

Experimental and numerical study of screening currents in REBCO high temperature superconducting magnets Guillaume Dilasser

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Communauté UNIVERSITÉ Grenoble Alpes

THÈSE

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Etude expérimentale et numérique des courants d'écrantage dans les aimants supraconducteurs à haute température critique REBCO

Thèse soutenue publiquement le **11 Octobre 2017**, devant le jury composé de :

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The old adage says that we should not judge a book by its cover. Indeed, one would be mistaken to believe that this Ph.D. thesis is the work of a single individual by only looking at the one author name written on it. In fact, its content is far more the product of a research environment and many interactions with a number of great people.

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Chapter 1

Screening currents in REBCO HTS coils

This chapter is intended as a general introduction to the problem of screening currents in REBCO superconducting magnets. After a few considerations about the characteristics of superconductivity in section 1.1, REBCO high-temperature superconductors are presented in 1.2. Section 1.3 highlights the interesting features of these materials, leading to a short description of its various applications in the field of magnet design in 1.4. Then, in section 1.5, the behavior of REBCO coated conductors under magnetic field is discussed and the phenomenon of screening currents is introduced. Its problematic consequences on the magnetic field of REBCO HTS magnets are shown in 1.6. Eventually, this introductory chapter ends in 1.7 with the outline of the present document.

1.1 Characteristics of the superconducting state

1.1.1 The two properties of superconductivity

Superconductivity was first discovered in 1911 by H. Kamerlingh Onnes during an attempt at measuring the electrical resistivity of mercury at low temperatures [54]. During his experiment, as the temperature of the mercury was brought just under 4 K, Kamerlingh Onnes was surprised to see the resistance of the material drop to a value so low that he was unable to measure it with his equipment. Later, he carefully stated that, without prejudice, the electrical resistance of mercury seemingly vanishes just above 4 K.

Superconductivity designates the state obtained for certain materials under a *critical temperature* T_c , in which the electrical resistivity drops to zero. For example, in the case of mercury, the critical temperature is just about 4 K. Above T_c , superconductors transition to their so-called *normal* state where behave like ordinary resistive materials. The first interesting consequence is therefore that a large amount of current can be transported through a superconductor cooled below T_c without causing any heat generation and thus without losing energy.

Later in 1933, W. Meissner and R. Ochsenfeld discovered a second characteristic of the superconducting state: the magnetic field is expelled out of the volume of a superconductor [71]. In the *Meissner state*, a superconducting material presented with an external magnetic field spontaneously develops *Meissner currents* in a shallow skin depth near its surface that effectively shield the inner volume from the external field. This discovery by

Meissner and Ochsenfeld separates the behavior of a superconductor from that of a *perfect* conductor.

This difference can be illustrated with the experiment of field cooling depicted in figure 1.1. A sample of material, either superconducting (top row) or perfectly conducting (bottom row), is cooled under T_c in the presence of a small, static magnetic field. In the superconducting material, Meissner currents develop spontaneously as the temperature drops under T_c to prevent the external field from penetrating inside the sample. This is the *Meissner effect*.

On the contrary, when repeating the same process with a perfectly conducting sample, no current arises. Indeed, the latter follows *Faraday's law* for induction $de + \partial_t b = 0$ (see 3.2.2) which states that the electric field e, and by extension the electric current, is caused by time variations in the external magnetic induction $-\partial_t b$. However, during the field cooling experiment, the external field remains constant, so no current is ever generated in the perfect conductor.



Figure 1.1: Field cooling experiment for a perfect conductor (top row) and a superconductor (bottom row)

The field cooling experiment shows that superconductors are not merely perfect conductors. The Meissner effect as been explained in 1934 by the brothers Fritz and Heinz London by considering that the whole superconducting bulk behaves like one giant diamagnetic atom. They proposed the equations that model this phenomenon, which are now known as the *London equations* [64]. These will be discussed in greater details in chapter 3, when we tackle the modeling of superconducting coils.

1.1.2 Limits of the superconducting state

It was soon demonstrated that, for a material to be superconducting, not only must the temperature be lower than a certain T_c , but the magnetic induction must also be lower than a *critical field* B_c . In 1957, a theoretical analysis by Abrikosov [1], established a distinction between two *types* of superconductors regarding their behavior when the magnitude of the external magnetic field rises.

Type I superconductors possess a single critical field value B_c . B_c is always a decreasing function of temperature T and $B_c \rightarrow 0$ T as $T \rightarrow T_c$. For $T \rightarrow 0$ K, B_c peaks at his maximum value, which is at most 0.1 T amongst known type I superconductors. As the external field is raised above the B_c limit, superconductivity collapses at once and the material immediately reverts back to the normal, dissipative state. This behavior is represented in figure 1.2a. Most type I superconductors are pure elements such as mercury, tin or lead.

Type II superconductors on the other hand have essentially two critical induction values, namely the lower critical induction $B_{c,1}$ and the upper $B_{c,2}$. Under $B_{c,1}$, which has about the same order of magnitude than B_c for the type I, the second type behaves just like the first. Between $B_{c,1}$ and $B_{c,2}$ however, type II superconductors enter a *mixed state*, where superconducting and normal conducting regions coexist. For short, the magnetic field is allowed to partially penetrate into the material while the electrical resistivity is still zero. This is a simplified explanation and the characteristics of the mixed state are discussed in further details in subsection 1.5.2. Eventually, when the magnetic induction is raised above $B_{c,2}$, superconductivity is lost in the bulk of the material. The surface of the material can however remain superconducting a bit longer, up to a third value $B_{c,3}$. The behavior of the second superconductor type is shown in figure 1.2b.



Figure 1.2: B - T phase diagrams for the two types of superconductors.

The very small value of B_c for type I superconductor (≈ 0.1 T) is an issue as is means that the transition to the normal state happens quickly as soon as there is just a little magnetic field. When current is supplied to a type I material, the self-field that it generates can even be enough to provoke the normal transition. Because of this, the practical interest of type I superconductors is limited. But interestingly, in type II materials, $B_{c,2}$ can often reach several tens, or even hundreds of Teslas; much larger than B_c for the type I. This renders the second type suitable to carry large current densities under high magnetic inductions and makes it highly interesting for applications requiring to generate magnetic field. A large portion of type II superconductors are metal alloys such as NbTi (Niobium-Titanium) or Nb₃Sn (Niobium-Tin), but the REBCO compounds, to be described in section 1.2, also belongs to this category.

Finally, the non-dissipative domain for a given material is also capped by a maximum current density *critical current density* J_c . Above it, the material once again reverts back to the normal state and starts dissipating energy as heat. J_c has however a less fundamental aspect than the first two critical values. Indeed, the transition between the superconducting state and the normal, dissipative one when J is raised is rather smooth. J_c is thus always set by convention to agree with an arbitrary criterion of energy dissipation.

There are intertwined dependencies between the critical temperature T_c , the critical field(s) B_c (or $B_{c,1}$ and $B_{c,2}$) and the critical current density J_c for any given superconducting material. Thereby, the operating domain of superconductors is often represented in the (T,B,J)-space as bounded by a *critical surface*. That surface separates the regions where the material is non-dissipative, under the surface, from those where it is dissipative (above the surface). One should remind itself that the critical surfaces possess a conventional aspect due to the definition of J_c . Also, note that consider here (T, B, J) because they are the most significant properties but the mechanical stress and strain level have an influence on the critical surface as well. A few examples are represented for some common superconductors in figure 1.3.



Figure 1.3: Critical surface plots for some common superconductors: NbTi (red), Nb₃Sn (blue) and MgB₂ (green).

1.1.3 An example of superconducting material: Niobium-Titanium

One notable example of superconductor is the *Niobum-Titanium* (NbTi) alloy, which is by far the most extensively used today. It is a type II ductile metal alloy that can be manufactured in long unit lengths through classical metallurgical processes. The critical temperature for Niobium-Titanium cables is 9.5 K and the upper critical field $B_{c,2}$ at zero Kelvin is 14 T [86]. The shape of the critical surface for Niobium-Titanium can be found on figure 1.3.

It is an alloy appreciated for its low cost, its well-known behavior and its resilience to mechanical stress. Niobium-Titanium is also superconducting as soon as it is produced unlike some other materials that require a final heat treatment to acquire their superconducting properties. As a consequence, it has been extensively used up to now in a wide range of applications, from accelerator magnets [24] to Magnetic Resonance Imaging [89], including large detector magnets [43]. Yet, it has today been pushed to its last limits and the quest for superconductors with improved T_c and larger transport current capabilities under field has led to considering completely new materials.

1.2 High-temperature superconductivity

1.2.1 The discovery of high-temperature superconductors

High-temperature superconductivity has been discovered for the first time in 1986 by J.G. Bednorz and K.A. Müller as they were investigating the electrical properties of ceramics formed from transition metal oxides [10]. They reported the discovery of superconductivity in a Lanthanum-Barium-Copper-Oxide (LaBaCuO) compound up to temperatures of 35 K, a full 12 K higher than the previous record in T_c . Following this discovery, superconductivity was also found in numerous similar ceramics. Compared to previously known superconductors, which were re-labeled as low-temperature superconductors or LTS, these high-temperature superconductors (HTS) obviously have much higher critical temperatures, of up to a hundred Kelvin, but also increased current transport capabilities under magnetic field at low temperatures.

Even more recently, high-temperature superconductivity has been discovered in a very wide range of materials, like *Magnesium Diboride* in 2001 [74] or *Iron Pnictide* compounds in 2008 [55].

1.2.2 Two main examples of HTS: BSCCO and REBCO

Among all the new kinds of HTS materials recently discovered, only a few are commercially available today in forms that are suitable for the design of magnets: the *Bismuth Strontium Calcium Copper Oxide* and *Rare Earth Barium Copper Oxide*, or respectively *BSCCO* and *REBCO* for short.

BSCCO superconductors

The BSCCO compounds, of precise formula $Bi_2Sr_2Ca_{n-1}Cu_nO_{2n+4}$, were discovered in 1988 [66]. Alternate versions of this compound exist for different values of n; for us however



Figure 1.4: Example of structure for a BSCCO 2212 round wire

n = 2 (so-called *BSCCO 2212*) and n = 3 (*BSCCO 2223*) are the most interesting. BSCCO 2212 for example is superconducting up to 85 K and has upper critical field $B_{c,2}$ of about 200 \pm 25 T [35]. BSCCO 2223 remains superconducting at even higher temperatures, up to 110 K.

BSCCO can be manufactured using conventional metallurgic powder-in-tube processes. In the case of BSCCO 2212, this even allows to produce round wires that can be bundled into cables. But for its rather easy fabrication, BSCCO has also some significant drawbacks. First, the wires need to be reacted in order to form the superconducting phase. This process is very demanding, requiring heat treatment of the cables at up to 900 ° C, with very precise constraints on temperature homogeneity, pressures up to 100 bars and partial O_2 atmosphere [60]. Besides, the matrix holding the BSCCO filaments together is usually made of silver to let the oxygen penetrate in the cable during the heat treatment. As a consequence, the cost of the cables is mostly driven by the price of the silver matrix, which is unlikely to ever drop low. Additionally, the mechanical resistance of the wires is rather poor as silver is a very soft metal. Figure 1.4 shows a schematic view of a BSCCO 2212 superconducting wire.

REBCO superconductors

One the other hand, REBCO compounds are a family of superconducting materials of similar chemical compositions: REBa₂Cu₃O_{7-x}, where RE can denote Yttrium (Y), Gadolinium (Gd), or other rare earth elements, hence the letters RE- in the name. The crystal lattice of the REBCO compounds is depicted in figure 1.5. The figure highlights the CuO planes that are a common features for every superconductor of that group. Up to now, the precise mechanism allowing superconductivity to arise in REBCO materials is unknown but these CuO planes are thought to play an important role in it [6].

The first REBCO, YBCO was discovered in 1987 [93]. It has a critical temperature of 93 K and an upper critical field $B_{c,2}$ at 0 K of about 140 T [69]. As opposed to BSCCO, REBCO cannot be fabricated using conventional metallurgic techniques but rather through *thin film deposition techniques*.



Figure 1.5: Crystal lattice of the REBCO superconducting compounds

1.2.3 The manufacturing of REBCO tapes

The fabrication of REBCO tape is a process that is a lot different from classical metallurgy. The production starts with a tape of *substrate*. It is usually a tape of Nickel alloy or hastelloy, measuring 2 to 12 millimeters in width and 30 μ m to 100 μ m in thickness. That substrate is the baseline on top of which other layers of materials are progressively stacked through *epitaxial growth*. It is the thickest layer in the tape composition and its mechanical properties mostly drive those of the finished product, at least in the longitudinal direction [8].

To generate a superconducting phase with a high critical current J_c , it is required to precisely control the alignment of its crystallographic lattice [22]. Otherwise, highly-angled grain boundaries within the REBCO layer cause the tape performances to drop quickly. Therefore, the REBCO layer has to be grown onto a bi-axially texture support to obtain a correct orientation and reach the best performances. A handful of techniques have been developed to achieve this. Overall, there are two main routes represented by the *Ion-Beam Assisted Deposition (IBAD)* and the *Rolling-Assisted-Biaxially Textured-Substrate* (*RABiTS*) techniques.

The IBAD route often starts with an hastelloy tape. Several bi-axially textured ceramic buffers are grown on top of it through Ion-Beam Assisted Deposition to progressively generate a clean, controlled lattice suitable for the REBCO deposition. The initial technique used for example a dual ion beam sputtering technique, with an Ar ion sputtering source directed at an *Yttria-stabilized Zirconia (YSZ)* buffer target and an assisting, etching, Ar + O ion beam directed at the substrate. The sputtered YSZ is deposited on the substrate to form a thin layer (about 1 μ m-thick) with an lattice orientation suitable enough for the REBCO layer. It was developed by Iijima at Fujikura Company in Japan [46]. Several variations have been developed over the years, like for example the interesting *Inclinate Substrate Deposition* technique, which is able to eliminate the etching ion beam by inclining the substrate [42]. The advantages of IBAD and its variants are the rather low cost and scalability, which makes it the preferred technique among industrial tape manufacturers. One the other hand, this route show rather slow film growth, of the order of 0.1 nm.s⁻¹.

The alternate RABiTS route was developed at Oak Ridge National Laboratory in 1994 [37]. The substrate is rolled between highly polished rolls and recrystallized with a heat treatment to obtain a cube-textured metal, with a controlled orientation. It the substrate is

made of silver, the RABiTS process can even allow to grow the REBCO superconducting layer directly on top of it because silver has only benign chemical interaction with the REBCO ceramic. Despite this, the preferred substrates for this route are rather Nickel alloys because of the sharpness of texture obtained, the lower cost of the material and better mechanical strength. For Ni-alloy, the rolling and recrystallization operation has to be complemented by additional buffer layers of YSZ, Cerium oxide CeO_2 or noble metals to prevent chemical reaction of the REBCO with the substrate. Some issues can also result from the use of certain Ni-based alloys. They can indeed be ferromagnetic, which leads to increased hysteresis losses in AC applications.

Once a suitable alignment has been achieved, the REBCO layer is deposited through epitaxial growth. Here again, several techniques exist, either using using chemical processes or physical ones. The group of chemical processes can for example be represented by the *Metal Organic Chemical Vapor Deposition (MOCVD)* technique. In MOCVD of REBCO, metal organics of the rare earth, barium and copper are individually evaporated, transported via an inert gas, mixed to the oxidant O_2 and finally decomposed on the heated substrate to form a thin REBCO layer [95]. Variants of MOCVD include the *Chemical Vapor Decomposition (CVD)* technique that uses commonly available salts of the constituent cations of REBCO instead of metal organics as precursors. Anyway, these techniques do not easily allow for a precise control over the stoichiometry of the reaction and require to special safety procedure to deal with the toxic chemical vapors. However, chemical techniques allow for fast deposition rates, rendering them interesting for large-scale, industrial processes.

On the other hand, the group of physical techniques can be represented by the *Pulsed Laser Decomposition*. A high-power pulsed laser is aimed inside a vacuum chamber to strike at target of the material to deposit. The latter is vaporized and forms a plasma plume which deposits on the heated substrate [75]. The use of PLD ensures the production of a high quality superconducting film but at the cost of very slow growth rates.

When the superconducting layer is finished, it is completed with a cap layer made of silver, which has almost no chemical interaction with the REBCO. Eventually, the whole assembly is wrapped with electroplated or laminated copper that helps shielding the materials from oxidation and enhancing the conductor stability by providing an alternate route for the current, in parallel to the superconductor. In the end, REBCO tapes typically measure a few millimeters in width and about a hundred micrometers in thickness. Figure 1.6 shows a schematic view of such a tape.



Figure 1.6: Generic structure of a REBCO coated conductor.

At present, there are about ten manufacturers of REBCO coated conductors worldwide, in the United States, Europe, Japan, Russian Federation, China and Korea. Each uses a mix of these different manufacturing processes, leading to superconducting tapes with various performances. Figure 1.7 illustrates these discrepancies by showing the critical current densities obtained for different REBCO tapes produced by several manufacturers (data from Barth *et al.* [8]).



Figure 1.7: Comparison of the engineering current densities of several REBCO tapes produced by different manufacturers.

1.3 Motivations to use REBCO tapes

From now on, this document will solely focus on REBCO tapes and their applications. This short section is entitled to explain the reasons for that choice.

1.3.1 Advantages of REBCO tapes

Performances for high-field operation

First of all, our interest for REBCO coated conductors is driven by their outstanding performances in terms of J_c under high magnetic field, which outclass every other known kinds of superconductor. To illustrate this, figure 1.8 shows a graph of the engineering critical current density versus the applied magnetic field for different kinds of superconductors at 4.2 K. Under magnetic inductions up to ≈ 20 T, one can see that REBCO tapes provide better performances than most earlier low-temperature superconductors. More

importantly however, under magnetic inductions above 25 T, all the known LTS have transitioned to the normal state and thus, the only remaining superconductors are the HTS BSCCO and REBCO. Therefore, REBCO coated conductors are prime candidates for the design of next-generation high-field magnets.



Figure 1.8: Comparison of the engineering critical current density of several types of conductors versus applied field. Data compiled by P. Lee at NHMFL [62]

Additionally, REBCO coated conductors open a vast new field of possible applications due to their high critical temperature T_c . Namely, the T_c for REBCO tapes being 93 K, it becomes feasible to design systems cooled by liquid nitrogen (LN₂) for which the boiling temperature under atmospheric pressure is 77 K. This can lead to new designs, both cheaper and more user-friendly because the cryogenic cooling does not require liquid Helium. These are however out of the scope of this document, which mostly focuses on applications where REBCO are used at low temperature (<10 K) to obtain the maximum performance out of the material. Some example of LN₂-cooled or cryogen-free systems can however be found in [33, 45, 21].

Mechanical properties

REBCO tapes also have interesting mechanical properties in the axial direction. They can be engineered with high strength substrates, which give them a very good resistance to tensile efforts. The Young modulus, elastic limit constraint and other longitudinal properties of the finished tape are indeed very close of those of the substrate. This is a significant upside when designing high-field magnets where there are intense tensile efforts in the windings. Figure 1.9 illustrates this by showing the tensile properties of various REBCO coated conductors produced by several manufacturers. However, REBCO tapes remain excessively fragile to delamination. Constraints as low as 20 MPa can be enough to peel the deposited superconducting layer from the substrate [88], which irredeemably damage the properties of the tape.





1.3.2 Drawbacks of REBCO tapes

Form factor

To begin with, the flat, tape-like shape of REBCO coated conductors generates new constraints on the design of both cables and magnet windings. Indeed, tapes are impossible to bend in the *hard way* (i.e. in the tape plane) so the conductor path in a REBCO winding must only feature bends in the easy way. Additionally, unlike round wires, flat conductors and cannot easily be bundled into a multi-stranded transposed cables. A few architectures have been proposed to solve this problem, which are covered in 1.4.1

Conductor cost

One of the main drawback limiting the use of REBCO tapes up to now is their cost. In 2017, the average cost for REBCO coated conductors is of the order of 100 $A^{-1}.m^{-1}$ when considering tape performances at 77 K under self magnetic field. At lower temperature and under magnetic field, the cost-to-performance figures are more favorable but still cannot compete with the cheap prices of NbTi ($\approx 1 A^{-1}.m^{-1}$ at 4.2 K) or Nb₃Sn (< 8 $A^{-1}.m^{-1}$ at 4.2 K). Nevertheless, REBCO prices are steadily decreasing with progresses made in the production processes. Besides, new manufacturers entering the market, like the Chinese company Shanghai Superconductors (SCSC), are also pushing the prices down. It can therefore be expected that the cost of REBCO coated conductors will continue to decrease, rendering them even more appealing over time.

Low quench propagation velocities

A second issue with current REBCO tapes comes from the slow propagation velocity of the quenches inside the material. The *quench* designates an irreversible normal transition from a part of the conductor. That transition is initiated by a deficiency of the cooling, a random energy deposit on the conductor or local weakness of the superconductor properties (e.g. a J_c locally below average). After a local transition has been initiated, the transited zone starts dissipating energy through Joule effect. If the cooling system is not able to absorb the dissipated power directly, a domino effect happens as the transited zone heats up and starts quenching the neighboring regions as well. This is a quench. The phenomenon continues to spread until all the energy of the system has been converted to heat or until safety systems have been triggered.

During a quench, the temperature of the transitioned zone rises, forming what is called a *hotspot*. The main danger is for that hotspot to reach harmful temperatures, that may cause irreversible damages to it. Hopefully, in previous LTS, the quench propagation velocities ranged from 1 to 100 m/s [50, 25]. With such high propagation speeds, when a quench happened, the transitioned zone was expending fast and the energy was converted as heat is a large volume of conductor. Consequently, the high quench propagation velocity helps mitigating the maximum temperature of the hotspot in LTS.

On the other hand, quench propagation velocities in REBCO HTS are of about 1 to 10 mm/s [87, 38] so 1 to 3 orders of magnitude slower than in LTS. As a consequence, when a REBCO conductor quenches, the transitioned zone remains rather localized, the energy is dissipated in a much smaller volume and eventually the temperature of the hotspot can rise much higher. The handling of quenches is therefore one of the main concerning problematic with REBCO HTS magnets. It is however beyond the scope of this document and we will only briefly return to that topic in section 4.3.1.

Field response

It should also be noted from the chart 1.8 that the behavior of REBCO coated conductors is influenced by the relative orientation between the magnetic field and the flat surface of the tape. A magnetic field orthogonal to the REBCO surface tends to limit the performances of the superconductor more quickly than a parallel field. In addition, the monolithic tape

geometry also requires a new approach to field quality, as the rest of the present document will demonstrate.

Typical characteristics of REBCO tapes have been discussed and one emphasized on their advantages and drawbacks with regards to other kind of superconductors. To summarize, the main upside to REBCO superconductors is that they allow to reach magnetic fields higher than any other known materials. Hence, they are currently unavoidable when envisioning future high field magnet designs. The next section introduces now some applications areas in which REBCO tapes have started diffusing and gives practical examples of realizations.

1.4 Applications of REBCO superconducting tapes

1.4.1 High-current cables

The main type of applications that we will focus on in the next subsections are magnets. For now though, we want to start with an excursion outside of that topic and talk about high-current cables.

High-current cables are superconducting cables assembled from several REBCO tapes. The main interest for bundling multiple tapes together is to increase the current that can be transported by adding more superconducting cross-sectional area. This is interesting in various scenarios and will discuss why as they are tackled later along this section.

Flat REBCO tapes are however not very practical to design cables, so a number of different architecture proposals have emerged along time. The most widespread kinds of REBCO high-current cables are summarized in figure 1.10.



Figure 1.10: Architecture of some common types of REBCO high-current cables

The most straightforward design of high-current cables that can be manufactured from REBCO tapes is the *twisted stack* [84]. It consists in several stacked tapes held together and twisted along the cable path. The twisting helps with reducing the length of the screening current loops, thus limiting the cable magnetization and AC losses. This aspect

is covered in greater details in section 1.5. The main advantage of the twisted stack is the easy fabrication but on the other hand, it also leads to a cable geometry that is rather awkward to work with. Designing a magnet out of twisted stack cables leads to a challenging optimization of the conductor path [44].

Next is the *Ræbel cable* [34]. A Ræbel cable is a type of multi-stranded fully transposed cable assembled out of several tapes of REBCO coated conductors, usually around 13 to 17. The production starts with multiple REBCO tapes of the same width that are cut in a zig-zag pattern, either by punching or laser cutting. Then, the tapes are weaved together to produce the Ræbel cable. A great upside with respect to the twisted stack is that the cable has now a somewhat rectangular cross-section which is easier to work with. Nonetheless, Ræbel cables remain more expensive and more complex to manufacture. Some of the material is also wasted during the fabrication because of the cuttings that are made in the REBCO tapes.

Eventually, a last, well-established design is the *CORC cable*, or *Conductor On Round Core* cable [91]. It is made out of several REBCO tapes wrapped around a cylindrical core, possibly onto multiple layers. The round cross-section of the resulting cable is convenient. Depending on the core diameter, the number of tapes and layers, CORC cables can have very different properties. The early designs featuring a large core had for example poor engineering current densities. Additionally, until recent developments that have allowed to make thinner cables, CORCs also suffered from very large minimum bending radii [5].

The first possible use for REBCO high-current cables is electrical networks to replace conventional high-voltage copper distribution lines. Indeed, compared to copper cables of the same transport current capability, REBCO-based power cables are lighter and more compact. Also, for very solicited power lines, REBCO cables lead to lower the energy losses because there suppress Joule heating. However, one should not forget that REBCO power grids have to be cooled down, meaning that there is an energy cost to refrigerate the installations. This cost is at their disadvantage when the power line is not used because it still needs to be refrigerated. Nevertheless, with efficient cooling and well-insulated cables, it is possible to render some sections of power grids more efficient with the use of REBCO cables. Several power lines using superconducting cables have already be demonstrated, for example in Long island, U.S.A. [79] or Incheon, Korea [82].

1.4.2 High magnetic field generation

As previously mentioned, REBCO coated conductors are able to withstand current densities far superior to any other kind of superconducting material in the presence of magnetic field. This makes them highly interesting to design high-field magnets. Currently the world's most powerful all-REBCO magnet is a 26 T prototype manufactured by SuNAM [102]. Besides, a flourish of new *hybrid* designs have emerged to combine the advantages of HTS, LTS and sometimes also resistive materials into one system. In hybrid designs, several concentric coils made out of different conductors are nested into one another, to try to use the most efficient material under each magnetic field condition. Examples of such design can be found in the 32 T project of NHMFL [70] or in the 30 T Nougat project of LNCMI [11]. With this type of application, the focus is on the reachable field magnitude. Constraints on the field homogeneity or time stability are relegated to the background.

1.4.3 Nuclear Magnetic Resonance

Another specific application where high-field magnets are interesting is Nuclear Magnetic Resonance and its medical counterpart, Magnetic Resonance Imagery. Here, a very homogeneous and very stable magnetic field is paramount for the measure to be possible. NMR magnets are indeed expected to deliver a field with spatial homogeneity of the order of 1-10 ppm peak-to-peak in the volume to measure [65]. Concerning the stability, the field drift rate over long periods of time is expected to be lower than 0.1 ppm/h. Eventually, a high field magnitude is also of interest as it allows to increase the resolution of the captured images. Overall, MNR and MRI magnets are challenging mostly because of the strict constraints on the field quality. This not only influences the conception of the system but also its fabrication, which must be very meticulous to avoid introducing manufacturing errors.

As of today, a few projects have started investigating the use of REBCO coated conductors for high-field NMR magnets. The most notable one is certainly the 1.3 GHz NMR magnet project designed by the Francis Bitter Magnet Laboratory team at MIT. This magnet takes advantage of the hybrid approach with LTS and HTS elements and to reach a center field of about 30,5 T [9]. Similarly, Bruker Corporation has been working on a 1.2 GHz hybrid NMR magnet with Nb₃Sn and REBCO elements that should release on the market in 2017.

1.4.4 Accelerator magnets

Superconducting magnets are also used in the context of synchrotron particle accelerators in order to drive the beam circulating inside the machine. Several types of magnets are needed for that purpose. The two most important ones are dipole magnets, used to bend the particle beam trajectory, and quadrupole magnets, that allow to focus the beam along one transverse direction. Each of these generates a field dominated by a certain *multipolar component* that dictate it influence on the beam dynamic (the dipolar component for the dipole, quadrupolar for the quadrupole, etc.). A comprehensive introduction to the computation of these multipolar components is given in [72]. For an accelerator to properly effect the beam, it is therefore needed that minor, unwanted multipolar components are made as small as possible with respect to the main one. In practice, the usual constraints ask that the amplitude of the minor components be about 10^{-4} smaller than the main one. This shows that there are some strong requirements on the quality of the field generated by accelerator magnets.

An other characteristic of accelerator magnets is that they need to be supplied with a ramped current during beam injection to accommodate with the increasing energy of the particles. To avoid dealing with large inductive voltages during the current ramps, one therefore wishes to shrink the magnet inductance. Besides, lower inductances are also appealing to enhance the protection against quenches. Indeed, the time constant for the decay of the current in the magnet $\tau \propto L/R$ is proportional to it. With lower L, the energy stored in the magnet can be dumped faster, therefore leaving less time for the quench hotspot to reach potentially harmful temperatures.

Naturally, to decrease the inductance while still keeping the same magnetic field, accelerator magnets have to operate at high current densities. This is the reason why they are designed with high current cables. Currently, to develop the use of REBCO in accelerator magnets, CERN is focusing on the Rœbel cable (see 1.4.1). Several tens of meters have already been produced in the framework of the european $EuCARD^2$ project and the integration in an actual magnet prototype is under investigation [59].

1.5 Magnetization currents in REBCO tapes

The latter section introduced some practical applications of REBCO tapes. In each case we also highlighted the main expectations on the magnetic field quality. In the present section, one introduces the notions of screening currents and flux creep and shows why these cause issues when dealing with REBCO magnets.

1.5.1 Screening currents

Screening currents designate the induced currents that arise in the bulk of a Type II superconductor in response to an external magnetic field between $B_{c,1}$ and $B_{c,2}$. They are also sometimes referred to as *shielding currents* or *magnetization currents*.

As mentioned in 1.1, a superconducting material responds to the external magnetic field as a perfectly diamagnetic medium. It shields itself from the external field by developing spontaneously *supercurrents*, i.e. currents that do not decay in time because they flow without resistance. The latter flow in shallow skin layer near the superconductor surface to effectively cancel the total magnetic field in its inner volume. This holds true both for type I superconductor under B_c and for type II under $B_{c,1}$. These surface shielding currents are referred to as *Meissner currents*.

For a type II superconductor in a magnetic field comprised between $B_{c,1}$ and $B_{c,2}$, the medium is not fully diamagnetic anymore. The external field starts to penetrate into the volume of the material and so does the supercurrents density. Since the protective currents flow now in the bulk of the material rather than on the surface only, they are given the different name of *screening currents*.

We keep for now a macroscopic point of view and focus on the response of a REBCO tape to the external field. When exposed to a magnetic field, screening currents arise in the tape to generate an opposed reaction field. Here, the flat geometry of the REBCO tapes plays a significant role, see figure 1.11. Namely, when the field is oriented orthogonally to the tape surface as in figure 1.11b, the screening current loops induced are wide and the reactive field they generate is intense. This reactive field is also referred to as the tape magnetization. On the contrary, when the exterior field is parallel to the tape as in figure 1.11a, the screening currents only develop in the thickness of the REBCO layer, which results in a much lower magnetization. Consequently, the orientation of the field with respect to the tape normal plays a major role in the field response of REBCO tapes.

These screening currents, associated with the tape magnetization, cause field disturbances in practical applications (see section 1.6).

1.5.2 Flux vortices in REBCO superconductors

The latter explanation can be developed in greater details by investigating the microscopic behavior of superconductors. During the mid-thirties, Abrikosov founded the theory de-



Figure 1.11: Local screening currents induced in a flat REBCO tapes by an external field as a function of its orientation

scribing the field structure inside of a type II superconductor between $B_{c,1}$ and $B_{c,2}$ [1]. The main point is that the magnetic field is only allowed to penetrate in the superconductor through "tubes" named *flux vortices*. Figure 1.12 shows the structure of a single flux vortex.



Figure 1.12: Internal structure of an isolated flux vortex

It is composed of a normal core where the superconducting electron pair density n_s (plotted in blue) quickly drops to zero within the characteristic coherence length ξ . ξ typically ranges from 1 nm to 500 nm for most superconducting material s[69]. Simultaneously, the magnetic peaks at the core of the vortex and decays over a short distance denoted λ . λ also usually varies around a few hundreds of nanometers. The integrated magnetic flux passing through one vortex is equal to:

$$\Phi_0 = \frac{h}{2e} \approx 10^{-15} T.m^2 \tag{1.1}$$

Where h is Planck's constant and e the electric charge of the electron. Φ_0 is also called the magnetic flux quantum.

In REBCO superconductors, flux vortices are altered by the fact that the material is very anisotropic. Indeed, superconductivity arises in the CuO planes that are separated from each other by the heavy rare earth / barium layers. Figure 1.13 shows a comparison between the aspect of flux vortices in an isotropic material and in a REBCO. Rather than



Figure 1.13: Comparison of the shape of flux vortices in isotropic and anisotropic (layered) superconductors like the REBCO compounds



Figure 1.14: Illustration of a flat strip of type II superconductor in the Abrikosov state. The external magnetic field is able to penetrate in the material through flux vortices.

being a single continuous tunnel spanning through the whole superconductor, they are instead formed by a stack of weakly-coupled pancakes. Given the proper excitation, parts of the vortex can de-solidarize themselves and slide past each other.

In the mixed state, a superconductor is thus populated with many flux vortices, a situation depicted in figure 1.14. The density is usually larger on the edges on the superconductor, where the field penetrates first and is the most intense. Inside the material, vortices are carrying same-sign flux quanta and thus tend of repel each other. Thereby, they spontaneously organize themselves into a lattice in order to minimize the repulsion forces. In a defect-free superconductor of ellipsoidal shape, this result in a 2D hexagonal lattice, called the Abrikosov lattice [101].

The lattice is not entirely set in stone however. In the presence of an external current density J, vortices also experience an external force :

$$f_{\Psi} = \mathbf{J} \times \Phi \mathbf{u} \tag{1.2}$$



Figure 1.15: Illustration of some possible defects for a 2D crystal lattice. (A) Edge dislocation; (B) Precipitate of impurity atoms; (C) Interstitial atom; (D) Vacancy.

where u is the unit vector parallel to the vortex axis and oriented with the field, Φ is the magnetic flux through the vortex, J the averaged local current density and \times denotes the vector product (see [69], Chap. 7). Given its expression, \mathbf{f}_{ϕ} is often time called "Lorentz force" yet its origin is different and still subject to questions. Under the influence of this force, vortices can be set in motion in the material and therefore, even for temperature lower than T_c and field magnitudes lower than $B_{c,1}$, the lattice can *melt* because of the transport current density. In this regime, called *flux flow*, the wide-scale order is then lost and the vortices begin to slide past each other, behaving like a two-dimensional fluid.

The Abrikosov lattice is a useful theory but it lacks a major consideration needed to understand experimental results: the fact that every superconducting material has defects. Defects in the crystallographic structure of the superconductor can be of many types. There are punctual defects, such as atomic defects or inclusions; 1-dimensional defects, like columnar defects or 2-dimensional defects, like dislocations of the superconductor or grain boundaries. Figure 1.15 illustrates some of these possibilities.

Defects in superconductors are interesting because they act as pinning centers for the vortices in the material. A vortex that should move under the driving "Lorentz force" can instead be anchored into some defect. Thereby, the material imperfections help locking the vortex lattice in place and preventing its motions. This has a positive influence on the transport current capability of the material as a moving vortex lattice dissipates energy and causes field instabilities. Thus, *dirty* superconductors have better performances than *clean* ones. Also as a consequence, the vortex lattice in a practical superconductor is not perfectly hexagonal, as described by Abrikosov, but instead warped by the pinning centers. Since there is no large-scale order anymore the phase is called a *vortex glass*. The glass can still melt given a large enough current density but it is overall more resilient than the unpinned Abrikosov lattice.

Given that they enhance the tape performance by anchoring the vortex lattice, defects are added on-purpose into the material by the manufacturer. Most recent techniques involve doping of the superconducting layer with nanoparticles so as to generate columnar defects [36].

1.5.3 Thermally activated flux creep

Eventually, these elements put us in position to understand the origin of the magnetic relaxation in superconductors. When the non-equilibrium flux vortex distribution is rendered quasi-stable by the presence of anchoring defects, a sufficient energy input can allow it to jump out of the pinning centers and continue to relax. This slow evolution of the pinned flux vortex density towards equilibrium is called *flux creep*. It is made of successive jumps of the lattice, form one pinned configuration to the next. Each jump causes a shift in the averaged current density in the sample and therefore also changes the magnetization.

Most of the time, the energy required to jump out of pinning center is brought to the vortex lattice through thermal excitation, the phenomenon is then called *Thermally Activated Flux Creep (TAFC)*. The first theory for magnetic relaxation in superconductors was first developed by Anderson and Kim, who were working on LTS between 1962 and 1964 [4]. Together, they introduced this concept of thermally-assisted relaxation of flux vortices out of pinning sites. They assumed that the hopping attempt time t for a flux vortex bundle is expressed in terms of a potential energy barrier U, the Bolzmann constant k and the temperature T through an Arrhenius's law:

$$t = t_0 . \exp\left(\frac{U}{kT}\right) \tag{1.3}$$

The effective hopping time t_0 is a parameter of the model, that can differ from the actual hopping time t by several orders of magnitude.

The movement of flux lines in the superconductor is assisted in the presence of current density J by the pseudo-Lorentz force f_{Ψ} (1.2). Therefore, the height of the energy barrier U is assumed to decrease with J. Anderson and Kim chose the simplest dependance possible, an affine U(J) relation:

$$U = U_0 \left(1 - \frac{J}{J_{c,0}} \right) \tag{1.4}$$

where parameter U_0 is the energy barrier height in absence of driving force and $J_{c,0}$ is the current density required to fully suppress the barrier.

Anderson and Kim illustrated the consequences of their theory is the simple case of an infinite superconducting slab exposed to a external magnetic field H large enough to fully penetrate the medium. In this case, combining the previous equations (1.3) and (1.4) with the quasi-static Maxwell equations lead to the classic flux creep expression:

$$J = J_{c,0} \left[1 - \frac{kT}{U_0} \ln\left(\frac{t}{t_0}\right) \right]$$
(1.5)

The theory predicts that the decay of the current density in the slab depends on the logarithm of time. Consequently, the slab magnetization, which is proportional to J also

drifts with the logarithm of time. This results was indeed experimentally verified shortly later [58].

Anderson and Kim's theory has the advantage of being rather simple with very few parameters, which is why we will often refer to it to analyze experimental results. Yet, a large number of issues were found with it. For HTS, careful studies of the magnetic relaxation over many orders of magnitude of t, have shown that the shape of magnetization decay is deviating from the predicted logarithmic rate [85]. This is a marker for the fact that the U(J) relation is more complex than a linear law. Also, is was discovered that the flux creep phenomenon does not vanish as ultra-low temperature (of the order of the milliKelvin), where thermal excitation disappears. Instead, a residual, temperature-independent decay was observed, which lead to the development of the idea of quantum tunneling of flux vortices [80]. A number of extensions to the initial idea of Anderson and Kim's have thereby been developed refined by numerous authors. All of these theories are consequently more complex with more parameters.

So, a precise description of the flux creep in HTS requires more advanced theories, as we will see in 2.3. However, one difference between LTS and HTS can still be introduced now with the simple Anderson and Kim theory: the very large magnitude of the screening effect decay in HTS. First, note that the temperature multiplies the time-dependent term in (1.5). Potentially higher operating temperatures for HTS therefore increase the magnitude of the magnetization drift. Additionally, the barrier height U_0 can be evaluated based on the condensation energy per unit length of the vortex $H_{c,2}^2 \xi^4 / 16\pi \lambda^2$ [101]. It strongly depends on the coherence length ξ , which is small in the case of HTS, leading to relatively small values of U_0 and also contributing to increase the amplitude of the decay. Those two factors combines in (1.5) to generate larger a drift amplitude, leading talk about giant flux creep in HTS.

1.6 Impact of screening currents for magnets

Origin of the screening currents and their relaxation in REBCO coated conductors have previously been discussed. The phenomenon has problematic consequences on the field generated by magnets designed with such tapes. One can isolate three main effects: the lowering of the produced magnetic field amplitude, the alteration of the field quality and finally, the slow time drift.

As mentioned in subsection 1.5.1, screening currents develop in REBCO coated conductors in opposition to the external field at the local scale. Considering the case of a REBCO magnet, the self-field obtained by suppling current to it acts just like the external field in the previous examples. It causes the apparition of screening currents at the local level in the REBCO winding. Each portion of it reacts to the global coil field by developing a local magnetization. When those contributions are integrated over the whole REBCO magnet, one obtains a global reactive *Screening-Current Induced Field*, also abridged SCIF. The SCIF oppose to the main coil field variation and so it lowers the maximum amplitude the global field can reach. This effect is summarized in figure 1.16.

Like explained in 1.5.1, the regions in the winding which develop the strongest magnetization are those where the magnetic field is close from being orthogonal to the REBCO tapes. For common solenoids and stacked double-pancakes, this means the coil ends develop the strongest local SCIF, as the coil global field has its largest radial component there. Nev-



Figure 1.16: Illustration of the SCIF effect in a REBCO coil. The coil self-field hits the REBCO winding at an angle, inducing local screening currents. Summed over the whole magnet, the global SCIF opposes to the initial field.

ertheless, they are also the furthest parts from the coil center. So, depending on the coil shape, the magnetization can be compensated by the distance. This is why the loss in field magnitude due to the SCIF varies considerably with the coil shape [99]. However, the reduction of the field amplitude can easily reach several percents of the expected value for most magnets. This first effect is already problematic but it is however not the worst. It could certainly be anticipated for certain high-field applications by simply oversizing the magnet right from the start.

The presence of screening currents also causes an alteration of the field quality a REBCO superconducting magnet can produce. The actual current density in a REBCO magnet is indeed the overlap of the transport current density, meaning the current density that was purposefully injected in the superconductor with the intent of generating a given field, and the screening current density. Therefore, the REBCO magnet will not generate a field with theoretical quality, the discrepancies being caused by the effect of the screening currents.

This is a major issue for a lot of applications. Namely, we just showed earlier that most applications, like NMR (see 1.4.3) or accelerator magnets (see 1.4.4), require to generate fields with precise features. Amemiya *et al.* for example made experiments on a small RE-BCO dipoles that highlighted the degradation of the dipolar field caused by the screening effect [3]. The issue is also critical in Nuclear Magnetic Resonance as the field homogeneity is paramount for the device function properly. Generating such homogeneous field using a REBCO magnet prove to be very challenging [2]. These are the only the most blatant cases were the issue arises but overall, the degradation of the field quality is bound to be a problem for every kind of applications. If the SCIF was stable over time, it would still have been possible to optimize the field of REBCO magnets by taking it into account in the design. But sadly, magnetic relaxation as explained in 1.5.3 causes it to redistribute over time. Therefore the overall current density drifts over time, inducing slow changes into the total magnet field. This is again problematic in applications where precise field has to be maintained over long periods of time, such as in NRM. Indeed, usual the usual Nuclear Magnetic Resonance requirements on magnetic field stability are of about 0.1 ppm/h maximum drift. This threshold is challenging to meet with REBCO tapes exhibiting very pronounced magnetic relaxation.

1.7 Outline of the present manuscript

The present document focuses on answering two main questions. First, it proposes a simulation model to anticipate the effects of screening currents on the field of REBCO magnets. This aim at providing magnet designers with the tools for picturing future REBCO coated conductor-based systems. On the other hand, it also explores the possible techniques for compensating, in full or in part, the effects of the screening currents on existing magnets. Both numerical simulation and experimental practice are used to answer these questions.

First, an experiment has been designed to measure the screening current-induced field in a small REBCO coil. This allowed to experimentally study the magnitude and the time evolution of the screening current induced field in simple scenarios that were to be later simulated. The design of the setup, the experimental setup and the acquired results are discussed in chapter 2.

A numerical model for simulating the behavior of the current density in REBCO coils has then been developed. It was thoroughly tested using the comparison with the experimental results from chapter 2 and later used in order to simulate the screening current effects in a real-size magnet, namely the EuCARD dipole. The magnetodynamical model employed, its implementation within the framework of the software GetDP and the eventual simulation results are discussed in chapter 3.

Moving to the second aspect of handling the screening currents into existing magnets, chapter 4 describes a second experiment achieved to test the applicability of different techniques. The principles of those are discussed in details, followed by the presentation of the new experimental setup. Finally, the results obtained with each technique are presented.

At last, chapter 5 addresses the general conclusion of this document.
Chapter 2

Measurement of the screening currentinduced field in REBCO coils

The present chapter describes the design, assembly and results of an experiment measuring the screening current-induced field intensity and decay in a small REBCO coil. Its aim is to assess the influence of different operating conditions like the temperature, magnetic field level or presence of transport current. The chapter starts by introducing the setup designed for the purpose of the experiment in section 2.1. Then, section 2.2 covers the various types of tests that were achieved on this system and shows for each one the final SCIF data measured. Eventually, section 2.3 comes back on the analysis of those experimental results using the theory for magnetic flux relaxation previoulsy introduced in section 1.5. The panel of measurements acquired in this chapter is later used as reference material in order to calibrate numerical simulations in chapter 3.

2.1 Description of the experimental setup

2.1.1 Local vs. global SCIF measurement techniques

As mentioned beforehand, the goal of this first experiment is to measure the screening current-induced effects in a small REBCO test coil. In essence, those are *local* phenomena: induction tends to create screening currents at any point in the winding based on the field conditions at that point. On the other hand, integrating the local SCIF contributions on the whole system allows to define the *global* effects on the generated magnetic field. Consequently, there are two approaches that one can take to investigate experimentally the screening effect in REBCO coils. The *local* approach consists in measuring the local magnetization of the REBCO tapes forming the superconducting winding; the second, *global* approach, measures the influence of the integrated SCIF on the magnetic field generated by the whole system.

To adopt the first approach requires to use a local imaging technique, like for example the Scanning Hall probe Microscopy [76]. With this technique, a field sensitive probe, like a Hall probe for example, sweeps the conductors less than 1 μ m above their surface to measure the local magnetization. This technique allows to map the magnetization with a spatial resolution better than 1 μ m. Afterward, it is also possible to obtain the local

current density in the superconducting sample, based on the magnetization, by solving an inverse problem.

This route is suited for small sized sample but not practical at all for studying a whole magnet system. Indeed, scanning a complete magnet would require a mechanism to move the probe across the magnet, accommodating large clearance, accurate positioning and fast displacement speed. Even so, the majority of the conductors cannot be accessed directly as they are packed inside the winding. In fact, only the inner and outer layers of the coil can easily be scanned, which limits the amount of information. Additionally, with only an incomplete scan of the coil, one is not able to reconstruct the current density in the full magnet. Lastly, should we have enough data, reverse-computation of the complete current density at the scale of a real magnet would still represent a tremendous computational effort.

Since local imaging techniques are only suited for the study of small samples, we turned to the global approach in order to analyze a full REBCO coil. The global route is also more in line with our objective to study the overall effects of the SCIF, that are the one limiting in the scope of practical applications. It eventually leads to a more simple experiment design, only using a few field sensitive probes to measure the magnetic field amplitude at selected locations.

2.1.2 Overview of the experimental setup



Figure 2.1 gives a schematic overview of the experimental setup we developed.

The center element of the system is the instrumented REBCO superconducting coil. When

Figure 2.1: Overview of the experiment

it is powered, its own magnetic field causes the appearance of screening currents in the winding, which in turn, cause disturbances on the initial field. Two test coils have been used with this experimental setup, they are described later in 2.1.3. In the rest of this section, the auxiliary systems of the setup are reviewed first.

To begin with, the superconducting coil has to be cooled down in order to keep it under the critical temperature of 89 K. During most experiments, a *liquid nitrogen* (LN_2) bath was used to cool the coil at 77 K, but some tests were also achieved at 4.2 K with cooling from a *liquid Helium* (LHe) bath. The tests in liquid nitrogen could have been performed in almost any kind of tank as the latent heat of vaporization of LN_2 allow to keep a fairly stable bath for hours under atmospheric pressure in an environment at 300 K. Nonetheless, we needed to use a proper cryostat in this experiment for the tests at 4.2 K using liquid Helium. It would have been impossible to keep a LHe cooling bath during several hours otherwise. Thus, we used a already existing *MERIC* cryostat that proved to be perfectly suitable for our application. It featured vacuum insulated double-walls with an outer auxiliary tank that could be filled with LN_2 to act as a thermal shield protecting the main bath. The cryostat is visible in figure 2.2e.

During some tests in liquid nitrogen, the SCIF drift was recorded during a long period of time, up to 10^5 s, which corresponds to about 1 day 4 hours. To sustain the cooling bath during this time frame, a simple level regulation loop was set in place. Two temperaturesensitive carbon resistors were positioned inside the cryostat to establish detection thresholds for the minimum and maximum LN₂ level. Their signals were interpreted by an *Air Liquide RN24* controller piloting the LN₂ inlet valve to maintain the cryogenic bath level in the acceptable range. The RN24 controler is shown in figure 2.2c.

The REBCO test magnet was connected outside of the cryostat to an Oxford Instrument IPS superconducting magnet power supply, capable at delivering a maximum of 120 A under 10 V. During all the experiments, the screening currents were induced in the REBCO winding by the mean of the supplied transport current. The IPS has several features that make it especially suited for the use with REBCO magnets. First, it featured front-panel controls to program current ramps with prescribed rate. Then, it also provided settings to configure a minimal quench protection system for the coil. Namely, the overvoltage detection threshold was programed to bring the output current of the power supply to 0 A when tripped. The same power supply was used both for the tests with LN₂ and LHe cooling. In the first case, its maximum output current was of the same order of magnitude than the critical current of the test REBCO coils (120 A and 82 A, respectively). In LHe however, the coil critical current was about eleven time greater than in LN₂ which means that the power supply was largely undersized. The IPS power supply is visible in figure 2.2a.

The coil was instrumented in order to monitor its behavior. To measure the magnetic field, we used two Arepoc S.R.O. cryogenic Hall probes, calibrated for use at 77 K. They showed high sensitivity of about 170 mV/T with an extremely clean linear response. Depending on the field level, the Hall probe voltages were either read with a National Instrument NI9238 ± 0.5 V input acquisition module or a Keithley 2182 nanovoltmeter (see figure 2.2b), in case of lower field levels. The second device was mostly used for measurements at the remanent state, when the overall field level is almost zero because there is no transport current anymore. Additionally, a set of Platinium RTDs was installed on the cryostat insert to measure various temperatures. Voltages taps inserted in the coil winding and a calibrated shunt resistor in the superconducting coil circuit were used to monitor the

coil voltage and current. The acquired data was eventually plotted and saved using a custom-made LabView program that interfaced with the acquisition devices.



(e) MERIC Cryostat

Figure 2.2: Equipment used for the SCIF measurement experiment

2.1.3 Details on the REBCO test coils used

Selection of the superconducting tape

To elect the REBCO tape used in our test coils, we compared the products of two manufacturers: the american *SuperPower* and russian *SuperOx*. We finally opted for *SuperOx* because the manufacturer was the only one to propose off-the-shelf insulated tapes and we preferred an insulated coil for this experiment. The reason is, non-insulated coils [96] or coils using metal-as-insulation [61] show transient current redistribution effects, as explained in 4.3.1. After charging such coils or discharging them, the current density needs some time to fully conform to the azimuthal path along the tapes. The transient current redistribution causes variations in the magnetic field produced and is eventually susceptible to interfere with the SCIF decay we want to measure. The already insulated conductors were chosen as they would save time with the winding process by removing the need to add inter-turn insulation.

The tape we eventually employed is SuperOx's 2G HTS 04-20Ag-20Cu-60H-PI. It is a 4 mm-wide 140 μ m-thick REBCO tape which includes an all-round dip-coated 20 μ m-thick polyimide insulation. The insulating coating is rather unusual but the structure of the rest of the tape is fairly common, made of a 60 μ m Hastelloy substrate, about 1 μ m of superconducting GdBCO compound, 2 μ m of silver cap layer and 20 μ m of surround electroplated copper for stabilization. We bought only 50 m of the tape, with an average critical current of 140 A at 77 K, self-field according to the manufacturer. This value was effectively validated by short sample I_c measurements.

Test coil design

For the design of the first REBCO test coil, configurations that lead to easier and less time-consuming fabrication were favored. Also, we decided to use only about half of the 50 m of superconducting tape for the first test coil. This way, should it be damaged, a second coil could be fabricated with the remainder of the tape to replace it. The inner radius of the test coils also had to be large enough to fit the Arepoc Hall probes and the probe holders that were re-used from a previous experiment. Consequently, we chose to make our first test coil a simple single-pancake with a 59 mm interior radius and 68 and a half turns total, accounting for the diametrically opposed positions of the currents leads. Thereby, the single pancake used a total of about 27.5 m of REBCO tape, saving enough for a potential second coil.

In the end, this precaution proved to be worth it. As esplained in 2.2.1, the first REBCO test coil was indeed severely damaged by a quench, due to insufficient cooling during a test at 77 K. After this incident, the first coil was unwound and so we re-used the coil mandrel to make a second coil with the remaining tape length (about 21.5 m). In the following, the coils are referred as either "A" for the initial coil or "B" for the second one.

Fabrication of the coils

Coil A was fabricated at CEA Saclay in August 2015 and its replacement Coil B in February 2016.

The main issue encountered during fabrication was to remove the insulating polyimide coating from the extremities of the tape in order to be able to make joint solders. Different chemical etching agents were tried that were either unable to remove the polyimide or worse, removed it but damaging the tape superconducting properties in the process. In particular, we were not able to reproduce the process recommended by the manufacturer, using a warmed-up amine solution, as the required chemical were not available to us. We eventually resorted to mechanical abrasion using fine sandpaper, in order to gently clear the ends of the conductor.

The innermost turn of the single pancake was glued on the G10 mandrel and then soldered onto a copper piece to make the positive termination. We did so using a indium-tin mix with a low melting point of 120°C in order to limit the risk of damaging the superconductor through excessive heating. Sixty-eight and a half turns of winding were made with a small pre-stress tension of 10 N applied on the superconductor. Nothing had to be co-wound with the REBCO tape to account for inter-turn insulation, since the conductor itself was already polyimide-coated. The outermost turn was then pulled out tangentially after the last turn and soldered onto the copper outer termination. Two voltage taps were added to the coil, just before the junctions with the current leads for means of reading the coil voltage. Finally, the Hall probes were soft-glued to the mandrel so as to measure the axial field in the mid-plane of the coil, both on the axis and 39 mm away from it. Figure 2.3 illustrates for instance the winding process of coil A and figure 2.4 shows a picture of the two finished coils, mounted on the experimental rig.



(a) Winding equipment used for the fabrication of the REBCO coils

(b) Preparation of the outer coil termination

Figure 2.3: Fabrication of REBCO test coil A

Our design had some flaws that can be discussed now. First, from the angle of the mechanics, the winding pack is eventually only held together by the solder on the external termination. If that one fails, the coil breaks and the conductor unwinds. Moreover, since this solder is made on the REBCO side of the tape, it is prone to delamination, which

makes it even more fragile. The mechanical resistance of the test coils was therefore quite poor. Hopefully, we did not planned to make tests under external magnetic field, which would have generated additional Lorentz force very susceptible to break the winding apart.

A second design flaw is to have made the coil junctions by soldering the superconducting tape directly onto copper terminations. Indeed, even with high-purity copper and a large soldered section, it means that the superconductor may locally heat up close to the terminations because of the Joule effect in the copper. The Joule heating in the inner junction creates an increased risk of quenching the magnet.



(a) Coil A

(b) Coil B

Figure 2.4: REBCO test coils used for the SCIF measurement experiments

Coil characteristics

The two single-pancakes A and B we made slightly differed in the total conductor length, number turns and inductance. These differences are highlighted in Table 2.1. Two coil constants are given, the first being for a position on the coil axis, in its mid-plane (hence the position of the center Hall probe) and the latter in the mid-plane but deported 39 mm off-axis (position of Hall probe 2). Also, the critical current values are computed using a 10^{-4} V/m criterion on the voltage of each turn.

Now that the experimental setup has been properly introduced, the different tests that were performed on that system and their results are presented in the next section.

2.2 Experiments of SCIF measurement

This section presents the test results gathered using the experimental setup previously introduced. Subsection 2.2.1 starts with the first SCIF measurement at 77 K, remanent field. Subsection 2.2.2 then develops on the campaign at 77 K with transport current supplied to the coil and eventually subsection 2.2.3 describes the last experiments achieved at 4.2 K with LHe cooling.

Parameter [unit]	Value for coil A	Value for coil B
Winding type	single-pancake	single-pancake
Number of turns	68.5	51.5
Inner / outer radius [mm]	$59 \ / \ 69$	$59 \ / \ 66$
Height [mm]	4	4
Total tape length [m]	27.5	21.5
Inductance [mH]	1.15	0.68
Coil constant at center [mT/A]	0.675	0.518
Coil constant deported [mT/A]	0.973	0.733
Critical current ¹ [A]	81.2	83.5

Table 2.1: Characteristics of the REBCO test coils for the SCIF measurement

¹Computed based on the measurements at 77 K, self-field.

2.2.1 SCIF relaxation at the remnant state at 77 K

Experimental protocol

For the first measurement campaign, we decided to measure the SCIF relaxation of the test coil at the remanent state, i.e. after having charged it to a nominal transport current and discharged it. Measuring the remanent SCIF at 0 A overall current has the advantage that the asymptotic value for the coil field is known to be 0 T. Any measured field value different than zero can thus be attributed with certainty to the presence of screening currents in the winding. On the other hand, it also has some drawbacks. First, the magnitude of the magnetic field to measure is low and thus the Hall probe signals to acquire are also low. This was however addressed using a *Keithley 2182* nanovoltmeter to read the probe signals, as mentioned in 2.1.2. Moreover, in the absence of magnetic field drift for 10^5 s (about 1 day, 4 hours) after the return to 0 A. This time frame was chosen because it was large enough to capture the decay rate of the SCIF, without being too overwhelming.

With the first campaign, we wanted to assess the effects of the maximum value and the ramping rate of the transport current on the measured SCIF. All the experiments presented in this Subsection were achieved using the test coil A. We recall that the coil I_c was estimated at 81 A at 77 K, self-field, and so we tested combinations of the three different transport current values 50, 60 and 70 A and two different ramping rates : 2 A/s and 10 A/s. Eventually, we operated in accordance to the following protocol :

- The test coil was cooled from a temperature above T_c to 77 K by filling the cryostat with LN₂. This allowed to start the experiment with the superconductor in a virgin state completely free from current density.
- The acquisition program was started and offsets on signals were computed.
- The coil current was ramped up from 0 A to its final value with a given ramping rate.
- A 30 s plateau was observed on the nominal current.
- The current was ramped down back to 0 A with the same ramping rate.

- The SCIF relaxation was then recorded for the next 10^5 s while the LN₂ level regulation kept a steady level of the cryogenic bath in the cryostat.
- Eventually, the liquid nitrogen was pumped out of the cryostat and the coil was warmed up above its critical temperature to suppress the trapped screening currents.

Figure 2.5 summarizes the protocol used in the form of a timeline.



Figure 2.5: Protocol for the measurement of the remanent SCIF at 77 K

Measured remanent SCIF in the test coil at 77 K

During the campaign, it appeared that the center Hall probe, which was recovered instrumentation from the R3B-Glad project, was not sensitive enough to sense the SCIF drift. More importantly, its signal displayed seemingly random jumps of a few 1 μ V several times during each experiment. We identified the origin of the signal jumps in faulty contacts in the SUB-D 9-pin connector the probes leads were connected to. As a consequence, we will only consider here the SCIF measurements obtained with the second Hall probe, deported off-axis. The second probe outputted a larger signal because it was exposed to a stronger field, being located closer to the winding. It also had soldered connections that prevented the kind of problem that happened on the center probe.

Figure 2.6 summarizes the gathered results. The screening current-induced field absolute intensity is plotted versus the elapsed time since the end of the coil discharge, using a semilog scale. Three clusters of two curves each can be isolated, each one corresponding to one value of maximum charge current. The higher the transport current supplied to the coil, the stronger the remanent SCIF. One can also note that, for a given charge current value, the ramping rate does not matter. Eventually, we can highlight that the observed decay rate appears to be logarithmic in the considered time frame. For the very beginning of the tests, for about t < 2 s, data are missing because the slow acquisition rate of the Keithley nanovoltmeter.

Quench of the REBCO test coil

As we were moving to the second campaign of the measurements using test coil A, it quenched at 70 A. This quench under the expected critical current of the coil was caused



Figure 2.6: Measured remanent SCIF decay over time at 77 K for coil A

by insufficient cooling. After the incident, we found that the LN_2 level regulator was giving erroneous indication of the bath level. We suspect that the value of the carbon resistors used as level probes shifted over the thermal cycling. The coil had been charged up to 70 A while it was supposedly submerged into the liquid nitrogen bath, but it was in fact only cooled by the nitrogen vapors. Quickly after the current rose up to 70 A, coil A quenched due to the insufficient cooling. Immediately, voltage rose up to 10 V and the over-voltage protection circuit tripped, shutting the power supply down, see figure 2.7.

After the quench, the coil was carefully re-tested and proved to be severely damaged. For currents as low as 35 A, it already showed 10 μ V resistive voltage. In this situation, coil A could not be used for further tests, so we decided to unwind it and build a second one. This was also an opportunity to examine the damaged conductor and try to locate possible faulty spots.

It appeared that the polyimide insulation surrounding the conductor was damaged in many places, as shown in figure 2.8. Probably due to the heat of the quench, it had become very brittle and could easily be scratched off. Other spots showed more preoccupying damages, in some places both the Kapton and the electroplated copper layers seemed to have disappear One could also notice that the damaged portions were roughly aligned along two radii : either in front of the inner junction or opposite to it. Hence, we supposed that the quenched was initiated in the first turns by the Joule heating of the copper inner junction. The heat propagated radially in the winding leading to the pattern of aligned Kapton damages. Also, because the coil insufficiently cooled, it must have quenched almost as a bulk.



Figure 2.7: Quench of the test coil "A"

2.2.2 SCIF relaxation at 77 K with transport current

Experimental protocol for the second measurement campaign

During the second campaign, the aim was to assess the effect of transport current on the relaxation rate of the coil SCIF. Therefore, the SCIF was this time measured on the current plateau, i.e. while a steady transport current was flowing into the coil. As a consequence, the signal values picked up by the Hall probes were significantly higher than they were at the remnant state like in the first campaign. The Keithley 2182 nanovoltmeter could be replaced by the NI 9238 ± 0.5 V input card which was precise enough and much more convenient to use.

Nonetheless, making the experiment on the current plateau also had drawbacks. First, the time frame of the acquisition was shortened to 10^4 s (2 hours, 46 minutes and 40 seconds). Indeed, the previous 10^5 s time frame (more than one day) was considered too long because the coil power supply could not be left to operate without surveillance during the night. The first campaign showed however that the SCIF decay was already tangible in the first 10^4 s so it was not a major issue. Moreover, the presence of self-field was expected to accelerate the SCIF decay, thus making the shortened time frame even less problematic.

Secondly, the presence of transport current poses a problem to precisely determine the asymptotic value for the magnetic field of the coil. It can in theory be computed given values for the coil constant and the Hall probe ratios. In reality, the uncertainty on these parameters meant that we could not compute a reliable value of the theoretical field. The absolute SCIF value, computed as the difference between the expected coil field and the



Figure 2.8: Damages observed on the unwound HTS tape from test coil 1

measured one, was therefore not reliable. Eventually, we decided to switch to relative measurement to get rid of that problem. Each dataset was then normalized with respect to the field value at the beginning of the acquisition.

The experimental protocol used to perform the test during this campaign is summarized in the timeline figure 2.9.



Figure 2.9: Protocol for the measurement of the field drift for the charged coil at 77 K

SCIF decay measurement in a single-pancake coil with transport current

To be consistent with the results presented for the first experiment, only the field measured by Hall probe 2 is presented here.

Figure 2.10 summarizes the results of the second experiment campaign. The normalized field is plotted against the logarithm of the duration since the beginning of the current plateau. The normalized field itself is computed as the ratio between the measured magnetic field at a given time and the measured field at the same location for t = 1 s. Each curve of a different color represents one experiment at a different nominal current.



Figure 2.10: Drift of the normalized field for the charged coil B at 77 K

2.2.3 Remanent SCIF relaxation at 4.2 K

To conclude the experiment campaigns on the first setup, we made a few tests at 4.2 K, using a bath of LHe for cooling. The supply in LHe was provided by a 250 L dewar. To lower the volume of LHe required to cool the experiment, voids in the cryostat were filled with extruded polystyrene. It allowed to cool the experiment with an average of 50 L of LHe per experiment. The gaseous Helium was also recovered at the cryostat exhaust in order to be re-liquefied, recycling this expensive element.

A few other modifications were also performed on the setup prior to the tests. The temperature probes equipping the experiment were notably replaced. Indeed, Platinum RTD are not sensitive anymore under 30 K and the RN24 level regulator used for LN_2 was not deemed as trustworthy anymore after the quench at 77 K. So, previous temperature sensors were removed and replaced by a LHe level gauge and the corresponding readout device, presented in figure 2.2d. Eventually, the same IPS power supply was kept for the tests at 4.2 K.

The same experimental protocol was used for this campaign as in the first one at 77 K: the remanent field of the superconducting coil was measured after charge and return to zero current, the only difference being the temperature of the tests. The experimental protocol is therefore close to the one presented in figure 2.5.

The few tests performed under liquid Helium cooling did not allow to measure the SCIF decay at 4.2 K during the time frame observed.

2.3 Analysis of the SCIF measurements

In section 1.5.3, we introduced the basic theory explaining the Thermally Assisted Flux Creep phenomenon: the Anderson-Kim theory. It predicted a decay of the magnetization of a infinite superconducting slab with logarithmic rate in time. Our experiments on the superconducting coils tend to agree with this prediction, as shown on figures 2.6 and 2.10. However, the geometry of the systems are very different in the two cases. Also, Anderson and Kim made their prediction for the critical state of a superconducting slab while we measured the logarithmic decay for remanent state of the coil (see 2.2.1). It leads to think that the logarithmic decay is a more universal characteristic of the TAFC and not simply a result of the Anderson-Kim theory.

First, let us consider for example the case of an thin REBCO superconducting film exposed to an external magnetic field aligned with its normal axis. This case is a good representation of a single REBCO tape submitted to an external field. Gurevich and Brandt have solve the non-linear, non-local problem corresponding to the time evolution of the magnetization of this tape in [39]. They found that the magnetization evolution is separated into two regimes.

During the initial part of the evolution, the magnetization M can be written as :

$$M(t) = M(0) - M_1 \ln\left(1 + \frac{t}{\tau}\right)$$
(2.1)

where, τ depends on the temperature, magnetic field level, sample geometry and ramping rate of the external field. When the relaxation is established on the other hand, i.e. for $t \gg \tau$, M(t) becomes independent of the initial conditions and writes :

$$M(t) = M_c - M_1 \ln\left(\frac{t}{t_0}\right)$$
(2.2)

The shape of the long-term relaxation in the case of the thin film is the same as for the slab. The parameter values and their interpretation are different nonetheless. Also, the curves in figure 2.10 show a different behavior for t < 10 s that can be explained by a initial decay regime like the one they found.

Eventually, Gurevich and Küpfer [40] generalized that form for the relaxation. The initial portion of the evolution, where the relaxation is not yet logarithmic, is caused by the transient redistribution of the magnetic flux over the sample cross-section after the stepwise change in \dot{B}_{ext} at t = 0 s. It plotted against the logarithm of time, the magnetization presents a plateau there. This plateau is however only the result of the compression of the *t*-axis through the logarithm action, there is no real plateau in M(t) for small t.

Lastly, the results that were obtained at 4.2 K can be explained by referring to the simple Anderson-Kim model. The effect of the lower temperature is prevalent, as seen in (1.3), the characteristic time for flux line hopping increases exponentially with the inverse of T. Also, at 4.2 K the critical current density of the REBCO tape employed is about eleven time greater than at 77 K. The low current used during the tests compared to the actual capability of the material means that $J/J_{c,0} \ll 1$ in (1.4). The current density does not

help the relaxation phenomenon by reducing the energy barrier to overcome. For those reasons, the effective rate of magnetic relaxation at 4.2 K was too low to be measured with this experiment.

The results of our experiments quantitatively match established results form the literature. However, it is not possible to analyze precisely our results using only analytical formula as previously cited authors have done, because of the size and complex shape of the system. To go beyond requires to establish a numerical model for the coil and that is precisely the objective of the next chapter.

Chapter 3

Magnetodynamical simulation of REBCO superconducting coils

This chapter presents the development of a simulation model for the computation of screening current-induced effects in REBCO superconducting magnets. The first two sections 3.1 and 3.2 introduce the objective of our simulations and the mathematical framework they use. The numerical model is then built progressively around three examples: a single, straight tape of REBCO coated conductor 3.4; a small axisymmetrical REBCO single pancake 3.5 and eventually the full-sized EuCARD dipole in 3.6. Each example builds on the previous one by adding further layers of complexity. For every case, the results of the simulations are presented and benchmarked against experimental data whenever possible or against a different simulation software otherwise.

3.1 Objectives for the numerical model

Superconducting magnets are interesting objects from the modeling and simulation point of view because of the number of coupled physical domains involved. These physical domains are also called *compartments* in this document.

Of course, the compartments of electromagnetism is the first to come to mind when dealing with the simulation of a magnet behavior. Nonetheless, the simulation of the cryogenic cooling of a superconducting magnet, for example, also requires to take thermal aspects into account. The thermal compartment is equally necessary for the computation of quenches. In the same way, the mechanical compartment needs to be introduced if one wants to compute the stresses and strains into the winding. Mechanical simulation is namely a crucial aspect in magnet design to make sure that the system is able to withstand the stress created by the Lorentz forces. Overall, because superconducting magnets are such intricate objects, one must carefully elect which physical aspects to take into account in their model given their target objective.

In our case, the goal is to make computational predictions of field quality in the case of full-size REBCO magnets. Therefore, the modeling of the system is simplified by making hypotheses to discard the least impacting compartments.

First, thermal aspects will not be taken into account in our simulations. The reason is

that temperature has only an indirect influence on the calculated magnetic field. T only acts through the variations of some material properties that depend on it. In our case, the prime example would be the electrical resistivity of the REBCO superconductor. Indeed, for $T < T_c$ the REBCO is superconducting and its resistivity therefore equals to zero whereas for $T \ge T_c$ it becomes dissipative and the resistivity is non-zero. We choose to simplify this aspect however by removing here all those dependencies on temperature in the properties of the simulated materials. Overall, this is equivalent to the assumption that the whole system is isothermal and set to a temperature T_{sim} . We obviously choose here to have $T_{sim} < T_c$, so that the REBCO material is in its superconducting state.

For the same kind of reason, we also discard the mechanical aspects. Mechanical stresses can have an effect on the field a magnet generates if one considers a winding with stressdependent properties. Additionally, when stresses are sufficiently large to cause displacements of the conductors, then the altered path of the current also causes a shift in the field. But as with temperature, that influence of the mechanical compartment on the field computation is only indirect. It would take a model coupling electromagnetism with mechanic to properly capture those effects. We do not consider that kind of complexity in the first place and we simply discard the mechanical compartment. Thus, we do in our simulations as if the materials have infinite stiffness and therefore there cannot be any winding deformations.

In the end, we mostly keep the electromagnetic compartment as our objective is to compute magnetic fields for design purposes. In the last example of the EuCARD dipole (see Section 3.6), the electromagnetic compartment is coupled with electrical networks to establish relations between electrical quantities of the model.

3.2 Model presentation

3.2.1 Simplification of the modeling domain

Essentially, the systems we want to model consist in a set of REBCO superconducting tapes arranged within an infinite air region. At first, there is no outer boundary that should limit the field extent of the magnetic field, so the computation should be performed over the entire euclidean 3D space E_3 (see appendix A).

In our case however, we are interested in magnet systems, for which the ordered layout of the REBCO tapes allows to simplify the modeling domain. The main technique is to consider symmetries in order to reduce the dimension of the domain from 3D to 2D. Dipoles magnets for example can be modeled as infinitely long objects by considering a endless extrusion of their cross-section along the orthogonal axis. Let us choose here the extrusion direction to be the z-axis.

The advantage of a 2D infinitely long model is that it eliminates the dependency of all the simulated quantities over the space coordinate associated with the extrusion direction, z in our case. Additionally, vector quantities become either aligned to the z-axis, like the electrical current density j, or contained in the orthogonal xy-plane, like the magnetic field h. Thus, this kind of model also reduces the number of unknown quantities to compute, as one or two coordinates of each vector quantity are already known to be zero $(j_x, j_y \text{ and } h_z \text{ for example})$. Overall, the 2D infinitely long approach allows to make more lightweight and more efficient simulations. This is why the 2D approximation is very common in

dipole magnet design, where it is quite systematically used for the initial optimization of the cross-section layout.

A drawback for this approximation on the other hand is that the topology of the modeled system is not identical to the one of the original. Indeed, a single conductor that would pass through the *xy*-plane multiple times in different directions is now represented as a set of disconnected ones. In a dipole magnet for example, the winding essentially consists in a single conductor moving back and forth along the *z*-direction with half-turns at each coil end. By contrast, when extruding a dipole cross-section, one obtains multiple conductors parallels to one another, which are separated and have no electrical connections between them. In the model of a dipole magnet, the consequence of the 2D approximation is that the coil ends are missing and thus, the magnetic field they should create is not computed.

For solenoid or double-pancake coils, we can proceed in a similar fashion by considering a 2D cross-section but this time extruded by a full 360 degrees rotation around the coil axis. We choose here that direction to be our y-axis. This way, all the simulated quantities do not depend on the azimuthal coordinate z anymore. Like previously, vector quantities are also either oriented along to the z-axis, corresponding to the azimuthal direction, (e.g. the current density j) or in the xy-plane (e.g. the magnetic field h). The advantages and drawbacks of the 2D axisymmetrical approach are still the same. Here again, the approximation alters the topology of the modeled object. For example, a single-pancake coil modeled with a 2D axisymmetrical approach is indeed not a spiral of conductor anymore but a set of disconnected concentric rings instead.

Eventually, after reducing the dimension from 3D to 2D, we end up with a modeling domain $\Omega = E_2$. We partition it into a *conducting region* Ω_c and its complement, an *insulating region* Ω_c^c such that $\Omega = \Omega_c \cup \Omega_c^c$. As the name imply, the conducting region Ω_c is where the electrical current is expected to flow whereas j is assumed to be uniformly zero in Ω_c^c , creating a perfectly dielectric behavior. The boundary of the conducting region is denoted $\partial\Omega_c$.

We note that Ω is still infinite in the *xy*-plane at this point as there is still no physical boudary to limit the radial extend of the domain. Thus, additional hypothesis will have to be made (see 3.2.5) in order to reduce it to a finite region on which a numerical simulation can be performed.

The schematic view of the generic problem is given on figure 3.1.

3.2.2 The Maxwell equations

We recall here the governing equations for the electromagnetic aspect of the model. The *Maxwell equations* are the fundamental equations that govern the physical compartment of electromagnetism. We prefer to use here the formalism of differential forms, described for example by A. Bossavit in chapter 1 of [73], because we think that it allows for the clearest and most uniform notations. We recommend the previous book to the interested reader but, alternately, a short introduction to this formalism is also available in Appendix B.

 Ω denotes the modeling domain, for the examples treated in this chapter we have $\Omega = E_2$. The space of differential k-forms on Ω is denoted $\mathcal{F}^k(\Omega)$. Accounting for the symmetry with regard to the z coordinate, some quantities make also use of the space of 1-forms orthogonal to the xy-plane. Thus, we denote this space $\mathcal{F}^1(\Omega_{\perp})$



Figure 3.1: Generic 2D modeling domain

The Maxwell equations express dependencies between the differential forms:

- $b \in \mathcal{F}^1(\Omega) \wedge \mathcal{F}^1(\Omega_\perp)$, the magnetic induction;
- $h \in \mathcal{F}^1(\Omega)$, the magnetic field;
- $d \in \mathcal{F}^2(\Omega)$, the electric displacement field;
- $e \in \mathcal{F}^1(\Omega_{\perp})$, the *electric field* and finally
- $j \in \mathcal{F}^2(\Omega)$, the current density.

They also make use of two tensor fields μ and ϵ , mapping $\mathcal{F}^1(\Omega) \to \mathcal{F}^1(\Omega)$ and $\mathcal{F}^1(\Omega_{\perp}) \to \mathcal{F}^1(\Omega_{\perp})$, respectively. μ is called *magnetic permeability*, ϵ is the *electric permittivity* and both depend on the local space properties.

Finally, concerning the operators, d symbolizes the *exterior derivative*. On a 1-form $\omega^1 \in \mathcal{F}^1(E_3)$, it acts similarly to the Curl $(\nabla \times)$ operator of vector fields, returning a differential 2-form $\nu^2 = d\omega^1 \in \mathcal{F}^2(E_3)$. ∂_t denotes the partial time derivative. Finally, \star denotes the Hodge operator, which is the canonical mapping of k-forms $\omega^k \in \mathcal{F}^k(E_3)$ onto (3-k)-forms $\nu^{(3-k)} = \star \omega^k \in \mathcal{F}^{(3-k)}(E_3)$. All these operators are defined in more details in appendix A.

With this, the Maxwell equations state that:

$$dh - \partial_t d = j$$
 (3.1a) $de + \partial_t b = 0$ (3.1b)

$$b = \star \mu h$$
 (3.1c) $d = \star \epsilon e$ (3.1d)

When using differential forms, some of the equations that we are used to in the vector field formalism vanish. This is for example the case with the constraint $\nabla B = 0$. With differential forms, that property of the magnetic field is already contained in the 2-form b as a mathematical object. The integral of a 2-form like b over a any closed 2-manifold is zero.

In a similar fashion, there is no need for specific interface conditions between materials with different electromagnetic properties (i.e. different ϵ or μ). For example, consider an interface between two materials 1 and 2 of different electrical permittivities. The interface is defined by the normal vector n_{12} . The vector field formulation gives us that the electrical field E on both sides of the interface are related through $n_{12} \times (E_2 - E_1) = 0$. This corresponds to the continuity of the tangential component electrical field across the interface between the materials. With differential forms, that kind of relation is not required anymore as e, modeled by a 1-form, automatically satisfies the continuity of the tangential component while not assuming the continuity of the normal one. These are some examples showing the interest of the formalism using differential forms.

The present chapter however makes use of a modified version of the Maxwell equations. It consists in neglecting the displacement current term $(-\partial_t d)$ in equation (3.1a), leading to only:

 $\mathrm{d}h = j \tag{3.2}$

which is known as *Ampere's theorem*. The set of Maxwell equations in which (3.2) replaces equation (3.1a) defines the so-called *eddy current problem*.

This simplification is legitimate in the scope of multi-turns coils as long as the total conductor length l is negligible compared to radiation wavelength λ [49]. Setting for example a criterion of $l < \lambda/100$, one find that the displacement remains negligible for frequencies up to 3 MHz and 3 kHz for conductor lengths of 1 m and 1 km, respectively. In the following examples, the values of l being of at most a few hundred meters and the rate of variation of the current being low, of about 1 A.m⁻¹, the eddy currents model makes a valid approximation.

3.2.3 Ohm's law

As previously introduced, j represents the local current density in a material.

In some situations, the current density j circulating inside some conducting regions can be assumed to be equal to a known 2-form j^d . This approach is often used to model stranded inductors supplied with a current I^d controlled by the operator. In that case, j^d simply boils down to $(I^d/\mathcal{A}) s$, where s is a 2-form on Ω_c and \mathcal{A} is the total area of the cross-section measured with that 2-form.

On one hand, this technique leads to easy calculations because the current density has a known, pre-computed value in the inductor. So, it can become handy to simplify simulations, as when one wants to generate a background field inside of a calculation. But, on the other hand, the fixed j^d is not physically correct because the current density j in the conducting region has not been let free to distribute spontaneously. Because of this, the magnetic field computed with a constant j^d is certainly not going to be the same as the one obtained through a physical process.

Instead, in this chapter, the local j in the conducting regions Ω_c is related to the electrical field e through a generalized *Ohm's law*:

$$e = \rho \star j \tag{3.3}$$

where ρ is a given tensor field $\mathcal{F}^1(\Omega_{\perp}) \to \mathcal{F}^1(\Omega_{\perp})$ representing the local *electrical resistivity* at any point in Ω_c . ρ can depend on various parameters and, in our model in particular,

the superconductor is assigned a tensor $\rho(h, dh)$ that depends non-linearly on the local current density j = dh and magnetic field h. The precise form for ρ will be discussed later in section 3.3

3.2.4 The H- φ formulation

To merge equations (3.1b), (3.1c), (3.1d), (3.2) and (3.3), one first needs to elect a physical quantity with which to write the problem down. In the case of the present document, one chooses to focus on the magnetic field h. Our model is then expressed in terms of a *field* variable as opposed to other formulations in *potential* that are centered around the quantities that the fields derive from. As an example, the same systems could have been modeled in terms of the magnetic vector potential $a \in \mathcal{F}^0(\Omega)$ (b = da), resulting in a formulation in potential.

At this stage, we still have an issue with the use of Ohm's law (3.3). One can conceivably apply it to the conducting regions Ω_c of the model, where the current density is allowed to flow. On the contrary, in Ω_c^c where the media are insulating, one would rather always see j = 0. To be consistent, insulating regions should not be assigned the same behavior law as the conducting materials. This is however what a purely *h*-based formulation does and this has often been encountered in practice.

Indeed, a widespread technique consists in assigning to Ω_c^c an arbitrary value of $\rho_{\Omega_c^c}$ such that $\rho_{\Omega_c^c} \gg \max_{\Omega_c}(\rho)$. By doing so, all the materials in the model become effectively conducting, but the current density tends to avoid the "insulating" regions because of their very large resistivity. The technique can certainly work [23], but it is also susceptible to cause major simulation errors. Stenvall *et al.* have for example shown that it could lead to non-negligible currents flowing through normally insulating air regions. In turn, this caused significant errors on the computed AC losses for their system [83]. Because of its unreliability, we prefer here the alternative approach introduced in the same article.

The partitions of Ω are handled in different ways. Ohm's law is directly applied without any changes in Ω_c . Then, in insulating regions, where one want j to be equal to zero, we note that dh = j = 0 through Ampere's theorem (3.2). h is therefore a closed differential form on Ω_c^c (see appendix A, a differential form is "closed" on a domain when its exterior derivative is zero there). Provided that de Rham's theorem is satisfied, Poincarré's Lemma tells us that closed differential forms are also exact in simply connected domains. Thus, in such domains, a 0-form $\varphi \in \mathcal{F}^0(\Omega)$ exists such that $h = -d\varphi$.

This leads to the $h-\varphi$ decomposition. The initial field variable h is kept as the unknown in Ω_c and its potential φ is used in Ω_c^c instead [18]. In the end, the system of PDE equations that we want to solve for the $(h-\varphi)$ -formulation is:

$$dh = j \text{ in } \Omega_c \qquad (3.4a) \qquad h = -d\varphi \text{ in } \Omega_c^c \qquad (3.4b)$$

$$de = -\partial_t b \tag{3.4c} \qquad b = \star \mu h \tag{3.4d}$$

$$e = \rho \star j \text{ in } \Omega_c \tag{3.4e}$$

This problem is not well-posed yet as the boundary conditions are not expressed.

First, let us see how we can handle the case of the boundaries present inside Ω . There is indeed the issue of the frontier between Ω_c and Ω_c^c , which we already denoted $\partial \Omega_c$. The physical argument concerning that interface is that the current density should not be able to cross it, as it would correspond to current "leaking" into the insulating region. The constraint we therefore want to enforce is $t_{\partial \Omega_c} j = t_{\partial \Omega_c} dh = 0$. It uses the trace operator t briefly introduced in appendix A.

Then, we have a second issue arising from the fact that infinite surrounding air domain is not bounded.

3.2.5 Treatment of the infinite space

We need a method to handle the infinite surrounding air region that is a part of Ω_c^c . The issue does not come from the z direction. Indeed, even if the modeled system is infinite along the z-axis, that is already addressed in a 2D model by using the symmetries of the unknown variables $h - \varphi$. The problem arises from the fact that the system also currently has an infinite radial extend in the xy-plane, which we cannot tolerate.

The reason is that in the Finite Element Method, the air needs to be meshed and the unknown functions need to be computed inside of it. So, for obvious reasons of storage into finite computer memory, Ω_c^c cannot remain unbounded and must be restricted to a finite domain in the *xy*-plane. However, the truncation of $\partial \Omega_c^c$ raises the issue of the boundary conditions that should be applied in order to best replicate the original infinite media.

Open boundary problems like our example are common in electromagnetism and thereby a large number of work-arounds have been developed over time [20]. In this thesis, we use a geometrical transformation technique that was first introduced in [47] and was subsequently extended by various authors. Brunotte, for example, introduced a variant suitable for problem with a large aspect ratio [16].

Here, we detail how this method works in the case of the 2D infinitely long models. This corresponds exactly to what is applied during the processing of the single tape and EuCARD dipole examples (see section 3.4 and 3.6, respectively). For the REBCO coil in the second example (see section 3.5), which is handled by an axisymmetrical model, the mathematics are slightly different but the principle remains identical.

First, a virtual frontier is introduced at an arbitrary distance R_c from the z-axis. The portion of Ω located inside the circular frontier is denoted Ω_{int} . R_c must be chosen large enough so that all the current density existing in the model flows inside Ω_{int} . The intersection between the insulating domain Ω_c^c and Ω_{int} is a region denoted Ω_a that is bounded along the radial direction. The infinite air region outside of the boundary at R_c is $\Omega_i = \Omega_c^c \setminus \Omega_a$. That way, the initial Ω_c^c has just been partitioned into Ω_a and Ω_i , with the two regions being connected on the boundary $\partial \Omega_a = \partial \Omega_i$.

For now, Ω_i is still infinite in the radial direction so we apply a spherical shell transformation to it. We choose a second arbitrary radius R_{∞} such that $R_c < R_{\infty}$. Then, Ω_i is mapped onto a finite domain Ω'_i using a coordinate transformation. Every point $X = (x, y)^T \in \Omega_i$ is mapped to $X' = (x', y')^T \in \Omega'_i$, with:

$$X' = \left\{ \frac{R_{\infty}}{|X|} - \frac{R_c \left(R_{\infty} - R_c\right)}{|X|^2} \right\} .X$$
(3.5)



Figure 3.2: Treatment of the infinite air domain. The initial open domain is divided in two using an arbitrary boundary at a radius R_c . A spherical shell transformation is then applied to the infinite domain to turn it into a spherical shell between radii R_c and R_∞ .

The transformation, depicted in figure 3.2, turns the infinite air domain outside of the frontier at R_c into a circular crown located between radii R_c and R_{∞} .

Doing so, the initial infinite air domain Ω_c^c is transformed into the union of two bounded ones $\Omega_c^{c'} = \Omega_a \cup \Omega_i'$. The full modeling domain Ω becomes $\Omega' = \Omega_c \cup \Omega_a \cup \Omega_i'$. The open-boundary problem that consisted in computing h and φ on Ω is turned into a closedboundary one.

The only influence of this transformation on our computations is in the way integration needs to be performed over Ω'_i , namely:

$$\int_{\Omega_i} f(X) \, dX = \int_{\Omega'_i} f(X) \, \frac{\partial X}{\partial X'} \left(X' \right) \, dX' \tag{3.6}$$

Computing integrals of quantities over Ω'_i now requires to factor in the coordinate change and the jacobian matrix of the transformation.

3.2.6 Handling non-simply connected regions

In figure 3.1, one should have noticed that $\Omega_c^{\ c'}$ is not simply connected. This means that not every cycle entirely contained in $\Omega_c^{\ c'}$ can be continuously contracted to a single point. Namely, a loop around the "hole" created by the conducting region Ω_c cannot be continuously transformed that way. Going back to what was stated in 3.2.4, Poincarré's lemma can not be applied on Ω_a because of this, which should deprive us from the possibility to use φ as variable. This is known as de Rham's first theorem:

$$\left(\mathrm{d}h=0 \text{ in } \Omega_{c}^{c} \Rightarrow \exists \varphi \in \mathcal{F}^{0}(\Omega_{c}^{c}), \ h=-\mathrm{d}\varphi\right) \Longleftrightarrow \left(\int_{z_{1}} h=0, \forall z_{1} \in H_{1}(\Omega_{c}^{c})\right)$$
(3.7)

 $H_1(\Omega_c^c)$ is the 1-homology space of Ω_c^c , which essentially contains all the loops z_1 around the conducting region (a more precise definition is given in appendix A).



Figure 3.3: Edge support of a generator for the discrete cohomology group $H^1(\Omega_c^c)$.

The right-hand side of the equivalence has a simple physical interpretation. The integral of h over a closed path like z_1 (its circulation along that path) is equal to the net current passing through the loop. De Rham's theorem states therefore that the $h - \varphi$ approach can only be used in non-simply connected domains when there is no current...

Obviously, this is an issue because we would like to constrain the current in the simulated system to take non-zero values. So, to restore the possibility of using the $h-\varphi$ formulation, one need to introduce a cohomology basis function Ψ . The aim for Ψ is to allow a possible non-zero circulation of h along certain cycles, corresponding to a non-zero current flowing into conducting sub-regions in Ω_c . In the context of Finite Element, Ψ sometimes referred to as a *thick-cuts*. It is indeed an edge-based basis function which slices through the insulating domain. Figure 3.3 gives a schematic representation of the support of the discrete cohomology generator Ψ . In order to keep the model construction focused, the more in-depth definition of (co-)homology and the detailed construction of Ψ are relegated to appendix A.

3.2.7 Weak-form of the problem

Now that the problem is expressed on a bounded domain and that the non-simply connected regions have been dealt with, it becomes possible to write the weak-form of the problem associated with the $h - \varphi$ formulation.

We look for a solution h such that:

$$h \in \mathcal{H}(\Omega') = \begin{cases} h \in \mathcal{F}^{1}(\Omega'), \\ \exists \varphi \in \mathcal{F}^{0}(\Omega_{a} \cup \Omega'_{i}), h = -d\varphi \text{ in } \Omega_{a} \cup \Omega'_{i}, \\ t_{\partial\Omega_{c}}dh = 0, \\ t_{\partial\Omega'}(\star \mu h) = 0 \end{cases}$$
(3.8)

that is a solution of the weak-form problem:

$$\begin{cases} \int_{\Omega'} \partial_t \star \mu h \wedge h' + \int_{\Omega_c} \rho \star dh \wedge dh' + I.\Psi = 0, \ \forall h' \in \mathcal{H} \left(\Omega' \right) \\ \int_{\Omega'} \star dh \wedge h' = 0, \ \forall h' \in \mathcal{H} \left(\Omega' \right) \end{cases}$$
(3.9)

where Ψ is a basis for the cohomology space $H^1(\Omega_c^{c'})$ and I is the the net current flowing through Ω_c .

In the next section, we cover one of the most important elements of the model, that is used to represent the behavior of the superconducting REBCO, the resistivity tensor ρ .

3.3 Definition of the material properties

3.3.1 A model about normal conductors

One must highlight now that (3.9) should not normally allow to simulate superconducting materials. Indeed, Ohm's law is not the proper material law to use in order to represent the properties of superconductors. It should even be counter-intuitive to apply this relation, usually describing a resistive behavior, to a material which main feature is precisely to have no resistance. Instead, one should have been using from the beginning the second London's equation [64]:

$$d(\mu \star j) = -\frac{1}{\lambda_L}b \tag{3.10}$$

In this thesis however, we do not use it and rely instead on a non-linear function for ρ to reproduce the properties of the superconducting material.

Therefore, we will not model here the REBCO as a superconducting material, not even as perfect conductor for which ρ would be uniformly zero. In our simulations, the superconductor is indeed assigned an electrical resistivity function ρ that is often going to take strictly positive values, which means that the REBCO is in fact modeled as a normal, resistive conductor. Nonetheless, the values of the REBCO resistivity function ρ are going to be very small, which makes the material quite close to a perfect conductor in practice.

As we saw in the introduction, see 1.1.1 on the experiment of field cooling, superconductors and perfect conductors respond to an external magnetic field in different ways. The modeled REBCO follows the behavior of the perfect conductor, meaning that it reacts to the magnetic field variations (through Faraday's law (3.1b)) but not to the field itself as a true superconductor would. Thus, the simulated material does not replicate all the properties of a true superconductor. Therefore additional hypotheses must be added to make sure that the modeled material still approximates the behavior of the real superconductor in a certain extend.

In fact, the Meissner effect we described in the introduction is truly what separates the superconductor from the perfect conductor. Because of this, what the model cannot simulated is the spontaneous generation of screening currents in the material when the temperature drops below T_c in the presence of a static external field. Nonetheless, we recall it is already established that our simulations would be isothermal with $T_{sim} < T_c$. As such, we are already not in the optic of simulating the field cooling experiment. So, in our case, discarding the London equations for Ohm's law to model the REBCO is acceptable as long as the initial condition corresponds to a material cooled below T_c without magnetic field. Technically, the problem (3.9) has to be initialized with h = 0 in all Ω' and j = 0 in Ω_c . Provided that initial condition, a normal conductor with the appropriate resistivity function ρ should have the same behavior as an authentic superconductor.

3.3.2 Forms for the superconductor resistivity

The power law

The most widespread relation used to emulate the properties of a superconductor using a variant of Ohm's law is the *power law*. This relation unites e and j through:

$$e = \left[\frac{E_0}{J_c} \left(\frac{|j|}{J_c}\right)^{n-1}\right] \star j = \rho\left(|j|\right) \star j \tag{3.11}$$

 E_0 is a arbitrary criterion on the electric field. It corresponds to the threshold separating the e-j curve into the "superconducting" domain (below E_0) and the dissipative domain (above E_0). One should note nonetheless that the power law is never going to give ρ exactly equal to 0. Thus, the material will always remain somewhat dissipative and there is strictly speaking no real "superconducting" state when using the power law. E_0 is therefore more closely related to the dissipated power in the material. The aim is to set up the E_0 criterion so that, below the value, the energy dissipation level is low enough that the material can be considered "superconducting". To summarize, the electrical field criterion has no concrete physical meaning, its choice depends entirely on what one defines as a negligible dissipation level. A common value used for E_0 in HTS is 10^{-4} V.m⁻¹.

The two other parameters of the law, J_c and n, are then defined by the choice of E_0 . J_c is the critical current density of the material, expressed in A.m⁻². It corresponds to the current density that flowing through the material when the electrical field e reaches E_0 . As an order of magnitude, J_c can often reach 10^{10} A.m⁻² for REBCO conductors used at 4.2 K under self-field.

n is a unitless exponent that characterizes the sharpness of the transition between the superconducting regime, for $|j| < J_c$, and the dissipative one, when $|j| \ge J_c$. For n = 1, the power law is equivalent to the usual linear Ohm's law (a case where there is no transition per se) and for increasing values of n the transition becomes gradually sharper, as seen in figure 3.4. In the limit of $n \to \infty$, the power law eventually become equivalent to critical state model [7]. In practical REBCO superconductors, n often ranges between 20 and 30 or even higher, which represent a very strong non-linear behavior.

In a first approximate, J_c and n are often taken as constants whereas in practice, they would vary with numerous parameters: temperature T, magnetic induction b, mechanical stress σ , etc. Associating functions to the material law parameters rather than using constant values is required to achieve accurate simulations. That topic is developed in more details in the next subsection 3.3.2.

The power law is especially suited to simulate high-temperature superconductors. Namely, it appears naturally when one combines the expression of the electric field generated by the flux creep (3.12) with the model proposed by Zeldov for the energy barrier U(j) (3.13) [103]. Equation (3.12) describing the flux creep regime is valid in REBCO superconductors over a wide range of current density and even for $j \approx J_c$ because of the very strong pinning forces. A different relation, describing the flux flow regime is only required when j is significantly larger than J_c , which shall not be the case in our simulations. On the other hand, the logarithmic decrease of U with the current density was also experimentally verified by Maley [68].



Figure 3.4: Plot of the e - j power law for different values of the n exponent.

$$e \propto b \exp\left(-\frac{U(j)}{kT}\right)$$
 (3.12) $U(j) = U_0 \ln\left(\frac{j}{J_c}\right)$ (3.13)

The second relation that will be used in this document is the percolation law, which this time allows for a true perfectly conducting state.

The percolation model

Proposed by Yamafuji [94], the percolation model links e and j through:

$$e = \begin{cases} 0 & \text{if } |j| < J_{c,0} \\ \left[\frac{E_c}{A} \left(\frac{|j|}{J_{c,0}} - 1\right)^{n_0}\right] \frac{\star j}{|j|} & \text{if } |j| \ge J_{c,0} \end{cases} \quad \text{with } A = \left(\frac{J_c}{J_{c,0}} - 1\right)^{n_0} \tag{3.14}$$

This model has overall a very similar shape compared to the power law, as shown in figure 3.5 but the notable difference is that it gives the material a true zero resistivity for $|j| < J_{c,0}$. The percolation model also keeps the J_c parameter from the power law (with $J_c > J_{c,0}$), which still corresponds to the current density flowing in the material when $|e| = E_0$. With the percolation model, there is this time a true perfectly conducting domain, for $|j| < J_{c,0}$, and a dissipative one, above $J_{c,0}$. Again, simulations using the percolation model become more accurate with the use of functions for J_c , $J_{c,0}$ and n_0 that depend on temperature, magnetic induction, etc.

These non-linear resistivity functions are used in the next sections to model the electrical behavior of the REBCO superconductors.



Figure 3.5: Comparison of the power law and the percolation model. Insert figure shows that the electrical field is uniformly zero when $|j| < J_{c,0}$ in the case of the percolation model.

Dependencies of the material law parameters

Accounting for the variations of the material law parameters (like J_c and n) is always necessary to achieve accurate simulations. Indeed, they depend in practice on a number of quantities, the most important ones being the temperature T and the local magnetic field B, both through its magnitude and orientation. There are at least two ways to implement these dependencies.

A first method is to rely on established analytical formulas. An interesting list of $J_c(T, B)$ relations is for example available the appendix of [27]. Analytical formulas are relatively easy to implement and can already help to get more accurate computations. On the other hand, they are generic and so, some tedious tweaking might be need in order to closely match the behavior of one precise type of superconductor.

The second method is to fit the material law onto a characterization of the REBCO tape that one wants to model. This leads to the most reliable results but it is also very demanding. Indeed, performing a characterization of a REBCO tape requires to make many measurements of its properties under various operating conditions. The more parameters the material law should take into account, the more experiments are needed. Should it be performed by hand, it would soon become a crippling task. Hopefully, that process is repetitive so some laboratories have equipped themselves with automated test stands, that can perform a characterization with little to no human input.

We went with the second approach during this Ph.D. To have the dependencies of the

material law parameters in the specific case of our REBCO tapes, we sent a short sample of it to Prof. Enric Pardo at the Slovak Academy of Science to perform a characterization. That process is explained in greater details in Appendix B. As a result of it, we were able to use precise $J_c(h)$ and n(h) parameters in our simulations.

3.3.3 Magnetic permeability

Apart from the paramount electrical resistivity ρ , the only other material property playing a role in the model is the magnetic permeability μ (T.m.A⁻¹). In the simulations we performed, we always consider that $\mu = \mu_0 = 4\pi 10^{-7}$ in the whole domain Ω . Doing so we discard the influence that eventual magnetic materials present in the system would have on the field. For the first two examples described in the next sections, this is not an issue because there are no such magnetic materials in the systems. This has yet some effect in the case of the last EuCARD dipole model because of the presence of an iron pole, as is explained in section 3.6.1.

3.4 Simulation of a single isolated superconducting tape

This section applies the generic problem established earlier to the concrete case of an isolated REBCO tape. The section starts with a description of the simulated system in 3.4.1, followed by the application of the generic problem to it in subsection 3.4.2. For this first example, the steps leading to the final discretized weak-form of the magnetodynamical problem are detailed in great length. Subsection 3.4.3 describes the resolution algorithm used to solved the discrete problem and eventually the last subsection 3.4.4 shows some simulated results for this very simple scenario.

3.4.1 Description of the system

The first example corresponds to a single, straight REBCO tape, measuring 4 mm in width and surrounded by an infinite air domain. The problem is studied in the cross-section with a 2D model.

Only the thin superconducting layer of the tape is modeled as a conducting material, all of the other materials are modeled as insulating. The current density can thereby only flow in the REBCO layer and never through the other components of the tape, like the substrate or the copper stabilizer (see 1.6). This is a strong assumption but in practice, the resistivity of the superconducting layer being zero, the current would effectively flow through it preferably. It would only start to spread into the parallel resistive layers when the current is so high that the REBCO is becoming resistive, at a phenomenon that is called *current sharing*. The limit of our hypothesis is that this phenomenon can never appear. Therefore, the model should be used for values of the current density low enough that they should not trigger current sharing. Hopefully, this also corresponds to the usual operating conditions for practical REBCO conductors. Above the current is forced to flow in the REBCO layer when it could be partially diverted into the other normal conducting layers. Apart from this, the hypothesis does not have any influence of the simulated field far from the tapes.



Figure 3.6: Modeled 2D domain of an single infinite tape

To summarize, the conducting region Ω_c is the REBCO superconducting layer alone and the insulating domain Ω_c^c is made of the air and all the other materials constituting the superconducting tape. The transformation technique introduced in 3.2.5 is then used to limit the radial extent of the surrounding air region, which lets us with the final modeled system depicted in figure 3.6.

3.4.2 Application of the model

Problem (3.9) is then applied to the modeled domain. For this first example, the different material laws introduced in section 3.3.2 are all tested: the basic power law and percolation model with constant parameters and the power law with parameters fitted onto the SuperOx tape characterization.

To now turn the weak form problem into a system of linear equations the computer can solve, we proceed to the discretization of both space and time.

Discretization of space

A finite element mesh of domain Ω' (figure 3.7) is obtained via the free and open-source software Gmsh [32, 31]. Gmsh combines a 3D finite element mesh generator with a lightweight CAD engine and a built-in post-processor. It provides fast and user-friendly meshing tools with parametric input and a large panel of visualization options. It can be operated either interactively using the graphical user interface or from the command line by supplying instructions within an ASCII text-file using Gmsh's own scripting language. Finally, Gmsh is able to interface with several finite element solvers using the ONELAB interface protocol.



Figure 3.7: Finite-element mesh used for the modeling of the first example. Surface (B) corresponds to the air domain Ω_a , the central black line (A) is the superconducting REBCO layer Ω_c (thickness not visible) and region (C) is the transformed air region Ω'_i image the infinite domain.

A structured mesh of quadrangular elements is preferred inside Ω_c because fits better the flat rectangular shape of the superconducting layer. We mostly used one layer of rectangular elements to mesh the thickness of the REBCO deposit and 60 to 150 elements to mesh its width. Meshing the REBCO layer with triangles would have either lead to very elongated elements or would have required to use a very small characteristic length (of about the tape thickness), resulting in a very large number of elements. Also, since we know that the current density is going to appear first on the edges of the REBCO layer, we can adjust the width of the quadrangular elements to make them progressively smaller closer to the edges. A simple triangular mesh is otherwise used in the insulating regions.

Figure 3.7 shows the type of mesh used, this example comprising of about 2100 nodes and 4100 elements, including 4000 triangles and 100 quadrangles.

Starting from here, $\mathcal{N}(\Omega')$ and $\mathcal{E}(\Omega')$ denote the set of mesh nodes and mesh edges in Ω' , respectively. Additionally, the set of mesh edges inside the conducting region is denoted $\mathcal{E}(\Omega_c \setminus \partial \Omega_c)$. The basis forms used to represent the solution on Ω' are the Whitney forms (see. Chap. 3 in [13]). In particular, a Whitney 0-form n_i is attached to every node in $\mathcal{N}(\Omega')$ and a Whitney 1-form e_i to each edge in $\mathcal{E}(\Omega_c \setminus \partial \Omega_c)$.

The next point that requires our attention is the determination of a generator for cohomology group $H^1(\Omega_c^{\ c'})$. $\Omega_c^{\ c'}$ is here a 2-manifold looking like a disk bored by a rectangular hole that corresponds to the imprint of the REBCO layer Ω_c . A coset of a 1-cycle looping around that hole defines a basis for the cohomology group. Gmsh conveniently allows to compute an approximation of this 1-cohomology basis function via its built-in discrete (co-)homology solver [77]. A basis function Ψ is automatically computed and expressed as a linear combination of the edge-based shape functions e_i in $\mathcal{E}(\Omega_c^{\ c'})$. The coefficients of that decomposition are denoted $z_i \in \mathbb{Z}$, they are computed once and for all at the during the construction of the mesh. An approximate solution for h in Ω' is then a differential form written as:

$$h = \sum_{i \in \mathcal{N}(\Omega')} \varphi_i \mathrm{d}n_i + \sum_{j \in \mathcal{E}(\Omega_c \setminus \partial \Omega_c)} h_j e_j + I. \underbrace{\sum_{k \in \mathcal{E}(\Omega_c^{c'})} z_k e_k}_{\Psi}$$
(3.15)

The real coefficients φ_i for $i \in \mathcal{N}(\Omega)$ and h_j for $j \in \mathcal{E}(\Omega_c \setminus \partial \Omega_c)$ are the unknowns resulting from the discretization. They can be stored inside vectors to be more easily handled: $\boldsymbol{h} = (h_j)^T, j \in \mathcal{E}(\Omega_c \setminus \partial \Omega_c)$ and $\boldsymbol{\varphi} = (\varphi_i)^T, i \in \mathcal{N}(\Omega)$.

According to Ampere's theorem (3.2), the pre-factor I multiplying the cohomology basis function Ψ corresponds to the value of the total current flowing through the superconducting tape. For this example, I is constrained to follow a pre-determined waveform I(t). This simulates the REBCO tape being supplied a controlled current using an external power supply.

Discretization of time

Time is also discretized and the progression from one time step to the next is achieved through the *backward Euler method*. This method was chosen because it is unconditionally stable, making it suitable for the resolution of stiff equations [48]. Thus, the solution at time index n + 1 is computed based on the solution at time index n using:

$$\left. \int_{\Omega'} \star \mu \cdot \frac{h(t_{n+1}) - h(t_n)}{t_{n+1} - t_n} \wedge h' + \int_{\Omega_c \setminus \partial \Omega_c} \rho(h(t_{n+1})) \star dh(t_{n+1}) \wedge dh' + [I(t_{n+1}) - I(t)] \cdot \Psi \right\} = 0 , \ \forall h' \in \mathcal{H}(\Omega) \cap \mathcal{W}^1(\Omega)$$

$$(3.16)$$

The second integral of equation (3.16) features a non-linear dependance in the unknown variable $h(t_{n+1})$. It is processed to recover a linear problem using the method presented in [53]. The electrical field $e(t_{n+1})$ is computed via the first-order approximation at t_n :

$$e(t_{n+1}) = \rho(h(t_{n+1})) \star dh(t_{n+1})$$

$$\approx \underbrace{\rho(h(t_n)) \star dh(t_n)}_{e(t_n)} + \frac{\partial e}{\partial j}(t_n) \star d(h(t_{n+1}) - h(t_n))$$
(3.17)

where $\partial e/\partial j$ is the Jacobian matrix of the function e(j). We then change the computed variable to use $\Delta h = h(t_{n+1}) - h(t_n)$, the variation of h between the two time steps.

Combining the discretizations of space and time, we finally obtain a linear system of equation. We denote \boldsymbol{H}_n the column vector resulting from the concatenation of the \boldsymbol{h} and $\boldsymbol{\varphi}$ vectors at time t_n . $\Delta \boldsymbol{H}_n^{n+1} = \boldsymbol{H}_{n+1} - \boldsymbol{H}_n$ is then the vector describing the change in the field between the two time steps t_n and t_{n+1} . The linearized problem is translated

into a block system of equations using this last vector:

$$\int_{\Omega} *\mu \cdot \frac{h(t_{n+1}) - h(t_n)}{t_{n+1} - t_n} \wedge h' \longrightarrow \mathbf{A} \cdot \Delta \mathbf{H}_n^{n+1} \\
\int_{\Omega_c \setminus \partial \Omega_c} \rho(h(t_n)) \star dh(t_n) \wedge dh' \longrightarrow \mathbf{R}_{1n} \cdot \mathbf{H}_n \\
\int_{\Omega_c \setminus \partial \Omega_c} \frac{\partial e}{\partial j}(t_n) \star d(h(t_{n+1}) - h(t_n)) \wedge dh' \longrightarrow \mathbf{R}_{2n} \cdot \Delta \mathbf{H}_n^{n+1} \\
[I(t_{n+1}) - I(t_n)] \cdot \Psi \longrightarrow \Delta I_n^{n+1} \cdot \mathbf{Z}$$
(3.18)

The equations the computer is eventually going to solve are:

$$\boldsymbol{A}.\Delta \boldsymbol{H}_{n}^{n+1} + \boldsymbol{R}_{1n}.\boldsymbol{H}_{n} + \boldsymbol{R}_{2n}.\Delta \boldsymbol{H}_{n}^{n+1} + \Delta \boldsymbol{I}_{n}^{n+1}.\boldsymbol{Z} = 0$$

$$\iff [\boldsymbol{A} + \boldsymbol{R}_{2n}].\Delta \boldsymbol{H}_{n}^{n+1} = -\boldsymbol{R}_{1n}.\boldsymbol{H}_{n} - \Delta \boldsymbol{I}_{n}^{n+1}.\boldsymbol{Z}$$
(3.19)

3.4.3 Resolution algorithm

The transient problem is solved using a simple adaptive time step algorithm. To compute the solution at time for time step n, we first try to compute ΔH_{n-1}^n . To do this, the linear problem (3.19) is solved multiple times, until its residual drops under an absolute convergence criterion α_{abs} . For most simulations, we used $\alpha_{abs} = 10^{-10}$. Based on the number of iterations needed to converge to the solution, the next time step Δt_n^{n+1} is determined by a simple multiplicative rule.

If the convergence is fast enough, i.e. if the solution at time t_n has been computed with few iterations, the new time step Δt_n^{n+1} is equal to $a.\Delta t_{n-1}^n$ with a > 1.0 (usually a = 1.5was used). Otherwise Δt_n^{n+1} is left unchanged with respect to the previous step. To prevent Δt_n^{n+1} from progressively becoming too large, we also capped it to a maximum value Δt_{max} . This notably limits the risk of the simulation suddenly starting to diverge because of a time step too large.

On the other hand, the algorithm can sometimes fail to compute $\Delta \boldsymbol{H}_{n-1}^{n}$ properly. It can happen either because the simulation starts diverging or because the solver is unable to reach the tolerance on the residual α_{abs} in less than a prescribed maximum number of iterations N_{max} . In that case, $\Delta \boldsymbol{H}_{n-1}^{n}$ is re-computed using a new time step decreased to $b \Delta t_{n-1}^{n}$, with b < 1 (usually we took b = 0.4).

Ultimately, it is also possible that the algorithm utterly fails to compute the solution ΔH_{n-1}^n and keeps reducing the time step to infinitesimal values. To avoid this, a lower threshold Δt_{min} is implemented. Whenever Δt_{n-1}^n becomes lower than this criterion, the whole computation is aborted and an error is returned.

3.4.4 Simulation results

The simulations have been carried out with the different kinds of material properties introduced in 3.3.2. Table 3.1 summarizes the values that were taken for the various

parameters of the laws. The total current I(t) supplied to the superconducting tape is the same in every case. It consists in a ramp from 0 A to 100 A with a rate of 2 A.s⁻¹.

Law type	$J_c \; [{\rm A.m^{-2}}]$	n	$J_{c,0} \; [{\rm A.m^{-2}}]$
Power law	$4 10^{10}$	35.81	n/a
SuperOx fit	see Appendix B	see Appendix B	n/a
Percolation law	$4 10^{10}$	11	$2.8 10^{10}$

Table 3.1: Parameters used in the material law describing the superconductor

These simulations take an average of 30 s on a Dell Precision T5510 workstation with an Intel Xeon E5-2650v2 CPU and 128 Go of RAM.

Figure 3.8 show the simulated current density for the different material laws at the end of the current ramp. For every case, the value which is plotted corresponds to the current density j averaged over the thickness of the tape in A.m⁻². Variations of j across the tape thickness are not computed in the present model as there is only one layer of quadrangular elements to mesh the REBCO thickness. With more layers, these variations could have been computed as well. The simulated values are compared with the predictions of the analytical formulas from E. Zeldov [104].



Figure 3.8: Simulated current density across the width of the tape for different material laws

The black line plot corresponds to the analytical reference computed with Zeldov's formula. The latter relies on the critical state model to characterize the superconducting behavior. With this model, the extremities of the REBCO layer which are completely penetrated by the magnetic flux carry a uniform J_c .

Unlike in the reference case, the average current density never completely reaches the

critical value J_c in any of the simulated cases. This is caused by the use of smooth e - j relations to model the superconducting behavior. With those, the electrical field appears progressively as $j \to J_c$ and therefore the current density also stabilizes earlier, for a value below J_c . To keep the total current in the tape constant, the difference at the extremities is compensated by a slightly deeper penetration of the current density inside the material. These discrepancies tend to vanish for higher values of the exponent, since increasing the value of n brings the power law model closer to the critical state model. This is illustrated in figure 3.9.

The third simulation uses the superconducting material law fitted onto the SuperOx characterization (see Appendix B). With this law, J_c and n become dependent on the local magnetic field. To be more precise, increasing the magnitude of field tends to lower the performances of the REBCO by reducing the local J_c and n. Additionally, in regions where the field is oriented close to the tape normal, the value of those parameters are further reduced. For our flat rectangular REBCO layer, the tape edges are the regions where the self-field reaches its maximum amplitude, meaning that those are the regions with the most degraded properties. This is why the simulated current density at the edges of the tape is lower for this case compared to the power law with constant parameters.

Nevertheless, the agreement between the simulated results and the analytical reference is very satisfactory and encourages us to further develop the model with the next example.



Figure 3.9: Simulated current density across the width of the tape for a power law with different exponents
3.5 Simulation of a REBCO single-pancake coil

3.5.1 Description of the system

For this second example, we want to model the superconducting test coil "B" that was used during the SCIF measurements described in chapter 2. It is a small REBCO single-pancake coil with 51 turns of coated conductors and inner / outer radii of 59 mm and 66 mm, respectively (its detailed characteristics are summarized in table 2.1). A picture of the physical object is shown in figure 2.4b.

To simplify, we model the coil in 2 dimensions as an axisymmetrical object. Essentially, the only difference between the present example and the previous 2D infinitely long case is the jacobian used for integration and coordinate changes. This time, we use an axisymmetrical jacobian instead of the usual cartesian jacobian from the first example.

We take advantage of the axisymmetry to only model the right half of the coil, hence dividing by two the size of the mesh and the number of unknowns to solve for. This introduces a new boundary on the revolution axis, the *y*-axis, so additional boundary conditions have to be defined. Since the *y*-axis is the revolution axis of the modeled coil, *h* must parallel to it, which translates into having h = th there. In fact, there is no specific constraint to set up to have *h* tangent the outer boundary of the modeling domain Ω . This is indeed the standard behavior of a the 1-form *h*. Therefore, the revolution axis does not pose any particular problem as far as boundary conditions are concerned.

Just like we did in the first example, we consider that only the REBCO layers are constituting the conducting region Ω_c . All the other materials in the windings plus the air region are grouped together into the insulating domain Ω_c^c . This model has thereby still the same issue of not being able to represent the current sharing phenomenon.

Finally, we use the same kind of geometrical transformation that was introduced in 3.2.5 in order to emulate an infinite air region surrounding the coil. The final modeled system is depicted in figure 3.10. The finite element mesh computed with Gmsh counts about 35000 nodes and 67000 elements, including 4100 quadrangles and the rest of triangles. Then, we simply apply the equation 3.9 to that new model.

3.5.2 Current constraints

The only remaining issue with the modeling of the second example is the definition of a generator for the cohomology group $H^1(\Omega_c^c)$.

A consequence of the axisymmetrical model is that the topology of the windings changes, as mentioned in 3.2.1. Instead of the spiral of conductor the real single-pancake is, the modeled coil is a set of concentric spires with no electrical connections between each other. Since the turns are not physically connected in series anymore, we have to add constraints to ensure that the same total current I is flowing inside each turn.

Contrary to the first example, there are now multiple "holes" in the insulating domain. To be specific, there are 51 such holes in Ω_c^c , one for each winding turn. So, there are now multiple candidates for the generator of $H^1(\Omega_c^c)$.

Let us focus on an easy example for now by considering 2 turns instead of the full 51, an illustration for this case is given in figure 3.11. A generator for the discrete homology group



(a) Global view of the simulated domain



 $H_1(\Omega_c^c)$ is made of a set of 1-cycles (loops) that are not boundaries of surfaces contained in Ω_c^c . As seen on figure 3.11, there are three possibilities to define the generator, each using two cycles $\{z_a, z_b\}$. The first two have one loop enclosing both holes and the second loop enclosing either one of them. On the other hand, the last possibility has its two loops each enclosing only one of the hole. Both solutions are generators of the same homology group.

The generator for the cohomology group $H^1(\Omega_c^c)$ consists in a set of two linear maps $\{z^1, z^2\}$ that assign real-values to the 1-cycles in the generator of $H_1(\Omega_c^c), \{z_a, z_b\}$. Therefore, we also have here three possibilities. To choose the most convenient, one focuses on the physical meaning of the 1-cochains in each generator. In the first two cases, the return value associated with z^a is the sum $I_1 + I_2$ of the current flowing into the two turns. The result of z^b is either I_1 or I_2 , depending on the enclosed turn. In the latter cases, the return value associated with $\{z^a, z^b\}$ are the currents $\{I_1, I_2\}$ in the first and second turn, respectively. We decide to use the second solution because it is more symmetrical.

The same principle is used with the complete 51-turns coils. We choose the generator of the cohomology group that is composed of 51 1-forms each related to the current in one and only one turn. It becomes then easy to ensure that the same current flows inside each turn by setting the coefficients associated with each form to $I_1 = \ldots = I_{51} = I(t)$.

With the generator for $H^1(\Omega_c^c)$ chosen and the current constraints set up, the rest of the calculation is essentially the same as in the previous example (see sections 3.4.2 and 3.4.3).



Figure 3.11: Three possibilities to define a generator for the homology group $H_1(\Omega_c^c)$.

3.5.3 Simulation results

With the model of coil "B", we simulate the experiments of the second SCIF measurement campaign described in section 2.2.2. To replicate as close as possible the characteristics of the real tape, we use the SuperOx-fitted power law to represent the behavior of the superconducting tape (see 3.3.2). For each simulation, we set the current in the coil to be equal to:

$$I(t) = \begin{cases} \frac{t \cdot I_{max}}{\tau} & \text{for } 0 \le t \le \tau\\ I_{max} & \text{for } t > \tau \end{cases}$$
(3.20)

Figure 3.12 plots as solid lines the simulated time-relaxation of the magnetic induction calculated at the center of the coil (subfigure 3.12a) and 39-mm off its axis (subfigure 3.12b). These positions correspond to locations of the Hall probes during the SCIF experiment (see section 2.1.3). We recall that the center probe did not allow to measure any field drift, likely because it was to far from the conductors for such a small coil and also because of faulty connections in its wiring. Therefore, we focus on the second, deported Hall probe for our comparison with the measurements.

As one can see, the simulation reproduces the overall tendency of the field drift but fails to give the precise value. The possible explanation for this is that the material law used to model the REBCO superconductor is not accurate enough. It might seem paradoxical at first because we precisely used a power law fitted onto a characterization of the tape to obtain the maximum fidelity. However, it should be noted that this fit was extracted from measurement data in the range 0-200 mT (see subsection 3.3.2). Yet in our simulations, we find that the magnetic induction on the tapes sometimes exceeds 200 mT, thus the software is sometimes extrapolating the material properties J_c and n outside of the range of the measured data, which we believe is the main cause for the discrepancies.

We also compared our simulation results with some benchmarks provided by other programs. In particular, many thanks to Messrs. F. Trillaud and E. Berrospe, who kindly accepted to reproduce our simulation of the magnet center field with Comsol for comparison. Mr. P. Fazilleau also provided a simulation code developed in Matlab that we used as an other comparison point. Whenever available, the results for the other simulation software are plotted onto the graphs in figure 3.12. Our model outputs a results that closely resembles the one obtained with Comsol by F. Trillaud and E. Berrospe. On the contrary, the shape of the field drift computed with Matlab seem especially off. Eventually, the comparison with both our experimental data and the Comsol simulations is rather positive.



(a) Coil center, position of Hall probe 1



(b) Deported 39 mm off-axis in the coil mid-plane, position of Hall probe 2

Figure 3.12: Time drift of the magnetic induction simulated for the modeled of coil "B" (see 2.1.3) on the current plateau and at different positions

3.6 Simulation of the Eucard dipole magnet

3.6.1 The EuCARD dipole magnet

For the last example, we want to test our model on a full scale magnet. We choose to work on the EuCARD dipole, a prototype dipole designed by CERN in collaboration with CEA.

The EuCARD program is the European initiative to develop magnet technologies for future particle accelerators reaching collision energies of about 100 TeV [105]. To build this machine, one proposal is to use 20 T bending dipoles inside a 80 km-long circular tunnel. However, 20 T is not a field level that can be achieved exclusively with LTS superconductors. Indeed, in the context of accelerator magnets where high current densities are required, these materials reach their limits for fields of 14-16 T. Thereby, the EuCARD program includes a study on REBCO inserts able to produce 4-5 T that can be employed within a background field of 13-15 T generated by an outer LTS magnet to possibly climb up to the expected 20 T. More precisely, we are looking here at the first prototype of REBCO insert that was developed in the scope of the EuCARD program.

At the time of writing this thesis, the fabrication of this prototype is completed and the magnet is undergoing its test phase. It will be operated in two configurations, first in stand-alone operation and then as an insert in a background field. The stand-alone tests have already started at CEA Saclay. For the later experiments in a background field, EuCARD will be inserted inside the aperture of the 13 T Nb₃Sn dipole Fresca 2 [26]. With the 13 T Fresca 2 is expected to produce, the assembly has been designed reach about 18.5 T under nominal operating conditions.

The following sections describe the important features of the EuCARD magnet before the modeling work starts in 3.6.2.

The conductor of the EuCARD dipole

EuCARD uses a specifically-designed high-current cable assembled from four REBCO tapes. As we mentioned in the introduction (see 1.4.4), the main reason to use high-current cables in the context of accelerator magnets is to lower the system inductance L. That way, in the event of a quench, the time constant of the current decay $\tau \propto L/R$ is lower. It allows to dump magnet energy faster, leaving less time for the hot-spot to reach harmful temperatures. Additionally, to lower the inductance allows to better control the inductive voltages generated during the current transients, like when the magnet is charging or when it quenches. In turn, lower voltages limit the risk of damaging the electrical insulation of the coil.

Figure 3.13 schematizes the cross-sectional structure of the EuCARD cable. It is made of two identical insulated conductors in parallel. Each conductor consists in two 12 mm-wide REBCO tapes soldered face-to-face on the sides of a 60 μ m-thick copper tape that acts as stabilizer. So, one conductor is already itself made of two parallel REBCO tapes. Additionally, two 100 μ m-thick CuBe layers are soldered on the outer hastelloy layers, providing enhanced thermal stabilization and mechanical strength. 30 μ m of polyester film is finally glued on each sides of the conductors to make the inter-turn insulation. The

full cable measures about 12 mm in width, 460 μm in thickness and has critical current of 3.6 kA at 4.2 K under 9.6 T perpendicular external field [12].



Figure 3.13: Cross-section of one winding turn of the EuCARD dipole, consisting in two parallel conductors.

EuCARD coil geometry

The initial objective for the EuCARD project was to build a REBCO dipole capable at generating a center field of 6 T. Besides, the magnet had to measure at most 99 mm in diameter so that it can be inserted inside the \emptyset 100 mm bore of the Fresca 2 dipole. A first design was achieved but mechanical studies showed that it was operating dangerously close to the mechanical limits of the supporting structure [12]. Thus, the target center field was revised downwards to 5.4 T and the dipole was re-designed with a reinforced support structure.

A schematic view of the dipole cross-section is given in figure 3.14. The magnet essentially consists in three double-racetrack coils wound around an iron pole. The middle coil is made of two stacked racetrack coils of 30 turns each, which represent 120 individual conductor turns and 240 REBCO layers in the cross section. It is supplemented by two additional coils for the top and bottom poles, both are made of one racetrack layer of 24 turns and the second of 10 turns. The entire dipole including its mechanical reinforcements measures 99 mm in diameter and about 700 mm in length. The operating of the dipole is 2.8 kA.

As previously explained, each turn in the winding of EuCARD is made of two conductors in parallel, which are only in electrical contact at the magnet terminations. The overall magnet is therefore constituted of two separate electrical circuits in which the supplied current is free to distribute. It should split evenly between the two circuits if their impedance are the same but, simply for geometrical reasons, this should never occur. The electrical



Figure 3.14: Simplified cross-section of the EuCARD dipole

circuit formed by the conductor the closest to the pole is indeed always going to have an impedance slightly lower than circuit formed by the outer one.

To try to better balance the current distribution in the parallel circuits, the two conductors in the EuCARD cable are transposed once in the winding, during the layer jump between the racetracks of the center coil. That way, the relative inner / outer position of the conductors are switched between the top and bottom halves of the dipole, which should balance the impedances. This solution works in the hypothesis that the parallel conductors are otherwise perfectly equivalent.

Lastly, we should note that the EuCARD dipole is designed to develop the technologies for accelerator REBCO magnets and is not production model at all. Therefore, some constraints that a real accelerator magnet would have to fulfill were not taken into account in its design. Most notably, one can see in figure 3.14 that it does not have an aperture for the particle beam. Besides, EuCARD had an objective in term of the center magnetic field but no constraints were set on the field quality. An actual accelerator dipole would have had requirements to meet regrading the multipolar components of the generated field. These more advanced aspects are being addressed by the follow-up program EuCARD² [59].

3.6.2 Modeling of the EuCARD dipole

Reduction of the simulated domain

We model the EuCARD dipole in 2 dimensions, in the magnet cross-section. By doing so, one eliminates the coil ends and considers only straight, infinitely long conductors. Discarding the ends of the coil is a strong hypothesis but it is usual in practice during magnet design and its legitimacy has been demonstrated through many experiments.

The full dipole cross-section is then reduced using symmetries. The first one we consider is a anti-reflection of the current density with respect to the y-axis. It means that for every

conductor in the right-hand half of the coil, it exists a returning conductor in the left-hand half carrying the same current density but in the opposite direction along z. This is an approximation because some elements present in the right half of the magnet cross-section are not exactly mirrored in the left half. For example, the layer transitions in the stacked racetrack coils are only located on one side of the y-axis. Still, we decide to neglect the few asymmetrical elements with regard to the overall symmetry of the winding.

The reflection with respect to the y-axis allows to easily half the number of elements in the model. Also, like in the previous example, it is very straightforward to implement. On the y-axis, the magnetic field has to be tangent to the boundary to enforce the reflection constrain. However, we saw that it is already the default behavior using the $h - \varphi$ formulation so there is no need to implement new constraints.

In a second time, we would also like to consider a symmetry of the current density with respect to the x-axis. This is however more complicated because, in practice, there is no such symmetry between the top and bottom half of the coil. Indeed, we recall that the inner and outer conductors are transposed at the center of the coil to try to balance the parallel circuit inductances. This implies that the image of one conductor from the top half of the coil through the reflection with respect to the x-axis is not the same conductor in the bottom half, but the one that was in parallel with it. This is depicted in figure 3.15. The conductor directly below (a, b) is not its mirrored image but instead (c, d). Therefore, the current density in the lower part of the magnet is not the reflection of the one in the upper half.



Figure 3.15: Structure of the winding in the EuCARD dipole. A different color is assigned to each tape of the two conductors constituting a winding turn. a and b represent the first conductor, c and d the second. The left half of the dipole is the anti-reflection of the right half through the symmetry of axis y. However, the lower half is not symmetrical with the top half because of the transposition between conductors (a, b) and (c, d).

Nevertheless, there is one scenario for which it is the case. If we consider that the impedances of the two parallel circuits are identical, then the current distributes evenly between the two. In practice, this is a very strong hypothesis because it means not only that the center transposition works perfectly but also for example that the conductors in parallel have exactly the same resistance. If we take that hypothesis, a pseudo-symmetry

is re-established: the image of one conductor from the top pole by the reflection is still not the same conductor in the lower pole but at least, it is an equivalent one that carries the same current density, which is technically sufficient to apply the symmetry.

We decided to use that symmetry because it halves again the size of the model. To implement it we have to enforce two conditions. First, we need to constrain the supplied current so that it distributes evenly between the two parallel conductors. Therefore, rather that supplying the nominal current of 2.8 kA to the EuCARD cable, we disconnect the two conductors that make it and supply 1.4 kA to each.

Additionally, we need to mirror the current sources from the top-right corner of the coil into the bottom-right corner. Contrary to the first domain reduction, which used an antisymmetry (i.e. the direction of the current density in the image is inverted), this time we want to enforce a true symmetry (same direction of the current density). To implement that, we need to assign a constant value to the magnetic scalar potential φ over the x-axis.

The counterpart with our hypothesis is that we are assuming the equivalence of the parallel circuits in the magnet winding. Thus we cannot simulate scenarios where there should be imbalanced currents in the two circuits because that would break the symmetry. Under our assumption, we are allowed to only consider the top-right corner of the dipole only and halve a second time the number of elements in the mesh. Eventually, we used the same spherical shell transformation technique as in the previous examples to restrict the radial extent of the modeling domain.

The final, transformed, modeling domain Ω' that we worked on is depicted on figure 3.16.



Figure 3.16: Finite element mesh of the EuCARD dipole

Material properties

We also make some strong hypotheses on the behavior of the materials used in the Eu-CARD model. Like in the first two examples, we only consider the superconducting REBCO layers to be part of the conducting domain Ω_c . Their superconducting behavior is described by a simple power law with constant J_c and n parameters. All the other parts of the EuCARD conductor (including the hastelloy, copper and CuBe layers, etc.) are merged with the air into the insulating region Ω_c^c . Just as in the former cases, our present model is thus unable to properly simulate the current sharing phenomenon.

Most importantly, in our simulation the iron pole of the magnet is assigned the same magnetic permeability as the air μ . Normally, one should have used a non-linear and possibly hysteretic behavior $\mu(h)$ law to properly model iron. However, and despite some efforts to make it work, our computations became very unstable when the non-linearity arising from ferromagnetism was added to the one linked to the superconducting behavior with $\rho(j)$. In the end, we chose to neglect the iron pole, disregarding the iron effects to stay focus on the simulation of the REBCO superconductors. We simply do as if the conductors are wound around a complete nonmagnetic pole. Obviously, this must have an impact on the simulated field, which will supposedly be lower than in the real object, where the center field is enhanced by the iron pole. This is something to keep in mind for the comparison of the simulation results with the experimental data from the stand-alone tests.

Coupling with an electrical network

Finally, in this third model, the electrical connections between the different REBCO tapes is a lot more intricate than in the first two. In the simulation of the small REBCO coil in 3.5 for example, we simply had to express that all turns were carrying the same current because they were technically connected in series.

Here, there are two REBCO tapes that are soldered together in each conductor. From the modeling point of view, it means that they are connected in parallel through the copper layer. Also, the initial cable is made of this pair of electrically insulated conductors that are only connected at the terminations of the magnets. Earlier, we already established that, in our model, we were to disconnect those and consider them independently supplied with half the magnet nominal current. That way we were able use the reflection with respect to the *x*-axis. Eventually, as in the previous example, we also have to to enforce constraints to emulate the serial connections between the consecutive turns in the windings.

The schematic 3.17 illustrates the various electrical connections between the winding turns in the EuCARD dipole.

First, the two insulated conductors that make the EuCARD cable are separated. They are represented by the two separated electrical networks in columns in figure 3.17. Each conductor is supplied with half the nominal current the real magnet, to ensure that the symmetry with regard to the x-axis can be exploited.

Then, each conductor is formed by two branches in which the impedances of the successive turns of REBCO tapes are connected in series. The red and green impedances (with subscript a and b) refer to the two REBCO tapes soldered together in the first conductor and the yellow and blue impedances (c and d) to the tapes in the second one.



Figure 3.17: Electrical network used to enforce the dependencies between the electrical quantities computed in the electromagnetic model. The four columns (colored) correspond to the four REBCO tapes in the EuCARD conductor; the lines (gray background) to the winding turns. Each quantity denoted $Z_{N,x}$ is coupled with the electromagnetic model to find the voltage and current in the branch.

Each variable impedance in figure 3.17 refers to one turn of REBCO tape in the magnet winding. During the computations, the current in the $Z_{N,x}$ and the voltage drop across them are the quantities coupled between the magnetodynamic model and the electrical circuit.

 $Z_{N,x}$ is linked with the impedances $Z_{N-1,x}$ and $Z_{N+1,x}$ that correspond to the previous and the next turns in series. The rows highlighted with a gray background therefore represent the cable turns in the windings. Also, between each turn, the two tapes from one conductor are connected through a R_c resistance that emulated parallel connection through the copper layer and the solder joints.

During the simulations, we experimented with the value of R_c by testing 0 Ω and 1 n Ω . The first value corresponds to the two REBCO tapes completely in parallel. To compute the second one, we considered a volume resistivity of $\rho_{Cu} = 1 \ n\Omega$.m for the copper and a surface resistivity of $\rho_{lap} = 5 \ p\Omega$.m² for one lap joint solder. We assumed that the surface of the solder joint was of the order of the tape width w = 12 mm times the dipole length L = 0.7 m and we used the known copper sheet thickness $t = 60 \ \mu$ m. Thereby, the order of magnitude for R_c was assumed to be of:

$$R_c = 2 \cdot \frac{\rho_{lap}}{wL} + \frac{\rho_{Cu}t}{wL} \approx 1 \ n\Omega \tag{3.21}$$

This electrical network model is built automatically inside of GetDP and coupled with the resolution of the electromagnetic model. The drive used during the simulation simply becomes the output current of the virtual power supplies. They are set to output identical ramps from zero to half the nominal operating current on the magnet.

3.6.3 Simulation results

The behavior of the EuCARD dipole has been simulated during a current ramp from 0 A to 2.8 kA, with a ramp rate of 10 A.s⁻¹, which correspond to the conditions of the stand alone tests of the magnet at 4.2 K that have taken place at CEA Saclay. In the simulations, we considered $J_c = 1.25 \times 10^{10}$ A.m⁻² and n = 25.

To make this simulation work, we had to modify slightly the source code of GetDP. To be more precise, we increased the size of the storage array used by the program to keep register the basis functions used in the problem. With the standard pre-compiled versions of GetDP, the maximum number of basis functions expected in one problem is 81. In our case however, we have one basis function associated with every cohomology cut in the air domain $\Omega_c^{\ c'}$, so 256 in total. We increase the storage size in consequence and was re-compiled the modifies code afterward.

Because of the difficulty to cross-compile for Windows, we eventually opted to produce a Linux OS executable. Due to this, we simulated the EuCARD model with the modified GetDP running inside of a Linux virtual machine executing on our Dell Precision T-5610 workstation. The virtual machine was authorized access to 8 cores and 32 Go of RAM of the host.

The overall computation time for the EuCARD model was of about 2 days. We suspect this could be reduced by removing the Linux emulation layer and running the modified version of GetDP directly on a native Linux host.

Figure 3.18 shows the simulated magnetic field h around the magnet at the end of the current ramp (right) and the current density j_z/J_c at the same time (left). The width of each REBCO tape has been amplified in place by a factor 30 on the left part of the figure to make the current density more visible.

Figure 3.19 also details how j redistributes over time during the ramp in a small portion of the windings corresponding to the third, topmost coil block. On the latter, one can clearly notice that current loops are generated inside each pair of superconducting tapes soldered together in response to the overall magnet field. These induced currents quickly saturate the REBCO tapes with $\pm J_c$. The current flowing in the outer tape is almost uniformly in the positive direction along the z-axis and in negative direction for the inner tape. The sum of the total current flowing into the pair is however well kept constrained at the expected value I(t). Only the extremities of the width contribute to the screening of the magnetic field normal to the conductors. At one point in the ramp, the innermost tape for each conductor is completely saturated with $-J_c$. So, to continue raising the overall conductor current I, j in the outer REBCO tape starts changing direction, reverting from $+J_c$ to $-J_c$.

The effect of this non-trivial current distribution on the center field is important. We took a reference case in which the current density j would distribute uniformly into all the REBCO tapes in the winding. The magnet field in that scenario is purely proportional to the supplied current I(t). That case was simply computed using a magnetostatic model using the same geometry for the 2D cross-section. The reference gives a center field of about 4.45 T. This value is far from the 5.4 T the dipole is expected to generate. The



Figure 3.18: Simulated magnetic induction (right) and current density in the REBCO tapes (left) at the end of a 2.8 kA ramp of the EuCARD magnet

discrepancy is certainly to be related with the lack of iron pole in the simulation model, which would have enhanced the center field if it were present.

If we now compare our EuCARD simulation with this reference case (see 3.20), we can see that the effect of the current redistribution generates a decrease of the center field from 4.45 T to 4.0 T. This corresponds to a considerable 9.9% field loss at coil center.

The simulation therefore shows the large influence the screening currents can have on the field of practical magnet. We focused on the center field here because it was the main design objective of the EuCARD dipole. We should however not forget that, in possible future REBCO HTS dipoles, the quality of the generated field, mostly in terms of multipolar components, will also be important elements of concern.



Figure 3.19: Normalized current density in the top-most coil block of the EuCARD dipole at different point of the current ramp



Figure 3.20: Center field

Chapter 4

Suppression of screening current induced effects in REBCO superconducting coils

This chapter tackles two techniques that can be used in order to attenuate or even to suppress the effects of the screening current-induced field in superconducting REBCO magnets. First, section 4.1 briefly recalls the problematic effects that screening currents generate on the field of REBCO superconducting magnets. Then, section 4.2 describes in detail a panel of techniques that can be used in order to deal with the SCIF. In particular, we focus on the vortex shaking and the current overshoot techniques. We present in 4.3 our experimental setup designed with the aim of testing the feasibility of those. Section 4.4 summarizes the experimental results that were obtained by using the two techniques. Sadly, we did not have time to perform all the tests we wanted in the time frame of the Ph.D. work, thereby section 4.5 develops on the perspectives of additional tests that would be interesting to perform with the experimental setup fabricated.

4.1 Reminder of the issues caused by the SCIF

As mentioned in section 1.6, the phenomenon of screening currents inside REBCO superconducting magnets is responsible for three problematic consequences on the field quality. As for example illustrated by the magnetodynamical simulation of the EuCARD dipole in 3.6, screening currents lower the field a magnet can generate. In the case of the Eu-CARD dipole, the decrease was even of about 10 % of the expected field. Secondly, the experiment described in chapter 2 has illustrated that the slow time relaxation of the generated magnetic field. For applications in which very stable magnetic fields are required, like NMR, this phenomenon greatly restrains the use REBCO coated conductors. Finally, screening currents also affect the field quality. Magnet are indeed designed so that a theoretical current density flowing through their winding allows for a targeted field quality, but because of the interference of the screening currents, that is not the case. To be more specific, it is for example an issue in accelerator magnets that are expected to produce a field dominated by certain harmonics (dipolar field, quadripolar field and such), or more simply in NMR that requires a very homogeneous field.

To summarize, screening currents are up-to-now a real obstacle to the use of REBCO

coated conductors in many applications. Given their negative impacts, a few different approaches have been imagined in order to deal with the screening current-induced effects, to reduce them or ideally suppress them entirely. The following section proposes a review of a panel of techniques that were developed for that purpose.

4.2 Countermeasures to the screening current effect

4.2.1 Tape striation

As explained in 1.5.1, the width of the REBCO tapes plays a significant role in the magnitude of their magnetization. Wider tapes indeed accommodate wider screening current loops, that in turn cause a more intense magnetization. In the early days of LTS, the issue of magnetization was also encountered but it was solved by evolving from monolithic superconductors to cables containing thousands of thin filaments incorporated in a normal resistive matrix [92]. This allowed to reduce the width of the screening currents loops while keeping the same total superconducting cross-sectional area in the conductor. Additionally, the filaments were also twisted together inside the cable to homogenize the current density distribution and reduce the length of the induced screening current loops. Thin filaments bundled together and twisted have been the recipe for efficient superconducting cables for many years with LTS; unfortunately because of the different manufacturing techniques used for HTS coated conductors (see 1.2.3), this cannot be directly transposed to REBCO tapes. However, it has still inspired the *striation technique* [19].

In the striation technique, the REBCO layer deposited on each tape is subdivided into multiple parallel filaments. There are three main methods to create those. One can work before the REBCO layer deposition by preparing the underlying buffer layers through mechanical scribing [67] or inkjet printing of insulating barriers [17]. That way, the REBCO deposit generates several parallel "tracks" instead of just one sheet expanding on the full width of the tape. Alternately, one can also create the filaments after the REBCO coating, by using laser ablation to subdivide the superconducting layer [63].

Note that striation process causes a lowering of the tape I_c . Indeed, that process implies a reduction of the total superconducting cross-section for a tape of given width. It has also been noted that striation can further reduce the tape I_c by introducing new defects in the filaments [78]. Besides, if striation helps reducing the tape magnetization by shrinking the width of screening current loops, the created filaments remain still in contact between one another through the resistive buffer layers, the substrate and possibly copper jacket. Hence, there can be coupling interactions between filaments, with screening currents potentially flowing in one filament and crossing the resistive materials to return through another one. Filament coupling partially negates the effect of the subdivision. From a mechanical point of view, striation also weakens the tape and renders it more prone to delamination. Eventually, it adds another step to the already complex manufacturing process of the coated conductors (see 1.2.3). Striated REBCO tapes would therefore be more expensive than regular ones.

We said that the striated tapes should also be twisted along their path to restrict the length of the magnetization loops and reduce the coupling effect. However, this is scarcely done because twisted flat tapes are very inconvenient for practical applications. Furthermore, twisting can cause further degradation of I_c [81]. Lastly, twisting also a reduce I_c because

some parts of the conductor become orthogonal to the magnetic field, in the situation where the local J_c is the lowest.

This striation technique has been shown to allow a reduction of the screening effect in REBCO tape-based coils, for instance in [100]. It suffers however from many different drawbacks and cannot be used to its full extend because it is not practical to twist the striated tape in most practical applications.

4.2.2 Tape alignment

REBCO tapes develop the largest magnetization when they are exposed to an external field oriented along the normal to their flat surface (see 1.5.1). Therefore, one can minimize the magnitude of the tape magnetization by carefully orienting the tapes along the direction of the local magnetic field: the *tape alignment* technique. In the case of a REBCO magnet alone, this is not manageable, because there are coil section that cannot be aligned on the self-field properly. However, it becomes more feasible with insert magnets that have to operate in the background field of another magnet, generally either resistive or LTS. In that case, the magnetic field orientation is for the most part dictated by the outer magnet, so optimization can be performed to align the REBCO winding with the field.

The aim of the optimization is not to assure that the magnet field is parallel to the REBCO tapes at every point in the winding and for every supplied current. The constraints are more relaxed, one wants to determine an orientation of the tapes that works for a specific operating current, given a known external field. This approach is one of the baseline designs for the EuCARD² 5 T dipole project, under the name of the align block design [59]. An illustration for it is given in figure 4.1. On the latter, one can see that the winding blocks are slanted to conform with the field lines when the magnet is used in a 13 T background field.



Figure 4.1: Simulation of the magnetic field generated by the EuCARD² dipole in stand-alone configuration (left) and in 13 T background field (right). Courtesy of Mr Jeroen Van Nugteren.

These first two techniques have the same major drawback: they need to be taken into account early in the design process of a new magnet. The striation technique requires it to be wound from processed REBCO tapes and the alignment technique drives the whole optimization of the coil cross-section. There are a few more techniques for reducing



Figure 4.2: Current overshoot procedure

screening current-induced effects that do not suffer from the same issue and can even potentially be used on already existing REBCO coils.

4.2.3 Transport current cycling

It is possible to use the transport current supplied to a REBCO magnet to partially attenuate the effects of the screening currents. Various kind of procedures have been imagined, the most straightforward one being the *Current Sweep Reversal* [98], also called here the *Overshoot* technique. It simply consists in charging the REBCO magnet to its nominal value with an initial overshoot of a few percents, as illustrated in figure 4.2. This technique does not alter the time relaxation of the current density in the REBCO windings. It is simply used to generate a more favorable initial condition for the current density, so that the later relaxation has a less pronounced effect on the tape magnetization. Provided that the overshoot amount is chosen correctly, the technique allows to reduce the field drift for a REBCO magnet but on the other hand it does not tackle the issue of the field reduction. We have tested this method with our experimental setup, and some results are summarized in section 4.4.1.

Alternately, one can proceed to a more complex *demagnetization* procedure that consists in several cycles of the supplied current with decreasing amplitude converging to the target nominal value [97]. By ramping up and down the current multiple times, one creates multiple screening current loops in the material, that cancel each other out in the end. It is possible by that mean to attenuate the time drift of the magnetic field and to reduce the amplitude of the field loss. The procedure takes a lot of time though and most importantly, requires that the operating point of the coil has very broad safety margins that can be exploited. The supplied current must indeed be raised far above the nominal current during the first cycle of the demagnetization procedure.

These two techniques have the advantage of only manipulating the supplied current, so they do not require any operations on the superconductor itself. Therefore, they can be applied applied to already existing REBCO magnets without alterations. Conversely, both require the current to be raised above its nominal value at one point. One therefore has to ensure that the coil has enough operating margins to withstand the temporary overcurrent and the increased Lorentz forces on the winding.

4.2.4 Vortex shaking

Lastly, *vortex shaking* is a common name used to designate a technique using an external AC magnetic field to suppress the magnetization of the REBCO tapes. It takes advantage the *abnormal transverse field effect*: an alternating magnetic field applied parallel to the flat surface of a REBCO coated conductor accelerates the relaxation of the tape magnetization. This effect was described on Niobium-Titanium in 1982 within a series of article by Funaki *et al.* [30, 28, 29].

The main requirement on the AC "shaking" field is its orientation, which has to be orthogonal to the tape magnetization, so parallel to the coated conductors. For solenoid coils or stacked pancakes coils, this means the field can either be axial (in this case one speaks of *transverse shaking*) or azimuthal (*longitudinal shaking*), both directions being illustrated in figure 4.3. With the transverse shaking, the AC field is applied orthogonally to the current density in the tape, whereas the shaker field and current density are aligned in the longitudinal case. On one hand, the former is the most appealing as it can easily be achieved using an auxiliary, concentric coil with the REBCO magnet in its bore. This allows for instance to use an existing outsert coil as the shaker. The latter configuration on the other hand requires to wind a dedicated toroidal coil around the REBCO magnet. It lead to a far more intricate coil setup, with the toroidal shaker even possibly getting in the way of the REBCO coil cooling. Thus, the preferred configuration is obviously the transverse one.



Figure 4.3: Possible directions for the vortex shaking field

Whatever the chosen orientation, the magnitude of the AC shaking field does not need to

be very high. In the case of a single, isolated tape, E.H. Brandt pointed out that it only needs to be able to fully penetrate the thickness of the REBCO tape, meaning [15]:

$$|H_{Sk}| \ge \frac{j_{c,\perp}.d}{2} \tag{4.1}$$

where $j_{c,\perp}$ is the critical current density of the tape under normal field and d is the thickness of the superconducting layer. For typical values of $j_{c,\perp} \approx 10^{10} \text{ A.m}^{-2}$ and $d \approx 10^{-6} \text{ m}$, the minimal shaking field $|H_{Sk}|$ is of about 10^4 A.m and $|B_{Sk}| \approx 10 \text{ mT}$.

Eventually, the frequency of the shaking field can vary greatly. For insulated windings, nothing prevents from using high frequencies like 10 or 50 Hz. Only with the emerging types of insulation like non-insulated winding or metal-as-insulation (see infra 4.3.1) must the frequency be lowered to compose with the transient current redistribution effects. Overall, the higher the AC field frequency, the shorter the shaking process. Namely, Funaki and Brandt showed separately that the number of AC field cycles that is relevant quantity, with a few tens of cycles being enough to achieve a full SCIF elimination [30], [15].

Up to now, vortex shaking has barely been used on REBCO coil but a few experiments suggest that the shaking process can lead to a complete relaxation of the SCIF in only 10 to 100 s [52, 56]. The usual vortex shaking procedure when working with REBCO coils consist in charging first the main superconducting coil to its nominal current. After that, the shaking field is fired for a short period of time to suppress the screening effect. When the shaking field is stopped, the current density in the REBCO coil has been forcibly pushed toward the thermodynamical equilibrium. The magnetic field drift over time is thus eliminated and the loss in generated field is recovered.

One upside of the vortex shaking technique is that it does not require any alteration of the superconducting tape itself and can therefore be used with readily available commercial tapes. Moreover, it is a process that does not require a lot of time. It is only a setup stage that needs to be performed once when the REBCO magnet is powered and can be completely forgotten afterward. The main limitation is that the technique requires an auxiliary "shaker" magnet. Fortunately, this is not always a problem, like in the case of hybrid magnets for example, where a REBCO insert as a natural shaker magnet: the outer, already existing LTS or resistive coil. This promising technique is also experimentally investigated in the following sections on the chapter.

4.2.5 Summary of the contermeasures to the SCIF effect

Overall, the techniques presented in the section are summarized in the table 4.1.

In the remainder of this chapter, one presents an experiment that has been assembled in order to tests the feasibility of the last two techniques.

4.3 Description of the experimental setup

The aim of this experiment is to test the possibility of reducing the screening currentinduced effects in small REBCO coils using the current overshoot and the vortex shaking techniques. One wants to assess the influence that the parameters of each technique have

	1	1	1
Technique	Implementation	Effects	Pre-requisite
Striation	subdivision of the	reduction of mag-	alteration of the
	conductor width	netization current	tape and twisting
		loops size	
Conductor align-	tape alignment	prevents induction	external field source
ment	with the field lines	from field perpen-	
		dicular to the tapes	
Overshoot	overshoot of the	initial condition	operating margins
	transport current	mitigating the	
		SCIF decay effects	
Demagnetization	transport current	cancellation of the	large operating
	cycling	tape magnetization	margins
Vortex shaking	external AC mag-	forced SCIF relax-	external field source
	netic field	ation towards ther-	
		modynamical equi-	
		librium	

Table 4.1: Summary of existing countermeasures against the SCIF effects in REBCO HTS magnets

on the SCIF reduction efficiency, for example the shaking frequency or the overshoot amount.

Figure 4.4 shows a global overview of the experimental setup.

The following subsections detail the different elements constituting the experiment.

4.3.1 **REBCO test coils**

The experimental setup is built around a set of concentric coils, with a test REBCO coil nested between two copper shaker solenoids. We focus on the REBCO test coil to begin with. As in the SCIF measurement experiment (see 2.1.3), we chose to work here again with single-pancake coils as they are the easiest to fabricate. Besides, considering the growing interest for unconventional *insulation types* in REBCO-based coils, we also wanted to perform our tests with various kinds of inter-turn insulation.

Unconventional insulation schemes

Unconventional types of winding insulation are a recent development in REBCO magnet studies. They were imagined as a response to the difficult task of protecting REBCO-based coils against quench damages. Namely, quench propagation velocities in REBCO HTS are of about 1 to 10 mm/s [87, 38], 1 to 3 orders of magnitude slower than in LTS [50, 25]. Thus, previously existing protection schemes were not always sensitive enough to detect the quench initiation through the arising resistive voltage and trigger the magnet safety systems in time, before irreversible damage are caused to the hot-spot in the windings.

The need was therefore to find replacements to the active quench detection-protection schemes. In this context, Hahn *et al.* showed that it was possible to use *non-insulated* (NI) windings to design self-protected REBCO magnets [41]. In NI coils, the current is



Figure 4.4: Schematic overview of the vortex shaking experiment

able to bypass a normal zone by switching to the neighboring tapes. That design allows to create very robust magnets, able to withstand several quenches and even temporarily work with current over I_c without damage. A side consequence of the NI scheme is that it allows to increase the engineering current density in the coils by removing the space that was occupied by the inter-turn insulation.

The issue with the NI technique is that the coil responds to changes in the supplied current with a delay [90]. The supplied current density indeed splits between a radial component that bypasses the coils turns, and an azimuthal component, flowing along the tapes. In the long term, all the current is guided along the azimuthal path because it is superconducting when the radial one requires to cross resistive layers. However, when the supply current changes, it takes some time for the radial component to conform to the azimuthal path. During that time, the radial component of the current induces a resistive coil voltage and causes losses. Eventually, since only the azimuthal current really contributes to the magnet field, the NI coil field reacts with a delay to the current changes.

The concept of non-insulated windings was extended and developed by different groups and gave birth to a number of variants. The most interesting one is the *metal-as-insulation* (MI) technique [61, 57]. This time, the superconducting REBCO tape is co-wound with a resistive tape, like usually a stainless steel tape. As stainless steel is a bad conductor, it prevents the current to flow radially when the REBCO tapes are superconducting, guiding it along the azimuthal path. On the other hand, when a resistive hot-spot arises, the current density can jump through the stainless steel along the radial direction to reach the neighboring tapes. Like the NI technique, the MI thus allows for quench-resilient windings. But also, it helps acknowledging the issue of the NI windings, reducing the charging delay by a factor 10 to 100 and lowering the losses caused by the radial component of the current as well. It can even enhances the mechanical properties of the coil through the addition of a high-strength co-winding material. The drawback in comparison with the no-insulation technique is that the engineering current density is lowered because of the added co-winding.

For our experiment, we chose to compare a classical, Kapton-insulated coil with a coil using a stainless steel tape as metallic insulation.

Design and fabrication of the test coils

To wind the two REBCO test coils, we used two pieces of a SuperPower 6 mm-wide coated conductor. The tapes measured both about 75 μ m in thickness, with 50 μ m of hastelloy substrate, about 1 μ m of REBCO deposit and a total of 20 μ m of electroplated copper wrapping. The two pieces did not have the same critical current as they were not from the same batch. The best length had a minimum I_c of 187 A at 77 K under self-field and the other one a minimum of 169 A in the same conditions.

The second tape length was used to make a single-pancake with metal-as-insulation using for the co-winding a 30 μ m-thick tape of stainless steel. The MI single-pancake obtained has 160 turns, an inner radius of 30 mm and and outer radius of 47 mm. It uses approximately 34.4 m of REBCO tape. To make the junctions at the ends of the winding, the REBCO tape was soldered onto solid brass terminations doubled with two parallel strands of 6-mm REBCO coated conductor. By doing so, the energy dissipation is reduced in the vicinity of the coil, which helps to avoid inducing accidental quenches. A few extra turns were made with the stainless steel tape soldered on itself on the outside of the coil, so as to create a mechanical support hoop. Lastly, the MI coil was also equipped with voltage taps in order to measure the voltage across the coil and at the two soldered junctions.

The first spool of superconductor was used in order to fabricate the insulated singlepancake, with the aim of having about the same characteristics as the MI coil. This time, the turns were insulated from each other using a 75 μ m-thick, 1/4 inch-wide adhesive Kapton tape. It was anticipated that the Kapton, and especially the glue layer, would squeeze under the conductor tension during the winding process so that its final thickness would be less than 75 μ m. Nevertheless, it still remained thicker on average than the stainless steel used for the MI coil and eventually we decided to stop the winding with fewer turns than in the previous MI coil, namely 133 turns instead of 160.

If our initial intent was to have the same number of turns in both coils, we realized it was still more important that the two have about the same outer diameter. Otherwise the outer coil terminations would have been on different diameters and it would have been more difficult to install both coils in the same experimental setup because of the fixed distance between the current lead attached to the cryostat. That is why we preferred to have about the same outer diameter rather than the same number of turns. The finished insulated coil used a total of 33 m of REBCO tape and has an inner and outer radii of 30 mm and 48.8 mm, respectively.

Figure 4.5b shows a picture of the REBCO test coils with metal-as-insulation. Also, table 4.2 summarizes the main characteristics of the two test coils.

Parameter [unit]	Insulated coil	MI coil
Winding type	single-pancake	single-pancake
Number of turns	133.5	160.5
Inner / outer radius [mm]	30 / 48.8	30 / 47
Height [mm]	6	6
Total tape length [m]	33.0	34.4
Inductance [mH]	1.80	2.77
Coil constant at center [mT/A]	2.06	2.56

Table 4.2: Characteristics of the REBCO test coils for the vortex shaking experiments

4.3.2 Shaker coils

Apart from the REBCO test coils, the experiment also uses a pair of concentric, seriallyconnected shaker coils. Those are used to generate the AC magnetic field on the REBCO winding, triggering the vortex shaking relaxation.

For budgetary reasons, we used $\otimes 1.59$ mm enameled copper wire for the two shaker coils. The drawback associated with copper instead of superconductor is that the coils would heat up through Joule effect. Note that, because they are actually supplied in AC current, superconducting shaker coils would also have dissipated heat through hysteretic and possibly coupling losses in the material. Nevertheless, the energy loss could still have been reduced by the use of superconductor. In our case, by choosing to go for copper shaker coils, we knew that we were unlikely to be able to perform tests under liquid Helium cooling. Indeed, the heat output of the shaker coil would have quickly vaporized the coolant bath. Thus, we restricted ourselves to experiments using liquid nitrogen cooling.

The outer, larger coil Sk_2 has an inner radius of 73.7 mm, large enough so that the REBCO test coil can be fitted inside its bore. It is a solenoid wound in the counterclockwise direction with 4 layers of 52 turns each. It results in an outer radius of 80.1 mm and the total height of the coil of about 83 mm. Sk_1 is a smaller solenoid with an inner radius of 16.6 mm and with 2 layers of 52 turns, wound in the clockwise direction. Sk_1 has an outer radius of 19.8 mm and is also 83 mm tall. The geometrical and electrical information on the shaker coils are summarized in table 4.3.

Parameter [unit]	Value for Sk_1	Value for Sk_2
Winding type	solenoid	solenoid
Direction	clockwise	counter-clockwise
Number of layers	2	4
Turns per layer	52	52
Inner / outer radii [mm]	16.6 / 19.8	73.7 / 80.1
Height [mm]	83	83
Resistance $[m\Omega]$	14	117
Inductance [mH]	not measurable ¹	5.9

Table 4.3: Characteristics of the shaker coils Sk_1 and Sk_2

 $^1\mathrm{In}$ the assembly, the self-inducted voltage on Sk_1 is almost perfectly canceled by the voltage induced by $\mathrm{Sk}_2.$

 Sk_1 and Sk_2 were wound on separate nonmagnetic stainless steel mandrels, that were later



(a) Hall probe assembly



(b) REBCO test coil (with metal as insulation)



(c) Shaker coils



(d) Complete assembly of the vortex shaking experiment



(e) Detail of the coil assembly

Figure 4.5: Assembly of the experimental setup

bolted together, as depicted on figure 4.5c. The solenoids were then connected in series by soldering their current leads. The final assembly of the shaker coil is designed so that the REBCO single-pancake can be loaded from the bottom without the need to disassemble any part. This simplified greatly the operations to change between the REBCO test coils.

The main function of the Sk pair is to generate the shaking field on the volume occupied by the REBCO windings. With our design, the field generated by Sk_2 inside its bore is raised up by the fringe field generated by Sk_1 . However, the main reason for using two concentric coils instead of just Sk_2 is to cancel the shaking AC field inside the bore of Sk_1 . The radii, number of layers and number of turns for the two coils were indeed optimized in order to cancel (on RMS average) the field inside the bore of Sk_1 , in the median plane of the coils. Naturally, the outer radius for Sk_1 and the inner radius for Sk_2 were constrained by the geometry of the REBCO coils, because it had to fit between the two shaker coils.

Figure 4.6 gives the result of that optimization. The norm of the magnetic field generated by the pair of coils Sk_1+Sk_2 is plotted against the distance from the assembly axis, inside the Sk coils median plane. One can see that, in the bore of Sk_1 , the magnetic field of the coil pair cancels. That way, whenever a Hall probe is used inside the bore of Sk_1 , it measures the field generated by the REBCO test coil with only a minimal disturbance created by the shaker coils.



Figure 4.6: Amplitude of the axial component of the field generated by the shaker coil assembly computed along a radius, in the coil mid-plane.

4.3.3 Auxiliary systems

All the coils in the experiment were cooled at 77 K using a bath of LN_2 inside of the same cryostat as the previous experiment (see 2.2e).

The experimental assembly consists in two independent electrical circuits: one with the REBCO coil and the second with the pair of shaker coils. Thereby, the experiment uses two power supplies, one to feed independently each circuit. The superconducting coil is connected to an Ametek-Sorensen SGA 10-400 4 kW 1-quadrant power supply, able to deliver up to 400 A. A freewheel diode was added to the back of the power supply in order to protect it from harmful inductive feedback voltages (see figure 4.7b). Also, because the SGA power supply has only very basic controls from the front panel, it was remotely controlled by a supervising computer running LabView and sending SCPI instructions over the serial port.

The shaker coils on the other hand were connected to a Kepco BOP-MG 25-40 1 kW power supply. The Kepco BOP is a 4-quadrants power supply with a square characteristic, thus able to deliver 40 A at up to 25 V. Also, it features better integrated controls than the the Ametek-Sorensen. It has namely its own embedded waveform generator that was used to program the sequence of AC current cycles to inject into the circuit. Both power supplies can be seen on figure 4.7a with the Kepco BOP on top and the Ametek-Sorensen below it.

Lastly, the experiment was instrumented with Hall sensors for the magnetic field, CLTS and Platinum RTDs for the temperatures, current measurements on calibrated shunts and voltage taps. The magnetic field is measured at two positions in the experiment: at the center of the assembly and between the two shaker coils. The center Hall probe is the main one in the experiment. As explained in 4.3.2, it is shielded from the AC field generated by the pair of shaker coils and thus it is used to monitor only the field generated by the REBCO single-pancake. The second Hall probe, positioned between the shaker coils, is used to monitor the AC field generated during the vortex shaking sequence.

The other temperature, current and voltage probes are mainly ways to monitor the experiment and detect possible issues. Two CLTS temperature sensors were glued on both shaker coil to monitor their temperature, ensuring that they do not overheat because of the Joule losses. One platinum RTD was also glued on the REBCO coil mandrel to have an indication of the test coil temperature and last one served as a LN_2 level probe above the experiment.

The currents in the two electrical circuits, the REBCO circuit and the shaker coil circuit, were measured using the voltage drop across calibrated shunt resistors. Lastly, the voltages of the superconducting coil and its resistive junctions, as well as the voltage on the two shaker coils were measured via voltage taps.

All signals were acquired using National Instruments cDAQ modules and the data processing was performed through a custom LabView program.

4.4 Experimental results

4.4.1 Current overshoot tests

Experimental protocol

The setup detailed previously was used in order to perform some test of the current overshoot technique.





(b) Shunts and freewheel diode

(a) Power supplies: Kepco BOP-MG 25-40 (top) and Ametek-Sorensen SGA 10-400 (bottom)

Figure 4.7: Electrical equipment used in the current overshoot and vortex shaking experiment

As in the SCIF measurement experiment (see section 2.2), the coil assembly was warmed up above 100 K and then cooled back down to 77 K between each test. This was to ensure that each test was performed with the REBCO coil in the same initial "virgin" state, with no supercurrent remaining in the winding. The rest of the protocol is straightforward: the coil was charged up to $(1 + \epsilon)I_n$ ($0 < \epsilon \le 0.2$), above the coil nominal current, and then it was lowered to reach the final value I_n . The ramping rate for all the transitions is the same, we used here 2 A.s⁻¹. After the current has reached the nominal value I_n , the center field of the REBCO coil is measured for the next 10^3 s.

The measurement was performed with transport current in the coil. The known drawback for doing so is that the true asymptotic limit of the magnetic field generated by the coil is harder to determine with certainty. Indeed, the theoretical value of the asymptotic center field depends on the geometry of the coil at 77 K and the precise current delivered by the power supply. Moreover, since the field amplitude is measured in our experiment through the voltage on a Hall sensor, the probe conversion rate must be calibrated with high precision to be able to take advantage of the absolute value of the signal. Lastly, when the measure is made of the plateau with the power supply is still on, the fluctuations on the output current are susceptible to disturb the measurement of the TAFC-induced magnetic field drift.

Nevertheless, we still preferred to proceed with the tests at nominal current rather than at the remanent state. In the latter case, the protocol was more difficult to define and the choice would have been confusing. To better explain that point, let us assume for a moment we decided to make the measurements at the remanent state. There would be two current ramps to perform: one to charge the REBCO magnet and the second to discharge it. In that case, we would have three possibilities to setup the overshoot: at the end of the first ramp, at the end of the second or both. None of these three possibilities appears as more natural than the others. Overall, this is the reason why we preferred to carry on the measurements with transport current. In this case, there is only one ramp up so the determination of the protocol is completely straightforward

Results of the current overshoot tests

Figure 4.8a shows the results obtained during the current overshoot experiment for the coil with metal-as-insulation. The Hall voltage of the center probe is plotted against the time elapsed since the beginning of the plateau, i.e. after the overshoot has been performed.

The center Hall probe voltage show a large drift of the magnetic field over time. However, this large drift is questionable. Indeed, the field evolution is largely correlated with the drift of the power supply current. Therefore, we computed a linear correlation between the measured power supply current and the center magnetic field and used it to compensate the drift of the power supply. After this operation, one obtains the result in figure 4.8b.

In the latter, the amplitude of the decay is significantly lower. Also, one can see the influence of the overshoot technique, which allows to negate the initial drift of the center field for only a small overshoot value of about 5 %.

4.4.2 Vortex shaking tests

Experimental protocol

Therewith, the same experimental setup was used for the tests of the vortex shaking technique.

For those, we decided to work at the remanent state like during the SCIF measurements presented in chapter 2. The reason is always the same: working with zero remaining transport current in the coil ensures that any magnetic field measured can only be caused by the screening current-induced effects. Therefore, the protocol starts by ramping up the current in the REBCO superconducting coil from 0 A to 40 A in 60 s, then a 60 s plateau is observed on the target current before ramping back down to 0 A in 60 s. Once back to 0 A, the Ametek-Sorensen is removed from the circuit by opening a switch inside the power supply that disconnects the rear-panel output. That way, when current is induced in the REBCO coil through the AC shaking field, it can not loop in the external circuit and through the back-panel electronics of the power supply. At 0 A, a pause of 60 s is marked before firing the shaker coils. The beginning of the SCIF relaxation under classical thermally-assisted flux creep is recorded during that time frame so as to establish the reference level for the magnetic field before shaking. The AC field is then applied, with a set amount of field cycles N_{Sk} at a given amplitude A_{Sk} and frequency f_{Sk} . After the procedure has been carried through, the center field of the REBCO coil is finally monitored for at least the next 600 s.

Our goal with this experiment is not only to test the feasibility of the vortex shaking technique for small REBCO coils but also to look at the influence of various parameters on its efficiency. Namely, we looked at the effect of the shaker field amplitude A_{Sk} and the number of shaking cycles performed N_{Sk} . Additionally, as we mentioned in 4.3.1, the tests are repeated on two REBCO single-pancake coils with different kinds of insulation, Kapton and metal-as-insulation, to see their influence. The results of the tests are presented in the next section. These are incomplete however, due to the lack of time remaining at the end of this Ph.D. to perform to all the tests we anticipated.





Figure 4.8: Results of the of the current overshoot tests on the MI test single-pancake. 4.8a plots the original measurements and 4.8b the corrected results after compensation of the drift of the power supply current.

Results of the vortex shaking tests

Figures 4.9 show the result obtained in reducing the SCIF with the vortex shaking technique in the case of the Kapton-insulated coil (top) and the coil with metal-as-insulation (bottom). The measured voltage on the center Hall probe, which is proportional to the magnetic field a the center of the the REBCO coil, is plotted against the time elapsed since the end of the application of the shaker field. For reference, the black curve indicate the decay of the coil SCIF under classical thermally-assisted flux-creep, without any use of vortex shaking.







(b) REBCO single-pancake with metal-as-insulation

Figure 4.9: Effect of the vortex shaking technique on the center magnetic field generated by two REBCO test coils

For both insulation types, the time decay of the SCIF is canceled by the application of the shaking field. Also, the magnitude of the remaining SCIF has been reduced, of about 14 % in the case of $I_{Sk} = 20$ A ($|H_{AC}| \approx 36$ mT on the REBCO windings) and 40 % in the case of $I_{AC} = 40$ A ($|H_{AC}| \approx 72$ mT). Most experiment also suggest that increasing

the number of cycles helps further reducing the trapped SCIF. Only the experiment at $I_{Sk} = 20$ A on the MI coil does not agree with this overall tendency. Lastly, if the number of shaking cycles is confirmed to have an effect, then the SCIF reduction does not scale linearly with it. The first cycles definitely are the most efficient ones, with the increase from 60 to 120 only leading to a very small impact on the field.

Sadly, only a few tests could be carried on during the time frame of this thesis work. The last section 4.5 of this chapter will discuss the additional tests that we think would be interesting to perform on the same experimental rig in order to complement our study in the vortex shaking technique.

4.5 Discussion of the results and perspectives

To begin with, we should emphasize on the small number of experiments that were performed to obtain those results. It would have been needed to proceed to a more thorough study but the lack of time prevented us to do so.

There is not much to add concerning the current overshoot procedure. It would still have been interesting to perform additional tests on the insulated coil for comparison with the MI one.

Considering the vortex shaking technique, it is surprising to see that the SCIF reductions we measured are significantly lower than what could have been anticipated based on previous reports. Up to now, there has only been few experiments using vortex shaking on REBCO coils, most of them by Kajikawa *et al.* [52, 51, 56]. Their tests however indicate that a full relaxation of the SCIF is obtain in a short time (about 50 s) when working with an AC field at 10 Hz and with a minimal amplitude of 28 mT on the REBCO tapes. On the contrary, we only witnessed partial reductions of the SCIF in our experiments, despite of a shaking field of greater amplitude. We have two hypotheses to explain that.

The first hypothesis is that the amplitude of the shaking field used in our experiment was not enough to lead to a full SCIF relaxation. Namely, as can be seen in [52] or computed by Brandt in [14] in the case of a single isolated REBCO taped, there is a threshold value for the shaking field amplitude. If the latter is lower than the threshold, the SCIF reduction is never going to be complete, no matter how large the number of AC shaking cycles applied. Only with a field amplitude above the threshold can the reduction be total, provided enough cycles are made. For a single tape, Brandt indicates that the threshold value is $j_c.d/2$, where j_c is the REBCO tape critical current density in A.m⁻² and d the thickness of the REBCO layer in m. In coils, were multiple turns of coated conductor can interact, the threshold is possibly different but no analytical formula is yet available to compute it. This assumption would probably have to be verified using numerical simulation, possibly using the magnetodynamic model described in 3. Therefore, the first explanation for the incomplete SCIF reduction we measured is that the shaking field we used was not large enough to exceed the threshold amplitude associated with our test coils.

The second hypothesis is simply that the amplitude of the shaking field was sufficient, but we did not make enough cycles to reach the complete SCIF reduction.

To test those two hypotheses, it is either required to raise the current in the shaker coils, thus increasing the shaking field amplitude to try the first one; or to make more shaking cycles in order to try the second one. In both cases, we are unfortunately held back by the equipment at our disposal.

Indeed, it is not possible to raise the current in the shaker coils with our current Kepco power supply, which we already pushed up to its maximum output current of 40 A. On the other hand, if we increase the number of shaking cycles, the duration of the shaking process would also increase unless we compensate by raising the AC frequency accordingly. However, when the Kepco is programmed to deliver a sine current of 40 A at 1 Hz to the shaker coils, the output voltage is already of about 6 V. This implies that, should we stay at 40 A, the frequency can only be multiplied by a factor slightly above 4 before the voltage on the shaker coils reaches 25 V, the maximum the power supply can produce. This does not leave us a considerable margin to increase the shaking field frequency. Consequently, increasing the number of cycles will very quickly imply that the duration of the shaking process rises accordingly.

A consequence to the solutions associated with both hypotheses is that more energy is going to be dissipated in the LN_2 bath. In the first case, the current is raised, which enhances the dissipated power as the square power; in the other case, the duration of the shaking process is increased, which extends the time frame for the integration of the dissipation. Both ideas lead thus to more ebullition in the coolant bath and a possible deterioration of the coils cooling.

Those reflexions connect with the main interrogation about the experiments that we already performed: is it possible that a degradation of the coil cooling is responsible for the reduction of the SCIF observed? Indeed, all the tests from the reported measurement campaigns lasted for one or two minutes. This may represent enough time for ebullition to start taking place in the LN_2 bath, possibly forming trapped pouches of nitrogen gas and lowering the quality of the cooling of the REBCO coil. In the worst case amongst the tests performed, i.e. the maximum current (40 A) and largest number of cycles (120), it was indeed observed using the CLTS sensors glued to the shaker coils that the temperature at their surface rose from about 3 K during the test. Therefore, we can imagine that the REBCO also changed somewhat, inducing a change in the electrical properties of the REBCO and parasiting the measurement of the vortex shaking phenomenon.

A way to verify if the cooling is not degrading would be to proceed to shorter tests, using higher shaking frequencies. That way, heat would be dissipated in the cooling bath for a shorter period of time, and ebullition would have less time to intervene. However, as we saw, we could not do this because of the voltage limitation of the Kepco power supply.

An other idea would be to perform tests at 65 K with a subcooled nitrogen bath. To do this, a pump would have to be connected to the cryostat in order to extract the nitrogen gas and lower the pressure of the bath. The benefit of the subcooled bath is that it has to be first raised up to 77 K before ebullition starts appearing. This would have have allowed to perform our experiment with more confidence in the coil cooling.

As we tried to show in this last section, the experiments we made with the vortex shaking were merely initial explorations. There are still numerous experiments to perform to properly assess the feasibility of the vortes shaking technique on full-scale magnets.
Chapter 5

General conclusion and outlook

Our Ph.D. thesis has been focusing on the treatment of screening current-induced effects generated in REBCO HTS magnets. This last chapter includes first a short reminder of the issues related to the screening current in REBCO tapes and highlights then the main achievements of this Ph.D.. Eventually, it proposes some directions in which to investigate in the continuation of the present work.

REBCO coated conductors are extremely interesting products in the domain of high-field magnet design. The main reason is because they possess the best transport current capacities under high magnetic inductions amongst commercially available superconductors. Besides, the coated conductors feature good mechanical properties in response to tensile efforts, which help them withstanding the intense Lorentz forces that apply within high-field magnet windings. Therefore, REBCO tapes are currently one of the most promising superconductor types to use in projects of magnets required to generate about 20 T or more.

Nonetheless, the spread of REBCO coated conductors is still held back by several issues, some of which are economical, some others technical. The phenomenon of screening currents belongs to the latter and it is the reason why it was investigated during this Ph.D.. Screening currents designate supercurrents induced in the REBCO material by in response to an external magnetic field and which generate a tape magnetization opposed to the initial field variation. On the global scale, in a REBCO magnet, the screening current effect causes a reduction of the total field produced by the system as well as a loss in the field quality and stability over time. These parasitic effects represent a serious issue for most types of applications, like NMR and accelerator multipoles.

To tackle the problematic of screening currents, our approach was twofold. On one hand, we contributed to the development of magnetodynamical simulation methods that can help magnet designers to anticipate the impact of screening currents during the conception of new systems. On the other hand, we also investigated experimentally some techniques that can be applied to REBCO magnets to reduce or even suppress the screening effects. The objective with both research directions remains to provide ways to deal with the phenomenon and hopefully to foster the use of REBCO tapes in high-field magnet applications.

The main achievement to the first axis of this thesis is the implementation of a magnetodynamical model capable at simulating real-size REBCO magnets. However, the first step to come up with this numerical model has been to design and fabricate an experimental setup with which to obtain reference data on the screening current induced effects. This setup was mostly operated at 77 K to make measurements of the SCIF amplitude and decay over time on small REBCO single-pancakes. Tests were performed at the remanent state, after the coil had been charged and discharged, to obtain the trapped SCIF and with fixed transport current to monitor the magnetic field drift caused by flux creep. The experiments showed that the evolution over time of the REBCO coil magnetic field in the horizon of $10^4 - 10^5$ s follows a logarithmic behavior that is well in line with the Anderson-Kim theory. The experimental data we gathered thanks to this first experiment were then used as benchmark cases during the development of the numerical model.

Concerning the numerical simulations, we used a classical approach based on the Maxwell equations to model quasi-static magnetodynamical phenomena. We used an $h - \varphi$ formulation and relied on homology theory to enforce current constraints in the models. The implementation of the models were done in the framework of the free and open-source software Gmsh and GetDP. Several scenarios were tested, from which this thesis describes three: a single isolated REBCO tape, the 51-turns REBCO single pancake of the previous experiment and eventually the EuCARD prototype dipole. Our numerical results successfully compare with established analytical formulas, results from other simulation software like Comsol and our experimental data. Eventually, we also managed to demonstrate the capability of this model to tackle real-size REBCO magnet by performing a simulation of a stand-alone test of the EuCARD dipole.

For the second axis of this Ph.D., we focused on two techniques that have the potential to reduce the impact of the screening current effects in REBCO magnets: the current overshoot and vortex shaking techniques. Our interest was drawn to these because they can be implemented on already existing magnets. To investigate their effects, we designed and fabricated a second experimental setup consisting in a small REBCO single-pancake coil inside of a pair of copper shaker coils. With it, we were able to study the feasibility of the techniques at 77 K on two REBCO test coils with different insulation types: Kapton insulation and metal-as-insulation. The current overshoot technique allowed to compensate for the initial drift of the center magnetic field for overshoot values of only 5 %. Besides, due to its easy implementation, it is a technique that has the potential to be commonly used with REBCO high-field magnets.

In as second time, we also tried the vortex shaking technique in the longitudinal configuration on the two coils at 77 K and under remanent field conditions. It showed to be capable at lowering the trapped magnetization of the coil by up to 40 % while also canceling the visible time drift of the magnetic field. The vortex shaking technique proved to be an interesting approach for the handling of screening currents in REBCO magnets as it is able to lower the tape magnetization within a short period of time (of the order of 1 minute) and needs to be performed only once, as part of an initialization routine. The technique however requires an external coil in order to generate the shaking field. This is why we think vortex shaking is probably not going to be the technique of choice for most magnet systems but could still be very relevant with REBCO insert in high-field hybrid magnets for example.

Lastly, we want to highlight some of the aspects that could be focused on in the continuation of this Ph.D. work. About the numerical simulations first, magnetodynamical computations of basic superconducting REBCO coils has been achieved but our model could still be improved. The implementation of ferromagnetic materials should definitely be the first point to address here. The presence of ferromagnetic elements are indeed extremely usual in magnets, be it for field enhancement of shimming, and thus, the inability to simulate those is problematic. To go beyond magnetodynamics, it would then be interesting to implement within GetDP coupled models with the thermal compartment of the mechanical compartment. That would allow to simulate new aspects of the REBCO coil behavior, like cooling and quenches.

On the experimental aspect of this Ph.D. work, our primary experiments on the vortex shaking technique could certainly be further developed. Upgrading the shaker coils power supply should be tried to see whether a full relaxation of the trapped magnetization can be performed at the remanent state. Afterward, one should move to shaking tests on a charged REBCO coil. Thereby, we could assess the feasibility of the technique for a larger prototype.

Appendix A

Introduction on differential forms

A.1 Elements of topology

A.1.1 Euclidean space E_3

 E_3 is a structure $\{A_3, \cdot, \text{Or}\}$ regrouping an *affine space* A_3 , to which is added a *scalar* product \cdot and an *orientation* Or on the associated vector space V_3 .

The affine space A_3 is the set of points in the 3D space. In the context of affine spaces, one separates *free vectors* from *bound vectors*. The former are elements of the associated vector space V_3 , the latter are couples $\{a, v\}$ made of a point $a \in A$ and a free vector v. Bound vectors corresponds to the intuitive idea of the vector arrow with its tail in a and its head in the translated of a by v. It makes sense to subtract two points of A_3 , which gives a bound vector, but not to add them.

The scalar product \cdot is a positive definite bilinear operation on V_3^2 , \cdot : $(u, v) \in V_3^2 \rightarrow u \cdot v \in \mathbb{R}$. \cdot allows to define the *norm* of elements of V_3 and, as a consequence, it allows to compute distances between points of A_3 . The scalar product is said to give a *metric* to the euclidean space.

The orientation Or is an equivalence class of basis of V_3 . Choosing Or designates the basis of V_3 that are considered positively and negatively oriented. In E_3 , the "right-hand rule" is often invoked to define positively oriented basis.

Lastly, euclidean spaces are not limited to 3 dimensions and the previous definition can easily be adapted to derive any other *n*-dimensional euclidean space E_n . n = 3 is however especially important as it is the go-to context to model most of the classical physics phenomena. In particular, we use here E_3 to model the physical 3D space where the electromagnetic phenomena happen. Note that, with that choice, time t is considered a parameter rather than a coordinate, like in it would for example be the case in Minkowskiś space-time.

A.1.2 Manifolds

A manifold M of dimension p, called a p-manifold, is a topological space that locally resembles the euclidean space of dimension $p E_p$. More precisely, this means that every point $m \in M$ has an open neighborhood that is homeomorphic to an open neighborhood of the affine space A_p . The homeomorphisms creating the mapping between the manifold Mand the A_p are called *charts*. In this document, the word manifold is used in a very loose manner, mostly as a convenient way to designate either points (manifolds of dimension 0), smooth curves or surfaces (1- and 2-manifolds, respectively) or simply-connected volumes (3-manifolds).

More complex objects are also manifolds like Möbius's ring or Klein's bottle which both are 2-manifolds. They are however not relevant in the scope of this document. The *p*manifolds that interest us are those that can be *embedded* in E_3 , meaning they can be formed into E_3 without self-intersections. Figure A.1 illustrates some manifolds formed into E_3 .



Figure A.1: Two examples of 2-manifolds formed into E_E : the sphere (left) can be embedded in E_3 whereas Klein's bottle (right) cannot.

The boundary operator ∂ works on *p*-manifolds, $p \geq 1$ and returns (p-1)-manifolds. As its name implies, ∂ extracts the boundary of *p*-manifold, that can be a curve (1-manifold) for a surface (2-manifold), a surface (2-manifold) for a volume (3-manifold), etc. Some manifolds have an empty boundary. In that case, either they are not *compact*, like an infinite straight line, or they are compact but *closed*, like a circle for example.

Manifolds are used to represent integration domains in our physical model. Say we want for example to compute the flux of a certain quantity through a surface, then that surface is modeled with 2-manifold. Another example, to compute the energy of the electromagnetic field in a given volume, one integrates the energy density over a 3-manifold. Consequently, all the integral relations we use in electromagnetism are written as equalities between quantities integrated over manifolds.

A.1.3 Tangent space

For a non-singular point m in a p-manifold M one can define its *tangent space*, the set of all tangent vectors at that point, denoted $T_m M$. $m + T_m M$ is then the *affine tangent space* to M in m. It denotes the space of points $x \in A_3$ for which the translation vector turning m into x is in $T_m M$. The latter refers to the intuitive idea of tangency: $m + T_m M$ is for example the line tangent to smooth curve passing through m whereas $T_m M$ is only the direction of that line.

The tangent space of the whole space $T_x E_3$ is also interesting. $x + T_x E_3$ is the space $\{\{x, v\}, v \in V_3\}$ of all vectors bound to point x. This is precisely the type of objects suited for the definition of *vector fields*. Indeed, a vector field is a function $f : x \in A_3 \rightarrow f(x) \in T_x E_3$ meaning a function that associates to x a vector bound to that point.

A.1.4 Chains

A *p*-chain is a is finite collection \mathcal{M} of linearly independent *p*-manifolds $\mathcal{M} = \{M_i, i \in [1, k]\}$ carrying weights $\mu_i, i \in [1, k]$. Manifolds M_i are called the *components* of the chain. The weights μ_i are taken from a ring of coefficients, which in the vast majority of cases is the space of signed integers \mathbb{Z} . Chains are often written as formal sums $\sum_i \mu_i M_i$, where the "+" sign is not to be misinterpreted as the ordinary addition.

On the other hand, chains themselves can also be added. The sum of $\sum_{i} \mu_{i} M_{i}$ and $\sum_{j} \nu_{j} N_{j}$ is the chain with the set of components $\mathcal{M} \cup \mathcal{N}$ weighted by coefficients that are the sums of the coefficients of each component in the two chains.

The interest of chains comes from the possibility to assemble them to create complex objects and vice-versa, to divide intricate manifolds into more simple components. In the context of computational electromagnetism, chains are employed to combine mesh elements. A surface that has been approximated by a triangle mesh is for example modeled as the chain of the 2-dimensional triangular manifolds with a unit coefficient on each component.

A.2 Differential forms

A.2.1 Lengths, areas and volumes

To introduce the notion of differential forms, let us start with a few examples concerning the measurement of lengths, areas and volumes.

First, suppose that we want to measure the projected length of a vector v along the direction given by a unit vector e. For that, we need a function α , with $\alpha : v \in V_3 \rightarrow \alpha(v) \in \mathbb{R}$. This is easily done by taking the scalar product between v and e. That function α assigning values to vectors is an example of *1-form*.

One often likes to work in E_3 with an orthonormal frame $\mathcal{R} = (O, e_1, e_2, e_3)$. $O \in A_3$ is an affine point called the origin of space and $\mathcal{B} = \{e_1, e_2, e_3\}$ is an orthonormal basis of the vector space V_3 . With what we just introduced, such a frame allows us to define vector coordinates. This is done by having three functions α_i that measure the length of that vector along the set of directions e_i .

We then consider the parallelogram constructed on the pair of vectors $(u, v) \in V_3^2$. We would like to measure its projected area onto the plane generated by two orthogonal unit vectors $\{e_1, e_2\}$. The projection of (u, v) on the plane gives the coordinates $(\alpha_1(u), \alpha_2(u))$ and $(\alpha_1(v), \alpha_2(v))$, respectively. We know the signed, projected area of the parallelogram is given by the cross product, $(\alpha_1(u), \alpha_2(v)) - (\alpha_1(v), \alpha_2(u))$. That composition of 1form is call *external product* and denote \wedge (hence its other name *wedge product*). \wedge is therefore an operation that turns two 1-forms α and β into an application $\alpha \wedge \beta : (u, v) \in$ $V_3^2 \rightarrow \alpha \wedge \beta(u, v) \in \mathbb{R}$. $\alpha \wedge \beta$ is an example of 2-form, an application that assigns a value to a pair of vectors.

Eventually, we can also want to measure the volume of a parallelepiped build on three vectors $(u, v, w) \in V_3^3$. Given the three 1-forms α_i associated with the vectors $e_i, i \in [1, 3]$, the wedge product $\alpha_1 \wedge \alpha_2 \wedge \alpha_3$ is that 3-form that computes the algebraic value of that volume. Here, $\alpha_1 \wedge \alpha_2 \wedge \alpha_3 (u, v, w)$ acts the same as the *triple product* between the components of the vectors u, v and w. It is an example of 3-form, an application that assigns a value to a set of three vectors.

A.2.2 Definition

Let M be a smooth manifold and $m \in M$, a k-form on that manifold is then a linear, alternating map $\omega_m : (T_m M)^k \to \mathbb{R}$. Complex-valued differential forms exist but they are disregarded in the scope of this appendix. To begin with, note that k-forms only operate with manifolds of the same dimension k. The result of ω_m depends both on the point $m \in M$ but also on the shape of M near m through the tangent space $T_m M$. To make it more explicit, the characteristics of the k-forms ω^k for $0 \le k \le 3$ are discussed here.

A 1-form ω_m^1 assigns a real value to vectors in the tangent space $T_m M$ to a 1-manifold M (so a curve). In electromagnetism, this echoes the notion of circulation of a quantity along an infinitesimal line element. Consider a electrical circuit: its layout can be modeled by a 1-manifold M embedded in E_3 . If we are to compute the e.m.f. generated on that circuit by the electrical field E, we need its circulation along a infinitesimal line element at a point m. That precisely depends on the shape of the circuit near m, which is given by the tangent vector in m. Thus, a 1-differential form e depending on both m and $T_m M$ is suited to model the circulation of E.

A 2-form ω_m^2 assigns a real value to pairs of vectors from $T_m M$. This means is associates a algebraic value to a small, oriented surface element tangent to M in m. That behavior is exactly what is needed to define the flux of a quantity through a surface. For that reason, in electromagnetism the flux of the magnetic induction B is a good quantity to be represented with a 2-form b.

3-forms assign real values to sets of three vectors from $T_m M$. It therefore takes a parallelepiped volume located in m and constructed on those vectors, and returns an algebraic value. This is the kind of behavior expects for a function representing a density. In the context of electromagnetism, the density of electric charges ρ is for example well modeled via a 3-form.

Finally, 0-form are the exception. Normally, we would expect a 0-form ω_m^0 to be a linear differential map that takes 0 vectors as arguments and returns a real value. By convention, ω_m^0 is rather defined as a function $\omega_m^0 : \mathbb{R} \to \mathbb{R}$. Anyway, one can note that a 0-form is suited to model a scalar field, namely a function that depends of the position m. The additional degree of freedom that the argument $x \in \mathbb{R}$ introduces might or might not be used as a weighting coefficient for the field. The scalar electric potential Φ is an example of physical quantity that is suited to be modeled with a 0-form on E_3 .

Regarding the practical use, differential forms are almost all of the time used inside integrals. The integration of the k-forms ω^k takes place over k-manifolds or k-chains $C_k = \sum_i \mu_i M_i$. In the latter case, the integral is simply given by $\int_{C_k} \omega^k = \sum_i \mu_i \int_{M_i} \omega^p$.

As we saw, the main interest of differential forms to model electromagnetic phenomenon is that they pull towards the real physical observables. The electrical field E is indeed only measurable through the electromotive force that it generates along test circuits. In the same way, the magnetic induction B can only be measured via its magnetic flux through test surfaces. The vector-field formalism of electromagnetism places a considerable weight on fields like E or B that are by-products of the true measurable quantities. On the other hand, the formalism using differential forms focuses on the real observable quantities, fluxes and circulations. An e.m.f. \mathcal{E} is simply computed like the integral $\int_C e$ of a 1-form e over a 1-manifold C modeling the test circuit. Likewise, a magnetic flux Φ is just $\int_S b$, integral of b over the test surface represented by a 2-manifold S.

The second advantage to the use of differential forms is that they allow to write equations using easy, unified notations.

A.2.3 Exterior calculus

This sections describes the most important operations that can be performed with differential forms.

Earlier we started to introduce the *exterior product* \wedge in the context of the measuring the length of vectors, the areas of surfaces, etc. It can however be used for far more than that. The wedge product is an operation between a k_1 -form $\alpha_m^{k_1}$ and a k_2 -form $\beta_m^{k_2}$ which return a $(k_1 + k_2)$ -form $\alpha_m^{k_1} \wedge \beta_m^{k_2} : (T_m M)^{k_1+k_2} \to \mathbb{R}$. In the scenario where $(k_1 + k_2)$ is superior to the dimension p of the manifold M, the result returns uniformly 0 because of the alternating property of differential forms. In all the other cases, the return values are computed with the same formula introduced in A.2.1. In E_3 , the components of a wedge product between the 0-form α_m^0 and the k-form β_m^k $(k_i 3)$ are $\alpha_m^0 . \beta_m^k$. For two 1-forms they are $\alpha_m^1 \times \beta_m^1$ and finally, for a 1-form and a 2-form, $\alpha_m^1 \cdot \beta_m^2$. Regarding the physical interpretation, the exterior product between 1 and 2-forms has often the signification of an energy. For example, the energy of the magnetic field in a volume \mathcal{V} is $\frac{1}{2}$. $\int_{\mathcal{V}} b \wedge h$.

Related to the wedge product is the *Hodge operator*. Denoted *, it is a linear operator mapping k-forms ω^k on a manifold M of dimension p (p > k) into (p - k)-forms $\beta^{(p-k)} = *\alpha^k$. The Hodge star of ω_m^k is the $*\omega_m^k$ such that $\alpha \wedge *\omega_m^k = \langle \alpha, \omega_m^k \rangle \mu, \forall \alpha. \langle , \rangle$ denotes the inner product and μ is the preferred unit vector. In the euclidean space, this means for example that components of $*\omega^1$ are given by $\nu^2(x) = A(x)\omega^1(x)$ where A(x) is the identity matrix of A_3 . For more general manifolds, the Hodge operator is obtained from the metric tensor of that manifold.

The Exterior derivative d is the operation that transforms the k-form ω^k into the (k+1)-form $d\omega^k : (T_m M)^{k+1} \to \mathbb{R}$ such that for all (k+1)-manifold M, $\int_M d\omega^k = \int_{\partial M} \omega^k$. When compared to the vector field formalism, one finds that $d\omega^k$ for $p \in \{0, 1, 2\}$ has the same action as the gradient, curl and divergence operators, respectively. Like the wedge product, the interest of the exterior derivative is thereby that it encapsulates in a single operator three operations that were of different natures for vector fields.

The *trace* operator "restricts" the action of a differential form to a subdomain of the euclidean space. Given a 1-manifold M, the trace operator applied to a differential 1-

form ω^1 evaluated in $m \in M$ returns the tangential part of $\omega^1(m)$. For a 2-form and a 2-manifold, the trace operator returns the component of $\omega^2(m)$ normal to M.

As one can see after the definition of the operators, the second interesting element about the differential formalism is that it allows for unified and simplified notations. Compared to the vector field formalism, this leads to a more streamlined expression of the fundamental laws of electromagnetism.

A.3 Homology and cohomology

This section introduces the concepts of discrete (co-)homology, meaning the (co-)homology on finite element meshes.

We consider a manifold M (see A.1.2) and a *simplicial mesh* approximating M: \mathcal{M} . A mesh is said to be simplicial when it is only consisting of *simplices*, that is to say points, lines, triangles and tetrahedrons (these elements are the 0-dimensional to 3-dimensional simplices). Based on the elements of the mesh \mathcal{M} , *p*-chains c_p of simplices can be defined (see A.1.4). For example, let us denote $\mathcal{F}(\mathcal{M})$ the set of facets (triangles) in the mesh. A 2-chain c_2 can be written as the formal sum $c_2 = \sum_{\sigma_i^2 \in \mathcal{F}(\mathcal{M})} \mathbf{c}_i \sigma_i^2$ where the \mathbf{c}_i are real coefficients. The vector $\mathbf{C} = (\mathbf{c}_1 \ \mathbf{c}_2 \ \dots \ \mathbf{c}_{N_2})^T$ is a way to represent that chain using its coefficients.

The discrete boundary operator ∂_k is the operator mapping (k)-chains onto (k-1)-chains that mimics the behavior of the smooth boundary operator (see A.1.2). The discrete boundary operator ∂_k is associated with the incidence matrix of the k-simplices of the mesh, denoted \mathbf{D}_k . Namely, if $\mathcal{E}(\mathcal{M})$ is the set of edges on the mesh, $\partial_2 c_2$ is the 1-chain $\sum_{\sigma_i^1 \in \mathcal{E}(\mathcal{M})} (\mathbf{D}_2 \mathbf{c})_i \sigma_i^1$. This is a sum of mesh edges, weighted by the original coefficients of the 2-chain c_2 and the incidence matrix of the facets on the edges. All the edges that are shared between the components of c_2 get canceled out in the summation and it only remains the edges that are located on the boundary of c_2 . Eventually, the result is the 1-chain describing the contour of c_2 .

A k-chain whose boundary is the empty (k-1)-chain is called a k-cycle and is often denoted z_k . Also, a k-chain that is the boundary of a (k+1)-chain is called a k-boundary. Thus, the set of the k-chains represent the kernel of the boundary operator ∂_k (ker (∂_k)) and the set of the k-boundaries, the image of the operator ∂_{k+1} (im (∂_{k+1})). Two k-cycles are then called homologous if they only differ by a k-boundary. This is an equivalence relation that can be used to define equivalence classes of homologous k-cycles.

A k-cochain c^k is a linear map that assign a real values to a k-chain c_k . It can be represented with a vector of real coefficients \mathbf{c}^k such that, for any k-chain c_k , we have $c^k(c_k) = (\mathbf{c}^k)^T \mathbf{c}_k$. The coboundary operator δ_k is the operator mapping k-cochains onto (k + 1)-cochains and defined such that $\delta_k c^k(c_{k+1}) = c^k(\partial_{k+1}c_{k+1})$. Therefore the matrix representation of the δ_k operator is the transpose of the incidence matrix for the k-simplices of the mesh: \mathbf{D}_k^T . The elements of the spaces ker (δ_k) are called k-cocycles. It consists in the set of linear maps c^k that returns always zero when applied to a k-boundary. Conversely, the set im (δ_{k-1}) regroups the k-coboundaries. Eventually, the discrete *homology* and *cohomology* spaces are the quotient spaces:

$$H_k(\mathcal{M}) = \ker(\partial_k) / \operatorname{im}(\partial_{k+1}) = \{c_k + \operatorname{im}(\partial_{k+1}), c_k \in \ker(\partial_k)\}$$
(A.1a)

$$H^{k}(\mathcal{M}) = \ker(\delta_{k}) / \operatorname{im}(\delta_{k-1}) = \{ c^{k} + \operatorname{im}(\delta_{k-1}), c^{k} \in \ker(\delta_{k}) \}$$
(A.1b)

 $H_k(\mathcal{M})$ is therefore represented by k-chains that are k-cycles but not (k+1)-boundaries. c is a generator of the homology space $H_k(\mathcal{M})$. On the other hand, $H^k(\mathcal{M})$ is represented by the k-cocycles that are not k-coboundaries.

Appendix B

Short sample characterization of a RE-BCO tape

B.1 Objectives of the characterization

A characterization of the SuperOx tape that we used to wind the test coils for the first experiment (see 2.1.3) has been achieved by the team of Prof. Enric Pardo at the Institute of Electrical Engineering at the Slovak Academy of Science. The aim of that characterization was to obtain a material law able to represent with accuracy the behavior of the SuperOx tape during numerical simulations, like the ones described in chapter 3.

A tape characterization is obtained by performing a series of experiments on a short sample of REBCO coated conductor. Obviously, one has to assume that the properties measured on the sample are representative of that of the whole tape. Alternately, one can enhance the reliability by repeating the characterization process on several samples and averaging the results. Nevertheless, it also requires to perform even more experiments. So, in our case, just a single sample was tested and we assumed that it was representative for the rest of the tape.

The properties of the REBCO materials depend on various parameters, the two most important being the operating temperature T and the magnetic induction b, both through its magnitude and orientation relative to the tape surface. For our simulations specifically, we were not interested in the dependencies of the SuperOx tape characteristics on temperature. Eliminating this factor conveniently allows to reduce the number of experiments to perform. In the end, the characterization that has been performed is only to represent the variations of the tape properties with the applied magnetic field.

B.2 Characterization protocol

In practice, a straight sample of REBCO tape of about 20 cm in length is soldered between two current leads. Voltage taps are then soldered on the coated conductor, between the current leads and on the REBCO-side of the tape, to measure the voltage drop. The assembly is then cooled at the desired temperature, so in our case, all the experiments



Figure B.1: Characterization of the REBCO tape

were performed at 77 K, the boiling temperature of liquid nitrogen under atmospheric pressure.

An external magnetic field is applied in the tape in the plane orthogonal to its axis, a situation represented in figure B.1. The amplitude of the magnetic field is the first variable parameter in this characterization The tested values ranged from 0 mT (no field at all) to about 200 mT. Also, the field hits the tape at angle with regard to its normal, that angle is the second parameter of the characterization. All the configurations from 0 degrees to 360 degrees were tested with a step of 9 degrees between each measurement.

When the external field has stabilized, the current in the REBCO sample is raise progressively while monitoring the voltage drop. That way, a U - I characteristic of the tape can be drawn. Eventually, the test is stopped when the voltage drop across the tape exceeds a predefined criterion U_{max} and the supplied current is brought back to 0 A. U_{max} needs to be large enough so that the transition between the superconducting and dissipative regimes is clearly visible. On the other hand, it should not be too large otherwise the conductor risks to suffer irreversible damage during a test, which would ruin the entire work. This process of drawing the U - I tape characteristic is repeated several times while changing the applied field.

Eventually, one has to fit a selected model onto the gathered data to extract the material law parameters they are looking for. In our case, a power law model was fitted onto the data. Therefore, we were able to determine the $I_c(h)$ and n(h) parameters of the law.

B.3 Results of the characterization

The results are plotted in figure B.2, with the plot for I_c on the left and n on the right.

The measurements of I_c are used to deduce the variations of the tape J_c with the local magnetic field. In our simulations, those data are then interpolated to obtain the critical current as a function of given local magnetic field. The end result is that we use the interpolated function $J_c(h)$ inside the power law.

When we consider the experimental data for n, we must note that they are very scattered



(a) Tape I_c vs. magnetic field orientation





Figure B.2: Characterization of the SuperOx GdBCO tape performed by Prof. Enric Pardo at the Slovak Academy of Science. The field angle with respect to the tape is defined on figure B.1.



Figure B.3: Dependency of the power law *n* exponent over the amplitude of the magnetic field for the SuperOx tape

for a given field magnitude. It is not clear whether the field angle relative to the tape has a real influence on n. The reason why the measurements are so dispersed is related to the experimental protocol used.

n is the parameter that controls the sharpness of the transition between the superconducting and the dissipative regimes of the power-law. To compute a good estimate of n, it is necessary to know the U - I relation over the widest range possible. But during the characterization experiments, there is however a practical limit for the maximum voltage that one should not exceed. Indeed, as we already mentioned, the sample could suffer irreversible damage if the voltage was allowed to raise too high and the REBCO was allowed to go too far in the dissipative regime. This is the why the experiments are stopped as soon as the conductor voltage reaches U_{max} . The frame inside which the U - I curve is measured is thereby restricted and eventually, the computation of n is made more imprecise.

We considered that the data across the tape orientation for n were too scattered to be serviceable and we only considered a dependency the magnitude of the field |h|. To do so, we averaged all the measurements performed at the same field level for every orientation. The resulting dependency of n over |h| is represented in figure B.3. This curve was implemented in the simulations to interpolate n in the power law.

Appendix C

Résumé de la thèse

C.1 Courants d'écrantage dans les supraconducteurs REBCO

C.1.1 Caractéristiques de l'état supraconducteur

La supraconductivité désigne l'état de certains matériaux dont la résistivité électrique tombe à zéro lorsqu'ils sont employés à des températures inférieures à une température seuil, appelée température critique T_c . Les matériaux supraconducteurs possèdent également une seconde caractéristique qui les distinguent d'un simple conducteur parfait : le champ magnétique est expulsé en dehors de leur volume par un phénomène appelé l'effet Meissner. En présence d'un champ magnétique extérieur, des courants se créent en effet spontanément dans une épaisseur de peau au niveau de la surface du supraconducteur. Ces courants, les courants Meissner, contrebalancent le champ magnétique extérieur et l'annulent dans le volume intérieur du matériau, transformant alors celui-ci en diamagnétique parfait.

L'état supraconducteur ne peut en revanche exister que sous certaines conditions pour un matériau donné. Si celles-ci ne sont plus remplies, celui-ci transite alors vers l'état *normal*, dissipatif. La première contrainte, déjà mentionnée, est que la température du milieu supraconducteur doit rester inférieure à sa température critique T_c .

Deuxièmement, l'intensité du champ magnétique doit également demeurer inférieure à un certain seuil. Il existe cependant ici deux *types* de supraconducteurs, possèdant des comportements différents. Pour un matériau de *type I*, la supraconductivité disparait brutalement dès que l'induction magnétique b dépasse l'induction critique B_c , le matériau redevient alors normal. Pour les matériaux de *type II* en revanche, la transition s'effectue plus progressivement, entre deux valeurs $B_{c,1}$ et $B_{c,2}$. En-deçà de $B_{c,1}$, le milieu est parfaitement supraconducteur ; au-delà de $B_{c,2}$, il est complètement dissipatif et enfin, entre $B_{c,1}$ et $B_{c,2}$, il présente un mélange des deux phases, ce que l'on appelle l'état mixte ou l'état Shubnikov. Enfin, il faut noter que si les inductions critiques B_c (pour les types I) et $B_{c,1}$ (pour les types II) sont toujours faibles, de l'ordre de 100 mT, la seconde induction critique $B_{c,2}$ des matériaux de type II peut atteindre plusieures dizaines voire centaines de Teslas, ce qui leur confère beaucoup d'intérêt pratique.

Enfin, il existe aussi une transition de l'état supraconducteur vers l'état normal lorsque la densité de courant devient trop importante. Cette transition est néanmoins progressive

et ne possède pas véritablement de valeur seuil ayant un sens physique, c'est pourquoi la densité de courant critique J_c d'un supraconducteur possède un sens très conventionnel.

On représente ainsi souvent la frontière de l'état supraconducteur dans l'espace (T, B, J) par une *surface critique*. Des exemples de telles surfaces pour quelques supraconducteurs répandus sont visibles sur la Figure C.1.



Figure C.1: Exemples de surfaces critiques pour quelques matériaux supraconducteurs répandus : le NbTi (en rouge), le Nb₃Sn (en bleu) et le MgB₂ (en vert).

C.1.2 Les supraconducteurs à haute température critique de type REBCO

Les *REBCO* sont une famille de céramiques supraconductrices à haute température critique dont les membres possédent des compositions chimiques similaires : $REBa_2Cu_3O_{7-x}$. Le symbole RE- désigne dans cette formule une terre rare (RE pour *Rare Earth* en anglais), le plus souvent de l'Yttrium (Y) ou du Gadolinium (Gd). Les REBCO suscitent beaucoup d'intérêt en raison de leurs températures critiques élevées, pouvant atteindre 90 K voire 110 K, lorsque tous les supraconducteurs connus auparavant avaient $T_c < 20$ K. Par ailleurs, les REBCO présentent aussi des performances inégalées sous champs magnétiques intenses, comme le souligne la Figure C.2.

Les supraconducteurs REBCO sont donc les candidats de prédilection pour les aimants à fort champ de nouvelle génération, capables de produire 25 T et plus. Leurs bonnes propriétés mécaniques en tension sont un avantage supplémentaire pour ce type d'applications car elles permettent aux conducteurs de mieux encaisser aux contraintes dans les bobinages. Les REBCO sont donc ainsi à l'étude pour des projets de câbles à fort courant [84, 34, 91], d'aimants haut champ [70, 11], pour l'imagerie par résonnance magnétique nucléaire [9] et enfin pour les aimants d'accélérateurs [59].

Toutefois, la mise en œuvre concrète de ces nouveaux matériaux prometteurs est encore limitée à l'heure actuelle en raison d'un certain nombre de contraintes technicoéconomiques. Les conducteurs REBCO souffrent en effet aujourd'hui d'un prix élevé (≈ 100 \$.kA⁻¹.m⁻¹ à 77 K en champ propre), de la forte anisotropie de leurs propriétés par rapport au champ magnétique et d'une faible vitesse de propagation de quench, rendant la protection des aimants difficile [87, 38].

Enfin, les REBCO sont sujet au problème des courants d'écrantage, qui constituent l'objet principal de cette étude.



Figure C.2: Comparaison des densités de courant critique hors-tout J_e pour différents types de conducteurs en fonction du champ magnétique appliqué. Données compilées par P. Lee au NHMFL [62]

C.1.3 Le phénomène des courants d'écrantage

Le terme de *courants d'écrantage* désigne les courants induits dans le volume des REBCO (des matériaux de type II) en réponse à un champ compris entre $B_{c,1}$ et $B_{c,2}$. Ces courants étendent donc en quelque sorte la notion de courants Meissner au-delà de $B_{c,1}$ et jusque dans l'état mixte. Dans ce dernier, le volume intérieur du supraconducteur n'est plus parfaitement écranté et une partie du champ externe est donc capable d'y pénétrer.

Une particularité des conducteurs REBCO réside dans le fait qu'ils prennent la forme de rubans très aplatis, mesurant typiquement 10 mm de largeur pour seulement 100 μ m d'épaisseur. Ce type de géométries est la conséquence des procédés de fabrications employés. Le fort rapport d'aspect des conducteurs fait que le phénomène d'écrantage est alors d'autant plus important lorsque le champ externe est appliqué perpendiculairement

à la surface du ruban (voir Figure C.3). Ces courants d'écrantage, parce qu'ils entraînent une aimantation du conducteur, induisent alors des perturbations sur le champ magnétique total généré dans les applications pratiques.



Figure C.3: Aperçu des courants d'écrantage induits localement dans un ruban supraconducteur REBCO en fonction de l'orientation du champ magnétique.

Les aimants fabriqués à base de conducteurs REBCO possèdent de plus un problème de stabilité de leur champ magnétique produit. En effet, la densité de courant se redistribue au fil du temps dans les matériaux REBCO, ce qui entraîne une lente décroissance de l'aimantation des conducteurs et, par conséquent, une dérive du champ magnétique. La dynamique de la densité de courant dans les REBCO ne provient pas de l'effet Joule, puisque le matériau est supraconducteur, mais d'un phénomène appelé *flux creep*.

Le flux creep dans les supraconducteurs REBCO

Comme mentionné précédamment, le champ magnétique est capable de pénétrer dans un REBCO lorsque son intensité devient supérieure à $B_{c,1}$. Cela se produit sous la forme quantifiée de vortex de flux : à partir des bords du matériau, des régions normales apparaîssent, isolées de la phase supraconductrice par des courants s'enroulant autour d'elles et qui donnent leur nom aux vortex (voir Figure C.4). Chaque vortex laisse passer un quantum de flux au travers du matériau, le champ b macroscopique est alors finalement obtenu par moyennage sur la distribution. De la même façon, la densité de courant d'écrantage macroscopique correspond à la moyenne des courants autour des vortex.

Les vortex de flux subissent plusieurs forces et notamment une répulsion de la part de leurs voisins. Ainsi, les interactions mutuelles tendent à homogénéiser la distribution présente dans le milieu, en amenant les vortex des zones de forte densité vers celles où la densité est plus faible. En général, cela correspond à un mouvement global depuis les bords vers le cœur du matériau supraconducteur.

Néanmoins, la relaxation de la densité de courant est ralentie en pratique par les défauts présents dans le matériau et qui constituent des *sites d'ancrage* restreignant les déplacements des vortex. Ces défauts peuvent être de différentes natures : inclusion d'impuretés, dislocation du réseau cristallin, etc. La redistribution de la densité de vortex ne se produit donc pas de manière continue mais plutôt par une succession discrète de sauts. A chacun de ces sauts, un ou plusieurs vortex sont délogés de leurs sites d'ancrage et repoussés en cascade vers les régions à faible densité jusqu'à être de nouveau piégés par d'autres défauts



Figure C.4: Illustration d'un ruban de supraconducteur de type II dans l'état mixte de *Shubnikov*. Le champ magnétique externe traverse le matériau sous la forme de vortex portant chacun un quantum de flux.

du matériau. En pratique, dans le cas des REBCO, les sauts sont surtout provoqués par l'agitation thermique qui donne occasionnelement à un vortex suffisamment d'énergie pour sortir de son site d'ancrage. Le relaxation est également facilitée en présence de densité de courant car celle-ci tend à abaisser la hauteur des barrières d'énergie associées aux sites d'ancrage.

Une des théories essentielles permettant de décrire le phénomène du flux creep a été formulée par Anderson et Kim entre 1962 et 1964 [4]. Cette dernière prédit notamment que l'aimantation décroit de manière logarithmique dans le temps. Elle sera largement employée dans la suite pour l'analyse de nos résultats.

Impact des courants d'écrantages sur les aimants REBCO

Considérons maintenant le cas pratique d'un aimant bobiné à partir de rubans REBCO. Lorsque celui-ci est alimenté en courant de transport, le champ propre qui est généré induit dans les bobinages des courants d'écrantage parasites. Ceux-ci créent en retour une aimantation des conducteurs qui s'oppose à la fois localement et globalement au champ propre de l'aimant. On nomme *SCIF* (pour *Screening Current Induced Field* en anglais) le champ global obtenu en intégrant sur l'ensemble de l'aimant les courants d'écrantage induits (c.f. Figure C.5).

Le SCIF est la cause de plusieurs problèmes importants du point de vue de la qualité de champ dans les aimants REBCO. Tout d'abord, il s'oppose au champ propre et conduit donc à une diminution de l'amplitude du champ total généré. Le SCIF cause également des perturbations sur la qualité spatiale du champ, ce qui rend plus difficile de répondre à des spécifications précises en terme d'homogénéité ou de développement en multipôles, par exemple. Enfin, le flux creep est responsable d'une relaxation lente de la densité de courant dans le bobinage, ce qui engendre une dérive temporelle indésirable du champ.

Le phénomène des courants d'écrantage soulève donc, dans les aimants supraconducteurs REBCO, des problèmes de qualité et de stabilité du champ magnétique produit dont les amplitudes sont en général incompatibles avec les exigences des applications concrètes



Figure C.5: Illustration de l'effet du SCIF dans une bobine REBCO. Le champ propre de l'aimant induit des courants d'écrantages dans les conducteurs de son bobinage. Intégrée globalement, l'aimantation locale donne alors naissance au SCIF (*Screening Current Induced Field*) qui s'oppose au champ initial.

visées.

C.1.4 Plan de l'étude

Cette thèse propose d'étudier le problème posé par les courants d'écrantage dans les aimants supraconducteurs REBCO ainsi que ses possibles solutions. Elle se consacre à cette question en suivant deux directions principales. Dans un premier temps, la simulation numérique du phénomène, dans l'optique de développer des outils de calcul permettant aux concepteurs d'aimants d'anticiper ses effets dans le cadre de futurs projets. Deuxièmement, l'étude des techniques de suppression et de compensation, qui peuvent permettent de pallier aux conséquences problèmatiques du SCIF dans des aimants REBCO déjà existants.

C.2 Mesure expérimentale de courants d'écrantage dans des bobines REBCO

C.2.1 Description du dispositif expérimental

Dans un premier temps, une expérience visant à mesurer les effets du SCIF sur le champ magnétique généré par de petites bobines supraconductrices a été conçue et fabriquée.

L'objectif de cette expérience était de collecter une base de résultats pouvant ensuite servir de cas-tests lors du développement du code de simulation.



Katrin 400 A current leads

Coil 1 : 68 turns (R_{int} = 59 mm ;

Coil 2 : 51 turns (R_{int} = 59 mm ;

SuperOx 4 mm-wide insulated tape, $I_c = 150 \text{ A} @ 77 \text{ K} \text{ s.-f.}$

R_{ext} = 69 mm)

R_{ext} = 66.2 mm)

AREPOC Hall sensors
Pt100 RTD temperature probes

Instrumentation:

Voltage taps

Coil:

Le dispositif expérimental est présenté dans son ensemble sur la Figure C.6.

Figure C.6: Vue d'ensemble du dispositif expérimental pour les essais de mesures du SCIF

L'élément central de l'installation est une petite galette simple supraconductrice, bobinée à partir d'un ruban REBCO isolé fourni par SuperOx et possédant un courant critique moyen I_c de 140 A à 77 K, sous champ propre. Deux galettes simples de géométrie similaires ont en fait été fabriquées pour les essais (notées "A" et "B"). En effet, la première bobine REBCO (A) a été endommagée par un quench lors d'un des tests et il a donc fallu préparer une bobine de remplacement afin de pouvoir terminer toutes les expériences prévues. Les deux galettes simples sont visibles sur les Figures C.7a et C.7b et leurs caractéristiques sont résumées dans la Table C.1

La bobine de test est placée dans un cryostat permettant de la refroidir sous sa température critique T_c jusqu'à 77 K en utilisant un bain d'azote liquide (LN₂), voire jusqu'à 4,2 K avec un bain d'Hélium liquide (LHe). Pour simplifier les longues mesures de SCIF, qui peuvent s'étendre sur des durées allant jusqu'à 10^5 s (soit environ 1 jour et 4 heures), une boucle de régulation du niveau d'azote liquide a été implémentée au moyen d'un contrôleur Air Liquide RN24.

Dans cette expérience, c'est le champ propre de la bobine, lorsque qu'elle est alimentée en courant de transport, qui induit dans les conducteurs REBCO des courants d'écrantages et créée le SCIF. La galette simple est donc connectée à l'extérieur du cryostat à une alimentation *Oxford Instrument IPS* afin de piloter le courant de transport entre 0 A et 120 A.

La bobine est instrumentée au moyen de sondes de Hall de précision afin de pouvoir

NI acquisition modules:

9217 (temperatures)

(low voltage readings)

Monitoring:

9265 (Hall current supply).

Keithley 2182 n-voltmeter

Desktop PC with LabView

Custom acquisition software

9238, 9239, 9269 (voltages),

 \bigcirc



(a) Bobinette A

(b) Bobinette B

Figure C.7: Galettes simples REBCO utilisées pendant les expériences de mesure de SCIF.

Paramètre [unité]	Valeur pour la bobine A	Valeur pour la bobine B
Type de bobinage	galette simple	galette simple
Nombre de tours	68.5	51.5
Rayon int. $/ \text{ ext. } [\text{mm}]$	59 / 69	$59 \ / \ 66$
Hauteur [mm]	4	4
Longueur de conducteur [m]	27.5	21.5
Inductance [mH]	1.15	0.68
Constante au centre $[mT/A]$	0.675	0.518
Constante déportée ¹ $[mT/A]$	0.973	0.733
Courant critique ² $[A]$	81.2	83.5

¹Constante théorique de la bobine dans le plan médian, pour une position radiale éloignée de l'axe de 39 mm correspondant à l'emplacement de la sonde sonde de Hall. ${}^{2}I_{c}$ calculé sur la base des mesures effectuées à 77 K en champ propre.

Table C.1: Caractéristiques des bobinettes REBCO employées pour les essais de mesures de SCIF

mesurer le champ magnétique qu'elle produit. Deux sondes ont été employées, la première positionnée au centre de la bobine et la seconde déportée plus proche des conducteurs, à une distance radiale de 39 mm par rapport à l'axe. En regard de la petite taille de l'aimant et donc du faible champ produit, les signaux provenant des sondes sont lus au travers d'appareils de précision : des cartes d'acquisition National Instrument NI9238 ± 0.5 V, voire un nanovoltmètre Keithley 2182.

Pour finir, la bobine REBCO est également instrumenté au moyen de sondes RTD afin d'en mesurer la température. De prises de potentiel insérées dans le bobinage au niveau des jonctions permettent de calculer la tension à ses bornes et un shunt calibré en série dans le circuit permet d'obtenir le courant.

C.2.2 Mesures de SCIF réalisées

Relaxation du SCIF à la rémanence à 77 K

La première séquence d'essais réalisée consistait en une mesure de la dérive temporelle du champ rémanent de la bobine REBCO à 77 K.

Le choix de réaliser la mesure à la rémanence présente l'avantage que la valeur théorique du champ est parfaitement connue puisqu'il n'y a plus de courant de transport dans la bobine : 0 T. Ainsi, tout valeur mesurée différente de zéro est imputable en totalité aux courants d'écrantages induits dans les bobinages REBCO. En revanche, cette même approche souffre d'un incovénient majeur : le champ à mesurer est extrêmement faible et sa dérive est également très lente, ce qui oblige à effectuer la mesure de champ avec des appareils très sensibles.

Le protocole pour cette série de test commence donc par le refroidissement à 77 K de la galette simple REBCO. Cette dernière est ensuite chargée jusqu'à un courant I_{max} avec une pente dI/dt, puis après un plateau de 60 s, déchargée jusqu'à 0 A à la même vitesse. A la rémanence, l'acquisition du champ magnétique de la bobine démarre et se poursuit pour les 10^5 s suivantes. A la fin de l'essai la bobine est réchauffée au dessus de sa température critique avant d'être de nouveau refroidie pour le test suivant. Ce cyclage thermique permet de supprimer toute densité de courants encore piégée dans les conducteurs REBCO en faisant repasser temporairement le bobinage à l'état dissipatif. Ainsi, la bobine REBCO commence chaque essai dans un état vierge de toute densité de courant, ce qui permet de comparer facilement les différents tests.

Les résultats de cette première campagne de tests sont résumés sur la Figure C.8. L'amplitude de l'induction magnétique produite par l'aimant est tracée en fonction du temps écoulé depuis la fin de la décharge de la bobine avec une échelle logarithmique.

Naturellement, plus le courant de charge I_{max} de la bobine est élevé, plus l'aimantation des rubans REBCO est importante, ce qui accroit le SCIF initial. Pour les deux valeurs testées (2 et 10 A.s⁻¹), la vitesse de charge de la bobine dI/dt n'a en revanche aucun effet visible sur la relaxation mesurée. Enfin, on notera surtout que le champ présente dans tous les cas une lente décroissance en fonction du temps avec une vitesse logarithmique. Ce type de décroissance est cohérent avec la théorie du flux creep de Anderson-Kim.

Relaxation du SCIF à 77 K avec courant de transport

La seconde campagne de test a été effectué en utilisant la galette simple B, la première bobine (A) ayant été endommagée par un quench. Lors de cette campagne, la décroissance du champ magnétique créé par la bobine-test est mesuré pendant qu'un courant de transport constant circule dans son bobinage. Par rapport à la première campagne, ce protocle permet donc d'étudier l'influence du courant de transport sur la vitesse de relaxation du SCIF. En revanche, puisque les mesures sont réalisées en courant, la durée du test a dû être raccourcie à 10^4 s afin de ne pas laisser l'alimentation en fonctionnement sans surveillance.

Le protocole pour cette seconde série de mesure débute donc une nouvelle fois par le refroidissement de la galette simple REBCO. Celle-ci est ensuite chargée jusqu'à un courant cible I_{max} . Une fois ce dernier atteint, le courant dans la bobine est stabilisé pour 10^4 s durant lesquelles le champ magnétique est mesuré. Pour terminer, le courant de transport



Figure C.8: Décroissance temporelle du SCIF mesuré à la rémanence, à 77 K, pour la bobinette A.

est enfin abaissé jusqu'à 0 A et un cycle thermique est effectué de sorte à supprimer les courants induits dans les rubans REBCO.

Les réultats pour cette seconde campagne de mesure sont représentés sur la Figure C.9. Celle-ci montre la variation, en fonction du temps passé sur la plateau de courant, de l'induction magnétique normalisée par la valeur initiale mesurée. Cette normalisation est nécessaire pour pouvoir comparer plus facilement les vitesses de relaxation obtenues dans des essais conduits à différents niveaux de champ.

La Figure C.9 montre qu'un courant de transport plus élevé accélère la dérive du champ. Cela est expliqué par la théorie du flux creep. En effet, J y entraîne une diminution de la hauteur des barrières de potentiel associés aux les sites d'ancrages, ce qui favorise la mobilité des vortex et résulte en une relaxation accélérée des courants d'écrantage et du SCIF.

Relaxation du SCIF à la rémanence à 4,2 K

Pour conclure les mesures de décroissance de SCIF, quelques essais ont également été menés à 4,2 K en utilisant un bain d'Hélium liquide à pression atmosphérique pour refroidir la bobine de test B. Néanmoins, ces tests n'ont pas permis de mesurer la plus infime relaxation du champ à 4,2 K.

Cela est également expliqué par la théorie d'Anderson-Kim. La vitesse de relaxation des vortex obéit en effet selon celle-ci à une loi d'Arrhenius faisant intervenir $\exp(1/T)$. Ainsi, lors du passage de 77 K à 4,2 K, l'assistance apportée par l'agitation thermique diminue exponentiellement et la vitesse de relaxation résultante devient trop lente pour pouvoir être mesurée.

Les résultats de mesure obtenus à l'occasion de cette première expérience ont ensuite été



Figure C.9: Dérive temporelle du champ magnétique généré par la bobine B en charge, à 77 K

employés comme référence pour tester le code dont le développement est abordé dans la partie suivante.

C.3 Simulation des courants d'écrantage dans les aimants REBCO

Dans le cadre de cette thèse, un modèle numérique a été développé, qui permet notamment de simuler les courants d'écrantage dans des aimants REBCO. L'objectif avec ce modèle est de proposer un outil à l'usage des ingénieurs de conception permettant d'anticiper les effets du SCIF dans de futurs projets d'aimants REBCO.

C.3.1 Construction du modèle

Simplifications communes aux systèmes modélisés

Afin de simplifier la modélisation des aimants supraconducteurs, qui sont intrinsèquement des systèmes multiphysiques très riches, seul le *compartiment* de l'électromagnétisme a été pris en compte. Ainsi, tous les phénomènes physiques liés, par exemple, à la thermique ou à la mécanique sont écartés. Les simulations réalisées considèrent donc des supraconducteurs infiniment rigides, refroidis en permanence à une température fixe $T_{sim} < T_c$.

La géométrie des systèmes étudiés est également simplifiée. Les objets qui nous intéressent pouvant en effet se modéliser avec précision en 2D ou en 2D axisymétrique (respectivement pour les aimants d'accélérateurs dans leur section transverse et solénoïdes), nous avons opté pour le développement d'un modèle électromagnétique purement 2D. Le passage d'une description complète 3D à la simplification 2D s'accompagne d'une réduction de la taille du système à résoudre qui améliore à la fois les temps de calcul et la consommation en mémoire. En revanche, cette transformation soulève aussi de nouveaux probèmes liés à la topologie, traités dans la Section C.3.1.

Le domaine 2D finalement considéré est noté Ω . A ce stade, Ω est un domaine infini puisqu'il n'y a pas *a priori* de frontière extérieure pour limiter l'extension du champ magnétique. Ω est ensuite partitionné en deux avec un premier sous-domaine "conducteur" Ω_c , dans lequel on attend que la densité de courant j puisse circuler, et un second sous-domaine "isolant" Ω_c^c , où j = 0 A.m⁻².

Equations fondamentales

Le travail consiste dorénavant à résoudre le système formé par les équations de Maxwell décrivant le comportement des champs électriques et magnétiques dans le domaine infini Ω .

Compte-tenu des faibles fréquences de variations des champs, on se permet de négliger le terme de courant de déplacement dans l'équation de Maxwell-Ampère, ce qui nous amène à considérer le problème des courants de Foucault (C.1). Enfin, on complète ces équations en y adjoignant une Loi d'Ohm décrivant la dépendance entre e et j: $e = \rho \star j$. Dans cette dernière, ρ est un tenseur dépendant non-linéairement de j.

$$dh = j$$
 (C.1a) $de + \partial_t b = 0$ (C.1b)

$$b = \star \mu h$$
 (C.1c) $d = \star \epsilon e$ (C.1d)

Formulation $H-\varphi$

Notre approche pour résoudre ce problème met en application la formulation $h - \varphi$.

Le champ magnétique h est d'abord choisi comme l'inconnue principale. Ensuite, suivant le domaine considéré (conducteur Ω_c ou isolant Ω_c^c), on emploie une décomposition de h sur des fonctions de base différentes. Ainsi, dans Ω_c , le champ magnétique s'écrit comme une combinaison linéaire des éléments de Whitney d'arrête e_i . En revanche, dans le domaine isolant, puisque l'on souhaite contraindre j = dh = 0 alors on a $\exists \varphi \in \mathcal{F}^0(\Omega_c^c)$, $h = d\varphi$. L'existence de φ est garantie par le Lemme de Poincarré pourvu que le premier théorème de de Rham soit rempli (ce qui requiert généralement que Ω_c^c soit simplement connexe, point discuté dans la Section C.3.1).

Le principal avantage de cette formulation est de forcer la densité de courant j à ne circuler que dans le domaine conducteur, éliminant ainsi un risque d'artefacts numériques, comme démontré dans [83].



Figure C.10: Traitement du domaine Ω . Le domaine initialement infini est divisé en deux par une frontière arbitraire à une distance R_c de l'origine. Le changement de coordonnées d'enveloppe sphérique est appliqué aux points situés plus loin que R_c afin de transformer l'espace infini en une couronne comprise entre les rayons R_c et R_f .

Traitement de l'espace infini

Le point suivant à aborder est le traitement de la frontière libre du domaine Ω . Ω est en effet infini mais il doit être réduit à un domaine d'extension finie afin de pouvoir être simulé numériquement.

Pour ce faire, nous avons utilisé la technique de la transformation d'enveloppe sphérique introduite dans [47]. Cette méthode consiste à appliquer une transformation des coordonnées des points situés au-delà d'une frontière arbitraire à R_c afin de contracter l'espace infini en une enveloppe sphérique finie comprise entre deux rayons R_c et R_{∞} . On note par la suite avec une apostrophe ' les domaines images par la transformation appliquée. Cette transformation est illustrée sur la Figure C.10.

Traitement des problèmes de topologie

Enfin, un dernier élément important du modèle que nous avons développé réside dans la manière de prendre en compte les parties non-simplement connexes du modèle.

Comme mentionné dans le paragraphe C.3.1, la décomposition $h-\varphi$ ne peut être employée que sous les conditions définies par le *premier théorème de de Rham*, c'est-à-dire pour résumer, dans les domaines non-simplement connexes. Or, le sous-domaine isolant Ω_c^c n'est jamais simplement connexe dans les cas qui nous intéressent puisque chacun des conducteurs REBCO présents dans le modèle y créé un "trou". Ceci devrait par conséquent nous empêcher d'employer la formulation $h-\varphi$.

Néanmoins, ce problème peut-être résolu en faisant intervenir dans l'écriture de h un où plusieurs termes additionnels représentant une base du groupe de cohomologie de Ω_c^c (c.f. Annexe A). Un terme supplémentaire est nécessaire pour chaque conducteur (chaque

"trou" de Ω_c^c). Les termes sont précalculés une fois pour toute avant le début de la simulation électromagnétique et, au cours de celle-ci, un coefficient leur est associé à chacun, qui possède une interprétation en terme de courant dans les conducteurs.

Forme faible du probème étudié

Le problème à résoudre s'écrit enfin à la forme faible de la manière suivante, rechercher :

$$h \in \mathcal{H}(\Omega') = \begin{cases} h \in \mathcal{F}^{1}(\Omega'), \\ \exists \varphi \in \mathcal{F}^{0}(\Omega_{a} \cup \Omega'_{i}), h = -d\varphi \text{ in } \Omega_{a} \cup \Omega'_{i}, \\ t_{\partial\Omega_{c}}dh = 0, \\ t_{\partial\Omega_{a}}h = t_{\partial\Omega'_{i}}h \end{cases}$$
(C.2)

solution du problème faible :

$$\begin{cases} \int_{\Omega'} \partial_t \star \mu h \wedge h' + \int_{\Omega_c} \rho \star dh \wedge dh' + I.\Psi = 0, \ \forall h' \in \mathcal{H} \left(\Omega' \right) \\ \int_{\Omega'} \star dh \wedge h' = 0, \ \forall h' \in \mathcal{H} \left(\Omega' \right) \end{cases}$$
(C.3)

où Ψ est une base du groupe de cohomologie $H^1(\Omega_c^{c'})$ et I est le courant algébrique circulant dans Ω_c .

C.3.2 Modélisation des propriétés du matériau supraconducteur

Le comportement des matériau supraconducteurs dans notre modèle est simulé au moyen de la relation $e = \rho \star j$ (*loi d'Ohm*), avec un tenseur ρ dépendant non-linéairement de j et parfois d'autres paramètres.

Concrètement, cette relation seule n'est pas suffisante pour caractériser le comportement d'un véritable supraconducteur car elle ne permet en effet pas de simuler l'effet Meissner. Néanmoins, en choisissant correctement les conditions initiales des problèmes simulés, on peut parvenir à faire coincider le comportement du matériau $e = \rho \star j$ avec celui d'un supraconducteur. Pour cela, il est impératif que dans l'état initial à t = 0 s de toutes les simulations, le champ magnétique h soit uniformément nul dans Ω et que la densité de courant j vaille également 0 dans tout Ω_c .

Le choix de la forme du tenseur ρ détermine ensuite le type de comportement du matériau supraconducteur simulé. Dans notre cas, nous avons majoritairement employé l'expression en *loi de puissance* (C.4), omniprésente dans la littérature. Tel que démontré dans [103], cette forme décrit bien le phénomène de flux creep dans les supraconducteur à haute température critique REBCO.

$$e = \left[\frac{E_0}{J_c} \left(\frac{|j|}{J_c}\right)^{n-1}\right] \star j = \rho\left(|j|\right) \star j \tag{C.4}$$

Alternativement, la loi de percolation (C.5) a également été employée dans quelques simulations à des fins de comparaison. La principale différence entre celle-ci et la précédente loi est que, cette fois, un régime véritablement supraconducteur e = 0 existe lorsque $j < J_{c,0}$.

$$e = \begin{cases} 0 & \text{if } |j| < J_{c,0} \\ \left[\frac{E_c}{A} \left(\frac{|j|}{J_{c,0}} - 1\right)^{n_0}\right] \frac{\star j}{|j|} & \text{if } |j| \ge J_{c,0} \end{cases} \quad \text{with } A = \left(\frac{J_c}{J_{c,0}} - 1\right)^{n_0} \tag{C.5}$$

Enfin, pour pouvoir simuler avec précision le comportement des matériaux supraconducteur réels, il est indispensable de prendre en considérations les dépendances des paramètres tels que J_c et n en fonction du champ magnétique h. Le ruban SuperOx employé pour les bobinettes des expériences de la Section C.2 a donc été caractérisé sous champ avec l'aide du Prof. Enric Pardo de l'Académie des Sciences de Slovaquie (c.f. Annexe B). Les variations des paramètres J_c et n de la loi de puissance en fonction de l'amplitude et l'orientation du champ magnétique h ont ensuite été extraites de ces mesures et implémentées dans les simulations.

C.3.3 Simulation d'un ruban supraconducteur isolé

Pour débuter, le modèle précédemment décrit a été testé sur le cas simple d'un unique ruban supraconducteur REBCO isolé dans un domaine d'air infini.

Application du modèle

Ce système est modélisé en 2D dans la section du conducteur. Seul la couche supraconductrice REBCO est considérée comme porteuse de courant (faisant partie de Ω_c), tous les autres matériaux constituant le ruban, comme par exemple le cuivre du stabilisant, sont regroupés avec l'air dans le domaine isolant Ω_c^c . C'est une hypothèse de modélisation forte, qui implique notamment que le modèle ne pourra pas décrire le *partage du courant* entre le supraconducteur et son stabilisant. Néanmoins, comme notre intention est exclusivement de simuler le comportement électromagnétique du système et non la thermique ou les phénomènes de quench, cette approximation se justifie. Le modèle final, réduit par la transformation d'enveloppe sphérique, est présenté sur la Figure C.11a.

Discrétisation du problème

Un maillage du domaine Ω' est obtenu au moyen du logiciel gratuit et open-source Gmsh [32, 31], voir Figure C.11b. Des éléments quadrangulaires réguliers sont employés pour mailler la couche supraconductrice Ω_c afin de bien se conformer à la forme de celle-ci. Le domaine isolant Ω_c^c quant à lui est simplement maillé au moyen de triangles non-structurés.

L'inconnue du problème, le champ magnétique h, est alors décomposée de la manière suivante :

$$h = \sum_{i \in \mathcal{N}(\Omega')} \varphi_i \mathrm{d}n_i + \sum_{j \in \mathcal{E}(\Omega_c \setminus \partial \Omega_c)} h_j e_j + I. \underbrace{\sum_{k \in \mathcal{E}(\Omega_c^c)} z_k e_k}_{\Psi}$$
(C.6)



Figure C.11: Modélisation 2D d'un ruban REBCO infini.

La première somme correspond à la décomposition de h dans le domaine isolant sur la base des 0-formes de Whitney n_i ; la seconde à la décomposition dans le domaine conducteur Ω_c sur les 1-formes. Enfin, la dernière somme est une décomposition précalculée d'une fonction de cohomologie du domaine $\Omega_c^{c'}$ sur la base des 1-formes de Withney. Le coefficient I en facteur de cette dernière somme correspond au courant total circulant dans le ruban, en Ampère. Dans notre cas, cette valeur est contrainte par une fonction du temps I(t) qui dicte le courant de transport injecté dans le ruban.

Le problème est ensuite discrétisé en temps au moyen d'un schéma d'Euler implicite. Le système déquations obtenu est comporte une partie non-linéaire engeandré par le terme $\rho(h(t_{n+1})) \star dh(t_{n+1})$. Celui-ci est donc traité en utilisant la méthode introduite dans [53] afin de se ramener à un système linéaire.

Combinant toutes les discrétisations, en temps et en espace, ainsi que la linéarisation nous parvenons à un système d'équations (C.7) soluble par un ordinateur.

$$\boldsymbol{A}.\Delta \boldsymbol{H}_{n}^{n+1} + \boldsymbol{R}_{1n}.\boldsymbol{H}_{n} + \boldsymbol{R}_{2n}.\Delta \boldsymbol{H}_{n}^{n+1} + \Delta \boldsymbol{I}_{n}^{n+1}.\boldsymbol{Z} = 0$$
$$\iff [\boldsymbol{A} + \boldsymbol{R}_{2n}].\Delta \boldsymbol{H}_{n}^{n+1} = -\boldsymbol{R}_{1n}.\boldsymbol{H}_{n} - \Delta \boldsymbol{I}_{n}^{n+1}.\boldsymbol{Z}$$
(C.7)

 H_n est le vecteur champ magnétique formé par la concaténation de tous les coefficients φ_i et h_j au temps t_n . ΔH_n^{n+1} est l'inconnue du problème, correspondant la variation du champ magnétique entre t_n et t_{n+1} , lorsque le courant dans le ruban varie de ΔI_n^{n+1} . A est une matrice par blocs constante, les matrices R_{1n} et R_{2n} doivent quant à elles être recalculées à chaque itération.

Algorithme de résolution

En pratique, ce système d'équations est résolu dans le domaine temporel à l'aide d'un algorithme à pas de temps adaptatif. Le pas Δt_n^{n+1} est simplement calculé en fonction de Δt_{n-1}^n au moyen d'heuristiques basées sur la vitesse de convergence de l'algorithme.

Pour chaque t, des itérations internes de Newton-Raphson sont effectuées afin d'obtenir la convergence du terme non-linéaire.

Rsultats du simulation

Pour ce premier exemple, nous avons notamment simulé la densité de courant j dans la largeur du ruban au terme d'une rampe de courant de transport depuis 0 A jusqu'à 100 A. Les résutats pour différentes lois de matériaux sont présentés sur la figure C.12 et comparés avec une référence donnée par une expression analytique de j par E. Zeldov [104].



Figure C.12: Densité de courant *j* simulé dans la largeur d'un ruban REBCO isolé pour différente lois de matériaux.

C.3.4 Simulation d'une galette simple REBCO

Dans un second temps, le même modèle a été employé afin de simuler le SCIF créé par les bobinettes REBCO dans les expériences présentées dans la section C.2.

Cette fois, le système étant une galette simple REBCO, il est modélisé en 2D axisymétique sous la forme d'un assemblage de spires concentriques. La taille du modèle peut ainsi être divisée par deux en exploitant la symétrie par rapport à l'axe de la bobine.

Ce modèle constitue une évolution du précédent, notamment en ce qui concerne l'utilisation de la cohomologie pour fixer les contraintes en courant. En effet, pour ce second système, le générateur du groupe de cohomologie de $\Omega_c^{c'}$ devient plus complexe car il se constitue d'autant de fonctions de base qu'il y a de tours dans la bobine modélisée. Nous avons choisi de travailler avec le générateur de $H_1(\Omega_c^c)$ dans lequel chaque fonction de base n'entoure qu'un seul conducteur (qu'une seule entité connexe de Ω_c). Ce faisant, le coefficient attribué a chaque fonction de base correspond au courant total à l'intérieur du conducteur associé, ce qui simplifie la mise en place des contraintes I(t).

Pour ce second système, nous avons cherché à simuler les expériences de mesures de SCIF réalisée lors de la deuxième campagne de test, c'est-à-dire en présence de courant de transport. Par conséquent, lors de chaque simulation, le courant dans la bobine est contraint à suivre un profil formé d'une rampe jusqu'au courant I_{max} , puis d'un plateau de 10^3 s pendant lequel est simulé la relaxation du champ central sous l'influence du flux creep.

Les résultats de ces simulations sont illustrés sur la figure C.13 et comparés aux données expérimentales tirés de la Section C.2 et à des benchmarks réalisés sous Comsol (par MM. F. Trillaud et E. Berrospe) et Matlab (par M. P. Fazilleau).



(a) Centre de la bobine, position de la sonde de Hall 1

(b) Point déporté de 39 mm par rapport à l'axe de la bobine, position de la sonde de Hall 2



C.3.5 Simulation du dipôle EuCARD

Pour terminer, nous avons testé les capacités de notre modèle sur un exemple d'aimant concret.

Nous avons donc opté pour le dipôle REBCO 5 T EuCARD développé en collaboration par le CERN et le CEA. Cet aimant prototype a pour but de développer les techniques de bobinage des conducteurs REBCO en vue de futurs aimants d'accélérateur hybrides capables de produire des champs de plus de 15 T. En exploitant les symétries de la section d'EuCARD (c.f. Figure C.14), on se ramène alors à l'étude du seul quadrant supérieur droit, présenté sur la Figure C.15.

EuCARD est un aimant bobiné autour d'un pôle en fer pour amplifier le champ produit. Cependant, ce pôle ferromagnétique n'est pas pris en compte dans notre modélisation à cause des difficultés pour faire converger un calcul mélangeant les non-linéarités issues de la supraconductivité et du ferromagnétisme.

Une spécificité de l'aimant EuCARD réside dans son conducteur. Le bobinage est en effet formé de deux circuits parallèles transposés une seule fois au centre du bobinage de sorte à équilibrer leurs impédences. Le conducteur de chacun des circuits parallèle est lui-même constitué de deux rubans REBCO soudés en face-à-face et avec des rubans additionnels



Figure C.14: Coupe de la section du dipôle EuCARD



Figure C.15: Maillage du domaine réduit pour la simulation du dipôle EuCARD.

de cuivre et de cupro-beryllium pour stabilisant. Tous ces circuits parallèles impliquent que des redistributions de la densité de courant sont possibles entre les différents rubans REBCO. Cette fois, le modèle électromagnétique 2D est donc couplé avec un système d'équations électriques de réseaux qui autorisent la prise en compte des redistributions.

En utilisant notre modèle, nous avons alors simulé le test de l'aimant EuCARD en standalone qui a été réalisée au CEA en Septembre 2017. Le courant dans l'aimant est monté de 0 A jusqu'au courant nominal de 2,8 kA tout en calculant les valeurs de champ au centre du diple. La figure C.16 montre ainsi le champ au centre simulé incluant l'effet des courants d'écrantage (en rouge) par rapport à la situation idéale (en noir).



Figure C.16: Simulation du champ eu centre du dipôle EuCARD lors d'une rampe de courant de O à 2,8 kA. En noir, la référence sans courants d'écrantage, obtenue en magnétostatique en considérant une distrbution uniforme de la densité de courant des les rubans REBCO. En rouge, le résultat obtenu avec notre modèle. La baisse du champ au centre est imputée aux courants d'érantages.

C.4 Suppression du SCIF dans les aimants supraconducteurs REBCO

Comme expliqué dans le Section C.1.3, le phénomène des courants d'écrantages engendre des problèmes de qualité et de stabilité temporelle du champ magnétique dans les aimants supraconducteurs REBCO. Pour pallier à ces problèmes, différentes techniques ont été imaginées en vue de réduire, voire de supprimer, les effets du SCIF. Ici, nous ne ferons que mentionner brièvement la *striation des rubans* [19] et l'*alignement des blocs de conducteurs* [59]. Nous nous étendrons en revanche plus longuement sur les deux approches que nous avons étudié expérimentalement : le *overshoot en courant* et le *vortex shaking*.
C.4.1 Méthodes pour la compensation du SCIF

Manipulations du courant de transport

Il existe plusieurs méthodes de réduction du SCIF dans les bobines REBCO qui sont basées sur la manipulation du courant d'alimentation. Pour atteindre leur objectif, toutes ces techniques reposent sur le même principe : utiliser un profil de charge spécifique permettant d'induire dans les rubans REBCO plusieurs boucles de courants d'écrantage de sens opposés, dont les effets au niveau de l'aimantation se compensent.

Dans notre cas, nous avons étudié la technique du *current overshoot*, qui consiste à charger la bobine REBCO à son courant nominal avec un dépassement temporaire de quelques pourcent. Cette approche n'affecte nullement la relaxation des courants d'écrantage au travers du flux creep. En revanche, l'inversion temporaire de la pente du courant d'alimentation permet de créer, à l'intérieur les rubans REBCO, une distribution initiale favorable de la densité de courant, à partir de laquelle la relaxation naturelle du SCIF par le flux creep est moins perceptible.

Simplement par un choix adapté du dépassement initial, il est donc possible de stabiliser le champ magnétique sans devoir apporter aucune modification sur l'aimant lui-même. Cette approche demande par contre de monter transitoirement le courant d'alimentation au-delà de sa valeur nominale. Elle requiert donc que le point de fonctionnement normal de l'aimant possède une marge suffisante sur la droite de charge pour supporter le dépassement.

Vortex Shaking

La technique du *vortex shaking* consiste quant à elle à forcer la relaxation de la densité de courant présente dans le supraconducteur en lui appliquant temporairement un champ magnétique alternatif de faible amplitude [30, 28, 29].

Pour cette technique, le champ d'excitation, aussi appelé champ de *shaking*, doit être appliqué dans le plan orthogonal à la direction de l'aimantation du ruban, c'est-à-dire parallèlement à la surface du ruban. L'amplitude du champ de shaking h_{sk} n'a pas besoin d'être très importante. Ainsi d'après E.H. Brandt, de l'ordre de 10⁴ A.m, soit $|b_{sk}| \approx 10 \text{ mT}$, sont suffisant pour forcer une relaxation totale de la densité de courant dans un ruban REBCO isolé [15]. Enfin, la fréquence du champ d'excitation ne joue *a priori* aucun rôle dans le fonctionnement la méthode, elle peut donc être choisie librement, notamment en fonction de l'aimant.

La principal handicap de cette approche est qu'elle demande de disposer d'un aimant auxiliaire pour générer le champ de shaking sur le volume occupé par la bobine principale REBCO. Le protocole est alors de charger dans un premier temps la bobine supraconductrice principale, puis d'activer temporairement le champ de shaking jusqu'à obtenir la relaxation complète du SCIF, après quoi l'aimant REBCO peut être utilisé tout-à-fait normalement. Le vortex shaking est tout de même une technique très intéressante car elle permet en théorie d'obtenir une relaxation rapide des courants d'écrantage, avec une vitesse exponentielle en temps au lieu de logarithmique avec le mécanisme naturel du flux creep.



Figure C.17: Vue d'ensemble du dispositif experimental réalisé pour les tests de vortex shaking

C.4.2 Réalisation d'un nouveau dispositif expérimental

Afin de tester les techniques de réduction du SCIF, un nouveau dispositif expérimental a été conçu et réalisé. Ce dernier est schématisé sur la Figure C.17. Le cœur du dispositif est constitué d'un assemblage de plusieurs bobines coaxiales : une galette simple REBCO constituant l'aimant principal entourée par une paire de solénoïdes cuivre produisant le champ de shaking.

Bobines de test REBCO

Lors des tests, on cherche à observer l'effet des techniques de réductions présentés sur le SCIF d'une petite galette simple REBCO. Deux exemplaires de la bobinette de test ont été fabriqués en utilisant des principes d'isolation différents. La première galette simple est isolée tour-à-tour tandis que, dans la seconde, le ruban REBCO est cobobiné avec de l'acier inoxidable. Ceci permet d'étudier la faisabilité du vortex shaking dans le cas des bobines utilisant une *isolation métallique (metal-as-insulation* en anglais), une solution qui se répend pour la protection des aimants REBCO contre les quenchs [61, 57]. En dehors de leur isolation, les deux bobinettes de test sont très similaires : elles utilisent le même type de ruban REBCO SuperPower et possèdent des caractéristiques proches résumées dans la table C.2.

Paramètre [unité]	Bohine isolée	Bohine MI
Type de bobinage	galette simple	galette simple
Nombre de tours	133.5	160.5
Rayon int. / ext. [mm]	30 / 48.8	30 / 47
Hauteur [mm]	6	6
Longueur de conducteur [m]	33.0	34.4
Inductance [mH]	1.80	2.77
Constante au centre $[mT/A]$	2.06	2.56

Table C.2: Caractéristiques des bobines REBCO utilisées pour les essais de vortex shaking

Bobines de shaking

La bobine de test est encadrée par une paire de bobines de *shaking* servant, lors des essais de vortex shaking, à générer le champ d'excitation alternatif sur le volume occupé par le bobinage REBCO.

Les bobines de shaking sont deux solénoides de cuivre $(Sk_1 \text{ et } Sk_2)$ bobinés en suivant des directions opposées puis connectés en série. Les caractéristiques des deux solénoïdes sont récapitulées dans la Table C.3. Cette paire de bobine est capable de créer un champ axial d'une amplitude maximale d'environ 80 mT sur le volume occupé par la bobine REBCO. Les dimensions et nombres de tours des bobines de shaking ont par ailleurs été optimisés pour annuler leur contribution au champ magnétique au centre de l'expérience. Ainsi, la mesure du SCIF de l'aimant principal REBCO n'est impactée qu'au minimum par la présence des bobines auxiliaires.

L'utilisation du cuivre pour ces bobines s'est imposée pour des raisons de coût. Ce choix nous a malheureusement privé de la possibilité de réaliser des tests à 4,2 K en utilisant de l'Hélium liquide pour le refroidissement. Les bobines de shaking étant résistives, elles auraient en effet dissipé un puissance importante dans le bain d'Hélium, ce qui aurait entraîné son évaporation rapide.

Paramètre [unité]	Valeur pour Sk_1	Valeur pour Sk_2
Type de bobinage	solénoïde	solénoïde
Sens de bobinage	horaire	trigonométrique
Nombre de couches	2	4
Tours par couche	52	52
Rayon int. / ext. [mm]	16.6 / 19.8	73.7 / 80.1
Hauteur [mm]	83	83
Résistance $[m\Omega]$	14	117
Inductance [mH]	non mesurable ¹	5.9

 1 Une fois Sk₁ et Sk₂ assemblées, la tension auto-induite aux bornes de Sk₁ est annulée quasi-parfaitement par la tension due à la mutuelle avec Sk₂.

Table C.3: Caractéristiques des bobines de shaking Sk₁ et Sk₂

Systèmes auxiliaires

Les nouvelles bobines ont été installées dans le même cryostat que celui utilisé lors des essais de mesure de SCIF (voir Section C.2). En revanche, tous les tests ont, cette fois, été pratiqués à 77 K en bain d'azote liquide.

Deux alimentations électriques ont été employées pour pouvoir alimenter indépendemment les circuits de la bobine REBCO (Ametek-Sorensen SGA 4 kW 10 V - 400 A) et des bobines de shaking (Kepco BOP 1 kW 25 V - 40 A).

L'assemblage est instrumentée par une sonde à effet Hall de précision Arepoc afin de mesurer le champ central. Des capteurs de température Pt100 et CLTS sont également employés afin de suivre les températures de l'expérience à l'intérieur du cryostat. Enfin, les tensions dans les deux circuits électriques sont surveillées au moyen de prises de potentiels et les courants par lecture de la résistance sur des shunts calibrés.

L'ensemble des signaux sont finalement traités par un programme d'acquisition ${\it LabView}$ dédié.

C.4.3 Essais d'overshoot

Le nouveau dispositif expérimental a d'abord été utilisé pour quelques essais de la technique de *current overshoot* (voir Section C.4.1). En particulier, toutes les expérience ont ici été menées sur la bobinette MI.

Chaque test effectué commence, comme d'habitude, par un cyclage thermique afin d'éliminer tous les courants induits et de restaurer l'état vierge du supraconducteur. Le courant de transport est ensuite monté jusqu'à sa valeur cible avec un dépassement initial d'une amplitude déterminée. Une fois le plateau de courant atteint, l'évolution du champ au centre de la bobine REBCO est alors mesurée pendant les 10^3 s suivantes.

La Figure C.18 présente les résultats obtenus avec cette méthode. Par rapport au cas de référence présenté en noir, la charge de la bobine suivant un profil avec un dépassement de quelques pourcent (< 5%) permet bien de réduire la dérive initiale du champ magnétique central. En particulier, il existe une valeur de dépassement optimale, ici environ 5% de la valeur nominale du courant, qui permet de supprimer la dérive visible du SCIF. En revanche, lorsque l'amplitude de l'*overshoot* devient trop importante (par exemple ici 10%), on constate alors qu'une dérive réapparait avec une pente opposée. Cela est signe que la relaxation des boucles de courants induites sur les bords des conducteurs REBCO lors de l'inversion de la pente du courant d'alimentaion est devenue prédominante.

Ainsi, l'expérience montre que la technique de current overshoot permet simplement de stabiliser la dérive du champ des aimants REBCO causé par le SCIF. Néanmoins, pour optimiser l'effet de cette méthode, l'amplitude du dépassement doit être déterminé avec précision, ce qui difficile *a priori*. Par ailleurs, si cette méthode permet de limiter la dérive temporelle du champ créé, elle ne permet pas réellement de supprimer les courants d'écrantage induits.



Figure C.18: Dérive temporelle du champ au centre de la bobine de test REBCO mesuré pour différentes valeur d'overshoot en courant.

C.4.4 Essais de vortex shaking

Le même equipement a donc ensuite été employé afin de réaliser des essais de vortex shaking sur les deux bobinettes supraconductrices.

Cette fois, le protocole consiste à effectuer les essais à la rémanence, une fois la bobine REBCO déchargée de son courant de transport. Comme dans la Section C.2.2, l'avantage est que dans ces circonstances, le champ mesuré est totalement imputable aux courants d'écrantage induits dans les bobinages REBCO, puisque le courant de transport est nul.

La bobine REBCO est donc chargée, partant d'un état vierge, jusqu'au courant cible de 40 A puis déchargé. Une fois de retour à 0 A, le champ de shaking est appliqué pour un nombre de cycles N_{sk} avec une amplitude $|b_{sk}|$ et une fréquence f_{sk} prédéfinies. Après la phase de shaking, le champ magnétique rémanent de la bobine test REBCO est mesuré pendant les 500 s suivantes pour en acquérir la dérive.

Les résultats des expériences réalisées avec ce protocole sont présenté sur les Figures C.19a pour la bobine de test isolée et C.19b pour la galette MI (isolation métallique). Par rapport au cas de référence en noir, sur lequel la tendance à la décroissance logarithmique du champ est bien visible, la mise en pratique de la technique du *vortex shaking* permet non seulement d'éliminer la dérive du champ mais également de supprimer jusqu'à 40% du SCIF au centre.

Malheureusement, seuls une poignée de test ont pu être conduit dans le temps imparti pour cette thèse. Il serait notamment intéressant de poursuivre les essais en utilisant des fréquences d'excitation f_{sk} ou des amplitudes $|b_{sk}|$ plus élevés de sorte à vérifier qu'une

suppression compète du SCIF rémanent puisse être observé.



Figure C.19: Effet du *vortex shaking* sur le champ magnétique généré par deux galettes simples REBCO en leur centre.

C.5 Conclusion générale

Les rubans supraconducteurs REBCO sont des produits extrêment intéressants pour le domaine des aimants à haut champ magnétique. Leurs excellentes performances en terme de densité de courant critique J_c sous champ en font en effet le conducteur de prédilection pour les projets d'aimants devant générer 20 T ou plus.

Cependant, l'utilisation pratique des rubans REBCO est freinée par un certain nombre de problèmes, au rang desquels on compte le phénomène des courants d'écrantage. Ces courants parasites sont induits de le matériau supraconducteur par les variations du champ magnétique (propre ou externe) et créent en réponse un SCIF s'opposant à la variation initiale. Dans le contexte des aimants REBCO, le SCIF occasionne des problèmes de qualité spatiale et de stabilité temporelle du champ magnétique.

Cette thèse s'est focalisé sur l'étude des courants d'écrantage dans les aimants REBCO en suivant deux axes : l'anticipation du phénomène au moyen de simulations numériques et l'élimination du SCIF au travers de deux techniques testées expérimentalement.

Sur le versant de la simulation, un modèle numérique a été développé qui permet le calcul des courants d'écrantage dans des aimants de taille réelle. La validité de ce modèle a été éprouvée par la comparaison de ses résultats avec nos propres mesures expérimentales et par des benchmarks avec d'autres logiciels commerciaux. A l'avenir, cet outil pourra donc venir compléter les moyens à disposition des ingénieurs de conception pour les futurs projets d'aimants haut champ REBCO.

Dans la seconde partie de cette thèse, deux techniques de réduction du SCIF ont été étudiées expérimentalement : le *current overshoot* et le *vortex shaking*. Un dispositif experimental dédié a été conçu et assemblé afin d'expérimenter ces techniques. Nous avons montré que le *current overshoot* permet, pourvu que la valeur du dépassement soit correctement choisie, de stabiliser la dérive du champ causée par la relaxation du SCIF. Pour le *vortex shaking*, nous avons montré que cette technique permettait non seulement de stabiliser le champ à la rémanence mais également de réduire l'amplitude du SCIF piégé. A l'avenir, certains aspects des travaux préentés dans cette thèse pourraient être complétés. Concernant les simulations, l'implémentation du comportement ferromagnétique dans le modèle proposé serait un avantage pour celui-ci. On pourra éventuellement aussi envisager l'étude du couplage multiphysique pour, par exemple, se diriger vers un modèle magnétothermique capable de simuler des quenchs.

Enfin, concernant l'étude du *vortex shaking*, de nombreux tests complémentaires sont envisageables avec le dispositif établi. Il pourrait notamment s'agir de tester le *shaking* à plus haute fréquence ou plus grande amplitude, afin de voir quelles sont les conditions requises pour obtenir une relaxation complète du SCIF.

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