



Optimal transition to clean technologies

Baris Nevzat Vardar

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To my family, my teachers and my friends

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Abstract

This dissertation investigates the economic mechanisms underlying the transition to clean technologies and examines policy approaches to achieve the socially optimal path. It studies various policy measures aiming to deal with climate change, such as adaptation and taxation of non-renewable resources. Furthermore, it examines the policy instruments that target increasing the use of efficient technologies and identifies cases in which the policy reaches its objectives or not. It also analyzes the role of heterogeneity in society on agents' willingness to support a pollution tax.

The first chapter studies the energy transition by using an optimal growth model in which non-renewable and renewable natural resources are imperfect substitutes in providing energy services necessary for production. We characterize the social optimum and show that the economy converges to the clean state in the long run. In a general equilibrium framework, we show that the decentralized economy converges to the same state even if there is no regulation, but with higher damages to the environment. We then investigate the properties of taxation trajectory that induces the laissez-faire economy to follow the optimal path, and show that it can be either increasing or decreasing depending on the initial and final states of the economy. If the renewable resources have the capacity to provide all energy services in the long run by themselves and there is economic growth ahead, then the optimal tax is initially set low and it increases over time. In contrary, if the renewable resources are not able to substitute non-renewables in many activities and there is a need for degrowth, then it is set high in the beginning and it decreases over time.

The second chapter studies the role of adaptation policy on the transition to a low-carbon economy. It incorporates adaptation policy into the problem of optimal non-renewable resource extraction with pollution externalities, by focusing on the capital nature of adaptation measures. We characterize the social optimum with general functional forms under economically reasonable assumptions. Due to adaptation policy, a possibility of a simultaneous resource use regime -a transition- arises within a model of two perfect substitute resources. The optimal transition path depends on the initial levels of pollution and adaptation capital, and can follow different sequences of non-renewable and renewable resource use regimes. We identify the conditions that distinguish the optimal path and explore their economic significance. Then we examine the properties of optimal path for different combinations of initial pollution and adaptation. Finally, we emphasize the role of durability of adaptation measures on the optimum, which is represented by the depreciation rate of adaptation capital.

The third chapter focuses on the problem of adopting new technologies in a micro-economic framework. It studies the behavior of firms when they face a decision to invest either in a cheap but inefficient production capacity or in an

expensive but efficient one, by taking into account the presence of a financial constraint. We present a two-period dynamic game between two firms and show that the equilibrium behavior is to invest in a mixture of both types of capacity. Furthermore, under duopoly competition, we show the existence of a symmetric equilibrium and two asymmetric equilibria with preemption. Accordingly, in the equilibrium outcome, we may have either identical firms in terms of size and technology mix, or a preempting firm being inefficient and large and its opponent being efficient and small. We study different policy instruments aiming to increase the use of efficient technologies and show the conditions under which the instruments are successful or not.

The fourth and last chapter investigates the distributional impacts of a pollution tax by considering a society in which wealth is distributed heterogeneously among households. We present a static general equilibrium model in which firms produce with dirty and/or clean technologies, and show novel results on the effect of a pollution tax on factor prices. When dirty technologies are more capital intensive, pollution tax leads to a reallocation of production factors towards cleaner technology, changing the factor prices in favor of workers. As a result, richer people in the society, who own a larger share of capital, lose a higher proportion of their income compared to the low income households. Consequently, the loss in their well-being due to the fall of income outweighs the benefits of a better environment, and their support for a pollution tax declines. These results propose a theoretical explanation for the question of why the rich may prefer a low pollution tax.

Résumé

Cette thèse étudie les mécanismes économiques concernant la transition vers des technologies propres et examine les approches politiques pour atteindre le sentier de transition socialement optimal. Elle examine les politiques économiques visant à faire face au changement climatique, telles que l'adaptation et la taxation des ressources non-renouvelables. En outre, elle examine les politiques économiques visant à accroître l'utilisation de technologies efficaces et identifie les cas pour lesquels la politique atteint ses objectifs ou non. Elle analyse également l'impact des inégalités de richesse sur le soutien politique aux taxes environnementales.

Le premier chapitre étudie la transition énergétique en utilisant un modèle de croissance optimal dans lequel les ressources non-renouvelables et renouvelables sont des substituts imparfaits. Nous caractérisons l'optimum social et montrons que l'économie converge vers l'état stationnaire propre sur le long terme. Dans un cadre d'équilibre général, une économie décentralisée converge vers le même état bien qu'il n'y ait pas de régulation, mais avec plus de dommages environnementales. Ensuite, nous étudions les propriétés de la trajectoire d'imposition qui amène l'économie du "laissez-faire" à suivre le sentier optimal. Par cette analyse, il se révèle que la trajectoire peut être toujours croissante ou toujours décroissante selon les états initiaux et finaux de l'économie. Si les ressources renouvelables ont la capacité de fournir tous les services d'énergie à long terme par eux-mêmes, et si la croissance économique à long terme est assurée, alors la taxe optimale est plus faible au début et elle augmente au fil du temps. En revanche, si elles ne disposent pas de capacité de faire fonctionner toutes les activités et il y a un besoin de décroissance, la taxe est plus élevée au début et diminue ensuite au fil du temps.

Le deuxième chapitre étudie le rôle de la politique d'adaptation sur la transition vers une économie propre. Il intègre la politique d'adaptation dans le problème de l'extraction optimale des ressources non-renouvelables avec des externalités de pollution, en mettant l'accent sur la politique d'adaptation en étant une variable de stock. Nous caractérisons l'optimum social avec des fonctions générales, accompagnées d'hypothèses économiquement raisonnables. En raison de la politique d'adaptation, la possibilité d'exploiter simultanément les ressources renouvelables et non-renouvelables -une transition- s'avère possible. Le sentier de transition optimale dépend du niveau initial de la pollution et celui du capital d'adaptation, et peut suivre différentes séquences de régimes d'exploitation des ressources non-renouvelables et renouvelables. Nous identifions les conditions qui caractérisent le sentier optimal et proposons des explications économiques. Ensuite, nous examinons comment le sentier optimal change en fonction de la pollution initiale et d'adaptation. Enfin, nous soulignons le rôle de la durabilité des mesures d'adaptation sur l'optimum, qui est représenté par le taux d'amortissement du capital d'adaptation.

Le troisième chapitre se concentre sur le problème de l'adoption des nouvelles technologies dans un cadre micro-économique. Il regarde le comportement des

entreprises qui font face à une décision d'investir : soit dans une capacité de production bon marché mais inefficace, soit dans une capacité plus chère mais efficace, lorsqu'on prend en compte la présence d'une contrainte financière. Nous présentons un jeu dynamique de deux périodes entre deux entreprises et nous montrons que le comportement d'équilibre est d'investir dans une combinaison des deux types de capacités. Dans un cadre de duopole, nous montrons l'existence d'un équilibre symétrique et de deux équilibres asymétriques avec préemption. En conséquence, à l'équilibre, nous pouvons avoir soit des entreprises identiques en termes de taille et un mélange de technologies, soit une entreprise étant inefficace mais grande par préemption et son adversaire étant efficace et petit. Nous étudions les différents instruments politiques visant à inciter l'utilisation des technologies efficaces et donnons les conditions dans lesquelles les instruments ont réussi ou non.

Le quatrième et dernier chapitre examine les effets distributifs d'une taxe sur la pollution en considérant une société dans laquelle la richesse est répartie de manière hétérogène entre les ménages. Nous présentons un modèle d'équilibre général dans lequel la production peut se faire à partir des technologies polluantes et/ou des technologies plus propres. Nous montrons de nouveaux résultats par rapport à la littérature concernant l'effet d'une taxe sur la pollution sur les prix des facteurs. Lorsque les technologies polluantes sont plus intensives en capital, une taxe sur la pollution conduit à une réallocation des facteurs de production vers les technologies propres, en changeant les prix des facteurs en faveur des travailleurs. Les plus riches de la société, qui possèdent une plus grande part du capital, perdent une plus grande partie de leur revenu par rapport aux ménages à faible revenu. Par conséquent, la perte de leur bien-être en raison de la baisse des revenus l'emporte sur les avantages d'une meilleure qualité environnementale. Ces résultats proposent une explication théorique à la question de savoir pourquoi les riches peuvent préférer une taxe sur la pollution plus faible.

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Chapter 1

Introduction

This dissertation investigates the economic principles underlying the transition to clean technologies and examines policy approaches to achieve the socially optimal trajectory. It studies various policy measures to deal with climate change, such as adaptation and taxation of non-renewable natural resources. Moreover, it examines the policy instruments that target to incentivize using efficient technologies and identifies the conditions under which the policy is successful or not. It also focuses on the issue of heterogeneity among households in terms of their wealth and analyzes its implications on agents' willingness to support a pollution tax.

It contributes the areas of research on environmental economics, energy and natural resource economics and economics of technological change by using the theoretical tools of natural resource economics, macroeconomics, microeconomics and industrial organization.

Motivation and objectives

The recent scientific evidence shows the consequences of the greenhouse effect on the Earth's temperature. The emphasis is put on the role of carbon dioxide and humanity's part in contributing its accumulation in

the atmosphere. Accordingly, emissions of CO₂ from fossil fuel combustion and industrial processes contributed about 78% of the total greenhouse gas emissions (IPCC, 2014). The global temperature increases as these gasses accumulate in the atmosphere. In a related manner, the adverse effects of climate change on social welfare, economic performance and ecosystem have been a main research focus in recent decades (Nordhaus, 1994; Stern, 2007). The consensus is the necessity of a proactive policy action that applies necessary measures to prevent climate change and enables resilience to it.

The main approaches to deal with climate change are mitigation policy and adaptation policy. These two approaches differ in their main objectives. Mitigation policy aims to reduce the emission of greenhouse gasses by incentivizing utilization of cleaner and more efficient technologies. Taxation of non-renewable resources, increasing the costs of polluting and inefficient technologies, and subsidizing the use of renewable resources and efficient technologies are some examples of mitigation policy instruments.

Adaptation policy aims to reduce the adverse effects of climate change by improving the infrastructure and investing in specific capital. Some examples of adaptation measures include adapting buildings to extreme weather events, building flood defenses, raising the levels of dykes and using scarce water resources more efficiently; which mostly require large investments in capital and infrastructure.

The common point of both approaches is their dynamic nature, as it requires time to adopt new technologies and change the way of production, resource utilization and infrastructure. Therefore, a transition process is necessary to reach their objectives. In this dissertation, we analyze both policy approaches by focusing on their roles on the transition to a low-carbon economy.

Transition from an economy that is based on the utilization of non-renewable resources towards an economy that is based on renewable resources (the

energy transition), green growth, adoption of new and cleaner technologies and the policy challenges related to those subjects are currently at the center of ongoing academic debates (Acemoglu et al., 2012, 2016; Golosov et al., 2014; van der Ploeg and Withagen, 2012a, 2014).

Furthermore, adaptation policy has recently become a heavily discussed subject as a measure to deal with climate change. The growing literature on this subject studies the relationship between adaptation and mitigation policies in the long run (Bréchet et al. (2013)), the effect of catastrophic risks (Zemel (2015)), the strategic effects (Buob and Stephan (2011) and Bréchet et al. (2016)) and the optimal use of adaptation with carbon capture and storage (Moreaux and Withagen (2015)).

There are many open question that are still under discussion within these ongoing debates. What is the optimal transition path to clean technologies when we consider imperfect substitution between natural resources, and what are the consequences of not regulating the transition? What is the optimal time profile of taxes on non-renewable resources to induce the decentralized economy to follow the socially optimal transition path? What is the role of adaptation policy on the transition? Should we pollute more and adapt more, or the reverse? What are the consequences of market structure on the behavior of firms when they face a decision to adopt more efficient technologies, and do they bring any policy issues to take into account? And what is the role of heterogeneity in society on the agents' willingness to support an environmental policy? These questions are the main focus of study in this dissertation.

The objective of this dissertation is to investigate the economic mechanisms underlying the optimal transition, to analyze the two main policy approaches in detail and to contribute these debates by investigating them within a framework of transition to clean technologies. The chapters present results from different perspectives that include macro- and micro-economic frameworks as well as consideration of heterogeneity within the

society. The following section presents the frameworks used in these analysis.

Methodology and framework

The framework used in the first two chapters is the model of optimal resource extraction with pollution externalities. It addresses the problem of how to extract a scarce resource when its consumption brings benefit but also generates pollution which in turn inflicts damages to society. It uses the methods of optimal control theory. The foundation of this framework goes back to the infamous work of Hotelling (1931) which presents a simple and powerful framework to analyze optimal extraction of a non-renewable resource over time. It has been studied and extended broadly by many economists within almost a century. Some important contributions are on the theory of exhaustible resources (Dasgupta and Heal, 1974; Solow, 1974; Stiglitz, 1974), on the role of damages of pollution generated by non-renewable resource use (Withagen (1994)), on the role of natural decay rate of pollution (Tahvonen (1997)), and on the relationship between growth, transition to renewable resources and optimal carbon tax (van der Ploeg and Withagen (2014)).

The first two chapters take this framework as a benchmark to study the optimal transition path to renewable resources, the optimal taxation of non-renewable resources which is the main mitigation policy instrument, and the role of adaptation policy on the transition by taking into account capital nature of adaptation measures. The first chapter focuses on the optimum and general equilibrium by also implementing the tools of optimal growth theory, whereas the second chapter focuses only on the optimum of the resource extraction problem with pollution externalities. As the framework is defined in a general manner, the theoretical analysis yields results on the economic mechanisms underlying the socially optimal transition and the optimal trajectories of policy instruments.

The last two chapters depart from this framework to focus on the different dimensions of the issue. The third chapter uses the tools of industrial organization theory to address the problem of technology adoption under imperfect competition. Some pioneering works that use these tools are Reinganum (1981) and Fudenberg and Tirole (1985), which address similar issues such as a game theoretical approach to diffusion of new technologies and preemption in technology adoption. These tools use the dynamic game theory as methodology and they enable to study the behavior of firms when they face a technology adoption decision in a market structure that is imperfectly competitive. As one of the main objectives of mitigation policy is to incentivize the utilization of more efficient technologies, this framework brings results on the impacts of these policy instruments at the micro-economic level by studying the equilibrium firm behavior.

The fourth chapter uses the framework developed in international trade theory which is known as Heckscher-Ohlin theory. It originally addresses the problem of specialization and factor allocation between two countries. This framework has been applied to many other issues such as the incidence of the corporation income tax (Harberger (1962)), the relationship between trade, growth and environment (Copeland and Taylor (2004)) and the incidence of environmental taxes (Fullerton and Heutel (2007)). Applying this framework to the problem of factor allocation between clean and dirty technologies allows to analyze the effect of a pollution tax on factor prices. In a general equilibrium framework that includes households, firms, and the government, identifying these effects in turn yields results on the distributional impacts of a pollution tax in a society that consists of heterogeneity among households.

This dissertation uses these theoretical frameworks as benchmarks to address its research questions, and it presents novel results on the economic mechanisms and on the policy analysis. The next section briefly goes through each chapter and presents their contributions.

Contributions

The first chapter, which is a revised version of (Vardar, 2014), studies the economic mechanisms underlying the energy transition by using a dynamic general equilibrium framework. It takes into account the imperfect substitutability of non-renewable and renewable resources in providing the energy services necessary for production. Imperfect substitution between resources arises due to technical and geographical constraints, as some resources require certain conditions to operate and they cannot be used in certain economic activities. Imperfect substitution was considered by Michielsen (2014) in a partial equilibrium framework with two-periods. The present work departs from this study by considering a general equilibrium framework with an infinite time horizon. The benchmark model used in this chapter is the one studied by van der Ploeg and Withagen (2014), which investigates the relationship between growth and transition to renewable resources, as well as the path of optimal carbon tax. The problem of the social planner is to decide on the paths of resource utilization, consumption and investment in production capital when the utilization of scarce non-renewable resources generates pollution that irreversibly accumulates in the atmosphere. We present novel results on the transition path to renewable resource, on the trajectory of the optimal tax rate on fossil fuels, and on the roles of renewable resources and the degree of substitution by taking into account the imperfect substitution between resources.

We characterize the social optimum and decentralized equilibrium of the economy that consists of households, resource extraction firms and final good producing firms. We show that, due to the scarcity of non-renewable resources, the economy will eventually reach a clean state in which it uses only renewable resources. This is the case for both the social optimum and the decentralized equilibrium. However, considering the damages caused by pollution accumulation, what matters for welfare is the speed at which the economy reaches the clean state. As the firms in a decentralized equilibrium

do not internalize the negative externality of pollution, they extract more and pollute more compared to the socially optimal levels, therefore the transition takes longer and ends up with higher damages without policy intervention. A tax on the use of non-renewable resources can correct this market failure and induce the decentralized equilibrium to follow the socially optimal transition path.

Furthermore, we study the optimal trajectory of a carbon tax which is considered to be the main mitigation policy instrument to deal with climate change. We show that the optimal carbon tax can be increasing or decreasing over time depending on the initial state of the economy as well as the properties of the final green state that it will eventually reach. This means that the level of current economic development, the cost efficiency of renewable resources and the degree of substitution between non-renewable and renewable resources play an important role on the determination of the optimal time profile of a policy action. If the renewable resources have the capacity to provide all energy services in the long run by themselves and there is economic growth ahead, then the optimal tax is initially set low and it increases over time. In contrary, if they are not able substitute non-renewable resources in many activities and there is a need for degrowth, then it is set high in the beginning and it decreases over time. The first chapter presents results that contribute the line of research by Ulph and Ulph (1994), van der Ploeg and Withagen (2012a), Golosov et al. (2014) and van der Ploeg and Withagen (2014).

The second chapter (Bréchet and Vardar, 2016) incorporates adaptation policy into the problem of optimal non-renewable resource extraction with pollution externalities. It focuses on the capital nature of adaptation measures, as these measures mostly require large investments in infrastructure and specific capital. The damage of pollution can be reduced by increasing the stock of adaptation capital. This is done by investment in adaptation capital. These investments has an increasing opportunity cost to society

which raises due to allocating efforts to adaptation policies rather than other uses that would provide benefit. We take into account the presence of a renewable resource that is a perfect substitute to the non-renewable one, and study the optimal trajectories of investment in adaptation and resource use. The benchmark framework we use is the one studied in detail and extended by Withagen (1994), Tahvonen (1997), van der Ploeg and Withagen (2012a). We include adaptation policy as a stock variable in that framework by using the vulnerability function that has been introduced by Bréchet et al. (2013). As the way we consume fossil fuels (oil, coal, natural gas, etc.) directly relates to the adaptation policy, making the link between resource extraction and adaptation brings novel results on the economic mechanism underlying adaptation policy and its role on the transition to a low-carbon economy.

We study the optimal transition path with general functional forms under economically reasonable assumptions. As adaptation policy affects the marginal damage of pollution, the driver formula for the marginal cost of non-renewable resource (the modified Hotelling rule) significantly departs from the model without adaptation. The results show that, due to adaptation policy, a possibility of a simultaneous resource use regime -a transition- arises within a framework of two perfect substitute resources. This is because of the fact that adaptation policy reduces the marginal damage of pollution which in turn drags the total marginal cost of the non-renewable resource, making the non-renewable resource still beneficial to extract even though it is as costly as the renewable resource. The damage of pollution that is generated by resource extraction is compensated by increasing the stock of adaptation capital. However, this compensation mechanism cannot go on forever because investment in adaptation has an increasing opportunity cost. For a given level of pollution, there exists an efficient level of adaptation capital that equalizes the marginal cost of maintaining that level to the marginal benefit gained by reduction in damage of pollution. Above this level, it is no more beneficial to adapt more and extract

more. Showing this trade-off is a new result in the literature.

We characterize the optimum and show the existence of steady state. At the steady state, the rent of resource is equalized to its marginal damage and the stock of adaptation capital maintained at its level efficiently. We then identify different regions of the behavior of the optimal path depending on the initial states of pollution and adaptation capital. The optimal path can follow different sequences of resource use regimes which are only non-renewable use, simultaneous use of resources and only renewable use. For example, if the initial levels pollution and adaptation are low, then it is optimal to use only non-renewable resource and increase adaptation capital by large investments in the beginning. At a certain date, the cost of non-renewable reaches that of renewable and a gradual transition starts. Following a path of resource extraction on which the marginal damage is compensated by increased adaptation, the optimal path reaches the steady state and stays there indefinitely by using only renewable resource. In other initial situations such as high pollution and low adaptation, it is never optimal to extract the non-renewable resource. Only renewable resource will be used and adaptation capital will be adjusted to its efficient level.

We also put emphasis on the role of the depreciation rate of adaptation capital, which can differ due to the nature of capital and damages, on the efficient level of adaptation as well as on the transition to the renewable resource. When adaptation investments are oriented towards to less durable capital (which means a high depreciation rate), the benefits of adaptation are going to be limited due to high cost of maintenance. On the contrary, more durable adaptation investments decreases its cost which in turn leads to a longer transition to the renewable resource and larger amount of resource extraction. These theoretical findings are contributions to the literature that studies optimal extraction of non-renewable resources and adaptation policy measures such as Withagen (1994), Tahvonen (1997), van der Ploeg and Withagen (2012a), Bréchet et al. (2013), Zemel (2015)

and Moreaux and Withagen (2015).

The third chapter (Fagart and Vardar, 2016) studies the adoption of a new production tool technology in an imperfectly competitive market. Production tool innovation (new machines, robots, vehicles etc.) differs from process innovation (efficient management techniques, new recipes for processing inputs etc.), as firms have to purchase and install new production capacity that embody the new-efficient technology. We investigate the behavior of firms when they face a decision to invest either in a cheap but inefficient production capacity or in an expensive but efficient one, by taking into account imperfect competition between firms and the presence of a financial constraint in the investment opportunities of firms. The issue of technology adoption is a heavily studied subject on the literature of industrial organization and environmental economics, and our work contributes to these literature by studying production tool innovation under imperfect competition.

We present a dynamic game theoretical model of capacity investment between two firms that includes two periods. Firms compete in the level of production capacity *à la Cournot*. We show that, when firms have no interest to delay investment (meaning not financially constrained) they would invest as soon as possible. This investments are only in the capacity using the most efficient technology, and they invest in the quantities that are optimal for given market parameters. However, when firms are financially constrained, investing in the technology with the cheapest purchasing price allows firms to grow faster in the short run. Accordingly, firms may wish to invest in this inefficient technology in order to increase their short run profits, even though it reduces their future profits by increasing their production costs. The inefficient technology also generates a strategic effect: it allows one firm to preempt its opponent, building more capacity in the short run and reducing the future investment incentives of its opponent. Showing these effects are new results in the literature on technology adop-

tion.

We characterize the equilibrium strategies and show that there may exist symmetric and asymmetric equilibria when the firms are financially constrained. In the symmetric equilibrium, firms invest in a mixture of two technologies that maximize their discounted total profits. Each firm has the same mixture of efficient and inefficient technology as its competitor, and the same market share. An important result is that the total quantity of production does not change when firms have access to larger credits, an increase in financial endowment instead induces firms to produce the same quantity but with more efficient capacity. There may also exist other equilibria that are asymmetric, in which one of the firms overinvests in the inefficient capacity in the first period. This allows the firm to increase its total capacity in the short run and commit itself to a larger production in the long run. The opponent reacts to this preemption by investing less in the first period, focusing on the efficient capacity. The preempting firm finishes with a larger market share, producing mostly with the inefficient technology whereas the preempted firm stays smaller but more efficient.

When we compare the outcomes of monopoly and duopoly cases, we see that firms in a duopoly competition use more old capacity compared to a monopoly when there is a financial constraint. Moreover, we show that some policy instruments such as increasing the investment price of inefficient technologies may not yield the desired outcomes. Indeed, when the price of inefficient capacity increases, firms have to decrease their total capacity since they are financially constrained. To avoid a too large reduction of their total capacity, they decrease their investment in efficient capacity to purchase more of the inefficient one. Hence, an increase in the purchasing price of inefficient capacity may lead to an increase in its quantity used. Other policy instruments work as expected, increasing the marginal cost of inefficient technology (for example a pollution tax), and decreasing the price of efficient technologies incentive firms to increase their investments

in efficient capacity. These results contribute some strands of literature in industrial organization, operation research, energy economics and corporate investment (Fazzari et al., 1988; Feichtinger et al., 2008; Fudenberg and Tirole, 1985; Gaimon, 1989; Meunier, 2010; Milliou and Petrakis, 2011; Reinganum, 1981).

The fourth chapter (Arabzadeh and Vardar, 2015) investigates the policy challenges in implementation of a pollution tax within a framework that uses the modeling approach of international trade theory. We study the distributional impacts of a pollution tax by considering a society in which the wealth of households is heterogeneously distributed. The framework we use is a static general equilibrium model that includes households, firms, and the government. We study an economy in which firms produce a generic good by using two different technologies, dirty and clean, with each of them using capital, labor and pollution as an input to produce the final output. This way of modeling production structure is called as joint production technology and it has been used in some works such as Copeland and Taylor (2004), Fullerton and Heutel (2007). In this framework, the factor prices of capital and labor are determined endogenously in the equilibrium, for a given level of pollution tax that is determined by the government. The household revenues depend on the factor prices. Therefore, in the general equilibrium, the effect of a pollution tax on household income is going to vary when the factor endowments are distributed heterogeneously among households.

The results show that when the dirty technologies are more capital intensive, a pollution tax leads to a reallocation of production factors towards cleaner technology, changing the factor prices in favor of workers. This is because the reallocation of factors in more labor intensive technologies decreases the productivity of capital, thus the interest rate always declines with the pollution tax. But what happens to wage depends on two effects. On the one hand, the increase in cost of pollution pushes the wage down-

ward because a higher pollution tax induces a decline in factor productivity. On the other hand, factor reallocation towards the labor intensive technology pushes the wage upward since labor productivity rises. We show that the dominating effect depends on the comparison of the relative intensities of pollution and capital between the two production technologies. In particular, we show that the wage rate increases when the relative pollution intensity respect to capital is higher in the dirty technology, and *vice versa*. These results differ from many studies in the literature that study the effects of a pollution tax on factor prices (Copeland and Taylor (2004), Fullerton and Heutel (2007), Dissou and Siddiqui (2014)).

On the household side, we investigate the household's decision about its preferred pollution tax and we show the trade-off that they face between a higher consumption and a better environmental quality. We identify the two opposing effects that determine the role of wealth on households' willingness to support a pollution tax. The first one is the satiation effect, which says that the households with high wealth consume more and their marginal utility of consumption is low, thus they would be more willing to sacrifice from their consumption for a better environmental quality. The second channel is the income burden effect, which says that the households with high wealth have large capital investments in the market, thus, when the return of capital falls their revenues are going to be reduced more by the pollution tax compared to low-income households. Accordingly, whether the pollution tax increases or decreases with wealth depends on which one of these effects dominates. We show that, in fact, it depends on the pollution tax elasticity of consumption that is determined by the pollution tax elasticities of the factor prices.

In the general equilibrium, we show that the richer people in the society who own a larger share of capital lose a higher proportion of their income compared to the low income households. Consequently, the loss in their well-being due to the fall of income outweighs the benefits of a better envi-

ronment, and their support for a pollution tax declines. These results propose a theoretical explanation for the question of why the rich may prefer a low pollution tax, and they contribute the line of research by Copeland and Taylor (2004), Fullerton and Heutel (2007), Chiroleu-Assouline and Fodha (2014) and Dissou and Siddiqui (2014).

Chapter 2

Optimal energy transition and taxation of non-renewable resources

BARIS VARDAR

2.1 Introduction

Energy transition refers to the process in which renewable resources are substituted for non-renewables over time and the economy eventually reaches a green, no-polluting state. Due to the dynamic nature of the problem, what matters for energy transition is not only the current level of policy measures but also their planned time paths. When policies are suboptimal, in terms of their levels and paths, the speed of transition can be too slow or too rapid. The consequence of a too slow transition is that the environment will be damaged more than the socially optimal level. Similarly, when the transition is too rapid, the substitution costs -which depend on the degree of substitution between resources- will offset the environmental benefits. These issues raise the importance of studying the optimal policy measures for a decentralized economy that goes through the course of

energy transition.

What is the optimal transition path to clean technologies and what are the consequences of not regulating the transition? What is the optimal time profile of taxes on non-renewable resources to induce the decentralized economy to follow the socially optimal transition path? Should we tax low at the beginning and tighten it over time so that we let the economy develop while giving the firms increasingly higher incentive to substitute the renewables? Or should we tax very high and loosen it over time so that we initially stimulate the use of renewables? In this study, we address these classical questions by first characterizing the optimal transition path to clean technologies and identifying its determinants. Second, we characterize the optimal path of taxation on the non-renewable resources and we identify the main channels that it depends on by comparing the social optimum with laissez-faire equilibrium. As a novel approach, we take into account the imperfect substitution between non-renewable and renewable resources in a general equilibrium setting and we analyze the role of the degree of substitution between them.

The debate on the optimal taxation path of non-renewable resources was pioneered by Sinclair (1994) and Ulph and Ulph (1994). Both studies argue that the time profile of taxation is all that matters for regulating non-renewable resource extraction. On the one hand, Sinclair claims that the ad-valorem tax rate has to be initially set to a high value and should fall over time to postpone current extractions and to smooth consumption of non-renewables. On the other hand, Ulph and Ulph show cases where the tax rate should first rise and then fall. Other studies such as Chakravorty et al. (2006) pointed out that the time path of optimal carbon tax is inverted-U shaped when we consider an admissible carbon stabilization cap. Recently, van der Ploeg and Withagen (2014) identify the conditions under which the optimal tax rate rises or decreases by establishing four different regimes of energy use depending on the initial stocks of oil and capital. They also

discuss the role of a renewable resource which is a perfect substitute of oil. Golosov et al. (2014) also contribute the debate by stating that the optimal tax rate should be proportional to output and they show that whether the optimal tax rises or falls depends on the output growth rate and the increase rate of non-renewable resource price.¹ But these studies do not focus on the course of gradual and smooth transition to a clean economy as well as the presence of the renewable resources that are used simultaneously with non-renewables at all times.²

The framework in this work is an optimal growth model with capital and energy services as factors of production, which is similar to the one in van der Ploeg and Withagen (2014). We consider an economy that consists of households, final good producing firms and resource extraction firms. The firms use capital and energy services to produce the final good. Energy services are provided by non-renewable and renewable resources, which are imperfect substitutes. The marginal cost of renewable resource is exogenously given and constant while the shadow price of non-renewable resource is endogenously determined. The extraction cost of non-renewable resource increases as the firms extract more of it (à la Heal (1976)). Moreover, utilization of non-renewable resource damages the environment in an irreversible way.³

The results show that it is always optimal to use both non-renewable and renewable resources simultaneously. As these resources are imperfect substitutes, there are situations in which a resource is rational to use even

¹Under some specific assumptions on the utility, production and damage functions and on the accumulation dynamics of capital and pollution.

²For example, van der Ploeg and Withagen (2014), by considering non-renewable and renewable resources as perfect substitutes, show transition towards a cleaner economy with simultaneous use of resources but under specific conditions. Golosov et al. (2014) consider many non-renewable and renewable resources being used simultaneously and in their numerical example they use a CES-type of energy production, but they do not investigate the optimal transition path and the role of renewable resources.

³See Tahvonen and Withagen (1996) and Toman and Withagen (2000) for a comparison of policy outcomes with reversible and irreversible pollution accumulation dynamics.

though it is the expensive one.^{4,5} Furthermore, as extraction cost increases over time, it is optimal to reduce the share of non-renewables and substitute renewables in production. When the utilization of non-renewable resource approaches zero, the optimal path converges to a steady state in which only the renewable resource is used. An important result is that the decentralized economy converges to the same steady state in terms of capital and consumption whether there is a regulator intervention or not. What matters for welfare, however, is the speed at which the economy approaches the clean state - the energy transition. The optimal energy transition depends on the level of environmental damages that the society is willing to accept in the long run.

Even though the profit-motivated firms do not internalize the environmental damages of non-renewable resource extraction in the laissez-faire economy, the equilibrium path also converges to the clean state in the long run. On this path, households consume more and firms extract the non-renewable resource more rapidly, thus damage the environment more and faster compared to the optimal transition path. The energy transition is slower than the optimal one in the absence of regulation. The policy maker can correct this market failure by taxing non-renewable resource extraction.

The optimal tax rate is equal to the present value of all future marginal damages caused by one unit of non-renewable resource extraction. Its formula includes the endogenous net rental rate of capital, the marginal damages to the environment and the marginal utility of consumption. Accordingly, the optimal tax rate depends on the endogenous variables such as the capital stock, consumption, cumulative extraction and the shadow

⁴Imperfect substitution captures the technical or geographical constraints in substitution possibilities. For instance, some resources require specific geographical properties and the firms cannot use them even though they are the cheapest. Similarly, in some industries it may take time to adopt new technologies, hence the cheaper resource cannot be used immediately.

⁵This is consistent with the historical data on resource use. See Mattusch (2008) which shows that non-renewable resource use for energy services dates back to 371 and 287 BC and until industrial revolution nearly all energy services were provided by renewable resources.

price of non-renewable resource, as well as the exogenous factors such as the marginal cost of renewable resource and the degree of substitution between non-renewable and renewable resources. We investigate how these different factors affect the optimal tax rate. For example, a larger capital stock leads to a lower interest rate -hence a higher value of the future- which in turn makes the environmental quality more valuable and thus raises the optimal tax rate. A larger capital stock also gives the firms more incentive for extraction to fuel a larger economy that has to be corrected by a higher tax rate. Similarly, a higher level of consumption will lead to a lower marginal utility of consumption, more satiated households will care more about the environmental damages and thus the optimal tax rate will rise.

The time profile of optimal taxation depends on the initial state of the economy that is defined by the level of capital stock, the level of cumulative non-renewable resource extraction and the level of pollution, and it is either always increasing or always decreasing.⁶ This is consistent with the recent studies such as van der Ploeg and Withagen (2014), Golosov et al. (2014). Moreover, we emphasize another point that is not shown in the literature: the optimal taxation path also depends on the properties of the final (clean) state that the optimal path converges which is determined by the marginal cost of renewables and the degree of substitution between non-renewable and renewable resources.

In the final state of the economy, when only renewable resources are used, the production suffers from the technological characteristics that make non-renewable and renewable resources imperfect substitutes. These technological characteristics are the technical constraints, the geographical constraints and the differences in the opportunity costs of using renewable

⁶Despite that this result is similar to Farzin and Tahvonen (1996), the mechanism leading to it is different. Farzin and Tahvonen consider a depreciating carbon stock in the atmosphere together with irreversible carbon accumulation which leads to different taxation profiles. In the present paper, however, we only consider that non-renewable resource extraction damages the environment in an irreversible way but we may have different taxation profiles according to different initial and final levels of capital stock and pollution.

resources in production. A question arises at this point: are the renewable resources capable of sustaining today's level of economic activities by themselves? On the one hand, if renewable resources are good substitutes for non-renewables then economic growth can keep on, the capital stock and consumption can increase over time as it is considered by many studies. In this case the optimal tax rate has an increasing time profile. On the other hand, if renewable resources are not good substitutes for non-renewables there will be a need for degrowth - meaning that we may need to reduce the level of capital stock, production and consumption over time to sustain a clean economy in the long run. In this case, the optimal tax rate is initially set to a very high level and it decreases over time.⁷

The renewable resource plays a crucial role on both the level and the time profile of optimal taxation. Expensive renewable resources increase the incentive of firms to extract more non-renewables. Therefore, the optimal tax rate rises and its time profile shifts up when the cost of renewable resource becomes higher. The role of the degree of substitution between non-renewable and renewable resources depends on the time period. A strong degree of substitution makes the economy to benefit from the cheaper resource by allocating it in high proportions, and when the shadow price of non-renewable resource exceeds that of renewable, a rapid substitution of the renewable resource takes place.⁸ Consequently, when the degree of substitution is high, the optimal tax rate is initially set to a high value and its time profile remains higher in the short and medium run but lower in the long run compared to the case of low degree of substitution.

The results of present work differentiates from the ones in van der Ploeg and Withagen (2014) in several aspects. Consideration of imperfect substi-

⁷A similar comparison can be made for two countries with different capital stocks. For the country with a capital stock that is lower than its long run (clean state) value, the optimal tax is initially low and increasing, hence not creating a burden for growth while stimulating energy transition over time. However, for the country which accumulated capital higher than its long term value, hence over-producing and over-consuming, the optimal tax is initially set to a high value and decreases over time.

⁸As in the Herfindahl (1967) principle.

tution reduces the optimal path to include only a regime of simultaneous resource use. This eliminates the cases in which there is an instantaneous switch to the renewable resource. The convergence to a clean state and using non-renewable resources (even though in small quantities) in the long run is a realistic outcome, and it allows us to compare the optimal path with the path of transition without regulation. Moreover, it allows us to emphasize the roles of renewable resources and the degree of substitution between resources on the optimal transition path as well as on the trajectory of the optimal taxation path. Accordingly, as the cost and substitutability of renewable resource determine the final state, they have significant effect on the paths of resource use and regulation. In addition, Michielsen (2014) also considered imperfect substitution between resources in a partial equilibrium framework with two-periods, and studied the issues of Green Paradox and spatial carbon leakage. The present work departs from this study by considering a general equilibrium framework with an infinite time horizon and focusing on the issue of transition to a clean economy.

This work also relates to the wide literature on the optimal taxation of non-renewable resources as well as on the transition to a renewable resource. Early studies such as Withagen (1994) showed that the socially optimal extraction path consumes less than the laissez-faire path. Hoel and Kverndokk (1996) considered increasing extraction costs and pollution with natural absorption and showed that the tax rate should first rise then fall. Farzin (1996) and Farzin and Tahvonen (1996) showed the taxation path may either be arbitrary, rising or first falling then rising over time. In the last decade, studies such as Goulder and Mathai (2000), Schou (2002), van der Zwaan et al. (2002), Grimaud and Rougé (2005), Groth and Schou (2007), Grimaud and Rougé (2008), Lafforgue et al. (2008) and Grimaud et al. (2011) investigated the roles of technical progress, directed technical change, energy substitution, carbon sinks, innovation, learning-by-doing and endogenous growth. Belgodere (2009) emphasized that the time path of optimal tax may differ and the replacement of renewables

may change the outcome dramatically. There were also other approaches such as Groom et al. (2005) on the role of discount rate and Daubanes and Grimaud (2010) on the role of international heterogeneities. More recently, Aghion et al. (2012) stated that increasing taxes are needed to allow the clean technologies to overtake the dirty ones. van der Ploeg and Withagen (2012a,b, 2013) and van der Ploeg (2014) studied the relationship between taxes, backstop technology and the Green Paradox. Rezai et al. (2012) presented a comparison of the results on taxation with additive and multiplicative damages and Gaudet and Lasserre (2013) provided an analytical overview of the different types of taxes on non-renewable resources.

The remainder of the chapter is structured as follows: Section 2 introduces the model framework, preferences and technology. Section 3 characterizes the social optimum. Section 4 presents the decentralized framework, characterizes the equilibrium path and establishes the optimal taxation rule. Section 5 presents the results of the simulations and Section 6 concludes.

2.2 The model

Time is continuous and infinite. There is an infinitely-lived representative household that gains utility by consuming the final good. The economy has two sectors: final good production and resource extraction. In the final good production sector, capital is used with non-renewable and renewable resources which are imperfect substitutes. The extraction of the non-renewable resource damages the environment in an irreversible way which in turn reduces the total welfare.

The instantaneous total welfare ($V(\cdot)$) consists of the household's utility from consumption ($U(\cdot)$) and non-renewable resource extraction's damage to the environment ($D(\cdot)$). We consider the following additively separable form:

$$V(C, Z) = U(C) - D(Z)$$

where C denotes consumption of the final good and Z denotes cumulative extraction of the non-renewable resource ($Z_t = Z_0 + \int_{s=0}^t E_{ds} ds$ with E_{dt} is the instantaneous quantity of extraction). The utility of consumption is increasing and strictly concave in C ($U_C(C) > 0$ and $U_{CC}(C) < 0$) and the damage of cumulative extraction is increasing and strictly convex in Z ($D_Z(Z) > 0$ and $D_{ZZ}(Z) > 0$)⁹.

Production of the final good requires capital (K) and energy (E). Energy is obtained from non-renewable (dirty) (E_d) and renewable (clean) (E_c) resources which are imperfect substitutes. The function $H(\cdot)$ captures the imperfect substitution and it is in CES form, $H(E_d, E_c) = (\gamma E_d^{1-\frac{1}{\epsilon}} + (1 - \gamma) E_c^{1-\frac{1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}}$. In this specification, $\gamma \in (0, 1)$ is a structural parameter that depends on the shares of non-renewable and renewable resources in ability to provide energy services. A value of γ higher than 0.5 would mean that non-renewable resources have a better ability in providing energy services with the current technology compared to that of renewables. The parameter ϵ denotes the degree of substitution; as we consider imperfect substitution it requires to assume $\epsilon > 1$.

The use of CES functional specification is an adequate way to capture imperfect substitution between resources because it leads to different marginal productivities for resources. For example, making a unit investment in renewables by installing a new solar panel in a region where there is not enough sunshine and making a unit investment in non-renewables by building a new well in a resource-rich region will not have the same effect on energy gain as well as on aggregate production. The function $H(\cdot)$ allows us to capture these differences. In CES specification, these technological characteristics are embodied in the technological share parameter (γ) and the degree of substitution parameter (ϵ).

⁹In the rest of the text the subscript for a function denotes its derivative respect to a variable or argument of the function. For example, $f_1(\cdot)$ denotes the derivative of function f respect to its first argument and $f_x(\cdot)$ denotes the derivative of function f respect to the variable x .

The production function $F(\cdot)$ is Cobb-Douglas, $F(K, E) = K^\alpha E^\beta$ where $\alpha > 0$ and $\beta > 0$ are the output elasticities of capital and energy in production respectively, and $\alpha + \beta \leq 1$. Embedding the different types of resources for energy services in the production function leads to the following form:

$$F(K, H(E_d, E_c)) = K^\alpha ((\gamma E_d^{1-\frac{1}{\epsilon}} + (1-\gamma) E_c^{1-\frac{1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}})^\beta$$

The extraction of non-renewable resource is costly. This cost depends on the level of cumulative extraction (Z). This approach, which was adopted by Heal (1976), Hoel and Kverndokk (1996), d'Autume (2012) and many others before, emphasizes that non-renewable resources are not limited by the nature but there are economic limitations in the long run. As more of the resource is extracted, the average extraction cost will increase over time. This cost consists of both direct and indirect effects of cumulative extraction such as searching costs for new resources and technical innovation expenditures to harvest deeper deposits. $G(Z)$ denotes the “average extraction cost” and we assume that it is increasing and strictly convex in Z ($G_Z(Z) > 0$ and $G_{ZZ}(Z) > 0$).¹⁰

2.3 Social optimum

The social planner solves the following problem:

$$\max_{\{C_t, E_{dt}, E_{ct}\}} \int_{t=0}^{\infty} e^{-\rho t} (U(C_t) - D(Z_t)) dt$$

$$\dot{K}_t = F(K_t, H(E_{dt}, E_{ct})) - G(Z_t)E_{dt} - \pi_c E_{ct} - C_t \quad (2.1)$$

$$\dot{Z}_t = E_{dt} \quad (2.2)$$

$$C_t, E_{dt}, E_{ct} \geq 0 \quad \forall t$$

with $K_0 > 0$ and $Z_0 > 0$ are given.

¹⁰Throughout the text we refer $G(Z)$ as “extraction cost” to keep the text simple.

where the marginal cost of renewable resource is exogenously given and denoted as π_c . The current-value Hamiltonian function associated to this problem is:

$$\mathcal{H}_t^{SO} = U(C_t) - D(Z_t) + \lambda_t (F(K_t, H(E_{dt}, E_{ct})) - G(Z_t)E_{dt} - \pi_c E_{ct} - C_t) - \mu_t E_{dt}$$

where λ_t denotes the co-state variable associated to capital and is interpreted as the shadow value of capital. Similarly, μ_t denotes the co-state variable associated to cumulative extraction.

The necessary conditions for an optimum are:

$$U_C(C_t) = \lambda_t \tag{2.3}$$

$$E_{dt} \geq 0, \quad \lambda_t (F_2(K_t, H(E_{dt}, E_{ct}))H_{E_d}(E_{dt}, E_{ct}) - G(Z_t)) - \mu_t \leq 0 \tag{2.4}$$

$$E_{ct} \geq 0, \quad F_2(K_t, H(E_{dt}, E_{ct}))H_{E_c}(E_{dt}, E_{ct}) - \pi_c \leq 0 \tag{2.5}$$

$$\dot{\lambda}_t = (\rho - F_1(K_t, H(E_{dt}, E_{ct}))) \lambda_t \tag{2.6}$$

$$\dot{\mu}_t = \rho \mu_t - \lambda_t G_Z(Z_t)E_{dt} - D_Z(Z_t) \tag{2.7}$$

$$\lim_{t \rightarrow +\infty} e^{-\rho t} \lambda_t = 0 \tag{2.8}$$

$$\lim_{t \rightarrow +\infty} e^{-\rho t} \mu_t = 0 \tag{2.9}$$

together with the equations (2.1) and (2.2). In order to proceed on resolution, we define the shadow price of non-renewable resource as follows:

$$\pi_{dt} = G(Z_t) + \mu_t / \lambda_t \tag{2.10}$$

This definition states that the shadow price of non-renewable resource is endogenously determined by the average extraction cost and the shadow value of non-renewable resource in capital units, in other words, the rent of resource. By taking the time derivative of (2.10) and using (2.2), (2.3),

(2.6) and (2.7), we obtain the law of motion of π_d given by

$$\dot{\pi}_{dt} = F_1(K_t, H(E_{dt}, E_{ct})) (\pi_{dt} - G(Z_t)) - D_Z(Z_t)/U_C(C_t). \quad (2.11)$$

Equation (2.11) is the modified Hotelling rule. Using (2.10) we can rewrite condition (2.4) as

$$E_{dt} \geq 0, \quad F_2(K_t, H(E_{dt}, E_{ct})) H_{E_d}(E_{dt}, E_{ct}) - \pi_{dt} \leq 0. \quad (2.4')$$

Conditions (2.4') and (2.5) are complementary slackness (c.s.) conditions and they show that a type of resource will be used if its marginal productivity is equal to its marginal cost. As non-renewable and renewable resources are imperfect substitutes, on the optimal path both of them is always used simultaneously, that is, the equalities in conditions (2.4') and (2.5) always hold. This property allows us to solve $F_2(\cdot)H_{E_d}(\cdot) = \pi_d$ and $F_2(\cdot)H_{E_c}(\cdot) = \pi_c$ and obtain the optimal quantities of non-renewable and renewable resources ($E_{dt}^*(K_t, \pi_{dt}, \pi_c)$ and $E_{ct}^*(K_t, \pi_{dt}, \pi_c)$). Using these results together with conditions (2.1 – 2.11) we find the differential equation system in $(K_t, Z_t, C_t, \pi_{dt})$. Optimal trajectories should satisfy the following:

$$\dot{K}_t = F(K_t, H(E_{dt}^*(\cdot), E_{ct}^*(\cdot))) - G(Z_t)E_{dt}^*(\cdot) - \pi_c E_{ct}^*(\cdot) - C_t \quad (2.12)$$

$$\dot{Z}_t = E_{dt}^*(K_t, \pi_{dt}, \pi_c) \quad (2.13)$$

$$\dot{C}_t/C_t = \sigma(F_1(K_t, H(E_{dt}^*(\cdot), E_{ct}^*(\cdot))) - \rho) \quad (2.14)$$

$$\dot{\pi}_{dt} = F_1(K_t, H(E_{dt}^*(\cdot), E_{ct}^*(\cdot))) (\pi_{dt} - G(Z_t)) - D_Z(Z_t)/U_C(C_t) \quad (2.15)$$

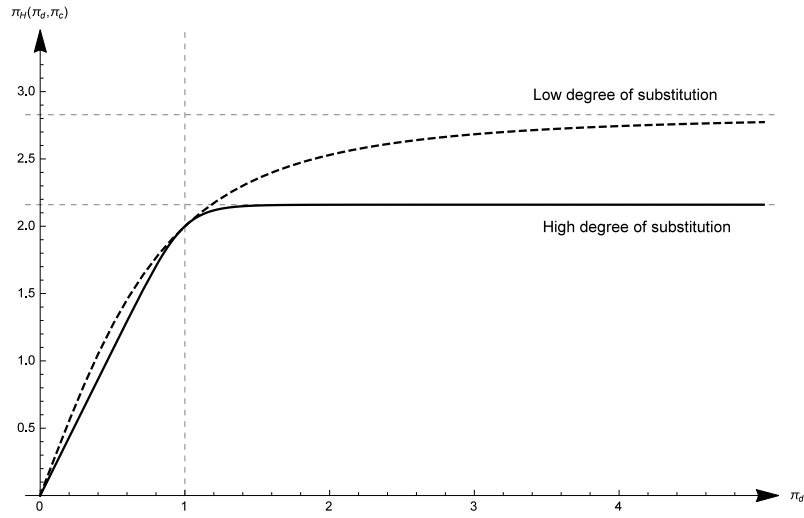
As the shadow price of non-renewable resource increases, the economy reduces its share in energy services, and it eventually converges to a regime in which only the renewable resource is used. To find the state that the optimal path converges, we need to compute the marginal productivity of energy services as the optimal resource allocation and its path depends on it. In the optimum, it reduces to $F_2(K_t, H(E_{dt}^*(\cdot), E_{ct}^*(\cdot))) = (\gamma^\epsilon \pi_{dt}^{1-\epsilon} + (1-\gamma)^\epsilon \pi_c^{1-\epsilon})^{\frac{1}{1-\epsilon}}$ which leads us to define the *energy price index*

as follows:

Definition 2.1. Let π_H be the energy price index given by:

$$\pi_{Ht}(\pi_{dt}, \pi_c) = (\gamma^\epsilon \pi_{dt}^{1-\epsilon} + (1-\gamma)^\epsilon \pi_c^{1-\epsilon})^{\frac{1}{1-\epsilon}}$$

The energy price index has a limit for a given marginal cost of renewable. As the shadow price of non-renewable increases it tends to a constant, $\lim_{\pi_{dt} \rightarrow +\infty} \pi_{Ht}(\pi_{dt}, \pi_c) = \pi_c(1-\gamma)^{\frac{\epsilon}{1-\epsilon}}$, that is illustrated in fig. 2.1.



Note: For $\pi_c = 1$, $\gamma = 0.5$, high degree of substitution $\epsilon = 10$ and low degree of substitution $\epsilon = 3$.

Figure 2.1 – The energy price index with respect to the shadow price of non-renewable resource

Fig. 2.1 shows that lower degree of substitution results as a higher value of the energy price index for any given non-renewable resource cost (with the exception of the case when the two marginal costs are equal). Energy price index is the marginal productivity of energy services, by definition, and the marginal productivity of energy services is decreasing in their utilization. Therefore, a lower degree of substitution results as lower utilization of energy services compared to the high degree of substitution case.

In addition, the closer the energy price index to its limit, the lower the utilization of non-renewable resources. As can be seen from the figure, in

the case of high degree of substitution the energy price index approaches its limit right after the cost of non-renewables exceeds that of renewables. This means that renewable resources are substituted as quickly as possible when they became the cheaper resource, thanks to the high substitution possibility. In contrary, lower degree of substitution case shows that the cost of non-renewables has to increase much more to have the energy price index get closer to its limit, meaning that much more of non-renewable resource would be used even if they have very high cost compared to the renewables.

In the following we will investigate the limit case and investigate the state that the optimal path converges in the long run. The economy that simultaneously uses the non-renewable and renewable resources will asymptotically converge to the regime in which it uses only the renewable resource with the following conditions:

$$\lim_{t \rightarrow +\infty} \dot{\pi}_{dt} = 0 \quad (2.16)$$

$$\lim_{t \rightarrow +\infty} \pi_{Ht}(\pi_{dt}, \pi_c) = \pi_c(1 - \gamma)^{\frac{\epsilon}{1-\epsilon}} \quad (2.17)$$

$$\lim_{t \rightarrow +\infty} C^S(K_t, Z_t) - C^R(K_t) = 0 \quad (2.18)$$

Condition (2.16) is derived from the definition of shadow price of non-renewable resource (2.10). It means that the shadow value of non-renewable resource must approach zero, thus the resource rent must vanish over time. Condition (2.17) ensures that the energy price index approaches its limit, thus the non-renewable utilization approaches zero. In condition (2.18), $C^S(.)$ and $C^R(.)$ denote the optimal consumption as a function of the state variables in simultaneous use regime and only renewable use regime respectively. This condition is to ensure that the state (K, Z) and co-state (λ, μ) variables of the optimal control problem cannot jump, thus the trajectories of consumption, capital stock, pollution and resource use will be continuous over time.

In the regime that the optimal path converges, the production function reduces to

$$F(K, H(E_d = 0, E_c)) = \hat{F}(K, E_c) = K^\alpha E_c^\beta (1 - \gamma)^{\frac{\beta\epsilon}{\epsilon-1}}. \quad (2.19)$$

Equation (2.19) contains the features resulting from the consideration of imperfect substitution. Had we considered perfect substitution between resources, this production function would be $\hat{F}(K, E_c) = K^\alpha E_c^\beta$. The additional term $((1 - \gamma)^{\frac{\beta\epsilon}{\epsilon-1}})$ reduces production for given quantities of capital and renewable resource and it reflects the limitation of renewable resources in the absence of utilization of non-renewables.

Using condition (2.5), we obtain the optimal amount of renewable resource use, $E_c^*(K)$. Finally, conditions (2.3, 2.5 and 2.6) allow us to obtain the differential equation system in (K, C) . Optimal trajectories of the asymptotic clean regime should satisfy the following:

$$\dot{K}_t = F(K_t, H(0, E_{ct}^*(K_t))) - \pi_c E_{ct}^*(K_t) - C_t \quad (2.20)$$

$$\dot{C}_t / C_t = \sigma(F_1(K_t, H(0, E_{ct}^*(K_t))) - \rho) \quad (2.21)$$

$$\dot{Z}_t = 0 \quad (2.22)$$

$$\dot{\pi}_{dt} = 0 \quad (2.23)$$

As the extraction of non-renewable resource approaches zero, the dynamics of endogenous variables in equations (2.12 – 2.15) will approach the above differential equation system (2.20 – 2.23). This system has a stationary point (K^{ss}, C^{ss}) which can be obtained by solving the following equations:

$$F_1(K^{ss}, H(0, E_c^*(K^{ss}))) = \rho \quad (2.24)$$

$$C^{ss} = F(K^{ss}, H(0, E_c^*(K^{ss}))) - \pi_c E_c^*(K^{ss}) \quad (2.25)$$

The system given in equations (2.20 – 2.21) has a unique trajectory that leads to the steady state (K^{ss}, C^{ss}) . This unique trajectory allows us to

find the optimal consumption rule in the asymptotic regime, $C^R(K)$, which we referred in condition (2.18). We can see that the steady state depends on the level of production and the marginal productivity of capital when only renewable resource is used. This level highly depends on function $H(0, E_c^*(K^{ss}))$, which is the amount of energy services that can be provided by only renewable resource. If the degree of substitution is low, this amount will decrease which in turn lead to a decline in the steady state level of capital. This decline will modify the optimal path. Moreover, the steady state does not depend on the pollution due to the assumption of additive separability between utility of consumption and damage of pollution. In the contrary case, as the marginal utility of consumption depends on the level of pollution, the steady state levels of capital and consumption changes taking into account the level of pollution. In this analysis, we only focus on the case additively separable utility.

Following proposition states the properties of the optimum.

Proposition 2.1. *For a given level of pollution sufficiently low, in the optimum, the share of the non-renewable resource gradually reduces and the optimal path converges to a regime in which only the renewable is used. There exists a unique optimal path $\{K_t, Z_t, C_t, \pi_{dt}\}_{t=0}^{\infty}$ starting from any initial state $K_0 > 0, Z_0 > 0$ that follows the dynamics in equations (2.12-2.15) and satisfies conditions (2.16-2.18). This path converges to the steady state (K^{ss}, C^{ss}) given in (2.24, 2.25).*

Proof. See Appendix A1. □

According to Proposition 1, the optimal path of non-renewable and renewable resource use is $\{E_{dt}^*(K_t, \pi_{dt}, \pi_c), E_{ct}^*(K_t, \pi_{dt}, \pi_c)\}_{t=0}^{\infty}$. Consequently, the optimal path of renewable use converges to $\{E_c^*(K_t)\}_{t=0}^{\infty}$.

2.4 Equilibrium analysis

Let us turn to the equilibrium analysis. We assume that there are large number of final good producing firms and resource extraction firms which produce with the same technology and there is perfect competition in all markets. We first investigate the optimal behavior of agents and then characterize the equilibrium. This will allow us to study the effects of taxes on the decentralized economy as well as to obtain the optimal taxation rule that leads to the socially optimal transition path.

2.4.1 Household

The representative household solves:

$$\begin{aligned}
 & \max_{\{C_t\}} \int_{t=0}^{\infty} e^{-\rho t} (U(C_t)) dt \\
 & s.t. \quad \dot{K}_t = r_t K_t + \Pi_t + T_t - C_t \\
 & \quad C_t \geq 0 \quad \forall t \\
 & \quad \text{with } K_0 > 0 \text{ is given.}
 \end{aligned} \tag{2.26}$$

where r_t is the net rental rate of capital, Π_t profits from the ownership of final good producing firms and resource extraction firms. T_t denotes the government transfers and it is equal to the total tax revenues.

The current-value Hamiltonian function associated to this problem is given by:

$$\mathcal{H}_t^H = U(C_t) + \lambda_t (r_t K_t + \Pi_t + T_t - C_t)$$

First order conditions for optimality are:

$$U_C(C_t) = \lambda_t \quad (2.27)$$

$$\dot{\lambda}_t/\lambda_t = \rho - r_t \quad (2.28)$$

$$\lim_{t \rightarrow +\infty} e^{-\rho t} \lambda_t = 0 \quad (2.29)$$

Solving (2.27) and (2.28) gives the following well-known Ramsey rule for consumption:

$$\dot{C}_t/C_t = \sigma(r_t - \rho) \quad (2.30)$$

2.4.2 Final good producing firm

The representative final good producing firm aims to maximize its profits for given input prices. The programme of the firm is:

$$\max_{\{K_t, E_{dt}, E_{ct}\}} F(K_t, H(E_{dt}, E_{ct})) - r_t K_t - \pi_{dt} E_{dt} - \pi_c E_{ct}$$

In the case where the resource prices are positive and finite ($\pi_{dt} \in (0, +\infty)$ and $\pi_c \in (0, +\infty)$) the firm uses both type of resources simultaneously. The first order conditions are:

$$F_1(K_t, H(E_{dt}, E_{ct})) = r_t \quad (2.31)$$

$$F_2(K_t, H(E_{dt}, E_{ct})) H_{E_d}(E_{dt}, E_{ct}) = \pi_{dt} \quad (2.32)$$

$$F_2(K_t, H(E_{dt}, E_{ct})) H_{E_c}(E_{dt}, E_{ct}) = \pi_c \quad (2.33)$$

By solving (2.32) and (2.33) we obtain the optimal amount of non-renewable and renewable resources ($E_{dt}^*(K_t, \pi_{dt}, \pi_c)$ and $E_{ct}^*(K_t, \pi_{dt}, \pi_c)$). As a result, the condition for firm profit maximization reduces to:

$$F_1(K_t, H(E_{dt}^*(.), E_{ct}^*(.))) = r_t \quad (2.34)$$

2.4.3 Non-renewable resource extracting firm

The representative non-renewable resource extracting firm maximizes the discounted value of its intertemporal profits by taking into account the tax rate and the extraction cost which increases by cumulative extraction. We introduce the taxation of non-renewable resource as the amount paid per unit of extraction. This application can be considered as a wedge on non-renewable resource extraction in this decentralized economy. The extraction firm solves the following problem:

$$\begin{aligned}
 \max_{\{E_{dt}\}} & \int_{t=0}^{\infty} e^{-R_t} ((\pi_{dt} - \tau_t) E_{dt} - G(Z_t) E_{dt}) dt \\
 \text{s.t.} & \quad \dot{Z}_t = E_{dt} \\
 & \quad E_{dt} \geq 0 \quad \forall t \\
 & \quad \text{with } Z_0 > 0 \text{ is given.}
 \end{aligned} \tag{2.35}$$

where τ_t denotes the per unit tax rate of the resource at time t and R_t denotes cumulative interest rate as $R_t = \int_0^t r_s ds$. The current-value Hamiltonian function associated to this problem is:

$$\mathcal{H}_t^{EX} = (\pi_{dt} - \tau_t) E_{dt} - G(Z_t) E_{dt} - \mu_t E_{dt}$$

The first order conditions for optimality will be as follows:

$$\pi_{dt} = G(Z_t) + \tau_t + \mu_t \tag{2.36}$$

$$\dot{\mu}_t = r_t \mu_t - G_Z(Z_t) E_{dt} \tag{2.37}$$

$$\lim_{t \rightarrow +\infty} e^{-R_t} \mu_t = 0 \tag{2.38}$$

In equation (2.36), the taxation appears as a driver of the non-renewable resource price together with the extraction cost and resource rent. We solve

(2.36) and (2.37) to obtain the optimal law of motion of π_d :

$$\dot{\pi}_{dt} = \dot{\tau}_t + r_t(\pi_{dt} - G(Z_t) - \tau_t) \quad (2.39)$$

The relationship in equation (2.39) is the Hotelling rule for the market economy. It depends on the net rental rate of capital, extraction cost and taxation. It also shows that the regulator has to determine both the level and the time profile of taxation in order to control the resource extraction. Note that it reduces to the standard Hotelling rule in the absence of taxation and extraction cost.

2.4.4 Equilibrium

The competitive equilibrium definition we consider is as follows:

Definition 2.2. *Given the time profile of taxation $\{\tau_t\}_{t=0}^\infty$, initial capital stock (K_0) and cumulative extraction (Z_0) , the intertemporal competitive equilibrium is such that*

- i the time profiles of consumption $\{C_t\}_{t=0}^\infty$, capital stock $\{K_t\}_{t=0}^\infty$ and net rental rate of capital $\{r_t\}_{t=0}^\infty$ maximize the discounted value of household's intertemporal utility, thus (2.26) and (2.30) hold for each t ,*
- ii the time profiles of capital stock $\{K_t\}_{t=0}^\infty$, non-renewable resource price $\{\pi_{dt}\}_{t=0}^\infty$, net rental rate of capital $\{r_t\}_{t=0}^\infty$, non-renewable resource use $\{E_{dt}\}_{t=0}^\infty$ and renewable resource use $\{E_{ct}\}_{t=0}^\infty$ maximize the final good producing firm's profit at each instant, thus (2.34) holds for each t ,*
- iii the time profiles of net rental rate of capital $\{r_t\}_{t=0}^\infty$, non-renewable resource price $\{\pi_{dt}\}_{t=0}^\infty$, non-renewable resource use $\{E_{dt}\}_{t=0}^\infty$ and cumulative extraction $\{Z_t\}_{t=0}^\infty$ maximize the discounted value of extraction firm's intertemporal profits, thus (2.35) and (2.39) hold for each t .*

Using equations (2.26, 2.30, 2.34, 2.35, 2.39) we obtain the differential equation system in (K, Z, C, π_d) that denotes the law of motion for endogenous variables in the intertemporal equilibrium. This system is given by:

$$\dot{K}_t = F(K_t, H(E_{dt}^*(.), E_{ct}^*(.))) - G(Z_t)E_{dt}^*(.) - \pi_c E_{ct}^*(.) - C_t \quad (2.40)$$

$$\dot{Z}_t = E_{dt}^*(K_t, \pi_{dt}, \pi_c) \quad (2.41)$$

$$\dot{C}_t/C_t = \sigma(F_1(K_t, H(E_{dt}^*(.), E_{ct}^*(.)))) - \rho \quad (2.42)$$

$$\dot{\pi}_{dt} = \dot{\tau}_t + F_1(K_t, H(E_{dt}^*(.), E_{ct}^*(.)))(\pi_{dt} - G(Z_t) - \tau_t) \quad (2.43)$$

Let us first consider the decentralized equilibrium in the absence of taxation. We define the laissez-faire economy as the tax rate on non-renewable resource being zero for all t ($\tau_t = 0 \forall t$).

Proposition 2.2. *In the laissez-faire economy, there exists a unique equilibrium path that is given by $\{K_t, Z_t, C_t, \pi_{dt}\}_{t=0}^\infty$ starting from any initial state $K_0 > 0, Z_0 > 0$ that follows the dynamics in equations (2.40-2.43) and satisfies conditions (2.16-2.18). This path converges to the steady state (K^{ss}, C^{ss}) given in (2.24, 2.25).*

Proof. See Appendix A2. □

As the price of non-renewable resource increases, the firms gradually reduce the share of non-renewable resource and the dynamics of endogenous variables in equations (2.40-2.43) approaches (2.20-2.23). Proposition 2 shows that the final state is not affected by the absence of regulation. Sooner or later the market economy converges to the clean production state as well. However, the speed of transition, which also determines the level of environmental damages, is driven by the regulator intervention.

This is an interesting result concerning the welfare implications of public policy on the energy transition. The market economy rationally responds to the increasing extraction cost of non-renewable resource, therefore the

firms reduce non-renewable resource extraction and allocate more renewables in production over time also on the equilibrium path. On this equilibrium path, as the negative externalities of extraction are not internalized, the households consume more and the firms extract the non-renewable resource faster and thus damage the environment more and faster in the absence of regulation. The regulator can correct this market failure by introducing taxation on non-renewable resource extraction. Even though the final state that the optimal path converges is identical, public policy induces the decentralized economy to the optimal transition path, hence leads to a higher welfare level.

2.4.5 Optimal taxation of non-renewable resource

We characterized the equilibrium path of the decentralized economy in the absence of taxation in Proposition 2. The optimal path of taxation is the one that induces this equilibrium path to be equivalent to the social optimum which we characterized in Proposition 1. In order to obtain the optimal taxation rule, we consider the social optimum given in equations (2.12-2.15) and compare it with the decentralized equilibrium given in equations (2.40-2.43). The first three equations are equivalent in both system of differential equations. The taxation scheme, therefore, is optimal if (2.43) is equivalent to (2.15), that is:

$$\dot{\tau}_t^* + F_1(\cdot)(\pi_{dt} - G(Z_t) - \tau_t^*) = F_1(\cdot)(\pi_{dt} - G(Z_t)) - D_Z(Z_t)/U_C(C_t) \quad (2.44)$$

Rearranging (2.44) gives the following law of motion for the optimal tax rate:

$$\dot{\tau}_t^* = F_1(K_t, H(E_{dt}^*(\cdot), E_{ct}^*(\cdot)))\tau_t^* - D_Z(Z_t)/U_C(C_t) \quad (2.45)$$

Now we can write the optimal tax rate which is given in Proposition 3 as follows:

Proposition 2.3. *The optimal tax rate of non-renewable resource at a*

given time t is:

$$\tau_t^* = \int_t^\infty e^{-\int_t^s F_1(K_u, H(E_{du}^*(K_u, \pi_{du}, \pi_c), E_{cu}^*(K_u, \pi_{du}, \pi_c))) du} D_Z(Z_s) / U_C(C_s) ds \quad (2.46)$$

When the tax profile is $\{\tau_t^*\}_{t=0}^\infty$, there exists a unique equilibrium path which is identical to the optimal path that converges to the steady state (K^{ss}, C^{ss}) given in (2.24, 2.25).

Proof. Equation (2.46) is a direct conclusion of solving equation (2.45). The fact that the equilibrium path is identical to the optimal path is guaranteed by the comparison of (2.12-2.15) and (2.40-2.43) and taking into account (2.44). The rest of the proof follows the same procedure of Proposition 1 in Appendix A1. $\square$

Proposition 3 shows that the optimal tax rate is a forward-looking variable. One unit of non-renewable resource should be taxed at a rate which is equivalent to the present value of all future marginal damages occurred by itself. The determinants of the optimal tax rate are the net rental rate of capital (marginal productivity of capital), the marginal damage to the environment and the marginal utility of consumption. The formula for the optimal taxation on the non-renewable resource (2.46) has several similarities to the well-known literature. It, however, has new components (such as the degree of substitution between resources and the resource prices) that provide useful and new insights about the different channels that affect the optimal taxation.

There are four endogenous channels that affect the optimal tax rate of the non-renewable resource. We call the first channel as the *fulfillment effect*: as the household gets more satisfaction in consumption, she will care more about the environment. The household's marginal utility of consumption falls as the level of consumption rises which in turn increases the household's care for the environment for a given level of cumulative

extraction. As a result, the optimal tax rate of non-renewable resource increases with the level of consumption.

The second channel is the *discounting effect*: changes in the net rental rate of capital alters the interest rate. The net rental rate of capital falls as the stock of capital increases which in turn leads to a lower interest rate. The value of the future becomes higher, thus the care about the environment rises as well. Therefore the optimal tax rate increases with the stock of capital.

One remark about the role of the capital stock on the optimal tax rate is worth to be mentioned. An economy with a larger stock of capital indeed requires a higher amount of energy to fuel the production, thus there will be stronger incentive for the firms to extract more non-renewable resource. To correct this incentive, the regulator should tighten the wedge on extraction and thus increase the tax rate on the non-renewable resource.

The third channel is the *direct environment effect*: more cumulative extraction makes the marginal damage to the environment to be higher. The value of one unit of extraction's marginal damage rises which in turn increases the optimal tax rate. The more the cumulative extraction is, the worse the environmental status is, and therefore the higher the optimal tax rate is.

The fourth channel is the *non-renewable price effect*: an increase in the non-renewable price decreases the net rental rate of capital thus the interest rate falls. The value of future rises due to the lower discounting, which is similar to the discounting effect. Therefore the optimal tax rate increases with the price of non-renewable resource.

These results show that the time profile of optimal tax rate is either always increasing or decreasing depending on the initial state and the final state of the economy. We know that the capital stock and consumption both rise over time if the initial capital stock is less than its steady state value,

or vice versa. Besides, the non-renewable resource price and cumulative extraction are always increasing by definition. Therefore, the optimal tax rate is always increasing over time if the initial capital stock is less than its steady state value. If the initial capital stock is too large (greater than its steady state value), however, the tax rate will have a decreasing time profile. We shall investigate the different time profiles of optimal taxation in the numerical analysis section.

Let us now turn to the effect of the exogenous factors. A strong degree of substitution between non-renewables and renewables allows the economy to benefit from cost differences and allocate the cheaper resource in higher amounts in production. In the optimum, for a given stock of capital and resource costs, the total energy use rises if the degree of substitution is stronger. This leads the marginal productivity of capital to rise and so does the interest rate. As a consequence of the discounting effect, the initial optimal tax rate falls if the degree of substitution is stronger.

Higher renewable price, in contrast, reduces the net rental rate of capital hence the interest rate falls. It also gives incentive to firms to extract a larger amount of the non-renewable resource, hence the wedge on extraction should be tightened. Therefore, again due to the discounting effect, the optimal tax rate increases with the price of renewable resource.

2.5 Numerical analysis

This section illustrates the theoretical results we obtained in the previous sections. The aim is to investigate the differences between the social optimum and the laissez-faire equilibrium paths, the different time profiles of optimal taxation according to the initial state of the economy, the role of the renewable resource and the role of the degree of substitution between non-renewable and renewable resources.

For the analytical specification of the functions, we consider that $U(C) = \frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$ with $\sigma > 0$, $D(Z) = \frac{\phi_d}{2}Z^2$ with $\phi_d > 0$ and $G(Z) = \frac{\phi_g}{2}Z^2$ with $\phi_g > 0$ which satisfy the properties given in Section 2.

2.5.1 Calibration

The elasticity of capital in production is set to $\alpha = 0.2$ and the elasticity of energy services in production to $\beta = 0.1$. We set the technology parameter $\gamma = 0.5$, meaning that non-renewable and renewable resources have same ability in providing energy services. The discount rate is set to $\rho = 0.02$ and the elasticity of intertemporal substitution to $\sigma = 0.5$, hence the relative risk aversion coefficient to be $1/\sigma = 2$. The renewable resource price is $\pi_c = 1$, the parameter of marginal cost of extraction is $\phi_g = 0.1$ and the parameter of marginal damages to the environment is $\phi_d = 0.002$. Finally, the degree of substitution is $\epsilon = 3$ when we consider it low, and $\epsilon = 10$ when we consider it high.¹¹

2.5.2 Social optimum vs. Laissez-faire

We investigate the differences between the social optimum and laissez-faire equilibrium path which are illustrated in fig. 2.2.¹² In the laissez-faire, the damages of extraction are not internalized by the firms and there is no regulator intervention. The firms thus extract a larger amount of the non-renewable resource compared to the social optimum which results in less renewable use over time (fig. 2.2(a,d)). Indeed, the reason of higher amount of extraction is that the price of non-renewable resource is lower and rises slower due to the absence of taxation (fig. 2.2(b,e)). In addition,

¹¹This calibration setting is similar to the one in van der Ploeg and Withagen (2014).

¹²In particular, the solid lines represent the case which is characterized in Proposition 1, and the dashed lines represent a specific case of the equilibrium paths without taxation on the non-renewable resource, which is characterized in Proposition 2.

lower price and higher extraction cost leads to a lower rent of non-renewable resource over time in the laissez-faire case (fig. 2.2(e)).

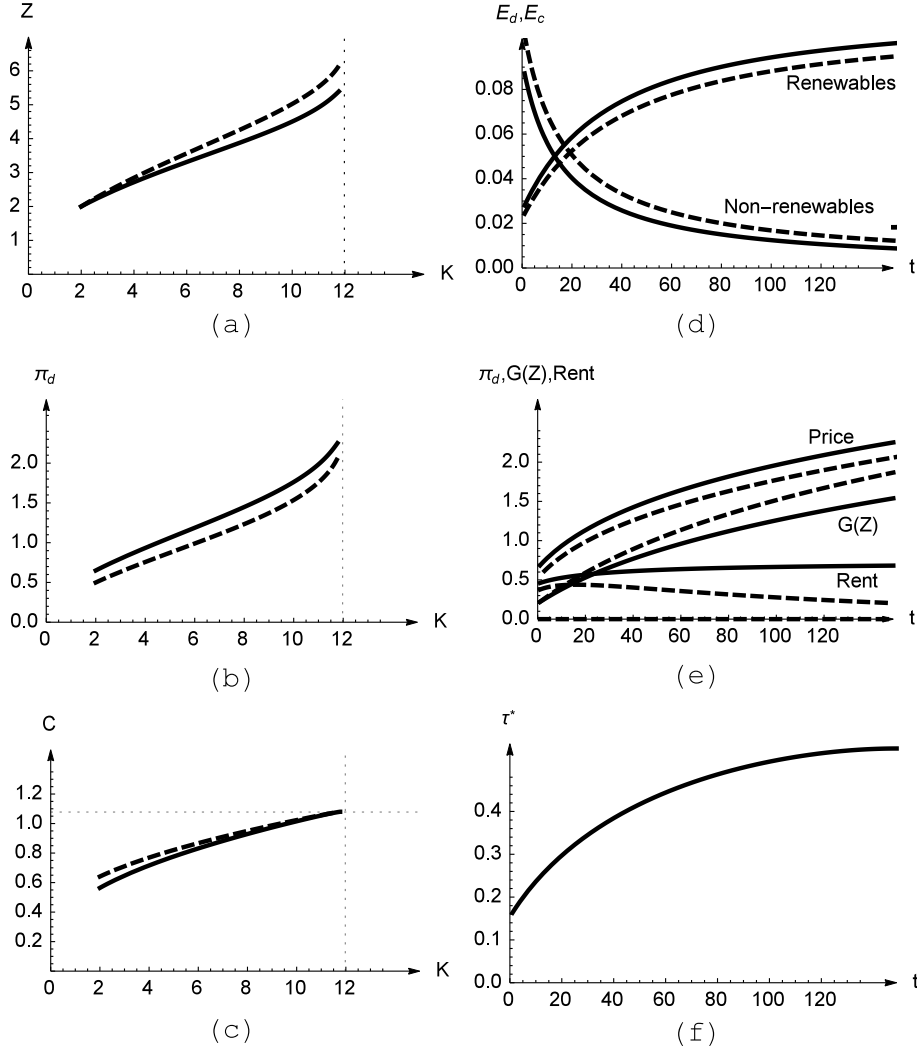


Figure 2.2 – Social optimum (solid lines) vs. Laissez-faire (dashed lines)

Despite the fact that there is no regulation, the market economy converges to the same steady state as the socially optimal path. The consumption is higher in the short and medium run, but converges to the socially optimal level in the long run (fig. 2.2(c)). Notwithstanding larger amount of extraction in the laissez-faire case, the firms reduce the share of non-renewable resource and the market economy also converges to a steady state in which it uses only the renewable resource. The speed of this transition, however, is

slower compared to the socially optimal one (fig. 2.2(a,c,d)). Accordingly, the damages to the environment are higher which leads to a lower level of welfare. This is an illustration of the fact that taxation of non-renewable resources does not affect the final state of the economy. However, it drives the speed at which the decentralized economy approaches the clean state - the energy transition.

2.5.3 Time profile of optimal taxation

The speed of transition to the clean state, which depends on the taxation profile, determines the level of environmental damages hence the welfare outcome. For the specific example of the initial state in fig. 2.2(f), the optimal tax rate monotonically increases and approaches a constant value. But is this the case for all possible initial states? The answer is no. The optimal tax rate can be either increasing or decreasing depending on the initial capital stock and cumulative extraction and also depends on the final state that the optimal path approaches.

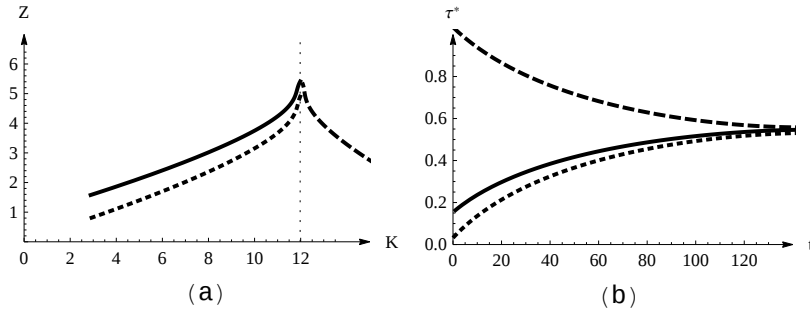


Figure 2.3 – Different time profiles of the optimal tax rate

Fig. 2.3 illustrates some examples of different taxation profiles. The initial state of the economy plays a crucial role on the time profile of the optimal tax rate. In the case where the initial capital stock is lower than its steady state level, the economy is always accumulating capital and increasing consumption. The four (positive) channels, which were introduced in Proposition 4, affect the optimal tax rate and thus it always increases over

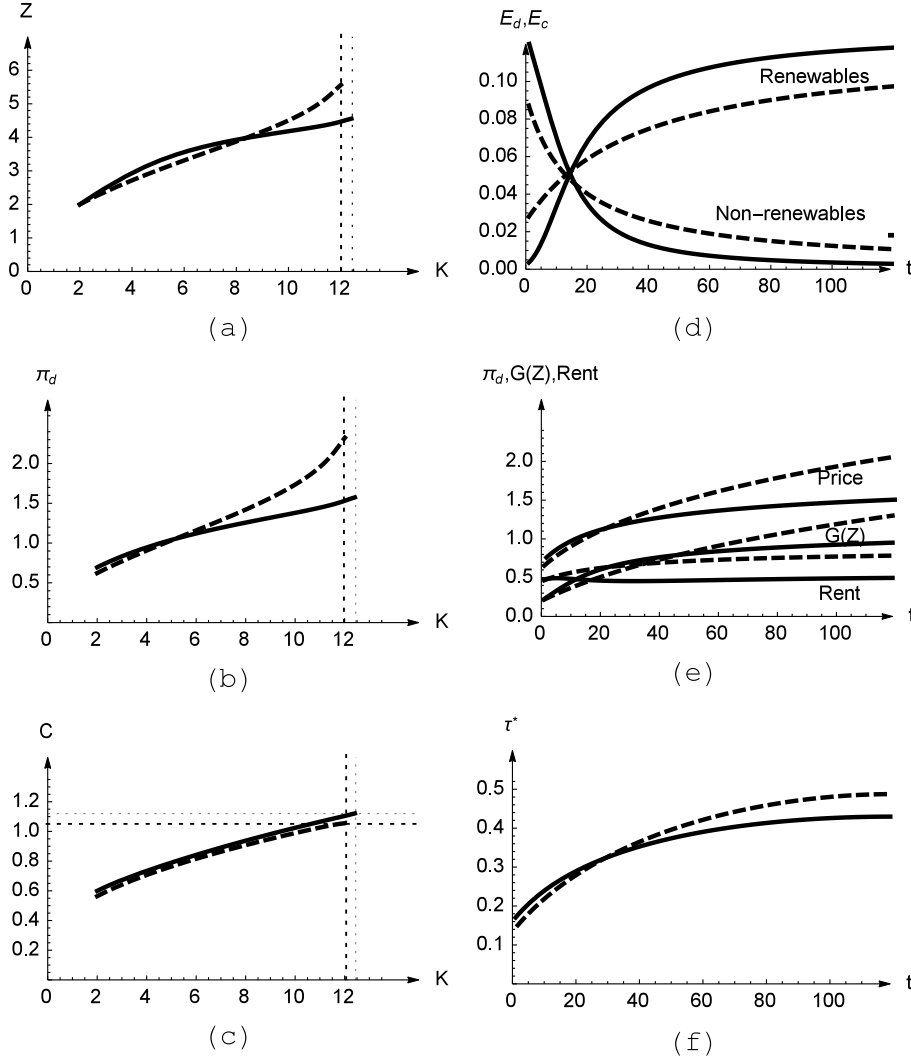
time. The initial optimal tax rate and its time profile rises if the initial capital stock as well as the initial cumulative extraction are larger, and vice versa. In contrast, if the initial capital stock is greater than its steady state value, the households consume the over-accumulated capital which leads to a decreasing capital and consumption over time. The initial consumption level is too high and the initial net rental rate of capital is too low. Thus the optimal tax rate is initially high due to the fulfillment and discounting effects. As the capital stock melts away and consumption decreases, the optimal tax rate also decreases over time. Another point to mention is that the optimal tax rate approaches a unique constant value for any given initial state (as the optimal path converges to the unique steady state in the long run).

2.5.4 The role of the degree of substitution between resources

We investigate the role of the degree of substitution by using fig. 2.4. We depict two cases: strong degree of substitution (the solid lines) and weak degree of substitution (the dashed lines).

When the degree of substitution is strong, the firms extract a greater amount of non-renewable resource until its price reaches the price of renewable, then they use more renewable resource compared to the weak degree of substitution case. Eventually the market economy converges to a steady state with a larger capital stock and higher consumption (fig. 2.4(a,c,d)). The effect on the price of non-renewable resource depends on the time period. In the case of strong degree of substitution, the price of non-renewable is initially set to a higher value and rises more rapidly in the short and medium run. But it rises slower in the long run as the extraction reduces swiftly after the renewable becomes the cheaper resource. Consequently, the non-renewable resource is cheaper in the long run when

the degree of substitution is strong (fig. 2.4(b,e)).



Note: The solid lines represent the strong degree of substitution ($\epsilon = 10$) and the dashed lines represent the weak degree of substitution case ($\epsilon = 3$).

Figure 2.4 – The role of the degree of substitution on optimal taxation

Due to these dynamics, the effect of the degree of substitution on the time profile of optimal taxation also depends on the time period. With strong degree of substitution, greater initial consumption and higher non-renewable price increases the initial optimal tax rate due to the aforementioned channels. These channels dominate the negative effect of low degree of substitution. Accordingly, when the degree of substitution is high, the optimal tax rate is higher in the short and medium run but lower in the long run

(fig. 2.4(f)).

These results underline the differences in the outcome between the cases of perfect and imperfect substitution. Consideration of perfect substitution between non-renewable and renewable resources arises the possibility of different resource use regimes with switches and one including a simultaneous use of resources, as shown by van der Ploeg and Withagen (2014). With imperfect substitution, the optimal path always contains simultaneous use of resources while making a gradual transition to the renewable resource (considering the initial level of pollution being sufficiently low). Furthermore, in the case of perfect substitution, the steady state level of capital is going to be at its highest, as there is no loss of energy services due to imperfect substitution in the final state. When the degree of substitution is low, the steady state level of capital and consumption diminishes, which in turn changes the optimal transition path. Finally, the trajectory of optimal tax rate highly shifts with the degree of substitution. When the resources are perfect substitutes, the optimal tax rate is going to be set much higher in the beginning compared to the case of imperfect substitution. This is due to the fact that the damage of pollution is higher when only non-renewable resource is used, and the initial tax rate becomes the highest to correct the negative externality. The contrary occurs in the long run when only renewable resource is used. With imperfect substitution, as less non-renewable resource is used in the beginning, the optimal tax rate is lower and it increases steeper compared to the case of perfect substitution. In the long run, it remains higher as the non-renewable resource is still being extracted in small quantities.

2.6 Conclusion

It is well known that the time profile of taxation on non-renewable resources is as important as its current level. The regulator has to decide on both to

control the resource extraction. But there is still an ongoing debate on the shape of the time profile of these policy measures.

In this paper we developed an optimal growth model to investigate the optimal taxation of a non-renewable resource which is an imperfect substitute to a renewable resource. We consider a framework with additively separable welfare function in terms of utility from consumption and damages of pollution. The damages of pollution to the environment are taken into account to be irreversible. This framework allowed us to investigate the determinants of the optimal tax rate and its time profile in a decentralized economy that goes through a gradual transition to a clean, zero-carbon state.

We showed that the decentralized economy converges to the clean state in terms of capital and consumption (the same state as the socially optimal one) in the long run whether there is a regulator intervention or not. What matters for welfare, however, is the speed at which the economy approaches there - the energy transition, which determines the level of environmental damages. The policy problem is to induce the firms in the economy to follow the paths leading to the level of environmental damages that the society is ready to accept in the long run. In the *laissez-faire* economy, the profit motivated firms do not internalize the environmental damages that they are causing, thus they extract a greater amount of the non-renewable resource and the households consume more compared to the optimal transition path. Consequently, the speed of transition to clean economy is slower in the absence of regulation. The regulator can correct this market failure by introducing taxation on non-renewable resource extraction.

We identified the factors that affect the optimal tax rate such as the capital stock, consumption, cumulative extraction, the resource prices and the degree of substitution between non-renewable and renewable resources. The endogenous net rental rate of capital, the marginal utility of consumption and the marginal damages to the environment are the key components of

the optimal tax rate. On the time profile of optimal taxation, the initial state of the economy is all that matters. If the initial capital stock is lower than its long term value then the optimal tax is always increasing over time. However, if the initial capital stock is too large then the optimal tax will have a decreasing time profile as the over-accumulated capital is consumed and the capital stock and consumption diminish. Accordingly, the social cost of suboptimal policies can be in many forms. For instance, let's take the example of a constant tax rate. It can initially be too high that it becomes an obstacle for short term development. In the long term, however, it can remain insufficient for leading the renewable resources to overtake the non-renewables.

The results also showed that the renewable resources play a crucial role on the taxation of non-renewables. More expensive renewables will require the regulator to increase the tax rate on non-renewables. In contrast, the technological improvements in the renewable technologies that reduce the costs will lead the regulator to loosen the policy measures on non-renewable resource use. Furthermore, if the degree of substitution between non-renewable and renewable resources becomes higher, the optimal tax rate rises in the short and medium run, but it will be lower in the long run.

The framework in the present paper can be extended towards several directions. Further research includes taking into account technological progress that improves the efficiencies of both non-renewable and renewable resources as well as changes the degree of substitution between resources. In addition, considering reversible pollution together with irreversible pollution, a more realistic global carbon cycle, will improve the results. Finally, incorporating the present modeling into empirical applications can lead to more realistic policy suggestions.

Appendix

2.A Proof of Proposition 2.1

The Jacobian of the system in (2.12 – 2.15) can be written as:

$$\mathbf{J} = \begin{bmatrix} F_K - G(Z)E_{dK}^* - \pi_c E_{cK}^* & -G_Z E_d^* & -1 & F_{\pi_d} - G(Z)E_{d\pi_d}^* - \pi_c E_{c\pi_d}^* \\ E_{dK}^* & 0 & 0 & E_{d\pi_d}^* \\ \sigma C F_{1K} & 0 & \sigma(F_1 - \rho) & \sigma C F_{1\pi_d} \\ F_{1K}(\pi_d - G(Z)) & -(F_1 G_Z + D_{ZZ}/U_C) & D_Z U_{CC}/(U_C)^2 & F_{1\pi_d}(\pi_d - G(Z)) + F_1 \end{bmatrix} \quad (\text{A1.1})$$

We want to prove that the steady state given in (2.24, 2.25) has the saddle point properties for the system in (2.12 – 2.15). For that, first we evaluate the Jacobian at the steady state.

$$\mathbf{J}^{ss} = \begin{bmatrix} \rho & 0 & -1 & 0 \\ E_{dK}^* & 0 & 0 & E_{d\pi_d}^* \\ \sigma C^{ss} F_{1K} & 0 & 0 & \sigma C^{ss} F_{1\pi_d} \\ 0 & -(F_1 G_Z + D_{ZZ}/U_C) & D_Z U_{CC}/(U_C)^2 & \rho \end{bmatrix} \quad (\text{A1.2})$$

The characteristic equation associated with the Jacobian $\mathbf{J}^{ss}$ is given by:

$$\xi^4 - (\text{Tr} \mathbf{J}^{ss}) \xi^3 + M_2 \xi^2 - M_3 \xi + \det \mathbf{J}^{ss} = 0 \quad (\text{A1.3})$$

where M_2 and M_3 are the sum of all diagonal second and third order minors of $\mathbf{J}^{ss}$, respectively. One can show that:

$$\text{Tr} \mathbf{J}^{ss} = 2\rho \text{ and } -M_3 + \rho M_2 - \rho^3 = 0 \quad (\text{A1.4})$$

Theorem 1 in Dockner (1985) shows that if the equations in (A1.4) are satisfied then one can write the four roots of the characteristic equation in

(A1.3) as follows:¹³

$$\xi_{1,2,3,4} = \frac{\rho}{2} \pm \sqrt{\left(\frac{\rho}{2}\right)^2 - \frac{\Omega}{2} \pm \frac{1}{2}\sqrt{\Omega^2 - 4\det \mathbf{J}^{ss}}} \quad (\text{A1.5})$$

where Ω is the sum of all diagonal second order minors and for $\mathbf{J}^{ss}$ it can be written as:

$$\Omega = \begin{vmatrix} \partial \dot{K}/\partial K & \partial \dot{K}/\partial C \\ \partial \dot{C}/\partial K & \partial \dot{C}/\partial C \end{vmatrix} + \begin{vmatrix} \partial \dot{Z}/\partial Z & \partial \dot{Z}/\partial \pi_d \\ \partial \dot{\pi}_d/\partial Z & \partial \dot{\pi}_d/\partial \pi_d \end{vmatrix} + 2 \begin{vmatrix} \partial \dot{K}/\partial Z & \partial \dot{K}/\partial \pi_d \\ \partial \dot{\pi}_d/\partial Z & \partial \dot{\pi}_d/\partial \pi_d \end{vmatrix} \quad (\text{A1.6})$$

In Theorem 3 of Dockner (1985) and Theorem of Tahvonen (1991) it is stated that when the determinant of the Jacobian is positive and Ω is negative the stationary point has the saddle point properties. Therefore, to prove that the steady state is a (local) saddle point, it is now sufficient to show that $\det \mathbf{J}^{ss} > 0$ and $\Omega < 0$. First, let us compute the determinant:

$$\det \mathbf{J}^{ss} = \underbrace{(F_1 G_Z + D_{ZZ}/U_C)}_{>0} \underbrace{(E_{d\pi_d}^* \sigma C^{ss} F_{1K})}_{>0} - \underbrace{(E_{dK}^* \sigma C^{ss} F_{1\pi_d})}_{<0} > 0 \quad (\text{A1.7})$$

Equation (3.29) shows that the determinant of $\mathbf{J}^{ss}$ is always positive due to the assumptions on $U(\cdot)$, $F(\cdot)$, $D(\cdot)$, $G(\cdot)$, as well as the optimal non-renewable use $E_d^*(\cdot)$ which is obtained by (2.4') and (2.5). To complete the proof, we compute the value of Ω given in (A1.6) as follows:

$$\Omega = \underbrace{\sigma C F_{1K}}_{<0} + \underbrace{E_{d\pi_d}^* (F_1 G_Z + D_{ZZ}/U_C)}_{<0} - \underbrace{\sigma C^{ss} F_{1\pi_d}}_{<0} \underbrace{D_Z U_{CC}/(U_C)^2}_{>0} < 0 \quad (\text{A1.8})$$

The result in (A1.8) which shows that Ω is always negative, together with the result in (3.29), ensures that the characteristic equation in (A1.3) consists of two roots with positive real parts and two roots with negative real

¹³See Dockner (1985) and Dockner and Feichtinger (1991) Appendix 1 for derivation.

parts. There are two two-dimensional manifolds which contain the steady state with one of them being stable. If the solution starts on this manifold then the path will asymptotically approach the steady state. For a given initial state $K_0 > 0$ and $Z_0 > 0$, it is possible to choose initial values C_0 and π_{d0} such that the corresponding paths approach the steady state as $t \rightarrow \infty$. Moreover, when the conditions in Mangasarian sufficiency theorem (Mangasarian (1966)) are satisfied, the saddle point path is the optimal infinite time solution. The concavity of Hamiltonian is clearly satisfied due to the assumptions on $U(\cdot)$, $F(\cdot)$, $D(\cdot)$ and $G(\cdot)$, therefore, the path leading to the saddle point is the optimal infinite time solution.

2.B Proof of Proposition 2.2

In the absence of taxation, the Jacobian of the equilibrium system in (2.40–2.43) evaluated at the steady state can be written as:

$$\hat{\mathbf{J}}^{ss} = \begin{bmatrix} \rho & 0 & -1 & 0 \\ E_{dK}^* & 0 & 0 & E_{d\pi_d}^* \\ \sigma C^{ss} F_{1K} & 0 & 0 & \sigma C^{ss} F_{1\pi_d} \\ 0 & -F_1 G_Z & 0 & \rho \end{bmatrix} \quad (\text{A1.9})$$

One can show that $\hat{\mathbf{J}}^{ss}$ in (A1.9) also satisfies the equations in (A1.4). By following the same procedure in the proof of Proposition 1, we can compute $\det \hat{\mathbf{J}}^{ss} > 0$ and $\hat{\Omega} < 0$, hence show that the steady state is a (local) saddle point for the equilibrium system as well as the path leading to the saddle point is the optimal infinite time solution.

Chapter 3

The role of adaptation on the transition to a low-carbon economy

THIERRY BRÉCHET AND BARIS VARDAR

3.1 Introduction

Adaptation policy has recently become a keystone in the ongoing debates to deal with climate change. Broadly defined, adaptation is the capacity to avoid the adverse effects of climate change. Still, the way we consume fossil fuels (oil, coal, natural gas, etc.) directly relates to the adaptation policy: should we pollute more and adapt more, or the reverse? The contribution of this paper is to link this on-going debate about adaptation to the issue of the energy transition, i.e. the move from an exhaustible-fossil based economy to a renewable carbon-free one.

Some examples of adaptation measures include adapting buildings to extreme weather events, constructing flood defenses, raising the levels of dykes and using scarce water resources more efficiently. These measures mostly

require large investments in capital and infrastructure. Therefore, durability and cost efficiency of adaptation measures play important roles on their impacts. In this work, we focus our analysis on the capital nature of adaptation measures.

We consider the problem of a social planner that maximizes the discounted value of total welfare by deciding on the paths of resource utilization and investment in adaptation capital. The problem of optimal resource extraction without adaptation policy is investigated in detail and extended by Withagen (1994), Tahvonen (1997), van der Ploeg and Withagen (2012a) and van der Ploeg and Withagen (2014). These works studied the impact of damages caused by non-renewable extraction, the role of natural decay rate of pollution, the existence of Green Paradox, and the relationship between growth, transition to renewable resources and optimal carbon tax. This work uses the framework studied in these articles to investigate the role of adaptation policy on the transition to a low-carbon economy.

There is a growing literature on the role of adaptation policy. Authors focus on its relationship with mitigation policy on the long run equilibrium (Bréchet et al. (2013)), the effect of catastrophic risks (Zemel (2015)), the strategic effects (Buob and Stephan (2011) and Bréchet et al. (2016)) and the optimal use of adaptation with carbon capture and storage (Moreaux and Withagen (2015)).

The present study contributes to both streams of the literature by investigating adaptation policy with the presence of exhaustible resources. The model includes a non-renewable resource and its average extraction cost increases as the cumulative quantity of extraction rises (as in Heal (1976)). This cost arises due to the necessity to develop technologies that will enable reaching deeper deposits. We also consider a renewable resource that is a perfect substitute to the non-renewable one, its marginal cost is exogenously given and constant. Utilization of non-renewable and renewable resources provides benefit, but extraction and consumption of non-renewable

resource generates pollution which accumulates in the atmosphere irreversibly. The accumulated pollution inflicts damages, which are increasing with the level of pollution. To include adaptation policy into the analysis, we consider the vulnerability function that is introduced by Bréchet et al. (2013). The damage of pollution can be reduced by investing in adaptation capital, and the possibility of reduction in damages can have limitations. Adaptation capital has the stock property, its quantity depreciates unless it is maintained by investments. Investment in adaptation capital has an opportunity cost that increases with its level. We will see that this property does affect the energy transition path.

As we consider the social optimum, mitigation policy can be viewed as implicitly included into the analysis. This is because the total marginal cost of non-renewable resource contains the marginal damage it inflicts as well as its scarcity rent. Therefore, increasing pollution makes the non-renewable resource more costly together with rising extraction cost, hence decreases its utilization. Considering the presence of perfect substitute renewable resource allows us to investigate the consequences of adaptation policy on the transition to renewable resource.

We characterize the optimal transition path with general functional forms under economically reasonable assumptions. As adaptation policy affects the marginal damage of pollution, the driver formula for the marginal cost of non-renewable resource (the modified Hotelling rule) sharply departs from the model without adaptation. An important issue is the possibility for a simultaneous use regime -a transition- within a model of two perfect substitute resources. This possibility arises when the marginal cost of non-renewable resource (which includes the scarcity rent and marginal damages) reaches the level of renewable's marginal cost. At this level, it is still possible to continue extraction, because being able to adapt more and reduce the damage of pollution drags the total marginal cost of the non-renewable resource. The fact is that reducing the damages with adaptation

makes the non-renewable resource beneficial. As investments in adaptation becomes more costly, the benefit cannot go on forever. For a given level of pollution, there exists an efficient level of adaptation capital that equalizes the marginal cost of maintaining that level to the marginal benefit gained by reduction in damage of pollution. Above this level, it is no more beneficial to adapt more and extract more. Showing such a trade-off is new in the literature.

The levels of pollution and adaptation capital that equalize the rent of resource to its marginal damage (at the level of renewable's marginal cost) when the adaptation capital maintained at its level efficiently defines the unique steady state of the simultaneous use regime. The two boundaries in the adaptation level-pollution level set determine the behavior of the optimal path. One boundary equalizes the rent of resource to the marginal damage of pollution when the cost of non-renewable resource equals to that of renewable. This boundary defines the highest level of pollution at which resource extraction is possible, for a given level of adaptation. Above that level it is optimal to use only renewable resource. The other boundary is characterized by the efficient level of adaptation for a given level of pollution. If adaptation capital is too high with respect to the level of pollution, then it is not optimal to maintain it at that level as its marginal cost outweighs the marginal benefit it brings. Its stock will then decrease with lower investments with respect to the depreciation level. When the adaptation capital is too low, the benefit it brings by reduction in damage is larger than its cost, so the level of adaptation should increase. The steady state occurs when these two boundaries are reached, and after that it is optimal to use only renewable resource indefinitely.

The properties of the optimal path heavily depend on the initial state levels of pollution and adaptation stocks with respect to these two boundaries. If the initial levels pollution and adaptation are low, then it is optimal to use only non-renewable resource and increase adaptation capital by large

investments in the beginning. At a certain date, the cost of non-renewable reaches that of renewable and a gradual transition starts. Following a path of resource extraction on which the marginal damage is compensated by increased adaptation, the optimal path reaches the steady state and stays there indefinitely by using only renewable resource. When the initial pollution is too high but adaptation is low, it is never optimal to extract the non-renewable resource. Only renewable resource will be used and adaptation capital will be adjusted to its efficient level. Another case is low pollution and high level of adaptation. In that case, it is optimal to extract non-renewable resource from the beginning and to decrease the level of adaptation by investing less than its depreciating rate. At a certain date, the optimal path reaches a steady state that can contain a higher level of pollution compared to the one previously explained. Empirically, one may assume that our adaptation capital stock is below the optimal steady state one and that the same holds for the pollution stock associated to a 2 degree increase with respect to the pre-industrial level.

Indeed, the optimum is determined by the properties of underlying functions which are utility, damage of pollution, extraction cost, vulnerability and opportunity cost of investment in adaptation. If the cost of investment in adaptation is too high, then its efficient level will decline. On the contrary, large damages of pollution will push up adaptation efforts. The limitations of adaptation are important. If adaptation is not capable of reducing damages enough, then its efficient level reduces. The durability of adaptation capital also plays a crucial role in the optimum solution. When adaptation investments are oriented towards less durable capital (which means a high depreciation rate), the benefits of adaptation are going to be limited due to high cost of maintenance. On the contrary, more durable adaptation investments decreases its cost which in turn leads to a longer transition to the renewable resource and larger quantity of resource extraction.

The following section presents the framework and the social planner's problem, Section 3.3 goes through the different regimes of transition, Section 3.4 presents the optimum and discusses its properties, Section 3.5 analyzes the role of adaptation capital's depreciation rate, Section 3.6 illustrates the simulation results and Section 3.7 concludes.

3.2 The model

The model incorporates adaptation policy into the problem of resource extraction with pollution externalities. We consider a non-renewable resource for which the extraction cost increases with the cumulative quantity of extraction. There is a renewable resource that is a perfect substitute to the non-renewable one and its marginal cost is given and constant. Utilization of non-renewable and renewable resources provides benefit to society, but non-renewable resource generates pollution as a by-product, which accumulates irreversibly in the atmosphere. Accumulated pollution inflicts damages to society that can be reduced by investing in adaptation capital. Adaptation capital depreciates over time at a constant rate.

The utility of using the resource flow R is denoted by $U(R)$. We assume the following:

Assumption 3.1. $U(R)$ is increasing and strictly concave in R ($U'(R) > 0, U''(R) < 0$).

The extraction cost of exhaustible resource (can be oil, coal, natural gas etc.) depends on the total amount of resource that has been extracted by time t which is denoted by Z_t ($Z_t = Z_0 + \int_{s=0}^t R_s ds$). The average extraction cost rises with cumulative extraction because of increasing efforts to develop technologies that are necessary to reach deeper deposits. It is denoted by $G(Z)$ and we assume the following property for $G(\cdot)$:

Assumption 3.2. $G(Z)$ is increasing and strictly convex in Z ($G'(Z) > 0$, $G''(Z) > 0$).

There is also a backstop resource (b) available (can be solar, wind power etc.) which is a perfect substitute to the exhaustible resource, does not pollute and have a constant marginal cost p_b .

The exhaustible resource flow R generates φR amount of pollution (φ is the pollution content of one unit of resource) and it adds up to the pollution stock. We consider that pollution accumulation is irreversible ($P_t = P_0 + \int_{s=0}^t \varphi R_s ds$) and therefore we can write the pollution stock in terms of cumulative extraction ($P_t = P_0 + \varphi(Z_t - Z_0)$).¹ Furthermore, this allows us to define the damages of pollution as a function of cumulative extraction which is denoted by $D(Z)$. We assume:

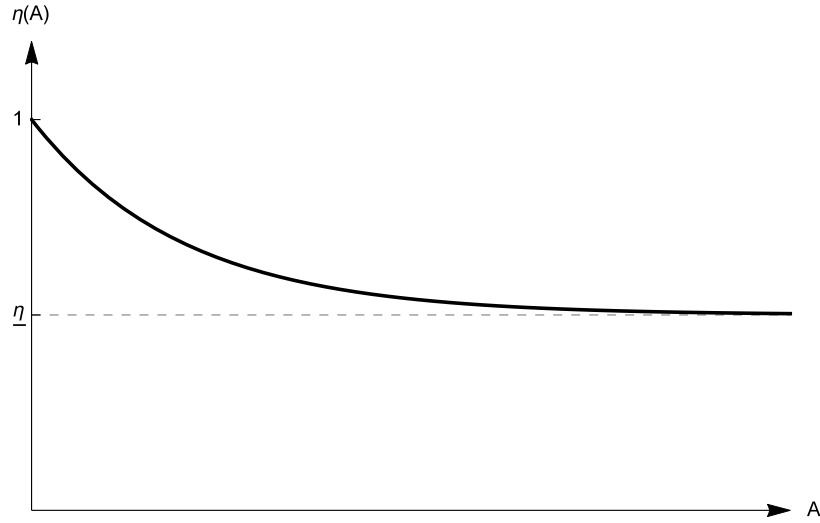
Assumption 3.3. $D(Z)$ is increasing and convex in Z ($D'(Z) > 0$, $D''(Z) > 0$).

The damages of pollution can be reduced by installing adaptation capital A . Adaptation capital has a stock property which means that in the absence of maintenance it depreciates at a rate δ . The function $\eta(A)$ denotes the proportional reduction in damages which we call as *vulnerability function*. This functional specification of adaptation is similar to the one in Bréchet et al. (2013). We assume the following:

Assumption 3.4. (a) $\eta(A)$ is decreasing and convex in A ($\eta'(A) < 0$, $\eta''(A) > 0$). (b) $\eta(0) = 1$, (c) $\eta(+\infty) = \underline{\eta}$ with $\underline{\eta} \in [0, 1)$, (d) $\eta'(0) = -\varphi$ with $\varphi \in (0, +\infty)$ and (e) $\eta'(+\infty) = 0$.

Since $\eta(A)$ stands for the proportional reduction in damages, the effective damages -realized impact of pollution- is denoted as $\eta(A)D(Z)$. This is the reason for assuming $\eta(0) = 1$, meaning that when there is no adaptation

¹Due to this relationship, we will refer to Z as cumulative extraction or pollution whenever it is appropriate in the rest of the text.

Figure 3.1 – Vulnerability function ($\eta(A)$)

the impact of pollution will be solely $D(Z)$. Moreover, it is appropriate to assume that the reduction in damages has a limit, as reducing the adverse effects of pollution entirely by making adaptation expenditures is not a realistic consideration. Assumption 4(d) says that the first unit of adaptation spending reduces the damages at φ proportion. It can be finite or infinite, but it is economically more plausible to consider that it is finite.

Investment in adaptation capital I has a cost to society $H(I)$. This cost raises due to allocating efforts to adaptation policies rather than other uses that would provide benefit. Higher investment in adaptation capital will increase its marginal cost to society. Therefore we assume:

Assumption 3.5. $H(I)$ is increasing and convex in I ($H'(I) > 0, H''(I) > 0$).

The social planner maximizes the discounted value of total welfare given

by the following programme:

$$\max_{\{R_t, b_t, I_t\}} \int_{t=0}^{\infty} e^{-\rho t} (U(R_t + b_t) - \eta(A_t)D(Z_t) - R_t G(Z_t) - p_b b_t - H(I_t)) dt \quad (3.1)$$

subject to

$$\dot{Z}_t = R_t$$

$$\dot{A}_t = I_t - \delta A_t$$

with $Z_0 \geq 0$ and $A_0 \geq 0$ given.

The current value Hamiltonian associated to (3.1) is

$$H_t = U(R_t + b_t) - \eta(A_t)D(Z_t) - R_t G(Z_t) - p_b b_t - H(I_t) - \lambda_t R_t + \gamma_t (I_t - \delta A_t) \quad (3.2)$$

where λ is the co-state variable associated to cumulative extraction and pollution. It can also be interpreted as the opportunity cost of one more unit of extraction. The term γ is the shadow value of adaptation capital. The necessary conditions for an optimum are:

$$R_t \geq 0, \quad U'(R_t + b_t) \leq G(Z_t) + \lambda_t \quad (3.3)$$

$$b_t \geq 0, \quad U'(R_t + b_t) \leq p_b \quad (3.4)$$

$$H'(I_t) = \gamma_t \quad (3.5)$$

$$\dot{\lambda}_t = \rho \lambda_t - \eta(A_t)D'(Z_t) - G'(Z_t)R_t \quad (3.6)$$

$$\dot{\gamma}_t = (\rho + \delta)\gamma_t + \eta'(A_t)D(Z_t) \quad (3.7)$$

$$\lim_{t \rightarrow +\infty} e^{-\rho t} \lambda_t Z_t = 0 \text{ and } \lim_{t \rightarrow +\infty} e^{-\rho t} \gamma_t A_t = 0 \quad (3.8)$$

where (3.3) and (3.4) are complementary slackness (c.s.) conditions. Condition (3.5) allows us to find the optimal investment in adaptation as a function of adaptation capital's shadow value: $I^*(\gamma_t) = H'^{-1}(\gamma_t)$.

As non-renewable and renewable resources are perfect substitutes, the cheaper one is going to be utilized at the optimum. To proceed, we define

the total marginal cost of non-renewable resource as follows:

Definition 3.1. *Let p_t be the total marginal cost of non-renewable resource given by*

$$p_t := G(Z_t) + \lambda_t. \quad (3.9)$$

Taking the time derivative of p_t and using (3.6) yields the optimal law of motion as $\dot{p}_t = \rho(p_t - G(Z_t)) - \eta(A_t)D'(Z_t)$. With the newly defined variable, complementary slackness conditions (3.3) and (3.4) become the following:

$$R_t \geq 0, \quad U'(R_t + b_t) \leq p_t \quad (3.10)$$

$$b_t \geq 0, \quad U'(R_t + b_t) \leq p_b \quad (3.11)$$

Conditions (3.10) and (3.11) show that there are three possible regimes: the regime in which only non-renewable resource is used ($R > 0$ and $b = 0$), the regime in which both resources are used simultaneously ($R > 0$ and $b > 0$) and the regime in which only renewable resource is used ($R = 0$ and $b > 0$). The following section investigates these cases.

3.3 Transition regimes

The optimum can contain three different regimes: only non-renewable resource use, simultaneous use of non-renewable and renewable resources, and only renewable resource use. In this section we study each case in detail.

Regime 1: Only non-renewable resource use ($R > 0$ and $b = 0$)

In this regime, the total marginal cost of non-renewable resource is lower than that of renewable ($p_t < p_b$). The optimal resource consumption is obtained by (3.10), $R_1^*(p_t) = U'^{-1}(p_t)$. The optimal trajectories must satisfy

the following differential equation system in (Z, A, p, γ) :

$$\dot{Z}_t = R_1^*(p_t) \quad (3.12)$$

$$\dot{A}_t = I^*(\gamma_t) - \delta A_t \quad (3.13)$$

$$\dot{p}_t = \rho(p_t - G(Z_t)) - \eta(A_t)D'(Z_t) \quad (3.14)$$

$$\dot{\gamma}_t = (\rho + \delta)\gamma_t + \eta'(A_t)D(Z_t) \quad (3.15)$$

First two equations are the evolution of state variables with the optimal amounts of resource extraction and investment in adaptation capital. Third equation drives the marginal cost of resource. The term $p_t - G(Z_t)$ is the resource rent and it is always non-negative, and the term $\eta(A_t)D'(Z_t)$ is the effective marginal damages. Hence, equation (3.14) can be viewed as the modified Hotelling rule for this problem, and it contains the information on both the scarcity rent and the social cost of carbon. The presence of adaptation in equation (3.14) plays a crucial role in the optimum. The reduction in marginal damage of pollution affects the optimal path of the marginal cost of non-renewable resource, which determines the time span of resource use as well as the level and accumulation of pollution. The last equation is the evolution of adaptation capital's shadow value and we see that its path is driven by marginal reduction in damages as well as the depreciation rate of adaptation capital.

This regime is optimal as long as the marginal cost of non-renewable resource is lower than that of renewable ($p_t < p_b$).

Regime 2: Simultaneous use and transition to renewable resource
($R > 0$ and $b > 0$)

When the total marginal cost of non-renewable resource reaches to the marginal cost of renewable ($p_t = p_b$), a regime of simultaneous use of resources may arise. This is due to the possibility of increasing adaptation capital and therefore reducing marginal damages to the environment. This

makes the non-renewable resource still beneficial to use, as reducing the social cost of pollution will push the total marginal cost of non-renewable resource downward, which could be used as an opportunity to extract more resource.

By conditions (3.10) and (3.11) we have $U'(R_t + b_t) = p_b$ thus $R_t + b_t = U'^{-1}(p_b)$ is a given constant. Now we need to find the values for R_t and b_t .

In this regime p_t has to stay constant at the marginal cost of renewable resource, therefore

$$\dot{p}_t = \rho(p_b - G(Z_t)) - \eta(A_t)D'(Z_t) = 0. \quad (3.16)$$

Equation (3.16) represents the curve on (A, Z) plane on which regime 2 can exist. Observe that there is no possibility of violating equation (3.16) in problem (3.1) without adaptation. Damage of pollution will stay constant, that's why there is going to be an instantaneous switch to the renewable resource in that case. However, with adaptation it is possible to reduce the effective marginal damage (the second term in equation (3.16)), which would make the non-renewable resource beneficial to extract once again. The damage due to marginal unit of extraction is going to be compensated by increased adaptation capital, and vice versa. We search for the quantity of extraction that equalizes these opposite effects.

Total marginal cost (p) has to stay constant ($\dot{p}_t = 0$), and ($\ddot{p}_t = 0$) should hold true as well. Taking the time derivative of (3.16) yields $-\rho G'(Z_t)\dot{Z}_t - (\eta'(A_t)D'(Z_t)\dot{A}_t + \eta(A_t)D''(Z_t)\dot{Z}_t) = 0$. Replacing $\dot{Z}_t = R_t$ and $\dot{A}_t = I^*(\gamma_t) - \delta A_t$ and solving for R_t gives the optimal non-renewable resource use in Regime 2 as a function of pollution (and marginal extraction cost), the stock of adaptation capital and the shadow price of adaptation $R_2^*(Z,$

A, γ):

$$R_2^*(Z_t, A_t, \gamma_t) = \begin{cases} \frac{-\eta'(A_t)D'(Z_t)(I^*(\gamma_t) - \delta A_t)}{\rho G'(Z_t) + \eta(A_t)D''(Z_t)} & \text{if } I^*(\gamma_t) > \delta A_t \\ 0 & \text{if } I^*(\gamma_t) \leq \delta A_t \end{cases} \quad (3.17)$$

Equation (3.17) shows that simultaneous use of resources can occur only if the investments in adaptation are exceeding the quantity that depreciates ($\dot{A} > 0$), so adaptation capital must be increasing in regime 2. The optimal value of renewable resource use is $b_2^*(Z_t, A_t, \gamma_t) = U'^{-1}(p_b) - R_2^*(Z_t, A_t, \gamma_t)$. In regime 2, the optimal trajectories must satisfy the following differential equation system in (Z, A, γ) :

$$\dot{Z}_t = R_2^*(Z_t, A_t, \gamma_t) \quad (3.18)$$

$$\dot{A}_t = I^*(\gamma_t) - \delta A_t \quad (3.19)$$

$$\dot{\gamma}_t = (\rho + \delta)\gamma_t + \eta'(A_t)D(Z_t) \quad (3.20)$$

$$\dot{p}_t = \rho(p_b - G(Z_t)) - \eta(A_t)D'(Z_t) = 0 \quad (3.21)$$

The use of non-renewable resource is null if the investment in adaptation capital is equal to the quantity of depreciation ($R_2^*(Z_t, A_t, \gamma_t) = 0$ if $I^*(\gamma_t) = \delta A_t$ by equation (3.17)). Using this, we can study the existence of a steady state in regime 2 by the following system of equations:

$$I^*(\gamma^{ss}) = \delta A^{ss} \quad (3.22)$$

$$(\rho + \delta)\gamma^{ss} = -\eta'(A^{ss})D(Z^{ss}) \quad (3.23)$$

$$\rho(p_b - G(Z^{ss})) = \eta(A^{ss})D'(Z^{ss}) \quad (3.24)$$

At the steady state, the level of pollution reaches to a level such that it is no more beneficial to adapt more and extract more of the resource. Equation (3.22) maintains adaptation capital at its level, (3.23) equalizes the opportunity cost of investing in adaptation capital to the marginal benefit gained by reduction in effective damage, and (3.24) equalizes the

marginal benefit of extraction to effective marginal damage. At a level of pollution higher than Z^{ss} , there may exist a level of adaptation capital that equalizes rent to damage, but its cost would outweigh its benefit. It cannot be kept at a steady level efficiently. That's why an optimal path containing regime 2 could not have a higher pollution. With the help of the following definition, we will look for the levels of pollution and adaptation capital that are optimal and constant.

Definition 3.2. *Let $\hat{Z}(A)$ be the level of pollution such that equations (3.22 and 3.23) hold true ($\dot{A} = 0$ and $\dot{\gamma} = 0$). That is*

$$\hat{Z}(A) := D^{-1}((\rho + \delta)H'(\delta A)/(-\eta'(A))). \quad (3.25)$$

At this level of pollution, investments in adaptation capital equals to the quantity that depreciates, and the marginal cost of investment equals to the marginal benefit gained by reduction in damages. The function $\hat{Z}(A)$ can be also interpreted as the level of pollution such that it will be efficient to keep adaptation capital steady at A . Indeed, this level depends on the properties of damage (D), adaptation (η) and opportunity cost of investment in adaptation (H) functions, as well as the social discount rate ρ and the depreciation rate of adaptation capital δ . The following lemma shows how this level of pollution changes with the stock of adaptation capital.

Lemma 3.1. *$\hat{Z}(A)$ is increasing in adaptation capital ($\hat{Z}'(A) > 0$).*

Proof. See Appendix 3.A. □

Substituting $\hat{Z}(A)$ in equation (3.24) allows us to reduce (3.22-3.24) to a single equation in A given by

$$\rho(p_b - G(\hat{Z}(A))) = \eta(A)D'(\hat{Z}(A)). \quad (3.26)$$

Assumption 3.6. *The effective marginal damage is increasing in A at the steady state*

$$(d(\eta(A)D'(\hat{Z}(A)))/dA > 0).$$

Assumption 3.6 states that, at the steady state, the increase in marginal damage due to the extraction made by adapting is higher compared to the level of reduction in marginal damage made by a marginal increase in adaptation.

The contrary case to Assumption 3.6 is $d(\eta(A)D'(\hat{Z}(A)))/dA < 0$. This would mean that the effective marginal damage is decreasing in A at the steady state. Then it would be optimal to increase adaptation capital and extract more resource that will increase Z , keeping $\dot{p} = 0$ in (3.21). In this case, there is no possibility of having a solution to (3.26) which has a positive value for the rent of resource. This would violate the necessary optimality conditions, and we do not treat this case as it is not economically meaningful.

The system in (3.18-3.21) has a steady state and it is unique when Assumption 3.6 is satisfied. Left hand side (LHS) of (3.26) is a positive constant (ρp_b) when $A = 0$ and it is decreasing in A because $\hat{Z}'(A) > 0$ as shown in Lemma 3.1. Right hand side (RHS) is null when $A = 0$. Then the existence and uniqueness of the steady state that contains positive values for LHS and RHS depends on how RHS of (3.26) changes with A . If $d(\eta(A)D'(\hat{Z}(A)))/dA > 0$ then RHS is increasing in A , therefore we can conclude that (3.26) has a unique solution given by a positive constant $A^{ss} > 0$. This is illustrated in figure 3.1.

Regime 3: Only renewable resource use ($R = 0$ and $b > 0$)

In this regime $p > p_b$ and only renewable resource is used at quantity $b^{ss} = U'^{-1}(p_b)$. There is no extraction ($R = 0$) hence $\dot{Z} = 0$, the problem reduces to a system in (A, γ) given in (3.19-3.20) for given Z . The optimal

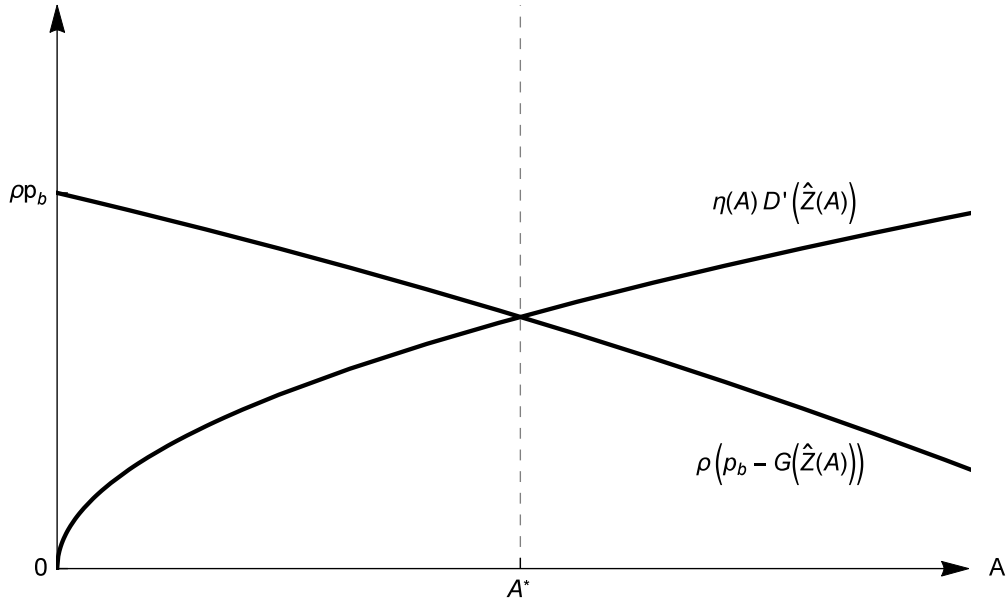


Figure 3.1 – Existence and uniqueness of the steady state in regime 2

trajectories will depend on the initial states of pollution and adaptation capital.

3.4 The optimum

The following proposition presents the solution to the problem given in (3.1):

Proposition 3.1. *Under assumptions 3.1 to 3.6, the problem in (3.1) attains a unique solution. For a given (A_0, Z_0) there exists a unique optimal path $\{Z_t, A_t, p_t, \gamma_t\}_{t=0}^T$ that reaches a steady state. Equation (3.16) characterizes the boundary between regimes 1 and 3 in state space (A, Z) . The level of steady state and the occurrences of different regimes depend on the initial state:*

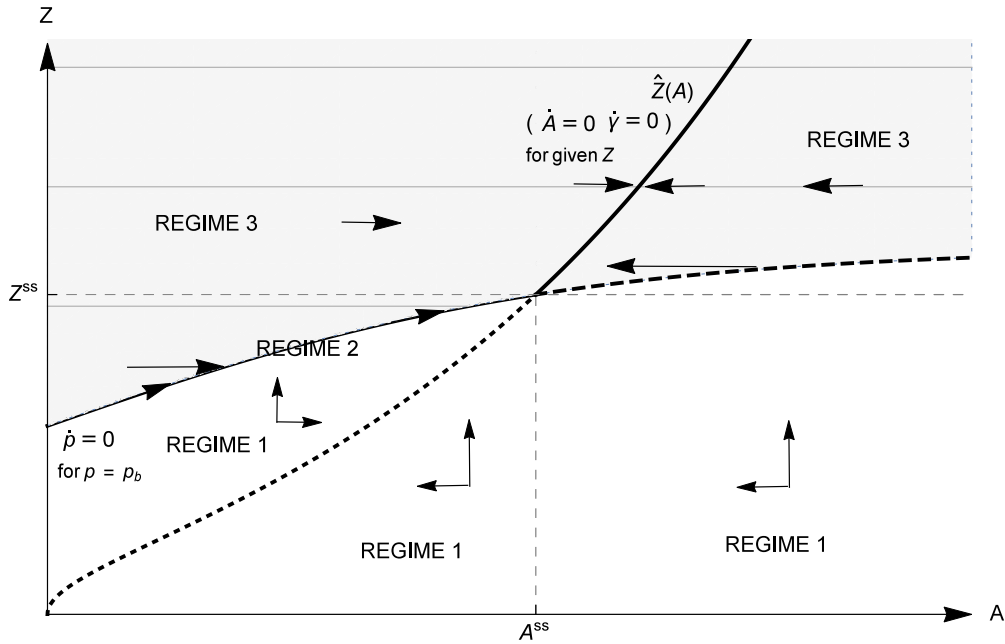
- if $A_0 \leq A^{ss}$ and $Z_0 \leq Z^{ss}$ then the steady state is (A^{ss}, Z^{ss}) that is characterized by the system of equations in (3.22-3.24). Depending on

the initial state, the optimal path can be one of the following sequences of regimes: $(1 \rightarrow 2 \rightarrow 3)$, $(1 \rightarrow 3)$, $(2 \rightarrow 3)$, $(3 \rightarrow 2 \rightarrow 3)$, (3) ,

- if $A_0 \leq A^{ss}$ and $Z_0 > Z^{ss}$ then the optimal path contains only regime 3 and reaches the steady state determined by (3.22 and 3.23) for given Z_0 ($Z_0 = \hat{Z}(A'^{ss})$),
- if $A_0 > A^{ss}$ then the steady state can be higher than Z^{ss} ($Z'^{ss} \geq Z^{ss}$) and the optimal path can be either $(1 \rightarrow 3)$ or (3) .

Proof. See Appendix 3.B. □

The occurrence of different regimes depends on the initial state and on the properties of underlying functions.



Note: $\hat{Z}(A)$ is the level of pollution given in Definition 3.2 and $\dot{p} = 0$ for $p = p_b$ is the curve given by equation (3.16).

Figure 3.1 – Phase diagram in (A, Z) plane

Diagram 3.1 shows the direction of optimal path for a given initial state (A_0, Z_0) , and it depicts the occurrences of different sequences of regimes. Having all loci presented on (A, Z) plane allowed us to determine these

directions. The stability of system in (3.12-3.15) can also be observed in this diagram. The two curves, equation (3.16) and $\hat{Z}(A)$, intersect only once at the steady state (as analyzed by equation (3.26)) and they divide the (A, Z) plane to four regions. As their importance on the optimum, we will now look into these two curves in more detail and interpret their economic meanings.

Equation (3.16) ($\dot{p} = 0$ for $p = p_b$) illustrates the highest level of pollution at which non-renewable resource extraction is possible for a given level of adaptation capital. At this level, in order to have beneficial extraction (marginal benefit of extraction being at least as higher as the marginal damage it inflicts), the total marginal cost of non-renewable must be set at ($p = p_b$). For a given level of adaptation, a higher level of pollution will require a higher p in order to have the resource still beneficial to extract. However, when the non-renewable resource is too costly compared to the renewable ($p > p_b$), it is optimal to use only the renewable resource due to perfect substitution between them. At lower levels of pollution, the marginal damage is lower which drags the total marginal cost of non-renewable resource down ($p < p_b$). In that case, it is optimal to use only non-renewable resource. We can see that this curve is increasing in adaptation capital, meaning that the highest level of pollution that resource extraction can take place is higher when there is too much adaptation capital deployed.

The curve $\hat{Z}(A)$ illustrates the most efficient level of adaptation capital for a given level of pollution. On that curve, the marginal cost of keeping adaptation capital on its level equals to the marginal benefit gained by reduction in pollution damage. For a given level of pollution, if adaptation capital is higher than this level, it is too costly to maintain it as its marginal cost outweighs the benefit it provides. In this case, investments in adaptation capital are lower than the quantity that depreciates, thus adaptation capital will decrease. If the level of adaptation capital is lower,

the benefit it provides is greater than its cost, thus its quantity will be increased by large investments.

As Proposition 3.1 states, the optimal path can follow different sequences of regimes depending on the initial state. All cases presented in Proposition 3.1 can be viewed from Diagram 3.1. In the following, we explore the properties of the optimal path for each case.

The first item in Proposition 3.1 is the case of low pollution and low adaptation. In this case, the optimal path can be of two types. In one type, it is optimal to start with using only non-renewable resource and increase adaptation capital. The level of pollution will rise and at a certain time the cost of non-renewable resource will be equalized to that of renewable ($p = p_b$), then a simultaneous use of resources and a gradual transition to renewable will commence. In this phase, resource extraction continues at a pace such that the marginal damage it inflicts is compensated by marginal increase in adaptation, following the path on curve $\dot{p} = 0$ for $p = p_b$. This compensation mechanism can go on until pollution reaches a level such that it is no more beneficial to extract more and adapt more. Indeed, this level highly depends on the properties of opportunity cost of investment in adaptation, as well as on the depreciation rate of adaptation capital (which will be discussed later on). At that level, it is optimal to stop extraction, to use only renewable resource and to maintain adaptation capital at its level indefinitely.

In another type, when the initial level of pollution is too high, but still lower than its steady state level ($Z_0 < Z^{ss}$), the optimal path initially starts with only renewable resource and deploys more adaptation capital over time, decreasing the marginal damage of pollution. At a certain time, adaptation capital reaches a level that makes the non-renewable resource beneficial to use together with the renewable. Following a gradual transition, the optimal path reaches the steady state and stays there indefinitely.

The second item in Proposition 3.1 is the case of high pollution and low

adaptation. In this case, the level of pollution is so high that it will never be optimal to extract resource by increasing adaptation capital. Only renewable resource will be used, and adaptation capital will be brought to its efficient level.

The third item in Proposition 3.1 is the case of too high initial adaptation. In that case, simultaneous use and gradual transition cannot exist. If the level of pollution is low enough, it is optimal to start by using only non-renewable resource, and to invest less in adaptation to let it depreciate towards its efficient quantity. For too high levels of initial adaptation, pollution could be increased to a level that is higher compared to its steady state value. Increasing pollution and decreasing adaptation will induce a rise in marginal damage, and at a certain date the non-renewable resource will no longer be rentable to extract. Following an instantaneous switch, adaptation capital will be decreased to its efficient while using only renewable resource indefinitely. If the initial level of pollution is too high, then it is never optimal to extract non-renewable resource, and adaptation capital is drawn down to its efficient level.

Indeed, the properties of all underlying functions (utility, damage, extraction cost, vulnerability and opportunity cost of investment in adaptation) play a crucial role on the characteristics of the optimum. In addition, the depreciation rate of adaptation capital (δ) is also an important parameter as it affects the cost of maintaining adaptation capital efficiently. Adaptation capital is a specific type of capital for which the rate of depreciation can be subject to differ. In the following, we study the role of depreciation rate and see how it affects the optimum.

3.5 The role of depreciation rate of adaptation capital

In this subsection we discuss the role of depreciation rate of adaptation capital (δ) and its implications on the outcome of the model. It is not common to study the depreciation rate of a capital variable as it is usually considered to be a technology parameter given exogenously. However, adaptation capital is a specific type of capital and its durability can differ due to the nature of damages that it involves. Difficulties in implementation and enforcement of right adaptation policies can also have impact on their durability. The rate of depreciation of adaptation capital will be determined by taking into account all of these technological characteristics underlying adaptation policy. The aim of this analysis is to investigate the impact of a high rate of depreciation on the optimum characterized in Proposition 3.1.

We first look at how the function $\hat{Z}(A)$ changes with the depreciation rate of adaptation capital.

Lemma 3.2. *For a given stock of adaptation capital (A), $\hat{Z}(A; \delta)$ increases with the depreciation rate of adaptation capital ($d\hat{Z}(A; \delta)/d\delta > 0$).*

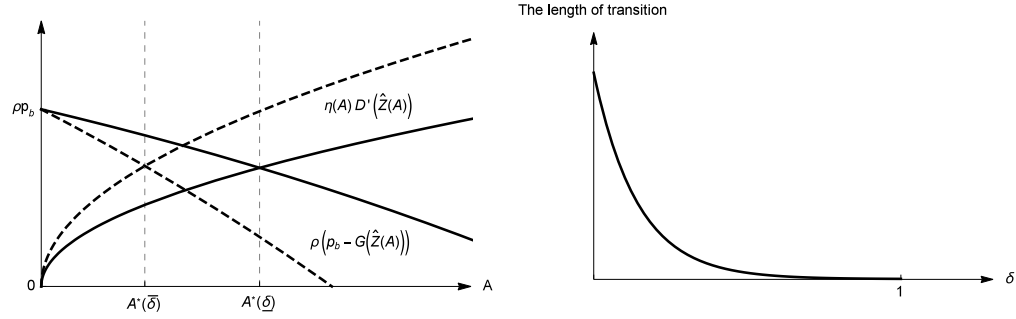
Proof. See Appendix 3.C. □

Indeed, a much larger proportion of adaptation capital depreciates when the rate of depreciation is higher, which makes it more costly to maintain at a certain level. Then it would require the level of pollution high enough to make it worth to keep a certain quantity of adaptation capital. If the level of pollution is lower than this level, the marginal cost of maintaining adaptation capital at its level is too high compared to the marginal benefit it brings, hence it is not efficient. As a result of Lemma 3.2, the curve $\hat{Z}(A)$ on (A, Z) plane in Diagram 3.1 is going to be steeper in the case of a high

rate of depreciation. The following proposition presents its impacts on the outcome.

Proposition 3.2. *When the depreciation rate of adaptation capital (δ) is high, the steady state values of adaptation capital and pollution are low. For a given initial state (A_0, Z_0) , the length of transition is also shorter when δ increases.*

Proof. See Appendix 3.D. □



Note: Solid lines are for a low value of δ , dashed ones are for a high value.

Figure 3.1 – The effect of depreciation rate of adaptation capital (δ) on the steady state and length of transition

Figure 3.1 illustrates Proposition 3.2 with two values of depreciation rate where $\underline{\delta} < \bar{\delta}$. It shows a decreasing relationship between δ and the length of transition. As the depreciation rate increases, it becomes more costly to maintain the adaptation capital installed, therefore the simultaneous regime in which extraction continues due to increasing adaptation capital takes shorter time for a given path that contains regime 2 within. This can also be observed by making $\hat{Z}(A)$ curve steeper in Diagram 3.1.

Accordingly, more durable adaptation capital increases the possibility of a simultaneous use of two perfect substitutes resources, by extending the region that it can exist, and it increases the time span of transition in the cases that it occurs. In contrary, less durable adaptation capital increases the cost of maintenance, which leads to a shorter transition and a lower

level of steady state in terms of adaptation capital and pollution, as well as the total quantity of resource extraction.

3.6 Numerical analysis

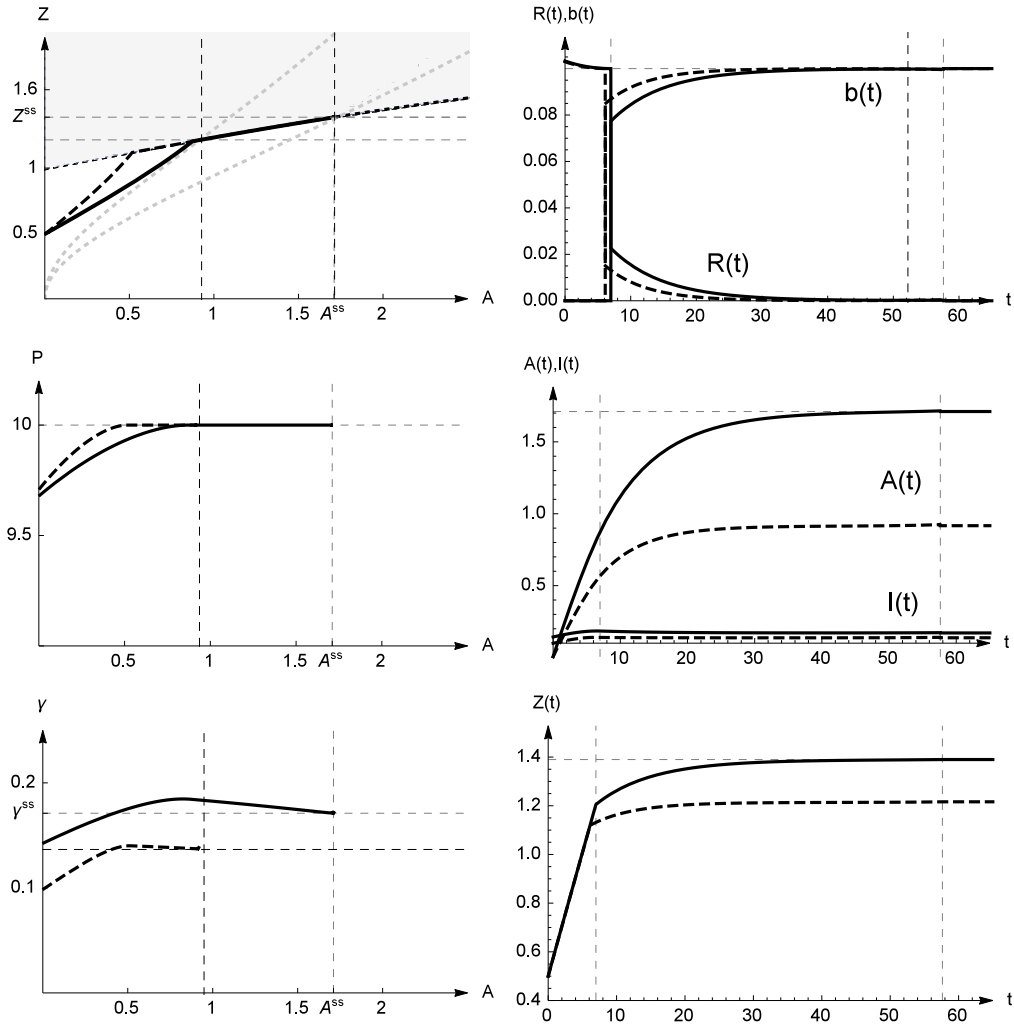
This section presents the numerical analysis based on the theoretical results of Section 3.2. Our aim is to illustrate an optimal path that includes all regimes within, study its properties, and provide a comparison with a case in which the depreciation rate of adaptation capital is high.

The functional specifications we use are as follows: $U(R) = \log(R)$, $G(Z) = \varphi_g Z^2/2$, $D(Z) = \varphi_d Z^2/2$, $H(I) = \varphi_i I^2/2$ and $\eta(A) = \underline{\eta} + (1 - \underline{\eta})e^{-\phi_A A}$. These specifications satisfy assumptions 3.1 to 3.6. In vulnerability function η , the parameter $\underline{\eta}$ determines the limit reduction of damages, and the parameter ϕ_A determines the curvature of the function.

We use a simple calibration: $p_b = 10$, $\rho = 0.02$, $\varphi_g = 0.1$, $\varphi_d = 0.2$, $\varphi_i = 1$, $\underline{\eta} = 0.5$ and $\phi_A = 0.5$. To present the comparison, we set $\delta = 0.1$ and $\delta = 0.15$ for the depreciation rate of adaptation capital. We set the initial pollution $Z_0 = 0.5$ and the initial adaptation stock $A = 0.005$ and search for the optimal path that reaches the steady state. The model is simulated by using an iteration program that calculates the numerical results of the differential equation systems (regime 1 and 2) with boundary conditions. It searches for the right initial values of marginal cost of non-renewable resource (p_0) and shadow value of adaptation capital (γ_0) that will lead to a minimum distance from the steady state, and then it iterates until convergence.

Figure 3.1 depicts the optimal path for the initial state $Z_0 = 0.5$ and $A_0 = 0.005$. The left side plots the phase diagrams in (A, Z) , (A, p) and (A, γ) planes, and the right side plots the optimal trajectories.

We first focus on the properties of the solid lines (for $\delta = 0.1$). (A, Z)



Note: Solid lines are for $\delta = 0.1$, dashed lines are for $\delta = 0.15$.

Figure 3.1 – Optimal paths with different rate of depreciation of adaptation capital

diagram shows that the initial state is contained in the area of regime 1, which is shown in Diagram 3.1. On the optimal path, the levels of pollution and adaptation capital increase until the path reaches to the curve $\dot{p} = 0$ for $p = p_b$. On this curve it is optimal to use both resources simultaneously, which is done by decreasing the damage of pollution by more adaptation. This path proceeds to the steady state (the intersection point of $\hat{Z}(A)$ and $\dot{p} = 0$ for $p = p_b$ curves), in which it is no more optimal to adapt more and extract more. (A, p) diagram shows the optimal path of adaptation and

marginal cost of non-renewable resource. It shows that the initial marginal cost of non-renewable is determined low and it increases as the extraction cost and marginal pollution damage rises. It reaches the marginal cost of renewable ($p = p_b$) and stays there indefinitely. (A, γ) diagram illustrates the optimal path of adaptation capital and its shadow value. The path of γ can also be viewed as of investments as investment is increasing in γ . It shows an increasing path of investments until the simultaneous regime begins. Then the investments are gradually decreases and becomes equal to the quantity of depreciation at the steady state.

The optimal trajectories of $R(t)$ and $b(t)$ show that initially only non-renewable resource is used. The quantity of extraction decreases as the marginal cost of non-renewable rises. Simultaneous use of two resources begins when the marginal cost reaches that of renewable. At this date, non-renewable resource extraction is adjusted to a level such that the marginal damage it inflicts is compensated by a marginal increase in adaptation capital. As adaptation capital rises and its marginal benefit decreases, and as the level of pollution increases, the amount of resource extraction declines and reaches zero at the steady state. After that, only renewable resource is used indefinitely. The trajectory of pollution (also cumulative extraction) ($Z(t)$) shows a rapidly increasing pollution caused by utilization of only non-renewables. When the simultaneous use regime starts, the level of pollution rises more slowly and stabilizes at its steady state level.

Finally, we focus on the role of adaptation capital's depreciation rate (the dashed lines). (A, Z) diagrams shows that the slope of $\hat{Z}(A)$ curve had risen due to high rate of depreciation. It intersects with $\dot{p} = 0$ for $p = p_b$ curve at a lower level of pollution and adaptation capital, hence their steady state quantities are lower. The optimal path follows a steeper increase in pollution compared to the case of low depreciation rate. It reaches to simultaneous use regime at a lower level of adaptation, and follows the boundary curve to reach its steady state. (A, P) diagram shows that, when the rate

of depreciation is higher, the marginal cost of non-renewable resource is set at a higher level for any given adaptation capital. With too large depreciation, it is too costly to maintain and increase the stock of adaptation capital. Thus the marginal reduction in damage is lower when δ is higher. This effect reflects itself in the marginal cost of non-renewable resource as the marginal damage caused by extraction is larger. (A, γ) diagram shows that investments in adaptation are much lower when δ is higher. Resource use trajectories show that simultaneous use of resources begins earlier and lasts much shorter with a high δ . Due to the high cost of adaptation, a lower amount of non-renewable resource is extracted while using a larger quantity of renewables. The level of adaptation capital stabilizes at a lower level compared to the case of low depreciation rate.

3.7 Conclusion

This work studies the role of adaptation policy on the problem of optimal non-renewable resource extraction with pollution externalities. It emphasizes the capital nature of adaptation by considering it as a stock that depreciates unless it is maintained. The investments, which are costly, increase the stock of adaptation capital. This in turn decreases the damage of pollution that is generated by the use of non-renewable resources. To analyze the transition to a low-carbon economy, the model includes a renewable resource that is a perfect substitute to the non-renewable one. For given initial levels of pollution and adaptation capital, we present the problem of social planner in a general form and characterize the optimal paths of resource uses and investments in adaptation capital.

As adaptation policy affects the marginal damage of pollution, it appears in the driver formula for the marginal cost of non-renewable resource (modified Hotelling rule), which brings different consequences with respect to the model without adaptation. An important one is the possibility of a si-

multaneous use regime within a model of two perfect substitute resources. This is due to the possibility of compensating the marginal damage of generated pollution by increasing the stock of adaptation, which will keep the marginal cost of non-renewable resource equal to that of renewable. However, this cannot go on forever as investments in adaptation are costly. There is an efficient level of adaptation that equalizes the marginal benefit gained by reduction in damages to the marginal cost of investment that maintains the stock at its level. A steady state exists when the adaptation capital is at that level, and the marginal cost of non-renewable resource equals to that of renewable. The optimal path reaches this steady state and after that it uses only renewable resource indefinitely.

The unique optimal path can be one of many different types depending on the initial state. For example, for a low level of pollution and low adaptation, it is optimal to start with using only non-renewable resource, then reach the steady state by following a gradual transition to the renewable resource. If pollution is too high and adaptation is low, then it is never optimal to use the non-renewable resource, and adaptation capital adjusts to its efficient level. When the initial adaptation is too high, there is no possibility for the simultaneous use regime to appear. In this case, adaptation capital will be decreased by low investments, only non-renewable resource is going to be used and at a certain date there will be an instantaneous switch to the renewable resource. The final level of pollution can be higher if the initial adaptation capital is too high.

Indeed, the optimum highly depends on the properties of underlying functions such as utility, damage, extraction cost, vulnerability and opportunity cost of investment in adaptation. The efficient level of adaptation decreases when the investment cost of adaptation is too high, whereas larger damages will lead to a higher adaptation. If adaptation's capability of reducing damages is low, then the steady state levels of adaptation and pollution will decrease. Another parameter of interest is the depreciation rate of

adaptation capital. When it is too high, meaning less durable adaptation, the cost of maintaining adaptation capital rises which in turn lowers the steady state levels for pollution and adaptation capital and reduces the time span of transition to renewable resource.

The analysis in this work focuses on the capital aspect of adaptation, and it abstracts from some factors that could be interest of further research. Indeed, relaxing the irreversible pollution assumption would change the characteristics of the optimum. Moreover, the damage function is defined to be continuous. Consideration of a catastrophic outcome when the pollution level rises too high, or a cap on pollution accumulation would change the optimum as well. The transition in the outcome is always beneficial, when it exists. However, if the transition itself brings costs due to the change of resource utilization then the compensation mechanism during the transition would work differently. Finally, taking into account the regional differences in pollution damages and cost of adaptation would provide more extended results.

Appendix

3.A Proof of Lemma 3.1

Let $\tilde{D}$ be the inverse of damage function that is presented in Assumption 3.3 ($\tilde{D} := D^{-1}$). It is increasing and concave in its argument ($\tilde{D}' > 0$ and $\tilde{D}'' < 0$). Taking the derivative of $\hat{Z}(A)$ with respect to A yields

$$\hat{Z}'(A) = \tilde{D}'((\rho + \delta)H'(\delta A)/(-\eta'(A)))(\rho + \delta) \left(\frac{\delta H''(\delta A)}{-\eta'(A)} + \frac{H'(\delta A)\eta''(A)}{\eta'(A)^2} \right), \quad (3.27)$$

which is positive due to Assumptions 3.3 to 3.5.

3.B Proof of Proposition 3.1

We first investigate if the system in (3.12-3.15) has saddle path property. The Jacobian of (3.12-3.15) is

$$\mathbf{J}^{ss} = \begin{bmatrix} 0 & 0 & R'(p) & 0 \\ 0 & -\delta & 0 & I'(\gamma) \\ -(\rho G'(Z) + \eta(A)D''(Z)) & -\eta'(A)D'(Z) & \rho & 0 \\ \eta'(A)D'(Z) & \eta''(A)D(Z) & 0 & \rho + \delta \end{bmatrix} \quad (3.28)$$

The trace of Jacobian is $\text{Tr}\mathbf{J}^{ss} = 2\rho$. We will use the theorem in Dockner (1985) that presents two conditions for the saddle path property of canonical four dimensional systems. These conditions depend on the sign of determinant of the Jacobian. The first case is $\det(\mathbf{J}^{ss}) < 0$ that ensures the saddle point property. To check that, we compute the determinant of

$\mathbf{J}^{ss}$ written as follows:

$$\det \mathbf{J}^{ss} = \underbrace{(-R'(p))}_{>0} \underbrace{(\eta(A)D''(Z) + \rho G'(Z))}_{>0} \underbrace{(\delta(\rho + \delta) + \eta''(A)D(Z)I'(\gamma))}_{>0} - \underbrace{(\eta'(A)D'(Z))^2 I'(\gamma)}_{>0} \quad (3.29)$$

If the value of third term in (3.29) offsets the second term then the sign of determinant is negative ($\det(\mathbf{J}^{ss}) < 0$). In this case the characteristic equation associated to $\mathbf{J}^{ss}$ has two positive and two negative roots. Then the system has the saddle path property in this case.

In the contrary case, if the sign of determinant is positive ($\det(\mathbf{J}^{ss}) > 0$), the saddle path property of the system can be investigated by calculating the sum all diagonal second order minors of $\mathbf{J}^{ss}$. Define Ω to be this sum written as follows:

$$\Omega := \begin{vmatrix} \partial \dot{Z} / \partial Z & \partial \dot{Z} / \partial p \\ \partial \dot{p} / \partial Z & \partial \dot{p} / \partial p \end{vmatrix} + \begin{vmatrix} \partial \dot{A} / \partial A & \partial \dot{A} / \partial \gamma \\ \partial \dot{\gamma} / \partial A & \partial \dot{\gamma} / \partial \gamma \end{vmatrix} + 2 \begin{vmatrix} \partial \dot{Z} / \partial A & \partial \dot{Z} / \partial \gamma \\ \partial \dot{\gamma} / \partial A & \partial \dot{\gamma} / \partial \gamma \end{vmatrix} \quad (3.30)$$

The last term is null and the value of Ω is given by

$$\Omega = \underbrace{(R'(p))}_{<0} \underbrace{(\eta(A)D''(Z) + \rho G'(Z))}_{>0} - \underbrace{(\delta(\rho + \delta) + \eta''(A)D(Z)I'(\gamma))}_{>0}. \quad (3.31)$$

The sign of Ω is negative ($\Omega < 0$). As the theorem in Dockner (1985) states, if $\det(\mathbf{J}^{ss}) > 0$ and $\Omega < 0$ then the system satisfies the conditions to have saddle path property.

Therefore, we verify that conditions for the saddle path property are satisfied in both cases for the sign of determinant. We conclude that the system in (3.12-3.15) has the saddle path property.

The existence of the steady state is already shown and discussed by equation (3.26). Indeed, the occurrence of this steady state depends on the initial levels of pollution (Z_0) and adaptation capital (A_0). To proceed on identification of different possibilities, we will study two boundaries in (A, Z) space that are characterizing the optimal behavior.

One boundary is characterized by the curve given in equation $\dot{p}_t = \rho(p_b - G(Z)) - \eta(A)D'(Z) = 0$ (3.16). Consider a point on this curve in (A, Z) space by fixing the level of adaptation capital (A) . For a given A , if the level of pollution is higher, $(Z' > Z)$ then $\rho(p_b - G(Z')) < \eta(A)D'(Z')$, meaning that the marginal benefit of one unit of extraction is lower than the marginal effective damage it inflicts when $p = p_b$. In order to make the resource beneficial p must increase. But when $p > p_b$ there is no extraction ($R = 0$ and $\dot{Z} = 0$) due to conditions (3.10) and (3.11). Hence regime 3 is going to be optimal for the values of Z that are above this curve. Below that curve, for $(Z'' < Z)$, p takes lower values ($p < p_b$), hence regime 1 is optimal ($\dot{Z} > 0$).

To analyze the dynamics of pollution and adaptation capital, and occurrences of different regimes, we need to study the behavior of the system around the other loci ($\dot{A} = 0$ and $\dot{\gamma} = 0$). These loci are given by function $\hat{Z}(A)$ that is presented in Definition 3.2. Consider a point on this curve in (A, Z) space, that is $(A, Z) = (A, \hat{Z}(A))$. On this point $\dot{\gamma} = 0$, hence $\gamma = -\eta'(A)D(Z)/(\rho + \delta)$ and $\dot{A} = I^*(-\eta'(A)D(Z)/(\rho + \delta)) - \delta A = 0$. An increase in Z , $(Z' > Z)$ will lead to an increase in $I^*(.)$ since $I^{*'} > 0$ and $\partial I^*(.)/\partial Z = I^{*'}(.)(-\eta'(A)D''(Z)/(\rho + \delta)) > 0$. Then for a given A around point $(A, \hat{Z}(A))$, $\dot{A} > 0$ for the values of Z that are higher than $\hat{Z}(A)$ and $\dot{A} < 0$ for the lower values.

These results are presented in diagram 3.1, which shows different possibilities of optimal paths presented in Proposition 3.1. It can be seen that regime 2 can exist only on curve (3.16), and only for the values below $Z < Z^{ss}$ that is characterized by (3.22-3.24). If $Z \geq Z^{ss}$, then there is no possibility of increasing Z and keeping the adaptation capital at a steady level efficiently. In this case, an optimal path arriving to curve $\dot{p}_t = \rho(p_b - G(Z)) - \eta(A)D'(Z) = 0$ will instantaneously switch to regime 3 if $A > A^{ss}$ and $Z \geq Z^{ss}$.

Consequently, we conclude that there exists a unique path for any given

initial state (Z_0, A_0) that leads to the steady state.

3.C Proof of Lemma 3.2

As in Lemma 3.1, let $\tilde{D}$ denote the inverse of damage function D . When the stock of adaptation capital is given, the change in $\hat{Z}(A; \delta)$ with respect to the parameter δ is

$$\frac{d\hat{Z}(A; \delta)}{d\delta} = \tilde{D}'((\rho + \delta)H'(\delta A)/(-\eta'(A))) \left(\frac{H'(\delta A) + (\rho + \delta)H''(\delta A)A}{-\eta'(A)} \right), \quad (3.32)$$

which is positive due to Assumptions 3.3 to 3.5.

3.D Proof of Proposition 3.2

First we will prove the first part of proposition. Recall that the steady state value of adaptation capital is determined by equation (3.26). We are interested in how the intersection point of two curves in LHS and RHS change with the rate of depreciation. To check that, we look how the value of each side changes with δ for a given A . We have:

$$d\text{LHS}/d\delta = -\rho G''(\hat{Z}(A; \delta)) \frac{d\hat{Z}(A; \delta)}{d\delta} < 0 \quad (3.33)$$

$$d\text{RHS}/d\delta = \eta(A) D''(\hat{Z}(A; \delta)) \frac{d\hat{Z}(A; \delta)}{d\delta} > 0 \quad (3.34)$$

Equation (3.33) shows that the value of LHS is lower when δ is higher. This means that LHS decreases faster in A when δ is higher. Similarly, equation (3.34) shows that the value of RHS is increasing in δ for a given A , which implies that RHS increases faster in A as δ increases. Consequently, the intersection point of two curves is going to occur at a lower value of A , which proves that A^{ss} is decreasing in δ . This is shown in figure 3.1.

As A^{ss} is decreasing in δ , the pollution level of the steady state (Z^{ss}) must also decrease in δ to keep equation (3.24) hold true. This completes the first part of the proof.

A conclusion of the first part is that the steady state levels of pollution and adaptation capital (Z^{ss}, A^{ss}) are going to be at their highest when $\delta = 0$. There is no cost of maintaining adaptation capital as it is irreversible and it remains indefinitely when installed. In this case, the curve $\hat{Z}(A)$ is the flattest (by Lemma 3.2), and $\dot{p} = 0$ for $p = p_b$ remains on its position (as it does not depend on δ) in Diagram 3.1. Thus the intersection point of two curves will be at farthest right, bringing the steady state levels of pollution and adaptation at their highest level. For a path that contains simultaneous regime within, the length of transition will be longest in this case. Then the level of steady state and the time spent on simultaneous regime are related for a path that contains regime 2. When δ is higher, the curve $\hat{Z}(A)$ is going to be steeper and the intersection point will move below. For a path that contains regime 2, the time of arrival to the steady state will get shortened and it will be shortest when $\delta = 1$.

Chapter 4

The role of capacity building on technology adoption under imperfect competition

THOMAS FAGART AND BARIS VARDAR

4.1 Introduction

New technologies develop rapidly and firms constantly face adoption decisions. The industrial organization and environmental economics literatures usually consider that the adoption of a new technology allows firms to decrease their marginal cost of production. This is the case when the new technology corresponds to a more efficient management technique, a new software or a new method to process inputs. However, technological progress also often takes the form of the creation of new production tools such as a new robot capable of producing a piece more quickly or a new aircraft consuming less kerosene. In that case, firms have to buy new production tools to benefit from this new technology. The cost reduction implied by the new technology is therefore only effective for the production

done using these new tools, and the rest of the production of the firms remains with the same efficiency that they had before adoption.

This paper studies how the adoption of production tool technology differs from the adoption of classical marginal cost reducing technology, and how it changes the way firms compete. It shows the existence of symmetric and asymmetric equilibria in which firms may use different technologies at the same time.

For example, in the commercial aircraft market, Airbus sells the A321 model with two different engine options: current engine option (CEO) and new engine option (NEO). CEO's price is 114.9 million US dollars and NEO's price is 125.7 million US dollars.¹ NEO is more expensive to buy but it reduces the fuel burn per seat by 20 percent (and also improves payload capacity and range).² The problem of the airline companies is to decide to invest in which type of aircraft at which quantity. The orders and deliveries report shows airline companies ordering either the CEO, or the NEO, or even the two altogether. For instance, in 2015 Frontier Airlines ordered 10 CEOs, Air Lease Corp. ordered 30 NEOs, and ANA Holdings ordered 4 CEOs and 3 NEOs.³ We are interested in the economic mechanisms underlying this kind of problem and observation.

When firms have no interest to delay investment, they would invest as soon as possible, and only in the capacity using the most efficient technology, i.e. the technology with the lowest inter-temporal cost (purchasing price plus the discounted cost of production). In such case, investments are done as if the technology was a cost margin reducing technology. However, when firms are financially constrained, investing in the technology with the cheapest purchasing price allows the firm to grow faster. Firms then may wish to invest in this inefficient technology in order to increase their short

¹New Airbus aircraft list prices for 2016, Airbus S.A.S, 12 January 2016.

²A321 state-of-the-art capabilities and technical details, Airbus S.A.S, retrieved on 17 January 2016

³Airbus orders and deliveries spreadsheet, Airbus S.A.S, 30 November 2015.

run profits, even though it reduces their future profits by increasing their production costs. The inefficient technology also generates a strategic effect: it permits one firm to preempt its opponent, building more capacity in the short run and reducing the future investment incentives of its opponent. These mechanisms explain why firms may use different technologies at the same time, and imply that the adoption of a production tool technology is slower than the adoption of a classic marginal cost reducing technology.

More precisely, we develop a two-period model in which firms' production is determined by their level of capacities. There are two types of capacity embodying two different types of technology. One type has a purchasing price higher than the other one, but it produces at a lower cost. This technology is also assumed to be more efficient, meaning that the inter-temporal cost of unit production is inferior for the capacity with the higher purchasing price. Firms compete *à la Cournot*. In the first period, firms are considered as entrepreneurs and have no initial capacity, but they possess an initial amount of funds in order to enter the market. Their capacity investment is then limited by their initial endowments. In the second period, firms have access to a perfect credit market and can invest as they wish to increase their capacities.

The monopoly faces a tradeoff between investing in the efficient capacity in the first period but growing slowly, and investing in the inefficient capacity and growing faster but facing a larger production cost in both periods. The optimal solution is then a mixture of both types of capacity, and the total capacity of the mixture does not depend on the financial constraint.

When there is a duopoly in the market, there may exist two different types of equilibrium: symmetric and asymmetric. In the symmetric equilibrium, firms invest in the same way as the monopoly, but adjusted to duopoly levels. Each firm has the same mixture of efficient and inefficient technology as its competitor, and the same market share. There may also exist other equilibria that are asymmetric, in which one of the firms overinvests in the

inefficient capacity in the first period. This allows the firm to increase its total capacity above the final total capacity of the symmetric case, committing itself to a larger production for the second period. The opponent reacts to this preemption by investing less in the first period, focusing on the efficient capacity. In the second period the preempted firm is the only firm to invest, but it does not catch up its rival. The preempting firm finishes with a larger market share, producing mostly with the inefficient technology whereas the preempted firm stays smaller but more efficient.

These results lead to two unexpected recommendations for the policy maker. First, when an industry is financially constrained, a duopoly uses more old technology compared to a monopoly, thus competition makes the industry to use more inefficient capacity. In the case where the old technology generates a negative externality, the increase of competition may lead to a lower welfare if welfare loss due to the externality exceeds the usual welfare gain due to competition. Second, an increase in the price of old technology may increase its utilization. Indeed, when the price of old capacity increases, the firm has to decrease its total capacity since it is financially constrained. To avoid a too large reduction of its total capacity, the firm can substitute its investment in new capacity by old one, and an increase in the price of old technology may lead to an increase in the quantity of old capacity used. Other comparative statics are as expected.

The next subsection reviews the related literature. Section 4.2 presents the model framework. Section 4.3 studies the decision of a monopoly and Section 4.4 studies the duopoly behavior. Section 5.9 concludes.

4.1.1 Related literature

This work is related to several strands of literature in industrial organization, operation research and corporate investment.

In industrial organization, since the pioneering works of Reinganum (1981),

Fudenberg and Tirole (1985) and Gaimon (1989), there has been a large literature studying technology adoption. Authors have considered the impact of learning, timing, uncertainty, environmental impacts and competition. However, to our knowledge, all papers consider a marginal cost reducing technology. We differ from this assumption by modeling production tool technologies.

For example, Stenbacka and Tombak (1994) study the timing of adoption of a new technology with uncertainty and they emphasize that the level of uncertainty can affect the dispersion between the equilibrium timings of adoption. Hoppe (2002) provides a survey of theoretical results and empirical evidence on the timing of adoption of new technologies. Huisman and Kort (2004) study the adoption decision in the case where firms take into account possible future technological improvements and Hoppe and Lehmann-Grube (2005) emphasize the role of R&D costs of process innovation and product innovation that generate a second-mover advantage in technology adoption games. Milliou and Petrakis (2011) investigate the timing of adoption with a focus on product market competition and they present results showing that different market features, such as the type and toughness of competition, can change the incentives for adoption. About the environmental impacts, Sanin and Zanjaj (2011) study the influence of technology adoption on the prices of tradable emission permits. These few examples give a view of the diversity on the literature on technology adoption.

In operation research, our model is similar to some studies regarding the electricity generation markets. These studies question whether to invest in inefficient and cheap generation capacity (e.g. a base-load technology like coal-fired generator (CFG)) or to invest in efficient and expensive generation capacity (e.g. combined cycle gas turbines (CCGT)), knowing that there will be demand or supply uncertainties in the future. For example, Murphy and Smeers (2005), Tishler et al. (2008), Meunier (2010) and Mil-

stein and Tishler (2012) study generation capacity mixture and expansion in different models of investment. They investigate the roles of different competition structures and show the possibilities of underinvestment or precautionary investment in electricity markets. The present work abstracts from the role of uncertainty, showing that the presence of a financial constraint is enough to induce the firms to invest in different types of capacity.

The presence of financial constraints is investigated by some studies in corporate investment literature. Fazzari et al. (1988) stress that internal funds and external finance are not perfect substitutes due to asymmetric information and capital market imperfections. They empirically show that the financial constraint is particularly active in the short run, and for the start-up ventures or small sized firms. Carpenter and Petersen (2002) discuss and empirically verify the reasons underlying the financial constraints in high-tech industry. More recently, Almeida and Campello (2007) show that firms with low level of asset tangibility are financially constrained in their investment decisions. Feichtinger et al. (2008) study the differences of disembodied and embodied technical progress when the firms have financial constraints. Differences in investment decisions in new or used capital is studied by Eisfeldt and Rampini (2007). They show that the firms are attracted to invest in used capital due to the financial constraints. The financial constraint introduced in this paper is in line with these findings: It constraints only the small firms, only in the short run. Our work contributes to this branch of literature by showing that the presence of financial constraints in imperfectly competitive markets can lead to the use of inefficient technologies as well as asymmetric outcomes in terms of technology and market shares.

4.2 The framework

4.2.1 The Model

We consider a two-period model of competition in production capacity. At each period, firms first invest in new units of capacity then determine their level of production. We assume irreversible investment and full utilization of capacity.⁴ The price is determined by the total quantity of the industry. In the first period, firms start with no initial capacity and face a financial constraint which limits their investment opportunities. In the second period, firms are free to invest as they wish.

There are two different types of capacity available in the market that embody two different kinds of technology. The purchasing prices of the two capacities are p and $\tilde{p}$ and the unit costs of production by using the two capacities are c and $\tilde{c}$, respectively. We call the more efficient technology as the new technology and it has a lower cost of production ($\tilde{c} < c$) but more expensive to purchase ($\tilde{p} > p$). Thereafter we will speak of old (new) capacity to name the capacity using the old (new) technology.

We make the following assumption:

Assumption 4.1. *The new capacity is more efficient than the old one:*
 $\tilde{p} + \tilde{c} < p + c$.

Assumption 4.1 ensures that firms have incentives to invest in new capacity. It means that the cost of buying the capacity to produce one unit of output

⁴Including the possibility of underutilization of capacity would make the model more realistic but at the cost of computational complexity. This will change our result in two different ways. If firms prefer to use their old capacity than buying new one in the long run ($\tilde{p} + \tilde{c} > c$), then assuming capacity underutilization reduces the possibility of the existence of asymmetric equilibrium given in Proposition 3, but it still may exist. If not ($\tilde{p} + \tilde{c} < c$), the firms always prefer to invest in new capacity than using their old one in the long run, therefore no preemption using old capacity is possible. In that case, the asymmetric equilibrium vanishes.

with new capacity is lower compared to that of old capacity. Under this assumption, a firm facing no constraint would invest only in new capacity.

In the monopoly case, we denote k_t and $\tilde{k}_t$ the level of old and new capacity at time t . In the duopoly case, we denote k_t^i and $\tilde{k}_t^i$ the capacities of firm i , with $i \in \{A, B\}$. Let $\mathcal{K}$ be the total capacity of the industry. The profit of firm i at time t is then:

$$\Pi_t^i = (k_t^i + \tilde{k}_t^i)P(\mathcal{K}_t) - ck_t^i - \tilde{c}\tilde{k}_t^i - p(k_t^i - k_{t-1}^i) - \tilde{p}(\tilde{k}_t^i - \tilde{k}_{t-1}^i) \quad (4.1)$$

under the capacity constraints

$$k_t^i \geq k_{t-1}^i \text{ and } \tilde{k}_t^i \geq \tilde{k}_{t-1}^i. \quad (4.2)$$

For simplicity, assume that the price is linear, $P(\mathcal{K}) = 1 - \mathcal{K}$, and the unit production cost of new capacity is zero ($\tilde{c} = 0$).

We introduce the financial constraint of the first period as follows:

$$pk_1^i + \tilde{p}\tilde{k}_1^i \leq G \quad (4.3)$$

where G denotes the initial endowment of firm i . This constraint implies that the purchasing cost of capacity in the first period cannot exceed the given initial endowment.

Firm i aims to maximize its discounted total profit:

$$\Pi^i = \Pi_1^i + \delta\Pi_2^i \quad (4.4)$$

where $\delta \in (0, 1)$ denotes the discount rate. In this setup, investments in the first period can be viewed as short run decisions while the second period represents the long run.⁵ We focus on sub-game perfect equilibria. Let $\vec{K}_t =$

⁵This two-period game is equivalent to an infinite horizon game with a discount factor β if firms' capacities are assumed to remain constant after the second period and

$(k_t^i, \tilde{k}_t^i, k_t^j, \tilde{k}_t^j)$ be the vector of capacities at time t . By backward induction, $(\vec{K}_1^*, \vec{K}_2^*)$ is a sub-game perfect equilibrium if $\vec{K}_2^*(\vec{K}_1)$ is a mapping which verifies:

$$\left(k_2^{i*}, \tilde{k}_2^{i*}\right) = \max_{k_2^i, \tilde{k}_2^i} \Pi_2^i \left(\vec{K}_1, k_2^i, \tilde{k}_2^i, k_2^{j*}, \tilde{k}_2^{j*}\right) \text{ under (4.2),} \quad (4.5)$$

and $\vec{K}_1^*$ verifies:

$$\left(k_1^{i*}, \tilde{k}_1^{i*}\right) = \max_{k_1^i, \tilde{k}_1^i} \Pi^i \left(k_1^i, \tilde{k}_1^i, k_1^{j*}, \tilde{k}_1^{j*}, \vec{K}_2^*(k_1^i, \tilde{k}_1^i, k_1^{j*}, \tilde{k}_1^{j*})\right) \quad (4.6)$$

under (4.2) and (4.3).

The sub-game perfect equilibrium path is then $(\vec{K}_1^*, \vec{K}_2^*(\vec{K}_1^*))$. In order to emphasize the role of the financial constraint, the next subsection describes firms' choices when there is no financial constraint in the first period.

4.2.2 The role of financial constraint

In the absence of a financial constraint, firms would invest in the first period, as any capacity installed in the first period allows to produce in both periods. The choice between old and new capacity then reduces to a simple cost-benefit analysis and firms invest only in new capacity.

Result 4.1. *When firms are not financially constrained, firms invest only in the most efficient capacity and only in the first period.*

The profit of the firm in (4.1) can be rewritten as

$$\Pi_t^i = (k_t^i + \tilde{k}_t^i) [P(\mathcal{K}_t) - (\tilde{p} + \tilde{c})] + [(\tilde{p} + \tilde{c}) - (p + c)] k_t^i + p k_{t-1}^i + \tilde{p} \tilde{k}_{t-1}^i. \quad (4.7)$$

$\delta = \frac{\beta}{1-\beta}$. In the infinite horizon game in which capacities are not assumed to remain constant after the second period, there exist equilibria equivalent to the ones defined in Proposition 2 and 3, but also other equilibria may appear due to punishment scheme (as trigger strategies).

Given a constant total capacity ($k_t^i + \tilde{k}_t^i$), the profit in (4.7) is decreasing in k_t^i , as $(\tilde{p} + \tilde{c} < p + c)$ due to assumption 4.1. Since old and new capacities are perfect substitutes in production, in this case firms have interest to invest only in new (the most efficient) capacity. Moreover, firms invest only in the first period as every unit of capacity invested in the first period is also utilized in the second period.

Accordingly, when there is no financial constraint, the introduction of different types of capacities to model the technology choice has no impact on the firm's decision as the firm always has an incentive to invest immediately and in only one type of capacity. In reality, however, firms often delay their investment decisions due to financial constraints that arise from capital market imperfections and asymmetric information, as discussed in Section 4.1.1. In the following, we see how the interest to delay investment generates an incentive to invest in both types of capacities.

4.3 Monopoly

In this section we consider that there is only one firm in the market. The investment decision of the firm is highly dependent on the financial constraint and on the differences between the two technologies. Investing only in new capacity allows to produce at lower cost (in both periods), but limits the first period production, as the new capacity is more expensive to purchase compared to the old one. On the contrary, investing only in old capacity increases the first period production, but also increases the cost of production in both periods. The optimal strategy of the firm in the first period is then to invest in a mixture of the two capacities, balancing the tradeoff between rapid growth and long run cost efficiency.

When the financial constraint is binding and the firm invests in the second period, its profit in (4.1) can be rewritten as a function of the total capacity

and old capacity installed in the first period:

$$\Pi = \left(1 - \tilde{p} + \delta\tilde{p} - (k_1 + \tilde{k}_1)\right) (k_1 + \tilde{k}_1) + \delta \left(\frac{1 - \tilde{p}}{2}\right)^2 - (p + (1 + \delta)c - \tilde{p})k_1. \quad (4.8)$$

The first and second terms of (4.8) represent the profit that the firm would make if its total capacity was composed only of new capacity.⁶ The third term represents the cost of using old capacity instead of new one for a given total capacity. If there were no links between the level of old capacity and total capacity, then equation (4.8) shows that the firm would never invest in old capacity. However, due to the financial constraint, if the firm wants to increase its total capacity, it has to reduce its level of new capacity in order to purchase more of old capacity. Rewriting (4.3) yields the level of old capacity as a function of the total capacity:

$$k_1 = \frac{\tilde{p}(\tilde{k}_1 + k_1) - G}{\tilde{p} - p}. \quad (4.9)$$

Replacing the level of old capacity in equation (4.8) by (4.9) allows to express the profit of the firm as a function of total capacity. As we consider the case in which the financial constraint is binding, all the initial endowment must be spent by investing only in old capacity, only in new capacity or in a mixture of them. Thus the total capacity belongs to the interval $\left[\frac{G}{\tilde{p}}, \frac{G}{p}\right]$. The objective of the firm then reduces to a simple one dimensional maximization problem where the decision variable is the total capacity. Let Ψ_M be the interior solution of this problem given by:

$$\Psi_M = \frac{1 - \tilde{p} - (1 + \delta)\tilde{p} \left(\frac{c}{\tilde{p} - p} - 1\right)}{2} \quad (4.10)$$

This interior solution equalizes the marginal revenue of total capacity and

⁶More precisely, the first period profit is $(1 - \tilde{k}_1)\tilde{k}_1 - \tilde{p}\tilde{k}_1$ whereas the discounted second period profit is $\delta((\frac{1 - \tilde{p}}{2})^2 + \tilde{p}\tilde{k}_1)$, as the capacity purchased in the first period is also used in the second one.

the marginal cost of changing the composition of capacity mixture. It does not depend on the financial endowment. When Ψ_M is outside of the feasible interval, the solution lies on the boundaries:

$$(k_1^*, \tilde{k}_1^*) = \begin{cases} \left(\frac{G}{p}, 0\right) & \text{if } \Psi_M > \frac{G}{p} \\ \left(\frac{\tilde{p}}{\tilde{p}-p}(\Psi_M - \frac{G}{p}), \frac{p}{\tilde{p}-p}(\frac{G}{p} - \Psi_M)\right) & \text{elsewhere} \\ \left(0, \frac{G}{\tilde{p}}\right) & \text{if } \Psi_M < \frac{G}{\tilde{p}} \end{cases} \quad (4.11)$$

The above analysis assumes that the firm invests in the second period. This is the case when $\frac{1-\tilde{p}}{2} \leq \frac{G}{\tilde{p}}$. Indeed, the capacity maximizing second period profit is $\frac{1-\tilde{p}}{2}$, as the firm in the second period is not financially constrained. When the firm invests in old capacity (fully in old capacity or in a mixture), its first period total capacity is always inferior to the second period's optimal capacity.⁷ When the firm uses only new capacity, it invests in the second period if its financial endowment is sufficiently low ($\frac{G}{\tilde{p}} \leq \frac{1-\tilde{p}}{2}$). In the other case, the firm invests only in the first period, only in new capacity, and to the level $\frac{G}{\tilde{p}}$.

Finally, when the firm is not financially constrained, investments are made only in the first period, only in new capacity, and to the level $\frac{1}{2} \left(1 - \frac{\tilde{p}}{1+\delta}\right)$. This behavior is optimal as long as the financial constraint is not binding $\left(\frac{G}{\tilde{p}} \geq \frac{1}{2} \left(1 - \frac{\tilde{p}}{1+\delta}\right)\right)$.

The following proposition sums up the monopoly outcome.

Proposition 4.1. *Assume 4.1. Then,*

- *If $\frac{G}{\tilde{p}} < \frac{1-\tilde{p}}{2}$, the first period decision of the monopoly is given by the pair $(k_1^*, \tilde{k}_1^*)$ described in equation (4.11). In the second period, the monopoly installs a total capacity $k_2^* + \tilde{k}_2^* = \frac{1-\tilde{p}}{2}$ and invests only in new capacity ($k_2^* = k_1^*$).*

⁷As the first period total capacity in a mixture is $\Psi_M < \frac{1-\tilde{p}}{2}$.

- If $\frac{1-\tilde{p}}{2} \leq \frac{G}{\tilde{p}} < \frac{1}{2} \left(1 - \frac{\tilde{p}}{1+\delta}\right)$, the first period decision of the monopoly is to invest only in new capacity, to the level $\frac{G}{\tilde{p}}$. In the second period, the monopoly does not invest.
- If $\frac{G}{\tilde{p}} \geq \frac{1}{2} \left(1 - \frac{\tilde{p}}{1+\delta}\right)$, the first period decision of the monopoly is to invest only in new capacity, to the level $\frac{1}{2} \left(1 - \frac{\tilde{p}}{1+\delta}\right)$. In the second period, the monopoly does not invest.

Proof. See Appendix. □

Proposition 1 shows that the monopoly behavior is highly dependent on its financial endowment. Figure 4.1 illustrates the role of the financial endowment on the choice of capacities (total capacity, old capacity, and new capacity). When the financial endowment is low, the firm wishes to grow as fast as possible and thus invests only in old capacity. For a higher financial endowment, the firm balances the tradeoff between the efficiency of new capacity and the size advantage of old capacity, investing in a mixture of the two technologies. The total capacity of the firm is constant as long as it invests in both capacities. However, for a large financial endowment, the firm invests only in new capacity, and an increase in its endowment once again increases its total capacity. Finally, when the financial endowment is too high, the firm behaves as if there is no financial constraint and invests at a level that is the optimum of the problem without a financial constraint.

The contrast between Result 4.1 and Result 4.2 emphasizes the role of financial constraint.

Result 4.2. *There exists a range of financial endowment such that:*

- *The monopoly invests in a mixture of the two capacities.*
- *An increase in the initial endowment (G) increases the share of new capacity and decreases the share of old one, but does not impact the total capacity of the firm.*

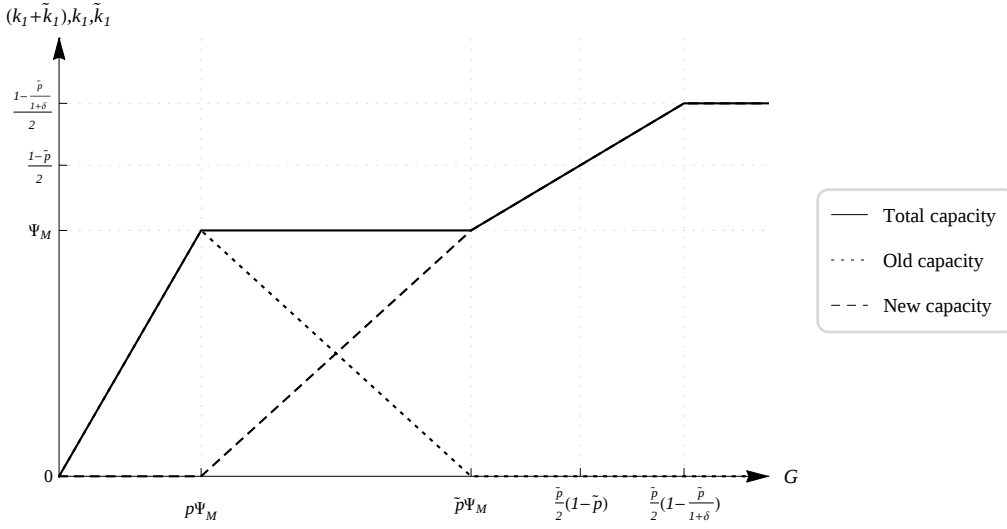


Figure 4.1 – First period capacity investment with respect to the financial endowment

The rest of the section presents the comparative static analysis done on this monopoly behavior. We focus on the range of financial endowment where the firm invests in a mixture of capacities. We consider that consumers are only affected by the price, and ignore any externality arising from the utilization of one or the other technology.

The next result exhibits the differences in outcomes when one of the technologies is not present in the market, by comparing the mixture outcome with the cases in which there is only old or only new technology in the market.

Result 4.3. *The profit of the firm is higher with two technologies than with only one technology. Furthermore:*

- *The introduction of a new technology is harmful for the consumer in the short run, but beneficial in the long run.*
- *The prohibition of the old technology is harmful for the consumer in the short run and neutral in the long run.*

If there is only old technology in the market, the firm would use all of its endowment to install old capacity in the first period $\left(\frac{G}{p}\right)$, before reaching a long run capacity $\left(\frac{1-c-p}{2}\right)$ that is inferior to the final total capacity of the case with two technologies $\left(\frac{1-\tilde{p}}{2}\right)$. This is due to the inefficiency of old capacity (see Assumption 4.1). On the contrary, the total capacity of the firm in the first period is superior with only old technology. Indeed, old capacity is cheaper than new one, and more of it can be installed with a given endowment. As consumer surplus increases with the level of production, this states Result 4.3.

When the old technology is prohibited, the firm invests all of its endowment in new capacity, leading to a first period level $\left(\frac{G}{\tilde{p}}\right)$ inferior to the total capacity of the two technology case (Ψ_M) . In the two technology case, the long run total capacity is determined only by the features of the new technology, and the prohibition of old technology has no impact on the consumer.

The next result discusses the impact of a change in the price of old capacity. One may expect that an increase in the price of old capacity would diminish its utilization by the firm. However, our result is more ambiguous.

Result 4.4. *The effect of a variation in the price of old capacity (p) on the percentage and quantity of old capacity used in the technology mixture depends on p :*

- *for a low value of p , an increase in p increases the utilization of old capacity (both in the short run and in the long run),*
- *for a high value of p , an increase in p reduces the utilization of old capacity (both in the short run and in the long run).⁸*

Moreover, an increase in the price of old capacity always decreases the total capacity in the short run (and has no impact in the long run).

⁸See Appendix for the analytic thresholds.

This unexpected result comes from the fact that an increase in the price of old capacity has two effects. First, for a given total capacity, the firm wants to increase its share of new capacity and to reduce its share of old one, as the new capacity becomes relatively cheaper to purchase. This is the price effect. However, when the price of old capacity increases, the total capacity of the monopoly would reduce due to the financial constraint and the price effect (the price of new capacity remains larger than the price of old). This increases the marginal profit of total capacity (as the profit is a concave function of total capacity), and make the firm willing to invest more in old capacity, i.e. willing to sacrifice more of its long run efficiency to increase its first period size, which is the substitution effect.

When the difference between the prices of two capacities is sufficiently large, the substitution effect dominates the price effect and induces the firm to increase its investment in old capacity. In the contrary, when the prices are too close, the price effect offsets the substitution effect, and the firm increases its share of new capacity.

There is no such ambiguity for the other policy tools: the price of new capacity and the marginal cost of production using old capacity. When the price of new capacity rises, the price effect and the substitution effect incentivize the firm to increase its investment in old capacity. Indeed, the price increase leads to a decline in total capacity, which increases the marginal profit of total capacity and induces the firm to install more old capacity. The substitution effect then works in the same direction as the price effect.

Result 4.5. *We have:*

- *An increase in the price of new capacity ($\tilde{p}$) decreases the percentage and the quantity of new capacity used in the technology mixture, both in the short run and in the long run. Moreover, it increases the total capacity in the short run, but decreases it in the long run.*

- *An increase in the marginal cost of production using old capacity (c) decreases the percentage and the quantity of old capacity used in the technology mixture, both in the short run and in the long run. Moreover it decreases the total capacity in the short run (and has no impact in the long run).*

To complete the comparative statics of the monopoly case, we discuss the effect of the time preference of the firm (δ). When the firm is more patient (a higher δ), the firm increases its level of new capacity and reduces the level of old capacity. The firm also decreases the total capacity in the short run (and makes no change in its long run choice of capacities). Indeed, the firm values more the long run efficiency of the new technology than the short run growth provided by the old technology.

4.4 Equilibria in the duopoly case

In this section, there are two entrepreneurs, A and B , present in the market. Two different equilibria may arise: symmetric and asymmetric. To investigate these equilibria, let us first focus on what happens in the second period.

4.4.1 Behavior of firms in the second period

In this period firms are not financially constrained and thus invest only in the most efficient capacity, the new one. The investment choice of a firm depends on the level of capacity held by its rival. If the rival's total capacity of the first period is inferior to the Cournot outcome ($k_1^j + \tilde{k}_1^j < \frac{1-\bar{p}}{3}$) then the firm increases its investment to take a larger share of the market until it reaches the Cournot outcome. In the contrary case ($k_1^j + \tilde{k}_1^j > \frac{1-\bar{p}}{3}$), the opponent is committed itself to a large production due to the level

of capacities installed in the first period, and the firm adapts its capacity according to the opponent's first period choice.

This is resumed formally as follows. Let k_1^i be the level of old capacity of firm i in the first period and $\tilde{k}_2^i$ the level of new capacity of firm i in the second period. Then the optimal investment decision of firm i is:

$$(k_2^{*i} + \tilde{k}_2^{*i}) = \left\{ \begin{array}{ll} \max \left\{ \frac{1-\tilde{p}}{3}, k_1^i + \tilde{k}_1^i \right\} & \text{if } k_1^j + \tilde{k}_1^j < \frac{1-\tilde{p}}{3} \\ \max \left\{ \frac{1-\tilde{p}-K_1^j-\tilde{K}_1^j}{2}, \tilde{k}_1^i + k_1^i \right\} & \text{if } k_1^j + \tilde{k}_1^j > \frac{1-\tilde{p}}{3} \end{array} \right\} \quad (4.12)$$

where $k_2^{*i} = k_1^i$ because the firm always invests in new capacity in the second period. We summarize the optimal investment decision of the firm in the second period in the following lemma:

Lemma 4.1. *Assume 4.1. For any vector of first period capacities $\vec{K}_1$, the equilibrium of the second period is to invest only in new capacity in order to reach the total capacity given in (4.12).*

Proof. See Appendix. □

Lemma 4.1 shows that the space of total capacities can be separated in three regions, as presented in Figure 4.1.⁹

In the area named *the no-move zone* (which is marked by the gray mesh) firms have no interest to invest in more capacity. In fact, in that area, the marginal value of an additional capacity is positive but inferior to the price of investment. So the firm wishes to produce more, but the return does not compensate the cost of investment. As we consider that the firms start with no capacity, no equilibria will take place inside this area.

When the first period total capacity of each firm is inferior to the Cournot outcome ($\frac{1-\tilde{p}}{3}$), the equilibrium strategy of period two is, for both firms,

⁹The formal description of these regions can be found in Appendix.

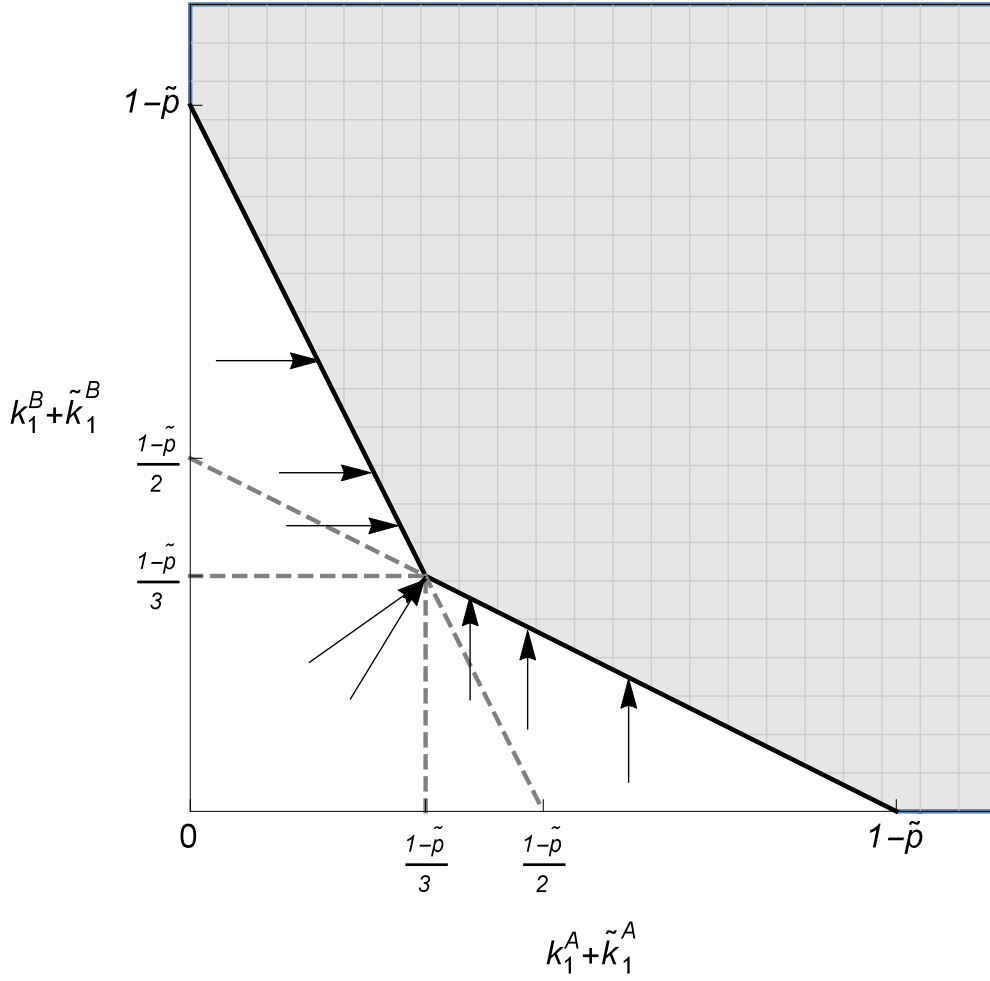


Figure 4.1 – The second period investment regions for duopoly

to invest until the Cournot outcome. We name this area *the symmetric zone*. This leads to a possible symmetric equilibrium in the first period. In this equilibrium, firms invest in a mixture of old and new capacities due to the tradeoff between investing as soon as possible in the first period and focusing on long run efficiency.

When one firm has a first period total capacity larger than the Cournot outcome and its opponent has a lower capacity, the equilibrium investment choice in the second period is for the smaller firm to invest and for the larger firm to do nothing. We name this area *the asymmetric zone*. The firm may reach this area if it preempts its opponent by investing mostly

in old capacity in the first period, in order to gain an advantage in the second period. This leads to the existence of an asymmetric equilibrium¹⁰, in which one of the firm (thereafter called *the preempting or the leader*¹¹) invests mainly in old capacity, in order to have a first period total capacity higher than the Cournot outcome. In this case, the best response of the other firm (thereafter called *the preempted or the follower*) is to invest less than its opponent in the first period, mostly based on new capacity, before getting closer to its rival in the second period. The preempted firm remains smaller than its opponent.

Figure 2 illustrates the different equilibrium paths corresponding to these symmetric and asymmetric outcomes in the industry. The next subsections investigate these cases in detail.

4.4.2 Case of symmetric equilibrium

Symmetric equilibrium can exist only if firms are not be able to reach the Cournot outcome by investing only in new capacity in the first period. In the contrary case the equilibrium is straightforward: firms invest only in the first period, and only in new capacity.

When their financial endowments are sufficiently low, firms face a tradeoff between short run growth and long run efficiency. The total capacity that the firms wish to install in the symmetric equilibrium is given by:

$$\Psi_D = \frac{1 - \tilde{p} - (1 + \delta) \tilde{p} \left(\frac{c}{\tilde{p} - p} - 1 \right)}{3} \quad (4.13)$$

If the financial endowment is too low that the firms cannot reach Ψ_D then the firms invest only in old capacity at the maximum possible level. If the

¹⁰There exists some parameter values such that the asymmetric or the symmetric equilibria do not exist.

¹¹This terminology has no relation with the Stackelberg game, as firms take their action simultaneously.

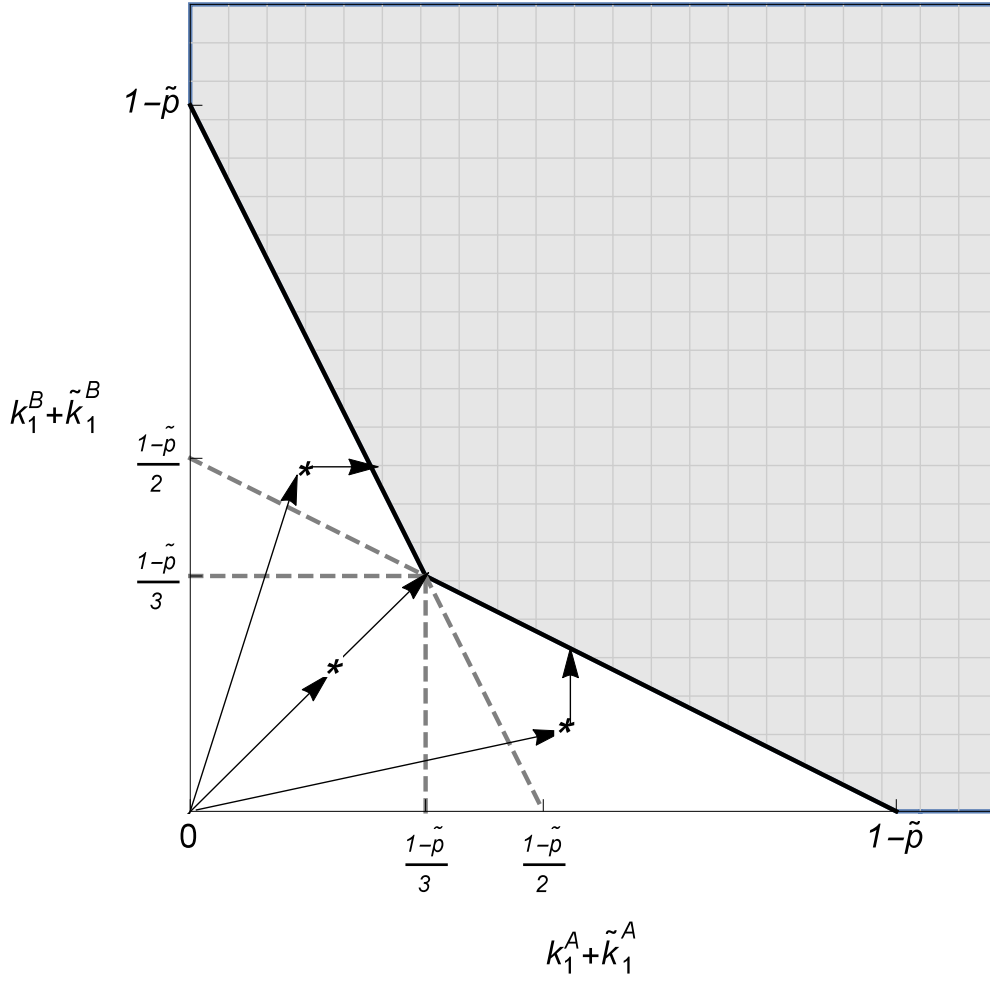


Figure 4.2 – Potential equilibrium paths

financial endowment is too high that the firms can reach Ψ_D using only new technology then the firms invest only in new capacity and reach a higher level than Ψ_D . We define k_1^{sym} and $\tilde{k}_1^{sym}$, the old and new capacity of the firm in the first period as:

$$(k_1^{sym}, \tilde{k}_1^{sym}) = \left\{ \begin{array}{l} \left(\frac{G}{p}, 0 \right) \text{ if } \Psi_D > \frac{G}{p} \\ \left(\frac{\tilde{p}}{\tilde{p}-p} \left(\Psi_D - \frac{G}{\tilde{p}} \right), \frac{p}{\tilde{p}-p} \left(\frac{G}{p} - \Psi_D \right) \right) \text{ elsewhere} \\ \left(0, \frac{G}{\tilde{p}} \right) \text{ if } \Psi_D < \frac{G}{\tilde{p}} \end{array} \right\} \quad (4.14)$$

The strategy which consists for each firm to invest $(k_1^{sym}, \tilde{k}_1^{sym})$ in the first period is a local equilibrium, meaning that there is no profitable deviation

inside the symmetric zone. To verify that (4.14) is an equilibrium strategy, we have to ensure that no firm has an incentive to deviate to an asymmetric strategy profile when its opponent invests $(k_1^{sym}, \tilde{k}_1^{sym})$. To characterize this profile, let $\Psi_{Asym}^{BR} = \frac{1}{2} \left[1 - \frac{1}{(1+\frac{\delta}{2})} \left(k_1^{sym} + \tilde{k}_1^{sym} + (1+\delta) c \frac{\tilde{p}}{\tilde{p}-p} - \delta \frac{\tilde{p}}{2} \right) \right]$, then the asymmetric strategy profile that is the best response to $(k_1^{sym}, \tilde{k}_1^{sym})$ given by:

$$(k_1^{BRasym}, \tilde{k}_1^{BRasym}) = \left\{ \begin{array}{l} \left(\frac{G}{p}, 0 \right) \text{ if } \Psi_{Asym}^{BR} > \frac{G}{p} \\ \left(\frac{\tilde{p}}{\tilde{p}-p} \left(\Psi_{Asym}^{BR} - \frac{G}{p} \right), \frac{p}{\tilde{p}-p} \left(\frac{G}{p} - \Psi_{Asym}^{BR} \right) \right) \text{ elsewhere} \end{array} \right\} \quad (4.15)$$

To ensure that $(k_1^{sym}, \tilde{k}_1^{sym})$ is an equilibrium, firms must be worse-off by deviating to the asymmetric best response strategy. Formally, the following condition must hold:

$$\Pi \left\{ (k_1^{sym}, \tilde{k}_1^{sym}); (k_1^{sym}, \tilde{k}_1^{sym}) \right\} \geq \Pi \left\{ (k_1^{BRasym}, \tilde{k}_1^{BRasym}); (k_1^{sym}, \tilde{k}_1^{sym}) \right\} \quad (4.16)$$

The following proposition characterizes the symmetric equilibrium in the duopoly case:

Proposition 4.2. *Assume 4.1. Then,*

- If $\frac{G}{p} \geq \frac{1-\tilde{p}}{3}$, there exists a unique sub-game perfect equilibrium of the game, given by: $\tilde{k}_1^{*i} = \min \left(\frac{G}{p}, \left(1 - \frac{\tilde{p}}{1+\delta} \right) \right)$, $\tilde{k}_2^{*i} = \tilde{k}_1^{*i}$ and $k_2^{*i} = k_1^{*i} = 0$.
- If $\frac{G}{p} < \frac{1-\tilde{p}}{3}$, there exists a symmetric sub-game perfect equilibrium if and only if condition (4.16) is true. In that case, the first period equilibrium capacities are given by (4.14) and the second period capacities are $\left(\frac{1-\tilde{p}}{3}, \frac{1-\tilde{p}}{3} \right)$.

Proof. See Appendix. □

Proposition 4.2 shows that the symmetric equilibrium strategy of the firms is similar to the one of monopoly case, but adjusted to duopoly levels. Therefore, results 2 to 5 of the monopoly hold for the symmetric equilibrium.

Result 4.6. *In the symmetric equilibrium of the duopoly, we have:*

- *When there is a financial constraint, the duopoly invests in a mixture of the two capacities.*
- *A decrease in the price of new capacity ($\tilde{p}$) or an increase in the marginal cost of production (c) or an increase in the discount rate (δ) reduce the percentage and the quantity of old capacity used in the industry.*
- *An increase in the price of old capacity (p) can increase or decrease the utilization of old capacity depending on the prices of capacities.*

4.4.3 Case of asymmetric equilibria

Besides the equilibrium previously considered, there is another possible behavior of the industry. One of the firms can overinvest in old technology, in order to increase its total capacity above the Cournot outcome, and commit itself to a larger production in the next period. In reaction to this strategy, its opponent reduces its total capacity in the first period, focusing on the efficient capacity. In the second period, the follower is the only firm to invest, only in new capacity, but it does not catch up its rival.

The equilibrium depends on the initial financial endowment available to the entrepreneurs. When this amount is too low, the firms cannot reach the Cournot outcome even by investing only in old capacity. In this case no preemption is possible. For larger amounts of financial endowment, when both firms invest in a mixture of capacity, the total capacity of the preempting firm is $\Psi_L^{nc} = \frac{(1+\delta)-(1+\delta)c\frac{\tilde{p}}{\tilde{p}-p}}{(3+2\delta)}$ and the total capacity of the preempted one is $\Psi_F^{nc} = \frac{(1+\frac{\delta}{2})-(1+\delta)^2c\frac{\tilde{p}}{\tilde{p}-p}+(\frac{3}{2}+\delta)\delta\tilde{p}}{(3+2\delta)}$. If the firms cannot reach these outcomes then firms have to invest only in one kind of capacity, in the same way as Proposition 2. When the preempting firm invests only in old capacity, the total capacity of the preempted firm is

$\Psi_F^c = \frac{1}{2} \left(1 - \frac{G}{p} - (1 + \delta) c \frac{\tilde{p}}{\tilde{p} - p} + \delta \tilde{p} \right)$, and when the preempted firm invests only in new capacity, the preempting one installs a total capacity of $\Psi_L^c = \frac{(1 + \frac{\delta}{2}) - \frac{G}{\tilde{p}} - (1 + \delta) \frac{c \tilde{p}}{\tilde{p} - p} + \frac{\delta}{2} \tilde{p}}{2 + \delta}$. This yields the level of capacities of the asymmetric local equilibrium (the vector of capacities such that there is no profitable deviation inside the asymmetric zone).

$$(k_1^L, \tilde{k}_1^L) = \left\{ \begin{array}{l} \left(\frac{\tilde{p}}{\tilde{p} - p} \left(\Psi_L^{nc} - \frac{G}{\tilde{p}} \right), \frac{p}{\tilde{p} - p} \left(\frac{G}{p} - \Psi_L^{nc} \right) \right) \text{ if } \Psi_L^{nc} \leq \frac{G}{p} \text{ and } \Psi_F^{nc} \geq \frac{G}{\tilde{p}} \\ \left(\frac{\tilde{p}}{\tilde{p} - p} \left(\Psi_L^c - \frac{G}{\tilde{p}} \right), \frac{p}{\tilde{p} - p} \left(\frac{G}{p} - \Psi_L^c \right) \right) \text{ if } \Psi_L^{nc} \leq \frac{G}{p} \text{ and } \Psi_F^{nc} < \frac{G}{\tilde{p}} \\ \left(\frac{G}{p}, 0 \right) \text{ if } \Psi_L^{nc} > \frac{G}{p} \end{array} \right\}, \quad (4.17)$$

and

$$(\tilde{k}_1^F, k_1^F) = \left\{ \begin{array}{l} \left(\frac{\tilde{p}}{\tilde{p} - p} \left(\Psi_F^{nc} - \frac{G}{\tilde{p}} \right), \frac{p}{\tilde{p} - p} \left(\frac{G}{p} - \Psi_F^{nc} \right) \right) \text{ if } \Psi_L^{nc} \leq \frac{G}{p} \text{ and } \Psi_F^{nc} \geq \frac{G}{\tilde{p}} \\ \left(\frac{\tilde{p}}{\tilde{p} - p} \left(\Psi_F^c - \frac{G}{\tilde{p}} \right), \frac{p}{\tilde{p} - p} \left(\frac{G}{p} - \Psi_F^c \right) \right) \text{ if } \Psi_L^{nc} > \frac{G}{p} \text{ and } \Psi_F^{nc} \geq \frac{G}{\tilde{p}} \\ \left(0, \frac{G}{\tilde{p}} \right) \text{ if } \Psi_F^{nc} < \frac{G}{\tilde{p}} \end{array} \right\} \quad (4.18)$$

Moreover, as in the case of symmetric equilibrium, we have to ensure that firms have no incentive to deviate from the asymmetric equilibrium strategies. Let $\Psi_{sym}^{BR} = \frac{1}{2} (1 - k_1^F - \tilde{k}_1^F - (1 + \delta) c \frac{\tilde{p}}{\tilde{p} - p} + \delta \tilde{p})$. Preempting firm's best response symmetric strategy when its opponent acts following the asymmetric strategy is as follows:

$$(k_1^{BRsym}, \tilde{k}_1^{BRsym}) = \left\{ \begin{array}{l} \left(\frac{\tilde{p}}{\tilde{p} - p} \left(\Psi_{sym}^{BR} - \frac{G}{\tilde{p}} \right), \frac{p}{\tilde{p} - p} \left(\frac{G}{p} - \Psi_{sym}^{BR} \right) \right) \text{ elsewhere} \\ \left(0, \frac{G}{\tilde{p}} \right) \text{ if } \Psi_{sym}^{BR} < \frac{G}{\tilde{p}} \end{array} \right\} \quad (4.19)$$

The preempting firm does not have any incentive to deviate from the asymmetric strategy if

$$\Pi \left\{ (k_1^L, \tilde{k}_1^L); (k_1^F, \tilde{k}_1^F) \right\} \geq \Pi \left\{ (k_1^{BRsym}, \tilde{k}_1^{BRsym}); (k_1^F, \tilde{k}_1^F) \right\} \quad (4.20)$$

or

$$\Psi_{sym}^{BR} > \frac{1 - \tilde{p}}{3} \quad (4.21)$$

hold true. Now we can characterize the asymmetric equilibrium with the following proposition:

Proposition 4.3. *Assume 4.1. If $\frac{G}{p} > \frac{1-\tilde{p}}{3}$, and (4.20) or (4.21) hold true, then there exists an asymmetric sub-game perfect equilibrium which consists of one firm to install $(k_1^L, \tilde{k}_1^L)$ in the first period, and for the other one to install $(k_1^F, \tilde{k}_1^F)$, before investing as described in Lemma 4.1.*

Proof. See Appendix. □

Even though the form of the asymmetric equilibrium differs from the symmetric one, most of the comparative static results remain valid. Indeed, results 2 to 5 of the monopoly hold for the asymmetric equilibrium.

Result 4.7. *In the asymmetric equilibrium of the duopoly, we have:*

- *When there is a financial constraint, the duopoly invests in a mixture of the two capacities.*
- *A decrease in the price of new capacity ($\tilde{p}$) or an increase in the marginal cost of production (c) reduce the percentage and the quantity of old capacity used in the industry.*
- *An increase in the price of old capacity (p) can increase or decrease the utilization of old capacity depending on the prices of capacities.*

However, in this case, the impact of the discount rate (δ) is ambiguous. Two effects are in place. When the discount rate increases, firms value the future more and prefer to invest more in new capacity. This is the direct effect. There is also a competition effect. When a firm decreases its total capacity, its opponent wishes to increase its own capacity to recuperate the abandoned market share. The direct effect is more important for the

preempted firm than the preempting one, as it invests more in new capacity, aiming for efficiency. Therefore, the competition effect is more pronounced for the preempting firm. Consequently, when the discount rate increases, the preempting firm can increase its total capacity by investing more in old technology, while the preempted firm invests more in new capacity.¹²

4.4.4 Impact of competition

This subsection compares the outcomes of the monopoly and of the duopoly symmetric and asymmetric equilibria. To make a reasonable comparison, we assume that the financial endowment of the monopoly is twice the financial endowment of each firm in the duopoly. In that way, the total financial endowment of the industry remains constant. The first result compares the level of each type of capacity depending on the strength of competition.

Result 4.8. *The effect of competition on the level of capacities is:*

- *The level of old capacity is inferior under monopoly than under symmetric competition, and inferior under symmetric competition than under asymmetric competition (both in the short run and long run).*
- *In the long run, the level of new capacity is higher under symmetric competition than under monopoly or asymmetric competition. The comparison between monopoly and asymmetric competition is ambiguous.*

The first period total capacity of the asymmetric duopoly is superior to the one of the symmetric duopoly which is superior to the one of monopoly. The difference between the monopoly and the symmetric duopoly is an expected competition effect, whereas the difference between symmetric and

¹²The competition effect does not always dominate as the derivative of the leader's total capacity is: $\frac{\partial \Psi_L^{nc}}{\partial \delta} = \frac{\bar{p}-p-c\bar{p}}{(\bar{p}-p)(3+2\delta)^2}$, and the follower's: $\frac{\partial \Psi_F^{nc}}{\partial \delta} = \frac{-\bar{p}(c+p-\bar{p})-(\bar{p}-p)\frac{\partial \Psi_L^{nc}}{\partial \delta}}{(\bar{p}-p)(3+2\delta)^2} < 0$.

asymmetric duopoly is due to preemption. As firms are financially constrained in the first period, the level of old capacity increases with the strength of competition in the short run. In the long run, this result remains valid as there is no more investment in old capacity.

Due to the financial constraint, the level of new capacity in the short run decreases with the strength of competition. In the long run, the total capacity of the firm increases with the strength of competition. To install a higher total capacity than the monopoly, the symmetric duopoly invests more in the second period (and only in new) than the monopoly, and ends up with a higher level of new capacity. This result is reversed for the comparison between the asymmetric and the symmetric equilibria. The asymmetric duopoly invests so much in old capacity in the first period, that even if the total capacity is larger in the long run, its level of new capacity does not catch up the one of the symmetric duopoly. In fact, the level of new capacity of asymmetric duopoly may even be inferior to the one of the monopoly.

These results allow us to discuss the impact of competition on consumers and firms, assuming that there is no technological externality.¹³

Result 4.9. *For consumers, asymmetric competition is better than symmetric competition and symmetric competition is better than monopoly, both in the short run and in the long run. This ordering is reversed for the industry profit.*

The strength of competition decreases the price, as it increases the total capacity of the industry (both in the short and long run). Consumers are then better off with competition. Furthermore, the strength of competition also increases the level of old capacity, and the profit of the industry is reduced due to a higher intertemporal cost of capacity (and a lower market price).

¹³Consumers solely care about the price, and firms solely care about their profits.

4.5 Conclusion

This paper studies the adoption of a new production tool technology when firms are financially constrained. In the short run, firms face a trade-off between investing in capacity using the old technology and growing rapidly, and investing in new capacity and producing efficiently. The optimal decision of the monopoly is then to install a mixture of capacities. For the duopoly, two different types of equilibrium may arise. In the symmetric equilibrium, the duopoly also invests in a mixture at a level of total capacity larger than the monopoly, due to competition. The duopoly therefore installs more old capacity. In the asymmetric equilibrium, one of the firms preempts its opponent by investing more in old capacity, increasing its short run total capacity. The opponent reacts by focusing on the efficient technology and reducing its total capacity. In the long run, the preempted firm is the only investing firm, only in the new technology. It does not catch up its rival. The utilization of old technology in the industry is thus higher under asymmetric equilibrium than symmetric equilibrium.

The present work does not model any externality arising from the utilization of technologies, such as pollution. However, such externalities are often in mind of the policy maker. Our results allow to stress some implications. For example, let's consider that the old technology generates more negative externality than the new one. In that case, an increase of competition may not be desirable as it increases the utilization of old technology. To produce more, financially constrained industry will invest more in cheap and inefficient capacity. The welfare loss due to the externality may exceed the usual welfare gain due to competition.

When we consider policy instruments, such as taxes or subsidies on capacity prices, cost of production (carbon tax) or financial constraint, most of them work as expected. Indeed, an increase in the financial constraint does not change the total capacity of the firms (within a certain range), but it

increases the share of new technology in the mixture. The consumption price does not change, but the total welfare increases due to the efficiency of new technology. In addition, increasing the old technology's marginal cost of production or reducing the price of the new technology reduces the utilization of old technology.

Finally, we show that an increase in the price of old technology may increase its utilization. Indeed, when the price of old capacity increases, the firm has to decrease its total capacity since it is financially constrained. When the investment cost of new technology is too high compared to the old one, increasing the use of new technology can result as a large reduction production capacity. To avoid this too large reduction, the firm can substitute its investment in new capacity by old one. In that case, an increase in the price of old technology may lead to an increase in the quantity of old capacity used. The policy maker should then be careful if it decides to use a tax or subsidy on the investment price of old capacity.

The present work can be extended in many directions. Capacity prices can evolve over time, due to exogenous innovation processes or endogenous learning effects. Marginal production cost of using capacities can also vary over time. These may change the technology mixture and the possibility of preemption. Moreover, in several markets, as the electricity generation market, both demand and supply uncertainties play an important role in investment decisions. One possible research direction is to combine our framework with these uncertainties. Finally, for more applied research, where the externalities of the technologies are known and measurable, our framework can be expanded for market-based policy recommendations.

Appendix

4.A Proof of Proposition 4.1:

To solve the problem of the firm, we proceed by backward induction. In the second period, the firm is not financially constrained and it invests only in the most efficient technology as seen in the proof of Result 1. The problem of the firm in the second period is thus to maximize:

$$\Pi_2 = (1 - k_1 - \tilde{k}_2) (k_1 + \tilde{k}_2) - ck_1 - \tilde{p} (\tilde{k}_2 - \tilde{k}_1). \quad (\text{C.1})$$

with respect to $\tilde{k}_2$, taking the first period choice of capacities as given, under the irreversibility constraint:

$$\tilde{k}_2 > \tilde{k}_1. \quad (\text{C.2})$$

If the total capacity of the first period, $k_1 + \tilde{k}_1$, is inferior to $\frac{1-\tilde{p}}{2}$ the irreversibility constraint (C.2) is not binding, and the optimal level of capacity in the second period is:

$$k_2^* = k_1 \text{ and } \tilde{k}_2^* = \frac{1-\tilde{p}}{2} - k_1. \quad (\text{C.3})$$

When $k_1 + \tilde{k}_1$ is greater than $\frac{1-\tilde{p}}{2}$, the firm is constrained by its first period total capacity and it does not invest in the second period. Thus, the discounted total profit can be written as a function of the first period total

capacity:

$$\Pi = \begin{cases} (1 + \delta) \left(1 - \frac{\tilde{p}}{1 + \delta} - k_1 - \tilde{k}_1\right) (k_1 + \tilde{k}_1) - (p + (1 + \delta)c - \tilde{p}) k_1 \\ , \text{ if } k_1 + \tilde{k}_1 > \frac{1 - \tilde{p}}{2} \\ \left(1 - \tilde{p} + \delta\tilde{p} - k_1 - \tilde{k}_1\right) (k_1 + \tilde{k}_1) + \delta \left(\frac{1 - \tilde{p}}{2}\right)^2 - (p + (1 + \delta)c - \tilde{p}) k_1 \\ , \text{ if } k_1 + \tilde{k}_1 \leq \frac{1 - \tilde{p}}{2} \end{cases} \quad (\text{C.4})$$

The problem of the firm is then to maximize (C.4) under the financial constraint (4.3).

When the firm is not financially constrained, it does not invest in old capacity ($k_1 = 0$ due to Result 1) and (C.4) is a concave function that is maximized at $\tilde{k}_1 = \frac{1 - \frac{\tilde{p}}{1 + \delta}}{2}$. Indeed, the second line of (C.4) is a concave function that is maximized at $\tilde{k}_1 = \frac{1 - \tilde{p} + \delta\tilde{p}}{2} > \frac{1 - \tilde{p}}{2}$. The second line of (C.4) is then increasing until $\frac{1 - \tilde{p}}{2}$. The first line of (C.4) is a concave function that is maximized at $\tilde{k}_1 = \frac{1 - \frac{\tilde{p}}{1 + \delta}}{2} > \frac{1 - \tilde{p}}{2}$ as $\delta > 0$. Therefore Π is a concave function that is maximized at $\tilde{k}_1 = \frac{1 - \frac{\tilde{p}}{1 + \delta}}{2}$.

When the firm is financially constrained $\left(\frac{1 - \frac{\tilde{p}}{1 + \delta}}{2} > \frac{G}{\tilde{p}}\right)$, (C.4) is a concave function that is maximized at $k_1 + \tilde{k}_1 = \frac{1}{2} \left(1 - \frac{(1 + \delta)\tilde{p}}{\tilde{p} - p} c + \delta\tilde{p}\right)$. The financial constraint (4.3) can be written as follows:

$$k_1 = \frac{\tilde{p} (k_1 + \tilde{k}_1) - G}{\tilde{p} - p}, \quad (\text{C.5})$$

and (C.5) can be replaced in the intertemporal profit (C.4):

$$\Pi = \begin{cases} (1 + \delta) \left(1 - \frac{c\tilde{p}}{\tilde{p} - p} - (k_1 + \tilde{k}_1)\right) (k_1 + \tilde{k}_1) + \frac{p + (1 + \delta)c - \tilde{p}}{\tilde{p} - p} G \\ \text{if } k_1 + \tilde{k}_1 > \frac{1 - \tilde{p}}{2} \\ \left(1 + \delta\tilde{p} - \frac{(1 + \delta)c\tilde{p}}{\tilde{p} - p} - (k_1 + \tilde{k}_1)\right) (k_1 + \tilde{k}_1) + \delta \left(\frac{1 - \tilde{p}}{2}\right)^2 + \frac{p + (1 + \delta)c - \tilde{p}}{\tilde{p} - p} G \\ , \text{ if } k_1 + \tilde{k}_1 \leq \frac{1 - \tilde{p}}{2} \end{cases} \quad (\text{C.6})$$

The first line of (C.6) is a concave function that is maximized at $k_1^* + \tilde{k}_1^* =$

$\frac{1}{2} \left(1 - \frac{c\tilde{p}}{\tilde{p}-p}\right) < \frac{1-\tilde{p}}{2}$ due to Assumption 4.1. So the first line of (C.6) is decreasing for the values greater than $\frac{1-\tilde{p}}{2}$. The second line of (C.6) is a concave function that is maximized at $k_1^* + \tilde{k}_1^* = \frac{1}{2} \left(1 - \frac{(1+\delta)\tilde{p}}{\tilde{p}-p}c + \delta\tilde{p}\right) \leq \frac{1-\tilde{p}}{2}$ due to Assumption 4.1. Therefore, Π is a concave function maximized at $k_1^* + \tilde{k}_1^* = \frac{1}{2} \left(1 - \frac{(1+\delta)\tilde{p}}{\tilde{p}-p}c + \delta\tilde{p}\right)$. Using (C.5), we can determine the technology mixture associated with this level of total capacity. If $\frac{1}{2} \left(1 - \frac{(1+\delta)\tilde{p}}{\tilde{p}-p}c + \delta\tilde{p}\right) > \frac{G}{p}$ then the firm invests only in old capacity to a level $\frac{G}{p}$, as the firm cannot have a negative amount of one capacity. For the same reason, if $\frac{1}{2} \left(1 - \frac{(1+\delta)\tilde{p}}{\tilde{p}-p}c + \delta\tilde{p}\right) < \frac{G}{\tilde{p}}$ then the firm invests only in new capacity. This establishes (4.11) when $\frac{G}{\tilde{p}} < \frac{1-\tilde{p}}{2(1+\delta)}$. Finally, the firm does not invest in the second period when $\frac{G}{\tilde{p}} < \frac{1-\tilde{p}}{2}$.

4.B Proof of Lemma 4.1:

In the second period the profit of firm i is:

$$\Pi_2^i = \left(k_2^i + \tilde{k}_2^i\right) \left(1 - \left(k_2^i + \tilde{k}_2^i + k_2^j + \tilde{k}_2^j\right)\right) - ck_2^i - p(k_2^i - k_1^i) - \tilde{p}(\tilde{k}_2^i - \tilde{k}_1^i). \quad (\text{C.7})$$

As in the case of monopoly, firm i invests only in new technology and $k_2^i = k_1^i$, as in Result 1. Then, maximizing the above profit with respect to the level of new capacity in the second period yields the best response of firm i as follows:

$$\tilde{k}_2^i + k_1^i = \max \left\{ \frac{1 - \tilde{p} - (k_2^j + \tilde{k}_2^j)}{2}, \tilde{k}_1^i + k_1^i \right\}. \quad (\text{C.8})$$

If the first period capacities of both firms is inferior to $\frac{1-\tilde{p}}{3}$, the equilibrium is $\frac{1-\tilde{p}}{3}$ (the Cournot outcome). If they are both superior to $\frac{1-\tilde{p}}{3}$ then the equilibrium is not to invest for both firms. When the capacity of firm j is superior to $\frac{1-\tilde{p}}{3}$ and the capacity of firm i is inferior to $\frac{1-\tilde{p}}{3}$, firm j does not

invest and firm i invests only if $\tilde{k}_1^i + k_1^i < \frac{1-\tilde{p}-(\tilde{k}_1^j+k_1^j)}{2}$.

4.C Proof of Propositions 4.2 and 4.3:

Lemma 4.1 separates the set of first period capacities in three regions with different firm behavior in the second period (see Figure 2):

- the no-move zone:

$$\left\{ \left(k_1^A, \tilde{k}_1^A, k_1^B, \tilde{k}_1^B \right) \mid \tilde{k}_1^i + k_1^i \geq \frac{1-\tilde{p}-(\tilde{k}_1^j+k_1^j)}{2} \text{ for each } i \in \{A, B\} \right\}, \quad (\text{C.9})$$

- the symmetric zone:

$$\left\{ \left(k_1^A, \tilde{k}_1^A, k_1^B, \tilde{k}_1^B \right) \mid \tilde{k}_1^i + k_1^i < \frac{1-\tilde{p}}{3} \text{ for each } i \in \{A, B\} \right\}, \quad (\text{C.10})$$

- the asymmetric zone:

$$\left\{ \left(k_1^A, \tilde{k}_1^A, k_1^B, \tilde{k}_1^B \right) \mid \frac{1-\tilde{p}}{3} \leq \tilde{k}_1^i + k_1^i < \frac{1-\tilde{p}-(\tilde{k}_1^j+k_1^j)}{2} \text{ for each } i \in \{A, B\} \right\}. \quad (\text{C.11})$$

The aim is to determine the sub-game perfect equilibria of the game. In the first step, we search for potential equilibria in each region, i.e. if there are some vectors of capacity without any profitable deviation inside the region. In the second step, we verify if potential equilibria are Nash by studying the possibility of a deviation to other regions.

Case 1: No-move zone

When the first period capacities are inside the no-move zone, firms do not invest in the second period and their profits are therefore:

$$\Pi^i = (1 + \delta) \left(1 - \frac{\tilde{p}}{1 + \delta} - k_1^i - \tilde{k}_1^i - k_1^j - \tilde{k}_1^j \right) (k_1^i + \tilde{k}_1^i) - (p + c - \tilde{p}) k_1^i. \quad (\text{C.12})$$

If the firms are not financially constrained, the equilibrium is then to invest only in new capacity, and to the level $\tilde{k}_1^i = \frac{1}{3} \left(1 - \frac{\tilde{p}}{1 + \delta} \right)$. The firms are financially constrained when $\frac{1}{3} \left(1 - \frac{\tilde{p}}{1 + \delta} \right) \geq \frac{G}{\tilde{p}}$. This states the first part of Proposition 2 when $\frac{G}{\tilde{p}} > \frac{1}{3} \left(1 - \frac{\tilde{p}}{1 + \delta} \right)$.

If the firms are financially constrained, we can rewrite their profit by using (C.5):

$$\begin{aligned} \Pi^i = (1 + \delta) \left(1 - \frac{\tilde{p}}{1 + \delta} - k_1^i - \tilde{k}_1^i - k_1^j - \tilde{k}_1^j \right) (k_1^i + \tilde{k}_1^i) \\ - (p + c - \tilde{p}) \frac{\tilde{p} (k_1 + \tilde{k}_1) - G}{\tilde{p} - p} \end{aligned} \quad (\text{C.13})$$

and the equilibrium total capacity is therefore $k_1^i + \tilde{k}_1^i = \frac{1}{3} \left(1 - \frac{(1 + \delta)\tilde{p}}{\tilde{p} - p} c + \delta\tilde{p} \right) = \Psi_D$. There are several possibilities.

- When $\frac{G}{\tilde{p}} \geq \frac{1 - \tilde{p}}{3}$, $\frac{G}{\tilde{p}} \geq \Psi_D$ as $1 - \frac{(1 + \delta)\tilde{p}}{\tilde{p} - p} c + \delta\tilde{p} < 1 - \tilde{p}$ due to Assumption 4.1. In that case firms never reach the optimal mixture, as they cannot invest in a negative amount of old capacity. Firms then invest only in new capacity to the level $\frac{G}{\tilde{p}}$. This states the first part of Proposition 4.2 when $\frac{1}{3} \left(1 - \frac{\tilde{p}}{1 + \delta} \right) \geq \frac{G}{\tilde{p}} \geq \frac{1 - \tilde{p}}{3}$.
- When $\frac{1 - \tilde{p}}{3} > \frac{G}{\tilde{p}} \geq \Psi_D$, firms invest only in new capacity to the level $\frac{G}{\tilde{p}}$. However $\frac{G}{\tilde{p}} < \frac{1 - \tilde{p}}{3}$, and firms optimal first period capacity does not belong to the no-move zone.
- When $\frac{G}{\tilde{p}} \geq \Psi_D > \frac{G}{\tilde{p}}$, firms invest in a mixture of capacity, to a level

of total capacity Ψ_D and firms optimal first period capacity does not belong to the no-move zone.

- When $\Psi_D > \frac{G}{p}$, firms invest only in old capacity, to a level of total capacity $\frac{G}{p}$. Since $\frac{G}{p} < \Psi_D < \frac{1-\tilde{p}}{3}$, firms optimal first period capacity does not belong to the no-move zone.

Case 2: Symmetric zone

Assume that the first period capacities are inside the symmetric zone. Then, firms invest to a level $\frac{1-\tilde{p}}{3}$ in the second period, and only in new capacity. The intertemporal profit of firm i is then:

$$\begin{aligned} \Pi^i = & \left[\left(1 - k_1^i - \tilde{k}_1^i - k_1^j - \tilde{k}_1^j - \tilde{p} + \delta\tilde{p} \right) \left(k_1^i + \tilde{k}_1^i \right) - ((1 + \delta)c + p - \tilde{p}) k_1^i \right] \\ & + \delta \left[\left(1 - 2\tilde{k}_C \right) \tilde{k}_C - \tilde{p}\tilde{k}_C \right]. \quad (\text{C.14}) \end{aligned}$$

If firm i is not financially constrained, its best response in the first period is $\tilde{k}_1^i = \frac{1}{2} \left(1 - \tilde{p} + \delta\tilde{p} - k_1^j - \tilde{k}_1^j \right)$ (and no investment in old capacity). If both firms are not constrained at the equilibrium, then $\tilde{k}_1^{i*} = \frac{1}{3} (1 - \tilde{p} + \delta\tilde{p}) > \frac{1-\tilde{p}}{3}$. In this case the first period capacities do not belong to the symmetric zone anymore. This rules out the possibility of any equilibrium in the symmetric zone such that none of the firms is financially constrained.

Let then i be the firm which is financially constrained. Using (C.5) we can rewrite its profit as follows:

$$\begin{aligned} \Pi^i = & \left[\left(1 - k_1^i - \tilde{k}_1^i - k_1^j - \tilde{k}_1^j - \frac{(1 + \delta)c\tilde{p}}{\tilde{p} - p} + \delta\tilde{p} \right) \left(k_1^i + \tilde{k}_1^i \right) \right] \\ & + \frac{(1 + \delta)c + p - \tilde{p}}{\tilde{p} - p} G + \delta \left[\left(1 - 2\tilde{k}_C \right) \tilde{k}_C - \tilde{p}\tilde{k}_C \right]. \quad (\text{C.15}) \end{aligned}$$

This leads the following best response:

$$k_1^i + \tilde{k}_1^i = \frac{1}{2} \left[1 - k_1^j - \tilde{k}_1^j - (1 + \delta)c \frac{\tilde{p}}{\tilde{p} - p} + \delta\tilde{p} \right]. \quad (\text{C.16})$$

As the firm cannot invest in a negative amount, the complete best response of firm i is then:

$$k_1^i + \tilde{k}_1^i = \begin{cases} \frac{G}{p} & \text{if } \frac{1}{2} \left(1 - k_1^j - \tilde{k}_1^j - (1 + \delta) \tilde{p} \frac{c}{\tilde{p} - p} + \delta \tilde{p} \right) > \frac{G}{p} \\ \frac{1}{2} \left(1 - k_1^j - \tilde{k}_1^j - (1 + \delta) \tilde{p} \frac{c}{\tilde{p} - p} + \delta \tilde{p} \right) & \text{elsewhere} \\ \frac{G}{\tilde{p}} & \text{if } \frac{1}{2} \left(1 - k_1^j - \tilde{k}_1^j - (1 + \delta) \tilde{p} \frac{c}{\tilde{p} - p} + \delta \tilde{p} \right) < \frac{G}{\tilde{p}} \end{cases} \quad (\text{C.17})$$

In the case where both of the firms are financially constrained, the equilibrium is

$$k_1^i + \tilde{k}_1^i = \begin{cases} \frac{G}{p} & \text{if } \frac{1}{3} \left(1 - (1 + \delta) \tilde{p} \frac{c}{\tilde{p} - p} + \delta \tilde{p} \right) > \frac{G}{p} \\ \frac{1}{3} \left(1 - (1 + \delta) \tilde{p} \frac{c}{\tilde{p} - p} + \delta \tilde{p} \right) & \text{elsewhere} \\ \frac{G}{\tilde{p}} & \text{if } \frac{1}{3} \left(1 - (1 + \delta) \tilde{p} \frac{c}{\tilde{p} - p} + \delta \tilde{p} \right) < \frac{G}{\tilde{p}} \end{cases}. \quad (\text{C.18})$$

To verify that firm j is also financially constrained, suppose that it is not. In that case, its best response would be:

$$\tilde{k}_1^j = \frac{1}{2} \left(1 - \tilde{p} - k_1^i - \tilde{k}_1^i \right), \quad (\text{C.19})$$

leading to a total capacity in the industry:

$$k_1^i + \tilde{k}_1^i + k_1^j + \tilde{k}_1^j = \frac{1}{3} \left(2 - (1 - \delta) \tilde{p} - (1 + \delta) \frac{c \tilde{p}}{\tilde{p} - p} \right). \quad (\text{C.20})$$

The capacity of the unconstrained firm at the equilibrium is therefore:

$$\tilde{k}_1^j = \frac{1}{3} \left(1 - (2 + \delta) \tilde{p} + (1 + \delta) \frac{c \tilde{p}}{\tilde{p} - p} \right), \quad (\text{C.21})$$

This can be rewritten as:

$$\tilde{k}_1^j = \frac{1}{3} \left(1 - \tilde{p} + (1 + \delta) \tilde{p} \left(\frac{c}{\tilde{p} - p} - 1 \right) \right). \quad (\text{C.22})$$

However, $\frac{1}{3} (1 - \tilde{p}) > \frac{G}{\tilde{p}}$ and $c + p > \tilde{p}$ by Assumption A1. Thus $\tilde{k}_1^j > \frac{G}{\tilde{p}}$, which is impossible.

The study of no-move zone and symmetric zone proves Proposition 4.2.

Case 3: Asymmetric zone

When the first period capacities of firms are inside the asymmetric zone, the profit of the firms are:

- for the preempting firm,

$$\begin{aligned} \Pi^L = & \left(\left(1 - k_1^L - \tilde{k}_1^L \right) \left(1 + \frac{\delta}{2} \right) - \tilde{p} + \delta \frac{\tilde{p}}{2} - \left(k_1^F + \tilde{k}_1^F \right) \right) \left(k_1^L + \tilde{k}_1^L \right) \\ & - (p + (1 + \delta) c - \tilde{p}) k_1^L, \end{aligned} \quad (\text{C.23})$$

- and the preempted firm:

$$\begin{aligned} \Pi^F = & \left(1 - k_1^F - \tilde{k}_1^F - k_1^L - \tilde{k}_1^L - \tilde{p} + \delta \tilde{p} \right) \left(k_1^F + \tilde{k}_1^F \right) \\ & - (p + (1 + \delta) c - \tilde{p}) k_1^F + \delta \left(\frac{1 - \tilde{p} - k_1^L - \tilde{k}_1^L}{2} \right)^2. \end{aligned} \quad (\text{C.24})$$

The scheme of the proof is the following: first we assume that both firms are financially constrained and we express the best response of the leader and of the follower. Then we verify that there is no possible equilibrium such that one of the firm is not financially constrained.

Assume that both firm are financially constrained. Then, using (C.5), its profit can be rewritten:

$$\begin{aligned} \Pi^L = & \left(\left(1 - k_1^L - \tilde{k}_1^L \right) \left(1 + \frac{\delta}{2} \right) - \frac{(1 + \delta) c \tilde{p}}{\tilde{p} - p} + \delta \frac{\tilde{p}}{2} - \left(k_1^F + \tilde{k}_1^F \right) \right) * \\ & \left(k_1^L + \tilde{k}_1^L \right) + \frac{p + (1 + \delta) c - \tilde{p}}{\tilde{p} - p} G. \end{aligned} \quad (\text{C.25})$$

This give us a best response:

$$k_1^L + \tilde{k}_1^L = \frac{1}{2} \left[1 - \frac{1}{(1+\frac{\delta}{2})} \left(k_1^F + \tilde{k}_1^F + (1+\delta) c \frac{\tilde{p}}{\tilde{p}-p} - \delta \frac{\tilde{p}}{2} \right) \right]. \quad (\text{C.26})$$

As the firm cannot invest a negative amount of capacity, the complete best response function is:

$$k_1^L + \tilde{k}_1^L = \left\{ \begin{array}{l} \frac{1}{2} \left[1 - \frac{1}{(1+\frac{\delta}{2})} \left(k_1^F + \tilde{k}_1^F + (1+\delta) c \frac{\tilde{p}}{\tilde{p}-p} - \delta \frac{\tilde{p}}{2} \right) \right] \text{ elsewhere} \\ \frac{G}{p} \text{ if } \frac{G}{p} < \frac{1}{2} \left[1 - \frac{1}{(1+\frac{\delta}{2})} \left(k_1^F + \tilde{k}_1^F + (1+\delta) c \frac{\tilde{p}}{\tilde{p}-p} - \delta \frac{\tilde{p}}{2} \right) \right] \end{array} \right\}. \quad (\text{C.27})$$

Remark than we did not include the possibility for the leader to have a total capacity equal to $\frac{G}{\tilde{p}}$. This is because the equilibrium total capacity of the leader has to be higher than the follower's total capacity in an equilibrium in the asymmetric zone. It is therefore not necessary to take into account a best response possibility for the leader to have a total capacity of $\frac{G}{\tilde{p}}$. For the same reason, the total capacity of the follower has to be inferior to $\frac{G}{p}$.

Similarly, using (C.5), the profit of the preempted firm can be rewritten:

$$\begin{aligned} \Pi^F = & \left(1 - k_1^F - \tilde{k}_1^F - k_1^L - \tilde{k}_1^L - \frac{(1+\delta) c \tilde{p}}{\tilde{p}-p} + \delta \tilde{p} \right) (k_1^F + \tilde{k}_1^F) \\ & + \frac{p + (1+\delta) c - \tilde{p}}{\tilde{p}-p} G + \delta \left(\frac{1 - \tilde{p} - k_1^L - \tilde{k}_1^L}{2} \right)^2, \end{aligned} \quad (\text{C.28})$$

which leads to the best response:

$$k_1^F + \tilde{k}_1^F = \frac{1}{2} \left[1 - k_1^L - \tilde{k}_1^L - (1+\delta) c \frac{\tilde{p}}{\tilde{p}-p} + \delta \tilde{p} \right]. \quad (\text{C.29})$$

The complete best response function of the follower is thus:

$$k_1^F + \tilde{k}_1^F = \left\{ \begin{array}{l} \frac{1}{2} \left(1 - k_1^L - \tilde{k}_1^L - (1+\delta) c \frac{\tilde{p}}{\tilde{p}-p} + \delta \tilde{p} \right) \\ \text{if } \frac{G}{\tilde{p}} < \frac{1}{2} \left(1 - k_1^L - \tilde{k}_1^L - (1+\delta) c \frac{\tilde{p}}{\tilde{p}-p} + \delta \tilde{p} \right) \\ \frac{G}{\tilde{p}} \text{ if } \frac{G}{\tilde{p}} > \frac{1}{2} \left(1 - k_1^L - \tilde{k}_1^L - (1+\delta) c \frac{\tilde{p}}{\tilde{p}-p} + \delta \tilde{p} \right) \end{array} \right\}. \quad (\text{C.30})$$

These can be rewritten as follows:

$$k_1^F + \tilde{k}_1^F = \begin{cases} \frac{1}{2} \left(1 - k_1^L - \tilde{k}_1^L - (1 + \delta) c_{\frac{\tilde{p}}{\tilde{p}-p}} + \delta \tilde{p} \right) \\ \text{if } \left(k_1^L + \tilde{k}_1^L \right) < 1 - 2\frac{G}{\tilde{p}} - (1 + \delta) c_{\frac{\tilde{p}}{\tilde{p}-p}} + \delta \tilde{p} \\ \frac{G}{\tilde{p}} \text{ if } \left(k_1^L + \tilde{k}_1^L \right) > 1 - 2\frac{G}{\tilde{p}} - (1 + \delta) c_{\frac{\tilde{p}}{\tilde{p}-p}} + \delta \tilde{p} \end{cases}. \quad (\text{C.31})$$

Combining (C.27) and (C.31) yields Proposition 3.

We now have to verify that the financial constraint is binding for both firms. Due to the same reason for the symmetric zone, at least one firm is financially constrained. As the leader produces more than the follower, the leader is necessarily financially constrained. Furthermore, if the preempted firm is not financially constrained, the maximization of its profit gives the following best response:

$$k_1^F + \tilde{k}_1^F = \tilde{k}_1^F = \frac{1}{2} \left[1 - k_1^L - \tilde{k}_1^L - (1 - \delta) \tilde{p} \right]. \quad (\text{C.32})$$

However, as the preempting firm does not invest in the second period, the second period profit of the preempted firm is

$$\pi_2^F = \left(1 - k_1^L - \tilde{k}_1^L - k_1^F - \tilde{k}_2^F \right) \left(k_1^F + \tilde{k}_2^F \right) - \tilde{p} \left(\tilde{k}_2^F - \tilde{k}_1^F \right) - ck_1^F, \quad (\text{C.33})$$

which leads to a second period optimal capacity of the preempted firm:

$$\left(k_1^F + \tilde{k}_2^F \right) = \frac{1}{2} \left(1 - k_1^L - \tilde{k}_1^L - \tilde{p} \right), \quad (\text{C.34})$$

which is inferior to the first period best response given by (C.32), meaning that the preempted firm does not invest in the second period. This contradicts the fact that the potential equilibrium is in the asymmetric zone.

4.D Proof of Result 4.4:

Most of the results are directly obtained by taking the derivatives of capacity amounts given in propositions. However, the effect of the price of old capacity is less straightforward. The following proves the result for the monopoly.

The percentage of old capacity is given by:

$$\%_{Old} = \frac{k_1^*}{k_1^* + \tilde{k}_1^*} = \frac{\tilde{p}}{\tilde{p} - p} \frac{\Psi_M - \frac{G}{\tilde{p}}}{\Psi_M}. \quad (C.35)$$

The derivative of the percentage is positive if and only if:

$$\frac{\partial \%_{Old}}{\partial p} > 0 \Leftrightarrow (\Psi_M)^2 > \frac{G(1 + \delta\tilde{p})}{2\tilde{p}}. \quad (C.36)$$

This condition can be rewritten:

$$\frac{\partial \%_{Old}}{\partial p} > 0 \Leftrightarrow p < \tilde{p} - \frac{c(1 + \delta)\tilde{p}}{1 + \delta\tilde{p} - \sqrt{2\frac{G(1 + \delta\tilde{p})}{\tilde{p}}}}. \quad (C.37)$$

The same approach works for the quantity of old capacity:

$$\frac{\partial k_1^*}{\partial p} > 0 \Leftrightarrow p < \tilde{p} - \frac{2c(1 + \delta)\tilde{p}}{2G - \tilde{p} - (\tilde{p})^2}. \quad (C.38)$$

The same approach can be used to prove this result for the symmetric duopoly. For the asymmetric duopoly, the proof is given in the following.

In the asymmetric equilibrium, the total capacity of the industry can be written as:

$$\Psi_L^{nc} + \Psi_F^{nc} = \frac{6\Psi_D + 3\delta\Psi_D + \delta\left(\frac{1}{2} - \frac{1}{2}\tilde{p}\right)}{3 + 2\delta}. \quad (C.39)$$

The percentage of old capacity in the industry is therefore:

$$\%_{old} = \frac{\frac{\tilde{p}}{\tilde{p}-p} \left(\Psi_L^{nc} + \Psi_F^{nc} - \frac{2G}{\tilde{p}} \right)}{\Psi_L^{nc} + \Psi_F^{nc}}. \quad (C.40)$$

The derivative of the total capacity according to the price of the old capacity is decreasing as $\frac{\partial \Psi_D}{\partial p} < 0$. Furthermore, the derivative of the old percentage gives:

$$\frac{\partial \%_{old}}{\partial p} = \frac{\tilde{p}}{(\tilde{p}-p)^2} - \frac{4G(3+2\delta)(4+\delta(3+\tilde{p}(3+2\delta)))}{(4(c\tilde{p}+p-\tilde{p})+3\delta(p+2c-1-\tilde{p})+2\tilde{p}(c+p-\tilde{p})\delta^2)^2}. \quad (C.41)$$

Asymmetric equilibrium exists only when condition (4.21) exists. When the total capacity of the follower is Ψ_F^{nc} , this condition can be rewritten:

$$\frac{1}{2} \left(1 - \Psi_F^{nc} - (1+\delta) \frac{c\tilde{p}}{\tilde{p}-p} + \delta\tilde{p} \right) > \frac{1-\tilde{p}}{3}. \quad (C.42)$$

This is equivalent to:

$$c < \frac{\tilde{p}-p}{\tilde{p}} \frac{1 + \frac{3}{2}\delta + \tilde{p}}{(1+\delta)(2+\delta)}. \quad (C.43)$$

We will see that this condition implies that

$$4(c\tilde{p}+p-\tilde{p})+3\delta(p+2c-1-\tilde{p})+2\tilde{p}(c+p-\tilde{p})\delta^2 \quad (C.44)$$

is negative. Indeed, by using (C.43) in (C.44) we obtain:

$$(C.44) < -(\tilde{p}-p)(2+\tilde{p}(2+\delta)(2\delta-1)) \quad (C.45)$$

Since $2+\tilde{p}(2+\delta)(2\delta-1) < 0$ for any $\tilde{p} < 1$ and $\delta \geq 0$, (C.44) is negative.

This implies that $\frac{\partial \%_{old}}{\partial p} > 0$ if and only if:

$$4(c\tilde{p}+p-\tilde{p})+3\delta(p+2c-1-\tilde{p})+2\tilde{p}(c+p-\tilde{p})\delta^2 - (\tilde{p}-p) \sqrt{\frac{4G(3+2\delta)(4+\delta(3+\tilde{p}(3+2\delta)))}{\tilde{p}}} < 0 \quad (C.46)$$

Therefore,

$$p < \frac{4(\tilde{p} - c\tilde{p}) + 3\delta(1 + \tilde{p} - 2c) + 2\tilde{p}(\tilde{p} - c - p)\delta^2 + \tilde{p}\sqrt{\frac{4G(3+2\delta)(4+\delta(3+\tilde{p}(3+2\delta)))}{\tilde{p}}}}{(4 + 3\delta + 2\tilde{p} + \sqrt{\frac{4G(3+2\delta)(4+\delta(3+\tilde{p}(3+2\delta)))}{\tilde{p}}})}. \quad (\text{C.47})$$

As the total capacity is decreasing, the ambiguity of the percentage proves the ambiguity of the total capacity.

4.E Proof of Result 4.9:

Let $\tilde{k}_2^M$ be the level of new capacity in second period for the monopoly:

$$\tilde{k}_2^M = \frac{1 - \tilde{p}}{2} - \frac{\tilde{p}}{\tilde{p} - p} \left(\Psi_M - \frac{2G}{\tilde{p}} \right).$$

Let $\tilde{k}_2^S$ be the level of new capacity in second period for the symmetric duopoly:

$$\tilde{k}_2^S = \frac{2(1 - \tilde{p})}{3} - 2\frac{\tilde{p}}{\tilde{p} - p} \left(\Psi_D - \frac{G}{\tilde{p}} \right).$$

Then, $\tilde{k}_2^M < \tilde{k}_2^S$ if and only if:

$$\frac{1 - \tilde{p}}{2} - \frac{\tilde{p}}{\tilde{p} - p} \Psi_M < \frac{2(1 - \tilde{p})}{3} - 2\frac{\tilde{p}}{\tilde{p} - p} \Psi_D$$

As $\Psi_D = \frac{2}{3}\Psi_M$, this is always the case.

Let $\tilde{k}_2^{As}$ be the level of new capacity in second period for the asymmetric duopoly:

$$\tilde{k}_2^{As} = \frac{1 - \tilde{p} - \Psi_L}{2} - \Psi_F - \frac{\tilde{p}}{\tilde{p} - p} \left(\Psi_L - \frac{G}{\tilde{p}} \right) - \frac{\tilde{p}}{\tilde{p} - p} \left(\Psi_F - \frac{G}{\tilde{p}} \right)$$

The difference between the level of new capacity for the asymmetric duopoly and for the symmetric duopoly is given by:

$$\tilde{k}_2^{As} - \tilde{k}_2^S = \frac{(1 + \delta)(3 + 2\delta)\tilde{p}^2 - p\delta - \tilde{p}p(3 + 2\delta) - \frac{c\tilde{p}}{\tilde{p} - p}(1 + \delta)(\tilde{p}(3 + 2\delta) - 3p)}{6(\tilde{p} - p)(3 + 2\delta)}.$$

Due to assumption A1, $\frac{c}{\tilde{p}-p} > 1$, therefore:

$$\tilde{k}_2^{As} - \tilde{k}_2^S < \frac{-p\delta - \tilde{p}p(3+2\delta) + 3p\tilde{p}(1+\delta)}{6(\tilde{p}-p)(3+2\delta)},$$

so:

$$\tilde{k}_2^{As} - \tilde{k}_2^S < \delta \frac{p\tilde{p} - p}{6(\tilde{p}-p)(3+2\delta)}.$$

As $\tilde{p} < 1$ (if not, no firm will enter in the market),

$$\tilde{k}_2^{As} < \tilde{k}_2^S.$$

The difference between the level of new capacity for the asymmetric duopoly and for the monopoly is given by:

$$\tilde{k}_2^{As} - \tilde{k}_2^M = \frac{p \left(\frac{c\tilde{p}}{\tilde{p}-p} - 1 \right) (1+\delta)}{6(\tilde{p}-p)(3+2\delta)}.$$

If the cost of adding and using during a single period a capacity is similar for the old and new technology ($c+p \simeq \tilde{p}$), then $\tilde{k}_2^{As} < \tilde{k}_2^M$ as $\tilde{p} < 1$. When the cost difference is important, $\tilde{k}_2^{As} > \tilde{k}_2^M$. ■

Chapter 5

Why the rich may want a low pollution tax?

HAMZEH ARABZADEH AND BARIS VARDAR

5.1 Introduction

Who is willing to give more support for environmental protection? And what are the sources of differences among households that lead them to prefer different levels for environmental policy tools such as pollution taxes? In this study, we focus on these questions and particularly on the dimension of heterogeneity in wealth and its implications on the preferred pollution tax of the households. Environmental policies may affect the households with a higher wealth differently than the ones with a lower wealth because of the fact that they have more capital invested in the market and their consumption levels are not the same.¹ May the richer people in society prefer a lower pollution tax than the poorer ones? If yes, then why? This question is the main interest of this study.

¹Throughout the text we treat capital ownership and wealth as identical terms. This equivalence relies on the assumption that all wealth owned by the households are lent to the firms in the economy and thus employed in production.

The rich, owning a higher amount of capital than the poor, are in control of a larger proportion of production factors that are employed by the firms. Firm decisions can be affected by the policies that aim to reduce the level of pollution through the adjustment of factor demands. Therefore these policies can have important impacts on the factor prices such as the wage and the interest rate. In a general equilibrium setting, changes in factor prices affect the household revenues. Indeed, when factors are unevenly distributed within the society, these impacts can lead to differences in the preferred pollution taxes of households.

There has been a few works, mainly empirical, that study the distributional impacts of environmental policies. Most of the studies consider a partial equilibrium framework by focusing only on the uses side of income, which means the impact of environmental policies on the commodity prices. The common result is that the pollution taxes are regressive because the dirty commodities constitute a larger share of the poor households' expenditures. Besides, there is a growing literature that consider a general equilibrium framework and thus taking into account the sources side of income as well, which are more closer to our framework. For example, Fullerton and Heutel (2007) study the incidence of environmental taxes in a general equilibrium framework and they take into account general forms of substitution among the factors. They show the importance of elasticity of substitution between dirty and clean goods in both production and consumption sides. Furthermore, using this framework, they identify the impact of a pollution tax on the factor prices as well as on the prices of the final goods. In more recent works, Rausch et al. (2011) and Dissou and Siddiqui (2014) show that the pollution tax can be progressive by considering the sources side of income by using a similar approach.

The incidence of environmental taxes can also be studied by considering the heterogeneities among the households in terms of labor income, transfer income or time preferences. For example, Chiroleu-Assouline and Fodha

(2014), Fullerton and Monti (2013) and Marsiliani and Rengström (2002) study the heterogeneity in terms of labor income, Fullerton and Heutel (2010) and Rausch et al. (2011) study the heterogeneity in terms of transfer income and Borissov et al. (2014) could be given as an example that study the heterogeneity in the discount rates of the households. In this paper we abstract from these and we consider only the case of heterogeneity in terms of capital endowment.

The income data of the U.S. economy from the 2007 Survey of Consumer Finances (SCF) show that the revenues from capital constitute 25% of the total overall income. Moreover, as shown by Fullerton and Heutel (2010), the fraction of income coming from capital is increasing over income deciles.² For example, the fraction of income that comes from capital is 5.7% for the lowest income decile, 7.8% for the fifth income decile and 45.6% for the highest income decile. Accordingly, neglecting the heterogeneity in capital revenues generates a significant gap in the theoretical analysis.

In a recent empirical study, by using micro data from European Value Survey (EVS), Ercolano et al. (2014) show an inverted U-shaped relationship between income of the households and their willingness to monetary contribute to protection of the environment. This means that for households in the low and middle income deciles, the higher income they have, the more they are willing to pay for a better environment. However, for the highest income percentiles, the willingness to pay for environmental protection decreases with the income. What distinguishes the highest percentile income households from the others is the fact that the share of income coming from wealth, as well as heterogeneity in wealth, is more pronounced for them, as shown in data from the SCF. The combination of these two observations makes us to question if heterogeneity in wealth is the determinant factor to explain the negative relationship between income and support for pollution taxes among the very high percentile income households.

²With the exception that the lowest income decile has slightly higher share of capital in their income compared to the next decile.

Heterogeneity in wealth is taken into account in a few previous studies. For example, Rausch et al. (2011) and Dissou and Siddiqui (2014) consider it but they do not conduct in depth theoretical analysis of its implications on the households' preferred pollution taxes. Furthermore, Kempf and Rossignol (2007) study the relationship between wealth inequality and environmental protection in a theoretical framework and address the questions that are similar to ours. By using an endogenous growth model, they show that the richer households prefer a higher environmental tax and correspondingly inequality is harmful for the environment. But this result is based on the fact that the relative price of labor to capital is independent from the environmental tax since their model does not incorporate alternative cleaner production technologies. This dimension is indeed the main focus of our paper and it makes our framework, and thus our results, significantly different from theirs.

Our aim in this study is two folds. First, to investigate the effects of a pollution tax on the firm behavior and factor prices in the partial competitive equilibrium and identify the determinants of these effects. Second, in a general equilibrium setting, to relate these findings with the households' preferred pollution taxes and eventually identify the cases in which the pollution tax is regressive or progressive in terms of households' welfare.³

We develop a static general equilibrium model by taking into account households, firms and the government. Households have different wealth endowments and their utility depends on their consumption level and the level of environmental quality. The level of environmental quality depends negatively on the level of pollution. The production side of the model is inspired by the works of Harberger (1962), Copeland and Taylor (2004), Fullerton and Heutel (2007) and many others that apply the international trade framework of Heckscher-Ohlin. We study an economy with firms that produce a generic good by using two different technologies, namely dirty

³In this paper, we use the progressivity and regressivity terms always in terms of welfare.

and clean, with each of them using capital, labor and pollution as an input to produce the final output.⁴ The factor prices of capital and labor are determined endogenously in the equilibrium, the government determines the pollution tax and uses its revenues for government spending purposes.

Our results show that the impact of a pollution tax on the factor prices depends on the characteristics of the production technologies utilized by the firms in the economy. We find that the relative price of factor that is more intensively used in the dirty technology will decrease as a response to an increase in the pollution tax - which is a well-known result in the literature. Moreover, when we consider the dirty technology is more capital intensive than the clean one, the interest rate always decreases with the pollution tax. But, whether the wage increases or decreases depends on the comparison of the relative intensities of pollution and capital between the production technologies. In particular, we show that the wage increases when the relative pollution intensity respect to capital is higher in the dirty technology, and *vice versa*. These results, which we summarize in Table (5.1), differ from the many studies in the literature (for example Copeland and Taylor (2004), Fullerton and Heutel (2007)). These findings are based on the fact that in our setting, contrary to theirs, the clean technology also pollutes thus its pollution intensity matters.

On the household side, we investigate the household's decision about its preferred pollution tax and we identify the trade-off that they face between a higher consumption and a better environmental quality. At this point, this paper differs from the ones in the literature (such as Fullerton and Heutel (2007), Dissou and Siddiqui (2014) in two ways. First, we consider the utility of household depends also on the environmental quality that leads to the trade-off that we mentioned above. Second, this paper does not address the uses side effects of the pollution tax. The reason is that our

⁴The use of pollution as an input in the production process is a well-established modeling approach in the environmental economics literature and the motivation behind is explained in Section 2.1.

model constitute a closed economy in which the firms produce a generic good by using alternative technologies.⁵ In this setting, pollution tax has no effect on the commodity prices. On the contrary, the models presented by those papers are consistent with a closed economy with two sectors. Therefore, the pollution tax increases the relative price of the dirty good to the clean one and thus causes the uses side effect.

Having only the sources side in the setting leads us to find the effect of wealth on a household's preferred pollution tax which depends on two opposite channels. We call the first one as the *satiation effect*. It says that households with a higher wealth consume more and their marginal utility of consumption is lower, thus they would be more willing to sacrifice from their consumption for a better environmental quality. And we call the second channel as the *income burden effect*. It says that households with a higher wealth have larger capital investments in the market, thus, when the return of capital falls their revenues are more reduced by the pollution tax. Accordingly, whether the pollution tax increases or decreases with wealth depends on which one of these effects dominates. We show that, in fact, it depends on the pollution tax elasticity of consumption that is determined by the pollution tax elasticities of the factor prices.

By using these results, in the general equilibrium, we show that if the firms are operating with a single production technology then the richer households prefer a higher pollution tax, hence the tax is regressive. On the contrary, if the firms are using the dirty and clean technologies simultaneously, the pollution tax leads to a reallocation of resources in the clean technology. In this case, when the dirty technology is more capital intensive, the richer households lose more from their consumption in percentage terms which means that they would prefer a lower pollution tax. In other words, when the economy operates on two technologies the tax is progres-

⁵Our model can also be interpreted as a small open economy with two sectors in which the production factors are mobile across sectors but immobile across countries. In this type of setting, the country engages in goods trade but has an isolated financial market. This setting is suitable for some of the developing countries today.

sive.

The following section presents the model. Section 3 presents the firm decision and analyzes the impact of a pollution tax on the factor prices, Section 4 explains the role of the government and how the proceeds from the pollution tax are used, Section 5 presents the household decision, Section 6 characterizes the general equilibrium for this economy and Section 7 shows the conditions for the impact of the wealth on the preferred pollution tax of an household. Then Section 8 discusses the implications of the cases when some of the assumptions that we made are relaxed. Finally Section 9 concludes.

5.2 The framework

Within a static framework, we analyze a closed economy that consists of households, firms and the government. We consider a continuum of households indexed by $i \in (0, 1)$ with each of them supplying one unit of labor inelastically. Each household i has an initial capital (*wealth*) endowment k_i , and the total capital in the economy is $\bar{K} = \int_0^1 k_i di$.

Household's utility $V(c, E)$ depends on consumption of the generic good (c) and the level of environmental quality (E) that decreases with the level of pollution (z). The firms produce the generic good in a perfectly competitive market by using capital (k), labor (l) and pollution. The factor prices of capital and labor (r and w) are determined endogenously in the equilibrium. The government determines the unit price of pollution (τ) and uses the collected tax revenue for its expenditures.

In the following sections we explain the aims and the decision making processes of the firms, the government and the households in detail and study the outcome in a general equilibrium framework.

5.3 Production

The production of the generic good is a function of capital (k), labor (l) and pollution (z). We consider the price of the generic good as numeraire. In line with Siebert et al. (1980), Copeland and Taylor (1994), Copeland and Taylor (2004), Fullerton and Heutel (2007), we take into account pollution as an input in the production process. This approach for modeling production is usually called as “joint production technology”.

One way of motivating this is to think about two production processes: the first one is the production of the final good and the second one is the abatement of pollution. The first production process uses capital and labor as inputs and produces the final good as well as pollution as a by-product. The second one also employs capital and labor to produce equipment which are used to reduce the level of pollution that is generated by the first production process. These two production processes can be transformed into a joint production technology, which is depicted in figure 5.1. Jouvet et al. (2005) also shows a similar exercise of this transformation and conclude by obtaining a production function homogenous of degree one of capital, labor and pollution.

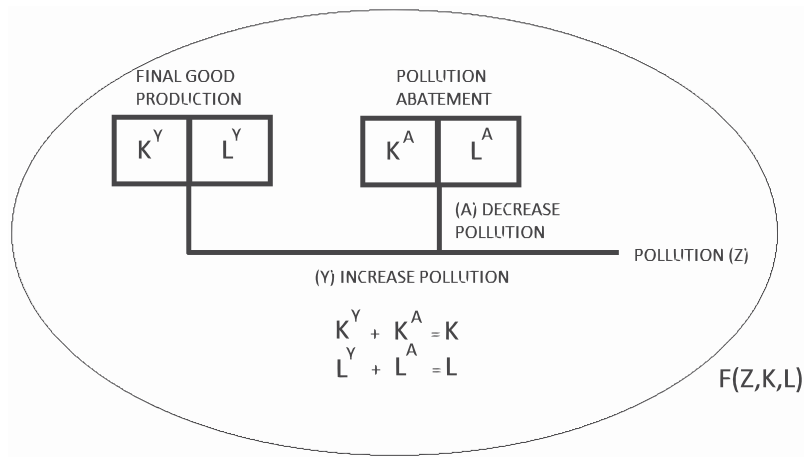


Figure 5.1 – Joint production technology

We assume functional separability between pollution and the physical in-

puts in the joint production technology. Hence, the production function is denoted as $F(z, G(k, l))$ where the first argument of $F(., .)$ is pollution (z) and the second argument is the conjoint physical input of capital and labor ($G(k, l)$). This way of specification is similar to and more general than the one in Copeland and Taylor (2004).⁶ Functional separation implicitly assumes that the relative factor demands are identical in both final good production process and the pollution abatement process.⁷ As will be shown later on, this restriction is necessary to analyze the single production technology (Section 2.1.1) while it is not necessary for multiple production technologies (Section 2.1.2). We prefer to keep this form to maintain consistency throughout the text.

This nested structure for production function captures the fact that the physical inputs for production (capital and labor) are having a bilateral elasticity of substitution between them and pollute to operate the production process. Moreover, the conjoint physical input of capital and labor has an elasticity of substitution with pollution. The shapes of $F(., .)$ and $G(., .)$ determines the substitutability (or complementarity) of each input respect to the others. We assume the following properties for the production function:

Assumption 5.1. *The production function satisfies the following proper-*

⁶Copeland and Taylor (2004) assumes that the production function is Cobb-Douglas in pollution and conjoint physical input of capital and labor, that is $x = z^\alpha (F(K_x, L_x))^{1-\alpha}$.

⁷See Appendix 5.K for details. Note that this certain assumption is necessary just for this motivation of the production function and it does not have any role in our results.

ties.^{8,9}

(i) $F(.,.)$ and $G(.,.)$ are homogenous of degree one.

(ii) $F_1(.,.) > 0$, $F_{11}(.,.) < 0$, $F_2(.,.) > 0$, $F_{22}(.,.) < 0$, $F_{12}(.,.) > 0$

(iii) $G_1(.,.) > 0$, $G_{11}(.,.) < 0$, $G_2(.,.) > 0$, $G_{22}(.,.) < 0$, $G_{12}(.,.) > 0$

Assumption 5.1 means that the production technology embodies constant returns to scale. It also implies that each factor's marginal productivity is positive and decreasing in its amount and is increasing in other factors' amounts.¹⁰

We proceed step by step for the decision making process of the firms. Our aim is to analyze the effect of a change in the pollution tax on the prices of capital and labor and on the allocation of resources in the economy. We first investigate a simple case in which there is only a single production technology available. Then we study the case in which there are two alternative production technologies with different factor intensities. We will show that these two cases may have contrasting results depending on the characteristics of the production technologies.

⁸Throughout the text we use the following notations for a derivative of a function: $f'(x) = \partial f / \partial x$, $f''(x) = \partial^2 f / \partial x^2$, $f_i(x, y) = \partial f / \partial i$ and $f_{ij}(x, y) = \partial^2 f / \partial i \partial j$ where i and j denote the order of the arguments of f . For example, $f_1(x, y) = \partial f / \partial x$, $f_2(x, y) = \partial f / \partial y$, $f_{11}(x, y) = \partial^2 f / \partial x^2$ and $f_{12}(x, y) = \partial^2 f / \partial x \partial y$.

⁹These assumptions on the production function are satisfied by most commonly used production functions such as Cobb-Douglas and CES. We consider to proceed on the analysis by using the general form in order to cover a larger family of functional forms.

¹⁰The assumptions on capital and labor are straightforward and standard, however, the ones on pollution still need to be justified. Total output increases if we increase pollution keeping the amount of capital and labor constant ($F_1(.,.) > 0$). One can think that in this case the amount of capital and labor allocated for abatement activities are reallocated in the production of the final good. Therefore pollution will increase due to decreased abatement and total output will increase due to higher amount of capital and labor employed in the final good production process. Of course a technology is more *dirty* if it needs more amount of capital and labor relocated from final good production to the pollution abatement for having a unitary decrease in pollution.

5.3.1 Single production technology

In this framework there is only one production technology available. The firms take the prices of input factors as given and minimize their cost by deciding on their factor demands $(\alpha_z, \alpha_k, \alpha_l)$ for producing one unit of the output. The problem of the representative firm is:

$$\min_{\{\alpha_z, \alpha_k, \alpha_l\}} \{\tau\alpha_z + r\alpha_k + w\alpha_l\} \quad (5.1)$$

$$\text{subject to } F(\alpha_z, G(\alpha_k, \alpha_l)) = 1 \quad (5.2)$$

$$\text{and } 0 \leq \alpha_j \text{ for } j \in \{z, k, l\}$$

where r , w and τ denote the interest rate, wage and unit pollution tax respectively. The cost minimization problem in (5.1) yields the following first order conditions:

$$F_1(\alpha_z, G(\alpha_k, \alpha_l)) = \tau \quad (5.3)$$

$$F_2(\alpha_z, G(\alpha_k, \alpha_l))G_1(\alpha_k, \alpha_l) = r \quad (5.4)$$

$$F_2(\alpha_z, G(\alpha_k, \alpha_l))G_2(\alpha_k, \alpha_l) = w \quad (5.5)$$

Since marginal productivity of each factor is always positive and we assume perfect competition among the firms, capital and labor will be employed at their highest quantities ($\bar{K}$ and $\bar{L}$) in the equilibrium. Constant returns to scale property of the production function implies that the relative intensity of capital to labor is fixed by the factor endowment in the economy.

$$\frac{\alpha_k}{\alpha_l} = \frac{\bar{K}}{\bar{L}} \quad (5.6)$$

Equations (5.2 to 5.6) allow us to obtain factor intensities and the prices of capital and labor as a function of the pollution tax $(\alpha_z(\tau), \alpha_k(\tau), \alpha_l(\tau), w(\tau), r(\tau))$. Furthermore, by taking into account the fact that $\bar{K} = \alpha_k(\tau)F(\alpha_z(\tau), G(\alpha_k(\tau), \alpha_l(\tau)))$ or $\bar{L} = \alpha_l(\tau)F(\alpha_z(\tau), G(\alpha_k(\tau), \alpha_l(\tau)))$ we can

determine the equilibrium level of output.

In the equilibrium, an increase in pollution tax decreases the pollution intensity of production ($\alpha'_z(\tau) < 0$). A lower pollution intensity reduces the marginal productivity (and hence the price) of conjoint physical input ($F_2(\cdot, G(\cdot))$). Moreover, the relative price of capital and labor will not change since the relative intensity of capital to labor is fixed by the total endowment (eq. (5.6)). As a result, the prices of labor and capital will decrease at the same rate.

Proposition 5.1. *When firms operate by using a single production technology, in the partial equilibrium, the wage and the interest rate are decreasing in the pollution tax ($w'(\tau) < 0$, $r'(\tau) < 0$). Moreover, both has the same elasticity respect to the pollution tax, $\epsilon_{w,\tau} = \epsilon_{r,\tau} < \epsilon_{R,\tau} < 0$.¹¹ where R denotes the gross interest rate.¹²*

Proof. See Appendix 5.A. □

To summarize, in this basic framework the interest rate and the wage decreases with the same elasticity as a response to an increase in the pollution tax. This result relies on the following assumptions: (i) only one technology is available in the economy, (ii) the production function is constant returns to scale and it is separable between pollution and conjoint physical input of capital and labor, (iii) the endowment of capital and labor is fixed in the economy, (iv) labor supply is inelastic.

In the following subsection, we will relax the first assumption and we investigate how the results will change. More specifically, we will investigate how the responses of factor prices to an increase in pollution tax will change when an alternative production technology is available to use.

¹¹The term $\epsilon_{x,y}$ denotes the elasticity of x respect to y ($\frac{\partial x / \partial y}{x/y}$)

¹²Here we also report the differences respect to the elasticity of gross capital return because they will be useful for the analysis of the household's problem.

5.3.2 Two production technologies: dirty and clean

In this framework, we consider that the generic good can be produced by using two different technologies: dirty (X) and clean (Y).¹³ The two technologies both require the use of capital (k), labor (l) and pollution (z) and they are denoted as $X = F^X(z_x, G^X(k_x, l_x))$ and $Y = F^Y(z_y, G^Y(k_y, l_y))$. The functions $F^i(\cdot)$ and $G^i(\cdot)$ for $i \in \{X, Y\}$ satisfy the properties given in Assumption 5.1.

The representative firm takes the factor prices as given and minimizes its unit cost of production for each technology with the following programme:

$$\min_{\{\alpha_z^X, \alpha_k^X, \alpha_l^X, \alpha_z^Y, \alpha_k^Y, \alpha_l^Y\}} \{\tau(\alpha_z^X + \alpha_z^Y) + r(\alpha_k^X + \alpha_k^Y) + w(\alpha_l^X + \alpha_l^Y)\} \quad (5.7)$$

$$\text{subject to } F^i(\alpha_z^i, G^i(\alpha_k^i, \alpha_l^i)) = 1 \text{ for } i \in \{X, Y\} \quad (5.8)$$

$$\text{and } 0 \leq \alpha_j^i \text{ for } i \in \{X, Y\} \text{ and } j \in \{z, k, l\}$$

The cost minimization problem leads to the following first order conditions:

$$F_1^X(\alpha_z^X, G^X(\alpha_k^X, \alpha_l^X)) = F_1^Y(\alpha_z^Y, G^Y(\alpha_k^Y, \alpha_l^Y)) = \tau \quad (5.9)$$

$$F_2^X(\alpha_z^X, G^X(\alpha_k^X, \alpha_l^X))G_1^X(\alpha_k^X, \alpha_l^X) = F_2^Y(\alpha_z^Y, G^Y(\alpha_k^Y, \alpha_l^Y))G_1^Y(\alpha_k^Y, \alpha_l^Y) = r \quad (5.10)$$

$$F_2^X(\alpha_z^X, G^X(\alpha_k^X, \alpha_l^X))G_2^X(\alpha_k^X, \alpha_l^X) = F_2^Y(\alpha_z^Y, G^Y(\alpha_k^Y, \alpha_l^Y))G_2^Y(\alpha_k^Y, \alpha_l^Y) = w \quad (5.11)$$

where $\{\alpha_z^i, \alpha_k^i, \alpha_l^i\}$ for $i \in \{X, Y\}$ are the derived demands of pollution, capital and labor, respectively, for producing one unit of output by using technology i . The six first order equations in (5.9 - 5.11) allow us to obtain the

¹³Studying only two technologies case is not too restrictive because even if we had taken into account an economy with n technologies, in this framework, the firms would utilize maximum two of them. This assertion is valid in the case where $F^i(\cdot)$ and $G^i(\cdot)$ for $i \in \{1, \dots, n\}$ are homogenous of degree one. See Appendix 5.J for details.

unit factor demands as a function of the factor prices: $\{\alpha_z^i(r, w, \tau), \alpha_k^i(r, w, \tau), \alpha_l^i(r, w, \tau)\}$. In fact, when we consider profit maximization problem which is the dual of problem (5.7), we have the same first order conditions and the same functions for factor demands. Substituting the factor demands into the iso-unit cost function leads to an implicit relationship between the factor prices such that $C(r, w, \tau) = 1$. This implicit relationship is the factor price frontier. Indeed, it corresponds to the minimum value of the cost in (5.7) under all technical conditions including the constraint of one unit of production given in (5.8). In the following, we show that at least one factor price (w and/or r) decreases as a response to an increase in the pollution tax. That is consistent with the factor price frontier.

Replacing the factor demands we obtained before ($\{\alpha_z^i(r, w, \tau), \alpha_k^i(r, w, \tau), \alpha_l^i(r, w, \tau)\}$) into the two equations in (5.8), we can find wage and interest rate as a function of pollution tax ($w(\tau), r(\tau)$). Hence, we find the intensities of all factors in each sector and prices of capital and labor as a function of pollution tax. Note that contrary to the single technology framework, factor intensities, wage and interest rate are independent from the total resource endowment ($\bar{K}$ and $\bar{L}$).

We define the technology with higher pollution intensity as the dirty one and we assume no factor intensity reversal to ensure that the dirty technology, according to this definition, always remains as the dirty one. Moreover, we assume that the dirty technology is more capital intensive as well. Recently, Fullerton and Heutel (2010) calculated the factor intensities of the US economy in clean and dirty sectors, in which they defined petroleum refining, electricity and transportation industries as the dirty sector, and all remaining industries as the clean one. They showed that relative intensity of capital with respect to labor in dirty industries is $\frac{\alpha_k^X(\tau)}{\alpha_l^X(\tau)} = 1.28$ whereas the same indicator for the clean industries is $\frac{\alpha_k^Y(\tau)}{\alpha_l^Y(\tau)} = 0.60$. Therefore assuming the dirty technology being more capital intensive is reasonable. Moreover, our framework contains only capital and labor as production

factors, and it does not include energy and fossil-fuel use. These two factors are among the main contributors of pollution, and they are intensively used in dirtier industries. One can think that these factors are included in the capital variable in this model, which will imply that the dirty technology is more capital intensive than the clean one. Formally, we assume the following:

Assumption 5.2. *The dirty technology (X) is assumed to be more capital intensive than the clean technology (Y):*

$$\alpha_z^X(\tau) > \alpha_z^Y(\tau), \quad \alpha_k^X(\tau) > \alpha_k^Y(\tau) \quad \alpha_l^X(\tau) < \alpha_l^Y(\tau).$$

Note that in Assumption 5.2 we compare the factor intensities ($\frac{z_x}{X} > \frac{z_y}{Y}$, $\frac{k_x}{X} > \frac{k_y}{Y}$, $\frac{l_x}{X} < \frac{l_y}{Y}$) between the technologies to define the type of production technology. This approach is equivalent to the comparison of factor shares in production ($\frac{\tau z_x}{X} > \frac{\tau z_y}{Y}$, $\frac{rk_x}{X} > \frac{rk_y}{Y}$, $\frac{wl_x}{X} < \frac{wl_y}{Y}$).

As we stated before, the factor intensities and the factor prices are independent from the aggregate level of capital and labor. However, the allocation of resources between the two technologies will depend on the total resources. The total demand for factor j in technology a can be computed by multiplying the unit demand for that factor and the total production of that technology. Therefore, the total resource constraint implies the following:

$$X\alpha_k^X(\tau) + Y\alpha_k^Y(\tau) = \bar{K} \tag{5.12}$$

$$X\alpha_l^X(\tau) + Y\alpha_l^Y(\tau) = \bar{L} \tag{5.13}$$

where X and Y represent total production by the dirty and clean technology respectively. Solving these two equations for total output of each

technology (X and Y) yields to the following relations:

$$F^X(z_x, G^X(k_x, l_x)) = X(\tau) = \frac{\alpha_l^Y(\tau)\bar{K} - \alpha_k^Y(\tau)\bar{L}}{\alpha_k^X(\tau)\alpha_l^Y(\tau) - \alpha_l^X(\tau)\alpha_k^Y(\tau)} \quad (5.14)$$

$$F^Y(z_y, G^Y(k_y, l_y)) = Y(\tau) = \frac{\alpha_l^X(\tau)\bar{K} - \alpha_k^X(\tau)\bar{L}}{\alpha_k^Y(\tau)\alpha_l^X(\tau) - \alpha_l^Y(\tau)\alpha_k^X(\tau)} \quad (5.15)$$

Using equations (5.14) and (5.15) we can obtain the allocation of each factor between the technologies, that is, $z_x(\tau) = X(\tau)\alpha_z^X(\tau)$, $k_x(\tau) = X(\tau)\alpha_k^X(\tau)$, $l_x(\tau) = X(\tau)\alpha_l^X(\tau)$, $z_y(\tau) = Y(\tau)\alpha_z^Y(\tau)$, $k_y(\tau) = Y(\tau)\alpha_k^Y(\tau)$, $l_y(\tau) = Y(\tau)\alpha_l^Y(\tau)$.

Now that we obtained all the factor intensities, the factor prices, the amounts of each factor employed in each technology and the total amounts of production made by using each technology, we can characterize the partial competitive equilibrium:

Definition 5.1. *For a given pollution tax (τ), the unique partial competitive equilibrium for this economy is characterized by the vector of factor intensities in each technology $\{\alpha_z^X, \alpha_k^X, \alpha_l^X, \alpha_z^Y, \alpha_k^Y, \alpha_l^Y\}$, the vector of labor and capital prices $\{w, r\}$, the vector of the factors amounts employed in each technology $\{z_x, k_x, l_x, z_y, k_y, l_y\}$ and the total production in each technology $\{X, Y\}$ such that:*

- (i) *The firms minimize their costs, thus (5.8 to 5.11) hold.*
- (ii) *The markets clear, thus the resource constraints (5.14 and 5.15) hold.*

By using the definition above, we determine the level of total output and allocation of factors between the two technologies, as well as the factor intensities and the factor prices at the equilibrium as a function of the pollution tax. So how does the pollution tax affects these variables, in particular the prices of capital and labor?

An increase in the pollution tax makes pollution more expensive as an input. Hence both sectors will use pollution less intensively which causes

an adverse effect on the productivities of labor and capital. Since the dirty technology is more pollution intensive, an increase in the tax affects the use of this technology at most. It will be more profitable for the firms to use the clean technology, thus, some of the resources that are used in the dirty technology will be reallocated in the clean one. Consequently, the share of the clean technology, which is more labor intensive, will increase in aggregate production. This leads to an increase in relative productivity of labor respect to capital.

Accordingly, a rise in the pollution tax affects the factor prices from two channels: (i) a decline in pollution intensity and (ii) reallocation of capital and labor from the dirty technology to the clean one. Both channels impose a negative impact on the interest rate while they push the wage in two opposite directions. On the one hand, less pollution intensity pushes the wage downward, and on the other hand, factor reallocation from capital intensive technology to the labor intensive one pushes it upward. Whether the wage increases or decreases depends on which one of these effects dominates.

In the following proposition we show that in fact it depends on the relative intensity of pollution and capital between the two technologies:

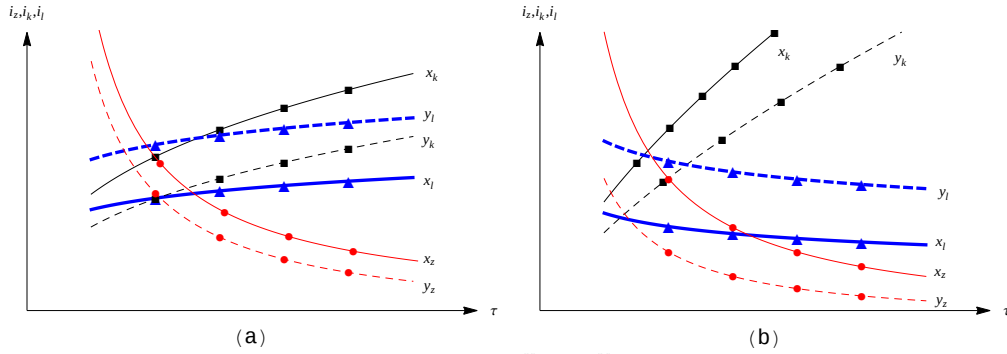
Proposition 5.2. *When the economy operates using both technologies, the interest rate decreases in the pollution tax ($r'(\tau) < 0$). However, the change in the wage ($w'(\tau) \lesseqgtr 0$) depends on the technologies' relative pollution intensities respect to capital.*

- (i) if $\frac{\alpha_z^X}{\alpha_k^X} < \frac{\alpha_z^Y}{\alpha_k^Y}$ then $r'(\tau) < 0, w'(\tau) < 0$ and $\epsilon_{r,\tau} < \epsilon_{R,\tau} < \epsilon_{w,\tau} < 0$
- (ii) if $\frac{\alpha_z^X}{\alpha_k^X} = \frac{\alpha_z^Y}{\alpha_k^Y}$ then $r'(\tau) < 0, w'(\tau) = 0$ and $\epsilon_{r,\tau} < \epsilon_{R,\tau} < \epsilon_{w,\tau} = 0$
- (iii) if $\frac{\alpha_z^X}{\alpha_k^X} > \frac{\alpha_z^Y}{\alpha_k^Y}$ then $r'(\tau) < 0, w'(\tau) > 0$ and $\epsilon_{r,\tau} < \epsilon_{R,\tau} < 0 < \epsilon_{w,\tau}$

Proof. See Appendix 5.B. □

The comparison of the two cases ((i) and (iii)) in Proposition 5.2 is illus-

trated in fig.(5.2). As it is clear from the figure, pollution will be used less intensively in both technologies when the pollution tax increases. Besides, as Proposition 5.2 asserts, the interest rate declines in both cases as a response to an increase in the pollution tax. This makes firms to use capital more intensively in both technologies. However, the wage can increase or decrease once the pollution tax rises. When the relative pollution intensity of the dirty technology to the clean one ($\frac{\alpha_z^X}{\alpha_z^Y}$) is lower than the relative capital intensity ($\frac{\alpha_k^X}{\alpha_k^Y}$) then the wage decreases and so labor is employed more intensively in both technologies. (fig.(5.2,a)). In the contrary case ($\frac{\alpha_z^X}{\alpha_z^Y} > \frac{\alpha_k^X}{\alpha_k^Y}$), higher pollution tax leads to an increase in the wage, therefore more environmental protection leads to a decline in labor intensities of both technologies. fig.(5.2,b).



Note: Panel (a) illustrates the case where $\frac{\alpha_z^X}{\alpha_k^X} < \frac{\alpha_z^Y}{\alpha_k^Y}$ and panel (b) illustrates the case where $\frac{\alpha_z^X}{\alpha_k^X} > \frac{\alpha_z^Y}{\alpha_k^Y}$. The solid curves are for the dirty technology (X) and the dashed curves are for the clean one (Y). The squares, triangles and circles mark the unit factor demand curves for capital, labor and pollution respectively.

Figure 5.2 – Example unit factor demands respect to the pollution tax

Whether the relative intensity of capital to labor increases or decreases in the two technologies depends on how their relative price changes with the pollution tax. Proposition 5.2 implies that the relative price of capital to labor will decrease as a response to higher pollution tax. Therefore, more environmental protection makes the firms to use capital more intensively. This leads us to the following proposition:

Proposition 5.3. *If the economy operates using both technologies, and if*

Assumption 5.1 and Assumption 5.2 hold, then higher pollution tax will increase relative intensity of capital to labor in both technologies.

$$\frac{d(\alpha_k^i(\tau)/\alpha_l^i(\tau))}{d\tau} > 0 \quad \text{for } i \in \{X, Y\} \quad (5.16)$$

where, α_j^i is the unit-demand for factor j in technology i .

Proof. See Appendix 5.C. □

As it can be seen in Appendix 5.B, functional separability between pollution and physical inputs is not necessary for Proposition 5.2. But, Proposition 5.3 is conditional on that assumption.

Proposition 5.3 implies two extreme cases: In one extreme case, when the pollution tax is sufficiently high, all the resources will be allocated only in the clean technology and at this point capital/labor ratio in the clean technology equals to the ratio between total capital and total labor in the economy. As the tax decreases, the resources will be reallocated in the dirty technology and both technologies will become more labor intensive. In the other extreme case, the tax will be low enough such that all resources will be allocated only in the dirty technology. Obviously, in this case the capital/labor ratio in the dirty technology equals to the ratio of their total endowments in the economy.

Accordingly, we can define two thresholds for the pollution tax: (i) the dirty threshold and (ii) the clean threshold. In the case where the pollution tax is lower than the dirty threshold only the dirty technology is used and if it is greater than the clean threshold the firms operate by using only the clean technology. When the tax is between these thresholds, the firms will operate by using both of the technologies simultaneously in production.

Proposition 5.4. *If τ_{dirty} and τ_{clean} satisfy $\frac{\alpha_k^X(\tau_{dirty})}{\alpha_l^X(\tau_{dirty})} = \frac{\bar{K}}{\bar{L}}$ and $\frac{\alpha_k^Y(\tau_{clean})}{\alpha_l^Y(\tau_{clean})} =$*

$\frac{\bar{K}}{\bar{L}}$, then:

(i) if $\tau \leq \tau_{dirty}$ then firms use only dirty technology, $k_x = \bar{K}, l_x = \bar{L}$

$k_y = 0, l_y = 0$.

(ii) if $\tau_{dirty} < \tau < \tau_{clean}$ then firms use dirty and clean technologies

simultaneously $k_x > 0, l_x > 0, k_y > 0, l_y > 0$ with $k_x + k_y = \bar{K}, l_x + l_y = \bar{L}$

(iii) if $\tau \geq \tau_{clean}$ then firms use only clean technology, $k_x = 0, l_x = 0,$

$k_y = \bar{K}, l_y = \bar{L}$.

Proof. See Appendix 5.D. □

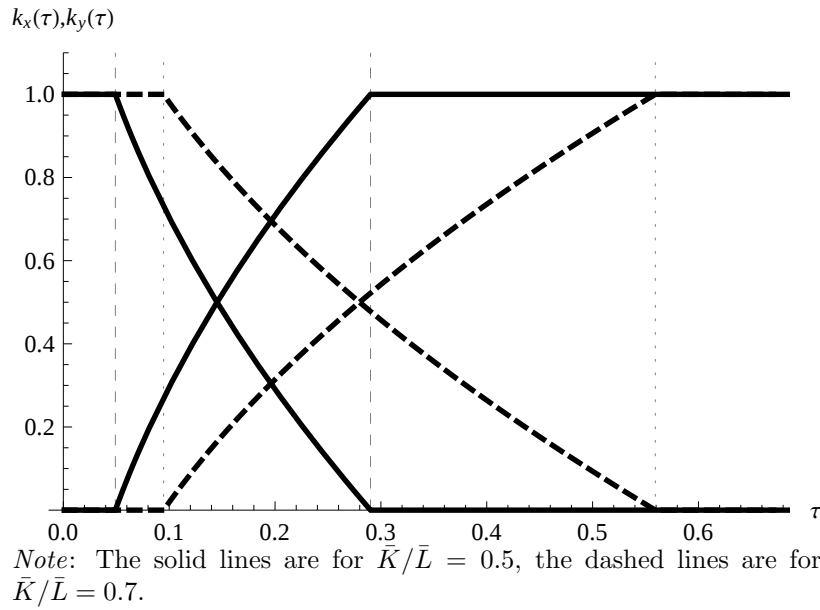


Figure 5.3 – Illustration of Proposition 5.4

As it is shown in Proposition 5.4, τ_{dirty} and τ_{clean} depend only on the relative endowment of capital and labor in the economy. Using the definition of these thresholds and equation (5.16), we can show that both of the thresholds are increasing in $\frac{\bar{K}}{\bar{L}}$. For a given amount of labor force, the more capital endowed in the economy is, the more profitable the dirty technology would be compared to the clean one. Therefore, it would require a higher

pollution tax to induce the firms to use the cleaner technology. This is illustrated in figure 5.3. The following corollary presents this result.

Corollary 5.1. *τ_{dirty} and τ_{clean} are both increasing in the ratio of total capital and labor in the economy, $\partial\tau_{dirty}/\partial(\bar{K}/\bar{L}) > 0$ and $\partial\tau_{clean}/\partial(\bar{K}/\bar{L}) > 0$.*

Proof. See Appendix 5.E. □

Table (5.1) summarizes the results of Proposition 5.1 and Proposition 5.2 that show the impact of an increase in the pollution tax on the prices of capital and labor.

	Single technology	Dirty&clean technologies	
		$\alpha_z^X/\alpha_k^X < \alpha_z^Y/\alpha_k^Y$	$\alpha_z^X/\alpha_k^X > \alpha_z^Y/\alpha_k^Y$
Interest rate	$r'(\tau) < 0$	$r'(\tau) < 0$	$r'(\tau) < 0$
Wage	$w'(\tau) < 0$	$w'(\tau) < 0$	$w'(\tau) > 0$
Elasticities	$\epsilon_{r,\tau} = \epsilon_{w,\tau} < \epsilon_{R,\tau} < 0$	$\epsilon_{r,\tau} < \epsilon_{R,\tau} < \epsilon_{w,\tau} < 0$	$\epsilon_{r,\tau} < \epsilon_{R,\tau} < 0 < \epsilon_{w,\tau}$

Table 5.1 – Impact of an increase in pollution tax on factor prices and their tax elasticities

We can conclude the analysis of production side by stating that the effects of an increase in the pollution tax on factor prices depend on the characteristics of the production technologies available and utilized by the firms in the economy. When the production technologies satisfy the properties given in Assumption 5.1 and Assumption 5.2, meaning that the technologies embody constant returns to scale and the dirty technology is more capital intensive than the clean one, the impact of an increase in the pollution tax on the factor prices will be as shown in Table (5.1) in the equilibrium.

5.4 Government

The government collects the pollution tax and uses it to finance its expenditure. Note that static nature of the model implies that government will

not save and its budget must be balanced ($G = \tau Z$). To avoid mixing fiscal policy and climate policy, we consider that government expenditure does not include any kind of redistribution neither in the form of public services nor in the form of transfer to the households. This assumption allows us to keep our focus on households' trade-off between consumption and environmental quality and to abstract from redistributive impacts of fiscal policy. Besides, it provides analytical tractability and convenience. Therefore, in line with Harberger (1962), Chiroleu-Assouline and Fodha (2006), Fullerton and Heutel (2007) and others, we consider that the government uses the collected tax revenues to buy the goods from the market which has no effect on the households' utility.

5.5 Households

Household i 's utility $V(c_i, E)$ depends on its level of consumption (c_i) and the level of environmental quality (E).¹⁴ We impose the following assumptions for the utility function:

Assumption 5.3. *The utility function $V(c_i, E)$ is additively separable in c_i and E ($V_{cE}(\cdot) = 0$), increasing and concave in c ($V_c(\cdot) > 0$ and $V_{cc}(\cdot) < 0$) and increasing and concave in E ($V_E(\cdot) > 0$ and $V_{EE}(\cdot) < 0$). We assume that:*

$$V(c_i, E) = v(c_i) + h(E) \quad (5.17)$$

These assumptions about the effects of consumption and environmental quality on utility are standard and widely used in the literature. However, the assumption on the additive separability is rather restrictive. In Section 5.8.1, we study the impact of relaxing this assumption but, for the rest of this section, we abstract from the cross relationship between consumption

¹⁴See Michel and Rotillon (1995) and Weitzman (2010) for a detailed discussion of this type of preferences.

and environmental quality in the household's utility. This leads us to have a more clear analytic resolution.

Environmental quality is a decreasing function of pollution ($E(z)$ with $E'(z) < 0$). Thus we can rewrite the utility function as $V(c_i, E(z)) = U(c_i, z)$ where $U(\cdot)$ is increasing and concave in c_i ($U_c(\cdot) > 0$ and $U_{cc}(\cdot) < 0$) and decreasing and concave in z ($U_z(\cdot) < 0$ and $U_{zz}(\cdot) < 0$). Thereafter we will use the utility function $U(\cdot)$ in our analysis.

Due to the static nature of our framework, households that maximize their utility will consume all of their revenue which consists of the wage and the gross return of their capital. In Section 5.3, we showed that the wage and the interest rate are determined by the pollution tax in the partial competitive equilibrium. Therefore, in the general equilibrium, the consumption level of the household i will depend on the pollution tax and its wealth, that is

$$c_i(\tau, k_i) = w(\tau) + (1 + r(\tau))k_i \quad (5.18)$$

The following section characterizes the general equilibrium in this economy.

5.6 General Equilibrium

We first start by studying the goods market equilibrium, which implies that total consumption (public and private) must be equalized to total production:

$$Y(\tau) + X(\tau) = C(\tau) + G(\tau) = ((1 + r(\tau))\bar{K} + w(\tau)\bar{L}) + \tau Z(\tau) \quad (5.19)$$

The left hand side of equation (5.19) is the aggregate production in terms of numeraire price and the right hand side denotes total private and public consumption. Now we can investigate the effect of pollution tax on aggregate production, private and public consumption by looking at the

derivative of equation (5.19) with respect to the pollution tax:¹⁵

$$Y'(\tau) + X'(\tau) = (r'(\tau)\bar{K} + w'(\tau)\bar{L}) + Z(\tau) + \tau Z'(\tau) = \tau Z'(\tau) < 0 \quad (5.20)$$

An increase in the pollution tax decreases the total private consumption due to the decrease in the factor revenues. This holds true even in the case where the wage increases in the pollution tax because the effect of the decrease in the interest rate on total private consumption dominates the gains from the increase in the wage.¹⁶ Moreover, the aggregate production is also decreasing in the pollution tax. Hence there is no room for double dividend in this model. The impact on government revenue remains ambiguous since an increase in pollution tax leads to a decrease in the tax base.

Now we can characterize the general equilibrium in this economy:

Definition 5.2. *For a given pollution tax (τ) , the unique general equilibrium for this economy is characterized by the vector of factor intensities in each technology $\{\alpha_z^X, \alpha_k^X, \alpha_l^X, \alpha_z^Y, \alpha_k^Y, \alpha_l^Y\}$, the vector of labor and capital prices $\{w, r\}$, the vector of the factors amounts employed in each technology $\{z_x, k_x, l_x, z_y, k_y, l_y\}$, the total production in each technology $\{X, Y\}$, the government spending $\{G\}$, the consumption level of each household $\{c_i\}_{i=0}^1$*

¹⁵See Appendix 5.F for the proof.

¹⁶Note that in the case where the wage is increasing in the pollution tax, there may exist some households with a very low wealth such that their consumption increases in the pollution tax. Total consumption of the households, however, is always decreasing in pollution tax.

and the total consumption $\{C = \int c_i\}$ such that:

- (i) The firms minimize their costs, thus the eight equations in (5.8 to 5.11) hold.
- (ii) The markets clear, thus the resource constraints (5.12 and 5.13) hold.
- (iii) The government budget is balanced ($G = \tau(z_x + z_y)$) hold.
- (iv) Households consume all their revenue. (5.18) holds for each i

5.7 Preferred pollution tax of households

This section aims to investigate preferred pollution tax of households which is defined as the level of tax that maximizes household i 's utility. Then we will examine how it is affected by capital endowment of households. In this paper, we consider progressivity and regressivity of the tax always in terms of welfare. Hence, the pollution tax progressive if it harms (favors) the poor less (more) than the rich. Then, households with a higher capital endowment will prefer a lower pollution tax if the tax is progressive (*vice versa* for regressivity):

$$\text{Pollution tax is progressive} \iff \frac{\partial(\frac{\partial u(\tau, k_i)}{\partial \tau})}{\partial k_i} < 0 \Rightarrow \text{sign}(\frac{\partial \tau_i^*(k_i)}{\partial k_i}) < 0$$

Therefore, all of our results about the impact of capital endowment on preferred pollution tax can be equivalently interpreted as progressivity/regressivity of the pollution tax in terms of welfare.

To find the preferred pollution tax of a household we consider the following maximization programme:

$$\max_{\{\tau \mid \tau \geq 0\}} \{U(c_i(\tau, k_i), z(\tau))\} \quad (5.21)$$

which leads to the following first order condition:

$$\frac{\partial U(c_i(\tau_i^*, k_i), z(\tau_i^*))}{\partial \tau_i^*} = U_c(\cdot) \frac{\partial c_i(\tau_i^*, k_i)}{\partial \tau_i^*} + U_z(\cdot) \frac{\partial z(\tau_i^*)}{\partial \tau_i^*} = 0 \quad (5.22)$$

Condition (5.22) clearly reflects the trade-off between higher consumption and better environmental quality. On the one hand, the pollution tax has an adverse effect on consumption due to its impact on factor prices which decreases the revenue of the household (the first term in the RHS of eq. (5.22)). This effect indeed has a negative impact on the household's utility. On the other hand, it decreases the level of pollution hence has a positive effect on the utility from the environmental well-being channel (the second term in the RHS of eq. (5.22)). Therefore one may expect that there is a preferred pollution tax for a household that balances these opposite effects.

In Proposition 5.2 we showed that when the firms are operating by using dirty and clean technologies, we may have a case such that the wage is increasing in the pollution tax ($w'(\tau) > 0$). In this case, the pollution tax may increase the total revenues of some households which have a low wealth because the increase in wage may dominate the loss from their gross capital return. Thus, the pollution tax will not impose a trade-off as in equation (5.22) for these households and their utility will obviously increase in tax. However, as shown in Proposition 5.4, there exists a threshold for pollution tax above which only the clean technology is used. Above this threshold, independent of their wealth, the trade-off in equation (5.22) will be valid for all households because when the firms are operating by using a single technology the wage decreases in pollution tax ($w'(\tau) < 0$) as shown in Proposition 5.1.

To proceed further, we assume the following:

Assumption 5.4. *Once τ_i^* exists for household i , its marginal utility is*

decreasing with respect to the pollution tax (τ) at this tax level, that is

$$\frac{\partial^2 U(c_i(\tau_i^*, k_i), z(\tau_i^*))}{\partial \tau_i^{*2}} < 0 \quad (5.23)$$

This assumption implies that the utility of household reaches a peak when the equation (5.22) holds. Note that while for discussing about preferred pollution tax we need the assumptions on the sign of the second derivative of utility function as well as on the existence of preferred pollution tax, we do not need any of these assumptions to analyze the progressivity/regressivity of the tax.

Equation (5.22) shows that household's preferred pollution tax depends on its wealth. To investigate the effect of an increase in the household's wealth on its preferred pollution tax, we take the derivative of equation (5.22) and solve it for $\partial \tau_i^* / \partial k_i$ which yields the following result:¹⁷

$$\text{sign}\left(\frac{\partial \tau_i^*(k_i)}{\partial k_i}\right) = \text{sign}\left(\underbrace{U_{cc}(\cdot) \frac{\partial c_i(\tau_i^*, k_i)}{\partial k_i} \frac{\partial c_i(\tau_i^*, k_i)}{\partial \tau_i^*}}_{>0 ; \text{ Satiation effect}} + \underbrace{U_c(\cdot) \frac{\partial^2 c_i(\tau_i^*, k_i)}{\partial \tau_i^* \partial k_i}}_{<0 ; \text{ Income burden effect}}\right) \quad (5.24)$$

The first term in the RHS(5.24), which has a positive sign, can be called as the *satiation effect*. When a household is richer, its level of consumption is relatively higher and thus its marginal utility of consumption is lower. This results in a lower marginal rate of substitution between consumption and environmental quality. In other words, richer households care less about the loss from their consumption due to the pollution tax. Therefore, through this channel richer households would prefer a higher pollution tax.

The second term in the RHS(5.24), which has negative sign, can be called as the *income burden effect*. It reflects the fact that, in absolute terms, richer households lose more from their consumption due to an increase in pollution tax. This is because of the fact that richer households have

¹⁷See See Appendix 5.G.

greater amount of capital invested in the market and so their revenue is more affected by the decline in return to capital. Consequently, through this channel richer households will prefer a lower pollution tax. Therefore, whether the households with higher capital endowment would prefer a higher or a lower pollution tax will depend on which one of these two effects dominates.

In the case that $v(c)$ in household utility has logarithmic form, we can analytically show that the dominating effect depends only on the pollution tax elasticity of consumption.

Proposition 5.5. *If the household's utility satisfies the properties given in Assumption (5.3) and assumption (5.4), and moreover $v(c_i) = \log(c_i)$, then the preferred pollution tax of a household is increasing in its wealth if and only if the pollution tax elasticity of consumption is increasing in wealth. Formally:*

$$\text{sign}\left(\frac{\partial \tau_i^*(k_i)}{\partial k_i}\right) = \text{sign}\left(\frac{\partial \epsilon_{c_i, \tau}}{\partial k_i}\right) \quad (5.25)$$

Proof. See Appendix 5.H. □

Proposition 5.5 shows that, for the logarithmic form of utility, the richer households want a higher environmental protection if and only if their percentage loss in consumption due to the pollution tax is lower than the poorer households. Since our framework is static and households consume all and only the revenues from their factor supplies, the pollution tax elasticity of consumption is decreasing in wealth if and only if the ratio of gross capital return to wage ($\frac{R}{w}$) decreases with respect to the pollution tax. In this case, the richer households will experience a higher percentage loss from their consumption due to an increase in the tax compared to the poorer households. This fact, combined with the assertion in Proposition 5.5 leads to the following result:

Proposition 5.6. *If the household's utility satisfies the properties given in Assumption (5.3) and assumption (5.4), and moreover $v(c_i) = \log(c_i)$, the*

preferred pollution tax is increasing in the household's wealth if and only if the pollution tax elasticity of gross interest rate is greater (less negative) than the one of the wage. Formally:

$$\text{sign}\left(\frac{\partial \tau_i^*(k_i)}{\partial k_i}\right) = \text{sign}(\epsilon_{R,\tau} - \epsilon_{w,\tau}) \quad (5.26)$$

Proof. See Appendix 5.I. □

From Section 5.3 we know that $(\frac{R}{w})$ is increasing with respect to the tax in the case where firms operate by using a single technology and it is decreasing in the two-technology case. Combining these results with Proposition 5.6 leads us to the central claims of this subsection.

Proposition 5.7. *When firms operate using a single production technology, the preferred pollution tax of an household is increasing in its wealth and the tax is regressive, $\frac{\partial \tau_i^*(k_i)}{\partial k_i} > 0$.*

Proof. Direct conclusion of Proposition 5.1 and Proposition 5.6. □

Proposition 5.8. *When firms operate using dirty and clean production technologies which satisfy the properties in Assumption 5.2, the preferred pollution tax of an household is decreasing in its wealth and the tax is progressive, $\frac{\partial \tau_i^*(k_i)}{\partial k_i} < 0$.*

Proof. Direct conclusion of Proposition 5.2 and Proposition 5.6. □

Proposition 5.8 shows that when the pollution tax leads to a reallocation of factors in cleaner technologies, which are more labor intensive, the rich prefers a lower pollution tax compared to the low-income households. Therefore pollution tax is progressive in this case. The richer people in the society who own a larger share of capital lose a higher proportion of their income compared to the low income households. Consequently, the loss in their well-being due to the fall of income outweighs the benefits of a better environment, and their support for a pollution tax declines.

In the following section, we will discuss the outcome when some of the model assumptions are relaxed.

5.8 Discussion

5.8.1 The case of non-separable utility function

The assumptions on the utility function have crucial effects on the results presented in the previous section. An important one is the additive separability of utility of consumption and disutility of pollution, meaning $U_{cz} = 0$ in our framework. Michel and Rotillon (1995) studied the cases in which the utility function is non-separable, naming the case of $U_{cz} < 0$ as the “distaste effect” and the case of $U_{cz} > 0$ as the “compensation effect”. They study the impact of these assumptions on the outcome within an endogenous growth framework. In this section, we will discuss how our results could differ when we consider a non-separable utility function.

Additive separability of the utility function with respect to consumption and environmental quality have two implications: (i) the marginal utility of consumption does not depend on pollution and (ii) the marginal utility of environmental quality is independent from the level of consumption. When this assumption is relaxed, the households’ preferred pollution taxes will vary as the marginal utility of consumption depends on the environmental quality.

To evaluate the effect of $U_{cz}(\cdot)$ on the household’s preferred pollution tax and, hence, on progressiveness of the pollution tax, we rewrite equation (5.24) for the case in which $U_{cz}(\cdot) \neq 0$:

$$\begin{aligned}
\text{sign}\left(\frac{\partial \tau_i^*(k_i)}{\partial k_i}\right) = & \underbrace{\text{sign}\left(U_{cc}(\cdot) \frac{\partial c_i(\tau_i^*, k_i)}{\partial k_i} \frac{\partial c_i(\tau_i^*, k_i)}{\partial \tau_i^*}\right)}_{>0 ; \text{ Satiation effect}} + \underbrace{U_c(\cdot) \frac{\partial^2 c_i(\tau_i^*, k_i)}{\partial \tau_i^* \partial k_i}}_{<0 ; \text{ Income burden effect}} \\
& + U_{cz}(\cdot) \underbrace{\frac{\partial c_i(\tau_i^*, k_i)}{\partial k_i}}_{>0} \underbrace{Z'(\tau)}_{<0} \quad (5.27)
\end{aligned}$$

We can see that another term is added in (5.24), which played crucial role in propositions 5.5 to 5.8. Equation 5.27 shows that the value of the new term will be added in either satiation effect or income burden effect depending on its sign.

When we consider the distaste effect ($U_{cz}(\cdot) < 0$), which means that the marginal utility of consumption decreases in the level of pollution, the sign of last term will be positive and the cross effect of consumption and pollution is going to be added to the satiation effect. In this case, higher pollution tax improves the utility of households not only by enhancing environmental quality, but also by improving the marginal utility of consumption. As the rich consume more, the latter effect is more pronounced for them. From this channel, the rich wants a higher pollution tax. Considering all the effects that we discussed previously, taking into account the distaste effect makes the tax less progressive. In extreme cases where the distaste effect is very strong, it can even make the tax regressive compared to the separable utility case.

In the contrary case in which there is a compensation effect ($U_{cz}(\cdot) > 0$), a higher consumption decreases the disutility of pollution. This makes the sign of the last term to be negative and it contributes in the income burden effect. This in turn makes the pollution tax more progressive compared to the separable utility case.

5.9 Conclusion

We showed that the households with uneven wealth endowments prefer different levels of pollution tax. This is due to the fact that wealth inequality implies two distinctions between the rich and the poor households: (i) their consumption levels are not the same and (ii) the amounts of capital that they invest in the market are different. In fact, these differences correspond to the channels that we identified as the determinant of the household's preferred pollution taxes which we called as the satiation effect and the cost of pollution tax effect. The satiation effect means that the marginal utility of consumption is lower for the richer households, henceforth, they are more willing to sacrifice from their consumption for a better environmental quality. The cost of pollution tax effect refers to the fact that the revenue of the rich is more reduced by the pollution tax due to their higher capital investment in the market. Furthermore, we showed that the effect that dominates depends on the pollution tax elasticity of consumption. This means that the effect of household's wealth on its preferred pollution tax depends on its percentage and not on its absolute loss from consumption due to the tax. Moreover, the tax elasticity of consumption obviously depends on how the revenues of the households are affected by the increase in the pollution tax.

By using a general equilibrium framework, we showed that the impact of the pollution tax on the household revenue (which comes from the wage and the interest rate) depends on the characteristics of the production technologies employed by the firms. We identified the cases in which the wage and the interest rate move in the same or different direction as a response to an increase in the pollution tax. When the firms operate by using only one production technology, the pollution tax elasticity of the wage and the interest rate are identical which makes the rich to lose less than the poor from their consumption in percentage terms. Thus, in this case, the rich prefer a higher pollution tax and the tax is regressive. This

result changes when the firms operate by using two technologies: (i) dirty and more capital intensive and (ii) cleaner and more labor intensive. In this case, an increase in the pollution tax leads to a reallocation of factors from the dirty technology to the clean one. This reallocation leads to a relatively higher decrease in the returns of capital. Consequently, in this case, the rich loses more than the poor from their consumption in percentage terms and thus they prefer a lower pollution tax and the tax is progressive.

Our results suggest that the pollution tax always decreases the wealth inequality in the economy since the rich always loses more from their consumption in absolute terms. This is due to the fact that we abstracted from the wage inequality and the redistributive effects of the pollution tax. Further research could include these dimensions. For example, the labor supply side of the model can be improved to allow heterogeneities in labor income and the government transfers that are not neutral can be considered.

We discussed about relaxing a few of the assumptions that we made throughout the text and in the discussion section. Relaxing the other key assumptions of this framework can lead to further research on this subject. For example, transforming the model into the dynamic framework will allow to investigate intertemporal effects of environmental policies in the existence of wealth inequality. Introducing consumer preferences towards dirty and clean products will allow to study both the sources and the uses sides of income. Moreover, the extension of the model for multiple countries will provide benefits that are many fold. A simple model of two countries with different wealth distributions, factor endowments and production technologies would allow to analyze concepts such as pollution havens as well as to identify patterns of factors in response to environmental policies. Furthermore, imperfections in capital mobility and labor mobility can result with different implications.

Moreover, our setting is more compatible with the reality compared to other

studies in the literature since we consider that the clean technology also pollutes. Using this framework will allow to have more robust results in the empirical research on this subject. Finally, this study provided a potential benchmark for further analysis in political economics research concerning environmental policies and wealth inequalities.

Appendix

5.A Proof of Proposition 5.1

We use the first order conditions given in (5.3 to 5.5). First we use (5.3) to obtain:

$$z(\tau) = F_1^{-1}(\tau; G(\bar{K}, \bar{L})) \quad (5.28)$$

Note that since $G(\bar{K}, \bar{L})$ is given and constant, it affects $z(\tau)$ as a parameter. By using the properties of the production function given in Assumption 1, we know that $F_{11}^{-1}(\cdot, \cdot) < 0$ hence

$$z'(\tau) < 0 \quad (5.29)$$

Now that we have $z(\tau)$, we replace it in equations (5.4 and 5.5) to get the following:

$$F_2(z(\tau), G(\bar{K}, \bar{L}))G_1(\bar{K}, \bar{L}) = r \quad (5.30)$$

$$F_2(z(\tau), G(\bar{K}, \bar{L}))G_2(\bar{K}, \bar{L}) = w \quad (5.31)$$

We can now compute the wage and interest rate as a function of pollution tax and how they change according to that.

$$r'(\tau) = z'(\tau)F_{21}(z(\tau), G(\bar{K}, \bar{L}))G_1(\bar{K}, \bar{L}) < 0 \quad (5.32)$$

$$w'(\tau) = z'(\tau)F_{21}(z(\tau), G(\bar{K}, \bar{L}))G_2(\bar{K}, \bar{L}) < 0 \quad (5.33)$$

since $z'(\cdot) < 0$, $F_{21}(\cdot) > 0$, $G_1(\cdot) > 0$ and $G_2(\cdot) > 0$ which completes the first part of the proof.

The elasticities of wage and interest rate respect to the pollution tax are:

$$\begin{aligned}\epsilon_{r,\tau} &= \frac{r'(\tau)}{r(\tau)/\tau} = \frac{z'(\tau)F_{21}(z(\tau), G(\bar{K}, \bar{L}))G_1(\bar{K}, \bar{L})\tau}{F_2(z(\tau), G(\bar{K}, \bar{L}))G_1(\bar{K}, \bar{L})} \\ &= z'(\tau) \frac{F_{21}(z(\tau), G(\bar{K}, \bar{L}))\tau}{F_2(z(\tau), G(\bar{K}, \bar{L}))} < 0\end{aligned}\quad (5.34)$$

$$\begin{aligned}\epsilon_{w,\tau} &= \frac{w'(\tau)}{w(\tau)/\tau} = \frac{z'(\tau)F_{21}(z(\tau), G(\bar{K}, \bar{L}))G_2(\bar{K}, \bar{L})\tau}{F_2(z(\tau), G(\bar{K}, \bar{L}))G_2(\bar{K}, \bar{L})} \\ &= z'(\tau) \frac{F_{21}(z(\tau), G(\bar{K}, \bar{L}))\tau}{F_2(z(\tau), G(\bar{K}, \bar{L}))} = \epsilon_{r,\tau} < 0\end{aligned}\quad (5.35)$$

$$\epsilon_{R,\tau} = \frac{R'(\tau)}{R(\tau)/\tau} = \frac{r(\tau)}{R(\tau)} \frac{r'(\tau)}{r(\tau)/\tau} = \frac{r(\tau)}{1 + r(\tau)} \epsilon_{r,\tau} \quad (5.36)$$

which completes the second part of the proof.

Note that this property implies the following relationships:

$$\frac{w(\tau)}{r(\tau)} = \frac{w'(\tau)}{r'(\tau)} = \frac{w''(\tau)}{r''(\tau)} \quad (5.37)$$

Equation (5.37) can be obtained as follows:

$$\frac{r'(\tau)}{r(\tau)} = \frac{w'(\tau)}{w(\tau)} \quad (5.38)$$

$$\Rightarrow \text{Log}(r'(\tau)) - \text{Log}(r(\tau)) = \text{Log}(w'(\tau)) - \text{Log}(w(\tau)) \quad (5.39)$$

$$\Rightarrow \frac{r''(\tau)}{r'(\tau)} - \frac{r'(\tau)}{r(\tau)} = \frac{w''(\tau)}{w'(\tau)} - \frac{w'(\tau)}{w(\tau)} \quad (5.40)$$

$$\Rightarrow \frac{w''(\tau)}{r''(\tau)} = \frac{w'(\tau)}{r'(\tau)} = \frac{w(\tau)}{r(\tau)} \quad (5.41)$$

5.B Proof of Proposition 5.2

We use the first order conditions (5.9 to 5.11) of the cost minimization problem in (5.7) to obtain the derived unit production demands for factors in both of the two technologies. For the dirty technology we have $\{\alpha_z^X(\tau), \alpha_k^X(\tau), \alpha_l^X(\tau)\}$ and for the clean technology we have $\{\alpha_z^Y(\tau), \alpha_k^Y(\tau), \alpha_l^Y(\tau)\}$. From now on we will drop functional arguments (τ) for notational

simplicity.

Let $\eta_x = \alpha_k^X / \alpha_l^X$, $\eta_y = \alpha_k^Y / \alpha_l^Y$, $\zeta_x = \alpha_z^X / \alpha_l^X$ and $\zeta_y = \alpha_z^Y / \alpha_l^Y$. By Definition 1 ($\alpha_z^X > \alpha_z^Y$, $\alpha_k^X > \alpha_k^Y$ and $\alpha_l^X < \alpha_l^Y$) we have $\eta_x > \eta_y$ and $\zeta_x > \zeta_y$. Perfect competition implies:

$$\tau \alpha_z^X + r(\tau) \alpha_k^X + w(\tau) \alpha_l^X = \bar{p} \quad (5.42)$$

$$\tau \alpha_z^Y + r(\tau) \alpha_k^Y + w(\tau) \alpha_l^Y = \bar{p} \quad (5.43)$$

where $\bar{p}$ is the price of the generic good and we take it as numeraire hence $\bar{p} = 1$. Now we will compute how the unit cost changes with the pollution tax. For that we take the derivative of equations (5.42 and 5.43) respect to τ . Note that all the derived demands depend on the pollution tax, however, they are obtained from the cost minimization problem which means that when we apply the envelope theorem we will have $\tau a'_z(\tau) + r(\tau) a'_k(\tau) + w(\tau) a'_l(\tau) = 0$ for $i \in \{x, y\}$. Applying this to the derivative of equations (5.42 and 5.43):

$$\alpha_z^X + r'(\tau) \alpha_k^X + w'(\tau) \alpha_l^X = 0 \quad (5.44)$$

$$\alpha_z^Y + r'(\tau) \alpha_k^Y + w'(\tau) \alpha_l^Y = 0 \quad (5.45)$$

We divide (5.44) by α_l^X and (5.45) by α_l^Y to obtain:

$$\zeta_x + r'(\tau) \eta_x + w'(\tau) = 0 \quad (5.46)$$

$$\zeta_y + r'(\tau) \eta_y + w'(\tau) = 0 \quad (5.47)$$

Subtracting (5.47) from (5.46) gives:

$$r'(\tau) = -\frac{\zeta_x - \zeta_y}{\eta_x - \eta_y} < 0 \text{ by Definition 1} \quad (5.48)$$

Furthermore, we multiply (5.47) by η_x / η_y and subtract the resulting equation

tion from (5.46) to obtain:

$$w'(\tau) = \frac{\zeta_x \eta_y - \zeta_y \eta_x}{\eta_x - \eta_y} \quad (5.49)$$

The sign of $w'(\tau)$ depends on the relative factor intensities between the two technologies. We have:

$$w'(\tau) > 0 \text{ if } \frac{\zeta_x}{\eta_x} > \frac{\zeta_y}{\eta_y} \Leftrightarrow \frac{\alpha_z^X}{\alpha_k^X} > \frac{\alpha_z^Y}{\alpha_k^Y} \quad (5.50)$$

$$w'(\tau) = 0 \text{ if } \frac{\zeta_x}{\eta_x} = \frac{\zeta_y}{\eta_y} \Leftrightarrow \frac{\alpha_z^X}{\alpha_k^X} = \frac{\alpha_z^Y}{\alpha_k^Y} \quad (5.51)$$

$$w'(\tau) < 0 \text{ if } \frac{\zeta_x}{\eta_x} < \frac{\zeta_y}{\eta_y} \Leftrightarrow \frac{\alpha_z^X}{\alpha_k^X} < \frac{\alpha_z^Y}{\alpha_k^Y} \quad (5.52)$$

which completes the first part of the proof. For the elasticities, we can rewrite equations (5.42) and (5.43) as follows:

$$\zeta_x \tau + \eta_x r(\tau) + w(\tau) = \frac{\bar{p}}{\alpha_l^X} \quad (5.53)$$

$$\zeta_y \tau + \eta_y r(\tau) + w(\tau) = \frac{\bar{p}}{\alpha_l^Y} \quad (5.54)$$

Multiplying equation (5.53) by ζ_y and equation (5.54) by ζ_x and subtracting the latter from the former, we get:

$$(1+r(\tau))(\zeta_y \eta_x - \zeta_x \eta_y) + w(\tau)(\zeta_y - \zeta_x) = \bar{p} \left(\frac{\zeta_y}{\alpha_l^X} - \frac{\zeta_x}{\alpha_l^Y} \right) = \frac{\bar{p}}{\alpha_l^X \alpha_l^Y} (\alpha_z^Y - \alpha_z^X) < 0 \quad (5.55)$$

Dividing LHS of inequality (5.55) by $(\eta_x - \eta_y)$ and using equations (5.48) and (5.49), we can show:

$$-w'(\tau)r(\tau) + w(\tau)r'(\tau) < 0 \quad (5.56)$$

Therefore:

$$\frac{r'(\tau)}{r(\tau)} < \frac{w'(\tau)}{w(\tau)} \Leftrightarrow \epsilon_{r,\tau} < \epsilon_{w,\tau} \quad (5.57)$$

Moreover, since $r' < 0$, $\epsilon_{r,\tau} < \epsilon_{R,\tau} < 0$.

Finally, equation (5.50) define the conditions for the sign of $\epsilon_{w,\tau}$ and it completes the second part of the proof.

5.C Proof of Proposition 5.3

From equations (5.10) and (5.11), we have:

$$r = F_2^a(\alpha_z^X, G^a(\alpha_k^X, \alpha_l^X))G_1^a(\alpha_k^X, \alpha_l^X) \quad (5.58)$$

$$w = F_2^a(\alpha_z^X, G^a(\alpha_k^X, \alpha_l^X))G_2^a(\alpha_k^X, \alpha_l^X) \quad \text{for } a \in \{x, y\} \quad (5.59)$$

Dividing equation (5.58) by (5.59) we get:

$$\frac{r}{w} = \frac{G_1^a(\alpha_k^X, \alpha_l^X)}{G_2^a(\alpha_k^X, \alpha_l^X)} \quad (5.60)$$

Proposition 3 implies that $\frac{d(r/w)}{d\tau} < 0$ and so:

$$\frac{d\left(\frac{G_1^a(\alpha_k^X, \alpha_l^X)}{G_2^a(\alpha_k^X, \alpha_l^X)}\right)}{d\tau} < 0 \Leftrightarrow \frac{d\left(\frac{a_k}{a_l}\right)}{d\tau} > 0 \quad \text{for } a \in \{x, y\} \quad (5.61)$$

5.D Proof of Proposition 5.4

Resource Constraints for capital and labor imply that:

$$X\alpha_k^X + Y\alpha_k^Y = \bar{K} \quad (5.62)$$

$$X\alpha_l^X + Y\alpha_l^Y = \bar{L} \quad (5.63)$$

Solving equations (5.62) and (5.63) for X and Y will result in the followings:

$$X = \frac{\alpha_l^Y \bar{K} - \alpha_k^Y \bar{L}}{\alpha_k^X \alpha_l^Y - \alpha_l^X \alpha_k^Y} \quad (5.64)$$

$$Y = \frac{\alpha_l^X \bar{K} - \alpha_k^X \bar{L}}{\alpha_l^X \alpha_k^Y - \alpha_k^X \alpha_l^Y} \quad (5.65)$$

Therefore:

$$X = 0 \Leftrightarrow \frac{\alpha_k^Y(\tau_{clean})}{\alpha_l^Y(\tau_{clean})} = \frac{\bar{K}}{\bar{L}} \quad (5.66)$$

$$Y = 0 \Leftrightarrow \frac{\alpha_k^X(\tau_{dirty})}{\alpha_l^X(\tau_{dirty})} = \frac{\bar{K}}{\bar{L}} \quad (5.67)$$

The denominator in RHS of equation (5.66) is positive. Since $\frac{d(\frac{\alpha_k}{\alpha_l})}{d\tau} > 0$ for $a \in \{x, y\}$, if pollution tax is higher than τ_{clean} , then the production in dirty technology will be negative which is not possible. Therefore, for pollution tax higher than τ_{clean} , economy will use only the clean technology. With the same method, it is easy to show that for the pollution tax lower than τ_{dirty} the economy will operate only by the dirty technology.

5.E Proof of Corollary 5.1

From equations (5.66) and (5.67), we know that:

$$\frac{d(\frac{\alpha_k^Y(\tau_{clean})}{\alpha_l^Y(\tau_{clean})})}{d(\frac{\bar{K}}{\bar{L}})} = 1 > 0 \quad (5.68)$$

$$\frac{d(\frac{\alpha_k^X(\tau_{dirty})}{\alpha_l^X(\tau_{dirty})})}{d(\frac{\bar{K}}{\bar{L}})} = 1 > 0 \quad (5.69)$$

And from Proposition 5.4 we know that $\frac{d(a_k(\tau)/a_l(\tau))}{d\tau} > 0$ for $a \in \{x, y\}$. Therefore:

$$\frac{d(\tau_{clean})}{d(\frac{\bar{K}}{\bar{L}})} > 0 \quad (5.70)$$

$$\frac{d(\tau_{dirty})}{d(\frac{\bar{K}}{\bar{L}})} > 0 \quad (5.71)$$

5.F Proof of equation (5.20)

Multiplying equation (5.44) by total production of the dirty technology (X), and multiplying equation (5.45) by total production of the clean technology (Y) results in the following:

$$Z_x + r'(\tau)k_x + w'(\tau)l_x = 0 \quad (5.72)$$

$$Z_y + r'(\tau)k_y + w'(\tau)l_y = 0 \quad (5.73)$$

By adding the two last equations, we have:

$$Z = -(r'(\tau)\bar{K} + w'(\tau)\bar{L}) = -C''(\tau) \quad (5.74)$$

Using equation (5.74) in the RHS of the first equality in equation (5.20), will lead to the second equality of that equation. Moreover, since $Z > 0$, total private consumption is decreasing in pollution tax.

5.G Proof for equation (5.24)

We start from the first order condition resulted from household's maximization programme given in equation (5.21):

$$\frac{\partial U_i(c_i(\tau_i^*, k_i), z(\tau_i^*))}{\partial \tau_i^*} = U_c(.) \frac{\partial c_i(\tau_i^*, k_i)}{\partial \tau_i^*} + U_z(.) \frac{\partial z(\tau_i^*)}{\partial \tau_i^*} = 0 \quad (5.75)$$

To find $\frac{\partial \tau_i^*(k_i)}{\partial k_i}$ we take the derivative of (5.75) with respect to k_i at $\tau_i^*(k_i)$:

$$\begin{aligned}
& U_{cc}(\cdot) \frac{\partial c}{\partial k_i} c_1(\tau, k_i) + U_{cz}(\cdot) z'(\tau) \frac{\partial \tau^*}{\partial k_i} c_1(\tau, k_i) + U_{cc}(\cdot) (c_1(\tau, k_i))^2 \frac{\partial \tau^*}{\partial k_i} \\
& + U_c(\cdot) c_{11}(\tau, k_i) \frac{\partial \tau^*}{\partial k_i} + U_c(\cdot) c_{12}(\tau, k_i) \\
& + U_{cz}(\cdot) c_2(\tau, k_i) z'(\tau) + U_{cz}(\cdot) c_1(\tau, k_i) \frac{\partial \tau^*}{\partial k_i} z'(\tau) \\
& + U_{zz}(\cdot) (z'(\tau))^2 \frac{\partial \tau^*}{\partial k_i} + U_z(\cdot) z''(\tau) \frac{\partial \tau^*}{\partial k_i} = 0
\end{aligned} \tag{5.76}$$

Setting $U_{cz}(\cdot) = 0$ (by Assumption 3) and collecting $\frac{\partial \tau^*}{\partial k_i}$ we obtain:

$$\frac{\partial \tau^*(k_i)}{\partial k_i} = -\frac{S_1}{S_2} \tag{5.77}$$

$$\text{where } S_1 = U_{cc}(\cdot) c_2(\tau, k_i) c_1(\tau, k_i) + U_c(\cdot) c_{12}(\tau, k_i) \tag{5.78}$$

$$\begin{aligned}
S_2 = & U_{cc}(\cdot) (c_1(\tau, k_i))^2 + U_c(\cdot) c_{11}(\tau, k_i) + U_z(\cdot) z''(\tau) \\
& + U_{zz}(\cdot) (z'(\tau))^2
\end{aligned} \tag{5.79}$$

Equation (5.79), S_2 , corresponds to the second order condition and it is negative ($S_2 < 0$) by Assumption 4. Therefore S_1 determines the sign of $\frac{\partial \tau^*}{\partial k_i}$.

5.H Proof for Proposition 5.5

If $v(c_i) = \log(c_i)$, then, $U_c(\cdot) = \frac{1}{c_i}$ and $U_{cc}(\cdot) = \frac{-1}{c_i^2}$. By replacing these two equations in equation (5.78), we will have:

$$s_1 = -\frac{1}{c_i^2} \frac{\partial c_i}{\partial k_i} \frac{\partial c_i}{\partial \tau} + \frac{1}{c_i} \frac{\partial^2 c_i}{\partial k_i \partial \tau} \tag{5.80}$$

And equivalently:

$$s_1 = \frac{\partial(\frac{1}{c_i} \frac{\partial c_i}{\partial \tau})}{\partial k_i} = \frac{1}{\tau} \frac{\partial \epsilon_{c_i, \tau}}{\partial k_i} \tag{5.81}$$

5.I Proof for Proposition 5.6

$$\epsilon_{c_i, \tau} = \frac{\partial c_i}{\partial \tau} \frac{\tau}{c_i} = \frac{r'(\tau)k_i + w'(\tau)}{(1 + r(\tau))k_i + w(\tau)} \tau \quad (5.82)$$

Therefore:

$$\begin{aligned} \frac{\partial \epsilon_{c_i, \tau}}{\partial k_i} &= \frac{r'(\tau)c_i - (1 + r(\tau))c'_i}{c_i^2} \tau = \frac{\tau}{c_i} (r'(\tau)w(\tau) - (1 + r(\tau))w'(\tau)) \\ &= \frac{(1 + r(\tau))w(\tau)}{c_i^2} (\epsilon_{R, \tau} - \epsilon_{w, \tau}) \end{aligned} \quad (5.83)$$

Using equation 5.83 and equation 5.81, we can get:

$$s1 = \frac{(1 + r(\tau))w(\tau)}{\tau c_i^2} (\epsilon_{R, \tau} - \epsilon_{w, \tau}) \quad (5.84)$$

Which establishes the proof for Proposition 5.7.

5.J The case of n technologies

We claim that in our framework, where the economy is open and operating in n -sectors (thus, prices in all the sectors are fixed), or equivalently, where the economy is closed but producing and consuming only one generic good with n -technologies, the economy will operate using maximum two sectors/technologies.

We have endowment constraints:

$$\sum_{i=1}^n k^i = \bar{K} \quad (5.85)$$

$$\sum_{i=1}^n l^i = \bar{L} \quad (5.86)$$

For each sector i , we have: $Q_i = F^i(z^i, G^i(k^i, l^i))$ which has a market price p_Q^i that is exogenously given. The prices of capital and labor (r and w) are

endogenously determined, however, the price of z (τ) is exogenously given (by the government). The firms solve the following problem:

$$\max_{\{z^i, k^i, l^i\}} \left\{ \sum_{i=1}^n (p_Q^i F^i(z^i, G^i(k^i, l^i)) - rk^i - wl^i - \tau z^i) \right\}$$

subject to (5.85), (5.86) and $z^i \geq 0 \forall i$

First order conditions for an interior solution are:

$$p_Q^i F_1^i(z^i, G^i(k^i, l^i)) = \tau \quad (5.87)$$

$$p_Q^i F_2^i(z^i, G^i(k^i, l^i)) G_1^i(k^i, l^i) = r \quad (5.88)$$

$$p_Q^i F_2^i(z^i, G^i(k^i, l^i)) G_2^i(k^i, l^i) = w \quad (5.89)$$

Therefore, we have:

$\{5.85, 5.86, 5.87, 5.88, 5.89\} \Rightarrow 3n + 2$ equations and $\{\{k^i, l^i, z^i\}, r, w\} \Rightarrow 3n + 2$ variables.

Now we will show that if the functions $F(\cdot)$ and $G(\cdot)$ are homogeneous of degree 1 then these equations are not independent when $n > 2$. Therefore the solution for $n > 2$ does not exist. In other words, it is not possible that the economy operates with more than two technologies. To show that, we define:

$$\eta^i = \frac{k^i}{l^i} \quad (5.90)$$

$$\zeta^i = \frac{z^i}{l^i} \quad (5.91)$$

Using the property of homogenous of degree 1 for $F(\cdot)$ and $G(\cdot)$, we can rewrite equations (5.87) to (5.89) as follows:

$$p_Q^i F_1^i\left(\frac{\zeta^i}{G^i(\eta^i, 1)}, 1\right) = \tau \quad (5.92)$$

$$p_Q^i F_2^i\left(\frac{\zeta^i}{G^i(\eta^i, 1)}, 1\right) G_1^i(\eta^i, 1) = r \quad (5.93)$$

$$p_Q^i F_2^i\left(\frac{\zeta^i}{G^i(\eta^i, 1)}, 1\right) G_2^i(\eta^i, 1) = w \quad (5.94)$$

For n sectors, we have $\{\{\eta^i, \zeta^i, k^i, l^i, z^i\}, r, w\} \Rightarrow 5n + 2$ variables and (5.85, 5.86, 5.90, 5.91, 5.92, 5.93, 5.94) $\Rightarrow 5n + 2$ equations. At this point, the number of equations equals the number of variables and, thus, the system of equations seems to have a solution. However, a subset of this equation system, equations (5.92, 5.93, 5.94) contain $3n$ equations with $2n + 2$ variables. Therefore, if $n > 2$ then the number of equations is greater than the number of variables. This fact concludes that the system of equations are not independent. Hence there is no solution for $n > 2$ when all of the n -technologies are being operated by the economy. In other words, the economy will use maximum two technologies for a given τ .

In fact, we can generalize the results above. Consider an economy with n -technologies (sectors) where all of the technologies are homogenous of degree 1 and they use m factors as inputs. In the case where the prices of s factors are given, meaning that $m - s$ factors' prices are determined endogenously (and their total amount must be constrained by endowment or ceiling constraints), we can conclude that maximum $m - s$ technologies will be operated by the economy.

5.K An alternative setting: pollution as a byproduct

In this alternative setting the firms are involved in two processes. In the first process, they hire capital and labor (k^P, l^P) to produce the final good. Pollution (z) is byproduct of this process. Since we assume that the pollution is taxed (τ), the firms will get involved in the abatement activities in which they use capital and labor (k^A, l^A) to produce equipment that is used to reduce pollution. Therefore, in this alternative setting, pollution is a function of final good production ($H(k^P, l^P)$) and abatement process ($B(k^A, l^A)$):

$$z = \Phi(H(k^P, l^P), B(k^A, l^A))$$

$$\text{Where: } \Phi_1(\cdot) > 0, \Phi_2(\cdot) < 0, \Phi_{11}(\cdot) > 0, \Phi_{22}(\cdot) > 0$$

where j^P and j^A are demands of factor j for production of final good and for pollution abatement respectively. Since factor prices and pollution tax are given to the firms, their cost-minimization problem for producing one unit of final good is as follows:

$$\min_{\{a_z, a_k, a_l\}} \{ (a_k^P + a_k^A)r + (a_L^P + a_L^A)w + \Phi(H(a_k^P, a_L^P), B(a_k^A, a_L^A))\tau \} \quad (5.95)$$

$$\text{subject to: } H(a_k^P, a_L^P) = 1 \quad (5.96)$$

Here, a_j^P and a_j^A are demand of factor j for unit production of final good and for corresponding pollution abatement respectively. Factor demands in our main setting a_z, a_k, a_l can be translated to this setting as follows:

$$a_z = \Phi(H(a_k^P, a_L^P), B(a_k^A, a_L^A)) \quad (5.97)$$

$$a_k = a_k^P + a_k^A \quad (5.98)$$

$$a_l = a_l^P + a_l^A \quad (5.99)$$

Constant returns to scale form assumption for $F(\cdot)$ and $G(\cdot)$ in our main setting can be translated to constant returns to scale property of $H(\cdot)$, $B(\cdot)$ and $\Phi(\cdot)$ in this alternative setting. Firms' minimization problem leads to the following first order conditions:

$$\begin{aligned} r &= H_1(a_k^P, a_L^P)(1 - \Phi_1(H(a_k^P, a_L^P), B(a_k^A, a_L^A))\tau) \\ &= \tau \Phi_2(H(a_k^P, a_L^P), B(a_k^A, a_L^A))B_1(a_k^A, a_L^A) \end{aligned} \quad (5.100)$$

$$\begin{aligned} w &= H_2(a_k^P, a_L^P)(1 - \Phi_1(H(a_k^P, a_L^P), B(a_k^A, a_L^A))\tau) \\ &= \tau \Phi_2(H(a_k^P, a_L^P), B(a_k^A, a_L^A))B_2(a_k^A, a_L^A) \end{aligned} \quad (5.101)$$

Besides, resource constrains imply:

$$Y(a_k^P + a_k^A) = \bar{K} \quad (5.102)$$

$$Y(a_l^P + a_l^A) = \bar{L} \quad (5.103)$$

$$\text{Where: } Y = H\left(\frac{a_k^P \bar{K}}{a_k^P a_k^A}, \frac{a_l^P \bar{L}}{a_l^P a_l^A}\right) \quad (5.104)$$

Equations (5.100) to (5.104) provides seven equations and seven variables: $\{a_k^P, a_L^P, a_k^A, a_L^A, w, r, Y\}$. Therefore, factor demands and input prices can be found as a function of pollution tax (τ). For the sake of notation simplicity, in the following, we don't write (τ) knowing that all these variables are function of this variable.

As we explained in Section 2.1, the assumption of functional separability directly implies that, once there is only one technology used in the economy,

wage and interest rate will have the identical pollution tax elasticity. Now, we can investigate the implication of this result in this alternative setting. Below, we will prove that, in this alternative setting, relative price of wage to interest rate remains unchanged, if and only if, production process and pollution abatement process have identical relative factor intensity.

Dividing equation (5.100) by equation (5.101) results in:

$$\frac{r}{w} = \frac{H_1(a_k^P, a_L^P)}{H_2(a_k^P, a_L^P)} = \frac{B_1(a_k^A, a_L^A)}{B_2(a_k^A, a_L^A)} \quad (5.105)$$

Thus:

$$\frac{d(\frac{r}{w})}{d\tau} = 0 \Rightarrow \begin{cases} \frac{d(\frac{H_1(a_k^P, a_L^P)}{H_2(a_k^P, a_L^P)})}{d\tau} = 0 \\ \frac{d(\frac{B_1(a_k^A, a_L^A)}{B_2(a_k^A, a_L^A)})}{d\tau} = 0 \end{cases} \Rightarrow \begin{cases} \frac{d(\frac{a_k^P}{a_L^P})}{d\tau} = 0 \\ \frac{d(\frac{a_k^A}{a_L^A})}{d\tau} = 0 \end{cases} \quad (5.106)$$

Since $H(\cdot)$ is constant returns to scale and by definition $H(a_k^P, a_L^P) = 1$, the first equality in equation (5.106) implies that a_k^P and a_L^P are constant. Therefore:

$$a_k^{P'} = a_L^{P'} = 0 \quad (5.107)$$

Moreover, resource constraint and CRS property of production function implies the following:

$$\frac{a_k^P + a_k^A}{a_L^P + a_L^A} = \frac{\bar{K}}{\bar{L}} \quad (5.108)$$

Making derivative from equation (5.108) and applying equation (5.107) leads to the following:

$$a_k^{A'}(a_L^P + a_L^A) = a_L^{P'}(a_k^P + a_k^A) \underbrace{\Rightarrow}_{\text{by eq. (5.106)}} a_k^A a_L^P = a_L^A a_k^P \Rightarrow \frac{a_k^A}{a_L^A} = \frac{a_k^P}{a_L^P} \quad (5.109)$$

The intuition behind this observation is that if pollution tax increases, firms will hire more capital and labor for abatement process. In overall, hence, the input hired in production process will decrease while that hired in pollution abatement process will increase. Consequently, if, compared to the former process, the latter uses one factor relatively more intensively than the other one, the price of that factor will increase relatively. Hence, relative price of factors will remain constant only if both process employ the factors with the same relative intensity.

Finally, we can investigate what dirty and clean technology mean when our main setting is transformed to this alternative one: If two production technologies, $(H^d(.), H^c(.))$, are available, $H^d(.)$ is dirty if and only if the pollution it generates to produce one unit of final good is more than the pollution that $H^c(.)$ generates for producing the same amount of final good.

Chapter 6

Conclusion

This dissertation investigated the economic mechanisms underlying the transition to clean technologies and showed the roles of different policy measures aiming to deal with climate change, such as adaptation and taxation of non-renewable resources. It presented novel results on the optimal transition path to a clean economy, on the optimal trajectory of a carbon tax, on the effects of adaptation measures on transition, on the issue of technology adoption of firms when they face financial constraints in imperfectly competitive market structures, and on the role of heterogeneity in wealth on agents' willingness to support a pollution tax. These results contribute to the literature on environmental economics, energy and natural economics and economics of technological change.

Each chapter has its own further research directions. The first chapter can be expanded for a better understanding of the role of renewable resources and technical change on energy transition. This analysis can be done by including the different features of the type of renewable resources such as wind power, solar power and biofuels. Furthermore, a novel structural decomposition approach can be applied to the production function that is used in this chapter. These extensions would lead to a useful tool to provide accurate estimations of crucial parameters as well as market-based policy instruments. Moreover, taking into account technological progress that

improves the efficiencies of both non-renewable and renewable resources as well as changes the degree of substitution between resources would provide new results.

On the role of adaptation measures, relaxing the irreversible pollution assumption would change the characteristics of the optimum. Moreover, the damage function is defined to be continuous. Consideration of a catastrophic outcome when the pollution level rises too high, or a cap on pollution accumulation would change the optimum as well. The transition in the outcome is always beneficial, however, if the transition itself brings costs due to the change of resource utilization then the compensation mechanism during the transition would work differently. Finally, taking into account the regional differences in pollution damages and cost of adaptation is one of the potential future extensions.

At the firm level analysis of technology adoption, the present work can be extended by including the uncertainties in energy supply and demand sides. This extension would allow the modeling approach to suit better to the electricity generation markets, which is a mainly studied subject at the intersection of energy economics, industrial organization and operation research fields. Another potential future research direction is to take into account negative externalities that arise due to utilization of inefficient production technologies (such as pollution). This would allow to investigate a social optimum and study the optimal policy instruments that will induce decentralized equilibrium to reach the socially optimal production capacity and technology mixture.

On the policy challenges in implementation of a carbon tax due to wealth inequality, further research includes the transformation of the model into a dynamic framework. This would allow to investigate the intertemporal effects of environmental policies in the existence of wealth inequality. Moreover, the extension of the model for multiple countries would allow to analyze issues such as pollution havens as well as to identify patterns of

factor movements in response to environmental policies.

Bibliography

- Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hemous (2012). The Environment and Directed Technical Change. *American Economic Review* 102(1), 131–66.
- Acemoglu, D., U. Akcigit, D. Hanley, and W. Kerr (2016). Transition to clean technology. *Journal of Political Economy* 124(1), 52–104.
- Aghion, P., A. Dechezleprêtre, D. Hemous, R. Martin, and J. V. Reenen (2012). Carbon taxes, path dependency and directed technical change: Evidence from the auto industry. CEP Discussion Papers No: 1178, Centre for Economic Performance, LSE.
- Almeida, H. and M. Campello (2007). Financial Constraints, Asset Tangibility, and Corporate Investment. *Review of Financial Studies* 20(5), 1429–1460.
- Arabzadeh, H. and B. Vardar (2015). Why the rich may want a low pollution tax? Working paper.
- Belgodere, A. (2009). On the path of an oil Pigovian tax. *The Manchester School* 77(5), 632 – 649.
- Borissov, K., T. Bréchet, and S. Lambrecht (2014). Environmental policy in a dynamic model with heterogeneous agents and voting. In E. Moser, W. Semmler, G. Tragler, and V. M. Veliov (Eds.), *Dynamic Optimization in Environmental Economics*, Volume 15 of *Dynamic Modeling and Econometrics in Economics and Finance*. Springer Berlin Heidelberg.

- Bréchet, T., N. Hritonenko, and Y. Yatsenko (2016). Domestic environmental policy and international cooperation for global commons. *Resource and Energy Economics* 44, 183 – 205.
- Bréchet, T. and B. Vardar (2016). The role of adaptation on the transition to a low-carbon economy. Working paper.
- Bréchet, T., N. Hritonenko, and Y. Yatsenko (2013). Adaptation and mitigation in long-term climate policy. *Environmental and Resource Economics* 55(2), 217–243.
- Buob, S. and G. Stephan (2011). To mitigate or to adapt: How to confront global climate change. *European Journal of Political Economy* 27(1), 1–16.
- Carpenter, R. E. and B. C. Petersen (2002). Capital market imperfections, high-tech investment, and new equity financing. *Economic Journal* 112(477), F54–F72.
- Chakravorty, U., B. Magné, and M. Moreaux (2006). A hotelling model with a ceiling on the stock of pollution. *Journal of Economic Dynamics and Control* 30(12), 2875 – 2904.
- Chiroleu-Assouline, M. and M. Fodha (2006). Double dividend hypothesis, golden rule and welfare distribution. *Journal of Environmental Economics and Management* 51(3), 323–335.
- Chiroleu-Assouline, M. and M. Fodha (2014). From regressive pollution taxes to progressive environmental tax reforms. *European Economic Review* 69, 126 – 142.
- Copeland, B. R. and M. S. Taylor (1994). North-south trade and the environment. *The Quarterly Journal of Economics* 109(3), 755–787.
- Copeland, B. R. and M. S. Taylor (2004). Trade, growth, and the environment. *Journal of Economic Literature* 42(1), 7–71.

- Dasgupta, P. and G. Heal (1974). The optimal depletion of exhaustible resources. *The Review of Economic Studies* 41, 3–28.
- Daubanes, J. and A. Grimaud (2010). Taxation of a polluting non-renewable resource in the heterogeneous world. *Environmental and Resource Economics* 47(4), 567 – 588.
- d’Autume, A. (2012). The user cost of natural resources and the optimal exploitation of two non-renewable polluting resources. Documents de travail du centre d’economie de la sorbonne no: 12037.
- Dissou, Y. and M. S. Siddiqui (2014). Can carbon taxes be progressive? *Energy Economics* 42, 88 – 100.
- Dockner, E. J. (1985). Local stability analysis in optimal control problems with two state variables. In G. Feichtinger (Ed.), *Optimal control theory and economic analysis*, Volume 2. North-Holland, Amsterdam.
- Dockner, E. J. and G. Feichtinger (1991). On the optimality of limit cycles in dynamic economic systems. *Journal of Economics* 53(1), 31 – 50.
- Eisfeldt, A. L. and A. a. Rampini (2007). New or used? Investment with credit constraints. *Journal of Monetary Economics* 54, 2656–2681.
- Ercolano, S., G. L. Gaeta, and O. Romano (2014). Environmental tax reform and individual preferences: An empirical analysis on european micro data. *Journal of Behavioral and Experimental Economics* 51, 1 – 11.
- Fagart, T. and B. Vardar (2016). The role of capacity building on technology adoption under imperfect competition. CORE discussion paper 2016/11.
- Farzin, Y. (1996). Optimal pricing of environmental and natural resource use with stock externalities. *Journal of Public Economics* 62(1-2), 31 – 57.

- Farzin, Y. H. and O. Tahvonen (1996). Global carbon cycle and the optimal time path of a carbon tax. *Oxford Economic Papers* 48(4), 515 – 536.
- Fazzari, S. M., R. G. Hubbard, and B. C. Petersen (1988). Financing Constraints and Corporate Investment. *Brookings Papers on Economic Activity* 1(1), 141–206.
- Feichtinger, G., R. F. Hartl, P. M. Kort, and V. M. Veliov (2008). Financially constrained capital investments: The effects of disembodied and embodied technological progress. *Journal of Mathematical Economics* 44(5-6), 459–483.
- Fudenberg, D. and J. Tirole (1985). Preemption and rent equalization in the adoption of new technology. *The Review of Economic Studies* 52(3), 383 – 401.
- Fullerton, D. and G. Heutel (2007). The general equilibrium incidence of environmental taxes. *Journal of Public Economics* 91(3-4), 571–591.
- Fullerton, D. and G. Heutel (2010). Analytical general equilibrium effects of energy policy on output and factor prices. *The B.E. Journal of Economic Analysis & Policy* 10(2).
- Fullerton, D. and H. Monti (2013). Can pollution tax rebates protect low-wage earners? *Journal of Environmental Economics and Management* 66(3), 539 – 553.
- Gaimon, C. (1989). Dynamic game results of the acquisition of new technology. *Operations Research* 37(3), 410 – 425.
- Gaudet, G. and P. Lasserre (2013). The taxation of nonrenewable natural resources. CIREQ Cahiers de recherche 15-2013.
- Golosov, M., J. Hassler, P. Krusell, and A. Tsyvinski (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica* 82(1), 41 – 88.

- Goulder, L. H. and K. Mathai (2000). Optimal CO₂ abatement in the presence of induced technological change. *Journal of Environmental Economics and Management* 39(1), 1 – 38.
- Grimaud, A., G. Lafforgue, and B. Magné (2011). Climate change mitigation options and directed technical change: A decentralized equilibrium analysis. *Resource and Energy Economics* 33(4), 938 – 962.
- Grimaud, A. and L. Rougé (2005). Polluting non-renewable resources, innovation and growth: welfare and environmental policy. *Resource and Energy Economics* 27(2), 109 – 129.
- Grimaud, A. and L. Rougé (2008). Environment, directed technical change and economic policy. *Environmental and Resource Economics* 41(4), 439 – 463.
- Groom, B., C. Hepburn, P. Koundouri, and D. Pearce (2005). Declining discount rates: The long and the short of it. *Environmental and Resource Economics* 32(4), 445 – 493.
- Groth, C. and P. Schou (2007). Growth and non-renewable resources: The different roles of capital and resource taxes. *Journal of Environmental Economics and Management* 53(1), 80 – 98.
- Harberger, A. C. (1962). The incidence of the corporation income tax. *Journal of Political Economy* 70(3), 215–240.
- Heal, G. (1976). The relationship between price and extraction cost for a resource with a backstop technology. *The Bell Journal of Economics* 7(2), 371 – 378.
- Herfindahl, O. C. (1967). Depletion and economic theory. In M. Gaffney (Ed.), *Extractive resources and taxation*. Madison : University of Wisconsin Press.

- Hoel, M. and S. Kverndokk (1996). Depletion of fossil fuels and the impacts of global warming. *Resource and Energy Economics* 18(2), 115 – 136.
- Hoppe, H. C. (2002). The timing of new technology adoption: theoretical models and empirical evidence. *The Manchester School* 70(1), 56 – 76.
- Hoppe, H. C. and U. Lehmann-Grube (2005). Innovation timing games: a general framework with applications. *Journal of Economic Theory* 121(1), 30–50.
- Hotelling, H. (1931). The economics of exhaustible resources. *The journal of political economy*, 137–175.
- Huisman, K. J. and P. M. Kort (2004). Strategic technology adoption taking into account future technological improvements: A real options approach. *European Journal of Operational Research* 159(3), 705 – 728.
- IPCC (2014). *Climate Change 2014: Synthesis Report. Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*. IPCC, Geneva, Switzerland.
- Jouvet, P.-A., P. Michel, and G. Rotillon (2005). Optimal growth with pollution: how to use pollution permits? *Journal of Economic Dynamics and Control* 29(9), 1597–1609.
- Kempf, H. and S. Rossignol (2007). Is inequality harmful for the environment in a growing economy? *Economics & Politics* 19(1), 53 – 71.
- Lafforgue, G., B. Magné, and M. Moreaux (2008). Energy substitutions, climate change and carbon sinks. *Ecological Economics* 67(4), 589 – 597.
- Mangasarian, O. L. (1966). Sufficient conditions for the optimal control of nonlinear systems. *SIAM Journal on Control* 4(1), 139 – 152.
- Marsiliani, L. and T. I. Rengström (2002). Inequality, environmental protection and growth. W. Allen Wallis Institute of Political Economy Working Paper No: 35/2002, University of Rochester.

- Mattusch, C. (2008). Metalworking and tools. In J. P. Oleson (Ed.), *The Oxford Handbook of Engineering and Technology in the Classical World*, pp. 418 – 438. Oxford University Press.
- Meunier, G. (2010). Capacity choice, technology mix and market power. *Energy Economics* 32(6), 1306–1315.
- Michel, P. and G. Rotillon (1995). Disutility of pollution and endogenous growth. *Environmental and Resource Economics* 6(3), 279 – 300.
- Michielsen, T. O. (2014). Brown backstops versus the green paradox. *Journal of Environmental Economics and Management* 68(1), 87 – 110.
- Milliou, C. and E. Petrakis (2011). Timing of technology adoption and product market competition. *International Journal of Industrial Organization* 29(5), 513–523.
- Milstein, I. and A. Tishler (2012). The inevitability of capacity underinvestment in competitive electricity markets. *Energy Economics* 34(1), 62–77.
- Moreaux, M. and C. Withagen (2015). Optimal abatement of carbon emission flows. *Journal of Environmental Economics and Management* 74, 55 – 70.
- Murphy, F. H. and Y. Smeers (2005). Generation capacity expansion in imperfectly competitive restructured electricity markets. *Operations research* 53(4), 646–661.
- Nordhaus, W. D. (1994). *Managing the Global Commons: The Economics of Climate Change*, Volume 31. The MIT Press, Cambridge, MA.
- Rausch, S., G. E. Metcalf, and J. M. Reilly (2011). Distributional impacts of carbon pricing: A general equilibrium approach with micro-data for

- households. *Energy Economics* 33, Supplement 1, S20 – S33. Supplemental Issue: Fourth Atlantic Workshop in Energy and Environmental Economics.
- Reinganum, J. F. (1981). On the diffusion of new technology: A game theoretic approach. *The Review of Economic Studies* 48(3), 395 – 405.
- Rezai, A., F. van der Ploeg, and C. Withagen (2012). The optimal carbon tax and economic growth: Additive versus multiplicative damages. OxCarre Working Papers 093.
- Sanin, M.-E. and S. Zanjaj (2011). A note on clean technology adoption and its influence on tradeable emission permits prices. *Environmental and Resource Economics* 48(4), 561 – 567.
- Schou, P. (2002). When environmental policy is superfluous: growth and polluting resources. *The Scandinavian Journal of Economics* 104(4), 605 – 620.
- Siebert, H., J. Eichberger, R. Gronych, and R. Pethig (1980). *Trade and Environment: A Theoretical Inquiry*. Amsterdam: Elsevier/North Holland.
- Sinclair, P. J. N. (1994). On the optimum trend of fossil fuel taxation. *Oxford Economic Papers* 46, 869 – 877.
- Solow, R. M. (1974). Intergenerational equity and exhaustible resources. *The review of economic studies* 41, 29–45.
- Stenbacka, R. and M. M. Tombak (1994). Strategic timing of adoption of new technologies under uncertainty. *International Journal of Industrial Organization* 12(3), 387 – 411.
- Stern, N. (2007). *The Economics of Climate Change: the Stern Review*. Cambridge University Press.

- Stiglitz, J. (1974). Growth with exhaustible natural resources: efficient and optimal growth paths. *The review of economic studies* 41, 123–137.
- Tahvonen, O. (1991). On the dynamics of renewable resource harvesting and pollution control. *Environmental and Resource Economics* 1(1), 97 – 117.
- Tahvonen, O. (1997). Fossil fuels, stock externalities, and backstop technology. *Canadian Journal of Economics* 30(4), 855–874.
- Tahvonen, O. and C. Withagen (1996). Optimality of irreversible pollution accumulation. *Journal of Economic Dynamics and Control* 20(9-10), 1775 – 1795.
- Tishler, A., I. Milstein, and C.-K. Woo (2008). Capacity commitment and price volatility in a competitive electricity market. *Energy Economics* 30(4), 1625–1647.
- Toman, M. and C. Withagen (2000). Accumulative pollution, clean technology, and policy design. *Resource and Energy Economics* 22(4), 367 – 384.
- Ulph, A. and D. Ulph (1994). The optimal time path of a carbon tax. *Oxford Economic Papers* 46, pp. 857–868.
- van der Ploeg, F. (2014). Untapped fossil fuel and the Green Paradox. OxCarre Working Papers 119.
- van der Ploeg, F. and C. Withagen (2012a). Is there really a green paradox? *Journal of Environmental Economics and Management* 64(3), 342 – 363.
- van der Ploeg, F. and C. Withagen (2012b). Too much coal, too little oil. *Journal of Public Economics* 96(1-2), 62 – 77.
- van der Ploeg, F. and C. Withagen (2013). Global warming and the green paradox. OxCarre Working Papers 116.

- van der Ploeg, F. and C. Withagen (2014). Growth, renewables, and the optimal carbon tax. *International Economic Review* 55(1), 283 – 311.
- van der Zwaan, B., R. Gerlagh, G. Klaassen, and L. Schrattenholzer (2002). Endogenous technological change in climate change modelling. *Energy Economics* 24(1), 1 – 19.
- Vardar, B. (2014). Optimal energy transition and taxation of non-renewable resources. CORE Discussion Paper Series 2014/21.
- Weitzman, M. L. (2010). What is the damages function for global warming - and what difference might it make? *Climate Change Economics* 1(01), 57 – 69.
- Withagen, C. (1994). Pollution and exhaustibility of fossil fuels. *Resource and Energy Economics* 16(3), 235 – 242.
- Zemel, A. (2015). Adaptation, mitigation and risk: An analytic approach. *Journal of Economic Dynamics and Control* 51, 133 – 147.

Abstract

This dissertation investigates the economic mechanisms underlying the transition to clean technologies and examines policy approaches to achieve the socially optimal path. It studies various policy measures aiming to deal with climate change, such as adaptation and taxation of non-renewable resources. Furthermore, it examines the policy instruments that target increasing the use of efficient technologies and identifies cases in which the policy reaches its objectives or not. It also analyzes the role of heterogeneity in society on agents' willingness to support a pollution tax.

The first chapter studies the energy transition by using an optimal growth model in which non-renewable and renewable natural resources are imperfect substitutes in providing energy services necessary for production. The second chapter studies the role of adaptation policy on the transition to a low-carbon economy. It incorporates adaptation policy into the problem of optimal non-renewable resource extraction with pollution externalities, by focusing on the capital nature of adaptation measures. The third chapter focuses on the problem of adopting new technologies in a micro-economic framework. It studies the behavior of firms when they face a decision to invest either in a cheap but inefficient production capacity or in an expensive but efficient one, by taking into account the presence of a financial constraint. The fourth and last chapter investigates the distributional impacts of a pollution tax by considering a society in which wealth is distributed heterogeneously among households.

Keywords: energy transition, climate change, optimal taxation, non-renewable resource, renewable resource, mitigation policy, adaptation policy, capacity building, technology adoption, market structure, financial constraint, heterogeneity in wealth, distributional impacts.

Résumé

Cette thèse étudie les mécanismes économiques concernant la transition vers des technologies propres et examine les approches politiques pour atteindre le sentier de transition socialement optimal. Elle examine les politiques économiques visant à faire face au changement climatique, telles que l'adaptation et la taxation des ressources non-renouvelables. En outre, elle examine les politiques économiques visant à accroître l'utilisation de technologies efficaces et identifie les cas pour lesquels la politique atteint ses objectifs ou non. Elle analyse également l'impact des inégalités de richesse sur le soutien politique aux taxes environnementales.

Le premier chapitre étudie la transition énergétique en utilisant un modèle de croissance optimal dans lequel les ressources non-renouvelables et renouvelables sont des substituts imparfaits. Le deuxième chapitre étudie le rôle de la politique d'adaptation sur la transition vers une économie propre. Il intègre la politique d'adaptation dans le problème de l'extraction optimale des ressources non-renouvelables avec des externalités de pollution, en mettant l'accent sur la politique d'adaptation en étant une variable de stock. Le troisième chapitre se concentre sur le problème de l'adoption des nouvelles technologies dans un cadre micro-économique. Il regarde le comportement des entreprises qui font face à une décision d'investir: soit dans une capacité de production bon marché mais inefficace, soit dans une capacité plus chère mais efficace, lorsqu'on prend en compte la présence d'une contrainte financière. Le quatrième et dernier chapitre examine les effets distributifs d'une taxe sur la pollution en considérant une société dans laquelle la richesse est répartie de manière hétérogène entre les ménages.

Mots-clés: transition énergétique, changement climatique, taxation optimale, ressource non-renouvelable, ressource renouvelable, politique d'abattement, politique d'adaptation, capacité, adoption de technologie, structure du marché, contrainte financière, hétérogénéité de la richesse, effets distributifs.

