Scheduling Handling Resources: Robotic Flowshops with Circular Layouts and a Practical Railway Problem

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Jury members

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UMR 5272

Scheduling and transportation

Scheduling: assigning *tasks* to *resources* submitted to *constraints* to optimize a *performance criteria*

Physical handling or transportation between resources: connectivity, layout, additional specific handling resources

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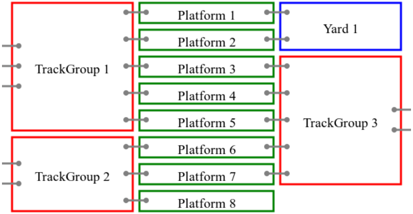
Two contexts:

- **Manufacturing management**

- Railway management

Railway management

“Trains don’t vanish”...except when they do.
EURO/Roadef Challenge 2014, SNCF



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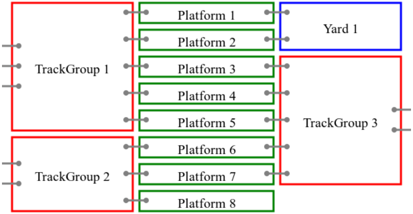

- Resource capacity
- Resource compatibility
- Conflict between trains
- ...

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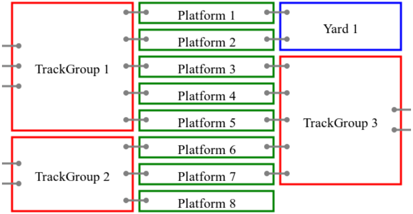

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Optimize departure coverage and performance costs

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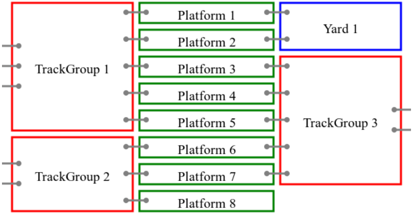

Horizon: 14 days
200–2000 departures/arrivals
30–90 resources

- Resource capacity
- Resource compatibility
- Conflict between trains
- ...

Optimize departure coverage and performance costs

Railway management

Joint work with H. Joudrier [Joudrier and T. 17]

- Simplify the station's graph by grouping similar resources
- Sophisticated routing algorithm
- (very) simple assignment algorithm
- Up to 40% covered departures...

... And the 1st place in the junior category!

Manufacturing management

Flowshop: the parts must be processed on machine M_1, then M_2...

$\boxed{M_1}$ $\boxed{M_2}$ $\boxed{M_3}$

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Sufficient model?

Manufacturing management

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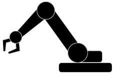

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Sufficient model? Operator, tools, **handling resources...**

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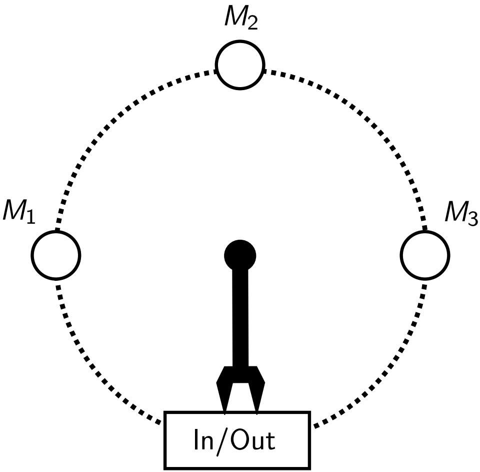

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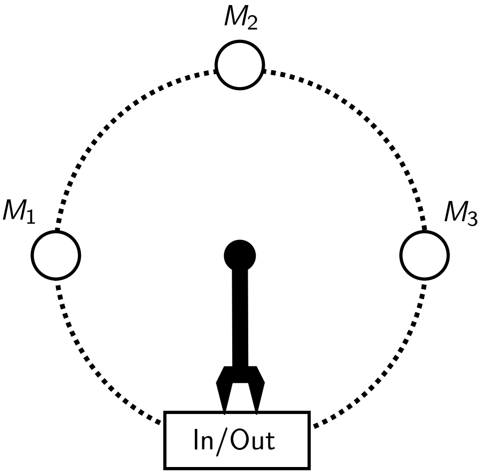

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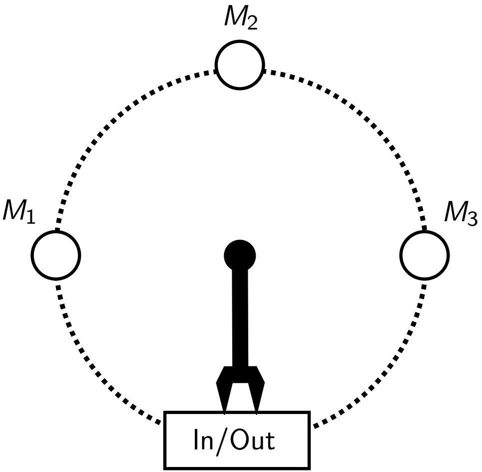

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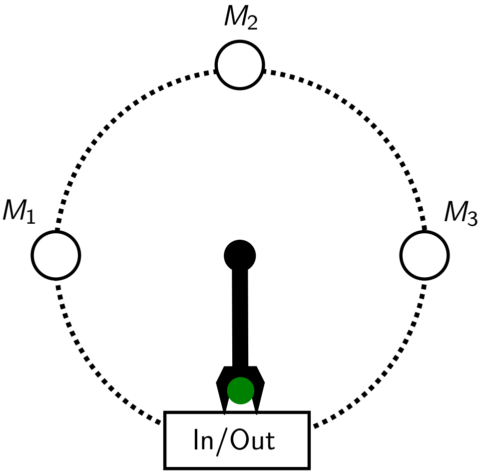

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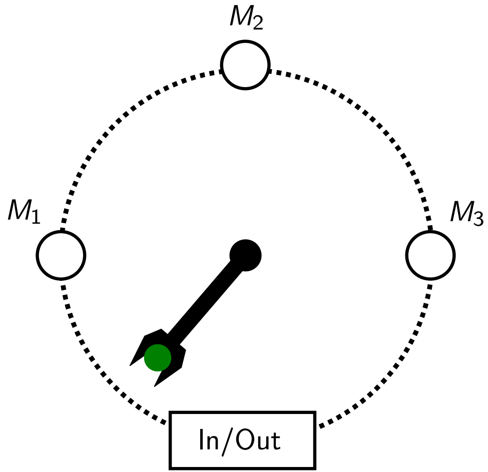

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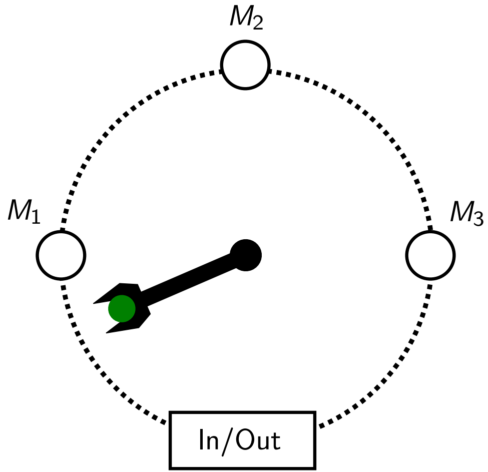

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G-SCOP

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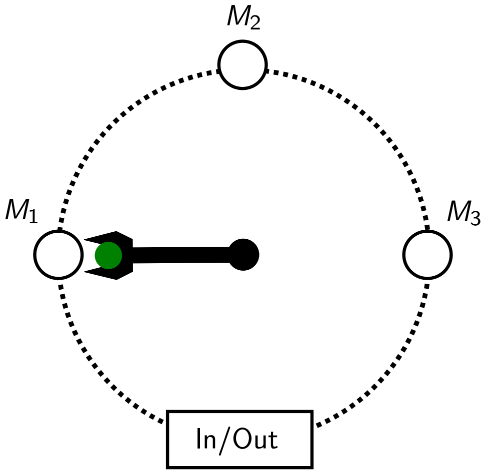

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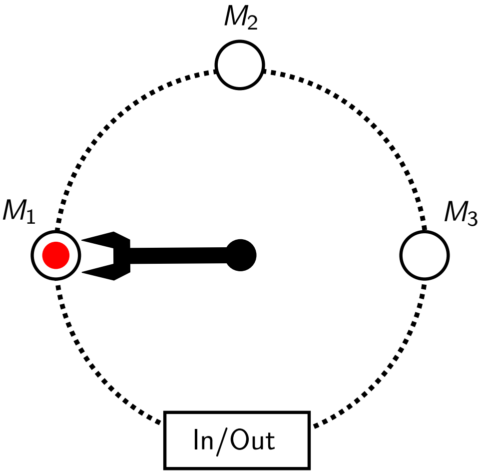

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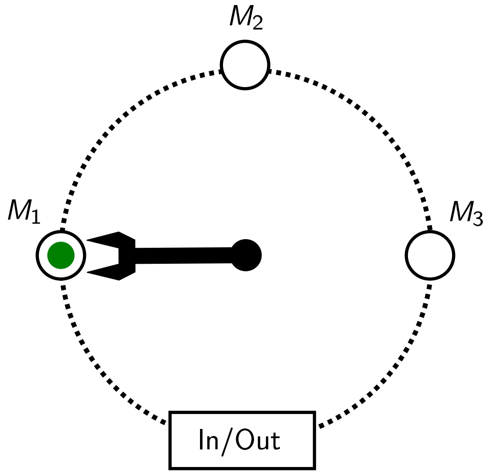

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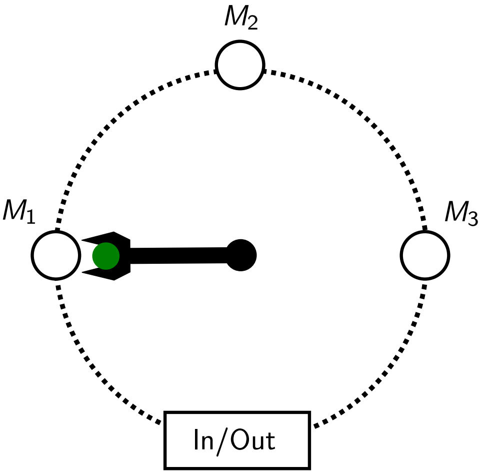

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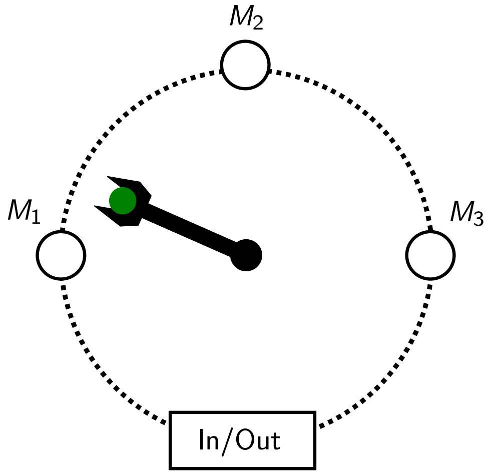

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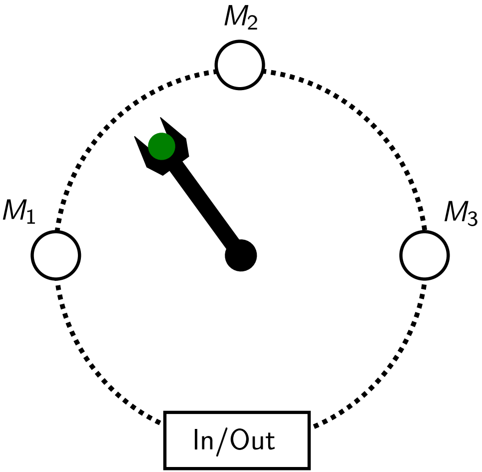

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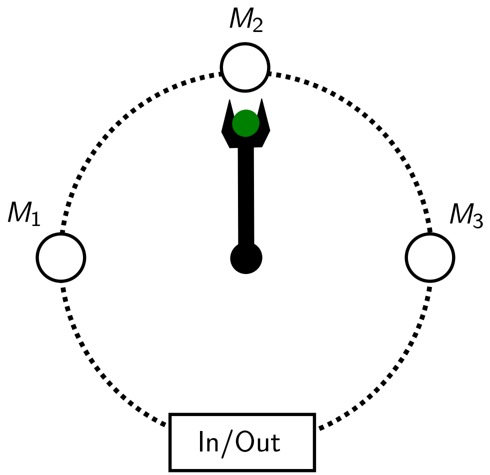

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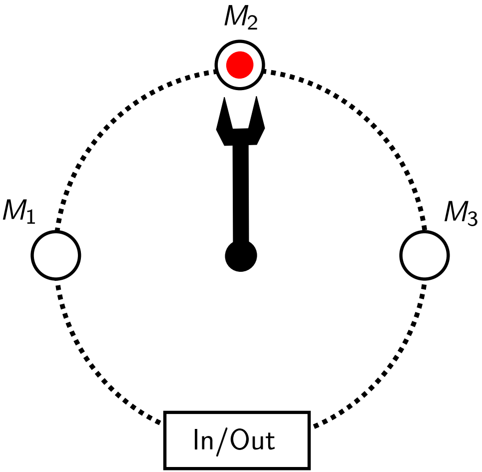

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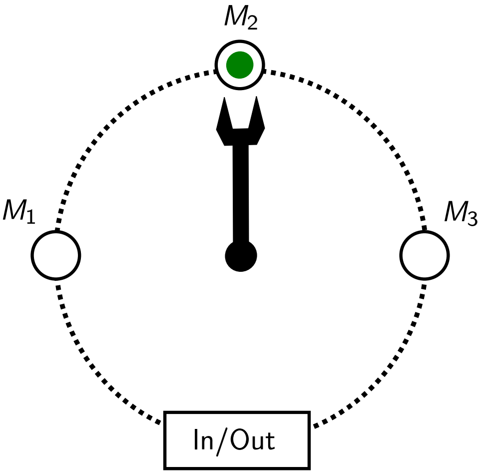

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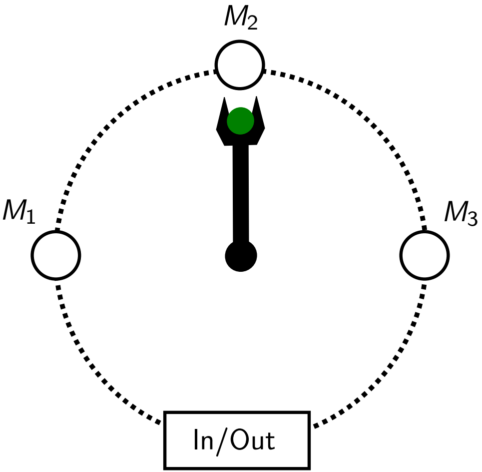

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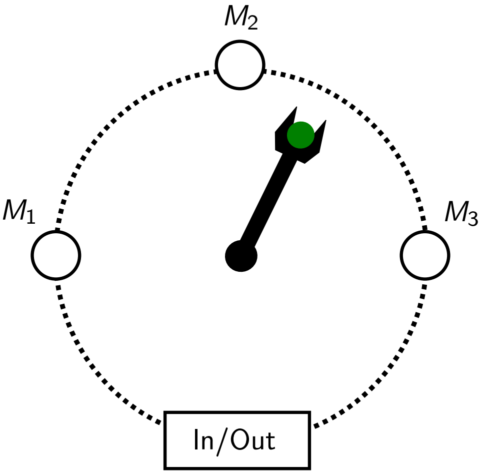

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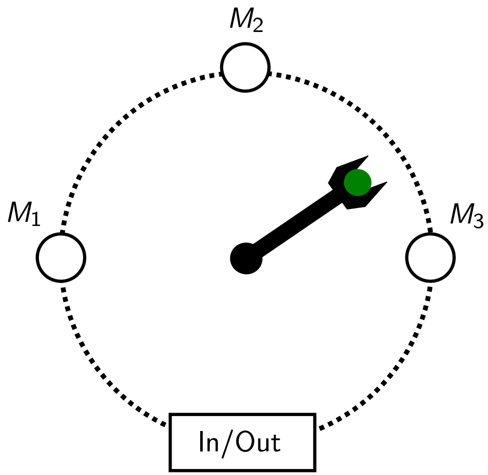

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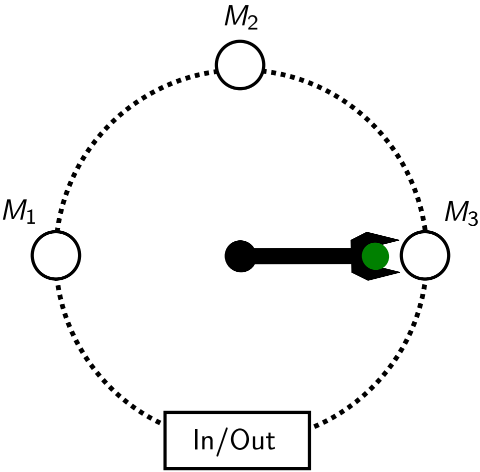

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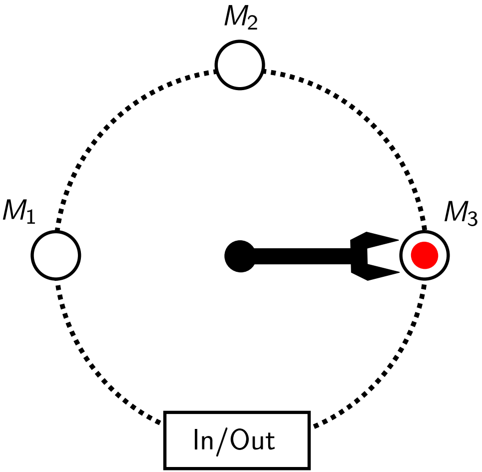

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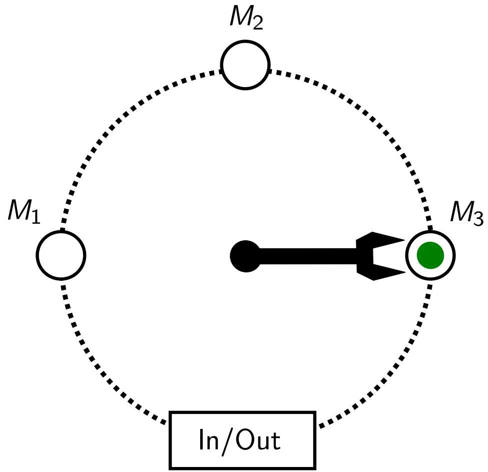

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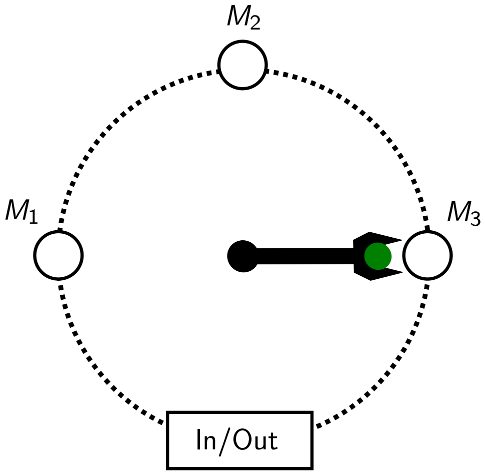

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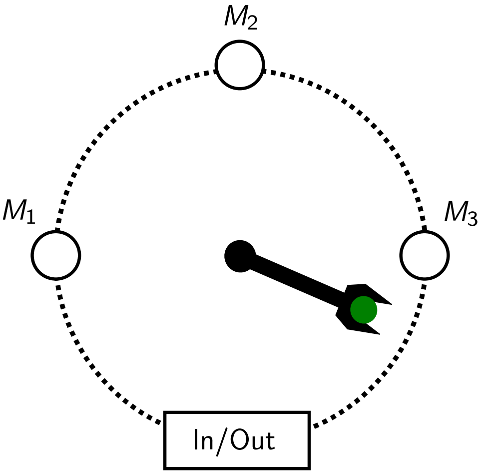

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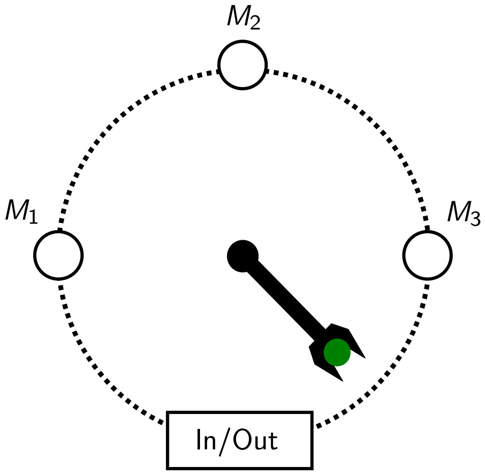

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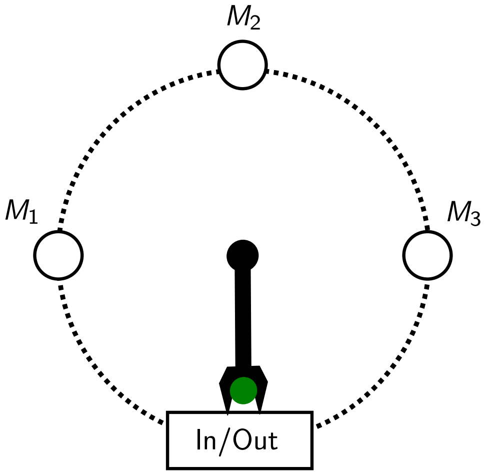

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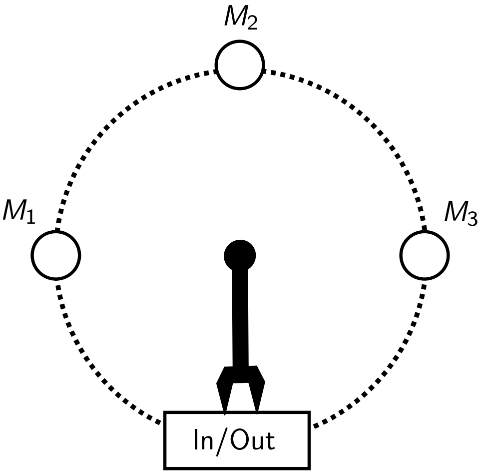

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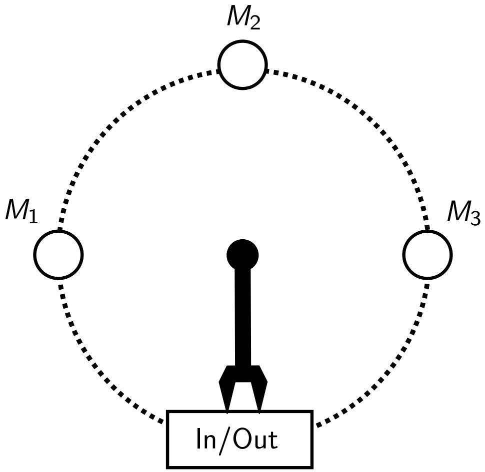

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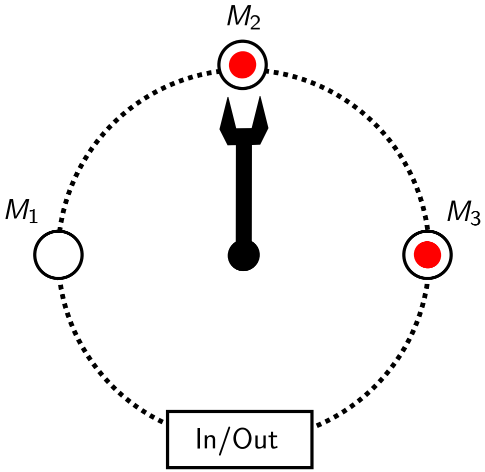

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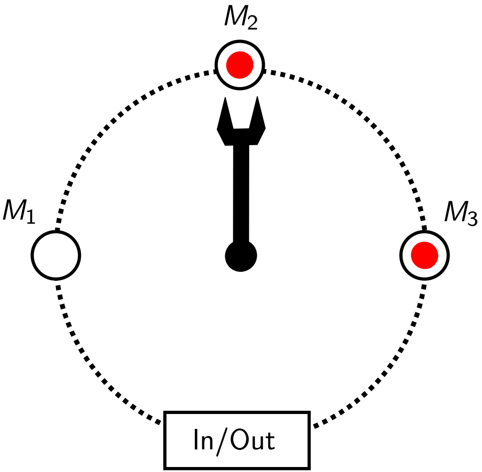

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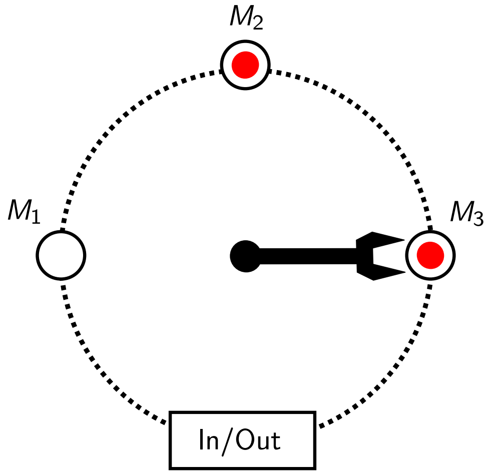

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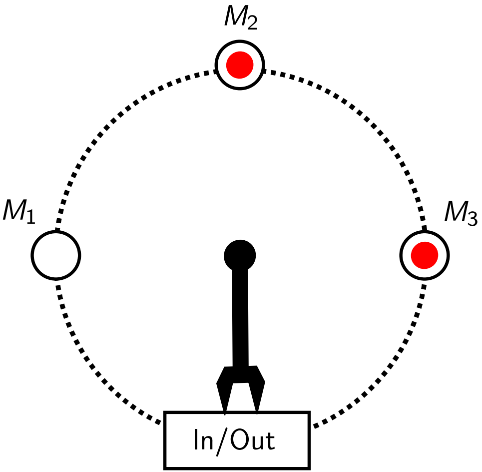

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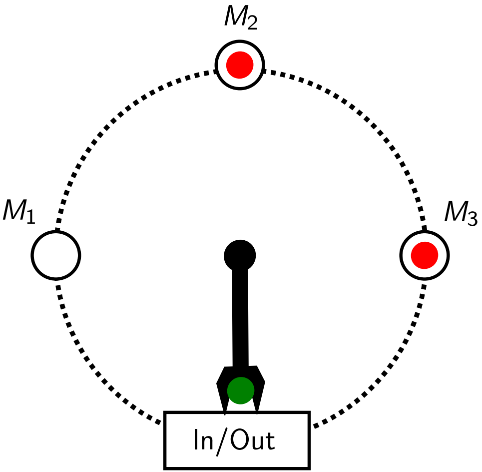

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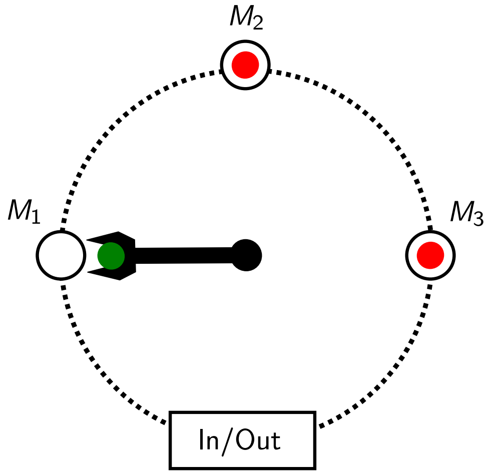

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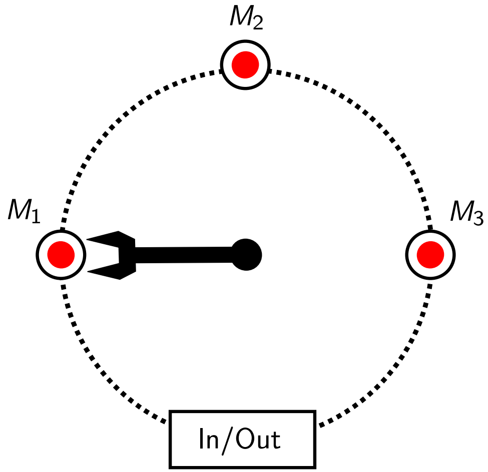

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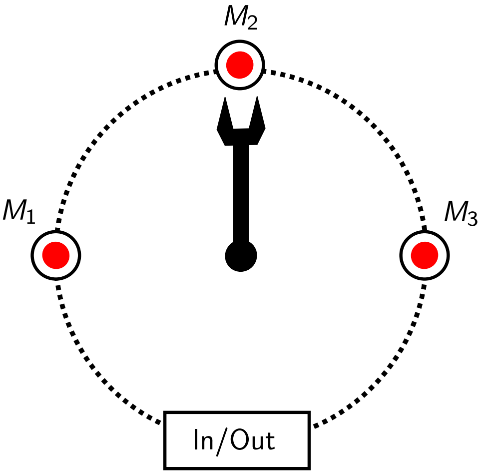

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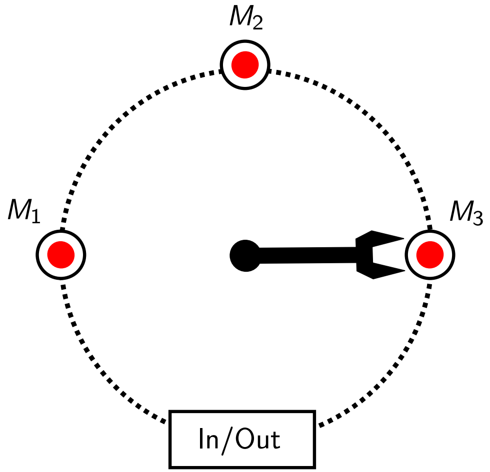

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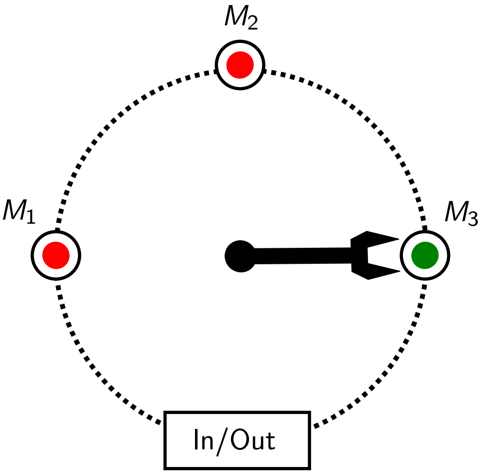

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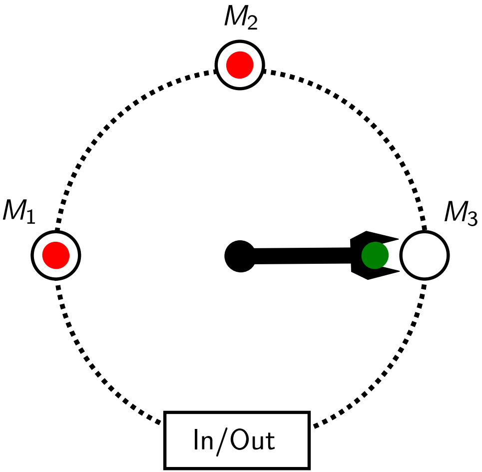

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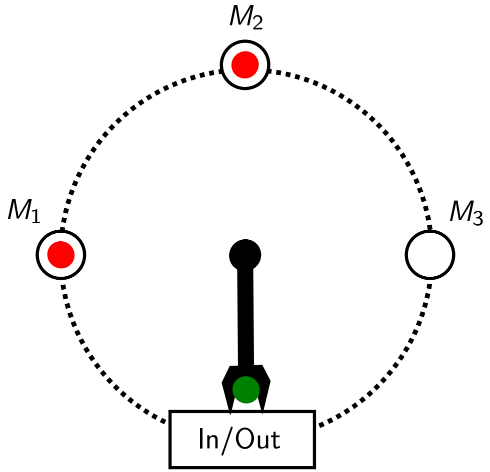

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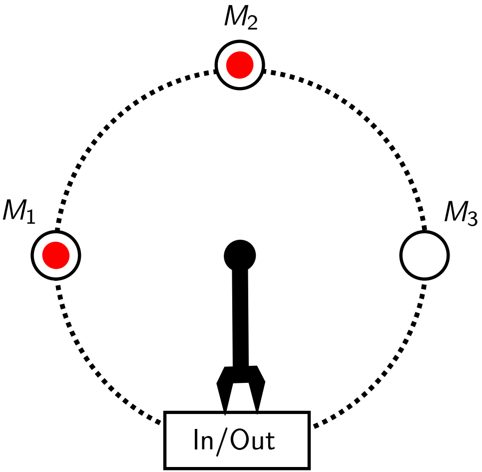

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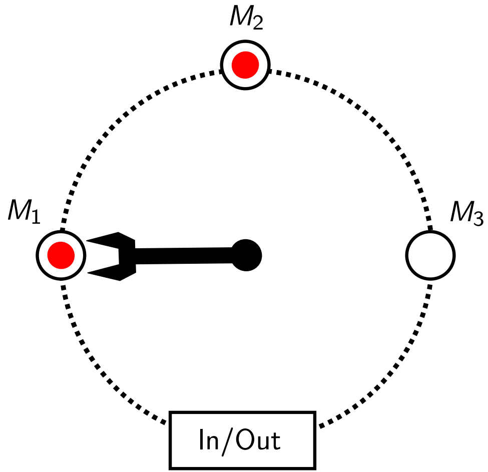

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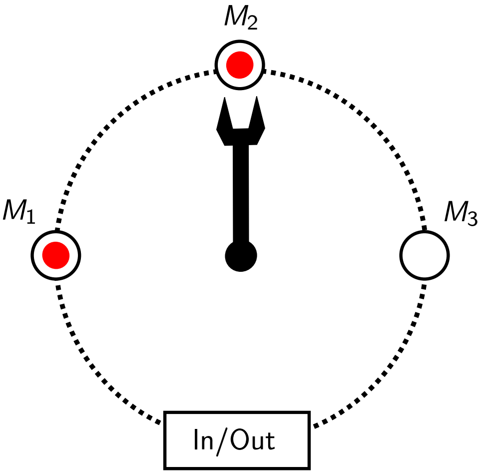

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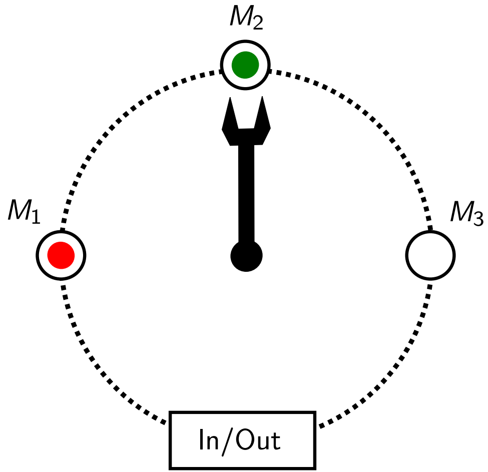

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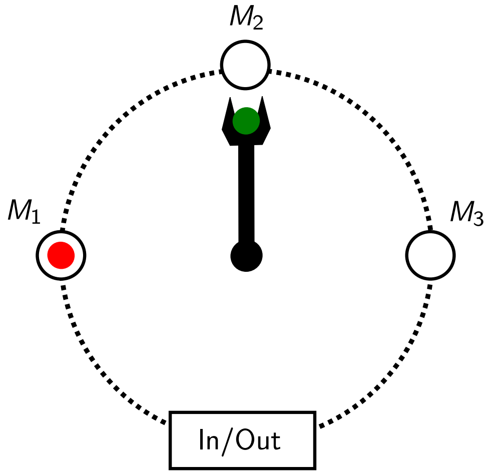

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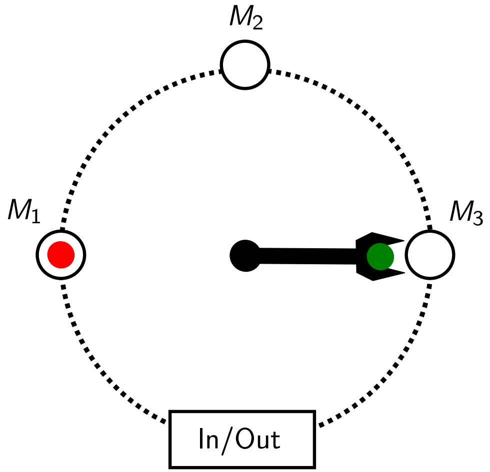

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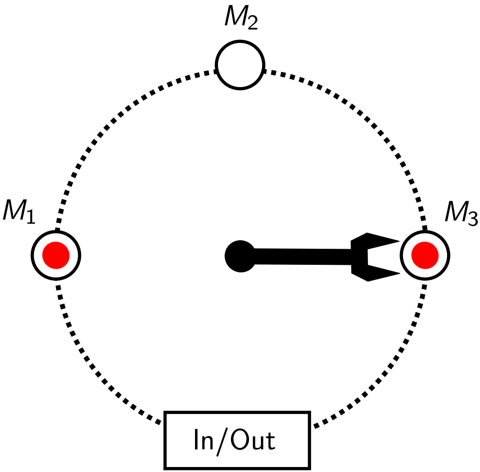

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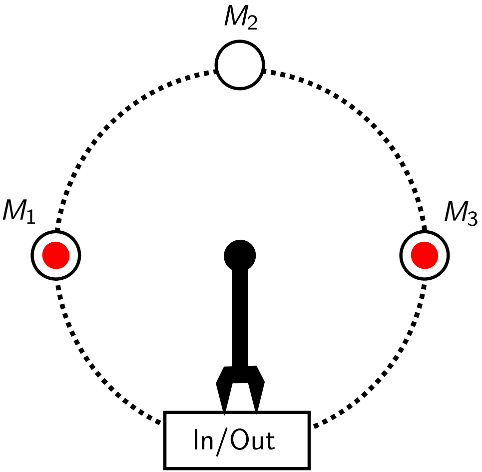

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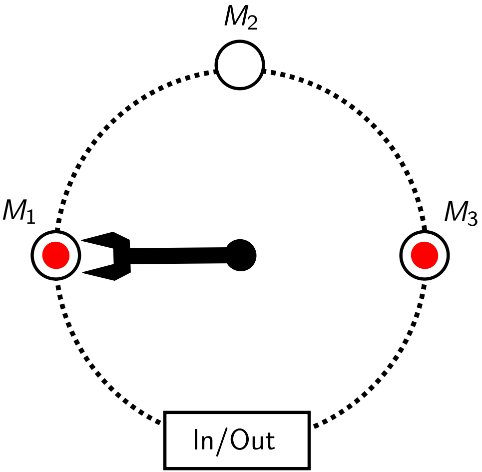

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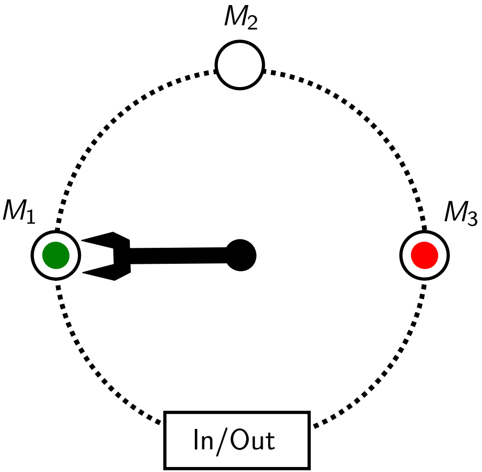

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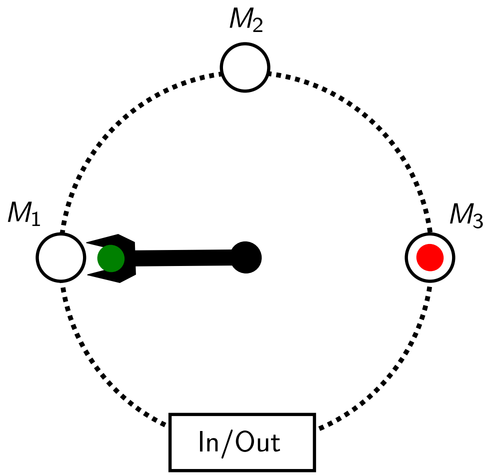

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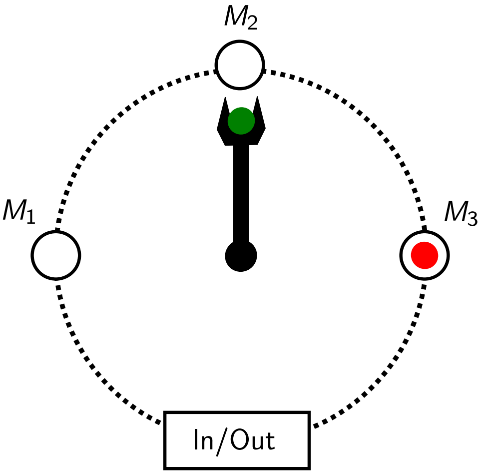

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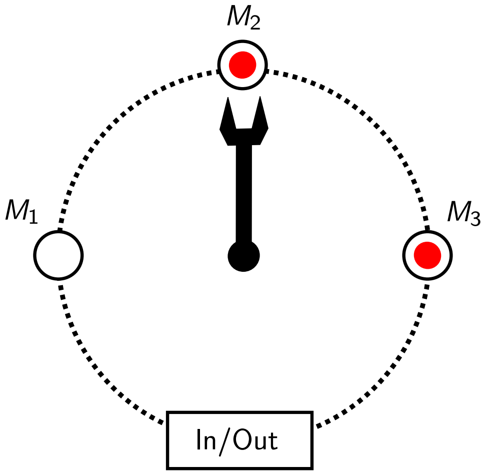


"Long processing times, fast robot"

And in-between?

That's where the problem is

Balancing

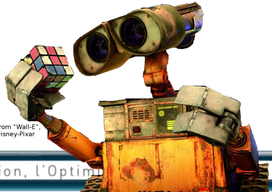
- Total travel time
- Waiting time

Objective

Given the *processing times* and *travel time between machines*, find a robot programmation which optimizes the ***throughput***:

$$\text{Maximize } \frac{\text{k}}{\text{Time to produce } k \text{ parts}}$$

$$\text{Minimize } \frac{\text{Time to produce } k \text{ parts}}{\text{k}}$$

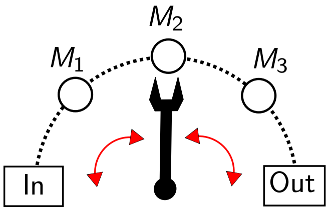


From "Wall-E", Disney-Pixar

G-SCOP

Models: Layouts

Linear



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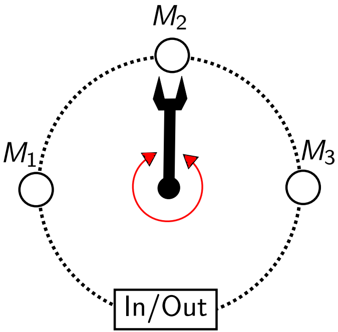


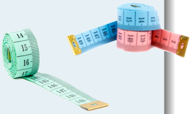
Models: cell parameters

Waiting policy

- No-wait [Agnetis, 00 ; Kats *et al* ; 09, Che *et al*, 12]
- Unbounded [Crama *et al*, 97 ; Rajapakshe *et al.*, 11]
- Time-window (HSP) [Dawande *et al*, 09; Zhou *et al*, 12]

Travel metric

- General
- Additive
  | regular
- Constant



Processing times

Identical parts

- General
- Balanced

Machine and robot capacity

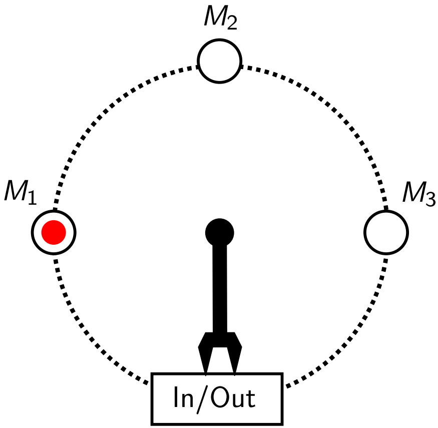
Classical model (blocking)

Few studies for the circular layout [Rajapakshe *et al.*, 11]

Relaxing the blocking constraint

- Dual-gripper
  [Sethi *et al.*, 01 ; Jung *et al.*, 15 ; Drobouchevitch *et al.*, 06]
- Swapping [Jolai *et al*]
- Machine buffers [Drobouchevitch *et al.*, 06]

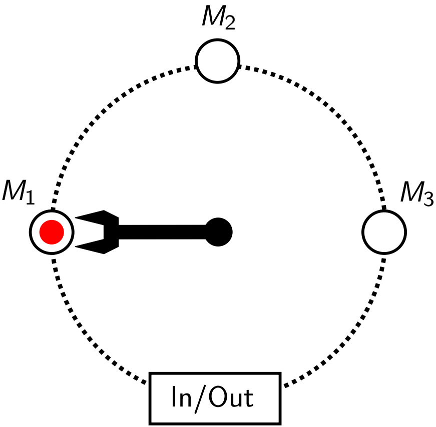
Describing the robot moves: Activities



[Crama *et al*, 97]

Activity A_i

Describing the robot moves: Activities



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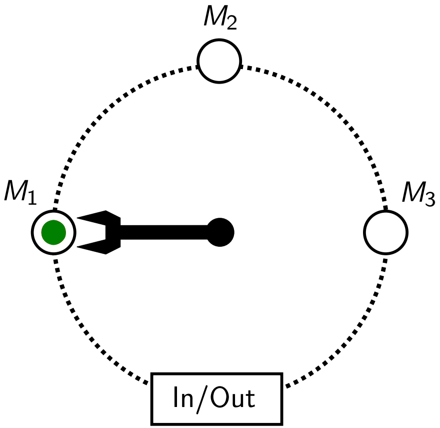
Activity A_i

- Go to M_i

Describing the robot moves: Activities

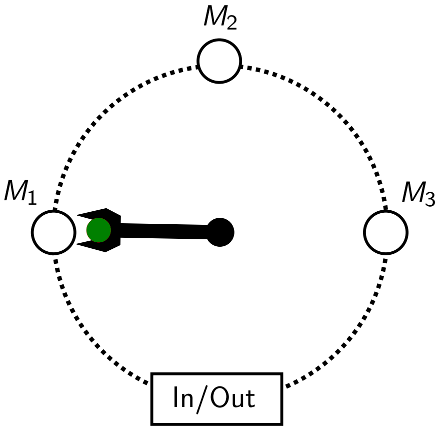


[Crama *et al*, 97]

Activity A_i

- Go to M_i
- Wait?

Describing the robot moves: Activities

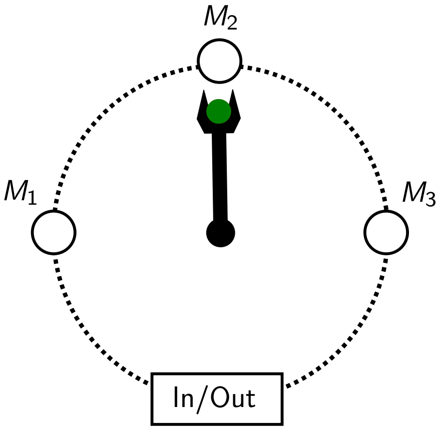


[Crama *et al*, 97]

Activity A_i

- Go to M_i
- Wait?
- Unload M_i

Describing the robot moves: Activities

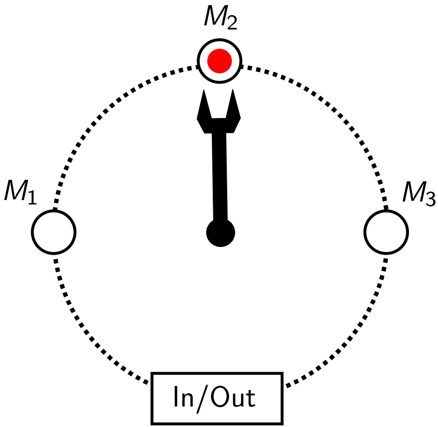


[Crama *et al*, 97]

Activity A_i

- Go to M_i
- Wait?
- Unload M_i
- Go to M_{i+1}

Describing the robot moves: Activities



[Crama *et al*, 97]

Activity A_i

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- Go to M_{i+1}
- Load M_{i+1}

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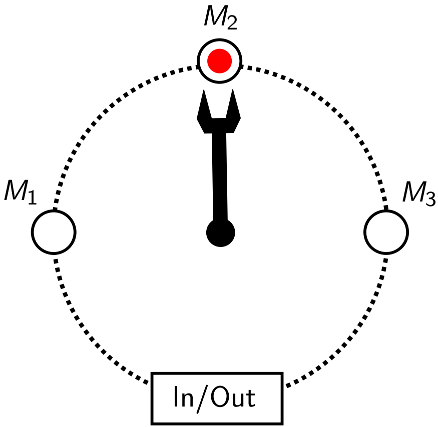

[Crama *et al*, 97]

Activity A_i

- Go to M_i
- Wait?
- Unload M_i
- Go to M_{i+1}
- Load M_{i+1}

$A_0, A_1, \ldots, A_m$

Cyclic programmation

Dominance [Dawande *et al*, 05]

Cycle

- Feasible sequence of activities
- Leaves the cell in the same state

G-SCOP

Cyclic programmation

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- k-cycle: produces k parts
- 1-cycle: produces 1 part

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1-cycles $\Leftrightarrow$ permutations of the m activities $A_1 \ldots A_m$

G-SCOP

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- k-cycle: produces k parts
- 1-cycle: produces 1 part

1-cycles $\Leftrightarrow$ permutations of the m activities $A_1 \dots A_m$

Objective function

max throughput $\longleftrightarrow$ min $\frac{\text{cycle time}}{\text{number of parts produced}}$ $\longleftrightarrow$ min $\frac{T(C_k)}{k}$

Best 1-cycle problem

1-cycles are...

- easy to describe
- easy to implement
- better known and more studied

But not necessarily optimal!

G-SCOP

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1-cycles are...

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But not necessarily optimal!

Finding the best 1-cycle...

- ...in linear regular cells: P [Crama *et al.*, 97] (balanced [Brauner *et al.*, 99])
- ...in circular regular cells: NP-hard [Rajapakshe *et al*, 11] ($\frac{5}{3}$ approx).

G-SCOP

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- ...in circular regular balanced cells: ??

Dominance?

1-cycle conjecture

1-cycles dominate all cycles [Sethi *et al.*, 92]

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- Valid for regular cells, 2- to 3-machine [Crama *et al.*, 97]

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Circular layout

???

G-SCOP

Dominance?

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- Valid for regular cells, 2- to 3-machine [Crama *et al.*, 97]
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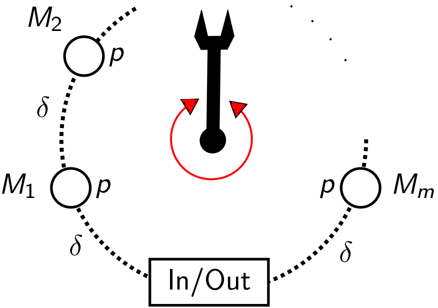
Circular layout

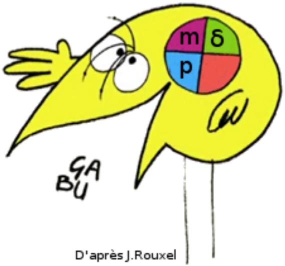
Counter-example for regular balanced cells with 6 machines

[T. *et al.* MIM 16]

Our problem

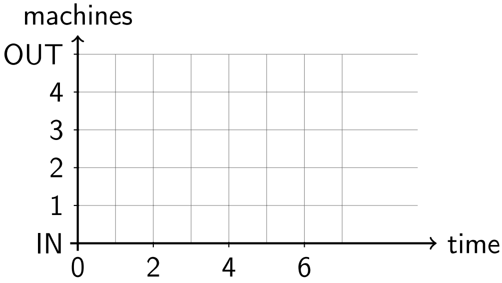
Circular, identical parts, **regular**, **balanced**: (m, δ, p)

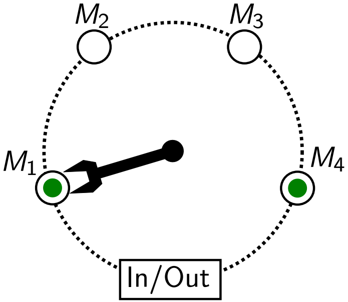




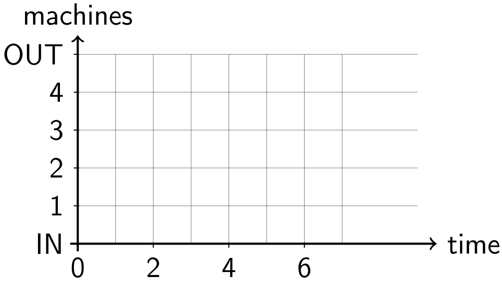
Small enough to fit in a Shadok's head!

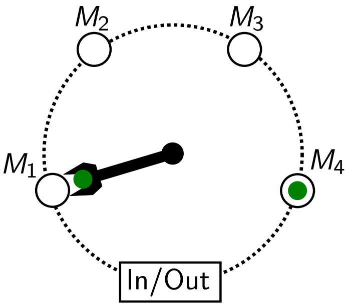
Representing the robot moves





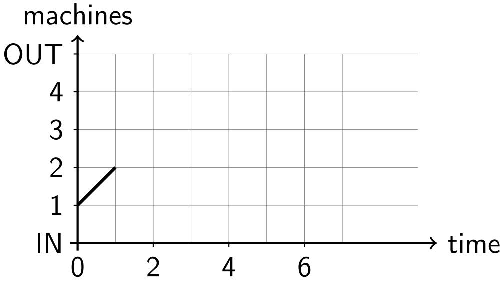
Representing the robot moves

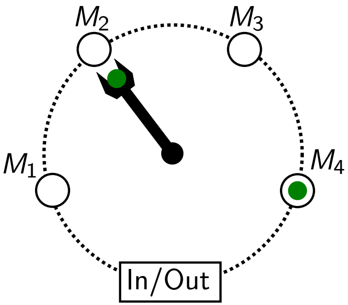




A_1

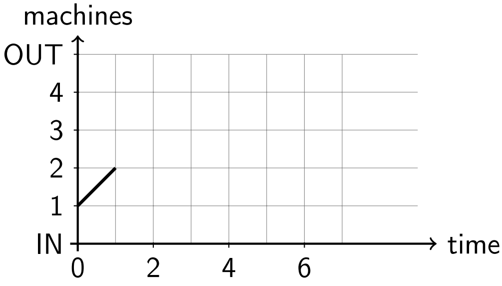
Representing the robot moves

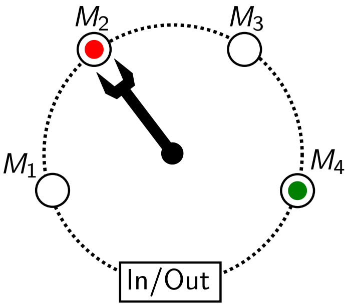




A_1

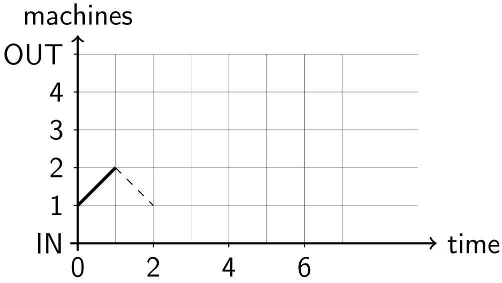
Representing the robot moves

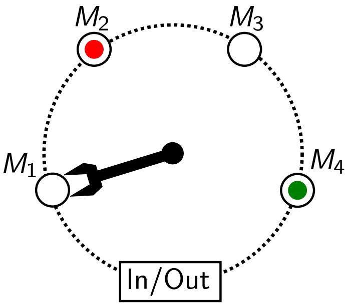




A_1

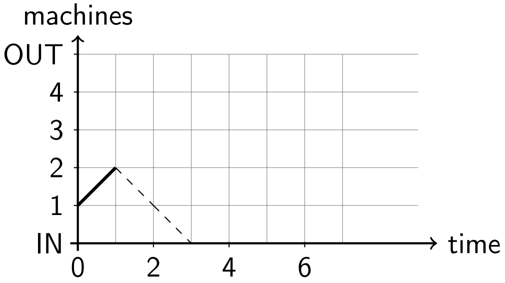
Representing the robot moves

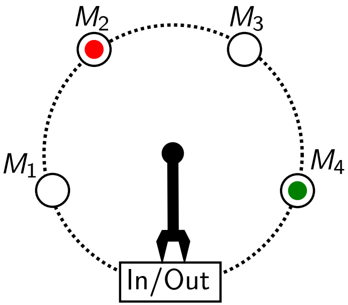




$A_1 A_0$

Representing the robot moves





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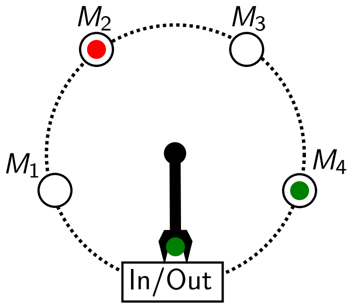
Representing the robot moves



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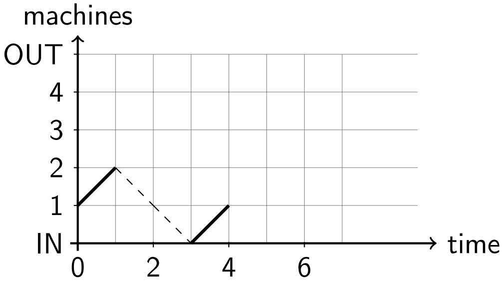
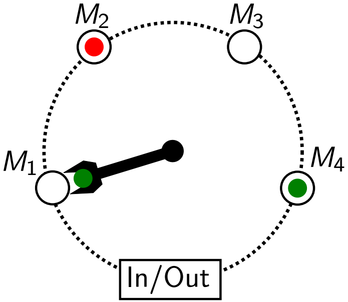
Representing the robot moves





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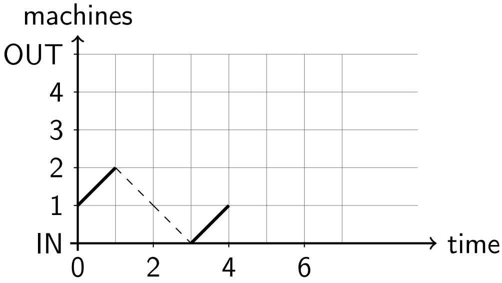
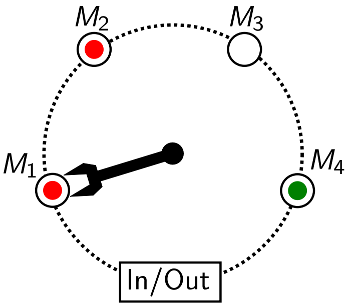
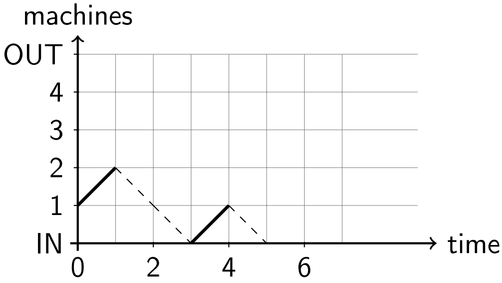
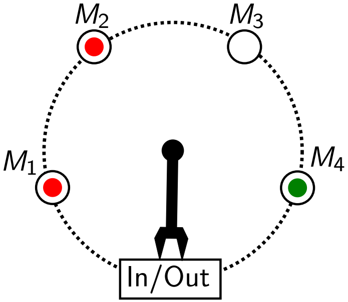




$A_1 A_0$

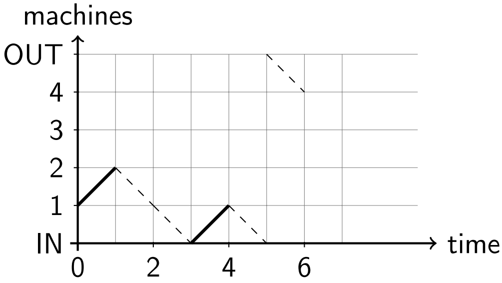
Representing the robot moves

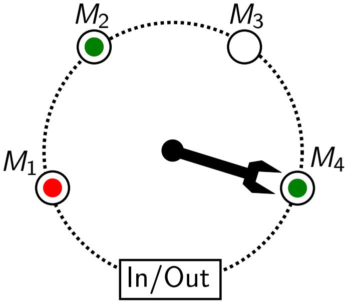




$A_1 A_0 A_4$

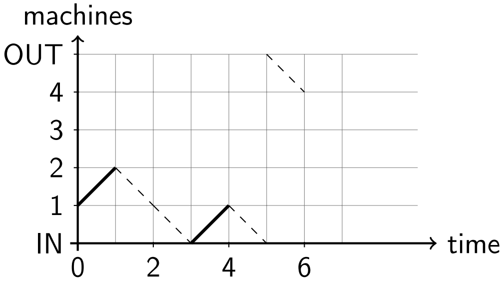
Representing the robot moves

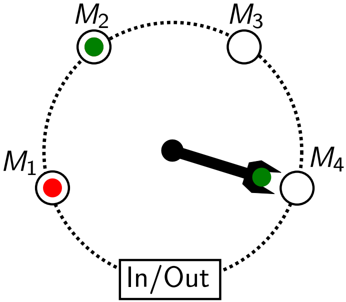




$A_1 A_0 A_4$

Representing the robot moves





$A_1 A_0 A_4$

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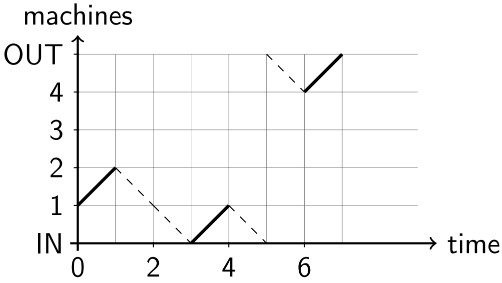
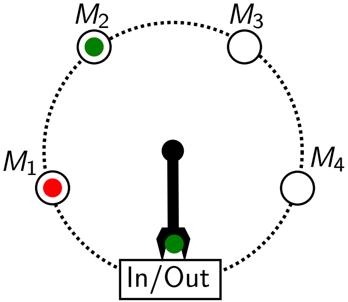
Representing the robot moves





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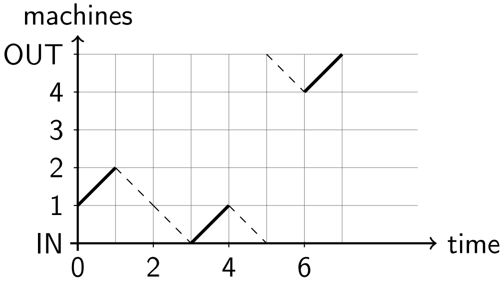

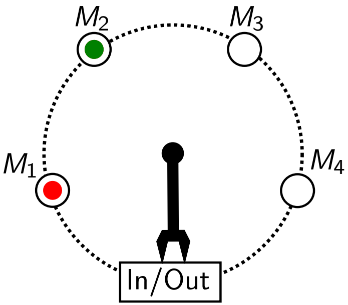

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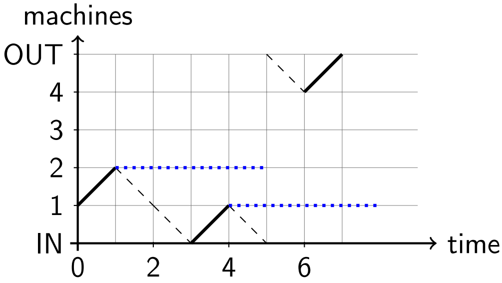
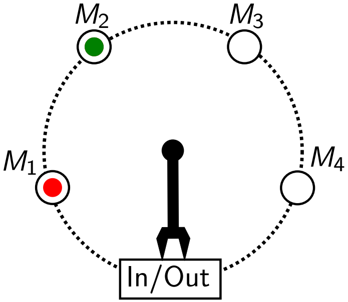
Representing the robot moves



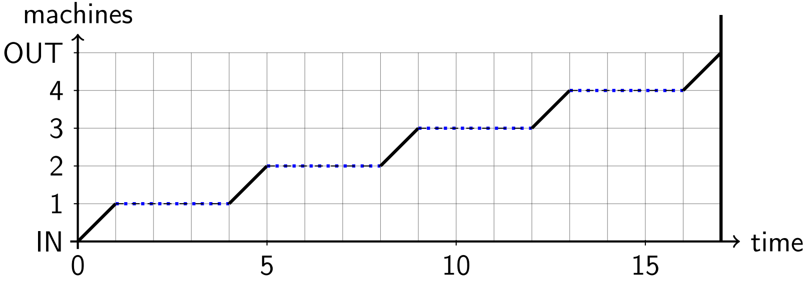


$A_1 A_0 A_4$

Some usual 1-cycles: Identity cycle

Identity cycle π_{id}

π_{id}: $A_0 A_1 \ldots A_m$



Cycle time

$T(\pi_{id}) = (m+1)\delta + mp$

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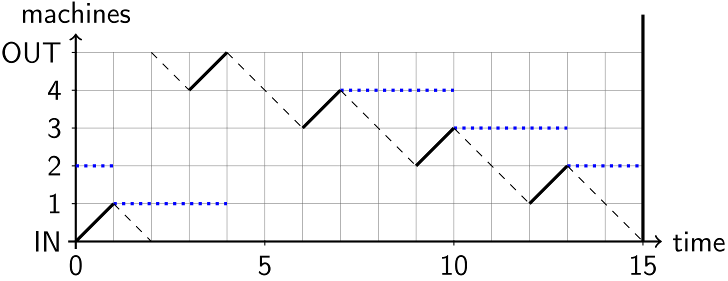
Some usual 1-cycles: Downhill cycle

Downhill cycle π_d

π_d: $A_0 A_m A_{m-1} \ldots A_1$



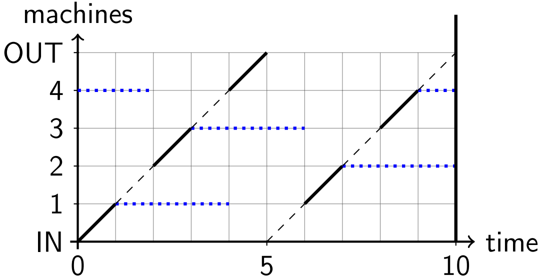
Cycle Time

$$T(\pi_d) = 3(m+1)\delta + \max(0, p - (3m-1)\delta)$$

Some usual 1-cycles: Odd-even cycle

Odd-Even cycle π_{oe}

π_{oe}: $A_0 A_2 \ldots A_1 A_3 \ldots$

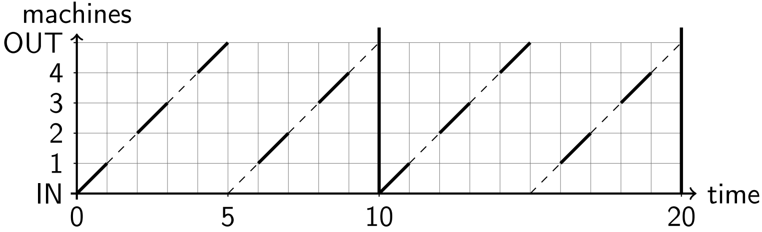


Cycle time

$T(\pi_{oe}) = \quad ??$

Odd-even's cycle time

$$T(\pi_{oe}) = 2(m+1)\delta + \max\left(0, \frac{2\alpha-1}{\alpha}\left(p - (m+1)\delta\right)\right) \quad \alpha = \lfloor \tfrac{m+1}{2} \rfloor$$
$$\leq 2p$$



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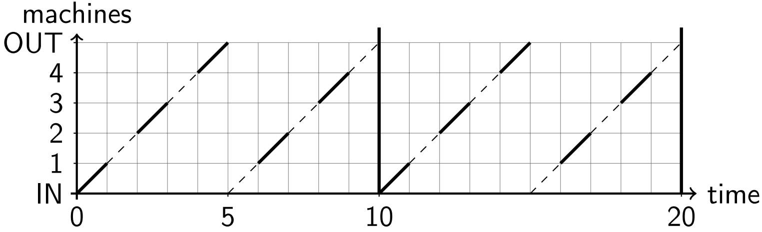

$20 +$ ______

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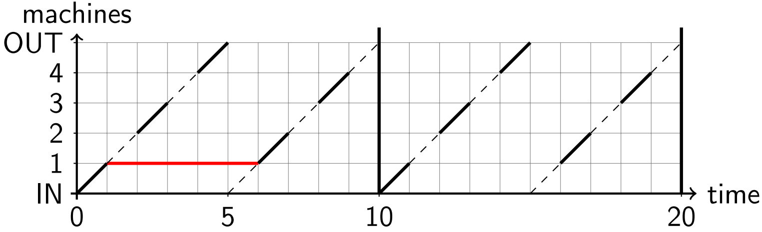

$20 + \underline{\hspace{6cm}}$

Odd-even's cycle time

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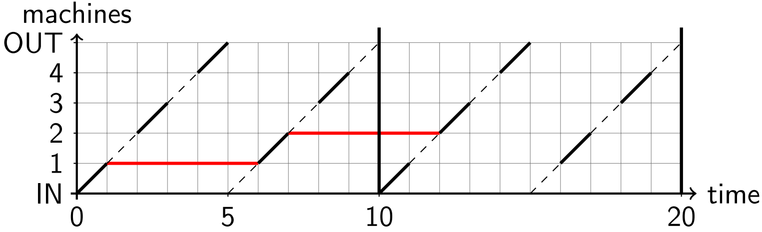

$20 +$

Odd-even's cycle time

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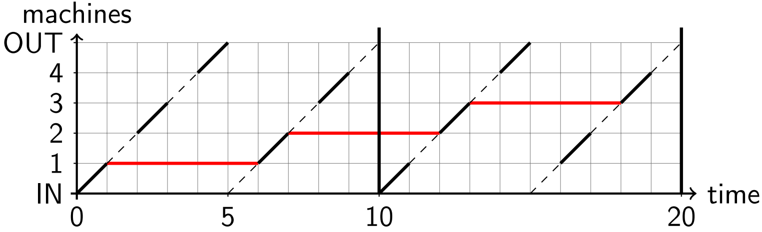
$$\leq 2p$$



$20 +$ ______________

Odd-even's cycle time

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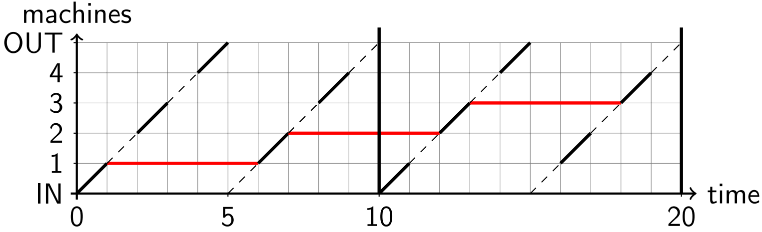
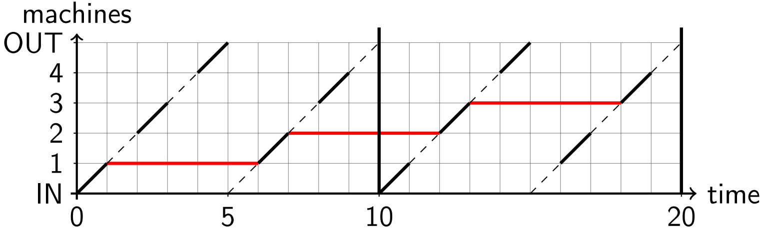
$$\leq 2p$$



$$\underline{20 + 3\max\left(0, (p-5\delta)\right)}$$

Odd-even's cycle time

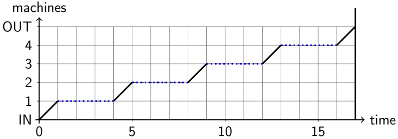
$$T(\pi_{oe}) = 2(m+1)\delta + \max\left(0, \frac{2\alpha-1}{\alpha}\left(p-(m+1)\delta\right)\right) \quad \alpha = \lfloor \frac{m+1}{2} \rfloor$$
$$\leq 2p$$



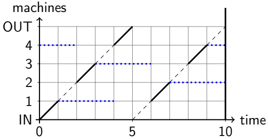
$$\frac{20 + 3\max\left(0, (p-5\delta)\right)}{2}$$

Finding the best 1-cycle

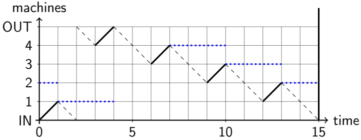
Identity



Odd-Even



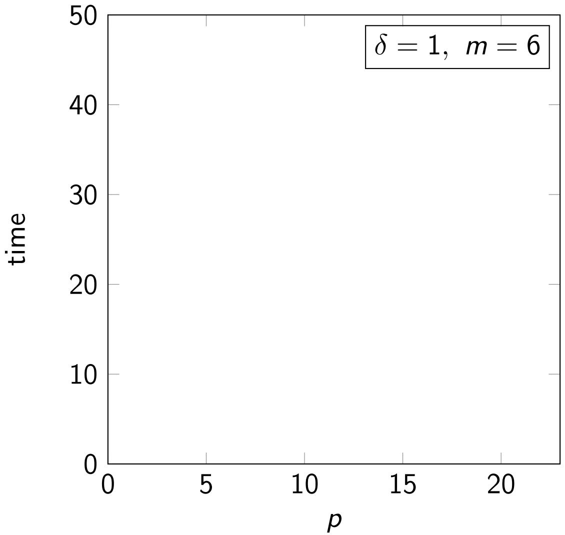
Downhill



?

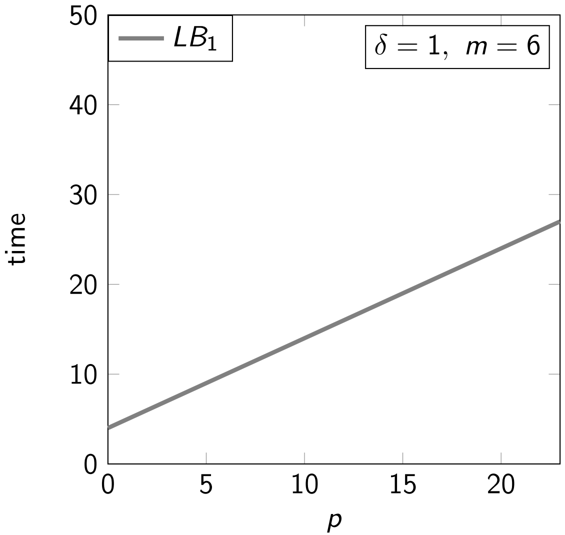
Which is the best 1-cycle?

Best 1-cycles?



Lower bounds

Best 1-cycles?



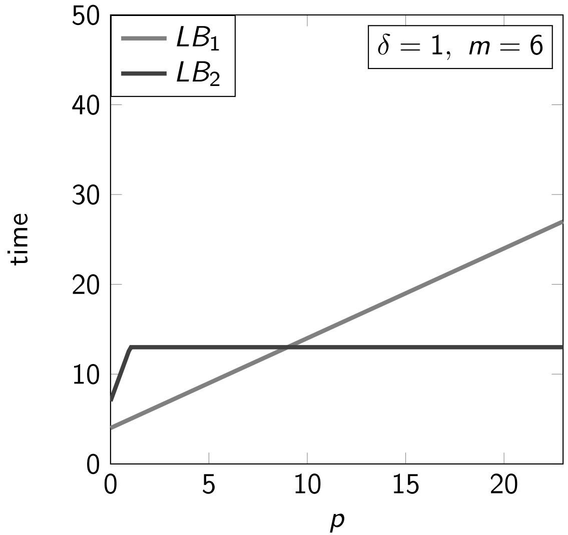
Lower bounds

LB_1 [Crama, 97]

$T(\pi) \geq p + 4\delta$

G-SCOP

Best 1-cycles?



Lower bounds

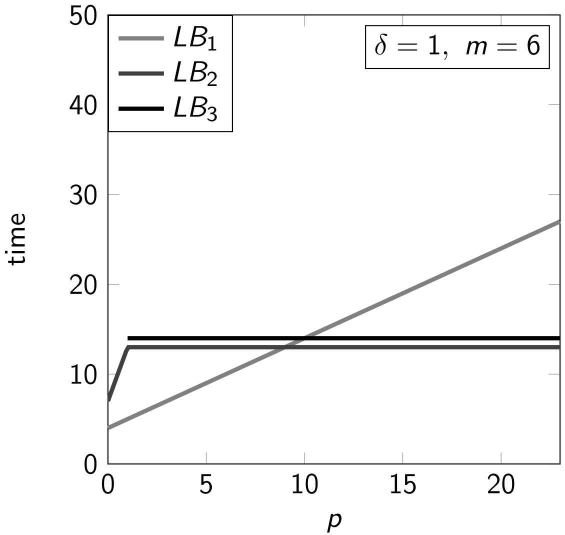
LB_1 [Crama, 97]

$$T(\pi) \geq p + 4\delta$$

LB_2 [Dawande *et al*, 02]

$$T(\pi) \geq (m+1)\delta + m \min(\delta, p)$$

Best 1-cycles?



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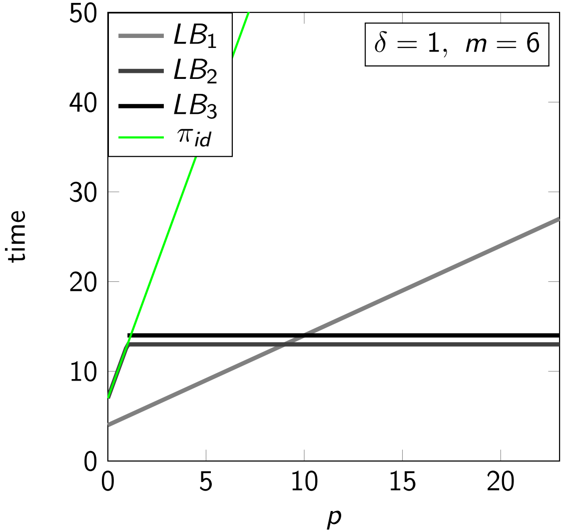
LB_3 (1-cycles only)

$T(\pi) \geq 2(m+1)\delta$

Best 1-cycles?



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G-SCOP

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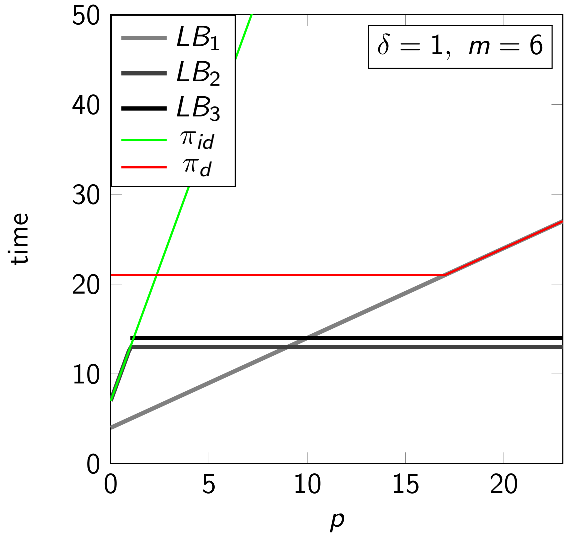

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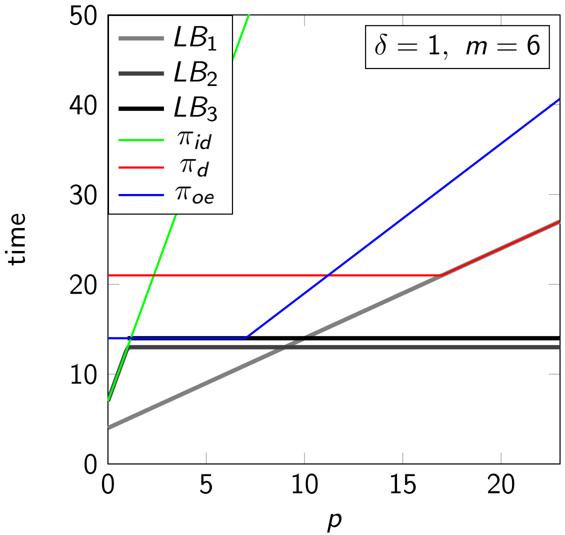

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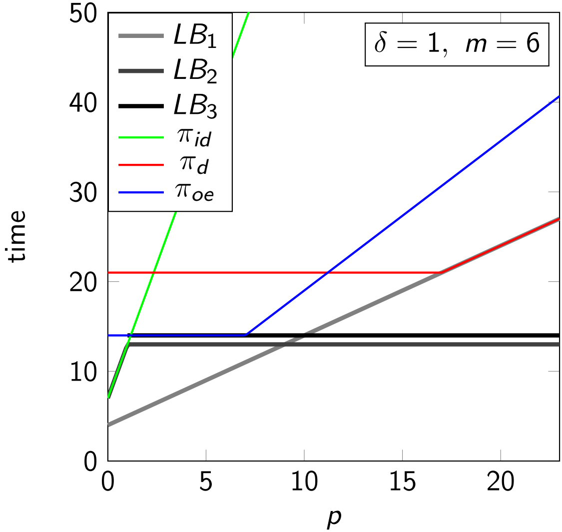
LB_3 (1-cycles only)

$T(\pi) \geq 2(m+1)\delta$

The unknown region

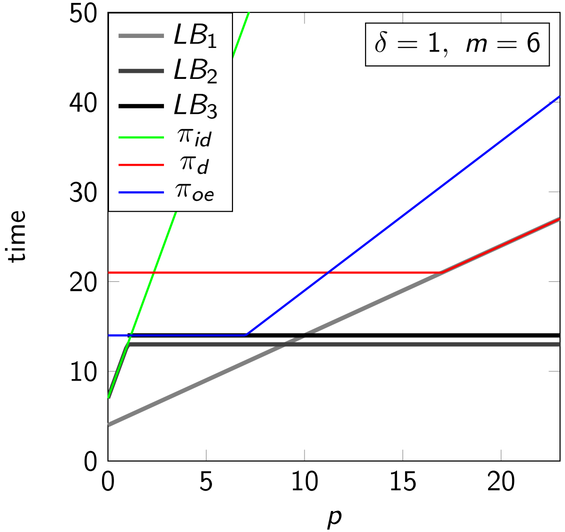


Best 1-cycle

- $p \leq \frac{m+1}{m}\delta$

  $\boldsymbol{\pi_{id}}$

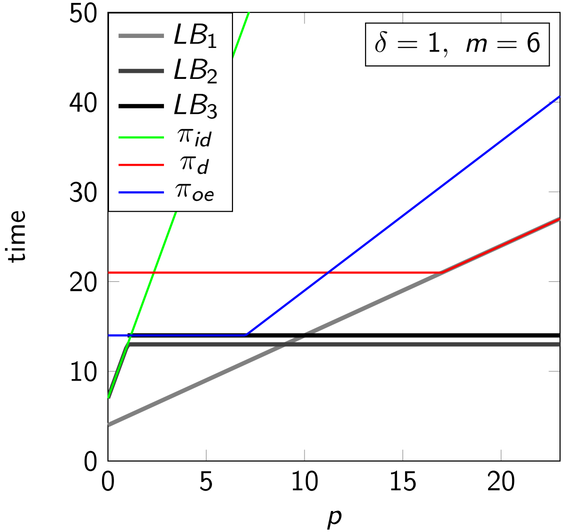
The unknown region



Best 1-cycle

- $p \leq \frac{m+1}{m}\delta$
  $\boldsymbol{\pi_{id}}$
- $p > \frac{m+1}{m}\delta$
  $p \leq (m+1)\delta$
  $\boldsymbol{\pi_{oe}}$

The unknown region



Best 1-cycle

- $p \leq \frac{m+1}{m}\delta$
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- $p > \frac{m+1}{m}\delta$
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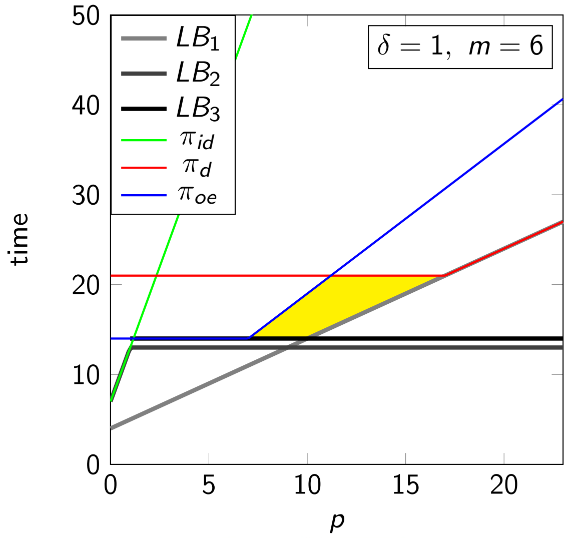
  $\boldsymbol{\pi_{oe}}$
- $p \geq (3m-1)\delta$
  $\boldsymbol{\pi_d}$

The unknown region



Best 1-cycle

- $p \leq \frac{m+1}{m}\delta$
  π_{id}
- $p > \frac{m+1}{m}\delta$
  $p \leq (m+1)\delta$
  π_{oe}
- $p \geq (3m-1)\delta$
  π_d

between $(m+1)\delta$ and $(3m-1)\delta$?

G-SCOP

Exploring the unknown region

Best 1-cycles : $\{\pi_{id},\ \pi_{oe},\ \pi_d\}\ +\ ??$

Introducing π^*

- π^* is a 1-cycle
- π^* crosses the yellow area
  (meaning: for some $(m+1)\delta \leq p \leq (3m-1)\delta$,
  π^* does strictly better than both π_{oe} and π_d)

Exploring the unknown region

Best 1-cycles : $\{\pi_{id}, \pi_{oe}, \pi_d\}$ + ??

Introducing π^*

- π^* is a 1-cycle
- π^* crosses the yellow area
  (meaning: for some $(m+1)\delta \leq p \leq (3m-1)\delta$,
  π^* does strictly better than both π_{oe} and π_d)

Who is π^*?

What can we say about it? Does it exist?

from "Up", Disney - Pixar

Is there anybody in there?

Size of a minimum dominant set within 1-cycles:

m	3	4	5	6	7	8	9	10	11	12	13	14
cardinal	3	4	4	3	3	3	4	4	4	5	4	4

proven

computed

$$\{\pi_{id},\ \pi_{oe},\ \pi_d\} \cup ?$$

Is there anybody in there?

Size of a minimum dominant set within 1-cycles:

m	3	4	5	6	7	8	9	10	11	12	13	14
cardinal	3	4	4	3	3	3	4	4	4	5	4	4

proven

computed

$\{\pi_{id},\ \pi_{oe},\ \pi_d\} \cup ?$

[Brauner, 99]

Is there anybody in there?

Size of a minimum dominant set within 1-cycles:

m	3	4	5	6	7	8	9	10	11	12	13	14
cardinal	3	4	4	3	3	3	4	4	4	5	4	4

proven

computed

$\{\pi_{id},\ \pi_{oe},\ \pi_d\} \cup ?$

Properties on the total travel time

G-SCOP

First properties

Notations (last ones...)

$\Delta(\pi)$: total travel time

$d_i(\pi)$: travel time between the loading and unloading of machine M_i

$d_{min}(\pi)$: $\min d_i(\pi)$ e.g $d_{min}(\pi_{oe}) = (m+1)\delta$

Lower bound $LB_4(\pi)$

$$T(\pi) \geq \underbrace{\Delta(\pi)}_{\text{Total travel time}} + \underbrace{\max(0, p - d_{min}(\pi))}_{\text{Minimum waiting time}}$$

d_{min} is the minimum value of p for which waiting is necessary.

First properties

If π^* exists:

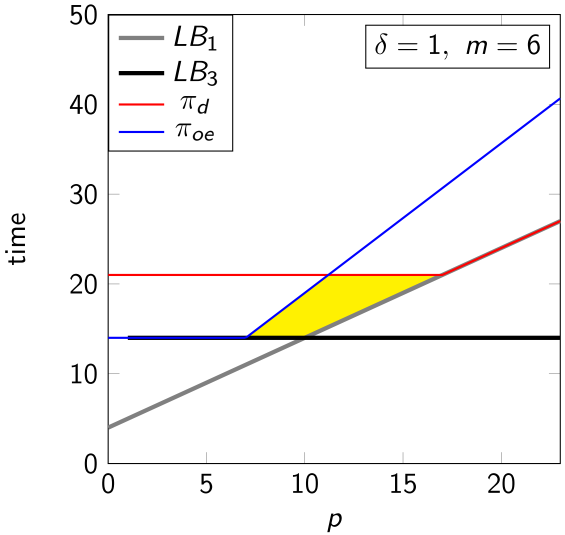
- The robot travels between 2 and 3 times the size of the cell:

$$2(m+1)\delta < \Delta(\pi^*) < 3(m+1)\delta$$

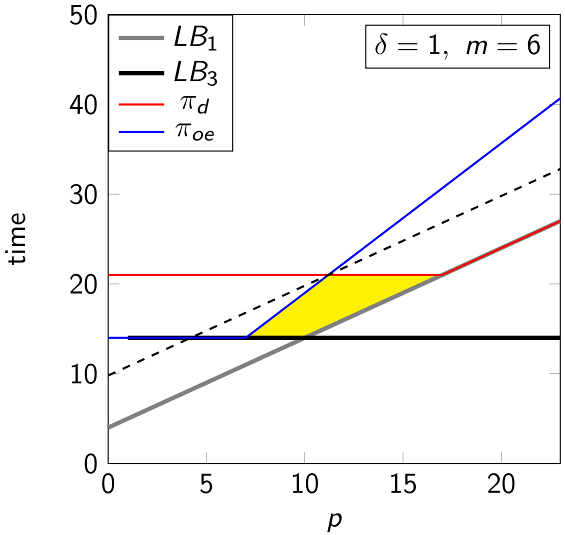
- No waiting time if p is “small” relatively to the travel time:

$$d_{min}(\pi^*) > \Delta(\pi^*) - \frac{3\alpha - 2}{2\alpha - 1}(m+1)\delta$$

First properties



First properties



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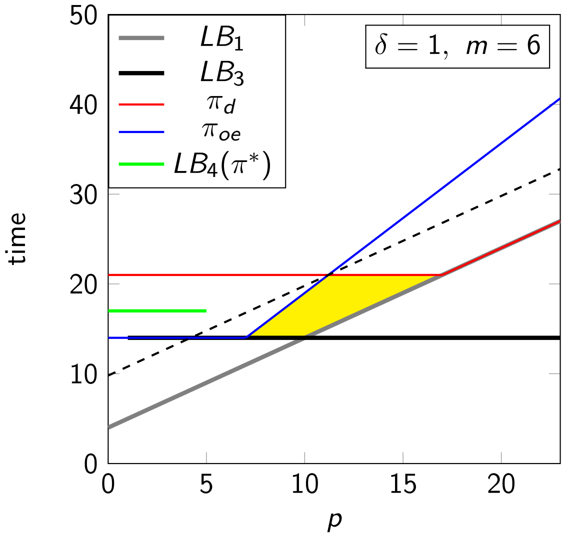

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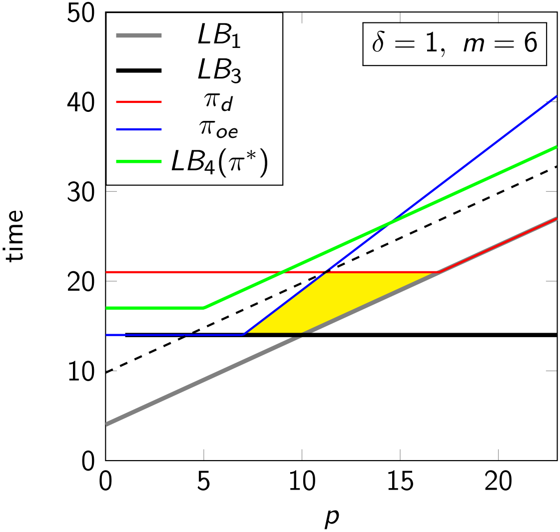

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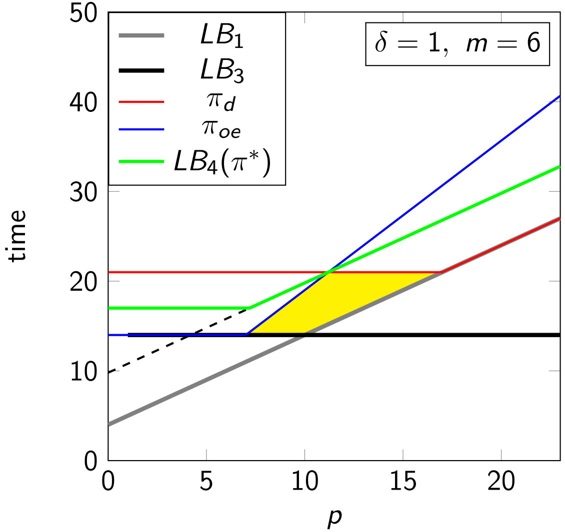

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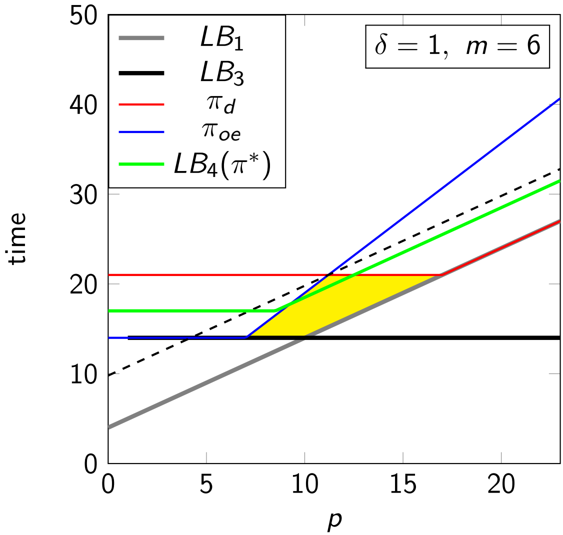

And for $m > 8$?

Cardinal of a minimum dominant set within 1-cycles:

m	3	4	5	6	7	8	9	10	11	12	13	14
cardinal	3	4	4	3	3	3	4	4	4	5	4	4

proven

computed

G-SCOP

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m	3	4	5	6	7	8	9	10	11	12	13	14
cardinal	3	4	4	3	3	3	4	4	4	5	4	4

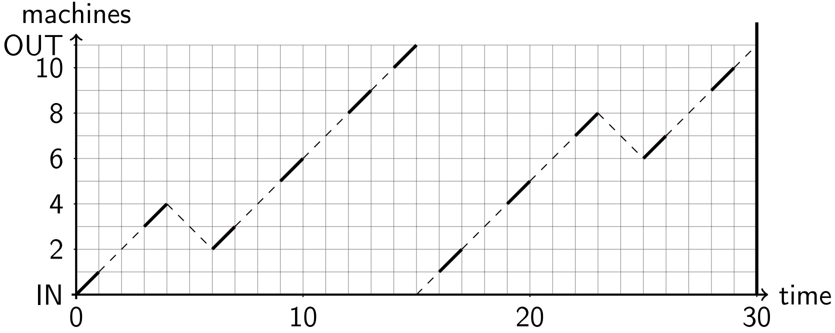
proven

computed

G-SCOP

Two wavelets (π_{2w})...

For $m > 8$, this guy...



...always (slightly) crosses the yellow area

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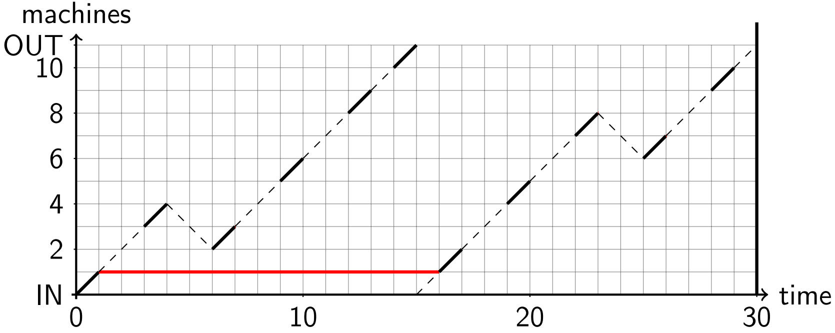

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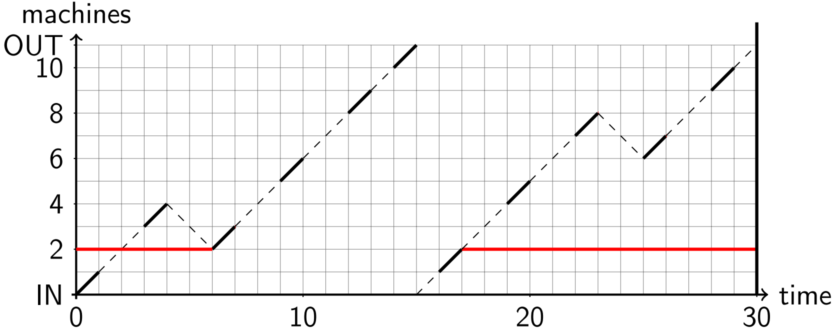

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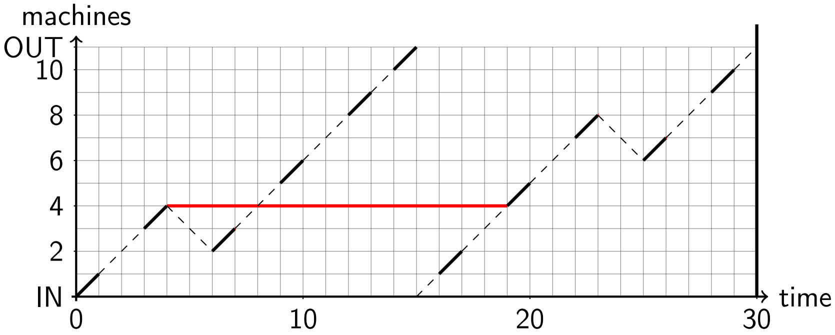

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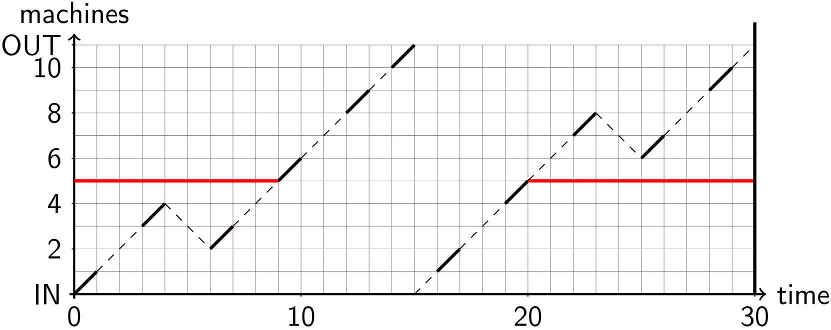

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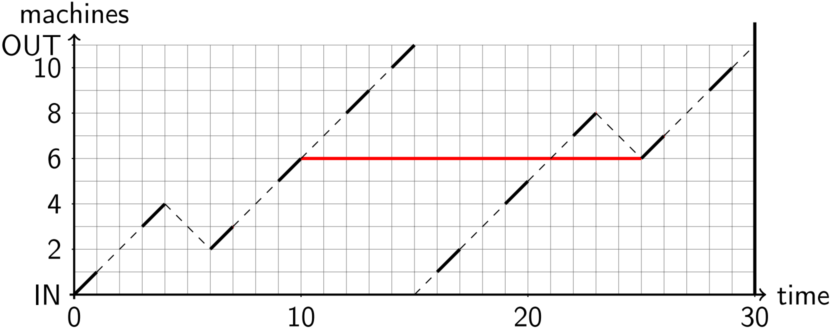

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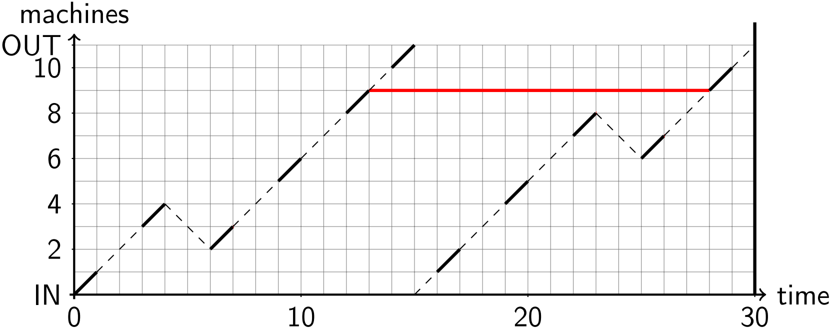

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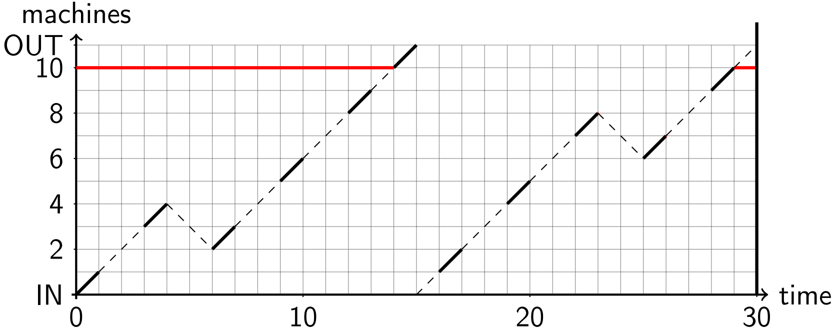

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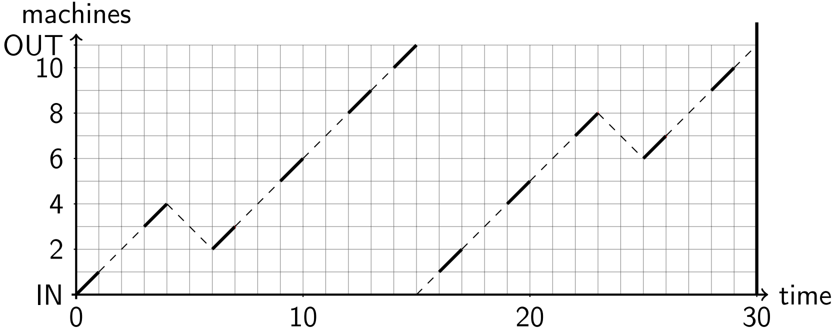
G-SCOP

Two wavelets (π_{2w})...

For $m > 8$, this guy...

$d_{min}(\pi_{2w}) = (m+5)\delta$
(One turn and one wave)



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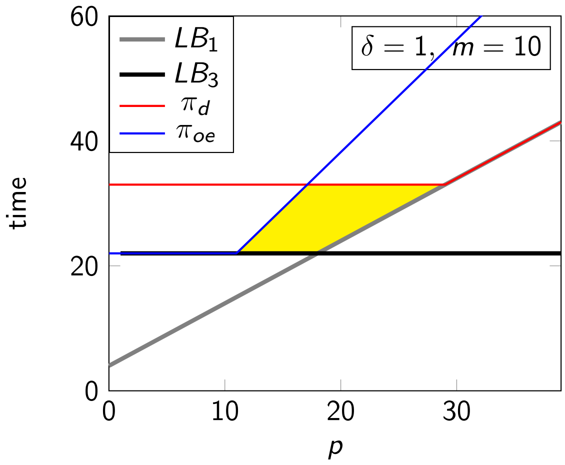

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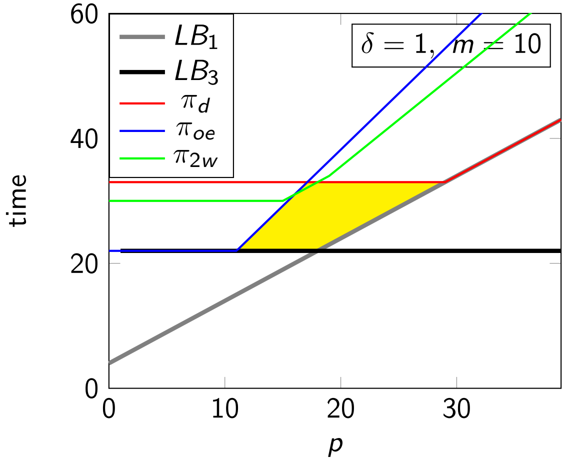
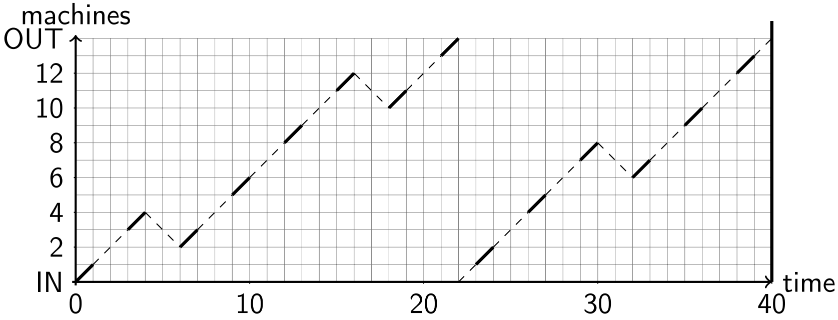


G-SCOP

Three wavelets (π_{3w})...

For $m = 12$ and $m = 13$...



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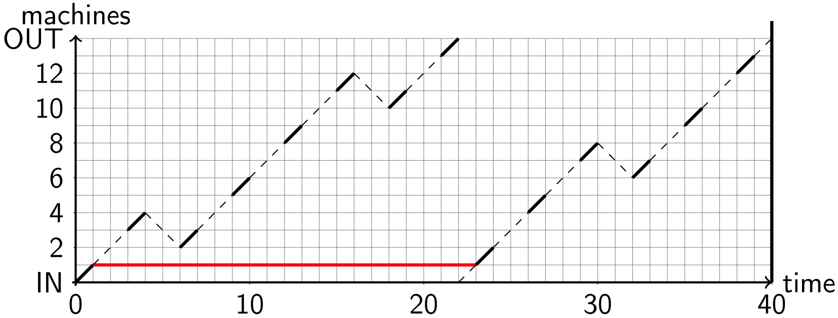

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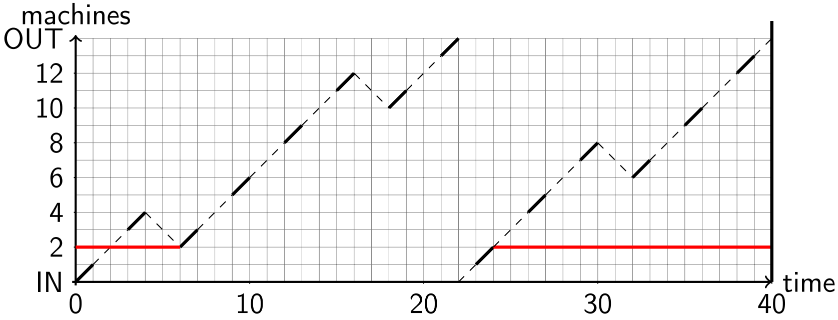

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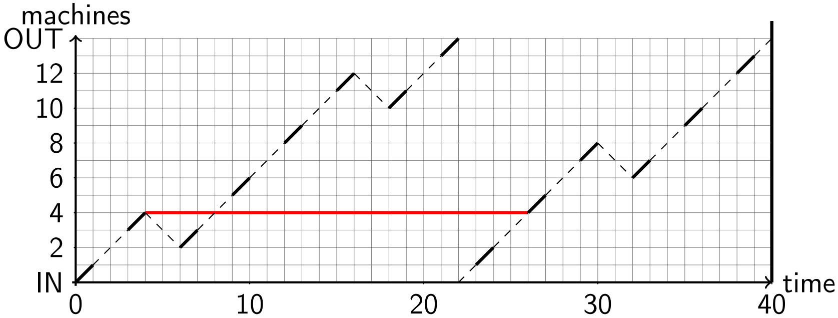

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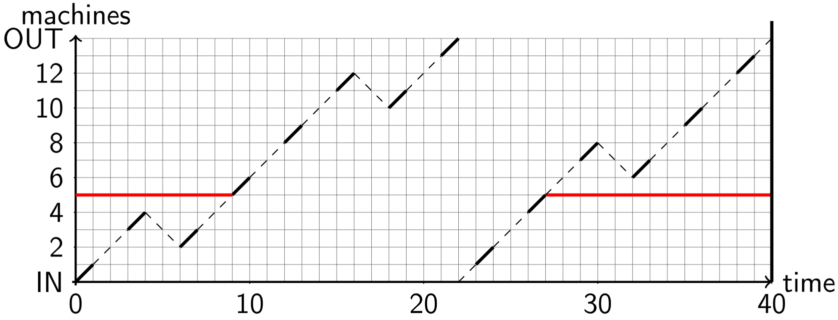

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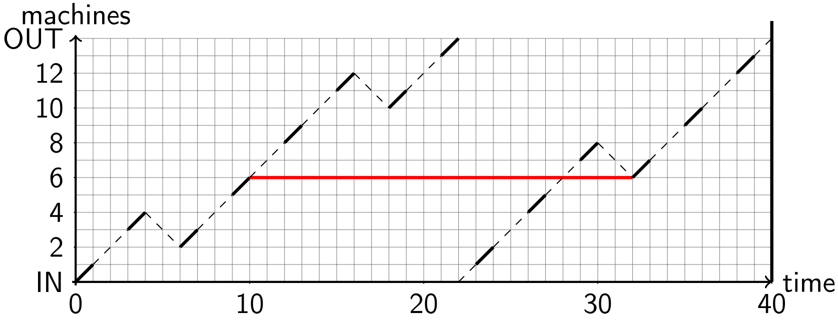

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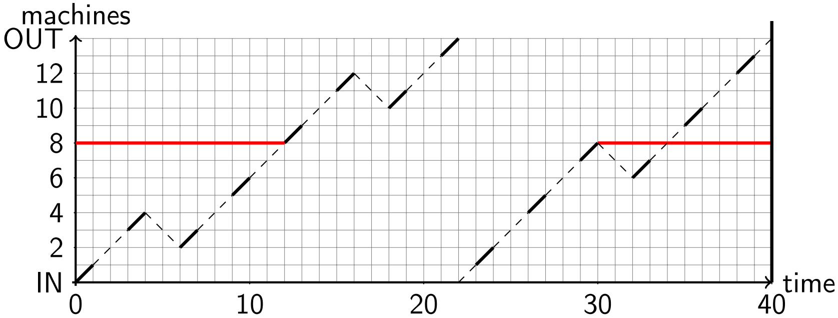

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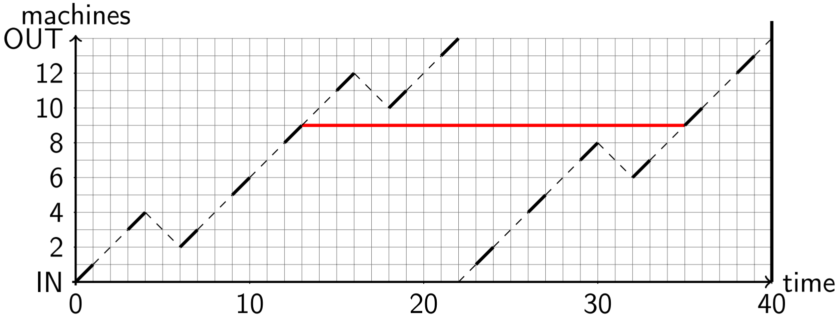

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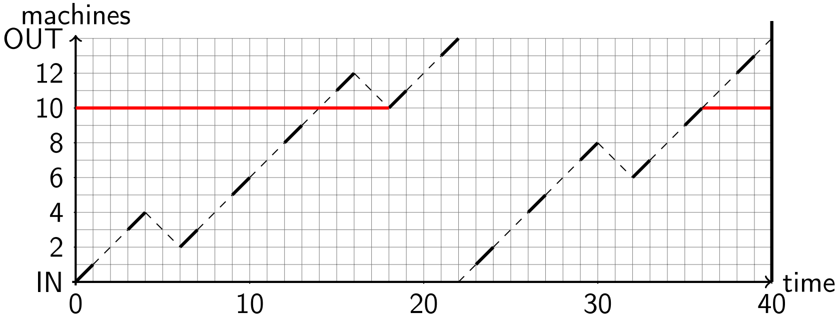
For $m = 12$ and $m = 13$...



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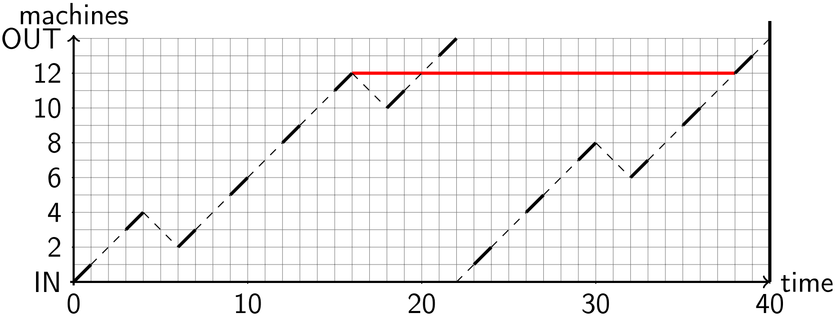

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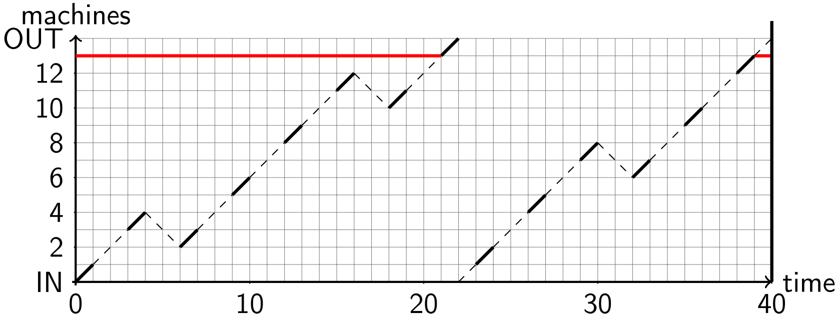
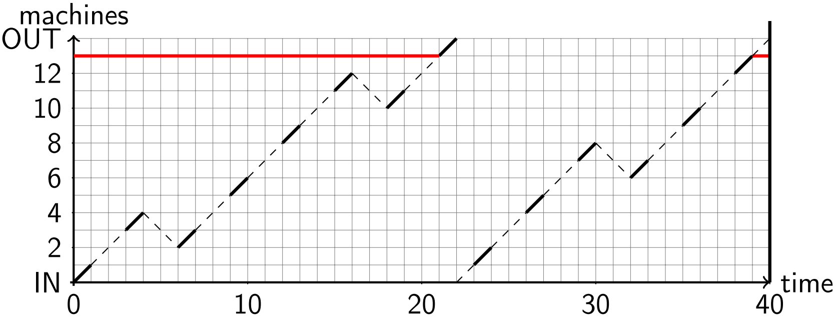


Three wavelets (π_{3w})...

For $m = 12$ and $m = 13$...

$d_{min}(\pi_{3w}) = (m+9)\delta$
(One turn and two waves)



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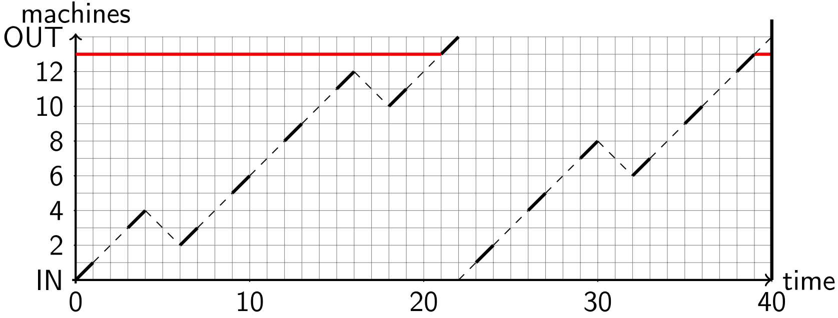
Three wavelets (π_{3w})...

For $m = 12$ and $m = 13$...

False for $m \geq 14$!

$d_{min}(\pi_{3w}) = (m + 9)\delta$

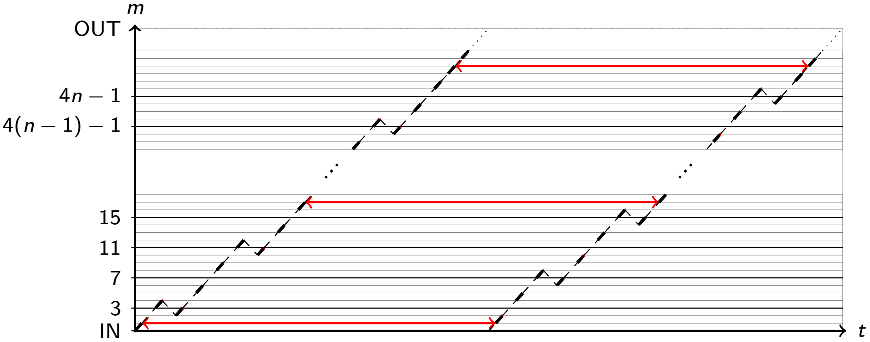
(One turn and two waves)



...n wavelets (π_{nw})

(here for n even)

$d_{min}(\pi_{nw}) = (m + 2n + 1)\delta$
(One turn and $\frac{n}{2}$ waves)



$m \geq 8$: A proposition and a conjecture

Proposition: best cycles within $\{\pi_{id}, \pi_d, \pi_{oe}\} \cup (\pi_{nw})_n$

- **Three** classical 1-cycles $\pi_{id}, \pi_d, \pi_{oe}$
- **+One** π_{nw} with the highest **even** value of n possible

  And/or
- **+One** π_{nw} with the highest **odd** value of n possible

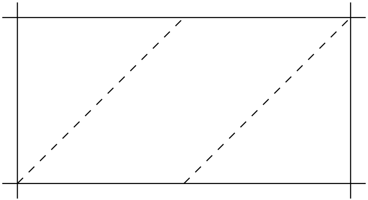
Conjecture: best 1-cycle

These also dominate **all 1-cycles**

Proven for $m \leq 11$

Proof ideas

- 2 “turns”

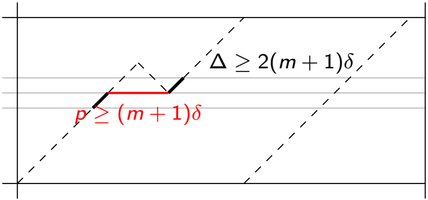


$T(\pi^*) < 2p$
(otherwise, dominated by π_{oe})
$T(\pi^*) < 3(m+1)\delta$
(otherwise, dominated by π_d)

G-SCOP

Proof ideas

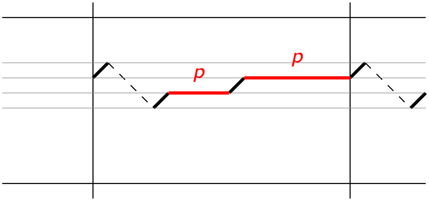
- 2 “turns”
- A_i and A_{i+1} can't be on the same turn in that order



$T(\pi^*) < 2p$
(otherwise, dominated by π_{oe})
$T(\pi^*) < 3(m+1)\delta$
(otherwise, dominated by π_d)

Proof ideas

- 2 "turns"
- A_i and A_{i+1} can't be on the same turn in that order
- no subsequence $A_i A_{i-2}$ (rules out bigger alterations up to 11 machines)



$T(\pi^*) < 2p$
(otherwise, dominated by π_{oe})
$T(\pi^*) < 3(m+1)\delta$
(otherwise, dominated by π_d)

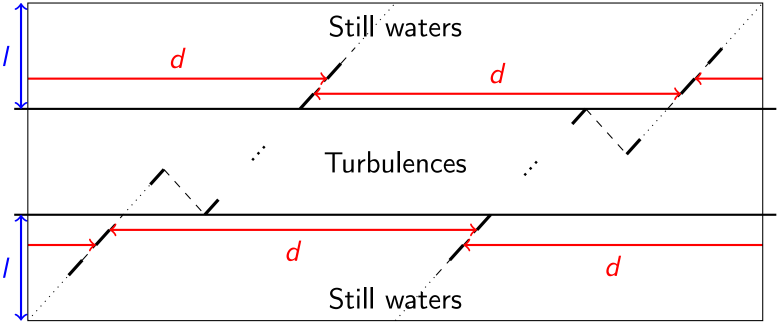
Best 1 cycles for $m \leq 11$ ...

Cardinal of a minimum dominant set within 1-cycles:

m	3	4	5	6	7	8	9	10	11	12	13	14
cardinal	3	4	4	3	3	3	4	4	4	5	4	4

$6 \leq m \leq 8$: $\{\pi_{id}, \pi_{oe}, \pi_d\}$
$9 \leq m \leq 11$: $\{\pi_{id}, \pi_{oe}, \pi_d, \pi_{2w}\}$

And for any m?

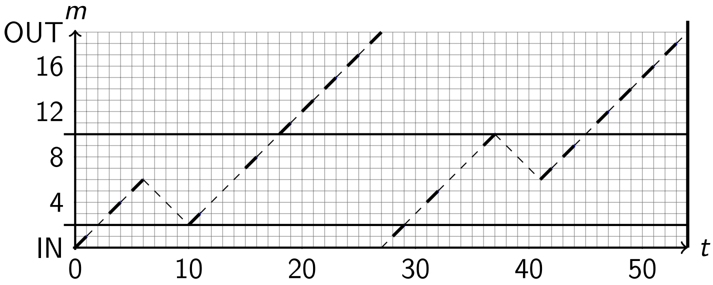


Lower bound on the waiting time: $\nearrow$ $\boldsymbol{l}$, $\searrow$ $\boldsymbol{d}$
(tight for p_{nw} cycles)

And for any m?

Not proven Regular disposition of the waves in the "turbulence" areas: seems intuitive but...

Proven Wavelets preferable to big waves:



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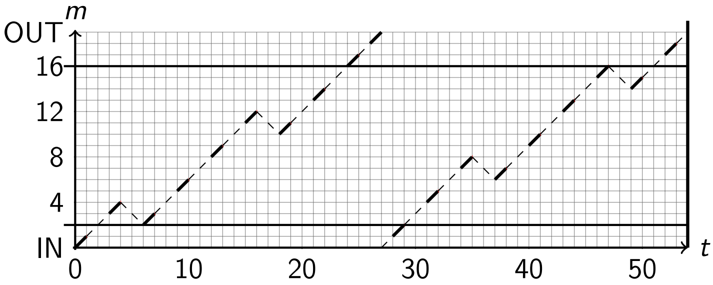
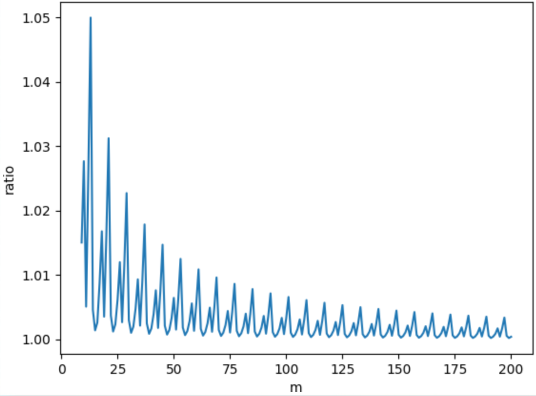


If the conjecture is valid...

If the conjecture is valid, then...

- The best 1-cycle problem in circular, regular balanced cell would be **polynomial**
- Performance ratio of the usual cycles:



$$\frac{min\left(T(\pi_{oe}), T(\pi_d)\right)}{min_n\left(T(\pi_{oe}), T(\pi_d), T(\pi_{nw})\right)}$$

(as a function of m)

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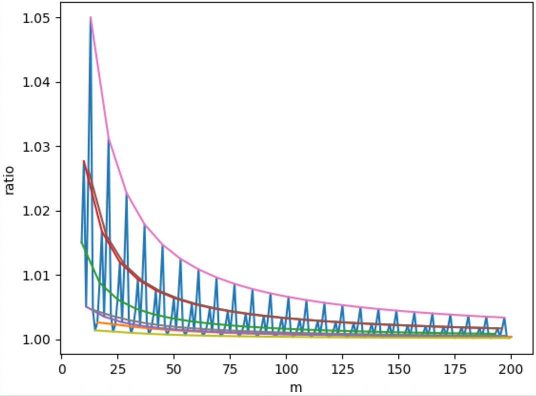

$$\frac{min(T(\pi_{oe}), T(\pi_d))}{min_n(T(\pi_{oe}), T(\pi_d), T(\pi_{nw}))}$$

(as a function of m)

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Conclusions...

Throughput optimization in robotic cells with **circular layout** was less studied and poorly understood so far. We:

- proposed new tools and specific cycle structures...
- ... leading to a **conjecture** on the best 1-cycle problem.

	Best 1-cycle	1-cycle conjecture
Linear	P	$m \leq 3$: valid $m = 4$: false
| balanced	P	$m \leq 15$: valid
Circular	NP-hard	
| balanced	$m \leq 11$: P $m \geq 12$: also P?	$m = 6$: false

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… and perspectives

(Well, aside from settling the conjecture)

Other types of production constraints:

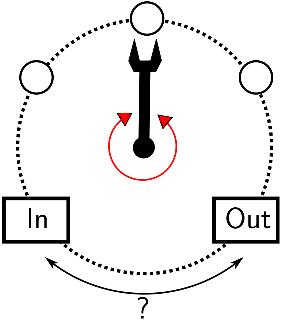
- Non-balanced case: Improving existing approximation…
- Proportionate flow-shop

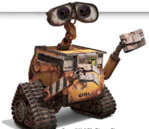
Open questions for regular balanced cells:

- Best 1-cycle for $m > 11$…
- 1-cycle conjecture for $m \leq 5$
- Cycle function for $m \geq 6$

Relationships with other layouts:

- Comparisons of layouts
- Generalization of the circular layout:

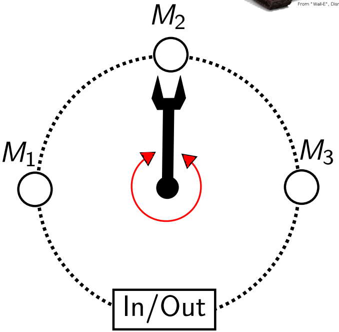




From "Wall-E", Disney-Pixar

Thank you!

Brio



D'après J.Rouxel

from "Up", Disney - Pixar

Third party content

- This presentation features some tributes to the following works (short excerpts):
  - *Wall-E* (Pixar Animation Studios, 2008), slides 1, 4, 8 and 43;
  - *Up* (Pixar Animation Studios & Walt Disney Pictures, 2009), slides 25 and 43;
  - *Les Shadoks* (Jacques Rouxel), slides 16 and 43.
- The picture of a toy train featured on slides 1, 2, 3 and 43 is extracted from a Brio commercial.
- Other clipart images (slides 4 and 10) are either public domain or released under a CC0 license.
- The template and backgrounds belong to G-SCOP laboratory.