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Junfei Chu

To cite this version:

Junfei Chu. Improvement methods for data envelopment analysis (DEA) cross-efficiency evaluation. Other. Université Paris Saclay (COmUE); University of science and technology of China, 2018. English. NNT : 2018SACLC096 . tel-02015680

HAL Id: tel-02015680

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université PARIS-SACLAY

中国科学技术大学 University of Science and Technology of China

CentraleSupélec

NNT : 2018SACLC096

Improvement methods for data envelopment analysis cross-efficiency evaluation

Thèse de doctorat de l'Université Paris-Saclay et de University of Science and Technology of China (USTC), préparée à CentraleSupélec

École doctorale n°573 approches interdisciplinaires, fondements, applications et innovation (Interfaces)
GRADUATE SCHOOL of USTC
Spécialité de doctorat : sciences et technologies industrielles

Thèse présentée et soutenue à Hefei, le 21 décembre 2018, par

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Thèse de doctorat

Acknowledgements

I am very glad to have this chance to express my heartful gratitude to those who helped, supported, and accompanied me during my five-years' PhD study.

This is a joint PhD training program between CentraleSupélec, Université Paris-Saclay (France) and the University of Science and Technology of China (USTC) (China). Thanks to the Laboratoire Génie Industriel (LGI) at CentraleSupélec and the School of Management at the USTC for having provided me with this excellent training program and wonderful working conditions. Also, I would like to thank the China Scholarship Council for the financial support during my academic stay in France.

My deep appreciation goes to all members of my thesis defense commission. They are Professor Yingming Wang, Professor Imed Kacem, Professor Vicent Mousseau, Professor Gongbing Bi, Professor Jie Wu, Professor Chengbin Chu, and Dr. Pengzhen Yin. It is my great honor to have you all participating in this PhD thesis defense. Thanks a lot for your positive responses and efforts in helping me to complete this last step for the doctoral diploma. Special thanks go for the reviewers, Professor Yingming Wang and Professor Imed Kacem for professional evaluation of the manuscript and for all the constructive and insightful comments.

Particularly, I am grateful to my two supervisors, Prof. Chengbin Chu at CentraleSupélec and Prof. Jie Wu at the USTC. They have taught me a lot of professional knowledge, skills, and scientific research experiences which are very helpful to my research and would also be meaningful for my future career.

Prof. Chu is an academic leader. He is rigorous, very eager to learn, and always striving for excellence. He is always willing to help me with scientific research. During my two-years' study and research with him, I cannot remember how many times we have discussed my research questions in his office and the parks, on the telephone and the WeChat, etc. Also, I cannot remember how many times my proofs were overthrown and reconstructed by his rigorous inference. On him, I saw the literacy and quality of a successful researcher. He is the master that I would pursue and learn from in the rest of my life.

Prof. Wu has strong enthusiasm and passion for scientific research. He is very

brilliant, confident, and with very broad research horizon. Prof. Wu is the mentoring teacher of my scientific research road. He taught me how to be a qualified independent researcher. I have always kept in my mind your inculcation that “spur with long accumulation” which has also become a criterion that I have adhered on my road of scientific research.

I would like to express my gratitude to other professors who helped me during my PhD study: Profs Liang Liang, Gongbin Bi, Bernard Yannou, Vincent Mousseau, Yongjun Li, Shaofu Du, Yu Pan, Malin Song, etc.

My warmest thanks to Delphine Martin, Sylvie Guillemain, Corinne Ollivier, Suzanne Thuron, Zhengsi Peng, Honglin Huang, and Xiaowen Ye, secretaries of the LGI and the School of Management at the USTC. They have always been helpful and lovely. I would also like to thank my current and previous collogues at the LGI and the school of management at the USTC for their help, support, and companionship. Heartful thanks go to Zhixiang, Qingxian, Jiasen, Pengzhen, Xiang, Dong, Beibei, Qingyuan, Yuwei, Yafei, Hongwei, Haoliang, Pingfang, Jessie, Tianjun, Huanhuan, Tasneem, Phuong, Hichame, Selemen, Haythem, Zhiguo, Yiping, Mengfei, Xiangyu, Jinduo, and Hongping. I have been so lucky and so happy to work with these wonderful people.

Finally, I would like to show my special thanks to my parents and my brother who have been there to support and help me in any circumstances. Their love and encouragements have always been my driving force to carry on.

Abstract

Data envelopment analysis (DEA) cross-efficiency evaluation has been widely applied for efficiency evaluation and ranking of decision-making units (DMUs). However, two issues still need to be addressed: non-uniqueness of optimal weights attached to the inputs and outputs and non-Pareto optimality of the evaluation results. This thesis proposes alternative methods to address these issues.

We first point out that the cross-efficiency targets for the DMUs in the traditional secondary goal models are not always feasible. We then give a model which can always provide feasible cross-efficiency targets for all the DMUs. New benevolent and aggressive secondary goal models and a neutral model are proposed. A numerical example is further used to compare the proposed models with the previous ones.

Then, we present a DEA cross-efficiency evaluation approach based on Pareto improvement. This approach contains two models and an algorithm. The models are used to estimate whether a given set of cross-efficiency scores is Pareto optimal and to improve the cross-efficiency scores if possible, respectively. The algorithm is used to generate a set of Pareto-optimal cross-efficiency scores for the DMUs. The proposed approach is finally applied for R&D project selection and compared with the traditional approaches.

Additionally, we give a cross-bargaining game DEA cross-efficiency evaluation approach which addresses both the issues mentioned above. A cross-bargaining game model is proposed to simulate the bargaining between each pair of DMUs among the group to identify a unique set of weights to be used in each other's cross-efficiency calculation. An algorithm is then developed to solve this model by solving a series of linear programs. The approach is finally illustrated by applying it to green supplier selection.

Finally, we propose a DEA cross-efficiency evaluation approach based on satisfaction degree. We first introduce the concept of satisfaction degree of each DMU on the optimal weights selected by the other DMUs. Then, a max-min model is given to select the set of optimal weights for each DMU which maximizes all the DMUs' satisfaction degrees. Two algorithms are given to solve the model and to ensure the

uniqueness of each DMU's optimal weights, respectively. Finally, the proposed approach is used for a case study for technology selection.

Keywords: Data envelopment analysis (DEA), Decision-making units, Cross-efficiency evaluation.

Résumé

L'évaluation croisée d'efficacité basée sur la *data envelopment analysis* (DEA) a été largement appliquée pour l'évaluation d'efficacité et le classement des unités de prise de décision (decision-making units, DMUs). A l'heure actuelle, cette méthode présente toujours deux défauts majeurs : la non-unicité des poids optimaux attachés aux entrées et aux sorties et la non Pareto-optimalité des résultats d'évaluation. Cette thèse propose des méthodes alternatives pour y remédier.

Nous montrons d'abord que les efficacités croisées visées dans les modèles traditionnels avec objectifs secondaires ne sont pas toujours atteignables pour toutes les DMUs. Nous proposons ensuite un modèle capable de toujours fournir des objectifs d'efficacité croisée atteignables pour toutes les DMUs. Plusieurs nouveaux modèles avec objectifs secondaires bienveillants ou agressifs et un modèle neutre sont proposés. Un exemple numérique est utilisé pour comparer les modèles proposés à ceux qui existent dans la littérature.

Nous présentons ensuite une approche d'évaluation croisée d'efficacité basée sur l'amélioration de Pareto. Cette approche est composée de deux modèles et d'un algorithme. Les modèles sont utilisés respectivement pour estimer si un ensemble donné de scores d'efficacité croisée est Pareto-optimal et pour améliorer l'efficacité croisée de cet ensemble si cela est possible. L'algorithme est utilisé pour générer un ensemble Pareto-optimal de scores d'efficacité croisée pour les DMUs. L'approche proposée est finalement appliquée pour la sélection de projets de R&D et comparée aux approches traditionnelles.

En outre, nous proposons une approche d'évaluation croisée d'efficacité qui traite simultanément les deux problématiques mentionnées ci-dessus. Un modèle de jeu de négociation croisée est proposé pour simuler la négociation entre chaque couple de DMUs au sein du groupe afin d'identifier un ensemble unique de poids à utiliser pour le calcul de l'efficacité croisée entre eux. De plus, un algorithme est développé pour résoudre ce modèle via une suite de programmes linéaires. L'approche est finalement illustrée en l'appliquant à la sélection des fournisseurs verts.

Enfin, nous proposons une évaluation croisée d'efficacité basée sur le degré de satisfaction. Nous introduisons d'abord la nation de degré de satisfaction de chaque

DMU sur les poids optimaux sélectionnés par les autres. Ensuite, un modèle max-min est fourni pour déterminer un ensemble des poids optimaux pour chaque DMU afin de maximiser tous les degrés de satisfaction des DMUs. Deux algorithmes sont ensuite développés pour résoudre le modèle et garantir l'unicité des poids optimaux de chaque DMU, respectivement. Enfin, l'approche proposée est appliquée sur une étude des cas pour la sélection de technologies.

Mots clés: Analyse d'enveloppement des données (DEA), Unités de prise décision, Évaluation de l'efficacité croisée.

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Chapter 1 Introduction

In this introduction, we present the motivation of this work, summarize our contributions, and outline the structure of this thesis.

1.1 Motivation

Data envelopment analysis (DEA), originally proposed by Charnes et al. (1978), is a non-parametric programming method for efficiency evaluation of a group of homogenous decision-making units (DMUs) in which multiple inputs are consumed to produce multiple outputs (Cook et al., 2009; Thanassoulis et al., 2011; Cook et al., 2013; Yang et al., 2014; Imanirad et al., 2015). The main idea of DEA is to generate a set of optimal weights for each DMU to maximize the ratio of its sum of weighted outputs to its sum of weighted inputs while keeping all the DMUs' ratios at most 1. This maximum ratio is defined as the efficiency of the DMU under evaluation (Wang and Chin, 2010; Ghasemi et al., 2014). For its effectiveness in identifying the best-practice frontier and ranking the DMUs, DEA has been widely applied in benchmarking and efficiency evaluation of schools (Charnes et al., 1994), hospitals (Mitropoulos et al., 2014), bank branches (Wang et al., 2014; Paradi et al., 2011), and so on.

However, traditional self-evaluated DEA models with total weight flexibility may evaluate many DMUs as DEA-efficient and cannot make any further distinction among them. Therefore, one of the main shortfalls of the traditional DEA models (CCR and BCC models) is their inability to discriminate among DMUs that are all deemed efficient (Wang and Chin, 2010). To improve the power of DEA in discriminating among efficient DMUs, Sexton et al. (1986) incorporated the concept of peer evaluation into DEA and proposed the cross-efficiency evaluation method. In cross-efficiency evaluation, each DMU defines its most favorable weights associated with the inputs and outputs for self-efficiency evaluation. Using these weights, it can also evaluate the efficiencies of the other DMUs, which gives rise to peer-evaluated efficiencies. For each DMU under evaluation, we can obtain a final efficiency by aggregating its self-evaluated efficiency and its efficiencies peer-evaluated by the others. Cross-efficiency evaluation presents at least three main advantages. Firstly, it almost always ranks the

DMUs in a unique order (Doyle and Green, 1995). Secondly, it eliminates unrealistic weight schemes, such as zero weight or disproportionate weights, without incorporating weight restrictions (Anderson et al., 2002). Finally, it effectively distinguishes good performers from poor ones among the DMUs (Boussofiane et al., 1991). Due to these advantages, cross-efficiency evaluation has been extensively applied in performance evaluation of nursing homes (Sexton et al., 1986), preference ranking and project selection (Green et al., 1996), selection of flexible manufacturing systems (Shang and Sueyoshi, 1995), judging suitable computer- or numerically-controlled machines (Sun, 2002), determining efficient operators and measuring labor assignment in cellular manufacturing systems (Ertay and Ruman, 2004), performance ranking of countries in the Olympic Games (Wu et al., 2009a), supplier selection in public procurement (Macro et al., 2012), portfolio selection in the Korean stock market (Lim et al., 2014), energy efficiency evaluation for airlines (Cui and Li, 2015), and so on.

In spite of its advantages and wide applications, there are still some shortcomings in DEA cross-efficiency evaluation. One main deficiency is the non-uniqueness of optimal weights. Specifically, the optimal set of weights obtained for self-evaluation may not be unique, which may result in situations where the set of cross-efficiency scores for the DMUs cannot be uniquely identified since different optimal weights of any DMU lead to different peer-evaluated efficiencies for the others and therefore their final efficiency scores. To reduce the non-uniqueness of optimal weights, Doyle and Green (1994) proposed to use secondary goal models. That is to shrink the region to search for optimal weights, even guarantee the uniqueness, by selecting the weights that achieve some new goals under the condition that the self-evaluated efficiency of each DMU is guaranteed to be at the optimal level. Inspired by this idea, scholars have proposed many secondary goal models. The most representative secondary goal models are the benevolent and aggressive secondary goal models proposed by Doyle and Green (1994). These models have been widely applied and extended ever since (Liang et al. 2008a; Wang and Chin, 2010a). However, in the benevolent and aggressive models and their extensions, the ideal efficiency points, used as targets for the DMUs, are not always achievable (or feasible). Additionally, to the best of our knowledge, no existing study theoretically guarantees the uniqueness of optimal weights in DEA cross-efficiency evaluation although many proposed secondary goal models have the ability to limit the occurrences of non-uniqueness of optimal weights.

Most of existing studies on DEA cross-efficiency evaluation only focus on developing methods for reducing the non-uniqueness of optimal weights. Few of them have considered whether the DMUs will be satisfied with the evaluation result and accept it. Specifically, selecting different sets of optimal weights for DMUs lead to different sets of cross-efficiency scores. In addition to guaranteeing the uniqueness of the optimal set of cross-efficiency scores, we also need develop appropriate theories to make the evaluation result likely to be accepted by all the DMUs. For instance, a typical problem is that the generated average cross-efficiency scores for the DMUs are generally not Pareto optimal (Wu et al., 2011), which is to say at least one DMU can improve its cross-efficiency score without reducing those of the others. To some extent, this drawback makes the evaluation result unacceptable to the DMUs, especially for those whose cross-efficiency scores can be improved.

The above discussions show that DEA cross-efficiency evaluation has wide applications. The two disadvantages both indicate that a lot of work is still needed to fill the gaps in DEA cross-efficiency evaluation. Therefore, it is meaningful to propose new cross-efficiency evaluation methods or models to surmount the deficiencies of existing DEA cross-efficiency evaluation methods. This is what has motivated this thesis. We focus on developing new DEA cross-efficiency evaluation methods to generate more acceptable cross-efficiency evaluation result.

1.2 Research contributions

This study has brought at least four contributions to DEA cross-efficiency evaluation. Firstly, it proposes a series of secondary goal models which consider both desirable and undesirable targets and always use reachable cross-efficiency targets for the DMUs. Secondly, a Pareto-improvement DEA cross-efficiency evaluation approach is proposed which can guarantee the Pareto-optimality of the cross-efficiency scores for the DMUs. Additionally, in some special cases, the approach generates an evaluation result that unifies the self-evaluation, peer-evaluation, and common-weight evaluation where the weights of inputs and outputs are not DMU-specific, which makes the evaluation result even more acceptable to all the DMUs. Thirdly, the Nash bargaining game theory is incorporated into DEA cross-efficiency evaluation and a cross-bargaining game DEA cross-efficiency evaluation approach is proposed. The

proposed approach can not only solve the non-uniqueness of optimal weights by providing the DMUs with a unique set of cross-efficiency scores but also provide a Pareto optimal evaluation result. Finally, the concept of satisfaction degree is incorporated into DEA cross-efficiency evaluation, and a cross-efficiency evaluation approach based on satisfaction degree is proposed. This approach has not only the ability to maximize all the DMUs' satisfaction degrees but also guarantees the uniqueness of the optimal weights for each DMU.

1.3 Structure of the Thesis

The rest of this thesis is organized as follows. In Chapter 2, we illustrate some basic concepts of DEA and DEA cross-efficiency evaluation. Then, we review the literature about DEA ranking methods and DEA cross-efficiency evaluation. Then, we summarize the limits of existing works. Chapter 3 proposes some extended secondary goal models for weights selection in DEA cross-efficiency evaluation. Chapter 4 presents a new DEA cross-efficiency evaluation approach based on Pareto improvement. In Chapter 5, we provide a cross-bargaining game DEA cross-efficiency evaluation approach and apply it for green supplier selection. Chapter 6 gives a new cross-efficiency evaluation approach based on satisfaction degree. This approach is then applied for technology selection. Finally, in Chapter 7, we conclude this thesis and discuss some further research directions.

Chapter 2 Literature review

2.1 A brief introduction of DEA and cross-efficiency evaluation

In this Section, we give a brief introduction of basic concepts and models of DEA and DEA cross-efficiency evaluation. Then we illustrate the research problem with a numerical example. We first introduce the following notation which will be used throughout this thesis.

n: number of DMUs

m: number of inputs

s: number of outputs

x_{ij}: the ith $(i = 1, \dots, m)$ input of DMU j $(j = 1, \dots, n)$

y_{rj}: the rth $(r = 1, \dots, s)$ output of DMU j $(j = 1, \dots, n)$

w_{id}: weight attached by DMU d $(d = 1, \dots, n)$ to the ith $(i = 1, \dots, m)$ input

u_{rd}: weight attached by DMU d $(d = 1, \dots, n)$ to the rth $(r = 1, \dots, s)$ output

In this notation, $w_{id}, \forall i, d$, and $u_{rd}, \forall r, d$ are decision variables. The input and output data of the DMUs are known and assumed to be all positive. The inputs and outputs of DMU j form vectors X_j and Y_j, respectively. Similarly, the weights attached to the inputs and the outputs of DMU d form vectors W_d and U_d, respectively. In the remainder, we often use vectorial representation, especially, vectorial products. The vectorial product of two p-entry vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, denoted as $\boldsymbol{a} \cdot \boldsymbol{b}$, is defined as follows.

$$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}^T \boldsymbol{b} = \boldsymbol{b}^T \boldsymbol{a} = \sum_{k=1}^{p} a_k b_k \tag{2.1}$$

2.1.1 Basics of DEA

According to Charnes et al. (1978), a standard definition of data envelopment analysis (DEA) can be given as follows.

Definition 2.1 Data envelopment analysis (DEA) is a non-parametric method for efficiency evaluation of a group of homogeneous decision-making units (DMUs) in which multiple inputs are consumed to produce multiple outputs.

From the definition, firstly, we know that DEA is a non-parametric method. It does not require any predetermined information on the production function of the production entities before evaluation. The evaluation results are directly derived from the input and output data. Secondly, the application of DEA needs the data of a group of DMUs. In addition, the DMUs are required to be homogeneous, i.e., input and output indicators of the DMUs should be the same for all DMUs. The efficiency evaluation result is obtained by comparing the production of each DMU with those of the others. Of course, the number of DMUs in the group are not necessarily very large. Finally, DEA is used to model the production process of multiple inputs and multiple outputs. Now, we give the illustration of some basic concepts of DEA.

Decision-making units (DMUs)

The structure of a DMU is illustrated in Figure 2.1.

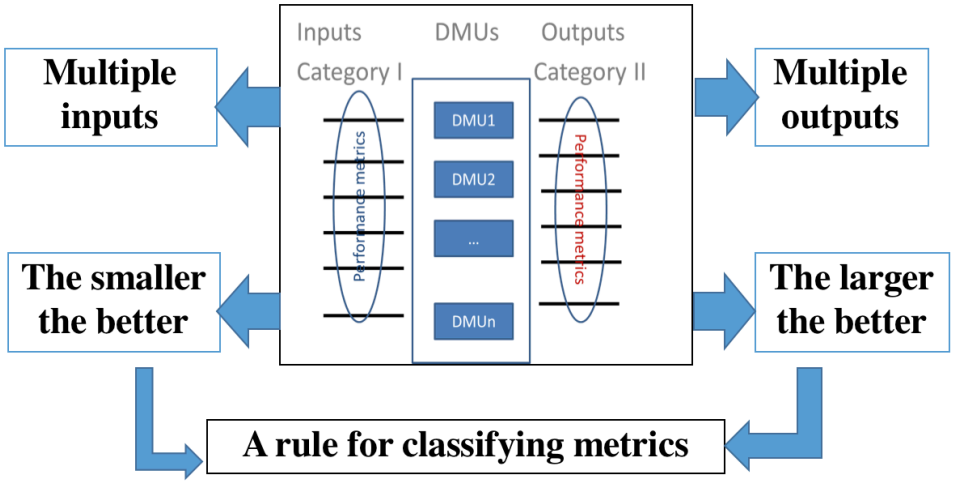

Figure 2.1 The structure of a DMU

This figure shows that a DMU can be seen as a production entity containing some performance metrics. These performance metrics can be classified as the larger the better for outputs, and the smaller the better for the inputs. Then, each DMU can be seen as a production entity in which multiple inputs are used to produce multiple outputs. A DMU can be a not-for-profit organization as well as a for-profit organization. Examples of DMUs can be schools, hospitals, manufacturing systems, and so on. In these DMUs, the inputs can be capitals, labors, fixed cost, etc.; the outputs can be product yields, profits, etc.

When applying DEA for the evaluation of DMUs, we assume that three aspects are the same for all the DMUs: production environment, the input and output indicators, and production process.

Based on the input and output data of a group of homogenous DMUs, we can construct the production possibility set (PPS). Here, we introduce the PPS as follows.

Production possibility set (PPS)

In DEA, the production possibility set can be defined as follows.

Definition 2.2. $T = \{(X, Y) | X \text{ can produce } Y\}$ is defined as the production possibility set (PPS) constituted of all the DMUs' production activities, where X and Y are input and output vectors, respectively.

Before giving the detailed mathematical formulation of the production possibility set, we need to introduce the following axioms of PPS given in Charnes et al. (1978).

Axiom 1. Feasibility: All the observed production activities of any DMU j belong to PPS, i.e., $(X_j, Y_j) \in T$.

Axiom 2. Free disposability: $(X, Y) \in T$, $X' \geq X$, and $Y' \leq Y$ imply that $(X', Y') \in T$.

Axiom 3. Convexity: The production possibility set is convex.

Axiom 4. Cone-Convexity: $(X, Y) \in T$ and $k > 0$ imply $(kX, kY) \in T$.

Axiom 5. Minimum extrapolation: T is the intersection of all the productions $T' \in R_+^{m+s}$ that satisfy the above Axioms 1-4.

Based on the above Axioms 1-5, the production possibility set under constant-returns to scale (CRS) can then be mathematically formulated as follows.

$$T = \{(X, Y) | \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_i, \forall i \quad \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_r, \forall r \quad \lambda_j \geq 0, \forall j\} \tag{2.2}$$

In (2.2), λ_j denotes the intensity variable attached to DMU j. The PPS under the assumptions of increasing-, decreasing-, and variable-returns to scale (noted as IRS, DRS, and VRS, respectively) can be obtained by adding to (2.2) the constraints

$\sum_{j=1}^{n} \lambda_j \geq 1$, $\sum_{j=1}^{n} \lambda_j \leq 1$, and $\sum_{j=1}^{n} \lambda_j = 1$, respectively. The following Figure 2.2 gives the production possibility sets under different returns to scales with a simple numerical example with 3 single-input-single-output DMUs denoted as A, B, and C, respectively.

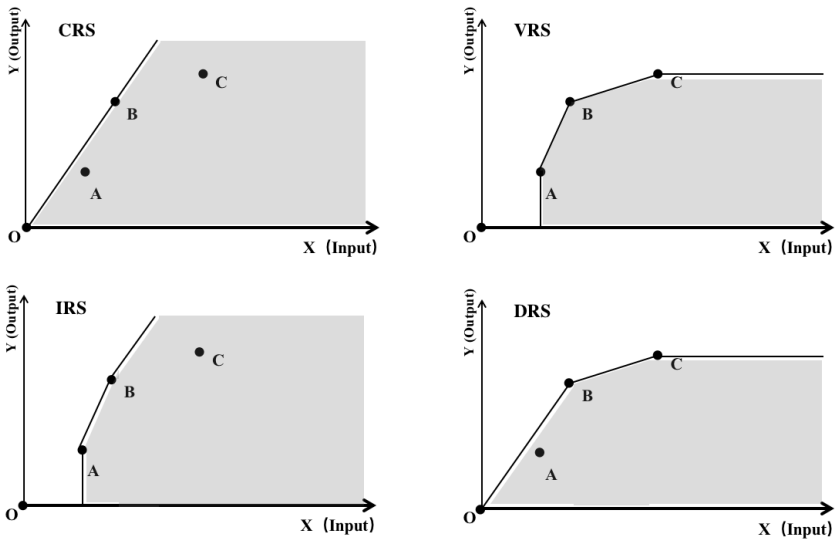

Figure 2.2 The PPS under different returns to scale

In Figure 2.2, the shaded parts show the production possibility sets. In the remainder of this study, we focus on the PPS under CRS, since cross-efficiency evaluation is usually based on the evaluation results generated by the Charnes-Cooper-Rhodes (CCR) model, while the CCR model is built under the CRS assumption.

Efficiency and Basic model

Charnes et al. (1978) give a definition of efficiency, called Pareto-Koopmans efficiency, which is described as follows.

Definition 2.3. (Pareto-Koopmans efficiency) A DMU $(X, Y) \in T$ is said to be fully efficient if there is no $(X', Y') \in T$ such that $x_i > x_i'$ for some i or $y_r < y_r'$ for some r.

When DMU d is under evaluation, based on the PPS discussed above, the first DEA model, called CCR model, given by Charnes et al. (1978) can be shown in its input-oriented envelopment format as follows.

$$
\begin{aligned}
\min \quad & E_d \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le E_d x_{id}, \forall i \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rd}, \forall r \\
& \lambda_j \ge 0, \forall j
\end{aligned}
\tag{2.3}
$$

The objective of model (2.3) is to minimize E_d, i.e., the equi-proportionate contraction of the inputs of DMU *d*. Actually, that is to project DMU *d* on the Pareto-efficient frontier to see how much its inputs can be reduced at maximum. The following Figure 2.3 shows a simple example with 6 two-input-one-output DMUs. The outputs of the DMUs are assumed to be the same. In the figure, we can see that when evaluating the efficiency of DMU E, we project it to E_1 on the frontier. Then we have the efficiency of DMU E as OE_1/OE.

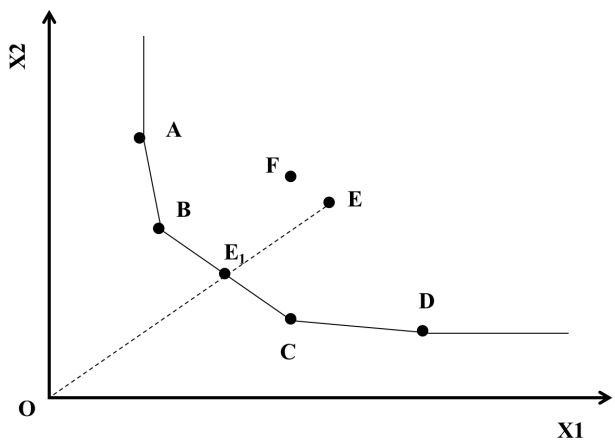

Figure 2.3 Efficiency measurement and frontier under CRS assumption

Let $(E_d^*, \boldsymbol{\lambda}^*)$ be an optimal solution of model (2.3). E_d^* is called the efficiency score (also called CCR efficiency) of DMU *d*. Then, we have the following definition of weak DEA-efficiency.

Definition 2.3. (Weak DEA-efficiency) A DMU *d* is said to be weakly DEA-efficient if $E_d^* = 1$.

It can be seen that in a weakly DEA-efficient DMU, it is impossible to reduce

(respectively increase) all its inputs (respectively outputs) simultaneously in the same proportion without decreasing (respectively increasing) its outputs (respectively inputs). The same conclusion for the output-oriented CCR in envelopment format can be obtained using the words between brackets. Note that if a DMU is Pareto-Koopmans efficient, it must be weakly DEA-efficient. However, a weakly DEA-efficient DMU is not necessarily Pareto-Koopmans efficient. To define a Pareto-Koopmans efficient DMU, the following model (2.4) is used.

$$\begin{aligned} \text{Max } & E_d - \epsilon\left(\sum_{i=1}^{m} s_i^+ + \sum_{r=1}^{s} s_r^-\right) \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} = E_d x_{id} - s_i^+, \forall i \\ & \sum_{j=1}^{n} \lambda_j y_{rj} = y_{rd} + s_r^-, \forall r \\ & \lambda_j \geq 0, \forall j \\ & s_i^+, s_r^- \geq 0, \forall i, r \end{aligned} \tag{2.4}$$

In model (2.4), ϵ is a small-enough positive value. $s_i^+, s_r^-, \forall i, r$ are slacks corresponding to the inputs and outputs, respectively. It can be seen that this model contains two goals. The primary goal is to minimize variable E_d to see how much the inputs can be simultaneously contracted at most. Then, the secondary goal is to minimize the sum of the slacks to see whether it is possible to reduce some inputs or increase some outputs. Model (2.4) has the ability to define Pareto-Koopmans efficient, also called strongly DEA-efficient, DMUs.

Let $(E_d^*, \boldsymbol{\lambda}^*, \boldsymbol{s}^{+*}, \boldsymbol{s}^{-*})$ be an optimal solution of model (2.4). We then have the following Definition 2.4.

Definition 2.4. (Strong DEA-efficiency) A DMU is said to be strongly DEA-efficient if $E_d^* = 1$, $s_i^{+*} = 0, \forall i$ and $s_r^{-*} = 0, \forall r$.

Strongly DEA-efficient DMUs are those that are Pareto-Koopmans efficient. These DMUs are located on the Pareto-efficient frontier. For instance, in Figure 2.3, DMUs A, B, C, and D are strongly DEA-efficient. They are located on the Pareto-efficient frontier.

The dual model of model (2.3) is shown as model (2.5), which is also called the

multiplicative-form output-oriented CCR model.

$$\begin{aligned} E_d^* = \quad & \max U \cdot Y_d \\ s.t. \quad & W \cdot X_d = 1 \\ & U \cdot Y_j - W \cdot X_j \leq 0, \forall j \\ & U, W \geq 0 \end{aligned} \tag{2.5}$$

In model (2.5), the first constraint is used to avoid trivial solutions. The second group of constraints is to ensure that all cross-efficiencies, including DMU *d's* self-evaluated efficiency is no larger than 1. In this multiplicative form, the ratio of the total weighted output to the total weighted input is used to measure the efficiency of each DMU. In this thesis, we are mainly concerned with the multiplicative model, since we consider the peer-evaluation mechanism and cross-efficiency evaluation in which optimal weights of the DMUs are used for efficiency evaluation.

2.1.2 DEA cross-efficiency evaluation and non-uniqueness of optimal weights

From the analysis in the previous paragraphs, we know that the CCR model can only discriminate the DMUs into weakly DEA-efficient ones and inefficient ones. It cannot make any further discrimination among the weakly DEA-efficient DMUs, since they all get an efficiency score of 1. This will make it unsuitable in situations where a decision maker needs to choose the best one among all the DMUs. For instance, an investor needs to select the best project proposal from a group of candidates to make investment.

To address this issue, Sexton et al. (1986) proposed to use DEA cross-efficiency evaluation. Let (U_d^*, W_d^*) be an optimal solution to model (2.5), which are actually the most favorable weights of DMU d, attached to the outputs and the inputs, respectively. The ***cross efficiency***, denoted as E_{dj}, of any DMU j evaluated by the most-favorable weights of DMU d, can be calculated as follows.

$$E_{dj} = \frac{U_d^* \cdot Y_j}{W_d^* \cdot X_j} \tag{2.7}$$

Note that $E_{d,d} = E_d^*$.

Then, the ***cross-efficiency score***, denoted as E_j^c, of any DMU j can be calculated

as follows.

$$E_j^c = \frac{1}{n}\sum_{d=1}^{n} E_{dj} \tag{2.8}$$

E_j^c is also called the original cross-efficiency score of DMU j. It is simply the average of the cross efficiencies evaluated by all the DMUs, including DMU j itself.

However, as mentioned above, the optimal solution to model (2.5) may not be unique, which will lead to non-uniqueness of the result of DEA cross-efficiency evaluation. Specifically, different selections of optimal weights for the DMUs will generate different evaluation results. This is called non-uniqueness of optimal weights in DEA cross-efficiency evaluation. Mathematically, for each DMU d whose optimal weights from the CCR model are not unique, we need to choose a suitable set of weights for it in the following weight possibility set WS_d.

$$\begin{aligned} WS_d = \{(U,W) | & U \cdot Y_d = E_d^* \\ & W \cdot X_d = 1 \\ & U \cdot Y_j - W \cdot X_j \leq 0, \forall j \\ & U, W \geq 0\} \end{aligned} \tag{2.9}$$

Now, we use a small example taken from Liang et al. (2008a) to illustrate the issue. The example contains 5 DMUs, each with 3 inputs and 2 outputs. The raw data of this numerical example is shown in the following Table 2.1.

Table 2.1 Raw data of the numerical example

DMUs	Inputs			Outputs	
	X1	X2	X3	Y1	Y2
DMU1	7	7	7	4	4
DMU2	5	9	7	7	7
DMU3	4	6	5	5	7
DMU4	5	9	8	6	2
DMU5	6	8	5	3	6

We evaluate the DMUs using the CCR model (2.5) and the arbitrary cross-efficiency calculation (2.7). The results are listed in columns 2 and 3 in Table 2.2,

respectively. We also generate two sets of different cross-efficiency scores for the DMUs, which are listed in columns 4 and 5, respectively. These two sets of cross-efficiency scores are generated by using the benevolent and aggressive cross-efficiency evaluation models (Doyle and Green, 1994) which we will give more details in Chapter 3.

From the cross-efficiency evaluation results listed in Table 2.2, firstly, we can see that the CCR model evaluates DMUs 2 and 3 as weakly DEA-efficient. It cannot make any further discrimination between these two DMUs.

Then, we observe that different selections of optimal weights lead to different cross-efficiency evaluation results. This raises the problem that the decision makers do not know which set of efficiency scores they should refer to. This non-uniqueness of optimal weights is one of the main issues discussed in DEA cross-efficiency evaluation.

Table 2.2 The results of the numerical example

DMUs	CCR efficiency	Arbitrary	Benevolent	Aggressive
DMU1	0.6857	0.4743	0.5616	0.4473
DMU2	1.0000	0.8793	0.9295	0.8895
DMU3	1.0000	0.9856	1.0000	0.9571
DMU4	0.8571	0.5554	0.6671	0.5843
DMU5	0.8571	0.5587	0.5871	0.5186

Additionally, through comparison, we can see that the cross-efficiency scores of each DMU generated by the aggressive and arbitrary strategies are smaller than their counterparts obtained with the benevolent strategy. That is to say that the two former sets of cross-efficiency scores are dominated by the latter one. Actually, the DMUs should be willing to accept cross-efficiency evaluation results that provide them with higher cross-efficiency scores. However, using different weight selection strategies, we generate the results shown in columns 2 and 4 which may not gain the favor of the DMUs. Therefore, in addition to guaranteeing the uniqueness of the optimal set of cross-efficiency scores, we also need build suitable methods and theories to improve the acceptance of the evaluation result by the DMUs.

2.2 Research on DEA-based ranking methods

As we mentioned above, the traditional DEA models (CCR and BCC models) have the shortcoming in discriminating among the weakly DEA-efficient DMUs. To overcome this problem, scholars have proposed some guidelines and extended DEA-based methods.

Selecting a suitable number of references: Cooper et al. (2007) suggested that the number of DMUs (references) should be no smaller than the maximum between $m * s$ and $3 * (m + s)$ if a good discrimination is deemed to be achieved in the evaluation results, where m and s are the numbers of inputs and outputs of the DMUs, respectively.

The super-efficiency evaluation method: Andersen and Petersen (1993) proposed a super-efficiency evaluation model which can be used to further discriminate among strongly DEA-efficient DMUs. In this model, the DMU under evaluation will be removed from the reference set, which will result in situations where the strongly DEA-efficient DMUs obtain efficiency scores larger than 1. So, the strongly DEA-efficient DMUs can be further discriminated. However, this method still has other shortcomings, such as, infeasibility and usefulness in discriminating the weakly (but not strongly) DEA-efficient DMUs. To address these issues, some extensions and discussions are provided. More details can be seen in Zhu (1996), Zhu (1999), Khodabakshi (2007), Jahanshahloo et al. (2011a), Chen et al. (2013), Du et al. (2015), Chu et al. (2016).

DEA common-weight evaluation method: Cook et al. (1990) and Roll et al. (1991) proposed the DEA common-weight evaluation method. Unlike the traditional DEA method in which each DMU uses its own most favorable weights for efficiency evaluation, the DEA common-weight evaluation method uses a set of weights which is common to all the DMUs. The main problem facing this method is how to determine the set of common weights for efficiency evaluation. Further work on DEA common-weight evaluation can be seen in Kao and Hung (2005), Zohrehbandian et al. (2010), Ramezani-Tarkhorani et al. (2014), Sun et al. (2013), and Wu et al. (2016a).

Benchmark ranking method: The idea was given by Sueyoshi (1990), Lu & Lo (2009), Sinuany-Stern et al. (1994). It is used for discriminating the strongly DEA-efficient DMUs. The main idea is to see how often the DMUs are regarded as benchmarks. The more often a DMU is regarded as a benchmark, the more approved it

is by the other DMUs; thus, the better ranking it gets.

DEA cross-efficiency evaluation: Sexton et al. (1986) proposed to use cross-efficiency evaluation to rank the DMUs. The main idea is to use a peer-evaluated mechanism to replace the self-evaluated mechanism. We will describe this method in detail in the next subsection.

Besides the above-discussed methods, there are other DEA-based ranking methods such as the multi-criteria decision-making methodologies (Li & Reeves, 1999; Strassert and Prato, 2002; Wang and Jiang, 2012; Mousseau et al.2018; Bisdorff et al. 2015), and the context-dependent DEA method (Seiford & Zhu, 2003; Chen et al., 2005).

2.3 DEA cross-efficiency evaluation

In this section, we discuss in detail about the current research on DEA cross-efficiency evaluation, from the following perspectives: the classic methods addressing the non-uniqueness of optimal weights, extended cross-efficiency evaluation models, research on cross-efficiency aggregation, and applications of DEA cross-efficiency evaluation.

2.3.1 Secondary goal models

As we mentioned in the introduction, Doyle and Green (1994) pointed out that the set of optimal weights generated by the CCR model for a DMU may not be unique, which in turn causes situations where different selections of optimal weights generate different cross-efficiency scores for the DMUs. This is called the non-uniqueness of optimal weights. Up to now, most of the studies on DEA cross-efficiency evaluation focus on solving this problem. To address this problem, Sexton et al. (1986) further proposed to use secondary goal models. Inspired by this idea, many secondary goal models have been proposed for optimal weights selection.

Secondary goal models to address the issue of non-uniqueness of optimal weights are based on two principles. Firstly, each DMU selects a single set of optimal weights for both self-evaluation and peer-evaluation (*one weight set* for short). Secondly, the optimal set of weights selected by each DMU will maintain its self-evaluated efficiency at the CCR efficiency level (*efficiency optimality* for short). Under these two principles,

secondary goals are proposed to limit the non-uniqueness of optimal weights or to generate a unique set of optimal weights for each DMU. In the following, we divide the secondary goal models into several groups and discuss them respectively.

The benevolent and aggressive models: Doyle and Green (1994) proposed two of the most famous secondary goal models. They are called benevolent and aggressive cross-efficiency evaluation models, respectively. The core idea of aggressive (respectively benevolent) model is to select a set of optimal weights by making the efficiencies of the other DMUs as small (respectively large) as possible under the two principles given above. Liang et al. (2008a) extended the benevolent and aggressive models proposed by Doyle and Green (1994). They use the efficiency score of 1 as the cross-efficiency target of the other DMUs when selecting the optimal weights for a specific DMU. They then proposed some alternative secondary goal models and gave specific application environments for each of them. Similar ideas also appeared in Wang and Chin (2010a) which presents other secondary goal models in which the cross-efficiency target (efficiency score of 1) of each DMU in Liang et al. (2008) model is replaced by its CCR efficiency. The aggressive and benevolent models were further investigated by Wu et al. (2016b). They pointed out that cross-efficiency targets used in traditional benevolent and aggressive models are not always reachable for all DMUs. Accordingly, they proposed a model to identify reachable desirable and undesirable cross-efficiency targets for the DMUs. They then proposed new benevolent and aggressive models using the new identified cross-efficiency targets of the DMUs and considered the DMUs' willingness to get close to the desirable cross-efficiency targets while avoiding the undesirable ones. Another work based on the benevolent and aggressive strategies is by Lim et al. (2012). When selecting optimal weights for a DMU, their benevolent model maximizes the minimum cross efficiency of the others, while the aggressive model tries to minimize the maximum cross efficiencies of the others.

The neutral models: This kind of models only focus on the DMU under evaluation without caring about the impact on the cross efficiencies of the others. Wang and Chin (2010b) proposed a neutral model in which the efficiency of each output of the DMU is maximized while maintaining the whole DMU's efficiency at the maximum (CCR) efficiency level. Wang et al. (2011a) presented some other neutral cross-efficiency evaluation models based on the ideal and anti-ideal DMUs. In their models, when

selecting optimal weights for a DMU, the secondary goals maximize its distance from the anti-ideal DMU, minimize the distance from the ideal one, and maximize the distance between the ideal DMU and the anti-ideal DMU. There are also studies that investigate the ranking ranges of the DMUs. For example, Alcaraz et al. (2013) proposed two models to identify the best and the worst ranking positions of each DMU, respectively. The DMUs are then ranked by analyzing the ranking ranges of the DMUs. A similar idea can also be seen in Yang et al. (2012).

Weight-balanced models: Another problem raised in DEA cross-efficiency evaluation, because of the non-uniqueness of optimal weights and total weight flexibility in weights selection, is the use of unrealistic weights. For instance, the optimal weights selected for a DMU might contain a lot of zero weights. To address this problem, scholars proposed alternative secondary goal models in order to avoid the selection of zero optimal weights. Ramón et al. (2010) proposed the concept of similarity among the input and output weights of each DMU. Then, they proposed a model to identify the maximized minimum similarity among the DMUs' optimal weights considering only the strongly DEA-efficient DMUs. A model is then given to reselect optimal weights for those DMUs that are not strongly DEA-efficient. The proposed approach avoids zero weights in DEA cross-efficiency evaluation. Ramón et al. (2011) proposed another approach for avoiding zero optimal weights while reducing the differences between the optimal weight sets of the DMUs as much as possible. A similar idea can also be seen in Wang and Jiang (2012). Wang et al. (2011b) and Wu et al. (2012a) proposed a weight-balanced model in which the authors avoided zero optimal weights by maximizing the minimum weighted input or output. However, unlike Ramón et al. (2010) and Ramón et al. (2011), Wang et al. (2011b) and Wu et al. (2012)'s methods cannot theoretically guarantee the absence of zero optimal weights. Nevertheless, their models have the ability to reduce zero optimal weights in practical applications.

Other secondary goal models: Some other studies use alternative secondary goals. For instance, Wu et al. (2009b), Contreras (2012), Maddahi et al. (2014), and Liu et al. (2017) proposed secondary goal models by using the goal of optimizing the ranking position of the DMU under evaluation. Jahanshahloo et al. (2011b) presented a secondary goal model in which the symmetric technique is incorporated and the secondary goal is to select a set of symmetric weights for each DMU. More recently,

Wu et al. (2016c) proposed a new DEA cross-efficiency evaluation approach in which the secondary goal is to maximize the DMUs' satisfaction degrees on the selected optimal weights.

Of course, different secondary goal models have different application scenarios. We will discuss this later in Chapter 3 when presenting our models and comparing them with these traditional models.

2.3.2 Extended cross-efficiency evaluation models

The studies introduced above generally hold the two classic principles: one weight set and efficiency optimality. Some other studies partially relax these principles in order to generate cross-efficiency evaluation results with more special properties. For example, Liang et al. (2008a) proposed to incorporate game theory in DEA cross-efficiency evaluation. The DMUs are seen as players in a non-cooperative game who are competing each other in the evaluation to obtain higher cross-efficiencies. They have the authority to use different evaluation criteria with respect to different players (DMUs). Therefore, their method allows each DMU to use different weight sets to calculate cross efficiencies for different DMUs. Additionally, they also allow each DMU to reduce its self-evaluated efficiency. Then, they proposed a DEA game cross-efficiency evaluation model and developed a corresponding algorithm. Their method can finally generate a set of cross-efficiency scores that constitute a Nash equilibrium solution. The DEA game cross-efficiency evaluation approach was extended to a form of variable-returns to scale by Wu et al. (2009). Cook and Zhu (2014) proposed another method. They also break the *one weight set* principle. They proposed a units-invariant multiplicative DEA model. Their model can directly generate DMUs' maximum cross-efficiency scores, which means there is no need to select a unique set of optimal weights.

Wu et al. (2016d) relaxed the *efficiency optimality* principle. They proposed a cross-efficiency evaluation method based on Pareto improvement. Their method can generate cross-efficiency scores that constitute a Pareto optimal solution. Additionally, in some special cases, their method unifies self-evaluation, peer-evaluation, and common-weight evaluation in DEA cross-efficiency evaluation, which makes the evaluation results easier to be accepted by all the DMUs.

2.3.3 Cross-efficiency aggregation

In traditional cross-efficiency evaluation, each DMU's cross-efficiency score is calculated by averaging its self-evaluated efficiency (CCR efficiency) and the peer-evaluated efficiencies. However, as some scholars pointed out, calculating each cross-efficiency score by simply averaging the efficiencies neglects the DMUs' preferences on its CCR efficiency and peer-evaluated efficiencies. Additionally, the average cross-efficiency scores generally cannot constitute a Pareto optimal solution (Wu et al. 2008). To address this issue, scholars proposed alternative cross-efficiency aggregation methods.

Wu et al. (2008) considered the DMUs as players in a cooperative game. Then, they calculated the final cross-efficiency scores based on the nucleolus solution of the game. By also considering the DMUs as players in a cooperative game, Wu et al. (2009b) proposed to aggregate the cross efficiencies of the DMUs using the Shapley value of the game. Wu et al. (2011) and Wu et al. (2013) incorporated the TOPSIS technique to determine the final weights to aggregate cross efficiency scores for the DMUs. Wu et al. (2011) and Wu et al. (2012b) focused on the aggregation process of the cross-efficiency matrix. They further used the Shannon entropy for cross-efficiency aggregation. Considering the decision maker's optimism level to the best relative efficiencies, Wang and Chin (2011) proposed to use the ordered weighted averaging operator to determine the aggregation weights for cross-efficiency aggregation. José and Sirvent (2012) selects cross-efficiency aggregation weights considering the disequilibrium in optimal weight sets of the DMUs. The approach has the following ability: a cross efficiency is obtained with the more zero optimal weights, the lower aggregation weight is attached to it. Yang et al. (2013) proposed an evidential-reasoning approach for cross-efficiency aggregation: They provided a new procedure for aggregating the cross efficiencies of each DMU based on the distributed assessment framework and the evidence combination rule of Dempster–Shafer (D–S) evidence theory (Sentz and Ferson, 2002).

2.3.4 Application of DEA cross-efficiency evaluation

After the DEA cross-efficiency evaluation was proposed, it has been utilized in many applications. For instance, Green et al. (1996) proposed to use DEA cross-

efficiency evaluation to R&D project selection. In this instance, the proposals are evaluated and ranked using the cross-efficiency scores. Then, they are selected according to the ranking position and under the constraint of total budget. Alternative studies of applying DEA for R&D project selection can be seen in Liang et al. (2008b), Wu et al. (2016d). Wu et al. (2009a) and Wu et al. (2009c) proposed to use DEA cross-efficiency evaluation for performance evaluation and benchmarking of countries in Summer Olympics. They mainly extended DEA cross-efficiency evaluation by considering the ordered weights for the importance of different medals. Yu et al. (2010) proposed to use DEA cross-efficiency evaluation for analyzing the supply chain performance with different information-sharing scenarios. Falagario et al. (2012) proposed to use DEA cross-efficiency evaluation for top supplier selection in public procurement tenders. In this application, the suppliers with multiple performance metrics are evaluated by the DEA cross-efficiency evaluation method. Then, the best supplier is selected. Similar applications can be seen in the advanced manufacturing technology selection (Baker and Talluri, 1997; Wu et al., 2016b). More recently, Lim et al. (2014) used DEA cross-efficiency evaluation for portfolio selection. They pointed out that the DEA cross-efficiency evaluation method will generally select a portfolio in which the selected funds (DMUs) are relatively robust to the risk of change in weights. Apart from using DEA cross-efficiency evaluation for ranking and benchmarking the DMUs, there are also other applications. For instance, Du et al. (2014) extended DEA cross-efficiency evaluation for fixed cost allocation and resource allocation.

2.4 Research gaps

From the above analysis, we identify the following research gaps.

Firstly, in the traditional benevolent and aggressive models, the efficiency targets (the CCR efficiencies or the ideal targets 1) is not always reachable for the DMUs. Additionally, the traditional benevolent and aggressive models only consider the desirable cross-efficiency targets as referenced efficiencies for all DMUs while neglecting the fact that the undesirable cross-efficiency targets are also important indicators that the DMUs need to consider (Baumeister et al., 2001; Wang and Chin, 2011a; Dotoli et al., 2015).

Secondly, although alternative secondary goal models have been proposed to

reduce the non-uniqueness of optimal weights in DEA cross-efficiency evaluation, there still lack studies theoretically guaranteeing the uniqueness of optimal weights or uniqueness of the final evaluation result.

Additionally, there are no studies considering the DMUs' acceptance, satisfaction degree, or preference on evaluation result. For instance, the final set of cross-efficiency scores is generally not Pareto optimal, which makes the evaluation unconvincing, especially for those DMUs whose efficiency scores can be improved.

Chapter 3 Extended secondary goal models for weight selection in DEA cross-efficiency evaluation *

To reduce the non-uniqueness of optimal weights in DEA cross-efficiency evaluation, Doyle and Green (1994), Liang et al. (2008a), and Wang and Chin (2010a) proposed to use the benevolent and aggressive models. However, the ideal targets given for the DMUs in the traditional benevolent and aggressive models are not always reachable. Additionally, these traditional models only consider the desirable cross-efficiency targets (1 or the CCR efficiency) as reference efficiencies for all DMUs. However, as Baumeister et al. (2001), Wang and Chin (2011a), and Dotoli et al. (2015) rightly pointed out, undesirable targets are also important indicators that the DMUs need to consider.

Aiming at addressing these issues, in this chapter, we first incorporate a target identification model to get reachable targets for all DMUs. Then, several secondary goal models are proposed for weights selection considering both desirable and undesirable cross-efficiency targets of the DMUs. Compared with the traditional secondary goal models, cross-efficiency targets are improved in the sense that all targets are always reachable for the DMUs. In addition, the proposed models consider the DMUs' willingness to get close to their desirable cross-efficiency targets and to avoid their undesirable ones simultaneously while the traditional secondary goal models considered only the ideal targets. Finally, our models are compared with the traditional methods on a numerical example: efficiency evaluation of six nursing homes.

The rest of this chapter is organized as follows. Section 3.1 briefly discusses the traditional benevolent and aggressive models. Section 3.2 describes the target identification model. Section 3.3 proposes new benevolent and aggressive models and a neutral model. Further, in Section 3.4, a numerical example and the application of R&D project selection are provided. Finally, Section 3.5 concludes this chapter.

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* This chapter is primarily referenced from: Jie Wu, Junfei Chu, Jiasen Sun, Qingyuan Zhu, and Liang Liang. (2016). Extended secondary goal models for weights selection in DEA cross-efficiency evaluation. *Computers & Industrial Engineering*, *93*, 143-151.

3.1 The traditional benevolent and aggressive models

To reduce the non-uniqueness of optimal weights in DEA cross-efficiency, Doyle and Green (1994) proposed to use secondary goal models. They further proposed the famous benevolent and aggressive DEA cross-efficiency evaluation models, which are shown in (3.1) and (3.2) for the selection of weights U and W for DMU d, respectively.

$$\begin{aligned} \max \quad & U \cdot \sum_{j=1, j\neq d}^{n} Y_j \\ s.t. \quad & W \cdot \sum_{j=1, j\neq d}^{n} Y_j = 1 \\ & U \cdot Y_d - E_d^* \times W \cdot X_d = 0 \\ & U \cdot Y_j - W \cdot X_j \leq 0, \forall j \\ & U, W \geq 0 \end{aligned} \tag{3.1}$$

and

$$\begin{aligned} \min \quad & U \cdot \sum_{j=1, j\neq d}^{n} Y_j \\ s.t. \quad & the\ same\ as\ those\ in\ model\ (3.1) \end{aligned} \tag{3.2}$$

In models (3.1) and (3.2), E_d^* is the CCR efficiency of DMU *d* obtained by model (2.5). It can be seen from model (3.1) (model (3.2)) that when selecting weights for DMU *d*, the model strives to maximize (minimize) the cross efficiency of the aggregation of the other DMUs while maintaining DMU *d*'s efficiency at the optimal CCR efficiency level. This is why the model is called benevolent (aggressive) cross-efficiency evaluation model. Based on the above models (3.1) and (3.2), Liang et al. (2008a) proposed a new pair of benevolent and aggressive models which are shown in models (3.3) and (3.4), respectively.

$$\begin{aligned} min \quad & \sum_{j=1}^{n} s_j \\ s.t. \quad & W \cdot X_d = 1 \\ & U \cdot Y_d = E_d^* \\ & U \cdot Y_j - W \cdot X_j + s_j = 0, \forall j \end{aligned} \tag{3.3}$$

$$U, W \geq 0$$
$$s_j \geq 0, \forall j$$

and

$$\begin{aligned} max \quad & \sum_{j=1}^{n} s_j \\ s.t. \quad & the\ same\ as\ those\ in\ model\ (3.3) \end{aligned} \tag{3.4}$$

In models (3.3) and (3.4), $s_j = W \cdot X_j - U \cdot Y_j,\ \forall j$ denotes DMU j's deviation from its ideal efficiency score 1. It can be seen that, the smaller s_j is, the closer the efficiency of DMU j is to 1. In model (3.3) (model (3.4)), when DMU d selects a set of optimal weights, it keeps its efficiency at the CCR efficiency level. Then, it strives to make the sum of other DMUs' deviations from their ideal efficiency 1 as small (large) as possible.

However, Wang and Chin (2010a) observed that the ideal efficiency score 1 is not realizable for the DEA-inefficient DMUs. They further improved the models of Liang, et al. (2008a) by replacing the cross-efficiency target from the ideal point 1 to the CCR efficiency. The improved benevolent and aggressive models are shown in models (3.5) and (3.6), respectively.

$$\begin{aligned} min \quad & \sum_{j=1}^{n} s_j \\ s.t. \quad & U \cdot Y_d - E_d^* \times X_d = 0 \\ & U \cdot \sum_{j=1}^{n} Y_j + W \cdot \sum_{j=1}^{n} X_j = n \\ & U \cdot Y_j - E_j^* \times W \cdot X_j + s_j = 0, \forall j \\ & U, W \geq 0 \\ & s_j \geq 0, \forall j \end{aligned} \tag{3.5}$$

and

$$\begin{aligned} min \quad & \sum_{j=1}^{n} s_j \\ s.t. \quad & the\ same\ as\ those\ in\ model\ (3.5) \end{aligned} \tag{3.6}$$

In models (3.5) and (3.6), $s_j = E_j^* \times W \cdot X_j - U \cdot Y_j,\ \forall j$ defines DMU j's

deviation from its target efficiency E_j^*. Wang and Chin (2010a) also made another change in their models by adding a new constraint $U \cdot \sum_{j=1}^{n} Y_j + W \cdot \sum_{j=1}^{n} X_j = n$ to avoid trivial solutions. They consider that this new constraint is fixed and does not vary from one DMU to another.

As can be seen from the above discussions, the traditional benevolent and aggressive models suffer two main deficiencies. Firstly, the cross-efficiency targets (the ideal point 1 or the CCR efficiency) in their models are not always reachable for all DMUs (a fact that will be proved in Theorem 3.1 in Section 3.2). Secondly, the traditional models consider only the desirable targets (1 or CCR efficiency scores) as reference efficiencies for all the DMUs, which ignores the DMUs' unwillingness to get close to their undesirable targets.

3.2 A target identification model

In the models of Liang, et al. (2008a) and Wang and Chin (2010a), the target efficiencies (the ideal point 1 and CCR efficiency) are not always reachable. In this section, we propose a target identification model to calculate the desirable and undesirable targets for each DMU. Compared with traditional target efficiencies, the generated desirable and undesirable targets are always reachable and realizable for the DMUs in the cross-efficiency evaluation. The proposed model is shown as (3.7).

$$\begin{aligned} E_{dj}^{max}/E_{dj}^{min} = \quad & \max U \cdot Y_j \ / \min U \cdot Y_j \\ s.t. \quad & W \cdot X_j = 1 \\ & U \cdot Y_d - E_d^* \times W \cdot X_d = 0 \\ & U \cdot Y_k - W \cdot X_k \leq 0, \forall k \\ & U, W \geq 0 \end{aligned} \tag{3.7}$$

Model (3.7) calculates the maximum and minimum cross efficiencies of DMU j corresponding to DMU d, which are denoted as E_{dj}^{max} and E_{dj}^{min}, respectively. Here, we give the following Definitions 3.1 and 3.2 and Theorem 3.1.

Definition 3.1. E_{dj}^{max} is defined as the desirable cross-efficiency target for DMU j relative to DMU d.

Definition 3.2. E_{dj}^{min} is defined as the undesirable cross-efficiency target for DMU j

relative to DMU d.

Theorem 3.1. For any DMU j, we have $E_j^* \geq E_{dj}^{max}, \forall d$.

Proof. By definition, for any DMU j, E_j^* can be obtained by solving model (3.7) with the second constraint removed and by maximizing the objective function. In other words, calculating E_j^* consists of solving a relaxed model of (3.7). As a consequence, $E_{dj}^{max} \leq E_j^*, \forall d$. Q.E.D.

From Theorem 3.1, we know that the ideal point 1 or CCR efficiency may not always be a reachable target for the DMUs if the weights selected must keep the efficiency of a corresponding DMU at its optimal level. Specifically, when $E_{dj}^{max} < E_j^*$ and the unreachable situation may appear.

3.3 New weight selection models

In this section, we propose some new benevolent and aggressive models and a neutral model based on the desirable and undesirable cross-efficiency targets discussed in the above section.

3.2.1 New benevolent and aggressive models

In the traditional secondary goal models, the weights are selected only with the consideration that the efficiencies of the DMUs are as close to their desirable targets as possible, ignoring the DMUs' unwillingness to get close to their undesirable targets. What is more, as mentioned above, the desirable targets (1 or the CCR efficiency) in traditional models are not always realizable for the DMUs. In order to overcome these issues, we propose the following weights selection model (3.8) based on the desirable and undesirable cross-efficiency targets ($E_{dj}^{max}$and E_{dj}^{min}).

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} (s_j - \varphi_j) \\ s.t. \quad & U \cdot Y_d = E_d^* \\ & W \cdot X_d = 1 \\ & U \cdot Y_j - E_{dj}^{max} \times W \cdot X_j + s_j = 0, \forall j, j \neq d \\ & U \cdot Y_j - E_{dj}^{min} \times W \cdot X_j - \varphi_j = 0, \forall j, j \neq d \end{aligned} \tag{3.8}$$

$$U, W \geq 0$$
$$s_j, \varphi_j \geq 0, \forall j$$

In model (3.8), E_d^* is the CCR efficiency of DMU d. E_{dj}^{max} and E_{dj}^{min} are respectively the desirable and undesirable cross-efficiency targets. s_j denotes the deviation of DMU j from its desirable cross-efficiency target and φ_j represents the distance of DMU j from its undesirable cross-efficiency target. Note that we omit the constraints $U \cdot Y_j - W \cdot X_j \leq 0, \forall j$ in model (3.8). This is because these constraints become redundant due to the existence of $U \cdot Y_j - E_{dj}^{max} \times W \cdot X_j + s_j = 0, \forall j$ and $s_j \geq 0, \forall j$. As we can see from model (3.8), the first and second constraints ensure that when selecting weights for a particular DMU d, its efficiency is guaranteed to be at its CCR efficiency level. The third and fourth constraint groups in this model mean that the cross efficiency of each DMU j with respect to DMU d should be constrained between its desirable cross-efficiency target E_{dj}^{max} and undesirable cross-efficiency target E_{dj}^{min}. The objective function of the model shows that the model strives to make the other DMUs' deviations from the desirable cross-efficiency targets as small as possible and the distances from the undesirable cross-efficiency targets as large as possible, when selecting weights for a given DMU d. As a result, model (3.8) makes the cross efficiencies of the DMUs close to their desirable cross-efficiency targets and away from their undesirable cross-efficiency targets.

To reduce large differences between the cross efficiencies of the DMUs determined by the weights of DMU d, we minimize the maximum difference between deviations from desirable and undesirable cross-efficiency targets of the DMUs. The model is shown as the following model (3.9).

$$\begin{aligned}
\min \quad & \beta \\
s.t. \quad & U \cdot Y_d = E_d^* \\
& W \cdot X_d = 1 \\
& U \cdot Y_j - E_{dj}^{max} \times W \cdot X_j + s_j = 0, \forall j, j \neq d \\
& U \cdot Y_j - E_{dj}^{min} \times W \cdot X_j - \varphi_j = 0, \forall j, j \neq d \\
& s_j - \varphi_j \leq \beta, \forall j, j \neq d \\
& U, W \geq 0 \\
& s_j, \varphi_j \geq 0, \forall j
\end{aligned} \tag{3.9}$$

Model (3.9) aims at minimizing the maximum difference between the deviation from desirable cross-efficiency target and the distance from the undesirable cross-efficiency target among the DMUs. Actually, the original objective function of this model should be written as $\min \max_{j \neq d, \forall j} (s_j - \varphi_j)$, but doing so would lead to a nonlinear program. To solve this problem, we let β to represent $\max_{j \neq d, \forall j} (s_j - \varphi_j)$ and use the constraints $s_j - \varphi_j \leq \beta, \forall j, j \neq d$ to transform the model into an equivalent linear program shown in model (3.9). Observing from the efficiency point of view, it is easy to see that model (3.9) intends to find out the set of weights that maximizes the minimum cross efficiency of the DMUs relative to DMU d. In doing so, the selected weights have the potential to reduce large differences between cross efficiencies of all the DMUs. Specifically, in order to show the best possible efficiency level of the worst performer, the cross efficiencies of other DMUs (better performers) may be decreased, thus resulting in situations where the differences between the DMUs' cross efficiencies are smaller than when model (3.8) is used.

In models (3.8) and (3.9), DMU d selects its weights to maximize the other DMUs' cross efficiencies while keeping its own efficiency at its optimal level. Therefore, models (3.8) and (3.9) are benevolent. Model (3.8) can be transformed into an aggressive model by maximizing the objective function as in model (3.10).

$$\begin{aligned} \max \quad & \sum_{j=1}^{n} (s_j - \varphi_j) \\ s.t. \quad & \textit{the same as those in model } (3.8) \end{aligned} \tag{3.10}$$

The aggressive model corresponding to model (3.9) is shown as the following model (3.11).

$$\begin{aligned} \max \quad & \beta \\ s.t. \quad & U \cdot Y_d = E_d^* \\ & W \cdot X_d = 1 \\ & U \cdot Y_j - E_{dj}^{max} \times W \cdot X_j + s_j = 0, \forall j, j \neq d \\ & U \cdot Y_j - E_{dj}^{min} \times W \cdot X_j - \varphi_j = 0, \forall j, j \neq d \\ & s_j - \varphi_j \geq \beta, \forall j, j \neq d \\ & U, W \geq 0 \\ & s_j, \varphi_j \geq 0, \forall j \end{aligned} \tag{3.11}$$

3.2.2 A neutral model

The above models consider the aggressive and benevolent strategies of the DMUs. But sometimes the DMU under evaluation does not care to maximize or minimize the cross efficiencies of the others. We define this situation as neutral strategy when each DMU optimizes its self-evaluated efficiency. Considering from the neutral point of view, we propose the following neutral secondary model (3.12).

$$\begin{aligned} \min \quad & \eta \\ s.t. \quad & U \cdot Y_d = E_d^* \\ & W \cdot X_d = 1 \\ & U \cdot Y_j - E_{dj}^{max} \times W \cdot X_j + s_j = 0, \forall j, j \neq d \\ & U \cdot Y_j - E_{dj}^{min} \times W \cdot X_j - \varphi_j = 0, \forall j, j \neq d \\ & s_j - \varphi_j \leq \eta, \forall j, j \neq d \\ & s_j - \varphi_j \geq -\eta, \forall j, j \neq d \\ & U, W \geq 0 \\ & s_j, \varphi_j \geq 0, \forall j \end{aligned} \tag{3.12}$$

In model (3.12), we minimize the maximum $|s_j - \varphi_j|, \forall j$, i.e., we try to make the values of $|s_j - \varphi_j|, \forall j$ as close to zero as possible. The optimal weights for DMU d by this model are chosen to make the efficiency of DMU j, $\forall j$ as close as possible to the midpoint between E_{dj}^{max} and E_{dj}^{min} (which is exemplified in **Theorem 3.2**). Therefore, we regard it as a neutral secondary model.

Theorem 3.2. Let E_{dj} be the cross efficiency for DMU j corresponding to DMU d obtained by model (3.12). If $s_j - \varphi_j = 0$, then, we have $E_{dj} = \frac{E_{dj}^{max} + E_{dj}^{min}}{2}$.

Proof. In model (3.12), we have $U \cdot Y_j - E_{dj}^{max} \times W \cdot X_j + s_j = 0, \forall j, j \neq d$ and $U \cdot Y_j - E_{dj}^{min} \times W \cdot X_j - \varphi_j = 0, \forall j, j \neq d$. By adding these two equations, we have $2 \times U \cdot Y_j - \left(E_{dj}^{max} + E_{dj}^{min}\right) \times W \cdot X_j + s_j - \varphi_j = 0$. Since $s_j - \varphi_j = 0$, we can then get $E_{dj} = \frac{U \cdot Y_j}{W \cdot X_j} = \frac{E_{dj}^{max} + E_{dj}^{min}}{2}$. Q.E.D.

From Theorem 3.2, we can see that model (3.12) is intended to make the other DMUs' cross efficiencies as close as possible to the midpoint between E_{dj}^{max} and

E_{dj}^{min}.

By using the proposed weights selection models, we can obtain a set of optimal weights, denoted as (U_d^*, W_d^*), for each DMU d. Then, we can calculate the cross-efficiency scores (E_j^c) for the DMUs as in equation (2.8).

It should be noted that any of the proposed secondary goal models can be used for weights selection in cross-efficiency evaluation. None of these models is a clear winner or loser in any circumstances. The decision-makers can choose different models to fit different applications. Models (3.8) and (3.9) are applicable when the DMUs are cooperative. Model (3.8) aims at maximizing the efficiency of the whole system made of the DMUs. In addition, the efficiency of the DMU which uses model (3.8) to select its optimal weights will be kept at its maximum level (CCR efficiency). Nakabayashi and Tone (2006) suggested that such a model can be used to solve benefit-sharing problems. For example, all the departments in a university will cooperate with each other to maximize the efficiency of the university and make the university prestigious, thereby winning it more funding. At the same time, each department will maximize its own efficiency to share more of the funds.

Model (3.9) seeks to maximize the minimum cross efficiency among all the DMUs, which will lead to a situation in which the variations between the DMUs' cross efficiencies are smaller than those in the evaluation results of model (3.8). This model is applicable in the situation where a more cooperative atmosphere exists among the DMUs. Specifically, in such a cooperative atmosphere, some better performers are willing to sacrifice their own efficiencies to help the worst performer to achieve its best efficiency level. Such an example can be seen in Walker et al. (2008), in which a supply chain involves a set of sectors. In the supply chain, every sector is very important. The supply chain will suffer a disadvantageous competitive position if any sector of the chain performs badly, which will lead to the consequence of failure. Therefore, the worst performing sector in the supply chain should be given its best efficiency level in the evaluation so as to show a better performance of the whole chain.

Models (3.10) and (3.11) are suitable for evaluation of DMUs which are competitive with each other. Similar to the case of models (3.8) and (3.9), if the relationships among the DMUs are rather competitive, model (3.11) is a more suitable model than model (3.10). Model (3.12) is proposed from a neutral point view and is suitable for situations where the DMUs do not care if they benefit or harm the others'

cross efficiencies.

3.4 A numerical example

In this section, we provide a classical numerical example: the efficiency evaluation of six nursing homes, which was also used in Liang et al. (2008a) and Wang and Chin (2010a). The example serves to compare the proposed models with the traditional secondary models listed in Section 3.

As shown in Table 3.1, each nursing home has two inputs (X1 and X2) and two outputs (Y1 and Y2) (Sexton et al., 1986).

StHr (X1): staff hours per day, including nurses, physicians, etc.

Supp (X2): supplies per day, measured in thousands of dollars.

MCPD (Y1): total Medicare-plus Medicaid-reimbursed patient days.

PPPD (Y2): total privately paid patient days.

Table 3.1 Input and output data of nursing homes

DMU	Inputs		Outputs	
	StHr(X1)	Supp(X2)	MCPD(Y1)	PPPD(Y2)
A	1.50	0.20	1.40	0.35
B	4.00	0.70	1.40	2.10
C	3.20	1.20	4.20	1.05
D	5.20	2.00	2.80	4.20
E	3.50	1.20	1.90	2.50
F	3.20	0.70	1.40	1.50

We evaluate and rank the DMUs using the CCR model, the traditional benevolent and aggressive models (3.1-3.6), and our proposed models (3.8-3.12). The results are listed in Tables 3.2 and 3.3. Through comparing the results, several findings are identified. Firstly, the CCR model cannot further distinguish between the efficient DMUs, but every model proposed in this chapter can effectively discriminate among the DMUs and give a unique ranking position for each DMU.

Secondly, for each DMU, the average cross-efficiencies obtained from models (3.8) and (3.9) are larger than those from model (3.12), and efficiencies from models (3.10)

and (3.11) are smaller than those obtained from model (3.12). These results show that the proposed benevolent (or aggressive) model has a good ability to maximize (or minimize) other DMUs' cross efficiencies and the proposed neutral model (3.12) has the neutral characteristic that it does not care to minimize or maximize the cross-efficiency scores of other DMUs.

Thirdly, compared with model (3.8), model (3.10) and the traditional benevolent and aggressive models, the average cross-efficiencies of DMUs obtained by model (3.9) or (3.11) are closer in value. This finding reveals that models (3.9) and (3.11) could reduce large differences between the cross efficiencies of the DMUs.

Fourthly, differences are found in the ranking orders between our benevolent and aggressive models. This suggests that they are sensitive to the evaluation strategies that the DMUs choose, illustrating the fact that different strategies result in different ranking orders.

Table 3.2 Aggressive average cross-efficiency and their rankings

DMU	Doyle and Green's model	Liang *et al.*'s model	Wang and Chin's model	Proposed models	
	Model (3.4)	Model (3.5)	Model (3.6)	Model (3.10)	Model (3.11)
1	0.7639 (1)	0.7639 (1)	0.7639 (1)	0.7639 (1)	0.8023 (2)
2	0.7004 (3)	0.7004 (3)	0.7004 (3)	0.7004 (3)	0.7681 (4)
3	0.6428 (5)	0.6428 (5)	0.6428 (5)	0.6428 (5)	0.6850 (5)
4	0.7184 (2)	0.7184 (2)	0.7184 (2)	0.7184 (2)	0.8071 (1)
5	0.6956 (4)	0.6956 (4)	0.6956 (4)	0.6956 (4)	0.7799 (3)
6	0.6081 (6)	0.6081 (6)	0.6081 (6)	0.6081 (6)	0.6730 (6)

Table 3.3 Benevolent average cross-efficiency and their rankings

DMU	CCR efficiency	Doyle and Green's model	Liang *et al.*'s model	Wang and Chin's model	Proposed models		Neutral model	Arbitrary strategy
		Model (3.1)	Model (3.2)	Model (3.3)	Model (3.8)	Model (3.9)	Model (3.12)	Equation (2.8)
1	**1.0000 (1)**	**1.0000 (1)**	**1.0000 (1)**	**1.0000 (1)**	0.9163 (4)	0.8763 (3)	0.8655 (1)	0.8330 (1)
2	**1.0000 (1)**	0.9773 (3)	0.9547 (4)	0.9773 (3)	0.9773 (2)	0.8622 (4)	0.8217 (3)	0.7617 (3)
3	**1.0000 (1)**	0.8580 (5)	0.8864 (5)	0.8580 (5)	0.7886 (6)	0.8122 (5)	0.7612 (5)	0.7072 (5)
4	**1.0000 (1)**	**1.0000 (1)**	**1.0000 (1)**	**1.0000 (1)**	1.0000 (1)	0.9425 (1)	0.8607 (2)	0.7747 (2)
5	0.9775 (5)	0.9758 (4)	0.9742 (3)	0.9758 (4)	0.9714 (3)	0.9097 (2)	0.8361 (4)	0.7565 (4)
6	0.8675 (6)	0.8570 (6)	0.8465 (6)	0.8570 (6)	0.8462 (5)	0.7692 (6)	0.7253 (6)	0.6687 (6)

Fifthly, the traditional benevolent models (3.1), (3.3), and (3.5) cannot give any further distinction between nursing homes 1 and 4 (their cross-efficiency scores are both equal to 1). On the other hand, our models (3.8) and (3.9) do give them different cross-efficiency scores and rank them in different positions. This result indicates that our benevolent models generally have stronger power in discriminating the DMUs.

Sixthly, except for the results of model (3.10) (which are identical to those in Wang and Chin (2010)), the average cross-efficiencies and the ranking orders obtained from our models are different from those generated by the traditional models. This indicates the desirable and undesirable targets in the proposed models have influences on the results for all DMUs. Therefore, the decision-makers have more flexibility in choosing their preferable models according to their different decision preferences.

Finally, the efficiency score of each DMU generated by model (3.9) is smaller than that generated by traditional benevolent model (model (3.1)), and the efficiency score of each DMU generated by model (3.11) is larger than that generated by aggressive model (model (3.2)). This indicates that the benevolent and aggressive powers of the proposed models are relatively weaker than the traditional models. Decision-makers could choose the proposed or traditional models based on their preference degree.

As can be seen from the above discussions, the proposed models can effectively discriminate the DMUs and evaluate the DMUs using different strategies, allowing more choices based on the characteristics of the DMUs.

3.5 Conclusions

As an effective method for evaluating and ranking the DMUs, cross-efficiency evaluation has been applied in a wide variety of areas. However, the problem of the non-uniqueness of optimal weights reduces the usefulness of the cross-efficiency evaluation method. In order to solve this problem, we propose a series of new secondary goal models. Compared with the traditional secondary goal models, our models not only use the cross-efficiency targets that are always reachable for the DMUs but also consider both positive and negative aspects of these targets, that is, the DMUs' simultaneous goals to get close to desirable targets and away from undesirable ones. A numerical example is used to illustrate the proposed models. The results show that our secondary goal models not only have strong power in discriminating among DMUs,

but also show strong applicability which provides more choices for the decision-makers. Therefore, the proposed secondary goal models in this chapter can be seen as improvements and extensions to the traditional secondary goal models, which makes them meaningful contributions to DEA cross-efficiency evaluation.

This work can be extended in at least two directions. On the one hand, nonlinear combinations of the deviations in the objective function may generate some more appropriate weights for the DMUs. But before that can be done, a linearization method needs to be firstly proposed to guarantee the nonlinear programs can be solved. On the other hand, our models are not applicable when the input and/or output data are stochastic, as they are in some real-world applications. Some further extensions might consider this problem and propose suitable methods to address it based on stochastic DEA or fuzzy DEA methodologies.

Chapter 4 DEA cross-efficiency evaluation based on Pareto improvement *

In the traditional cross-efficiency evaluation approaches, the generated cross-efficiency scores may not be Pareto optimal, which reduces the effectiveness of this method. To fix this issue, we propose in this Chapter a cross-efficiency evaluation approach based on Pareto improvement, which contains two models (Pareto optimality estimation model and cross-efficiency Pareto improvement model) and an algorithm. The Pareto optimality estimation model is used to estimate whether given cross-efficiency scores of DMUs constitute a Pareto-optimal solution. If they do not, the Pareto improvement model is then used to improve them into a Pareto optimal solution. In contrast to other cross-efficiency approaches, our approach always obtains cross efficiencies that constitute a Pareto-optimal solution under the predetermined weight selection principles. More importantly, under some conditions, the evaluation result generated by our approach unifies self-evaluation, peer-evaluation, and common-weight-evaluation in DEA cross-efficiency evaluation. Specifically, the self-evaluated efficiency and the peer-evaluated efficiency converge to the same common-weight-evaluated efficiency. This will make the evaluation results more likely to be accepted by all the DMUs.

The rest of this chapter is organized as follows. Section 4.1 proposes the Pareto-optimal cross-efficiency evaluation models which contain the Pareto optimality estimation model and the cross-efficiency Pareto improvement model. An algorithm and related discussions of common weights are presented in Section 4.2. In Section 4.3, we compare the proposed approach with the existing studies through three instances. Finally, conclusions and further research directions are given in Section 4.4.

4.1 Pareto-optimal cross-efficiency evaluation models

Although the alternative secondary goal models can reduce the number of possible optimal solutions (i.e. more likely to give a unique optimal solution), the results in

* This chapter is primarily referenced from: Jie Wu, Junfei Chu, Jiasen Sun, Qingyuan Zhu, and Liang Liang. (2016). DEA cross-efficiency evaluation based on Pareto improvement. *European Journal of Operational Research*, *248*(2), 571-579.

general are not Pareto optimal, which may not be acceptable for all DMUs. In this section, to obtain Pareto-optimal cross efficiencies, we propose a Pareto optimality estimation model to estimate whether given cross-efficiency scores for the DMUs are susceptible to be improved. Then, a cross-efficiency Pareto improvement model is proposed to bring the cross-efficiency scores for the DMUs closer to Pareto optimality.

4.1.1 A Pareto optimality estimation model

When improving the DMUs' cross-efficiency scores, each DMU needs to attach new weights to inputs and outputs. This improvement requires consideration of all the DMUs because the CCR optimality says that no DMU can improve its efficiency alone and in general all DMUs must be considered if the goal is Pareto optimality.

For the sake of simplification, we often use n-entry vectors to represent sets of cross-efficiency scores of DMUs. We also talk of self-evaluated efficiency and peer-evaluated efficiencies. To be more specific, $E_{j,k} = \frac{U_j \cdot Y_k}{W_j \cdot X_k}$ is called the self-evaluated efficiency of DMU j if $k = j$, and the peer-evaluated efficiency of DMU j by DMU k, otherwise.

Here, we state the following two basic weight selection principles, both of which are implicitly required by the Pareto optimality in this chapter.

Principle 4.1. Given cross-efficiency scores for the DMUs, when new weights are to be selected for a DMU to improve the cross-efficiency scores, these new weights must guarantee that the DMU's new self-evaluated efficiency is no smaller than its current cross-efficiency score.

Principle 4.2. Given cross-efficiency scores for the DMUs, when new weights are to be selected for a DMU to improve the cross-efficiency scores, these new weights must guarantee that the other DMUs' peer-evaluated efficiencies using the new weights are no smaller than their current cross-efficiency scores.

These two principles are required for all the DMUs because they all have the intention of setting lower bounds for their cross efficiencies. Sometimes, the DMUs may even require the lower bounds to be the CCR efficiencies. Similar principles have also appeared in the models of Liang, Wu, Cook, and Zhu (2008a), Wang and Chin

(2010a), and Du et al. (2014). To better illustrate our method, we give the following Definition 4.1.

Definition 4.1. Given a vector $E^c = \{E_j^c, \forall j\}$ of cross-efficiency scores, E^c is said to be ***Pareto optimal***, if it is impossible to find another vector $E^{c\prime} = \{E_j^{c\prime}, \forall j\}$ of cross-efficiency scores such that $E_j^{c\prime} \geq E_j^c$, for any DMU j ($1 \leq j \leq n$), with at least one inequality being strict.

Based on Pareto-optimality theory, the DMUs may consider whether new weights can be selected to improve their cross-efficiency scores without decreasing any of them. For the DMUs, to see whether a given vector of cross-efficiency scores is Pareto-optimal, we propose the following Pareto-optimality estimation model (4.1). As will be seen later, a non-zero optimal objective value indicates that the given set of cross-efficiencies is Pareto-optimal and a zero optimal objective value is a reasonable (but not conclusive) evidence that it is not.

$$\begin{aligned} min \quad & \gamma \\ s.t. \quad & W \cdot X_d = 1 \\ & U \cdot Y_d \geq E_d^c \\ & U \cdot Y_j - W \cdot X_j \leq 0, \forall j \\ & U \cdot Y_j - E_j^c \times W \cdot X_j + s_j = 0, \forall j, j \neq d \\ & s_j \leq \gamma, \forall j, j \neq d \\ & \gamma \geq 0 \end{aligned} \tag{4.1}$$

In model (4.1), E_j^c ($1 \leq j \leq n$) is the given cross-efficiency score of DMU j to be evaluated. They form vector E^c. d can be any arbitrary DMU. When DMU d selects its weights, it keeps its own efficiency no less than its given cross-efficiency score (i.e., it respects the weight selection principle 4.1) and, subject to that priority, it then strives to make the other DMUs' cross efficiencies as large as possible.

In the remainder, let $(U_d^*, W_d^*, S_d^*, \gamma_d^*)$ denote an optimal solution to model (4.1) corresponding to an arbitrary DMU d, where S_d^* is the vector made of the optimal values of $s_j, \forall j, j \neq d$.

In the context of Pareto-optimality estimation model (4.1), the following theorems can be proven.

Theorem 4.1. If $\gamma_d^* = 0$, then we have $\gamma_k^* = 0$ for any k such that $1 \leq k \leq n$.

Proof. Let $v = (W_d^* \cdot X_k)$, $W_k = W_d^*/v$, $U_k = U_d^*/v$, $S_k = S_d^*/v$, and $\gamma_k = \gamma_d^*$. It is easy to verify that $(U_k, W_k, S_k, \gamma_k)$ is a feasible solution to model (4.1) when solving the model for any DMU *k*. So, we get $\gamma_k^* \leq \gamma_k = \gamma_d^* = 0$. Because it is known that $\gamma_k^* \geq 0$, we must have $\gamma_k^* = 0$. Q.E.D.

Theorem 4.2. If there is some DMU *d* such that $\gamma_d^* = 0$, all DMUs have the potential to improve their cross-efficiency scores without reducing the cross-efficiency scores of any other.

Proof. According to Theorem 4.1, if $\gamma_d^* = 0$, we have $\gamma_k^* = 0, \forall k$. So, from the fifth constraint group of model (4.1), we can get $s_{kj}^* \leq 0, \forall k, j$. From the fourth constraint group of model (4.1), we have $E_j^c \leq \frac{U_k^* \cdot Y_j}{W_k^* \cdot X_j}, \forall k, j$. Consequently, we have $E_j^c \leq \frac{1}{n}\sum_{k=1}^{n} \frac{U_k^* \cdot Y_j}{W_k^* \cdot X_j} \triangleq E_j^{c\prime}$, $\forall j$. Therefore, the DMUs have the potential to improve their cross-efficiency scores to accomplish Pareto improvement without reducing the cross-efficiency score of any other. Q.E.D.

It can be seen from Theorems 4.1 and 4.2 that model (4.1) can be used to estimate whether a given set of cross-efficiency scores are Pareto-optimal. If $\gamma_d^* > 0$, then none of the given cross-efficiency scores for DMUs can be strictly improved without reducing at least one of the others under the predetermined two principles. The given vector is Pareto optimal. If $\gamma_d^* = 0$, then the DMUs have the potential to improve their cross-efficiency scores by Pareto improvement, so these cross-efficiency scores may not be Pareto-optimal and need to be further checked.

4.1.2 Cross-efficiency Pareto-improvement model

By using the Pareto-optimality estimation model (4.1), we can determine whether the DMUs have the potential to make their own cross-efficiency scores better off without making any DMU's cross-efficiency score worse off. To make Pareto improvement for the cross-efficiency scores which do not constitute a Pareto-optimal solution, we propose the following cross-efficiency Pareto-improvement model (4.2).

$$\begin{aligned} max \quad & U \cdot Y_d \\ s.t. \quad & W \cdot X_d = 1 \end{aligned} \tag{4.2}$$

$$U \cdot Y_j - W \cdot X_j \leq 0, \forall j$$
$$U \cdot Y_j - E_j^c \times W \cdot X_j \geq 0, \forall j$$
$$U, W \geq 0$$

We know that there is always a feasible solution to model (4.2), if $\gamma_j^* = 0, \forall j$ in the solution to model (4.1). Furthermore, when cross-efficiency improvement is made for DMU *d*, it is intended to maximize the efficiency of DMU *d* while keeping all DMUs' cross efficiencies no less than their current cross-efficiency scores.

In the remainder, let (U_d^*, W_d^*) be an optimal solution of model (4.2) with respect to DMU *d*. By solving model (4.2) for each DMU d, it gets new optimal input and output weights (U_d^*, W_d^*). These weights are used for both self-evaluation and peer-evaluation. By averaging the self-evaluated and peer-evaluated efficiency scores for each DMU *d* we can obtain the corresponding Pareto-improved cross efficiency for the DMUs as defined in (4.3).

Definition 4.2. For each DMU *j*,

$$E_j^{PI} = \frac{1}{n} \sum_{d=1}^{n} \frac{U_d^* \cdot Y_j}{W_d^* \cdot X_j} \tag{4.3}$$

is defined as its ***Pareto-improved cross efficiency***.

4.2 Algorithm and common weights

In this section, we first propose an algorithm to get the Pareto-optimal cross efficiencies for the DMUs. Then, we discuss the existence of common weights.

4.2.1 Algorithm

We propose an iterative procedure to get Pareto-optimal cross efficiencies for the DMUs. The basic idea of the algorithm is to start with solving the traditional CCR model to get the original cross-efficiency scores for the DMUs. Then, we solve model (4.1) for an arbitrary DMU *d* to see whether the DMUs have the potential to make Pareto improvements in their cross-efficiency scores. If the DMUs have the potential to improve their cross-efficiency scores, we solve model (4.2) to get Pareto-improved cross efficiency for each DMU by (4.3). After this, the Pareto-improved cross efficiencies will be evaluated again by model (4.1), and this process is repeated as many

times as needed. When the change in Pareto-improved cross efficiency from one iteration to the next one becomes very small for all DMUs, or the Pareto-improved cross efficiencies are revealed to be Pareto-optimal by model (4.1), the algorithm terminates, and we get a vector of Pareto optimal cross-efficiency scores. The details are shown as follows.

Algorithm 4.1

Begin

Step 1: Solve the CCR model and obtain a vector of cross-efficiency scores defined by (2.8) for the DMUs. Let $t = 1$ and $E_j^{PO} = E_j^{c,1} = E_j^c, \forall j$.

Step 2: Solve model (4.1) for an arbitrary DMU d. If $\gamma_d^* > 0$ in the optimal solution, then stop. Otherwise, solve model (4.2) to select new optimal weights (U_d^{t*}, W_d^{t*}) for each DMU d, and let $E_j^{PO} = E_j^{c,t+1} = \frac{1}{n}\sum_{d=1}^{n} \frac{U_d^* \cdot Y_j}{W_d^* \cdot X_j}, \forall j$.

Step 3: If we have $\left|E_j^{c,t} - E_j^{c,t+1}\right| \geq \epsilon$ for some j, let $t := t + 1$ and go Step 2. Otherwise stop.

End

Let (U_d^{t*}, W_d^{t*}) be an optimal solution to model (4.2) when DMU d is considered in the tth iteration of the algorithm. Let E^* be the vector of the CCR (self-evaluated) efficiencies. We claim that when the proposed algorithm stops, the obtained E^{PO} (denoted as the vector of the pareto optimal cross-efficiency scores E_j^{PO}) is a Pareto-optimal vector of cross efficiency scores as defined in Definition 4.1. Concerning this algorithm, we present the following Theorems 4.3-4.7.

Theorem 4.3. For any DMU j, $E_j^{c,t}$ are nondecreasing with t, and we have $E_j^c \leq E_j^{c,t} \leq E_j^*$, where E_j^c and E_j^* are the original cross-efficiency score and the CCR (self-evaluated) efficiency (generated by model (2.3)) of DMU j, respectively.

Proof. From the constraints in model (4.2), for each DMU j, we have $E_j^{c,t} \leq \frac{U_d^{t*} \cdot Y_j}{W_d^{t*} \cdot X_j}$.

Therefore $E_j^{c,t} \leq \frac{1}{n}\sum_{d=1}^{n} \frac{U_d^{t*} \cdot Y_j}{W_d^{t*} \cdot X_j} = E_j^{c,t+1}$. It is easy to see that $(\frac{U_d^{t*}}{W_d^{t*} \cdot X_j}, \frac{W_d^{t*}}{W_d^{t*} \cdot X_j})$ is a feasible solution to the CCR model (2.5) corresponding DMU j. So, for any d, we have

$\frac{U_d^{t*}\cdot Y_j}{W_d^{t*}\cdot X_j} = \frac{U_d^{t*}}{W_d^{t*}\cdot X_j}\cdot Y_j/(\frac{W_d^{t*}}{W_d^{t*}\cdot X_j}\cdot X_j) \le E_j^*$, which implies $E_j^{c,t+1} = \sum_{d=1}^{n}\frac{U_d^{t*}\cdot Y_j}{W_d^{t*}\cdot X_j} \le E_j^*$. Thus, we get $E_j^c \le E_j^{c,t} \le E_j^{c,t+1} \le E_j^*, \forall j, t = 1,2, \ldots$. Q.E.D.

Theorem 4.4. For any t, if $E^{c,t}$ is not Pareto-optimal, there is some j such that $E_j^{c,t+1} > E_j^{c,t}$.

Proof. If $E^{c,t}$ is not Pareto-optimal, according to Definition 4.1, there must be some DMUs d and j such that $E_j^{c,t} < \frac{U_d\cdot Y_j}{W_d\cdot X_j}$, where (U_d, W_d) is a feasible solution to model (4.2). Then it is easy to see that $(\frac{U_d}{W_d\cdot X_j}, \frac{W_d}{W_d\cdot X_j})$ is also a feasible solution to model (4.2) when DMU *j* is being improved. This means we have $\frac{U_d\cdot Y_j}{W_d\cdot X_j} = \frac{U_d}{W_d\cdot X_j}\cdot Y_j/(\frac{W_d}{W_d\cdot X_j}\cdot X_j) \le \frac{U_j^{t*}\cdot Y_j}{W_j^{t*}\cdot X_j}$. Therefore, we have $E_j^{c,t} < \frac{U_d\cdot Y_j}{W_d\cdot X_j} \le \frac{U_j^{t*}\cdot Y_j}{W_j^{t*}\cdot X_j}$. By definition, we have $E_j^{c,t} \le \frac{U_k^{t*}\cdot Y_j}{W_k^{t*}\cdot X_j}, \forall k \ne j$, so we get $E_j^{c,t} < \frac{1}{n}\left[\frac{U_j^{t*}\cdot Y_j}{W_j^{t*}\cdot X_j} + \sum_{k=1,k\ne j}^{n}\frac{U_k^{t*}\cdot Y_j}{W_k^{t*}\cdot X_j}\right] = E_j^{c,t+1}$. Q.E.D.

Theorem 4.5. For any DMU *d,* the self-evaluated efficiency E_{dd}^t is nonincreasing with t where E_{dd}^t is defined as $E_{dd}^t = U_d^{t*}\cdot Y_d$. We also have $E_d^{c,t} \le E_{dd}^t \le E_d^*$.

Proof. As was identified in Theorem 4.3, for any $E_j^{c,t} \le E_j^{c,t+1}$ for any DMU j. This means that in model (4.2) for any DMU *d*, any solution which is feasible at iteration $t+1$ is also feasible at iteration t. So, (U_d^{t+1*}, W_d^{t+1*}) is a feasible solution to model (4.2) corresponding to DMU *d* in the tth iteration of the algorithm. It is easy to see that for any *t*, (U_d^{t*}, W_d^{t*}) is also a feasible solution to the CCR model (2.5). Thus, we have $E_{dd}^{t+1} = U_d^{t+1*}\cdot Y_d \le U_d^{t*}\cdot Y_d = E_{dd}^t \le E_d^*$. Also, it is easy to figure out that for any *t* and *k*, we have $\frac{U_k^{t*}\cdot Y_d}{W_k^{t*}\cdot X_d} \le U_d^{t*}\cdot Y_d$, which implies $E_d^{c,t} \le E_d^{c,t+1} = \frac{1}{n}\sum_{k=1}^{n}\frac{U_k^{t*}\cdot Y_d}{W_k^{t*}\cdot X_d} \le U_d^{t*}\cdot Y_d = E_{dd}^t$. Therefore, based on the above inferences, we have $E_d^{c,t} \le E_{dd}^t \le E_d^*$. Q.E.D.

Theorem 4.6. If the algorithm terminates at step 3 in the tth iteration of the algorithm, for any *d,* the Pareto-improved cross efficiency and the self-evaluated efficiency of DMU *d* converge to the same value, i.e. $E_d^{c,t} = E_{dd}^t$.

Proof. If the algorithm converges in this iteration; i.e., $E_d^{c,t} = E_d^{c,t+1} = E_d^{PO}$, then we can show that $E_d^{c,t} = U_d^{t*} \cdot Y_d = E_{dd}^t$. This can be proved by absurdity. If $E_d^{c,t} < U_d^{t*} \cdot Y_d$ in model (4.2), then by adding $E_d^{c,t} \leq U_k^* \cdot Y_d/(W_k^* \cdot X_d)$, for any $k \neq d$, we would have $E_d^{c,t} < \frac{1}{n}\left[U_d^{t*} \cdot Y_d + \sum_{k=1,k\neq d}^{n} U_k^{t*} \cdot Y_d/(W_k^{t*} \cdot X_d)\right] = E_d^{c,t+1}$, which would be in contradiction with the fact that $E_d^{c,t} = E_d^{c,t+1}$. Therefore, we have $E_d^{c,t} = U_d^{t*} \cdot Y_d = E_{dd}^t$. Q.E.D.

Theorem 4.7. The final Pareto-improved cross efficiencies for the DMUs constitute a Pareto-optimal solution.

Proof. The algorithm terminates at either step 2 or step 3. If the algorithm stops at step 2, no new weights can be selected by the algorithm to improve any of the DMUs' cross-efficiency scores without making at least one of the DMUs' cross-efficiency score worse off. Thus, by the definition of Pareto-optimality, the final generated Pareto-improved cross efficiency scores constitute a Pareto-optimal solution. If the algorithm converges at step 3, we have $E_d^{c,t} = E_d^{c,t+1}$ for all DMU *d*. But if the final generated Pareto-improved cross efficiencies were not Pareto optimal, we know from Theorem 4.4 that $E_d^{c,t} < E_d^{c,t+1}$ for some DMU *d*. This would be contrary to the convergence condition that $E_d^{c,t} = E_d^{c,t+1}$for all DMU *d*. Therefore, the final Pareto-improved cross efficiencies for the DMUs constitute a Pareto-optimal solution. Q.E.D.

Theorem 4.3 shows that the Pareto-improved efficiency $E_j^{c,t}$ for each DMU j is non-decreasing and located between its original cross-efficiency score E_j^c and CCR self-evaluated efficiency E_j^* during the process. Theorem 4.4 shows that the Pareto-improved efficiency $E_j^{c,t}$ for each DMU j is increasing until it reaches the Pareto-optimal cross efficiency. These theorems ensure that the algorithm terminates within a finite number of steps and that the cross-efficiency scores of the DMUs are improved as the algorithm proceeds. Theorem 4.5 shows that the self-evaluated efficiencies of the DMUs are non-increasing during the process. The self-evaluated efficiencies of the DMUs are not smaller than their peer-evaluated efficiencies (Pareto-improved cross efficiencies) in each iteration of the algorithm. Theorem 4.6 points out that the self-evaluated efficiency and the peer-evaluated efficiency of each DMU will converge to the same efficiency score if the algorithm stops at step 3 (and Theorem 4.7 implies that this score is also the DMU's Pareto-optimal cross-efficiency score). We can view the convergence of the algorithm as a kind of tradeoff, in which the self-evaluated

efficiency of each DMU is reduced to improve its peer-evaluated efficiency. This kind of tradeoff stops when the self-evaluated efficiency and peer-evaluated efficiency converge to the same value (see Figure 4.1).

Theorem 4.7 shows that the final cross-efficiency scores for the DMUs constitute a Pareto-optimal solution. Additionally, if the algorithm stops at its step 3, the self-evaluated efficiency and peer-evaluated efficiency of each DMU will converge to the same Pareto-optimal cross efficiency. This is of interest because the Pareto optimality of the results and the unification of self-evaluation with peer-evaluation for the DMUs will make the evaluation results more acceptable to the DMUs, i.e. the DMUs are more likely to believe in the fairness of the evaluation.

4.2.2 Common weights

In this part, we show that common weights appear for the DMUs if the proposed algorithm terminates at step 3. Firstly, we propose the following Theorem 4.8 and Corollary 4.1.

Theorem 4.8. If the algorithm terminates at step 3 in the tth iteration of the algorithm, the peer-evaluated efficiencies of any DMU d by the other DMUs are all equal to its Pareto-optimal cross efficiency, i.e. for any d and k, we have $E_d^{PO} = E_d^{c,t} = E_{kd}^{c,t} = U_k^* \cdot Y_d / W_k^* \cdot X_d$.

Proof. This can be proved using reduction to absurdity. Since the algorithm converges at the tth iteration, for any d, we have $E_d^{c,t} = E_d^{c,t+1} = E_d^{PO}$. For any k, we have $E_d^{c,t} \leq \frac{U_k^* \cdot Y_d}{W_k^* \cdot X_d}$ from model (4.2). If $E_d^{c,t} < \frac{U_k^* \cdot Y_d}{W_k^* \cdot X_d}$ for some DMU k, we would have $E_d^{c,t} < E_d^{c,t+1} = \frac{1}{n}\sum_{k=1}^{n} \frac{U_k^* \cdot Y_d}{W_k^* \cdot X_d}$, which would be in contradiction with the fact that $E_d^{c,t} = E_d^{c,t+1}$. So, we have $E_d^{PO} = E_d^{c,t} = E_{kd}^{c,t}$ for any d, k. Q.E.D.

From the proof of Theorem 4.8, the common optimal weight result follows easily.

Corollary 4.1. If the algorithm terminates at step 3 of the algorithm, there must exist a pair of vectors of common weights (U, W), such that $E_j^{PO} = \frac{U \cdot Y_j}{W \cdot X_j}, \forall j$.

Theorem 4.8 and Corollary 4.1 show that all the DMUs use the same weights to peer-evaluated the others if the algorithm stops at step 3. The optimal weights of every DMU can be seen as a set of common weights which can generate Pareto-optimal cross efficiencies. But it should be noted here that the DMUs use different constraints ($W_d \cdot X_d = 1$) to avoid trivial solutions in model (4.2). This may lead to the situation that the set of optimal weights selected by model (4.2) for each DMU is a multiple of the set of common weights, i.e. $(U_d^*, W_d^*) = \alpha(U, W)$ for some $\alpha > 0$. Fortunately, this does not affect the common-weight property of the optimal weights because the optimal weights in DEA reflect only the relative importance a DMU attaches to its corresponding inputs and outputs when evaluating efficiency (Charness & Cooper, 1962). Therefore, we can give the common weights of the DMUs by standardizing the optimal weights of any DMU *d* as follows.

$$U = \frac{U_d^*}{\sum_{i=1}^{m} w_{id}^* + \sum_{r=1}^{s} u_{rd}^*} \tag{4.4}$$

$$W = \frac{W_d^*}{\sum_{i=1}^{m} w_{id}^* + \sum_{r=1}^{s} u_{rd}^*} \tag{4.5}$$

4.2.3 A Numerical example

To illustrate the Pareto-optimal cross-efficiency evaluation models and the proposed algorithm, we use a small numerical example from Liang et al. (2008a) involving five DMUs. Each DMU has three inputs and two outputs. The raw data of this numerical example can be found in Table 2.1, Chapter 2.

We evaluate the DMUs by the CCR model, the original cross-efficiency scores, benevolent model (3.1), aggressive model (3.2), and the proposed algorithm. The results of the evaluations are listed in Table 4.1. Additionally, we standardize the optimal weights of all the DMUs by (4.4) and (4.5) when the algorithm terminates, with the results reported in Table 4.2.

From the results, the cross-efficiency scores for the DMUs generated from equation (2.8) and models (3.1) and (3.2) are not Pareto optimal, for they can be further improved. To proceed, we use the proposed model to improve the cross-efficiency scores of all DMUs, and Figure 4.1 shows the process of efficiency improvement.

Table 4.1 The results of the numerical example

DMUs	CCR efficiency	Pareto-optimal cross efficiency*	Arbitrary	Benevolent	Aggressive
	Model (1)	The proposed algorithm	Equation (2.8)	Model (3.1)	Model (3.2)
DMU1	0.6857	0.5715	0.4743	0.5616	0.4473
DMU2	1.0000	1.0000	0.8793	0.9295	0.8895
DMU3	1.0000	1.0000	0.9856	1.0000	0.9571
DMU4	0.8571	0.7500	0.5554	0.6671	0.5843
DMU5	0.8571	0.5999	0.5587	0.5871	0.5186

Table 4.2 The common weights

Inputs & Outputs	Weights
X1	0.00005
X2	0.00016
X3	0.49978
Y1	0.50001
Y2	0.00000

As can be seen from Figure 4.1, the proposed algorithm obtains the Pareto-optimal cross efficiency scores for all DMUs after 8 iterations. Take DMU 1 as an example, its Pareto-improved cross efficiency increases and its self-evaluated efficiency decreases during the process. At the end of the algorithm, the cross-efficiency is identical to its self-evaluated efficiency (0.5715) which is defined as the Pareto-optimal cross efficiency for DMU 1. Additionally, a common set of weights is obtained for the Pareto-optimal cross efficiencies of the DMUs when the algorithm terminates.

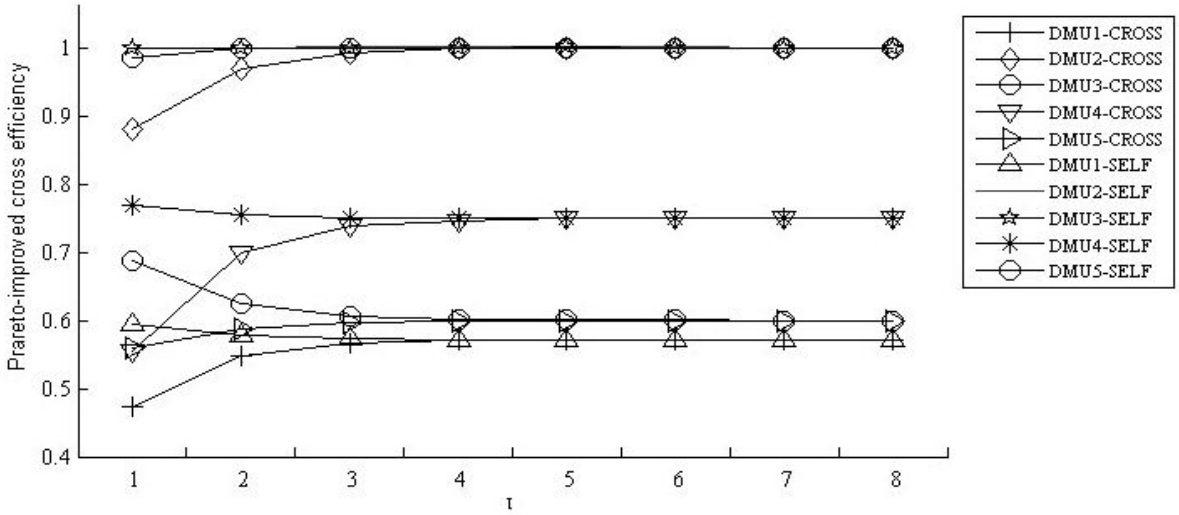

Figure 4.1 Calculation process of the proposed algorithm

Note that in the optimal common weights of the DMUs, we find a zero-output weight. Although it is recognized that zero weights should be avoided in DEA common-weight evaluation, this result is normal, since it is a group-decision result by all the DMUs, i.e., the zero weight is determined based on all the DMUs' favor. The zero weights in such a group-decision mechanism only demonstrates that the selected indicator is not in favor for any of the DMUs and should be removed from the performance metrics.

4.3 Application to R&D project selection and efficiency evaluation of nursing homes

In this section, we illustrate our method by applying it to R&D project selection and efficiency evaluation of nursing homes.

4.3.1 R&D project selection

Thirty-seven R&D projects, each involving one input and five outputs, are used, which are documented in Table 4.3 (example from Oral et al. 1991). It should be noted here that the DMUs are in competition with each other because they all want to get the funding to support their projects (Liang et al., 2008a), so we need to get a set of cross-efficiency scores that each DMU can admit as true and therefore accept.

X1: Budgets

Y1: Indirect economic contribution

Y2: Direct economic contribution

Y3: Technical contribution

Y4: Social contribution

Y5: Scientific contribution

The input budgets are in monetary units. The outputs are adjusted average scores obtained using Delphi Method. More details of the input and outputs can be seen in Oral et al. (1991).

Table 4.4 shows the evaluation results of the proposed algorithm and the alternative traditional models. As can be seen from the results, each DMU's cross-efficiency score generated from equation (2.8) is larger than that from model (3.2) and smaller than that from model (3.1). This is in accordance with the arbitrary, benevolent, and aggressive characteristics of the traditional models. Additionally, compared with the results of the traditional models, the proposed algorithm generates higher cross-efficiency scores for the DMUs. This is because of the Pareto optimality of the cross-efficiency scores, for they cannot be further improved without reducing some other DMU's cross-efficiency score. Also, when the algorithm stops, we get a common set of weights for Pareto-optimal cross-efficiencies for the DMUs, which is listed in Table 4.5.

Table 4.3 The common weights

Inputs & Outputs	Weights
X1	0.54804
Y1	0.05945
Y2	0.22369
Y3	0.04892
Y4	0.04988
Y5	0.07003

Table 4.4 Input and output data of 37 R&D projects

Project	Input	Output				
	X1	Y1	Y2	Y3	Y4	Y5
1	84.2	67.53	70.82	62.64	44.91	46.28
2	90	58.94	62.86	57.47	42.84	45.64
3	50.2	22.27	9.68	6.73	10.99	5.92
4	67.5	47.32	47.05	21.75	20.82	19.64
5	75.4	48.96	48.48	34.9	32.73	26.21
6	90	58.88	77.16	35.42	29.11	26.08
7	87.4	50.1	58.2	36.12	32.46	18.9
8	88.8	47.46	49.54	46.89	24.54	36.35
9	95.9	55.26	61.09	38.93	47.71	29.47
10	77.5	52.4	55.09	53.45	19.52	46.57
11	76.5	55.13	55.54	55.13	23.36	46.31
12	47.5	32.09	34.04	33.57	10.6	29.36
13	58.5	27.49	39	34.51	21.25	25.74
14	95	77.17	83.35	60.01	41.37	51.91
15	83.8	72	68.32	25.84	36.64	25.84
16	35.4	39.74	34.54	38.01	15.79	33.06
17	32.1	38.5	28.65	51.18	59.59	48.82
18	46.7	41.23	47.18	40.01	10.18	38.86
19	78.6	53.02	51.34	42.48	17.42	46.3
20	54.1	19.91	18.98	25.49	8.66	27.04
21	74.4	50.96	53.56	55.47	30.23	54.72
22	82.1	53.36	46.47	49.72	36.53	50.44
23	75.6	61.6	66.59	64.54	39.1	51.12
24	92.3	52.56	55.11	57.58	39.69	56.49
25	68.5	31.22	29.84	33.08	13.27	36.75
26	69.3	54.64	58.05	60.03	31.16	46.71
27	57.1	50.4	53.58	53.06	26.68	48.85
28	80	30.76	32.45	36.63	25.45	34.79
29	72	48.97	54.97	51.52	23.02	45.75
30	82.9	59.68	63.78	54.8	15.94	44.04
31	44.6	48.28	55.58	53.3	7.61	36.74
32	54.5	39.78	51.69	35.1	5.3	29.57
33	52.7	24.93	29.72	28.72	8.38	23.45
34	28	22.32	33.12	18.94	4.03	9.58
35	36	48.83	53.41	40.82	10.45	33.72
36	64.1	61.45	70.22	58.26	19.53	49.33
37	66.4	57.78	72.1	43.83	16.14	31.32

Table 4.5 Evaluation results of the projects

Project	CCR efficiency	Pareto-optimal cross efficiency*	Arbitrary	Benevolent	Aggressive
	Model(2.5)	The proposed algorithm	Equation (2.8)	Model (3.1)	Model (3.2)
1	0.6543	0.6155	0.6055	0.6127	0.6004
2	0.5512	0.5212	0.5124	0.5181	0.5080
3	0.3360	0.1738	0.1707	0.1724	0.1697
4	0.5283	0.4546	0.4457	0.4540	0.4436
5	0.5064	0.4581	0.4495	0.4562	0.4468
6	0.6148	0.5225	0.5121	0.5231	0.5101
7	0.5060	0.4323	0.4243	0.4324	0.4226
8	0.4204	0.4103	0.4047	0.4087	0.4004
9	0.5177	0.4433	0.4334	0.4410	0.4319
10	0.5431	0.5247	0.5184	0.5226	0.5119
11	0.5618	0.5440	0.5373	0.5416	0.5307
12	0.5525	0.5282	0.5222	0.5261	0.5153
13	0.5045	0.4650	0.4573	0.4632	0.4533
14	0.6539	0.6121	0.6015	0.6098	0.5967
15	0.6518	0.5327	0.5209	0.5315	0.5194
16	0.8542	0.7758	0.7672	0.7713	0.7568
17	1.0000	1.0000	0.9779	0.9785	0.9677
18	0.7618	0.7108	0.7020	0.7085	0.6930
19	0.5179	0.4835	0.4766	0.4806	0.4708
20	0.3523	0.3036	0.3003	0.3003	0.2951
21	0.6022	0.5656	0.5571	0.5608	0.5502
22	0.5068	0.4746	0.4667	0.4694	0.4613
23	0.6754	0.6576	0.6477	0.6541	0.6412
24	0.5003	0.4785	0.4704	0.4737	0.4651
25	0.4024	0.3565	0.3519	0.3530	0.3465
26	0.6633	0.6318	0.6234	0.6287	0.6163
27	0.7420	0.7135	0.7034	0.7089	0.6949
28	0.3478	0.3327	0.3271	0.3291	0.3233
29	0.5784	0.5596	0.5519	0.5568	0.5454
30	0.5505	0.5365	0.5304	0.5359	0.5242
31	0.9459	0.8535	0.8468	0.8549	0.8356
32	0.6393	0.6020	0.5945	0.6029	0.5882
33	0.4299	0.4015	0.3970	0.4005	0.3919
34	0.7973	0.6865	0.6774	0.6911	0.6728
35	1.0000	1.0000	0.9881	1.0000	0.9772
36	0.7708	0.7583	0.7487	0.7569	0.7402
37	0.7391	0.6789	0.6690	0.6805	0.6639

Table 4.6 Selection results of different methods

Project	Pareto-optimal cross efficiency	Oral et al.(1991)	Green et al. (1996)	Our approach	Budget
35	1.0000	yes	yes	yes	36
17	1.0000	yes	yes	yes	32.1
31	0.8535	yes	yes	yes	44.6
16	0.7758	yes	yes	yes	35.4
36	0.7583	yes	yes	yes	64.1
27	0.7135	yes	yes	yes	57.1
18	0.7108	yes	yes	yes	46.7
34	0.6865	yes	yes	yes	28
37	0.6789	yes	yes	yes	66.4
23	0.6576	yes	yes	yes	75.6
26	0.6318	yes	yes	yes	69.3
1	0.6155	yes	yes	yes	84.2
14	0.6121	yes	yes	yes	95
32	0.6020		yes	yes	54.5
21	0.5656	yes	yes	yes	74.4
29	0.5596	yes	yes	yes	72
11	0.5440				
30	0.5365				
15	0.5327				
12	0.5282		yes	yes	47.5
10	0.5247				
6	0.5225				
2	0.5212				
19	0.4835				
24	0.4785				
22	0.4746				
13	0.4650				
5	0.4581	yes			75.4
4	0.4546				
9	0.4433				
7	0.4323				
8	0.4103				
33	0.4015				
25	0.3565				
28	0.3327				
20	0.3036				
3	0.1738				
Budget sum		956.3	982.9	982.9	1058.3

The R&D project selection results using the Pareto-optimal cross efficiencies are shown in Table 4.6. We also give the results generated by the method of Green, Doyle, and Cook (1996) and Oral et al. (1991). Based on the project selection rule in Green et al. (1996), the projects are chosen by decreasing values of the cross-efficiency scores of the DMUs subject to the requirement that the total budget of the projects cannot exceed 1,000. As can be seen from the results, our method selects the same 17 projects as Green et al.'s. The total budget for this selection is 982.9. Compared to the results obtained by Oral et al.'s method, our method selects project 32 (Pareto-optimal cross efficiency, 0.6020) and project 12 (0.5282) while their method selects project 5 (0.4581) instead. Therefore, within the residual budget, our method can achieve the goal of project selection with projects that have slightly higher cross-efficiency scores than the traditional method proposed by Oral et al. (1991). Furthermore, the total budget of our method is higher than Oral et al.'s method and one more project is given the chance to take part in the program. This indicates that more chances are provided and more resources are used for the candidates in our solution. Additionally, our approach finally evaluates the DMUs with a common set of weights. This will make the evaluation results and the R&D project selection results more likely to be accepted by all the DMUs.

4.3.2 Efficiency evaluation of nursing homes

As listed in Table 4.7, each nursing home has two inputs (X1 and X2) and two outputs (Y1 and Y2) (example from Sexton et al., 1986).

StHr (X1): staff hours per day, including nurses, physicians, etc.

Supp (X2): supplies per day, measured in thousands of dollars.

MCPD (Y1): total Medicare-plus Medicaid-reimbursed patient days.

PPPD (Y2): total privately paid patient days.

Table 4.8 shows the evaluation results of the six nursing homes. Accompanied with it, we show the common weights for the Pareto-optimal cross efficiencies of the DMUs in Table 4.9. As can be seen from the results, the proposed algorithm has improved the original cross-efficiency scores and finally generated larger cross-efficiency scores (in fact, Pareto-optimal cross efficiencies) for all the DMUs.

Table 4.7 Input and output of nursing homes

DMU	Inputs		Outputs	
	StHr(X1)	Supp(X2)	MCPD(Y1)	PPPD(Y2)
A	1.50	0.20	1.40	0.35
B	4.00	0.70	1.40	2.10
C	3.20	1.20	4.20	1.05
D	5.20	2.00	2.80	4.20
E	3.50	1.20	1.90	2.50
F	3.20	0.70	1.40	1.50

Table 4.8 Evaluation results of the six nursing homes

DMUs	CCR efficiency	Pareto-optimal cross efficiency*	Arbitrary	Benevolent	Aggressive
	Model (2.5)	The proposed algorithm	Equation (2.8)	Model (3.1)	Model (3.2)
A	1.0000	1.0000	0.8330	1.0000	0.7639
B	1.0000	0.9848	0.7617	0.9773	0.7004
C	1.0000	0.8464	0.7072	0.8580	0.6428
D	1.0000	1.0000	0.7747	1.0000	0.7184
E	0.9775	0.9765	0.7565	0.9758	0.6956
F	0.8675	0.8607	0.6687	0.8570	0.6081

Table 4.9 The common weights

Inputs & Outputs	Weights
X1	0.11177
X2	0.48178
Y1	0.11596
Y2	0.29050

Furthermore, apart from DMU C, all the other DMUs' Pareto-optimal cross efficiencies are larger than those generated from the benevolent model (3.1). This shows that the efficiency-improving power of the benevolent model (3.1) is weaker than the proposed algorithm. Additionally, the DMUs finally use a common set of weights to make efficiency evaluation in our algorithm, which makes the evaluation

results more acceptable to all the DMUs.

4.4 Conclusions

Because of its good ability in evaluation and ranking of DMUs, DEA cross-efficiency evaluation has been widely applied in various areas. However, not all the DMUs are ready to accept these cross-efficiency scores as their efficiency measurement; they may refuse to admit the scores' validity, because the traditional cross-efficiency scores generally do not constitute Pareto-optimal solutions. To fix this issue, we first proposed a Pareto-optimality estimation model to estimate whether a given set of cross-efficiency scores is Pareto-optimal under the predetermined weight selection principles. We then introduced a cross-efficiency Pareto improvement model to improve the cross-efficiency scores of the DMUs to a Pareto-optimal solution. Finally, based on these two models, an algorithm was proposed to generate cross efficiencies which are proved to be Pareto optimal for the DMUs and cannot be further improved.

Our method brings at least four advantages to cross-efficiency evaluation. Firstly, because of the Pareto optimality of the generated cross efficiencies, they will be more acceptable to all the DMUs. Secondly, the numerical examples show that the proposed algorithm has good power to improve the cross-efficiency scores of the DMUs. Thirdly, if the proposed algorithm stops at step 3, the proposed approach will generate Pareto-optimal cross efficiencies that unify self-evaluation, peer-evaluation, and common-weight evaluation. To be specific, the self-evaluated efficiency and peer-evaluated efficiency for each DMU converge to the same common-weight evaluated efficiency which is a Pareto-optimal cross efficiency. Finally, if the algorithm terminates at step 3, then a common set of weights can be determined which generates Pareto-optimal cross efficiencies, which will give an additional reason for all the DMUs to accept the evaluation results.

We suggest two further research directions to build upon our result. Firstly, the Pareto-optimal cross efficiencies are generated while maintaining two basic principles 4.1 and 4.2. We believe that better cross efficiencies might be found without the constraints of these principles. Secondly, the unification (under certain conditions) of self-evaluation, peer-evaluation, and common-weight-evaluation seen in this paper provides a new research path for cross-efficiency evaluation.

Chapter 5 Cross-bargaining game DEA cross-efficiency evaluation *

This Chapter proposes a cross-bargaining game data envelopment analysis (DEA) cross efficiency evaluation approach to address the non-uniqueness of optimal weights and non-Pareto-optimality of the evaluation result in DEA cross-efficiency evaluation. A cross-bargaining game model is given to obtain optimal weights for calculating cross efficiencies for each pair of DMUs among the group. An algorithm is further provided to transform the proposed model into a series of linear programs. Compared with traditional cross-efficiency evaluation approaches, the proposed approach can not only guarantee the uniqueness of the optimal cross efficiency scores, but also ensures the Pareto-optimality of the evaluation result. Finally, the proposed approach is applied to green supplier selection and the results are compared with those from previous studies.

The rest of this Chapter is organized as follows. Section 5.1 proposes the cross-bargaining game model. An algorithm is given to solve the cross-bargaining game model in Section 5.2. An application to green supplier selection of the proposed approach is presented in Section 5.3. Finally, Section 5.4 concludes this Chapter.

5.1 The cross-bargaining game model

From the benevolent model (3.1), we observe that the optimal weights selection process of each DMU is like a bargaining process. The other DMUs are bargaining to maximize their aggregated efficiency while maintaining the efficiency of the specific DMU at the optimal level. In this chapter, we propose a new cross-efficiency evaluation model, called cross-bargaining model, in which each pair of DMUs among the group bargain with each other to determine a common set of weights for calculating their cross efficiencies. Each DMU needs to negotiate with the other $n-1$ DMUs respectively to determine $n-1$ set of weights for calculating the cross efficiencies for itself and these $n-1$ DMUs. In fact, in each negotiation, the two DMUs involved can be

* This chapter is primarily referenced from: Junfei Chu, Jie Wu, Chengbin Chu. (2018). Cross-bargaining game DEA cross-efficiency evaluation: An application to R&D project selection. Under review at Computers and Industrial Engineering.

regarded as two players in a Nash bargaining game. Therefore, we can incorporate the Nash bargaining game for cross-bargaining of cross efficiencies between every pair of DMUs. Before introducing our model, we give a brief introduction of Nash bargaining game.

Denote the two participants in the bargaining as $N = \{1, 2\}$, the payoff vector of the participants is an element of the payoff space defined as a two-dimensional Euclidean space. Denote S as the set of possible strategies, which is a subset of the payoff space, and b as the breakdown (or disagreement) point which is an element of the payoff space. Then, the bargaining problem can be formulated as a triple (N, S, b) containing the participants, the feasible set, and the breakdown point. Nash (1950, 1953) pointed out that a solution of the two-individual bargaining game should have the properties of Pareto efficiency, invariance with respect to affine transformation (IAT), independence of irrelevant alternatives (IIA), and symmetry. Nash (1950, 1953) proposed the following model which can be used to obtain the unique solution which satisfies all the above-mentioned properties when the feasible set S is convex and compact.

$$\max_{v \in S, v \geq b} \prod_{i=1}^{2} (v_i - b_i) \tag{5.1}$$

In (5.1), v is the payoff vector of the participants, v_i and b_i are the ith elements in vector v and b, respectively.

In the remainder, (U_k^b, W_k^b) denotes the optimal solution of the benevolent model (3.1) when solving it corresponding to DMU k.

In our case, we regard two DMUs, which intend to determine the optimal weights for calculating cross efficiencies, as two players in a bargaining game, the cross efficiency as each DMU's payoff, and the weights for calculating cross efficiencies as strategies. For any pair of DMUs d and k ($k \neq d$), among the group, the cross-bargaining game model is proposed as follows.

$$max \quad \left(\frac{U \cdot Y_d}{W \cdot X_d} - E_{d,k}^{bre}\right)\left(\frac{U \cdot Y_k}{W \cdot X_k} - E_{k,d}^{bre}\right)$$

$$
\begin{aligned}
s.t. \quad & \frac{U \cdot Y_j}{W \cdot X_j} \leq 1, \forall j \\
& \frac{U \cdot Y_d}{W \cdot X_d} \geq E_{d,k}^{bre} \\
& \frac{U \cdot Y_k}{W \cdot X_k} \geq E_{k,d}^{bre} \\
& U, W \geq 0
\end{aligned}
\tag{5.2}
$$

In model (5.2), (U, W) is the set of weights attached to the inputs and outputs. $E_{d,k}^{bre}$ (resp. $E_{k,d}^{bre}$) is the breakdown efficiency of DMU *d* (resp. DMU *k*) corresponding to DMU *k* (resp. DMU *d*). The breakdown efficiency of a DMU is defined as follows.

Definition 5.1. In the two-DMU bargaining process, for any DMU *d*, its breakdown efficiency corresponding to a DMU *k*, $k \neq d$, is defined as

$$
E_{d,k}^{bre} = \frac{U_k^b Y_d}{W_k^b X_d} \tag{5.3}
$$

where U_k^b and W_k^b are the weights selected by DMU *k* when it uses the benevolent strategy.

Definition 5.1 defines the breakdown efficiency of a DMU corresponding to another as its cross efficiency generated by the latter's optimal weights obtained by solving the benevolent model (3.1). This is to say that a DMU would rather accept the other DMU's benevolent cross efficiency if its new cross efficiency given by this DMU is smaller. From model (5.2), we can see that, in the bargaining process, each DMU's efficiency should be kept no smaller than the breakdown efficiency and then the two DMUs bargain with each other to maximize their respective efficiencies.

With respect to model (5.2), we have the following lemmas.

Lemma 5.1. There is always a solution to model (5.2).

Proof. Accompanied with Definition 5.1, it is easy to verify that (U_d^b, W_d^b) is a feasible solution to model (5.2). Q.E.D.

Lemma 5.2. The feasible region of model (5.2) is compact and convex.

Proof. The proof of this lemma is similar to the proof of lemma 1 in Du et al. (2011). We omit the proof here. Q.E.D.

Lemma 5.1 shows that there is always a feasible solution to model (5.2). Lemma 5.2 demonstrates that model (5.2) is compact and convex, which indicates that model (5.2) always provides a unique Nash bargaining solution. With respect to the four properties of the Nash bargaining solution, the solution of model (5.2) has: (i) Pareto efficiency which means that for the pair of DMUs bargaining in model (5.2), no one can improve its cross efficiency corresponding to the other without decreasing the other DMD's; (ii) IAT which indicates that if both the breakdown efficiencies and the feasible region of model (5.2) satisfy an affine transformation in the payoff space, the cross efficiencies of the two DMUs are also subjected to this affine transformation; (iii) IIA which reveals that the cross efficiencies generated by model (5.2) will not change when the feasible region of model (5.2) decreases while the bargaining solution still remains in; (iv) symmetry which demonstrates that if the feasible region and the breakdown efficiencies of the two DMUs are symmetric, then the cross efficiencies obtained by model (5.2) for the two participating DMUs are the same.

In the remainder, let $(U_{j,k}^*, W_{j,k}^*)$ be the optimal solution of model (5.2) for the pair of DMUs j and k and (U_j^*, W_j^*) the optimal solution of the CCR model with respect to DMUj. We can define the cross-bargaining cross-efficiency score as follows.

Definition 5.2. For each DMUj,

$$E_j^{cbcro} = \frac{1}{n}\left(\frac{U_j^* \cdot Y_j}{W_j^* \cdot X_j} + \sum_{k=1,k\neq j}^{n} \frac{U_{j,k}^* \cdot Y_j}{W_{j,k}^* \cdot Y_j}\right) \tag{5.4}$$

is defined as its ***cross-bargaining cross-efficiency score***.

Similar to the traditional cross-efficiency evaluation approach, the cross-bargaining cross-efficiency score of each DMU is obtained by averaging the self-evaluated efficiency (CCR efficiency) and the peer-evaluated efficiencies (cross efficiencies). However, unlike the traditional cross-efficiency evaluation approach in which each DMU uses its most favorable weights for calculating the cross efficiencies of the other DMUs, our evaluation model obtains the cross efficiencies of a pair of DMUs corresponding to each other through bargaining between them. This indicates that each DMU may use different sets of weights for calculating the other DMUs' cross efficiencies. In addition, the set of weights used for calculating the cross efficiencies between two DMUs are the same, i.e., they both use the optimal weights, obtained by

solving model (5.2) corresponding to them, to calculate the other DMU's cross efficiencies. This kind of evaluation mode is on a certain level more acceptable to the DMUs, because each pair of DMUs have reached an agreement on the set of weights to be used for calculating each other's cross efficiencies. With respect to model (5.2), we have the following theorems.

Theorem 5.1. For each DMU j, its cross-bargaining cross-efficiency score E_j^{cbcro} is unique.

Proof. According to the Nash bargaining theorem, if the feasible set of model (5.2) is convex, there exists only one solution $\frac{U_{j,k}^{*} \cdot Y_j}{W_{j,k}^{*} \cdot X_j}$, when solving model (5.2) with any DMU k, that satisfies the properties of Nash solutions. Therefore, with respect to any DMU k, $\frac{U_{j,k}^{*} \cdot Y_j}{W_{j,k}^{*} \cdot X_j}$ is unique. In addition, the CCR efficiency of each DMU is also unique according to model (1). Therefore, E_j^{cbcro} defined in (5.4) is unique. Q.E.D.

Theorem 5.2. The solution composed of E_j^{cbcro}, $1 \leq j \leq n$, is Pareto optimal.

Proof. This theorem can be easily proved based on Pareto efficiency of the Nash solution of Nash bargaining games. We omit the proof here. Q.E.D.

Theorem 5.1 indicates that our approach obtains a unique cross-bargaining cross-efficiency score for each DMU, which makes it unnecessary to consider the non-uniqueness of optimal weights as in the traditional cross-efficiency evaluation method. Theorem 5.2 reveals that the final cross-bargaining cross-efficiency scores obtained for the DMUs constitute a Pareto optimal solution. This means that no DMU can improve its cross-bargaining cross-efficiency score without reducing that of at least one of the others. The Pareto optimality of the evaluation results makes the evaluation results more satisfactory and acceptable to all the DMUs.

5.2 An algorithm and a numerical example

In this section, we first give an algorithm to solve the proposed model (5.2) by solving a series of linear programs. Then a numerical example is provided to illustrate the proposed approach.

5.3.1 An algorithm to solve model (5.2)

Note that model (5.2) is a nonlinear program which is hard to be solved directly. We now propose an algorithm to transform it into a series of linear programs. With respect to the notation in model (5.2), let $t = \frac{1}{W \cdot X_d}$, $M = tU$, $\Omega = tW$, and $\alpha = \frac{M \cdot Y_k}{\Omega \cdot X_k}$. Then, model (5.2) can be transformed into a series of linear programs parameterized by $\alpha \in [E_{k,d}^{bre}, E_k^{ccr}]$ defined in (5.5).

$$
\begin{aligned}
F(\alpha) = \quad & \max \left(\alpha - E_{k,d}^{bre}\right) M \cdot Y_d - \alpha E_{d,k}^{bre} + E_{k,d}^{bre} E_{d,k}^{bre} \\
s.t. \quad & M \cdot Y_j - \Omega \cdot X_j \leq 0, \forall j \\
& M \cdot Y_d \geq E_{d,k}^{bre} \\
& \Omega \cdot X_d = 1 \\
& M \cdot Y_k - \alpha \Omega \cdot X_k \geq 0 \\
& M, \Omega \geq 0
\end{aligned}
\tag{5.5}
$$

Note that the constraint $M \cdot Y_k - E_{k,d}^{bre} \Omega \cdot X_k \geq 0$ is omitted, because it becomes redundant when $M \cdot Y_k - \alpha \Omega \cdot X_k = 0$ and $\alpha \in [E_{k,d}^{bre}, E_k^{ccr}]$. To avoid infeasibility of for some values of α, constraint $M \cdot Y_k - \alpha \Omega \cdot X_k \geq 0$ is used instead of the equality constraint.

The optimal objective value of (5.5) is a function of α. Therefore, solving model (5.2) is equivalent to finding out a value of α so that $F(\alpha)$ is maximized. It should be noted here that it is possible that $E_{k,d}^{bre} = E_k^{ccr}$ for some DMU *k*. This is to say that DMU *d* gives DMU *k* a cross efficiency that is equal to its CCR efficiency. For such a pair of DMUs *d* and *k*, their cross efficiencies corresponding to each other are both at the most favorable CCR efficiency level. So, there is no need to solve model (5.2) for such a pair. Now, we give the following algorithm which can be used to find the optimal solution of model (5.2) based on model (5.5).

Algorithm

Begin

Step 1: Let $t = 0$, $\alpha = E_{k,d}^{bre}$, $\epsilon = \frac{E_k^{ccr} - E_{k,d}^{bre}}{N}$, where N is a parameter of the algorithm.

Solve model (5.5) with the current value of α and obtain the optimal objective value $F^t(\alpha)$ and the optimal solution $(M'_{d,k}, \Omega'_{d,k})$. Let $F^* = F^t(\alpha)$, $\left(M_{d,k}^*, \Omega_{d,k}^*\right) =$

$(M'_{d,k}, \Omega'_{d,k})$.

Step 2: Let $e = \frac{M^*_{d,k}Y_k}{\Omega^*_{d,k}X_k}$. If $e > \alpha$, let $t = t + [e - \alpha]/\epsilon$, $\alpha = E^{bre}_{k,d} + t\epsilon$. If $e = \alpha$, let $t = t + 1$, $\alpha = E^{bre}_{k,d} + t\epsilon$.

Step 3: Solve model (5.5) to calculate $F^t(\alpha)$ and the optimal solution $(M'_{d,k}, \Omega'_{d,k})$. If $F^t(\alpha) > F^*$, let $F^* = F^t(\alpha)$ and $\left(M^*_{d,k}, \Omega^*_{d,k}\right) = (M'_{d,k}, \Omega'_{d,k})$.

Step 4: If $t \geq N$, stop. The obtained $\left(M^*_{d,k}, \Omega^*_{d,k}\right)$ is the optimal solution of model (5.2). Otherwise, go step 2.

End

In this work, we set $N = 10{,}000$. It is easy to see from the algorithm that it will always find an optimal solution to model (5.2) with a proper small error (which depends on the parameter N in the algorithm), although we relax a constraint in the transformation. It can be seen that the algorithm needs to only solve linear programs to obtain the optimal weights of model (5.2).

5.3.2 A numerical example

Now, we give a small numerical example to illustrate the proposed model and algorithm. The numerical example (Liang et al. 2008b) contains five DMUs. Each DMU uses three inputs to produce two outputs. Please find the raw data of this numerical example in Table 2.1 in Chapter 2.

We use the CCR model, the traditional cross-efficiency evaluation method, the benevolent and aggressive model, and the proposed cross-bargaining game model to evaluate the DMUs. The results are listed in the following Table 5.1.

As it can be seen from the evaluation result, firstly, the CCR model cannot make a full discrimination among the DMUs. It evaluates both DMUs 2 and 3 as DEA efficient and cannot make any further discrimination among them. Secondly, all the cross-efficiency evaluation methods make full discrimination among the DMUs. They all rank the DMUs in a unique order. Thirdly, consistent with the different strategies of the models, the benevolent cross-efficiency score generated for each DMU is larger than its aggressive cross-efficiency score, and the arbitrary cross-efficiency score is between the aggressive and benevolent cross-efficiency scores for each DMU. Fourthly,

for each DMU, its cross-bargaining cross-efficiency score generated by our model is generally larger than the cross-efficiency scores generated by other models. This is because our new cross-bargaining cross-efficiency evaluation model allows each pair of DMUs to bargain with each other for the Pareto optimal cross efficiencies. Additionally, our approach ensures the uniqueness of the cross-bargaining cross-efficiency scores, which makes the results more acceptable to all DMUs.

Table 5.1 Results of the numerical example

DMUs	CCR efficiency	Cross-bargaining game	Arbitrary	Benevolent	Aggressive
	Model (2.5)	Model (5.2)	Equation (2.8)	Model (3.1)	Model (3.2)
DMU1	0.6857	0.6104	0.5453	0.5616	0.4473
DMU2	1.0000	0.9714	0.8629	0.9295	0.8895
DMU3	1.0000	1.0000	1.0000	1.0000	0.9571
DMU4	0.8571	0.8166	0.5767	0.6671	0.5843
DMU5	0.8571	0.7479	0.5614	0.5871	0.5186

To see how the proposed algorithm works, we take the calculating process of model (5.2) for DMUs 4 and 5 as an example and show the details in Figure 5.1.

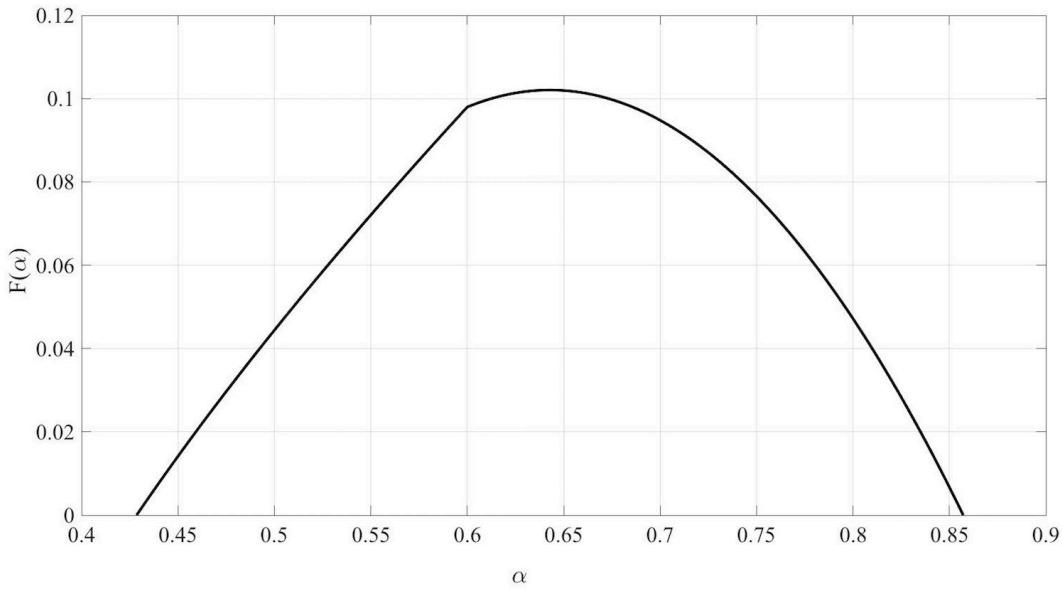

Figure 5.1 Calculating process of the algorithm

It can be seen from Figure 5.1 that the optimal objective value $F(\alpha)$ of model (5.2) varies with the gradual increment of α from 0.5871 to 0.8571. The objective value achieves the peak when $\alpha = 0.6429$. At this time, we obtain the optimal solution to model (5.2) corresponding to DMUs 4 and 5.

5.3 Application to green supplier selection

Recently, DEA has been applied for green supplier selection in supply chain management. The main idea is to regard the multiple management criteria and green criteria of the suppliers as inputs and outputs, respectively, and then evaluate and rank the suppliers using alternative DEA models to select the best suppliers. Some studies have also considered the situations with imprecise data and fuzzy data. Relative studies can be seen in Karsak and Dursun (2014), Dobos and Vörösmarty (2014), Azadeh and Zarrin(2016), Fallahpour et al. (2017), and Dobos and Vörösmarty (2018). In this chapter, we also apply the proposed cross-bargaining game cross-efficiency evaluation approach for green supplier selection and compare it with the previous studies.

5.3.1 Case background and data

The case is taken from Dobos and Vörösmarty (2018), where 18 suppliers need to be evaluated. Each supplier is measured by three management criteria and two green criteria. The management criteria and two green criteria are regarded as inputs and outputs, respectively. Additionally, Dobos and Vörösmarty (2018) also incorporated one more input, inventory-related cost (or EOQ-related cost), which is calculated according to the lot size. The input and output indicators of the suppliers are listed as follows.

Inputs (Management criteria):

X1: Lead time (hours);

X2: Product quality (%);

X3: Quoted price (monetary unit);

X4 (q): The inventory cost when the lot size is q.

Outputs (Green criteria):

Y1: Reusability level (%);

Y2: CO2 emissions (g/unit product).

As it is known in DEA calculations, the inputs and outputs are the smaller the better and the larger the better, respectively. While input X2 (product quality) is the larger the better and the undesirable output Y2 (CO2 emissions) is the smaller the better. To handle this, Dobos and Vörösmarty (2018) used data transformation method (using the inverse of the data corresponding to these indicator). Here, we use the same data transformation method for the convenience of comparing the results with their model's. The data and descriptive statistical analysis results are listed in the following Table 5.2.

Table 5.2 Raw data and descriptive statistical analysis of the suppliers

Supplier	Inputs							Outputs	
1	X1	1./X2	X3	X4(50)	X4(100)	X4(150)	X4(200)	Y1	1./Y2
1	48	0.0125	20	90	120	163	210	70	0.0333
2	24	0.0143	30	155	190	252	320	50	0.1000
3	72	0.0111	50	245	310	415	530	60	0.0667
4	36	0.0118	10	65	70	88	110	40	0.0500
5	60	0.0133	25	202.5	195	234	285	65	0.0286
6	48	0.0105	40	220	260	340	430	90	0.0400
7	72	0.0125	15	137.5	125	146	175	75	0.0667
8	36	0.0118	35	267.5	265	323	395	85	0.0500
9	24	0.0143	35	147.5	205	283	365	55	0.1000
10	60	0.0133	40	180	240	327	420	45	0.1000
11	84	0.0111	25	182.5	185	228	280	80	0.0400
12	48	0.0154	15	137.5	125	146	175	50	0.0500
13	72	0.0118	30	235	230	278	340	75	0.0667
14	36	0.0143	45	252.5	295	384	485	85	0.0500
15	24	0.0154	20	170	160	190	230	75	0.0667
16	48	0.0143	50	265	320	422	535	80	0.1000
17	24	0.0111	10	105	90	102	120	85	0.0667
18	72	0.0118	25	182.5	185	228	280	75	0.0500
Max	84	0.0154	50	267.5	320	422	535	90	0.1000
Min	24	0.0105	10	65.00	70	88	110	40	0.0286
Average	49.33	0.0128	28.89	180.00	198.33	252.59	315.83	68.89	0.0625
Std.dev	19.69	0.0016	12.78	60.32	74.62	102.29	132.43	15.39	0.0236

5.3.2 Evaluation result and discussion

To compare the proposed models with the previous models, we consider the situation where the lot size is 50 and evaluate the suppliers using the CCR model, the benevolent and aggressive cross-efficiency evaluation models and the proposed cross-bargaining game model. The evaluation and ranking results are listed in Table 5.3.

Table 5.3 Evaluation and ranking results of the suppliers

Supplier	E_j^*	E_j^{cbcro}	Arbitrary	Benevolent	Aggressive
	Model (2.5)	Model (5.2)	Equation (2.8)	Model (3.1)	Model (3.2)
1	0.9608 (8)	0.7650 (14)	0.5841 (15)	0.5928 (16)	0.5245 (16)
2	1.0000 (1)	0.9230 (4)	0.7373 (5)	0.7721 (4)	0.7175 (3)
3	0.8888 (13)	0.7986 (12)	0.6153 (14)	0.6816 (10)	0.5416 (14)
4	1.0000 (1)	0.7895 (13)	0.6161 (13)	0.6077 (15)	0.5620 (12)
5	0.6136 (17)	0.5487 (17)	0.4339 (18)	0.4497 (18)	0.3917 (18)
6	1.0000(1)	0.8980 (5)	0.7053 (6)	0.7506 (6)	0.6323 (7)
7	0.8569 (14)	0.8222 (9)	0.7387 (4)	0.7653 (5)	0.6656 (5)
8	0.9010 (10)	0.8513 (7)	0.6753 (8)	0.7178 (9)	0.6198 (8)
9	1.0000 (1)	0.9266 (2)	0.7612 (2)	0.7980 (3)	0.7357 (2)
10	1.0000 (1)	0.8868 (6)	0.6660 (9)	0.7260 (8)	0.5897 (10)
11	0.8917 (12)	0.8053 (10)	0.6317 (12)	0.6617 (13)	0.5577 (13)
12	0.5596 (18)	0.5245 (18)	0.4636 (17)	0.4711 (17)	0.4300 (17)
13	0.8988 (11)	0.8413 (8)	0.6902 (7)	0.7391 (7)	0.6089 (9)
14	0.7628 (16)	0.7171 (16)	0.5787 (16)	0.6133 (14)	0.5363 (15)
15	0.9311 (9)	0.8007 (11)	0.6591 (10)	0.6749 (11)	0.6439 (6)
16	1.0000 (1)	0.9234 (3)	0.7389 (3)	0.8059 (2)	0.6763 (4)
17	1.0000 (1)	1.0000 (1)	0.9888 (1)	1.0000 (1)	0.9468 (1)
18	0.7993 (15)	0.7535 (15)	0.6354 (11)	0.6682 (12)	0.5634 (11)

In Table 5.3, the numbers in brackets are the ranking positions of the suppliers. From the evaluation results, we can see that similar results can be obtained for comparisons of different methods as those obtained in the numerical example. All the cross-efficiency evaluation methods can fully discriminate all the suppliers and rank them in different positions. The cross-bargaining cross efficiency score of each supplier is the highest among all cross-efficiency scores generated by different models. This is

because of the new cross-efficiency evaluation model and the Pareto-optimality of the evaluation results in our method.

However, some suppliers get different ranking positions with different methods. For example, supplier 2 is ranked the 4th based on the scores generated by our method, while it is ranked the 5th, 4th, and 3th with the arbitrary, benevolent, and aggressive strategies, respectively. To get further insights into the ranking results of different cross-efficiency evaluation methods and to see whether these ranking results correlate with each other, we make Spearman's rank correlation coefficient test among the ranking results of the different methods. The results of this test are shown in Table 5.4.

Table 5.4 Correlation analysis results

Methods	Our method	Arbitrary	Benevolent	Aggressive
Our method	1.0000	0.9319	0.9340	0.8947
Arbitrary		1.0000	0.9649	0.9670
Benevolent			1.0000	0.9298
Aggressive				1.0000

According to the results listed in Table 5.4, we can see that the cross-efficiency ranking results of different methods are positively and highly correlated with each other. This indicates that all the cross-efficiency evaluation methods generate similar ranking results for the suppliers and they can all be used for evaluating and ranking them. However, it is known that the benevolent, aggressive, and arbitrary cross-efficiency evaluation strategies cannot guarantee the uniqueness of the evaluation result while our method provides a unique cross-bargaining cross-efficiency score for each supplier. In addition, the final cross-bargaining cross-efficiency scores generated by our approach constitute a Pareto-optimal solution, which makes our evaluation and ranking results more acceptable to all the suppliers.

Finally, if we refer to the cross-efficiency evaluation results of the cross-efficiency methods, we can see that all the methods rank supplier 17 as the best. Therefore, according to these results, supplier 17 should be selected.

5.3.3 Green supplier selection considering lot size

Dobos and Vörösmarty (2018) proposed to use the CCR efficiency to rank the suppliers to select the best. They further define the lot size as a parameter to see whether the selected suppliers would change or not. The suppliers always ranked at the first position with different lot sizes will be selected as the best supplier. Here, we also apply our method to do this and compare the selection results with Dobos and Vörösmarty's. The evaluation and selection results of the CCR efficiencies and the cross-bargaining game cross-efficiency scores of the suppliers with different lot sizes are listed in Tables 5.5 and 5.6, respectively.

Table 5.5 Evaluation and selection result of Dobos and Vörösmarty (2018)

Supplier	Lot sizes (units)			
	50	100	150	200
1	0.9608	0.7243	0.7177	0.7152
2	1.0000	1.0000	1.0000	1.0000
3	0.8888	0.8875	0.8875	0.8875
4	1.0000	0.9638	0.8628	0.8178
5	0.6136	0.6130	0.6130	0.6130
6	1.0000	1.0000	1.0000	1.0000
7	0.8569	0.8578	0.8578	0.8578
8	0.9010	0.9045	0.9045	0.9045
9	1.0000	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000
11	0.8917	0.8923	0.8923	0.8923
12	0.5596	0.5410	0.5379	0.5370
13	0.8988	0.9019	0.9019	0.9019
14	0.7628	0.7643	0.7643	0.7643
15	0.9311	0.9311	0.9311	0.9311
16	1.0000	1.0000	1.0000	1.0000
17	1.0000	1.0000	1.0000	1.0000
18	0.7993	0.8029	0.8029	0.8029

Table 5.6 Evaluation result of our approach

Supplier	Lot sizes (units)			
	50	100	150	200
1	0.7650	0.7676	0.6498	0.6483
2	0.9230	0.9232	0.9231	0.9174
3	0.7986	0.7976	0.7966	0.7903
4	0.7895	0.7912	0.7090	0.7062
5	0.5487	0.5477	0.5457	0.5472
6	0.8980	0.8982	0.9001	0.9002
7	0.8222	0.8014	0.8261	0.8260
8	0.8513	0.8548	0.8573	0.8571
9	0.9266	0.9274	0.9261	0.9257
10	0.8868	0.8877	0.8963	0.8962
11	0.8053	0.8162	0.8046	0.8085
12	0.5245	0.5250	0.5128	0.5116
13	0.8413	0.8435	0.8536	0.8539
14	0.7171	0.7146	0.7183	0.7179
15	0.8007	0.8037	0.8047	0.8044
16	0.9234	0.9289	0.9342	0.9343
17	1.0000	1.0000	1.0000	1.0000
18	0.7535	0.7605	0.7583	0.7583

From the evaluation results listed in Tables 5.5 and 5.6, we can see that when the lot size changes, the efficiencies generated for each supplier by each method does not vary very much. Dobos and Vörösmarty's approach consistently rank suppliers 2, 6, 9, 10, 16, and 17 as efficient and the first-ranked suppliers. Their approach cannot further select the best among these suppliers. While our approach always ranks supplier 17 as the best although the lot size changes values. Additionally, we can see that no matter how large the lot size is, our approach can always fully rank all the suppliers. Therefore, even if the decision makers need to select more than 1 supplier, they can always identify the corresponding suppliers with better performance at specific lot sizes. While Dobos and Vörösmarty's approach cannot do this if fewer suppliers than considered as efficient need to be selected.

5.4 Conclusions

Aiming at addressing the non-uniqueness and non-Pareto-optimality of evaluation results in DEA cross-efficiency evaluation, we proposed a cross-bargaining game DEA cross-efficiency evaluation approach. Firstly, we introduce a new cross-efficiency evaluation model in which each pair of DMUs determine a common set of weights for computing cross efficiencies corresponding to each other. Secondly, we incorporated the Nash bargaining game theory and proposed a cross-bargaining game model which can be used to determine the set of weights for calculating the cross-efficiency scores of the DMUs. In addition, an algorithm was presented to solve the cross-bargaining game model by solving a series of linear programs. Finally, the proposed approach was applied to green supplier selection.

Our approach brings at least three advantages to DEA cross-efficiency evaluation. Firstly, in the new cross-efficiency evaluation model, each pair of DMUs can reach an agreement on the set of weights to be used for calculating each other's cross efficiencies, which makes the evaluation results more acceptable to the DMUs. Secondly, our approach guarantees the uniqueness of the set of cross-bargaining cross-efficiency scores, which makes it unnecessary to consider the non-uniqueness of optimal weights problem as in the traditional DEA models. Finally, the set of cross-bargaining cross-efficiency scores constitutes a Pareto-optimal solution, which once more increases the DMUs' motivation to accept the evaluation results.

Two further research directions can be drawn from this study. Firstly, the proposed approach assumes that the DMUs are cooperative with each other, and the model is presented from a cooperative game perspective. Further studies may consider competitions among the DMUs and propose cross-efficiency evaluation methods from non-cooperative game perspective. Secondly, our approach addresses the non-uniqueness of the optimal weights by directly determining a unique set of cross-efficiency scores, which provides a new perspective that can be further explored.

Chapter 6 DEA cross-efficiency evaluation based on satisfaction degree: an application to technology selection *

Existing studies on DEA cross-efficiency evaluation mainly focus on the non-uniqueness of optimal weights. Few studies have considered the DMUs willingness to accept the cross-efficiency evaluation results. To address this issue, in this Chapter, we introduce the concept of the satisfaction degree of a DMU toward a set of optimal weights for another DMU. Then, a new DEA cross-efficiency evaluation approach, which contains a max-min model and two algorithms, is proposed based on the satisfaction degrees of the DMUs. Our max-min model and algorithm 1 can obtain an optimal set of weights for each DMU that maximizes the least satisfaction degree among all the other DMUs. Further, our algorithm 2 can then be used to guarantee the uniqueness of the optimal weights for each DMU. Finally, our approach is applied to a real-world case study on technology selection.

The rest of this Chapter is organized as follows. Section 6.1 defines the concept of satisfaction degree. The max-min weights selection model is given in Section 6.2. Two algorithms and a numerical example are given in Section 6.3. Further, a case study on technology selection is discussed in Section 6.4. Finally, Section 6.5 concludes this Chapter.

6.1 The satisfaction degree

In this section, we introduce the concept of *satisfaction degree* of a DMU toward (i.e. in relation to) a set of optimal weights selected by the other DMUs. For each DMU *d*, if its optimal weights selected by the CCR model is not unique, its possible optimal weights set can be defined as the following WS_d as mentioned in Chapter 2.

$$WS_d = \{(U, W) | W \cdot X_d = 1 \\ U \cdot Y_d - E_d^* \times W \cdot X_d = 0 \tag{6.1}$$

* This chapter is primarily referenced from: Jie Wu, Junfei Chu, Qingyuan Zhu, Pengzhen Yin, and Liang Liang. (2018). DEA cross-efficiency evaluation based on satisfaction degree: an application to technology selection. *International Journal of Production Research*, *54*(20), 5990-6007.

$$U \cdot Y_j - W \cdot X_j \leq 0, \forall j$$
$$U, W \geq 0\}$$

Based on the possible optimal weight sets WS_d of DMU d, we can calculate the maximum and minimum cross-efficiencies of any other DMU k, respectively denoted as E_{dk}^{max} and E_{dk}^{min}, corresponding to DMU d using the following model (6.2) and (6.3).

$$\begin{aligned} E_{dk}^{max} &= \max U \cdot Y_k \\ s.t. \quad & W \cdot X_k = 1 \\ & U \cdot Y_d - E_d^* \times W \cdot X_d = 0 \\ & U \cdot Y_j - W \cdot X_j \leq 0, \forall j \\ & U, W \geq 0\} \end{aligned} \tag{6.2}$$

and

$$\begin{aligned} E_{dk}^{min} = \quad & \min U \cdot Y_k \\ s.t. \quad & the\ same\ as\ those\ in\ model\ (6.2) \end{aligned} \tag{6.3}$$

In models (6.2) and (6.3), we change the constraint which is used to avoid the trivial solution from $W \cdot X_d = 1$ to $W \cdot X_k = 1$. This will not affect the weights selection for the DMUs because the optimal weights in DEA reflect only the relative importance a DMU attaches to its inputs and outputs when making efficiency evaluation (Charnes and Cooper, 1962). Based on the calculation results of models (6.2) and (6.3), we can equivalently transform the possible optimal weights set WS_d to the following (6.4).

$$\begin{aligned} WS_d^{trans} = \{(U, W) | & U \cdot Y_d - E_d^* \times W \cdot X_d = 0 \\ & W \cdot X_d = 1 \\ & U \cdot Y_j - W \cdot X_j \leq 0, \forall j \\ & U \cdot Y_j - E_{dj}^{max} \times W \cdot X_j + s_{dj} = 0, \forall j, j \neq d \\ & U \cdot Y_j - E_{dj}^{min} \times W \cdot X_j - \varphi_{dj} = 0, \forall j, j \neq d \\ & U, W \geq 0 \\ & s_{dj}, \varphi_{dj} \geq 0, \forall j\} \end{aligned} \tag{6.4}$$

Note that the constraint $U \cdot Y_j - W \cdot X_j \leq 0, \forall j$ can be omitted since it becomes redundant because of constraints $U \cdot Y_j - E_{dj}^{max} \times W \cdot X_j + s_{dj} = 0, \forall j, j \neq d$ and $U \cdot Y_d - E_d^* \times W \cdot X_d = 0$. We keep this redundant constraint here because in what follows

the transformation of WS_d^{trans} will require it. It is easy to see that WS_d^{trans} is equivalent to WS_d because the cross-efficiencies of any DMU j generated by DMU d's optimal weights will be between the E_{dj}^{min} and E_{dj}^{max}.

When DMU d tries to select a set of optimal weights from WS_d^{trans}, any other DMU j will prefer that its cross-efficiency corresponding to DMU d, generated by the newly selected set of optimal weights, be close to the maximum possible value E_{dj}^{max}. Based on this observation, we give a definition to characterize the degree to which DMU j is satisfied with the set of optimal weights selected by DMU d.

Definition 6.1. The ***satisfaction degree*** of DMU j toward the set of optimal weights $(U, W) \in WS_d^{trans}$ is defined as

$$SD_{dj} = \frac{U \cdot Y_j / W \cdot X_j - E_{dj}^{min}}{E_{dj}^{max} - E_{dj}^{min}}, \quad \forall j: E_{dj}^{max} \neq E_{dj}^{min} \tag{6.5}$$

It can be seen from (6.5) that $SD_{dj} \in [0,1]$. If the new set of optimal weights of DMU d generates for DMU j its maximum cross-efficiency E_{dj}^{max}, then $SD_{dj} = 1$. If the new set of optimal weights of DMU d generates for DMU j its minimum cross-efficiency E_{dj}^{min}, then $SD_{dj} = 0$.

It should be noted here that it is possible that $E_{dj}^{max} = E_{dj}^{min}$ for some DMU j corresponding to DMU d. This indicates that the cross-efficiency of DMU j corresponding to DMU d will be unchanged no matter which set of optimal weights is selected by DMU d. There is no need for DMU d to consider the cross-efficiencies of such DMUs when selecting a new set of optimal weights. In the remainder of this Chapter, let $\Omega_d = \{j \neq d | E_{dj}^{max} \neq E_{dj}^{min}\}$.

6.2 The max-min weights selection model based on satisfaction degree

In this section, we introduce a max-min model which is used for optimal weights selection for the DMUs. Firstly, we give the following Theorem 6.1.

Theorem 6.1. For any $(U, W) \in WS_d^{trans}$, we have $SD_{dj} = \frac{U \cdot Y_j / W \cdot X_j - E_{dj}^{min}}{E_{dj}^{max} - E_{dj}^{min}} = \frac{\varphi_{dj}}{s_{dj} + \varphi_{dj}}$, $\forall j \in \Omega_d$ where φ_{dj} and s_{dj} are the slacks defined in (6.4).

Proof. From the first and second constraint groups in (6.4), we have that $U \cdot Y_j -$

$E_{dj}^{max} \times W \cdot X_j + s_{dj} = 0, \forall j, j \neq d$ and $U \cdot Y_j - E_{dj}^{min} \times W \cdot X_j - \varphi_{dj} = 0, \forall j, j \neq d$. Then, we can obtain through transformation that (a) $E_{dj}^{max} = \frac{U \cdot Y_j + s_{dj}}{W \cdot X_j}$ and (b) $E_{dj}^{min} = \frac{U \cdot Y_j - \varphi_{dj}}{W \cdot X_j}$. In Definition 6.1, we have (c) $SD_{dj} = \frac{U \cdot Y_j / W \cdot X_j - E_{dj}^{min}}{E_{dj}^{max} - E_{dj}^{min}}, \forall j \in \Omega_d$. So, by substituting (a) and (b) into (c) we can get $SD_{dj} = \frac{\varphi_{dj}}{s_{dj} + \varphi_{dj}}$. Q.E.D.

From Theorem 6.1, we know that the satisfaction degree of DMU j toward DMU d's optimal weights can be compactly presented as $SD_{dj} = \frac{\varphi_{dj}}{s_{dj} + \varphi_{dj}}, \forall j \in \Omega_d$.

In this chapter, we propose to select optimal weights for the DMUs in order to enhance all the DMUs' satisfaction degree. Therefore, when a DMU selects a unique optimal set of weights, it tries to maximize the satisfaction degrees of all the other DMUs, although it is generally impossible for a DMU d to select a set of optimal weights that can make all the satisfaction degrees equal to 1. Also, we believe that the new set of optimal weights selected by each DMU d should not generate satisfaction degrees that have large differences from each other. This is because large differences among the satisfaction degrees of the DMUs will reduce their willingness in accepting the set of optimal weights for peer-evaluation, especially for those DMUs whose satisfaction degrees are relatively low. Therefore, we propose to use the following model (6.7) for optimal weights selection for each DMU d.

$$
\begin{aligned}
& \max_{(U,W)} \min_{j \in \Omega_d} \frac{\varphi_{dj}}{s_{dj} + \varphi_{dj}} \\
s.t. \quad & U \cdot Y_j - E_{dj}^{max} \times W \cdot X_j + s_{dj} = 0, \forall j \in \Omega_d \\
& U \cdot Y_j - E_{dj}^{min} \times W \cdot X_j - \varphi_{dj} = 0, \forall j \in \Omega_d \\
& U \cdot Y_d - E_d^{*} \times W \cdot X_d = 0 \\
& W \cdot X_d = 1 \\
& U \cdot Y_j - W \cdot X_j \leq 0, \forall j \\
& U, W \geq 0 \\
& s_{dj}, \varphi_{dj} \geq 0, \forall j \in \Omega_d
\end{aligned}
\tag{6.7}
$$

In model (6.7), the first and second constraint groups guarantee that the cross-

efficiency of each DMU $j \in \Omega_d$ is between its maximum and minimum cross-efficiencies corresponding to DMU d. The third and fourth constraint groups ensure that the efficiency of the DMU d under evaluation must reach its CCR efficiency level. As can be seen from model (6.7), when DMU d determines its set of optimal weights, it maximizes the least satisfaction degree among all the other DMUs while keeping its own efficiency at its CCR efficiency level. Therefore, model (6.7) can be also regarded as a weights selection model that maximizes all the other DMUs' satisfaction degrees when DMU d selects optimal weights (Li, et al., 2013).

The min operation in the objective function of model (6.7) makes it complex. Therefore, we let $SD = \min_{j \in \Omega_d} \frac{\varphi_{dj}}{s_{dj}+\varphi_{dj}}$ and transform it into the following model (6.8).

$$
\begin{aligned}
\max \quad & SD \\
s.t. \quad & U \cdot Y_j - E_{dj}^{max} \times W \cdot X_j + s_{dj} = 0, \forall j \in \Omega_d \\
& U \cdot Y_j - E_{dj}^{min} \times W \cdot X_j - \varphi_{dj} = 0, \forall j \in \Omega_d \\
& U \cdot Y_d - E_d^* \times W \cdot X_d = 0 \\
& W \cdot X_d = 1 \\
& U \cdot Y_j - W \cdot X_j \leq 0, \forall j \\
& \frac{\varphi_{dj}}{s_{dj} + \varphi_{dj}} \geq SD, \forall j \in \Omega_d \\
& U, W \geq 0 \\
& s_{dj}, \varphi_{dj} \geq 0, \forall j \in \Omega_d
\end{aligned}
\tag{6.8}
$$

By solving model (6.8) for each DMU d, then we can generate for each DMU d an optimal set of weights that maximizes the minimum of the DMUs' satisfaction degrees.

6.3 The algorithms

In this section, we present two algorithms. The first algorithm is used for solving model (6.8) linearly. The second one can be used for ensuring that the final optimal set of weights for each DMU d is unique.

6.3.1 An algorithm to solve (6.8) linearly

Model (6.8) is still a nonlinear program. We propose Algorithm 1 below to solve

it by solving a linear program. Firstly, we give the following model (6.9).

$$
\begin{aligned}
\min \quad & \beta \\
s.t. \quad & U \cdot Y_j - E_{dj}^{max} \times W \cdot X_j + s_{dj} = 0, \forall j \in \Omega_d \\
& U \cdot Y_j - E_{dj}^{min} \times W \cdot X_j - \varphi_{dj} = 0, \forall j \in \Omega_d \\
& U \cdot Y_d - E_d^* \times W \cdot X_d = 0 \\
& W \cdot X_d = 1 \\
& U \cdot Y_j - W \cdot X_j \leq 0, \forall j \\
& \varphi_{dj} - SD \times \left(s_{dj} + \varphi_{dj}\right) + \eta_{dj} = 0, \forall j \in \Omega_d \\
& \eta_{dj} \leq \beta, \forall j \in \Omega_d \\
& U, W \geq 0 \\
& s_{dj}, \varphi_{dj} \geq 0, \forall j \in \Omega_d
\end{aligned}
\tag{6.9}
$$

Let SD_d^* denote the optimal objective value of model (6.8). From models (6.8) and (6.9), we can obtain the following theorems. Together these theorems let us use binary search to find a solution for model (6.8), where each step of the search involves solving model (6.9) once.

Theorem 6.2. Let β_d' be the optimal objective value of model (6.9) for a value $SD_d' \in [0,1]$ for SD. $SD_d^* \geq SD_d'$ if and only if $\beta_d' \leq 0$.

Proof. Assume that the optimal solution of model (6.9) is $(U_d', W_d', s_{dj}', \varphi_{dj}', \eta_{dj}', \beta_d', j \in \Omega_d)$ when solving it with $SD = SD_d'$.

We first prove that if $\beta_d' \leq 0$, $SD_d^* \geq SD_d'$. From the seventh constraint group of model (6.9), we have $\eta_{dj} \leq 0, \forall j \in \Omega_d$. Then, from the sixth constraint group of model (6.9), we have $\frac{\varphi_{dj}'}{s_{dj}'+\varphi_{dj}'} \geq SD_d' - \frac{\eta_{dj}'}{s_{dj}'+\varphi_{dj}'}, \forall j \in \Omega_d$. Since $s_{dj}' + \varphi_{dj}' > 0$ and $\eta_{dj}' \leq 0, \forall j \in \Omega_d$, we get $\frac{\varphi_{dj}'}{s_{dj}'+\varphi_{dj}'} \geq SD_d' - \frac{\eta_{dj}'}{s_{dj}'+\varphi_{dj}'} \geq SD_d', \forall j \in \Omega_d$. Then, it is easy to verify that $(U_d', W_d', s_{dj}', \varphi_{dj}', \eta_{dj}', \beta_d', \forall j \in \Omega_d)$ is also a feasible solution to model (6.8). Therefore, we have $SD_d^* \geq SD_d'$.

We now prove that if $SD_d^* \geq SD_d'$ then $\beta_d' \leq 0$. From the sixth constraint group of

model (6.8), we get $\frac{\varphi_{dj}^*}{s_{dj}^*+\varphi_{dj}^*} \geq SD_d^* \geq SD_d', \forall j \in \Omega_d$. Therefore, we have $-\varphi_{dj}^* + SD_d' * \left(s_{dj}^* + \varphi_{dj}^*\right) \leq 0, \forall j \in \Omega_d$. Then we let $\eta_{dj}' = -\varphi_{dj}^* + SD_d' * \left(s_{dj}^* + \varphi_{dj}^*\right) \leq 0, \forall j \in \Omega_d$ and $\beta_d = \max_{j \in \Omega_d} \eta_{dj}' \leq 0$. Then it is easy to verify that $(U_d^*, W_d^*, s_{dj}^*, \varphi_{dj}^*, \eta_{dj}', \beta_d, \forall j \in \Omega_d)$ is a feasible solution to model (6.9) when $SD = SD_d'$. We then must have $\beta_d' \leq \beta_d \leq 0$. Q.E.D.

Theorem 6.2 indicates that if we have $\beta_d' \leq 0$, $SD_d^* \geq SD_d'$, i.e., SD_d' is a lower bound on SD_d^*. Otherwise, SD_d' is an upper bound on SD_d^*.

We then give algorithm 6.1 to solve model (6.9) based on theorem 6.2. We know the optimal satisfaction degree SD_d^* of each DMU d is in the range $[0,1]$. The basic idea of the algorithm is for each iteration of the algorithm to halve the width of the possible of values until we reach a sufficiently narrow range, i.e. a sufficiently accurate satisfaction degree.

Algorithm 6.1.

Begin

Step 1: Let $SD_d^u = 1$, $SD_d^l = -0.001$, and $SD_d' = \frac{SD_d^u + SD_d^l}{2}$.

Step 2: Solve model (6.9) by setting $SD_d = SD_d'$, and obtain the optimal solution $(U_d', W_d', s_{dj}', \varphi_{dj}', \eta_{dj}', \beta_d', \forall j \in \Omega_d)$. If $\beta_d' \leq 0$, let $SD_d^l = SD_d'$, $SD_d^* = SD_d'$, $SD_d' = \frac{SD_d^u + SD_d^l}{2}$, and $(U_d^*, W_d^*, s_{dj}^*, \varphi_{dj}^*, \forall j \in \Omega_d) = (U_d', W_d', s_{dj}', \varphi_{dj}', \forall j \in \Omega_d)$.

Go step 3. If $\beta_d' > 0$, let $SD_d^u = SD_d'$ and $SD_d' = \frac{SD_d^u + SD_d^l}{2}$.

Step 3: If $\left|SD_d^u - SD_d^l\right| < \epsilon$, stop and output $(U_d^*, W_d^*, s_{dj}^*, \varphi_{dj}^*, \forall j \in \Omega_d)$ as the optimal solution of model (6.8). If $\left|SD_d^u - SD_d^l\right| \geq \epsilon$, go step 2.

End

In algorithm 6.1, ϵ is a very small positive value; in this work we set it to 0.0001. Note that we assign values to the optimal solution of model (6.8) only if we get $\beta_d' \leq 0$, because it is the necessary and sufficient condition for $(U_d', W_d', s_{dj}', \varphi_{dj}', \forall j \in \Omega_d)$ to be a feasible solution to model (6.8). Also note that we start the algorithm with $SD_d^l = -0.001$ and not with $SD_d^l = 0$. This is because if the algorithm starts with

$SD_d^l = -0.001$ (actually with SD_d^l equaling any negative number like -0.001), we have $\beta_d' \leq 0$ in at least one iteration of the algorithm, which helps to ensure that SD_d^* and $(SD_d^*, U_d^*, W_d^*, s_{dj}^*, \varphi_{dj}^*, \forall j \in \Omega_d)$ are always assigned the optimal values before the algorithm stops.

It is easy to confirm the convergence of our algorithm 6.1 because it is built based on the dichotomy method. Algorithm 6.1 only needs to solve some linear programs to get an optimal solution to model (6.8). Additionally, it should be noted here that algorithm 6.1 converges very fast. For example, if an error tolerance of $1/2^{14}$ is adopted, only 14 iterations are needed (i.e., 14 linear programs need to be solved) for each DMU.

6.3.2 An algorithm to get unique optimal weights

Although the proposed model (6.8) can be solved using algorithm 6.1, there are still chances that the optimal solution to model (6.8) is not unique. Therefore, in this part, we give an algorithm 6.2 to generate a unique set of optimal weights using model (6.8).

Algorithm 6.2.

Begin

Step 1: Let $t = 1$, solve model (6.8) using algorithm 6.1 and get the optimal solution $(SD_d^{1*}, U_d^{1*}, W_d^{1*}, s_{dj}^{1*}, \varphi_{dj}^{1*}, \forall j \in \Omega_d)$. Calculate the satisfaction degree of the DMUs using $SD_{dj}^1 = \frac{\varphi_{dj}^{1*}}{s_{dj}^{1*} + \varphi_{dj}^{1*}}, \forall j \in \Omega_d$. Then we divide the DMUs into the following two groups.

$$J_1 = \{j | SD_{dj}^1 = SD_d^*, \forall j \in \Omega_d\} \tag{6.10}$$

and

$$J_L = \{j | SD_{dj}^1 > SD_d^*, \forall j \in \Omega_d\} \tag{6.11}$$

Denote the number of DMUs in J_1 as n_1. If we have $n_1 = m + s - p - 1$, where p is the number of DMUs in $J_{uc} = \{j \neq d | E_{dj}^{max} = E_{dj}^{min}\}$, then stop.

Step 2: Let $t = t + 1$. Solve the following model (6.12) and get the optimal solution $(SD_d^{t*}, U_d^{t*}, W_d^{t*}, s_{dj}^{t*}, \varphi_{dj}^{t*}, \forall j \in \Omega_d)$.

$$\max \quad SD$$

$$
\begin{aligned}
s.t. \quad & U \cdot Y_j - E_{dj}^{max} \times W \cdot X_j + s_{dj} = 0, \forall j \in \Omega_d \\
& U \cdot Y_j - E_{dj}^{min} \times W \cdot X_j - \varphi_{dj} = 0, \forall j \in \Omega_d \\
& U \cdot Y_d - E_d^* \times W \cdot X_d = 0 \\
& W \cdot X_d = 1 \\
& U \cdot Y_j - W \cdot X_j \leq 0, \forall j \\
& \frac{\varphi_{dj}}{s_{dj} + \varphi_{dj}} = SD_d^{1*}, j \in J_1 \\
& \ldots \\
& \frac{\varphi_{dj}}{s_{dj} + \varphi_{dj}} = SD_d^{t-1*}, j \in J_1 \\
& \frac{\varphi_{dj}}{s_{dj} + \varphi_{dj}} \geq SD, j \in J_L \\
& U, W \geq 0 \\
& s_{dj}, \varphi_{dj} \geq 0, \forall j \in \Omega_d
\end{aligned}
\tag{6.12}
$$

Calculate the satisfaction degrees of the DMUs using $SD_{dj}^1 = \frac{\varphi_{dj}^{1*}}{s_{dj}^{1*} + \varphi_{dj}^{1*}}, j \in J_L$.

Then, let

$$J_T := \{j | SD_{dj}^1 = SD_d^*, J \in J_L\} \tag{6.13}$$

and

$$J_L := \{j | SD_{dj}^1 > SD_d^*, J \in J_L\} \tag{6.14}$$

Also, we denote the number of DMUs in J_L as n_t. If $\sum_{\tau=1}^{t} n_\tau = m + s - p - 1$, stop. Otherwise, repeat step 2.

End

It is easy to see that model (6.12) can be solved similarly to model (6.8) using algorithm 6.1. When the algorithm stops, the generated $(SD_d^{1*}, U_d^{t*}, W_d^{t*}, s_{dj}^{t*}, \varphi_{dj}^{t*}, \forall j \in \Omega_d)$ is the unique optimal solution to model (6.9) (the proof of uniqueness can be seen in the following Corollary 6.1), which means that (U_d^{t*}, W_d^{t*}) is the unique set of optimal weights determined for DMU *d*.

Theorem 6.3. In algorithm 6.2, if $n_1 = m + s - p - 1$, then (U_d^{1*}, W_d^{1*}) is the unique set of optimal weights of DMU *d*.

Proof. For each DMU $j \in J_{uc}$, we have $\frac{U_d^{1*} \cdot Y_j}{W_d^{1*} \cdot X_j} = E_{dj}$ (a), where E_{dj} is the cross-efficiency of DMU j corresponding to DMU d calculated with the traditional CCR model. We also have the equation in model (6.8) that $W_d \cdot X_d = 1$ (b). From algorithm 6.2, we know that $SD_{dj}^1 = \frac{\varphi_{dj}^{1*}}{s_{dj}^{1*} + \varphi_{dj}^{1*}} = (\frac{U_d^{1*} Y_j}{W_d^{1*} X_j} - E_{dj}^{min})/(E_{dj}^{max} - E_{dj}^{min}), j \in J_1$ (c). It is known that in $(a), (b)$ and (c) form a system of $p + n_1 + 1 = m + s$ equations containing $m + s$ variables (U_d^{t*}, W_d^{t*}). Since the vectors $(X_j, Y_j), \forall j$ are mutually linearly independent, therefore (U_d^{t*}, W_d^{t*}) is the unique set of optimal weights for DMU d. From the first and second constraint group of model (6.8), we know that s_{dj}^{1*} and φ_{dj}^{1*} can then be uniquely calculated as $s_{dj}^{1*} = E_{dj}^{max} \times W_{dj}^{1*} \cdot X_j - U_d^{1*} \cdot Y_j$ and $\varphi_{dj}^{1*} = -E_{dj}^{min} \times W_{dj}^{1*} \cdot X_j + U_d^{1*} \cdot Y_j, \forall j \in \Omega_d$.

Therefore, $(SD_d^{1*}, U_d^{1*}, W_d^{1*}, s_{dj}^{1*}, \varphi_{dj}^{1*}, \forall j \in \Omega_d)$ is the unique set of optimal solution of model (6.8) and (U_d^{t*}, W_d^{t*}) is unique. Q.E.D.

From the proof of Theorem 6.3, the following Corollary 6.1 holds.

Corollary 6.1. In algorithm 6.2, if $\sum_{\tau=1}^{t} n_\tau = m + s - p - 1$, then (U_d^{t*}, W_d^{t*}) is the unique set of optimal weights of DMU d.

It should be noted that it is possible that $n_t > m + s - p - 1 - \sum_{\tau=1}^{t-1} n_\tau$ in the last iteration of the algorithm, although the possibility is very small. In this situation, we would have more than one group of $m + s$ equations with the input and output weights as variables and the coefficient matrix mutually linearly independent. So, selecting different group of equations might result in different optimal weights for the DMU. In this case, we always select for the DMU the set of optimal weights which generates the highest minimum cross-efficiency among the DMUs whose cross efficiencies have not been identified yet.

Therefore, our Algorithms 6.1 and 6.2 can be used to solve the proposed model (6.8) and generate for each DMU d a unique set of optimal weights. We denote the unique set of optimal weights generated for each DMU d as (U_d^*, W_d^*). Then, for each DMU j, we define its ***satisfactory cross-efficiency*** corresponding to DMU d as the following (6.15).

$$E_{dj}^{satis} = \frac{U_d^* \cdot Y_j}{W_d^* \cdot X_j} \tag{6.15}$$

Then, the ***satisfactory cross-efficiency score*** of a DMU j can be calculated as the following (6.16).

$$E_j^{satis} = \frac{1}{n}\sum_{d=1}^{n} \frac{U_d^* \cdot Y_j}{W_d^* \cdot X_j} \tag{6.16}$$

Note that the DEA cross-efficiency evaluation mechanism has good discriminatory power for the ranking of DMUs. Since our procedure is proposed under such an efficiency evaluation mechanism, we believe that in practical applications the satisfactory cross-efficiency scores generated by our procedure can usually give a complete ranking of all the DMUs. If in some special cases it unfortunately generates the same satisfactory cross-efficiency score for two or more DMUs, we think the maximized-minimum satisfaction degree generated by each DMU for the others can be used as secondary ranking indicator. Specifically, for any pair of DMUs with the same satisfactory cross-efficiency score, the DMU which provides the larger maximized-minimum satisfaction degree for the others ranks first. This secondary criterion is reasonable because the larger the maximized-minimum satisfaction degree that a DMU provides for the others, the more its evaluation results satisfy the others, thus the more likely these DMUs are willing to accept it being ranked above others with the same satisfactory cross-efficiency score.

6.3.3 A numerical example

In this part, we provide a small numerical example (from Liang et al. 2008a) to compare the proposed model with the traditional benevolent and aggressive models to see their similarities and differences. The numerical example includes 5 DMUs, each DMU having 3 inputs and 2 outputs. The data of this numerical example are listed in Table 2.1 of Chapter 2.

We evaluate the DMUs by the CCR model, the arbitrary strategy, benevolent cross-efficiency evaluation model (Doyle and Green, 1994), the aggressive cross-efficiency evaluation model (Doyle and Green, 1994), and our approach. The efficiencies of the DMUs and their rankings are listed in the following Table 6.1. From

Table 6.1, several conclusions can be drawn. Firstly, the CCR model cannot fully discriminate the DMUs while all the cross-efficiency methods fully rank all the DMUs in different positions. Secondly, the ranking results of the proposed model is consistent with those of the traditional models. Thirdly, the benevolent property of our model can be seen because the cross-efficiency score of each DMU generated from our approach is larger than that generated from the arbitrary strategy and aggressive model (3.2). Finally, compared with the benevolent model (3.1), the benevolent power of the proposed model is slightly weaker, which provides the decision makers with more choices in efficiency evaluation.

Next, we give the satisfaction degree matrices generated by the previous models, and our model (6.8), which can be seen in Table 6.2. The value in the dth row and j^{th} column of the sub-tables represents DMU j's satisfaction degree toward the optimal weights of DMU d selected by the corresponding models. Symbol "\" means that for such DMU j, its maximum and minimum cross-efficiencies corresponding to DMU d have the same value.

As can be seen from Table 6.2, firstly, our approach generally obtains higher satisfaction degrees for the DMUs than those generated from the aggressive and arbitrary models. For example, DMU3's satisfaction degree toward DMU2's optimal weights generated by our approach is 1.0000. In contrast, DMU3's satisfaction degree toward DMU2's optimal weights generated by the aggressive model and the arbitrary model are 0.0000 and 0.9676 which are smaller than that generated by our approach.

Table 6.1 Evaluation and ranking results

DMUs	CCR efficiency	Arbitrary	Benevolent	Aggressive	Our approach
	Model (2.5)	Equation (2.8)	Model (3.1)	Model (3.2)	Model (6.8)
1	0.6857 (4)	0.4743 (5)	0.5616 (5)	0.4473 (5)	0.5529 (5)
2	1.0000 (1)	0.8793 (2)	0.9295 (2)	0.8895 (2)	0.9143 (2)
3	1.0000 (1)	0.9856 (1)	1.0000 (1)	0.9571 (1)	1.0000 (1)
4	0.8571 (3)	0.5554 (3)	0.6671 (3)	0.5843 (3)	0.6453 (3)
5	0.8571 (3)	0.5587 (4)	0.5871 (4)	0.5186 (4)	0.5829 (4)

Table 6.2 Satisfaction degree matrices

Our model					
DMUs	1	2	3	4	5
1	\	\	\	\	\
2	1.0000	\	**1.0000**	**0.6299**	**0.6299**
3	**0.5604**	0.7710	\	0.6171	**0.5604**
4	1.0000	\	1.0000	\	1.0000
5	\	\	\	\	\

Aggressive model					
DMUs	1	2	3	4	5
1	\	\	\	\	\
2	0.0000	\	**0.0000**	1.0000	0.0000
3	0.0000	0.4000	\	0.0737	0.3333
4	0.0000	\	0.0000	\	0.0000
5	\	\	\	\	\

Benevolent model					
DMUs	1	2	3	4	5
1	\	\	\	\	\
2	1.0000	\	1.0000	**0.3750**	1.0000
3	0.6818	1.0000	\	0.8421	**0.4000**
4	1.0000	\	1.0000	\	1.0000
5	\	\	\	\	\

Arbitrary model					
DMUs	1	2	3	4	5
1	\	\	\	\	\
2	0.2426	\	**0.9676**	0.2717	0.4411
3	0.1237	0.2466	\	0.0451	0.4938
4	0.3095	\	0.3591	\	0.3433
5	\	\	\	\	\

Secondly, our model and the benevolent model both generate weights that give the DMUs high satisfaction degrees. However, this result is not surprising because both models intend to maximize the other DMUs' cross-efficiencies when selecting the optimal weights. But we can see that large differences exist among the satisfaction degrees of the DMUs in the results generated by the benevolent model. Take DMU2 as an example: the largest satisfaction degree it generates is 1.000 (for DMU1, DMU3, and DMU5) while the smallest satisfaction it generates is 0.3750 (for DMU4) which is much smaller than 1.000. Large differences among the satisfaction degrees of the DMUs will bring a sense of unfairness and make the evaluation unacceptable by the DMUs. Compared to the results generated by the benevolent model, the satisfaction degree differences among DMUs generated from our model are significantly smaller. Also taking DMU2 as an example: the highest satisfaction degree model (6.8) generates is also 1 (for DMU1, DMU3) while the lowest satisfaction it brings is 0.6229 (for DMU4 and DMU5).

Table 6.3 Optimal weight matrices

Our model (11)					
DMUs	X1	X2	X3	Y1	Y2
1	*0.0000*	*0.4545*	*0.0000*	*0.5455*	*0.0000*
2	0.1101	0.1101	0.2798	0.5000	**0.0000**
3	**0.0000**	**0.0000**	0.5198	0.3809	0.0992
4	0.2500	0.2500	**0.0000**	0.5000	**0.0000**
5	*0.0000*	*0.0000*	*0.5833*	*0.0000*	*0.4167*

Aggressive model (5)					
DMUs	X1	X2	X3	Y1	Y2
1	*0.0000*	*0.4545*	*0.0000*	*0.5455*	*0.0000*
2	0.4677	0.0431	0.0499	0.3525	0.0869
3	0.3726	0.0901	0.1436	0.0031	0.3905
4	0.4689	0.0858	**0.0000**	0.4453	**0.0000**
5	*0.0000*	*0.0000*	*0.5833*	*0.0000*	*0.4167*

Benevolent model (4)					
DMUs	X1	X2	X3	Y1	Y2
1	*0.0000*	*0.4545*	*0.0000*	*0.5455*	*0.0000*
2	**0.0000**	**0.0000**	0.5000	0.5000	**0.0000**
3	**0.0000**	**0.0000**	0.5000	0.5000	**0.0000**
4	0.2500	0.2500	**0.0000**	0.5000	**0.0000**
5	*0.0000*	*0.0000*	*0.5833*	*0.0000*	*0.4167*

Arbitrary model (1)					
DMUs	X1	X2	X3	Y1	Y2
1	*0.0000*	*0.4545*	*0.0000*	*0.5455*	*0.0000*
2	0.4677	0.0431	0.0499	0.3525	0.0869
3	0.3726	0.0901	0.1436	0.0031	0.3905
4	0.4689	0.0858	**0.0000**	0.4453	**0.0000**
5	*0.0000*	*0.0000*	*0.5833*	*0.0000*	*0.4167*

Additionally, it can be easily identified from the calculation process of Algorithm 6.2 that the final satisfaction degrees of the DMUs generated by each DMU will constitute a Pareto-optimal solution. Such characteristics will make the evaluation results of our model more acceptable by all the DMUs.

Finally, Table 6.3 shows the final DMU optimal weights matrices generated by different models. To make the optimal weights comparable with each other, we standardized them to ensure that we have $\sum_{r=1}^{s} u_{rd} + \sum_{i=1}^{m} w_{id} = 1$.

From Table 6.3, several conclusions can be seen. Firstly, the benevolent strategy generally obtains optimal weights that include more zero weights than the aggressive and arbitrary strategies. Secondly, although both the benevolent cross-efficiency model and our model use benevolent strategy, our model (6.8) generates fewer zero weights than the benevolent model. Finally, our approach guarantees the uniqueness of the optimal weights while the traditional models cannot.

6.4 Application to technology selection

DEA was incorporated for an application of technology selection by Knouja (1995). He used the CCR model to identify the robot with the best performance. Baker and Talluri (1997) noted some deficiencies of the work of Knouja (1995), and they suggested using DEA cross-efficiency evaluation for technology selection. The model used in their research is the aggressive model proposed by Doyle and Green (1994). In addition, Karsak and Ahiska (2005) proposed a DEA common-weight method for technology selection. This research was further extended by Amin, Toloo, and Sohrabi (2006), and Karsak and Ahiska (2008). In this section, the method proposed in this chapter is applied to a real case study of server selection for a company which plans to incorporate an enterprise resource planning (ERP) system.

6.4.1 Case background

Many previous studies have suggested that information technology can enhance the performance of organizations by improving operational efficiency and innovation (Dewett and Jones, 2001; Kwak et al. 2012). The ERP system is one of the most preferred technologies because of its good ability to integrate material, financial, and information flows for decision support of the organizations (Yao and He, 2000; Wei et al., 2005). *Chuangxian* Industrial Co. Ltd. (CICL) is producing rubber bands in Anhui, China. Most of its products are sold in the Occident, Southeast Asia, and Middle East regions. Recently, it has planned to incorporate an ERP system to confront the highly dynamic markets and to enhance its competitive advantage. One important problem that CICL faced was to select a suitable server to support the ERP system.

As in the above-mentioned methods for technology selection, here the servers are regarded as DMUs and the proposed method is used to evaluate all the candidates to select the best server for the company. One input and five outputs are selected for measuring the efficiencies of the servers. The detailed input and output variables of the DMUs are listed in Table 6.4.

Table 6.4 Variables of the servers

Type	Variable	Notation	Unit
Input	Price	X1	10,000 yuan
Outputs	CPU frequency	Y1	GHz
	Maximum number of CPUs	Y2	Piece
	Maximum memory capacity	Y3	GB
	Maximum hard drive capacity	Y4	TB
	After-sale service	Y5	Year

Table 6.5 Raw data and descriptive statistical analysis of the servers

DMUs	X1	Y1	Y2	Y3	Y4	Y5
1	5.29	2.00	2	768	21.6	3
2	5.32	2.00	4	512	12.0	3
3	5.60	2.20	4	1500	8.0	3
4	5.94	2.20	4	1500	16.0	3
5	8.01	2.10	4	512	20.0	3
6	9.29	2.30	2	1024	6.4	3
7	7.48	2.40	8	384	18.0	4
8	8.80	2.00	4	3072	16.0	3
9	7.10	1.87	8	1000	16.0	3
10	6.50	2.30	4	1000	32.0	3
11	7.89	2.00	4	512	20.0	4
12	6.50	2.20	4	768	24.0	4
13	5.99	2.00	4	512	16.0	3
14	5.50	2.30	4	1000	32.0	3
15	5.00	1.80	2	512	32.0	3
Max	9.29	2.40	8	3072	32	4
Min	5	1.80	2	384	6.4	3
Average	6.68	2.11	4.13	971.73	19.33	3.20
Std.dev	1.35	0.18	1.77	679.26	8.04	0.41

CICL specifies two basic principles in the selection of the server. Firstly, the maximum number of CPUs in the server should be no smaller than two. Secondly, the budget should be no larger than 100,000 yuan. Based on these two principles 15 servers are identified as candidates from the market. The input and output data and the descriptive statistical analysis of the 15 candidates are listed in Table 6.5.

6.4.2 Evaluation and selection results

It should be noted that the servers all want to be ranked the highest and to be selected. Therefore, it is essential to give the servers efficiency scores that seem reasonable to them all. We evaluate the servers using the CCR model, the arbitrary model, and our model proposed in this chapter. For comparison, we also give the evaluation results based on the technology selection methods of Baker and Talluri (1997) and Karsak and Ahiska (2005). The results are shown in Table 6.6.

Table 6.6 Evaluation and ranking results of the servers

DMUs	CCR efficiency	Arbitrary	Baker and Talluri (1997)	Karsak and Ahiska (2005)	Our approach
1	0.9850(10)	0.7880(7)	0.7487(7)	0.9009(4)	0.8856(8)
2	0.9975(8)	0.7736(8)	0.7262(8)	0.8852(5)	0.9059(6)
3	1.0000(1)	0.8394(3)	0.7960(3)	0.9586(2)	0.9830(2)
4	0.9898(9)	0.8316(4)	0.7933(4)	0.9046(3)	0.9424(3)
5	0.6710(14)	0.5499(14)	0.5190(14)	0.6175(12)	0.6179(14)
6	0.5934(15)	0.433(15)	0.4053(15)	0.5925(14)	0.5161(15)
7	1.0000(1)	0.7690(9)	0.7243(9)	0.7504(10)	0.8970(7)
8	1.0000(1)	0.6581(12)	0.6419(12)	0.5925(14)	0.7457(12)
9	1.0000(1)	0.7387(10)	0.7049(10)	0.6345(11)	0.8413(10)
10	0.8457(12)	0.7925(6)	0.7577(6)	0.8457(7)	0.8423(9)
11	0.8245(13)	0.6220(13)	0.59024(13)	0.5974(13)	0.7397(13)
12	1.0000(1)	0.8120(5)	0.7728(5)	0.8038(8)	0.9415(4)
13	0.8862(11)	0.7068(11)	0.6661(11)	0.7864(9)	0.8121(11)
14	1.0000(1)	0.9371(1)	0.8960(1)	1.0000(1)	0.9960(1)
15	1.0000(1)	0.8432(2)	0.8082(2)	0.8509(6)	0.9108(5)

From the evaluation results, several conclusions can be drawn. Firstly, the CCR model cannot effectively discriminate the DEA efficient DMUs. It selects seven servers as the best performer: servers 3, 7, 8, 9, 12, 14, and 15. Secondly, although the arbitrary method and Baker and Talluri's method generate different cross-efficiency scores for the DMUs, the complete ranking results of these two methods are the same. Thirdly, the satisfactory cross-efficiency scores generated from our approach for the DMUs are

higher than those generated from the arbitrary and Baker and Talluri's (1997) models. This is because our approach uses the benevolent strategy which aims at maximizing the other DMUs' cross-efficiencies. Fourthly, large differences appear in the ranking results of the servers between the CCR model and the other models. For example, server 7 ranks first in the CCR evaluation results, but it ranks 9th, 9th, 10th, and 7th in the evaluation results of the arbitrary model, Baker and Talluri's model, Karsak and Ahiska's model, and our approach, respectively. Fifthly, although Karsak and Ahiska's method successfully identifies the best performer, it cannot rank all the DMUs into unique positions. Specifically, servers 6 and 8 both obtain an efficiency score of 0.5925 and they cannot be further distinguished. Finally, except for the CCR model, all the other methods can effectively distinguish all the DEA efficient servers and rank them in different positions. In addition, they all identify server 14 as the best performer. Therefore, server 14 should be selected by the company.

6.4.3 Further comparisons of the different methods

Although the methods identify the same best performer, some other servers (for instance, server 7) do get different ranking positions with the different methods. To get further insights into the ranking results, we conduct Spearman's rank correlation test among the ranking results of the different approaches. The results are listed in Table 6.7.

According to Anderson et al. (2013), positive correlation exists between the two rankings if the correlation coefficient of the two rankings is larger than the benchmark value ($r_{s,\alpha}$). For $s = 12$ and $\alpha = 0.05$, the benchmark level $r_{s,\alpha}$ is 0.497. Then, according to the Spearman's rank correlation coefficients listed in Table 6.7, we obtain that all the rankings have positive correlations with each other. This means that ranking results generated by these methods are similar to each other, and they all can be used for ranking the servers and selecting the best one. However, it is known that the arbitrary model is used without considering the multiple optimal solution problem of the CCR model. Further, Baker and Talluri (1997) used the aggressive model to evaluate the servers and it is known that this method cannot guarantee the uniqueness of the optimal weights either. In addition, Karsak and Ahiska's (2005) algorithm sometimes fails to detect the most efficient DMUs (Amin, Toloo, and Sohrabi 2006).

Because of these deficiencies, these other methods are sometimes not suitable for applications in technology selection. Our method can not only effectively discriminate all the servers and identify the best performer but also generate for each DMU its unique optimal set of weights which can maximize the satisfaction degrees of all the DMUs. Therefore, the evaluation and ranking results of our approach are more satisfactory to all the DMUs, which makes the technology selection results more convincing and acceptable.

Table 6.7 Correlation analysis results

Methods	Arbitrary	Baker and Talluri (1997)	Karsak and Ahiska (2005)	Our approach
Arbitrary	1.0000	1.0000	0.8888	0.9464
Baker and Talluri (1997)		1.0000	0.8888	0.9464
Karsak and Ahiska (2005)			1.0000	0.8888
Our approach				1.0000

6.5 Conclusions

In this chapter, we propose a new DEA cross-efficiency evaluation approach. Firstly, we incorporated the concept of satisfaction degree of a DMU toward the optimal weights of another DMU. Then, a max-min model was proposed to select for each DMU a set of optimal weights based on the satisfaction degrees of all the other DMUs. To solve our max-min model linearly and to ensure the final optimal set of weights for each DMU is unique, we further proposed two algorithms. Finally, our approach was applied to an example of technology selection.

Our approach brings at least three advantages to DEA cross-efficiency evaluation. Firstly, the concept of satisfaction degree is incorporated into DEA cross-efficiency evaluation, and the DMUs' satisfaction degrees are maximized in weights selection, which makes the evaluation results more satisfactory and more acceptable to all the DMUs. Secondly, the numerical example shows that the discrimination power has been improved compared to some previous methods when our approach is used for evaluation. Thirdly, the set of final determined optimal weights for each DMU is

guaranteed to be unique.

Three further research directions are suggested based on this chapter. Firstly, the proposed optimal weights selection model is a benevolent model, but it can be easily transformed into an aggressive model by inverting its objective function for specific application scenarios. Secondly, the proposed approach can also be extended for addressing the problem of determining common weights in research on DEA common-weight evaluation. Thirdly, we suggest that it might be useful to set for each DMU a minimum acceptable value for its satisfaction degree, and to develop suitable weights selection strategies incorporating these restrictions.

Chapter 7 Conclusions and Perspectives

In this Chapter, we conclude the work of this thesis, illustrate the application domains of the proposed methods, and give some directions for further study.

7.1 Summary of the results and contributions

As a well-established extension of data envelopment analysis (DEA), DEA cross-efficiency evaluation has been widely applied in performance evaluation and ranking of the decision-making units (DMUs). However, the problems of non-uniqueness of optimal weights and non-Pareto optimality of the efficiency evaluation result have reduced the usefulness of this powerful method. In this thesis, we try to address these issues and apply the newly proposed approaches for applications like R&D project selection, technology selection, green supplier selection, etc. The main work and contributions can be concluded as the follows.

First, we pointed out that the cross-efficiency targets for the DMUs in the traditional benevolent and aggressive models are not always reachable. We then gave a target-identification model which can provide the DMUs with efficiency targets that are always feasible. Then, alternative new secondary goal models were proposed considering both desirable and undesirable cross-efficiency targets of the DMUs. The contributions of this study lie in: (I) It discussed more appropriate cross-efficiency targets for the DMUs; (II) It provided alternative secondary goal models which consider the DMUs' both willingness of getting close to the desirable cross-efficiency targets and away from the undesirable ones.

Second, we proposed a DEA cross-efficiency evaluation approach based on Pareto improvement. Our approach contains two models and an algorithm. Firstly, we proposed a Pareto optimality estimation model to see whether a given set of cross-efficiency scores is Pareto optimal. Then, a cross-efficiency Pareto-improvement model was given to make Pareto-optimal a set of initially non-Pareto-optimal cross-efficiency scores. Finally, an algorithm was proposed to provide a calculating process based on the two models and finally generate a set of Pareto-optimal cross-efficiency scores for the DMUs. The main contributions of this study lie in: (I) The proposed approach

always generates a set of cross-efficiency scores for DMUs that is Pareto optimal; (II) The result unifies self-evaluation, peer-evaluation, and common-weight evaluation in some special cases, which makes the evaluation result more acceptable to all the DMUs.

Third, we proposed a cross-bargaining cross-efficiency evaluation approach which addresses both the non-uniqueness of optimal weights and non-Pareto-optimality of the evaluation results of classical methods. We introduced a new cross-efficiency evaluation mode, cross-bargaining mode, in which each pair of DMUs in the group bargain with each other to determine a set of common weights for calculating their corresponding cross efficiencies. The Nash bargaining game was incorporated to construct the model. Additionally, an algorithm was presented to ensure the cross-bargaining game model can be solved by solving linear programs. The main contribution of this study lies in that the approach always provides a unique set of Pareto-optimal cross-efficiency scores for the DMUs.

Finally, we proposed the concept of satisfaction degree of a DMU on one of the other DMUs optimal weights in DEA cross-efficiency evaluation. Then, a model was provided which can maximize all the DMUs satisfaction degrees on the efficiency evaluation result. Additionally, two algorithms are provided to solve the proposed model linearly and to guarantee the uniqueness of the efficiency evaluation result, respectively. The main contributions of this study lie in: (I) It considered the DMUs willingness of accepting the evaluation result and introduced a concept of satisfaction degree; (II) The proposed approach maximized all the DMUs satisfaction degrees and the uniqueness of the evaluation result can be guaranteed.

7.2 Application domains of the proposed approaches

We proposed several methods to address the non-uniqueness of optimal weights and the non-Pareto optimality of evaluation result in DEA cross-efficiency evaluation. The DEA cross-efficiency evaluation method uses a group-decision mechanism to make efficiency evaluation for the DMUs. It is necessary to consider to optimize the efficiency evaluation result so as to make it acceptable to all the DMUs. Therefore, in spite of making improvements for the traditional benevolent and aggressive models, we have proposed several new DEA cross-efficiency evaluation methods to generate cross-

efficiency evaluation results with properties like Pareto-optimality, Nash equilibrium, and maximization of the DMUs' satisfaction degrees. These properties will enhance the stability of the evaluation results so as to make the evaluation result more acceptable to all the DMUs.

It should be emphasized here again that none of the proposed methods is a clear winner or loser in any circumstances. They might have some commonalities while they can be distinguished and have different application scenarios. We have illustrated the detailed application scenarios of the proposed benevolent, aggressive and neutral models. Here, we further give some guidelines for the other three cross-efficiency evaluation methods proposed in this thesis. The three methods are all applicable when the DMUs are cooperative since they all have the intention of maximizing the other DMUs' efficiency scores. According to the cooperative and competitive level, we classify all the methods and models into levels listed in Table 7.1.

Table 7.1 Classification of different methods

Cooperative/Competitive level	Methods
4	Cross-bargaining game method
3	Pareto improvement method
2	Model (3.9), Satisfaction degree maximization method
1	Model (3.8)
0	Model (3.12)
-1	Model (3.11)
-2	Model (3.10)

In Table 6.8, positive levels indicate the DMUs are cooperative, negative levels mean they are competitive, and level- "0" denotes that the DMUs are neither cooperative nor competitive. The higher (resp. lower) the level of the method, the more cooperative (resp. competitive) it is with respect to the positive (resp. negative) levels.

The cross-bargaining game DEA cross-efficiency evaluation approach is classified as the most cooperative method because it allows each DMU to have close bargaining

with the other DMUs one by one. Each DMU not only has the willingness to sacrifice its self-evaluated efficiency (i.e., the self-evaluated efficiency of it can be smaller than its CCR efficiency) to improve the cross-efficiencies of the other DMUs but also it is to use different optimal weights to make cross-efficiency calculation for the other DMUs. That is to say, the DMUs have very close contacts with each other. They regard each DMU as an independent entity and negotiate with it separately for the cross-efficiency evaluation result. Such a method can be used in the situation when the DMUs have very strong cooperative relationships, especially, in the situation when the DMUs have bargaining game relationship. For instance, the evaluation of green suppliers. On the one hand, the green suppliers would be willing to cooperate with each other to enhance the reputation of their sector to show as a whole they have very good operational and environmental efficiencies. On the other hand, the suppliers may also have direct contact with each other. For instance, they may trade their CO2 emission allowances among the group. This kind of contact would make the green suppliers have even stronger link and cooperation. The green suppliers may have the intention of bargaining with each of the other suppliers to determine the weights for cross-efficiency calculation so as to make more improvements on their cross-efficiency evaluation result.

The DEA cross-efficiency evaluation method based on Pareto improvement is classified as level- "3" cooperative. This is because it allows each DMU to sacrifice its self-evaluated efficiency to make cross-efficiency improvement while it does not let each DMU use different weights to evaluate the DMUs like the cross-bargaining game DEA cross-efficiency evaluation approach. Also, this approach has the property to generate a stable Pareto-optimal evaluation result. Therefore, this approach should be used when a set of DMUs are centrally evaluated by a supervisor. The supervisor needs to provide an evaluation result that is stable and convincing among all the DMUs. For instance, R&D project selection and preference voting.

The DEA cross-efficiency evaluation method based on satisfaction degree is classified as the same cooperative level as model (3.9). This is because this method generates the evaluation result that maximizes the minimum satisfaction degree of the DMUs which is similar to model (3.9)'s intention of maximizing the minimum cross efficiency among all the DMUs. While both in this method and model (3.9), each DMU's self-evaluated efficiency is required to be maintained at the CCR efficiency level and is not allowed to be reduced. However, the method and model (3.9) still have

some differences. Model (3.9) focuses on the efficiency of the DMUs while the method emphasizes the DMUs' satisfaction degrees on the evaluation result. The decision makers may choose one method from these two according to their practical focus point when evaluating the DMUs. For instance, when evaluating the sectors in a supply chain, if the evaluation is made externally to show the public the comprehensive strength of the supply chain, model (3.9) might be more suitable. Otherwise, if the evaluation is made internally (like to evaluate the sectors to allocate performance rewards among them), the method based on satisfaction degree might be more suitable since the evaluation should focus on making all the sectors satisfied.

The application scenarios of the other benevolent, aggressive and neutral models listed in Table 6.8 have been clearly discussed in 3.2.2.

7.3 Perspectives

We believe this thesis has brought numerous advantages to DEA cross-efficiency evaluation, especially in addressing the non-uniqueness of optimal weights and the non-Pareto-optimality of the evaluation results of existing approaches. However, there are still some limitations in DEA cross-efficiency evaluation which are worthy of further study.

First, it can be seen that scholars have proposed many approaches to address the non-uniqueness of optimal weight problem in DEA cross-efficiency evaluation. However, there still lacks formal criteria to adjust whether a proposed cross-efficiency evaluation approach is good or not or when and in which situation a proposed approach might be suitable. Actually, we will obtain different efficiency evaluation results when we chose to use different approaches. The decision makers might be confused in selecting the approaches when applying them in different scenarios. In further studies, scholars may investigate the properties of the existing DEA cross-efficiency evaluation approaches and classify them into different groups to discuss their suitable application circumstances.

Second, the Pareto optimality of the cross-efficiency scores in this thesis was discussed under two given weight selection principles. In the further studies, scholars may have a deeper exploration on the Pareto optimality in DEA cross-efficiency evaluation under less restrictive assumptions. For example, regardless of the introduced

principles, the DMUs might generate more generous Pareto-optimal cross-efficiency scores. Additionally, the unification of self-evaluation, peer-evaluation, and common-weight evaluation might be always guaranteed if we discuss such a situation.

Third, we introduced the concept of satisfaction degrees of DMUs on the efficiency evaluation results. Further researches should pay more attention on the DMUs' acceptance on the efficiency evaluation result in DEA cross-efficiency evaluation. For instance, similar to but not identical to the concept of satisfaction degree, quantifying the level of satisfaction, affinity or preference of the DMUs for cross-efficiency evaluation results can also be defined to measure the DMUs' willingness in accepting these results.

Finally, as we can see in DEA cross-efficiency evaluation, each DMU needs to solve at least two linear programs to obtain the final efficiency evaluation results. This indicates that the approaches will be time-consuming if the number of the DMUs is very large. Therefore, another research direction we note here is exploring the possibility to propose suitable algorithms to accelerate the calculating process of DEA cross-efficiency evaluation.

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Academic achievements during the PhD study

Publications: (Jie Wu is his advisor at USTC)

[1] Jie Wu, **Junfei Chu**, Jiasen Sun, Qingyuan Zhu (2016). DEA cross-efficiency evaluation based on Pareto improvement. *European Journal of Operational Research*, 248(2), 571-579. (IF: 3.428, Q1).

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[12] **Junfei Chu**, Jie Wu, Qingyuan Zhu, Jiasen Sun (2016). Resource scheduling in a private cloud environment: an efficiency priority perspective. *Kybernetes*, 45(10), 1524-1541. (IF:0.98, Q3).

Co-authored publications :

[13] Jie Wu, Qingyuan Zhu, Xiang Ji, **Junfei Chu** (2016). Two-stage network processes with shared resources and resources recovered from undesirable outputs. *European Journal of Operational Research,* 251(1), 182-197. (IF: 3.428, Q1).

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Working papers:

[26] **Junfei Chu,** Jie Wu, Chengbin Chu. DEA-based fixed cost allocation among two-stage systems: leader-follower and satisfaction degree bargaining game approaches. Accept after minor revision at *Omega-International Journal of Management Science*

[27] **Junfei Chu,** Jie Wu, Chengbin Chu. A New Common-Weight Multi-Criteria Decision-Making Approach for Technology Selection. Major revision at *International Journal of Production Research.*

[28] Pengzhen Yin, **Junfei Chu***. A DEA-based multi-objective approach for hotel

performance evaluation: a viewpoint from marketing. Major revision at to *Omega-International Journal of Management Science.*

[29] Fangqing Wei, **Junfei Chu** *. A cross-bargaining game approach for cross-directional efficiency evaluation in DEA. Minor revision at *OR spectrum.*

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[31] Jiasen Sun, Wei Dai , **Junfei Chu** *, Jie Wu. DEA altruism and exclusiveness cross-efficiency evaluation models. Minor revision at *Scientia Iranica.*

[32] **Junfei Chu,** Jie Wu, Chengbin Chu. A bargaining-game DEA based fixed cost allocation approach with more equitable weight profiles. Under review *at European Journal of Operational Research.*

[33] **Junfei Chu,** Jie Wu, Chengbin Chu. Cross-bargaining game DEA cross-efficiency evaluation: An application to R&D project selection. Under review at *Computers & Industrial engineering*.

[34] **Junfei Chu**. Fixed cost allocation based on the utility: A DEA common-weight approach. Under Review at *Annals Of operations Research.*

[35] **Junfei Chu***, Jie Wu. Estimating the optimal production scale size of strongly context-dependent DMUs: DEA-based approaches. Under review at *Resource, Conservation & Recycling*.

[36] **Junfei Chu.** A bargaining game approach for equilibrium fixed cost allocation under DEA common-weight framework. Under review at *Expert systems with Applications.*

Projects :

[1] **DEA Theories, Methods, and Applications for Efficiency Evaluation and Improvement Considering Non-Homogeneity Decision-making Units** (Participate) 2015-2018. Sponsored by Natural Science Foundation of China (NSFC) for RMB 493,000. I am responsible for data collection, mathematical modelling, and analyzing sub research problem.

[2] **Study on the Utilization Efficiency and Cultivating Path of New Type of Logistics Talents** (Participate) 2016-2017. Sponsored by Anhui Social Science Funding for RMB 5,000. I am responsible for data collection, evaluating the utilization

efficiency of the new type of logistics talents in the project.

Awards：

[1] Graduate Scholarship, USTC 2013-2018

[2] National Scholarship for Graduate Student, USTC, (Award Rate: 3%) 2015

[3] Scholarship from State Scholarship Fund, CSC, (Award Rate: 1%) 2016-2018

[4] National Scholarship for Graduate Student, USTC, (Award Rate: 3%) 2016

[5] Chinese Academy of Sciences Zhu Liyuehua scholarship, (Award Rate: 1%) 2017

[6] National Scholarship for Graduate Student, USTC, (Award Rate: 3%) 2017

[7] Chinese Academy of Science Dean Award, USTC, (Award Rate: 1%) 2018

Academic activities：

[1] May 23 to 26, 2016: Attend the ***14th International Conference of Data Envelopment Analysis*** held in Jianghan University, Wuhan, China as a speaker. Title of the speech: "DEA cross-efficiency evaluation based on Pareto-improvement".

[2] June 26 to 29, 2017: Attend the ***15th International Conference of Data Envelopment Analysis*** held in University of Economics, Prague in the Czech Republic, on June 26 to 29, 2017 as a speaker. Title of the speech: "A New Common-Weight Multi-Criteria Decision-Making Approach for Technology Selection"

[3] Reviewer for European Journal of Operations Research, Annals of Operations Research, Journal of the Operational Research Society, Journal of Cleaner Production, Ecological Indicators, Energy Efficiency, International Journal of Contemporary Hospitality Management, Management of Environmental Quality.

université PARIS-SACLAY

ÉCOLE DOCTORALE
INTERFACES
Approches Interdisciplinaires : Fondements, Applications et Innovation

Titre: Méthodes d'amélioration pour l'évaluation de l'enveloppement des données évaluation de l'efficacité croisée

Mots clés: Analyse d'enveloppement des données, Unités de prise décision, Évaluation de l'efficacité croisée.

Résumé: L'évaluation croisée d'efficacité basée sur la *data envelopment analysis* (DEA) a été largement appliquée pour l'évaluation d'efficacité et le classement des unités de prise de décision (decision-making units, DMUs). A l'heure actuelle, cette méthode présente toujours deux défauts majeurs : la non-unicité des poids optimaux attachés aux entrées et aux sorties et la non Pareto-optimalité des résultats d'évaluation. Cette thèse propose des méthodes alternatives pour y remédier. Nous montrons d'abord que les efficacités croisées visées dans les modèles traditionnels avec objectifs secondaires ne sont pas toujours atteignables pour toutes les DMUs. Nous proposons ensuite un modèle capable de toujours fournir des objectifs d'efficacité croisée atteignables pour toutes les DMUs. Plusieurs nouveaux modèles avec objectifs secondaires bienveillants ou agressifs et un modèle neutre sont proposés. Un exemple numérique est utilisé pour comparer les modèles proposés à ceux qui existent dans la littérature. Nous présentons ensuite une approche d'évaluation croisée d'efficacité basée sur l'amélioration de Pareto. Cette approche est composée de deux modèles et d'un algorithme. Les modèles sont utilisés respectivement pour estimer si un ensemble donné de scores d'efficacité croisée est Pareto-optimal et pour améliorer l'efficacité croisée de cet ensemble si cela est possible. L'algorithme est utilisé pour générer un ensemble Pareto-optimal de scores d'efficacité croisée pour les DMUs. L'approche proposée est finalement appliquée pour la sélection de projets de R&D et comparée aux approches traditionnelles. En outre, nous proposons une approche d'évaluation croisée d'efficacité qui traite simultanément les deux problématiques mentionnées ci-dessus. Un modèle de jeu de négociation croisée est proposé pour simuler la négociation entre chaque couple de DMUs au sein du groupe afin d'identifier un ensemble unique de poids à utiliser pour le calcul de l'efficacité croisée entre eux. De plus, un algorithme est développé pour résoudre ce modèle via une suite de programmes linéaires. L'approche est finalement illustrée en l'appliquant à la sélection des fournisseurs verts. Enfin, nous proposons une évaluation croisée d'efficacité basée sur le degré de satisfaction. Nous introduisons d'abord la nation de degré de satisfaction de chaque DMU sur les poids optimaux sélectionnés par les autres. Ensuite, un modèle max-min est fourni pour déterminer un ensemble des poids optimaux pour chaque DMU afin de maximiser tous les degrés de satisfaction des DMUs. Deux algorithmes sont ensuite développés pour résoudre le modèle et garantir l'unicité des poids optimaux de chaque DMU, respectivement. Enfin, l'approche proposée est appliquée sur une étude des cas pour la sélection de technologies.

Title: Improvement methods for data envelopment analysis (DEA) cross-efficiency evaluation

Keywords: Data envelopment analysis, Decision-making units, Cross-efficiency evaluation.

Abstract: Data envelopment analysis (DEA) cross-efficiency evaluation has been widely applied for efficiency evaluation and ranking of decision-making units (DMUs). However, two issues still need to be addressed: non-uniqueness of optimal weights attached to the inputs and outputs and non-Pareto optimality of the evaluation results. This thesis proposes alternative methods to address these issues. We first point out that the cross-efficiency targets for the DMUs in the traditional secondary goal models are not always feasible. We then give a model which can always provide feasible cross-efficiency targets for all the DMUs. New benevolent and aggressive secondary goal models and a neutral model are proposed. A numerical example is further used to compare the proposed models with the previous ones. Then, we present a DEA cross-efficiency evaluation approach based on Pareto improvement. This approach contains two models and an algorithm. The models are used to estimate whether a given set of cross-efficiency scores is Pareto optimal and to improve the cross-efficiency scores if possible, respectively. The algorithm is used to generate a set of Pareto-optimal cross-efficiency scores for the DMUs. The proposed approach is finally applied for R&D project selection and compared with the traditional approaches. Additionally, we give a cross-bargaining game DEA cross-efficiency evaluation approach which addresses both the issues mentioned above. A cross-bargaining game model is proposed to simulate the bargaining between each pair of DMUs among the group to identify a unique set of weights to be used in each other's cross-efficiency calculation. An algorithm is then developed to solve this model by solving a series of linear programs. The approach is finally illustrated by applying it to green supplier selection. Finally, we propose a DEA cross-efficiency evaluation approach based on satisfaction degree. We first introduce the concept of satisfaction degree of each DMU on the optimal weights selected by the other DMUs. Then, a max-min model is given to select the set of optimal weights for each DMU which maximizes all the DMUs' satisfaction degrees. Two algorithms are given to solve the model and to ensure the uniqueness of each DMU's optimal weights, respectively. Finally, the proposed approach is used for a case study for technology selection.

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