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Pierre Aldama

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UNIVERSITÉ PARIS 1 PANTHÉON-SORBONNE
UFR de Sciences économiques
Centre d'Economie de la Sorbonne

THÈSE

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Essays on fiscal policy and public debt sustainability

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*À tous mes professeurs de Sciences Economiques et Sociales,
et à mon père, le premier d'entre eux.*

“La science, dans son besoin d'achèvement comme dans son principe, s'oppose absolument à l'opinion. S'il lui arrive, sur un point particulier, de légitimer l'opinion, c'est pour d'autres raisons que celles qui fondent l'opinion ; de sorte que l'opinion a, en droit, toujours tort. L'opinion pense mal ; elle ne pense pas : elle traduit des besoins en connaissances. En désignant les objets par leur utilité, elle s'interdit de les connaître. On ne peut rien fonder sur l'opinion : il faut d'abord la détruire.”

Gaston Bachelard, La formation de l'esprit scientifique

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Introduction

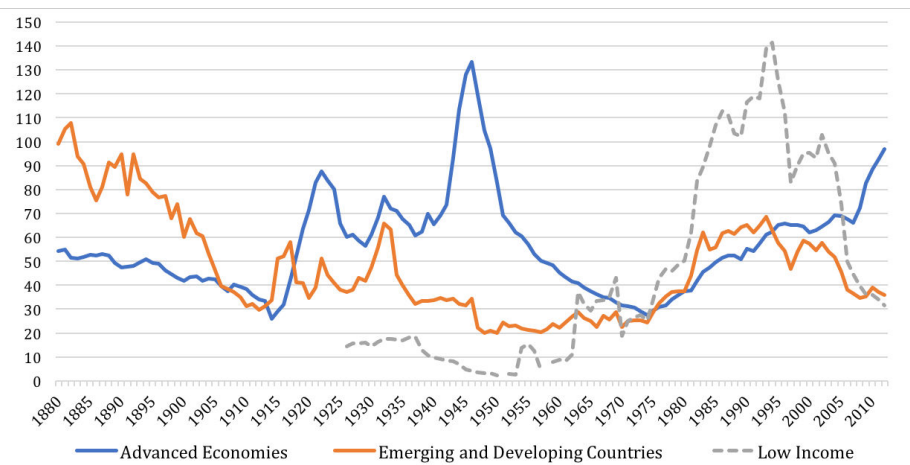
This thesis aims to contribute to the analysis of fiscal policy and public debt sustainability in macroeconomics. This research's first motivation is inevitably empirical. Following the Global Financial Crisis and the Great Recession, advanced economies have experienced an *historically significant* increase in their public debt-to-GDP levels by 30 percentage points, between 2007 and 2012, see figure 1.¹ The gradual increase of public debt-to-GDP ratios in advanced countries actually started well before 2007, at the end of the 1970s. At the time, most of advanced economies' central banks turned progressively toward disinflation policies, notably implying a sharp increase of *ex post* real (long-term) interest rates while the growth rate of real GDP generally stayed steady or even slowed. As a result, fiscal requirements to stabilize the level of public debt² mechanically increased while fiscal policies not necessarily met them, mainly explaining the gradual build-up in public debts in advanced economies.

There is no coincidence that the sustainability of public debt became a *specific* research agenda in macroeconomics as well in public economics at about the same time. For instance, the seminal paper by Hamilton and Flavin (1986) in fiscal sustainability analysis was first published as a NBER working paper in June 1985, while there was a growing concern whether current US fiscal policy was on a sustainable path. In the context of the Cold War refreeze, following the invasion of Afghanistan by the USSR in 1979, US military spending significantly increased, leading to persistent primary deficits *coupled with* higher growth-adjusted real interest rates and causing an increase in the Federal debt-to-GDP ratio. Following the approach of Hamilton and Flavin, Wilcox (1989) made this concern explicit in his paper:

¹Still, it is worth noting that emerging and developing as well as low income countries did not experienced such a surge in public debt levels, which are rather progressively decreasing since the 1990s.

²That is the debt-stabilizing primary surplus, easily calculated as the growth-adjusted real interest rate $(r_t - y_t)b_{t-1} / (1 + y_t)$ multiplied by the stock of public debt-to-GDP.

FIGURE 1 – Public debt-to-GDP, weighted average levels in percentage of PPP-GDP (1880-2012)



Source: Historical public debt database, IMF, 2017.

(...) the experience of the 1980s stands out as unprecedented in peacetime history, and raises the issue of sustainability: can the Federal government continue to operate the current fiscal policy indefinitely? (p. 291)

The same remark would probably apply to the literature on strategic default *à la* Eaton and Gersovitz (1981) following the sovereign debt crisis in emerging economies which started in the late 1970s, or to the academic interest for the *Fiscal Theory of Price-Level* in the 1990s at the time the need of Maastricht Treaty's fiscal rules for inflation stability in the EMU was discussed (Sims, 1999; Woodford, 1996). Hence, the research agenda in the macroeconomics of public debt and fiscal policy cannot be separated from historical and political developments and, naturally, this thesis makes no exception.

More generally speaking regarding policymaking and economic debate outside the academic circle, this thesis was also motivated by the personal conviction that issues about public debt and fiscal policy are still too often *primarily* treated from a moral and/or political standpoint. Increasing public debts, deficits are too often seen as dangerous or immoral, which lead some policymakers and politicians to support the adoption of strictly balanced-budget rules ("zero-deficit" rules), even if applied macroeconomic research has provided strong theoretical arguments against such fiscal rules. *Normative* judgments about public debts and deficits come too often before, and sometimes even evict, *positive* questions. A positive approach would start with the following questions: what are the requirements for

fiscal sustainability? How does it depend on the current monetary-fiscal policy mix, on the dynamic (in)efficiency of the economy? Do governments generally meet them, according to the data? What could happen if they do not?

Both theoretical and empirical researches have shown how these questions are "*darned hard*", to use the words of Leeper (2015). Fiscal requirements can be, to some extent, significantly weaker than commonly accepted. In particular, this thesis builds on the idea that governments can –for bad or good reasons– deviate from them in the short-run, without necessarily violating them in the long-run. Going to the data, statistical identification of fiscal policy stance and objectives remains imperfect and, specifically regarding the empirical part of this research, we would never claim to be exempt from any endogeneity bias in our empirical strategy, but we rather fully acknowledge it. In our defense, econometric techniques used in this research (mainly, Markov switching dynamic regressions) do not have a clear and well-established framework to deal with endogenous regressors, contrary to constant-parameters models.³ Finally, this research shares the general statement made by Leeper (2010) about respective approaches of monetary and fiscal policies. While monetary policy receives a systematic, consistent and evidence-based analysis of its objectives, instruments and effects –which Leeper calls "science"–, fiscal policy is still too often alchemy, based on unsystematic and politically-grounded analyses. And this is especially true regarding the issue of public debt.

First, we start with a general overview of the literature about fiscal policy and public debt sustainability. Then, we briefly present the motivations, contributions and results of each chapter.

General overview of the literature on fiscal policy and public debt sustainability

Fiscal sustainability analysis starts from the government present-value budget constraint (PVBC, henceforth), under the assumption of a dynamic efficient economy. Hence, the

³In fact, there is only one reference on the subject which proposes a two-step method based on the control function approach, see Kim (2010).

PVBC implies that initial stock of public debt *must be paid back* by future expected present-value primary surpluses, in a stochastic framework, that is formally:

$$\text{Public debt}_{t-1} \leq \sum_{k=0}^{\infty} \text{EPV}(\text{Primary budget surplus})_{t+k} \quad (1)$$

Equation (1) implies a transversality condition: in a stochastic infinite-horizon economy, it means the expected present value of public debt must be zero or negative in the long-run, that is:

$$\lim_{T \rightarrow \infty} \text{EPV}(\text{Public debt})_{t+T} \leq 0 \quad (2)$$

This transversality condition is generally called the No-Ponzi Game condition. Bohn (1995) notably shows equations (1) and (2) must hold with equality in presence of rational optimizing agents, to prevent both lenders and government from playing a Ponzi Scheme against each other. But above all, he provides important clarifications about the correct choice of *discount factor* to write intertemporal budget constraint in a stochastic economy, as we will see further below.

Regarding dynamic (in)efficiency, Diamond (1965) has famously shown that equations (1) and (2) are no longer binding constraints on fiscal policy in a dynamically inefficient economy, when the marginal productivity of capital is lower than the output growth rate. Hence, evidence of dynamic inefficiency should lead us to conclude that public debts are sustainable and should increase until the economy becomes dynamically efficient. In their seminal paper, Abel et al. (1989) provided a methodology to assess dynamic efficiency. First, they argue dynamic efficiency should be assessed by comparing the *risky* real interest rate with the growth-rate of real GDP in a stochastic economy; in particular, dynamic efficiency would imply a *positive* growth-adjusted risky real return on capital. This is worth noting since the growth-adjusted *safe* real interest rate on public debt is quite often *negative* and we could be tempted to conclude that Ponzi Schemes are possible and optimal because the economy has over-accumulated capital. Second, to answer the puzzle about choosing the *right* real rate, they provide an operational testing framework by comparing gross investment to gross capital income: a dynamic efficient economy should imply that investment is *lower* than capital income. Finally, they found evidence that seven advanced OECD

economies were dynamically efficient.

This result has recently been overturned by Geerolf (2013). Using a larger and richer dataset for OECD economies, Geerolf provides empirical evidence that "the condition for dynamic efficiency is not verified for any advanced economy" and that South-Korea and Japan have clearly over-accumulated capital. Following Diamond (1965), we could argue this "world savings glut" could make the case for a global increase of public debts. Geerolf (2013) argues it would justify the implementation of pay-as-you-go pension schemes for instance. At least, these findings suggest that the concerns about public debt overhangs in advanced economies may be, to some extent, *overstated* in the context of an excess in global savings.

Still, under the implicit assumption our economies are dynamically efficient, first attempts to test fiscal sustainability aimed at deriving sufficient constraints on macroeconomic fiscal variables such that the government's intertemporal budget constraint is satisfied in the long-run. In their seminal paper on fiscal sustainability, Hamilton and Flavin (1986) build their testing procedure on the present-value budget constraint (PVBC) in the following framework:

$$B_{t-1} = \sum_{i=t}^N \frac{\mathbb{E}_t S_i}{(1+r)^{i-t}} + \frac{(1+r)^t \mathbb{E}_t B_N}{(1+r)^N} \quad (3)$$

where B_t is the real stock of outstanding debt, S_t the real primary surplus including seigniorage revenue and r is the real safe interest rate on public debt. Hamilton and Flavin define

$$\lim_{N \rightarrow \infty} \mathbb{E}_t \frac{B_N}{(1+r)^N} \equiv A_0$$

then rewrite the PVBC as:

$$B_t = \sum_{i=t}^{\infty} \frac{\mathbb{E}_t S_i}{(1+r)^{i-t}} + (1+r)^t A_0 \quad (4)$$

Thus, the transversality condition should imply $A_0 = 0$. Thus, Hamilton and Flavin propose three different procedures to test for $A_0 = 0$. The first one is unit-root testing: assuming that primary surplus is stationary, then a non-stationary public debt implies $A_0 > 0$ and violates the PVBC. Then they propose two additional specifications, based on Flood and Garber (1980)'s testing procedures for self-fulfilling bubbles. Hamilton and Flavin's

findings concluded US federal fiscal policy was sustainable, against the common wisdom. Hence, unit-root and stationarity tests were firstly motivated by the No-Ponzi Game condition and not by additional considerations such as, for instance, fiscal limits.

Answering to Hamilton and Flavin, Wilcox (1989) shows that a non-stationary primary surplus, and a non-stationary public debt though, does not necessarily violate the PVBC. Rather than testing the stationarity of public debt, Wilcox proposes to test if *discounted* public debt is a stationary autoregressive process with unconditional mean equal to zero. Using actual interest rates on public debt as discount factor, Wilcox relaxes two restrictive assumptions made by Hamilton and Flavin. First, he assumes real interest rate to be stochastic. Secondly, he assumes that violations of the PVBC can be stochastic, while there were supposed to be non-stochastic in Hamilton and Flavin. Thus, Wilcox aims to test directly if discounted debt converges to zero in the long-run, i.e. the exact implication of the PVBC.

Trehan and Walsh, 1988; Trehan and Walsh, 1991 extend this analysis in a different way, testing if the with-interest deficit is a stationary variable (i.e. the case $B_t \sim I(1)$), or equivalently if government spending, inclusive of interest and government revenue, inclusive of seignorage revenue are cointegrated. Quintos (1995) weakens the econometric requirements of the PVBC: he shows that a difference-stationary with interest deficit (i.e. the case $B_t \sim I(2)$) is sufficient for the PVBC.

We label this research agenda the "econometric analysis" of fiscal sustainability. This approach takes advantage of being easily reproducible, as long as good data are available. A very nice example of the econometric analysis of fiscal sustainability is Afonso (2005). Unfortunately such a general definition may be far too general to dismiss unsustainable fiscal policies. One reason might be the lack of economic theory. Most criticisms of this research agenda come from Bohn (1995, 1998, 2007, 2008).

First, the econometric analysis always requires restrictive assumptions on real interest rate, even in Wilcox's analysis. Bohn (1995) shows that, in any stochastic economy, the PVBC *should* be written using the model-based *stochastic discount factor* which is the common pricing kernel for all financial assets, under the complete market hypothesis. Bohn's main argument against the econometric analysis is that it often use a constant safe interest rate (Hamilton and Flavin, 1986; Quintos, 1995; Trehan and Walsh, 1988; Trehan and Walsh,

1991) or the actual interest rate (Wilcox, 1989) rather than the stochastic discount factor to test for sustainability. These assumptions are restrictive in the sense it is equivalent to assume that lenders are risk-neutral and/or there is no uncertainty. Incorrect discounting, Bohn argues, can lead to absurd results. For instance, in an economy where the safe real interest rate is always below the real rate of growth, a fiscal policy which maintains G_t/Y_t and B_t/Y_t constant violates its PVBC if one uses the safe interest rate rather than the model-based stochastic discount factor. Indeed, a constant debt-to-GDP ratio would imply that public debt grows faster than the safe interest rate, that is, fiscal policy apparently running a Ponzi Scheme.⁴ For these reasons, Bohn (2008) and Mendoza and Ostry (2008) label this analysis "*ad hoc* sustainability".

Second, unit-root tests on public debt-to-GDP and cointegration tests between spending and revenues are probably misleading, for two reasons. Bohn (1998) argues that ignoring cyclical components of primary surplus induces an omitted variable bias in unit-root testing. Basically, based on Barro (1979) tax-smoothing model, fiscal policy intertemporally smooths the tax rate to minimize tax collection costs. As a consequence, primary surplus has two main cyclical components: output gap and government cyclical spending (war-time spending, or simply the cyclical component of government spending). Then, from the instantaneous flow budget constraint, it appears that standard Dickey-Fuller or Phillips-Perron regressions may have an omitted variable bias as long as output gap and cyclical government spending are not used as exogenous regressors. Controlling for these variables, Bohn shows that it is possible to reject the unit-root hypothesis on US long-run data: he provides evidence that US public debt-to-GDP is actually a mean-reverting process.

Moreover, Bohn (2007) formulates a more general and powerful criticism: usual econometric restrictions⁵ are not even necessary conditions for the PVBC. For any arbitrary high m , a m -th order integrated debt-to-GDP ratio is still satisfying the PVBC. The proof is rather simple and immediate: for any discount factor $\rho < 1$, the transversality condition is ex-

⁴Actually, when the safe real interest rate on public is below the growth-rate of real GDP, a stable debt-to-GDP does not provide any information about the sustainability of public debt, in a dynamically efficient economy. If the growth-adjusted safe real rate is sufficiently negative, a government can very well run a Ponzi Scheme against its creditors *and* stabilize its public debt level.

⁵Restrictions such that: the public debt must be stationary or difference-stationary (i.e. stationary with-interests deficit, cointegration between revenues and spending) or at least integrated of order two (weak sustainability).

ponential in the time horizon n and the conditional expectation of a m -th order integrated variable is at most a polynomial of order m . Since exponential growth dominates polynomial growth in the long-run, given $\rho < 1$, then the transversality would hold. Therefore, unit-root or cointegration tests are *incapable* of rejecting sustainability. Still, this proposition is a kind of "absurdly weak" sustainability because debt-to-GDP would violate any upper bound, implied by a fiscal limit on primary surplus for instance, as Bohn (2007) acknowledges. As a result, stationary public debt-to-GDP ratios and cointegration restrictions find new justifications (Daniel and Shiamptanis, 2013).

To overcome some of these caveats of the econometric analysis of public debt sustainability and, somehow, to fill the gap with macroeconomic theory, Bohn (1998) has proposed to rather study linear fiscal policy rules (also called fiscal reaction functions). In particular, by modeling the behavior of fiscal policy through a feedback rule similar to monetary Taylor rules, Bohn established a simple, elegant condition on the feedback effect of initial public debt on primary surplus, such that the No-Ponzi Game condition holds. A detailed presentation of Bohn's contribution, labeled Model-Based Sustainability (MBS, henceforth) will be proposed at the beginning of chapter 1.

More generally speaking, the literature on fiscal sustainability and fiscal policy rules has progressively closed the gap with macroeconomics of sovereign default risks and monetary-fiscal interactions, see Bi (2012), Bi and Traum (2012), Daniel and Shiamptanis (2012, 2013), Davig et al. (2011), Guillard and Kempf (2017), Leeper (2013), and Uribe (2006, among others). This thesis starts from Bohn (1998) and recent developments about nonlinearities in fiscal policy behavior and fiscal limits.

Outline of the thesis

This thesis is composed of four chapters; each can be read independently from the others and they are not necessarily linked to each other, except the fact they all contribute to the macroeconomics of fiscal policy and public debt sustainability. Each of them include a specific literature review which completes the previous general overview; in particular, chapter 4 is an extensive survey of the literature about fiscal sustainability, monetary-fiscal

interactions, macroeconomic effects of fiscal policy shocks and the economics of monetary unions. Chapters 1 and 2 are closely related, hence we present them in a unique section; yet, they can nonetheless be read independently. Chapters 3 and 4 are mutually independent and will be presented in separated sections.

Why fiscal regimes matter for fiscal sustainability

Chapters 1 and 2 are respectively the theoretical and empirical sections of Aldama and Creel (2017). They contribute to generalize Bohn's sustainability condition to a specific non-linear framework and present an empirical application of the Regime-Switching Model-Based Sustainability. This condition was already known to be robust to non-linear specifications, such as quadratic or kinked functions (Bohn, 1998) or time-varying and regime-switching specifications (Canzoneri et al., 2001). Still, regarding time-varying specifications, there was no formal and explicit testing framework for sustainability; the closest paper to this idea was Canzoneri et al. (2001) but, as argued in chapter 1, they considered a quite particular case, making fiscal sustainability requirements very weak, and did not propose an explicit testing framework.

Hence, chapter 1 starts from the idea that governments cannot (or may not willing to) follow a sustainable fiscal policy, that is, satisfying Bohn's condition that primary surplus increase with the level of initial outstanding public debt. Hence, non-linearities arise from recurrent and persistent episodes of local unsustainability of fiscal policy. This could be, for instance, an additional explanation of prolonged periods of public debt build-ups. The analysis starts in the worst-case scenario for fiscal policy: we assume a real stochastic, purely Ricardian economy in which government issues real debt and cannot expect debt repudiation through inflation. Of course, this thesis does not deny that monetary policy would play a significant role on fiscal sustainability in a more general framework, but rather builds the hardest benchmark for regime-switching fiscal sustainability we could think of. As we will argue further, any empirical evidence in favor of sustainability, based on this benchmark, could be considered as credible and robust evidence, *all other things equal*. Finally, we assume dynamic efficiency through the assumption of finite present-value of output; as we discussed earlier, dynamic inefficiency would make sustainability irrelevant.

We specify a regime-switching fiscal policy rule that admits two regimes: an unsustainable regime during which fiscal policy deviate from Bohn's principle and a sustainable regime. We consider two alternative concepts of sustainability: the No-Ponzi Game condition, derived from the government PVBC and the Debt-Stabilizing condition, justified by the stationarity requirements for public debt when primary surplus admits an exogenous fiscal limit on its primary surplus. We will argue the latter condition is not a substitute to the former, which reinforces the criticism of testing the stationarity of public debt-to-GDP ratio as a sufficient measure of fiscal sustainability. This chapter makes two main propositions. Our contribution is to propose *explicit* and *testable* sufficient conditions, for each concept of sustainability, on regime-specific feedback parameters associated to the level of public debt *but also* on the ergodic probabilities of each regimes⁶. An intuitive interpretation of these conditions can be made in terms of expected duration of regimes. In particular, the longer unsustainable regimes –relatively to sustainable regimes– the larger the required reaction of primary surplus to initial debt. Consequently, *local* unsustainability does not necessarily imply *global* (or long-run) unsustainability.

From the empirical point of view, recurrent and persistent unsustainability regimes may be good candidates to explain gradual and persistent build-ups of public debts, which often lead to conclude to the non-stationarity of public debt in empirical studies.

The existence of fiscal regimes have significant consequences on the empirical properties of public debt-to-GDP and primary surplus-to-GDP ratios. Prolonged periods of explosive public debt dynamics coupled with stationary primary surplus may not necessarily imply fiscal unsustainability, as we show in chapter 1. Moreover, neglecting regime-switching could eventually result in biased estimates of fiscal reaction functions because of econometric misspecification; this was already suggested by Favero and Monacelli (2005)'s empirical work. A real difficulty in empirical studies of fiscal reaction functions is that primary surplus and public debt do not generally have the same *persistence*, or worse, are not integrated of same order (Lamé et al., 2014). As a result, constant-parameter estimates usually have trouble to find a positive and significant correlation between primary surplus and lagged

⁶Theoretically, there would be no restriction to assume n regimes: it would not change the following analysis since we could still consider an *aggregated* unsustainable regime of k regimes associated to an aggregated ergodic probability, and respectively the same for the aggregated sustainable regime.

public debt, in country-specific estimates⁷.

Chapter 2 suggests taking into account fiscal regimes may be a solution. The idea is the following. During unsustainable regimes, there would not be any cointegration between primary surplus and lagged debt or possibly a *negative* cointegration relation. Yet, under sustainable regimes, we argue *both* time series would likely be stationary and positively correlated. Allowing for non-linearities in the correlation of these time series may help statistical inference about the long-run correlation between the two fiscal variables. Hence, we consider the case of France's fiscal policy from 1962 to 2013 which is often discussed as being potentially unsustainable, given persistent primary deficits and increasing public debt and despite historically low real interest rates. We estimate various specifications of standard fiscal policy rules *à la* Bohn. The baseline estimate yields no evidence of fiscal sustainability. Trying to overturn initial results, we control for non-linearities due to the level of public debt with a quadratic specification and take into account the exceptional degradation of primary surplus since 2009 using a dummy variable. None of these specifications yield significant evidence in favor of sustainability, hence confirming some previous empirical studies about the unpleasant dynamics of France's public debt.

Taking these constant-parameter estimates as a benchmark, we estimate the baseline fiscal policy rule using a Markov-switching dynamic regression allowing for two regimes. We identify a significantly sustainable regime and a virtually unsustainable one, in which primary surplus is not significantly correlated to lagged public debt. Identified regimes are quite plausible given historical knowledge about France's fiscal policy: while the 1980s are identified as a prolonged period of unsustainability, the period starting from 1996 with the convergence toward SGP fiscal requirements prior to EMU creation until the financial crisis (2008) is identified as a sustainable regime. The recent degradation of primary surplus (2009-2013) is identified as an unsustainable regime. Over the whole sample, 1965-2013, estimated Markov-switching fiscal policy rule for France successfully passes the Regime-Switching MBS test, both for No-Ponzi Game and Debt-Stabilizing conditions.

⁷To some extent, this problem can be overcome using panel-data techniques (Daniel and Shiamptanis, 2013; Weichenrieder and Zimmer, 2014). Yet, by estimating panel-data models, we lose the ability to conclude about the sustainability or unsustainability of a specific country. Panel-data techniques would be relevant, for instance, to test for the overall fiscal sustainability of a multiple-country monetary union without federal debt, such as the EMU

These results calls two comments at least. First, long-run average public debt-to-GDP ratio can be really high, given these estimates, because of the high persistence of the unsustainable regime. Concerns about French public debt reaching high levels, at which sovereign default risk becomes significant, may be justified if France's fiscal policy continues to experience long lasting periods of unsustainable fiscal policy in the future. Second, these estimates do not deal with the potential endogeneity biases –and this problem should be addressed in future research. Still, in our defense, econometric techniques allowing to deal with endogenous regressors in dynamic regression models are recent and not yet well established. Furthermore, the most serious potential bias is likely the simultaneity bias between primary surplus and output gap, due to positive and significant fiscal multipliers. Hence if one agrees on this, the Keynesian effects of primary surplus on output gap would most likely induce a downward bias in the estimates of the feedback effect of public debt. In a nutshell, controlling for endogeneity in the output gap may play in favor of fiscal sustainability.

Fiscal sustainability, default and long-term debt: The longer, the safer?

Chapter 3 contributes to the literature on non-strategic sovereign defaults. This approach evaluates fiscal sustainability through the concept of "fiscal space", the financial leeway a government has to react to bad macroeconomic shocks. The literature on fiscal sustainability progressively moved toward sovereign default analysis in the light of the sovereign debt crisis in the EMU. The reason might be that both econometric and Model-Based Sustainability approaches did not provide an operational framework to gauge fiscal sustainability in highly indebted advanced countries. Even Greece might, to some extent, have satisfy the No-Ponzi Game condition and even a Debt-Stabilizing condition (Collignon, 2012; Lamé et al., 2014; Mendoza and Ostry, 2008). Hence, this chapter builds on the idea governments face a fiscal limit on their primary surplus such that

$$s_t \leq s_t^{max} \tag{5}$$

which is formally justified from distortionary taxation and the existence of a dynamic Laffer curve⁸. As a result, government would eventually reach its fiscal limit and would no longer be able to stabilize public debt if bad macroeconomic shocks hit the economy. This is the general idea behind most of the recent papers about non-strategic sovereign defaults (Bi, 2012; Bi and Leeper, 2013; Bi and Traum, 2012; Daniel and Shiamptanis, 2013). An alternative to assuming a structural fiscal limit is to consider a non-linear fiscal rule displaying "fiscal fatigue" as suggested by Ghosh et al. (2013a,b); but this chapter rather embraces the former.

Regarding debt limits, does longer debt maturity increase fiscal space and public debt sustainability? Policymakers dealing with sovereign debt restructuring generally advocate to increase the maturity of public debt to help the country resolve its insolvency. In practice, it has the double advantage of smoothing the fiscal burden over time and lowering the gross financing needs of government while avoiding a straight nominal haircut on bonds for private investors. From a general perspective, we ask whether a longer maturity can help to prevent default by increasing the debt limit.

As far as we are aware of, Kim (2015) is the only one to treat this question specifically. In a partial equilibrium model with risk-neutral investors, he shows that debt maturity increases the debt limit when the primary balance is uncertain; but once primary balance is deterministic debt limits for one-period debt and long-term debt are strictly equal. Kim explains his findings by saying potential good shocks on *future* primary balance increase the *current* price of long-term debt and reduce the pace of public debt accumulation with respect to one-period debt.

This chapter extends the analysis of debt limits to longer maturities of public debt, following the work of Guillard and Kempf (2017). In the model, a representative investor faces a government who strictly seeks to stabilize the post-default public debt-to-GDP ratio. Bad productivity shocks (or potential growth shocks, equivalently) eventually make the fiscal limit bind. In this *constrained regime*, default would be a demand-driven market event when gross financing needs are larger than the market value of long-term debt, which is the total amount the government can borrow from the market. Following Guillard and Kempf we

⁸Additional justifications may argue that the inertia of public spending also imply a minimum level of expenditures/GDP, see chapter 4.

solve for the maximum market value of long-term debt in function of the recovery rate and the maturity of long-term debt, using the Euler relation on long-term debt. Our results are twofold. First, we confirm Guillard and Kempf's findings that the maximum market value of debt is time-invariant and depends positively of the recovery rate. Second, we show that this maximum is *independent* from the maturity of public debt. As a consequence, we show the equilibrium stochastic default threshold does not depend on the maturity structure of long-term debt. We interpret this result as a clarification of the benefits of long-term regarding fiscal sustainability. While long-term debt may help the government to insulate itself from bad fiscal shocks on the primary balance, as in Kim (2015), longer maturities of public debt do not provide any insulation against bad potential growth shocks, in a frictionless economy with perfect financial markets.

Public debts and fiscal policies in the European Monetary Union

Chapter 4 is a survey on public debt sustainability and fiscal policy the context of the European Monetary Union; it is an augmented and revised version of Aldama (2017).

Over the last three decades, both macroeconomists and policymakers involved in fiscal policy have mainly focused on long-run issues. The consensus was government should adopt rules ensuring the long-run sustainability of public finance, and let an independent central bank take charge of controlling inflation and stabilizing GDP growth and unemployment. The design of the EMU and its fiscal union directly hinges on this paradigm. But to a large extent, the Great Recession has challenged this view, and (partially) restored fiscal policy as a powerful macroeconomic stabilization instrument, while insisting on long-run fiscal sustainability requirements.

This chapter builds on the idea that a sharing a currency always implies a fiscal union because of the multiplicity and specificity of fiscal requirements. For instance, fiscal sustainability is no longer uniquely considered at country-level but also at the Monetary Union level. Whether inter-national or federal transfers exist do not change this fact: the consolidated monetary union intertemporal budget constraint (IBC) implies that primary deficits in some countries must be financed by primary surpluses in some others; transfers, whether inter-national or federal, would be additional tools to ensure the consolidated monetary

union IBC holds. If not, the whole fiscal sustainability of the monetary union could be jeopardized: this mechanism could explain contagion effects, or inflation instability at the monetary union level according the FTPL. First, the chapter presents fiscal sustainability requirements in monetary unions, depending on fiscal union's architecture. We notably consider three possibilities: (i) an "ordoliberal" monetary union with no transfers and only strict fiscal rules (ii) a inter-national fiscal union with transfers and fiscal rules but no federal fiscal authority and finally (iii) a fiscal federation where federal transfers replace international transfers. We also extend the discussion to the monetary-fiscal interactions and the Fiscal Theory of Price-Level, which has intensively discussed fiscal requirements for the control of inflation in monetary unions. The main takeaway of the FTPL's analysis is that fiscal sustainability constraints matter for the control of inflation by an independent central bank in normal times *but* also in *bad times*. In particular, the FTPL makes a strong case for *using aggressively* fiscal policy to fight deflation. Lastly, we draw the implications of these various analyses for the design of fiscal rules, as well as recent developments about fiscal limits and "prudent" fiscal rules and finally present the main theoretical results about the optimal monetary-fiscal policy mix in monetary unions.

In a second part of the chapter, we propose a critical appraisal of the European fiscal framework, based on theoretical and empirical research. We start by briefly reviewing the history of European fiscal rules. Then, we ask whether European fiscal rules are sufficient to ensure fiscal sustainability or not, from a theoretical and empirical perspective. In our opinion, the worst problem of these rules are the implied procyclicality in the effective stance of European fiscal policies, despite the will of the Euro-Area policymakers to allow for more flexibility and countercyclicality. As a result, we argue the European fiscal framework may be both too tight and too loose: too tight because fiscal sustainability requirements are lower than those embodied in the SGP, too loose since procyclicality would result in less stabilizing policies and, as a result, in higher debt-to-GDP ratios. We conclude by discussing the causes of the European Sovereign debt crisis and the proposal of Eurobonds to complete the European fiscal union.

Chapter 1

Fiscal Regimes and Public Debt Sustainability: A Regime-Switching Model-Based Sustainability Test

Co-authored by Jérôme Creel

1.1 Introduction

Fiscal policy rules describing the reaction of primary balance to the initial level of public debt have long been used to analyze fiscal sustainability. According to Bohn (1998)'s seminal contribution, primary public balance must increase after an increase of the public debt-to-GDP ratio to ensure sustainability, as defined by the respect of the government intertemporal budget constraint. This chapter is motivated by the empirical evidence of fiscal episodes during which public debt-to-GDP is non-stationary and generates no improvement in primary public balance. Under these episodes, fiscal policy *periodically* violates Bohn's sustainability condition and thus raises critical questions on the long-run fiscal sustainability: is a periodically unsustainable fiscal policy a threat to long-run sustainability of public finance? How long can fiscal policy be periodically unsustainable without violating its sustainability constraints in the long-run?

To our knowledge, only a few papers have addressed a regime-switching (or time-varying) fiscal policy rule while also proposing a testing framework for long-run sustainability. In

their seminal contribution Canzoneri et al. (2001) consider a time-varying fiscal policy rule and derive a necessary and sufficient condition such that the government intertemporal budget constraint holds in the long-run. Davig (2005) extends Wilcox (1989)'s unit-root testing procedure to a Markov-switching framework in which discounted debt can be periodically expanding. Finally, there is a literature on regime-switching monetary and fiscal policy rules that has successfully identified *local* equilibria in the data where fiscal policy (or monetary policy) is either "active" or "passive", following Leeper (1991). Still, these papers do not test whether fiscal policy *globally* satisfies the intertemporal budget constraint or the debt-stabilizing criterion in the long-run. Based on a Markov-switching monetary policy rule, Davig and Leeper (2007b) have proposed a long-run Taylor principle such that the price-level is *globally* determined despite periodic violations of the short-run Taylor principle; but there is no equivalent proposition for a *globally* sustainable fiscal policy. In contrast, we derive a formal test of global fiscal sustainability which depends on fiscal regimes' transition probabilities and on their respective durations.

This chapter introduces a Regime-Switching Model-Based Sustainability (RS-MBS) test for fiscal policy, building on Bohn's Model-Based Sustainability (MBS) framework and on the literature on Markov-switching fiscal policy rules. We assume a Markov-switching fiscal policy rule that stochastically switches between sustainable and unsustainable regimes. We define unsustainable regimes by periodic and persistent *negative or null* feedback effect of initial public debt on primary surplus, i.e. violating Bohn's sustainability condition. Consequently, the public debt-to-GDP ratio becomes periodically and persistently explosive during unsustainable regimes. We demonstrate how fiscal regimes matter for global (in opposition with local) fiscal sustainability analysis.

This chapter addresses the two usual concepts of long-run fiscal sustainability: the No-Ponzi game condition (related to the transversality condition) and the debt-stabilizing condition (related to the stationarity of the debt-to-GDP ratio). For each concept of fiscal sustainability, we derive the necessary and sufficient conditions for long-run (or global) fiscal sustainability which depend on regime-specific feedback coefficients of the Markov-switching fiscal policy rule and on expected durations (or persistence) of fiscal regimes. We show that fiscal policy can be *locally* unsustainable, with a periodically explosive public-

debt-to-GDP ratio, and still be *globally* sustainable¹.

The rest of the chapter is organised as follows. Section 1.2 reviews the literature on fiscal sustainability. Section 1.3 presents the extension of the Model-based approach of sustainability to regime switches and develops a new condition for fiscal sustainability. Section 1.4 discusses the implications of fiscal regimes on public debt-to-GDP dynamics. Section 1.5 concludes.

1.2 Related literature

Bohn (1998) builds a Model-Based Sustainability (MBS) framework to analyze fiscal sustainability through the lens of fiscal policy rules (or fiscal reaction functions) in a simple general equilibrium model, as an alternative to the econometric analyses *à la* Hamilton-Flavin². Basically, Bohn assumes the following framework composed of a linear fiscal rule (1.1)

$$s_t = \gamma b_{t-1} + \mu_t \quad (1.1)$$

where s_t is the primary surplus-to-GDP ratio, b_t is the end-of-period public debt-to-GDP ratio and finally μ_t is a vector including all cyclical components of primary surplus (e.g. output gap, temporary public spending), plus a constant and an error term. Thus, Bohn finds that a strictly positive feedback effect $\gamma > 0$ satisfies the No-Ponzi Game (NPG) condition³.

Under a stricter sustainability condition⁴, like a debt-stabilizing fiscal policy rule, the

¹Episodes of a locally-explosive debt which does not lead to global unsustainability or default are theoretically investigated in Blot et al. (2016).

²Seminal empirical investigations on fiscal sustainability proposed a testing framework based on the present-value budget constraint and the transversality condition, drawing on stationarity or cointegration properties of fiscal data (Hamilton and Flavin, 1986; Quintos, 1995; Trehan and Walsh, 1988; Trehan and Walsh, 1991; Wickens and Uctum, 1993; Wilcox, 1989). Still, the econometric analysis of fiscal sustainability has raised a number of issues and led to important criticisms by Bohn (1995, 1998, 2007).

³The Non-Ponzi Game condition states that the present-value of public debt tends to zero in the long-run, which means that the government must pay back at least a part of the interest charges.

⁴Bohn (2007) acknowledges that an upper bound on primary surplus, i.e a fiscal limit, requires a stationary public debt-to-GDP for fiscal sustainability to hold. Research about the upper-bound of primary surplus has been recently explored by Bi (2012), Bi and Traum (2012), Daniel and Shiamptanis (2013), and Davig et al. (2011). Daniel and Shiamptanis show that stationarity and cointegration restrictions are necessary for fiscal sustainability when assuming existence of a fiscal limit. Existence of a fiscal limit (i.e. an upper bound on primary balance-to-GDP and on public debt-to-GDP) requires a sustainability criterion ensuring that public debt must be stable around a long-run value compatible with fiscal limit.

feedback effect should be larger than the growth-adjusted real average interest rate on public debt, that is $\gamma > (r - y)/(1 + y)$ ⁵. MBS analysis has been shown to be empirically powerful in the case of US fiscal policy on long-run data (Bohn, 1998, 2008). On international panel data, Mendoza and Ostry (2008) find evidence that fiscal policy is "responsible" (i.e. there is evidence of a strictly positive feedback rule).

Two types of nonlinear specifications of the fiscal rules exist. On the one hand, fiscal rules are polynomial functions of public debt-to-GDP ratio, i.e. include quadratic and cubic terms (Bohn, 1998). This specification is motivated by the idea that primary surplus may either react more to lagged public debt or on the contrary may become "flatter" at higher public debt levels. This approach has been followed by Ghosh et al. (2013a,b) to account for "fiscal fatigue" where they derive debt limits as the maximum level of public debt beyond which primary balance can no longer adjust to stabilize debt. On the other hand, fiscal rules are time-varying. The assumption that simple linear policy rules (either monetary or fiscal) are constant over time is not convincing regarding multiple evidence of "structural breaks" or "regime changes". In particular, empirical literature on regime-switching fiscal rules has produced evidence that fiscal rules may be better described by "fiscal regimes", see Afonso and Toffano (2013), Bianchi (2012b), Burger and Marinkov (2012), Chung et al. (2007), Davig and Leeper (2007a, 2011), and Favero and Monacelli (2005)⁶. This literature generally identifies sub-periods during which fiscal policy does not stabilize public debt, and sometimes even displays a negative feedback effect of initial public debt on primary surplus.

The literature on regime-switching monetary and fiscal rules builds on Leeper's seminal contribution (Leeper, 1991), which developed a set of formal conditions for *local* equilibrium determinacy stemming from the properties of the monetary and fiscal rules. Fiscal policy is *passive* under the debt-stabilizing condition, *active* otherwise⁷. Recent research on

⁵If one considers a fiscal rule with variables in absolute level rather than as share of GDP, then this feedback should be larger than the real average interest rate on public debt. This is basically what Leeper (1991) finds when describing the stability conditions of an active monetary/passive fiscal regime.

⁶For monetary policy, see Auerbach (2002), Clarida et al. (2000), and Lubik and Schorfheide (2004), among others.

⁷Condition on monetary policy is the Taylor Principle: monetary policy is labeled "active" (A) when it reacts aggressively to inflation (i.e. the Taylor principle holds) and "passive" (P) otherwise. From these two conditions, Leeper (1991) identify four local regimes: Monetary regime (AM/PF), Fiscal regime (PM/AF), Indeterminacy regime (PM/PF) and Explosive regime (AM/AF).

fiscal policy (Bi, 2012; Bi and Leeper, 2013) explores regime-switching fiscal policies to derive an endogenous and stochastic fiscal limit. This literature analyzes fiscal sustainability as the sovereign default probability, computed from the fiscal limit distribution, rather than as generalized conditions on the regime-switching fiscal rule⁸. Davig and Leeper (2007b) define the long-run Taylor monetary principle, based on a Markov-switching Taylor rule, allowing for periodic (or local) violations of the short-run Taylor principle. But, to our knowledge, none has proposed and tested analogous conditions on a regime-switching fiscal rule such that NPG and debt-stabilizing conditions hold in the long-run. In this respect, this chapter's motivation is similar to Davig and Leeper (2007b) but applied to fiscal policy.

Finally, this chapter is also responding to two important contributions in the field of fiscal sustainability analysis. Canzoneri et al. (2001) investigate theoretically a particular time-varying fiscal policy rule in which public debt feedback effect on primary surplus is positive or null. They show primary surplus only has to react positively to public debt on an infrequent basis but "infinitely often" in order to satisfy the government intertemporal budget constraint. This analysis is restrictive in at least two respects. First, assuming primary surplus does not react negatively to initial public debt is a critical assumption, at odds with some empirical evidence on regime-switching policy rules (Afonso and Toffano, 2013; Davig and Leeper, 2007a, 2011; Favero and Monacelli, 2005). Second, the sustainability condition does not ensure a stationary public debt-to-GDP ratio, which is probably the relevant fiscal sustainability condition when the economy faces a fiscal limit. Alternatively, Davig (2005) proposes a unit-root testing framework using a Markov-switching model which accounts for episodes of periodically expanding discounted public debt. This approach is inherently subject to the criticisms addressed by Bohn (1995, 2007) to the econometric analysis of fiscal sustainability. In particular, unit-root testing does not provide any information about fiscal policy behavior since it does not involve an explicit model of fiscal policy.

⁸Fiscal limit distributions are obtained by numerical approximation of the decision rule in calibrated or sometimes estimated Real business cycle models.

1.3 A Regime-Switching Model-Based Sustainability Test

We assume a stochastic real endowment and cashless economy composed of a representative rational household and a government. By assuming a real cashless economy, we implicitly assume that monetary policy has full control over the price-level and inflation dynamics. Using the terminology of the Fiscal Theory of Price-Level (Cochrane, 2005; Leeper, 1991; Sims, 1994; Woodford, 1995), we only consider Ricardian equilibria for which the government intertemporal budget constraint must hold for any path of the price level. Thus we assume that fiscal policy is the only game in town, and we study the worst-case scenario in which fiscal authorities are left without monetary support to ensure public debt sustainability: what are then the fiscal sustainability requirements and can we reject the null hypothesis of unsustainability (i.e. violation of these requirements)? Rejection of unsustainability in this "worst-case" scenario may be interpreted as credible evidence of sustainability.

1.3.1 Model

Stochastic real endowment. Total output Y_t is following a unit-root with drift:

$$Y_t = Y_{t-1}(1 + y + \varepsilon_t^y) \quad (1.2)$$

where $y > 0$ is the long-run growth rate of output and ε_t^y is an i.i.d random shock to the growth rate.

Representative household. Household's preferences are represented by the utility function $u(\cdot)$ which is strictly increasing ($u'(\cdot) > 0$) and concave ($u''(\cdot) < 0$) and a subjective discount factor β . At each period, consumer chooses consumption C_t and buys public bond B_t at a price $(1 + r_t)^{-1}$ in order to maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$$

subject to the following budget constraint:

$$C_t + (1 + r_t)^{-1}B_t = B_{t-1} + Y_t - T_t$$

and transversality condition:

$$\lim_{T \rightarrow +\infty} \mathbb{E}_t \frac{B_{t+T}}{1 + r_{t,T+1}} \geq 0$$

with $(1 + r_{t,T+1})$ being the $T + 1$ -period ahead real interest rate. First order conditions of the representative consumer's maximization program yield the standard Euler equation:

$$(1 + r_t)^{-1} = \beta \mathbb{E}_t \frac{u'(C_{t+1})}{u'(C_t)} \quad (1.3)$$

Equation (1.3) evaluates the stochastic discount factor $Q_{t,1} \equiv \beta \frac{u'(C_{t+1})}{u'(C_t)}$ at the optimal solution of the representative consumer's program, which is the common pricing kernel of any asset in the economy. Hence, a j -period public bond has a price $(1 + r_{t,j})^{-1} = \mathbb{E}_t Q_{t,j}$ with $Q_{t,j} = \beta^j \frac{u'(C_{t+j})}{u'(C_t)}$.

Government. Government spends G_t and collects lump-sum taxes T_t . At each start of period t , government carries one-period public bonds B_{t-1} and it will issue B_t at a price $(1 + r_t)^{-1}$ at end of period. Thus, government faces the following one-period budget constraint:

$$(1 + r_t)^{-1}B_t = (G_t - T_t) + B_{t-1} \quad (1.4)$$

with $S_t \equiv T_t - G_t$ representing the primary budget balance. Under balanced growth, all variables in level grow at rate y_t , thus we rewrite the government budget constraint in terms of output ratios:

$$b_t = \frac{1 + r_t}{1 + y_t} b_{t-1} - (1 + r_t) s_t \quad (1.5)$$

where b_t is the end-of-period debt-output ratio, s_t is the primary surplus-output ratio, r_t and y_t are respectively the real interest rate and the growth rate of real output.

Preventing government from running a Ponzi scheme against its creditor implies the following Present-Value Budget Constraint (PVBC). Following Bohn (1995), we write the PVBC using the stochastic discount factor in order to account for uncertainty and con-

sumer's risk-aversion:

$$B_{t-1} = \sum_{i=0}^{+\infty} \mathbb{E}_t [Q_{t,i} S_{t+i}] \quad (1.6)$$

which is equivalent to the following transversality condition (TC):

$$\lim_{T \rightarrow +\infty} \mathbb{E}_t [Q_{t,T+1} B_{t+T}] = 0 \quad (1.7)$$

Both the PVBC and TC must hold with equality since the representative consumer cannot run a Ponzi Scheme against government (Bohn, 1995).

We assume the following Markov-switching fiscal policy rule:

$$s_t = \gamma(z_t) b_{t-1} + \mu_t(z_t) \quad (1.8)$$

Regime-switching parameter $\gamma(z_t)$ represents the feedback effect of the initial public debt-output ratio b_{t-1} on primary surplus-output ratio *conditional on* fiscal regime z_t . Fiscal regimes are then defined as:

$$\gamma(z_t) = \begin{cases} \gamma_S > 0 & \text{if } z_t = 1 \quad (\text{Sustainable Regime}) \\ \gamma_{NS} \leq 0 & \text{if } z_t = 0 \quad (\text{Unsustainable Regime}) \end{cases} \quad (1.9)$$

During sustainable regimes ($\gamma_S > 0$) primary balance improves following a debt increase while it does not improve or even worsens during unsustainable regimes ($\gamma_{NS} \leq 0$)⁹. Finally, we define $\mu_t(z_t)$ by:

$$\mu_t(z_t) = \alpha(z_t) + \alpha_y(z_t) \hat{y}_t + \alpha_g(z_t) \hat{g}_t + \sigma(z_t) \varepsilon_t^s \quad (1.10)$$

where \hat{y}_t is the output gap, \hat{g}_t is temporary public spending, $\alpha(z_t)$ is a regime-switching constant, $\sigma(z_t)$ is the regime switching standard-error associated to an i.i.d distributed shock $\varepsilon_t^s \sim \mathcal{N}(0, 1)$. We assume regime-switching to be stochastic and exogenous, following a hid-

⁹Canzoneri et al. (2010, p.959) discuss empirical results of Davig and Leeper (2007a, 2011) and note that a negative coefficient on lagged debt in the fiscal rule may be difficult to interpret since "regardless of whether the fiscal rule is Ricardian or non-Ricardian, we would expect a positive estimated coefficient". Indeed, Cochrane (2001) shows there exists a positive correlation between primary surplus and initial debt at *equilibrium*, even when fiscal policy is active (with primary surplus following an AR(1) process). Still, empirical research on regime-switching fiscal policy rules provides some evidence of periodic *negative* feedback effect, see Davig and Leeper (2011) and Afonso and Toffano (2013) for instance; these empirical results motivate our specification of unsustainable fiscal regimes by $\gamma_{NS} \leq 0$.

den two-state Markov process z_t describing fiscal regimes. The use of a Markov switching model rather than endogenous or threshold switching models represents an agnostic way of modelling regime changes of fiscal policy without making any critical assumption about what drives fiscal regime shifts. In addition, given our economy is purely Ricardian, we also assume that fiscal regime z_t is independent of real output's growth rate.

Define $\gamma = (\gamma_S \ \gamma_{NS})$ a row-vector containing regime-specific parameters and $Z_t = (z_t \ 1 - z_t)^T$ a column-vector associated to the Markov process z_t . Hence, we can define the scalar $\gamma(z_t)$ by:

$$\gamma(z_t) \equiv \gamma Z_t = \begin{pmatrix} \gamma_S & \gamma_{NS} \end{pmatrix} \times \begin{pmatrix} z_t \\ 1 - z_t \end{pmatrix} \quad (1.11)$$

Markov process z_t is associated to a transition matrix P whose elements are $p_{ij} \equiv \mathbb{P}(z_t = i | z_{t-1} = j)$ for all $(i, j) \in \{0, 1\}$ such that:

$$Z_t = P Z_{t-1} + v_t \quad \text{with} \quad v_t \equiv Z_t - \mathbb{E}_{t-1}[Z_t] \quad (1.12)$$

We assume z_t to be an ergodic Markov process¹⁰ implying that $\mathbb{E}_t Z_{t+j} = P^j Z_t$ converges to a unique ergodic distribution π :

$$P^j Z_t \xrightarrow{j \rightarrow +\infty} \pi \quad (1.13)$$

where $\pi = (\pi_S \ \pi_{NS})^T$ is the column-vector of ergodic probabilities associated to each fiscal regime. Ergodic probabilities are defined by:

$$\pi_i = \frac{1 - p_{jj}}{(1 - p_{ii}) + (1 - p_{jj})} \quad (1.14)$$

for all $(i, j) \in \{0, 1\}$. Hence, using equations (1.11) and (1.13), the conditional expectation at time t of feedback parameter $\gamma(z_t)$ converges toward its *unconditional* expectation, i.e. ergodic (or long-run) value:

$$\mathbb{E}_t[\gamma(z_{t+j})] = \gamma P^j Z_t \xrightarrow{j \rightarrow +\infty} \gamma \pi \quad (1.15)$$

¹⁰Any Markov process is ergodic as long as $p_{ii} < 1$ and $p_{ii} + p_{jj} > 0$ for all $(i, j) \in \{0, 1\}$ (Hamilton, 1994, Chap. 22), meaning there is no absorbing state.

1.3.2 No-Ponzi Game condition

Following Bohn (1998), we derive sufficient condition on the sequence $\{\gamma(z_{t+i})\}_{i=0}^{\infty}$ such that Present-Value Budget Constraint (1.6) and Transversality condition (1.7) hold. Denoting the j -periods growth-adjusted stochastic discount factor by

$$\tilde{Q}_{t,j} \equiv Q_{t,j} \prod_{i=0}^{j-1} (1 + y_{t+i}) \quad (1.16)$$

allows us to rewrite Transversality condition (1.7) in terms of debt-output ratio by:

$$\lim_{T \rightarrow +\infty} \mathbb{E}_t [\tilde{Q}_{t,T+1} b_{t+T}] = 0 \quad (1.17)$$

Then, using the regime-switching fiscal policy rule (1.8) and iterating on the flow budget constraint of government (1.5) up to date $t + T$, we obtain an expression for expected present-value debt-output ratio $\mathbb{E}_t [\tilde{Q}_{t,T+1} b_{t+T}]$ which explicitly depends on $\{\gamma(z_{t+i})\}_{i=0}^{\infty}$. Finally, we find a sufficient condition on the regime-switching fiscal policy rule to satisfy the No-Ponzi Game condition, that allows us to conclude to the following proposition.

Proposition 1 (No-Ponzi Game) *In a dynamically efficient economy, and provided that $\mu_t(z_t)$ is bounded, a sufficient condition that transversality condition (1.17) holds is*

$$\gamma\pi > 0 \quad (1.18)$$

with $\gamma\pi \equiv \gamma_S\pi_S + \gamma_{NS}\pi_{NS}$ being the unconditional expectation of $\gamma(z_t)$. Using the definition of ergodic probabilities (1.14) and denoting expected duration of regimes by $d_i = \frac{1}{1-p_{ii}}$, we can express condition (1.18) by

$$\gamma_S > |\gamma_{NS}| \frac{d_{NS}}{d_S} \quad (1.19)$$

Proof 1 See appendix A.1.

To understand this condition, let us consider the following approximation of transversality condition when $T \rightarrow +\infty$:

$$\mathbb{E}_t [\tilde{Q}_{t,T+1} b_T] \approx (1 - (1 + y)\gamma\pi)^T b_t \quad (1.20)$$

Following Bohn (2008), consider a Ponzi Scheme such that $\{s_t\}_{t=0}^{\infty} = 0$. This Ponzi Scheme implies debt-output ratio growing at a rate $\frac{r_t - y_t}{1 + y_t}$. As a consequence the limit value of future discounted debt-output ratio is equal to initial debt-output ratio (which violates the transversality condition):

$$\mathbb{E}_t[\tilde{Q}_{t,T+1}b_{t+T}] = b_t \quad (1.21)$$

Thus, $\gamma\pi > 0$ implies the reduction of $\mathbb{E}_t[\tilde{Q}_{t,T+1}b_{t+T}]$ by a factor $(1 - (1 + y)\gamma\pi)^T$ relative to a Ponzi Scheme. Saying it differently: the average growth rate of debt-output ratio is reduced by a factor $(1 - (1 + y)\gamma\pi) > 0$.

Condition (1.18) states that a regime-switching fiscal policy has to satisfy the NPG condition *on average*, that is, sustainable regimes have to be frequent enough to balance unsustainable regimes in the long-run. Ruling out a Ponzi Scheme means that the longer unsustainable regimes *vis-à-vis* duration of sustainable regimes, the larger primary deficits during unsustainable regimes, then the larger the required reaction of primary surplus to debt during sustainable regimes. Still, provided (1.19) holds, fiscal policy can be periodically unsustainable *while* satisfying its present-value budget constraint (PVBC).

1.3.3 Debt-stabilizing condition

A stronger constraint on fiscal policy would require that debt-output ratio must be stationary at a sufficiently low level, below a "fiscal limit" defined as follows. Assume an exogenous upper-bound on the primary surplus-output ratio such that $s_t \leq s^{max}$. This assumption can be justified by tax evasion, following Daniel (2014) or more generally by the political inability and/or unwillingness to reduce public spending and increase taxes, following Daniel and Shiamptanis (2013)¹¹. This directly implies the existence of a maximum level of debt-output ratio, i.e. a fiscal limit, such that:

$$b^{max} = s^{max} \sum_{i=0}^{+\infty} \mathbb{E}_t[\tilde{Q}_{t,i}] \quad (1.22)$$

¹¹In a framework with distortionary taxation, the fiscal limit would arise endogenously from the existence of a dynamic Laffer curve, see Bi (2012) and Bi and Leeper (2013).

Thus, for $b_t > b^{max}$ fiscal policy would be necessarily running a Ponzi Scheme against creditors. Since Proposition 1 does not rule out explosive path for the debt-output ratio, a necessary and sufficient condition for fiscal sustainability, in presence of a fiscal limit on the debt-output ratio, would be a debt-stabilizing fiscal rule around a steady-state level below the fiscal limit.

Therefore, a regime-switching fiscal rule implies that debt-output ratio follows a Markov-switching autoregressive process, defined by equations (1.5) and (1.8):

$$b_t = \phi(z_t)b_{t-1} + u_t(z_t) \quad (1.23)$$

where

$$\phi(z_t) = \frac{1+r_t}{1+y_t}(1 - (1+y_t)\gamma(z_t)) \quad \text{and} \quad u_t(z_t) = -(1+r_t)\mu_t(z_t).$$

A sufficient condition for (strict) stationarity of stochastic processes like (1.23) is given by Kesten (1973), from which we deduce the following proposition.

Proposition 2 (Debt-stabilizing condition) *A sufficient condition for a (strictly) stationary debt-output ratio is*

$$\gamma\pi > \frac{r-y}{1+y} \quad (1.24)$$

which can be expressed in terms of expected durations

$$\gamma_S > |\gamma_{NS}| \frac{d_{NS}}{d_S} + \frac{r-y}{1+y} \frac{d_S + d_{NS}}{d_S} \quad (1.25)$$

Proof 2 See appendix A.2.

Provided conditions (1.24) or (1.25) hold, then public debt-output has an ergodic mean:

$$\mathbb{E}[b_t] = \frac{-\mathbb{E}[(1+r_t)\alpha(z_t)] + \text{Cov}(\phi(z_t), b_{t-1})}{\mathbb{E}[1-\phi(z_t)]} \quad (1.26)$$

where $\mathbb{E}[\alpha(z_t)] < 0$ is the ergodic value of $\alpha(z_t)$.

As long as the growth-adjusted real interest rate is positive, a debt-stabilizing condition is stricter than the NPG condition. During sustainable regimes, the required reaction of primary surplus to initial debt must be large enough to compensate for both primary

deficits during unsustainable regimes, weighted by the ratio of expected durations, and the growth-adjusted real interest rate, weighted by the inverse fraction of (expected) time spent in sustainable regimes. On the contrary, when $r < y$, condition (1.25) could eventually imply government is violating NPG condition (1.19) which is the minimum requirement for fiscal sustainability. Since history provides numerous examples of $r < y$, this illustrates why testing stationarity of debt-output ratio may sometimes be misleading as a test of fiscal sustainability. As a result, NPG condition and debt-stabilizing condition would be complements rather than substitutes: a stationary public debt-output ratio does not always rule out Ponzi Schemes.

1.4 Discussion

The assumption of the existence of different fiscal regimes may, in general, imply that public debt-output ratio can periodically follow an explosive path. To see why, let us consider the example of Canzoneri et al. (2001) and assume $\gamma_{NS} = 0$. We find exactly the same proposition they made: based on equation (1.19), any infrequent $\gamma_S > 0$ would be sufficient to rule out Ponzi Schemes. Yet this equilibrium does not ensure a stable debt-output ratio, that is public debt is $I(1)$. For a stable debt-output ratio, assuming $r - y > 0$ and $\gamma_{NS} = 0$, a regime-switching fiscal policy must satisfy the following condition, from equation (1.25):

$$\gamma_S > \frac{r - y}{1 + y} \frac{d_S + d_{NS}}{d_S} \quad (1.27)$$

For $\gamma_{NS} < 0$ the condition on γ_S is stronger. Under a regime-switching debt-stabilizing fiscal policy, debt-output ratio becomes periodically explosive, and explosive regimes can be really frequent without necessarily implying debt-output is globally non-stationary. As a consequence, standard unit-root and stationarity tests may be weak at discriminating unsustainable from sustainable fiscal policies, in addition to Bohn's criticisms (Bohn, 2007).

Periodic explosive dynamics of public debt has critical consequences on regime-switching policy rules, not only on $\gamma(z_t)$ but also on the constant $\alpha(z_t)$. Rewriting equation (1.8) in terms of deviations of primary balance and public debt from their respective steady-state

values $s^*(z_t) = (s_S^*, s_{NS}^*)$ and $b^*(z_t) = (b_S^*, b_{NS}^*)$ yields:

$$s_t - s^*(z_t) = \gamma(z_t)(b_{t-1} - b^*(z_t)) + \alpha_y(z_t)\hat{y}_t + \alpha_g(z_t)\hat{g}_t + \sigma(z_t)\varepsilon_t^s \quad (1.28)$$

from which we deduce that $\alpha(z_t)$ is equal to:

$$\alpha(z_t) = s^*(z_t) - \gamma(z_t)b^*(z_t) \quad (1.29)$$

In a sustainable regime, primary surplus-output ratio s_t and debt-output ratio b_t must admit steady-state values¹². Provided condition (1.25) holds, we would expect s_S^* to be equal to the debt-stabilizing primary surplus ratio, for a stationary debt-output target ratio b_S^* :

$$s_S^* = \frac{r-y}{1+y}b_S^* \quad (1.30)$$

which implies:

$$\alpha_S = \left(\frac{r-y}{1+y} - \gamma_S\right)b_S^* < 0 \quad (1.31)$$

provided that condition (1.24) holds, which would account for negative estimates of α_S but also $\mathbb{E}[\alpha(z_t)] = \pi_S\alpha_S < 0$ if $\gamma_{NS} < 0$. As a consequence, insofar as $b_t < b_S^*$ fiscal policy can run primary deficits without necessarily jeopardizing fiscal sustainability.

1.5 Conclusions

This chapter introduces a Regime-Switching Model-Based Sustainability test for fiscal policy, building on Bohn's Model-Based Sustainability (MBS) framework and on the literature on Markov-switching fiscal policy rules.

We assume a Markov-switching fiscal policy rule that stochastically switches between sustainable and unsustainable regimes, where by unsustainable regime we mean a periodic

¹²Under the unsustainable regime and periodic explosive dynamics of public debt, the time series properties of s_t can be twofold, depending on the value of γ_{NS} . When $\gamma_{NS} < 0$ we expect α_{NS} to be equal to zero. Explosive debt-output ratio dynamics are not compatible with any steady-state debt-output level, hence $b_{NS}^* = 0$. Then, primary balance would be necessarily non-stationary since the two variables would be negatively cointegrated with $\{b_t\}$ being non-stationary, implying $s_{NS}^* = 0$. Otherwise, if $\gamma_{NS} = 0$, $\{s_t\}$ could be stationary and then s_{NS}^* would be eventually significantly different from zero.

and persistent *negative or null* feedback effect of initial public debt on primary surplus, i.e. violation of Bohn's sustainability condition. Consequently, the public debt-to-GDP ratio becomes periodically and persistently explosive during unsustainable regimes, and fiscal regimes thus matter for fiscal sustainability analysis. We prove formally that global fiscal sustainability differs from local sustainability. The former depends on the relative sensitivity of the fiscal rule to the debt-to-GDP ratio from one regime to another, and also on the relative duration and persistence of both regimes.

Future research in this area should aim at generalizing this condition in a monetary economy in which the central bank can eventually lose the control of inflation and thus bridging the gap between this research and the Fiscal Theory of Price-Level, as in Ascari et al. (2017).

Chapter 2

Is France's Public Debt Sustainable? An Application of the Regime-Switching MBS Test

Co-authored by Jérôme Creel

2.1 Introduction

This chapter proposes an empirical application of the Regime-Switching Model-Based Sustainability (RS-MBS) test on French annual data from 1965 to 2013. The choice to investigate France's fiscal policy sustainability is motivated by two main reasons. As a Euro Area member state, France has neither a domestic monetary policy nor a lender of last resort. Both features make the issue of fiscal sustainability very acute. First, the French government cannot expect a domestic accommodative monetary policy when or after it implements a non-Ricardian fiscal policy. Second, sustainability issues cannot be disregarded and left to the management of the lender of last resort. As a result, we focus exclusively on Ricardian equilibria for which the government intertemporal budget constraint must hold for any path of the price-level.¹ The mere observation of French sovereign interest rates, which have been historically low during the European sovereign-debt crisis, conveys some information about lenders' seemingly expectations that France's fiscal policy is on a sustainable

¹Another consequence of the first feature is methodological: the Leeper (1991) and Davig and Leeper (2011)'s policy interaction framework is not applicable to France.

path. However empirical investigation has given rise to contradictory outcomes: Afonso (2005), Lamé et al. (2014), and Schoder (2014) did not find evidence of a sustainable fiscal policy in France whereas Afonso and Jalles (2016), Chen (2014), Fincke and Greiner (2012), and Weichenrieder and Zimmer (2014) reached mixed evidence; in contrast, Greiner et al. (2007) found that fiscal policy was sustainable.

This chapter argues this contradiction may be attributed to the lack of account for regime-switching fiscal policies. To assess this argument, we develop a two-stage empirical strategy. First, we estimate fiscal rules following Bohn's MBS tests. From these tests, we conclude that French public debt is not sustainable and thus confirming some of the former empirical analyses. Second, we estimate a Markov-switching fiscal rule and perform a Regime-Switching MBS test. The latter outcomes challenge the former results obtained with standard techniques: the existence of a locally unsustainable regime cannot be automatically interpreted as global unsustainability. We conclude that omitting fiscal regime-switches may lead to reject mistakenly French sustainability. Another advantage with the RS-MBS approach is that it dates sub-periods of sustainability and unsustainability in France. Thus, it permits to check whether these sub-periods fit the history of French public finances.

Our results are threefold. First, we estimate different specifications of Bohn's constant parameters fiscal policy rule. These estimates do not allow to reject unsustainability: the feedback coefficient on public debt-to-GDP is rarely positive and never significant, according to standard MBS tests. Second, we estimate a Markov-switching fiscal policy rule. We identify two different fiscal regimes over the period: one regime is sustainable, with a strong positive and significant feedback effect of lagged public debt-to-GDP on primary surplus-to-GDP, while the second one is unsustainable with no significant feedback effect. In addition, identified fiscal regimes are found to be strongly persistent. In particular, our findings support the view that the Maastricht Treaty and the Stability and Growth Pact (SGP) actually made France's fiscal policy more sustainable despite being under an Excessive Deficit Procedure from 2003 to 2007. Third, we perform RS-MBS tests for No-Ponzi Game and Stationary debt-output ratio. We reject the null hypothesis of a Ponzi Scheme as well as the null of an explosive public debt-to-GDP ratio.

The chapter is organized as following. Section 2.2 briefly summarizes and recall the theo-

retical proposition of the Regime-Switching MBS test. Section 2.3 presents statistical sources and the dataset. Section 2.4 produces standard Model-Based Sustainability tests on France. Section 2.5 presents the Regime-Switching MBS tests. Section 2.6 discusses the implications of our findings on monetary-fiscal interactions in France. Section 2.7 concludes.

2.2 Theory

The Regime-Switching MBS test builds on (Bohn, 1998) and considers a fiscal policy feedback rule

$$s_t = \gamma(z_t)b_{t-1} + \underbrace{X_t'\beta(z_t) + \sigma(z_t)\varepsilon_t}_{\mu_t(z_t)} \quad \text{with } \varepsilon_t \text{ i.i.d. } \mathcal{N}(0,1) \quad (2.1)$$

where s_t represents the primary surplus-to-GDP ratio, b_{t-1} the end-of-period debt-to-GDP ratio and the vector X gathers control variables such as output gap, cyclical public spending and an intercept. Coefficients γ , β and standard-error σ are subject to recurring and persistent switches between two values, according an hidden exogenous two-state Markov process z_t with transition probabilities p_{ii} for all $i \in \{S, NS\}$, where S denotes a sustainable regime ($\gamma_S > 0$) and NS denotes an unsustainable one ($\gamma_{NS} \leq 0$).

In Chapter 1, we derive sufficient conditions on regime-specific feedback coefficients γ_S , γ_{NS} and expected durations of regimes $d_i = 1/(1 - p_{ii})$ for $i = \{S, NS\}$ such that fiscal policy satisfies the No-Ponzi Game (NPG, henceforth) condition and the Debt-Stabilizing condition; hence, we briefly recall Propositions 1 and 2.

The NPG condition requires that the initial public debt-to-GDP ratio b_{t-1} would be backed by the sum of future expected and discounted real primary surpluses-to-GDP, implying the following transversality condition

$$\lim_{T \rightarrow +\infty} \mathbb{E}_t [\tilde{Q}_{t,T+1} b_{t+T}] = 0 \quad (2.2)$$

where $\tilde{Q}_{t,j}$ the j -periods growth-adjusted stochastic discount factor, in a general equilibrium setup. Proposition 1 states that, in a dynamically efficient economy, and provided that $\mu_t(z_t)$

is bounded, a sufficient condition that transversality condition (2.2) holds is

$$\gamma\pi > 0 \quad (2.3)$$

with $\gamma\pi \equiv \gamma_S\pi_S + \gamma_{NS}\pi_{NS}$ being the unconditional expectation of $\gamma(z_t)$. Using the definition of ergodic probabilities $\pi_i = (1 - p_{jj}) / (2 - p_{ii} - p_{jj})$ and denoting expected duration of regimes by $d_i = \frac{1}{1 - p_{ii}}$, we can express condition (2.3) by

$$\gamma_S > |\gamma_{NS}| \frac{d_{NS}}{d_S} \quad (2.4)$$

The NPG condition *per se* does not impose any stationarity restriction, see Bohn (2007). As a result, an ever-increasing public debt-to-GDP ratio will eventually reach the fiscal limit. Thus, a stronger sustainability condition would require a stable public debt-to-GDP ratio around a long-run value with a sufficient safety margin with respect to the fiscal limit. Hence, under a regime-switching fiscal policy rule and using the flow budget constraint of government, b_t follows a Markov-switching autoregressive process

$$b_t = \phi(z_t)b_{t-1} + u_t(z_t) \quad (2.5)$$

where

$$\phi(z_t) = \frac{1 + r_t}{1 + y_t} (1 - (1 + y_t)\gamma(z_t)) \quad \text{and} \quad u_t(z_t) = -(1 + r_t)\mu_t(z_t).$$

We derive the Debt-Stabilizing condition (Proposition 2) from the *strict* stationarity condition of process (2.5). Hence, a sufficient condition for a (strictly) stationary debt-output ratio is

$$\gamma\pi > \frac{r - y}{1 + y} \quad (2.6)$$

which can be expressed in terms of expected durations

$$\gamma_S > |\gamma_{NS}| \frac{d_{NS}}{d_S} + \frac{r - y}{1 + y} \frac{d_S + d_{NS}}{d_S} \quad (2.7)$$

where r and y are the real interest rate and the growth rate of real GDP. Provided conditions

(2.6) or (2.7) hold, then public debt-output has an ergodic mean

$$\mathbb{E}[b_t] \simeq \frac{-(1+r) \mathbb{E} \alpha(z_t)}{(1+r)\gamma\pi - \frac{r-y}{1+y}} \quad (2.8)$$

neglecting covariance terms, following Bohn (1998, 2008) and Mendoza and Ostry (2008). In the following sections, we will test propositions 1 and 2 after having estimated a Markov-Switching fiscal policy rule.

2.3 Dataset

The choice of annual data is guided by two arguments: availability on a long time span and consistency with the fiscal institutional process. First, fiscal sustainability can only be appreciated in the long-run: PVBC or stationarity might only be satisfied in the long-run –over half a century, or more. Regarding data availability for France, we are forced to renounce using *true* quarterly data which are available from 1995-Q4 only for public debt². Still, a second argument prevents us from using quarterly data: fiscal decisions are taken on an annual basis in the law of finance, despite some infra-annual adjustments. Using quarterly data may result in spurious results as it may add noise to the *true* response of primary balance to the initial stock of debt.

This paper uses the longest time series available for French public debt. Indeed, because of changes in national accounts systems, it is relatively hard to find historical data on French public debt. Most of available time series (in particular, those using Maastricht debt definitions) start by 1978. The IMF Historical Public Debt Database (HPDD) proposes a long-run time series for public debt, but still with missing observations for years 1978 and 1979, because of national accounting issues. Thus, regarding public debt, we use the OECD government total gross financial liabilities rather than the Maastricht definition of gross public debt since the OECD series goes back to 1969. As in Auerbach and Gorodnichenko (2017), we complete this series by backward interpolation using the budget identity $B_t = B_{t-1} + DEF_t$ where B_t is the nominal stock of debt and DEF_t is the nominal with-interests deficit. As a

²It is possible to build a quarterly measure for public debt using interpolation methods and quarterly government budget balance. Indeed Lamé et al. (2014) report the use of recalculated quarterly data of net French public debt, though on a shorter time span (1980Q1-2007Q4) than the one used in this paper.

result, for $t < 1969$, public debt at time $t - 1$ is equal to public debt at time t minus the government overall budget balance at time t . This backward interpolation implicitly assumes that there were no stock-flow adjustments between 1963 and 1968. This is not a strong assumption on this period. Stock-flow adjustments are more important under large financialisation of public assets and liabilities and when public debts can be denominated in a foreign currency. Financialisation in France has started in the 1980s and public debt remains almost entirely denominated in the domestic currency. Regarding time convention in national accounts, public debt stock is the *end-of-period* stock of debt.

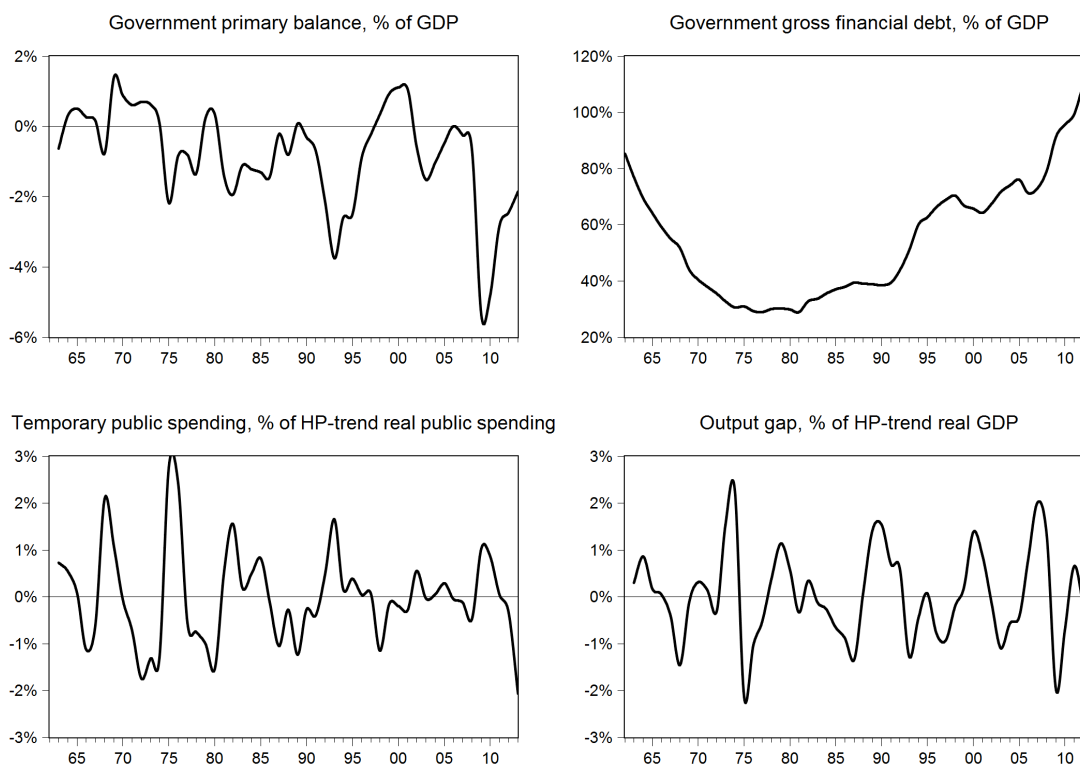
Overall budget balance and primary budget balance (budget balance *minus* interests paid) are taken from OECD database for years 1977-2013; observations for years 1963 to 1977 are completed using data collected by Creel and Le Bihan (2006), from French National Institute for Statistics and Economic Studies (INSEE). We build time series for output gap and temporary government spending by detrending and removing the cyclical component of real GDP and real government spending using the HP filter. Regarding the estimation of output gaps, many competing techniques are available and their relative strengths and weaknesses still discussed (see Cotis et al. (2005), for a survey of estimation methods). Our choice of the HP-filtered method has been motivated by its easiness, fastness and recent use by Fincke and Greiner (2012) and, with more sophistication, by Borio et al., 2014. To address the end point bias problem of the HP filter, we add univariate 3-year ahead forecasts for each series, using ARIMA models, prior to filtering and then dropping the last 3 observations³. Such a "mechanic" correction of the end point bias is applied, for instance, by the European Commission (Havik et al., 2014), when using the HP filter. Finally, our dataset covers 51 years of annual data, from 1963 (1962, for gross public debt) to 2013. Data are shown in figure 1.

2.4 Model-Based Sustainability Tests

We estimate different specifications of a standard fiscal policy rule and use constant parameter estimates as a benchmark for comparison with Regime-Switching estimates. We

³We also drop the first 3 observations at the beginning of filtered series which are affected by the end point bias.

FIGURE 2.1 – Dataset overview, France (1962-2013)



specify the following fiscal rule:

$$s_t = \gamma b_{t-1} + X_t' \beta + \varepsilon_t \quad (2.9)$$

where the dependent variable s_t is the primary balance-to-GDP ratio, b_{t-1} is the public debt-to-GDP ratio at end of period $t - 1$ and X_t is a vector of control variables. It includes a constant, output gap \hat{y}_t , and cyclical government spending \hat{g}_t as suggested by Bohn (1998). Then we include a dummy variable FinCrisis_t equal to one for years 2008–2013 in order to account for severe crisis years. To account for potential non-linearities regarding the level of debt, we also estimate fiscal rules as polynomial functions of debt-to-GDP ratio following Bohn (1998) and Mendoza and Ostry (2008). Finally, we account for a potential deterministic time trend, as suggested by unit-root and stationarity tests, see appendix B.1. In presence of serial correlation in the residuals, we correct for serially correlated residuals of order one or two, depending on the model estimated.

Table 2.1 presents the results. Based on these estimates of constant-parameters fiscal

TABLE 2.1 – Constant-parameters Fiscal policy rules

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Initial Debt b_{t-1}	-0.0121 (-0.71)	-0.0058 (-0.35)	0.0283 (0.93)	0.0300* (-1.72)	0.0962 (1.50)	0.0547 (0.86)	0.0735 (1.38)
Quadratic debt b_{t-1}^2	-0.0555 (-1.18)	-0.0429 (-0.87)	-0.0367 (-0.86)
Constant	-0.0025 (-0.24)	-0.0052 (-0.57)	0.0014 (0.16)	-0.0065 (-0.96)	-0.0190 (-1.01)	-0.0219 (-1.15)	-0.0179 (-1.19)
Output gap \hat{y}_t	0.4190*** (3.38)	0.3807*** (3.23)	0.4800*** (3.91)	0.4527*** (3.56)	0.4565*** (3.64)	0.4163*** (3.21)	0.4360*** (3.38)
Temporary spending \hat{g}_t	-0.4053*** (-3.18)	-0.3667*** (-3.09)	-0.3448*** (-2.73)	-0.3763*** (-2.91)	-0.3754*** (-2.85)	-0.4147*** (-3.08)	-0.3982*** (-2.98)
FinCrisis _t	.	-0.0179*** (-2.95)	.	-0.0160** (-2.25)	.	-0.0112 (-1.37)	-0.0131 (-1.70)
Trend	.	.	-0.0009 (-1.55)	-0.0006** (-2.16)	-0.0008 (-1.61)	.	-0.0006** (-2.06)
DW	1.98	1.99	1.70	1.87	1.68	1.81	1.83
Adj. R^2	0.70	0.75	0.72	0.73	0.72	0.70	0.72
Observations	49	49	50	50	50	50	50

Notes: t-stats are in parentheses. Results are significant at 1% level (***), 5% level (***) and 10% level (*). Models (1)–(2) are controlling for second-order serial correlation in the residuals. Model (3)–(7) control for first-order serial correlation in the residuals.

policy rules, we find no evidence of fiscal sustainability⁴. Models (1)–(2) give no positive feedback effect, but rather negative though non-significant estimates for γ . We do not find evidence of a polynomial specification of the fiscal policy rule, since coefficients on debt b_{t-1} and quadratic debt b_{t-1}^2 are never significant. Still, point estimates for polynomial specifications would imply a "flattening" of the fiscal policy rule for high debt-output ratio.

Unit-root and stationarity tests conclude to the potential presence of deterministic time trends respectively negative in s_t and positive in b_t . Thus we control for a deterministic trend in equation (2.9), in models (3)–(5) and (7) of Table 2.1. When estimating the fiscal rule with a time trend, the feedback coefficient on initial debt turns out to be positive, but never significant at 5% level. Only model (4) shows a positive but weakly significant (at 10% level) feedback response of primary surplus to initial debt. Moreover, deterministic trends enter negatively in all equations, which would imply $\lim_{t \rightarrow +\infty} s_t = -\infty$, thus obviously violating the PVBC.

⁴This result contrasts with Fincke and Greiner (2012) who find a significant positive reaction of the primary surplus to debt. Two differences with our approach are worth mentioning. First, Fincke and Greiner do not strictly reproduce Bohn's fiscal rule: they limit cyclical public spending to spending related to the social insurance system though some of these expenditures may be structural; second, their sample is shorter (1970–2008) than ours.

2.5 Regime-Switching Model-Based Sustainability Test

We estimate the following Markov-switching fiscal rule by direct maximisation of the log likelihood (Hamilton, 1989):

$$s_t = \gamma(z_t)b_{t-1} + \alpha(z_t) + \alpha_y(z_t)\hat{y}_t + \alpha_g(z_t)\hat{g}_t + u_t \quad (2.10)$$

where except the autoregressive residuals and the error variance⁵, all remaining parameters can periodically shift between two values, according to a hidden two-state Markov-process z_t .

Numerical optimization of the log likelihood function is raising identification issues, so we choose the following estimation strategy. We randomize the estimation algorithm by drawing 500 starting values and running initial ML estimations with 100 iterations on each draw, in order to reduce the dependence of the ML algorithm on starting values and thus the risk of reaching a local maximum of the log likelihood function; the main estimation algorithm begins using the starting values for which the maximization algorithm reached the highest value of the log likelihood function among the 500 initial random draws. Regarding model specification, we start estimating the most general model, allowing *all parameters, including error variance* to switch between regimes 1 and 2, thus being agnostic on the true structural form of the regime-dependent fiscal rule. At this stage, if the maximization algorithm converges, we can already appreciate how precise the resulting estimates are, both across regimes and in the long-run through the computation of the ergodic value of each parameter. This can be achieved through basic t-statistics and F-statistics analysis. We also look carefully at estimated regimes' properties: transition probabilities associated to the Markov process and filtered and smoothed regime probabilities. We check, in particular, if they are consistent with historical knowledge on fiscal policy shifts, and if they are sufficiently persistent, regarding the timing of fiscal policy.

If any subset of parameters were non-significantly different from zero in *both regimes* or if they were not taking significantly different values across regimes it would be a strong

⁵To account for first-order serial correlation in the data, we assume: $(1 - \rho)u_t = \sigma\varepsilon_t$ with an i.i.d error term $\varepsilon_t \sim \mathcal{N}(0, 1)$

motivation to estimate a restricted model in which this subset of parameters would be regime-invariant. Thus, if any restricted model can be successfully estimated, that is, if the maximization algorithm successfully converges, then the same procedure as described before can be applied.

As a result of our estimation strategy, equation (2.10), without regime heteroskedasticity, seems to be the best specification of the Markov-switching fiscal policy rule⁶. We also estimated a model with a regime-invariant deterministic trend. We conclude to a non-significant (at 5% level) deterministic trend, while other parameters' estimates do not change dramatically with respect to the baseline model, so we choose to exclude the deterministic trend from our baseline specification; results are presented in section 2.5.1.

Given the short length of the sample, we acknowledge that ML estimates must be considered with caution. Yet, given the potential presence of unit-root in the debt-to-GDP ratio, with stationary primary balance-to-GDP ratio, estimates of a *constant-parameters* fiscal policy rule would be equally dubious. But this paper builds on the idea that a non-linear fiscal policy behavior implies periodical explosive dynamics of public debt-to-GDP, without necessarily implying either instability of public debt-to-GDP ratio, or Ponzi schemes, in the *long run*.

Table 2.2 presents estimation results of equation (2.10). We report estimated parameters for each regime and we also compute implied long-run estimates of regime-switching parameters using ergodic probabilities. Standard deviations of long-run estimates are obtained using standard deviations and covariance of regime-specific parameters: for any regime switching parameter $\alpha(z_t)$ which takes two values (α_1, α_2) , with associated standard deviations $(\sigma_{\alpha_1}, \sigma_{\alpha_2})$ and covariance $\text{Cov}(\alpha_1, \alpha_2)$, we compute the long-run (ergodic) estimate α using ergodic probabilities (π_1, π_2) by:

$$\alpha \equiv \alpha_1\pi_1 + \alpha_2\pi_2$$

⁶We have also estimated an alternative specification with regime heteroskedasticity. While the MLE successfully converged, our results appeared a posteriori to be highly dependent on initial values for estimation algorithm and they might be a local maximum of the log likelihood function. That is the main reason why we increased the number of random draws at the start of the estimation process. After having randomized the estimation algorithm, we no longer obtain successful convergent ML estimates of an equation with regime heteroskedasticity.

and with standard deviation:

$$\sigma_\alpha \equiv \sqrt{(\pi_1\sigma_{\alpha_1})^2 + (\pi_2\sigma_{\alpha_2})^2 + 2\pi_1\pi_2\text{Cov}(\alpha_1, \alpha_2)}$$

The results raise some comments. First, France's fiscal policy is well described by a two-state Markov-switching policy rule. One regime is sustainable with a strongly positive and significant correlation between primary balance s_t and initial debt b_{t-1} , implying a stable debt-to-GDP ratio, while the other one shows a non-significant positive correlation. As expected, the constant is significantly negative in the sustainable regime, which is consistent with a debt-stabilizing fiscal policy, while non-significant in the unsustainable regime. Second, both regimes appear to be strongly persistent with respective expected durations of 8.1 and 11.9 years, respectively for sustainable and unsustainable regimes. This would explain why OLS estimates were inconclusive about the long-run correlation between primary surplus and initial debt in table 2.1.

TABLE 2.2 – Estimated Markov-switching fiscal rule for France (1965–2013)

Regime-switching parameters	Regime 1	Regime 2	Long-run estimates
Debt b_{t-1}	0.0889*** (3.08)	0.0017 (0.07)	0.0370 (1.51)
Constant	-0.0608* (-1.90)	-0.0256 (-0.84)	-0.0399 (-1.30)
Output gap \hat{y}_t	0.4214*** (3.30)	0.2894** (2.39)	0.3429*** (4.08)
Temporary spending \hat{g}_t	-0.0637 (-0.50)	-0.5491*** (-5.33)	-0.3524*** (-4.20)
Regime-invariant parameters			
AR(1)	0.9443*** (13.10)	0.9443*** (13.10)	-
Standard-error σ	0.0046*** (8.34)	0.0046*** (8.34)	-
Regimes properties			
	Transition probability p_{ij}	Ergodic probability π_i	Exp. durations (years) d_i
i=1	0.8770	0.4051	8.1
i=2	0.9162	0.5949	11.9
Durbin-Watson statistic	1.7724	Akaike info criterion	-6.8723
Log likelihood	180.3709	Schwarz criterion	-6.4090
Number of observations	49	Hannan-Quinn criter.	-6.6965

Notes: t-stats are in parentheses. Results are significant at 1% level (***), 5% level (***) and 10% level (*). We control for regime-invariant first-order serial correlation in the residuals. Basically, estimates for $\hat{\sigma}$ were obtained as $\log \hat{\sigma}$: consequently, standard errors and t-statistics are obtained applying the Delta method. For regime-switching parameters we compute "long-run estimates" as defined earlier. We report estimates for regime-invariant parameters twice in columns "Regime 1" and "Regime 2", for clarity purposes since they are constant in each regime-specific equation.

FIGURE 2.2 – Estimated sustainable regime, France (1965-2013)

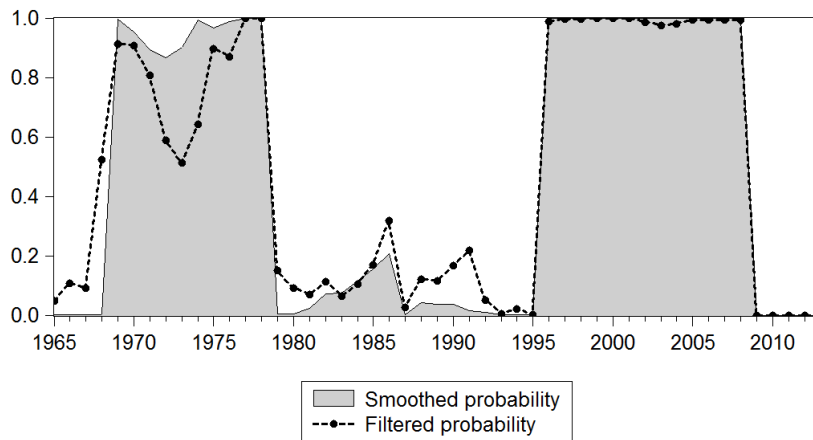


Figure 2.2 represents estimated smoothed and filtered probabilities for regime 1 which we label as sustainable. Results show a succession of periods of unsustainable or sustainable fiscal policies with marked decades. Public finances in the 1970s were sustainable over the most part. In sharp contrast, France's fiscal policy has been mostly unsustainable during the 1980s. Still, filtered probabilities show a small and transitory increase in the probability of being in a sustainable regime during the so-called "Tournant de la rigueur" of 1983-1986 when the Socialist government turned to disinflation and deficit-reduction policies. Overall, results are consistent with a comprehensive and historical analysis of France's fiscal policy. In the 1990s, results report that France's fiscal policy became gradually sustainable (or passive to use Leeper's terminology) and actually so by 1996, until 2008 and the advent of the Great Recession. This finding supports the view that the Maastricht Treaty and the Stability and Growth Pact (SGP) actually made France's fiscal policy more sustainable, despite it being under an Excessive Deficit Procedure from 2003 to 2007. In contrast with Weichenrieder and Zimmer (2014) who show that Euro membership of France has reduced the responsiveness of the primary surplus to debt, our results show that the 1999-2011 period (Euro membership years in Weichenrieder and Zimmer) was heterogeneous as regards fiscal responsiveness: it was positive until 2008 and then negative.

The long-term estimate of $\gamma\pi$ is positive, equal to 0.037 but non-significant (with a p-value equal to 0.1394). Still, this result raises two comments. First, the long-run estimate of $\gamma\pi$ appears non-significant mainly from the fact that the estimate of γ_{NS} is strongly non-

TABLE 2.3 – Regime-Switching MBS: unilateral versus bilateral tests

Student tests for...	t-stat	Bilateral test p-value	Unilateral test p-value
No-Ponzi Game condition (1.19)	3.0841	0.0039	0.0020
Stable long-run debt-to-GDP ratio (1.25) $\frac{r-y}{1+y} = 0.33\%$	2.8008	0.0080	0.0041

Notes: these Student tests assume γ_{NS} is virtually equal to 0. Real interest rate is the ex-post real 10-year yield on French public bonds, obtained using the implicit GDP deflator from OECD Economic Outlook database.

significant (i.e. with a large estimated standard-error), and thus might be considered as virtually equal to 0. Second, significance tests are not appropriate to test for the No-Ponzi Game condition and debt-stabilizing conditions on $\gamma\pi$ since they are *bilateral* tests. On the contrary, Propositions 1 and 2 call for *unilateral* tests for which critical values are lower with respect to bilateral tests⁷.

Assuming that γ_{NS} is virtually equal to 0, we find significant evidence that France's fiscal policy not only satisfies the No-Ponzi Game condition (Proposition 1) but also the Debt-stabilizing condition (Proposition 2). In other words, given past history of French fiscal policy and fiscal regimes, we find significant evidence that France's fiscal policy has been sustainable overall the period 1965-2013, despite a prolonged period of unsustainability from 1979 to 1995.

Using point estimates reported in table 2.2 and historical average for the real interest rate and real GDP growth rate, table 2.4 reports the expected debt-to-GDP ratios, neglecting the covariance terms, under two alternative scenarios. In scenario 1, we suppose sustainable regimes last longer and we increase their expected duration (or persistence) while keeping the expected duration of unsustainable regimes constant and equal to their estimated value. In scenario 2, we suppose unsustainable regimes are shorter and we decrease their expected duration while keeping the expected duration of sustainable regimes constant and equal to their estimated value.

Our computations indicate France's gross public debt-to-GDP ratio would reach an average value of 121% across fiscal regimes, which may be interpreted as too high to prevent

⁷For instance, a bilateral test of the NPG condition on the parameter $\gamma\pi$ is build upon the null hypothesis $\gamma\pi = 0$ against the alternative $\gamma\pi \neq 0$, while the unilateral test is build upon the null hypothesis $\gamma\pi = 0$ against the alternative $\gamma\pi > 0$ which is a more adequate testing hypothesis in the sustainability context.

TABLE 2.4 – Expected regime durations and Debt-GDP ratios using market long-term interest rate

<i>Scenario 1: Increasing expected duration of sustainable regime</i>						
d_S	π_S	π_{NS}	$\gamma\pi$	NPG condition	Stable debt-GDP ratio	$\mathbb{E}[b_t]$
2	0.14	0.86	1.27%	Satisfied	Yes	313%
4	0.25	0.75	2.23%	Satisfied	Yes	178%
7	0.37	0.63	3.28%	Satisfied	Yes	129%
8.1	0.40	0.60	3.59%	Satisfied	Yes	121%
15	0.56	0.44	4.94%	Satisfied	Yes	97%
30	0.72	0.28	6.35%	Satisfied	Yes	84%
60	0.83	0.17	7.41%	Satisfied	Yes	77%
∞	1.00	0.00	8.88%	Satisfied	Yes	71%
<i>Scenario 2: Decreasing expected duration of unsustainable regime</i>						
d_{NS}	π_S	π_{NS}	$\gamma\pi$	NPG condition	Stable debt-GDP ratio	$\mathbb{E}[b_t]$
50	0.14	0.86	1.24%	Satisfied	Yes	322%
30	0.21	0.79	1.89%	Satisfied	Yes	207%
15	0.35	0.65	3.12%	Satisfied	Yes	134%
11.9	0.41	0.59	3.60%	Satisfied	Yes	121%
6	0.58	0.42	5.11%	Satisfied	Yes	95%
3	0.73	0.27	6.49%	Satisfied	Yes	83%
1	0.89	0.11	7.91%	Satisfied	Yes	75%
0	1.00	0.00	8.88%	Satisfied	Yes	71%

Notes: Debt-output ratios are computed from equation (2.8) neglecting covariance terms. For scenarios 1 and 2, we use average market long-term interest rate $r = 3\%$, average real growth rate $y = 2.68\%$ and $r - y = 0.32\%$ (sample: 1963-2013). In scenario 1, we compute expected debt-output ratios under various values of d_S and for $d_{NS} = 11.9$. In scenario 2, we compute expected debt-output ratios under various values of d_{NS} and for $d_S = 8.1$. All others parameters are constant and equal to point estimates obtained in table 2.2, except γ_{NS} which is set to 0.

sovereign default. First, this approach does not pretend sovereign default would be ruled out with certainty by a debt-stabilizing fiscal policy rule⁸. Using regime-switching models, this paper proposes a new non-linear test to discriminate between obviously unsustainable fiscal policies and most probable sustainable ones, given taking into account fiscal policy can periodically deviates from sustainability requirements. But we do not propose any measure of "fiscal space" or "fiscal vulnerability". Second, this expected debt-to-GDP ratio cannot be interpreted as a long-run steady-state ratio, in the usual sense. It represents a long-run average between a regime where public debt follows stable dynamics and a regime with explosive public debt. In particular, assuming $d_S \rightarrow +\infty$ or equivalently $d_{NS} = 0$, we obtain the underlying debt-to-GDP target ratio $b_S^* = 71\%$ towards which public debt converges during sustainable regimes⁹.

⁸We agree with Daniel and Shiamptanis (2013, p.2308) who argue that "a country following a responsible fiscal rule could still encounter solvency problems due to negative shocks or due to future plans which are insolvent. However, a country following a fiscal rule which is not responsible will encounter solvency problems with certainty."

⁹This level cannot be compared to Maastricht criterion of 60% of gross public debt. Indeed, we used the OECD's gross government financial liabilities in our estimates rather than Maastricht gross public debt, for

TABLE 2.5 – Growth-adjusted real rates and Debt-GDP ratios

$\frac{r-y}{1+y}$	Stable debt-GDP ratio	$E[b_t]$
2.5%	Yes	334%
2.0%	Yes	235%
1.5%	Yes	183%
1.0%	Yes	150%
0.5%	Yes	127%
0.0%	Yes	111%
-0.5%	Yes	98%
-1.0%	Yes	89%
-1.4%	Yes	81%
-1.9%	Yes	74%
-2.4%	Yes	69%
-2.8%	Yes	64%

Notes: Debt-output ratios are computed from equation (2.8) neglecting covariance terms. We use point estimates of γ_S , α_S , α_{NS} , except for γ_{NS} which is set to 0, and we use expected durations of regime d_S and d_{NS} from table 2.2. Then, we set $r = 3\%$ and compute expected debt-output ratios for various real GDP growth rate.

Finally, we show how the debt-to-GDP ratio vary with the level of the growth-adjusted real interest rate given our point estimates, in table 2.5. Our results indicate that a modest increase (resp. decrease) in the growth-adjusted real interest rate would result in a significant increase (resp. decrease) of the long-run average public debt-to-GDP ratio.

2.5.1 Robustness checks

In earlier estimates of constant-parameter fiscal policy rules, the (rare) significance of a negative deterministic trend, coupled with a non-significant positive reaction of primary surplus to public debt, raised concerns about fiscal sustainability. We check whether this finding remains in Markov-switching estimates. We re-estimate the Markov-switching fiscal rule (2.10) allowing for a time-invariant deterministic trend. Estimates are shown in table 2.6 and regime probabilities in figure 2.3.

First, regarding parameters common to each specification, estimates are not significantly different from the baseline estimates in table 2.2. This is particularly true regarding the estimated feedback parameter of public debt in regime 1 –the sustainable regime–, to a lesser extent of regime 2. Yet as in the baseline model, the feedback parameter of public debt in the unsustainable regime is far from being significant, and should probably be considered

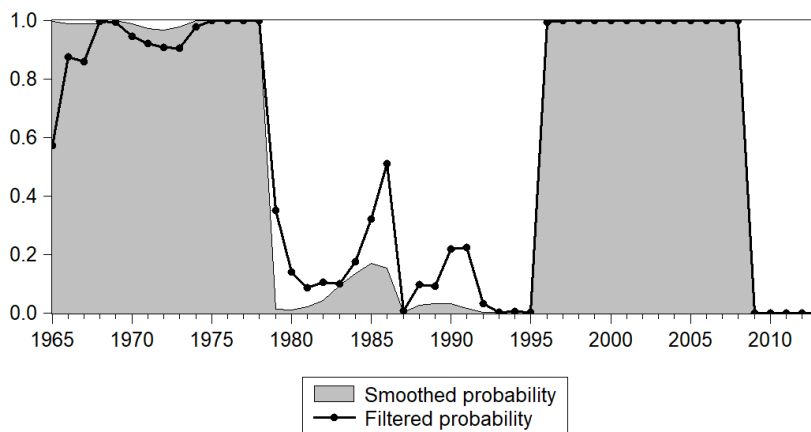
data availability reasons. These two measures of gross public debt differ in terms of debt instruments and valuation methods. As a result, Maastricht debt is generally much lower than gross government financial liabilities.

TABLE 2.6 – Estimated Markov-switching fiscal rule for France with regime-invariant deterministic trend (1965-2013)

Regime-switching parameters	Regime 1	Regime 2	Long-run estimates
Debt b_{t-1}	0.0908** (2.49)	0.0152 (0.57)	0.0574* (1.83)
Constant	-0.0194** (-2.23)	0.0100 (1.12)	-0.0065 (-0.89)
Output gap \hat{y}_t	0.4825*** (4.17)	0.3415*** (2.61)	0.4202*** (4.97)
Temporary spending \hat{g}_t	-0.0594 (-0.54)	-0.7532*** (-5.36)	-0.3658*** (-4.12)
Regime-invariant parameters			
Deterministic trend	-0.0011* (-1.86)	-0.0011* (-1.86)	–
AR(1)	0.7242*** (6.29)	0.7242*** (6.29)	–
Standard-error σ	0.0042*** (8.28)	0.0042*** (8.28)	–
Regimes properties	Transition probabilities p_{ij}	Ergodic probabilities π_i	Expected durations (years) d_i
i=1	0.9330	0.5584	14.9281
i=2	0.9153	0.4416	11.8034
Durbin-Watson statistic	1.7624	Akaike info criterion	-7.0826
Log likelihood	186.5228	Schwarz criterion	-6.5807
Number of observations	49	Hannan-Quinn criter.	-6.8921

Notes: t-stats are in parentheses. Results are significant at 1% level (***), 5% level (***) and 10% level (*). We control for regime-invariant first-order serial correlation in the residuals. Basically, estimates for $\hat{\sigma}$ were obtained as $\log \hat{\sigma}$: consequently, standard errors and t-statistics are obtained applying the Delta method. For regime-switching parameters we compute "long-run estimates" as defined earlier. We report estimates for regime-invariant parameters twice in columns "Regime 1" and "Regime 2", for clarity purposes since they are constant in each regime-specific equation.

FIGURE 2.3 – Estimated sustainable regime, model with deterministic trend, France (1965-2013)



as virtually equal to zero. Consequently, accounting for a potential deterministic trend *does neither* overturn the finding of a strongly sustainable regime *nor* change the point estimate of the feedback parameter in the sustainable regime. One exception is the constant term

in the sustainable regime which is significantly lower in the baseline estimates; we will discuss this point below. Regarding estimated regimes, we observe only a few changes after accounting for a deterministic trend with respect to the baseline estimates. The sustainable regime is more persistent than in the baseline model and fiscal policy in the second-half of the 1960s is identified as sustainable.

Second, the deterministic trend is weakly significant at 10% level and negative. This result is linked to the difference in point-estimates of the constant in the sustainable regime. Recall from equation (2.8) that the constant term determines *ceteris paribus* the level of the long-run expected debt-to-GDP ratio. Hence we may here interpret both results –weakly significant negative deterministic trend and higher estimated constant in the sustainable regime– as the inability for the estimated model to capture the possible structural change in the level of steady-state debt-to-GDP ratio b_S^* between the late 1970s and the 1990s.

To sum up, the weak significance of the deterministic trend coupled with relatively similar point-estimates between the two models as well as similar fiscal policy estimated regimes lead us to conclude that the baseline model is an acceptable representation of France’s fiscal policy. A richer specification with a time-varying (stationary) steady-state debt-to-GDP ratio would probably account for the weakly significant deterministic time trend. Yet, this could not be achieved endogenously using Markov-switching dynamic regressions¹⁰.

2.6 Discussion

In the following, we investigate the stability of the monetary-fiscal policy mix in France since 1965. It is well-known that the estimation of a Bohn-type fiscal rule is not informative on the monetary-fiscal policy mix (Bai and Leeper, 2017; Leeper and Li, 2017). Following Leeper’s (1991) terminology, fiscal and monetary policies can be either active or passive. Consequently, a Bohn-type fiscal rule where the reaction of the fiscal instrument towards public debt is positive –the result we achieved overall– will ultimately be a stable regime of monetary dominance if and only if monetary policy is actively targeting inflation, otherwise

¹⁰It would rather require to use a nonlinear Kalman filter and would be much more data-intensive.

the monetary-fiscal regime is indeterminate. Episodes of active fiscal policies —when fiscal policy's reaction to debt is low or null— lead to a stable regime of fiscal dominance if and only if monetary policy does not actively target inflation, otherwise the monetary-fiscal regime is unstable.

Drawing inferences on the monetary-fiscal regime thus requires studying fiscal and monetary reaction functions. In contrast with Bianchi (2012a) and Chen et al. (2015) who estimate both reaction functions simultaneously on US data, we confront our results for the French fiscal rules with former estimations of the French monetary reaction function.

The design of French monetary policy over our time sample, i.e. between 1965 and 2013, has not been invariant, quite the contrary, and evaluations of the French monetary policy rule gave rise to contrasting results. Monnet (2014) shows that between 1948 and 1973, the main instrument of monetary policy by Banque de France was not the interest rate but a mix of quantitative controls on liquidity (rediscount ceilings) or on bank credit (credit ceilings). He identifies monetary policy shocks with a narrative approach and shows that restrictive episodes of monetary policy produced decreases in industrial production and inflation. Although he does not estimate a monetary policy rule per se, his results nicely fit short episodes of monetary policy aiming at limiting inflation: he notably shows that quantitative controls had negative effects on inflation and GDP growth. Figure 2.4 mixes the fiscal regimes (active or passive) stemming from our results until 1973 with the restrictive monetary regimes identified by Monnet (2014) using narrative analysis¹¹. Figure 2.5 merges the information from narrative analysis by Monnet with our quantitative results to identify monetary-fiscal policy mix; yet, while we use the classification of Davig and Leeper (2007a), Davig et al. (2011), and Leeper (1991), these results *are not* obtained through regime-switching regressions. During this short period, France may have experienced each of the four monetary-fiscal regimes: fiscal dominance from mid-1965 to the end of 1968, then a mix of monetary dominance and indeterminacy between 1969 and the end of 1974. Over the entire period however, Monnet (2015) recalls that the use of monetary policy by Banque de France to fight (double-digit) inflation officially started in 1977, with the explicit use of monetary targets (M2), but was experimented since 1973. They were however very often

¹¹The dummy variable for liquidity and credit controls is constructed on a monthly basis by Monnet (2014) and available at <https://www.aeaweb.org/articles?id=10.1257/mac.6.4.137>.

exceeded. All in all, it is reasonable to state that monetary policy in France between 1965 (the start of our sample) and 1979 was probably not actively targeting inflation –at least clearly until 1969.

FIGURE 2.4 – Liquidity and credit controls (Monnet, 2014) and estimated sustainable regime probability (1965-1973)

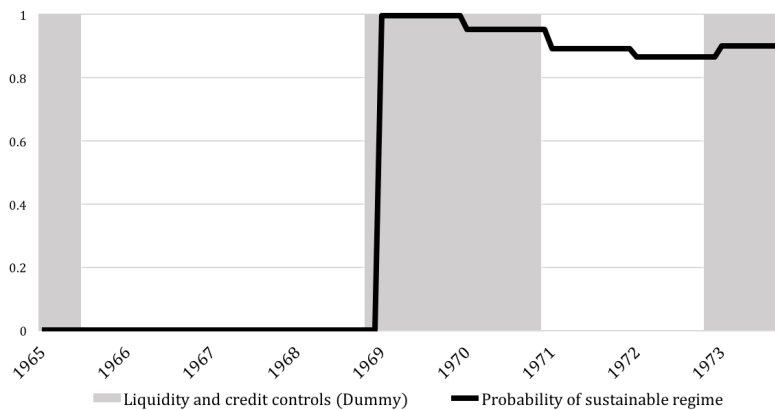
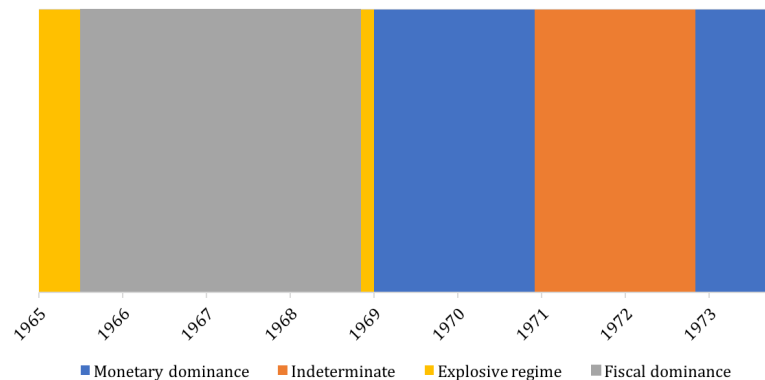


FIGURE 2.5 – Monetary-fiscal regimes in France (1965-1973)



France joined the Exchange Rate Mechanism (ERM) in 1979 and adopted and participated in the European Currency Unit (ECU). The asymmetry of this regime has long been studied and corroborated in the empirical literature. The conclusion has been that France's monetary policy became anchored to Germany's monetary policy under the ERM. Smets (1997) shows that between 1979 and 1996, French monetary policy depended on the ECU exchange rate: monetary policy was driven by the requirement of stabilising the French Franc in the ERM. Unsurprisingly, he does not find any impact of the ECU exchange rate on German monetary policy. Artus et al. (1991) also found evidence of asymmetry in monetary policymaking between the different member states of the ERM. German short-run interest

rate acted as an anchor for French monetary policy. Bec et al. (2002) report a high sensitivity of the French policy rate towards inflation and the German policy rate. The reaction towards real output is not statistically significant. They also show some non-linearities in the French policy rule. The sensitivity of the policy rate towards the German rate (resp. domestic inflation) is higher during expansions (resp. recessions). It has two implications. First, the period of “competitive disinflation” –sharp policy aimed to fight inflation– that started at the end of the 1980s is clearly visible in the reaction function. Second, whatever the period, expansion or recession, monetary policy was actively fighting inflation, either directly or indirectly by applying the German disinflation preference. Consequently, monetary dominance nicely depicts the monetary-fiscal interactions in France between 1979 and 1998.

In 1999, France adopted the Euro. Linear specifications of the ECB monetary policy are usually consistent with the Taylor-rule principle (Castro, 2011; Gorter et al., 2008; Surico, 2007, among others). Consequently, France has gone through two different monetary-fiscal regimes during this period: before 2009, monetary dominance prevailed, whereas after 2009, an unstable regime emerged.

However, if we make use of our main result on global sustainability, we can argue that France has gone through a monetary dominance regime since the late 1970s without exception: public finances were globally sustainable (or passive) whereas monetary policy was inflation-oriented (or active).

2.7 Conclusions

In this chapter, we apply the Regime-Switching MBS test to French data over a 51-year horizon and compared to standard MBS tests. Our results are threefold.

First, we estimate different specifications of Bohn's constant parameters fiscal policy rule. These estimates do not allow to reject unsustainability of France's fiscal policy: the feedback coefficient on public debt-to-GDP is rarely positive and never significant, according to standard MBS tests. Second, we estimate a Markov-switching fiscal policy rule. While standard MBS tests conclude to reject fiscal sustainability, even when taking into account potential

non-linearities in fiscal policy rules estimates, a Markov switching fiscal policy rules tends to identify persistent sustainable and unsustainable regimes. Instability in the fiscal policy rule could explain why standard techniques could fail to identify the true response of primary surplus-to-GDP ratio to lagged public debt-to-GDP ratio. We identify two different fiscal regimes over the period: one regime is sustainable, with a strong positive and significant feedback effect of lagged public debt-to-GDP on primary surplus-to-GDP, while the second one is unsustainable with no significant feedback effect. In addition, identified fiscal regimes are found to be strongly persistent. In particular, our findings support the view that the Maastricht Treaty and the Stability and Growth Pact (SGP) actually made France's fiscal policy more sustainable, and notably, despite being under an Excessive Deficit Procedure from 2003 to 2007. Third, we perform RS-MBS tests for No-Ponzi Game and Stationary debt-output ratio. They reject the null hypothesis of a Ponzi Scheme as well as the null of an explosive public debt-to-GDP ratio.

Regarding empirical application, this research should be extended in at least two ways. Empirical research on linear fiscal policy rules usually ignore potential endogeneity problems: reverse causality between primary balance and output gaps through fiscal multipliers or simultaneity bias as argued recently by Leeper and Li (2017) based on the Fiscal Theory of Price-Level. A first way of improving this RS-MBS framework should be to adopt the Control Function approach for Markov-switching dynamic regressions developed by Kim (2010). Given the multiple evidence on time-varying fiscal multipliers (e.g. Auerbach and Gorodnichenko (2012a) and Riera-Crichton et al. (2015)), effects of fiscal policy on economic activity can be neglected during expansions but cannot be ignored during recessions, when fiscal multipliers are likely positive. Hence, estimates of primary surplus response to public debt would likely be biased downward during recessions. In this respect, our empirical test can be interpreted as a lower bound for fiscal sustainability. Another way of dealing with endogeneity and simultaneity biases is to estimate regime-switching policy rules in empirical DSGE or VAR models following the suggestion of Leeper and Li (2017) and allowing to impose cross-equation restrictions to correctly identify policy behavior.

Chapter 3

Long-term Debt and the Sovereign Default Threshold: Does Maturity Matter?

3.1 Introduction

Increasing the maturity of public debt is always considered among the first instruments to increase debt sustainability in public debt restructuring plans. Indeed, extending the maturity of public debt on existing debt contract allows to avoid a straight default on the principal and to smooth the fiscal burden over time, reducing the gross financing needs of government. This was, for instance, one of the recommendation of the IMF's Debt-Sustainability Analysis for Greece, in July 2015 (IMF, 2015). So, does longer maturity of public debt result in a higher debt limit and increase fiscal sustainability? A recent paper by Kim (2015) argues it does in partial equilibrium model. He shows that uncertainty about future level of primary balance makes debt maturity matter for fiscal space: the longer the maturity of public debt, the larger fiscal space.

This chapter studies the effect of long-term debt on the sovereign default threshold in a fully microfounded general equilibrium model: does an higher maturity of public debt increase the equilibrium sovereign default threshold? We consider a real economy under complete financial markets, with a representative risk-averse household and a government, following Guillard and Kempf (2017). Government issues long-term debt, spends

a constant fraction of total output and collects a tax on labor income. It follows a simple tax rule such that it stabilizes the post-default (or redeemed) level of public debt-to-output. Still, distortionary taxation gives birth to a dynamic Laffer curve and implies the existence a fiscal limit on labor tax rate. Finally, we assume an exogenous debt-recovery and default rule such that government can eventually default on its total outstanding debt and then repays a constant fraction of the maximum debt level.

This economy admits two steady states depending on whether fiscal policy is constrained by the fiscal limit or not. Hence, when fiscal policy reaches the fiscal limit, it enters the constrained regime and primary balance reaches its maximum level. In this regime, the economy admits an unstable steady state which can be interpreted as the deterministic solvency ratio and which independent from the maturity structure of public debt. Then, sovereign default can be endogenized as a market event when the gross financing needs of government exceeds the maximum level it could borrow from the bond market, given the probability distribution of labor productivity shocks. We solve for the bond market equilibrium and derive the maximum market value of public bonds: we find that the equilibrium default threshold is independent from the maturity structure. We suggest this result might not be seen as necessarily contradictory with Kim (2015). In particular, we would argue it clarifies the benefits of long-term debt (with respect to short-term debt) regarding fiscal sustainability issues. Long-term debt would not allow governments to insulate themselves from bad potential growth shocks, in our framework, but rather from bad fiscal policy shocks as in Kim, 2015.

The rest of the chapter is organized as follows. Section 3.2 reviews the literature about long-term debt and sovereign default. Section 3.3 presents and solves the model. Section 3.4 defines the deterministic solvency ratio. Section 3.5 solves for the equilibrium default threshold and shows it is independent from the maturity structure of public debt. Section 3.6 discusses the results. Section 3.7 concludes.

3.2 Related literature

Empirical literature generally shows debt-maturity can play a significant role in the dynamics of public debt. A recent major contribution in this area is Abbas et al. (2014) who study the composition of sovereign debt in 13 advanced economies using an historical dataset ranging from 1900 to 2011. They find that episodes of large debt accumulation, prior to the 1980's, were mainly "absorbed by short-term debt, foreign-currency denominated and banking-system-held debt". Since the 1980's, it became more "skewed toward long-term, local-currency debt" which consequence they interpret as being the result of the "capital account liberalization in advanced economies, the emergence of a large contractual saving sector, and innovative sovereign debt products" (Abbas et al., 2014). In particular, longer maturity of public debt can increase the erosion of debt through inflation, as shown by Aizenman and Marion (2011) for the US debt-to-GDP ratio after WWII. Recently, Equiza-Goñi (2016) has produced counterfactual simulation suggesting that "extending debt-maturity in 2013-2015 would result in lower debt-to-GDP ratios by 2022" and "that higher (lower) inflation in Euro Area countries would lower (raise) their fiscal burden much more than in the US".

From a theoretical point of view, the seminal paper by Lucas and Stokey (1983) shows, under the assumption of complete markets, that an optimal fiscal policy with long-term debt (or debt management) can allow the government to improve welfare by minimizing the cost of distortionary taxation. Extending this analysis to incomplete markets, Angeletos (2002) and Buera and Nicolini (2004) contribute to the concept of *fiscal insurance*, according which debt maturity can help the government to reduce financing needs in case of negative macroeconomic shocks. Recently, and with respect to the question of debt liquidation through inflation, Lustig et al. (2008) and Faraglia et al. (2013) both show longer maturities of public debt contributes to the liquidation of a significant amount of public debt through inflation in an optimal policy framework. Close to these papers, Leeper and Zhou (2013) conclude that inflation's role in optimal fiscal financing increases with the maturity of public debt.

The question of sovereign default can be addressed in two distinct ways, whether default is considered strategic or non-strategic. The literature on strategic default begins with Eaton and Gersovitz (1981). This approach analyses sovereign default as the result of the

strategic decision of a fully rational government. Hence, strategic defaults occur when the government has more incentive to default than to repay its obligations. In this research area, Calvo (1988) proposes a two-period model with a benevolent government maximizing the household utility and issuing public debt with potential repudiation (or default). He shows that, even in a perfect foresight framework, the existence of public debt with possible default lead to multiple equilibria and indeterminacy. Cole and Kehoe (2000) study financial crises due the loss of confidence in the government, namely "self-fulfilling debt crises". In particular, they characterize the values of government debt and debt maturity that are more likely to be subject to such crises and characterize the optimal policy response to self-fulfilling crises. They show that increasing the debt maturity permits to reduce the crisis zone hence reducing the risk of self-fulfilling debt crises. For extensive reviews of the literature in the area of sovereign default, the reader may refer to Eaton and Fernandez (1995), Aguiar and Amador (2014) and D'Erasmus et al. (2015).

Non-strategic defaults, on the contrary, are not resulting of a strategic choice by a rational government but rather the consequence of market events, when government fails to borrow a sufficient amount to cover its gross financing needs. This approach generally allows to study the debt limit (or sometime "fiscal limit"¹). Bi (2012), Bi and Traum (2012) and Bi and Leeper (2013) develop a new methodology to simulate and estimate debt limit distributions based on numerical methods in flexible price models. They compute the debt limit using the intertemporal budget constraint, evaluated at the maximum tax rate of the dynamic Laffer curve that arises from distortionary taxation, and conditional on current and future shocks on productivity and explosive fiscal expenditures and transfers' regimes. This approach has been extended to the analysis of monetary-fiscal interactions by Davig et al. (2011) in the framework of the Fiscal Theory of Price-Level.

Ghosh et al. (2013b) extend Bohn's analysis to sovereign default and consider the effect of "fiscal fatigue" on public debt sustainability which is defined as "fiscal space" that is, the difference between actual debt level and the debt limit, or alternatively, the financial leeway a government has to face adverse macroeconomic shocks. Based on empirical evidence from a panel of 23 advanced economies, they assume the government follows a non-linear

¹But this can be misleading since it refers sometimes to the maximum primary surplus or tax rate and sometimes to the maximum debt level.

fiscal policy rule displaying "fiscal fatigue", i.e. the inability of government to increase sufficiently its primary surplus-to-GDP ratio to stabilize public debt-to-GDP ratio when it becomes too high. They estimate empirical non-linear fiscal policy rules where primary surplus is a cubic function of initial public debt and consequently where the feedback effect of public debt on primary surplus-to-GDP ratio "flattens" at high public debt-to-GDP ratio levels. From these estimates, they compute debt limits and fiscal space for each country.

Finally, this chapter is particularly close to the following papers. Lorenzoni and Werning (2013) study a dynamic model where non-strategic default is driven by insolvency and show that multiple equilibria arise. In their model, the government follows a fiscal rule to stabilize public debt but is also subject to shocks. Generalizing to long-term debt, they show that shorter maturities increase "the exposure to self-fulfilling high interest rates" by contrast with longer maturities which are "less sensitive to the interest rate", which adds to Cole and Kehoe's arguments in favor of long-term debt as an instrument to reduce fiscal vulnerability. But they do not explore the link between debt maturity and the default threshold (or debt limit). More recently, Kim, 2015 builds on Ghosh et al.'s framework to investigate whether the debt maturity can increase the debt limit or not. When primary balance is deterministic, Kim finds that the debt limit is independent from the maturity structure of public debt. But once the primary surplus is stochastic and uncertain, he finds that the debt limit would increase with the maturity structure of public debt, hence increasing "fiscal space" and reducing sovereign risk. Yet, neither Lorenzoni and Werning (2013) nor Kim (2015) develop a microfounded general equilibrium model.

We build extensively on Guillard and Kempf (2017). The authors consider a real economy where labor productivity's growth rate is stochastic and where distortionary taxation on labor generates a dynamic laffer curve. The representative household, in particular, can invest in two assets: an Arrow-Debreu contingent asset and a one-period public bond, which can be subject to default. On the other hand, the government follows a simple and non-stochastic fiscal rule such that the tax rate adjusts to stabilize the redeemed (or post-default) debt-to-output ratio as long as it is lower than the fiscal limit (e.g. the Laffer maximum tax rate for instance). Once fiscal policy reaches this fiscal limit, it enters the "constrained regime" in which default, at some point, will become possible. They assume a recovery

rule allowing the lenders to get a "constant debt recovery ratio"². Such a recovery rule ensures that the government is able to re-enter the bond market after default, without ruling out the possibility of future defaults. Combining the recovery rule with the Euler equation for one-period public bonds, Guillard and Kempf (2017) show that the equilibrium default threshold is locally unique, time-invariant, and critically depends on the constant debt recovery ratio. In particular, they show that the default threshold is always lower or equal to the solvency threshold implied by the government intertemporal budget constraint and the fiscal limit on the labor tax rate. They further derive new sustainability conditions on the debt-to-output ratio such that default can be ruled out (or not) with certainty, depending of the growth-rate of the economy.

3.3 The Model

We consider a real economy with complete financial markets with a representative risk-averse household and a government that uniquely aims at stabilizing the level of debt-to-output. A dynamic Laffer curve arises from distortionary taxation on labor such that government may eventually enter a "constrained" regime in which the tax rate reaches its maximum and default becomes possible.

3.3.1 Private sector

The household The representative household has logarithmic preferences on consumption $u(C_t) = \ln C_t$ and Frisch labor supply $v(L_t) = \frac{1}{\eta} \frac{L_t^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}}$ where σ is the Frisch-elasticity of labor supply. The household chooses, at each period, consumption C_t , labor L_t and a portfolio $\{\{D_{t+1}\}, B_t^S, B_t^M\}$ to maximize its intertemporal utility

$$U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u(C_t) - v(L_t)) \quad (3.1)$$

²By "constant debt recovery ratio", the authors means creditors would recover a constant fraction h of the debt limit.

subject to the flow-budget constraint

$$C_t + q_t^S B_t^S + q_t^M B_t^M + \mathbb{E}_t[Q_{t,t+1} D_{t+1}] \leq (1 - \tau_t)(W_t L_t + \Pi_t) + D_t + h_t B_{t-1}^S + (1 + \rho q_t^M) h_t B_{t-1}^M \quad (3.2)$$

and to an intertemporal constraint on wealth

$$h_{t+1}(B_t^S + (1 + \rho q_{t+1}^M) B_t^M) \leq -\mathbb{E}_{t+1} \sum_{s=t+1}^{\infty} Q_{t+1,s} (1 - \tau_s)(W_s L_s + \Pi_s) \quad (3.3)$$

where W_t is the real wage rate and Π_t are profits. Total income is taxed at a proportional rate τ_t . We assume perfect and complete financial markets since $\{D_{t+1}\}$ is a portfolio of Arrow-Debreu state-contingent assets. In addition, the household can save using one-period public bonds B_t^S and a portfolio of long-term bonds B_t^M . We model long-term bonds using Woodford (2001)'s model of zero-coupon perpetuities with a decay factor ρ . The parameter $\rho \in [0, 1]$ determines the average duration $\mathcal{M}(\rho)$ of long-term bonds, which is

$$\mathcal{M}(\rho) = (1 - \rho\beta)^{-1}$$

When $\rho = 0$, B_t^M becomes a one-period bond; on the other hand, when $\rho = 1$, it is a consol (or perpetual) bond. Since we focus on long-term public debt, we will assume without loss of generality that $\{D_{t+1}\}$ and B_t^S are in zero net supply at equilibrium. Finally, h_t is the recovery ratio (in case of sovereign default) on short-term and long-term public bonds.

First order conditions of the representative household maximization problem are:

$$(1 - \tau_t)W_t = \frac{L_t^{1/\sigma} C_t}{\eta} \quad (3.4)$$

$$Q_{t,t+1} = \beta \frac{C_t}{C_{t+1}} \quad (3.5)$$

$$q_t^S = \mathbb{E}_t Q_{t,t+1} h_{t+1} \quad (3.6)$$

$$q_t^M = \mathbb{E}_t Q_{t,t+1} h_{t+1} (1 + \rho q_{t+1}^M) \quad (3.7)$$

with transversality condition

$$\lim_{T \rightarrow +\infty} \mathbb{E}_t Q_{t,T} (h_T (B_{T-1}^S + (1 + \rho q_T^M) B_{T-1}^M) + D_T) = 0 \quad (3.8)$$

holding with equality. In particular, equation (3.4) is the intratemporal optimal condition between labor and supply (i.e. the labor supply equation). Equation (3.5) evaluates the stochastic discount factor at the optimal solution of the household maximization problem and equations (3.6) and (3.7) determine the prices of short-term and long-term public bonds respectively.

The good market We assume a perfectly competitive good market with a constant return-to-scale production function with only labor as input. Then the production technology is given by

$$Y_t \leq A_t N_t \quad (3.9)$$

where Y_t denotes production, N_t is the quantity of labor and A_t is the marginal productivity of labor. Profit maximization of the representative firm implies the labor demand equation

$$W_t = A_t \quad (3.10)$$

with $\Pi_t = 0$ and $Y_t = A_t N_t$.

Following Guillard and Kempf, 2017, we make the following assumption on productivity A_t :

Assumption 1 (Productivity shock) We assume A_t is described by

$$A_t = a_t A_{t-1} \quad (3.11)$$

where a_t is an i.i.d. shock, with cumulative distribution function $G(a)$ and density function $g(a)$ which satisfies the following properties:

1. $g(a) \in [a_{inf}; a^{sup}]$ and $0 < a_{inf} < 1 < a^{sup}$ with

$$\mathbb{E}(a_t) = 1 \quad \text{and} \quad \beta \mathbb{E}(1/a_t) < 1$$

2. $g(a) > 0$ and $\lim_{a \rightarrow a^{sup}} g(a) = \lim_{a \rightarrow a^{inf}} g(a) = \varepsilon$ with ε arbitrarily small
3. Elasticity of the density function $g(a)$ satisfies $\frac{g'(a)}{g(a)}a > -1$

3.3.2 Fiscal policy

We assume total public expenditures $G_t = gY_t$ are a constant fraction of output. Total tax revenues are defined by

$$T_t = \tau_t Y_t \quad (3.12)$$

Budget constraint We assume B_t^S is in zero net supply. Then, the flow budget constraint of government is

$$q_t^M B_t^M = h_t(1 + \rho q_t^M) B_{t-1}^M - (\tau_t - g)Y_t \quad (3.13)$$

where $q_t^M B_t^M$ is the market value of issued debt at period t , $(\tau_t - g)Y_t$ is the government primary surplus and B_{t-1}^M is the amount of long-term bonds issued at period $t - 1$. Because of long-term debt, the outstanding level of government debt at the beginning of period t is no longer B_{t-1}^M but rather $(1 + \rho q_t^M) B_{t-1}^M$. Consequently, in order to compare one-period debt with longer maturity debt for different values of ρ on the same conceptual basis, we define the outstanding public debt at beginning of period t by

$$\mathcal{L}_t \equiv (1 + \rho q_t^M) B_{t-1}^M \quad (3.14)$$

and rewrite (3.13)

$$\mathcal{L}_{t+1} = \frac{1 + \rho q_{t+1}^M}{q_t^M} \left(h_t \mathcal{L}_t - (\tau_t - g)Y_t \right) \quad (3.15)$$

where $(1 + \rho q_{t+1}^M)/q_t^M$ is the gross return on total outstanding government liabilities. We also denote the outstanding public debt-to-output ratio by $\ell_t = \mathcal{L}_t/Y_{t-1}$.

Fiscal Rule Fiscal authority seeks to stabilize the post-default level of public debt-to-output ratio $\omega_t \equiv h_t \ell_t / y_t$ where $y_t = Y_t/Y_{t-1}$ is the gross growth rate of real output. Since the labor income tax is distortionary, τ_t admits an upper bound $\hat{\tau}$ ³. As result, we assume

³A good candidate for $\hat{\tau}$ would be the top of the Laffer curve $\tau^{max} \equiv \frac{1+\sigma}{1+2\sigma}$.

the following fiscal rule:

$$\tau_t = \min\{\phi(\omega_t - \bar{\omega}) + \bar{\tau}; \hat{\tau}\} \quad (3.16)$$

where $\bar{\omega}$ is the steady-state level, $\phi > (1 - \beta)$ and $\bar{\tau} = g + (1 - \beta)\bar{\omega}$. In addition, we assume $h_t = 0$ when $\omega_t = \bar{\omega} = \bar{\ell}$ ⁴. This fiscal rule implies the existence of $\hat{\omega}$ such that $\tau_t = \hat{\tau}$ and defined by

$$\hat{\omega} \equiv \bar{\omega} + \frac{\hat{\tau} - \bar{\tau}}{\phi} \quad (3.17)$$

It is worth noting that the fiscal policy rule does not include any stochastic exogenous component.

The recovery rule Let denote the sovereign default threshold by Ω_t^{max} . Because of long-term debt, we assume default is triggered when total outstanding public debt at beginning of period t is larger than the default threshold, that is:

$$\mathcal{L}_t > \Omega_t^{max} \quad (3.18)$$

Then, following Guillard and Kempf, 2017, we choose the following specification for the recovery rule h_t :

$$h_t = \begin{cases} h \frac{\Omega_t^{max}}{\mathcal{L}_t} < 1 & \text{if } \mathcal{L}_t > \Omega_t^{max} \\ 1 & \text{else} \end{cases} \quad (3.19)$$

which implies post-default total outstanding debt is a fraction h of Ω_t^{max} if $\mathcal{L}_t > \Omega_t^{max}$. As noted by Guillard and Kempf, such a rule displays two important features. First, it ensures the government can re-enter the bond market after default, such that an equilibrium exists after default. Second, it does not rule out future default *a priori*.

3.3.3 Equilibrium conditions

At this stage, we define the equilibrium conditions of this economy taking as *given* the stochastic sequence of default thresholds $\{\Omega_t^{max}\}_{t=0}^{\infty}$ at each period. We will endogenize this default threshold in presence of long-term debt in section 3.5.

⁴At steady-state, remark that the growth rate of Y_t is 1 directly from Assumption 1.

A competitive equilibrium is a sequence of prices $\{W_t, Q_{t,t+1}, q_t^S, q_t^M\}_{t=0}^\infty$, policy instruments $\{\tau_t, h_t\}_{t=0}^\infty$, and quantities $\{Y_t, C_t, N_t, D_{t+1}, B_t^S, B_t^M, G_t\}_{t=0}^\infty$ such that for all possible sequences of exogenous realizations $\{A_t\}_{t=0}^\infty$ and default thresholds $\{\Omega_t^{max}\}_{t=0}^\infty$ the representative household and firms solve their optimization programs, the flow-budget constraint of government, the fiscal rule and the recovery rule hold and all markets clear:

$$\{D_{t+1}\} = \{0\} \quad (3.20)$$

$$B_t^S = 0 \quad (3.21)$$

$$C_t = (1 - g)Y_t \quad (3.22)$$

$$L_t = N_t \quad (3.23)$$

Since neither the household nor the government can run a Ponzi Scheme against each other (Bohn, 1995), fiscal policy satisfies the following transversality condition with equality:

$$\lim_{T \rightarrow +\infty} \beta^T \mathbb{E}_t \omega_T = 0 \quad (3.24)$$

Equation (3.4) along with (3.9), holding with equality, and (3.10) yields:

$$Y_t = \left(\eta \frac{1 - \tau_t}{1 - g} \right)^{\frac{\sigma}{1+\sigma}} A_t \quad (3.25)$$

Using conditions (3.5), (3.6) and (3.7), we obtain the no-arbitrage conditions on short and long-term public bonds:

$$q_t^S = \beta \mathbb{E}_t \frac{h_{t+1}}{y_{t+1}} \quad (3.26)$$

$$q_t^M = \beta \mathbb{E}_t \frac{h_{t+1}}{y_{t+1}} (1 + \rho q_{t+1}^M) \quad (3.27)$$

Iterating forward on (3.27), we can derive the pricing equation of long bonds in t

$$q_t^M = \mathbb{E}_t \sum_{k=0}^{\infty} \rho^k \left(\prod_{i=0}^k Q_{t,t+i} h_{t+1+i} \right) = \beta \mathbb{E}_t \sum_{k=0}^{\infty} (\rho\beta)^k \left(\prod_{i=0}^k \frac{h_{t+1+i}}{y_{t+1+i}} \right) \quad (3.28)$$

Last equation implies the long-term bond's price is determined by weighted average of

expectations of future realizations of the stochastic discount factor.

3.4 The deterministic solvency ratio

In this section, we characterize the steady state properties of this economy. From the existence of a fiscal limit $\hat{\tau}$ on τ_t , this economy admits two different deterministic steady states for the level of post-default outstanding public debt-to-GDP ratio ω_t . This can be shown by scaling the government flow-budget constraint (3.15) by output Y_t so we can get a dynamic equation for the outstanding public debt-to-output ratio ℓ_{t+1}

$$q_t^M \ell_{t+1} = (1 + \rho q_{t+1}^M) \left(\frac{h_t \ell_t}{y_t} - (\tau_t - g) \right) \quad (3.29)$$

and finally, using the equilibrium condition (3.7), rearranging and taking the expectation in t yields

$$\mathbb{E}_t \omega_{t+1} = \beta^{-1} \omega_t - \beta^{-1} (\tau_t - g) \quad (3.30)$$

As in Guillard and Kempf, 2017, replacing τ_t using (3.16), this dynamic equation for $\mathbb{E}_t \omega_{t+1}$ has a kink in $\hat{\omega}$ such that

$$\mathbb{E}_t \omega_{t+1} = \begin{cases} (1 - \phi) \beta^{-1} \omega_t + (1 - (1 - \phi)) \beta^{-1} \bar{\omega} & \text{for } \omega_t \leq \hat{\omega} \\ \beta^{-1} \omega_t - \beta^{-1} (\hat{\tau} - g) & \text{for } \omega_t > \hat{\omega} \end{cases} \quad (3.31)$$

As one can see, because of the fiscal limit $\hat{\tau}$ and the kink it creates in the dynamic equation of expected post-default public debt-to-output ratio, this model admits two deterministic steady states for ω_t . The first one $\bar{\omega}$ is exogenous and defined by the fiscal policy rule (3.16); it is stable as long as $\phi > 1 - \beta^5$.

From now on, as long as $\tau_t > \hat{\tau}$ and $\omega_t > \hat{\omega}$, fiscal policy becomes constrained by the fiscal limit, so we'll refer to this situation as the "constrained regime", following Guillard and Kempf. Then, the second steady-state ω^{sup} defined by

$$\omega^{sup} \equiv \frac{\hat{\tau} - g}{1 - \beta} \quad (3.32)$$

⁵This condition is actually slightly different but close to Leeper's passive fiscal policy condition which would write here $\phi > \frac{1}{\beta} - 1$ with $\frac{1}{\beta} - 1$ being the risk-free steady state interest rate.

is obviously unstable since $\beta^{-1} > 1$ and endogenous to the model, defined by the fiscal limit $\hat{\tau}$, the level of public expenditures as a share of output and the discount factor β . As a result, ω^{sup} is interpreted as the maximum level of post-default outstanding public debt such that the No-Ponzi game condition holds *at steady state*, that is when the government satisfies its intertemporal budget constraint:

$$\omega^{sup} \equiv \sum_{k=0}^{\infty} \beta^k (\hat{\tau} - g) \quad (3.33)$$

Hence it follows from equation (3.33) that for any $\omega_t > \omega^{sup}$ fiscal policy would violate its transversality condition (3.24); ω^{sup} should then be interpreted as the deterministic *solvency* ratio, which is distinct from the stochastic default threshold ω_t^{max} (expressed in percentage of output, see section 3.5). From what precedes, the following proposition comes straightforward:

Proposition 3 *For a given discount factor $\beta \in (0, 1)$, the deterministic solvency ratio ω^{sup} does not depend on ρ and as a result neither on $\mathcal{M}(\rho)$ the average maturity of long-term public bonds portfolio.*

Proof 3 *From equation (3.33), it is straightforward to see that ω^{sup} is independent from ρ .*

This result is not surprising since one would expect the benefits of long-term debt with respect to short-term debt to rely on the stochastic and transitory dynamics of this economy, not on the properties of its deterministic steady-state. As a consequence, if long-term debt has any effect at all on the debt limit, it should be on the stochastic default threshold.

3.5 The equilibrium default threshold with long-term debt

In this section, we endogenize the stochastic default threshold ω_t^{max} . Following closely Guillard and Kempf, 2017, we define sovereign default as a "market event" when government cannot borrow a sufficient amount $q_t b_t$ on the bond market to cover its gross financing needs, given fiscal policy is in the constrained regime at the maximum primary surplus $\hat{\tau} - g$.

From now on, we will restrict the analysis of default to market events when the govern-

ment is the constrained regime, that is when fiscal policy has reach its fiscal limit, implying the following assumption:

Assumption 2 *The economy in period t is such that:*

1. *Fiscal policy is constrained by the fiscal limit since period $t - 1$:*

$$\min(\omega_{t-1}, \omega_t) > \hat{\omega}$$

2. *Given it remains in the constrained regime in period $t + 1$, i.e. $\omega_{t+1} > \hat{\omega}$, we assume it still exist some $\tilde{\omega}_t > \hat{\omega}$ such that the conditional probability of sovereign default in $t + 1$ is zero:*

$$\mathbb{P}(h_{t+1} < 1 | \omega_{t+1}, \tilde{\omega}_t) = 0.$$

We define $b_t^M \equiv B_t^M / Y_t$ as the quantity of long-term bonds scaled by output and issued at time t . The stochastic default threshold as a share of output is then defined by $\omega_t^{max} \equiv \Omega_t^{max} / Y_t$. Hence, to endogenize ω_t^{max} , we study the bond market equilibrium, described by the following equations:

$$q_t^M = \beta \mathbb{E}_t \frac{h_{t+1}}{a_{t+1}} (1 + \rho q_{t+1}^M) \quad (3.34)$$

$$q_t^M b_t^M = h_t \frac{\ell_t}{a_t} - \hat{\tau} + g \quad (3.35)$$

$$h_{t+1} = \begin{cases} h \frac{a_{t+1} \omega_{t+1}^{max}}{(1 + \rho q_{t+1}^M) b_t^M} < 1 & \text{if } \frac{(1 + \rho q_{t+1}^M) b_t^M}{\omega_{t+1}^{max}} > a_{t+1} \\ 1 & \text{else} \end{cases} \quad (3.36)$$

where total outstanding public debt-to-output ratio is defined by $\ell_t \equiv (1 + \rho q_t^M) b_{t-1}^M$. Equation (3.34), which is the Euler equation for long-term bonds, implicitly characterizes the *demand* for public long-term bonds from rational lenders. The right-hand side of equation (3.35) with $h_t = 1$ determines the *supply* of long-term bonds the government wants to issue to cover its gross financing needs, assuming it did not default on its total outstanding debt at time t . Finally, the recovery rule (3.36) rewritten in terms of scaled-by-output variables determines whether the government defaults or not. These three equations fully characterize the equilibrium on bond market.

3.5.1 The demand for long-term bonds: the valuation function

First, we derive the demand equation for long-term bonds, or valuation function henceforth. To solve for the equilibrium, as in Guillard and Kempf, 2017, the following conjecture is needed:

Conjecture 1 *The stochastic default threshold at time $t + 1$ is known at time t :*

$$\mathbb{E}_t \omega_{t+1}^{max} = \omega_{t+1}^{max}$$

Let denote the pricing function for long-term bonds by

$$q_t^M \equiv \tilde{q}\left(b_t^M, \omega_{t+1}^{max}, q_{t+1}^M; \mathbf{h}, \rho\right)$$

Under the conjecture that ω_{t+1}^{max} is known in t and that a_t is an i.i.d. shock from assumption 1, the pricing function writes as follows:

$$q_t^M = \beta \begin{cases} \mathbb{E}_t \frac{1 + \rho q_{t+1}^M}{a_{t+1}} & \text{if } \delta_t(\rho) \leq a_{inf} \\ h \frac{\omega_{t+1}^{max}}{b_t^M} G(\delta_t(\rho)) + \int_{\delta_t(\rho)}^{a^{sup}} \frac{\mathbb{E}_t(1 + \rho q_{t+1}^M)}{a_{t+1}} dG(a_{t+1}) & \text{if } \delta_t(\rho) \in (a_{inf}, a^{sup}) \\ h \frac{\omega_{t+1}^{max}}{b_t^M} & \text{if } \delta_t(\rho) \geq a^{sup} \end{cases} \quad (3.37)$$

where $\delta_t(\rho)$ denotes the conditional expectation in t of total outstanding debt at beginning of period $t + 1$, that is

$$\delta_t(\rho) \equiv \frac{\mathbb{E}_t \ell_{t+1}}{\omega_{t+1}^{max}} = \frac{\mathbb{E}_t(1 + \rho q_{t+1}^M) b_t^M}{\omega_{t+1}^{max}} \quad (3.38)$$

Then, total demand for long-term bonds in t can be defined as the market value of public bonds issued in t and is represented by the following valuation function:

$$v_t \equiv \tilde{v}\left(\delta_t(\rho), \omega_{t+1}^{max}; \mathbf{h}, \rho\right) = \tilde{q}\left(b_t^M, \omega_{t+1}^{max}, q_{t+1}^M; \mathbf{h}, \rho\right) b_t^M \quad (3.39)$$

and using (3.37) we can derive an explicit expression for equation (3.39):

$$v_t = \beta \begin{cases} \mathbb{E}_t \frac{1}{a_{t+1}} \delta_t(\rho) \omega_{t+1}^{max} & \text{if } \delta_t(\rho) \leq a_{inf} \\ h\omega_{t+1}^{max} G(\delta_t(\rho)) + \int_{\delta_t(\rho)}^{a^{sup}} \frac{\delta_t(\rho) \omega_{t+1}^{max}}{a_{t+1}} dG(a_{t+1}) & \text{if } \delta_t(\rho) \in (a_{inf}, a^{sup}) \\ h\omega_{t+1}^{max} & \text{if } \delta_t(\rho) \geq a^{sup} \end{cases} \quad (3.40)$$

The valuation equation for long-term bonds is a non-monotonic function of $\delta_t(\rho)$, hence also non-monotonic in total outstanding debt ℓ_t for any ω_t^{max} :

- For all $\delta_t(\rho) \leq a_{inf}$, the default risk premium on long-term bonds is zero and v_t is an increasing function of ℓ_t .
- When $\delta_t(\rho) \in (a_{inf}, a_{sup})$, sovereign default becomes possible depending on the realization of the productivity shock a_{t+1} . The default risk premium hence is positive and v_t becomes a non-monotonic function of ℓ_t .
- Finally for all $\delta_t(\rho) \geq a_{sup}$, sovereign default is certain and v_t is constant and equal to $h\omega_{t+1}^{max}$ with

$$h\omega_{t+1}^{max} \leq \mathbb{E}_t \frac{1}{a_{t+1}} \delta_t(\rho) \omega_{t+1}^{max}$$

As a result, v_t must admit a maximum v_t^{max} for $\delta_t(\rho) \in (a_{inf}, a_{sup})$. A first step in solving for the equilibrium default threshold ω_t^{max} is to solve for v_t^{max} the maximum amount the government could borrow on the bond market in function of $\delta_t(\rho)$ and h . This leads us to the following proposition:

Proposition 4 *Given ω_{t+1}^{max} and the maturity structure of public debt $\mathcal{M}(\rho)$ the valuation function reaches a unique maximum v_t^{max} for an expected ratio $\mathbb{E}_t \ell_{t+1}^{max}$ such that:*

1. Both v_t^{max} and $\mathbb{E}_t \ell_{t+1}^{max}$ are linearly increasing in h such that

$$v_t^{max} = x_h(\rho) \omega_{t+1}^{max} \quad \text{and} \quad \mathbb{E}_t \ell_{t+1}^{max} = \delta_h(\rho) \omega_{t+1}^{max}$$

with $x_h(\rho) \in (0, \beta]$ and $\delta_h(\rho) \in (1, a^{sup}]$ for any $h \in [0, 1]$.

2. In addition, both x_h and δ_h are independent of ρ since

$$\frac{\partial \delta_h(\rho)}{\partial \rho} = \frac{\partial x_h(\rho)}{\partial \rho} = 0$$

for all $\rho \in [0, 1]$ and maturity structure $\mathcal{M}(\rho)$.

Proof 4 See appendix C.1.

Strikingly, the maturity structure of public debt does not affect the maximum market value of long-term bonds v_t^{max} and this proposition is exactly equivalent to Guillard and Kempf's proposition 1 when government issues one-period debt. This will also imply that the equilibrium default threshold ω_t^{max} is independent from the maturity structure as well.

3.5.2 The equilibrium default threshold

Given the maximum demand for long-term bonds, we are able to solve for the equilibrium default threshold. Sovereign default occurs when the gross financing needs of the government is larger than the total value it could borrow from rational and risk averse rational lenders v_t^{max} , that is

$$\frac{\ell_t}{a_t} - (\hat{\tau} - g) \geq v_t^{max} \quad (3.41)$$

and since the default condition is written as $\frac{\ell_t}{a_t} > \omega_t^{max}$, then the default threshold ω_t^{max} is necessarily equal to

$$\omega_t^{max} = v_t^{max} + (\hat{\tau} - g) \quad (3.42)$$

And finally using Proposition 1 and replacing v_t^{max} by $x_h \omega_{t+1}^{max}$ yields a forward looking equation for ω_t^{max}

$$\omega_t^{max} = x_h \omega_{t+1}^{max} + (\hat{\tau} - g) \quad (3.43)$$

which admits a locally unique but unstable equilibrium:

$$\omega_t^{max} = \omega_h^{max} \equiv \frac{1 - \beta}{1 - x_h} \omega^{sup} \quad (3.44)$$

where $\hat{\tau} - g = (1 - \beta)\omega^{sup}$. Hence, we make the following proposition:

Proposition 5 The equilibrium default threshold ω_h^{max} is constant, determined by the recovery rate

h and independent of the maturity structure of public debt $\mathcal{M}(\rho)$.

Proof 5 From Propositions 3 and 4, both x_h and ω^{sup} are independent from ρ , hence ω_h^{max} is also independent of ρ and $\mathcal{M}(\rho)$.

Whatever the maturity structure $\mathcal{M}(\rho)$, the equilibrium default threshold will remain the same which seems *a priori* to contradict the result by Kim (2015) who concludes that the debt limit is higher with long-term debt than with one-period debt. We also check this result in a simple particular case when default is only possible at $t + 1$ but never after, which allow us to use simple expressions for bond's prices at t and $t + 1$, see appendix C.2.

3.6 Discussion

The result that debt maturity has no impact on the equilibrium default threshold calls for some explanation. We would expect *a priori* long-term debt to smooth the effect of productivity shocks on bond's price, such that the effect of bad productivity shock on the risk-premium is lower when government's debt has a long maturity. Actually, the smoothing effect of long-term debt is effectively at work here, through the Euler equation (3.34): a shock on a_{t+1} would affect both q_t^M and q_{t+1}^M when $\rho > 0$. Yet we have shown that, despite this smoothing effect, an increase of ρ is neutral on the equilibrium default threshold ω_h^{max} . This result is most probably driven by the no-arbitrage condition that leads to equation (3.28).

In comparison with (Kim, 2015), we both use the model of geometrically decaying zero-coupon bonds hence the difference between our results cannot be explained by the modelling choice of long-term bonds. Lorenzoni and Werning (2013) also use a model of geometrically decaying coupon bonds. Yet they do not investigate the effect of public debt maturity structure on the debt limit but rather its effect on equilibrium determinacy and fiscal vulnerability. As already mentioned, Kim (2015) and Lorenzoni and Werning (2013) do not build fully microfunded general equilibrium models, which is an important difference with the sovereign default model build by Guillard and Kempf (2017).

Particularly, the main difference between Kim's model and ours is the *source* of stochastic shocks that causes government to default. In this framework, default is triggered by the de-

mand side of the bond market: government defaults when its gross financing needs exceeds the maximum level it could borrow on the bond market, *given the probability distribution of productivity shocks*, assuming fiscal policy is constrained by the fiscal limit. On the contrary, stochastic shocks in Kim's framework affect the primary balance, and not the pricing equation of long-term bonds through productivity shocks. In particular, Kim shows that the debt limit does not depend on debt maturity when primary balance is deterministic. Actually, our result may not be contradictory to his since, in our framework, the equilibrium default threshold (or debt limit) is defined when the economy is at the fiscal limit $\hat{\tau}$: hence primary balance ($\hat{\tau} - g$) is deterministic.

Thus, we suggest interpreting proposition 5 as a generalization of Kim's findings that the debt limit does not depend on public debt maturity structure, when primary balance is deterministic, in a microfunded general equilibrium model with stochastic productivity shocks. If one agrees to this interpretation, this would imply longer maturity of public debt does not allow government to insulate itself from bad productivity shocks and demand-driven sovereign defaults whereas it is an efficient tool to insulate itself from bad fiscal shocks to primary balance, which somehow may confirm the concept of fiscal insurance (or fiscal hedging).

3.7 Conclusions

Does a longer maturity of public debt result in a higher debt limit and a larger fiscal space? A recent paper by Kim (2015) argues it does. In this chapter, we consider a real economy with complete financial markets following Guillard and Kempf (2017). Government issues long-term debt and collect taxes on wages such that a dynamic Laffer curve arises, implying a fiscal limit on labor tax rate. As a result, when primary balance reaches its maximum level, sovereign default can be endogenized as a market event, at the point the gross financing needs of government exceeds the maximum level it could borrow from the bond market, given the probability distribution of labor productivity shocks. We show that the maturity structure of public debt does not affect neither the deterministic solvency ratio nor the equilibrium default threshold. We suggest these results might not be seen as neces-

sarily contradictory with Kim (2015). In particular, we would argue it clarifies the benefits of long-term debt regarding fiscal sustainability issues. Long-term debt would not allow governments to insulate themselves from bad potential growth shocks, in our framework, but rather from bad fiscal policy shocks as in Kim, 2015.

Regarding future research, one can argue these results may be mainly driven by: the modelling choice of long-term debt, the assumption made on productivity shocks or the flexible-price framework. Regarding the assumption on productivity shock, the question would be whether shocks on labor productivity are independent and identically distributed *over time*, or not. If labor productivity growth actually follows a unit-root with drift process, hence it may well be a realistic assumption since shocks to the long-run growth rate would be i.i.d. in that case. The modelling choice of long-term debt may have much more impact on the result. In our framework, long-term bonds are modelled as perpetuities with a constant decaying factor, so government rolls over the same fraction of total outstanding debt each period, in addition to newly issued debt to cover for primary deficits (if any). A richer debt maturity structure—in particular the co-existence of bonds with different maturities—may not yield the same result. Yet modelling richer maturity structure for public debt entails a significant cost in terms of tractability; that is essentially why Woodford (2001)'s model of long-term is often used. Finally, as shown by empirical research, long-term debt plays a significant role in debt-erosion through inflation. Hence, studying the effect of long-term debt on sovereign default without taking into account inflation dynamics with sticky-prices might be a strong limitation to this analysis.

Chapter 4

Fiscal Policy and Public Debt Sustainability in a Monetary Union: An Appraisal of the European Monetary Union

4.1 Introduction

Over the last three decades, under what is usually called the “Great Moderation,” both macroeconomists and policymakers involved in fiscal policy mainly focused on long-run issues. The consensus was that discretionary fiscal policy was mostly inefficient relative to monetary policy, to say the least. As a consequence, government should adopt rules ensuring the long-run sustainability of public finance, and let an independent central bank take charge of controlling inflation and stabilizing gross domestic product (GDP) growth and unemployment. This consensus was recently challenged following the experience of 2008’s subprime crisis and the following “Great Recession”. To some extent, fiscal policy has been restored as a powerful macroeconomic stabilization instrument during deep recessions, especially when monetary policy can no longer decrease the nominal short-run interest rate. But it also stressed the need of fiscal rules ensuring the long-run sustainability of public debt.

Nevertheless, the need for fiscal rules ensuring fiscal sustainability should not be seen

as necessarily contradicting the short-run stabilization motives of fiscal policy. One of the lessons of European sovereign debt crisis may be that, in face of strong and negative demand shocks, a government must have enough “fiscal space” to use fiscal policy aggressively when needed (Blanchard et al., 2010).

What are these fiscal sustainability requirements? Despite being an (almost) infinitely lived-agent, government faces an intertemporal budget constraint like any other economic agent: it is expected to pay back its debts with future (present-value) primary surpluses; if not, it will at some point default—directly or indirectly—and lose access to financial markets as long as its borrower’s credibility is not restored. Fiscal rules and monitoring of fiscal policy precisely aim at preventing government from engaging upon an unsustainable path; that is, violating its intertemporal budget constraint and eventually defaulting on its debt.

When this occurs, violations of the government intertemporal budget constraint may take different forms, depending on institutional framework: direct default on public debt repayments, monetization by the Central Bank and/or through an increase in present and unexpected future inflation, which are actually indirect forms of default, through an inflation tax. At some point, from a theoretical point of view, violations of a government’s sustainability constraint may result in some “unpleasant monetarist arithmetic,” to use the words of (Sargent and Wallace, 1981). Thus monetary–fiscal interactions’ effects on inflation, and more broadly speaking on macroeconomic stability, provide another set of theoretical arguments in favour of fiscal policy rules. Moreover, fiscal rules and fiscal surveillance are of great importance within a monetary union without federal budget. Uncoordinated national fiscal policies may have a significant impact on monetary policy’s ability to control inflation; but it also makes room for countercyclical fiscal policy, since common monetary policy cannot react to country-specific or asymmetric shocks.

The issue of public debt sustainability and fiscal policy rules has been at the centre of European macroeconomic debate since the Maastricht Treaty (1992), the Stability and Growth Pact (SGP, 1997) and the creation of the European Monetary Union (EMU, 1999). The European fiscal framework has been intensively criticized since the beginning of the 1990s. Its detractors regularly denounce the economic growth and employment costs of alleged procyclical European fiscal rules, while its promoters argue that sound public finances and

financial stability are the sine qua non for strong and sustainable economic growth and therefore full employment. Based on theoretical and empirical research on fiscal policy, fiscal sustainability and monetary–fiscal interactions, we propose a critical appraisal of the European fiscal framework.

This chapter is organized as following. In section 4.2, we develop and present the requirements of fiscal sustainability in a monetary union. Sharing a common currency has strong implications for national fiscal policies. In particular, we show how the government intertemporal budget constraints can be mutually dependent, depending on the architecture of the underlying fiscal union. This mutual interdependence particularly materialized at the monetary level, through the consolidated monetary union intertemporal budget constraint. Following the presentation of fiscal sustainability constraints, we turn to monetary-fiscal interactions' analysis. According the Fiscal Theory of Price-Level, an independent monetary policy needs the appropriate fiscal backing to fulfill its mandate –and by appropriate we mean whether fiscal policy satisfy (or not) its sustainability requirements. As they become more demanding in a large monetary union, we will see the particular implication for fiscal policies in a MU. Finally, we address the question of adequate fiscal rules in a monetary union. We show that the design of these rules would critically depend on: (i) the type of fiscal union in place, (ii) on the risk of reaching the fiscal limit, which lead to the concept of "prudent" fiscal rules and finally (iii) on the trade-off between fiscal sustainability requirements and macroeconomic stabilization.

In section 4.3 we propose a critical survey on the European Fiscal Framework based on empirical and theoretical research presented earlier. We argue that European fiscal rules may be both too tight and too loose: too tight because European fiscal rules are a priori much stricter than what would be required according to fiscal sustainability analysis; too loose because they induce a procyclical bias that, in addition to economic growth and employment costs, may be counterproductive in ensuring fiscal sustainability. In particular we present the debate about the causes of the European sovereign debt crisis, which was at first interpreted as the result of irresponsible fiscal policies and therefore called for a tightening of fiscal rules. We show that recent research results have significantly challenged this view. A new consensus narrative recently emerged which significantly changes the diagnosis as

well as the economic policy responses it calls for. Finally, we discuss the on-going reflexion about moving toward a more integrated fiscal union in Europe, in particular with the proposal of Eurobonds.

Section 4.4 draws some general conclusions about fiscal policy in a monetary union and more specific conclusions about the EMU and the European fiscal framework.

4.2 Fiscal sustainability in a Monetary Union

The existence of a monetary union mechanically the interdependence of fiscal policies and, as a result, we agree with Kempf (2013) saying that a monetary union is *always* a fiscal union. In this section, we present and review the fiscal sustainability requirements of a monetary union of n economies. We derive the budget constraints for each authority: national, federal but also at the monetary union level. We show how fiscal sustainability requirements became specifically more complex once countries decide to share the same currency. Fiscal requirements, in addition to be necessary *per se*, are also key to the understanding of monetary-fiscal interactions and inflation determination in a monetary union. Consequently, we present the main mechanisms of the Fiscal Theory of Price-Level and its specific implications for the economics of Monetary Unions. Finally, we present the policy implications in terms of fiscal rules.

4.2.1 Fiscal sustainability requirements in a Monetary Union

Following Kempf (2017), let's consider a monetary union composed by n national fiscal authorities indexed by $i = 1, \dots, n$, a federal fiscal authority indexed by F and a common Central Bank. We make the following assumptions:

- In the most general case, each national government and the federal fiscal authority can issue national and federal *nominal* debts, respectively denoted by B_t^i and B_t^F . For the sake of simplicity, we assume national and federal debts pay the same level of nominal interest rate i_t .
- We denote by G_t^i and T_t^i respectively non-interest spending and revenue and by $S_t^i =$

$T_t^i - G_t^i$ primary balance of fiscal authority i . We will also denote by G_t^F , T_t^F and S_t^F respectively the non-interest spending, revenue and primary balance of the federal fiscal authority.

- Each national government i receives a net transfer Z_t^{ij} from country j and a real net transfer Z_t^{iF} from the federal government which can be positive or negative¹.
- Finally, for clarity and simplification purposes, we assume the Central Bank do not monetize public deficits, implying no seigniorage revenues for national and federal authorities.

In a monetary union, fiscal sustainability requirements are heavily dependent on the architecture of the monetary union and particularly on the form of the fiscal union. Each national government as well as the federal government face specific budget constraints. But there is an additional fiscal sustainability constraint as one considers the monetary union *consolidated* budget constraint. Finally, the role of inter-national and federal transfers, hence the nature of the fiscal union and whether the federal government has a full fiscal capacity or not, matter for the sustainability constraints of the monetary union.

National and federal budget constraints

Let us start with the national budget constraint of government i , in nominal terms:

$$B_t^i + T_t^i + \sum_{j=1}^n Z_t^{ij} + Z_t^{iF} = (1 + i_t)B_{t-1}^i + G_t^i \quad (4.1)$$

where $\sum_{j=1}^n Z_t^{ij}$ is the total net transfer received by country i from all the other members of the monetary union. By construction, the sum of total net transfers between national governments is necessarily equal to zero

$$\sum_{i=1}^n \left(\sum_{j=1}^n Z_t^{ij} \right) = 0 \quad (4.2)$$

¹It implies $Z_t^{ii} = 0$, by construction.

In a similar way, the federal government budget constraint writes, in nominal terms:

$$B_t^F + T_t^F = (1 + i_t)B_{t-1}^F + G_t^F + \sum_{i=1}^n Z_t^{iF} \quad (4.3)$$

where the total net federal transfers toward national government, i.e. $\sum_{i=1}^n Z_t^{iF}$, enters on the right hand-side of the budget constraint since Z_t^{iF} (if positive) is equivalent to a federal spending.

We now rewrite precedent flow budget constraints in terms of (respectively) national and federal nominal GDP. First, let $P_t^i Y_t^i$ denote the nominal GDP of country i where P_t^i is the price-level and Y_t^i is the real GDP. We define the inflation rate $\pi_t^i \equiv P_t^i / P_{t-1}^i - 1$ and the growth-rate of real GDP $y_t^i \equiv Y_t^i / Y_{t-1}^i - 1$. Hence, at the monetary union level, we denote nominal GDP by $P_t^{MU} Y_t^{MU}$, the gross inflation rate by π_t^{MU} and the growth-rate of real GDP by y_t^{MU} , which are weighted average of national inflation and real-growth rates $\pi_t^{MU} = \sum_{i=1}^n \psi_t^i \pi_t^i$ and $y_t^{MU} = \sum_{i=1}^n \psi_t^i y_t^i$ where $\psi_t^i \equiv P_t^i Y_t^i / P_t^{MU} Y_t^{MU}$ denotes the relative size of country i in the monetary union at time t . Hence, the national budget constraint of country i in terms of nominal GDP-scaled variables² can be expressed as follows:

$$b_t^i + s_t^i + \sum_{j=1}^n z_t^{ij} + z_t^{iF} = \frac{1 + i_t}{(1 + \pi_t^i)(1 + y_t^i)} b_{t-1}^i \quad (4.4)$$

where s_t^i is the primary surplus-to-GDP ratio of country i . Similarly the federal government budget constraint:

$$b_t^F + s_t^F = \frac{1 + i_t}{(1 + \pi_t^{MU})(1 + y_t^{MU})} b_{t-1}^F + \sum_{i=1}^n \psi_t^i z_t^{iF} \quad (4.5)$$

where s_t^F is the primary surplus-to-GDP ratio of the federal authority. Equations (4.4) and (4.5) calls some preliminary comments. First, in addition to fiscal spillovers effects, these equations show the direct interdependence of fiscal policies in a monetary union because of inter-national and federal fiscal transfers: a primary deficit in country i could be financed by a positive net transfer from country j or from the federal government. Second, the budget constraint of the federal government critically depends on the nature of the underlying

²For any variable X_t^i , the corresponding GDP-scaled variable is defined by $x_t^i \equiv S_t^i / P_t^i Y_t^i$. For federal variables, we will use the monetary union nominal GDP.

fiscal union. If the federal government has no fiscal capacity, i.e. $s_t^F = 0$, then the federal government cannot issue federal debt and can only redistribute net federal transfers between member states of the monetary union. In that particular case, the budget constraint becomes:

$$\sum_{i=1}^n \psi_t^i z_t^{iF} = 0 \quad (4.6)$$

in absence of seigniorage revenue from the central bank³.

Sustainability conditions and intertemporal budget constraints

Studying sustainability of public debt and deficits requires examining the intertemporal budget constraint of government, which is obtain by iterating forward equations (4.4) and (4.5) and by imposing a "transversality condition" or No-Ponzi Game condition (NPG, henceforth). The NPG condition states that a solvent government cannot roll over debt plus interests forever but needs to cover at least a small amount of its debt-service with primary surpluses. This is equivalent to say that the average rate of growth of public debt must be strictly lower than the average interest rate (Bohn, 2007; Hamilton and Flavin, 1986). As a consequence, NPG condition implies long-run present-value public debt–GDP ratio must be equal to zero:

$$\lim_{T \rightarrow +\infty} \mathbb{E}_t \tilde{Q}_{t,T+1}^i b_{t+T}^i = 0 \quad (4.7)$$

Hence for any national government i , when equation (4.7) holds, the government intertemporal budget constraint (GIBC, henceforth) writes:

$$b_{t-1}^i = \mathbb{E}_t \sum_{k=0}^{\infty} \left[\tilde{Q}_{t,k}^i \left(s_{t+k}^i + z_{t+k}^{iF} + \sum_{j=1}^n z_{t+k}^{ij} \right) \right] \quad (4.8)$$

where $\tilde{Q}_{t,k}^i$ is the growth-adjusted stochastic discount factor for country i and satisfies, in equilibrium, $\tilde{Q}_{t,k}^i = \prod_{j=0}^k \left(\frac{1+i_{t+j}}{(1+\pi_{t+j}^i)(1+y_{t+j}^i)} \right)^{-1}$. Equation (4.8) shows that country i benefits from inter-national and federal transfers to meet its intertemporal budget constraint. As a result, the initial outstanding level of national debt could eventually being backed not only by national primary surplus but also by net transfers from other members of the monetary

³In presence of seigniorage revenue, total net federal transfers should be equal to the seigniorage revenue federal government gets from the Central Bank, as noted by Kempf (2017).

union and the federal government.

Similarly if we apply the NPG condition to the federal government, the GIBC at the federal level writes:

$$b_{t-1}^F = \mathbb{E}_t \sum_{k=0}^{\infty} \left[\tilde{Q}_{t,k}^{MU} \left(s_{t+k}^F - \sum_{i=1}^n \psi_{t+k}^i z_{t+k}^{iF} \right) \right] \quad (4.9)$$

where $\tilde{Q}_{t,k}^i$ is the growth-adjusted stochastic discount factor for the whole Monetary Union and satisfies, in equilibrium, $\tilde{Q}_{t,k}^{MU} = \prod_{j=0}^k \left(\frac{1+i_{t+j}}{(1+\pi_{t+j}^{MU})(1+y_{t+j}^{MU})} \right)^{-1}$. Equation (4.9) states that the outstanding level of federal debt-to-GDP must be backed by federal primary surplus-to-GDP *minus* total net federal transfers to national governments, illustrating the interdependence between national fiscal policies and federal fiscal policy.

The Monetary Union consolidated intertemporal budget constraint

In addition to national and federal fiscal sustainability constraints, we can also consider the consolidated intertemporal budget constraint at the Monetary Union level, by summing all budget constraints. Let b_t^{MU} be the consolidated outstanding level of public debt-to-GDP ratio:

$$b_t^{MU} = b_t^F + \sum_{i=1}^n \psi^i b_t^i \quad (4.10)$$

Hence, by summing GIBCs of national governments and federal government (i.e. equations (4.8) and (4.9)), we obtain the consolidated monetary union GIBC:

$$\begin{aligned} b_t^{MU} &= \mathbb{E}_t \sum_{k=0}^{\infty} \left[\tilde{Q}_{t,k}^{MU} \left(\sum_{i=1}^n \psi_{t+k}^i \left(s_{t+k}^i + z_{t+k}^{iF} \right) + s_{t+k}^F - \sum_{i=1}^n \psi_{t+k}^i z_{t+k}^{iF} \right) \right] \\ &= \mathbb{E}_t \sum_{k=0}^{\infty} \left[\tilde{Q}_{t,k}^{MU} \left(\sum_{i=1}^n \psi_{t+k}^i s_{t+k}^i + s_{t+k}^F \right) \right] \end{aligned} \quad (4.11)$$

since we know from equation (4.2) that $\sum_i (\sum_j z_{t+k}^{ij}) = 0$, for all $t+k$. Equation (4.11) show the financial interdependence within members states: directly, since countries running a primary surplus could eventually compensate countries running primary deficits, or indirectly through the federal government primary surplus. As we will see below in section

4.2.3, if a country i no longer adjusts its primary surplus to its outstanding level of debt b_t^i and if no member country of the MU or the federal government adjust their primary surplus to b_t^i , then equation (4.11) would not be satisfied and the whole monetary union would not meet its sustainability requirements.

Fiscal sustainability of MUs under alternative forms of fiscal union

Fiscal sustainability requirements in a monetary union are summarized by equations (4.8), (4.9) and (4.11). How these three constraints interact depend on the architecture of fiscal policies within the monetary union, that is, the type of *fiscal union* backing the monetary union. Using the typology proposed by Kempf (2013), fiscal unions can be classified according two criteria:

1. It can be either *horizontal* or *vertical*. A fiscal union would be horizontal if it consists in rules and institutions organizing the cooperation between member countries. It would be vertical if a federal fiscal government exists.
2. It can be either *negative* or *positive*. A negative fiscal union would only consist, for instance, in strict fiscal rules preventing national governments from running excessive deficits without any fiscal cooperation and transfers. A positive fiscal union, on the contrary, would imply fiscal cooperation and transfers.

Kempf (2013) applies this typology to the European Monetary Union and use it to describe three competing views of the European fiscal union. The "German view" –which is also the current state of the EMU, embodied in the TSCG or "Fiscal Compact"– would be a negative and horizontal fiscal union, based on strict fiscal rules with no significant fiscal transfers between members states of the monetary union. The "French view" would be a positive and horizontal fiscal union, implying fiscal cooperation (i.e. coordination of national fiscal policies and transfers between member states) without an autonomous fiscal authority. Finally, the "European Commission view" would be both a positive and vertical union, implying a complete fiscal federalism.

What would be the consequences of each of these views on fiscal sustainability requirements presented earlier? Taking the German view, and actual situation of the EMU, it im-

plies that national governments cannot expect neither inter-national nor federal transfers to back national public debt. Hence, the national GIBC of country i collapses to:

$$b_{t-1}^i = \mathbb{E}_t \sum_{k=0}^{\infty} \left[\tilde{Q}_{t,k}^i s_{t+k}^i \right] \quad (4.12)$$

since $z_t^{ij} = 0$ and $z_t^{iF} = 0$ for all t . Equation (4.12) implies the consolidated monetary union GIBC –i.e. equation (4.11)– is satisfied *if and only if* each national government meets its own GIBC. As a result, it explains the importance of a strict compliance with fiscal rules at the national level. In that situation, there are as many sustainability constraints as national governments, and the consolidated monetary union GIBC is *redundant*: as long as equation (4.12) holds for all i , then equation (4.11) holds. Taking the French view, the existence of inter-national transfers without federal transfers would make the consolidated monetary union GIBC matter, since primary deficits in country j could be compensated by positive net transfers from countries $i \neq j$, to the extent that primary surpluses in the rest of the monetary union are sufficient to make equation (4.11) hold. Similarly, in the case of fiscal federalism⁴ and for a given level of federal transfers, fiscal sustainability in the monetary union would require each national government as well as the federal government to meet their respective GIBC.

As we have seen, fiscal requirements in a monetary union are multiple and critically depends on the architecture of the underlying fiscal union. We will develop implications of these intertemporal budget constraints in terms of fiscal rules, in section 4.2.3.

4.2.2 Monetary-Fiscal interactions in a Monetary Union

Since the very beginning of EMU, while the Maastricht Treaty was being negotiated, negative externalities coming from unsustainable fiscal policy at national level received a lot of attention (Buiter et al., 1993; Wyplosz, 1991). Expansionary fiscal policy generally boosts demand and increases the real interest rate and the inflation rate. Outside a monetary union, in a flexible exchange rate regime, these effects would be partially or totally offset through adjustment in the nominal exchange rate. On the contrary, within a monetary

⁴For simplicity, we assume that federal transfers would replace inter-national transfers.

union adjustment occurs entirely through prices, wages and real interest rate (Cooper et al., 2014; Woodford, 1996) without any insulation possible through monetary policy, because of the interdependence of national fiscal policies. Thus “excessive deficits” of one member country of the monetary union may affect the real interest rate and the inflation rate of all member countries, in proportion to its relative size in the monetary union.

Concerns about undesirable effects of “excessive deficits” mostly focused on the monetary and financial instability that they could imply (Buiters et al., 1993). The motivation for preventing “excessive deficits” and unsustainable national fiscal policies was to ensure (nominal) convergence among members of EMU. What is the rationale behind fiscal rules as requirements for price-level stability?

There are two main approaches of monetary–fiscal interactions to the explanation of why fiscal policy should be constrained in order to control inflation stability: Sargent and Wallace’s (1981) “unpleasant monetarist arithmetic” and the Fiscal Theory of Price Level (Cochrane, 2001, 2005; Leeper, 1991; Sims, 1994, 1999; Woodford, 1995, 1996, 2001). Both approaches focus on GIBC and link the need for fiscal rules that ensure the sustainability of public debt to achieve inflation stability, in particular in a monetary union.

A primer on monetary-fiscal interactions: Sargent and Wallace’s “unpleasant monetarist arithmetic” (1981)

In their seminal paper, Sargent and Wallace show that strategic interactions between fiscal and monetary authorities can jeopardize a central bank’s ability to stabilize inflation, even in a purely monetarist economy. What matters is which authority moves first, the monetary or the fiscal authority. If fiscal policy decides to run excessive deficits, implying “fiscal dominance”, then it will accumulate public debt until it reaches its maximum sustainable level, given the demand for public bonds. Thus, even when the central bank follows a strict monetarist rule, controlling money supply growth and inflation in the short run, it will be forced to monetize public debt and increase the money supply when public debt hits its maximum level. So here is the main result of Sargent and Wallace: “tighter money now can eventually mean higher inflation tomorrow” if fiscal policy is dominant and even if monetary policy is tight today.

It is important that Sargent and Wallace's model does not depart from the quantity theory of money, since higher inflation arises from the fact that the monetary authority is forced to monetize public debt, which is to increase money supply and seigniorage revenue of government. As a result, the GIBC can affect the inflation rate significantly when fiscal policy dominates monetary policy, i.e. when the central bank loses control of the money creation and is forced to monetize public debt. Consequently, achieving inflation stability requires credible and binding policy rules for each authority: the central bank must credibly commit to inflation stability and cannot monetize public debt while outstanding public debt must be credibly backed by future expected primary surpluses. In practice, it supports the introduction of a no bail-out clause between monetary and fiscal authorities. Still, the credibility—and desirability—of such a clause remains questionable, in the light of the recent sovereign debt crisis in Europe and the positive impact of the Outright Monetary Transaction (OMT) on sovereign debt spreads after summer 2012 (Afonso et al., 2017).

The FTPL and its implications for MUs

The Fiscal Theory of Price Level (FTPL) is more radical than Sargent and Wallace's "unpleasant monetarist arithmetic". It basically states that "monetary policy alone does not provide the nominal anchor for an economy" and it is "a particular pairing of monetary policy and fiscal policy" which provides the nominal anchor and stabilizes inflation (Canzoneri et al., 2010). According to the FTPL, even in the absence of seigniorage revenue, binding rules on excessive deficits and public debt are necessary to achieve price stability.

The FTPL starts from the assumption that government issues nominal debt rather than real debt and then rewrites the intertemporal budget constraint:

$$\frac{B_{t-1}}{P_t} = \mathbb{E}_t \sum_{k=0}^{+\infty} \frac{S_{t+k}}{(1+r)^k} \quad (4.13)$$

where we assume, for the sake of simplicity, that the discount factor is constant and equal to $(1+r)^{-1}$ where r is the real interest rate and where B_{t-1} , S_t and P_t denote respectively the end-of-period stock of nominal debt, the real primary surplus and the price-level.

Fiscal theory considers the government's IBC as an *ex post* equilibrium condition rather

TABLE 4.1 – Leeper’s (1991) classification of monetary-fiscal interactions

	Passive Monetary policy (PM)	Active Monetary policy (AM)
Passive Fiscal policy (PF)	Indeterminacy of inflation and public debt dynamics Multiple equilibria	Regime M Inflation determined by monetary policy Unique equilibrium
Active Fiscal policy (AF)	Regime F Inflation determined by fiscal policy Unique equilibrium	Explosive dynamics of inflation and public debt No equilibrium

than an *ex ante* budget constraint on fiscal policy. Then, if government does not adjust its fiscal policy to make this constraint hold *ex ante*, then price level will have to adjust *ex post* to make it hold in equilibrium. Within FTPL’s framework, two polar cases for fiscal policy arise. First, fiscal policy is Ricardian and future primary surplus adjusts such that GIBC holds *ex ante*; monetary authority can have full control over the price level through a standard interest rate rule. Second, fiscal policy is not Ricardian and does not satisfy its GIBC *ex ante*; GIBC is no longer a constraint for fiscal policy but a valuation equation for real public debt such that price-level P_t adjusts in order to equalize *ex post* the real value of public debt to the sum of future primary surpluses. In this case, monetary policy loses control of the price level, even in absence of seigniorage revenue. Cochrane (2001) extends the FTPL in a framework with long-term debt. In that case, the maturity of public debt determines whether the adjustment occurs through current or future inflation.

Leeper’s typology (1991) of monetary and fiscal interactions is a more restrictive definition of the FTPL. He studies different sets of monetary and fiscal policies achieving both stable inflation dynamics and stable nominal public debt dynamics, which requirements are stronger than the GIBC. He assumes monetary policy follows a Taylor Rule and fiscal policy follows a tax rule such that tax rate reacts to debt level. He characterizes monetary and fiscal policies as “active” and “passive”:

- Monetary policy is said *active* if it satisfies the Taylor principle; if not, it is “passive” and it reacts less aggressively to inflation.
- Fiscal policy is said *passive* if the tax rate reacts to public debt more than the average interest rate, such it stabilizes debt; if not, it is “active” and does adjust taxes to debt.

Consequently, Leeper describes four combinations possible for monetary and fiscal policies, see table 4.1. Two combinations of monetary and fiscal policies—Regime M and Regime

F—lead to a unique macroeconomic equilibrium, implying stable inflation and public debt dynamics along the balanced growth path. In regime M, the central bank determines the inflation rate following Taylor Rule while the government stabilizes its public debt. In regime F, the government no longer stabilizes public debt but determines the price-level and the inflation rate through its intertemporal budget constraint, as long as the central bank reacts passively to inflation (i.e. deviates from the Taylor Principle). One combination (PM/PF) leads to indeterminacy and multiple equilibria: in this case, the economy is subject to self-fulfilling dynamics. The last case (AM/AF) leads to explosive dynamics of both inflation and public debt.

The FTPL has particular implications for monetary unions as it emphasizes how fiscal sustainability requirements *in each country* matter for the control of inflation *in the whole monetary union*. Woodford (1996) specifically addresses this question and develops a simplified two-country monetary union model with a common central bank following a non-inflationary monetary policy. He shows that it only suffices that one country deviates from a Ricardian fiscal policy to affect "inflation, real interest rate and output in *both* countries". In that case, the only way for country 2 to offset the macroeconomic effects of a non-Ricardian policy in country 1 is to adjust its primary deficit inversely with that of country 1: that is, to fully "cooperate" and finance its primary deficits. We can see why directly in equation (4.11), in absence of any transfer and of any binding fiscal rule on national deficits. Ensuring the consolidated monetary union GIBC hold would require that primary deficits from non-Ricardian governments be totally compensated by *additional* primary surpluses from Ricardian governments⁵. Refusal to "cooperate" would result in a even higher inflation and output instability. As a result, sharing a common currency increases the incentive for national governments to follow a non-Ricardian policy because it allows to redistribute wealth from households of countries following a Ricardian policy toward those of countries following non-Ricardian policies (Sims, 1999; Woodford, 1996).

While Sims (1999) does not deny the risk of "fiscal free-riding" and the need of credible and binding fiscal rules, he emphasizes the specific implications of the FTPL for central bank independence and fiscal policies' coordination in MUs. Following Sims, it would be

⁵By "additional" we mean: in addition to what is required to back the outstanding level of public debt.

mistaken to think of central bank independence only as *the absence of relationship* between the monetary and fiscal authorities. For instance, by ruling out debt-monetization or introducing a strict bail-out clause between monetary and fiscal authorities, as suggested by Sargent and Wallace's view of monetary-fiscal interactions. On the contrary, a truly independent central bank is such that it can *reach* its inflation and economic activity objectives, under *both* inflationary *and* deflationary stress, with the adequate fiscal policy backing, according the FTPL's view (Sims, 1999). In particular, under deflationary stress in a monetary union, Sims argues that national fiscal authorities *should* run active (or non-Ricardian) fiscal policies to avoid the risk of self-fulfilling dynamics and help the central bank fulfilling its mandate. Recently, Jarociński and Maćkowiak (2017) somehow confirmed Sims (1999) and argued the current Euro-area policy-mix –with both monetary and national fiscal policies being passive following the crisis– leads to multiple equilibria and self-fulfilling dynamics, as suggested by the FTPL, and preventing the ECB to achieve its 2% inflation mandate.

In that situation, the FTPL implies fiscal policy can eventually have large and significant real effects on the economy. In a Regime F, debt-financed expansionary fiscal policy actually boosts aggregate demand through a positive wealth effect because Ricardian equivalence no longer holds and households expect that current deficits will not be financed through future taxes. As a result, government spending and tax multipliers are significantly higher when monetary policy is passive and fiscal policy active (Davig and Leeper, 2011). When monetary policy becomes passive as it is constrained by the Zero Lower Bound (ZLB) on nominal interest rate, then fiscal multipliers would likely be much higher and inflation would be determined by fiscal policy.

As a result, the FTPL provides strong arguments to implement an active fiscal policy when monetary policy is constrained by the ZLB and unable to reach its inflation objective, as well as it emphasize the importance of binding fiscal rules in the control of inflation when monetary policy is not constrained.

Criticisms of the FTPL and empirical validity discussion

FTPL's detractors such as Buiter (2002) strongly criticized this interpretation of GIBC as an equilibrium condition. In his view, GIBC is a real constraint on government behaviour and

GIBC must hold for any price level. As a result, macroeconomic equilibria described by the FTPL are “invalid” in Buiter’s view. On the contrary, Woodford (2001) considers that government knows it can affect equilibrium price level and interest rates, which is not possible for other economic agents. Another question is the empirical validity of the FTPL: is there evidence of “fiscal inflation” episodes? Empirical literature has not reached any consensus yet. Canzoneri et al. (2001) show that fiscal sustainability imposes very weak restrictions, such that observed data on public debt and primary surplus would be consistent with GIBC and, as a result, making it difficult to distinguish between Ricardian and non-Ricardian fiscal policies. They show that US post-Second World War data may be well explained by the Ricardian regime. Creel and Le Bihan (2006) extend Canzoneri et al.’s method using cyclically adjusted balance data and find no evidence supporting the FTPL, using an international dataset that includes the USA, Germany, Italy, France and the UK.

Yet, using regime-switching techniques to estimate feedback policy rules for monetary and fiscal authorities, Favero and Monacelli (2005), Davig and Leeper (2007a, 2011), Afonso and Toffano (2013) and Cevik et al. (2014) provide evidences of recurring changes in monetary and fiscal policy rules. Both monetary and fiscal policies periodically switch from active to passive (or passive to active). In a New-Keynesian DSGE model solved using non-linear methods, Davig and Leeper (2007a, 2011) show the “expectation effect” regime-switching fiscal and monetary policies. Since monetary and fiscal authorities are subject to recurring changes, agents expect with a positive probability that the economy could switch toward a regime F. As a result, the economy displays non-Ricardian features, suggesting the FTPL would be effectively at work. Jarociński and Maćkowiak (2017) also supports the FTPL in the sense the recent “Euro-area malaise” could be explain by a bad coordination of passive monetary and fiscal policies, resulting in sunspot equilibria and expectation-driven persistently low inflation and output growth.

4.2.3 The design of Fiscal Rules in a Monetary Union

In a monetary union, fiscal rules should have two objectives: fiscal sustainability motives, as it must guarantee compliance with intertemporal budget constraints at national, federal and union levels simultaneously, but also inflation *and* output growth stability. We will

present sufficient conditions on simple linear fiscal rules, based on Bohn (1998) Model-Based Sustainability (MBS, henceforth) analysis; we will focus on the three competing views for the European fiscal union identified by Kempf (2013) and presented in section 4.2.1. Second, it appears that fiscal rules *à la* Bohn would not be compatible with the existence of a fiscal limit, that is a maximum level of primary surplus-to-GDP. Hence, we will review the literature on fiscal limits and present its consequences on a general fiscal rule. Finally, we discuss the trade-offs between macroeconomic stabilization and fiscal sustainability in a monetary union.

Sufficient fiscal rules for transversality conditions in MUs

Analogous to monetary feedback policy rules, such as the Taylor Rule that relates the short-term interest rate to the current (or past) inflation rate and output gap, Model-Based Sustainability analysis proposes to study fiscal sustainability using fiscal feedback policy rules. Following Bohn (1998, 2008) such fiscal rules are generally specified as follows:

$$s_t = \alpha + \gamma b_{t-1} + \beta_x x_t + \beta_g s g_t + \varepsilon_t \quad (4.14)$$

These feedback rules basically assume that fiscal policy's instrument –in general, primary surplus-to-GDP ratio⁶– reacts to:

- Initial level of public debt–GDP b_{t-1} , to account for fiscal sustainability motives.
- Contemporaneous output gap x_t , defined as the gap between actual and potential (or trend) real GDP, to account for “automatic stabilizers” and countercyclical fiscal policy.
- Cyclical fluctuations in real public expenditures $s g_t$, defined as the difference between actual and trend real expenditures, to account for spending reversals.
- The constant term α would be different from zero and negative, accounting for the fact fiscal policy is not required to run primary surpluses all the time, if the fiscal policy rule is satisfying a debt-stabilizing criterion (see below).

⁶The same analysis would apply to any fiscal instrument: taxes and net transfers should behave in the exact same way as the primary surplus while spending should behave *inversely*.

In a dynamic general equilibrium model, Bohn (1998) shows a sufficient condition on these fiscal policy rules to satisfy GIBC on public debt is such that primary surplus-to-GDP must increase with public debt-to-GDP:

$$\gamma > 0 \quad (4.15)$$

This sustainability criterion can be generalized to any fiscal instruments and measure of public debt, as well as to any non-linear specifications of fiscal policy rule (Bohn, 2008).

In monetary unions, fiscal rules may be defined both at national and federal level, depending on the form of the underlying fiscal union. Let f_U denotes the fiscal rule linking each fiscal instrument –namely s^i and s^F – to the level of outstanding national debt b^i , federal debt b^F or consolidated monetary union debt b^{MU} and to the level of output gap at the national level x^i or at the monetary union level x^{MU} .⁷ Each rule is indexed by the type U of fiscal union $U \in \{GER, FRA, EC\}$, where *GER* stands for "German", *FRA* stands for "French" and *EC* for "European Commission".

Ordoliberalism and strict binding fiscal rules: the German view In the case of an horizontal and negative fiscal union, national fiscal policies cannot expect inter-national transfers to back national public debt. As a result, net transfers (if they exist) z^{ij} are independent of any measure of national public debt b^i . Hence, each national government then must credibly commit to run sufficient future national primary surpluses to meet its GIBC. Building on Bohn (1998), it comes that each member state i of the MU must follow the following fiscal rule:

$$s_t^i = f_{GER}(b_{t-1}^i, x_t^i) + \varepsilon_t^i \quad (4.16)$$

where ε_t^i is a fiscal policy shock and satisfy the following condition on the reaction of primary surplus to national debt:

$$\frac{\partial f_{GER}}{\partial b_{t-1}^i} > 0 \quad (4.17)$$

If each national government simultaneously satisfy this condition, hence the consolidated GIBC at the monetary union (4.11) would hold; but if only one government deviates from it, the monetary union consolidated GIBC would no longer hold.

⁷We abstract here from spending reversals for simplicity and clarity purposes

Fiscal cooperation without federal authority: the French view The French view complicates slightly the analysis. Inter-national net transfers within the monetary union may eventually depend on national debt, implying financial solidarity between member states. In that case, national primary surplus should not only react to the national level but also to the monetary union consolidated level of outstanding public debt, implying the following fiscal rule:

$$s_t^i = f_{FRA}(b_{t-1}^i, b_{t-1}^{MU}, x_t^i) + \varepsilon_t^i \quad (4.18)$$

where the consolidated monetary union outstanding public debt $b_{t-1}^{MU} = \sum_{i=1}^n \psi^i b_{t-1}^i$ denotes a weighted average of national public debts. Two cases are possible. First, if all member states satisfy the following condition

$$\frac{\partial f_{FRA}}{\partial b_{t-1}^i} > 0 \quad (4.19)$$

the monetary union would meet its consolidated GIBC, even when members states' primary surplus do not react to b^{MU} . Second, if k countries among the n fiscal authorities of the monetary union do not satisfy condition (4.19), hence the $n - k$ other fiscal authorities must now satisfy the additional condition:

$$\frac{\partial f_{FRA}}{\partial b_{t-1}^{MU}} > 0 \quad (4.20)$$

such that the monetary union meets its GIBC as a whole.

Fiscal federalism: the European Commission view Now we assume a federal authority exists with a real fiscal capacity, hence the existence of federal debt b^F . For simplicity, we assume federal transfers would replace inter-national transfers. Last, ensuring the sustainability of the monetary union would likely be the mission of the federal government. Thus, two fiscal rules should coexist. One at the national level:

$$s_t^i = f_{EC}(b_{t-1}^i, x_t^i) + \varepsilon_t^i \quad (4.21)$$

and another at the federal level:

$$s_t^F = f_{EC}(b_{t-1}^F, b_{t-1}^{MU}, x_t^{MU}) + \varepsilon_t^F \quad (4.22)$$

In any case, the federal government should satisfy

$$\frac{\partial f_{EC}}{\partial b_{t-1}^F} > 0 \quad (4.23)$$

to ensure the sustainability of federal public debt. As in the French view, regarding national governments, the possibility that one (or more) member state could deviate from condition (4.19) implies that the federal authority should be eventually forced to make the monetary union consolidated GIBC hold by satisfying the following condition:

$$\frac{\partial f_{EC}}{\partial b_{t-1}^{MU}} > 0 \quad (4.24)$$

Moving toward a more integrated, positive, fiscal union undoubtedly increase the complexity of fiscal requirements and fiscal rules in a monetary union. At some point each national government or the federal government would be forced to consider not only its outstanding level of debt but also the consolidated level of debt at the monetary union level. On the other hand, in the case of a negative union, it also appears that deviations of one country from its fiscal rule requirement would not be compensated by any institutional mechanism or federal fiscal buffer.

Prudent fiscal rules in presence of fiscal limits

Satisfying the GIBC imposes very weak requirements per se (Bohn, 2007). Theoretically, as long as government can roll over its debts on financial markets, it could accumulate an ever-increasing amount of public debt–GDP, provided that this ratio grows at a rate lower than the real interest rate adjusted for real GDP growth rate. As a consequence, GIBC does not imply per se any upper bound on public debt–GDP ratio, raising questions whether GIBC and TC are really sufficient to ensure “fiscal sustainability”. Yet additional considerations on fiscal policy would be required to justify bounded debt–GDP ratios, which would be

a stronger definition of fiscal sustainability. A prudent answer could be that they are the minimum requirements for sustainability, but still they do not exclude sovereign default, if government was not able to roll over debt on financial markets.

There are two main arguments to justify an upper bound on public debt–GDP ratio. One approach is structural, using simulated or estimated DSGE models, and relies on the assumption of an upper limit on primary surplus–GDP ratio (Bi, 2012; Bi and Leeper, 2013; Bi and Traum, 2012). The upper boundary for primary surplus–GDP ratio is justified by two main reasons:

1. The existence of a “Laffer curve” owing to distortionary taxation: there should be an optimal tax rate which maximizes tax revenue (Trabandt and Uhlig, 2011);
2. The fact government may not be capable of decreasing public spending–GDP ratio beyond some level for political reasons (Daniel and Shiamptanis, 2013).

Given that $s_t \leq s_t^{max}$ and using GIBC, one can define a maximum public debt–GDP ratio, called the “fiscal limit,” at which government defaults. The following equation combines the GIBC and the assumption made about $s_t^{max} \equiv \tau_t^{max} - g_t^{min}$ to yield an analytic expression for the “fiscal limit” b_{t-1}^{max} :

$$b_{t-1}^{max} = \sum_{k=0}^{\infty} \mathbb{E}_t \tilde{Q}_{t,t+k}^{max} \left(\tau_{t+k}^{max} - g_{t+k}^{min} \right) \quad (4.25)$$

where $\tilde{Q}_{t,t+k}^{max}$ represents the growth-adjusted stochastic discount factor evaluated at the Laffer maximum tax rate.

The fiscal limit is the maximum level of public debt–GDP ratio that could be backed by expected future present-value primary surpluses; beyond this level, fiscal policy would be necessarily playing a Ponzi Game against its creditors. A complete presentation of this concept accounts for uncertainty and effects of aggregate productivity shocks, or fiscal policy regime shifts on the future maximum primary surpluses (Bi, 2012; Bi and Leeper, 2013). Accounting for uncertainty implies the fiscal limit would not be deterministic but rather stochastic. Consequently, in a stochastic economy, sovereign default could occur at very various levels of public debt–GDP, even relatively low levels if the economy faces very adverse macroeconomic shocks and/or if a government is engaged on an unsustainable path,

running persistent primary deficits.

Another approach suggested by Ghosh et al. (2013b) accounts for the “fiscal fatigue” phenomenon. Using panel data on 23 advanced economies and covering the years 1970–2007, Ghosh et al. found a non-linear relationship between primary balance and public debt such that, at high debt levels, fiscal policy is no longer able to increase sufficiently its primary balance to stabilize public debt. Facing risk-neutral international investors, government hits the fiscal limit when primary surplus–GDP can no longer offset public debt’s snowball effect $(r_t - y_t)b_{t-1}/(1 + y_t)$ for high levels of public debt. The concept of fiscal limit leads to a definition of “fiscal space”, which is the difference between the actual level of public debt and its estimated maximum sustainable level. Fiscal space offers an alternative and complementary measure for fiscal sustainability as the financial leeway of a government that allows it to face very adverse macroeconomic shocks.

Daniel and Shiamptanis (2013) show that, in the presence of “fiscal limits”, a relevant fiscal sustainability criterion would be a debt-stabilizing rule around prudent public debt–GDP ratio –with sufficient fiscal space to face with adverse macroeconomic shocks. Considering the general fiscal rule (4.14), such a debt-stabilizing rule requires that, on average, the reaction of primary surpluses to be greater than the average growth-adjusted real interest rate; that is:

$$\gamma > \frac{r - y}{1 + y} \quad (4.26)$$

where r denotes the average real interest rate on outstanding public debt and y denotes the average growth-rate of real GDP. Under a debt-stabilizing fiscal policy rule, it is straightforward to show that:

$$\alpha = \left(\frac{r - y}{1 + y} - \gamma \right) b^* \quad (4.27)$$

with b^* being the targeted level of debt–GDP (or steady-state) which also defines the debt-stabilizing primary surplus–GDP:

$$s^* = \frac{r - y}{1 + y} b^* \quad (4.28)$$

As long the debt-stabilizing condition holds, α would be negative, as is usually found in the data. Thus, one can provide a comprehensive interpretation of linear–fiscal policy rules in

terms of deviations from steady-state values:

$$s_t - s^* = \gamma(b_{t-1} - b^*) + \beta_x x_t + \beta_g g_t + \varepsilon_t \quad (4.29)$$

Linear-fiscal policy rules do not imply that government must always run primary surpluses but only when its debt-GDP ratio is above its reference long-run value b^* .

Considering these long-run ratios, fiscal policy rules show themselves being useful theoretical and empirical tools both for fiscal sustainability analysis and the design of numerical reference values for fiscal variables—what we generally label “fiscal rules”. Suppose policymakers take economic environment $(r - y)/(1 + y)$ as given (which may be at some points a very restrictive assumption) and set reference values for b^* , then they can deduce how much fiscal policy must react to public debt γ and what must be the long-run average debt-stabilizing primary surplus s^* .

Fiscal sustainability analysis based on GIBC and fiscal policy rules yields important lessons on what constraints are needed for sustainability. A "prudent" fiscal policy should probably ensure convergence of public debt-GDP ratios towards prudent levels (Fall et al., 2015; Fournier and Fall, 2015), with sufficient fiscal space in order to face adverse macroeconomic shocks, such as the 2008 financial crisis and the following Great Recession. Unfortunately, this does not definitively prevent government from hitting its fiscal limit when facing extremely adverse macroeconomic shocks, even if it is committed to a strongly sustainable fiscal policy rule (i.e. the debt-stabilizing rule). Fiscal discipline cannot reduce fiscal risk to zero, and this fact may support the view that a central bank should act as a lender of last resort.

Macroeconomic stabilization and fiscal sustainability trade-offs in a MU

Fiscal sustainability analysis *à la* Bohn makes the general claim that public debt sustainability requirements are not antagonistic to macroeconomic stabilization purpose of fiscal policy. Indeed, fiscal policy rules like equation (4.14) are sufficiently flexible to allow for counter-cyclical fiscal policy while ensuring that the government intertemporal budget constraint holds. According (Bohn, 1998), fiscal sustainability is a long-run requirement and

fiscal numerical rules should account for the effects of automatic stabilizers or temporary public expenditures (or spending reversals); in practice, it supports fiscal numerical rules specified in terms of structural (or cyclically adjusted) balance. Yet, Bohn's analysis does not apply explicitly to monetary unions nor it specifies the interactions of fiscal and monetary policies in a MU.

In a monetary union, the loss of monetary independence induces welfare costs resulting of insufficient macroeconomic stabilization at the national level and thus creating a trade-off between transaction costs reduction and stabilization losses. In absence of national fiscal policies, Optimal Currency Area theory (Mundell, 1961) shows that transaction costs reduction dominate stabilization losses only if shocks are sufficiently correlated. Cooper and Kempf (2004) argues this condition on shock correlation mainly results from ignoring the role of national fiscal policies. When properly accounting for it, benefits from transaction costs dominate as welfare costs associated to the loss of monetary independence are compensated by fiscal instruments at the national level. Still, Cooper and Kempf do not deal with the sustainability of public debt at national and monetary union levels since national deficits are financed by seigniorage revenue from the central bank. In a New-Keynesian framework, Galí and Monacelli (2008) consider a multi-country model of a monetary union with independent national fiscal authorities⁸ with imperfectly correlated, idiosyncratic shocks to labor productivity. In a linear-quadratic framework, they show that an optimal monetary–fiscal policy mix in a currency union would require the independent central bank stabilizes inflation rate at the monetary union level, while national fiscal policies should react in counter-cyclical way to national output gaps. It is worth noting that Galí and Monacelli's analysis also abstracts from public debt and hence from fiscal sustainability requirements.

Ferrero (2009) incorporates public debt and distortionary taxation in a two country monetary union model and solves the optimal policy-mix problem in a linear quadratic framework. He confirms Galí and Monacelli's result that the central bank should focus on inflation targeting at the MU level and stabilizing the monetary union output gap. Regarding, Ferrero finds national fiscal policies can stabilize the national economy's output gap as long

⁸They do not consider the case of a federal fiscal authority.

as they do not push up inflation pressures at the MU level, i.e. satisfying its fiscal sustainability requirements according the FTPL (see section 4.2.2). Derived from optimal policy analysis, the author considers simple monetary and fiscal rules and compares welfare under a strict fiscal rule (i.e. balanced-budget rule) and a flexible rule in which the real public debt target reacts negatively to output gap in national economy. Simulations show that a flexible fiscal rule allowing for macroeconomic stabilization increases welfare by 50% with respect to a balanced-budget rule. They also show that a flexible rule better mimics the dynamics of government debt according optimal policy analysis. As a result, it strengthens the case for flexible fiscal policy rules both allowing for macroeconomic stabilization while ensuring the sustainability of public debt at the national level.

4.3 European Fiscal Rules: Too Tight? Too Loose? Or Both?

Fiscal rules embedded in the Maastricht Treaty and the SGP have been intensively discussed over the last two decades. Are these rules sufficient to ensure fiscal sustainability and flexible enough to allow countercyclical fiscal policies? Some argued the European fiscal framework, both the preventive and the corrective arm, were far too tight in regard to fiscal sustainability requirements. While at the beginning of the 2000s some argued there was no clear evidence that national fiscal policies had lost their ability to follow countercyclical stabilization objectives, recent research suggests the opposite: European national fiscal policies became more procyclical after the implementation of the SGP. More recently, the financial and economic crisis of 2008 and the following European sovereign debt crisis in 2010 raised concerns about the ability of European fiscal rules to prevent excessive deficits and debts within the EMU.

This section proposes a critical appraisal of the European fiscal framework. First, we start by a brief history of the evolution of European fiscal rules since the Maastricht treaty was signed to the ratification of the Treaty on Stability, Cooperation and Governance (TSCG) in 2013. Then, we discuss whether European fiscal rules are necessary and/or sufficient to ensure the sustainability of public debts in Europe. We argue the fiscal requirements specified within the SGP are probably too tight and that fiscal policies in the EMU have actually been

sustainable since 1992. Yet, fiscal policies are usually found to be too much procyclical in the EMU. This procyclical bias, we argue, is the principal threat to fiscal sustainability, in addition to evident costs in terms of economic activity and employment, particularly during the crisis. Thus, we turn to the question of the causes of the European sovereign debt crisis and argue that excessive deficits in southern Euro Area countries are probably not the main culprits, while nevertheless playing an important role. Finally, we address the ongoing reflexion about further improvements of the European fiscal union and particularly the proposal of Eurobonds.

4.3.1 A brief history of the European Fiscal Framework

Since the ratification of the Maastricht Treaty, the European fiscal framework has known three major reforms, in 2005, 2011 and 2013. We briefly review the initial specification of the initial Stability and Growth Pact and amendments, following European Commission (2013).

The initial Stability and Growth Pact

The Maastricht Treaty on the European Union (1992) and the following Stability and Growth Pact (SGP, 1997) implemented numerical fiscal policy rules at European Union (EU) level, divided into a preventive arm and a corrective arm. These rules were explicitly designed to ensure macroeconomic convergence and stability among EU member states, and in particular conditioning future participation to the EMU. Policymakers considered that sustainable fiscal policies were required to prevent both spillover effects among member states and inflationary effects of fiscal policy while monetary policy could successfully ensure price stability and promote economic growth.

In the initial SGP, the preventive arm only specified the objective of a balanced-budget rule ("close-to-balance or in surplus budget", to be exact) yet without being specific about adjustment path or enforcement mechanism. The corrective arm and the Excessive Deficit Procedure (EDP), specified procedures to correct deviations from the Maastricht Treaty's reference values of 60% of gross public debt-to-GDP and 3% of deficit-to-GDP. While these

values were *a priori* arbitrarily specified, in particular the 60% threshold, it is possible to provide an *ex post* rationale for the 3% deficit-to-GDP threshold, see below in section 4.3.2. Regarding the adjustment path under the EDP, no predetermined rule was specified; fiscal adjustment was supposed to be determined within each country's EDP, on a case-by-case basis. Finally, a country would be allowed to breach the 3% deficit limit without being placed under an EDP *only under "exceptional circumstances"* which was defined by the Treaty as a 2% annual contraction of real GDP.

The 2005 reform: More flexibility and more compliance

During the 2000s, European policymakers tried to deal with two identified flaws and shortcomings of the initial European fiscal framework. First, rules specified by the initial specification of the SGP were judged too much uniform and insufficiently flexible, to allow counter-cyclical fiscal policy for instance. Second, there was a growing concern about the credibility of European fiscal rules after France and Germany were exempted from sanctions while being under Excessive Deficit Procedures in 2003.

As a result, the 2005 reform of the SGP aimed at two objectives: increasing the flexibility of the European fiscal rules and reinforcing compliance of EMU member states. First, the preventive arm was modified to include a country-specific Medium-Term Objective (MTO). The MTO would be based on the level of gross government debt and would take into account population's ageing effect on pension systems. It would include a "safety margin against breaching the 3% deficit limit" and imply a "rapid progress toward sustainability". The MTO should allow "room for budgetary manoeuvre" within a maximum *structural* deficit of 1% of GDP. In order to increase compliance with the rules, the SGP became more precise about the adjustment path. Annual adjustment toward the MTO was set to be equal to an annual reduction of structural deficit larger than 0.5% of GDP in good times, lower than 0.5% in bad times. The reform also introduced the possibility to deviate from adjustment path in case of "major reforms with verifiable impact on long-term sustainability".

Second, regarding the corrective arm, the concept of exceptional circumstances was broadened to "to either a negative output growth or an accumulated loss of output due to protracted period of growth below potential". The adjustment path to exit the EDP was also

specified as an annual reduction of structural deficit of 0.5% of GDP –without possibility to reduce fiscal adjustment in bad times. Some flexibility was still introduced with the possibility of a *deadline extension* if "effective action has been taken" and if an unexpected deterioration of the economic situation, "beyond the control of the government", has occurred.

The 2011 and 2013 reforms: the Six-Pack, Two-Pack and Fiscal Compact

The last set of reforms came at the early stages of the European sovereign debt crisis which was interpreted as the consequence of fiscal profligacy in "Southern European" countries and competitiveness divergence and large macroeconomic imbalances among EA member states. The Six-Pack reform aimed at improving fiscal rules and the European economic governance with the implementation of the European Semester, in order to synchronize and harmonize the process of budget surveillance. It also implemented the Macroeconomic Imbalance Procedure (MIP), following the SGP pattern with a preventive arm and a corrective arm, in order to correct competitiveness and macroeconomic imbalances.

First, according to the idea that the European fiscal framework was not credible enough because sanctions were never decided against profligate governments, the 2011 reform aimed at reinforcing the enforcement mechanism of the SGP, for both preventive and corrective arms. Regarding the preventive arm, the Six-Pack reform notably toughened the enforcement mechanism. It introduced a procedure to correct "significant deviation" from the country-specific MTO and relative adjustment path, with the possible sanction of an interest bearing deposit of 0.2% of GDP in case of repeated non-compliance with fiscal adjustments requirements specified. The Excessive Deficit Procedure enforcement mechanism was reinforced as well with an "early and gradual sanction system to be activated at each stage of the EDP procedure". It also introduced an expenditures benchmark, globally insuring that non-discretionary expenditures do not grow faster than potential output, and thus, preventing EA member states against the risk of breaching the 3% deficit limit.

The 2011 reform toughened the structural adjustment (higher than 0.5% per year) for member states with debt–GDP ratios above 60% for both preventive and corrective arm. Following the Great Recession, it was also specified that member states could deviate from

their MTOs or obtain a deadline extension within the EDP “in case of severe economic downturn in the euro area or the union as a whole”.

Finally, Two-Pack reform and the "Fiscal Compact" (Treaty on Stability, Coordination and Governance, TSCG) further strengthened the European Semester and the economic surveillance procedure at the EA level. In particular, the Fiscal Compact brings the MTO and adjustment path toward it into national law and reinforce independent bodies charged of monitoring compliance with fiscal rules at the national level.

Yet, the growing complexity of European fiscal rules, their reliance on complex methods –when estimating potential output and structural fiscal balance in particular– may also reduce rather than increase compliance with rules (Eyraud and Wu, 2015; Eyraud et al., 2017). As a result, calls for a simplification of the European fiscal rules have multiplied in the recent years, see for instance Andrieu et al. (2015) and Claeys et al. (2016).

4.3.2 Are European fiscal rules ensuring the sustainability of public debts?

We argue that European fiscal rules embedded in the preventive arm and the corrective arm may be generally sufficient and sometimes not necessary, to ensure that national fiscal policies are sustainable, both from a theoretical and empirical perspective.

First, the preventive arm appears to be largely ineffective. It is worth noting this was precisely the motivation of the tightening of adjustment rules and enforcement mechanisms following the implementation of the Six-Pack in 2011. During the 1990s and the move toward the creation of the EMU, most EU member states and future EA members focused on the Maastricht reference values more than on the medium-term objective of a close-to-balance or surplus budget position. Collignon (2012) estimates empirical fiscal rules matching the European fiscal framework and shows, using rolling-regressions, that implicit deficit targets of Euro-Area member states converged toward a 3% deficit-to-GDP and *not* toward a close-to-balance or surplus fiscal policy, as specified by the preventive arm of the SGP. As a matter of fact, the corrective arm (the EDP) obviously dominated the preventive arm. What could be the rationale behind this? Many have argued it mainly results from an insufficient specification of the preventive arm, which implies consequently a lack

of credibility (European Commission, 2013). Still, a competing view could point out that the nominal balanced-budget objective defined in the initial SGP does not find any economic justification. A nominal balanced-budget rule would imply a constantly decreasing debt-to-GDP ratio, which is a far too strong requirement for fiscal sustainability according both No-Ponzi Game and Debt-stabilizing conditions. In addition, from a more general point of view, macroeconomic theory has shown that balanced-budget rules are likely to increase aggregate economic instability (Schmitt-Grohe and Uribe, 1997) in addition to induce welfare costs as shown by Ferrero (2009).

While the preventive arm's objective of a nominal balanced-budget rule seems neither necessary nor desirable, the corrective arm could, to some extent, make more sense from a fiscal sustainability analysis. Buitter et al. (1993) find an *ex post* rationale for the 3% deficit-to-GDP rule, given specific assumptions about nominal growth and a reference level of 60% for debt-to-GDP. Consider the variation of debt-to-GDP ratio Δb_t described by

$$\Delta b_t = -\frac{\tilde{y}_t}{1 + \tilde{y}_t} b_{t-1} + def_t \quad (4.30)$$

where \tilde{y}_t is the growth-rate of nominal GDP and def_t the deficit-to-GDP ratio. Hence, the debt-stabilizing deficit equals

$$def_t^* = \frac{\tilde{y}_t}{1 + \tilde{y}_t} b_{t-1}^* \quad (4.31)$$

Indeed, given a nominal growth rate of 5% and a reference value of 60% gross debt-to-GDP ratio, we find that fiscal policy would stabilize the debt-to-GDP ratio at 60% by setting the deficit to 3% of GDP.

This analysis calls several remarks. First, the Excessive Deficit Procedure's reference values rely heavily on assumptions made on real GDP growth rate and inflation rate, respectively 3% and 2%. As pointed out by Buitter et al. (1993), countries with higher real growth rate and inflation could support a higher deficit-to-GDP ratio. As a matter of fact, EA member states diverged in terms of real GDP growth and inflation rates, reinforcing the criticism of a uniform nominal deficit reference value as a useful guideline for fiscal surveillance. Second, the reference value for "excessive deficits" remains partly arbitrary since it depends on the reference value of public debt-to-GDP which is completely arbitrary and

may not be justified as an excessive public debt level. For instance, research on sovereign default risk rather suggests maximum debt limits are generally (well) above 100% of debt-to-GDP (Ghosh et al., 2013a) and that sovereign default risk for debt-to-GDP levels lower than 100% is limited in advanced economies. Consequently, it is questionable whether a 60% debt-to-GDP level is a relevant threshold to determine excessive deficits. Third and last, a debt-stabilizing fiscal rule *does not require* the deficit to be always equal to def_t^* defined by equation (4.31), but only on average, over a sufficiently long time length. All in all, a nominal (with interest) deficit guideline would not seem the most appropriate way to monitor sound fiscal policy, as requirements in terms of debt-stabilizing (with-interests) deficits would heavily depend on real-growth and inflation rates. The bottom line is: one size *does not* fit all.

On the contrary, a debt-stabilizing deficit rule may not always be the proof of a sustainable fiscal policy: if the real growth rate of GDP is to exceed the long real interest rate on a government's bonds (i.e. $r < y$), fiscal policy may well be non-sustainable, that is violating Bohn's condition for NPG, and still stabilizing its public debt-to-GDP ratio. As a result, the relevant condition would rather be the NPG condition, that is a positive average response of primary surplus to the initial level of public debt. In theory, it would be more efficient to impose a positive average (structural) primary surplus—over a sufficiently large time length—, which is the relevant fiscal indicator for sustainability analysis. Indeed, while a permanent deficit could still be consistent with (strong) fiscal sustainability (i.e. stable debt-to-GDP ratio), fiscal policy should run primary surplus on average, over the business cycle, if the real interest rate exceeds real-growth $r > y$ in the long-run.

From an empirical perspective, there have been mixed evidence in favour of fiscal sustainability of European countries since the implementation of the SGP in 1997 and the creation of the EMU in 1999. In the early 2000s, Afonso (2005) described what he called the "Unpleasant European Case." Despite their stabilizing of debt-GDP ratios by the end of the 2000s, he found many European countries were likely to be at risk regarding the sustainability of public finance. Yet Afonso's dataset stopped in 2003, which did not provide enough data to evaluate the impact of the SGP on fiscal sustainability. In contrast with Afonso's results, more recent papers found evidence that European fiscal policies became

more responsible during the 2000s, after the implementation of the SGP. Collignon (2012) follows closely the European fiscal framework to specify fiscal reaction functions. Hence, he estimates the following fiscal rule for each Euro Area and European Union member states:

$$\Delta \tilde{s}_t = \alpha(def_{t-1} - def^*) + \beta(b_{t-1} - b^*) + \varepsilon_t \quad (4.32)$$

where \tilde{s}_t is the structural primary surplus-to-GDP, def_{t-1} denotes the deficit-to-GDP ratio and b_{t-1} the debt-to-GDP ratio. The parameters def^* and b^* represents the implicit policy objectives followed by national governments. His results suggest that the implementation of the SGP rules may have induced a regime change, in terms of policy responses and objectives. Using both panel-data and country-specific cointegration techniques, Daniel and Shiamptanis (2013) estimate empirical fiscal rules for eleven EMU countries and test for a *debt-stabilizing condition*. First, they find statistically significant evidence in favour of debt-stabilizing fiscal rules in panel-data estimates while not always significant in country-specific estimates. This latter result is most probably an econometric issue due to a short sample size rather than a proof of unsustainability. Second, they find evidence that the EMU *increased fiscal sustainability of member states*, in panel-data estimates. Weichenrieder and Zimmer (2014) also provide evidence in favour of a significant positive effect of Euro-membership on fiscal sustainability. While baseline estimates seem to confirm the popular view that EA member states became less sustainable after they joined the EMU, they show these results are not robust to the exclusion of Greece from the panel⁹. Overall, they conclude that fiscal policies have been sustainable in the EMU, since 1992.

These empirical results suggest the European fiscal framework was sufficient to promote responsible fiscal policies in terms of primary surplus responsiveness to public debt, and despite excessive deficit procedures engaged against several EA member states, including France and Germany, during the first decade of EMU.

⁹They also show these results are not robust to the exclusion of crisis years 2009-2011.

4.3.3 Procyclical bias in European fiscal policy rules

Beside fiscal sustainability issues, an important question was about the alleged “procyclical bias” of the European fiscal policy rules. This point was already made by Buiter et al. (1993): as they noted, the SGP was really ambiguous about whether countercyclical deficits in excess of 3% were acceptable. Actually, in the initial SGP’s specification, these excessive deficits were supposed to be exceptional and temporary, only in case of a 2% recession in term of real GDP, which supports the view that the SGP induced de facto a procyclical bias in European fiscal policy –until the definition of “exceptional circumstances” was modified in 2005 (see section 4.3.1).

In the early 2000s, Galí and Perotti (2003) produced empirical evidence against the conventional view that “the Maastricht Treaty and then Stability and Growth Pact have impaired the ability of EU governments to conduct a stabilizing fiscal policy and to provide an adequate level of public infrastructure.” Using annual data from the Organization for Economic Cooperation and Development (OECD) Economic Outlook, ranging from 1980 to 2002, they estimated a linear fiscal policy rule linking the structural primary deficit–GDP ratio to output gap, initial debt–GDP ratio and past primary deficit–GDP. They found fiscal policy in EMU “has become more counter-cyclical over time, following what appears to be a trend that affects other industrialized countries.” Regarding the decline in public investment, they found “industrialized regions not subject to the SGP have experienced an even greater decline.” Still, they noted that deep, severe recessions have been rare in the post-Maastricht period, implying the SGP fiscal rules were not really binding. They concluded that the impact of the SGP could be different in the future.

Recently, Huart (2013) found no significant evidence of procyclical fiscal policy between 1999-2009, at the Euro-Area level using both panel data analysis and country-specific regressions. But at the same time the results also indicate that discretionary fiscal policy was mostly acyclical in most EA members states and significantly countercyclical only in France, Ireland and the Netherlands.

Yet some recent empirical research challenge these findings. Beetsma and Giuliadori (2010) distinguish two stages of fiscal policy implementation –the planning stage and the

implementation stage— using real-time data and estimate fiscal reaction functions. They use panel data running from 1995 to 2006, for EU-14 plus the USA, Canada, Japan, Norway and Australia. Their results are twofold. First, they found planned fiscal policy was acyclical in EU countries but countercyclical in non-EU countries. Second, they provide evidence that EU countries react procyclically to unexpected changes in the output gap while non-EU countries react acyclically during the implementation stage. Collignon (2012) also provides empirical evidences that fiscal policy became more procyclical in the EU countries than in the non-EU countries.

These results have been confirmed by Eyraud and Wu (2015) and Eyraud et al. (2017). Interestingly the authors shows that, if European fiscal policy had been more countercyclical in the first decade of the EMU (1998–2008), it would have entered the crisis in a far stronger fiscal position (see figure 5, p. 13, 2015), hence increasing fiscal sustainability during the crisis. It well illustrates the complementarity between the requirements of long-run sustainability of public debt *and* the need for a countercyclical fiscal policy. The lack of flexibility and the quasi-exclusive focus on fiscal sustainability within the European fiscal framework, which likely induce the procyclical bias observed in the data, would eventually threaten the long-run sustainability of public debt. More recently, using panel data analysis at the EA level, Eyraud et al. (2017) provide evidences of a procyclical bias and a deficit bias, confirming that procyclical fiscal policy in good times prevented government to consolidate their fiscal position and to use countercyclical sufficiently aggressively after 2011.

The European Commission claims that recent reforms of the SGP have increased the flexibility of fiscal rules, in particular with respect to macroeconomic stabilization objectives (European Commission, 2013). Still, Creel et al. (2013) contradict this claim. They developed a medium-scale New Keynesian DSGE model to compare three different rules: the Maastricht Treaty (3% of deficit–GDP), the Fiscal Compact framework and a public investment rule. Their simulations show that the Fiscal Compact is likely to be more deflationary and recessionary than both the status quo and the public investment rule. The public investment rule displays the lowest output cost. Creel et al. conclude by saying that “such a drastic consolidation strategy [i.e. the Fiscal Compact] embedded into EU constitutional laws threaten future macroeconomic performances of Eurozone countries.”

Detractors of the SGP have been pointing out to the procyclical bias of fiscal policy rules since the early 1990s; and to some extent, European policymakers tried to improve the European fiscal framework and reduce this bias –some would say "too timidly". The reason was essentially because fiscal multipliers were underestimated (at least, under some circumstances). While the consensus before the Great Recession was that fiscal multipliers were low –probably close to 0.5 or lower (Blanchard et al., 2010; Blanchard and Leigh, 2013)–, both empirical and theoretical researches have recently challenged the common wisdom of low fiscal multipliers.

Empirical research has shown the size of fiscal multipliers can vary a lot according to the state of the economy, and reach values well above 1 or even 2 in some cases. For instance, fiscal multipliers appear to be larger during recessions than expansions (Auerbach and Gorodnichenko, 2012a,b). Riera-Crichton et al. (2015) show that fiscal policy has asymmetric effects depending on the state of the economy (expansion versus recession) and on the stance of fiscal policy (procyclical versus countercyclical) using a panel dataset of OECD countries. Two main results emerge from their analysis. First, estimated countercyclical fiscal multipliers are very large: the long-run multiplier is 2.3 in normal recessions and 3.1 in extreme recessions. Second, while the austerity motto "*short-run pain, long-run gain*" may be correct in normal recessions, it is no longer the case in extreme recessions, as they conclude: "applied to the current debate on austerity in the Eurozone, this would imply that debt-to-GDP ratios would increase in response to cuts in fiscal spending."

Regarding the debate on austerity in Europe, Blanchard and Leigh (2013) produce empirical evidence that professional forecasters (including the IMF) have underestimated the size of fiscal multipliers in the years following the Great Recession and the sovereign debt crisis: while these multipliers were probably about 0.5 before the crisis, their results for European countries, in 2010–2011, indicate they were significantly above 1 in the early stage of the sovereign debt crisis.

Theoretical research also provides new explanations for larger fiscal multipliers. New Keynesian DSGE models with imperfect competition and staggered price-setting did not produce fiscal multipliers above 1 for one fundamental reason: in these models, the Ricardian equivalence holds, and therefore fiscal spending shocks induce negative wealth effects

for consumers, thus having a crowding-out effect on private consumption (being at odds with most empirical findings). This puzzle has been solved in many different ways. Relaxing some fundamental hypothesis of DSGE models dramatically changes the value of fiscal multipliers and produce a crowding-in effect in private consumption. For instance, taking into account Limited Asset Market Participation makes the Ricardian equivalence fall as a fraction of consumers are credit constrained and cannot smooth consumption over time (Bilbiie, 2008). Another way to solve the puzzle is to assume that consumers have non-separable preferences between consumption and labour such that hours worked and private consumption both increase after a positive government spending shock (Bilbiie, 2011; Monacelli and Perotti, 2008). Still one of the most important theoretical propositions is the analysis of fiscal policy when monetary policy is at the ZLB. Building on the old (Keynesian) wisdom that fiscal policy is more “effective” when monetary is accommodative, many theoretical papers have shown fiscal multipliers are far above 1 when the nominal interest rate is at the ZLB (Christiano et al., 2011; Corsetti et al., 2010; Denes et al., 2013; Eggertsson and Krugman, 2012). And as already mentioned, an alternative monetary/fiscal policy mix can also make the Ricardian equivalence property fall and imply bigger fiscal multipliers (Davig and Leeper, 2011).

Yet the “sovereign risk channel” (i.e. the effect on private sector funding costs of sovereign default risk) can substantially reduce the size (and even invert the sign) of fiscal multipliers, suggesting that fiscal stimulus could eventually be self-defeating in countries in which sovereign financial distress tends to increase private sector funding costs (Corsetti et al., 2013). This last result is currently being discussed by empirical research. Nickel and Tudyka (2014) estimate a bayesian panel VAR in which parameters vary continuously with the level of public debt-to-GDP ratio. They found evidence in favor of reduced or even negative fiscal multipliers for high level of public debt. A recent paper by Auerbach and Gorodnichenko (2017) find a negative but non-significant average response of real GDP to government spending shock when debt-to-GDP is larger than 100%, suggesting negative fiscal multipliers for high debt.

Still, this result may not well account the asymmetric and non-linear effects of fiscal policy during expansions and recessions. Indeed, Auerbach and Gorodnichenko find a positive

and strongly significant impact response of real GDP to government spending shocks *during recessions*, even when public debt is high; the average response of real GDP to spending shocks is still positive but weakly significant. All in all, if fiscal multipliers may be negative for high public debt level in expansions, the positive effects of fiscal stimulus seem to dominate the negative effects of public debt overhang during recessions.

To summarize, both theoretical research and empirical evidence rather suggest that fiscal sustainability is a long-run requirement, allowing for deficit-financed fiscal stimulus during recessions, on the condition that fiscal policy must tighten during expansions. Interestingly, the claim that fiscal stimulus would increase fiscal risks and lower fiscal sustainability may not be as stronger as it seems. Auerbach and Gorodnichenko (2017) find no evidence that government spending shocks would increase debt-to-GDP ratio or interest rates, especially during recessions, supporting the claim by DeLong and Summers (2012) that expansionary fiscal policies in deflationary depressions could be self-financed. Furthermore, they do not find significant evidence of a negative effect of fiscal stimulus on fiscal sustainability *during recessions*, when public debt-to-GDP is above 100% of GDP. Even more, increasing the counter-cyclicality of fiscal policy would even increase fiscal sustainability since it would imply larger primary surplus-to-GDP ratios in good times –and lower debt-to-GDP ratios (Eyraud and Wu, 2015; Eyraud et al., 2017)–, in addition to positive effects on welfare (Ferreiro, 2009; Galí and Monacelli, 2008). Recent experience of non-EA countries with respect to EA countries shows that the first could both stabilize their debt–GDP ratio and reduce the output gap quicker than the latter while undergoing less austerity or, at least, not too soon following the Great Recession and with an accommodative (or passive) monetary policy.

4.3.4 Was the European Debt Crisis the result of irresponsible fiscal policies?

The European sovereign debt crisis revived the debate about fiscal policy rules in the EU and the EMU. It opposes two antagonist views of fiscal policy. The first is the orthodox view promoting balanced-budget rules and decreasing debt–GDP ratios, and is based on the Expansionary Fiscal Contraction (EFC) hypothesis, following the seminal paper by Giavazzi and Pagano (1990) and the work of Alberto Alesina and Silvia Ardagna (Alesina et al., 2015; Alesina and Ardagna, 2009). This approach follows from the political economy

of public debt and relies heavily on the so-called “confidence effect” of fiscal consolidations. Taking the contrary view, the second one puts emphasis on new empirical evidences of state-dependent and time-varying fiscal multipliers as well as new theoretical results on fiscal multipliers in new Keynesian DSGE models. It also contradicts EFC supporters on empirical grounds, arguing for an upward bias in Alesina and Ardagna’s estimates of expansionary effect of fiscal consolidation (Guajardo et al., 2014; Jorda and Taylor, 2015).

Guajardo et al. (2014) follow Romer and Romer (2010) and use narrative analysis to identify exogenous fiscal policy shocks, i.e. non-related to short-term economic developments. They find little evidence in favour of the EFC; fiscal austerity rather have, on average, contractionary effect on private consumption, investment and GDP. They notably find that episodes of expansionary fiscal consolidations are usually associated with accommodative monetary policy. Jorda and Taylor (2015) argue that even the narrative analysis and the IV approach of the IMF used by Guajardo et al. (2014) are not totally controlling for endogeneity in the identification of fiscal consolidation plans. Using the same dataset, they replicate both Alesina and Ardagna (2009) and Guajardo et al. (2014) methods and results. Then, they apply a new method consisting in estimating the probability for a country to implement fiscal consolidation, in order to estimate the *average treatment effect* of fiscal consolidation. Their results show fiscal consolidations are even more contractionary, during slumps, with respect to Guajardo et al. findings.

Yet, despite serious criticisms of it, the EFC hypothesis obviously won the political battle in Europe at the very beginning of the European sovereign debt crisis. The early narrative of this crisis found irresponsible (or imprudent) fiscal policies in southern European countries were the main culprits, rather than excessive current account deficits and excessive private borrowing in the periphery countries. As mentioned earlier, it explains the strong tightening of the European fiscal rules after the Six-Pack, Two-Pack and Fiscal Compact reforms, and the relative disconnection between the SGP and the MIP; see Bénassy-Quéré and Ragot (2015).

On the contrary, five years after the beginning of the European sovereign debt crisis, another consensus narrative emerged among macroeconomists. Lane (2012) had already suggested the so-called European sovereign debt crisis was not primarily caused by exces-

sive deficits in the early 2000s, but rather by original flaws in the EMU architecture (absence of banking union, federal buffer mechanisms), leading to large current account imbalances and excessive private borrowing within the EMU. More recently, a panel of economists from the CEPR (Centre for Economic Policy and Research) proposed a new consensus narrative of the European crisis (Baldwin and Giavazzi, 2015), claiming it was primarily a sudden-stop crisis, not a Sovereign debt crisis as claimed by the EFC hypothesis supporters. According to this narrative, financial fragility, excessive private borrowing in non-productive sectors and current account imbalances were the source of the crisis, when the sudden stop occurred following 2008–2009's global crisis; and the Sovereign debt crisis would rather be a consequence of the Global Financial crisis and the Great Recession. This narrative also stresses the "causes of the causes" of the Eurozone crisis: "policy failures that allowed the imbalances to get so large," "lack of institutions to absorb shocks at the Eurozone level" and "crisis mismanagement" (Baldwin and Giavazzi, 2015). To some extent, this narrative supports the view that the European fiscal framework (the SGP and in particular the "no bail-out" clause) were probably not credible enough to prevent both excessive current account and public deficits. In particular, they did not prepare the European Union and the EMU to deal with a sudden-stop crisis, which was likely to cause a banking crisis and a sovereign debt crisis.

Recently, Martin and Philippon (2017) have proposed a model to disentangle the "usual suspects" of the Eurozone crisis: fiscal profligacy, excessive private leverage and competitiveness divergence due to fixed exchange rates, using counterfactual simulations. Precisely, they look at the performance of employment during the crisis in 4 of 5 PIIGS countries (Greece, Ireland, Portugal and Spain) relatively to the rest of the Eurozone under four alternative scenarios:

- Fiscal conservatism during the boom, i.e. years previous to the crisis.
- Macroprudential policy during the boom.
- Fiscal conservatism *and* macroprudential policy during the boom.
- No financial segmentation through an early intervention of the ECB on sovereign bond markets (i.e. Draghi's "Whatever it Takes" in 2008).

They argue that each factor has played an important role, depending on the country. Fiscal indiscipline in years previous the crisis seems to have dramatically worsen the negative effect on employment in Greece and in Portugal; for Ireland and Spain, the fiscal mechanism seems "unrealistic since they require buying back almost the entire stock of public debt"¹⁰. The author notably argue avec macroprudential policy (to avoid excessive private leverage) would not have reduced the effect on employment by itself, but rather in coordination with fiscal conservatism in years previous to the crisis. As a result, they claim fiscal conservatism and macroprudential policy are complements rather than substitutes. Regarding the sudden-stop factor and the financial segmentation it induced, Martin and Philippon argue that an earlier intervention of the ECB would have experienced "a boom-and-bust cycle similar to the one in the United States" without avoiding the large build-up of public debt, though.

4.3.5 Toward a Fiscal Union? The case of Eurobonds

Following the Great Recession and the European sovereign debt crisis, the idea that EMU's fiscal union was incomplete has become widely accepted among economists and policy-makers. In this section, we argue that strict fiscal rules were insufficient both to ensure the viability of the monetary union and to yield the adequate policy-mix both at national and monetary union levels, both before and after the crisis. As a result, calls for a more integrated fiscal union and the creation of Eurobonds have multiplied since.

As shown in section 4.2.1, we agree with Kempf (2013, 2017) to say that a monetary union is *always* a fiscal union because of the interdependencies –hence the *constraints*– it creates between national fiscal policies and monetary policy. We have argued why fiscal rules are *minimum requirements* to ensure fiscal sustainability at national and monetary union level, price-level stability and optimal functioning of the monetary union. Still, the design of a *negative* and *horizontal* fiscal union, exemplified within the TSCG may not be sufficient at ensuring the long-run viability of the EMU. The credibility of the *no bail-out* clause now appears questionable since EMU countries were forced to bail-in the Greek government to

¹⁰Both Ireland and Spain would have entered the crisis with almost very low public debt-to-GDP levels, which seems unrealistic.

put an end to the panic on sovereign bond markets and the contagion to other southern EMU countries. There may be several reasons to this.

First of all, a *negative* fiscal union which only consists in strict fiscal rules at national level would put the fiscal sustainability of the whole monetary union at risk. Indeed, we have seen that the consolidated monetary union GIBC holds as long as *each and every* member of the union fulfills *simultaneously* its sustainability condition. Hence, the overall fiscal sustainability would be in jeopardy because of *one* free-rider—at least in theory. Then, in case of fiscal free-riding of one or several member states, the absence of any federal government—which could eventually undertake corrective actions at the monetary union level if it existed—, would force the central bank to intervene to preserve the existence of the monetary union, as it happened when Draghi pronounced his "Whatever It Takes" speech (Kempf, 2013). As a result, the current design of the EMU as a negative and horizontal fiscal union seems not viable, in our view. The nature of monetary union and the constraints it puts on fiscal policies would mechanically lead toward a greater integration, in terms of fiscal transfers and policy coordination.

The incompleteness of the European fiscal union also relates to the question of the excessive procyclicality of fiscal policies in the EMU and their lack of coordination. Fiscal policies in the Euro Area have probably been too much expansionary before the crisis while being excessively restrictive since 2010. The absence of fiscal transfers among member states probably worsen the effect of Sovereign debt crisis, since fiscal consolidation in southern countries could not be eased by net fiscal transfers from northern countries, for instance through a European unemployment benefit scheme. As a matter of fact, national fiscal rules were not capable to deliver neither the necessary macroeconomic stabilization following the Great Recession nor fiscal sustainability efforts before 2008, both at national and monetary union levels (Jarociński and Maćkowiak, 2017; Martin and Philippon, 2017).

While moving toward a fiscal federation seems politically unrealistic, at least in the medium run, the need to overcome the incompleteness and flaws of the EMU lead to the proposal of creating Eurobonds without federal fiscal authority. Eurobonds has been intensively discussed by Beetsma and Mavromatis (2012), Bofinger et al. (2012), Frankel (2012), Gros (2011), Philippon and Hellwig (2011), and Weizsäcker and Delpla (2010). Beetsma

and Mavromatis (2012) summarize the pros and cons of Eurobonds. According to them, Eurobonds would likely reduce the financial segmentation and the vulnerability of national public debt to speculative attacks. But they would also likely create a moral hazard problem by reinforcing the incentives for fiscal free-riding from national governments.

The most popular proposal of blue and red debts has been made by Weizsäcker and Delpla (2010) with the main objective to put an end to financial instability and stress on the European sovereign debt market. National public debts under 60% of GDP would be pooled at the EMU level into a Eurobond backed by all member states of the EMU: the Blue debt. While excess debts with respect to 60% of GDP would be issued by national governments: the Red debt. As a result, the existence of blue debt would allow stressed countries to benefit from a decrease in borrowing costs, while the increase in borrowing costs on red debt would reinforce market discipline and reduce moral hazard, according to the authors. Against Eurobonds, Philippon and Hellwig (2011) argue the EMU needs Eurobills –short-term debt– rather than long-term debt. In their view, Eurobills would be sufficient to promote financial stability and liquidity at the EMU level while minimizing moral hazard. Finally, Gros (2011) remains skeptical on the possibility to create Eurobonds because of political and legal reasons: he points out the fact Eurobonds cannot be viable without a strong political union.

Still, Eurobonds could lead to a better coordination and policy mix. In the context of a liquidity trap, Jarociński and Maćkowiak (2017) realize a policy experiment in a New-Keynesian DSGE two-country model showing how Eurobonds could help achieving the ECB's mandate and restoring higher economic growth. They introduce a non-defaultable Eurobond linked to a common fund, which is backed by national fiscal surpluses –hence without federal fiscal policy. Each national government follows a passive fiscal policy: primary surplus satisfies debt-stabilizing condition both on national debt *and* Eurobond. Yet, the aggregated primary surplus does no longer react to any measure of debt, implying active fiscal policy is coupled with passive monetary policy. As a result, the EMU economy would escape from indeterminacy and multiple equilibria. Finally, in their model, moral hazard from national fiscal authorities would be dealt through market discipline: non-compliance with fiscal rules would lead to default on national debt and inflation at the

MU level. To some extent, inflationary effects of default by EMU member states could be dampened if the fund had the possibility to tax households of each country.

Although a further integrated fiscal union, through fiscal cooperation or fiscal federalism, is highly desirable from an economic point of view, its political feasibility remains quite doubtful as argued by Gros (2011). To some extent, Eurobonds without a federal and political authority may be an intermediate solution to increase the viability and stability of the EMU –conditional on the requirement to be appropriately designed, in particular regarding moral hazard from national government.

4.4 Conclusions

When sharing a common currency, national governments mechanically become mutually interdependent and must commit to fiscal rules ensuring the sustainability of their respective public debts as well as the consolidated debt of the monetary union. We have shown it implies additional constraints on fiscal policies, depending on the fiscal union's architecture. In addition to being required *per se*, this is also needed to allow the common central bank to fulfill its mandate and achieve inflation stability in the monetary union. But it also imply that, facing adverse macroeconomic shocks such as a deflationary depression and a liquidity trap, national fiscal policies must also commit to be *unsustainable* or *active* to appropriately back monetary policy.

Following this analysis, we argue the European fiscal framework is simultaneously too tight and too loose to ensure fiscal sustainability. In our opinion, the biggest flaw remains its serious procyclical bias and the incompleteness of the fiscal union, which jeopardizes both fiscal sustainability objectives and economic growth and stability. In particular, the recent reforms (Six-Pack, Two-Pack and Fiscal Compact) are not likely to reduce the procyclical bias of fiscal policy, in our view. And while the case for creating Eurobonds is serious, its implementation require a careful understanding of fiscal sustainability requirements in a monetary union: Eurobonds are not a substitute to fiscal discipline.

Finally, a broader approach of economic surveillance now includes current account imbalances and private debt through the MIP, which is in our view the most important im-

provement in the European economic surveillance procedure. Further reforms should aim at simplifying European fiscal rules, reducing the procyclical bias (in particular in the implementation stage of fiscal policy) and giving a more important role to the analysis of current account imbalances.

Conclusion

First, this thesis aims to extend the Model-Based Sustainability analysis to a regime switching fiscal policy rule. The theoretical literature had already considered the effects of regime switching fiscal policy rules on macroeconomic dynamics (Canzoneri et al., 2001; Chung et al., 2007; Davig and Leeper, 2007a, 2011, among others) while the empirical literature had used regime-switching or time-varying models to identify structural breaks or shifts in empirical policy rules (Afonso and Toffano, 2013; Burger and Marinkov, 2012; Favero and Monacelli, 2005, for fiscal policy, among others). Still, the question whether prolonged episodes of *local* fiscal unsustainability threaten the *long run* sustainability of public debt remained unanswered: this thesis aims at filling this gap. We have derived and proposed an explicit testing framework for *global* sustainability based on regime switching fiscal policy. Regarding fiscal sustainability requirements on policy behavior, we show that the question is not only about "How much?" primary surplus reacts to public debt (i.e. feedback parameters γ_S and γ_{NS}) but also "How long?" each regime lasts (i.e. expected durations d_S and d_{NS}).

This research is both of theoretical and empirical interest. Theoretically, it implies that a government could very well be *globally* Ricardian despite being *locally* non-Ricardian from time to time. Hence, it remains unclear whether these recurring episodes are sufficient to make the Ricardian equivalence fall. Regarding this question, Davig and Leeper (2007a) and Davig et al. (2011)'s results would let us think that it does: agents would expect with a positive probability that fiscal policy can switch to a non-Ricardian regime, hence that current deficits won't be financed through higher future expected present-value surpluses.¹¹ How

¹¹Davig and Leeper (2007a) explains: "Shocks to (lump-sum) taxes always affect aggregate demand, even when the rules in place at a given moment would suggest that Ricardian equivalence should hold if regime were fixed. The fiscal theory is operating whenever it is possible for fiscal policy to become active. Then a cut in current taxes, financed by sales of nominal government debt, does not generate an expectation that future taxes will rise by at least enough to service the new debt. The tax reduction leaves households feeling wealthier, at initial prices and interest rates, and they perceive they can raise their consumption paths. (...) Chung et al.

could we reconcile this theoretical property with *global* sustainability? One solution might be that Barro (1974)'s result is overturned when fiscal policy stochastically switches from Ricardian (sustainable) to non-Ricardian (unsustainable) regimes. To be precise, the Ricardian equivalence property would fall despite the government PVBC still holds in the long-run, with infinitely-lived agents. Saying it differently, the Ricardian equivalence would rather be the consequence of a constant (Ricardian) policy regime than the implication of the PVBC *per se*.

Empirically, this thesis argues the Regime-Switching MBS test might better discriminate between unsustainable and fiscal policies than former methods. As Bohn (2008) remarks, public debt-to-GDP cycles are low-frequency, near-integrated stochastic processes, which would likely explain why it is usually hard to reject the unit-root hypothesis in unit-root tests.¹² The RS-MBS test may be a solution to that puzzle. In our framework, debt dynamics can follow a near unit-root stochastic process from the existence of potentially *persistent* unsustainable regimes during which primary surplus do not stabilize public debt. Empirical models which avoid to account for this feature would necessarily be misspecified and unable to detect sub-periods of debt-stabilizing fiscal policy. Applied to France's fiscal policy, we show our framework can overturn ambiguous previous results and conclude that fiscal policy was actually sustainable overall the period 1965-2013, despite a prolonged period of unsustainability during the 1980s.

Of course, as any richer econometric model with respect to simple constant-parameters models, regime switching modeling comes at the cost of harder statistical identification. Future research should probably use semi-annual or even quarterly data in order to increase estimation's robustness. Unfortunately, harmonized quarterly measures of public debt are rarely available for years prior to the 1990s –except for the US. Hence it would imply to estimate interpolated quarterly time series for public debt. Estimates should also be improved regarding potential endogeneity biases in fiscal policy rules. As already argued, methods to control for endogenous regressors in regime-switching dynamic regressions are not yet well-established. For instance, Kim (2010) suggests a two-step method based on the

(2007) show that in a regime-switching environment, the fiscal theory is always at work, as long as agents believe there is a positive probability of moving to a regime with active fiscal policy. In this paper, that belief is governed by the long-run properties of the estimated policy process."

¹²ADF tests are well-known to be of low power when time series are near-integrated but still stationary.

control function approach. Applied to fiscal policy rules and the potential reverse causality between primary surplus and output gap, it would be equivalent to use a proxy for the unexpected-component of output gap as exogenous regressor in the fiscal policy rule.

This thesis also contributes to the growing literature on debt limits and sovereign risk. Theoretical and empirical research about what public debt instruments and structure could help mitigating sovereign insolvency risks. Recently, the benefits of GDP-linked sovereign bonds are particularly discussed, see Benford et al. (2016) and Pienkowski (2017) for recent examples. Regarding long-term debt, there is a large literature which has investigated the optimal maturity structure, see section 3.2 for a discussion. Still, effects of longer maturities of public debt on debt limits were not considered until very recently. Building on Ghosh et al. (2013a)'s methodology to derive debt limits, Kim (2015) finds that longer maturities increase fiscal space and debt limits, i.e. reduce fiscal risk, when primary balance is stochastic. In Kim (2015)'s framework *future* expected positive shocks on primary balance increase the price of bonds *today*, hence reduce the interest rate and public debt's accumulation, which in turn also reduce future risk of default.

In this thesis, we study the effect of long-term debt on the stochastic default threshold in a microfounded dynamic general equilibrium frictionless model build by Guillard and Kempf (2017). In that framework, sovereign default is triggered by the existence of a fiscal limit. Distortionary taxation on labor gives birth to a dynamic Laffer curve which admits a maximum tax rate. Hence sovereign default can happen when fiscal policy enters a constrained regime in which it can no longer increase the level of tax revenues and primary surpluses. As a result, default is essentially driven by the demand of public bonds through the Euler equation. At some point, the gross financing needs of government will be higher than the maximum amount it can borrow from the bond markets, which depends on aggregate productivity shocks' probability distribution, and government won't be able to roll over maturing debt. This framework has two main advantages. First, both creditors' behavior and recovery rule have explicit microfoundations. Second, as a consequence, it allows us to derive an *endogenous* stochastic default threshold level of public debt.

Extending this model to long-term debt, we seek to determine whether longer maturities of public debt can increase the stochastic default threshold. When government defaults

on its *outstanding level of public debt*, we show the maturity of public debt does not affect the stochastic default threshold. This result holds even if the price of long-term bonds depends not only on the next period productivity shock but also on all future expected productivity shocks. While this result seems to contradict Kim (2015)'s findings, one should rather insist on differences between the models, in particular the explicit microfoundations of our framework and the source of shocks. A natural extension of this work should aim to solve this non-linear model using numerical techniques. It could allow to simulate whether this result is robust to other shocks, in particular fiscal policy shocks or serially correlated shocks.

Finally, this thesis proposes a critical appraisal of the European fiscal rules, based on a survey of theoretical and empirical literature. In a monetary union with independent national fiscal policies, requirements for public debt sustainability and inflation stability become really demanding. Theoretically, only one fiscal free-rider could eventually violates the monetary union consolidated intertemporal budget constraint. This might explain why a *negative* and *horizontal* fiscal union –like the current state of the EMU– is fundamentally exposed to a fiscal crisis and contagion effects. A more integrated fiscal union could allow to implement a corrective mechanism in order to guarantee that fiscal free-riding would not destabilize the whole monetary union while building an institutional framework for a better coordination of national fiscal policies.

Regarding the current state of the EMU, we argue the European fiscal rules are simultaneously too tight and too loose. Fiscal requirements embedded in the preventive arm are far from being necessary for sustainability and are largely sub-optimal regarding macroeconomic stabilization. While the corrective arm is conceptually based on a debt-stabilizing rule, its specification is far too strong for fiscal sustainability. Finally, both remains fundamentally procyclical despite several reforms of the SGP in 2005, 2011 and 2013. While European fiscal policies may have been *actually* sustainable since the EMU was created, according Model-Based Sustainability empirical analyses, the procyclical bias in Euro-Area fiscal policies still may increase the risk of a fiscal crisis *ceteris paribus*.

Appendix A

Appendix of chapter 1

A.1 Proof of Proposition 1 (No-Ponzi Game)

We show that a strictly positive long-run feedback effect, i.e. (1.18)

$$\gamma\pi > 0$$

is a *sufficient* condition for the NPG (1.17) to hold, in a dynamically efficient economy and a bounded innovation process $\mu(z_t)$, following Bohn (1998, see online appendix). Using (1.8) and iterating of (1.5) yields:

$$b_{t+T} = \prod_{i=0}^T \frac{1+r_{t+i}}{1+y_{t+i}} (1 - (1+y_{t+i})\gamma(z_{t+i})) b_{t-1} - \sum_{k=0}^T (1+r_{t+k}) \left(\prod_{j=k+1}^T \frac{1+r_{t+j}}{1+y_{t+j}} (1 - (1+y_{t+j})\gamma(z_{t+j})) \right) \mu_{t+k}(z_{t+k}) \quad (\text{A.1})$$

Then, multiplying by (1.16), one gets an expression for the discounted debt-output ratio at time $t+T$:

$$\mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T} = \mathbb{E}_t \prod_{i=0}^T (1 - (1+y_{t+i})\gamma(z_{t+i})) b_{t-1} - \mathbb{E}_t \sum_{k=0}^T \left(\prod_{j=k+1}^T (1 - (1+y_{t+j})\gamma(z_{t+j})) \right) a_{t,k} \quad (\text{A.2})$$

with $a_{t,k} = (1 + y_{t+k})\tilde{Q}_{t,k}\mu_{t+k}(z_{t+k})$. Taking the absolute value¹ of (A.2) and using triangle inequality yields:

$$\begin{aligned} |\mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T}| &\leq \mathbb{E}_t \left| \prod_{i=0}^T (1 - (1 + y_{t+i})\gamma(z_{t+i})) b_{t-1} \right| \\ &\quad + \underbrace{\mathbb{E}_t \left| \sum_{k=0}^T \left(\prod_{j=k+1}^T (1 - (1 + y_{t+j})\gamma(z_{t+j})) \right) a_{t,k} \right|}_{W_t} \end{aligned} \quad (\text{A.3})$$

and applying the triangle inequality on W_t allow us to give an upper bound to the absolute value of (A.2):

$$\begin{aligned} |\mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T}| &\leq \mathbb{E}_t \prod_{i=0}^T |1 - (1 + y_{t+i})\gamma(z_{t+i})| |b_{t-1}| \\ &\quad + \mathbb{E}_t \sum_{k=0}^T \left| \prod_{j=k+1}^T (1 - (1 + y_{t+j})\gamma(z_{t+j})) \right| |a_{t,k}| \end{aligned} \quad (\text{A.4})$$

An important step is to give a tractable expression for

$$\mathbb{E}_t \prod_{i=0}^T |1 - (1 + y_{t+i})\gamma(z_{t+i})| \quad (\text{A.5})$$

in order to study the limit property of equation (A.2). Thus remark that:

$$\begin{aligned} \mathbb{E}_t \prod_{i=0}^T |1 - (1 + y_{t+i})\gamma(z_{t+i})| &= \mathbb{E}_t \left[\exp \left(\ln \prod_{i=0}^T |1 - (1 + y_{t+i})\gamma(z_{t+i})| \right) \right] \\ &= \mathbb{E}_t \left[\exp \left(T \times \frac{1}{T} \sum_{i=0}^T \ln |1 - (1 + y_{t+i})\gamma(z_{t+i})| \right) \right] \end{aligned} \quad (\text{A.6})$$

where $\frac{1}{T} \sum_{i=0}^T \ln |1 - (1 + y_{t+i})\gamma(z_{t+i})|$ is the Lyapunov exponent associated to the present-value debt-output ratio. Since both $(1 + y_t)$ and z_t are stationary-ergodic, then we know that:

$$\lim_{T \rightarrow +\infty} \frac{1}{T} \sum_{i=0}^T \ln |1 - (1 + y_{t+i})\gamma(z_{t+i})| = \mathbb{E} \left[\ln |1 - (1 + y_t)\gamma(z_t)| \right] \quad (\text{A.7})$$

¹Note that $f(x) = |x|$ is convex, then Jensen inequality yields for any random variable X :

$$|\mathbb{E}[X]| \leq \mathbb{E}[|X|]$$

which is measurable at time t . If one assumes $(1 + y_t)\gamma(z_t) < 1^2$ then it yields

$$\ln|1 - (1 + y_t)\gamma(z_t)| = \ln(1 - (1 + y_t)\gamma(z_t))$$

Applying Jensen's inequality on the logarithm function and the expectation operator yields an upper-bound for

$$\mathbb{E}_t \ln(1 - (1 + y_t)\gamma(z_t)) \leq \ln(1 - \mathbb{E}(1 + y_t)\gamma(z_t)) \quad (\text{A.8})$$

From what precedes³, we deduce it exists an arbitrarily high $N \in \mathbb{N}$ such that:

$$\forall T \geq N, \quad \mathbb{E}_t \prod_{i=0}^T |1 - (1 + y_{t+i})\gamma(z_{t+i})| \leq \exp \left[\ln(1 - \mathbb{E}(1 + y_t)\gamma(z_t))^T \right] \quad (\text{A.9})$$

which allows us to conclude

$$\mathbb{E}_t \prod_{i=0}^T |1 - (1 + y_{t+i})\gamma(z_{t+i})| \leq (1 - \mathbb{E}(1 + y_t)\gamma(z_t))^T \quad (\text{A.10})$$

Finally, we define the following upper bound for equation (A.5):

$$\mathbb{E}_t \prod_{i=0}^T |1 - (1 + y_{t+i})\gamma(z_{t+i})| \leq (1 - (1 + y)\gamma\pi - (\gamma_S - \gamma_{NS}) \text{Cov}(y_t, z_t))^T \quad (\text{A.11})$$

where $\text{Cov}(y_t, z_t)$ is the unconditional covariance of y_t and z_t .

At this stage, we need two assumptions to proceed further.

Assumption 3 *Following Bohn (1998), we assume dynamic efficiency which implies present-value of income is finite:*

$$\lim_{T \rightarrow +\infty} Y_t \sum_{i=1}^T \mathbb{E}_t \tilde{Q}_{t,i} = \bar{Y}$$

implying $\lim_{T \rightarrow +\infty} \mathbb{E}_t \tilde{Q}_{t,T} = 0$, by convergence of the serie $\sum_{i=1}^T \mathbb{E}_t \tilde{Q}_{t,i}$.

²This assumption is actually purely technical, since it mainly relies on the assumption $|\gamma(z_t)|$ is close to zero, about the size of a small interest rate and $(1 + y_t)$ is close to 1.

³In particular, Jensen inequality implies that:

$$\frac{1}{T} \sum_{i=0}^T \ln|1 - (1 + y_{t+i})\gamma(z_{t+i})| \leq \ln \left(\frac{1}{T} \sum_{i=0}^T |1 - (1 + y_{t+i})\gamma(z_{t+i})| \right)$$

and allows to define an upper-bound for $\mathbb{E}_t \prod_{i=0}^T |1 - (1 + y_{t+i})\gamma(z_{t+i})|$.

Assumption 4 Following Bohn (1998), we assume the innovation process $\mu_t(z_t)$ is bounded $|\mu_t(z_t)| \leq M$.

Assumptions 1-2 jointly imply $\lim_{T \rightarrow +\infty} \mathbb{E}_t a_{t,k} = 0$ ⁴ that is:

$$\forall \delta > 0, \quad \exists K \in \mathbb{N} \quad / \quad \forall k > K, \quad |\mathbb{E}_t a_{t,k}| \leq \delta \quad (\text{A.12})$$

Then, using assumptions 1-2 along with equation (A.11) yields:

$$\begin{aligned} |\mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T}| &\leq (1 - (1+y)\gamma\pi - (\gamma_S - \gamma_{NS}) \text{Cov}(y_t, z_t))^T |b_{t-1}| \\ &\quad + \Omega (1 - (1+y)\gamma\pi - (\gamma_S - \gamma_{NS}) \text{Cov}(y_t, z_t))^{T-K} \\ &\quad + \sum_{k=K}^T (1 - (1+y)\gamma\pi - (\gamma_S - \gamma_{NS}) \text{Cov}(y_t, z_t))^{T-k} \delta \end{aligned} \quad (\text{A.13})$$

where $\Omega = \sum_{k=0}^{K-1} E_t \prod_{j=k+1}^{K-1} |1 - (1+y_{t+i})\gamma(z_{t+i})| |\mathbb{E}_t a_{t,k}|$ is finite. Finally, rearranging the last expression allows us to write:

$$\begin{aligned} |\mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T}| &\leq (1 - (1+y)\gamma\pi - (\gamma_S - \gamma_{NS}) \text{Cov}(y_t, z_t))^T |b_{t-1}| \\ &\quad + \Omega (1 - (1+y)\gamma\pi - (\gamma_S - \gamma_{NS}) \text{Cov}(y_t, z_t))^{T-K} \\ &\quad + \frac{\delta}{(1+y)\gamma\pi + (\gamma_S - \gamma_{NS}) \text{Cov}(y_t, z_t)} \end{aligned} \quad (\text{A.14})$$

Assumption 5 In a purely Ricardian economy, we assume the fiscal regime z_t is independent of the real growth rate of the economy y_t , i.e. $\text{Cov}(y_t, z_t) = 0$.

Therefore, under assumption 3, a sufficient condition for the NPG condition only requires:

$$\gamma\pi > 0 \quad (\text{A.15})$$

which implies $(1+y)\gamma\pi > 0$. Therefore, we find that

$$\forall \hat{\epsilon} > 0, \quad \exists K \in \mathbb{N} \quad / \quad \forall T \geq K \quad |\mathbb{E}_t \tilde{Q}_{t,T+1} b_T| < \hat{\epsilon}$$

⁴Given that $\lim_{T \rightarrow +\infty} \mathbb{E}_t \tilde{Q}_{t,T} = 0$ also implies $\lim_{T \rightarrow +\infty} \mathbb{E}_t (1+y_T) \tilde{Q}_{t,T} = 0$

provided one sets $\hat{\varepsilon} = \frac{\delta}{|(1+y)\gamma\pi|}$, from which we conclude that:

$$\lim_{T \rightarrow +\infty} \mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T} = 0 \quad (\text{A.16})$$

Discussion. In a more general framework with $\text{Cov}(y_t, z_t) \neq 0$, a sufficient condition to rule out Ponzi schemes, given a Markov-switching fiscal rule such as (1.8), would be:

$$\gamma\pi > -\frac{(\gamma_S - \gamma_{NS})}{1+y} \text{Cov}(y_t, z_t) \quad (\text{A.17})$$

and would critically depends on the covariance term $\text{Cov}(y_t, z_t)$. If positive (i.e. if sustainable regimes are positively correlated to higher growth), it implies that a strictly positive $\gamma\pi$ would not be required to rule out Ponzi schemes; if negative, on the contrary, it would not be sufficient. Still, our empirical results provide an *ex post* validation for assuming $\text{Cov}(y_t, z_t) = 0$, since the estimated unconditional covariance between smoothed probabilities of a sustainable regime (i.e. the empirical counterpart of z_t) and the growth rate of real GDP is non-significantly different from zero, with a positive point estimate.

A.2 Proof of Proposition 2 (Debt-stabilizing condition)

Using the sufficient condition for a *strictly* stationary Markov-switching autoregressive process of order one, we show a strictly larger feedback effect than the average growth-adjusted real interest rate, i.e. (1.24), is a sufficient condition for the debt-output ratio process (1.23) to be strictly stationary and fluctuate around its ergodic mean (1.26).

Considering stochastic processes $\{x_t\}$ described by:

$$x_t = \phi_0 + \phi(z_t)x_{t-1} + \varepsilon_t \quad (\text{A.18})$$

where z_t is a discrete-time Markov process, defined on the state-space $z(\Omega)$. We know from Kesten (1973) that a sufficient condition for *strict stationarity* is:

$$\mathbb{E}[\ln|\phi(z_t)|] \equiv \sum_{i \in z(\Omega)} \ln|\phi(i)|\pi(i) < 0 \quad (\text{A.19})$$

which means that a globally stationary process $\{x_t\}$ can be locally (or periodically) non-stationary. This condition ensures that $\{x_t\}$ is strictly (or strongly) stationary implying its joint-probability distribution does not change over time. Strict stationarity only implies $\{x_t\}$ has a finite mean but does not imply necessarily a finite variance. Since weak stationarity requires finite variance, this condition is not sufficient for weak stationarity. For a finite variance, this process must verify a stronger condition. Define $\Phi \equiv \text{diag}(\phi(i), \forall i \in z(\Omega))$ and $\rho(M)$ the spectral radius of any square-matrix M . Then, for this strictly stationary process to admit a unique stationary solution at second-order, it must satisfy the following condition:

$$\rho(\Phi^2 P) < 1 \quad (\text{A.20})$$

where P is the transition matrix of the underlying Markov-chain.

Applying condition (A.19) to equation (1.23) yields the following condition:

$$\mathbb{E}[\ln|\phi(z_t)|] = \mathbb{E}\left[\ln\left|\frac{1+r_t}{1+y_t}\right| + \ln|1 - (1+y_t)\gamma(z_t)|\right] < 0 \quad (\text{A.21})$$

Hence, using usual approximation $\ln(1+x) \sim x$ when $x \rightarrow 0$ and taking unconditional expectations of r_t, y_t and $\gamma(z_t)$, we find a sufficient condition for strict stationarity of process $\{b_t\}$ is:

$$\gamma\pi > \frac{r-y}{1+y} \quad (\text{A.22})$$

assuming that $\text{Cov}(y_t, z_t) = 0$.

Therefore, process $\{b_t\}$ has an ergodic mean equal to

$$\begin{aligned} \mathbb{E}[b_t] &= \frac{-\mathbb{E}[(1+r_t)\alpha(z_t)] + \text{Cov}(\phi(z_t), b_{t-1})}{\mathbb{E}[1 - \Phi(z_t)]} \\ &= \frac{-(1+r)\mathbb{E}\alpha(z_t) - (\alpha_S - \alpha_{NS})\text{Cov}(r_t, z_t) + \text{Cov}(\phi(z_t), b_{t-1})}{(1+r)\gamma\pi + (\gamma_S - \gamma_{NS})\text{Cov}(r_t, z_t) - \frac{r-y}{1+y}} \end{aligned} \quad (\text{A.23})$$

which we approximate by

$$\mathbb{E}[b_t] \simeq \frac{-(1+r)\mathbb{E}\alpha(z_t)}{(1+r)\gamma\pi - \frac{r-y}{1+y}} \quad (\text{A.24})$$

neglecting covariance terms, following Bohn (1998, 2008) and Mendoza and Ostry (2008).

Appendix B

Appendix of chapter 2

B.1 Unit-root and stationarity tests

First, we perform unit-root (Augmented Dickey-Fuller and Phillips-Perron) and stationarity (KPSS) tests on public debt-to-GDP ratio and primary balance-to-GDP ratio. Results are reported in tables [B.1](#) to [B.4](#).

We find that French public debt is $I(1)$ and does not converge toward a stable debt-to-GDP ratio. Still, according to Bohn (2007), a difference-stationary public debt is sufficient to satisfy the PVBC. Hence, unit-root and stationarity tests are inconclusive on fiscal sustainability in France. At least, these results can be interpreted under the assumption that a "fiscal limit" does exist: without a bounded debt-to-GDP ratio, France may be forced to default if the public deficit hits the "fiscal limit". Turning to primary balance, both Augmented Dickey-Fuller and Phillips-Perron tests reject the unit-root hypothesis for primary balance, at 5% level for both unit-root and unit-root with drift and only at 10% for a unit-root with deterministic trend, see table [B.3](#). KPSS stationarity test rejects at 10% level the hypothesis of a stationary primary balance GDP ratio and cannot reject the null hypothesis for the case of a trend-stationary primary balance ratio, see table [B.4](#). Results suggest the presence of a deterministic trend component for the primary balance which can explain why KPSS test rejects at 10% level the null of a stationary primary balance ratio around a constant. Still, regarding results from unit-root tests, and given the low level of confidence at which we reject stationarity in KPSS test, we conclude to a stationary (or potentially trend-stationary¹) primary balance GDP ratio.

¹We will account for a deterministic trend when performing OLS estimation of Bohn's fiscal policy rule.

TABLE B.1 – Unit-root tests for gross debt-to-GDP ratio b_t

Variable: b_t	Augmented Dickey-Fuller		Phillips-Perron	
	t-statistic	p-value	Adjusted t-statistic	p-value
H_0 : unit-root	0.8978	0.8987	0.3870	0.7920
H_0 : unit-root with drift	0.0648	0.9598	-0.1998	0.9316
H_0 : unit-root with deterministic trend	-2.0649	0.5521	-2.6394	0.2654

Variable: Δb_t	Augmented Dickey-Fuller		Phillips-Perron	
	t-statistic	p-value	Adjusted t-statistic	p-value
H_0 : unit-root	-3.1898***	0.0020	-3.0537***	0.0029
H_0 : unit-root with drift	-3.2706**	0.0217	-3.1123**	0.0320
H_0 : unit-root with deterministic trend	-4.4382***	0.0046	-4.3718***	0.0055

Reported probabilities are MacKinnon (1996) one-sided p-values. Results are significant at 1% level (***), 5% level (***) and 10% level (*). Number of observations: 50. Lag-order selection based on SIC criterion. For ADF regressions: one lag included.

TABLE B.2 – KPSS stationarity tests for gross debt-to-GDP ratio b_t

Variable: b_t	LM-stat	Asymptotic critical values		
		10% level	5% level	1% level
H_0 : stationary	0.5385**	0.3470	0.4630	0.7390
H_0 : trend-stationary	0.2304***	0.1190	0.1460	0.2160

Results are significant at 1% level (***), 5% level (***) and 10% level (*). Number of observations: 50. Lag-order selection based on SIC criterion.

TABLE B.3 – Unit-root tests for primary balance-to-GDP ratio s_t

Variable: s_t	Augmented Dickey-Fuller		Phillips-Perron	
	t-statistic	p-value	Adjusted t-statistic	p-value
H_0 : unit-root	-2.7436**	0.0143	-2.5884**	0.0106
H_0 : unit-root with drift	-2.9558**	0.0462	-3.0947**	0.0334
H_0 : unit-root with deterministic trend	-3.4330*	0.0584	-3.5910**	0.0408

Reported probabilities are MacKinnon (1996) one-sided p-values. Results are significant at 1% level (***), 5% level (***) and 10% level (*). Number of observations: 50. Lag-order selection based on SIC criterion.

TABLE B.4 – KPSS stationarity tests for primary balance-to-GDP ratio s_t

KPSS stationarity test in level	LM-stat	Asymptotic critical values		
		10% level	5% level	1% level
H_0 : stationary	0.3790*	0.3470	0.4630	0.7390
H_0 : trend-stationary	0.0658	0.1190	0.1460	0.2160

Results are significant at 1% level (***), 5% level (***) and 10% level (*). Number of observations: 50. Lag-order selection based on SIC criterion.

Following Bohn (1998), we perform ADF tests accounting for cyclical components. Drawing on the fiscal rule specified in equation

$$s_t = \gamma b_{t-1} + \mu_t$$

and substituting s_t into the standard public debt dynamics equation²

$$b_t = \frac{1+r}{1+y} b_{t-1} - (1+r)\mu_t$$

where debt is expressed as end-of-period value, we obtain:

$$\Delta b_t = \left(\frac{1+r}{1+y} (1 - (1+y)\gamma) - 1 \right) b_{t-1} - (1+r)\mu_t \quad (\text{B.1})$$

Equation (B.1) suggests to estimate modified ADF regressions controlling for cyclical components in order to check whether public debt-to-GDP is mean-reverting or not, i.e. if $\gamma > r - y$ ³. Still, a strictly positive γ but lower than $r - y$ would satisfy the PVBC without any upper bound on public debt; in this case, primary balance s_t should be non-stationary to ensure that the PVBC holds. Excluding these cyclical components from ADF regressions may result in an omitted variable bias that could explain why public debt-to-GDP ratio is always found to be non-stationary. In addition, to account for the effects of 2008's financial crisis and the Great Recession at the end of our sample, we run a second modified ADF regression with both cyclical components and a dummy for years 2008-2013. Basically, we estimate the following ADF equation by OLS, with one lag:

$$\Delta b_t = \alpha + \beta t + \phi b_{t-1} + \rho \Delta b_{t-1} + \Phi Z_t + \varepsilon_t \quad (\text{B.2})$$

where Z_t is a vector of exogenous regressors, including output gap \hat{y}_t , temporary government spending \hat{g}_t and a dummy variable FinCrisis_t , depending on model specification. Results are reported in table B.5. We only reject the null of a unit-root in the case of a deterministic time trend and including all exogenous regressors in the equation. For all other

²Let's assume for simplicity that $r_t = r$ and $y_t = y$ respectively the average long-term real interest rate and the average growth rate of real output.

³The exact stationarity condition is actually $\gamma > \frac{r-y}{(1+r)(1+y)} \approx r - y$, with $r - y > \frac{r-y}{(1+r)(1+y)}$.

TABLE B.5 – Modified ADF unit-root tests for gross debt-to-GDP ratio b_t

Variable: b_t	Augmented Dickey-Fuller	
Exogenous regressors: \hat{y}_t, \hat{g}_t	t-statistic	p-value
H_0 : unit-root	0.9396	0.9052
H_0 : unit-root with drift	0.1577	0.9670
H_0 : unit-root with deterministic trend	-2.4435	0.3538
Variable: b_t	Augmented Dickey-Fuller	
Exogenous regressors: $\hat{y}_t, \hat{g}_t, \text{FinCrisis}_t$	t-statistic	p-value
H_0 : unit-root	-0.2808	0.5798
H_0 : unit-root with drift	-1.4208	0.5648
H_0 : unit-root with deterministic trend	-4.1551***	0.0099

Reported probabilities are MacKinnon (1996) one-sided p-values. Results are significant at 1% level (***), 5% level (***) and 10% level (*). Number of observations: 50.

specifications, we cannot reject the unit-root hypothesis. Controlling for cyclical components of primary balance and for effects of financial and economic crisis since 2008 does not allow to conclude to a stationary public debt-to-GDP ratio. Still, these results do not imply necessarily a negative or null response of primary balance to public debt. Estimated $\hat{\phi}$ are negative, implying $\gamma > r - y$, in four models out of six (when t-statistics are negative). And even in the two remaining cases, a positive $\hat{\phi}$ does not necessarily imply a negative γ .

As a result, primary balance-to-GDP ratio would most probably be stationary while public debt-to-GDP might have a unit-root. Still, given the short range of our time sample and the low power of unit-root tests against near integrated processes, we cannot definitely exclude that public debt may be a mean-reverting process. Furthermore, if public debt is globally stationary but periodically explosive, standard unit-root analysis is unable to distinguish between a unit-root process and a periodically explosive but globally stationary process.

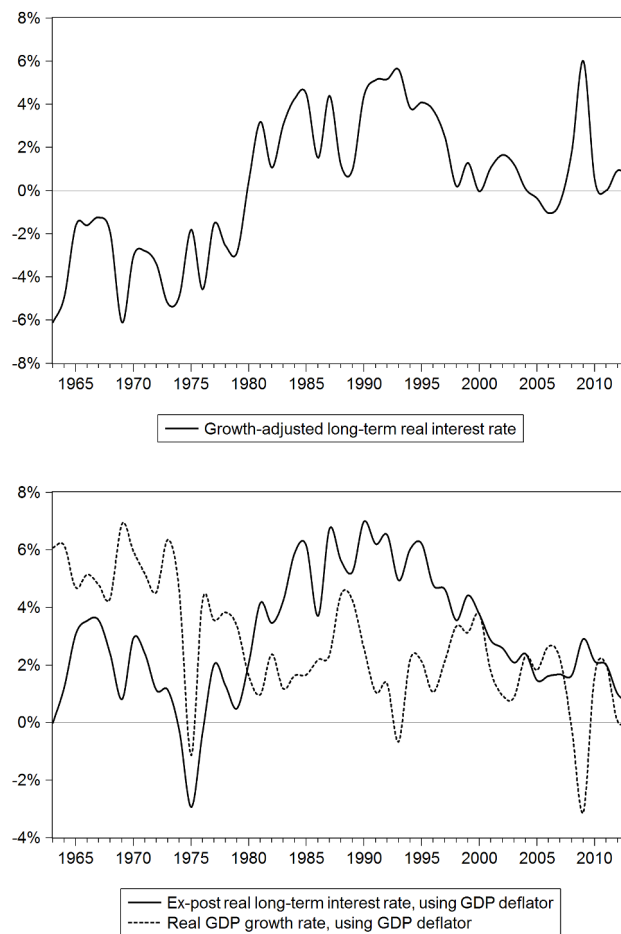
B.2 Data on real interest rates and real GDP growth rate

Table B.6 presents descriptive statistics on long-run ex-post real interest rate (using the yield on 10-year public bonds) and real GDP growth. Figure B.1 plots the growth-adjusted real interest rate and each time series separately.

TABLE B.6 – Descriptive statistics on real interest rates and real GDP growth, 1963-2013

	Long-term real rate	Real GDP growth rate
Mean	3.00%	2.68%
Median	2.86%	2.31%
Maximum	6.99%	6.91%
Minimum	-2.94%	-3.11%
Std. Dev.	2.2%	2.1%
Observations	51	51

FIGURE B.1 – Growth-adjusted real interest rate, real interest rates and real GDP growth rate



Appendix C

Appendix of chapter 3

C.1 Proof of proposition 4

Equation (3.39) is actually very close to the valuation equation (see equations A.1 and A.2 in Guillard and Kempf (2017)), so the present proof follows closely theirs. The market value (as a share of output) of long-term bonds issued in t can be expressed as a function of ω_{t+1}^{max} , which is known in t and of a function x_t

$$v_t = x_t \omega_{t+1}^{max} \quad (\text{C.1})$$

where

$$x_t = \beta \begin{cases} \mathbb{E}_t \frac{1}{a_{t+1}} \delta_t(\rho) & \forall \delta_t(\rho) \leq a_{inf} \\ \chi(\delta_t(\rho), \mathbf{h}) & \forall \delta_t(\rho) \in (a_{inf}, a^{sup}) \\ \mathbf{h} & \forall \delta_t(\rho) \geq a^{sup} \end{cases} \quad (\text{C.2})$$

is non-monotonic in $\delta_t(\rho)$.

Drop the time subscript for convenience and let focus on $\chi(\delta(\rho), \mathbf{h})$ for all $\delta(\rho) \in (a_{inf}, a^{sup})$:

$$\chi(\delta(\rho), \mathbf{h}) = \delta(\rho) \mathbb{E} \frac{1}{a} - \int_{\delta(\rho)}^{\delta(\rho)} \left(\frac{\delta(\rho)}{a} - \mathbf{h} \right) dG(a) \quad (\text{C.3})$$

Let denote the derivative of $\chi(\delta(\rho), \mathbf{h})$ with respect to $\delta(\rho)$ by $\Phi(\delta(\rho), \mathbf{h})$:

$$\Phi(\delta(\rho), \mathbf{h}) = \mathbb{E} \frac{1}{a} - \int_{\delta(\rho)}^{\delta(\rho)} \left(\frac{1}{a} dG(a) - (1 - \mathbf{h})g(\delta(\rho)) \right) \quad (\text{C.4})$$

Assume there exists a $\delta_h(\rho)$ such that:

$$\Phi(\delta_h(\rho), \mathbf{h}) = 0 \quad (\text{C.5})$$

Hence using (C.4) evaluated at $\delta_h(\rho)$ in (C.3), we find an extremum for $\chi(\delta(\rho), \mathbf{h})$ such that

$$\chi(\delta_h(\rho), \mathbf{h}) = \mathbf{h}G(\delta_h(\rho)) + (1 - \mathbf{h})\delta_h(\rho)g(\delta_h(\rho)) \quad (\text{C.6})$$

which is effectively a local maximum since the second derivative of $\chi(\delta_h(\rho), \mathbf{h})$ with respect to δ is strictly negative

$$\frac{\partial^2 \Phi}{\partial \delta^2}(\delta_h(\rho), \mathbf{h}) = -\frac{1}{\delta_h(\rho)} \left(g(\delta_h(\rho)) + (1 - \mathbf{h})\delta_h(\rho)g'(\delta_h(\rho)) \right) < 0 \quad (\text{C.7})$$

recalling from Assumption 1 that $ag'(a)/g(a) > -1/(1 - \mathbf{h})$ for all $\mathbf{h} \in [0, 1]$, implying

$$g(\delta_h(\rho)) + (1 - \mathbf{h})\delta_h(\rho)g'(\delta_h(\rho)) > 0$$

Following Guillard and Kempf (2017, see Appendix A.1), we can prove the exact same way that

$$\frac{\partial \delta_h(\rho)}{\partial \mathbf{h}} > 0 \quad (\text{C.8})$$

implying that $\delta_h(\rho)$ is increasing in \mathbf{h} . We can also prove that $\delta_h(\rho) \in (1, a^{sup}]$ and $x_h(\rho) \in (0, \beta]$, for all $\mathbf{h} \in [0, 1]$. Finally, directly using (C.3) we can prove that x_h is also increasing in \mathbf{h} :

$$\frac{\partial x_h(\rho)}{\partial \mathbf{h}} = \beta \frac{\partial \chi(\delta_h(\rho), \mathbf{h})}{\partial \mathbf{h}} = \beta G(\delta_h(\rho)) > 0 \quad (\text{C.9})$$

So I refer to their proof here.

Now, let's prove x_h and δ_h do not depend on ρ . First, remark that the derivative of $\chi(\delta_h(\rho))$ with respect to ρ is equal to

$$\frac{\partial \chi(\delta_h(\rho), \mathbf{h})}{\rho} = \frac{\partial \delta_h(\rho)}{\partial \rho} \left(g(\delta_h(\rho)) + (1 - \mathbf{h})\delta_h(\rho)g'(\delta_h(\rho)) \right) \quad (\text{C.10})$$

with $g(\delta_h(\rho)) + (1 - \mathbf{h})\delta_h(\rho)g'(\delta_h(\rho)) > 0$ for all $\mathbf{h} \in [0, 1]$. As a consequence $\frac{\partial \chi(\delta_h(\rho))}{\rho}$ is of

the same sign that $\frac{\partial \delta_h(\rho)}{\partial \rho}$.

Now, from the definition of $\delta_h(\rho)$, we know that

$$\frac{\partial \delta_h}{\partial \rho} \left(\frac{g(\delta_h(\rho))}{\delta_h(\rho)} + (1-h)g'(\delta_h(\rho)) \right) = 0 \quad (\text{C.11})$$

and since $g(\delta_h(\rho)) + (1-h)\delta_h(\rho)g'(\delta_h(\rho)) > 0$ for all $h \in [0, 1]$, thus it comes necessarily that:

$$\frac{\partial \delta_h(\rho)}{\partial \rho} = 0 \quad (\text{C.12})$$

and maturity structure of long-term debt does not impact δ_h neither x_h .

C.2 A special case

In this section, we check whether the result obtained in appendix C.1 holds in a simple and particular case: assume the economy stays forever in the constrained regime since period $t-1$ and government can default in $t+1$ but *never after*. Then one can easily check that, despite longer maturity smoothes the impact of productivity shocks on long-term bond's price, this does not imply a higher equilibrium default threshold.

Using (3.28) and since a_t is i.i.d (Assumption 1), we get:

$$q_t^M = \beta \sum_{k=0}^{\infty} (\rho\beta)^k \left(\mathbb{E} \frac{1}{a} \right)^k \mathbb{E}_t \frac{h_{t+1}}{a_{t+1}} = \frac{\beta}{1 - \rho\beta \mathbb{E} \frac{1}{a}} \mathbb{E}_t \frac{h_{t+1}}{a_{t+1}} \quad (\text{C.13})$$

with $\rho\beta \mathbb{E} \frac{1}{a} < 1$ and:

$$q_{t+1}^M = \beta \mathbb{E}_t \sum_{k=0}^{\infty} (\rho\beta)^k \left(\mathbb{E} \frac{1}{a} \right)^{k+1} = \frac{\beta \mathbb{E} \frac{1}{a}}{1 - \rho\beta \mathbb{E} \frac{1}{a}} \quad (\text{C.14})$$

Using the recovery rule (3.19), we can get an expression for q_t^M , the valuation equation of

long-term bonds:

$$q_t^M = \frac{\beta}{1 - \rho\beta \mathbb{E} \frac{1}{a}} \begin{cases} \mathbb{E}_t 1/a_{t+1} & \text{if } \delta_t(\rho) \leq a_{inf} \\ h(1 - \rho\beta \mathbb{E} \frac{1}{a}) \frac{\omega_{t+1}^{max}}{b_t^M} G(\delta_t(\rho)) + \int_{\delta_t(\rho)}^{a^{sup}} \frac{1}{a_{t+1}} dG(a_{t+1}) & \text{if } \delta_t(\rho) \in (a_{inf}, a^{sup}) \\ h(1 - \rho\beta \mathbb{E} \frac{1}{a}) \frac{\omega_{t+1}^{max}}{b_t^M} & \text{if } \delta_t(\rho) \geq a^{sup} \end{cases} \quad (\text{C.15})$$

where $\delta_t(\rho) = \mathbb{E}_t(1 + \rho q_{t+1}^M) b_t^M / \omega_{t+1}^{max}$ and $\mathbb{E}_t(1 + \rho q_{t+1}^M) = (1 - \rho\beta \mathbb{E} \frac{1}{a})^{-1}$.

Hence, defining the valuation equation of long-term bonds, $v_t = q_t^M b_t^M$, we have:

$$v_t = x_t \omega_{t+1}^{max} \quad (\text{C.16})$$

where

$$x_t = \beta \begin{cases} E_t 1/a_{t+1} \delta_t(\rho) & \text{if } \delta_t(\rho) \leq a_{inf} \\ hG(\delta_t(\rho)) + \int_{\delta_t(\rho)}^{a^{sup}} \frac{\delta_t(\rho)}{a_{t+1}} dG(a_{t+1}) & \text{if } \delta_t(\rho) \in (a_{inf}, a^{sup}) \\ h & \text{if } \delta_t(\rho) \geq a^{sup} \end{cases} \quad (\text{C.17})$$

As one can see in this simple particular case, equation (C.17) has the exact same form as equation (3.40) in the general case: as a result, v_t^{max} will necessarily be independent from ρ . Hence we confirm that the maturity structure of public debt has no effect on the equilibrium default threshold ω_h^{max} .

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Résumé français

Cette thèse est consacrée à la politique budgétaire et à la soutenabilité de la dette publique en macroéconomie. Le point de départ de l'analyse de la soutenabilité des dettes publiques est le concept de contrainte budgétaire intertemporelle (CBI). Dans la mesure où l'Etat a théoriquement¹ une durée de vie infinie, celle-ci implique une condition terminale, appelée condition de *No-Ponzi Game*, selon laquelle la valeur actualisée espérée (VAE) de la dette publique ne peut être positive à l'horizon infini, c'est-à-dire :

$$\lim_{T \rightarrow \infty} \text{VAE}(B_{t+T}) \leq 0 \quad (1)$$

où B_t représente le stock de dette publique à la fin de la période t . Lorsque cette condition est vérifiée, alors l'Etat vérifie sa contrainte budgétaire intertemporelle, c'est-à-dire qu'il assure que la dette publique initiale soit *a minima* couverte par la somme de ses excédents budgétaires primaires futurs en VAE :

$$B_{t-1} \leq \sum_{i=0}^{\infty} \text{VAE}(S_{t+i}) \quad (2)$$

où S_t représente le montant des excédents budgétaires primaires, c'est-à-dire la différence entre les recettes et les dépenses hors transferts financiers nets (c'est-à-dire la différence entre intérêts reçus et payés sur les actifs et les dettes de l'Etat).

¹Cette hypothèse peut sembler grossièrement fautive, compte-tenu des multiples exemples historiques de disparition, annexion, etc. Elle tire sa justification du fait qu'un Etat, à la différence de tout autre agent économique (ménage, entreprise) n'a pas de durée de vie finie *a priori*, ce qui implique —par exemple— qu'il puisse émettre perpétuellement des titres de dette publiques pour se refinancer sur les marchés financiers. Cependant, elle n'implique absolument pas qu'un Etat ne puisse pas être contraint de faire défaut sur sa dette publique, si cette contrainte n'était pas vérifiée.

Notons par ailleurs que, pour cette même raison, la notion de faillite ne s'applique pas à un Etat : un ménage, une entreprise ou une banque peuvent faire faillite au sens où leur incapacité à rembourser leurs dettes entraîne généralement leur liquidation, pour pouvoir rembourser en partie les crédateurs. Au contraire, en affirmant qu'un Etat ne peut pas faire faillite, on signifie en réalité qu'un Etat ne peut pas être liquidé, pour des raisons politiques et juridiques évidentes.

Plusieurs remarques s'imposent à ce stade. Tout d'abord, les équations (1) et (2) ne sont effectivement contraignantes, d'un point de vue théorique, que dans le cas où l'économie est en situation d'*efficience dynamique*, c'est-à-dire lorsque le taux de croissance de l'économie est supérieur à la productivité marginale du capital (Diamond, 1965). Dans le cas contraire d'*inefficience dynamique*, Diamond montre qu'un jeu de Ponzi serait optimal dans le sens où il permettrait de réduire l'accumulation du capital et de rétablir la règle d'or pour ainsi maximiser la consommation intertemporelle de l'ensemble des générations présentes et futures. Par ailleurs, en situation d'inefficience dynamique le surcroît de dette publique n'aurait pas besoin d'être remboursée par des surplus primaires futurs du fait que le taux d'intérêt sur la dette publique serait toujours inférieur au taux de croissance de l'économie.

Cependant, il est à noter qu'un taux d'intérêt sur la dette publique inférieur au taux de croissance n'implique pas systématiquement une situation inefficience dynamique. Abel et al. (1989) montrent que, dans une économie stochastique, cette condition s'applique au taux d'intérêt sur le capital risqué, et non pas sur le taux d'intérêt de la dette publique est un actif (relativement) sans risque. D'autre part, ils montrent également que la mesure de ce taux risqué est difficile et proposent une condition suffisante pour tester la condition d'efficience dynamique : il s'agit alors de comparer si les revenus bruts du capital sont supérieurs à l'investissement brut dans toute l'économie (en pourcentage de produit intérieur brut). Dans leur article, Abel et al. trouvent que la plupart des économies avancées de l'OCDE, dont les Etats-Unis, étaient en situation d'efficience dynamique à la fin des années 1980. Mais plus récemment, Geerolf (2013) a montré, à partir de leur méthodologie et de données plus récentes, que la condition d'efficience dynamique n'était peut-être plus vérifiée dans les économies avancées, ce qui pèse en faveur de l'hypothèse de l'excès d'épargne mondial (*global savings glut*).

Bohn (1995) fonde notamment sa critique de l'approche économétrique de la soutenabilité budgétaire sur la question du choix de facteur d'escompte dans l'écriture de la contrainte budgétaire intertemporelle de l'Etat. Dans une économie stochastique, il montre que l'utilisation du taux d'intérêt moyen sur la dette publique (Hamilton et Flavin, 1986), ou du taux d'intérêt effectif (Wilcox, 1989) peut aboutir à des conclusions erronées quant à la soutenabilité de la dette publique. Par opposition aux tests standards, Bohn suggère d'utiliser les

propriétés théoriques du facteur d'escompte stochastique (FES) et d'en déduire des conditions générales de soutenabilité sur une règle de comportement de la politique budgétaire, ce qui fonde l'analyse *Model-Based Sustainability* (MBS) de la politique budgétaire.

Par "approche économétrique" de la soutenabilité budgétaire, nous faisons référence au programme de recherche initié par Hamilton et Flavin (1986). Dans leur article fondateur, Hamilton et Flavin cherchent à déduire des *restrictions économétriques* sur les séries temporelles à partir de l'analyse de la contrainte budgétaire intertemporelle (2). Ils montrent notamment que des conditions de stationarité sur les séries temporelles de solde primaire et de dette publique sont *suffisantes* pour garantir que la politique budgétaire satisfait sa contrainte budgétaire intertemporelle, sous l'hypothèse que le facteur d'escompte est le taux d'intérêt réel moyen sur la dette publique. Wilcox (1989) étend l'approche économétrique de Hamilton et Flavin en proposant une procédure de test de la condition de transversalité (1). Après avoir construit une série de dette publique en valeur actualisée, à partir du taux d'intérêt réel effectif, Wilcox cherche à déterminer si la série temporelle obtenue suit bien un processus stationnaire autorégressif de *moyenne nulle* ; dans le cas où elle serait stationnaire de moyenne non-nulle, alors cela signifierait que la politique budgétaire joue un jeu de Ponzi contre ses créanciers.

Par la suite, Trehan et Walsh (1988) et Trehan et Walsh (1991) ont étendu l'approche économétrique de la soutenabilité à l'analyse des séries temporelles non-stationnaires, après avoir déduit de la contrainte budgétaire intertemporelle des restrictions de cointégration sur les séries de dépenses et de recettes. Ces restrictions sont d'ailleurs semblables à une condition de stationarité du déficit total (incluant les intérêts nets). Finalement, Quintos (1995) montre que la stationarité en différence du déficit total serait même suffisante pour garantir que la contrainte budgétaire intertemporelle est vérifiée.

Davig (2005) a repris et étendu l'approche de Wilcox à des modèles à changements de régime Markoviens afin de tenir compte du fait que la politique budgétaire pourrait jouer un jeu de Ponzi de manière périodique et à court terme sans toutefois violer la condition de transversalité à long terme. Cette thèse reprend notamment l'intuition de Davig (2005) mais dans le cadre d'analyse proposé par Bohn (1998) plutôt que celui d'une approche économétrique *stricto sensu*.

Cette approche économétrique a le principal mérite d'être rapidement et facilement reproductible, dès lors qu'un économètre dispose de données suffisamment longues et de bonne qualité. Cependant, elle a fait l'objet de fortes critiques par Bohn (1995, 1998, 2007, 2008) qui la qualifie d'analyse de la soutenabilité *ad hoc*. En premier lieu, l'approche économétrique est souvent contrainte de faire des hypothèses non-justifiées sur le choix du facteur d'escompte. Bohn (1995) montre comment le choix du taux d'intérêt sans risque sur la dette publique comme facteur d'escompte peut paradoxalement mener à conclure (à tort) quant à l'insoutenabilité de la dette publique. Supposons que le taux d'intérêt sans risque est inférieur au taux de croissance alors que l'économie est par ailleurs en situation d'efficacité dynamique ; dans ce cas, Bohn (1995) montre qu'une politique budgétaire qui maintiendrait son ratio dette/PIB constant à chaque période violerait la contrainte budgétaire intertemporelle *ad hoc*, et quand bien même une telle politique est de façon évidente (très fortement) soutenable. Il montre d'ailleurs que l'utilisation du facteur d'escompte stochastique invalide la conclusion initiale d'insoutenabilité.²

Par conséquent, Bohn (1998) propose une approche alternative pour l'analyse de la soutenabilité de la dette publique fondée sur un modèle d'équilibre général dynamique et stochastique (*Model-Based Sustainability*). Ce cadre d'analyse est dit "*model-based*" par opposition à l'approche économétrique dans la mesure où il déduit une règle de comportement de la politique budgétaire telle que la condition de transversalité (1) soit respectée, à partir de l'équation d'Euler qui détermine la demande d'obligation publique dans un modèle d'équilibre général. Bohn suggère de tester *indirectement* les conditions de CBI (2) et transversalité (1) en estimant la règle budgétaire suivante :

$$s_t = \gamma b_{t-1} + \mu_t \quad (3)$$

où s_t représente le solde primaire (en pourcentage de PIB) à la période t , b_{t-1} le ratio dette publique/PIB à la fin de la période $t - 1$. μ_t est un terme qui rassemble l'ensemble des autres déterminants du solde primaire. Il inclut généralement :

- l'écart de production, pour tenir compte de l'effet des stabilisateur économique et de

²Il est nécessaire de préciser ici que la démarche de Bohn ne dépend pas de la forme fonctionnelle du FES, mais seulement de son évaluation comme solution du programme de maximisation d'un prêteur rationnel à horizon de vie infini.

la composante discrétionnaire de la politique budgétaire ;

- une mesure des dépenses cycliques, l'écart des dépenses réelles par rapport à leur tendance de long-terme ;
- un terme constant qui capture implicitement le solde primaire stabilisant et la cible stationnaire de dette/PIB, si la politique budgétaire suit une règle de stabilisation à la Leeper (1991).

Enfin, μ_t inclut également un terme d'erreur ε_t que l'on suppose généralement indépendant et identiquement distribué. En utilisant les propriétés de la relation d'Euler et du facteur d'escompte stochastique, Bohn démontre qu'une réaction positive du solde primaire au niveau d'endettement public, c'est-à-dire :

$$\gamma > 0 \tag{4}$$

garantit que l'Etat vérifie, à long-terme, la condition de transversalité (1) et la CBI (2). Il est également possible d'en déduire une condition plus forte :

$$\gamma > \frac{r - y}{1 + y} \tag{5}$$

où $r - y$ est le taux d'intérêt réel ajusté du taux de croissance du PIB en volume, et telle que le ratio dette/PIB soit stable à moyen terme. Cette condition est identique à celle proposée par Leeper (1991). Cette approche aboutit également à douter de la pertinence des tests de racines unitaires et de stationnarité sur les séries de dettes publiques. Bohn (1998) montre en particulier qu'un test de racine unitaire ADF est sujet à un biais de variables omises dans la mesure où la variation de la dette Δb_t est impactée par le terme μ_t (principalement les composantes cycliques du solde primaire). Après avoir intégré les variables d'écart de production et de dépenses cycliques dans l'équation du test ADF, Bohn (1998) aboutit au rejet de l'hypothèse de racine unitaire pour le ratio dette/PIB aux Etats-Unis.

Enfin, Bohn (2007) adresse une critique probablement plus fondamentale à l'encontre de l'approche économétrique de la soutenabilité budgétaire. Il démontre que les conditions de stationnarité ou de cointégration usuellement testées ne sont pas en réalité nécessaires

pour assurer que la politique budgétaire vérifie sa contrainte budgétaire intertemporelle. Il conclut que ces tests économétriques ne sont tout simplement pas capables de distinguer entre une politique budgétaire soutenable et non-soutenable, à l'inverse de l'approche par l'estimation de fonctions de comportements (ou règles budgétaires). Enfin, il suggère que la condition de stationarité du ratio de dette/PIB est nécessaire non pas du fait de la CBI en soi mais du fait de l'existence d'une limite budgétaire (*fiscal limit*), c'est-à-dire un niveau maximal d'excédent primaire qu'un Etat pourrait dégager.

L'approche *Model-Based Sustainability* a l'avantage d'être suffisamment flexible pour pouvoir être applicable à différentes spécifications de la règle budgétaire, notamment non linéaires : par exemple, pour tenir compte d'une sur-réaction à un haut niveau d'endettement public, par l'ajout d'un terme quadratique de la variable dette/PIB (Bohn, 1998, 2008) ou au contraire pour modéliser le phénomène de "fatigue budgétaire" (Ghosh et al., 2013a) par l'ajout d'un terme cubique.

Théoriquement, il est aussi possible d'envisager que les paramètres de la règle budgétaire (3) varient en fonction du temps.³ Cette idée a notamment été explorée par Afonso et Toffano (2013), Bianchi (2012a,b), Burger et Marinkov (2012), Chung et al. (2007), Davig et Leeper (2007a, 2011) et Favero et Monacelli (2005). Ces travaux ne tirent cependant aucune conclusion de l'existence de régimes budgétaires quant à la question de savoir si l'Etat satisfait (ou non) sa contrainte budgétaire intertemporelle à long-terme. À notre connaissance, seuls Canzoneri et al. (2001) explorent théoriquement l'implication d'une règle budgétaire variant en fonction du temps sur les conditions de soutenabilité (i.e. le respect de la CBI), dans le cas particulier où $\gamma_t \geq 0$. Ils montrent alors qu'il est possible que la politique budgétaire vérifie toujours sa contrainte budgétaire intertemporelle, sous la condition que γ_t soit *infinitement souvent* positif. Cependant, ils ne proposent pas de procédure de test de leur critère ni ne définissent un critère garantissant la stabilité du ratio dette/PIB.

La recherche sur la soutenabilité de la dette publique s'est récemment tournée vers la question du défaut souverain, des limites budgétaires et des ratio maximums de dette/PIB. En particulier, Daniel et Shiamptanis (2013) réhabilite les restrictions de cointégration entre solde primaire et dépenses du fait de l'existence d'une limite budgétaire sur le solde pri-

³De ce point de vue, il est possible d'envisager une variation *continue* ou *discrète* des paramètres de la règle budgétaire ; dans le second cas, on parlera de changement de régimes budgétaires".

maire. Plus généralement, la littérature sur les limites budgétaires à la Bi (2012), Bi et Leeper (2013) et Bi et Traum (2012) ou à la Fall et al. (2015), Fournier et Fall (2015) et Ghosh et al. (2013a) affirme qu'une règle *prudente* de politique budgétaire doit avoir pour objectif la stabilité du ratio dette/PIB, à un niveau offrant suffisamment d'espace budgétaire (*fiscal space*) pour permettre à la politique budgétaire de réagir activement suite à un choc macroéconomique négatif d'ampleur.

Ce programme de recherche s'est notamment intéressé à la question des instruments budgétaires (obligations contingentes indexées sur le PIB, maturités longues de la dette publique, endettement domestique ou international) qui peuvent permettre d'augmenter l'espace fiscal, c'est-à-dire d'accroître le seuil maximal de dette soutenable. En particulier, Kim (2015) s'est intéressé à la question de l'effet de la maturité de la dette publique sur le seuil de défaut d'équilibre. Dans un modèle d'équilibre partiel, il montre que l'allongement de la maturité permet d'accroître le seuil de défaut lorsque le solde primaire est sujet à des chocs aléatoires. Dans ce cas-là, la dette de long-terme est marginalement moins coûteuse car le prix des obligations ne dépend pas seulement de l'état de l'économie à la période suivante mais aussi de toutes les périodes suivantes. Ainsi, la probabilité d'un ajustement budgétaire *futur* (i.e. choc positif sur le solde primaire) affecte positivement le prix *actuel* des obligations et réduit le taux d'intérêt. Kim (2015) en conclut que la dette de long-terme est, à la marge, moins coûteuse que la dette de court-terme. Cependant, cette analyse ne repose pas sur un cadre d'équilibre général micro-fondé et ne considère pas l'effet d'un choc fondamental (par exemple, sur la productivité du travail).

Cette thèse se compose de quatre chapitres pouvant être lus indépendamment les uns des autres, mais peuvent entretenir des liens assez forts. En particuliers, les deux premiers chapitres sont extraits de Aldama et Creel (2017). Le premier propose une procédure de test à partir d'une règle budgétaire à changements de régime Markoviens. Le second chapitre propose une application empirique au cas de la politique budgétaire française. Le troisième chapitre traite de l'effet de la maturité de la dette publique sur le seuil de défaut d'équilibre dans un modèle d'équilibre général dynamique et stochastique simplifié. Enfin, le quatrième chapitre traite de la soutenabilité des dettes publiques en Union Monétaire et propose une appréciation critique du cadre budgétaire de l'UEM.

Régimes budgétaires et soutenabilité de la dette publique

Le premier chapitre établit formellement des critères de soutenabilité de la dette publique, lorsque que la règle budgétaire admet l'existence d'un régime localement insoutenable. Celle-ci s'écrit formellement :

$$s_t = \gamma(z_t)b_{t-1} + \mu_t(z_t) \quad (6)$$

où $z_t = \{0, 1\}$ représente une chaîne de Markov à deux états, ergodique et irréductible, dont les probabilités de transition sont représentées par $p_{ij} = \mathbb{P}(z_t = i | z_{t-1} = j)$ pour tout $(i, j) \in \{S, NS\}$.⁴ Dans ce cas, le paramètre de réaction du solde primaire à la dette publique est décrit par :

$$\gamma(z_t) \equiv \gamma Z_t = \begin{pmatrix} \gamma_S & \gamma_{NS} \end{pmatrix} \times \begin{pmatrix} z_t \\ 1 - z_t \end{pmatrix} \quad (7)$$

Nous identifions un régime localement soutenable par une réaction strictement positive du solde primaire à la dette publique, i.e. $\gamma_S > 0$, et inversement un régime soutenable par une réaction nulle voire négative, i.e. $\gamma_{NS} \leq 0$.

À partir de la règle budgétaire (6), nous cherchons des conditions sur le vecteur des paramètres de réaction à la dette publique $\gamma = (\gamma_S, \gamma_{NS})$ mais également sur le vecteur des probabilités ergodiques (ou inconditionnelles) $\pi = (\pi_S, \pi_{NS})$, où $\pi_i = \frac{1-p_{jj}}{2-p_{ii}-p_{jj}}$, telles que la politique budgétaires vérifient ses contraintes de soutenabilité. Dans un premier temps, nous généralisons le critère de (Bohn, 1998) pour la condition de transversalité (No-Ponzi Game). Dans une économie en situation d'efficience dynamique, sous l'hypothèse que les composantes cycliques du solde primaires soient bornées, nous montrons que

$$\gamma\pi \equiv \gamma_S\pi_S + \gamma_{NS}\pi_{NS} > 0 \quad (8)$$

est une condition suffisante telle que la politique budgétaire décrite par (6) vérifie les contraintes (1) et (2). Cette condition peut être exprimée de façon plus intuitive en fonction des *durées*

⁴Une chaîne de Markov qui admettrait un régime absorbant aurait des conséquences triviales du point de vue de l'analyse de la soutenabilité budgétaire : la politique budgétaire serait globalement (in)soutenable si le régime absorbant était lui-même (in)soutenable.

espérées de chaque régime notées $d_i = 1/(1 - p_{ii})$:

$$\gamma_S > |\gamma_{NS}| \frac{d_{NS}}{d_S} \quad (9)$$

Nous montrons que la réaction de la politique budgétaire au niveau d'endettement en régime soutenable (i.e. γ_S) doit être d'autant plus agressive que la dérive du solde primaire en régime insoutenable (i.e. $|\gamma_{NS}|$) est forte et que les régimes insoutenables sont *relativement plus longs* que les régimes soutenables (i.e. d_{NS}/d_S).

Nous considérons également un critère de soutenabilité plus strict. Dans la mesure où l'Etat fait face à une limite budgétaire sur le niveau d'excédent primaire qu'il peut dégager, ce qui implique un niveau de dette maximal via la contrainte budgétaire intertemporelle (2), un critère de soutenabilité plus robuste serait celui d'une règle stabilisatrice du ratio dette/PIB. A partir de la condition de *stationnarité stricte* d'un processus AR(1) à changement de régime markovien, nous montrons que

$$\gamma\pi > \frac{r - y}{1 + y} \quad (10)$$

est une condition suffisante pour que le ratio dette/PIB soit strictement stationnaire à long-terme. En exprimant cette condition à partir des durées espérées, nous obtenons la condition suivante :

$$\gamma_S > |\gamma_{NS}| \frac{d_{NS}}{d_S} + \frac{d_S + d_{NS}}{d_S} \frac{r - y}{1 + y} \quad (11)$$

Il est à noter qu'une politique stabilisatrice du ratio dette/PIB sera plus stricte que la condition de No-Ponzi si et seulement si le taux d'intérêt réel est supérieur au taux de croissance du PIB réel $r > y$. Dans le cas inverse, et si l'économie est dynamiquement efficiente⁵, la condition de soutenabilité sera bel et bien la condition de No-Ponzi. Ces deux conditions (9) et (11) formalisent l'intuition selon laquelle la politique budgétaire peut périodiquement et de façon persistante suivre une politique budgétaire insoutenable sans toutefois nécessairement violer à long terme sa contrainte budgétaire intertemporelle ou déstabiliser de *façon permanente* son ratio dette/PIB, sous condition de vérifier la plus stricte des deux conditions.

⁵Ce qui n'est pas nécessairement contradictoire avec le fait que $r < y$, comme nous l'expliquons plus haut en discutant du critère d'efficience dynamique.

Cette approche de la soutenabilité de la dette publique par régimes budgétaires a d'importantes conséquences du point de vue empirique. Une des grandes difficultés des analyses économétriques de la soutenabilité provient généralement des différences d'ordre d'intégration des séries temporelles de solde primaire et de dette publique. Alors que le solde primaire est souvent jugé stationnaire, il est généralement difficile voire impossible de rejeter l'hypothèse d'une racine unitaire pour les séries temporelles de dette publique ; ce qui explique notamment pourquoi les estimations de règle budgétaire à la Bohn sont souvent peu concluantes puisqu'il est théoriquement impossible de trouver une relation stable entre deux variables d'ordres d'intégration différents. Cependant, la prise en compte des régimes budgétaires pourrait apporter une réponse à cette difficulté. L'existence d'un régime budgétaire insoutenable implique que les séries temporelles de solde primaire et de dette publique puissent périodiquement diverger en termes d'ordre d'intégration ou alors avoir une relation de cointégration *négative*, ce qui correspondrait à une politique budgétaire fortement insoutenable. À l'inverse, dans un régime budgétaire soutenable, solde primaire et dette publique auraient le même ordre d'intégration et on retrouverait une relation de cointégration strictement positive entre les deux séries ; en particulier, la dette publique serait probablement stationnaire par sous-période, caractérisée par un phénomène de retour à la moyenne (*mean-reverting*).⁶ Ainsi, l'existence de régimes budgétaires expliquerait pourquoi les tests usuels, qui sont fondés sur des modèles linéaires à paramètres constants, sont fondamentalement mal-spécifiés, puisqu'ils ne modélisent pas la possibilité de ruptures structurelles et récurrentes dans la dynamique de la dette publique.

La dette publique française est-elle soutenable ?

Le second chapitre propose d'illustrer cet argument par une application du test de soutenabilité avec régimes budgétaires (*Regime-Switching MBS test*) au cas de la France. La dette publique française soulève beaucoup d'interrogations : sa dynamique semble à première vue explosive depuis la fin des années 1970, sans que l'on puisse observer une tendance nette à dégager des excédents primaires –à l'exception de quelques épisodes, notamment la période allant du milieu des années 1990 au début 2008. Pourtant, dans le même temps, les

⁶C'est une conséquence directe des conditions (9) et (11).

taux d'intérêts réels et les primes de CDS sur la dette publique française ne donnent aucun signe que les marchés financiers la jugent insoutenable.

À partir d'une base de données annuelles couvrant les années 1963-2013, nous proposons une analyse en deux étapes. Dans un premier temps, nous estimons différentes spécifications à paramètres constants de la règle budgétaire suggérée par Bohn. Nous tentons notamment de prendre compte une possible non-linéarité dans la réaction du solde primaire à la dette publique (en introduisant un terme quadratique de dette publique), en introduisant une variable indicatrice pour contrôler l'effet spécifique de la récession de 2009 et de son impact durable sur le solde primaire. Cependant aucune spécification ne permet de conclure à la soutenabilité de la politique budgétaire française. Au contraire, nos estimations montrent une tendance déterministe significativement *négative* dans l'équation de règle budgétaire, ce qui tend à indiquer une forte insoutenabilité de la politique budgétaire.

Dans un deuxième temps, nous estimons une règle budgétaire dont les paramètres varient en fonction d'une chaîne de Markov inobservée, d'après le filtre proposée par Hamilton (1989). Nous identifions deux régimes, soutenable et insoutenable, tous les deux fortement persistants. Le régime insoutenable, caractérisé par l'absence de corrélation entre le solde primaire et la dette publique, apparaît être le plus persistant avec une durée espérée de presque 12 ans, tandis que le régime soutenable se caractérise par une réaction fortement significative du solde primaire à la dette publique, pour une durée espérée d'environ 8 ans. Le modèle permet également d'identifier (par filtrage et lissage) la probabilité que la politique budgétaire se trouve dans le régime soutenable, à une date donnée. De ce point de vue, nos résultats indiquent une longue période d'insoutenabilité de la politique budgétaire française, de la fin des années 1970 jusqu'au milieu des années 1990 suivi d'un ajustement budgétaire significatif, à partir de 1995 jusqu'à la récession de 2008/2009. Ces estimations appuient l'analyse selon laquelle la politique budgétaire française n'a pas fourni un niveau suffisamment fort de consolidation budgétaire dans les années 1980 jusqu'au début des années 1990, alors que l'effet boule de neige de la dette publique était historiquement fort (comme le montre le taux d'intérêt réel ajusté du taux de croissance, cf. figure B.1). L'entrée dans l'UEM a permis un ajustement budgétaire suffisamment fort pour garantir la soutenabilité de la politique budgétaire sur la période considérée, et ce en dépit de la Procédure de

Déficit Excessif engagée contre la France, de 2003 à 2007.

En plus d'identifier des régimes budgétaires par sous-périodes, nous procédons au test des conditions (9) et (11) à partir de tests de Student unilatéraux. Ces tests concluent au rejet à la fois de l'hypothèse nulle d'un jeu de Ponzi et de celle d'une dette publique explosive à long-terme. Ces résultats doivent cependant être nuancés, dans la mesure où l'espérance *ergodique*⁷ du ratio dette/PIB est élevée, de l'ordre de 110% —pour une cible stationnaire du ratio dette/PIB, associée au régime soutenable, d'environ 70%. Sous l'hypothèse que la politique budgétaire française est correctement représentée par l'équation estimée, ces résultats apportent une réponse nuancée à la question de la soutenabilité de la dette publique française : si celle-ci satisfait les deux critères de soutenabilité, elle est cependant exposée au risque d'atteindre des niveaux élevés, pour lesquels la limite budgétaire serait probablement atteinte, compte tenu de la persistance du régime insoutenable.

La maturité de la dette publique a-t-elle un impact sur le seuil de défaut d'équilibre ?

Dans le troisième chapitre, nous explorons l'effet de la maturité de la dette publique sur le seuil de défaut d'équilibre dans un modèle d'équilibre général dynamique et stochastique stylisé, à partir des travaux de Guillard et Kempf (2017). L'unique source des chocs dans cette économie provient de la productivité du travail, pour laquelle nous faisons l'hypothèse qu'elle suit un processus racine unitaire dont les chocs sont indépendants et identiquement distribués.

Dans cette économie, à l'instar de Bi (2012), Bi et Leeper (2013) et Bi et Traum (2012), la présence d'un impôt distorsif sur le travail implique l'existence d'une courbe de Laffer dynamique et donc d'une limite budgétaire. Nous supposons désormais que l'Etat s'endette à long terme, en émettant des obligations dont les paiements décroissent de façon géométrique au taux ρ , suivant la modélisation suggérée par Woodford (2001). Cette représentation a le grand avantage de représenter de façon simplifiée une structure complexe

⁷Cette espérance est difficile à interpréter car le processus de dette publique est alors un MS-AR(1) dont l'un des deux régimes est explosif et n'admet pas d'espérance.

de dette publique, permettant d'aboutir à des résultats analytiques. L'introduction d'une obligation de long-terme implique cependant d'introduire une variable supplémentaire désignant l'encours total de la dette. On suppose que le gouvernement suit une règle budgétaire déterministe par laquelle il stabilise le niveau de dette publique post-défaut, tant que le taux d'imposition est inférieur au taux maximal. Une fois la limite budgétaire atteinte, la politique budgétaire devient contrainte et le défaut devient possible. Lorsque le défaut survient, la règle de recouvrement qui lui est associée fait l'hypothèse que le taux de recouvrement h_t sera suffisant pour garantir que le niveau de dette post-défaut soit inférieur au niveau maximal, i.e. le seuil de défaut d'équilibre. Cette hypothèse garantit qu'un équilibre existe après la survenance d'un défaut souverain sans cependant écarter la possibilité que l'Etat puisse faire de nouveau défaut.

Dans le régime contraint, c'est-à-dire quand la politique budgétaire a atteint sa limite budgétaire, nous montrons d'abord que la maturité de la dette publique n'impacte pas le seuil de solvabilité de l'Etat ω^{sup} , défini comme le niveau de maximum de dette publique satisfaisant, à l'état stationnaire, la contrainte budgétaire intertemporelle. Nous suivons ensuite la démarche de Guillard et Kempf (2017) pour déterminer de façon endogène le seuil de défaut d'équilibre ω^{max} . Dans ce modèle, le défaut est envisagé comme un événement de marché : il survient lorsque le besoin de financement brut de l'Etat est supérieur à la valeur maximale des obligations qu'il pourrait émettre sur les marchés obligataires, c'est-à-dire de la demande quantité-prix maximale v_t^{max} . Après avoir décrit l'équilibre du marché obligataire, nous dérivons l'expression de la valeur maximale des obligations de long-terme puis nous déterminons le seuil de défaut d'équilibre. Nous étendons les résultats de Guillard et Kempf (2017) pour la dette de long-terme : sous les hypothèses du modèle (en particulier, celles de chocs de productivité i.i.d.), le seuil de défaut d'équilibre est constant dans le temps, toujours inférieur au seuil de solvabilité et dépend de la valeur du facteur h qui caractérise la règle de recouvrement de la dette publique. Cependant, nous montrons que la maturité de la dette publique n'a aucun impact sur v_t^{max} et donc sur le seuil de défaut d'équilibre ω^{max} .

Ce résultat s'explique principalement à la relation d'Euler et l'équation de prix des obli-

gations longues que l'on peut en déduire :

$$q_t^M = \mathbb{E}_t \sum_{k=0}^{\infty} \rho^k \left(\prod_{i=0}^k Q_{t,t+i} h_{t+1+i} \right) = \beta \mathbb{E}_t \sum_{k=0}^{\infty} (\rho\beta)^k \left(\prod_{i=0}^k \frac{h_{t+1+i}}{y_{t+1+i}} \right) \quad (12)$$

qui définit le prix d'une obligation longue comme la moyenne pondérée (du produit) des facteurs d'escompte stochastiques futurs $Q_{t,t+i}$, ajusté du risque de défaut h_{t+1+i} à chaque période. La forme de cette équation implique que, quelle que soit la maturité de la dette et pour un encours total de dette donné, les investisseurs ont déjà intégré dans le prix la distribution des chocs futurs sur le taux de croissance y_{t+1+i} . Par conséquent, un accroissement de la maturité de la dette ne permet pas d'accroître, à la marge, la valeur de marché des obligations et donc le montant maximal v_t^{max} que l'Etat peut espérer emprunter pour couvrir son besoin de financement brut. Il est à noter que nous utilisons le même modèle de dette de long terme que Kim, aussi la différence de résultat ne peut pas être attribuée au choix de modélisation de la dette de long terme.

Cependant notre résultat pourrait ne pas être nécessairement interprété de façon contradictoire avec celui de Kim (2015). En effet, Kim étudie une situation dans laquelle le défaut provient non pas d'un choc négatif sur le taux de croissance de l'économie mais d'un choc négatif sur le solde primaire –lorsque celui-ci est supposé stochastique. D'autre part, il montre que lorsque le solde primaire est déterministe, alors obligations de court terme et de long terme sont parfaitement équivalentes du point de vue du seuil maximal de dette publique. Aussi, nous proposons d'interpréter ces résultats comme le fait que si la dette de long terme peut permettre à un gouvernement de s'assurer contre des chocs négatifs de politique budgétaire, elle ne peut pas lui permettre de s'assurer contre des chocs négatifs sur le taux de croissance potentiel.

Soutenabilité des dettes publiques en Union Monétaire : Une appréciation critique de l'UEM

Le quatrième chapitre propose une revue de littérature critique sur les questions de dette publique et de politique budgétaire en union monétaire, ainsi qu'une appréciation des

règles budgétaires de l'UEM.⁸

Les trente dernières années ont vu la politique budgétaire reléguée au second plan, derrière la politique monétaire. Selon le consensus caractéristique de la *Grande Modération*, la banque centrale se charge de stabiliser l'inflation et la croissance, tandis que la politique budgétaire se cantonne à la production de biens publics et de laisser jouer les stabilisateurs automatiques au cours du cycle macroéconomique, tout en respectant strictement sa contrainte budgétaire intertemporelle ; la politique budgétaire discrétionnaire est alors jugée inefficace. De ce point de vue, la conception de l'UEM et des règles budgétaires qui la caractérisent sont les purs produits de ce consensus sur les rôles respectifs des politiques monétaires et budgétaires. Cependant, la crise financière de 2008 et la Grande Récession ont largement bouleversé ce consensus macroéconomique. La politique budgétaire discrétionnaire a été, dans une certaine mesure, réhabilitée en même temps que l'on a insisté sur l'importance de la contrainte budgétaire et du risque de défaut souverain, compte tenu des niveaux historiquement élevés de dette publique *en temps de paix*.

Ce chapitre se fonde sur l'idée qu'une union monétaire implique toujours une union budgétaire (Kempf, 2013), en raison de la multiplicité et de la spécificité des contraintes budgétaires impliquées par le partage d'une monnaie unique. En particulier, nous montrons que la soutenabilité budgétaire ne peut plus être considérée uniquement au niveau des pays-membres mais doit également l'être au niveau de l'union monétaire. L'existence de transferts internationaux ou fédéraux ne modifie d'ailleurs pas ce résultat : la contrainte budgétaire intertemporelle consolidée de l'union monétaire implique que les déficits primaires dans certains pays soient financés par les excédents primaires d'autres pays ; les transferts, qu'ils soient inter-nationaux ou fédéraux, seraient seulement des outils supplémentaires pour garantir le respect de la contrainte budgétaire intertemporelle consolidée de l'union monétaire. Si un seul pays-membre dévie de sa contrainte budgétaire alors la soutenabilité budgétaire de l'ensemble de l'union monétaire peut être menacée : c'est d'ailleurs un mécanisme qui pourrait expliquer les effets de contagion ou l'instabilité de l'inflation au niveau de l'union monétaire selon la Théorie Budgétaire du Niveau des Prix (FTPL, en Anglais) en cas de défaut souverain d'un membre de l'union monétaire.

⁸Ce chapitre est une version augmentée et révisée de Aldama (2017).

Tout d'abord, le chapitre présente les contraintes de soutenabilité budgétaire en union monétaire, en fonction de l'architecture de l'union budgétaire sous-jacente. Nous retenons trois possibilités : (i) une union monétaire "ordolibérale" sans transferts et seulement fondée sur des règles budgétaires strictes, (ii) une union budgétaire inter-nationale avec des transferts et des règles budgétaires mais pas d'autorité budgétaire fédérale et enfin (iii) une fédération budgétaire où les transferts fédéraux remplacent les transferts internationaux. Nous étendons également la discussion aux interactions des politiques monétaires et budgétaires et à la Théorie Budgétaire du Niveau des Prix, qui a été intensivement discutée au sujet des restrictions budgétaires nécessaires pour garantir la stabilité des prix en union monétaire. Le principal point à retenir de l'analyse de la FTPL est que le respect des contraintes budgétaires n'est pas seulement nécessaire en soi, mais aussi en raison de l'effet de la politique budgétaire sur l'inflation. Cependant, la FTPL plaide fortement en faveur d'un policy-mix alternatif, lorsque la politique monétaire est contrainte par la ZLB (*Zero-Lower Bound*) suite à un choc déflationniste. Elle suggère alors que la politique budgétaire devienne agressive pour lutter contre la déflation : c'est-à-dire qu'elle devrait s'engager de façon crédible à *ne pas respecter* sa contrainte budgétaire intertemporelle, afin de dissiper les risques déflationnistes et restaurer la capacité d'action de la politique monétaire. Enfin, nous tirons les implications de ces différentes analyses pour la conception des règles budgétaires, ainsi que les évolutions récentes sur la limite budgétaire et les règles «prudentes» et présentons enfin les principaux résultats théoriques en matière de policy-mix optimal en union monétaire.

Dans une seconde partie, nous proposons une évaluation critique du cadre budgétaire européen, à partir de la littérature théorique et empirique. Nous commençons par un bref rappel historique de la mise en place et des différentes réformes du Pacte de Stabilité et de Croissance (PSC). Ensuite, nous traitons la question de savoir si les règles budgétaires européennes sont suffisantes pour assurer la soutenabilité des dettes publiques européennes ou non, d'un point de vue théorique et empirique. Selon nous, le problème le plus grave des règles budgétaires européennes reste la procyclicité des politiques budgétaires, malgré la volonté affichée par les décideurs économiques et politiques de le réduire, à l'occasion des différentes réformes du PSC. En conséquence, nous soutenons que le cadre budgétaire européen peut être à la fois trop strict et trop souple : trop strict parce que les règles budgétaires européennes sont bien trop fortes en regard de l'analyse théorique de la soutenabilité

budgétaire, mais également trop souples du fait de leur caractère procyclique qui accroît l'instabilité du cycle macroéconomique et en particulier du ratio dette/PIB. Nous terminons ce chapitre en discutant des causes de la crise de la dette souveraine européenne et de la proposition de créer des euro-obligations pour compléter et stabiliser l'union budgétaire européenne.

Essays on fiscal policy and public debt sustainability

Abstract

This thesis contributes to the analysis of public debt sustainability and fiscal rules. It starts from the multiple empirical evidence that points to the existence of unsustainable fiscal regimes during which fiscal policy does not increase its primary surplus following an increase of public debt. Do unsustainable fiscal regimes necessarily threaten the long-run sustainability of public debt? If not, how long can fiscal policy be *periodically* unsustainable without being *globally* unsustainable? The first chapter answers theoretically this question and proposes a Regime-Switching Model-Based Sustainability (RS-MBS) test. We study a Markov-switching fiscal policy rule, which displays an unsustainable fiscal regime, and derive sufficient conditions for the No-Ponzi Game condition and for a globally stable public debt-to-GDP ratio. The second chapter proposes an empirical application of the RS-MBS to France's fiscal policy between 1962 and 2013. It shows that taking into account regime switches can overturn former results and conclude that France's public debt has been sustainable overall the period. The third chapter considers another case of unsustainable regime, when fiscal policy is constrained by the fiscal limit, and studies the effect of public debt maturity on the debt limit. We show that longer debt maturities do not increase the stochastic default threshold when sovereign default is triggered by bad productivity shocks. Finally, the fourth chapter proposes a critical appraisal of the fiscal architecture of the EMU, based on a literature survey about fiscal sustainability, monetary-fiscal interactions and fiscal rules in monetary unions.

Keywords : Fiscal policy, Public debt, Sustainability constraints, Fiscal regimes

Essais sur la politique budgétaire et la soutenabilité de la dette publique

Résumé

Cette thèse contribue à l'analyse de la soutenabilité de la dette publique et des règles budgétaires en macroéconomie. Elle tire sa motivation des multiples preuves empiriques de l'existence de régimes budgétaires insoutenables durant lesquels le solde primaire ne s'accroît pas suite à un accroissement de la dette publique. Ces régimes insoutenables menacent-ils nécessairement la soutenabilité de la dette publique à long-terme ? Si non, combien de temps la politique budgétaire peut-elle rester *périodiquement* insoutenable sans être *globalement* insoutenable ? Le premier chapitre apporte une réponse théorique à cette question et propose un test de type "Model-Based Sustainability" étendu aux changements de régimes (RS-MBS). Nous étudions une règle budgétaire à changement de régime Markovien, dont l'un des régimes est insoutenable, et définissons des conditions suffisantes pour exclure un Jeu de Ponzi et pour garantir la stabilité du ratio dette/PIB à long terme. Le second chapitre propose d'appliquer le test RS-MBS à la politique budgétaire française entre 1962 et 2013. Il montre que la prise en compte des changements de régime peut inverser les résultats empiriques précédents et conclure à la soutenabilité de la dette publique française sur l'ensemble de la période. Le troisième chapitre traite d'un autre cas de régime insoutenable, quand la politique budgétaire est contrainte par sa limite fiscale, et étudie l'effet de la maturité de la dette sur le seuil d'endettement public maximal. Nous montrons que l'allongement de la maturité de la dette n'accroît pas le seuil de défaut stochastique quand le défaut survient à cause de chocs négatifs sur la productivité. Enfin, le quatrième chapitre propose une appréciation critique de l'architecture budgétaire de l'UEM, à partir d'une revue de la littérature traitant de la soutenabilité budgétaire, de l'interaction des politiques monétaires et budgétaires et des règles budgétaires en union monétaire.

Mots-clés : Politique budgétaire, Dette publique, Conditions de soutenabilité, Régimes budgétaires