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# Distributed tracking control of nonholonomic multi-robot formation systems

Xing Chu

► **To cite this version:**

Xing Chu. Distributed tracking control of nonholonomic multi-robot formation systems. Automatic. Ecole Centrale de Lille, 2017. English. NNT : 2017ECLI0035 . tel-01818078

**HAL Id: tel-01818078**

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N° d'ordre: 

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**CENTRALE LILLE**

**THÈSE**

présentée en vue  
d'obtenir le grade de

**DOCTEUR**

En

*Spécialité : Automatique, Génie Informatique, Traitement du Signal et Image*

par

**Xing CHU**

**DOCTORAT DELIVRE PAR CENTRALE LILLE**

Titre de la thèse :

**Commande distribuée, en poursuite, d'un système multi-robots  
non holonomes en formation**

**Distributed Tracking Control of Nonholonomic Multi-Robot  
Formation Systems**

---

Soutenue le 13 Décembre 2017 devant le jury d'examen :

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*À mes parents,  
à ma femme,  
à mes professeurs,  
à mes chère(s) ami(e)s.*

## Acknowledgements

In the end of program, my sincere thanks go to many people for their enthusiastic help and constant support during my PhD journey. They are my advisor, colleagues, friends, and most importantly, my families.

First and foremost, I would like to express my sincere appreciation to my advisor Professor Ahmed Rahmani for his continuous as well as effective guidance and support throughout the whole process of forming this thesis. He was always patient to discuss with me about all my research topics critically and rigorously yet with full of wise humor. He helped me improve my research results and figured out the tiny but crucial flaws in my work. He shaped carefully my writing type for theoretical paper. At the same time, he gave me enough freedom to work in my own way. Finally, I would also like to give my gratitude to Professor Rahmani for his valuable comments and proofreading for this thesis.

Besides my advisor, I also would like to acknowledge all the members of jury for their valuable time to review this thesis. Their invaluable comments and objective criticisms which shed a light on my drawbacks and the research road in future. Meanwhile, I would like to thank Mr. Belkacem Ould Bouamama, who is the leader of the group MOCIS in CRISAL Lab, for offering many opportunities for me to exchange academic ideas with other group members and to collaborate with each other.

Particularly, my great gratitude should be given to Prof. Guogunag Wen and Prof. Zhaoxia Peng, they have indeed mentored me so much which boosts my rapid progress in the distributed control and robotics fields. Their rigorous and excelsior research attitude and continuous encouragement have deeply affected me. Without their aid and collaboration, I surely can not achieve the current accomplishments. Also, I am grateful that I were surrounded by so many excellent and brilliant colleagues in our group. They are Jing Bai, Youwei Dong, Yan Wei, Wei Jiang, Wei Hu, Yue Yao, Yinqi Liu, Thanh Binh Do, Joel, etc. Especially, I were very enjoyable for the significant discussions on the general control theory in daily life, with the visiting scholar Prof. Yunhe Meng, who was from Chinese National University of Defense Technology. Unforgettably, I were extremely grateful for the encouragement and support from Prof. Yongguang Yu, who was the deputy dean for the school of science in Beijing Jiaotong University, when I temporarily

lost myself during my PhD journey. Also, I am grateful to Mrs. Vanessa Fleury for her kind help to handle my administrative matters, Mr. Terrier Patrick for all his technical help to enable me work effectively, and the cleaners for creating a neat office for me.

Finally, I wish to express my heartfelt thanks to my master advisor, Prof. Guilin Wen, which is the recipient of "National Outstanding Youth Fund". He has ever gave me critical support in my most difficult time. I will also take this opportunity to thank my teachers when I was in senior high school, Mr. Chun Yang and Mr. Mingxiang Hu. Without them, I'm not possible to stand in the defence platform today. They have vastly extended the length and width of my life, I will never forget their grace. Meanwhile, special gratitude goes to the Chinese Scholarship Council for supporting my PhD research in France.

Lastly but also most importantly, I would like to acknowledge my great parents, Mr. Guolin Chu and Mrs. Jili Bao, due to their adamant support for my ideal and career ambition. Particularly, special gratitude will be dedicated to my wife Mrs. Suqin Mao, I will not be able to finish my PhD research successfully without her encouragement and love, which is my most precious gem forever.

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Winter, 2017

## English Abstract

The main aim of this thesis is to study the problem of distributed tracking control of multi-robot formation systems with nonholonomic constraints. The control objective is to drive a team of unicycle-type mobile robots to adopt a desired formation configuration with its centroid moving along a dynamic reference trajectory, which can be specified by a virtual leader. In this context, we consider several problems, ranging from finite-time stability and fixed-time stability, event-triggered communication and control mechanism, kinematics and dynamics, to continuous-time systems and hybrid systems. The tracking control problem is solved in this thesis through the design of different practical controllers with faster convergence rates, higher control accuracy, stronger robustness, explicit and independent estimation for the upper bound of settling time, less communication cost and energy consumptions rather than most existing results in literature.

In the first part of the thesis, we first study the problem of finite-time stability for multi-robot formation systems in Chapter 2. A distributed observer-based controller is developed for each robot. Finite-time stability of the combined observer and controller is analyzed using Lyapunov direct method, algebraic graph theory, and matrix analysis. A formula for the upper bound estimation of the settling time, which strongly depends on the initial conditions, is derived. Furthermore, to remove this unexpected dependence, a novel class of finite-time controllers, also called fixed-time controllers, is proposed in Chapter 3. A less conservative theoretical estimation for the upper bound of the settling time is obtained, which is independent of the initial conditions and affords the capability to control the convergence time more precisely and independently of initial conditions. In addition, in order to investigate the effect of dynamics of nonholonomic mobile robots, which can describe the characteristics of robots more completely, we propose a fixed-time controller for the closed-loop dynamical systems derived by dynamic linearization technique in Chapter 4. A set of distributed fixed-time controllers and corresponding sufficient conditions to guarantee the fixed-time stability, are derived with the aid of sliding mode techniques and Lyapunov theory.

In the second part, we study event-triggered communication and control mechanism for nonholonomic multi-robot formation tracking control in Chap-

ter 5 based on continuous-time sampling. Firstly, a novel type of distributed event-triggered controller is proposed both for fixed and switching communication topologies. An associated event condition, which only needs intermittent communication amongst neighboring robots for event detection and control update, is designed to aid in the implementation of the distributed controllers. With the proposed event condition, it becomes possible to reduce the communication cost, energy consumption and mechanical abrasion of the multi-robot formation systems, especially when the number of robots is extremely large. Moreover, with a view to develop a digital implementation scheme, we propose another class of periodic event-triggered controllers based on fixed-time observers in Chapter 6. Two different types of event conditions are analyzed in detail. Unlike most continuous-time strategies described in the literature, only periodic or aperiodic wireless communication is required for the control updates and event detections. The nature of the sampled-data method adopted excludes Zeno-Behavior for all the robots. Compared with most existing results on multiple robots formation control, in the new proposed method the control input avoids continuous updates and aperiodic communications can be realized by choosing appropriate event conditions. This results in a significant reduction in communication costs and energy consumptions for multi-robot formation systems.

**Keywords:** Multi-robot systems, Nonholonomic constraints, Distributed coordination, Tracking control, Formation control, Finite-time stability, Fixed-time stability, Event-triggered control, Sampled-data control, Non-smooth analysis, Algebraic graph theory, Matrix theory, Lyapunov theory.

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**II Multi-Robot Formation Systems: Event-Triggered Com-**

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# Chapter 1

## Introduction

### 1.1 Background and motivations

Over past two decades, along with the rapid advances of communication, sensing, embedded techniques, robotics and control theory, the researches on multi-robot coordination have attracted tremendous attention from different fields due to its broad applications, such as surveillance, transportation, cooperative construction, artistic performance, search and rescue, forest fire monitoring, and flood fighting, just name a few. Some corresponding application scenarios are provided in the Figures.1.1-1.3.

In this context, fruitful results have been continuously obtained for various multi-robot coordination tasks. And a flood of papers have appeared on the

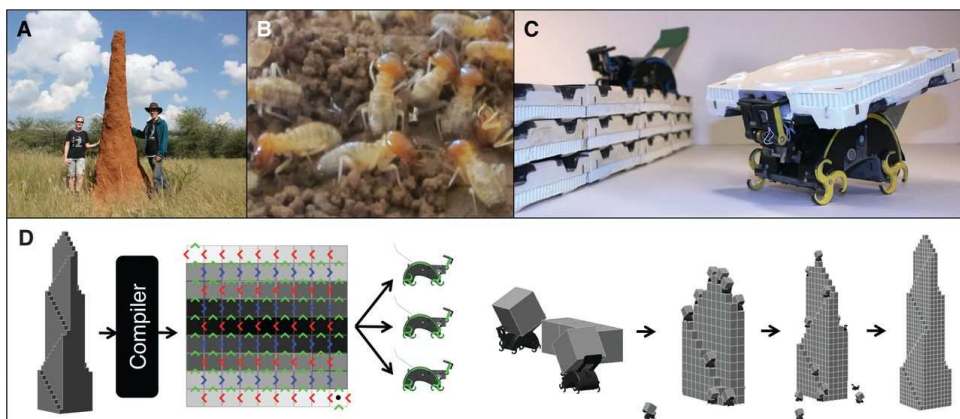


Figure 1.1: From science, 343(6172), 754-758

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Figure 1.2: From TED, presented by Prof.V.Kumar      Figure 1.3: Amazon logistics systems

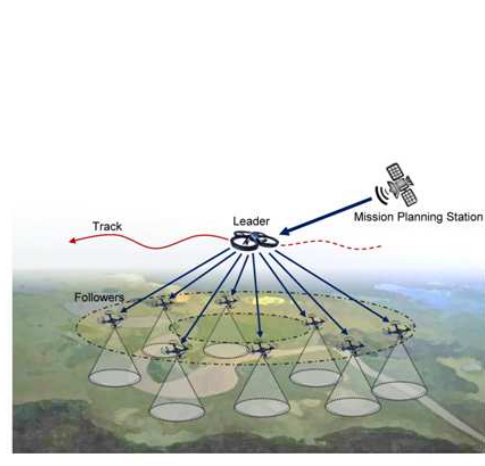
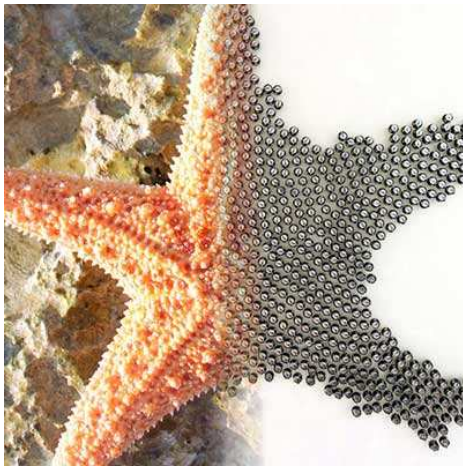


Figure 1.4: From science, 345(6198),      Figure 1.5: Multiple UAV surveillance 795-799

formation control problem, which plays the essential role for the coordination of multi-robot systems. The corresponding applications are shown in Figures.1.4-1.7.

Generally speaking, multi-robot systems consists of a group of unmanned ground robots (or aerial vehicles and underwater robots) subject to certain types of interaction. Along with the rapid development of distributed control theory, many typical distributed formation control approaches have been proposed and investigated based on different sensing manners, such as consensus based method, distance based method and bearing based method, etc. We refer the interested readers to the extensive surveys [Bayat et al. \(2017\)](#); [Knorn et al. \(2016\)](#); [Oh et al. \(2015\)](#); [Ye et al. \(2016\)](#). Note that in this thesis, we do not involve the centralized formation control approaches, hence we omit its background knowledge, and focus on the distributed approaches.

## 1.1 Background and motivations

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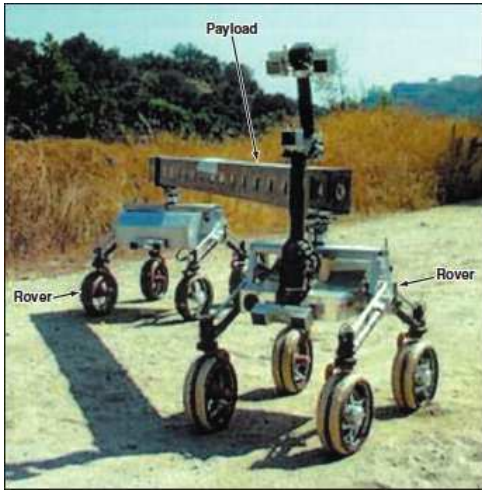


Figure 1.6: Payload transport, NASA



Figure 1.7: Multi-robot patrol

Distance based formation control method has been investigated frequently in recent years as it allows reduced requirement on the sensing capability based on local coordinate system of each robot. Further the robots do not need to have a common sense of orientation and the desired formation is invariant to the combination of translation and rotation. And attractive theoretical challenges induced by nonlinearity are also important motivations. Partial interesting results can be found in references [Anderson \*et al.\* \(2017\)](#); [Chen \*et al.\* \(2017a\)](#); [Mou \*et al.\* \(2016\)](#); [Sun \*et al.\* \(2015b, 2016, 2017a,b\)](#). Generally speaking, multiple equilibriums problem (including correct equilibrium points, incorrect equilibrium points and degenerate equilibrium points), robustness problem, global convergence problem and complex formation target shape control problem under various sensing topologies and node dynamics have constituted the core research interests, while leaving numerous challenging open problems for the future. For the moment, only the control mechanism of simple target formation shape such as triangle or rectangle in 2D and tetrahedron in 3D have been thoroughly understood. However, it is difficult to extend current results to more complex geometric shape. Another, although this class of formation control method is called distance based, most of existing results still need the relative position measurements. Also, from the practical point of view, one of the troublesome issues of this technique lies in the recognition for the labels, or identifications of its neighboring robots in complex and uncertain environment (there will exist unexpected disturbances for the target robot recognition). In other words, it is far away from a simple

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practical problem as theoretical assumption, which supposes that each robot can identify its neighbouring robots for granted. In addition, most of current papers assume that the robots can always obtain the accurate distance measurements corresponding to its target robots. However, in practice, the mismatch and disturbance problem of range sensor are inevitable. To this question, it was revealed by the work [Mou \*et al.\* \(2016\)](#) that slightly different understandings on the desired distance or mismatched distance measurements between robots will lead to the formation converge exponentially fast to a closed circular orbit in two-dimensional space which is traversed periodically at a constant angular speed. Afterwards, [Sun \*et al.\* \(2017a\)](#) extended the result [Mou \*et al.\* \(2016\)](#) into the three dimensional space, and observed a similar phenomenon.

Along with the rapid development of distance based formation control method, most recently, bearing based method also has been studied extensively due to the fact that the bearings are invariant to the translation and scaling of the target formation shape, thus this method can easily achieve translational and scaling motion of multi-robot formation systems. On the other hand, bearing measurements are often more accessible than position measurements by using onboard sensors, for example, bearing based method can be applied in the GPS-denied scenarios. To this end, much attention has been paid on this method whereas many theoretical and technical problems for this topic are still remain open. From the view of applications, bearing based method demonstrates many superiorities under the precondition of accurate recognition and measurement for the target robots and no blocks between robot and target robots. Accordingly, it often assumes that the available detection range is large enough and the designated connection relation is always preserved. However, ideal identifications and accurate measurements are luxury for real multi-robot systems in practice. Meanwhile, it is often required that every robot has a common compass base for the bearing based method. In other words, slight errors in the understanding of true north will lead to the distortion of actual formation compared to the desired one. However, this kind of errors is inevitable in practice due to the perturbation of geomagnetic field and sensor error, etc.

As well known that huge theoretical progresses have made for the multi-agent systems over past two decades, in which the consensus problem has been extensively studied for different continuous or discrete node dynamics under fixed or

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switching topologies Cao & Ren (2014); Olfati-Saber & Murray (2004); Wang *et al.* (2015a,b); Wen *et al.*, 2014b, 2015), we refer interested readers to the comprehensive surveys Cao *et al.* (2013a); Olfati-Saber *et al.* (2007); Qin *et al.* (2016); Ren *et al.* (2007). Researchers have also systematically investigated various practical constraints such as time delays, disturbances, uncertain parameters, input saturation, quantization error, etc. In order to improve control accuracy and robustness, decrease the communication cost and energy consumption, the finite-time stability and event-triggered method were introduced in the controller development. In this context, the consensus based formation control method is emerging as a promising alternative due to its flexibility and simplicity during the process of controller design and implementation, its capability for the tracking control of arbitrary formation shape with large numbers of robots, its scalability and global convergence property, etc. We note that the problem about the global stability and complex shape stabilization in arbitrary dimensions for the distance based and bearing based methods are extremely difficult and unsolved. Along with the great advance of global position systems and various indoor localization techniques, it is feasible as well as reliable to obtain the high accuracy position and orientation information in most scenarios, unless in other planets or under water.

Based on above observations, synthesizing various advantages with shortcomings, this thesis mainly devotes to the consensus based formation control method from the practical point of view. The author aims to investigate the formation tracking problem relatively thoroughly through considering different practical aspects in applications. Under the guidance of this objective, we propose a general closed-loop consensus based formation control framework, based on which some key engineering problems for the nonholonomic multi-robot formation systems are tackled. In summary, the author of this thesis intends to present some practical techniques which are really feasible, reliable and applicable in reality, on the basis of rigorous theoretical analysis and proofs using diverse mathematical tools. One important motivation of this thesis mainly stems from the enthusiasm for developing the commercial products to boost the industrialisation process of multi-robot formation systems, which can satisfy the urgent demands both in civilian and military areas. At the same time, these techniques are predicted to promote the rapid development of human life and industrial production while

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playing a key role to assist us in exploring more unknown world. Meanwhile, these technical practices, in turn, will refine the original theoretical results, and finally help it form a technical science about the distributed tracking control for the nonholonomic multi-robot formation systems.

### 1.2 Literature overview

In this section, some relevant and recent results for each research subject of this thesis will be briefly reviewed, and a more detailed literature review will be provided in the beginning of each main chapter for specific research topic.

#### 1.2.1 Formation control problems

In general, the problems of formation control of multi-robot systems can be divided into two phases: *formation stabilization* and *formation tracking*. When a group of robots start from their initial shape to gradually form a desired formation shape while maintaining the shape regardless of external disturbances and uncertainties, this type of task is commonly called formation stabilization. Furthermore, persons may be more interested in guiding the formation to move as a rigid body, along a reference trajectory that is specified by one practical or virtual leading robot. In this case, we refer to it as the formation tracking. In practice, formation control techniques have broad applications, such as surveillance, localization, the deployment of wireless sensors network, intelligent transportation, artistic performances, mines clearance, and so forth.

In this thesis, the author mainly focuses on the formation tracking problem due to its greater application potential. In many practical scenarios, people always expect that the formation can track a desired trajectory while maintaining the precise shape. This motivates the author to further investigate the formation tracking problem of nonholonomic multi-robot systems.

#### 1.2.2 Finite-time stability of multi-robot formation systems

Some real-time applications need to strictly control the settling time of multi-robot systems in practice. For example, formation shape must be formed in a



finite time to perform surveillance or track a forest fire in time. However, most of currently existing control laws for formation control problem of multi-robot systems can only be proven to converge asymptotically fast. In other words, we are not sure that when the formation can be formed while tracking the reference trajectory steadily. Yet rather than an accurate instant, most current work of multi-robot systems can't assure at least an upper bound of the settling time.

As one type of crucial property for multi-robot systems, finite-time stability can trace its history back to classical optimal control theory, such as bang-bang control. This stability theory was formally proposed in the paper [Bhat & Bernstein \(2000a\)](#). Compared to common asymptotic stability and exponential stability, the most distinguished feature of it is that the equilibrium can be attained at an explicit time instant, rather than asymptotically over an infinite time-horizon. This property has proved very useful in practice, and stronger robust against uncertain disturbances and parameters variation than the other two kinds of convergence characteristics. Moreover, some complex systems can be effectively decoupled rely on finite-time stability so as to facilitate related theoretical analysis. This theory is also important for observer design which often requires the observed states converge to the real ones as soon as possible.

In general, finite-time stability is closely related to the homogeneity property of the system. The papers [Zhao & Duan \(2015\)](#); [Zhao \*et al.\* \(2013, 2014, 2015b\)](#) have studied the finite-time stability for tracking control problem of multiple Euler-Lagrange dynamics and multi-agent systems with different influence factors, like disturbance, bounded unknown velocity and acceleration, saturated control input, etc. However, in all the aforementioned work utilized the homogeneity property of the systems to prove the stability, the explicit upper bound of settling time does not be estimated let alone adjusted. Although the formation tracking issue [Liu \*et al.\* \(2015c\)](#) was addressed with an lower bound estimate for the settling time. To overcome this drawback that can not provide explicit estimation for the upper bound of convergence instant, we will devote one chapter in this thesis to develop the finite-time formation tracking controller with an explicit estimate of the upper bound of settling time.

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### 1.2.3 More practical convergence characteristics: fixed-time stability

In the last subsection, we devote special attention to the problem of improving the convergence speed of formation systems, which is an important performance index of the controller for distributed coordination of multi-robot systems. However, based on finite-time stability theory, the convergence time instant (settling time) strongly depends on the initial positions of all the robots, which is usually unavailable in reality for single robot. Note that this is a type of global information for individual robot. Assume that initial positions are bounded but very large, then the convergence speed will even be slower than exponential stability during the rise time of control systems.

In order to improve this type of problem, another important concept, fixed-time stability, was introduced by [Polyakov \(2012\)](#). This kind of stability theory can off-line specify the upper bound of convergence time in advance, which is regardless of the initial positions. In other words, the settling time function of fixed-time stability always has a fixed upper bound. This property affords the engineers the possibility to realize more accurate control for the settling time of systems.

Motivated by these facts, various results based on this new stability concept have appeared recently, mainly focusing on designing distributed controller with guaranteed fixed upper bound of settling time for the consensus problem of multi-agent systems [Fu & Wang \(2016\)](#); [Parsegov \*et al.\* \(2013b\)](#); [Zuo \(2015\)](#). We will contribute two chapters to introduce this practical theory into the development of a novel class of formation tracking controllers for multi-robot systems. Due to the nonlinearity of the multi-robot formation systems and controllers, it's not easy to directly extend the existing controllers for linear multi-agent systems to the nonholonomic multi-robot formation systems.

On the other hand, preview studies in this thesis only investigate the kinematics of robots. However, the dynamics of systems can provide more information for the motion characteristics of robots in complex environment. In the meantime, the disturbances are inevitable in practice, we must consider its negative effect on multi-robot formation systems and incorporate the disturbance rejection



property into the controllers. For this point, Chapter 4 will provide a satisfactory solution.

### 1.2.4 Event-triggered mechanism beyond time-triggered communication and control

In addition to the convergence rate of the controller, communication and control actuation frequency also play key roles in the whole performance of multi-robot systems. In most of the work on formation control, the assumption that the communication between neighboring robots is time-triggered, has been widely accepted. From a practical point of view, time-triggered information exchange mechanism may be somewhat conservative. From a theoretical perspective, continuous wireless communication will occupy a large amount of limited bandwidth and uninterrupted control input update will lead to excess energy consumption and mechanical wear. In view of these practical issues, event based (also called event-triggered) control methods were revisited. It has been shown in many papers [Dimarogonas & Johansson \(2009\)](#); [Dimarogonas \*et al.\* \(2012a\)](#); [Fan \*et al.\* \(2013a, 2015\)](#); [Nowzari & Cort es \(2014\)](#); [Seyboth \*et al.\* \(2013a\)](#); [Tabuada \(2007\)](#) on multi-agent systems that event based method can indeed effectively improve the above issues through intermittent communication and controller updates. However, substantially less work has been devoted to designing the event based strategy for nonholonomic multi-robot systems. For this reason, the author intends to link the theory with practical application, and devise an applicable event based formation control system to truly reduce communication and control update frequency.

Furthermore, most current controllers are digital in real world, then sampled-data control method is introduced naturally to address the real engineering requirements [Meng & Chen \(2013\)](#); [Postoyan \*et al.\* \(2015\)](#). In general, sampled-data method can be divided into more conservative periodic sampling and more real aperiodic sampling. Compared to event based only method, the introduction of sampled-data method will further reduce the communication frequency and control input update. Moreover, the Zeno-Behavior (a phenomenon that an infinite number of events accumulate in a finite time interval) can be excluded in theory,

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this is also the core request except for stability in the design of event based formation controller. Hence, we will contribute two successive Chapters 5 and 6 to address these problems.

### 1.3 Mathematical tools

#### 1.3.1 Algebraic graph theory

Before formally introducing the core concepts of algebraic graph theory, we will first explain the reasons to employ it. Considering how information is exchanged among multiple robots, the mutual interaction can be divided into several classes. Here, we just list some commonly used sensing methods, see Figure.1.8. In mode (A), robots exchange information such as absolute position, absolute velocity, heading angle and so on, through wireless communication techniques. In mode (B), a camera is used by robot to obtain the relative distance and orientation information with respect to the target robot. In mode (C), the robot applies the 360° Lidar to measure the distance and bearing associated with the target robot. Then how to describe these information exchange manner is one crucial problem in the research of multi-robot formation systems. Popularly, algebraic graph theory is naturally adopted in most of the existing work. We can model the different manners for information exchange by using undirected and directed graphs or/and fixed and switching graphs. Specifically, an undirected graphs can be used to describe bidirectional information flow between neighbouring robots, whereas directed graphs only allow robot to send/receive information to/from the neighbouring robots. In other words, the information transmission is unidirectional and asymmetric in directed graphs. We also recall that in a so-called fixed graph, the information links are fixed over time, whereas a switching graphs allow for the change of inter-robot links due to limited sensing range, environmental disturbances, and so forth. We here provide an example for an undirected and fixed sensing graph. Consider the robot group (D) in the Figure.1.8, where the robots exchange information based on certain sensing methods of (A), (B) or (C), in order to generate desired collective behavior by means of local interaction. If robot  $i$  can mutually exchange information with robot  $j$ , an undirected dot line connecting two robots will be used to describe this kind of information link.

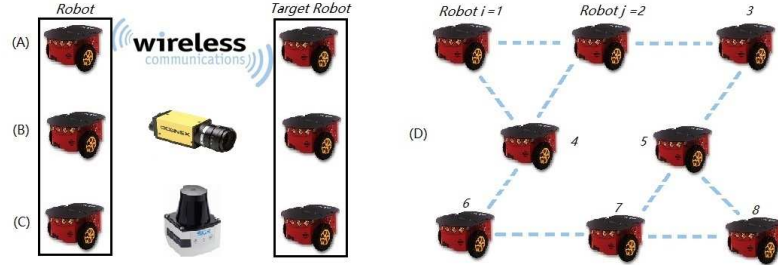


Figure 1.8: Different interaction manners

Based on the preview statement, a weighted graph  $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E}, \mathcal{A})$  will be employed to describe the information exchange relation among robots, where  $\mathcal{V} = 1, 2, \dots, N$  and  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$  denote the nodes set and edges set, respectively. The weighted adjacency matrix  $\mathcal{A} \in \mathbb{R}^{N \times N}$  is defined as

$$\mathcal{A} = \begin{cases} a_{ij} = 1, & \text{for } (v_j, v_i) \in \mathcal{E} \quad \text{and} \quad i \neq j, \\ a_{ii} = 0, & \text{otherwise,} \end{cases}$$

where  $a_{ij} = 1$  denotes robot  $i$  can receive information from robot  $j$ , and  $a_{ij} = 0$  otherwise. Take the Figure.1.8 (D) as an example, the weighted adjacency matrix can be obtained as follows

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

For an undirected graph,  $a_{ij} = a_{ji}$ , that is, robot  $i$  and  $j$  can sense each other. If  $(v_j, v_i) \in \mathcal{E}$ , then node  $j$  is called a neighbor of node  $i$ . The graph Laplacian  $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$  is given by

$$\mathcal{L} = [l_{ij}] = \begin{cases} l_{ij} = -a_{ij}, & i \neq j, \\ l_{ii} = \sum_{j=1, j \neq i}^n a_{ij}, & \text{otherwise,} \end{cases}$$

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and satisfies that  $\sum_{j=1}^N l_{ij} = 0$ . Let  $\mathcal{D} = \text{diag}(d_1, d_2, \dots, d_N)$  be the diagonal in-degree matrix of the graph, where  $d_i$  is equal to the number of neighbors for robot  $i$ , so that one has  $\mathcal{L} = \mathcal{D} - \mathcal{A}$ . Taking Figure.1.8 (D) as an example, we have

$$\mathcal{L} = \mathcal{D} - \mathcal{A} = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Hence,

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 3 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 3 & 0 & -1 & 0 & 0 & -1 \\ -1 & -1 & 0 & 3 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 3 & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix}$$

Note that the graph Laplacian  $\mathcal{L}$  of an undirected graph is symmetric and positive semi-definite, and all the nonzero eigenvalues are positive real numbers. We further define diagonal matrix  $\mathcal{B} = \text{diag}(b_1, b_2, \dots, b_N)$  as follows to represent the communication relationship between a leader and its followers,  $b_i = 1$  if the leader's information is available to the follower  $i$ , and 0 otherwise.

$$\mathcal{B} = \begin{bmatrix} b_1 & & & \\ & b_2 & & \\ & & \ddots & \\ & & & b_n \end{bmatrix} \in \mathbb{R}^{N \times N}$$

Based on Figure.1.8 (D), if we let one virtual leader transmits the information to

the robot 1, one has

$$\mathcal{B} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Here, we define another useful matrix  $\mathcal{H} = \mathcal{L} + \mathcal{B}$  which is frequently used in the subsequent theoretical analysis. Then, the matrix  $\mathcal{H}$  of communication graph Figure.1.8 (D) is finally obtained as follows

$$\mathcal{H} = \begin{bmatrix} 3 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 3 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 3 & 0 & -1 & 0 & 0 & -1 \\ -1 & -1 & 0 & 3 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 3 & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix}$$

In an undirected graph, if there is a link between  $i$  and  $j$ , then nodes  $i$  and  $j$  can exchange information with each other. An undirected graph  $\mathcal{G}$  is called connected if and only if there exists an undirected path from arbitrary distinct node  $v_i$  to  $v_j$ , ( $i, j = 1, 2, \dots, N$ ). For instant, see Figure.1.8 (D). The connection between connected undirected graph and the algebraic characteristics is established: "*An undirected graph is connected if and only if the matrix  $\mathcal{L}$  has a simple minimum zero eigenvalue corresponding to the eigenvector  $\mathbf{1}_N$* ". The second smallest eigenvalue of  $\mathcal{L}$  is called the algebraic connectivity, which is positive if and only if the undirected graph is connected.  $N_i$  denotes the neighbors set of robot  $i$ . In the thesis, we let  $\lambda_{min}$  and  $\lambda_{max}$  be the minimum and maximum eigenvalues of matrix  $\mathcal{H} \otimes I_2$ , respectively. Note that  $\lambda_{min}(\mathcal{H} \otimes I_2) = \lambda_{min}(\mathcal{H})$  and  $\lambda_{max}(\mathcal{H} \otimes I_2) = \lambda_{max}(\mathcal{H})$ . We refer interested readers to the reference [Mesbahi & Egerstedt \(2010\)](#) for more details. Moreover, matrix  $\mathcal{H}$  is positive-definite symmetric and invertible for a connected undirected graph, which means that its

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eigenvalues are positive real numbers.

### 1.3.2 Methods

#### 1.3.2.1 Nonsmooth analysis

When discontinuous terms are included in the models of dynamic systems, some tools such as nonsmooth analysis and differential inclusions are needed to analyze these systems. Consider the differential equation

$$\dot{x}(t) = f(x(t)), \quad x(t_0) = x_0. \quad (1.1)$$

**Definition 1.1 (Cortes (2008))** For a vector field  $f(x(t)) : \mathbb{R} \times \mathbb{R}^p \rightarrow \mathbb{R}^p$ , the Filippov set-valued map is defined by

$$\mathcal{K}[f](x(t)) \triangleq \bigcap_{\delta > 0} \bigcap_{\mu(N)=0} \bar{co}f(t, \mathcal{B}(x, \delta) \setminus N), \quad (1.2)$$

where the  $\bigcap_{\mu(N)=0}$  denotes the intersection over all sets of Lebesgue measure zero, and  $\mathcal{B}(x, \delta)$  represents the open ball of radius  $\delta$  centered at  $x$ .  $\bar{co}$  refers to the convex closure and  $N \in \mathbb{R}^p$  ( $p \in \mathbb{N}$ ).

**Definition 1.2 (Filippov (1960))** A function  $x(t)$  is called a Filippov solution of the above differential equation over  $[t_0, t_1]$ , if  $x(t)$  is absolutely continuous and for almost all  $t \in [t_0, t_1]$

$$\dot{x}(t) \in \mathcal{K}[f](x(t)). \quad (1.3)$$

**Theorem 1.3 (Ceragioli (1999))** If  $f(x)$  is measurable and locally essentially bounded, then there exists at least one Filippov solution of the differential equation starting from any initial condition.

**Definition 1.4 (Cortes (2008))** Consider the vector differential equation  $\dot{x}(t) = f(x(t))$ , a set-valued map  $\mathcal{K} : \mathbb{R}^p \rightarrow \mathcal{B}(\mathbb{R})$ , the set-valued Lie derivative of  $V$  with respect to  $\dot{x}(t) = f(x(t))$  is defined as

$$\dot{V} \triangleq \bigcap_{\xi \in \partial V} \xi^T \mathcal{K}[f](x(t)). \quad (1.4)$$

### 1.3.2.2 Finite-time stability theory

Finite-time stability is a kind of time optimal theory. Let us consider the system

$$\begin{aligned} \dot{x}(t) &\in F(t, x(t)), \\ x(0) &= x_0, \end{aligned} \tag{1.5}$$

where  $x \in \mathbb{R}^n$  is the state,  $F : \mathbb{R}^+ \times \mathbb{R}^n \rightarrow \mathbb{R}^n$  is an upper semi-continuous convex-valued mapping, such that the set  $F(t, x(t))$  is non-empty for any  $(t, x(t)) \in \mathbb{R}^+ \times \mathbb{R}^n$  and  $F(t, 0) = 0$  for  $t > 0$ . The solution of (1.5) is understood in the Filippov sense [Filippov \(2013\)](#). Two commonly used Lemmas in the research on closed-loop systems with finite-time stability are given as follows.

**Lemma 1.5** *Bhat & Bernstein (2000a)* Suppose there exists a positive definite continuous function  $V(t) : [0, \infty) \rightarrow [0, \infty)$  which is differentiable such that the following conditions holds:

$$\dot{V}(t) \leq -KV(t)^a,$$

where real numbers  $K > 0$  and  $0 < a < 1$ , then  $V(t)$  will converge to zero at finite time instant  $t^* = \frac{1}{K(1-a)}V(0)^{1-a}$ .

**Lemma 1.6** *Parsegov et al. (2013b)* Suppose that there exists a continuously differentiable positive definite and radially unbounded function  $V : \mathbb{R}^n \rightarrow \mathbb{R}^+$  such that

$$\sup_{t>0, y \in F(t, x)} \frac{\partial V(x)}{\partial x} y \leq -aV^p(x) - bV^q(x) \quad \text{for } x \neq 0,$$

where  $a, b > 0, p = 1 - \frac{1}{\mu}, q = 1 + \frac{1}{\mu}, \mu \geq 1$ . Then the origin of the differential inclusion (1.5) is globally fixed-time stable with the following settling time estimate:

$$T(x_0) \leq T_{max} = \frac{\pi\mu}{2\sqrt{ab}}.$$

### 1.3.2.3 Event-triggered mechanism

Distributed formation control usually needs significant exchange of information between neighboring robots such that each robot can properly compute its control input. In a multi-robot systems for example, computation of the controller of

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an robot requires the states of neighbors which must be updated regularly. Compared to one single robot, communication is incorporated to the entire systems. That's why controlling multi-robot formation systems with limited communication resources is a challenging task. And transmitting and receiving electromagnetic signal will directly result in consumptions in limited energy for each robot. Besides, the regular control updates in time-triggered manner not only lead to energy consumptions, but also lead to severe abrasion. Others associated issues include communication bandwidth saturation, congestion, packet loss, time delays and loss of stealth. Thus, event-triggered mechanism as an alternative is proven a feasible way to deal with these problems.

The history of event-triggered control method can be traced back to 1960s, see references [Bekey & Tomovic \(1966\)](#); [Gupta \(1963\)](#); [Liff & Wolf \(1966\)](#); [Mitchell & McDaniel \(1969\)](#); [Tomovic & Bekey \(1966\)](#), which is opposite with the familiar time-triggered control mechanism. Since [Tabuada \(2007\)](#) introduced the theoretical framework for the application of event based method in the linear systems, including the synthesis of controller and event condition function and the method to exclude the Zeno-Behavior, a mass of papers emerged to apply event-triggered communication and control in the distributed coordination of networked systems ranging from single integrators to general linear systems or nonlinear systems, from continuous time models to discrete time models, from fixed undirected topology to switching directed topologies or stochastic graph. Meanwhile, different practical constraints such as time delays, quantization effects, disturbances, uncertainties and input saturations, etc., were extensively considered in this context. The early work introducing this method to the robotic field is the paper [Postoyan \*et al.\* \(2015\)](#) on the time-varying trajectory tracking of nonholonomic robot. However, we notice that related work mainly focuses on the consensus of linear agents except for few results, for instance, [Sun \*et al.\* \(2015a\)](#) proposed an event based rigid formation control scheme recently.

Generally speaking, in event-triggered mechanism, the communications and control updates only happen when necessary, which implies the reduction of conservatism in communications and control updates compared to time-triggered pattern. For example, when the systems approach the instability boundary, or when some performance indexes can't be guaranteed anymore, the event will be



triggered, and new actions will be activated. That is to say, as long as a well-constructed event equation is violated, an event is triggered. A possible mode is that when certain event is triggered, robot will update its own states or outputs in the distributed controllers, while transmitting the states to the neighbouring robots to update the controllers of neighbors. This process involves the monitoring of event equations, updates of control inputs, and communications which are linked to the computations in microprocessors, the actions of actuators and transmit or receive signals with certain energy intensity, respectively. However, the consumptions of computation resources, mechanical abrasion, limited energy and communication bandwidth are reduced dramatically compared with time-triggered mechanism.

#### Notations

Firstly denote by  $t$  the continuous time variable, then we let  $t_{k_i}^i$  be the  $k_i$ th event time instant for the robot  $i$  ( $i = 1, 2, \dots, N$ ,  $k_i = 0, 1, \dots$ ), i.e.,  $t_0^i, t_1^i, \dots, t_{k_i}^i$  is a separate sequence of event instants. Moreover, for the situation of periodical sampling, let  $T$  represent the sampling period while  $t_{k_i}^i + nT$  represents current sampling instant of the robot  $i$  in the time interval between  $t_{k_i}^i$  and  $t_{k_i+1}^i$ . Thus, the next event instant can be defined as  $t_{k_i+1}^i = t_{k_i}^i + T \text{Inf}[n : f_i(e_i, C_i) > 0]$ , and the event instant of robot  $j$  is thus defined as  $t_{k_j}^j = \max\{t | t \in \{t_{k_j}^j, k_j = 0, 1, \dots\}, t \leq t_{k_i}^i + nT\}$ ,  $j \in \mathcal{N}_i$ . Note that  $t_0^i=0$  is the initial moment and the event instants  $\{t_0^*, t_1^*, \dots\} \subseteq \{0, T, 2T, \dots, nT\}$ , which means that the inter-event interval  $\tau = t_{k_*+1}^* - t_{k_*}^*$  of each robot is equal or greater than sampling period  $T$ . It implies that all the robots can avoid the Zeno-Behavior of event-triggered formation systems in theory, which is extremely hard to be proven in continuous time case. Then what is Zeno-Behavior? The Zeno-Behavior corresponds to cases when an infinite number of discrete transitions is made in a finite time interval.

#### Triggering condition

The idea of event-triggered control is that controllers are not updated continuously, but only at specific moments which are not necessarily periodic. Therefore, the information of neighbors will be transmitted to the robot at appropriate time when they are needed to update the distributed controller. The rules to determine this appropriate time is the most crucial. How to design it to balance the

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tradeoffs between reduced information exchange and high performances of controller. For instance, robot can judge that whether or not the designed event function is satisfied so as to guarantee the asymptotic stability of closed-loop systems, through which the event instants can be determined. Generally speaking, the event condition is of the following form

$$f_i(\cdot) = 0 \tag{1.6}$$

where  $f_i$  is called an event function. In most situations, the function  $f_i$  associated with a robot  $i$  depends on the measurement error and the sum of relative errors with respect to neighbors. And the measurement error can be defined as the discrepancy between the state at last event instant and the current state. In case of distributed controller,  $f_i$  is locally judged as every robot decides the event instants by itself.

### 1.3.3 Models of nonholonomic mobile robot

#### 1.3.3.1 Nonholonomic constraint

In general, the types of constraints on wheeled mobile robot can be divided into three classes: holonomic, nonholonomic and others, which use  $f = (q_1, \dots, q_n, t) = 0$ ,  $f = (q_1, \dots, q_n, \dot{q}_1, \dots, \dot{q}_n, t) = 0$  and  $f = (q_1, \dots, q_n, t) < 0$  to describe these constraints, respectively.

Recall that the constraint of the robot in Figure.1.9 satisfies

$$\dot{x}_i \sin \theta_i - \dot{y}_i \cos \theta_i = 0. \tag{1.7}$$

Obviously, the constraint acting on the unicycle-type mobile robot is nonholonomic, which will introduce considerable difficulties for the controller design and theoretical analysis. From constraint (1.7), one can learn that the  $i$ -th robot can only move in the direction normal to the axis of the driving wheels, i.e., the wheeled mobile robot satisfies the conditions of pure rolling and non slipping.

#### 1.3.3.2 Kinematics and dynamics

The nonholonomic robot considered in this thesis is shown in Figure.1.9, of which the structure is a widely used commercial platform, such as the iRobot, Pioneer

### 1.3 Mathematical tools

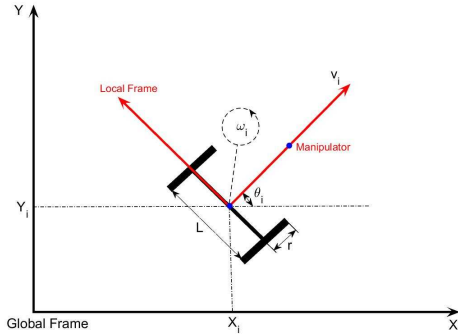


Figure 1.9: Structural sketch of robot



Figure 1.10: iRobot



Figure 1.11: Pioneer 3 DX



Figure 1.12: Kiva robot of amazon

3 DX, Kiva Systems, etc., see the Figures.1.10-1.12. It is assumed that the robot moves in the planar without any longitudinal or lateral slipping. Then, the kinematics of the  $i$ -th robot can be described as follows

$$\begin{aligned}\dot{x}_i &= v_i \cos \theta_i, \\ \dot{y}_i &= v_i \sin \theta_i, \\ \dot{\theta}_i &= \omega_i,\end{aligned}\tag{1.8}$$

where  $q_i = (x_i, y_i, \theta_i)^T$  represents the posture of the  $i$ -th robot in the cartesian coordinate frame, of which  $(x_i, y_i)$  denotes the position of the center of driving

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axis of the robot and  $\theta_i$  is the heading angle of the  $i$ -th robot. In addition,  $v_i$  and  $\omega_i$  denote the control inputs, namely linear velocity and angular velocity, respectively.

Next, we consider the dynamical model of nonholonomic robot, thus the "perfect speed track" assumption is no longer required in this case. The mathematical model used in [Lawton \*et al.\* \(2003\)](#) is revisited of the state space form

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \\ \dot{v}_i \\ \dot{\omega}_i \end{bmatrix} = \begin{bmatrix} \nu_i \cos \theta_i \\ \nu_i \sin \theta_i \\ \omega_i \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1/m_i & 0 \\ 0 & 1/J_i \end{bmatrix} \begin{bmatrix} F_i \\ \tau_i \end{bmatrix}, \quad (1.9)$$

where  $\tau_i$  denotes the torque generated by the differential wheels and  $F_i$  refers to the force.  $m_i$  and  $J_i$  represent the mass and moment of inertia of mobile robot  $i$ .

### 1.4 Synopsis

This thesis presents various practical distributed methods for formation control of nonholonomic multi-robots systems under different communication topologies. The main contributions are summarized as follows.

**Chapter 2:** studies the control problem for the formation tracking of multiple nonholonomic wheeled robots via distributed manner which means each robot only needs local interaction. A class of general states and inputs transformation is introduced to convert the formation tracking issue of multi-robot systems into the state tracking problem of chain systems with time-varying reference. The distributed observer-based protocol with nonlinear dynamics is developed for each robot to achieve the state tracking of chain systems, which namely means a group of nonholonomic mobile robots can form the desired formation shape with its centroid moving along the predefined reference trajectory. The finite-time stability of observer and control law is analyzed rigorously by using Lyapunov direct method, algebraic graph theory and matrix analysis. Numerical examples are finally provided to illustrate the effectiveness of the theoretical results, which have been published in *International Journal of Control*.

**Chapter 3:** considers the fixed-time formation tracking problem of multi-robot systems with nonholonomic constraints. A new type of distributed nonlinear controller for each robot is designed. Some corresponding sufficient conditions are derived by using algebraic graph theory, matrix analysis and fixed-time stability theory. In addition, an upper bound of the settling time for the multi-robot systems is also explicitly given. It is shown that the obtained upper bound of settling time is regardless of initial errors of systems, which implies that it can facilitate the pre-design of the convergence time off-line. Numerical example is provided to illustrate the effectiveness of the present theoretical results. This work has been conditionally accepted by Neurocomputing.

**Chapter 4:** investigates the robust fixed-time consensus tracking problem of second-order multi-agent systems under fixed topology. A novel type of nonlinear protocol and the corresponding sufficient conditions for achieving robust fixed-time consensus tracking are proposed with the aid of sliding mode technique and Lyapunov theory. Compared to finite-time consensus tracking, the convergence time of the tracking errors is globally bounded for any initial conditions of the agents, which is also the global information for each agent. Furthermore, the results obtained for second-order multi-agent systems are also extended to deal with the fixed-time formation tracking problem for unicycle-type robots. Extensive numerical simulations are performed to illustrate the effectiveness of the present theoretical results. This work has been published in IET Control Theory and Applications.

**Chapter 5:** investigates the distributed tracking problem of multi-robot formation systems with nonholonomic constraint via event-triggered approach. A variable transformation is firstly given to convert the formation tracking problem into the consensus-like issue. Then a novel type of distributed event-triggered control strategy is proposed under fixed topology and switching topology, which can guarantee multi-robot systems to produce desired geometric configuration from arbitrary initial positions and orientations for each robot, while the centroid of formation can follow one dynamic reference trajectory. Moreover, the novel event-triggering conditions under fixed topology and switching topology, which only need intermittent interaction between neighboring robots, are designed to assist the execution of distributed controllers. Based on the designed event-triggering conditions, the robot systems can effectively reduce the communication

## 1. INTRODUCTION

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cost, energy consumption and mechanical wear, especially when the quantity of robots is huge. Finally, the effectiveness of theoretical results is illustrated by some numerical examples. This work has been published in Neurocomputing.

**Chapter 6:** addresses the distributed formation tracking control problem for multi-vehicle systems with nonholonomic constraints, by the aid of event-triggered and sampled-data control methods. Two classes of event-triggered communication and control strategies with fixed sampling period are considered. By designing different event conditions, the communications amongst neighboring vehicles are allowed at each sampling time instant in the first strategy, whereas the control input of each vehicle is updated only when its own or neighbors' event conditions are violated. Furthermore, both communication and control update are allowed only when the events of itself or neighbors are triggered in another strategy. To this end, an unified event-triggered and distributed observer-based controller with globally asymptotic convergence rate is proposed. And corresponding sufficient conditions are derived regarding to two types of event conditions, based on Lyapunov technique, matrix analysis and algebraic graph theory. It is worth noting that the Zeno-Behavior of systems with the presented controllers and event conditions is naturally avoided for all the vehicles due to the advantageous property of sampled-data control. Finally, simulations are provided to verify the effectiveness of the obtained theoretical results. This work has been conditionally accepted by International Journal of Control.

## 1.5 Notations

$\mathbb{R}$	the set of real numbers
$\mathbb{R}^n$	the $n$ -dimensional Euclidean real vector space
$\mathbb{R}^{m \times n}$	the $m \times n$ real matrix space
$\ x\ _p$	$(\sum_{i=1}^n  x_i ^p)^{1/p}$ , for any vector $x \in \mathbb{R}^n$ and $p > 0$
$I_n$	identity matrix with $n \times n$ dimensions
$\mathbf{1}_n$	$[1, 1, \dots, 1]^T$ with compatible dimensions
$\lambda_{min}(M)$	minimum eigenvalue of matrix $M$
$\lambda_{max}(M)$	maximum eigenvalue of matrix $M$
$M^T$	the transpose of matrix $M$
$M^{-1}$	the inversion of square matrix $M$
$diag()$	the diagonal matrix
$sgn(x)$	signum function of variable $x$
$ x $	absolute value of variable $x$
$sig(x)^\epsilon$	$sgn(x) x ^\epsilon$ , $\epsilon \in [0, +\infty)$
$max(\cdot)$	the maximum value of number series
$Inf$	the infimum of a set

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## Part I

# Multi-Robot Formation Systems: Finite-Time Stability



## Chapter 2

# Distributed Finite-Time Tracking control of Multi-Robot Formation Systems with Nonholonomic Constraint

### 2.1 Problem setup

This chapter addresses the problem of designing distributed control laws, which can guarantee that *nonholonomic multi-robot formation systems* achieve *formation tracking* (see related definitions in Chapter 1) in a finite time from arbitrary initial positions. In contrast with asymptotic stability which only can make systems converge to the equilibrium as time tends to infinite in theory, shown in Figure.2.1, the property of finite-time stability can ensure that the formation tracking is attained at an finite time instant, which is dependent of the initial positions, rather than asymptotically over an infinite time-horizon.

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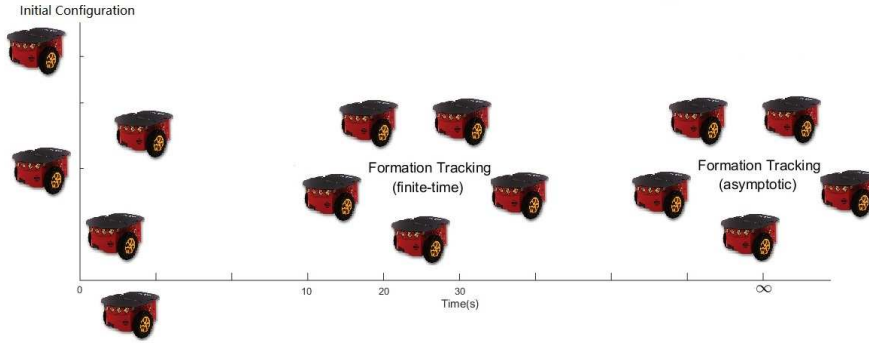


Figure 2.1: Rough comparison between the finite-time and asymptotic stability.

After, we will reformulate the problem to be solved in a more rigorous manner. Mathematically, the problem of this chapter is formulated to design the control inputs  $v_i$  and  $\omega_i$  for each robot  $i$  only based on local information exchange such that a group of mobile robots can form the desired formation shape in a finite time convergence rate. Meanwhile, the orientation of each robot can track a reference value  $\theta_0$ , and the centroid of the formation follows a desired trajectory  $(x_0, y_0)$  satisfactorily, see Figure.2.1. That is to say, the states of robot  $i$ ,  $i = 1, 2, \dots, N$ , should satisfy the constraints as below

$$\lim_{t \rightarrow +\infty} \begin{bmatrix} x_i - p_{xi} \\ y_i - p_{yi} \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}, \quad (2.1)$$

$$\lim_{t \rightarrow +\infty} \left( \sum_{i=1}^N \frac{x_i}{N} - x_0 \right) = 0, \quad \lim_{t \rightarrow +\infty} \left( \sum_{i=1}^N \frac{y_i}{N} - y_0 \right) = 0, \quad (2.2)$$

$$\lim_{t \rightarrow t^*} (\theta_i - \theta_0) = 0. \quad (2.3)$$

Here, we recall the kinematics of robot given in Chapter 1, where  $t^*$  represents the finite convergence time instant,  $(x_i, y_i)$  denotes the position in 2D space, while  $\theta_i$  represents the orientation. The notation  $(x_0, y_0, \theta_0)$  denotes the posture of the virtual leader 0. Suppose that the desired formation shape  $\mathcal{Q}$  of  $N$  robots is defined by displacement  $(p_{xi}, p_{yi})$ , which satisfies

$$\sum_{i=1}^N p_{xi} = p_{x0}, \quad \sum_{i=1}^N p_{yi} = p_{y0},$$

where  $(p_{x0}, p_{y0})$  is the center of the geometric shape  $\mathcal{Q}$ . Without loss of generality,

assume that  $p_{x0} = 0, p_{y0} = 0$  in the orthogonal coordinates. In order to generate a reasonable reference trajectory for robots, the centroid position  $(x_0, y_0)$  and the heading angle  $\theta_0$  can be considered as the posture of a virtual nonholonomic robot  $R_0$ , which is specified by

$$\begin{aligned}\dot{x}_0 &= v_0 \cos \theta_0, \\ \dot{y}_0 &= v_0 \sin \theta_0, \\ \dot{\theta}_0 &= \omega_0.\end{aligned}\tag{2.4}$$

## 2.2 Literature overview

Consensus-based formation control technique was developed in recent years along with the progress of consensus theory [Cao & Ren \(2014\)](#); [Liu \*et al.\* \(2015c\)](#); [Olfati-Saber & Murray \(2004\)](#); [Sun & Geng \(2015\)](#); [Wang \*et al.\* \(2015a,b\)](#); [Wen \*et al.\*, 2014b, 2015](#)); [Zhao \*et al.\* \(2016b\)](#). The most crucial characteristic of this approach is the decentralized manner. A specific transformation is given to convert the formation control problem into consensus tracking issue in [Dong \(2012\)](#). In this way, two observer-based distributed control algorithms are obtained to govern formation tracking while each robot only needs to interact with local neighbors instead of the global information. In [Peng \*et al.\* \(2015\)](#), another two discontinuous distributed consensus-based formation control laws are proposed, in which at least exponential convergence of the whole system is strictly guaranteed via Lyapunov technique and matrix analysis.

In general, the convergence rate of closed-loop system can be roughly categorized as asymptotic, exponential and finite-time. In engineering application field, the last one is preferred because of its robustness against the uncertainty, better disturbance rejection and the advantage of high accuracy control, see [Bhat & Bernstein \(2000a\)](#). For these reasons, the research of finite-time consensus theory has become a hot topic attracting numerous scholars' attention in recent years. In [Cortes \(2008\)](#), the sign function based control laws are proposed to achieve finite-time consensus with undirected topology. The distributed finite-time formation control problem is addressed in [Xiao \*et al.\* \(2009a\)](#). However, each node in above networks is treated as single-integrator. However, many dynamical systems will be converted into second-order form in reality by several techniques. So double

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integrators started to attract continued attention. In [Zhao & Duan \(2015\)](#), the authors investigate the finite-time containment control problem for the multi-agent systems with second-order dynamics. A novel approach is introduced to deal with the containment problem of linear multi-agent systems under directed topology in [Wen \*et al.\* \(2016\)](#). An unknown inherent nonlinear dynamics item of system is studied in [Cao & Ren \(2014\)](#). A continuous nonlinear control algorithm is proposed to address the finite-time consensus problem by a comparison-based stability analysis approach. However, most of the existing results rely on the facts that the nodes are just viewed as linear systems, even though [Zhao \*et al.\* \(2016a\)](#) considers the finite-time tracking problem of nonlinear system and [Wen \*et al.\* \(2014a\)](#) investigates the consensus tracking of nonlinear multi-agent systems with switching directed topology by using M-matrix approach. In this chapter, more realistic and complicated nonlinear characteristics in engineering will be taken into account.

### 2.3 Contributions

Inspired by the recent papers, especially the works in [Dong \(2012\)](#); [Peng \*et al.\* \(2015\)](#), the contributions of this chapter are mainly threefold. Firstly, a general state and input transformation is proposed to convert the formation tracking problem into consensus-like issue. Secondly, a well-designed distributed control law is proposed through observer-based method. It's worth noting that the proposed observer can converge to the real estimated state in finite time. Thirdly, compared with the existing results provided in [Dong \(2012\)](#); [Peng \*et al.\* \(2015\)](#), one novel decentralized finite-time control laws with nonlinear dynamics is investigated, which can guarantee multiple nonholonomic robots to produce the predefined geometric configuration and follow the reference as a whole. Moreover, it can avoid some defects which may be caused by discontinuous controller, like chattering. The most appealing feature of the control scheme is that each robot only needs to interact with local neighbors rather than obtaining global information. This is very natural in the engineering applications, where the sensor and the communication device can only work in a limited range. Our method can also deal with the problem when some nodes are removed from or added to the

original topology which means the graph can be variable to some extent, because of malfunction or task switching.

## 2.4 Preliminaries

### 2.4.1 Variables transformation

In order to achieve the formation control objectives for multi-robot systems, this subsection provides the state and input transform to convert the original problem into a states tracking problem. It is well known for us that the nonlinear model of nonholonomic robot can be transferred into the single-integrators through the feedback linearization method, unfortunately it will loss the orientation information [Cao \*et al.\* \(2011\)](#). Nevertheless in engineering applications, the heading angle of the robot is very vital for operational precision. Therefore, a class of state and input transformation is introduced in this chapter to convert the formation tracking problem into a states tracking problem while preserving controllability of the orientation. After considering the different specific variable transformations in [Dong \(2012\)](#); [Dong & Farrell \(2008\)](#) and [Peng \*et al.\* \(2015\)](#), the **states transformation** are given as follows

$$z_{1i} = \theta_i, \tag{2.5}$$

$$z_{2i} = (x_i - p_{xi}) \cos \theta_i + (y_i - p_{yi}) \sin \theta_i + r, \tag{2.6}$$

$$z_{3i} = (x_i - p_{xi}) \sin \theta_i - (y_i - p_{yi}) \cos \theta_i, \tag{2.7}$$

where  $z_{1i}, z_{2i}, z_{3i}$  are the states of the new system,  $i = 0, 1, \dots, N$ .  $r$  is a function of state and input to be determined afterwards. By introducing **inputs transformation**  $u_{1i} = \omega_i$  and  $u_{2i} = \dot{r} + v_i - u_{1i}z_{3i}$ , the evolution of new system is obtained as

$$\dot{z}_{1i} = u_{1i}, \tag{2.8}$$

$$\dot{z}_{2i} = u_{2i}, \tag{2.9}$$

$$\dot{z}_{3i} = u_{1i}z_{2i} - u_{1i}r. \tag{2.10}$$

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Define  $Z_i = (z_{1i}, z_{2i})^T$ ,  $U_i = (u_{1i}, u_{2i})^T$ , system (9) and (10) could be rewritten as one single-integrator

$$\dot{Z}_i = U_i. \quad (2.11)$$

In the following, one lemma will show that the above state and input transform are feasible for the formation tracking issue.

**Lemma 2.1** *If the equations (2.12)-(2.13) hold for  $0 < i \leq N$ , then a group of nonholonomic mobile robots can converge to the target formation configuration  $\mathcal{Q}$ , i.e., the equations (2.1)-(2.3) can be satisfied.*

$$\lim_{t \rightarrow \infty} (z_{li} - z_{l0}) = 0, l = 1, 2, 3 \quad (2.12)$$

$$\lim_{t \rightarrow \infty} (u_{1i} - u_{10}) = 0. \quad (2.13)$$

**Proof:** Based on the above states and inputs transformation, it yields

$$\begin{aligned} \lim_{t \rightarrow \infty} (x_i - p_{xi}) &= \lim_{t \rightarrow \infty} [(z_{2i} - r) \cos(\theta_i) + z_{3i} \sin(\theta_i)] \\ &= [x_0 \cos(\theta_0) + y_0 \sin(\theta_0) + r] \cos(\theta_0) - r \cos(\theta_0) \\ &\quad + [x_0 \sin(\theta_0) - y_0 \cos(\theta_0)] \sin(\theta_0) \\ &= x_0, \end{aligned}$$

$$\begin{aligned} \lim_{t \rightarrow \infty} (y_i - p_{yi}) &= \lim_{t \rightarrow \infty} [(z_{2i} - r) \sin(\theta_i) - z_{3i} \cos(\theta_i)] \\ &= [x_0 \cos(\theta_0) + y_0 \sin(\theta_0) + r] \sin(\theta_0) - r \sin(\theta_0) \\ &\quad - [x_0 \sin(\theta_0) - y_0 \cos(\theta_0)] \cos(\theta_0) \\ &= y_0. \end{aligned}$$

For the second original control objective,

$$\begin{aligned} \lim_{t \rightarrow \infty} \left( \sum_{i=1}^N \frac{x_i}{N} - x_0 \right) &= \lim_{t \rightarrow \infty} \left( \frac{1}{N} \sum_{i=1}^N x_i - x_0 \right) = \frac{1}{N} \sum_{i=1}^N (p_{xi} + x_0) - x_0 \\ &= \frac{1}{N} \sum_{i=1}^N x_0 - x_0 = 0. \end{aligned}$$



Similarly,

$$\lim_{t \rightarrow \infty} \left( \sum_{i=1}^N \frac{y_i}{N} - y_0 \right) = 0.$$

Since  $z_{1i} \rightarrow z_{10}$ , which means  $\theta_i \rightarrow \theta_0$ . Thus, the proof is completed.

**Remark 2.2** *Based on the state and input transform and above proof, the formation tracking problem for nonholonomic multi-robot systems has become a consensus-like issue. In the next section, some novel nonlinear finite-time control law will be proposed for each robot based on the consensus theory.*

### 2.4.2 Assumptions and lemmas

In order to facilitate the theoretical analysis in next section, the following reasonable assumptions and lemmas are needed.

**Assumption 2.3** *There exists only one virtual leader, from which at least one robot can directly obtain information.*

**Remark 2.4** *The virtual leader can provide a desired trajectory and orientation for its neighbors. Actually, these reference information could be a preprogram in the control codes of those robots who can directly access the virtual leader. Besides, this kind of reference information can also be transmitted by a real robot or a human in the practical applications.*

In reality, the multi-robot systems might encounter the situation, in which some nodes are removed because of malfunction, and sometimes new robots are introduced into the original network. In order to address these requirements, the mild assumption is made as follows

**Assumption 2.5** *The communication graph  $\mathcal{G}$  is undirected, fixed and connected.*

**Remark 2.6** *Since the information exchange mechanism between neighbors in Chapter 2-6 relies on wireless communication technology. Considering the statement in paper [Kumar \(2001\)](#), "We should mention here that every problem in wireless networks is considerably exacerbated if the links are not bidirectional", undirected communication topology is reasonable to be applied in Chapters 2-6.*

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**Assumption 2.7** *Suppose the first-order derivative of reference trajectory is bounded, i.e.,  $|\dot{z}_{10}| < \gamma_1$  and  $|\dot{z}_{20}| < \gamma_2$ ,  $\gamma_1$  and  $\gamma_2$  are positive constants,  $\gamma = (\gamma_1, \gamma_2)^T \in \mathbb{R}^2$ .*

**Assumption 2.8** *The  $\theta_i$  ( $0 \leq i \leq N$ ) is bounded,  $\omega_0$  is persistently exciting and  $|\omega_i| \leq c_1$ ,  $c_1$  is a positive constant.*

**Assumption 2.9** *The external disturbance  $d_i$  for the observer satisfies the following condition:  $d_i$  is bounded, namely, there exists an upper bound  $d_u \in \mathbb{R}^2$  such that  $|d_i| < d_u$ .*

**Lemma 2.10** (*Feng & Long (2007)*) *Let  $\xi_1, \xi_2, \dots, \xi_n \geq 0$ , and  $0 < p \leq 1$ , then*

$$\left(\sum_{i=1}^n \xi_i\right)^p \leq \sum_{i=1}^n \xi_i^p \leq n^{1-p} \left(\sum_{i=1}^n \xi_i\right)^p. \quad (2.14)$$

**Lemma 2.11** *Let  $x \in \mathbb{R}^n$  be a column vector. Then,  $x^T \text{sig}(x)^\alpha \geq \|x\|_2^{\alpha+1}$  when  $0 < \alpha < 1$ . Recall the function  $\text{sig}(x)^\alpha = \text{sgn}(x)|x|^\alpha$ , where  $\text{sgn}(\cdot)$  denotes the sign function.*

**Proof:** Let  $x = (x_1 \ x_2 \ \dots \ x_n)^T$  be a column vector, thus

$$\begin{aligned} x^T \text{sig}(x)^\alpha &= [x_1 \ x_2 \ \dots \ x_n] [\text{sig}(x_1)^\alpha \ \text{sig}(x_2)^\alpha \ \dots \ \text{sig}(x_n)^\alpha]^T \\ &= x_1 \text{sig}(x_1)^\alpha + x_2 \text{sig}(x_2)^\alpha + \dots + x_n \text{sig}(x_n)^\alpha \\ &= |x_1|^{\alpha+1} + |x_2|^{\alpha+1} + \dots + |x_n|^{\alpha+1}. \end{aligned}$$

If  $0 < \alpha + 1 < p$ , it follows

$$x^T \text{sig}(x)^\alpha \geq \left\{ \sum_{i=1}^n |x_i|^p \right\}^{\frac{\alpha+1}{p}},$$

let  $p = 2$ ,

$$\begin{aligned} x^T \text{sig}(x)^\alpha &\geq \left\{ \sum_{i=1}^n |x_i|^2 \right\}^{\frac{\alpha+1}{2}} \\ &= \|x\|_2^{\alpha+1}. \end{aligned}$$

The proof is completed.

## 2.5 Main results

In this section, we will propose and analyze a type of new distributed observer-based controllers. First, a distributed observer is developed to estimate the states of virtual leader.

### 2.5.1 Development of distributed finite-time observer

Since the information of virtual leader is not accessible for each robot, a distributed observer is needed. It is well known that both the stability and convergence rate are very important performance indexes for observer design. In order to guarantee the formation precision of multi-robot systems, this subsection first proposes a distributed finite-time observer as follows

$$\begin{aligned}\dot{\mathcal{P}}_i &= \rho \sum_{j \in \mathcal{N}_i} (\mathcal{P}_j - \mathcal{P}_i) - \rho b_i (\mathcal{P}_i - \mathcal{P}_0) \\ &\quad + \kappa \operatorname{sgn} \left( \sum_{j \in \mathcal{N}_i} (\mathcal{P}_j - \mathcal{P}_i) - b_i (\mathcal{P}_i - \mathcal{P}_0) \right) + d_i,\end{aligned}\tag{2.15}$$

where  $\mathcal{P}_i = (\varrho_{1i}, \varrho_{2i})^T$  is the state of observer for robot  $i$ , and  $\mathcal{P}_0 = (\varrho_{10}, \varrho_{20})^T = (z_{10}, z_{20})^T$  is the state of virtual leader. Besides,  $d_i = (d_{1i}, d_{2i})^T$  is the external disturbance that affects the observer. There exists one positive constant vector  $\gamma$  making  $|\dot{\mathcal{P}}_0| < \gamma$  based on Assumption 2.7.  $\operatorname{sgn}(\cdot)$  is the sign function with the fact that  $x^T \operatorname{sgn}(x) = \|x\|_1$ .  $\rho$  and  $\kappa$  are positive constants. In the next step, we will prove that this distributed observer converges in a finite time.

**Lemma 2.12** *Under Assumptions 2.3, 2.5, 2.7, 2.8 and 2.9, the distributed observer (2.15) is stabilized in a finite time if  $\kappa > \max(\gamma) + \max(d_u)$ . That is  $\lim_{t \rightarrow \bar{T}} (\mathcal{P}_i - \mathcal{P}_0) = 0$  when  $t \geq \bar{T}$ , with*

$$\bar{T} = \frac{\sqrt{\lambda_{\max} \tilde{\mathcal{P}}(0)^T (\mathcal{H} \otimes I_2) \tilde{\mathcal{P}}(0)}}{(\kappa - \max(\gamma) - \max(d_u)) \lambda_{\min}}.\tag{2.16}$$

Particularly,  $\mathcal{P}_i(t) = \mathcal{P}_0(t)$  for any  $t \geq \bar{T}$ .

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**Proof:** Define the state error as  $\tilde{\mathcal{P}}_i = \mathcal{P}_i - \mathcal{P}_0$ , take the derivative of  $\tilde{\mathcal{P}}_i$  as

$$\begin{aligned}\dot{\tilde{\mathcal{P}}}_i &= \dot{\mathcal{P}}_i - \dot{\mathcal{P}}_0 \\ &= \rho \sum_{j \in \mathcal{N}_i} (\tilde{\mathcal{P}}_j - \tilde{\mathcal{P}}_i) - \rho b_i \tilde{\mathcal{P}}_i \\ &\quad + \kappa \text{sgn} \left( \sum_{j \in \mathcal{N}_i} (\tilde{\mathcal{P}}_j - \tilde{\mathcal{P}}_i) - b_i \tilde{\mathcal{P}}_i \right) + d_i - \dot{\mathcal{P}}_0,\end{aligned}$$

then the compact form of above error dynamics can be rewritten as

$$\begin{aligned}\dot{\tilde{\mathcal{P}}} &= -\rho(\mathcal{L} \otimes I_2)\tilde{\mathcal{P}} - \rho(\mathcal{B} \otimes I_2)\tilde{\mathcal{P}} + \kappa \text{sgn}(-(\mathcal{L} \otimes I_2)\tilde{\mathcal{P}} - (\mathcal{B} \otimes I_2)\tilde{\mathcal{P}}) + d - (\mathbf{1}_N \otimes I_2)\dot{\mathcal{P}}_0 \\ &= -\rho(\mathcal{H} \otimes I_2)\tilde{\mathcal{P}} - \kappa \text{sgn}((\mathcal{H} \otimes I_2)\tilde{\mathcal{P}}) + d - (\mathbf{1}_N \otimes I_2)\dot{\mathcal{P}}_0,\end{aligned}$$

where  $\tilde{\mathcal{P}} = (\tilde{\mathcal{P}}_1, \tilde{\mathcal{P}}_2, \dots, \tilde{\mathcal{P}}_N)^T$  and  $d = (d_1, \dots, d_N)^T$ . Consider the Lyapunov function candidate as

$$V_1 = \frac{1}{2} \tilde{\mathcal{P}}^T (\mathcal{H} \otimes I_2) \tilde{\mathcal{P}}. \quad (2.17)$$

Based on the properties of  $\mathcal{K}[\cdot]$ , the set-valued Lie derivative of  $V_1$  can be derived as follows

$$\begin{aligned}\tilde{L}_F V_1 &= \bigcap_{\xi \in \partial V(\tilde{\mathcal{P}})} \xi^T \mathcal{K}[-\rho(\mathcal{H} \otimes I_2)\tilde{\mathcal{P}} - \kappa \text{sgn}((\mathcal{H} \otimes I_2)\tilde{\mathcal{P}}) + d - (\mathbf{1}_N \otimes I_2)\dot{\mathcal{P}}_0] \\ &= \mathcal{K}[-\rho \tilde{\mathcal{P}}^T (\mathcal{H} \otimes I_2)^2 \tilde{\mathcal{P}} - \kappa \|(\mathcal{H} \otimes I_2)\tilde{\mathcal{P}}\|_1 + \tilde{\mathcal{P}}^T (\mathcal{H} \otimes I_2)(d - (\mathbf{1}_N \otimes I_2)\dot{\mathcal{P}}_0)] \\ &= -\rho \tilde{\mathcal{P}}^T (\mathcal{H} \otimes I_2)^2 \tilde{\mathcal{P}} - \kappa \|(\mathcal{H} \otimes I_2)\tilde{\mathcal{P}}\|_1 + \tilde{\mathcal{P}}^T (\mathcal{H} \otimes I_2)(d - (\mathbf{1}_N \otimes I_2)\dot{\mathcal{P}}_0),\end{aligned}$$

where  $\partial V_1(\tilde{\mathcal{P}})$  is the generalized gradient of  $V_1$  at  $\tilde{\mathcal{P}}$ ,  $\tilde{L}_F V$  is a singleton. Therefore, based on Assumptions 2.7 and 2.9, it follows that

$$\begin{aligned}\max \tilde{L}_F V_1 = \dot{V}_1 &\leq -\rho \tilde{\mathcal{P}}^T (\mathcal{H} \otimes I_2)^2 \tilde{\mathcal{P}} - \kappa \|(\mathcal{H} \otimes I_2)\tilde{\mathcal{P}}\|_1 + \max(\gamma) \|(\mathcal{H} \otimes I_2)\tilde{\mathcal{P}}\|_1 \\ &\quad + \max(d_u) \|(\mathcal{H} \otimes I_2)\tilde{\mathcal{P}}\|_1\end{aligned}$$

When  $\rho \geq 0$ , since item  $\tilde{\mathcal{P}}^T(\mathcal{H} \otimes I_2)^2\tilde{\mathcal{P}} > 0$ , one has

$$\begin{aligned}
 \max \tilde{L}_F V_1 = \dot{V}_1 &\leq -(\kappa - \max(\gamma) - \max(d_u))\|(\mathcal{H} \otimes I_2)\tilde{\mathcal{P}}\|_1 \\
 &\leq -(\kappa - \max(\gamma) - \max(d_u))\|(\mathcal{H} \otimes I_2)\tilde{\mathcal{P}}\|_2 \\
 &= -(\kappa - \max(\gamma) - \max(d_u))\sqrt{\tilde{\mathcal{P}}^T(\mathcal{H} \otimes I_2)^2\tilde{\mathcal{P}}} \\
 &\leq -(\kappa - \max(\gamma) - \max(d_u))\lambda_{\min}\|\tilde{\mathcal{P}}\|_2 \\
 &\leq -(\kappa - \max(\gamma) - \max(d_u))\frac{\sqrt{2}\lambda_{\min}}{\sqrt{\lambda_{\max}}}\sqrt{V_1},
 \end{aligned}$$

where  $\lambda_{\min}$  and  $\lambda_{\max}$  are the minimum and maximum eigenvalues of  $\mathcal{H} \otimes I_2$ , respectively. If the gain parameter satisfies  $\kappa > \max(\gamma) + \max(d_u)$ , the Lyapunov function will be negative definite. Then under Lemma 1.5, the present observer (2.15) is finite time stabilized, and the settling time can be estimated as follows

$$\bar{T} = \frac{\sqrt{\lambda_{\max}\tilde{\mathcal{P}}(0)^T(\mathcal{H} \otimes I_2)\tilde{\mathcal{P}}(0)}}{(\kappa - \max(\gamma) - \max(d_u))\lambda_{\min}}. \quad (2.18)$$

Therefore,  $\mathcal{P}_i(t) = \mathcal{P}_0(t)$  when  $t \geq \bar{T}$ .

### 2.5.2 Design of distributed finite-time controller

Among the existing control laws for consensus tracking problem of system (2.11), it is commonly presented in linear form. However, what the practical application needs is that the output of controller be nonlinear with respect to the error signal, with the objective of making it very sensitive to small errors and not so responsive to larger ones. In addition, the actuators usually have saturation constraints, which must be taken into account during the design and synthesis process of control strategies. Based on the above observation, the following decentralized control law can be proposed to deal with the formation tracking issue based on local interaction

$$U_i = \mathcal{P}_i + \eta \text{sig}\left[\sum_{j \in \mathcal{N}_i} (Z_j - Z_i) - b_i(Z_i - Z_0)\right]^\epsilon, \quad (2.19)$$

where  $Z_i = (z_{1i}, z_{2i})^T$ ,  $Z_0 = (z_{10}, z_{20})^T$ ,  $U_i = (u_{1i}, u_{2i})^T$ ,  $\eta$  is a positive constant,  $\text{sig}(x)^\phi = \text{sgn}(x)|x|^\epsilon$ . To improve the sensitivity of controller to tiny relative state errors, in order to guarantee finite-time convergence rate, the parameter  $\phi$  is

## 2. DISTRIBUTED FINITE-TIME TRACKING CONTROL OF MULTI-ROBOT FORMATION SYSTEMS WITH NONHOLONOMIC CONSTRAINT

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chosen in the range  $(0, 1)$  in this chapter. The dynamics of  $\mathcal{P}_i$  obeys the observer (2.15). Define the state error as

$$\tilde{Z}_i = Z_i - Z_0.$$

Herein, the first main result is obtained.

**Theorem 2.13** *Consider the subsystem (2.11), Assumptions 2.3, 2.5, 2.7 and 2.8 are satisfied. When  $0 < \epsilon < 1$ , then  $\lim_{t \rightarrow t^*} (z_{li} - z_{l0}) = 0$ ,  $l=1, 2$  and  $\lim_{t \rightarrow t^*} (u_{1i} - u_{10}) = 0$ , where  $t^* = 2 \frac{\sqrt{\frac{1}{2} \tilde{Z}(\bar{T})^T (\mathcal{H} \otimes I_2) \tilde{Z}(\bar{T})^{1-\epsilon}}}{K_1(1-\epsilon)}$ ,  $K_1 = \eta [\lambda_{\min} \sqrt{2/\lambda_{\max}}]^{\epsilon+1}$ , which means the closed-loop system satisfies finite-time stability under the control law (2.19) with the distributed observer (2.15).*

**Proof:** when  $t \geq \bar{T}$ , computing the derivative of  $\tilde{Z}$  based on Lemma 2.12 yields

$$\dot{\tilde{Z}} = -\eta \text{sig}((\mathcal{H} \otimes I_2) \tilde{Z})^\epsilon, \quad (2.20)$$

where  $\tilde{Z} = (\tilde{Z}_1, \dots, \tilde{Z}_N)^T$ . Choosing the Lyapunov function candidate as follows

$$V_2 = \frac{1}{2} \tilde{Z}^T (\mathcal{H} \otimes I_2) \tilde{Z} \leq \frac{1}{2} \lambda_{\max} \|\tilde{Z}\|_2^2. \quad (2.21)$$

Differentiate  $V_2$  with respect to time, based on Lemma 2.11, it yields

$$\begin{aligned} \dot{V}_2 &= \tilde{Z}^T (\mathcal{H} \otimes I_2) \dot{\tilde{Z}} \\ &= -\eta \tilde{Z}^T (\mathcal{H} \otimes I_2) \text{sig}((\mathcal{H} \otimes I_2) \tilde{Z})^\epsilon \\ &\leq -\eta \|(\mathcal{H} \otimes I_2) \tilde{Z}\|_2^{\epsilon+1} \\ &= -\eta \sqrt{\tilde{Z}^T (\mathcal{H} \otimes I_2)^2 \tilde{Z}}^{\epsilon+1} \\ &\leq -\eta (\lambda_{\min} \|\tilde{Z}\|_2)^{\epsilon+1} \\ &\leq -\eta [\lambda_{\min} \sqrt{2/\lambda_{\max}}]^{\epsilon+1} V_2^{\frac{\epsilon+1}{2}}. \end{aligned}$$

Let  $K_1 = \eta [\lambda_{\min} \sqrt{2/\lambda_{\max}}]^{\epsilon+1}$ , if  $K_1 > 0$ , the closed-loop system converges to equilibrium within finite time by the Lemma 1.5. Further, The settling time can be estimated as  $t^* = 2 \frac{\sqrt{\frac{1}{2} \tilde{Z}(\bar{T})^T (\mathcal{H} \otimes I_2) \tilde{Z}(\bar{T})^{1-\epsilon}}}{K_1(1-\epsilon)}$ .

**Theorem 2.14** *Consider the coupled system (2.10), choosing  $r = k_0 \text{sig}(u_{1i})^\epsilon z_{3i}$ , where  $k_0$  is a positive constant, and  $0 < \epsilon < 1$ . Then, it has  $\lim_{t \rightarrow \infty} (z_{3i} - z_{30}) = 0$  under the finite-time distributed consensus protocol (2.19) based on the observer (2.15).*

**Proof:** Let  $\tilde{z}_{3i} = z_{3i} - z_{30}$ . Take the derivative of  $\tilde{z}_{3i}$  as

$$\begin{aligned}\dot{\tilde{z}}_{3i} &= \dot{z}_{3i} - \dot{z}_{30} \\ &= -k_0|u_{1i}|^{\epsilon+1}\tilde{z}_{3i} + x_2(t),\end{aligned}\tag{2.22}$$

where  $x_2(t) = u_{1i}\tilde{z}_{2i} + (u_{1i} - u_{10})z_{20} - k_0(|u_{1i}|^{\epsilon+1} - |u_{10}|^{\epsilon+1})z_{30}$ . when  $\lim_{t \rightarrow t^*}(z_{1i} - z_{10}) = 0$ ,  $\lim_{t \rightarrow t^*}(z_{2i} - z_{20}) = 0$ ,  $\lim_{t \rightarrow t^*}(u_{1i} - u_{10}) = 0$ , the solution of the differential equation (2.22) is given as follows

$$\tilde{z}_{3i}(t) = e^{\int_0^t -k_0|u_{1i}|^{\epsilon+1}d\tau}\tilde{z}_{3i}(0) + \int_0^t e^{\int_\tau^t -k_0|u_{1i}|^{\epsilon+1}d\nu}x_2(\tau)d\tau.\tag{2.23}$$

According to Theorem 2.13, state error  $\tilde{Z}_i$  asymptotically converges to zero, and  $u_{1i}$  asymptotically reaches to  $u_{10}$ . Then,  $x_2(t)$  also asymptotically converges to zero. Hence, according to the stability in the sense of Lyapunov, for a arbitrary positive value  $\sigma > 0$ , it exists  $o(\sigma) > 0$ , when the  $|x_2(0)| < o(\sigma)$ , it has  $|x_2(t)| < \sigma$ .

From the Assumptions 2.5, and the input transform  $u_{1i} = \omega_i$ , and  $1 < \epsilon + 1 < 2$ , one has  $|u_{1i}|^{\epsilon+1} \leq c_1$ . The solution (2.23) satisfies the inequality

$$\begin{aligned}\tilde{z}_{3i}(t) &= e^{\int_0^t -k_0|u_{1i}|^{\epsilon+1}d\tau}\tilde{z}_{3i}(0) + \int_0^t e^{\int_\tau^t -k_0|u_{1i}|^{\epsilon+1}d\nu}x_2(\tau)d\tau \\ &\leq e^{-k_0c_1t}|\tilde{z}_{3i}(0)| + \int_0^t e^{-k_0c_1(t-\tau)}|x_2(\tau)|d\tau \\ &\leq e^{-k_0c_1t}|\tilde{z}_{3i}(0)| + e^{-k_0c_1t} \int_0^t e^{k_0c_1\tau}|x_2(\tau)|d\tau \\ &\leq e^{-k_0c_1t}|\tilde{z}_{3i}(0)| + \frac{\sigma k_0c_1 - \sigma k_0c_1e^{-k_0c_1t}}{k_0c_1} \\ &= \sigma + e^{-k_0c_1t}(|\tilde{z}_{3i}(0)| - \sigma).\end{aligned}$$

Hence, when  $t \rightarrow +\infty$ ,  $|\tilde{z}_{3i}(t)| \leq \sigma$ . Since  $\sigma$  is a arbitrary small positive value, from the definition of asymptotic stability, the  $\tilde{z}_{3i}(t)$  is asymptotically stabilized to the arbitrary small neighborhood of origin. The proof is completed.

**Remark 2.15** Combining Theorem 2.13 with 2.14, the consensus objectives (2.12)-(2.13) are achieved, which means the formation tracking objectives (2.1)-(2.3) of original system are all reached based on Lemma 2.1.

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### 2.5.3 From theory to practice

In this subsection, we will provide a control diagram to show how to apply the proposed theoretical results to the real multi-robot systems. In Figure.2.2,  $j \in \mathcal{N}_i$ , the closed-loop control flow is as follows:

- (1) Robot  $i$  obtains its real-time position and orientation through sensing measurements;
- (2) After variables transformation, the signals in (1) and desired relative position of robot  $i$  are converted into new state variables;
- (3) Through communication robot  $i$  can exchange information with robot  $j$ , thus new state variables of robots  $i$  and  $j$  will be sent to the distributed observer-based controller  $i$ ;
- (4) After the variables inverse transformation, the control inputs are converted into the linear and angular speeds while being further converted into the speed commands of right and left wheels;
- (5) The speed commands will be sent to the motors and drive the motion of robot  $i$ . Return to the step (1).

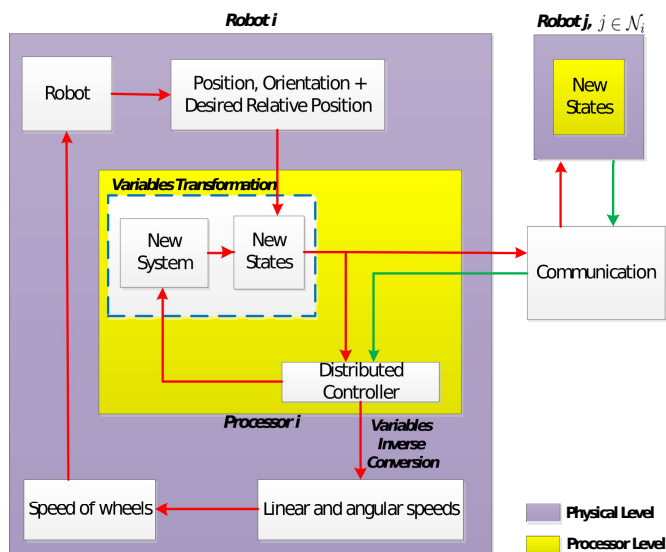


Figure 2.2: Control diagram.



## 2.6 Numerical example

To verify the theoretical results obtained in the preview sections, the following simulation experiments are implemented based on the control diagram. We consider a multi-robot systems with nonholonomic constraints consisting of ten robots denoted by  $F_1 \sim F_{10}$  and one virtual leader denoted by  $L_0$ . The interaction topology is designed by Figure.2.3 due to the limited communication range, in which each robot only needs local information exchange. The corresponding adjacency matrix is provided by (2.24). We consider an extreme condition, only one robot  $F_1$  can access the reference information, then the matrix  $\mathcal{B} = \text{diag}(1, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T$  is obtained. Generally speaking, the more robots can access the reference, the stronger robustness against link failure the formation systems hold.

The desired formation configuration  $\mathcal{Q}$  is predefined in orthogonal coordinates as  $(p_{x1}, p_{y1}) = (0, 1)$ ,  $(p_{x2}, p_{y2}) = (0.95, 0.31)$ ,  $(p_{x3}, p_{y3}) = (0.59, -0.81)$ ,  $(p_{x4}, p_{y4}) = (-0.59, -0.81)$ ,  $(p_{x5}, p_{y5}) = (-0.95, 0.31)$ ,  $(p_{x6}, p_{y6}) = (0, -0.38)$ ,  $(p_{x7}, p_{y7}) = (-0.36, -0.12)$ ,  $(p_{x8}, p_{y8}) = (-0.22, 0.31)$ ,  $(p_{x9}, p_{y9}) = (0.22, 0.31)$ ,  $(p_{x10}, p_{y10}) = (0.36, -0.12)$ , shown in Figure.2.4. These coordinate values determine the geometric configuration with respect to the relationship of distance and bearing between robots which depends on the practical task requirement of applications. For example, in an automobile assembly workshop, it depends on the shape and measurement of the component to transport. Another common example is that when a group of robots traverse an area filled with obstacles, they need to change the shape or scale in real time to avoid collision. In this example, the trajectory of the virtual robot  $R_0$  is briefly chosen as

$$x_0 = 5 \sin(0.05t), y_0 = -5 \cos(0.05t)$$

for the surveillance task or the military intimidation. The control parameters are chosen based on Lemma 2.12 and Theorem 2.13, i.e.,  $k_0 = 2$ ,  $\rho = 0.5$ ,  $\kappa = 0.8$ ,  $\eta = 1.3$ . In order to demonstrate the robustness of our controller, the identical disturbances  $d_{1i} = 0.02 \sin(t)$  and  $d_{2i} = 0.1 \cos(0.5t)$  are introduced into the input of each robot. The  $\lambda_{\max}(\mathcal{H} \otimes I_2) = 5.7367$  and  $\lambda_{\min}(\mathcal{H} \otimes I_2) = 0.0630$  based on the topology in the Figure.2.3. From the Figure.2.5, it shows the evolution of ten robots (circles with different colours) at certain instants under finite-time

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control law (2.19) and the distributed finite-time observer (2.15) with exogenous disturbances. The arbitrary initial positions of ten robots are represented by squares. It is observed that after a short period of evolution, robots gradually produce the desired shape while keeping the fixed geometric structure in the next maneuvering, i.e., Eq.(2.1) is verified. The centroid (its trajectory is blue line) of ten robots tracks the trajectory of reference (red line), i.e., Eq.(3) is verified. The heading angle tracking errors is shown by Figure.2.6,  $\theta_i - \theta_0$  converges to zero over time, i.e., Eq.(2.3) is verified. It can be seen from Figure.2.7 and Figure.2.8 that the control input signals do not demonstrate the chattering phenomenon, which is often the drawback of sliding mode control. From Figure.2.5 to Figure.2.6, the original formation tracking objectives (2.1)-(2.3) are all reached.

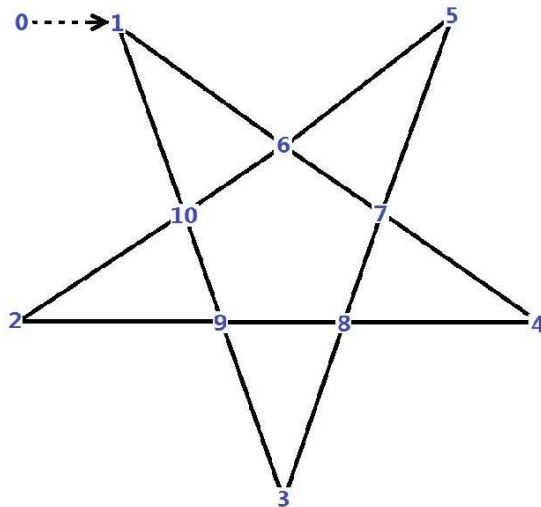


Figure 2.3: Communication topology based on nearest neighbors rule.

## 2.6 Numerical example

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (2.24)$$

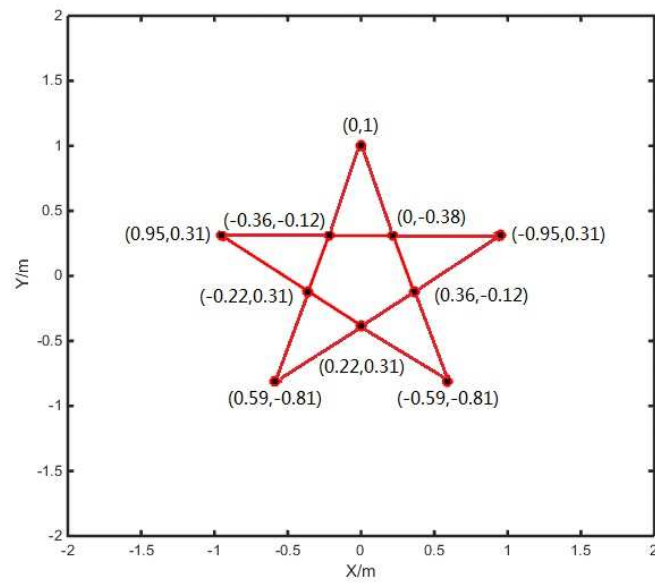


Figure 2.4: Desired formation shape specified by displacements with respect to the centroid.

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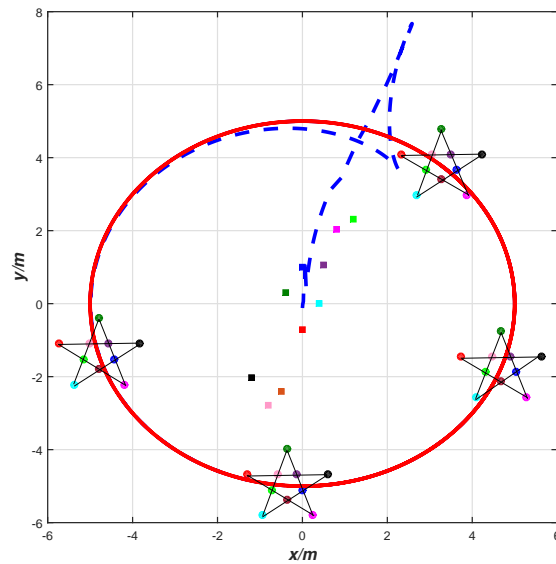


Figure 2.5: Result under control law (2.19): formation evolution of ten robots at certain instant, the red and blue line are the trajectory of reference and centroid of configuration, respectively.

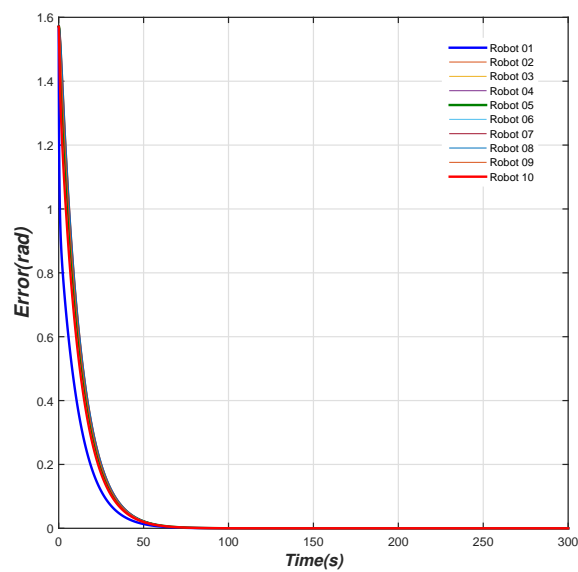


Figure 2.6: The orientation error between  $\theta_i$  and  $\theta_0$  under control law (2.19).

## 2.6 Numerical example

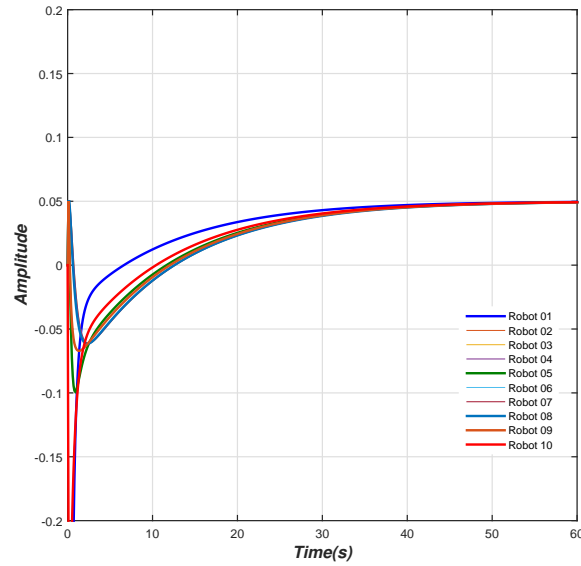


Figure 2.7: The control input  $u_{1i}$  ( $i = 1, 2, \dots, N$ ) under control law (2.19).

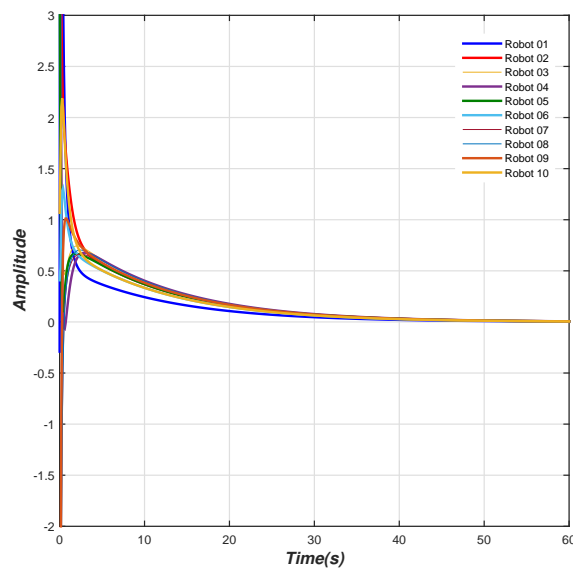


Figure 2.8: The control input  $u_{2i}$  ( $i = 1, 2, \dots, N$ ) under control law (2.19).

## **2.7 Conclusions**

This chapter solve the problem of how to design and analyze a type of observer based finite-time controller for the formation tracking of multi-robot systems with nonholonomic constraint in distributed manner. The finite-time stability of resulting closed-loop formation systems was analyzed rigorously with the help of algebraic graph theory, matrix analysis and Lyapunov techniques. Numerical example is provided to verify the effectiveness of the present controller.

Nevertheless, it is worth noting that the settling time of formation systems strongly depends on the initial conditions in the results of this chapter, which are often unavailable in practice. Besides, the initial conditions which are one kind of global information for single robot. Furthermore, when the initial errors are large enough, the convergence rate will be slower than exponential rate in the rise-time. In order to overcome these problems, in the next chapter, the concept of fixed-time stability will be introduced so as to overcome the shortcomings of the current controller.

## Chapter 3

# Distributed Fixed-Time Tracking Control of Multi-Robot Formation Systems with Nonholonomic Constraint

### 3.1 Problem setup

The problem will be addressed in this chapter is that how to design a type of *distributed control laws*, which can guarantee that *nonholonomic multi-robot systems* achieve *formation tracking* (see related definitions in Chapter 1) in a fixed time from an arbitrary initial configuration. Compared to the finite-time stability in which the upper bound  $T_2$  of settling time (the moment that system converges to the equilibrium) is strongly dependent of the initial positions, as shown in Figure.3.1, the upper bound  $T_1$  of settling time of fixed-time stability, which is regardless of the initial positions thus can be flexibly prescribed off-line.

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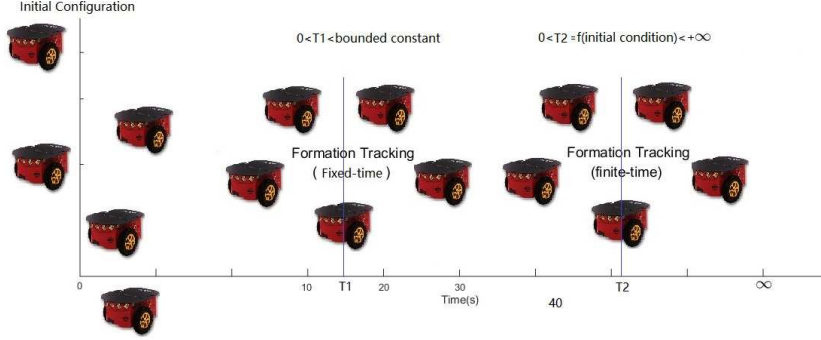


Figure 3.1: Rough comparison between the fixed-time and finite-time stability.

The control problem of this chapter is posed in a rigorous manner as follows: design the control input  $v_i$  and  $\omega_i$  for the robot  $i$  by using  $(q_i, \dot{q}_i)$ ,  $(p_{xi}, p_{yi})$ , and  $(q_j, \dot{q}_j)$ ,  $j \in N_i$ , such that the control objectives

$$\lim_{t \rightarrow +\infty} \begin{bmatrix} x_i - p_{xi} \\ y_i - p_{yi} \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}, \quad (3.1)$$

$$\lim_{t \rightarrow +\infty} \left( \sum_{i=1}^N \frac{x_i}{N} - x_0 \right) = 0, \quad \lim_{t \rightarrow +\infty} \left( \sum_{i=1}^N \frac{y_i}{N} - y_0 \right) = 0, \quad (3.2)$$

$$\lim_{t \rightarrow T_{max1}} (\theta_i - \theta_0) = 0 \quad (3.3)$$

are reached.

**Remark 3.1** *In our design, for each robot the first step is to determine the desired relative position  $(p_{xi}, p_{yi})$  with respect to the virtual leader, which might depend on the types of tasks and requirements of the specific applications. It is easy to observe that the shape and scale of formation can be arbitrary adjusted through modifying the desired relative position  $(p_{xi}, p_{yi})$  for each robot. In practice, the geometric configuration of formation often needs to satisfy the specific requirements of different tasks, such as surveillance, materials transport, traversing a specific area, transport payloads, etc.*

## 3.2 Literature overview

Over the past two decades, the distributed control mechanism has become a hot topic that received great attention in broad areas. This technique has gradually



infiltrated into distributed computation, ground/aerial/underwater multiple vehicles cooperative control, attitude calibration of satellites, formation stabilization and maneuver of aircrafts, cooperative source searching, disaster surveillance and rescue and multi-sensor information fusion. For detailed instance, authors in [Li \*et al.\* \(2012\)](#) studied the distributed kinematic control of multiple redundant manipulators via recurrent neural networks. Furthermore, reference [Li \*et al.\* \(2017\)](#) investigated the distributed cooperative control of manipulators with a game-theoretic perspective. Reference [Li \*et al.\* \(2013b\)](#) proposed the first distributed protocol to deal with winner-take-all problem in networks via Lyapunov theory. And reference [Cao \*et al.\* \(2013b\)](#) studied the hunting problem of multi-robot systems by a distributed approach. In addition, distributed filter under directed switching topologies via consensus theory are proposed and analyzed in [Li & Guo \(2015\)](#). Thus, when we naturally introduce distributed control perspective to the decision and control of a group of autonomous robots, traditionally centralized control mechanism gradually faded. Based on the characteristics of distributed control approach, each robot with built-in micro distributed controller only needs to interact with its neighbors or the leader, and the collective formation behaviors will be produced to finally complete the complex task.

Also, the formation tracking problem for the distributed multi-robot systems have been considered widely in various communities due to its broad applications against single mobile robot, except for the cases referred above, it is very suitable for the forest fire monitoring and huge component transport, etc. Employing distributed control mechanism to solve the formation tracking problem of multi-robot systems with nonholonomics constraints is a promising direction recently. For this point, there have existed numerous interesting results, the partial results can be found in references [Hu & Shao \(2016\)](#); [Peng \*et al.\* \(2013b\)](#); [Sun \*et al.\* \(2017a\)](#); [Wang & Wu \(2012\)](#); [Wang \*et al.\* \(2016\)](#). However, it still leaves many compelling challenges such as communication security problem, resource optimization problem, convergence rate problem, etc [Liu \*et al.\* \(2015a\)](#); [Wen \*et al.\* \(2017a\)](#).

Among these urgent problems, convergence rate problem serves as a key performance indicator of multi-robot systems, which has attracted considerable attention from relevant areas. The convergence rate problem was specially discussed

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in [Cheng \*et al.\* \(2016\)](#). According to convergence speed, the stability of multi-robot systems could be roughly categorized as asymptotic stability, exponential stability, finite-time stability and fixed-time stability, respectively. Note that finite-time stability is better than the former two since it can make systems reach equilibrium point within a finite settling time (convergence time) rather than infinite. Up to now, many results about finite-time stability have been obtained. Early work [Bhat & Bernstein \(2000a\)](#) defined finite-time stability for equilibria of continuous but non-Lipschitzian autonomous systems. In [Zhao \*et al.\* \(2016a\)](#), the authors investigated the distributed finite-time consensus tracking problem for a group of autonomous agents modeled by multiple non-identical second-order nonlinear systems. In [Peng \*et al.\* \(2015\)](#), formation tracking problem of multi-robot systems was considered, and the finite-time controllers were proposed. A saturated protocol was proposed for the finite-time consensus of the networked agents with second-order integrators in [Zhao \*et al.\* \(2015a\)](#). In [Liu \*et al.\* \(2015b\)](#), distributed exponential finite-time containment control and consensus of multi-agent systems were considered. In [Du \*et al.\* \(2015\)](#), the authors considered the finite-time formation control issue of multi-robot systems which were transformed into the chain systems via linear feedback technique. The finite-time stabilization problem for a class of nonholonomic feedforward systems was investigated in [Gao \*et al.\* \(2016\)](#), in which the input saturations were considered. Furthermore, in [Bayat \*et al.\* \(2016\)](#), finite-time tracking problem for chain systems with unknown disturbances was addressed. In the meantime, the finite-time fault-tolerant formation control issue for spacecrafts was studied based on dual-quaternion in [Dong \*et al.\* \(2016a\)](#). Most recently, the authors in [Chu \*et al.\* \(2016\)](#) investigated the finite-time control strategies for nonholonomic multi-robot formation tracking problem.

The researchers working on neural networks also express strong interests to finite-time stability. Reference [Li \*et al.\* \(2013b\)](#) solved the quadratic programming problems in a finite time and the optimality of proposed neural network is proven in theory. And reference [Li \*et al.\* \(2013a\)](#) utilized a non-trivial sign-bi-power activation function to endow the finite-time convergence rate to Zhang neural network for solving Sylvester equation. Meanwhile, the upper bound of the global convergence time is explicitly derived in this work. One of the important significance of this work is to obtain an upper bound of convergence time using

## 3.2 Literature overview

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finite-time stability theory, which reveals that there is close relationship between finite-time and fixed-time stability. More recently, the work about finite-time stability in [Huang \*et al.\* \(2017\)](#) claimed that it can derive the minimum estimated value of settling time. However, finite-time stability still cannot be equated with fixed-time stability that will be introduced in the next paragraph. As well known that there have been proven that many controllers can attain stability in a finite time based on homogeneous theory, especially in multi-agent domain, nevertheless can't explicitly derive a convergence time estimate function. On one hand, most finite-time controllers can only obtain an unbounded time estimate function in theory. On the other hand, some finite-time controllers can succeed to obtain an upper bound via selecting appropriate parameters, just as [Li \*et al.\* \(2013a\)](#). However, in fixed-time stability theory, an explicit upper bound of convergence time estimate function which is regardless of the initial conditions, can always be guaranteed.

Generally speaking, it is worthy to notice that the flaws of most results of finite-time control approaches can't be neglected. In finite-time case, the convergence rate strongly relies on the initial errors of multi-robot systems, this is one kind of unbounded global information. If the initial errors are sufficiently large, its convergent capability might even be slower than exponential rate during the rise time of the response. In addition, in order to pre-design the settling time for the multi-robot systems, the global information of the initial robots conditions is needed for each robot, which makes pre-design convergence time off-line impractical. To overcome the flaws of finite-time stability, a new stability theory, called fixed-time stability, was recently developed in [Polyakov \(2012\)](#). According to the fixed-time stability theory, more and more interesting results appeared in multi-agent systems domain. In [Defoort \*et al.\* \(2015a\)](#), fixed-time consensus tracking problem for first-order multi-agent systems with unknown inherent non-linear dynamics was considered. In [Parsegov \*et al.\* \(2013a\)](#), the fixed-time average-consensus of first-order systems was addressed under a weighted undirected graph. In [Zuo \(2015\)](#), authors studied the fixed-time consensus tracking issue for second-order multi-agent systems with directed topology. In [Liu \*et al.\* \(2016\)](#), a class of distributed fixed-time algorithms was developed for the multi-agent systems with double-integrator dynamics by using a motion-planning approach. More recently, the authors in [Chu \*et al.\* \(2017c\)](#) studies the fixed-time

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consensus tracking problem for double integrators using slide mode technique, and related results are further extended to formation tracking of unicycle-type mobile robots.

### 3.3 Contributions

Motivated by aforementioned observation, this chapter aims to solve the formation tracking problem via designing distributed controllers with fixed-time properties for each unicycle-type robot so as to emerge desired collective formation tracking behaviors. In summary, the main contributions of this work lie in the following aspects. First, a new type of distributed fixed-time control protocol for the network of robots is devised. Meanwhile, the corresponding sufficient conditions are derived by using algebraic graph theory, matrix analysis and fixed-time stability theory. A specific form inequality of the time derivative of Lyapunov function has been deduced. It thus proves that robots can converge to the desired behaviors in a fixed time based on fixed-time stability theory. Accordingly, an upper bounded settling time formula for the multi-robot systems with nonholonomic constraints is explicitly derived. Finally, numerical simulations are given to illustrate the effectiveness of the present theoretical results. When compared with the existing results for the formation tracking problem of multiple nonholonomic mobile robots, this chapter has the following advantages. Firstly, to the best of our knowledge, this chapter is the first time to employ fixed-time stability to distributively solve kinematic formation tracking problem for unicycle-type multi-robots systems with the controlled position located in the center of driving axis. Secondly, in contrast to [Cai & Xiang \(2015\)](#); [Shi \(2015\)](#) and Chapter 2, the derived settling time of this chapter is independent of the global initial information. Moreover, the settling time can be pre-designed off-line through tunable control gains, graph Laplacian, total amount of robots and boundary value of reference state. This is a crucial feature in reality, especially when the requirement of convergence performance is rigorous, such as disaster rescue tasks or initial conditions are lacked in practice.

### 3.4 Main results

In this chapter, we will use connected undirected topology  $\mathcal{G}$  to characterize the communication relation amongst robots. The item  $\chi_1(t) = k_0 \text{sig}(u_{1i})^\epsilon z_{3i}$  in the variables transformation is investigated,  $k_0 > 0$ , and  $0 < \epsilon < 1$ . Thus, the subsystem (2.10) in Chapter 2 can be rewritten as follows

$$\dot{z}_{3i} = u_{1i} z_{2i} - k_0 |u_{1i}|^{\epsilon+1} z_{3i}. \quad (3.4)$$

**Theorem 3.2** *Consider the subsystem (2.8) under connected topology with Assumptions 2.3 and 2.7, and adopt the distributed fixed-time controller for mobile robot  $i$ , ( $1 \leq i \leq N$ ), which is given by*

$$u_{1i} = \alpha_1 \text{sig}\left[\sum_{j=0}^N a_{ij}(z_{1j} - z_{1i})\right]^2 + \beta_1 \text{sgn}\left[\sum_{j=0}^N a_{ij}(z_{1j} - z_{1i})\right], \quad (3.5)$$

where  $\alpha_1$  and  $\beta_1$  are positive constants, and  $\beta_1 > \gamma_1$ . Then, the subsystem (2.8) of the mobile robot globally converges in a fixed time, i.e.  $\lim_{t \rightarrow T_{max1}}(z_{1i} - z_{10}) = 0$ ,  $\lim_{t \rightarrow T_{max1}}(u_{1i} - u_{10}) = 0$ , where the upper bound of settling time is given by

$$T_{max1} = \frac{\pi}{\frac{2\lambda_{min}^2}{\lambda_{max}} \sqrt{\alpha_1 N^{-\frac{1}{2}} (\beta_1 - \gamma_1)}}.$$

**Proof:** Define the consensus error as  $\varepsilon_1 = z_1 - \mathbf{1}_n z_{10}$ ,  $\varepsilon_1 = (\varepsilon_{11}, \dots, \varepsilon_{1N})^T$ ,  $z_1 = (z_{11}, \dots, z_{1N})^T$ . Taking the time derivative of consensus error and rewriting it in compact form yields

$$\begin{aligned} \dot{\varepsilon}_1 &= \dot{z}_1 - \dot{z}_{10} \mathbf{1}_n \\ &= -\alpha_1 \text{sig}(H\varepsilon_1)^2 - \beta_1 \text{sgn}(H\varepsilon_1) - \dot{z}_{10} \mathbf{1}_n. \end{aligned}$$

Choose the Lyapunov function candidate

$$V_1 = \frac{1}{2} \varepsilon_1^T H \varepsilon_1.$$

Taking the time derivative of the Lyapunov function gives

$$\begin{aligned} \dot{V}_1 &= (H\varepsilon_1)^T \dot{\varepsilon}_1 \\ &= -\alpha_1 (H\varepsilon_1)^T \text{sig}(H\varepsilon_1)^2 - \beta_1 (H\varepsilon_1)^T \text{sgn}(H\varepsilon_1) - \dot{z}_{10} (H\varepsilon_1)^T \mathbf{1}_n \\ &\leq -\alpha_1 N^{-\frac{1}{2}} \|H\varepsilon_1\|_2^3 - \beta_1 \|H\varepsilon_1\|_1 + \gamma_1 \|H\varepsilon_1\|_1. \end{aligned}$$

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Next, the first and second term in right hand side of above inequality are separately analyzed

$$\begin{aligned}
-\alpha_1 N^{-\frac{1}{2}} \|H\varepsilon_1\|_2^3 &= -\alpha_1 N^{-\frac{1}{2}} (\|H\varepsilon_1\|_2^2)^{\frac{3}{2}} \\
&\leq -\alpha_1 N^{-\frac{1}{2}} (\lambda_{min}^2 \|\varepsilon_1\|_2^2)^{\frac{3}{2}} \\
&= -\alpha_1 N^{-\frac{1}{2}} \left(\frac{2\lambda_{min}^2}{\lambda_{max}}\right)^{\frac{3}{2}} V_1^{\frac{3}{2}}.
\end{aligned} \tag{3.6}$$

Since  $\beta_1 > \gamma_1$ , this yields

$$\begin{aligned}
(\gamma_1 - \beta_1) \|H\varepsilon_1\|_1 &\leq (\gamma_1 - \beta_1) \|H\varepsilon_1\|_2 \\
&= (\gamma_1 - \beta_1) (\|H\varepsilon_1\|_2^2)^{\frac{1}{2}} \\
&\leq (\gamma_1 - \beta_1) (\lambda_{min}^2 \|\varepsilon_1\|_2^2)^{\frac{1}{2}} \\
&\leq (\gamma_1 - \beta_1) \left(\frac{2\lambda_{min}^2}{\lambda_{max}}\right)^{\frac{1}{2}} V_1^{\frac{1}{2}}.
\end{aligned} \tag{3.7}$$

Combining inequality (3.6) with (3.7), it follows that

$$\dot{V}_1 \leq (\gamma_1 - \beta_1) \left(\frac{2\lambda_{min}^2}{\lambda_{max}}\right)^{\frac{1}{2}} V_1^{\frac{1}{2}} - \alpha_1 N^{-\frac{1}{2}} \left(\frac{2\lambda_{min}^2}{\lambda_{max}}\right)^{\frac{3}{2}} V_1^{\frac{3}{2}}. \tag{3.8}$$

Since  $\beta_1 > \gamma_1$  and  $\alpha_1 > 0$ , it can easily conclude that  $\dot{V}_1 < 0$ , which implies that the closed-loop subsystem (2.8) with controller (3.5) for  $1 \leq i \leq N$  is asymptotically stable. Moreover, according to Lemma 1.6 and the form of inequality (3.8), we can obtain  $p = \frac{1}{2}$ ,  $q = \frac{3}{2}$  and  $\mu = 2$ . Hence, it follows from Lemma 1.6 that the consensus error  $\varepsilon_1$  converges to zero in a fixed time, and the upper bound of settling time can be derived by

$$T_{max1} = \frac{\pi}{\frac{2\lambda_{min}^2}{\lambda_{max}} \sqrt{\alpha_1 N^{-\frac{1}{2}} (\beta_1 - \gamma_1)}}.$$

The Proof is completed.

Similar, the second Theorem is given.

**Theorem 3.3** *Consider the subsystem (2.9) under connected topology with the Assumptions 2.3 and 2.7, the distributed fixed-time control law for mobile robot*

$i, (1 \leq i \leq N)$ , is designed as

$$u_{2i} = \alpha_2 \text{sig} \left[ \sum_{j=0}^N a_{ij} (z_{2j} - z_{2i}) \right]^2 + \beta_2 \text{sgn} \left[ \sum_{j=0}^N a_{ij} (z_{2j} - z_{2i}) \right], \quad (3.9)$$

where  $\alpha_2$  and  $\beta_2$  are positive constants, and  $\beta_2 > \gamma_2$ . Then, the subsystem (2.9) of the mobile robot can globally converge in a fixed time, i.e.  $\lim_{t \rightarrow T_{max2}} (z_{2i} - z_{20}) = 0$ . Moreover, the upper bound of settling time is derived as

$$T_{max2} = \frac{\pi}{\frac{2\lambda_{min}^2}{\lambda_{max}} \sqrt{\alpha_2 N^{-\frac{1}{2}} (\beta_2 - \gamma_2)}}.$$

**Proof:** The proof follows the same line with the Theorem 3.2, hence it is omitted here to save space.

**Remark 3.4** According to Theorem 3.2 and Theorem 3.3, for  $1 \leq i \leq N$ , we have proved that the states  $z_{1i}$ ,  $z_{2i}$  and input  $u_{1i}$ , respectively, converge to  $z_{10}$ ,  $z_{20}$  and  $u_{10}$  in a fixed time under the proposed control laws (3.5) and (3.9). In the following Theorem 3.5, the convergence result of  $z_{3i}$  will be analyzed.

**Theorem 3.5** Consider the subsystem (3.4) under connected topology with Assumptions 2.3 and 2.7, then state  $z_{3i}$  can globally converge asymptotically fast to  $z_{30}$  under the distributed fixed-time control laws (3.5) and (3.9), i.e.  $\lim_{t \rightarrow \infty} (z_{3i} - z_{30}) = 0$ .

**Proof:** Let  $\tilde{z}_{3i} = z_{3i} - z_{30}$ . Take the time derivative of  $\tilde{z}_{3i}$ , one has

$$\begin{aligned} \dot{\tilde{z}}_{3i} &= \dot{z}_{3i} - \dot{z}_{30} \\ &= -k_0 |u_{1i}|^{\epsilon+1} \tilde{z}_{3i} + x_2(t), \end{aligned} \quad (3.10)$$

where  $x_2(t) = u_{1i} \tilde{z}_{2i} + (u_{1i} - u_{10}) z_{20} - k_0 (|u_{1i}|^{\epsilon+1} - |u_{10}|^{\epsilon+1}) z_{30}$ . The solution of the differential equation (3.10) is given as follows

$$\begin{aligned} \tilde{z}_{3i}(t) &= e^{\int_0^t -k_0 |u_{1i}|^{\epsilon+1} d\tau} \tilde{z}_{3i}(0) \\ &+ \int_0^t e^{\int_\tau^t -k_0 |u_{1i}|^{\epsilon+1} d\nu} x_2(\tau) d\tau. \end{aligned} \quad (3.11)$$

According to Theorem 3.2,  $\varepsilon_i$  asymptotically converges to zero, and  $u_{1i}$  asymptotically converges to  $u_{10}$ . It then follows the definition of  $x_2(t)$  that  $x_2(t)$  also

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asymptotically converges to zero. Hence, according to the definition of asymptotic stability, for an arbitrary positive value  $\sigma > 0$ , it exists  $o > 0$ , when the  $|x_2(0)| < o$ , it has  $|x_2(t)| < \sigma$ .

From the Assumption 2.8, the  $u_{1i}$  is bounded, and  $u_{1i} = w_i$ ,  $1 \leq i \leq n$ . Hence,  $|u_{1i}|^{\epsilon+1} \leq c_1$ .

The solution of the differential equation (3.10) satisfies the inequality

$$\begin{aligned} \tilde{z}_{3i}(t) &= e^{\int_0^t -k_0|u_{1i}|^{\epsilon+1}d\tau} \tilde{z}_{3i}(0) + \int_0^t e^{\int_\tau^t -k_0|u_{1i}|^{\epsilon+1}d\nu} x_2(\tau) d\tau \\ &\leq e^{-k_0 c_1 t} |\tilde{z}_{3i}(0)| + \int_0^t e^{-k_0 c_1 (t-\tau)} |x_2(\tau)| d\tau \\ &\leq e^{-k_0 c_1 t} |\tilde{z}_{3i}(0)| + e^{-k_0 c_1 t} \int_0^t e^{k_0 c_1 \tau} |x_2(\tau)| d\tau \\ &\leq e^{-k_0 c_1 t} |\tilde{z}_{3i}(0)| + \frac{\sigma k_0 c_1 - \sigma k_0 c_1 e^{-k_0 c_1 t}}{k_0 c_1} \\ &= \sigma + e^{-k_0 c_1 t} (|\tilde{z}_{3i}(0)| - \sigma). \end{aligned}$$

Hence, when  $t \rightarrow +\infty$ ,  $|\tilde{z}_{3i}(t)| \leq \sigma$ . Since  $\sigma$  is a arbitrary small positive value, in terms of the Lyapunov definition of asymptotic stability, the  $\tilde{z}_{3i}(t)$  is asymptotically stable. This proof is completed.

Based on the Theorems 3.2, 3.3 and 3.5, we can conclude the following results.

**Theorem 3.6** *Consider the multi-robot systems in (1.8). Assume the kinematics of reference signal is same, and Assumptions 2.3 and 2.7 are satisfied. Then, the control objectives of the multi-robot system (3.1)-(3.3) can be realized under the distributed fixed-time protocols (3.5) and (3.9).*

**Proof:** Under the Assumptions 2.3 and 2.7, the distributed fixed-time control law (3.5), (3.9) are proposed for the nonholonomic multi-robot systems under fixed topology. The new chain system have achieved states tracking satisfactorily



based on Theorems 3.2, 3.3 and 3.5, i.e.,

$$\begin{aligned}\lim_{t \rightarrow T_{max1}} (z_{1i} - z_{10}) &= 0, \\ \lim_{t \rightarrow T_{max2}} (z_{2i} - z_{20}) &= 0, \\ \lim_{t \rightarrow T_{max1}} (u_{1i} - u_{10}) &= 0, \\ \lim_{t \rightarrow \infty} (z_{3i} - z_{30}) &= 0.\end{aligned}$$

And thus the control objectives (3.1)-(3.3) are realized based on Lemma 2.1. The proof is completed.

### 3.5 Numerical Example

In order to intuitively verify the effectiveness of the present theoretical results, a numerical simulation has been performed based on the control diagram proposed in Chapter 2. Let's consider a group of differential driving robots consisting of three followers and one virtual leader. The communication topology is given by Figure 3.2 with the corresponding matrix  $H$  (3.12). Thus, we can calculate that  $\lambda_{min}(H) = 0.2679$ ,  $\lambda_{max}(H) = 3.7321$ . The graph is obviously connected and each robot only needs local interactions, which implies that the control strategy is distributed. In this simulation case, only robot  $F_1$  can receive the information of virtual leader  $L$ .

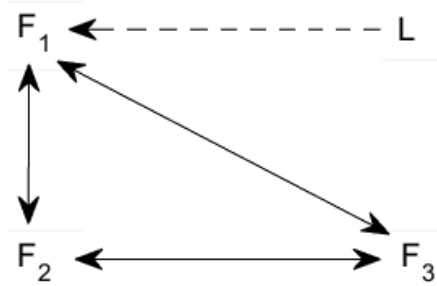


Figure 3.2: Communication topology of three followers and one virtual leader.

$$\mathcal{H} = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \quad (3.12)$$

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The desired formation shape  $\mathcal{F}$  is pre-defined by orthogonal coordinates as  $(p_{x1}, p_{y1}) = (0, 0.2)$ ,  $(p_{x2}, p_{y2}) = (-0.15, -0.1)$ ,  $(p_{x3}, p_{y3}) = (0.15, -0.1)$  shown in Figure 3.3, and the initial states of the robots in Cartesian Frame are randomly chosen. To conduct a monitoring task around one target, the trajectory of reference can be planned as

$$x_0 = \sin(0.05t), y_0 = -\cos(0.05t).$$

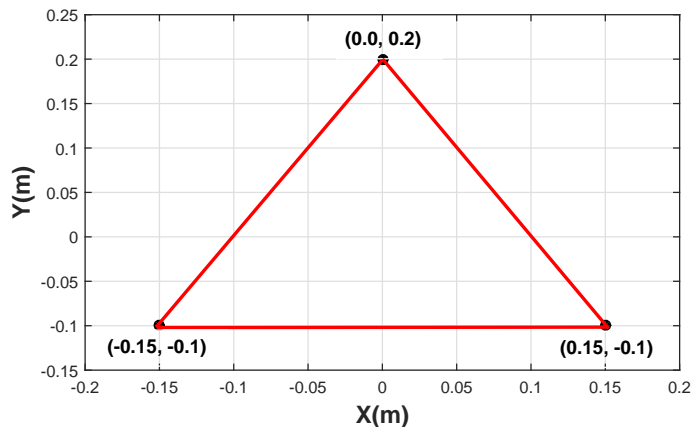


Figure 3.3: Desired shape of formation.

The parameters of the simulation are chosen as follows: based on Theorems 3.2 and 3.3, let  $\epsilon = 0$ ,  $\alpha_1 = \alpha_2 = 23 > 0$ ,  $\beta_1 = \beta_2 = 3.05 > \max\{\gamma_1, \gamma_2\} = \max\{|\dot{z}_{10}|, |\dot{z}_{20}|\} = 0.05$ ; then the upper bound of convergence time can be calculated as  $T_{max1} = 12.9284s$ ,  $T_{max2} = 12.8220s$  based on Theorem 3.2 and Theorem 3.3. The reference states of  $z_{1i}$ ,  $z_{2i}$ ,  $z_{3i}$  can be calculated as  $z_{10} = 0.05t$ ,  $z_{20} = k_0 = 2$ ,  $z_{30} = 1$ , respectively, based on variable transformation and the reference state of virtual leader. We first can see in Figure 3.4 that three non-holonomic robots start from different initial positions in the plane, and gradually form the desired shape. Meanwhile, the centroid of formation satisfactorily tracks the reference trajectory of virtual leader. Note that the formation is not converged at 3s because  $z_{2i}$  and  $z_{3i}$  do not attain the equilibrium although  $z_{1i}$  has converged at less than 2s. Compared with the situation at 3s, when time exceeds 50s which means that all the states  $z_{1i}$ ,  $z_{2i}$ ,  $z_{3i}$  just reach the stable points, it is observed that the desired formation has been created by three robots through local interactions. Afterwards, the formed shape keeps invariant up to 120s which implies

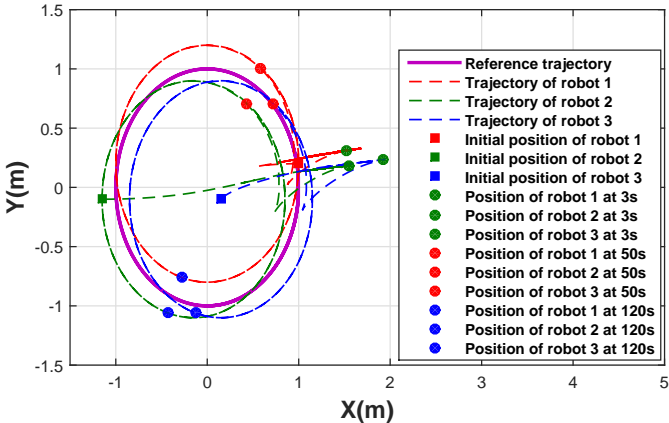


Figure 3.4: Evolution of the three following robots at certain time instants, the purple line is reference trajectory and the dotted lines are trajectories of 3 robots, respectively. The squares denote the initial position for each robot, and the circles are real-time positions

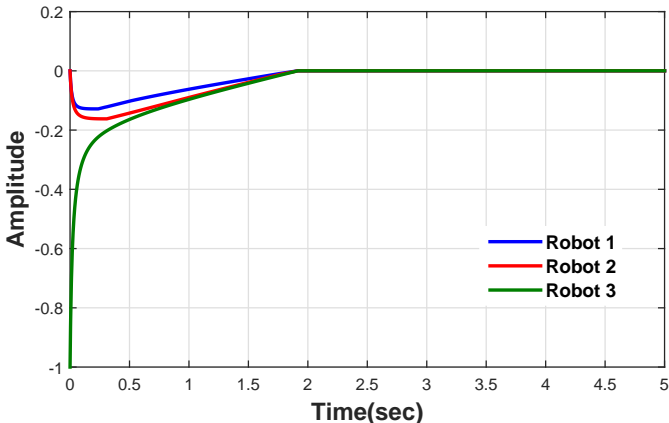


Figure 3.5: State error  $z_{1i} - z_{10}$  under controller ( $i=1-3$ : robot 1-3)

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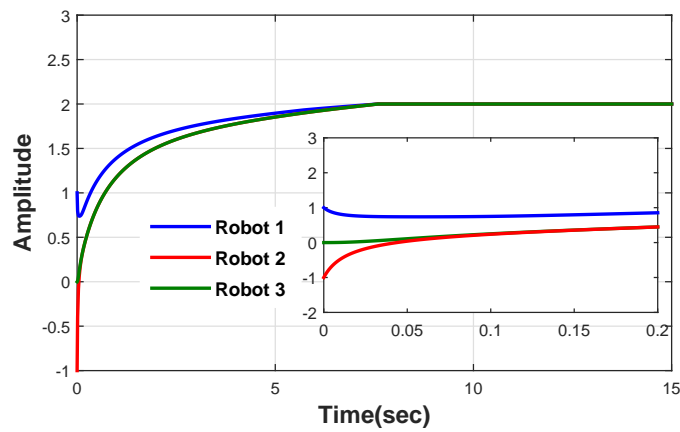


Figure 3.6: State error  $z_{2i} - z_{20}$  under controller ( $i=1-3$ : robot 1-3)

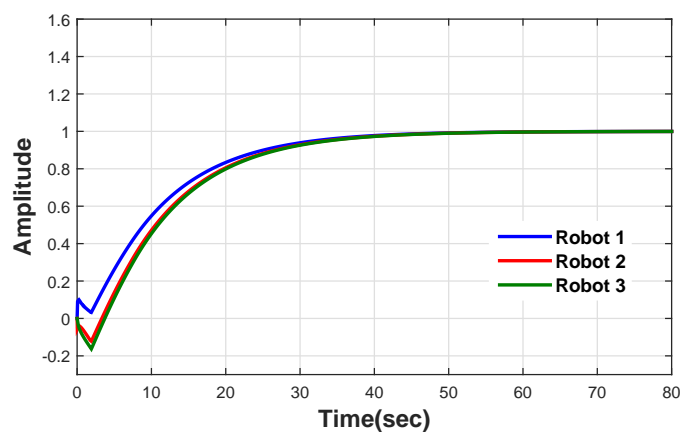


Figure 3.7: State error  $z_{3i} - z_{30}$  under controller ( $i=1-3$ : robot 1-3)

that the proposed controllers are stable. Besides, it can be observed that the convergence time of  $z_1$  and  $z_2$  are 1.9s and 7.5s in Figure 3.5 and Figure 3.6. And both the convergence time is less than  $T_{max1}$  and  $T_{max2}$ , respectively. The differences between the upper bound of estimate time and actual convergence time are influenced by many factors. For example, one of important reasons is the application of Lemma 1.6. Also, it applies many inequalities during the calculation, which further enlarges the time estimate error. In order to be more closed to the upper bound of convergence time, one available way is to choose sufficiently large initial error. Meanwhile, it can be found that the convergence time of  $z_3$  is about 50s in Figure 3.7, which demonstrates its asymptotic convergence rate as theoretical analysis. Overall, the simulation results from Figure 3.4 to Figure 3.7 illustrate that the original formation tracking objectives described by equations (3.1)-(3.3) are all reached.

## 3.6 Conclusions

This chapter presents a novel distributed fixed-time control protocol for the formation tracking problem of multi-robot systems with nonholonomic constraints. Under the proposed control laws, multi-robot systems converge to the desired formation shape, while the centroid of the formation tracks the dynamic reference trajectory. Since the fixed-time stability theory is applied to the development of control laws, we can pre-design off-line a specific upper bound for the convergence time of partial subsystems regardless of global initial conditions. However, it can be observed that the asymptotic convergence speed of state  $z_{3i}$  has affected the whole convergence performance of multi-robot systems, this is a promising direction to pursuit in the future research. Moreover, it is inevitable in reality that robots will be influenced by external disturbances. Meanwhile, when the velocities of robot is sufficiently fast such that the dynamics can not be ignored again. Hence, the exogenous disturbances and robotic dynamics will be further considered in next chapter.

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## Chapter 4

# Robust Fixed-Time Consensus Tracking with Application to Tracking Control of Unicycles Formation

### 4.1 Problem setup

Consider a group of  $N$  followers labeled as  $1, 2, \dots, N$ . Their dynamics are given by equations (4.1).

$$\begin{aligned}\dot{x}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t) + d_i(t), \quad i \in \mathcal{V},\end{aligned}\tag{4.1}$$

where  $x_i(t) \in \mathbb{R}^m$  and  $v_i(t) \in \mathbb{R}^m$  denote the position and velocity of agent  $i$ , respectively. And the  $u_i(t) \in \mathbb{R}^m$  represents the control input. Moreover,  $d_i(t) \in \mathbb{R}^m$  denotes the exogenous matched disturbance. To facilitate the following theoretical analysis, the following mild Assumptions are made.

Furthermore, the dynamics of the leader is given by

$$\dot{x}_0(t) = v_0(t), \quad \dot{v}_0(t) = u_0(t),\tag{4.2}$$

where  $x_0, v_0, u_0 \in \mathbb{R}^m$  denote respectively the position, velocity and control input of the leader. It is assumed that only a subgroup of followers need access to the leader's states. An assumption for the input  $u_0$  is given as follows.

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Here, the distributed fixed-time robust consensus tracking problem is defined as below.

### Definition 4.1 (*Problem Statement*)

For arbitrary initial conditions of followers (4.1) with exogenous matched disturbance, find a distributed control law  $u_i(t)$  only based on local information feedback, such that the positions and velocities of followers can track the leader(4.2)'s in a fixed time as follows

$$\lim_{t \rightarrow T} \|x_i(t) - x_0(t)\|_2 = 0, \quad (4.3)$$

$$\lim_{t \rightarrow T} \|v_i(t) - v_0(t)\|_2 = 0, \quad \forall i \in \mathcal{V}, \quad (4.4)$$

and  $x_i(t) = x_0(t), v_i(t) = v_0(t)$  as long as  $t \geq T$  for all the followers in the presence of disturbances. Furthermore, the upper bound of settling time can be pre-defined.

## 4.2 Literature overview

In recent years, multi-agent systems have attracted the increasing attention of many researchers, due to their widely applications such as flocking control, satellite formation, sensor network Cheng *et al.* (2016); Hu & Shao (2016); Hu *et al.* (2016); Wang & Wu (2012). Consensus tracking as a very important research issue regarding multi-agent systems has been also widely studied Cheng *et al.* (2017); Wang *et al.* (2017); Wen *et al.* (2017a,b); Yu *et al.* (2017a). Consensus tracking means that the states of agents achieve agreement on a reference trajectory via local information exchange under certain protocols. For consensus tracking issues, convergence rate always plays an important role for the performance judgement of proposed protocol. The early work Olfati-Saber & Murray (2004) has established the connection between algebraic connectivity of communication topology and asymptotic convergence rate. However, the high accuracy requirement for the control of convergence time can't be satisfied. In order to improve the convergence characteristics, finite-time stability theory has been proposed in Bhat & Bernstein (2000b), in which the explicit estimate of convergence time for continuous autonomous systems has been given. A continuous finite-time control scheme was considered for rigid robotic manipulators in Yu *et al.* (2005). Furthermore, in Cortés (2006), a discontinuous protocol was developed



for finite-time stability of first-order multi-agent systems by using signum. Work in Wang & Xiao (2010); Xiao *et al.* (2009a); Zhang *et al.* (2013) further enriched the results of finite-time stability for first-order multi-agent systems.

It is well known that all aforementioned results for first-order agents are non-trivial to be expanded to second-order systems straightforwardly Yu *et al.* (2017b). Despite these difficulties, some significant progress still has been made recently. The authors in Zhang & Yang (2013); Zhao *et al.* (2015b) studied the finite-time consensus tracking and containment control problems of multi-agent systems with second-order dynamics by using homogeneity principle. And in Khoo *et al.* (2009), the authors extended the terminal sliding mode technique to address the finite-time consensus problem of the second-order multi-robot systems, while Zhao *et al.* (2014) investigated the distributed finite-time consensus problem of double integrators without velocity measurements. The finite-time consensus tracking of double integrators with bounded control input under fixed and switching jointly reachable digraphes was analyzed in Lu *et al.* (2013).

Note that in the aforementioned work, the settling (or convergence) time function derived by the stability analysis strongly depends on the initial condition of multi-agent systems. Moreover, the required initial condition is usually global information for each agent, that is to say, to estimate the convergence time for each agent, it needs to know all initial state tracking errors of agents. It is well known that in many practical applications, the knowledge of initial tracking errors of multi-agent systems are usually not available in advance. Moreover, when the initial tracking errors of multi-agent systems are extremely large, the convergence rate of the whole systems will be slower than an exponential rate during the rise time. In addition, some results based on homogeneity theory couldn't determine the explicit settling time. To overcome these drawbacks, a new class of stability concept, called fixed-time stability was introduced in Polyakov (2012). In contrast to finite-time stability, the upper bound of the convergence time can be specified in advance, which is independent of initial tracking errors of the whole systems. Then the resulting upper bound can be used as one index to evaluate the proposed controller and determine whether the controller is suitable for the specific applications. Recently, Defoort *et al.* (2015b); Zuo (2015); Zuo & Tie (2016) proposed some fixed-time nonlinear protocols for a network of single integrators. However, to the best of the authors knowledge, there are few work

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to consider fixed-time consensus tracking problem for second-order multi-agent systems [Fu & Wang \(2016\)](#); [Zuo \(2015\)](#).

### 4.3 Contributions

Motivated by above-mentioned discussions, this chapter aims to investigate the robust fixed-time consensus tracking problem of second-order multi-agent systems under fixed topology. The main contributions of this chapter can be stated as follows: Firstly, compared to the work in [Zuo & Tie \(2016\)](#) for single integrators, our work investigates the consensus tracking problem for second-order multi-agent systems. We propose a novel type of fixed-time nonlinear protocol with the aid of fixed-time stability theory and sliding mode technique, in which an effective sliding mode manifold is well constructed. Secondly, the explicit estimation for the upper bound of the convergence time is obtained, which is regardless of initial global information and can be pre-designed off-line. Thirdly, compared to [Zuo \(2015\)](#) in which the singularity is eliminated, this chapter analyzes the disturbance rejection property of control law against exogenous disturbances. Finally, the results obtained for second-order multi-agent systems have been applied to the fixed-time formation tracking problem for a simplified nonholonomic multi-robot dynamical systems, where a corresponding protocol is derived for the multi-robot systems. Compared to [Chu \*et al.\* \(2016, 2017b\)](#); [Defoort \*et al.\* \(2016\)](#) in which only kinematic controllers are considered, in this chapter the dynamic controllers for robots are considered.

### 4.4 Preliminaries

In this section, some Assumptions, which act the basis of the work of this chapter, will be given.

**Assumption 4.2** *There exists at least one follower that can receive information from the leader, i.e., at least one diagonal element of matrix  $\mathcal{B}$  is equal to 1. The communication graph without the leader is undirected and connected.*

**Assumption 4.3** *The exogenous disturbance term  $d_i(t)$  is uniformly bounded by a positive constant, i.e.,  $\|d_i(t)\|_\infty \leq d_{max} \in \mathbb{R}, i \in \{1, \dots, N\}$ .*

**Assumption 4.4** *It is assumed that the control input  $u_0$  of the leader is bounded by a known positive constant, i.e.  $\|u_0\|_\infty \leq \kappa$ .*

## 4.5 Main results

In this section, we will design a distributed fixed-time control law to address the consensus tracking problem of multi-agent systems subject to second-order dynamics (4.1) with an external leader (4.2). It is well known that the linear control laws only can stabilize the closed-loop systems asymptotically or exponentially fast, hence this chapter employs nonlinear continuous function  $\text{sig}(x)^a = \text{sgn}(x)|x|^a$  and non-smooth signum  $\text{sgn}(x)$  to constitute the distributed fixed-time protocol for the system (4.1) as below

$$\begin{aligned}
 u_i = & - \alpha_i \text{sig}\left\{(v_i - z_i) + \text{sig}\left[\sum_{j=0}^N a_{ij}(x_i - x_j)\right]^\phi\right\}^2 \\
 & - \beta_i \text{sgn}\left\{(v_i - z_i) + \text{sig}\left[\sum_{j=0}^N a_{ij}(x_i - x_j)\right]^\phi\right\} \\
 & - \phi \left|\sum_{j=0}^N a_{ij}(x_i - x_j)\right|^{\phi-1}
 \end{aligned} \tag{4.5}$$

with the distributed fixed-time observer given by

$$\begin{aligned}
 \dot{z}_i & = \hat{\alpha}_i \text{sig}\left[\sum_{j=0}^n a_{ij}(z_j - z_i)\right]^2 + \hat{\beta}_i \text{sgn}\left[\sum_{j=0}^n a_{ij}(z_j - z_i)\right], \\
 z_0 & = v_0,
 \end{aligned} \tag{4.6}$$

where  $0 < \phi < 1$ , the constraints of control gains  $\alpha_i, \hat{\alpha}_i, \beta_i, \hat{\beta}_i$  are positive constants and will be derived later. Since all robots cannot obtain the value of  $v_0$  of the leader in real time, they have to estimate it throughout the process. Here, both the followers and the leader are modelled by double integrators, but the observer is of the first-order. In fact, it is desired to construct a one-dimensional reduced-order observer (4.6) rather than second-order observer corresponding to the double integrators. Since it will probably lead to technical difficulties in constructing Lyapunov function for the higher-order system in the following theoretical analysis. Denote by  $z_i$  the observation value, it is shown that  $z_i$  will

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converge to time-varying observed state  $v_0$  which is the velocity of leader. To this end, the distributed observer (4.6) is first investigated, then the following Lemma can be obtained.

Note that the control gains and observer parameters are distributed, which means that each robot can determine its own parameters and performance. It does't need the same gains for the whole agents. Let  $\alpha = \text{diag}(\alpha_1, \dots, \alpha_N) \in \mathbb{R}^{N \times N}$ , and  $\beta = \text{diag}(\beta_1, \dots, \beta_N) \in \mathbb{R}^{N \times N}$ .  $\alpha_{\min} = \min\{\alpha_i | 1 \leq i \leq N\}$ ,  $\beta_{\min} = \min\{\beta_i | 1 \leq i \leq N\}$ . And  $\hat{\alpha} = \text{diag}(\hat{\alpha}_1, \dots, \hat{\alpha}_N) \in \mathbb{R}^{N \times N}$ ,  $\hat{\beta} = \text{diag}(\hat{\beta}_1, \dots, \hat{\beta}_N) \in \mathbb{R}^{N \times N}$ .  $\hat{\alpha}_{\min} = \min\{\hat{\alpha}_i | 1 \leq i \leq N\}$ ,  $\hat{\beta}_{\min} = \min\{\hat{\beta}_i | 1 \leq i \leq N\}$ ,  $1 \leq i \leq N$ .

**Lemma 4.5** *Consider the distributed fixed-time observer (4.6) for the time-varying velocity  $v_0$  of the leader. Then, the observer (4.6) will come to the steady state in a fixed time  $T_1$  for any initial condition, if  $\hat{\alpha}_{\min} > 0$  and  $\hat{\beta}_{\min} > \kappa$  are satisfied. That is,  $z_i = v_0$  when  $t \geq T_1$ , where the upper bound of settling time can be pre-designed based on the flowing equation*

$$T_1 = \frac{\pi \lambda_{\max}}{2\lambda_{\min}^2 \sqrt{\hat{\alpha}_{\min} N^{-0.5} (\hat{\beta}_{\min} - \kappa)}}.$$

**Proof:** Define the observation error as  $\varepsilon = z - 1_N \otimes z_0$ . Let  $\varepsilon = (\varepsilon_1^T, \dots, \varepsilon_N^T)^T$ , and  $z = (z_1^T, \dots, z_N^T)^T$ . Then, taking the time derivative of observation error and rewriting it to compact form yields

$$\begin{aligned} \dot{\varepsilon} &= \dot{z} - 1_N \otimes \dot{v}_0 \\ &= -(\hat{\alpha} \otimes I_m) \text{sig}[(H \otimes I_m)\varepsilon]^2 - (\hat{\beta} \otimes I_m) \text{sgn}[(H \otimes I_m)\varepsilon] \\ &\quad - 1_N \otimes u_0. \end{aligned}$$

Choose a Lyapunov function candidate as

$$V_1 = \frac{1}{2} \varepsilon^T (H \otimes I_m) \varepsilon \tag{4.7}$$

Taking the time derivative of (4.7), one obtains

$$\begin{aligned}
\dot{V}_1 &= [(H \otimes I_m)\varepsilon]^T \dot{\varepsilon} \\
&= -[(H \otimes I_m)\varepsilon]^T (\hat{\alpha} \otimes I_m) \text{sig}[(H \otimes I_m)\varepsilon]^2 \\
&\quad - [(H \otimes I_m)\varepsilon]^T (\hat{\beta} \otimes I_m) \text{sgn}[(H \otimes I_m)\varepsilon] \\
&\quad - [(H \otimes I_m)\varepsilon]^T 1_N \otimes u_0 \\
&\leq -\hat{\alpha}_{min} N^{-\frac{1}{2}} \|(H \otimes I_m)\varepsilon\|_2^3 \\
&\quad - \hat{\beta}_{min} \|(H \otimes I_m)\varepsilon\|_1 \\
&\quad + \kappa \|(H \otimes I_m)\varepsilon\|_1 \\
&\leq -\hat{\alpha}_{min} N^{-\frac{1}{2}} (\|(H \otimes I_m)\varepsilon\|_2^2)^{\frac{3}{2}} \\
&\quad - (\hat{\beta}_{min} - \kappa) \|(H \otimes I_m)\varepsilon\|_2 \\
&\leq -\hat{\alpha}_{min} N^{-\frac{1}{2}} \left(\frac{2\lambda_{min}^2}{\lambda_{max}}\right)^{\frac{3}{2}} V_1^{\frac{3}{2}} \\
&\quad - (\hat{\beta}_{min} - \kappa) (\lambda_{min}^2 \|(H \otimes I_m)\varepsilon\|_2^2)^{\frac{1}{2}} \\
&\leq -\hat{\alpha}_{min} N^{-\frac{1}{2}} \left(\frac{2\lambda_{min}^2}{\lambda_{max}}\right)^{\frac{3}{2}} V_1^{\frac{3}{2}} \\
&\quad - (\hat{\beta}_{min} - \kappa) \left(\frac{2\lambda_{min}^2}{\lambda_{max}}\right)^{\frac{1}{2}} V_1^{\frac{1}{2}}.
\end{aligned}$$

Based on Lemma 1.6,  $p = \frac{1}{2}$ ,  $q = \frac{3}{2}$ ,  $\mu$  is equal to 2. The observation error  $\varepsilon$  will converge to the origin in a fixed time. Furthermore, the settling time can be estimated by

$$T_1 = \frac{\pi \lambda_{max}}{2\lambda_{min}^2 \sqrt{\hat{\alpha}_{min} N^{-0.5} (\hat{\beta}_{min} - \kappa)}}.$$

The proof is completed.

**Remark 4.6** *Note that the observers proposed here have a fixed-time fast convergence rate, and the convergence time is upper bounded that means explicit time control is allowed by engineers. Actually, the employed observer essentially belongs to sliding mode observer with mild disturbances rejection capability.*

**Remark 4.7** *It is observed that the implementation of the proposed distributed controller does not require information on the velocities of the neighbours, which is extremely difficult to obtain the accurate measurements in practice. For each*

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follower, the strategy depends only on the relative-position and its own velocity information. Only a few of followers have to receive the velocity information from the leader for the distributed observers.

Define the tracking errors as  $e_{xi} = x_i - x_0$ ,  $e_{vi} = v_i - v_0$ , and differentiate them to obtain

$$\begin{aligned}\dot{e}_{xi} &= \dot{x}_i - \dot{x}_0 = v_i - v_0 = e_{vi}, \\ \dot{e}_{vi} &= \dot{v}_i - \dot{v}_0 = u_i + d_i - u_0.\end{aligned}$$

Under the consensus tracking protocol (4.5), and according to the definition of matrix  $H$ , after  $t \geq T_1$ , the closed-loop error system can be written of the following form

$$\begin{aligned}\dot{e}_v = & - (\alpha \otimes I_m) \text{sig}\{e_v + \text{sig}[(H \otimes I_m)e_x]^\phi\}^2 \\ & - (\beta \otimes I_m) \text{sgn}\{e_v + \text{sig}[(H \otimes I_m)e_x]^\phi\} \\ & - \phi |(H \otimes I_m)e_x|^{\phi-1} + d - 1_N \otimes u_0,\end{aligned}\quad (4.8)$$

where  $e_x = (e_{x1}^T, e_{x2}^T, \dots, e_{xN}^T)^T$ ,  $e_v = (e_{v1}^T, e_{v2}^T, \dots, e_{vN}^T)^T$  and  $d = (d_1^T, d_2^T, \dots, d_N^T)^T$ . Next, the sliding mode technique is employed to analyze the stability of the closed-loop error system (4.8). The main lemma of this chapter is addressed as follows.

**Lemma 4.8** *Consider the sliding mode manifold  $S = e_v + \text{sig}[(H \otimes I_m)e_x]^\phi$ ,  $0 < \phi < 1$ . If  $\beta_{min} - d_{max} - \kappa > 0$ ,  $\alpha_{min} > 0$ , then the tracking errors  $e_x$  and  $e_v$  will reach the sliding mode surface in a fixed time  $T_2$  under the local control input (4.5). The settling time can be pre-designed based on the following formula*

$$T_2 = \frac{\pi}{2\sqrt{\alpha_{min}N^{-0.5}(\beta_{min} - d_{max} - \kappa)}}.$$

**Proof :** Substituting the sliding mode manifold  $S$  into system (4.8), one has

$$\begin{aligned}\dot{e}_v = & - (\alpha \otimes I_m) \text{sig}(S)^2 - (\beta \otimes I_m) \text{sgn}(S) \\ & - \phi |(H \otimes I_m)e_x|^{\phi-1} + d - 1_N \otimes u_0,\end{aligned}\quad (4.9)$$

Take the time derivative of  $S$  with respect to velocity error system (4.9) as

$$\dot{S} = -(\alpha \otimes I_m) \text{sig}(S)^2 - (\beta \otimes I_m) \text{sgn}(S) + d - 1_N \otimes u_0.\quad (4.10)$$

Based on the above discussion, we can prove that the position error  $e_x$  and velocity error  $e_v$  will converge to the sliding mode surface  $S$  in a fixed time and after stay there. Here, choose the Lyapunov function candidate  $V_2$  as follows

$$V_2 = \frac{1}{2} S^T S.$$

Taking the time derivative of  $V_2$  yields

$$\begin{aligned} \dot{V}_2 &= S^T \dot{S} \\ &= -\alpha_{min} S^T sig(S)^2 - \beta_{min} S^T sgn(S) + S^T d - S^T \mathbf{1}_N \otimes u_0 \\ &\leq -\alpha_{min} N^{-\frac{1}{2}} \|S\|_2^3 - (\beta_{min} - d_{max} - \kappa) \|S\|_1. \end{aligned}$$

Force that  $\beta_{min} - d_{max} - \kappa > 0$ , namely,  $\beta_{min} > d_{max} + \kappa$ . Here,  $\|d_i(t)\|_\infty \leq d_{max} \in \mathbb{R}, i \in \{1, \dots, N\}$  and  $\|u_0\|_\infty \leq \kappa$  are used. Due to  $\beta_{min} - d_{max} - \kappa > 0$ , the following inequality can be obtained based on the fact that  $\|S\|_2 < \|S\|_1$

$$\dot{V}_2 \leq -\alpha_{min} N^{-\frac{1}{2}} \|S\|_2^3 - (\beta_{min} - d_{max} - \kappa) \|S\|_2.$$

Additionally, because of the fact  $\|S\|_2 = \sqrt{2V_2}$ , there holds

$$\begin{aligned} \dot{V}_2 &\leq -\alpha_{min} N^{-\frac{1}{2}} (\|S\|_2^2)^{\frac{3}{2}} - (\beta_{min} - d_{max} - \kappa) \sqrt{2V_2} \\ &= -\alpha_{min} N^{-\frac{1}{2}} \sqrt{8} V_2^{\frac{3}{2}} - (\beta_{min} - d_{max} - \kappa) \sqrt{2} V_2^{\frac{1}{2}}. \end{aligned}$$

Finally, based on Lemma 1.6, the explicit estimate for upper bound of convergence time for system (4.10) can be obtained as follows

$$T_2 = \frac{\pi}{2\sqrt{\alpha_{min} N^{-0.5} (\beta_{min} - d_{max} - \kappa)}}.$$

That is to say, the position and velocity tracking errors  $e_x, e_v$  can reach the sliding mode surface  $S$  in a fixed time  $T_2$ . Afterwards,  $S = 0$  can be always held. The proof is completed.

**Remark 4.9** According to Lemmas 4.5 and 4.8, the observer will firstly attain equilibrium as long as  $t \geq T_1$ . Afterwards, the trajectories of errors  $e_x$  and  $e_v$  are proved to reach the well-constructed sliding mode surface  $S$  after  $t \geq T_1 + T_2$ . Note that either  $T_1$  or  $T_2$  is not a function of the initial conditions, which means that the present observer-based controller can guarantee closed-loop agents systems

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to globally converge in a fixed time. Namely, large initial errors can not result in lengthy convergence time. This point is a critical advantage for the control technique proposed in this Chapter.

In the next Lemma, it will be shown that the errors  $e_x$  and  $e_v$  will reach the origin in a finite time.

**Lemma 4.10** *After the tracking errors reach the sliding mode manifold considered in Lemma 4.8, the tracking errors  $e_x$  and  $e_v$  will slide along the surface to the origin in a finite time  $T_3$  which can be explicitly estimated by*

$$T_3 = 2 \frac{\sqrt{0.5e_x(T_1 + T_2)^T H e_x(T_1 + T_2)}}{(1 - \phi)[\lambda_{\min} \sqrt{2/\lambda_{\max}}]^{\phi+1}}.$$

**Proof :** Since  $e_x, e_v$  attain the sliding mode surface so as to  $S = 0$ , which leads to  $e_v = -\text{sig}[(H \otimes I_m)e_x]^\phi$ . In order to reveal  $e_x, e_v$  will slide to the origin in a finite time, choose the Lyapunov function candidate as

$$V_3 = \frac{1}{2} e_x^T (H \otimes I_m) e_x. \quad (4.11)$$

Differentiating it with respect to time yields

$$\begin{aligned} \dot{V}_3 = e_x^T (H \otimes I_m) e_v &= -[(H \otimes I_m)e_x]^T \text{sig}[(H \otimes I_m)e_x]^\phi \\ &\leq -\|(H \otimes I_m)e_x\|_2^{\phi+1} \\ &= -\sqrt{e_x^T (H \otimes I_m)^2 e_x}^{\phi+1} \\ &\leq -[\lambda_{\min} \sqrt{2/\lambda_{\max}}]^{\phi+1} V_3^{\frac{\phi+1}{2}}. \end{aligned} \quad (4.12)$$

Since  $[\lambda_{\min} \sqrt{2/\lambda_{\max}}] > 0$  and  $0 < \phi < 1$ , then based on Lemma 1.5, tracking errors  $e_x$  and  $e_v$  will converge to zero in a finite time with the estimated convergence time  $T_3$  as

$$T_3 = 2 \frac{\sqrt{0.5e_x(T_1 + T_2)^T (H \otimes I_m) e_x(T_1 + T_2)}}{(1 - \phi)[\lambda_{\min} \sqrt{2/\lambda_{\max}}]^{\phi+1}}.$$

To here, the proof is completed.

**Remark 4.11** *Since the initial time  $T_1 + T_2$  of the error item  $e_x$  in the formula is regardless of the initial condition based on fixed-time stability, that is to say, the convergence time of the entire multi-agent systems is also regardless of the initial condition, on which the finite-time controller strongly depends.*



**Theorem 4.12** Consider the second-order multi-agent systems (4.1) with the leader (4.2) under fixed topology. If Assumptions 4.2, 4.3 and 4.4 are satisfied, and  $\beta_{\min} - d_{\max} - \kappa > 0$ , then using the distributed protocol (4.5) with an observer (4.6), for any initial condition, the fixed-time consensus tracking can be achieved when  $t \geq T$ , where the total upper bound of settling time can be derived as  $T = T_1 + T_2 + T_3$ .

**Proof :** The proof can be obtained directly combining Lemmas 4.5, 4.8 and 4.10. Specifically, in the process of addressing the fixed-time consensus tracking problem in this Chapter, a distributed fixed-time observer has been first developed. Afterwards, by using a sliding mode technique, a novel type of distributed fixed-time nonlinear protocol is proposed, and a new sliding mode manifold is constructed. Based on sliding mode control mechanism, the sliding surface  $S = 0$  will be reached from anywhere in the phase plane in a fixed time  $T_2$  after the distributed observer of each agent first comes to a steady state in a fixed time  $T_1$  following the Lemmas 4.5 and 4.8. Once the sliding surface  $S = 0$  is reached, the errors  $e_x, e_v$  will reach the origin in a finite time  $T_3$  following the Lemma 4.10. The proof is completed.

## 4.6 Applications

### 4.6.1 Formation tracking of unicycles with dynamics

When angular and linear velocities are sufficiently fast in practice, a kinematics based controller can not satisfy the perfect tracking requirement any more, which calls for the introduction of a dynamical model. In this section, a simplified dynamical model of the unicycle-type robot applied in Lawton *et al.* (2003) is recalled in the state space form as system (1.9) and linear feedback technique is applied to transform system (1.9) into the following double integrator (4.16). For the details, see Lawton *et al.* (2003). Consider the equations of motion

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \\ \dot{v}_i \\ \dot{\omega}_i \end{bmatrix} = \begin{bmatrix} v_i \cos \theta_i \\ v_i \sin \theta_i \\ \omega_i \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1/m_i & 0 \\ 0 & 1/J_i \end{bmatrix} \begin{bmatrix} F_i \\ \tau_i \end{bmatrix},$$

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where  $\mathbf{p}_i = [x_i, y_i]^T$  represents the inertial position of the nonholonomic mobile robot  $i$ ,  $\theta_i$  is the orientation,  $\nu_i$  and  $\omega_i$  denote the linear velocity and angular velocity, respectively.  $\tau_i$  is the torque generated by the differential wheels.  $F_i$  refers to the force,  $m_i$  and  $J_i$  are the mass and moment of inertia of mobile robot  $i$ .

Define the point  $\mathbf{h}_i = [h_{xi}, h_{yi}]^T \in \mathbb{R}^2$  which is located in the line that is perpendicular to the wheel axis and holds a distance  $L_i$  to the intersects  $\mathbf{p}_i$ ,

$$\mathbf{h}_i = \begin{bmatrix} h_{xi} \\ h_{yi} \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} + L_i \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix}. \quad (4.13)$$

Then, take the second time derivative of (4.13) as

$$\begin{aligned} \ddot{\mathbf{h}}_i &= \begin{bmatrix} -\nu_i\omega_i \sin(\theta_i) - L_i\omega_i^2 \cos(\theta_i) \\ \nu_i\omega_i \cos(\theta_i) - L_i\omega_i^2 \sin(\theta_i) \end{bmatrix} \\ &+ \begin{bmatrix} \cos \theta_i/m_i & -L_i \sin \theta_i/J_i \\ \sin \theta_i/m_i & L_i \cos \theta_i/J_i \end{bmatrix} \begin{bmatrix} F_i \\ \tau_i \end{bmatrix}. \end{aligned} \quad (4.14)$$

By means of output feedback linearization techniques, the following control inputs  $\mathbf{u}_i = [F_i, \tau_i]^T$  are proposed

$$\begin{aligned} \mathbf{u}_i &= \begin{bmatrix} \cos \theta_i/m_i & -L_i \sin \theta_i/J_i \\ \sin \theta_i/m_i & L_i \cos \theta_i/J_i \end{bmatrix}^{-1} \\ &* \begin{bmatrix} \mu_{xi} + \nu_i\omega_i \sin \theta_i + L_i\omega_i^2 \cos \theta_i \\ \mu_{yi} - \nu_i\omega_i \cos \theta_i + L_i\omega_i^2 \sin \theta_i \end{bmatrix}. \end{aligned} \quad (4.15)$$

Thus, (4.14) can be transformed into the following second-order multi-agent systems with one exogenous disturbance item.

$$\ddot{\mathbf{h}}_i = \boldsymbol{\mu}_i, \quad (4.16)$$

where  $\boldsymbol{\mu}_i = [\mu_{xi}, \mu_{yi}]^T \in \mathbb{R}^2$ .

Note that the above system is consistent with (4.1). In order to achieve fixed-time formation tracking of nonholonomic multi-robot dynamic systems, we can

apply the variant of fixed-time protocol (4.5) as follows

$$\begin{aligned}
\mu_i = & - \alpha_i \text{sig}\{(\nu_i - \dot{h}_i^*) + \text{sig}[\sum_{j=1}^N a_{ij}((h_i - h_i^*) \\
& - (h_j - h_j^*))]^\phi\}^2 - \beta_i \text{sgn}\{(\nu_i - \dot{h}_i^*) \\
& + \text{sig}[\sum_{j=1}^N a_{ij}((h_i - h_i^*) - (h_j - h_j^*))]^\phi\} \\
& - \phi |\sum_{j=1}^N a_{ij}((h_i - h_i^*) - (h_j - h_j^*))|^{\phi-1}
\end{aligned} \tag{4.17}$$

where  $h_i^* = [h_{x_i}^*, h_{y_i}^*]^T$  specifies the position of manipulator on the robotic platform. Note that  $h_i - h_i^*$ ,  $\nu_i$  and  $\dot{h}_i^*$  play the roles of  $x_i$ ,  $v_i$ ,  $z_i$  in the control law (4.5). Then  $h_i - h_i^* \rightarrow 0$ ,  $\nu_i - \dot{h}_i^* \rightarrow 0$  in a fixed time if the conditions in Theorem 4.12 are satisfied.

**Remark 4.13** *In order to obtain  $h_i^*$ , the first step is to employ the distributed fixed-time observer (4.6) such that the position  $h_0^*$  of the manipulator of virtual leader can be observed by each robot in a fixed time. Afterwards,  $h_i^*$  can be calculated combining the desired relative position of robot  $i$  with the  $h_0^*$ .*

## 4.6.2 From theory to practice

In this subsection, we will provide a control diagram to show how to apply the proposed theoretical results to the real multi-robot systems. In Figure.4.1,  $j \in \mathcal{N}_i$ , the closed-loop control flow is as follows:

- (1) Robot  $i$  obtains its real-time position and orientation in the plane through sensing measurements;
- (2) After variables transformation, the signals in (1) and desired relative position of robot  $i$  are converted into new state variables;
- (3) The new state variables of robot  $i$  and  $j$  will be sent to the distributed observer-based controller;
- (4) After the variables inverse transformation, the control inputs are converted into the force and torque and also further converted into the voltage commands of right and left wheels;
- (5) The speed commands will be sent to the motors and drive the motion of robot  $i$ . Return to the step (1).

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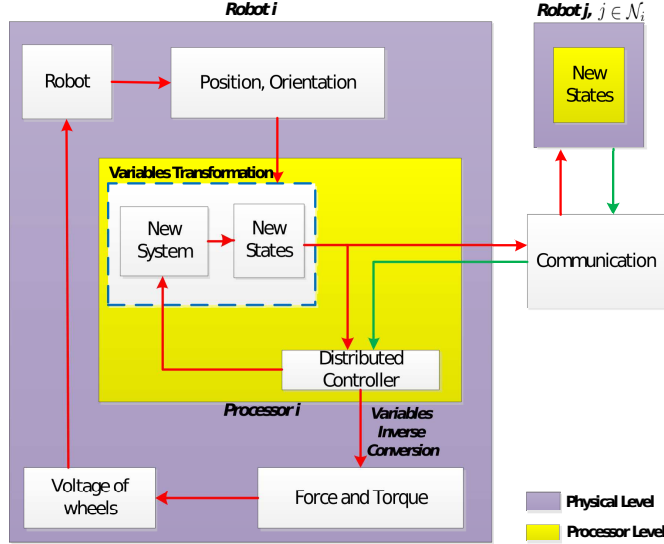


Figure 4.1: The control diagram.

## 4.7 Simulations

### 4.7.1 Example 1

In this subsection, consider a group of multi-agent systems consisting of one leader and three followers, whose communication topology is shown in Figure.4.2. Suppose the state of leader is  $x_0 = \sin(0.5t)$ , thus implying  $u_0 = 0.25 \sin(0.5t)$ . Hence, let  $\kappa = 0.25 \geq |u_0|$ . Set the initial states of the followers as  $x(0) = [x_1(0), x_2(0), x_3(0)]^T = [3, -1, 5]^T$  and  $v(0) = [v_1(0), v_2(0), v_3(0)]^T = 0 * 1_n$ . The initial condition of the observers is set to zero. In order to verify the robustness of our algorithm, let the exogenous disturbances be  $d_1 = 1.3 \sin(2t)$ ,  $d_2 = 0.9 \cos(t)$ ,  $d_3 = 1.8 \sin(2t)$ . For simplify, based on Lemmas 4.5, 4.8 and 4.10, choose the same control gains and design parameters for each agent as  $\alpha = 10$ ,  $\hat{\alpha} = 23.22$ ,  $\beta = 3 > d_{max} + \kappa = 2.05$ ,  $\hat{\beta} = 26.77 > \kappa = 0.25$ ,  $\phi = 0.6$  for  $1 \leq i \leq 3$ , that is to say, the condition of Lemmas are satisfied. Furthermore,  $\lambda_{min} = 0.2679$ ,  $\lambda_{max} = 3.7321$  can be obtained from the communication topology in Figure.4.2. Hence, the upper bound of settling times for each stage can be computed as  $T_1 = 4.3320s$ ,  $T_2 = 0.6707s$ ,  $T_3 = 11.7358s$ . Thus, we can obtain the total upper bound of convergence time  $T = 16.7385s$  which is the sum of  $T_1, T_2, T_3$  based on Theorem 4.12. The numerical results in Figures.4.3 and 4.4

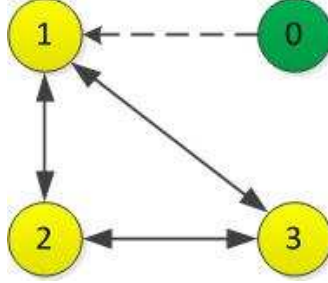


Figure 4.2: Communication topology of multi-agent systems in Example 1.

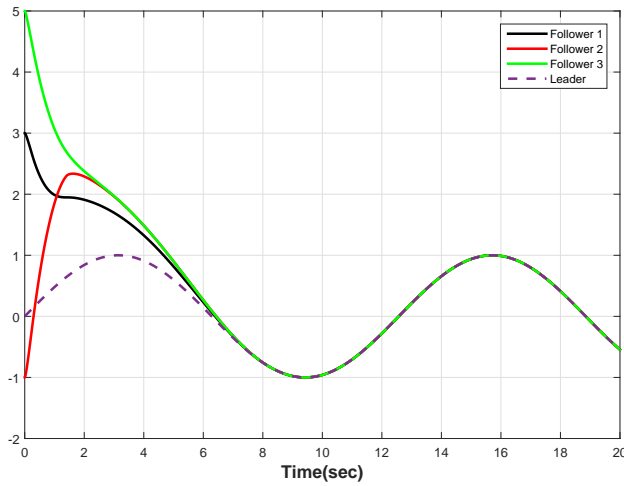


Figure 4.3: Consensus tracking results for position  $x$ .

demonstrate the consensus tracking for positions and velocities of multi-agent systems, respectively. It is observed that consensus tracking problem has been solved by the fixed-time protocol (4.5) and observer (4.6). Note that the settling time in the simulation is about  $10s < T = 16.7385s$ , which illustrates the conservatism of the estimate for the convergence time. In order to further approach the upper bound of convergence time, one can choose sufficiently large initial errors for agents, by which the convergence time in simulation can be more close to the specific upper bound.

### 4.7.2 Example 2

In this subsection, the theoretical results about fixed-time formation tracking of multi-robot dynamic systems with exogenous disturbances will be illustrated.

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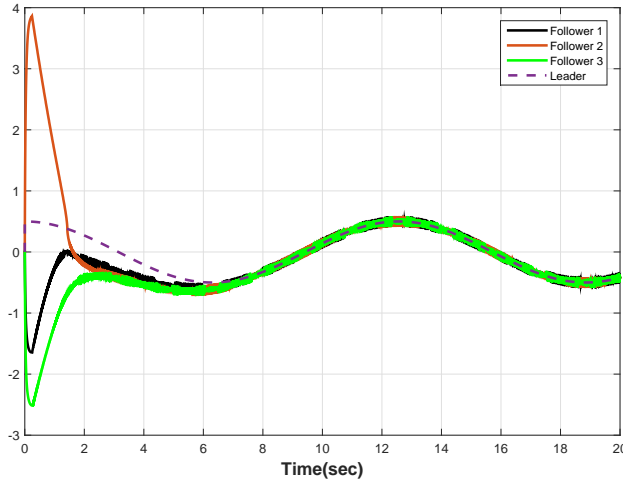


Figure 4.4: Consensus tracking results for velocity  $v$ .

Suppose the mechanical parameters of three robots are identical, i.e.,  $L = 0.12m$ ,  $m = 10.1kg$ ,  $J = 0.13kg \cdot m^2$ . Without loss of generality, choose the same related gains of formation control law (4.17) as Example 1. The desired formation displacements for each robot are defined as  $[0, -\sqrt{3}/15, -2\sqrt{3}/15, 0, 2\sqrt{3}/15, \sqrt{3}/15]$  and  $[0.2, 0, -0.2, -0.2, -0.2, 0]$ . Note that the displacement can be designed off-line or generated on-line to adapt to the uncertain environment. The reference trajectory generated by the virtual leader is given by  $c^*(t) = (c_x^*(t), c_y^*(t)) = (\sin(0.5t), 0.5t)$ . The communication topology is given as Figure.4.5 and the evolution of formation configuration can be found in Figure.4.6, in which the squares with different colors denote the initial positions of six robots, and the circles denote the actual positions update in certain instants, and the dotted curve represents the actual motion trajectories of robots. From Figure.4.6, it is observed that the multi-robot systems forms a rectangle formation shape, and tracks the reference trajectory generated by the virtual leader. Figures.4.7, 4.8 and 4.9 demonstrates the linear velocities, angular velocities and heading angles of six robots. From Figures.4.7 and 4.8, the linear velocities and angular velocities of robots converge to the common values. From Figure.4.9, it is observed that the heading angles among robots converge to common value or hold a  $2\pi$  difference, that is to say, all robots reach to the same orientation. Hence, based on the above simulation, the theoretical results obtained in this chapter have been verified.

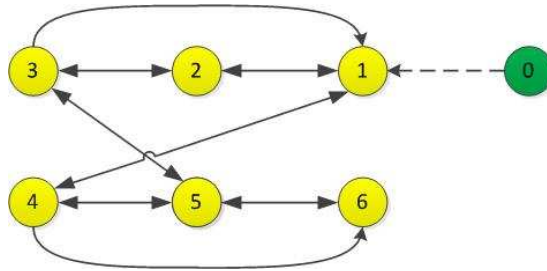


Figure 4.5: Communication topology of multi-robot systems in Example 2.

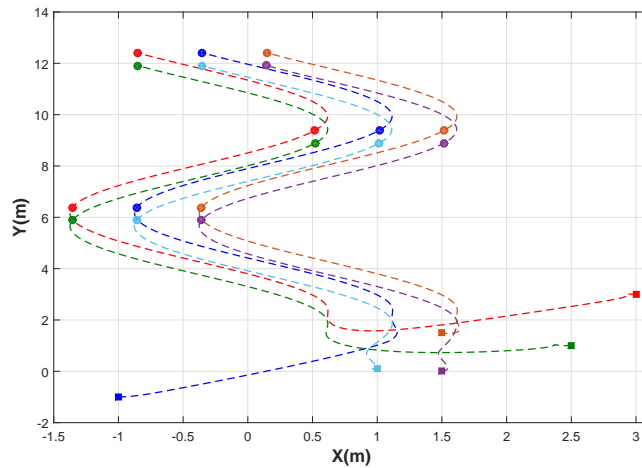


Figure 4.6: The evolution of formation configuration (different colors represent various robots, squares denote their initial positions, circles denote their real-time positions, curves refer to their actual motion trajectories).

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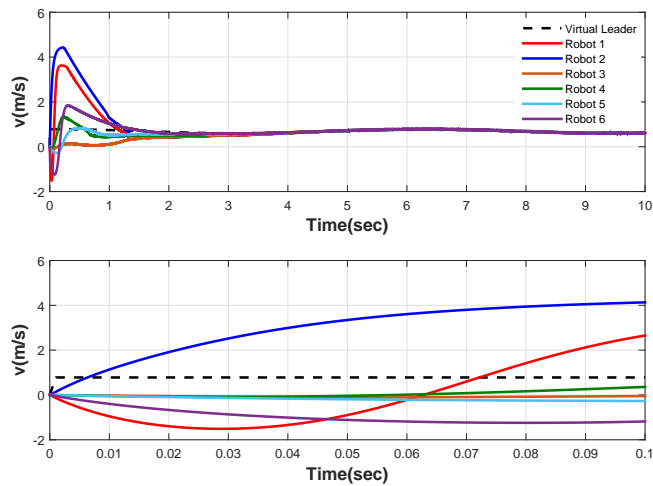


Figure 4.7: Linear velocities of robots (the bottom graph is the snapshot of top graph in time period  $[0, 0.1]$ ).

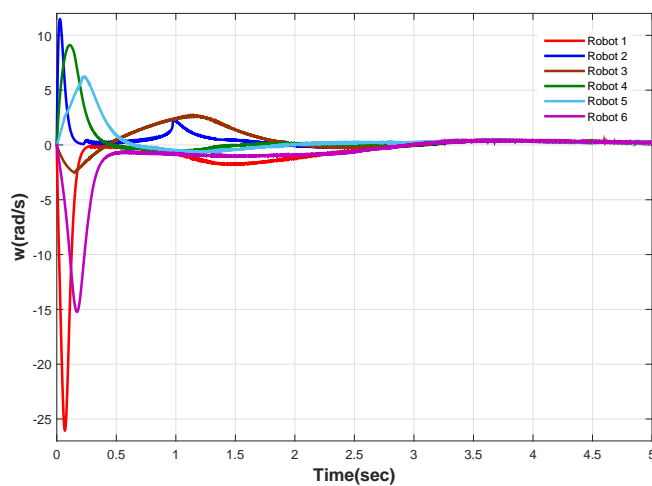


Figure 4.8: Angular velocities of robots.



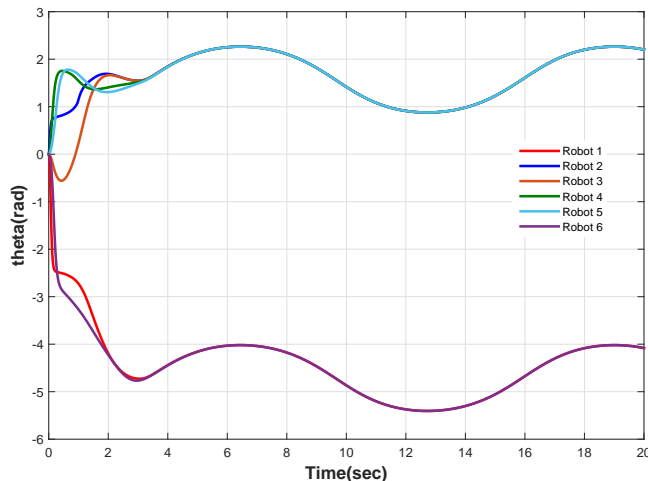


Figure 4.9: Heading angles of robots.

## 4.8 Conclusions

In this chapter, the robust fixed-time consensus tracking problem was solved for second-order multi-agent systems under a fixed communication topology. A novel type of distributed nonlinear protocol was developed to drive each follower to track the dynamic leader via local information exchange in a fixed time. Meanwhile, a distributed fixed-time observer for the velocity of leader was designed, and the sliding mode manifold was well constructed so as to govern the tracking errors reach the sliding mode surface in a fixed time. It is confirmed that the tracking errors can slide along the surface to reach the origin in a finite time based on the theoretical analysis. In particular, the convergence time of the closed-loop multi-agent systems can be designed a priori by properly choosing the control parameters, due to the fact that upper bound of convergence time is independent of the initial global information and hence bounded based on fixed-time stability theory. Furthermore, the proposed protocol has been extended to deal with fixed-time formation tracking problem of nonholonomic multi-robot dynamical systems. Future studies will concentrate on extensions to directed fixed and switching topologies, communication time-delays.

Nevertheless, the work in this Part assume continuous communication and control update which are a little bit conservative in reality, and sampling of states for each robot is not continuous in practice, certain sampling period will

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be imposed, which means that the multi-robot formation systems is actually a class of hybrid systems. In the next Part of the thesis, we will investigate this new class of systems by using event-triggered method and sampled-data control, and quantify the relation and constraints among sampling period, parameters of event conditions and control gains.

## Part II

# Multi-Robot Formation Systems: Event-Triggered Communication and Control



## Chapter 5

# Distributed Event-Triggered Tracking Control of Multi-Robot Formation Systems with Nonholonomic Constraint

### 5.1 Problem setup

This chapter aims to apply the event-triggered mechanism for solving the tracking control problem of nonholonomic multi-robot formation systems. The time instant of information exchange and control update is determined by certain event conditions so as to guarantee the stability of closed-loop systems. In other words, only when some specific event conditions are violated, the communication and/or control actions are triggered, yielding so-called event-triggered systems. This mechanism is quite different from the time-triggered approaches using in Chapters 2-4. We consider in detail two aspects of event-triggered method for the multi-robot formation systems, that is, communication triggering and control update triggering mechanisms.

Recall the kinematics of multi-robot systems (1.8) and virtual leader (2.4). The aim of this Chapter is to study the piecewise continuous control input  $v_i$  and  $\omega_i$  and intermittent communication mechanism for the robot  $i$ , of which the occurring of control update and information exchange strictly depend on whether or not the associated event condition is violated, such that the following control

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objectives

$$\lim_{t \rightarrow \infty} \begin{bmatrix} x_i - p_{xi} \\ y_i - p_{yi} \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}, \quad (5.1)$$

$$\lim_{t \rightarrow \infty} \left( \sum_{i=1}^N \frac{x_i}{N} - x_0 \right) = 0, \quad \lim_{t \rightarrow \infty} \left( \sum_{i=1}^N \frac{y_i}{N} - y_0 \right) = 0, \quad (5.2)$$

$$\lim_{t \rightarrow \infty} (\theta_i - \theta_0) = 0 \quad (5.3)$$

can be reached.

**Remark 5.1** *In our design, the first step is to determine the relative position  $(p_{xi}, p_{yi})$  which depend on the types of tasks and requirements of some specific applications. It is easy to observe that shape, orientation and scale of a robotic formation can be changed flexibly by modifying two parameters  $(p_{xi}, p_{yi})$  of each robot. In practice, the geometric configuration of formation often needs to match the specific requirement of different tasks, such as executing surveillance, moving huge components, traversing an area filled with obstacles, etc.*

The rough comparison on communication mechanism between time-triggered and event-triggered controllers is demonstrated as Figure.5.1. Note that the communication situation doesn't represent the actual status of the implementing of the controller proposed in this Chapter, but provides only a graphical representation about the communication mode among neighboring robots.

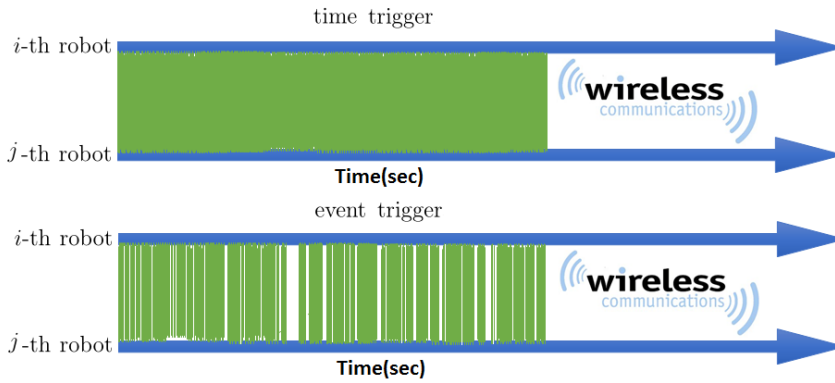


Figure 5.1: Rough comparison of time-triggered communication and event-triggered communication mechanism.

## 5.2 Literature overview

Over the past 20 years, the topic of cooperative control of nonholonomic multi-robot systems has attracted significant attention from the academic community to the industry area, due to its wide applications in both civilian and military areas, such as intelligent transportation, material handling, mine clearance, surveillance, military deterrence, construction, multi-sensor/actuator networks, and so forth [Rubenstein \(2014\)](#); [Werfel \*et al.\* \(2014\)](#). In general, the subject of cooperative control mainly includes consensus, rendezvous, formation control [Cepeda-Gomez & Olgac \(2016\)](#); [Liu & Tian \(2009\)](#); [Luo \*et al.\* \(2011\)](#), flocking/swarming and circumnavigation, etc. Among these themes, formation control plays a core role in numerous robotic applications. In order to achieve formation control task, there already exist a variety of methods such as leader-follower approach, behavior-based approach, artificial potential approach, virtual structure approach, consensus-based approach [Balch & Arkin \(1998\)](#); [Lewis & Tan \(1997b\)](#); [Peng \*et al.\* \(2013b\)](#). In general, current control frameworks can be categorized roughly as two categories: centralized and decentralized, from the point of view of communication and computation. It is worth noting that most of the above control methods belong to the first category. However, different from centralized strategies, the distributed control approach has inherent superiority with higher robustness, flexibility, maintainability and economical efficiency. It is specially suitable for the control task of large amount of robot networks under limitative communication costs and computing capabilities. As aforementioned, the consensus-based formation control method with decentralized nature is a promising direction for the multiple robots control in the future along with the continuous advances of consensus theory [Li \*et al.\* \(2010\)](#); [Wen \*et al.\* \(2016\)](#); [Zhao \*et al.\* \(2016a,b\)](#).

Except for above aspects, communication is also a key factor in the coordination behavior. Along with increasing quantity of robots, the burden of information channel with limited communication bandwidth and rate will become an extremely important problem. Current consensus-based formation control methods [Dong \(2012\)](#); [Peng \*et al.\* \(2015, 2016\)](#) for multi-robot systems always require continuous state feedback. In order to reduce the communication frequency and increase the efficiency of networks, the event-triggered paradigm has been revived

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in the control community over the past decade. In this way, the energy consumption of sensors will be reduced and the average updating period of the actuators will be larger. Thus, it might decrease mechanical wear, and improve the overall performance of system.

An original event-triggered theory framework for the problem of scheduling stabilizing control tasks was proposed in [Tabuada \(2007\)](#). Moreover, the positive lower boundary of inter-events interval was guaranteed to avoid Zeno-Behavior. In [Dimarogonas & Johansson \(2009\)](#), authors studied the event based average consensus problem both in centralized and decentralized cases dispensing with any knowledge of the initial average. Although their controllers are updated discretely, the continuous monitoring for the states of its neighbors is also needed to check the triggered condition. In order to overcome this kind of problem, [Dimarogonas \*et al.\* \(2012a\)](#) studied the average consensus problem based on self-triggered control. Following this work, [Seyboth \*et al.\* \(2013a\)](#) discussed the distributed average consensus problem of multi-agent systems based on triggered condition with exponential decay rate, which could avoid the continuous communication problem among neighbors. Both single-integrators with communication delay and double-integrators were investigated. [Fan \*et al.\* \(2013a\)](#) proposed an iterative event-triggered algorithm to avoid consecutive local information exchange. Both continuous and intermittent communication strategies were discussed in [Zhang \*et al.\* \(2015a\)](#), the states converged to a ball centered spot at the average consensus under the event-triggered control law. In addition, the average consensus was investigated again via intermittent information exchange by [Nowzari & Cortés \(2014\)](#), the time-varying topologies were covered. Through self-triggered control, a Zeno-Free consensus algorithm was proposed in [Fan \*et al.\* \(2015\)](#), which has resemblance to the work in [Cheng \*et al.\* \(2014\)](#), in which the leader-following consensus is achieved based on general linear system through event-triggered control. An state estimate method was introduced to design controller and event condition.

In the literature, most studies on event-triggered distributed control of multi-agent systems only involve linear system, but the majority of robots in industrial applications are nonlinear and nonholonomic. It is noted that the results of above works could not be directly extended to the formation tracking problem of multi-robot systems with nonholonomic constraints.



## 5.3 Contributions

Inspired by the above-mentioned papers, especially the results in [Dimarogonas \*et al.\* \(2012a\)](#), [Dong \(2012\)](#), [Peng \*et al.\* \(2015, 2016\)](#), this Chapter focuses on solving the formation tracking problem of identical nonholonomic multi-robot systems by using decentralized control strategy and event-triggered mechanism.

The main contributions of the present work are threefold. First, a distributed control law with fully intermittent local information exchange is designed to ensure that the desired formation configuration will be produced asymptotically by a group of robots from arbitrary initial positions, while the centroid of formation can track a time-vary reference trajectory. This control method can effectively reduce the communication cost and mechanical wear compared with previous results in [Tabuada \(2007\)](#), [Dimarogonas & Johansson \(2009\)](#), [Dong \(2012\)](#), [Peng \*et al.\* \(2015, 2016\)](#), especially when the quantity of robots is large. Second, one kind of event conditions only needs fully intermittent communication is designed, and the determination of parameters is very concise. Third, the event condition is designed through rigorous stability analysis by using Lyapunov techniques, algebraic graph theory and matrix analysis, which completely relax the requirement for continuous local communication to monitor the event condition, compared with the approaches in [Tabuada \(2007\)](#) and [Dimarogonas & Johansson \(2009\)](#). It is worth noting that both the protocol update and event monitoring only need intermittent local interaction in this chapter, which consequently guarantees that the frequencies of communication and control update are reduced, thus the energy consumption and mechanical abrasion will be decreased simultaneously.

## 5.4 Preliminaries

Recall the class of nonholonomic multi-robot systems (1.8) defined in Chapter 1, of which robots are labelled as  $1, \dots, N$ , and move in the plane. For simplicity, the kinematics of each robot  $i$  is assumed identical and can be described of the form

$$\dot{q}_i = \begin{bmatrix} \cos \theta_i & 0 \\ \sin \theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix},$$

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where  $q_i = [x_i, y_i, \theta_i]^T$  is the position and orientation of robot  $i$  in **Cartesian Frame**;  $v_i$  and  $w_i$  represent the linear velocity and angular speed, respectively. From equations (1.8), it follows that the  $i$ -th robot can only move in the direction normal to the axis of the driving wheels, i.e. the wheeled mobile robot satisfies pure rolling and non-slipping.

Before we proceed, an assumption used throughout this Chapter is given.

**Assumption 5.2** *There exists at least one robot that can access the reference information.*

### 5.5 Main results

#### 5.5.1 Distributed formation tracking under fixed topology

**Definition 5.3** *A discrete event moment  $t_{k_i}^i$  at which robot  $i$  updates states information of its neighbors and itself for controller implementation and event condition, is defined as*

$$t_{k_i}^i = \inf[t > t_{k_i-1}^i | f_i(e_i, C_i) = 0], i \in \mathcal{V}, \quad (5.4)$$

where  $1 \leq i \leq N$  represents the  $i$ -th robot,  $t_0^i, t_1^i, \dots, t_{k_i}^i$  is a sequence of event instants, and  $f_i(e_i, C_i)$  is the event condition to be designed in the following.

**Remark 5.4** *For most existing event-triggered control strategies, they can only assure that the communication for the control update is intermittent. However, in order to monitor whether the event is triggered or not, agent often needs continuously communicate with its neighbors, thus the original purpose of event-triggered mechanism is lost. Hence, when event-triggered control strategies are designed, we should make the communications intermittent both for the controller and the event condition. In this chapter, we will analyze skillfully this problem by using Lyapunov techniques, algebraic graph theory and matrix analysis.*

In this chapter, following the choice of Chapters 2 and 3,  $r = k_0 \text{sig}(u_{1i})^\epsilon z_{3i}$  is considered, where  $k_0 > 0$ ,  $0 < \epsilon < 1$ . By employing the same variables transformation (2.5)-(2.7) in Chapter 2, the kinematics (1.8) of multi-robot systems can

be converted into the following chained system form

$$\begin{aligned}\dot{z}_{1i} &= u_{1i}, \\ \dot{z}_{2i} &= u_{2i}, \\ \dot{z}_{3i} &= u_{1i}z_{2i} - k_0|u_{1i}|^{\epsilon+1}z_{3i}.\end{aligned}$$

By using an undirected graph  $\mathcal{G}$  to describe the communication topology among robots, we let  $Z_i = [z_{1i}, z_{2i}]^T$  and  $U_i = [u_{1i}, u_{2i}]^T$ . Systems (2.8) and (2.9) can be expressed of the compact form

$$\dot{Z}_i = U_i.$$

Define the consensus tracking error  $\varepsilon_i(t) = Z_i(t) - Z_0(t)$ . Let  $e_i(t) = Z_i(t_{k_i}^i) - Z_i(t)$ ,  $t \in [t_{k_i}^i, t_{k_i+1}^i)$  be state measurement error and  $C_i(t) = \sum_{j \in \mathcal{N}_i} (Z_i(t_{k_i}^i) - Z_j(t_{k_j}^j)) + b_i(Z_i(t_{k_i}^i) - Z_0(t))$ . Then the above equations can be rewritten as

$$C = (\mathcal{H} \otimes I_2)\varepsilon + (\mathcal{H} \otimes I_2)e, \quad (5.5)$$

where  $C = [C_1(t), \dots, C_N(t)]^T$ ,  $\varepsilon = [\varepsilon_1(t), \dots, \varepsilon_N(t)]^T$  and  $e = [e_1(t), \dots, e_N(t)]^T$ . Thus  $\varepsilon = (\mathcal{H} \otimes I_2)^{-1}C - e$  and  $\varepsilon^T = C^T(\mathcal{H} \otimes I_2)^{-1} - e^T$  can be obtained, respectively.

According to our control objectives, the distributed event-triggered controllers are proposed as follows

$$\begin{aligned}U_i(t) &= -\alpha \left[ \sum_{j \in \mathcal{N}_i} (Z_i(t_{k_i}^i) - Z_j(t_{k_j}^j)) + b_i(Z_i(t_{k_i}^i) - Z_0(t)) \right] \\ &\quad - \beta \operatorname{sgn} \left[ \sum_{j \in \mathcal{N}_i} (Z_i(t_{k_i}^i) - Z_j(t_{k_j}^j)) + b_i(Z_i(t_{k_i}^i) - Z_0(t)) \right], \\ &\quad t \in [t_{k_i}^i, t_{k_i+1}^i)\end{aligned} \quad (5.6)$$

where  $\alpha, \beta$  are positive constant,  $Z_i(t_{k_i}^i)$  denotes the state of robot  $i$  at the  $k_i$ -th event instant  $t_{k_i}^i$ , refers to the Definition 5.3. The zero-order hold is used for state keeping during inter-event interval  $[t_{k_i}^i, t_{k_i+1}^i)$ ,  $Z_0(t)$  is the real-time state of virtual leader, and  $\operatorname{sgn}(\cdot)$  represents the sign function.

**Remark 5.5** *In the controller (5.6), each robot only needs the communications at event instants to update the control inputs of neighbors while updating its own.*

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*That is to say, the proposed controller works in an intermittent communication manner. Since the trajectory of virtual leader could be a preprogram in the micro-processors of robots who are assigned to directly access the virtual leader, which means that the item  $Z_0(t)$  in (5.6) can be updated consecutively during the implementation process of controller. Thus, the control inputs are discrete except for the ones who can directly access the virtual leader. Also because the majority of robots do not need to directly receive data from the virtual leader, so most of robots hold intermittent actuation. In this way, the mechanical abrasion and energy consumption would be further cut down.*

In order to analyze the stability for controller (5.6), the following assumption is needed.

**Assumption 5.6** *The communication topology  $\mathcal{G}$  corresponds to a fixed, undirected and connected graph.*

After some manipulations, (5.6) can be rewritten in a compact form as

$$U = -\alpha((\mathcal{H} \otimes I_2)\varepsilon + (\mathcal{H} \otimes I_2)e) - \beta \text{sgn}C, \quad (5.7)$$

where  $U = [U_1(t), \dots, U_N(t)]^T$ . Taking the time derivative of the consensus tracking error yields

$$\dot{\varepsilon} = -\alpha((\mathcal{H} \otimes I_2)\varepsilon + (\mathcal{H} \otimes I_2)e) - \beta \text{sgn}C - (\mathbf{1}_N \otimes I_2)\dot{Z}_0, \quad (5.8)$$

Since the right-hand side of (5.8) is discontinuous, differential inclusions and non-smooth analysis will be employed to deal with the stability issue. The Filippov solution for the above equation exists because the signum function is measurable and locally essentially bounded due to Theorem 1.3. Then, (5.8) can be rewritten as

$$\dot{\varepsilon} \in^{a.e.} \mathcal{K}[-\alpha((\mathcal{H} \otimes I_2)\varepsilon + (\mathcal{H} \otimes I_2)e) - \beta \text{sgn}C - (\mathbf{1}_N \otimes I_2)\dot{Z}_0], \quad (5.9)$$

By Assumption 2.7,  $|\dot{Z}_0| \leq \gamma$  holds,  $\gamma$  is a bounded positive constant vector. The first theorem is given as follows.

**Theorem 5.7** *Consider the single-integrator  $\dot{Z}_i = U_i$  with the distributed event-triggered control law (5.6) under Assumptions 5.2 and 2.7. Then,  $\lim_{t \rightarrow \infty} (Z_i - Z_0) = 0$  can be achieved with the event condition*

$$f_i(e_i, C_i) = \kappa_1 \|C_i\|_2 - \|e_i\|_2 > 0. \quad (5.10)$$

where  $0 < \kappa_1 < 1/\lambda_{\max}((\mathcal{H} \otimes I_2))$ , the control gains can be chosen as  $\alpha > 0$ ,  $\beta > |\dot{Z}_0|$ , respectively.

**Proof:** Considering the following Lyapunov function candidate  $V = \frac{1}{2}\varepsilon^T(\mathcal{H} \otimes I_2)\varepsilon$ , then the set-valued Lie derivative of  $V$  is obtained as

$$\dot{V}(\varepsilon) \triangleq \bigcap_{\xi \in \partial V(\varepsilon)} \xi^T \mathcal{K}[-\alpha((\mathcal{H} \otimes I_2)\varepsilon + (\mathcal{H} \otimes I_2)e) - \beta \text{sgn}C - (\mathbf{1}_N \otimes I_2)\dot{Z}_0]$$

Because the set-valued Lie derivative  $\dot{V}$  is a singleton, which means  $\dot{V}$  is the only element, then

$$\begin{aligned} \max \dot{V} &= \dot{V} \\ &= (C^T - e^T(\mathcal{H} \otimes I_2))[-\alpha((\mathcal{H} \otimes I_2)\varepsilon + (\mathcal{H} \otimes I_2)e) - \beta \text{sgn}C - (\mathbf{1}_N \otimes I_2)\dot{Z}_0] \\ &\leq -\alpha\|C\|_2^2 - \beta\|C\|_2 + |C^T(\mathbf{1}_N \otimes I_2)\dot{Z}_0| + \alpha((\mathcal{H} \otimes I_2)e)^T C \\ &\quad + \beta |((\mathcal{H} \otimes I_2)e)^T \text{sgn}C| + |((\mathcal{H} \otimes I_2)e)^T(\mathbf{1}_N \otimes I_2)\dot{Z}_0|, \end{aligned}$$

where  $|\dot{Z}_0| \leq \gamma$  according to Assumption 2.7,  $\gamma$  is a positive constant vector. By using the well known inequality  $x^T y \leq \frac{m}{2}\|x\|_2^2 + \frac{1}{2m}\|y\|_2^2$ ,  $x, y$  is vectors,  $m > 0$ , it follows that

$$\begin{aligned} \dot{V} &\leq -\alpha\|C\|_2^2 - \beta\|C\|_2 + \max(\gamma)\|C\|_2 + \beta\|(\mathcal{H} \otimes I_2)e\|_2 \\ &\quad + \alpha\left(\frac{m}{2}\|(\mathcal{H} \otimes I_2)e\|_2^2 + \frac{1}{2m}\|C\|_2^2\right) + \max(\gamma)\|(\mathcal{H} \otimes I_2)e\|_2 \\ &= -\alpha\|C\|_2^2 + \frac{\alpha}{2m}\|C\|_2^2 + (\max(\gamma) - \beta)\|C\|_2 \\ &\quad + \frac{\alpha m}{2}\|(\mathcal{H} \otimes I_2)e\|_2^2 + (\max(\gamma) + \beta)\|(\mathcal{H} \otimes I_2)e\|_2 \\ &\leq \left(-\alpha + \frac{\alpha}{2m}\right)\|C\|_2^2 + \frac{\alpha m}{2}\lambda_{\max}^2\|e\|_2^2 \\ &\quad + (\max(\gamma) - \beta)\|C\|_2 + (\max(\gamma) + \beta)\lambda_{\max}\|e\|_2. \end{aligned}$$

Define

$$\begin{aligned} f_1(t) &= \left(-\alpha + \frac{\alpha}{2m}\right)\|C\|_2^2 + \frac{\alpha m}{2}\lambda_{\max}^2\|e\|_2^2, \\ f_2(t) &= (\max(\gamma) - \beta)\|C\|_2 + (\max(\gamma) + \beta)\lambda_{\max}\|e\|_2. \end{aligned}$$

Enforcing the condition  $f_1(t) < 0$  and  $f_2(t) < 0$  are both satisfied, the  $\dot{V}$  will be strictly negative definite, which means  $Z_i \rightarrow Z_0$  as  $t \rightarrow \infty$ . Thus the following

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condition holds

$$\frac{\|e\|_2}{\|C\|_2} < \sqrt{\frac{2m-1}{m^2\lambda_{max}^2}},$$

and

$$\frac{\|e\|_2}{\|C\|_2} < \frac{\beta - \max(\gamma)}{(\max(\gamma) + \beta)\lambda_{max}},$$

where  $\alpha > 0$ ,  $m > \frac{1}{2}$  and  $\beta > \max(\gamma)$ . Simple mathematical manipulations show that  $0 < \frac{2m-1}{m^2} \leq 1$ ,  $0 < \frac{\beta - \max(\gamma)}{\max(\gamma) + \beta} < 1$ . Thus, the two conditions above can be unified as

$$\frac{\|e\|_2}{\|C\|_2} < \frac{1}{\lambda_{max}}.$$

Hence, the event-triggered condition can be designed as

$$f_i(e_i, C_i) = \kappa_1 \|C_i\|_2 - \|e_i\|_2 > 0.$$

If  $0 < \kappa_1 < 1/\lambda_{max}((\mathcal{H} \otimes I_2))$ , then  $\|e\|_2/\|C\|_2 < \kappa_1 < 1/\lambda_{max}$ . The proof is completed.

**Remark 5.8** *Note that the event-triggered condition (5.10) is concise and easily computed, compared with the aforementioned existing results in Dimarogonas & Johansson (2009). Besides, it does not need continuous monitoring for its neighbors' information during the interval between two event instants. This would immensely improve the communication efficiency, while saving the limited energy of the micro robots. It is notable that this chapter establishes the explicit relation between the event triggering frequency and the maximum eigenvalue  $\lambda_{max}$  of the incidence matrix  $\mathcal{H}$  of multi-robot systems. In fact, the event condition implies that the event triggering frequency only depends on the latter. It is feasible to decrease  $\lambda_{max}$  through various methods, for example, by adjusting the weights of communication graph, so as to reduce the triggering frequency. However, a smaller  $\lambda_{max}$  will lead to slower convergence rate for the systems. Thus, there exists a trade-off between control performance and communication cost. This fact is also in agreement with our intuition: the less the information, the slower the convergence rate.*

**Remark 5.9** *The upper boundary of the largest eigenvalue of matrix  $(\mathcal{H} \otimes I_2)$  can be obtained as  $\lambda_{max} \leq 2(N-1)$  by the results in Grone & Merris (1994a), thus the*

parameter of event condition (5.10) can be chosen as  $0 < \kappa_1 < 1/2(N-1)$ , where  $N$  is the total number of robots in networks. It further relaxes the requirement about the global topology information when design the event condition. Note that the trigger frequency will increase as the quantity of robots arises based on the event condition (5.10).

**Remark 5.10** It is obvious that the distributed event-triggered control law (5.6) can be directly applied as the inputs of first-order systems (2.8) and (2.9), which make  $z_{1i} \rightarrow z_{10}$ ,  $z_{2i} \rightarrow z_{20}$ , and  $u_{1i} \rightarrow u_{10}$  as  $t \rightarrow \infty$ .

**Theorem 5.11** Consider the nonlinear subsystem in (2.10) under Assumptions 5.2 and 2.7, thus  $\lim_{t \rightarrow \infty} (z_{3i} - z_{30}) = 0$  asymptotically fast, under the distributed event-triggered control law (5.6) and event condition (5.10).

**Proof:** Let  $\tilde{z}_{3i} = z_{3i} - z_{30}$ . Take the derivative of  $\tilde{z}_{3i}$  as

$$\begin{aligned} \dot{\tilde{z}}_{3i} &= \dot{z}_{3i} - \dot{z}_{30} \\ &= -k_0 |u_{1i}|^{\epsilon+1} \tilde{z}_{3i} + x_2(t). \end{aligned} \quad (5.11)$$

where  $x_2(t) = u_{1i} \tilde{z}_{2i} + (u_{1i} - u_{10})z_{20} - k_0(|u_{1i}|^{\epsilon+1} - |u_{10}|^{\epsilon+1})z_{30}$ . The solution of the differential equation (5.11) is given as follows

$$\begin{aligned} \tilde{z}_{3i}(t) &= e^{\int_0^t -k_0 |u_{1i}|^{\epsilon+1} d\tau} \tilde{z}_{3i}(0) \\ &+ \int_0^t e^{\int_\tau^t -k_0 |u_{1i}|^{\epsilon+1} d\nu} x_2(\tau) d\tau. \end{aligned} \quad (5.12)$$

According to Theorem 5.7,  $\varepsilon_i$  asymptotically converges to zero, and  $u_{1i}$  asymptotically converges to  $u_{10}$ . It then follows the definition of  $x_2(t)$  that  $x_2(t)$  also asymptotically converges to zero. Hence, according to the definition of asymptotic stability, for a arbitrary positive value  $\sigma > 0$ , it exists  $o > 0$ , when the  $|x_2(0)| < o$ , it has  $|x_2(t)| < \sigma$ .

From the Assumption 2.7, the  $u_{1i}$  is bounded, and  $u_{1i} = w_i$ ,  $1 < \epsilon + 1 < 2$ , Hence,  $|u_{1i}|^{\epsilon+1} \leq c_1$ .

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The solution of the differential equation (5.11) satisfies the inequality

$$\begin{aligned}
\tilde{z}_{3i}(t) &= e^{\int_0^t -k_0|u_{1i}|^{\epsilon+1}d\tau} \tilde{z}_{3i}(0) + \int_0^t e^{\int_\tau^t -k_0|u_{1i}|^{\epsilon+1}d\nu} x_2(\tau) d\tau \\
&\leq e^{-k_0 c_1 t} |\tilde{z}_{3i}(0)| + \int_0^t e^{-k_0 c_1 (t-\tau)} |x_2(\tau)| d\tau \\
&\leq e^{-k_0 c_1 t} |\tilde{z}_{3i}(0)| + e^{-k_0 c_1 t} \int_0^t e^{k_0 c_1 \tau} |x_2(\tau)| d\tau \\
&\leq e^{-k_0 c_1 t} |\tilde{z}_{3i}(0)| + \frac{\sigma k_0 c_1 - \sigma k_0 c_1 e^{-k_0 c_1 t}}{k_0 c_1} \\
&= \sigma + e^{-k_0 c_1 t} (|\tilde{z}_{3i}(0)| - \sigma).
\end{aligned}$$

Hence, when  $t \rightarrow +\infty$ ,  $|\tilde{z}_{3i}(t)| \leq \sigma$ . Since  $\sigma$  is an arbitrary small positive value, from the definition of asymptotic stability, the  $\tilde{z}_{3i}(t)$  is asymptotically stabilized to the neighborhood of origin. This proof is completed.

Based on the Theorems 5.7 and 5.11, we can conclude the following theorem.

**Theorem 5.12** *Consider the multi-robot systems in (1.8). Assume the kinematics of reference is described by the same model. Suppose that Assumptions 5.2, 2.7 are satisfied. Then the original control objectives (5.1)-(5.3) can be reached asymptotically fast with the distributed protocols (5.6) under event-triggered condition (5.10).*

**Proof:** Under the Assumptions 5.2, 2.7, the distributed event-triggered control strategy (5.6) under the event-triggered condition (5.10) are proposed for the dynamic systems (2.8)-(2.10). The formation tracking objectives  $\lim_{t \rightarrow \infty} (z_{1i} - z_{10}) = 0$ ,  $\lim_{t \rightarrow \infty} (z_{2i} - z_{20}) = 0$ ,  $\lim_{t \rightarrow \infty} (z_{3i} - z_{30}) = 0$  and  $\lim_{t \rightarrow \infty} (u_{1i} - u_{10}) = 0$  hold by Theorems 5.7 and 5.11.

**Remark 5.13** *In this chapter, the control objectives (5.1)-(5.3) are transferred into states tracking problem. The convergence property is analyzed rigorously. Combining the analysis results of each subsystem, it follows that the new systems can achieve states tracking asymptotically fast, which means the original control objectives can also be guaranteed in terms of Lemma 2.1.*



### 5.5.2 Distributed formation tracking under switching topologies

In practice, formation reconfiguration or collision avoidance might change the communication topology. Herein, the multiple robots system under a switching topology will be considered in this subsection. Before moving on, we need to give the following Assumption.

**Assumption 5.14** *The communication topology  $\mathcal{G}_l, l \in \mathbb{R}^+$  switches among a finite set of possible connected undirected graphs given by  $\mathbf{G} = \{\mathcal{G}_1, \mathcal{G}_1, \dots, \mathcal{G}_l\}$ .*

Then the following theorem can be derived.

**Theorem 5.15** *Consider the single-integrator  $\dot{Z}_i = U_i$  with the distributed event-triggered controller (5.6) under Assumptions 5.2 and 2.7. Then  $\lim_{t \rightarrow \infty} (z_{1i} - z_{10}) = 0$ ,  $\lim_{t \rightarrow \infty} (z_{2i} - z_{20}) = 0$  and  $\lim_{t \rightarrow \infty} (u_{1i} - u_{10}) = 0$  achieve asymptotically. The corresponding event triggered conditions are derived as (5.15).*

**Proof:** Similarly to the event based protocol (5.6) and the triggered condition (5.10) under fixed topology, the same Lyapunov function candidate for the proof of Theorem 5.7 is chosen as follows

$$V = \frac{1}{2} \varepsilon^T (\mathcal{H}(\mathcal{G}_i) \otimes I_2) \varepsilon, \quad i = 1, \dots, l, \quad (5.13)$$

where  $(\mathcal{H} \otimes I_2)(\mathcal{G}_i) = \mathcal{L}(\mathcal{G}_i) + \mathcal{B}(\mathcal{G}_i)$ ,  $\mathcal{G}_i$  is defined in Assumption 5.14, then it follows

$$\begin{aligned} \dot{V} \leq & (-\alpha + \frac{\alpha}{2m}) \|C\|_2^2 + \frac{\alpha m}{2} \lambda_{max}^2(\mathcal{H}(\mathcal{G}_i) \otimes I_2) \|e\|_2^2 \\ & + (max(\gamma) - \beta) \|C\|_2 + (max(\gamma) + \beta) \lambda_{max}(\mathcal{H}(\mathcal{G}_i) \otimes I_2) \|e\|_2 \end{aligned} \quad (5.14)$$

with

$$\frac{\|e_i\|_2}{\|C_i\|_2} < \frac{1}{\lambda_{max}(\mathcal{H}(\mathcal{G}_i) \otimes I_2)}.$$

It is analogous to the results of fixed undirected graph, it can be proved  $\dot{V}$  is negative definite if the event condition is designed as

$$f_i(e_i, C_i) = \kappa_2 \|C_i\|_2 - \|e_i\|_2 > 0, \quad (5.15)$$

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where  $0 < \kappa_2 < 1/\max\{\lambda_{\max}(\mathcal{H}(\mathcal{G}_i) \otimes I_2)\}$ . The proof is completed.

It is easy to obtain the following theorem based on the results above.

**Theorem 5.16** *Consider the multi-robot systems described in (1.8) with the virtual leader governed by kinematics (2.4). Suppose that Assumptions 5.2, 2.7 and 5.14 are satisfied, then the original formation tracking objectives (5.1)-(5.3) can be achieved asymptotically using the distributed event-triggered control strategy (5.6) with event-triggered condition (5.15) in switching topologies case.*

**Proof:** Under Assumptions 5.2, 2.7 and 5.14, the distributed event-triggered control strategies (5.6) under the event-triggered condition (5.15) in switching topologies case are proposed for the dynamic system (2.8)-(2.10). The formation tracking objectives  $\lim_{t \rightarrow \infty}(z_{1i} - z_{10}) = 0$ ,  $\lim_{t \rightarrow \infty}(z_{2i} - z_{20}) = 0$ ,  $\lim_{t \rightarrow \infty}(z_{3i} - z_{30}) = 0$  and  $\lim_{t \rightarrow \infty}(u_{1i} - u_{10}) = 0$  hold by Theorems 5.11 and 5.15. Thus, the original control objectives (5.1)-(5.3) are reached based on Lemma 2.1. The proof is completed.

### 5.5.3 From theory to practice

In this subsection, we will provide another control diagram to show how to apply the proposed event-triggered methods to the real multi-robot systems. In Figure 5.2,  $j \in \mathcal{N}_i$ , and the closed-loop control flow is as follows:

- (1) Robot  $i$  obtains its real-time position and orientation in the plane through sensing measurements;
- (2) After variables transformation, the signals in (1) and desired relative position of robot  $i$  are converted into new state variables;
- (3) The new state variables of robot  $i$  will be sent to the event condition;
- (4) Based on event condition, robot  $i$  judges whether or not send its own new states to the robot  $j$ , while updating its new states of the controller at event instants;
- (5) After the variables inverse transformation, the control inputs are converted into the linear and angular speeds and also further converted into the speed commands of right and left wheels;
- (6) The speed commands will be sent to the motors and drive the motion of robot  $i$ . Return to the step (1).

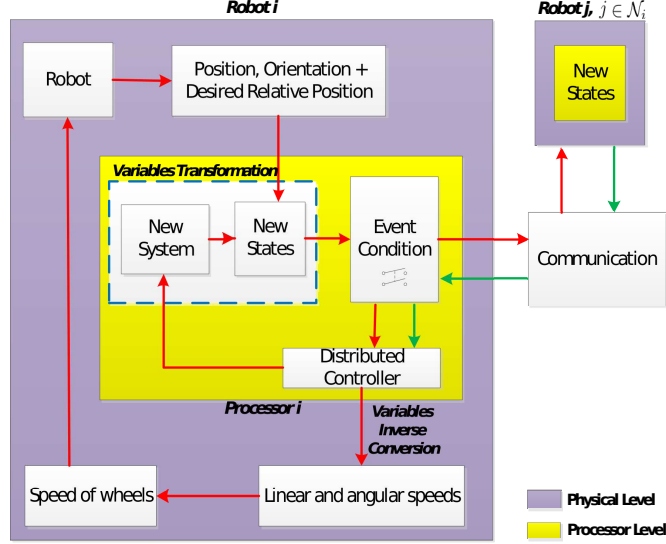


Figure 5.2: The control diagram.

## 5.6 Numerical examples

To verify the theoretical results obtained in this chapter, two simulation experiments are provided to demonstrate the effectiveness of decentralized event based control law for formation tracking task of multi-robot systems. Consider a group of multiple differential driving mobile robots (see Figure.1.9 for individual robot) consists of three followers and one virtual leader.

### 5.6.1 Example 1

The topology of communication used in this simulation is given by associated  $\mathcal{H}$  matrix (5.16). It is connected obviously and each robot only needs local communication, then the control strategy is distributed.

$$\mathcal{H} = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \quad (5.16)$$

The desired formation configuration  $\mathcal{F}$  is predefined by orthogonal coordinates as  $(p_{x1}, p_{y1}) = (0, 0.2)$ ,  $(p_{x2}, p_{y2}) = (-0.15, -0.1)$ ,  $(p_{x3}, p_{y3}) = (0.15, -0.1)$  shown

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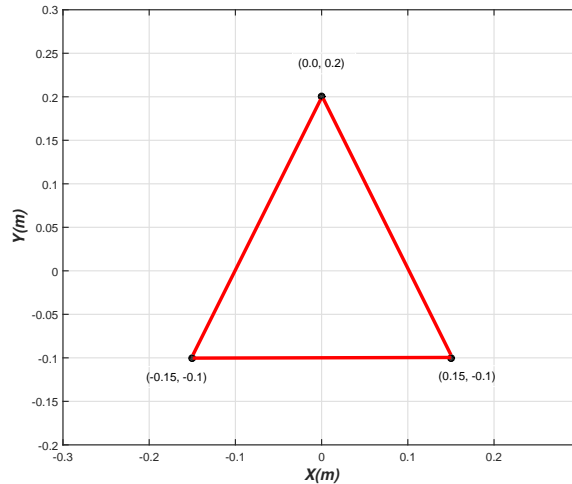


Figure 5.3: Desired geometric pattern of formation.

in Figure.5.6.1, and the initial positions and orientations of the robots are chosen randomly. To conduct a monitoring task around one target, the trajectory of virtual leader can be planed as

$$x_0 = \sin(0.05t), \quad y_0 = -\cos(0.05t).$$

The parameters of the simulation are chosen as follows: based on Theorem 5.7,  $\alpha = 1 > 0$ ,  $\beta = 0.06 > \gamma > \max\{|\dot{z}_0|\} = 0.05$ ; it can be calculated that  $\lambda_{max} = 3.7321 \leq 2(N - 1) = 4$ ,  $N = 3$ , then  $\kappa_1$  should less than  $1/\lambda_{max} = 0.268$ , thus the triggered condition is designed as  $f_i(e_i, C_i) = 0.267\|C_i\| - \|e_i\|$ . Here, in order to compare with Peng *et al.* (2015, 2016),  $\epsilon = 0$ . Figure.5.4 is the evolution of the three following robots (three circles with different color) at certain instants, the squares denote initial position. After a short period of evolution, robots produce asymptotically the desired configuration while satisfactorily track the reference trajectory of the virtual leader.

The states  $z_1, z_2, z_3$  of different robot agree with  $z_{10} = 0.05t$ ,  $z_{20} = k_0 = 2$ ,  $z_{30} = 1$  in Figure.5.5. By the event-triggered schedule, it decreases exactly unnecessary information exchange and requires fewer controller updates. Besides, it is easy to find that the Zeno-Behavior never happens. Moreover, the measurement

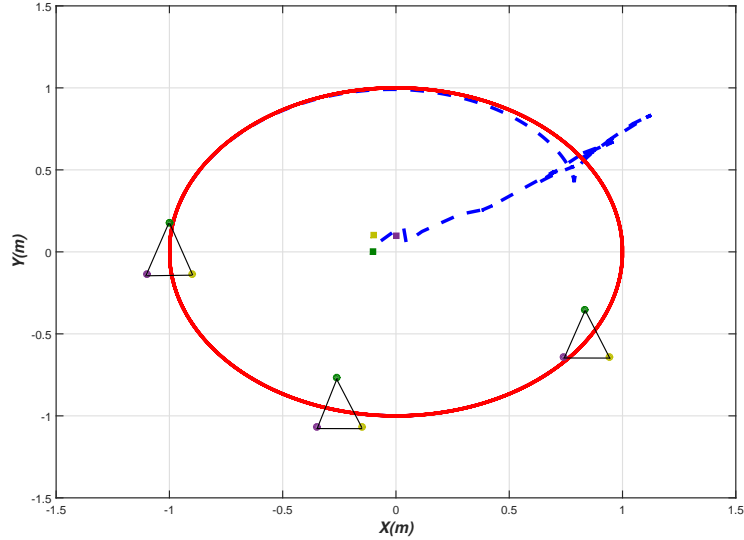


Figure 5.4: Evolution of the three following robots at certain time, the red and blue line are the trajectory of virtual leader and centroid of configuration, respectively.

error norm of three robots are given in Figure 5.6, which stay below the specified state-dependent threshold (5.10). From Figure 5.4 to 5.6, the formation control objectives described by equations (5.1)-(5.3) are all reached.

### 5.6.2 Example 2

In this case, we will verify the effectiveness of proposed control algorithm under the situation in which three possible communication topologies switch alternately at certain instant due to obstacles or instability of communication. All the possible topologies amongst robots are denoted by the adjacency matrix  $\mathcal{A}$  and Laplacian matrix  $\mathcal{L}$  without virtual leader. Obviously, we note that all of topologies are connected.

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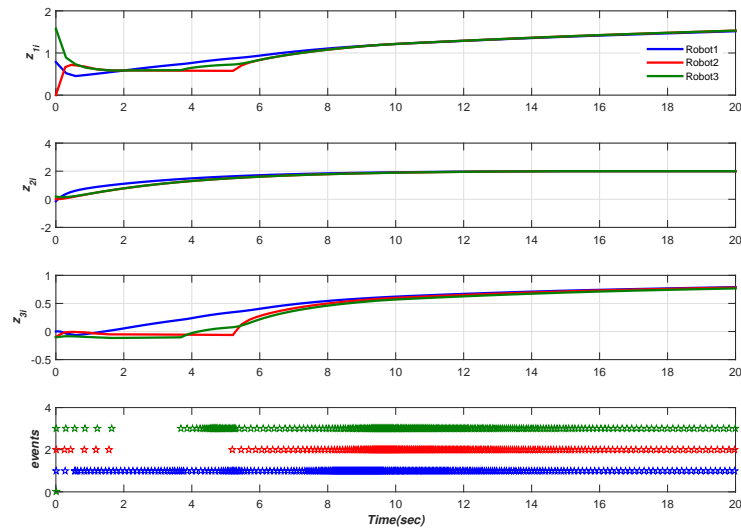


Figure 5.5: State consensus and event instant under fixed topology (1-3: robot1-3)

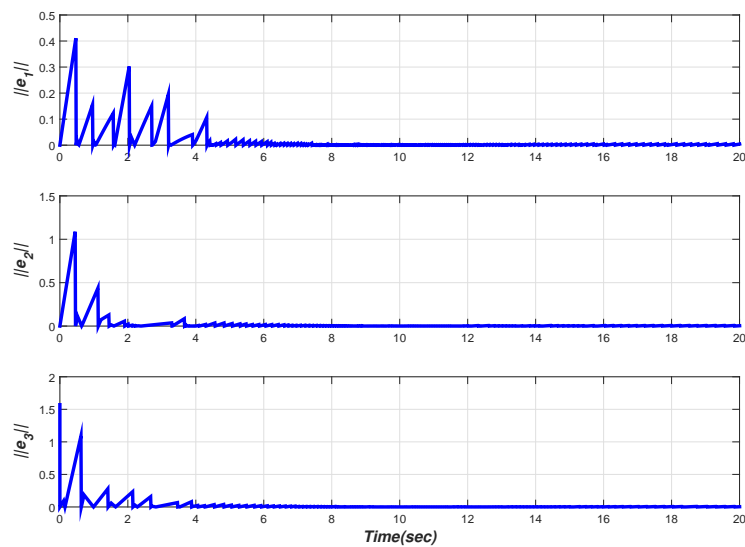


Figure 5.6: The measurement errors under fixed topology.

$$\begin{aligned} \mathcal{A}_{\mathcal{G}_1} &= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, & \mathcal{A}_{\mathcal{G}_2} &= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, & \mathcal{A}_{\mathcal{G}_3} &= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \\ \mathcal{L}_{\mathcal{G}_1} &= \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, & \mathcal{L}_{\mathcal{G}_2} &= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \\ \mathcal{L}_{\mathcal{G}_3} &= \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{bmatrix}. \end{aligned}$$

Computing the  $\max\{\lambda_{\max}(\mathcal{G}_i)\} = \max\{3.7321, 3.2470, 3.2470\} = 3.7321$ . Based on Theorem 5.15,  $0 < \kappa_2 < 1/3.7321$ . In order to compare with the results of **Example 1**, the control gains, system parameters, desired formation geometric configuration and initial states are chosen to be same with **Example 1**. The initial topology is  $\mathcal{G}_1$ , and afterwards it switches randomly among three possible graphs with a stochastic dwell time on the range  $[1, 5]$ . The evolution of formation configuration by three following robots are shown in Figure.5.7. It is shown that our event triggered controllers work well under switching topology. The convergence results of states  $z_1, z_2, z_3$  and event triggered instants of three robots are shown in Figure.5.8. And the norms of measurement errors vanish asymptotically in Figure.5.9. Specially, the Zeno-Behavior does not appear.

## 5.7 Conclusions

This chapter presents a novel distributed event-triggered protocols for the formation tracking problem of multi-robot systems with nonholonomic constraint. Under the proposed control law, the entire formation systems converge asymptotically to the desired configuration, while the centroid of the formation tracks satisfactorily the reference trajectory. Due to only the intermittent local communication is needed for the controller update and event detection, the high communication cost which is mainly caused by the large number of information links, is eased to some extent. Furthermore, the actuation updating frequency is also reduced vastly, which might improve the mechanical wear of multi-robot systems, prolong the life-span of components.

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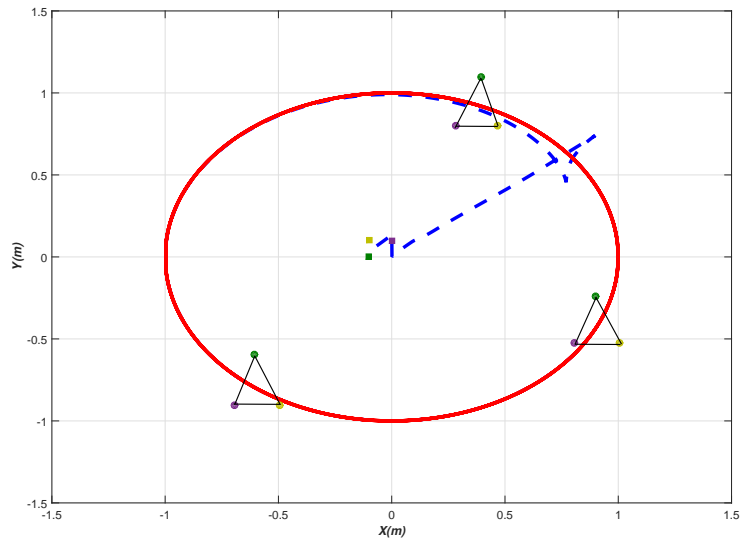


Figure 5.7: Evolution of the three following robots at certain time, the red and blue line are the trajectory of virtual leader and centroid of configuration, respectively

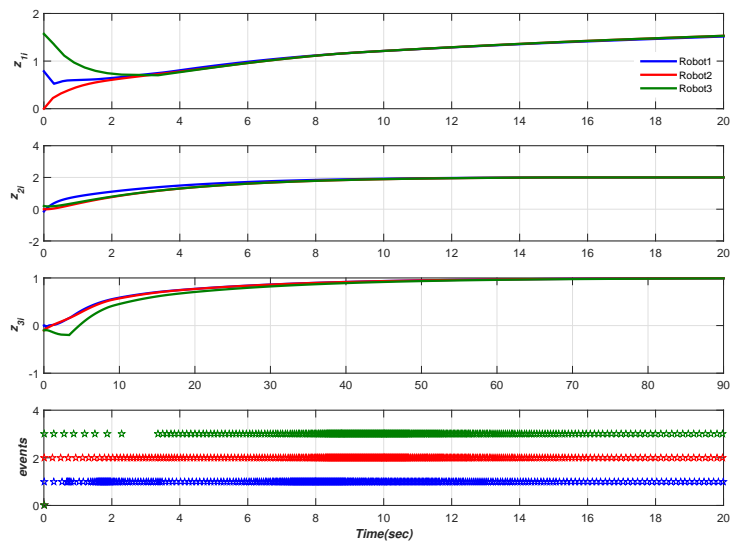


Figure 5.8: State consensus and event instant under switching topology (1-3: robot1-3)



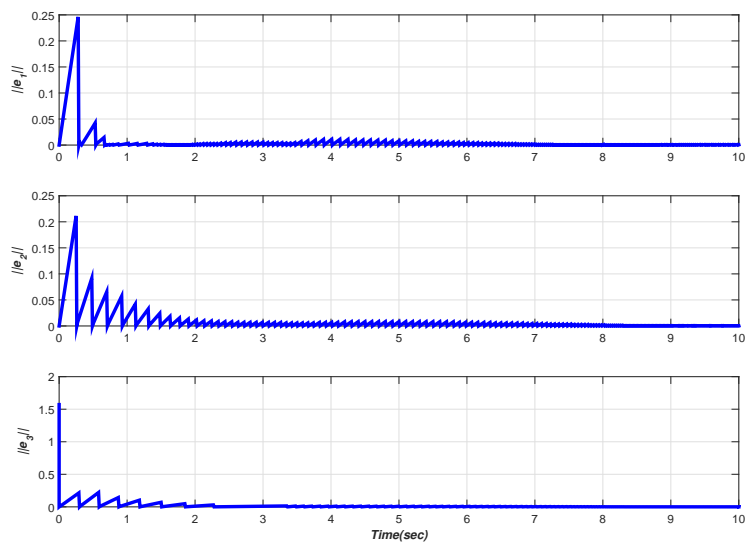


Figure 5.9: The measurement errors under switching topology

However, continuous sampling is against the reality, it is natural to introduce the sampled-data approach into event based formation controller. The extra advantage of this combination is to guarantee that the Zeno-Behavior of event-triggered systems can be excluded for all the robots in theory. In next chapter, we will investigate this class of hybrid systems.

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# Chapter 6

## Distributed Tracking Control of Nonholonomic Multi-Robot Formation Systems via Periodically Event-Triggered Method

### 6.1 Problem setup

#### 6.1.1 Mathematic modeling of nonholonomic mobile robot

Recall a group of nonholonomic autonomous robots labeled as  $1, \dots, N$ , which move on the planar without sideslip and slipping. The kinematic model of robot  $i$  is given by

$$\dot{q}_i(t) = S(q_i(t))\bar{v}_i(t), \quad (6.1)$$

where  $q_i(t) = [x_i(t), y_i(t), \theta_i(t)]^T$  is the position and orientation of robot  $i$  in **Cartesian Coordination**;  $\bar{v}_i(t) = [v_i(t), w_i(t)]^T$ ,  $v_i(t)$  and  $w_i(t)$  represent the linear and angular speeds, respectively. Let

$$S(q_i(t)) = \begin{bmatrix} \cos \theta_i(t) & 0 \\ \sin \theta_i(t) & 0 \\ 0 & 1 \end{bmatrix}.$$

In order to plan a reasonable reference trajectory for robots formation, the virtual leader 0 that is located in the centroid  $[x_0(t), y_0(t), \theta_0(t)]$  of prescribed

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formation shape is required and shares the same mathematical model (6.1) with the real ones.

### 6.1.2 Original control objectives

For each robot  $i$ , the objectives are to design the control input  $\bar{v}_i(t)$  by using its state  $q_i(t)$ , the neighbors' state  $q_j(t)$  and the specific constant displacement  $[p_{xi}(t), p_{yi}(t)]$  with respect to the position  $(x_0(t), y_0(t))$  of virtual leader such that

$$\begin{aligned} \lim_{t \rightarrow \infty} \begin{bmatrix} x_i(t) - p_{xi}(t) \\ y_i(t) - p_{yi}(t) \end{bmatrix} &= \begin{bmatrix} x_0(t) \\ y_0(t) \end{bmatrix}, \\ \lim_{t \rightarrow \infty} \left[ \sum_{i=1}^N \frac{x_i(t)}{N} - x_0(t) \right] &= 0, \quad \lim_{t \rightarrow \infty} \left[ \sum_{i=1}^N \frac{y_i(t)}{N} - y_0(t) \right] = 0, \\ \lim_{t \rightarrow \infty} [\theta_i(t) - \theta_0(t)] &= 0 \end{aligned} \quad (6.2)$$

are reached.

### 6.1.3 Variables transformation

Here, we will introduce a class of global invertible variables transformation to convert the above formation tracking problem into the consensus tracking issue of a chain system. Let's define the new states and inputs as follows

$$\begin{aligned} z_{1i}(t) &= \theta_i(t), \\ z_{2i}(t) &= [x_i(t) - p_{xi}(t)] \cos \theta_i(t) + [y_i(t) - p_{yi}(t)] \sin \theta_i(t) \\ &\quad + \chi_1(t), \\ z_{3i}(t) &= [x_i(t) - p_{xi}(t)] \sin \theta_i(t) - [y_i(t) - p_{yi}(t)] \cos \theta_i(t), \\ u_{1i}(t) &= \omega_i(t), \\ u_{2i}(t) &= \dot{\chi}_1(t) + v_i(t) - u_{1i}(t) z_{3i}(t), \end{aligned} \quad (6.3)$$

where  $z_{1i}(t)$ ,  $z_{2i}(t)$  and  $z_{3i}(t)$  are new states,  $u_{1i}(t)$  and  $u_{2i}(t)$  are new inputs,  $\chi_1(t) = f(z_{1i}(t), z_{2i}(t), z_{3i}(t), u_{1i}(t), u_{2i}(t))$ . Note that  $p_{xi}(t)$  and  $p_{yi}(t)$  are constant functions, the values of which depend on specific task requirements in prac-

## 6.1 Problem setup

tice. Define  $\delta_{1i}(t) = z_{1i}(t) - z_{10}(t)$ ,  $\delta_{2i}(t) = z_{2i}(t) - z_{20}(t)$ ,  $\delta_{3i}(t) = z_{3i}(t) - z_{30}(t)$ ,  $\delta_{4i}(t) = u_{1i}(t) - u_{10}(t)$ , the original control objectives (6.2) become

$$\lim_{t \rightarrow \infty} (\delta_{ji}(t)) = 0; j = 1, 2, 3, 4; i = 1, 2, \dots, N \quad (6.4)$$

based on the following Lemma.

**Lemma 6.1** *If the equation (6.4) holds for  $1 \leq i \leq N$ , then  $N$  mobile robots can converge to the prescribed formation pattern predefined by  $[p_{xi}(t), p_{yi}(t)]$ , i.e., the equations (6.2) can be satisfied.*

**Proof:** Based on the variables transformation (6.3), in the case of the equation (6.4) being satisfied, we have

$$\begin{aligned} & \lim_{t \rightarrow \infty} (x_i - p_{xi}) \\ &= \lim_{t \rightarrow \infty} [(z_{2i} - \chi_1(t)) \cos(\theta_i) + z_{3i} \sin(\theta_i)] \\ &= [x_0 \cos(\theta_0) + y_0 \sin(\theta_0) + \chi_1(t)] \cos(\theta_0) \\ & \quad + [x_0 \sin(\theta_0) - y_0 \cos(\theta_0)] \sin(\theta_0) - \chi_1(t) \cos(\theta_0) \\ &= x_0, \end{aligned}$$

$$\begin{aligned} & \lim_{t \rightarrow \infty} (y_i - p_{yi}) \\ &= \lim_{t \rightarrow \infty} [(z_{2i} - \chi_1(t)) \sin(\theta_i) - z_{3i} \cos(\theta_i)] \\ &= [x_0 \cos(\theta_0) + y_0 \sin(\theta_0) + \chi_1(t)] \sin(\theta_0) \\ & \quad - [x_0 \sin(\theta_0) - y_0 \cos(\theta_0)] \cos(\theta_0) - \chi_1(t) \sin(\theta_0) \\ &= y_0. \end{aligned}$$

For the second original control objective, we have

$$\lim_{t \rightarrow \infty} \left( \sum_{i=1}^N \frac{x_i}{N} - x_0 \right) = \lim_{t \rightarrow \infty} \left( \frac{1}{N} \sum_{i=1}^N x_i - x_0 \right) = \frac{1}{N} \sum_{i=1}^N (p_{xi} + x_0) - x_0 = \frac{1}{N} \sum_{i=1}^N x_0 - x_0 = 0.$$

Similarly,

$$\lim_{t \rightarrow \infty} \left( \sum_{i=1}^N \frac{y_i}{N} - y_0 \right) = 0.$$

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Since  $z_{1i} \rightarrow z_{10}$  means  $\theta_i \rightarrow \theta_0$ , thus the third original control objective is also satisfied. All the original control objectives (6.2) are achieved by this point. The proof is completed.

### 6.2 Literature overview

Over the past two decades, coordination control of multi-robot systems which can achieve better performance than a single robot, has attracted considerable attention due to its broad applications, including exploration, surveillance, rescue, search and transport, just name a few. As a critical topic of the coordination of multi-robot systems, the formation tracking problem, for which recent years have witnessed dramatic advances, with various solutions (Desai *et al.* 2001; Dong & Hu 2016, 2017; Dong *et al.* 2015, 2016b; Lewis & Tan 1997a; Liu & Geng 2015; Peng *et al.* 2013a,b, 2014, 2015, 2016; Werfel *et al.* 2014; Xiao *et al.* 2009b; Yamaguchi 2003). In order to address this kind of problems (Cao 2015; Chu *et al.* 2016; Shames *et al.* 2012), certain control strategies are requested to be proposed for a group of robots with inexpensive cost and simple structure so as to drive them to reach a desired formation shape from arbitrary initial positions and heading angles, while driving the centroid of formation to move along with a desired reference trajectory. In practice, formation tracking is widely applied in cooperative transport, monitoring targets, localization, and so forth. Generally speaking, there are various standards of classification for the formation control strategies of multi-robot systems, this chapter roughly divides them into two categories, of which the distributed control policy based on local interaction, with many superiorities in diverse aspects such as higher flexibility and robustness, nice maintainability and scalability, low cost and high efficiency, etc., is more practical than another one-centralized control strategy especially for large-scale robot swarm.

To deal with the formation tracking problem in a distributed manner, communication strategies and controller design are two main aspects to be considered. From the information exchange point of view, distributed control architecture can significantly reduce the usage of communication channel while avoiding the existence of one central coordination unit. However, it is still a severe issue regarding to the limited resources of multi-robot systems, along with the increasing number

of robots and communication links in the real-world. In order to further decrease the communication cost and control updates, event-triggered paradigm revived in the past decade (Dimarogonas *et al.* 2012b; Fan *et al.* 2013b; Mahmoud & Sabih 2014; Seyboth *et al.* 2013b; Yin *et al.* 2016; Zhang *et al.* 2014a,b, 2015b).

It is well known that the numerical simulation in the computer is often implemented based on time-varying or fixed step size, which leads to the fact that most existing continuous time controllers can merely be validated approximately. Specifically, for the validation of event-triggered schemes, common digital computer can't accurately detect the event so as to conduct related action in time when the event condition is violated, although this problem can be regarded as the effect of time delay which can be tackled by many existing methods for time delay problems. Therefore, most existing event-triggered controllers based on continuous sampling are emulated in a digital manner. Another, even though this kind of controller designed based on continuous time feedback is applied in the real robot system, real-time states or outputs feedback is approximated by a periodic feedback process actually. According to these considerations, the sampled-data control is introduced in this chapter. That is to say, the controlled systems are continuous whereas the control update can only occur at periodic sampling time instants, which actually results in a hybrid systems based on local states feedback (Chen & Francis 2012; Gao *et al.* 2009). This kind of multi-robot systems can be more precisely simulated in digital computer while being easily realized in practice by using low-cost AD/DA convertor and digital microprocessor.

Based on the above discussions, the average consensus problem of single integrator robots under event-triggered and sampled-data controllers was investigated in the typical work Meng & Chen (2013) over fixed and switching undirected connected graphs, which is the early paper to combine the sampled-data control with event-triggered mechanism. Meanwhile, references (Heemels & Donkers 2013; Heemels *et al.* 2013) defined this type of method as periodic event-triggered control. Most recently, the authors studied the synchronization problem of linear multi-agent systems with communication time delays in Garcia *et al.* (2017) by using periodic event-triggered control approach. And reference Yin *et al.* (2016) considered input saturation consensus problem by using adaptive periodic event-triggered controller. However, a well-known fact in most existing work is that the robot dynamics is strictly restricted to the single and double integrators or

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general linear systems. The investigations to the sampled-data event-triggered mechanism for the nonholonomic robot are still critically lacking. As the early experimental investigations in (Postoyan *et al.* 2013, 2015), where the event-triggered mechanism was applied to control the nonholonomic robot to track a time-varying reference trajectory. In above work, a controller was implemented in a remote PC while control input was transmitted back to robot through wireless network. Besides, a camera in the ceiling is used to capture the posture of robot. However, this chapter only investigated the event-triggered method for the tracking problem of single mobile nonholonomic robot. Actually, its control architecture is essentially centralized. Also, it is worth noticing that reference Chu *et al.* (2017b) provided a solution for the formation tracking problem of nonholonomic multi-robot systems based on event-triggered intermittent communication and control update with continuous sampling.

### 6.3 Contributions

Motivated by aforementioned observations, the main contributions of this chapter are summarized as follows: first, a modified variables transformation is given to cast the formation tracking problem into a states tracking problem; second, a unified distributed observer-based controller with two types of event conditions are designed to guarantee the time derivative of corresponding Lyapunov function negative definite, while sufficient conditions are derived through theoretical analysis. Note that each robot only needs local interaction with its neighbors; third, numerical examples are provided to illustrate the effectiveness of the obtained theoretical results.

The current paper has the following advantages. Compared with the most existing event-triggered controllers with continuous time sampling like the typical work in Seyboth *et al.* (2013b), the periodic sampling is further investigated in this chapter. Compared with the continuous communication manner, the periodic and aperiodic communication amongst neighboring robots are realized with the aid of sampled-data technique. Compared with periodic communication for the event monitoring in the representative work Meng & Chen (2013), the second event condition of this chapter only requires aperiodic communication, which



immensely reduces the amount of information exchange. Compared with continuous time systems with event-triggered control approach, which is difficult to thoroughly prove the avoidance of Zeno-Behavior for all the robots, whereas the lower bound of the inter-event interval for each robot is guaranteed to be the sampling period due to the characteristics of sampled-data control in this chapter. That is to say, the Zeno-Behavior is excluded for all the robots naturally. Compared with the real-time feedback in most existing distributed controllers like [Chu \*et al.\* \(2016\)](#), the control update only takes place at the event-triggered time instants which dramatically reduces the actuation frequency and energy consumption. Compared with the most recent paper [Chen \*et al.\* \(2017b\)](#), we further investigate the case where the position of the center of drive axis is the controlled state variable, which is also required to study in many application scenarios. In addition, directly controlling the position of the center of drive axis is more challenging than the manipulator position, since the kinematics of system can't be linearized into two linear single integrators. Moreover, we extensively consider the situation with a dynamical reference trajectory for the desired formation beyond the rendezvous and formation stabilization problem considered in [Chen \*et al.\* \(2017b\)](#). At last, the different variables transformation proposed by our paper leads to the diverse control framework.

## 6.4 Preliminaries

Before we proceed, the following assumptions are reasonable and useful for theoretical analysis.

**Assumption 6.2** *The communication topology  $\mathcal{G}$  among robots is fixed, undirected and connected, while there exists at least one robot that can directly access the information of the virtual leader.*

**Assumption 6.3**  *$x_i, y_i$  and  $v_i, \omega_i$  are bounded,  $i = 0, 1, \dots, N$ ,  $\omega_0$  is a persistently exciting signal due to the fact that the system is nonholonomic.*

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### 6.5 Main results

In this section, a distributed event based sampled-data controller is designed, and two different event conditions are well constructed, which lead to two totally diverse communication mechanisms. The asymptotic convergence rate of multiple nonholonomic autonomous robots can be guaranteed by rigorous stability analysis, related parameters constraints are derived.

Taking the time derivative of variables (6.3) yields

$$\dot{z}_{1i} = u_{1i}, \quad (6.5)$$

$$\dot{z}_{2i} = u_{2i}, \quad (6.6)$$

$$\dot{z}_{3i} = u_{1i}z_{2i} - u_{1i}\chi_1(t). \quad (6.7)$$

Through making  $z_i = (z_{1i}, z_{2i})^T \in \mathbb{R}^2$  and  $u_i = (u_{1i}, u_{2i})^T \in \mathbb{R}^2$ , the subsystems (6.5) and (6.6) could be rewritten as one single-integrator form

$$\dot{z}_i(t) = u_i(t). \quad (6.8)$$

Define the measurement error as  $e_i(t_{k_i}^i + lT) = z_i(t_{k_i}^i) - z_i(t_{k_i}^i + lT)$ . In what follows, let  $nT$  be the abbreviation of  $t_{k_i}^i + lT$ , so one gets

$$e_i(nT) = z_i(t_{k_i}^i) - z_i(nT). \quad (6.9)$$

Before moving on, the following Assumption is made and a distributed fixed-time observer is employed to design the observer-based controller.

**Assumption 6.4**  $(|\check{z}_{10}|, |\check{z}_{20}|)^T < (|\gamma_1|, |\gamma_2|)^T = \gamma (1 \leq i \leq N)$ ,  $\gamma$  is positive constant column vector.

**Lemma 6.5** [Chu et al. (2017a)] Under Assumptions 6.2 and 6.4, we employ the following distributed fixed-time observer as follows

$$\begin{aligned} \dot{\zeta}_i(t) &= \alpha \operatorname{sig}\left[\sum_{j=0}^N a_{ij}(\zeta_j(t) - \zeta_i(t))\right]^2 + \beta \operatorname{sgn}\left[\sum_{j=0}^N a_{ij}(\zeta_j(t) - \zeta_i(t))\right], \\ \dot{\zeta}_i(t) &= 0, t \in (t^*, \infty), \end{aligned}$$

where parameters  $\alpha$  and  $\beta$  are positive and constant. Let  $\zeta_0(t) = \mathbf{1}_N \otimes \dot{z}_0(t)$  and  $|\dot{\zeta}_0| < \gamma$ . Then, if  $\beta \geq \gamma$ ,  $\zeta_i(t) = \zeta_0(t) = u_0(t)$  for any  $t \geq t^*$ , where  $t^*$  is the upper bound of settling time for the observer as follows

$$t^* = \frac{\pi \lambda_{max}}{2\lambda_{min}^2 \sqrt{\alpha N^{-0.5}(\beta - \gamma)}}.$$

**Remark 6.6** In this chapter, it is not difficult to show that the observed state vector  $\dot{z}_0(t)$  is constant based on the calculation by substituting the model of virtual leader into the variables transformation 6.3. Thus, the observer can be shut down to avoid continuous communication after it has converged in a fixed time. Once new velocity commands are sent to the virtual leader, the robots can be informed to restart their observer for another cycle.

### 6.5.1 Periodic information exchange

Define the sum of relative state errors for each robot as

$$z_i(nT) = \sum_{j \in \mathcal{N}_i} [z_i(nT) - z_j(nT)] + b_i [z_i(nT) - z_0(nT)]. \quad (6.10)$$

Herein, the event condition for robot  $i$  can be designed as follows

$$\|e_i(nT)\|_2^2 \leq c_i \|z_i(nT)\|_2^2, \quad (6.11)$$

where  $c_i$  is a positive scalar.

Define the periodic state tracking error as

$$\varepsilon_i(nT) = z_i(nT) - z_0(nT), \quad (6.12)$$

and corresponding real-time state tracking error as

$$\varepsilon_i(t) = z_i(t) - z_0(t). \quad (6.13)$$

The equation (6.10) can be further rewritten in a stacked form as

$$z(nT) = (H \otimes I_2)\varepsilon(nT),$$

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where  $H = \mathcal{L} + \mathcal{B}$ , thus

$$z(nT)^T z(nT) = \varepsilon(nT)^T (H \otimes I_2)^2 \varepsilon(nT).$$

Based on the event condition (6.11), one has

$$e(nT)^T e(nT) \leq c_{max} z(nT)^T z(nT),$$

where  $e(nT) = [e_1(nT), \dots, e_N(nT)]^T$  and  $z(nT) = [z_1(nT), \dots, z_N(nT)]^T$ ,  $c_{max} = \max\{c_1, \dots, c_N\}$ . Then, one has

$$\begin{aligned} e(nT)^T (H \otimes I_2) e(nT) &\leq \lambda_{max} e(nT)^T e(nT) \\ &\leq c_{max} \lambda_{max} z(nT)^T z(nT) \\ &= c_{max} \lambda_{max} \varepsilon(nT)^T (H \otimes I_2)^2 \varepsilon(nT) \\ &\leq c_{max} \lambda_{max}^2 \varepsilon(nT)^T (H \otimes I_2) \varepsilon(nT), \end{aligned} \quad (6.14)$$

where  $\lambda_{max}$  is the abbreviation of  $\lambda_{max}(H \otimes I_2)$ , which will be used in the sequel if no confusion takes place. Note the fact that  $\lambda_{max}(H \otimes I_2) = \lambda_{max}(H)$ .

With the aid of event condition (6.11), by using fixed undirected graph  $\mathcal{G}$  to describe the communication topology among robots, the distributed event-triggered sampled-data controller of the mobile robot  $i$  ( $1 \leq i \leq N$ ) for  $t \in [t_k^i + lT, t_k^i + lT + T)$  is proposed based on its state and neighbors' states feedback

$$\begin{aligned} u_i(t) &= \zeta_i(t) - \sum_{j \in \mathcal{N}_i} [z_i(t_{k_i}^i) - z_j(t_{k_j}^j)] \\ &\quad - b_i [z_i(t_{k_i}^i) - z_0(nT)], \end{aligned} \quad (6.15)$$

where  $\zeta_i = (\zeta_{1i}, \zeta_{2i})^T$  and  $z_0 = (z_{10}, z_{20})^T$ .

**Remark 6.7** *In the periodic information exchange case, each pair of neighboring robots will communicate with each other periodically, while checking the event condition. Once the event condition is violated, we can say that, one event is triggered. Then, it will send its state to the neighbors, update its states in controller and reset measurement error  $e_i(t_{k_i}^i + lT)$  to zero again. Meanwhile, by using zero-order holder one keeps the state in the controller until next event is triggered by itself or neighbors. It can be observed that the smallest inter-event interval is lowest bounded by the sampling period, which means the Zeno-Behavior can be excluded for all the robots.*

**Theorem 6.8** Consider the subsystem (6.8) under the distributed controller (6.15) with the distributed fixed-time observer in Lemma 6.5 and event condition (6.11) with the sum of relative errors (6.10). If Assumptions 6.2, 6.3 are satisfied, then  $\delta_{ji}(t) = 0; j = 1, 2, 4; i = 1, 2, \dots, N$  is achieved asymptotically fast if the following constraints can be satisfied

$$0 < T < \frac{1}{4\lambda_{max}} \quad \text{and} \quad 0 < c_{max} < \frac{1 - 4T\lambda_{max}}{\lambda_{max}^2 + 4T\lambda_{max}^3}.$$

**Proof:** Substituting the periodic state tracking error (6.12) and measurement error (6.9) into the controller (6.15) with kinematics (6.8), yields

$$\dot{z}(t) = \zeta(t) - (H \otimes I_2)e(nT) - (H \otimes I_2)\varepsilon(nT). \quad (6.16)$$

When  $t \geq t^*$ , based on Lemma 6.5,  $\zeta(t) = \mathbf{1}_N \otimes \dot{z}_0(t)$ ,  $\varepsilon(t) = [\varepsilon_1(t)^T, \varepsilon_2(t)^T, \dots, \varepsilon_N(t)^T]^T$ ,  $e(t) = [e_1(t)^T, e_2(t)^T, \dots, e_N(t)^T]^T$  and  $\zeta(t) = [\zeta_1(t)^T, \zeta_2(t)^T, \dots, \zeta_N(t)^T]^T$ . Differentiating  $\varepsilon(t)$  with respect to time, we have

$$\begin{aligned} \dot{\varepsilon}(t) &= \dot{z}(t) - \mathbf{1}_N \otimes \dot{z}_0(t) \\ &= -(H \otimes I_2)e(nT) - (H \otimes I_2)\varepsilon(nT). \end{aligned} \quad (6.17)$$

Choose the Lyapunov function candidate as

$$V(t) = \frac{1}{2}\varepsilon(t)^T \varepsilon(t). \quad (6.18)$$

Differentiating (6.18) with respect to  $t \in [nT, nT + T)$ , yields

$$\begin{aligned} \dot{V}(t) &= \varepsilon(t)^T \dot{\varepsilon}(t) \\ &= \varepsilon(t)^T [-(H \otimes I_2)e(nT) - (H \otimes I_2)\varepsilon(nT)]. \end{aligned}$$

Since  $\dot{\varepsilon}(t) = [\varepsilon(t) - \varepsilon(nT)]/(t - nT) = -(H \otimes I_2)e(nT) - (H \otimes I_2)\varepsilon(nT)$ , and

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$0 \leq t - nT < T$ , one has

$$\begin{aligned}
\dot{V}(t) &= -\varepsilon(nT)^T (H \otimes I_2) (e(nT) + \varepsilon(nT)) \\
&\quad + (t - nT) (e(nT) + \varepsilon(nT))^T (H \otimes I_2)^2 (e(nT) + \varepsilon(nT)) \\
&\leq -\varepsilon(nT)^T (H \otimes I_2) e(nT) - \varepsilon(nT)^T (H \otimes I_2) \varepsilon(nT) \\
&\quad + T (e(nT) + \varepsilon(nT))^T (H \otimes I_2)^2 (e(nT) + \varepsilon(nT)) \\
&= -\varepsilon(nT)^T (H \otimes I_2) e(nT) - \varepsilon(nT)^T (H \otimes I_2) \varepsilon(nT) \\
&\quad + T e(nT)^T (H \otimes I_2)^2 e(nT) + 2T \varepsilon(nT)^T (H \otimes I_2)^2 e(nT) \\
&\quad + T \varepsilon(nT)^T (H \otimes I_2)^2 \varepsilon(nT) \\
&\leq (2T\lambda_{max} - \frac{1}{2}) \varepsilon(nT)^T (H \otimes I_2) \varepsilon(nT) \\
&\quad + (2T\lambda_{max} + \frac{1}{2}) e(nT)^T (H \otimes I_2) e(nT).
\end{aligned}$$

Due to the second term in the right side of above inequality is positive definite, in order to make the derivative of Lyapunov function negative definite, based on inequality (6.14),  $\dot{V}(t)$  can be bounded as follows

$$\begin{aligned}
\dot{V}(t) &\leq (2T\lambda_{max} + \frac{1}{2}c_{max}\lambda_{max}^2 + 2Tc_{max}\lambda_{max}^3 \\
&\quad - \frac{1}{2}) \varepsilon(nT)^T (H \otimes I_2) \varepsilon(nT).
\end{aligned}$$

Forcing  $2T\lambda_{max} + \frac{1}{2}c_{max}\lambda_{max}^2 + 2Tc_{max}\lambda_{max}^3 - \frac{1}{2}$  be negative definite, then the constraints can be derived

$$0 < T < \frac{1}{4\lambda_{max}} \quad \text{and} \quad 0 < c_{max} < \frac{1 - 4T\lambda_{max}}{\lambda_{max}^2 + 4T\lambda_{max}^3}.$$

Thereby

$$\dot{V}(t) < 0.$$

Therefore,  $V(t)$  converges to 0 asymptotically fast. The proof is completed.

**Remark 6.9** From Theorem 6.8, it can be observed that the maximum sampling period is less than  $\frac{1}{4\lambda_{max}}$  in order to guarantee the stability of closed-loop systems. It can also be found that the choices of sampling period  $T$  and parameter of event condition  $c_{max}$  depend on the global information about the communication topol-

ogy. Based on the results in Grone & Merris (1994b), the upper bound on  $\lambda_{max}$  can be obtained as

$$\lambda_{max} \leq 2(N - 1).$$

Thereby, the sampling period and parameter of event condition can be chosen conservatively by

$$0 < T < \frac{1}{8(N - 1)} \quad \text{and} \quad 0 < c_{max} < \frac{1 - 8T(N - 1)}{4(N - 1)^2 + 32T(N - 1)^3}.$$

Meanwhile, it reveals that the allowed sampling period will be close to zero along with the number of robots tending to infinite and the discrete sampling will degenerate into continuous sampling.

**Remark 6.10** *In this subsection, the update frequency of control input has been decreased dramatically. However, since synchronized communications are required for the event monitoring in each period, the heavy communication congestion might be resulted at each sampling time instants.*

### 6.5.2 Aperiodic information exchange

In the last subsection, the event-triggered controller and event condition are developed to guarantee globally asymptotic stability, and the Zeno-Behaviors are avoided for all the robots, while the update times of actuators are decreased dramatically and continuous communications are averted. However, it can be seen that robots need to interact with neighbour in each sampling period to check the event conditions. That is to say, when sampling period is set to one small value with certain reasons, the communication burden is still heavy. Motivated by this observation, the event condition with aperiodic information exchange are investigated in this subsection. Design the sum of relative errors  $z_i(nT)$  in the event condition (6.11) as

$$z_i(nT) = \sum_{j \in \mathcal{N}_i} [z_i(t_{k_i}^i) - z_j(t_{k_j}^j)] + b_i [z_i(t_{k_i}^i) - z_0(nT)]. \quad (6.19)$$

**Remark 6.11** *In the aperiodic information exchange case, robots still check their event condition in each sampling period, but do not communicate with the neighbors. Only when its own or neighbors' event is triggered, the information is transmitted to or received from its neighbors, depicted as Figure.6.1.*

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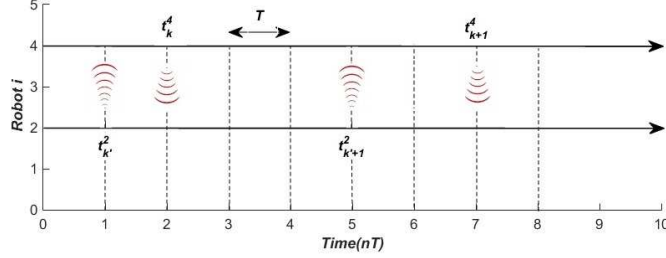


Figure 6.1: The schedule of aperiodic communication.

In the following, an event-triggered algorithm associated with the results of this subsection is given in the Table 6.1 and  $\bar{t}$  is the task termination time instant. Meanwhile, the second main result is presented to guarantee the availability of the proposed algorithm.

**Theorem 6.12** *Consider the subsystem described by (6.8) under the distributed controller (6.15) with distributed fixed-time observer in Lemma 6.5 and event condition is redesigned as (6.11) with the sum of relative errors (6.19). If the Assumptions 6.2 and 6.3 are satisfied, then  $\delta_{ji}(t) = 0; j = 1, 2, 4; i = 1, 2, \dots, N$  can be achieved asymptotically fast if the following constraints are satisfied*

$$0 < T < \frac{1}{4\lambda_{max}} - c_{max}\lambda_{max}, \quad \text{and} \quad 0 < c_{max} < \frac{1}{4\lambda_{max}^2}.$$

**Proof:** Similar to the calculation of last subsection, it has obtained the following inequality

$$\begin{aligned} \dot{V}(t) &\leq (2T\lambda_{max} - \frac{1}{2})\varepsilon(nT)^T(H \otimes I_2)\varepsilon(nT) \\ &\quad + (2T\lambda_{max} + \frac{1}{2})e(nT)^T(H \otimes I_2)e(nT). \end{aligned}$$

Based on new proposed event conditions, one has

$$\begin{aligned} e(nT)^T(H \otimes I_2)e(nT) &\leq \lambda_{max}e(nT)^T e(nT) \\ &\leq c_{max}\lambda_{max}z(nT)^T z(nT) \\ &\leq 2c_{max}\lambda_{max}^2\varepsilon(nT)^T(H \otimes I_2)\varepsilon(nT) \\ &\quad + 2c_{max}\lambda_{max}^2e(nT)^T(H \otimes I_2)e(nT). \end{aligned} \quad (6.20)$$



Table 6.1: Distributed formation cooperative control algorithm for each robot.

**Algorithm 1** Event-Triggered Control for Robot  $i$  with Aperiodic Communication

**Initialization**

$\zeta_i(0) \leftarrow$  arbitrary value;  
 $[x_i(0), y_i(0), \theta_i(0)] \leftarrow$  Initialization;  
 $(p_{xi}, p_{yi}) \leftarrow$  Initialization;  
**if** robot  $0 \in \mathcal{N}_i$  **then**  
     $[x_0(0), y_0(0), \theta_0(0)] \leftarrow$  Initialization  
     $(p_{x0}, p_{y0}) \leftarrow$  Initialization;  
**end if**

**Iteration**

01: **while**  $0 \leq t \leq \bar{t}$  **do**  
02:   run finite-time observer (10);  
     $\dot{\zeta}_i(t) = \alpha \sum_{j=0}^N a_{ij}(\zeta_j(t) - \zeta_i(t)) + \beta \text{sgn}[\sum_{j=0}^N a_{ij}(\zeta_j(t) - \zeta_i(t))]; t \in [t_m, t_{m+1});$   
03:   **if**  $t \geq t^*$  **then**  
04:     stop finite-time observer (10);  
05:      $\dot{\zeta}_i(t) = 0, t \in [t_m + t^*, t_{m+1}); m = 0, 1, 2, \dots;$   
06:   **end if**  
07:   **Variables Transformation;**  
08:      $z_{1i}(t) \leftarrow \theta_i(t);$   
09:      $z_{2i}(t) \leftarrow [x_i(t), y_i(t), \theta_i(t)]; [p_{xi}(t), p_{yi}(t)];$   
10:      $z_{3i}(t) \leftarrow [x_i(t), y_i(t), \theta_i(t)]; [p_{xi}(t), p_{yi}(t)];$   
11:     run event detector (16);  
12:     **if**  $\|e_i(nT)\|_2^2 > c_i \|z_i(nT)\|_2^2$  **then**  
13:       send information to neighbors, update own event condition and controller;  
14:       run distributed controller (15);  
         $u_i(t) = \zeta_i(t) - \sum_{j \in \mathcal{N}_i} [z_i(t_{k_i}^i) - z_j(t_{k_j}^j)] - b_i [z_i(t_{k_i}^i) - z_0(nT)];$   
15:     **else if**  
16:       stop information sending to neighbors, open listening and invoke zero-order holder;  
17:     **end if**  
18:     **Inverse Variables Transformation;**  
19:      $\omega_i(t) \leftarrow u_{1i}(t);$   
20:      $v_i(t) \leftarrow u_{1i}(t)z_{3i}(t) + u_{2i}(t) - \dot{\chi}_1(t);$   
21:      $[x_i(t), y_i(t), \theta_i(t)] \leftarrow$  robot  $i \leftarrow [v_i(t), \omega_i(t)];$   
22:     **return** step 7  
23: **end while**

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Then, it is not difficult to obtain the inequality as follows

$$e(nT)^T(H \otimes I_2)e(nT) \leq \frac{2c_{max}\lambda_{max}^2}{1 - 2c_{max}\lambda_{max}^2}\varepsilon(nT)^T(H \otimes I_2)\varepsilon(nT)$$

with  $c_{max} < \frac{1}{2\lambda_{max}^2}$ . Using above inequality,  $\dot{V}(t)$  can be bounded as

$$\begin{aligned} \dot{V}(t) &\leq (2T\lambda_{max} - \frac{1}{2})\varepsilon(nT)^T(H \otimes I_2)\varepsilon(nT) \\ &\quad + \frac{(4T\lambda_{max} + 1)}{2} \cdot \frac{2c_{max}\lambda_{max}^2}{1 - 2c_{max}\lambda_{max}^2}\varepsilon(nT)^T(H \otimes I_2)\varepsilon(nT) \\ &= -(0.5 - 2T\lambda_{max} - 2c_{max}\lambda_{max}^2)\varepsilon(nT)^T(H \otimes I_2)\varepsilon(nT). \end{aligned}$$

Hence,  $\dot{V}(t) < 0$  if the following inequality is satisfied

$$0 < T < \frac{1}{4\lambda_{max}} - c_{max}\lambda_{max}.$$

Also, in order to guarantee that the upper bound of  $T$  is strict larger than zero, an additional condition  $\frac{1}{4\lambda_{max}} - c_{max}\lambda_{max} > 0$  must be satisfied, based on which the condition  $c_{max} < \frac{1}{4\lambda_{max}^2}$  can be derived. According to the above calculations, the final constraints can be obtained as below

$$0 < T < \frac{1}{4\lambda_{max}} - c_{max}\lambda_{max}, \quad \text{and} \quad 0 < c_{max} < \frac{1}{4\lambda_{max}^2}.$$

The proof is completed.

**Remark 6.13** *Based on the parameter constraints in the Theorem 6.12, the maximum allowed sampling period is less than  $\frac{1}{4\lambda_{max}}$ , which is the upper bound of sampling period for the periodic communication scheme. The allowed sampling period  $T$  is inversely proportional to the coefficient of event condition  $c_{max}$  and the determination of  $c_{max}$  is regardless of  $T$ . Compared with the results in Meng & Chen (2013), the present method in this subsection doesn't need to communicate with neighbors in each sampling period, but only needs to communicate at the event instants while update the control input. This kind of event-triggered communication and control mechanism tremendously decreases the communication burden, calculated amounts, mechanical abrasion and energy consumption in contrast to the continuous solutions.*

### 6.5.3 Convergence analysis of the entire multi-robot systems

Combining the results obtained in the previous analysis, in this subsection, we will prove the global stability for the entire multi-robot systems.

**Theorem 6.14** *Consider the subsystem (6.7) with  $\chi_1(t) = k_0 \text{sig}(u_{1i})^\epsilon z_{3i}$  ( $0 < \epsilon < 1$ ) under Assumptions 6.2 and 6.3, thus  $z_{3i} - z_{30} = 0$  can be achieved asymptotically fast under the distributed controller (6.15) with distributed fixed-time observer in Lemma 6.5 and event condition (6.11) with the sum of relative errors (6.10) or (6.19).*

**Proof:** Let  $\tilde{z}_{3i} = z_{3i} - z_{30}$ . Take the derivative of  $\tilde{z}_{3i}$  with respect to the time

$$\begin{aligned} \dot{\tilde{z}}_{3i} &= \dot{z}_{3i} - \dot{z}_{30} \\ &= -k_0 |u_{1i}|^{\epsilon+1} \tilde{z}_{3i} + x_2(t), \end{aligned} \quad (6.21)$$

where  $x_2(t) = u_{1i} \tilde{z}_{2i} + (u_{1i} - u_{10}) z_{20} - k_0 (|u_{1i}|^{\epsilon+1} - |u_{10}|^{\epsilon+1}) z_{30}$ . The solution of the differential equation (6.21) is given as follows

$$\tilde{z}_{3i}(t) = e^{\int_0^t -k_0 |u_{1i}|^{\epsilon+1} d\tau} \tilde{z}_{3i}(0) + \int_0^t e^{\int_\tau^t -k_0 |u_{1i}|^{\epsilon+1} d\nu} x_2(\tau) d\tau. \quad (6.22)$$

According to Theorem 6.8 and 6.12,  $\delta_{ji}(t); j = 1, 2, 4; i = 1, 2, \dots, N$  asymptotically converges to zero, it then follows the definition of  $x_2(t)$  that  $x_2(t)$  also asymptotically converges to zero. Hence, according to the definition of asymptotic stability, for arbitrary positive constant  $\sigma > 0$ , it exists  $o > 0$ , when the  $|x_2(0)| < o$ , it has  $|x_2(t)| < \sigma$  as  $t \rightarrow \infty$ .

From the Assumptions 6.3, the  $u_{1i}$  is bounded, and  $u_{1i} = w_i$ ,  $1 < \epsilon + 1 < 2$ . Hence,  $|u_{1i}|^{\epsilon+1} \leq c_1$ .

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Furthermore, the solution (6.22) of the differential equation satisfies the following inequality

$$\begin{aligned}
 \tilde{z}_{3i}(t) &= e^{\int_0^t -k_0|u_{1i}|^{\epsilon+1}d\tau} \tilde{z}_{3i}(0) + \int_0^t e^{\int_\tau^t -k_0|u_{1i}|^{\epsilon+1}d\nu} x_2(\tau) d\tau \\
 &\leq e^{-k_0 c_1 t} |\tilde{z}_{3i}(0)| + \int_0^t e^{-k_0 c_1 (t-\tau)} |x_2(\tau)| d\tau \\
 &\leq e^{-k_0 c_1 t} |\tilde{z}_{3i}(0)| + e^{-k_0 c_1 t} \int_0^t e^{k_0 c_1 \tau} |x_2(\tau)| d\tau \\
 &\leq e^{-k_0 c_1 t} |\tilde{z}_{3i}(0)| + \frac{\sigma k_0 c_1 - \sigma k_0 c_1 e^{-k_0 c_1 t}}{k_0 c_1} \\
 &= \sigma + e^{-k_0 c_1 t} (|\tilde{z}_{3i}(0)| - \sigma).
 \end{aligned}$$

Hence, when  $t \rightarrow \infty$ ,  $|\tilde{z}_{3i}(t)| \leq \sigma$ . Since  $\sigma$  is an arbitrary small positive value, from the definition of asymptotic stability, the  $\tilde{z}_{3i}(t)$  is asymptotically stabilized to the neighborhood of origin. This proof is completed.

**Theorem 6.15** *Consider the nonholonomic multi-robot systems (6.1), if Assumptions 6.2, 6.3 and 6.4 are satisfied, with the aid of Lemma 6.5 with Theorems 6.8 or 6.12, the original control objectives (6.2) can be reached asymptotically fast subject to the control law (6.15) and event condition (6.11) with the sum of relative errors (6.10) or (6.19).*

**Proof:** Combining the Theorems 6.8, 6.12 with 6.14, the Theorem 6.15 can be obtained naturally. The proof is completed.

## 6.6 Numerical examples

The simulation results are provided for the proposed schemes with periodic and aperiodic information exchange strategies based on the control diagram proposed in Chapter 5, to illustrate the effectiveness of the present theoretical results in above sections.

### 6.6.1 Example 1: periodic information exchange

In this example, we first verify the effectiveness of controller with periodic communication strategy. Consider a group of nonholonomic mobile robots composed

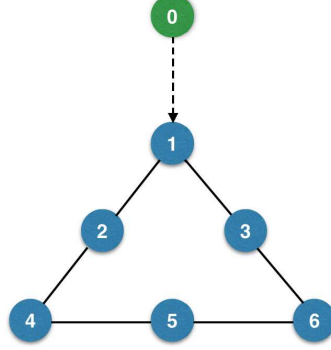


Figure 6.2: Communication topology (solid and dotted black lines for the robots and virtual leader, respectively) and six robots (the blue ones denote robots and the green one is the virtual leader).

of six followers  $F_1 - F_6$  and one dynamical virtual leader  $L_0$ . An undirected graph in Figure.6.2 is used to describe the communication topology and robots. Note that the topology of followers  $F_1 - F_6$  is connected, while only a subset of followers need access the information of virtual leader. Here, the number is just one. The rounded data of desired formation shape is given by Figure.6.3 with the displacements  $(p_{x1}, p_{y1}) = (0, 0.2)$ ,  $(p_{x2}, p_{y2}) = (-\sqrt{3}/15, 0)$ ,  $(p_{x3}, p_{y3}) = (\sqrt{3}/15, 0)$ ,  $(p_{x4}, p_{y4}) = (-2\sqrt{3}/15, -0.2)$ ,  $(p_{x5}, p_{y5}) = (0, -0.2)$ ,  $(p_{x6}, p_{y6}) = (2\sqrt{3}/15, -0.2)$ . The trajectory of virtual leader is chosen as

$$x_0 = 3 \sin(0.05t), y_0 = -3 \cos(0.05t).$$

According to communication links in Figure.6.2,  $\lambda_{max} = 4.2784$  and  $\lambda_{min} = 0.1088$  can be calculated. In terms of Theorem 6.8,  $0 < T < 0.0584$  should be satisfied. If  $T$  is chosen to be 0.002, then event condition parameter should satisfy  $0 < c_{max} < 0.051$ . Then  $c_{max} = 0.013$  is chosen. The evolution of formation shape for six robots at certain time instants is given in Figure.6.4, in which the squares denote the initial positions and the circles denote the actual positions. The orientation of each robot converges to the time-varying heading angle of virtual leader. From Figure.6.4, it is observed that all the original formation tracking objectives characterized by equations (6.2) are attained. Besides, the norm of measure errors of the robot  $F_2$ , which is selected randomly, is depicted by Figure.6.5. And event triggered time instants of six robots are given in Figure.6.6. It can be observed that the control input updates are dramatically decreased. In addition, it can be

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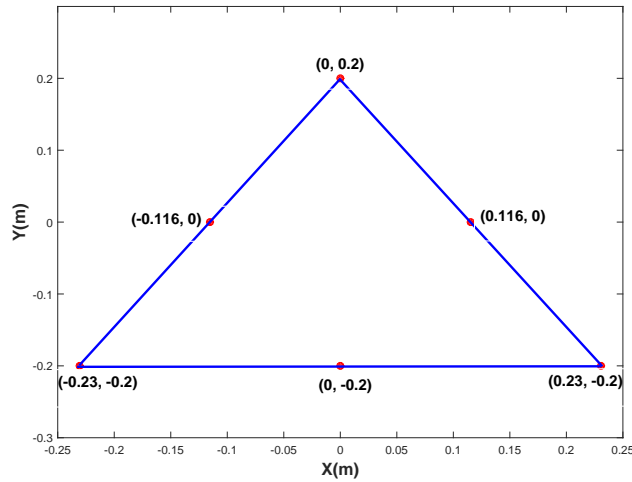


Figure 6.3: The desired shape.

seen that the Zeno-Behaviour is avoided due to the nature of sampled-data control method, i.e.,  $\tau = t_{k+1}^* - t_k^* \geq 0.002s, k = 0, \dots, n$ , in this case. As a result, the superiority of the present distributed event based sampled-data controller with periodic communication is clearly demonstrated.

### 6.6.2 Example 2: aperiodic information exchange

In this example, to facilitate the comparison with the results of periodic communication case, the same communication graph and desired formation shape are used. The event condition parameter should satisfy the constraint  $0 < c_{max} < 0.0137$  based on Theorem 6.12. When  $c_{max} = 0.013$  is chosen,  $0 < T < 0.0028$  should be satisfied. In order to keep consistent with case 1, the sampling period is still chosen as  $T = 0.002s$ . The evolution of formation shape is given by Figure.6.7. And Figure.6.8 depicts the norm of measurement errors along with time for the robot  $F_2$ . Note that the convergence rate of the measurement error  $e_{12}$  is faster than the periodic communication approach. The event-triggered time instants are shown in Figure.6.9. The performance comparison between periodic and aperiodic information exchange mechanism are reported in Table 6.2 with the totally same sampling period  $T$  and event condition parameter  $c_{max}$ . It is observed that

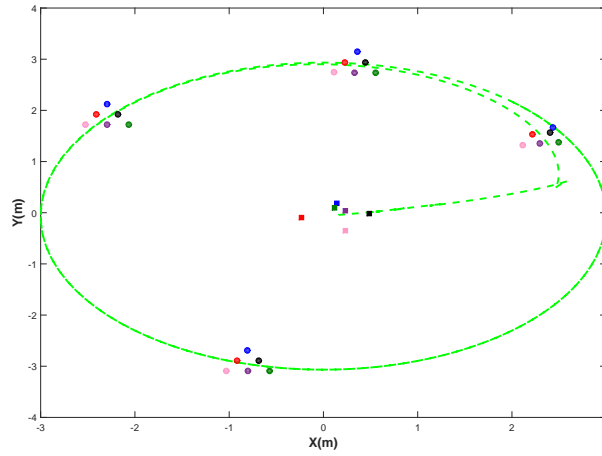


Figure 6.4: The evolution of formation shape for six robots at certain time instants, the green dashed line represents the trajectory of the centroid, squares denote the initial positions of robots and circles refer to the actual positions.

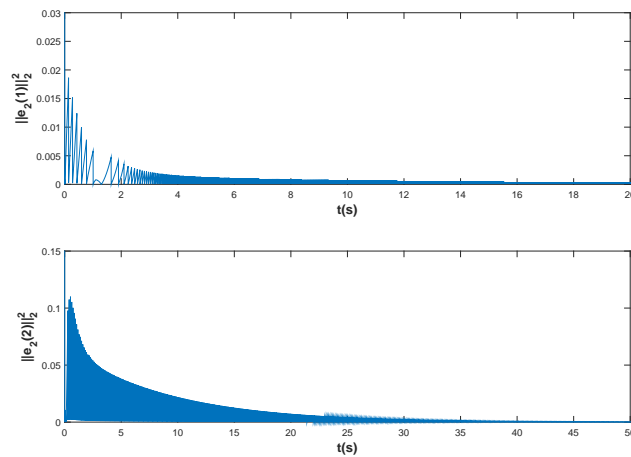


Figure 6.5: Measurement errors of robot 2.

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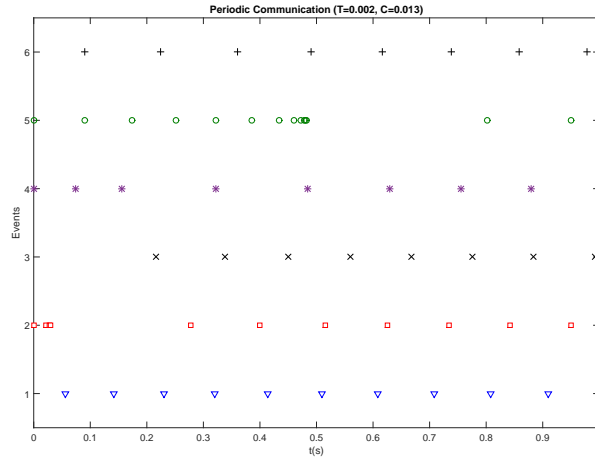


Figure 6.6: Event instants for robot  $i$ ,  $i = 1, \dots, 6$ .

the communication times for Case 2 are dramatically reduced compared to Case 1, whereas event triggering times for two cases is analogical in the interval 0-1s.

Table 6.2: Comparison between Case 1 and Case 2 (0-1s).

Case	1	2
$T$	0.002	0.002
$c_{max}$	0.013	0.013
Events of Robot 1	10	9
Events of Robot 2	11	8
Events of Robot 3	8	7
Events of Robot 4	9	10
Events of Robot 5	13	9
Events of Robot 6	8	8
Total Communication Times	3000	51

## 6.7 Conclusions

This chapter investigated the distributed event based sampled-data control strategies for formation tracking problem of nonholonomic multi-robot systems. An unified distributed controller with intermittent communication and input update was introduced to drive multiple nonholonomic mobile robots to converge towards and maintain certain desired formation shape, while tracking a dynamical



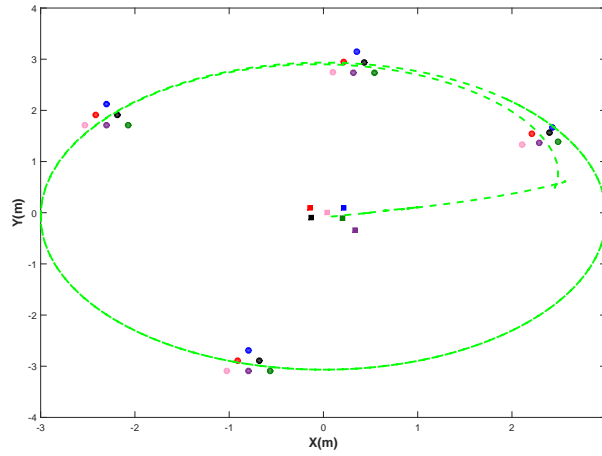


Figure 6.7: The evolution of formation shape for six robots at certain time instants, the green dashed line represents the trajectory of the centroid, squares denote the initial positions of robots and circles refer to the actual position.

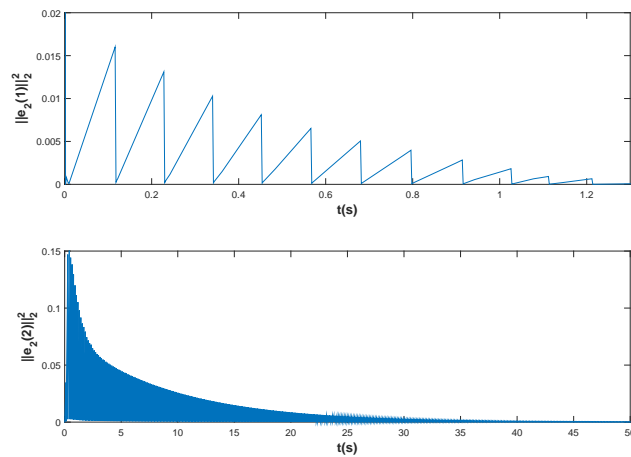


Figure 6.8: Measurement errors of robot 2.

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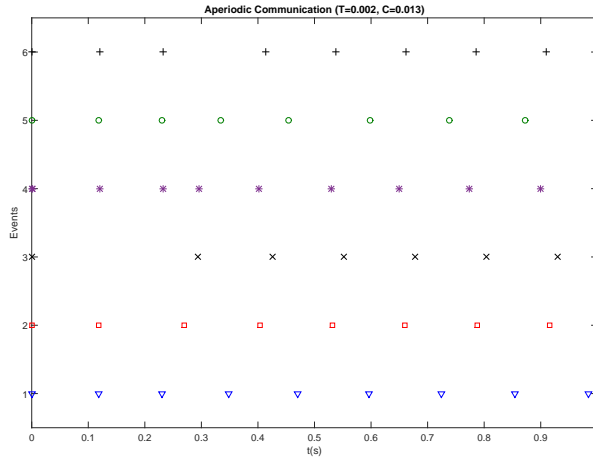


Figure 6.9: Event instants for robot  $i$ ,  $i = 1, \dots, 6$ .

reference trajectory. Two classes of event conditions, which only need periodic and aperiodic communication, were developed. The constraint for the sampling period and event condition parameter was derived using Lyapunov technique and matrix theory. Furthermore, the Zeno-Behavior is naturally excluded for all the robots, due to the property of sampled-data control. Particularly, the significance of these theoretical results lies in assisting real-world multi-robot systems in decreasing the consumption of limited resources such as communication bandwidths, computation loads, and energy, etc., through combining the superiorities of distributed control, event-triggered mechanism, and sampled-data technique. Meanwhile, our research provides a digital implementable framework of event-triggered controller with the potential applications to real multi-robot systems. As a consequence, our controller does not depend on dedicated hardware for continuous event monitoring and control update, and the total costs of multi-robot systems are cut.

In the future, many open problems in our framework must still be addressed, such as collision avoidance, connectivity preserving, disturbance rejection, and time-varying sampling period, time delays, etc. Also, validating the present theoretical results in real multi-robot systems is the ongoing work. In the meantime, the asynchronous periodic event-triggered controller for multi-robot systems should be further considered due to the difficulties of clock synchronization in practice.



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# Chapter 7

## Conclusions and Future Work

In this chapter, we summarize all the work presented in this thesis. Furthermore, significant research directions and open problems in future are suggested.

### 7.1 Thesis summary and contributions

This thesis mainly considers the distributed tracking control problem for non-holonomic multi-robot formation systems, mainly involving two typical targets: formation stabilization and formation maneuver. Some theoretical and practical issues are addressed in this point, ranging from controller synthesis and stability analysis, kinematics and dynamics of robots, certain practical considerations in order to realize engineering applications (such as finite-time convergence rate, disturbances rejection, event-triggered communication and control mechanism, sampled-data control, etc.). In the end of thesis, we make a brief summary of the main results and contributions of the present work.

#### 7.1.1 Multi-robot formation systems: finite-time stability

In the first part, we mainly explore the possibility to realize global formation tracking in a finite time rather than in infinite time in Chapter 2. The convergence time, which is strongly dependent on initial conditions, is derived by using finite-time stability theory. Nevertheless, if initial errors are very large, the boundary of settling time can be very large too. In the meantime, the initial errors are a kind of global information which is not the situation we expect. Hence, in Chapter

## 7. CONCLUSIONS AND FUTURE WORK

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3, we try to fix this flaws by employing the fixed-time stability theory, devise another class of finite time controller, of which the upper bound of settling time can be flexibly tuned off-line. In other words, the upper bound of settling time of formation tracking systems is regardless of any global initial conditions. Some sufficient conditions for fixed-time stability are derived, and explicit formula for upper bound of settling time is obtained. Next, beyond kinematics model, when the velocities of robots are sufficiently large, in Chapter 4, the more practical dynamics of robots is considered. In addition, since disturbances are inevitable in reality, the disturbance rejection is also a requirement for our controller synthesis.

### 7.1.2 Multi-robot formation systems: event-triggered communication and control

In the second part, we mainly focus on the problems about how to decrease the number of communication times and control update frequency so as to further reduce the communication cost, computation load, energy consumption and excessive mechanical abrasion. To this end, we revisit the once-sleepy event-triggered control method. In Chapter 5, a novel event based communication and control update mechanism is devised. It is efficient as shown in numerical examples. Both the communication and control update frequencies are decreased dramatically while the original formation tracking performance is not affected significantly. Furthermore, consider that the microprocessor is digital, in order to develop a set of digital implement solutions, the sampled-data control method is combined with our event-triggered formation systems, while the periodic and aperiodic information exchange methods are proposed simultaneously in Chapter 6. The sufficient conditions are established and the constraints for sampling period, control gains, event condition parameters are also derived.

All the theoretical results obtained from those two parts can be validated repeatedly and scientifically by numerical simulations.

## 7.2 Future work

In this section, we list some interesting topics for the future research.

### 7.2.1 Theoretical extension of present work

- Throughout the whole thesis, the interaction graph is assumed to be undirected, although we have considered the switching graphs in Chapter 5 and the authors have practical consideration of technique realization. However, it is interesting to expand the current results to directed graph.
- The time delays are inevitable since computation, actuation and information exchange need to consume time. It is very urgent to thoroughly investigate the possible emerging results in the presence of different kinds of time delays. How to utilize the positive effect of time delay is also of significance.
- The quantization error will be produced in the wireless communication devices and computer systems, hence we need to further consider the negative effect of quantization error to match the reality.
- For multi-robot formation systems, the collision might take place. It is also worthwhile to propose modified controller to avoid the collision amongst robots and obstacles.
- In this thesis, the author merely considers the most commonly used but extremely challenging type of nonholonomic mobile robot, differential driving mobile robot (is also simply called unicycle). The extension of our currently proposed approaches to broader classes of nonholonomic robot models is promising.

### 7.2.2 Engineering applications of present work

- In our thesis, except the Chapter 4, we do not thoroughly deal with robustness problems. Although the controllers proposed in Chapter 3 and 5 also have the potential to reject the disturbances to some extent. As is well known, position and orientation information obtained from global positioning system, indoors positioning system or inertial navigation system inevitably include noises and uncertainties in practice. Moreover, the wireless communication signal is possibly crippled. Meanwhile, communication loads and time delays are also crucial issues. All of these practical constraints need to be tackled in our future endeavors.

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- Currently, we have conducted some partially separate validations. The ongoing work is to implement the proposed distributed formation controllers in the TurtleBot2 robotic platform in INRIA, Lille, France. In the next step, we look forward to obtain rich data from the testbed, and thoroughly verify our theoretical results while further enhancing our understanding for the multi-robot formation systems. Besides, the most precious aspects include the discoveries of new problems and bottlenecks which are not predicted by the theory analysis.
- How to guarantee the same performance of multi-robot formation systems when they are pushed to mass production, is worth to be further considered. And the issue of how to guarantee the reliability is another practical challenge.



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**Résumé étendu:** Ce travail de thèse se situe dans la continuité du travail mené dans notre équipe depuis de nombreuses années autour de la commande distribuée d'un système multi-robots. Cette thèse s'intéresse plus particulièrement au cas d'un groupe de robots mobiles non-holonomes de type unicycle évoluant en formation. L'objectif étant de créer la formation désirée, avec son centriole se déplaçant le long d'une trajectoire de référence dynamique et pouvant être spécifié par un leader virtuel ou réel, et de la maintenir durant le déplacement. Dans ce travail, de nombreux problèmes ont été considérés, notamment la stabilité en temps fini, la stabilité en temps fixe, la communication et le mécanisme de contrôle de l'évènement déclenché (Event Triggered Control). Cette thèse se décompose de 5 chapitre.

Le premier chapitre est dédié à l'introduction, contexte, motivations et un état de l'art concernant le contrôle en formation, la stabilité d'un système multi robots. Enfin des outils mathématiques relatifs à la théorie des graphes ou encore la modélisation des robots mobiles unicycles sont rappelés. La suite du mémoire est scindée en deux parties, la première est consacrée à la stabilité des systèmes multi-robots en formation.

Dans le chapitre 2 on étudie la stabilité à temps fini des systèmes multi-robots en formation. Un contrôleur basé sur un observateur distribué est développé pour chaque robot. La

stabilité à temps fini de l'observateur et du contrôleur est analysée en utilisant la méthode directe de Lyapunov, la théorie des graphes algébrique et l'analyse matricielle. L'estimation du temps de stabilisation proposée dépend fortement des conditions initiales.

Afin de supprimer cette dépendance inattendue, une nouvelle classe de contrôleur à temps fini est proposée dans le chapitre 3, également appelé contrôleur à temps fixe. Nous avons pu obtenir une estimation théorique moins conservatrice de la limite supérieure du temps de stabilisation indépendamment des conditions initiales. Ce qui a permis de pouvoir contrôler le temps de convergence de façon plus précise et indépendante.

Afin d'étudier l'effet dynamique pour les systèmes multi-robots en formation, nous étudions les systèmes dynamiques de suivi de formation de multi-robots non holonomiques dans le chapitre 4. Un ensemble de contrôleurs à temps fixe distribués et quelques conditions suffisantes sont dérivés à l'aide de la technique du mode coulissant et de la théorie de Lyapunov.

Dans la deuxième partie on s'intéresse au mécanisme de communication et de contrôle déclenché par l'événement (Event Triggerred Communication and Control) d'une formation multi-robots non-holonomes. Tout d'abord, un nouveau type de contrôleur déclenché par événement distribué est proposé sous topologie de communication inter robots fixe puis variable. Puis, une condition d'événement associée, qui n'a besoin que d'une communication

intermittente entre les robots voisins pour la détection d'événement et la mise à jour du contrôle, est conçue pour faciliter l'exécution des contrôleurs distribués proposés. En fonction de l'état de l'événement, les systèmes de formation des multi-robots réduisent efficacement les coûts de communication et la consommation d'énergie et l'usure mécanique, en particulier quand la formation comporte un nombre élevé de robots.

De plus, afin de développer un schéma d'implémentation numérique, nous proposons une autre classe de contrôleur périodique événementiel basée sur un observateur à temps fixe est proposé dans le chapitre 6. Il convient de noter que deux types différents de conditions d'événements sont étudiés en détail. Contrairement à la plupart des conditions continues existantes, seule la communication sans fil périodique ou aperiodique est nécessaire pour la mise à jour de contrôle et la détection d'événements. Cette approche entraîne une diminution significative des couts de communication et de la consommation d'énergie.

Enfin une conclusion est donnée en reprenant les contributions et en proposant quelques perspectives de travail.

## **Commande distribuée, en poursuite, d'un système multi-robots non holonomes en formation**

**Résumé:** Cette thèse s'intéresse plus particulièrement au cas d'un groupe de robots mobiles non-holonomes de type unicycle évoluant en formation. Dans ce travail, de nombreux problèmes ont été considérés, notamment la stabilité en temps fini, la stabilité en temps fixe, la communication et le mécanisme de contrôle de l'évènement déclenché (Event Triggered Control). Tout d'abord, on étudie la stabilité à temps fini des systèmes multi-robots en formation. Un contrôleur basé sur un observateur distribué est développé pour chaque robot. La stabilité à temps fini de l'observateur et du contrôleur est analysée en utilisant la méthode directe de Lyapunov, la théorie des graphes algébrique et l'analyse matricielle. Mais l'estimation du temps de stabilisation proposée dépend fortement des conditions initiales. Afin de supprimer cette dépendance inattendue, une nouvelle classe de contrôleur à temps fini est proposée, également appelé contrôleur à temps fixe. Afin d'étudier l'effet dynamique pour les robots, nous étudions les systèmes dynamiques de suivi de formation de multi-robots non holonomiques. Pour des raisons pratiques, on s'intéresse au mécanisme de communication et de contrôle déclenché par l'évènement d'une formation multi-robots non-holonomes. De plus, afin de développer un schéma d'implémentation numérique, nous proposons une autre classe de contrôleur périodique événementiel basée sur un observateur à temps fixe est proposé. Il convient de noter que deux types différents de conditions d'évènements sont étudiés en détail.

**Mots-Clefs:** Systèmes multi-robots, Contraintes non-holonomes, Contrôle de formation, Stabilité à temps fini, Contrôle déclenché par l'évènement, Théorie des graphes, Lyapunov.

## **Distributed Tracking Control of Nonholonomic Multi-Robot Formation Systems**

**Abstract:** The main aim of this thesis is to study the control problem of distributed formation tracking for nonholonomic multi-robot systems. In this context, we consider several problems, ranging from finite-time stability and fixed-time stability, event-triggered communication and control mechanism, kinematics and dynamics, to continuous-time systems and hybrid systems. The formation tracking control problem is solved in this thesis through the design of different practical controllers with faster convergence rates, higher control accuracy, stronger robustness, more explicit and independent estimation for the upper bound of settling time, less communication costs and energy consumptions rather than most existing results in literature. To this end, we first study the problem of finite-time stability for multi-robot systems. The distributed observer-based controller is developed for each robot. A formula for the upper bound estimation of the settling time, which strongly depends on the initial conditions, is derived. Furthermore, to remove this unexpected dependence, a novel class of finite-time controllers is proposed. In addition, in order to investigate the effect of robotic dynamics, we solved the design problem of fixed-time controller with respect to the dynamical models of robots. Further, for practical reasons, we went on to study event-triggered communication and control mechanism of cooperative multiple nonholonomic mobile robot team based on continuous-time sampling. Moreover, with a view to develop a digital implementation scheme, we propose another class of periodic event-triggered controllers based on fixed-time observers. Two different types of event conditions are analyzed in detail.

**Keywords:** Multi-robot systems, Nonholonomic constraints, Formation control, Finite-time stability, Event-triggered control, Algebraic graph theory, Matrix theory, Lyapunov theory.

