

Scheduling Handling Resources: Robotic Flowshops with Circular Layouts and a Practical Railway Problem

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Scheduling and transportation

Scheduling: assigning *tasks* to *resources* submitted to *constraints* to optimize a *performance criteria*

Physical handling or transportation between resources: connectivity, layout, additional specific handling resources

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Two contexts:

- **Manufacturing management**

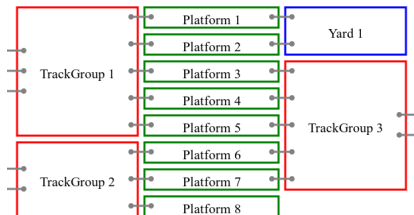


- **Railway management**



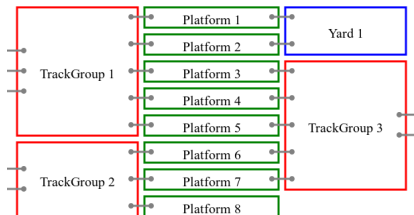
Railway management

“Trains don't vanish”...except when they do.
EURO/Roadef Challenge 2014, SNCF



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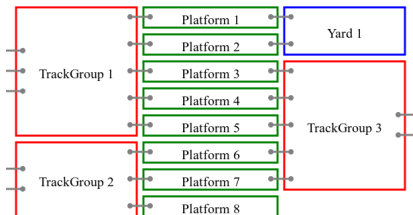


- Resource capacity
- Resource compatibility
- Conflict between trains
- ...



Railway management

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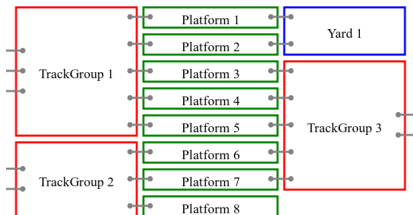
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- ...

Optimize departure coverage and performance costs



Railway management

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EURO/Roadef Challenge 2014, SNCF



Horizon: 14 days

200–2000 departures/arrivals

30–90 resources

- Resource capacity
- Resource compatibility
- Conflict between trains
- ...

Optimize departure coverage and performance costs



Railway management

Joint work with H. Joudrier [[Joudrier and T. 17](#)]

- Simplify the station's graph by grouping similar resources
- Sophisticated routing algorithm
- (very) simple assignment algorithm
- Up to 40% covered departures...

... And the 1st place in the junior category!



Manufacturing management

Flowshop: the parts must be processed on machine M_1 , then M_2 ...



Manufacturing management

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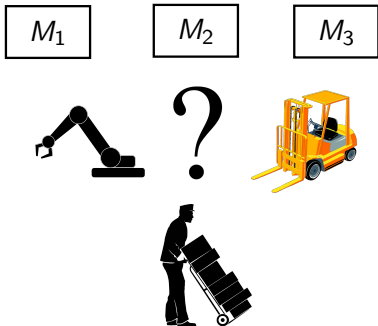


Sufficient model?



Manufacturing management

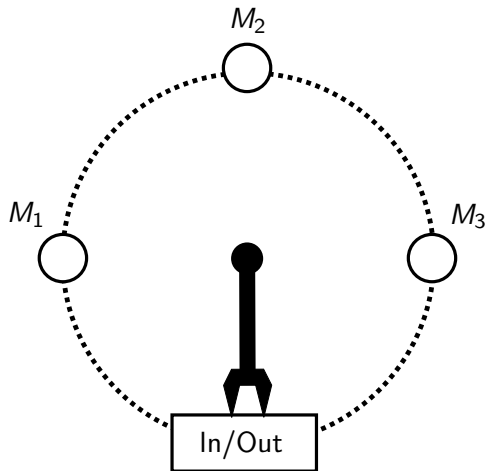
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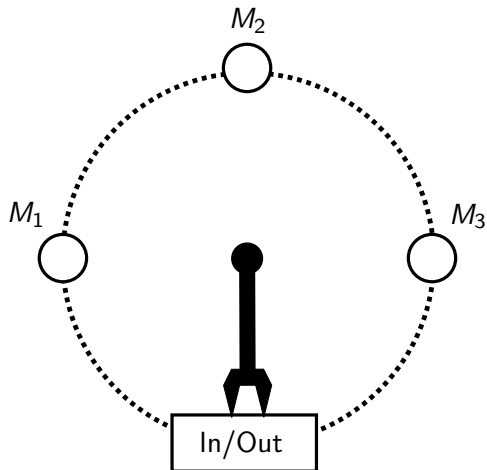
Sufficient model? Operator, tools, **handling resources**...



Robotic cell – Example

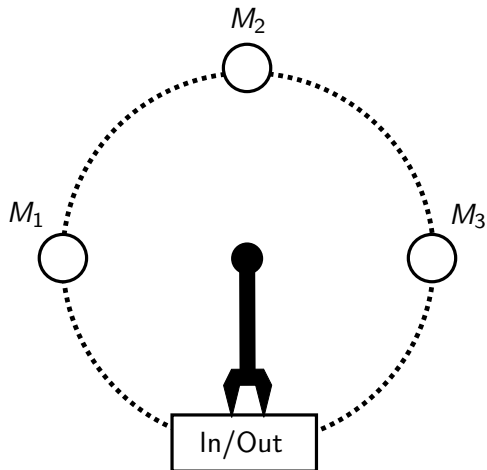


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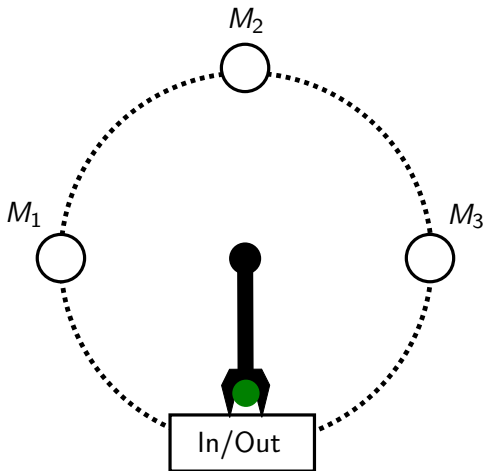
“Short processing times, slow robot”

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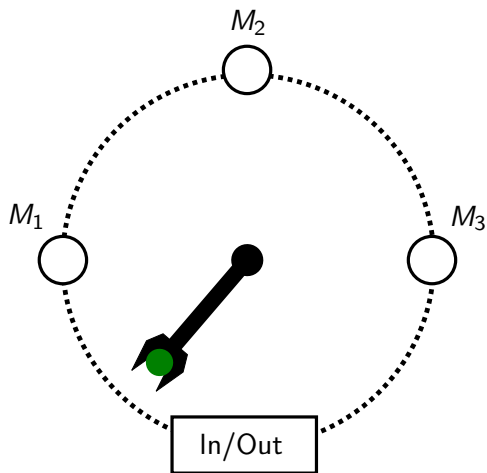
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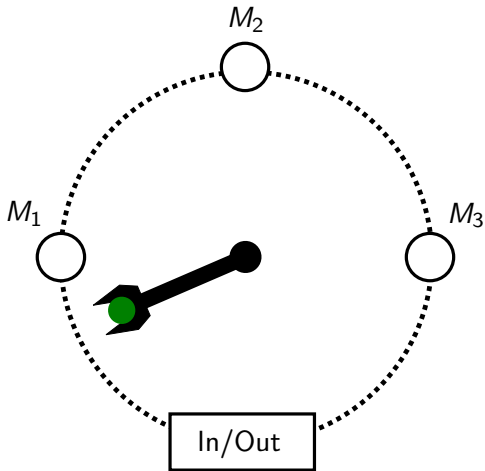
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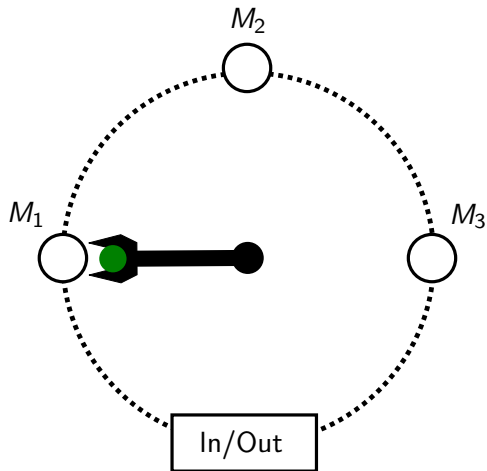
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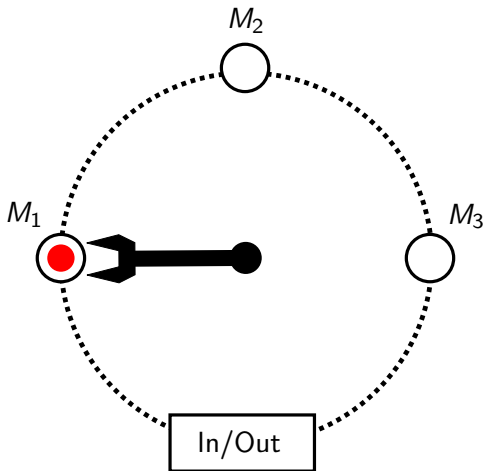
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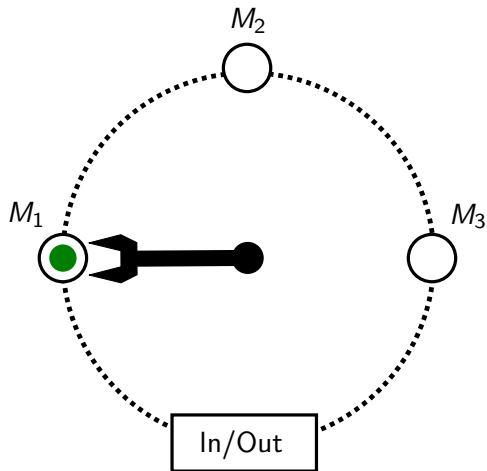
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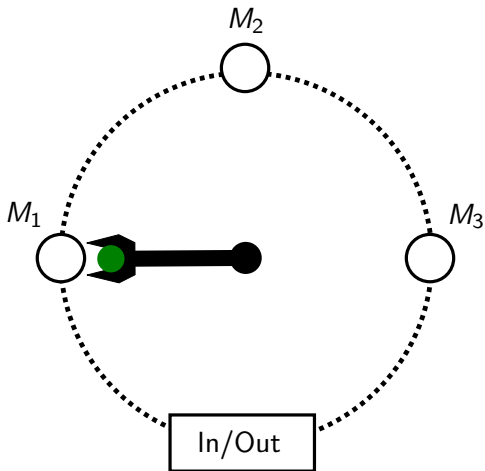
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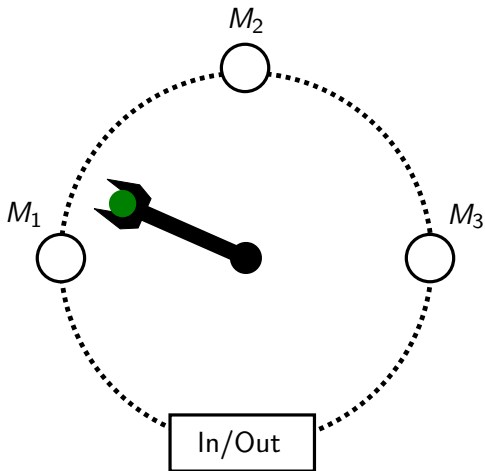
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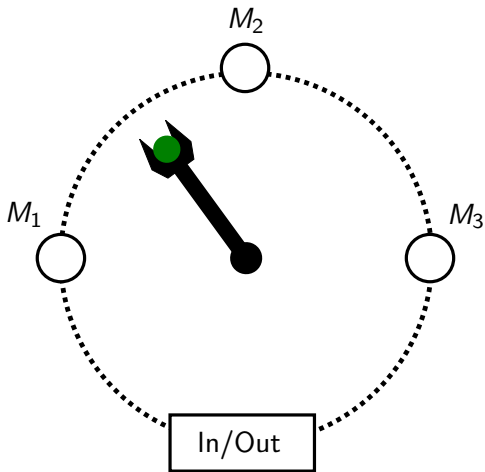
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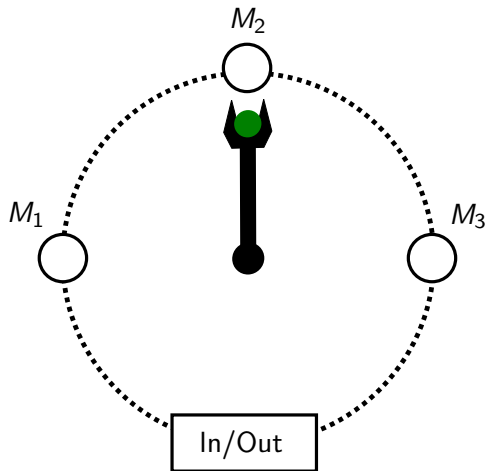
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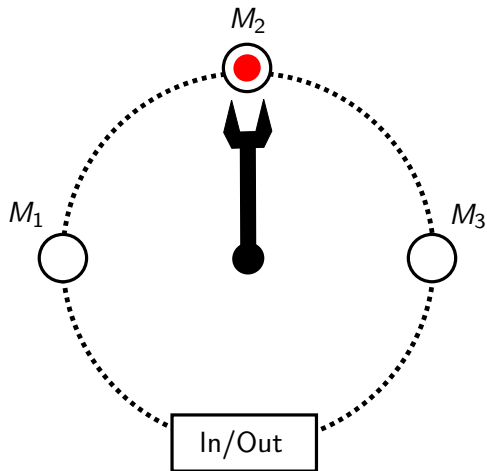
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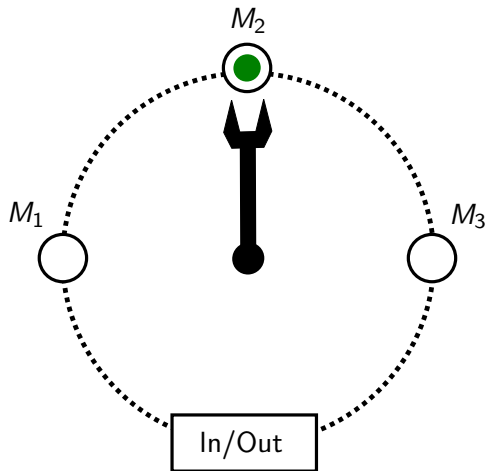
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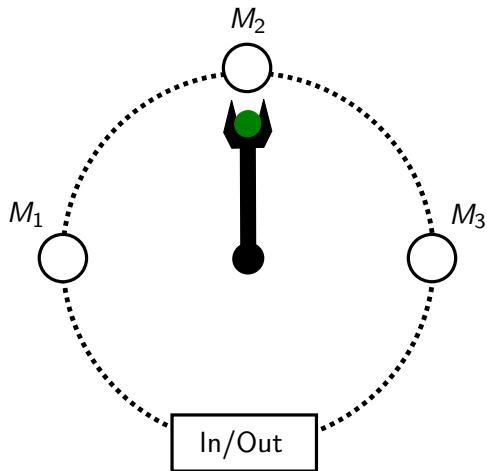
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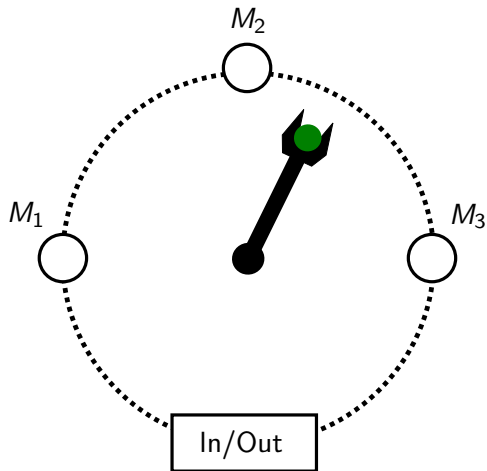
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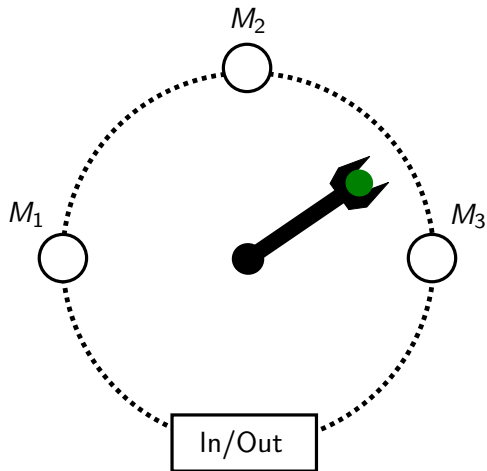
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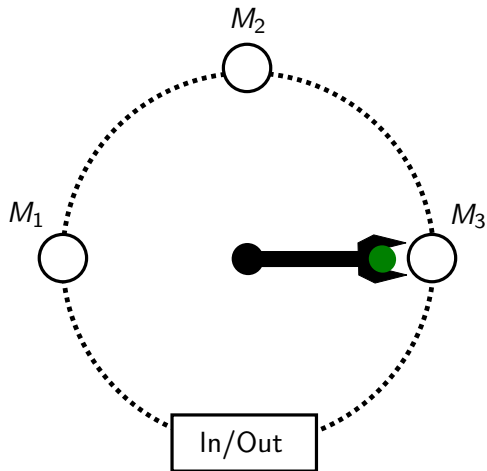
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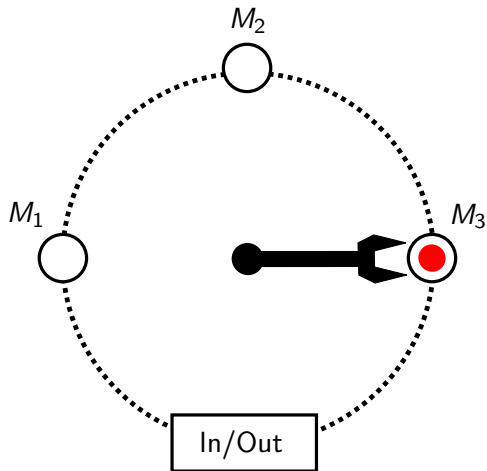
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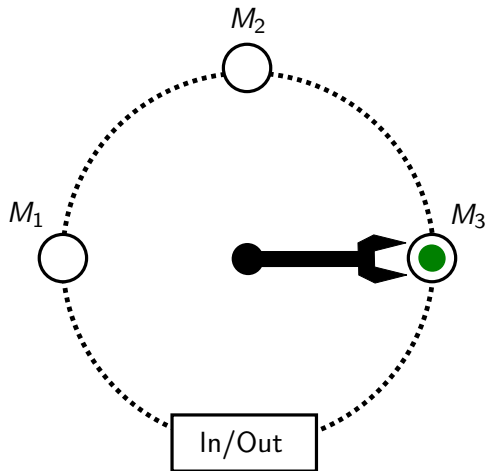
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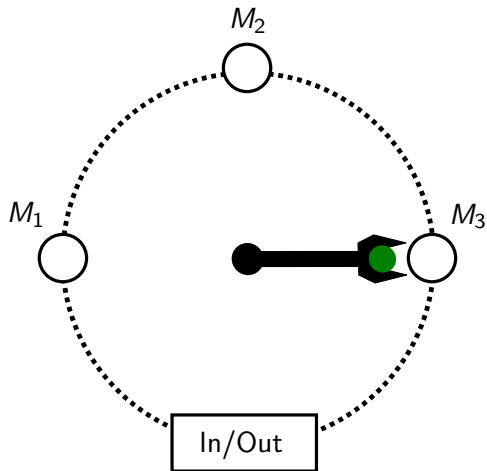
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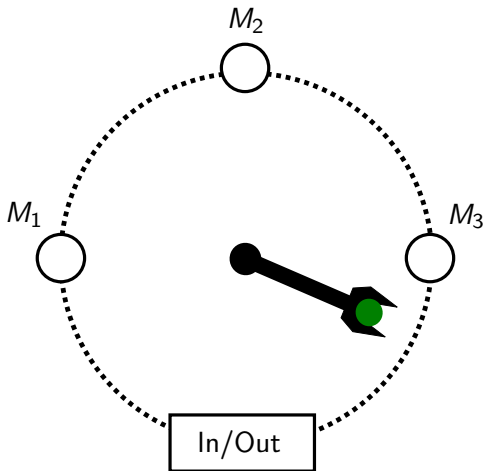
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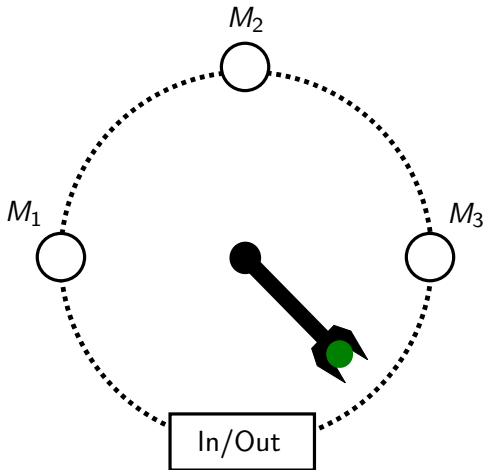
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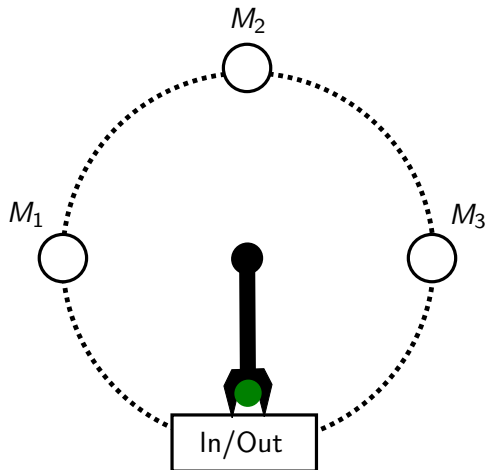
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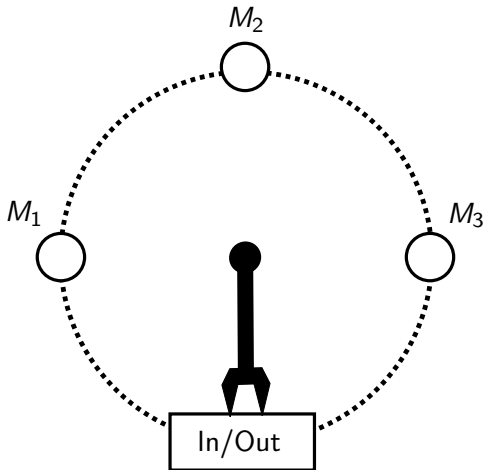
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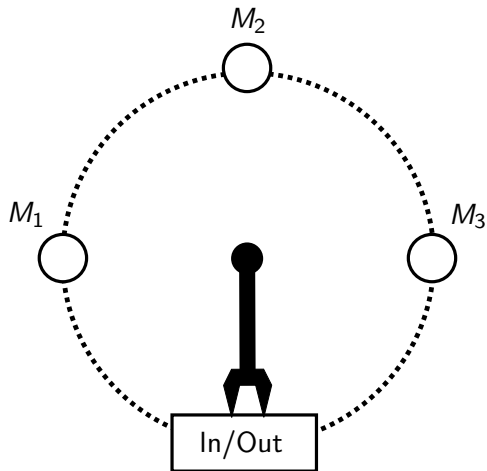
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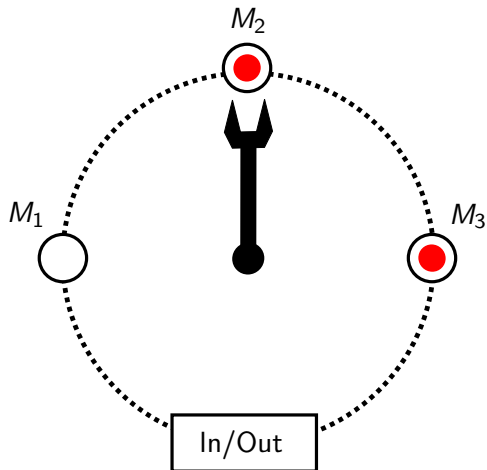
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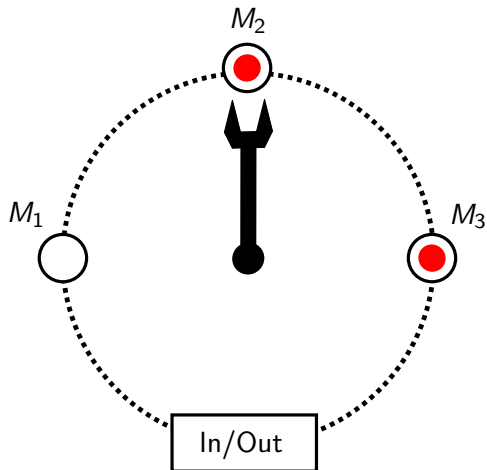
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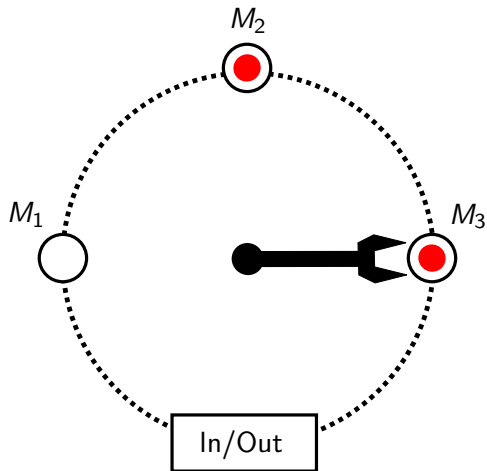
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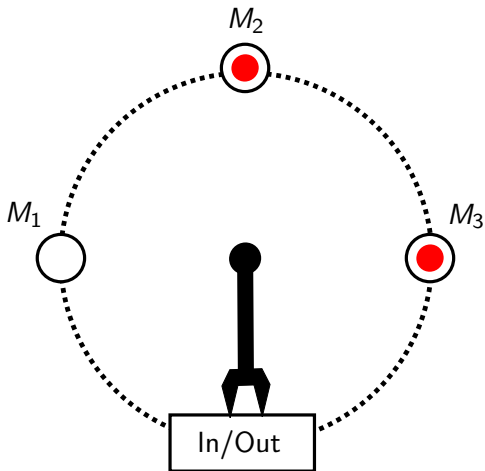
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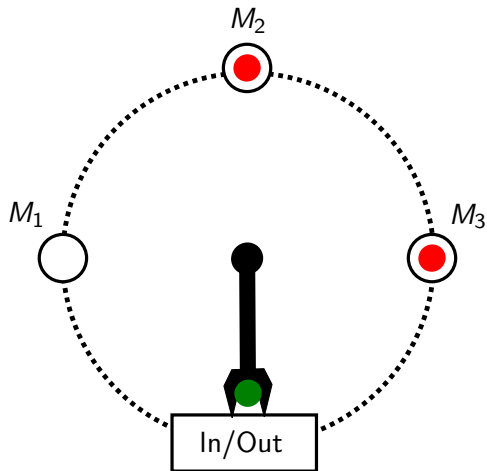
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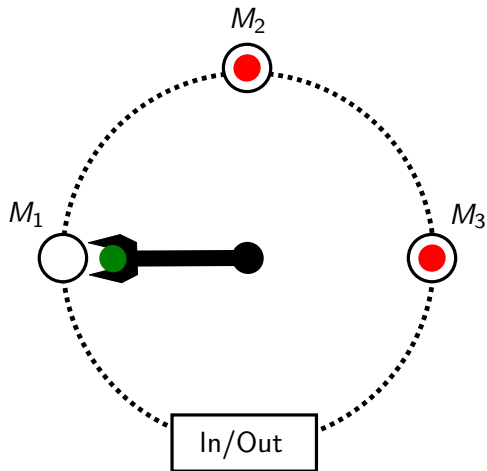
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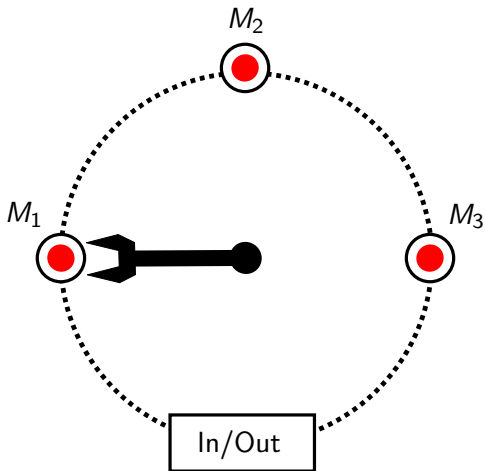
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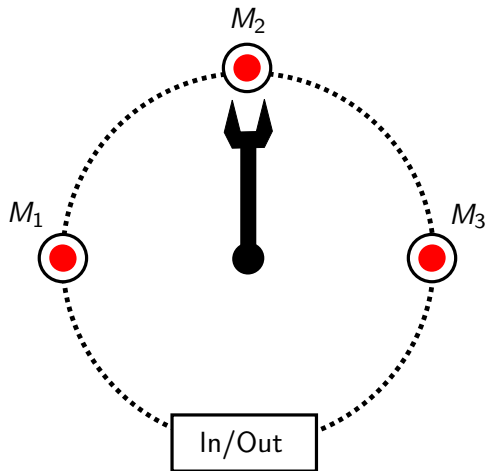
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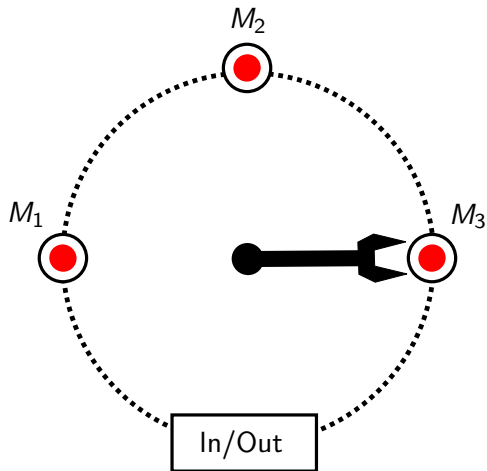
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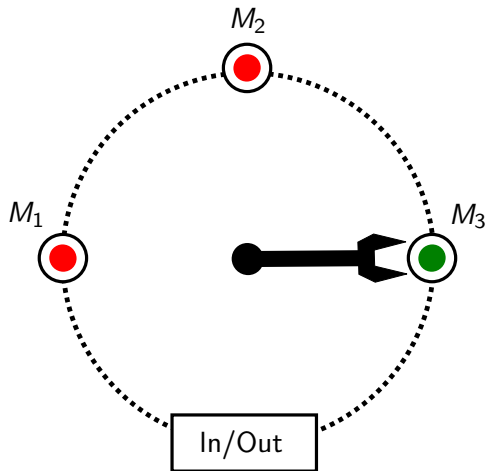
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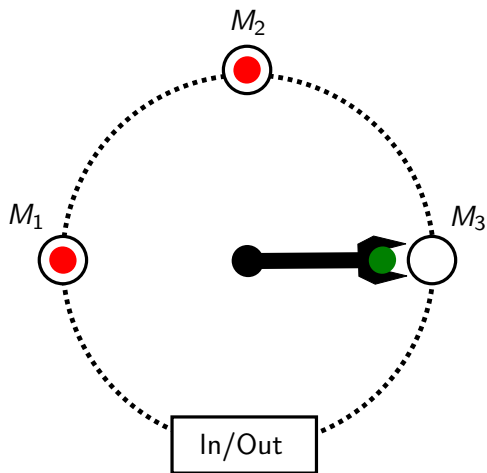
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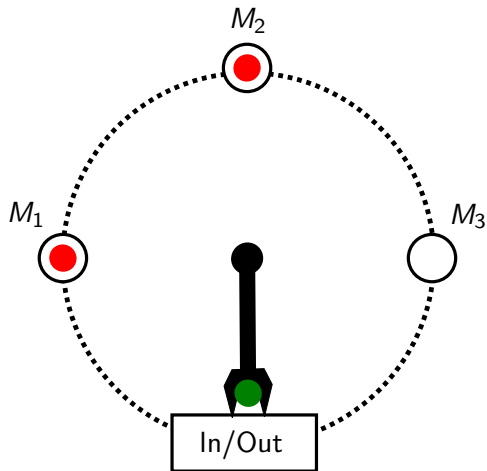
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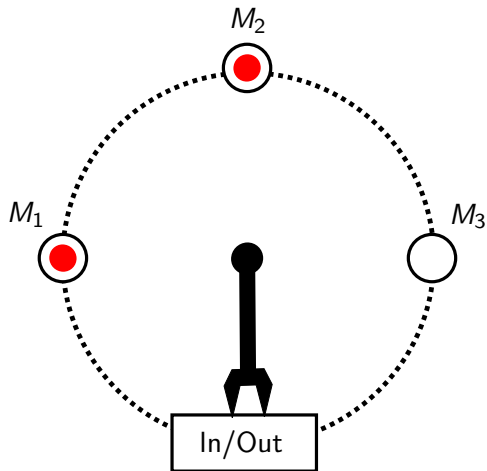
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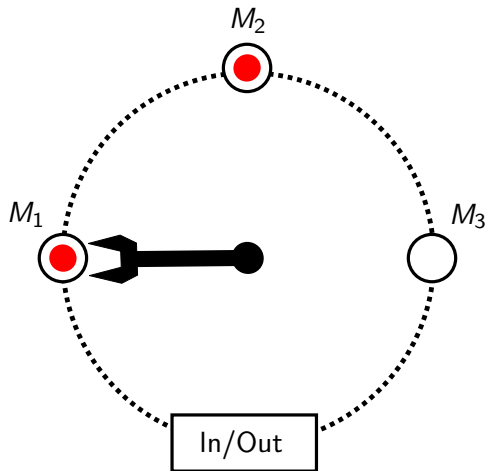
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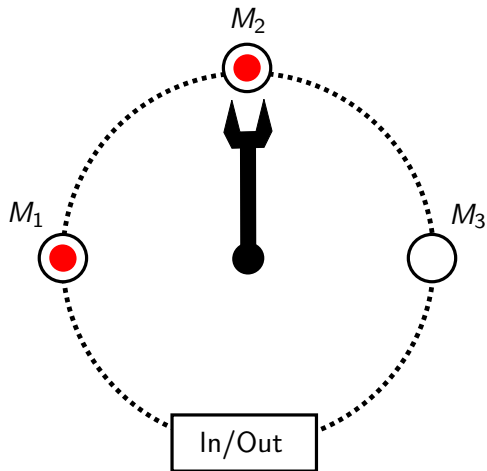
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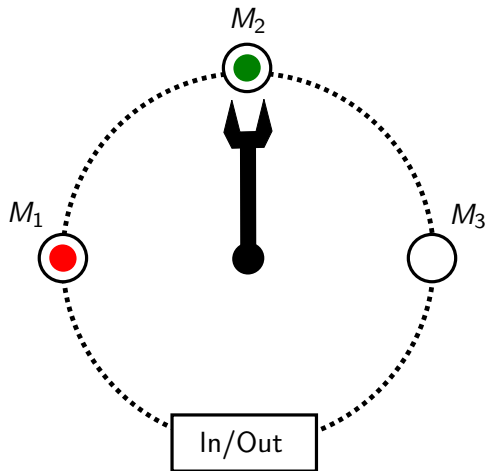
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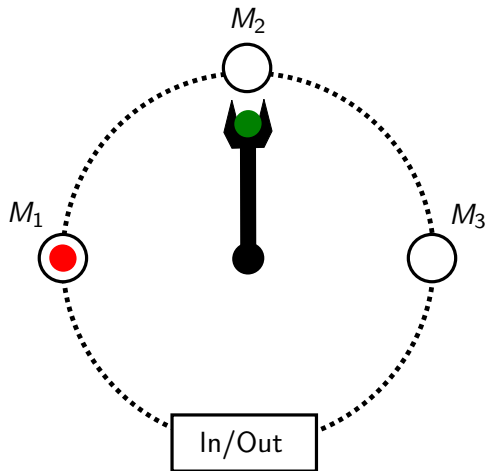
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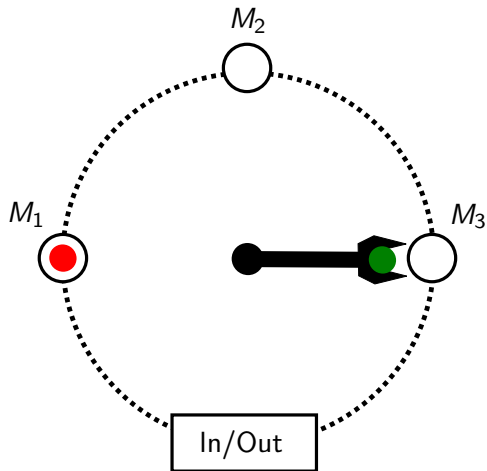
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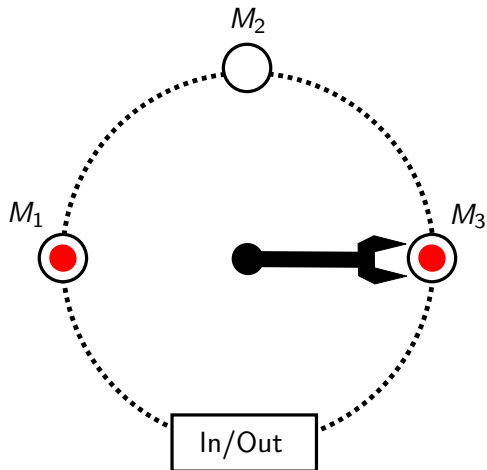
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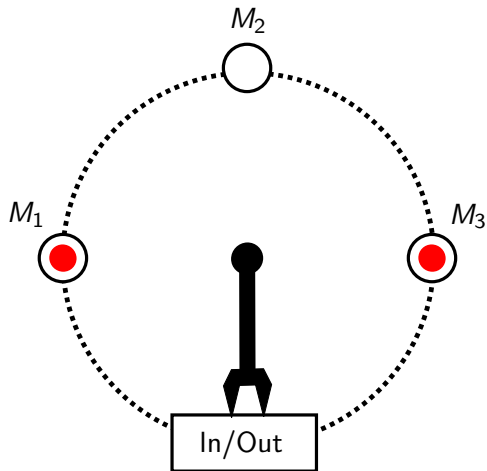
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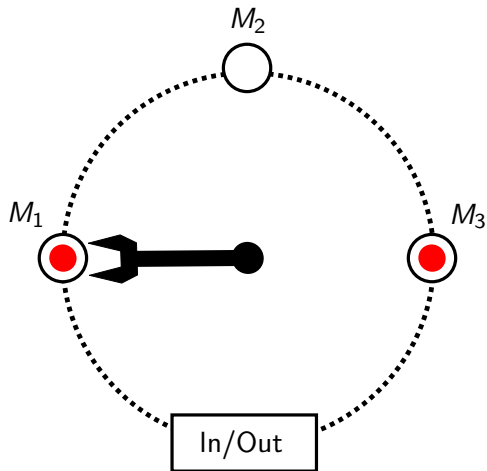
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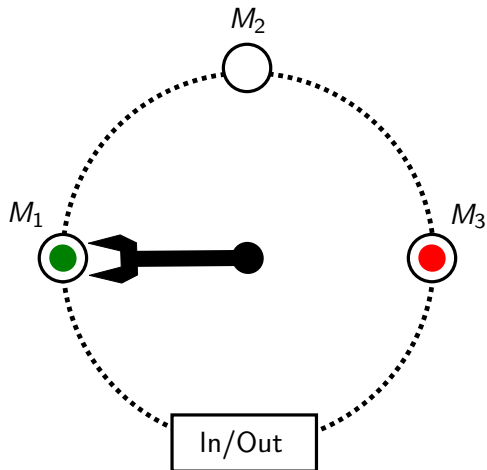
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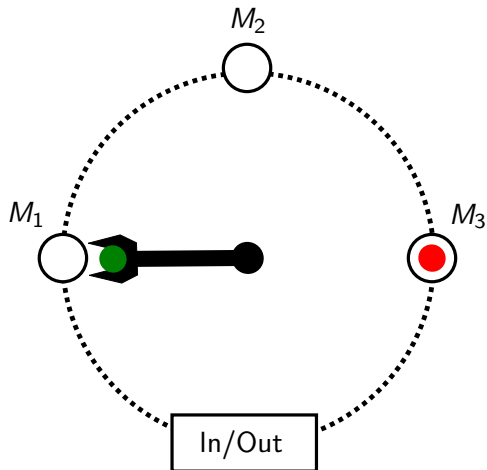
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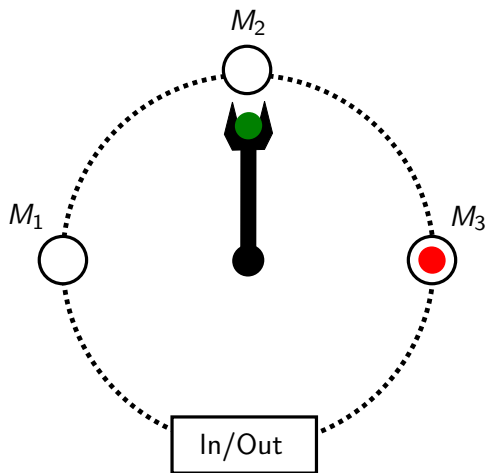
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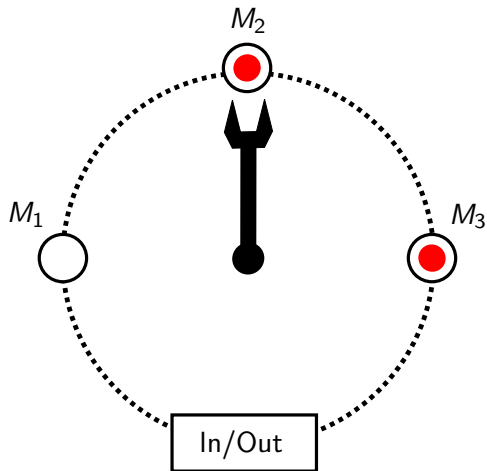
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And in-between?

That's where the problem is

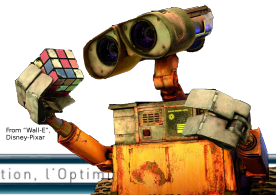
Balancing

- Total travel time
- Waiting time

Given the *processing times* and *travel time between machines*, find a robot programming which optimizes the **throughput**:

$$\text{Maximize } \frac{k}{\text{Time to produce } k \text{ parts}}$$

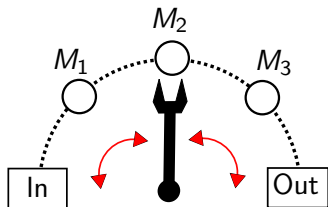
$$\text{Minimize } \frac{\text{Time to produce } k \text{ parts}}{k}$$



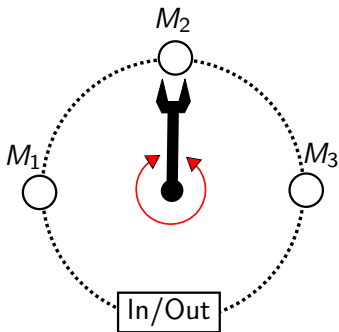
From "WALL-E"
Disney-Pixar

Models: Layouts

Linear



Circular



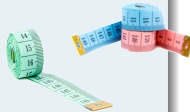
Models: cell parameters

Waiting policy

- No-wait [Agnetis, 00 ; Kats *et al* ; 09, Che *et al*, 12]
- **Unbounded** [Crama *et al*, 97 ; Rajapakshe *et al.*, 11]
- Time-window (HSP) [Dawande *et al*, 09; Zhou *et al*, 12]

Travel metric

- General
- **Additive**
| regular
- Constant



Processing times

Identical parts

- General
- **Balanced**



Machine and robot capacity

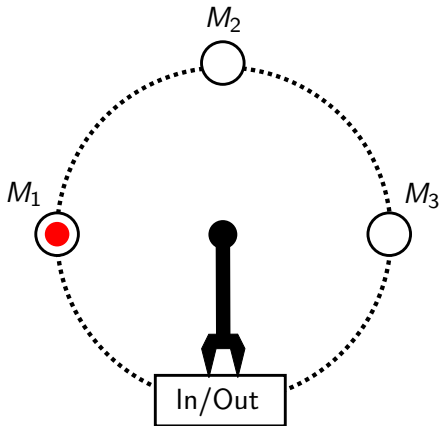
Classical model (blocking)

Few studies for the circular layout [Rajapakshe *et al.*, 11]

Relaxing the blocking constraint

- Dual-gripper
[Sethi *et al.*, 01 ; Jung *et al.*, 15 ; Drobouchevitch *et al.*, 06]
- Swapping [Jolai *et al.*]
- Machine buffers [Drobouchevitch *et al.*, 06]

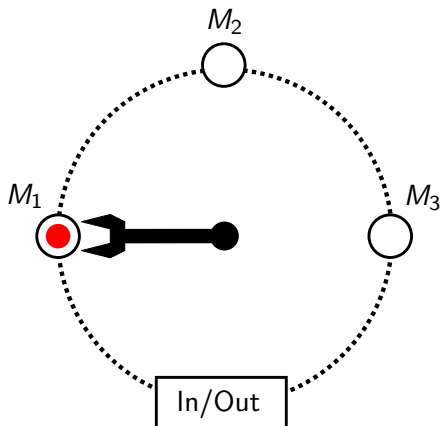
SCOP Describing the robot moves: Activities



[Crama *et al*, 97]

Activity A_i

SCOP Describing the robot moves: Activities

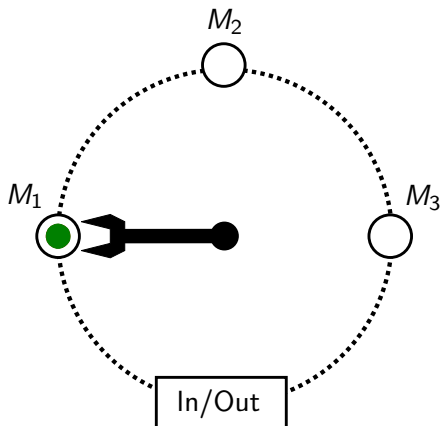


[Crama *et al*, 97]

Activity A_i

- Go to M_i

SCOP Describing the robot moves: Activities

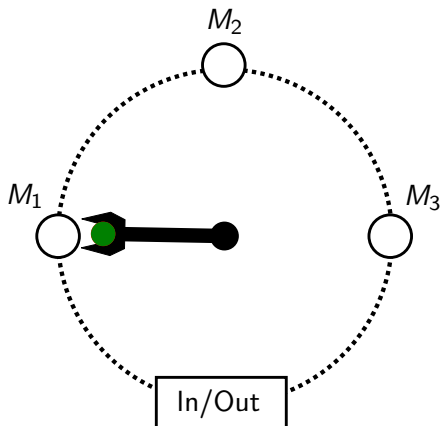


[Crama *et al*, 97]

Activity A_i

- Go to M_i
- Wait?

SCOP Describing the robot moves: Activities

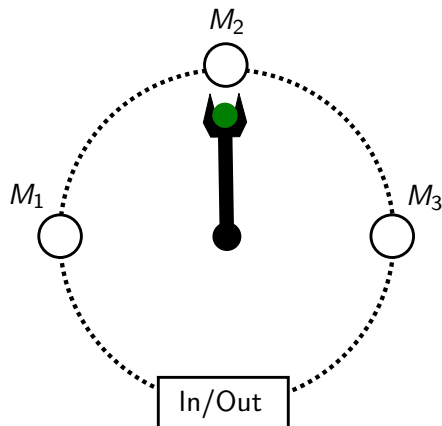


[Crama *et al*, 97]

Activity A_i

- Go to M_i
- Wait?
- Unload M_i

SCOP Describing the robot moves: Activities

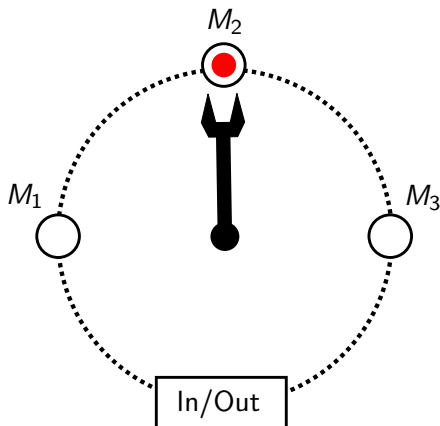


[Crama *et al*, 97]

Activity A_i

- Go to M_i
- Wait?
- Unload M_i
- Go to M_{i+1}

SCOP Describing the robot moves: Activities

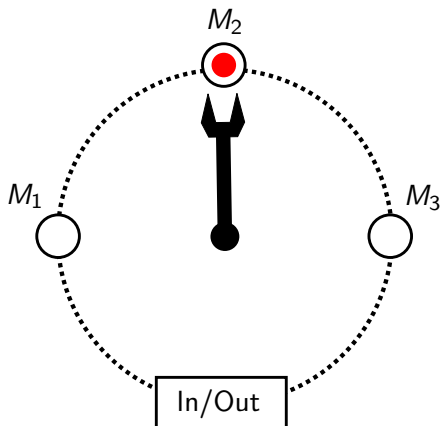


[Crama *et al*, 97]

Activity A_i

- Go to M_i
- Wait?
- Unload M_i
- Go to M_{i+1}
- Load M_{i+1}

SCOP Describing the robot moves: Activities



[Crama *et al*, 97]

Activity A_i

- Go to M_i
- Wait?
- Unload M_i
- Go to M_{i+1}
- Load M_{i+1}

A_0, A_1, \dots, A_m

Cyclic programmation

Dominance [Dawande *et al*, 05]

Cycle

- Feasible sequence of activities
- Leaves the cell in the same state

Cyclic programming

Dominance [Dawande *et al*, 05]

Cycle

- Feasible sequence of activities
- Leaves the cell in the same state
- k -cycle: produces k parts
- **1-cycle**: produces 1 part

Cyclic programming

Dominance [Dawande *et al*, 05]

Cycle

- Feasible sequence of activities
- Leaves the cell in the same state
- k -cycle: produces k parts
- **1-cycle**: produces 1 part

1-cycles \Leftrightarrow permutations of the m activities $A_1 \dots A_m$

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1-cycles \Leftrightarrow permutations of the m activities $A_1 \dots A_m$

Objective function

max throughput \longleftrightarrow min $\frac{\text{cycle time}}{\text{number of parts produced}}$ \longleftrightarrow min $\frac{T(C_k)}{k}$

Best 1-cycle problem

1-cycles are...

- easy to describe
- easy to implement
- better known and more studied

But not necessarily optimal!

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- ...in linear regular cells: P [Crama *et al.*, 97]
(balanced [Brauner *et al.*, 99])
- ...in circular regular cells: NP-hard [Rajapakshe *et al.*, 11]
($\frac{5}{3}$ approx).

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($\frac{5}{3}$ approx).

- ...in circular regular balanced cells: ??

Dominance?

1-cycle conjecture

1-cycles dominate all cycles

[Sethi *et al.*, 92]

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- Valid for regular cells, 2- to 3-machine

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[Crama *et al.*, 97]

[Brauner *et al.*, 01]

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Circular layout

???

Dominance?

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Linear layout

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- False for regular cells, 4-machine [Brauner *et al.*, 01]
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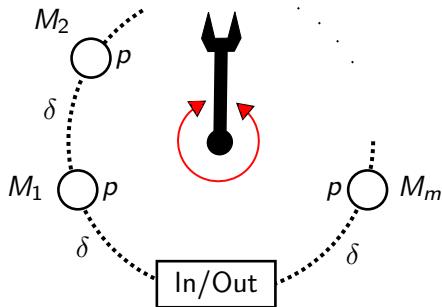
Circular layout

Counter-example for regular balanced cells with 6 machines

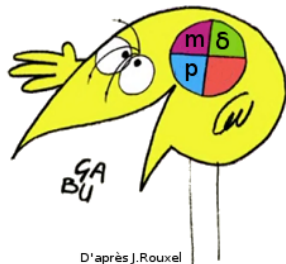
[T. *et al.* MIM 16]

Our problem

Circular, identical parts, **regular, balanced**: (m, δ, p)

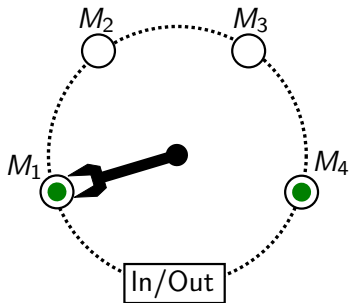
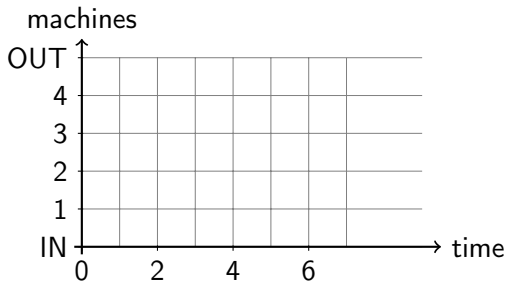


Small enough to fit in a Shadok's head!

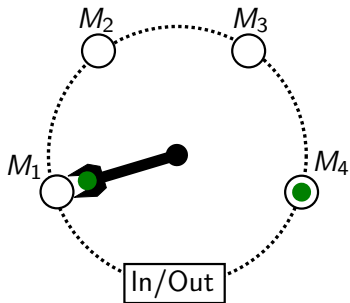
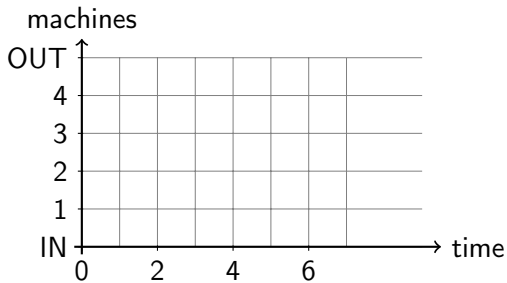


D'après J. Rouxel

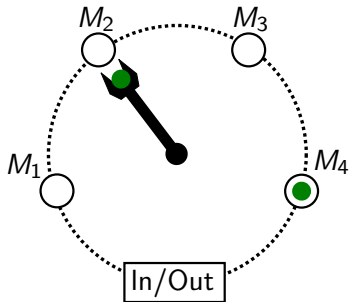
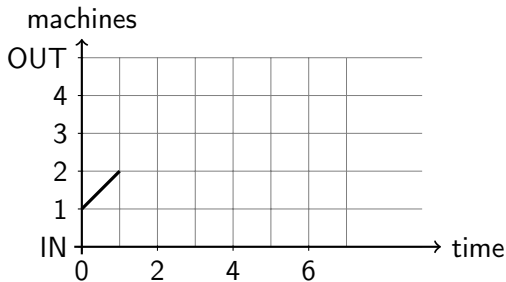
Representing the robot moves



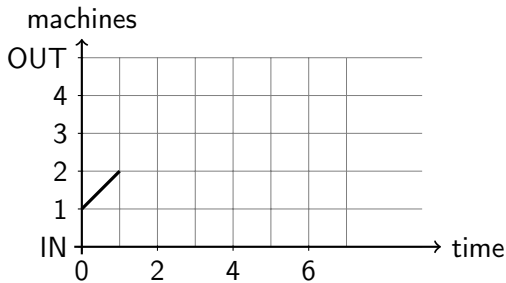
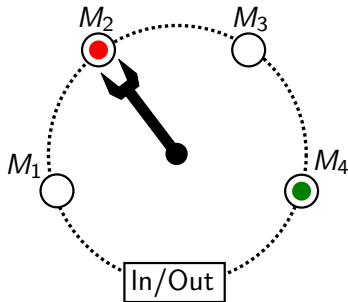
Representing the robot moves

 A_1

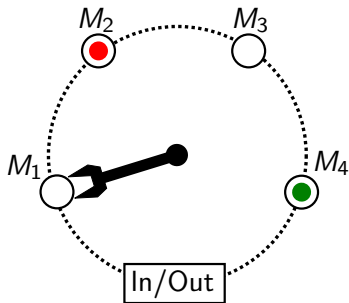
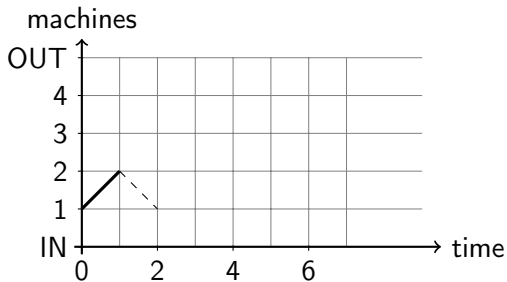
Representing the robot moves

 A_1

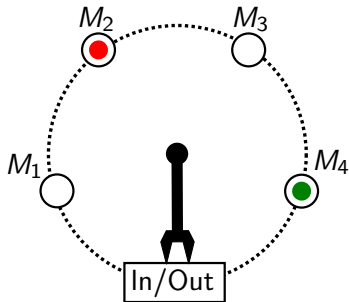
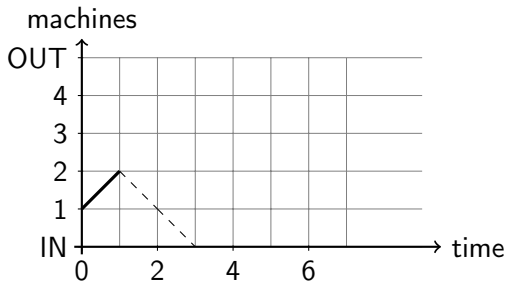
Representing the robot moves

 A_1 

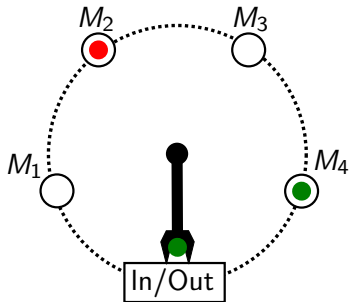
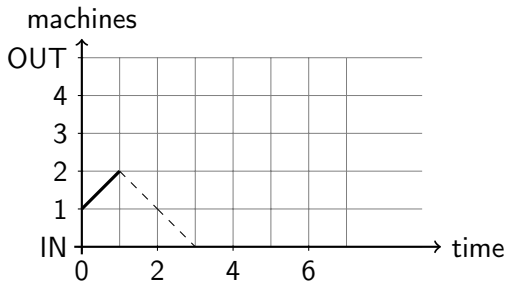
Representing the robot moves

 $A_1 A_0$

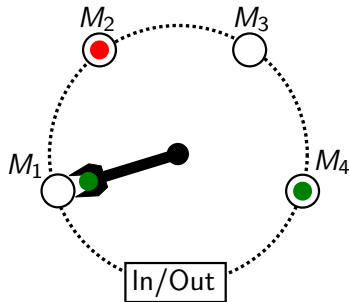
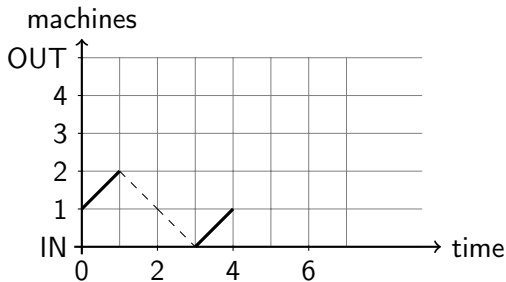
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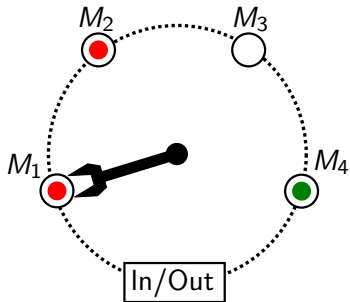
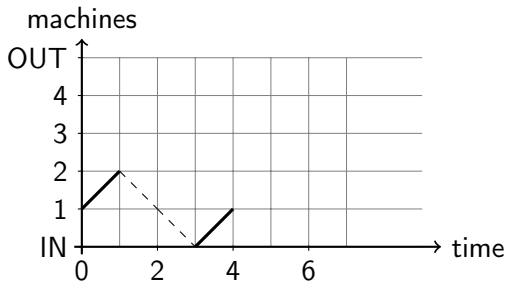
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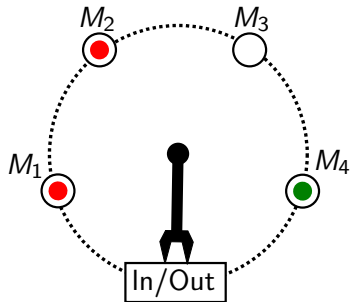
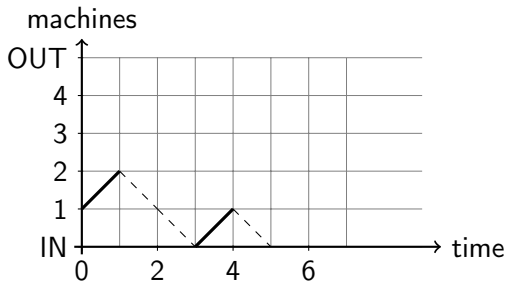
Representing the robot moves

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Representing the robot moves

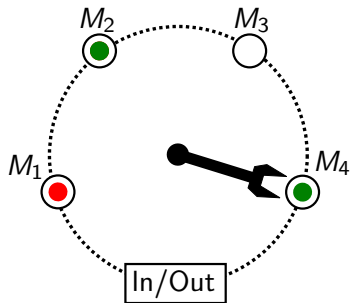
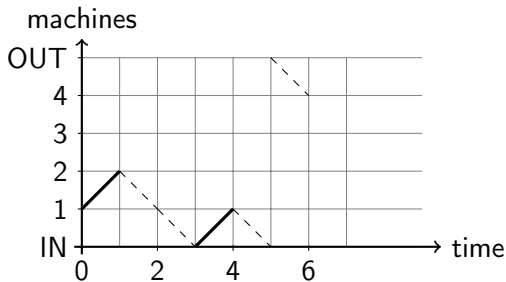
 $A_1 A_0$

Representing the robot moves

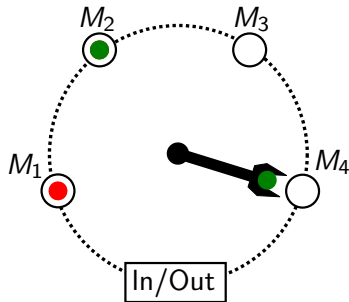
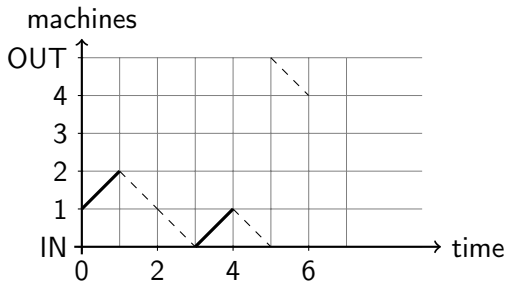


$A_1 A_0 A_4$

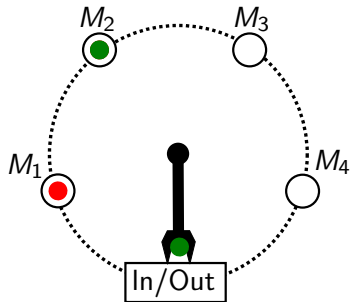
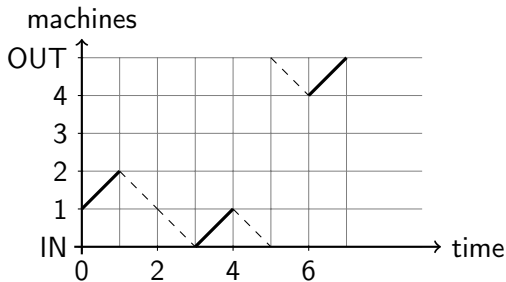
Representing the robot moves


 $A_1 A_0 A_4$

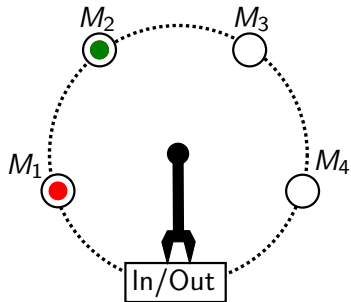
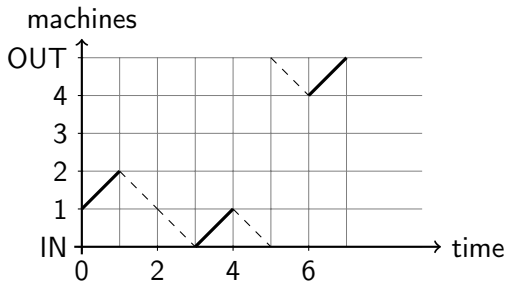
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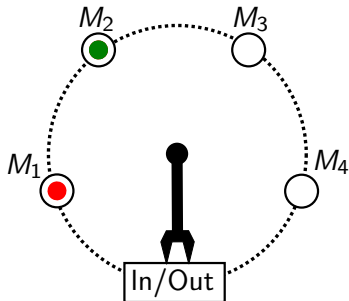
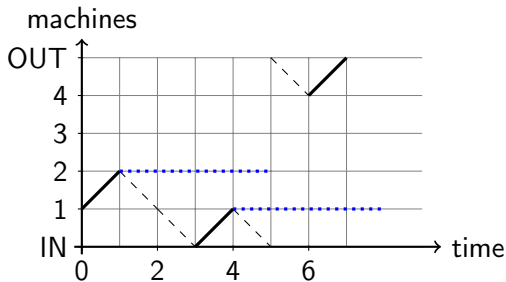
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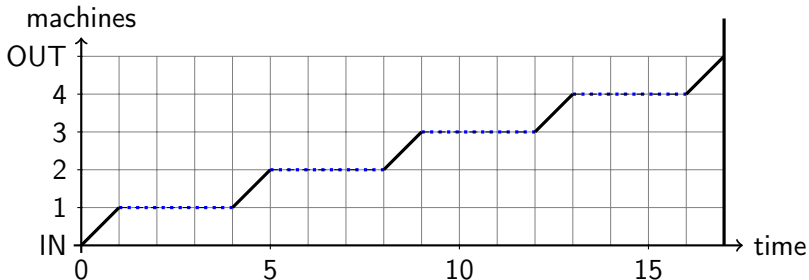
Representing the robot moves


 $A_1 A_0 A_4$

Some usual 1-cycles: Identity cycle

Identity cycle π_{id}

$$\pi_{id}: A_0 A_1 \dots A_m$$



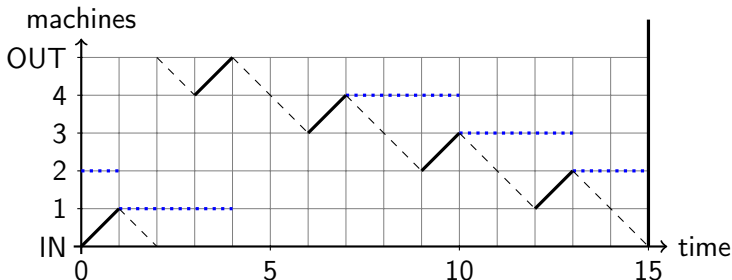
Cycle time

$$T(\pi_{id}) = (m + 1)\delta + mp$$

Some usual 1-cycles: Downhill cycle

Downhill cycle π_d

$$\pi_d: A_0 A_m A_{m-1} \dots A_1$$



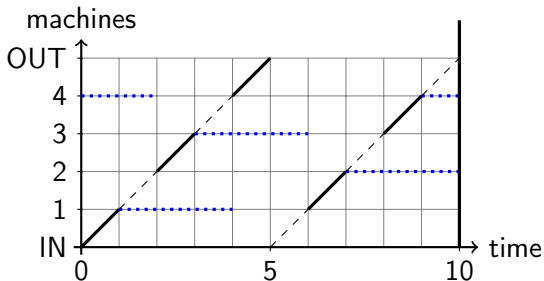
Cycle Time

$$T(\pi_d) = 3(m+1)\delta + \max(0, p - (3m-1)\delta)$$

Some usual 1-cycles: Odd-even cycle

Odd-Even cycle π_{oe}

$\pi_{oe}: A_0A_2 \dots A_1A_3 \dots$



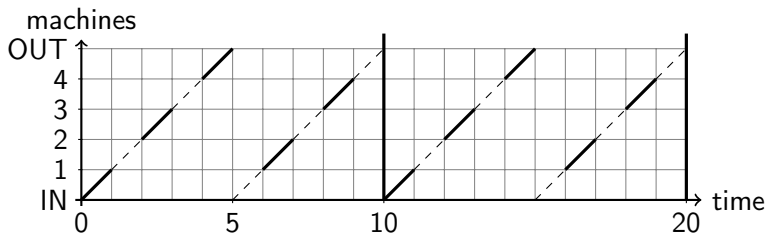
Cycle time

$T(\pi_{oe}) = ??$

Odd-even's cycle time

$$T(\pi_{oe}) = 2(m+1)\delta + \max\left(0, \frac{2\alpha-1}{\alpha}(p - (m+1)\delta)\right) \quad \alpha = \lfloor \frac{m+1}{2} \rfloor$$

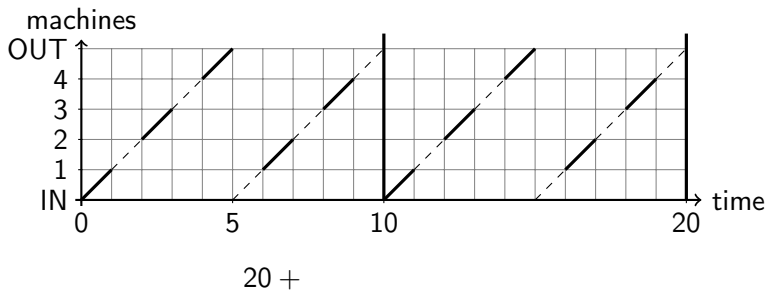
$$\leq 2p$$



Odd-even's cycle time

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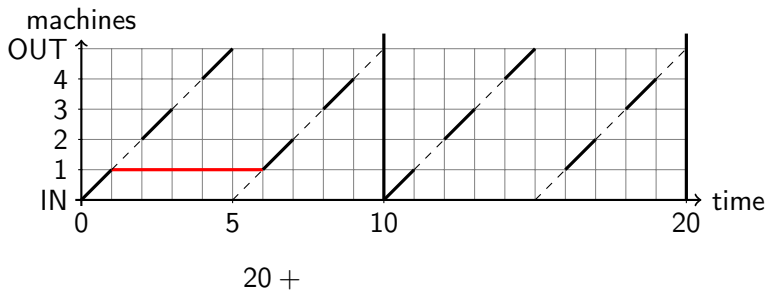
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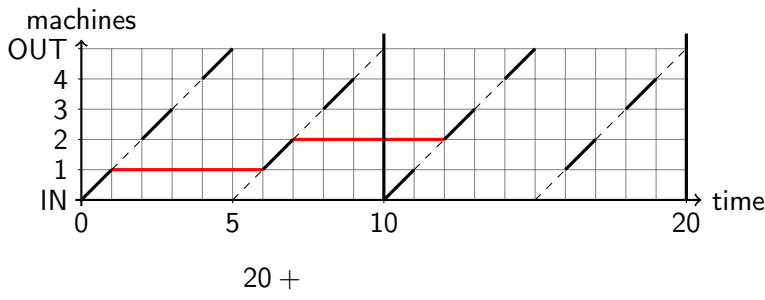
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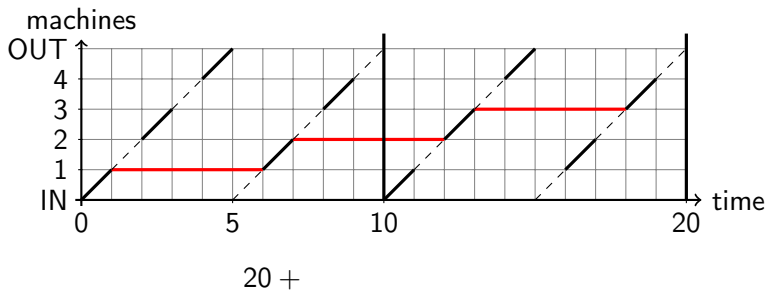
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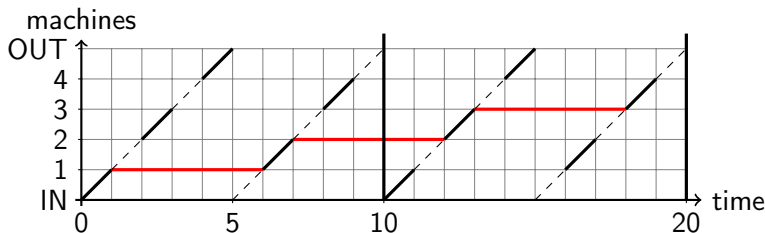
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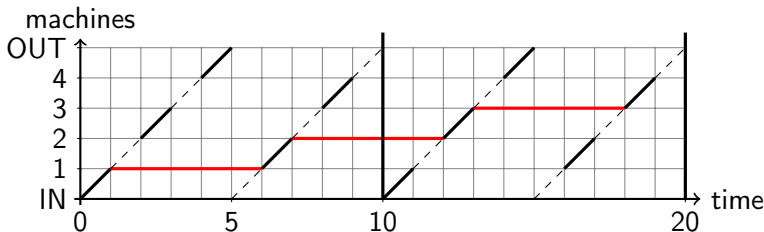


$$\underline{20 + 3 \max(0, (p - 5\delta))}$$

Odd-even's cycle time

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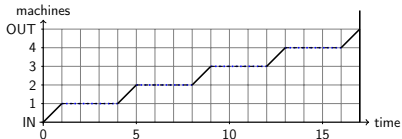
$$\leq 2p$$



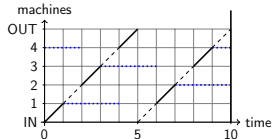
$$\frac{20 + 3 \max(0, (p - 5\delta))}{2}$$

Finding the best 1-cycle

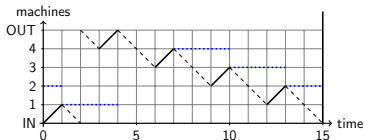
Identity



Odd-Even



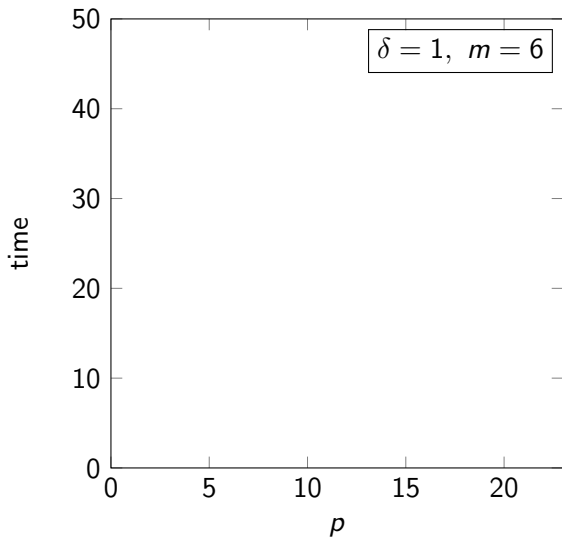
Downhill



?

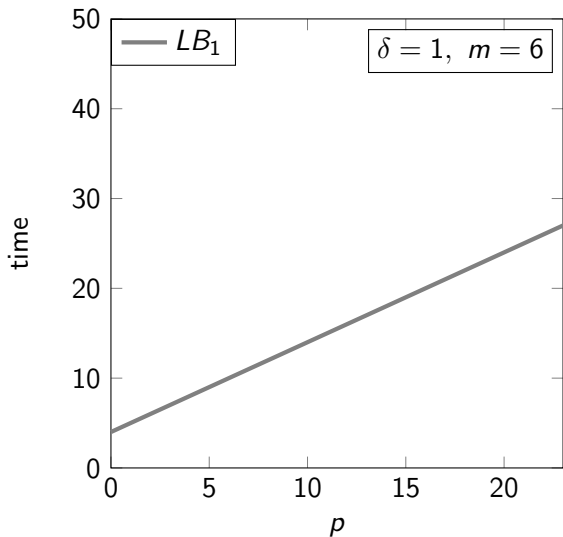
Which is the best 1-cycle?

Best 1-cycles?



Lower bounds

Best 1-cycles?

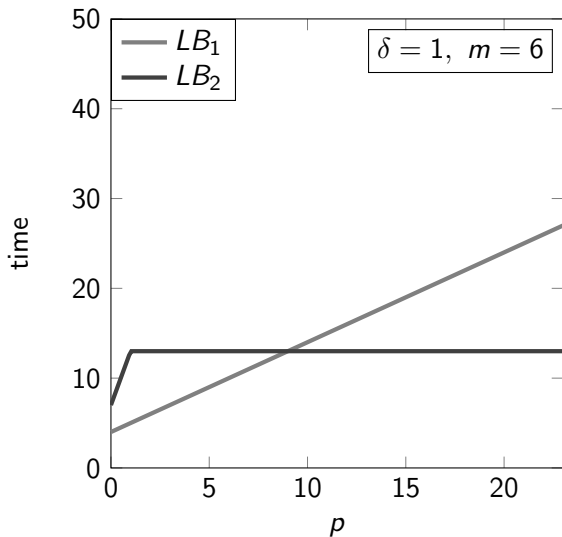


Lower bounds

LB_1 [Crama, 97]

$$T(\pi) \geq p + 4\delta$$

Best 1-cycles?



Lower bounds

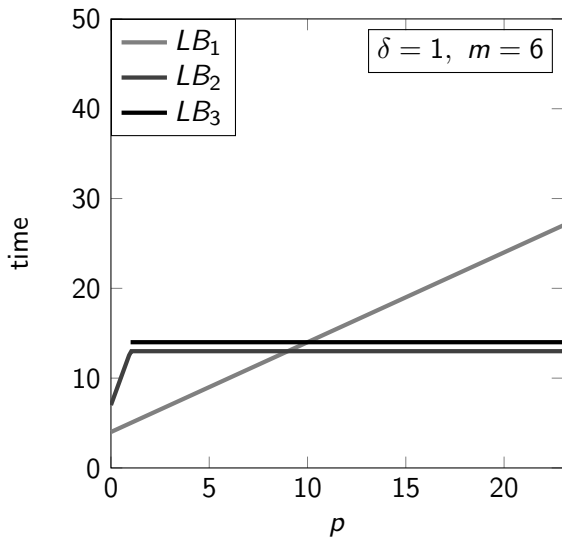
LB_1 [Crama, 97]

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LB_2 [Dawande et al, 02]

$$T(\pi) \geq (m + 1)\delta + m \min(\delta, p)$$

Best 1-cycles?



Lower bounds

LB₁ [Crama, 97]

$$T(\pi) \geq p + 4\delta$$

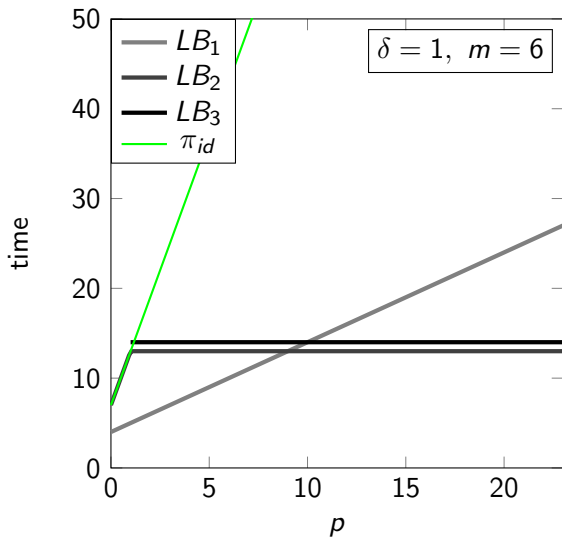
LB₂ [Dawande et al, 02]

$$T(\pi) \geq (m + 1)\delta + m \min(\delta, p)$$

LB₃ (1-cycles only)

$$T(\pi) \geq 2(m + 1)\delta$$

Best 1-cycles?



Lower bounds

 LB_1 [Crama, 97]

$$T(\pi) \geq p + 4\delta$$

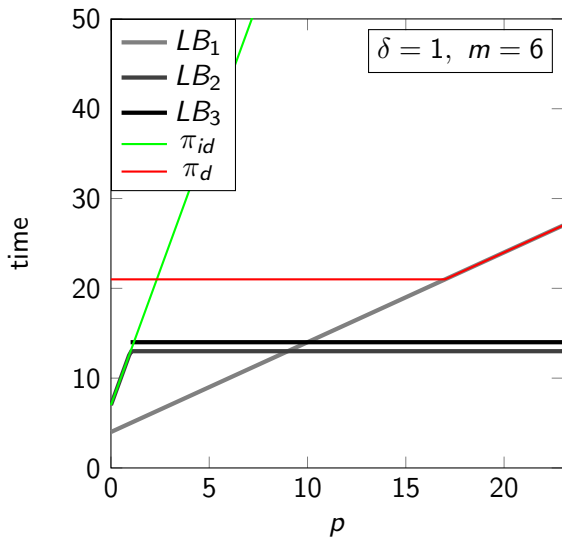
 LB_2 [Dawande et al, 02]

$$T(\pi) \geq (m + 1)\delta + m \min(\delta, p)$$

 LB_3 (1-cycles only)

$$T(\pi) \geq 2(m + 1)\delta$$

Best 1-cycles?



Lower bounds

LB_1 [Crama, 97]

$$T(\pi) \geq p + 4\delta$$

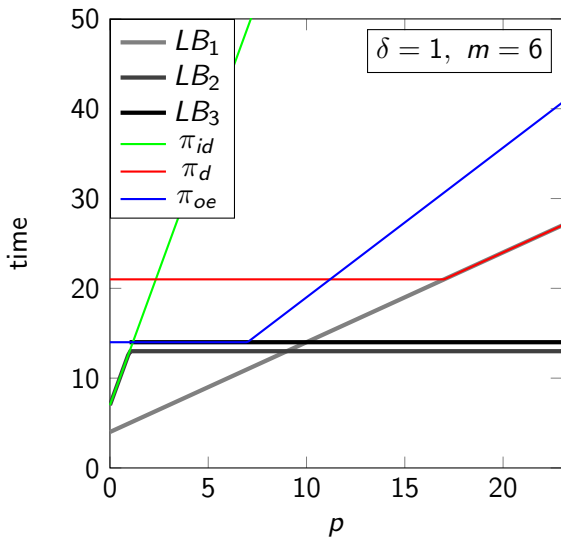
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Best 1-cycles?



Lower bounds

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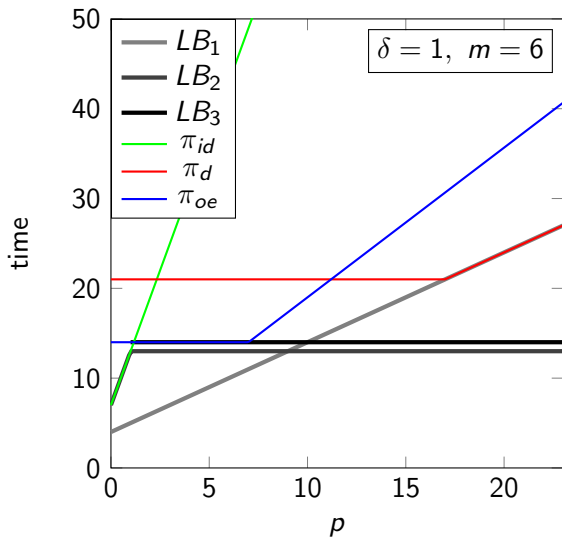
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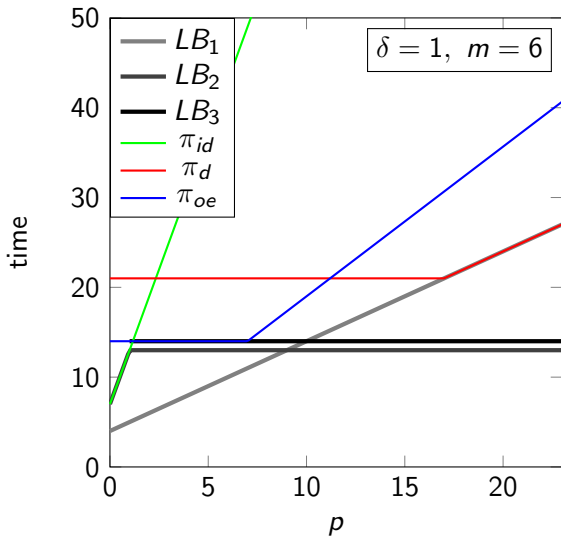
The unknown region



Best 1-cycle

- $p \leq \frac{m+1}{m} \delta$
 π_{id}

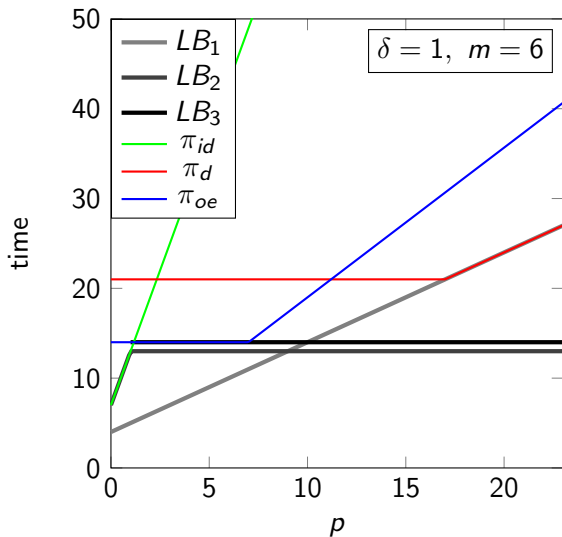
The unknown region



Best 1-cycle

- $p \leq \frac{m+1}{m}\delta$
 π_{id}
- $p > \frac{m+1}{m}\delta$
 $p \leq (m+1)\delta$
 π_{oe}

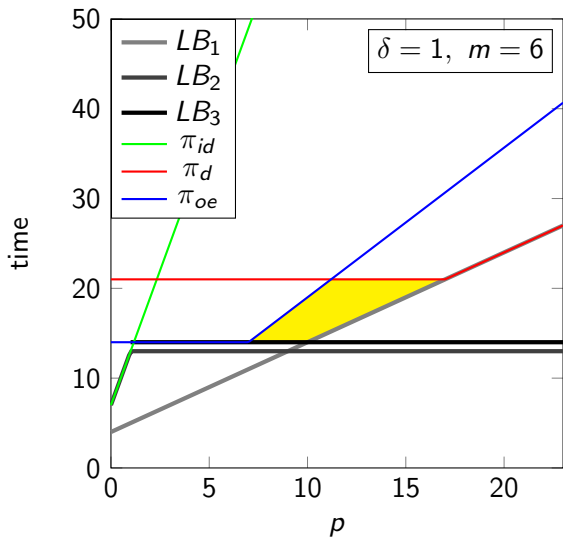
The unknown region



Best 1-cycle

- $p \leq \frac{m+1}{m}\delta$
 π_{id}
- $p > \frac{m+1}{m}\delta$
 $p \leq (m+1)\delta$
 π_{oe}
- $p \geq (3m-1)\delta$
 π_d

The unknown region



Best 1-cycle

- $p \leq \frac{m+1}{m}\delta$
 π_{id}
- $p > \frac{m+1}{m}\delta$
 $p \leq (m+1)\delta$
 π_{oe}
- $p \geq (3m-1)\delta$
 π_d

between $(m+1)\delta$
and $(3m-1)\delta$?

Exploring the unknown region

Best 1-cycles : $\{\pi_{id}, \pi_{oe}, \pi_d\} + ??$

Introducing π^*

- π^* is a 1-cycle
- π^* crosses the yellow area
(meaning: for some $(m + 1)\delta \leq p \leq (3m - 1)\delta$,
 π^* does strictly better than both π_{oe} and π_d)

Exploring the unknown region

Best 1-cycles : $\{\pi_{id}, \pi_{oe}, \pi_d\} + ??$

Introducing π^*

- π^* is a 1-cycle
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(meaning: for some $(m + 1)\delta \leq p \leq (3m - 1)\delta$,
 π^* does strictly better than both π_{oe} and π_d)

Who is π^* ?

What can we say about it? Does it exist?



from "Up", Disney - Pixar

Is there anybody in there?

Size of a minimum dominant set within 1-cycles:

<i>m</i>	3	4	5	6	7	8	9	10	11	12	13	14
<i>cardinal</i>	3	4	4	3	3	3	4	4	4	5	4	4

proven

computed

$$\{\pi_{id}, \pi_{oe}, \pi_d\} \cup ?$$

Is there anybody in there?

Size of a minimum dominant set within 1-cycles:

<i>m</i>	3	4	5	6	7	8	9	10	11	12	13	14
<i>cardinal</i>	3	4	4	3	3	3	4	4	4	5	4	4

proven

computed

$$\{\pi_{id}, \pi_{oe}, \pi_d\} \cup ?$$

[Brauner, 99]

Is there anybody in there?

Size of a minimum dominant set within 1-cycles:

<i>m</i>	3	4	5	6	7	8	9	10	11	12	13	14
<i>cardinal</i>	3	4	4	3	3	3	4	4	4	5	4	4

proven

computed

$\{\pi_{id}, \pi_{oe}, \pi_d\} \cup ?$

Properties on the total travel time

First properties

Notations (last ones...)

$\Delta(\pi)$: total travel time

$d_i(\pi)$: travel time between the loading and unloading of machine M_i

$d_{min}(\pi)$: $\min d_i(\pi)$ e.g. $d_{min}(\pi_{oe}) = (m + 1)\delta$

Lower bound $LB_4(\pi)$

$$T(\pi) \geq \underbrace{\Delta(\pi)}_{\text{Total travel time}} + \underbrace{\max(0, p - d_{min}(\pi))}_{\text{Minimum waiting time}}$$

d_{min} is the minimum value of p for which waiting is necessary.

First properties

If π^* exists:

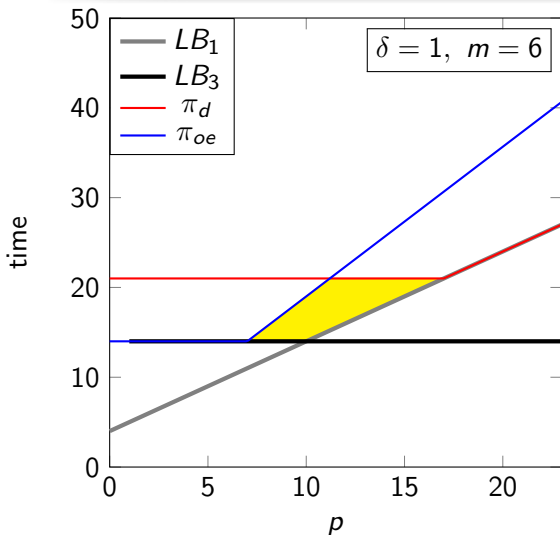
- The robot travels between 2 and 3 times the size of the cell:

$$2(m+1)\delta < \Delta(\pi^*) < 3(m+1)\delta$$

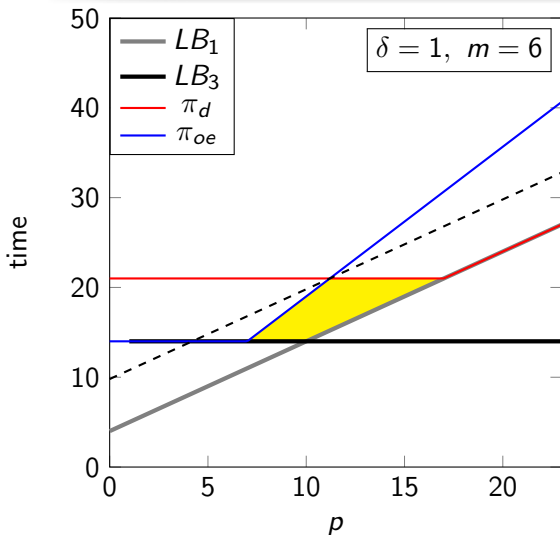
- No waiting time if p is “small” relatively to the travel time:

$$d_{min}(\pi^*) > \Delta(\pi^*) - \frac{3\alpha - 2}{2\alpha - 1}(m+1)\delta$$

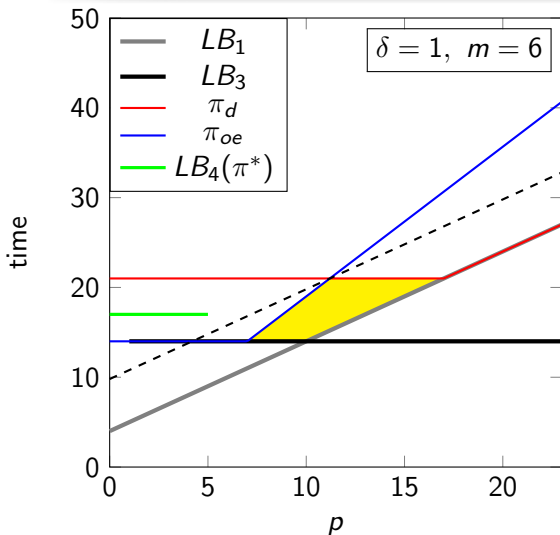
First properties



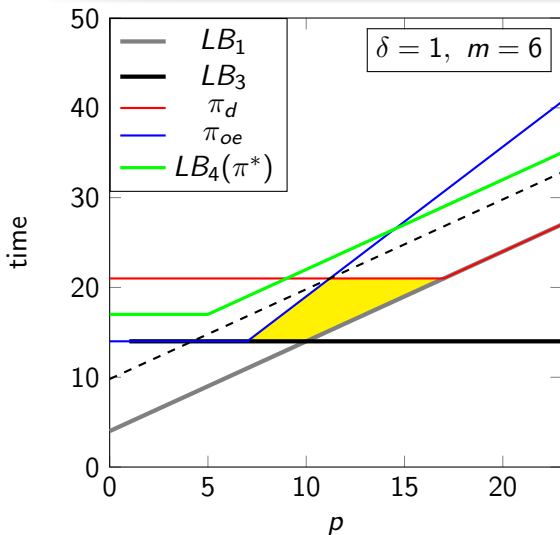
First properties



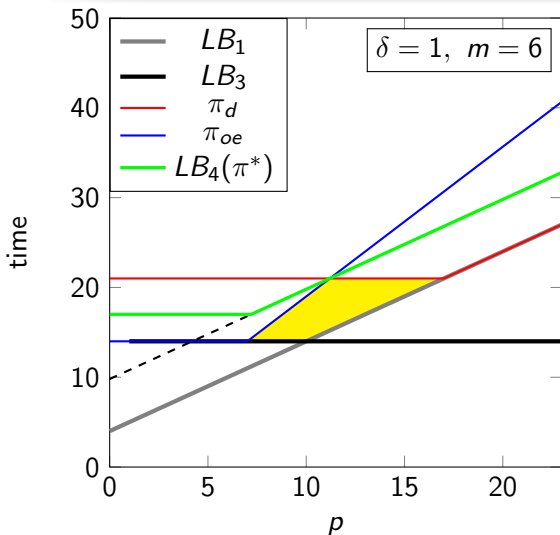
First properties



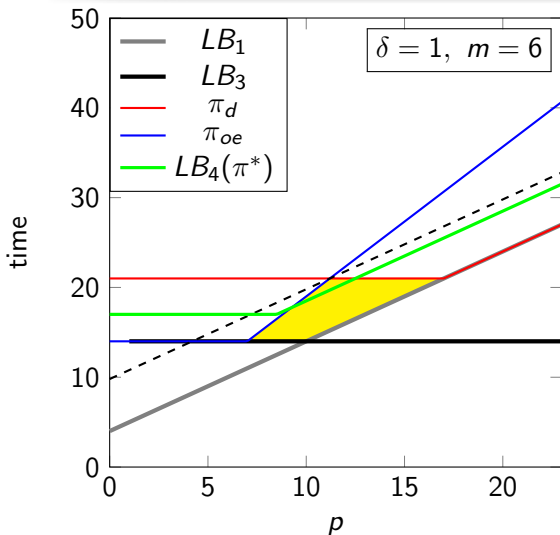
First properties



First properties



First properties



And for $m > 8$?

Cardinal of a minimum dominant set within 1-cycles:

m	3	4	5	6	7	8	9	10	11	12	13	14
<i>cardinal</i>	3	4	4	3	3	3	4	4	4	5	4	4

proven

computed

And for $m > 8$?

Cardinal of a minimum dominant set within 1-cycles:

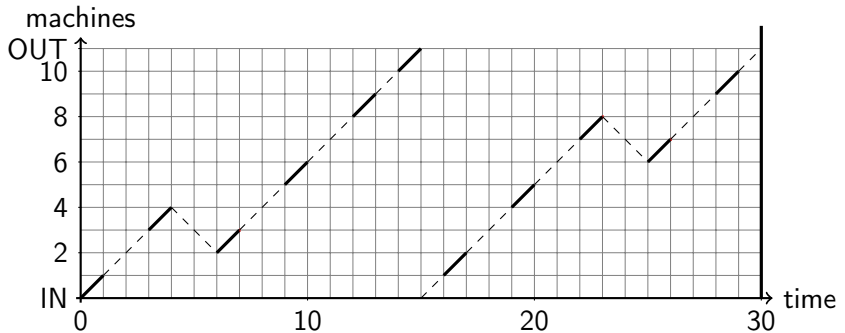
m	3	4	5	6	7	8	9	10	11	12	13	14
<i>cardinal</i>	3	4	4	3	3	3	4	4	4	5	4	4

proven

computed

Two wavelets (π_{2w})...

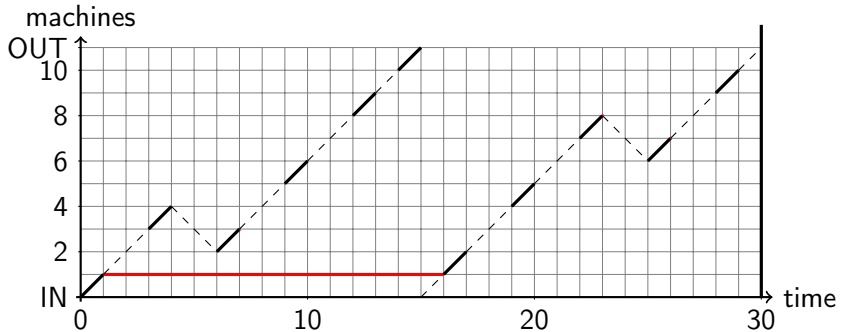
For $m > 8$, this guy...



...always (slightly) crosses the yellow area

Two wavelets (π_{2w})...

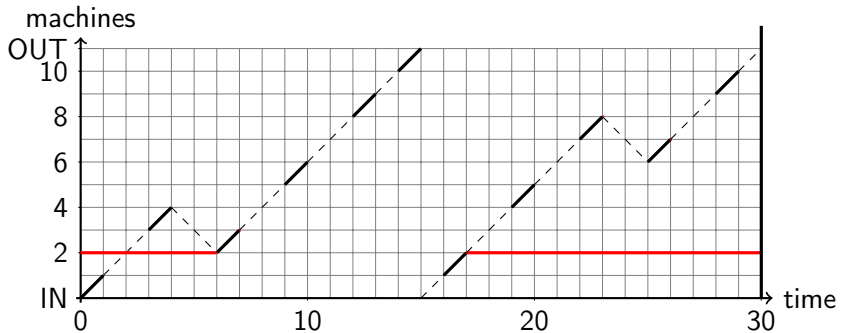
For $m > 8$, this guy...



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Two wavelets (π_{2w})...

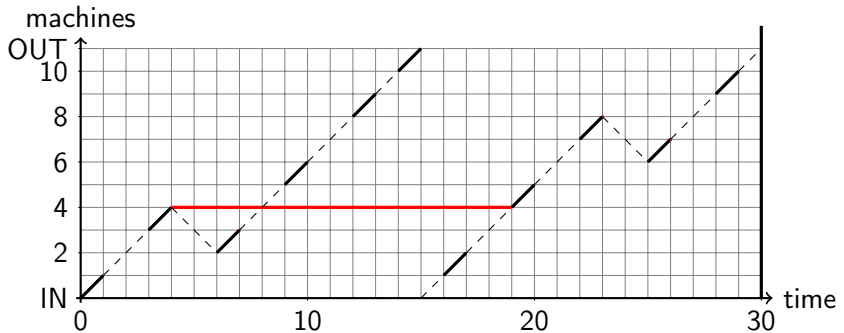
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Two wavelets (π_{2w})...

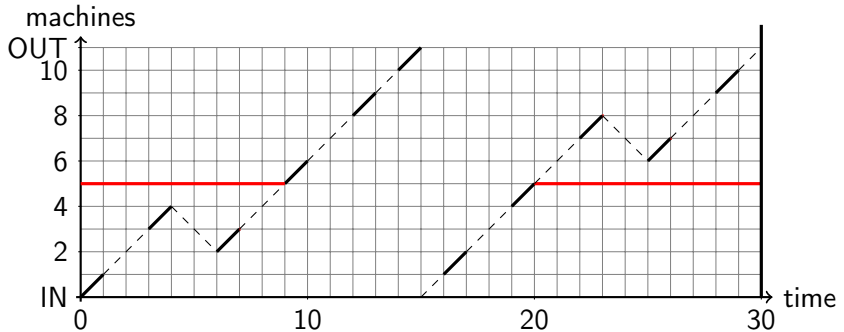
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Two wavelets (π_{2w})...

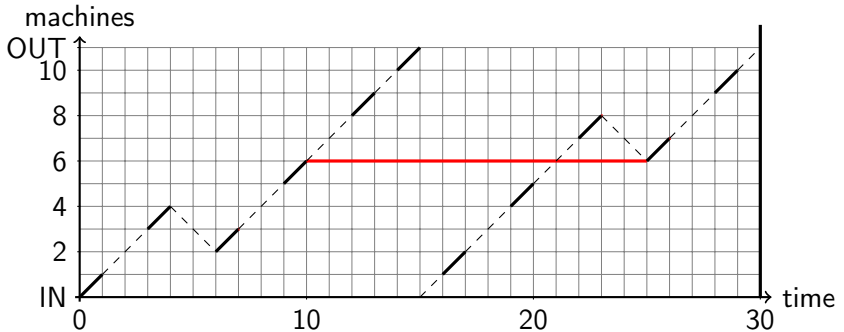
For $m > 8$, this guy...



...always (slightly) crosses the yellow area

Two wavelets (π_{2w})...

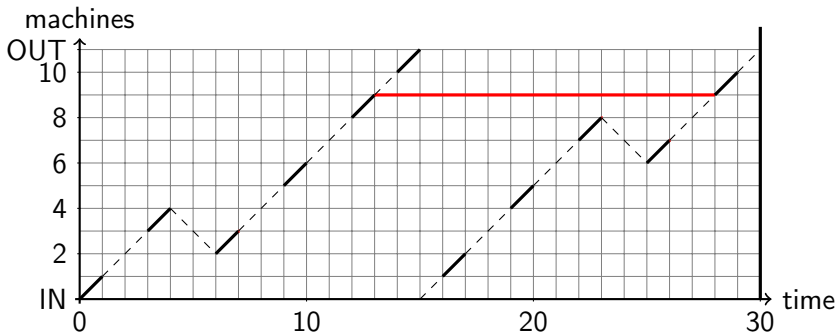
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Two wavelets (π_{2w})...

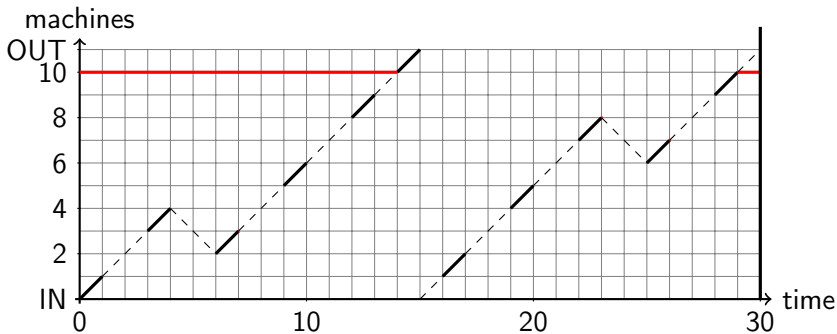
For $m > 8$, this guy...



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Two wavelets (π_{2w})...

For $m > 8$, this guy...



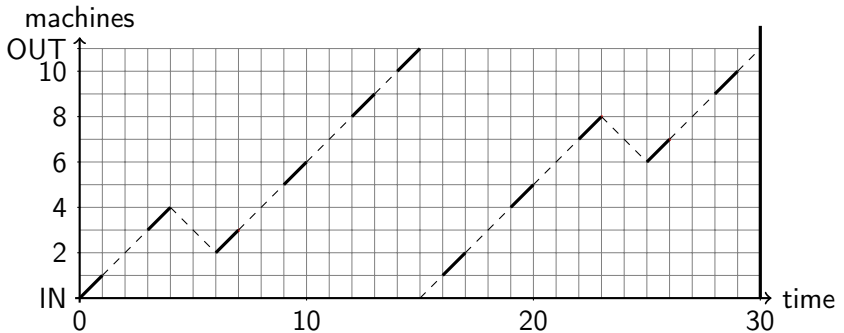
...always (slightly) crosses the yellow area

Two wavelets (π_{2w})...

For $m > 8$, this guy...

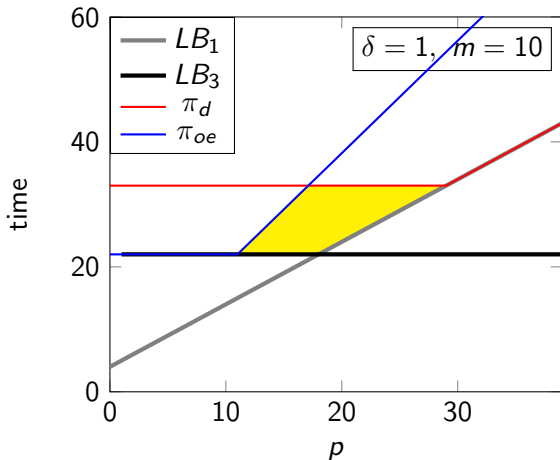
$$d_{\min}(\pi_{2w}) = (m + 5)\delta$$

(One turn and one wave)

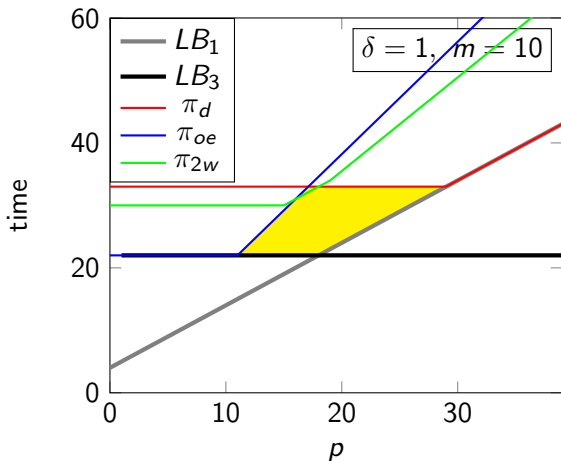


...always (slightly) crosses the yellow area

Two wavelets (π_{2w})...

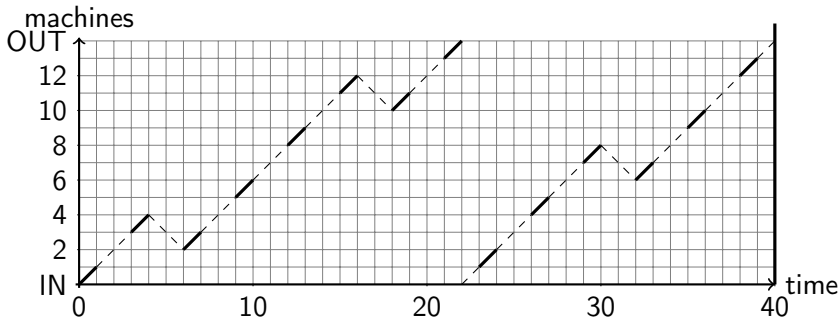


Two wavelets (π_{2w})...



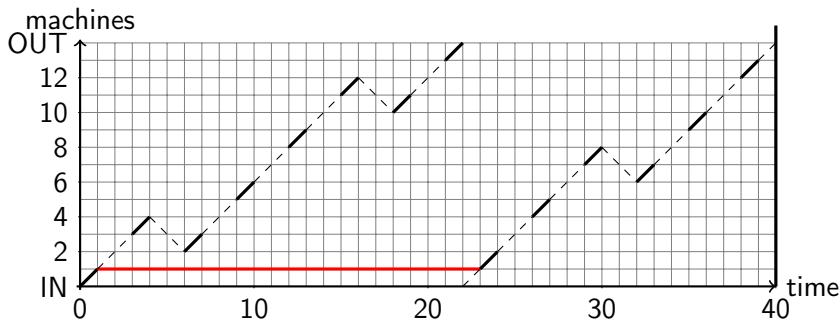
Three wavelets (π_{3w})...

For $m = 12$ and $m = 13$...



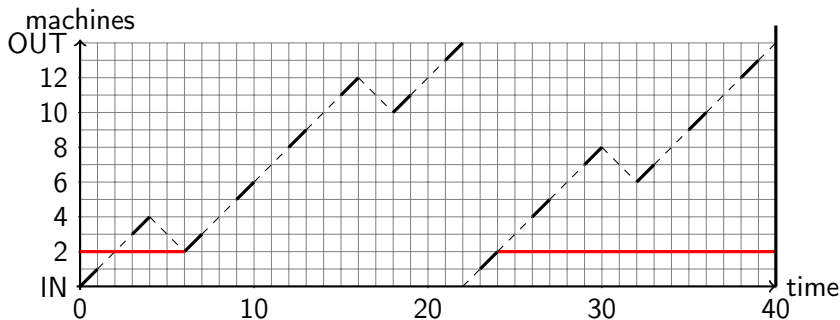
Three wavelets (π_{3w})...

For $m = 12$ and $m = 13$...



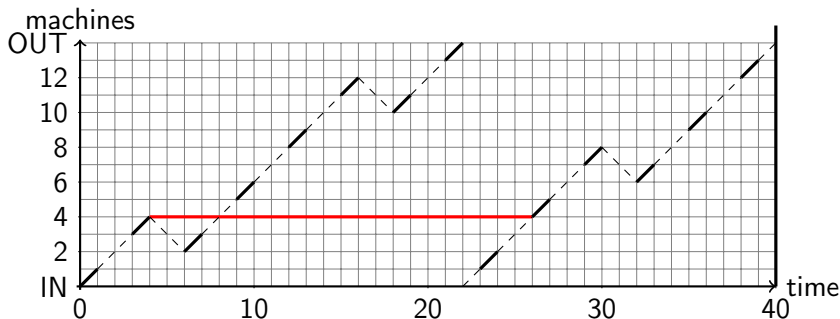
Three wavelets (π_{3w})...

For $m = 12$ and $m = 13$...



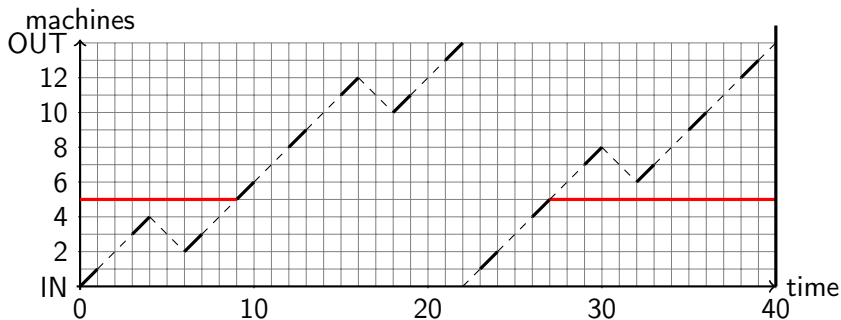
Three wavelets (π_{3w})...

For $m = 12$ and $m = 13$...



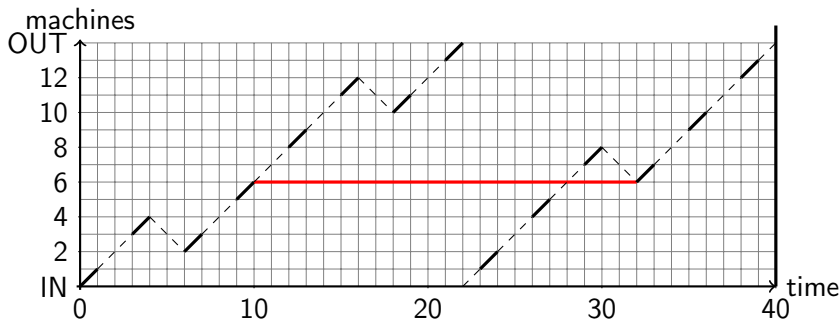
Three wavelets (π_{3w})...

For $m = 12$ and $m = 13$...



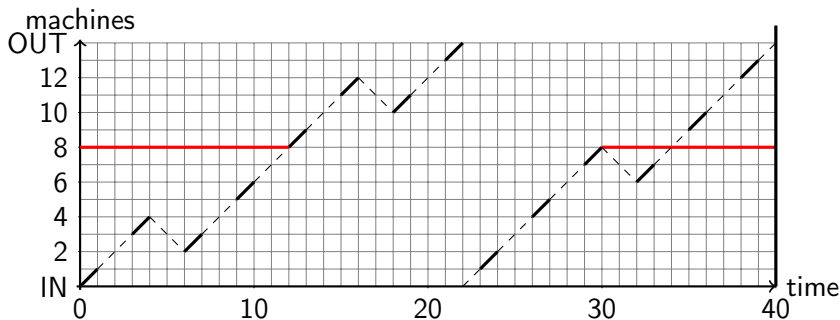
Three wavelets (π_{3w})...

For $m = 12$ and $m = 13$...



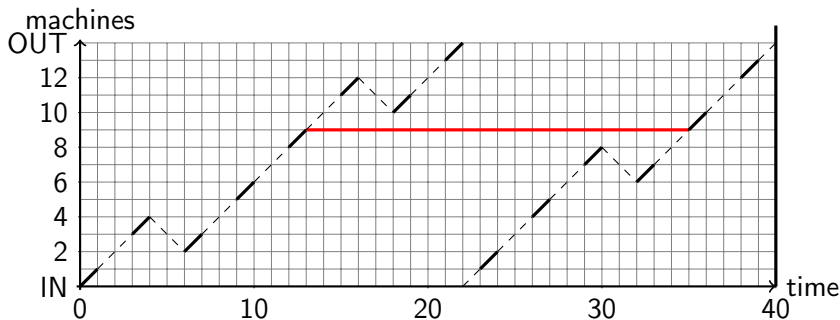
Three wavelets (π_{3w})...

For $m = 12$ and $m = 13$...



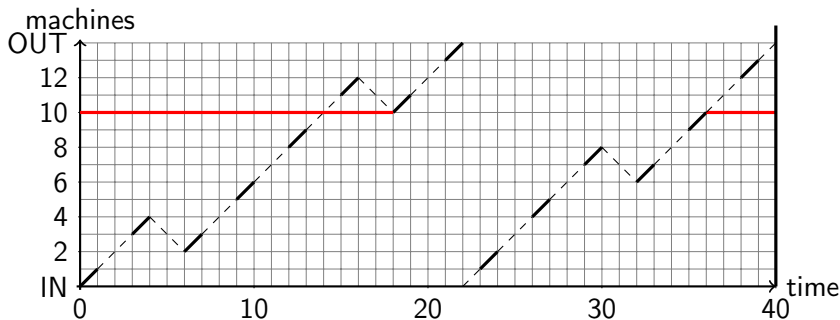
Three wavelets (π_{3w})...

For $m = 12$ and $m = 13$...



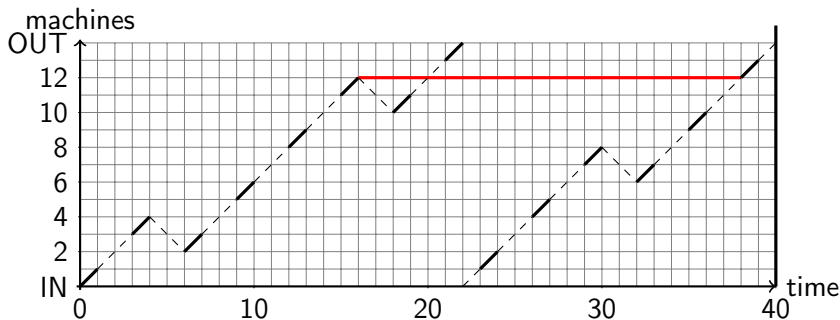
Three wavelets (π_{3w})...

For $m = 12$ and $m = 13$...



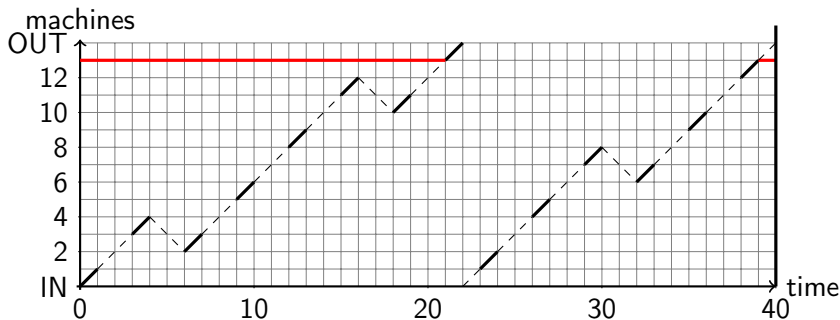
Three wavelets (π_{3w})...

For $m = 12$ and $m = 13$...



Three wavelets (π_{3w})...

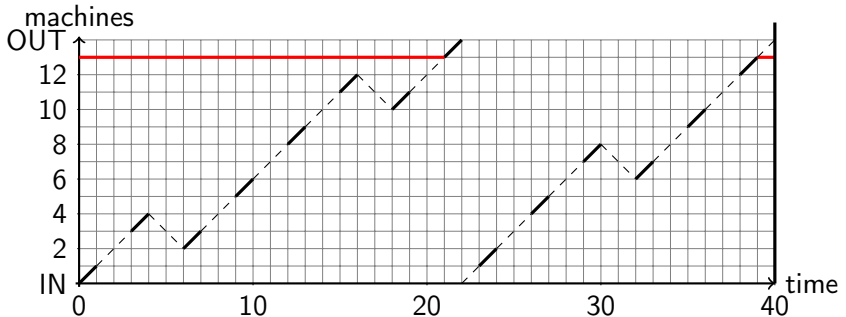
For $m = 12$ and $m = 13$...



Three wavelets (π_{3w})...

For $m = 12$ and $m = 13$...

$d_{min}(\pi_{2w}) = (m + 9)\delta$
(One turn and two waves)

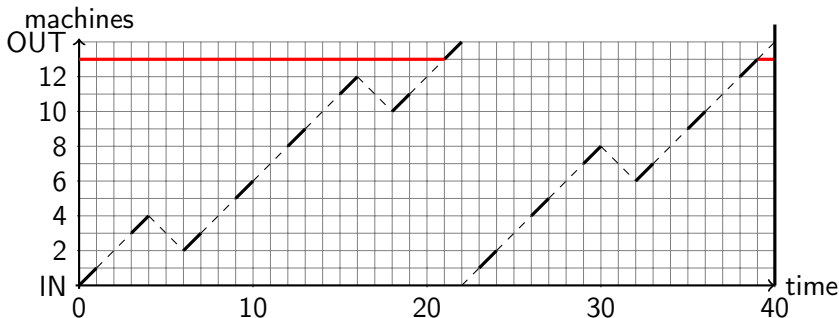


Three wavelets (π_{3w})...

For $m = 12$ and $m = 13$...

False for $m \geq 14$!

$d_{min}(\pi_{2w}) = (m + 9)\delta$
(One turn and two waves)

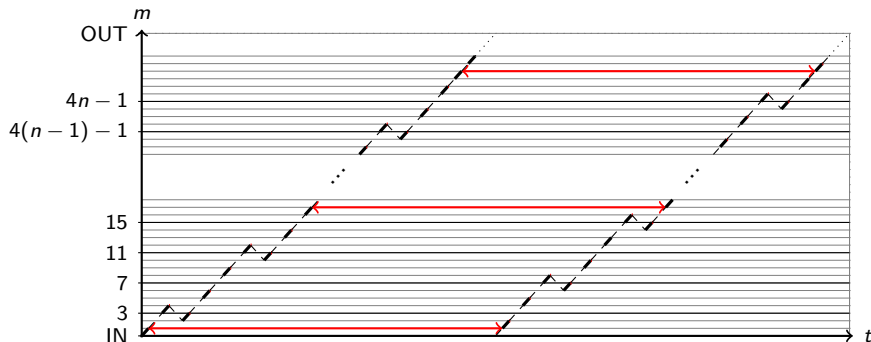


... n wavelets (π_{nw})

(here for n even)

$$d_{min}(\pi_{nw}) = (m + 2n + 1)\delta$$

(One turn and n waves)



G-SCOP $m \geq 8$: A proposition and a conjecture

Proposition: best cycles within $\{\pi_{id}, \pi_d, \pi_{oe}\} \cup (\pi_{nw})_n$

- **Three** classical 1-cycles $\pi_{id}, \pi_d, \pi_{oe}$
- **+One** π_{nw} with the highest **even** value of n possible
And/or
- **+One** π_{nw} with the highest **odd** value of n possible

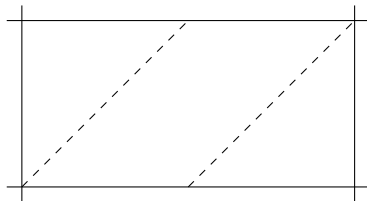
Conjecture: best 1-cycle

These also dominate **all** 1-cycles

Proven for $m \leq 11$

Proof ideas

- 2 “turns”



$$T(\pi^*) < 2p$$

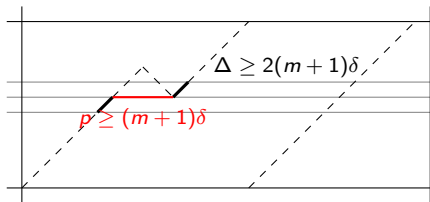
(otherwise, dominated by π_{oe})

$$T(\pi^*) < 3(m+1)\delta$$

(otherwise, dominated by π_d)

Proof ideas

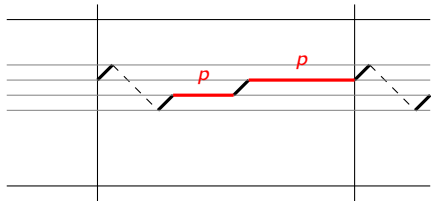
- 2 “turns”
- A_i and A_{i+1} can't be on the same turn in that order



$T(\pi^*) < 2p$
 (otherwise, dominated by π_{oe})
 $T(\pi^*) < 3(m+1)\delta$
 (otherwise, dominated by π_d)

Proof ideas

- 2 “turns”
- A_i and A_{i+1} can't be on the same turn in that order
- no subsequence $A_i A_{i-2}$
(rules out bigger alterations up to 11 machines)



$$T(\pi^*) < 2p$$

(otherwise, dominated by π_{oe})

$$T(\pi^*) < 3(m+1)\delta$$

(otherwise, dominated by π_d)

Best 1 cycles for $m \leq 11 \dots$

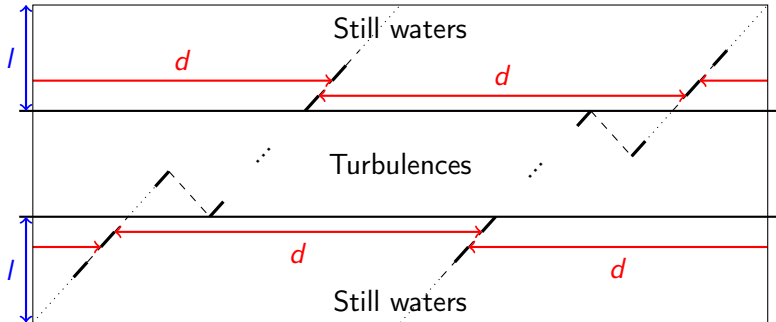
Cardinal of a minimum dominant set within 1-cycles:

m	3	4	5	6	7	8	9	10	11	12	13	14
<i>cardinal</i>	3	4	4	3	3	3	4	4	4	5	4	4

$$6 \leq m \leq 8: \{\pi_{id}, \pi_{oe}, \pi_d\}$$

$$9 \leq m \leq 11: \{\pi_{id}, \pi_{oe}, \pi_d, \pi_{2w}\}$$

And for any m ?

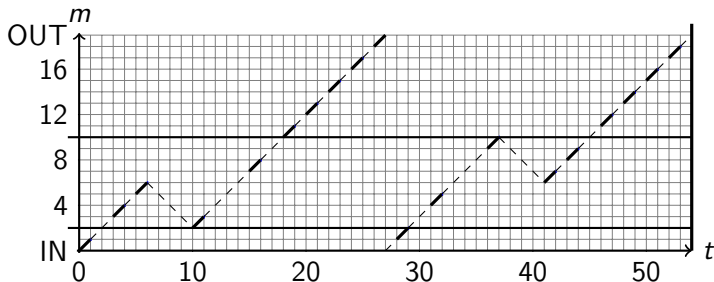


Lower bound on the waiting time: $\nearrow l$, $\searrow d$
 (tight for p_{nw} cycles)

And for any m ?

Not proven Regular disposition of the waves in the “turbulence” areas: seems intuitive but...

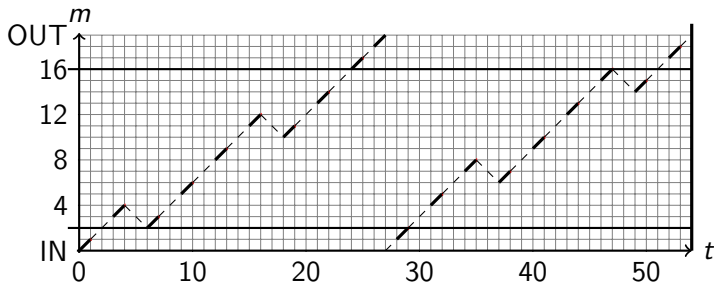
Proven Wavelets preferable to big waves:



And for any m ?

Not proven Regular disposition of the waves in the “turbulence” areas: seems intuitive but...

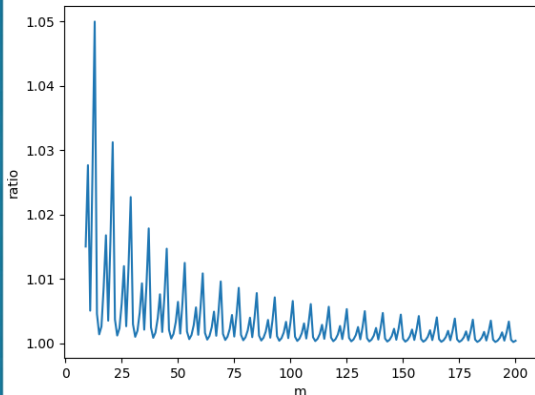
Proven Wavelets preferable to big waves:



If the conjecture is valid...

If the conjecture is valid, then...

- The best 1-cycle problem in circular, regular balanced cell would be **polynomial**
- Performance ratio of the usual cycles:



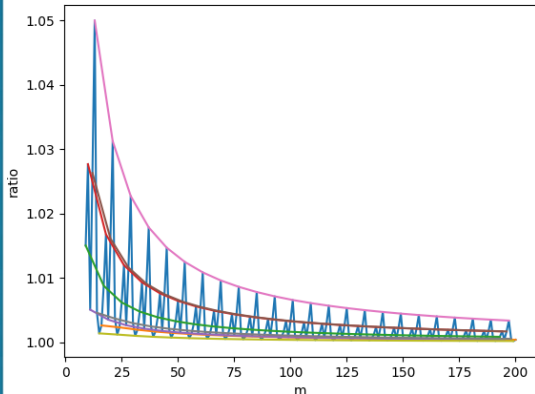
$$\frac{\min(T(\pi_{oe}), T(\pi_d))}{\min_n(T(\pi_{oe}), T(\pi_d), T(\pi_{nw}))}$$

(as a function of m)

If the conjecture is valid...

If the conjecture is valid, then...

- The best 1-cycle problem in circular, regular balanced cell would be **polynomial**
- Performance ratio of the usual cycles:



$$\frac{\min(T(\pi_{oe}), T(\pi_d))}{\min_n(T(\pi_{oe}), T(\pi_d), T(\pi_{nw}))}$$

(as a function of m)

Conclusions...

Throughput optimization in robotic cells with **circular layout** was less studied and poorly understood so far. We:

- proposed new tools and specific cycle structures...
- ... leading to a **conjecture** on the best 1-cycle problem.

	Best 1-cycle	1-cycle conjecture
Linear	P	$m \leq 3$: valid $m = 4$: false
balanced	P	$m \leq 15$: valid
Circular	NP-hard	
balanced	$m \leq 11$: P $m \geq 12$: also P?	$m = 6$: false

(Well, aside from settling the conjecture)

Other types of production constraints:

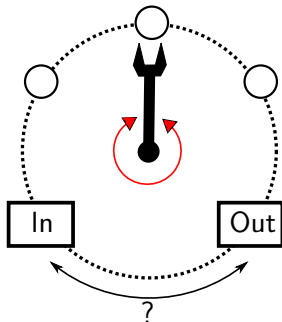
- Non-balanced case:
Improving existing approximation...
- Proportionate flow-shop

Open questions for regular balanced cells:

- Best 1-cycle for $m > 11$...
- 1-cycle conjecture for $m \leq 5$
- Cycle function for $m \geq 6$

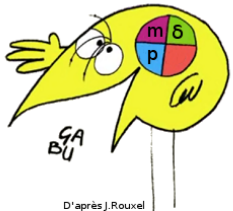
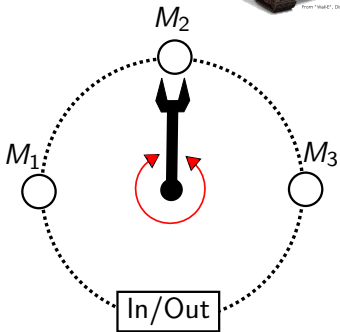
Relationships with other layouts:

- Comparisons of layouts
- Generalization of the circular layout:





Thank you!



Third party content

- This presentation features some tributes to the following works (short excerpts):
 - *Wall-E* (Pixar Animation Studios, 2008), slides 1, 4, 8 and 43;
 - *Up* (Pixar Animation Studios & Walt Disney Pictures, 2009), slides 25 and 43;
 - *Les Shadoks* (Jacques Rouxel), slides 16 and 43.
- The picture of a toy train featured on slides 1, 2, 3 and 43 is extracted from a Brio commercial.
- Other clipart images (slides 4 and 10) are either public domain or released under a CC0 license.
- The template and backgrounds belong to G-SCOP laboratory.