

Collisionless shocks in the context of Laboratory Astrophysics

Anna Grassi

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présentée par

Anna Grassi

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Collisionless shocks in the context of Laboratory Astrophysics

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devant le jury composé de :

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CHAPTER 1

Introduction

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Tremendous progress in the development of high-energy and/or highintensity laser facilities has made it possible to recreate, in the laboratory, conditions relevant to the study of various astrophysical scenarios. These developments have paved the way to the field of study central to this thesis, the so-called *Laboratory Astrophysics*, which aims at advancing our understanding of the universe through the experimental exploration of astrophysically relevant processes as well as by promoting our capabilities for theoretical and numerical modeling of the physics at hand.

This field is undergoing a phase of growing activity. It can shed light on the dynamics of particles and electromagnetic fields in remote astrophysical environments. A deep and detailed understanding of the physics behind the most extreme astrophysical processes is of crucial importance for both the astrophysical systems, for which *in situ* measurements are beyond our reach, and a full interpretation of the astronomical observations. In addition, laboratory experiments offer the unique possibility to have access to astrophysical phenomena in a controlled and well-diagnosed environment. This is a fundamental step for the validation of the theoretical models as well as numerical simulation tools.

Over the last two decades, laboratory astrophysics has thus been the drive and focus of a wide range of studies ranging from warm dense matter for the investigation of the interiors of planets (Koenig et al., 2005) to radiative hydrodynamics for our understanding of the physics of supernovae (Remington et al., 1997) and radiative shocks (Drake, 2005).

Due to the extremely large differences in the spatial and temporal scales involved in the astrophysical processes and those obtained in the laboratory, these studies rely on similarity criteria to ensure that the conditions produced in the experiments are equivalent to those of the original astrophysical scenario. Hence, similarity criteria have been identified by Ryutov et al. (1999); Bouquet et al. (2010), and are based on a proper scaling of various dimensionless parameters (Mach number, Reynolds number, etc.). The equivalence of astrophysical and laboratory plasmas is even more easily achieved when considering collisionless, non-radiative processes (Ryutov et al., 2012). This is the case of, for instance, magnetic reconnection (Treumann and Baumjohann, 2013), and the so-called collisionless shocks, the latest being central to this work.

Collisionless shocks have been demonstrated to be ubiquitous in a wide-range of astrophysical environments that involve a tremendous energy release over short timescales (e.g. in active galaxy nuclei, pulsar wind nebulae, supernovae remnants, etc.). They are also held responsible for the production of non-thermal particles (Cosmic Rays) and high-energy radiation (e.g. Gamma Ray Bursts), (Kirk and Duffy, 1999).

At a macroscopic level, a shock appears as a discontinuity in the macroscopic plasma state developing in the presence of fast plasma flows. In hydrodynamic shock waves, this discontinuity is mediated by collisional effects that dissipate irreversibly the kinetic energy of the fluid crossing the shock. The fluid behind the discontinuity region is therefore compressed, isotropized and strongly heated. In the absence of particle collisions, plasma micro-instabilities can be responsible for the dissipation of the flow kinetic energy into thermal energy leading to shock formation. Several instabilities can develop, following the interaction of a fast plasma flow with the surrounding medium. Among these, the Weibel instability (Weibel, 1959; Fried, 1959) has been identified to play a key role in various phenomena (Medvedev and Loeb, 1999). In fact, it leads to the development of strong magnetic fluctuations that eventually lead to the flow isotropization and, at later times, to particle energization via Fermilike acceleration mechanism (Fermi, 1949, 1954). This mechanism is the foundation of the well accepted paradigm for Cosmic Rays (CRs) production: the so-called *Diffusive Shock Acceleration* (DSA). This model relies on the presence of strong magnetic turbulence, produced in an initially unmagnetized or weakly magnetized environment, that scatter the nonthermal particles and make them cross repeatedly the shock front, the particles gaining energy at each crossing, see Sec. 1.1.2.

In this Chapter we provide a concise overview of the recent investigations on Weibel-mediated collisionless shocks. At first we describe the astrophysical context (Sec. 1.1), presenting the relevant astrophysical scenarios in Sec. 1.1.1, a simplified model for the particle acceleration mechanism in Sec. 1.1.2 and describing the CRs spectrum in Sec. 1.1.3. Then we present the major results of recent experimental campaigns aiming at producing such collisionless shocks on high-energy laser facilities (Sec. 1.2). In Sec. 1.3, we finally provide a brief overview of the structure of this thesis.

1.1 Astrophysical context

1.1.1 Relevant astrophysical scenarios

As already mentioned, collisionless shocks are very common in various astrophysical environments. The origin of these structures is connected with some of the most powerful and energetic sources currently known: relativistic jets from Active Galactic Nuclei (AGN), Gamma Ray Bursts (GRB), SuperNova Remnants (SNRs), etc. The significant efforts pursued in understanding the physics of collisionless shocks is motivated by the identification of these structures as responsible for the acceleration of CRs and for the emission of high-energy radiation from the X- to γ -range.

The material emitted by SNRs generally propagates at non-relativistic velocity (up to 10% of the speed of light), while in the other abovementioned astrophysical contexts, the shocks is driven by relativistic and even ultra-relativistic flows. Despite this difference, a common description accurately captures the early phase of shock formation. Indeed, in all these systems, the interaction of the plasma jets with the surrounding medium leads to the development of electromagnetic instabilities. The latter can produce magnetic turbulence strong enough to isotropize the flows and eventually drive shock formation.

In these extreme scenarios, insight into the accelerated particle dynamics can be obtained via *indirect* astrophysical observations, relying on the spectrum of the radiation emitted by the high-energy particles, and by the measurements of the CRs spectrum on Earth (described in Sec. 1.1.3). We now briefly summarize some of the most typical observations that provide evidences of the formation of collisionless shocks and the subsequent particle acceleration.

Evidences of jet emission from AGN are shown in Fig. 1.1: on the left we show the galaxy Centaurus A and on the right the galaxy Cygnus A, one of the strongest radio sources in the sky. In both examples, a fast jet produced by the super-massive black hole at the galaxy core is observed in the radio wavelengths range. The radiation in the MHz-GHz frequency range has been associated with synchrotron emission from non-thermal relativistic electrons having MeV-GeV energies and propagating in a magnetic field of intensity much larger than the weak interstellar field $B_{\rm ISM} \simeq 2 \div 3 \,\mu \text{G}$. This suggests that the radiating electrons have been accelerated at the shock front and that the magnetic field has been locally amplified, most likely by plasma instabilities. The theoretical and numerical validation of this explanation has been given in Lemoine (2013), where a synchrotron-like spectrum is predicted for particles accelerated at the shock front, up to relativistic energies, and interacting with the



Figure 1.1: a) The galaxy Centaurus A at different wavelengths. Figure from: X-ray (NASA/CXC/M. Karovska et al.); Radio 21cm image (NRAO/VLA/Schminovich et al.), Radio continuum image (NRAO/VLA/J. Condon et al.); Optical (Digitized Sky Survey, U. K. Schmidt Image/STScI). b) The galaxy Cygnus A at different wavelengths. Figure from: X-ray: NASA/CXC/SAO; Optical: NASA/STScI; Radio: NSF/NRAO/AUI/VLA.

magnetic field micro-turbulence ahead of the shock front.

The observation of this kind of emission is not restricted to shocks produced by the AGN-jets. Most of the information available have been provided by SNRs and evidences of a synchrotron spectrum are exceptionally clear in Young and Type I SNRs. Figure 1.2 reports the X-ray emission in the range $\simeq 1 \div 6$ keV, from the Tycho SNR. The thermal component of the ejected material is visible mostly in the center of the image in green, and the non-thermal one, i.e. accelerated at the shock front, corresponds to the blue rims at the edge of the expanding shock. The observed X-ray emission requires electrons with energy $\simeq 20 \div 30$ TeV streaming in a magnetic field with amplitude $B \simeq 100 \,\mu$ G, i.e. much larger than the interstellar magnetic field $B_{\rm ISM}$. Other evidences (Reynolds et al., 2012) suggest that in young SNRs the magnetic field strength may approach \sim mG levels. Furthermore, the presence of heavy ions in the CRs accelerated at the edge of SNRs has been confirmed by the detected γ -rays emission as discussed by Abdo et al. (2009); Tavani et al. (2010). This



Figure 1.2: Tycho SNR emission as measured by the X-ray Chandra satellite, Ref. (Warren et al., 2005), the energy levels of the X-ray emission have been assigned to colors as: Red 0.95-1.26 keV, Green 1.63-2.26 keV, Blue 4.1-6.1 keV.

radiation is supposed to have an hadronic origin, being associated with the production of neutral pions in the interaction of the CR-ions with the surrounding molecular cloud, and the following decay $\pi^0 \rightarrow 2\gamma$.

A very different range of parameters can be investigated considering GRBs, that represent the largest known explosions in the universe after the Big Bang and whose emitted material reaches a Lorentz factor $\gamma_{\rm jet} \gtrsim 100$. Most of the observed GRBs consist of an initial very bright emission in the γ -range, with duration up to tens of seconds, and a successive afterglow emitted at longer wavelengths, that can last for hours after the main event. The origin of GRBs is uncertain, but an accredited description relies on the so-called *Fireball model* (Piran, 1999). This description envisions the presence of several internal shocks, produced by relativistic electron-positron shells emitted by a central compact engine, such as a supernova collapsing in a neutron star or even the binary collision between two neutron stars. Within this model, the detectable initial burst of γ -rays (Inverse Compton and synchrotron emission) is considered a consequence of the interaction between these internal shocks, arising from their different propagation velocities. The afterglow is considered to originate at the front of a shock propagating outward (usually called *external shock*), due to the interaction of the ejected material with either the interstellar medium or a progenitor wind. This interaction causes an emission with a longer duration and at frequency ranging from radio to soft X-rays. The registered afterglow spectrum has been successfully described as synchrotron emission by shock-accelerated electrons streaming

in a locally amplified magnetic field (Sari et al., 1998).

The level of magnetic field at the shock front, extracted from each of the above-mentioned indirect observations, indicates that the local magnetic field was substantially larger than the interstellar magnetic field, and this suggests that it has been amplified by the micro-instabilities produced by the streaming plasma. A first phase of amplification might be intrinsically connected with the shock front formation, in particular by the development of the Weibel instability. A successive stage of amplification might be due to secondary streaming instabilities, driven by the current of the shock-accelerated particles themselves, which propagate ahead of the shock front and thus interact with the weakly magnetized interstellar medium ($B_{\rm ISM} \sim \mu G$). For a review on CRs driven instabilities see Bykov et al. (2013). Two main types of instability have been identified: a resonant amplification of Alfvén waves with wavelength equal to the CRs Larmor radius (Skilling, 1975; Achterberg, 1983), and a nonresonant instability, commonly named Bell instability (Bell, 2004, 2005), that produces high amplitude waves with shorter wavelengths and circular polarization. The investigation on the magnetic field amplification is still an active area of research, since the level of magnetic turbulence has important implications in both particle acceleration and particle transport across the Universe.

Up to now we have presented observations related with the magnetic fields self-generated in the region corresponding to the shock front. However, large-scale, ordered, magnetic fields might already be embedded in the jet itself. For instance, Pulsar Wind Nebulas (PWNs), such as the one in the famous Crab Nebula, are intrinsically highly magnetized and the signature of an efficient particle acceleration is identified at the termination shock. In PWNs, the jet is composed mostly of a pair plasma, emitted with relativistic energy, up to Lorentz factor $\gamma_{\text{wind}} \simeq 10^6$. The typical magnetization, defined as the ratio of the Poynting flux to the kinetic energy flux in the region ahead of the shock front, is $\sigma \gtrsim 0.01$, i.e. several orders of magnitude larger than in the interstellar medium $\sigma_{\text{ISM}} \simeq 10^{-9}$. The magnetic field morphology in these outflows can be investigated by means of polarization measurements (Montier, L. et al.,



Figure 1.3: Simplified schematic representation of the characteristic shock formation mechanisms and dominant particle acceleration processes as a function of the flow magnetization and velocity. In green the typical astrophysical scenarios where these shocks take place.

2015). Several observations have demonstrated that in most of the situations the magnetic field is mainly perpendicular to the jet flows, while Pudritz et al. (2012) suggest that AGN jets propagate with a magnetic field parallel to the jet axis at the center of the expelled material and a surrounding helical field towards the jet edges.

A strong magnetization can play a determinant role in the formation and evolution of the jets and can also lead to different shock formation mechanisms. The inclusion of an ordered macroscopic magnetic field has been the topic of several numerical investigations, considering both electron-positron plasmas (Sironi and Spitkovsky, 2009; Sironi et al., 2013; Bret et al., 2017; Plotnikov et al., 2017) and electron-ion plasmas (Sironi and Spitkovsky, 2011a; Gargaté and Spitkovsky, 2012; Sironi et al., 2013; Caprioli and Spitkovsky, 2014a; Lemoine et al., 2016). A major result of the numerical studies is that highly magnetized relativistic shocks (as the termination shocks of PWNs) do not produce the level of magnetic turbulence required for Fermi-like acceleration. Thus alternative acceleration mechanisms or alternative ways to produce the magnetic turbulence (not intrinsic to the shock formation) are currently investigated (Lyubarsky, 2003; Camus et al., 2009; Sironi and Spitkovsky, 2011b).

In Fig. 1.3, we sketch a summary of the main mechanisms that lead to shock formation as a function of $\gamma_{\rm sh} v_{\rm sh}$, with $v_{\rm sh}$ the shock velocity and $\gamma_{\rm sh} = 1/\sqrt{1 - v_{\rm sh}^2/c^2}$ the shock Lorentz factor, and the magnetization σ . On the one hand, at low magnetizations (σ \lesssim 10^{-3} and σ \lesssim 3×10^{-5} for pair and electron-ion relativistic plasmas, respectively), the dominant role in shock formation is played by the Weibel instability, and efficient acceleration (up to 10% of the flow kinetic energy) via Fermi-like mechanism has been demonstrated by means of numerical simulations. On the other hand, in strongly magnetized plasmas ($\sigma \gtrsim 0.1$), the physics changes significantly. The shock is sustained by a magnetic barrier (often referred to as a *magnetic reflection* shock) that provides the local dissipation and flows isotropization mechanism through Maser Synchrotron Instability (Alsop and Arons, 1988; Hoshino and Arons, 1991). In this type of shocks, Fermi mechanism is suppressed and the acceleration relies on the so-called Shock Drift Acceleration (Begelman and Kirk, 1990; Sironi and Spitkovsky, 2009) in which particles gain energy from the motional electric field driven by the moving magnetic barrier and are found to drift along the shock front surface. An analogous acceleration mechanism (Shock Surfing Acceleration) has been observed in electrostatic shocks developing in magnetized electron-ion flows. In Fig. 1.3, we also report the dominant acceleration mechanism at play and the corresponding astrophysical systems. Note however that this simplified representation does not take into account many parameters, such as the macroscopic field orientation, that have been demonstrated to affect the acceleration mechanism by Sironi and Spitkovsky (2009); Caprioli and Spitkovsky (2014a).

Numerical simulations have been a central tool for obtaining information regarding the early stage of shock formation, which is not accessible by astrophysical measurements. Despite the impossibility to reproduce in fully kinetic simulations the large-scale astrophysical system in which jets are produced and propagate, they provide important constrains on the magnetic field amplification and on the acceleration (on the accelerated species, fraction of particles, dependence on the external field, etc.) from first principles. As such, they will also be a central tool for the present thesis.

1.1.2 Fermi acceleration mechanism

The most acknowledged model for CRs acceleration relies on the so-called Fermi mechanism (Fermi, 1954). In his original model, Fermi considered an already mildly relativistic particle, with 4-momentum $p_{\mu} = (E/c, \mathbf{p})$, interacting with a galactic cloud streaming toward the particle. Following the derivation presented by Vietri (2008), we assume the galactic cloud to have a mass $M \gg m$, where m is the particle mass, and velocity $\mathbf{V} = -V\hat{\mathbf{x}}$, forming an angle θ with the particle direction of motion. In the rest frame of the cloud, the collision can be considered elastic, so that the particle exits the cloud with unchanged energy and reflected momentum. Note that the particle-cloud collision does not involve real binary collisions with the cloud constituents, but rather collective effects. In particular, the elastic reflection can be the result of repeated deflection from magnetic turbulence moving with the cloud. In the cloud reference frame, henceforth identified by the index (c), the particle energy and momentum p_x read

$$E_1^{(c)} = \gamma^c \left(E + pV \cos \theta \right) ,$$
 (1.1.1)

$$p_{x,1}^{(c)} = \gamma^c \left(p \cos \theta + \frac{VE}{c^2} \right) , \qquad (1.1.2)$$

with $\gamma^c = (1 - V^2/c^2)^{-1/2}$ and $p = |\mathbf{p}|$. After the elastic collision with the cloud, we obtain $E_2^{(c)} = E_1^{(c)}$ and $p_{x,2}^{(c)} = -p_{x,1}^{(c)}$. For an observer in the external reference frame, i.e. where the cloud moves with velocity \mathbf{V} , the final particle energy and momentum are

$$E_2 = \gamma^c \left(E_2^{(c)} - V p_{x,2}^{(c)} \right), \qquad (1.1.3)$$

$$p_{x,2} = \gamma^c \left(p_{x,2}^{(c)} + \frac{VE_2^{(c)}}{c^2} \right) \,. \tag{1.1.4}$$

By expressing the energy E_2 as a function of the initial energy E and of the absolute value of the initial velocity v, we find

$$E_2 = (\gamma^c)^2 E\left(1 + \frac{2Vv\cos\theta}{c^2} + \frac{V^2}{c^2}\right).$$
 (1.1.5)

Equation (1.1.5) demonstrates the possibility to gain energy from the interaction with propagating magnetic turbulence. In particular, considering a typical cloud velocity $V \ll c$ and averaging over all possible values of θ , due to the random orientation of the particle-cloud relative velocities, the second term in brackets vanishes and the increase of the energy is $\propto (V/c)^2$. For this reason this process is commonly referred to as the second order Fermi mechanism. Note that the purely magnetic nature of the scattering process has been assumed in the cloud rest frame, where the particle does not gain energy. However, in the external frame, where electric fields are present due to the coupling of the electric and the magnetic fields in the Lorentz transformations, an energy gain is possible. In this way, part of the magnetized cloud energy is transformed into kinetic energy of the individual charged particle.

Generally, given the low velocity of the particle cloud, the role of second order Fermi acceleration is negligible. A more efficient mechanism, that may account for the extremely high energy observed in the CRs spectrum is the *first order Fermi mechanism*, first discussed by Bell (1978); Blandford and Eichler (1987). This model extends the second order Fermi mechanism to a system where a shock propagates through an unperturbed plasma. Analyzing this situation from the shock front rest frame (S), considering it as stationary, we observe an incoming flow crossing the shock front with velocity v_u^s and a downstream shocked plasma that moves away from the shock front with velocity v_d^s . In presence of a particle separated from its thermal background and moving from the upstream to the downstream with a marginally relativistic energy, the shocked plasma acts as the cloud in the previous model. Hence, once the particle has bounced back in the upstream region, its energy is increased as reported in Eq. (1.1.5), where $V = v_u^{s} - v_d^{s}$. The particle crosses the shock front and starts the acceleration process only if $\cos \theta > 0$, contrarily to the second order Fermi mechanism. Note that the system is completely symmetric, considering a particle moving from the downstream to the upstream, for which the condition on the particle propagation direction is $\cos\theta \leq 0$. In conclusion, in the presence of a propagating shock front the accelerated particle always see a converging flow, while in the cloud-particle system the cloud could also move away from the particle.

This constrain ensures that the average over all the acceptable angles in Eq. (1.1.5) does not vanish, so that the energy gain at each front crossing is $\propto V/c$. For this reason, this mechanism is named *first order Fermi* acceleration. More recent studies (Gallant and Achterberg, 1999) have demonstrated that for ultra-relativistic shocks, i.e. $\gamma^c \gg 1$ in Eq. (1.1.5), the energy gain predicted by Eq. (1.1.5), $\Delta E \simeq (\gamma^c)^2 E$, is achieved only during the first crossing of the shock front, while in the successive ones the gain is reduced to $\Delta E \simeq E$.

The estimate of the largest attainable energy is currently a subject of deep investigation. An obvious limit is given by the interval of time for which the assumption on the stationarity of the shock front is valid, corresponding to the maximum duration of the acceleration process. However, a more restrictive limitation relies on the possibility that the particle escapes from the discontinuity region, instead of crossing back the shock front. A simple model to estimate the probability that a particle leaves the acceleration region has been first given by Bell (1978). This model considers that, on the one hand, the diffusive motion due to magnetic field turbulence tends to bring the non-thermal particles back into the shock front, while, on the other hand, the advection with the background fluid in the downstream region tends to bring them away from the shock front. The probability that the particle crosses the shock front leads to the convergence of the accelerated particle spectrum to a power-law $f(\gamma) \propto \gamma^{-p}$, with $p \simeq 2.2$ in ultra-relativistic shocks, as analytically derived by Kirk et al. (2000); Keshet and Waxman (2005); Blasi (2013). This power-law index originates from the assumption of isotropic magnetic turbulence in both the upstream and the downstream of the shock. This simplified hypothesis does not consider the self-consistent development of the magnetic fluctuations during the phase of shock formation and propagation, that appears in Particle-In-Cell (PIC) simulations (Niemiec et al., 2006; Spitkovsky, 2008; Martins et al., 2009; Haugbølle et al., 2013; Sironi et al., 2013). Moreover, the isotropic turbulence assumption has been relaxed in Monte-Carlo simulations (Lemoine and Pelletier, 2003; Lemoine et al., 2006), demonstrating the production of a slightly different slope of the non-thermal distribution. Yet, numerical studies are in rather good agreement with the theoretical values, showing a slope of the non-thermal

spectrum ranging from 2.1 to 2.7.

One of the main open questions with respect to the models proposed in the literature is related to the so-called *injection problem*. This concerns the onset of the acceleration mechanism, i.e. the production of mildly relativistic particles with velocity large enough to start the acceleration cycle. The existence of these particles is the starting point of the models based on the Fermi acceleration mechanism. The most supported solution of this problem, at least for ions, relies on the so-called *thermal leakage*. This model assumes that high energy particles in the tail of the Maxwellian distribution produced in the downstream region close to the shock front, have a Larmor radius large enough to cross the shock and thus start the Fermi process (see Blasi, 2013). However, the characterization of the injection process is strictly connected with the physics at the shock front, that is not yet well understood, and mostly relies on fully kinetic simulations.

1.1.3 Cosmic Rays

The observation of CRs, by many experimental facilities, provides the spectrum shown in Fig. 1.4. CRs have been detected over approximately twelve decades of energy: from fractions of GeV up to 10^{11} GeV. The single-particle energy may exceed of several order of magnitude what is achievable using current accelerators, as pointed out in Fig. 1.4.

CRs are mainly composed of protons with a small fraction (< 10%) of helium and an even smaller amount of heavier elements. The spectrum is dominated by light ions up to ~ 10⁶ GeV, while, at larger energies, heavy nuclei seem to give a stronger contribution. Indeed, the clear steepening around ~ 5×10⁶ GeV [from $f(E) \propto E^{-2.7}$ to $f(E) \propto E^{-3.1}$], the so-called *knee*, could be produced by the superposition of the spectral cut-offs of the different elements. Precisely, this explanation relies on the assumption of a rigidity dependent acceleration mechanism (i.e. with maximum energy $E_{\rm max} \propto Z$ the ion charge), so that if a proton is accelerated up to 5×10⁶ GeV, a fully ionized iron nucleus ($Z_{\rm Fe} = 26$) might reach $\simeq 10^8$ GeV. This is supported by the analytical and numerical works of Caprioli et al. (2011, 2017) that demonstrate a heavier chemical composition above the knee. At even larger energies, a transition from galactic to extra-galactic



Figure 1.4: All particle CR flux measured by different experiments. The energies reached by Tevatron and LHC accelerators are reported for comparison. (Source: Telescope Array Project Commons)

CRs dominated-spectrum appears, even if this is still the topic of intensive investigation. Note that for energy ≤ 10 GeV the spectrum bends down, since the interaction with the magnetized solar wind prevents the low energy CRs to reach the Earth. See Refs. (Hörandel et al., 2006; Gaisser et al., 2013; Blasi, 2013) for a more detailed discussion.

Many studies currently aim at the interpretation of the CRs spectrum, for both the modelization of such an efficient acceleration mechanism and the description of CRs propagation through the Universe. Collisionless shocks formed in SNRs are believed to be the sources of most of the Galactic CRs. However, Fermi-like mechanisms seem not efficient enough to describe the spectrum of CRs above 10^8 GeV. Indeed, in the framework of Fermi-like acceleration, protons are expected to reach energies up to the knee level, and heavier elements up to $\simeq 10^8$ GeV. Therefore, a model accounting for non-linear secondary effects beyond the simple Fermi acceleration mechanism has been proposed to explain the observed particles with much larger energies. For instance, Blasi (2013) has taken into account the effect of the pressure of the accelerated particles on the shock compression and the amplification of the magnetic field turbulence by means of secondary instabilities driven by the accelerated particles themselves. Indeed, in the presence of stronger magnetic fields, a more efficient confinement of the particles at the shock front would allow for a longer acceleration stage. Even considering these secondary effects, the production of the so-called Ultra-High Energy CRs $(E > 10^9 \text{ GeV})$ can not be explained, even if AGN and GRBs are considered as probable sources, because of their exceptional luminosity (see Waxman, 1995; Vietri, 1995; Blasi et al., 2000; Caprioli, 2015).

When the accelerated CRs escape from the shock front region, they propagate, before being detected, for a traveling time that can be inferred from the measurement of the secondary-to-primary-nuclei ratio, usually Boron over Carbon nuclei (B/C) (Strong et al., 2007). This estimate is based on the classical assumption of diffusive particle motion. Nevertheless, recent investigations seem to confirm that this is accurate only far from the CRs source, where the diffusive coefficient depends only on the particle energy. Close to the CRs source, the description of CRs propagation have to take into account CRs-driven secondary electromagnetic instabilities (Amato and Blasi, 2017). Numerical characterizations of these secondary instabilities, presented by (Riquelme and Spitkovsky, 2009; Caprioli and Spitkovsky, 2014b), have been performed to investigate on the maximum level of additional magnetic field amplification. Furthermore, a system in which current-dominated flows propagate in a plasma at rest with an ambient magnetic field can be created in the laboratories with the state-of-the-art laser facilities. This experimental study is intended to bring a deeper insight into competing processes in the development of the instability and their role in the slow down and trapping of high-energy particles close to the sources.

1.2 Laser-plasma experiments

1.2.1 Short laser pulse technology

On Earth plasmas are seldom found in nature but can be produced under ad hoc experimental conditions. Among them the most important for this work is the interaction of gas or solid targets with intense lasers. Already at modest laser intensities, of the order of $10^{15} \div 10^{16} \,\mathrm{Wcm^{-2}}$, the laser electric field is sufficiently large to ionize an atom in a single laser period, and in ultra-high intensity laser-matter interaction even solid targets rapidly become plasmas.

The evolution of laser technologies had a drastic change of paradigm in the mid 1980s with the introduction of the *Chirped Pulse Amplification* (CPA). This is, up to now, at the basis of the most advanced technology for the production of ultra-intense pulses. Specifically it enables for the production of femtosecond-duration pulses with intensity above 10^{21} Wcm⁻². Classical technologies for the production of a high intensity laser pulse require a series of active media, i.e. several consecutive amplification stages, and are necessarily limited by the damage threshold of the gain medium. CPA efficiently overcomes this issue. Indeed, as represented in Fig. 1.5, the laser pulse is stretched out in time by means of two gratings, before entering in the gain medium. Due to the gratings, the pulse that undergoes amplification is *chirped*, i.e. the spectral components of the pulse are dispersed in time according to their frequency.



Figure 1.5: Schematic representation of the Chirped Pulse Amplification technology. Taken from LLNL S&TR.

Hence, the stretched pulse is amplified without producing any damage to the optical component, since it has an intensity much lower than the original one. Finally, the amplified laser pulse is compressed back to its original time duration (or close to it), exploiting two more gratings placed in a reverse configuration to the previous one.

Today, the record of intensity has been obtained at the HERCULES Petawatt Laser at the University of Michigan USA (Yanovsky et al., 2008), where a CPA-based Ti:Sapphire laser system reached 2×10^{22} Wcm⁻² for a pulse duration of $\simeq 10$ fs, for an energy of the order of kiloJoule. The CPA technology is currently used in many other facilities, such as the OMEGA-EP laser at the University of Rochester's Lab and the Gekko Petawatt laser at the Institute of Laser Engineering in Osaka. Other ongoing projects, such as the Vulcan Petawatt Upgrade at the Rutherford Appleton Laboratory's, the APOLLON in France, plan to reach the multi-Petawatt regime, with an advanced scheme (Optical Parametric Chirped Pulse Amplification) based on CPA method (Dubietis et al., 1992; Ross et al., 1997).

1.2.2 Towards collisionless shock experiments

Laboratory experiments are invaluable to gain insights on astrophysical phenomena (Remington et al., 1999, 2006; Drake and Gregori, 2012). They can allow to improve our understanding, in particular, of the early phase of shock formation for which no information are available from astrophysical observations. They are also fundamental for testing theoretical models and numerical results.

Pioneering experimental investigations on collisionless shocks have been carried out in the late '80s (Bell et al., 1988), making use of a plasma ablated from a dense target irradiated by an intense laser pulse. In these studies, the limited flow velocity of the laser-ablated plasma allowed the development of electrostatic shocks only. Lately (Kuramitsu et al., 2012; Ahmed et al., 2013; Morita et al., 2016, 2017), the formation and evolution of these electrostatic structures have been confirmed and investigated in more detail, thanks to both the development of laser technology and improved experimental diagnostics.

Yet, the production of Weibel-mediated collisionless shocks demands extremely high-power laser facilities, present or soon-available, and has to this day remained unconfirmed. The typical experimental set-up for the production of Weibel-mediated shocks consists of two interpenetrating flows, created by irradiating two solid targets. This configuration has been theoretically and numerically demonstrated to favor the development of the Weibel instability with a typical growth rate $\propto \omega_p$, the plasma frequency of the species involved in the instability (Bret et al., 2010b). The interaction region of the two counter-streaming plasmas, located in the middle of the two laser-irradiated solid foils, will eventually become Weibel-unstable and potentially drive the development of two collisionless shock fronts propagating away from each other with the shocked (downstream) plasma in between.

This configuration has been first investigated by Woolsey et al. (2001); Courtois et al. (2004) at the Vulcan laser facility, with the additional inclusion of an external magnetic field. The typical parameters obtained in these experiments should match the phase of SNR shock propagation at roughly 100 years after the supernova event, following the scaling laws proposed by Ryutov et al. (1999). However, these experimental results evidenced the necessity of larger laser facilities, in order to produce collisionless Weibel-mediated shocks driven by electron-ion flows. Indeed, shock formation requires the development of the ion-Weibel instability to efficiently stop the electron-ion flows in the interaction region. This instability develops on large time scales, due to both the large ion mass and the usually low density of the counter-propagating flows. The plasmas have thus to overlap over distances of several centimeters. The resulting laser energy that needs to be transmitted into these large-scale flows is thus large and available only on today's most energetic laser systems.

For this reason nowadays, most of the experimental results, aimed at probing the early phase of instability development, have been obtained at National Ignition Facility at the Lawrence Livermore National Laboratory and at OMEGA Laser Facility, using high-energy multi-kiloJoule nanosecond duration lasers operating at modest intensities $I \lesssim 10^{16} \,\mathrm{W/cm^2}$.

Magnetic field generation via the Weibel instability was demonstrated at the OMEGA laser facility (Fox et al., 2013; Huntington et al., 2015; Park et al., 2015), where interpenetrating flows with density $\simeq 5 \times 10^{18} \text{ cm}^{-3}$ and velocity $\simeq 0.3\%$ of the speed of light were created with the configuration shown in Fig. 1.6. The interaction region is probed with proton radiography, demonstrating the filamentary structure typical of the early stage of the Weibel instability. Experiments currently carried out at NIF allow for higher flow density, up to 10^{20} cm^{-3} , and velocities, potentially up to 1% of the speed of light. These higher density and flow velocity are expected to speed up the development of the Weibel instability, but the possibility to preserve the collisionless regime up to shock formation is still under investigation (Park et al., 2012; Ross et al., 2017).

Ultra-intense laser systems with $I \gtrsim 10^{18} \,\mathrm{W/cm^2}$ could allow to alleviate the limitations of the NIF laser class (multi-kJ and nanosecond duration) on the accessible density and flow velocity. They might provide us with a complementary path towards the creation of collisionless, ultra-fast and high density plasma flows, as will be discussed in Chap. 4.

A configuration analogous to the one investigated at the Vulcan laser facility by Woolsey et al. (2001); Courtois et al. (2004), has been further



Figure 1.6: Experimental configuration used at OMEGA laser facility to drive two counter-streaming plasmas (Huntington et al., 2015) by irradiating two plastic targets with kJ-ns laser pulses. The interaction region is probed with proton radiography.

studied in ultra-intense (above 10^{19} W/cm²) laser system (Titan Laser Facility at the Lawrence Livermore National Laboratory) by Higginson et al. (2015). In these works, the two counter-propagating fast plasma flows are produced at the rear side of the target irradiated by an ultra-intense laser, by means of the Target Normal Sheath Acceleration (TNSA) (Wilks et al., 2001). However, in this configuration and with the use of such an intense laser beam, the density of the expanding plasma is $\leq 10^{17}$ cm⁻³, which yields a very long instability growth time. This complicates reaching the late stage of the instability development and in particular the shock formation.

All the above mentioned experimental configurations aimed at probing the compenetration of non-relativistic or mildly relativistic electron-ion flows. The typical parameters of these experiments might well match with SNRs, by making use of the hydrodynamic similarity proposed by Ryutov et al. (1999). For instance, the typical scale of the experimental campaign performed at the OMEGA laser facility, i.e. flow velocity $v_{\text{LAB}} \simeq 3 \times 10^{-3} c$ and timescale $t_{\text{LAB}} \simeq 1$ ns, might correspond to the emitted material of a SNR, streaming at $v_{\text{SNR}} \simeq 10^4$ km/s for approximately 1 years. A region of extension 100 μ m in the laboratory would thus correspond to 0.3 light-years in the astrophysical system. This scaling has been done in order to match the SuperNova 1987A at roughly 13 years from its explosion (Ryutov et al., 1999).

In addition to these preliminary experimental results for electron-ion flows, the production in the laboratory of pair plasmas collisionless shocks is currently under investigation. Indeed, the generation of neutral and high-density electron-positron plasmas has been recently demonstrated by Sarri et al. (2015). Numerical studies (Lobet et al., 2015; Chen et al., 2015; Zhu et al., 2016) have then confirmed that soon-available laser facilities might be able to test pair plasma collisionless astrophysics.

1.3 Outline of the thesis

This thesis aims at describing the processes related to the development of the Weibel instability and of Weibel-mediated collisionless shocks in the context of laboratory astrophysics. We will therefore present analytical models supported by numerical studies for the processes involved in the instability and shocks development. Fully relativistic kinetic simulations are presented exploring the feasibility of collisionless shocks in the laboratory. Finally the investigation of astrophysical relevant phenomena will be discussed. The manuscript is organized as follows:

- Chapter 2 introduces the basic processes as well as summarizes the analytic and numerical tools exploited throughout this thesis. Starting from the Vlasov-Maxwell description, we will review the derivation of relativistic fluid equations, used to describe the linear theory of the Weibel instability, along with a discussion on the accuracy of this approach. We give a general description of collisionless shocks, computing the Rankine-Hugoniot conditions to define the shock front. We also provide a brief description of the Particle-In-Cell (PIC) numerical method that has been central to this project.
- Chapter 3 presents our study on the Weibel instability driven by two symmetric counter-streaming electron plasmas. We consider relativistic flows in a uniform flow-aligned external magnetic field, under both cold plasma assumption and considering thermal effects. Both the linear and non linear stages of the instability are investigated using analytical modeling and PIC simulations. The different

mechanisms responsible for the saturation are discussed in detail in the relativistic cold fluid framework considering a single unstable mode. Multi-mode and temperature effects are also investigated. Differences arising due to the dimensionality of the simulations are highlighted.

- Chapter 4 presents the study of a suitable configuration for laboratory investigations of the ion-Weibel instability driven by a fast quasi-neutral plasma flow launched into the target via the radiation pressure of an ultra-high-intensity laser pulse ("Hole-Boring" process). After an introduction on the necessary background notions of laser-plasma interaction, we identify the optimal configuration for driving the ion Weibel instability and we demonstrate its development in a fully 3D simulation. We discuss the origin and the effect on the Hole-Boring-produced flow of an additional electron instability developing at the laser-plasma interaction surface, providing an analytical model to describe the processes correlated with its mitigation. Conditions for the late time evolution of the ion-Weibelunstable system into a collisionless shock are also investigated.
- Chapter 5 faces the numerical challenges of PIC simulation, when ultra-relativistic flows are studied. We report on the techniques that we have implemented in the PIC code SMILEI during this thesis with the goal of correctly describing the dynamics of ultra-relativistic pair plasmas. We illustrate the main numerical effects yielding unphysical results in simulations of relativistic flows and compare the efficiency of the implemented method in reducing these numerical artifacts.
- Chapter 6 investigates collisionless shocks in ultra-relativistic pair plasmas. In particular, we focus on describing the mechanisms and timing for shock formation. A proper identification of the shock formation time is of major interest for future experiments. We consider counter-propagating flows both unmagnetized and in the presence of a perpendicular magnetic field, being the most interesting field configuration in the ultra-relativistic case.
- Chapter 7 presents our conclusions and perspectives for future work.

CHAPTER 2

Analytical and numerical methods

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In this chapter we present the basic theoretical tools that will be used throughout this manuscript to describe relativistic plasmas and ultraintense laser-plasma interaction.

The most complete description of the plasma dynamics relies on a microscopic kinetic approach, presented in Sec. 2.1. The theoretical models proposed in this thesis are however based on a relativistic fluid approach, i.e. on a macroscopic quantities description, and systematically compared with the kinetic approach. The fundamental equations of the macroscopic approach are derived, starting from the kinetic description, in Sec. 2.3. We discuss in Sec. 2.2 the distribution functions that represent the equilibrium state of a relativistic plasma, then in Sec. 2.4 the linear theory adequate to characterize linear waves in a plasma, as well as the growing phase of streaming plasma instabilities. Furthermore, we describe (Sec. 2.5) the main features of collisionless shocks and we derive the basic relations for the identification of shock formation, the so-called *Rankine-Hugoniot conditions*, that relate the macroscopic state of the shocked plasma with the unperturbed one.

Details on the numerical methods used in this work are given in Sec. 2.6. Since this thesis is mainly focused on the physics of relativistic plasma flows, the limitations of the classical numerical methods will be discussed in more detail in Chapter 5, along with a description of the techniques that have been implemented and allow for an accurate treatment of relativistic particle dynamics for the cases of interest here.

2.1 Kinetic description

2.1.1 From Klimontovich to Vlasov equation

The most complete kinetic approach relies on the knowledge of the position and the momenta of each particle composing the plasma. This description requires to study the evolution in time of the distribution function of N particles in total, defined as

$$f_{\mathrm{N}}(t, \mathbf{x}_{1}, \mathbf{p}_{1}, \dots, \mathbf{x}_{\mathrm{N}}, \mathbf{p}_{\mathrm{N}}) = \sum_{i=1\dots\mathrm{N}} \delta\left[\mathbf{x} - \mathbf{x}_{i}(t)\right] \delta\left[\mathbf{p} - \mathbf{p}_{i}(t)\right], \qquad (2.1.1)$$

in the approximation of point-like particles with position $\mathbf{x}_i(t)$ and momentum $\mathbf{p}_i(t)$, and δ the Dirac function. This is the first step toward the derivation of the so-called *Klimontovich-Dupree equation* that describes the evolution in time of $f_N(t, \mathbf{x}_1, \mathbf{p}_1, \ldots, \mathbf{x}_N, \mathbf{p}_N)$, see Krall and Trivelpiece (1986). Despite the Klimontovich-Dupree equation representing the exact microscopic description of the plasma, it is technically too demanding to solve the equations of motion for $\mathbf{x}_i(t)$ and $\mathbf{p}_i(t)$, for the large number of particle composing any physical system, for the actual computational resources. Therefore the kinetic description that we will exploit in the following, relies on the knowledge of the ensemble averaged particle distribution function $f(t, \mathbf{x}, \mathbf{p})$. This distribution function represents, at a given time, the particle number density in a volume of the six-dimensional phase-space d**x** d**p**. With this definition, at a fixed time, the distribution function is normalized as

$$N = \int_{V} f(\mathbf{x}, \mathbf{p}) \, \mathrm{d}\mathbf{x} \, \mathrm{d}\mathbf{p} \,, \qquad (2.1.2)$$

where N is the total number of particles in the volume V of the 6Dphase space. Assuming the volume V to move with the particle flow, the conservation of the number of particles gives

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \int_{V} \frac{\partial f}{\partial t} \,\mathrm{d}\mathbf{x} \,\mathrm{d}\mathbf{p} + \oint_{\partial V} f \,\mathbf{U} \cdot \mathrm{d}\mathbf{S} = 0\,, \qquad (2.1.3)$$

where the second integral has to be computed on the surface ∂V that underlies the volume V, and $\mathbf{U} = \{\frac{d\mathbf{x}}{dt}, \frac{d\mathbf{p}}{dt}\} = \{\dot{\mathbf{x}}, \dot{\mathbf{p}}\}$, where $\mathbf{x}(t)$ and $\mathbf{p}(t)$ are the single particle position and momentum, so that \mathbf{U} corresponds to the velocity in the 6D-space. Using Gauss's theorem to simplify the second integral, we obtain

$$\int_{V} \left(\frac{\partial f}{\partial t} + \nabla \cdot (f\mathbf{U}) \right) \, \mathrm{d}\mathbf{x} \, \mathrm{d}\mathbf{p} = 0 \,, \qquad (2.1.4)$$

where $\nabla = \{\partial_{\mathbf{x}}, \partial_{\mathbf{p}}\}$. Since relation (2.1.4) does not depend on the chosen volume, the quantity inside the integral has to vanish. This leads to

$$\frac{\partial f}{\partial t} + \nabla \cdot (f\mathbf{U}) = \frac{\partial f}{\partial t} + \dot{\mathbf{x}} \cdot \partial_{\mathbf{x}} f + \dot{\mathbf{p}} \cdot \partial_{\mathbf{p}} f = 0, \qquad (2.1.5)$$

assuming that $\dot{\mathbf{x}}$ and $\dot{\mathbf{p}}$ are independent from \mathbf{x} and \mathbf{p} , respectively. This is accurate in a classical (non-quantum) plasma, for which the force exerted on a particle does not depend on its momentum. For an ultra-relativistic particle, emitting electromagnetic radiation, we should consider the backreaction force on the particle due to the electromagnetic emission, the so-called Radiation Reaction force.

In a classical (non-quantum) plasma, equation (2.1.5) can be simplified considering that $\dot{\mathbf{p}}$ is equal to the Lorentz force

$$\mathbf{F} = q \left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) \,, \tag{2.1.6}$$

where the fields might be externally imposed or self-generated by the average current and density of the plasma itself. Note that CGS units are used throughout this manuscript. Equation (2.1.5) can then be rewritten in the form of the Vlasov equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \partial_{\mathbf{x}} f + q \left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) \cdot \partial_{\mathbf{p}} f = 0.$$
 (2.1.7)

Note that this description of the plasma assumes that the dynamics is dominated by collective effects rather than by local interactions between neighboring particles. Additional effects due to particle-particle interaction can be accounted for in an *ad hoc* term appearing on the right side of the Vlasov equation $(\partial_t f)_{coll}$. The latter can be interpreted as a collisional effect that produces a random scattering of the particles. Here we assume that its contribution to the particle dynamics is negligible in comparison with the coherent motion in the mean fields. This is accurate if the typical scale-lengths of the physical mechanisms of interest are much smaller than the collisional mean-free-path. From a temporal point of view, this means that all the characteristic frequencies of the system are larger than the collision frequency $\nu_{\rm coll}$. The latter is proportional to the cross section of Coulomb scattering σ_{coll} . In particular, $\nu_{\text{coll}} = n\sigma_{\text{coll}}v$, where n is the plasma density, v is the typical particles velocity and $\sigma_{\rm coll} \propto E^{-2}$ with E the particle kinetic energy, so that $\nu_{\rm coll} \propto E^{-3/2}$. The collisional frequency decreases with increasing particle energy.

Therefore, the *collisionless* assumption is accurate for most of the

ultra-intense laser produced plasmas, since the strong electromagnetic fields of the laser give such a high energy to the particles that the rate of collisions is drastically quenched. The collisionless regime is even more easily achieved in the astrophysical situations considered for this work, in which high-velocity plasma flows interact with the very diluted plasma of the interstellar medium.

2.1.2 The Vlasov-Maxwell model

In order to study the evolution of the distribution function, the Vlasov equation (2.1.7) has to be coupled with Maxwell's equations for the description of the electromagnetic fields

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \qquad (2.1.8)$$

$$\nabla \cdot \mathbf{B} = 0, \qquad (2.1.9)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \qquad (2.1.10)$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \,. \tag{2.1.11}$$

The source terms, charge and current densities, are given by

$$\rho(t, \mathbf{x}) = \sum_{s} q_s n_s(t, \mathbf{x}), \qquad (2.1.12)$$

$$\mathbf{J}(t,\mathbf{x}) = \sum_{s} q_{s} n_{s}(t,\mathbf{x}) \mathbf{V}_{s}(t,\mathbf{x}), \qquad (2.1.13)$$

where the sum runs over all the plasma species. The total density n_s and the average velocity \mathbf{V}_s are obtained averaging over the distribution function, i.e. by integrating over the momentum space as

$$n_s = \int f_s(t, \mathbf{x}, \mathbf{p}) \,\mathrm{d}\mathbf{p} \,, \qquad (2.1.14)$$

$$\mathbf{V}_s = \frac{1}{n_s} \int \mathbf{v} f_s(t, \mathbf{x}, \mathbf{p}) \,\mathrm{d}\mathbf{p} \,. \tag{2.1.15}$$

The resulting coupled nonlinear system of equations, Eqs. (2.1.7)-(2.1.11), constitutes the so-called *Vlasov-Maxwell system*.

2.2 Equilibrium distribution functions

According to the *principle of maximum entropy*, any system tends to evolve in time towards the configuration that maximizes the specific entropy

$$S = K_{\rm B} \int f(\mathbf{x}, \mathbf{p}) \, \ln(f(\mathbf{x}, \mathbf{p})) \, \mathrm{d}\mathbf{p} \,, \qquad (2.2.1)$$

where $K_{\rm B}$ is the Boltzmann constant.

From Eq. (2.2.1), we can derive the distribution function that represents the equilibrium state of our system. The maximization has to be done consistently with some known constraints expressed in terms of average values of one or more physical quantities. In the following the physical constraints are given by the number of particles, as in Eq. (2.1.2), the average momenta and the average energy. To perform the calculations, each of these quantities should be associated to a Lagrange multiplier, in the following α , β , η . The maximum of the entropy is obtained imposing $\delta S = 0$ as

$$\delta \int \left[\ln(f(\mathbf{x}, \mathbf{p})) + \alpha + \boldsymbol{\beta} \cdot \mathbf{p} + \eta \sqrt{1 + \frac{\mathbf{p}^2}{m^2 c^2}} \right] f(\mathbf{x}, \mathbf{p}) \, \mathrm{d}\mathbf{p} = 0. \quad (2.2.2)$$

Imposing that this relation is verified for every possible choice of $\delta f(\mathbf{x}, \mathbf{p})$ leads to a distribution function of the form

$$f(\mathbf{x}, \mathbf{p}) \propto e^{\left(-\eta \sqrt{1 + \frac{\mathbf{p}^2}{m^2 c^2}} - \beta \cdot \mathbf{p}\right)}.$$
 (2.2.3)

The values of η and β can be obtained from the knowledge of the average momenta and energy, as well as the normalization factor can be derived from Eq. (2.1.2).

In the following we provide the explicit forms of the non-relativistic and relativistic distribution functions used in this thesis.

Non-relativistic distribution function

The plasma equilibrium for a species with single particle mass m_s and temperature T_s , in the non-drifting and non-relativistic case, i.e. with

temperature $T_s \ll m_s c^2$, can be calculated from Eq. (2.2.3) taking the limit $v \ll c$. In this regime, the equilibrium is described by the Maxwell-Boltzmann distribution function, defined as

$$f_s(\mathbf{v}) = n_0 \left(\frac{m_s}{2\pi T_s}\right)^{D/2} e^{-\frac{1}{2}\frac{m_s \mathbf{v}^2}{T_s}}, \qquad (2.2.4)$$

where the temperature is in units of $m_s c^2$ and D is the dimensionality of the momentum phase-space. We have assumed a uniform distribution in space, corresponding to the density n_0 . Therefore we do not specify any dependence on \mathbf{x} of the distribution function.

Equation (2.2.4) describes a plasma with no drift velocity or in its own rest frame. The straightforward generalization to the case with drift velocity \mathbf{V}_0 reads

$$f_s(\mathbf{v}) = n_0 \left(\frac{m_s}{2\pi T_s}\right)^{D/2} e^{-\frac{1}{2}\frac{m_s(\mathbf{v} - \mathbf{V}_0)^2}{T_s}}.$$
 (2.2.5)

Maxwell-Jüttner distribution function

In this section we consider a plasma at equilibrium with an arbitrary large average kinetic energy, i.e. exceeding the rest mass energy of the particles. This model concerns thermal plasmas with temperature $T_s \gtrsim m_s c^2$ and plasmas drifting with relativistic velocity. In both cases, the Maxwell-Boltzmann distribution functions, Eqs. (2.2.4) and (2.2.5), no longer give an appropriate description, since they predict particles with velocity exceeding the speed of light.

The relativistically accurate distribution function can be calculated starting from Eq. (2.2.3). As demonstrated by Juttner (1911); Wright and Hadley (1975), the resulting equilibrium distribution takes the form of the so-called *Maxwell-Jüttner function*, that in the rest frame of the plasma, i.e. where the plasma has vanishing mean drift velocity, reads

$$f_s(\mathbf{p}) = n_0 \frac{\mu_s}{4\pi K_2(\mu_s)} e^{-\mu_s \sqrt{1 + \frac{\mathbf{p}^2}{m_s^2 c^2}}}, \qquad (2.2.6)$$

¹The normalization factor in 2D is $n_0 2\pi e^{-\mu_s} \mu_s / (\mu_s + 1)$ and in 1D $n_0 2K_1(\mu_s) / \mu_s^2$.

where the normalization factor has been computed for a 3D momentum phase-space¹, $\mu_s = m_s c^2/T_s$ with T_s the plasma temperature, K_n is the modified Bessel function of the second kind of order n and n_0 is the plasma density.

Considering a plasma moving with drift velocity \mathbf{V}_0 , the distribution function (2.2.6) can be generalized to

$$f_{s}(\mathbf{p}) = n_{d} \frac{\mu_{s}}{4\pi K_{2}(\mu_{s})\Gamma_{0}} e^{-\mu_{s}\Gamma_{0} \left[\sqrt{1 + \frac{\mathbf{p}^{2}}{m_{s}^{2}c^{2}}} - \frac{\mathbf{v}_{0}\cdot\mathbf{p}}{m_{s}c^{2}}\right]},$$
(2.2.7)

where $\Gamma_0 = \left[1 - (\mathbf{V}_0/c)^2\right]^{-1/2}$ and n_d is the density in the frame where the plasma is streaming with velocity \mathbf{V}_0 . The relation between n_d and n_0 , the density of the plasma in its proper frame, is simply $n_d = n_0 \Gamma_0$.

In some cases (see e.g. Chap. 6), it can be useful to express the Maxwell-Jüttner distribution (2.2.6) as function of $\gamma = \left[1 + \mathbf{p}^2/(m_s^2 c^2)\right]^{1/2}$. This simply requires a change of variable that depends on the dimensionality of the system, since $\mathbf{p}^2 = p_x^2 + p_y^2$ in the 2D case and $\mathbf{p}^2 = p_x^2 + p_y^2 + p_z^2$ in 3D. This change of coordinates in Eq. (2.2.6) leads to

$$f_s^{2D}(\gamma) = n_d \frac{\mu_s^2}{\mu_s + 1} \gamma e^{-\mu_s(\gamma - 1)} , \qquad (2.2.8)$$

$$f_s^{3D}(\gamma) = n_d \frac{\mu_s}{K_2(\mu_s)} \gamma \sqrt{\gamma^2 - 1} e^{-\mu_s \gamma} . \qquad (2.2.9)$$

Note that the dimensionality of the system is defined by the effective degree of freedom over which the distribution function is isotropized in the momentum phase-space.

Average quantities

In this paragraph we describe the method used to calculate the average value of a generic microscopic quantity over a drifting distribution function. This is necessary to express the relativistic fluid equations that will be derived in the next section.

The average of a microscopic quantity $g^{\rm RF}(\mathbf{p}_0)$ in the plasma rest

frame (RF) is defined as

$$\langle g^{\rm RF} \rangle_{\rm RF} = \frac{1}{n_0} \int d\mathbf{p}_0 \, g^{\rm RF}(\mathbf{p}_0) \, f_0(\mathbf{p}_0) \,, \qquad (2.2.10)$$

where \mathbf{p}_0 is the momentum in the plasma rest frame and the distribution function $f_0(\mathbf{p}_0)$ corresponds to the non-drifting Maxwell-Jüttner, Eq. (2.2.6), for which the index of species *s* has been dropped for simplicity. Following Melzani et al. (2013), we can relate $\langle g^{\rm DF} \rangle_{\rm DF}$, the average in the frame where the plasma drifts with velocity \mathbf{V}_0 (DF), with its average value in the plasma rest frame. Expressing the single particle momenta in the drifting frame \mathbf{p}_d in terms of \mathbf{p}_0 , by means of the Lorentz transformations, and exploiting the relation $d\mathbf{p}_d/\gamma_d = d\mathbf{p}_0/\gamma_0$, where $\gamma_{0,d} = [1 + |\mathbf{p}_{0,d}|^2/(mc)^2]^{1/2}$, we obtain

$$\langle g^{\rm DF} \rangle_{\rm DF} = \frac{1}{n_d} \int \mathrm{d}\mathbf{p}_d \, g^{\rm DF}(\mathbf{p}_d) \, f_d(\mathbf{p}_d) = \frac{1}{n_0 \Gamma_0} \int \mathrm{d}\mathbf{p}_0 \frac{\gamma_d}{\gamma_0} \, g^{\rm DF}(\mathbf{p}_0) \, f_0(\mathbf{p}_0) \,.$$
(2.2.11)

This leads to the extremely useful relation

$$\langle g^{\rm DF} \rangle_{\rm DF} = \frac{1}{\Gamma_0} \left\langle \frac{\gamma_d}{\gamma_0} g^{\rm DF}(\mathbf{p}_0) \right\rangle_{\rm RF},$$
 (2.2.12)

which enables us to calculate all average quantities from their average in the plasma rest frame, by performing the integral over the distribution function Eq. (2.2.6), which turns out to be analytically more tractable than using Eq. (2.2.7).

As an example, we exploit this relation to obtain the expression of the average momentum of the drifting Maxwell-Jüttner, denoting by xthe direction of the drift

$$P_{x,d} = \langle p_{x,d} \rangle_{\rm DF} = \frac{1}{\Gamma_0} \Big\langle \frac{\Gamma_0^2 \left[(p_{x,0} + V_0 \gamma_0 m) \left(\gamma_0 + V_0 p_{x,0} / (mc^2) \right) \right]}{\gamma_0} \Big\rangle_{\rm RF},$$
(2.2.13)

where we used the Lorentz transformations for $p_{x,d}$ and γ_d , and $V_0 = \langle v_{x,d} \rangle_{\text{DF}}$. Since in the plasma rest frame $\langle p_{x,0} \rangle_{\text{RF}} = 0$, we obtain

$$\mathbf{P}_d = h_0(\mu) m \Gamma_0 \mathbf{V}_0 \,, \tag{2.2.14}$$
where $h_0(\mu)$ is the normalized enthalpy in the rest frame of the plasma defined as

$$h_0(\mu) = \frac{n_0 \langle \gamma_0 \rangle_{\rm RF} mc^2 + \mathcal{P}}{n_0 mc^2} = \langle \gamma_0 \rangle_{\rm RF} + \frac{1}{\mu}, \qquad (2.2.15)$$

where we have assumed the thermal pressure tensor component

$$\underline{\mathbf{\Pi}}_{xx} = n_0 \langle v_{x,0} p_{x,0} \rangle_{\rm RF} \tag{2.2.16}$$

to reduce to the scalar $\mathcal{P} = n_0 T$, since the Maxwell-Jüttner distribution function is isotropic. Calculating analytically the expression for the average value of γ_0 over a 3D Maxwell-Jüttner, we obtain $h_0(\mu) = K_3(\mu)/K_2(\mu) = k_{32}(\mu)$. Note that, from the definition of the average momentum Eq. (2.2.14), the drift relativistic factor $\Gamma_0 \neq \sqrt{1 + \mathbf{P}_d^2/(mc)^2}$.

Assuming the system to behave as an ideal gas $(\mathcal{P} = n_0 T_0)$ with adiabatic equation of state, the relation between pressure and internal energy of the system e_0 can be written as

$$\mathcal{P} = (\Gamma_{\rm ad} - 1)e_0 = (\Gamma_{\rm ad} - 1)(\langle \gamma_0 \rangle_0 - 1)n_0 mc^2, \qquad (2.2.17)$$

where e_0 corresponds to the kinetic energy, since the perfect gas assumption involves non-interacting particles, and $\Gamma_{\rm ad}$ is the adiabatic index. Note that in the non-relativistic limit ($T \ll mc^2$) the enthalpy takes the value $h(\mu) \simeq 1 + 5T/(2mc^2)$, corresponding to an adiabatic index $\Gamma_{\rm ad} = 5/3$, while in the ultra-relativistic limit $T \gg mc^2$, the enthalpy tends to $4T/(mc^2)$ and $\Gamma_{\rm ad} = 4/3$.

Note that the pressure is a Lorentz invariant $\mathcal{P}_0 = \mathcal{P}_d$, therefore in the drifting frame the apparent temperature becomes $T_d = T_0/\Gamma_0$. The other useful average quantities are listed in Table 2.2.1.

2.3 Relativistic hydrodynamic equations

The hydrodynamic approach is based on a description of the plasma in terms of macroscopic quantities, such as density, temperature, mean velocity, etc. Therefore, this description is accurate for distribution function close to the equilibrium condition over which the average quantities are computed.

Quantities	$\langle \cdot angle_{ m RF}$	$\langle \cdot angle_{ m DF}$
Enthalpy h	$k_{32}(\mu)$	$\Gamma_0 k_{32}(\mu)$
Energy γ	$k_{32}(\mu) - \mu^{-1}$	$\Gamma_0 k_{32}(\mu) - (\Gamma_0 \mu)^{-1}$
Velocity	0	\mathbf{V}_0
Momentum	0	$k_{32}(\mu)m\Gamma_0\mathbf{V}_0$
$\Gamma_{ m ad}$	$1 + (\mu k_{32}(\mu) - \mu - 1)^{-1}$	$1 + \left(\mu \Gamma_0^2 k_{32}(\mu) - \mu \Gamma_0 - 1\right)^{-1}$

Table 2.2.1: Average quantities on a 3D system in the rest frame of the plasma $(\langle \cdot \rangle_{\rm RF})$ and in a frame drifting with velocity \mathbf{V}_0 $(\langle \cdot \rangle_{\rm DF})$. Here $k_{32} = K_3(\mu)/K_2(\mu)$, with K_n the modified Bessel function of the second kind of order n.

We can derive the relativistic hydrodynamic equations starting from the Vlasov equation, Eq. (2.1.7), and computing the successive moments, as detailed in what follows. For a derivation in terms of 4-vector follows Groot (1980).

Conservation of the number of particles

The zero order moment is obtained by integrating the Vlasov equation, Eq. (2.1.7), over all momentum components as

$$\partial_t \int d\mathbf{p} f(\mathbf{x}, \mathbf{p}, t) + \partial_\mathbf{x} \int d\mathbf{p} \, \mathbf{v} f(\mathbf{x}, \mathbf{p}, t) + \int d\mathbf{p} \, \left(\mathbf{F} \cdot \partial_\mathbf{p}\right) f(\mathbf{x}, \mathbf{p}, t) = 0,$$
(2.3.1)

being **x** and **v** independent variables, and **F** the Lorentz force, Eq. (2.1.6). The first integral is simply the definition of the density, Eq. (2.1.12), while, in the second one, we recognize the average velocity, Eq. (2.1.13). The last term can be integrated by parts and gives no contribution as $\partial_{\mathbf{p}} \mathbf{F} = 0$. In conclusion, from Eq. (2.3.1) we obtain

$$\partial_t n + \partial_{\mathbf{x}} \left(n \mathbf{V} \right) = 0, \qquad (2.3.2)$$

that corresponds to the conservation of the particle number.

Conservation of momentum

The conservation of momentum follows from the calculation of the first moment of the Vlasov equation, Eq. (2.1.7). Thus, we multiply Eq. (2.1.7) for **p** and integrate over d**p** as

$$\partial_t \int d\mathbf{p} \, \mathbf{p} f(\mathbf{x}, \mathbf{p}, t) + \partial_{\mathbf{x}} \int d\mathbf{p} \, \underline{\mathbf{pv}} f(\mathbf{x}, \mathbf{p}, t) + \int d\mathbf{p} \, \mathbf{p} \left(\mathbf{F} \cdot \partial_{\mathbf{p}} \right) f(\mathbf{x}, \mathbf{p}, t) = 0$$
(2.3.3)

where the underlined notation represents the tensor $\underline{\mathbf{pv}}_{ij} = p_i v_j$ and the derivative of a tensor is defined as

$$\left(\partial_{\mathbf{x}}\underline{\mathbf{pv}}\right)_{i} = \sum_{j} \partial_{j}\underline{\mathbf{pv}}_{ij} \,. \tag{2.3.4}$$

The first integral in Eq. (2.3.3) corresponds to the average momentum **P**, as defined in Eq. (2.2.14). The second represents the total pressure tensor and can be decomposed as $n\mathbf{PV} + \mathbf{\Pi}$, the sum of the kinetic pressure tensor, concerning the overall drift motion of the species, and the thermal pressure tensor, following from the random motion of the particles around their average drift velocity. The third integral of Eq. (2.3.3) can be performed by parts and leads to

$$\partial_t (n\mathbf{P}) + \partial_{\mathbf{x}} (n\mathbf{\underline{PV}} + \mathbf{\underline{\Pi}}) = nq \left(\mathbf{E} + \frac{\mathbf{V}}{c} \times \mathbf{B}\right).$$
 (2.3.5)

Using the conservation of the number of particles, Eq. (2.3.2), the previous equation becomes

$$\left(\partial_t + \mathbf{V} \cdot \partial_{\mathbf{x}}\right) \mathbf{P} = -\frac{\partial_{\mathbf{x}} \underline{\Pi}}{n} + q \left(\mathbf{E} + \frac{\mathbf{V}}{c} \times \mathbf{B}\right) \,. \tag{2.3.6}$$

As previously mentioned, the relation between the average momentum and the average velocity is not straightforward, see Eq. (2.2.14). The latter equation allows us to write Eq. (2.3.6) as

$$h_0(\mu)m\Gamma_0\left(\partial_t + \mathbf{V}\cdot\partial_{\mathbf{x}}\right)\mathbf{V} = -\frac{\partial_{\mathbf{x}}\mathbf{\Pi}}{n} + q\left(\mathbf{E} + \frac{\mathbf{V}}{c}\times\mathbf{B}\right). \quad (2.3.7)$$

This equation can be expressed in a proper conservation form, by writing the electromagnetic contribution in a different way. We exploit the conservation of momentum density of the electromagnetic fields, represented by the so-called Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{B} (c/4\pi)$, as

$$\partial_t \left(\frac{\mathbf{S}}{c^2} \right) - \partial_{\mathbf{x}} \underline{\mathbf{T}} = -\left(\rho \mathbf{E} + \frac{\mathbf{J} \times \mathbf{B}}{c} \right) ,$$
 (2.3.8)

where \mathbf{T} is the Maxwell's stress tensor

$$\underline{\mathbf{T}}_{ij} = \frac{1}{4\pi} \left[E_i E_j - \frac{\mathbf{E}^2 \delta_{ij}}{2} + B_i B_j - \frac{\mathbf{B}^2 \delta_{ij}}{2} \right].$$
(2.3.9)

Combining Eqs. (2.3.5) and (2.3.8), and considering $\mathbf{J} = nq\mathbf{V}$, we obtain

$$\partial_t \left(n\mathbf{P} + \frac{\mathbf{S}}{c^2} \right) + \partial_{\mathbf{x}} \left(n\underline{\mathbf{PV}} + \underline{\mathbf{\Pi}} - \underline{\mathbf{T}} \right) = 0,$$
 (2.3.10)

that has the form of a continuity equation for the total momentum density.

Conservation of the energy

The equation for energy conservation is obtained by multiplying the Vlasov equation, Eq. (2.1.7), for the energy (γmc^2) and by integrating over all momenta as

$$\partial_t \int d\mathbf{p} \,\gamma f(t, \mathbf{x}, \mathbf{p}) + \partial_{\mathbf{x}} \int d\mathbf{p} \,\gamma \mathbf{v} f(t, \mathbf{x}, \mathbf{p}) + \int d\mathbf{p} \,\gamma \left(\mathbf{F} \cdot \partial_{\mathbf{p}}\right) f(t, \mathbf{x}, \mathbf{p}) = 0$$
(2.3.11)

The first and second integrals are by definition $n\langle\gamma\rangle$ and $n\langle\gamma\mathbf{v}\rangle$, respectively. Performing the third integral by parts, we find the energy dissipation due to the electric field, as expected. To summarize, equation (2.3.11) becomes

$$\partial_t \left(\langle \gamma \rangle mc^2 \right) + \partial_{\mathbf{x}} \left(\langle \gamma \mathbf{v} \rangle mc^2 \right) = q \langle \mathbf{E} \cdot \mathbf{v} \rangle , \qquad (2.3.12)$$

meaning that the variation in time of the total kinetic energy is balanced by the flux of the kinetic energy (second term on the left hand side) and by the energy dissipated by the electric field. The energy flux clearly vanishes for a non-drifting thermal plasma at equilibrium, for which $\langle \gamma \mathbf{v} \rangle = \langle \mathbf{p} \rangle = 0$, while for a drifting equilibrium distribution, energy is transported by the average flow, see Eq. (2.2.14). In a more general situation, in which the distribution function deviates from a Maxwellian, the energy flux does not vanish $\langle \gamma \mathbf{v} \rangle \neq 0$ and it accounts for the total kinetic energy transport, carried by both average or thermal motion. While in the non-relativistic case it is possible to separate the term corresponding to the thermal energy transported by the thermal motion (the so-called *heat flux*) and the terms relative to the average motion, in Eq. (2.3.12) both terms are included in the γ -factor. Nevertheless, it is possible to find an expression analogous to the non-relativistic one computing the moment of the tensor \mathbf{pv} , appearing in Eq. (2.3.3), necessary to express the pressure tensor $\mathbf{\Pi}$.

The term on the right side of Eq. (2.3.12) can be written in a conservative form exploiting the equation for the conservation of the electromagnetic field energy density

$$\partial_t \left(\frac{\mathbf{E}^2}{8\pi} + \frac{\mathbf{B}^2}{8\pi} \right) + \partial_\mathbf{x} \left(c \, \frac{\mathbf{E} \times \mathbf{B}}{4\pi} \right) = -\mathbf{E} \cdot \mathbf{J} = -nq \langle \mathbf{E} \cdot \mathbf{v} \rangle \,. \tag{2.3.13}$$

Note that here the Poynting vector \mathbf{S} appears in the second term on the left as the flux of the electromagnetic energy density. Combining Eqs. (2.3.12) and (2.3.13), we obtain

$$\partial_t \left(n \langle \gamma \rangle mc^2 + \frac{\mathbf{E}^2}{8\pi} + \frac{\mathbf{B}^2}{8\pi} \right) + \partial_{\mathbf{x}} \left(n \langle \gamma \mathbf{v} \rangle mc^2 + c \, \frac{\mathbf{E} \times \mathbf{B}}{4\pi} \right) = 0 \,, \quad (2.3.14)$$

where we can insert the average values listed in Table 2.2.1, for the case of a Maxwell-Jüttner distribution.

Closure of the hydrodynamic equations

The hydrodynamic equations, Eqs. (2.3.2)- (2.3.10) and (2.3.14), derived from the kinetic description by taking the moments of the Vlasov equation, do not constitute a closed system. Indeed, each moment of the Vlasov equation is coupled to the higher order one. Therefore, an additional equation is required.

In the following of this chapter, we will make use of the equation of

state which defines the relation between internal energy e_0 , pressure \mathcal{P} and density n.

Obviously in a cold plasma², the simple closure is $\mathcal{P} = 0$, while for an isothermal process $\mathcal{P} = nT$, with constant temperature. The closure, that we will demonstrate to be the most accurate for our investigation, is the adiabatic one, in which the system is considered isolated, hence it evolves without exchanging energy with any external source. Within this approximation a truncation of the hierarchy of the Vlasov equation moments, is often exploited, assuming high order moments, and in particular the heat flux, to be vanishing. This corresponds to a closure such as $T \propto n^{\Gamma ad-1}$, so that $\mathcal{P} \propto n^{\Gamma ad}$, with Γ_{ad} the adiabatic index, as already discussed in Eq. (2.2.17).

2.4 Linear Waves and Instabilities

In this paragraph we derive the key quantity for the description of the plasma instabilities: the susceptibility tensor. This quantity can be calculated with a perturbative approach, starting from either a kinetic or a hydrodynamic description of the species constituting the plasma. Here we will exploit the relativistic fluid equations derived in Sec. 2.3 (for a kinetic treatment refer to Krall and Trivelpiece (1986)). Note that the use of the fluid equations, in which the average quantities are calculated over a drifting Maxwell-Jüttner distribution, allows for much simpler calculations with respect to a kinetic approach with the same distribution function, in particular in the presence of an external magnetic field as will be considered in Chap. 3. We will discuss the limit of validity of this approach in Sec. 2.4.3, by means of a direct comparison with the kinetic results.

We consider that all space-/time-dependent physical quantities can be written in the form

$$\epsilon(\mathbf{x},t) = \epsilon_0 + \epsilon_1 e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})}, \qquad (2.4.1)$$

 $^{^{2}}$ We should not confuse the cold approximation with the regime of low-temperature plasmas, e.g. in electrical discharges. In the context of this thesis, a *cold plasma* has a negligible thermal spread so that effectively the momentum distribution is a Dirac function.

with $|\epsilon_1| \ll |\epsilon_0|$. Maxwell's equations (2.1.10) and (2.1.11) become

$$i\mathbf{k} \times \mathbf{E}^{(1)} = \frac{i\omega}{c} \mathbf{B}^{(1)},$$
 (2.4.2)

$$i\mathbf{k} \times \mathbf{B}^{(1)} = \frac{4\pi}{c} \mathbf{J}^{(1)} - \frac{i\omega}{c} \mathbf{E}^{(1)}, \qquad (2.4.3)$$

where (1) refers to the quantity of the order of ϵ_1 . The first order perturbation of the current density reads

$$\mathbf{J}^{(1)} = \sum_{s} q_s (n_{0s} \mathbf{V}_s^{(1)} + n_s^{(1)} \mathbf{V}_{0s}) \,. \tag{2.4.4}$$

Combining Eqs. (2.4.2) and (2.4.3), by taking the curl of Eq. (2.4.2), we obtain

$$\mathbf{k}(\mathbf{k}\cdot\mathbf{E}^{(1)}) - |\mathbf{k}|^2\mathbf{E} + \frac{\omega}{c}(\frac{4\pi i}{c}\mathbf{J}^{(1)} + \frac{\omega}{c}\mathbf{E}^{(1)}) = 0.$$
 (2.4.5)

This represents a tensorial relation whose non trivial solution is obtained as det $\Xi = 0$, where the susceptibility tensor Ξ is

$$\Xi = \begin{bmatrix} \frac{\omega^2}{c^2} \varepsilon_{xx} - k_y^2 - k_z^2 & \frac{\omega^2}{c^2} \varepsilon_{xy} + k_x k_y & \frac{\omega^2}{c^2} \varepsilon_{xz} + k_x k_z \\ \frac{\omega^2}{c^2} \varepsilon_{yx} + k_x k_y & \frac{\omega^2}{c^2} \varepsilon_{yy} - k_x^2 - k_z^2 & \frac{\omega^2}{c^2} \varepsilon_{yz} + k_y k_z \\ \frac{\omega^2}{c^2} \varepsilon_{zx} + k_x k_z & \frac{\omega^2}{c^2} \varepsilon_{zy} + k_y k_z & \frac{\omega^2}{c^2} \varepsilon_{zz} - k_x^2 - k_y^2 \end{bmatrix}, \quad (2.4.6)$$

with the permittivity tensor component $\varepsilon_{ij} = \left[\delta_{ij} + \frac{4\pi i}{\omega}\sigma_{ij}\right]$, and $J_i = \sigma_{ij}E_j$, with $\underline{\sigma}$ the conductivity tensor.

In order to find an explicit expression for the components of the conductivity tensor $\underline{\sigma}$, we linearize Eq. (2.3.2) as

$$-i\omega n_s^{(1)} + i\mathbf{k} \cdot (n_s^{(1)}\mathbf{V}_{0s} + n_{0s}\mathbf{V}_s^{(1)}) = 0, \qquad (2.4.7)$$

assuming an initial flow velocity \mathbf{V}_{0s} for the species s. Hence, the perturbation in the density becomes

$$n_s^{(1)} = n_{0s} \frac{\mathbf{k} \cdot \mathbf{V}_s^{(1)}}{\omega - \mathbf{k} \cdot \mathbf{V}_{0s}} \,. \tag{2.4.8}$$

Linearizing Eq. (2.3.6), we get

$$i \left[\mathbf{V}_{0s} \cdot \mathbf{k} - \omega \right] \mathbf{P}_s^{(1)} = q_s \left(\mathbf{E}^{(1)} + \frac{\mathbf{V}_{0s}}{c} \times \mathbf{B}^{(1)} + \frac{\mathbf{V}_s^{(1)}}{c} \times \mathbf{B}_0 \right) - \frac{i \mathbf{k} \Gamma_{\mathrm{ad}} n_s^{(1)} T_s}{n_{0s}},$$
(2.4.9)

where \mathbf{E}_0 and \mathbf{B}_0 are external fields that might be present at time t = 0, the pressure tensor has been considered isotropic, so that it reduces to $\mathcal{P} = nT$ in the perfect gas approximation, and we consider an adiabatic closure. This assumption will be discussed further in Sec. 2.4.3.

Exploiting Eqs. (2.4.8) and (2.4.9), we can calculate the first order current density (2.4.4) and insert it into Eq. (2.4.5). The solution of det $\Xi = 0$ provides the dispersion relation of all the waves allowed in the system. In particular, a solution with a complex frequency $\omega = \omega_{\rm R} + i\Gamma$, from Eq. (2.4.1) reads

$$\epsilon_1(\mathbf{x}, t) = \epsilon_1 e^{\Gamma t} e^{-i(\omega_{\rm R} t - \mathbf{k} \cdot \mathbf{x})} . \tag{2.4.10}$$

For $\Gamma < 0$, this corresponds to an exponentially damped solution, while, for $\Gamma > 0$, it gives an exponentially growing mode, i.e. an instability.

2.4.1 Linear waves in a cold plasma

In this paragraph we provide a simple description of the propagation of electromagnetic waves in a cold ($\mathcal{P} = 0$), non-magnetized ($B_0 = 0$) plasma at rest, as done in several textbooks (see Jackson, 1962; Krall and Trivelpiece, 1986). Neglecting relativistic effects, equation (2.4.9) reduces to

$$-i\omega m_s \mathbf{V}_s^{(1)} = q_s \mathbf{E}^{(1)} . \qquad (2.4.11)$$

Therefore, all off-diagonal terms in Ξ , Eq. (2.4.6), vanish and det $\Xi = \prod_i \Xi_{ii}$. Looking for electromagnetic modes propagating along the *x*-direction $\mathbf{k} = k_x \hat{\mathbf{x}}$, both components Ξ_{yy} and Ξ_{zz} give

$$\omega^2 = \sum_s \omega_{ps}^2 + k_x^2 c^2 \,, \qquad (2.4.12)$$

with $\omega_{ps}^2 = 4\pi n_{0s} q_s^2/m_s$, known as *plasma frequency* of the *s*-species. Relation (2.4.12) gives the threshold value for the lowest frequency of the electromagnetic waves allowed to propagate in a plasma with density n_{0s} . In the case of electron-proton plasma, since $m_i \gg m_e$ we can neglect the contribution of ω_{pi} in the sum in Eq. (2.4.12). Hence, electromagnetic waves with $\omega > \omega_{pe}$ propagates, while for $\omega < \omega_{pe}$ the waves are evanescent, namely the fields decreases exponentially as $\sim e^{-k_x x}$, considering a plasma filling the region x > 0.

Note that the component Ξ_{xx} gives a different type of solutions. In particular, longitudinal electrostatic modes with frequency $\omega = \omega_{ps}$.

2.4.2 Electron Weibel or Filamentation instability

As already mentioned in the previous chapter, the Weibel or Filamentation instability has been demonstrated to be crucial for collisionless shock formation in astrophysical plasmas. It has been also often encountered in studies related to laser-plasma interaction (Huntington et al., 2015; Göde et al., 2017). In this paragraph, we explain its behavior and we derive its growth rate in a simple configuration.

A Weibel unstable initial condition requires an anisotropy in the distribution function, that can be produced by a strong temperature anisotropy (the scenario originally envisioned by Weibel (1959)), or by counterstreaming flows (also referred to as current-filamentation instability). In both situations the instability transfers energy from the particles to the magnetic field and tends to isotropize the particle distribution function.

The Weibel instability has an electromagnetic nature and it can be triggered by small amplitude electromagnetic fluctuations. As charged particles get deflected by any fluctuation of the magnetic field perpendicular to their initial velocity, particles initially moving in opposite directions will concentrate in spatially separated current filaments, hence amplifying the initial magnetic field perturbation (linear phase), as sketched in Fig. 2.1. As the self-generated magnetic field amplitude grows, the particle dynamics is strongly modified by the fields (non-linear phase), and various saturation mechanisms may set in, as will be discussed in detail in Chapter 3.



Figure 2.1: Schematic description of the Weibel instability development. The particles (in this case electrons) propagating along the x-direction, enter in a region with magnetic field fluctuations B_z (positive in blue and negative in red) and get deflected along the y-direction toward the zero of the magnetic field. Particles accumulate and create current filaments J_x that increase the initial magnetic fluctuations.

This instability has been the focus of several works considering different configurations and unmagnetized plasmas (Pegoraro et al., 1996; Califano et al., 1997, 1998; Bret et al., 2010a). Two-dimensional spectra of the unstable modes have been provided by Bret et al. (2010b), with particular attention to the competition between transverse, oblique and longitudinal (driving the so-called *two-stream instability*) modes. In the systems investigated in this thesis, the unstable spectrum is dominated by electromagnetic modes.

For this reason, we derive here the Weibel instability dispersion relation in one of the simplest configuration, that of two cold counterstreaming electron beams with respective densities $n_0/2$ and drift velocities $\mathbf{V}_0 = \pm V_0 \hat{\mathbf{x}}$, in a background of neutralizing ions. We also neglect the external electromagnetic fields $\mathbf{E}_0 = \mathbf{B}_0 = 0$ and we look for purely transverse modes, i.e. $\mathbf{k} = k_y \hat{\mathbf{y}}$, in order to simplify the presentation. We can model the linear phase of the instability, namely the phase of exponential growth, using the linearized relativistic cold fluid equations [Eqs. (2.4.8)-(2.4.9) in the limit of enthalpy h = 1 and pressure $\mathcal{P} = 0$]. To do so, we need to relate the variation of momentum and velocity, as

$$m_s \mathbf{V}_s^{(1)} = \frac{\mathbf{P}_s^{(1)}}{\Gamma_{0s}} - \frac{\mathbf{P}_{0s}}{\Gamma_{0s}^3} \frac{\mathbf{P}_{0s} \cdot \mathbf{P}_s^{(1)}}{m_s^2 c^2}, \qquad (2.4.13)$$

so that the current perturbation in Eq. (2.4.4) becomes

$$\mathbf{J}^{(1)} = \frac{en_0}{2m_e\Gamma_0} \sum_{s} \left[\mathbf{P}_s^{(1)} - \frac{\mathbf{P}_{0s}}{\Gamma_0^2} \frac{P_{0xs}P_{xs}^{(1)}}{m_e^2 c^2} + \mathbf{V}_{0s} \frac{k_y P_{ys}^{(1)}}{\omega} \right].$$
 (2.4.14)

Inserting Eq. (2.4.14) into Eq. (2.4.5) and expressing the dependence of the momenta perturbation on the electric field $\mathbf{E}^{(1)}$, the tensor Ξ becomes

$$\Xi = \begin{bmatrix} \frac{\omega^2}{c^2} - k_y^2 - \frac{\omega_{pe}^2}{c^2 \Gamma_0} \left(\frac{1}{\Gamma_0^2} + \frac{k_y^2 V_0^2}{\omega^2} \right) & 0 & 0 \\ 0 & \frac{\omega^2}{c^2} - \frac{\omega_{pe}^2}{c^2 \Gamma_0} - k_y^2 & 0 \\ 0 & 0 & \frac{\omega^2}{c^2} - \frac{\omega_{pe}^2}{c^2 \Gamma_0} \end{bmatrix},$$
(2.4.15)

where the non-diagonal terms vanish thanks to the symmetry of the problem. Since det $\Xi = \prod_i \Xi_{ii}$, we can solve for each component $\Xi_{ii} = 0$. For $\Xi_{yy} = 0$ and $\Xi_{zz} = 0$, we obtain stable waves, while Ξ_{xx} allows for a purely imaginary solution $\omega = i\Gamma$ as

$$\Gamma(k_y) = \frac{1}{\sqrt{2}} \left[\sqrt{\left(k_y^2 c^2 + \frac{\omega_{pe}^2}{\Gamma_0^3}\right)^2 + 4\frac{\omega_{pe}^2}{\Gamma_0}k_y^2 V_0^2} - \left(k_y^2 c^2 + \frac{\omega_{pe}^2}{\Gamma_0^3}\right) \right]_{(2.4.16)}^{1/2}.$$

This solution corresponds to the growth rate of electron-Weibel transverse modes with wavenumber k_y , leading to a magnetic field B_z .

As a first generalization, we can account for a finite temperature $T \ll m_e c^2$. In Eqs. (2.4.8)-(2.4.9), we should include the effect of the normalized enthalpy $h(\mu)$, with $\mu = m_e c^2/T_0$ and T_0 the rest frame plasma temperature, and the contribution of the first order perturbation of the pressure term $\mathcal{P} = nT$, in a perfect gas assumption. Proceeding as for



Figure 2.2: Electron-Weibel instability growth rate Γ as a function of the wavenumber k, for electron streaming velocity $V_0 = 0.4 c$ and temperature $T_0 = 0$ (dashed dark green line) and $T_0 = 5 \times 10^{-3} m_e c^2$ (light green line).

the cold case, we obtain the dispersion relation for the purely transverse electron Weibel modes

$$\frac{\omega^2}{c^2} - k_y^2 - \frac{\omega_{pe}^2}{c^2 \Gamma_0 h(\mu)} \left(\frac{1}{\Gamma_0^2} + \frac{k_y^2 V_0^2}{\omega^2 - \frac{\Gamma_{ad} T_0 k_y^2}{m_e \Gamma_0^2 h(\mu)}} \right) = 0, \qquad (2.4.17)$$

and consequently we can derive the growth rate of the instability. In the limit $T_0 = 0$, we recover the prediction of the cold fluid theory, i.e. the Ξ_{xx} component in Eq. (2.4.15).

A detailed description of the instability growth, of the role of the temperature and of the application of external fields will be presented in Chap. 3, as well as the development of the ion instability will be investigated in Chap. 4. However, in order to have a qualitative idea of the instability features in this linear phase of growth, we report in Fig. 2.2, $\Gamma(k)$ for two counter-streaming beams with $V_0 = 0.4 c$ and temperature $T_0 = 0$ (dashed dark green line). In this cold case the fastest growing modes have $k \gtrsim \omega_{pe}/c$ (small wavelengths) and the asymptotic growth rate is $\Gamma_{\text{asymp}} = (V_0/c\sqrt{\gamma_0}) \omega_{pe} \simeq 0.38 \omega_{pe}$. In Fig. 2.2, we also show the growth rate in the case of beam temperature $T_0 = 5 \times 10^{-3} m_e c^2$ (light plain green line). This value has been chosen because we expect

the growth of modes with $k \gtrsim \Gamma_{\rm asymp}/v_{\rm th}$, with $v_{\rm th} \sim \sqrt{T_0/m_e}$, to be drastically modified. In fact, the transverse motion associated with the introduction of a temperature, leads to the stabilization of the large-k (small wavelengths), while does not affect much the growth of the small wavenumbers.

2.4.3 Discussion on the limit of the fluid approach

The limit of validity of the fluid description is discussed in this section, in order to justify the use of this approach. We provide here a direct comparison with the dispersion relation of the Weibel instability derived from a kinetic approach. For simplicity, we restrict ourselves to the nonrelativistic limit. We consider the same geometry of the previous section, with initially two counter-streaming electron flows with drifting-Maxwellian distribution function, Eq. (2.2.5).

Following Davidson et al. (1972), the dispersion relation for purely transverse mode ($\mathbf{k} = k_y \hat{\mathbf{y}}$) reads

$$\frac{\omega^2}{c^2} - k_y^2 - \frac{\omega_{pe}^2}{c^2} + \frac{\omega_{pe}^2}{c^2} \left[a_e + 1\right] \left[1 + \sqrt{\frac{m_e}{2T_0}} \frac{\omega}{k_y} \mathcal{Z}\left(\sqrt{\frac{m_e}{2T_0}} \frac{\omega}{k_y}\right)\right] = 0, \quad (2.4.18)$$

where $a_e = m_e V_0^2/T_0$ measures the anisotropy of the beams and we assume $a_e \gg 1$ for the case of interest in the following of this work $\mathcal{Z}(\xi)$ is the so-called *plasma dispersion function* (Fried et al., 1960) defined as

$$\mathcal{Z}(\xi) = \int_{-\infty}^{+\infty} \mathrm{d}t \, \frac{e^{-t^2}}{t-\xi} \,. \tag{2.4.19}$$

Note that for the description of the Weibel instability, ξ is purely imaginary and equal to $i\sqrt{m_e/2T_0}\Gamma/k_y$, with $\Gamma > 0$ the growth rate. The dispersion relation calculated with the fluid approach $[\Xi_{xx} = 0$ in Eq. (2.4.15)] is recovered in the limit $|\xi| \gg 1$. This means that the thermal velocity has to be much smaller than the typical quantity Γ/k , analogous of the phase velocity for the case of propagating waves. Indeed, for $\xi \gg 1$ the function $\mathcal{Z}(\xi)$ becomes

$$\mathcal{Z}(\xi) \simeq i\sqrt{\pi}\sigma e^{-\xi^2} - \frac{1}{\xi} \left(1 + \frac{1}{2\xi^2} + \frac{3}{4\xi^4} + \dots \right) \,, \tag{2.4.20}$$

where σ takes finite values in the range [0, 2], (see Fried and Conte, 1961). Neglecting the last term on the right hand side of Eq. (2.4.20) and inserting this development in Eq. (2.4.18), we obtain the dispersion relation of the cold fluid approximation.

Note that the limit to recover the fluid dispersion relation $\Gamma/k \gg \sqrt{2T_0/m_e}$ corresponds to the condition for the adiabaticity of the system, so that $T \propto n^{\Gamma_{\rm ad}-1}$ with $\Gamma_{\rm ad}$ the adiabatic index defined in Eq. (2.2.17). This means that, during a typical interval of time $t \sim \Gamma^{-1}$, a particle moving at the thermal velocity is not able to explore a filament wavelength. The adiabatic closure leads to a pressure term dependence on the density as $\mathcal{P} \propto n^{\Gamma_{\rm ad}}$, so that its first order perturbation becomes $\mathcal{P}^{(1)} = \Gamma_{\rm ad} n^{(1)} T$. This factor is in fact considered in the last term on the right hand side of Eq. (2.4.9).

The presence of the multiplicative factor Γ_{ad} in the pressure term is confirmed by the kinetic approach. Indeed, considering the last term on the right hand side in the expansion (2.4.20) and inserting it in Eq. (2.4.18), we obtain the dispersion relation

$$\frac{\omega^2}{c^2} - k_y^2 - \frac{\omega_{pe}^2}{c^2} - \frac{\omega_{pe}^2}{c^2} \frac{k_y^2 V_0^2}{\omega^2} \left[1 + \frac{3T_0 k_y^2}{m_e \omega^2} \right] = 0.$$
 (2.4.21)

Comparing it with the results of the fluid description, Eq. (2.4.17), we obtain the same dispersion relation once performing an additional development in the last term of Eq. (2.4.17) $[(1 - \epsilon)^{-1} \simeq 1 + \epsilon, \text{ with } \epsilon \ll 1]$. Indeed, within the assumption of purely transverse mode, it is reasonable to assume that the adiabatic index takes its 1D value, i.e. $\Gamma_{\text{ad}} = 3$, since the instability compresses the flow along the k-direction and all modes evolve independently in the linear phase.

In the opposite limit $|\xi| \ll 1$, the quantity Γ/k is smaller than the thermal velocity, hence it corresponds to a region of the phase-space where the distribution function is non-vanishing. Therefore, kinetic effects are

expected to play an important role. This is usually referred to as *kinetic* regime of the instability. In this regime, the fluid description is not applicable.³ For the sake of completeness, we recall the expansion of the function $\mathcal{Z}(\xi)$ in the limit $|\xi| \ll 1$

$$\mathcal{Z}(\xi) \simeq i\sqrt{\pi} e^{-\xi^2} - 2\xi \left(1 - \frac{2\xi^2}{3} + ...\right).$$
 (2.4.22)

The leading term of the expansion $(\mathcal{Z}(\xi) \simeq i\sqrt{\pi}e^{-\xi^2})$ that is responsible for the instability growth, derives from the integration around the pole of the dispersion function. For this reason the instability can not be recovered within the fluid approach in the limit $|\xi| \ll 1$. Using Eq. (2.4.22), the dispersion relation for $\omega = i\Gamma$ becomes a simple second order equation

$$\Gamma^2 + c^2 k_y^2 + \omega_{pe}^2 - \omega_{pe}^2 \left(a_e + 1\right) \left(1 - \sqrt{\frac{\pi m_e}{2T_0}} \frac{\Gamma}{k_y}\right) = 0.$$
 (2.4.23)

As a consequence of this purely kinetic instability regime, the value of the cut-off wavenumber, maximum unstable mode, is slightly larger than predicted by the fluid approach.

2.5 Collisionless Shocks

Hydrodynamic shocks are quite well understood and extensively described in the literature, while there are still many open questions regarding collisionless shocks. While in the previous Chap. 1 we focused on the description of the late stage dynamics of the particles accelerated at Weibelmediated shock fronts, here we discuss mainly the initial phase of shock formation. This leads to a first general classification of the different type of shocks developing in collisionless plasmas.

2.5.1 General considerations

Common hydrodynamic shock waves, such as those following detonations or those produced by supersonic aircraft, can be described as discontin-

 $^{^{3}\}mathrm{In}$ case one computes, by means of the fluid theory, the Weibel instability growth

uous steady solutions of the ideal fluid equations which propagate with velocity larger than the speed of sound in the fluid. The propagation of such a wave involves an abrupt change in the macroscopic state of the fluid mediated by collisional effects. In particular, the fluid behind the discontinuity region (the so-called shock front) is found to be compressed and strongly heated, hence reaching a high pressure state. Indeed, collisions produce an irreversible conversion of the kinetic energy of the fluid crossing the shock front into thermal energy. The relations between the macroscopic quantities that describe the unperturbed fluid and the ones that characterize the shocked plasma, are the well known Rankine-Hugoniot conditions (Blandford and McKee, 1976). They can be derived exploiting the conservation of particle number, momentum and energy at the shock front, as detailed in Sec. 2.5.2.

In principle, the formation of shocks in the collisionless regime is not allowed within the fluid description, since no dissipative effects can justify the macroscopic change of state. However, a shock wave solution becomes possible in the presence of kinetic effects acting, at the shock front, as an effective dissipation. Indeed, many numerical and experimental works have demonstrated the existence of a discontinuous propagating wave in collisionless plasmas, see (Marcowith et al., 2016) and references therein. While in usual hydrodynamics shocks, the region where the variation of macroscopic state takes place has an extension of the order of some λ_{mfp} , the collisional mean-free-path, in collisionless plasma the discontinuity appears as a transition layer $l_{\rm f} \ll \lambda_{\rm mfp}$.

Two main classes of collisionless shocks can be identified: *laminar* or *turbulent*, as first discussed by Tidman and Krall (1971). Laminar shock waves are formed due to the steepening of finite amplitude waves propagating in the medium. With the linearized fluid equations, described in Sec. 2.4, we can identify the small amplitude waves allowed to propagate in the plasma and their characteristic velocities, such as the ion-acoustic, magnetosonic speed, etc. For a wave with finite amplitude, non-linear terms in the fluid equations play a fundamental role. Indeed, these terms allow the crests of the wave to move faster than the zeros, leading to a

rate, in a system with parameters typical of this kinetic regime, the configuration would be erroneously predicted to be stable.

progressive steepening of the wave profile. This process creates regions where the density gradient is so large that dispersive effects are not negligible and can balance the ongoing steepening. While in hydrodynamic shocks, collisions at the shock front mediated the macroscopic change of state, in the laminar collisionless shocks the effective dissipation is due to the collective interaction between particles and the laminar electromagnetic fields of the non-linear wave. Therefore, a classification of the different types of laminar shocks relies on the nature of the fields at the shock front. We discriminate between electrostatic shocks, where only an electrostatic potential mediates the discontinuity, such as in the nonlinear evolution of ion-acoustic waves (Grassi et al., 2016), and strongly magnetized shocks, in which a magnetic barrier forms as a consequence of the non-linear evolution of, for instance, magnetosonic waves. In both cases the dissipation mechanism relies on the reflection of part of the ion population and on the trapping of part of the electron population entering in the shock front, as described by Tidman and Krall (1971).

In addition to these class of laminar shocks, turbulent shocks have been identified in many astrophysical systems. As already mentioned in the previous Chapter 1, in this case the shock front is the result of plasma instabilities that generate and amplify turbulent electromagnetic fields. For instance, the strong magnetic fields produced by the Weibel instability (hence, the name Weibel-mediated collisionless shocks) can lead to the dissipation of the flow kinetic energy and consequently to the isotropization of the incoming flow. The dissipation of the flow kinetic energy, not only involves an increase of the temperature, but also an increase of the density, since the flow is drastically slowed down. This produces a narrow region, of the order of tens of c/ω_p , the skin depth of the particles involved in the instability, where the density and the temperature change abruptly. The level of density accumulation in a non-propagating structure is limited by fluid constrains on momentum and energy conservation. Therefore, this front might start propagating once enough density has been accumulated. At this point, the shock is considered fully formed. The characteristic density and temperature jump, and the expected shock front velocity, will be discussed in the following paragraph.

While the phase of the instability development has been analytically

well characterized, a theoretical description of the shock formation stage is still lacking. Hence, an extensive numerical investigation has been performed in the last decades (Spitkovsky, 2008; Sironi and Spitkovsky, 2009, 2011a; Stockem et al., 2014; Caprioli and Spitkovsky, 2014a; Stockem-Novo et al., 2015; Bret et al., 2016; Novo et al., 2016; Ruyer et al., 2016; Bret et al., 2017). These works have successfully described the physics behind shock formation and late stage of shock propagation in both electronion and pair plasma in unmagnetized and magnetized conditions, while the timing for shock formation is still an open discussion. Part of this thesis (Chap. 6) focuses on the identification and measurements of the shock formation time in PIC simulations of relativistic pair plasma flows in unmagnetized and perpendicular shocks.

2.5.2 Rankine-Hugoniot conditions at the shock front

In this section we will exploit the relativistic hydrodynamics equations to derive the Rankine-Hugoniot conditions.

The derivation of these relations does not require the knowledge of the physical mechanisms acting at the shock front. The region where the discontinuity takes place is usually considered as a "black box". The plasma outside of this region is assumed at the equilibrium and characterized by macroscopic quantities such as density, temperature, flow velocity etc. Making use of the number of particle, momentum and energy conservations across the shock front, in the rest frame of the shock, Eqs. (2.3.2), (2.3.10) and (2.3.14) become

$$\partial_{\mathbf{x}} \left(n \mathbf{V} \right) = 0 \,, \tag{2.5.1}$$

$$\partial_{\mathbf{x}} \left(n \underline{\mathbf{PV}} + \underline{\mathbf{\Pi}} - \underline{\mathbf{T}} \right) = 0,$$
 (2.5.2)

$$\partial_{\mathbf{x}} \left(n \langle \gamma \mathbf{v} \rangle m c^2 + c \, \frac{\mathbf{E} \times \mathbf{B}}{4\pi} \right) = 0 \,,$$
 (2.5.3)

where we have assumed that the shock front is stationary ($\partial_t = 0$) in this frame. In the following, we will refer to the unperturbed region (upstream) with the index u, and to the shocked one (downstream) with d. We aim at computing the density jump $n_{\rm d}/n_{\rm u}$ and the shock velocity $v_{\rm sh}$. We consider an incoming flow with momentum $\mathbf{P}_{u} = P_{u}\hat{\mathbf{x}}$ and temperature T_{u} , in the presence of an external magnetic field that forms an angle θ with the axis flow. In order to have an initial equilibrium condition, we assume the existence of an electric field that satisfy $\mathbf{E} + (\mathbf{V} \times \mathbf{B})/c = 0$. Without loss of generality we can assume $\mathbf{B} = (B_x, 0, B_z)$ and $\mathbf{E} = (0, E_y, 0)$. Considering a shock front moving along the x-direction and all quantities to be spatially uniform along the y-z directions (planar shock), so that their variation is only a function of x, equations (2.5.1)-(2.5.3) become

$$n_{\rm u}V_{\rm u} = n_{\rm d}V_{\rm d}\,,\qquad(2.5.4)$$

$$n_{\rm u}P_{\rm u}V_{\rm u} + \mathcal{P}_{\rm u} + \frac{E_{y,{\rm u}}^2 + B_{z,{\rm u}}^2 - B_{x,{\rm u}}^2}{8\pi} = n_{\rm d}P_{\rm d}V_{\rm d} + \mathcal{P}_{\rm d} + \frac{E_{y,{\rm d}}^2 + B_{z,{\rm d}}^2 - B_{x,{\rm d}}^2}{8\pi}$$
(2.5.5)

$$n_{\rm u}h_{0,\rm u}\Gamma_{\rm u}V_{\rm u}mc^2 + \frac{V_{\rm u}B_{z,\rm u}^2}{4\pi} = n_{\rm d}h_{0,\rm d}\Gamma_{\rm d}V_{\rm d}mc^2 + \frac{V_{\rm d}B_{z,\rm d}^2}{4\pi}.$$
(2.5.6)

Note that, following the definition of the derivative of a tensor given in Eq. (2.3.4), in the presence of both B_x and B_z , an additional equation has to be considered for $T_{zx} = B_z B_x/4\pi$, the x-z component of the Maxwell's stress tensor defined in Eq. (2.3.9). This equation, following from Eq. (2.5.2), involves either that the shock front moves along the oblique field line ($\underline{\mathbf{PV}}_{zx} \neq 0$) or that the pressure tensor has a nonvanishing off-diagonal component $\underline{\Pi}_{zx}$.

In order to close the system we need an equation for the magnetic field. In particular, the conservation of the magnetic flux across the shock front reads

$$\partial_t \mathbf{B} - \nabla \times (\mathbf{V} \times \mathbf{B}) = 0, \qquad (2.5.7)$$

derived from Maxwell-Faraday's equation (2.1.10) imposing $\mathbf{E} = \frac{\mathbf{V}}{c} \times \mathbf{B}$.

In the case of a magnetic field along the plasma streaming direction $(\mathbf{B} = B_x \hat{\mathbf{x}} \text{ and } \mathbf{E} = 0)$, equation (2.5.7) predicts that B_x is unchanged passing from the upstream to the downstream region. This means that from a fluid point of view a longitudinal magnetic field does not affect the structure of the shock. This point will be discussed further at the end of

this section.

Since the fluid predictions for the purely longitudinal case can be recovered in the unmagnetized limit, in the following we consider a purely transverse external magnetic field. The variation, across the shock front, of the magnetic field component perpendicular to the front propagation direction is

$$B_{z,u}V_u = B_{z,d}V_d$$
. (2.5.8)

The same compression factor appears in the density, Eq. (2.5.4).

To obtain the analytical solution of the system of equations (2.5.4)-(2.5.6) and (2.5.8), as will be used in Chap. 6, we express all quantities in the reference frame of the downstream plasma, by means of Lorentz transformations. In this way equations (2.5.4)-(2.5.6) and (2.5.8) become

$$n_{0,u}\gamma_{u}^{d}\left(V_{u}^{d}+v_{\rm sh}^{d}\right) = n_{0,d}v_{\rm sh}^{d}, \qquad (2.5.9)$$

$$B_{z,u}^{d}\gamma_{u}^{d}\left(V_{u}^{d}+v_{sh}^{d}\right) = B_{z,d}^{d}v_{sh}^{d}, \qquad (2.5.10)$$

$$\left(\gamma_{\mathrm{u}}^{\mathrm{d}}\right)^{2} \left(V_{\mathrm{u}}^{\mathrm{d}} + v_{\mathrm{sh}}^{\mathrm{d}}\right) \left[w_{0,\mathrm{u}} + \left(B_{z,\mathrm{u}}^{\mathrm{d}}\right)^{2}/4\pi\right] - v_{\mathrm{sh}}^{\mathrm{d}} \left[\mathcal{P}_{\mathrm{u}} + \left(B_{z,\mathrm{u}}^{\mathrm{d}}\right)^{2}/8\pi\right]$$
$$= v_{\mathrm{sh}}^{\mathrm{d}} \left[w_{0,\mathrm{d}} - \mathcal{P}_{\mathrm{d}} + \left(B_{z,\mathrm{d}}^{\mathrm{d}}\right)^{2}/8\pi\right], \qquad (2.5.11)$$

$$\left(\gamma_{\mathrm{u}}^{\mathrm{d}}\right)^{2} V_{\mathrm{u}}^{\mathrm{d}} \left(V_{0}^{\mathrm{d}} + v_{\mathrm{sh}}^{\mathrm{d}}\right) \left[w_{0,\mathrm{u}} + \left(B_{z,\mathrm{u}}^{\mathrm{d}}\right)^{2} / 4\pi\right] + \left[\mathcal{P}_{\mathrm{u}} + \left(B_{z,\mathrm{u}}^{\mathrm{d}}\right)^{2} / 8\pi\right]$$
$$= \mathcal{P}_{\mathrm{d}} + \left(B_{z,\mathrm{d}}^{\mathrm{d}}\right)^{2} / 8\pi, \qquad (2.5.12)$$

where the apex d identifies the quantity in the downstream rest frame, $v_{\rm sh}$ is the shock front velocity, $n_{0,i}$ and $w_{0,i}$ are the rest frame density and the rest frame non-normalized enthalpy $w_0 = n_0 h_0 mc^2$, with h_0 defined in Eq. (2.2.15), for the plasma in the region i = u, d. Equations (2.2.15) and (2.2.17) can be recast in the more useful form

$$w_0 = n_0 m c^2 + \frac{\Gamma_{\rm ad}}{\Gamma_{\rm ad} - 1} \mathcal{P}, \qquad (2.5.13)$$

where the enthalpy is directly related to the rest mass energy and to the pressure, by means of the adiabatic index Γ_{ad} . Making the approximation

of cold upstream plasma $\mathcal{P}_{\rm u} = 0$ and of strong shock $\gamma_{\rm u}^{\rm d} \gg 1$, so that we can neglect the second terms on the left of Eqs. (2.5.11) and (2.5.12), we obtain a second order equation for $v_{\rm sh}^{\rm d}$ whose solution reads

$$\frac{v_{\rm sh}^{\rm d}}{c} = \frac{A + \sqrt{8 \left(\gamma_{\rm u}^{\rm d} V_{\rm u}^{\rm d}/c\right)^2 \sigma (1+\sigma)(2-\Gamma_{\rm ad}) + A^2}}{4(1+\sigma)\gamma_{\rm u}^{\rm d} V_{\rm u}^{\rm d}/c} , \qquad (2.5.14)$$

where $A = 2(\gamma_{\rm u}^{\rm d} - 1)(\Gamma_{\rm ad} - 1) + \Gamma_{\rm ad}\gamma_{\rm u}^{\rm d}\sigma$, $\Gamma_{\rm ad}$ refers to the downstream plasma and $\sigma = (B_{z,{\rm u}}^{\rm d})^2/4\pi w_{0,{\rm u}}$ is the upstream magnetization.

The compression ratio, computed using the Eq. (2.5.9), reads

$$\Delta n = \frac{n_{\rm d}^{\rm d}}{n_{\rm u}^{\rm d}} = \frac{n_{0,\rm d}}{\gamma_{\rm u}^{\rm d} n_{0,\rm u}} = 1 + \frac{V_{\rm u}^{\rm d}}{v_{\rm sh}^{\rm d}} \,. \tag{2.5.15}$$

The variation in the magnetic field is the same in virtue of Eq. (2.5.10).

In the unmagnetized case ($\sigma = 0$), equation (2.5.14) gives a shock velocity $v_{\rm sh}^{\rm d}/c = (\gamma_{\rm u}^{\rm d} - 1)(\Gamma_{\rm ad} - 1)/(\gamma_{\rm u}^{\rm d}V_{\rm u}^{\rm d}/c)$, so that the density jump becomes

$$\Delta n = 1 + \frac{\gamma_{\rm u}^{\rm d} + 1}{(\Gamma_{\rm ad} - 1)\gamma_{\rm u}^{\rm d}}, \qquad (2.5.16)$$

in agreement with previous works (Gallant et al., 1992; Lemoine et al., 2016; Novo et al., 2016). Hence, we recover the classical result $\Delta n \simeq 3$ for a perfect 2D gas in both the non-relativistic limit ($\gamma_u^d \simeq 1$ and $T_d \ll mc^2$) for which $\Gamma_{ad} = 2$ and in the relativistic case ($\gamma_u^d \gg 1$ and $T_d \gg mc^2$) for which $\Gamma_{ad} = 3/2$. In a 3D geometry, we obtain $\Delta n \simeq 4$ with both $\Gamma_{ad} = 5/3$, typical of the non-relativistic plasma, and $\Gamma_{ad} = 4/3$ for the relativistic one. For any intermediate situations in 3D, the adiabatic index can be calculated as reported in Table 2.2.1.

In addition, we can derive the expected downstream temperature as

$$\frac{T_{\rm d}^{\rm d}}{\gamma_{\rm u}^{\rm d}mc^2} = \frac{v_{\rm sh}^{\rm d}V_{\rm u}^{\rm d}}{c^2}(1+\sigma) - \frac{V_{\rm u}^{\rm d} + v_{\rm sh}^{\rm d}}{v_{\rm sh}^{\rm d}}\frac{\sigma}{2}, \qquad (2.5.17)$$

that in the unmagnetized limit becomes $T_{\rm d}^{\rm d} = (\Gamma_{\rm ad} - 1)(\gamma_{\rm u}^{\rm d} - 1) mc^2$.

In a strongly magnetized case $\sigma \simeq (\gamma_u^d)^2 \gg 1$, the terms in Eqs. (2.5.11) and (2.5.12) that we have previously neglected, should

be taken into account. In this way, solving a third order equation for $v_{\rm sh}^{\rm d}$ leads to a shock velocity that tends asymptotically to c and a compression ratio that tends to $\Delta n = 2$.

The fulfillment of the Rankine-Hugoniot jump conditions is considered an essential signature for shock formation, even in the collisionless regime. Since their derivation is based on a fluid approach, we should in principle apply these relations only if the plasma, before and after the shock, has reached an equilibrium. This is necessary for a proper definition of quantities such as the temperature and the adiabatic index. In the case of a Weibel-mediated shock the magnetic fields at the shock front produce isotropization, but thermalization to the equilibrium condition requires a much longer time due to the absence of collisions. Nevertheless these conditions are verified in most of the examples studied in the literature and in this thesis.

Another criticism on the application of these relations is that no kinetic effects are included in the energy balance. A few works on Weibelmediated collisionless shocks have tried to theoretically include, in the energy continuity equation, an *ad hoc* term corresponding to the high energy non-Maxwellian tail of particles accelerated by means of first order Fermi mechanism. However, in previous studies the energy transferred to the accelerated particles has been demonstrated to remain smaller than $\sim 10\%$ of the initial kinetic energy flow (Sironi et al., 2013). Therefore we can consider this effect as a negligible contribution. Other kinetic effects will be discussed in Chap. 6.

Regarding Weibel-mediated pair plasma shocks, the impact of the different kinetic effects, such as the self-consistent evolution of the upstream plasma dynamics, the high energy non-Maxwellian distribution function as well as the energy of the Weibel-generated magnetic turbulence, has been investigated in a more systematic way by means of numerical simulations by Stockem et al. (2012). Indeed, numerical simulations based on a kinetic approach are of extreme importance to reveal a departure from the purely hydrodynamic formulation of the RH conditions, as observed by Bret et al. (2017). This means that, if kinetic effects are important, the RH conditions marginally apply despite a shock may exist.

2.6 Numerical Methods

Numerical simulations have become a fundamental tool for the study of complex systems and non-linear phenomena in many fields of physics. Regarding plasma physics, several numerical approaches have been developed in order to describe phenomena in very different regimes, from the laboratory to astrophysics. In this section we briefly introduce the two major kinetic numerical approaches (Sec. 2.6.1) and then describe the Particle-In-Cell method (Sec. 2.6.2), that will be central for the study performed in this thesis. In Sec. 2.6.2, the numerical algorithms commonly implemented in PIC codes are presented, while for a more detailed description of the code used in this thesis we refer to Dérouillat et al. (2017) and to Chap. 5.

2.6.1 Kinetic numerical approaches

A kinetic description of the plasma is necessary for a complete characterization of most of the physical phenomena of interest in this thesis. Therefore, a numerical code that couples the relativistic Vlasov equation (2.1.7)with the Maxwell's equations (2.1.8)-(2.1.11) is the most suitable, in particular as collisional effects are negligible in most of the scenarios studied in this work⁴.

Two major approaches have attracted most of the attention so far: the so-called *Vlasov approach* (Cheng and Knorr, 1976), which performs a direct integration of the Vlasov-Maxwell system of equations discretized on a phase-space grid (Eulerian approach), and the *Particle-In-Cell method* (PIC) (Birdsall and Langdon, 2004), in which the distribution function of each species is sampled using a collection of macro-particles (Lagrangian approach).

The main advantage of the Vlasov codes is the extremely high resolution of the low density region of the distribution functions, allowing for a detailed description for instance of acceleration and trapping of a small fraction of the plasma particles (Grassi et al., 2014, 2016). The price for this is a great numerical complexity of the algorithms and a consider-

⁴Particle collisions can however be introduced in Particle-In-Cell codes (see Pérez et al., 2012; Dérouillat et al., 2017)

able amount of required computational resources. Moreover, numerical artifacts as the filamentation of the distribution function into small scale structures, need to be treated with some care, (see Büchner, 2007). For these reasons, nowadays most of the kinetic simulations are performed with PIC codes. This method is based on a discretization of the distribution function that entails a great simplification in term of numerical algorithms with respect to the Vlasov code. Indeed, instead of finding a direct solution of the Vlasov partial differential equation, the code solves a series of ordinary differential equations. As will be clarified in the next section, the PIC method solves the Vlasov equation following the evolution of the distribution function along the characteristics of the Vlasov equation. Nevertheless, because of the sampling of the distribution function, PIC codes are affected by a higher level of statistical noise with respect to Vlasov codes. This may put some limits in the description of low density region and possibly non-thermal tails of the distribution function.

2.6.2 Particle-In-Cell code

A Particle-In-Cell code solves the Vlasov-Maxwell system of equations (2.1.7)-(2.1.11), with the charge and the current density defined by Eqs. (2.1.12)-(2.1.13). The fundamental idea of this approach is the discretization of the distribution function $f_s(t, \mathbf{x}, \mathbf{p})$, with s index of the species, as a sum of N_s macro-particles, as

$$f_s(t, \mathbf{x}, \mathbf{p}) = \sum_{p=1}^{N_s} w_p S\left(\mathbf{x} - \mathbf{x}_p(t)\right) \delta\left(\mathbf{p} - \mathbf{p}_p(t)\right), \qquad (2.6.1)$$

where \mathbf{x}_p and \mathbf{p}_p are the macro-particle position and momentum, δ is the Dirac distribution and $S(\mathbf{x})$ is the shape-function of the macro-particle centered in its position $\mathbf{x}_p(t)$. Note that \mathbf{x}_p and \mathbf{p}_p denote the trajectory of the *p*-th particle in the continuous phase-space. The quantity w_p is the macro-particle "weight", defined as

$$w_p = \frac{n_s \left(\mathbf{x}_p(t=0)\right)}{N_s \left(\mathbf{x}_p(t=0)\right)},$$
 (2.6.2)

where n_s is the initial density of the plasma at the *p*-th particle position. Clearly a larger number of particle N_s involves a more accurate description of the local density variation.

Inserting the discrete distribution function, Eq. (2.6.1), into the Vlasov equation (2.1.7) and considering all macro-particles independently, we obtain that each *p*-th particle satisfies the relativistic equations of motion

$$\frac{\mathrm{d}\mathbf{x}_p}{\mathrm{d}t} = \frac{\mathbf{u}_p}{\gamma_p}, \qquad (2.6.3)$$

$$\frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} = \frac{q_s}{m_s} \left(\mathbf{E}_p + \frac{\mathbf{u}_p}{\gamma_p} \times \mathbf{B}_p \right), \qquad (2.6.4)$$

where we have introduced $\mathbf{u}_p = \mathbf{p}_p/m_s$ the macro-particle reduced momentum and the corresponding relativistic factor is $\gamma_p = [1 + |\mathbf{u}_p|^2/c^2]^{1/2}$. It clearly appears that the PIC code integrates the Vlasov equation along the continuous trajectories of the macro-particles. The electromagnetic fields, appearing in Eq. (2.6.4), are interpolated at the particle position as

$$\mathbf{E}_p = \int \mathrm{d}\mathbf{x} \, S(\mathbf{x} - \mathbf{x}_p) \, \mathbf{E}(\mathbf{x}) \,, \qquad (2.6.5)$$

$$\mathbf{B}_p = \int \mathrm{d}\mathbf{x} \, S(\mathbf{x} - \mathbf{x}_p) \, \mathbf{B}(\mathbf{x}) \,. \tag{2.6.6}$$

While the trajectories of the macro-particles are followed in a continuous phase-space, the electromagnetic fields are defined on a discrete spatial grid, and are computed integrating the Maxwell's equations onto the grid. Two popular integration schemes can be implemented: the Spectral method and the Finite Difference Time Domain (FDTD) method. The PIC code SMILEI, used throughout this work, relies on the FDTD method (Taflove, 2005). This allows for a high computational efficiency and for an advanced parallelization strategy. Different schemes for the numerical discretization and integration of Maxwell's equations will be described in Chap.5. Each of these is subject to its own Courant-Friedrich-Lewy (CFL) condition to ensure the algorithm stability. For the standard solver (so-called *Yee scheme*), the CFL condition requires the time-step to be smaller than

$$c\Delta t_{\rm CFL} = \sum_{i} \frac{1}{\sqrt{\Delta x_i^{-2}}},\qquad(2.6.7)$$

where Δx_i is the spatial resolution of the grid in the *i*-th direction and the sum runs on the spatial directions resolved in the simulation.

The most serious drawback of this integration is a non-exact numerical dispersion relation for the electromagnetic waves in vacuum. This point will be discussed in detail in Chap.5, along with different types of solutions, which have been implemented during this thesis in SMILEI to overcome this issue.

In contrast, spectral methods provide a procedure to ensure the dispersion relation to be $\omega = c|\mathbf{k}|$. A transformation of the electromagnetic fields in their spatial Fourier components is performed and these quantities are then advanced in time before being transformed back into the real space. However, the use of a global Fourier transform poses some difficulties in the code optimization for high-performance computing, for an efficient parallelization strategy and regarding the scalability on a large number of processors. Recently a technique to overcome this issue has been proposed by Vay et al. (2013).

The PIC loop

In a typical PIC simulation, during the stage of the initialization, the user defines for t = 0 the spatial profiles for the particle density n_s , the mean velocity \mathbf{V}_s and the temperature T_s of each species s. The particle loading then consists in creating in each cell N_s particles (with N_s the user-defined number of macro-particles per cell), with positions \mathbf{x}_p randomly distributed within the cell, and with momenta \mathbf{p}_p randomly sampled from the requested distribution function. Once all particles in the simulation domain have been created, the total charge and current densities $\rho(t = 0, \mathbf{x})$ and $\mathbf{J}(t = 0, \mathbf{x})$ are computed onto the grid and the initial electric fields are computed from $\rho(t = 0, \mathbf{x})$ by solving Poisson's equation (2.1.8). At the end of the initialization stage, all macro-particles in the simulation have been loaded and the electromagnetic fields have been computed over the whole simulation grid.

After this initialization stage, the so called PIC-loop is applied over a fixed number N of time-steps, each with duration Δt . The typical PIC loop over a time-step Δt proceeds as illustrated in Fig. 2.3. Each time-steps consists in (i) interpolating the electromagnetic fields at the particle positions, (ii) computing the new particle velocities and positions, (iii) projecting the new charge and current densities on the grid, and (iv) computing the new electromagnetic fields on the grid.

For a complete description of these four steps necessary to advance from time-step n to time-step n + 1, refer to (Birdsall and Langdon, 2004; Dérouillat et al., 2017). Here we limit ourselves to a brief description of the particle pusher commonly used to advance particle positions in time. One of the widely used is the so-called Boris pusher (Boris, 1970), a second-order accuracy leap-frog solver. This means that the particle positions are defined at integer time-steps n, while the velocities are computed at half-integer n - 1/2 time-steps. Knowing, for each macroparticle, the electromagnetic fields at its position from the interpolation Eqs. (2.6.5)-(2.6.6), the advanced particle momentum and position are computed according to

$$\mathbf{u}_{p}^{(n+\frac{1}{2})} = \mathbf{u}_{p}^{(n-\frac{1}{2})} + \frac{q_{s}}{m_{s}} \Delta t \left[E_{p}^{(n)} + \frac{\mathbf{v}_{p}^{(n+\frac{1}{2})} + \mathbf{v}_{p}^{(n-\frac{1}{2})}}{2} \times B_{p}^{(n)} \right], \quad (2.6.8)$$

$$\mathbf{x}_{p}^{(n+1)} = \mathbf{x}_{p}^{(n)} + \Delta t \, \frac{\mathbf{u}_{p}^{(n+2)}}{\gamma_{p}} \,. \tag{2.6.9}$$

Lately, Vay (2008) proposed an alternative leap-frog scheme, that reduces the numerical error introduced by the Boris method when solving relativistic particle trajectories in the case of $\mathbf{E} + \mathbf{V} \times \mathbf{B}/c = 0$.

Two main methods have been developed to calculate charge and current projection onto the grid: an energy-conserving and a chargeconserving algorithm. In the code used for this thesis charge and current deposition is performed using the charge-conserving algorithm proposed by Esirkepov (2001). This strategy ensures global and local conservation of the total charge within the numerical error, at the expense of a



Figure 2.3: Scheme of the typical Particle-In-Cell loop to advance from time-step (n) to time-step (n + 1).

non-perfect energy conservation. Nevertheless violation on energy conservation, due to the deposition scheme implemented, is always negligible in the simulations performed for this thesis. The other approach forces energy conservation but the non-exact local charge conservation might lead to the appearance of non-physical effects, due to the local deviation from Gauss law.

Boundary conditions

For each time-step, after having computed the macro-particle positions and velocities, boundary conditions are applied to each macro-particle that has reached the limits of the spatial grid and may be located outside of the box. Each species may have a different condition for each boundary of the simulation box: the macro-particles can either loop around the box (periodic), be stopped (momentum set to zero), suppressed (removed from memory), reflected (momentum and position follow specular reflection rules) or thermalized. In the latter case, the macro-particle is set back inside the simulation box, and its new momentum is randomly sampled in a Maxwellian distribution (Spohn, 1991) with a given temperature and drift velocity, both specified by the user.

Boundary conditions are as well applied to the electromagnetic fields. We can apply injecting (or absorbing) conditions (Barucq and Hanouzet, 1997), able to inject an electromagnetic wave (e.g. a laser) in the simulation box, and to absorb outgoing electromagnetic waves. In contrast, the reflective electromagnetic boundary conditions reflect any outgoing electromagnetic wave reaching the simulation boundary. As already defined for the particles, periodic boundary conditions correspond to applying the fields from one boundary of the box to the opposite one.

Parallelization strategies

The PIC method presented above is intrinsically suitable for large scale parallelization, as it relies only on local computations (in contrast with spectral methods). The most promising parallelization strategy is based on a domain decomposition technique, used in all state-of-the-art PIC codes. The simulation box is split into several small domains, each one associated to a different processor. Once a particle crosses the border between two domains, communications between different processors are required in order to exchange the information relative to the particle positions and momenta. In addition, information on the electromagnetic fields at the boundaries of the domain must be exchanged.

The standard protocols, used for the parallelization of most of the available PIC codes, are the Message Passing Interface (MPI) and the Open Multi-Processing (openMP) interface that supports shared memory multiprocessing programming. Recently, a hybrid MPI + openMP parallelization strategy have been demonstrated to enhance the code performance. In particular, MPI can be used to communicate data between distinct nodes, while openMP interface is exploited to deal with the computational load within a single node.

This hybrid MPI-OpenMP parallelization has been exploited in SMILEI. A first standard domain decomposition is associated with the MPI processes. In addition, a second level of decomposition in smaller sub-domains (usually referred to as *patches*) is managed by the threads owned by the MPI process. This improves data locality for a fast memory access and helps in minimizing load imbalance. Indeed, with the use of

the openMP dynamic scheduler, if a thread is still busy, treating a patch with a high number of macro-particles, other threads can in the meanwhile handle the remaining patches. Moreover, SMILEI balances the load between MPI processes by exchanging patches between neighbor MPI processes, as detailed in Dérouillat et al. (2017). A good scalability of this parallelization and balancing strategy has been demonstrated up to several hundred of thousands processors.

CHAPTER 3

Electron Weibel Instability in magnetized plasmas

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3.A Alfvén limit in the presence of an external				
magnetic field				

In this chapter we discuss the study of the Weibel instability driven by two symmetric counter-streaming relativistic electron plasmas in a constant and uniform external magnetic field aligned with the plasma flows. Understanding the instability evolution in both its linear and nonlinear phases, and the prediction of the amplitude of the Weibel-generated magnetic fields are of primary importance. In particular, these points are still unclear in the relativistic and magnetized flows configuration considered in this chapter.

On the one hand, this study gives us a deeper insight into various astrophysical events where a macroscopic magnetic field might be present, such as AGN or Microquasar outflows. On the other hand, recent laserplasma experiments have been able to produce magnetized flows. In this experimental context, the introduction of a flow-aligned magnetic field has been proposed as a way to control and/or direct the high-energy plasma flows. However, a general concern is related with the stabilizing effect of the external field on the development of the Weibel instability. In this chapter, we demonstrate that the strength of the Weibel magnetic field at saturation is not affected by the external magnetic field. This result might renew the interest on this experimental investigation.

Part of the results presented in this Chapter have lead to the publication on Physical Review E (Grassi et al., 2017b).

This Chapter is structured as follows. We first give, in Sec. 3.1, a brief summary of the extensive works that have been done on the study of the Weibel instability in different unstable initial conditions. In Sec. 3.2, we consider a cold relativistic plasma model that allows the study of the evolution of a single unstable mode. The analytic predictions are confirmed by 1D PIC simulations. In this section, we first present the linear phase of the instability (Sec. 3.2.1), in which the particles are deflected by the small amplitude electromagnetic fluctuations perpendicular to their initial velocity, so that current filaments are produced and amplify the initial magnetic field perturbations. As soon as the particle dynamics is strongly modified by the Weibel-generated fields (non-linear phase) various saturation mechanisms may set in. The different mechanisms responsible for the saturation are discussed in detail in Sec. 3.2.2, for both unmagnetized and magnetized cases. The generalization of the previously proposed saturation mechanisms, to account for the presence of an external magnetic field, is verified by testing against simulations in Sec. 3.2.2.2. We show that the magnetic field strength at saturation for a given wavenumber is weakly affected by the external magnetic field.

In the following Sec. 3.3, temperature and multi-mode effects are investigated in 1D PIC simulations, by seeding the instability from the electromagnetic fluctuations of a thermal plasma. The linear phase is described within the framework of the relativistic warm fluid theory in Sec. 3.3.1.1 and theoretical predictions for the growth rate are compared to simulations in Sec. 3.3.1.2. In Sec. 3.3.2, we confirm with 1D PIC simulations that, at large temperatures, the saturation level is unaffected by the presence of the external magnetic field as in the single-mode analysis, while at lower (but finite) temperature the competition between different modes in the presence of an external magnetic field leads to a saturation level slightly lower with respect to the unmagnetized case. On a longer timescale, 1D simulations show, in most of the cases discussed in Sec. 3.3.3, that filaments of parallel currents tend to attract each other and merge, forming larger filaments (late merging phase).

In Sec. 3.4, we compare the results of 1D and 2D simulations. In the 2D geometry we investigate two possible configurations, i.e. counterstreaming plasmas with velocity in the simulation plane or perpendicular to it. We show (Sec. 3.4.2) that the linear phase is not affected by the different geometry, confirming the results of the 1D study. Indeed, the main interest of 2D simulations is related with the non-linear stage of the instability. We studied the late-stage behavior in Secs. 3.4.2 and 3.4.3, highlighting that saturation and especially the coalescence processes are affected by the dimensionality. In the perpendicular flows configuration, saturation is independent from the external field for all temperatures, and the additional dimension increases the efficiency of the coalescence processes with respect to the 1D case. On the contrary, in the in-plane flows configuration, longitudinal modes develop and disrupt the filaments. In addition, the development of a secondary Weibel instability, appearing in one particular 2D configuration (flows perpendicular to the simulation plane), and the reduced stabilizing effect of the external field for warm plasmas are investigated in Secs. 3.4.4 and 3.4.5.

Finally, Section 3.5 presents our conclusions.

3.1 State of the art

The Weibel instability has been at the center of several works and different configurations have been investigated for unmagnetized plasmas, considering as initial unstable conditions both counter-streaming plasmas (Pegoraro et al., 1996; Califano et al., 1997, 1998; Bret et al., 2010b,a) or a plasma distribution function with temperature anisotropy (Morse and Nielson, 1971; Davidson et al., 1972; Palodhi et al., 2009; Stockem et al., 2010). More recent studies focus on magnetized scenarios, considering counter-streaming flows in a macroscopic external magnetic fields, perpendicular (Novo et al., 2016) or parallel to the plasma flows (Ji-Wei and Wen-Bing, 2005; Stockem et al., 2007, 2008), and with a general orientation (Bret, 2014; Bret and Dieckmann, 2017). In magnetized plasmas, the linear phase of the instability in the cold beam approximation is well characterized. It is known that the instability growth rate is reduced by the external magnetic field. The effect of the temperature in the linear phase of the instability has been investigated in the magnetized non relativistic regime by Bornatici and Lee (1970); Tautz and Schlickeiser (2006) and in the unmagnetized relativistic one by Bret et al. (2010a).

Regarding the saturation mechanisms at play in the magnetized case, previous works leave many open questions. In the absence of an external magnetic field, it has been well established in the literature (Davidson et al., 1972; Yang et al., 1994; Medvedev and Loeb, 1999; Achterberg et al., 2007) that, two different mechanisms lead to the saturation of the Weibel instability. Saturation is expected when the particles trajectory is drastically affected by the Weibel-generated magnetic field, either producing a gyroradius of the order of the Weibel unstable wavelength (the so-called Larmor/Alfvén mechanism) or making the particle bounce in the current filament (the so-called trapping mechanism), as will be discussed in detail in the following. The generalization of these mechanisms to the magnetized plasma case is however not straightforward. Indeed, Stockem et al. (2008) suggested that trapping can not be the saturation mechanism in the magnetized case and a detailed investigation of different processes that might be responsible for saturation has been carried out by Dieckmann (2009); Dieckmann et al. (2009); Dieckmann and Bret (2010). Moreover, the possibility to reach saturation via the Larmor/Alfvén mechanism in magnetized plasma has been questioned by Bret (2016).

Our studies demonstrate, by properly generalizing the above mentioned mechanisms, that the same processes lead to saturation in the unmagnetized systems and in the one with a flow-aligned magnetic field. Moreover, saturation mechanisms that are equivalent in the unmagnetized case (i.e. trapping saturation estimate can be derived equating the bouncing frequency in the filament with the Weibel instability growth rate, or the deflection toward the filament with the filament wavelength), differ with the application of the external magnetic field. However, the strength of the magnetic field at saturation is found to be weakly affected by the external field. We provide an analytical prediction for the saturation level, in good agreement with PIC simulations.

Very few works investigated the saturation phase in finite temperature counter-streaming plasmas. In the non-relativistic regime, Novo et al. (2015) provides a quasi-linear theory to describe the plasma parameters evolution up to saturation. Here, we will focus in particular on the interplay between different modes in an initial multi-mode configuration, highlighting the difference between an initial broad spectrum versus a quasi-single mode one.

3.2 Single unstable mode in cold relativistic flows

3.2.1 Linear phase

3.2.1.1 Relativistic cold fluid theory

We start by an analytical description of the linear phase of the Weibel instability. This phase can be studied by taking ions at rest, providing a uniform neutralizing background for the two counter-streaming electron species (with respective densities $n_0/2$ and drift velocities $\mathbf{v}_0 = \pm v_0 \hat{\mathbf{z}}$),
modeled using the relativistic cold-fluid model described in Sec. 2.3. We consider a uniform external magnetic field $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$ parallel to the initial electron drift velocity.

Linearizing the governing equations and considering all space-/timedependent quantities $\phi(t, \mathbf{x}) \simeq \phi_0 \exp\left[-i(\omega t - \mathbf{k}_{\perp} \cdot \mathbf{x})\right]$, where \perp refers to the direction perpendicular to the flows, we obtain the dispersion relation for the purely transverse modes, as derived by Ji-Wei and Wen-Bing (2005); Stockem et al. (2007),

$$\frac{\omega^2}{c^2} - \mathbf{k}_{\perp}^2 - \frac{\omega_{pe}^2}{c^2 \gamma_0} \left(\frac{1}{\gamma_0^2} + \frac{\mathbf{k}_{\perp}^2 v_0^2}{\omega^2 - \Omega_0^2} \right) = 0, \qquad (3.2.1)$$

where ω and \mathbf{k}_{\perp} are the frequency and the wavevector of the considered modes, $\omega_{pe} = \sqrt{4\pi e^2 n_0/m_e}$ is the plasma frequency associated with the total density n_0 , and $\Omega_0 = -eB_0/(\gamma_0 m_e c)$ is the cyclotron frequency of an electron in the external magnetic field \mathbf{B}_0 .

The growth rate of the instability is found from the dispersion relation, Eq. (3.2.1), looking for solutions with $\omega = i\Gamma$ and $\Gamma > 0$. One then obtains

$$\Gamma(k) = \frac{1}{\sqrt{2}} \left[\sqrt{\left(k^2 c^2 + \frac{\omega_{pe}^2}{\gamma_0^3} - \Omega_0^2\right)^2 + 4\frac{\omega_{pe}^2}{\gamma_0}k^2 v_0^2} - k^2 c^2 - \frac{\omega_{pe}^2}{\gamma_0^3} - \Omega_0^2 \right]^{1/2},$$
(3.2.2)

where $k = |\mathbf{k}_{\perp}|$. In the limit of large wavenumbers $c^2 k^2 \gg \Omega_0^2 + \omega_{pe}^2/\gamma_0^3$, the growth rate takes the maximum and asymptotic value

$$\Gamma_{\rm max} = \sqrt{\frac{v_0^2}{c^2} \frac{\omega_{pe}^2}{\gamma_0} - \Omega_0^2} \,. \tag{3.2.3}$$

From this, we clearly see that Γ is reduced by the external magnetic field, as $\Omega_0 > 0$. Moreover, from Eq. (3.2.2) we find that, in the presence of an external magnetic field, filaments with size larger than $\lambda_{\text{stab}} = 2\pi/k_{\text{stab}}$, with

$$k_{\rm stab} = \gamma_0^{-1} \left(\frac{v_0^2}{\Omega_0^2} - \gamma_0 \frac{c^2}{\omega_{pe}^2} \right)^{-1/2} , \qquad (3.2.4)$$

cannot be created. Note that $k_{\text{stab}} = \gamma_0^{-1} \left(r_{\text{L}}^2 - d_e^2 \right)^{-1/2}$ with $r_{\text{L}} = v_0/\Omega_0$ the Larmor radius of an electron with velocity v_0 transverse to the external magnetic field and $d_e = c_{\sqrt{\gamma_0}}/\omega_{pe}$ the relativistic skin-depth. The growth rate indeed vanishes for $k \leq k_{\text{stab}}$ and only oscillatory solutions are admitted.

From the above Eqs. (3.2.3) and (3.2.4), we easily find that there is a critical value of the external magnetic field above which the instability is quenched. The critical value is found by imposing $\Omega_0 = v_0 \omega_{pe}/(c\sqrt{\gamma_0})$ or $r_{\rm L} = d_e$, for which $\Gamma_{\rm max}$ goes to zero and $k_{\rm stab}$ goes to infinity. The so-called critical magnetic field is given by

$$B_c = \sqrt{\gamma_0} \frac{v_0}{c} \frac{m_e \omega_{pe} c}{e} \,. \tag{3.2.5}$$

Notice also that for $B_0 > B_c$ the period of the electron gyration around B_0 is faster than the growth time of the instability computed in the absence of the external magnetic field. For values of the magnetic field $0 < B_0 < B_c$, the formation of the filaments is slowed down. This can be explained considering that, once a particle is deflected in the direction perpendicular to the initial flow, toward the center of the filament, it starts gyrating around the external magnetic field. Similar considerations explain the stabilization of modes with large wavelengths (small-k modes), Eq. (3.2.4).

Figure 3.1 shows the growth rate $\Gamma(k)$ for electron flows with velocity $v_0 = \pm 0.9 c$ and external magnetic field $B_0 = 0$ (light green line) and $B_0 = 0.75 B_c$ (dark purple line). For $B_0 = 0.75 B_c$, corresponding to a cyclotron frequency $\omega_c = eB_0/m_e\gamma_0c \simeq 0.45 \omega_{pe}$, no unstable solutions are found for $k < k_{\text{stab}} \simeq 0.33 \omega_{pe}/c$, as predicted by Eq. (3.2.4). We recall that without external magnetic field (light green line), the growth rate in the limit of small wavenumber $k^2 c^2 \ll \omega_{pe}^2/\gamma_0^3$ increases linearly as $\Gamma(k) \simeq v_0 \gamma_0 k$.

In the rest of the chapter, we always consider this large value of the external magnetic field $B_0 = 0.75 B_c$, in order to show that even if the growth rate is strongly reduced the saturation is not significantly affected. Notice that for a given value of the external magnetic field, the maximum growth rate Eq. (3.2.3) still depends on the electron drift velocity (or γ_0)



Figure 3.1: Growth rate of the instability as a function of the wavenumber for the cold plasma case (zero temperature). Analytical values for the unmagnetized (light green line) and magnetized $B_0 = 0.75 B_c$ (dark purple line) cases are computed from Eq. (3.2.2). Circles (squares) correspond to the growth rate measured in 1D3V PIC simulations with a single-mode seeded perturbation and $B_0 = 0$ ($B_0 = 0.75 B_c$).

and is reduced in the relativistic domain with increasing flow velocity. The growth rate takes its largest value for $\gamma_0 = b_0^2 + \sqrt{3 + b_0^4}$, with $b_0 = eB_0/(m_e c \omega_{pe})$. In the unmagnetized case $(b_0 = 0)$, this corresponds to $v_0 \simeq 0.82 c$.

In addition to the amplification of the magnetic field fluctuations, the linear theory predicts that E_z , the inductive component of the electric field in the flow direction, grows as fast as the magnetic field B_y , see Eq. (2.4.2). Its amplitude is proportional to $\Gamma(k)B_y/k$ and it is phase-shifted with respect to the magnetic field (the maxima of E_z being located at the nodes of B_y). At the first order, there is no total density perturbation, and the electric field E_x due to charge separation appears as a second order term.

3.2.1.2 Simulation set-up and comparison with theory

The analytical predictions of Sec. 3.2.1.1 for the linear phase of the instability are confirmed by a series of 1D3V PIC simulations. These are carried out in Cartesian geometry $[\mathbf{x} = (x, y, z)$ and considering $\nabla = \mathbf{\hat{x}}\partial_x$



Figure 3.2: a) Evolution in time of the magnetic energy (plain line) and energies associated with the E_z field (dash-dotted line) and E_x field (dashed line) for the simulation with seeded mode $k = 2.0 \omega_{pe}/c$. Light green lines refer to $B_0 = 0$ and dark purple lines to $B_0 = 0.75 B_c$. All energies are normalized to the total initial flow energy U_{k0} . b) Spatial distribution of the magnetic field B_y (plain line), electric field E_z (dashdotted line) and E_x (dashed line) for the simulation without external magnetic field in the linear phase $t \simeq 12 \omega_p^{-1}$.

in 1D3V]. In the simulations we reach the non-linear phase, discussed in the following Sec. 3.2.2.2.

We consider two symmetric cold counter-streaming electron beams with initial drift velocities $\mathbf{v}_0 = \pm v_0 \hat{\mathbf{z}}$, with $v_0 = 0.9 c$ ($\gamma_0 \simeq 2.3$, hereafter mildly relativistic case). Simulations with $\gamma_0 = 50$ (hereafter highly relativistic case) have also been performed. The mildly relativistic case is representative of both situations, unless specified. The system has initially no net current. A population of immobile ions is taken into account in order to neutralize the total charge. In this 1D geometry, the Weibel instability amplifies the perturbations with wavevector $\mathbf{k} = k\hat{\mathbf{x}}$, magnetic field $\mathbf{B} = B_y \hat{\mathbf{y}}$ and inductive electric field $\mathbf{E} = E_z \hat{\mathbf{z}}$.

In this Section, a single-mode is seeded as initial condition. This is done by imposing, at t = 0, a magnetic field perturbation $B_{y0}(x) = \delta \sin(kx)$, with $\delta = 10^{-3}$ and $\lambda = 2\pi/k$ the wavelength of the seeded mode. We consider wavenumbers in the range $0.2 < kc/\omega_{pe} < 15$. The extension of the simulation box is $L_x = 10 \lambda$ and periodic boundary conditions are used. The resolution in space is $\Delta x = \lambda/200$ and in time is set to the 95% of the CFL condition $(c\Delta t = 0.95 \Delta x)$. The number of macro-particlesper-cell is $N_p = 200$ for each species.

Figure 3.2 a) shows the evolution in time of the energy in the magnetic field B_y (plain lines), electric fields E_z (dash-dotted lines) and E_x (dashed lines) for the simulation initialized with $k = 2 \omega_{pe}/c$. Both unmagnetized $(B_0 = 0, \text{ light green lines})$ and magnetized $(B_0 = 0.75 B_c, \text{ dark purple})$ lines) cases are presented. The phase of linear growth of the magnetic energy can be clearly identified in the interval $t = (10 \div 18) \omega_{pe}^{-1}$ [$t = (15 \div 28) \omega_{pe}^{-1}$] for the unmagnetized [magnetized] case. The values of the corresponding growth rates are reported in Fig. 3.1. A very good agreement with the theory is obtained over the whole range of investigated k values, for both the unmagnetized and magnetized cases. In particular, the growth rate of the instability is found to be reduced as B_0 is increased. Similar agreement has been found for $\gamma_0 = 50$ (not shown).

Figure 3.2 a) also demonstrates the mainly magnetic nature of the Weibel instability despite E_z growing as fast as the magnetic field B_y $[E_z \sim \Gamma(k)B_y/k]$. During the linear phase, the space-charge electric field E_x appears as a second order quantity. Indeed, it starts growing at a later time with respect to the magnetic component and it grows with twice the growth rate of the instability, see Fig. 3.2 a). The generation of this electrostatic field is a nonlinear effect, the onset of its growth corresponding to the formation of the current filaments resulting in a charge separation (Palodhi et al., 2009; Dieckmann, 2009). Therefore, it has a characteristic wavelength equal to half of the λ of the unstable seeded mode, as shown in Fig. 3.2 b), for the unmagnetized case. Figure 3.2b) also shows that the E_z component of the electric field is in counter-phase with the Weibel generated magnetic field. Therefore, it tends to reduce the current of the filaments slowing the particles down, as predicted from the linear theory.

3.2.2 Nonlinear phase and saturation

3.2.2.1 Theoretical considerations

In what follows, we will make use of a single particle dynamics approach. This allows us to retrieve the saturation level predicted in the absence of an external magnetic field, while providing a better understanding of how these saturation mechanisms operate, and helps us to generalize these results to the magnetized case.

Let us consider the single particle dynamics in the fields developed during the linear stage of the instability. Despite the instability having a dominantly magnetic nature in its linear phase (see Sec. 3.2.1.1), we will consider the electron dynamics governed by the total magnetic field as well as by the inductive electric field E_z ,

$$\mathbf{B}(t,x) = B_{y0}\sin(kx)e^{\Gamma t} \,\,\mathbf{\hat{y}} + B_0 \,\,\mathbf{\hat{z}} \,, \qquad (3.2.6)$$

$$\mathbf{E}(t,x) = -E_{z0} \frac{\Gamma}{ck} \cos(kx) e^{\Gamma t} \,\,\hat{\mathbf{z}} \,, \qquad (3.2.7)$$

where $\Gamma = \Gamma(k)$ and $E_{z0} \sim B_{y0}$. The equations of motion of an electron in the fields given by Eqs. (3.2.6) and (3.2.7) read

$$\frac{dx}{dt} = \gamma_0 v_0 \frac{\hat{p}_x(t)}{\gamma(t)}, \qquad (3.2.8)$$

$$\frac{d\hat{p}_x}{dt} = -\hat{v}_z(t)\,\Omega_{y0}\sin(kx)e^{\Gamma t} + \hat{v}_y(t)\Omega_0\,,\,\,(3.2.9)$$

$$\frac{d\hat{p}_y}{dt} = -\hat{v}_x(t)\,\Omega_0\,,\qquad(3.2.10)$$

$$\frac{d\hat{p}_z}{dt} = +\hat{v}_x(t)\,\Omega_{y0}\sin(kx)e^{\Gamma t} + \mathcal{E}_{z0}\,\cos(kx)e^{\Gamma t}\,,\qquad(3.2.11)$$

where $\Omega_{y0} = -eB_{y0}/(\gamma_0 m_e c)$, $\mathcal{E}_{z0} = eE_{z0}\Gamma/(m_e c\gamma_0 v_0 k)$. In this section, momentum and velocities have been normalized such that $\hat{p}_i = p_i/(m_e \gamma_0 v_0)$ and $\hat{v}_i = v_i/v_0$ where the velocity v_0 is by definition positive $v_{z0} = v_z(t=0) = \pm v_0$. No general analytical solution can be given for this system of equations. Therefore, we first solve the system numerically, then we derive analytical solutions valid under some approximations.

Typical electron trajectories obtained by numerically solving the system of Eqs. (3.2.9)-(3.2.11) are given in Fig. 3.3 for the unmagnetized case $\gamma_0 = 2.3$, considering two values of the wavenumber $k = 0.35 \omega_{pe}/c$ (Fig. 3.3 a) henceforth referred to as the small-k case and $k = 2 \omega_{pe}/c$ (Fig. 3.3 b) henceforth referred to as the large-k case, corresponding to



Figure 3.3: Typical trajectories of electrons with initial velocity $\mathbf{v}_0 = +v_0 \hat{\mathbf{z}}$ ($\gamma_0 = 2.3$) in the electromagnetic fields developed during the linear stage of the instability: a) $k = 0.35 \omega_{pe}/c$ (small-k), b) $k = 2 \omega_{pe}/c$ (large-k). The trajectories are obtained numerically solving Eqs. (3.2.8)-(3.2.11). No external magnetic field is considered ($B_0 = 0$). In the top panel the red-blue color map highlights the spatial distribution of the Weibel generated magnetic field. Blue (red) area corresponds to regions of positive (negative) B_y .



Figure 3.4: Typical trajectories of electrons with initial velocity $\mathbf{v}_0 = +v_0 \mathbf{\hat{z}}$ ($\gamma_0 = 50$) in the electromagnetic fields developed during the linear stage of the instability: a) $k = 0.125 \,\omega_{pe}/c$ (small-k), b) $k = 0.25 \,\omega_{pe}/c$ (large-k). The trajectories are obtained numerically solving Eqs. (3.2.8)-(3.2.11). The external magnetic field is $B_0 = 0.75 B_c$. In the top panel the red-blue color map highlights the spatial distribution of the Weibel generated magnetic field. Blue (red) area corresponds to regions of positive (negative) B_y .

two different saturation mechanisms.

In both cases, we show the trajectories of the electrons with an initially positive velocity $v_{z0} = v_0 > 0$, under the effect of the fields given by Eqs. (3.2.6) and (3.2.7). These electrons will be mainly deflected toward the magnetic node in $kx = \pi$ and form a filament, the center of which being located at $kx = \pi$. The numerical results are valid up to the saturation time $t = t_{sat}$, at which $B_{sat} = B_{y0}e^{\Gamma(k)t_{sat}}$. In Fig. 3.3, the dashed areas correspond to $t > t_{sat}$ as deduced in the following section.

The two different behaviors of the particle dynamics depending on their k values are highlighted in Fig. 3.3. In the small-k case, the electrons located at the center of the filament $kx \sim \pi$ see their longitudinal velocity v_z decreased, even vanishing then changing sign. In contrast, in the largek case, all particles reach the center of the filament $kx = \pi$ with their velocity along the z-direction mainly unchanged $v_z \sim v_0$. The situation is totally symmetric if we consider particles with initial velocity $-v_0$, and the filaments form around $kx = 0, 2\pi$.

The same difference between the large-k and the small-k saturation mechanisms is recovered for the magnetized case. For reasons that will be clarified in the following section, in Fig. 3.4 we report the trajectories of electrons with initially $\gamma_0 = 50$ (instead of $\gamma_0 = 2.3$ as in Fig. 3.3) with seeded modes $k = 0.125 \omega_p/c$ (small-k, Fig. 3.4 a) and $k = 0.25 \omega_p/c$ (large-k, Fig. 3.4 b). The same features observed and discussed previously characterize the two mechanisms.

Note that in both the unmagnetized and magnetized case, at small-k, for which the saturation is reached when the particle velocity vanishes, the particles whose velocity approach the zero are initially located at $kx = \pi$, i.e. around the zero of the initial magnetic field fluctuation. This suggests that the inductive electric field E_z has a dominant role in slowing the particles down. Indeed, Bret (2016) concludes that, in the presence of a strong external field, the Weibel-generated filaments are not able to stop the flow, since a single particle dynamics study has been performed assuming a steady saturation stage with fields amplitude $B_y = B_{\text{sat}}$ and $E_z = 0$. The disagreement with our results is caused by the assumption in (Bret, 2016) on the fields stationarity. On the contrary, our numerical integration takes into account the exponential growth of

both the Weibel magnetic field B_y and the inductive field E_z , so that the saturation mechanism for which the particles velocity is strongly reduced can be observed.

Saturation mechanism in the small-k limit

In the small-k limit, saturation will be reached because particles inside the filament see their longitudinal velocity strongly reduced, hence decreasing the total current in the filament. In the absence of the external magnetic field, the saturation level can be recovered by equating the characteristic size of a filament k^{-1} with the Larmor radius $r_{\rm L} = v_0/|\Omega_{y,{\rm sat}}|$ of an electron with velocity $\pm v_0$ in the Weibel generated magnetic field. Similar estimates have already been derived in the literature considering that the saturation arises due to the Alfvén limitation of current (Alfvén, 1939). Indeed, as described in Appendix 3.A, there exists a maximum value of the current (Alfvén current) beyond which the longitudinal velocity v_z of a particle initially at the border of the filament vanishes while crossing the center of the filament, and then reverses, due to the effect of the self-generated magnetic field. This estimate of the Alfvén limit however does not account either for the fact that, in the Weibel scenario, the magnetic fields are continuously and exponentially building up, nor for the effect of the resulting inductive electric field. However by considering both these effects, we can show that we obtain the same saturation value than the (static) Alfvén limit. From Eq. (3.2.7), we see that the inductive electric field is in counter-phase with the Weibel generated magnetic field, and has its maximum at the center of the filament. The dynamics of a particle initially located at the center of the filament $kx \sim \pi$ plays a central role in the saturation of the instability as shown in Fig. 3.3. This position corresponds to a node of the magnetic field B_{y} , so that the particle dynamics will be marginally affected by the magnetic field. It will be governed by the electric field E_z , leading to the reduced equation of motion, from Eq. (3.2.11)

$$\frac{d\hat{p}_z}{dt} = -\mathcal{E}_{z0} e^{\Gamma t} \,. \tag{3.2.12}$$

Solving Eq. (3.2.12) and taking for the saturation time the moment in which the longitudinal momentum vanishes $\hat{p}_z = 0$, allows one to derive the strength of the magnetic field at saturation as

$$B_{\text{sat}}^{k\ll} = \gamma_0 \, \frac{v_0}{c} \, \frac{ck}{\omega_p} \, \frac{m_e \omega_p c}{e} \,. \tag{3.2.13}$$

This is exactly the same value as obtained from the Alfvén current limitation or Larmor radius saturation. The single particle trajectory numerical solution shows that taking into account only the temporal growth of the magnetic field B_y and neglecting the induction field E_z , would overestimate the saturation level. Indeed, Eq. (3.2.13) is valid only if one considers both the fields B_y and E_z , thus finally justifying the use of the static condition.

The static Alfvén picture can be generalized to the case with an external flow-aligned magnetic field. The calculations that we have performed in this configuration show that the saturation level increases with respect to the unmagnetized case. Considering a sinusoidal profile for the current and the magnetic field, in a 1D configuration, and calculating the field that corresponds to $\hat{p}_z = 0$ for a particle moving toward the center, the predicted saturation value is

$$B_{\text{sat}}^{k\ll} = f(A)\gamma_0 \frac{v_0}{c} \frac{ck}{\omega_p} \frac{m_e \omega_p c}{e}, \qquad (3.2.14)$$

with $f(A) = [\cos(\pi/2(1-A))]^{-1} > 1$ for A < 1, and f(A) = 1 for $A \ge 1$, with $A = v_0/|\Omega_0 x_0|$ and $x_0 = \lambda/4$ the particle initial position. Equation (3.2.13) is recovered in the limit $A \gg 1$. The detailed derivation is given in Appendix 3.A.

Saturation mechanism in the large-k limit

In the large-k limit, the particle longitudinal velocity is mainly unchanged $v_z \sim v_0$ and thus the saturation follows from a different mechanism with respect to what was discussed for the small-k modes. Saturation is expected once all particles have been injected inside the filament, whose center is in $kx = \pi$ and has an extension of the order of $\simeq \pi/2$ (see

Figs. 3.3 and 3.4). Thereafter no additional particles can be found to increase the current and contribute to the instability growth. The current of all the particles with velocity v_0 in a filament of diameter $\sim \lambda/2$ remains much smaller that the Alfvén limit in the large-k limit. Moreover, the contribution of the inductive field E_z can be neglected in the limit of large k for which $\Gamma(k)/ck \ll 1$ [see Eq. (3.2.7)]. Indeed, numerically solving Eqs. (3.2.8)-(3.2.11) with or without E_z (not shown) does not affect the particle trajectories. In the unmagnetized case, neglecting the effect of the electric field and considering $v_z \sim \pm v_0$, the system of Eqs. (3.2.9)-(3.2.11) leads to an ordinary differential equation for the normalized particle position $\xi(t) = kx(t)$

$$\frac{d^2\xi}{d\tau^2} = -\alpha \,\sin(\xi)\,\exp(\tau)\,,\qquad(3.2.15)$$

with $\tau = \Gamma t$ and $\alpha = \operatorname{sgn}\{v_{z0}\} v_0 k \Omega_{y0} / \Gamma^2$, with initial conditions $\xi(\tau = 0) = kx(t=0) = \xi_0$ and $d\xi/d\tau|_{\tau=0} = 0$.

Considering a particle initially located at a maximum/minimum of the magnetic field $\xi_0^{\pm} = \pi \pm \pi/2$, leads to

$$\xi^{\pm}(t) = \xi_0^{\pm} \mp \alpha \, \left[\exp(\tau) - \tau - 1 \right]. \tag{3.2.16}$$

The particle sees its velocity $v_x \propto e^{\tau}$ exponentially increasing with time, and depending on the sign of v_{z0} , the particle will head toward one or the other node of the magnetic field, hence spatially segregating particles with opposite velocities in well separated currents of opposite directions. Taking the limit $\tau \gg 1$, one can extrapolate the time $\tau^* \sim \ln(\pi/|2\alpha|)$ at which the particle reaches the node of the magnetic field, and infer from this the corresponding magnetic field amplitude at saturation $B_{\text{sat}} = B_{y0} \exp(\tau^*)$, for the unmagnetized case, leading to

$$B_{\rm sat}^{k\gg} = \frac{\pi}{2} \, \frac{\gamma_0 \Gamma_0^2}{v_0 k} \, \frac{m_e c}{e} \,. \tag{3.2.17}$$

In the literature (Davidson et al., 1972; Achterberg et al., 2007), the magnetic field strength at saturation for large-k modes has been also computed by equating the so-called bouncing frequency ω_b in the magnetic

field at saturation with the growth rate of the instability. Computing the bouncing frequency of an electron in the saturation field given by Eq. (3.2.17) would indeed give

$$\omega_b = \left(\frac{ev_0 k B_{\text{sat}}^{k\gg}}{\gamma_0 m_e c}\right)^{1/2} \simeq \Gamma.$$
(3.2.18)

While Eq. (3.2.17) leads a prediction similar to Eq. (3.2.18), it highlights that saturation is obtained because all particles are injected and trapped into the filament.

The criterion on the bouncing frequency can be generalized in the presence of an external magnetic field. The bouncing frequency in this case becomes

$$\Omega_b = \sqrt{\omega_b^2 + \Omega_0^2} \,, \tag{3.2.19}$$

with ω_b defined in Eq. (3.2.18). Considering that saturation is reached when the bouncing frequency equates the growth rate of the instability (Γ approximately the asymptotic value reported in Eq. (3.2.3) for large k), the expected saturation level would depend on the strength of the external magnetic field (see Stockem et al., 2008).

To generalize the result of Eq. (3.2.17) in the case of an external flow-aligned magnetic field, we calculate the magnetic field strength by considering that saturation occurs when all the electrons participate to the current filament. As a consequence, we can show that the saturation level of the instability is independent of the external magnetic field. We proceed as in the unmagnetized case: (i) we assume that the particle velocity is not drastically reduced at saturation $v_z(t) \sim \pm v_0$, and (ii) we neglect the effect of the longitudinal field E_z on the particle motion, as $\Gamma(k)/k \ll 1$ in the large-k limit. Both assumptions are found to be in good agreement with the numerical solution of Eqs. (3.2.8)-(3.2.11), even in the presence of E_z . One can write the equations of motion for a particle initially close to the maximum of the magnetic field, using $\sin(kx) \simeq 1$, in the form

$$\frac{d^2 v_x(t)}{dt^2} = v_0 \Gamma \Omega_{y0} e^{\Gamma t} - \Omega_0^2 v_x \,. \tag{3.2.20}$$

Looking for exponentially growing solution $v_x = v_{x0}e^{\Gamma t}$, as inferred from

the unmagnetized case, the particle displacement $\delta x = x - x_0$ reads $\delta x = v_{x0}e^{\Gamma t}/\Gamma$. The saturation level is obtained for $\delta x(t_{\text{sat}}) \simeq \lambda/4$, leading to

$$B_{\text{sat}}^{k\gg} = \frac{\pi}{2} \frac{\gamma_0 \left(\Gamma^2 + \Omega_0^2\right)}{v_0 k} \frac{m_e c}{e}.$$
(3.2.21)

In the limit $B_0 = 0$, we recover the result of Eq. (3.2.17). Moreover, Eq. (3.2.21) predicts that the saturation level does not depend on the application of the external magnetic field for large-k. Indeed, the growth rate $\Gamma(k)$ decreases with the application of the external magnetic field Eq. (3.2.3), but this variation is exactly compensated by the term Ω_0^2 in Eq. (3.2.21), since the maximum value of the growth rate is $\Gamma^2 \sim \Gamma_0^2 - \Omega_0^2$, with Γ_0 the growth rate in the absence of external magnetic field. This is in contradiction with the estimate obtained considering the bouncing frequency but it is found to be confirmed by PIC simulations, as will be shown in the following Sec. 3.2.2.2.

3.2.2.2 Saturation phase in the PIC simulations

In this section we compare the theoretically predicted saturation level with the 1D3V PIC simulations presented in Sec. 3.2.1.2. The expected field strength at saturation is shown in Fig. 3.5 as a function of the wavenumber, for two initial velocities corresponding to $\gamma_0 \simeq 2.3$ (mildly relativistic case) and $\gamma_0 = 50$ (ultra-relativistic case). We recall that these 1D3V simulations account for a single mode seeded at early time. In order to measure the saturation level, we perform a Fourier spectrum of $B_y(x,t)$ and consider the maximum magnetic field of the given k mode. Each unstable mode saturates because of the mechanism that predicts the lowest saturation value. The maximum magnetic field is found at the intersection between the curves corresponding to the Alfvén limit Eq. (3.2.14) and the trapping mechanism Eq. (3.2.21). This can be analytically computed in the unmagnetized case as

$$\frac{ck^*}{\omega_{pe}} = \left[\frac{\pi}{2\gamma_0} \left(1 - \frac{2}{\pi\gamma_0^2}\right) \frac{1}{\left(1 + \frac{2v_0^2}{\pi c^2}\right)}\right]^{1/2}.$$
 (3.2.22)



Figure 3.5: Magnetic field strength at saturation. Values predicted by the "trapping mechanism" Eq. (3.2.21) are shown as dashed line. Values predicted by the Alfvén limitation mechanism Eq. (3.2.14) are shown as plain lines for the unmagnetized (light green) and for the magnetized (dark purple) cases. Circles (squares) are the values measured in PIC simulations seeded with a single-mode perturbation and $B_0 = 0$ ($B_0 =$ $0.75 B_c$). Two initial flow velocities are considered: a) $\gamma_0 = 2.3$ (mildly relativistic case), b) $\gamma_0 = 50$ (ultra-relativistic case).

For the mildly relativistic case this value is $k^* \simeq 0.63 \omega_{pe}/c$ and for the ultra-relativistic case $k^* \simeq 0.14 \omega_{pe}/c$. In the presence of an external magnetic field, the wavenumber at the intersection between the two curves has been computed numerically and give $k_{B_0}^* \simeq 0.60 \omega_{pe}/c$ and $k_{B_0}^* \simeq 0.13 \omega_{pe}/c$ for the mildly and ultra-relativistic case, respectively. It is clear that the Alfvén limit cannot be the dominant saturation mechanism for large wavenumbers. This can be easily understood as the magnetic energy (increasing with k) would exceed the total kinetic energy of the beams. The saturation would appear for lower values due to the trapping mechanism. Nevertheless for the modes with small k, the Alfvén mechanism is responsible for the saturation of the instability.

Figure 3.5 reports the measured saturation level for different unstable modes, for the two initial velocities in unmagnetized plasma and with $B_0 = 0.75 B_c$. The trapping saturation mechanism is the dominant one for $k > k^*$. In this regime the theoretical predictions of Eq. (3.2.21) show a very good agreement with the simulations, confirming the independence of the saturation level from the external magnetic field. For wavenumbers

 $k < k^*$ and $B_0 = 0$, the Alfvén limit accurately reproduces the data. In the magnetized case two different behaviors are observed for highly relativistic ($\gamma_0 = 50$) and mildly relativistic ($\gamma_0 \simeq 2.3$) flows. In the first case the saturation level is slightly increased, as predicted by the generalized Alfvén limit in a magnetized plasma, Eq. (3.2.14). On the contrary in the mildly relativistic case Fig. 3.5 a), the saturation level decreases with the application of B_0 .

The discrepancy between Eq. (3.2.14) and the numerical simulations in the mildly relativistic case is due to the fact that the single-mode analysis does not hold anymore. With the application of the external magnetic field, the growth rate is decreased, so that the time required to reach saturation is increased. This results in the harmonics of the initial k becoming important before the considered seeded mode reaches its saturation. In particular, we observe the growth of the third harmonic with a growth rate close to three times the one of the seeded mode \sim $3\Gamma(k)$. This prevents the seeded mode reaching its own (independent) saturation level. This effect is strongly reduced in the ultra-relativistic limit where in the simulations a much weaker signal for the third harmonic is observed. Analysis of the mildly relativistic simulations confirms that the saturation via the Alfvén limit is not reached: the velocity along the flow direction does not vanish. The total energy that is expected to be transferred to the magnetic field is instead distributed in the two modes, the seeded one with wavenumber k and the harmonic at 3k.

In presence of harmonics, the single mode saturation criterion Eq. (3.2.14) cannot be applied. However we can consider that saturation is associated to a redistribution of kinetic energy into magnetic field energy, so that the overall level of conversion into one mode and its harmonics has to be roughly the same as in the single mode case. It is then useful to calculate the ratio of the magnetic energy density over the kinetic energy. Indeed, the Alfvén limit Eq. (3.2.13) can also be interpreted as an energy equipartition relation for the most unstable mode $(kc/\omega_{pe} \sim 1)$, the equipartition condition being defined as

$$\frac{B_{\rm sat}^2/8\pi}{n_0(\gamma_0 - 1)m_ec^2} = \frac{1}{2}.$$
(3.2.23)



Figure 3.6: Spatial distribution of the total current J_z (dashed lines) in the initial direction of the flows, Weibel-generated magnetic field B_y (plain line) and flow-aligned magnetic field B_z (dash-dotted lines) for the mildly relativistic ($\gamma_0 = 2.3$) simulation in the small-k limit $k = 0.35 \omega_{pe}/c$ in the linear phase. a) unmagnetized case $B_0 = 0$ at $t \simeq 12.5 \omega_{pe}^{-1}$, b) magnetized case $B_0 = 0.75 B_c$ at $t \simeq 26.5 \omega_{pe}^{-1}$.

Actually, the saturation level for the most unstable k saturating via the Alfvén mechanism in Fig. 3.5, gives an energy ratio Eq. (3.2.14), smaller than 15% for the mildly relativistic case and 10% for the ultra-relativistic one, roughly independent from the external magnetic field. Similar levels of equipartition were already observed in simulations (Califano et al., 1998; Medvedev and Loeb, 1999). The predicted equipartition level, calculated with $B_{\rm sat}$ from Eq. (3.2.14), in the mildly relativistic case for $k = 0.35 \omega_{pe}/c$ (representative of the small-k limit) is ~ 2%. This is much larger than the value one would obtain considering the single mode saturation (value of $B_{\rm sat}$ measured in the simulation, reported in Fig. 3.5a) but it is comparable (1.8%) if the contributions of the two modes (seeded and third harmonic) are considered.

Since the harmonic is weaker in the ultra-relativistic case, the agreement with the theoretical curve is significantly improved, Fig. 3.5 b). This confirms that Eq. (3.2.14) is only valid for single mode. In the presence of higher harmonics the current filament profile evolves from a sinusoidal shape to a double peaked structure, see Fig. 3.6 b), where current fila-



Figure 3.7: (x, p_z) -phase space distribution at the saturation for the simulations initialized with a single mode in the mildly relativistic case $\gamma_0 = 2.3$. In the small-k limit, $k = 0.35 \omega_{pe}/c$: a) $B_0 = 0$, b) $B_0 = 0.75 B_c$. In the large-k limit, $k = 2.0 \omega_{pe}/c$: c) $B_0 = 0$, d) $B_0 = 0.75 B_c$.

ments are formed of two consecutive maxima or minima. The electron density has the same profile as the current J_z , meaning that the particles are concentrated in the two spikes at the edge of the filament, and the hypothesis of sinusoidal profile used to derive Eq. (3.2.14) breaks down. The competition between different modes will be addressed in Sec. 3.3.

The signature of the two different saturation mechanisms can be clearly observed in the PIC simulations. Figure 3.7 shows the phase space $x \cdot p_z$ for the simulations with $\gamma_0 = 2.3$, for a small-k mode ($k = 0.35 \omega_{pe}/c$) and for a large one ($k = 2 \omega_{pe}/c$) with and without external magnetic field, at the time corresponding to their own saturation. We chose two modes that saturate at the same value of B_y but for the two different mechanisms, the Alfvén limit for small-k and the trapping mechanism for the large-k. With large wavenumber, Figs. 3.7 c) and 3.7 d), the flow kinetic energy associated with the motion along the z-direction is still large at saturation, and the value of p_z is close to the initial one $p_z(t=0) \simeq \pm 2.1 m_e c$, typical of the trapping mechanism. In the case of small-k and $B_0 = 0$, the particles responsible for the saturation lie in the region $p_z \simeq 0$, Fig. 3.7 a), as expected from the Alfvén limit. As already discussed, in the mildly relativistic case, adding the external magnetic field, the harmonics of the seeded mode set in before the mode saturates. These harmonics are clearly seen in Fig. 3.7 b). The gain of momentum along the z-direction up to twice the initial value, observed in Fig. 3.7 for all simulations, is associated with the small fraction of particles trapped in the region occupied by the filaments flowing in the opposite direction, as previously observed by D'Angelo et al. (2015) in the case of counterpropagating electron-positron plasmas.

3.3 Temperature and multi-mode effects

The introduction of an initial temperature has two effects. On the one hand it affects the single mode growth rate, on the other hand it allows for the growth of a broad spectrum of magnetic perturbations from the intrinsic electromagnetic fluctuations of a thermal plasma. In a PIC code these two effects can not be separated. The modification of the growth rate can be studied for a single mode and compared with PIC simulations in the linear phase when all modes grow independently. However, saturation of the instability most often involves multi-mode evolution. We present the studies of the linear and non-linear phases in the following sections.

3.3.1 Linear phase

3.3.1.1 Relativistic warm fluid theory

For the sake of analytical tractability, we use the relativistic fluid approach described in Sec. 2.3, including the thermal pressure of the relativistic plasma flows. We remind that the limit of validity of this approach corresponds to the ratio Γ/k being larger than the thermal velocity. This limit involves the necessity of an adiabatic closure for the system of fluid

equations, as discussed in Sec. 2.4.3.

We consider two symmetric counter-propagating beams and an external magnetic field \mathbf{B}_0 aligned with the flow velocity. In order to properly describe a plasma with arbitrary flow velocity and temperature, we use the Maxwell-Jüttner distribution function defined in Eq. (2.2.7). We recall that the normalized enthalpy is $h(\mu) = k_{32}(\mu) = K_3(\mu)/K_2(\mu)$, with $\mu = T_0/m_e c^2$ the normalized temperature in the plasma rest frame, see Table (2.2.1). One proceeds as in Sec. 3.2.1 and obtains the dispersion relation for the purely transverse Weibel modes

$$\frac{\omega^2}{c^2} - k^2 - \frac{\overline{\omega}_{pe}^2}{c^2 \gamma_0} \left(\frac{1}{\gamma_0^2} + \frac{v_0^2 k^2}{\omega^2 - \overline{\Omega}^2(k)} \right) = 0, \qquad (3.3.1)$$

where $\overline{\omega}_{pe}^2 = \omega_{pe}^2/h(\mu)$, $\overline{\Omega}^2(k) = \overline{\Omega}_0^2 + \gamma_0^{-1}\Gamma_{\rm ad}(\mu)\overline{v}_{th}^2k^2$ with $\overline{\Omega}_0^2 = \Omega_0^2/h^2(\mu)$ and $\overline{v}_{th} = [\mu h(\mu)]^{-1/2}$. From Eq. (2.4.17), we derive the growth rate of the instability

$$\Gamma(k) = \frac{1}{\sqrt{2}} \left[\sqrt{\left(k^2 c^2 + \frac{\overline{\omega}_{pe}^2}{\gamma_0^3} - \overline{\Omega}^2(k)\right)^2 + 4\frac{\overline{\omega}_{pe}^2}{\gamma_0}k^2v_0^2} - \left(k^2 c^2 + \frac{\overline{\omega}_{pe}^2}{\gamma_0^3} + \overline{\Omega}^2(k)\right) \right]^{1/2}.$$
(3.3.2)

In the limit $T_0 = 0$, we recover the prediction of the cold fluid theory, Eq. (3.2.2). From Eqs. (2.4.17) and (3.3.2), we can deduce the range of unstable wavenumbers. The main effect of the temperature is to strongly reduce the instability growth rate at large k. The instability is completely quenched for wavenumbers larger than

$$c^{2}k_{\text{cut-off}}^{2} = \frac{\gamma_{0}}{2\Gamma_{\text{ad}}\overline{v}_{th}^{2}} \left[\frac{\overline{\omega}_{pe}^{2}v_{0}^{2}}{\gamma_{0}c^{2}} - \frac{\overline{\omega}_{pe}^{2}\Gamma_{\text{ad}}\overline{v}_{th}^{2}}{\gamma_{0}^{4}c^{2}} - \overline{\Omega}_{0}^{2} + \sqrt{\left(\frac{\overline{\omega}_{pe}^{2}v_{0}^{2}}{\gamma_{0}c^{2}} - \frac{\overline{\omega}_{pe}^{2}\Gamma_{\text{ad}}\overline{v}_{th}^{2}}{\gamma_{0}^{4}c^{2}} - \overline{\Omega}_{0}^{2}} \right)^{2} - 4\frac{\overline{\Omega}_{0}^{2}\overline{\omega}_{pe}^{2}\Gamma_{\text{ad}}\overline{v}_{th}^{2}}{\gamma_{0}^{4}c^{2}} \right]. \quad (3.3.3)$$



Figure 3.8: Growth rate of the instability as a function of the wavenumber. Theoretical predictions are computed from Eq. 3.3.2 assuming adiabatic closure (plain lines). a) Quasi-cold $B_0 = 0$ (light green lines) and $B_0 = 0.75 B_c$ (dark purple lines). b) Warm cases $B_0 = 0$ (light green lines) and $B_0 = 0.75 B_c$ (dark purple lines). PIC simulations with $B_0 = 0$ (circles) and $B_0 = 0.75 B_c$ (squares).

Indeed, the thermal motion of the particles in the direction transverse to the flow prevents their confinement in the filaments. Figure 3.8 shows the growth rate of the instability as a function of the wavenumber. The adiabatic closure has been exploited, with adiabatic index $\Gamma_{\rm ad} = 3$ for the unmagnetized case and $\Gamma_{\rm ad} = 2$ for the magnetized one. Indeed, with the introduction of a flow-aligned magnetic field, isotropization is achieved in the two transverse directions (2D adiabatic index) while in the unmagnetized case the flow is isotropized only along the direction of $\mathbf{k} = k_x \hat{\mathbf{x}}$ (1D adiabatic index).

The range of modes amplified by the instability is clearly dependent on the temperature: the higher the temperature, the smaller the value of $k_{\text{cut-off}}$. Qualitatively, the difference between unmagnetized/magnetized systems can be explained considering that the growth rate of the instability decreases with the introduction of B_0 . In order to allow the instability to grow, a particle should remain in the region where the filament forms for a time of the order of Γ^{-1} . The larger the external magnetic field, the longer the required interval of time. Hence with equal temperatures, small filaments are less likely to form in the magnetized case, and the value of $k_{\text{cut-off}}$ decreases. Note that in Eq. (3.3.1) the magnetic field appears in $\Omega_0^2/h^2(\mu)$. Therefore qualitatively a large temperature, increasing the enthalpy, can reduce the efficiency of the external field in quenching the instability. To properly investigate this effect, a kinetic relativistic approach should be required, since for $\Gamma \simeq 0$ and finite temperature the fluid approach is not accurate. In the following Sec. 3.4.5, we provide a description based on the simulation results.

3.3.1.2 Simulation set-up and comparison with linear theory

In order to investigate the temperature effects and the interplay between the various growing modes, we present a series of 1D3V simulations with, at initial time, a broad spectrum of modes seeded exploiting the intrinsic electromagnetic fluctuations of a finite-temperature plasma at equilibrium. The two electron populations are uniformly distributed in space and have a Maxwell-Jüttner distribution function in the momentum space. The implementation in the PIC code of the relativistic drifting Maxwell-Jüttner distribution follows the algorithm presented by Zenitani (2015). Two series of simulations are carried out with temperature $T_0 \simeq 3.2 \times 10^{-4} m_e c^2$ [correspondingly $T_{\rm L} \simeq 10^{-4} (\gamma_0 - 1) m_e c^2$ in the laboratory frame] and $T_0 \simeq 0.1 m_e c^2 [T_{\rm L} = 3.3 \times 10^{-2} (\gamma_0 - 1) m_e c^2]$ referred to in the following as quasi-cold case and warm case, respectively. The length of the simulation box is $L_x \simeq 50 c/\omega_{pe}$ and the cell extension is $\Delta x = \lambda_{\rm De}/2$, where $\lambda_{\rm De}$ is the Debye length $\lambda_{\rm De} = \sqrt{T_{\rm L}/(4\pi n_0 e^2)}$. The time resolution is $c\Delta t = 0.95\Delta x$ and we use 2000 macro-particles-per-cell per species.

The growth rate of different modes has been extrapolated from PIC simulations performing a Fourier analysis and measuring the growth of each mode independently. Results are reported in Fig. 3.8 for the two temperatures (quasi-cold and warm cases), with and without external magnetic field ($B_0 = 0$ and $B_0 = 0.75 B_c$). Theoretical predictions from Eq. (3.3.2) are also shown considering the appropriate adiabatic closure (solid lines). A fairly good agreement is found between PIC simulations and theory. The deviation from the predicted values in the magnetized quasi-cold case, appears in a region of the Γ -k space at the limit of validity of our fluid approach, where kinetic effects might play an important



Figure 3.9: a) Evolution of the magnetic energy of the field B_y . b) Spectrum of B_y at $t = 30 \omega_{pe}^{-1}$. c) Spectrum of B_y at the end of the simulations $t = 120 \omega_{pe}^{-1}$. Quasi-cold simulations (plain lines), warm simulations (dashed lines), with $B_0 = 0.0$ (light green lines) and $B_0 = 0.75 B_c$ (dark purple lines). Spectra are shown after the application of Savitzky-Golay filter (Savitzky and Golay, 1964) in k-space to reduce the noise.

role, as investigated by Sarrat et al. (2016). Our results nevertheless suggest that the proposed relativistic fluid approach, which gives tractable solutions for the growth rate, is relevant to model the Weibel instability in the regimes discussed here.

3.3.2 Nonlinear phase and saturation

We now focus on the non-linear phase and saturation of the instability. Figure 3.9 a) shows the evolution of the energy U associated with the Weibel generated magnetic field B_y normalized to the total initial flow energy U_{k0} . In the quasi-cold simulations (plain lines), the saturation level is modified by the application of the external field B_0 . For the magnetized plasma (dark purple line), saturation is reached at $t_{\text{sat,B}_0} \simeq$ $25 \,\omega_{pe}^{-1}$, identified by the clear change in the slope in Fig. 3.9 a). This stage corresponds to the saturation of the modes with large wavenumbers $k \gtrsim$ $10 \,\omega_{pe}/c$, that grow with the largest rate, see Fig. 3.8 a), and saturate with a low level of the magnetic field, as predicted by the trapping mechanism, see Fig. 3.5 a). Indeed, at that time, the amplitude of the oscillations of the magnetic field $B_y \simeq 0.12 \,m_e \omega_{pe} c/e$, is consistent with the saturation predicted for those modes. The saturation of these modes occurs at the same level of U for the simulation with $B_0 = 0$ (light green line) around $t^* \simeq 16 \, \omega_{pe}^{-1}$. Note that $t^* < t_{\text{sat,B}_0}$ as expected due to the larger growth rate in the absence of the external magnetic field.

After this first saturation stage, the magnetized and unmagnetized cases evolve differently. In Fig. 3.9 a), the two plain lines do not reach the same level. Even at late time (not shown here), the slow rise in the magnetized curve ceases, the energy reaches an asymptotic value slightly lower than the unmagnetized one and remains constant after $t \simeq 300 \, \omega_{pe}^{-1}$.

In the unmagnetized plasma, once the large wavenumber modes have reached saturation, the small-k modes keep growing up to their own saturation level. The growth of small k filaments involves a rearrangement in large structures of the particles with opposite flow velocity. In the magnetized case, in order to create filaments with small k, not only the currents should be redistributed but also the external magnetic field lines, that during the linear phase are compressed inside the filaments. This process entails a slowdown in the growth of small-k modes, hence the very low slope in Fig. 3.9 a). Thus, with the introduction of the external magnetic field the large-k modes remain stable after their saturation and this affects the growth of the modes not yet saturated. This is clearly shown in the spectra of the magnetic field B_y reported in Fig. 3.9 b) for all four simulations at $t = 30 \omega_{pe}^{-1}$ and at $t = 120 \omega_{pe}^{-1}$. In the quasi-cold magnetized case the spectrum is dominated by the large-k modes, while in the unmagnetized case there is a dominant mode with $k \simeq 1.2 \omega_{pe}/c$.

The increase of the initial temperature, limits the range of unstable wavenumbers due to the temperature effect of stabilizing the large-k modes, Fig. 3.8 b). In this way, the saturation level becomes again independent from the external magnetic field, Fig. 3.9 a) (dashed lines). The spectrum of B_y at the saturation is peaked around $k \simeq 0.7 \omega_{pe}/c$ for the unmagnetized case (dashed light green line) and $k \simeq 0.9 \omega_{pe}/c$ for the magnetized case (dashed dark purple line). The peak values are in good agreement with the k predicted to have the highest saturation level in the cold single-mode model $k^* = 0.63 \omega_{pe}/c$, Fig. 3.5 a).

To summarize, with initially large temperature the saturation level does not depend on the application of the external flow-aligned magnetic field and the spectra are peaked around the optimal value found in the cold case, while at low temperature the energy transfer towards small-k filaments is hampered by the external magnetic field, resulting in a lower saturation amplitude and a wider distribution in k.

3.3.3 Late merging phase

At later times, after the saturation of the instability, $t > 30 \,\omega_{pe}^{-1}$ referring to the simulations presented in Fig. 3.9, the so-called merging or coalescence of filaments phase governs the dynamics of the system. During this phase the total energy in the magnetic field remains roughly constant, see Fig. 3.9 a). The merging of two filaments is the result of the attractive force between filaments of parallel current. Regarding the spectrum of the Weibel generated magnetic field, the coalescence of filaments involves a shift toward small wavenumber modes as it creates structures of increased transverse size in the current and accordingly in the magnetic field. Simplified models for the coalescence of filaments in cylindrical geometry have been presented by Medvedev et al. (2005); Achterberg et al. (2007). In our 1D geometry, the merging of filaments could be quite unexpected. Indeed, in order to observe the coalescence, the attractive force between two filaments of parallel current should overcome the repulsive force due to the filament of opposite current in the middle of them. Thus, a series of equal positive and negative current filaments would produce a stable situation, the attractive and repulsive force balancing each other. This is the case in single-mode simulations, as illustrated in Fig. 3.10, where the evolution in time of the current J_z of the two counter-streaming electron beams and the Weibel generated magnetic field B_y are shown for $k = 2 \omega_{pe}/c$. However, in the case of an initial broad spectrum of unstable modes, merging can occur as this balance is not achieved due to (i) the intrinsic irregularity in the filament spatial distribution and (ii) the effect of the inductive electric field, as detailed below.

In Fig. 3.11 the evolution in time of J_z and the B_y are shown for the unmagnetized quasi-cold simulation. At time t = 0 the total current vanishes, then the filaments start to develop and the saturation is reached at $t \simeq 30 \omega_{pe}^{-1}$. The magnetic energy in Fig. 3.9 a) remains constant after saturation and events of coalescence are clearly shown in Fig. 3.11.

Coalescence processes, even if in a 1D configuration can be explained



Figure 3.10: a) Evolution of the current J_z of the two counter-streaming beams. b) Evolution of the Weibel generated magnetic field B_y , for the unmagnetized case with seeded mode $k = 2\omega_{pe}/c$.



Figure 3.11: a) Evolution of the current J_z of the two counter-streaming beams. b) Evolution of the Weibel generated magnetic field B_y , for the unmagnetized quasi-cold case.



Figure 3.12: Spectrum of B_y (plain line) and E_z (dashed line) in the linear phase $t = 10 \omega_{pe}^{-1}$, for the unmagnetized quasi-cold case. Spectra are shown after the application of Savitzky-Golay filter (Savitzky and Golay, 1964) in k-space to reduce the noise.

as follows. In this series of simulations, the instability starts from a broad spectrum of modes. As already pointed out this leads to an intrinsic irregularity (randomness) in the filament spatial distribution. Furthermore this entails a difference in the spectrum of the Weibel-generated magnetic field B_y (plain line) and the spectrum of inductive electric field E_z (dashed line), as shown in Fig. 3.12. This difference in the k-space can be explained considering a broad spectrum in the magnetic field B_y in the linear phase of the instability as

$$B_y(x,t) = \sum_n \tilde{B}_{y0,n} \sin(k_n x) e^{\Gamma(k_n)t}, \qquad (3.3.4)$$

the sum running over all wavenumbers. Using Maxwell-Faraday equation to compute the inductive electric field E_z , we obtain

$$E_z(x,t) = \sum_n \tilde{B}_{y0,n} \frac{\Gamma(k_n)}{ck_n} \cos(k_n x) e^{\Gamma(k_n)t} . \qquad (3.3.5)$$

We assume the same amplitude for each mode at early time, so that $B_{y0,n}$ is independent from k_n . Due to the factor $\Gamma(k)/k$, considering $\Gamma(k)$ as

calculated from Eq. (3.3.2), the inductive electric field E_z vanishes at large k, so that its spectrum peaks at small k.

Despite the amplitude of E_z is smaller than the amplitude of B_y , it can play a key role due to the different spectrum with respect to B_y . Since the electric field has a peak in the spectrum at small k, i.e. large wavelengths, it can have opposite effect on two neighbor filaments with opposite current, corresponding to a mode with large k. E_z accelerates the particles of one filament while decelerating the other. The unbalance produced in the current allows for the merging of the filaments. The attractive force between two filaments of positive currents, whose particles are accelerated by E_z , exceeds the repulsive force due to the negative filament in the middle, for which E_z is decelerating, resulting in the coalescence of the positive currents. In the simulations with single seeded mode, E_z and B_y have the same periodicity, see Eqs. (3.2.6) and (3.2.7), so that E_z tends to slow the electrons of both the counter-streaming beams down. The filaments form a regular structure of identical positive and negative filaments, and merging is not be observed, see Fig. 3.10.

At saturation, in all simulations, except in the magnetized quasi-cold case, the spectra of the magnetic field B_y have a peak for $k \leq \omega_{pe}/c$, as shown in Fig. 3.9 b). The corresponding spectra at the end of the simulation $t = 120 \,\omega_{pe}^{-1}$, show that the peak is increased, narrower and slightly shifted toward a lower k. After the saturation the energy in the magnetic field is constant, Fig. 3.9 a), thus the evolution of the peak is a signature of the merging events, that transfer energy to the modes with large wavelengths. In the quasi-cold simulation, the presence of the external magnetic field produces a broad spectrum of modes at the saturation, that remains much broader than in the other cases, also at the end of the simulation. The coalescence of filaments is hampered by the external magnetic field, since an additional energy is required to move the magnetic field lines. This can be deduced from the comparison between Fig. 3.13 and Fig. 3.14, corresponding to the simulations initialized with warm plasma in the unmagnetized and magnetized cases, respectively. Merging events are observed in the unmagnetized case around $t \simeq 40 \,\omega_{pe}^{-1}$, after the saturation stage, while the filamentary structure remains unvaried in the magnetized case.



Figure 3.13: a) Evolution of the current J_z of the two counter-streaming beams. b) Evolution of the Weibel generated magnetic field B_y , for the unmagnetized warm case.



Figure 3.14: a) Evolution of the current J_z of the two counter-streaming beams. b) Evolution of the Weibel generated magnetic field B_y , for the magnetized warm case.

3.4 2D simulations

In this section we present the results of a series of two-dimensional simulations that allow to both confirm and extend the main findings of the previous 1D study.

Two different 2D configurations can be investigated. Considering x-y as the simulation plane, the electron counter-streaming beams can either propagate in the direction perpendicular to the simulation plane $\mathbf{v}_0 = \pm v_0 \,\hat{\mathbf{z}}$, or have their drift velocity into it $\mathbf{v}_0 = \pm v_0 \,\hat{\mathbf{x}}$. In the first configuration, both B_x and B_y components of the magnetic field are amplified by the Weibel instability. The wavevector lies in the simulation plane, so that only purely transverse modes can be investigated, as in the 1D simulations. In the other case, Weibel modes produce magnetic field modulations B_z , with wavevector $\mathbf{k} = k_y \,\hat{\mathbf{y}}$. In addition, this configuration accounts for the development of longitudinal modes, with wavevector parallel to the flow direction. This might be useful to investigate in the late stage of the instability the competition between transverse, longitudinal and oblique instabilities, as studied analytically in the linear phase by Bret et al. (2010b).

We expect major differences with the 1D study in the late non-linear phase, after saturation of the instability. Especially the process of coalescence is known to critically depend on the system geometry.

3.4.1 Simulation set-up

The simulations presented in this section have been carried out in a 2D geometry and both the above described configurations (corresponding to initial flow velocity $\mathbf{v}_0 = \pm v_0 \hat{\mathbf{x}}^1$ or $\mathbf{v}_0 = \pm v_0 \hat{\mathbf{z}}$, with $v_0 = \pm 0.9 c$) have been tested, in the unmagnetized case and with flow-aligned magnetic field $B_0 = 0.75 B_c$. In order to make a comparison with the results of Sec. 3.3, we use the same temperatures $T_1 \simeq 3.2 \times 10^{-3} m_e c^2$ (quasi-cold case) and $T_2 \simeq 0.1 m_e c^2$ (warm case). The resolution in space for the

¹This configuration can be affected by *numerical Cherenkov* emission, as will be described in Chap. 5. In order to remove this unphysical effect, we used the Friedman filter with $\theta = 0.05$ and 2-passes current filtering, see Sec. 5.3. The efficiency of these techniques on this system is discussed in Sec. 5.4.

simulation initialized with temperature T_1 is $(c/\omega_{pe})/128$, corresponding to $\simeq 0.43 \lambda_{\text{De},1}$, and for the simulation initialized with temperature T_2 is $(c/\omega_{pe})/32$, corresponding to $\simeq 0.1 \lambda_{\text{De},2}$. The time resolution is $c\Delta t = 0.5 \Delta x$ and 50 particles-per-cell per species have been used. The simulation box extends over $(32 c/\omega_{pe} \times 32 c/\omega_{pe})$ and periodic boundaries are used for both particles and fields.

3.4.2 Linear phase and saturation

The linear phase of the instability, and in particular the value of the growth rate, is not affected by the different geometry, as shown in Fig. 3.15, where the evolution in time of the Weibel-generated magnetic field energy density is reported for the quasi-cold (left frame) and warm (right frame) simulations, initialized with flow perpendicular to the simulation plane (plain lines) and in the simulation plane (dashed lines), for both the unmagnetized (light green lines) and magnetized (purple lines) cases. As a reference, we report also the 1D magnetic energy density (dash-dotted lines). The growth rate values measured in the 2D simulations differ from the corresponding 1D cases by less than 5%.

No evidence of longitudinal modes or instabilities, such as the twostream instability, has been found. Indeed, the purely transverse Weibel instability is expected to be the dominant mode in the unstable spectrum for symmetric relativistic beams (Bret et al., 2010b).

The warm simulations, Fig. 3.15 b), confirm the 1D result that the saturation level does not depend on the external magnetic field (compare green and violet lines with the same style). Indeed, at saturation, happening between $(20 \div 30) \omega_{pe}^{-1}$ depending on the configurations, the same level of Weibel-generated magnetic energy is achieved. Moreover, the magnetic field strength at the time corresponding to saturation does not depend on the geometry (compare plain and dashed lines).

The quasi-cold simulations have different evolution in the unmagnetized and magnetized case, as already discussed in Sec. 3.3.2 for the 1D case, and some differences appear depending on the beams propagation direction, Fig. 3.15 a). In the unmagnetized cases, all configurations lead to the same magnetic field energy level at saturation. In the presence of the external magnetic field, a change of slope in the energy growth is



Figure 3.15: a) Evolution in time of the energy associated with the Weibel-generated magnetic field in 2D simulations. a) Quasi-cold case. b) Warm case. Light green lines refer to $B_0 = 0$ and dark purple lines to $B_0 = 0.75 B_c$. Plain lines corresponds to initial flows along z-direction, dashed lines along the x-direction and dash-dotted lines the 1D cases. All energies are normalized to the total initial flow kinetic energy U_{k0} .

evident for U/U_{k0} in the range $10^{-3} \div 10^{-2}$. This stage corresponds to the saturation of large-k modes, excited in the initial broad spectrum of modes characteristic of a low but finite temperature, as in the 1D simulations. The magnetic field amplitude at this time is $\simeq 0.1 m_e \omega_{pe} c/e$, consistent with the saturation of modes with $k \gtrsim 10 \omega_{pe}/c$, as shown in the 1D spectrum in Fig. 3.9. After that, modes with higher saturation level, i.e. small-k, can keep growing. The effect of the external magnetic field in impeding the redistribution of the currents, required for the growth of the small-k modes, is maximum in the 1D configuration. Indeed, the magnetic energy is still growing at late time and it does not reach the unmagnetized level. Regarding the 2D simulations, the effect of the external field is stronger in the case of initial velocity in the simulation plane ($\mathbf{v}_0 = \pm v_0 \, \hat{\mathbf{x}}$). The saturation level (purple dashed line $t \simeq 50 \,\omega_{pe}^{-1}$ is slightly smaller than in the unmagnetized case (green dashed line $t \simeq 30 \, \omega_{pe}^{-1}$). In the other 2D configuration $(\mathbf{v}_0 = \pm v_0 \, \hat{\mathbf{z}})$ the two curves reach the same level. Indeed, in this latter configuration, the redistribution of the current filaments is facilitated by the possibility to move in the x-y plane, while the other corresponds to a quasi-1D geometry, in which the redistribution take place only along the y-direction. Note that, as in 1D, in the warm simulations, modes with $k \gtrsim 10 \omega_{pe}/c$ are stable. Hence, we do not observe any change of slope in the magnetic energy density growth.

Comparing the saturation levels in Fig. 3.15a-b), we see that the fraction of initial kinetic energy U_{k0} transferred to the Weibel-generated magnetic field energy U is, with good accuracy, independent from the temperature.

In conclusion, we confirm the main result discussed in the previous 1D study: the temperature and a strong external magnetic field do not drastically affect the saturation level, despite reducing the growth rate.

3.4.3 Late merging phase

The main differences between the two 2D configurations appear in the late phase of the simulations, when merging events are usually considered to govern the dynamics of the system.

For longitudinal beams, the energy in the Weibel-produced magnetic field decreases after saturation, while it remains constant in the perpendicular flows case. This different behavior is approximately independent from the temperature, see Fig. 3.15 a-b). As discussed by Medvedev et al. (2005), the coalescence of two filaments involves a redistribution of the current filaments and of the magnetic field structures that does not change the total energy in the electromagnetic fields. This is indeed what happens in the 2D configuration with perpendicular flows and in 1D, as shown in Fig. 3.15 by the energy plateau at $t \gtrsim 50 \,\omega_{pe}^{-1}$. Moreover, the formation of large structures is highlighted in Fig. 3.16, where we compare the current filament distribution $J_z(x,y)$ at saturation $(t \simeq 30 \,\omega_{pe}^{-1})$ and at the end of the simulation $(t = 100 \,\omega_{pe}^{-1})$, for the warm magnetized case, being representative of all the other cases in which merging events give the dominant contribution. The limit on the increase of the filament extension is given by the duration of the simulation and by the box dimension. Comparing the filament dimension at $t = 100 \,\omega_{pe}^{-1}$, in the 1D case Fig. 3.14 b) with this 2D configuration Fig. 3.16 b), we clearly see that the efficiency of the coalescence is enhanced by the 2D geometry, es-



Figure 3.16: Current filament distribution J_z in the magnetized warm case initialized with $\mathbf{v}_0 = \pm v_0 \hat{\mathbf{z}}$, at saturation $(t \simeq 30 \, \omega_{pe}^{-1})$ and at the end of the simulation $(t = 100 \, \omega_{pe}^{-1})$.



Figure 3.17: Current filament distribution J_x in the magnetized warm case initialized with $\mathbf{v}_0 = \pm v_0 \hat{\mathbf{x}}$, at saturation $(t \simeq 30 \, \omega_{pe}^{-1})$ and at the end of the simulation $(t = 100 \, \omega_{pe}^{-1})$.

pecially for the magnetized case for which no redistribution was observed in the 1D simulation.

In the longitudinal flow configuration, the current filaments are affected by kink-like instabilities, as discussed by Milosavljević and Nakar (2006). The presence of these longitudinal modes at saturation is shown in Fig. 3.17 a). This leads to the disruption of the filaments themselves at late times, Fig. 3.17 b), and to the decrease of the total electromagnetic energy, as shown in Fig. 3.15.

3.4.4 Secondary Weibel instability

A secondary Weibel instability, growing during the late linear phase of the main instability, has also been identified. It grows in the case of initially unmagnetized flows $(B_0 = 0)$ and amplifies the magnetic field component along the electrons propagation direction.

The only simulation that allows for this observation is the one performed in the 2D configuration with flows perpendicular to the simulation plane ($\mathbf{v}_0 = \pm v_0 \hat{\mathbf{z}}$). The main instability amplifies the magnetic field components B_x and B_y , while the secondary instability the magnetic field B_z . The exponential growth of U_{B_z} , the energy density associated with the flow-aligned magnetic field, is illustrated in Fig. 3.18 (plain dark green lines), along with the electrostatic energy density (dashed dark green line) and $U_{B_{x,y}}$ associated with the Weibel-generated magnetic field (light green line), in the unmagnetized case for both temperatures.

The growth rate measured for U_{B_z} is in good agreement with twice that of $U_{B_{x,y}}$, the main Weibel instability. Indeed, as particularly clear in the warm case, Fig. 3.18 b), U_{B_z} has the same growth than the electrostatic energy, produced as a second order effect because of charge separation. This suggests that second order effects might also be responsible for the growth of the flow-aligned magnetic field. Looking at B_z and at the J_x and J_y distributions in the simulation plane at $t = 60 \omega_{pe}^{-1}$, Fig. 3.19, a similar pattern is found. We can therefore describe this mode as a secondary Weibel instability driven by the transverse motion of the particles, following from the deflection produced by the primary Weibel instability. From an analytical point of view, we can extend the linearization performed in Sec. 2.4, developing the fluid equations up to the second order.



Figure 3.18: Evolution in time of the energy density associated with the B_z component (plain dark green line), the electrostatic field components E_x and E_y (dashed dark green line) and in the Weibel-generated magnetic fields B_x and B_y (light green line) for the 2D simulations with initial plasma flows along the z-direction. a) The unmagnetized quasi-cold case. b) The unmagnetized warm case.

While the current perturbation at first order appears only along the flow direction (here $J_z^{(1)}$), at the second order we obtain $J_x^{(2)} = J_y^{(2)} \neq 0$. In the Maxwell-Ampère equation, Eq. (2.4.3), this second order current couples the electrostatic field, growing at twice the primary Weibel growth rate, with the flow-aligned magnetic field B_z , thus explaining the growth observed in Fig. 3.18.

This secondary instability is not observed in any of the other unmagnetized configurations since the reduced geometry of the simulations does not allow it. Indeed, to observe the growth of the flow-aligned magnetic field in the 2D longitudinal flows configuration ($\mathbf{v}_0 = \pm v_0 \,\mathbf{\hat{x}} \parallel B_x \,\mathbf{\hat{x}}$), a 2nd order current J_z with modulations along y should be present. However the main Weibel-instability deflects the particle only along the y-direction ($\mathbf{k} = k_y \,\mathbf{\hat{y}}$), thus producing a second order current $J_y^{(2)}$. Similarly the growth of B_z is prevented in the 1D cases.

In all magnetized simulations the secondary instability does not develop. Indeed, the initial energy associated with the flow-aligned external magnetic field exceeds the saturation value of the secondary instability


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Figure 3.19: a) Flow-aligned magnetic field B_z produced by the secondary Weibel instability. b) Current filament distribution J_x . c) Current filament distribution J_y . In the unmagnetized warm simulation initialized with $\mathbf{v}_0 = \pm v_0 \, \hat{\mathbf{z}}$, during the linear phase of growth of the secondary Weibel instability, $t \simeq 60 \, \omega_{pe}^{-1}$.

observed here. However, in the magnetized simulations, the external magnetic field get compressed inside the filaments, following the accumulation of the particle density. This was clearly observed in 1D simulations with single seeded mode, where modulations at twice the seeded k appear, as shown in Fig. 3.6.

3.4.5 Critical magnetic field

The value of the critical magnetic field for which the Weibel instability is quenched has been derived in Sec. 3.2.1.1 starting from the cold plasma dispersion relation, Eq. (3.2.5). In this section, we investigate the dependence of this parameter on the temperature. Indeed, as mentioned at the end of Sec. 3.3.1.1, the external magnetic field appears in the dispersion relation for finite temperature beams, Eq. (3.3.1), as $B_0/h(\mu)$, so that a high temperature, increasing the enthalpy, reduces the effective value of the external field. Note that the dispersion relation (3.3.1) can not be used to extract analytically the critical value $B_c(\mu)$ for which the growth rate vanishes ($\Gamma = 0$) in finite temperature plasma, since the fluid description is accurate in the limit $\Gamma/k \gg v_{th}$, where v_{th} is the thermal



Figure 3.20: Weibel-generated magnetic field energy density with $B_0 = B_c$ and initial temperature $T = 5 \times 10^{-3} m_e c^2$ (dash-dotted purple line), $T = 5 \times 10^{-2} m_e c^2$ (dashed purple line) and $T = 0.5 m_e c^2$ (plain purple line). a) 1D simulations. b) 2D simulations.

velocity. To do so, a relativistic kinetic description of magnetized relativistic Weibel-instability would be required. To our knowledge, this has not been derived. Here, we evaluate the dependence of this threshold on the temperature in the range $[0, 0.5] m_e c^2$, by means of 1D and 2D simulations, keeping unchanged all other parameters.

In Fig. 3.20 we show the results of simulations performed in 1D (left frame) and 2D with transverse flows (right frame) with external field equal to the cold case critical field $B_c \simeq 1.36 m_e \omega_{pe} c/e$, for temperatures $T_0 = 5 \times 10^{-3}$, 5×10^{-2} , $0.5 m_e c^2$ in the plasma rest frame. With low temperature ($T_0 = 5 \times 10^{-3} m_e c^2$, dash-dotted purple lines) the system is stable, while the Weibel instability is found to grow in the case of higher initial temperature. The growth rate of the instability in the case of $T_0 = 0.5 m_e c^2$, extracted from both 1D and 2D simulations, is $\Gamma \simeq$ $0.15 \omega_{pe}$, in fairly good agreement with the fluid prediction for which the maximum growth rate is $\Gamma \simeq 0.18 \omega_{pe}$. The fluid theory is not applicable at temperature $T_0 = 5 \times 10^{-2} m_e c^2$, since with this temperature the limit $\xi \gg 1$ is not satisfied. We remind that the parameter ξ does not depend exclusively on the temperature, see Sec. 2.4.3. With the inappropriate use of the fluid theory we would predict the system to be stable.

3.5 Conclusions

The electron Weibel instability driven by two symmetric counterstreaming relativistic electron beams in the presence of a flow-aligned magnetic field has been investigated using both analytical modeling and PIC simulations.

The linear stage of the instability is modeled using a relativistic fluid approach accounting for the effect of the electron pressure in the case of finite temperature plasma flows. This fluid model proves to give tractable solutions for the growth rate which are found to be in good agreement with the PIC simulations.

The saturation (non-linear phase) of the instability has also been investigated. Considering a single growing mode, the mechanisms responsible for saturation in the unmagnetized case have been clarified and a proper analytical generalization to predict the saturation level in the presence of the external magnetic field has been obtained. At small wavenumbers the dominant role of the Alfvén current-limitation is highlighted. We show that, in this range, the external magnetic field can slightly increase the field amplitude at saturation. In the large wavenumber limit, the trapping mechanism (revisited as the injection of particles in the filaments) leads to the saturation of the instability. The saturation level for large k modes is predicted to be independent of the strength of the external magnetic field, as long as the latter remains smaller than the well-known critical field above which the instability is quenched. Our analysis also explains why the bouncing frequency argument, which is shown to hold in the absence of the external magnetic field, does not lead to a correct prediction for the trapping mechanism saturation level in the presence of a flow-aligned magnetic field. These theoretical results are in good agreement with 1D PIC simulations seeded with a single mode.

The saturation and late merging stages have also been numerically investigated by seeding the instability from broadband thermal fluctuations, in 1D and 2D PIC simulations. In a low temperature plasma, the average saturation level in the 1D geometry is decreased by the application of an external magnetic field, since after the saturation of large-kmodes the external magnetic field hinders the redistribution of energy towards small k. For this reason, the Weibel magnetic field spectrum in a magnetized plasma is much broader than in the unmagnetized case. In the 2D geometry, especially with perpendicular flows, a similar saturation level is achieved in the unmagnetized and magnetized case and the capability of the external field in keeping stable the saturated large-k modes is much weaker. This highlight the limits of the 1D numerical study and the importance of multi-dimensional simulations for the non-linear stage of the instability. Increasing the initial flow temperature, the saturation level is found to be independent from the external magnetic field in all configurations. The Weibel magnetic field spectra are found to be peaked around the wavenumber predicted to have the maximum saturation level by single mode analysis.

At late times, merging processes have been found to govern the dynamics in 1D and 2D simulations with perpendicular flows. The mechanisms that allow for the filament coalescence observed in multi-mode 1D simulations (and absent in the single-mode ones) have been explained as following from both, the irregular distribution of filaments growing from thermal fluctuations and the effect of the small-k inductive electric field. Filament coalescence is found to be inhibited by the external magnetic field in 1D simulations, while it is weakly affected in the 2D case when considering perpendicular flows.

The late time evolution of the 2D in-plane flows configuration is dominated by kink-like instability that disrupts the filaments and involves a redistribution of the magnetic energy. This instability can not be observed in the other configurations in which merging is found to have a dominant role.

Our results can be applied to astrophysical systems where the Weibel instability develops in magnetized plasmas and drive the formation of collisionless shocks. In the context of relativistic laboratory astrophysics, one of the major experimental issue to investigate the development and the late stage of the Weibel instability is related with the low density of the counter-propagating plasma flows that can be produced by means of laser-plasma interaction. Indeed, a low density involves long time scale and large interaction length of the plasma flows. A guiding magnetic field

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has been suggested as a way to maintain a high plasma density. A major concern was thus the potential effect of the external field in stabilizing the Weibel instability development and, in particular, its effect on the strength of the produced magnetic field. Our results prove that using a guiding external magnetic will not strongly modify the level of Weibelgenerated magnetic fields, so that it might be used in experiment to keep a high density flows, hence fastening the development of plasma instabilities and potentially of the formation of collisionless shocks.

Appendix

3.A Alfvén limit in the presence of an external magnetic field

The Alfvén limit defines the maximum (critical) current that a beam of charged particles can sustain before the particle trajectories, in the self-generated magnetic field, start limiting the current itself due to the reduction and/or inversion of particle motion in the flow direction (Alfvén, 1939). The maximum current can be defined in different ways (Alfvén, 1939; Hammer and Rostoker, 1970; Honda, 2000) that, within a factor, give very similar results. In this Appendix, we follow more closely the original approach proposed by Alfvén.

The derivation presented here considers a 1D3V geometry and a given sine-like profile for the current density, in order to be consistent with our PIC simulations. Generalization to the more realistic 2D(r, z) geometry and arbitrary profile is straightforward. The critical current for a uniform cylindrical current in 2D(r, z) geometry is given at the end of this Section.

We assume a sinusoidal profile for the current density J_z , for $-\pi/2 \le kx \le \pi/2$, as

$$J_z(x) = -J_0 \cos(kx), \qquad (3.A.1)$$

and, consistently with the Ampère's law, the magnetic field

$$B_y(x) = -2\pi I_{1D}^{(0)}/c\,\sin(kx)\,,\qquad(3.A.2)$$

with $I_{1D}^{(0)} = 2 J_0/k$ the absolute value of the total (areal) current.

Considering this magnetic field and the external (guiding) magnetic field $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$ as time-independent, three constants of motion allow for the description of the dynamics of an electron in these fields: the electron energy (Hamiltonian) $\mathcal{H} = m_e c^2 \sqrt{1 + \mathbf{p}^2/(m_e^2 c^2)}$, and the two components of the electron canonical momentum $\mathbf{\Pi} = \mathbf{p} - e\mathbf{A}/c$ lying in the *y*-*z* plane. The vector potential \mathbf{A} is computed inverting the relation $\mathbf{B} = \nabla \times \mathbf{A}$, leading to

$$A_y(x) = B_0 x \,, \tag{3.A.3}$$

$$A_z(x) = 2\pi \frac{I_{1D}^{(0)}}{kc} \left[1 - \cos(kx)\right], \qquad (3.A.4)$$

where we have taken $A_y(0) = A_z(0) = 0$. Considering an electron initially located at the border of the filament $kx_0 = \pi/2$, with initial momentum $m_e \gamma_0 v_0 \hat{\mathbf{z}}$ (correspondingly $\mathcal{H}_0 = \gamma_0 m_e c^2$), one gets

$$p_x^2 = m_e^2 c^2 (\gamma_0^2 - 1) - p_y^2 - p_z^2, \qquad (3.A.5)$$

$$p_y = -\frac{eB_0 x_0}{c} \left(1 - x/x_0\right), \qquad (3.A.6)$$

$$p_z = m_e \gamma_0 v_0 - 2\pi e \frac{I_{1D}^{(0)}}{kc^2} \cos(kx) \,. \tag{3.A.7}$$

The critical current $I_{1D}^{(c)}$ is then defined as the minimum current for which the longitudinal momentum Eq. (3.A.7) vanishes leading to

$$I_{1D}^{(c)} = \frac{1}{2\pi} \frac{m_e c^2}{e} \min\left\{\frac{\gamma_0 v_0 k}{\cos(kx)}\right\}.$$
 (3.A.8)

In the absence of the external magnetic field $(B_0 = 0, \text{ so that } p_y = 0 \text{ at all times})$, this minimum is reached for x = 0, i.e. when the electron longitudinal momentum vanishes on-axis, leading to

$$I_{1D}^{(c)} = \frac{1}{2\pi} \frac{m_e c^2}{e} \gamma_0 v_0 k , \qquad (3.A.9)$$

that corresponds, using Eq. (3.A.2), to the magnetic field B_y strength

$$B_{\max} = \gamma_0 v_0 k \, \frac{m_e c}{e} \,, \tag{3.A.10}$$

given by Eq. (3.2.13).

In the presence of a guiding magnetic field $(B_0 \neq 0)$, the electron initially at the border of the current may not reach its center x = 0before being turned back under the effect of the guiding magnetic field. As a consequence, the critical current Eq. (3.A.8) has to be computed taking $x = x^*$, with $x^* = 0$ if the electron can reach the center of the current, and $x^* > 0$ the turning point of the electron when it cannot reach x = 0. For large enough external magnetic field $[B_0 > (\gamma_0 v_0/x_0) m_e c/e,$ correspondingly $A \equiv v_0/|\Omega_0 x_0| < 1]$, one obtains x^* as the point for which $p_x = p_z = 0$ (all the electron momentum is in p_y) leading to $x^* = x_0 (1 - A)$. For lower values of the external magnetic field $(A \ge 1)$, the electron will eventually reach the center of the filament so that $x^* = 0$. This leads to the critical current

$$I_{1\mathrm{D}}^{(c)} = \frac{1}{2\pi} f(A) \, \frac{m_e c^2}{e} \, \gamma_0 v_0 k \,, \qquad (3.\mathrm{A.11})$$

with $f(A) = [\cos(\pi(1-A)/2)]^{-1}$ for A < 1, and f(A) = 1 otherwise, corresponding to the magnetic field strength

$$B_{\max} = f(A) \gamma_0 v_0 k \frac{m_e c}{e}, \qquad (3.A.12)$$

as in Eq. (3.2.14).

A similar derivation can be done in the case of a uniform cylindrical current with radius R, in 2D(r, z) geometry. The constants of motions are then given by the Hamiltonian, z-component of the canonical momentum and canonical angular momentum. One then obtains the critical current

$$I^{(c)} = I_0 \, \frac{\gamma_0 v_0/c}{1 - r^*/R} \,, \tag{3.A.13}$$

with $I_0 = m_e c^3 / e \simeq 17$ kA, and for which r^* plays the same role as x^* in

the 1D geometry and depends on the external magnetic field as

$$r^* = \frac{R}{2} \left(\sqrt{4 + A^2} - A \right) ,$$
 (3.A.14)

with $A = v_0/|\Omega_0 R|$ and Ω_0 the cyclotron frequency of an electron in the external magnetic field B_0 . In the absence of external magnetic field $A \to \infty$ ($r^* = 0$), one recovers the well-known result by Alfvén.

Notice that both Eqs. (3.A.11) and (3.A.13) predict an increase of the critical current with the application of a guiding magnetic field. The possibility to exceed the Alfvén limit by applying an external magnetic field along the flow direction has been already mentioned in (Peratt, 1992, Par.2.5.2-6).

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CHAPTER 4

Radiation Pressure Driven Ion Weibel Instability and Collisionless Shocks

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In this Chapter we propose a scheme to reproduce, in the laboratory, the ion Weibel instability and the subsequent collisionless shock formation, by means of laser-plasma interaction at Ultra-High-Intensity (UHI).

Up to now, most of the studies have focused on the use of high-energy (multi-kJ and NIF-LMJ class) laser facilities operating at modest intensities ($\leq 10^{16}$ W/cm²) (Huntington et al., 2015; Park et al., 2015; Ross et al., 2017), whose main findings have been summarized in Sec. 1.2.2. On such laser systems, the resulting plasma flows are created by ablation of dense targets, which limits the accessible flow density and velocity (typically $\leq 0.5\%$ of the speed of light). As a result, the characteristic length and time over which shock formation can be expected are large (few centimeters and tens of nanoseconds). This has potentially two drawbacks. First, it requires the use of large laser systems such as NIF or LMJ. Second, the effect of particle collisions over such lengths/times may not be completely negligible. In contrast, UHI laser systems, with peak intensities beyond 10^{18} W/cm², could allow to alleviate these limitations by providing a complementary path toward the creation of collisionless, ultra-fast and high-density electron-ion flows.

This work has led to the publication on Physical Review E (Grassi et al., 2017a).

This Chapter is structured as follows. We first introduce the basic concepts of intense laser-plasma interaction in Secs. 4.1.1 and 4.1.2, and then, in Sec. 4.1.3, we briefly describe the energy absorption mechanisms that take place under the investigated conditions. Our proposed scheme relies on the production of quasi-neutral flows generated by the so-called Radiation Pressure Acceleration (RPA) mechanism, described in Sec. 4.1.4. The main findings of a scheme previously proposed by Fiuza et al. (2012) are summarized in Sec. 4.2 in order to highlight the differences with our proposition.

In order to optimize the energy transfer from the laser pulse to the fast ion beam, circular polarized light is commonly considered to be the optimal driver for RPA. This configuration is indeed expected to reduce the electron laser energy absorption (Macchi et al., 2005; Robinson et al., 2009; Schlegel et al., 2009), as reminded in Sec. 4.1.3. In contrast, by means of PIC simulations, we demonstrate that an S-polarized laser beam irradiating the target at oblique incidence is the optimal configuration. In particular, the usual configuration of normal incidence and circularly polarized light is affected by surface instabilities which increase drastically the production of hot electrons and thus modify the characteristics of the ion flow in the target. A detailed comparison between the different interaction configurations is presented in Sec. 4.3. Furthermore, we notice, in PIC simulations, that a current generated along the surface, as can be obtained at non-normal incidence in linear polarization, is always correlated with the mitigation of the surface instabilities (Secs. 4.3.1 and 4.3.3). We provide, in Sec. 4.3.2, an analytical model for the description of the surface current and field profiles.

Motivated by these results, in Sec. 4.4, we consider an S-polarized laser with oblique incidence and we demonstrate with a full 3D simulation that it is possible to drive fast, dense and quasi-cold flows into the target, and enter in a regime governed by the Ion-Weibel-Instability (IWI). The possibility to produce, at later times, a collisionless shock is then investigated in Sec. 4.5.

4.1 UHI laser-plasma interaction

In order to describe the regime of intense laser-plasma interaction, we start from the definition of the laser intensity I as the cycle-averaged value of the Poynting vector **S** modulus:

$$I = \langle |\mathbf{S}| \rangle = \langle \frac{c}{4\pi} |\mathbf{E} \times \mathbf{B}| \rangle = \frac{c}{8\pi} |E_0|^2, \qquad (4.1.1)$$

where $E_0 = B_0$ are the amplitudes of the laser fields. The relativistic regime sets in when an electron gains a momentum $\gtrsim m_e c$ in a single oscillation in the laser fields. This can be quantified by means of the dimensionless vector potential a_0 , defined as

$$a_0 = \frac{eE_0}{m_e c\omega_{\rm L}} \,, \tag{4.1.2}$$

where $\omega_{\rm L}$ is the laser frequency. The relativistic laser-plasma interaction regime requires $a_0 > 1$. To directly relate the laser characteristics with the parameter a_0 , we can exploit the following relation:

$$a_0 = 0.85 \sqrt{\frac{I\lambda_{\rm L}^2}{10^{18} {\rm W cm}^{-2} \mu {\rm m}^2}},$$
 (4.1.3)

where $\lambda_{\rm L}$ is the laser wavelength.

For the reader's convenience, we remind that, at oblique incidence, a linearly S-polarized laser pulse has electric field in the plane of the plasma surface. On the contrary, P-polarized light impinging at an angle has magnetic field parallel to the surface. In the following of this Chapter, the parameter δ representing the polarization, is defined so that the vector potential associated with the fields of a wave propagating along the *x*direction, reads

$$\mathbf{A}(x,t) = A_0 e^{+ikx} \left(\delta \cos(\omega_{\rm L} t) \, \hat{\mathbf{y}} + \sqrt{1 - \delta^2} \sin(\omega_{\rm L} t) \, \hat{\mathbf{z}} \right) \,, \qquad (4.1.4)$$

where linear polarization is obtained for $\delta = 0$ and $\delta = 1$, circular for $\pm 1/\sqrt{2}$. Other values correspond to an elliptical polarization. As a convention, we will distinguish between S-polarized ($\delta = 0$) and P-polarized ($\delta = 1$) light even at normal incidence. This is commonly done in 2D PIC simulations to refer to the laser polarization in the simulation plane (henceforth considered P-polarization) and out-of-plane (henceforth considered S-polarization).

4.1.1 Transparent and Opaque Regime

The first parameter that is of interest to describe the interaction of an intense laser pulse with a gas or a solid, is the cut-off density that allows for the reflection or the penetration of the laser pulse in the target.

In Section 2.4.1, we have already defined the condition for a wave to

propagate in a cold, unmagnetized plasma, namely $\omega_{\rm L} > \omega_{pe}$. Hence, we can define the so-called *critical density* n_c as the plasma density for which $\omega_{\rm L} = \omega_{pe}$. In practical units, this condition gives

$$n_c = \frac{m_e \omega_{\rm L}^2}{4\pi e^2} = 1.1 \times 10^{21} {\rm cm}^{-3} \left(\frac{\lambda_{\rm L}}{1\mu {\rm m}}\right)^{-2} .$$
 (4.1.5)

A plasma with density $n_e < n_c$ is said *underdense*, while in the opposite case *overdense*. As already mentioned in Section 2.4.1, in the case of overdense plasma filling the region x > 0 and laser pulse propagating along the *x*-direction, the laser fields are exponentially damped $\sim e^{-kx}$ and the characteristic length of penetration, along the normal to the plasma surface, is

$$\ell = \frac{c}{\sqrt{\omega_{pe}^2 - \omega_{\rm L}^2}},\tag{4.1.6}$$

that for a strongly overdense plasma reduces to $\ell \simeq c/\omega_{pe}$.

This derivation is accurate only in the non-relativistic regime. In the presence of an intense laser pulse $(a_0 \gg 1)$, relativistic effects become important from the early phase of the interaction and the response of the plasma can be strongly modified by non linear effects. Indeed, in the relativistic regime, the dispersion relation (hence the propagation) of the electromagnetic wave depends on both the plasma parameters and the wave amplitude itself. However, one of the main features of the relativistic laser-plasma interaction can be obtained for circularly polarized plane wave irradiating a cold overdense target at normal incidence. In this configuration, the relativistic dispersion relation for electromagnetic waves is equivalent to Eq. (2.4.12) with the replacement $m_e \mapsto m_e \gamma_e$, where γ_e is the relativistic factor related with the electron motion in the laser fields, as first derived by Akhiezer A.I. (1956). This is however a great simplification and should be used with some care. The value of the relativistic factor γ_e can be derived by noticing that during the interaction with a circularly polarized plane wave γ_e remains constant. The main contribution to the relativistic factor is given by the transverse momentum \mathbf{p}_{\perp} , i.e. $\gamma_e \simeq \left[1 + \mathbf{p}_{\perp}^2 / (m_e c)^2\right]^{1/2}$. Exploiting the conservation

of the transverse canonical momentum \mathbb{P} , that reads

$$\frac{d\mathbb{P}}{dt} = \frac{d}{dt} \left[\mathbf{p}_{\perp} - \frac{e\mathbf{A}}{c} \right] = 0, \qquad (4.1.7)$$

and assuming no laser fields as initial condition $\mathbf{p}_{\perp}(0) = \mathbf{A}(0) = 0$, we obtain

$$\mathbf{p}_{\perp} = \left(eA_0/\sqrt{2}c\right)\left[\cos\phi,\,\sin\phi\right]\,,\tag{4.1.8}$$

with $\phi = \mathbf{k}_{\mathrm{L}} \cdot \mathbf{x} - \omega_{\mathrm{L}} t$. Equation (4.1.8) leads to $\gamma_e = \sqrt{1 + a_0^2/2}$, using the definition of the vector potential given in Eq. (4.1.4) with $\delta = \pm 1/\sqrt{2}$ for circular polarization, in normalized units. Therefore, from the relativistic dispersion relation, whose validity is limited to a circularly polarized plane wave at normal incidence, the relativistic critical density becomes $n_c^{\mathrm{REL}} = \gamma_e n_c$. This means that a relativistically intense laser pulse can propagate through a nominally overdense plasma ($n_c < n_e < n_c^{\mathrm{REL}}$), a process known as *Relativistic Self-Induced Transparency* (RSIT). By contrast, in the case of linear polarization, the electron relativistic factor γ_e is not constant and its rapidly oscillating component leads to the generation of harmonics. Despite the analysis being more complicated, it was demonstrated by Sprangle et al. (1990); Lefebvre and Bonnaud (1995) that at least for the main frequency a good estimation of the effective critical density is given by $n_c^{\mathrm{REL}} = \gamma_e n_c$ with $\gamma_e = \sqrt{1 + \langle a_0^2/2 \rangle}$, where the brackets represent the average on the laser period.

For any realistic laser pulse, i.e. finite spot size and finite duration, the value of the critical density needs to be evaluated self-consistently with the density profile modification and pulse shaping at the laser-plasma interaction surface. Refined models to predict the critical density can be found in Refs. (Goloviznin and Schep, 2000; Siminos et al., 2012, 2016). In particular, Cattani et al. (2000) found a value of the critical density significantly increased once considering that, at the laser-plasma interaction surface, an electron density peak is produced by means of the ponderomotive force described in the following Sec. 4.1.2, so that the penetration of the laser pulse is drastically reduced.

4.1.2 Ponderomotive Force

In this paragraph, we introduce a fundamental concept for the study of the laser-plasma interaction, the so-called *Ponderomotive Force*, namely the time-averaged force exerted by an electromagnetic wave on a target. As we explicitly derive in the following, this force is slowly varying with respect to the fast oscillations of the laser electromagnetic fields and allows to describe the evolution of a single particle cycle-averaged position and velocity.

We assume the laser pulse to be described by a monochromatic wave with frequency $\omega_{\rm L}$, modulated with a spatial and a temporal profile as

$$\mathbf{E}(\mathbf{x},t) = \Re \left(\tilde{\mathbf{E}}(\mathbf{x},t) e^{-i\omega_{\mathrm{L}}t} \right), \qquad (4.1.9)$$

$$\mathbf{B}(\mathbf{x},t) = \Re \left(\tilde{\mathbf{B}}(\mathbf{x},t) e^{-i\omega_{\mathrm{L}}t} \right), \qquad (4.1.10)$$

where \Re identifies the real part, and the envelope functions $\dot{\mathbf{E}}(\mathbf{x},t)$ and $\tilde{\mathbf{B}}(\mathbf{x},t)$ vary on a time-scale slower than the laser period $t_{\rm L} = 2\pi/\omega_{\rm L}$ and on a typical length larger than the laser wavelength $\lambda_{\rm L} = ct_{\rm L}$. Assuming the existence of two characteristic time-scales, we can decompose the single particle motion in the laser fields as

$$\mathbf{x}(t) = \mathbf{x}_q(t) + \mathbf{x}_s(t), \qquad (4.1.11)$$

where $\mathbf{x}_q(t)$ is the fast oscillating component that describes the quiver motion around the position of the oscillation center, corresponding to the slowly varying term $\mathbf{x}_s(t)$. We derive now the dynamic equation for the slow component $\mathbf{x}_s(t)$, that leads us to the definition of the ponderomotive force in the non-relativistic regime.

Thanks to the assumption of a smooth laser profile in comparison with the laser wavelength, the electric field of the laser pulse, up to the first order perturbation, can be expressed as

$$\mathbf{E}(\mathbf{x},t) \simeq \mathbf{E}(\mathbf{x}_s,t) + (\mathbf{x}_q(t) \cdot \nabla) \mathbf{E}(\mathbf{x}_s,t).$$
(4.1.12)

Considering \mathbf{x}_s constant at the lowest perturbation order, the first term on the right side acts on the fast oscillating component of the single particle motion as

$$\frac{d^2 \mathbf{x}_q}{dt^2} = \frac{q}{m} \mathbf{E}(\mathbf{x}_s, t), \qquad (4.1.13)$$

whose solution is simply

$$\tilde{\mathbf{x}}_q = \frac{-q}{m\omega_{\rm L}^2} \tilde{\mathbf{E}}(\mathbf{x}_s) \,, \tag{4.1.14}$$

where $\tilde{\mathbf{x}}_q$ follows the notation used in Eqs. (4.1.9)-(4.1.10). To study the evolution of the slow motion $\mathbf{x}_s(t)$, we consider the averaged Lorentz force

$$m\frac{d^2\mathbf{x}_s}{dt^2} = q\left\langle \mathbf{E}(\mathbf{x}, t) + \frac{\mathbf{v}}{c} \times \mathbf{B}(\mathbf{x}, t) \right\rangle.$$
(4.1.15)

The electric field contribution reads

$$\langle \mathbf{E}(\mathbf{x},t) \rangle \simeq \langle (\mathbf{x}_q(t) \cdot \nabla) \mathbf{E}(\mathbf{x}_s,t) \rangle =$$

= $\frac{-q}{4m\omega_{\rm L}^2} \left(\tilde{\mathbf{E}}^*(\mathbf{x}_s,t) \cdot \nabla \right) \tilde{\mathbf{E}}(\mathbf{x}_s,t) + \text{c.c.},$ (4.1.16)

where $\tilde{\mathbf{E}}^*$ is the complex conjugate (c.c.) of the envelope function and we used the solution of the equation of motion for \mathbf{x}_q , Eq. (4.1.14). The magnetic term of the Lorentz force at the first order perturbation gives

$$\left\langle \frac{\mathbf{v}}{c} \times \mathbf{B}(\mathbf{x}, t) \right\rangle \simeq \left\langle \frac{\mathbf{v}_q(t)}{c} \times \mathbf{B}(\mathbf{x}, t) \right\rangle =$$
$$= \frac{-q}{4m\omega_{\rm L}^2} \tilde{\mathbf{E}}^*(\mathbf{x}_s, t) \times \left(\nabla \times \tilde{\mathbf{E}}(\mathbf{x}_s, t) \right) + \text{c.c.}, \qquad (4.1.17)$$

where we used the Maxwell-Faraday equation (2.1.10), that in this case becomes $\nabla \times \tilde{\mathbf{E}} = i\omega_{\rm L}\tilde{\mathbf{B}}/c$. In conclusion, summing the two contributions Eqs. (4.1.16) and (4.1.17), equation (4.1.15) becomes

$$m\frac{d^2\mathbf{x}_s}{dt^2} = \frac{-q^2}{4m\omega_{\rm L}^2}\nabla|\tilde{\mathbf{E}}^*(\mathbf{x}_s,t)|^2 = \frac{-q^2}{4m\omega_{\rm L}^2}\nabla\langle\mathbf{E}(\mathbf{x}_s,t)^2\rangle \equiv \mathbf{f}_p.$$
(4.1.18)

This provides the definition of the ponderomotive force \mathbf{f}_p .

From Eq. (4.1.18), we see that the ponderomotive force tends to expel from the region where the laser fields are stronger, particles with both positive and negative charge. Obviously the electrons respond much faster than the ions, due to their large inertia. Thus we usually consider the direct effect of the ponderomotive force on the ions negligible.

The definition of a relativistic ponderomotive force is not straightforward and an extensive discussion has been provided by Mulser and Bauer (2010). Here we just consider the simple case of a laser pulse defined as in Eqs. (4.1.9)-(4.1.10), for which we can introduce the concept of ponderomotive potential ϕ_p , related with the ponderomotive force in the standard way $\mathbf{f}_p = -\nabla \phi_p$. Hence, ϕ_p corresponds to the cycle-averaged single particle oscillation energy. Describing the laser pulse propagating along the *x*-direction by means of the potential vector $\mathbf{A}(x, t)$, as in Eq. (4.1.4), and exploiting the conservation of canonical momentum Eq. (4.1.7), we obtain

$$\mathbf{f}_p = -mc^2 \nabla \left[1 + \left\langle \left(\frac{q\mathbf{A}}{mc^2} \right)^2 \right\rangle \right]^{1/2}, \qquad (4.1.19)$$

consistent with the discussion of the previous Sec. 4.1.1.

4.1.3 Electron Heating in overdense target

A research topic that has been matter of intense investigation for the last 50 years, concerns the laser energy absorption in the collisionless regime. Due to the scaling of the collision frequency with the electron kinetic energy as $\nu_{coll} \propto E_{kin}^{-2}$, the collisional absorption is completely negligible for intense laser pulses (Wilks and Kruer, 1997; Mulser and Bauer, 2010). For this reason, in this paragraph, we briefly describe the main mechanisms of collisionless energy absorption, restricting ourselves to overdense targets. Despite the laser being reflected back in the vacuum during the interaction with an overdense target, a significant fraction of its energy may be transferred to the electrons located at the laser-target interaction surface. Different processes, depending on the laser-plasma interaction configuration, can directly convert the laser energy into electron kinetic energy. Much research effort is still devoted to the investigation of the optimal absorption conditions. In particular, electron heating is strongly dependent on the plasma density profile and on laser properties, such as intensity, polarization and angle of incidence. This topic has been investigated both experimentally (Popescu et al., 2005; Singh et al., 2015) and theoretically (Forslund et al., 1975; Kruer and Estabrook, 1985; Brunel, 1987; Wilks et al., 1992; Gibbon, 1994; Cialfi et al., 2016). The study of Gibbon (1994) identifies the optimal configuration to maximize collisionless electron heating as P-polarized laser with angle of incidence $\simeq 45^{\circ}$ for intensities above $I\lambda_{\rm L}^2 \sim 10^{17} {\rm W cm}^{-2} \mu {\rm m}^2$.

The typical order of magnitude of the energy acquired by an electron interacting with an intense laser pulse $(a_0 > 1)$ is given by the so-called *ponderomotive scaling* as

$$E_{h.e.} \simeq \left(\sqrt{1 + \frac{a_0^2}{2}} - 1\right) m_e c^2,$$
 (4.1.20)

where the factor under the square-root has been already encountered in Sec. 4.1.1 and in Eq. (4.1.19). This estimation predicts the production of MeV electrons already at modest intensities widely achievable with the current laser facilities ($a_0 \simeq 4$, i.e. $I\lambda_{\rm L}^2 \simeq 2 \times 10^{19} \,{\rm Wcm}^{-2} \mu{\rm m}^2$), and electrons with hundreds of MeV are commonly observed in both simulations and UHI laser experiments.

In the case of an intense laser pulse interacting with a solid target, as considered in the following of this Chapter, we can assume the plasma to have a steep profile, since the ponderomotive force can prevent the thermal expansion of the electrons in the vacuum region. This happens if the laser intensity is high enough that the radiation pressure exceeds the plasma thermal pressure, thus it depends on both the laser intensity and the plasma temperature. Due to the assumption of a step-like profile, we do not discuss in detail the *resonant absorption* mechanism (Forslund et al., 1975), that takes place in the region where the electron density $n_e \simeq n_c \ll n_0$ with n_0 the solid target density. In that region, the laser fields are responsible for the excitation of resonances, in particular of electrostatic plasma modes, since $\omega_{pe} \simeq \omega_{\rm L}$. In a finite temperature plasma, these modes propagate in the direction outward from the target, where $n_e \leq n_c$, and along their propagation direction they accelerate a small fraction of electrons in phase with the wave itself. Note that this mechanism requires an electric field component along the density gradient, i.e. a P-polarized laser pulse irradiating the target at nonnormal incidence.

Two dominant mechanisms to produce fast and hot electrons, propagating inside the target, will be observed in the simulations of this Chapter: the so-called *Vacuum Heating* (or Brunel mechanism) and the $\mathbf{J} \times \mathbf{B}$ *Heating.* A simple model for the vacuum heating was proposed by Brunel (1987). This model considers the dynamics of a single electron dragged out into the vacuum for about half of a laser period by means of the electric field component perpendicular to the surface. The electron is then injected back into the target, with the energy acquired during the oscillation in the vacuum region. Since the laser electromagnetic fields are evanescent at the laser-plasma interaction surface, thus penetrate only in a layer of the order of the skin depth $\simeq c/\omega_{pe}$, the high energy electron does not feel a strong-enough field to be dragged back into the vacuum and it freely propagates deep inside the target. This mechanism accounts for the production of hot electrons as bunches, generated with a frequency equal to the laser one. In the case of a planar target, in order to have a component of the electric field perpendicular to the target surface, the laser has to be P-polarized and have oblique incidence.

At high laser intensities the magnetic component of the Lorentz force exerted on an electron in the laser fields, becomes comparable to the one associated with the electric field. Therefore, a heating mechanism analogous to the previous one is found considering the $\mathbf{v} \times \mathbf{B}$ component normal to the plasma surface. Indeed, an electron can be dragged across the vacuum-plasma interface by the magnetic force (thus, the name $\mathbf{J} \times \mathbf{B}$ Heating (Kruer and Estabrook, 1985)), and get accelerated in the vacuum region, as explained before. This mechanism is the dominant one for linear polarization at normal incidence, for which vacuum heating gives no contribution, as well as for S-polarization at both normal and oblique incidence. We now report a simple argument to show that the driving force of the $\mathbf{J} \times \mathbf{B}$ heating is suppressed for circular polarization and normal incidence.

We consider for simplicity a cold plasma with a step-like density profile, with surface corresponding to the y-z plane at x = 0. The vector potential of a plane wave propagating along the x-direction can be expressed for x > 0, i.e. inside the target, as

$$\mathbf{A}(x,t) = A_0 e^{-x/\ell_s} \left(\delta \cos(\omega_{\rm L} t) \hat{\mathbf{y}} + \sqrt{1 - \delta^2} \sin(\omega_{\rm L} t) \hat{\mathbf{z}} \right) , \qquad (4.1.21)$$

where δ represents the polarization, as in Eq. (4.1.4), and $\ell_s \simeq c/\omega_{pe}$ is the evanescent penetration length. The Lorentz force, expressed as a function of the potential vector, reads

$$F_x(x,t) = -\frac{q^2}{m_e \gamma_e c^2} \frac{\partial}{\partial x} \left(\frac{|\mathbf{A}(x,t)|^2}{2} \right) =$$
$$= \frac{q^2}{m_e \gamma_e c^2} \frac{A_0^2}{2\ell_s} e^{-2x/\ell_s} \left(1 + \left[2\delta^2 - 1 \right] \cos(2\omega_{\rm L} t) \right) \,. \tag{4.1.22}$$

From the last term in the previous equation, it is clear that this force accounts for the generation of electron bunches at a frequency $2\omega_{\rm L}$. Note that the bunches at $\omega_{\rm L}$ produced by the vacuum heating can not be found due to the assumption of normal incidence. Furthermore, in the case of circular polarization ($\delta = \pm 1/\sqrt{2}$) only the ponderomotive average term appears, and the $\mathbf{J} \times \mathbf{B}$ heating is suppressed. This involves a very different laser-plasma coupling between linear and circular polarization at normal incidence, as will be clarified in Sec. 4.1.4. Performing a time-average on the laser period, the term oscillating at $2\omega_{\rm L}$ vanishes and we recover the ponderomotive force, Eq. (4.1.18), which is independent from the polarization.

Experimental confirmation of these collisionless heating mechanisms relies on the measurement of the Coherent Transition Radiation (CTR) emitted from the rear surface of the target when the fast electron bunches cross it (Popescu et al., 2005). These observations reveal also that the temporal coherence of the electron bunches is conserved during the propagation through the target up to $\simeq 1 \text{ mm}$ depth.

Moreover, an intensive area of research is dedicated to the study of target-shaping effects, that have been demonstrated to be able to strongly increase both the energy and the fraction of the produced fast electrons (Gaillard et al., 2010; Fedeli et al., 2015).

4.1.4 Radiation Pressure Acceleration

We now focus on the ion dynamics during the interaction of an intense laser with an overdense target. In the previous section, the ion dynamics has been neglected since the ion typical response time-scale is $\omega_{pi}^{-1} \gg \omega_{pe}^{-1}$. Therefore, we can assume that the ions are not affected by the fast oscillating laser fields, while they respond to the slowly varying fields generated, for instance, by large charge separations. This electrostatic field can be produced either by the fast electrons escaping at the rear target surface (driving the so-called *Target Normal Sheath* ion acceleration mechanism (Wilks et al., 2001)) or by the ponderomotive force that pushes the electrons inward at the laser-plasma interaction surface. The latter situation is the topic of this section and it is schematic represented in Fig. 4.1. The laser ponderomotive force pushes the electrons lying in a region of extension $\leq c/\omega_{pi}$ and accumulates them, creating a doublelayer structure with the ions following the electrons because of the strong charge-separation electric field (Schlegel et al., 2009). Thus, the ponderomotive force applies directly on the electrons but it is effectively exerted on the whole target. The total pressure exerted by the a plane wave on the target surface (x = 0) can be computed as the integral of the force on the skin layer of extension c/ω_{pe} , that gives

$$P_{\rm rad} = \int_0^{+\infty} n_0 f_{p,x} dx = \int_0^{+\infty} n_0 \frac{m_e c^2}{\ell_s} \frac{\omega_{\rm L}^2}{\omega_{pe}^2} a_0^2 e^{-2x/\ell_s} dx = \frac{2I}{c} \,, \quad (4.1.23)$$

where n_0 is the plasma density and we exploit the average of Eq. (4.1.22) in the non-relativistic limit to express the ponderomotive force. The potential vector at the surface $\mathbf{A}(0)$ has been computed in the approximation of perfectly reflecting medium by means of Fresnel formula, giving $|\mathbf{A}(0)| = 2a_0(m_ec^2/q)(\omega_{\rm L}/\omega_{pe})$. From Eq. (4.1.1), the intensity can be written as $I = m_ec^3n_ca_0^2/2$. Note that the integral on the skin layer c/ω_{pe} can be extended along the whole x-axis since the fields vanish deeper inside the target. The quantity $P_{\rm rad}$ is known as radiation pressure and it accounts for the electromagnetic momentum transferred to the target. Indeed, it is usually defined as the flow of delivered momentum per unit of time and surface and computed using the momentum conservation theorem Eq. (2.3.8), if the fields are know at the laser-target interaction surface. We can easily generalize Eq. (4.1.23) to the case of non-perfectly reflective medium and oblique incidence, by taking into account the reflectivity R of the target, its transmissivity T and its absorption coefficient A, related as R + T = 1 - A in order to ensure energy conservation. We obtain

$$P_{\rm rad} = (1 + R - T) \frac{I\cos^2(\theta)}{c} = (2R + A) \frac{I\cos^2(\theta)}{c}, \qquad (4.1.24)$$

with θ angle of incidence of the wave with respect to the normal of the target surface.

Due to the effect of the radiation pressure the target surface is put in motion. If the target has a thickness $L_x \gg c/\omega_{pi}$, the surface velocity is found to be a constant and the ions at the front surface are accelerated via the so-called *Hole Boring* (HB) mechanism (Daido et al., 2012; Macchi et al., 2013). The constant surface recession velocity (HB velocity in the following) can be estimated by balancing the flux of ion momentum with the laser radiation pressure (Schlegel et al., 2009; Robinson et al., 2009). Since the HB velocity becomes easily a fraction of c, this balance has to be computed in the rest frame of the plasma surface.

Following the relativistic model presented by Robinson et al. (2009), we consider, in the laboratory frame L, the plasma surface (y-z plane at x = 0) to move with velocity $v_{\rm HB}$ along the x-direction, under the action of a plane wave with intensity I. In the surface rest frame L', the ions of the target are seen to move with velocity $-v_{\rm HB}$ toward the surface, where they are reflected by the electrostatic potential associated with the charge separation electric field, created by the ponderomotive force. In order to balance the momentum flux associated with the reflected ions, with the radiation pressure we exploit the Lorentz's transformations for the wave electromagnetic fields, so that the intensity of the plane wave becomes $I' = I(1 - \beta)/(1 + \beta)$, with $\beta = v_{\rm HB}/c$. The balance thus reads

$$\frac{2I}{c}\frac{1-\beta}{1+\beta} = 2\gamma_{\rm HB}m_i n_i v_{\rm HB}^2 \,, \qquad (4.1.25)$$

with $\gamma_{\rm HB} = [1 - \beta^2]^{-1/2}$. The HB velocity can be analytically extracted

from the previous equation and reads

$$\frac{v_{\rm HB}}{c} = \frac{\sqrt{a_0^2 \frac{m_e}{m_i} \frac{n_c}{n_0}}}{1 + \sqrt{a_0^2 \frac{m_e}{m_i} \frac{n_c}{n_0}}}.$$
(4.1.26)

This expression has been confirmed by several numerical studies.

The charge separation electrostatic field created at the laser-plasma interaction surface, not only set in motion the laser-plasma interaction surface, but it is also responsible for ions acceleration. Indeed, in the surface rest frame L', the HB reflected ions propagate back inside the target with velocity $v_{\rm HB}$. This means that, in the laboratory frame L, the ions are accelerated at velocity $\simeq 2v_{\rm HB}$, assuming non relativistic HB velocity. This simple argument approximately explains the acceleration mechanism at play.

In the following Sec. 4.1.4.1, we provide an original extension to the usual calculation of the HB velocity for the case of laser pulse with oblique incidence. Then we describe the dynamical non-relativistic model of HB acceleration presented by Macchi et al. (2009) in order to clarify some aspects of the HB accelerated ions distribution, not accounted for in the stationary model of Schlegel et al. (2009); Robinson et al. (2009).

4.1.4.1 Hole Boring velocity

In this paragraph, we present the detailed calculation of the Hole-Boring velocity $v_{\rm HB}$ for the case of a plane wave of intensity I and frequency ω , irradiating a perfectly reflecting target with an angle θ with respect to the surface normal. We assume the wavevector to be $\mathbf{k} = (k_x, k_y, 0)$ and the plasma surface (y-z plane at x = 0) to move with velocity $v_{\rm HB}$ along the x direction, in the laboratory frame L. Therefore, in the frame co-moving with the target surface L', the incident wave wavevector becomes

$$k'_{x} = \gamma(k_{x} - \beta\omega/c) = \gamma k(\cos\theta - \beta), \qquad (4.1.27)$$

$$k_y' = k_y = k\sin\theta, \qquad (4.1.28)$$

where $k = \omega/c$, $\beta = v_{\rm HB}/c$, $\gamma = (1 - \beta^2)^{-1/2}$. In L' the incidence angle θ' is thus given by

$$\tan \theta' = \frac{k'_y}{k'_x} = \frac{\sin \theta}{\gamma(\cos \theta - \beta)}.$$
(4.1.29)

In L' a plane wave of intensity I' interacts with an immobile plasma with incidence angle θ' . Thus, from Eq. (4.1.24), the radiation pressure is

$$P'_{\rm rad} = \frac{2I'}{c} \cos^2 \theta' \,.$$
 (4.1.30)

Exploiting the Lorentz's transformations for the wave electromagnetic fields, we obtain $I' = I(1-\beta)/(1+\beta)$. In the frame L, since the pressure is a relativistic invariant $P'_{rad} = P_{rad}$, we have

$$P_{\rm rad} = \frac{2I}{c} \frac{1-\beta}{1+\beta} \frac{1}{1+\tan^2 \theta'} = \frac{2I}{c} \frac{1-\beta}{1+\beta} \frac{\gamma^2 (\cos \theta - \beta)^2}{\gamma^2 (\cos \theta - \beta)^2 + \sin^2 \theta} \,. \quad (4.1.31)$$

The HB velocity can now be estimated by balancing the laser radiation pressure with the flux of ion momentum $P_i = n_0 \gamma \beta c (2m_i \gamma \beta c)$. Solving numerically for $v_{\rm HB} = \beta c$ yields the Hole-Boring velocity as a function of θ . Note that, from Eq. (4.1.29), we obtain that for $\beta = \cos \theta$ the wave propagates parallel to the surface in the L' frame, meaning that $c \cos \theta$ appears has a natural upper limit for the Hole-Boring velocity. From the balance of the radiation pressure Eq. (4.1.31) and P_i , in the case of normal incidence ($\theta = 0$), we recover the well-known result of Eq. (4.1.26).

4.1.4.2 Hole Boring acceleration mechanism

We now present the non-relativistic model for the ion acceleration proposed by Macchi et al. (2009) that takes into account the ion dynamics in the charge separation layer.

At the initial stage of the laser-plasma interaction Fig. 4.1a), ions are still at rest, while electrons have penetrated in the target for a distance x_d creating the charge separation field E_x . Assuming the simplified profiles of Fig. 4.1a) for the surface electric field E_x and for the density of both ions and electrons, the maximum value of E_x from Poisson's equation (2.1.8) is $E_{\text{max}} = 4\pi e n_0 x_d$, with x_d the position of the electron surface. In a steadystate condition, the ponderomotive force is balanced by the electrostatic field in the region $x_d < x < x_s$, with x_s defined in Fig. 4.1a), so that $f_{p,x} = eE_x$ and $2I/c = E_{\max}en_{p0}(x_d - x_s)/2$, with n_{p0} the peak density of the electron compressed layer. Moreover, imposing charge conservation, we obtain $n_0x_s = n_{p0}(x_s - x_d)$. Note that the penetration length of the ponderomotive force corresponds to $x_d - x_s \simeq c/(2\omega_{pe})$. To study the dynamics of the ions in the region $x_d < x < x_s$, we simply have to solve the equation of motion for the single particle position $x_i(t)$:

$$m_i \frac{d^2 x_i}{dt^2} = e E_{\max} \left(1 + \frac{x_i(0) - x_d}{x_s - x_d} \right) \,. \tag{4.1.32}$$

Since the trajectories of the ions do not intersect each other, the electric field is defined by the initial condition. The straightforward solution of Eq. (4.1.32) reads

$$x_i(t) = x_i(0) + \frac{eE_{\max}}{2m_i}t^2$$
. (4.1.33)

This solution predicts all the ions locate in the region $x_d < x < x_s$ to accelerate and accumulate in the layer where the laser fields are evanescent. As soon as the ions move toward x_s , the electrons follow in order to keep the balance between the ponderomotive force and E_x , as sketched in Fig. 4.1b). The trajectory described by Eq. (4.1.33) shows that all ions reach the position $x = x_s$ at the same time $t_s = [2(x_s - x_d)m_i/eE_{\text{max}}]^{1/2}$, causing a singularity in the ion density, see Fig. 4.1c), and a subsequent wave-breaking, i.e. a break down of the hydrodynamic description, leading to injection of the ions inside the target. In addition, the ions reaching $x = x_s$ have a flat velocity distribution in the interval $0 < v_x < 2v_{\text{HB}}$ and numerical simulations have confirmed that the fastest ions penetrate in the target as a bunch with velocity $\simeq 2v_{\rm HB}$. After the injection of the accelerated ion bunch, the quasi-equilibrium condition of Fig. 4.1a) is restored and the process repeats itself. Note that the ions initially in the depletion layer $0 < x < x_d$ are as well accelerated by the charge separation field but they do not reach the ion singularity for $t = t_s$.

The clear signatures of this acceleration mechanism are the large number of high energy ions produced (theoretically a density equal to n_0) and the very narrow energy spectrum (within the static model, all particles



Figure 4.1: Schematically representation of the Hole-Boring acceleration mechanism, (see Macchi et al., 2009).

get $\simeq 2v_{\rm HB}$) in comparison with other acceleration schemes. These features arose the interest on this mechanism for envisioned applications where the localized ion energy deposition (at the so-called Bragg peak) is required.

Despite this mechanism seems to be the dominant one only for intensity $I\lambda^2 \gtrsim 10^{23} \,\mathrm{Wcm}^{-2}\mu\mathrm{m}^2$ (Esirkepov et al., 2004), a regime dominated by the HB acceleration with present-day laser systems has been achieved using circularly polarized pulses. This configuration is expected to suppresses electron heating, as discussed in Sec. 4.1.3, avoiding competing acceleration mechanisms. Indeed, a first experimental proof of this acceleration mechanism has been given by Palmer et al. (2011), using weakly overdense hydrogen gas jet targets and circularly polarized CO₂ laser pulses with intensity $I \sim 5 \times 10^{15} \,\mathrm{Wcm}^{-2}$. The observed accelerated ions spectra show a scaling $\propto I/n_0$, a narrow energy spread $\simeq 4\%$ around the peak energy values and a large number of accelerated particle ($\gtrsim 10^{12} \,\mathrm{protons/MeV/sr}$), consistent with HB acceleration.

In the previous model, many effects of the intense laser-plasma interaction have been neglected. For instance, electron heating can drastically reduce the efficiency of the HB mechanism. Indeed, due to the electron energy absorption, the total radiation pressure on the target is reduced and the formation of the ion density spike can be counteracted by the electron thermal pressure. This effect will be highlighted in the PIC simulations presented in Sec. 4.2. As already said, the use of a circularly polarized laser pulse at normal incidence is commonly proposed to overcome this issue. The limited efficiency of this solution in the case of UHI lasers will be discussed in Sec. 4.3.

Another critical point is the possibility to maintain the opacity of

the target over a long time, since the propagation of the laser pulse in the plasma would change completely the interaction regime and would prevent HB acceleration. As mentioned in Sec. 4.1.1, RSIT may set in during the interaction with a high-intensity laser pulse. For a deep investigation of additional effects that might cause the target transparency during the laser-target interaction, such as electron heating and ion motion, see Refs. (Siminos et al., 2012, 2016). However, as demonstrated by Cattani et al. (2000), the non-linear dynamic at the surface, leads to the production of an electron density peak, which suppresses the laser penetration and significantly increases the value of the critical density, i.e. increase the intensity necessary to reach the induced transparency regime.

Note that operating at the lowest acceptable density n_0 , hence close to the RSIT threshold, and with light ions, is beneficial for the acceleration mechanism since the HB velocity, and therefore the energy of the HB reflected ions, decreases with increasing n_0 and ion mass, as clear from Eq. (4.1.26). For this reason, in our study, we will use a density slightly above the critical one. We take into account relativistic effects on n_c , as discussed in Sec. 4.1.1, so that we consider as effective critical density $n_c^{\text{REL}} = n_c \sqrt{1 + \langle a_0^2/2 \rangle}$, where the brackets represent the average on the laser period.

4.2 Previous works on UHI laser-driven Weibel instability

We have already stressed in Chap. 1 the importance of investigating in the laboratory the Weibel instability and reproducing the phase of collisionless shock formation. Apart from the experimental configuration under investigation at NIF and OMEGA facilities presented in Sec. 1.2.2, an alternative proposition, that relies on the use of UHI laser facilities, has been discussed by Fiuza et al. (2012). The authors consider an intense $(I\lambda_{\rm L}^2 \sim 10^{20} \div 10^{22} \, {\rm Wcm}^{-2} \mu {\rm m}^2)$ picosecond-duration linearly polarized laser interacting with an overdense target at normal incidence. They demonstrate that the system becomes Weibel unstable due to the hot electrons propagating into the target and the cold return current, produced by induction to maintain charge neutrality and the total electron current smaller than the Alfvén limit. A first stage in which an electrostatic shock propagates inside the target has been observed to evolve in a Weibel-mediated shock, once the Weibel-produced magnetic turbulence become strong enough to overcome the electrostatic potential at the shock front, (see Ruyer et al., 2015).

The configuration investigated by Fiuza et al. (2012); Ruyer et al. (2015), i.e. linear polarization and normal incidence, aimed at maximizing the hot electrons production. This is not the optimal scheme to produce HB, that will be the focus of our investigation. However, in order to highlight the main features of this interaction configuration and the differences with our proposition, we first describe the results obtained by Fiuza et al. (2012); Ruyer et al. (2015). To do that, we reproduce a 2D simulation with the same parameters of Fiuza et al. (2012); Ruyer et al. (2015). The same parameters will be used for our study on the optimal configuration to drive HB, that will be achieved by changing the laser-plasma interaction scheme (e.g. angle of incidence and polarization). This allows for a direct comparison between the different laser configurations discussed in the following Sec. 4.3.

We consider a linearly polarized laser pulse (here a plane wave, i.e. no transverse laser profile), with the electric field lying in the plane of the simulation ($\mathbf{E}_{\rm L} = E_0 \, \hat{\mathbf{y}}$), of intensity $I\lambda_{\rm L}^2 \sim 5 \times 10^{21} {\rm Wcm}^{-2} \mu {\rm m}^2$, corresponding to $a_0 = 60$, irradiating at normal incidence ($\theta = 0$) an overdense target with density $n_0 = 49 n_c$ and temperature $T_e = T_i = 1 {\rm keV}$. The target is assumed completely ionized and constituted of electrons and protons ($m_i = 1836 m_e$). At t = 0, the plasma fills the region $x \ge 4 \lambda_{\rm L}$, up to the right boundary of the simulation box, and has a step-like profile, while the laser has \sin^2 -like ramp of $5 t_{\rm L}$ duration, with $t_{\rm L}$ the laser period. The spatial resolution is $\Delta x = \Delta y = \lambda_{\rm L}/320 \simeq (c/\omega_{pe})/7$ and the box extension $L_x = 32 \lambda_{\rm L}$ and $L_y = 4 \lambda_{\rm L}$. The resolution in time is $c\Delta t = 0.5 \Delta x$. We use 49 particles per cell per species, for a total of $\simeq 10^9$ particles.

The hot electron bunches, produced by the $\mathbf{J} \times \mathbf{B}$ mechanism, have relativistic momentum $p_x \gtrsim 100 \, m_e c$, as shown in Fig. 4.2a). The Weibel instability magnetic field turbulence develops in the region where these



Figure 4.2: Interaction with a linearly polarized laser pulse at normal incidence, at $t = 25 t_{\rm L}$. a) Electron $x - p_x$ phase-space. b) Ion $x - p_x$ phase-space. c) Electron density. d) Ion density. e) Magnetic field B_z and zoom in the region $12.5 \lambda_{\rm L} < x < 17.5 \lambda_{\rm L}$ and $\lambda_{\rm L} < y < 3 \lambda_{\rm L}$, f) Electric field E_x . Both B_z and E_x have been averaged over the laser period.

fast electrons have driven a return current, see the inset in Fig. 4.2e) corresponding to $12.5 \lambda_{\rm L} < x < 17.5 \lambda_{\rm L}$ and $\lambda_{\rm L} < y < 3 \lambda_{\rm L}$. Note that the strong magnetic structures close to the target surface $x \simeq (6.5 \div 8) \lambda_{\rm L}$, have a different origin. They are related with an additional instability developing at the surface, as will be discussed in Sec. 4.3.

The electrostatic shock front is identified by the strong positive E_x component at $x \simeq 8 \lambda_{\rm L}$, reported in Fig. 4.2f). Indeed, at that position, the typical reflection of a small fraction of ions is observed in the ion phase space $x - p_x$, Fig. 4.2b). These ions have momentum $p_x \simeq 660 m_e c \simeq 0.36 m_i c$, consistent with the reflection from the electrostatic shock front propagating with velocity $v_{\rm es} \simeq 0.174 c$, as measured in the simulation. However the density of the reflected ions, obtained in-

tegrating the ion distribution function for $p_x \gtrsim 500 m_e c$, remains always smaller than $5 n_c$. This is approximately 1/10 of what we can achieve with the HB mechanism on a target with density $n_0 = 49 n_c$, therefore we do not expect the ion-Weibel instability to be efficiently triggered by the ions reflected at the electrostatic shock front. Note that the electrostatic shock must propagate inside the target with a velocity larger than the recession velocity of the target surface, located at $\simeq 6.5 \lambda_{\rm L}$ in Fig. 4.2. Indeed, the HB velocity measured in the simulation is $v_{\rm HB} \simeq 0.12 \, c < v_{\rm es}$, in good agreement with the theoretical prediction $v_{\text{HB}}^{\text{th}} = 0.124 c$. At this stage the electrostatic energy governs the transition between the upstream region $(x \gtrsim 8 \lambda_{\rm L})$ and the downstream one $(6.5 \lambda_{\rm L} \lesssim x \lesssim 8 \lambda_{\rm L})$. The two regions are clearly separated by the abrupt density jump in both electron and ion density, Fig. 4.2c-d), respectively. The transition to a Weibel-mediated shock occurs around $t \simeq 50 t_{\rm L}$ (not shown), consistent with Ruyer et al. (2015), when the magnetic energy at the shock front becomes larger than the electrostatic potential.

The Weibel filaments, of size $\leq 0.2 \lambda_{\rm L}$, observed in both magnetic field B_z (see inset of Fig. 4.2) and electron density n_e in the upstream region $(x \geq 8 \lambda_{\rm L})$, are mainly created by the electron currents. The modulations in the ion density follow from the charge separation electrostatic field (here E_y) driven by the electron filaments formation, and do not involve any ion current filaments instability. This is shown in Fig. 4.3a), where a 1D cut of the current density J_x as a function of y is reported for the electrons (blue line) and for the ions (red line). On the contrary the density modulations, as shown in Fig. 4.3b), are even more peaked in the ions than in the electrons. Both density and current density are taken at $x = 12.5 \lambda_{\rm L}$ for $t = 25 t_{\rm L}$, the same time shown in Fig. 4.2.

In conclusion, this configuration seems suitable for the investigation of Weibel-mediated shocks propagation and potentially particle acceleration via e.g. Fermi-like mechanism, while the shock formation follows from a non-purely Weibel mechanism, i.e. an initial transient stage of electrostatic shock. However, probing the shock front fields, in a dedicated experiment, would be a challenging task. In particular, it would be complicated to distinguish between the laser fields and the instabilityproduced fields, due to the proximity of the shock front and the laser-



Figure 4.3: Interaction with a linearly polarized laser pulse at normal incidence, at $t = 25 t_{\rm L}$. a) Current density J_x . b) Density. For the electrons (blue lines) and ions (red lines) at $x = 12.5 \lambda_{\rm L}$ (ahead of the shock front), see Fig. 4.2.

plasma interaction surface. Moreover, as mentioned above and investigated by Ruyer et al. (2015), ion currents in the region where the Weibel instability develops, are much weaker than the electronic ones, meaning that the dynamics is totally governed by the electrons. This contrasts with the standard astrophysical scenarios in which the ion instability is considered a key phase of shock formation, (see Lyubarsky and Eichler, 2006). This motivates the investigation of an alternative configuration able to reproduce fast and neutral electron-ion flows, in which the phase of ion-Weibel instability can be investigated. The optimal scheme for that entails a minimization of the hot electron production, otherwise governing the dynamics as in the presented system, that can be achieved with a linear S-polarized laser irradiating the target at large incident angle, as will be demonstrated in the following.

4.3 Comparison between different laser-plasma interaction configurations

We demonstrate in this section that the optimal laser configuration to create a quasi-neutral flow in a dense target by means of the HB acceleration mechanism, is obtained with linear S-polarization and large incident an-

gle. This contrasts with the usual choice of normally incidence circularly polarized laser pulse that would be the first candidate for Hole Boring, as it is commonly considered to suppress electron heating, as discussed in Sec. 4.1.3. Indeed, the long time scale of our processes entails a strong electron heating even with a circularly polarized laser pulse, due to the deformation of the interaction-surface. Once relativistic hot electrons are produced, the system follows the same evolution as presented in the previous Sec. 4.2. As observed in Fig. 4.2, strong magnetic field structures develop at the surface and can scatter the HB reflected ions, preventing their penetration deep into the target. In addition, the energy spread of the HB reflected ions has been demonstrated to increase as the electron heating increases (Paradkar and Krishnagopal, 2016). This has important implications in the ion Weibel instability development, i.e. the instability growth rate decreases with the increase of the HB ions energy spread. The ideal configuration is given by S-polarization and angle of incidence $\theta = 45^{\circ}$, as it is the configuration that minimize the hot electron production and in which the surface instability is efficiently quenched. We demonstrate that the suppression of the surface instability is linked to the appearance of a transverse slowly varying electron current layer and we provide a simple analytical model to explain its generation, Sec. 4.3.2.

The main results of this section are confirmed by 2D and 3D PIC simulations. Due to the more complicated analysis in the 3D geometry, we focus first on the 2D simulations in Sec. 4.3.1, then we present the generalization to the 3D in Sec. 4.3.3.

4.3.1 2D simulations

We present a comparison between 2D3V simulations considering circular polarization (at normal incidence) and linear polarizations (both P and

¹To describe the propagation along x of a plane wave with an angle $\theta \neq 0$, using periodic boundary condition on y, we need to impose $L_y \sin \theta = N\lambda_L$, with N integer number. Therefore, we use a slightly different box dimension L_y for each case. In detail, at normal incidence $L_y = 4 \lambda_L$ and N = [1, 2, 3] have been used in the previous relation for $\theta = [15^\circ, 30^\circ, 45^\circ]$, respectively.

²Due to the non-perfect open boundary conditions on the electromagnetic fields at the boundary from which the laser enters in the box in SMILEI, the effective amplitude of the laser pulse is $a_0 = [60, 61, 64.2, 70.3]$ for $\theta = [0^\circ, 15^\circ, 30^\circ, 45^\circ]$, respectively.

Pol	θ	$v_{ m HB}^{ m th}/c$	$v_{ m HB}/c$	$\Gamma_{\rm surf} t_{\rm L}^{-1}$	$T_h \mathrm{MeV}$	$U_h n_c \mathrm{MeV}$	n_h/n_c
С	0°	0.124	0.115	1.8	13.9	1.64	0.12
S	0°	0.124	0.118	4.1	16.3	3.82	0.23
\mathbf{S}	15°	0.121	0.114	3.6	15.4	1.95	0.13
\mathbf{S}	30°	0.112	0.109		17.0	0.41	0.024
\mathbf{S}	45°	0.098	0.098		12.0	0.06	0.005
Р	0°	0.124	0.119	3.9	14.1	5.42	0.38
Р	15°	0.121	0.130	3.5	17.5	5.01	0.29
Р	30°	0.112	0.135		26.9	3.93	0.15
Р	45°	0.098	0.127		52.4	1.05	0.02

Table 4.3.1: Theoretical $v_{\text{HB}}^{\text{th}}$, Sec. 4.1.4.1, and measured v_{HB} HB velocity. Growth rate of the surface instability Γ_{surf} . Hot electron temperature T_h and energy density U_h measured at $t = 25 t_{\text{L}}$ in the interval $x \simeq (8 \div 12.5) \lambda_{\text{L}}$. The density is computed as $n_h = U_n/T_h$, in order to compare with the simulation results shown in Fig. 4.9c-d).

S, varying the angle of incidence). The parameters are the same than in the simulation presented in the previous Sec. 4.2, namely $a_0 = 60$ and plasma density $n_0 = 49 n_c^{1,2}$. The main quantities of interest for the comparison of the different configurations are listed in Table 4.3.1, and will be discussed in the following paragraphs. We first provide a description of the numerical observations and then a discussion on the main features for the different configurations, focusing on the surface instability and on the electron heating.

Circular polarization at normal incidence

After $25 t_{\rm L}$ of interaction with a circularly polarized laser pulse, the electron density and the ion surface density profile give evidence of the generation of strong corrugations at the surface, as shown in Fig. 4.4a). The corrugation implies a non-exactly local normal incidence of the laser pulse, thus a non-negligible contribution of vacuum and $\mathbf{J} \times \mathbf{B}$ heating. This leads to the production of a large amount of hot electrons that propagate at relativistic velocity into the target as shown in the electron x- p_x phase-space in Fig. 4.4c), even if at early times the hot electron production is weaker than in the linear case at normal incidence, Figs. 4.5e) and 4.6e).



Figure 4.4: After $25 t_{\rm L}$ of interaction with the circularly polarized laser pulse: a) electron density and surface profile of the ion density (red line), b) magnetic field B_z averaged on a laser period and surface profile of the ion density (black line), c) $x-p_x$ electron phase space, the dashed red lines identify the x-interval plotted in a-b), the dotted red line corresponds to the average position of the ion surface.

Indeed, the amount of energy transferred to the fast electrons increases with time, as the surface corrugations become stronger: in Fig. 4.4c) the hot electrons located deep inside the target, hence produced at early time, have lower density and momentum, with respect to those closer to the surface. The correspondence of the surface rippling with the magnetic field structures inside the target is highlighted in Fig. 4.4b) and will be discussed in the following. The surface magnetic field reaches a large amplitude, the absolute value is $B_{z,surf} \simeq 40 \, m_e \omega_{\rm L} c/e$. For comparison we recall that the laser magnetic field amplitude is $B_{\rm L} \simeq 42.4 \, m_e \omega_{\rm L} c/e$, corresponding to $\simeq 4.2 \times 10^9 \, {\rm G}$ for $\lambda_{\rm L} = 1 \, \mu {\rm m}$.

Linear polarization

We present in Figs. 4.5 and 4.6 the results of the simulations performed with an S-polarized and a P-polarized plane wave respectively, with incidence angle $\theta = 0^{\circ}$, 15° , 30° , 45° , at $t = 25 t_{\rm L}$.

At normal incidence (Figs. 4.5a and 4.6a) the magnetic structures in B_z at the surface form quickly and a strong electron heating is observed (Figs. 4.5i and 4.6i), as expected. The correspondence of the mag-



Figure 4.5: After $25 t_{\rm L}$ of interaction with a linearly S-polarized laser pulse at angle of incidence a,e,i) $\theta = 0^{\circ}$, b,f,l) $\theta = 15^{\circ}$, c,g,m) $\theta = 30^{\circ}$, d,h,n) $\theta = 45^{\circ}$: a-d) Magnetic field B_z averaged on a laser period and surface profile of the ion density (black line), e-h) x-p_x electron phase space, i-n) $x - p_x$ ion phase-space.

netic field structures and the ion density modulations is highlighted in Fig. 4.5a (black line). Increasing the angle of incidence (from left to right in Figs. 4.5a-d and Figs. 4.6a-d), the surface magnetic structures become weaker. In particular, at large angles $\theta = 30^{\circ}, 45^{\circ}$, the development of the magnetic filaments is completely prevented and a strong positive magnetic field is created in a surface layer of thickness $\simeq 0.1 \lambda_{\rm L} \simeq 4.4 c/\omega_{pe}$ for S-polarization and $\simeq \lambda_{\rm L} \simeq 44 c/\omega_{pe}$ for P-polarization. At this time, the unipolar magnetic field amplitude is $B_{z,\rm unip} \simeq 10 m_e \omega_{\rm L} c/e$ and $B_{z,\rm unip} \simeq 40 m_e \omega_{\rm L} c/e$ for S- and P-polarization, respectively, corresponding to $B_{z,\rm unip} \simeq (1 \div 4) \times 10^9$ G for $\lambda_{\rm L} = 1 \,\mu$ m.

With the increase of the angle of incidence, there is an evident decrease


Chapter 4. RPA driven Ion Weibel Instability and Collisionless Shocks

Figure 4.6: After $25 t_{\rm L}$ of interaction with a linearly P-polarized laser pulse at angle of incidence a,e,i) $\theta = 0^{\circ}$, b,f,l) $\theta = 15^{\circ}$, c,g,m) $\theta = 30^{\circ}$, d,h,n) $\theta = 45^{\circ}$: a-d) Magnetic field B_z averaged on a laser period, e-h) $x - p_x$ electron phase space, i-n) $x - p_x$ ion phase-space.

of the fast electron production for both S- and P-polarizations, as shown in the electron x- p_x phase-spaces in Figs. 4.5e-h) and 4.6e-h).

From the ion $x - p_x$ phase-spaces, Figs. 4.5i-n) and 4.6i-n), we see that the highest density and the minimum momentum spread of the HB reflected ions, is obtained for S-polarization and $\theta = 45^{\circ}$. However, due to the decrease of the HB velocity at large angles, as listed in Table 4.3.1, the reflected ion momentum is lower than at normal incidence.

Discussions on the origin of the surface instability

The formation of the surface density ripplings in circular polarization at normal incidence, highlighted in Fig. 4.7d), is at the origin of the strong electron heating that makes us reject this configuration for the long term

production of the HB ion beam. Understanding the instability origin is therefore a key step to clarify how we can suppress it.

The correspondence of these corrugations with the strong magnetic field structures inside the target, at late times, has been evidenced in Figs. 4.4b) and 4.5a), and is reported for convenience in Fig. 4.7c-d) for the circularly polarized case.

At small angle of incidence $\theta = 0^{\circ}, 15^{\circ}$, the growth of the surface magnetic filaments is associated with an instability whose typical growth rate is $\Gamma_{\rm surf} \simeq 3.5 t_{\rm L}^{-1} \simeq 0.08 \,\omega_{pe}$ (see Table 4.3.1 for the detailed values), measured as described in the following. Since the target interaction surface is moving at the velocity $v_{\rm HB}$, listed in Table 4.3.1, the growth rate has been measured extracting at different times the total transverse magnetic energy in a region of extension $0.15 \,\lambda_{\rm L}$ moving with $v_{\rm HB}$. The growth rate follows from the linear fit of the data, as shown in Fig. 4.8. The instability quickly saturates, after $\simeq 6 t_{\rm L}$, with field amplitude $B_{z,{\rm surf}} \simeq 30 \, m_e \omega_{\rm L} c/e$. The dimension of the magnetic structures evolves from $\simeq c/\omega_{pe}$ to $\simeq c/\omega_{pi}$.

We propose that these magnetic fields correspond to Weibel modes, produced by the strongly anisotropic electron distribution function at the laser-plasma interaction surface. Within this model, the surface density modulations can be seen as a consequence of the current filament formation. Indeed, the electrons in the filament of positive current, i.e. moving outward from the target, can not escape in vacuum because of the laser piston effect, thus accumulating in the high density peak of the modulations. The position of these density peaks actually corresponds to the node of $B_{z,surf}$ with positive magnetic field on the top and negative on the bottom, where a positive current filament is expected. The valleys of the density modulations, thus correspond to the position of filaments with particles moving in the positive x-direction. Note that this density surface corrugation should not be confused with the density modulations intrinsic of the Weibel instability, that has a wavelength equal to one half of the Weibel-generated magnetic field (i.e. a density peak for each current filament, see Fig. 3.2). On the contrary, the surface modulations (black line in Fig. 4.7d) have wavelength equal to the magnetic field B_z , as expected from our explanation. The density corrugation is therefore a consequence of the development of the electron Weibel instability.

The simulation at P-polarization and normal incidence, reported in Fig. 4.7a-b), makes us exclude the possibility that the strong magnetic field modulations are a result of the density modulations. Indeed, in that case, the density modulations are smoothed by the laser electric fields oscillations at the surface and the creation of an expanding layer of hot electrons. However, the magnetic field structures are observed with the same characteristics (intensity, growth rate and wavelength) than in the other cases.

In many previous numerical studies, interested on the electron transport in dense targets, strong magnetic structures at the target surface have been observed, (see Lasinski et al., 1999; Sentoku et al., 2000, 2002; Adam et al., 2006). These structures have been associated with the Weibel instability driven by the hot electrons and resulting cold return current. However, this model can not explain many characteristic features observed in our simulations: (i) the confinement of these strong magnetic fields close to the surface, (ii) the much faster growth rate and larger amplitude of the surface magnetic fields with respect to the Weibel modes driven by the hot electrons and resulting cold return current deep into the target (corresponding to the inset in Fig. 4.2), (iii) the formation of these magnetic structures in the case of a circularly polarized laser pulse, for which one would theoretically expect a strong reduction of electron heating.

To highlight the difference between the surface instability and the one developing deep into the target along the hot electrons propagation, it is useful to compare the values of their growth rates. For the surface instability, i.e. the magnetic structures shown in Figs. 4.5a-b) and 4.6a-b), we measured in PIC simulations in linear polarization $\Gamma_{\rm surf} \gtrsim 3.5 t_{\rm L}^{-1}$, corresponding to the fit reported in Fig. 4.8. Ruyer et al. (2015) found a growth for the filaments forming deep inside the target $\Gamma_{\rm in-depth} < 0.22 t_{\rm L}^{-1}$, with linearly P-polarized laser pulse at normal incidence. The surface instability grows more than 10 times faster than the in-depth one. This implies that the mechanism acting at the surface is different from the one driving the instability deep into the target. An analytical prediction of the anisotropy rate would rely on a detailed investigation of the electron



Figure 4.7: Interaction with: (Top panels) linearly P-polarized laser pulse at normal incidence (as discussed in Sec. 4.2), (Central panels) circularly polarized laser pulse at normal incidence (same as in Fig. 4.4), (Bottom panels) S-polarized laser pulse at $\theta = 45^{\circ}$ (same as in Fig. 4.5 right column). a-c-e) Electron density. b-d-f) Magnetic field B_z averaged on a laser period. Red and black lines represent to the ion density profile.



Figure 4.8: Growth of the surface transverse magnetic energy density, associated with the surface instability, for normal incidence laser pulse $\theta = 0^{\circ}$ (blue dots) and for $\theta = 15^{\circ}$ (red dots). Linear fit of the growing phase (blue and red lines). a) S polarization. b) P polarization.

dynamics in the compressed layer. This is a longstanding open problem in laser-plasma physics and goes beyond the scope of this study. We note (not shown) that the anisotropy is stronger for linear polarization (LP) than in circular polarization (CP), resulting in a growth rate $\Gamma_{\rm surf,CP} \simeq 0.5 \Gamma_{\rm surf,LP}$.

In previous works, analyzing simulations performed at normal incidence with circular polarization, the origin of the surface density modulations has been associated with different types of instabilities. Here, we give a brief description of them in order to illustrate why they should not apply to our study.

The existence of a Rayleigh-Taylor-like instability, on the surface of a thin foil (thickness $\leq \lambda_{\rm L}$) interacting with an intense laser, has been demonstrated and theoretically described by Sgattoni et al. (2015); Eliasson (2015). By means of 3D PIC simulations at normal incidence, it was shown that in circular polarization the instability leads to a pattern of hexagonal-like structures, while in linear polarization the density ripplings are smoothed along the polarization direction. Indeed, the motion of the electrons oscillating in the laser electric field "washes out" the instability, leaving elongated structures with wavevector perpendicular to the laser electric field. This mechanism can not apply to our observations for the following reasons. Even though this instability explains the formation of the surface density rippling, it does not provide a motivation for the magnetic structures formation at the surface. Moreover, in our case the development of $B_{z,\text{surf}}$ is not quenched by the electron oscillations in the laser field, i.e. in Ppolarization at normal incidence Fig. 4.6a). In addition, the typical scale of the rippling in the Rayleigh-Taylor-like instability is $\simeq \lambda_{\text{L}}$, while our simulations suggest that the structures keep increasing their dimension from c/ω_{pe} up to c/ω_{pi} . For the parameters used in the simulations presented up to now $\lambda_{\text{L}} \simeq c/\omega_{pi}$, therefore in order to discriminate between the two scales at late time, a simulation with density $n_0 = 16 n_c$ has been performed. The surface structures reach extension $\simeq 1.7 \lambda_{\text{L}} \simeq c/\omega_{pi}$. The model of Rayleigh-Taylor-like instability requires the foil to accelerate under the radiation pressure action (being intended for accelerating thin target in the light sail regime) and its generalization to the constant HB velocity case is not straightforward.

In another recent proposition, Wan et al. (2016) provides a description of the surface density modulations as a result of the coupling between the transverse oscillations of the electrons in the laser electric field and the quasi-static ion density fluctuations. This model is not intended for the high intensity exploited in our simulations, and does not involve the formation of any magnetic field structures. Note that this mechanism comes into play only if the trajectories of the particles under the action of the laser electric field component is described in the simulation, hence it can not be observed in 2D simulations in S-polarization for which $\mathbf{E}_{\rm L} = E_z \hat{\mathbf{z}}$. This argument makes clear that the surface instability observed in our simulation has a different origin.

In conclusion, we exclude these two surface instabilities as dominant mode in our systems. There might be however a complex interplay between them and the Weibel instability driven by the surface distribution function anisotropy.

Discussions on electron heating

In the case of S-polarization, the laser field components responsible for the $\mathbf{J} \times \mathbf{B}$ heating, i.e. E_z and B_y , have a maximum amplitude at normal incidence. Thus we expect the amount of laser energy transferred to the electron population to decrease at large angles, consistent with the simulation results, see Fig. 4.5. In P-polarization, contrarily to what we observe in Fig. 4.6, considering the laser field components responsible for the electron heating, we expect the hot electron production to increase at large angles of incidence. Indeed, in previous studies (Gibbon, 1994), the heating in P-polarization was demonstrated to have a maximum around $\theta \simeq 40^{\circ} \div 50^{\circ}$, due to a combination of radiation pressure and ablative expansion that create a smooth density profile at the laser-plasma interaction surface. We should note that the intensity of the laser pulse considered for our purposes is much larger than the one used by Gibbon (1994), i.e. $I\lambda_{\rm L}^2 \leq 10^{18} \,{\rm Wcm}^{-2}\mu{\rm m}^2$. Furthermore, in a 2D simulation performed with the parameters investigated by this author ($a_0 = 0.27$ and density $n_0 = 2 n_c$) no surface instability producing the strong magnetic structures observed in our case, is found.

This leads us to the investigation of a possible correlation of the surface instability, and its stabilization at large incidence angles, with the hot electron production. To do that, we measure the hot electron temperature, energy density and density, as reported in Fig. 4.9 and Table 4.3.1. We consider an electron distribution $\propto e^{-\gamma/T_h}$, with T_h the hot electron temperature. The values reported in Table 4.3.1 correspond to fit for $\gamma \gtrsim 20$, averaging in the interval $8 \lambda_{\rm L} < x < 12.5 \lambda_{\rm L}$, as shown in Fig. 4.9a-b). Analogously, the hot electron energy density has been computed integrating in the same $x-\gamma$ region. The hot electron densities reported in Table 4.3.1 have been extracted assuming $U_h \simeq n_h T_h$, and are in good agreement with the curves shown in Fig. 4.9c-d) in the region $8 \lambda_{\rm L} < x < 12.5 \lambda_{\rm L}$. The ponderomotive scaling Eq. (4.1.20) would predict a hot electron temperature $\simeq 20 \div 25 \,\mathrm{MeV}^3$, fairly consistent with our observations. In both S- and P-polarizations, the energy density transferred to the electron decreases with the angle of incidence, despite the temperature increases for P-polarization. Indeed, the hot electrons density n_h (for $\gamma \gtrsim 20$) is strongly reduced at large angles, as reported in Figs. 4.9c-d).

³The value of the electron temperature depends on the parameter a_0 , that, in the simulations, varies with the angle of incidence in the range [60, 70.3], due to numerical issues with the boundary conditions, as stated in the footnote at the beginning of Sec. 4.3.1.



Figure 4.9: (Top row) Electron energy distribution as a function of γ at $t = 25 t_{\rm L}$ averaged in the interval $8 \lambda_{\rm L} < x < 12.5 \lambda_{\rm L}$. (Bottom row) Density of the hot electron population ($\gamma_0 \gtrsim 20$). a-c) S-polarized plane wave. b-d) P-polarized plane wave. With $\theta = 0^{\circ}$ (blue lines), $\theta = 15^{\circ}$ (red lines), $\theta = 30^{\circ}$ (green lines), $\theta = 45^{\circ}$ (yellow lines). Circular polarized plane wave at normal incidence is reported in all frames (blue dashed lines.) Black dotted lines correspond to the typical fit to extract the temperature reported in Table 4.3.1.

These measurements, as well as the differences with respect to Gibbon (1994), suggest an active role of the surface instability enhancing the electron heating at normal incidence in P-polarization, where the instability-produced magnetic field can sum up with the laser one increasing the $\mathbf{J} \times \mathbf{B}$ efficiency. On the contrary, $B_{z,\text{surf}}$ is not expected to increase the heating at normal incidence in S-polarization, since the laser field responsible for the $\mathbf{J} \times \mathbf{B}$ heating are E_z and B_y . The decrease of hot electron density in the case of large angle of incidence and P-polarization



Figure 4.10: a) Electron current density $J_{y,e}$. b) Ion current density $J_{y,i}$. c) Electric field E_y averaged on a laser period. After 93 $t_{\rm L}$ of interaction with the linearly S-polarized plane wave at $\theta = 45^{\circ}$.

can be associated with the unipolar surface magnetic field. Indeed, electrons with $\gamma < 50$ can remain trapped in the unipolar magnetic field $B_{z,\text{unip}}$ at the surface, having a Larmor radius $(r_{\rm L} < 0.2 \lambda_{\rm L})$ smaller than the extension of $B_{z,\text{unip}}$ ($\simeq \lambda_{\rm L}$). Note that, despite $B_{z,\text{unip}}$ has roughly the same intensity of $B_{z,\text{surf}}$ created by the instability at normal incidence, in the unipolar magnetic field particles remain trapped more easily than in the modulated one.

To conclude, we observe that with S-polarization and $\theta = 45^{\circ}$, the hot electron production is weaker (both density and temperature have lower values) in comparison with the case of circularly polarized light, reported as dashed lines in Fig. 4.9.

Discussions on the surface stabilization

Another point to be further discussed is related to the stabilization of the surface. In order to avoid the formation of the surface magnetic structures, a large angle of incidence is required, as highlighted in Figs. 4.7e-f) for S-polarization and $\theta = 45^{\circ}$ (see also Figs. 4.5 and 4.6).

The suppression of the surface Weibel-like instability is correlated with the growth of a transverse electron current J_y . In the fast ignition context, the formation of a current sheet along the interaction surface has been already observed in PIC simulations at non-normal incidence (Vshivkov et al., 1998; Nakamura et al., 2004; Kato et al., 2004). The stabilizing role of this current is clear in a framework in which the surface structures correspond to Weibel-amplified field $B_{z,surf}$ with wavevector k_y . In this situation, a coherent motion along the y-direction would prevent the particles confinement in the filaments, removing the feedback mechanism for the instability growth.

We consider the 2D simulation performed with an S-polarized laser pulse irradiating the target at $\theta = 45^{\circ}$, reported in Figs. 4.7e-f). As shown in Fig. 4.10a), the electrons form a double layer current J_y , negative toward the vacuum and positive slightly deeper inside. A net motion along the y-direction, excludes the formation of the localized filaments of electronic current J_x required to amplify Weibel modes at the surface. The electron currents J_y produces and confines the positive magnetic field in the thin layer at the surface, as in Fig. 4.7f). In turn, an electric field E_{y} is generated by induction, Fig. 4.10c). This tends to slow the electrons down and is responsible for the ion motion in the positive y-direction, as shown by the $J_{y,i}$ component in Fig. 4.10b). Note that the polarity of the surface fields depends on the choice of the laser propagation direction, determined by $\pm \theta$. Indeed, as will be clarified by the analytic model presented in Sec. 4.3.2, the electron dynamics at the surface is related to the absorption of electromagnetic momentum in the direction parallel to the target surface.

4.3.2 Model for the current at the surface

Considering the model case of an electromagnetic plane wave with intensity I(t) obliquely incident at an angle θ on the planar x-y surface of a medium with reflectivity $R \leq 1$, filling the x > 0 region, the flow of electromagnetic momentum **P** through the surface has two components:

$$(P_x, P_y) = \left((1+R)\frac{I'}{c}\cos^2\theta' , \ (1-R)\frac{I'}{c}\sin\theta'\cos\theta' \right) , \qquad (4.3.1)$$

where the prime refers to the frame moving with the surface. These relations are obtained starting from Fresnel formulas for the laser electromagnetic fields at the surface, by calculating the flow of momentum at the surface using Maxwell's stress tensor. The longitudinal component P_x corresponds to the standard radiation pressure on the surface which drives the Hole Boring action, Eq. (4.1.24), while the P_y component describes the transfer of momentum to electrons in the direction parallel to the surface, giving rise to a current in the skin layer. In turn, this current generates a magnetic field and, by induction, an electric field which counteracts the acceleration of the electrons along the surface and transfers the absorbed momentum to ions.

In order to provide a model to describe the current and field distributions, we introduce the ponderomotive force in the skin layer $\mathbf{f}_p \simeq \mathbf{P} \exp(-2x/\ell_s)/(\ell_s/2)$, where ℓ_s is an appropriate screening length for the electromagnetic fields in the plasma, and we use cold fluid equations for the plasma electrons that reads

$$\partial_t J_{sy} = \frac{\omega_{pe}^2}{4\pi} E_{sy} - \frac{e}{m_e} f_{py}, \qquad (4.3.2)$$

$$\partial_x B_{sz} = -\frac{4\pi}{c} J_{sy}, \qquad (4.3.3)$$

$$\partial_x E_{sy} = -\frac{1}{c} \partial_t B_{sz} , \qquad (4.3.4)$$

where the suffix "s" means that all fields are slowly varying on the temporal scale of the pulse profile, so that the displacement current is negligible. We also neglected the contribution of the ion current to B_{sz} because of the large mass difference with the electrons.

Combining the previous equations, we obtain an inhomogeneous Helmholtz equation for the electric field component E_{sy} :

$$\left(\partial_x^2 - \frac{\omega_{pe}^2}{c^2}\right) E_{sy} = -\frac{4\pi e}{m_e c^2} f_{py} \,. \tag{4.3.5}$$

The particular solution of Eq. (4.3.5) can be obtained as a Laplace transform in space:

$$\hat{E}_{sy}(s,t) = -\frac{4\pi e}{m_e c^2} \frac{1}{s^2 - \omega_{pe}^2/c^2} \hat{f}_{py}(s,t), \qquad (4.3.6)$$

$$E_{sy}(x,t) = \int_0^\infty \hat{E}_{sy}(s,t) e^{sx} ds.$$
 (4.3.7)

In the definition of the ponderomotive force, we consider $\ell_s \simeq c/\omega_{pe}$, see Eq. (4.1.6), consistent with the relatively weak absorption in a highly overdense plasma, and a general time-dependence $f_{py}(0,t)$. Therefore, adding the homogeneous solution to the one computed from Eq. (4.3.7), we obtain

$$E_{sy}(x,t) = -\frac{4\pi e}{3m_e\omega_{pe}^2} f_{py}(0,t) \left(e^{-2\omega_{pe}x/c} - \frac{3}{2} \left(1-C\right) e^{-\omega_{pe}x/c} \right), \quad (4.3.8)$$

where C is a constant that will be fixed by the boundary condition and we have reasonably assumed the same time-dependence in both particular and homogeneous solutions.

Solving for the magnetic field B_{sz} and for the current J_{sy} , exploiting Eqs. (4.3.3) and (4.3.4), we obtain

$$B_{sz} = \frac{4}{3} \frac{\omega_{pe}^2}{e n_0 c} \left(e^{-\omega_{pe} x/c} - e^{-2\omega_{pe} x/c} \right) \int_0^t f_{py}(0, t') dt', \quad (4.3.9)$$

$$J_{sy} = \frac{4}{3} \frac{e\omega_{pe}}{m_e c} \left(e^{-\omega_{pe} x/c} - 2e^{-2\omega_{pe} x/c} \right) \int_0^t f_{py}(0, t') dt', \quad (4.3.10)$$

where we have assumed at the surface $B_{sz}(x = 0) = 0$, imposing C = -1/3. This corresponds to a positive magnetic field B_z , an electric field E_y for x > 0 and a bipolar current structure. These profiles are coherent with the simulation results.

A more general boundary condition, for the electric and magnetic fields at the surface (x = 0), corresponds to a radiative boundary $\mathbf{E}(x = 0) = \mathbf{B}(x = 0)$, i.e. considering the emission of an electromagnetic wave propagating in vacuum. This case can be solved with the same approach presented before, just replacing the integration constant C with a function of time C(t). The solution, for $t \gtrsim \omega_{pe}^{-1}$, corresponds to vanishing electric field E_{sy} and magnetic field B_{sz} at x = 0, thus justifying the assumption made in the presented model.

The maximum value of the magnetic field B_{sz} , from the solution obtained in Eq. (4.3.9), is found at $x = (c/\omega_{pe}) \ln 2$, and considering a flat-top profile $I(t) = I_0 \Theta(t)$, it reads

$$B_{sz}^{\max} \simeq \frac{\pi}{6} a_0^2 \frac{t}{t_{\rm L}} (1-R) \sin(2\theta_i) \frac{m_e \omega_{\rm L} c}{e} \,. \tag{4.3.11}$$

Even for very small absorption $(1-R \leq 10^{-2})$, this value becomes comparable with the simulation results. Moreover, this simple estimate shows that already at sub-relativistic intensities $a_0 \simeq 1$ the field may rapidly reach values comparable to the laser field $B_{\rm L} = a_0 m_e \omega_{\rm L} c/e$. However, saturation will occur when the cyclotron frequency $\omega_c \simeq \omega_{pe}^{\rm rel} = \omega_{pe}/\sqrt{\gamma_e}$, which for a relativistic electron corresponds to the gyroradius becoming smaller than the relativistic skin depth (a criterion similar to the Alfvén limit for a current beam). This leads to the saturation value

$$B_{zs}^{\text{sat}} \simeq \sqrt{\gamma_e} \frac{m_e \omega_{pe} c}{e} = \sqrt{\gamma_e} \sqrt{\frac{n_0}{n_c}} \frac{m_e \omega_{\text{L}} c}{e} .$$
(4.3.12)

For our parameters and considering $\gamma_e \simeq a_0$, this yields $B_{zs}^{\text{sat}} \simeq 58 \, m_e \omega_{\text{L}} c/e$, fairly close to the value observed in the simulation.⁴

Notice that this purely fluid model can not account for kinetic effects, such as the small amount of fast electrons that leave the surface and propagate freely inside the target. However, the fluid motion of the surface has a dominant effect in our configuration, and the main features of the fields distribution at the surface are well described.

4.3.3 3D simulations

The analysis of the previous Secs. 4.3.1 and 4.3.2 suggests S-polarized laser at 45° of incidence as optimal configuration. The 2D simulations however confirm the stabilization of the surface instability only along the direction in which the current is produced (in our configuration J_y). A 3D study is therefore required to investigate the possible instability development along the other transverse direction (here z). In this sections, we comment on the 3D simulations carried out in the optimal configuration (S-polarized laser at 45°) and in circular polarization at normal incidence,

 $^{^4\}mathrm{At}$ the timestep presented in Figs. 4.5 and 4.6 the surface magnetic field has not yet reached its maximum amplitude.



Figure 4.11: Magnetic field component at the laser plasma interaction surface at $t = 36 t_{\rm L}$. For a S-polarized plane wave with $\theta = 45^{\circ}$, a) B_y at $x \simeq 6 \lambda_{\rm L}$, in the electron compressed layer, b) B_z component averaged along the z-direction. For a circularly polarized plane wave with $\theta = 0^{\circ}$, c) B_y and d) B_z at $x \simeq 6.7 \lambda_{\rm L}$, in the electron compressed layer.

to confirm our previous findings. The simulations have been performed with the same physical parameters than used in the 2D simulations. We reduce the box extension to $(48 \times 2.5 \times 2.5) \lambda_{\rm L}$, the spatial resolution to $dx = dy = dz = \lambda_{\rm L}/64$ and the time resolution to $dt = 0.95 dt_{\rm CFL}$, defined in Eq. (2.6.7). We use 8 particles-per-cell per species, for a total of $\simeq 1.3 \times 10^9$ particles. The target surface at t = 0 corresponds to the *y*-*z* plane located at $x = 2 \lambda_{\rm L}$. The angle θ is defined with respect to the *z*axis. Periodic boundary conditions are used along the *y* and *z* directions for fields and particles. Using a circularly polarized plane wave at normal incidence strong magnetic field modulations appear at the surface. This is illustrated in Fig. 4.11c-d), where we show the y-z plane located at $x \simeq 6 \lambda_{\rm L}$ for $t = 36 t_{\rm L}$. Due to the 3D geometry, the modulations have wavevector $\mathbf{k} =$ $(0, k_y, k_z)$ and both magnetic field components B_y and B_z are amplified by the surface instability. Surface modes are mostly transverse, meaning that the B_y component has a dominant wavevector k_z , and the same for B_z - k_y , as deducible from Fig. 4.11c-d). On the contrary, for S-polarization and $\theta = 45^{\circ}$, the current along the y-direction stabilizes the modes with wavevector k_y , producing the positive magnetic fields B_z , shown in the xy plane in Fig. 4.11b) for $t = 36 t_{\rm L}$. The instability in the B_y component is still present, Fig. 4.11a), but the field strength is somewhat smaller than in circular polarization.

The stabilization in both y- and z-directions might be achieved using two lasers, in order to drive the currents J_y and J_z . However, simulations performed with two lasers are found to be very sensitive to the relative phase between the two pulses. In addition, to remain closer to a possible experimental configuration, the use of two intense lasers should be avoided.

In conclusion, the 3D simulations confirm the 2D result that S-polarized laser incidence at 45° is the optimal condition to stabilize the surface.

4.4 Ion-Weibel Instability in solid target

In this Section, we demonstrate the possibility to investigate the ion-Weibel instability (IWI) driven by a fast quasi-neutral plasma flow launched into the target via the radiation pressure of an UHI laser pulse (Hole-Boring process, see Sec. 4.1.4.2).

In the frame co-moving with the surface, the background plasma and the Hole-Boring reflected beam constitute two neutral counterpropagating beams with velocity $\simeq (\pm v_{\rm HB})$ and ideally equal density n_0 . The large plasma density entails a fast growth rate, while the large drift velocity, $v_{\rm HB} \gtrsim 0.1c$, ensures the Weibel instability to be the dominant mode in the unstable spectrum (Bret et al., 2010b). In the presence of two counter-streaming electron-ion plasmas, the instability evolution follows a first phase in which electrons turn unstable while the large inertia of the ions keeps their trajectories weakly affected by the magnetic turbulence. On a longer timescale, IWI develops in a background of warm electrons, and the magnetic field grows up to an amplitude large enough to efficiently deflect the ions. The progressive deceleration of the ion flow produces an increase of the density that eventually leads to the formation of a shock front, as will be demonstrated in Sec. 4.5.

We first present an analytical derivation of the IWI growth rate in Sec. 4.4.1, and a study of the IWI saturation in Sec. 4.4.2. Motivated by the possibility of an experimental investigation, we present a full 3D simulation of laser-plasma interaction up to the development of the IWI, in Sec. 4.4.3. The results are compared with the analytical studies of the Secs. 4.4.1 and 4.4.2.

4.4.1 IWI growth rate

The linear phase of the IWI can be investigated using a relativistic fluid approach, as discussed in Sec. 2.4.2 and tested in Secs. 3.2 and 3.3.

Here, we consider two counter-streaming proton beams with velocity $\mathbf{v} = \pm v_0 \hat{x}$, density $n_{\pm} = n_0/2$ and temperature $T_{\pm} = T_0$, so that $\mu(T_{\pm}) = m_i c^2/T_{\pm} = \mu_0$. The neutralizing background is provided by thermalized electrons with density n_0 , zero average velocity and temperature T_e , with the corresponding enthalpy $h(\mu_e)$, with $\mu_e = m_e c^2/T_e$. Linearizing the fluid equations coupled with Maxwell's equations, as discussed in Sec. 2.4.2, for both the electron and the ion populations, and looking for purely transverse unstable modes, we obtain the dispersion relation

$$\frac{\omega^2}{c^2} - k_y^2 - \frac{\omega_{pi}^2}{h(\mu_0)\gamma_0^3 c^2} - \frac{\omega_{pi}^2}{h(\mu_0)\gamma_0 c^2} \frac{k_y^2 v_0^2}{\omega^2 - \frac{T_0 k_y^2}{m_i h(\mu_0)\gamma_0}} - \frac{\omega_{pe}^2}{h(\mu_e)c^2} = 0,$$
(4.4.1)

where $\omega_{p\,e,i}^2 = 4\pi n_0 q_{e,i}^2 / m_{e,i}$ and for simplicity we assumed $\mathbf{k} = k_y \hat{y}$.

As already stated in Sec. 2.4.3, the use of fluid equations is accurate if the thermal velocity is smaller than Γ/k , with Γ the instability growth



Figure 4.12: Growth rate as a function of the wavenumber, for the ion-Weibel-instability driven by two ion flows with velocity $v_0 = \pm 0.11 c$ and temperature $T_0 = 1 \text{ keV}$. The electrons are assumed thermalized, so that $v_{0,e} = 0$, with temperature $T_e = T_0$ (red line) and $T_e \gg 500 \text{ keV}$ (blue line).

rate and k the wavenumber. In many cases the electron population does not satisfy this condition. This is particularly true for the electrons heated by an intense laser pulse. Hence, in this case, the use of a kinetic approach is required for the electrons. Therefore, we derive the dispersion relation, where the contribution of the ions is computed within a fluid approach [limit of $\xi \gg 1$ in Eq. (2.4.18), leading to Eq. (2.4.21)] and the electrons contribution is obtained by means of the kinetic theory [limit $\xi \ll 1$ in Eq. (2.4.18) leading to Eq. (2.4.23)]. Thus, from the dispersion relation in the kinetic regime Eq. (2.4.23), recalling that $a_e = m_e v_0^2/T_0 = 0$ for a zero average velocity and that the last term in bracket vanishes in the limit of a large temperature, we obtain:

$$\frac{\omega^2}{c^2} - k_y^2 - \frac{\omega_{pi}^2}{h(\mu_0)\gamma_0^3 c^2} - \frac{\omega_{pi}^2}{h(\mu_0)\gamma_0 c^2} \frac{k_y^2 v_0^2}{\omega^2 - T_0 k_y^2 / (m_i h(\mu_0)\gamma_0)} = 0. \quad (4.4.2)$$

This is similar to Eq. (4.4.1), but the electron contribution has disappeared from the dispersion relation, as already discussed by Lyubarsky and Eichler (2006); Achterberg and Wiersma (2007).

In order to highlight the role of the thermalized electron background,

both dispersion relations, Eqs. (4.4.1) and (4.4.2), have been numerically solved for $\omega = i\Gamma$, with $\Gamma > 0$, to obtain the IWI growth rate Γ_{IWI} . Figure 4.12 shows $\Gamma_{IWI}(k)$, for protons streaming with velocity $v_0 = \pm 0.11 c$, with initial temperature $T_0 = 1$ keV, and electrons with $T_e = T_0$ (red line) and $T_e \gg 500 \,\mathrm{keV}$ (blue line). If the electrons have a temperature large enough that a kinetic description is required [leading in the limit $\xi \ll 1$ to Eq. (4.4.2), whose solution corresponds to the blue line in Fig 4.12, the growth rate is larger with respect to the case of quasi-cold electron background, correctly treated with the fluid approach [red line, solution of Eq. (4.4.1)]. This can be explained as follows. During the linear phase of growth of the ion instability, the electrons are put in motion, along the ion streaming direction, by the electric field generated by induction due to the Weibel-magnetic field amplification. As already explained in Sec. 3.2, in the linear phase of the instability, the inductive electric field tends to slow down the particles responsible for the instability, in this case the ions, thus accelerating the electrons. Therefore, the electron background forms current filaments that tend to screen the ion currents, thus slowing down the growth of the instability. The efficiency of the screening depends on the size of the filaments and on the electron temperature. In particular, the screening increases at small wavenumbers and for low electron temperatures. Indeed, in Sec. 3.2.2.1, we pointed out that the inductive electric field component is stronger for small-k modes (large wavelengths), thus producing a stronger screening current and in turns reducing the growth rate, consistent with Fig. 4.12. In addition, we expect a less efficient screening if the background of electrons has a high temperature. Indeed, an electron with large thermal velocity can easily escape from the filament, while the electrons of a cold background would remain confined in the ion filaments, producing a current that efficiently screens the ion one. Moreover, this involves a shift toward low wavenumbers of the mode with the maximum growth rate, with the increase of the electron temperature. These qualitative arguments are in agreement with Fig. 4.12.

Note that cutoff value of the unstable k (at $k \simeq 43 \omega_{pi}/c$ in Fig. 4.12), is determined by the ion temperature and roughly independent from the electron one, as can be calculated from the dispersion relation imposing



Figure 4.13: Magnetic field strength at saturation as a function of the wavenumber, for the ion-Weibel-instability driven by two cold ion flows with velocity $v_0 = \pm 0.11 c$. Alfvén mechanism (green line) and trapping mechanism (darkest red line). Predicted saturation level accounting for the screening currents of the electron thermalized background, i.e. trapping mechanisms with ion current multiplied by the κ factor in Eq. (4.4.3), with $T_e = [1, 10, 10^2, 5 \times 10^2, 10^3, 5 \times 10^3]$ keV (from blue to red). The dots report the values measured in 2D simulations with counter-streaming cold ions and background electrons with temperature equal to the line of the same color. The black diamond corresponds to the magnetic field obtained at the end of the 3D simulation presented in Sec. 4.4.3.

 $\omega = 0.$

4.4.2 IWI saturation

In this paragraph we briefly discuss the saturation of the ion Weibel instability. The same physical mechanisms, that have been investigated in depth for the electron Weibel instability in Sec. 3.2.2, can be assumed to take place. A straightforward generalization, with the substitution $m_e \longrightarrow m_i$ in Eqs. (3.2.13) and (3.2.17), gives the green and darkest red curves in Fig. 4.13. The green curve corresponds to the Alfvén saturation mechanism, for which the forward velocity of the particles contributing to the instability approximately vanishes at saturation. The darkest red curve represents the trapping mechanism, which corresponds to the injection of all ions into the current filaments. These saturation conditions however apply only for a purely ion driven instability, completely neglecting the role of the electrons in the instability evolution and saturation. This might be correct if the electrons are warm enough that no screening of the ion currents is produced.

At lower temperature, a simplified analytic model to take into account the electron screening effect, in cylindrical current filaments, was proposed by Achterberg et al. (2007). The inductive electric field responsible for the electron motion is estimated by means of the linearized fluid theory. It is found that the reduction of the current in a filament is quantified by the *screening factor*:

$$\kappa = \frac{I_{i,e}}{I_i} = 2I_1 \left(\frac{\omega_{pe}}{ch(\mu_e)}r_0\right) K_1 \left(\frac{\omega_{pe}}{ch(\mu_e)}r_0\right) , \qquad (4.4.3)$$

where $I_{i,e}$ and I_i are the total and the ion currents, respectively, I_1 and K_1 are the modified Bessel functions and r_0 is the filament radius.

In order to test this model and assess the efficiency of the screening at saturation, a series of 2D simulations have been performed varying the temperature of the electron background. In Fig. 4.13, we report the saturation values (dots) measured in the simulations with electron temperature $[0, 1, 10, 10^2, 5 \times 10^2, 10^3, 5 \times 10^3]$ keV and initially cold ions, streaming with velocity $v_0 = \pm 0.11 c$. These values are compared with the theoretical prediction of the trapping mechanism, multiplying the ion currents by the screening factor κ of Eq. (4.4.3), (curves and dots of the same color correspond to the same electron temperature). In all these tests the dominant mode is in the range $k \simeq 10 \div 25 \omega_{pi}/c$, where saturation is expected to occurs via the trapping mechanism. Increasing the electron temperature (from blue to red) the dominant mode shift toward lower k, coherent with the shift of the fastest growing mode discussed in the previous Sec. 4.4.1. As predicted by the trapping mechanism, at saturation the ions velocity is not drastically reduced and the region in between two filaments is depleted, i.e. complete injection of the ions in the filaments is achieved. The largest saturation value is obtained for the highest temperature, i.e. for a minimum screening by the electrons. However, even in this case, the magnetic field strength is slightly lower than in the purely ion situation (no electron contribution, corresponding to the darkest red line). The reported saturation values are in good agreement with the theoretical prediction, once accounting for the electron screening.

Therefore, referring to Fig. 4.13, we can consider the Alfvén limit (green line) and the trapping mechanism (darkest red curve) as the upper limits for the field strength at saturation that can be achieved in any realistic situations.

4.4.3 IWI in solid target

Motivated by the results of the previous Secs. 4.3.1 and 4.3.3, we performed a full 3D simulation considering a S-polarized laser irradiating the target at 45°. Due to the efficient quenching of the surface instability with this large angle of incidence, the surface profile remains approximately flat and HB can be driven for a long time.

We present here the results of the 3D simulation carried out with plane wave intensity $I\lambda_{\rm L} \simeq 6.8 \times 10^{21} \,\mathrm{W cm^{-2} \mu m^{2}}$, corresponding to a normalized laser vector potential $a_0 = 70$, for duration such that the IWI fully develops. All parameters have been defined in Sec. 4.3.3. This corresponds to an experimental set-up for soon available laser facilities. The configuration investigated in this work is pictured in Fig. 4.14a).

Figure 4.14b) highlights the filamentary structures in both the magnetic field B_z and the ion current $J_{x,i}$ at $t = 65 t_{\rm L}$, in the region of overlapping between the reflected HB beam and the background plasma, corresponding to $8 \lambda_{\rm L} \leq x \leq 16 \lambda_{\rm L}$. The density in this region is roughly $\simeq 2 n_0$, as shown in Fig. 4.14c). We define the laser-plasma interaction surface as the first x > 0 where $\langle n_i(x) \rangle_{y,z} = 2 n_0$, with $\langle n_i(x) \rangle_{y,z}$ the ion density averaged along the y and z directions. The surface moves with velocity $v_{\rm HB}^{\rm sim} \simeq 0.11 c$, in good agreement with the theoretical value of $v_{\rm HB} \simeq 0.098 c$.

The growth rate of the magnetic energy $U_B \propto e^{2\Gamma t}$ has been measured in the simulation considering a layer in the overlapping region of extension $\simeq 0.2 \lambda_{\rm L}$ moving at $v_{\rm HB}^{\rm sim}$. For the growth rate and the dominant mode, we



Figure 4.14: a) Schematic presentation of the investigated set-up. The incident angle θ of the S-polarized laser pulse is defined in the *x-y* plane. b) Ion Weibel instability from radiation pressure driven flows in a 3D PIC simulation with realistic ion mass. Magnetic field B_z and slice in the *y-z* plane at $x = 14 \lambda_L$ of the ion current $J_{x,i}$ at $t = 65 t_L$. c) Ion density $\langle n_i(x) \rangle_{y,z}$ averaged over the *y-z* plane. The laser pulse (not shown) propagates along the x > 0 direction, and the laser-plasma interaction surface at this time is located at $x \simeq 10 \lambda_L$.

obtain

$$\Gamma_{\rm IWI}^{\rm sim} \simeq 0.034 \, t_{\rm L}^{-1} \; ; \; k_{\rm IWI}^{\rm sim} \simeq 2 \, \omega_{\rm L}/c \, , \qquad (4.4.4)$$

corresponding to a filamentary structure with wavelength $\lambda_{\text{IWI}}^{\text{sim}} \simeq 0.5 \,\lambda_{\text{L}}$. The resolution in the spectrum depends on the extension of the box, in this simulation we have $\Delta k_{y,z}^{\text{sim}} = 2\pi/L_{y,z} = 0.5 \,\omega_{\text{L}}/c$.

These observations are consistent with the development of the IWI. To compare with the theoretical model discussed in Sec. 4.4.1, we need first to measure the temperature of both electron and ion populations. To characterize the thermalized electron background, we report in Fig. 4.15b) the electron distribution as a function of the transverse momentum $f_e(p_y)$, in the overlapping region at early time of interaction, $t = 14 t_{\rm L}$. Three temperatures can be identified: from the fit we obtain $T_{e,1} \simeq 530 \,{\rm keV}$, $T_{e,2} \simeq 1.4 \,{\rm MeV}$ and $T_{e,3} \simeq 14.5 \,{\rm MeV}$. Similar values are found along p_x and p_z . The HB ions transverse temperature is measured by means of a



Figure 4.15: a) Ion distribution function $f_i(p_y)$ of the HB reflected beam. b) Electron distribution function $f_e(p_y)$. Integrated in the overlapping region $3.2 \lambda_{\rm L} < x < 3.8 \lambda_{\rm L}$ at $t = 14 t_{\rm L}$. Dashed red lines correspond to the fit performed to extract the plasma temperature.

Gaussian fit as shown in Fig. 4.15a). We obtain $T_{i,\text{HB}}^{\text{sim}} \simeq 14.2$ keV. The ion background temperature corresponds to the initial one, $T_i = 1$ keV.

With these high temperatures, the electrons need to be treated with the kinetic approach in the limit of $\xi \ll 1$, discussed in Sec. 4.4.1. In order to compare the value of the dominant unstable mode observed in the 3D simulation, Eq. (4.4.4), with the analytical prediction reported in Fig. 4.12 (blue line), we express $k_{\text{IWI}}^{\text{sim}}$ in units of the ion skin depth as $k_{\text{IWI}}^{\text{sim}} \simeq 8.7 \omega_{pi}/c$, where $\omega_{pi}^2 = 4\pi (2n_0)e^2/m_i$ is the plasma frequency in the overlapping region, where the density is $2n_0$, and $\Delta k^{\text{sim}} \simeq 2 \omega_{pi}/c$. Therefore, the $k_{\text{IWI}}^{\text{sim}}$ is approximately the one with maximum growth rate, see Fig. 4.12 (blue line).

The growth rate reported in Fig. 4.12 (blue line), obtained considering two counter-propagating ion beams both with temperature $T_i = 1 \text{ keV}$ (initial temperature), would predict for the dominant mode observed in the simulation $\Gamma(k_{\text{IWI}}^{\text{sim}}) \simeq 0.076 \,\omega_{pi} \simeq 0.11 \, t_{\text{L}}^{-1}$. Nevertheless, the observed growth in the 3D simulation, Eq. (4.4.4), is ~3 times slower than theoretically expected. This is due to the assumption of equal low temperature $(T_i = 1 \text{ keV})$ of the two ion counter-propagating beams, while, as we already said, the HB reflected beam has a temperature larger than the initial one. Note that the instability would be completely quenched in the case of two counter-streaming beams with $T_i \simeq 14.2 \text{ keV}$, as confirmed by PIC simulations. To our knowledge no relativistic analytical theory exists for co-penetrating plasmas with different temperatures. Therefore, to obtain the value of the growth rate in this configuration, we performed an additional 2D simulation with counter-streaming ions with temperatures $T_{i1} = 1$ keV and $T_{i2} = 14.2$ keV and thermalized background of hot electrons. This simulation confirms the result of the 3D analysis. The dominant mode is $k \simeq 2 \omega_{\rm L}/c$ and the growth rate $\Gamma \simeq 0.043 t_{\rm L}^{-1}$, in good agreement with the values reported in Eq. (4.4.4).

In Sec. 3.3.1.2, we have demonstrated that increasing the beam temperature the growth rate of the modes with $k \ll k_{\text{max}}$, where k_{max} is the mode with the maximum growth rate, is weakly affected, but the maximum value of Γ can be drastically reduced. Therefore, we expect the dominant mode to grow slower than predicted by the theory with low temperature ($T_i = 1 \text{ keV}$) ion beams. Moreover, Tzoufras et al. (2006) have associated the decrease of the growth rate, in a configuration with counter-streaming electron beams with two different temperatures, with the different pinching of the high/low temperature beam, under the action of the Weibel-generated magnetic field.

Our analysis confirms that the structures highlighted in Fig. 4.14b) follow from the IWI driven, deep inside the dense target, by the interaction of the HB accelerated ion flow with the background plasma.

In the 3D simulation at $t \simeq 65 t_{\rm L}$, the IWI-generated magnetic field strength is $B_z^{\rm sim} \simeq 3 m_e \omega_{\rm L} c/e$ ($B_z^{\rm sim} \simeq 3 \times 10^8 \,{\rm G}$ for $\lambda_{\rm L} = 1 \,\mu{\rm m}$). To compare it with the theoretical prediction shown in Fig. 4.13, we recall that the dominant mode in units of ion skin depth is $k_{\rm IWI}^{\rm sim} \simeq 8.7 \,\omega_{pi}/c$ and the magnetic field amplitude is $B_z^{\rm sim} \simeq 7 \times 10^{-3} m_i c \omega_{pi}/e$, see the black diamond in Fig. 4.13. This value is close to the magnetic field amplitude at saturation predicted for electron temperature $T_e = 500 \,{\rm keV}$. This means that we are already in a late stage of the instability, close to saturation, with a significant part of the ions trapped in the filaments whose current is partially screened by the thermalized electrons.

4.4.4 Finite spot 2D simulation

An additional 2D simulation has been performed at the optimal configuration (S-polarization and $\theta = 45^{\circ}$), considering a Gaussian transverse laser profile⁵ in order to investigate the finite laser spot size effects. This is clearly highly relevant for an experimental investigation. We expect the target surface under the effect of the laser radiation pressure to get bent as the radiation pressure onto the target is not uniform. This changes the local angle of incidence, with possible effects on the production of hot electrons. Moreover, the radiation pressure decreases radially around the laser propagation axes being proportional to the intensity of the laser pulse, thus produces HB accelerated ions with different velocity at different position. The spread in the ion energy spectrum can be detrimental for the development of the IWI. As discussed in the previous Sec. 4.4.3, a high temperature of the reflected beam decreases the growth rate of the instability. Nevertheless, the simulation with Gaussian transverse laser profile gives similar results to the plane-wave one and confirms (i) the generation of the surface current, with the corresponding static magnetic field and the stabilizing effect on the surface instability, as well as (ii) the formation of current and magnetic filaments in the overlapping region.

The simulation has been performed with the same physical parameters of Sec. 4.3.1. The simulation box has been increased to $64 \times 128 \lambda_{\rm L}$, the resolution has been reduced to $dx = dy = \lambda_{\rm L}/64$. We used 49 particlesper-cell per species, for a total of $\simeq 2.5 \times 10^9$ particles. The laser focal spot is $10 \,\mu{\rm m}$ (defined as $1/{\rm e}^2$ in intensity).

In the electron current $J_{y,e}$, shown in Fig. 4.16a), we observe the bipolar structure predicted by Eq. (4.3.10). This produces and confines the positive magnetic field in the skin layer, as shown in Fig. 4.17. The ion current $J_{y,i}$, see Fig. 4.16b), is directed along the positive y-direction, i.e. in the direction predicted by momentum conservation, except in the region $y < 45 \lambda_{\rm L}$, where the intensity gradient in the transverse direction, due to the finite spot causes the local ponderomotive force to be in the negative y direction.

The IWI-driven magnetic filaments inside the target are clearly visible in the insert of Fig. 4.17. The amplitude and the characteristic wavelength confirm the observation of the previous 3D simulation.

⁵The corresponding 3D simulation would have been extremely costly from a com-



Figure 4.16: a) Electron current $J_{y,e}$, and zoom of the highlighted region. b) Ion current $J_{y,i}$. After 85 $t_{\rm L}$ of interaction with S-polarized laser with transverse spot size 10 μ m and angle of incidence $\theta = 45^{\circ}$.



Figure 4.17: Magnetic field B_z (not averaged) and zoom of the highlighted region. After 85 $t_{\rm L}$ of interaction with S-polarized laser with transverse spot size 10 μ m and angle of incidence $\theta = 45^{\circ}$.



Figure 4.18: 2D3V simulation with reduced ion mass $m_i = 100 m_e$ of collisionless shock formation. a-b) Ion distribution in p_x - p_y at $t = 30t_{\rm L}$ and $t = 515t_{\rm L}$, respectively. c) Ion density $n_i(x, y)$ and d) $\langle n_i(x) \rangle_y$ at $t = 515 t_{\rm L}$.

4.5 Weibel-mediated shock in solid target

The situation described above is known to be the early stage of Weibelmediated collisionless shock formation. After the saturation phase, the Hole Boring reflected ions may remain confined in the overlapping region where the magnetic field has developed strong and large-wavelength modulations. As more and more ions get injected and then trapped in this region, its density starts increasing. With this mechanism the density in the overlapping region evolves from $\sim 2n_0$ to the expected value calculated from the Rankine-Hugoniot conditions (RH), see Sec. 2.5.2. The fulfillment of these conditions and the isotropization of the downstream plasma identify the shock front formation.

To be able to reach shock formation, with reasonable computational

putational point of view.

resources, we performed a 2D simulation with artificially reduced ion to electron mass ratio $m_i = 100 m_e$ which allows us to follow the system evolution for longer times while clearly separating the electron and ion time-scale. It was shown that while it affects the time of shock formation, this does not affect the overall physics at hand (see Spitkovsky, 2008; Stockem-Novo et al., 2015; Ruyer et al., 2016). Reducing the ion mass while keeping unchanged the other parameters, gives a Hole Boring velocity $v_{\rm HB} = 0.28 c$, in good agreement with that measured in the simulation $v_{\rm HB}^{\rm sim} \simeq 0.29 c$.

Figures 4.18a-b) show, at $30 t_{\rm L}$ and $515 t_{\rm L}$ respectively, the phase space $p_x p_y$ of a region with extension $\simeq \lambda_{\rm L}$ initially close to the surface, moving at $v_{\rm HB}^{\rm sim}$. In Fig. 4.18a) (before shock formation) we clearly identify the background plasma distribution, centered around $p_x \simeq 0$, and the HB-reflected beam, centered around $p_x \simeq 71 m_e c$, corresponding to ~ $2v_{\rm HB}^{\rm sim}$. Figure 4.18b) shows at $t = 515 t_{\rm L}$ isotropization of the ion distribution function around the momentum $p_x \simeq 30 m_e c$ corresponding to $v_{\rm HB}$. Full thermalization is not yet reached as the measured ion temperature $T_i^{\text{sim}} \simeq 1.1 \, m_e c^2$ is lower than $T_i = (\gamma_{\text{HB}} - 1) m_i c^2 \simeq 4.5 \, m_e c^2$, obtained considering that all the drift kinetic energy is dissipated into thermal energy, and derived in Eq. (2.5.17). Nevertheless, a density jump (up to $3n_0$), consistent with the Rankine-Hugoniot prediction for a nonrelativistic two-dimensional flow, is observed in Figs. 4.18c-d), suggesting that the shock is formed. Furthermore, in Figs. 4.18c-d), the shock front located around $x \simeq 250 \lambda_{\rm L}$ and with characteristic width $\simeq 50 \lambda_{\rm L}$, corresponding to ~ 50 ion skin depths, moves with a velocity $v_{\rm sh}^{\rm sim} = 0.42 c$, consistent with RH prediction $v_{\rm sh} \simeq 0.43 c$.

Note that in this configuration the shock front is created deeper inside the target with respect to the configuration previously investigated by Fiuza et al. (2012); Ruyer et al. (2015) (see Sec. 4.2), far from the target surface, so that there is a clear distinction between the role of the laser-plasma interaction and the evolution of the IWI up to the resulting shock formation.

A theoretical investigation of the shock formation time for mildly relativistic electron-ion flows can be found in (Ruyer et al., 2016), while other works focused on the relativistic regime (relativistic factor of the ion flows $\gamma > 10$) (Stockem-Novo et al., 2015; Bret et al., 2016). Following Ruyer et al. (2016), we can define an isotropization parameter

$$a_{\rm iso} = \frac{\langle v_x p_x \rangle}{\langle v_y p_y \rangle} - 1 \,, \tag{4.5.1}$$

where $\langle v_i p_j \rangle$ correspond to the averaged pressure tensor components, Eq. (2.2.16), accounting for both the average flow contribution and the thermal spread. In (Ruyer et al., 2016) the time for which a good isotropization level ($a_{iso} = 1$) is achieved is theoretically predicted and assumed to be the shock formation time t_f . However, their measurements of the shock formation time in PIC simulations of symmetric counterstreaming flows suggest that a shock is fully formed for $a_{iso} \leq 0.1$, so that their model marginally underestimate the shock formation time. Indeed, for our flow velocity and density, with this model, a shock is expected to be formed ($a_{iso} = 1$) after $\simeq 130 t_L$ of interaction between the HB reflected beam and the background plasma. At this time, we measure $a_{iso}^{sim} \simeq 1.5$, in good agreement with the theoretical model. We can therefore consider this t_f as a lower limit for the shock formation time, even if in the simulation the isotropization parameter becomes $a_{iso}^{sim} \lesssim 0.1$ only at $t_f^{sim} \simeq 350 t_L$.

4.5.1 Range of parameters to reach shock formation

For a future investigation on the scaling down of the parameters used in this Chapter to observe IWI and IWI-driven collisionless shocks, we provide here a discussion on the range of acceptable parameters, with realistic ion mass. As mentioned in Sec. 4.4, a HB velocity $v_{\rm HB} \gtrsim 0.1$ is advisable, so that the IWI is the dominant mode in the unstable spectrum. This gives a first constrain: the laser field normalized amplitude a_0 and the target density n_0 should lye on the right of the black plain line shown in Fig. 4.19a), where we report the value of the HB velocity, obtained in Eq. (4.1.26), for the case of normal incidence laser pulse as a function of (a_0, n_0) . Moreover, to avoid the target transparency to the laser light, we need a density above the effective critical density (black dashed line in Fig. 4.19a) computed following the model of Cattani et al. (2000), that as



Figure 4.19: a) Hole Boring velocity as a function of the laser fields normalized amplitude a_0 and of the target density n_0 , for the case of normal incidence laser pulse, obtained in Eq. (4.1.26). The plain black line corresponds to $v_{\rm HB} = 0.1 c$ and the dashed black line to the effective critical density as derived by Cattani et al. (2000). b) Shock formation time, following the model of Ruyer et al. (2016). Both quantities are computed for an electron-proton target ($m_i = 1836 m_e$).

mentioned in Sec. 4.1.1, take into account the ponderomotive-produced electron density spike at the laser-plasma interaction surface.

In the literature most of the simulation of Weibel-mediated electronion collisionless shocks driven by two counter-streaming beams exploit an artificially low ion to electron mass ratio. The correct scaling to recover the time formation with realistic ion mass is still a matter of debate (see Kato and Takabe, 2008; Stockem-Novo et al., 2015; Ruyer et al., 2016; Bret et al., 2016). We assume the scaling proposed by Ruyer et al. (2016), since it was tested for mildly relativistic flows. This predicts $t_f \propto (m_i/m_e)^{0.9} t_{\rm L}$. Thus, considering the realistic proton mass $(m_i = 1836 m_e)$ the shock observed in Fig. 4.18 should form after $\simeq 4 \times 10^3 t_{\rm L}$, corresponding to $\simeq 13$ ps for $\lambda_{\rm L} = 1 \,\mu$ m.

In conclusion, from Fig. 4.19b), we see that the duration of the laser pulse required to achieve shock formation, in the range of acceptable laser-target parameters (in between the two black lines) is in the range $(2 \div 6) \times 10^3 t_{\rm L}$, corresponding to $(6.5 \div 19.5)$ ps for a laser wavelength $\lambda_{\rm L} = 1 \,\mu{\rm m}$.

4.6 Conclusions

In this Chapter, we have demonstrated the possibility to efficiently drive the ion-Weibel instability in the collisionless regime on UHI laser facilities. Motivated by an experimental investigation, the ion-Weibel instability development has been identified in realistic mass ratio simulation, in a 3D geometry and accounting for transverse laser finite-size effects. We have also provided a detailed characterization of the ion-Weibel instability driven by the HB beam by means of comparisons with analytical models and additional simplified numerical studies, for both the growth rate and the Weibel-generated magnetic field amplitude at saturation.

We have identified the optimal experimental configuration that (i) minimize the production of hot electrons, (ii) mitigate the surface instability and (iii) maximize the time over which the Hole Boring process is efficient. These conditions are required in order to obtain a dynamics dominate by the ion-Weibel instability and are best satisfied with the use of an S-polarized laser pulse irradiating the target at a large angle of incidence (45°) .

We have evidenced the nature of the surface instability as electron Weibel-like modes driven by the anisotropy in the electron compressed layer. Then we have correlated the instability mitigation with a slowlyvarying laser-driven electron surface current, described by means of an analytical fluid model. The suppression of the instability leads to an increase of the time during which the laser can act as a piston, and to a remarkable reduction of the hot electron energy, already after a few laser cycles.

The development of the surface instability has important implications for Hole Boring and Radiation Pressure Acceleration experiments, which are not limited to the focus of this work on collisionless astrophysics. Indeed, despite the current belief that, with circular polarization and normal incidence, these acceleration mechanisms would benefit from UHI (i.e. large a_0 , to increase the energy of the accelerated particles), we have demonstrated that maintaining the Hole Boring process over a long time is ensured only if the surface instability is efficiently suppressed.

The physical situation obtained with our configuration is dominated

by ion instabilities and it has been demonstrated to potentially lead to Weibel-mediated collisionless shocks. This situation relates to most of the astrophysical scenarios where neutral electron-ion flows are emitted from central sources.

The ion-Weibel instability stage will be accessible and will give important information on the stage of shock formation, with soon available laser systems, e.g. L4 for ELI in Czech Republic (over 1 kJ in 150 fs), or the LFEX in Japan and Petal in France (both delivering multi-kJ picosecond pulses), while the full shock formation is still out of reach experimentally. In particular, an experimental observation and characterization of the development of ion-Weibel instability, will allow to test the current theoretical models and to estimate the shock formation time, shock magnetization level and finally extract information regarding the particle acceleration, relevant to Cosmic Rays physics. Chapter 4. RPA driven Ion Weibel Instability and Collisionless Shocks

CHAPTER 5

Numerical Cherenkov attenuation methods

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This chapter is devoted to the presentation of some numerical techniques that we have studied and implemented in the PIC code SMILEI to correctly describe the dynamics of ultra-relativistic particles. The standard PIC method indeed faces strong limitations when trying to model plasma flows drifting at relativistic speed in the simulation box. The advanced techniques discussed in this Chapter help to overcome these limitations. They are therefore of extreme importance for the characterization of the physics relevant to ultra-energetic astrophysical events such as Gamma-Ray-Bursts and jets from Active Galactic Nuclei, or the dynamics of particles accelerated in collisionless shocks. Furthermore, a relativistic approach is necessary for the investigation of UHI laser-matter interaction. Indeed, with the advent of UHI laser systems, a broad class of relativistic non-linear phenomena need to be characterized in detail by means of PIC simulations. For instance, the Laser Wake-Field Acceleration (LWFA) mechanism (Esarey et al., 2009) requires an accurate description of the high-energy electrons injected in the accelerating field. Actually, many of the techniques presented in this chapter have been developed or inspired from simulations of LWFA.

These developments are in part presented in Dérouillat et al. (2017).

This Chapter is structured as follows. We begin with a general presentation of the numerical effects that lead to unphysical results in simulations of relativistic streaming plasmas (Sec. 5.1), along with a brief overview of the numerical methods proposed in the literature to reduce these numerical artifacts and improve the stability and accuracy of the simulation. Then in Sec. 5.2, we detail the classical FDTD method (Yee scheme) and a refined FDTD scheme (4th-order scheme, different from what is usually proposed in the literature) developed as a part of this thesis work. Both schemes have been implemented in SMILEI. The discrepancy between the dispersion relations for electromagnetic waves on the numerical grid and the theoretical value in a continuum becomes extremely important in simulations of UHI laser-created plasma and/or relativistic flows. The major effects of this numerical issue and of the improved FDTD scheme are highlighted using simulations of relativistic streaming plasmas in Sec. 5.2.3.

Complementary filtering techniques, that have been demonstrated in several numerical studies to enhance the simulation accuracy, are described in Sec. 5.3. We have implemented them in SMILEI and tested in PIC simulations (Sec. 5.3.3), in order to find the optimal value of the filter parameters and FDTD scheme for the simulations of interest for this work. In Sec. 5.4, we provide a study of the different methods on a system composed of two counter-propagating ultra-relativistic electron-positron beams, as investigated in Chap. 6, to ensure the absence of numerical artifacts on the Weibel instability development. Finally we present our conclusions in Sec. 5.5.

5.1 Numerical grid-Cherenkov instability

The Finite-Difference Time-Domain (FDTD) method (Taflove, 2005) (presented in the next Sec. 5.2) is often employed in PIC codes as a standard numerical solver for the Maxwell's equations. In particular, the classical Yee scheme, with a second order discretization of the numerical derivative on the computational grid, is appropriate for the description of a wide range of physical systems, but it is affected by numerical dispersion especially for the electromagnetic waves propagating along the main axes.

In general, in FDTD schemes, the numerical dispersion gives an unphysical *reduction* of the electromagnetic wave phase velocity in vacuum, that is in addition not isotropic, as will be illustrated in Sec. 5.2.2. As a consequence, in the simulation, ultra-relativistic particles might have a velocity larger than the numerical phase velocity of electromagnetic waves. This leads to the unphysical emission of Cherenkov-like radiation. This is a purely numerical artifact, generally known as numerical Cherenkov instability (NC) (Godfrey, 1974), which arises from the discretization of the Maxwell's equations onto the computational grid 1 . The numerical Cherenkov radiation can be emitted in the presence of relativistic streaming particles in 2D simulations, when the particle velocity is in the simulation plane, or in the 3D geometry. Note that 1D simulations are intrinsically Cherenkov-free, as the Cherenkov emission propagates obliquely with respect to the relativistic particles stream. In particular, numerical effects are stronger for high-frequency small-wavelength electromagnetic waves, for which the discretization leads to larger numerical errors.

In addition to the Cherenkov-like radiation emission, the time and

¹Note that in the limit of infinitely small cell size, this instability is completely suppressed and the numerical dispersion relations tends to the physical one.
space discretization leads to the development of the so-called *finite-grid instability*. Unstable modes arise at the intersection in the (ω -**k**) space of the plasma characteristic modes with the electromagnetic waves. These intersections do not exist in the physical dispersion relation and might appear due to both aliased longitudinal modes and numerical dispersion of the electromagnetic waves. A detailed analysis of the coupling between electrostatic, both admitted and aliased, modes and electromagnetic modes, for drifting plasmas, is reported in (Xu et al., 2013; Meyers et al., 2015).

A strong Cherenkov radiation emission and the appearance of unphysical amplified modes can drastically affect the dynamics of the system and the physical results of the simulation. Several techniques to inhibit the growth of this instability (henceforth considered as *grid-Cherenkov instability*) have been proposed.

Several methods based on high order algorithms for the FDTD Maxwell solver have been demonstrated to delay the development of the instability (Pukhov, 1999; Vay et al., 2011; Cowan et al., 2013; Lehe et al., 2013) (for a review see (Nuter et al., 2014)). Since the electromagnetic waves numerical dispersion relation is strictly connected with the numerical stencil used in the FDTD solver, the emission of radiation and the position in the $(\omega - \mathbf{k})$ space of the unstable modes can be modified by implementing different schemes. These techniques are usually associated with filtering strategies in order to dump the unphysical emitted radiation and/or smooth the current density profile at small wavelengths (as described in Sec. 5.3). It will be shown that the instability can be efficiently mitigated, by shifting, with the adequate FDTD solver, the unphysical radiation in a region of the $(\omega - \mathbf{k})$ where the filters can be applied without impacting the physics. Moreover, the so-called *magic timestep* has been identified by Vay et al. (2011); Xu et al. (2013). It corresponds to the optimal value of the time resolution that drastically reduces the instability growth rate for different schemes. In the 2D simulations performed with SMILEI, this magic timestep corresponds to $c\Delta t = 0.5 \Delta x$.²

²Note that this value is obtained for the so-called uniform field interpolation, independently from the type of FDTD solver, see Sec. 5.2. With other interpolation schemes, a magic timestep still exists but it takes a different value (see Godfrey and Vay, 2013).



Figure 5.1: Representation of the staggered Yee-grid. The location of all fields and current densities follows from the standard convention to define charge densities at the cell nodes.

Filtering methods and the use of the magic timestep can efficiently suppress radiation emitted at large wavenumbers. However, Cherenkov instability with lower growth rate might appear in the small- \mathbf{k} region, where filtering methods risk to alter the physics. Some advanced techniques have been proposed by Godfrey and Vay (2014); Li and other (2017), inspired by spectral code methods, but will be not discussed here.

5.2 The Finite Difference Time Domain solver

5.2.1 Computational stencil

As already mentioned, Maxwell's equations are solved in SMILEI using the FDTD approach, as well as refined methods based on this algorithm (Nuter et al., 2014). In these methods, the electromagnetic fields are discretized onto a staggered grid, the Yee-grid, that allows for spatial-centering of the discretized curl operators in Maxwell-Faraday and Maxwell-Ampère equations, Eqs. (2.1.10) and (2.1.11). Figure 5.1 summarizes at which points of the Yee-grid the electromagnetic fields, as well as charge and currents density, are defined. Similarly, time-centering of the time-derivative in Eqs. (2.1.10) and (2.1.11), is ensured by considering the electric fields as defined at integer time-steps n and magnetic fields at half-integer time-steps $(n + \frac{1}{2})$. Note that recomputing the magnetic fields at integer time-step (n) is however necessary for diagnostic purposes, and most importantly when computing the Lorentz force acting on the macroparticles. It should also be noted, as briefly discussed in Sec. 2.6.2, that a *leap-frog* scheme is used to advance the particles in time, so that their positions and velocities are defined at integer (n) and half-integer $(n - \frac{1}{2})$ time-steps, respectively.

To advance the electromagnetic fields, Maxwell-Ampère equation, Eq. (2.1.11), is solved first, giving the advanced electric fields

$$\mathbf{E}^{(n+1)} = \mathbf{E}^{(n)} + \Delta t \left[\left(\nabla \times \mathbf{B} \right)^{(n+\frac{1}{2})} - \mathbf{J}^{(n+\frac{1}{2})} \right], \qquad (5.2.1)$$

where Δt is the time resolution. Then, Maxwell-Faraday equation, Eq. (2.1.10), is computed, leading to the advanced magnetic fields

$$\mathbf{B}^{(n+\frac{3}{2})} = \mathbf{B}^{(n+\frac{1}{2})} - \Delta t \ (\nabla \times \mathbf{E})^{(n+1)} \ . \tag{5.2.2}$$

Before discussing the discretization of the curl-operator in more details, it is worth noting that solving Eqs. (2.1.10) and (2.1.11) is sufficient to get a complete description of the advanced electromagnetic fields. Indeed, it can be shown that this conserves a divergence-free magnetic field if Gauss' equation, Eq. (2.1.9), is satisfied at time t = 0. Similarly, Poisson's equation, Eq. (2.1.8), is verified as long as it is satisfied at time t = 0 and the charge deposition algorithm fulfills the charge conservation equation

$$\partial_t \rho + \nabla \cdot \mathbf{J} = 0. \tag{5.2.3}$$

This motivated the use of Esirkepov's projection scheme mentioned in Sec 2.6.2, (see Esirkepov, 2001).

We now discuss in more details the discretization of the curl-operators in Eqs. (5.2.1) and (5.2.2). To do so, let us focus on the equations for

the electric and magnetic fields E_x and B_x discretized on the staggered Yee-grid

$$\frac{(E_x)_{i+\frac{1}{2},j,k}^{(n+1)} - (E_x)_{i+\frac{1}{2},j,k}^{(n)}}{\Delta t} = (J_x)_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}} + (\partial_y B_z)_{i+\frac{1}{2},j,k}^{(n+\frac{1}{2})} - (\partial_z B_y)_{i+\frac{1}{2},j,k}^{(n+\frac{1}{2})},$$
(5.2.4)

$$\frac{(B_x)_{i,j+\frac{1}{2},k+\frac{1}{2}}^{(n+\frac{3}{2})} - (B_x)_{i,j+\frac{1}{2},k+\frac{1}{2}}^{(n+\frac{1}{2})}}{\Delta t} = (\partial_z^* E_y)_{i,j+\frac{1}{2},k+\frac{1}{2}}^{(n+1)} - (\partial_y^* E_z)_{i,j+\frac{1}{2},k+\frac{1}{2}}^{(n+1)} .$$
(5.2.5)

The partial derivatives in space are discretized as follows. In the Maxwell-Ampère equation, the partial derivative in x (similarly in y and z) reads

$$(\partial_x F)_{i,j,k} = \frac{F_{i+\frac{1}{2},j,k} - F_{i-\frac{1}{2},j,k}}{\Delta x}, \qquad (5.2.6)$$

where Δx is the cell extension. Equation (5.2.6) corresponds to the usual curl-operator discretization of the FDTD method. In the Maxwell-Faraday equation, the partial derivatives can be modified using an extended stencil. The spatial derivative in the x-direction (similarly in the y and z directions) reads

$$\begin{aligned} (\partial_x^* F)_{i,j,k} &= \alpha_x \frac{F_{i+\frac{1}{2},j,k} - F_{i-\frac{1}{2},j,k}}{\Delta x} + \eta_x \frac{F_{i+\frac{3}{2},j,k} - F_{i-\frac{3}{2},j,k}}{\Delta x} \quad (5.2.7) \\ &+ \beta_{xy} \left[\frac{F_{i+\frac{1}{2},j+1,k} - F_{i-\frac{1}{2},j+1,k}}{\Delta x} + \frac{F_{i+\frac{1}{2},j-1,k} - F_{i-\frac{1}{2},j-1,k}}{\Delta x} \right] \\ &+ \beta_{xz} \left[\frac{F_{i+\frac{1}{2},j,k+1} - F_{i-\frac{1}{2},j,k+1}}{\Delta x} + \frac{F_{i+\frac{1}{2},j,k-1} - F_{i-\frac{1}{2},j,k-1}}{\Delta x} \right], \end{aligned}$$

the set of parameters α_x , η_x , β_{xy} and β_{xz} depends of the type of solver used. The standard FDTD solver is recovered for $\alpha_x = 1$, $\eta_x = \beta_{xy} = \beta_{xz} = 0$. Figure 5.2 shows the computational stencil used to compute the



Figure 5.2: Computational stencil used to advance the field F solving the Maxwell-Faraday equation Eq. (5.2.7), considering an x-y plane at fixed z position identified by the index k. The parameter β_{xz} is not present, since it appears in the planes with $k \pm 1$.

derivative of the field F, in order to highlight the role of the parameters in Eq. (5.2.7). Note that the extended stencil is applied only to the Maxwell-Faraday equation, so that the charge-conserving scheme, related with the resolution of the Maxwell-Ampère equation, can be easily implemented. A comparison between different extended stencils, proposed by Pukhov (1999); Vay et al. (2011); Cowan et al. (2013); Lehe et al. (2013), has been provided by Nuter et al. (2014).

In some of the proposed numerical schemes the electromagnetic waves propagate at velocity larger than the light speed. This is done by Lehe et al. (2013) with a particular setting of the parameters in Eq. (5.2.7) or, as in (Nuter and Tikhonchuk, 2016), explicitly in the Maxwell-Ampère equation, keeping the standard Yee scheme. This removes completely numerical Cherenkov, since the light speed appears anyway as the upper limit for the particle velocity. It might however cause unphysical effects in the plasma-wave coupling.

For our purpose we will compare the Yee and Cowan scheme, for which the parameters in Eq. (5.2.7) are listed in Table 5.2.1 assuming a 2D geometry and an equal resolution along the axes $\Delta x = \Delta y$. Moreover, we implemented and tested an additional scheme, henceforth referred to as 4th-order scheme, with $\alpha_x \neq 0$, $\eta_x \neq 0$ (whose values are reported in Table 5.2.1), $\beta_{xy} = 0$ and $\beta_{xz} = 0$. The reason of these choices and the difference with the 4th-order method presented in the literature will be clarified in the following Sec. 5.2.2.

5.2.2 Numerical dispersion relation

In this section we derive the 2D numerical dispersion relation for a monochromatic plane wave, propagating in the x-y plane, so that the wavevector is $\mathbf{k} = (k_x, k_y, 0)$, with fields described as

$$(A)_{\mathrm{I,J}}^{\mathrm{N}} = A_0 e^{i(\omega N \Delta t - k_x I \Delta x - k_y J \Delta y)}, \qquad (5.2.8)$$

where A_0 is the field amplitude and ω is the frequency. Solving the discretized Maxwell's equations for the propagation of this electromagnetic wave in vacuum, we obtain

$$\frac{\sin^2\left(\frac{\omega\Delta t}{2}\right)}{c^2\Delta t^2} = \frac{\sin^2\left(\frac{k_x\Delta x}{2}\right)}{\Delta x^2} \Big[\alpha_x + 2\beta_{xy}\cos(k_y\Delta y) + \eta_x(1+2\cos(k_x\Delta x))\Big] \\ + \frac{\sin^2\left(\frac{k_y\Delta y}{2}\right)}{\Delta y^2} \Big[\alpha_y + 2\beta_{yx}\cos(k_x\Delta x) + \eta_y(1+2\cos(k_y\Delta y))\Big].$$
(5.2.9)

Taking the limit $(\Delta t, \Delta x, \Delta y) \longrightarrow 0$, equation (5.2.9) leads to the correct dispersion relation $\omega = c|\mathbf{k}|$. Moreover, developing at first order in $\Delta t, \Delta x$ and Δy , the parameters in Eq. (5.2.9) are found to satisfy $\alpha + 2\beta + 3\eta = 1$, in order to avoid unphysical damping of the waves.

From Eq. (5.2.9), we derive $\omega(k_x, k_y)$ as

$$\omega(k_x, k_y) = \frac{2}{\Delta t} \sin^{-1} \left[\Delta t \sqrt{\frac{\sin^2\left(\frac{k_x \Delta x}{2}\right)}{\Delta x^2}} \delta_x + \frac{\sin^2\left(\frac{k_y \Delta y}{2}\right)}{\Delta y^2} \delta_y \right], \quad (5.2.10)$$

where δ_x and δ_y correspond to the first and second terms in the square brackets in Eq. (5.2.9), respectively. Note that equation (5.2.10) corresponds to the numerical dispersion relation of electromagnetic waves in

Scheme	α	β	η	$\Delta t_{ m CFL}$	
Yee	1	0	0	$1/\sqrt{2}$	
Cowan	3/4	1/8	0	1	
$4^{\rm th}$ order (diagonal)	$\frac{5-2\sigma}{4}$	0	$\frac{2\sigma-1}{12}$	$1/\sqrt{2}$	
$4^{\rm th}$ order (on axes)	$\frac{5-\sigma}{4}$	0	$\frac{\sigma-1}{12}$	$\left \left[2 - \sqrt{5/2} \right]^{1/2} \right $	

Table 5.2.1: Parameters to compute the space derivative in Eq. 5.2.7, considering $\Delta x = \Delta y = \Delta$, so that we can drop the index as $\alpha_x = \alpha_y = \alpha$ for all parameters. In the 4th order scheme $\sigma = \Delta t^2 / \Delta^2$.

vacuum.

The parameters for the 4th order scheme, reported in Table 5.2.1, have been chosen in order to achieve 4th order accuracy in the phase velocity $v_{\phi} = \omega/|\mathbf{k}|$. Assuming $\Delta x = \Delta y = \Delta$ in Eq. (5.2.9), and developing for $\omega \Delta t \ll 1$, $k_x \Delta \ll 1$ and $k_y \Delta \ll 1$ up to the 4th order, we obtain

$$\frac{\omega^2}{k_x^2 + k_y^2} \simeq 1 + \frac{1}{12}\omega^2 \Delta t^2 - \left(\frac{1}{12} + \eta\right) \frac{k_x^4 + k_y^4}{k_x^2 + k_y^2} \Delta^2 - 2\beta \frac{k_x^2 k_y^2}{k_x^2 + k_y^2} \Delta^2 .$$
(5.2.11)

From Eq. (5.2.11), we can compute the values of α , β and η so that the terms at 2nd order vanish. To do so with the lowest computational cost, we assume $\beta = 0$, so that $\alpha + 3\eta = 1$, and the parameters for the 4th-order scheme are derived for both propagation along the diagonal ($k_x = k_y$) and along the main axes ($k_x \neq 0$ and $k_y = 0$, or vice-versa), see Table 5.2.1. Note that with the parameters optimized for wave propagation along the axes, the dispersion relation allows off-axes superluminal numerical electromagnetic waves propagation ($v_{\phi} \gtrsim c$).

With the analogous choice of $\beta \neq 0$ and $\eta = 0$, it would have been possible to achieve 4th accuracy along the diagonal but not along the main axes. Considering both $\beta \neq 0$ and $\eta \neq 0$, other solutions at 4th order can be found. However, this would increase the computational cost of each derivative, since it would add 4 points for the computation of each derivative with respect to the proposed scheme. Moreover, additional cells (usually called *ghost cells*) have to be added at the border of the simulation domain, or of the parallel domain treated by each processor, in order to ensure that the full macro-particle charge and current densities are correctly projected onto the simulation grid. The number of ghost cells is defined by the order of the Maxwell's equations discretization scheme, and by the shape function of the macro-particles. ³Increasing the number of ghost cells and the number of computation for each derivative might cause a significant increase of the simulation time.

As reported in Table 5.2.1, the parameters for the 4th order scheme depend on the ratio $\Delta t/\Delta$. Therefore, in each simulation, they will be defined after the choice of the time and space resolution.

In the literature, the higher order accuracy in the FDTD scheme is obtained considering only the derivative in the curl of the numerical resolution of the Maxwell-Faraday equation, Eq. (5.2.7). Therefore, the already tested 4th-order scheme (Xie et al., 2002; Wilson et al., 2008) is achieved with parameters $\alpha = 9/8$, $\eta = 1/24$ and $\beta = 0$. This is different from what we propose here, since we obtain the 4th-order accuracy in the phase velocity expression and our parameters are optimized for each choice of $\Delta t/\Delta$.

As already mentioned in Sec 2.6.2, FDTD solvers are subject to the Courant-Friedrich-Lewy (CFL) condition. For the standard Yee solver, the CFL condition has been reported in Eq. (2.6.7). For the other schemes, it can be derived from Eq. (5.2.9), imposing $\sin^2\left(\frac{\omega\Delta t}{2}\right) < 1$ for every choice of the wavevector. For the schemes that will be exploited in the following, the CFL conditions are listed in Table 5.2.1.

In Fig. 5.3, we compare the numerical phase velocity v_{ϕ} of electromagnetic waves propagating in vacuum as a function of the wavevector, as obtained for the different schemes, using Eq. (5.2.10). We consider the propagation along the main axes k_x (Fig. 5.3a) and along the diagonal (Fig. 5.3b) for the Yee scheme (dark red lines), the Cowan scheme (green lines) and the 4th-order scheme (dark blue lines). The parameters of the 4th-order scheme have been optimized for propagation along the main axes or along the diagonal, accordingly. We consider time-resolution $\Delta t = 0.95\Delta t_{\rm CFL}$ (dash-dotted lines) and the magic time-step $c\Delta t = 0.5\Delta$ (plain lines).

For propagation along the main axes, the Cowan scheme is found to

 $^{^{3}}$ The standard Yee scheme, for instance, uses 4 ghost cells per dimension (2 on each side), while the 4th order scheme implemented here requires 6 ghost cells per dimension.



Figure 5.3: Numerical phase velocity v_{ϕ} for electromagnetic waves propagating in vacuum, computed from Eq. (5.2.10), for the Yee scheme (dark red lines), the Cowan scheme (green lines) and the 4th-order scheme (dark blue lines). With $\Delta t = 0.95\Delta t_{\rm CFL}$ (dash-dotted lines) and $\Delta t = 0.5\Delta$ (plain lines). a) Propagation along the main axes. b) Propagation along the diagonal. Note that for the 4th-order scheme we used the corresponding optimal parameters for each case.

have the lowest deviation from the speed of light, close to the CFL. However, its efficiency is reduced by decreasing the time resolution. Indeed, using the magic timestep Yee and Cowan schemes give the same results (green and red lines are overlapped). With resolution equal to the magic timestep, the 4th-order scheme is the optimal of the investigated schemes.

For propagation along the diagonal, it is well known that the Cowan scheme is not competitive with the classical Yee scheme. The 4th-order gives a slight improvement close to the CFL and a significant one at the magic timestep. Notice that the 4th-order scheme is weakly sensitive to the variation of time resolution, since the parameters are optimized once the values of Δt and Δ have been set in the simulation.

Despite the deviation of the numerical phase velocity from the theoretical one increases while decreasing the time resolution, as will be shown in the next Sec. 5.2.3, the grid-Cherenkov instability growth rate decreases at $\Delta t = 0.5\Delta$, meaning that the value of the time-step has a stronger impact than the choice of the FDTD stencil.

5.2.3 PIC simulations of a streaming thermal plasma

In order to compare the strength of the numerical artifacts that affect the different computational schemes, a series of 2D3V PIC simulations has been performed considering a thermal plasma flowing at relativistic velocity.

We consider an electron-positron beam, with initial temperature $T = 10^{-4} m_e c^2$, moving along the x-direction with $\gamma_0 = 10$, corresponding to $v_0 \simeq 0.995 c$, in a periodic box of extension $(256 \times 64) c^2/\omega_p^2$, with ω_p the plasma frequency of a single species with density n_0 . We used 16 particle-per-cell per species and a resolution $\Delta x = \Delta y = (1/8) c/\omega_p \simeq 2 \lambda_{\rm De}$, where the Debye length in normalized units is $\lambda_{\rm De} = \sqrt{T/n_0}$. We tested different time resolutions equal to $\Delta t = 0.95 \Delta t_{\rm CFL}$ and $c\Delta t = (0.4, 0.5, 0.6) \Delta x$. As shown in Fig. 5.4a) for the Yee scheme and Fig. 5.4b) for the 4th-order scheme, using $c\Delta t = 0.5 \Delta x$ (plain lines), the growth of the electromagnetic energy density due to the numerical instability is significantly slowed down, in agreement with Vay et al. (2011); Xu et al. (2013). Both larger and smaller time resolutions lead to a faster development of the instability.

In Fig. 5.4c-d) the spectrum of the magnetic field component B_z is shown for the Yee and 4th-order schemes, respectively, using $c\Delta t = 0.5 \Delta x$, at $t = 130 \omega_p^{-1}$, when the emitted radiation reaches its maximum. To verify that these modes correspond to the numerical Cherenkov emission, we highlight (green lines) the modes that are excited by the particle streaming at v_0 , i.e. the modes with phase velocity $v_{\phi} = \omega/k_x = v_0$. These curved are thus obtained by looking at the modes that satisfy Eq. (5.2.10) with $\omega = v_0 k_x$.

The interaction of the streaming plasma with the numerical instability-produced radiation, yields to the unphysical heating of the plasma flows. In the context of astrophysical studies, such as investigated in Chap. 6, the corresponding spread in the distribution function drastically affects the development of streaming instability and in particular the formation of collisionless shocks. Despite the amplitude of the NC radiation is approximately the same for the two FDTD schemes, see Fig. 5.4, using the 4th-order one the radiation is emitted at larger values of k_x . Therefore, we expect a small-wavelength filter to reduce the insta-



Figure 5.4: Electromagnetic energy density evolution in time in 2D3V simulations of streaming pair plasma ($\gamma_0 = 10$) for : a) the Yee scheme (red lines) and b) 4th-order scheme (blue lines) with different time resolution $\Delta t = 0.4 \Delta x$ (dotted lines), $\Delta t = 0.5 \Delta x$ the magic timestep (plain lines), $\Delta t = 0.6 \Delta x$ (dash-dotted lines) and $\Delta t = 0.95 \Delta t_{\rm CFL}$ (dashed lines). Spectrum of the magnetic field B_z generated via the NCI at $t = 130 \, \omega_p^{-1}$ in the simulations with timestep $\Delta t = 0.5 \, \Delta x$ for: c) the Yee scheme, d) the 4th-order scheme. Green lines are the modes predicted to be NC unstable, i.e. satisfying Eq. (5.2.10) with $\omega = v_0 k_x$.

bility more efficiently in this case than with the Yee scheme, as will be verified in the following section.

This suggests that the combined use of the 4th-order scheme and filtering techniques, with appropriate parameters, is the optimal choice to reduce the NC emission and consequently the unphysical heating of the streaming flow.

5.3 Time and Current Filtering

Filtering techniques have been proposed and demonstrated in many works to improve the stability of the simulation. In particular, in SMILEI PIC code we make use of a time-filtering on the electric fields, as suggested by Greenwood et al. (2004), and a spatial-filtering on the current density, as in (Vay et al., 2011). These filters reduce the radiation at large k and high frequency, thus they should not alter the physics of interest in the simulations.

Both filtering methods can be used together or separately and have allowed to satisfactorily reduce the numerical grid-Cherenkov instability when dealing with relativistic drift of electron-positron plasmas in the framework of collisionless shock studies (see Chap. 6).

5.3.1 Implementation in Smilei of the Friedman filter

Specifically we implemented in SMILEI the Friedman temporal filter (Friedman, 1990), that consists in replacing the electric field in the Maxwell-Faraday solver Eq. (5.2.2) by a time-filtered field

$$\mathbf{E}^{n} = \left(1 + \frac{\theta}{2}\right)\mathbf{E}^{n} - \left(1 - \frac{\theta}{2}\right)\mathbf{E}^{n-1} + \frac{1}{2}\left(1 - \theta\right)^{2}\theta\bar{\mathbf{E}}^{n-2},\qquad(5.3.1)$$

where $\bar{\mathbf{E}}^{n-2} = \mathbf{E}^{n-2} + \theta \bar{\mathbf{E}}^{n-3}$ and θ is a tuning parameter that determines the strength of the filter. Its value can be chosen in the range $0 \le \theta \le 1$, with the unfiltered scheme recovered for $\theta = 0$. Note that the Maxwell-Ampère equation remains unchanged and the particle pusher uses the original unfiltered fields.

The dispersion relation computed in Sec. 5.2.2 is modified by the time



Figure 5.5: Temporal Friedman filter implemented in SMILEI. Filter response for increasing filter strength (from red to blue lines) in the range $\theta = [0.025, 0.4]$, from Eq. (5.3.3): a) amplitude, b) phase.

filtering operation. In particular, Eq. (5.2.9) becomes

$$\frac{\sin^2\left(\frac{\omega\Delta t}{2}\right)}{c^2\Delta t^2} = H(\omega)f(k_x, k_y), \qquad (5.3.2)$$

where $f(k_x, k_y)$ corresponds to the left hand side of Eq. (5.2.9). The function $H(\omega)$ describes the response of the time-filtering:

$$H(\omega) = 1 - \frac{2\theta \sin^2(\omega \Delta t/2)}{e^{i\omega \Delta t} - \theta}.$$
 (5.3.3)

From Eq. (5.3.2), we obtain that the phase velocity of the waves depends on $|H(\omega)|$, i.e. the amplitude of the filter response, while the phase of the filter introduces a negative imaginary part in $\omega(k)$, which describes the attenuation of the large-k modes. Therefore, the optimal filter would have an amplification factor equal to the unity and no phase shift in the low-k region. These conditions are well satisfied by the Friedman filter, for which the response is shown in Fig. 5.5 for different values of θ .

Since we have modified the dispersion relation, as reported in Eq. (5.3.2), the CFL conditions must be consistently recomputed. A detailed study has been performed by Greenwood et al. (2004). It was



Figure 5.6: Current density filter implemented in SMILEI, corresponding to $N = 1 \div 6$ (from dark red line to dark blue line) successive applications of the binomial filter of Eq. (5.3.7).

shown that the stability of the system is ensured by $\Delta t_{\rm CFL}^{\rm FF}$, the CFLtimestep in the presence of Friedman filter, equal to

$$\Delta t_{\rm CFL}^{\rm FF} = \sqrt{\frac{1+\theta}{1+3\theta}} \,\Delta t_{\rm CFL} \,, \tag{5.3.4}$$

with $\Delta t_{\rm CFL}$ the CFL condition for the applied stencil. Despite that, the filter behavior changes drastically at a time resolution below the stability threshold given by Eq. (5.3.4). Greenwood et al. (2004) introduce the so-called *practical limit*, maximum time-resolution to be used, is $\Delta t_{\rm CFL}^{\rm FF} = f_{\rm prac-lim}\Delta t_{\rm CFL}$ with

$$f_{\rm prac-lim}^2 = \begin{cases} \frac{1+\theta}{1+3\theta} & \text{for } \theta \lesssim 0.1168 \,, \\ \frac{(\theta-1)(\theta-4-3\sqrt{2-\theta})}{2(\theta+2)^2} & \text{for } 0.1168 \lesssim \theta \lesssim 0.3028 \,, \\ \frac{1}{2} & \text{for } 0.3028 \lesssim \theta \lesssim 0.5 \,. \end{cases}$$
(5.3.5)

5.3.2 Implementation in Smilei of the current filter

A multi-pass bilinear filter on the current density has also been implemented, following Vay et al. (2011). Each pass consists in a 3-point spatial averaging of the current in all spatial dimensions. The filtered current density J_i^f , defined at location *i* on a one-dimensional grid, is computed before solving Maxwell's equation as

$$J_i^f = \frac{1}{2}J_i + \frac{J_{i+1} + J_{i-1}}{4}.$$
 (5.3.6)

To characterize the filter effect, we assume the current $J_i \propto e^{ikx}$ and the filtered current $J_i^f \propto g(k)e^{ikx}$, where g(k) is the *filter gain*, i.e. the filter response as a function of the wavenumber. Inserting these definition in Eq. (5.3.6), we obtain the filter gain

$$g(k) = \frac{1}{2} + \frac{1}{2}\cos(k\Delta x) \simeq 1 - \frac{1}{4}(k\Delta x)^2 + \mathcal{O}\left((k\Delta x)^4\right) \,. \tag{5.3.7}$$

The response for successive applications of the binomial filter [from 1, dark red line, to 6, dark blue line] is shown in Fig. 5.6b).

Note that sharper cutoff in k might be obtained with a *compensation* factor depending on the number of applications, as detailed in (Birdsall and Langdon, 2004) and tested by Vay et al. (2011). However the use of the standard multi-pass binomial filter proved sufficient for our study, see next Sec. 5.3.3.

5.3.3 PIC simulations of a streaming thermal plasma

The same simulations presented in Sec. 5.2.3 have been performed applying the temporal and current filtering techniques described in the previous Secs. 5.3.1 and 5.3.2. We present here the results obtained with initial time resolution $c\Delta t = 0.5 \Delta x$, i.e. considering the magic time-step.

In Fig. 5.7, we compare the total electromagnetic energy emitted because of NCI with the use of the Yee scheme (left frame) and the $4^{\rm th}$ -order scheme (right frame). We apply the Friedman filter with $\theta = 0, 0.01, 0.025, 0.05$ (from dark to light color plain lines). The dashed lines correspond to the simulations performed with 2-pass current filter with $\theta = 0$ (no Friedman filter, dark dashed lines), and $\theta = 0.025$ (light dashed lines).

With the use of the 4th-order scheme the time filtering is more efficient and the instability growth is substantially delayed. Already with a weak filter ($\theta = 0.05$), it is possible to study the evolution of a streaming



Figure 5.7: Electromagnetic energy density growth due to NCI in 2D3V simulations of streaming pair plasma ($\gamma_0 = 10$) for : a) the Yee scheme and b) 4th-order scheme, with time resolution $\Delta t = 0.5 \Delta x$, and time-filter parameter $\theta = 0, 0.01, 0.025, 0.05$ (from dark to light lines). Dashed lines correspond to $\theta = 0$ (dark lines) and $\theta = 0.025$ (light lines), with 2-pass binomial current filter.

plasma with $\gamma_0 = 10$ up to a couple of $10^3 \omega_p^{-1}$, without being affected by numerical unphysical effects.

The application of the current filter further improves the stability, in particular considering the 4th-order scheme. As already mentioned, we expect both filtering techniques to reduce the instability growth of the 4th-order scheme more than of the Yee scheme, since the emission in that case is confined at larger k, as evidenced in the spectra in Fig. 5.4c-d).

5.4 Test on the Weibel Instability

In this section we test the impact of the different numerical techniques on a physical system of interest for this thesis, i.e. the development of the Weibel instability. In particular, we want to verify that the filtering methods do not modify the Weibel instability growth rate as well as the saturation time and field amplitude.

We present a series of 2D3V simulations performed with two counterpropagating relativistic pair plasmas, streaming along the x-direction



Figure 5.8: Electromagnetic energy density evolution in time in 2D3V simulations of two counter-streaming relativistic pair plasmas ($\gamma_0 = 10$) during the linear and saturation stage of the Weibel instability. Black dashed lines correspond to the theoretical growth rate. a) Yee scheme and b) 4th-order scheme, with time resolution $\Delta t = 0.5 \Delta x$, and temporal-filter parameter $\theta = 0, 0.01, 0.025, 0.05$ (from dark to light lines) with no current filter, $\theta = 0$ (dark red and blue dashed lines) and $\theta = 0.025$ (light red and blue dashed lines) with 2-passes binomial current filter.

with $\gamma_0 = 10$ and temperature $T = 10^{-4} m_e c^2$. Each species has density $n_0 = 0.5$ and time is in units of the plasma frequency is $\omega_p = [4\pi (2n_0)e^2/m_e]^{1/2}$. The numerical parameters are the same than Sec. 5.2.3. We used 16 particles-per-cell per species and a resolution $\Delta x = \Delta y = (1/8) c/\omega_p$, the periodic box has extension $(256 \times 64) c^2/\omega_p^2$. This is the physical configuration investigated in Chap. 6.

Figure 5.8 shows the evolution of the electromagnetic energy. The Weibel instability develops much faster than the grid-Cherenkov one. It reaches saturation after $\simeq 40 \omega_p^{-1}$, when the radiation emitted via Cherenkov instability is expected to be still very weak, as seen in Fig. 5.7 where the maximum of the numerical instability, in the absence of any filtering, is obtained at $t \simeq 130 \omega_p^{-1}$.

The higher level of electromagnetic energy at $t \leq 20 \omega_p^{-1}$, in the case without filtering (dark red and blue lines), is due to the presence of numerical Cherenkov, and these electromagnetic fluctuations are reduced

with the filters application. These small differences appearing at early time, do not affect the development of the Weibel instability the typical growth rate (dashed black lines) is recovered with all applied filters. Moreover, both saturation level and time do not change with the application of the filters. Note that the two dashed curves, obtained with the 2-pass current filter and $\theta = 0$ and $\theta = 0.025$, are overlapped.

Analogous tests have been performed with the parameters considered in Chap. 3, in particular $\gamma_0 = 2.3$, confirming the accuracy of the filtering techniques in those simulations.

5.5 Conclusions

The main limitations of the standard PIC method in describing relativistic streaming plasma have been highlighted, along with the techniques that have been implemented in SMILEI to reduce these numerical artifacts.

Both the proposed advanced stencil for the numerical resolution of the Maxwell's equations and the temporal and spatial filters improve the simulation stability. Even though more advanced techniques exist and are still developed, this rather simple approach proves sufficiently accurate for our studies.

We have discussed the impact of these techniques on two physical benchmarks: a free streaming thermal plasma and two counterpropagating flows. We have been able to properly describe the dynamics of an ultra-relativistic streaming pair plasma with $\gamma_0 = 10$ up to a couple of thousand plasma times. Moreover, in simulations initiated with overlapped beams, we verified that both numerical Cherenkov instability and filters do not alter the development of the Weibel instability. Indeed, with and without filtering, the theoretically expected growth rate is recovered, the saturation time and field amplitudes are not modified.

These filtering techniques together with the proposed advanced FDTD scheme have proved to be particularly helpful in mitigating numerical Cherenkov in the relativistic flows configuration presented in this thesis (in both Chaps. 3 and 6) and have made possible for SMILEI to operate in the regime of relativistic astrophysics, as will be presented in the next

Chap. 6 and in (Plotnikov et al., 2017).

In particular these techniques allow to properly describe both a simulation initialized with relativistic streaming plasma and a configuration with laser-produced relativistic flows, and follow in both cases the development of instabilities and shocks.

Tests considering an external flow aligned magnetic field have shown a strong growth of the numerical instability. With the parameters of interest for the simulations presented in Sec. 3.4, we have been able to efficiently remove the numerical artifacts with the implemented filtering techniques. However, we are currently working on the implementation of complementary techniques. In particular, the Godfrey modified interpolation scheme (Godfrey and Vay, 2014) could help to perform flow aligned magnetic field simulations over long times, which is challenging with the current version of the code.

In addition, a collaboration between the SMILEI development team and the Lawrence Berkeley National Laboratory has been recently started to couple SMILEI with the open source library PICSAR (Vincenti and Vay, 2017). This coupling will allow SMILEI to use HPC-relevant spectral methods to solve Maxwell's equations.

CHAPTER 6

Collisionless shocks in magnetized relativistic pair plasmas

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In many astrophysical scenarios that have been considered relevant sources of high-energy radiation and particle acceleration, such as PWNs, microquasar jets and GRBs, collisionless shocks are believed to develop following from the propagation of relativistic pair plasma flows. As discussed in Chap. 1, only indirect observations are up to now available to confirm the existence of shock waves and accelerated particles, and theoretical investigations as well as numerical simulations are central to this field of study. This has in particular motivated several works presenting first principle simulations of counter-streaming pair plasma flows (Gallant et al., 1992; Nishikawa et al., 2005; Lemoine et al., 2006; Spitkovsky, 2008; Sironi and Spitkovsky, 2009; Sironi et al., 2013; Bret et al., 2014; Lemoine et al., 2016). The main findings of these studies will be summarized in Sec. 6.1.

In parallel, the development of the laser facilities might lead in a near future to the production of collisionless shocks in pair plasmas. Up to now, the main experimental results on the investigation of the Weibel instability and Weibel-mediated collisionless shocks have been obtained in electron-ion flows, as summarized in Sec. 1.2.2. However a recent experiment (Sarri et al., 2015) was able to demonstrate the generation of neutral and high-density electron-positron plasmas and numerical works testing the optimal experimental configuration on the future-available laser facilities have already started (Chen et al., 2015; Lobet et al., 2015; Zhu et al., 2016).

The experimental and numerical approaches give complementary insights into the physics of relativistic collisionless shocks, with respect to the astrophysical observations. In particular, in experimental and numerical studies, we can directly investigate the amplification and/or generation of magnetic fields in magnetized or initially unmagnetized plasmas, so as to extract information related to the production of accelerated particles.

The study presented in this Chapter aims at numerically identifying the shock formation time in relativistic pair plasma shocks, both initially unmagnetized and considering an external field perpendicular to the plasma flows. This latter configuration is the most interesting when considering ultra-relativistic shocks due to Lorentz transformations and field compression at the shock front.

The knowledge of the shock formation time is especially important for the design of experiments, and also for the planning of large-scale 3D simulations. The definition of the time of shock formation is not straightforward. In contrast with previous studies (Bret et al., 2014), our definition of the formation time follows from the investigation of the temporal evolution of two complementary quantities: (i) the compression factor of the downstream region, (ii) the anisotropy of the downstream plasma phase-space. As shock formation implies both a compression consistent with the Rankine-Hugoniot (RH) conditions and isotropization of the flows, the shock formation time is here defined as the maximum of the two measurements.

An extensive paper discussing, but not limited to, the results presented in this Chapter has been submitted (Plotnikov et al., 2017).

This chapter is structured as follows. After a brief overview of previous numerical and theoretical works on relativistic pair plasma shocks (Sec. 6.1), we present in Sec. 6.2 the set-up of the simulations presented in this chapter. We then consider the unmagnetized case in Sec. 6.3 and the magnetized one in Sec. 6.4. For both configurations, we first describe the mechanism behind shock formation (Secs. 6.3.1 and 6.4.1), paying special attention to the measurements of the jump condition and its comparison with the RH predictions. We then discuss the measurement of the shock formation time (Secs. 6.3.2 and 6.4.2). Finally we briefly comment on the acceleration efficiency (Secs. 6.3.3 and 6.4.3) to benchmark the code capabilities in treating the late stage of the shock evolution. We present our conclusions in Sec. 6.5.

6.1 Brief overview of previous works

In this section we summarize recent investigations on pair plasma collisionless shocks, focusing mainly on the early stage of shock formation. For a detailed review see (Marcowith et al., 2016) and references therein.

For unmagnetized counter-streaming pair plasma flows the Weibel instability has been identified as responsible for shock formation in various numerical works, e.g. (Nishikawa et al., 2005; Spitkovsky, 2008). Once the Weibel instability reaches the saturation stage, both species (electrons and positrons) can be drastically slowed down, e.g. via the Alfvén saturation mechanism discussed in Chap. 3. Particles start accumulating in the unstable region producing a density larger than the overlapping value. Bret et al. (2014) suggest that a simple estimate of the shock formation time t_f might be given by the time required to reach the RH density jump, assuming that all particles are effectively stopped in the overlapping region at the instability saturation. Within this picture, at saturation the density in the overlapping region is simply twice the initial upstream density and a compression of $\simeq 3$ in a 2D geometry and $\simeq 4$ in a 3D geometry (as predicted by the RH conditions for relativistic flows) should be achieved after twice (three times) the saturation time $t_{\rm sat}$ for a 2D (3D) case. This gives the estimate of the shock formation time proposed by Bret et al. (2014): $t_f \simeq N_{\rm DIM} t_{\rm sat}$, with $N_{\rm DIM}$ the dimensionality of the system.

The estimate of the saturation time requires the knowledge of the initial magnetic field fluctuation amplitude B_{init} , the magnetic field at saturation $(t = t_{\text{sat}})$ being

$$B_{\rm sat} = B_{\rm init} {\rm e}^{\Gamma t_{\rm sat}} \Longrightarrow t_{\rm sat} = \frac{1}{2\Gamma} \ln \left(\frac{B_{\rm sat}^2}{B_{\rm init}^2} \right) ,$$
 (6.1.1)

where Γ is the Weibel instability growth rate. Even if the growth rate is a function of the wavenumber, in the following we will consider its maximum asymptotic value as derived in Eq. (3.2.3). Generalizing it for the case of two counter-streaming pair plasma with density n_0 , velocity v_0 and Lorentz factor $\gamma_0 = [1 - v_0^2/c^2]^{-1/2}$, we obtain $\Gamma \simeq (2v_0/c\sqrt{\gamma_0}) \omega_{pe}$, with ω_{pe} the single species plasma frequency at the upstream density n_0 . The analysis of the thermal plasma magnetic field fluctuation amplitude and spectrum has been carried out by Bret et al. (2013). This study provides an estimate of the initial fluctuation B_{init} , leading to a saturation time

$$t_{\rm sat} = \frac{1}{2} \frac{\sqrt{\gamma_0}}{2v_0/c} \ln\left[\frac{4}{15} \sqrt{\frac{6}{\pi}} n_0 \left(\frac{c\sqrt{\gamma_0}}{\omega_{pe}}\right)^3 \frac{mc^2}{T_0}\right] \omega_{pe}^{-1}, \qquad (6.1.2)$$

where we remind that T_0 is the temperature defined in the upstream plasma rest frame. From this relation, the shock formation time can be computed, as mentioned above, using $t_f \simeq N_{\text{DIM}} t_{\text{sat}}$.

The level of electromagnetic fluctuations depends on the number of particles in a volume related to the electron skin-depth $\propto (c/\omega_{pe})^3$ in Eq. (6.1.2). In PIC codes, this level is much larger than in any real physical systems because of the reduced number of macro-particles. To account for this, the following replacement has to be considered in Eq. (6.1.2):

$$n_0 \left(\frac{c}{\omega_{pe}}\right)^3 \to N_{\rm ppc} \left(\prod_i^{N_{\rm DIM}} \Delta \tilde{x}_i\right)^{-1},$$

where $N_{\rm ppc}$ is the number of particle-per-cell and $\Delta \tilde{x}_i$ is the space resolution along the *i*-th direction normalized to c/ω_{pe} . Note that this relation does not account for the effect of the order of the shape function, used during the interpolation/projection phase of the PIC loop. Neither does it account for the effect of filtering methods often necessary to run long time simulations with relativistically drifting plasmas (see Chap. 5). Both high order interpolation and filtering indeed reduce the initial thermal noise in PIC simulations, thus increasing the saturation time. Nevertheless, as this factor appears with a logarithmic dependency, equation (6.1.2) actually gives a good estimate for the saturation time of the Weibel instability.

Considering the shock formation time $t_f \simeq N_{\text{DIM}} t_{\text{sat}}$ then gives an estimate of the minimum required time to build-up the RH predicted density jump. In fact, as we will highlight by means of PIC simulations in the following Secs. 6.3.2 and 6.4.2, the typical formation time is significantly longer than this value.

In the case of magnetized plasmas, the physics behind shock formation depends critically on the initial magnetization of the upstream region

$$\sigma = \frac{B_0^2}{8\pi n_0 \gamma_0 mc^2},\tag{6.1.3}$$

where $m = m_{e^-} = m_{e^+}$ is the electron/positron mass, B_0 is the external magnetic field, n_0 is the unperturbed upstream plasma density and γ_0 is its Lorentz factor. All quantities are defined here in the downstream rest frame (here corresponding to the simulation frame, as will be make clear

in Sec. 6.2).

Several studies have investigated the dependence of the shock formation mechanism and of the late acceleration stage on the field orientations considering both pair plasmas (Sironi and Spitkovsky, 2009) and electronion plasmas (Caprioli and Spitkovsky, 2014a). However, a configuration with the external magnetic field transverse to the plasma flows has major interest for ultra-relativistic shocks. Indeed, in the downstream rest frame, due to the compression of the perpendicular magnetic field component at the shock front [as described by the RH conditions, Eq. (2.5.8)] and to the Lorentz transformation, the magnetic field seen by the upstream flow is mostly perpendicular. In the perpendicular pair plasma shock configuration, as demonstrated by means of PIC simulations (Gallant et al., 1992; Sironi and Spitkovsky, 2009; Sironi et al., 2013), the Weibel instability is responsible for shock formation for magnetization $\sigma \lesssim 10^{-3}$ while at large value ($\sigma \gtrsim 0.1)$ the macroscopic magnetic field, compressed in the overlapping region, is strong enough to make the particles gyrate and accumulate in the downstream region, thus isotropizing the flow and forming a shock front by means of the Maser-Synchrotron Instability (Alsop and Arons, 1988; Hoshino and Arons, 1991). The difference between the two processes will be illustrated in Sec. 6.4.1, along with a comparison against the unmagnetized case, discussed in Sec. 6.3.

The Weibel instability in the presence of a perpendicular magnetic field should be studied in more detail, focusing on the investigation of both the linear growth and the saturation level. Indeed, as we have shown in Chap. 3, the generalization of the unmagnetized saturation mechanisms to the magnetized case is not straightforward. However, since the Weibel instability is responsible for shock formation only in the weakly magnetized cases, we expect a marginal modification of the instability development with respect to the unmagnetized case, for which growth rate and saturation level are known. Therefore, Novo et al. (2016) suggests that shock formation in the weakly magnetized case should be reached at the same time than predicted by Bret et al. (2014) using Eq. (6.1.2). Regarding the highly magnetized case, Novo et al. (2016) suggests that the shock is formed after several gyrations of the particles located at the leading edge of the overlapping region in the compressed magnetic field. The particle dynamics seems however more complicated than suggested by these authors, so that a deeper analysis is still lacking.

The late phase of pair plasma shocks propagation and particle acceleration was also investigated by means of PIC simulations (Spitkovsky, 2008; Sironi and Spitkovsky, 2009; Sironi et al., 2013). As already discussed in Sec. 1.1.2, the Diffusive Shock Acceleration paradigm, based on the first order Fermi acceleration mechanism, is believed to give a dominant contribution to the accelerated particle spectrum in several astrophysical scenarios. Indeed, a non-thermal component with power-law ($\frac{dN}{d\gamma} \propto \gamma^{-2.5}$), in good agreement with the analysis of the astrophysical observations ($\frac{dN}{d\gamma} \propto \gamma^{-2.7}$), has been often observed in PIC simulations of unmagnetized and weakly magnetized shocks. Fermi-like acceleration is suppressed for $\sigma \gtrsim 0.1$, as the shock formation is not accompanied by the generation of turbulent magnetic fields required to trigger the Fermi acceleration cycle, as will be further discussed in Sec. 6.4.3.

In this Chapter, we will consider the same configuration than investigated in these previous studies (Spitkovsky, 2008; Sironi and Spitkovsky, 2009; Sironi et al., 2013). We will however focus our attention on the early phase of shock formation, as well as on the fulfillment of the Rankine-Hugoniot conditions once the shock is formed. The shock formation time as well as the downstream density, temperature and shock velocity are indeed important observable when considering the production of collisionless shocks in laboratory experiments.

6.2 Simulation set-up

The series of 2D3V PIC simulations presented in this Chapter aims at investigating collisionless shocks driven by two counter-streaming relativistic electron-positron plasma flows.¹ The simulation method consists

¹One of these simulations, for unmagnetized flows was also presented as a physics highlight in (Dérouillat et al., 2017), presenting the PIC code SMILEI.



Figure 6.1: Sketch of the simulation set-up. The upstream pair plasma drifts in the positive x-direction. An external homogeneous magnetic field \mathbf{B}_0 is initialized in the out-of-plane direction. An electric field \mathbf{E}_0 is as well initialized, in the simulation box in the direction perpendicular to the flow. The produced shock propagates from the right to the left side of simulation box.

in initializing a single plasma drifting in the positive x-direction. A reflecting boundary condition is applied at the right border of the simulation box for both fields and particles, hence creating a counter-penetrating reflected flow. The reflected beam mimics a flow with velocity $-v_0 \hat{\mathbf{x}}$. This allows to greatly reduce the computational cost of the simulation.

The simulation set-up is schematically presented in Figure 6.1. The interaction between the incoming and wall-reflected flow eventually produces a collisionless shock that propagates in the negative x-direction. In this way, the shocked (downstream) plasma has no average velocity, as it corresponds to the center-of-mass frame of the system composed of the incoming and wall-reflected flow. Therefore, the simulations are performed in the downstream rest frame, allowing for the direct use of the Rankine-Hugoniot conditions, derived in Sec. 2.5.2.

We consider a plasma flow with velocity $v_0 \simeq 0.995 c$, corresponding to $\gamma_0 = 10$, and rest-frame temperature $T_0 = 10^{-4} mc^2$. A first set of simulations run up to $500 \,\omega_{pe}^{-1}$, with ω_{pe} the plasma frequency of a single species with density n_0 . We consider a box extension $L_x = 1216 c/\omega_{pe}$ (from $-L_x/2$ to $+L_x/2$) and $L_y = 128 c/\omega_{pe}$ (from $-L_y/2$ to $+L_y/2$). The plasma fills the entire box. The reflecting right boundary of the simulation is located at the right edge $x_{\text{wall}} = L_x/2 = 608 c/\omega_{pe}$. The spatial resolution is set to $\Delta x = \Delta y = (1/4) c/\omega_{pe}$ and the timestep to $c\Delta t = 0.5 \Delta x$. All relativistic scales are properly resolved since $\omega_{pe} > \omega_{pe}^{\text{rel}} = \omega_{pe}/\sqrt{\gamma_0}$, and the transverse box size was chosen in order to observe several relativistic skin depth $c/\omega_{pe}^{\text{rel}}$. We use 25 particles-per-cell per species, leading to a total of $\simeq 10^8$ particles. We present simulations performed with unmagnetized flows and with magnetization in the range $\sigma = [10^{-5}, 10]$.

A second set of simulations has been performed, for a reduced number of cases, up to a longer time $t = 2 \times 10^3 \omega_{pe}^{-1}$, in order to study particle acceleration and its dependence on the magnetization. We have performed three simulations with $\sigma = [0, 10^{-4}, 0.1]$. In these cases, we consider a simulation box of $(2048 c/\omega_{pe} \times 64 c/\omega_{pe})$, we increase the resolution $\Delta x = \Delta y = (1/16) c/\omega_{pe}$ and we use 16 particles-per-cell per species for a total of $\simeq 10^9$ particles. With these parameters the number of particles in a region of extension c/ω_{pe} is larger than in the first set of simulations, so that these longer simulations better resolve the dynamics of the accelerated particles.

For the magnetized simulations, the external uniform magnetic field \mathbf{B}_0 is imposed in the out-of-plane z-direction. As demonstrated by Sironi et al. (2013), this configuration allows to describe the 3D particle dynamics in a 2D3V PIC simulation, in particular the scattering and acceleration process. The analogous configuration with $\mathbf{B}_0 = B_0 \hat{\mathbf{y}}$, would not properly resolve the motion of the particles across different magnetic field lines, thus underestimating the accelerating efficiency in the weakly magnetized case. To provide an initial equilibrium condition, an electric field $\mathbf{E}_0 = -(\mathbf{v}_0/c) \times \mathbf{B}_0$ is imposed at t = 0 in the whole box.

In order to avoid spurious effects (e.g. upstream heating) due to the grid-Cherenkov numerical instability, the temporal Friedman filter (with $\theta = 0.1$) and the binomial current filter (using 3 passes), described in Sec. 5.3, have been applied. A discussion on the effects of the filtering on the shock structure is reported in Appendix 6.A.

6.3 Unmagnetized shock

We present here the results of the unmagnetized simulations. We describe first the mechanism driving shock formation (Sec. 6.3.1), then we discuss the shock formation time (Sec. 6.3.2) and we conclude showing the production of accelerated particles (Sec. 6.3.3). This will be our reference case in order to understand the effect of the external perpendicular magnetic field. It also allows to introduce the definition of all important quantities.

6.3.1 Shock structure

Figure 6.2 presents the characteristic magnetic field B_z (panel a) and electron density (panel b) structures of a fully formed Weibel-mediated shock, at $t = 500 \,\omega_{pe}^{-1}$, as well as the evolution in time of the density profile $\langle n_e \rangle_y$ averaged along the y-direction (panel c), between $t = 50 \,\omega_{pe}^{-1}$ (yellow line) and $t = 500 \,\omega_{pe}^{-1}$ (black line). At early time (yellow line), the overlapping region of the incoming and the reflected flows has density $n_e \simeq 2 n_0$. This region becomes Weibel-unstable which results in the creation of magnetic field structures that eventually become strong enough to stop the incoming flow and lead to an increase of the downstream density (from orange to dark lines). The density jump reaches $n_{\rm d}^{\rm sim} \simeq 3.18 \, n_0$, measured in the downstream at $t = 500 \,\omega_{pe}^{-1}$, averaging over a region starting from $x_{\text{wall}} - 108 c/\omega_{pe}$ to $x_{\text{wall}} - 8 c/\omega_{pe}$. The filamentary structures in both the magnetic field (panel a) and the electron density (panel b), characteristic of the development of the Weibel instability, are still present at late time $(t = 500 \,\omega_{pe}^{-1})$ in the region before the shock front, that is located at $x \simeq 400 c/\omega_{pe}$ (black line in panel c). Note that closer to the shock front the filaments become turbulent. Their size and field amplitude increase, while in the downstream their amplitude decays due to the flow isotropization. In the simulation, the shock propagates toward the left with a velocity $v_{\rm sh}^{\rm sim} \simeq 0.46 c$. The RH conditions predict a shock velocity $v_{\rm sh} = c (\Gamma_{\rm ad} - 1)(\gamma_0 - 1)/(\gamma_0 v_0) \simeq 0.45 c \ [\text{Eq. } (2.5.14)]$ with adiabatic index $\Gamma_{\rm ad} = 3/2$ appropriate for a 2D isotropization at relativistic temperature, in good agreement with the value observed in the simulation. The density jump measured in the simulation $(n_{\rm d}^{\rm sim}/n_0\simeq 3.18)$ is also in



Figure 6.2: Shock structure for initially unmagnetized flows ($\sigma = 0$) at $t = 500 \,\omega_{pe}^{-1}$. a) Weibel-generated magnetic field B_z . b) Electron density. c) Electron density averaged along the *y*-direction at $t = [50, 100, 200, 300, 400, 500] \,\omega_{pe}^{-1}$, light to dark lines.

good agreement with the RH conditions, Eq. (2.5.15), that would predict $n_{\rm d}/n_0 = 1 + (\gamma_0 + 1)/[\gamma_0(\Gamma_{\rm ad} - 1)] = 3.2.$

6.3.2 Shock formation time

As shock formation involves both a compression consistent with Rankine-Hugoniot conditions and isotropization of the flow, we define the shock formation time as the maximum of two measurements

$$t_f^{\rm sim} = \max\{t_{\rm dens}^{\rm sim}, t_{\rm iso}^{\rm sim}\}, \qquad (6.3.1)$$

where t_{dens} is the compression time for which the density jump in the downstream has reached a value consistent with the RH conditions, and t_{iso} is the isotropization time. To compute t_{iso} , we introduce the

isotropization parameter

$$a_{\rm iso} = \frac{\langle v_x p_x \rangle}{\langle v_y p_y \rangle} - 1 \,, \tag{6.3.2}$$

where $\langle v_i p_j \rangle$ corresponds to the pressure tensor component averaged over a given spatial region in the downstream, Eq. (2.2.16), accounting for both the average flow contribution and the thermal spread. In the simulation, we consider that isotropization is achieved if, for all time $t > t_{\rm iso}^{\rm sim}$, $|a_{\rm iso}(t)| < 0.04$.

The measure of $t_{\rm dens}^{\rm sim}$ is given by the time for which the density jump in the downstream has reached the 95% of the value measured in the downstream at the end of the simulation. We average on the measurements obtained considering the downstream layers of extension [20, 15, 10, 5] c/ω_{pe} located $8 c/\omega_{pe}$ to the left of the right boundary of the simulation box. For the unmagnetized simulation, we obtained $t_{\rm dens}^{\rm sim} = (146 \pm 5) \omega_{pe}^{-1}$.

In order to measure t_{iso}^{sim} , we study the time evolution of the p_x - p_y electron phase-space, presented in Fig. 6.3 for $t = [20, 30, 90, 500] \omega_{pe}^{-1}$ (from the left to the right) for the electrons located close to the right boundary of the simulation box, in the region $580 c/\omega_{pe} < x < 600 c/\omega_{pe}$. The initial stage for which two counter-streaming beams with $p_x \simeq \pm 10 \, mc$ are still distinguishable, is shown for $t = 20 \,\omega_{pe}^{-1}$. At $t = 30 \,\omega_{pe}^{-1}$, the upstream flow is still centered around $p_x \simeq 10 \, mc$ but starts spreading in the y-direction, i.e. acquiring a momentum $p_y \neq 0$ due to the development of the Weibel instability. The reflected beam is centered around $p_x \simeq -10 \, mc$ and it has a larger spread in both p_x and p_y , having traveled through the unstable region for a longer time. At this stage the isotropization parameter defined by Eq. (6.3.2) is still large $a_{\rm iso} \simeq 5.5$ and we are far from shock formation. A high level of isotropization $a_{\rm iso} \simeq 0.09$, is reached at $t \simeq 90 \,\omega_{pe}^{-1}$, as shown in Fig. 6.3 c), with the electron downstream distribution function almost centered around zero. Indeed, looking at the evolution of $a_{\rm iso}(t)$, we obtain $t_{\rm iso}^{\rm sim} \simeq 100 \omega_{pe}^{-1}$.

Following the definition in Eq. (6.3.1), the shock formation time is $t_f^{\text{sim}} \simeq 145 \, \omega_{pe}^{-1}$.



Figure 6.3: Isotropization process in unmagnetized counter-streaming pair plasma flows: $p_x p_y$ phase-space of the downstream electron in the region $580 c/\omega_{pe} < x < 600 c/\omega_{pe}$ for $t = [20, 30, 90, 500] \omega_{pe}^{-1}$ (from left to right).

At later times, the isotropization parameter $a_{\rm iso}$ keeps decreasing. At the end of the simulation $t = 500 \,\omega_{pe}^{-1}$, corresponding to Fig. 6.3 d), we measure $a_{\rm iso} = 0.014$ and the temperature is $T^{\rm sim} \simeq 4.53 \,mc^2$. This value is in excellent agreement with the prediction of the RH conditions, $T = \frac{1}{2}(\gamma_0 - 1) \,mc^2 \simeq 4.5 \,mc^2$, and we conclude that thermalization is achieved at $t = 500 \,\omega_{pe}^{-1}$. Note that analogous results are obtained for the positron population (not shown).

The shock formation time predicted by Eq. (6.1.2), making use of the relation to apply it for these 2D simulation parameters, is $t_f = 2 t_{\text{sat}} \simeq 30 \, \omega_{pe}^{-1}$. This substantial underestimates the value extracted from the simulation. Indeed in the model proposed by Bret et al. (2014), all particles are considered to be trapped in the overlapping region at the time in which the instability saturates, so that at $\simeq 2 t_{\text{sat}}$ the RH density jump is predicted to be achieved in a 2D simulation. As discussed in Sec. 3.2.2, in order to efficiently slow down the particles contributing to the instability, the Alfvén saturation mechanism has to take place. This mechanism has been demonstrated to be the dominant one for wavenumbers $k < k^*$, Eq. (3.2.22) and Fig. 3.5, which, for the flow velocity investigated in this section, corresponds to $k^* \simeq 0.44 \, \omega_{pe}/c$.

In order to measure the wavenumber at saturation $k_{\text{sat}}^{\text{sim}}$, we performed an additional 2D simulation with two counter-streaming pair plasma flows, already overlapped at t = 0, with otherwise identical parameters. The instability reaches saturation at $t_{\text{sat}}^{\text{sim}} \simeq 20 \, \omega_{pe}^{-1}$, with dominant wavenum-

ber $k_{\rm sat}^{\rm sim} \simeq 0.8 \, \omega_{pe}/c$. The flow velocity along the *x*-direction at $t_{\rm sat}$ is still large, meaning that saturation is achieved via the trapping mechanism described in Sec. 3.2.2. Therefore to efficiently stop the flow in the overlapping region, an additional amount of time is required to allow for merging events, that decrease the wavenumber down to $k \leq k^{*,2}$ Note that the saturation time observed in the simulation $(t_{\rm sat}^{\rm sim} \simeq 20 \, \omega_{pe}^{-1})$ is however in fairly good agreement with the prediction of Eq. (6.1.2), $t_{\rm sat} \simeq 15 \, \omega_{pe}^{-1}$. Yet, with our method to extract the shock formation time, an interval longer than $2 \, t_{\rm sat}$ is required to build-up a proper downstream region.

6.3.3 Particle acceleration

Finally, a longer simulation, up to $t = 2 \times 10^3 \, \omega_{pe}^{-1}$, has been performed to observe the production of a supra-thermal tail in the downstream particle energy distribution, as shown in Fig. 6.4. Around $t = 500 \,\omega_{pe}^{-1}$, the downstream distribution function has relaxed to an isotropic thermal distribution, that closely follows the 2D Maxwell-Jüttner distribution, Eq. (2.2.8), with temperature predicted by the RH condition (dashed black line). This stage corresponds also to Fig. 6.3d). At later times, a supra-thermal tail, characteristic of first order Fermi acceleration at the shock front, appears in the energy spectrum, Fig. 6.4. The non-thermal component, in the far downstream region, is found to follows a power-law steeper than observed in previous works $\frac{dN}{d\gamma} \propto \gamma^{-2.5}$ (Spitkovsky, 2008), corresponding to the guide-line reported in the figure (dash-dotted black line). However, the spectrum (dashed green line) at the latest timestep $(t = 2 \times 10^3 \,\omega_{pe}^{-1})$ closer to the shock front $(500 \, c/\omega_{pe} < x < 600 \, c/\omega_{pe})$ is in good agreement with the expected power-law. This means that particles are still being accelerated at the shock front, so that additional time is required to observe the good power-law dependence in the far downstream. Indeed, in the previous numerical work (Spitkovsky, 2008) the simulations run for much longer time (up to $10^4 \,\omega_{pe}^{-1}$) and the slope has

²Note also that the phase space reported in Fig. 3.7a) reveals that a small amount of particles keep a forward velocity (up to $\simeq 2 v_0$) even if saturation follows the Alfvén mechanism. An estimate of the time to reach the RH density jump should also account for these particles that can escape from the overlapping region.



Figure 6.4: Electron energy distribution in the downstream region $(800 c/\omega_{pe} < x < 900 c/\omega_{pe})$ for different times $t > 450 \omega_{pe}^{-1}$. The shock is already formed and the thermal part of the energy distribution closely follows the (2-dimensional) thermal Maxwell-Jüttner distribution (Eq. (2.2.8)) with temperature $T = 4.5 mc^2$ expected from RH conditions (dashed black line). The non-thermal component in the electron energy distribution at $t = 2 \times 10^3 \omega_{pe}$ closer to the shock front $500 c/\omega_{pe} < x < 600 c/\omega_{pe}$ (green dashed line) is in good agreement the power law $\gamma^{-2.5}$ (dot-dashed guide line).

been observed to flatten with time, up to the achievement of the scaling $\propto \gamma^{-2.5}.$

6.4 Magnetized perpendicular shock

We now present a series of 2D simulations performed considering an external magnetic field perpendicular to the plasma flows and to the shock propagation direction. We aim at identifying the shock formation time, and its dependence on the magnetization.

To do so, we first describe the mechanisms leading to shock formation for different magnetizations in Sec. 6.4.1, and we compare the theoretically predicted jump conditions with the PIC simulation results. This will reveal kinetic effects that the hydrodynamic formulation, used to derive the RH conditions, does not take into account, such as the emission of



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Figure 6.5: Shock structure for initially weakly magnetized flows $(\sigma = 10^{-4})$ at $t = 500 \,\omega_{pe}^{-1}$. a) Weibel generated magnetic field B_z . b) Electron density. c) Electron density averaged along the *y*-direction at $t = [50, 100, 200, 300, 400, 500] \,\omega_{pe}^{-1}$, from light to dark lines.

electromagnetic waves in the strongly magnetized case. Section 6.4.2 discusses the shock formation time, following the same procedure than used for the unmagnetized simulation of the previous Sec. 6.3.2. Following the evolution of the shocks up to late times, we investigate the efficiency of the acceleration mechanism varying the initial magnetization in Sec. 6.4.3.

6.4.1 Formation mechanisms and jump conditions

The mechanism behind shock formation is different in the unmagnetized and strongly magnetized cases.

The Weibel instability has been confirmed to be responsible for the strong magnetic fields observed in the early phase of shock formation, in the absence of external magnetic field, in Sec. 6.3 and in many previous numerical studies (Nishikawa et al., 2005; Spitkovsky, 2008). This mechanism is found to be efficient for magnetization lower than $\sigma = 10^{-3}$, for ultra-relativistic pair plasma flows. Indeed, the structures of the magnetic field and density, shown in Fig. 6.5 for the case of $\sigma = 10^{-4}$, are similar to those observed in the unmagnetized case reported in Fig. 6.2. In particular, in this weakly magnetized case, the average density compression Fig. 6.5c) follows the same evolution than in the unmagnetized case, Fig. 6.2c). Weibel-generated filaments in the magnetic field B_z and in the electron density are clear in Fig. 6.5a-b). The region where these filamentary structures are observed, usually referred to as the *precursor* of the shock front, is shorter than in the unmagnetized case. This is because the external magnetic field bends the particle trajectories in the precursor, thus reducing their penetration in the upstream region, ahead of the shock front. This bending also leads to Weibel-generated magnetic filaments with a stronger oblique component than in the unmagnetized case. The extension of the precursor and the intensity of the magnetic field in that region have important implications on the Fermi-like acceleration mechanism, that requires a particle to repeatedly cross the shock front thanks to its diffusive motion in the magnetic field turbulence. A detailed discussion on the precursor extension is reported in (Plotnikov et al., 2017).

At large magnetization ($\sigma \gtrsim 0.1$), as soon as a particle gets reflected at the right boundary of the simulation box, it starts gyrating around the external and compressed magnetic field. The dissipation, intrinsic to shock formation, follows from the emission of large-amplitude electromagnetic waves by means of the Maser-Synchrotron Instability (Sprangle et al., 1977; Hoshino and Arons, 1991). These waves are emitted from the leading edge of the cyclotron ring at the shock front, due to the coherent bunching of the particles during their Larmor gyration. The magnetic field and electron density structures are shown in Fig. 6.6a-b). The radiation emitted at the shock front corresponds to electromagnetic extraordinary modes (i.e. $\mathbf{k} \perp \mathbf{B}_0$ and $\mathbf{E} \perp \mathbf{B}_0$) and produces the modulations observed in the magnetic field ahead of the shock front. These




Figure 6.6: Shock structure for initially highly magnetized flow ($\sigma = 0.1$) at $t = 500 \,\omega_{pe}^{-1}$. a) Weibel generated magnetic field B_z . b) Electron density. c) Electron density averaged along the *y*-direction at $t = [50, 100, 200, 300, 400, 500] \,\omega_{pe}^{-1}$, from light to dark lines.

modes have a plane wave-like structure far from the shock front (corresponding to the waves emitted at early times) while, closer to the shock front, their structure is less regular. This is the result of the filamentation of the electron density ahead of the shock front and of the corrugation of the shock front itself. The filamentary structures observed in the electron density corresponds to the reaction of the incoming background plasma to the electromagnetic large-amplitude waves emitted from the shock front. This filamentation does not indicate the Weibel instability

³Note that the magnetic field that is theoretically predicted to suppress Weibel purely transverse modes, derived by Novo et al. (2016), corresponds to a magnetization $\sigma \simeq 200$, much larger than the transition value ($\sigma \simeq 10^{-2}$) above which the shock is driven by the compression of the external field.



Figure 6.7: a) Density compression ratio n_d/n_0 , b) shock front velocity $v_{\rm sh}$ as a function of the upstream flow magnetization σ . Theoretical predictions from Eqs. (2.5.14) and (2.5.15) (black lines) and values measured in PIC simulations at $t = 500 \, \omega_{pe}^{-1}$ (red dots).

development. Indeed, the external magnetic field \mathbf{B}_0 is strong enough to suppress the precursor beam. Almost no particle of the reflected beam are found ahead of the shock front, hence avoiding the amplification of Weibel modes³. The precursor, in the highly magnetized case, has a purely electromagnetic nature.

The evolution in time of the average density $\langle n_e \rangle_y$, shown in Fig. 6.6c), is different from the unmagnetized and weakly magnetized cases. Indeed, the shock front is sharper than in the Weibel-mediated shocks and it is characterized by an overshoot region, where a density hump is formed, followed by a flat density profile that well satisfies the RH conditions, as will be now discussed into more details.

We now compare the RH predictions for the density jump $n_{\rm d}/n_0$ Eq. (2.5.15) (black line in Fig. 6.7 a), and shock propagation velocity $v_{\rm sh}$ (black line in Fig. 6.7 b) with the results of PIC simulations (red dots). In the simulations, the downstream density $n_{\rm d}$ has been measured at $t = 500 \, \omega_{pe}^{-1}$, considering the average along the x-direction in the region $500 \, c/\omega_p < x < 600 \, c/\omega_p$ and along the whole y-axes. For $\sigma \lesssim 10^{-3}$, the compression ratio keeps its unmagnetized limit $n_{\rm d}/n_0 \simeq 3.2$ and the simulations results are in very good agreement with the ideal RH predictions. A gradual deviation occurs for intermediate magnetizations $(10^{-3} < \sigma < 5 \times 10^{-2})$, for which the measured compression ratio is approximately 3% lower than theoretically expected.

The slightly larger value at $\sigma = 0.1$ is related with the strong x-modes emission at the shock front, that reduces the energy transferred to the downstream plasma. This is not accounted for in the fluid picture of the RH conditions and it has been demonstrated to increase the density compression and slow down the shock front in comparison with the RH predictions, (see Gallant et al., 1992). However, in the 1D configuration investigated by Gallant et al. (1992), the wave emission is much stronger, and consequently the deviation from the RH density jump is more important, than in the 2D case tested here, where two-dimensional effects reduces the coherence of the cyclotron loop responsible for the waves emission. While the wave emission can affect the density jump, the upstream filamentation seems to have no significant influence on the shock front, at least up to $t = 500 \,\omega_{pe}^{-1}$. A good agreement with the ideal RH prediction is recovered for large magnetization ($\sigma > 0.1$).

Similarly, the shock velocity has been measured in the simulations following the position of the shock front $x_{\rm sh}$, defined by $n_e(x_{\rm sh}) = 2.8 n_0.^4$ These measurements are in good agreement with the theoretical predictions, as shown in Fig. 6.7b). The shock propagates slightly faster than theoretically expected, in the intermediate regime $(10^{-3} < \sigma < 5 \times 10^{-2})$ for which the density is found to be lower than predicted by the RH conditions.

6.4.2 Shock formation time

In order to quantify the shock formation time, we first discuss the evolution of the p_x - p_y phase-space, then measure the shock formation time with the same method discussed in Sec. 6.3.2 for the unmagnetized case.

In Fig. 6.8, we report the downstream $p_x p_y$ electron phase-space for $\sigma = 10^{-4}$, 5×10^{-3} , 1, at $t = [20, 30, 90, 500] \omega_{pe}^{-1}$. We remind that the

⁴Note that this density value is reached at the shock front even in the highly magnetized cases, for which the RH predicted density jump is smaller than $2.8 n_0$, thanks to the density overshoot observed in Fig. 6.6.

external magnetic field is in the z-direction, transverse to the plasma flow that is along the x-axes. The case with $\sigma = 10^{-4}$ ($\sigma = 1$) is representative of all other weak (strong) magnetizations investigated in this work, while in the simulation with $\sigma = 5 \times 10^{-3}$ we can identify a mixed behavior characteristic of the transition between the two shock formation mechanisms.

For $\sigma = 10^{-4}$ and $\sigma = 5 \times 10^{-3}$ (left and middle columns), two counter streaming cold beams are present at $t = 20 \,\omega_{pe}^{-1}$. A slight shift in the p_y direction seen for the case $\sigma = 5 \times 10^{-3}$ is the result of the beam rotation in the external field B_0 , while both beams are centered in $p_y \simeq 0$ for $\sigma = 10^{-4}$. This configuration quickly becomes Weibel unstable, and at $t = 30 \,\omega_{pe}^{-1}$ the phase-space distribution of the two beams appear to be strongly modified, and in particular broader. However, two distinct beams can still be identified and the anisotropy parameter remains large, as reported in the figure. At $t = 90 \,\omega_{pe}^{-1}$, the distribution is almost fully isotropized, and the anisotropy parameter has strongly decreased down to $a_{\rm iso} \simeq 0.036$ and $a_{\rm iso} \simeq 0.13$ for $\sigma = 10^{-4}$ and $\sigma =$ 5×10^{-3} , respectively. At much larger times, $t = 500 \,\omega_{pe}^{-1}$, the p_x - p_y phase space assumes a Maxwell-Jüttner like distribution and thermalization has been achieved. The temperature measured at the end of the simulation $(t = 500 \,\omega_{ne}^{-1})$ is $T^{\rm sim} \simeq 4.58 \, mc^2$ for the weakly magnetized case, in good agreement with the RH prediction $T\,\simeq\,4.50\,mc^2.$ For $\sigma\,=\,5\,\times$ 10^{-3} the final temperature is $T^{\rm sim} \simeq 4.78 \, mc^2$. This is slightly higher than expected from the RH conditions $(T \simeq 4.48 \, mc^2)$ and confirms the departure from the RH conditions observed earlier as a deviation in the density compression observed at intermediate magnetization.

The evolution is different in the highly magnetized case ($\sigma = 1$). At $t = 20 \,\omega_{pe}^{-1}$, the particles start gyrating in the compressed magnetic field and create a cold ring-shaped structure in the $p_x p_y$ phase-space, which quickly turns unstable with respect to the Maser-Synchrotron Instability (Hoshino and Arons, 1991). This instability quickly heats up the flows and a much broader ring-like structure is observed at $t = 30 \,\omega_{pe}^{-1}$. Yet, the anisotropy parameter for these two early times remains larger than the threshold value considered for shock formation ($|a_{iso}| < 0.04$). The distribution function keeps evolving up to isotropization ($t_{iso}^{sim} \simeq 65 \,\omega_{pe}^{-1}$),



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Figure 6.8: Isotropization process in counter-streaming magnetized pair plasma flows: p_x - p_y electron phase-space for simulations with magnetization $\sigma = 10^{-4}$, 5×10^{-3} , 1, at $t = [20, 30, 90, 500] \omega_{pe}^{-1}$ (from top to bottom).

achieved before the time corresponding to the third frame $(t = 90 \,\omega_{pe}^{-1})$ for which $|a_{\rm iso}| \simeq 0.01$. This value keeps decreasing at later times. Notice that the distribution function maintains a ring-like structure, i.e. the region around $p_x \simeq p_y \simeq 0$ in the phase-space is somewhat depleted of particles and a complete relaxation toward a Maxwellian in not observed in the simulation. This means that, even if isotropization is quickly achieved, thermalization requires a longer times than in the unmagnetized and weakly magnetized cases. However, the temperature measured at the end of the simulation is $T^{\rm sim} \simeq 4.0 \, mc^2$, just slightly larger than the one predicted by the RH conditions $T \simeq 3.7 \, mc^2$.

We now discuss the shock formation time extracted from our PIC simulations. In Fig. 6.9, we report the measurements of $t_{\rm dens}^{\rm sim}$, the time to achieve 95% of the late time density jump, and $t_{\rm iso}^{\rm sim}$, the time after which $|a_{\rm iso}| < 0.04$, for all magnetizations. The gray area highlights the shock formation time as $t_f^{\rm sim} = \max\{t_{\rm dens}^{\rm sim}, t_{\rm iso}^{\rm sim}\}$.

For weakly magnetized counter-propagating flows, the results are in good agreement with the unmagnetized case. In particular, for $\sigma < 10^{-3}$, the shock formation time is determined by the time required for the density to accumulate in the downstream, since $t_{\text{dens}} > t_{\text{iso}}$, while isotropization is achieved at earlier times ($t_{\text{iso}}^{\text{sim}} \leq 100 \, \omega_{pe}^{-1}$). Note that the observed $t_{\text{dens}}^{\text{sim}}$ is systematically larger than the value predicted by Eq. (6.1.2), corresponding to the dashed line in Fig. 6.9, for the reasons explained in Sec. 6.3. However, its constant behavior is well reproduced.

In the strongly magnetized cases, on the contrary, the downstream density accumulation predicted by the RH conditions is reached very quickly $t_{\rm dens}^{\rm sim} \simeq (30 \div 50) \omega_{pe}^{-1}$, due to the particles gyration in the compressed magnetic field and to the lower value of the density jump, see Fig. 6.7. However isotropization requires longer times. Notice that the scaling of the shock formation time with the magnetization, for $\sigma > 10^{-2}$, is weaker than $\sigma^{-2/3}$, as would be expected considering the Maser-Synchrotron Instability growth rate (Sprangle et al., 1977). It is also weaker than $\sigma^{-1/2}$, as expected considering the gyration time of the particles in the external field, i.e. the inverse of the cyclotron frequency $\omega_c = eB_0/m\gamma_0 c$. This highlights an open question that deserves further



Figure 6.9: Shock formation time t_f as a function of the magnetization σ . Values measured in the PIC simulations (red dots), considering the accumulation of the density up to 95% of the RH prediction, Eq. (2.5.15), averaging on the values measured for different extensions of the downstream region, equal to [20, 15, 10, 5] c/ω_{pe} located $8 c/\omega_{pe}$ to the left of the right boundary of the simulation box. The dashed black line corresponds to the theoretical expectation from Eq. (6.1.2), (Bret et al., 2014; Novo et al., 2016).

investigations.

In conclusion in the weakly magnetized regime, the shock formation is determined by the measurements on the RH density jump, while in the case of highly magnetized flows, the strong external magnetic field accounts for a fast accumulation of particles in the downstream, and isotropization is achieved only at later times, determining the time of shock formation.

6.4.3 Particle acceleration

We now discuss the efficiency of particle acceleration in perpendicular shocks. The isotropization process, intrinsic to shock formation, leads to a thermal spectrum with temperature close to the one predicted by the RH conditions. At late times, as already observed in the unmagnetized case Sec. 6.3.3, if Fermi-like acceleration takes place, a high-energy non-

thermal component appears in the spectrum, with the typical power-law $dN/d\gamma \propto \gamma^{-2.5}$. The acceleration efficiency of the shocks as a function of the magnetization can be measured by the extent of the non-thermal tail and its high-energy cut-off. This topic has been widely investigated in the literature and the results presented here are in good agreement with Sironi et al. (2013). A more detailed study on the kinetic aspects of particle transport and acceleration will be presented in (Plotnikov et al., 2017).

We present now the downstream electron energy spectra, well behind the shock front, considering the downstream region $800 c/\omega_{pe} < x < 900 c/\omega_{pe}$. These results are extracted from the additional simulations, described in Sec. 6.2, that run up to $2 \times 10^3 \omega_{pe}^{-1}$. In Fig. 6.10 the evolution in time of the electron spectrum is shown for $\sigma = 10^{-4}$, representative of the weakly magnetized case, and $\sigma = 0.1$, representative of the highly magnetized one.

In the weakly magnetized case, at the earliest time shown, the distribution function is well fitted by a 2D Maxwell-Jüttner distribution, defined in Eq. (2.2.6), with a temperature in good agreement with the value expected from the RH jump conditions (black dashed line), as already found in the shorter-time simulation presented in Sec. 6.4.2. At later times, a non-thermal power-law component develops. The dot-dashed guide-line follows the $dN/d\gamma \propto \gamma^{-2.5}$ scaling, that closely reproduce the accelerated particle spectrum. The maximum energy of the accelerated particles extends with time. At the end of the simulation, it reaches $\gamma_{\rm end}^{\rm sim} \simeq 150$, for a total energy transmitted to the accelerated particles equal to the 2% of the total energy in the downstream. It has been already demonstrated by Sironi et al. (2013) that the maximum energy increases with time as $\gamma_{\rm max} \propto \sqrt{t}$, due to the magnetic field scattering involved in the acceleration process. The saturation of this maximum level weakly depends on the magnetization, as $\gamma_{\rm max} \propto \sigma^{-1/4} \div \sigma^{-1/2}$, (see Sironi et al., 2013; Plotnikov et al., 2017). Longer simulation times would be necessary to allow a more precise investigation of the saturation process in the weakly magnetized case.

At high magnetization ($\sigma = 0.1$), in Fig. 6.10, the width of the thermal part of the distribution function narrows with respect to the weakly



Chapter 6. Collisionless shocks in magnetized relativistic pair plasmas

Figure 6.10: Evolution in time $(t > 450 \, \omega_{pe}^{-1})$ of the electron energy distribution in the downstream region $(800 \, c/\omega_{pe} < x < 900 \, c/\omega_{pe})$ for $\sigma = 10^{-4}$ (left frame) and $\sigma = 0.1$ (right frame). The shock is already formed for both magnetizations. The 2D thermal Maxwell-Jüttner distributions [Eq. (2.2.8)] with temperature $T \simeq 4.50 \, mc^2$ and $T \simeq 4.27 \, mc^2$ expected from RH conditions for $\sigma = 10^{-4}$ and $\sigma = 0.1$, respectively, are reported (black dashed lines). A supra-thermal tail appears at later times, in the weakly magnetized case, the dot-dashed guide line goes as the power law $\gamma^{-2.5}$.

magnetized case, as predicted by the RH conditions, Eq. (2.5.17). No supra-thermal component is observed. This follows from the mechanism behind shock formation, i.e. particles gyration in the compressed magnetic field and x-modes emission, which does not allow for the production of the magnetic field turbulence required for the Fermi acceleration mechanism.

In conclusion, we confirm that relativistic pair plasma shocks are good accelerators only below the magnetization threshold $\sigma \simeq 10^{-3}$, while some open questions have been identified and more detailed ongoing studies on particle acceleration will be presented in (Plotnikov et al., 2017).

6.5 Conclusions

In this Chapter we focused on the early phase of shock formation in relativistic pair plasma. A proper measurements of the shock formation time is important for the planning of future experiments and large-scale simulations. Both initially unmagnetized flows and the case with an external magnetic field perpendicular to the flows have been investigated. A systematic survey of the Rankine-Hugoniot conditions has been performed for magnetization $\sigma \leq 10$. This work is included in the more extensive study presented in (Plotnikov et al., 2017).

We have identified a new operating definition for the shock formation time, that requires both compression and isotropization of the flows as predicted by the RH conditions. With this method the shock formation time is found to be longer than previously stated by Bret et al. (2014); Novo et al. (2016), for all magnetizations. The measurements of the shock formation time reveal that, in the unmagnetized and weakly magnetized cases, the isotropization requires longer time than the compression up to the RH density jump, while the opposite is observed at large magnetizations.

This Chapter also demonstrates the ability of the PIC code SMILEI to correctly treat ultra-relativistic astrophysical scenarios, thanks to the development presented in Chap. 5. The accuracy of the presented simulations has been verified by comparing our results with the theoretical RH predictions and with previous works on particle acceleration in relativistic pair shocks.

The astrophysical applications of the present study range from AGN, GRBs and PWNs. We confirm that at large magnetization, as expected in PWNs, Fermi-like acceleration is not efficient enough to explain the astrophysical observations, while at low magnetizations, as the one of the interstellar medium, the observed non-thermal tails are coherent with the astrophysical spectra.

Appendix

6.A Effect of the numerical filtering techniques

The relativistic stream of particles along the x-axes in a two-dimensional simulation performed in the x-y plane produces spurious grid-Cherenkov instability, as described in Chap. 5. This can critically affect the results of the simulation.

The tests performed in the previous Sec. 5.4 (two counter-streaming already overlapped pair plasma flows) show that, for the parameters considered in this Chapter (in particular $\gamma_0 = 10$), the development of the Weibel instability is much faster than the typical growth of the numerical instability. The tests shown in Fig. 5.8 seem to suggest that no filtering techniques are required, since no difference in the Weibel evolution are observed with/without the application of the filters. However, in the simulation of this chapter, the upstream plasma flows freely for long time before interacting with the counter-propagating reflected beam, similar to the test performed in Sec. 5.2.3 (note that the moving injector technique is not yet available in SMILEI). Therefore, a strong Cherenkov-like emission can be produced and can dramatically heat up the upstream flow before it interacts with the counter-propagating beam. Following the results of the previous Chapter, in all the simulations presented in this Chapter, we applied a Friedman filter with $\theta = 0.1$ and 3-passes of current filtering. Section 5.4 has demonstrated that these filtering do not affect the Weibel evolution, both the typical growth rate and the magnetic filed amplitude at saturation, while efficiently quenching the Cherenkov-like emission in the upstream streaming plasma, as demonstrated in Sec. 5.2.3. In this Section, we compare the results of unmagnetized simulations performed without filtering and with $\theta = 0.3$, i.e. a stronger Friedman filter than the one applied in Sec. 6.3. Figure 6.11 summarizes the main findings, at a time much after shock formation $t = 500 \,\omega_{pe}^{-1}$, for the simulation without filtering (green lines), with Friedman filter with $\theta = 0.1$ (blue lines) and $\theta = 0.3$ (red lines). In the presence of Cherenkov radiation (i.e. without the application of the filter) the density jump is reduced and the shock propagates slightly faster than in the other cases, as shown



Figure 6.11: Effect of the Cherenkov radiation and of the Friedman filter on the shock front and plasma distribution function: a) Electron density $\langle n_e \rangle_y$. b) Electron distribution function $f_e(p_x)$ in the upstream. c) Electron distribution function $f_e(p_x)$ in the downstream. At $t = 500 \, \omega_{pe}^{-1}$, for the simulation performed with no filtering (green lines), Friedman filter with $\theta = 0.1$ (blue lines), Friedman filter with $\theta = 0.3$ (red lines).

in the average electron density (left frame). This is a consequence of the strong heating of the upstream region, as shown by comparing the distribution functions $f_e(p_x)$ (central frame) for the three cases. The upstream heating changes the interaction conditions of the two counter-propagating flows. The spread in p_x of the downstream distribution is however un-affected (right frame). The same result has been observed for the other momentum component in the downstream (not shown). The increase of the parameter θ of the Friedman filter does not alter substantially the dynamics.

In the case of highly magnetized shock, the amplitude of the waves emitted at the shock front are drastically reduced by the application of strong filters. Figure 6.12 presents the results of the simulation performed with $\sigma = 0.1$ and Friedman filter with $\theta = 0.3$, so that it can be compared with Fig. 6.6. The strong filtering reduces the modulation of the magnetic fields ahead of the shock front and completely suppress the density filamentation. The overshoot in the density is more peaked.

Plotnikov et al. (2017) performed a systematic comparison of PIC simulations performed with SMILEI (FDTD-based) and Shockapic, the latter relying on pseudo-spectral methods to solve Maxwell's equations. In particular, a comparison between SMILEI and Shockapic results is pre-



Figure 6.12: Damping effect of the filtering techniques on the electromagnetic wave precursor of a magnetized shock ($\sigma = 0.1$) at $t = 500 \,\omega_{pe}^{-1}$. a) Weibel generated magnetic field B_z . b) Electron density. c) Electron density averaged along the *y*-direction at $t = [50, 100, 200, 300, 400, 500] \,\omega_{pe}^{-1}$, from light to dark lines. Friedman filter $\theta = 0.3$ and 3-passes current filter. To be compared with Fig. 6.6, obtained with a weaker Friedman filter ($\theta = 0.1$).

sented for $\sigma = 2 \times 10^{-3}$. The level of electromagnetic waves in the precursor has larger amplitude in the simulation performed with the spectral code Shockapic than in the simulation performed with SMILEI for both $\theta = 0.1$ and $\theta = 0.3$.

Even if the early shock formation stage and the evolution up to $t = 500 \,\omega_{pe}^{-1}$ does apparently not depend on the presence of the emitted waves, they should not be removed by the filtering techniques in order to capture their potential effects on the shock long-term evolution. In particular, this electromagnetic precursor has been demonstrated to be important in the

case of electron-ion flows (see Lyubarsky, 2006; Sironi and Spitkovsky, 2011b).

Nevertheless an overall good agreement was found in between the two codes over a wide range of magnetizations.

APPENDIX

CHAPTER 7

Conclusions and perspectives

7.1 Conclusions

The work presented in this thesis belongs to the general framework of *Laboratory astrophysics*. Motivated by the present or forthcoming experimental capabilities, we addressed various aspects of the physics related, but not restricted, to collisionless shocks driven by mildly and ultra-relativistic plasma flows, in configurations of interest for both the astrophysics and the laser-plasma interaction (LPI) communities.

The approach used throughout this thesis relied on both analytical modeling and high-performance, first principles kinetic numerical simulations. On the analytical side we have developed different models to improve our understanding and prediction capabilities of various processes relevant to astrophysics and LPI, from the Weibel instability to the generation of large sheath magnetic field at the laser-plasma interaction surface when using a laser at oblique incidence. On the numerical simulation side, we have made an extensive use of the Particle-In-Cell code SMILEI, and also developed and implemented new modules to enhance its capabilities in dealing with ultra-relativistic flows.

We have investigated three physical configurations: (i) two counterstreaming electron beams in an external flow-aligned magnetic field (Chap. 3), (ii) two counter-streaming ultra-relativistic pair plasma flows (Chap. 6) and (iii) a regime dominated by the ion-Weibel-instability driven by the Radiation-Pressure on a dense target (Chap. 4).

The interest toward the first configuration follows from recent experiments that consider counter-streaming electron-ion flows in the presence of an initially external magnetic field perpendicular to the plasma flows (Higginson et al., 2015). The analogous configuration with flowaligned magnetic field has not received much experimental attention up to now due to the common belief that the stabilizing effect of the external field would decrease the Weibel-generated magnetic field strength. The investigated configuration (symmetric and homogeneous electron beams in a neutralizing ion background with flow-aligned magnetic field) has been modeled by means of a relativistic fluid approach, accounting for temperature effects. This method has proved able to predict the linear phase of the development of the Weibel instability, in good agreement with the results of fully kinetic (PIC) simulations. The limit of validity of this approach, beyond which a kinetic treatment is required, has been discussed. The use of a fluid model has the advantage to give more tractable solutions with respect to the analytical kinetic approach, in particular in the presence of an external magnetic field. In parallel to the analytical investigation, we relied on a systematic comparison between the analytical and the numerical simulation results to confirm the accuracy of the fluid model.

This study helped shading new light on the processes responsible for the instability saturation. While this has been done in this thesis in detail (with and without external B-field) for the electron Weibel instability, we are currently generalizing our approach to also clarify the processes behind the saturation of the ion-Weibel instability.

We provided the analytical generalization of the existing models for the unmagnetized electron Weibel instability saturation, to account for the external field, and, as for the linear phase of the instability, we confirmed our predictions by comparing them with PIC simulations.

Following these results, our study suggests that a guiding magnetic field could be used to focus the plasma flows in a situation of counterpropagating ablative or TNSA plasmas to observe a faster growth of Weibel-generated magnetic fields. Indeed, it would be optimal for an experimental investigation to keep a high plasma density, hence speeding up the development of the instabilities and potentially the formation of collisionless shocks, while not strongly modifying the amplitude of the Weibel magnetic fields at saturation. Further numerical studies should investigate the required strength of the external field to efficiently collimate the flows.

Clearly, PIC simulations appear as a central tool to describe most of the UHI laser-plasma interaction processes as well as the non-linear relativistic physics behind shock formation. The PIC code SMILEI, used and further developed during this work, is a recent¹ open-source project codeveloped by both physicists and high-performance computing experts. This tight collaboration led to a versatile tool that benefits from the most advanced parallelization techniques. We have presented (Chap. 5) our main contribution to the development of SMILEI, i.e. the implementation of techniques that allow for the correct treatment of relativistic particle flows, mitigating the so-called *numerical Grid-Cherenkov instability*. The use of advanced FDTD scheme combined with temporal and spatial filtering strategies, has allowed for an accurate description of the dynamics of ultra-relativistic streaming pair plasmas. More advanced techniques and HPC-relevant spectral methods are currently under development in SMILEI, so that the code will be able to deal with configurations with stronger numerical artifacts.

Our developments have opened SMILEI to the ultra-relativistic astrophysics community. In particular, we initiated a collaboration with astrophysicists from the *Institut de Recherche en Astrophysique et Planétologie* (Toulouse, France). Chapter 6 presents part of the results obtained during this collaboration. In this study we investigated the interaction between counter-streaming ultra-relativistic pair plasmas. We considered both initially unmagnetized and magnetized flows, focusing on the so-called perpendicular shocks, of particular importance for relativistic astrophysical shocks such as in the Pulsar Wind Nebulae. Our study particularly focused on the identification and measurement of the shock formation time, defined as the time for which both compression

¹Development started on 2013, slightly before the beginning of this thesis.

and isotropization of the flows are achieved. Even though the mechanisms behind shock formation and the late stage of particle acceleration were previously investigated, a reliable definition for the shock formation time was not presented before. The knowledge of this quantity is of interest for the planning of laboratory and numerical investigation and our constraining definition led to a shock formation time longer than the one proposed by previous studies. We further highlight that these two conditions are reached at different times and, depending on the flow magnetization, either isotropization or compression is established first.

Most of present-day experiments of laser-generated plasmas are only capable of producing counter-propagating electron-ion flows. The weakly relativistic velocity, characteristic densities and temperatures of these flows make them properly scalable to the typical SNRs emitted material. In this work, we proposed a complementary approach that relies on the use of UHI laser facilities, such that the produced flows are faster and denser than in current experiments. These flow characteristics involve a larger growth rate and stronger saturation magnetic fields, beneficial for the experimental investigation.

By means of large-scale 3D simulations, we have demonstrated that mildly relativistic, dense quasi-neutral plasma flows can be produced via the Radiation Pressure Acceleration, and open up a regime governed by the ion-Weibel instability. The required laser energy of the order of kJ will be achievable on soon-available facilities such as ELI in Czech Republic, LFEX in Japan and PETAL in France. This study could also provide information regarding to the open questions related to the magnetic field generation or amplification by means of Weibel instability in astrophysical conditions.

Particular care has been taken in the identification of the experimental configuration most relevant to the astrophysical scenario, i.e. dominated by the ion-Weibel instability. We have identified the optimal experimental configuration to limit the development of competing instabilities, as linear S-polarized laser irradiating the target at large-angle oblique incidence. This configuration allows for the mitigation of a surface instability, i.e. the electron-Weibel instability at the laser-plasma interaction surface. We have observed the correlation of the suppression of the surface instability with the generation of a strong transverse current and associated magnetic field in the skin layer, which we identified as following from the transverse momentum absorption at oblique incidence.

The identification of the optimal laser-plasma configuration has a much broader interest than the collisionless astrophysics investigated in this thesis, e.g. it is relevant to Hole Boring and Radiation Pressure Acceleration experiments. In particular, using UHI laser pulses, only in this interaction configuration can the Hole Boring acceleration process operate over a long time.

With our scheme, we can investigate the ion-Weibel instability driven by electron-ion flows with densities and flow velocities different from those currently studied on high-energy (low intensity) laser systems, such as OMEGA, NIF or LMJ. Having two complementary approaches can also be useful for the validation of the numerical simulations and the theoretical models in different regimes. In both cases, an experimental assessment of the early stage of shock formation will be extremely helpful to extract information regarding the shock magnetization level and finally the particle acceleration efficiency with application to Cosmic Rays physics.

7.2 Future perspectives

This PhD work focused on collisionless laboratory astrophysics, and in particular, to the production of Weibel-mediated collisionless shocks using laser-produced plasma flows. The exploration of the fundamental microphysics behind collisionless shock formation and magnetic field amplification is one of today's hot topics. Indeed, it is at the center of a *Discovery Science* program² which aims at reproducing astrophysically relevant plasmas on the high-energy, nanosecond, laser system NIF (Lawrence Livermore National Laboratory, CA) in the US. At the French level, reduced experiments have been carried out on laser facilities at LULI and an experimental campaign at LMJ has been designed (even though it was not granted laser time at this date).

Clearly, the production of this type of collisionless shocks would be

 $^{^{2}}$ goo.gl/v2XYmt

an experimental breakthrough. A first step in this direction was the possibility to drive (and diagnose) the early phase of ion-Weibel instability in counter-streaming plasmas (Fox et al., 2013; Huntington et al., 2015). PIC simulations of counter-streaming flows suggest that the stage of shock formation might be achievable using high-energy facilities. These simulations however relied on artificially reduced electron-to-ion mass ratio in order to perform the simulation up to shock formation with a reasonable amount of computer resources. The recent work performed by Ruyer et al. (2016) has however shown that the generalization to the realistic mass ratio is not straightforward, and may strongly delay the time of shock formation. As a result, centimeter-scale plasma flows evolving on several nanoseconds need to be considered. This places NIF as the best, if not unique, facility on which collisionless shock formation may be obtained. At the same time, this opens questions about the possibility to maintain the collisionless regime over such a long interaction duration. A recent study has been started at the CEA DAM Ile de France to investigate this issue (Gremillet et al., 2017).

Let us note also that the use of external magnetic fields in experiments planned at NIF or LMJ is currently being considered. To our knowledge, most configurations envisioned today consider a perpendicular orientation of the external magnetic field. Our work on the Weibel instability suggests that using a longitudinal, guiding magnetic field could help maintain a higher expanding plasma density, hence shortening the time of shock formation without impacting much the strength of the Weibel-generated magnetic field, required to mediate shock formation.

In this work, we also investigated a complementary approach that, instead of relying on high-energy NIF-LMJ class laser systems, makes use of UHI picosecond laser facilities. Our study proved that probing the ion-Weibel instability might be possible on soon-available multi-kJ, sub-picosecond UHI facilities. Achieving this regime would already be a tremendous experimental achievement, and is proved to rely on a good control of the laser-plasma interaction physics in order to reduce electron heating and prevent electron-driven instabilities to govern the overall plasma dynamics. To reach shock formation would however require longer interaction times (of the order of few tens of picoseconds). The required laser energy would be beyond what will be achievable in a near future, in particular due to the use of oblique incidence that put stringent constraints on the laser focal spot size. It is our understanding that only a deeper knowledge of the surface instability, taking place during the initial stage of the laser-driven Hole-Boring, can help us alleviate these constraints.

A more detailed investigation of this surface instability has been started over the last few months. This study is of extreme importance not only for laboratory astrophysics, but for laser-plasma interaction in general. Indeed, both electron heating and kinetic effects at the laser-plasma interaction surface have proved to have great impact on, for instance, the relativistic self-induced transparency (Siminos et al., 2012, 2016). These studies have however been conducted in a 1D geometry and do not account for the effect of the surface instability. Our ongoing work on the surface instability is therefore relevant to various applications, from ion acceleration to fast-ignition.

Let us note also that, in the scenario proposed here, the effect of particle collisions should be more carefully addressed. As a first attempt, the use of Radiation-Pressure-driven flows, allowing to reach much higher flow velocities (×30 with respect to what is achievable at NIF) with a flow density increase of the order of ×50 the density achievable at NIF, would suggest that the collisionality of our system is much less than that investigated at NIF, due to the scaling of the collision mean-free-path $\lambda_{mfp} \propto v^4/n$. However, a recent work by Bhadoria et al. (2017) demonstrated that collisions might play a non negligible role in Radiation-Pressure-driven Hole-Boring. These collisions arise from the interplay between the ions and the cold and dense electron return current. While this return current should be strongly suppressed in the scenario envisioned in this work (hot electron production is strongly reduced in our laser-plasma interaction configuration), a more careful study of particle collisions under the conditions presented here should be considered.

The possibility to produce stable counter-propagating pair plasma flows, in future experimental facilities, opens the Laboratory Astrophysics to a wide range of new studies, with many astrophysical implications. In these future experimental investigations our measurements of the shock formation time could be tested and important information regarding open questions highlighted in our study could be answered. For instance, the maximum energy reached by the particles accelerated at the shock front, that still represents a challenging numerical task, could be investigated as a function of the magnetization and compared with the astrophysical observations.

Throughout this thesis, beyond the theoretical and analytical efforts done, extensive use of massively parallel, high-performance simulations has been made. Our simulation capabilities have strongly relied on the use and development of the open-source SMILEI PIC code.

An on-going effort has allowed to open SMILEI to the relativistic astrophysics community. Furthermore developments are still carried on, in particular, aiming at providing the user community with additional filtering techniques (see Godfrey and Vay, 2014) and advanced physics modules, such as a moving particle injector, Monte-Carlo routines to account for strong field QED processes, etc.

However, one of the major weaknesses of the present available PIC codes, for simulations of interest for this work, is that electron-ion collisionless shocks are still computationally challenging to reproduce, due to the large difference of temporal scale for the two populations. In our simulations, with the available computing power, we could reach the phase of shock formation from the early stage of the instability only using an artificially reduced ion-to-electron mass ratio. As previously stated, the reliability of this type of studies is still a matter of intense numerical and theoretical investigation. One possible solution relies on the recent development of high-performance hybrid codes, which treat the ions kinetically, while electrons are assumed to be a neutralizing fluid. The hybrid method has allowed the investigation of larger and longer simulations with respect to the full PIC approach (Gargaté and Spitkovsky, 2012; Caprioli and Spitkovsky, 2014a). The shock propagation has been followed up to the formation of the accelerated particle non-thermal spectrum and up to the development of secondary instabilities driven by the accelerated particles themselves. Indeed, a collaboration initiated in France over the last three years (the MACH project³) aims at providing the community

³goo.gl/isbsvG

with a massively parallel MHD code coupled to a PIC treatment of the kinetic species.

However, while these hybrid tools are most interesting to investigate the late stage of the shock evolution, they are not able to capture the physics at the laser-plasma interaction surface, where, as demonstrated in this work, kinetic effects on the electron dynamics play a fundamental role. Therefore, the planning of experimental campaigns should mostly rely on massively parallel PIC simulations. In that sense, the strong effort made to provide the scientific community with an HPC-relevant, multi-purpose, and reliable PIC code is still necessary.

Communications related with this thesis

Parts of this work have been presented to the scientific community. In the following there is a list of the publications in peer-reviewed journals and a list of oral presentations during conferences and seminars.

List of publications

[6] I. Plotnikov, A.Grassi, M. Grech, *Perpendicular relativistic shocks in magnetized pair plasma*, ArXiv 1712.02883, submitted.

[5] A. Grassi, M. Grech, F. Amiranoff, A. Macchi, and C. Riconda, *Radiation Pressure Driven Ion Weibel Instability and Collisionless Shocks*, Phys. Rev. E, **96**, 033204 (2017)

[4] J. Dérouillat, A. Beck, F. Pérez, T. Vinci, M. Chiaramello, A. Grassi, M. Flé, G. Bouchard, I. Plotnikov, N. Aunais, J. Dargent, C. Riconda and M. Grech, *SMILEI: a collaborative, open-source, multi-purpose particle-in-cell code for plasma simulation*, Comput. Phys. Commun. 222,351-373 (2017)

[3] A. Grassi, M. Grech, F. Amiranoff, F. Pegoraro, A. Macchi and C. Riconda, *Electron Weibel Instability in Relativistic Counter-Streaming Plasmas with Flow-Aligned External Magnetic Fields*, Phys. Rev. E, 95, 023203 (2017)

 [2] A. Grassi, L. Fedeli, A. Sgattoni and A. Macchi, *Vlasov simulation* of laser-driven shock acceleration and ion turbulence, Plasma Phys. Control. Fusion 58, 034021 (2016)

[1] A. Grassi, L. Fedeli, A. Macchi, S.V. Bulanov and F. Pegoraro, *Phase space dynamics after the breaking of a relativistic Langmuir wave in a thermal plasma*, The European Physical Journal D **68**(6):178, (2014)

List of presentations

[1] May 2017– *PLAS@PAR Young Researcher's day*, Paris, France. (Oral Presentation)

[2] Mar 2017– *8ème Forum Laser et Plasma*, Aussois, France. (Poster Presentation)

[3] Dec 2016– Seminar at Institut Jean Lamour, Université de Lorraine, Nancy, France. (Oral Presentation)

[4] Nov 2016– Seminar at SLAC National Accelerator Laboratory, Menlo Park CA, USA. (Oral Presentation)

[5] Nov 2016– 58th Annual Meeting of the APS Division of Plasma Physics, San Jose CA, USA. (Poster Presentation)

[6] Oct 2016– 1st Chalmers Workshop on Laser Plasma Theory, Gothenburg, Sweden. (Invited Presentation)

[7] Sept 2016– *PRACE Autumn School "Modern HPC Development for Scientists and Engineers"*, Hagenberg, Austria. (Poster Presentation)

[8] May 2016– 11th International Conference on High Energy Density Laboratory Astrophysics, Stanford CA, USA. (Poster Presentation)

[9] Jan 2016– *Scientific Day of PLAS@PAR*, Paris, France. (Oral Presentation)

 [10] Jun 2015– 42th European Physical Society Conference on Plasma Physics, Lisbon, Portugal. (Poster Presentation)

[11] Jun 2015– 7ème Forum Laser et Plasma, Île de Porquerolles, France.(Poster Presentation)

[12] Aprl 2015- Seminar at the Centre Lasers Intenses et Applications, Bordeaux, France. (Oral Presentation)

[13] Oct 2015– *Collisionless shock workshop*, Paris, France. (Oral Presentation)

[14] Sept 2015– 100th National Congress of the Italian Physical Society, Pisa, Italy. (Oral Presentation) Chapter 7. Publications and presentations related with this thesis

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