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## THESE

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(Diplôme National - Arrêté du 25 mai 2016)

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Présentée par: ROBIN SEBASTIAN

NUMERICAL SIMULATION OF ACOUSTIC PROPAGATION IN A TURBULENT CHANNEL FLOW WITH AN ACOUSTIC LINER

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Soutenue le 26 Novembre 2018 devant la Commission d'Examen

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## CONTENTS

List of Figures ..... ix
List of Tables ..... xiv
Listings ..... xvi
Nomenclature ..... xvii
Acronyms ..... xix
1 INTRODUCTION ..... 1
1.1 Overview on Acoustic liners ..... 2
1.2 Overview on Wall Turbulence ..... 6
1.3 Overview on Large Eddy Simulation ..... 9
1.4 Motivations and Objectives ..... 10
1.5 Organisation of Thesis ..... 13
2 EQUATIONS AND SIMULATION TECHNIQUES ..... 15
2.1 Equations ..... 15
2.2 Flow configuration ..... 17
2.3 Boundary conditions ..... 19
2.3.1 Periodicity ..... 19
2.3.2 Isothermal rigid wall ..... 20
2.4 Computational grid ..... 21
2.5 Numerics ..... 22
2.5.1 Spatial first-derivative ..... 22
2.5.2 Spatial second-derivative ..... 24
2.5.2.1 Modified second-derivative for DNS ..... 25
2.5.2.2 Modified second-derivative for ILES ..... 26
2.5.3 Time integration ..... 30
2.5.4 Stability Criteria ..... 31
2.6 Parallel implementation and scalability ..... 33
2.7 Validation - DNS of temporal supersonic channel flow ..... 34
3 MODIFIED FINITE DIFFERENCE SCHEME FOR IMPLICIT LARGE EDDY SIMULATION ..... 41
3.1 Parametric study of modified spatial 2nd-derivative Finite Difference Scheme ..... 41
3.1.1 Channel flow at $\operatorname{Re}_{\tau} \approx 400$ and $\mathcal{M}=0.5$ ..... 42
3.1.2 High $\operatorname{Re}_{\tau}$ channel flows at $\mathcal{M}=0.5$ ..... 47
3.1.3 Supersonic channel flows ..... 49
3.2 Grid requirement for wall-resolved ILES ..... 52
3.2.1 Simulation parameters ..... 52
3.2.2 Results ..... 53
3.2.2.1 Mean velocity ..... 53
3.2.2.2 Reynolds stress and rms of vorticity ..... 54
3.2.2.3 Velocity correlation and spectra ..... 56
3.2.2.4 Vorticity correlation ..... 59
3.2.2.5 Velocity skewness and kurtosis ..... 61
3.2.3 Conclusion ..... 63
4 SCALING TECHNIQUES FOR COMPRESSIBLE TURBULENT CHANNEL FLOW AT MACH NUMBER UP TO 3 ..... 65
4.1 Review of scaling techniques ..... 65
4.1.1 Incompressible transformation ..... 65
4.1.2 Compressibility effects ..... 66
4.1.3 Van Driest transformation ..... 67
4.1.4 Semi-local transformation ..... 69
4.1.5 Trettel's transformation ..... 70
4.2 Numerical test ..... 71
4.2.1 Simulation parameters ..... 72
4.2.2 Results ..... 73
4.2.2.1 Mean velocity ..... 74
4.2.2.2 Rms profiles of velocity and vorticity ..... 74
4.2.2.3 Velcity correlation ..... 78
4.2.2.4 Velocity skewness and kurtosis ..... 8o
4.3 Conclusion ..... 82
5 INNER-OUTER LAYER SCALES-INTERACTION ..... 83
5.1 Overview on large-scale structures in wall-bounded turbulent flow ..... 83
5.2 Grid requirement ..... 85
5.3 Simulation at high Reynolds \& Mach number ..... 87
5.3.1 Validation of the simulations ..... 87
5.3.2 Evidence of Large Scale Structures and their effect on near-wall scales ..... 89
5.3.3 Procedure to extract large-scale structures ..... 93
5.3.4 Organisation of large-scale structures ..... 96
5.3.5 Quantification of large-scale influence ..... 102
5.4 Conclusion ..... 108
6 TEMPORAL SIMULATION OF CHANNEL FLOW WITH ACOUSTIC LINER 109
6.1 Boundary condition - Impedance wall ..... 109
6.1.1 Validation ..... 111
6.2 Grid requirement for channel flow with acoustic liner ..... 112
6.3 Flow statistics ..... 115
6.3.1 Effect of the liner resistance $R$ ..... 116
6.3.2 Effect of the resonance frequency $\omega_{\text {res }}$ ..... 117
6.3.3 Effect of the flow Mach number $\mathcal{M}$ ..... 119
6.4 Existence of a wave along the acoustic liner ..... 120
6.5 Drag ..... 126
6.6 Linear stability analysis ..... 128
6.6.1 Instability due to impedance boundary ..... 128
6.6.2 Comparison with the numerical simulations ..... 131
6.7 Conclusion ..... 133
7 SPATIAL SIMULATION OF CHANNEL FLOW AND SOUND ATTENUATIONI 35
7.1 Boundary conditions ..... 135
7.1.1 Subsonic inflow boundary condition ..... 135
7.1.1.1 Based on Local One Dimensional Inviscid (LODI) relations ..... 136
7.1.1.2 Based on Navier-Stokes equations ..... 137
7.1.1.3 Non-reflecting characteristics boundary condition ..... 137
7.1.2 Subsonic outflow boundary condition ..... 138
7.1.2.1 Sponge zone ..... 139
7.2 Wave extraction using global minimization ..... 141
7.3 Validation ..... 145
7.3.1 Poiseuille flow ..... 145
7.3.1.1 Flow configuration ..... 145
7.3.1. 2 Results ..... 147
7.3.2 Poiseuille flow with acoustic wave ..... 148
7.3.2.1 Multiple plane acoustic waves ..... 150
7.3.2.2 Plane and transverse acoustic waves ..... 152
7.4 Spatial turbulent channel flow ..... 154
7.4.1 Turbulent inflow ..... 154
7.4.2 Turbulent flow statistics ..... 155
7•4.2.1 Simulation set-up ..... 155
7.4.2.2 Results ..... 156
7.4.3 Interaction between sound and turbulence ..... 161
7.4.3.1 Simulation setup ..... 161
7.4.3.2 Results ..... 163
7.5 Conclusion ..... 167
8 SIMULATION OF FLOW TUBE WITH ACOUSTIC LINER ..... 169
8.1 Simulation setup ..... 170
8.2 Flow statistics ..... 172
8.3 Instability over the acoustic liner ..... 176
8.4 Turbulence-liner-acoustic interaction ..... 177
8.5 Effect on sound attenuation ..... 179
8.6 Conclusion ..... 182
9 CONCLUSION AND PERSPECTIVES ..... 183
9.1 Future works ..... 184
Appendix
A BRIEF HISTORY OF CODE "COMPACT3D" AND COEFFICIENTS FOR COMPACT SCHEMES ..... 189
A. 1 History of Compact3D ..... 189
A. 2 Coefficient of compact schemes ..... 190
B GRID STRETCHING ..... 193
B. 1 Wall-normal grid ..... 193
B.1.1 Tanh stretching ..... 193
B.1.2 Incompact stretching ..... 194
B. 2 Stream-wise grid ..... 194
C COMPARISON BETWEEN DNS AND ILES ..... 197
D COMPRESSIBLE RELATIONSHIP ..... 201
E LINEARISED 2D NAVIER-STOKES EQUATIONS AND BOUNDARY CON- DITIONS ..... 203
F TARGET SOLUTION FOR SPONGE ZONE ..... 207
BIBLIOGRAPHY ..... 209
Index ..... 230

## LIST OF FIGURES

1 Introduction
Figure 1.1 Illustration of engine noise treatment ..... 2
Figure 1.2 Schematic of grazing flow ..... 3
Figure 1.3 Regions of TBL ..... 7
Figure 1.4 Illustration of DNS, LES and RANS ..... 9
Figure 1.5 Objectives of the thesis ..... 11
2 Equations and Simulation techniques
Figure 2.1 Flow configuration for the temporal channel flow simulation ..... 17
Figure 2.2 Characteristics waves entering and leaving the computational do- main for a subsonic flow ..... 20
Figure 2.3 Illustration of wall boundary treatment ..... 20
Figure 2.4 Modified wave-numbers for spatial first-derivative ..... 23
Figure 2.5 Spatial discretisation for first-derivative in non-periodic boundary ..... 24
Figure 2.6 Modified wave-numbers for spatial second-derivative ..... 24
Figure 2.7 Modified wave-numbers of spatial second-derivative scheme used for DNS ..... 26
Figure 2.8 Shape of SVV kernel ..... 28
Figure 2.9 Modified wave-numbers of spatial second-derivative kernels used for ILES ..... 28
Figure 2.10 Spatial discretisation for the second-derivative in non-periodic boundary ..... 30
Figure 2.11 Constraint on explicit time stepping due to $\operatorname{Re}_{\mathrm{b}}$ and $\mathcal{M}$ ..... 32
Figure 2.12 Time constraint due to CFL and Fourier constraint with different mesh size at the wall ..... 33
Figure 2.13 2D pencil domain decomposition using 2DECOMP library ..... 33
Figure 2.14 Scalability curves for Compact3D ..... 34
Figure 2.15 Comparison of mean velocity profile ..... 36
Figure 2.16 Comparison of rms velocity profiles ..... 36
Figure 2.17 Comparison of mean profile of scalar quantities ..... 37
Figure 2.18 Comparison of rms profile of scalar quantities ..... 38
Figure 2.19 Comparison of stress profiles ..... 38
Figure 2.20 Comparison of stream-wise and span-wise correlations ..... 39
3 Modified Finite Difference scheme for Implicit Large Eddy Simulation
Figure 3.1 Comparison of mean, rms velocity profiles and spectra from ILES of channel flow with fine grid at $\operatorname{Re}_{\tau} \approx 400$ and $\mathcal{M}=0.5$ ..... 44
Figure 3.2 Comparison of mean and rms velocity profiles from ILES of chan- nel flow with coarse grid at $\operatorname{Re}_{\tau} \approx 400$ and $\mathcal{M}=0.5$ ..... 45
Figure 3.3 Comparison of spectra from ILES of channel flow with coarse grid at $\operatorname{Re}_{\tau} \approx 400$ and $\mathcal{M}=0.5$ ..... 46
Figure 3.4 Comparison of mean and rms velocity profiles for ILES of channel flow for $\operatorname{Re}_{\tau}=640$ and 1000 at $\mathcal{M}=0.5$. ..... 48
Figure 3.5 Comparison of mean and rms velocity profiles for ILES of channel flow at $\mathcal{M}=1.5$ and $\operatorname{Re}_{\mathrm{b}}=3000$ ..... 50
Figure 3.6 Comparison of mean and rms velocity profiles for ILES of channel flow at $\mathcal{M}=3$ and $\operatorname{Re}_{\mathrm{b}}=4880$ ..... 50
Figure 3.7 Comparison of mean and rms velocity profile for analysing the grid requirement for wall-resolved ILES ..... 54
Figure 3.8 Comparison of Reynolds stress profiles for analysing the grid re- quirement for wall-resolved ILES ..... 55
Figure 3.9 Comparison of $\sqrt{\overline{\omega^{\prime 2}}}$ profiles for analysing the grid requirement for wall-resolved ILES ..... 55
Figure 3.10 Comparison of velocity correlation in stream-wise and span-wise direction at $\mathrm{y}^{+}=10$ for analysing the grid requirement for wall- resolved ILES ..... 57
Figure 3.11 Comparison of stream-wise velocity spectra at $y=0.5 \mathrm{H}$ for analysing the grid requirement for wall-resolved ILES ..... 58
Figure 3.12 Comparison of $\omega$ correlation in stream-wise and span-wise direc- tion at $y^{+}=10$ for analysing the grid resolution for wall-resolved ILES ..... 60
Figure 3.13 Comparison of skewness and kurtosis of velocity components for analysing the grid requirement for wall-resolved ILES ..... 62
4 Scaling techniques for compressible turbulent channel flow at Mach num- BER UP TO 3
Figure 4.1 Mean velocity profiles for incompressible and compressible chan- nel with cold walls ..... 69
Figure 4.2 Local friction length scale as a function of wall-distance ..... 70
Figure 4.3 Raw and transformed mean velocity profiles ..... 74
Figure 4.4 Raw and transformed rms velocity profiles ..... 75
Figure 4.5 Rms velocity profiles for showing Reynolds number effect ..... 76
Figure 4.6 Raw and transformed rms vorticity profiles ..... 77
Figure 4.7 Raw and transformed near-wall span-wise correlation ..... 78
Figure 4.8 Transformed near-wall stream-wise correlation ..... 79
Figure 4.9 Velocity skewness profiles ..... 80
Figure 4.10 Velocity flatness profiles ..... 81
5 INNER-OUTER LAYER SCALES-INTERACTION
Figure 5.1 2D pre-multiplied stream-wise velocity spectra at $y^{+} \approx 15$ ..... 86
Figure 5.2 2D pre-multiplied stream-wise velocity spectra at $y / H=0.5$ ..... 86
Figure 5.3 Mean and rms of stream-wise velocity profiles ..... 88
Figure 5.4 Instantaneous visualisation of $u^{\prime *}$ on the wall parallel plane close to the wall ..... 89
Figure 5•5 Pre-multiplied spectra of stream-wise velocity and qualitative rep- resentation of scales-interaction in the inner-outer region ..... 90
Figure 5.6 Tilting angle of the large-scale structures ..... 91
Figure 5.7 Decoupling procedure for quantifying inner-outer scales interaction ..... 92
Figure 5.8 Amplitude modulation covariance map ..... 93
Figure 5.9 Rms of the small- and large-scale velocity fluctuation ..... 95
Figure 5.10 Example of feature extraction for large-scale structures ..... 96
Figure 5.11 Conditionally averaged LMLSS in the $y-z$ plane ..... 97
Figure 5.12 Example of detected large-scale structures after different thresh- olding criteria ..... 99
Figure 5.13 Conditionally averaged 3D large-scale structure ..... 100
Figure 5.14 Comparison of large-scale footprint at different $\mathcal{M}$ with detection based on conventional wall-units ..... 101
Figure 5.15 Comparison of large-scale footprint at different $\mathcal{M}$ with detection based on semi-local units ..... 101
Figure 5.16 Organisation of large-scale structures ..... 102
Figure 5.17 Conditional statistic for the high- and low-momentum large-scale structures ..... 106
Figure 5.18 Conditional average of rms of stream-wise velocity for the high- and low-momentum large-scale structures ..... 107
Figure 5.19 Conditional average of rms of stream-wise velocity for the high- and low-momentum large-scale structures for $\operatorname{Re}_{\tau}^{*}=590$ and 640 ..... 107
6 Temporal simulation of channel flow with acoustic liner
Figure 6.1 Validation curves for impedance boundary condition ..... 111
Figure 6.2 Grid convergence for impedance wall test cases ..... 113
Figure 6.3 Comparison of channel flow simulation with Scalo et al. (2015) ${ }^{[240]} 114$
Figure 6.4 Results for varying liner resistance ..... 117
Figure 6.5 Change in drag as a function of liner resistance ..... 117
Figure 6.6 Mean velocity profile for varying resonance frequency of the liner ..... 118
Figure 6.7 Turbulent statistics for varying liner resonance frequency ..... 119
Figure 6.8 Change in drag as a function of $\mathcal{M}$ number ..... 120
Figure 6.9 Velocity spectra for varying liner resonance frequency ..... 121
Figure 6.10 Instantaneous visualization of turbulent structures for channel flow with acoustic liner ..... 123
Figure 6.11 Slice of instantaneous flow field for varying liner resonance fre- quency ..... 124
Figure 6.12 Amplitude of wave along the liner ..... 125
Figure 6.13 Phase averaged drag ..... 126
Figure 6.14 Phase averaged Reynolds stress ..... 127
Figure 6.15 Complex phase speed spectrum for parabolic base flow ..... 129
Figure 6.16 Complex phase speed spectrum with MSD wall tuned to desta- bilise A-branch mode ..... 130
Figure 6.17 Eigenfunction for parabolic and steeper base flow with bottom MSD wall ..... 131
Figure 6.18 Comparison between simulation and temporal linear stability anal- ysis for test-case ACo2 ..... 132
Figure 6.19 Comparison of eigenfunction from simulation and linear stability analysis ..... 133
7 Spatial simulation of channel flow and sound attenuation
Figure 7.1 Sponge layer for a spatial channel flow simulation ..... 140
Figure 7.2 Up-stream and down-stream travelling acoustic waves and the measurement configuration for the TMM ..... 141
Figure 7.3 Comparison of raw and span-wise averaged signal ..... 143
Figure 7.4 Example of transverse acoustic modes ..... 144
Figure 7.5 Flow configuration for Poiseuille flow validation ..... 146
Figure 7.6 Time evolution of inlet and outlet mass flow rate ..... 147
Figure 7.7 Mass flow rate and percentage change in pressure and tempera- ture in stream-wise direction ..... 148
Figure 7.8 Flow field for the low Reynolnds number poiseuille flow ..... 149
Figure 7.9 Comparison of simulation results with analytical solution ..... 150
Figure 7.10 Perturbation field with two plane waves ..... 151
Figure 7.11 Comparison of wave profiles from simulation and modal analysis ..... 152
Figure 7.12 Instantaneous acoustic pressure field with plane and transverse wave ..... 153
Figure 7.13 Comparison of transverse wave profile from simulation and modal analysis ..... 153
Figure 7.14 Flow configuration for turbulent spatial channel flow simulation ..... 155
Figure 7.15 Mean flow field of turbulent channel flow ..... 157
Figure 7.16 Stream-wise distribution of mean flow variables ..... 158
Figure 7.17 Evolution of mean pressure and temperature ..... 159
Figure 7.18 Profiles of mean stream-wise velocity and temperature ..... 159
Figure 7.19 Comparison of Reynolds stresses profiles ..... 160
Figure 7.20 Comparison of rms profiles of thermodynamic quantities ..... 161
Figure 7.21 Simulation setup to study sound attenuation in a turbulent chan- nel flow ..... 162
Figure 7.22 Schematic of acoustic and turbulent boundary layers ..... 162
Figure 7.23 Instantaneous pressure perturbation field in turbulent channel with multiple plane waves ..... 163
Figure 7.24 Frequency spectra of wall-pressure ..... 164
Figure 7.25 Comparison of down-stream travelling acoustic wave profiles ..... 165
Figure 7.26 Comparison of wave profiles from simulation and modal analysis ..... 166
Figure 7.27 Damping coefficient for the down-stream and up-stream travel- ling waves in a turbulent channel flow ..... 166
8 Simulation of flow tube with acoustic liner
Figure 8.1 Configuration for flow tube with acoustic liner ..... 169
Figure 8.2 Instantaneous flow visualisation at bottom wall of the flow tube with lined section ..... 171
Figure 8.3 Contours of mean spatial flow-field for the flow tube with lined section ..... 172
Figure 8.4 Evolution of mean velocity over the lined surface ..... 173
Figure 8.5 Stream-wise distribution of mean flow variables ..... 173
Figure 8.6 Evolution of mean pressure and temperature in the flow tube with acoustic liner ..... 174
Figure 8.7 Comparison of Reynolds stress profiles ..... 175
Figure 8.8 Instantaneous wall-normal velocity on the impedance surface ..... 176
Figure 8.9 Spatial and temporal spectra over the acoustic liner ..... 176
Figure 8.10 Eigen function over acoustic liner ..... 177
Figure 8.11 Linear Stability Analysis (LSA) of experiment of Marx et al. (2010) ${ }^{[181]}$ (liner corresponding to test-case ACoi) ..... 178
Figure 8.12 Turbulence-acoustic interaction for experiment of Marx et al. (2010) ${ }^{[181]}{ }_{178}$
Figure 8.13 LSA of simulation with liner corresponding to test-case ACo2 ..... 179
Figure 8.14 Turbulence-acoustic interaction for simulation test-case ACoz ..... 179
Figure 8.15 Pressure signal and SPL on the top wall ..... 180
Figure 8.16 Stability analysis for varying resistance of the liner ..... 181
Figure 8.17 Comparison of complex wave-number over liner against the modal analysis ..... 181
Figure 8.18 Sound attenuation in the flow-tube with liner ..... 182
C Comparison between DNS and ILES
Figure C. 1 Comparison of velocity spectra obtained from different simula- tion techniques for channel flow at $\mathrm{Re}_{\mathrm{b}}=3000$ and $\mathcal{M}=1.5$ ..... 198
Figure C. 2 Comparison of mean velocity profiles ..... 199
Figure C. 3 Comparison of rms velocity profiles ..... 200
Figure C. 4 Comparison of Reynolds stress profiles ..... 200
D Compressible relationship
Figure D. 1 Evolution of density and viscosity ratio as the function of $\mathcal{M}$ number ..... 201
Figure D. 2 Evolution of $\bar{\rho}_{w} \mathfrak{u}_{\tau}$ as the function of $\mathcal{M}$ number ..... 202
F Target solution for sponge zone
Figure F. 1 Schematic for computing the target field for the sponge zone ..... 207

## LIST OF TABLES

2 Equations and Simulation techniques
Table 2.1 Spatial discretisation for first-derivative ..... 23
Table 2.2 Spatial discretisation for second-derivative ..... 29
Table 2.3 Parameters of validation test-case for DNS of supersonic channel flows ..... 35
Table 2.4 Validation results ..... 35
3 Modified Finite Difference scheme for Implicit Large Eddy Simulation
Table 3.1 Computational grids used for the parametric study of modified spatial 2nd-derivative FDS at $\mathrm{Re}_{\tau} \approx 400$ and $\mathcal{M}=0.5$ ..... 42
Table 3.2 SVV kernels used for the parametric study of modified spatial $2^{\text {nd }}$-derivative FDS for ILES of channel flow at $\operatorname{Re}_{\tau} \approx 400$ and $\mathcal{M}=0.5$ ..... 43
Table 3.3 Mean flow variables and errors in their prediction with different SVV kernels for ILES of channel flow at $\operatorname{Re}_{\tau} \approx 400$ at $\mathcal{M}=0.5$ ..... 43
Table 3.4 Simulation parameters and results for the parametric study of modified spatial $2^{\text {nd }}$-derivative FDS for ILES of high $\operatorname{Re}_{\tau}$ channel flows ..... 48
Table 3.5 Computational grids used for the parametric study of modified spatial $2^{\text {nd }}$-derivative FDS for ILES of supersonic channel flows ..... 49
Table 3.6 Mean flow variables and their errors with different SVV kernels for ILES of supersonic channel flows ..... 51
Table 3.7 Test-cases for analysing grid requirement for ILES of channel flows ..... 53
4 Scaling techniques for compressible turbulent channel flow at Mach num- BER UP TO 3
Table 4.1 Simulation test-cases for analysing the scaling techniques ..... 73
Table 4.2 Mean flow variables for simulation test-cases for analysing scal- ing techniques ..... 73
5 INNER-OUTER LAYER SCALES-INTERACTION
Table 5.1 Simulation test-cases for studying inner-outer scales interaction at different $\mathcal{M}$ numbers ..... 87
Table 5.2 Area occupied by the large-scale structures ..... 103
Table 5.3 Change in drag due to the large-scale structures ..... 104
Table 5.4 Contribution to skin-friction from the large-scale structures ..... 104
6 TEMPORAL SIMULATION OF CHANNEL FLOW WITH ACOUSTIC LINER
Table 6.1 Grid convergence analysis with impedance boundary condition ..... 112
Table 6.2 Simulation test-cases with impedance boundary condition ..... 115
Table 6.3 Characteristics of the wave along the liner ..... 122
Table 6.4 Comparison between simulation and linear stability analysis ..... 132
7 Spatial simulation of channel flow and sound attenuation
Table 7.1 Comparison of wave properties computed from simulation and modal analysis for multiple plane acoustic waves ..... 152
Table 7.2 Comparison of wave properties computed from simulation and modal analysis for plane and transverse acoustic waves ..... 154
Table 7.3 Simulation parameters to study wave attenuation in turbulent channel flow ..... 163
Table 7.4 Details of signal used for post-processing ..... 164
Table 7.5 Properties of acoustic waves in turbulent channel flow ..... 167
A Brief history of code "Compact3D" and coefficients for compact schemes
Table A. 1 Compact schemes for spatial first-derivative ..... 190
Table A. 2 Compact schemes for spatial second-derivative ..... 191
C Comparison between DNS and ILES
Table C. 1 Computational grids for testing DNS and ILES ..... 197
Table C. 2 Computational grid for comparing the benchmark channel flow test-case with other solvers ..... 199

## B Grid stretching

Listing $1 \quad$ Wall-normal grid stretching ..... 193
Listing 2 Stream-wise grid stretching ..... 194

## Physical quantities

$\eta \quad$ Kolmogorov scale
$\gamma \quad$ Ratio of specific heats
$\mu \quad$ Dynamic fluid viscosity
q Heal flux
$\omega \quad$ Angular frequency
$\rho \quad$ Fluid density
R Resistance
$\tau \quad$ Stress
$r_{u v}$ Reynolds stress
s Entropy
$\xi \quad$ Damping ratio
T Temperature
c Speed of sound
t Time
$c_{f} \quad$ Drag coefficient
$u ; u_{1}$ Stream-wise velocity
$c_{p}$ Adiabatic specific heat
H Channel half-height
h Enthalpy
$w ; \mathfrak{u}_{3}$ Span-wise velocity
K Stiffness
$x ; x_{1}$ Stream-wise coordinate
$\mathrm{K}_{\mathrm{t}} \quad$ Thermal conductivity
$y ; x_{2}$ Wall-normal coordinate
$l_{v} \quad$ Viscous length-scale
$z ; x_{3}$ Span-wise coordinate

## Non-dimensional numbers

| $\kappa$ | von Kármán constant | Pr | Prandtl number |
| :--- | :--- | :---: | :--- |
| $\mathcal{M}$ | Mach number | $\operatorname{Re}$ | Reynolds number |
| $\mathrm{B}_{\mathrm{q}}$ | Heat flux | C | Log-law intercept |

## Numerics

$\alpha ; \beta ; a ; b ; c ; d$ FDS coefficients
v Spectral viscosity
$v_{0} \quad$ Spectral viscosity at cut-off
$\sigma_{i} \quad$ stability footprint on imaginary axis
$\sigma_{r} \quad$ stability footprint on real axis
k Spatial wave-number

Simulation parameters
$\Delta_{x, y, z, t}$ Discretisation $\quad N_{x, y, z}$ Number of grid points
$L_{x, y, z}$ Domain length

Other
F Flux vector
$\mathcal{F} \quad$ Flatness
$\mathcal{S}$ Skewness
$\bar{A} ;<A>$ Mean of quantity A
$A^{\prime} \quad$ Fluctuation of quantity $A$
q Solution vector $\quad X^{v, w} ; \gamma^{u, w} ; Z^{u, v}$ Vorticity waves

Subscripts
$A_{\mathrm{b}} \quad$ Bulk quantity A
$A_{c} \quad$ Value of quantity $A$ at centre of the channel
$A_{t} \quad$ Turbulent quantity of A
$A_{w} \quad$ Value of quantity $A$ at the wall $A_{\tau} \quad$ Friction quantity $A$

## Superscripts

| $\tilde{A}$ | Dimensional quantity A | $A^{+}$ | Quantity A scaled with viscous <br> scales |
| :--- | :--- | :--- | :--- |
| $A^{*}$ | Quantity A after compressible <br> transformation |  |  |

ACRONYMS

| 2DOF | Two Degree Of Freedom | LSA | Linear Stability Analysis |
| :--- | :--- | :--- | :--- |
| BPF | Blade Passing Frequency | LSM | Large Scale Motion |
| CAA | Computational Aero Acoustics | LSM | Least Square Method |
| CFD | Computational Fluid Dynamics | MMM | Mode Matching Method |
| CFL | Courant-Friedrichs-Lewy | MPI | Message Passing Interface |
| DGM | Discontinuous Galerkin Method | MSD | Mass-Spring-Damper |
| DNS | Direct Numerical Simulation | NSE | Navier-Stokes Equations |
| FDS | Finite Difference Scheme | RANS | Reynolds Averaged Navier Stokes |
| HMLSS | High Momentum Large Scale | SDOF | Single Degree Of Freedom |
|  | Structure | SFM | Straight Forward Method |
| HPC | High Performance Computation | SGS | Sub-Grid Scale |
| IBC | Impedance Boundary Condition | SNR | Sound to Noise Ratio |
| IBM | Immersed Boundary Method | SPL | Sound Pressure Level |
| ILES | Implicit Large Eddy Simulation | SVV | Spectral Vanishing Viscosity |
| LEE | Linearised Euler Equations | SWR | Standing Wave Ratio |
| LES | Large Eddy Simulation | TBL | Turbulent Boundary Layer |
| LODI | Local One Dimensional Inviscid | TMM | Two Microphone Method |
| LMLSS | Low Momentum Large Scale | TPM | Two Port Matrix |
|  | Structure | VLSM | Very Large Scale Motion |
|  |  |  |  |

## INTRODUCTION

Aircraft and engine manufacturers are always under the continuous pressure of growing demand for quieter aircraft. This is due to the increasing quality of life and due to the necessity to compensate air traffic growth and the encroachment of neighbouring communities for larger airport infrastructures. Air traffic is increasing because of the time efficiency of aircraft, and it is speculated to double in the next 20 years. But at the same time aircrafts are noisy and the well-known issues associated with elevated aircraft noise level includes stress, sleep depreciation and hypertension among others. Governments and other related institutions are taking necessary actions to control the noise emission and protect people. Major source of aircraft noise can be broadly classified into (a) engine, (b) airframe and (c) mechanical vibrations. The growth in the theoretical description of many aeroacoustic mechanisms in the past sixty years has led to a progressive reduction of aircraft noise.

Engine noise is a major contributor to the aircraft noise. The first generation of aircrafts in 1960 s were very noisy, which led to the introduction of high by-pass ratio turbofan engines and effective nacelle acoustic treatment. These type of engines not only reduced the sound level by $25-30 \mathrm{~dB}$, but they were also very fuel efficient. This makes turbofan engines the only choice for commercial aircrafts. However, since the 1980 os the noise reduction trend has not been so significant. Therefore, any further noise reduction is very difficult to be achieved without affecting the aircraft operating cost. Due to the entry of ultra high-bypass ratio turbofans into service and novel noise control devices on modern civil aircraft, the engine noise is expected to be comparable and even lower than the airframe noise generated by the high-lift devices and by the undercarriage. A great deal of interest have been devoted during the last years to the rediscovery and improvement of analytical models for both airframe and engine noise prediction using multi-disciplinary/multi-objective optimisation processes which relies on fast numerical methods ${ }^{[7,8,170,171]}$.

A turbofan engine produces both tonal and broadband noises. Tonal noise is generated due to the fan, and the broadband noise is due to the turbulence. Various technologies have been developed to reduce the engine noise, such as scarf inlet, forward swept fans, trailing-edge blowing, among others acoustic treatment. Acoustic liners are key technology to absorb sound in ducts such as turbofan engines, and it is the passive method for noise reduction. Acoustic liner is a sandwich panel and work as a group of Helmholtz resonators. A single degree of freedom liner panel will include the perforated face-sheet, honeycomb structure and the solid back-plate. Some liners have multiple layers of honeycomb and perforated face-sheet for multiple degrees of freedom. In many practical applications they are subjected to high-speed flows and turbulence. In turbofan engines, liners are usually applied on the internal walls of the engine nacelle to absorb the radiated acoustic energy (see figure 1.I for an example of acoustic liner.) Many studies have been devoted to understand the behaviour of acoustic liner, owing to the importance of


Figure 1.1: Illustration of engine noise treatment.
noise reduction. In the past, most of the research was done with the help of experimental techniques. Experimental techniques have several advantages, but the major drawback is the intrusive measurement of the flow. This indeed limits access to the major part of the flow, where one might find interesting physics. On the other hand numerical simulation gives access to every point in space, but the major setback is the truncation of the computational domain and high computational cost. With the recent developments in the computer technology, High Performance Computing (HPC) has entered the reality of petascale computing. Currently, HPC has deep consequences for scientific research as it open doors for solving complex turbulence problems, which were once considered impossible. The major objective of this Ph.D. thesis is to study how acoustic liners affect the turbulent flow and noise attenuation with the help of numerical simulations. This is an attempt to study the interaction within a complex flow physics, therefore this topic offers wide range of possibility for research. The thesis was conceived as a 6 step approach, therefore before presenting the motivations and the multiple objectives (in Section 1.4) a brief introduction on acoustic liners, wall-turbulence and the numerical simulation is given in Sections 1.1-1.3. Finally, the organisation of the thesis is presented in Section 1.5 .

### 1.1 OVERVIEW ON ACOUSTIC LINERS

Acoustic liner covers most of the available surface in turbofan engines depending on the installation of other engine systems. The typical acoustic liners are locally reacting and the acoustic impedance of the liner will depend on properties of the lining, grazing flow and frequency of the sound. In the future the bypass ratio of the turbofan engine will in-
crease and the inlet length will not be scaled with the same factor. Since the performance of the liner is limited to its length, these changes will make liners less effective. Different methods of optimisation can focus on the variations in the depth of the honeycomb core, core-cell dimensions and number of layers. However from the engineering point of view, manufacturers should also keep in mind that the liner will increase the overall engine weight.

Since 1970's acoustic liner with adjustable impedance that has potential application in the active control of duct noise and unstable flow in turbo-machines has received some attention ${ }^{[62,305]}$. It is well known that the grazing flow changes the acoustic impedance of the liner, thereby affecting the noise attenuation in the duct. A better understanding of this so called grazing flow effect is necessary for the design of efficient acoustic liners ${ }^{[102,148,204]}$. During the past decades, considerable research has been carried out to understand the effect of grazing flow on the acoustic impedance of the liner ${ }^{[138]}$. The acoustic resistance of perforated face-sheet linearly increases and the acoustic reactance slightly decreases with increasing grazing flow velocity ${ }^{[57, ~ 96, ~ 102, ~ 138, ~ 221] ~ . ~ T h e ~ v i s u a l i s a t i o n ~}$ and the measurements of the flow details in the vicinity of an orifice reveal that vortices are generated from leading edge of the orifice under the acoustic excitation and convected down-stream by the mean grazing flow ${ }^{[208,295]}$. The vortical and acoustic flow at an orifice in the presence of grazing flow interacts strongly, and this leads to the change of the orifice acoustic impedance. Several investigations have been concentrated on de-


Figure 1.2: Schematic of grazing flow.
scribing how sound energy is dissipated in the near-field of the orifice and theoretical models have been developed ${ }^{[108,116,143,236,288]}$. Although several models has been proposed with different assumption and in order to make the model flexible the problem has been generally over simplified. Theoretical modelling of the acoustic behaviour of perforates subjected to grazing flow is mainly dependent on the experimental data. Mea-
surement of the impedance of perforated plates is generally restricted to a single orifice, and it has been assumed that the results could be extrapolated to multi-orifice perforated plates via the percentage open area or porosity of the perforate ${ }^{[148]}$. Ronneberger (1972) ${ }^{[235]}$, Goldman and Panton (1976) ${ }^{[96]}$ and Rao and Munjal (1986) ${ }^{[228]}$ among others measured the impedance of the orifice within a thin boundary layer and found that the boundary layer thickness affects the impedance of the orifice. Goldman and Chung (1982) ${ }^{[97]}$ found that the orifice impedance was affected only by the inner-region of the boundary layer. Later Cummings (1986) ${ }^{[57]}$ concluded that boundary layer turbulence is an important parameter in the measurement of perforate impedance and it is necessary to measure the perforates under conditions similar to those in which they are to be used.

The acoustic liners have millimetre-size perforated holes. These holes are too small to simulate accurately in most of the computational codes, therefore their analysis is restricted to theoretical modelling, measurement and high fidelity simulations. The majority of modelling work has been carried out in the frequency domain. Howe (1979) ${ }^{[114]}$ modelled the acoustic energy dissipated by periodic vortex shedding for a single orifice in a high Reynolds number flow. Followed by experimental and numerical investigation of Jing and Sun (1999) ${ }^{[139]}$ and Jing and Sun (2000) ${ }^{[140]}$ on the effect of the orifice thickness and the bias flow rate, showing that an appropriate bias flow rate can significantly increase damping and that the orifice thickness is important. Tam et al. (2001) ${ }^{[267]}$ carried out Direct Numerical Simulation (DNS) of a separate aperture and showed that vortex shedding was the dominant damping mechanism for large-amplitude incident waves. Mendez and Eldredge (2009) ${ }^{[189]}$ performed compressible Large Eddy Simulation (LES) to study the aeroacoustic characteristics of orifice. Burak et al. (2009) ${ }^{[40]}$ also used LES to solve linearised Navier-Stokes Equations (NSE) to study the damping performance of an acoustic liner (without meshing the holes) in the presence of grazing flow. With the DNS, Zhang and Bodony (2012) ${ }^{[302]}$ investigated the acoustic behaviour of a honeycomb liner and found that the orifice boundary layer played a critical role. Since it is easier to measure near the tiny orifice, experimental investigations focus on measuring the liner impedance or power absorption coefficient ${ }^{[303, ~ 304]}$. Ingård and Labate (1950) ${ }^{[131]}$ experimentally visualised that the incident sound amplitude, frequency, the orifice diameter and thickness affected the induced motion of the fluid near the orifice. Hughes and Dowling (1990) ${ }^{[121]}$ showed that the sound incident on a perforated liner with a bias flow might be completely absorbed, if the flow speed and the liner geometry were chosen properly. Experiment of Jing and Sun (1999) ${ }^{[139]}$ confirmed that the orifice thickness and the bias flow Mach number played dominant roles in affecting the liner's damping performance. Tam et al. (2014) ${ }^{[271]}$ conducted experiments to study the grazing flow effect on the damping performance of acoustic liners. They showed that the acoustic liner could generate self-noise, which might result from a feedback resonance mechanism driven by a Kelvin-Helmholtz instability wave of the free shear layer spanning the openings of the liner cavity. Furthermore the drag was found to increase by about $4 \%$ for an acoustic liner with a $10 \%$ open area ratio in comparison to the turbulent boundary layer drag over a flat wall.

The key parameter in the liner optimisation process is the acoustic impedance. This is comprised of a real part, the resistance, and an imaginary part, the reactance. The first step in the liner design consists in estimating the values of the liner resistance and reactance that ensure the maximum sound attenuation for a prescribed duct modal content, over the frequency range of interest ${ }^{[46]}$. The second step consists in selecting a liner class
that matches as close as possible the optimal resistance and reactance values for each frequency band of interest ${ }^{[45]}$. The acoustic liners used in turbofan engines consist of one or two sandwich layers. A Single Degree Of Freedom (SDOF) panel is constituted of a porous face-sheet, a honeycomb core, and a solid backplate. A Two Degree Of Freedom (2DOF) panel is constituted of a porous face-sheet, two layers of honeycomb separated by a porous septum, and a solid backplate. Both SDOF and 2DOF liners are effective over narrow frequency ranges and must be tuned on one or two fan tones, respectively. 2DOF has larger band-width than SDOF. Typically, the acoustic properties of this class of treatment shows linear behaviour at low Sound Pressure Level (SPL) and with no flow and do not depend on the amplitude of the incident acoustic wave. For a high value of the incident sound pressure level the liner resistance starts exhibiting non-linear behaviour and dependence on the incident wave amplitude. Melling (1973) ${ }^{[188]}$ first recognised this behaviour and argued that, in the linear regime, the micro flow in the orifice is laminar and the dissipative (resistive) losses maybe of Poiseuille type or Helmholtz type. In both cases the losses are due to viscous dissipation in the shear layer. This hypothesis have been partially confirmed by Tam et al. (2000) ${ }^{[272]}$. In the linear regime, a jet-like flow close to the orifice openings and a strongly oscillatory boundary layer. In the nonlinear regime, Melling $(1973)^{[188]}$ argued that a turbulent jet takes place at the mouth of the resonator and the primary dissipation mechanism is turbulence. This mechanism was not confirmed by the numerical analysis by Tam et al. (2000) ${ }^{[272]}$ who observed a vortex-shedding mechanism taking place at certain acoustic frequency bands, which is responsible for the conversion of acoustic energy into kinetic energy and further viscous dissipation into heat.

In many practical situations, liners are subject to high speed flows and turbulence, and much effort has been devoted for studying the effect of grazing flow on the liner impedance. It is for example, well known that as a result of the interaction between the acoustic and vortical modes in the holes of the perforated face-sheet liner properties can be modified. Conversely, the liner may modify the flow and turbulence in its vicinity, compared to a rigid wall. An effect of this is drag increase, ${ }^{[271,294]}$ especially for small liner porosity. Another effect is the flow instability observed in the vicinity of a low resistance acoustic liner ${ }^{[18]]}$. Several numerical simulations in flow ducts with liners have been performed in connection with this topic. The objective of many simulations has been to study sound propagation in lined ducts with a known base flow using the linearised Euler equations. A difficulty is then to impose a well-posed impedance boundary condition, especially in the time-domain solvers ${ }^{[86,169,183,212,213,230,233,264,306]}$. These simulations neglect both the effect of the grazing flow on the impedance and the effect of the impedance on the flow. Other simulations are based on the full non-linear NSE and the flow is computed together with the acoustic field ${ }^{[211,240,267,270,301,302]}$. Among these simulations, some include the liner back cavity and the face sheet perforations, ${ }^{[267, ~ 270, ~ 301, ~ 302] ~}$ so as to include all possible flow-acoustics interactions. Others use an impedance boundary condition with a given impedance, ${ }^{[211,240]}$ which means that the effect of the flow on the impedance is not a part of the computation. Olivetti et al. (2015) ${ }^{[211]}$ computed the sound propagation in a lined pipe, a simple model for a nozzle, in order to suppress resonant modes in the duct which have a strong impact on the noise produced by the jet outside the nozzle. Scalo et al. (2015) ${ }^{[240]}$ studied the turbulent flow in a compressible periodic channel with an impedance boundary condition and described how the structure of turbulence gets modified as the resistance of the liner decreases. Scalo et al. (2015) ${ }^{[240]}$
set the resonance frequency of the liner so that it corresponds to some typical time scale of the flow. As a result the liner resonance frequency was rather high, and larger than typical frequencies encountered in aeroacoustic applications. Apart from sound damping in ducts, there is a growing interest in passive methods for aeroacoustic and flow control, ${ }^{[307]}$, and a better knowledge of the behaviour of the turbulent flow in the vicinity of non-rigid wall is interesting in general.

### 1.2 OVERVIEW ON WALL TURBULENCE

Wall-bounded turbulent flows have received continuous attention due to its immense practical importance. Turbulence has remained one of the greatest unsolved problem of classical physics since the pipe flow experiment of Reynolds. Many challenges are present in physical understanding, theoretical interpretation, experimental techniques and numerical simulations. In the past few decades, high Reynolds number wall-bounded turbulent flows have become a very active area of research. The research on wall-turbulence can be categorised into: (a) scaling and (b) coherent structures of wall-turbulence.

Initially, the scaling was mainly focussed on the mean velocity profile and the Turbulent Boundary Layer (TBL) was categorised into two distinct regions: (a) near-wall region and (b) outer-region. Viscosity is dominant in the near-wall region and not in the outer-region ${ }^{[51,103]}$. In the near-wall region, friction velocity and viscous length scale are used as the inner-variables for scaling, and in the outer-region, friction velocity and characteristics flow length scale are used. Recent advances in computational and experimental capabilities have expanded the understanding about turbulence. For example, the inability of the present scaling techniques to collapse the distribution of stream-wise turbulence intensity. Stream-wise turbulence intensity near the wall in inner-variables show a Reynolds number dependence. Moreover, the emergence of a secondary peak in the outer-region distribution of turbulence intensity is also found at higher Reynolds number.

An example of the law-of-the-wall with different regions of TBL is shown in figure 1.3. Viscosity is dominant in the near-wall region up to $y^{+} \approx 5$ ( $y^{+}$is the wall-normal distance scaled with friction length-scale), and this region is known as viscous sub-layer. Log-law is valid from approximately $y^{+}=30$ up to $y=0.2 \mathrm{H}$. The region which blends the viscous sub-layer and log-region is known as buffer layer. Generally, part of the TBL within the log-region is called as inner-layer and above $y=0.2 \mathrm{H}$ is called as outer-layer. Definition of $y^{+}$and $\bar{u}^{+}$will be given in equation 2.51.

The second category of wall-turbulence research was inspired by the observation of coherent structures in TBL. Theodorsen (1952) ${ }^{[275]}$ identified the horse shoe vortices, followed by the discovery of near-wall streaks and their role in the turbulence production cycle by Hama et al. (1957) ${ }^{[104]}$ and Kline et al. $(1967)^{[151]}$. Townsend (1961) ${ }^{[280]}$ introduced the idea of active and inactive motion to distinguish the motions that contribute to wall-normal velocity fluctuations and the momentum transport. Other works on coherent structures in wall-turbulence can be found in Bradshaw (1967) ${ }^{[33]}$, Townsend (1976) ${ }^{[281]}$, Head and Bandyopadhyay (1981) ${ }^{[107]}$ and Perry and Chong (1982) ${ }^{[220]}$, among others. Review on this topic have been presented by Cantwell (1981) ${ }^{[44]}$ and Robinson (1991) ${ }^{[234]}$.


Figure 1.3: Regions of turbulent boundary layer. (a) $---\bar{u}^{+}=y^{+},(b)-\cdots \bar{u}^{+}=1 / k \ln \left(y^{+}\right)+C$.

The advancement in experimental and computational facilities have encouraged the research on high Reynolds number flows. The findings revisited questions on the universality of turbulence and the influence of outer-region structures. A summary of the current state-of-art research on scaling and coherent structures can be found in McKeon and Sreenivasan (2007) ${ }^{[187]}$.

Coherent structures are generally considered as organised motions that are continuous in time and space and contribute significantly to the transport of heat, mass and momentum ${ }^{[179]}$. The self-sustaining or regeneration mechanism of the near-wall turbulence can be related to these structures ${ }^{[105,137]}$. Panton (2001) ${ }^{[215]}$ reviewed the ideas "why wall-turbulence is self-sustaining?", and the process can be classified into two categories, based on: (a) instability and transient growth mechanism and (b) vortex structure regeneration mechanism ${ }^{[4,248]}$. The main coherent structures can be categorised as: (1) near-wall streaks associated with the near-wall cycle and scale on inner-variables; (2) Least Square Method (LSM) which are related to outer-layer bulge and the vortex packets as discussed by Head and Bandyopadhyay (1981) ${ }^{[107]}$ and Adrian (2007) ${ }^{[4]}$ which scale in $\mathcal{O}(\mathrm{H})$ and (3) Very Large Scale Motion (VLSM) is the concatenated packets of vortices and/or meandering superstructures with stream-wise length scale of $\mathcal{O}(10 \mathrm{H})$, where H is the characteristic flow length scale. While large stream-wise structures in wall turbulence has been observed several decades ago ${ }^{[153,281]}$, the dynamical importance of the these structures had not been acknowledged until recently. The measurements of spectra revealed the presence of large-scale motions and found that these structures contribute about half of the total energy to the spectra ${ }^{[12,178]}$. These structures are not yet fully understood, but they are generally associated with a peak in the pre-multiplied spectra of stream-wise velocity. Guala et al. (2006) ${ }^{[101]}$, Adrian (2007) $)^{[4]}$ and Balakumar and Adrian $(2007)^{[12]}$ have reported that even though there is no evidence of extremely long scales in the wall-normal velocity spectra due to the presence of wall, the superstructures must be considered to be active in the sense that they carry a significant part of the shear stress.

A popular example of external wall-bounded flow is a boundary layer flow, and channel and pipe for the internal flows. The main differences in these flows are the geometrical confinement. The inner-region of the TBL is invariant for all these flow types, but the outer-region large-scale structures will depend on the flow geometry. Monty et al. (2009) ${ }^{[196]}$ compared a channel, pipe and boundary layer flow at the same friction Reynolds number and found VLSM energy in pipes and channels agrees well, but resides in longer wave-length and further away from the wall than in boundary layers.

The main problem with the detection of large-scale structures is that their transverse scale is of the order of the flow characteristic length-scale and their lateral arrangement can lead to overestimation of width, if the neighbouring lateral structures are grouped and considered as a single structure ${ }^{[242]}$. Currently, high Reynolds number simulations (for investigating large-scale structures) can be performed with larger computational domain up to certain extent, but there is still a long way to go. On the other hand large amount of experimental data on the large-scale structures are gathered using the Taylor's hypothesis ${ }^{[274]}$. The validity of Taylor's hypothesis to convert the temporal data to spatial data remains a big concern. Taylor's hypothesis becomes worse towards the wall because convection velocity of the large-scale structure is of the order of centreline velocity rather the local mean velocity ${ }^{[65]}$.
taylor's hypothesis: If turbulence level were low, the time variation in the velocity $u$ observed at a fixed point in the flow would be approximately the same as those due to the convection of an unchanged spatial pattern past the point with a constant convection velocity $u_{\text {conv }}$.

$$
\mathfrak{u}(\mathrm{x}, \mathrm{t})=\mathfrak{u}\left(\mathrm{x}-\mathrm{u}_{\mathrm{conv}} \mathrm{t}, \mathrm{o}\right)
$$

where $x$ is the distance measures down-stream in the mean flow direction and $t$ is the time respectively.

The study of coherent structures also revealed that the turbulent motion in the nearwall region interacts with the outer-region large-scale motions. The inner-outer interaction was documented earlier by Rao et al. (1971) ${ }^{[227]}$ and Wark and Nagib (1991) ${ }^{[289]}$, but their objective was investigating Reynolds number scaling and large-scale structures. Due to the unavailability of experimental facilities and computational resources for high Reynolds number flows in those periods, most of the research was primarily focussed on low Reynolds number flows. Enough scale separation does not exist at lower Reynolds number and different scales of turbulence overlap with the small-scales of near-wall cycle ${ }^{[111]}$. An important consequence of this interaction between these structures is observed via the Reynolds number dependent peak of the stream-wise intensity in the buffer-layer. Computational studies by Spalart (1988) ${ }^{[262]}$, Toh and Itano (1999) ${ }^{[278]}$, Abe et al. (2004) ${ }^{[1]}$, Hoyas and Jiménez (2006) ${ }^{[117]}$, among others tried to analyse the influence of large-scale structures at the wall. Several studies including Jiménez et al. (1999) ${ }^{[135]}$, Del Álamo and Jiménez (2003) ${ }^{[64]}$, Hutchins and Marusic (2007) ${ }^{[125]}$ among others have shown that outer-region large-scale influence on the near-wall region becomes increasingly noticeable with the increasing Reynolds number. With high Reynolds number TBL experiments Hutchins and Marusic (2007) $)^{[125,126]}$ showed the evidence of the influence of large-scale structures or superstructures on the near-wall small-scales. They used instantaneous time series which was converted to spatial data with the help of Taylor's hypothesis. By analysing the low-pass filtered time-series with the Hilbert transform of
stream-wise velocity Tardu (2008) $)^{[273]}$, Mathis et al. (2009) ${ }^{[186]}$ found the inner-outer interaction was similar to the amplitude modulation of near-wall small-scale structures by the outer-region large-scale structures. The influence of the outer-region large-scale structure increases with the Reynolds number ${ }^{[125]}$, however the near-wall streaks which are typical coherent structures close to the wall remains unchanged even at higher Reynolds number. Mathis et al. (2009) ${ }^{[186]}$ investigated the location of the outer-region spectral peak and found that the geometric centre of the logarithmic region coincides with the reversal of the phase relationship between large-scale and small-scale turbulent intensities.

### 1.3 OVERVIEW ON LARGE EDDY SIMULATION

Currently numerical simulations are gaining importance and more companies are coming forward for simulation driven engineering or product development. This is due to the cost-effectiveness of the numerical simulations, and their ability to provide information about the complete flow-field. It was mentioned in the previous section that wall-turbulence involves different scales with non-linear energy cascade. Hence, the Computational Fluid Dynamics (CFD) approaches for wall-turbulence can be classified into three: (a) DNS, (b) LES and (c) Reynolds Averaged Navier Stokes (RANS) simulations based on the quantity of flow scales that are discretised. In comparison with DNS, LES is computationally cheap and applicable to high Reynolds number flow, whereas RANS fails to reproduce the small-scale dynamics accurately (see figure 1.4). The idea of LES


Figure 1.4: Differences between - — DNS, - - - LES and ---- RANS.
is to reduce the number of degrees of freedom to save the computational resources. This is done by introducing the low-pass filter to define the large-scales and the unresolved part is known as Sub-Grid Scale (SGS). The equations for the resolved scales are solved by introducing the SGS tensor within the framework of a closure problem. One restrictive assumption in establishing the equations concerns the commutation between the filtering operator and the spatial derivatives ${ }^{[59,91-93,239]}$. The commutation error is always ignored and it is actually negligible if the distortion of computational grid is very weak, however the contrary is not true.

Another widespread LES approach is to define modelling strategy. This involves developing SGS models purely based on theory and physics. The SGS model ensure the dissipation for LES, but has considerable truncation error at the mesh cut-off as the kinetic energy is still significant at the cut-off wave-number. These numerical errors can become superior compared to the SGS contribution even with the high-order scheme. The errors from aliasing and differentiation can be comparable or even outweigh SGS modelling terms in LES ${ }^{[48,93]}$. The role of discretisation and modelling errors have also been investigated directly by LES through a posteriori tests ${ }^{[190-192,286]}$.

The restrictive formalism of LES combined with the numerical errors lead to the mismatch between theory and practice. Therefore the motivation behind another approach known as ILES was to overcome this mismatch using formulation based on numerical and physical consideration ${ }^{[28,73,85,152,177,277]}$. The source of regularisation was the discretisation of the governing equations and/or through an additional discrete operator to selectively damp or filter the smallest scale ${ }^{[9,14,22,25,26,133,265]}$. This type of relaxation model can be used alone ${ }^{[185,243]}$ or in conjunction with a deconvolution model ${ }^{[70,71,109,110]}$. An overview can be found in Grinstein et al. (2007) ${ }^{[100]}$.

Dairay et al. (2014) ${ }^{[58]}$ found dynamic Smagorinsky model ${ }^{[90,259]}$ or the WALE ${ }^{[209]}$ model were unable to prevent the production of small-scale oscillation. With artificial dissipation significant improvement was found in the LES by damping the small-scale oscillation. This led to smooth solution as expected in the formalism based on filtering procedure. Example of mismatch between practice and theory was shown by Dairay et al. (2014) ${ }^{[58]}$, where a SGS model (based on filtering in practice) cannot ensure filtering effect, and on the other hand the use of artificial dissipation free from rigorous formalism provides filtering effect that is favourable for the accuracy of the calculation. To overcome the weakness of classical LES, a selective action is carried out on the small-scales. This selective action can be based on a numerical stabilisation procedure ${ }^{[149,172,216-218, ~ 256, ~ 284, ~ 297] ~}$ that can be combined with SGS model particularly in the framework of Variational Multiscale method ${ }^{[99,122-124]}$. The numerical parameter driving the regularisation depend on the physical SGS model because the selective action on small scales and the SGS model are combined together. Dairay et al. (2017) ${ }^{[59]}$ presented a new way to calibrate the numerical dissipation based on the explicit SGS modelling through a physical scaling. Due to this, the numerical dissipation becomes a substitute for the SGS model, thus the difference between artificial dissipation and SGS modelling becomes less worthwhile. The implicit dissipation is controlled explicitly and Dairay et al. (2017) ${ }^{[59]}$ introduces the notion controlled ILES to describe this approach.

This is an alternative way to perform LES based on targeted numerical dissipation introduced by the discretisation of the diffusion terms in NSE. Regularisation technique is equivalent to the use of SVV. The flexibility of this method ensures high-order accuracy while controlling the level and spectral features of excess artificial numerical viscosity. Dairay et al. (2017) ${ }^{[59]}$ used a Pao-like spectral closure based on physical arguments to scale the numerical viscosity a priori, and found this approach more efficient and accurate. The main benefit of this approach is the possibility to correctly calibrate the numerical dissipation at the smallest scale. This model can be viewed as (a) ILES because numerical error is the source of artificial dissipation and (b) as explicit SGS model because of the equivalence with spectral viscosity prescribed on a physical basis.

### 1.4 MOTIVATIONS AND OBJECTIVES

In many practical situations, acoustic liners are subject to high speed flows and turbulence. It is well known that the acoustic liner interacts with the flow and vice-versa. Several experimental studies have been performed to understand the liner behaviour and properties, but due to the limitations in the experimental diagnostics it is not yet fully
understood. Hence, the main objective of this thesis is to understand the flow physics: acoustic propagation in turbulent flow in the presence of acoustic liners with the help of numerical simulation. This is a very broad topic which offers the possibility to dive into many other directions. A brief outline of the thesis is presented in figure 1.5 . Motivations behind each objective will be presented in the following.


Figure 1.5: Objectives of the thesis.

To accomplish the objectives the thesis, sound propagation at moderately higher subsonic Mach numbers has to be simulated. This requires solving compressible NSE, and this led to the development of parallel ${ }_{3} \mathrm{D}$ compressible NSE solver "Compact3D". The solver bears many similarity with the incompressible solver "Incompact3D" (which was developed with the collaboration between the Institut PPRIME and Imperial College, London). The solver uses high-order compact Finite Difference Schemes (FDS) for the spatial discretisation and 2D pencil domain decomposition. Compact3D was initially developed in early 2000 at the Institut PPRIME for the direct computation of sound ${ }^{[78,79]}$. Initially the solver was sequential, which has then been constantly updated to simulate different flow configuration. This is the first version of Compact3D which (a) allows for parallel computation and (b) includes wall-bounded flow configuration. A more detailed introduction about Compact3D is provided in Chapter 2 and Appendix A.I.

Solving compressible NSE facilitates solving both sound generation and propagation. For a turbulent flow the scale-separation increases with the Reynolds number. In direct computation one has to discretise all scales of the flow which will lead to the usage of a very large number of grid points. This will have a direct impact on the computational cost of the numerical simulation. To circumvent this issue, LES is performed where only the large-scales of the flow are resolved and the unresolved SGS are modelled. With proper modelling, one can account for the unresolved small-scales and reproduce solution similar to the DNS. The advantage of LES is that, one can find accurate solution with a reduced computational cost. There are several LES techniques for instance; low-pass filtering filtering, SGS modelling, numerical dissipation and SGS modelling combined with numerical dissipation. When the numerical dissipation is used it is called as Implicit Large Eddy Simulation (ILES), but the challenge with this technique is that if the dissipation is not calibrated properly then the model would not behave as expected. Hence, in the thesis controlled ILES is performed where the information from the turbulence physics is used to find the amount of excess dissipation that needs to be introduced with the discretisation scheme for ILES. The thesis mainly includes ILES of channel flows with different wall boundary conditions. A detailed note on different LES techniques is given in Section 1.3.

As mentioned earlier, the primary objective of this thesis is to study the interaction between three entities: turbulence, acoustic liner and sound propagation. These subjects could be investigated standalone by themselves, but here they are considered to be interacting with each other. This complicates the situation, thus first and foremost an attempt is made to look into these entities separately, because it will give better insight into the flow physics. First of all, large-scale motions of the wall-bounded turbulent flows were investigated. The large-scale motions have gained huge interest in the past decade due to the advancements in experimental and computational facilities. These structures were identified long ago, but they were considered inactive. The recent research with the incompressible flow found that these structures are energy containing eddies and plays a significant part in the high Reynolds number wall-bounded turbulent flows. A brief review on wall-turbulence is given in Section 1.2. The effect of compressibility was not explored in this direction, therefore the influence of large-scale motions in high Reynolds number supersonic channel flows up to Mach 3 is investigated in this thesis.

It is well-known that the liner modifies the flow and turbulence in its vicinity compared to a rigid wall. An effect of this is the drag increase, especially for small liner porosity. Another effect is the flow instability in the vicinity of a low resistance liner. A brief overview on acoustic liners is given in Section 1.1. In the numerical simulations acoustic liners are generally modelled with the Impedance Boundary Condition (IBC) without cavity meshing. In the past several researchers have worked in this direction. A difficulty is then to impose a well-posed IBC, especially in the time domain. These simulations neglect both the effect of grazing flow on the impedance and the effect of the impedance on the flow. Another technique is to use the IBC with a given impedance, which means the effect of the flow on the impedance is not part of the computation. In the current case, non-linear NSE are solved with the latter technique for the IBC. The objective is to study the behaviour of turbulent flow in the vicinity of non-rigid wall, to see how the introduction of such boundary condition modifies the flow.

Flow-acoustic interaction in a duct is an extensively researched topic in the past with the help of experiments. The major limitation with the experiments was the intrusive
measurement techniques, therefore measurements were done only at the duct walls. Currently, with the numerical simulation one has access to all the points in space. This gives an opportunity to accurately examine the interaction. In this thesis, plane and first-order transverse waves were studied.

Without the mean flow the impedance of the liner can be easily educed, whereas with the mean flow it is very difficult. Similarly, in the past with the help of experiments it was found that the acoustic liner creates self-noise. Therefore, the final objective of the thesis will be to understand the interaction between the flow, acoustic liner and sound propagation.

### 1.5 ORGANISATION OF THESIS

The numerical method for simulating channel flow is presented in Chapter 2. The compressible NSE are written in the characteristic form presented of Sesterhenn (2000) ${ }^{[255]}$. In the first part of the thesis (Chapter $2-6$ ), stream-wise periodic channel flow will be studied (the flow configuration is given in Section 2.2). A brief review of the compact Finite Difference Scheme (FDS) was performed and the time-integration scheme was presented. Since this is the first parallel version of compressible NSE solver "Compact3D", the parallel implementation is presented. Finally, the numerical schemes and boundary conditions are validated by performing a DNS of supersonic temporal channel flow.

In Chapter 3, first of all a parametric study of the "controlled ILES" technique used in this thesis was performed. Since this technique was calibrated with physical arguments of isotropic turbulence, it was very important to check the technique before using it for simulating channel flows. Secondly, the grid requirement analysis for wall-resolved ILES was studied on a moderately high Reynolds number channel flow. Various degree of grid coarsening was tested to access the accuracy of the simulation technique.

At higher Mach number the compressibility effects will be manifested through the mean property variations of density and temperature across the channel, moreover due to the isothermal walls, heat transfer through the walls increase at higher Mach number. Therefore, a review of the compressible scaling techniques was performed in Chapter 4. Another interest in the compressible scaling techniques was to use it in the development of an algorithm for detecting large-scale structures of wall-bounded turbulent flow in Chapter 5.

In Chapter 5 , first of all a grid requirement analysis for accurately reproducing the flow physics of large-scale motions was performed. ILES of high Reynolds number channel flow at subsonic and supersonic Mach numbers were performed. A feature extraction algorithm was developed to detect the large-scale motions. Conditional averaging was computed based on the detected structures to analyse the inner-outer scales-interaction.

The boundary condition for acoustic liner is presented in Chapter 6. Boundary condition is validated and the grid requirements for the channel with acoustic liners was investigated. Finally a series of simulations was performed by changing the parameters of the boundary condition, to access the flow-liner interaction. Phase-averaging procedure
was introduced and the effect of the surface-wave due to acoustic liner was quantified. LSA was also performed to complement the findings of the simulation.

In Chapter 7 spatial simulation of channel flow was performed and the attenuation of the sound wave was studied. The boundary conditions for the subsonic inflow and outflow were discussed, and the wave extraction procedure was presented. In Chapter 7 and 8 periodic boundary condition in the stream-wise direction is not considered (flow configuration is presented in Section 7.4.1). The boundary conditions were validated, before simulating turbulent channel flow. Realistic inlet turbulence data for the spatial simulation was generated using a simultaneous precursor simulation. Plane and transverse acoustic waves were propagated to study their attenuation in a turbulent channel flow.

Results for the ILES of acoustic propagation in a turbulent flow tube with acoustic liner is presented in Chapter 8. Effect of acoustic liner on the turbulent flow modification was discussed by analysing the flow statistics in Section 8.2. Wall-transpiration on the impedance surface was analysed in Section 8.3, and an attempt was made to connect it to instability found with the periodic box simulation in Chapter 6. Turbulence-lineracoustic interaction was investigated in Section 8.4. Finally, the effect of the instability on the sound attenuation was analysed in Section 8.5.

The conclusions of this thesis are summarised and recommendations for future work are presented in Chapter 9. The dissertation also includes several appendix where one can find additional or supplementary information to follow the thesis. In Appendix A, a brief history of the code Compact3D is presented, and the coefficients of compact FDS (useful to follow Chapter 2). Grid stretching functions are presented in Appendix B. Comparison of channel flow results from different DNS and LES techniques is discussed in Appendix C. Compressible relations required to find an approximate bulk Reynolds number for a given friction Reynolds number and Mach number is presented in Appendix D. Bulk Reynolds number and Mach number is required to prescribe the operating point for the simulation. Linearised 2D NSE used for the stability analysis of channel flow with acoustic liner is given in Appendix E. The new technique to compute the target solution for the sponge zone for the spatial simulations is presented in Appendix F.

## 2

EQUATIONS AND SIMULATION TECHNIQUES

This chapter presents the equations and the simulation techniques which are used in this thesis. The objective was to prepare the numerical code "Compact3D" for parallel computation of channel flow with different wall boundary conditions (rigid and acoustic liner) for studying wall turbulence and sound propagation in a channel. Compact3D was initially developed as a sequential code for simulating sound radiation from free shear flows. This is the first version of Compact3D which is capable of parallel computation and used for simulating wall-bounded flows. See Appendix A.I for a brief history of Compact3D. Compressible NSE which were solved is given in Section 2.1. The channel flow configuration is presented in Section 2.2, and boundary conditions in Section 2.3. The numerical method, which includes the spatial derivatives and time-integration schemes are introduced in Section 2.5 . The parallel implementation is presented in Section 2.6. Finally in Section 2.7 the equations and numerical methods are validated by performing a DNS of a supersonic channel flow with rigid walls.

### 2.1 EQUATIONS

The primary interest of this thesis is to study the sound propagation in a turbulent channel flow with acoustic liner. Which involves studying the interaction between sound wave, turbulence and acoustic liner. Moderately higher subsonic Mach numbers will be used for investigating such problems and the necessity to also include the acoustic wave demands the usage of compressible NSE. This equations are solved in the characteristiclike form presented by Sesterhenn (2000) ${ }^{[255]}$. There are two advantages for the characteristic formulation (equation $2.1-2.5$ ). Firstly, it makes it possible to use upwind schemes ${ }^{[3]}$ in the characteristic directions, which allows introducing some dissipation to stabilize the computation (this technique was used in previous versions of Compact3D, see Appendix A.1). Secondly, the form is ready-to-use for the implementation of boundary conditions, such as non-reflecting boundary conditions. Equations were written for a Cartesian mesh in the normalised form (the coordinates were denoted either by $(x, y, z)$ or $\left(x_{1}, x_{2}, x_{3}\right)$ in the following):

$$
\begin{align*}
\frac{\partial u}{\partial t} & =-\left(\frac{1}{2}\left(X^{+}-X^{-}\right)+Y^{u}+Z^{u}\right)+\frac{1}{\operatorname{Re}} \frac{1}{\rho} \frac{\partial \tau_{1 j}}{\partial x_{j}}+F  \tag{2.1}\\
\frac{\partial v}{\partial t} & =-\left(X^{v}+\frac{1}{2}\left(Y^{+}-Y^{-}\right)+Z^{v}\right)+\frac{1}{\operatorname{Re}} \frac{1}{\rho} \frac{\partial \tau_{2 j}}{\partial x_{j}}  \tag{2.2}\\
\frac{\partial w}{\partial t} & =-\left(X^{w}+Y^{w}+\frac{1}{2}\left(Z^{+}-Z^{-}\right)\right)+\frac{1}{\operatorname{Re}} \frac{1}{\rho} \frac{\partial \tau_{3 j}}{\partial x_{j}}  \tag{2.3}\\
\frac{\partial s}{\partial t} & =-\left(X^{s}+Y^{s}+Z^{s}\right)+\frac{(\gamma-1)}{\gamma} \frac{1}{\operatorname{Re}} \frac{1}{p}\left(\Phi+\frac{1}{\operatorname{Pr}} \nabla \cdot\left(K_{t} \nabla T\right)\right) \tag{2.4}
\end{align*}
$$

$$
\frac{\partial p}{\partial t}=-\frac{\rho c}{2}\left[\left(X^{+}+X^{-}\right)+\left(Y^{+}+Y^{-}\right)+\left(Z^{+}+Z^{-}\right)\right]+\frac{1}{\operatorname{Re}}(\gamma-1)\left[\Phi+\frac{1}{\operatorname{Pr}} \nabla \cdot\left(\mathrm{~K}_{\mathrm{t}} \nabla \mathrm{~T}\right)\right] \text { (2.5) }
$$

where $X^{ \pm}, Y^{ \pm}, Z^{ \pm}$are the rate of change of the characteristics acoustic wave amplitudes; $X^{s}, Y^{s}, Z^{s}$ are the rate of change of the entropy waves; and $Y^{u}, Z^{u}, X^{v}, Z^{v}, X^{w}, Y^{w}$ are the rate of change of the vorticity waves. These characteristics waves are given by the following relations:

$$
\begin{array}{ll}
X^{ \pm}=(u \pm c)\left(\frac{1}{\rho c} \frac{\partial p}{\partial x} \pm \frac{\partial u}{\partial x}\right) & Y^{ \pm}=(v \pm c)\left(\frac{1}{\rho c} \frac{\partial p}{\partial y} \pm \frac{\partial v}{\partial y}\right) \quad Z^{ \pm}=(w \pm c)\left(\frac{1}{\rho c} \frac{\partial p}{\partial z} \pm \frac{\partial w}{\partial z}\right)(2.6) \\
Y^{u}=v \frac{\partial u}{\partial y} \quad Z^{u}=w \frac{\partial u}{\partial z} & X^{v}=u \frac{\partial v}{\partial x} \quad Z^{v}=w \frac{\partial v}{\partial z} \quad X^{w}=u \frac{\partial w}{\partial x} \quad Y^{w}=v \frac{\partial w}{\partial y}
\end{array}
$$

The velocity components along $x, y$, and $z$ directions were denoted by $u, v, w\left(\right.$ or $u_{1}$, $\left.u_{2}, u_{3}\right) . p$ is the pressure, $\rho$ is the density, $s$ is the entropy, $F$ is the dynamic forcing term to ensure constant mass flux (when periodic boundary conditions were used in the stream-wise direction) and $c$ the sound speed given by:

$$
\begin{equation*}
c^{2}=\frac{\gamma p}{\rho} \tag{2.9}
\end{equation*}
$$

The viscous stress tensor is given by:

$$
\begin{equation*}
\tau_{i j}=\mu\left(\frac{\partial u_{i}}{\partial x_{j}}+\frac{\partial u_{j}}{\partial x_{i}}\right)+\left(\mu_{b}-\frac{2}{3} \mu\right)\left(\frac{\partial u_{k}}{\partial x_{k}}\right) \delta_{i j} \tag{2.10}
\end{equation*}
$$

and the viscous dissipation is:

$$
\begin{equation*}
\Phi=\tau_{i j}\left(\frac{\partial u_{i}}{\partial x_{j}}\right) \tag{2.11}
\end{equation*}
$$

In addition the state equation reads:

$$
\begin{equation*}
p=\frac{(\gamma-1)}{\gamma} \rho T \tag{2.12}
\end{equation*}
$$

and the thermodynamic relation is given by:

$$
\begin{equation*}
\rho=p^{1 / \gamma} e^{-s} \tag{2.13}
\end{equation*}
$$

where $\gamma=1.4$ is the ratio of specific heats. The dependence of viscosity on temperature is given by the power law:

$$
\begin{equation*}
\mu=T^{0.7} \tag{2.14}
\end{equation*}
$$

These equations were normalized using the following reference scales for a channel flow. In the following the tilde $\widetilde{()}$ indicates a dimensional quantity: the viscosity $\widetilde{\mu}_{w}$ and thermal conductivity $\widetilde{\mathrm{K}}_{\mathrm{t}, w}$ at the walls of the channel (with isothermal boundary conditions imposed), the speed of sound $\widetilde{c}_{w}$ at wall temperature, the half-height of the channel $\widetilde{H}$ and the adiabatic specific heat $\widetilde{c}_{p}$. The bulk density $\widetilde{\rho}_{b}$, Prandtl number $\operatorname{Pr}$ and Reynolds number Re to be defined in the following section.

### 2.2 FLOW CONFIGURATION

The compressible NSE introduced in the previous section are solved numerically to simulate channel flows, which is one of the geometrically simple flow configuration to study wall-bounded flow. The length, height and breadth of the channel is denoted with $\widetilde{\mathrm{L}}_{x}$, $\widetilde{\mathrm{L}}_{y}$ and $\widetilde{\mathrm{L}}_{z}$ in the dimensional form. The isothermal walls of the channel were located at $y=-\widetilde{H}$ and $+\widetilde{H}$, and the channel always has the height $\widetilde{\mathrm{L}}_{y}=2 \widetilde{\mathrm{H}}$. The walls of the channel can be rigid or non-rigid. An example of the flow configuration can be found in figure 2.1. Span-wise periodicity was considered for all the channel flow simulations in this thesis. Presently, the channel flow is considered to be periodic in the stream-wise direction with constant mass flow rate. This is ensured by the dynamic forcing term $F$ in equation 2.1 which preserves the stream-wise homogeneity. Unlike pressure-gradient, body-force is not uniform along the wall-normal direction. With such condition in the stream-wise direction, one will simulate a temporal channel flow, where basically the frame of reference moves along with the flow or one simulate the temporal evolution of turbulence in a channel flow. Currently, the top and bottom walls are considered to be rigid (no-slip:


Figure 2.1: Flow configuration for the temporal (stream-wise periodic) channel flow simulation.
$u=v=w=0$ ) and isothermal ( $\mathrm{T}_{w}=$ constant $)$. The characteristic isothermal rigid wall boundary condition will be presented in the following section. The initial conditions for temporal channel flow simulations are: (a) Parabolic mean flow with random noise and stream-wise rolls proposed by Waleffe (1997) ${ }^{[287]}$, (b) constant pressure ( $p=1 / \gamma$ ) and (c) zero entropy ( $s=0$ ).

It is important to characterise the flow physics with non-dimensional numbers, because they have a deeper physical significance and they are generally a ratio of two quantities. $\operatorname{Re}$ and $\operatorname{Pr}$ are the two non-dimensional numbers used in the NSE in Section 2.1. The thermal conductivity was related to the viscosity through the Prandtl number:

$$
\begin{equation*}
\operatorname{Pr}=\frac{\widetilde{\mu}_{w} \widetilde{c}_{p}}{\widetilde{\mathrm{~K}}_{\mathrm{t}, w}} \tag{2.15}
\end{equation*}
$$

and $\operatorname{Pr}=0.7$. The Reynolds number resulting from the aforementioned normalization was:

$$
\begin{equation*}
\operatorname{Re}=\frac{\widetilde{\mu} \widetilde{\rho}_{\mathrm{b}} \widetilde{\mathrm{c}}_{w}}{\widetilde{\mu}_{w}} \tag{2.16}
\end{equation*}
$$

The bulk density used for the normalization was defined by:

$$
\begin{equation*}
\widetilde{\rho}_{\mathrm{b}}=\frac{1}{2 \widetilde{H}} \int_{-\widetilde{H}}^{\widetilde{H}} \overline{\widetilde{\rho}} d \widetilde{\mathrm{y}} \tag{2.17}
\end{equation*}
$$

where $\overline{()}$ is the average over time and stream- and span-wise directions. The bulk velocity is:

$$
\begin{equation*}
\widetilde{\mathrm{u}}_{\mathrm{b}}=\frac{\frac{1}{2 \widetilde{\mathrm{H}}} \int_{-\widetilde{\mathrm{H}}}^{\widetilde{\mathrm{H}}}(\overline{\widetilde{\rho} \widetilde{\mathrm{u}}}) d \widetilde{\mathrm{y}}}{\widetilde{\rho}_{\mathrm{b}}} \tag{2.18}
\end{equation*}
$$

In a channel flow with stream-wise periodicity, $\widetilde{\rho}_{\mathrm{b}}$ is constant since no mass escapes the channel, and in normalised form $\rho_{b}=1$. The velocity $\mathfrak{u}_{b}$ was also forced to keep a constant value, which was ensured ${ }^{[52]}$ by dynamically changing the uniform force $F$ appearing in equation 2.1. Note that due to normalisation, $\mathfrak{u}_{\mathrm{b}}=\mathcal{M}$ where:

$$
\begin{equation*}
\mathcal{M}=\frac{\widetilde{\mathfrak{u}}_{\mathrm{b}}}{\widetilde{\mathfrak{c}}_{w}} \tag{2.19}
\end{equation*}
$$

is the Mach number based on the bulk velocity and the sound speed at the wall. This is another non-dimensional number which can be used to characterise the flow. The bulk Reynolds number is defined by:

$$
\begin{equation*}
\operatorname{Re}_{\mathrm{b}}=\frac{\widetilde{\rho}_{\mathrm{b}} \widetilde{\mu}_{\tilde{u}_{\mathrm{b}}}}{\widetilde{\mu}_{w}} \tag{2.20}
\end{equation*}
$$

It is commonly used to specify the operating point of a channel flow. Hence, in the following $\mathrm{Re}_{\mathrm{b}}$ and $\mathcal{M}$ will be prescribed, from which the Reynolds appearing in the equations can be derived using:

$$
\begin{equation*}
\operatorname{Re}=\frac{\operatorname{Re}_{\mathrm{b}}}{\mathcal{M}} \tag{2.21}
\end{equation*}
$$

The following quantities are computed from the results of the simulation. Generally for wall-bounded flows, friction velocity is used as the viscous velocity scale which is defined by:

$$
\begin{equation*}
\widetilde{\mathfrak{u}}_{\tau}=\sqrt{\frac{\widetilde{\tau}_{w}}{\overline{\tilde{\tilde{\rho}}}_{w}}} \tag{2.22}
\end{equation*}
$$

where the shear-stress is:

$$
\begin{equation*}
\widetilde{\tau}_{w}=\left.\overline{\widetilde{\mu}}_{w} \frac{\partial \overline{\widetilde{u}}}{\partial \widetilde{\mathrm{y}}}\right|_{w} \tag{2.23}
\end{equation*}
$$

and the viscous length-scale:

$$
\begin{equation*}
\tilde{\mathrm{I}}_{v}=\frac{\overline{\tilde{\mu}}_{w}}{\overline{\tilde{\rho}}_{w} \widetilde{\mathrm{u}}_{\tau}} \tag{2.24}
\end{equation*}
$$

Friction Reynolds number is another important non-dimensional number generally used to characterize the wall-bounded turbulent flows, and it is defined as:

$$
\begin{equation*}
\operatorname{Re}_{\tau}=\frac{\widetilde{\rho}_{w} \widetilde{H} \widetilde{u}_{\tau}}{\widetilde{\mu}_{w}} \tag{2.25}
\end{equation*}
$$

non-dimensional heat-flux through the walls:

$$
\begin{equation*}
\mathrm{B}_{\mathrm{q}}=-\left.\frac{\widetilde{\mathrm{K}}_{\mathrm{t}, w}}{\overline{\bar{\rho}}_{w}{\widetilde{\mathfrak{c}_{p}}}^{\mathrm{u}_{\tau}} \overline{\widetilde{T}}_{w}} \frac{\partial \overline{\widetilde{T}}}{\partial \widetilde{\mathrm{y}}}\right|_{w} \tag{2.26}
\end{equation*}
$$

and friction Mach number:

$$
\begin{equation*}
\mathcal{M}_{\tau}=\frac{\widetilde{\mathfrak{u}}_{\tau}}{\widetilde{\mathfrak{c}}_{w}} \tag{2.27}
\end{equation*}
$$

In Section 5.3.1, results from the validation using DNS of temporal supersonic channel flow with isothermal walls will be presented. In Chapter 3, 4 and 5 results for the temporal channel will be presented to address different aspects of compressible channel flows with rigid walls. In Chapter 6 results for the channel flows with acoustic liner (non-rigid wall) will be discussed. The lined non-rigid wall will be modelled with an impedance wall boundary condition in Section 6.1. Later in Chapter 7, the periodicity in the streamwise direction will be discarded to perform the spatial evolution of turbulence in the channel. The flow configuration and the necessary inlet/outlet boundary conditions for such case will be addressed in Section 7.1.

### 2.3 BOUNDARY CONDITIONS

In the first part of the thesis, channel flow which is periodic in stream- and span-wise direction will be simulated with the Compact3D. Since characteristics type formulation was chosen, the boundary condition should also have the same form. Before, one begin it is always good to have a brief introduction about different characteristic waves. For simplicity, lets consider 1D problem, $\mathrm{X}^{+, \nu, w, s}$ are the characteristics which travels downstream. $\mathrm{X}^{+}$ travels with velocity $(u+c)$, whereas others characteristics waves travel downstream at velocity $u$. Characteristics $X^{-}$travels at the velocity $(u-c)$. For a subsonic(supersonic) Mach number, $X^{-}$travels upstream(downstream). Characteristics waves entering and leaving the computational domain for a subsonic flow is shown in figure 2.2. Whereas for a supersonic Mach number all characteristics travels downstream. In the first part of the thesis (Chapter $1-5$ ), only two boundary conditions are used: (a) periodic boundary condition in the stream- ans span-wise direction and (b) isothermal no-slip condition at the walls. These conditions are presented below.

### 2.3.1 Periodicity

With the periodic boundary condition, the information which leave the domain through one face (or boundary surface) will enter the computational domain through the opposite


Figure 2.2: Characteristics waves entering and leaving the computational domain for a subsonic flow.
face. This is one of the most simple and straightforward boundary conditions, because one only has to fill the matrix with the right set of coefficients before matrix inversion. This is not true while simulating a spatial channel flow. One will have to impose a welldefined inflow and outflow section. This will be addressed for spatial channel flows later in Section 7.1.

### 2.3.2 Isothermal rigid wall

For a rigid isothermal wall with $u=v=w=0$ and $T_{w}=$ constant, the boundary conditions to apply have been given by Lechner et al. (2001) ${ }^{[164]}$. For the no-slip condition at the bottom wall for example, $\mathrm{Y}^{-}$is the known characteristic flux coming from the interior of the domain, and the reflected characteristic flux $\mathrm{Y}^{+}$should be calculated. The situation is reversed at the top wall (see figure 2.3).


Figure 2.3: Illustration of wall boundary treatment.

Lechner et al. (2001) ${ }^{[164]}$ gave (adapted to the present notation/normalisation):

$$
\begin{equation*}
\mathrm{Y}^{+}=\mathrm{Y}^{-}+2 \frac{1}{\operatorname{Re}} \frac{1}{\rho} \frac{\partial \tau_{2 j}}{\partial x_{j}} \quad \text { (bottom wall) } \quad \mathrm{Y}^{-}=\mathrm{Y}^{+}-2 \frac{1}{\operatorname{Re}} \frac{1}{\rho} \frac{\partial \tau_{2 j}}{\partial x_{j}} \quad \text { (top wall) } \tag{2.28}
\end{equation*}
$$

The isothermal condition needs to be enforced using the thermodynamic relationship:

$$
\begin{equation*}
\frac{\partial \widetilde{T}}{\partial \widetilde{t}}=\left.\frac{\partial \widetilde{T}}{\partial \widetilde{p}}\right|_{\widetilde{s}} \frac{\partial \widetilde{p}}{\partial \widetilde{t}}+\left.\frac{\partial \widetilde{T}}{\partial \widetilde{s}}\right|_{\widetilde{p}} \frac{\partial \widetilde{s}}{\partial \widetilde{t}} \tag{2.29}
\end{equation*}
$$

by substituting $\left.\frac{\partial \widetilde{T}}{\partial \widetilde{p}}\right|_{\widetilde{s}}=\frac{(\gamma-1)}{\gamma} \frac{\widetilde{T}}{\widetilde{p}}$ and $\left.\frac{\partial \widetilde{T}}{\partial \widetilde{s}}\right|_{\widetilde{p}}=\frac{\widetilde{T}}{\widetilde{\mathcal{F}_{\mathfrak{p}}}}$ one finds:

$$
\begin{equation*}
\frac{\partial \widetilde{\mathrm{T}}}{\partial \widetilde{\mathrm{t}}}=\frac{(\gamma-1)}{\gamma} \frac{\widetilde{\mathrm{T}}}{\widetilde{\mathrm{p}}} \frac{\partial \widetilde{\mathrm{p}}}{\partial \widetilde{\mathrm{t}}}+\frac{\widetilde{\mathrm{T}}}{\widetilde{\widetilde{c}_{p}}} \frac{\partial \widetilde{\mathrm{~s}}}{\partial \widetilde{\mathrm{t}}} \tag{2.30}
\end{equation*}
$$

after normalisation and rearrangement one will obtain:

$$
\begin{equation*}
\frac{\partial s}{\partial t}=\frac{1}{T} \frac{\partial T}{\partial t}-\frac{(\gamma-1)}{\gamma} \frac{1}{p} \frac{\partial p}{\partial t} \tag{2.31}
\end{equation*}
$$

For isothermal condition at the wall $\frac{\partial T}{\partial t}=0\left(T_{w}=\right.$ constant $)$, which leaves one with:

$$
\begin{equation*}
\frac{\partial s}{\partial t}=-\frac{(\gamma-1)}{\gamma} \frac{1}{p} \frac{\partial p}{\partial t} \tag{2.32}
\end{equation*}
$$

The isothermal boundary condition was obtained by replacing the pressure and entropy equations at the wall with:

$$
\begin{gather*}
\frac{\partial p}{\partial t}=-\frac{p}{2 c}\left(Y^{+}+Y^{-}\right)  \tag{2.33}\\
\frac{\partial s}{\partial t}=\frac{(\gamma-1)}{\gamma} \frac{1}{2 c}\left(\mathrm{Y}^{+}+\mathrm{Y}^{-}\right) \tag{2.34}
\end{gather*}
$$

Equation 2.34 was obtained by substituting equation 2.33 in to equation 2.32. Later in Chapter 6, the aforementioned isothermal rigid wall boundary conditions will be modified for an isothermal non-rigid wall which will be modelled as an impedance wall boundary condition.

### 2.4 COMPUTATIONAL GRID

The numerical code is based on the structured Cartesian grid, with collocated arrangement of all flow variables. Moreover, for a rectangular channel geometry, it is rather straightforward to generate a computational grid. For the temporal channel flow simulation, uniform grid spacing is chosen in the stream- and span-wise direction. Since, there is turbulent boundary layer in the wall-normal direction, stretched grid is used. The stretching ensures that close to the wall more points are used to discretise the inner-layer of the boundary layer where strong gradients exist and fewer grid points away from the wall. The stretching functions are integrated to Compact3 $D$, and the code generates the required computational grid. The stretching functions are introduced in Appendix B. The
code uses the compact FDS (see Section 2.5.1 and 2.5.2) with the traditional tri-diagonal matrix inversion algorithm to compute the derivatives. The selection of 2D pencil decomposition of the computational domain (see Section 2.6) with the structured grid and the FDS for the derivatives were found to be a very reasonable choice. The use of simplified mesh offers the opportunity to implement high-order numerical schemes, additionally the domain decomposition method is suited for the the compact FDS. In Chapter 7, stretched grid will be used in the stream-wise direction for the spatial channel flow simulation.

### 2.5 NUMERICS

DNS and LES need accurate numerical method, to produce result, which represents the physics. For the spatial derivatives, spectral schemes are preferred for performing such simulations due to their accuracy, but spectral schemes are restrictive to simple domains and boundary conditions. Lele (1992) ${ }^{[167]}$ proposed compact FDS with spectral-like resolution, which represent small scales with less error and, are accurate up to $10^{\text {th }}$ order for first- and second-derivative, with both periodic and non-periodic boundary conditions.

In this section, compact schemes will be introduced, and numerical schemes used in "Compact3D" for the spatial first- and second-derivative will be presented. Compact3D is a parallel compressible NSE solver with high-order compact schemes for simulating $3^{D}$ flows. The time integration scheme and the stability criteria for explicit computation will be addressed in the later part of the section.

### 2.5.1 Spatial first-derivative

Compact FDS are spectral-like, but still not free from discrepancy. In table A. 1 (see Appendix A.2) high-order compact FDS and their coefficients for spatial first-derivative are presented. Modified wave-number for the schemes listed in table A.I is computed using equation 2.35, and presented in figure 2.4. The modified wave-number for the firstderivative is given as follows:

$$
\begin{equation*}
k^{\prime} \Delta x=\frac{a \sin (k \Delta x)+b / 2 \sin (2 k \Delta x)+c / 3 \sin (3 k \Delta x)}{1+2 \alpha \cos (k \Delta x)+2 \beta \cos (2 k \Delta x)} \tag{2.35}
\end{equation*}
$$

The modified wave-number concept helps to clearly distinguish, between the wellresolved scales (for which the modified wave-number is close to exact wave-number) and the erroneous scales. $k$ is the wave-number, and $\Delta x$ is the grid size. $\alpha, \beta, a, b$ and $c$ are the coefficients of the FDS (see table A.1). Currently, uniform grid is only considered. Figure 2.4 compares the resolution properties of different schemes for the spatial first-derivative. Modified wave-number for the first-derivative spectral scheme is linear ( $k^{\prime} \Delta x=k \Delta x$ ). From the figure it is clear, that with increasing order of accuracy, compact FDS moves close to exact differentiation, thus behaving like a spectral scheme. Compact3D uses $6^{\text {th }}$ order centered compact FDS (6TriD in table A.1) for computing the first-derivative. $6^{\text {th }}$ order centered FDS is accurate up to $k / k_{c}=0.5$, where $k_{c}$ is the cut-off wave-number.


Figure 2.4: Modified wave-numbers for spatial first-derivative. $k_{\max }^{\prime}$ is the maximum value of the modified wave-number for a particular scheme. $k_{\max }^{\prime}$ will be used in Section 2.5.4 while discussing stability criteria.

The numerical schemes used in Compact3D for computing the spatial first-derivatives are summarised in the table 2.1. The schemes are taken from Lele (1992) ${ }^{[167]}$, and reference for the schemes are also cited in the table. In the periodic direction $6^{\text {th }}$ order compact scheme was utilised. $3^{\text {rd }}$ order upwind scheme was used at the non-periodic boundary, $4^{\text {th }}$ order centred compact scheme at next point to the boundary and $6^{\text {th }}$ order centred compact scheme at the interior nodes. Figure 2.5 shows the order of usage of different boundary schemes for computing the spatial first-derivative.

|  |  | SCHEME | COEFFICIENTS |
| :---: | :---: | :---: | :---: |
| Non-periodic boundary | $\mathrm{id} x=1$ | $\begin{aligned} & f_{1}^{\prime}+\alpha f_{2}^{\prime} \\ & =\frac{1}{\Delta x}\left(a f_{1}+b f_{2}+c f_{3}\right) \end{aligned}$ | $3^{\text {rd }}$ order upwind <br> Lele (1992) ${ }^{[167]}$ Eq. (4.1.3) $\alpha=2 ; a=-\frac{5}{2} ; b=2 ; c=\frac{1}{2}$ |
|  | idx $=2$ | $\beta f_{i-2}^{\prime}+\alpha f_{i-1}^{\prime}+f_{i}^{\prime}+\alpha f_{i+1}^{\prime}+\beta f_{i+2}^{\prime}$ | $4^{\text {th }}$ order centered compact Lele (1992) ${ }^{[167]}$ Eq. (2.1.6) $\alpha=\frac{1}{4} ; \beta=0 ; a=\frac{3}{2} ; b=0$ |
|  | Interior nodes | $\begin{aligned} & b \frac{1}{4 \Delta x}\left(f_{i+2}-f_{i-2}\right) \\ & +a \frac{1}{2 \Delta x}\left(f_{i+1}-f_{i-1}\right) \end{aligned}$ | $6^{\text {th }}$ order centered compact <br> Lele (1992) ${ }^{[167]}$ Eq. (2.1.7) |
| Periodic boundary |  |  | $\alpha=\frac{1}{3} ; \beta=0 ; a=\frac{14}{9} ; b=\frac{1}{9}$ |

Table 2.1: Spatial discretisation for first-derivative.


Figure 2.5: Spatial discretisation for first-derivative in non-periodic boundary. idx $=1$ is at the non-periodic boundary. Values from grid points in yellow are used to compute the first-derivative at grid point in orange.

### 2.5.2 Spatial second-derivative

Similarly, for the spatial second-derivative first of all a general introduction is given about the compact FDS, followed by the introduction of modified FDS which will over-dissipate at higher wave-numbers (Section 2.5.2.1 and 2.5.2.2). All the schemes used for computing the second-derivative was summarised in table 2.2.

The popular compact FDS and their coefficients for computing the spatial secondderivative is listed in table A. 2 in Appendix A.2. The modified wave-number of the compact FDS for the spatial second-derivative is given by:

$$
\begin{equation*}
k^{\prime \prime} \Delta x^{2}=\frac{2 a[1-\cos (k \Delta x)]+b / 2[1-\cos (2 k \Delta x)]+2 c / 9[1-\cos (3 k \Delta x)]+d / 8[1-\cos (4 k \Delta x)]}{1+2 \alpha \cos (k \Delta x)+2 \beta \cos (2 k \Delta x)} \tag{2.36}
\end{equation*}
$$

Modified wave-number for different schemes presented in table A. 2 is computed using equation 2.36, and the curves for different schemes are presented in figure 2.6. Resolution


Figure 2.6: Modified wave-numbers for spatial second-derivative. $\mathrm{k}^{\mathrm{\prime} \mathrm{\prime}}{ }_{\text {max }}$ is the maximum value of the modified wave-number for a particular scheme. $\mathrm{k}^{\prime 2}$ max will be used in Section 2.5.4 while discussing stability criteria.
properties of different schemes for the spatial second-derivative can be compared from
the figure. Actual wave-number for the second-derivative is a parabola $\left(k^{\prime \prime} \Delta x^{2}=(k \Delta x)^{2}\right)$. Similarly like figure 2.4, it was clear that for increasing order of numerical schemes, compact FDS behaves like a spectral scheme.

Error at small scales (higher wave-numbers), due to numerical scheme and aliasing, can lead to spurious oscillations. Refined meshes or some other numerical technique is used to control such grid-to-grid oscillations or wiggles. Mesh refinement will increase the computational cost, thus the most popular way is to use non-conservative methods such as upwind schemes, filters, etc ${ }^{[3,24,41,42,112,268,269]}$. The previous version of Compact3D was using upwind schemes, but these have now been replaced by centred schemes (see Appendix A. 1 for a brief history of Compact3D). Since the centred scheme is non-dissipative, the extra dissipation needed to stabilise the computations or serve for LES is introduced through the diffusive terms in equation 2.1-2.5 with the second-order spatial derivative (rather than with the convective terms using the first-order derivative), as will be explained in Section 2.5.2.1 and 2.5.2.2 ${ }^{[58,59,161]}$.

In this thesis, the modified FDS is used for computing the spatial second-derivative. It helps introducing targeted excess numerical dissipation selectively at different scales. It acts as an implicit filter for the LES and removes grid-to-grid oscillations for the DNS ${ }^{[58,59,161]}$. These schemes are centred and no extra upwinding is required. Excess dissipation is included in the diffusion terms through the modified FDS for computing second-derivative. Thus molecular and spectral viscosity are included in a single operator.

### 2.5.2.1 Modified second-derivative for DNS

High-order schemes are very attractive for DNS due to their ability to provide accurate results using reasonable degrees of freedom. From figure 2.4 and 2.6 one can notice that the $6^{\text {th }}$ order compact FDS provides an erroneous estimation of spatial-derivatives (underdissipative) at the cut-off ( $k \Delta x=\pi$ ). Hence for DNS, modified second-derivative scheme was used to control the wiggles, by introducing artificial dissipation only at the smallscales (higher wave-numbers) and leaving large-scales (lower wave-numbers) unaffected. Lamballais et al. (2011) ${ }^{[161]}$ presented the $6^{\text {th }}$ order modified second-derivative scheme which mimics the hyper-viscosity kernel. The scheme writes as follows:

$$
\begin{align*}
\alpha f_{i-1}^{\prime \prime}+f_{i}^{\prime \prime}+\alpha f_{i+1}^{\prime \prime} & =c \frac{1}{9 \Delta x^{2}}\left(f_{i+3}-2 f_{i}+f_{i-3}\right)+b \frac{1}{4 \Delta x^{2}}\left(f_{i+2}-2 f_{i}+f_{i-2}\right) \\
& +a \frac{1}{\Delta x^{2}}\left(f_{i+1}-2 f_{i}+f_{i-1}\right) \tag{2.37}
\end{align*}
$$

The scheme is of 3-7 stencil form, and the modified wave-number can be found using equation 2.36 with coefficients,

$$
\begin{array}{ll}
\alpha=\frac{272-45 k_{c}^{\prime \prime} \Delta x^{2}}{416-90 k_{c}^{\prime \prime} \Delta x^{2}}, & a=\frac{48-135 k_{c}^{\prime \prime} \Delta x^{2}}{164-360 k_{c}^{\prime \prime} \Delta x^{2}} \\
b=\frac{524-81 k_{c}^{\prime \prime} \Delta x^{2}}{208-45 k_{c}^{\prime \prime} \Delta x^{2}}, & c=-\frac{432+63 k_{c}^{\prime \prime} \Delta x^{2}}{1664-360 k_{c}^{\prime \prime} \Delta x^{2}} \tag{2.38}
\end{array}
$$

This scheme ensures targeted numerical dissipation near the mesh cut-off. At the cut-off the modified wave-number is, $k_{c}^{\prime \prime} \Delta x^{2}=n \pi^{2}$, and $n$ can be any value which determines


Figure 2.7: Modified wave-numbers of spatial second-derivative scheme used for DNS. $6^{\text {th }}$ order scheme mimicking hyper-viscosity kernel with different level of dissipation at cut-off wave-number $(1 \leq n \leq 10)$. Blue curve is for $n=1$, red is for $n=10$, and dotted line is the exact differentiation.
the level of dissipation required at the cut-off. This scheme overestimates the value of $k^{\prime \prime} \Delta x^{2}$, in the narrow high wave number range (see figure 2.7). With large values of $n$, the numerical dissipation will be more concentrated around higher wave-numbers and it will also cause strong constraint on explicit time-integration. Stability issues of the modified scheme will be addressed in Section 2.5.4. Lamballais et al. (2011) ${ }^{[161]}$ demonstrated the application of this modified scheme by simulating sound radiation from the mixing layer.

### 2.5.2.2 Modified second-derivative for ILES

LES generally introduces a low-pass filter to remove small (sub-grid) scales and computes only for resolved scale. Hence with LES, one can reduce several degrees of freedom and save computational resources. But results of LES are always questionable; aliasing, discretisation and modelling errors are encountered. For sub-grid scale models based on the eddy-viscosity, non-linearity in the equations creates extra aliasing errors. This errors can be controlled by choosing a mesh size smaller than the LES filter. Thus, it will significantly increase the computational cost. But usually, such practices are never followed. Here a modified spatial second-derivative FDS more suitable for the ILES will be presented.

Modified $6^{\text {th }}$ order scheme for second-derivative which mimics the Spectral Vanishing Viscosity (SVV) kernel writes as follows:

$$
\begin{align*}
\alpha f_{i-1}^{\prime \prime}+f_{i}^{\prime \prime}+\alpha f_{i+1}^{\prime \prime} & =d \frac{1}{16 \Delta x^{2}}\left(f_{i+4}-2 f_{i}+f_{i-4}\right)+c \frac{1}{9 \Delta x^{2}}\left(f_{i+3}-2 f_{i}+f_{i-3}\right) \\
& +b \frac{1}{4 \Delta x^{2}}\left(f_{i+2}-2 f_{i}+f_{i-2}\right)+a \frac{1}{\Delta x^{2}}\left(f_{i+1}-2 f_{i}+f_{i-1}\right) \tag{2.39}
\end{align*}
$$

This scheme is of 3-9 stencil form, for which the modified wave-number can be found using equation 2.36 with coefficients:

$$
\begin{align*}
& \alpha=\frac{1}{2}-\frac{320 k_{m}^{\prime \prime} \Delta x^{2}-1296}{405 k_{c}^{\prime \prime} \Delta x^{2}-640 k_{m}^{\prime \prime} \Delta x^{2}+144} \\
& a=-\frac{4329 k_{c}^{\prime \prime} \Delta x^{2} / 8-32 k_{m}^{\prime \prime} \Delta x^{2}-140 k_{c}^{\prime \prime} \Delta x^{2} k_{m}^{\prime \prime} \Delta x^{2}+286}{405 k_{c}^{\prime \prime} \Delta x^{2}-640 k_{m}^{\prime \prime} \Delta x^{2}+144} \\
& b=\frac{2115 k_{c}^{\prime \prime} \Delta x^{2}-1792 k_{m}^{\prime \prime} \Delta x^{2}-280 k_{c}^{\prime \prime} \Delta x^{2} k_{m}^{\prime \prime} \Delta x^{2}+1328}{405 k_{c}^{\prime \prime} \Delta x^{2}-640 k_{m}^{\prime \prime} \Delta x^{2}+144} \\
& c=-\frac{7695 k_{c}^{\prime \prime} \Delta x^{2} / 8+288 k_{m}^{\prime \prime} \Delta x^{2}-180 k_{c}^{\prime \prime} \Delta x^{2} k_{m}^{\prime \prime} \Delta x^{2}-2524}{405 k_{c}^{\prime \prime} \Delta x^{2}-640 k_{m}^{\prime \prime} \Delta x^{2}+144} \\
& d=-\frac{198 k_{c}^{\prime \prime} \Delta x^{2}+128 k_{m}^{\prime \prime} \Delta x^{2}-40 k_{c}^{\prime \prime} \Delta x^{2} k_{m}^{\prime \prime} \Delta x^{2}-736}{405 k_{c}^{\prime \prime} \Delta x^{2}-640 k_{m}^{\prime \prime} \Delta x^{2}+144} \tag{2.40}
\end{align*}
$$

One needs to specify the dissipation at the mesh cut-off and at an intermediate wavenumber, such that:

$$
\begin{align*}
k^{\prime \prime}\left(k_{c}\right) & =\left(1+\frac{v_{0}}{v}\right) k_{c}^{2} \\
k^{\prime \prime}\left(2 k_{c} / 3\right) & =\left(1+c_{1} \frac{v_{0}}{v}\right) \frac{4}{9} k_{c}^{2} \tag{2.41}
\end{align*}
$$

where $v_{0} / v$ is the excess spectral viscosity at the cut-off with:

$$
\begin{equation*}
c_{1}=\frac{1}{\mathrm{~N}_{\mathrm{fac}}} e^{-((\pi-2 \pi / 3) /(0.3 \pi-2 \pi / 3))^{2}} \tag{2.42}
\end{equation*}
$$

which is the parameter which defines the value of modified wave-number of secondderivative at $k=2 \pi / 3$ or the width of the kernel (see figure 2.9 b for different types of SVV kernel). Shape of the hyper-viscosity kernel in Section 2.5.2.1 was defined using a single parameter: target dissipation at the cut-off ${ }^{[161]}$, whereas shape of the SVV kernel is defined using two parameters: (a) target dissipation which increases with mesh coarsening and (b) the dissipation at an intermediate wave-number ${ }^{[58]}$. Or in other words, for the hyper-viscosity kernel in Section 2.5.2.1, one just need to impose the value of dissipation through $n$, whereas for the SVV kernels one has to specify the level of excess dissipation and the distribution of dissipation across different wave-numbers through parameters $v_{0} / v$ and $N_{\text {fac }}$. Here $v$ is the spectral viscosity, $v_{0}$ is the spectral viscosity at the cut-off and $k_{c}$ is the cut-off wave-number. $\eta$ is the Kolmogorov scale and $k_{\eta}=1 / \eta$ is the wave-number of the Kolmogorov scale ${ }^{[59]} . \pi k_{\eta} / k_{c}$ gives the ratio of $\Delta x / \eta$, or the measure of grid coarsening based on the Kolmogorov scale, where $k_{c} \Delta x=\pi$. Dairay et al. $(2017)^{[59]}$ computed the value of $v_{0} / v$ such that, it ensures that the same kinetic energy will be found at $k_{c}$ with the LES as at $\pi k_{\eta}$ for the DNS of isotropic turbulence. Predicted value of $v_{0} / v$ as a function of $\pi k_{\eta} / k_{c}$ is presented in figure 2.9a. Figure 2.8 shows an example of the filtering technique with the modified scheme for LES. Marker " $a$ " in the figure shows the level of dissipation at the cut-off and marker " b " shows the dissipation at the intermediate wave-number. By adjusting these markers, using the predicted value of $v_{0} / v$ and $N_{\text {fac }}$ the distribution of excess dissipation can be controlled. One can find the SVV-like kernel with $\mathrm{N}_{\text {fac }}=1$ and $\mathrm{c}_{1}=0.44$. The width of this kernel can be modified by changing the value of $\mathrm{N}_{\text {fac }}$, for $\mathrm{N}_{\text {fac }}>1$ the kernel width will decrease and the introduced excess numerical dissipation will be concentrated in a narrow high-wave number


Figure 2.8: Shape of SVV kernel. Parameters " $a$ " and " $b$ " are used to control the distribution of excess dissipation.
range. For $\mathrm{N}_{\mathrm{fac}}=2,8$ one will find, $\mathrm{c}_{1}=0.22,0.055$ for steep SVV and sharp SVV kernels. In Chapter 3, results from the parametric study of different types of SVV kernels will be presented. For $\pi k_{\eta} / k_{c}=4$ different SVV kernels were shown in figure 2.9 b. Sharper the kernels, stronger is the time-constraint for explicit time integration. It will be addressed in Section 2.5.4.


Figure 2.9: (a) $v_{0} / v$ as a function of $\pi k_{\eta} / k_{c}$ and (b) Modified wave-numbers of spatial secondderivative kernels used for ILES with $\pi k_{\eta} / k_{c}=4$. Symbol $\bigcirc$ is the value of $v_{0} / v$ for $\pi k_{\eta} / k_{c}=4$.

This scheme overestimates dissipation over a wide range of small-scales (higher wavenumbers), without adding extra dissipation at large-scales (lower wave-numbers). The usage of SVV-like kernel is similar to ILES, where the error is the source of artificial dissipation, but also like sub-grid scale modelling due to the similarity between spectral viscosity. With ILES, the truncation error of the discretisation scheme was used to model the effects of unresolved scales instead an explicit computation of the sub-grid tensor.

Lamballais et al. (2011) ${ }^{[161]}$ performed ILES of incompressible turbulent channel flow using $4^{\text {th }}$ order SVV-like kernel (previous version of the modified second-derivative operator). Dairay et al. (2014) ${ }^{[58]}$ compared the present numerical scheme with the eddyviscosity models, and found better regularization with SVV-like kernel for simulating
turbulent jet impinging on a heated wall. ILES can fail to represent the effects of sub-grid scale, unless the numerical scheme and discretisation parameters are calibrated properly. The main benefit of this method is, they do not create spurious noise and this method can be calibrated for producing right dissipation at small scales. With this method ILES dissipation is controlled explicitly ${ }^{[59]}$.

FDS used in Compact3D and their coefficients for computing the spatial second-derivative are summarised in table 2.2. In the periodic direction, modified FDS mentioned in Section 2.5.2.1 or 2.5.2.2 were used depending on the requirement. At the non-periodic boundary, $3^{\text {rd }}$ order upwind scheme was used, with $4^{\text {th }}$ order centered compact scheme at the next point to the boundary. For the $3^{\text {rd }}$ and $4^{\text {th }}$ point from the boundary, $6^{\text {th }}$ order compact scheme were used and for all the interior nodes modified schemes were used depending whether one wants to perform DNS or ILES. Figure 2.10 presents the order of usage of boundary schemes for computing the spatial second-derivative.

|  |  | SCHEME | CoEfficients |
| :---: | :---: | :---: | :---: |
| Non-periodic boundary | idx $=1$ | $\begin{aligned} & f_{1}^{\prime \prime}+\alpha f_{2}^{\prime \prime}= \\ & \frac{1}{\Delta x^{2}}\left(a f_{1}+b f_{2}+c f_{3}+d f_{4}\right) \end{aligned}$ | $3^{\text {rd }}$ order upwind <br> Lele (1992) ${ }^{[167]}$ Eq. (4.3.6) $\alpha=11 ; a=\frac{156}{126} ; b=-27 ; c=15 ; d=-1$ |
|  | idx $=2$ |  | $4^{\text {th }}$ order centered compact <br> Lele (1992) ${ }^{[167]}$ Eq. (2.2.6) $\alpha=\frac{1}{10} ; \beta=0 ; a=\frac{6}{5} ; b=0 ; c=0 ; d=0$ |
|  | idx $=3-4$ | $\beta f_{i-2}^{\prime \prime}+\alpha f_{i-1}^{\prime \prime}+f_{i}^{\prime \prime}+\alpha f_{i+1}^{\prime \prime}+\beta f_{i+2}^{\prime \prime}$ | $6^{\text {th }}$ order centered compact Lele (1992) ${ }^{[167]}$ Eq. (2.2.7) $\begin{aligned} & \alpha=\frac{2}{11} ; \beta=0 ; \\ & a=\frac{12}{11} ; b=\frac{3}{11} ; c=0 ; d=0 \end{aligned}$ |
|  | Interior nodes | $\begin{aligned} & =d \frac{1}{16 \Delta x^{2}}\left(f_{i+4}-2 f_{i}+f_{i-4}\right) \\ & +c \frac{1}{9 \Delta x^{2}}\left(f_{i+3}-2 f_{i}+f_{i-3}\right) \\ & +b \frac{1}{4 \Delta x^{2}}\left(f_{i+2}-2 f_{i}+f_{i-2}\right) \\ & +a \frac{1}{\Delta x^{2}}\left(f_{i+1}-2 f_{i}+f_{i-1}\right) \end{aligned}$ | $6^{\text {th }}$ order modified centered for DNS Lamballais et al. (2011) ${ }^{[161]}$ Eq. (3) $\begin{aligned} & \alpha=\frac{272-45 k_{c}^{\prime \prime} \Delta x^{2}}{416-90_{c}^{\prime \prime \prime} \Delta x^{2}} ; \beta=0 ; \\ & \mathrm{a}=\frac{48-135 k_{c}^{\prime \prime} \Delta x^{2}}{1664-360 \mathrm{k}^{\prime \prime} \prime x^{2}} ; \mathrm{b}=\frac{528-81 \mathrm{k}_{\mathrm{c}}^{\prime \prime} \Delta \mathrm{x}^{2}}{208-45 \mathrm{k}_{\mathrm{c}}^{\prime \prime} \Delta \mathrm{x}^{2}} ; \\ & \mathrm{c}=\frac{-432+63 \mathrm{k}_{\mathrm{c}}^{\prime \prime} \Delta \mathrm{x}^{2}}{1664-360 k_{\mathrm{c}}^{\prime \prime} \Delta \mathrm{x}^{2}} ; \mathrm{d}=0 \end{aligned}$ |
| Periodic boundary |  |  | $6^{\text {th }}$ order modified centered for ILES <br> Dairay et al. (2014) ${ }^{[58]}$ Eq. (16) |

Table 2.2: Spatial discretisation for second-derivative.


Figure 2.10: Spatial discretisation for the second-derivative in non-periodic boundary. $\mathrm{idx}=1$ is at the non-periodic boundary. Values from grid points in yellow are used to compute the second-derivative at grid point in orange.

### 2.5.3 Time integration

Time-integration of the compressible NSE (equation 2.1-2.5) was performed using $4^{\text {th }}$ order 4 -step Runge-Kutta Algorithm ${ }^{[129,193]}$. Lets assume $\Delta$ t is the time step and $\mathbf{q}^{n}=[u$ $v w s p]^{\mathrm{t}}$ is the solution vector at time $\mathrm{t}_{\mathrm{n}}=\mathrm{n} \Delta \mathrm{t}$. The 4 steps of the Runge-Kutta algorithm are:

1. Euler predictor:

$$
\begin{align*}
\mathbf{q}^{\prime} & \leftarrow \mathbf{F}\left(\mathbf{q}^{n} ; \mathrm{t}_{n}\right) \\
\widetilde{\mathbf{q}} & \leftarrow \mathbf{q}^{n}+\frac{\Delta t}{2} \mathbf{q}^{\prime}  \tag{2.43}\\
\widehat{\mathbf{q}} & \leftarrow \mathbf{q}^{n}+\frac{\Delta t}{6} \mathbf{q}^{\prime}
\end{align*}
$$

$\mathbf{q}^{\prime}, \widetilde{\mathbf{q}}$ and $\widehat{\mathbf{q}}$ are the intermediate solution vectors of the Runge-Kutta Algorithm.
2. Euler corrector:

$$
\begin{align*}
& \mathbf{q}^{\prime} \leftarrow \mathbf{F}\left(\widetilde{\mathbf{q}} ; \mathrm{t}_{\mathrm{n}}+\frac{\Delta \mathrm{t}}{2}\right) \\
& \widetilde{\mathbf{q}} \leftarrow \mathbf{q}^{n}+\frac{\Delta \mathrm{t}}{2} \mathbf{q}^{\prime}  \tag{2.44}\\
& \widehat{\mathbf{q}} \leftarrow \widetilde{\mathbf{q}}+\frac{\Delta \mathrm{t}}{3} \mathbf{q}^{\prime}
\end{align*}
$$

3. Leapfrog predictor:

$$
\begin{align*}
& \mathbf{q}^{\prime} \leftarrow \mathbf{F}\left(\widetilde{\mathbf{q}} ; \mathrm{t}_{n}+\frac{\Delta \mathrm{t}}{2}\right) \\
& \widetilde{\mathbf{q}} \leftarrow \mathbf{q}^{n}+\Delta t \mathbf{q}^{\prime}  \tag{2.45}\\
& \widehat{\mathbf{q}} \leftarrow \widehat{\mathbf{q}}+\frac{\Delta \mathrm{t}}{3} \mathbf{q}^{\prime}
\end{align*}
$$

4. Milne corrector:

$$
\begin{align*}
\mathbf{q}^{\prime} & \leftarrow \mathbf{F}\left(\widetilde{\mathbf{q}} ; \mathrm{t}_{\mathrm{n}}+\Delta \mathrm{t}\right) \\
\mathbf{q}^{\mathbf{n + 1}} & \leftarrow \widehat{\mathbf{q}}+\frac{\Delta t}{6} \mathbf{q}^{\prime} \tag{2.46}
\end{align*}
$$

### 2.5.4 Stability Criteria

A FDS is stable if the error at one time step of the computation do not cause the error to be magnified as the computation continues. A neutrally stable scheme is the one in which the error remains constant as the computation is carried forward. If the error decay and eventually vanishes, then the numerical scheme is said to be stable. Whereas, if the errors grow with time, the scheme is said to be unstable. The stability of numerical schemes can be investigated by performing von-Neumann stability analysis. For a time-dependent problem, the stability guarantees that the numerical method produces a bounded solution whenever the solution of the exact differential equation is bounded.

Courant-Friedrichs-Lewy (CFL) condition or/and Fourier number decides the global time-step required for explicit time-integration, while ensuring the stability of the simulation. The principle behind the condition is that, e.g., if a wave is moving across a discrete spatial grid and one wants to compute its amplitude at discrete time steps of equal duration, then this duration must be less than the time for the wave to travel to adjacent grid point. As a result, when the grid point separation is reduced, the upper limit for the time-step is also reduced. For the 1D convection equation, CFL condition is:

$$
\begin{equation*}
(1+\mathcal{M}) \frac{\Delta t}{\Delta x}<\frac{\sigma_{i}}{k_{\max }^{\prime}} \tag{2.47}
\end{equation*}
$$

where $\sigma_{i}$ is the footprint limit of the time-integration method, and $k_{\max }^{\prime}$ is the maximum of the modified wave-number of the spatial first-derivative scheme. For the Runge-Kutta scheme, $\sigma_{i}=2.83$, and for the compact $6^{\text {th }}$ order first-derivative scheme $k_{\max }^{\prime}=1.989$ (maximum value of $k^{\prime} \Delta x$ in figure 2.4 for compact FDS 6 TriD). For the $1 D$ case $\sigma_{i} / k_{\max }^{\prime}=$ 1.42, whereas for the ${ }_{3} \mathrm{D}$ case with all the convective fluxes an empirical value was found:

$$
\begin{equation*}
(1+\mathcal{M}) \frac{\Delta t}{\Delta x}<2.6 \tag{2.48}
\end{equation*}
$$

Similarly, 1D diffusion equation is stable when,

$$
\begin{equation*}
\frac{\Delta t}{\operatorname{Re} \Delta x^{2}}<\frac{\sigma_{r}}{k_{\max }^{\prime \prime 2}} \tag{2.49}
\end{equation*}
$$

where $\sigma_{r}$ is the footprint of the time-integration method and $k_{\max }^{\prime \prime 2}$ is the maximum value of the modified wave-number of the spatial second-derivative. For Runge-Kutta scheme $\sigma_{r}=2.785$ and for the compact $6^{\text {th }}$ order second-derivative $k_{\text {max }}^{\prime \prime 2}=6.857$ (maximum value of $k^{\prime \prime} \Delta x^{2}$ in figure 2.6 for compact FDS 6TriD). For 1D case $\sigma_{r} / k_{\text {max }}^{\prime \prime 2}=0.406$, whereas in practice for the ${ }_{3} \mathrm{D}$ case with both thermal and viscous diffusion terms, an empirical value was found to be:

$$
\begin{equation*}
\frac{\Delta t}{\operatorname{Re} \Delta x^{2}}<0.02 \tag{2.50}
\end{equation*}
$$

When the modified scheme was used for computing the spatial second-derivative (equation 2.37 or 2.39 ), then one should use the maximum value of the modified wave-number computed from equation 2.36 . Since the objective of the modified second-derivative FDS is to introduce excess numerical dissipation, the maximum value of the modified wavenumber of the modified schemes is always higher than the compact $6^{\text {th }}$ order FDS. Which means, by using the modified scheme one will have more stringent condition on the global time-step.


Figure 2.11: Constraint on explicit time stepping due to $\operatorname{Re}_{\mathrm{b}}$ and $\mathcal{M}$ with $y_{\min }^{+}=1$. Orange shade for limitation due to CFL condition and black shade for limitation due to Fourier number.

For a channel flow, equation 2.48 and 2.50 are evaluated for different pairs of $\operatorname{Re}_{\mathrm{b}}$ and $\mathcal{M}$. Result is shown in figure 2.11. For all $\operatorname{Re}_{\mathrm{b}}$ at low $\mathcal{M}$, constraint on global time-step is due to the CFL condition, whereas at higher $\mathcal{M}$, time-step is limited by the Fourier number. Usually while simulating wall-bounded flows, one uses finer mesh close to the wall to capture the wall dynamics accurately. The level of refinement increases with the Reynolds number. Recalling that, for smaller mesh size one will also have to reduce the time-step in order to have a stable computation. For a given Re and $\mathcal{M}$ number, the dependence of global time stepping is shown in figure 2.12. With finer mesh close to the wall (smaller values of $y^{+}$), Fourier condition decides the time step, whereas with larger mesh resolution CFL condition decides the time step.


Figure 2.12: Time constraint due to CFL and Fourier constraint for $\operatorname{Re}_{\mathrm{b}}=6900$ and $\mathcal{M}=0.1$ with different mesh size at the wall. (a) $\square$ is for Fourier constraint and (b) $\circ$ is for CFL constraint.

### 2.6 PARALLEL IMPLEMENTATION AND SCALABILITY

Compact3D is a parallel compressible NSE solver, which is written in FORTRAN 95 and uses Message Passing Interface (MPI) for inter-communication between the computational processes. FDS uses information from the neighboring grid points (see figure 2.5 and 2.10) to compute the derivatives. With the conventional domain decomposition technique, one will have to communicate the information about few grid points in order to compute the derivatives. Compact3D performs efficient domain decomposition using


Figure 2.13: 2D pencil domain decomposition with 2DECOMP library. (a) x-pencil; (b) y-pencil and (c) z-pencil (from Laizet and $\operatorname{Li}(2011)^{[160]}$ ).
the 2DECOMP library which performs 2D pencil decomposition of the computational domain as in Laizet and $\mathrm{Li}(2011)^{[160]}$. Figure 2.13 shows an example of domain decomposition using 12 computational processes. The 2D grid for the domain decomposition is of size $4 \times 3$. The benefit of 2D pencil decomposition is that, one can compute the derivatives without communicating between processes. For example, with the data stored in $x$-pencil each process will have all the information required to compute the derivative in $x$-direction. Similarly, with the data stored in y-pencil and z-pencil, one can compute
the derivatives in $y$ - and $z$-direction respectively. The library also has the feature to convert the data from one pencil type to another. In figure 2.13 (a) each coloured block is a x-pencil which is handled by each of the 12 processors. With the transpose function of the library, one can convert these x-pencils into the y-pencils, the colour of the pencil can be used as the reference to know what information is transferred to each processes. MPI communication occurs only while performing such pencil conversion, which makes the code very efficient for parallel processing.

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Figure 2.14: Scalability curves for Compact3D on supercomputer Curie.
up to 3000 computational cores with 134 million grid points (see figure 2.14). The code has $85 \%-90 \%$ parallel efficiency for strong scaling, thanks to the highly scalable 2D decomposition library "2DECOMP". Generally, the number of computational cores depends on the simulation size, but for a majority of the simulations 1000-2000 computational cores were used for the simulation.

### 2.7 VALIDATION - DNS OF TEMPORAL SUPERSONIC CHANNEL FLOW

Channel flow is one of the geometrically simple flow configuration to study wall-bounded flows. It is also considered as a benchmark simulation for validating the numerical code. Based on the $\mathcal{M}$ number, the flow regime is divided in to subsonic ( $\mathcal{M}<0.8$ ), transonic ( $0.8<\mathcal{M}<1.2$ ), supersonic $(1<\mathcal{M}<5)$ and hypersonic $(\mathcal{M}>5)$. The flow physics varies as one moves from one regime to another. For lower $\mathcal{M}$ numbers the compressibility effects are negligible, and they start showing up at higher $\mathcal{M}$ numbers. Generally for
low supersonic $\mathcal{M}$ number, the major compressibility effects are due to the mean property variation of density and temperature, and some connections can be drawn with the incompressible flow. Whereas at hypersonic $\mathcal{M}$ numbers, the flow physics is very different. This thesis will be focusing on non-hypersonic $\mathcal{M}$ number channel flows, hence Compact3D is validated against the supersonic turbulent channel flow simulation of Coleman et al. (1995) ${ }^{[52]}$. Temporal simulation or stream-wise periodic channel flow simulation is performed with isothermal rigid wall boundary conditions (equation 2.33 and 2.34) which behaves as a cold wall (see figure 2.1 for the flow configuration). Simulations are performed for test-case $\mathrm{A}($ at $\mathcal{M}=1.5)$ and $\mathrm{B}($ at $\mathcal{M}=3)$ of Coleman et al. $(1995)^{[52]}$ to validate Compact3D. Additional simulations for the same flow configuration with different techniques is presented in Appendix C.

DNS of temporal channel flow at $\mathcal{M}=1.5 ; \operatorname{Re}_{\mathrm{b}}=3000$ and $\mathcal{M}=3 ; \mathrm{Re}_{\mathrm{b}}=4880$ was performed using the modified second-derivative FDS mentioned in Section 2.5.2.1 with $n=6$. Simulation parameters are tabulated in Table 2.3. Coleman et al. (1995) $)^{[52]}$ used

| Case | $\mathrm{Re}_{\mathrm{b}}$ | $\mathcal{M}$ | $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}$ | $\mathrm{N}_{\mathrm{x}} \times \mathrm{N}_{\mathrm{y}} \times \mathrm{N}_{z}$ | $\Delta y_{\text {min }}^{+}$ | $\Delta y_{\text {max }}^{+}$ | $\Delta \chi^{+}$ | $\Delta z^{+}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 3000 | 1.5 | $4 \pi \mathrm{H} \times 2 \mathrm{H} \times \frac{4 \pi}{3} \mathrm{H}$ | $170 \times 150 \times 100$ | 0.99 | 8.88 | 16 | 9 |
| Ref ${ }^{[52]}$ (Case A spectral) |  |  |  | $144 \times 119 \times 80$ | 0.1 | 5.88 | 19 | 12 |
| B | 4880 | 3.0 | $4 \pi \mathrm{H} \times 2 \mathrm{H} \times \frac{4 \pi}{3} \mathrm{H}$ | $220 \times 181 \times 120$ | 0.99 | 25.07 | 26 | 16 |
| Ref ${ }^{[52]}$ (Case B spectral) |  |  |  | $144 \times 119 \times 80$ | 0.2 | 11.95 | 39 | 24 |

Table 2.3: Parameters of validation test-case for DNS of supersonic channel flows. Values taken from Coleman et al. $(1995)^{[52]}$.
quasi-spectral code for simulating supersonic channel flows. Difference between spectral and compact FDS was shown in figure 2.7 . Hence some discrepancy is expected when the results of Compact3D are compared to the results of Coleman et al. (1995) ${ }^{[52]}$. Statistics were computed over 10-15 flow through time after the transition to steady state. Table 2.4 present the results for the mean flow variables, and there is a very good agreement between the present simulation and the reference data.

| $\mathcal{M}$ | $\operatorname{Re}_{\mathrm{b}}$ | Case | $\operatorname{Re}_{\tau}$ | $\mathrm{u}_{\tau}$ | $-\mathrm{B}_{\mathrm{q}}$ | $\bar{\rho}_{w}$ | $\bar{\rho}_{\mathrm{c}}$ | $\overline{\mathrm{T}}_{\mathrm{c}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.5 | 3000 | A | 222.93 | 0.0542 | 0.050 | 1.369 | 0.99 | 1.373 |
|  |  | $\operatorname{Ref}^{[52]}$ | 222 | 0.0546 | 0.049 | 1.355 | 0.980 | 1.378 |
| 3 | 4880 | B | 456.04 | 0.0387 | 0.134 | 2.41 | 0.949 | 2.525 |
|  |  | $\operatorname{Ref}^{[52]}$ | 451 | 0.0386 | 0.137 | 2.388 | 0.952 | 2.490 |

Table 2.4: Validation results. Values taken from Coleman et al. (1995) ${ }^{[52]}$.

Mean velocity profile scaled using the friction velocity $u_{\tau}$ is presented in figure 2.15. Superscript ' + ' is used for quantities scaled with viscous velocity scale $u_{\tau}$ and length scale $l_{v}$, such that:

$$
\begin{equation*}
\bar{u}^{+}=\frac{\overline{\widetilde{u}}}{\widetilde{\mathfrak{u}}_{\tau}} ; \quad y^{+}=\frac{\widetilde{\mathrm{y}}}{\widetilde{\mathfrak{l}}_{v}} \tag{2.51}
\end{equation*}
$$



Figure 2.15: Comparison of mean velocity profile for (a) $\mathcal{M}=1.5$ and (b) $\mathcal{M}=3$. Reference curves taken from Coleman et al. (1995) ${ }^{[52]}$. Grey curves are for $\bar{u}^{+}=y^{+}$and $\bar{u}^{+}=1 / \kappa \ln y^{+}+$ C.

Viscous sub-layer close to wall is very well captured $\left(\bar{u}^{+}=y^{+}\right)$. Profile at the outer-layer log-region also collapses very well on the reference curve, but they have a higher log-law intercept value. Log-law for the incompressible flow can be written as $\bar{u}^{+}=1 / \kappa \ln y^{+}+C$, where von Kármán constant $\kappa=0.41$ and log-law intercept $C=5.2$. For supersonic flows with isothermal walls $C$ increases with the $\mathcal{M}^{[282]}$. The compressibility effects will be addressed later in Chapter 4.


Figure 2.16: Comparison of rms velocity profiles for (a) $\mathcal{M}=1.5$ and (b) $\mathcal{M}=3$. Reference curves taken from Coleman et al. (1995) ${ }^{[52]}$.

Root-mean-square velocities for both Mach numbers are plotted in figure 2.16. Rms of the velocity is computed from the fluctuation. If " $u$ " is the instantaneous velocity, then the fluctuation is:

$$
\begin{equation*}
u^{\prime}=u-\bar{u} \tag{2.52}
\end{equation*}
$$

Here $\overline{()}$ is the mean over stream-wise and span-wise directions and time. Except a slight over-prediction of $\sqrt{\overline{u^{\prime 2}}}$ peak at $\mathcal{M}=3$, all other curves collapse very well on the reference curves of Coleman et al. (1995) ${ }^{[52]}$. For $\mathcal{M}=3, \sqrt{\overline{u^{\prime 2}}}$ is larger in comparison to the result at $\mathcal{M}=1.5$. This necessarily does not mean there is enhanced turbulence at higher $\mathcal{M} . u_{\tau}$ which is used for normalizing the result was defined using $\tau_{w}$ and $\bar{\rho}_{w}$, and these quantities at the wall changes with the $\mathcal{M}$. With right scaling technique one can account for this mean property variation at higher $\mathcal{M}$. It will be discussed in detail in Chapter 4 dedicated to compressible scaling techniques.

(b)




Figure 2.17: Comparison of mean profile of (top) temperature; (center) pressure and (bottom) density for (a) $\mathcal{M}=1.5$ and (b) $\mathcal{M}=3$. Check figure 2.15 for legends. Reference curves taken from Coleman et al. (1995) ${ }^{[52]}$.

Profiles of mean pressure, temperature and density are plotted in figure 2.17. Good agreement is found with respect to the results of Coleman et al. (1995) ${ }^{[52]}$. Channel with isothermal walls behave differently compared to the ones with adiabatic walls, and this can have direct impact on the flow physics. Isothermal wall promote heat transfer through them in order to maintain constant wall temperature. From the figure it is clear that the walls are colder than the bulk of the flow because excess heat generated due to dissipation is transferred through the walls, whereas $\bar{\rho}$ behaves opposite to $\overline{\mathrm{T}}$. Maximum gradients are found in the buffer layer. Mean flow is mostly isobaric; $\overline{\mathrm{T}}$ reaches its maximum, and $\bar{\rho}$ its minimum in the core of the flow. When Mach number increases the rate of heat transferred through walls and the near-wall density and temperature gradients also become larger.

Curves for $\sqrt{\overline{T^{\prime 2}}}$ at both $\mathcal{M}$ collapse very well on the reference data. At $\mathcal{M}=1.5, \sqrt{\overline{p^{\prime 2}}}$ is slightly under-predicted. $\sqrt{\overline{\rho^{\prime 2}}}$ is slightly over-predicted across the channel for both $\mathcal{M}$. The real reason for this discrepancy is unknown, Coleman et al. (1995) ${ }^{[52]}$ mentioned about the presence of weak unrealistic acoustic waves in their simulation. Considering the over-dissipative nature of the compact FDS with respect to the spectral scheme, minor differences for the higher-order moments should be expected. At $\mathcal{M}=3, \sqrt{\overline{\rho^{\prime 2}}}{ }_{\max } \approx 0.11 \bar{\rho}$, which means that the major compressibility effects is due to the mean properties and not


Figure 2.18: Comparison of rms profile of (top) temperature; (center) pressure and (bottom) density for (a) $\mathcal{M}=1.5$ and (b) $\mathcal{M}=3$. Check figure 2.15 for legends. Reference curves taken from Coleman et al. (1995) ${ }^{[52]}$.
their fluctuations. Larger rms values for density and temperature is close to the wall, Coleman et al. (1995) ${ }^{[52]}$ show that this is the result of solenoidal passive-mixing across the mean gradient.


Figure 2.19: Comparison of stress profiles for (a) $\mathcal{M}=1.5$ and (b) $\mathcal{M}=3$. Reference curves taken from Coleman et al. $(1995)^{[52]}$.

Stress profiles for both $\mathcal{M}$ are presented in figure 2.19. Satisfactory agreement is found with Coleman et al. (1995) ${ }^{[52]}$. Close to the wall, a small kink in the stress profiles could be observed. Foysi et al. (2004) ${ }^{[80]}$ reported that anisotropy of Reynolds stress is due to near-wall pressure-strain. At $\mathcal{M}=3$, larger stress is found when scaled with friction velocity $\mathfrak{u}_{\tau}$.
(a)
(b)


Figure 2.20: Comparison of stream-wise and span-wise correlations for $\mathcal{M}=1.5$ at the center of the channel. Reference curves taken from Coleman et al. (1995) ${ }^{[52]}$.

The two-point correlation computed in the stream-wise and span-wise direction for $\mathcal{M}=1.5$ is presented in figure 2.20. For $R_{u u}$ in the stream-wise direction, the small kink around $\Delta x \approx 2.5$ is not captured, except for that all correlation curves collapse very well on the reference data.

In this chapter, parallel compressible NSE solver Compact3D was validated against the reference simulation of Coleman et al. (1995) ${ }^{[52]}$. The code solves the characteristics NSE with $6^{\text {th }}$ order FDS for the spatial derivatives. In order to account for the under-dissipation of the traditional compact scheme, a modified scheme which introduces excess dissipation at the mesh cut-off was utilised. The modified scheme was used for computing the spatial second-derivative and it was tested by performing the DNS of supersonic channel flow. The explicit time integration was performed using the $4^{\text {th }}$ order RungeKutta scheme. The code performs 2D pencil decomposition of the computational domain, which is very efficient and suitable for the FDS. In the following chapter the modified FDS for the spatial second-derivative will be tested for performing ILES of channel flows.

MODIFIED FINITE DIFFERENCE SCHEME FOR IMPLICIT
LARGE EDDY SIMULATION

In this chapter, the capability of the numerical solver to perform ILES of channel flow will be assessed. In channel flow, one encounters anisotropic turbulence and thus it is very important to assess the modified scheme given in Section 2.5.2.2. Therefore the objective was to first perform a parametric study to analyse the modified FDS by performing ILES using different computational grids, Reynolds and Mach numbers. This scheme has been extensively tested and used for simulating such flows in the incompressible regime. The modified scheme has been tested for incompressible flows and other flow configuration, some examples can be found in Dairay et al. (2014) ${ }^{[58]}$ and Dairay et al. (2017) ${ }^{[59]}$, among others and for sound radiation in Lamballais et al. (2011) ${ }^{[161]}$. This is the first time that this model is tested for ${ }_{3} \mathrm{D}$ compressible flow and channel configuration. Presently, the model has been tested for channel flows with $\mathcal{M} \leq 3$ and $\operatorname{Re}_{\tau} \leq 1000$.

Secondly, the grid requirement for wall resolved ILES was investigated. Since no wall model is used, it is very important to know the limit of grid coarsening for ILES because the numerical dissipation is added through the modified spatial second-derivative FDS and not by modelling the physics. As mentioned earlier, the level of dissipation is dependent on the level of grid coarsening. By coarsening beyond a certain limit, one will not be able to discretise some small-scale physics which are very important to the turbulence phenomenon. Various degrees of mesh coarsening was performed starting from a fine to a very coarse grid. Turbulent statistics up to $4^{\text {th }}$ order moments were computed and compared against the reference DNS to find the critical grid requirement for performing trustworthy ILES at a reduced computational cost.

### 3.1 PARAMETRIC STUDY OF MODIFIED SPATIAL 2ND-DERIVATIVE FINITE DIFFERENCE SCHEME

Extensive parametric study of the modified spatial second-derivative FDS was mainly performed at a moderate Reynolds number $\operatorname{Re}_{\tau} \approx 400$ for a subsonic channel flow at $\mathcal{M}=0.5$. Two mesh resolutions, and a variety of SVV kernels were chosen for the analysis. The critical test-cases for channel flow simulations at $\operatorname{Re}_{\tau} \approx 400$ and $\mathcal{M}=0.5$ were then tested for supersonic channel flows at $\mathcal{M}=1.5$ and 3 . SVV kernels were also tested for channel flows at higher Reynolds number of up to $\mathrm{Re}_{\tau} \approx 1000$.

### 3.1.1 Channel flow at $\mathrm{Re}_{\tau} \approx 400$ and $\mathcal{M}=0.5$

ILES was performed with fine $\left(\pi k_{\eta} / k_{c}=2\right)$ and coarse $\left(\pi k_{\eta} / k_{c}=4\right)$ grids. $\pi k_{\eta} / k_{c}$ is the measure of grid coarsening based on Kolmogorov scale $\eta$, and the model has been calibrated to produce right level of dissipation based on the level of grid coarsening (see Section 2.5.2.2 for detailed explanation). Fine(coarse) grid used for ILES is two(four) times coarser in stream- and span-wise direction, when compared to the grid used for the reference simulation by Moser et al. (1999) ${ }^{[203]}$. Reference DNS at $\operatorname{Re}_{\mathrm{b}}=6882$ (for which $\operatorname{Re}_{\tau} \approx 400$ ) was performed using a spectral code. Moser et al. (1999) ${ }^{[203]}$ used computational domain of size $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}=2 \pi \mathrm{H} \times 2 \mathrm{H} \times \pi \mathrm{H}$, which was discretised with $\mathrm{N}_{x} \times \mathrm{N}_{y} \times \mathrm{N}_{z}=256 \times 193 \times 192$ grid points. Here, low Mach number simulation at $\mathcal{M}=0.5$ was performed with $\mathrm{Re}_{\mathrm{b}}=6882$ using both computational grids. Compressibility effects at this $\mathcal{M}$ are expected to be negligible, hence the results can be directly compared with the incompressible DNS. For the present simulations larger computational domains were used, $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}=12 \mathrm{H} \times 2 \mathrm{H} \times 6 \mathrm{H}$. More details about the computational grids used for the parametric study can be found in table 3.1.

|  | $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}$ | $\mathrm{~N}_{x} \times \mathrm{N}_{y} \times \mathrm{N}_{z}$ | $\Delta x^{+}$ | $\Delta z^{+}$ | $\Delta y_{\min }^{+}$ | $\Delta y_{\max }^{+}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Ref $^{[203]}$ | $2 \pi \mathrm{H} \times 2 \mathrm{H} \times \pi \mathrm{H}$ | $256 \times 193 \times 128$ | 10 | 6.5 | 0.0295 | 6.5 |
| Fine ILES | $12 \mathrm{H} \times 2 \mathrm{H} \times 6 \mathrm{H}$ | $240 \times 157 \times 240$ | 19.75 | 9.875 | 1 | 9.7 |
| Coarse ILES | $12 \mathrm{H} \times 2 \mathrm{H} \times 6 \mathrm{H}$ | $120 \times 101 \times 120$ | 39.50 | 19.75 | 2 | 14 |

Table 3.1: Computational grids used for the parametric study of modified spatial secondderivative $\operatorname{FDS}$ at $\mathrm{Re}_{\tau} \approx 400$ and $\mathcal{M}=0.5$. Reference values from Moser et al. (1999) ${ }^{[203]}$.

From Section 2.5.2.2, it is clear that the degree of grid coarsening determines the level of dissipation and shape of the kernel determines the range of scales over which dissipation is applied. In the present study three different types of SVV kernels namely SVV-like, steep SVV and sharp SVV were tested. As mentioned earlier in Chapter $2, v_{0} / v$ is used to denote the excess spectral viscosity at the cut-off and $\pi k_{\eta} / k_{c}=\Delta x / \eta$ is the measure of grid coarsening based on Kolmogorov scale $\eta$. Predicted value of $v_{0} / v$ (based on isotropic turbulence) for the corresponding $\pi k_{\eta} / k_{c}$ for each grid is taken from the figure 2.9a, where $v_{0}$ is the spectral viscosity at the cut-off spatial wave-number. Kernels with $50 \%$ and $200 \%$ of the predicted value of $v_{o} / v$ for each kernel type were also tested. The idea was to vary the excess spectral viscosity (or numerical dissipation) while maintaining the SVV kernel type and check its effect. Kernels chosen and their coefficients for each computational grid is presented in table 3.2.

Mean flow variables and errors in their prediction are presented in table 3.3. With the fine grid, $\operatorname{Re}_{\tau}$ and $u_{\tau}$ were predicted very well. Error on these quantities with the fine grid was less than $1 \%$, when compared against the reference DNS. Results were independent of different percentage of $v_{0} / v$ and kernel types. For the coarse grid, least error was obtained with $50 \%$ of the $v_{0} / v$, and/or also for decreasing kernel width. Best results were obtained with sharp SVV, but with $50 \%$ of $v_{0} / v$ the simulation blew up due to the lack of numerical dissipation. SVV-like and steep SVV kernels that acts on wider wave-number range, dissipates more than required but this ensured that the simulation was stable and the errors were also within the acceptable range.

|  | FINE ILES $\left(\pi k_{\eta} / k_{c}=2\right)$ |  | COARSE ILES $\left(\pi k_{\eta} / k_{c}=4\right)$ |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | SVV-like | steep SVV | sharp SVV | SVV-like | steep SVV | sharp SVV |
| $100 \% \frac{v_{0}}{v}$ | 3.06 | 4.15 | 7.41 | 14.61 | 19.75 | 35.81 |
| $k^{\prime \prime}\left(k_{c}\right) / \pi^{2}$ | 4.06 | 5.15 | 8.41 | 15.61 | 20.75 | 36.81 |
| $k^{\prime \prime}\left(2 k_{c} / 3\right) / \pi^{2}$ | 1.04 | 0.85 | 0.62 | 3.30 | 2.37 | 1.31 |
| $50 \% \frac{v_{0}}{v}$ | 1.53 | 2.07 | 3.70 | 7.30 | 9.87 | 17.90 |
| $k^{\prime \prime}\left(k_{c}\right) / \pi^{2}$ | 2.53 | 3.07 | 4.70 | 8.30 | 10.87 | 18.90 |
| $k^{\prime \prime}\left(2 k_{c} / 3\right) / \pi^{2}$ | 0.74 | 0.64 | 0.53 | 1.87 | 1.41 | 0.88 |
| $200 \% \frac{v_{0}}{v}$ | 6.12 | 8.30 | 14.82 | 29.22 | 39.5 | 71.62 |
| $k^{\prime \prime}\left(k_{c}\right) / \pi^{2}$ | 7.12 | 9.3 | 15.82 | 30.22 | 40.5 | 72.62 |
| $k^{\prime \prime}\left(2 k_{c} / 3\right) / \pi^{2}$ | 1.64 | 1.25 | 0.80 | 6.15 | 4.30 | 2.19 |

Table 3.2: SVV kernels used for the parametric study of modified spatial second-derivative FDS for ILES of channel flow at $\operatorname{Re}_{\tau} \approx 400$ and $\mathcal{M}=0.5$.

|  |  |  | FINE GRID $\left(\pi \mathrm{k}_{\mathrm{\eta}} / \mathrm{k}_{\mathrm{c}}=2\right)$ |  |  | COARSE GRID $\left(\pi \mathrm{k}_{\mathrm{\eta}} / \mathrm{k}_{\mathrm{c}}=4\right)$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{Re}_{\tau}$ | $\operatorname{Ref}^{[203]}$ |  | SVV-like | steep SVV | sharp SVV | SVV-like | steep SVV | sharp SVV |
|  |  | 100\% $\frac{v_{0}}{v}$ | 397.8 | 398.0 | 400.0 | 382.3 | 387.7 | 397.1 |
|  | 395 | $50 \% \frac{v_{0}}{v}$ | 397.4 | 397.7 | 392.1 | 388.1 | 392.5 | - |
|  |  | 200\% $\frac{v_{0}}{v}$ | 395.8 | 394.9 | $397 \cdot 4$ | $377 \cdot 4$ | 382.6 | 390.3 |
| $\epsilon_{\operatorname{Re}_{\tau}}(\%)$ |  | 100\% $\frac{v_{0}}{v}$ | 0.72 | 0.77 | 1.26 | -3.21 | -1.83 | 0.54 |
|  |  | $50 \% \frac{v_{0}}{v}$ | 0.61 | 0.70 | -0.73 | -1.72 | -0.61 | - |
|  |  | 200\% $\frac{v_{0}}{v}$ | 0.21 | -0.01 | 0.62 | -4.45 | -3.13 | -1.17 |
| $\frac{u_{\tau}}{u_{\mathrm{b}}}$ | 0.056 | $100 \% \frac{v_{0}}{v}$ | 0.0556 | 0.0557 | 0.0559 | 0.0536 | 0.0543 | 0.0555 |
|  |  | $50 \% \frac{v_{0}}{v}$ | 0.0558 | 0.0558 | 0.0558 | 0.0544 | 0.0551 | - |
|  |  | $200 \% \frac{v_{0}}{v}$ | 0.0557 | 0.0557 | 0.0559 | 0.0530 | 0.0537 | 0.0548 |
| $\epsilon_{\frac{u_{\tau}}{u_{\mathrm{b}}}}(\%)$ |  | $100 \% \frac{v_{0}}{v}$ | -0.58 | -0.42 | -0.17 | -4.24 | -2.94 | -0.769 |
|  |  | $50 \% \frac{v_{0}}{v}$ | -0.25 | -0.26 | -0.34 | -2.84 | -1.54 | - |
|  |  | 200\% $\frac{v_{\text {o }}}{v}$ | -0.41 | -0.41 | -0.03 | -5.23 | -3.96 | -2.07 |

Table 3.3: Mean flow variables and errors in their prediction with different SVV kernels for ILES of channel flow at $\operatorname{Re}_{\tau} \approx 400$ at $\mathcal{M}=0.5$. Reference values from Moser et al. (1999) ${ }^{[203]}$.

Mean and turbulent statistics obtained for fine grid $\left(\pi k_{\eta} / k_{c}=2\right)$ is presented in figure 3.1. Fine grid is twice coarser in the stream- and span-wise direction compared to the reference DNS of Moser et al. (1999) ${ }^{[203]}$. Results with the SVV-like kernel is only presented in the figure. For the mean velocity profile, both the near-wall viscous sub-layer and outer-layer log-region were captured very well. Rms velocity profiles collapse perfectly on the reference curves. Peak values and their location were predicted accurately. From the figure, it was evident that there was no discrepancy for the mean and rms velocity profiles with, $50 \%, 100 \%$ and $200 \%$ of $v_{0} / v$ with SVV-like kernels. Different percentage of $v_{0} / v$ defines the target dissipation at cut-off. In figure 3.1 (c) the comparison of spectra is presented. The dashed vertical line is located at $k_{z}=k_{m}=2 k_{c} / 3$, generally for different types of SVV kernels, the distribution of excess dissipation is controlled by tuning the
amount of dissipation at this wave-number. In this particular case, the excess dissipation is varied by maintaining the SVV kernel type. Minor effects of varying $v_{0} / v$ can be seen in the spectra mainly in the region $k_{m} \leq k_{z} \leq k_{c}$, for lower wave-numbers the spectra collapsed very well on the reference curve of Moser et al. (1999) ${ }^{[203]}$. Similar observations were made with steep SVV and sharp SVV kernels (not presented here). This implies, for a fine grid which discretise almost all the scales of the flow, the distribution of excess dissipation will not affect the results. Hence for a well refined grid, modified spatial second-derivative FDS, mainly remove grid-to-grid oscillations by performing quasi DNS.


Figure 3.1: (a) Mean velocity profile, (b) rms velocity profiles and (c) stream-wise velocity spectra computed in span-wise direction, dashed vertical line is at $k_{m}=k_{z}=2 k_{c} / 3$. Results are for test-cases with SVV-like kernel with fine grid ( $\pi \mathrm{k}_{\mathrm{\eta}} / \mathrm{k}_{\mathrm{c}}=2$ ) at $\mathrm{Re}_{\tau} \approx 400$ and $\mathcal{M}=0.5$. Reference data from Moser et al. (1999) ${ }^{[203]}$.

In figure 3.2 mean and rms velocity profiles obtained with the coarse grid $\left(\pi k_{\eta} / k_{c}=4\right)$ are presented. With all the kernels, viscous sub-layer of the mean velocity was predicted very well. With SVV-like and steep SVV kernels, the log-region of the mean velocity pro-


Figure 3.2: ( $\mathrm{a}, \mathrm{c}, \mathrm{e}$ ) Mean and (b, d, f) rms velocity profiles computed from ILES of channel flow with coarse grid $\left(\pi k_{\eta} / k_{c}=4\right)$ using ( $\left.a, b\right)$ SVV-like, $(c, d)$ steep SVV and (e, f) sharp SVV kernel at $\operatorname{Re}_{\tau} \approx 400$ and $\mathcal{M}=0.5$. Reference data from Moser et al. (1999) ${ }^{[203]}$.
file did not collapse very well on the reference curve. In the same fashion, for the rms
velocity profiles, the peak of $\sqrt{\overline{\mathfrak{u}^{\prime 2}}}$ was over-predicted which is a characteristic feature of mesh coarsening, but the peak location was predicted accurately. Discrepancy in the mean and rms velocity were mainly observed with the SVV-like and steep SVV kernels, whereas with the sharp SVV kernel, there was better collapse of the results on the reference data. Sharp SVV kernel acts only on the high wave-numbers unlike SVV-like and steep SVV kernels. Similarly, increasing discrepancy for increasing percentage of the predicted value of $v_{0} / v$ was also observed with the SVV-like and steep SVV kernels, because it added more than required dissipation. Better collapse of the results were obtained for $50 \%$ of $v_{0} / v$ for SVV-like and steep SVV kernel. Satisfactory agreement with small dis-

(a)
(b)

(c)


Figure 3.3: Stream-wise velocity spectra in span-wise direction computed from ILES of channel flow with coarse grid $\left(\pi k_{\eta} / k_{c}=4\right)$ using (a) SVV-like, (b) steep SVV and (c) sharp SVV kernel at $\operatorname{Re}_{\tau} \approx 400$ and $\mathcal{M}=0.5$. Dashed vertical line is at $k_{m}=k_{z}=2 k_{c} / 3$. Reference data from Moser et al. (1999) ${ }^{[203]}$. Simulation blew up with $50 \% v_{0} / v$ for sharp SVV kernel.
crepancy close to cut-off wave-number was observed for the spectra presented in figure 3.3. For sharp SVV with $200 \%$ of the predicted value of $v_{0} / v$, slight over-dissipation was observed, whereas with $50 \%$ of predicted value lack of dissipation was observed. Lack of
dissipation resulted in grid-to-grid oscillations, which eventually led to simulation blowup. The effect of the SVV kernel type can also be noticed with the help of the dashed vertical line at $k_{m}=k_{z}=2 k_{c} / 3$. With the sharp SVV kernel the spectra stays close to the reference data of Moser et al. (1999) ${ }^{[203]}$ until the vicinity of the intermediate wavenumber ( $k_{m}$ ), whereas with the SVV-like kernel, the spectra departs from the reference data.

For the coarse grid, kernel sharper than the sharp SVV was also tested, for which most of the dissipation was concentrated in a narrow high wave-number band ( $\mathrm{N}_{\mathrm{fac}}=8$ and $c_{1}=0.0275$ ). Due to the highly concentrated dissipation restricted to only high wavenumbers, some instabilities due to collocated grid ${ }^{[21,207]}$ ) or due to non-conservative formulation of compressible $\mathrm{NSE}^{[75,154,197]}$ exists in the simulation which eventually leads to simulation blow-up. For InCompact $3 D^{[158-160]}$, (which solves incompressible NSE and uses same modified spatial second-derivative FDS as Compact3D), such problem is not encountered due to the conservation of kinetic energy even if the target dissipation is insufficient.

Generally similar trend in the results were found with different kernels and with different percentage of predicted value of $v_{0} / v$ with fine grid $\left(\pi k_{\eta} / k_{c}=2\right)$. Hence with increased grid resolution, one performs DNS (similar like using hyper-viscosity kernel introduced in Section 2.5.2.1), regardless the kind of modified second-derivative scheme used for extra dissipation. On the other hand, with coarser grid, the results were sensitive to the kernels. Better statistics were obtained with sharp kernels, for which all the excess dissipation was targeted only at the higher wave-number range. With the lack of dissipation, the simulation blew-up, plus sharper the kernel stronger the constraint on the time-step for explicit computation (see Section 2.5.4). As a practical approach, SVV-like and steep SVV were the desired choice for ILES. These kernels might introduce slightly more than required dissipation, but it also ensured that (a) the computation was stable; (b) the errors were within acceptable limits (see table 3.3), and (c) the time-step for explicit time-stepping remains reasonable.

### 3.1.2 High $\operatorname{Re}_{\tau}$ channel flows at $\mathcal{M}=0.5$

In this section, the modified spatial second-derivative FDS was used for the ILES of high $\operatorname{Re}_{\tau}$ channel flows. Simulation parameters and results for $\operatorname{Re}_{\tau}=640$ and 1000 are presented in table 3.4. Results for $\operatorname{Re}_{\mathrm{b}}=12214$ was compared against Abe et al. (2004) ${ }^{[1]}$, and $\mathrm{Re}_{\mathrm{b}}=20000$ was compared against Lee and Moser $(2015)^{[166]}$. Incompressible NSE were solved for both the reference test-cases. Present ILES were performed at $\mathcal{M}=0.5$, for which the compressibility effects are expected to be negligible, hence allows for direct comparison with the results from incompressible NSE solvers. Computational grid chosen for the ILES was about 4 times coarser $\left(\pi k_{\eta} / k_{c}=4\right)$ than the reference DNS ${ }^{[1,166]}$. 32 times less grid points was used for the ILES at $\operatorname{Re}_{b}=12214$, and 18 times less grid points for $\mathrm{Re}_{\mathrm{b}}=20000$. For both Reynolds number steep SVV kernel with $100 \%$ of $v_{0} / v$ was used. From Section 3.1.1 it was clear that SVV-like and steep SVV kernels were the reasonable choice, because they produce results with errors within the acceptable limits. Moreover in Section 2.5.4 it was shown that sharper kernels can have stringent constraint on global time-step.

| $\operatorname{Re}_{\mathrm{b}}$ |  | $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}$ | $\mathrm{~N}_{x} \times \mathrm{N}_{y} \times \mathrm{N}_{z}$ | $\Delta \mathrm{x}^{+}$ | $\Delta z^{+}$ | $\Delta y_{\min }^{+}$ | $\Delta y_{\max }^{+}$ | $\operatorname{Re}_{\tau}$ | $\mathrm{u}_{\tau} / \mathrm{u}_{\mathrm{b}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12214 | $\operatorname{Ref}^{[1]}$ | $4 \pi \mathrm{H} \times 2 \mathrm{H} \times 2 \pi \mathrm{H}$ | $1024 \times 256 \times 1024$ | 8 | 4 | 0.15 | 8.02 | 640 | 0.0522 |
|  | ILES | $4 \pi \mathrm{H} \times 2 \mathrm{H} \times 4 \pi \mathrm{H}$ | $200 \times 205 \times 400$ | 40 | 20 | 2 | 14 | 650 | 0.0516 |
| 20000 | Ref $^{[166]}$ | $8 \pi \mathrm{H} \times 2 \mathrm{H} \times 3 \pi \mathrm{H}$ | $2304 \times 512 \times 2048$ | 10.9 | 4.6 | 0.019 | 6.2 | 1000 | 0.0500 |
|  | ILES | $8 \pi \mathrm{H} \times 2 \mathrm{H} \times 3 \pi \mathrm{H}$ | $628 \times 235 \times 943$ | 40 | 10 | 2 | 14 | 1000 | 0.0494 |

Table 3.4: Simulation parameters and results for the parametric study of modified spatial secondderivative FDS for high $\operatorname{Re}_{\tau}$ channel flows. Reference values from Abe et al. (2004) ${ }^{[1]}$ for $\mathrm{Re}_{\mathrm{b}}=12214$ and from Lee and Moser $(2015)^{[166]}$ for $\mathrm{Re}_{\mathrm{b}}=20000$.


Figure 3.4: ( $\mathrm{a}, \mathrm{c}$ ) Mean and (b, d) rms velocity profiles computed from ILES of channel flow at $\mathcal{M}=0.5$ for $(\mathrm{a}, \mathrm{b}) \operatorname{Re}_{\mathrm{b}}=12214$ and $(\mathrm{c}, \mathrm{d}) \mathrm{Re}_{\mathrm{b}}=20000$. Reference data from Abe et al. $(2004)^{[1]}$ for $\operatorname{Re}_{\mathrm{b}}=12214$ and from Lee and Moser $(2015)^{[166]}$ for $\operatorname{Re}_{\mathrm{b}}=20000$.

The error on the global mean flow variables were around $1 \%$, which was within the acceptable limits. Mean and rms velocity profiles for both Reynolds number are presented in figure 3.4. Satisfactory collapse of the curves was obtained for both Reynolds numbers.

For the mean velocity profile, both the near-wall and outer-region of the boundary layer were captured very well. For the rms velocity profiles, except for a slight overestimation of the $\sqrt{\overline{u^{\prime 2}}}$ peak for $\mathrm{Re}_{\tau}=1000$, excellent collapse was found with the reference curves.

ILES high Reynolds number channel flow was performed using the modified FDS. Excellent agreement was found for the mean flow variables and first- and second-order statistics. Errors were within 1\%, compared to the reference DNS at both Reynolds number. The present approach gives the opportunity to perform high-order and accurate large Reynolds number simulation with a huge reduction in the computational cost. More to be discussed about high Reynolds number channel flows in Section 3.2 and Chapter 5.

### 3.1.3 Supersonic channel flows

The ability of the modified spatial second-derivative FDS to perform ILES of supersonic channel flows is evaluated in the following section. SVV-like, steep SVV and sharp SVV kernels with $100 \%$ of the predicted value of $v_{0} / v$ were used to introduce excess numerical dissipation. Channel flow at $\mathrm{Re}_{\mathrm{b}}=3000 ; \mathcal{M}=1.5$ and $\mathrm{Re}_{\mathrm{b}}=4880 ; \mathcal{M}=3$ were simulated and the results were compared against Coleman et al. (1995) ${ }^{[52]}$. Computational domain of size $L_{x} \times L_{y} \times L_{z}=4 \pi H \times 2 H \times \frac{4 \pi}{3} H$ was used for all the simulations. Only coarse mesh is tested because from Section 3.1.1 it was clear that, DNS like results were obtained with the refined grid . Details of the computational grids used for the analysis are presented in table 3.5. In the present ILES, 6.6 times less grid points were used than the reference DNS.

|  | $\mathrm{N}_{x} \times \mathrm{N}_{\mathrm{y}} \times \mathrm{N}_{z}$ | $\Delta \mathrm{x}^{+}$ | $\Delta z^{+}$ | $\Delta \mathrm{y}_{\min }^{+}$ | $\Delta \mathrm{y}_{\max }^{+}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\mathcal{M}=1.5$ | $71 \times 57 \times 51$ | 40 | 20 | 2 | 14 |
| $\operatorname{Ref}^{[52]}$ | $144 \times 119 \times 80$ | 19 | 12 | 0.1 | 5.88 |
| $\mathcal{M}=3$ | $71 \times 57 \times 51$ | 80 | 40 | 4 | 28 |
| $\operatorname{Ref}^{[52]}$ | $144 \times 119 \times 80$ | 39 | 24 | 0.2 | 11.95 |

Table 3.5: Computational grids used for the parametric study of modified spatial secondderivative FDS for supersonic channel flows. Reference values from Coleman et al. $(1995)^{[52]}$.

Mean and rms velocities for the supersonic channel flow at $\operatorname{Re}_{\mathrm{b}}=3000$ at $\mathcal{M}=1.5$ are presented in figure 3.5. Similar trends in the results were observed as in figure 3.2. Slight discrepancy was observed with the SVV-like kernel, whereas perfect collapse of curves on the reference data was observed with the sharp SVV kernel. Compressibility effects were visible in the plots, such as higher log-law intercept for the mean velocity profile. Law-of-the-wall is not valid for supersonic wall-bounded flows unless they are scaled properly to compensate the compressibility effects. The compressibility effects and scaling techniques will be discussed in details in Chapter 4.

Mean and rms velocity statistics for the channel flow simulation at $\operatorname{Re}_{\mathrm{b}}=4880$ and $\mathcal{M}=3$ is presented in figure 3.6. Stronger compressibility effects were visible at $\mathcal{M}=3$, with huge increase in the log-law intercept for the mean velocity profile in figure 3.6 (a)


Figure 3.5: (a) Mean and (b) rms velocity profiles computed from ILES of channel flow at $\mathcal{M}=1.5$ and $\operatorname{Re}_{\mathrm{b}}=3000$ with $100 \%$ of $v_{0} / v$. Reference data from Coleman et al. $(1995)^{[52]}$.


Figure 3.6: (a) Mean and (b) rms velocity profiles computed from ILES of channel flow at $\mathcal{M}=3$ and $\mathrm{Re}_{\mathrm{b}}=4880$ with $100 \%$ of $v_{0} / v$. Reference data from Coleman et al. (1995) ${ }^{[52]}$.
and enhanced rms velocity in figure 3.6 (b). Mean flow variables and their errors for both Mach numbers are presented in table 3.6. In the table, the friction Reynolds number and the friction Mach number obtained with different SVV kernels are listed. Friction Reynolds number is defined in equation 2.25, and the friction Mach number in equation 2.27. Results in the table shows the same trend as in table 3.3. Errors on the mean flow variables decrease as the kernel becomes sharper.

Overall satisfactory agreement was observed with the modified spatial second-derivative FDS for performing ILES. The ILES technique was tested for channel flows $\mathcal{M} \leq 3$ and $\operatorname{Re}_{\tau} \leq 1000$. Hence, the possibility to perform high-order and accurate ILES with a huge reduction in the computational cost using coarser grid for subsonic and supersonic channel flows using the modified spatial second-derivative FDS (introduced in Section 2.5.2.2)

|  |  | Ref $^{[52]}$ | SVV-like | steep SVV | sharp SVV |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathcal{M}=1.5$ | $\operatorname{Re}_{\tau}$ | 222.00 | 212.35 | 212.81 | 217.26 |
|  | $\mathcal{M}_{\tau}$ | 0.0820 | 0.0804 | 0.0802 | 0.0814 |
|  | $\epsilon_{\operatorname{Re}_{\tau}}(\%)$ |  | -4.34 | -4.13 | -2.13 |
|  | $\epsilon_{\mathcal{M}_{\tau}}(\%)$ |  | -1.83 | -2.01 | -0.54 |
|  | $\operatorname{Re}_{\tau}$ | 451.00 | 429.94 | 439.13 | 448.14 |
|  | $\mathrm{u}_{\tau}$ | 0.1160 | 0.1140 | 0.1155 | 0.1164 |
|  | $\epsilon_{\operatorname{Re}_{\tau}}(\%)$ |  | -4.66 | -2.63 | -0.63 |
|  | $\epsilon_{\mathcal{M}_{\tau}}(\%)$ |  | -1.29 | -0.25 | -0.51 |

Table 3.6: Mean flow variables and their errors with different SVV kernels for ILES of channel flow at $\mathrm{Re}_{\mathrm{b}}=3000 ; \mathcal{M}=1.5$ and $\mathrm{Re}_{\mathrm{b}}=4880 ; \mathcal{M}=3$. Reference values from Coleman et al. $(1995)^{[52]}$.
was successfully demonstrated. In the following section, the modified scheme will be used to find the grid requirements for ILES of channel flows.

### 3.2 GRID REQUIREMENT FOR WALL-RESOLVED ILES

Generally, the turbulent boundary layer of a wall-bounded flow can be subdivided into two regions, namely near-wall and outer region. The typical features in the near-wall region are the elongated streaks which are approximately 1000 viscous units long and 100 viscous units wide, and the stream-wise vortices which are about 200 viscous units long and 50 viscous units wide. These two features are the necessary ingredients to the self regeneration of turbulence mechanism close to the wall ${ }^{[105,234]}$. In the outer region, the typical feature is the large-scale structure of turbulence which scales in outer lengthscale (H). Mostly, these large-scale structures can be found only for sufficiently high Reynolds number flow.

In the previous section, results from the parametric study of the modified spatial second-derivative FDS were presented and discussed for $\mathrm{Re}_{\tau} \leq 1000$ and $\mathcal{M} \leq 3$. Since no wall-models were used for the ILES, here the objective is to check the grid requirements for the wall-resolved ILES. Hence it is expected that if one fails to discretise any part of the physics then the outcome of the simulation will be erroneous. Various degree of coarsened mesh were tested for simulating a moderately high Reynolds number channel flow. Turbulent statistics up to $4^{\text {th }}$ order moment were evaluated to understand the grid requirements for performing high-order and trustworthy ILES with reasonable computational cost.

### 3.2.1 Simulation parameters

Channel flow simulation for $\operatorname{Re}_{\tau} \approx 640$ was performed at $\mathcal{M}=0.5$ for studying the grid requirements for wall-resolved ILES. Simulation was performed in a computational domain of size $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}=4 \pi \mathrm{H} \times 2 \mathrm{H} \times 4 \pi \mathrm{H}$ with $\mathrm{Re}_{\mathrm{b}}=12214$. Simulation details can be found in table 3.7. Six grids were chosen for the simulation, starting from fine to very coarse grid. Results from the ILES were compared against the DNS of incompressible channel flow by Abe et al. (2001) ${ }^{[2]}$. Grid resolution in the table are scaled with the viscous length-scale of the reference DNS. For all the test-cases $\Delta y_{\text {min }}^{+}=2$ and $\Delta y_{\max }^{+}=14$. Modified spatial second-derivative kernels were adjusted according to the computational grid. Abe et al. (2001) ${ }^{[2]}$ used computational domain, which has dimensions $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}=4 \pi \mathrm{H} \times 2 \mathrm{H} \times 2 \pi \mathrm{H}$. Presently, computational domain which was twice wider than the reference simulation of Abe et al. (2004) ${ }^{[1]}$ was chosen, in order to capture more information about the large-scale features and the scales interaction between the inner- and outer-regions of the turbulent boundary layer. Large-scale features and scales-interaction will be discussed in detail in Chapter 5.

Test-cases in table 3.7 were named after the grid resolution, such as $\Delta x^{+} \times \Delta z^{+}$. For example test-case 20x10 means $\Delta x^{+}=20$ and $\Delta z^{+}=10$ with respect to the reference $\operatorname{Re}_{\tau}$.

| Category | Case | $N_{x} \times N_{y} \times N_{z}$ | $\Delta x^{+}$ | $\Delta z^{+}$ | $\Delta y_{\min }^{+}$ | $\Delta y_{\max }^{+}$ | $\operatorname{Re}_{\tau}$ | $u_{\tau} / u_{b}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\operatorname{Ref}^{[2]}$ | $1024 \times 256 \times 1024$ | 8 | 4 | 0.15 | 8.02 | 640 | 0.0522 |
| fine grid | $20 \times 10$ | $399 \times 205 \times 801$ | 20 | 10 |  |  | 650 | 0.0519 |
| test-cases | $50 \times 20$ | $200 \times 205 \times 400$ | 40 | 20 |  |  | 650 | 0.0516 |
|  | $50 \times 30$ | $161 \times 205 \times 400$ | 50 | 20 | 2 |  | 659 | 0.0523 |
| coarse grid | $70 \times 20$ | $115 \times 205 \times 400$ | 70 | 20 |  |  | 689 | 0.0501 |
| test-cases | 100x100 | $81 \times 205 \times 81$ | 100 | 100 |  |  | 982 | 0.0765 |

Table 3.7: Test-cases for analysing grid requirement for ILES of channel flows. Reference values from Abe et al. (2004) ${ }^{[1]}$.

### 3.2.2 Results

The mean flow variables are also presented in table table 3.7. Except for test-cases $70 \times 20$ and $100 \times 100$ mean flow variables were predicted with error less than $2 \%$. For test-case 70x20(100x100) large over-(under-) prediction of the mean flow variables was observed. Test-cases $70 \times 20$ and $100 \times 100$ were grouped as coarse grid test-cases, because they shared many similar observations. The rest of the test-cases were grouped as fine grid test-cases. The term "fine grid" was chosen just to differentiate the results from the coarse grid testcases. In the following the mean, rms, skewness and kurtosis statistics and correlations and spectra will be used to access the grid requirements for wall-resolved ILES.

### 3.2.2.1 Mean velocity

Mean velocity profiles computed from the ILES using six different computational grids are presented in figure 3.7 (a). In the viscous sub-layer all the test-cases collapsed on the reference data ${ }^{[2]}$. In the buffer-layer and the log-layer except for the coarse grid testcases, satisfactory collapse of the results on the reference data was found for the fine grid test-cases. The fine grid test-cases reproduced the law-of-the-wall with the first point from the wall being located at $y^{+}=2$. Mean velocity profiles of test-case $70 \times 20$ and 100x100 departed from the law-of-the-wall, and had higher values for the log-law intercept. Comparison of rms velocity profiles is presented in figure 3.7 (b, c, d). Similar trend as the mean velocity profile in figure 3.7 (a) was found. Over-prediction of the $\sqrt{\overline{u^{\prime 2}}}$ peak in the buffer-layer, which is a typical characteristic of the coarsened grid was found for the coarse grid test-cases. For the $\sqrt{\overline{u^{\prime 2}}}$, emergence of a weak secondary peak around $y^{+} \approx 200$ can be found. This peak is in the log-region of the turbulent boundary layer and the contribution is coming from the outer-layer large-scale structure ${ }^{[125,126,179]}$. The largescale contributions and the scales-interaction between the inner- and outer-layer will be discussed in detail in Chapter 5. For the fine grid test-cases, overall satisfactory agreement was obtained for $\sqrt{\overline{u^{\prime 2}}}, \sqrt{\overline{v^{\prime 2}}}$ and $\sqrt{\overline{w^{\prime 2}}}$ with the reference data of Abe et al. (2004) ${ }^{[1]}$.


Figure 3.7: Comparison of (a) $\overline{u^{+}}$, (b) $\sqrt{\overline{u^{+\prime 2}}}$, (c) $\sqrt{\overline{v^{+\prime 2}}}$ and (d) $\sqrt{\overline{w^{+\prime 2}}}$ profiles obtained with different computational grids. Reference data from Abe et al. (2004) ${ }^{[1]}$.

### 3.2.2.2 Reynolds stress and rms of vorticity

Figure 3.8 presents the Reynolds stress plotted against wall-normal coordinates scaled with H . Except for the coarse grid test-cases, the results collapsed on the reference curve by Abe et al. (2004) ${ }^{[1]}$. Test-case $70 \times 20$ was slightly over-predicted, probably because information related to the near-wall small-scale features were not captured entirely and this disturbed the near-wall turbulence regeneration mechanism. This will be shown later with the help of other turbulent quantities. Test-case $100 x 100$ was too coarse to discretise scales smaller than 100 viscous units in the stream- and span-wise direction. Recalling that the stream-wise streaks are 1000 viscous units long and 100 viscous units wide; and the stream-wise vortices are about 200 viscous units long and 50 viscous units wide. Therefore these near-wall features were not well discretised with the test-case $100 x 100$. This resulted in the under-prediction of Reynolds stress.


Figure 3.8: Comparison of Reynolds stress with wall-normal coordinate scaled with H obtained with different computational grids. Reference data from Abe et al. (2004) ${ }^{[1]}$.


Figure 3.9: Comparison of (a) $\sqrt{\overline{\omega_{x}^{\prime 2}}}$, (b) $\sqrt{\overline{\omega_{y}^{\prime 2}}}$, (c) $\sqrt{\overline{\omega_{z}^{\prime 2}}}$ and (d) $\sqrt{\overline{\omega_{\text {tot }}^{\prime 2}}}$ profiles obtained with different computational grids. Reference data from Abe et al. (2004) ${ }^{[1]}$.

Figure 3.9 present the rms of vorticity, for the $\sqrt{\overline{\omega_{x}^{\prime 2}}}$ all the fine grid test-cases were slightly under-predicted in the buffer-layer, but they all collapse on each other across the whole boundary layer. Test-cases $70 \times 20$ and $100 x 100$ were more erroneous. For $\sqrt{\overline{\omega_{y}^{\prime 2}}}$, increasing under-prediction with mesh coarsening was observed, which was because relevant information close to the wall were not captured by the simulations. Which also means, by coarsening the grid, somehow the near-wall turbulence regeneration mechanism ${ }^{[136,234]}$ and also the inner-outer layer interaction ${ }^{[125,186,299]}$ was disturbed (by not creating enough vortices). $\sqrt{\overline{\omega_{z}^{\prime 2}}}$ and $\sqrt{\overline{\omega_{\text {tot }}^{\prime 2}}}$ collapsed on the reference curve for all the test-cases except for test-case 100x100. From figure $3.7,3.8$ and table 3.7 it was clear that test-case $70 \times 20$ could not discretise all the flow physics, especially small-scale features close to the wall. Nevertheless, $\sqrt{\overline{\omega_{z}^{\prime 2}}}$ and $\sqrt{\overline{\omega_{\text {tot }}^{\prime 2}}}$ computed from test-case $70 \times 20$ collapsed on the reference data.

### 3.2.2.3 Velocity correlation and spectra

Velocity correlation at $y^{+}=10$ in the stream- and span-wise directions are presented in figure 3.10. For the $u$ component in the stream-wise direction, enhanced correlation was observed for the coarse grid test-cases. For the fine grid test-cases, stream-wise streaks has length $\lambda_{\chi}^{+} \approx 2000$, and in the span-wise direction the streak spacing was $\lambda_{z}^{+} \approx 120$. These observations are in line with the literature. For the test-case $70 \times 20$ streak spacing was $\lambda_{z}^{+} \approx 190$. Even though the streaks were well discretised in stream- and span-wise direction for test-case 70x20, erroneous results were obtained, because as mentioned earlier some relevant near-wall information were not reproduced correctly. Test-case $100 x 100$ was not capable of discretising a near-wall streak. These results are in line with the figure 3.9, where under-prediction of $\sqrt{\overline{\omega^{\prime 2}}}$ was observed for the coarse grid test-cases. From the stream-wise correlation of $v$ component, except for coarse grid test-cases, excellent agreement was found for the fine grid test-cases, and they collapsed very well on the reference data. From the span-wise correlation of $v$ component the diameter of the stream-wise vortices was found to be $\lambda_{z}^{+}=60-80$. Whereas for test-case $50 \times 30$ wider correlation were observed, with $\lambda_{z}^{+}=120$. Once again test-case $70 \times 20$ was resolved enough in the spanwise direction to discretise a stream-wise vortex, but it fails to provide better results. In the span-wise correlation of $w$ component minimum was found at $\lambda_{z}^{+} \approx 60$ except for the coarse grid test-cases. For test-cases $70 \times 20$ and $100 x 100$, the minimum were observed for higher values of $\lambda_{z}^{+}$. The minimum denotes the presence of counter-rotating vortex pair. This can be verified by plotting the correlation of $\omega$, which is shown in figure 3.12.
(a)

(c)

(e)

(b)

(d)

(f)


Figure 3.10: Comparison of correlation of ( $\mathrm{a}, \mathrm{b}$ ) u-component, ( $\mathrm{c}, \mathrm{d}$ ) $v$-component, (e, f) wcomponent in the ( $a, c, e$ ) stream-wise and ( $b, d, f$ ) span-wise direction at $y^{+}=10$ obtained with different computational grids. Reference data from Abe et al. (2004) ${ }^{[1]}$.


Figure 3.11: Comparison of ( $\mathrm{a}, \mathrm{b}$ ) energy spectra and ( $\mathrm{c}, \mathrm{d}$ ) pre-multiplied energy spectra of stream-wise velocity at $y=0.5 \mathrm{H}$ in the ( $a, c$ ) stream-wise and ( $b, d$ ) span-wise direction obtained with different computational grids. Reference data from Abe et al. $(2004)^{[1]}$.

In figure 3.11 the comparison of spectra for the stream-wise velocity at $y=0.5 \mathrm{H}$ is presented. Satisfactory collapse of result was observed for all the cases when the energy spectra was plotted against wave-numbers in the stream- and span-wise directions (figure 3.11 ( a and b )). For the pre-multiplied spectra in stream-wise direction results collapsed excellently for the test-case 20x10. Pre-multiplied spectra allows to identify the energy content on a specific wave number range. Slight over-prediction for the intermediate scales was observed for the test-cases $40 \times 20,50 \times 20$ and $50 \times 30$. The pre-multiplied spectra for the test-cases $70 \times 20$ and 100x100 were erroneous. Peak value of the pre-multiplied energy spectra in stream-wise direction was at $\lambda_{x}=3-4 \mathrm{H}$. The wave-length corresponding to the maxima of the pre-multiplied spectra is the average size of the structure which carries most of the energy. The location of maxima of the pre-multiplied spectra scales with the outer length-scale $H$. The peak at the higher values of $\lambda$ is the proof for the presence of large-scale structures away from the wall. All test-cases except the coarse grid test-cases,
predicted the peak value of pre-multiplied stream-wise spectra and the corresponding scale accurately. For the pre-multiplied spectra in span-wise direction excellent agreement was observed for the test-cases 20xio and 40x20. Slight over-(under-) prediction was found for the test-case $50 \times 20$ ( $50 \times 30$ ). Location of the peak value was at $\lambda_{z}=1.3 \mathrm{H}$, and the test-cases except 70x20 predicted the location accurately, with discrepancy in the peak value. For the reference simulation, Abe et al. (2004) ${ }^{[1]}$ also found the peak at same value of $\lambda_{z}$. Erroneous span-wise pre-multiplied spectra was found for test-cases $70 \times 20$ and $100 x 100$.

### 3.2.2.4 Vorticity correlation

In figure 3.12 correlation of $\omega_{x}, \omega_{y}$ and $\omega_{z}$ at $y^{+} \approx 10$ in the stream- and span-wise directions are presented. From all the previous figures it was clear that test-cases 20xio and $40 \times 20$ were very accurate (had excellent comparison with the reference DNS of [1]) and since no reference data was available for the following quantities test-case $20 x 10$ was considered as the reference. In the stream-wise direction $\omega_{x}$ correlation goes to zero around $\Delta x^{+} \approx 100$, except for the coarse grid test-cases, which is the typical size of the stream-wise vortices. For the test-case 70x20, stream-wise correlation of $\omega_{x}$ goes to zero by around $\Delta x^{+} \approx 1000$ (not shown here), which means the stream-wise vortices were not discretised. Instead test-case 70x20 generated some ambiguous flow physics (near-wall turbulence regeneration) which was not representative of the real wall-turbulence. Similarly, the correlation of $\omega_{x}$ in the span-wise direction was increasingly erroneous with mesh coarsening. For the correlation of $\omega_{y}$, increasing discrepancy with mesh coarsening was found in the span-wise direction, whereas in the stream-wise direction correlation goes to zero by around $\Delta x^{+} \approx 700$. Span-wise vortices has the size $\lambda_{x}^{+} \approx 400$ and $\lambda_{z}^{+} \approx 80-120$ (distance between zero-crossing on either side of the ordinate). In the stream-wise direction $\omega_{z}$ was correlated up to $\Delta x^{+} \approx 2000$ for the coarse grid test-cases.
(a)

(c)

(e)

(b)

(d)

(f)


Figure 3.12: Comparison of $(a, b) \omega_{x},(c, d) \omega_{y},(e, f) \omega_{z}$ correlation in the ( $a, c, e$ ) stream-wise and ( $b, \mathrm{~d}, \mathrm{f}$ ) in span-wise direction at $\mathrm{y}^{+}=10$ obtained from different computational grids. Test-case $20 x 10$ is the reference.

### 3.2.2.5 Velocity skewness and kurtosis

In figure 3.13 the skewness and kurtosis for velocity components are presented. Skewness is the measure of lack of symmetry in a probability distribution, whereas kurtosis is the measure of tailedness of a probability distribution or chances for extreme events. For any quantity " $a$ " skewness and kurtosis can be defined as:

$$
\begin{align*}
& \mathcal{S}_{a}=\frac{\overline{a^{\prime 3}}}{\sqrt{{\overline{a^{\prime 2}}}^{3}}}  \tag{3.1}\\
& \mathcal{F}_{a}=\frac{\overline{a^{\prime 4}}}{\sqrt{{\sqrt{a^{\prime 2}}}^{4}}} \tag{3.2}
\end{align*}
$$

These quantities help to understand the shape of the probability distribution. For a normal distribution skewness $\mathcal{S}=0$ and kurtosis $\mathcal{F}=3$. Skewness is the third moment so it can be either positive or negative, whereas kurtosis is always positive. Negative skewness means the left tail of the probability distribution is longer, and vice-versa for the positive skewness. $\mathcal{F}>3$ means the distribution produces more extreme events than the normal distribution, and vice-versa for $\mathcal{F}<3$.

Usually kurtosis is presented as $\mathcal{F}-3$ in order to compare with the normal distribution. Since there is no reference curve from the DNS of Abe et al. (2004) ${ }^{[1]}$, test-case $20 x 10$ was taken as the reference. Skewness of $u$ velocity component for all the test-cases collapsed on each other except test-case 100xioo. In the inner-layer $\mathcal{S}>0$, and above buffer-layer $\mathcal{S}<0$. Skewness for test-cases 40x20, 50×20, 50x30 were slightly over predicted compared to test-case $20 x 10$ across the boundary layer. For the kurtosis of $u$ component, the coarse grid test-cases were slightly under-predicted close to the wall, and completely erroneous for the test-case 100xioo. Skewness of the $v$ component had similar trend as the $u$ component. Test-cases $40 \times 20,50 \times 20,50 \times 30$ collapsed on each other across the boundary layer, with slight discrepancy in the log region when compared to test-case 20x10. Skewness of $v$ component computed from coarse grid test-cases, were over-predicted compared to the fine grid test-cases. For kurtosis of $v$ increasing discrepancy was found close to the wall with increasing mesh coarsening except for the coarse grid test-cases. Xu et al. (1996) ${ }^{[296]}$ reported higher kurtosis for wall-normal velocity component is due to events that are very rare in space and time. Hence, with coarsened mesh one can expect to have decreased kurtosis close to the wall, but test-cases 70x20 and 100x100 produced better kurtosis for the $v$ component, it is erroneous and the reason for such better estimates are unknown. For the kurtosis of $w$ component slight under-prediction with mesh coarsening was observed close to the wall. Test-cases 70x20 and 100x100 were erroneous.
(a)

(c)

(e)

(b)

(d)

(f)


Figure 3.13: Comparison of ( $a, c, e$ ) skewness and (b, d, f) kurtosis for ( $a, b$ ) stream-wise, ( $c, d$ ) wall-normal and (e,f) span-wise velocity component obtained from different computational grids. Test-case $20 \times 10$ is the reference.

### 3.2.3 Conclusion

Hence by analysing the correlation of velocity components and vorticity, it was clear that the coarse grid test-cases, $70 \times 20$ and 100x100 produced erroneous results. Since wallresolved ILES was performed here, these coarse grids failed to capture the near-wall dynamics (streaks and vortices), which has important role in the near-wall turbulence regeneration cycle and also in the inner-outer layer interaction. From the analysis, it was found that test-case $50 \times 30$ was the limiting resolution for performing ILES, such that one can capture the near-wall turbulence and its dynamics sufficiently well with reduced computational cost. For a fast computation one could choose a coarser mesh to approximately compute the first- and second-order moments. For a clean ILES, which can be compared against DNS, it is recommended to use grid resolution as good as test-case 40x20. The capability of the modified spatial second-derivative FDS to perform ILES (of a channel flow with Mach number up to 3 and friction Reynolds number up to 1000), which was accurate up to $4^{\text {th }}$ order moments with extremely reduced computational cost was successfully demonstrated.

SCALING TECHNIQUES FOR COMPRESSIBLE TURBULENT CHANNEL FLOW AT MACH NUMBER UP TO 3

In Chapter 2 and 3 examples of channel flow at Mach numbers up to 3 with isothermal walls were presented. The turbulent mean and rms velocity profiles for these channel flows were different from the traditional (incompressible) profiles (Section 5.3.1 and 3.1.3). The difference in the turbulent statistics arises mainly due to the flow Mach number and heat-transfer through the isothermal walls. Isothermal or cooled walls modify the wall-normal gradients of mean density and temperature, unlike the incompressible flows where density is considered to be constant.

In this chapter, scaling techniques for compressible channel flows (for $\mathcal{M} \leq 3$ ) with isothermal walls will be discussed and analysed. Compressible scaling techniques are very important because it helps non-dimensionalise and compare results for different conditions which can be used for making crucial connections between the incompressible and compressible wall-bounded turbulent flows. The objective was first to test and verify the scaling techniques in-order to use it later in the algorithm for detecting the largescale structures of the wall-bounded turbulent flow in Chapter 5. Literature review of the popular and the most recent advancement of the scaling technique is presented and discussed in detail. ILES of channel flows with isothermal walls at different $\mathcal{M}$ number were performed for checking the adequacy of scaling techniques. Results from the ILES were compared against the DNS of incompressible channel flow by Moser et al. (1999) ${ }^{[203]}$ at $\mathrm{Re}_{\tau} \approx 390$.

### 4.1 REVIEW OF SCALING TECHNIQUES

In the following, different scaling or non-dimensionalisation techniques will be presented. First of all, the conventional technique used for incompressible flows will be presented. It will serve as the basis for the scaling techniques for compressible flows. Following the incompressible scaling, compressibility effects for non-hypersonic flows will be discussed before presenting different scaling techniques for compressible flows.

### 4.1.1 Incompressible transformation

The important feature of the wall-bounded flow is the universality of the inner-layer of velocity profile when scaled with inner variables. This leads to the existence of law-of-thewall for the velocity profile ${ }^{[225,226,285]}$. Law-of-the-wall is well-established in the inner-layer
with small uncertainties ${ }^{[11,179]}$. For an incompressible, constant property flow, law-of-thewall is:

$$
\begin{align*}
& \frac{\overline{\bar{u}}}{\widetilde{\mathfrak{u}}_{\tau}}=f\left(\frac{\widetilde{\tilde{y}}}{\widetilde{\mathfrak{u}}_{\nu}}\right) \\
& \overline{\mathrm{u}}^{+}=\mathrm{f}\left(\mathrm{y}^{+}\right) \tag{4.1}
\end{align*}
$$

Like Chapter 2, accent $\widetilde{(\cdot)}$ is used for the dimensional quantities. In the above relation f is the universal function, $y$ is the wall-normal coordinate, $\widetilde{l}_{v}=\widetilde{\mu}_{w} / \widetilde{\rho}_{w} \widetilde{u}_{\tau}$ is the viscous length scale, $\bar{u}$ is the mean velocity and $\widetilde{\mathfrak{u}}_{\tau}=\sqrt{\widetilde{\tau}_{w} / \widetilde{\rho}_{w}}$ is the friction velocity. $\rho_{w}$ is the wall density, $\mu_{w}$ is the wall viscosity. $u_{\tau}$ and $l_{v}$ are the traditional inner variables used for scaling the statistics of incompressible turbulent wall-bounded flows. Quantities scaled with the inner variables are denoted with a " + " superscript.

Similarly, for an incompressible constant property flow, in a given geometry, it is known that there is no universal scaling for Reynolds stresses ${ }^{[11,117,179,200] \text {. The peak amplitude }}$ of the stream- and span-wise velocity fluctuations increases logarithmically with $\mathrm{Re}_{\tau}$, while the wall-normal velocity fluctuations increases sub-logarithmically ${ }^{[16]}$, thus one can write:

$$
\begin{align*}
& \frac{\overline{\tilde{u}_{i}^{\prime} \widetilde{u}_{j}^{\prime}}}{\widetilde{u}_{\tau}^{2}}=f\left(\frac{\widetilde{y}}{{\widetilde{\imath_{v}}}_{v}}, \operatorname{Re}_{\tau}\right) \\
& \overline{u_{i}^{\prime} u_{j}^{\prime}}{ }^{+}=f\left(y^{+}, \operatorname{Re}_{\tau}\right) \tag{4.2}
\end{align*}
$$

### 4.1.2 Compressibility effects

Compressible channel with isothermal wall gives rise to a flow which is strongly influenced by the wall-normal gradients of $\bar{\rho}$ and $\overline{\mathrm{T}}$ compared to the incompressible flow. Non-uniform mean density and temperature are due to the viscous heating. Therefore, the scaling laws (equation 4.1 and 4.2 ) mentioned above are no longer valid. Morkovin's hypothesis is valid up to $\mathcal{M}=5$, and statistical properties of turbulence are unaffected by compressibility if $\sqrt{\overline{\rho^{\prime 2}} / \bar{\rho}}<\mathcal{O}(1 / 10)^{[34,52,168]}$.

MORKOVIN'S HYPOTHESIS: In non-hypersonic boundary layers, the acoustic mode is negligible and the entropic mode is small. The large-scale motion should be statistically coupled to the thermal field almost exclusively through the mean values of $\rho$, $\mu, \lambda$, and generalized law of the wall so that with a variable lateral stretching factor, it may resemble the incompressible motion ${ }^{[199]}$.

The straightforward solution to this problem is to extend the law-of-the-wall to compressible flows ${ }^{[38,53,77,81,237,260,282,283,293,300]}$, with most of the research dedicated to transform the profiles of compressible flows to equivalent profiles of incompressible flows. Transformed mean velocity profile should satisfy law-of-the-wall, which generally involves scaling the velocity and wall-normal coordinates in equation 4.1.

For compressible flows with isothermal walls due to the heat transfer through the walls, the law-of-the-wall becomes ${ }^{[34]}$ :

$$
\begin{equation*}
\overline{\mathrm{u}}^{*}=f\left(\widetilde{\mathrm{y}}, \mathrm{~B}_{\mathrm{q}}, \mathcal{M}_{\tau}\right) \tag{4.3}
\end{equation*}
$$

where $\bar{u}^{*}$ is the transformed mean velocity, $\mathcal{M}_{\tau}=\widetilde{\mathfrak{u}}_{\tau} / \widetilde{\mathfrak{c}}_{w}$ is the friction Mach number and $\mathrm{B}_{\mathrm{q}}$ is the dimensionless heat flux at the isothermal or cold wall (see equation 2.26 in Chapter 2). Generally the $B_{q}$ and $\mathcal{M}_{\tau}$ increases with the Mach number. From the physical point of view the above relation is valid, but it is not straightforward to formulate a compressible transformation using this relation. Thus, equation 4.1 can be modified for the compressible flows together with considering the Morkovin's hypothesis as:

$$
\begin{align*}
\bar{u}^{*} & =f\left(\frac{\widetilde{\tilde{y}}}{\widetilde{\mathrm{~L}}_{v}}, \overline{\widetilde{\rho}(y)}, \widetilde{\widetilde{\mu}(y)}\right) \\
& =f\left(y^{*}\right) \tag{4.4}
\end{align*}
$$

where superscript "*" is used for the transformed mean velocity profile, for which the law-of-the-wall is valid. Equations 4.3 and 4.4 are both valid for compressible flows with isothermal wall. But the later is more suitable from the compressible transformation point of view. The transformed mean velocity profile is written as a function of wallnormal coordinate, viscous length-scale, mean density and viscosity (or temperature). The r.h.s. of the above relation is then finally written using a the modified wall-normal coordinates. With such way of writing, equations 4.1 and 4.4 have the same structure, where the former is for incompressible and the latter is for compressible flows. Similarly, for turbulent quantities one can write:

$$
\begin{align*}
{\overline{\widetilde{\mathfrak{u}}_{i}^{\prime} \widetilde{u}_{j}^{\prime}}}^{*} & =f\left(\frac{\widetilde{\tilde{u}}}{\widetilde{\mathrm{~L}}_{v}}, \operatorname{Re}_{\tau}, \overline{\tilde{\mathrm{p}}}, \overline{\widetilde{\mu}}\right) \\
& =f\left(y^{*}, \operatorname{Re}_{\tau}^{*}\right) \tag{4.5}
\end{align*}
$$

here $\mathrm{Re}_{\tau}^{*}$ is the modified or equivalent friction Reynolds number. For the two above mentioned relations to work as expected, the wall-normal coordinate, mean and turbulent statistics have to be scaled appropriately. In the following sections different scaling techniques will be introduced, and finally in Section 4.2 the variables used for scaling the results of the compressible channel flows will be summarised.

### 4.1.3 Van Driest transformation

The classical way of transforming the mean velocity profile of a compressible wallbounded flow is to replace the scaled velocity in equation 4.1 with Van Driest transformed velocity. But the original transformation proposed by Van Driest (1951) ${ }^{[283]}$ is an analytic trigonometric relation and it writes as follows (see equation 6.35 in Gatski and Bonnet (2013) ${ }^{[89]}$ ):

$$
\begin{array}{r}
\overline{\mathrm{u}}_{V D}^{*}=\sqrt{\mathrm{b}}\left[\sin ^{-1}\left(\frac{\mathrm{a}_{w}+\overline{\mathrm{u}}}{\sqrt{\mathrm{a}_{w}^{2}+\mathrm{b}}}\right)-\sin ^{-1}\left(\frac{\mathrm{a}_{w}}{\sqrt{\mathrm{a}_{w}^{2}+\mathrm{b}}}\right)\right] \\
\mathrm{a}_{w}=\operatorname{Pr}_{\mathrm{t}} \mathrm{q}_{w} / \tau_{w} ; \mathrm{b}=2 \mathrm{~h}_{w} / \operatorname{Pr}_{\mathrm{t}} \tag{4.6}
\end{array}
$$

The above mean velocity transformation is in line with equation 4.3 , but the popular form of Van Driest transformation is in the integral form. From the similarity argument for inner-layer the integral form of the Van Driest transformation can be found as:

$$
\begin{equation*}
\overline{\mathrm{u}}_{V D}^{*}=\int_{0}^{\bar{u}^{+}} \sqrt{\frac{\overline{\widetilde{\rho}}}{\frac{\tilde{\rho}_{w}}{}}} d \overline{\mathrm{u}}^{+} \tag{4.7}
\end{equation*}
$$

Hence, the transformation involves correcting the velocity gradient by the factor $\sqrt{\bar{\rho} / \rho_{w}}$ (to account for the compressibility effect) to obtain the Van Driest transformed mean velocity. Equation 4.7 partially has the form similar to equation 4.4 , in the r.h.s. $y^{+}$is used instead of $y^{*}$. The transformed velocity is used with the wall-normal coordinate scaled with viscous length-scale. The exact origin of the integral form of the Van Driest transformation is unknown, but it can be traced back to Dorrance (1962) ${ }^{[72]}$ and Danberg $(1964)^{[60]}$.

Derivation for the integral form of Van Driest transformation can be put down as follows. In the equilibrium part of the inner layer, where the turbulent motion concerned with turbulent energy production and dissipation is determined by the shear stress within that region, and independent of conditions outside, one can relate the velocity gradient and the Reynolds stress as ${ }^{[280]}$ :

$$
\begin{equation*}
\overline{\bar{\rho}} \overline{\tilde{u}^{\prime} \bar{v}^{\prime}}=\widetilde{\tau}_{w}=\overline{\widetilde{\mu}}_{t} \frac{\partial \overline{\widetilde{u}}}{\partial \widetilde{\mathrm{y}}} \tag{4.8}
\end{equation*}
$$

where $\mu_{\mathrm{t}}$ is the turbulent eddy viscosity, Reynolds stress is assumed constant and equal to $\tau_{w}$ (see Gatski and Bonnet (2013) ${ }^{[89]}$ ). With the Boussinesq assumption ${ }^{[29-31,246]}$, using mixing-length hypothesis ${ }^{[225]}$, turbulent eddy viscosity can be defined as:

$$
\begin{equation*}
\overline{\widetilde{\mu}}_{\mathrm{t}}=\overline{\tilde{\rho}} \widetilde{l}^{2} \frac{\partial \overline{\widetilde{u}}}{\partial \widetilde{\mathrm{y}}} \tag{4.9}
\end{equation*}
$$

where mixing-length is inferred as a function of the position in the flow, and is defined as $\widetilde{I}=\kappa \widetilde{y}$, and $\kappa$ is a constant. Substituting $\mu_{\mathrm{t}}$ into equation 4.8 and further developing, one finds:

$$
\begin{equation*}
\sqrt{\frac{\overline{\bar{\rho}}}{\tilde{\rho}_{w}}} \frac{\partial \overline{\widetilde{u}}}{\partial \widetilde{\mathrm{y}}}=\frac{\widetilde{\mathrm{u}}_{\tau}}{\kappa \widetilde{\mathrm{y}}} \Longrightarrow \quad \overline{\mathrm{u}}_{V D}^{*}=\frac{1}{\mathrm{~K}} \ln \mathrm{y}^{+}+\mathrm{C} \tag{4.10}
\end{equation*}
$$

integrating the above equation yields the Van Driest transformed velocity in the integral form as in equation 4.7. Van Driest transformed velocity is so successful due to its accuracy in turbulent boundary layers over adiabatic walls, up to $\mathcal{M}=20^{[74,76,222]}$.

For compressible flows with adiabatic walls, one obtains approximately the universal log-law, which implies the von Kármán constant $\kappa$ and the log-law intercept $C$ are constant. Whereas for the Van Driest transformed mean velocity profile of compressible flows with isothermal walls, an upward shift in the log-law intercept C, outward shift of the wall-normal coordinate and drop in the viscous sub-layer slope $S$ are observed ${ }^{[282]}$. Thus for the compressible flows with isothermal walls, k and C depends on the heattransfer through the walls. Disagreement of Van Driest transformed velocity profiles increases as the wall becomes increasingly non-adiabatic (or for increasing flow Mach number). Figure 4.1 presents the comparison between the Van Driest transformed velocity profile at $\mathcal{M}=3$ for a channel with cold walls (test-case $\mathrm{M}_{3} \cdot 0^{*}, \mathrm{Re}_{\tau}=1085$ in table 4.2) and the incompressible channel flow of Lee and Moser (2015) ${ }^{[166]}$ at $\operatorname{Re}_{\tau} \approx 1000$. Above mentioned differences in the mean velocity profile can be found with the Van Driest transformation.


Figure 4.1: Mean velocity profiles for incompressible and compressible channel with cold walls.
Gray solid lines used to represent $\bar{u}^{+}=y^{+}$and $\bar{u}^{+}=1 / \kappa \ln y^{+}+C$. Arrows are used to show the changes for compressible channel flows with cold walls. Reference data is for $\operatorname{Re}_{\tau}=1000$ taken from Lee and Moser (2015) ${ }^{[166]}$.

### 4.1.4 Semi-local transformation

For the turbulent quantities, Huang et al. (1995) ${ }^{[120]}$ proposed the semi-local scaling (which has the form of equation 4.5 ), where basically the turbulent quantities and the wall-normal coordinate were scaled using the local viscous velocity and length-scale. These local scales were defined using the local density and viscosity. The local viscous velocity and length-scale are:

$$
\begin{align*}
& \widetilde{u}_{\tau}^{*}(\widetilde{y})=\sqrt{\frac{\widetilde{\tau}_{w}}{\overline{\widetilde{\rho}(\widetilde{y})}}}  \tag{4.11}\\
& {\widetilde{\tau_{v}^{*}}}_{v}^{*}(\widetilde{y})=\frac{\widetilde{\widetilde{\mu}(\widetilde{y})}}{\widetilde{\widetilde{\rho}(\widetilde{y}) \widetilde{u}_{\tau}^{*}(\widetilde{y})}} \tag{4.12}
\end{align*}
$$

The semi-local scaling technique works very well for the turbulent quantities (such as rms of velocity and vorticity), and one can collapse the compressible curves over the incompressible data. Examples can be found in Huang et al. (1995) ${ }^{[120]}$, Coleman et al. (1995) ${ }^{[52]}$, Morinishi et al. (2004) $)^{[198]}$ and Modesti and Pirozzoli (2016) ${ }^{[194]}$, among others. The semi-local transformation has also been successfully used for variable density flows in Patel et al. $(2015)^{[219]}$.

An example for dependence of $l_{v}^{*}$ on wall-normal distance is shown in figure 4.2. Local friction length-scale is for test-case at $\mathcal{M}=3$ from table 5.1. At higher $\mathcal{M}$, conventional friction Reynolds number $\operatorname{Re}_{\tau}$ increases because $l_{\nu}^{*}$ at the wall decreases. For example in the figure, $l_{\nu}^{*}$ at the wall is twice smaller than the value at the channel centre.


Figure 4.2: Local friction length scale as a function of wall-distance.

### 4.1.5 Trettel's transformation

As mentioned in Section 4.1.3, Van Driest transformation only scales the mean velocity profile and leaves the wall-normal coordinate $\left(y^{+}\right)$untouched. Trettel and Larsson $(2016)^{[282]}$ proposed a very efficient transformation technique for the mean velocity profile. This scaling was used in the present work, because it overcomes the drawbacks of Van Driest transformation (violation of stress balance between the initial and transformed data).

Scaling proposed by Trettel and Larsson (2016) ${ }^{[282]}$ uses a differential approach like most of the other scaling techniques, but it tries to relate the transformation through velocity transformation kernel, coordinate transformation kernel and raw velocity gradient. Unlike other techniques, coordinate transformation is left unknown, and is derived in a later stage together with velocity and Reynolds stress transformation using log-law and stress-balance condition. With this transformation, one will recover the semi-local scaling. Hence, with this technique an unified coordinate transformation can collapse both the mean and turbulent quantities.

Their transformation assumes $\tau_{w}, \rho_{w}$ and $\mu_{w}$ stays constant for the raw and transformed states. Therefore, $u_{\tau}$ and $l_{v}$ are also same for both states. Transformed velocity gradient is multiplied and divided by both $d u$ and $d y$, upon rearrangement one will find:

$$
\begin{equation*}
\frac{d \overline{\widetilde{u}}^{*}}{d \widetilde{\widetilde{y}}^{*}}=\frac{d \overline{\widetilde{u}}^{*}}{d \overline{\bar{u}}} \frac{d \widetilde{\mathrm{y}}}{d \widetilde{\mathrm{y}}^{*}} \frac{d \overline{\tilde{u}}}{d \widetilde{\mathrm{y}}} \tag{4.13}
\end{equation*}
$$

The transformed velocity gradient is written as a function of velocity transformation kernel, coordinate transformation kernel and raw velocity gradient. Superscript "*" is used for the transformed quantities. As a first step, log-law condition is derived in the form of a velocity gradient for raw and transformed states from the dimensional analysis ${ }^{[35]}$ and upon simplification one will find:

$$
\begin{equation*}
\frac{d \overline{\widetilde{u}}^{*}}{d \widetilde{\mathrm{y}}^{*}}=\frac{\widetilde{\mathrm{y}}}{{\widetilde{\tilde{y}^{*}}}^{\frac{\overline{\widetilde{\rho}}}{\tilde{\tilde{\rho}}_{w}}} \frac{\partial \overline{\overline{\mathrm{u}}}}{\partial \widetilde{\mathrm{y}}}} \tag{4.14}
\end{equation*}
$$

Secondly, unlike the Van Driest transformation a stress balance condition between the raw and transformed states is derived (assuming $\tau_{w}$ is same for both states).

By assuming Morkovin's scaling ${ }^{[199]}$ is true, the turbulent shear stress is constant. In nondimensional form one will find the semi-local scaling of Huang et al. (1995) ${ }^{[120]}$ for the Reynolds stress:

$$
{\overline{u_{i}^{\prime} u_{j}^{\prime}}}^{*}=\frac{\overline{\tilde{\rho}} \overline{\widetilde{u}_{\mathfrak{u}}^{\prime} \widetilde{u}_{j}^{\prime}}}{\overline{\bar{\rho}}_{w} \widetilde{u}_{\tau}^{2}}
$$

The dimensional version of the above relation $\left(\widetilde{\rho}_{w}{\overline{\widetilde{u}}, \widetilde{u}_{j}^{\prime}}^{*}=\overline{\widetilde{\rho}} \overline{\widetilde{u}}_{i}^{\prime} \widetilde{\mathfrak{u}}_{j}^{\prime}\right)$ can be used to simplify equation 4.15, such that:

$$
\begin{equation*}
\widetilde{\mu}_{w} \frac{d \overline{\widetilde{u}}^{*}}{d \widetilde{\mathrm{y}}^{*}}=\overline{\widetilde{\mu}} \frac{\partial \overline{\widetilde{\mathrm{u}}}}{\partial \widetilde{\mathrm{y}}} \Longrightarrow \frac{d \overline{\widetilde{u}}^{*}}{d \widetilde{\mathrm{u}}^{*}}=\frac{\overline{\widetilde{\mu}}}{\widetilde{\mu}_{w}} \frac{\partial \overline{\widetilde{u}}}{\partial \widetilde{\mathrm{u}}} \tag{4.16}
\end{equation*}
$$

By equating the log-law condition and stress-balance condition (equation 4.14 and 4.16), one can derive the transformed wall-normal coordinate in dimensional form:

$$
\begin{equation*}
\widetilde{y}^{*}=\widetilde{y} \frac{\widetilde{\mu}_{w}}{\overline{\widetilde{\mu}}} \sqrt{\frac{\overline{\bar{\rho}}}{\widetilde{\rho}_{w}}} \tag{4.17}
\end{equation*}
$$

In non-dimensional form the wall-normal coordinate will be:

$$
y^{*}=\frac{\widetilde{y} \overline{\widetilde{\rho}(y)} \sqrt{\widetilde{\tau}_{w} / \overline{\widetilde{\rho}}(y)}}{\overline{\widetilde{\mu}(y)}}
$$

By differentiating equation 4.17 with respect to $\widetilde{y}$, and substituting in equation 4.14 or 4.16 and then integrating the resulting relation, the transformed velocity proposed by Trettel and Larsson (2016) ${ }^{[282]}$ can be derived:

$$
\bar{u}^{*}=\int_{0}^{\overline{u^{+}}} \sqrt{\frac{\overline{\widetilde{\rho}}}{\widetilde{\widetilde{\rho}}_{w}}}\left[1+0.5 \frac{1}{\overline{\bar{\rho}}} \frac{d \overline{\widetilde{\rho}}}{d \widetilde{y}} \widetilde{y}-\frac{1}{\overline{\tilde{\mu}}} \frac{d \overline{\widetilde{\mu}}}{d \widetilde{y}} \widetilde{y}\right] d u^{+}
$$

Scaling techniques relevant to this work were only discussed, but there are also other transformation techniques which were not presented here. A comparative study of the different scaling techniques can be found in Modesti and Pirozzoli (2016) ${ }^{[194]}$.

### 4.2 NUMERICAL TEST

In this section, numerical simulation of channel flows at different Mach numbers will be performed to test the scaling techniques. In summary, the semi-locally scaled wall-normal coordinate works for all quantities unlike other transformation techniques, where each quantity works well with some specific coordinate transformation. For example, Van Driest transformed velocity should be used with wall-normal coordinate scaled with viscous length-scale, whereas Reynolds stress should be used with wall-normal coordinate scaled with semi-local variables to collapse the curves on the incompressible data. The complete transformation which will be tested in this section is summarised below:

- At the centre of the channel, the modified or equivalent friction Reynolds number (in equation 4.5 ) is defined as:

$$
\begin{equation*}
\operatorname{Re}_{\tau}^{*}=\frac{\tilde{\mu} \overline{\widetilde{\rho}}_{c} \sqrt{\widetilde{\tau}_{w} / \overline{\widetilde{\rho}}_{c}}}{\overline{\widetilde{\mu}}_{c}} \tag{4.18}
\end{equation*}
$$

subscript c is used for quantities at the centre of the channel, $\mathrm{y}=\mathrm{H}$.

- Transformed coordinate (used in equation 4.4 and 4.5 ) can be defined as follows:

$$
\begin{equation*}
y^{*}=\frac{\widetilde{y} \overline{\widetilde{\rho}}(y)}{\sqrt{\widetilde{\tau}_{w} / \sqrt{\widetilde{\rho}(y)}}} \underset{\widetilde{\widetilde{\mu}(y)}}{ } \tag{4.19}
\end{equation*}
$$

- transformed mean velocity (in equation 4.4):

$$
\begin{equation*}
\bar{u}^{*}=\int_{0}^{\bar{u}^{+}} \sqrt{\frac{\overline{\widetilde{\rho}}}{\widetilde{\rho}_{w}}}\left[1+0.5 \frac{1}{\overline{\tilde{\rho}}} \frac{d \overline{\widetilde{\rho}}}{d \widetilde{y}} \widetilde{y}-\frac{1}{\overline{\tilde{\mu}}} \frac{d \overline{\widetilde{\mu}}}{d \widetilde{\tilde{y}}} \widetilde{\widetilde{y}}\right] d u^{+} \tag{4.20}
\end{equation*}
$$

- transformed Reynolds stress (in equation 4.5):

$$
\begin{equation*}
\overline{u_{i}^{\prime} u_{j}^{\prime}}{ }^{*}=\frac{\overline{\tilde{\rho}} \overline{\widetilde{u}_{i}^{\prime} \widetilde{u}_{j}^{\prime}}}{\widetilde{\tau}_{w}} \tag{4.21}
\end{equation*}
$$

### 4.2.1 Simulation parameters

Nine test-cases which were analysed in the present study $(0.1 \leq \mathcal{M} \leq 3)$ are listed in table 4.1. Simulations were performed in a computational domain of size $L_{x} \times L_{y} \times L_{z}=$ $2 \pi \mathrm{H} \times 2 \mathrm{H} \times \pi \mathrm{H}$, except for test-case Mo. 5 which uses larger domain, $12 \mathrm{H} \times 2 \mathrm{H} \times 6 \mathrm{H}$. One test-case (Mo.1DNS) was a DNS and others were ILES. For all the test-cases $1 \leq \Delta y^{*} \leq$ 10. Simulations were performed with constant $\mathrm{Re}_{\mathrm{b}}=6882$ (test-cases Mo.1DNS, Mo.1, Mo.3, Mo.5, Mo.7, M1.5 and M3.0) for which $\mathrm{Re}_{\tau} \approx 390$ in the incompressible regime. For a given $\mathrm{Re}_{\mathrm{b}}$ at supersonic $\mathcal{M}$ number, the equivalent friction Reynolds number $\mathrm{Re}_{\tau}^{*}$ decreases (see table 4.2). Hence, two additional simulations were performed at $\mathcal{M}=1.5$ and 3 with higher $\mathrm{Re}_{\mathrm{b}}$. One test-case each at $\mathcal{M}=1.5\left(\mathrm{M}_{1} .5^{*}\right)$ and $\mathcal{M}=3\left(\mathrm{M}_{3} . \mathrm{o}^{*}\right)$ were performed such that $\mathrm{Re}_{\tau}^{*} \approx 390$ for test-cases Mo.1DNS, Mo.1, Mo.3, Mo.5, Mo.7, M1.5* and $\mathrm{M}_{3} \cdot 0^{*}$. The receipe to find the higher $\mathrm{Re}_{\mathrm{b}}$ for $\mathrm{Re}_{\tau}^{*}=$ constant is detailed in Appendix D. The objective here is to verify the scaling techniques for compressible flows up to $\mathcal{M}=3$. Turbulent statistics up to $4^{\text {th }}$ order moment were computed and compared for different $\mathcal{M}$ number. Results for the present test-cases were compared with the DNS of Moser et al. (1999) ${ }^{[203]}$ for $\operatorname{Re}_{\tau} \approx 390$.

The non-dimensional skin-friction coefficient is also tabulated in the table, and it is defined as:

$$
\begin{equation*}
c_{f}=\frac{2 \widetilde{\tau}_{w}}{\widetilde{\rho}_{b} \widetilde{u}_{b}^{2}} \tag{4.22}
\end{equation*}
$$

| Case | $\mathcal{M}$ | $\mathrm{Re}_{\mathrm{b}}$ | $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}$ | $N_{x} \times N_{y} \times N_{z}$ | $\Delta x^{+}$ | $\Delta y_{\text {min }}^{+}$ | $\Delta y_{\text {max }}^{+}$ | $\Delta z^{+}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ref ${ }^{[203]}$ | - | $6882$ | $2 \pi \mathrm{H} \times 2 \mathrm{H} \times \pi \mathrm{H}$ | $256 \times 193 \times 192$ | 10 | 0.029 | 6.5 | 6.5 |
| Mo.1DNS | 0.1 |  |  | $251 \times 157 \times 251$ | 10 | 1 | 10 | 5 |
| Mo.1 | 0.1 |  |  | $125 \times 157 \times 125$ | 20 | 1 | 10 | 10 |
| Mo. 3 | 0.3 |  |  | $101 \times 251 \times 125$ | 24 | 1 | 10 | 10 |
| Mo. 5 | 0.5 |  | $12 \mathrm{H} \times 2 \mathrm{H} \times 6 \mathrm{H}$ | $276 \times 201 \times 276$ | 17 | 1 | 15 | 8.5 |
| Mo. 7 | 0.7 |  | $2 \pi \mathrm{H} \times 2 \mathrm{H} \times \pi \mathrm{H}$ | $85 \times 201 \times 123$ | 30 | 1 | 15 | 10 |
| M1. 5 | 1.5 |  |  | $125 \times 157 \times 125$ | 20 | 1 | 10 | 10 |
| M3.0 | 3.0 |  |  | $125 \times 157 \times 125$ | 20 | 1 | 10 | 10 |
| M1.5* | 1.5 | 8339 |  | $125 \times 157 \times 125$ | 20 | 1 | 10 | 10 |
| M3.0* | 3.0 | 13748 |  | $125 \times 157 \times 125$ | 20 | 1 | 10 | 10 |

Table 4.1: Simulation test-cases for analysing the scaling techniques. Reference data taken from Moser et al. (1999) ${ }^{[203]}$. Grid resolution mentioned in the table are based on the reference $\operatorname{Re}_{\tau}=390$.

| Case | $\operatorname{Re}_{\tau}$ | $\mathrm{Re}_{\tau}^{*}$ | $\mathrm{u}_{\tau} / \mathrm{u}_{\mathrm{b}}$ | $\rho_{w} / \rho_{\mathrm{b}}$ | $\mathcal{M}_{\tau}$ | $-\mathrm{B}_{\mathrm{q}}$ | $\mathrm{c}_{\mathrm{f}} \times 10^{-3}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ref $^{[203]}$ | 395 | - | 0.056 | - | - | - | 6.27 |
| Mo.1DNS | 388 | 388 | 0.0563 | 1.0012 | 0.00563 | 0.000196 | 6.36 |
| Mo.1 | 387 | 387 | 0.0561 | 1.0011 | 0.00561 | 0.000194 | 6.31 |
| Mo.3 | 389 | 385 | 0.0560 | 1.0093 | 0.01680 | 0.001725 | 6.33 |
| Mo.5 | 393 | 384 | 0.0560 | 1.0206 | 0.02800 | 0.004147 | 6.40 |
| Mo.7 | 392 | 381 | 0.0555 | 1.0271 | 0.03885 | 0.007154 | 6.32 |
| M1.5 | 455 | 313 | 0.0487 | 1.3585 | 0.07310 | 0.042282 | 6.45 |
| M3.0 | 628 | 208 | 0.0368 | 2.4763 | 0.11065 | 0.130130 | 6.73 |
| M1.5 $^{*}$ | 495 | 383 | 0.0479 | 1.2439 | 0.07187 | 0.035159 | 5.71 |
| M3.0 $^{*}$ | 1085 | 384 | 0.0341 | 2.3463 | 0.10250 | 0.113110 | 5.47 |

Table 4.2: Mean flow variables computed for simulation test-cases for analysing the scaling techniques. Reference data taken from Moser et al. (1999) ${ }^{[203]}$.

### 4.2.2 Results

The $\operatorname{Re}_{\tau}$ and $\mathfrak{u}_{\tau}$ predicted from the present DNS (test-case Mo.1DNS) in table 4.2, have error less than $2 \%$ in comparison with the reference data of Moser et al. (1999) ${ }^{[203]}$. For a constant $\operatorname{Re}_{\mathrm{b}}, u_{\tau} / u_{\mathrm{b}}$ decreases whereas friction Mach number $\mathcal{M}_{\tau}$ and normalised heat flux $B_{q}$ increases with the $\mathcal{M}$ number. For a constant $\operatorname{Re}_{b}, c_{f}$ increased with the $\mathcal{M}$ number, whereas for the case with constant $\operatorname{Re}_{\tau}^{*}, c_{f}$ decreased for higher $\operatorname{Re}_{b}$.

### 4.2.2.1 Mean velocity

Mean velocity profiles scaled with traditional inner variables $\widetilde{u}_{\tau}$ and $\widetilde{\mathcal{T}}_{\nu}$ (equation 4.1) and transformed scaling (equation 4.19 and 4.20 ) are presented in figure 4.3 . Results for $\mathrm{Re}_{\tau}^{*} \approx$ const are shown in the figure. With classical scaling, when the $\mathcal{M}$ number increases, velocity profiles departs from the incompressible profile (see figure 4.3 (a)). After velocity transformation using equation 4.20 , better collapse was observed for the velocity profiles in figure 4.3 (b). The discrepancy observed with the Van Driest transformation (see figure 4.1) were not observed with the scaling technique of Trettel and Larsson (2016) ${ }^{[282]}$. Both in the inner- and outer-layer, the curves collapsed on the reference curve of Moser et al. (1999) ${ }^{[203]}$ for incompressible channel flow at $\mathrm{Re}_{\tau} \approx 390$.


Figure 4.3: Mean velocity profiles with (a) conventional scaling and (b) Trettel's scaling. Reference data taken from Moser et al. (1999) ${ }^{[203]}$.

### 4.2.2.2 Rms profiles of velocity and vorticity

In figure 4.4 rms velocity profiles for different $\mathcal{M}$ number are presented. Since there is no universal scaling for Reynolds stresses and/or rms velocity ${ }^{[11,16,117,179,200]}$, cases at constant $\mathrm{Re}_{\tau}^{*}$ are presented in the figure (see equation 4.5). With classical scaling, outward shift of the wall-normal coordinates indicating wider buffer layer and enhancement of rms velocity for all the velocity components were observed for higher $\mathcal{M}$ number. The widening of the buffer layer or enhancement of the rms of velocity is not due to enhanced turbulence, rather it is just the limitation of scaling with traditional inner variables. With transformed scaling (semi-local scaling equation 4.19 and 4.21) the results collapsed on the reference data, except for slightly increasing over-prediction of $\sqrt{\overline{u^{\prime 2}}}$ peak value, for increasing $\mathcal{M}$ number. The slight over-prediction of $\sqrt{\overline{\mathfrak{u}^{\prime 2}}}$ peak was also observed by Modesti and Pirozzoli (2016) ${ }^{[194]}$ with their DNS (see figure 11 in Modesti and Pirozzoli (2016) $)^{[194]}$ ). They checked the transformation for up to $\operatorname{Re}_{\tau}^{*} \leq 670$ and $\mathcal{M} \leq 3$. Which suggests that this could be a limitation of the semi-local scaling, nevertheless excellent agreement was found for other velocity components.


Figure 4.4: Rms velocity profiles scaled in ( $\mathrm{a}, \mathrm{c}, \mathrm{e}$ ) traditional inner variables and, ( $\mathrm{b}, \mathrm{d}, \mathrm{f}$ ) semilocal variables. (a, b) $\sqrt{\overline{u^{\prime 2}}}$; (c, d) $\sqrt{\overline{v^{\prime 2}}}$ and (e, f) $\sqrt{\overline{w^{\prime 2}}}$. Reference data taken from Moser et al. (1999) ${ }^{[203]}$.

At higher $\mathcal{M}$ numbers, $\operatorname{Re}_{\tau}$ increases significantly due to enhanced heat-transfer through cold wall. Which is indirectly related to the variation of density across the channel height. Similarly $\mathfrak{u}_{\tau} / \mathfrak{u}_{\mathfrak{b}}\left(\operatorname{Re}_{\tau}\right)$ decrease(increase) because $\rho_{w} / \rho_{\mathfrak{b}}$ increases with the $\mathcal{M}$ number. With the scaling technique presented in Section 4.1.2, all the quantities are transformed to their equivalent values in the incompressible regime. As mentioned earlier, in equation 4.5, Reynolds stress is Reynolds number dependent ${ }^{[16,179]}$. In figure 4.5 results obtained
(a)

(b)


Figure 4.5: Rms velocity profiles scaled in (a) traditional inner variables and (b) semi-local variables. Reference data taken from Moser et al. (1999) ${ }^{[203]}$.
from simulations at constant $\mathrm{Re}_{\mathrm{b}}$ (test-cases M1.5 and M3.0) and constant $\mathrm{Re}_{\tau}^{*}$ (test-cases $\mathrm{M}_{1} .5^{*}$ and $\mathrm{M}_{3} .0^{*}$ ) at $\mathcal{M}=1.5$ and $\mathcal{M}=3$ are compared against the incompressible reference data. For instance, case M3.o has $\operatorname{Re}_{\tau}=628$ and $\operatorname{Re}_{\tau}^{*}=207$, here $\operatorname{Re}_{\tau}^{*}$ is almost half of the reference friction Reynolds number, $\operatorname{Re}_{\tau} \approx 390$, and it exhibits the low $\operatorname{Re}_{\tau}$ effect (underprediction of the outer-layer stream-wise internsity after transformation in figure $4.5(\mathrm{~b})$ ). At low $\operatorname{Re}_{\tau}$, there is no enough scale separation and $\sqrt{\overline{\mathfrak{u}^{\prime 2}}}$ has a single clear peak in the buffer-layer, whereas for higher $\operatorname{Re}_{\tau}$ a emergence of secondary peak can be observed in the log-region of the turbulent boundary layer due to the large-scale motions ${ }^{[125,179]}$. For $\mathrm{Re}_{\tau}^{*} \approx 390$, this secondary peak cannot be clearly witnessed due to the lack of sufficient scale separation. Nevertheless in figure 4.5 (b), $\sqrt{\overline{u^{\prime 2}}}$ for test-case M3.0 is under-predicted in the log-region compared to the $\sqrt{\overline{u^{\prime 2}}}$ for test-case $\mathrm{M}_{3} .0^{*}$. For test-case $\mathrm{M}_{1} .5, \mathrm{Re}_{\tau}^{*}=313$, for which a minor difference can be noticed away from the wall. Results for test-cases $M_{1} .5^{*}$ and $M_{3} .0^{*}$ collapse very well on the reference data since their $\mathrm{Re}_{\tau}^{*}$ is equivalent to the reference incompressible $\mathrm{Re}_{\tau}$.
$\sqrt{\overline{\omega^{\prime 2}}}$ profiles are presented in figure 4.6. When scaled with inner variables, weakening of $\sqrt{\overline{\omega^{\prime 2}}}$ in the buffer-layer was observed for increasing $\mathcal{M}$ number. When scaled with semi-local variables, better collapse was observed, except for the small difference in $\sqrt{\overline{\omega_{x}^{\prime 2}}}$ and $\sqrt{\overline{\omega_{y}^{\prime 2}}}$ close to the wall for test-case $M_{3} \cdot o^{*}$. Under-prediction for the rms of stream-wise and wall-normal components of vorticity was observed, which could be because results from the ILES were compared against the DNS. Nevertheless satisfactory agreement was found between the transformed and reference $\sqrt{\overline{\omega^{\prime 2}}}$.


Figure 4.6: Rms vorticity profiles scaled in ( $\mathrm{a}, \mathrm{c}, \mathrm{e}$ ) traditional inner variables and, ( $\mathrm{b}, \mathrm{d}, \mathrm{f}$ ) semilocal variables. $(\mathrm{a}, \mathrm{b}) \sqrt{\overline{\omega_{x}^{\prime 2}}}$; $(\mathrm{c}, \mathrm{d}) \sqrt{\overline{\omega_{y}^{\prime 2}}}$; and (e, f) $\sqrt{\overline{\omega_{z}^{\prime 2}}}$. Reference data taken from Moser et al. (1999) ${ }^{[203]}$.

### 4.2.2.3 Velcity correlation



Figure 4.7: Near-wall span-wise correlation scaled in (a, c, e) traditional inner variables at $y^{+}=$ 10 and, (b, d,f ) semi-local variables at $y^{*}=10$. (a, b) stream-wise velocity; (c, d) wall-normal velocity; (e, f) span-wise velocity. Reference data taken from Moser et al. (1999) ${ }^{[203]}$.


Figure 4.8: Near-wall stream-wise correlation scaled in semi-local variables at $y^{*}=10$. (a) streamwise velocity; (b) wall-normal velocity and (b) span-wise velocity. Reference data taken from Moser et al. (1999) ${ }^{[203]}$.

Auto-correlation in the span-wise direction close to the wall $\left(y^{+}=10\right.$ and $\left.y^{*}=10\right)$ is presented in figure 4.7 . For all the velocity component, location of the minimum of correlation increases with the $\mathcal{M}$ number when scaled with inner variables ${ }^{[52,290]}$. From $R_{u u}$ and $R_{v v}$ it implies, increased streak spacing and vortex diameter ${ }^{[146]}$. Coleman et al. (1995) ${ }^{[52]}$ used Van Driest transformation for the mean velocity and semi-local scaling for the turbulent statistics to collapse the curves of their supersonic channel. But they argued that the near-wall streaks becomes more coherent with the increase in $\mathcal{M}$ number. Widening of the streak and vortex in the span-wise direction and enhancement of streaks in the stream-wise direction ${ }^{[52]}$ (not shown here) is not due to strong modification in turbulence, rather it is due to the poor choice of scaling variables ${ }^{[198]}$. When scaled with right quantities (semi-local variables), the results at different $\mathcal{M}$ number collapsed on the reference data of Moser et al. (1999) ${ }^{[203]}$. Constant streak-spacing of $\lambda_{z}^{*} \approx 114$ and constant vortex diameter $\lambda_{z}^{*} \approx 38$ were observed from the correlations.

Near-wall auto-correlation in the stream-wise direction is presented in figure 4.8. Results for semi-locally scaled distance is only presented. Length of the near-wall streamwise streaks is around 2000 wall-units, and all the correlation curves collapse very well on the reference data.

### 4.2.2.4 Velocity skewness and kurtosis



Figure 4.9: Skewness profiles for ( $a, b$ ) stream-wise and ( $c, d$ ) wall-normal velocity plotted against wall-normal coordinate scaled with ( $a, c$ ) traditional viscous length-scale and (b,d) semi-local viscous length-scale. Reference data taken from Moser et al. (1999) ${ }^{[203]}$.

Skewness of the velocity component is presented in figure 4.9. Third moment used to compute the skewness is non-dimensionalised using the cubic powered rms. Therefore the results are plotted against wall-normal coordinate scaled in traditional viscous lengthscale $\left(l_{v}\right)$ and local viscous length-scale $\left(l_{v}^{*}(y)\right)$. Outward stretching of the wall-normal coordinate scaled with traditional viscous length-scale at higher $\mathcal{M}$ number is evident here like the previous figures. At $\mathcal{M}=0.1$, results of DNS were also plotted to check the effect of mesh coarsening on higher-order statistics. From the figure it was clear that,
skewness was not affected by mesh coarsening. For the stream-wise velocity component, close to the center of the channel slight discrepancy was observed among all the cases. In the log-region, all the transformed profiles collapse very well on each other, with slight under-prediction compared to the reference data. For the transformed wall-normal velocity component, slight under-prediction of the minima was observed for increasing $\mathcal{M}$ number. Skewness of the span-wise velocity component is almost zero, hence not presented here.
(a)

(b)

(c)


Figure 4.10: Velocity flatness profiles plotted against semi-local wall-normal coordinates. (a) Stream-wise and (b) wall-normal and (c) span-wise velocity component. Reference data taken from Moser et al. (1999) ${ }^{[203]}$.

Kurtosis of velocity component plotted against the semi-locally scaled wall-normal coordinate is presented in figure 4.10. Away from the wall, the present results for the kurtosis collapsed on the reference curve. Slight under-prediction of kurtosis close to wall was noticed for all the velocity components, except for wall-normal component, for which huge difference was found. For the test-case Mo.1DNS, discrepancy in the kurtosis was less compared to the ILES (test-case Mo.1) which was at same $\operatorname{Re}_{\mathrm{b}}$ and $\mathcal{M}$ number. Xu et al. (1996) ${ }^{[296]}$ reported higher kurtosis for wall-normal velocity component is due to
events that are very rare in space and time. Since ILES solves only for the resolved scales, these rare events were not captured very well. For the stream-wise velocity component slight discrepancy was found close to the centre of the channel. In general, it can be concluded that, especially close to the wall grid coarsening has some minor effects on the kurtosis.

### 4.3 CONCLUSION

Scaling techniques for compressible channel flows for $\mathcal{M} \leq 3$ were discussed and those scaling transformations were verified for channel flows up to $\mathcal{M} \leq 3$. Mean velocity transformation proposed by Trettel and Larsson (2016) ${ }^{[282]}$ was found to be very effective, since it overcomes the drawbacks of the Van Driest transformation for compressible channel flow with cold walls. Additionally, scaling proposed by Trettel and Larsson (2016) ${ }^{[282]}$ finds semi-local viscous length scale and friction velocity as the scaling variables. This allows to use a single coordinate transformation for mean and turbulent quantities. Turbulent quantities collapse very well with semi-local variables. Simulations were performed for constant $\operatorname{Re}_{\mathrm{b}}$ and $\mathrm{Re}_{\tau}^{*}$ at different $\mathcal{M}$ numbers. Due to heat-transfer through the cold walls, an equivalent friction Reynolds number $\mathrm{Re}_{\tau}^{*}$ was defined and results were compared for $\mathrm{Re}_{\tau}^{*}=$ const. Scaling techniques were tested for up to 4 th order moments and correlations, and better agreement was found with the reference incompressible DNS of Moser et al. (1999) ${ }^{[203]}$. Analysis was also performed for a moderately higher Reynolds number $\mathrm{Re}_{\tau}^{*} \approx 590$, and similar observations were made (not shown here). These scaling techniques will be used in Chapter 5 to develop an algorithm to detect the large-scale structures of turbulence in wall-bounded compressible flows.

## INNER-OUTER LAYER SCALES-INTERACTION

The objective of the present chapter is to investigate the influence of the large-scales on turbulence at different Mach numbers, in a range in which the effect of compressibility is to create density/temperature gradients (Morkovin's hypothesis satisfied). These gradients can be accounted for by using the scalings discussed in Chapter 4. Firstly, a short introduction on large-scale structures is given in Section 5.1. Secondly, the grid requirements for studying the large-scale structures with ILES will be addressed in Section 5.2, followed by the investigation on large-scale's influence in Section 5.3.

Numerical simulations of channel flow at Mach numbers 0.5 and 3.0 for $\operatorname{Re}_{\tau}^{*} \approx 950$ were performed using ILES technique presented in Section 2.5.2.2. The data for both Mach numbers have comparable turbulent statistics after a suitable scaling is applied that compensates for the effect of compressibility. The question is then to know whether the large scale structures have the same characteristics at different Mach numbers. To answer this question, conditional analysis for the high- and low- momentum large scale structures are performed and the results are compared for both Mach numbers. The threshold for conditional analysis is based on the local friction velocity. A straightforward detection algorithm to detect the large scale structures and study their effect at both Mach numbers is also presented.

### 5.1 OVERVIEW ON LARGE-SCALE STRUCTURES IN WALL-BOUNDED TURBULENT FLOW

In wall-bounded flows there exists a near-wall turbulence regeneration mechanism, which involves stream-wise vortices and near-wall streaks ${ }^{[234]}$. They are associated with ejections and sweeps, which transports momentum to and away from the wall ${ }^{[37]}$. Research has also revealed the existence of large scales in the form of long meandering structures for wall-bounded turbulent flows ${ }^{[98,279]}$. These structures consist of alternating low- and high speed regions away from the wall ${ }^{[1,88,125]}$. Such large scale structures are characteristics of high Reynolds number flows, characterized by a sufficient scale separation between the largest and the smallest scales of turbulence. Large scale structures scale with outer length scale $(\mathrm{H})$ of the flow. Structures that are $2 \mathrm{H}-3 \mathrm{H}$ long are termed $\operatorname{LSM}^{[147]}$, and structures that are up to $10 \mathrm{H}-20 \mathrm{H}$ long are called VLSM or superstructures $\left.{ }^{[101,} 125,147,261\right]$. The typical span-wise width of these structures is $\mathrm{H}-2 \mathrm{H}$ (see Abe et al. (2004) ${ }^{[1]}$, and Del Álamo and Jiménez (2003) ${ }^{[64]}$, among others). Near-wall turbulence in different wall bounded flows such as boundary layer, pipe and channel are similar, and the differences arise from the core region ${ }^{[195,196]}$, mainly due to geometric constraints.

It is well known that close to wall sweep motions due to strong stream-wise vortices create high skin friction ${ }^{[47,155]}$. Similarly, for turbulence away from the wall Adrian et al. (2000) ${ }^{[5]}$ proposed a model suggesting that hairpin vortices in the outer layer are regions of low speed fluid, with ejections between their legs and sweeps outside. Similar observations were also made by Ganapathisubramani et al. (2003) ${ }^{[87]}$ who found that groups of hairpin structures contribute $25 \%$ to the Reynolds stress while they occupy only $4 \%$ of the area in a boundary layer experiment. Lee and Sung (2011) ${ }^{[165]}$ showed that large scale structures greater than $3 \delta-4 \delta$ (where $\delta$ is the boundary layer thickness) occupy $40 \%$ of the area and contribute $45 \%$ to the total Reynolds shear stress in a turbulent boundary layer simulation. There is a top-down effect of outer-layer large scale structures on wall structures in high Reynolds number wall-bounded flows. Del Álamo and Jiménez (2003) ${ }^{[64]}$ plotted low pass filtered spectra to show that large scale structures span almost the entire channel height and penetrate into the buffer layer. Mathis et al. (2009) ${ }^{[186]}$ reported that large-scale structures away from the wall modulate the small-scales amplitude close to the wall. Recently Lee and Moser (2015) ${ }^{[166]}$ compared the large scale motions in turbulent pipe and channel flows at friction Reynolds number $\operatorname{Re}_{\tau} \approx 930$. They found that close to the wall high speed large scale motions have greater contributions to the turbulent intensities and Reynolds stress, whereas away from the wall low speed large-scale motions have a dominant effect. Dekou et al. (2016) ${ }^{[63]}$ characterize the size, intensity, and life-time of the large scale motions for an experimental turbulent boundary layer. Yoon et al. (2016) ${ }^{[299]}$ uses the FIK identity of Fukagata et al. (2002) ${ }^{[84]}$ to quantify the influence of large scale motions on drag reduction, and they report low(high) speed large scale motions contribute approximately $25 \%(20 \%)$ to the skin friction.

Most of the studies devoted to large-scale structures were performed for incompressible flows, but compressible wall-bounded flows are equally important in engineering applications. The first compressible channel flow simulation was performed by Coleman et al. (1995) ${ }^{[52]}$ for a low friction Reynolds number and Mach number up to 3. At these Mach numbers, Coleman et al. (1995) ${ }^{[52]}$ find that Morkovin's hypothesis is satisfied (that is, the rms of turbulent pressure is less than $10 \%$ of the mean pressure) and in such a case compressibility manifests itself mainly through gradients of temperature, density, or viscosity. At $\mathcal{M}=3$, Coleman et al. (1995) ${ }^{[52]}$ find an increased coherence of the nearwall streaks (see Section 1.2 for more information on different coherent structures) in the stream-wise and span-wise directions compared to the incompressible case. However, it is found that when scaled properly, near-wall streaks of the compressible wall-bounded flow have approximately the same characteristics as the incompressible wall-bounded flows (see for example figures 4.7 and figure 4.8 in Chapter 4, Morinishi et al. (2004) ${ }^{[198]}$, Patel et al. $(2015)^{[219]}$, and appendix in Sebastian et al. (2017) ${ }^{[249]}$ among others). This shows the importance of scaling the compressible data to have a one-to-one comparison with the incompressible counter part (see Chapter 4 for detailed information on compressible scaling).

### 5.2 GRID REQUIREMENT

One of the reasons for neglecting the large-scale structure originates from the Townsend's attached eddy hypothesis, which describes the large-scale structure as inactive, in the sense that they do not contain Reynolds stress. Del Álamo and Jiménez (2003) ${ }^{[64]}$ show that the large-scale structures are stream-wise elongated anisotropic motions similar to near-wall streaks, but of a size comparable to or even larger than the channel height. The practical difficulty with these structures lies in the fact that a large experimental set-up or a computational domain at sufficiently high Reynolds number conditions is necessary to properly capture their entire motions. This requirement has been the stumbling block to both experiments and DNS for large-scale studies. One-point measurements have fundamental difficulty in describing the spatial characteristics of large-scale structures. The validity of a spectrum converted from frequency to wave-number using Taylor's hypothesis for use in large-scale analysis should be considered with caution.

Considering the fact that the size of the large-scales in the outer layer is at least comparable to the channel half-height and assuming that their outer motions are detached from the wall and self-organized, it is expected that relatively coarse grids will be sufficient to resolve their motions. On the other hand if they originate from the small-scale organised structures close to the wall as proposed in the physical model of Adrian et al. $(2000)^{[5]}$, then fine grid which discretise the near-wall turbulence is required to reproduce the large-scale structures in the outer-layer. Large-scale structure penetrates deep into the buffer-layer, and appear only in the stream-wise velocity.

Therefore grid requirement to reproduce the large-scale structures was analysed. Testcase in table 3.7 in Chapter 3 is used here. $\operatorname{Re}_{\tau} \approx 640$ and this it the lower limit of the Reynolds number to observe such structures. The findings on Del Álamo and Jiménez $(2003)^{[64]}$ suggest that the structures in stream-wise velocity can be decomposed into two components. One which has maximum intensity close to the wall, and consists of anisotropic structures which scales in inner-variables. This structures widens, lengthens and becomes more isotropic in the outer-layer. The other structure is present in the outer-layer which has size $\lambda_{x} \leq 5 \mathrm{H}$ and $\lambda_{z} \leq 2 \mathrm{H}$ and penetrates into the buffer-layer. 2 D spatial pre-multiplied spectra ( $k_{x} k_{z} \overline{\mathrm{E}}_{\mathfrak{u}^{\prime} \mathfrak{u}^{\prime}} / \mathrm{u}_{\tau}^{2}$ ) of the stream-wise velocity component in the near-wall and outer-layer region are presented in figures 5.1 and 5.2. The energy spectrum multiplied by the wave-number in the logarithmic plot is the measure of the stream-wise energy in the wave-length interval centred at $\lambda_{x}, \lambda_{z}$, and the total area over the entire wave-number is equivalent to the turbulent energy of the stream-wise velocity component. In both figures, spectra computed from the fine grid test-cases have satisfactory collapse, whereas spectra computed from the coarse grid test-cases do not collapse. The near-wall structures widen as they elongate because they progressively separate from the 2 D isotropy close to the wall $\left(\lambda_{x}=\lambda_{z}\right)$. The outer-layer structure has two modes. The first one is a quasi-isotropic structure with $\lambda_{x}=2 \lambda_{z}$, and the other one is a long anisotropic structure whose size scale in H. Del Álamo and Jiménez (2003) $)^{[64]}$ showed that this anisotropic mode penetrates very deep in the wall-normal direction.

Del Álamo and Jiménez (2003) ${ }^{[64]}$ also showed that the large-scale structures in the wall-bounded turbulent flow originates from the wall and/or they penetrate deep into the buffer-layer and have some kind of non-linear interaction with the near-wall small-


Figure 5.1: Iso-contours of 2 D pre-multiplied stream-wise velocity spectra $\left(k_{x} k_{z} \bar{E}_{u^{\prime} u^{\prime}} / u_{\tau}^{2}\right)$ at $y^{+} \approx$ 15. Solid line $-\lambda_{x}=\lambda_{z}$, dash line $-\lambda_{x}^{+} \approx\left(\lambda_{z}^{+}\right)^{3}$.
(a)

(b)


Figure 5.2: Iso-contours of 2 D pre-multiplied stream-wise velocity spectra ( $k_{x} k_{z} \overline{\mathrm{E}}_{\mathfrak{u}^{\prime} u^{\prime}} / u_{\tau}^{2}$ ) at $y / H=0.5$. Solid line $-\lambda_{x}=\lambda_{z}$, dash line $-\lambda_{x}=2 \lambda_{z}$ and $\lambda_{z}=1.25 H$.
scale structures. Therefore, using a coarse mesh to investigate the large-scale features will under-resolve the near-wall small-scale structures. This will hinder the reproduction of large-scale flow features through simulation. Hence, the conclusions on the grid requirement is similar as in Chapter 3. Well-resolved computational grid which discretise the near-wall features must be used to correctly reproduce the outer-layer large-scale physics.

## 5.3 simulation at high reynolds \& mach number

### 5.3.1 Validation of the simulations

In this section, compressible channel flow simulations at $\mathcal{M}=0.5$ and 3.0 are performed using ILES. These simulations are compared to the incompressible reference DNS of Del Alamo et al. (2004) ${ }^{[66]}$ at $\mathrm{Re}_{\tau} \approx 950$ after a scaling transformation is used. The simulation details are summarised in table 5.1. The values of $\mathrm{Re}_{\mathrm{b}}$ were chosen to obtain an effective

| $\mathcal{M}$ | $\mathrm{Re}_{\mathrm{b}}$ | $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}$ | $\mathrm{N}_{x} \times \mathrm{N}_{\mathrm{y}} \times \mathrm{N}_{z}$ | $\mathrm{Re}_{\tau}$ | $\mathrm{Re}_{\tau}^{*}$ | $\mathcal{M}_{\tau}$ | $\mathrm{u}_{\mathrm{fi}} / \mathrm{u}_{\mathrm{b}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.5 | 20000 | $8 \pi \mathrm{H} \times 2 \mathrm{H} \times 3 \pi \mathrm{H}$ | $628 \times 235 \times 943$ | 1000 | 985 | 0.0247 | 0.0494 |
| 3.0 | 30000 |  |  | 2059 | 870 | 0.0993 | 0.0331 |
| 3.orefined |  |  | $628 \times 300 \times 943$ | 2108 | 850 | 0.0993 | 0.0331 |

Table 5.1: Simulation test-cases for studying inner-outer scales interaction at different $\mathcal{M}$ numbers.
$\mathrm{Re}_{\tau}^{*}$ close to 950 , see Appendix D to find an approximate $\mathrm{Re}_{\mathrm{b}}$ for a given $\mathrm{Re}_{\tau}^{*}$ and $\mathcal{M}$. The obtained $\operatorname{Re}_{\tau}^{*}$ is 985 for $\mathcal{M}=0.5$ and $\operatorname{Re}_{\tau}^{*}=870$ for $\mathcal{M}=3$ (this latter value corresponds to $\operatorname{Re}_{\tau}=2059$ ). Both simulations are performed using a large computational domain of size $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}=8 \pi \mathrm{H} \times 2 \mathrm{H} \times 3 \pi \mathrm{H}$ discretized with $\mathrm{N}_{x} \times \mathrm{N}_{y} \times \mathrm{N}_{z}=628 \times 235 \times 943$ grid points. The grid resolution is such that $\Delta x^{*} \approx 40, \Delta z^{*} \approx 10, \Delta y_{\text {min }}^{*} \approx 2$, and $\Delta y_{\text {max }}^{*} \approx 14$, where the * superscript here indicates that mesh sizes are scaled with the local viscous length scale $\tilde{l}_{v}^{*}(\widetilde{H})$ at the centre of the channel. Note that since the mesh size is kept constant in * units, the mesh size increases in traditional wall units as the Mach number is increased, due to a decrease in $l_{v}$. Indeed, one has for example $\Delta y^{*}=\Delta y R_{\tau}^{*}$ and $\Delta y^{+}=\Delta y \operatorname{Re}_{\tau}$, which gives $\Delta y^{+}=\Delta y^{*} \operatorname{Re}_{\tau} / \operatorname{Re}_{\tau}^{*}$. According to table 5.1, $\operatorname{Re}_{\tau}$ is multiplied by a factor of about 2 as $\mathcal{M}$ increases from 0.5 to 3.0 , while $\mathrm{Re}_{\tau}^{*}$ and $\Delta \mathrm{y}^{*}$ are kept constant by construction, and as a result $\Delta y^{+}$increases by a factor of 2 . The first point off the wall is at $y^{*}=2$ for both Mach numbers, which corresponds to $y^{+}=2$ for $\mathcal{M}=0.5$, and $y^{+}=4$ for $\mathcal{M}=3.0$. Also, the mesh is such that there are 10 points for $y^{*}<20$, which means 10 points for $y^{+}<20$ at $\mathcal{M}=0.5$, but only 5 points for $y^{+}<20$ at $\mathcal{M}=3.0$. Compared to typical grid resolutions for well resolved LES in incompressible flows (requiring about to points within $y^{+}=20$ ), it would appear that the present resolution at $\mathcal{M}=3.0$ is insufficient to describe this region of the flow, in which turbulence is produced and where large temperature gradients take place. However, quantities such as turbulent intensities, correlations, or buffer layer size scale in * units (see for example figure 5.3 below), which indicates that it is the discretisation in * units ( 10 points for $y^{*}=20$ ) that should be maintained, as is the case here. To make sure the grid is not too coarse at $\mathcal{M}=3.0$, another simulation with a refined grid in the wall-normal direction (called $\mathcal{M}=3$.orefined in table 5.1) has been performed. The refined grid has 300 points in the wall-normal direction, including 18 points to discretise $y^{+}<20$ with a first point at $y^{+} \approx 2$, meaning the grid in the buffer layer now matches the one at $\mathcal{M}=0.5$ in + units rather than in * units. This refinement changes the $\operatorname{Re}_{\tau}$ by only about $2 \%$, and the Reynolds stresses (in particular $\sqrt{\overline{u^{\prime 2}}}$ ) are unchanged. This is important because most of the processing to follow is based on $u^{\prime}$.

For a DNS at $\mathrm{Re}_{\tau} \approx 950$, one would require approximately $2.7 \times 10^{9}$ grid points ${ }^{[66]}$. For the present ILES approximately $0.14 \times 10^{9}$ grid points is used, which represents a reduction factor of 19 . In the $x$ and $z$ directions, the resolution is about 3-4 times coarser than for the DNS, which determines the coefficient of the scheme according to the recipe given in section 2.5.2.2. To save some computing time, the simulation at $\mathcal{M}=3.0$ was initiated from a converged field at $\mathcal{M}=0.5$ rescaled in the following way:

- the mean stream-wise velocity profile at $\mathcal{M}=0.5$ was stretched to $\mathcal{M}=3$;
- the stream-wise velocity fluctuations at $\mathcal{M}=0.5$ were added to the newly computed mean velocity profile;
- random noise was added to the turbulent wall-normal and span-wise velocities. No modification of the mean temperature profile was done but a rescaling would probably have made the simulation converge faster.

The mean and rms stream-wise velocity profiles computed from our two simulations are presented in figure 5.3. The two different normalizations introduced earlier (tradi-
(a)

(b)


Figure 5.3: (a) Mean and (b) rms stream-wise velocity profiles at $\mathcal{M}=0.5$ and 3. Reference curves are taken from Del Alamo et al. (2004) ${ }^{[66]}$.
tional wall units indicated with + , or compressible scaling indicated with *) are used in this figure. Specifically, on the $x$-axis, $y^{*}=y / l_{v}^{*}(y)$ is used as introduced in Chapter 4 . For scaling the mean flow in figure $5.3(\mathrm{a})$, the transformation proposed by Trettel and Larsson (2016) ${ }^{[282]}$ is used (see equation 4.20), it is well suited for isothermal walls. For scaling the rms velocity $\sqrt{\overline{\mathfrak{u}^{\prime 2}}}$ in Fig. 5.3(b) the semi-local scaling technique introduced by Coleman et al. (1995) ${ }^{[52]}$ is used (see equation 4.21).

For the incompressible reference, and also at $\mathcal{M}=0.5$ for which there is almost no compressible effect, there is no difference between the + and ${ }^{*}$ scalings, and these cases correspond to a single curve in the figures. At $\mathcal{M}=3$, there is a large difference, and the curve labelled $\mathrm{M}_{3} .0$ corresponds to + units while the curve labelled $\mathrm{M}_{3} .0^{*}$ corresponds to the * units.

At $\mathcal{M}=0.5$, there is an excellent agreement for the mean and rms velocity with the reference data in figure $5 \cdot 3$, except for the slight over prediction of the $\sqrt{\overline{{u^{\prime 2}}^{2}}}$ peak. This can be attributed to the coarse mesh used in the ILES. As expected, profiles at $\mathcal{M}=3.0$ (in + units) do not match the incompressible reference curves: the mean velocity profile has a higher log-law intercept value and a steeper slope close to the wall compared to the incompressible result, whereas the rms of stream-wise velocity is extremely overpredicted. The scaled curve $\mathrm{M}_{3} . \mathrm{o}^{*}$ is in much better agreement with the incompressible reference even if the buffer layer peak of $\sqrt{\overline{{u^{\prime 2}}^{2}}}$ is slightly over-predicted compared to the reference incompressible curve. As explained above, a grid refinement study has shown that this overshoot does not result from poor discretisation, the statistics of $\sqrt{{u^{\prime 2}}^{*}}$ being unchanged by a grid refinement in the buffer layer region. Modesti and Pirozzoli (2016) ${ }^{[194]}$ compared different compressible scaling techniques with DNS on a finer grid and observed similar over-prediction of the $\sqrt{{\overline{u^{\prime 2}}}^{*}}$ peak with semi-local scaling transformation (see Fig. 11 in Modesti and Pirozzoli (2016) ${ }^{[194]}$ ). Hence, the over-prediction of ${\sqrt{\mathrm{u}^{\prime 2}}}$ peak can be attributed to the unperfect semi-local scaling. Overall, a satisfactory agreement is obtained with the ILES using scaling transformations.

### 5.3.2 Evidence of Large Scale Structures and their effect on near-wall scales

In this section large-scales are evidenced in the simulations performed in the previous section, and their effect on the near-wall scales is also shown. An example of near-wall streaks is presented in figure 5.4 together with the contours of the projection of filtered large-scale structures. The procedure used to obtain these contours will be presented in detail in Section 5.3.3. Near wall streaks are approximately few 1000 semi-local wall units in length, with streak spacing of approximately 120 semi-local wall-units in span-wise direction. For test-case at $\mathcal{M}=0.5$ (in figure 5.4), $1000 l_{v} \approx \mathrm{H}$ which means streaks are 1H long. Within the contour of High Momentum Large Scale Structure (HMLSS) we find higher density of high-speed streaks (red structures), whereas within the contour of Low Momentum Large Scale Structure (LMLSS) we find higher density of low-speed streaks (blue structures). This is a visual expression of the amplitude modulation of near-wall structures by large scale structures away from the wall mentioned in Section 5.1. In the


Figure 5.4: Colour map of $u^{* *}$ at $y^{*}=15$ for an instantaneous velocity field at $\mathcal{M}=0.5$. This shows near-wall streaks (in blue(red) for low(high)-speed streaks). The contours of the High Momentum Large-Scale Structures (thick) and Low Momentum Large-Scale Structures (thin) are also indicated.
remaining of this section we study this modulation with methods similar to those initially
introduced by Mathis et al. (2009) ${ }^{[186]}$ and also use semi-local scaling of the coordinates in all the plots.

First, we need to separate the large scales from the small scales. Scale separation is traditionally seen in pre-multiplied spectra of turbulent velocity fluctuations. At the moderately high Reynolds number used here, scale separation is more obvious in the span-wise direction. The pre-multiplied spectrum of the stream-wise velocity in the span-wise direction, $k_{z} \bar{E}_{u^{\prime} u^{\prime}}\left(k_{z}, y\right)$, is given in figure $5 \cdot 5(a)$ for both Mach numbers. In this particular plot $k_{z} \bar{E}_{u^{\prime} u^{\prime}}\left(k_{z}, y\right)$ has been normalised by the rms of the velocity $\sqrt{\overline{u^{\prime 2}}}$ for each $y$ (this tends to strengthen the separation between scales and is also used for example by Abe et al. (2004) ${ }^{[1]}$ ). Here $k_{z}=2 \pi / \lambda_{z}$ is the wave-number in the span-wise direction, and $\lambda_{z}$


Figure 5.5: Pre-multiplied spectra, $k_{z} \bar{E}_{u^{\prime} u^{\prime}}\left(\lambda_{z}, y\right)$ obtained at several wall-normal distance $y$ and $\lambda_{z}^{*}=\frac{\widetilde{\lambda}_{z}}{\widetilde{\mathrm{~L}}_{v}(\widetilde{\mathrm{H}})}$. The color map corresponds to $\mathcal{M}=0.5$ and and the contours to $\mathcal{M}=$ 3.0 (contour lines value: 0.3 - dashed, and 0.4 - solid black). The vertical black solid line indicates the filter cut-off scale $\left(\lambda_{z, \text { cut-off }}=0.5 \mathrm{H}\right)$. Symbol $x$ in (a) indicate the inner- and outer-layer peak in the pre-multiplied spectra. Spatial $u^{\prime}$ in the stream-wise direction from the location of (b) outer-layer and (c) inner-layer spectral peak, and (d) is the comparison of the low-pass filtered signal in (b) and (c).
the corresponding wavelength. Upon using semi-locally scaled wall-normal distance and wave-length, the pre-multiplied spectra obtained at $\mathcal{M}=0.5$ and $\mathcal{M}=3$ compare well. The inner-layer peak around $y^{*} \approx 10-15$ and $\lambda_{z}^{*} \approx 120$ is due to the near-wall streaks, and the peak around $y \approx 0.3 \mathrm{H}$ and $\lambda_{z} \approx \mathrm{H}$ is due to the outer-layer large-scale structures. There is an obvious scale separation between the inner- and outer-layer, and the scale $\lambda_{z, \text { cut-off }}$ separating the two regions is: $\lambda_{z, \text { cut-off }}=0.5 \mathrm{H}$ (indicated by a solid vertical line in figure $5.5(\mathrm{a})$ ). Therefore, to obtain the large-scales, a low-pass Fourier filter is applied with cut-off at $\lambda_{z, \text { cut-off. }}$. Unless specified otherwise, in the following the filtering operation to obtain large-scales is applied in the span-wise direction to the stream-wise velocity, for all stream-wise and wall-normal locations. We will denote by $u_{L}^{\prime}(x, y, z, t)$ the large scale velocity obtained from $u^{\prime}(x, y, z, t)$ by applying the filter. By applying a high-pass filter with
cut-off scale $\lambda_{z, \text { cut-off }}$ instead of a low-pass one, we can of course obtain the small scales, which are denoted by $u_{s}^{\prime}(x, y, z, t)$ (and we have $u^{\prime}(x, y, z, t)=u_{L}^{\prime}(x, y, z, t)+u_{s}^{\prime}(x, y, z, t)$ ). An example of input and output of the low-pass filter is shown in Fig. 5.5(b-d), where a sample of stream-wise velocity along the stream-wise direction is shown. Figure $5.5(\mathrm{~b})$ and (c) show the unfiltered velocity at a wall distance corresponding to the outer-layer and inner-layer, respectively. The corresponding filtered data are shown in the same plot in figure $5.5(\mathrm{~d})$. Some correlation can be found between the two signals, which demonstrates that the outer-layer large-scale are felt close to the wall. The large scale structures are tilted in the forward direction, and the tilting angle will be needed below when we address the modulation of the small scales by the large scales. To compute this tilting angle the stream-wise correlation is computed between the low-pass filtered velocity away from the wall and the low-pass filtered velocity at a reference point in the inner-layer.

$$
\begin{equation*}
\mathrm{R}_{\mathbf{u}_{\text {Lref }}^{\prime *}}{ }_{\mathrm{L}}^{\prime *}(\delta x, y)=\frac{\mathcal{F}^{-1}\left[\operatorname{conj}\left(\mathcal{F}\left[\mathfrak{u}_{\text {Lref }}^{\prime *}\left(x, y_{\text {ref }}\right)\right]\right) \mathcal{F}\left[\mathfrak{u}_{\mathrm{L}}^{\prime *}(x, y)\right]\right]}{\sqrt{\mathfrak{u}_{\text {Lref }}^{\prime 2 *}} \sqrt{\overline{\mathfrak{u}_{\mathrm{L}}^{\prime 2 *}}}} \tag{5.1}
\end{equation*}
$$

where $\mathcal{F}$ is the Fourier transform and conj(•) is the complex conjugate. The cross-correlation is obtained from the inverse Fourier transform of the cross-spectrum which is statistically averaged in the span-wise direction and time. From the lag for maximal correlation as a function of wall distance, we can calculate that the inclination angle.

$$
\begin{equation*}
\theta=\arctan \left(\frac{y-y_{\text {ref }}}{\delta x}\right) \tag{5.2}
\end{equation*}
$$

The average inclination angle is between $13^{\circ}$ and $14^{\circ}$ for both Mach numbers. This result is consistent with the values reported in Marusic and Heuer (2007) ${ }^{[180]}$.


Figure 5.6: Tilting angle of the large-scale structures computed from the lag of the maximum cross-correlation and wall-normal displacement between two locations used for crosscorrelation. $\bigcirc$ is the reference location which corresponds to the inner-layer spectral peak.

Second, in order to show the modulation of the amplitude of the small-scales by the large-scales, a manifestation of which has been shown in figure 5.4 , we can apply the method of Mathis et al. (2009) ${ }^{[186]}$, who proposed to compute the correlation coefficient between the large-scale velocity and the envelop of small-scale to quantify the scales interaction. Schlatter and Örlü (2010) ${ }^{[24]}$ cautioned the use of the correlation coefficient introduced by Mathis et al. (2009) ${ }^{[186]}$ because the correlation coefficient is actually mainly determined by the skewness. To avoid being possibly misled by this problem Bernardini and Pirozzoli (2011) ${ }^{[15]}$ proposed to compute amplitude modulation effects using twopoint covariance, which is what is done here.

The covariance is computed from the following procedure (see figure 5.7):


Figure 5.7: Decoupling procedure for quantifying inner-outer scales interaction.

1. the raw $u^{\prime *}(x, y, z, t)$ is low- and high-pass filtered in the spanwise direction (with cut-off at $\left.\lambda_{z, \text { cut-off }}=0.5 H\right)$ to obtain $u_{L}^{\prime *}(x, y, z, t)$ and $u_{s}^{\prime *}(x, y, z, t)$, the large- and small-scale velocity fluctuations;
2. the Hilbert transform of the small-scale fluctuation field from step 1 is computed in the span-wise direction for all stream-wise and wall-normal locations to find the envelop of the small-scale fluctuation, denoted by $u_{s e}^{\prime *}(x, y, z, t)$;
3. the envelop of the small-scale fluctuations is again low-pass filtered (with cut-off at $\left.\lambda_{z, \text { cut-off }}=0.5 \mathrm{H}\right)$, which provides the large-scale envelope of the small scales, denoted $u_{\text {seL }}^{\prime *}(x, y, z, t)$;
4. the two-point amplitude modulation covariance is finally computed between the large-scale velocity fluctuation at position $y_{\mathrm{L}}^{*}$ and the large-scale envelop of smallscale velocity fluctuation at position $y_{s e L}^{*}$. It is given by:

$$
\begin{equation*}
\mathrm{C}\left(\mathrm{y}_{\mathrm{L}}^{*}, \mathrm{y}_{\mathrm{seL}}^{*}, \delta x\right)=\mathcal{F}^{-1}\left[\operatorname{conj}\left(\mathcal{F}\left[u_{\mathrm{L}}^{\prime *}\left(x, y_{\mathrm{L}}^{*}\right)\right]\right) \mathcal{F}\left[\mathrm{u}_{\text {se } \mathrm{L}}^{\prime *}\left(x, \mathrm{y}_{\mathrm{se}}^{*}\right)\right]\right] \tag{5.3}
\end{equation*}
$$

where $\mathcal{F}$ is the Fourier transform and conj $(\cdot)$ represents the complex conjugate. The covariance $C$ is obtained by inverse Fourier transform of the cross-spectrum (with
statistical averaging on $z$ and $t$ ). It depends on the axial shift $\delta x$ between the two locations $y_{\mathrm{L}}^{*}$ and $y_{\text {seL }}^{*}$, and in the following the shift is such that the correlated positions correspond to the inclination angle of $14^{\circ}$ of the large scale structure obtained above. However, we have verified that the covariance map is not extremely sensitive to the stream-wise shift and taking $\delta x=0$ (correlated positions on a vertical, wall-normal, line) would not change the result.

Figure 5.8 presents the two-point amplitude modulation covariance for both Mach numbers. The wall distance in these plots is the semi-local scaled wall-normal coordinate. For $y_{\mathrm{L}}^{*} \approx 100-150$ and $y_{\text {seL }}^{*} \approx 10$ we have a peak which is the evidence of top-down mechanism in the form of amplitude modulation of near-wall small scales (at $y_{s e L}^{*} \approx 10$ ) due to outer layer large scale structures (from $y_{\mathrm{L}}^{*} \approx 100-150$ ) from the lower part of the log-layer. With the scaling used the covariance maps at both Mach numbers are very similar, showing that the top-down mechanism is not dependent on the Mach number. These results indicate that we have similar base-line test cases for a near incompressible and supersonic channel flow to study large scale effects.


Figure 5.8: Two-point amplitude modulation covariance map $C\left(y_{\mathrm{L}}^{*}, y_{\text {seL }}^{*}, \delta x\right)$ for $\mathcal{M}=0.5$ (colour map) and $\mathcal{M}=3.0$ (contour lines at value (a) -0.1 - white, (b) 0.1 - solid black and (c) 0.25 - dashed black). $\delta x$ is such that the two points being correlated are along a inclination angle of the large-scale structures ( $14^{\circ}$ ).

### 5.3.3 Procedure to extract large-scale structures

In this section a feature extraction technique is proposed which can be used for detecting HMLSS and LMLSS. We have seen in Section 5.1 that depending on the Reynolds number the stream-wise length of the large scales is reported to be between 2 H and 20 H . For the channel flows considered presently, the average extent of the large scale structure computed from the spectra is found to be H in the span-wise direction, and about 46 H in the stream-wise direction, although there are a variety of large-scale structures in the flow field. These observations are consistent with Abe et al. (2004) ${ }^{[1]}$, Lee and Sung (2011) ${ }^{[165]}$ and Lee and Moser (2015) ${ }^{[166]}$, among others. The feature extraction procedure
used for the detection and projection of HMLSS and LMLSS for conditional averaging is based on the following steps:

1. a) The velocity fluctuation $u^{\prime}(x, y, z, t)$ is low-pass filtered in the span-wise direction using the same Fourier filter as in Section 5.3.2 to obtain the large scale fluctuation $\mathfrak{u}_{\mathrm{L}}^{\prime}(x, y, z, t)$. The filter cut-off scale is $\lambda_{z, \text { cut-off }}=0.5 \mathrm{H}$ as before.
b) The resulting field $u_{L}^{\prime}(x, y, z, t)$ is filtered with a Fourier low-pass filter in the stream-wise direction to obtain the large scale fluctuation $u_{\mathrm{LL}}^{\prime}(x, y, z, t)$. Several cut-off scales $\lambda_{x, \text { cut-off }}$ are used: $\mathrm{H}, 3 \mathrm{H}, 6 \mathrm{H}$, and 12 H and results obtained with each of them will be compared below.
c) The small scales are obtained by applying high-pass filters instead of low-pass ones. The result of the high-pass filter in the span-wise direction is denoted $u_{s}^{\prime}(x, y, z, t)$ as in Section 5.3.2, and a further application of the filter in the stream-wise direction provides the small scale fluctuation $\mathfrak{u}_{s s}^{\prime}(x, y, z, t)$.
2. The filtered field is thresholded to differentiate the large-scale structures into HMLSS and LMLSS for conditional averaging:

- $\mathfrak{u}_{\mathrm{LL}}^{\prime}<-\mathfrak{u}_{\text {threshold }}$ is a LMLSS
- $u_{\mathrm{LL}}^{\prime}>\mathrm{u}_{\text {threshold }}$ is a HMLSS

In incompressible flows the threshold value $u_{\text {threshold }}$ is commonly chosen to be a few times $\mathfrak{u}_{\tau}$ (see for example Yoon et al. (2016) ${ }^{[299]}$ ). However, for the present investigation where the flow is compressible with a large Mach number, the threshold value $u_{\text {threshold }}$ is computed relatively to $u_{\tau}^{*}(y)$. Results obtained for $u_{\text {threshold }}=u_{\tau}$ and $u_{\text {threshold }}=u_{\tau}^{*}(y)$ will be presented below and it will be shown that the latter choice warrants that the effect of the large scales on the wall remains the same for both Mach numbers.

Moreover, a precise choice for $u_{\text {threshold }}$ can be made by considering how energy is distributed between the small-scales and the large-scales. Semi-locally scaled values of $\sqrt{\overline{\mathrm{u}^{\prime 2}}}$. for the large- and small-scale velocity fluctuations are computed separately: that is, the quantities $\sqrt{{\overline{u_{\mathrm{LL}}^{\prime 2}}}^{*}}$ and $\sqrt{\overline{\bar{u}_{\mathrm{ss}}^{\prime 2}}}{ }^{*}$ are computed, and they are presented in figure 5.9 for both Mach numbers. The curves for the large- and small-scale collapse reasonably well on their respective counterpart for both Mach numbers (which we knew already from figure 5.3 for the total fluctuation, curves Mo.5 and M3.0*). $\sqrt{\overline{\mathrm{u}_{s s}^{\prime 2}}}{ }^{*}$ for the small-scale velocity (curves Mo.5S and M3.OS*) is dominant close to the wall and from the figure it is clear that the buffer layer peak is due to the small-scales. On the other hand $\sqrt{{\overline{u_{\mathrm{LL}}^{\prime 2}}}^{*}}$ for the large-scale (curves Mo.5L and $\mathrm{M}_{3} . \mathrm{LL}^{*}$ ) is dominant away from the wall. The Reynolds number is not high enough to have a distinct outer-layer peak, nevertheless the emergence of this peak is witnessed. Semi-local wall variables scales total $\sqrt{\overline{{u^{\prime 2}}^{2}}}$ and rms of small and large-scale separately. The horizontal dashed line at $\sqrt{{\overline{u^{\prime 2}}}^{*}}=1$ is the threshold chosen for the structure extraction. $u_{\text {threshold }}=u_{\tau}^{*}(y)$ is a reasonable choice since


Figure 5.9: Filtered ${\sqrt{\bar{u}^{\prime 2}}}^{*}$ for the large- and small-scale stream-wise velocity. Mo. 5 and $\mathrm{M}_{3} .0^{*}$ is the global ${\sqrt{{\overline{u^{\prime 2}}}^{2}}}^{*}$; Mo.5s, M3.0s* are the $\sqrt{{\overline{u_{\mathrm{ss}}^{\prime 2}}}^{*}}$ of small-scales; and Mo.5L, M3.0L ${ }^{*}$ are the $\sqrt{{\overline{u_{\mathrm{LL}}^{\prime 2}}}^{*}}$ of large-scale. The horizontal dashed line $\sqrt{{\overline{u^{\prime 2}}}^{*}}=1$ indicates the chosen threshold $u_{\text {threshold }}$ for the feature extraction.
${\sqrt{\bar{u}_{\mathrm{LL}}^{\prime 2}}}^{*}$ for the large-scale is greater than 1 from the buffer layer to the outer-layer of the flow.
3. Large-scale structures have vertical limits (will be shown later in Section 5.3.4) therefore, 2D images of the large-scale structures were produced by projecting all the detected structures above $y^{*}=20$ onto a plane $P(x, z)$ parallel to the flow. $P(x, z)$ is an auxiliary plane which is used to define where the large-scale structures are present, and of which type they are. Projection is computed according to the following rule:

$$
P(x, z)=\left\{\begin{array}{l}
+1 \text { for HMLSS } \\
-1 \text { for LMLSS } \\
0 \text { otherwise }
\end{array}\right.
$$

If the detected structures overlap each other in the wall-normal direction then the projection will be $P(x, z)=0$. It ensures that the HMLSS and the LMLSS are perfectly differentiated from each other. The current choice of projecting a large-scale structure is very simple and straightforward approach since it captures the main body of the structure, while ensuring that the detected structure do not overlap. The contour lines in figures 5.4 and $5.10(b)$ are the edges of the regions having $P(x, z)= \pm 1$.
4. Finally, conditional averaging is performed: the original unfiltered flow fields $u^{\prime}(x, y, z, t)$ are averaged in regions $P(x, z)=+1$ and $P(x, z)=-1$ to study the effect of HMLSS and LMLSS on the turbulence.

Figure 5.10 presents an example of flow-field before step 1 and after step 4 of the detection procedure. Before step 1 of the detection procedure, in figure 5.10(a), both HMLSS and LMLSS of various scales are found in the flow-field. In figure 5.10(b) contours of the projection of the filtered large-scale motions after step 4 of the detection procedure are presented. In this example, a low-pass filter with $\lambda_{x, \text { cut-off }}=6 \mathrm{H}$ was used. Thin(thick)


Figure 5.10: (a) Iso-contour of the HMLSS $\left(u^{\prime *}=3\right)$ in orange and LMLSS $\left(u^{\prime *}=-3\right)$ in blue and (b) Contour of the projection of the filtered HMLSS(thick) and LMLSS(thin) after the detection procedure and slice of stream-wise vorticity at $y^{*}=15$ (blue: negative, orange: positive) close to wall. Snapshot taken from test-case at $\mathcal{M}=0.5$
contour lines enclose LMLSS(HMLSS). A colormap of the stream-wise vorticity $\omega_{\chi}$ close to wall is also presented in figure 5.10 (b). High density of stream-wise vorticity is associated with the HMLSS, indicating a more active bursting process beneath these structures. Evidence of this can also be seen in figure 5.4, with more instabilities associated with the HMLSS, involving streak breakdown and enhanced regenerative cycle.

### 5.3.4 Organisation of large-scale structures

The objective in this section is to (a) educe the shape of the large-scale structures by conditional averaging and (b) see the effect of scaling on the educed structure. The idea is to check the robustness of the feature extraction technique and choice of scaling. The conditional averaging is performed as soon as a large-scale structure is detected, i.e., when the large-scale velocity fluctuation exceeds some threshold at a chosen reference wall-normal location $y_{\text {ref }}$.

2D and 3 D conditional analysis will be performed in this section to understand the large-scale structures. Firstly, the 2D conditional average is defined as:

$$
\begin{align*}
\left\langle\left.\mathfrak{u}^{\prime+}(y, \delta z)\right|_{\text {LMLSS }}\right\rangle & =\left\langle\mathfrak{u}^{\prime+}(x, y, z+\delta z) \mid\left(u_{\mathrm{L}}^{\prime+}\left(x, y_{\text {ref }}, z\right)<-1\right)\right\rangle  \tag{5.4}\\
\left\langle\left.\mathfrak{u}^{\prime *}(y, \delta)\right|_{\text {LMLSS }}\right\rangle & =\left\langle u^{\prime *}(x, y, z+\delta z) \mid\left(u_{\mathrm{L}}^{\prime *}\left(x, y_{\text {ref }}, z\right)<-1\right)\right\rangle \tag{5.5}
\end{align*}
$$



Figure 5.11: Conditionally averaged LMLSS (a, b, c, d) $\left\langle\left. u^{\prime *}(y, \delta z)\right|_{\text {LMLSS }}\right\rangle$ and (e, f) $\left\langle\left. u^{\prime+}(y, \delta z)\right|_{\text {LMLSS }}\right\rangle$ for ( $\mathrm{a}, \mathrm{b}$ ) $\mathcal{M}=0.5$ and ( $\left.\mathrm{c}, \mathrm{d}, \mathrm{e}, \mathrm{f}\right) \mathcal{M}=3.0$. Result plotted against distance scaled with ( $a, c$, e) characteristics flow length-scale $H,(b, d)$ semi-local scaled viscous length-scale $\left(a^{*}=\frac{\widetilde{a}}{\widetilde{L}_{v}(\widetilde{y}}\right)$ and (f) conventional viscous length-scale $\left(a^{+}=\frac{\widetilde{a}}{\tilde{\tau}_{v}}\right)$. Arrows show vector due to the conditional average of $v^{\prime}$ and $w^{\prime}$ based on respective conditional event for each figure. Symbol + in white is each figure is the location of the conditional event. Solid contour line is for $\left\langle u^{\prime *}(y, \delta z)\right|$ LMLSS $\rangle=0.3$ or $\left\langle\left. u^{\prime+}(y, \delta z)\right|_{\text {LMLSS }}\right\rangle=0.3$ and dashed contour line for $\left\langle\left. u^{\prime *}(y, \delta z)\right|_{\text {LMLSS }}\right\rangle=-0.2$ or $\left\langle\left. u^{\prime+}(y, \delta z)\right|_{\text {LMLSS }}\right\rangle=-0.2$.

Equation 5.4 is used when conventional wall-units are used for the threshold and equation 5.5 when semi-local scaling is used. $\rangle$ is the average of all conditional event detected in the stream- and span-wise directions and time. Results are plotted with the conventional scaling and semi-local scaling. Conditional event is from the location ( $y_{\text {ref }}$ ) of the outer-layer spectral peak in figure $5.5(\mathrm{a})$. Results of the 2 D conditional average is presented in figure 5.11 and, one can find that the LMLSS is flanked by HMLSS. Similar observation was also made in Fig. 7 of Hutchins and Marusic (2007) ${ }^{[126]}$. In the wall-normal direction contour of the averaged structure is mostly parallel. Therefore the choice of projecting the detected structure in the wall-normal direction as presented in Section 5.3.3 will not affect the results. Currently, conditional analysis for LMLSS is only presented, for HMLSS similar results were obtained (not shown here).

When conditional event was detected and scaled with semi-local friction velocity and the coordinates scaled in semi-local viscous length-scale (equation 5.5), at $\mathcal{M}=0.5$ and 3 similar results are obtained in figure $5.11(a-d)$ : averaged structures at different $\mathcal{M}$ have similar dimensions in the $y-z$ plane (figures $5.11(\mathrm{~b})$ and $5.11(\mathrm{~d})$ ). The structure width is about $500 l_{v}^{*}$. In Figures 5.11 (b) and $5.11(\mathrm{~d})$, both wall-normal and span-wise coordinates was scaled with the semi-local length scale, and skewed contour map was found (see figure $5.11(\mathrm{~d})$ ). Figure 4.2 shows the wall-normal dependence of semi-local length-scale $l_{v}^{*}$ at $\mathcal{M}=3$. At the wall $l_{v}^{*}$ is twice smaller compared the channel centre. At $\mathcal{M}=3$ conventional scaling and thresholding (equation 5.4) approach was also tested. With conventional scaling wider structure (figure $5.11(\mathrm{f})$ ) was found (approximately twice wider in the span-wise direction, 500 wall-units wide at $\mathcal{M}=0.5$ and 1000 wall-units wide at $\mathcal{M}=3$ ). Regardless of the choice of scaling for the conditional event the size of the structure in outer-variables was constant. However this does not mean scaling is not required for the feature extraction. The need for proper scaling is justified in the following.

Example for the thresholding based on different criteria is shown in figure 5.12. Raw stream-wise velocity at $\mathcal{M}=3$ in the $x-z$ plane at $y / H=0.3$ (location of outer-layer spectral peak in figure $5.5(\mathrm{a})$ ) is shown in the figure. Contour of the detected high- and low-momentum regions is shown after thresholding $u_{L L}^{\prime}$ based on $u_{\tau}$ and $u_{\tau}^{*}(y)$. Thresholding based on $u_{\tau}$ leads to more detection, due to improper scaling. For instance, in figure 5.9, rms of both large- and small-scale velocity fluctuations at different $\mathcal{M}$ number collapse satisfactorily after proper scaling. However, if similar results were computed with conventional scaling, then rms of both large- and small-scale velocity will be overpredicted at higher $\mathcal{M}$ (see for example figure $5 \cdot 3(\mathrm{~b})$ ). The need for scaling will be discussed further in the following section.

Another interesting feature which can be noticed in figure 5.12 is the size and organisation of large-scale structures. For example, within the yellow rectangle in the figure a long meandering low- and high-momentum regions of length approximately 20 H can be found. In the same region after thresholding, multiple low- and high-momentum regions of length $2 \mathrm{H}-3 \mathrm{H}$ was detected. This poses a question, whether the outer-layer structures are really very large, or are they group of multiple large-scale structures? Another interesting question is the organisation of these structures. These questions will be answered below.


Figure 5.12: Example of detected large-scale structures after different thresholding criteria at $\mathcal{M}=3$. Data is taken at the location of the outer-layer spectral peak in figure 5.5(a) $y / H=0.3$. Gray scale contour map is used for the raw stream-wise velocity field, black(white) colour is for low-(high-)momentum regions. Green colour for the criteria based on (a) $--u_{\mathrm{LL}}^{\prime}<-\mathfrak{u}_{\tau}$ and (b) $-\mathfrak{u}_{\mathrm{LL}}^{\prime}>\mathrm{u}_{\tau}$ and Magenta for criteria based on semi-local threshold (c) - - $-u_{\text {LL }}^{\prime}<-u_{\tau}^{*}(y=0.3 H)$ and (d) $-u_{L L}^{\prime}>u_{\tau}^{*}(y=0.3 H)$. Yellow rectangle is used to discuss the length and organisation of the large-scale structures.

The conditional averaging is now performed in ${ }_{3} \mathrm{D}$, and it is computed as:

$$
\begin{align*}
\left\langle\left. u^{\prime+}(\delta x, y, \delta z)\right|_{\text {HMLSS }}\right\rangle & =\left\langle u_{\mathrm{LL}}^{\prime+}(x+\delta x, y, z+\delta z) \mid u_{\mathrm{LL}}^{\prime+}\left(x, y_{\mathrm{ref}}, z\right)>1\right\rangle  \tag{5.6}\\
\left\langle\left. u^{\prime *}(\delta x, y, \delta z)\right|_{\text {HMLSS }}\right\rangle & =\left\langle u_{\mathrm{LL}}^{\prime *}(x+\delta x, y, z+\delta z) \mid u_{\mathrm{LL}}^{\prime *}\left(x, y_{\text {ref }}, z\right)>1\right\rangle \tag{5.7}
\end{align*}
$$

The conditional event was looked in the filtered field $u_{\mathrm{LL}}^{\prime}$ at $\mathrm{y} / \mathrm{H}=0.3$, and $\rangle$ is the average of all conditional event detected in the stream- and span-wise direction and time. Conditionally averaged 3 D HMLSS is presented in figure 5.13 . Slice from the figure 5.13 at $\delta x=0$ and $\delta z=0$ will give similar result as in figure 5.11. A large-scale structure will be flanked by a pair of large-scale structures of opposite sign (in figure 5.13 neighbouring LMLSS is not shown). Another example will be provided in figure 5.16. In figure 5.13, results obtained after averaging the filtered field is shown, similar results were obtained when the raw-field was averaged based on the conditional event from the filtered field (see figure 5.16 ). In figure $5.13(\mathrm{a}, \mathrm{b})$ results from $\mathcal{M}=0.5$ and 3 with semi-locally scaled threshold are presented. The dimensions of the averaged structures are more or less equal. Additionally in figure $5.13(b, c)$ conditional average based on $u_{\tau}$ and $u_{\tau}^{*}$ is presented at $\mathcal{M}=3$. The dimension of the averaged structure is almost equal with different threshold criteria.

Footprint of the averaged structure in figure 5.13 is presented in figures 5.14 and 5.15 . Footprint is estimated by projecting the averaged structure in the wall-normal direction on to an auxiliary plane (similar to the detection procedure in Section 5.3.3). Structure above $y^{+}>20$ or $y^{*}>20$ only is used for the projection. In figure 5.14 , the structure detection is based on $u_{\tau}$, and the footprint is scaled with $l_{v}$ and $H$ respectively. A clear difference can be seen in figure 5.14(a), when the footprint is scaled with $l_{v}$. At $\mathcal{M}=3, l_{v}$ is approximately half compared to the $\mathcal{M}=0.5$. When the same footprint is scaled with outer flow unit H , satisfactory agreement is observed.
(a)

(b)

(c)


Figure 5.13: Conditionally averaged 3 D HMLSS between size $3 \mathrm{H}-6 \mathrm{H}$ for (a) $\mathcal{M}=0.5$ and (b, c) $\mathcal{M}=3$. Conditional event is from the location of the outer-layer spectral peak in figure 5.5(a). Condition for $(a, b) u_{L L}^{\prime}\left(x, y_{\text {ref }}, z\right)>u_{\tau}^{*}(y=0.3 H)$, and (c) $u_{L L}^{\prime}\left(x, y_{\text {ref }}, z\right)>u_{\tau}$. Isosurface of $(\mathrm{a}, \mathrm{b})\left\langle\mathrm{u}^{\prime *}(\delta x, y, \delta z) \mid \mathrm{HMLSS}\right\rangle=0.5$ and (c) $\left\langle\left. u^{\prime+}(\delta x, y, \delta z)\right|_{\text {HMLSS }}\right\rangle=0.5$.

In figure 5.15, the threshold was based on $u_{\tau}^{*}(y)$. The estimated footprint of the largescale structure is scaled with semi-local length scale in figures 5.15(a) and H in figure 5.15(b). For figure 5.15(a), size of the structure was scaled with $l_{v}^{*}(y)$. Since $l_{v}^{*}(y)$ varies for $\mathcal{M}=3$ (twice lower at the walls compared to the channel centre, see figure 4.2 ), the structure is stretched near the wall (see figure $5.11(\mathrm{~d})$ for similar example). Nevertheless, reasonable agreement was found between the footprint at different $\mathcal{M}$ with both scaling techniques.

It can be concluded that, footprint of the large-scale structure has similar size when scaled in H regardless the thresholding criteria for different $\mathcal{M}$ (figures 5.14(b) and 5.15(b)). But more structures are detected when scaled with $l_{v}$ (see figure 5.12). Foot-
(a)

(b)


Figure 5.14: Comparison of large-scale footprint at different $\mathcal{M}$ with detection based on conventional wall-units. Size of the foot-print scaled in (a) conventional wall-units and (b) H . Solid curve for $\mathcal{M}=0.5$ and dashed line for $\mathcal{M}=3$.
(a)

(b)


Figure 5.15: Comparison of large-scale footprint at different $\mathcal{M}$ with detection based on semilocal units. Size of the foot-print scaled in (a) semi-local units and (b) H. Solid curve for $\mathcal{M}=0.5$ and dashed line for $\mathcal{M}=3$.
prints have similar size when the threshold was based on $u_{\tau}^{*}(y)$ at different $\mathcal{M}$ numbers, and larger size at higher $\mathcal{M}$ with threshold based on $u_{\tau}$.

Conditional average of HMLSS from the raw field based on the event at $y / H=0.3$ is defined as follows:

$$
\begin{equation*}
\left\langle\left. u_{\text {raw }}^{\prime *}(\delta x, y, \delta z)\right|_{\text {HMLSS }}\right\rangle=\left\langle u^{\prime *}(x+\delta x, y, z+\delta z) \mid u_{\text {LL }}^{\prime *}\left(x, y_{\text {ref }}, z\right)>1\right\rangle \tag{5.8}
\end{equation*}
$$

The result is presented in figure 5.16. Conditionally averaged HMLSS is surrounded by the LMLSS on all four sides. Similar results was obtained for conditional average of LMLSS (not shown here). By connecting figure 5.16 to figure 5.12 , one can find that away from the


Figure 5.16: Conditionally averaged 3 D HMLSS between size $3 \mathrm{H}-6 \mathrm{H}$ for $\mathcal{M}=3$. Conditional event is from the location of the outer-layer spectral peak in figure 5.5(a). Conditional event is $u_{\text {LL }}^{\prime}\left(x, y_{\text {ref }}, z\right)>u_{\tau}^{*}(y=0.3 H)$. Isosurface (a) orange $-\left\langle\left. u_{\text {raw }}^{\prime *}(\delta x, y, \delta z)\right|_{\text {HMLSS }}\right\rangle=$ 0.45 and (b) blue $-\left\langle\left. u_{\text {raw }}^{\prime *}(\delta x, y, \delta z)\right|_{\text {HMLSS }}\right\rangle=-0.3$. Dashed curves indicate the merging of large-scale structures away from the wall.
wall large-scale structures of the same type group together to form a long meandering structure (dashed curves in figure 5.16). This answers the above question about the length and organisation of the large-scale structures. These results in figures $5.11-5.16$ justifies the choice of (a) semi-local scaling, (b) projection of detected large-scale structure in the wall-normal direction and (c) outer-variable H to scale the large-scale structure size.

### 5.3.5 Quantification of large-scale influence

The detection procedure detailed in Section $5 \cdot 3 \cdot 3$ is used to perform conditional analysis based on the projection of HMLSS and LMLSS. Conditional analysis was performed for different cut-off scales in the stream-wise direction and the resulting percentage of area occupied by the projection of large-scale motions is presented in table 5.2. The percentage of change in the skin-friction coefficient compared to the overall skin-friction coefficient and the percentage of contribution to the drag coefficient, for both HMLSS and LMLSS,
are presented in tables 5.3 and 5.4 , respectively. The change in the drag coefficient is computed as

$$
\begin{equation*}
\Delta c_{f} \%=\frac{\left(c_{f}^{\text {cond }}-c_{f}^{\text {ref }}\right)}{c_{f}^{\text {ref }}} \times 100 \% \tag{5.9}
\end{equation*}
$$

where the drag coefficient is defined by:

$$
\begin{equation*}
c_{f}=\frac{2 \tau_{w}}{\rho_{\mathrm{b}} u_{\mathrm{b}}^{2}} \tag{5.10}
\end{equation*}
$$

In these equations, $\mathrm{c}_{\mathrm{f}}^{\text {ref }}$ is the global skin friction coefficient computed from all the unfiltered original flow fields of the considered configuration. For the HMLSS, $\mathrm{c}_{\mathrm{f}}^{\text {cond }}$ is computed from equation 5.10 in which $\tau_{w}$ is computed from the conditional average $\bar{u}_{\text {HMLSS }}(y)$ corresponding to this type of region. For the LMLSS, the same is done with $\tau_{w}$ computed from the conditional average $\bar{u}_{\text {LMLSS }}(y)$. The conditional $c_{f}^{\text {cond }}$ is then used to compute the change in drag coefficient with respect to the global drag coefficient $c_{f}^{\text {ref }}$.

The results in tables $5.2-5.4$, all depend on the detection procedure detailed in Section $5 \cdot 3.3$, and thus depend on the threshold value $u_{\text {threshold }}$ in step 2 of this procedure. For $\mathcal{M}=3.0$, two types of thresholding were tested: (a) thresholding based on conventional wall-variables $\mathfrak{u}_{\text {threshold }}=\mathfrak{u}_{\tau}$, and (b) thresholding based on semi-local scaling $u_{\text {threshold }}=$ $\mathrm{u}_{\tau}^{*}(\mathrm{y})$. Results obtained with conventional scaling are tabulated under column $\mathrm{M}_{3} .0$ and those obtained with semi-local thresholding are under column M3.0* in tables 5.2,5.3 and 5.4. As mentioned in previous section, span-wise cut-off for the low-pass Fourier filter was $\lambda_{z, \text { cut-off }}=0.5 \mathrm{H}$. Conditional analysis was performed with different $\lambda_{x, \text { cut-off }}=\mathrm{H}, 3 \mathrm{H}$, 6 H and 12 H in the stream-wise direction to understand the features of different largescale structures.

| $\lambda_{x, \text { cut-off }}$ | Mo. 5 |  | M3.0 |  | M3.0* |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | HMLSS | LMLSS | HMLSS | LMLSS | HMLSS | LMLSS |
| H | 27 | 28 | 35 | 38 | 28 | 30 |
| 3 H | 22 | 23 | 34 | 37 | 23 | 25 |
| 6H | 16 | 17 | 32 | 35 | 19 | 20 |
| 12H | 9 | 11 | 28 | 29 | 14 | 14 |

Table 5.2: Percentage of area occupied by the projection of HMLSS and LMLSS at M=0.5 and 3.0
From table 5.2 it is found that both HMLSS and LMLSS occupy equivalent area. With traditional scaling, reported under column $\mathrm{M}_{3} .0,20 \%-30 \%$ increase in the combined area (HMLSS + LMLSS) is found compared to the results obtained for $\mathcal{M}=0.5$, whereas with semi-local thresholding similar percentage of the area are found at $\mathcal{M}=0.5$ and $\mathcal{M}=3.0$ (columns Mo.5 and $\mathrm{M}_{3.0^{*}}$ in table 5.2). Hence by properly scaling the data at higher Mach number, we find that the large-scale structures are Mach independent except maybe at 12 H . As expected, with increasing $\lambda_{x, \text { cut-off, }}$ the number of structures detected is decreased: approximately $55 \%-60 \%$ of the area is occupied by structures greater than H including HMLSS and LMLSS, whereas approximately $20 \%-28 \%$ of the area is occupied by structures greater than 12 H . Earlier in figure 5.12 , it was shown that with thresholding based on conventional friction velocity more high- and low-momentum
large-scale regions were detected. From the results in the table it is now clear that this is an over-estimation due to poor scaling technique.

| $\lambda_{x, \text { cut-off }}$ | Mo. 5 |  | M3.0 |  | M3.0* |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | HMLSS | LMLSS | HMLSS | LMLSS | HMLSS | LMLSS |
| H | 4.5 | -5.3 | 3.7 | -3.7 | 3.7 | -3.9 |
| 3 H | 5.1 | -5.6 | 3.8 | -3.8 | 4.0 | -4.4 |
| 6 H | 6.2 | -6.3 | 4.0 | -4.0 | 4.6 | -5.1 |
| 12H | 6.8 | -7.5 | 4.3 | -4.6 | 5.6 | -6.4 |

Table 5.3: Percentage of $\Delta \mathfrak{c}_{f}$ for HMLSS and LMLSS at $M=0.5$ and 3.0 with respect to global $\mathfrak{c}_{f}^{\text {ref }}$.
$\Delta \mathfrak{c}_{f} \%$ for HMLSS and LMLSS at $\mathcal{M}=0.5$ and $\mathcal{M}=3$ is presented in table 5.3. As for the results reported in table 5.2 , values obtained for $\mathcal{M}=0.5$ and $\mathcal{M}=3.0$ are closer when a thresholding based on semi-local $\mathfrak{u}_{\tau}^{*}$ is used. A trend observable in the table is that the larger structures have higher influence on the skin friction coefficient. HMLSS greater than 12 H has about $7 \%$ higher skin friction, whereas LMLSS has $7 \%$ lower skin friction compared to the global value.

| $\lambda_{x, \text { cut-off }}$ | Mo. 5 |  | M3.0 |  | M3.0* |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | HMLSS | LMLSS | HMLSS | LMLSS | HMLSS | LMLSS |
| H | 28.2 | 26.5 | 36.6 | 36.6 | 29.6 | 29.2 |
| 3 H | 23.1 | 21.7 | 36.1 | 36.3 | 24.8 | 23.9 |
| 6 H | 16.9 | 15.9 | 33.6 | 33.9 | 20.3 | 18.7 |
| 12 H | 9.6 | 10.1 | 28.7 | 27.7 | 14.8 | 13.1 |

Table 5.4: Percentage of contribution to $\mathrm{c}_{\mathrm{f}}$ for HMLSS and LMLSS at $\mathrm{M}=0.5$ and 3 .
Percentage of contribution to $c_{f}$ is presented in table 5.4 (this table merely results from a combination of tables 5.2 and table 5.3). Again a thresholding based on semi-local scaling provides a better agreement between $\mathcal{M}=0.5$ and $\mathcal{M}=3.0$ compared with the thresholding based on $u_{\tau}$, although results do not match perfectly well for larger values of $\lambda_{x, \text { cut-off }}$ (see columns Mo. 5 and $\mathrm{M}_{3} .0^{*}$, for $\lambda_{x, \text { cut-off }}=12 \mathrm{H}$, in table 5.4). At $\operatorname{Re}_{\tau}=577$, Yoon et al. (2016) ${ }^{[299]}$ found that the large scale structures having sizes greater than H contribute $45 \%$ to the skin friction coefficient, with $25 \%$ coming from low-momentum structures and $20 \%$ coming from high-momentum structures. In the present work, we find that HMLSS and LMLSS have equivalent contributions. Structures greater than H contribute approximately $55 \%-60 \%$ (HMLSS + LMLSS) to the skin-friction, whereas structures grater than 12 H contribute $20 \%-25 \%$ to the skin friction (HMLSS + LMLSS). These result show the importance of large-scale structures for drag reduction.

Statistics computed from the conditional analysis are presented in figure 5.17 where $\sqrt{\overline{u^{\prime 2}}}, \overline{u^{\prime} v^{\prime}}$ and $\sqrt{\overline{\omega_{x}^{\prime 2}}}$ are plotted against wall-normal distance scaled with inner and outer units, where inner units are computed from the whole flow field. For $\mathcal{M}=3.0$, the inner units are based on semi-local (compressible) scaling in figure $5 \cdot 17$ (d-f) (thus the label $\mathrm{M}_{3} . \mathrm{o}^{*}$ for these figures) and on traditional wall scaling in figure $5.17(\mathrm{~g}-\mathrm{i})$ (label

M3.0). Accordingly, the thresholding in step 2 of the large scale detection procedure is based on $\mathfrak{u}_{\tau}^{*}$ for figure $5.17(\mathrm{~d}-\mathrm{f})$ and on $\mathfrak{u}_{\tau}$ for figure $5.17(\mathrm{~g}-\mathrm{i})$. Results for $\mathcal{M}=0.5$ are presented in figure $5 \cdot 17(\mathrm{a}-\mathrm{c})$ where inner units are based on semi-local scaling, but at such a low Mach number the difference between semi-local scaling and traditional wall scaling is insignificant. In the legend in figure 5.17, HSo1 and LSo1 refer respectively to HMLSS and LMLSS obtained with a stream-wise Fourier filter cut-off scale $\lambda_{x, \text { cut-off }}=H$, while HS12 and LS12 refer to HMLSS and LMLSS obtained with $\lambda_{x, \text { cut-off }}=12 \mathrm{H}$.

With traditional wall scaling at $\mathcal{M}=3, \sqrt{\overline{\mathfrak{u}^{\prime 2}}}$ and $\overline{\mathfrak{u}^{\prime} v^{\prime}}$ have higher peak values compared to $\mathcal{M}=0.5$ (compare figure 5.17 (a) vs $5.17(\mathrm{~g})$ and figure 5.17 (b) vs $5.17(\mathrm{~h})$ ) whereas $\sqrt{\overline{\omega_{\mathrm{x}}^{\prime 2}}}$ is weaker compared to $\mathcal{M}=0.5$ (figure 5.17 (c) vs 5.17 (i)). Similarly, peaks of $\sqrt{\overline{u^{\prime 2}}}$ and $\overline{\mathfrak{u}^{\prime} v^{\prime}}$ are found at $y^{+} \approx 30$ and 160 respectively with traditional wall scaling, values larger from those at $\mathcal{M}=0.5$, which results from coordinate stretching (because of higher $\operatorname{Re}_{\tau}$ at $\left.\mathcal{M}=3\right)$. On the contrary, semi-local scaled results at $\mathcal{M}=3.0\left(\mathrm{M}_{3} . \mathrm{o}^{*}\right)$ are very similar to the results obtained at $\mathcal{M}=0.5$. This shows that gross properties of large-scale structures do not depend too much on the Mach number (in the range investigated, that is, non hypersonic) as long as the proper scaling transformation is used to compensate for the compressibility effects (figure 5.17 (a) vs 5.17 (d); figure $5.17(\mathrm{~b})$ vs 5.17 (e); and figure 5.17 (c) vs $5.17(f)$ ).

Away from the wall in the outer part of the log-layer LMLSS have higher contribution to $\sqrt{\overline{u^{\prime 2}}}$ and $\overline{u^{\prime} v^{\prime}}$ compared to the global curve (see figure 5.17(a-f)), whereas near the wall and lower-part of the log-layer HMLSS have a dominant effect. For high Reynolds number flow, it is well known that the near-wall $\sqrt{\overline{u^{\prime 2}}}$ peak and the secondary peak away from the wall do no scale in wall units. From figure 5.9, it is already clear that the near-wall peak is due to the small-scales and the secondary peak is due to the large-scale contribution. From the conditional analysis it is clear that the LMLSS contributes more to the outer peak of the $\sqrt{\overline{\mathfrak{u}^{\prime 2}}}$ at higher Reynolds number and the HMLSS to the inner peak. A similar trend was observed for other velocity component as-well. This trend in the results is consistent with the findings of Lee and Moser (2015) ${ }^{[166]}$ at similar $\mathrm{Re}_{\tau}^{*}=930$. They disagree with the findings of Yoon et al. (2016) ${ }^{[299]}$ with no-slip wall simulations, who found increased turbulence for LMLSS across entire channel height. Close to wall increased Reynolds stress and enhanced $\sqrt{\overline{\omega_{x}^{\prime 2}}}$ is found for HMLSS, which supports the observations found in figures 5.4 and 5.10 (b), which both present enhanced turbulence activity associated with HMLSS due to bursting process leading to streak breakdown and strong vorticity which results in increased momentum transfer and higher turbulence intensity close to the wall. Kravchenko et al. (1993) ${ }^{[155]}$ have shown that the skin friction correlates with the stream-wise vortices near the wall, which explains the increased drag associated with HMLSS from the outer layer. For $\sqrt{\overline{u^{\prime 2}}}$ and $\overline{u^{\prime} v^{\prime}}$ the curves of HMLSS and LMLSS cross at around $y^{*}=250$. Away from the wall, LMLSS become more dominant, they carry more energy and enhanced Reynolds stress.

In figure 5.18, conditionally averaged results are scaled with their respective $u_{\tau}$ (one value of $\mathfrak{u}_{\tau}$ for HMLSS, and a different value of $\mathfrak{u}_{\tau}$ for LMLSS). Traditional wall units are sufficient at low Mach number, $\mathcal{M}=0.5$ ). Data in figure 5.18 are the same as data in figure 5.17 (a), scaled differently. Close to wall we find excellent collapse for the HMLSS and LMLSS on the global result. Near the wall even though both HMLSS and LMLSS have slightly


Figure 5.17: Represented in inner and outer scalings are: ( $a, d, g$ ) rms stream-wise velocity; (b, e, h) Reynolds stress; and (c, f, i) rms stream-wise vorticity. The Mach number is: (a-c) Mo.5; (d-f) M3.0* ; (g-i) M3.o. In ( $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}, \mathrm{f}$ ) inner units are based on semi-local scaling; in (g-i) inner units are based on conventional wall-units, where inner units (either traditional or semi-locally scaled) are computed from the whole flow field. In the legend, "Global" represents overall statistics for the whole flow field. Other labels correspond to conditional averaging: HSo1 and LSo1 stand for HMLSS and LMLSS with $\lambda_{x, \text { cut-off }}=H ; H S 12$ and LS12 stand for HMLSS and LMLSS with $\lambda_{x, \text { cut-off }}=12 \mathrm{H}$.
differentiable near-wall regeneration cycle, once they are scaled with their respective wall-friction they exhibit an equilibrium state which is similar to the global near-wall turbulence. Away from the wall, they do not collapse any more and LMLSS have a larger secondary peak value. This again suggests that the outer peak of $\sqrt{\overline{u^{\prime 2}}}$ traditionally
observed at high Reynolds numbers (see figure 5.9) is especially a manifestation of LMLSS away from the wall.


Figure 5.18: Conditional average of $\sqrt{\overline{\mathrm{u}^{\prime 2}}}$ for HMLSS and LMLSS at $\mathcal{M}=0.5$. Results for HMLSS and LMLSS are scaled using their respective $u_{\tau}$.

In the results presented above, we found similar trends as in Lee and Moser (2015) ${ }^{[166]}$ at an equivalent Reynolds number and domain size, but some differences on the results for HMLSS and LMLSS compared with Yoon et al. (2016) ${ }^{[299]}$ whose DNS at $\mathrm{Re}_{\tau}^{*}=577$ was performed in a larger domain of size $L_{x} \times L_{y} \times L_{z}=10 \pi \mathrm{H} \times 2 \mathrm{H} \times 3 \pi \mathrm{H}$. Hence, to check the validity of (a) large scale influence, (b) Reynolds number effect and (c) domain size, the conditional analysis was repeated for $\mathrm{Re}_{\tau}^{*}=590$ and 640 at $\mathcal{M}=0.1$ and 0.5. The simulation at $\mathrm{Re}_{\tau}^{*}=590$ was performed in a very small domain of size $2 \pi \mathrm{H} \times 2 \mathrm{H} \times \pi \mathrm{H}$, whereas the simulation at $\mathrm{Re}_{\tau}^{*}=640$ was performed in a domain of size $4 \pi \mathrm{H} \times 2 \mathrm{H} \times 4 \pi \mathrm{H}$. Similar mesh resolutions as reported in table 5.1 were used for the new simulations. For $\operatorname{Re}_{\tau}^{*}=590, \lambda_{x, \text { cut-off }}=3 \mathrm{H}$ and for $\operatorname{Re}_{\tau}^{*}=640, \lambda_{x, \text { cut-off }}=6 \mathrm{H}$ were chosen. The results


Figure 5.19: Conditional average of $\sqrt{\overline{u^{\prime 2}}}$ for $\mathrm{Re}_{\tau}^{*}=590$ at $\mathcal{M}=0.1$ and $\mathrm{Re}_{\tau}^{*}=640$ at $\mathcal{M}=0.5$
in figure 5.19 show that, compared to values of the global $\sqrt{\overline{\mathbf{u}^{\prime 2}}}$ for the whole flow field, higher values of $\sqrt{\overline{u^{\prime 2}}}$ are obtained close to the wall for HMLSS, and smaller values
are obtained away from the wall. The situation reverses for the LMLSS. The trends are thus similar to those reported above in figure 5.17 and the conclusions given above are expected to be valid at least in the range $\mathrm{Re}_{\tau}^{*}=590-950$ for Mach numbers up to 3 .

### 5.4 CONCLUSION

An ILES of a nearly incompressible channel flow at $\mathcal{M}=0.5$ and of a compressible channel at $\mathcal{M}=3$ were performed in order to study the effect of the large-scales together with the effect of compressibility. Both simulations were performed for the same effective Reynolds number $\mathrm{Re}_{\tau}^{*}=950$ so that similar turbulence statistics are obtained for the two Mach numbers. It has been verified that the outer-layer large scales modulate the amplitude of the near-wall small scales by straightforward flow visualization and by using the two-point covariance between the amplitude of the large scales and that of the low-pass filtered envelop of the small scales. To gain more insight into this effect, a simple large-scale detection procedure is proposed which involves obtaining a footprint on the wall of the 2 D filtered large-scale motions. Putting apart undetermined regions, this allows to segregate wall positions either into regions sitting below high-momentum large-scale structures or into regions sitting below low-momentum large-scale structures. Conditional averaging can then be performed for each of these regions. Semi-local thresholding was used in order to compensate the compressibility effects at high Mach number. In practice it means that thresholding in the large-scale detection procedure has to use a threshold based on a semi-local scale and not a threshold based on the traditional friction velocity. If this is done, gross quantities and statistics characterising large-scale structures have comparable values at both Mach numbers. Both high- and low momentum largescale structures occupy similar area. Below high-momentum large-scale structures and close to the wall there is an increased turbulent activity with momentum transfer, vorticity, and skin-friction larger than average. For low-momentum large-scale structures, it is in the outer-layer region that the turbulent activity is larger than average. The lowmomentum large-scale structures may thus contribute to the peak in turbulent energy arising in the outer layer at high Reynolds numbers. The large-scale structures have an important effect on the drag, since structures greater than 3 H contribute approximately $50 \%$ to the skin-friction. High(low)-momentum large-scale structures larger than 12 H have approximately $7 \%$ more (less) drag compared to the global drag. This supports the argument of controlling large-scale events in the outer-layer for efficient drag reduction ${ }^{[43,67, ~ 247]}$.

At the Mach numbers up to 3 investigated here, compressibility changes mainly the mean temperature and density gradients of the flow, while turbulent fluctuations are nearly incompressible. Thresholding the velocity based on the local friction velocity is only valid in these conditions. It would be interesting to study how this can be carried over to compressible fluctuations at much higher Mach numbers.

## TEMPORAL SIMULATION OF CHANNEL FLOW WITH ACOUSTIC LINER

The objective of this chapter is to investigate the turbulent channel flow in the presence of an acoustic liner, and observe what changes in the flow may result from such material (acoustic liner) compared to a channel with rigid walls. In many practical situations, liners are subject to high speed flows and turbulence, and much effort has been devoted for studying the effect of grazing flow on the liner impedance. It is for example, well known that as a result of the interaction between the acoustic and vortical modes in the holes of the perforated face sheet, the resistance increases linearly with the grazing flow speed, whereas the reactance tends to decrease ${ }^{[96,102,138]}$. Conversely, the liner may modify the flow and turbulence in its vicinity, compared to a rigid wall. An effect of this is drag increase, ${ }^{[294]}$ especially for small liner porosity. Another effect is the flow instability observed in the vicinity of a low resistance acoustic liner ${ }^{[181]}$.

First of all, the boundary condition used to model the acoustic liner will be presented in Section 6.1. In Section 6.2 grid requirements with the new wall boundary condition will be addressed. Later in Section 6.3 the flow - liner interaction will be investigated by performing a parametric study of acoustic liner's resistance and resonance frequency, and the flow $\mathcal{M}$ number. Scalo et al. (2015) ${ }^{[240]}$ performed similar study of the turbulent flow in a compressible periodic channel with an impedance boundary condition and described how the structure of turbulence gets modified as the resistance of the liner decreases. Scalo et al. (2015) ${ }^{[240]}$ set the resonance frequency of the liner so that it corresponds to some typical time scale of the flow. As a result the liner resonance frequency was rather high, and larger than typical frequencies encountered in aeroacoustic applications. Compared with Scalo et al. (2015) ${ }^{[240]}$ smaller resonance frequencies were considered in the present work, and only the bottom surface was lined in order to match the experiments of Marx et al. (2010) ${ }^{[181]}$. In Section 6.4, procedure to extract the surface wave due to the acoustic liner will be presented. In Section 6.5 it will be shown that this wave is responsible for higher drag and finally in Section 6.6 it will be shown that this surface wave was partially governed by a linear instability.

### 6.1 BOUNDARY CONDITION - IMPEDANCE WALL

Until now, isothermal rigid wall boundary conditions (see Section 2.3.2) were used for simulating channel flows. In this section, boundary conditions for the non-rigid wall will be presented, which will be used to simulate channel flow with acoustic liner. The conditions at the wall are $u=w=0$ and $\mathrm{T}_{w}=$ constant. Wall-normal velocity component at the wall is non-zero and the non-rigid wall is modelled with the impedance boundary condition. Generally speaking, an impedance boundary condition is a relation between $p$ and $v$ at a given position on the wall. An impedance wall of the Mass-Spring-Damper (MSD)
type has been used in here. This model is simple and accounts for one resonance/absorption frequency of the acoustic liner. This model was first introduced by Tam and Auriault (1996) ${ }^{[264]}$, and has been used several times since then, either in linearised codes ${ }^{[183]}$, or in NSE solvers ${ }^{[240]}$. At the bottom wall it reads:

$$
\begin{equation*}
M \frac{\mathrm{~d}^{2} v}{\mathrm{dt} \mathrm{t}^{2}}+\mathrm{R} \frac{\mathrm{~d} v}{\mathrm{dt}}+\mathrm{K} v=-\frac{\mathrm{dp}}{\mathrm{dt}} \tag{6.1}
\end{equation*}
$$

where $R$ is the resistance, $M$ the mass, and $K$ the spring constant. These quantities are respectively normalized with $\widetilde{\rho}_{b} \widetilde{\mathcal{c}}_{w}$, with $\widetilde{\rho}_{b} \widetilde{H}$, and with $\widetilde{\rho}_{b} \widetilde{c}_{w}^{2} / \widetilde{H}$. This equation can be recast into a first-order system:

$$
\begin{align*}
\frac{d v}{d t} & =Q  \tag{6.2}\\
\frac{d Q}{d t} & =\frac{1}{M}\left[-\frac{d p}{d t}-R Q-K v\right] \tag{6.3}
\end{align*}
$$

For known resistance $R$, mass $M$ and spring coefficient $K$, angular resonance frequency can be defined as:

$$
\begin{equation*}
\omega_{\mathrm{res}}=\sqrt{\frac{K}{M}} \tag{6.4}
\end{equation*}
$$

and damping ratio as:

$$
\begin{equation*}
\xi=\frac{R}{2 \omega_{\text {res }} M} \tag{6.5}
\end{equation*}
$$

of an acoustic liner can be modelled as the impedance boundary condition. In addition at the wall one still has the non-slip boundary conditions $\mathfrak{u}=0$ and $w=0$, as well as the isothermal wall condition $T=T_{w}$. All together, these are four conditions that need to be imposed in the characteristic formulation. Moreover, four characteristic quantities that need to be computed at the wall were, $\mathrm{Y}^{+}, \mathrm{Y}^{-}, \mathrm{Y}^{\mathrm{u}}, \mathrm{Y}^{\mathrm{s}}$.

First note that due to the boundary condition $u=0$, one has: $X^{+}=\frac{1}{\rho} \frac{\partial p}{\partial x}$ and $X^{-}=-\frac{1}{\rho} \frac{\partial p}{\partial x}$, and at the wall leading to

$$
X^{+}+X^{-}=X^{s}=X^{v}=X^{w}=0
$$

Due to the boundary condition $w=0$ at the wall, one has similarly:

$$
Z^{+}+Z^{-}=Z^{s}=Z^{v}=Z^{w}=0
$$

Accounting for these relations, and since one also has to satisfy $\partial u / \partial t=0$ at the wall, $Y^{u}$ can fixed from equation 2.1.

$$
Y^{u}=-\frac{1}{2}\left(X^{+}-X^{-}\right)+\frac{1}{\operatorname{Re}} \frac{1}{\rho} \frac{\partial \tau_{1 j}}{\partial x_{j}}
$$

In the same fashion $Y^{w}$ will be fixed from equation 2.3 to satisfy $\partial w / \partial t=0$.

$$
Y^{w}=-\frac{1}{2}\left(Z^{+}-Z^{-}\right)+\frac{1}{\operatorname{Re}} \frac{1}{\rho} \frac{\partial \tau_{3 j}}{\partial x_{j}}
$$

To obtain the reflected wave $\mathrm{Y}^{+}$at the bottom wall from the incident wave $\mathrm{Y}^{-}$one injects equation 6.2 into equation 2.2 to obtain:

$$
\begin{equation*}
Y^{+}=Y^{-}+2\left(\frac{1}{\operatorname{Re}} \frac{1}{\rho} \frac{\partial \tau_{2 j}}{\partial x_{j}}-Q\right) \quad \text { (bottom wall) } \tag{6.6}
\end{equation*}
$$

Note that for a rigid wall, the same relation holds with $\mathrm{Q}=0$, this is equation 2.28.

### 6.1.1 Validation

The implementation of the impedance boundary condition was validated against the reference solution of Zheng and Zhuang (2004) ${ }^{[306]}$ for the reflection of an initial Gaussian pressure pulse by a plane MSD wall. To remain in the linear inviscid regime in which the analytical solution has been derived, the amplitude of the pulse was small, and the thermo-viscous terms were neglected (the solver was then an Euler equations solver).

The computational domain was a square box with $(x, y, z) \in[-50: 50] \times[0: 100] \times[-50$ : 50]. The bottom MSD wall was at $y=0$, and on the other boundaries non-reflecting boundary conditions were used (to be defined in Section 7.1.2). The MSD characteristics, pulse size, and pulse-wall distance were the very same as those used by Zheng and Zhuang (2004) ${ }^{[306]}$. Hence, $\mathrm{R}=0.2 ; \mathrm{M}=2.0938 ; \mathrm{K}=0.4758$, and the initial pressure (mean + pulse) was given by: $p(x, y, z, t=0)=\frac{1}{\gamma}+p_{a} e^{-\ln (2) / 25\left(x^{2}+(y-30)^{2}+z^{2}\right)}$, where the amplitude $p_{a}=1 e-8$ is very small. A regular mesh size was used in all directions, with $\Delta x=\Delta y=$ $\Delta z$. Equal numbers of grid points were used in all directions, $N_{x}=N_{y}=N_{z}$. Figure 6.1 shows a comparison between the computed and the analytical solution along the $x$-axis at $t=30$, obtained with $\Delta t=0.5$ and $N_{x}=101$. The pressure from the simulation collapsed

(a)
(b)


Figure 6.1: (a) Numerical vs analytical pressure solution by Zheng and Zhuang (2004) ${ }^{[306]}$ on the $x$-axis at time $t=30$ for Gaussian pulse reflection at an impedance wall. (b) Error (symbols) vs number of grid points in one direction.
excellently on the reference data. A convergence study was performed by varying $\mathrm{N}_{x}$ (with $N_{y}=N_{z}=N_{x}$ ) with a smaller time-step $\Delta t=0.01$. Despite the boundary FDS being of order 3 , overall the error turns out to decrease as $1 / N_{x}^{6}$, which corresponds to the order of the scheme used for central points.

### 6.2 GRID REQUIREMENT FOR CHANNEL FLOW WITH ACOUSTIC LINER

The mesh size requirement in the vicinity of an impedance wall is not as well established as for a rigid wall. Hence, in this section a grid convergence study was performed to determine the grid requirements for accurate numerical simulations with impedance walls. The different configurations studied are summarized in table 6.1. Four test cases from table 6.2 were considered (ACo1, ACo2, ACo3, ACo5), which were those for which the impedance strongly affected the flow dynamics. All simulations share the same value for $\operatorname{Re}_{\mathrm{b}}=6900$, and low $\mathcal{M}$ number for which compressibility effects were negligible (see Chapter 4). For an incompressible channel flow with rigid walls (a configuration referred to as $\mathrm{M}_{395}$ ) at similar $\operatorname{Re}_{\mathrm{b}}$ one would have $\operatorname{Re}_{\tau} \approx 395^{[203]}$. Other useful quantities such as the drag coefficient and the percentage change in drag coefficient summarised in table 6.1 and 6.2 were computed as follows:

$$
\begin{align*}
c_{f} & =\frac{2 \widetilde{\tau}_{w}}{\widetilde{\rho}_{b} \widetilde{u}_{b}^{2}}  \tag{6.7}\\
\Delta c_{f} \% & =\frac{c_{f}-c_{f}^{\text {ref }}}{c_{f}^{\text {ref }}} \times 100 \tag{6.8}
\end{align*}
$$

The change in drag was computed with respect to the conventional channel flow, with $c_{f}^{r e f}$ being the drag coefficient for reference test-case M 395 . Thus, a positive value indicates a drag larger than for a channel flow with rigid walls. As it will see below, for

|  | $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}$ | $\mathcal{M}$ | R | $\omega_{\text {res }}$ | $\mathrm{N}_{\mathrm{x}} \times \mathrm{N}_{\mathrm{y}} \times \mathrm{N}_{z}$ | $\Delta \chi^{+}$ | $\Delta y_{\text {min }}^{+}$ | $\Delta z^{+}$ | $\Delta c_{f}^{\text {bot }} \%$ | $\Delta c_{f}^{\text {top }} \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ACoicc | $6 \pi \mathrm{H} \times 2 \mathrm{H} \times \pi \mathrm{H}$ | 0.3 | 0.23 | 0.185 | $351 \times 151 \times 125$ | 20 | 1 |  | 443 | 44 |
| ACoic |  |  |  |  | $351 \times 171 \times 125$ |  | 0.5 |  | 573 | 45 |
| ACoi |  |  |  |  | $351 \times 201 \times 125$ |  | 0.25 | 10 | 575 | 44 |
| ACo2c | $3 \pi \mathrm{H} \times 2 \mathrm{H} \times \pi \mathrm{H}$ |  |  | 0.367 | $185 \times 151 \times 125$ |  | 1 |  | 162 | 20 |
| $\mathrm{ACo2}$ |  |  |  |  | $185 \times 171 \times 125$ |  | 0.5 |  | 159 | 18 |
| $\mathrm{ACo3c}$ | $3 \pi \mathrm{H} \times 2 \mathrm{H} \times \pi \mathrm{H}$ |  |  | 0.738 |  | $20$ | 1 | $\begin{aligned} & 10 \\ & 20 \end{aligned}$ | $74$ | $7$ |
| $\mathrm{ACo} 3$ |  |  |  |  | $91 \times 151 \times 61$ | 40 |  |  | $78$ | -1 |
| ACo5 | $3 \pi \mathrm{H} \times 2 \mathrm{H} \times \pi \mathrm{H}$ |  |  | 2.960 | $185 \times 151 \times 125$ | 20 | 1 | 10 | O | -3 |
| AC05w | $3 \pi \mathrm{H} \times 2 \mathrm{H} \times 2 \pi \mathrm{H}$ |  |  |  | $185 \times 151 \times 251$ |  |  |  | -2 | -4 |
| Ro.1cc | $6 \mathrm{H} \times 2 \mathrm{H} \times 6 \mathrm{H}$ | 0.5 | 0.10 | 3.140 | $115 \times 151 \times 251$ | 20 | 1 | 10 | 161 | 188 |
| Ro.1c |  |  |  |  | $115 \times 171 \times 251$ |  | 0.5 |  | 176 | 176 |
| Ro. 1 |  |  |  |  | $115 \times 201 \times 251$ |  | 0.25 |  | 181 | 181 |

Table 6.1: Test-cases for grid convergence analysis with impedance boundary condition.
test-cases ACo1 and ACo2, a travelling 2D surface wave were observed in the domain, which strongly interacted with the flow due to blowing and suction at the impedance wall. Hence, the grid convergence study was mainly focused on refinement in the wallnormal direction. For test-case $\mathrm{ACo3}$ quasi-2D surface waves with ripples was observed in the span-wise direction, therefore the grid requirement in the stream- and span-wise directions were analysed. For test-case ACo 5 the influence of domain size in the spanwise direction was studied. In addition, a configuration taken from Scalo et al. (2015) ${ }^{[240]}$
was also computed to validate the computations and is referred to as Ro.1 in the table 6.1. In this configuration, the resistance $R=0.1$ is smaller than generally used for other test-cases, and both top and bottom walls had impedance boundary condition, whereas for test-cases ACo1-ACo5 only the bottom wall had an impedance boundary condition. Scalo et al. (2015) ${ }^{[240]}$ used $\mathcal{M}=0.5$, higher than used for other test-cases. All in all, configuration Ro.1 is not hugely different from the test case ACo9 (see table 6.2), except that for the latter only the bottom wall was lined. For impedance walls it is not possible to estimate a priori the friction Reynolds number or viscous length-scale. Hence, all the grid resolution mentioned in table 6.1 were based on the value of $\mathrm{Re}_{\tau}$ of the rigid wall simulation. Similarly $\mathrm{c}_{\mathrm{f}}$ and $\Delta \mathrm{c}_{\mathrm{f}} \%$ computed for the two walls were based on the velocity and viscous length scales computed for bottom and top boundaries separately.

Results from the grid convergence analysis are shown in figure 6.2, where the mean stream-wise velocity, as well as the rms of stream-wise and wall-normal velocity are presented. Unless specified all results were non-dimensionalised with inner variables of
(a)

(b)

(c)


Figure 6.2: Grid convergence for impedance wall test cases. Profiles of (a) mean velocity; (b) rms of stream-wise and (c) rms of wall-normal velocity scaled with inner variables for bottom impedance wall.
the bottom impedance wall. These results do not depend much on the grid resolution,
which was therefore deemed to be sufficient in all test cases. From this study the grid resolution that was required in the directions parallel to the wall was $\Delta x^{+}=20$ and $\Delta z^{+}=10$ (computed with friction Reynolds number $\operatorname{Re}_{\tau}=395$ for rigid-wall simulation). In the wall-normal direction, a minimal mesh size of $y_{\text {min }}^{+}=0.25-1$ was necessary. These values were about the same, or slightly more stringent than required for a rigid wall. In particular smaller the resistance of the MSD wall, smaller the $y_{\min }^{+}$should be.

The profiles for the mean velocity, rms of stream-wise and span-wise velocity for testcase Ro.1 were compared to the ones obtained by Scalo et al. (2015) ${ }^{[240]}$ for the same configuration in figure 6.3, where the rigid wall test-case M395 is also shown. Although
(a)

(b)

(c)


Figure 6.3: Comparison between the present simulation and that from Scalo et al. (2015) ${ }^{[240]}$ for test-case Ro.1. Profiles of (a) mean velocity; (b) rms of stream-wise and (c) rms of wall-normal velocity.
an exact agreement with the results of Scalo et al. (2015) ${ }^{[240]}$ was not obtained (their simulation was also a LES), both simulations provided close results. The difference seen in the mean velocity profile also shows up in the drag increase, where larger drag increase ( $\approx 180 \%$ ) was observed for test-case Ro.1, whereas Scalo et al. (2015) ${ }^{[240]}$ reported a value of $148 \%$. Nevertheless, it was clear that both simulations show similar changes compared to the channel flow with rigid wall.

### 6.3 FLOW STATISTICS

In this section the statistics of a compressible turbulent flow in a channel having an impedance boundary condition at the bottom wall and a rigid upper wall are presented. The different test-cases for which ILES have been carried out are summarized in table 6.2. Of particular interest is the dependence of the flow on the resonance frequency and re-

| Case | $\mathcal{M}$ | R | $\omega_{\text {res }}$ | $\xi$ | $\omega_{\text {flow }}$ | $\operatorname{Re}_{\tau}^{\text {bot }}$ | $\operatorname{Re}_{\tau}^{\text {top }}$ | $\Delta \mathrm{c}_{\mathrm{f}}^{\text {bot }} \%$ | $\Delta \mathrm{c}_{\mathrm{f}}^{\text {top }} \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ACo1 | 0.3 | 0.23 | 0.185 | 0.109 | 1.84 | 1031 | 476 | 575 | 44 |
| AC02 | 0.3 | 0.23 | 0.367 | 0.109 | 1.84 | 642 | 434 | 162 | 20 |
| AC03 | 0.3 | 0.23 | 0.738 | 0.109 | 1.84 | 523 | 411 | 74 | 7 |
| AC04 | 0.3 | 0.23 | 1.479 | 0.109 | 1.84 | 453 | 399 | 31 | 1 |
| AC05 | 0.3 | 0.23 | 2.960 | 0.109 | 1.84 | 397 | 392 | 0 | -3 |
| AC06 | 0.3 | 0.23 | 5.887 | 0.109 | 1.84 | 390 | 386 | -3 | -5 |
| AC07 | 0.3 | 0.23 | 2.959 | 0.045 | 1.84 | 392 | 390 | -2 | -3 |
| AC08 | 0.3 | 0.10 | 2.961 | 0.045 | 1.84 | 438 | 398 | 22 | 1 |
| AC09 | 0.4 | 0.10 | 2.961 | 0.045 | 2.51 | 514 | 413 | 66 | 7 |
| AC10 | 0.1 | 0.10 | 2.961 | 0.045 | 0.63 | 385 | 383 | -4 | -5 |
| AC11 | 0.3 | 0.50 | 0.185 | 0.237 | 1.84 | 396 | 390 | 1 | -2 |
| AC12 | 0.3 | 1.00 | 0.185 | 0.474 | 1.84 | 393 | 391 | -1 | -2 |

Table 6.2: Simulation test-cases with impedance boundary condition.
sistance of the liner and dependence of liner on the $\mathcal{M}$ number of the flow. Several Mach numbers have been used, but all of them remain small. $\operatorname{Re}_{\tau}$ and $\Delta \mathfrak{c}_{f}$ were computed separately for bottom and upper walls using their respective inner variables.

Scalo et al. (2015) ${ }^{[240]}$ performed channel flow simulations with impedance walls for $\mathcal{M} \leq 0.5$ and resistance ranging from $R=0.01-1$. In addition, they tuned the resonance frequency of the liner so that it matches the typical angular frequency of the flow, defined to be:

$$
\begin{equation*}
\omega_{\text {flow }}=2 \pi \mathcal{M} \tag{6.9}
\end{equation*}
$$

This frequency is typically high and much larger than the acoustic frequency that would be found in aero-engines. In the present work, $\omega_{\text {res }}$ is first taken to be smaller than $\omega_{\text {flow }}$ for case ACo1 and is progressively increased for the test-cases ACo2 - ACo6, while keeping $\operatorname{Re}_{b}, \mathcal{M}$, and $R$ constant. The resistance is then varied for test-cases ACo7-ACo8 and $\mathrm{AC}_{11}-\mathrm{AC}_{12}$. The Mach number is varied for test-cases ACo8-AC1o.

The baseline frequency and resistance for test-case ACo1 correspond to that for which an instability had been measured experimentally by Marx et al. (2010) ${ }^{[88]}$, and the MSD characteristics were chosen to fit the impedance law of the liner in the vicinity of the resonance frequency. The fit was the same as the one used in Marx (2012) ${ }^{[182]}$. In dimensional units, this fit provides: $\widetilde{R}=94.4 \mathrm{~kg} \mathrm{~m}^{-2} \mathrm{~s}^{-1}$; $\widetilde{M}=0.0685 \mathrm{~kg} \mathrm{~m}^{-2}$; and $\widetilde{K}=2.71 \times 10^{6} \mathrm{~kg}$ $\mathrm{m}^{-2} \mathrm{~s}^{-2}$. The resonance frequency is thus $\widetilde{f}_{\text {res }}=\widetilde{\omega}_{\text {res }} /(2 \pi) \sim 1 \mathrm{kHz}$, which corresponded to a realistic value for a liner. The normalized resistance $R \sim 0.23$ has a rather low value
(lower than what would be found in practice in most aero-engines). In the experiments of Marx et al. (2010) ${ }^{[181]}$, the half-height of the rectangular channel was $\widetilde{\mathrm{H}}=0.01 \mathrm{~m}$ and this value was used here to compute the normalised numbers. In the experiment the friction Reynolds number upstream of the liner was $\operatorname{Re}_{\tau} \sim 3000$, which is costly to compute at the moment, even with an ILES. The simulations are thus performed at $\operatorname{Re}_{b}=6900$, which corresponded to a smaller value of the friction Reynolds number $\operatorname{Re}_{\tau}=395$. In the following the results of the present simulations were compared with the DNS of an incompressible channel flow with rigid walls at $\operatorname{Re}_{\tau} \approx 395$ made by Moser et al. (1999) ${ }^{[203]}$.

Test-case ACoI was performed in a computational domain of size $L_{x} \times L_{y} \times L_{z}=6 \pi H \times$ $2 \mathrm{H} \times \pi \mathrm{H}$ taken to be long to match the long wavelengths of surface wave corresponding to the low resonance frequency. Other test-cases use $3 \pi \mathrm{H} \times 2 \mathrm{H} \times \pi \mathrm{H}$. For all the test-cases, the grid resolution was $\Delta x^{+}=20$ and $\Delta z^{+}=10$, based on the inner variables of the conventional channel at $\operatorname{Re}_{\tau}=395$. In the wall-normal direction, for test-case ACo1, $\Delta y^{+}$ varies between 0.25 at the walls and 10 in the channel centre, whereas for the other cases $\Delta y^{+}$varies between 1 and 10.

### 6.3.1 Effect of the liner resistance $R$

The effect of liner resistance was first investigated. Olivetti et al. $(2015)^{[211]}$ performed the simulation of a pipe flow with a liner having a resistance larger than 1 and reported that the turbulence statistics were not much modified by the liner, compared to a rigid wall. Scalo et al. $(2015)^{[240]}$ performed a series of simulations for resistance varying between 0.01 and 1. They observed important changes in the flow statistics for a low resistance value. The effect of resistance is presently investigated for 3 different values of the resistance with $\omega_{\text {res }}=0.185: R=0.23$ (test-case $A C 01$ ); $R=0.5$ (AC11); and $R=1$ (AC12). The mean velocity profile and the rms of the stream-wise velocity are plotted in figure 6.4. Also plotted is the $\mathrm{M}_{395}$ reference data for rigid walls. It is clear that for the lower resistance $R=0.23$ there were important changes in the flow statistics compared to the rigid wall, with a lower and broader peak of $\sqrt{\overline{u^{\prime 2}}}$, and a quasi-disappearance of the mean flow logarithmic region (that would be interesting to know if this remains true at higher values of the Reynolds number). The law of the wall is not followed either. For $R=0.5$ and 1 , the statistics are very close to the rigid wall channel. This is in line with the findings of Olivetti et al. $(2015)^{[211]}$ and Scalo et al. $(2015)^{[240]}$.

In figure 6.5, change in the percentage change of drag with higher and lower values of liner's resonance frequency are presented. Test-cases ACo7 and ACo8 have $\omega_{\text {res }} \approx 2.96$ and $0.1<R<0.23$, whereas test-cases ACo1, AC11 and AC12 have $\omega_{\text {res }}=0.185$ and $0.23<R<1.0$. All these test-cases were at $\mathcal{M}=0.3$. From the figures it was clear that, the flow was affected when the resistance decreased, or permeability through the impedance surface increased ${ }^{[211,240]}$. Low resonance frequency liners at low resistance will strongly affect the turbulent flow, theses configurations were difficult to simulate and tends to be unstable. Hence in the following, configurations corresponding to a comparatively small resistance ( $R=0.23$ ) were analysed.


Figure 6.4: Profile of (a) mean and (b) rms of the stream-wise velocity for varying resistance of the liner. Values of the resistance are $R=0.23$ (ACo1); $R=0.5$ ( AC 11 ); $\mathrm{R}=1$ (AC12). The straight dotted line indicates the log law, and the other dotted line the viscous sub-layer (law of the wall).


Figure 6.5: Percentage change in the drag as a function of liner resistance. Pink curve is for $\omega_{\text {res }} \approx$ 2.96 and blue curve is for $\omega_{\text {res }}=0.185$.

### 6.3.2 Effect of the resonance frequency $\omega_{\text {res }}$

In this section the influence of the resonance frequency of the liner on wall turbulence was analysed and compared to the rigid wall turbulence. Test-cases ACo1 - ACo6 were considered, which correspond to a constant value of $\operatorname{Re}_{\mathrm{b}}$ and $\mathcal{M}$, and a given low resistance value $R=0.23$. The resonance frequency varies between $\omega_{\text {res }}=0.184$ and $\omega_{\text {res }}=5.887$. The typical angular frequency of the flow was $\omega_{\text {flow }}=2 \pi \mathcal{M}=1.84$.

The mean velocity profile for the different test-cases is shown in figure 6.6. For low resonance frequency (test-cases ACo1 - ACo4), the mean profile departs from the reference profile M395 and the law-of-the-wall was not valid any more. The flow speed was smaller in the vicinity of the impedance bottom wall and due to flow-rate conservation
(a)

(b)


Figure 6.6: Mean velocity profile for varying resonance frequency of the liner, scaled with (a) outer units $c$ and $H$; (b) inner variables $u_{\tau}$ and $l_{v}$ from the bottom impedance wall. In (b) the straight dotted line indicates the log law, and the other dotted line the viscous sublayer (law of the wall).
and the top wall being rigid, the flow speed was larger in the upper half of the channel. For the high resonance frequency test-cases (ACo5 and ACo6), the mean velocity profile follows the reference curve M395 for the rigid channel. Hence, for resonance frequencies somewhat higher than the flow frequency, the MSD wall behaves as a rigid wall, even at low resistance.

Figure 6.7 presents the rms of stream-wise velocity, wall-normal velocity, Reynolds stress, and rms of stream-wise vorticity. For higher resonance frequencies (test-cases $\mathrm{ACo5}$ - $\mathrm{ACo6}$ ) the turbulent statistics do not differ significantly from the ones for the rigid channel M395, which was the same behaviour as for the mean velocity profiles. For the low frequency test-cases (ACo1 - ACo4) noticeable differences were seen in the profiles of all quantities compared to the rigid wall channel M395. A broader peak in $\sqrt{\overline{u^{\prime 2}}}$ was seen in the buffer layer (as has been already noticed in figure 6.4 (b) for test-case ACor), and it occurs at a different location. This peak results from turbulent production, meaning the turbulent production was strongly affected by a low resonance frequency acoustic liner. Figure 6.7 (c) shows that large values of the Reynolds stress were found close to wall for test-cases ACo1 - ACo4, which indicated an increased momentum transfer in the turbulent flow throughout the channel. This momentum transfer increases the drag as will be shown in Section 6.5. In addition, a drop in $\sqrt{\overline{\omega_{x}^{\prime 2}}}$ was observed in figure 6.7 (d) for these test-cases, and a similar drop of the other components of the vorticity (not shown here) was observed. It is well known that there exists a near-wall turbulence regeneration mechanism involving streaks and stream-wise vortices. This classical nearwall turbulence mechanism was strongly disturbed for test-cases ACor and ACo2, the flow dynamics being strongly affected by the acoustic liner and the non-vanishing wallnormal velocity at its surface. The value of $\sqrt{\overline{v^{\prime 2}}}$ at the wall $\left(\sqrt{\overline{v^{\prime 2}}}\right)_{w}$ does not vanish for a liner and decreases when $\omega_{\text {res }}$ increases, as seen in figure 6.7 (b) (see also table 6.3). It was found that $\left(\sqrt{\overline{v^{\prime 2}}}\right)_{w}$ was roughly inversely proportional to the resonance


Figure 6.7: Profiles of (a) rms stream-wise velocity; (b) rms wall-normal velocity; (c) Reynolds stress and (d) rms stream-wise vorticity for varying resonance frequency of impedance boundary condition.
frequency. For large resonance frequencies (test-cases $\mathrm{ACo}-\mathrm{ACo6}),\left(\sqrt{\overline{v^{\prime 2}}}\right)_{w} \rightarrow 0$, the liner behaves nearly as a rigid wall, and the statistics of turbulence were close to those for a rigid wall, even for low value of the liner resistance.

### 6.3.3 Effect of the flow Mach number $\mathcal{M}$

The effect of the $\mathcal{M}$ number was studied for lower values of resistance and higher resonance frequency (test-cases $\mathrm{ACo8}-\mathrm{AC} 10$ ). The percentage change in the drag at different $\mathcal{M}$ number is presented in figure 6.8. Drag increase was proportional to the change in $\mathcal{M}$ number, Scalo et al. $(2015)^{[240]}$ also had similar observations. For $\mathcal{M}=0.1$, a slight decrease is the drag was observed, whereas for $\mathcal{M}=0.4,60 \%$ increase in the drag was found. Liner with lower resonance frequency would interact more strongly with the flow
at higher $\mathcal{M}$ number. These configurations are difficult to simulate and becomes unstable.


Figure 6.8: Percentage change of drag as function of $\mathcal{M}$ number.

### 6.4 Existence of a wave along the acoustic liner

In the previous section, it had been shown that for a liner with a resistance sufficiently small and a resonance frequency not too large, the statistics differ from that of a turbulent channel flow with rigid walls. It was shown in this section that this is due to (or accompanied with) the presence of waves along the impedance wall. To evidence these waves the velocity spectra in the stream-wise direction are presented in figure 6.9. The stream-wise wave-number is denoted by $k_{x}$. The spectra were obtained at a position $y=0.015$ close to the impedance wall. The effect of the acoustic liner on the stream-wise spectra was clearly observed, as energy piles up at the resonance frequency, which resulted in the partial modification of the turbulent energy cascade. For test-cases ACor ACo5 the spikes in $E_{u u}$ and $E_{v v}$ were observed, whose wave-number corresponded to the resonance frequency of the acoustic liner (see figure 6.9 (a)). Harmonics were found for test-case ACo1. For test-cases $\mathrm{ACo}_{3}$ - ACo4, spikes in $\mathrm{E}_{w w}$ were observed. Spectra for test-case ACo6 were in good agreement with the spectra for the rigid wall (reference data $\mathrm{M}_{3} 95$ ). The angular frequency corresponding to the waves, $\omega_{\text {wave, }}$ has been computed from measurements in time made at a point in the flow and reported in table 6.3. Overall, the observed frequency corresponds to the resonance frequency of the acoustic liner. For low values of $\omega_{\text {res }}$, the convection speed for the surface wave, $c_{\text {wave }}$, was such that $\mathrm{c}_{\text {wave }} / u_{\mathrm{b}} \approx 0.6$. In order to assess if the vertical movement at the lined wall can perturb wall turbulence, the vertical displacement amplitude at the lined surface is estimated from $d_{a}=\left(\sqrt{v^{\prime 2}}\right)_{w} / \omega_{\text {wave }}$. In wall units it becomes $d_{a}^{+}=d_{a} \operatorname{Re}_{\tau}$, where the friction Reynolds number for a rigid wall is used here $\left(\operatorname{Re}_{\tau}=395\right)$. For case ACo1, $d_{a}^{+}$is more than 50, which means that the vertical displacement from the wall goes well beyond the turbulent production region well-known to be located at $y^{+} \approx 15$. For case ACo2, $d_{a}^{+}$ is also large. This explains why turbulence is so affected by the liner in these cases. As the resonance frequency increases the value of the vertical displacement decreases. For


Figure 6.9: Velocity spectra of (a) stream-wise; (b) wall-normal; (c) span-wise velocity, versus the stream-wise wave-number $k_{x}$, at a location close to the wall $(y=0.015)$, for several resonance frequencies (test-cases ACo1-ACo6).
case $\mathrm{ACo5}$ and ACo6, the displacement is less than $\mathrm{d}_{\mathrm{a}}^{+}=1$, and $\mathrm{y}^{+}=1$ is often taken to be the first grid point position off the wall in LES. Hence, for these two cases the wall displacement is too small to modify turbulence and turbulent statistics are similar tot he rigid wall channel (M395), as observed above.

Some flow visualizations are presented to illustrate the presence of the wave and show its effect on the flow. Instantaneous visualizations of the turbulent structures for the bottom half of the channel is shown in figure 6.10, where flow direction is from left to right. Among the displayed quantities is a slice of the wall-normal velocity component close to the wall (red colour for blowing with $v$ out of the impedance wall, and blue colour for suction with $v$ into the impedance wall). Also shown are iso-contours of Q2 events (ejections) and $\mathrm{Q}_{4}$ events (sweeps), coloured with wall-normal location (darker shade close to wall and lighter away from the wall, blue (red) shade is used for Q2 $\left(Q_{4}\right)$ events). Iso-contours of the Q -criterion are finally coloured in yellow to show the near-wall turbulent structures.

|  | ACo1 | AC02 | ACo3 | ACo4 | ACo5 | ACo6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\omega_{\text {res }}$ | 0.185 | 0.370 | 0.739 | 1.48 | 2.96 | 5.92 |
| $\omega_{\text {wave }}$ | 0.185 | 0.380 | 0.78 | 1.5 | 3.0 | 5.1 |
| $\mathrm{k}_{x}$ | 1 | 2 | 3 | 12.3 | 20.3 | 20.3 |
| $\mathrm{c}_{\text {wave }} / u_{\mathrm{b}}$ | 0.61 | 0.62 | 0.42 | 0.41 | 0.49 | 0.87 |
| $\left(\sqrt{\overline{v^{\prime 2}}}\right)_{w}$ | 0.024 | 0.0094 | 0.0068 | 0.0047 | 0.0016 | 0.0005 |
| $\mathrm{~d}_{\mathrm{a}}^{+}$ | 51 | 20 | 3.5 | 1.2 | 0.21 | 0.039 |

Table 6.3: Characteristics of the wave along the acoustic liner: $\omega_{\text {wave }}, k_{x}$, and $c_{\text {wave }}$ are the measured angular frequency, stream-wise wave-number, and phase speed of the wave along the impedance wall, while $\left(\sqrt{\overline{v^{\prime 2}}}\right)_{w}$ is the rms of the wall-normal velocity computed at the impedance surface. The quantity $\mathrm{d}_{\mathrm{a}}^{+}$is the estimated vertical amplitude of the displacement of a point on the lined surface in wall-units.

For the low resonance frequency test-case ACo1, a large scale 2D wave was clearly visible which propagates in the stream-wise direction, with alternating blowing and suction regions. This wave strongly modified and modulated the flow: structures (in yellow) were mainly present in the blowing regions and were absent in the suction regions, leading to an inhomogeneous distribution of turbulent structures. In addition the near-wall streaks which are characteristic of the rigid wall turbulence and an essential ingredient of the regeneration cycle were not present. Q2 ejections were logically found in the blowing region ( $v$ out of the impedance wall), since the fluid was pushed away from the wall, and $\mathrm{Q}_{4}$ sweeps were found in the suction regions ( $v$ into the impedance wall).

For the higher resonance frequency test-case ACo4, waves of $v$ at the impedance wall were observed that were not 2 D any more and undulations were present in the spanwise direction. The Q2 and $Q_{4}$ events were rooted in the impedance wall but tends to merge farther away from the impedance surface where their scale become larger than the wavelength. This indicated an interaction between the flow and wave when $\omega_{\text {flow }} \approx \omega_{\text {res }}$. Unlike the flow for test-case ACo1, the flow for test-case ACo4 was densely populated with the near-wall turbulent structures. For test-cases with $\omega_{\text {res }}>\omega_{\text {flow }}$ (not shown here) near-wall turbulent structures similar to rigid wall turbulence were observed. This may not be the case if the resistance was smaller than for the baseline configuration (test-cases ACo1 - ACo6). For example, figure 6.10 (c) corresponds to test-case ACo9, where $\omega_{\text {res }}$ is large (but not much larger than $\omega_{\text {flow }}$ ) and where the resistance is very small ( $R=0.1$ ). In that case there was a large interaction between the small scales in the flow and the liner, leading to small span-wise rollers. Parameters for test-case ACo9 were close to those used by Scalo et al. $(2015)^{[240]}$, and these authors observed very similar patterns along their impedance wall.

A slice of the instantaneous fluctuations of stream-wise velocity at wall distance $y=$ 0.03 , and slices of wall-normal velocity and pressure at the impedance surface are shown in figure 6.11. The computational domain being larger for test-case ACo1, only a half of it is shown in the stream-wise direction. For the low resonance frequency test-cases (ACor and $\mathrm{ACo2}$ ), waves were seen for all three variables. The waves were 2 D with a phase


Figure 6.10: Instantaneous visualization of turbulent structures for the test-cases: (a) ACo1, (b) ACo4 and (c) ACo9. Several quantities are shown: a color-map of $v$ at the wall (light blue is for negative $v$ corresponding to flow into the impedance wall, light red is for $v$ positive corresponding to flow out of the impedance wall); iso-contours of $\mathrm{Q}^{+}=-4$, shown in blue and corresponding to ejection events; iso-contours $\mathrm{Q}^{+}=-4$ shown in red and corresponding to sweep events; iso-contours of Q -criterion $\mathrm{Q}^{+}=3$ in yellow color.
difference of $\pi$ between stream-wise and wall-normal velocity. As $\omega_{\text {res }}$ increases the wave progressively became less 2D. This was clearly seen for test-case ACo4 ( $v$ component). For the high resonance frequency test-case ACo6, elongated streaks were found close to the wall (bottom slice for $\mathfrak{u}^{+}$), which is the feature of rigid wall flows. Waves for pressure were not observed.

Hence, for a low resonance frequency, a wave was present along the liner surface. This wave had a rather two dimensional character which was lost as the resonance frequency increased and the wave length approached the typical size of the structures in the flow. Phase averaging was now performed in order to obtain the wave spatial distribution. Phase averaging allows distinguishing between the effect of the wave and that of the random turbulence. Phase averaging relies on the following triple decomposition ${ }^{[229]}$ for any quantity a :

$$
\begin{equation*}
a=\bar{a}+a^{\prime}=\bar{a}+\widetilde{a}+a^{\prime \prime} \tag{6.10}
\end{equation*}
$$



Figure 6.11: Slices of instantaneous fluctuations of $u^{+}$(left) at wall distance $y=0.03$, and of $\nu^{+}$(centre) and $\mathrm{p} / R u_{\tau}$ (right) at the impedance wall for the test-cases ACo1, ACo2, ACo4, and ACo6. Blue (red) colour corresponds to negative (positive) values, and the considered range is $-6 \leq u^{+} \leq 6,-1 \leq v^{+} \leq 1$ and $-6 \leq p / R u_{\tau} \leq 6$ for all cases.
where $\bar{a}$ is the Reynolds average, $\widetilde{a}$ is the contribution from the wave, and $a^{\prime \prime}$ is the contribution from turbulence (note that depending on the context a tilde $\widetilde{(\overline{)}}$ is used either for a dimensional quantity or for a wave component). The extraction of the wave is based on phase averaging: $\widetilde{\mathrm{a}}(\phi)=<\mathrm{a}>_{\phi}$ where $<\cdot>_{\phi}$ is the phase average, i.e., the average over all the available samples corresponding to the same phase $\phi$, with $0 \leq \phi<2 \pi$. When the wave results from some periodic external triggering, the phase reference for phaseaveraging was provided by this external trigger. Here the wave was self excited and one cannot rely on such external reference. However, for test-cases where the wave was twodimensional its normal component $v$ at the impedance surface was not noisy (see figure 6.10 and 6.11) and sinusoidal; the phase of $v$ at the wall was thus taken as the phase reference. The procedure used to extract the wave (amplitude and phase angle) is the following:

1. For any flow field, average of $v$ at the impedance surface in the span-wise direction was computed to obtain a periodic 1D wave of $v$ in the flow direction. This is legitimate as long as the wave is 2 D ;
2. Define several phases within a period (in the present case 12 bins were used);
3. For any component $(u, v, \ldots)$ a phase bin was assigned to any stream-wise position. This phase bin was simply taken to be that of the 1D wave of $v$ which serves as the phase reference; Phase averaged components at each phase were computed, by averaging the samples corresponding to the same phase bin;
4. The global mean was subtracted to obtain wave profiles at each phase;
5. The amplitude and phase angle of the waves were computed using the wave profiles at each phase.

Several instantaneous flow fields were used for this process. The random components $\mathrm{a}^{\prime \prime}$ can then be obtained by subtracting the global mean $\overline{\mathrm{a}}$ and the phase average $\widetilde{\mathrm{a}}$ from instantaneous fields. 2D surface waves which go all the way up to the channel centre were obtained at low values of $\omega_{\text {res }}$. Hence, phase average was computed for test-cases ACo1 - ACo3. Satisfactory 2D surface wave were observed for test-cases ACo1 and ACo2. However, entirely trustworthy results were not expected from test-case ACo3, since the 2 D character of the wave was partially lost.

The amplitude of the phase-averaged stream-wise, wall-normal velocity, pressure and density are shown in figure 6.12. The stream-wise velocity component of the wave and
(a)

(c)

(b)

(d)


Figure 6.12: Amplitude of the wave along the liner: (a) stream-wise velocity component; (b) wallnormal velocity component; (c) pressure and (d) density.
density were larger close to the wall, and the maximum decreased as the resonance frequency was increased. For the wall-normal velocity component and pressure, the peak amplitude was obtained away from the impedance surface, and also decreased when $\omega_{\text {res }}$ was increased. Some characteristics of the wave (wavelength, phase speed) were given in table 6.3. The shapes in figure 6.12 were reminiscent of the wave measured and modelled by Marx and Aurégan (2013) $)^{[184]}$ (see their Fig. 14 where normalized eigenfunctions for
$u$ and $v$ are given). However, in the present case the wave was not in its linear regime. More details are given in Section 6.6.

### 6.5 DRAG

In this section it was shown that the modifications in the flow induced by the liner were associated with an increase in the drag, compared with the channel with rigid walls. The friction at the wall is classically measured by $\operatorname{Re}_{\tau}$ defined in equation 2.25. This is given in table 6.2 for both the bottom impedance surface and the top rigid wall. Table 6.2 shows that the friction at the bottom impedance surface was more significant than the top rigid wall in the test-cases for which the flow modifications were important. This was the case for low resistance and a not too large resonance frequency, that is for test-cases ACo1$\mathrm{ACo}_{3}$ and ACog. For three of these cases a flow visualization has been shown in figure 6.10. For test-case ACo1 the drag was increased by as much as $575 \%$.

The connection between the wave along the acoustic liner and the drag increase is now discussed. Using the phase-averaging process introduced in the previous section, it is possible to compute the drag increase at each phase of the wave. This is shown in figure 6.13. The phase averaged wall-normal velocity at the impedance surface, which was used as reference for phase averaging, is also presented in the figure. Phases $\Phi=0$ (and $2 \pi$ )


Figure 6.13: (a) Percentage change in drag computed after phase averaging and (b) reference curve at the impedance wall.
correspond to blowing ( $v$ out of the impedance wall) and phase $\Phi=\pi$ to suction ( $v$ into the impedance wall). During suction there was a tremendous increase in the drag, up to $\Delta c_{f} \% \approx 2200 \%$ for test-case ACo1, due to the strong impingement of the flow at the wall. Averaged over a period the drag increase is $575 \%$ (table 6.2). Remember from figure 6.10 that for test-case ACo1 no turbulent structures were present in the suction regions. Hence, the drag increase was not due to the effect of turbulent structures, but rather due to the wave, which brings in high speed fluid to the liner surface. Figure 6.13 also shows that there was a small drag reduction $\left(\Delta \mathrm{c}_{\mathrm{f}} \%<0\right)$ during blowing for test-cases ACor and ACo2.

Now the triple decomposition is used to decompose the total Reynolds stress $\overline{u^{\prime} v^{\prime}}$ into several contributions:

$$
\begin{align*}
\overline{\mathfrak{u}^{\prime} v^{\prime}} & =\overline{\left(\widetilde{u}+\mathfrak{u}^{\prime \prime}\right)\left(\widetilde{v}+v^{\prime \prime}\right)}  \tag{6.11}\\
& =\overline{\widetilde{u} \tilde{v}}+\overline{u^{\prime \prime} v^{\prime \prime}}+\widetilde{\tilde{u} v^{\prime \prime}+u^{\prime \prime} \widetilde{v}}
\end{align*}
$$

The 3 contributions are those from the wave, from the turbulence, and from cross terms. In the original triple decomposition ${ }^{[229]}$, these cross terms are null because the wave and the turbulence were supposed to be uncorrelated, but such an assumption cannot be made a priori in the present case when the wave can modulate the turbulence. Nevertheless, it has been verified that the contribution of these terms was very small. The contributions for the other two terms are shown for test-case ACo1 in figure 6.14. Inner


Figure 6.14: Contributions to the Reynolds stress $\overline{u^{\prime} v^{\prime}}$, scaled with inner variables from the bottom impedance wall for test-case ACor.
variables based on the bottom impedance surface were used for scaling the results. Therefore, the peak values close to the top rigid wall had a smaller magnitude. Close to the impedance surface, the major contribution to the Reynolds stress were from $\overline{\widetilde{u} \widetilde{v}}$, whereas away from the wall it was from $\overline{u^{\prime \prime} v^{\prime \prime}}$. Hence, close to the wall an increased momentum transfer was mainly due to the wave, and since $\overline{\widetilde{u} \widetilde{v}}$ is negative, the momentum was transferred to the wall, which contributes to increase in the drag. This was in agreement with the drag increase being correlated with the wave, as observed above in figure 6.13.

It is interesting to draw a parallel between the wave created here spontaneously in the vicinity of the liner and the waves that are sometimes imposed using blowing and suction (or some related forcing) in flow control ${ }^{[84,113,174]}$. In flow control it is well known that the surface waves resulting from blowing/suction should generally have a negative phase speed to obtain a drag reduction. For example Mamori and Fukagata (2014) ${ }^{[174]}$ performed simulations of a channel flow with a wave-like wall-normal body-force. For upstream travelling waves (with a propagation speed smaller than the bulk velocity) they found drag reduction of up to $40 \%$. They reported the presence of span-wise rollers which produced positive $\overline{u^{\prime} v^{\prime}}$ close to wall and which contributed to reducing the drag. Here in figure 6.14 the opposite behaviour was found since the wave had a positive phase speed and $\overline{\widetilde{u} \widetilde{v}}$ was negative, which led to drag increase. In these test-cases the wave was formed spontaneously by the interaction between the flow and the boundary condition, thus it was not a controlled configuration. Hœpffner and Fukagata (2009) ${ }^{[113]}$
studied wall actuation, such as wall deformation or wall blowing and suction. They argued that both actuation can be characterized as pumping and this pumping was strongly connected to drag reduction. Here the drag increased as a result of the phase difference between the $\widetilde{u}$ and $\widetilde{v}$ components of the wave, and this also correspond to some pumping by the wave. The acoustic liners produced a wave with the phase speed in the flow direction and magnitude of the order of $u_{b}$ and it is unlikely that they can be used for passive drag reduction. Nevertheless, a slight drag decrease was reported for some test-cases in table 6.2.

### 6.6 LINEAR STABILITY ANALYSIS

In this section it is shown that the wave observed above the liner can be connected to an unstable surface wave. In the literature there has been several investigations of the surface wave modes in flow duct acoustics ${ }^{[32,36,182,184,231,232]}$. All these investigations have been done in a spatial frame of work, in which the (real) mode frequency is given and the (complex) mode wave-number is computed. This is the traditional way of computing a wave-number spectrum in acoustics. In the present case, since the computational domain was periodic in the flow direction, a temporal analysis was more relevant: the (real) wavenumber $k_{x}$ was given, and the spectrum of the (complex) angular frequency $\omega=\omega_{r}+\mathrm{i} \omega_{i}$ was computed. Both the linearised Euler or NSE, possibly complemented with a turbulent eddy viscosity model, were encountered for modal analysis. Normally, an unstable surface mode was found by using an inviscid model, and including the dissipative phenomena provides a better estimation of its characteristics ${ }^{[184]}$. In the present work, the two-dimensional linearised NSE for compressible perturbations were employed. Given low $\mathcal{M}$ numbers were used for the numerical simulations, the shear base flow $\mathrm{U}_{0}(\mathrm{y})$ for the linearisation is almost incompressible, and the mean density and temperature were uniform. The linearised NSE are given in equation E. 1 to E. 5 in Appendix E, where the same normalization (see Section 2.1) as for the numerical simulation was used. These equations were discretized in the wall-normal direction in the same way as in Marx and Aurégan (2013) $)^{[184]}$, which leads to the eigenvalue matrix problem in equation E.6. The MSD boundary condition at the bottom wall $(y=-1)$ was easily included in this eigenvalue problem, see equation E. 16 to E.17. The top wall at $y=1$ was rigid. The solution of equation E. 6 relies on standard libraries ${ }^{[184]}$, and for each value of the wave-number $k_{x}$ an eigenvalue spectrum $\omega$ was provided. The solver has been validated against spatial solvers that have themselves been extensively validated ${ }^{[184]}$. In the following, the spectrum of standard canonical flows were first considered briefly to show how a MSD wall can lead to instability before the method is applied to the numerical simulation.

### 6.6.1 Instability due to impedance boundary

A parabolic mean flow $U_{0}(y)=U_{c}\left(1-y^{2}\right)$ for a channel with rigid walls was first considered, where $U_{c}=0.1$ is the velocity at the centre of the channel (which is related to the Mach number $\mathcal{M}=2 \mathrm{U}_{\mathrm{c}} / 3$ given the normalization with the speed of sound). For a Reynolds number based on the centre velocity of $\operatorname{Re}_{c}=2000\left(\operatorname{Re}_{b}=2 / 3 \operatorname{Re}_{c}\right)$ and $k_{x}=1$
this flow is known to be stable in the incompressible regime ${ }^{[245]}$, which is confirmed by the spectrum of the phase speed $\mathrm{C}=\omega / \mathrm{k}_{\mathrm{x}}=\mathrm{C}_{r}+\mathrm{iC}_{i}$ presented in figure 6.15 (a). Indeed, all eigenvalues have $C_{i}<0$ (that is, also $\omega_{i}<0$ ), meaning the flow is stable.


Figure 6.15: Complex phase speed spectrum (+) for a parabolic flow with rigid walls and $k_{x}=1$, $\operatorname{Re}_{c}=2000$. (a) Full spectrum. Symbols (ם) indicate the acoustic modes; The vertical dotted lines are at $C_{r} / U_{c}=1+1 / U_{c}$ and $C_{r} / U_{c}=1-1 / U_{c}$. (b) Zoom in on the non-acoustic modes. Symbols $(\bigcirc)$ are some reference values obtained for an incompressible flow.

Acoustic modes were indicated by square symbols. The inviscid limit for these modes in a uniform flow of Mach number $\mathcal{M}$ is given by:

$$
\begin{equation*}
\omega=k_{x} \mathcal{M} \pm \sqrt{k_{x}^{2}+\left(\frac{n \pi}{2}\right)^{2}} \quad \forall n=0,1,2, \cdots \tag{6.12}
\end{equation*}
$$

Two vertical lines indicate the speed $\mathrm{U}_{\mathrm{c}}-\mathrm{c}_{w}=\mathrm{U}_{\mathrm{c}}-1$ and $\mathrm{U}_{\mathrm{c}}+\mathrm{c}_{w}=\mathrm{U}_{\mathrm{c}}+1$, which are the propagation speeds of the upstream and downstream plane sound waves. The modes located outside the region comprised within the two vertical lines are all non-plane acoustic modes. Figure 6.15 (b) presents a zoom in of the region within the two vertical lines. This region consists of non-acoustic modes. It classically displays a Y-shaped spectrum with 3 branches denoted as A, P, and S. The A-branch modes are often designated as wall modes because their eigenfunctions are maximal close to the walls. The P modes are designated as centre modes, since their eigenfunctions reach their maxima close to the channel centre. Some reference values for an incompressible flow (given in the appendix A. 7 of Schmid and S Henningson (2001) ${ }^{[245]}$ ) were added to the plot. For the low Mach number value $\mathrm{U}_{\mathrm{c}}=0.1$ taken here, the agreement between the present results and the reference value was already quite good (the agreement can be reached at any order of accuracy by lowering compressibility, by reducing the value of $\mathrm{U}_{\mathrm{c}}$ ).

The effect of the MSD boundary condition on stability is now evidenced. The particular mode shown by an arrow in figure 6.15 (b) belongs to the A-branch for rigid walls and corresponds to $C / U_{c} \sim 0.31-0.020 i$ and $\omega \sim 0.031-0.0020$ i. The bottom wall of the channel was changed from rigid to MSD, and the MSD resonance frequency was taken to match the frequency of that particular mode, with $\omega_{\text {res }}=\sqrt{\mathrm{K} / \mathrm{M}} \sim 0.031$. The resistance
was given an arbitrary small value, $\mathrm{R}=0.0001$, all other parameters were unchanged. The corresponding spectrum is shown in figure 6.16 (a). A new mode, indicated by an arrow,


Figure 6.16: (a) Complex phase speed spectrum (+) for a parabolic flow with a top rigid wall and a bottom MSD wall tuned to resonance frequency 0.031, $\mathrm{k}_{\mathrm{x}}=1, \operatorname{Re}_{c}=2000$. The reference values for the incompressible flows with rigid wall are still indicated (○). (b) Comparison of the spectra obtained for a $U_{c}\left(1-y^{8}\right)$ base flow with a top rigid wall and: $(\bigcirc)$ a bottom rigid wall; $(+)$ a bottom MSD wall tuned at the correct resonance frequency. $\mathrm{k}_{x}=1, \mathrm{Re}_{\mathrm{c}}=2000$.
stood just above the original particular mode chosen. This new mode had a small positive value $C_{i} \sim 0.00002$, indicating that the flow was now unstable, other modes remaining approximately the same. By modifying the resonance frequency of the MSD wall, it was possible to render unstable any mode of the A-branch and many modes of the P-branch. However, the modes with the highest growth rate were those with a lower value of $C_{r}$. Hence, a MSD wall can prompt some modes to become unstable, particularly those of the A-branch. Unstable modes above liners are often called surface waves due their fast decrease away from the wall. This was in agreement with these modes arising from the A-branch of wall modes.

A mean flow with a steeper profile $\mathrm{U}_{0}(\mathrm{y})=\mathrm{U}_{\mathrm{c}}\left(1-\mathrm{y}^{8}\right)$ (let us call it "turbulent like") was now considered, again with $k_{x}=1, \operatorname{Re}_{c}=2000$. The spectrum for rigid walls was shown with $\bigcirc$ symbols in figure 6.16 (b). Overall, compared with the parabolic profile, the spectrum was shifted toward higher phase velocities. All eigenvalues were found in the bottom half plane, indicating that the flow was stable. The spectrum obtained by replacing the bottom rigid wall by a MSD wall is shown with + symbols in figure 6.16 (b). The resonance frequency was tuned to the frequency of the leftmost mode of the rigid wall case (corresponding to $\omega_{r} \sim 0.04$, or $\mathrm{C}_{\mathrm{r}} / \mathrm{U}_{\mathrm{c}} \sim 0.4$ ). With a bottom MSD wall, a slightly unstable mode (indicated with an arrow) was found on top of the leftmost stable mode. All other modes were unchanged. This was the same behaviour as for the parabolic profile. Only the modes pertaining to the A-branch could be destabilized in that way. The normalized norm of the stream-wise velocity eigenfunction is given in figure 6.17 for both the parabolic and eighth-power velocity profiles. For the parabolic flow, the maxima close to the rigid and MSD walls were about the same. For the steeper


Figure 6.17: Stream-wise velocity eigenfunction (normalized to have unit maximum) for the unstable mode for: - the parabolic mean velocity profile with $U_{0}(y)=U_{c}\left(1-y^{2}\right)$; __ the steeper profile $\mathrm{U}_{0}(\mathrm{y})=\mathrm{U}_{\mathrm{c}}\left(1-\mathrm{y}^{8}\right) . \mathrm{k}_{\mathrm{x}}=1, \mathrm{Re}_{\mathrm{c}}=2000$. The bottom wall at $y=-1$ is of the MSD type.
profile, the peakedness of the eigenfunction close to the MSD wall was more pronounced and resembles the shape of a surface mode.

In this subsection, the major ingredients for the temporal linear stability of a channel flow with a bottom MSD wall have been given. If the MSD wall was tuned to the frequency of the modes of the A-branch, these modes can be destabilized. The destabilization was more effective for the modes located on the left of the A-branch. This was also where the classical Tollmien-Schlichting unstable wave can be found at higher Reynolds numbers. There is thus some similarity between this wave and the liner-due surface modes in acoustics, although the latter can exist even without viscosity.

### 6.6.2 Comparison with the numerical simulations

The waves observed in the numerical simulations and reported in Section 6.4 for testcases ACo1-ACo3 were 2D, and their presence is explained by the same type of 2 D stability analysis as in the previous subsection. These waves are non-linearly saturated waves, not really prone to a linear stability analysis. To circumvent this limitation the following method that has been employed: the configurations ACo1, ACo2, and ACo3 were run with a bottom rigid wall rather than a MSD wall until a statistically stationary turbulent channel flow was obtained. Then at some instant chosen as the origin of time, $\mathrm{t}=0$, the bottom rigid wall was suddenly replaced by a MSD wall. Due to the flow being unstable, an instability developed in the numerical simulation in the vicinity of the bottom impedance wall and for some time it should be linear.

The time evolution of the amplitude of the dominant spectral component (spectral refers to Fourier transform in the $x$-direction) of the wall-normal velocity $v$ on the bottom impedance wall is shown in figure 6.18 (a), for test-case ACo2. The value of the axial wave-number $k_{x}$ corresponding to this maximal spectral amplitude is given in figure 6.18 (b). Before saturation starts at time $t \sim 150$ the amplitude corresponds to a constant value of $k_{x}=3.55$ and exhibits an exponential growth, typical of an instability. At later times, saturation leads to a final state that is the same as described in Section 6.4, and the wave-number corresponding to the final state ( $k_{x} \sim 2$ in figure 6.18 (b), see also table 6.3) differs from the wave-number for the initial instability $\left(k_{x}=3.55\right)$. The characteristics of


Figure 6.18: For test-case ACo2: (a) Time evolution of the spectral component of the wall-normal velocity at the impedance wall having the largest amplitude (the straight dashed line with symbol is a fit to the initial exponential growth); (b) Stream-wise wave-number of the spectral component having the largest amplitude; (c) Normalized phase speed spectrum (+) resulting from a linear stability analysis performed for $k_{x}=3.55$. The triangle is the phase speed computed from the numerical simulation.
this instability (wave-number, angular frequency, growth rate) were computed for t < 150 and compared to those predicted by the temporal linear stability analysis. The base flow $\mathrm{U}_{0}(\mathrm{y})$ for the stability analysis was taken to be the mean velocity profile of the turbulent channel flow with rigid walls at $\operatorname{Re}_{\mathrm{b}}=6900\left(\operatorname{Re}_{\tau}=395\right)$, which was the actual profile when the MSD wall is set up at $t=0$. The numerically calculated profile could be used but it is more convenient to use an analytical velocity profile that matches this mean flow. As in Marx and Aurégan (2013) ${ }^{[184]}$ (see equations (3-4) therein), Cess's mean velocity profile was used for that purpose, it was a very good approximation of the computed mean flow.

The characteristics of the wave extracted from the numerical simulations was compared to that of the stability analysis in table 6.4. For the numerical simulation the characteristics were the one of the observed wave. For the stability analysis, a search for the most unstable mode was done, and the reported value of $k_{x}$ is the one for which the largest value of $\omega_{i}$ was obtained. If not perfect, the agreement was satisfactory. For example, for test-case ACo2 the wave-number $k_{x}$ and the growth rate $\omega_{i}$ agree rather well. In any case the value of $\omega_{r}$ was slightly larger than the angular resonance frequency of the liner, $\omega_{\text {res }}=\sqrt{\mathrm{K} / \mathrm{M}}$. Figure 6.18 (b) shows the spectrum predicted by the stability

| Case | $\begin{aligned} & \omega_{\mathrm{res}} \\ &= \sqrt{\mathrm{K} / \mathrm{M}} \\ & \hline \end{aligned}$ | Numerical simulation |  |  | Stability Analysis |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $k_{x}^{\max }$ | $\omega_{r}$ | $\omega_{i}$ | $\mathrm{k}_{\mathrm{x}}$ | $\omega_{r}$ | $\omega_{i}$ |
| ACor | 0.185 | 2.3 | 0.22 | 0.028 | 1.65 | 0.210 | 0.0282 |
| ACo2 | 0.367 | 3.3 | 0.47 | 0.044 | 3.55 | 0.408 | 0.0445 |
| ACo3 | 0.738 | 6.7 | 0.79 | 0.073 | 7.05 | 0.787 | 0.0585 |

Table 6.4: Comparison between the characteristics of the instability wave observed in the numerical simulation during the growth period and the characteristics of the most amplified wave computed by using the linear stability analysis.
analysis, for test-case ACo2. It bears many similarities with that for the canonical eighthpower velocity flow considered previously. In particular, the unstable mode indicated by an arrow stood on the left side of the A-branch. The mode calculated from the numerical simulation was indicated with a triangle symbol. The eigenfunctions for the stream-wise and wall-normal velocities for test-case ACo2 were compared in figure 6.19 ( $a, b$ b) respectively. Both were normalized with the value of the wall-normal velocity eigenfunction at the wall.
(a)

(b)


Figure 6.19: (a) Stream-wise velocity eigenfunction. (b) Wall-normal velocity eigenfunction for test-case ACo2 ( $k_{x}=3.55$ ). __ Stability analysis; __ Numerical computation, during the exponential growth of the instability; - - Numerical computation, final saturated state.

However, the values were not exactly the same, especially for $\hat{u}$, the general trends were similar. The eigenfunctions for the saturated state obtained at larger time (which were already included in figure $6.12(\mathrm{a}, \mathrm{b})$ ) are also shown. Obviously, the saturated state will differ in many respects of the initial instability leading to this state. Nevertheless, some similarities exist between the eigenfunction in the two regimes, which tends to indicate that the dynamics of the wave observed in the vicinity of the lined wall in Section 6.4 was partially governed by a linear instability.

### 6.7 CONCLUSION

Numerical simulations of a compressible channel flow with an impedance boundary condition have been performed. When the liner resistance is small, and when its resonance frequency is not too large compared to a typical frequency of the flow, the turbulent statistics differ from those for a channel with rigid walls. Surface waves are also observed along the liner surface. The waves have a large wavelength compared to the turbulent structures. They modulate these structures and transport momentum toward the impedance wall, causing a drag increase. As the resonance frequency increases these waves progressively lose their spanwise coherence while their wavelength decreases to get close to the flow typical length scales, which may also results in a drag increase when the resistance is sufficiently small, as was also observed by Scalo et al. $(2015)^{[240]}$.

At low resonance frequencies the two-dimensional waves have a spatial distribution which resembles the waveforms observed in former experiments ${ }^{[181]}$, even if the conditions differ (the simulation are periodic in the stream-wise direction, the Reynolds in the simulation is lower).

A linear two-dimensional temporal stability analysis has been performed. A temporal analysis suits the stream-wise periodic configuration of the channel flow and offers a new perspective on unstable modes, compared with the more widespread spatial analysis of surface modes in acoustics. It has been shown that by tuning the resonance frequency of the liner to the frequency of a mode pertaining to the A-branch of wall modes, it is possible to destabilize this mode. Numerical simulations of a channel flow have been performed where the bottom rigid wall is suddenly replaced by an impedance wall. An instability is then observed with characteristics similar to those obtained from a linear stability analysis. This instability saturates and leads to a final state of the flow which corresponds to a waveform different from the initial instability, but is not completely different.

The liner was modelled by an impedance boundary condition corresponding to a mechanical oscillator. This has first been introduced by Tam and Auriault (1996) ${ }^{[264]}$ and has been recast here to match the characteristic form of the equations used in the solver. It is a simple model that accounts for a resonance of the liner. As virtually any other impedance model, it can be derived by supposing that the incident acoustic wave has a long wavelength compared to the perforations of the face sheet. This model has been used as is in the turbulent channel numerical simulations, but some turbulent scales may be so small that the model is not valid any more. Also it is not clear how the rugosity due to liner perforations can be accounted for by an impedance boundary condition. As a result, the low frequency waves certainly correspond to some reality (and have been observed experimentally) but the presence of small span-wise rollers at high frequency probably needs to be confirmed by experiments or simulations including the full geometry of the liner.

Chapter 8 focus on spatial simulations, with a well defined inlet and outlet and no periodicity assumption. This is closer to practical situations, and also allows introducing a sound wave into the domain. The triggering of the surface wave by an incoming wave is indeed important physically. In the present case, the periodic stream-wise boundary conditions probably acts as means to sustain the wave as an incoming wave would do in a spatial simulation.

## 7

## SPATIAL SIMULATION OF CHANNEL FLOW AND SOUND ATTENUATION

Until now, temporal (i.e., stream-wise periodic) ILES of a turbulent channel flow, with rigid and/or impedance wall boundary conditions were performed. In this chapter spatial simulation (non-periodic in stream-wise direction) of a turbulent channel flow is performed. This requires imposing well-defined inflow and outflow sections. This also allows introducing an incoming sound wave at the inflow. The objective is then to study the attenuation of the sound wave in a turbulent channel and its interaction with the turbulence.

In the following, the inflow and outflow boundary conditions (Section 7.1) will be presented, followed by the wave-extraction technique (Section 7.2). Then results from the validation test-cases will be presented (Section $7 \cdot 3$ ). Finally, results for the spatial turbulent channel flow will be presented, followed by the extraction of the sound-waves from the turbulent channel (Section 7.4).

### 7.1 BOUNDARY CONDITIONS

Simulations are always performed in truncated computational domain, with appropriate boundary conditions at the truncated boundary. Sometimes periodicity is assumed and periodic boundary conditions are used, which to some extent do not affect the computation unless the truncated domain is small. But for most of the practical application periodicity cannot be considered. In these computations the exact conditions at the truncated boundary are usually unknown, and they require some special treatment. Generally, for a simulation one will have an inflow and outflow boundaries, and these two boundary require special treatment such that spurious oscillations and reflections are not created inside the computational domain.

### 7.1.1 Subsonic inflow boundary condition

Theoretically for a fluid flowing from left to right at subsonic $\mathcal{M}$, four characteristics travel downstream $\left(X^{+}, X^{v}, X^{w}, X^{s}\right)$, whereas $X^{-}$characteristic wave travels upstream (see figure 2.2). Therefore at the inlet one has to prescribe downstream travelling characteristics, because they cannot be computed using the information from within the domain. Generally one would prefer to prescribe (a) three velocity components and one thermodynamic component at the inlet, and (b) use them to compute the four downstream travelling characteristics. Characteristics are ID approach, and for multidimensional problem there is no exact method to prescribe incoming characteristics, due to the limitation of
the characteristic formulation. Assuming $u, v, w$ and $T$ are the turbulent quantities one choose to impose, then the characteristics $\mathrm{X}^{+}, \mathrm{X}^{v}, \mathrm{X}^{w}, \mathrm{X}^{s}$ can be computed using either Local One Dimensional Inviscid (LODI) relations or NSE. One can find several examples for inlet characteristic boundary conditions in Sesterhenn (2000) ${ }^{[255]}$, Poinsot and Lele (1992) ${ }^{[223]}$, Yoo et al. (2005) ${ }^{[298]}$ and Lu and Sagaut (2007) $)^{[173]}$ among others. In this section, existing subsonic characteristics inlet boundary conditions will be presented and their pros and cons will be discussed.

### 7.1.1.1 Based on Local One Dimensional Inviscid (LODI) relations

As the name indicates, LODI relations are for one dimensional inviscid problem ${ }^{[223]}$. LODI relations are obtained from equations $2.1-2.5$ by neglecting viscous forces as well as transverse terms (see equations $7.1-7.5$ ). All terms on the r.h.s. of LODI relations are $x$-derivatives. These simplified relations can be used to estimate the incoming characteristics for multidimensional viscous problem. LODI equations writes as:

$$
\begin{align*}
\frac{\partial u}{\partial t} & =-\frac{1}{2}\left(X^{+}-X^{-}\right)  \tag{7.1}\\
\frac{\partial v}{\partial t} & =-X^{v}  \tag{7.2}\\
\frac{\partial w}{\partial t} & =-X^{w}  \tag{7.3}\\
\frac{\partial s}{\partial t} & =-X^{s}  \tag{7.4}\\
\frac{\partial p}{\partial t} & =-\frac{\rho c}{2}\left(X^{+}+X^{-}\right)
\end{align*}
$$

The thermodynamic relation given by equation 2.31 in Chapter 2 is rewritten once again for completeness.

$$
\begin{equation*}
\frac{\partial s}{\partial t}=\frac{1}{T} \frac{\partial T}{\partial t}-\frac{(\gamma-1)}{\gamma} \frac{1}{p} \frac{\partial p}{\partial t} \tag{7.6}
\end{equation*}
$$

If one specifies $u, v, w$ and $T$ at the inlet, then one does not have to solve equations 2.1 - 2.4. From the LODI relations the 4 unknown characteristic can be calculated by simply recasting the equations 7.1-7.4.

$$
\begin{align*}
& X^{+}=X^{-}-2 \frac{\partial u}{\partial t}  \tag{7.7}\\
& X^{v}=-\frac{\partial v}{\partial t}  \tag{7.8}\\
& X^{w}=-\frac{\partial w}{\partial t}  \tag{7.9}\\
& X^{s}=\frac{(\gamma-1)}{\gamma} \frac{1}{p} \frac{\partial p}{\partial t}-\frac{1}{T} \frac{\partial T}{\partial t} \tag{7•10}
\end{align*}
$$

Equation 7.10 was obtained by substituting equations 7.6 in equation 7.4. $X^{\nu}$ and $X^{w}$ only appear in the equations for $\frac{\partial v}{\partial t}$ (equation 7.2 ) and $\frac{\partial w}{\partial t}$ (equation 7.3 ) respectively, thus prescribing $v$ and $w$ is equivalent to substituting the $X^{v}$ and $X^{w}$ characteristics. $X^{+}$ and $X^{-}$are used for computing $\frac{\partial u}{\partial t}$ and $\frac{\partial p}{\partial t}$ (equation 7.1 and 7.5 ). $X^{-}$is known as it comes from inside the domain (see figure 2.2). Therefore $X^{+}$computed from the LODI
relation (equation 7.7 ) is substituted into equation 7.5 to solve for pressure. Then entropy is solved using equation 7.6 with $\frac{\partial p}{\partial t}$ from equation 7.5 and known $T$.

### 7.1.1.2 Based on Navier-Stokes equations

Similarly as it was done with the LODI relations, one can estimate the inlet characteristics using the NSE (equations $2.1-2.4$ ). Like for the LODI relations, $X^{v}$ and $X^{w}$ only appear in the equations for $\frac{\partial v}{\partial t}$ (equation 2.2) and $\frac{\partial w}{\partial t}$ (equation 2.3) respectively. Therefore prescribing $v$ and $w$ is equivalent to substituting the $X^{v}$ and $X^{w}$ characteristics, and $X^{+}$is the only characteristic which appears in multiple equations (equations 2.1 and 2.5). Equation 2.1 can be used to compute the $\mathrm{X}^{+}$characteristics.

$$
\begin{equation*}
X^{+}=X^{-}+2\left(-Y^{u}-Z^{u}+\frac{2}{\operatorname{Re}} \frac{1}{\rho} \frac{\partial \tau_{1 j}}{\partial x_{j}}\right)-2 \frac{\partial u}{\partial t} \tag{7.11}
\end{equation*}
$$

$\mathrm{X}^{+}$computed from the above relation can be substituted into equation 2.5 to solve for pressure. Then entropy is solved using equation 7.6 with $\frac{\partial p}{\partial t}$ from equation 2.5 and known inlet T . This relation has more physical information compared to the LODI relations because it maintains the transverse and viscous terms.

### 7.1.1.3 Non-reflecting characteristics boundary condition

As it is shown previously, it is straightforward to formulate the characteristics boundary conditions. Both of the above mentioned formulation of the inflow boundary conditions in Sections 7.1.1.1 and 7.1.1.2 were found to produce excellent results for laminar flows. But for turbulent flows, in practice these boundary conditions do not perform as expected. These boundary conditions were found to create spurious reflections, due to the strong formulation of the boundary conditions. Characteristic equations are meant for 1 D problems, hence 3 D turbulent flows are very difficult to simulate without proper boundary treatment. Yoo et al. $(2005)^{[298]}$ presented the boundary formulation with some relaxation terms, which accounts for multidimensional flow effects and eliminates spurious acoustic wave reflections. For simulating spatial turbulent channel flow, the boundary conditions proposed by Yoo et al. $(2005)^{[298]}$ were used at the inflow plane, and they produced better results compared to the boundary formulations in Sections 7.1.1.1 and 7.1.1.2.

The basic idea of this boundary treatment is to introduce a non-reflecting boundary condition by reducing the spurious wave reflections at the boundary. This kind of boundary condition can be applied at both inflow and outflow boundaries, but here the application of such boundary condition is limited to the inflow section. In general these boundary conditions are just the simple extension of LODI relations, which introduces a set of linear relaxation constraints between the inlet flow variables and their prescribed values.

$$
\begin{equation*}
X=\alpha\left(a-a_{\text {target }}\right) \tag{7.12}
\end{equation*}
$$

where $X$ is any characteristics one needs to specify, $\alpha$ is the relaxation term, $a$ is the flow variable and $a_{\text {target }}$ is the prescribed value at the inlet plane. This relaxation coefficient
allows for the soft control of the boundary variables and this coefficients can be tuned to reduce the reflections. With larger values for $\alpha$ one will find reflecting boundary conditions similar to the ones in Sections 7.1.1.1 and 7.1.1.2, whereas with smaller values for $\alpha$ the boundary condition will become non-reflecting, but with poor control of the inlet flow variables. Therefore the choice of the relaxation coefficient must be made a priori in order to have a better compromise between the control of reflection and simulation accuracy.

The inflow boundary condition given by Yoo et al. (2005) ${ }^{[298]}$ (adapted to the present notation/normalisation):

$$
\begin{align*}
& X^{+}=\alpha^{+} \frac{c\left(1-\mathcal{M}^{2}\right)}{2 \mathrm{~L}_{x}}\left(u-u_{\text {target }}\right)  \tag{7.13}\\
& X^{v}=\frac{\alpha^{v}}{\rho \mathrm{~L}_{\chi}}\left(v-v_{\text {target }}\right) \\
& X^{w}=\frac{\alpha^{w}}{\rho \mathrm{~L}_{x}}\left(w-w_{\text {target }}\right)  \tag{7.15}\\
& X^{s}=\alpha^{s} \frac{(\gamma-1)}{\gamma} \frac{1}{\mathrm{~L}_{\chi} \mathrm{c}^{2}}\left(\mathrm{~T}-\mathrm{T}_{\text {target }}\right) \tag{7.16}
\end{align*}
$$

where $\alpha^{+, \nu, \omega, s}$ are the relaxation coefficients for each flow variables and $L_{x}$ is the domain length. Target value is defined below in equation $7 \cdot 17$. Yoo et al. (2005) ${ }^{[298]}$ also introduced a relaxation coefficient for the transverse terms, which is not used here. With careful examination it can be found that by using the above boundary conditions with the LODI relation (equations $7.1-7.4$ ) yields the same boundary conditions as Yoo et al. $(2005)^{[298]} \cdot \mathrm{X}^{+}$computed using equation 7.13 is injected into equation 7.5 to solve the pressure equation.

In this thesis, the non-reflecting inlet boundary conditions (equations $7.13-7.16$ ) were used for performing spatial simulation of channel flow. It is rather straightforward to send an incoming sound wave with such boundary treatment.

$$
\begin{equation*}
a_{\text {target }}=a_{\text {flow }}+a_{\text {wave }} \tag{7.17}
\end{equation*}
$$

where $a_{\text {flow }}$ is the flow and $a_{\text {wave }}$ is the sound wave which one wants to send into the domain. This boundary condition produced better results compared to the previous boundary conditions mentioned in Sections 7.1.1.1 and 7.1.1.2, especially for the turbulent flow.

### 7.1.2 Subsonic outflow boundary condition

The objective of the outflow boundary condition is to allow the outgoing flow features to leave the domain without creating any reflections. There are several techniques, Colonius (2004) ${ }^{[54]}$ performed a review of several such boundary conditions. The most popular ones are the one based on characteristics ${ }^{[94,223,266]}$, sponge zone ${ }^{[19,55,82,132,175,176]}$, gridstretching ${ }^{[56,144]}$, and perfectly matched layers (PML) ${ }^{[118,119]}$.

The most challenging part of the characteristic boundary condition is to make sure that the imposed characteristics at the inlet and the one that leave the domain do not create spurious oscillations. Outflow boundary conditions have to ensure that all types
of characteristics waves which leaves the domain do not create reflections and spoil the simulation. The most straightforward way to define characteristics outflow boundary condition is to impose

$$
\begin{equation*}
\mathrm{X}^{-}=0 \tag{7.18}
\end{equation*}
$$

It ensures that no wave enters the domain through outlet, and thus no reflections are created ${ }^{[173,255,276]}$. Non-reflecting boundary condition mentioned above is ill-posed because some type of feedback is required in order to maintain the right level of pressure at the outlet. In the absence of this feedback there will be a drift in the mean pressure field, and the simulation might not converge. The problem will be well-posed if outlet pressure is specified, but this can lead to reflections ${ }^{[210]}$. Thus rather soft outlet boundary condition has to be imposed (similar to the inlet boundary condition mentioned in Section 7.1.1.3), so that the pressure at the outlet plane is close to some target pressure. When the pressure at the outlet plane is not close to the target pressure then incoming waves will enter the domain through outlet plane in order to bring the pressure close to the target pressure. Thus a partially reflecting outlet boundary condition with some relaxation can be defined as:

$$
\begin{equation*}
X^{-}=\frac{\sigma}{\rho H}\left(p-p_{\infty}\right) \tag{7.19}
\end{equation*}
$$

where $\sigma$ is the relaxation factor ${ }^{[223,238]}$, and $p_{\infty}$ is the outlet pressure. The above boundary condition is well-posed and avoid drift of the mean values. Such boundary condition was first introduced by Rudy and Strikwerda (1980) ${ }^{[238]}$, which was then upgraded by Poinsot and Lele (1992) ${ }^{[223]}$ by adding an asymptotic value for incoming characteristics to the above equation 7.19:

$$
\begin{equation*}
X^{-}=\frac{\sigma}{\rho H}\left(p-p_{\infty}\right)+X_{\text {exact }}^{-} \tag{7.20}
\end{equation*}
$$

where $X_{\text {exact }}^{-}$is the exact value of $X^{-}$found from some asymptotic method. The additional term is just a small correction to equation 7.19, and the absence of this term is not going to affect the results drastically.

### 7.1.2.1 Sponge zone

Another simple approach to treat the outflow boundary is to use a sponge zone. The sponge zone technique is very popular due to its simplicity, robustness, non-stiff nature and flexibility to handle complex geometries and unstructured grids ${ }^{[175]}$. In the literature one can find several examples for sponge zones for simulating mixing-layer ${ }^{[13,}$, 27] , jets ${ }^{[18,258]}$, cavity ${ }^{[95,162]}$, and bluff-body ${ }^{[20]}$ flows.

With this method a sponge layer is added to the computational domain (see figure 7.1 for computational domain with sponge zone for simulating channel flow), where the modified NSE (equation 7.21 ) are solved. If $\mathrm{L}_{x}$ is the length of the physical domain of the channel and $\mathrm{L}_{\text {sponge }}$ is the length of the sponge then length of the complete computational domain will be $\mathrm{L}_{\text {dom }}=\mathrm{L}_{x}+\mathrm{L}_{\text {sponge }}$.

If $\mathbf{q}=[u v w s p]^{t}$ is the solution vector, then the modified NSE in the sponge zone for a channel will be:

$$
\begin{equation*}
\frac{\partial \mathbf{q}}{\partial \mathrm{t}} \leftarrow \frac{\partial \mathbf{q}}{\partial \mathrm{t}}-\sigma(x)\left(\mathbf{q}-\mathbf{q}_{\mathrm{ref}}\right) \tag{7.21}
\end{equation*}
$$

$\sigma(x)\left(\mathbf{q}-\mathbf{q}_{\text {ref }}\right)$ is the unphysical term added to the NSE inorder to damp the flow field to a known reference solution. $\sigma$ is the damping function defined as:

$$
\begin{equation*}
\sigma(x)=\left(1-\frac{\mathrm{L}_{\text {dom }}-x}{\mathrm{~L}_{\text {sponge }}}\right)^{3} \tag{7.22}
\end{equation*}
$$

Inside the sponge zone $0 \leq \sigma(x) \leq 1$. Sponge zone is generally characterised by its length


Figure 7.1: Sponge layer for a spatial channel flow simulation.
( $\mathrm{L}_{\text {sponge }}$ ) and strength $(\sigma)$. For a fixed strength here, longer sponge zone will perform better than shorter sponge zone, because the former will damp the flow features more gradually. On the other hand, longer sponge zone means longer computational domains and this will increase the computational cost. Hence, one has to find a better compromise between the sponge zone characteristics and the computational cost in order to have an optimal sponge zone. This is a numerical technique and, in the literature one can find mathematical analysis about their well-possessedness and stability ${ }^{[19,}{ }^{132]}$. An optimal sponge zone is mostly designed based on the trial-and-error approach. Mani (2012) ${ }^{[176]}$ performed the analysis of sponge zone to better understand the interaction between the flow and sponge zone and provided a guideline to design sponge zone for practical CFD purposes.

One solution for longer sponge zone is to use grid-stretching complemented with the excess dissipation ${ }^{[23,27]}$. Grid-stretching allows for longer sponge zone without increased computational cost. But this has to be accompanied with the excess dissipation in order to prevent reflections from unresolved flow features in the sponge zone. One can either add excess viscous or numerical dissipation, and the choice of excess dissipation should not affect the results because the sponge zone requirements are not dependent on the high wave-number features. For the present case, in the physical domain uniform grid was used in the stream-wise direction and in the sponge zone the grid was stretched gradually with the coarsest grid at the end of the sponge zone. The coarsest grid was about $15 \Delta x-20 \Delta x$. In the whole computational domain the SVV kernel mentioned in Section 2.5.2.2 was used for the excess dissipation. Same set of coefficients were used for the SVV kernel in the physical domain and the sponge zone. Therefore in the physical domain where finer grids were used the SVV kernel will add dissipation in a narrow high wave-number region, whereas in the sponge zone the coarser grid will damp some flow features by not being able to discretise them, and the SVV kernel will remove grid-to-grid oscillations if any (due to coarser mesh). Other filtering techniques were also tested instead of the SVV in the sponge zone, better results were obtained with the SVV kernels. The final step is to find a reference solution as a target in the sponge zone. Usually one can use some asymptotic solutions, or the solution for the same problem. In the present case, weighted time averaged flow field taken from the end of the physical domain is
used as the reference solution. The procedure to compute the reference solution is detailed in the Appendix F.

### 7.2 WAVE EXTRACTION USING GLOBAL MINIMIZATION

One of the objective of this chapter is to extract acoustic wave from a turbulent flow in a duct and investigate the effect of turbulence on sound attenuation. In this section the plane wave decomposition technique which was used to determine the wave amplitude and complex wave-number is presented. Generally, the standard methods for measuring the acoustic properties of a wave in a duct are using the Standing Wave Ratio (SWR) and Two Microphone Method (TMM) ${ }^{[127,128]}$. The SWR method performs very well for stationary medium but it is very difficult to apply for a turbulent flow moreover it is very time consuming ${ }^{[214]}$. Whereas TMM can work with broadband acoustic wave much faster than the SWR. Several research has been devoted to the analysis of accuracy of the $\mathrm{TMM}^{[17,206,257,308]}$. Another technique is the least square method, which was introduced by Fujimori et al. (1984) ${ }^{[83]}$. Later Pope (1986) ${ }^{[224]}$ showed that the least square method using the transfer function between 2 microphone is similar to the TMM. Chu (1991) ${ }^{[50]}$ and Chu (1988) ${ }^{[49]}$ showed that the least square method using multiple microphones give more accurate results that the TMM. Jang and $\mathrm{Ih}(1998)^{[134]}$ found improved results with the least square method where multiple microphones which were placed at uniform distance within half wavelength. All these works assume that the complex wave-number is known based on theory or empirical formulas. With no mean flow, the wave-number for circular ducts can be calculated from the Kirchhoff solution for plane waves ${ }^{[150]}$. With the mean flow the situation is complicated ${ }^{[61,68,69,115,130,205]}$.

Acoustic wave of frequency f can be written as the sum of down- and up-stream travelling waves:

$$
\begin{equation*}
\widehat{p}(x)=\widehat{p}_{+} e^{-i k_{+} x}+\widehat{p}_{-} e^{i k_{-} x} \tag{7.23}
\end{equation*}
$$

$\widehat{p}$ is the Fourier transform of the acoustic pressure, $x$ is the stream-wise location, $k$ is the complex wave-number and $\pm$ is used for the down- and up-stream travelling waves with time dependence $e^{i 2 \pi f t}$. Here the objective is to extract the wave and compute its


Figure 7.2: Up-stream and down-stream travelling acoustic waves and the measurement configuration for the TMM. $a$ and $b$ are the microphones at positions $x_{a}$ and $x_{b}$.
complex wave-number and waveform. Firstly, the simple TMM is presented, using which
one can find $p_{ \pm}$knowing $k_{ \pm}$, thanks to the microphone measurements $p_{a}$ and $p_{b}$. See figure 7.2 for a schematic representation of acoustic field using the down- and up-stream travelling waves and measurement configuration for the TMM. For the TMM, the complex wave-number is computed from theoretical or empirical equations 7.38 and 7.41 . With the known value of $k_{ \pm}$, one can write a set of two equations for microphones $a$ and $b$ :

$$
\begin{align*}
& \text { at } x_{\mathrm{a}}: \widehat{p}\left(x_{\mathrm{a}}\right)=\widehat{\mathrm{p}}_{+} e^{-i k_{+} x_{a}}+\widehat{p}_{-} e^{i k_{-} x_{a}}  \tag{7.24}\\
& \text { at } x_{\mathrm{b}}: \widehat{p}\left(x_{\mathrm{b}}\right)=\widehat{\mathrm{p}}_{+} e^{-i k_{+} x_{\mathrm{b}}}+\widehat{p}_{-} e^{i k_{-} x_{b}} \tag{7.25}
\end{align*}
$$

The above set of equations can be written in the matrix form as:

$$
\begin{gather*}
\mathbf{b}=\mathbf{A p}  \tag{7.26}\\
\mathbf{b}=\left[\begin{array}{c|c|c}
\widehat{p}\left(x_{\mathrm{a}}\right) \\
\hline \widehat{p}\left(x_{\mathrm{b}}\right)
\end{array}\right] \quad \mathbf{A}=\left[\begin{array}{l|l}
e^{-i k_{+} x_{a}} & e^{i k_{-} x_{a}} \\
\hline e^{-i k_{+} x_{\mathrm{b}}} & \mathrm{e}^{i k_{-} x_{b}}
\end{array}\right] \quad \mathbf{p}=\left[\begin{array}{l}
\widehat{\mathrm{p}}_{+} \\
\hline \widehat{p}_{-}
\end{array}\right] \tag{7.27}
\end{gather*}
$$

the unknowns $\widehat{\mathrm{p}}_{ \pm}$can be found by solving for:

$$
\begin{equation*}
\mathbf{p}=\mathbf{A}^{-1} \mathbf{b} \tag{7.28}
\end{equation*}
$$

The above system of equations will not yield any results if the distance between two microphone $s=x_{b}-x_{a}$ is a multiple of wave-length. In that case matrix $\mathbf{A}$ is not invertible. In addition, even when $\mathcal{A}$ is invertible, it can be ill-conditioned leading to incorrect results. Bodén and Åbom (1986) ${ }^{[17]}$ showed that the TMM is least sensitive to errors if $\mathrm{ks}=\pi(1-$ $\left.\mathcal{M}^{2}\right) / 2$, and Åbom and Bodén (1988) ${ }^{[308]}$ found TMM should be restricted to $0.1 \pi(1-$ $\left.\mathcal{M}^{2}\right)<\mathrm{ks}<0.8 \pi\left(1-\mathcal{M}^{2}\right)$.

Above mentioned TMM requires known wave-number information, and this data is not available a priori for wave attenuation in turbulent flow. Therefore, another method to find the wave-number was proposed by Allam and Åbom (2006) ${ }^{[6]}$. The method by Allam and Åbom (2006) ${ }^{[6]}$ is very straightforward and simple. The method is based on the measurements from multiple microphones. For each microphone, the pressure field of frequency $f$ is:

$$
\begin{equation*}
\widehat{p}_{j}=\widehat{p}_{+} e^{-i k_{+} x_{j}}+\widehat{p}_{-} e^{i k-x_{j}} \tag{7.29}
\end{equation*}
$$

here $j$ is the microphone position.
Unlike the TMM, for this method the complex wave-number is considered unknown. This leaves one with 4 unknowns to solve: $p_{ \pm}$and $k_{ \pm}$. Equation 7.29 is written for each microphone $j=1,2, \ldots, N$. Since one has 4 unknowns, one should have minimum 4 microphone recording, and they are placed such that:

$$
\begin{equation*}
0.1 \pi\left(1-\mathcal{M}^{2}\right)<\mathrm{ks}<0.8 \pi\left(1-\mathcal{M}^{2}\right) \tag{7.30}
\end{equation*}
$$

Several microphones were placed in the stream-wise direction which recorded the data averaged in the span-wise direction. Span-wise averaging removes a part of the turbulence from the measurement assuming that the wave is plane in the span-wise direction (see figure 7.3 for an example). Then cross-spectrum is computed between the acoustic wave and the microphone measurement.

$$
\begin{equation*}
\mathrm{S}_{\mathrm{j}}=\mathrm{FT}_{\mathrm{j}}^{\mathrm{mic}} \operatorname{conj}\left(\mathrm{FT}^{\text {wave }}\right) \tag{7.31}
\end{equation*}
$$



Figure 7.3: Comparison of raw and span-wise averaged signal.

If the source is excited at a frequency $f$ then in the cross-spectrum one will find only a single peak at this particular frequency. By averaging the cross-spectrum computed over multiple blocks (like Welch's method), one can cancel the contribution from turbulence at frequency $f$ and only retain the contribution from acoustic wave ${ }^{[308]}$.

$$
\begin{equation*}
\widehat{\mathfrak{p}}_{j}=\frac{\sum_{1}^{\mathrm{nb}} \mathrm{blocks} \mathrm{~S}_{\mathrm{j}}}{\mathrm{nb}_{\text {blocks }} \operatorname{conj}\left(\mathrm{FT}^{\text {wave }}\right)} \tag{7.32}
\end{equation*}
$$

But this requires averaging over a large number of blocks of cross-spectra. When multiple wave modes are present at the same frequency, then each mode must be separated from $\widehat{p}_{j}$. Transverse modes has the shape $\cos (n(y-1) \pi / 2)$, where $n$ is the mode. Acoustic field with multiple modes can be written as,

$$
\begin{equation*}
\widehat{p}_{j}(y)=a_{0}+a_{1} \cos ((y-H) \pi / 2)+a_{2} \cos (2(y-H) \pi / 2)+\ldots \tag{7.33}
\end{equation*}
$$

here $k_{y}^{n}=n \pi / 2 H$ is the wave-number of the transverse wave, $a_{0}$ is the plane wave, $a_{1}$ is the amplitude of the first order mode and $a_{2}$ is the amplitude of the second order mode. The amplitude of mode $n$ can be computed by projecting the acoustic field on to the basis,

$$
\begin{align*}
\frac{1}{2 H} \int_{-H}^{H} \widehat{p}(y) \cos \left(k_{y}^{m}(y-H)\right) d y & =\sum_{n} a_{n} \frac{1}{2 H} \int_{-H}^{H} \cos \left(k_{y}^{n}(y-H)\right) \cos \left(k_{y}^{m}(y-H)\right) d y \\
& =a_{n} \delta_{n m} \tag{7.34}
\end{align*}
$$

An example of synthetic acoustic field with multiple transverse mode and random noise is presented in figure 7.4. Wave amplitude is $a_{n}=1$ and amplitude of random noise is 0.1. A synthetic non-uniform grid with 81 points in the wall-normal direction was used to define the wave profiles.

Microphone recording in the stream-wise direction is required to compute the complex wave-number, and microphone recording in the wall-normal direction to find the acoustic wave-profile. At each wall-normal location, acoustic wave can be split into plane and transverse modes. Hence, for each wave mode, one knows the l.h.s. of equation 7.29. Now one can minimise the residual $\mathbf{R}$ to find the unknowns ( $\mathrm{k}_{+}, \mathrm{k}_{-}, \widehat{\mathrm{p}}_{+}, \widehat{\mathrm{p}}_{-}$) using the set of equations written for different microphones as:

$$
\begin{equation*}
\mathbf{R}=|\mathbf{A}-\mathbf{B}| \tag{7.35}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathbf{A}=\widehat{p}_{+} e^{-i k_{+} x_{j}}+\widehat{p}_{-} e^{i k_{-} x_{j}} \quad \mathbf{B}=\widehat{p}_{j} \tag{7.36}
\end{equation*}
$$



Figure 7.4: Example of transverse acoustic modes. $|\widehat{\mathfrak{p}}|$ is the complete acoustic field with random noise of amplitude 0.1 and $|\widehat{\mathrm{p}}|_{i}$ is the different modes each with amplitude 1.

Presently, Global non-linear minimisation method "NOMAD" from the "OPTI" Matlab Toolbox was used for this purpose. NOMAD uses a Mesh Adaptive Direct Search algorithm to solve non-differentiable and global non-linear programs, it is a derivative free optimisation ${ }^{[10,163]}$. Other optimisation techniques were also tested using the NewtonGauss method ${ }^{[6]}$ and Simplex search method ${ }^{[157]}$. Better results were obtained with the global minimisation method.

As a starting point one needs to provide some initial guess for the 4 unknowns. This is done in 2 -steps. First, guess of $k_{ \pm}$is computed from asymptotic theory, and secondly this guess is used to find guess for $p_{ \pm}$. For no mean flow case, wave-number of the plane wave is given by Kirchhoff $(1868)^{[150]}$ modified for the rectangular duct is:

$$
\begin{equation*}
\frac{k_{0}}{\omega}=1+\frac{\sqrt{i}}{2 \sqrt{2} \operatorname{Sto}}\left[1+\frac{(\gamma-1)}{\sqrt{\operatorname{Pr}}}\right]+\frac{i}{4 \text { Sto }^{2}}\left[1+\frac{(\gamma-1)}{\sqrt{\operatorname{Pr}}}-\frac{1}{4}\left(1+\frac{(\gamma-1)}{\sqrt{\operatorname{Pr}}}\right)^{2}\right] \tag{7.37}
\end{equation*}
$$

where Sto is the non-dimensional Stokes number (ratio of duct half-height to the acoustic boundary layer thickness, to be defined later in equation 7.49), $\operatorname{Pr}=0.75$ is the Prandtl number and $\gamma$ is the ratio of specific heats. By accounting for the mean flow, Dokumaci (1995) ${ }^{[68]}$ proposed the up- and down-stream wave-numbers as:

$$
\begin{equation*}
k_{ \pm}=\frac{k_{0}}{\left(1 \pm k_{0} \mathcal{M}\right)} \tag{8}
\end{equation*}
$$

these values are used as the initial guess of $k_{ \pm}$for plane wave.
For uniform flow, angular frequency can be defined as,

$$
\begin{equation*}
\omega=k_{x} \mathcal{M} \pm \sqrt{k_{x}^{2}+\left(\frac{n \pi}{2}\right)^{2}} \tag{7.39}
\end{equation*}
$$

For the transverse wave $n=1$, the above relation will have the form,

$$
\begin{equation*}
\left(\omega-k_{x} \mathcal{M}\right)^{2}=k_{x}^{2}+\left(\frac{n \pi}{2}\right)^{2}=k_{x}^{2}\left(1-\mathcal{M}^{2}\right)+2 \omega \mathcal{M} k_{x}+\left(\frac{n \pi}{2}\right)^{2}-\omega^{2}=0 \tag{7.40}
\end{equation*}
$$

Wave-number $k_{x}$ can be found as the roots of the above quadratic equation given by,

$$
\begin{equation*}
k_{ \pm}^{n}=\frac{-\mathcal{M} \omega \pm \sqrt{\omega^{2}-\left(1-\mathcal{M}^{2}\right)\left(\frac{n \pi}{2}\right)^{2}}}{\left(1-\mathcal{M}^{2}\right)} \tag{7.41}
\end{equation*}
$$

For the transverse wave, dissipation is not accounted for in the guess value for $k_{ \pm}$given by equation $7 \cdot 41$. One will find transverse acoustic wave only for $\omega^{2}>\left(1-\mathcal{M}^{2}\right)\left(\frac{n \pi}{2}\right)^{2}$.

Results from equation 7.38 and 7.41 are substituted in to equation 7.29 and then equation 7.28 is solved for 2 unknowns $p_{ \pm}$. The second step is very similar to the TMM. These guesses are finally used for the global non-linear optimisation (equation 7.35).

In summary the steps, of the wave extraction procedure are:

1. Record data averaged in the span-wise direction using multiple microphones with appropriate spacing between them (see equation 7.30);
2. Compute acoustic amplitude by computing cross-spectra between the source and microphone reading over multiple blocks of data (see equation 7.32);
3. Extract required wave modes using equation 7.34 ;
4. Compute initial guess for $\mathrm{k}_{ \pm}$with equation 7.38 and 7.41;
5. Use $k_{ \pm}$from step 4 to find initial guess of $\widehat{p}_{ \pm}$using equation 7.28 ;
6. Use the initial guesses from stem 4 and 5 for the global non-linear optimisation (see equation 7.35 ).

### 7.3 VALIDATION

In this section the inflow and outflow boundary conditions will be validated by simulating a Poiseuille flow. The results are compared with the analytical solution. Then the wave extraction method will be tested by extracting the wave profile from the Poiseuille mean flow. The results of the wave extraction will be compared with the modal analysis.

### 7.3.1 Poiseuille flow

### 7.3.1.1 Flow configuration

Spatial simulation of laminar plane Poiseuille flow was computed using the inflow and outflow boundary conditions mentioned in Section 7.1. Computational domain of size $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}=10 \mathrm{H} \times 2 \mathrm{H} \times 0.5 \mathrm{H}$ was discretised with $\mathrm{N}_{x} \times \mathrm{N}_{y} \times \mathrm{N}_{z}=121 \times 81 \times 10$ grid points. Uniform discretisation was chosen in the stream-wise and span-wise direction, in
the wall-normal direction refined mesh was used near the walls. For the subsonic inflow, three velocity components and temperature were prescribed.

$$
\begin{align*}
u(1, y, z) & =\frac{3}{2} \mathcal{M}\left(1-y^{2}\right) \\
v(1, y, z) & =0 \\
w(1, y, z) & =0 \\
T(1, y, z) & =T_{w} \tag{7.42}
\end{align*}
$$

where $\mathcal{M}=0.05$ and $\operatorname{Re}=150$. This channel flow configuration is the same as in Poinsot and Lele (1992) ${ }^{[223]}$. For completeness the boundary conditions used for this particular


Figure 7.5: Flow configuration for Poiseuille flow validation.
test-case are presented once again. The fluid flows from left to right. Stream-wise velocity, pressure and entropy are interlinked with each other (see Section 2.1), hence at the inlet (left) incoming characteristics $\mathrm{X}^{+}$(downstream travelling characteristic wave) is:

$$
X^{+}=X^{-}+2\left(-Y^{u}-Z^{u}+\frac{2}{\operatorname{Re}} \frac{1}{\rho} \frac{\partial \tau_{1 j}}{\partial x_{j}}\right)-2 \frac{\partial u}{\partial t}
$$

$\mathrm{X}^{+}$computed from the above relation (this is equation 7.11) is used to compute pressure using equation 2.5. Computing $X^{v}$ and $X^{w}$ is equivalent to imposing wall-normal and span-wise velocity component at the inlet, because these characteristic terms appear only in their respective equations (equations 2.2 and 2.3). Isothermal inlet was considered

$$
\frac{\partial s}{\partial t}=\frac{1}{p} \frac{\partial p}{\partial t} \frac{(\gamma-1)}{\gamma}
$$

With this way of prescribing inflow conditions, one solves the compressible NSE at the inlet boundary. It was found that such boundary conditions based on LODI and/or NSE (see Sections 7.1.1.1 and 7.1.1.2) are suitable for laminar flows due to their simplicity. In the present case results obtained with characteristic boundary conditions based on NSE are only presented, with other formulations similar results were obtained. For turbulent flows, such inflow boundary conditions creates reflections, hence the non-reflecting boundary conditions introduced in Section 7.1.1.3 will be used. This will be addressed later in Section 7.4.1.

For the present case,

$$
X^{-}=\frac{\sigma}{\rho H}\left(p-p_{\infty}\right)
$$

will be used as the outflow boundary condition. $\sigma=0.58$ was used for the simulation. $\sigma=0.025$ to 2 was also tested. With smaller values of $\sigma$ mean flow variables were slightly over-predicted, results are not shown here. Another approach is to introduce a sponge layer (see Section 7.1.2.1), where all the turbulence and acoustic wave will be damped gradually without creating any reflections. The sponge layer approach will be used while simulating wave-propagation in Section 7.3.2 or turbulence in Section 7.4.

### 7.3.1.2 Results

In figure 7.6, the time evolution of mass flow-rate is presented to show the convergence of the simulation. Within the reduced time of $\mathfrak{t u}$ flow state was obtained.


Figure 7.6: Time evolution of inlet and outlet mass flow rate

Figure 7.7 presents the stream-wise evolution of mass flow-rate, and the percentage change in the pressure and temperature. Constant mass flow rate was found along the channel in the stream-wise direction. Due to normalisation the flow-rate should be equal to the Mach number. Percentage change in pressure and temperature close to the wall and at the channel centre is presented in figure 7.7 (b,c). For the pressure a linear curve was found, which indicates a constant pressure drop along the stream-wise direction. Moreover curves close to the wall and at the centre collapse one over the other, which indicates the pressure do not vary across the wall-normal direction. For temperature figure 7.7 (c), close to the wall, almost a constant temperature was found due to isothermal wall. At the channel centre, in the stream-wise direction, a slight drop in temperature was witnessed. At the outlet the drop in temperature was $0.08 \%$.

Visualisation of flow field at reduced time $\mathrm{tu}_{\mathrm{b}} / \mathrm{H}=70$ is presented in figure 7.8. For the velocity plane parallel flow along the stream-wise direction was found, which is consistent with the above observation in figure 7.7 (a). Gradual drop in the pressure is noticeable in figure 7.8 (b). For the temperature in figure 7.8 (c), at the inlet plane and the walls the temperature is prescribed to be constant. A slight drop is the temperature can be found towards the center of the channel and the stream-wise direction. Like the pressure, in figure 7.8 (d) density drops in the stream-wise direction.

The analytical solution for velocity and temperature can be found in Schlichting et al. $(1955)^{[244]}$ and Poinsot and Lele (1992) ${ }^{[223]}$ :

$$
\begin{align*}
u & =\frac{-1}{2 \mu} \frac{\partial p}{\partial x}\left(\mathrm{H}^{2}-\mathrm{y}^{2}\right)  \tag{7•43}\\
\mathrm{T}-\mathrm{T}_{w} & =-u_{c}^{2} \operatorname{Pr}\left(\frac{1}{2}+\frac{1}{2} y^{4}-\mathrm{y}^{2}\right) \tag{7.44}
\end{align*}
$$



Figure 7.7: (a) Mass flow rate, percentage change in (b) pressure and (c) temperature in streamwise direction.

In figure 7.9 , the velocity and temperature profiles from the simulation are compared with the analytical solution. Profiles to velocity and temperature are taken at various stream-wise locations. Profiles of velocity collapsed excellently over each other. The Poiseuille velocity profile is maintained along the channel. Constant temperature profile was imposed at the inlet, and the agreement with the analytical solution improves as the flow travels downstream.

### 7.3.2 Poiseuille flow with acoustic wave

Propagation of acoustic wave in Poiseuille flow was simulated using the boundary conditions in Section 7.1. Two simulations were performed, firstly plane acoustic waves at two frequency were excited at the inflow boundary. Secondly, plane and transverse acoustic waves were excited at the same frequency. The wave decomposition procedure introduced in Section 7.2 will be used to extract the acoustic wave from the flow. Simulations are performed at $\mathcal{M}=0.25$ and $\operatorname{Re}=9905$. At the inflow boundary, characteristic bound-
(a)

(b)

(c)

(d)


Figure 7.8: Visualisation of (a) stream-wise velocity, (b) pressure, (c) temperature and (d) density fields for the low Reynolds number Poiseuille flow. Slice taken at $\mathrm{z}=0$
ary conditions based on NSE were used. This is the same boundary treatment used for the validation test in Section 7.3.1. Since there is acoustic wave propagation in the computa-
(a)

(b)


Figure 7.9: Comparison of (a) velocity and (b) temperature to the analytical solution
tional domain, sponge zone introduced in Section 7.1.2.1 is used at the outflow boundary. Other simulation details will be presented for the respective cases below.

## 7•3.2.1 Multiple plane acoustic waves

Propagation of plane acoustic waves of frequency 800 and 1000 Hz are simulated. The boundary conditions for the flow at the inlet were similar as in Section 7.3.1. The acoustic wave is imposed through a power 4 profile excitation $u=u_{\text {Poiseuille }}+u_{\text {wave }}\left(1-y^{4}\right) \sin (\omega t)$, here $u_{\text {wave }}$ is the amplitude of the wave, and at the inlet it was fixed at $u_{\text {wave }}=5 \% \mathcal{M}$. The height of the channel is $\widetilde{\mathrm{H}}=0.0043702$, here $\widetilde{()}$ is used for the dimensional quantity. Physical computational domain of size $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}=800 \mathrm{H} \times 2 \mathrm{H} \times 0.5 \mathrm{H}$ was used for the simulation to accommodate $6-8$ wavelengths in the computational domain. Sponge zone of length $L_{\text {sponge }}=400 \mathrm{H}$ was added past the outflow boundary. The physical domain was discretised with $N_{x} \times N_{y} \times N_{z}=241 \times 81 \times 10$ (uniform grid in the stream-wise and spanwise direction). 40 grid points in the stream-wise direction was used to discretise the sponge zone with stretched grids. Refined grid was used in the wall-normal direction.

The perturbation field at a given time of the simulation (obtained by removing the mean) of the stream-wise and wall-normal velocities, pressure and temperature are presented in figure 7.10. Plane wave can be observed in the different perturbation fields. Due to the acoustic boundary layer close to the wall, waves in stream-wise velocity and temperature are not entirely plane across the channel height. For the temperature, close to the inflow, entropy modes are excited which are evanescent in nature. Hence they are damped quickly, leaving the plane waves to only exist downstream.

Approximately a minimum of 6 microphones per half-wavelength in the stream-wise direction was used to find the complex wave-number with the minimisation procedure presented in Section 7.2. At each stream-wise station, microphones were deployed in the wall-normal direction to estimate the wave profile. The acoustic wave profiles extracted using the decomposition technique in Section 7.2 is shown in figure 7.11 and the complex wave-number is presented in table 7.1. Since both frequencies are very close the wave
(a)

(b)

(c)

(d)


Figure 7.10: Example of perturbation field at the given time of the simulation with two plane waves at $\widetilde{f}=800$ and 1000 Hz . (a) Stream-wise velocity, (b) wall-normal velocity, (c) pressure and (d) temperature.
profiles also looks alike. The wave profiles are normalised with the amplitude of the stream-wise velocity component at the channel centre. The profile extracted from the simulation agree very well with the modal analysis. As found earlier in figure 7.10, the acoustic wave in the stream-wise velocity and temperature is not entirely planar across the channel height. Plane waves are present for the pressure, whereas transverse wave exists for wall-normal velocity.

The wave-number and damping coefficient are presented in table 7.1. The real part of the wave-number is $\mathfrak{R}(k)=2 \pi / \lambda$, where $\lambda$ is the wavelength. Wavelength of the acoustic


Figure 7.11: Comparison of wave profiles from simulation and modal analysis. (a) Stream-wise velocity, (b) wall-normal velocity, (c) pressure and (d) temperature.
wave at 800 Hz is about 122 H and for 1000 Hz is 98 H . The real part of the wave-number computed from the simulation data using the wave-extraction procedure matches the result obtained from the modal analysis. The damping coefficient (imaginary part of the wave-number $k$ ) is also predicted accurately.

| Case | 800 Hz |  |  | 1000 Hz |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\Re(k)$ | $\Im(k)$ |  | $\Re(k)$ | $\Im(k)$ |
|  | Simulation |  | 0.05107 | 0.0006492 |  |
|  | 0.0638 | 0.000730 |  |  |  |
| Modal | 0.0515 | 0.000658 |  | 0.064 | 0.00073 |

Table 7.1: Comparison of wave properties computed from the simulation and modal analysis.

## 7-3.2.2 Plane and transverse acoustic waves

Propagation of a plane and transverse wave at a constant frequency 20 kHz was simulated with the same channel height as in previous example, $\widetilde{H}=0.0043702$. First transverse mode is only present if the excitation is above the cut-on frequency 18.8 kHz (see equation 7.39). The objective was to test the detection procedure to extract different wave modes at same frequency. The inflow and outflow boundary treatment was similar like the previous test-case where two plane waves were excited at the channel inlet. The plane and transverse wave excitation has the form $u=u_{\text {Poiseuille }}+u_{\text {wave }} \cos ((y-1) \pi / 2) \sin (\omega t)+$ $u_{\text {wave }}\left(1-y^{4}\right) \sin (\omega t)$, and the wave amplitude $u_{\text {wave }}$ was set at 133 dB at the inlet. The physical computational domain has the dimension $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}=80 \mathrm{H} \times 2 \mathrm{H} \times 0.5 \mathrm{H}$, and it was discretised with $N_{x} \times N_{y} \times N_{z}=483 \times 81 \times 10$. Past the outflow boundary, a sponge zone of stream-wise length $L_{\text {sponge }}=120 \mathrm{H}$ (discretised with 300 ) points was used to damp the acoustic waves.

The acoustic pressure field is shown in figure 7.12. Both plane and transverse waves are visible in the pressure field. Plane waves have shorter wavelength, whereas the transverse wave have longer wave-length because they are just above the cut-on frequency at which the latter waves are not evanescent. The wave extraction procedure was used to separate
the plane and transverse waves using cosine projection. Approximately a minimum of 6 microphones per half wave-length were used in the stream-wise direction to find the complex wave-number. Microphones recording in the wall-normal direction was used to find the profiles of the acoustic wave using the wave-extraction procedure in Section 7.2.


Figure 7.12: Example of instantaneous acoustic pressure field with plane and transverse wave. The first probe in the stream-wise direction is located at $x_{p 1}=23$.

The comparison of the extracted transverse pressure wave and the result from the modal analysis is presented in figure 7.13. The curves are normalised with the amplitude of the transverse wave at the wall. The curves obtained from the detection procedure and the modal analysis collapse very well on each other. Current procedure with cosine projection will only work for uniform mean flow, or with plane acoustic wave. In the present case, due to the no-slip wall, there is a boundary layer in the near-wall region (e.g., see figure 7.11). However, plane mode exists for pressure, therefore transverse wave could be extracted only for one component.


Figure 7.13: Comparison of transverse wave profile from simulation and modal analysis.

The wave properties computed from the modal analysis and the extraction procedure are presented in table 7.2. The results agree very well with the modal analysis. The plane wave has wavelength approximately 4.9 H and the transverse wave has wavelength approximately 46 H . Plane wave has higher damping coefficient compared to the transverse wave.

| Case | PLANE WAVE |  |  | TRANSVERSE WAVE |  |
| :--- | :---: | :---: | :--- | :--- | :--- |
|  | $\mathfrak{R}(\mathrm{k})$ | $\Im(k)$ |  | $\Re(k)$ | $\mathfrak{I}(\mathrm{k})$ |
|  | Simulation | 1.2900 | 0.0030 |  | 0.1398 |
| Modal | 1.2817 | 0.0036 |  | 0.1325 | 0.00103 |

Table 7.2: Comparison of wave properties computed from the simulation and modal analysis.

### 7.4 SPATIAL TURBULENT CHANNEL FLOW

In Section 7.1, different types of inflow and outflow boundary conditions for subsonic spatial simulations were presented. Some of these boundary conditions were validated for laminar flows with and without waves in Section $7 \cdot 3$. For turbulent flows, the inflow characteristic boundary conditions defined in Section 7.1.1.1 or 7.1.1.2, and the outflow boundary conditions in equations 7.18 or 7.19 creates spurious reflections due to the complexity of this flow. Moreover, the channel is a closed domain and any unwanted oscillation or artefact generated will not leave the computational domain due to the strong boundary conditions. In this section, simulation of spatial turbulent channel flow will be performed using inflow and outflow boundary conditions in Sections 7.1.1.3 and 7.1.2.1. These conditions were found to be more suitable for turbulent flows and they do not create spurious reflections. Weak formulation of the inlet boundary conditions ensures the imposition of the boundary conditions itself does not create strong reflections and additionally any upstream travelling artifacts which exist in the domain will be allowed to exit the domain through the inlet plane. The sponge layer in the outlet gradually adds excess numerical viscosity or dissipation which explicitly damps the down-stream travelling turbulent structures and/or acoustic waves. The sponge layer is accompanied with the grid-stretching (see Appendix B.2) and it complements the damping in sponge zone by under-resolving the flow features. Finally, the most important aspect for simulating turbulent flow is the quality of inlet data. Currently, a simultaneous temporal precursor channel flow simulation is performed which is coupled to the spatial simulation for the inflow turbulence data. The procedure will be addressed in the following section.

### 7.4.1 Turbulent inflow

Generally for simulating spatial evolution of turbulent flow, laminar or turbulent profiles are used at the inlet. When one uses laminar inlet, the flow has to go through a transitional phase before becoming turbulent. For such strategy one will require to have a very long domain upstream the developed flow regime, which will eventually increase the computational cost. A simple solution to this problem is to forcefully trigger turbulence with some tripping, yet the flow has to go through a transitional phase which is sometimes very longer. Therefore the best approach to have fully developed turbulent flow is to use turbulent inflow. This could be done by (a) using the data from a precursor simulation ${ }^{[263]}$ or (b) synthetic turbulence. Generally, for the precursor simulation, one has to perform an additional simulation with stream-wise periodic boundary condition and save the time-resolved data on a plane. This procedure is straightforward, but it
requires large storage space to save the data from the precursor simulation. Currently, a simultaneous precursor simulation is performed for the inflow data for the spatial channel. Since, both simulations are coupled in order to exchange the data on a plane from the temporal simulation to the inflow section on the spatial channel, additional storage space is not required. This approach will indeed increase the computational cost because one has to perform 2 simulations simultaneously, thanks to the " 2 DECOMP" library which allows for efficient parallel computation. The flow configuration is presented in figure 7.14. The main advantage of such technique is that, one can provide realistic turbulent


Figure 7.14: Flow configuration for turbulent spatial channel flow simulation.
inflow data for the spatial simulation. In the following, an acoustic wave will be sent along with the turbulence through the inlet. With the weak inlet boundary conditions presented in Section 7.1.1.3, one can easily impose realistic inflow data without creating spurious reflections. At the outflow the sponge layer with grid stretching was used to damp the flow features.

In the following sections, firstly flow statistics for a spatial turbulent channel flow will be presented. Secondly, the interaction between the turbulent flow and acoustics will be addressed.

### 7.4.2 Turbulent flow statistics

### 7.4.2.1 Simulation set-up

Spatial simulation of turbulent channel flow at $\operatorname{Re}_{\tau} \approx 395$ and $\mathcal{M}=0.3$ was performed. The size of the physical domain was $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}=180 \mathrm{H} \times 2 \mathrm{H} \times 4 \pi / 3 \mathrm{H}$, and it was discretised with $N_{x} \times N_{y} \times N_{z}=1767 \times 101 \times 85$ grid points. In the stream-wise direction sponge zone of size $L_{\text {sponge }}=80 \mathrm{H}$ was discretised with stretched grid consisting of 300 points. In the physical domain the grid has size, $\Delta x^{+}=40, \Delta y_{\text {min }}^{+}=2, \Delta y_{\text {max }}^{+}=14$ and $\Delta z^{+}=20$. Due to the weak inlet formulation, the flow has to go through a transition to reach the fully turbulent state ( $\alpha^{+, \nu, w, s}=20$ is the value of the relaxation parameter in equations
7.13-7.16). Due to the lack of homogeneity in the stream-wise direction, the mean quantity $<()>$ is a function of stream-wise and wall-normal location, and averaged only in time and span-wise direction. In the same fashion other global quantities are function of $x$. The friction Reynolds number is:

$$
\begin{equation*}
\operatorname{Re}_{\tau}(x)=\frac{\widetilde{\rho}_{w}(x) \widetilde{H} \widetilde{u}_{\tau}(x)}{\widetilde{\mu}_{w}} \tag{7•45}
\end{equation*}
$$

the friction velocity is defined as:

$$
\begin{equation*}
\widetilde{u}_{\tau}(x)=\sqrt{\frac{\widetilde{\tau}_{w}(x)}{\widetilde{\rho}_{w}(x)}} \tag{7.46}
\end{equation*}
$$

the non-dimensional heat-flux through the walls:

$$
\begin{equation*}
\mathrm{B}_{\mathrm{q}}(\mathrm{x})=-\left.\frac{\widetilde{\mathrm{K}}_{\mathrm{t}, w}}{\overline{\widetilde{\rho}}_{w}(x) \widetilde{\mathrm{c}}_{\mathrm{p}} \widetilde{\mathrm{u}}_{\tau}(x) \overline{\widetilde{T}}_{w}} \frac{\partial \overline{\widetilde{\mathrm{~T}}}(x)}{\partial \widetilde{\mathrm{y}}}\right|_{w} \tag{7•47}
\end{equation*}
$$

and the friction coefficient is:

$$
\begin{equation*}
c_{f}(x)=\frac{2 \widetilde{\tau}_{w}(x)}{\widetilde{\rho}_{b}(x) \widetilde{u}_{b}(x)^{2}} \tag{7.48}
\end{equation*}
$$

## 7•4.2.2 Results

2D mean flow fields are presented in figure 7.15. Mean stream-wise velocity is roughly constant along the stream-wise direction. Mean pressure is independent of wall-normal location. Mean temperature and density varies along the stream-wise and wall-normal direction. Brun et al. (2011) ${ }^{[39]}$ performed LES of spatial channel flow at a lower Reynolds but higher $\mathcal{M}$ number. They found similar trend for the velocity and pressure distribution. Their fluid was always hot at the channel centre compared to the wall-temperature, whereas in the present case the fluid becomes cold and in the down-stream region the fluid is hotter in the near-wall region than in the core of the flow.

Stream-wise evolution of $\operatorname{Re}_{\tau}, \mathrm{c}_{\mathrm{f}}$ and $\mathrm{B}_{\mathrm{q}}$ are presented in figure 7.16. The transition length is about $20 \mathrm{H}-25 \mathrm{H}$. As it was mentioned earlier, the relaxation coefficient $\alpha^{+, v, w, s}$ determines the level of reflectivity of the inflow boundary. But one should find a better trade-off between the reflectivity and the simulation accuracy. The present choice of $\alpha^{+, \nu, w, s}=20$ is reasonable. Since the pressure drop is constant in the stream-wise direction, $\operatorname{Re}_{\tau}$ and $c_{f}$ are constant in the fully turbulent regime and equal to their respective values in the periodic simulations. Non-dimensional heat-flux decreases linearly after the transition to fully turbulent state.

The evolution of mean pressure and temperature at the first point from the wall $\left(y^{+}=2\right)$ and at the channel centre is presented in figure 7.17. After the initial transition, the pressure drops linearly in the stream-wise direction. The pressure drop is constant close to the wall and at the channel centre, and it does not depend on the wall-normal location. The mean temperature also drops linearly in the stream-wise direction once the flow becomes fully turbulent. The temperature gradient is not constant close to the wall and at the channel centre. The stream-wise mean temperature gradient is stronger at the channel
(a)

(b)

(c)

(d)


Figure 7.15: Mean field of (a) stream-wise velocity, (b) pressure, (c) temperature and (d) density fields.
centre, whereas due to constant wall-temperature boundary condition the temperature remains more or less constant. Brun et al. (2011) ${ }^{[39]}$ found the temperature and pressure gradient is independent of wall-normal location. The present results for the mean temperature disagree with the findings of Brun et al. (2011) ${ }^{[39]}$, and the exact reason for this difference is not known.
(a)

(b)

(c)


Figure 7.16: Stream-wise distribution of (a) $\operatorname{Re}_{\tau}$, (b) $c_{f}$ and (c) $B_{q}$.

Profiles of mean stream-wise velocity and temperature is shown in figure 7.18. Profiles are taken from different stream-wise locations $x / L_{x}=0: 0.25: 1$. The mean velocity profile at all stream-wise locations collapse very well on the mean profile from the temporal simulation, except at $x=0.25 \mathrm{~L}_{x}$. The profile at $x=0.25 \mathrm{~L}_{x}$ deviates slightly from other profiles because the flow is going through a transition. The mean temperature profile at the inlet and the one computed from the temporal channel collapse very well. The flow temperature keeps dropping especially at the core of the flow, which was earlier shown in figure 7.17 (b). Due to turbulent mixing close to the isothermal walls, the rate of drop in the fluid temperature is very low near the walls. Present result for the mean temperature disagree Brun et al. (2011) ${ }^{[39]}$, who found collapse of mean temperature profiles at different stream-wise locations.

Comparison of Reynolds stress profiles at different stream-wise stations are presented in figure 7.19. Due to the relaxation coefficient $\alpha^{+, \nu, w, s}$, the inlet turbulence level is very


Figure 7.17: Evolution of (a) mean pressure and (b) mean temperature close to the wall and at the channel centre.


Figure 7.18: Profiles of (a) mean stream-wise velocity and (b) temperature at different stream-wise station.
low, and this is the reason for the initial transition close to the inflow boundary. For higher values of $\alpha^{+, v, w, s}$, higher levels of inflow turbulence can be attained but, with the drawback that the boundary condition being more reflective. The profile at $x=0.25 \mathrm{~L}_{x}$ deviates slightly from rest of the profiles because the flow is undergoing transition in this region. The profiles from $x / L_{x}=0.5,0.75$ and 1 collapse very well on the profile computed from the periodic simulation of channel flow at similar $\operatorname{Re}_{\tau}$.

Comparison of rms of thermodynamic quantities are presented in figure 7.20. Like other quantities, at the inlet plane rms of thermodynamic quantity is under-predicted. Curves for rms of pressure at $x / L_{x}=0.5-1$ has satisfactory collapse, but they are underpredicted compared to the rms of pressure from the temporal channel flow simulation. Rms of temperature and density is enhanced ( $x / L_{x}=0.5-1$ ) compared to the statistics computed from the periodic channel flow simulation.


Present results suggests that the spatial development of turbulence in a duct is reproduced satisfactorily. Simultaneous temporal precursor simulation ensures that realistic turbulent data is fed to the spatial channel. Due to the weak formulation of the inflow boundary the flow goes through a transition before reaching the fully turbulent state. Weak formulation of the inflow boundary ensures the best compromise between the accuracy of the inflow turbulence data and the reflectivity of the boundary. Global flow variables such as, $\operatorname{Re}_{\tau}, \mathrm{c}_{\mathrm{f}}$ and pressure gradient remains constant in the fully developed region. Mean and rms velocity profiles also collapse very well in this region. Difference is mainly found for the mean temperature distribution. The fluid in the core of the channel cools in the down-stream region, and due to the turbulent mixing close to the isothermal wall, the drop in fluid temperature is very low. The sponge zone ensures no reflections are created by the flow features which exits the domain. Except for the disagreement on the mean temperature distribution, the present results agree with the findings of Brun et al. (2011) $)^{[39]}$. The reason for the difference in the mean temperature distribution is not clearly understood. Nevertheless, the present simulations describes the turbulent flow field accurately and the results are satisfying. In the following section, acoustic wave attenuation in a turbulent channel flow will be addressed.
(a)

(b)

(c)


Figure 7.20: Comparison of (a) $\sqrt{\overline{\mathrm{p}^{\prime 2}}}$, (b) $\sqrt{\overline{\mathrm{T}^{\prime 2}}}$ and (c) $\sqrt{\overline{\rho^{\prime 2}}}$ profiles.

## 7-4.3 Interaction between sound and turbulence

## 7•4.3.1 Simulation setup

In this section attenuation of sound wave in a turbulent channel flow will be addressed. Simulation of acoustic propagation in the turbulent channel flow in Section 7.4.2 is done here. The computational domain, resolution and simulation parameters are given in Section 7.4.2.1 Acoustic waves are injected into the domain through the inlet, together with the turbulence (see equation 7.17). Several probes were deployed in the stream-wise and wall-normal directions to record the temporal data averaged in the span-wise direction. The flow configuration is presented in figure 7.21. The rate of attenuation and profiles of the acoustic wave was found with the wave extraction technique presented in Section 7.2. Approximately a minimum of 6 microphones per half wave-length were used in the stream-wise direction to find the complex wave-numbers using the minimisation proce-


Figure 7.21: Simulation setup to study sound attenuation in a turbulent channel flow.
dure, and at each stream-wise station microphones were deployed in the wall-normal direction to estimate the wave-profile.

It is well-known that for wall-bounded flow there is a turbulent boundary layer, similarly there exists an acoustic boundary layer for the acoustic wave. Thickness of the acoustic boundary layer is denoted as $\delta_{v}$ (see figure 7.22). Non-dimensional Stokes number is used to relate the acoustic boundary layer thickness to the channel half-height:

$$
\begin{equation*}
\text { Sto }=\frac{\widetilde{H}}{\widetilde{\delta}_{v}}=\frac{\widetilde{H}}{\sqrt{\widetilde{v} / \pi \widetilde{f}}} \tag{7.49}
\end{equation*}
$$

The objective is to know if the gradients of the acoustic boundary layer interact with


Figure 7.22: Schematic of acoustic and turbulent boundary layers.
the turbulence. Experiments have shown that the behaviour if governed by the ratio:

$$
\begin{equation*}
\delta_{v}^{+}=\frac{\widetilde{\delta}_{v}}{\widetilde{\tau}_{v}} \tag{7.50}
\end{equation*}
$$

When the acoustic boundary layer remains smaller than the viscous sub-layer of the turbulent boundary layer then the acoustic wave will not interact with the turbulence. On the other hand when the acoustic boundary layer is thicker than the viscous sub-layer then turbulence will interact with the acoustic wave. This will lead to enhanced damping of acoustic waves. See figure 1.3 to see different regions in the turbulent boundary layer.

### 7.4.3.2 Results

Channel flow simulation at $\operatorname{Re}_{\tau} \approx 395$ and $\mathcal{M}=0.5$ was performed to study the effect of turbulence on sound attenuation. Plane sound waves at frequency varying between $195 \leq \widetilde{f} \leq 10000 \mathrm{~Hz}$ were introduced through the inlet. At $\widetilde{f}=10000 \mathrm{~Hz}$, a transverse acoustic wave was also excited. See Section 7.3.2 to find more information on acoustic wave excitation. The simulation parameters are presented in table 7.3 .

| Case | $\delta_{V}^{+}$ | $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}$ | $\mathrm{N}_{\mathrm{x}} \times \mathrm{N}_{\mathrm{y}} \times \mathrm{N}_{z}$ | $\Delta \chi^{+}$ | $\Delta z^{+}$ | $\Delta y_{\text {min }}^{+}$ | $\Delta y_{\text {max }}^{+}$ | $\widetilde{f}(\mathrm{~Hz})$ | Sto |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| WA25 | 25 | $120 \mathrm{H} \times 2 \mathrm{H} \times 4 \pi / 3 \mathrm{H}$ | $1179 \times 121 \times 85$ | 40 | 20 | 1 | 14 | 196 | 15.8 |
| WA20 | 20 |  |  |  |  |  |  | 306 | 19.75 |
| WA15 | 15 |  |  |  |  |  |  | 544 | 26.33 |
| WA12 | 12 |  |  |  |  |  |  | 850 | 32.91 |
| WAio | 10 |  |  |  |  |  |  | 1224 | 39.50 |
| WAo8 | 8 |  |  |  |  |  |  | 1912 | 49.37 |
| WAo5 | 5 | $60 \mathrm{H} \times 2 \mathrm{H} \times 4 \pi / 3 \mathrm{H}$ | $590 \times 155 \times 85$ |  |  | 0.24 |  | 4894 | 79 |
| WA3.5 | 3.5 |  |  |  |  |  |  | 9988 | 112.85 |

Table 7.3: Simulation parameters to study wave attenuation in turbulent channel flow.

Mainly two simulations were performed, one for low frequency waves (test-cases WA25 - WAo8) and for high frequency waves (test-case WAo5 and WA3.5). Test-cases WAXX correspond to $\delta_{v}^{+}=X X$. Multiple sound waves were excited together as shown previously in Section 7.3.2. An example of pressure perturbation field at a given time with multiple plane acoustic waves is shown in figure 7.23 . Noise in the perturbation field due to turbulence can be seen in the figure in comparison with the previous example of acoustic propagation in Poiseuille mean flow (see figure 7.10).


Figure 7.23: Instantaneous pressure perturbation field in turbulent channel with multiple plane waves (corresponding to test-cases WA25 - WAo8).

Frequency spectra of wall-pressure is presented in figure 7.24. Spikes corresponding to the excitation frequency can be clearly seen in the figure. Similar result was found for pressure spectra elsewhere in the channel. Spectra without the acoustic wave from the spatial channel at the same location is also presented for comparison. For the streamwise velocity spectra, spikes were visible only at lower excitation frequency due to better Sound to Noise Ratio (SNR). At higher frequency SNR is very poor for stream-wise velocity, therefore the spectral peaks does not stand out (not shown here). The spectral peaks
present in the figure is due to the up- and down-stream travelling waves. However the average reflection coefficient was about $\widehat{p_{r}} / \widehat{p_{i}} \sim 0.01$, therefore this may not reflect the situation for the up-stream travelling wave.


Figure 7.24: Frequency spectra of wall-pressure with (a) low frequency excitation (test-cases WA25 - WAo8) at $y=\mathrm{H}$ and (b) high frequency excitation (test-case WAo5 and WA3.5) at $\mathrm{y}=0.43 \mathrm{H}$. Vertical dashed lines are used for different excitation frequency.

Details of signal used for the minimisation procedure is given in table 7.4. Since acoustic waves at multiple frequencies was injected at once, the number of periods recorded for each frequency is different. For all the cases, Welch's method was used without overlap to cancel turbulence (see equation 7.32). Each block of signal used for the Welch's method was 2 period long. In the following, acoustic wave was extracted using the procedure presented in Section 7.2. Present minimisation method relies on sufficiently accurate guess values and the variable bounds for minimisation search, therefore extra care should be taken while post-processing the data. However, it is also possible to use Prony's method ${ }^{[106,141,145]}$ to extract the characteristics of the acoustic wave, and this method does not require any guesses. In the future it will be interesting in general to compare two different methods.

|  | WA3.5 | WA05 | WAo8 | WA10 | WA12 | WA15 | WA20 | WA25 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{nb}_{\text {period }}$ | 180 | 88 | 82 | 52 | 36 | 22 | 12 | 8 |
| $\mathrm{nb}_{\text {Welch }}$ | 90 | 44 | 41 | 26 | 18 | 11 | 6 | 4 |

Table 7.4: Details of signal used for post-processing

Profiles of the downstream travelling waves, extracted using the procedure in Section 7.2 are presented in figure 7.25. Span-wise averaged microphone reading was used for the minimisation procedure, this helps reducing turbulent noise from the signal up to some extent (see figure $7 \cdot 3$ ). For test-case WA3.5 about 80 period of the acoustic wave was recorded, and for test-case WA25 only 8 period was recorded. All wave profiles are scaled with the amplitude $\widehat{u}$ at the centre of the channel. For the pressure component, plane waves were recovered. Due to the isothermal no-slip wall acoustic waves in velocity component and temperature have a boundary layer. The overall shape of the wave


Figure 7.25: Down-stream travelling (a) $\widehat{p}$, (b) $\widehat{u},(c) \widehat{v}$ and (d) $\widehat{T}$ waves.
profiles are similar to the ones reported in Section 7.3.2. With the decreasing excitation frequency the size of the boundary layer increases.

Comparison of wave profiles computed from the simulation and modal analysis is presented in figure $7 \cdot 26$. Modal analysis was performed with regular turbulence model and frequency dependent turbulence model ${ }^{[291,292]}$. For case WA20 and WA25 (figure 7.26(b, c)) wave profiles obtained from different methods collapse satisfactorily. Especially in the near-wall region there is an excellent collapse, whereas away from the wall a slight discrepancy can be found between the three cases. However, the result from the simulation is very close to the result found from the modal analysis with frequency dependent turbulence model. For case WA1o (figure 7.26(a)) noticeable difference can be found between different cases. Profile computed from the modal analysis with regular turbulence model has a boundary layer close to the wall and the profile is approximately uniform thereafter, whereas the profile from the modal analysis with frequency dependent turbulence model undulate away from the wall. Similar undulation of the wave profile can be found for case WAio, but the level of undulation is high compared to the modal analysis (case $v_{f}$ vs WAio in figure $7.26(a)$ ). Exact reason for the such difference is not known, however this can be attributed to the amount of physics described by each method.

The damping coefficients for the down-stream and up-stream travelling waves ( $\alpha_{ \pm}=$ $\left.\mathfrak{I}\left(k_{ \pm}\right)\right)$were compared against the results from the modal analysis with (a) regular mixing-length turbulence $\operatorname{model}\left(\alpha_{ \pm}^{\text {modal }} ; v\right)$, (b) frequency dependent turbulence model ( $\alpha_{ \pm}^{\text {modal }} ; v_{\mathrm{f}}$ ), and (c) no turbulence ( $\alpha_{ \pm}^{\text {modal }}$; laminar). The result is presented in figure 7.27. The damping coefficients are scaled with $\alpha_{0}$ (damping coefficient in a quiescent flow).

$$
\alpha_{0}=\frac{1}{4 \text { Sto }}\left[1+\frac{(\gamma-1)}{\sqrt{\operatorname{Pr}}}\right]+\frac{1}{4 \text { Sto }^{2}}\left[1+\frac{(\gamma-1)}{\sqrt{\operatorname{Pr}}}-\frac{1}{4}\left(1+\frac{(\gamma-1)}{\sqrt{\operatorname{Pr}}}\right)^{2}\right]
$$

This equation is the imaginary part of equation 7.37 and is valid for large Stokes number. Waves at different excitation frequency interact differently with the turbulence. For smaller $\delta_{v}^{+}$(large frequencies), acoustic wave do not come in contact with turbulence. Hence they would encounter laminar damping. From the figure it is clear that for $\delta_{v}^{+}<10$, $\alpha_{ \pm}$computed from the simulation agrees well with the laminar solution. Similarly for
(a)

(b)

(c)


Figure 7.26: Comparison of $\widehat{u}$ wave profiles from simulation and modal analysis for (a) WA10, (b) WA20 and (c) WA25. Modal analysis with $v-$ regular turbulence model and $v_{f}-$ frequency dependent turbulence model.
larger values of $\delta_{v}^{+}$, turbulence interact with the acoustic wave and enhance the damping. For $\delta_{v}^{+}>10$, damping coefficients computed from the simulation follow the modal solution with turbulent eddy viscosity. With the frequency dependent turbulence model given by Weng et al. (2013) ${ }^{[291]}$ these regions can be blend, and there is a good level of agreement between the present results and those predicted by Weng's model. however, while agreement is obtained obtained on the value of $\mathfrak{I k}$, there is a discrepancy on the waveform at low $\delta_{v}^{+}$, as seen in figure 7.26(a).

Up-stream travelling waves are generally more damped compared to the down-stream travelling waves. For larger $\delta_{v}^{+}$wave attenuation is enhanced by the turbulence (see the difference between the laminar solution and present result in figure 7.27). Similar curves can be found in Howe (1995) ${ }^{[115]}$ and Allam and Åbom (2006) $)^{[6]}$.


Figure 7.27: Damping coefficient for the down-stream and up-stream travelling waves in a turbulent channel flow. Results are compared with the modal analysis.

|  | PLANE WAVE |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: |
|  | WA25 | WA20 | WA15 | WA12 | WA10 | WAo8 | WAo5 | WA3.5 | WA3.5 |  |
| $\Re\left(k_{+}\right)$ | 0.0258 | 0.0378 | 0.0670 | 0.1084 | 0.1633 | 0.2452 | 0.6187 | 1.7159 | 0.3897 |  |
| Modal | 0.0240 | 0.0373 | 0.0667 | 0.1041 | 0.1497 | 0.2338 | 0.5988 | 1.227 | 0.4160 |  |
| $\left(k_{-}\right)$ | 0.0752 | 0.1248 | 0.2055 | 0.3010 | 0.4159 | 0.7045 | 1.7801 | 5.0749 | - |  |
| Modal | 0.0746 | 0.1150 | 0.2036 | 0.4518 | 0.3156 | 0.7021 | 1.7928 | 3.7214 | - |  |

Table 7.5: Properties of acoustic waves in turbulent channel flow.

Comparison between the real part of the complex wave-number is presented in table 7.5. For the down- and up-stream travelling waves, satisfactory agreement was found with the results from the modal analysis. Wavelength can be estimated as $\lambda=2 \pi / \mathfrak{R}(k)$. For the plane waves, $\lambda$ decreases with increasing frequency, and up-stream travelling plane waves has shorter wave-lengths compared to the down-stream travelling waves. Transverse waves have larger wave-length compared to the plane waves at similar frequency. The damping coefficient for the transverse wave is 0.0056 (with modal analysis 0.00538). Unlike the plane wave (with frequency dependent acoustic boundary layer), transverse wave interacts with the turbulence in the core of the channel. For a better understanding, similar to the plane wave several frequency must be analysed.

### 7.5 CONCLUSION

Different inlet and outlet boundary conditions or treatment for a subsonic spatial channel flow were presented. Characteristics boundary conditions produced reflections for turbulent flow, due to their strong formulation. Successful spatial turbulent channel flow was simulated with a weak inlet formulation and sponge zone with grid stretching. The turbulent inflow data is obtained from a simultaneous precursor simulation with periodic boundary condition in the stream-wise direction. Due to relaxed inflow treatment, the flow has to go through a transition before reaching a fully turbulent state. The choice of relaxation coefficient affects the transition length and reflectivity of the the inflow plane. The outflow sponge zone damps the turbulent flow features and acoustic waves.

A wave decomposition procedure was used to extract the properties of the injected acoustic wave. It was shown that in figure $7 \cdot 27$, turbulence enhance the acoustic wave damping. Results of damping agreed with the models based on experiments at low $\mathcal{M}$, and the effect of turbulence on the wave attenuation is controlled by $\delta_{v}^{+}$. However, for smaller $\delta_{v}^{+}$significant difference was found between modal solutions computed with different turbulence model.

SIMULATION OF FLOW TUBE WITH ACOUSTIC LINER

In this chapter, we continue our study of liner - turbulence interaction. Earlier in Chapter 6, it was already shown that low resistance acoustic liner triggers 2D instability, which is responsible for enhanced flow modification eventually resulting in drag increase. However, the earlier simulation in Chapter 6 considered periodic boundary conditions in the stream-wise direction. This is not representative of the realistic flow configuration, moreover it is also important to verify if the observed instability is affected by the periodic boundary condition. Another limitation of the previous flow configuration in Chapter 6 was the inability to send an acoustic wave, and therefore to study its attenuation.

Hence, the objective of the present chapter is to use a realistic flow configuration to account for the above mentioned concerns. A schematic of the present flow configuration is presented in figure 8.1. It involves simulating the spatial development of turbulent
Turbulence


Acoustic wave


Figure 8.1: Configuration for flow tube with acoustic liner.
flow and acoustic propagation in a flow tube with an acoustic liner with a well-defined inflow and outflow section (see Chapter 7 for the inflow and outflow boundary conditions). Performing a spatial simulation will also allow to send an acoustic wave through the inflow plane and study the noise attenuation. Both walls are isothermal. The top wall is completely rigid, whereas the bottom surface has a limited lined section. This flow configuration is typical of flow ducts used to measure acoustic impedance of materials, e.g., Jing et al. (2008) ${ }^{[141]}$, Jones et al. (2013) $)^{[142]}$ and Zhou et al. (2014) ${ }^{[307]}$, among others. The flow configuration is same as in Section 7.4 , except the bottom wall boundary condition. The acoustic liner is modelled as an impedance boundary of the MSD type given in Section 6.1. Simulation with an unstable and a stable liner was performed to understand (a) liner induced flow-instability which leads to complex flow-liner-acoustic interaction and (b) sound attenuation.

### 8.1 SIMULATION SETUP

ILES of the turbulent flow and acoustic propagation in a flow tube with acoustic liner was simulated at $\mathcal{M}=0.3$ and $\operatorname{Re}_{\mathrm{b}}=6900\left(\omega_{\text {flow }}=2 \pi \mathcal{M}=1.884\right)$. For a channel with top and bottom rigid wall, it would yield $\mathrm{Re}_{\tau} \approx 390$. The size of the full computational domain (physical + sponge zone) was $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}=170 \mathrm{H} \times 2 \mathrm{H} \times \frac{4}{3} \mathrm{H}$, and it was discretised with $N_{x} \times N_{y} \times N_{z}=1183 \times 171 \times 171$ grid points. Sponge-zone was 80 H long and discretised with 300 grid points in the stream-wise direction with exponential stretching. In the physical domain uniform grid spacing was used in the stream-wise and span-wise direction, $\Delta x^{+}=40$ and $\Delta z^{+}=10$. Close to the wall, refined grid was used with $\Delta y_{\text {min }}^{+}=0.5$ and $\Delta y_{\max }^{+}=14$ at the channel centre. This grid resolutions were chosen based on the previous grid-convergence analysis in Section 6.2. The grid resolution is based on the friction Reynolds number $\operatorname{Re}_{\tau} \approx 390$ for rigid-wall simulation.

The physical domain was 90 H long and the acoustic liner (of length 15 H ) was placed between $x=35 \mathrm{H}$ and $x=50 \mathrm{H}$. Initially, we were interested reproducing the experiment of Marx et al. (2010) ${ }^{[881]}$, however due to numerical instability, simulation with liner corresponding to the experiment (test-case ACo1 in table 6.2) was not feasible. Therefore, acoustic liner for which less unstable surface wave is present was used for the simulation. The acoustic liner corresponding to test-case $\operatorname{ACoz}\left(R=0.23, \omega_{\text {res }}=2 \pi f_{\text {res }}=0.367\right.$ and $\xi=0.109$ ) in table 6.2 was used for the simulation. In the present simulation, the half height of the channel is considered to be $\widetilde{H}=0.01 \mathrm{~m}$, this corresponds to a resonance frequency $\widetilde{f}_{\text {res }}=2 \mathrm{kHz}$ ( $f_{\text {res }}=0.058$ ). Resonance angular frequency ( $\omega_{\text {res }}$ ) of the acoustic liner is well below the typical flow frequency ( $\omega_{\text {flow }}$ ). Previously in Chapter 6, it was found that a 2 D instability was triggered when the acoustic liner was introduced to the simulation ( $\omega_{\text {res }} \ll \omega_{\text {flow }}$ and smaller $R$ ), and the findings were verified by performing a LSA (see figure 6.18). As mentioned earlier, one of the interest of this chapter is to check if the detected instability was affected by the periodic boundary conditions (with the help of spatial simulation).

The different steps in the simulation are the following, firstly a simulation was performed with top and bottom rigid wall until the flow reached a steady state. Later the acoustic liner was introduced. Once the simulation of flow tube with the lined section reached a steady state, finally acoustic wave which matches the resonance frequency of the liner was excited at the inflow boundary. As it was mentioned earlier, acoustic liners are designed to damp the sound wave which matches $\omega_{\text {res }}$ of the liner. Therefore this will allow to check the performance of the acoustic liner for sound damping.

Instantaneous visualisation of the flow in the vicinity of the bottom surface of the physical domain is presented in figure 8.2. Due to the relaxation coefficient ( $\alpha^{+, v, w, s}$ ) in the inflow boundary conditions (given in Section 7.1.1.3), the flow has to go through a transition before reaching a fully developed turbulent state. The transition length in the present case is about $15 \mathrm{H}-20 \mathrm{H}$. At the interface of change in boundary condition (rigid-wall to impedance at $x=35 \mathrm{H}$ and impedance to rigid-wall at $x=50 \mathrm{H}$ ) a part of the acoustic wave is transmitted down-stream and reflected up-stream (see figure 8.1). Nonzero wall-normal velocity can be seen in the region of acoustic liner $(35 \mathrm{H}-50 \mathrm{H})$ and the modified turbulent structures can be found. Flow modification, instability over the liner and its effect on sound attenuation will be discussed in the following section.
ZAxis $\quad \infty$
Transition to fully developed turbulent state $(15 \mathrm{H}-20 \mathrm{H})$
Transition to fully developed turbulent state (15H-20H)

Figure 8.2: Instantaneous flow visualisation at bottom wall of the physical domain of the flow tube with lined section. Data on the slice is the wall-normal velocity at the bottom surface and structures in black are the iso-surface of the Q -criterion $(\mathrm{Q}=0.4)$. Red(blue) colour denote positive(negative) wall-normal velocity.

### 8.2 FLOW STATISTICS

The mean flow field in the flow tube with lined section is presented in figure 8.3. For the mean velocity field in figure 8.3(a), boundary layer development can be seen over the acoustic liner. The boundary layer is strongly pushed away from the bottom surface (around $x=40 \mathrm{H}-45 \mathrm{H}$ ) close to the leading edge of the acoustic liner. Past $x \approx 45 \mathrm{H}$, the effect of wall-transpiration on the mean flow is relatively weak. About 20H down-stream the trailing edge of the acoustic liner the mean flow returns to its original state. The comparison of mean velocity profile at different stream-wise locations is presented later in figure 8.7 (a). Qualitatively the mean pressure, temperature and density fields look similar to the mean field without liner (see figure 7-15).
(a)

(b)

(c)

(d)


Figure 8.3: Contours of mean (a) stream-wise velocity, (b) pressure, (c) temperature and (d) density fields for the flow tube with a lined section.

Evolution of mean velocity over the lined surface is presented in figure 8.4. Displacement of fluid close to the lined surface can be seen in the figure. Boundary layer over the lined surface is gradually evolving, with rapid development in the region close to the leading edge.


Figure 8.4: Evolution of the mean velocity over the lined surface. - Present simulation compared with the - - - rigid-wall.
(a)

(b)


Figure 8.5: Stream-wise distribution of (a) $\operatorname{Re}_{\tau}$ and (b) $\mathrm{c}_{\mathrm{f}}$ at the bottom surface of the flow tube with lined section.

The evolution of stream-wise local friction Reynolds number (equation 7.45 ) and skinfriction coefficient (equation 7.48) is presented in figure 8.5. Density and viscosity at the bottom surface are used to compute the $\operatorname{Re}_{\tau}(x)$ and $c_{f}(x)$. After the initial transient region up to $x=20 \mathrm{H}$, the flow statistics in figure 8.5 reaches a plateau until the leading edge of the acoustic liner. When the acoustic liner is introduced at $x=35 \mathrm{H}$, the $\operatorname{Re}_{\tau}$ and $\mathrm{c}_{f}$ increase abruptly. $\mathrm{Re}_{\tau}$ is increased twice, whereas the skin-friction coefficient is 4 times larger. Down-stream of the acoustic liner, noticeable effect on the flow statistic cannot be found. For test-case ACo2 (periodic box simulation) in table 6.2, $\mathrm{c}_{\mathrm{f}}$ was 2.5 times and $\operatorname{Re}_{\tau}$ was 1.6 times larger compared to the rigid-wall test-case. The increase in $\operatorname{Re}_{\tau}$ and $c_{f}$
over the lined section in the spatial flow configuration is relatively higher compared to the values found with the stream-wise periodic boundary condition (see table 6.2). There is a small influence on the top rigid-wall due to the acoustic liner (not presented here).

Stream-wise evolutions of mean pressure and temperature close to the bottom surface and at the channel centre are presented in figure 8.6. Curves for the mean pressure collapse very well, indicating that the pressure field is independent of the wall-normal location. Nevertheless, a sharp gradient can be found over the liner indicating drag increase (see figure $8.5(\mathrm{~b})$ ). Earlier in Chapter 7 , it was shown that the temperature field has 2D space dependence. Similar trend is seen in the present simulation as well, however the sharp gradient similar to the mean pressure is observed over the lined section. This is the result of enhanced heat transfer due to increased wall-permeability.


Figure 8.6: Evolution of (a) mean pressure and (b) mean temperature close to the bottom surface and at the centre of the flow tube with acoustic liner.

Profiles of the mean stream-wise velocity and Reynolds stresses are presented in figure 8.7. Profiles computed at various stream-wise locations are compared with the profile computed from the temporal simulation at similar Reynolds and Mach numbers with top and bottom rigid-walls and with test-case ACo2 from Chapter 6. $x=0.25 \mathrm{~L}_{x}\left(0.75 \mathrm{~L}_{x}\right)$ is $\operatorname{up}($ down $)$-stream the lined section. $x=0.5 \mathrm{~L}_{x}=45 \mathrm{H}$ is at the location of the lined section. For the mean velocity profiles, all the curves have a satisfactory collapse on the rigid-wall profile, except the curve at $x=0.5 \mathrm{~L}_{x}$. The boundary layer growth due to the acoustic liner can be clearly seen in the figure. The mean flow is pushed away from the bottom surface. In order to maintain the flow-rate, the displaced mean flow is accumulated in the upperhalf of the channel. Curve for $x=0.5 \mathrm{~L}_{x}$ and ACo2 do not collapse perfectly, however the general trend is similar. For the Reynolds stresses, only the curve at $x=L_{x}$ collapse over the profile computed from the temporal channel flow simulation with bottom and top rigid-wall boundary condition. Due to the relaxation coefficient in the inlet boundary conditions, Reynolds stresses are not accurately reproduced at the inflow plane ( $x / \mathrm{L}_{x}=$ 0 ). Acoustic liner allows wall-transpiration, and due to the non-zero wall-normal velocity component at the impedance surface conventional wall-turbulence is modified. But this effect is more pronounced down-stream the liner. Maybe with longer liner section, this effect could be witnessed over the acoustic liner. For the rms of the stream-wise velocity
component over the liner, turbulence intensity is not very different from the rigid-wall curve. Down-stream the lined section, $\sqrt{\overline{u^{\prime 2}}}$ is enhanced in the bottom half of the channel. For the $\sqrt{\overline{v^{\prime 2}}}$, similar observation was made for the curves at $x / L_{x}=0$ and 1 . Curve at $x=$ $0.5 \mathrm{~L}_{x}$ has non-zero value at the bottom surface due to the wall-transpiration. Comparison of Reynolds stress is presented in figure $8.7(\mathrm{~d})$. Close to the bottom surface, enhanced Reynolds stress was found over the acoustic liner $x=0.5 \mathrm{~L}_{x}$, and the curve matched the result for test-case ACo2. Away from the wall, enhanced Reynolds stress was found at $x=0.75 \mathrm{~L}_{\mathrm{x}}$.
(a)

(c)

(b)

(d)


Figure 8.7: Comparison of (a) $\sqrt{\overline{u^{\prime 2}}}$, (b) $\sqrt{\overline{v^{\prime 2}}}$, (c) $\sqrt{\overline{w^{\prime 2}}}$ and (d) $\overline{u^{\prime} v^{\prime}}$ profiles.

It can be concluded that, because of the strong wall-transpiration close to the leading edge of the acoustic liner the mean flow is modified in the vicinity of the impedance surface. This led to the boundary layer development. Conventional wall-turbulence was also modified over the liner, but its effect on the central part of the channel was felt down-stream the lined section. With longer lined section, it could be possible to witness turbulence modification in the core part of the flow over the acoustic liner.

### 8.3 Instability over the acoustic liner

In the previous section the turbulent flow field was discussed, and it was shown that the acoustic liner modifies the turbulent flow due to wall-transpiration. In this section, we will be looking at the wall-transpiration in the vicinity of acoustic liner in detail. In figure 8.8, instantaneous wall-normal velocity component at acoustic liner is presented (see figure 8.2 for the 3 D instantaneous visualisation). Close to the leading edge of the acoustic liner, the amplitude of the wall-normal velocity component increase because an instability is triggered. After an initial growth, the amplitude of the wall-normal velocity component decreases towards the trailing edge of the acoustic liner. The surface wave has shorter wave-length in the growth region compared to the decaying region. The transition from the growth to decaying region happens around $x-$ liner $_{\text {pos }}=5-7$. The drop in the value of $\operatorname{Re}_{\tau}$ and $c_{f}(x=40 \mathrm{H}-42 \mathrm{H})$ in figure 8.5 corresponds to this location.


Figure 8.8: Instantaneous wall-normal velocity on the impedance surface, liner $_{\text {pos }}=35 \mathrm{H}$. Dashed lines indicate the growth and decay regions.

Time averaged mean spatial spectrum of the wall-normal velocity data on the impedance surface is plotted in figure 8.9(a). Spectral peaks are observed at $k_{x}=2 \pi / \lambda=2.3 \rightarrow \lambda=2.73$ and $k_{x}=3.06 \rightarrow \lambda=2.07$. These wave-numbers correspond to the growth and decay regions of surface wave shown by dashed lines in figure 8.8. The instability close to the leading edge has growth rate $\sigma=1.6$. For the temporal case, there was an evolution of $k_{x}$ in time, which here corresponds to an evolution in space (see figures 6.18(b) and 8.9). Temporal spectra for each stream-wise location on the acoustic liner are presented in
(a)

(b)


Figure 8.9: (a) Spatial and (b) temporal spectra over the acoustic liner.
figure 8.9 (b). It can be clearly seen that the wall-normal velocity has the spectral speak
which corresponds to the resonant frequency ( $\mathrm{f}_{\text {res }}=0.058$ ) of the acoustic liner even though the surface wave has two wave-number components.

Eigen functions of stream-wise and wall-normal velocity component are shown in figure 8.10. Eigen functions are computed close to the leading and trailing edges of the acoustic liner. Due to some non-linear interaction or changing mean flow profile the eigenfunctions at two different locations are clearly distinguishable from each other. The shape is reminiscent of the instability profile in the temporal case (see figure 6.19).


Figure 8.10: Eigen function of stream-wise and wall-normal velocity component over acoustic liner.

### 8.4 TURBULENCE-LINER-ACOUSTIC INTERACTION

As it was mentioned earlier, due to numerical instability spatial simulation of flow-tube with liner corresponding to test-case ACo1 was not feasible. Since explicit computation was performed, the only solution was to increase the grid resolution and it increases the computational cost. Therefore, acoustic liner for which less unstable surface wave is present was used for the simulation (test-case ACo2). However, it is still possible to draw some conclusions on turbulence-liner-acoustic interaction by comparing with the experiment of Marx et al. (2010) ${ }^{[181]}$.

LSA for the experiment of Marx et al. (2010) ${ }^{[181]}$ is reproduced in figure 8.11. The major difference between the experiment and the numerical simulation is the one order magnitude difference in the $\mathrm{Re}_{\tau}$. Linearised NSE was solved using both molecular and turbulent eddy-viscosity. Upstream branch is only presented in the figure. During the experiments, convective(global) instability was found without(with) acoustic wave excitation. With turbulent eddy-viscosity included in the LSA, the trajectory of the unstable modes follows path which is reminiscent of convective instability. However, with just molecular viscosity, the unstable modes follows the path which is reminiscent of a global instability with branch exchange. Therefore it can be generalised that, the change in the stability behaviour of liner is originating from the interaction between turbulence and acoustic wave.

The observation made in figure 8.11, can be demonstrated using the schematic shown in figure 8.12. For the experiment due to higher Reynolds number, laminar part of the turbulent boundary layer was very thin, whereas the acoustic boundary layer was thick.


Figure 8.11: LSA of experiment of Marx et al. (2010) ${ }^{[181]}$ (liner corresponding to test-case ACo1).

This resulted in the enhanced dissipation of the acoustic wave due to the interaction of turbulence. Therefore, it can be concluded that the transition from the global to convective instability of the liner is due to turbulence.


Figure 8.12: Turbulence-acoustic interaction for experiment of Marx et al. (2010) ${ }^{[181]}$. Area within the red box represents the turbulence-acoustic interaction region.

LSA for the simulation with liner corresponding to test-case ACo2 is presented in figure 8.13. A pinch point was found at $k=3.5-4.7 \mathrm{i}$, where branch exchange occours. Global instability was observed in the simulation at resonance frequency of the liner, $\widetilde{f}_{\text {res }}=2 \mathrm{kHz}$. The instability has wave-number $\mathrm{k}=3-1.6 \mathrm{i}$.

From the previous example, it was shown that excess dissipation of the acoustic wave due to turbulence prevents liner from being globally unstable. In the present simulation,


Figure 8.13: LSA of simulation with liner corresponding to test-case ACo2.

Reynolds number is one order smaller and acoustic boundary layer is thinner compared to the experiment of Marx et al. (2010) ${ }^{[181]}$ (see figure 8.14).


Figure 8.14: Turbulence-acoustic interaction for simulation test-case ACo2.

## 8.5 effect on sound attenuation

In this section, the effect of the instability detected in the previous section on sound attenuation will be addressed. The primary objective of the acoustic liner is to damp the sound wave at a particular resonance frequency. Therefore acoustic wave whose frequency matches the resonance frequency of the liner was excited at the inflow plane. In figure $8.15(\mathrm{a})$, instantaneous pressure signal at the top rigid-wall is presented. The lined section is between $x=35 \mathrm{H}$ and $x=50 \mathrm{H}$. The amplitude of pressure fluctuations decreases in the lined section compared to the up-stream region. However, the wave am-
plitude gradually grows towards the trailing edge of the liner. In a duct with bottom and top rigid wall, one would except the wave-length of the acoustic wave to be approximately $(1+\mathcal{M}) / \mathrm{f}=(1+0.3) / 0.06=21.6 \mathrm{H}$. Up-stream and down-stream the lined section, the wave-length is approximately close to the above mentioned value. However, in the lined section, the wave-length of the acoustic wave has been modified.
(a)


Figure 8.15: (a) Pressure signal and (b) SPL on the top wall.

SPL computed at the top rigid wall is presented in in figure 8.15(b). Up-stream of lined section, a standing wave feature can be seen due to the reflection from the rigid wallacoustic liner interface at $x=35 \mathrm{H}$. In the leading edge part of the acoustic liner, SPL decreased from 120 dB to 100 dB . Due to the instability, SPL was enhanced and excess noise was created by the acoustic liner. SPL was increased from 100 dB to 110 dB in the lined region. A similar standing wave feature can be seen close to the trailing edge of the liner, due to the wave reflection from the acoustic liner-rigid wall interface at $x=50 \mathrm{H}$. Down-stream of lined region, SPL slightly oscillates around a constant mean value up to $x=70 \mathrm{H}$. This is due to the effect of acoustic liner in the down-stream region. Past $x=70 \mathrm{H}$, SPL is almost constant and noise attenuation is mainly due to turbulence (see Chapter 7).

Stability analysis for varying resistance of the liner was performed to determine the resistance values for a stable liner (see figure 8.16). Firstly simulation was performed with a liner which is globally unstable and then with a liner which is stable to check the effect of liner on sound attenuation.

Another simulation was performed for similar configuration as in Section 8.1, except the liner resistance. In the following simulation $R=1$, and multiple acoustic wave of frequency $\widetilde{f}_{\text {res }}=1400: 200: 2200 \mathrm{~Hz}$ was injected through the inflow plane. Comparison


Figure 8.16: Stability analysis for varying resistance of the liner.
for the complex wave-number of the acoustic wave over the liner is shown in figure 8.17. Due to lack of large number of acoustic period, slight discrepancy was found in the result, however the general trend in the result is recovered.

(b)


Figure 8.17: Comparison of (a) real and (b) imaginary part of the wave-number over liner against the modal analysis.

SPL computed for the stable liner is presented in figure 8.18. In the up-stream part of the liner, standing wave was found similar to figure 8.15. However, in the lined section, SPL drops up to 20 dB approximately with the stable liner (see figure $8.18(\mathrm{~b})$ ). With the low resistance liner $(R=0.23)$, sound production was observed due to the instability. For higher frequency acoustic wave, in the down-stream part of the liner, sharp decrease in the SPL was found at the trailing edge of the liner. Physical mechanism behind this drop in SPL is not known.


Figure 8.18: (a) SPL and (b) sound absorption in the flow-tube with stable liner.

## 8.6 conclusion

Simulation of flow tube with acoustic liner and incoming sound wave was performed. For an unstable liner, it was found that the turbulent flow was pushed away from the bottom surface by the lined section. The effect of the flow modification over the acoustic liner can be felt up to some extent in the down-stream region as well. Skin friction is about 4 times larger over the lined section, and this is complemented with a strong pressure gradient in the lined section.

A 2 D instability is triggered at the leading edge of the liner. The instability grows exponentially before decaying close to the trailing edge of the acoustic liner. Growth and decay regions of the instability have different wave-lengths at the same pulsation frequency. Existence of 2D instability was verified with the spatial simulation and the wave-number corresponds to the previous findings in Chapter 6. Comparison between the present simulation and experiment of Marx et al. (2010) ${ }^{[181]}$ shown that the transition from global to convective instability over the lined section is due to the turbulence-acoustic interaction. In the present simulation, global instability was observed, it generated excess noise affecting the performance of the acoustic liner.

Another simulation with a stable liner shown that, in the absence of flow instability better sound attenuation can be obtained with the liner.

## CONCLUSION AND PERSPECTIVES

In the context of this thesis, a detailed research was done to understand the flow physics of acoustic wave propagation in a turbulent flow in a channel with acoustic liner. As part of the PhD, a parallel compressible Navier-Stokes Equations (NSE) solver Compact3D for High Performance Computation (High Performance Computation (HPC)) was developed in FORTRAN with Message Passing Interface (MPI) protocols for inter-communication between different processors. Compact3D has $80 \%-90 \%$ parallel efficient for strong scaling. Computing facility at GENCI, TGCC, CINES and local Mesocentre was very crucial to the successful completion of the research presented in this thesis. The compressible NSE was solved in the characteristics form. Implicit Large Eddy Simulation (ILES) of channel flows were mainly performed with different boundary conditions. Excess numerical dissipation for the ILES was introduced through the discretisation of diffusion terms in the NSE. Parametric study of the modified discretisation operator for the diffusion terms showed that high-order accurate simulations can be performed at a reduced computational cost. However, when the excess dissipation was concentrated in a narrow wave-number range, the code became unstable. It was found that minimum grid resolution for a wall-resolved ILES with acceptable error was $\Delta x^{*}=50$ and $\Delta z^{*}=30$, with $\Delta y_{\text {min }}^{*}=2$. However, finer grid resolution is always preferred to have statistics accurate up to $4^{\text {th }}$ order moment.

For supersonic flows for which compressibility effects are mainly due to the density or temperature gradients scaling techniques which compensate the compressibility effects were tested. Simulations for up to $\mathcal{M}=3$ was simulated to test the scaling transformation and excellent results were obtained. An efficient and straightforward feature extraction algorithm was developed using the scaling technique to detect the large-scale structures from the turbulent channel flow. Conditional analysis based on the feature extraction revealed that close to the wall high-momentum large-scale structure has significant contribution to the turbulence intensity and Reynolds stress. Vorticity and streak breakdown underneath a High-Momentum Large Scale Structure (HMLSS) is enhanced compared to the Low-Momentum Large Scale Structure (LMLSS). Away from the wall, LMLSS has a dominant effect on the turbulence intensity and Reynolds stress. Large Scale Motions (LSM) are similar in the range $0.5 \leq \mathcal{M} \leq 3$.

With periodic boundary condtions in the stream-wise direction, when the acoustic liner with low resistance and resonance frequency is introduced, an unstable surface wave is triggered. This was verified by the Linear Stability Analysis (LSA). For a typical liner, the surface wave has larger wave-length compared to the turbulent structures and modulated these structures and transport momentum toward the impedance wall, causing drag increase. As the resonance frequency increases, the surface waves loose the span-wise coherence and effect on the turbulent flow decreases.

Inflow boundary conditions with weak formulation was used for non-reflecting inlet boundary condition. At the outflow sponge zone with grid stretching was used to damp
the turbulent flow features and acoustic waves. The choice of the relaxation coefficient for the inflow boundary condition decides the transition length (to reach fully turbulent state) and reflectivity of the inflow plane. A new technique to compute the target field for the sponge zone was introduced which has less memory effect compared to moving average. Realistic turbulent inflow data for the spatial channel was generated with a simultaneous periodic channel flow simulation. Successful spatial simulation of turbulent channel flow was performed with and without acoustic wave. The wave decomposition procedure was used to extract the properties of the acoustic wave. Span-wise averaged temporal signal was used for the wave decomposition, and this helped in reducing the turbulence in the signal. When the acoustic boundary layer is thicker than the viscous sub-layer of the Turbulent Boundary Layer (TBL), turbulence interacts with the acoustic wave leading to enhanced attenuation. The effect of turbulence on wave attenuation is essentially controlled by $\delta_{v}^{+}$.

Simulation of acoustic propagation in a flow duct with liner revealed that with a liner prone to instability, the turbulent flow was pushed away from the impedance surface and boundary layer developed over the liner. Effect of the flow modification can be felt downstream the liner as well. Drag increase over the liner in the flow duct was more compared to the one found with the periodic box simulation. A 2D spatial instability was triggered at the leading edge of the liner, whose wave-number matched the wave-number of the temporal instability. 2D spatial instability created excess noise, affecting the performance of the liner, whereas with a stable liner better sound attenuation was found. Comparison with the experiment, revealed that transition from global to convective instability over the liner is resulting from the excessive dissipation of acoustic wave due to turbulence.

### 9.1 FUTURE WORKS

In the framework of the thesis very efficient high order compressible Navier-Stokes Equation (NSE) solver was developed to study wall-turbulence, duct-acoustic and grazing flow over the liner. Several scientific, numerical and technical challenges were encountered, and most of them were addressed in this work. In this section, additional works that can be done in the future for better understanding wall-turbulence and aeroacoustic within the framework of this work will be addressed.

Currently, Compact3D uses collocated arrangement of the flow variables and characteristic formulation. However for $\mathcal{M}>3$, certain difficulties were encountered. One possible solution to this problem is to use the staggered arrangement and skew-symmetric formulation of the convective terms for enhanced stability. Another numerical issue encountered was the instability and code blow-up with highly concentrated excess dissipation in a narrow wave-number range. For explicit computation this can be solved only by reducing the global time-step. However, after a certain extent reducing the time-step will not be practical any more. Diffusion terms can be made implicit to relax the time-step constraint due to the modified second-derivative operator.

Compact3D has been updated over the past several years. Currently one can successfully perform high order accurate simulations of free-shear and wall-bounded flows. The next step would be to enable the code to simulate flow around complex geometry for
studying aerodynamics and bluff-body noise. Immersed Boundary Method (IBM) can be incorporated in to Compact3D for simulating such flows. High-order schemes for the solid-fluid boundary interpolation could be developed which uses the advantages of the 2D pencil decomposition used here.

Currently, LSM and their influence on flow dynamics was investigated for up to $\mathcal{M}=3$. In this Mach number range, compressibility effects are due to the variation of the mean property variation, and the feature extraction technique is developed with keeping this in mind. However, it is very interesting to study the wall-turbulence ay hypersonic Mach number. Effect of compressible fluctuations and LSM on the flow dynamics will provide deeper insights in the flow physics. Moreover, it will also be an opportunity to check the validity of the present feature extraction technique.

For simulating spatial evolution of turbulence, one can generally either choose to impose a laminar profile at the inflow plane and use some artificial tripping for flow transition or store the turbulence data from the precursor simulation. The main disadvantages of these techniques is the longer computational domain due to slow transition to turbulence and large storage memory. Therefore in this work, simultaneous precursor simulation was performed to generate the inflow turbulence data. This approach do not have the above issues, but the overall computational cost increases. Hence, the best solution to overcome this problem is to generate synthetic turbulence. This technique can overcome the limitations of the simultaneous precursor simulation technique.

Presently in this work, a minimisation problem is solved to find the attenuation of the sound wave in a turbulent duct flow. The present approach strongly relies on the guess values, thus it will be very interesting to calculate the attenuation rate using the method which is more robust for example Prony's method and compare the findings with the present approach.

It was shown that the modal analysis with the regular and frequency dependent turbulence model predicted different wave attenuation rate. For low Mach number, such discrepancy was not that profound. Moreover, even though with correct attenuation rate, the waveform is not accurately reproduced with the modal analysis. However, in the future it will be rewarding to investigate the discrepancy between regular and frequency dependent turbulence models by studying wave-attenuation at different $\mathcal{M}$ numbers.

Presently for the simulation of flow-tube, low resistance acoustic liner was used which triggered an instability and created noise. Most of the current research was focussed on the flow instability created by the liner. However, in the future an acoustic liner with higher resistance will be used to analyse the sound damping by the acoustic liner. For experiment sometimes the impedance of the liner is not known a priori, whereas for a simulation liner impedance is known. Therefore it will also be an opportunity to perform impedance eduction, and compare different techniques.

## APPENDIX

## A

BRIEF HISTORY OF CODE "COMPACT3D" AND COEFFICIENTS FOR COMPACT SCHEMES

## A. 1 HISTORY OF COMPACT3D

The code Compact3D was initially developed during the Ph.D. thesis of Fortuné (2000) ${ }^{[78]}$ for studying the sound radiation from temporal mixing-layer flows with the DNS in a computational domain large enough to include the acoustic field from the flow ${ }^{[79]}$. This was a sequential version of Compact3D, which used conservative formulation of compressible NSE with centred $6^{\text {th }}$ order compact schemes ${ }^{[167]}$.

Later the code was updated by replacing the conservative formulation with the characteristics type formulation of NSE in the Ph.D. thesis of Moser (2006) ${ }^{[202]}$. The numerical schemes were maintained as in the previous version in Fortuné (2000) ${ }^{[78]}$. This version of Compact3D was used to study the acoustic radiation from a spatially evolving 2 D mixing-layer flow ${ }^{[201]}$.

The third version of Compact3D was developed in the Ph.D thesis of Cabana (2008) ${ }^{[42]}$. Compact upwind scheme of Adams and Shariff (1996) ${ }^{[3]}$ was introduced to solve the NSE in characteristics form to simulate acoustic radiation from spatial evolving mixing-layer and jet flows ${ }^{[4]}$.

Recently, the code was updated by incorporating the $6^{\text {th }}$ order modified compact FDS for the spatial second-derivative (to mimic the hyper-viscosity kernel) ${ }^{[161]}$. This is the same scheme used for the validation of the present version of Compact in Section 5.3.1.

All the previous versions of Compact3D were sequential and used for studying acoustic radiation from free-shear flows. In the current version of Compact3D, rigid and nonrigid wall boundary conditions, non-reflecting inlet and outlet boundary conditions were introduced. This is the first time Compact3D is used for studying wall-bounded flows. Like the previous versions, characteristic type NSE were solved using the compact FDS. The new version of the code has the modified FDS which mimics the hyper-viscosity and SVV kernels to introduce excess numerical dissipation for DNS and ILES respectively. In this version, the code was parallelised using the 2 D pencil decomposition (with the " 2 DE COMP" library) of the computational domain. This is the first version of Compact3D which can perform parallel computation. With this smart way of domain decomposition, no inter communication between processes are required to compute the spatial derivatives. The present version of Compact3D is mainly used for simulating temporal and spatial channel flows with rigid and non-rigid (acoustic liner) wall boundary conditions, and to study the interaction between turbulence, acoustic propagation and lined wall. See Chapter 2, 6 and 7, to find detailed information about the present version of Compact3D.

## A. 2 COEFFICIENT OF COMPACT SCHEMES

Compact schemes introduced by Lele (1992) ${ }^{[167]}$ are very popular in CFD and CAA. This schemes have good resolution properties in the wave-number space. For higher order of accuracy, the schemes moves close to the spectral scheme (see Section 2.5.1 and 2.5.2). Compact schemes for the first- and second-derivatives and their coefficients are listed in table A.1 and A.2. Modified wave-number computed from the compact FDS in the tables are presented in figure 2.4 and 2.6

| SCHEME |  | COEFFICIENTS |
| :---: | :---: | :---: |
| Pade | $\begin{aligned} & \beta f_{i-2}^{\prime}+\alpha f_{i-1}^{\prime}+f_{i}^{\prime}+\alpha f_{i+1}^{\prime}+\beta f_{i+2}^{\prime} \\ & =c \frac{1}{6 \Delta x}\left(f_{i+3}-f_{i-3}\right) \\ & +b \frac{1}{4 \Delta x}\left(f_{i+2}-f_{i-2}\right) \\ & +a \frac{1}{2 \Delta x}\left(f_{i+1}-f_{i-1}\right) \end{aligned}$ | Pade scheme $\alpha=\frac{1}{4} ; \beta=0 ; a=\frac{3}{2} ; b=0 ; c=0$ |
| 6TriD |  | $6^{\text {th }}$ order tri-diagonal compact FDS $\alpha=\frac{1}{3} ; \beta=0 ; a=\frac{14}{19} ; b=\frac{1}{9} ; c=0$ |
| 8TriD |  | $8^{\text {th }}$ order tri-diagonal compact FDS $\alpha=\frac{3}{8} ; \beta=0 ; a=\frac{25}{16} ; b=\frac{1}{5} ; c=0$ |
| 8PentaD |  | $8^{\text {th }}$ order penta-diagonal compact FDS $\alpha=\frac{4}{9} ; \beta=\frac{1}{36} ; a=\frac{40}{27} ; b=\frac{25}{54} ; c=0$ |
| 10PentaD |  | $10^{\text {th }}$ order penta-diagonal compact FDS $\alpha=\frac{1}{2} ; \beta=\frac{1}{20} ; a=\frac{17}{12} ; b=\frac{101}{150} ; c=\frac{1}{100}$ |
| spectral like |  | $\begin{aligned} & 4^{\text {th }} \text { order spectral like compact FDS } \\ & \alpha=0.05771439 ; \beta=0.0896406 ; \\ & a=1.3025166 ; b=0.9935500 ; c=0.03750245 \end{aligned}$ |

Table A.1: Compact schemes for spatial first-derivative ${ }^{[167]}$.

| SCHEME |  | COEFFICIENTS |
| :---: | :---: | :---: |
| Pade | $\begin{aligned} & \beta f_{i-2}^{\prime \prime}+\alpha f_{i-1}^{\prime \prime}+f_{i}^{\prime \prime}+\alpha f_{i+1}^{\prime \prime}+\beta f_{i+2}^{\prime \prime} \\ & =c \frac{1}{9 \Delta x^{2}}\left(f_{i+3}-2 f_{i}+f_{i-3}\right) \\ & +b \frac{1}{4 \Delta x^{2}}\left(f_{i+2}-2 f_{i}+f_{i-2}\right) \\ & +a \frac{1}{\Delta x^{2}}\left(f_{i+1}-2 f_{i}+f_{i-1}\right) \end{aligned}$ | Pade scheme $\alpha=\frac{1}{10} ; \beta=0 ; a=\frac{6}{5} ; b=0 ; c=0$ |
| 6TriD |  | $6^{\text {th }}$ order tri-diagonal compact FDS $\alpha=\frac{2}{11} ; \beta=0 ; a=\frac{12}{11} ; b=\frac{3}{11} ; c=0$ |
| 8TriD |  | $8^{\text {th }}$ order tri-diagonal compact FDS $\alpha=\frac{9}{38} ; \beta=0 ; a=\frac{147}{152} ; b=\frac{51}{95} ; c=\frac{-23}{760}$ |
| 8PentaD |  | $8^{\text {th }}$ order penta-diagonal compact FDS $\alpha=\frac{344}{1179} ; \beta=\frac{23}{2358} ; a=\frac{320}{393} ; b=\frac{310}{393} ; c=0$ |
| 10PentaD |  | $10^{\text {th }}$ order penta-diagonal compact FDS $\alpha=\frac{334}{899} ; \beta=\frac{43}{1798} ; a=\frac{1065}{1798} ; b=\frac{1038}{899} ; c=\frac{79}{1798}$ |
| spectral like |  | $\begin{aligned} & 4^{\text {th }} \text { order spectral like compact FDS } \\ & \alpha=0.50209266 ; \beta=0.05569169 ; \\ & a=0.21564935 ; b=1.7233220 ; c=0.17659730 \end{aligned}$ |

Table A.2: Compact schemes for spatial second-derivative ${ }^{[167]}$.

## B

## GRID STRETCHING

The stretching functions used to generate the non-uniform grid for the non-periodic boundary conditions are presented in this appendix. For the flow configuration considered in this thesis, non-uniform grid is required in the wall-normal and/or stream-wise direction. While using non-uniform grid, one will have to compute the derivative based on the information from the uniform grid. It involves mapping the non-uniform grid on to a uniform grid.

## B. 1 WALL-NORMAL GRID

For wall-bounded flows, it is very important to discretise the near-wall region of the turbulent boundary layer where there are large gradients of flow and higher turbulence intensity. Hence, non-uniform grids are used in the wall-normal direction which can add more points in the vicinity of the wall and less points away from the wall. In this section two kind of grid stretching in the wall-normal direction will be presented.

## b.1. 1 Tanh stretching

This is the popular hyperbolic Tangent stretching function:

$$
\begin{equation*}
y=\frac{\tanh (\kappa s)}{\tanh (\kappa)} \tag{B.1}
\end{equation*}
$$

here k is the stretching parameter and $s$ is a uniform grid from $[-1: 1]$. The stretching function is available in Compact3D and one just has to provide $\kappa$. The coefficient $\kappa$ is estimated using a Matlab script. By giving (a) the target $\mathrm{Re}_{\tau}$, (b) the number of grid points that one would like to have in the wall-normal direction and (c) the size of the first grid size close to the wall, the Matlab script will give the corresponding value of k . See Listing I for the input data for the Matlab script to find к.

Listing 1: Wall-normal grid stretching

```
1 Retau=1000; % target friction Reynolds number
    ny=300; % required number of wall-normal grid points
    dyplus_wanted=1; % required smallest mesh size
```


## b.1.2 Incompact stretching

This version of stretching will add more points close to the wall compared to the hyperbolic Tangent stretching in the previous section, and it is more suitable for DNS. The stretching function is:

$$
\begin{equation*}
y=p \times \tan ^{-1}\left(q \times \tan \left(\frac{\pi s}{2}\right)\right) \tag{B.2}
\end{equation*}
$$

where parameters $p$ and $q$ are:

$$
\begin{gathered}
p=\frac{\sqrt{\beta}}{\sqrt{\alpha} \sqrt{(1+\alpha \beta)}} \\
q=\frac{\sqrt{(1+\alpha \beta)}}{\sqrt{\alpha \beta}}
\end{gathered}
$$

here $\alpha$ and $\beta$ are the stretching parameters, and they are related as:

$$
\begin{equation*}
\beta=\frac{\alpha}{\left(\frac{\pi}{2}\right)^{2}-\alpha^{2}} \tag{B.3}
\end{equation*}
$$

Like the hyperbolic Tangent stretching in the previous section, one just has to provide the stretching parameter computed from the Matlab script to Compact3D. The value of stretching parameter $\alpha$ can be found by providing the information in Listing 1 .

## B. 2 STREAM-WISE GRID

Stretched grid in the stream-wise direction is used for the spatial simulation. Uniform grid is used in the physical domain and non-uniform grid is used in the sponge zone. The grid-stretching in the sponge zone helps reduce the computational cost and damp the flow-features and acoustic wave by under-resolving them. The stretching function is:

$$
\begin{equation*}
x=a L_{\text {dom }} \frac{s}{s(n k)}-1+e^{\alpha\left(s-s(n k)^{\theta}\right)} \tag{B.4}
\end{equation*}
$$

where $a$ is the ratio of physical domain to the complete domain $L_{x} / L_{\text {dom }}$. $s$ is a uniform grid with spacing $d x_{\text {min }}$, and $n k$ is the position where $x(n k)=L_{x}$. $\theta$ and $\alpha$ are the stretching parameters. $\alpha$ and $n k$ can be found as:

$$
\begin{equation*}
\alpha=\frac{\ln \left(L_{\text {dom }}+1-\frac{a L_{\text {dom }} s(n x)}{s(n k)}\right)}{(s(n x)-s(n k))^{\theta}} \quad n k=\operatorname{int}\left(\frac{\alpha L_{\text {dom }}}{d x_{\text {min }}}\right)+1 \tag{B.5}
\end{equation*}
$$

The Matlab program will generate mesh using exponential stretching, for which the user should provide information about the length of the physical domain, sponge zone and the mesh size required in the physical domain. ${ }^{1}$

Listing 2: Stream-wise grid stretching
l_phy=40; \% length of physical domain

[^0]```
l_buf=40; % length of sponge zone
    dxmin=0.1019; % grid size in physical domain
nx_buf=153; %initial guess for number of points in sponge
epsilon=5e-1; %initial guess percentage increase for the fist mesh in sponge
```

4

For successful generation of good quality meshes the user has to follow 3 steps:

1. Provide an arbitrary initial guess for the (a) number of grid points in the sponge and (b) stretching percentage. Once executed, the script will provide the maximum number of points which ensures smooth mesh coarsening.
2. Use the result from step 1 while maintaining the initial guess for the stretching percentage. The script will provide the best value for the stretching percentage. This step will ensure that there is a smooth transition from the physical domain to sponge.
3. Use the optimum stretching value from step 2, and provide required number of grid points in the sponge zone. ${ }^{2}$

The script contains 3 Matlab while-loop for finding the optimum values for the stretching. After completing the three steps, the script will return the values for $a, n k, \theta$, which the user can use directly in Compact3D.

[^1]
## COMPARISON BETWEEN DNS AND ILES

Results from different simulation techniques are presented in this appendix. First of all, different simulation techniques are compared with each other and the reference DNS using the spectra. Secondly, mean and rms statistics computed using the present DNS and ILES techniques are compared against the spectral DNS of Coleman et al. (1995) ${ }^{[52]}$ and a very recent DNS of Modesti and Pirozzoli (2016) ${ }^{[194]}$.

Simulation techniques were introduced in Chapter 2 and 3. Simulations are performed with $\operatorname{Re}_{\mathrm{b}}=3000$ and $\mathcal{M}=1.5$, and compared with Coleman et al. (1995) ${ }^{[52]}$. All together 5 channel flow simulations are performed (see table C.1). First of all a very refined simulation is performed, where all the scales of the flow is discretised. Then 3 simulations are performed with the same computational grid as in the reference simulation of Coleman et al. (1995) ${ }^{[52]}$. The reference simulation was done with a spectral solver. Different techniques such as filtering, hyper-viscosity and SVV-like operators are used to damp the high wave-number oscillations in these three simulations. Filter was used with compact FDS and applied after each time-step. ${ }^{[22,156]}$ Finally, the computational grid is coarsened in the wall-normal direction and SVV-like kernel is used.

|  | Case | $\mathrm{N}_{\mathrm{x}} \times \mathrm{N}_{\mathrm{y}} \times \mathrm{N}_{z}$ | $\Delta y_{\text {min }}^{+}$ | $\Delta y_{\text {max }}^{+}$ | $\Delta \chi^{+}$ | $\Delta z^{+}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DNS | Refined | $170 \times 150 \times 100$ | 0.99 | 8.88 | 16 | 9 |
|  | Filter |  |  |  | 19 | 12 |
|  | Hyperviscosity | $144 \times 119 \times 80$ | 0.99 | 14.07 |  |  |
| ILES | SVV-like |  |  |  |  |  |
|  | SVV-like2 | $144 \times 100 \times 80$ | 1.99 | 10.07 |  |  |
| $\operatorname{Ref}^{[52]}$ (Case A spectral) |  | $144 \times 119 \times 80$ | 0.1 | 5.88 |  |  |

Table C.1: Computational grids for testing DNS and ILES. Reference values taken from Coleman et al. $(1995)^{[52]}$.

Spectra computed from different computational techniques and grids are compared in figure C.i. In the low wave-number range, spectra computed from all the test-cases collapse very well on the reference data of Coleman et al. $(1995)^{[52]}$. The difference are visible in the vicinity of the cut-off wave-number for each case. The refined DNS has more grid points compared to the spectral DNS of Coleman et al. (1995) ${ }^{[52]}$. The spectra extends to higher wave-number range, and smoothly drops to zero. When filter was used, the spectra was smoothly damped, whereas with the hyper-viscosity operator (introduced in Section $2 \cdot 5 \cdot 2.1$ ), the spectra stays close to the refined DNS before dropping to zero. As it is already addressed in Chapter 2, the excess numerical dissipation introduced
by the hyper-viscosity operator is concentrated in the high wave-number range close to the cut-off. The SVV-like operator acts on a wider wave-number range compared to the hyper-viscosity operator, and it is evident from the spectra. When SVV-like operator was used the spectra departs from the reference spectra earlier compared to the other techniques. But one has to recall from Chapter 2 that the SVV-like kernel is more suited for the ILES. A parametric study on operators from SVV-like kernel family was presented in Chapter 3. Computational grid coarser in the wall-normal direction was also used to simulate the channel flow with the SVV-like operator. It is worth noting that noticeable difference was not found between the coarse and fine grid simulation with the SVV-like operator.


Figure C.1: Comparison of velocity spectra obtained from different simulation techniques for channel flow at $\operatorname{Re}_{\mathrm{b}}=3000$ and $\mathcal{M}=1.5$ at channel center. Reference data from Coleman et al. (1995) ${ }^{[52]}$.

Secondly, this opportunity is taken to report a slight inconsistency in the results obtained for a similar channel flow configuration with other 2 solver. DNS and ILES of channel flows at $\mathrm{Re}_{\mathrm{b}}=3000$ and $\mathcal{M}=1.5$ and 3 was performed with Compact3D and the mean and rms velocity were compared with the DNS of Coleman et al. (1995) ${ }^{[52]}$ and Modesti and Pirozzoli (2016) ${ }^{[194]}$. The computational grids used for the simulations are given in table C.2. Coleman et al. (1995) ${ }^{[52]}$ used a spectral code and Modesti and Pirozzoli $(2016)^{[194]}$ used a high order Finite Volume code.

In figure C.2, the mean velocity profiles are compared for both configurations. In the inner-layer of turbulent boundary layer all the profiles collapse very well on each other. In the outer-region, a slight disagreement was found. The mean velocity profile computed from the present DNS is very close to the DNS of Coleman et al. (1995) ${ }^{[52]}$ and the present ILES is very close to the DNS of Modesti and Pirozzoli (2016) ${ }^{[194]}$.

The comparison of rms velocity profiles are presented in figure C.3. For the rms of wall-normal and span-wise velocity component the curves collapse on each other. Noticeable mismatch between the results was found for the rms of the stream-wise velocity, especially the turbulence production peak in the buffer layer. It is well-known that with

|  | Case | $\mathrm{L}_{x} \times \mathrm{L}_{y} \times \mathrm{L}_{z}$ | $\mathrm{N}_{\mathrm{x}} \times \mathrm{N}_{\mathrm{y}} \times \mathrm{N}_{z}$ | $\Delta y_{\text {min }}^{+}$ | $\Delta y_{\text {max }}^{+}$ | $\Delta x^{+}$ | $\Delta z^{+}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \mathrm{Re}_{\mathrm{b}}=3000 \\ & \mathcal{M}=1.5 \end{aligned}$ | DNS | $4 \pi \mathrm{H} \times 2 \mathrm{H} \times \frac{4 \pi}{3} \mathrm{H}$ | $170 \times 150 \times 100$ | 0.99 | 8.88 | 16 | 9 |
|  | ILES |  |  |  |  |  |  |
|  | Coleman |  | $144 \times 119 \times 80$ | 0.1 | 5.88 | 19 | 12 |
|  | Modesti | $6 \pi \mathrm{H} \times 2 \mathrm{H} \times 2 \pi \mathrm{H}$ | $512 \times 128 \times 256$ | 0.4 | 5.61 | 8 | 5.2 |
| $\begin{aligned} & \mathrm{Re}_{\mathrm{b}}=4880 \\ & \mathcal{M}=3.0 \end{aligned}$ | DNS | $4 \pi \mathrm{H} \times 2 \mathrm{H} \times \frac{4 \pi}{3} \mathrm{H}$ | $220 \times 181 \times 120$ | 0.99 | 8.88 | 16 | 9 |
|  | ILES |  | $71 \times 57 \times 51$ | 4 | 28 | 80 | 40 |
|  | Coleman |  | $144 \times 119 \times 80$ | 0.2 | 11.95 | 39 | 24 |
|  | Modesti | $6 \pi \mathrm{H} \times 2 \mathrm{H} \times 2 \pi \mathrm{H}$ | $1024 \times 256 \times 512$ | 0.56 | $5 \cdot 35$ | 8.2 | $5 \cdot 5$ |

Table C.2: Computational grid for comparing the benchmark channel flow test-case with other solvers. Reference values taken from Coleman et al. (1995) ${ }^{[52]}$ and Modesti and Pirozzoli (2016) ${ }^{[194]}$.


Figure C.2: Comparison of mean velocity profiles: (a) $\operatorname{Re}_{\mathrm{b}}=3000 ; \mathcal{M}=1.5$ and (b) $\operatorname{Re}_{\mathrm{b}}=4880$; $\mathcal{M}=3$. Reference data taken from Coleman et al. (1995) ${ }^{[52]}$ and Modesti and Pirozzoli $(2016)^{[194]}$.
the coarsening of the mesh in the buffer layer, the peak value of the $\sqrt{\overline{u^{\prime 2}}}$ will be overpredicted. With a close comparison between the present DNS and ILES this difference can be easily noticed. Similar to the mean velocity profile in figure C.2, the present DNS is very close to the curve by Coleman et al. (1995) ${ }^{[52]}$, and ILES to the Modesti and Pirozzoli (2016) ${ }^{[194]}$. It is unclear why the peak value of the $\sqrt{\overline{u^{\prime 2}}}$ reported by Modesti and Pirozzoli (2016) ${ }^{[194]}$ is over-predicted even though the computational grid used for the simulation is very refined close to the wall. Similar trend was found for the Reynolds stress shown in figure C.4. It has to be mentioned that in all these comparison mismatch increased with the $\mathcal{M}$ number.


Figure C.3: Comparison of rms velocity profiles: (a) $\operatorname{Re}_{\mathrm{b}}=3000 ; \mathcal{M}=1.5$ and (b) $\operatorname{Re}_{\mathrm{b}}=4880$; $\mathcal{M}=3$. Reference data taken from Coleman et al. (1995) ${ }^{[52]}$ and Modesti and Pirozzoli (2016) ${ }^{[194]}$
(a)

(b)


Figure C.4: Comparison of Reynolds stress profiles: (a) $\operatorname{Re}_{\mathrm{b}}=3000 ; \mathcal{M}=1.5$ and (b) $\operatorname{Re}_{\mathrm{b}}=4880$; $\mathcal{M}=3$. Reference data taken from Coleman et al. (1995) ${ }^{[52]}$ and Modesti and Pirozzoli $(2016)^{[194]}$.

From this comparison between results obtained from different solvers, one would prefer to favor the simulation of Coleman et al. (1995) ${ }^{[52]}$ because they were using a spectral solver for their simulation. The results presented for the DNS and ILES with the Compact3D are consistent and highly accurate.

## D

## COMPRESSIBLE RELATIONSHIP

In Chapter 4, compressibility effects and appropriate scaling techniques were discussed. It was shown that the results from the flow over isothermal walls can be compared with the incompressible results only at equivalent friction Reynolds number. At supersonic $\mathcal{M}$ number for which the Morkovin's hypothesis is valid, the compressibility effect due to the mean property variations of density and temperature is maximum. For a fixed $\operatorname{Re}_{\mathrm{b}}$ the equivalent friction Reynolds number $\mathrm{Re}_{\tau}^{*}$ will decrease (see table 4.2). In order to have similar $\operatorname{Re}_{\tau}^{*}$ at high $\mathcal{M}$, the bulk Reynolds number $\operatorname{Re}_{\mathrm{b}}$ has to be raised. For the present simulations, $\mathrm{Re}_{\mathrm{b}}$ and $\mathcal{M}$ number is the operating point for the channel flow, and at higher $\mathcal{M}$ number one cannot know a priori the required $\operatorname{Re}_{\mathrm{b}}$ to get a similar $\mathrm{Re}_{\tau}^{*}$. In this appendix the recipe to infer the $\mathrm{Re}_{\mathrm{b}}$ for a given $\mathcal{M}$ and target $\mathrm{Re}_{\tau}^{*}$ will be presented.

The idea is to relate the conventional and equivalent friction Reynolds number by substituting for $\widetilde{\mathrm{H}}$.The relation is:

$$
\begin{equation*}
\operatorname{Re}_{\tau}=\operatorname{Re}_{\tau}^{*} \sqrt{\frac{\bar{\rho}_{w}}{\bar{\rho}_{c}}}\left(\frac{\bar{\mu}_{c}}{\bar{\mu}_{w}}\right) \tag{D.1}
\end{equation*}
$$

The friction Reynolds number of the raw and the transformed state is related through the ratio of density and viscosity at the wall and core of the channel. In figure D.1, ratios of density and viscosity is presented for different $\operatorname{Re}_{\mathrm{b}}$ at various $\mathcal{M}$ number. An


Figure D.1: Evolution of (a) $\bar{\rho}_{w} / \bar{\rho}_{c}$ and (b) $\bar{\mu}_{c} / \bar{\mu}_{w}$ as the function of $\mathcal{M}$ number. Solid curve is the exponent law $e^{\alpha \mathcal{M}}$ which passes through the data points. Different exponents were used for $\mathcal{M} \leq 0.7$ and $\mathcal{M}>0.7$ for both case.
exponential law $e^{\alpha \mathcal{M}}$ was fitted to the data points in figure D.I. In the subsonic regime $\mathcal{M} \leq 0.7$ the exponent for $\bar{\rho}_{w} / \bar{\rho}_{c}$ was $\alpha=0.0441$ and for $\bar{\mu}_{c} / \bar{\mu}_{w} \alpha=0.0252$. Similarly in the transonic and supersonic regime the exponent $\alpha$ was 0.3995 and 0.2791 respectively.

At a given $\mathcal{M}$ number, for a required $\operatorname{Re}_{\tau}^{*}$ one can get the value of $\bar{\rho}_{w} / \bar{\rho}_{c}$ and $\bar{\mu}_{c} / \bar{\mu}_{w}$ using the exponential fit. These values of density and viscosity ratios can be used to find $\operatorname{Re}_{\tau}$ using equation D.I. Then one can use the following relationship to find $\mathrm{Re}_{\mathrm{b}}$.

$$
\begin{equation*}
\frac{\operatorname{Re}_{\mathrm{b}}}{\operatorname{Re}_{\tau}}=\frac{1}{\bar{\rho}_{w} u_{\tau}} \tag{D.2}
\end{equation*}
$$

The evolution of $\bar{\rho}_{w} \mathfrak{u}_{\tau}$ as a function of $\mathcal{M}$ number is presented in figure D.2. An exponential law is used to fit a curve through the data points like in the previous figures. The value of the exponent for $\mathcal{M} \leq 0.7$ was $\alpha=0.01314$ and for $\mathcal{M}>0.7$ was $\alpha=0.2037$. With the $\operatorname{Re}_{\tau}$ from equation D.I and the value of $\bar{\rho}_{w} \mathfrak{u}_{\tau}$ from the exponential fit in figure D.2, one can use equation D. 2 to find the $\mathrm{Re}_{\mathrm{b}}$.


Figure D.2: Evolution of $\bar{\rho}_{w} \mathcal{u}_{\tau}$ as the function of $\mathcal{M}$ number. Solid curve is the exponent law $e^{\alpha \mathcal{M}}$ which passes through the data points. Different exponents were used for $\mathcal{M} \leq 0.7$ and $\mathcal{M}>0.7$.

## LINEARISED 2D NAVIER-STOKES EQUATIONS AND BOUNDARY CONDITIONS

The linearised two-dimensional N-S! equations were considered for performing the linear stability analysis. For the linearisation all variables were written as a sum of a base flow indicated by subscript 0 , and a perturbation indicated with a prime: $\rho(x, y)=\rho_{0}+$ $\rho^{\prime}(x, y, t) ; u(x, y, t)=U_{0}(y)+u^{\prime}(x, y, t) ; v(x, y, t)=v^{\prime}(x, y, t) ; p(x, y, t)=p_{0}+p^{\prime}(x, y, t) ;$ $T(x, y, t)=T_{0}+T^{\prime}(x, y, t)$. The base flow was taken to be essentially a shear flow with a stream-wise component $U_{0}(y)$ that depend on $y$. For the present test-cases, the $\mathcal{M}$ number was small, as a result the base density $\rho_{0}$ and temperature $T_{0}$ were not dependent on $y$. The linearised equations for the perturbations were:

$$
\begin{align*}
& \frac{\partial \rho^{\prime}}{\partial t}+U_{0} \frac{\partial \rho^{\prime}}{\partial x}+\left(\frac{\partial u^{\prime}}{\partial x}+\frac{\partial v^{\prime}}{\partial y}\right)=0  \tag{E.1}\\
& \frac{\partial u^{\prime}}{\partial \mathrm{t}}+\mathrm{U}_{0} \frac{\partial u^{\prime}}{\partial x}+v^{\prime} \frac{\mathrm{dU}_{0}}{\mathrm{dy}}+\frac{\partial p^{\prime}}{\partial x}=\frac{1}{\operatorname{Re}} \mu_{T} \Delta u^{\prime}+\frac{1}{\operatorname{Re}}\left(\frac{\mu_{T}}{3}+\frac{\kappa}{\mu}\right) \frac{\partial}{\partial x}\left(\frac{\partial u^{\prime}}{\partial x}+\frac{\partial \nu^{\prime}}{\partial y}\right) \cdots \\
& +\frac{1}{\operatorname{Re}} \frac{\mathrm{~d} \mu_{\mathrm{T}}}{\mathrm{dy}}\left(\frac{\partial u^{\prime}}{\partial y}+\frac{\partial \nu^{\prime}}{\partial x}\right)  \tag{E.2}\\
& \frac{\partial v^{\prime}}{\partial t}+\mathrm{U}_{0} \frac{\partial v^{\prime}}{\partial x}+\frac{\partial p^{\prime}}{\partial y}=\frac{1}{\operatorname{Re}} \mu_{T} \Delta v^{\prime}+\frac{1}{\operatorname{Re}}\left(\frac{\mu_{T}}{3}+\frac{k}{\mu}\right) \frac{\partial}{\partial y}\left(\frac{\partial u^{\prime}}{\partial x}+\frac{\partial v^{\prime}}{\partial y}\right) \ldots \\
& -\frac{2}{3} \frac{1}{\operatorname{Re}} \frac{\mathrm{~d} \mu_{T}}{\mathrm{dy}} \frac{\partial u^{\prime}}{\partial x}+\frac{4}{3} \frac{1}{\operatorname{Re}} \frac{\mathrm{~d} \mu_{T}}{\mathrm{dy}} \frac{\partial v^{\prime}}{\partial y}  \tag{E.3}\\
& \frac{\partial p^{\prime}}{\partial t}+\mathrm{U}_{0} \frac{\partial p^{\prime}}{\partial x}+\left(\frac{\partial u^{\prime}}{\partial x}+\frac{\partial v^{\prime}}{\partial y}\right)=\frac{1}{\operatorname{RePr}}\left(1+\frac{v_{t} \operatorname{Pr}}{\gamma \operatorname{Pr}_{t}}\right) \Delta \mathrm{T}^{\prime}+\frac{1}{\gamma \operatorname{RePr}_{t}} \frac{d v_{t}}{d y} \frac{\partial T^{\prime}}{\partial y} \cdots \\
& +\frac{(\gamma-1)}{\operatorname{Re}} 2 \frac{d U_{0}}{d y}\left(\frac{\partial u^{\prime}}{\partial y}+\frac{\partial v^{\prime}}{\partial x}\right)  \tag{E.4}\\
& \mathrm{p}^{\prime}=\frac{1}{\gamma}\left(\mathrm{~T}^{\prime}+\rho^{\prime}\right) \tag{E.5}
\end{align*}
$$

These equations were normalized with the same reference quantities used in the numerical solver (see Section 2.1). Since the temperature was uniform, the speed of sound at the wall was simply the speed of sound anywhere. For completeness, a turbulent eddy-viscosity $\mu_{t}(y)$ depending on $y$ was retained. The result obtained with this eddyviscosity were slightly different but neither better or worse than the ones obtained by accounting only for the molecular viscosity. The total viscosity (molecular + turbulent) is $\mu_{\mathrm{T}}(y)=\mu+\mu_{\mathrm{t}}(y)$. The corresponding dynamic viscosity (obtained after dividing by the uniform $\left.\rho_{0}\right)$ is $v_{T}(y)=v+v_{t}(y)$.

For a temporal linear stability analysis, modal solutions of the form $u^{\prime}(y)=\hat{u}(y) e^{i\left(k_{x} x-\omega t\right)}$ are searched for, where $k_{x}$ is a real wave-number and $\omega$ is the complex angular frequency. This solution is injected into equation E. 1 to E.5, which were discretised on a y-grid. The
corresponding derivation matrix was denoted by D. Marx and Aurégan (2013) ${ }^{[184]}$ provided the generalized eigenvalue problem:

$$
\begin{equation*}
(\mathbf{A}+\omega \mathbf{B}) \boldsymbol{\Psi}=0 \tag{E.6}
\end{equation*}
$$

where:

$$
\begin{align*}
& \mathbf{A}=\left[\begin{array}{c|c|c|c|c}
\mathrm{i} \mathrm{k}_{x} \mathbf{U}_{\mathbf{0}} & \mathrm{i} k_{x} \mathbf{I} & \mathbf{D} & & \\
\hline & \mathrm{i} k_{x} \mathbf{U}_{\mathbf{0}}+\mathbf{A}_{\mathbf{1}} & \mathbf{U}_{\mathbf{0}}^{\prime}+\mathbf{A}_{\mathbf{2}} & & \\
\hline & \mathbf{A}_{3} & \mathrm{ik}_{x} \mathbf{U}_{\mathbf{0}}+\mathbf{A}_{4} & \mathbf{D} & \\
\hline & \mathrm{i} k_{x} \mathbf{I}+\mathbf{A}_{5} & \mathbf{D}+\mathbf{A}_{6} & \mathrm{ik}_{x} \mathbf{U}_{\mathbf{0}} & \mathbf{A}_{7} \\
\hline-\frac{1}{\gamma} \mathbf{I} & & & \mathbf{I} & -\frac{1}{\gamma} \mathbf{I}
\end{array}\right]  \tag{E.7}\\
& \mathbf{B}=\left[\begin{array}{c|c|c|c|c}
-\mathrm{i} \mathbf{I} & & & & \\
\hline & -\mathrm{i} \mathbf{I} & & & \\
\hline & & -\mathrm{i} \mathbf{I} & & \\
\hline & & & -\mathrm{i} \mathbf{I} & \\
\hline & & & &
\end{array}\right] \tag{E.8}
\end{align*}
$$

$$
\begin{align*}
& \mathbf{A}_{1}=\frac{1}{\operatorname{Re}}\left(\frac{4}{3} \mathbf{N}_{\mathbf{T}}+\mathbf{K}\right) \mathrm{k}_{x}^{2}-\frac{1}{\operatorname{Re}}\left(\mathbf{N}_{\mathbf{T}} \mathbf{D}^{2}+\mathbf{N}_{\mathbf{T}}^{\prime} \mathbf{D}\right)  \tag{E.9}\\
& \mathbf{A}_{2}=-\frac{1}{\operatorname{Re}}\left(\frac{1}{3} \mathbf{N}_{\mathbf{T}}+\mathbf{K}\right) \mathrm{ik} \mathrm{k}_{x} \mathbf{D}-\frac{\mathrm{i} k_{x}}{\operatorname{Re}} \mathbf{N}_{\mathbf{T}}^{\prime}  \tag{E.10}\\
& \mathbf{A}_{3}=-\frac{1}{\operatorname{Re}}\left(\frac{1}{3} \mathbf{N}_{\mathbf{T}}+\mathbf{K}\right) i \mathrm{ik}_{x} \mathbf{D}+\frac{2}{3 \operatorname{Re}} i \mathrm{k}_{x} \mathbf{N}_{\mathbf{T}}^{\prime}  \tag{E.11}\\
& \mathbf{A}_{4}=-\frac{1}{\operatorname{Re}}\left(\frac{4}{3} \mathbf{N}_{\mathbf{T}}+\mathbf{K}\right) \mathbf{D}^{2}+\frac{1}{\operatorname{Re}}\left(\mathrm{k}_{x}^{2} \mathbf{N}_{\mathbf{T}}-\frac{4}{3} \mathbf{N}_{\mathbf{T}}^{\prime} \mathbf{D}\right)  \tag{E.12}\\
& \mathbf{A}_{5}=-\frac{2(\gamma-1)}{\operatorname{Re}} \mathbf{U}_{\mathbf{0}}^{\prime} \mathbf{D}  \tag{E.13}\\
& \mathbf{A}_{6}=-\frac{2(\gamma-1)}{\operatorname{Re}} i k_{x} \mathbf{U}_{\mathbf{o}}^{\prime}  \tag{E.14}\\
& \mathbf{A}_{7}=\frac{1}{\operatorname{RePr}}\left(\mathbf{I}+\frac{\operatorname{Pr}}{\gamma \operatorname{Pr}_{t}} \mathbf{N}_{\mathbf{t}}\right)\left(\mathrm{k}_{x}^{2} \mathbf{I}-\mathbf{D}^{2}\right)-\frac{1}{\gamma \operatorname{RePr}_{t}} \mathbf{N}_{\mathbf{t}}^{\prime} \mathbf{D} \tag{E.15}
\end{align*}
$$

The vector of unknown is $\boldsymbol{\Psi}=[\mathbf{R} \mathbf{~ U ~ V ~ P ~ T}]^{t}$ where $\mathbf{U}$ for example contains the values of $\hat{u}$ at the grid points. $\mathbf{U}_{\mathbf{0}}$ and $\mathbf{U}_{\mathbf{0}}^{\prime}$ are matrices containing the velocity and velocity derivative at the grid points. $\mathbf{N}_{\mathrm{T}}$ is the square matrix containing the total viscosity $\left(v_{\mathrm{T}}\right)$ in its diagonal; $\mathbf{N}_{\mathrm{T}}^{\prime}$ is the square matrix containing the y -derivative of the total viscosity ( $\mathrm{d} v_{\mathrm{T}} / \mathrm{dy}$ ) in its diagonal; $\mathbf{N}_{\mathrm{t}}$ is the square matrix with the turbulent eddy-viscosity only $\left(v_{\mathrm{t}}\right)$ in its diagonal; $\mathbf{N}_{\mathrm{t}}^{\prime}$ is the square matrix with the $y$-derivative of the turbulent eddyviscosity only ( $\mathrm{d} v_{\mathrm{t}} / \mathrm{dy}$ ) in its diagonal.

Finally, the implementation of the impedance boundary condition is discussed. Contrary to the spatial stability case, in the general case when the impedance $Z(\omega)$ is a transcendental function of $\omega$ it would not be possible to easily insert the impedance boundary condition into equation E.6. As the present MSD boundary condition leads to
an algebraic function of $\omega$, there is no such difficulty. Taking the Fourier transform of equation 6.2 and 6.3 , one obtains:

$$
\begin{align*}
\mathrm{q}^{\prime}+\omega & \left(\mathrm{i} v^{\prime}\right)=0 \quad(\text { at } y=-1)  \tag{E.16}\\
\mathrm{k} v^{\prime}+\omega & \left(-\mathrm{i} M q^{\prime}-\mathrm{iR} v^{\prime}-\mathrm{ip} p^{\prime}\right)=0 \quad(\text { at } \mathrm{y}=-1) \tag{E.17}
\end{align*}
$$

To introduce this into equation E.6, one has to add one unknown $\mathrm{q}^{\prime}$ in the vector $\boldsymbol{\Psi}$, and add an extra line in equation E. 6 that corresponds to the first equation (equation E.16), in the former system. The second equation of the system (equation E.17), replaces the equation for $v^{\prime}$ at the bottom wall in equation E.6. Overall The matrices $\mathbf{A}$ and $\mathbf{B}$ are square of size $\left(5 N_{y}+6\right) \times\left(5 N_{y}+6\right)$, where $N_{y}$ is the number of discretisation points in the $y$-direction.

## F

## TARGET SOLUTION FOR SPONGE ZONE

The final step is to find a reference solution as a target in the sponge zone. Usually one can use some asymptotic solutions, or the solution for the same problem. In the present case, weighted time averaged flow field taken from the end of the physical domain is used as the target solution. For this, an auxiliary array of size $M$ is used, where the time averaged solutions are solved. The idea is to compute moving average with maximum $\mathrm{nb}_{\text {sam_per_box }}$ samples and store it progressively in the auxiliary array. Such that, moving average computed from the first $n b_{\text {sam_per_box }}$ samples and store it in $\mathfrak{i d x}=1$ of the array, then the moving average from next $\mathrm{nb}_{\text {sam_per_box }}$ samples in $\mathrm{idx}=2$, and so on. A simple schematic for storing the time average is shown in figure F.1. The reference solution was computed only for $u, v, w$ and $s$, and for the pressure $p_{\text {target }}=p_{\infty}$ was used. If $\mathrm{a}=\mathrm{u}, v, w$ and $s$ then the moving average can be defined as:

$$
\begin{equation*}
\operatorname{avg}(i d x)=\frac{n b_{\text {iter }} * \operatorname{avg}(i d x)+a}{n b_{\text {iter }}} \tag{F.1}
\end{equation*}
$$

where $i d x=1, . ., M$ and $n b_{\text {iter }}$ is the number of iteration with the limit being $n b_{i \text { iter }}=$ $n b_{\text {sam_per_box }}$. Usually the spatial simulation will go through a transition phase start-


Figure F.1: Schematic for computing the target field for the sponge zone.
ing from its initial state in the beginning of the simulation. Generally, with the traditional moving average, one uses samples from the start of the simulation to directly use as the reference solution. But the problem with the traditional moving average is the memory of the old samples remains in the reference solution (because latest samples used to compute moving average has the weight-age $1 / n b_{\text {iter }}$ ). Therefore, moving average is calculated and segregated for different time window. Then each segregated moving average is given appropriate weight-age, such that the history effect has minor effect while computing the reference solution. In figure F.1, for $\mathrm{id} x=1-3$, the moving average is already computed with $n b_{\text {sam_per_box }}$ (this can also be called as history), and for $\mathfrak{i d x}=4$ moving average is being computed (which means $n b_{\text {iter }} \leq n b_{\text {sam_per_box }}$ ). For this example, the reference solution for flow field a can be computed as follows:

$$
\begin{equation*}
\mathrm{a}_{\text {ref }}=\frac{\left(\sum_{\text {idx }=1}^{3} n \mathrm{nb}_{\text {sam_per_box }} \times \mathrm{avg}(\mathrm{idx})\right)+\mathfrak{n b _ { \text { iter } } \times a v g ( 4 )}}{3 \times n b_{\text {sam_per_box }}+n b_{\text {iter }}} \tag{F.2}
\end{equation*}
$$

Here the first three moving averages (or history) has equal weightage and for the present average the weightage varies according to the number of samples used for that particular moving average. By doing so, the weightage of the history is distributed among averages
computed from different time, thus the history effect is reduced. Additionally, the new samples are given more weightage compared to the traditional moving average. Thus the new method to find the reference solution for sponge zone using the segregated moving time average is more efficient than the traditional moving average technique. Moreover, once the auxiliary array is filled completely with the moving averages, then the old moving averages will be replaced with the new data. From the several tests performed, it was found that, one local moving average length should be at least $\mathrm{H} / \mathrm{u}_{\mathrm{b}}$ and the complete average should be at least 5-6 flow through time.

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Boussinesq eddy viscosity hypothesis

First DNS of
compressible supersonic channel

Modified $2^{\text {nd }}$ der. FDS for ILES
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Improved scaling technique for compressible flows.

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## INDEX

A
acoustic liner, 113

## B

boundary condition, 113, 203
boundary conditions, 20
bulk density, 18
bulk Reynolds number, 18
bulk velocity, 18

## C

CFL, 31
characteristics waves, 16,114
compact FDS, 22, 24
Compact3D, 33
compressibility effects, 49, 65
compressible N-S, 15
compressible scaling, 67
compressible transformation, 66

## D

damping ratio, 114
DNS, 25, 35, 47
drag, 120
drag coefficient, 116

## E

eddy-viscosity, 201
eigenvalue problem, 202

## F

flow modification, 120, 121, 125
flow-liner interaction, 120, 121, 125, 130
Fourier number, 32
friction Reynolds number, 19
friction velocity, 18

## G

grid convergence, 115, 116
grid requirement, 52, 116

## H

high $\operatorname{Re}_{\tau}, 47$
hyper-viscosity, 25, 35, 47

## I

ILES, 26
impedance boundary condition, 113,203
implicit filter, 26
inner variables, 18, 66
isothermal boundary condition, 20

## K

Kolmogorov scale, 27

## L

law-of-the-wall, 66, 120, 121
linearised N-S, 201

## M

Mach number, 18, 123
Mass-Spring-Damper, 113, 203
modified second-derivative, $25,26,41$
modified wave-number, 24
Morkovin's hypothesis, 66

## N

near-wall turbulence, 122, 125
non-dimensionalisation, 16, 117
non-reflecting boundary conditions, 115
numerics, 22

## P

parallel efficiency, 34
parametric study, 41, 119
phase average, 127, 131
phase-speed spectrum, 133

## R

Re effect, 66
resistance, 120
resonance frequency, 114, 121
Reynolds number, 18, 19
Reynolds stress, 66, 122, 131
rigid-wall boundary condition, 20
Runge-Kutta, 30

## S

scalability, 34
scaling techniques, 65
semi-local scaling, 71
skin-friction coefficient, 116
spatial discretisation, 23, 29
stability, 26, 28, 31, 120, 123
stability analysis, 132, 135
strong scaling, 34
supersonic, 49
surface wave, 124-126, 134
SVV, 26

T
thermodynamic relation, 21, 140
time constraint, 26, 28, 32
time-integration, 30
Trettel's transformation, 70
triple decomposition, 127, 131

## V

validation, 35, 115, 118
van Driest scaling, 67
viscous length-scale, 18

## W

wall-resolved ILES, 52


[^0]:    1 Note: the program generates uniform mesh in the physical domain.

[^1]:    2 Number of grid points in sponge should not exceed the value from step 1.

