} \Delta P^* \right)^2 + \left(\frac{\partial g}{\partial N_b^*} \Delta N_b^* \right)^2 + \left(\frac{\partial g}{\partial N_{max}} \Delta N_{max} \right)^2 \right)^{1/2}. \quad (19)$$

We now discuss the different terms.

ΔP^* is mainly related to the limited number of years for which we know the value of the district population. Different aspects contribute to the error ΔN_b^*

- limited availability of historical data in particular for the first years of the city growth. This brings moreover to underestimate the total number of buildings for the following years.
- there is a percentage of buildings for which we do not know the built year. The value of this percentage depends on the district.

Concerning ΔN_{max} , where N_{max} has been defined as the district area A divided by the average building footprint area a_1 . The contribution to the error come mainly from the error on a_1 and the fact that we did not suppose that at most a given fraction of the district area can be built.

Quantifying these errors is not straightforward. In order to have an idea about $\Delta(\Delta P)$ we estimate it for the borough of Manhattan. To simplify we suppose that we can neglect the errors on P^* . The error on N_{max} has been estimated as if at most 60% of the district area can be built and moreover we suppose that there is a total of 40% of buildings that are not taken into account bringing to an underestimation of N_b .

Using Eq. 19 we obtain $\Delta(\Delta P) \approx 144$, with an estimate of $\Delta P = 184.84$. (For a full list of the estimate values of ΔP see [134].) This rough estimate shows how these microscopic quantities are highly sensitive to the noise, and given the precision of our data, they just give an indication about the average growth of the different districts but it is difficult to go beyond this simple interpretation.

THE DISPERSAL MODEL

7.1 INTRODUCTION

7.1.1 *Urban sprawl*

Cities are getting bigger and one of the consequences of this growth is the expansion of their borders through suburban sprawl. This old phenomenon is the result of a balance between two forces that changed their relative weight over time: a centripetal one related to the agglomeration economies that pushes people together in cities and make cities grow around their peripheries and a centrifugal force related to crowding and to the desire of larger living space that brings new activities and people to locate far from the city center while remaining connected to it [82, 135]. Prior to about 1850, people traveled mainly by foot and cities needed to keep an highly centralized shape around waterways and railroads. Moving further from the city center became possible thanks to the advent of electric streetcars and trolleys that allowed commuters to locate in proximity of train stations. This decentralization pattern was successively strengthened by the construction of subways. Later with cars, individuals began to look for larger living space in the low rent areas between transportation lines that became accessible.

The weight of the 'centrifugal force' became larger with the improvement in transportation technologies and in the last decades communication technologies played an important role as well. Faster transportations and cheaper communications brought, especially in US, to a decentralization process that is taking a more polycentric form due to older villages that are incorporated in a larger city or little villages at transportation stations that are getting bigger and bigger [82, 136]. Even if the centrifugal force acquired importance in modern cities, we can have any balance between the two opposing forces depending upon planning policy, cultural and economic constraints and population growth. Moreover, the de-centralization process shapes the city differently if it is related to car or to public transportation. All this gives rise to a large variety of possible scenarios. "Urban sprawl is one name for many conditions" [137] and it can be characterised by many different properties, a deeper discussion about this and a classification of the sprawl processes can be found in [135, 137].

Concerning empirical data, a big effort has been done by the NYU Urban Expansion Program at the Marron Institute of Urban Management and the Stern School of Business of New York University, in

partnership with UN-Habitat and the Lincoln Institute of Land Policy. Their challenging goal is to monitor the quantitative and qualitative aspects of urban expansion, [138]. In particular, thanks to historical maps and contemporary satellite imagery they identified the urban extents of 30 cities from 1800 and 2014. Although we have empirical evidences of the sprawl process and a wide qualitative understanding of it, we are still not able to assess quantitatively the impact of the different factors influencing the process, and this represents with no doubt a difficult task. In the next section we focus on the study of a dispersal model that has been used extensively in the investigation of dispersions of animals in theoretical ecology [139–141], and also as a simplified model for the growth of cancerous tumours [142, 143]. This model represents a good candidate for describing the growth of an urban area based on a double process, the growth of surface area and the absorption of neighboring towns (See Fig. 15). We are interested in understanding and exploring the different behaviors the model can produce and this can help us to get a better insight on the variety of behaviors observed in data, that can be reinterpreted and analyze in light of the new acquired knowledge.

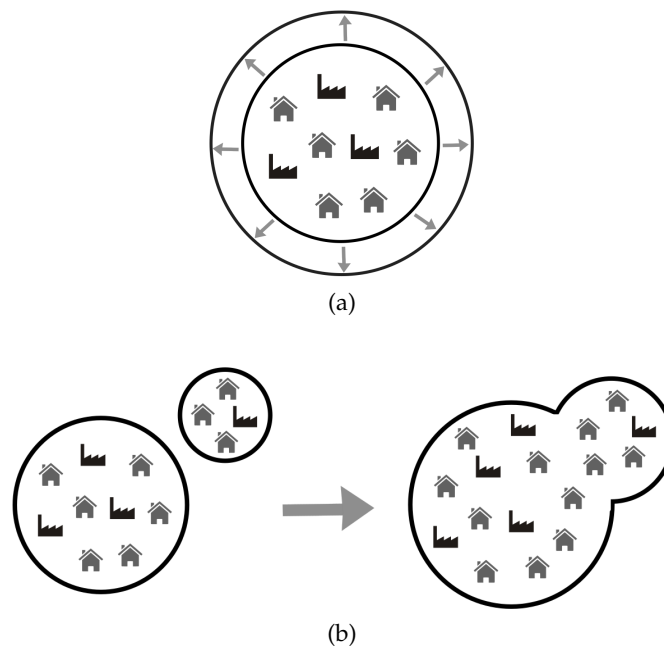


Figure 15: Illustration of the two processes described by the model.
(a) Growth of surface area. (b) Absorption of neighboring towns.

7.1.2 Dispersion models

Dispersion can occur in two different ways. It can follow privileged directions under the effect of a force secondary to wind, rivers, gravity or currents, if we focus on organisms dispersion. Concerning urban

sprawl it can be related to the transportation network, and in tumors privileged directions are given by blood vessels.

However, at least in a first phase, we neglect this possible directional bias and we focus on density-dependent dispersal, that is the one related to the population size, to city size or to tumor size. This kind of dispersal is isotropic.

A classical way to study dispersal is through the *dispersal kernel* representing the probability distribution of dispersal distances. Different models for dispersal kernels are widely discussed in [141]. A different approach has been introduced by Kawasaki and Shigesada in [139, 144] who proposed the use of simple models to tackle this challenging problem. In particular we are interested here in discussing the *coalescing colony model* to which we will refer in the following as the M_0 model. The model considers a primary colony that grows at radial velocity c (this represents the short-range dispersal) and has a probability per unit time $\lambda(r)$ to emit a secondary colony at a distance ℓ from its border (long-range dispersal). The variable ℓ is in general chosen from a probability distribution $P(\ell)$. Each secondary colony grows with the same radial speed c and does not emit other colonies. To express the possibility that the emission rate depends on the colony size, $\lambda(r)$ is defined as

$$\lambda(r) = \lambda_0 r^\theta, \quad (20)$$

with $\theta \geq 0$. We begin here with the investigation of the simpler model in which a secondary colony is emitted at a constant distance ℓ_0 from the boundary of the primary one. Hence, we can write

$$P(\ell) = \delta(\ell - \ell_0). \quad (21)$$

Coalescence happens when a secondary colony of radius r_2 intersects with the primary one, becoming part of this latter. When a secondary colony is absorbed, the primary colony keeps a circular shape (we will refer to it in the following as "circular approximation") and its new radius r' satisfies the equation

$$r'^2 = r^2 + r_2^2. \quad (22)$$

The Shigesada-Kawasaki model represents an interesting and alternative approach to tackle the dispersion problem based on simplifications. Nevertheless, a full quantitative understanding of the predictions of this model for the behavior of the radius $r(t)$ of the primary colony with time, is still lacking, and the discussion about the validity of the circular approximation for the shape of the primary colony is mainly qualitative. We propose in this chapter to discuss these problems and we present our preliminary results as well as the points where further work is needed.

In a first section we discuss a microscopic derivation for the Kawasaki-Shigesada equations in the mean-field approximation, and we study

its solutions depending on the parameter θ . In a second section we test our quantitative predictions through numerical simulations. Finally in a last section, we discuss a modified version of the model, referred to as " M_1 model". In it, when coalescence happens the secondary colony becomes part of the primary one changing the shape of this latter that will not be a circle anymore. See Fig. 16 for an illustration of the coalescence in the two models.

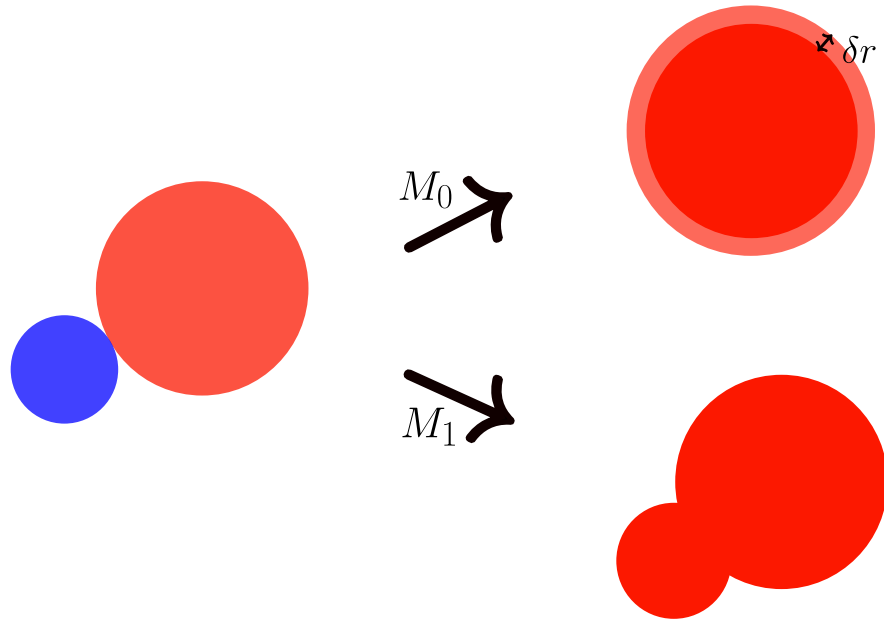


Figure 16: **Coalescence:** an example of coalescence in the model M_0 (top) M_1 (bottom).

7.2 NEGLECTING GEOMETRY (M_0)

7.2.1 Recovering Shigesada-Kawasaki equations

We focus here on the simpler case where the secondary colonies are emitted at fixed distance ℓ_0 from the boundary of the primary colony. The emission rate $\lambda(r)$ can depend instead on the radius of the primary colony. One has $\lambda(r) = \lambda_0 r^\theta$, with $\theta \geq 0$.

We define t_i' as the time of coalescence of the colonies emitted at time t_i and $\lambda(t_i)$ is the probability to emit a colony in the interval $[t_i, t_i + dt_i]$. This means that the following relation has to be verified

$$r(t_i') + ct_i' = \ell_0 + r(t_i) + ct_i . \quad (23)$$

We define the function ϕ as

$$\phi(\tau) : \tau \rightarrow r(\tau) + c\tau \quad (24)$$

and Eq. (23) can be written as

$$\phi(t_i') = \ell_0 + \phi(t_i) \quad (25)$$

which implies

$$t_i' = \phi^{-1}(\ell_0 + \phi(t_i)) = f(t_i) . \quad (26)$$

Being interested in understanding the possible behaviors of the evolution of the area of the primary colony with time, we write the following equation

$$\frac{dA}{dt} = 2\pi rc + \int dt_i \lambda(t_i) \delta(t - f(t_i)) \pi c^2 (t - t_i)^2 , \quad (27)$$

and knowing that

$$\delta(t - f(t_i)) = \delta(t_i - f^{-1}(t)) \left| [f^{-1}]'(t) \right| , \quad (28)$$

after few calculations, one obtains

$$\frac{dA}{dt} = 2\pi rc + \lambda(f^{-1}(t)) \left| [f^{-1}(t)]' \right| \pi c^2 (t - f^{-1}(t))^2 , \quad (29)$$

where in the right-hand side the first contribution is due to the short-range dispersion and the second one to the long-range dispersion.

We define $x(t)$ as the radius of the secondary colony absorbed at time t

$$x(t) = c(t - f^{-1}(t)) . \quad (30)$$

Replacing this latter expression in Eq. (29) one gets the Kawasaki-Shigesada equation

$$\frac{dA}{dt} = 2\pi rc + \lambda(t - x(t)/c) \left(1 - \frac{\dot{x}(t)}{c} \right) \pi x(t)^2 . \quad (31)$$

With

$$r(t) - r(t - x(t)/c) + x(t) = \ell_0 . \quad (32)$$

For the M_0 model we use a circular approximation, assuming that the primary colony can be described by a circular shape. Hence, $A = \pi r^2$ and writing the area as function of its radius we recover the Shigesawa-Kawasaki equations for the radius $r(t)$ ([139]):

$$\begin{cases} \frac{dr}{dt} = c + \frac{\lambda_0 r \left(t - \frac{x(t)}{c}\right)^\theta}{2\pi r(t)} \left(1 - \frac{x(t)}{c}\right) \pi x(t)^2 , \\ \ell_0 = r(t) - r\left(t - \frac{x(t)}{c}\right) + x(t) . \end{cases} \quad (33)$$

$$\ell_0 = r(t) - r\left(t - \frac{x(t)}{c}\right) + x(t) . \quad (34)$$

7.2.1.1 Constant emission rate: $\theta = 0$

We first solve Eq. (31) in the simpler case $\theta = 0$.

We suppose that in a first approximation $t \gg x(t)/c$ and we can write Eq. 33 as

$$\frac{dr}{dt} = c + \frac{\lambda_0 r^{\theta-1}}{2} x^{*2} , \quad (35)$$

with x^* the average radius of a secondary colony just before its coalescence, supposed constant in time. Solving the equation we get

$$r(t) \sim a + ct + \frac{\lambda_0 x^{*2}}{2c} \log\left(\frac{2cr}{\lambda_0 x^{*2}} + 1\right) \quad (36)$$

and at the first order one can write

$$r(t) \sim a + ct + \frac{\lambda_0 x^{*2}}{2c} \log\left(\frac{2c^2 t}{\lambda_0 x^{*2}} + 1\right) . \quad (37)$$

Using now Eq. (34) one has at the first order $x^* \simeq \ell_0/2$ and thus the previous equation becomes

$$r(t) \sim a + ct + \frac{\mathcal{C}}{c} \log\left(\frac{c^2 t}{\mathcal{C}} + 1\right) , \quad (38)$$

where we defined $\mathcal{C} = \frac{\lambda \ell_0^2}{8}$.

7.2.1.2 Size-dependent emission rate: $\theta \geq 1$

To get quantitative predictions of the solution of Eq. (31) with a size-dependent emission rate, we suppose the following behaviors for the average radius of the primary colony $r(t)$ and of the secondary colony at the moment of its coalescence $x(t)$:

$$r(t) \sim t^\beta , \quad x(t) \sim t^{-\alpha} . \quad (39)$$

From the system of Eqs. (33) and (34) at the dominant order one can write

$$Ar(t)^{\theta-1} \simeq r(t)^2 (r(t) - c) \quad (40)$$

with $A = \frac{\lambda c^2 \ell_0^2}{2}$. After some calculations one gets the following values for the two exponents.

$$\beta = \frac{3}{4-\theta}, \quad \alpha = \beta - 1. \quad (41)$$

We remark that for $\theta \rightarrow 4$, $\beta \rightarrow \infty$ and the radius growth is no more described by a power law but by an exponential. For $\theta = 1$ instead, we obtain $\alpha = 0$. This means that we can consider x^* independent of t and the behavior of the radius of the primary colony is linear. From Eq. (33) one can easily deduce that the radial velocity c' can be written as

$$c' = c + \frac{\lambda_0}{2} x^{*2} \quad (42)$$

and the value of x^* can be obtained solving Eq. (34) that can be written as

$$\frac{\lambda_0}{2c} x^{*3} + 2x^* - \ell_0 = 0. \quad (43)$$

This result for the specific case of $\theta = 1$ was already obtained by Kawasaki and Shigesada [139].

Let us now go a little bit further and discuss the range of validity of the power-law behavior $r(t) \sim t^\beta$, exploring the second order behavior. We write

$$r(t) \sim at^\beta + bt^{\beta'}. \quad (44)$$

After some calculations, a development at the first and second order of Eq. (40) bring to the following results

$$\beta' = 1, \quad (45)$$

$$a = \left(\frac{2\beta^3}{\ell_0^2 \lambda_0 c^2} \right)^{\frac{1}{\theta-4}}, \quad (46)$$

and

$$b = \frac{4-\theta}{15-6\theta}. \quad (47)$$

Being all the parameters determined we can deduce the value of the time $t_c = (b/a)^{1/(\beta-1)}$ starting from which the second order term begin to be smaller than the first order one. Hence, for $t \gg t_c$ we can write $r(t) \sim at^\beta$, neglecting the second order term. After some calculation one gets

$$t_c = f(\theta) \left(\frac{2c^{2-\theta}}{\lambda_0 \ell_0^2} \right)^{\frac{1}{\theta-1}} \quad (48)$$

with

$$f(\theta) = \left(\frac{4 - \theta}{15 - 6\theta} \right)^{\frac{4-\theta}{\theta-1}} \left(\frac{27}{(4 - \theta)^3} \right)^{\frac{1}{\theta-1}} . \quad (49)$$

We remark moreover that the Kawasaki-Shigesada equations are valid only if the coalescence of a colony does not cause the coalescence of another secondary colony. This means that the increasing in the radius at time t , $\delta r(t)$ has to be smaller than distance between two successively emitted secondary colonies. This means that the following relation has to be verified

$$\sqrt{r^2 + x^{*2}} - r < \frac{2c}{\lambda_0 r^\theta} . \quad (50)$$

From Eq. (34) one can write

$$x(t) \simeq \frac{c\ell_0}{r(t)} \quad (51)$$

and thus

$$x(t) \sim dt^{-\alpha} \quad (52)$$

with $\alpha = \beta - 1$ and $d = c\ell_0/(\beta a)$. After some calculations one can show that the Kawasaki-Shigesada equation is valid only for $t < t_{\max}$, with

$$t_{\max} = g(\theta) \left(\frac{2c^{2-\theta}}{\lambda_0 \ell_0^2} \right)^{\frac{1}{\theta-1}} \quad (53)$$

and

$$g(\theta) = 2^{\frac{4-\theta}{\theta-1}} \beta . \quad (54)$$

To summarize, we will be able to observe the power-law behavior given by $r(t) \sim t^\beta$ in the range of time for which $t_c \ll t < t_{\max}$. The size of the range of validity depends thus on the ratio between $g(\theta)$ and $f(\theta)$. This ratio decreases when θ increases as shown in Fig. 17.

AVALANCHE EFFECT We have just discussed that it exists a time t_{\max} over which the Kawasaki-Shigesada equation is not valid anymore. Indeed, for $t > t_{\max}$ avalanche effects arise. This is due to the high emission rate and means that not only we can have multiple coalescences, (that is more colonies absorbed in a single time step), but the increase in the radius produced by these coalescences can bring to other coalescences before moving to the next time step. Every time this happens we say that we observe an avalanche. In this situation the Kawasaki-Shigesada equations do not held and another treatment of the problem is necessary. This goes beyond the aim of this chapter, however we performed numerical simulations to highlight this phenomenon, with the choice of $\theta = 1.4$. At each time step $dt = 0.001$ we count the number of avalanches n_a and the number of total coalescences n_c happened during dt as consequence of the different

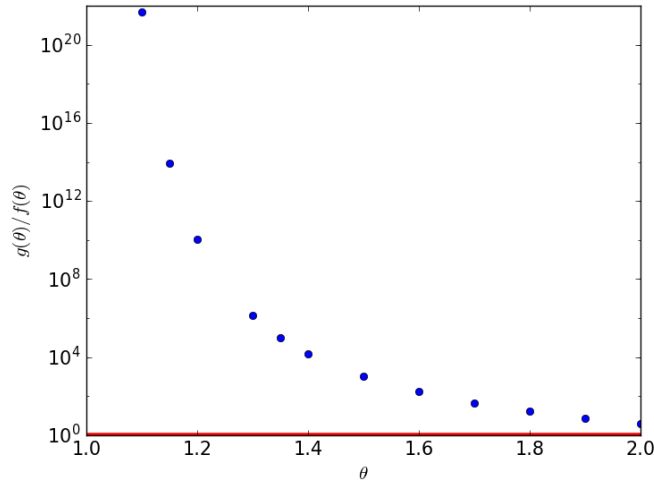


Figure 17: $g(\theta)/f(\theta)$ versus θ .

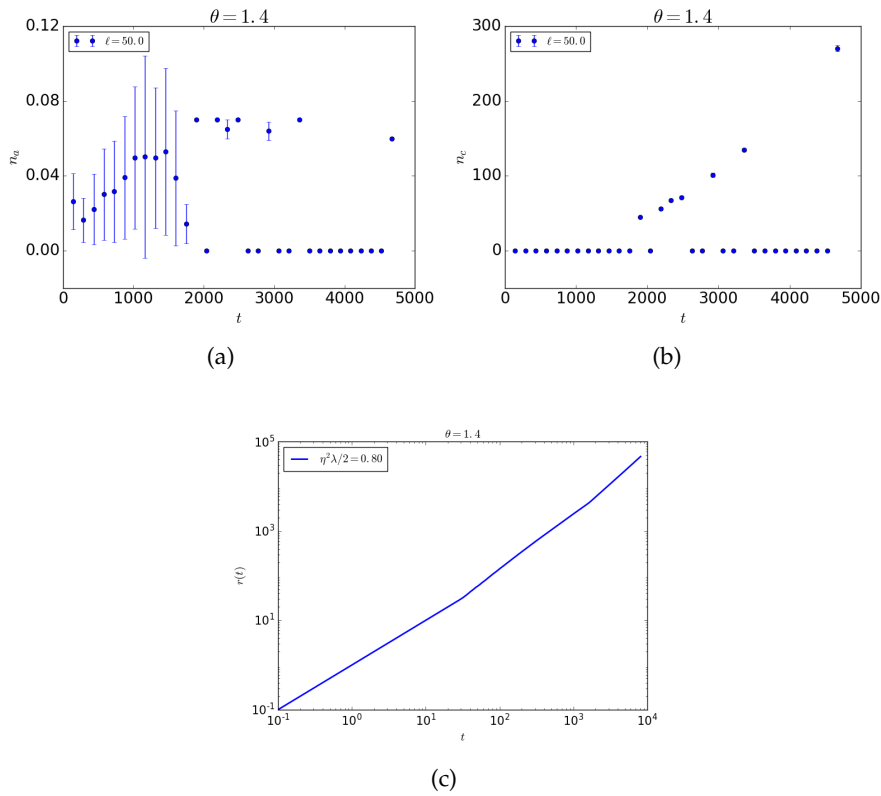


Figure 18: (a) n_a VS t (b) n_c VS t . (c) $r(t)$ VS t . The results are obtained averaging over 100 simulations. For $\theta = 1.4$, $\ell = 50$, $c = 1$, $\lambda = 0.001$.

avalanches.

The plots are shown in Fig. 18(a-b) where we observe that at a given time around $t \approx 2000$, the avalanche phenomenon change behavior

acquiring more relevance, and bringing to a change in the slope characterizing the behavior of $r(t)$ with time (see Fig. 18-c).

7.2.2 Simulation results M_0 model

7.2.2.1 Constant emission rate $\theta = 0$

We begin by considering the M_0 model with a constant emission rate $\lambda(r) = \lambda_0$. We obtained in the previous sections the following prediction for the behavior in time of the radius $r(t)$ of the primary colony:

$$r(t) \sim a + ct + \frac{\mathcal{C}}{c} \log \left(\frac{c^2 t}{\mathcal{C}} + 1 \right), \quad (55)$$

where we defined $\mathcal{C} = \frac{\lambda l_0^2}{8}$.

For the following analysis we introduce another important quantity $\eta = \frac{2c}{l_0 \lambda_0}$, which represents the ratio between the emission time $\tau_e = 1/\lambda_0$ and the coalescence time $\tau_c = l_0/(2c)$.

To test now the validity of the analytical prediction, we perform numerical simulations at constant $\lambda_0 = 0.5$ and $c = 1$, for different values of the emission distance l_0 .

In Fig. 19-left we plot the radius of the primary colony $r(t)$ versus t . We then perform on these data a two parameters fit with a function of the form

$$f(t) = a + ct + \frac{\mathcal{C}_{\text{simul}}}{c} \log \left(\frac{c^2 t}{\mathcal{C}_{\text{simul}}} + 1 \right), \quad (56)$$

where the parameters to fit are a and $\mathcal{C}_{\text{simul}}$. In Fig. 19-right we test

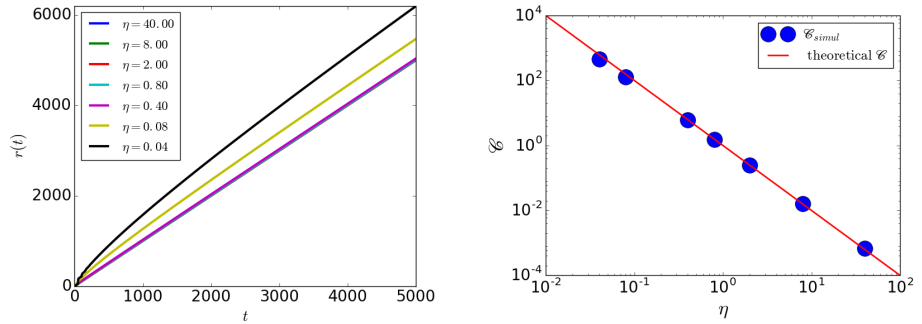


Figure 19: $\theta = 0$ (Left) We plot $r(t)$ versus t for different values of the parameter η , obtained averaging over 100 simulations. (Right) We plot $\mathcal{C}_{\text{simul}}$ versus η as obtained performing the empirical fit. In red is shown the theoretical prediction.

the results of the fit, comparing the estimated value $\mathcal{C}_{\text{simul}}$ obtained, with its theoretical value $\mathcal{C} = \frac{\lambda l_0^2}{8} = \frac{c^2}{2\lambda_0 \eta^2}$. We observe an excellent agreement, showing the validity of our theoretical calculations.

7.2.2.2 Emission rate proportional to the colony radius: $\theta = 1$

We now consider the model M_0 with an emission rate proportional to the radius of the primary colony. One has $\lambda(r) = \lambda_0 r$.

In this case, after some calculations (see section 7.2.1 or [139]) one can show that the radial expansion of the colony is linear, with a velocity $c' \neq c$.

In order to test the analytical predictions, we perform numerical simulations at constant $\lambda_0 = 0.3$ and $c = 1$, for different values of the emission distance ℓ_0 . In Fig. 20-left we plot the radius of the primary colony $r(t)$ versus t . Then, we perform a linear fit on the simulations data which allows us to obtain an estimation for the radial velocity c' that we compare in Fig. 20-right with the theoretical prediction got by solving Eq. (43). Here also, an excellent agreement is observed.

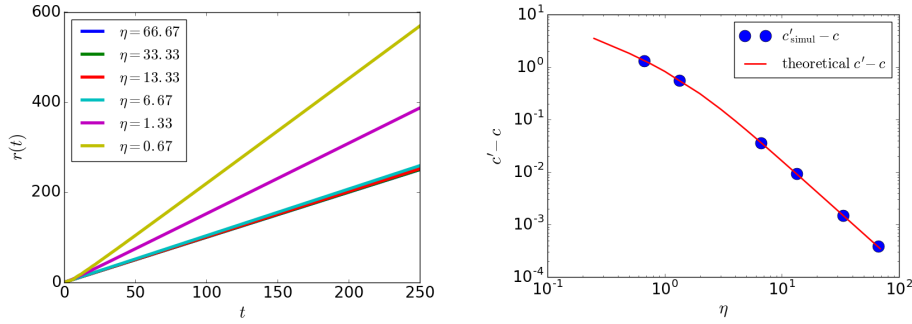


Figure 20: ($\theta = 1$) (Left) We plot radius versus time for different values of the parameter η , averaged over 100 simulations. (Right) We plot the quantity $c' - c$ versus η as obtained through the empirical fit. In red is shown the theoretical prediction.

7.2.2.3 Size-dependent emission rate with $\theta > 1$

We now consider the M_0 model characterized by an emission rate $\lambda(r) = \lambda_0 r^\theta$ with $\theta > 1$. The theory predicts a regime characterized by a power-law behavior

$$r(t) \sim t^\beta \quad \text{with} \quad \beta = \frac{3}{4 - \theta}. \quad (57)$$

Nevertheless, to observe this regime in the time range covered by our simulation results, the value of the combination of parameters $\eta^2 \lambda_0 / 2$ must be accurately chosen. Indeed, to observe this power law behavior we need $t_c \ll t < t_{\max}$ (see the discussion in section 7.2.1) and reaching this time range with numerical simulations is not always an easy task. We performed numerical simulations for $\theta = [1.1, 1.2, 1.3, 1.4, 1.6, 2.0]$, with the parameter $c = 1$ and $\lambda = 0.001$. Additional simulations with the parameter $\lambda = 0.005$ has been carried out for $\theta = 1.2$ to better confirm the role played by the parameter

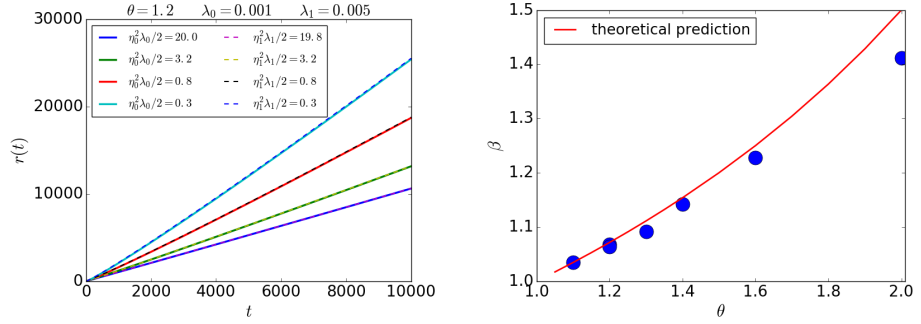


Figure 21: (Left **radius versus time**. $\theta = 1.2$. We plot the radius versus time averaged over 100 simulations, for different values of the parameter $\eta^2\lambda/2$ and different value of λ . (Right) **β versus θ** We plot the value of the exponent β versus θ , obtained from a power law fit on $r(t)$ versus t for the values of $\eta^2\lambda/2$ maximizing the range $[t_c, t_{\max}]$ in our simulation domain.

$\eta^2\lambda/2$ as we expect from the analysis conducting in section 7.2.1. In Table 1 we report, for each choice of θ the value of the parameter $\eta^2\lambda/2$ and the corresponding t_c and t_{\max} .

In Fig. 21-left, we plot the simulation results of the radius of the primary colony $r(t)$ versus t for $\theta = 1.2$, different values of ℓ and two values of λ_0 . The figure confirm that $\eta^2\lambda/2$ is the main parameter characterizing the evolution of the colony size in time, as expected.

In Fig. 21-right we plot the values of the exponents β obtained by power law fits and we compare it with the theoretical prediction in red. We observe a good agreement with some deviations for higher values of θ . We remark indeed that even if we performed the fit for the choice of $\eta^2\lambda/2$ for which we should observe a regime characterized by the behavior $r(t) \sim t^\beta$, this can be a quite small range in the time domain of our simulation (and it becomes smaller as θ increases). Moreover for all values of θ we performed a fit with $t > 10t_c$, while for $\theta = 2$ in order to be in the simulation range we had to choose $t > 5t_c$, this is another reason that could explain the larger deviation from the theoretical result.

	$\theta = 1.1$	$\theta = 1.2$	$\theta = 1.3$	$\theta = 1.4$	$\theta = 1.6$	$\theta = 2.0$
$\eta^2\lambda_0/2$	0.5	0.8	2.2	2.2	5	20
t_c	3×10^{-16}	5×10^{-7}	6×10^{-3}	5×10^{-2}	1.74	30
t_{\max}	1.5×10^6	5.7×10^3	8.1×10^3	7.7×10^2	2.9×10^2	1.2×10^2

Table 1: In the table we report for the different θ , the values of the variable $\eta^2\lambda_0/2$ for which we performed the fit, and the corresponding values of t_c , t_{\max} .

7.3 NON CIRCULAR GROWTH (M_1)

The Shigesada-Kawasaki model is based on the circular approximation that considers the shape of the primary colony as circular. We investigate here the validity of this hypothesis through an analysis of the simulation results obtained for the M_1 model, for which we do not have analytical predictions. The results and the analysis are at a preliminary stage, further and more accurate investigations are needed.

7.3.1 Constant emission rate $\theta = 0$

We present here the simulation results obtained for the M_1 model with a constant emission rate $\lambda(r) = \lambda_0$. In Fig. 22 we plot in a linear

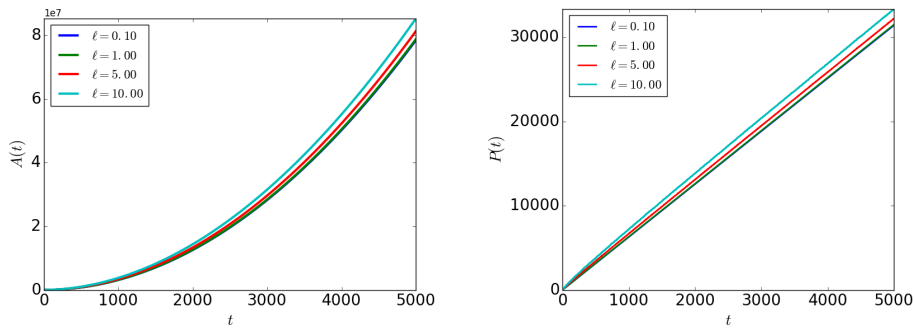


Figure 22: (Left) $A(t)$ VS t . (Right) $P(t)$ VS t . For different values of ℓ we plot the area and the perimeter of the primary colony versus time averaged over 10 simulations.

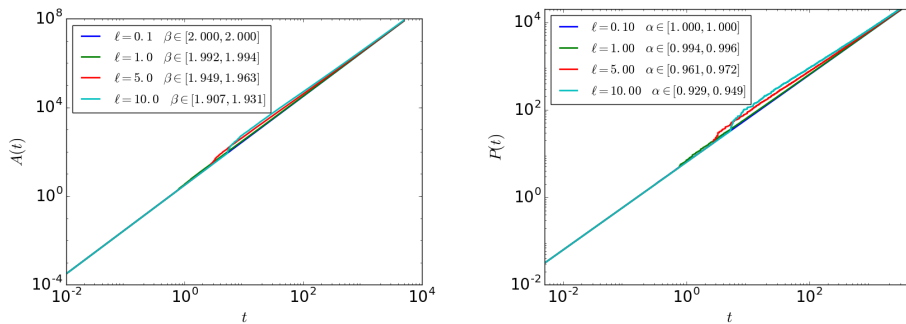


Figure 23: (Left) $A(t)$ VS t . (Right) $P(t)$ VS t . For different values of ℓ we plot in a log-log scale, the area and the perimeter of the primary colony versus time averaged over 10 simulations. We perform a power-law fit and the values of the exponents obtained are shown in the insets.

scale the area $A(t)$ and the perimeter $P(t)$ of the primary colony ver-

sus time, for different values of the emission distance ℓ . To get a first insight in the behavior of these quantities we suppose the following power-law behaviors

$$A(t) \sim t^\beta \quad P(t) \sim t^\alpha . \quad (58)$$

In Fig. 23 we perform a power-law fit on the empirical results for two different time regimes. This allows us to examine eventual finite-size effects: we choose $t > t_{fc}$ and then $t > 100t_{fc}$, with t_{fc} being the time at which the first coalescence happens. The values of the exponents α and β are shown in the insets of Fig. 23, with the higher value corresponding to the choice $t > 100t_{fc}$. This suggests that we can roughly write

$$\beta \approx 2 \quad \alpha \approx 1 . \quad (59)$$

In order to understand better these behaviors, we can ask if the dominant term describing the time evolution of the area and of the perimeter of the primary colony changes with respect to the M_0 model. To answer this question we plot in Fig. 24 the quantities $A(t)/(\pi c^2 t^2) - 1$ versus t (on the left) and $P(t)/(2\pi c t) - 1$ versus t (on the right). The plots show that these quantities go both to zero for large values of t , suggesting that at the dominant order the M_1 model behaves as the M_0 one, with

$$A(t) \sim \pi c^2 t^2 \quad P(t) \sim 2\pi c t . \quad (60)$$

Hence, for $\theta = 0$ and large value of t , the circular approximation seems to be appropriate. Then, to go a little bit further and deduce the

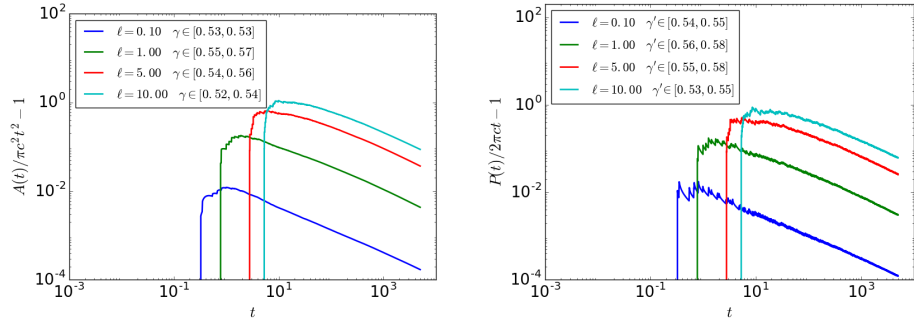


Figure 24: (Left) $A(t)/(\pi c^2 t^2) - 1$ VS t . (Right) $P(t)/(2\pi c t) - 1$ VS t . The results are obtained for different values of ℓ and are averaged over 100 numerical simulations.

first order correction to Eqs. (60) we assume the following behaviors

$$\frac{A(t)}{\pi c^2 t^2} - 1 \sim t^{-\gamma} \quad (61)$$

and

$$\frac{P(t)}{2\pi c t} - 1 \sim t^{-\gamma'} \quad (62)$$

and we perform a power-law fit on data for $t > t^*$ and $t > 10t^*$. The value of t^* and of the corresponding exponents are summarized in Table 2. The numerical results suggest

$$\gamma \sim 0.5, \quad \gamma' \sim 0.5. \quad (63)$$

This represent a preliminary result, further and more rigorous analysis of this behavior are necessary. In particular, we note that in the range $t > 10t^*$ the value of the exponents increases, going in the 'wrong' direction with respect to the value of $1/2$. Even if further investigations are needed, the second order term in the M_1 model seems to follow a power-law behavior, different from the logarithmic correction observed in the M_0 model.

	$\ell = 0.10$	$\ell = 1.0$	$\ell = 5.0$	$\ell = 10.0$
γ	0.53 – 0.53	0.55 – 0.57	0.54 – 0.56	0.52 – 0.54
γ'	0.54 – 0.55	0.56 – 0.58	0.55 – 0.58	0.53 – 0.55
t^*	10	30	70	100

Table 2: In the table we report for the different choice of ℓ , the values of t^* and of the exponents γ and γ' . For each value of ℓ the smaller value of the exponent correspond to the fit for $t > t^*$ and the larger value to the fit range $t > 10t^*$.

7.3.2 Emission rate proportional to the colony radius: $\theta = 1$

We now focus on the simulation results obtained for the M_1 model characterized by an emission rate λ proportional to the perimeter of the primary colony. One has

$$\lambda(t) = \lambda_0 P(t), \quad (64)$$

where $P(t)$ is the perimeter of the primary colony at time t . In order to make a comparison possible with the results of the M_0 model, we choose the same parameters. However, in the M_0 model the emission rate was proportional to the colony radius ($\lambda(r) = \lambda_0 r$), while now it is proportional to the perimeter. For this reason the value of λ_0 has been divided by 2π . We have

$$c = 1 \quad \lambda_0 = \frac{0.3}{2\pi}. \quad (65)$$

The simulations results for the area $A(t)$ and the perimeter $P(t)$ of the primary colony, obtained for different values of ℓ are shown in Fig. 25. As we did in the previous section for $\theta = 0$, to get a first insight in

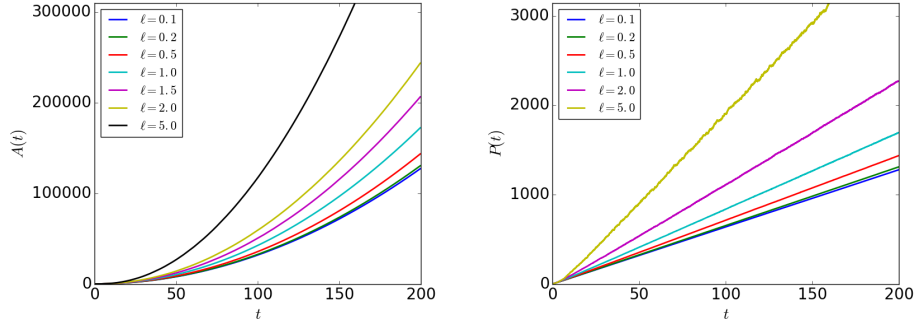


Figure 25: (Left) $A(t)$ VS t . (Right) $P(t)$ VS t . For different values of ℓ we plot the area and the perimeter of the primary colony versus time averaged over 100 simulations.

the behavior of these quantities we assume the following power-law behaviors

$$A(t) \sim t^\beta \quad P(t) \sim t^\alpha . \quad (66)$$

and we perform a fit on the empirical data. The values of the exponents obtained (for the time range $t > t_{fc}$ and $t > 100t_{fc}$, with t_{fc} the time at which the first coalescence happens) are shown in the insets of Fig. 26. The higher values correspond to the choice $t > 100t_{fc}$ and this suggests that, taking possible finite-size effects into account, one can write

$$\beta \approx 2 \quad \alpha \approx 1 . \quad (67)$$

Hence, at the first order the exponents are the same of the ones predicted by the M_0 model. We can go further and investigate the prefactor. We recall that for the M_0 model with $\theta = 1$, the radius of the primary colony increases with an effective radial velocity $c' \neq c$. In Fig. 27 we plot the quantities $A(t)/\pi c'^2 t^2 - 1$ and $P(t)/2\pi c' t - 1$; if the prefactor is the same of the M_0 model we should observe (as we did for $\theta = 0$) that these quantities tend to zero for large values of t . Nevertheless, in Fig. 27-(a)-(b) we can see that these two quantities tend to a constant $f(\ell)$ that is a function of ℓ , see Fig. 27-(c). This implies

$$A(t) = \pi c'^2 (1 + f_1(\ell)) t^2 \quad P(t) = 2\pi c' (1 + f_2(\ell)) t . \quad (68)$$

Although these are preliminary results and further investigations are needed to better understand these behaviors, the results seem to show that the circular approximation is not appropriate for $\theta = 1$, indeed its predictions are not recovered even for large values of t .

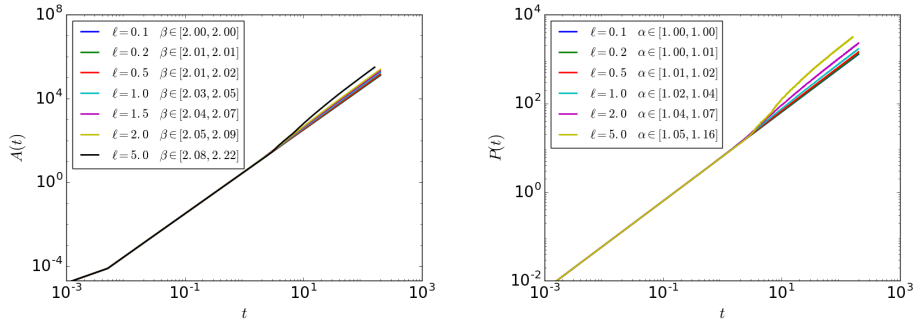


Figure 26: (Left) $A(t)$ VS t . (Right) $P(t)$ VS t . For different values of ℓ we plot in a log-log scale, the area and the perimeter of the primary colony versus time averaged over 100 simulations. We perform a power-law fit and the values of the exponents obtained are shown in the insets.

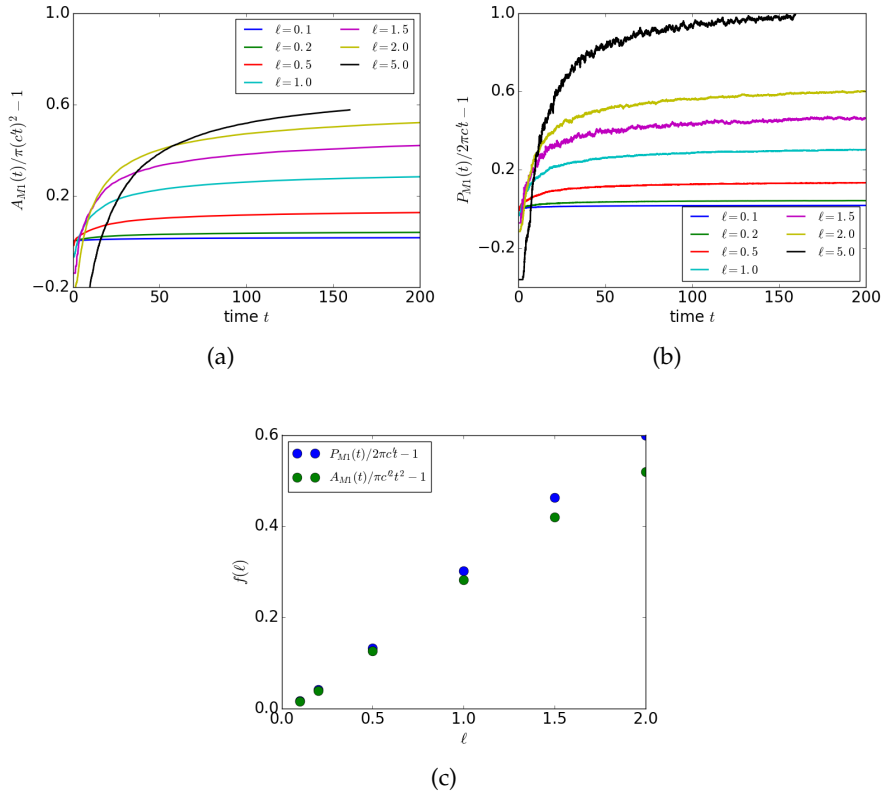


Figure 27: (a) $A(t)/(\pi c^2 t^2) - 1$ VS t for different values of ℓ . (b) $P(t)/(2\pi c t) - 1$ VS t for different values of ℓ . (c) We plot $f(\ell)$ for $t = 200$. The results are obtained averaging over 100 simulations.

7.3.3 Summary

Another important quantity at which is interesting to look at, to discuss the circular approximation used in the M_0 model is the following variable

$$S(t) = P(t)/(2\sqrt{\pi A(t)}) - 1. \quad (69)$$

One remarks that $S(t) \geq 0$ and is equal to zero when we have a perfect circle. It thus gives an indication of the "rugosity" we have if we do not use the circular approximation of the M_0 model. From Fig. 28-bottom we observe that for $\theta = 0$, $S(t)$ is larger than zero but tends to zero for large values of t as we expected from the previous discussion (see section 7.3.1). The approximation of the model M_0 for $\theta = 0$ seems appropriate. This is not the case for $\theta = 1$, indeed, in Fig. 28-top we observe that $S(t)$ is higher than one and tends to a constant for large t , consistently with the previous results (see section 7.3.2).

In the following two tables we summarize the differences between the M_0 and the M_1 model for $\theta = 0$ and $\theta = 1$.

	$M_0 \theta = 0$	$M_1 \theta = 0$
A	$\sim \pi c^2 t^2 + a t \log t$	$\sim \pi c^2 t^2 + a t^{3/2}$
P	$\sim 2\pi c t + a \log t$	$\sim 2\pi c t + a \sqrt{t}$
S	0	$\sim -\sqrt{t}$

	$M_0 \theta = 1$	$M_1 \theta = 1$
A	$= \pi c'^2 t^2$	$= \pi c'^2 (1 + f_1(\ell)) t^2$
P	$= 2\pi c' t$	$= 2\pi c' (1 + f_2(\ell)) t$
S	0	$\sim \ell$

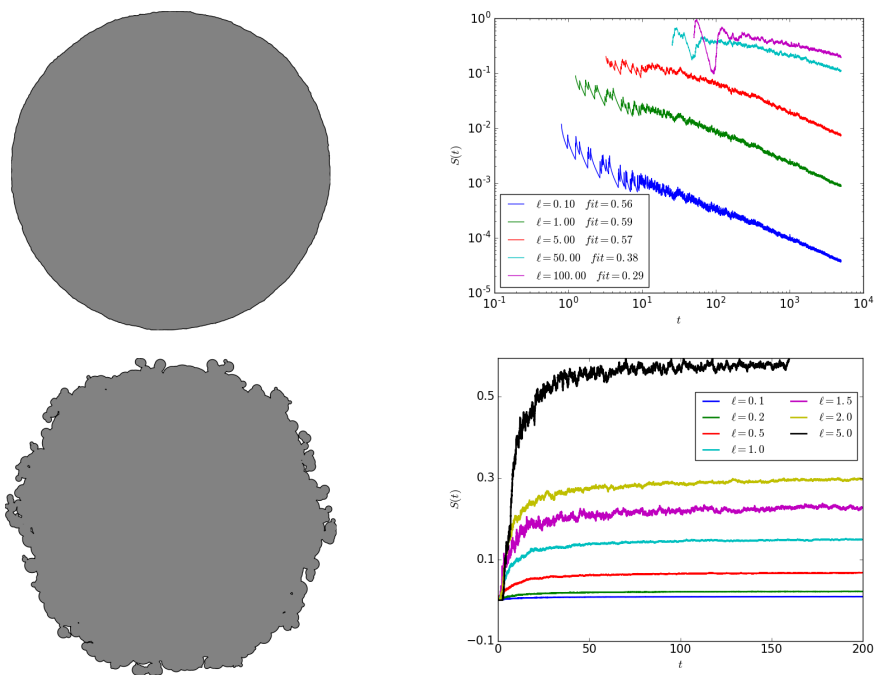


Figure 28: (left) an example of the shapes. (top-left) $\theta = 0, \ell = 10$. (top-right) $\theta = 1, \ell = 2$. (right) $S(t)$ versus t . On the top we have the behavior for $\theta = 0$ averaged over 10 simulations and on the bottom $\theta = 1$ averaged over 100 simulations.

Although we have a wide qualitative understanding of the sprawl process, we are still not able to assess quantitatively the impact of the different factors influencing it, and even a clear definition of the indicators defining it remain a challenging task. For this reason we decided here to leave aside data in a first step and to focus on the theoretical study of a dispersal model originally proposed by Shigesada and Kawasaki. This model could be a good candidate to describe the growth of an urban area based on a double process, the growth of surface area and the absorption of neighboring towns. It has been used in the investigation of the dispersion of animals and growth of cancerous tumors. It considers a primary colony that grows at radial velocity c (this represents the short-range dispersal) and has a probability per unit time $\lambda(r)$ to emit a secondary colony at a distance ℓ from its border (long-range dispersal). A full quantitative understanding of the predictions of this model for the behavior of the radius $r(t)$ of the primary colony with time, is still lacking, and in this chapter we proposed new preliminary results.

We began by the simpler problem where the emission distance ℓ is constant and we recover the Shigesada-Kawasaki equations in a mean-field framework. We then discuss the solutions of these equations for different values of θ and we test the analytical predictions against numerical simulations. We obtain a very good agreement for $\theta = 0$ and $\theta = 1$. For $\theta > 1$ we observe some deviations and we discussed the theoretical reasons for these. For the Shigesada-Kawasaki model, that we called the M_0 model, we thus gained an insight into the different possible behavior describing the growth of the primary colony thanks to mathematical equations describing the evolution of its radius.

In a second part, we modified the M_0 model to discuss the hypothesis that the shape of the primary colony can be consider circular. We do not have analytical predictions for this new model (the M_1 model) and we analyzed it with numerical simulations. This study is still at a preliminary stage and further and more rigorous analysis are needed. However the first results suggest that the circular approximation is appropriate for a constant emission rate ($\theta = 0$) but not for an emission rate proportional to the perimeter of the colony ($\theta = 1$). It is also worth discussing the behavior of $\theta = 2$, corresponding to an emission rate proportional to the colony area, and this work is in progress. Once all this will be clarified, it will be interesting to study other two different variants of the M_0 model: as first we can consider the emission distance ℓ to be a random number from a probability distribution $P(\ell)$. Second, we can introduce a field to analyzed the passive dispersal, that is the dispersal characterized by a privileged direction.

A major perspective of this theoretical work is its possible application to cities. Can it help us to understand the fluctuations we observe in empirical data? Is it possible to relate different trends to different values of θ ?

Part III

INDIVIDUAL BEHAVIORS AND SOCIO-ECONOMIC ASPECTS

This second part of the manuscript focuses on commuting patterns and their relation to the income. It thus concern the problem of individual behaviors at the time scale given by the day, and the importance of socio-economic aspects in the study of mobility in cities.

MODELING THE RELATION BETWEEN INCOME AND COMMUTING DISTANCE

Understanding mobility patterns and travel behavior is a central problem characterizing cities. It is probably one of the most important mechanisms that govern the structure and the evolution of cities and is a key aspect of demand analysis. Everyday individuals move from a set of origins O to a set of destinations D to buy, sell, work or to interact with other individuals. An important source of these flows is related to the labor market, a fundamental area of interest in economics, where the choice of work and residential locations determines the commuting. In order to build a simple model that gives quantitative predictions and allow us to gain a better understanding on these mobility patterns we need simplifications. We cannot consider all the wide range of possibly occurring situations: the individual has an house and seeks a job, or she got a job and she is looking for a place to live or even she is changing job or changing apartment, etc. In the spirit of many economical studies, we assume that the individual has a residence and is looking for a job, we thus focus on the job search process which has a direct impact on the spatial distribution of commuting trips.

8.1 INTRODUCTION

We begin by presenting two well-known models for the job search process that rely on two different frameworks: the McCall model that is an economic model based on an optimal strategy, and the radiation model that has been recently proposed by Simini in which the main ingredient influencing the choice is given by the intervening opportunities.

8.1.1 *Optimal control theory: the McCall model [78]*

Optimal control theory is a well known mathematical optimization method used to find policies that maximize the total benefit of a given process that lasts in time, it thus concerns optimization over time. The idea behind the optimization over time is that we put aside our desire to obtain the maximum possible profit immediately, and the best action we can take is the one that maximizes the sum of the cost incurred at the current moment and those will have to pay at the subsequent times, as a consequence of this decision. This is known as the "Principle of Optimality":

An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.[145]

Optimal control theory has been applied in many different areas [146–150], and to the job search problem in economics [78, 151, 152]. As a starting point, we will here consider the important McCall model [78] that has been used in many different forms and variants.

The model represents the choice behavior of individual job seekers and is an example of the stopping problem [153], where one has to choose the optimal time to take an action based on successive observations of a random variable. In the McCall model the job seeker decides the time at which accept the job. Specifically, a worker who is unemployed at time 0 reviews at every time step a random wage offer w drawn from a distribution with density f (and cumulative F). At each time step, the worker can either accept the current job offer and keep it forever, or she can pay a waiting cost c to discard the offer and wait for the next offer. The worker's income y_t at time t will thus be $y_t = w$ if she accepts the offer or $y_t = -c$ if she refuses it. The actual value of her total returns is the discounted sum of her future payoffs, assuming that the worker lives forever

$$\sum_{t=0}^{\infty} \mu^t y_t , \quad (70)$$

where the discount factor $\mu < 1$ takes into account that the value of a given amount of money is higher the earlier it is received. In this model, with an offer w at hand, the worker maximizes the expected value of her total return $v(w)$

$$v(w) = \left\langle \sum_{t=0}^{\infty} \mu^t y_t \right\rangle , \quad (71)$$

where the brackets denote the average over the offer distribution.

It accounts for the fact that a dynamic system may evolve stochastically and that key variables may be unknown or imperfectly observed. The classical way to solve this problem is to write the Bellman equation for this stopping process which reads [145]

$$v(w) = \max \left\{ \frac{w}{1-\mu}, -c + \mu \int v(w') f(w') dw' \right\} . \quad (72)$$

This equation has a simple interpretation. The value of the current offer $v(w)$ is the maximum of two terms: the first term is the total return if the current job offer is accepted, and the second term is the expected value of rejecting the current offer and waiting for the next. In the latter case, the worker pays the waiting cost c and evaluates the expectation of the value $v(w')$ of the next random offer w' . The optimal strategy that solves this equation is to accept the current offer

if it is larger than a threshold called *reservation wage* τ in this context and to refuse it if it is lower. This is shown in Fig. 29. We denote w^* the job offer received at time t , if it is accepted the total return $v_a(w^*)$ is represented by the red line, and if it is discarded $v_d(w^*)$ is represented by the light blue line. The choice that maximizes the profit is the one given by the dashed-black line, we accept the offer for $w^* > \tau$ for which $v_a(w^*) > v_d(w^*)$ and we discard it otherwise.

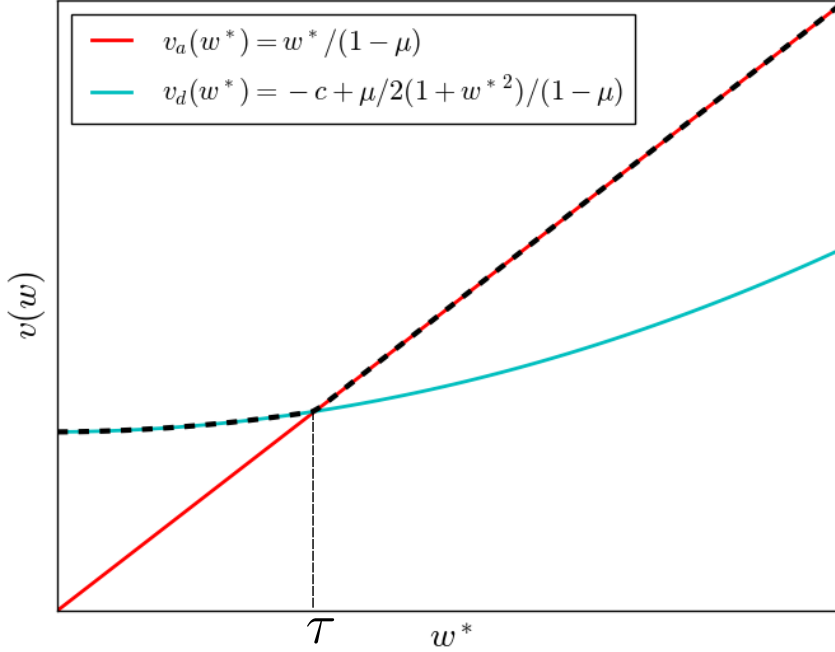


Figure 29: Representation of the Bellman equation. The red line gives the value of the total return if the offer is accepted. The light blue line gives the total return if it is rejected. The black dashed gives the solution of the Bellman equation.

This means that the reservation wage satisfies the following equation

$$\frac{\tau}{1 - \mu} = -c + \frac{\mu}{1 - \mu} \left[\tau F(\tau) + \int_{\tau}^{\infty} w' f(w') dw' \right], \quad (73)$$

so that the worker is indifferent between accepting the job for which $w = \tau$ or waiting for another offer. By solving this equation, we obtain a function τ that depends on the offer distribution. The probability p of accepting an offer is then

$$p = \int_{\tau}^{\infty} f(w) dw, \quad (74)$$

and the number of trials N before accepting a job offer thus follows a geometric distribution

$$P(N) = (1 - p)^{N-1} p. \quad (75)$$

Space is absent in the McCall model and in a following section will extend it in the simplest possible way to understand and discuss the implications of this model in the commuting patterns.

8.1.2 *Intervening opportunity: the radiation model*

As we discussed in the introduction, the importance of the distance in understanding mobility and interactions has been widely recognized since the 20th century [28, 29], when the gravitational law has been introduced. Later, in the sixties, Wilson [154] proposed a theoretical derivation of it based on the concept of entropy maximization. Since then, many different variants of the model exists and many different derivations have been proposed; an overview can be found in the book by Erlander and Stewart [155].

All these works highlight and recognize the dominant role played by the spatial distance in explaining commuting patterns and understand why people decide to get a particular job in a particular place. In 1940 instead, the sociologist Samuel A. Stouffer introduced the concept of *intervening opportunities* [156] to answer these questions. According to this idea, the number of individuals going from the origin i to destination j is directly proportional to the number of opportunities in location j and inversely proportional to the number of opportunities intervening between i and j . He argues then, that the main ingredient in the destination choice process is not the distance itself, but the spatial distribution of competing intervening opportunities and thus the accessibility of opportunities that satisfy the objective of the trip.

Based on the ideas proposed by Stouffer, Simini et al. [157] in 2012 proposed a new mobility framework whose main ingredient influencing the job choice is given by intervening opportunities. In this model, called *radiation model* [157], individuals are assumed to have a residence and seek a job. The job opportunities are characterized by a benefit z which encodes different factors such as income, working hours, conditions, etc. The number of job offers $n_j = m_j/n_{jobs}$ in each location j is proportional to its population m_j . Hence the best job offer in j is given by

$$\tilde{z}_j = \max\{z_1, z_2, \dots, z_{n_j}\} \quad (76)$$

where z_1, z_2, \dots, z_{n_j} are independent random numbers from a probability distribution $p(z)$ with cumulative $P(z)$ and represent the benefits of the job offers available in j .

Individuals living in i will choose the closest job to their home, whose benefit \tilde{z}_j is higher than \tilde{z}_i , the higher benefit available in her location

origin i . An individual living in i will work at location j with a probability given by

$$P(i \rightarrow j | m_i, m_j, s_{ij}) = \int_0^\infty dz P_{m_i}(z) P_{s_{ij}}(< z) P_{m_j}(> z), \quad (77)$$

where s_{ij} is the total population in the locations within the circle centered in i of radius r_{ij} (r_{ij} being the distance between i and j) and we choose for simplicity $n_{j\text{obs}} = 1$. $P_{m_i}(z)$ is the probability that $\tilde{z}_i = z$, $P_{s_{ij}}(< z)$ the probability that all the s_{ij} offers located between i and j have a benefit lower than z , and the last term in the integral is the probability that $\tilde{z}_i > z$, condition to accept the job in j . One can write

$$P_{m_i} = \frac{dP_{m_i}(< z)}{dz} = m_i P(z)^{m_i-1} \frac{dP(z)}{dz}, \quad (78)$$

$$P_{s_{ij}}(< z) = P(z)^{s_{ij}}, \quad (79)$$

$$P_{m_j}(> z) = 1 - P(z)^{m_j}. \quad (80)$$

After some calculations, Eq. (77) can be written as

$$P(i \rightarrow j | m_i, m_j, s_{ij}) = \frac{m_i m_j}{(m_i + s_{ij})(m_i + m_j + s_{ij})}. \quad (81)$$

From this expression, it is possible to compute the prediction for the average flow F_{ij} between i and j . The equation is invariant under rescaling by $n_{j\text{obs}} > 1$, and more importantly it does not depend on the particular choice of the probability distribution of benefits $p(z)$.

In the following we will extend this model to discuss the effect of the income in the job search process, and to propose an alternative framework to the generalized McCall model.

8.1.3 *Commuting patterns and income*

In this chapter, we focus on the commuting patterns and their relation to income. In a first part, we present an empirical analysis of the distribution of commuting distances for Denmark, the UK, and the US, exploring how the average commuting distance scales with income. In the following sections we will extend and modify the two job search models just presented (the McCall model and the radiation model), to investigate the quantitative predictions they give for the commuting patterns as function of the income. In particular, the standard model of job search [78] does not integrate space (some labour market studies do take space into account, see for example [158]). We introduce a spatial component in this model and derive the consequences for the distribution of the commuting distance. We show that the basic spatial extension of the McCall model [78] does not explain some fundamental statistical features observed in empirical data. This is the reason that brought us to question the relevance of optimal control theory as the main framework to explain mobility

and the behavior of living organisms. We propose a new stochastic model (based on the ideas proposed by Simini [157]) that rely on the idea that workers search through space, accepting the closest job to their residence judged to be good enough. We find excellent agreement between this new model and empirical data for Denmark, the UK, and the US.

8.2 EMPIRICAL ANALYSIS

We widely discussed through this thesis the recent availability of a big amount of urban data. Nevertheless socio-economic data at the individual level remain mainly related to surveys and administrative registers and the access is still limited, mainly for privacy reasons. Looking for available data represent an important and sometimes not so easy step in this type of studies. Here we use datasets for three different countries: Denmark, the UK [159], and the US [160] produced by national agencies and national household surveys.

We remark that in this empirical analysis we focus on the commuting distance and the income range. Investigations discerning different cities size, different transportation modes and interurban/intra-urban commuting could obviously be very interesting. However, at least at this stage, we consider average values without distinguish between different transportation modes or the geographical locations of the origin and destination of the trips.

8.2.1 Data description

8.2.1.1 UK data

We used data from the UK National Travel Survey (NTS) for the years 2002 – 2012 [159]. Each year's sample has a size of 15,048 addresses and was designed to provide a representative sample of households in the UK. A weighting methodology was developed to adjust for non-responses and drop-offs in the travel recording. Data collection is obtained from face-to-face interviews and a seven day travel record of individual daily travel activity.

We specifically exploit the *individual* and the *trip* files of this dataset. The *individual* file is used to determine the income category of each individual (data provides 23 income bands). The *trip* file allows us to link individuals to their weekly commuting trips for which we know the distance. In order to compute the average commuting distance as a function of the income class, we first average the commuting distance of each individual, including all transportation modes, over the number of commuting trips undertaken during the week. We then average these quantities over all individuals for each income category.

8.2.1.2 US data

We used data from the 1995, 2001 and 2009 national household travel survey (NHTS) [160], a survey of the civilian, non-institutionalized population of the United States. The NHTS datasets contain data for respectively 42,033, 26,032, and 150,147 households (with approximately 40,000 add-on interviews for the latest version). Weighting

factors are used in order to take into account nonresponses, under-coverage, and multiple telephones in a household.

These datasets allow us to associate an income category to each worker (this dataset indicates 18 different income bins) and the one-way distance to workplace. For the 2009 NHTS, the personal income is not provided, in this case we proxy personal income by the household income divided by the household size.

8.2.1.3 Danish data

The Danish data are derived from annual administrative register data from Statistics Denmark for the years 2001 – 2010. We observe the full population of workers, and for each year, we have information on the workers annual income and their commuting distance. We used the post-tax income. Commuting distances have been calculated using information on exact residence and workplace addresses using the shortest route in between. Note that for these data, no weighting methodology is required as we observe the full population of workers in the country.

8.2.2 Results

We first focus on the simplest quantity, the average commuting distance, and how it varies with income. The results for the three countries studied here are shown in Fig. 30. The basic equilibrium models of urban economics [13, 161, 162] predict, within a single city, that workers with higher incomes will have longer commuting distances. This prediction is confirmed for Denmark and the UK, while no particular trend can be detected for the US.

For Denmark, we observe an increasing range and a saturation at large income values, while for the UK we observe a plateau at low income values. To better characterize these behaviors, in the range where the increase is observed we can fit the data by a power law of the form

$$\bar{r}(Y) \sim Y^\beta \quad (82)$$

where Y is the individual income and the exponent β depends on the country considered. For the US, the fit gives an exponent $\beta \approx 0$ indicating that there is no clear trend. For the UK, the plateau around the commuting distance value $\bar{r} \approx 5$ miles occurs in the low income range $[10^2, 10^4]$ (GBP/year). The fit on UK data for incomes higher than 5,000 GBP (for all modes and all years) gives an exponent value $\beta \approx 0.5$ (in the range $[0.53, 0.66]$ when considering different years). In contrast, we observe for the Danish data a strong dependence with a large exponent of order 0.8 for yearly incomes larger than 250,000 DKK and smaller than 500,000 DKK (for lower incomes we observe

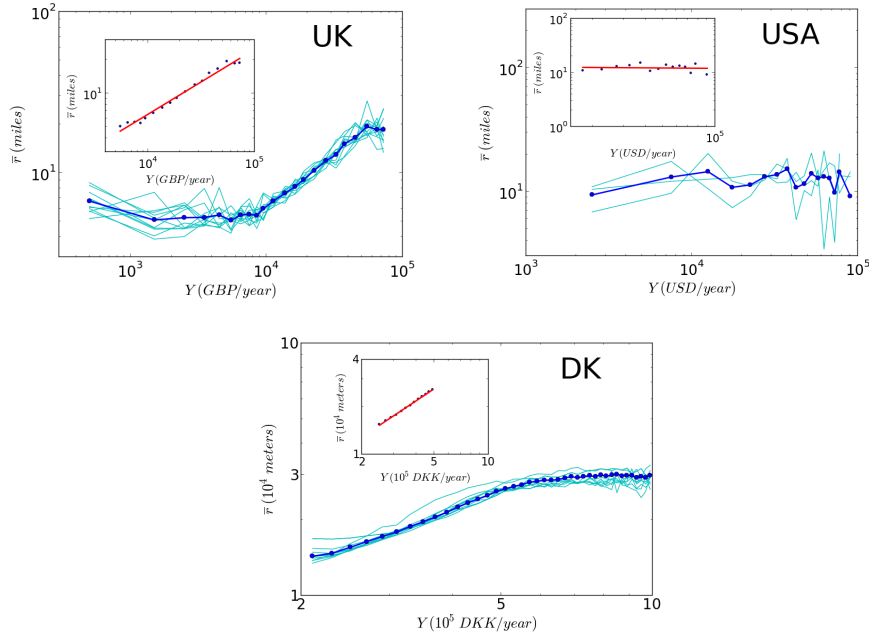


Figure 30: **Average commuting distance versus income for different years.** In dark blue, the commuting distance is averaged over all years. (Top-left) UK data. This loglog plot displays a plateau for small values of income followed by a regime, when fitted by a power law (see inset), gives an exponent $\beta \approx 0.5$ $[0.53, 0.66]$. In the inset the average commuting distance is averaged over all years and the power law fit gives an exponent $\beta = 0.58$. (Top-right) US data. In this loglog plot we do not observe an income dependence. Indeed, a power law fit gives an exponent $\beta \approx 0$. (Bottom) Danish data. The power law fit on the commuting distance averaged over all years (in the inset) gives an exponent $\beta = 0.77$.

a small plateau). Depending on the year considered, the exponent β varies in this case in the range $[0.61, 0.88]$.

We now consider the full distribution of the commuting distance, shown in Fig. 31 for different incomes for Denmark, the UK, and the US. There is some noise in the data and in particular we can note the bias for low income values for both UK and USA data which is very likely due to rounding effect in surveys, this is not observed for DK data coming from administrative registers. However, this bias does not change the order of magnitude of the commuting distance and thus it does not affect substantially the results.

Two important facts we can extract from these empirical observations. First, for all datasets studied here, the distribution is broad. This means that the variation range of commuting distances is extremely large. Indeed, we observe that with a non-negligible probability, individuals in Denmark, UK, and US are commuting on distances of the order of a few hundred. Second, the shape of the distribution and the large distance behavior are remarkably similar among the different

countries we have studied here. These non-trivial features are very important as they provide an opportunity to test for any model that aims to describe spatial commuting patterns.

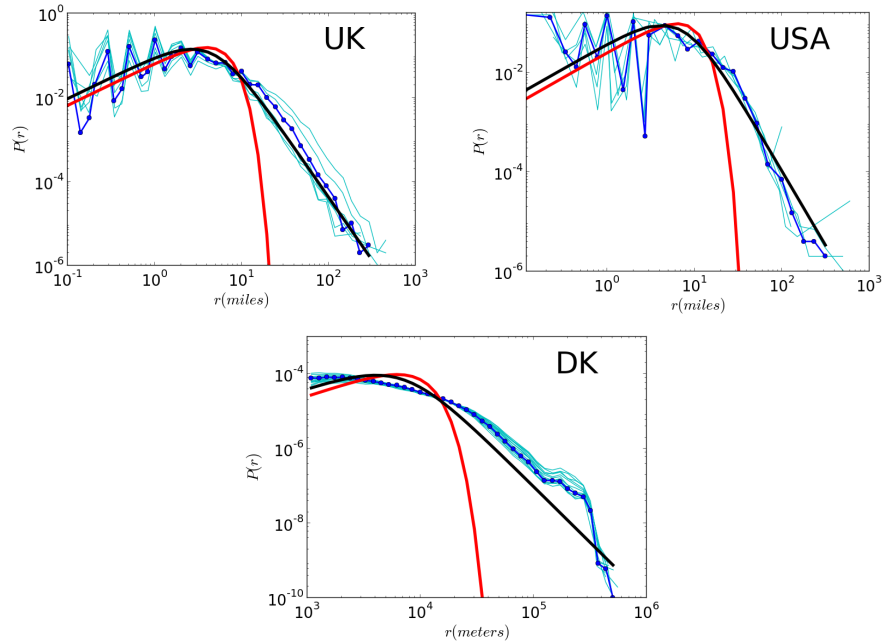


Figure 31: **Commuting distance distribution for different income classes.**

The probability distribution is shown for different income classes. In dark blue we show the distribution for a particular value of the income for which fits have been performed. In red, we show the one parameter fit with the analytical function predicted by the extended McCall model (Eq. 85), and in blue, the one parameter fit with the analytical function predicted by the closest opportunity model (Eq. 92). (Top-left) UK data (averaged over all available years). (Top-right) US data (averaged over all available years). (Bottom) Danish data (all years give the same result and we choose here to show the year 2008). In all cases we observe that the tail predicted by the extended McCall model (Eq. 85) decays too quickly and cannot fit the data for long distances. In contrast, the closest opportunity model is in excellent agreement with empirical observations.

8.3 THEORETICAL MODELING

The three datasets observed here display a slow increase of the average commuting distance with income and, more importantly, a slowly decaying tail for large distances. We would like to understand these two characteristics theoretically. We begin with a discussion of the modified McCall model and compare its predictions with our empirical observations. This will lead us to propose another model, the ‘closest opportunity’ model with predictions that are in much better agreement with the data at hand.

8.3.1 *The spatial optimal job search model*

As a starting point, we extend the McCall model including space. This can be done in a large number of different ways, and we decide to do it in the simplest possible. We assume that the individual reviews the job offers not sequentially in time anymore, but in the order of increasing distance from home. The first offer reviewed is the closest to her residence, the second one is the second closest and the n^{th} time step corresponds to the n^{th} closest job to the seeker residence. We simply move from the the question ‘when the individual gets a job?’ to the one ‘where the individual gets a job?’

As before, each random wage offer w is drawn from a distribution with density f (and cumulative F) and thus the probability that the individual accepts an offer is still given by Eq. (74). This means that the worker, starting from home, will examine the offer and will choose the first one that is above her reservation wage. To simplify the problem, we will assume that jobs are uniformly distributed in space with density ρ . If a worker has accepted the N^{th} offer, the probability that she has moved a distance r from its residence is given by a classical result for the N^{th} nearest neighbors in dimension $d = 2$ for uniformly distributed points [163]

$$P(R = r|N) = \frac{2}{(N-1)!} \frac{1}{r} (\rho\pi r^2)^N e^{-\rho\pi r^2} . \quad (83)$$

The distribution of the commuting distance R is then given by

$$P(R = r) = \sum_{N \geq 1} P(r|N)P(N) \quad (84)$$

and since the distribution of N is geometric (Eq. (75)), we obtain

$$P(R = r) = 2p\rho\pi r e^{-p\rho\pi r^2} . \quad (85)$$

where we remember that p is the probability of accepting a job offer. This distribution decreases as a gaussian over a scale of order $\sim 1/\sqrt{\rho p}$ where $1/\sqrt{\rho}$ corresponds to a typical interdistance between

different offers. We remark that the reservation wage τ and therefore the probability p depend on the income Y , and as a consequence the distance too. Moreover we note that the average commuting distance decreases if the spatial density of opportunities ρ increases. A decrease in the number of job openings during economic downturns then leads to increasing commuting distances.

To test the consistency of these result with empirical data, we fit in Fig. 31 empirical data using the prediction Eq. (85) of the extended McCall model. We observe that the best (one parameter) fit is reasonable for the short distance regime but is unable to reproduce the slow decay observed for large distances. As one could expect, an individual behavior guided by an optimal strategy will bring to an 'optimal' commuting distance and cannot reproduce a broad probability distribution. In the next section, inspired by the concept of the intervening opportunities we propose a model based on this different framework and compare its predictions with data.

8.3.2 *The closest opportunity model*

In this new model proposed here, we change two important assumptions of the McCall model. First, jobs are chosen based on some 'quality' aspect that could take into account many factors and not only on the wage (see for instance [164, 165]). More importantly, we change the framework used to study human behavior, and the reservation wage of the McCall model, which is the result of an optimal strategy, is replaced by a *reservation quality* representing the minimal job quality that meets worker expectations.

We still consider the problem of a worker who looks for a job starting from her residence (that we assume to be located at $r = 0$). Job offers are uniformly distributed across space with density ρ . The density of jobs ρ relevant for the worker depends on the income level Y and we assume that it is simply

$$\rho = \frac{\rho_0}{Y^\alpha} \quad (86)$$

such that higher income jobs are less dense than lower income jobs. The exponent α depends on the country under consideration and reflects many exogenous factors concerning job offers at a certain income level [164, 165]. This is a reasonable hypothesis that allow us to include the income in the model: indeed, the job density ρ is the only parameter that distinguish here different types of workers. The validity of Eq. 86 is not tested against data and could be further investigated. We also note that the framework introduced here for the income allows for many generalizations to other quantities such as the skill level for example.

The McCall model assumes that jobs are primarily characterized by the wage they offer. We depart from this and assume instead that each job is characterized by a random 'quality' X that encodes many

factors. The job quality is distributed according to f (with corresponding cumulative distribution F) and job qualities are independent. We further assume that a given worker has a *reservation quality* value τ (in the same spirit as the reservation wage), and she will keep expanding her search radius until this threshold is met. We denote by R the commuting distance and its cumulative thus reads

$$P(R \leq r|\tau) = P(X_{[0,r]} \geq \tau) = 1 - F(\tau)^{\rho\pi r^2}. \quad (87)$$

We now take into account that workers have different search costs and different expectations for a future job, which leads them to have different reservation qualities. We consider the reservation quality as random, distributed according to a density $g(\tau)$, and obtain the cumulative distribution of commute distances

$$P(R \leq r) = \int g(\tau)P(R \leq r|\tau)d\tau, \quad (88)$$

with corresponding density

$$\begin{aligned} P(R = r) &= \frac{dP(R \leq r)}{dr} \\ &= -2\rho\pi r \int g(\tau)F(\tau)^{\rho\pi r^2} \log F(\tau)d\tau. \end{aligned} \quad (89)$$

The first term in this integral is the probability that a worker has reservation quality τ , the second term is the probability that all offers are below τ in the disk of radius r , and the last term (the logarithm) corresponds to the probability that at least one offer is above τ in the circular band $[r, r + dr]$ (see Fig. 32 for a simple illustration of this process). A simple assumption that can be questioned and that is mainly motivated by the possibility to simplify mathematical equations is to suppose the distribution of the reservation quality τ equal to the distribution of job quality F : $g(\tau) \equiv f(\tau)$. In this way, Eq. (89) simplifies in a remarkable way and we can write

$$\begin{aligned} P(R = r) &= -2\rho\pi r \int f(\tau)F(\tau)^{\rho\pi r^2} \log(F(\tau))d\tau \\ &= -2\rho\pi r \int_0^1 x^{\rho\pi r^2} \log x dx \\ &= \frac{2\rho\pi r}{(1 + \rho\pi r^2)^2}. \end{aligned} \quad (90)$$

We observe that the distribution of commuting distances obtained does not depend on the distribution of job quality, an effect that was already observed in the specific case discussed in [157], and the model proposed here can then be considered as a microfoundation for this type of process. This also means that we may generalize the interpretation of the model: we may allow the distribution of job quality to

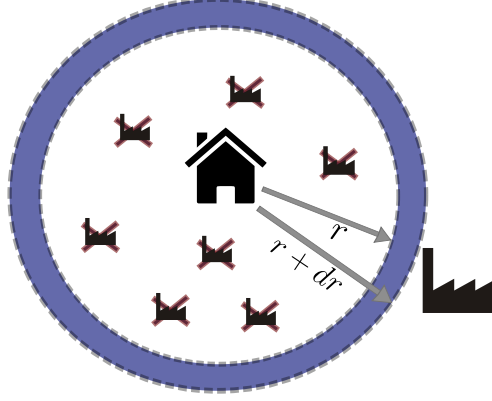


Figure 32: Illustration of the argument leading to Eq. (89).

be specific to each worker, since this has no consequence for the distribution of commuting distances; however it still has to be the same as $f(\tau)$.

In contrast to the McCall job-search model of the previous section that displayed a rapid gaussian decaying tail, we observe here that the distribution is slowly decaying as $P(R = r) \sim r^{-3}$ for large r . The average commuting distance is easily computed within the closest opportunity model and we find

$$\bar{r} = \frac{1}{2} \sqrt{\frac{\pi}{\rho}}. \quad (91)$$

Replacing ρ by ρ_0/Y^α , we find that the distribution of commute distance conditional on income is

$$P(R = r|Y) = \frac{2\rho_0\pi r Y^\alpha}{(Y^\alpha + \rho_0\pi r^2)^2} \quad (92)$$

and that the average commute distance is

$$\bar{r}(I) = \frac{1}{2} \sqrt{\frac{\pi}{\rho_0}} Y^{\alpha/2}, \quad (93)$$

which is a power law with exponent $\beta = \alpha/2$.

The theoretical result Eq. (92) also implies a simple scaling that can be checked empirically. Indeed, if we rescale the commuting distance by $Y^{\alpha/2}$, $u = r/\sqrt{Y^\alpha}$, all the curves for different incomes should collapse on the unique curve that depends on only one parameter and is given by

$$P(u) = \frac{2\pi\rho_0 u}{(1 + \rho_0\pi u^2)^2}. \quad (94)$$

In the next section, we compute these theoretical predictions against our data.

8.3.2.1 Comparison with empirical results

The closest opportunity model predicts that the average commuting distance varies with income as $\bar{r} \sim Y^{\alpha/2}$, where α depends on the country considered and it has been introduced in the expression of the skills dependent job density in Eq. (86). We will interpret our empirical results in terms of this relationship. For the US, we observe an exponent $\beta_{\text{US}} \approx 0$ indicating that the density of jobs is independent from the skill level in the US. For the UK and Denmark, we observe a non-zero exponent with $\beta_{\text{UK}} \approx 1/2$ for the UK and a larger value for Denmark $\beta_{\text{DK}} \approx 0.8$. These results indicate that the density of jobs decreases with the skill level, more in Denmark than in the UK. The observed difference between the US and two European countries in the spatial density of jobs at different income levels suggests a more general difference between Europe and the US (for a discussion in equilibrium theory about the spatial distribution of workers and skill levels, see for example [166]). It is interesting to note that there seems to be a correlation between the value of the exponent β and the size of the country. Further empirical studies are however needed in order to confirm this observation.

The crucial prediction allowing us to distinguish between models is the distribution of commuting distances and how it depends on income. Indeed, for the simple spatial extension of the McCall model presented here, the distribution of r decreases very quickly (Eq. (85)) and is not a broad distribution. In sharp contrast, in the closest opportunity model, we have a broad distribution of the form given by Eq. (92) and in Fig. 31 we display the one parameter fit with this form for a given income category. The agreement with data is very good for the UK and the US, but there are some discrepancies in the Danish case. It seems that for this Danish case there are other heterogeneity that are not taken into account in our model. In particular, Denmark is a small country with a large proportion of the population living in islands, imposing important constraints on commuting patterns.

An additional and very strong test of the validity of Eq. (92) is provided by the data collapse on the curve given by Eq. (94). In Fig. 33 (right column) we plot the rescaled commuting distance distribution for different income categories and we observe a very good collapse, except for the lower income category in the UK for which the square root behavior is not applicable. We remark that for the US $\beta = 0$, which implies that the probability distribution Eq. (92) does not depend on the income category so that the curves are automatically collapsed. We furthermore note that the agreement between the data and the closest opportunity model for Denmark is strongly reinforced by the data collapse predicted by our model and observed in the data (shown in Fig. 33).

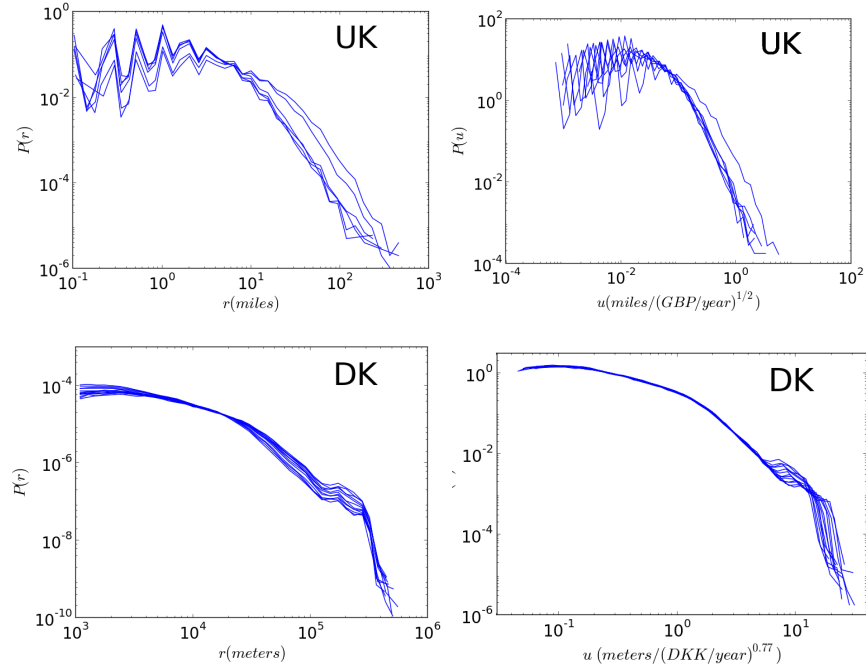


Figure 33: **Left column: Commuting distance distribution for different income classes.** The probability distribution is shown for different income classes. (Top) UK data (averaged over all available years). (Bottom) Danish data (all years give the same result and we choose here to show the year 2008). **Right column: rescaled probability distribution $P(u)$ for different income classes.** We observe a very good data collapse for both UK data (top), with $\beta = 0.5$ and averaged over all available years, and the Danish data (bottom) for $\beta = 0.77$ and for the year 2008.

8.3.2.2 Travel time distribution

Economists would argue that workers base their decisions on transport costs that depend mainly on accessibility and travel time. Small variants of the spatial McCall model and of the closest opportunity one that take into account distance through the transportation cost related to it, are discussed in [167]. We could have chosen, for example, to review job offers in travel time order, beginning with the quickest to reach, but we did the choice to do it sequentially across space, assuming that once the job is accepted the worker will choose the less costly (or the faster) way to reach its work place and this determines the travel time. We briefly discuss now the prediction of the closest opportunity model for the travel time distribution by slightly modifying the reasoning presented in [167], and we compare it to the data that we have for the UK and the US.

Denoting by r the commuting distance, the closest opportunity model predicts

$$d \log P(R = r) / d \log(r) = -3. \quad (95)$$

In general we may expect that the travel time is an increasing and concave function of distance, since travelers switch to faster modes for longer trips. Denoting the travel time by τ , we assume that $\tau \sim r^{1/\nu}$, where $\nu > 1$. In terms of travel time we then find that

$$\frac{d \log P(R=r)}{d \log \tau} = \frac{d \log P(R=r)}{d \log r} \frac{d \log r}{d \log \tau} = -3\nu. \quad (96)$$

Observing the travel time rather than commuting distance, we thus expect (according to the model) an exponent in the tail of the distribution larger than 3 in absolute value. This is consistent with the empirical data we have for the UK and the US (Fig. 34) where we observe an exponent around 4, giving $\nu \approx 4/3$.

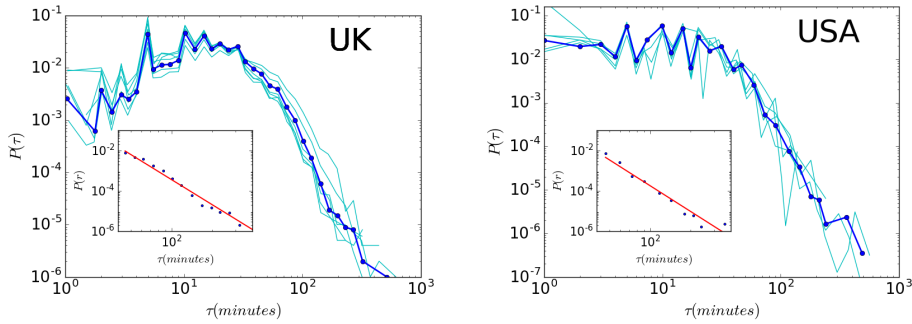


Figure 34: **Commuting distance distribution for different income classes.**

The probability distribution is shown for different income classes. In dark blue we show the distribution for a particular value of the income for which the fit on the tail is shown in the inset. (Left) UK data (averaged over all available years). The distribution displays a slow decaying tail and a power law fit on the tail ($\tau > 40$ minutes) give exponents in the range $[3.81, 4.45]$ (in the inset $\gamma\nu = 4.01$). (Right) US data (averaged over all available years). The power law fit on the tail ($\tau > 40$ minutes) gives values for the exponent in the range $[3.97, 6.35]$ (in the inset $\gamma\nu = 4.22$).

8.4 DISCUSSION AND PERSPECTIVES

With the increasing availability of always more precise and comprehensive data we can test a number of predictions of models for the urban structure and its processes. The goal of this chapter was to better understand the distribution of commuting distances and discuss its relation with income from a quantitative point of view.

Commuting patterns are related to the choice of the residence and of the job location; and a variety of different situations can occur in reality. Here we decided to focus on the job search process and to make the assumption that individuals have a residence and are looking for a job, and is the choice of the job that determines the home-work distance. We tackle the problem through two different frameworks. In both of them we make the assumption (that can be questioned) that the distance is the main 'ingredient'. Indeed, the job offers are reviewed sequentially across space, from the closest to the individual residence to the further, while they could have been reviewed by travel time or travel cost order, for example (this does not change dramatically the results [167]).

The main difference in the two frameworks is related to the job acceptance process. In the modified McCall model, the individuals will take the choice that maximizes their average benefit. In the closest opportunity model instead, the worker will accept the closest offer to its home that meets its expectation.

We showed that, while the optimal strategy framework is not in agreement with empirical evidences, the closest opportunity model is able to predict correctly the behavior of the average commuting distance with income in terms of the density of jobs offers, and also the form of the commuting distance distribution, its broad tail, and the data collapse predicted by its form. Although further studies on more countries are certainly needed, this framework would potentially find some applications in our understanding of foraging for example and other applications in ecology or finance where optimal control might be a too strong assumption.

Finally, we remark that in the empirical analysis we consider average values without distinguish between different transportation modes or the geographical locations of the origin and destination of the trips. It could be interesting to perform further investigations on the commuting patterns discerning different cities size, different transport means and interurban/intra-urban commuting, as well as a deeper analysis of their relation to travel time.

Part IV

CONCLUSIONS

8.5 CONCLUSIVE REMARKS

In this thesis we used a statistical physics approach to study urban systems. The idea is to focus on a particular phenomenon occurring in cities without wanting to describe it in all its microscopic details and to build up simple models that help us to understand which are the main mechanisms guiding the phenomenon. In particular, we want our models to provide mathematical relations between variables that can be tested against empirical data. As physicists our priority is to have quantitative models consistent with empirical evidences. Other than this, we do not have a strict and rigorous methodology to accomplish this goal, as it is the case for economists. I think that this sort of "freedom" in the process of 'exploration' for understanding, makes us more similar to butterflies (geographers) than lions (economists). Even if the mathematical framework and the idea of simplifying problems make us closer to economists.

In the first part of this thesis we focused on the spatial scale of the city and we investigated its physical structure at two different spatial resolutions: at the scale of the building lot in the first section and at a more coarse-grained scale in the second one.

In the first section we tackle the phenomenon of urbanization through the analysis of two important quantities: the building and the population. In particular we want to understand how these urban indicators evolve in time and how they relate to each other. Empirical data for 4 cities show that the number of buildings evolves with population in a possibly universal way that encompasses the peculiarity of the given city. Once this universal pattern has been determined, we propose a toy model based on simple mechanisms to contribute to the understanding of the empirical observations. Even if this model have some weaknesses, as the impossibility, at least at this stage, to predict future behaviors, these results bring evidences for the possibility of constructing a minimal model that could serve as a tool for understanding quantitatively urbanization and the future evolution of cities.

A different approach is used in the second section where we still focus on the evolution of the physical structure of the city, but at a more coarse-grained scale and staying at a theoretical level. We chose indeed to put data aside in a first moment, and to focus on the deep investigation of the mathematical behaviors resulting from the Shigesada-Kawasaki theoretical model and its variant. This model is suitable to describe urban sprawl as the result of a double process, the growth of surface area and the absorption of neighboring towns. The results presented are still at a preliminary stage and further analysis are in progress. Once all this will be clarified, we hope it will help to get a better insight on the variety of behaviors observed in the

growth of the city surface and that we will be able to reinterpret and analyze empirical data in light of the new acquired knowledges. In the second part of the manuscript we move from the scale to the city to the one of the individual and we introduce in our study socio-economic aspects such as the income class. We focus on commuting patterns and its relation to income. The general approach is the same used in the first section. We begin by looking at data, and this allows us to highlight some regularities that encompass city peculiarities. In a second step we aim to build a model consistent with these regularities. We did the choice to consider that individuals have a residence and look for a job, and it is the job choice that determines the commuting. We discuss two models built upon two different frameworks. The first one is an extension of the McCall model, an economic model for the job search process based on an optimal strategy. The second model called 'closest opportunity model', offers an alternative framework to study human or animal behavior, in which actions are taken not on the basis of an optimal strategy but on the first opportunity that is good enough for the individual. We studied the implications of these models for the spatial distribution of distances between residences and jobs as function of the income and we show that the McCall model is not supported by empirical data, while the closest opportunity model is able to correctly predict the form of the commuting distance distribution, its broad tail, and the data collapse predicted by its form. More generally, this study allowed us to question the relevance of optimal control theory as the main framework to explain mobility and individual behavior.

Three years ago when I began my PhD I slipped into a subject that was not the one I had studied until then. I still remember how long it took me to read my first economic article and interpret the language. I found interesting and challenging to progressively discover the different tools and approaches used for studying cities; and understand how I could contribute, as a physicist, bringing still other tools and approaches.

The quantitative understanding of urban systems is still at its infancy, this can be note for example if we look at the problems we met through the manuscript with the definitions of entities and processes. What is a city? What is urban? What do we mean by urbanization? And what do we mean by urban sprawl? However, the new availability of a big amount of data together with the success of the science of complexity brought a renewed interest on cities, and allowed a glimpse into the dawn of a new science of cities, interdisciplinary and based on data [8, 9]. I think that an important point is to improve the communication between the different fields. It is important to make an effort to understand what we can learn from each other

and how this new knowledge can be integrated and reinterpreted in each own framework. Using the words of Thisse [31] "it would be futile to expect an integrated theory of cities that would appeal equally to economists, regional scientists, geographers, and urban planners" (and physicists). Probably, one reason is that we simply ask different questions.

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EVOLUTION DES SYSTEMES URBAINS: UNE APPROCHE PHYSIQUE

RÉSUMÉ Plus de 50% de la population mondiale vit dans des zones urbaines et cette proportion devrait augmenter dans les prochaines décennies. Comprendre ce qui régit l'évolution des systèmes urbains est donc devenu d'une importance fondamentale. Ce renouveau d'intérêt combiné avec la disponibilité de données à grande échelle, permet d'entrevoir l'avènement d'une nouvelle science des villes, interdisciplinaire et basée sur les données. Des études récentes ont montré l'existence de régularités statistiques et de lois d'échelle pour plusieurs indicateurs socio-économiques, tels que la consommation d'essence, la distance moyenne parcourue quotidiennement, le coût des infrastructures, etc. Malgré plusieurs tentatives récentes, la compréhension théorique de ces résultats observés empiriquement demeure très partielle. Le but de cette thèse est d'obtenir une modélisation simplifiée, hors-équilibre de la croissance urbaine, en s'appuyant sur un petit nombre de mécanismes importants et qui fournit des prédictions quantitatives en accord avec les données empiriques. Pour cela, nous utiliserons des outils et des approches provenant de la physique statistique et nous nous inspirerons des études en géographie quantitative et en économie spatiale.

La thèse est organisée en deux parties. Dans la première partie, nous nous sommes concentrés sur l'échelle spatiale de la ville et nous avons étudié sa structure physique à deux résolutions spatiales différentes: à l'échelle du lot de construction dans la première section, et à une échelle plus grossière dans la seconde.

Dans la première section, nous abordons le phénomène de l'urbanisation à travers l'analyse de deux quantités importantes: le bâti et la population. En particulier, nous voulons comprendre comment ces indicateurs urbains évoluent dans le temps et comment ils sont corrélés. Les observations empiriques pour les quatre villes analysées montrent que l'évolution du nombre de bâtiments avec la population suit une courbe qui ne dépend pas des caractéristiques historiques et géographiques de la ville étudiée. Une fois que cette régularité empirique a été déterminée, nous proposons un modèle basé sur des mécanismes simples pour contribuer à la compréhension des observations empiriques. Même si ce modèle a des faiblesses, comme par exemple l'impossibilité, au moins à ce stade, de prédire des comportements futurs, les résultats obtenus apportent des preuves quant à la possibilité de construire un modèle minimal qui pourrait servir d'outil pour la compréhension quantitative de l'urbanisation et de la future évolution des villes.

Une approche différente est utilisée dans la deuxième partie où nous nous concentrons encore sur l'évolution de la structure physique de la ville, mais à une échelle plus grossière. Nous avons choisi de mettre les données de côté dans un premier temps, et de se concentrer sur une étude approfondie des comportements mathématiques résultant du modèle théorique de Shigesada-Kawasaki. Ce modèle représente un bon candidat pour décrire l'étalement urbain à la suite d'un double processus, la croissance de la surface et l'absorption des villes voisines. Les résultats présentés sont encore à un stade préliminaire et d'autres analyses sont en cours. Une fois que tout cela sera clarifié, nous espérons que ce modèle nous aidera à mieux comprendre la variété des comportements observés dans la croissance de la surface des villes et que nous pourrions réinterpréter et analyser les données empiriques à la lumière des nouvelles connaissances acquises.

Dans la deuxième partie de la thèse, nous passons de l'échelle de la ville à celle de l'individu et nous introduisons dans notre étude des aspects socio-économiques tels que le revenu. Nous étudions les déplacements domicile-travail et leur relation avec le revenu. L'approche générale est la même que celle utilisée dans la première section. Nous commençons par examiner les données empiriques, ce qui nous permet de mettre en évidence certaines régularités qui dépassent les particularités de la ville. Dans une deuxième étape, nous cherchons à construire un modèle cohérent avec ces régularités. Nous avons choisi de considérer le cas où les individus ont une résidence et cherchent un emploi, et c'est le choix de l'emploi qui détermine alors la distance domicile-travail. Nous abordons deux modèles basés sur deux cadres différents. Le premier est une extension du modèle de McCall, un modèle économique pour le processus de recherche d'emploi basé sur une stratégie optimale. Le deuxième modèle appelé «modèle de l'opportunité la plus proche», offre un cadre alternatif, pour étudier le comportement humain, dans lequel les actions sont effectuées non pas sur la base d'une stratégie optimale mais sur la première opportunité qui est satisfaisante pour l'individu. Nous avons étudié les implications de ces modèles pour la distribution spatiale des distances domicile-travail en fonction du revenu et nous montrons que le modèle McCall n'est pas vérifié par les données empiriques, tandis que "le modèle de l'opportunité la plus proche" est capable de prédire correctement la forme de la distribution de distances, sa queue large et le collapse des données prédit par sa forme. Plus généralement, cette étude nous a permis de nous interroger sur la pertinence de la théorie de la stratégie optimale comme cadre principal pour expliquer la mobilité et le comportement individuel.

EVOLUTION OF URBAN SYSTEMS : A PHYSICAL APPROACH

Keywords: complex systems, statistical physics, cities

Abstract: More than 50% of the world population lives in urban areas and this proportion is expected to increase in the coming decades. Understanding what governs the evolution of urban systems has thus become of paramount importance. This renewed interest combined with the availability of large-scale data, allows a glimpse into the dawn of a new science of cities, interdisciplinary and based on data. Recent studies have shown the existence of statistical regularities and scaling laws for several socio-economic indicators such as fuel consumption, average commuting distance, cost of infrastructure, etc., and despite several recent attempts, the theoretical understanding of these results empirically observed remains very partial. The purpose of this thesis is to obtain a simplified, out of equilibrium model of urban growth, based on a small number of important mechanisms and which provides quantitative predictions in agreement with empirical data. For this, we will draw on studies in quantitative geography and spatial economy and we will revisit some of these old models with a new approach that integrates the tools and concepts of physics.

EVOLUTION DES SYSTÈMES URBAINS: UNE APPROCHE PHYSIQUE

Mots clés: systèmes complexes, physique statistique, villes

Résumé: Plus de 50% de la population mondiale vit dans des zones urbaines et cette proportion devrait augmenter dans les prochaines décennies. Comprendre ce qui régit l'évolution des systèmes urbains est donc devenu d'une importance fondamentale. Ce renouveau d'intérêt combiné avec la disponibilité de données à grande échelle, permet d'entrevoir l'avènement d'une nouvelle science des villes, interdisciplinaire et basée sur les données. Des études récentes ont montré l'existence de régularités statistiques et de lois d'échelle pour plusieurs indicateurs socio-économiques, tels que la consommation d'essence, la distance moyenne parcourue quotidiennement, le coût des infrastructures, etc. Malgré plusieurs tentatives récentes, la compréhension théorique de ces résultats observés empiriquement demeure très partielle. Le but de cette thèse est d'obtenir une modélisation simplifiée, hors-équilibre de la croissance urbaine, en s'appuyant sur un petit nombre de mécanismes importants et qui fournit des prédictions quantitatives en accord avec les données empiriques. Pour cela, nous nous inspirerons des études en géographie quantitative et en économie spatiale et nous revisiterons certains de ces anciens modèles avec une nouvelle approche intégrant les outils et concepts de la physique.