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# Stress-test, produits structurés et gestion de bilan bancaire

Bünyamin Erkan

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**UNIVERSITE DE CERGY-PONTOISE E.D. ECONOMIE,  
MANAGEMENT, MATHEMATIQUES CERGY LABORATOIRE DE  
RECHERCHE THEMA**

**STRESS-TEST, PRODUITS STRUCTURES ET GESTION DE  
BILAN BANCAIRE  
THESE**

*pour l'obtention du titre de*

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à ma famille

## Résumé

Le but de cette thèse est de relier la valorisation des produits dérivés à l'analyse par scénario qui fait référence dans la gestion des banques. La valorisation des options est un domaine qui a donné lieu à une très large littérature avec un cadre mathématique bien défini, développé plutôt à destination des praticiens de la finance de marché dans l'optique d'une couverture par réplique. La gestion du risque de ces produits est donc analysée sous l'angle du fameux principe d'absence d'opportunité d'arbitrage. Selon ce principe, l'estimation pour un taux futur devient celle qu'anticipe les marchés dès aujourd'hui grâce à la présence d'instrument de couverture. D'un autre côté, le pilotage financier en entreprise se base sur une approche dite "par scénario". Différents scénarios qui semblent perspicaces et font sens économiquement pour les dirigeants de la banques sont retenus, et une projection pour chaque scénario est étudiée. Ces scénarios ne sont en général pas probabilisés; nous sommes dans une modélisation de l'incertain où la notion de probabilité et d'espérance n'intervient pas. La banque, qui possède des produits complexes liés à son activité, ne se couvre pas comme le fait un service lié à des activités de marché. C'est dans l'étude de ce cadre d'analyse que s'inscrit la thèse. Nous analysons ce besoin de projection par scénario sur le périmètre des produits complexes.

L'introduction établit le cadre bancaire et plus précisément la réglementation bancaire afin de situer la thèse dans ce contexte.

Le chapitre 1 est un chapitre préliminaire qui rappelle les notions fondamentales de valorisation d'options.

Le chapitre 2<sup>1</sup> étudie la valorisation des options bermudéennes "cross devises". Ces produits, avec options de remboursement anticipé, sont assez fréquents dans le bilan des banques. Nous proposons notamment une cal-

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<sup>1</sup>Le chapitre 2 a été présenté sous la forme d'un papier de recherche à la "32th International Conference of the French Finance Association" et à la "8th International Finance Conference" en 2015.

ibration adaptée aux produits de long terme, la maturité longue étant une autre des propriétés de ces produits de bilan. Une étude d'impacts finalise ce chapitre. Nous étudions l'impact de chaque marché d'options, qui sont des sources de risque, sur la valorisation de notre produit structuré.

Le chapitre 3 définit un cadre de projection des surfaces de volatilités implicites. Cette outil nous permet ainsi de relier les anticipations des marchés d'option avec un scénario de taux qui fait référence en pilotage bancaire.

Le chapitre 4 analyse les stress-tests avec une vision "grand angle". Nous énonçons les principes d'un stress-test et discutons des différences et problèmes de comptabilité de cet outil, très utilisé en pilotage bancaire, avec la modélisation risque neutre utilisée dans la valorisation d'options. Nous terminons notre étude en analysant les résultats des banques françaises du stress-test EBA 2016.



## Abstract

The aim of this thesis is to link the pricing of derivative products to the scenario based analysis used in banks management. Options' pricing has a wide literature and a well-defined mathematical framework focusing on the market finance problematics and in particular on delta hedging. The risk management of these products are thus analyzed with this point of view based on the famous absence of arbitrage opportunity principle. Under this principle, the best estimates for tomorrow are those anticipated by the market today due to the presence of hedging instruments. On the other side, corporate financial management is based on a scenario based approach. Senior managers select scenarios which seem both interesting and plausible and a projection for each scenario is analyzed. Generally, we do not attribute probability to the scenarios, which implies a modeling based on uncertainty where the notion of probability and mathematical expectation are not involved. Bank has complex products for the needs of its activity and it does not hedge them as a service related to market activities would do. We do our analyze in this framework. We analyze this need of scenario based projection for complex products.

The introduction defines the banking framework and more specially the banking regulatory and links our thesis to this context

Chapter 1 is a preliminary chapter which provides the basic notions on derivative pricing.

Chapter 2<sup>2</sup> details the pricing of cross-currency bermuda derivatives. These early redemption products are common on the balance sheet of the banks. We propose a calibration method for long term products. Indeed another property of this funding product is a long maturity. We end the chapter with an impact study where we analyze the impacts of the different volatility

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<sup>2</sup>The chapter 2 is a working paper that was presented at the 32th International Conference of the French Finance Association and at the 8th International Finance Conference in 2015.

markets, which we consider as risk factors, to the price of the product.

Chapter 3 proposes a projection model for implied volatility surface. This tool allows to link implied volatility projections with an interest rate scenario which is a key element of financial management.

Chapter 4 focuses on stress-testing. We define the principle of stress-testing and discuss the differences and incompatibility problems of this modeling (very common on banks management) with the risk neutral modeling (which is the common modeling in derivative pricing). We end our study with a part dedicated to 2016 EBA stress-test results of French banks.

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# Introduction: Pilotage Bancaire

## 1 Le Rôle de la Banque dans l'Économie

Le rôle principal de la banque reste le financement de l'économie réelle. La banque récolte les dépôts des clients qui sont liquides (disponibles à tout moment) et finance les crédits long terme. Elle s'engage donc à garantir le capital des dépôts à vue des clients. Elle subit donc le risque de crédit porté à son actif. Elle subit également par définition de son activité un risque potentiel de liquidité, potentiel car historiquement les dépôts à terme des clients ne s'écourent pas aussi rapidement. Elle s'engage à transformer en monnaie fiduciaire à tout moment les dépôts des clients. La gestion de ce risque de liquidité revient donc à la banque. En plus de ce risque de liquidité, la banque fait face à un risque de taux qui provient de la différence de taux entre les dépôts et les crédits. De plus, les banques françaises ont la spécificité d'accorder des prêts à taux fixe justifié par des dépôts majoritairement à taux fixe (plus précisément à taux zéro). Cette exposition à des taux fixes entraîne une sensibilité de la marge nette d'intérêt dégagée de l'activité de crédit-dépôts. Ces risques de bilan sont gérés par le service ALM des banques.

En plus de l'activité de banque de détail comme détaillée plus haut, les banques ont aussi des activités d'investissement et de financement (BFI). Ces activités de marché qui viennent s'ajouter au rôle de financement de l'économie réelle s'inscrivent dans la logique de banque universelle qui prédomine notamment en France.



## 2 Du Bilan et Compte de Résultat aux Ressources Rares Bancaires

Le bilan et le compte de résultat sont les deux éléments centraux de la banque. Ces deux jeux de données permettent un pilotage stratégique de la banque. Nous pouvons en effet en extraire beaucoup d'informations sur la banque et plus précisément sur ce que l'on appelle les ressources rares bancaires: rentabilité, solvabilité et liquidité. Ces indicateurs qui résument la situation financière de la banque sont au centre des préoccupations car ils permettent une analyse approfondie du business model et de l'appétit au risque de l'entreprise. C'est d'ailleurs sur les notions de solvabilité et de liquidité (qui sont les indicateurs reflétant la partie "risque") que les régulateurs vont mettre des contraintes afin de s'assurer d'une stabilité du système financier.

Le bilan détaille les emplois et les ressources de la banque ou en d'autres termes comment elle se finance et comment elle met à profit ce financement pour générer du résultat. Le bilan d'une banque est de ce fait capital car c'est en majorité du bilan que proviennent les risques, une partie importante du résultat, et la liquidité.

C'est également au travers du bilan que l'on peut influencer sur la notion de solidité financière de l'entreprise en jouant sur la décomposition dette - capital. Cet arbitrage dette-capital influe sur la solvabilité tout comme les risques. Il est bon de préciser qu'aujourd'hui une analyse plus fine du passif est réalisée dans le cadre des mesures concernant la résolution (TLAC, MREL, Bail-in). La classification du passif n'est plus binaire mais toute une subordination du passif se met en place.

Les risques issus du bilan sont matérialisés par le calcul de RWA qui se décompose en 3 catégories: risque de crédit, risque de marché et risque opérationnel (pour ce dernier il n'y a pas forcément de lien avec le bilan). La solvabilité est exprimée en fonction des RWA; elle représente le ratio de capitaux sur le

Table .1: Compte de résultat

Marge nette d'intérêt (1)
Commissions (2)
Autres revenus (3) (dont résultats des positions à la juste valeur)
Produit net bancaire (4) = (1) + (2) + (3)
Frais généraux (5) (dont salaires)
Résultat brut économique (6) = (4) - (5)
Coût du risque (7)
Impôts (8)
Résultat (9) = (6) - (7) - (8)

Table .2: Bilan

Actif	Passif
comptes banque centrale et assimilés	dettes banque centrales
portefeuille financier	dépôts clients et livrets
crédits clientels	dettes: titres et emprunts
autres (immobilisation, goodwill, ...)	fonds propres

risque.

C'est également via le bilan que l'on obtient les différentes métriques de liquidité réglementaire introduit par Bâle 3: LCR pour la liquidité court terme (1 mois) et le NSFR pour la liquidité long terme (1 an). Alors que le LCR impose d'avoir plus d'actifs liquides que de dépôts liquides, le NSFR impose d'avoir moins de besoins de ressources stables que de ressources disponibles.

Il est bon de préciser que nous avons un lien direct entre le bilan et le compte de résultat pour les revenus en marge nette d'intérêt. C'est en effet les encours des différents produits du bilan combinés avec les taux d'intérêt qui nous donnent le résultat issu de la marge d'intérêt.

Nous exposons dans la Table 1 et la Table 2 une description des grandes catégories de résultats et bilanciers (classées par liquidité décroissante).

## 3 La Réglementation Bancaire

### 3.1 Sur l'utilité d'une réglementation bancaire

La réglementation permet une surveillance des banques. Elle a pour objectif d'inciter les banques à une gestion des risques plus prudente[2]. Le rôle central des banques dans le financement de l'économie réelle place ces dernières dans un cadre assez spécial. Pour éviter la fuite des dépôts massive en temps d'incertitude économique, les gouvernements mettent en place des fonds de garantie des dépôts (jusqu'à 100 000 euros par client et par banque pour le Fonds de Garantie des Dépôts et de Résolution), ce qui présente un aléa moral : les déposants ne se soucient plus du risque de défaut et de ce fait de la gouvernance ainsi que de la bonne gestion des risques de leurs banques. C'est donc les superviseurs qui prennent le rôle de surveillance de la bonne gestion des banques. Différents acteurs sont impliqués dans cette surveillance. La Banque Centrale Européenne est le superviseur pour le contrôle prudentiel. Le Single Resolution Board a en charge la supervision de la résolution. L'European Banking Authority a un rôle plutôt sur l'aspect norme et méthodologie aussi bien pour le prudentiel que pour la résolution. De plus, chaque pays a une autorité tutelle locale qui fait le lien entre les banques du pays et les superviseurs européens. En France, ce rôle est assuré par le Autorité de Contrôle Prudentiel et de Résolution. Dans ce qui suit, nous décrivons le cadre actuel du contrôle prudentiel.

La supervision prudentielle est bâtie sur 3 piliers :

- Exigence minimale de fonds propres
- Processus de surveillance prudentielle
- Discipline de marché

## 3.2 Le Pilier 1

Le pilier 1 exige un niveau minimum de capital pour les 3 types de risques majeurs auxquels une banque est exposée :

- le risque de crédit
- le risque de marché
- le risque opérationnel

Il y a d'ailleurs eu des discussions entre le comité de Bâle et les banques sur l'inclusion du risque de taux d'intérêt dans le pilier 1. Au final, les banques ont eu gain de cause et le risque de taux du portefeuille bancaire (IRRBB) est resté dans le pilier 2 avec une méthodologie qui reste assez libre mais cependant avec un formalisme plus cadré qu'auparavant. Les trois risques pilier 1 sont donc comptabilisés par le biais des RWA. Le risque de crédit peut être calculé en méthode standard ou en modèle avec notations internes (IRB) qui lui-même se décompose en deux : IRB Fondation ou IRB avancée. Dans IRBF, seule la probabilité de défaillance est estimée par la banque alors qu'en IRBA tous les paramètres sont estimés par la banque par des modèles internes [1]. Comme le qualifie Brunel dans [1], IRB est un modèle externe avec un paramétrage interne.

Pour le calcul du risque de marché, la banque a le choix entre la méthode standard et ses modèles internes et notamment VaR, VaR stressée, IRC, CRM. Ainsi, les modèles internes permettent de capter les risques liés à des périodes de stress (VaR stressée) et les risques de crédit / migration du portefeuille de négociations (IRC, CRM). [3]

Le risque opérationnel peut être calculé par 3 méthodes :

- approche de base, forfaitaire par rapport au PNB de la banque;

Figure .1: SREP



- approche standard, forfaitaire par rapport au PNB par ligne de métier;
- approche avancée, reposant sur des modèles internes validés par les autorités de tutelle.

### 3.3 Le Pilier 2

L'objectif du pilier 2 est d'assurer une cohérence entre le profil de risque des banques, la gestion de leurs risques et la planification du capital. Deux documents sont à fournir par les banques dans le cadre du pilier 2 : l'ICAAP et l'ILAAP.

L'ICAAP traite de la capitalisation de la banque pour les différents risques supportés. Il met notamment en lien le business model, l'appétit au risque, le stress-testing afin d'en déduire un niveau de capital adéquat pour la banque.

L'ILAAP traite de la stratégie de gestion de la liquidité de la banque. La banque y expose une description détaillée de la gestion de la liquidité et du financement : plan de refinancement, circulation de la liquidité dans le groupe, indicateur quantitatif, gestion du risque de liquidité...

Le processus “Supervisory Review and Evaluation Process” (SREP) permet au régulateur de s’assurer de la bonne gestion de la banque (notamment sur les stratégies, processus, mécanismes, niveau de capital et gestion de la liquidité) par rapport aux risques encourus<sup>3</sup>. Il repose sur 4 éléments qui sont :

- “Business Model Assessment” : une analyse de la profitabilité de la banque et notamment une étude sur :
  - La viabilité du modèle d’entreprise qui correspond à la capacité de l’entreprise à générer du résultat sur une période d’un an;
  - La durabilité de la stratégie de l’entreprise qui correspond à la capacité à générer du résultat sur une période de 3 ans avec les hypothèses du plan stratégique et des prévisions financières internes.
- “Gouvernance and Risk Management Assessment” (Evaluation de la gouvernance interne et des contrôles): Evaluation de la conformité de la gouvernance interne aux standards et de la gestion des risques de la banque. Cette partie permet de s’assurer que la gouvernance interne et les contrôles au sein de l’institution sont adaptés à son profil de risque, son business model, sa taille et sa complexité. Les éléments clés de la notation sont :
  - La structure de gouvernance globale;
  - La culture corporate et culture risques;
  - L’organisation et fonctionnement des organes de direction;
  - les pratiques de rémunération;
  - Le contrôle interne;

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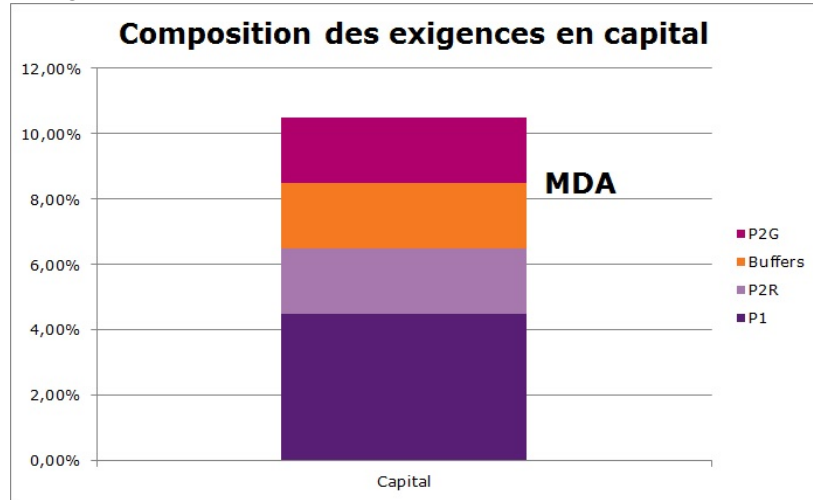
<sup>3</sup><http://www.eba.europa.eu/regulation-and-policy/supervisory-review-and-evaluation-srep-and-pillar-2>

- La structure de gestion des risques (incluant ICAAP et ILAAP);
  - Les systèmes d’information;
  - Le plan de résolution.
- “Assessment of risks to Capital”: adéquation du niveau de capital aux risques encourus (ICAAP) et leurs “benchmarking” face aux autres banques.
  - “Assessment of risks to Liquidity and Funding”: évaluation de la gestion de la liquidité et du financement (ILAAP). Différents éléments sur la liquidité (besoin intraday, court et long terme, “buffers”, stress-test liquidité réglementaire) et sur le financement (profil de refinancement, accès au marché) sont prises en considération dans cette partie.

Le régulateur utilise ces informations afin de calculer un score quantitatif en comparant notamment les résultats de la banque avec les résultats de ses paires. Une couche qualitative est ajoutée à ce score quantitatif afin d’obtenir le score final qui correspond à un niveau de capital supplémentaire à rajouter au niveau de capital exigé au titre du pilier 1. Ce niveau de capital au titre du SREP est annoncé par la BCE à la banque en laissant libre sa communication ou non au marché par la banque elle-même. Ce niveau est appelé lors de l’exercice du SREP 2016 le Pilier 2 Requirement. Il détermine le niveau de MDA (“Maximum Distribution Amount”). Ce niveau qui se place sur la couche pilier 1 et des buffers augmentés de la couche P2G entraîne des restrictions sur les distributions de dividendes, les paiements de coupons d’Additional Tier 1 et le paiement de bonus. Il est prévu qu’il soit déclenché dès que les exigences au titre du pilier 1 et P2G ne sont pas respectées.

Au dessus de cette couche P2G, nous plaçons les différents buffers. Au dessus de ces buffers, nous avons la couche Pilier 2 Guidance (P2G) qui correspond à un niveau de capital couvrant les risques de crises provenant des résultats

Figure .2: Composition des différentes couches de capital



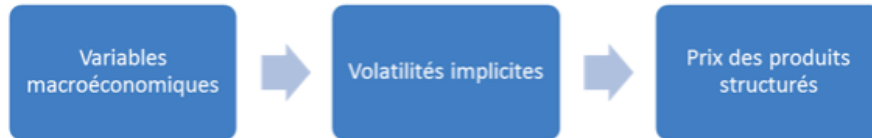
quantitatifs du stress-test EBA (les résultats qualitatifs provenant de la gestion du stress-test viennent alimenter le P2R). Ce niveau n'entraîne pas une réaction directe des régulateurs. Cependant la banque aura à justifier son non respect.

## 4 Objectif et Organisation de la Thèse

Dans ce contexte réglementaire, nous voyons donc clairement qu'il y a une séparation entre les exigences de capital qui viennent couvrir les risques encourus dans un régime de croisière (P1, buffers, P2R) et les exigences venant couvrir des risques survenant d'une situation de crise (P2G). C'est à la modélisation des crises extrêmes que la thèse s'intéresse. Dans ce contexte de projection des éléments du bilan et du résultat bancaire sur la base d'un scénario de crise, nous allons nous intéresser tout particulièrement à la projection de la valorisation des produits structurés qui sont construits sur une logique de risque-neutralité qui n'est pas naturellement compatible avec les approches par scénarios. La façon la plus naturelle revient à conserver la logique risque



Figure .3: les liens de causalité en valorisation de produits structurés



neutre de la valorisation tout en projetant les facteurs de risques tels que les taux d'intérêt et la volatilité implicite.

En général, dans le cadre de ces stress-tests, le méthodologie EBA impose une règle simplificatrice sur le remboursement anticipés des émissions. Lorsque l'option est à la main de la contrepartie, elle est supposée être exécutée dès la première date d'option alors que les options à la main de la banque ne sont jamais exécutées.

La thèse se penche sur cette problématique avec une approche de type "bottom-up". Dans notre thèse, l'échelon le plus fin correspond à la valorisation d'un produit structuré avec toutes les subtilités techniques qui y sont associées. Après un chapitre préliminaire rappelant quelques notions techniques, nous commençons par un chapitre (chapitre 2) qui s'intéresse à la méthodologie de valorisation d'un type assez commun de produits structurés. Ce chapitre est dédié à l'implémentation informatique et repose surtout sur les applications numériques.

Le chapitre 3 ajoute à l'aspect méthode de valorisation une approche économique qui permet déjà de faire un premier lien entre le pilotage financier bancaire et la valorisation des produits structurés. Cette partie contient également en majorité l'exploitation de résultats numériques.

Le chapitre 4 est celui qui se place au sommet de l'analyse. Nous étudions la notion de test de résistance (stress-test) et menons une réflexion sur les hypothèses de modélisation qui la sous-tendent afin de bien interpréter les résultats qui en seront issus. Une confrontation entre cette modélisation et celle issue de la valorisation nous permet de bien voir toutes les différences

entre ces deux approches et de mettre en lumière leurs incompatibilités en particulier les ordres de grandeurs. Nous clôturons ce chapitre et la thèse par une analyse des résultats du stress-test EBA de 2016 en examinant en particulier les banques françaises.

## Part I

# Interest Rates Models for Derivatives Pricing

## Introduction

This preliminary chapter recalls the notions on quantitative modeling of interest rates market and specially on derivatives pricing. In the spirit of [Brigo], we expose different elements of interest rates modeling. In fact in this chapter, we exclusively consider interest rate modeling in the framework of derivative pricing. It is important to notice this point because the absence of arbitrage opportunity framework that we study here is not the only way to model interest rates. This no arbitrage pricing was introduced by Bachelier [1] in 1900 and developed by Black and Scholes [2] in 1973 for non linear (payoff) products pricing. As it is perfectly adapted in pricing, it is not at all the perfect tool for all type of financial problems. No arbitrage modeling just takes as input market view and does not allow to inject into the model our own predictions on market fluctuations as market view is the best prevision because of the non arbitrage hypothesis.

There is a very large set of interest rates related products in the market and if the client is interested by a very exotic payoff non available in the market, the seller asks his structuration desk to create it. As our thesis focuses on balance sheet of banks and financial institution, we can hope a priori that the scope of products is more restrictive. Unfortunately it is not the case. If we take a look at the Euro Medium Term Notes (EMTN) program of different banks or financial institutions (these documents are public and available on internet), we can notice the wide number of payoffs.

We cannot give an exhaustive description of all interest rates market linked products. We focus on the most common products starting from the most basic to the most complex.

We briefly give the background to non arbitrage pricing models in interest rates market as the following chapters of the thesis require a minimum knowledge on this field. First of all, we present the different interest rates linked derivatives. We begin by introducing the most common products which are quoted on the market and which allows to extrapolate the price of non quoted products. Then we focus on a special class of options: the Bermuda and American options. This class of options is interesting because of the wide use of them in balance sheet management of banks. We finish by introducing structured debt products which are the funding instruments of banks.

## **1 Basic Interest Rates Options: Caps, floors and swaptions**

These products are what we can call the benchmark products. They are the most common products with very good quotation on market. Their extreme liquidity permits to obtain very precise price or more exactly volatility. Indeed, derivatives are quoted in terms of implied volatilities and not in terms of prices. The implied volatility is nothing else than the quantity to inject in the Black Scholes formula to obtain the price. On the implied volatility, Rebonato gives an interesting definition qualifying the implied volatility by “wrong number which, plugged into the wrong formula, gives the right answer.”[3]

As we said, two families of market are well quoted: cap/floor market and swaption market. All the others more complex derivatives are embedded in one of these two markets to get their prices. Derivatives pricing can be

defined roughly speaking by an interpolation method based on the quoted volatilities on caps, floors or swaptions market. Interpolation for non quoted vanilla products (caps, floors and swaptions with strike, maturity or tenor different from the strikes, maturities and tenors of the quoted products) is done by injecting the characteristic of the considered product in the Black Scholes formula. We now describe the three basic options: caps, floors and swaptions.

## 1.1 Caps and Floors

A cap (respectively floor) is a payer (respectively receiver) interest rate swap which has condition on each payment: for each payment date the exchange is effective if the owner of the option receives more than he pays. The discounted payoff of a cap is given by:

$$\sum_{i=\alpha+1}^{\beta} D(0, T_i) N \tau (F(T_{i-1}; T_{i-1}, T_i) - K)^+$$

and those of floor is then given by:

$$\sum_{i=\alpha+1}^{\beta} D(0, T_i) N \tau (K - F(T_{i-1}; T_{i-1}, T_i))^+$$

where:

$F(T_0; T_{i-1}, T_i)$ : the forward rate fixed at  $T_{i-1}$  and payed at  $T_i$  viewed at time  $T_0$ ;

$N$ : the nominal;

$\tau$ : the year fraction;

$K$ : strike of the option;

$D(0, T_i)$  : stochastic discount factor at time 0 for the maturity  $T_i$ .

The cap (resp. floor) option can be decomposed on caplets (floorlets), which can be priced independently of each other. This is a true advantage because it means that we can price each caplet in the well adapted universe of probability. Each caplet then has its own numeraire (in other world benchmark risk free asset).

Let us recall that the Black formula for a caplet expiring in  $T_1$  and maturing in  $T_2$  is given by:

$$C_0 = \tau P(0, T_2) F(0; T_1, T_2) \cdot N(d_1) - \tau P(0, T_2) \cdot K \cdot N(d_2)$$

with

$$d_1 := \frac{\ln\left(\frac{F(0; T_1, T_2)}{K}\right) + \frac{\sigma^2}{2} T_1}{\sigma \sqrt{T_1}},$$

$$d_2 := \frac{\ln\left(\frac{F(0; T_1, T_2)}{K}\right) - \frac{\sigma^2}{2} T_1}{\sigma \sqrt{T_1}}.$$

The analogous formula for the floorlet is given by

$$F_0 = \tau P(0, T_2) \cdot K \cdot N(-d_2) - \tau P(0, T_2) F(0; T_1, T_2) \cdot N(-d_1).$$

The best way to get intuitions on derivatives pricing is to focus on this formula. Then we expose the demonstration of the formula from caplet in what follows.

### **Proof of the Black formula for a cap**

The objective is to value the price of the call option with a payoff given by:

$$(F(T_1; T_1, T_2) - K)^+.$$

Let us define the forward rate

$$F(t; T_1, T_2) = \frac{\frac{P(t, T_1) - P(t, T_2)}{T_2 - T_1}}{P(t, T_2)} = \tau \frac{P(t, T_1) - P(t, T_2)}{\text{numeraire}}.$$

The price of the call is given by the mathematical expectation of the discounted payoff:

$$\begin{aligned} &= \mathbb{E}^{\mathbb{P}} \left[ \tau \exp \left( - \int_0^{T_2} r_s ds \right) (F(T_1; T_1, T_2) - K)^+ \right] \\ &= \mathbb{E}^{\hat{\mathbb{P}}} \left[ \tau \frac{B(0)}{B(T_2)} (F(T_1; T_1, T_2) - K)^+ \right] \\ &= \mathbb{E}^{\mathbb{Q}} \left[ \tau \frac{P(0, T_2)}{P(T_2, T_2)} (F(T_1; T_1, T_2) - K)^+ \right] \\ &= \tau P(0, T_2) \cdot \mathbb{E}^{\mathbb{Q}} [(F(T_1; T_1, T_2) - K)^+] \end{aligned}$$

with

0: valuation date;

$T_1$ : expiry date (reset date) of the forward rate;

$T_2$ : maturity date of the forward rate;

$K$ : strike of the option;

$\mathbb{P}$ : historical probability;

$\hat{\mathbb{P}}$ : risk neutral probability;

$\mathbb{Q}$ : forward neutral probability associated with the numeraire  $P(0, T_2)$ ;

$\tau$ : the year fraction;

$P(i, j)$ : the price at time  $i$  of a zero coupon giving 1€ at time  $j$ ;

$B(i)$ : money market account at time  $i$  with by definition:

$$B(i) := B(0) \exp \left( - \int_0^i r_s ds \right).$$

To price our derivative, we have to associate a diffusion process to our forward rate. The dynamic of our forward rate is directly related to the universe of probability in which we perform the calculation. In the case of caplet pricing, the adapted universe is the forward neutral probability universe with the zero coupon bond numeraire. The numeraire is the risk free account that we take as benchmark to value our derivative. The biggest advantage of this numeraire is that at time of pricing it is not stochastic. As the money account remuneration is not fixed ex ante, in the case of stochastic interest rate modeling it makes the numeraire being stochastic. To eliminate this stochastic characteristic of the numeraire, we consider the investment of our cash in zero coupon with fixed earning contrary to the investment of the money market account giving non determined earning. Passing from risk neutral universe to forward neutral universe impacts the dynamic of the discounted underlying as it has to be a martingale in the chosen universe.

In the forward neutral space, the dynamic of our forward rate is given by:

$$dF(t; T_1, T_2) = \sigma \cdot F(t; T_1, T_2) \cdot dW_t.$$

The Ito's formula gives

$$\begin{aligned} d \ln(F(t; T_1, T_2)) &= \ln'(F(t; T_1, T_2)) \cdot dF(t; T_1, T_2) + \frac{1}{2} \ln''(F(t; T_1, T_2)) \cdot d \langle F(t; T_1, T_2) \rangle \\ &= \frac{1}{F(t; T_1, T_2)} \cdot dF(t; T_1, T_2) + \frac{1}{2} \left( -\frac{1}{(F(t; T_1, T_2))^2} \right) \cdot d \langle F(t; T_1, T_2) \rangle \\ &= \frac{\sigma F(t; T_1, T_2)}{F(t; T_1, T_2)} dW_t - \frac{1}{2} \frac{1}{(F(t; T_1, T_2))^2} (\sigma F(t; T_1, T_2))^2 \cdot d \langle W \rangle_t \\ &= \sigma \cdot dW_t - \frac{1}{2} \cdot \sigma^2 \cdot dt. \end{aligned}$$



By integrating on both sides, we obtain:

$$\begin{aligned}
\int_0^{T_1} d \ln (F(t ; T_1, T_2)) &= \int_0^{T_1} \sigma dW_t - \frac{1}{2} \int_0^{T_1} \sigma^2 dt \\
\ln F(T_1 ; T_1, T_2) - \ln F(0 ; T_1, T_2) &= \int_0^{T_1} \sigma dW_t - \frac{1}{2} \int_0^{T_1} \sigma^2 dt \\
\ln \frac{F(T_1 ; T_1, T_2)}{F(0 ; T_1, T_2)} &= \int_0^{T_1} \sigma dW_t - \frac{1}{2} \int_0^{T_1} \sigma^2 dt \\
\frac{F(T_1 ; T_1, T_2)}{F(0 ; T_1, T_2)} &= \exp \left( \int_0^{T_1} \sigma dW_t - \frac{1}{2} \int_0^{T_1} \sigma^2 dt \right) \\
F(T_1 ; T_1, T_2) &= F(0 ; T_1, T_2) \cdot \exp \left( \int_0^{T_1} \sigma dW_t - \frac{1}{2} \int_0^{T_1} \sigma^2 dt \right).
\end{aligned}$$

If we go back to the price of our option, we get:

$$\begin{aligned}
\mathbb{E}^{\mathbb{Q}} \left[ (F(T_1 ; T_1, T_2) - K)^+ \right] &= \mathbb{E}^{\mathbb{Q}} \left[ \left( F(0 ; T_1, T_2) \cdot \exp \left( \int_0^{T_1} \sigma dW_t - \frac{1}{2} \int_0^{T_1} \sigma^2 dt \right) - K \right)^+ \right] \\
&= F(0 ; T_1, T_2) \cdot \mathbb{E}^{\mathbb{Q}} \left( \exp \left( \int_0^{T_1} \sigma dW_t - \frac{1}{2} \int_0^{T_1} \sigma^2 dt \right) 1_{\epsilon} \right) - K \cdot \mathbb{Q}(\epsilon) \\
&= F(0 ; T_1, T_2) \cdot \mathbb{E}^{\mathbb{Q}} \left( \exp \left( \sigma W_{T_1} - \frac{1}{2} \sigma^2 T_1 \right) 1_{\epsilon} \right) - K \cdot \mathbb{Q}(\epsilon);
\end{aligned}$$

with  $\epsilon$  the exercise region given by:

$$\begin{aligned}
\epsilon &= \left\{ F(0 ; T_1, T_2) \cdot \exp \left( \sigma W_{T_1} - \frac{1}{2} \sigma^2 T_1 \right) > K \right\} \\
&= \left\{ \frac{W_{T_1}}{\sqrt{T_1}} < \frac{\ln \left( \frac{F(0 ; T_1, T_2)}{K} \right) - \frac{\sigma^2}{2} T_1}{\sigma \sqrt{T_1}} \right\}.
\end{aligned}$$

Let us introduce the probability  $\hat{\mathbb{Q}}$  equivalent to  $\mathbb{Q}$  defined by:

$$\frac{d\hat{\mathbb{Q}}}{d\mathbb{Q}} \Big|_{\mathcal{F}_{T_1}} := Z_{T_1} = \exp \left( \sigma W_{T_1} - \frac{1}{2} \sigma^2 T_1 \right).$$

We have by definition:

$$\mathbb{E}^{\mathbb{Q}}(1_{\epsilon} Z_{T_1}) = \hat{\mathbb{Q}}(\epsilon).$$

The value of the option is then:

$$C_0 = F(0; T_1, T_2) \cdot \hat{\mathbb{Q}}(\epsilon) - K \cdot \mathbb{Q}(\epsilon).$$

By Girsanov theorem, the process  $\hat{W}$  defined by  $\hat{W}_t := W_t - \sigma t$  is a Brownian motion under  $\hat{\mathbb{Q}}$

We then have:

$$\begin{aligned} \epsilon &= \left\{ \frac{W_{T_1}}{\sqrt{T_1}} < \frac{\ln\left(\frac{F(0; T_1, T_2)}{K}\right) - \frac{\sigma^2}{2} T_1}{\sigma \sqrt{T_1}} \right\} \\ &= \left\{ \frac{\hat{W}_{T_1}}{\sqrt{T_1}} < \frac{\ln\left(\frac{F(0; T_1, T_2)}{K}\right) + \frac{\sigma^2}{2} T_1}{\sigma \sqrt{T_1}} \right\} \end{aligned}$$

with  $\frac{W_{T_1}}{\sqrt{T_1}}$  and  $\frac{\hat{W}_{T_1}}{\sqrt{T_1}}$  following respectively Gaussian laws  $N(0, 1)$  under  $\mathbb{Q}$  and  $\hat{\mathbb{Q}}$ . The price of the option is rewritten, including the discount factor and the year fraction, by:

$$C_0 = \tau P(0, T_2) F(0; T_1, T_2) \cdot N(d_1) - \tau P(0, T_2) \cdot K \cdot N(d_2);$$

with:

$$\begin{aligned} d_1 &:= \frac{\ln\left(\frac{F(0; T_1, T_2)}{K}\right) + \frac{\sigma^2}{2} T_1}{\sigma \sqrt{T_1}}, \\ d_2 &:= \frac{\ln\left(\frac{F(0; T_1, T_2)}{K}\right) - \frac{\sigma^2}{2} T_1}{\sigma \sqrt{T_1}}, \end{aligned}$$

which concludes our proof for the price of a caplet option. The floorlet option

formula is obtained with the same logic.

## 1.2 Swaptions

The swaptions are the second category of basic options on interest rates. These options give the right to enter into a payer or receiver interest rates swap at a future date (the maturity date of the swaption). The length of this underlying swap is called the tenor of the swaption. Compared to a cap option, the swaption can not be decomposed on different “one flow” options. The decision of exercising the option is taken at the maturity date and then we get the underlying on portfolio. The option depends then on a multitude of forward rates.

First we expose the discounted payoff of a payer swaption:

$$ND(0, T_\alpha) \sum_{i=\alpha+1}^{\beta} (P(T_\alpha, T_i) \tau (F(T_\alpha; T_{i-1}, T_i) - K))^+$$

which can be rewritten in function of the forward swap rate by:

$$ND(0, T_\alpha) (S_{\alpha, \beta}(T_\alpha) - K)^+ \sum_{i=\alpha+1}^{\beta} \tau P(T_\alpha, T_i)$$

where:

$$S_{\alpha, \beta}(T_\alpha) \equiv \frac{P(T_0, T_\alpha) - P(T_0, T_\beta)}{\sum_{i=\alpha+1}^{\beta} \tau P(T_\alpha, T_i)}.$$

The market prices this option with a Black formula based on swap rate and not forward rates. The numeraire under which the forward swap rate  $S_{\alpha, \beta}$  is a martingale is  $\sum_{i=\alpha+1}^{\beta} \tau P(0, T_i)$ . This numeraire corresponds to a portfolio of zero coupon bonds maturing at the different payment dates of the underlying swap.

As proved by Brigo, the expectation of this discounted payoff is given by:

$$\begin{aligned}
PS_0 &= E^{\mathbb{P}} \left( ND(0, T_\alpha) (S_{\alpha, \beta}(T_\alpha) - K)^+ \sum_{i=\alpha+1}^{\beta} \tau P(T_\alpha, T_i) \right) \\
&= E^{\hat{\mathbb{P}}} \left( N \frac{B(0)}{B(T_\alpha)} (S_{\alpha, \beta}(T_\alpha) - K)^+ \sum_{i=\alpha+1}^{\beta} \tau P(T_\alpha, T_i) \right) \\
&= E^S \left( N \frac{\sum_{i=\alpha+1}^{\beta} \tau P(0, T_i)}{\sum_{i=\alpha+1}^{\beta} \tau P(T_\alpha, T_i)} (S_{\alpha, \beta}(T_\alpha) - K)^+ \sum_{i=\alpha+1}^{\beta} \tau P(T_\alpha, T_i) \right) \\
&= N \sum_{i=\alpha+1}^{\beta} \tau P(0, T_i) E^S((S_{\alpha, \beta}(T_\alpha) - K)^+).
\end{aligned}$$

The market formula for this payoff is given by:

$$PS_0 = N \sum_{i=\alpha+1}^{\beta} \tau P(0, T_i) (S_{\alpha, \beta}(T_\alpha) \cdot N(d_1) - K \cdot N(d_2))$$

with:

$$\begin{aligned}
d_1 &:= \frac{\ln\left(\frac{F(0; T_1, T_2)}{K}\right) + \frac{\sigma^2}{2} T_1}{\sigma_{\alpha, \beta} \sqrt{T_1}} \\
d_2 &:= \frac{\ln\left(\frac{F(0; T_1, T_2)}{K}\right) - \frac{\sigma^2}{2} T_1}{\sigma_{\alpha, \beta} \sqrt{T_1}}.
\end{aligned}$$

The proof of this formula is done exactly with the same logic as in the caplet formula.

## 2 Bermudan / American Swaptions

Bermuda options related products are very frequent. The market uses them to structure different debt products as they offer to the holder the possibility to unload his commitment.

A Bermuda swaption is similar to a (European) swaption. Unlike European swaptions, Bermuda swaptions have multiple execution dates and not only one. The American options can be exercised at any time between two dates. The American option is approximated by a Bermuda swaption with multiple and close execution dates. The multiple execution dates characteristic adds complexity in the pricing of the product. Indeed, the presence of multiple exercise dates brings an optimal control problem in the pricing process.

$$u_{t_0} = \sup_{\tau \in \mathfrak{S}} E^{\mathbb{Q}} \left[ \frac{h_{\tau}}{B_{\tau}} | \mathcal{F}_{t_0} \right] \Leftrightarrow \begin{cases} u_T = \frac{h_T}{B_T} \\ u_{t_i} = \max \left( h_{t_i}, E^{\mathbb{Q}} \left[ \frac{B_{t_i}}{B_{t_{i+1}}} u_{t_{i+1}} | \mathcal{F}_{t_i} \right] \right) \end{cases}$$

with

$u_t$ : the value of the option at time  $t$ ;

$B_t$ : the numeraire associated to the measure  $Q$ ;

$h_t$ : payoff from exercise at time  $t$ ;

$\tau$ : stopping time taking value in a finite set  $\mathfrak{S} = \{t_1, t_2, \dots, T\}$ .

The whole process of the pricing is described in Chapter 3. We present here the Libor Market Model, model which we use to price Bermuda options in this thesis.

The Libor Market Model is based on the modeling of the forward Libor rate and not the forward swap rate as in the case of European swaption pricing. Basing the pricing of a swaptions related product on a modeling built on forward rates forces us to link the swaptions volatilities (as the

swaption market is quoted in terms of swaptions volatilities) to forward rates volatilities (needed in the dynamics of forward rates). The link is done by the well known Rebonato-Hull-White formula given by:

$$\left(\vartheta_{T_\alpha, T_\beta}^{LMM}\right)^2 \simeq \sum_{i,j=\alpha+1}^{\beta} \frac{\bar{\omega}_i(0) \bar{\omega}_j(0) F_i(0) F_j(0) \rho_{i,j}}{T_\alpha S_{\alpha,\beta}(0)^2} \sum_{h=0}^{\alpha} \sigma_{i,h+1} \sigma_{j,h+1}$$

where:

$$\omega_i(t) = \frac{P(t, T)}{\sum_{j=\alpha+1}^{\beta} P(t, T_j)}$$

$$\bar{\omega}_h(t) = \omega_h(t) + \sum_{i=\alpha+1}^{\beta} F_i(t) \frac{\partial \omega_i(t)}{\partial F_h}$$

$$\frac{\partial \omega_i(t)}{\partial F_h} = \frac{\omega_i(t)}{1 + F_h(t)} \left[ \frac{\sum_{k=h}^{\beta} \prod_{j=\alpha+1}^k \frac{1}{1+F_j(t)}}{\sum_{k=\alpha+1}^{\beta} \prod_{j=\alpha+1}^k \frac{1}{1+F_j(t)}} - \mathbf{1}_{\{i \geq h\}} \right].$$

We can precise that:

$\rho_{i,j}$  is the correlation between the non predictable parts (brownian parts) of the forward rates  $i$  and  $j$ ;

$\vartheta_{T_\alpha, T_\beta}^{LMM}$  is the quoted volatility for the swaption maturing at  $T_\alpha$  and with tenor  $T_\beta - T_\alpha$ ;

$\sigma_{i,t}$  is the volatility of the forward rate  $i$  at the time step  $t$  (we make the assumption of a piecewise constant volatility structure).

This step allows to pass from a triangular swaption volatilities matrix (see Table 1) to a forward rates volatilities matrix (see Table 2):

Once the LMM volatility are calibrated to the swaption matrix through a classical cascade calibration, we can use the dynamic of the different underlying forward rates to simulate our payoff. The dynamics of the forward rates

Table I.1: swaption volatilities

maturity/ tenor	1 year	2 years	3 years	...	10 years
1 year	$\vartheta_{1,2}$	$\vartheta_{1,3}$	$\vartheta_{1,4}$	...	$\vartheta_{1,11}$
2 years	$\vartheta_{2,3}$	$\vartheta_{2,4}$	$\vartheta_{2,5}$	...	-
...	...	...	...	...	...
9 years	$\vartheta_{9,10}$	$\vartheta_{9,11}$	-	...	...
10 years	$\vartheta_{10,11}$	-	-	...	-

Table I.2: forward rates volatilities

rates/ $t \in$	$]0, T_0]$	$]T_0, T_1]$	$]T_1, T_2]$	...	$]T_8, T_9]$
$F_1(t)$	$\sigma_{1,1}$	dead	dead	...	dead
$F_2(t)$	$\sigma_{2,1}$	$\sigma_{2,2}$	dead	...	dead
...	...	...	...	...	...
$F_{10}(t)$	$\sigma_{10,1}$	$\sigma_{10,2}$	$\sigma_{10,3}$	...	$\sigma_{10,10}$

are given in LMM by:

$$dF_k(t) = \sigma_k(t) F_k(t) \sum_{j=\beta(t)}^k \frac{\tau_j \rho_{j,k} \sigma_j(t) F_j(t)}{1 + \tau_j F_j(t)} dt + \sigma_k(t) F_k(t) dZ_k(t)$$

and the numeraire associated to the related spot LIBOR measure is given by:

$$B_d(t) = \frac{P(t, T_{\beta(t)-1})}{\prod_{j=0}^{\beta(t)-1} P(T_{j-1}, T_j)}$$

with

$F_k(t) \equiv F(t, k-1, k)$ : the forward rate between  $k-1$  and  $k$  view at time  $t$ ;

$\sigma_k(t)$ : the volatility of  $F_k(t)$ ;

$\rho_{j,k}$ : the correlation between  $F_j$  and  $F_k$ ;

$\tau_j$ : year fraction between  $j-1$  and  $j$ ;

$Z_k$ : a standard Brownian motion;

$P(t, T)$ : bond price at time  $t$  for the maturity  $T$ ;

$\beta(t)$ : the index of the first forward rate that has not expired by  $t$ .

The spot LIBOR numeraire is the benchmark portfolio which represents the investment of our cash. We can recall that:

- In the Black model for cap/floor the investment of the cash is done on a zero coupon bond maturing at the maturity date of the cap/floor
- In the Black model for swaption the investment of the cash is done on a portfolio of zero coupon bonds corresponding to the different payment dates of the swaption

In the LMM, it is common to choose the spot LIBOR numeraire which correspond to a discretely rebalanced account where dates corresponds to our discrete tenor dates. The investment of the cash is not resumed by a investment at the valuation date. Indeed, at each tenor date, we have to replace our cash in a zero coupon bond fixed at the placement date and maturing at the next tenor date. We roll our cash investment on zero coupon bond at each tenor date. This numeraire is obviously not known at the valuation date as it depends on forward starting (fixing) zero coupon bonds. We are facing stochastic numeraire as in the case of the risk neutral numeraire but into the space spanned by our discrete tenors.

### 3 Structured Debt Instruments

As exposed on previous section, derivatives are market instruments and can be traded for trading or hedging purposes. In addition to these derivatives, we have debt instruments. These debt instruments contrary to the derivatives imply exchanges of nominal as it is the principal aim of these products:



lending money. Debt instruments can be in the form of notes (market known product with by example an ISIN number) or in the form of other financial instruments.

The debt obligations can contain an embedded derivative component. Indeed the interest payment attached to the obligation can be more complex than a regular fixed or floating payment. These more complex payments are related to derivative products: the embedded derivative. In the case of a bond with embedded derivative component, we price the two elements separately.

The models presented in the previous sections are also useful to price debt instruments when they are structured with embedded derivatives. We present first derivatives but we can highlight the importance of structured bonds. Indeed these instruments are the principal need of the banks as they have a constant need to finance their activities. The derivatives are used to hedge their position in bonds. It is a way to change the nature of the payments (fixed to float, float to fixed, structured to fixed,...) generated by our bond.

An additional purpose on the pricing of debt instruments are the credit/liquidity risk. As these instruments include a payment of the nominal at the maturity, the owner of the bond is exposed to a non-negligible default risk. We have to include this risk in the pricing of our bond. A natural way is to change the risk free rate by a risky rate taking account of the risk attached to the issuer of the bond.

The presence of default/liquidity risk only on the bond causes an asymmetry on the prices of asset and swap. More particularly, the execution of the option is also impacted by this asymmetry component. Indeed, we have a link between the execution of the swap's option and bond's option. Consider the example of a bank which has a cancelable bond (ie embedded swaption) and hedges this bond cash flow with a swap which can be canceled by the counterpart. It is well known that swaptions are implicit options contrary to caps/floors. The exercise of the caps and floors is automatic and does not

need a special act from the option owner. The execution of swaption is not automatic and the owner of the option must signal his intention to execute. Thus, the exercise of this implicit option can depend on the intention of the owner. If we return to our example, the bank does not have interest to keep an open position (a bond without its hedge instrument). In other words, if the owner of the swaption on the swap exercises the option, the bank would exercise the option on the bond even if the payoff of the bond's option is not positive (because of the asymmetry). Given this specificity, the open question is: how the bank must price this bond option? It can price as an isolated product and take account of only the payoff of the option on bond. An alternative choice is to price this option by taking account of the execution zone of the swap's option and not of the bond's option, as the execution of the swap's option implies the execution of the bond's option.

The second modeling adds asymmetry on the pricing function of the option, it changes the natural option's price function:  $P \cdot 1_P$  by  $P \cdot 1_{\bar{P}}$

where:

$P$ : the payoff of the bond's option;

$\bar{P}$ : the payoff of the swap's option.

The calculation of this type of price function is more complex than standard function. A Monte Carlo method or change of variable technics can solve the problem.

Pricing this funding instruments are more complex because in addition to the pricing of the embedded option, we have to manage and incorporate others properties as the credit risk and sub-optimal execution of the option. All these specificities make the pricing of these products on pricing systems more difficult than a more common Bermuda option.

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## Part II

# About long-term (cross-currency) Bermuda swaption pricing

### Abstract

This paper details first, from a computational point of view, the pricing process of a Bermuda swaption and in a second stage a cross-currency Bermuda. Our aim is to examine the lengthy process that provides us a Bermuda swaption price with special attention to the method of justifying the quality of our price. We only consider long-term derivatives that lead to issues related to missing data and require adjustment on the calibration. We also deal with the sensitivity of the cross-currency swaption's price to the choice of the modeling. The standard model for pricing these kinds of options is a 3 factors hybrid model (typically a 2 Libor Market Model + Black Scholes or 2 Hull-White + Black Scholes). However we study the impact on the price of each one of the stochastic factors. This study gives us an idea of the relation between the cross-currency product and the volatility of each one of the three markets involved (domestic, foreign and the FX)

# 1 Introduction

Bermuda (or American) derivatives are among the most common exotic derivatives. Even though they are common, their prices are not easily determined. We do not have closed formulas for this class of product and therefore we need to complete a lengthy pricing process. The difficulty of the Bermuda option pricing comes from the determination of the optimal control problem's solution, which is a crucial step for obtaining the price of this product. We choose to work in the Libor Market Model (LMM) framework to model the pricing problem. The Hull and White (HW) model would also be an appropriate model but the LMM is more suitable for its compatibility with the Black model which is the common market model for vanilla products. LMM is detailed by Brigo and Mercurio [4]. They also consider the case of a Bermuda option pricing. The recent book by Guyon and Labordère [5] devotes an entire chapter to Bermuda option pricing. They give a good summary of an approach to the problem and also consider the dual problem associated to the optimal control. The duality of the problem is indeed a main element of the pricing if we want to give importance to the justification of the price. The dual gap can also be considered as a metric for comparing different methods/models and benchmarking their uncertainty. Andersen and Broadie [1] propose a method to solve the dual problem and provide a detailed algorithm for practical implementation. More generally, Bermuda options pricing has been largely studied in the financial literature. For mathematical aspects, we can refer to Rogers [11] and Schweizer [12]. Recently Borovykha, Pascucci and Oosterlee [3] explore an extension for local levy process. For numerical aspects, we can refer to Belomestny [2], Henrard [6] [7] and Zhang, C.W. Oosterlee [13].

Grounding our current work in all of these references, we summarize the pricing process of a Bermuda swaption. These part gives the necessary notions for a good understanding of the next sections. We face a problem of

interpolation on the tenor dimension of the quoted swaption matrix due to the long-term properties of this product. This makes us revisit the Rectangular Cascade Calibration with the Endogenous Interpolation Algorithm (RCCEIA) of Brigo and Mercurio [4]. We consider different hypotheses that can be used to make an interpolation. After a back-testing phase, we keep the hypothesis that provides the best results. This is the first contribution of our paper. We then consider the pricing of a cross-currency Bermuda swaption. We choose a hybrid three model to be the benchmark model. Our aim is to understand the sensitivities of the product to the different volatilities involved in the pricing. Thus, we can understand the importance of each market in the value of the product.

Our aim is to test the accuracy of this model by considering each variable one by one. The results of our tests allow us to understand the relationship between this cross-currency derivative and each market. Finally, we note the crucial place of the product's margin in this sensitivity analysis.

Our work is organized as follows. We focus on the general framework of Bermuda swaption pricing in the Section 2. In Section 3, we deal with the problems that are specific to long-term property of the product. In particular, it is this long-term property that makes us revisit the aspect of calibration, taking account of the problem of missing data in swaption volatility for long tenors. Finally, Section 4 presents the product and tests the relevance of the three involved markets in the pricing of the cross-currency Bermuda swaption.

## 2 About Bermuda swaption pricing

### 2.1 The model

First of all, we need to choose a model in which we can perform the pricing. The two most popular models in fixed income derivatives pricing are the Hull-White model and the Libor Market Model. As stated in the introduction, we choose the Libor Market Model. We consider the basic Libor Market Model without including a stochastic model on the dynamic of the volatility process (see Mercurio-Morini[10] and Labordère [8] for details on this subject). Indeed, a deterministic volatility model is enough for Bermuda options. Considering a stochastic volatility model will complicate the pricing without capturing important market effect for this class of product. The LMM gives the following dynamics to the forward Libor rates in the spot Libor measure:

$$dF_k(t) = \sigma_k(t) F_k(t) \sum_{j=\beta(t)}^k \frac{\tau_j \rho_{j,k} \sigma_j(t) F_j(t)}{1 + \tau_j F_j(t)} dt + \sigma_k(t) F_k(t) dZ_k(t)$$

and the numeraire associated to this measure is given by

$$B_d(t) = \frac{P(t, T_{\beta(t)-1})}{\prod_{j=0}^{\beta(t)-1} P(T_{j-1}, T_j)}$$

with

$F_k(t) \equiv F(t, k-1, k)$ : the forward rate between  $k-1$  and  $k$  view at time  $t$ ;

$\sigma_k(t)$ : the volatility of  $F_k(t)$ ;

$\rho_{j,k}$ : the correlation between  $F_j$  and  $F_k$ ;

$\tau_j$ : year fraction between  $j-1$  and  $j$ ;

$Z_k$ : a standard Brownian motion;

$P(t, T)$ : bond price at time  $t$  for the maturity  $T$ ;

$\beta(t)$ : the index of the first forward rate that has not expired by  $t$ .

For more details on the model (notations, hypothesis, proofs,..), we refer to Brigo-Mercurio [4].

The Libor Market Model models all the forward rates one by one. For numerical concerns, we can reduce the number of factors. In what follows, we consider a one factor model, all the forward rates being driven by an unique Brownian motion. In the case of Bermuda option, previous numerical tests prove that adding factors does not affect the price of the option. This assumption reveals that our option's price is mainly impacted by level move of the yield curve.

## 2.2 The calibration

Once the model chosen, we have to calibrate it to the data available in the market. The market data is the swaption volatility matrix while the parameters of the Libor Market Model is forward rates instantaneous volatilities. We assume that each rate has a volatility depending on the time. We make the assumption for a piecewise-constant volatility because combining this assumption with the Rebonato-Hull-White formula allows us to calibrate the model analytically. We use a revisited version of the Rectangular Cascade Calibration with the Endogenous Interpolation Algorithm of Brigo-Mercurio [4]. We return to this calibration algorithm in the next section of our paper.

## 2.3 The Monte Carlo simulation

Now, we have to simulate the different forward rates involved in the payoff function of our Bermuda swaption. The equations of our forward rates are given by our model and the parameter's values by the calibration. We choose the Euler schema for the discretization of stochastic processes.



The Libor Market Model is based on discrete tenors structure. Therefore to price a product that does not have the same tenors structure than the quoted products at the valuation day, we must apply a interpolation method. We choose to apply the drift interpolation method described in Brigo-Mercurio [4]. This method makes the interpolation in the Monte Carlo simulation. Once the interpolated forward rates obtained, we can construct the numeraire associated with the portfolio given by the rebalancement in the new tenor structure.

## 2.4 The optimal control problem

### 2.4.1 The primal problem

The crucial step for providing our Bermuda swaption price is the resolution of the optimal control problem. As mentioned in Anderson-Broadie [1], we are facing the following optimization problem:

$$u_{t_0} = \sup_{\tau \in \mathfrak{S}} E^{\mathbb{Q}} \left[ \frac{h_{\tau}}{B_{\tau}} | \mathcal{F}_{t_0} \right] \Leftrightarrow \begin{cases} u_T = \frac{h_T}{B_T} \\ u_{t_i} = \max \left( h_{t_i}, E^{\mathbb{Q}} \left[ \frac{B_{t_i}}{B_{t_{i+1}}} u_{t_{i+1}} | \mathcal{F}_{t_i} \right] \right) \end{cases}$$

with

$u_t$ : the value of the option at time  $t$ ;

$B_t$ : the numeraire associated to the measure  $Q$ ;

$h_t$ : payoff from exercise at time  $t$ ;

$\tau$ : stopping time taking value in a finite set  $\mathfrak{S} = \{t_1, t_2, \dots, T\}$ .

This primal problem can be solved by a Least-Squares Monte Carlo introduced by Longstaff-Schwartz [9].

Once the different paths of the payoff are simulated by our previous step, we can use the dynamic programming formulation (on the right side of the

previous implication) to solve the problem. We “just” have to define an exercising strategy. As suggested by Longstaff and Schwartz [9], we use a regression on basis functions for the estimation. We first estimate the exercising strategy (*ie* the coefficients of our regression) on learning paths and use independent paths for the pricing.

The primal solution is low biased due to the sub-optimality of our exercising strategy. We solve the problem numerically and obtain a maximum depending on the optimality of our exercising strategy, even if we converge to this theoretical maximum when we increase the number of simulations and basis functions. The solution is low biased. We need to solve the dual problem to obtain an interval.

#### 2.4.2 The dual problem

The dual problem is a minimization problem and gives a high biased estimation of the price. We use the algorithm suggested by Anderson-Broadie [1] and explained also by Guyon-Labordère [4] for solving this problem. The logic behind this method is to use the super-martingale property of the Bermuda swaption price process. As demonstrated in Anderson-Broadie [1], the dual formulation of the problem is:

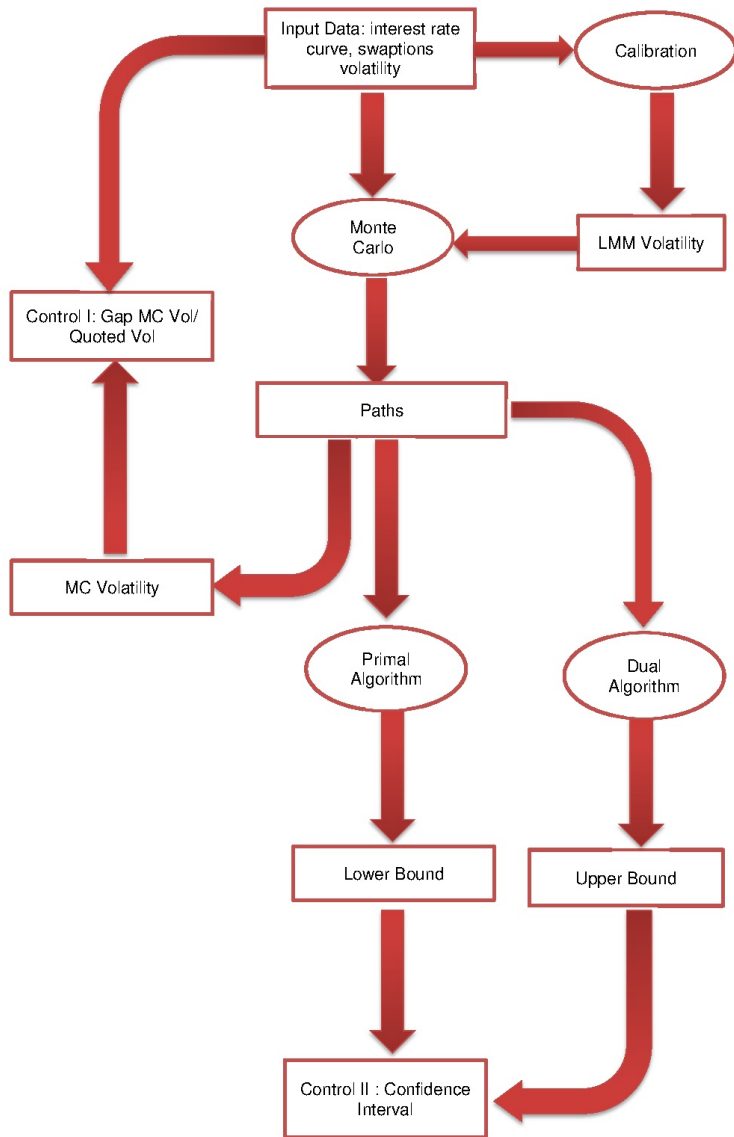
$$u_{t_0} = \inf_{\pi \in H} \left( \pi_{t_0} + E^{\mathbb{Q}} \left[ \max_{t_i \in \mathfrak{S}} \left( \frac{h_{t_i}}{B_{t_i}} - \pi_{t_i} \right) \middle| \mathcal{F}_{t_0} \right] \right).$$

We pass from a maximization problem in the space of stopping times to a minimization problem in the space of martingales. By using the Doob-Meyer decomposition, equalization  $\pi_t$  to the martingale part of our super-martingale (*theoretical* price process) gives us a null dual gap. The computation of this solution is “Monte Carlo in the Monte Carlo”. This solution requests a long computational time. Anderson and Broadie [1] show also the proportional link between the upper bound and lower bound. Our incertitude on the pri-

mal problem is compensated by the incertitude on the dual problem. Finally, we have non biased boundary for the expected solution.

## 2.5 The whole process

The following graphic resumes the pricing process for a Bermuda swaption. It exposes the different data, methods, results and the coherence control. We have two tests that help us to justify our price and alert us in case of error. The first one is just after our Monte Carlo simulation. We calculate the Monte Carlo implied volatility and compare it to the quoted implied volatility. This control tests the coherence of our calibration and simulation. If we do not have an abnormal spread between the MC and quoted volatility (the spread is due to the Monte Carlo error and is converging to zero with the number of paths), we can pass to the second test. This second test concerns the the relevance of the optimal control solution. Usually the primal solution is computed and used as the Bermuda swaption price but we have no indication on the veracity of our estimation. The second test consists in computing the upper bound and therefore the dual gap. Even if we use the lower bound (because of operational time constraint), the computation of the upper bound can be perform in a second stage or less frequently to confirm the level of incertitude of our price.



## 3 About long-term property

### 3.1 Calibration Algorithm

Brigo and Mercurio [4] suggest an analytical calibration with an endogenous interpolation (called RCCAEI in the book). Making an endogenous interpolation is an important step. Swaption matrix has missing data due to non-quoted products for certain maturities and tenors. We can do the interpolation (linear, functional,...) and calibrate our model or we can do the interpolation within the calibration algorithm. Brigo and Mercurio [4] opted for the second case due to the emergence of non-coherent (negative or complex) volatility in the case of linear interpolation. The RCCAEI algorithm makes endogenous interpolation for missing data only for the maturities. This algorithm assumes that our swaption matrix has no missing columns. This is indeed the case when we consider Bermuda swaption whose underlying swaption has a final maturity date at most in 10 years. In our case, we are pricing long-term products whose underlying may mature in 30 years. When we take a look at the swaption matrix taking into consideration the long tenors, we observe the multitude of missing volatilities for specific tenors. We need to re-adapt the RCCAEI algorithm for interpolating endogenously, even if it concerns a class of tenors.

The idea behind RCCAEI is to use the Rebonato-Hull-White formula:

$$v_{\alpha,\beta}^2 \simeq \frac{1}{T_\alpha} \sum_{i,j=\alpha+1}^{\beta} \frac{\bar{w}_i(0) \bar{w}_j(0) F_i(0) F_j(0) \rho_{i,j}}{S_{\alpha,\beta}(0)^2} \int_0^{T_\alpha} \sigma_i(t) \sigma_j(t) dt,$$

$$\bar{w}_i(t) = w_i(t) + \sum_{h=\alpha+1}^{\beta} F_h(t) \frac{\partial w_h}{\partial F_i},$$

$$\frac{\partial w_h}{\partial F_i} = \frac{w_h(t) \tau_i}{1 + \tau_i F_i(t)} \left[ \frac{\sum_{k=i}^{\beta} \tau_k \prod_{j=\alpha+1}^k \frac{1}{1 + \tau_j F_j(t)}}{\sum_{k=\alpha+1}^{\beta} \tau_k \prod_{j=\alpha+1}^k \frac{1}{1 + \tau_j F_j(t)}} - \mathbf{1}_{\{h \geq i\}} \right],$$

$$w_i(t) = \frac{\tau_i P(t, T_i)}{\sum_{j=\alpha+1}^{\beta} \tau_j P(t, T_j)}.$$

Combining this approximation with a piecewise constant volatility approximation (for the integral part of the first equation), we can get an analytical formula if and only if the swaption matrix has no missing data. When the matrix has missing rows (the maturity dimension of the matrix), RCCAEI solves the equation only for available data and makes a constant volatility hypothesis. In this way, when data is not available, we extend the “piece” of our piecewise constant volatility to keep an analytical calibration.

The solution of the problem is less obvious when we have missing columns. We can follow the same idea as RCCAEI. When we have missing swaption volatility, we solve the equation for the next swaption volatility available. “Next” means the closest tenor and the closest maturity for which swaption volatility is available. As we skip missing swaptions volatilities, we have missing forward rates volatility for this missing maturities and tenors. We keep the constant volatility hypothesis of RCCAEI to solve the problem for missing maturities. We have now missing data depending on the tenors between the last volatility for which we apply the algorithm and the current swaption volatility.

We have to set an hypothesis for missing columns as the RCCAEI do for missing rows. First, we can consider a linear interpolation for missing columns. This has the same disadvantage as interpolation row outside of the calibration algorithm. It gives us non coherent volatility. Another solution is to consider a homogeneity assumption (used also by Anderson-Broadie [1] for the volatilities which depends on non-quoted products) (i.e.  $\sigma_{i,k} = \sigma_{i-1,k-1}$ ). But this assumption is not enough dynamic. We have a lot of missing columns and this assumption makes our forward rates volatility constant on too much entries because of its recursive property. We can keep the same “frozen” value in too much loops. We need a dynamic interpolation for the columns as for the rows. The intuitive solution is to equalize all the missing forward rates

volatilities. This has the advantage to be dynamic but we lose the coherence of the interpolation. It is not truly a natural assumption unlike RCCAEI. More explicitly, RCCAEI makes an assumption of constancy of the volatility of the forward rates, but it concerns interpolation for the volatility of the same forward rate. It just extends the interval where the volatility is constant. When we make the assumption that all the missing forward volatilities are constant, we equalize volatilities of different rates on different time horizons. We make a dynamic calibration and this has the advantage to limit the emergence of non-coherent volatility. But we not give an economically interpretable hypothesis.

An important point is the emergence of non-coherent instantaneous forward rates volatility. We have the same problem of complex/negative volatility regardless the choice of the interpolation technique. We solve this problem through a change of value of the quoted swaption volatility. If we are in case of non-coherent volatility (i.e. the C quantity in the algorithm is positive), we change the value of the quoted swaption by calculating the nearest value to the quoted value that makes the volatility coherent (ie the C quantity negative). This makes a change in the quoted swaption volatility. We can consider this method as a smoothing method according the interpolation hypothesis we adopt. We can choose these criteria for analyzing the quality of our interpolation compared to the observation. Additionally, we minimize the spread between the original quoted swaption volatility and the adjusted swaption volatility. Our interpolation method is consistent with the observations. We define the distance between the two matrices by the  $L_1$  distance for the quoted and calibrated volatility (we calibrate only the upper triangular part of the matrix).

**Data:**  $F$  forward rates,  $P$  discount factor,  $v$  swaption volatility,  $\rho$  correlation

**Result:**  $\sigma$  forward rates volatility

$K \leftarrow$  missing rows of  $v$ ;

$H \leftarrow$  missing columns of  $v$ ;

$s \leftarrow$  last column/row of  $v$  included in the calibration;

$a \leftarrow 1$ ;

```

while ( $a \leq s$ ) do
  if  $a \in K$  then
     $\gamma \leftarrow a$ ;
     $a \leftarrow \min \{i \in \mathbb{Z} : a < i \leq s, i \notin K\}$ 
  else
     $\gamma \leftarrow a$ 
  end
   $b \leftarrow a + 1$ ;
  while ( $b \leq s + 1$ ) do
    if  $b - a \in H$  then
       $\delta \leftarrow b$ ;
       $b \leftarrow \min \{i \in \mathbb{Z} : b - a < i \leq s, i \notin H\} + a$ 
    else
       $\delta \leftarrow b$ 
    end
     $\bar{w}[a + 1 : b] \leftarrow \text{CalculateW}(F, P, a, b)$ 
    if  $\gamma \notin K$  then
      if  $\delta - a \notin H$  then
         $\sigma[b - 1 : a] \leftarrow \text{solveA}(\bar{w}, F, \rho, \sigma, a, b, \gamma, \delta)$ 
      else
        if  $b > s + 1$  then
          for  $i = \delta : s + 1$  do
            for  $j = \gamma : a$  do
               $\sigma[i : j] \leftarrow \sigma[\max(1, i - 2), \max(1, j - 1)]$ 
            end
          end
        else
           $\sigma[\delta : b - 1, a] \leftarrow \text{solveB}(\bar{w}, F, \rho, \sigma, a, b, \gamma, \delta)$ 
        end
      end
    else
      if  $\delta - a \notin H$  then
         $\sigma[b - 1, \gamma : a] \leftarrow \text{solveC}(\bar{w}, F, \rho, \sigma, a, b, \gamma, \delta)$ 
      else
         $\sigma[\delta : b - 1, \gamma : a] \leftarrow \text{solveD}(\bar{w}, F, \rho, \sigma, a, b, \gamma, \delta)$ 
      end
    end
     $b \leftarrow b + 1$ 
  end
   $a \leftarrow a + 1$ 
end
for  $i = 1 : \text{size}(K)$  do
   $\sigma[K[i], K[i]] = \sigma[K[i] - 1, K[i] - 1]$ ;
   $p \leftarrow 1$ ;
  while  $K[i] + p \in K$  do
     $\sigma[K[i] + p, K[i]] = \sigma[K[i] + p - 1, K[i] - 1]$ ;
     $p \leftarrow p + 1$ 
  end
end

```

**Algorithm 1:** Calibration



**Data:**  $F, P, a, b$   
**Result:**  $\bar{w}$   
 $w[a+1:b] \leftarrow \frac{P[a+1:b]}{\sum_{j=a+1}^b P[j]}$ ;  
**for**  $i = a+1 : b$  **do**  
    **for**  $h = a+1 : b$  **do**  
         $dw[i, h] \leftarrow \frac{w[i]}{1+F[h]} \left[ \frac{\sum_{k=h}^b \prod_{j=a+1}^k \frac{1}{1+F[j]}}{\sum_{k=a+1}^b \prod_{j=a+1}^k \frac{1}{1+F[j]}} - 1_{\{i \geq h\}} \right]$   
    **end**  
**end**  
**for**  $h = a+1 : b$  **do**  
     $\bar{w}[h] \leftarrow w[h] + \sum_{i=a+1}^b F[i] dw[i, h]$   
**end**

**Algorithm 2:** CalculateW

**Data:**  $\bar{w}, F, \rho, \sigma, a, b, \gamma, \delta$   
**Result:**  $\sigma$   
 $A \leftarrow \bar{w}_b^2 F_b^2$ ;  
 $B \leftarrow 2 \sum_{j=a+1}^{b-1} \bar{w}_b \bar{w}_j F_b F_j \sigma_{j-1, a} \rho_{b-1, j-1}$ ;  
 $C \leftarrow \bar{w}_b^2 F_b^2 \sum_{h=1}^{a-1} \sigma_{b-1, h}^2 + 2 \sum_{j=a+1}^{b-1} \bar{w}_b \bar{w}_j F_b F_j \rho_{b-1, j-1} \sum_{h=1}^{a-1} \sigma_{b-1, h} \sigma_{j-1, h} +$   
 $\sum_{i, j=a+1}^{b-1} w_i w_j F_i F_j \rho_{i-1, j-1} \sum_{h=1}^a \sigma_{i-1, h} \sigma_{j-1, h} - a S^2 v_{a, b-a}^2$ ;  
**if**  $C > 0$  **then**  
     $v_{a, b-a} \leftarrow \sqrt{\frac{C + a S^2 v_{a, b-a}^2}{a S^2}} + \epsilon$ ;  
    Re-Calculate C  
**end**  
 $\sigma[b-1, a] \leftarrow \frac{-B + \sqrt{B^2 - 4AC}}{2A}$

**Algorithm 3:** SolveA

**Data:**  $\bar{w}, F, \rho, \sigma, a, b, \gamma, \delta$

**Result:**  $\sigma$

$$A \leftarrow \bar{w}_b^2 F_b^2 + 2 \sum_{j=\delta}^{b-1} \bar{w}_b \bar{w}_j F_b F_j + \sum_{i=\delta}^{b-1} \sum_{j=\delta}^{b-1} \bar{w}_i \bar{w}_j F_i F_j \rho_{i-1, j-1};$$

$$B \leftarrow 2 \sum_{j=a+1}^{\delta-1} \bar{w}_b \bar{w}_j F_b F_j \sigma_{j-1, a} \rho_{b-1, j-1} + \sum_{i=\delta}^{b-1} \sum_{j=a+1}^{\delta-1} \bar{w}_i \bar{w}_j F_i F_j \rho_{i-1, j-1} \sigma_{j-1, a} + \sum_{i=a+1}^{\delta-1} \sum_{j=\delta}^{b-1} \bar{w}_i \bar{w}_j F_i F_j \rho_{i-1, j-1} \sigma_{i-1, a};$$

$$C \leftarrow \bar{w}_b^2 F_b^2 \sum_{h=1}^{a-1} \sigma_{b-1, h}^2 + 2 \sum_{j=a+1}^{b-1} \bar{w}_b \bar{w}_j F_b F_j \rho_{b-1, j-1} \sum_{h=1}^{a-1} \sigma_{b-1, h} \sigma_{j-1, h} + \sum_{i, j=a+1}^{b-1} w_i w_j F_i F_j \rho_{i-1, j-1} \sum_{h=1}^{a-1} \sigma_{i-1, h} \sigma_{j-1, h} + \sum_{i, j=a+1}^{\delta-1} \bar{w}_i \bar{w}_j F_i F_j \rho_{i-1, j-1} \sigma_{i-1, a} \sigma_{j-1, a} - a S^2 v_{a, b-a}^2;$$

**if**  $C > 0$  **then**

$$\left| \begin{array}{l} v_{a, b-a} \leftarrow \sqrt{\frac{C + a S^2 v_{a, b-a}^2}{a S^2}} + \epsilon; \\ \text{Re-Calculate } C \end{array} \right.$$

**end**

$$\sigma[\delta : b-1, a] \leftarrow \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

**Algorithm 4:** SolveB

**Data:**  $\bar{w}, F, \rho, \sigma, a, b, \gamma, \delta$

**Result:**  $\sigma$

$$A \leftarrow (a - \gamma + 1) \bar{w}_b^2 F_b^2;$$

$B \leftarrow$

$$2 \sum_{j=a+1}^{b-1} \bar{w}_b \bar{w}_j F_b F_j \sigma_{j-1, a} \rho_{b-1, j-1} + 2 \sum_{j=a+1}^{b-1} \bar{w}_b \bar{w}_j F_b F_j \rho_{b-1, j-1} \sum_{h=\gamma}^{a-1} \sigma_{j-1, h};$$

$$C \leftarrow \bar{w}_b^2 F_b^2 \sum_{h=1}^{\gamma-1} \sigma_{b-1, h}^2 + 2 \sum_{j=a+1}^{b-1} \bar{w}_b \bar{w}_j F_b F_j \rho_{b-1, j-1} \sum_{h=1}^{\gamma-1} \sigma_{b-1, h} \sigma_{j-1, h} + \sum_{i, j=a+1}^{b-1} w_i w_j F_i F_j \rho_{i-1, j-1} \sum_{h=1}^a \sigma_{i-1, h} \sigma_{j-1, h} - a S^2 v_{a, b-a}^2;$$

**if**  $C > 0$  **then**

$$\left| \begin{array}{l} v_{a, b-a} \leftarrow \sqrt{\frac{C + a S^2 v_{a, b-a}^2}{a S^2}} + \epsilon; \\ \text{Re-Calculate } C \end{array} \right.$$

**end**

$$\sigma[b-1, \gamma : a] \leftarrow \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

**Algorithm 5:** SolveC

**Data:**  $\bar{w}, F, \rho, \sigma, a, b, \gamma, \delta$

**Result:**  $\sigma$

$$\begin{aligned}
A &\leftarrow (a - \gamma + 1)\bar{w}_b^2 F_b^2 + 2 \sum_{j=\delta}^{b-1} \bar{w}_b \bar{w}_j F_b F_j + \sum_{i=\delta}^{b-1} \sum_{j=\delta}^{b-1} \bar{w}_i \bar{w}_j F_i F_j \rho_{i-1, j-1}; \\
B &\leftarrow \\
&2 \sum_{j=a+1}^{b-1} \bar{w}_b \bar{w}_j F_b F_j \sigma_{j-1, a} \rho_{b-1, j-1} + \sum_{i=\delta}^{b-1} \sum_{j=a+1}^{\delta-1} \bar{w}_i \bar{w}_j F_i F_j \rho_{i-1, j-1} \sigma_{j-1, a} + \\
&\sum_{i=a+1}^{\delta-1} \sum_{j=\delta}^{b-1} \bar{w}_i \bar{w}_j F_i F_j \rho_{i-1, j-1} \sigma_{i-1, a} + \\
&2 \sum_{j=a+1}^{b-1} \bar{w}_b \bar{w}_j F_b F_j \rho_{b-1, j-1} \sum_{h=\gamma}^{a-1} \sigma_{j-1, h}; \\
C &\leftarrow \bar{w}_b^2 F_b^2 \sum_{h=1}^{\gamma-1} \sigma_{b-1, h}^2 + 2 \sum_{j=a+1}^{b-1} \bar{w}_b \bar{w}_j F_b F_j \rho_{b-1, j-1} \sum_{h=1}^{\gamma-1} \sigma_{b-1, h} \sigma_{j-1, h} + \\
&\sum_{i, j=a+1}^{b-1} w_i w_j F_i F_j \rho_{i-1, j-1} \sum_{h=1}^{a-1} \sigma_{i-1, h} \sigma_{j-1, h} + \\
&\sum_{i, j=a+1}^{\delta-1} \bar{w}_i \bar{w}_j F_i F_j \rho_{i-1, j-1} \sigma_{i-1, a} \sigma_{j-1, a} - a S^2 v_{a, b-a}^2; \\
\text{if } C > 0 \text{ then} & \\
\quad \left| \begin{array}{l} v_{a, b-a} \leftarrow \sqrt{\frac{C + a S^2 v_{a, b-a}^2}{a S^2}} + \epsilon; \\ \text{Re-Calculate } C \end{array} \right. & \\
\text{end} & \\
\sigma[\delta : b-1, a] &\leftarrow \frac{-B + \sqrt{B^2 - 4AC}}{2A}
\end{aligned}$$

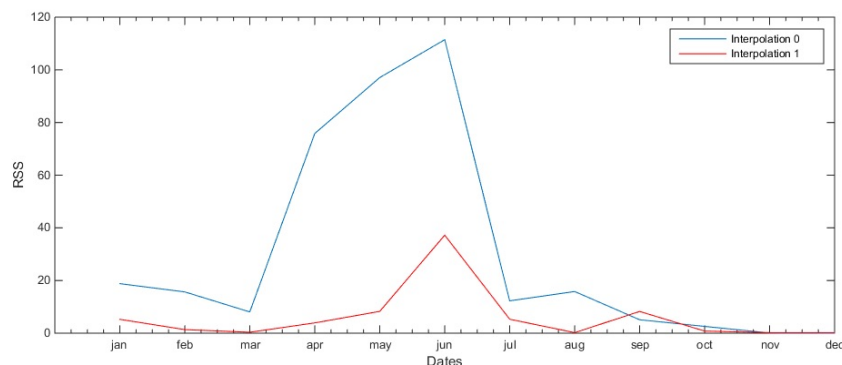
**Algorithm 6:** SolveD

## 3.2 Numerical Results

We test our interpolation method in the calibration algorithm (Interpolation 1) by benchmarking it with the basic linear interpolation for missing columns (Interpolation 0). We do our back-test from January 2014 to December 2014 with monthly intervals for LIBOR derivatives. We define our mispricing by the RSS (residual sum of squares) between the original swaption volatility matrix quoted in the market and the swaption volatility matrix obtained by our calibration method with the adjustment to keep coherent forward rates volatilities. Figure II.1 and Figure II.2 resume these results.

We see clearly the improvement made by Interpolation 1. We have a better fit to the observed prices. However, we can note that our method does not give us a perfect fit to the input data. This can be justified by two remarks. First, our endogenous interpolation is not necessarily an optimal interpolation method even if it combines an easy and analytical implementation with “not too bad”

Figure II.1: RSS between the theoretical and market volatility for LIBOR swaption market



results. The second point is that the mispricing may come from liquidity in the swaption matrix, specially for high tenors and maturities. Indeed, we can notice that non coherent volatilities and related adjustment come specially with the less liquid swaptions.

We can ask our-self about the impact of these differences in term of prices. We consider a 30Ync2Y+1Y, a 30 years final maturity Bermuda swaption callable every year after 2 years traded the 01/31/2014. The fixed rate is equal to 4% and the nominal is accreting with this same rate. The initial nominal is 40MM\$ and the final nominal is 129 735 900.40\$. We price this product with the volatility issued by Interpolation 0 and Interpolation 1. We keep the same path for the innovation part to isolate the impact of the change in volatility without the effect of the Monte Carlo error. We can remark that we choose a fixed rate OTM and this makes the optimal stop arrives more frequently in the first years. If we notice that the impacted swaption volatilities have long maturities, we can partially explain the relatively low impact on our pricing. It can differ if we consider an ITM fixed rate. Additionally we executed in early periods and less we are correlated to long maturity volatility (most affected volatilities in our calibration). To visualize the empirical distribution of the optimal stopping time, we can take a look to the Figure II.3 which

Figure II.2: Price impact for interpolation methods

	Price Interpolation 0	Price Interpolation 1	Differences in bps
01/31/2014	22.19%	22.15%	4
02/28/2014	22.74%	22.75%	-1
03/31/2014	23.69%	23.68%	1
04/30/2014	24.83%	24.86%	-3
05/30/2014	28.24%	28.28%	-4
06/30/2014	27.51%	27.55%	-4
07/31/2014	27.07%	27.11%	-4
08/29/2014	33.80%	33.86%	-6
09/30/2014	30.27%	30.28%	-1
10/31/2014	34.04%	34.00%	4
11/28/2014	38.61%	38.56%	5
12/31/2014	46.51%	46.47%	4

represents the empirical distribution for the 12/31/2014. We omit the final date which corresponds to the default stopping time if there is no execution before this last date.

We test the 05/30/2014 (date for which we have the biggest spread between Interpolation 0 et Interpolation 1 price). For more clear results, we price a non accreting Bermuda swaption which permits us to compare to have comparable price as the nominal final nominal is the same in all cases. We calculate this spread for different spread. Figure II.4 exposes the different prices and the relative spreads.

We can conclude that the spread depends heavily on the rate which affects the optimal stopping time empirical distribution. Nevertheless, the spread between these two methods is quite small in term of option prices.

Figure II.3: Empirical distribution of the optimal stopping time for the 4% fixed rate swaption

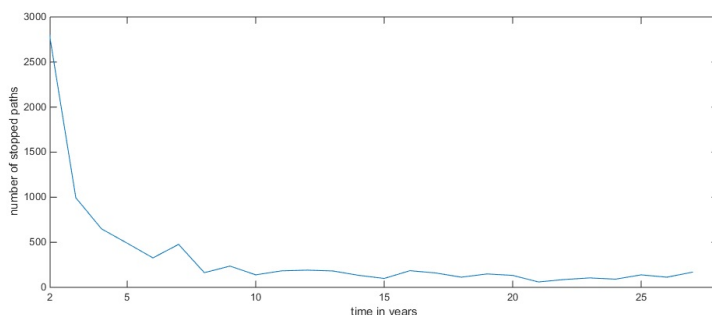


Figure II.4: Percentage variation between Interpolation 0 and Interpolation 1

fixed rate	percentage of variation
1.8%	-1.15%
2.8% ( $\approx$ ATM)	-0.17%
6%	0.00%

## 4 About cross-currency Bermuda swaption pricing

### 4.1 The product

The concerned product is a Bermuda callable swap. This type of swap is generally used for hedging a Bermuda callable bond. If the bond is issued in a foreign currency, it can be hedged by a cross-currency Bermuda callable swap to convert the risk to the domestic currency. Pricing a callable swap can be resumed to the pricing of related swaption (with the relation callable swap = swap +/- swaption, +/- depending on who gets the possibility of canceling the swap).

Let us detail first the callable swap. The swap has an accreting nominal. The swap begins with an exchange of initial domestic nominal versus initial foreign

nominal with the initial FX. It ends with the exchange of accreted domestic nominal versus accreted foreign nominal. The accretion is defined initially by a fixed rate. At the intermediate interest payment date, we have the exchange of accretion for the domestic and foreign nominal at a fixed rate. We also have an exchange of fixed foreign interest (the interest paid in the hedged bond) paid annually versus a floating domestic interest paid semi-annually. We can note that the fixed foreign interest payment is the opposite of the accretion of foreign nominal. These two series of cash-flow cancel each other. This swap is cancelable at any fixed interest payment date (so here annually). As mentioned above, this callable swap can be represented by a swap without cancellation option and a swaption whose underlying swap's cash-flows are the opposite of the initial swap. Thus activating the swaption allows to enter in a swap that cancels the existing swap in portfolio. The initial swap is cancelable at different dates, therefore the swaption is Bermudan. We have a series of underlying swaps whose length is depending on the date of call.

Now we can examine the cash-flows of the underlying swaps. When we decide to exercise the swaption, we enter in a swap that totally cancels the cash-flows in the initial swap. When the initial swap is canceled, we have an early redemption for the domestic and foreign nominal. Therefore the underlying swap has a exchange of current nominal at the call date. For all following payment dates, the underlying swap has domestic fixed and floating rate payment (in opposite direction of the initial swap's cash-flows). At the maturity of the contract the underlying swap has a exchange of final domestic and foreign nominal.

Now, for more precise analysis (for the direction of the cash-flows), we consider an issuer of the bond who takes engagement in a swap for hedging this bond. We have the option of early canceling on the bond and therefore we sell the early canceling option on the swap to our counterpart. The counterpart have a cancelable swap that pays the floating domestic rate and receives the fixed domestic rate at interest payment dates. At the final maturity date

(if no early cancellation), it pays domestic final nominal and receives the foreign final nominal. More precisely, he has a swaption with underlying swap pays fixed domestic rate and receives the floating domestic rate at interest payment dates. At the call date, the underlying swap pays the domestic current nominal and receives the foreign current nominal (early redemption cash-flows). Finally, at the maturity date, it pays the foreign final nominal and receives the domestic final nominal (cancellation of the final redemption of the initial swap).

As noticed by Piterbarg and Andersen, we simplify all the cash-flows in domestic currency. Indeed, if we consider only these domestic cash-flows, we have (1) an exchange of interest payments (payments of fixed rate and receptions of floating rate) summed up by an accreting swap receiving floating rate; (2) an exchange of nominal (payment of call date nominal and reception of final date nominal) resumed, according to the formula of Piterbarg and Andersen, by an accreting swap receiving fixed rate. The sum of these two swaps is equal to zero. In the hypothesis of absence of spread on the domestic floating rate leg, the option is not a cross-currency option. It can be resumed by an accreting swaption in foreign currency.

Nevertheless, usually the product has spread on these legs. The presence of spread gives to the option the cross-currency property. After defining the model used for the pricing in the next subsection, we numerically analyze how this spread affects the sensitivities of the product to the domestic option and FX option market.

## 4.2 The model

The product that we investigate depends on three variables: the domestic interest rate curve, the foreign interest rate curve and the FX rate between these two currencies. The standard model for pricing this kind of product is an hybrid model combining three distinct models. Generally, a combination



of 2 Libor Market Model and 1 Black-Scholes is proposed. We use the Libor Market Model described in the previous section to model the domestic and foreign forward rates. We use the foreign currency as the pivot currency. We convert all the cash-flows in the foreign currency. We use the foreign spot LIBOR measure as numeraire, implying the appearance of a quanto term in the dynamic of domestic forward rates. We use a Black-Scholes model for the evolution of the FX rate. We have to take care of the drift term of the FX rate dynamic which depends on the simulations of the domestic and foreign forward rates. The formula in the Monte-Carlo is given by (we separate the case  $t = 0$  and  $t > 0$ ):

$$\begin{aligned} \ln F_\chi(1) &= \ln F_\chi(0) + \ln \left( \frac{1 + F^f(1) \Delta s}{1 + F^d(1) \Delta s} \right) \Delta s - \frac{\sigma^{F_\chi}(s)^2}{2} \Delta s \\ &\quad + \sigma^{F_\chi}(s) (W_k(t + \Delta s) - W_k(t)), \end{aligned}$$

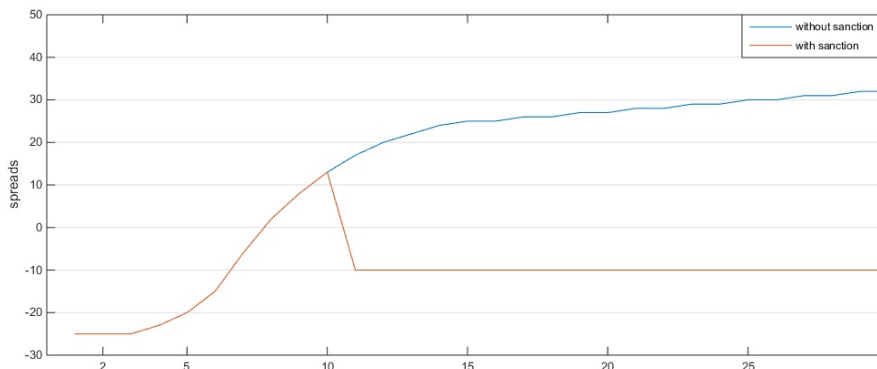
$$\begin{aligned} \ln F_\chi(t + \Delta t) &= \ln F_\chi(t) + \ln \left( \frac{1 + F^f(t, t, t + 1)}{1 + F^d(t, t, t + 1)} \right) \Delta t - \frac{\sigma^{F_\chi}(t)^2}{2} \Delta t \\ &\quad + \sigma^{F_\chi}(t) (W_k(t + \Delta t) - W_k(t)). \end{aligned}$$

For the domestic forward rates, the quanto adjustment changes the classic LMM formula by:

$$dF_k^d(t) = \sigma_k(t) F_k^d(t) \left( \sum_{j=\beta(t)}^k \frac{\tau_j \rho_{j,k} \sigma_j(t) F_j^d(t)}{1 + \tau_j F_j^d(t)} - \sigma^{F_\chi}(t) \right) dt + \sigma_k(t) F_k^d(t) dZ_k(t).$$

The remaining term (Libor Market Model dynamic for the foreign forward rates and numeraire) is unchanged.

Figure II.5: Spread Curves



### 4.3 Numerical Results

We consider a 30Ync2Y+1Y, a 30 years final maturity Bermuda swaption callable every year after 2 years traded the 01/31/2014. As in the first part, we choose a fixed rate (which is also the accretion rate of nominal) of 4%. We fix the dollar denominated nominal to 40MM\$. The initial fixing of the FX rate is 1.3488 (fixing of the trade date). The corresponding initial euro denominated nominal is 29 655 990.51€. These two nominal are accreting and reach respectively 129 735 900.40\$ and 96 186 165.78€ at the final maturity date. We consider two cases. In the first case, we assume a standard spread curve (as for a sovereign bond). In the second case, we consider that the issuer fixes a penalty spread in the the hedge swap after a certain year (it can be considered as a sanction price). It can happen that the issuer is not interested by a long maturity issuance. Typically, we will consider a negative spread in the twenty last years, which means that our issuer was not interested to issue for a bond longer than 10 years. We treat these two cases because of the importance of the spread curve in the emergence of the cross currency property.

We price our two swaptions monthly for the whole 2014 years with the benchmark two Libor Market Model + Black-Scholes hybrid model. Our preceding

Table II.1: Pricing of the standard spread swaption

	3 factors model	determinist €	determinist FX	LMM USD
01/31/2014	18.10%	19.37%	18.81%	19.70%
02/28/2014	18.39%	19.64%	19.00%	19.90%
03/31/2014	18.81%	20.01%	19.12%	20.55%
04/30/2014	20.05%	21.34%	20.79%	21.72%
05/30/2014	22.93%	24.11%	23.65%	24.46%
06/30/2014	22.11%	23.36%	22.70%	23.58%
07/31/2014	21.27%	22.41%	21.87%	22.76%
08/29/2014	27.48%	28.77%	28.12%	29.06%
09/30/2014	24.14%	25.20%	24.62%	25.41%
10/31/2014	28.32%	29.49%	28.90%	29.79%
11/28/2014	32.62%	33.61%	32.94%	33.74%
12/31/2014	40.60%	41.67%	41.01%	41.49%

sub-section tells us the importance of the USD volatility market. For measuring the impact of the other markets (the EUR volatility market and the foreign exchange volatility market), we choose to price with two factors model excluding first the stochastic modeling of the EUR forward rates and after the FX rate. We also price with a unique LMM for the USD forward rates. All these tests inform us about the relative importance of the EUR market and FX market coming from the spread. We use 5000 paths for the learning sample and 30 000 paths for the pricing sample. To eliminate the Monte Carlo error incertitude, we use the same paths in the four models.

We can observe that the EUR volatility market has a bigger impact than FX volatility impact on the price of the swaption in the two cases. In the first case (with no penalty), we can compare the magnitude of the impact level as shown in the Figure II.8 representing contribution. This Figure divides the impact of the passage of a three factor hybrid model to a Libor Market Model, the blue (respectively the green) level represents the percentage accorded to EUR volatility in the normal case (respectively in the penalty case) and the remaining is due to the FX level. In the case of penalty spread, the impact of

Table II.2: Pricing of the penalty spread swaption

	3 factors model	determinist €	determinist FX	LMM USD
01/31/2014	24.22%	23.73%	24.22%	23.75%
02/28/2014	24.58%	23.80%	24.56%	23.81%
03/31/2014	25.16%	24.73%	25.08%	24.69%
04/30/2014	27.09%	26.49%	27.07%	26.58%
05/30/2014	30.26%	29.43%	30.22%	29.46%
06/30/2014	29.08%	28.62%	29.05%	28.63%
07/31/2014	28.89%	28.45%	28.85%	28.53%
08/29/2014	36.24%	35.81%	36.21%	35.71%
09/30/2014	31.87%	31.33%	31.82%	31.82%
10/31/2014	35.39%	35.89%	36.33%	35.95%
11/28/2014	41.54%	40.89%	41.46%	40.77%
12/31/2014	49.88%	49.22%	49.83%	49.28%

Figure II.6: Prices in the standard case

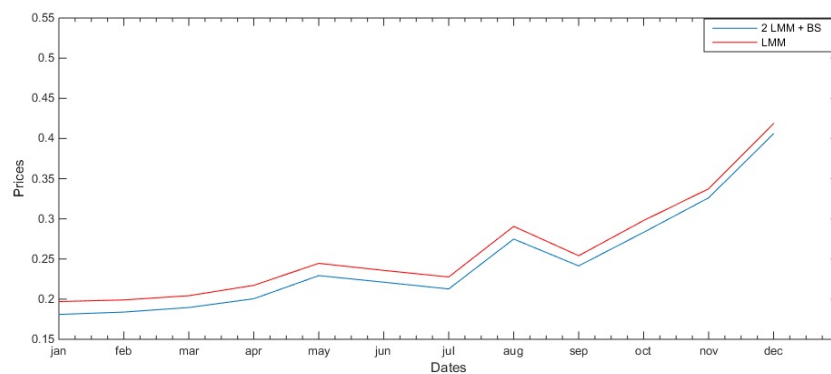


Figure II.7: Prices in the penalty case

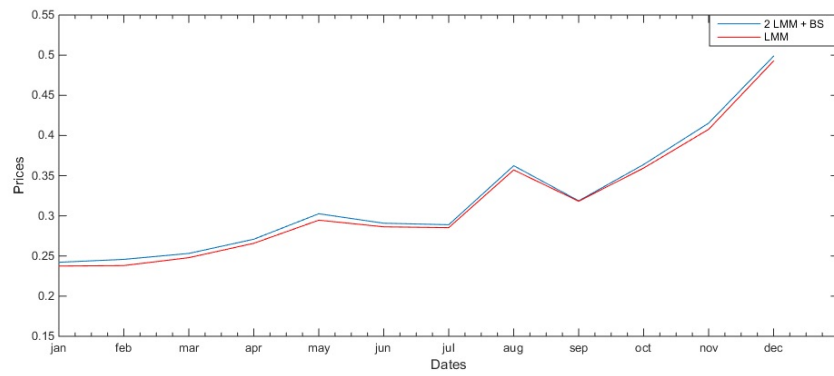
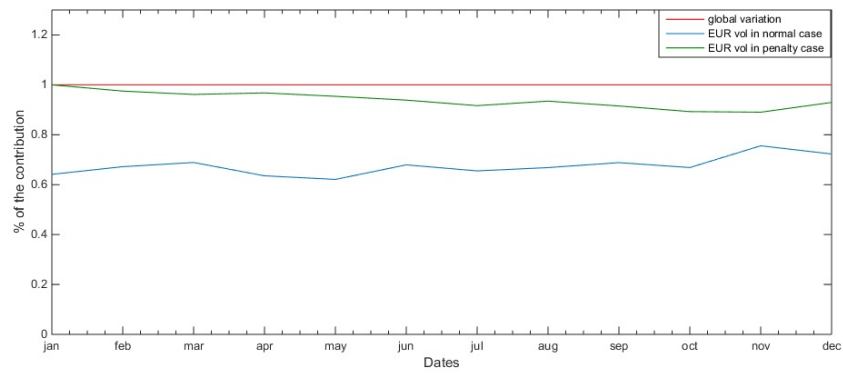


Figure II.8: Percentage of the contribution coming from EUR volatility market on the global impact



the FX volatility is almost null. On the other side, the impact of volatility of the EUR forward rates keep the same direction but with a smaller magnitude compared to the first case.

As expected in the case of a penalty spread curve, pricing with a hybrid model and pricing with an USD LMM is quite similar (cf Figure II.6). With a normal spread curve, the LMM model is overestimating the price of our Bermuda swaption as shown in the Figure II.6.

## 5 Conclusion

In this paper, after resuming the pricing process of Bermuda swaption, we focus on the two important properties of a long term cross-currency product. We propose an endogenous interpolation in the direct calibration algorithm based on Hull White Rebonato formula as done in Brigo-Mercurio [4] but we also include the interpolation for the missing tenors. This method is compared to the basic exogenous linear interpolation. Even if we lose the economic intuitive property in the interpolation hypothesis, we observe that our method gives better results when we examine the difference between theoretical price and market price. We also keep the advantage to have an analytical algorithm.

In the second part, we note that our product is cross-currency only because we have a spread on the floating domestic leg. A way of analyzing the relative importance of the dependence on the two supplementary volatility markets induced by this spread curve is to examine the pricing of our product in different models. The numerical results highlight the contribution of EUR volatility market induced by the apparition of this spread curve. The contribution of the FX volatility market is smaller than the EUR volatility market. This contribution analysis is different from a standard sensitivity analysis. With a sell side vision, we may be more interested by the sensitivity be-

cause of the presence of a dynamic hedge portfolio adjusted frequently. In a buy side vision, we may be more interested by how the determination of the spread curve during the issuance of an asset swap will impact the dependance of our product in the different volatility markets.

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## Part III

# Macro-Stressing the Pricing of Bermuda Swaptions

### Abstract

The aim of the paper is to analyze the possible evolutions of derivative products' prices based on predictions of the different risk factors. The projections of the risk factors are based on different macroeconomic scenarios. Our work is split into three parts. First, we give the different macroeconomic scenarios and the logic behind the construction of these previsions. In the second part, we project the implied volatility in the different macroeconomic environments. As we have a whole matrix of volatility to project with several entries, we compare the Factor Model approach and Principal Component Analysis to capture the information with a reduce number of variables. We use Kalman filter method as a de-noising method and backtest our previsions. Last part gives the results of the macro-stressing on a Bermuda swaption. In this part, we also present a Uncertain Parameters Model which takes account of the different macroeconomic scenarios and their probabilities.

## Introduction

The price of a derivative product is calculated using basically the no arbitrage assumption and the market quotation at the pricing date. For sell side, the most accrual way to measure the risk exposure on this kind of product is to calculate the Greeks. As the position on derivatives is hedged by a replication portfolio the actual risk exposure is coming from the daily (or intra-day if the hedge portfolio is rebalanced in intra-day) variations of the different risk factors. If we consider a buy side institution, we do not have this replication portfolio. The managers tack positions on the market because of the need of their activities, for example investment or funding requirements. The activity of the institution generates structural risk.

We can choose different modelings to analyze the risk exposure for a non hedged position. If we take a look at how insurance companies model their balance sheet risks for Solvency II, we find a “Monte Carlo on Monte Carlo simulation”. The first Monte Carlo is performed in the real world when the second is performed in the risk neutral world given the simulated real world factors (rates, implied volatility cube). As a “Monte Carlo on a Monte Carlo” is a time consuming method, least squared based method allows insurance companies to reduce the computing time (cf Devineau and Loisel [6]).

We can split the modeling into two parts: one to measure the risk exposure on market movement and another to obtain a price for an optional product. The second model is a strong tool to obtain a price of an hedgeable option given the hypothesis of the absence of arbitrage opportunity. As highlighted by El Karoui, Loisel, Prigent and Vedani [7] using risk neutral modeling needs strong hypothesis especially it needs the effective hedging by the holder which is true only locally in time and space (i.e. each parameter must be calibrated frequently and for a specific type of product). We discuss this point further in the next chapter. The modeling of risk exposure is more subject to discussion. As we have remarked, the insurance company bases its model on a real world

Monte Carlo, which corresponds to the calibration of a process to historical data. Another possible way to measure the risk exposure is to make future estimations of possible evolutions with few scenarios. A standard method is to use macroeconomic scenarios based on our economic visions. The readers can find references that resume the approach for scenario planning in the context of stress testing. We can mention BIS 2015 [2] as one of these papers. Once we have selected the possible different macroeconomic environments, we have to project the different variables according to these anticipations. This model is more based on a forward looking vision as it is not repeating the past (historical simulation) or repeating the parameter of the past (real world Monte Carlo). In what follows, we choose this scenario based modeling to project risk factors.

We expose the different macroeconomic scenarios and give the intuition on the projection of the different macroeconomic and financial (included interest rates) variables. We detail the projection of implied volatility. We suppose that all the needed information on the considered macroeconomic scenario is resumed by the projected interest rates. We have to project a high dimensional matrix with only few points of the interest curve. We use Factor Model and Principal Component Analysis (PCA) to reduce the number of variables. These methods are based on hidden variables. As the estimations of hidden variables by the models are noisy, we used Kalman filter to clean our observations. We backtest the previsions of our models and compare the outcomes of models with and without the de-noising technique. The pricing of our derivative product is performed in these different macroeconomic scenarios. We consider these valuations as future possible evolutions of the product based on our macroeconomic anticipations for the future. This different scenario-based prices give us the risk exposition on our portfolio. We also analyze the sensitivity of our risk exposition to the number of chosen factors in the projection of the implied volatility matrix.

We organize our work as follows. Section 1 is devoted to the different macroe-

conomic scenarios. We explain which scenarios we choose and why. This section also gives the general view on the projection method of the macroeconomic and financial variables. In Section 2, we focus on the implied volatility projection and the factor model. Finally, Section 3 gives the results of our macro-projected pricing. We use these projections to obtain the risk exposure of our product. We present the results resulting from the Uncertain Parameters Model.

# 1 Macroeconomic-Based Projections

We do the projections at the end of 2014 for the end of 2015. We consider three scenarios in a “what if” scenarios optic. The choice of the number of scenarios that we study is a central hypothesis. Three scenarios appear to be a quite optimal number of scenarios. It is indeed usual to consider three alternatives to make decision in finance. The number of scenarios in a stress-testing context ranges between 2 and 4. To give detailed examples, the EBA stress-test includes 2 scenarios (1 baseline, 1 adverse) and the recommendations for internal stress-testing are to take 4 scenarios (1 baseline, 2 adverses, 1 reverse). Considering more scenarios can make difficult the exploitation of the results. Nevertheless, in a more specific field, banks can choose to study more scenarios as it can be the case in asset and liability management where the only variable of interest can be the net interest income.

As we have mentioned, we consider three scenarios and in these three scenarios, we have baseline and adverse scenarios and all the scenarios have a non negligible probability to appear. They are “best guess” estimations. We have one central scenario which reflects the natural anticipations given today’s (end of 2014) macroeconomic context. This scenario is the most likely case for next year. The two alternative scenarios are deteriorated scenarios. We consider the two most realistic scenarios with opposite effects on the interest rates. These two scenarios are built on past historical scenarios. We detail the three scenarios in what follows.

## 1.1 Descriptions of the scenarios

### 1.1.1 Central Scenario

This scenario is based on three structural hypothesis. First, we suppose a fiscal consolidation in France, which affects the French growth but inverses

the debt/GPD dynamic. The second hypothesis concerns the low level of interest rates due to the economic policy. The last hypothesis is a decrease in the savings rate of French households supporting consumption.

This scenario is characterized by a slow recovery: the production is affected in level and growth rate. For the financial context, the low level of inflation and the expansionist monetary policy conducted by the ECB will keep the long rates low in France.

### **1.1.2 Deflation in the Eurozone Scenario**

This scenario is based on the Japanese Deflation. This scenario highlights the uncertainty of the recovery in GDP because of a negative confidence represented by a rise in the household savings to rate and a shock of competitiveness with a negative impact on the exports.

### **1.1.3 Bond Crash Scenario**

Based on the bond crash of 1994, this scenario's central hypothesis is a tightening of US monetary policy faster than expected. This policy implies a rise of US long interest rates and a contagion on the Eurozone. This scenario can be resumed by three consequences:

- Slowdown;
- Rise of the long term interest rates;
- Drop of all classes of asset prices.

## **1.2 Projected Interest Rates Curve**

Stress-testing interest rate risk supposes to project interest rate curve. Financial literatures present many ways to project interest rate curve in the

context of stress-testing. Abdymomunov and Gerlach [1] present different methods and compare the impacts of the choice of the method in stress-testing the economic value of equity of banks. Their paper highlights the advantage of the Nelson Siegel model generator in a multi-scenarios framework. Indeed, the proposed method calibrates a parametric curve at each historical observation and combines the different parameters (which stand for short term rate, slope and curvature) to generate hypothetical scenarios. They generate a large set of scenarios and obtain impact for each one. As mentioned earlier, our approach is based on pure historical scenario. So we need to generate interest rates curve for those already observed macroeconomic shocks in a deterministic approach. The naive methods can consist to apply historical change (in absolute or in relative) observed during the considered period of crisis to current interest rates. As highlighted by Abdymomunov and Gerlach [1], these naive methods have many drawbacks as negative forward rates, non smooth yields curve or non economic meaning yield curve given the current level of the yield curve. To obtain a more robust and economic meaning projection of interest rate curve, we use a projection coming from a Nelson Siegel Svensson Filipovic model. The shocks on the parameters of the parametric functional are applied to the parameters calibrated for the current interest rate curve. These applied shocks are observed during the considered period of historical crisis. These projected interest rate curve is not estimated by the author and is taken as an exogenous input in the following volatility projection model presented in this study. In the following table, we resume the different approach for generating stress-testing.



	Few scenarios	Many scenarios
Historical	<p><b>Deterministic:</b> after selecting of few crisis periods, we have to apply the historical shocks to the current level of interest rates and credit spreads.</p> <p><i>Method used in this chapter.</i></p>	<p><b>Deterministic:</b> we choose a large period of time and apply all the observed shocks of this large period to the current level of interest rates and spreads.</p> <p><b>Stochastic:</b> we choose the main variables of our modelling (directly interest rates or parameters of a Nelson-Siegel parametrization) and calibrate them to historical data. We can simulate many scenarios with the calibrated process.</p>
Hypothetical	<p><b>Deterministic:</b> we select risk factors given the current economic situation and deduce the impact of this hypothetic combination of risk factors on the level of interest rates and credit spreads (an intermediate step can be the impacts of risk factors on macroeconomics variables which affects financial variables).</p> <p><i>Method used in the EBA stress-test.</i></p>	<p><b>Deterministic:</b> we calibrate at each date of a large period the parameters of a Nelson-Siegel (ie the short term rate, the slope and the curvature). We extract the historical shocks on these parameters which give us a range of values for these three latent variables. With a 3 dimensional grid including different values of the parameters in the defined range, we can combine different values of factors. It permits to generate interest rates curve with multiple shocks on short term rate, slope and curvature which were not observed in the past.</p>

Figure III.1: Interest Rates Projections

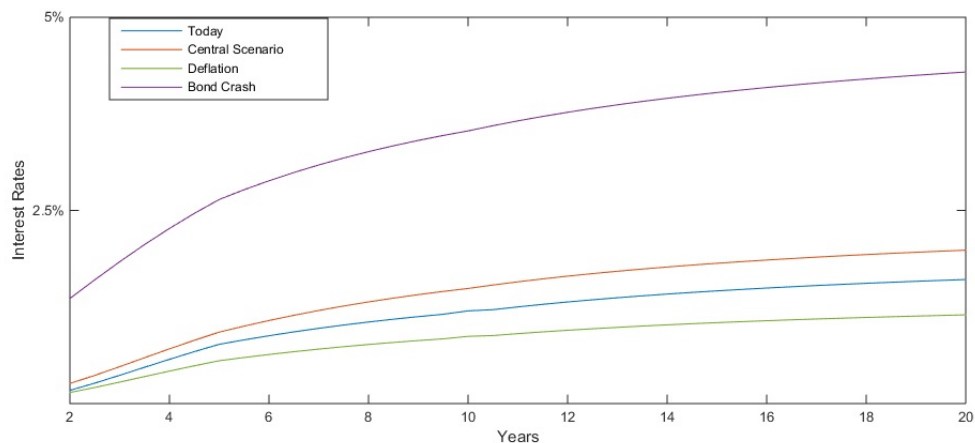


Figure III.1 illustrates the different interest rates curve compared to the curve at the current date. As we can remark on Figure III.1, the obtained curve scenarios are only parallel shift scenarios. The Central Scenario and Bond Crash Scenario are parallel shocks up (with a really more important amplitude for the Bond Crash Scenario) whereas the Deflation Scenario is a parallel shock down. We can notice that we are not considering cap and floor for the absolute value of the shocks. If we consider the level of the cap and the floor of the Document of Basel Committee on Interest Rate Risk on Banking Book [2], our scenarios are not affected by the cap of 500 basis points but two (Central and Deflation) of our three scenarios are lower than the floor of 100 basis points. This can express the fact that our two stressed scenarios are not enough stressed for a aim of calculating risk exposure and hedging it. Our objective is little different. We are not trying to calculate extreme risk exposure. We just have anticipations on the future with a significant probability of appear and we want to anticipate the value of our derivative products based on our intuition for the future macroeconomic context.

## 2 Projecting Implied Volatility with Dimension Reduction

A key risk factor to price a derivative is the implied volatility surface. As our Bermuda swaption is based on the market of (European) swaptions, the term “implied volatility” stands for swaptions implied volatility cube. The implied volatility is nothing else than the quoted price of the swaptions in term of volatility instead of price metric. In other words, it is the implied volatility that we have to put in the Black formula to obtain the price of the product. Market is quoted by implied volatility because it is the more intuitive scale compared to price. In the risk neutral world, implied volatility is the risk associated to the underlying as it represents its variability around its estimated value. This implied volatility can be define as a measure for analyzing market sentiment as highlighted by Kiesel and Rahe [10].

Our aim is to project the implied volatility, which is a key parameter of the risk neutral modeling, using projections coming from real world (or physical world). Christoffersen, Jacobs, Chang [4] and Kiesel and Rahe [10] analyze the causality in the other direction. They use information from risk neutral modeling (i.e. the implied volatility) to forecast the real world metrics as asset returns.

As we qualify the implied volatility as the sentiment of the market participants, we can use this sentiment to add information to the historical information to improve the predictions quality or we can use our anticipation on the real world variables (typically macroeconomic variables) to estimate future sentiment of market participant. In our study, we are clearly in this second logic as the final objective is to obtain stressed derivatives prices, i.e. what would be the sentiment of the derivative market participants in a stressed macroeconomic environment.

First of all, we suppose that all the informations about the scenario is resumed

by the interest rates. We use only the interest rate curve as predictor for estimating the implied volatility surface. We use some representative points of the interest rate curve: 2Y, 5Y, 10Y and 20Y points. Our swaption's implied volatility surface has 70 entries (10\*10 minus 3\*10 non quoted points) and we have only 4 predictor variables. We can naively project all the entries of our surface one by one. However we can image an important correlation between all this entry of our volatility surface. To take account of this strong link between the different volatility, we can use a Factor Model or a Principal Component Analysis. More details on these two methods can be found in [9] chapter 7. The aim of these models is to reduce the number of factors of our swaption matrix. With the N factors model, we extract the N more significant orthogonal variables/components for the whole matrix of data.

## 2.1 Factor Model

This model tries to explain  $p$  different variables in the dataset by  $m$  variables called factors, with of course  $m < p$ . The observed variables are linear functions of the common factors plus an error term.

We have:

$$\forall i \in \{1, \dots, p\}, x_i = \lambda_{i,1}f_1 + \dots + \lambda_{i,m}f_m + e_i;$$

where

$(x_i)_{i \in \{1, \dots, p\}}$  are the observed variables;

$(f_j)_{j \in \{1, \dots, m\}}$  the common factors;

$(\lambda_{i,j})_{i \in \{1, \dots, p\} \times j \in \{1, \dots, m\}}$  the constant factor loadings;

$(e_i)_{i \in \{1, \dots, p\}}$  the error terms.

We can rewrite this relation in matrix form:

$$x = \Lambda f + e.$$

We impose that all factors and errors  $(f_j)_{j \in \{1, \dots, m\}}$ ,  $(e_i)_{i \in \{1, \dots, p\}}$  are independent, normally distributed variables with zero means and unit variances. Then we have a multivariate normal distribution for the observed variables  $x$  (we can subtract the empirical means to obtain zero mean variables). The hypothesis of normality permits us to use the maximum likelihood estimation. The form of the model can remember us a simple linear regression. But when we look closer, we can observe that both  $\Lambda$  and  $f$  are unknown. We can use the maximum likelihood estimation to determine  $\Lambda$ ,  $E(ee') \equiv \Psi$  and in a second step we can estimate  $f$  with a regression.

We maximize the likelihood function of the sample by finding the optimal  $\Lambda$  and  $\Psi$  satisfying the following relation expressed in term of the covariance of our variables  $x$ :

$$\Sigma = \Lambda\Lambda' + \Psi.$$

We can point that this model fits the initial data by according a relative importance on the off-diagonal elements of  $\Sigma$ . It can be viewed as a correlation-focused approach. Its principal aim is to reproduce the inter-correlations among variables.

## 2.2 Principal Component Analysis

This method is similar to factor model in the sense that it tries to resume the information contained in a set of correlated variables with a set of uncorrelated variables. The new set of variables is a linear transformation of the initial set of variables.

If we denote by  $x$  the initial set of the correlated variables, by  $t$  the new set of uncorrelated variables and by  $w$  the loadings, we have the following relation:

$$t = xw.$$

The new set of variables called the principal component is classed in decreasing order of variance. An intuitive way for dimension reduction is to use the truncated transformation issued by the use of the first  $L$  loadings.

$$t_L = xw_L.$$

Contrary to the factor model, the PCA is variance-based method. It focuses the fitting on the diagonal term of  $\Sigma$ .

### 2.3 Denoising the Hidden Variables by Kalman Filter

The Kalman Filter is a strong tool to predict, filtrate and smooth the state (hidden) variables of a state-space representation. First of all, let us define this representation:

for a given  $(y_t)$  process which takes values in  $\mathbb{R}^m$  and a related process  $(\alpha_t)$  taking values in  $\mathbb{R}^n$ , we have:

$$\begin{cases} y_t &= M_t \alpha_t + d_t + Q_t u_t \\ \alpha_t &= T_t \alpha_{t-1} + c_t + R_t v_t \end{cases}$$

where we have the following deterministic parameters (which can be time dependent or not):

$$M_t \in \mathbb{R}^{m \times n};$$

$$d_t \in \mathbb{R}^m;$$

$$Q_t \in \mathbb{R}^{m \times m};$$

$$T_t \in \mathbb{R}^{n \times n};$$

$$c_t \in \mathbb{R}^n;$$

$$R_t \in \mathbb{R}^{n \times n};$$

and the two white noises:

$$u_t \in \mathbb{R}^m;$$

$v_t \in \mathbb{R}^n$ .

The PCA or FM models give an estimation of the observations of hidden variable ( $\alpha_t$ ). We can deduce from the observations of ( $y_t$ ) and estimation of ( $\alpha_t$ ) the different parameters of the state-space representation. As we have mentioned previously, Kalman filter allows us to obtain:

- A filtrated value:  $\alpha_{t|t} \equiv E(\alpha_t | y_1, \dots, y_t)$  which corresponds to our PCA or FM estimations that permit us to calibrate the parameters of the state-space representation;
- A predicted value:  $\alpha_{t|t-1} \equiv E(\alpha_t | y_1, \dots, y_{t-1})$ ;
- A smoothed value:  $\alpha_{t|T} \equiv E(\alpha_t | y_1, \dots, y_T)$ .

Our de-noising method consists in taking the predicted value instead of the filtrated value to estimate the factors. This method is based on the intuition that the theoretical model prevision provides a better estimate compared to the filtrated value.

## 2.4 The Projection Model

We assume that the swaption implied volatility is a function of the interest rates. This hypothesis seems to be coherent, in other words it is equivalent to assess that the uncertainty around the level depends on the level. Thus we have:

$$\sigma_{i,t} = f(r_{j=\{2,5,10,20\},t}).$$

We choose the 2Y, 5Y, 10Y and 20Y points of the interest rate curve to resume the information on the yield curve moves.  $r$  stands for the interest rates point,  $\sigma_{i,t}$  is the implied volatility  $i$  (corresponding to a pair of maturity and tenor) at time  $t$ .

As there are a lot of pairs  $i$  of maturity and tenor and only 5 points of interest rate curve for a point in time, we use dimension reduction techniques.

In the PCA or Factor Model, we can write the volatility matrix as a linear combination of factors and scores in matrix notation :

$$\sigma_{[1:I,1:T]} = a_{[1:I,1:I]} score_{[1:I,1:T]}.$$

As the factors are ordered by decreasing order of variance, we can set to 0 the factors after a rank  $n$  to eliminate the noise as the small variance scores are considered as noise.

$$\sigma_{[1:I,1:T]} = a_{[1:I,1:n]} score_{[1:n,1:T]}.$$

As the  $I$  implied volatilities are resumed by  $n$  scores, we search to project the  $n$  scores to obtain the projections of the  $I$  implied volatilities.

For each  $h \in [1, n]$ , we use linear regression to project the score. The PCA and FM give us the history of scores and therefore a linear regression is possible:

$$score_h = \beta_h \cdot r.$$

We can thus project implied volatility by first projecting the scores using linear regression parameters and our projected interest rates  $r$  for the projected date  $T_{proj}$ .

$$score_{h,T_{proj}} = \beta_h \cdot r_{T_{proj}}.$$

The Kalman Filter denoising can permit us to estimate a predicted score instead of a filtrated score.



## 2.5 Back-testing Methodology

To test the quality of our prediction, we back-test our projection model with historical data. We choose a number of observations which constitutes our learning sample, in which we perform PCA/FM and calibrate the linear regression parameters linking the scores and the interest rates (using only information available in the learning sample). Once the model calibrated, we inject the realization of the interest rates for a certain horizon (1 year) on the model to obtain an estimation of the implied volatility for this horizon. We can compare this projection with the realized implied volatility. We perform this procedure by shifting the time window of our learning sample.

## 2.6 Numerical Results

### 2.6.1 Resuming the data by reduced number of factors/components

In our paper, the initial set of correlated variables is the implied volatility matrix entries, as we said the 70 “well” quoted implied volatilities. Figure III.2 represents the correlation matrix of this 70 volatilities. The calculation of the correlation is based on daily data from 01/01/2010 to 12/31/2014. We choose a 5 years history because we have to get a wide spectrum of realizations in our learning data encompassing the projected values of the interest rates.

We can observe the high level of correlation between the different volatilities. The minimum correlation level is 68.10% and the average correlation for the off-diagonal elements is 93.11%. This high level of correlation confirms our initial intuition to use a dimension reduction technique. We plot the percentage variability explained by the different principal components in the Figure III.3 after performing a PCA. It is obvious that the number of significant principal components is quite small compared to the number of variables in the initial set (*ie* 70).

Figure III.2: Correlation of the quoted volatility

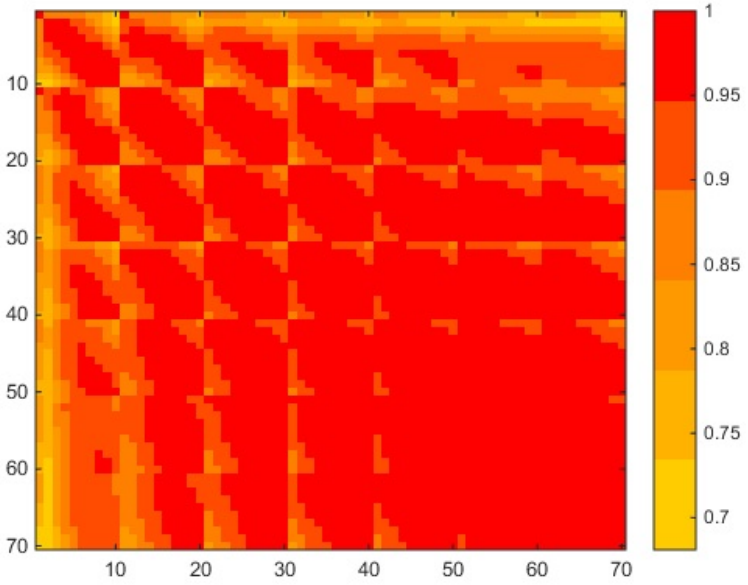


Figure III.3: Variability Explanation

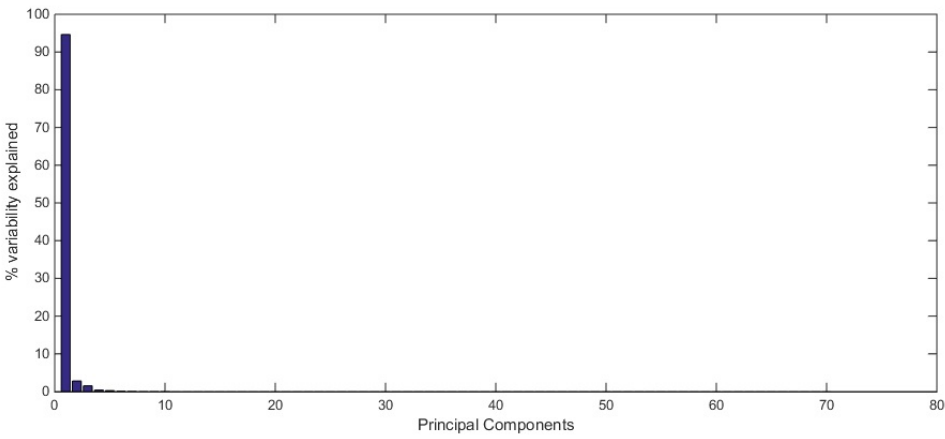
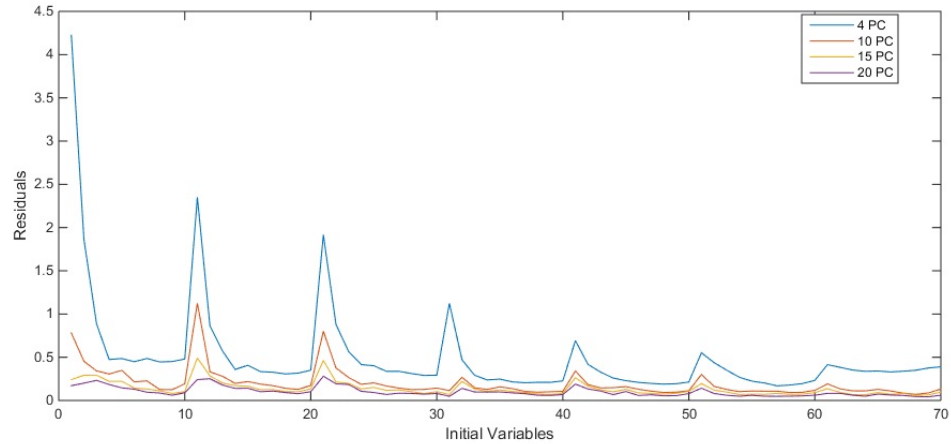


Figure III.4: Residuals of the PCA



In the following tests, we arbitrary choose four benchmark dimensions: 4, 10, 15 and 20.

In Figure III.4, we represent the historical average of the residuals for the 70 variables of the initial set. The residuals represent the absolute difference between the observations of variables and the reconstructed values of variables with a PCA of  $L$  factors.

We can also perform the same tests with the residuals when we use a Factor Model.

We can also compare the two methods for the same level of dimension reduction. Figure III.6 shows the differences for a reduction to 4 dimensions.

### 2.6.2 Projecting the data in the different scenarios

Our preceding subsection permits us to obtain the different values of the hidden factors and components. We apply a classical regression analysis on our data. We regress the values of the selected number of factors/components on the four points of our interest rate curve.

Figure III.5: Residuals of the FM

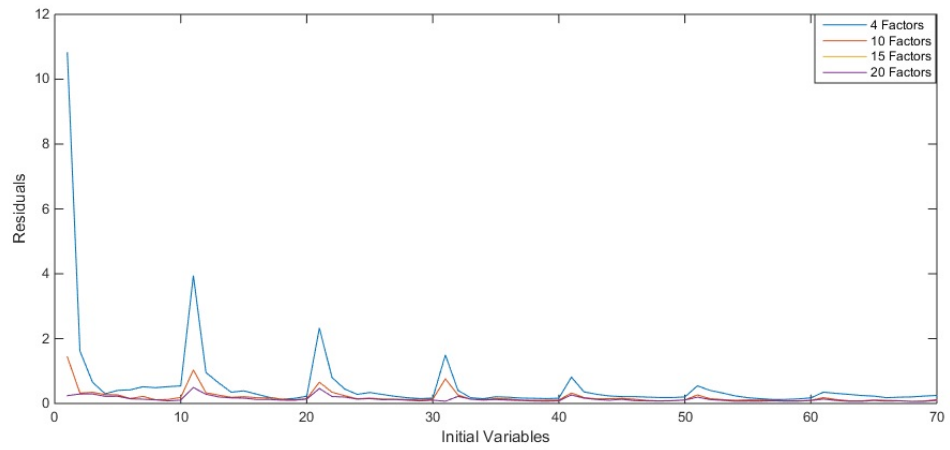


Figure III.6: Residuals of the PCA and Factor Model with 4 factors

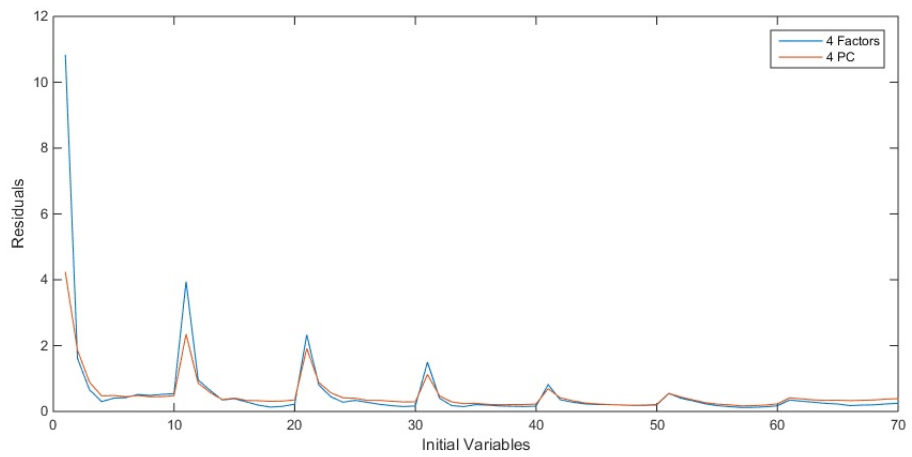
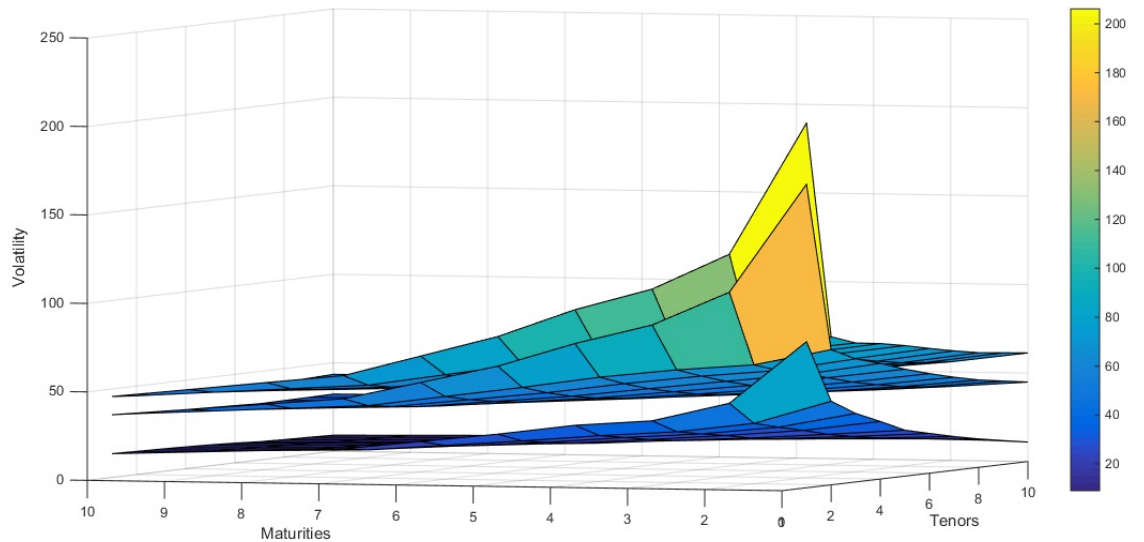


Figure III.7: Volatility Projections



Then we can predict the value of principal factors/components for the scenario based estimations of the interest curve. Once we have predicted the factors/component for each anticipated scenario, we can obtain the projected full volatility matrix with the 70 entries for each scenario. Figure III.7 illustrates the three projected volatility matrices (the results are those of the Factor Model with 4 factors). The hypothesis of parallel shifts of our interest rate impacts the shocks on implied volatility. The difference between the three implied volatility matrices are just explained by parallel shifts.

### 2.6.3 Back-testing of the prevision

We first focus on the average efficiency of our predictions and, to be able to compare the different models, we consider the same period of time: 150 last points of our back-testing which correspond to the period from June 2014 to December 2014. We compare the calculated indicators for different inputs of our model:

Table III.1: 2 factors

Sample	PCA	PCA Kalman	Factor Model	Factor Model Kalman
900	63.57	63.65	63.44	63.75
800	64.70	64.86	64.54	64.90
600	61.02	62.92	60.85	62.68
400	59.03	61.12	59.03	61.27
300	62.36	61.52	61.33	59.79

Table III.2: 3 factors

Sample	PCA	PCA Kalman	Factor Model	Factor Model Kalman
900	60.66	59.69	60.66	60.78
800	62.57	62.23	62.47	62.67
600	58.57	60.32	58.64	60.65
400	56.34	58.56	56.97	59.29
300	57.32	62.83	57.90	58.09

- Number of factors: 2 to 5;
- Number of observations in the learning sample of PCA/ factor model 300, 400, 600, 800 and 900 observations;
- The method: PCA or factor model;
- Kalman filter de-noising method or not.

Here we calculate the quality of the prevision by the L2 distance between the prevision given the realized interest rates and the realized volatility. The following tables present the different results.

The 4 factors model gives best results except for PCA Kalman when the number of observations is small (i.e. less than 600 observations).

Best results if we focus on the number of estimation dimension are obtained for 400 observations both for the PCA and factor model.

For factor model, for all considered numbers of factors, the Kalman de-noising method gives mostly less good results compared to the non-de-noised data.

Table III.3: 4 factors

Sample	PCA	PCA Kalman	Factor Model	Factor Model Kalman
900	58.96	55.06	59.21	60.50
800	60.51	56.19	60.66	61.50
600	56.81	60.64	56.96	58.40
400	53.25	64.10	53.57	56.39
300	55.91	69.49	55.74	55.56

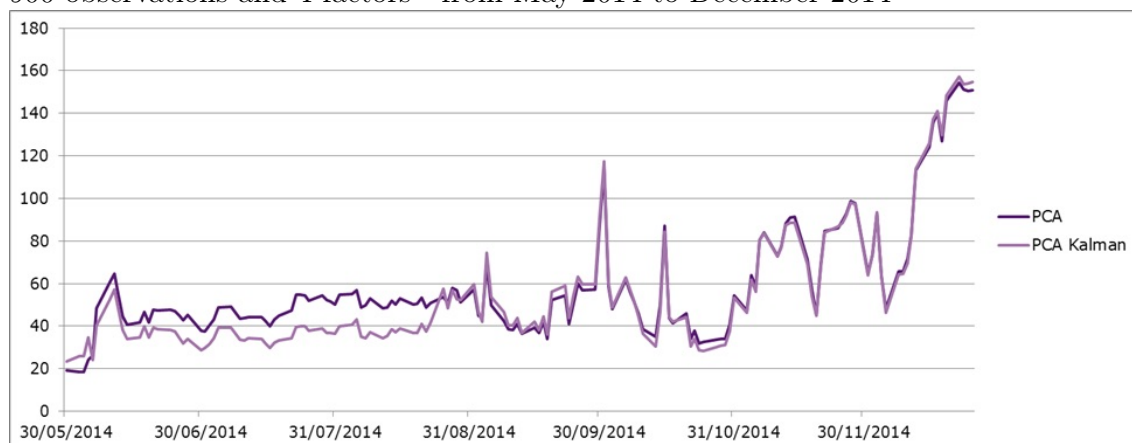
Table III.4: 5 factors

Sample	PCA	PCA Kalman	Factor Model	Factor Model Kalman
900	62.48	69.49	62.23	61.12
800	64.00	72.19	64.40	62.81
600	61.02	63.00	59.25	59.73
400	54.90	69.63	55.97	59.12
300	56.88	108.50	57.19	59.24

The results are not impacted by the chosen number of factors. The results are quite different when de-noising is applied to PCA. The worst results compared to non-denoised data are obtained for the 5 factors for all numbers of observation (the indicators are degraded from 30% with Kalman filter prediction). The Kalman de-noising method gives bad results for high numbers of PCA with large components. The method gives better results for 3 and 4 components PCA and only when the number of observations are quite high (900 – 800 on our data). The improvement on the predictability is large for the 4 components PCA; our indicator is 7 % lower when data are de-noised for 4 components PCA compared to 1% for the 3 components PCA given a large number of observations (900 and 800). The method does not improve the indicator for the 2 components PCA. We can resume by acting that the de-noising method improves our prevision only locally when the number of observations are quite large and for the most adapted numbers of components (ie 4 and accessory 3).

For the optimal number of factors 4, if we consider numbers of observations

Figure III.8: Prevision errors for PCA and Kalman PCA models based on 900 observations and 4 factors - from May 2014 to December 2014



and de-noising method together with PCA, we notice inverse effects on our prediction quality. As we have seen, the optimal number of observations is 400 points but the Kalman filter gives bad results for this number of observations. The positive effect of the de-noising method for 900 observations does not offset the negative effect from passing from 400 to 900 observations. It seems better to choose the optimal number of observation and to do not apply de-noising method.

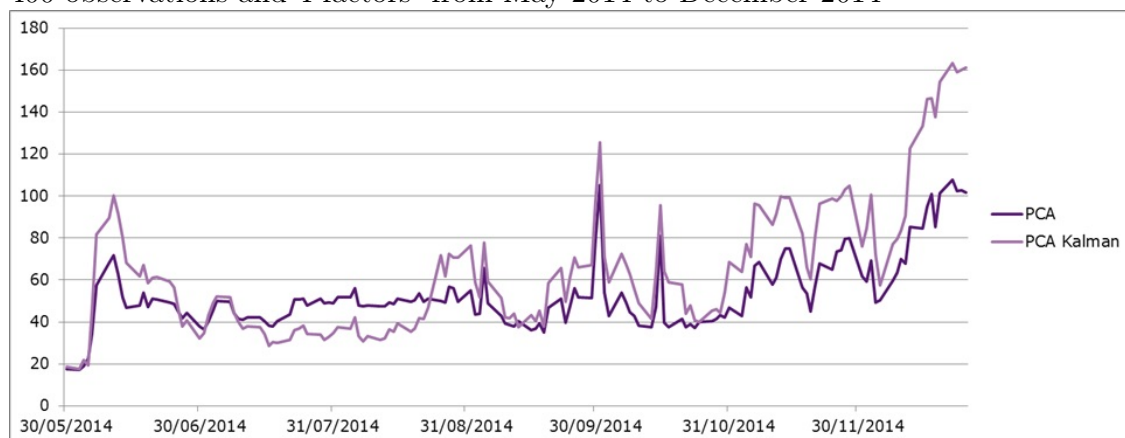
If we want to compare PCA and factor model, we notice that the two methods give quite similar results without de-noising method (as we have seen, the two methods react very differently to the Kalman de-noising). Even if the results are close, the PCA gives better results than factor model (few basis points better).

Let now go further for the 4 components/ factors model. We consider now the time series of the backtesting composed by 150 points corresponding to a 900 observations based model. Figure III.8 draws the historical value of our indicator.

We can notice that the de-noising method improves our predictions until



Figure III.9: Prevision errors for PCA and Kalman PCA models based on 400 observations and 4 factors- from May 2014 to December 2014



September 2014 only. After, we have a quite similar result between the method with Kalman Filter and without Kalman filter. If we draw the same historical but with a PCA based on 400 observations, we obtain Figure III.9.

As we expect, the general improvement brought by the Kalman filter is less than in the case of 900 observations based PCA. Nevertheless, we observe that the same 2 distinct periods with different results stay. From May 2014 to September 2014, the Kalman filter adds improvement to the model whereas from September 2014 to end of 2014 we observe the contrary.

We draw the interest rates historic to try to understand why we have two distinct time windows. Figure III.10 represents the 2 years point of the interest rates curve and Figure III.11 represents the 20 years point.

We can observe that there is a real break on September as a change of regime, a regime in which we observe a change of causality link between short term and long term rates. We can deduce that the de-noising method is not adapted to regime switching and gives low quality predictions after a regime switch. This break in the time series is caused by the negative rates applied by the ECB to banks funding.

Figure III.10: Historical Evolution of 2Y point of Interest Rate Curve - from May 2014 to December 2014

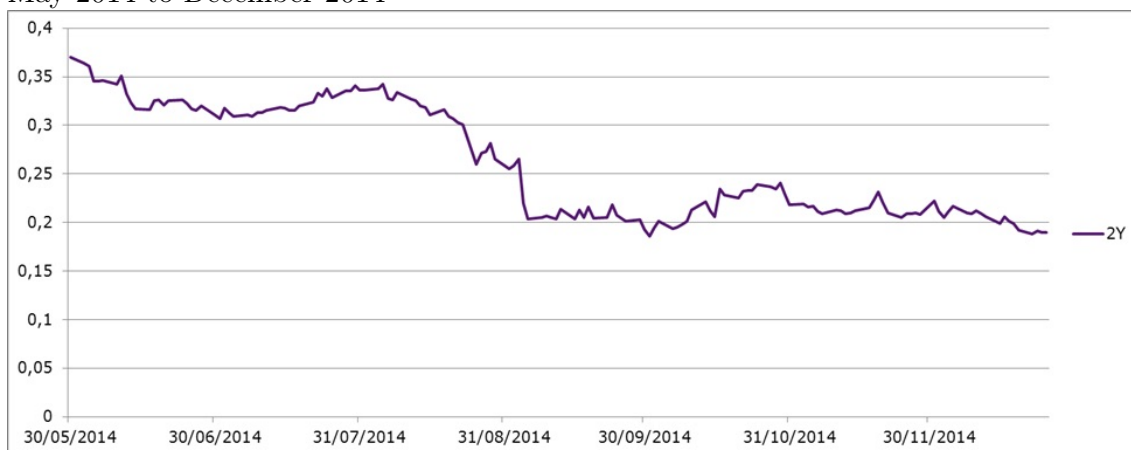


Figure III.11: Historical Evolution of 20Y point of Interest Rate Curve - from May 2014 to December 2014

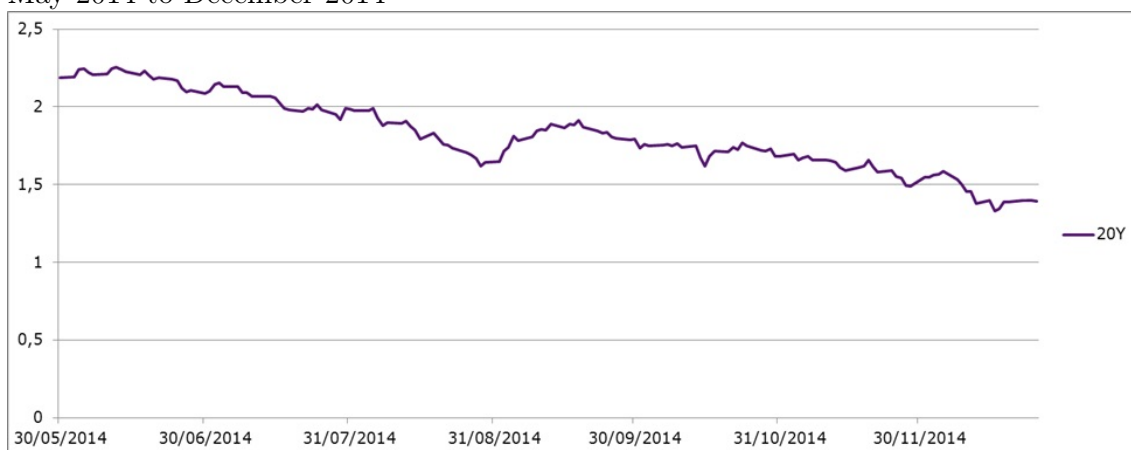
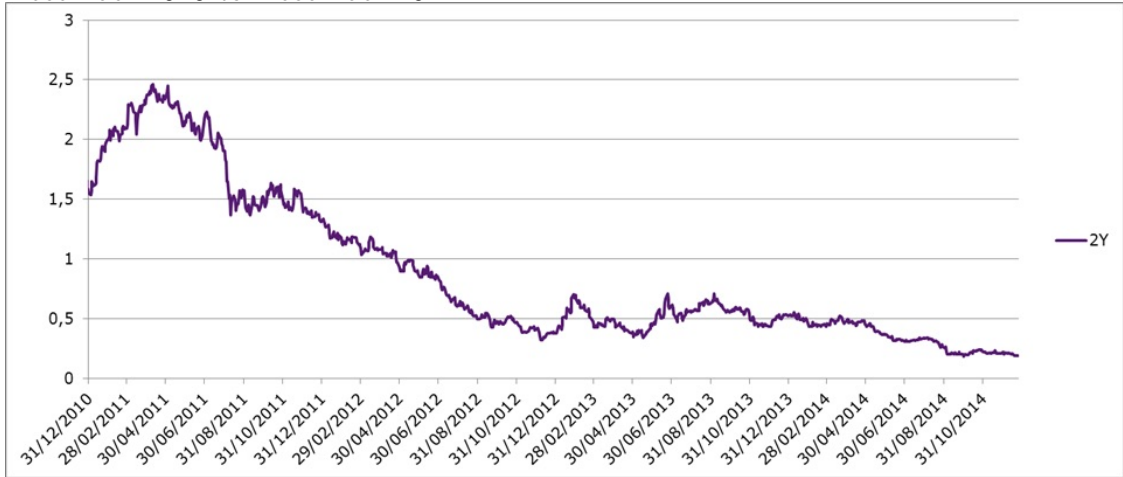


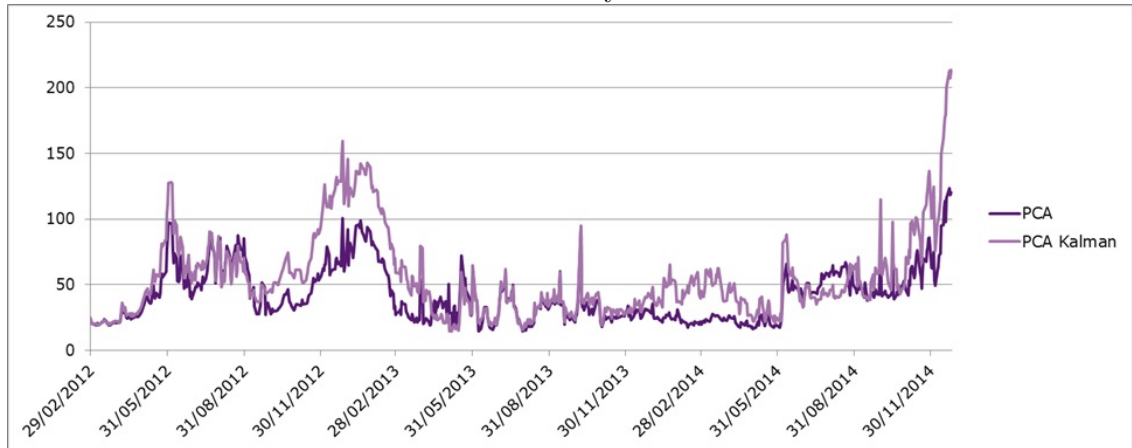
Figure III.12: Historical Evolution of 2Y point of Interest Rate Curve - from December 2010 to December 2014



To test the statement at another breaking point in time series, we have to go back in our historical data. To do so, we consider the 300 points based PCA which permits us to have a back-test for 750 points. We can notice that the quality of our predictions provided by Kalman Filter method for 300 points based on PCA is degraded compared to a higher number of observations based PCA; but, for a technical purpose, (i.e. initial number of points on our total dataset), we cannot analyze previous breaking point with a model based on a high number of observations.

We can observe in Figure III.12 two breaking points; one in September 2012 and another one in December 2013. Indeed the period until September 2012 can be resumed by a high negative trend and low volatility; the period between September 2012 and December 2013 has a flat trend a high volatility while the period starting in December 2013 has a negative trend and low volatility around this trend. Figure III.13 confirms our intuition on the poor capacity of adaptation of the de-noising method to regime switching as we can notice that September 2012 and December 2013 are followed by bad prevision provided by the de-noising method.

Figure III.13: Prevision errors for PCA and Kalman PCA models based on 300 observations and 4 factors- from February 2012 to December 2014

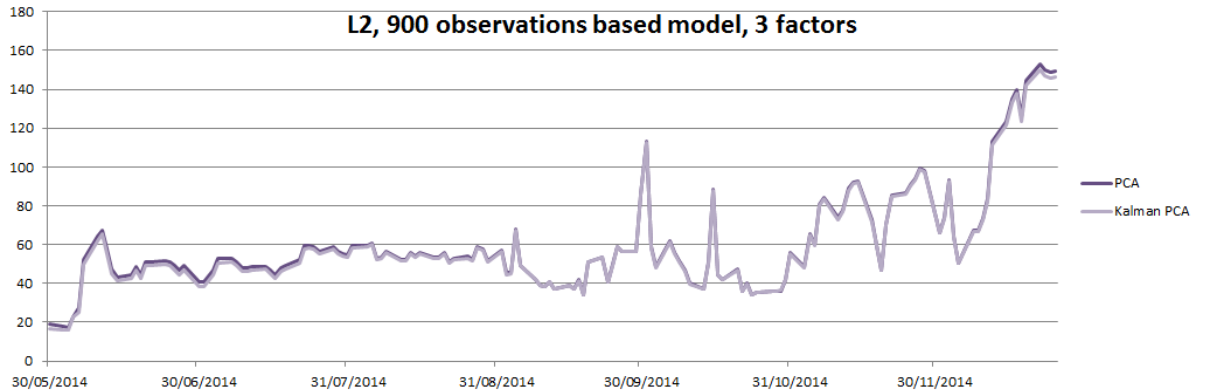


To conclude, the tested de-noising method can add information on the prediction process only if we use enough data points in the learning sample of PCA on a stable period without regime switching.

We focused on the evolution of the prevision error of the 4 factors PCA. We represent this evolution for the 3 and the 5 factors PCA for 400 observations and 900 observations to understand to impacts of adding components in the PCA.

In the 3 factors PCA, the performance of both methods (with Kalman and without) is similar for 900 observations learning sample and is quite similar for 400 observations (the naive PCA is slightly better in the post August 2014 period said regime 2 in what follows). For the 400 observations based models, when we add the 4th component on regime 1 (pre August 2014) the performance of the Kalman based method is highly improved. On regime 2, the impact is negative, the we lose performances. For the naive PCA, the impacts are positive (but less than in the Kalman PCA method) on regime 1 and neutral on regime 2. The effect 5th component is neutral on the regime 1 for Kalman PCA and naive PCA. On regime 2, the impact is negative for

Figure III.14: Prevision errors for PCA and Kalman PCA models based on 900 observations and 3 factors - from May 2014 to December 2014



the Kalman PCA whereas it has no effect for naive PCA.

For the 900 based models, the results are quite similar to those of the 400 based models. Mainly, we can observe that including the 4th component permits to improve the performance of the Kalman PCA compared to the naive PCA on regime 1 whereas the two methods performances are not impacted in regime 2 and so have same performance on this period as in the 3 factors models.

One of the input of our model that we have not consider above is the number of points of the interest rates curve that we choose. We compute all the different back-tests based on 4 points of the interest rates curve. As we also conclude that 4 factors models are the best fitting models, we can ask ourselves if the optimal number of factors is not linked to the number of points of the interest rates curve that we have selected. To obtain an answer to this question, we launch our back-testing algorithm with 3 points of interest rates curve as input: 2 years, 10 years and 20 years. We drop the 5 years point that seems to be the worst information provider. We fix 400 observations in the learning sample of PCA for a backtest window of 150 points (as considered above).

Figure III.15: Prevision errors for PCA and Kalman PCA models based on 900 observations and 4 factors - from May 2014 to December 2014

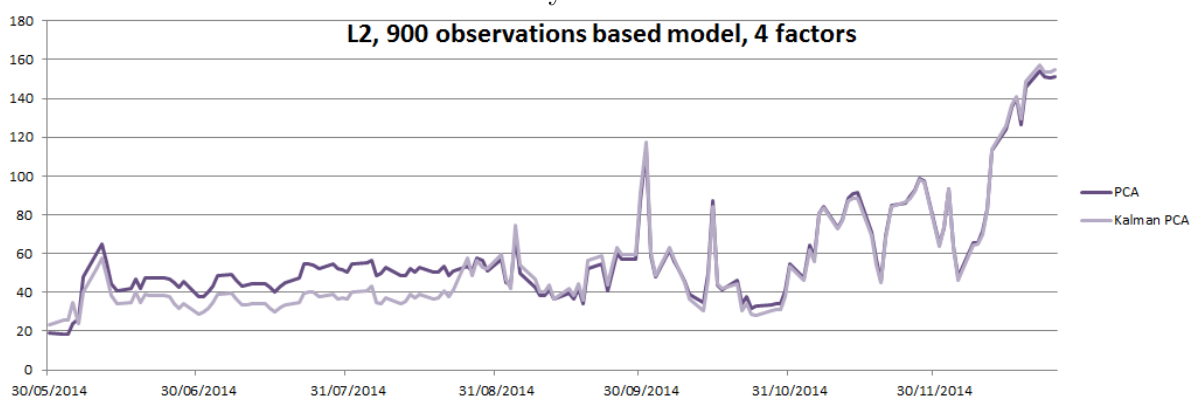


Figure III.16: Prevision errors for PCA and Kalman PCA models based on 900 observations and 5 factors - from May 2014 to December 2014

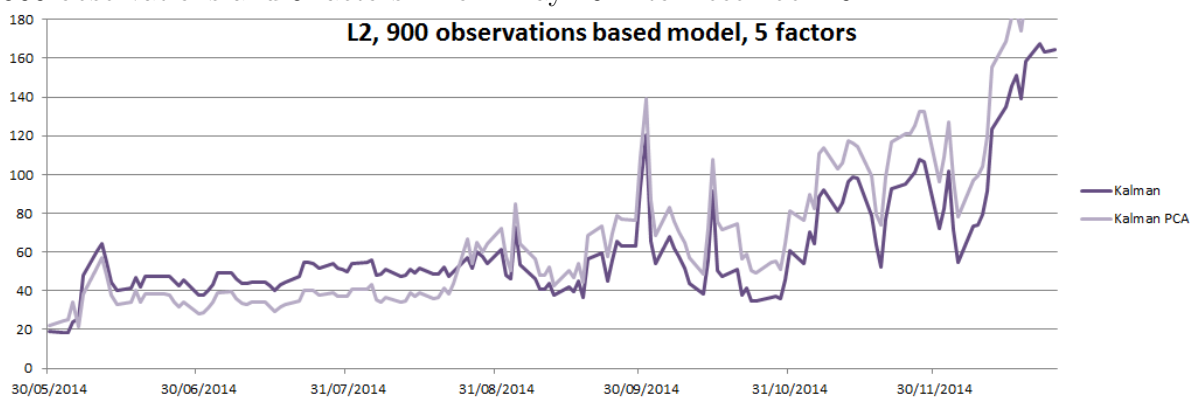


Table III.5: backtesting for the prevision based on 2y, 10y and 20y points of interest rates curve

Method	2 factors	3 factors	4 factors	5 factors
PCA	61.73	61.05	58.74	60.74
PCA Kalman	62.03	60.79	63.87	70.35

Figure III.17: Prevision errors for PCA and Kalman PCA models based on 400 observations and 3 factors - from May 2014 to December 2014

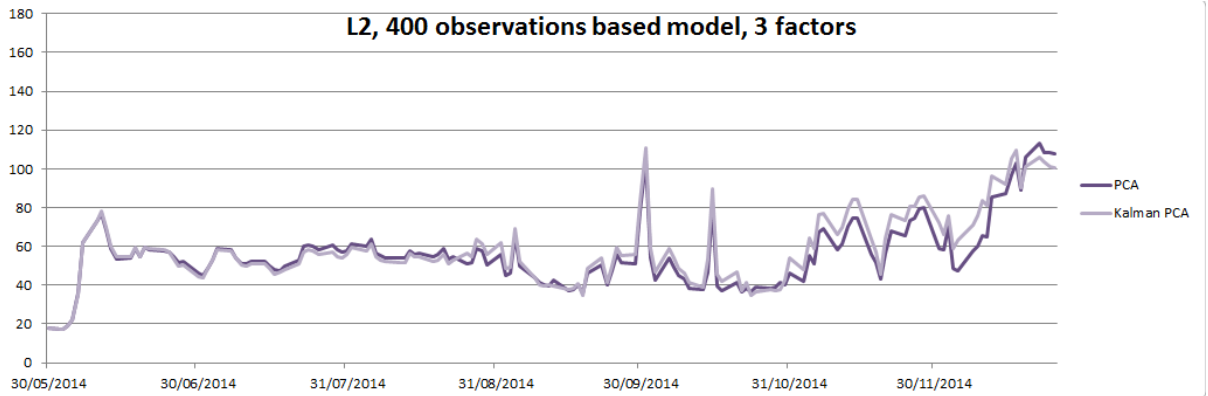


Figure III.18: Prevision errors for PCA and Kalman PCA models based on 400 observations and 4 factors - from May 2014 to December 2014

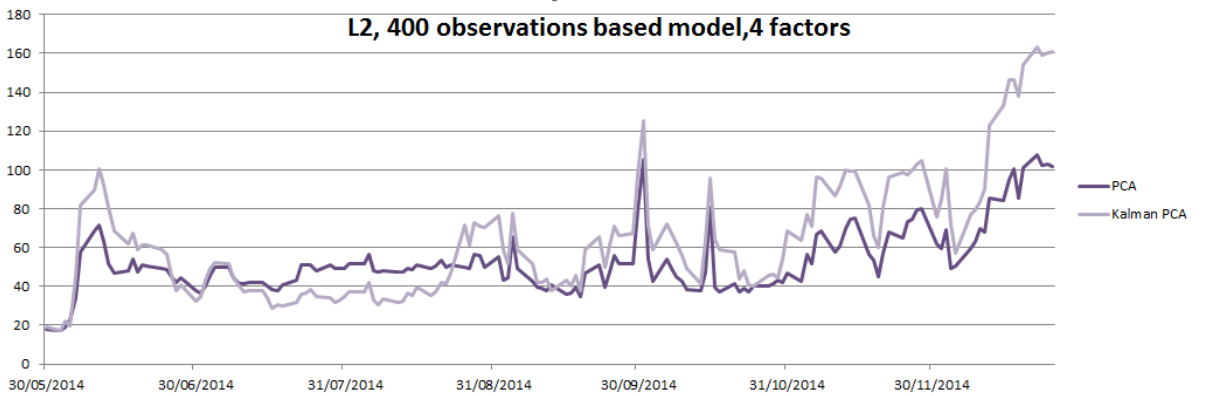
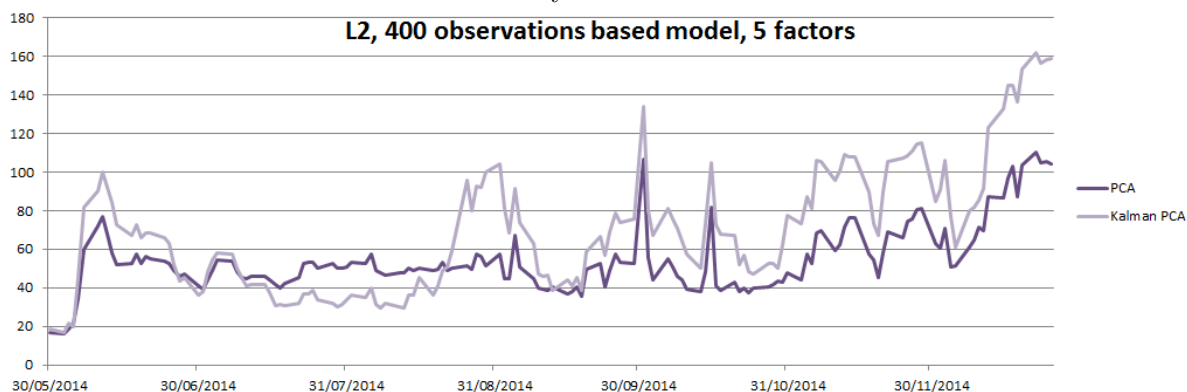


Figure III.19: Prevision errors for PCA and Kalman PCA models based on 400 observations and 5 factors - from May 2014 to December 2014



As shown by Table 5, the quality of the prevision is lower than in a 4 points of yield curve and the optimal number of factors is still 4 factors based PCA. This means that the selected number of points for representing the interest rates curve and the optimal number of factors in the PCA are not directly linked.

### 3 Macro-Stressed Pricing of Bermuda Swaptions

#### 3.1 About the Pricing of the Bermuda Swaption

For all the details on the pricing process of the Bermuda Swaption, we refer to the paper of Erkan and Prigent [8]. We just recall in this section the most important notions and definitions in order to make the reading of this paper possible without reading [8].

A Bermuda swaption is an option that permits us to enter in a swap at different dates. The difficulty of the pricing is coming from the presence of



different possible call dates. The payoff of the product is then depending on the optimal execution date. To calculate the expected value of this payoff, we have to solve an optimal control problem. The lecturer can ask “why this derivative option and not another one (by example like ranged accrual CMS) is concerned by this paper”. As we have specified at the beginning of the chapter, we do our study in the skill of a buy side actor. It is quite common for a buy-side actor to make a micro-hedging strategy with the use of asset swap. We buy/issue a bond with complex payoff and we are long a swap which cancels the complex cash-flows of our bond and pays/makes us pay a local floating rate if we have bought/issued the bond. This micro-hedging can preserve us from different complex interest rates of the bond but if the considered bond is a Bermuda concealable bond, we cannot actually hedge the risk coming from the undetermined redemption date. Then, it is interesting to study the price and the estimated call date in different preselected macro-scenarios.

We want to obtain the price of a Bermuda swaption callable every year and with final maturity of the underlying swap in 11 years. We are at 12/31/2014 and we want to estimate the price at 12/31/2015. The previous section gives us the estimation of our future risk factors. We perform our pricing in a LMM 1 factor model and compute the pricing with a Least-Square Monte Carlo (Longstaff-Schwartz). We use the different projected risk factors (interest rates and volatility) to price in each macroeconomic scenario.

## **3.2 Pricing for the Different Projected Risk Factors**

We expose the different prices for all the scenarios, reduction models and selected reduced dimensions. Our learning sample is composed by 5 000 paths and our valuation sample by 30 000 paths. Figure III.14 resumes all this information. We can observe that for each scenario the choice of the model and dimension does not impact significantly the price of our product.

Figure III.20: Price of the option

scenarios	FM			PCA		
	4 F	10 F	20 F	4 PC	10 PC	20 PC
Central Scenario	5.66%	5.69%	5.71%	5.66%	5.69%	5.69%
Deflation	10.00%	9.99%	9.98%	10.01%	10.02%	10.03%
Bond Crash	0.03%	0.04%	0.04%	0.03%	0.04%	0.04%

Figure III.21: Estimated option's maturity (if executed)

scenarios	FM			PCA		
	4 F	10 F	20 F	4 PC	10 PC	20 PC
Central Scenario	2.71	2.74	2.69	2.68	2.75	2.74
Deflation	1.73	1.72	1.76	1.69	1.73	1.73
Bond Crash	7.24	7.34	7.31	6.97	7.40	7.33

The difference between the different configuration is about few bps. We also represent the estimated optimal maturity date of the option for the different configurations in Figure III.15. We estimate it by the empirical average of the simulated execution dates over all the sample of our Monte Carlo simulations. We can assert that Factor Model or PCA gives similar results and in the context of dimension reduction of the implied volatility matrix the output will be independent of the chosen method. For the dimension, as the 4 factors/components reduction gives a very close price compared to higher dimensions, it seems more suitable to choose the lower dimension to explain the implied volatility matrix. This result highlights the uselessness of a high dimension (full dimension) model to explain a highly correlated implied volatility matrix. To resume, we can say that all the entries of our implied volatility matrix can be seen as a mixture of a few (here we choose randomly 4) independent variables without losing significant information about the data.

### 3.3 Uncertain Parameters Model

In this section, instead of pricing for different risk factors, we build an uncertain parameters model (in spirit of Brigo-Mercurio[3]). This model takes account of the uncertainty of the risk factors and models them by random variables. This implies to define probability for each scenario. This point is not so trivial. The logic and hypothesis of our model completely change when we span our scenarios in a probabilistic world. This implies indeed to pass from a notion of uncertainty to a notion of risk (c.f. Rebonato [11]). We discuss this point further in the next chapter. In what follows, we consider probabilized scenarios counter to the scenario based modeling of first part of this chapter.

The values of the risk factors are known immediately after time zero. For each path of our Monte Carlo simulation and before launching the simulation of the trajectory, we have to determine in which scenario we are. We have first to assign probability of occurrence to the different macro-scenarios: Central Scenario appear with a probability of  $p_1$ , Deflation Scenario with a probability of  $p_2$  and finally Bond Crash with a probability of  $p_3$ . We simulate an uniformly distributed variable  $U$  taking values in  $[0, 1]$ . If  $U$  is  $t$  in the interval  $[0, p_1]$ , we do the simulation with the risk factors of the Central Scenario, for a value of  $U$  in  $]p_1, p_2 + p_1]$  we consider the risk factors of the Deflation Scenario and, if  $U$  is inside the interval  $]p_2 + p_1, 1]$  it will be the risk factors of Bond Crash Scenario. Once we have the realization of this variable for all paths, the algorithm is the same than in the standard LMM. Each scenario uses specific risk factors (interest rates and volatility) depending on the realization of the state variable.

We price our product with 10 000 paths for the learning sample and 150 000 paths for the valuation sample. We use the PCA with 3 Principal Components for dimension reduction of the volatility matrix. We fix  $p_1 = 0.5$ ,  $p_2 = 0.35$  and  $p_3 = 0.15$ . The price of the product on our uncertain parame-

Figure III.22: Price in the uncertain parameters model and the weighted average price

	Central	Deflation	Bond Crash	Weighted Average	UPM
Prices	5.65%	10.01%	0.03%	6.33%	6.32%

ters model is 6.33%.

We can compare the price of our model with a naive average of the price in each scenario weighted by the probability of accuracy. We perform the pricing for each scenario with 10 000 paths for the learning sample and 50 000 paths for the valuation sample. As we can see on Figure III.16, the weighted average is equal to the price issued by the uncertain parameters model. The two methods give the same result. This means that the optimal control part of our Monte Carlo is not affected by a sample where there are different regimes and so the data are more volatile. This can be explained by the linearity of the scenario projection. The non-linearity coming from the optionality does not affect the choice of the methods (an ex-post mean or an uncertain parameters model) because this uncertainty is outside of the pricing process.

## 4 Conclusion

In this paper, we focus on a forward looking approach to estimate future value of a derivative product. When we price an option, we take care only about the information of the valuation date. The calibration step of the pricing allows us to fit the data of the current date. The different hypothesis behind derivatives pricing allow us to resume the information by the quotation of the day. As the calibrated variables change every day, this shows the non-stability of our modeling. The objective of our approach is to include our vision on the future in the estimation of future price of our option. As we know that our risk factors (here interest rates and volatility) are not constant and extreme values of these risk factors are non-negligible, we decide to estimate the price of our product for next year with macro projection of these risk factors.

In the first part, we detail the selected macroeconomic scenarios and how we project the implied volatility matrix according to this scenarios. As the dimension of the implied volatility matrix is quite high, we chose to use dimension reduction technique. We compare the Factor Model and Principal Component Analysis. We detail the numerical results for this two methods and for different dimensions. The two methods give similar results. We estimate the latent factors with the predicted value of the Kalman filter (instead of the filtered value) used as a de-noising method. We deduce from the numerical results that the de-noising method is not adapted to regime switching and gives low quality predictions after a regime switch. On the other hand, in a stable economic context, the Kalman method gives better results than the standard PCA method.

The second part of the paper applies the macro stressing to the pricing of a Bermuda swaption. Similarly to the results on the implied volatility, the price is not affected by the chosen model and dimension. First, we calculate distinctly the 3 prices for each scenario with the respective risk factors. We also build an Uncertain Parameters Model taking account of

the three scenarios with their probabilities of accuracy. This model gives us one price which aggregates our 3 anticipated macroeconomics scenarios. We compare this price with the probability weighted average of the 3 distinct valuations. The two prices (averaged and Uncertain Parameters Model) are similar. This means that learning algorithm is not affected by including uncertain parameters in the modeling. This similar price can be explained by the absence of non-linearity before the pricing of the option. The non-linear problem appears only on the second step: pricing of the option and not on the scenario generation.

We propose a method based on classical statistical tools (PCA or factor model) to project the implied volatility according to the interest rates scenarios. The easy computation (as easy as the other satellite models used in stress-testing) can permit the fast implementation of the method. Compared to actual treatment of the implied volatility in stress-tests (increase of 25% and deterministic short-cut), it has the advantage to give a result with an economic sense.

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## Part IV

# Stress-testing as a Core Tool for Financial Institutions Management

### Abstract

Stress-tests have become a major tool to manage the strategic orientations of a financial institution and especially banking Groups. The present paper analyzes the logic behind stress-testing. We present the different steps composing stress-testing from the scenarios definition to the impact on the activity of the Institution with a focus on the logic and hypothesis behind the choose of this tool. After this “tool focus” part, we analyze modeling hypothesis behind stress-testing. As a buy product, we bridge the gap between derivatives pricing grounded on risk-neutral modeling and stress-testing based on scenarios analysis.

## Introduction

Managing financial institutions is a quite complex issue. We are facing changing economic and financial environments. The different breakdowns of our environment make the risks and performance's management of on financial institution difficult. The banks have to adapt their business plans to take account of the specificity of the current economic situation (eg: low interest rates, low market index, deflation,...). Crisis can come very quickly compared to the reactivity of a bank to change the strategic orientation of its activities. Let us define specific macroeconomic and financial context as a regime. The risks, performance, capital and liquidity of an institution can be managed by the usual quantitative tools for a given regime. All these models base their calculations on the indicators of the considered regime. These tools can be based on a "pseudo" forward-looking logic as in the case of Monte Carlo VaR or derivative pricing. We qualify them as "pseudo" forward-looking because they do not actually have a forward-looking logic. If we consider the two examples that we have given, we simulate future realizations of the variables but only with past (VaR) or current (derivatives pricing) information. These models combine the input information with random simulations to produce estimations for the future. These tools are adapted to manage the activities of the institution for a given regime of the financial and economic environment. The question that we are facing here is: how managing the banking Groups taking the versatility of our world into account? The regime changes can make our usual tools becoming outdated.

Stress-testing quantifies the impact of breakdowns in our economy that are not been captured in the standard tools. The objective is to extend all these metrics in different regimes taking in consideration the associated scenarios. It is obviously difficult to consider all possible regimes. To make the analysis possible, it used to choose two or three scenarios that can appear in a defined time horizon. Generally, a small number of scenarios are build including one

baseline scenario which is different from the market forward scenario. The presence of baseline scenario as the benchmark scenario shows the difference of logic between other tools where the market anticipations are the reference. The stress-testing brings this forward-looking view with taking account of our intuition on the evolution of our environment, which is really different from repeating the past or following current market information. Nevertheless it is possible in the scenarios planning part of stress-testing to repeat the past crisis (we discuss with more details this point in the following section). One of the important features of stress-testing is that it takes account of the specific opinions of the stress-tester. We are not only mixing modeling with market information (current or past). Including stress-testers views in the engine allows us to add real plus-value into the results and in its interpretation/uses.

Stress-testing is widely used in the context of prudential supervisory as a part of Supervisory Review and Evaluation Process (SREP). It permits to make projections of the equity ratio in a regime built on extreme events. It is clearly a supervision of tail risks in the management of the solvency. Solvency is the central notion in the management of financial institutions. It reflects the ability for our corporation to honor its obligations and to continue its activities and expansions. Combining the capital adequacy analysis with the P&L of the bank gives strong information on the situation of the bank to the senior managers.

Stress-testing brings key elements of answers also for strategic and financial planning. It permits to analyze the deformation of P&L incomes and risk exposure in different regimes and/or strategic changes. These strategic changes can be the loss of one market (because of an offensive of a concurrent in a specific area) or a gain in another market (thanks to an acquisition).

As stress-testing requires the projection of balance sheet, we need to project in particular the debts of the bank. These debts contain structured securities and related derivatives. Projecting these complex products requires to

connect scenarios based projection and risk neutral modeling.

First, we expose the context in which the stress-testing is widely used for bank monitoring. In the second part, we present the whole process of stress-testing from the scenarios planning to the solvency ratio calculation. Section 3 is dedicated to the modeling hypothesis behind stress testing. The last section compares the compatibility of stress-testing with risk-neutral modeling to bring elements of analysis to structured notes and derivatives stressed projection.

## 1 Stress-Testing Contexts

### 1.1 Supervisory Framework

The regulators focus on two major three elements of banks: solvency, liquidity and resolution. When ECB focus on solvency and liquidity level, the SRB focus on the resolution. ECB fixes capital requirement to avoid any problem of failure which can impact the ratepayers by intermediate of the States, the creditors of the bank and the whole financial system (the contagion to others banks). If despite the effort on solvency the bank fails, to avoid a repercussion on the ratepayers the resolution have fix a level of MREL: a ratio of capital and bail-able debts. It permits more concretely to cancel the debts of bank (bail-in) following a rank of subordination of the creditors. The stress-test is the tool used on solvency calibration to take account of on capital requirement the consequences of a future crisis.

During recent years, stress-testing has gained importance in the supervisory of the banks, see Schuermann [15]. One of the main subject treated in management board comity is definitively solvency stress-testing. Indeed, the result stress-testing is crucial as it determines the equity ratio of the bank and then its capitalistic structure imposed by the regulator. The result

of (regulatory) stress-test affects also the market value of the Group as the impact is published by the ECB. In the case of bad results, the impact of the publication can be drastic if the Group owns quoted subsidiaries or are directly quoted. Indeed stressed solvency seems to be a good metric to assess coherent capital structure given the risk appetite of the bank. The notion of risk appetite is central notion in stress-testing because it permits to fix the probability of failure that the bank accepts as defined by Schulermann [15]. If the banks have “optimal” (not to say “high”) level of capital ratio, in case of losses they can honor their debts. More the funding comes from capital and less the bank are subject to propagate failures in the market. The idea of Basel is to identify the different risks and to adapt the level of capital to the calculated risk level. We calculate risks from different origins (essentially classed in credit, market, funding, operational and business) and deduce in counterpart the level of capital ratio as the ratio is calculated in terms of RWA (equity on RWA).

For solvency stress-testing, we can mention two stress-tests. We call external stress-test or top-down stress test, when the exercise is carried out by macro-prudential authorities. The assumptions and scenarios are common to the whole panel of banks. The aim of this stress test is to stress the macro-financial system in its entirety, that’s why it is important to have homogeneity for the assumptions, methodology and scenarios. This external stress-test is called EBA stress-test as it is designed by the EBA and its results are published by the ECB. The EBA stress test is a part of the Supervisory Review and Evaluation Process (SREP) as part of the Pillar 2 of Basel Accords. While the methodology and the scenarios are determined by EBA, the modeling challenge into the bank is less important. The biggest challenge is to adapt internal system and organization to be able to do the stress-test as defined by EBA.

The internal stress test is the second type of stress-test. It is a part of Internal Capital Adequacy Assessment Process (ICAAP). Indeed the ICAAP

document which describes its capital adequacy with a focus on risk that is not covered by the Pillar 1 capital requirements. The internal stress-test is one of the elements provided in ICAAP as a supplementary measure of risk. Banks fix the scenarios and methodology of this stress-test. It permits to be more confident for the results compared to those obtained in the EBA stress-test. The regulator has also the advantage to have both results and can compare them.

## 1.2 Internal Management

In addition to the regulatory needs, stress-testing and more generally scenario based analysis are also very useful for internal management of the scale resources of the bank. Indeed it is quite common to use scenarios analysis to determine the adequate strategic position of the bank. The usual examined variables are earnings and balance sheet based variables. As we usually do in finance, we want to report us in a profitability / risk couple and add the liquidity constraint.

For example in ALM, the net interest income is an indicator of profitability and the sensitivity of balance sheet's net present value is an indicator of risks. This NPV of the balance sheet can be determined in deterministic (as a sensitivity to a shock) or stochastic (as an economic capital) manner. Stressing this profitability - risk couple permits to take account of the sensitivity of the financial management to different macroeconomic shocks.

More generally in financial planning, a steering of scale resources is performed and the scenario based approach is the central tool. For the incomes, the realized incomes are compared to the expectation and if there are non negligible differences, the spread is explained given the non expected events. The capital management, in the sense of optimal allocation of capital, is also performed with scenario based approach.

The scenario based and stress-testing are also central for M&A purposes. To analyze the relevance of an acquisition, it is common to project different representative variable on profitability and solvability in different scenarios. We consider a baseline scenario, a worst case scenario and a good case scenario. The decision of the acquisition is taken considering the baseline and under the risk to fall in the worst case scenario. In others words, the return is those of baseline scenario and the risk associated is those of worst case scenario. The decision of the acquisition is to decide if the benefit in the baseline scenario is more important than the risk to fall in the worst case scenario. We can notice that this scenario based approach add a real economic logic to the modeling compared to a alternative method where we would simulate in a Monte Carlo method. The relevance of the method is widely based on the quality of the scenarios as the approach is judgment based. We can also remark the absence of probability as in the scenario based method it is quite difficult to add probability as we analyze in following sections.

Let's note that for insurance companies and financial institutions specialized on asset management, the related challenge is focus on strategic allocation given this profitability - risk couples in different scenarios as the business model of these companies are to invest in market.

To summary, stress-testing is adapted when we want to perform usual calculus on balance sheet position, incomes statements and risks in a stressed situation to take account of the variability of our environment. Stress-testing can be opposed to modeling as instead of modeling specific risks by an economic capital it is possible to proceed by stressing the whole system to take account of all possible risks.

Of course different stress-testing is possible. We expose in what follows the details of a solvency stress-testing to give an example of a wide stress-testing which needs to project almost all the variables of a bank. Solvency is very central and controlled by the regulator and investors since the crisis. Even if the solvency stays a very important indicators, we have to precise that

today the investors and regulator (which is the main focus of the business model analysis of SREP and so of solvency requirements) are increasingly interested by profitability. The low interest rates context, where the NII of banks are in difficulty, put this scale resource (profitability) more and more in foreground.

## **2 Solvency Stress-Testing: from scenario planning to impact analysis**

In this section, we expose all the steps to build stress-tests. Stress-testing is a quite long and transverse tool. It requires knowledge from different area: economics, risk, capital structure, prudential requirements, PnL (included asset and liability management). If we took the specific example of solvency stress-testing, we need to involve in the stress-tests many different teams from different departments of the Group. These high heterogeneity implies to attribute each step to a team of specialist in this specific domain.

### **2.1 Scenario Planning**

The first pillar that represents the grounding of all the following steps is the scenario planning also called macro-financial scenario design [5]. It is perhaps the most important part of the stress-test as the reliability and quality of the test is at best the reliability and quality of the designed scenarios. All the results will be linked to these scenarios and if we planned poor quality scenarios, the plus value of our stress test will be low even if we perform high-quality methods in the following parts.

We first have to identify macro-financial risks. There are two different ways to fix theses risks.



We can use historical risks that we already observe in the past and have affected our macro-financial system. To name just a few, we can list October 1987, Asiatic crisis, 11. September 2001, Lehman bankruptcy. The biggest lack of this approach is, as highlighted by Schuermann [15], that it not test for “anything new”.

The second way of risks identifying is to do it hypothetically. Given the current macro-financial situation, we (by we, I mean economists) select some risks (or just one) that we can face in the future (future being defined by the horizon of our stress-test). For example, at the writing date of this paper, we can mention Brexit or growing of shadow banking sector (this risk is one of the four risks identified by the EBA for the 2016 EBA stress-test [6]). The identified risks should be appropriated with consistent impacts but need to be plausible at all. This arbitrage between severity and plausibility is important for the use and reliability of the stress-test. We recall that generally not assign probabilities to the scenarios. We discuss this question of probabilities in details in the section 3.

Once we get the different systematic risks, we have to quantify the impacts on several directly related macro-financial variables. In other words, we have to pass from risks to shocks on core variables (here we use “core” in the sense of directly related to the planned risks). This shock profile (as named by ECB[5]) can be calibrated in different manners. It can be done without use of models, based on historical reactions of these variables to past crisis. A second method is the use of historical distribution in a VaR based calculation. And of course, we can use model to calibrate the size of the shocks, supposing that the variations explained by the model is the natural evolution and the variability coming from residuals is interpreted as shocks.

The next step consists in expending the perimeter to all macro-financial variables. We can mention two approaches.

In the first approach, we use simple linear regressions to project all the vari-

ables. The most subtle part of this method is the selection of explanatory variables. The number of explanatory variables must be consistent with the number of observations. This selection of these independent variables can be specified by the economist without a quantitative study or it can be done by an algorithm selection (eg. forward regression, lasso regression,...).

In the second approach, we use a dynamic macro-econometric models. These models are those that have been developed by regulators in the context of monetary policy. We can mention Global Vector Autoregressive Model or Dynamic Stochastic General Equilibrium Model as the most popular approaches. We will not enter on the details of these models, the interested lecturers can read ECB paper [4] for further details. The principal idea is that these models expand the perimeters of our variables and the projections are based on macroeconomic equilibrium assumptions. With this last step, we obtain the final output of our scenario planning: the projected values of all the macro-financial variables associated with the risks that we have identified.

## **2.2 Projecting Core Variables of the Bank by Top-Down Satellite Models**

The scenario planning permits us to obtain the state of our economy and financial market given the identified risk that we “think” plausible. This is, as we said, the first pillar of our stress-test. We now want to obtain the values of the different variables that represent the health of the bank. These “core variables” of the bank are the different risk exposures on its balance sheet, the profitability and the capital. In other words, we want to project the PnL/capital and the risk of the bank given the structure of its balance sheet. Indeed the balance sheet structure of the bank is a representation of its business model and resumes also its risk appetite. On one side we have the PnL generated by its activities and on the other side we have the risk

that it takes to generate this profit (if PnL is positive). Additionally to this PnL/Risk duality at a point in time, we also have the level of capital as a pillow to absorb shocks. We can do a little reminder and precise that the capital is essentially composed (without going into the restatements details) by the equity and the reserve, reserve in the sense of the accumulation of past PnL that we had not distributed to shareholders. We go further in all these notions in the next section dedicated to the consolidation and impact calculation.

The top-down models used to project these risks and PnL elements can be very heterogeneous and specific to the considered variables. We do not project the NII (net interest income) in the same manner than we project the LGD and PD for credit risk. The core variables projections are related to the different hypothesis and logic behind the initial modeling of the studied risk/variable.

When the macro-financial variables are direct risk factors on the modeling of the core variable (as it is the case in ALM for NII calculation), we can inject the projected values of the macro-financial variables in the modeling of our core variable instead of the current or baseline values of the initial model.

For other core variables, we can have no direct link between the macro-financial variables and the input of risk/PnL calculation models. In this case, we need to project core variables. These projections can be done by simple linear regression (with a special attention to explanatory variables selection) or as advised by ECB [5] with a Bayesian model averaging approach. This Bayesian method's logic consist to suppose that all the simple linear regression models are possible but uncertain. We do not select one model but use all the possible models and specify probabilities to all models. The probability of one model is related to its fitting quality (generally Bayesian Information Criterion: BIC). The models with high BIC have a high probability of occurrence and this implies that the estimated values of the regression's coefficients are more weighted by the values of the coefficients

in these more plausible models. For further details on Bayesian averaging models we refer to Sala-Doppelhofer-Miller [13].

As we expect, this part of the stress-testing process is calculated by the different risk and financial departments teams and each team focus on its specialty. Credit risks team is affected to the calculation of the credit risk component, ALM team the NII, market risk the stressed VaR, ...

In addition of these core variables, we need intermediate variables projections as balance sheet evolution in the case of a dynamic balance sheet assumption.

The different core variables that we need to projected are:

- Credit risk RWA
- Market risk RWA
- Operational risk RWA
- NII
- other PnL elements (as Commissions, fees, expenses,...)
- NTI (net trading income)

As the different calculation models can be complex and long to describe, we do not detail the modeling of these variables.

## 2.3 Solvency Calculation

Once we obtain the different core variables, we need to consolidate the projections and resume the health of the bank with an unique variable. As we said in the beginning of our paper, our stress-test is a solvency stress-test. The solvency can be viewed as a representative metric for the stability of the

bank. In fact, if the bank is solvent, it can continue its activities and honor its commitments.

The final output variable of our stress-testing is the core equity tier one ratio (CET 1 ratio) which stands as a metric of the solvency of the bank.

The following formula gives us the level of CET1 ratio:

$$CET1 = \frac{Capital + Net Operating Income}{RWA}.$$

The different modules projections give the inputs to calculate this ratio.

The CET 1 ratio is based on the core capital composed of equity and non distributed earnings. We also calculate solvency ratios for broader definition of capital. Indeed, we can include additional Tier 1 and Tier 2 capital in the calculation of the ratio.

More generally, the regulator fixes different ratio levels which regulate these different debt instruments. For example, the Pillar 2 of Basel is focus on core capital while the MREL is focus on T1, T2 and other "bail-in-able" instruments.

To resume, stress testing is based on the capitalistic structure of our balance sheet, balance sheet that generates profitability and risk exposure. Once we project our balance sheet, we have to measure the impact depending on the axis that we want to focus: solvency, capacity of bail-in, ...

### 3 Modeling Hypothesis behind Stress-Testing

We presented how we perform stress-testing in the previous section. Now we analyze the hypothesis behind this tool. To make good use of the outcome of our stress-testing and take appropriate decisions, we have to understand the fundamentals on which the tool is based. As highlighted by Rebonato

[12], stress-testing is grounded on the Keynesian uncertainty framework as opposed to the probabilistic worlds in which we use to model in finance. As a reminder, Keynes opposes risk and uncertainty. The notion of risk is employed when we have a perfect knowledge of the probabilities associated with future events while Keynes uses the term uncertainty when we do not know exactly these probabilities.

Financial modeling is generally based on the notion of risk with intensive use of probabilities. Rebonato [12] notices the quantitative potential of the framework set up in the Keynesian risk. Financial modeling is generally split in two areas. It is common to oppose Q world to P world [7]. The Q world based modeling is used to price derivatives. In this context, we use present information on quoted products (in other words products for which price is available) and interpolate other non quoted ( and more complex) products. Nevertheless, the use of historical information is possible if we have free parameters as correlations in certain cases. This Q world modeling is based on the famous hypothesis of “absence of arbitrage opportunity”. We are in a logic of interpolation and not prediction. The pricing of a financial product is given by the market quotations as the fair price viewed by the market. We just expend this logic to obtain the fair price of more complex and not directly quoted products. As highlighted by El Karoui-Loisel-Prigent-Vedani [9], the use of Q world is subject to different hypothesis as the systematic use of hedging strategic and a short term consideration. Indeed everyone knows the mismatching of the Q world modeling with the stylized facts, however the modeling gives acceptable result and can be consider as true locally in space and time. This implies that we need frequent (to take account the limit in time) calibrations for each type of product (to take account the limit in space).

As opposed to this Q world modeling, the P world modeling is used in asset management and risk management. The aim of this approach is to predict the future. We look at the past history and estimate the future based on the

information contained on this history. The hypothesis behind this approach is very different from the hypothesis in derivative pricing. In derivative pricing, we set the fact that estimation of future is present (because market permits us to buy forward contract on the underlying) and in asset/risk modeling we focus on predictions on the spread between present and future (because we focus on best estimation). Even if the hypothesis are very different we build in both cases probabilistic world in the sense of Keynesian risk.

On the other side, we have uncertainty and more specifically scenario analysis. This modeling is common in corporate finance, investment banking and accounting. As other tools of these areas, stress-testing is based on this scenario based approach. We do not assign probabilities on future possible realizations. We select plausible and interesting scenarios as described on the previous section. We focus on the outcomes given the specific scenario. The biggest difference with the risk based modeling is the central place of subjectivity. We can wonder on the plus value of these uncertainty based approach. The presence of subjectivity is the key element of the answer. As remarked by number of authors, the past does not repeat itself (cf Taleb [15]). The occurrence of non predictable events makes the prediction of the future based on past difficult. To fix this problem, we add subjectivity with scenario based approach. The biggest weakness of this approach is the lack of probability. As remarked by Rebonato [12], in the case of stress testing, we know how much we can lose but we do not have an idea on the probability associated to this loss. Giving a level of loss without giving the occurrence makes difficult the use of the outcome by the management body.

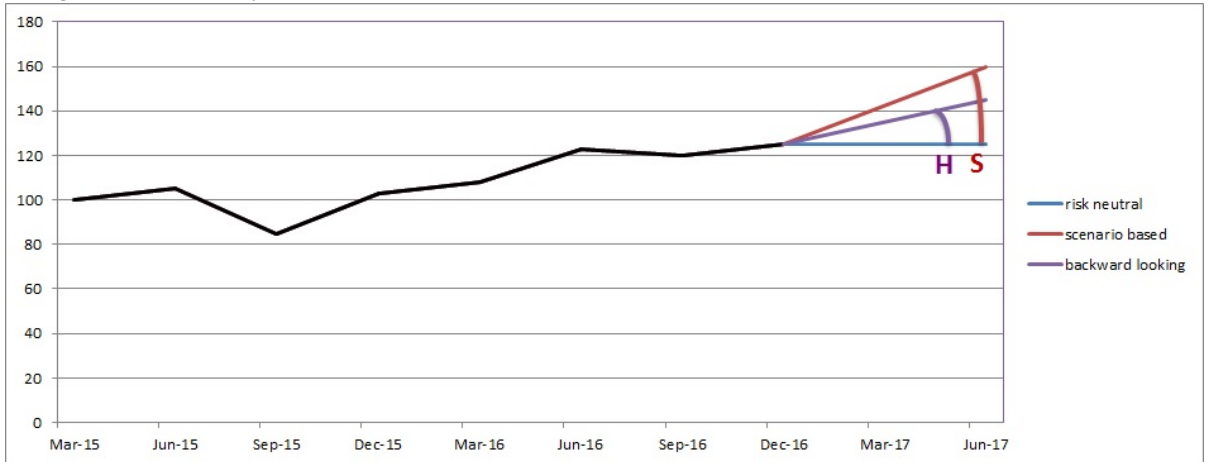
We can wonder on the assessment of probability on the two or three selected scenarios. The scenario based analysis is build on picked scenarios (scenarios that are plausible and with enough severity). Assigning probability on these scenarios are very difficult task as we are working with subjective probabilities and fictive scenarios that we can not calibrate with the past (as we do for risk based approach). Even if Rebonato explores Bayesian modeling to

assess probabilities on our stress-tests [12], the banking Groups do not use today probabilities when they perform stress-testing. The ECB does not claim probability neither in the context of supervisory stress-testing exercise defined by the EBA. In today's stressed scenarios management, we do not have a view on probability. First, we can find this approach debatable but we can defend it by remarking the conservative characteristic. For years, we observe stressed economic situations that come true one by one. We are not, in our scenario based tool, trying to analyze all possible futures event or even less to analyze an average situation given all future possibilities. We want to handle a crisis situation that can happen and so to analyze our capacity to absorb the impact. The question that we want to answer is: "if an other crisis would happen (and it will happen!), how would the robustness of my institution react?". If we focus on our soundness given the fact that a crisis happened, we do not really care about the probability of occurrence of this crisis. Is it better as we said above to assess probability to this event? Perhaps not in our angle analysis. It is better to have a knowledge for management body to attribute a adequate importance on the results of the stress-test even if we are aware of the poor quality of the estimation of this probability. It seem to be not necessary if we first of all want to analyze post crisis situation. We do not want to take account of the fact that perhaps we will not have a crisis situation tomorrow, we suppose that we will have and just observe our financial situation. Of course selecting a "good" scenario is crucial in this modeling. We have to pay attention on the scenario planning step of our stress-testing.

Stress-testing is based on crisis management with a conservative view because of the absence of probabilities. We can perhaps criticize this "what if" approach in different fields (ALM, M&A, ...) but the number of crisis that we have suffered and the conservative property coming by our focus on crisis management make the tool less debatable compared to other application areas.

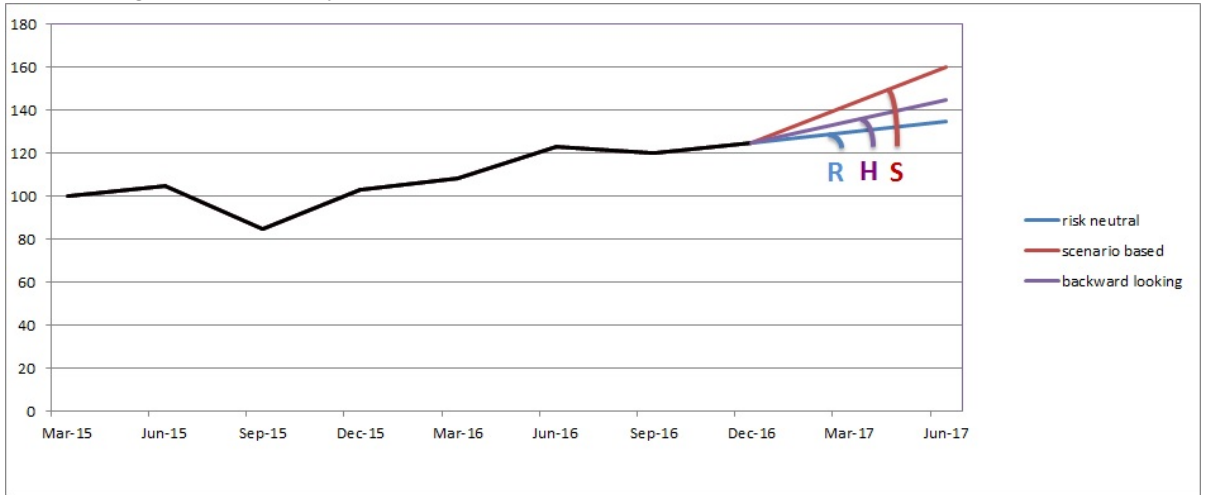


Figure IV.1: Projection of the discounted value of a tradable instrument



To resume, we can notice that three different approach are possible to project a variable (with all the properties stated above for each method) which gave three different output. We represent these three different estimations for a tradable instrument in Figure IV.1 and a discounted value of a tradable instrument in Figure IV.2. The discounted value is more adapted in fixed income market as future non known rates are estimated by forward rates which are a discounted zero coupon bond (a tradable asset). In equity market, we would consider the stock price and not necessarily the discounted value of stock price. For the risk neutral approach, the discounted value of the tradable instrument in the future must be estimated by the current value of this instrument as there is no possibility of free lunch on the market. This implies that the non discounted value of this instrument must increase and this increase has to correspond to the interest rate.

Figure IV.2: Projection of the value of a tradable instrument



## 4 Compatibility between risk neutral modeling and scenarios analysis

### 4.1 Context

We examined the logic behind stress-test compared to other alternative approaches. The multiplicity of modeling approaches is due to the existence of an heterogeneous panel of problems in finance. As highlighted by Demey-Franchot-Riboulet [3], the most telling example is the definition given to interest rate risk. A real misunderstanding can appear between an IR trader and an asset and liabilities manager on this subject. The trader wants to neutralize the impact of yield curve movements on the valuation of his product (fair value hedging) while the AL manager wants to neutralize the impact of yield curve movement on his future cash flows (cash flow hedging). We cannot do both at the same time. Nevertheless it is important to notice that a hedging position in ALM, which supposes a placement at fixed rate of deposit, can be classified in cash flow hedging or fair value hedging as it can be viewed as

- a hedging of future cash flows versus the floating short rate remuneration provided that we let the deposit in the money market
- a hedging of the fair value of the assets of the bank: loans providing fixed rate

These two alternative visions of the same hedging position highlight the specificity of the French banking system in which the asset and liability are fixed rate instruments. Fixed rate of position can then be viewed as fair value hedging when having a global vision the balance sheet: asset and liability.

The definition of the risk depends on the considered activity. It seems logic to secure the market price of a tradable product while it is more suitable to guarantee future incomes coming from an economic activity. When trader considers risky to move away from the market rate, the AL manager considers risky to suffer from the volatility of market rate and even more when the liabilities of the bank pay fixed rate.

Given the differences between these two fields, we understand the need of different models to manage the risk. When these two activities stay independent, we “just” have to apply the adequate modeling to the specific problem. The question is more complex when we have to manage problems combining the two fields. As we have already noticed, financial management of banks is based on forward looking modeling especially via stress-testing. As detailed on previous sections, we stress core variables of the bank to project a possible future situation given our selected scenario. These core variables incorporate also the liabilities of the bank. The liabilities are composed by different products. If we make a focus on liabilities, we can classify them in five categories:

- Deposits (non maturing and maturing)
- Securities

- Financial Liabilities
- Derivatives with negative fair values ( in IFRS, the derivatives appear in balance sheet).
- Capital

These categories are also those appearing in regulatory reporting on liabilities (eg. MREL reporting asked by the Single Resolution Board on April/June 2016).

We need to use derivatives pricing specially to valuate structured securities and all derivatives (those on assets side and liabilities side) when they are evaluated at fair value hedge. In fair value hedging, the carrying amount corresponds to the fair values. When the product is structured, we need to use absence of arbitrage opportunity hypothesis and derivatives pricing to obtain its price.

When stressing our balance sheet, we face therefore a problem which gathering risk neutral modeling and scenario analysis.

## 4.2 Fundamental differences

We need to price our derivatives on a projected scenario. We can see this combination as a two steps modeling: we first model variables with a scenario projection coherent with our intuition on future risks and our specific risk appetite and in a second step we model them with absence of arbitrage opportunity hypothesis given the effective scenario.

We can highlight the difference gap on numerical results coming from these two modeling because of the inherent hypothesis of each approach. When in the first modeling step, our scenario based projection gives a baseline and an adverse case projections, the risk neutral modeling would give the present value of the variable grounded on martingale hypothesis on the variables.

If we want to link the different models, we can compare the baseline scenario with the forward estimation of risk neutral modeling. The adverse scenario can be compared to a Value-at-Risk estimation for a given probability in the risk neutral world.

Let first consider the two central estimations. What we call baseline scenario is also very different from the risk neutral as also highlighted by El Karoui-Loisel-Prigent-Vedani [9]. We can expect a coherence between these two scenarios as one express the baseline situation (in the sense of most likely from the bank's or central bank's point of view) and the other express the value estimated by the market given quoted products prices. In practice, we have very different estimations given by these two approaches. The baseline estimation can be therefore very distant from the forward rate. The forward rate does not take account of important postulae on macroeconomics equilibrium even if the slope and the curvature of the interest rates curve reflect economic view of the market. The baseline estimation is very different from the current estimated level because baseline estimates most likely macroeconomic situation which differs from the current macroeconomic situation. In other word, we can observe a difference on time scale focus. The risk neutral estimation is focus on hypothesis on financial market evolution view given current macroeconomic context as considered the most likely whereas the baseline estimation is focus on most likely macroeconomic context given our knowledge on the different macroeconomic cycles. When the market reflects the current situation and reports it into future estimation, the baseline estimation (grounded on his knowledge on the variability of macroeconomic context) has a long term vision and estimates very different values compared to current values. The market estimation is short term vision which need frequent calibration for a specific product and have credibility only if we effectively hedge position with a replication portfolio, see El Karoui, Loisel, Prigent and Vedani [9] for more details. In [1] Bergomi highlights that risk neutral models are motivated by the capacity to measure the impacts (sign

and size) of different risks on the P&L of a derivative product and have not capacity of prediction. Kiesel and Rahe [10] define implied volatility as the market sentiment and use risk neutral information to analyze risk aversion evolution and forecast future returns. Using risk neutral information to forecast is also studied in depth in Christoffersen, Jacobs and Chang [2]. They highlight the importance to combine implied volatility to historical return to improve the quality of previsions in certain situations. However it requires a mapping of the risk neutral information into the “physical measure”. Their article [2] gives all the details on this mapping.

To additionally illustrate differences between risk neutral world and scenario based prevision, we can remark that in our current low (negative) rates environment the forward rates are also very low whereas the baseline scenarios anticipates a rise of interest rates levels. We can also add that when ECB choose to select a baseline where interest rates rise, it is also to have an unfavorable scenario for the assets of the banks.

The adverse scenario is based on identified economic risks and have no direct link with the quantile of probabilistic distribution as the VaR is. However it is possible to relate the adverse scenario to the risk neutral world. Indeed we can calculate the probability associate to the adverse scenario as a quantile of the risk neutral distribution of the variable. We thus have an estimation of the severity of our adverse scenario in a market perspective.

### **4.3 Mismatch on precision of the models**

We combine two models of projection. The first projections model is grounded on the uncertainty of Keynes and it gives global estimation. The precise value of this projection is not the real focus of the logic behind our scenario based approach. The plus value of the output is more the level of the estimation than the value itself. Indeed, the confidence interval of the projected value is large. The projection is a subjective value.

The situation is totally different in the case of derivatives pricing. We are facing to extremely precise tool giving a very accurate output. To provide very precise estimation, derivatives pricing use complex methods. For example, the construction of continuum of tenors in a Libor Market Model add complexity in pricing process then we could interpolate very baldly with a low loss of accuracy. An other example is the calculation of the solution of the dual problem in the case of bermuda or american option pricing. The calculation of this price is much more complicated than the initial price calculation, in fact it consists in a Monte Carlo in a Monte Carlo.

The two projection models are very different in term of accuracy. Clearly, combining these two models gives for our global projection model (ie the risk neutral price in the projected scenario) the accuracy of the less precise model.

A practical question is : “How appropriate is it to use a very complex and time consuming pricing method when the inputs of the pricing are global and not very precise?”

The answer would depend on the existence of robust pricing tools on IT systems and the possibility of stressing the parameters. If it is not the case, it is possible to make different short cuts. The noise introduced by “reasonable” short cuts would be lower than the initial incertitude around the projected inputs.

It is important to have in mind that we are pricing financial products for purposes of balance sheet management and not as usual to delta hedge risky position on derivatives. The requirement on accuracy is not the same.

## 5 An Example of Stress-Test Exercise: 2016 EBA EU-wide Stress-Testing

### 5.1 Context of the Stress-Test

The EBA Stress-Test results have been published on 29 July 2016. This publication concludes a quite long and dynamic process which lasted 5 months. A scope of 51 European banks, which covers 2/3 of the EU banking assets, is concerned by this exercise. The starting point of the stress is end-2015 and focus on 3 years period ending in end-2018. This stress-test corresponds to a macro-prudential stress: all the banks are faced to the same methodology and scenario. It permits to benchmark the different banks to each others and to test the banking sector's resilience.

The results of the test are expected to be a part of the 2016 SREP on the determination of the Pillar 2 level. The 2016 SREP is expected to be quite different from the 2015 SREP as communicated by the EBA as of 1 July 2016. This forthcoming SREP would split the Pillar 2 capital into Pillar 2 requirement and Pillar 2 guidance.

Banks are expected to meet the P2G which is situated on top of combined buffer but the non respect of this level would not imply an action of the supervisor and would not be used to determine the maximum distribution amount (MDA) trigger which imply restrictions on dividends, AT1 distributions or bonus.

As highlighted by the ECB, the qualitative outcome of the Stress-Test will be included in the determination of the P2R, especially in the element of risk governance; and the quantitative impact of the adverse Stress-Test will be one input factor into determining the level of P2G. Other more qualitative elements (bank specific risk profile, measure taken by the bank, ...) would be also taken account of in the determination of the P2G.



Then we would now have two additional capital positions, one to cover the risks not covered by the Pillar 1 (P2R) and one to cover the risks materialize into a stress scenario (P2G).

## **5.2 The Scenarios of the Stress-Test**

### **5.2.1 The Baseline Scenario**

The baseline scenario is based on the forecasts of the Directorate General for Economic and Financial Affairs (DG ECFIN) of the European Commission [8]. The DG ECFIN produce three scenarios each year, the 2016 EBA baseline scenario for most variables are extracted from the autumn forecasts of European Commission [7]. This publication projects macroeconomic variables for 2016 and 2017. Year 2018 is extrapolated for the needs of the stress test. The projections of the publication are obtained through a judgmental approach while the extrapolation of the thirist year is based on a model based approach. The European Commission justifies the use of technical assumption 2018 with the argument that the error associated with judgmental approach grows with the time horizon of projection. Projections of house prices for the full time horizon (2016/2018) are obtained also by a model based approach. The macroeconomic variables projections combine both judgmental and model based forecasts, the two major types of approaches. The interest rates are extracted by the market quotation, we use, as it's common in pricing, the forward rates to estimate future rates. In other words, we use the risk neutral estimation for the baseline scenario based estimation. This means that in our baseline estimation, we suppose that the non-arbitrage hypothesis is not violated.

### 5.2.2 The Adverse Scenario

As precised in previous sections, adverse scenarios are based on different risks. Here in the EBA 2016 Stress-Test, the ESRB General Board identifies 4 major risks (cf European Commission [7]). This four risks correspond to the most likely adverse event that can affect the stability of the financial system in European Union.

These four risks are the following:

- an abrupt reversal of compressed global risk premia amplified by low secondary market liquidity;
- weak profitability prospects for banks and insurers in a low nominal growth environment, amid incomplete balance sheet adjustments;
- rising of debt sustainability concerns in the public and non-financial private sectors, a mid low nominal growth;
- prospective stress in a rapidly growing shadow banking sector, amplified by spillover and liquidity risk.

The first risk is the main risk of the adverse scenario which affects the economic stability of our financial system. This risk is the source of the others three risk of the scenario. The realization of the first risk set off the others risks. This main risk implies a decreasing demand of long term fixed income instrument and automatically an increase of the long term interest rates as implied by the equilibrium of supply and demand.

We expose on the following the detailed consequences of the different risks as exposed by the EBA explanation paper [7].

- an abrupt reversal of compressed global risk premia amplified by low secondary market liquidity

- Rising long-term interest rates and risk premia in the United States and other non-EU advanced economies
- Global equity price shock
- Increase in the VIX volatility index and spillover to emerging market economies
- Foreign demand shocks in the EU via weaker world trade
- Exchange rate shocks
- Oil and commodity price shocks
- weak profitability prospects for banks and insurers in a low nominal growth environment, amid incomplete balance sheet adjustments
  - Investment and consumption demand shocks in EU countries
  - Residential and commercial property price shocks in EU countries
- rising of debt sustainability concerns in the public and non-financial private sectors, a mid low nominal growth
  - Country-specific shocks to sovereign credit spreads
  - Shocks to corporate credit spreads
- prospective stress in a rapidly growing shadow banking sector, amplified by spillover and liquidity risk
  - EU-wide uniform shock to interbank money market rates
  - Shocks to EU financial asset prices
  - Shocks to financing conditions in EU countries (via shocks to household nominal wealth and user cost of capital)

As we also precised, from these risks the stress-tester have to attribute shocks level. The different shocks are detailed in the EBA note.

Table IV.1: Scenarios of the main economic and financial variable

variables	scenario	2015	2016	2017	2018
CAC 40 variations	adverse		-28%	-26.5%	-17.6%
	baseline		0%	0%	0%
long term french IR	adverse	0.9%	1.6%	2%	2%
	baseline	0.9%	1.1%	1.3%	1.4%
GDP	adverse	1.1%	-0.6%	-1.1%	0.6%
	baseline	1.1%	1.4%	1.7%	1.6%
HICP	adverse	0.1%	-0.5%	0.5%	1%
	baseline	0.1%	0.9%	1.3%	1.6%
EUR 3m	adverse	0%	0.3%	0.3%	0.1%
	baseline	0%	-0.1%	0%	0%
EUR 10Y	adverse	0.9%	1.7%	2%	1.8%
	baseline	0.9%	1.2%	1.5%	1.6%

We expose in Table 1 the scenarios for the main economic and financial variables in the both baseline and adverse scenario.

### 5.3 The Results

An important element in the 2016 stress-test is the absence of a hurdle rates than banks have to meet. The 2016 stress-test does not result natively to a pass-fail outcome. Nevertheless the capital ratio of 5.5% + GSIB is considered as a benchmark for the stressed ratio. Only two banks are below this benchmark ratio: MPS and Barclays. Nevertheless there are specially important banks which are very close to this hurdles (here we consider very close when the difference is near 1%): BNP (stressed ratio:8.51% vs 7.5%) , SG (stressed ratio:7.5% vs 6.5%), HSBC (stressed ratio:8.76% vs 8%).

More globally, the banks passed the test without problems and the market highlights that the stress-test did not really stressed the banks.

We can take a focus on the french banking industry to analyze with more attention the results.

Figure IV.3: Projection of CAC 40

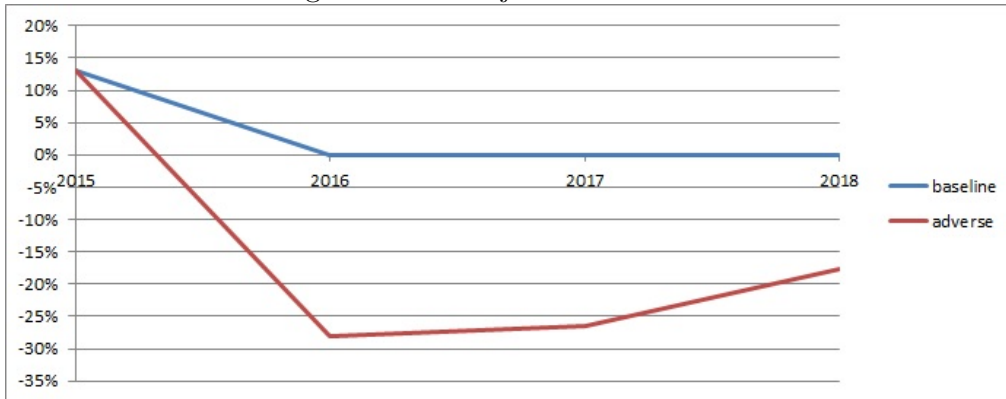


Figure IV.4: Projection of GDP



Figure IV.5: Projection of HIPC

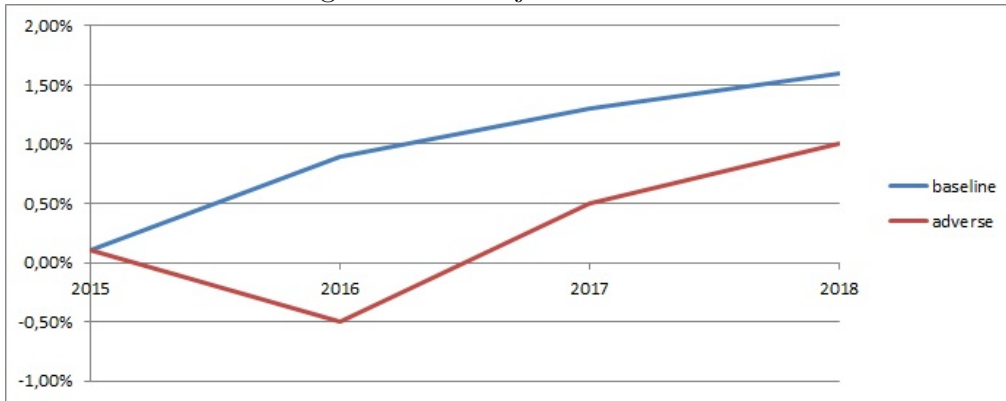


Figure IV.6: Projection of 3 months IR

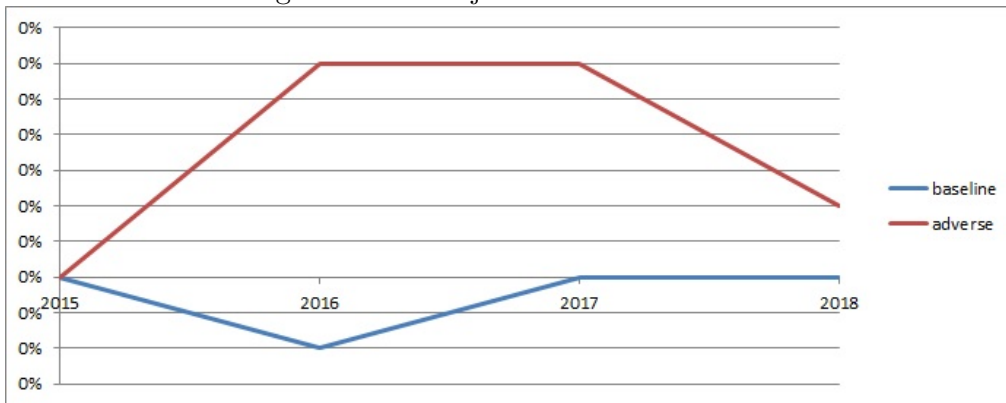


Figure IV.7: Projection of 10 years IR

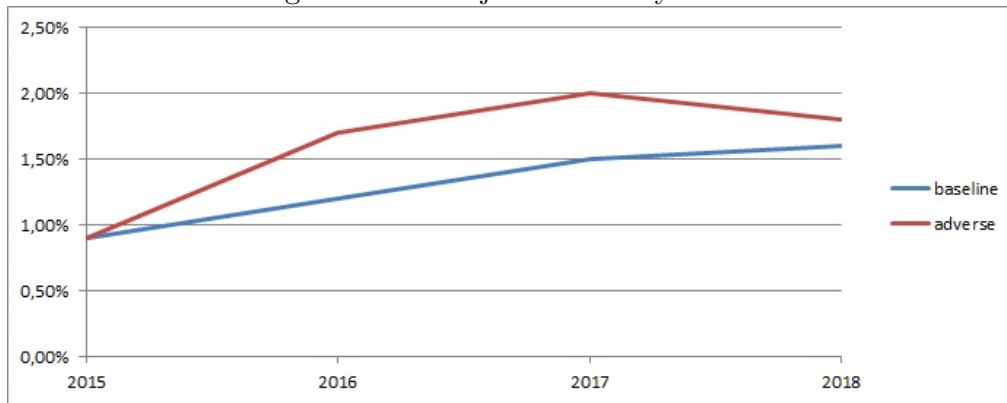


Table IV.2: CET1 Ratio Fully Loaded

Scenario	Banks	12-2015	12-2016	12-2017	12-2018
Adverse	Europe	12.6	9.8	9.3	9.2
Adverse	BNPP	10.9	8.7	8.5	8.5
Adverse	GCA	13.7	11.4	10.9	10.5
Adverse	BPCE	12.8	10.3	9.6	9.5
Adverse	SG	10.9	7.9	7.7	7.5
Adverse	Crédit Mutuel	15.6	13.8	13.6	13.4
Adverse	LBP	14.5	10.6	10.2	9.8
Baseline	Europe	12.6	13.0	13.5	13.8
Baseline	BNPP	10.9	11.3	11.7	12.1
Baseline	GCA	13.7	13.9	14.4	14.8
Baseline	BPCE	12.8	13.3	13.9	14.4
Baseline	SG	10.9	11.0	11.3	11.6
Baseline	Crédit Mutuel	15.6	16.0	16.3	16.6
Baseline	LBP	14.5	14.8	14.9	15.0

Table IV.3: Total operating income

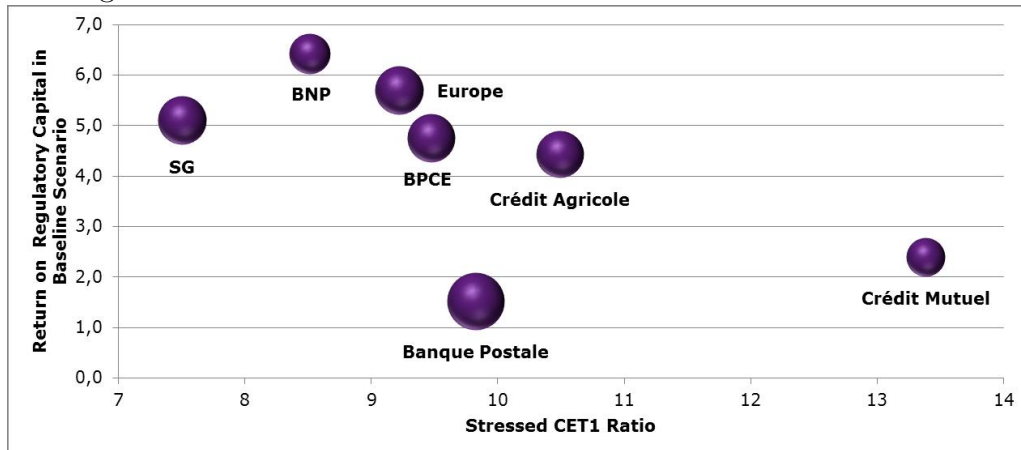
Scenario	Banks	12-2015	12-2016	12-2017	12-2018
Adverse	Europe	587 386	428 100	472 591	462 760
Adverse	BNPP	41 816	31 399	34 309	33 659
Adverse	GCA	29 846	23 271	24 667	23 413
Adverse	BPCE	22 670	14 363	17 220	18 511
Adverse	SG	27 514	17 057	22 098	21 765
Adverse	Crédit Mutuel	14 234	9 700	11 100	10 852
Adverse	LBP	5 705	4 859	5 016	4 787
Baseline	Europe	587 386	542 772	534 451	527 447
Baseline	BNPP	41 816	40 093	39 326	38 970
Baseline	GCA	29 846	27 117	26 449	25 957
Baseline	BPCE	22 670	21 615	21 516	20 988
Baseline	SG	27 514	24 869	24 869	24 869
Baseline	Crédit Mutuel	14 234	12 376	11 837	11 644
Baseline	LBP	5 705	5 218	5 010	4 851

The central variable is the capital ratio as the stress-test is before all a solvability stress-testing. The first Table shows the projection of the fully loaded CET 1 ratio in the baseline and adverse scenario. If we focus on the 4 systematic french banks, we can notice that the non listed banks (GCA and BPCE) have a better solvability ratio compared to the listed banks (BNPP and SG). This is due to the higher level ratio at the starting point for the cooperative banks. It is a fact that capitalistic banks have a more optimized solvability ratio (in other words less high ratio) because of the requirement of the shareholder. This optimization in current economic context gives less pillow for future stressed context. In the cooperative banks, the context is different. The owner of the “capital” are less implicated in the governance of the bank and the cost of the capital is less expensive than in a standard capitalistic structure with market quoted stocks.

Of course the solvability of banks is a key variable and we have to give attention to all these solvability outcomes, however solvability is not the



Figure IV.8: Outcomes for French Banks in 2016 EBA Stress-Test



only important indicator for banks. The profitability is also very important indicator to get a judgment on the wellness of the banks. Table IV.3 exposes the projections of total operating incomes. The evolution of total operating income widely depends on the decomposition between net interest income and fees and commissions. Others sources of revenue are negligible (as net trading incomes). In fact, the evolution of these two type of incomes can be perpendicular. If we take a look at the results of banks during 2015, we can notice the decrease of NII (because of difficult interest rates context) was offset by an increase in commissions.

We notice that even in the baseline scenario, the total incomes of the banks are down, it due to the unfavorable context of the baseline scenario for the NII combined with stress test constraint (no improve of NII is permit).

The Figure IV.8 represents a global vision of the stress test outcome in a profitability/risk logic. The successive crisis forced us to take special attention in risk but today with the deterioration of the incomes banks begin to consider also the profitability as a key resource. We expose the stressed CET1 ratio and the return on regulatory capital in baseline scenario at the end of the stress-test exercise. The size of the bubbles is related to the impact of the

stress on the CET1 (stressed point vs starting point).

We can conduct our analyze by considering 3 buckets of banks.

The first bucket is composed by Crédit Mutuel and Banque Postale. These two banks have low profitability compared to others (french and european banks). Nonetheless, Crédit Mutuel has the advantage to have a very high stressed CET1 ratio, almost 50% higher than the european average.

The two remaining buckets contain the biggest french banks in terms of total asset and also number of clients.

One of this bucket is composed by the non listed banks: Crédit Agricole and BPCE. Their stressed ratios are higher than european average while their profitabilities are less than the european banks. In the bucket, BPCE has a higher profitability than Crédit Agricole ( almost 10%) when Crédit Agricole has a higher stressed CET1 ratio (a little more than 10%).

The last bucket contains the listed banks: BNP and Société Générale (SG). These two banks are on the top left of the european average. The profitability is higher but the stressed ratio is lower. Even if we place these banks in the same bucket, the outcomes of these banks are very different. Indeed, the market considers that BNP showed a good performance while it remarks the low level of stressed ratio of SG. In addition of the higher stressed ratio of BNP, we can notice its higher profitability. It is important to notice for these two banks that the lower level of stressed ratio is caused by the optimized level of starting point. Banks, which obtain better stressed ratio, have a higher level compared to the “optimized” level even if this higher level implies a lower profitability.

The advantage of BNP compared to SG is the low sensitivity of its ratio to the stress combined with a high profitability. The low level of stressed ratio (low compared to the other french banking groups) of BNP does not really worry the analysts thanks to the potential of BNP to improve its ratio.

The opacity of the stress-test makes the understanding of this low sensitivity

for BNP difficult. In fact, it is quite difficult for analysts and others banks to explain this less sensitive scale resource compared to the market. As stress-testing results are the outcome of very long process with a high dimensional data, it is not possible to compare the elements of business model that provide the outcomes.

## 6 Conclusion

We present the context of stress-testing. Indeed after the several crisis periods which affect our economy it became a widely used tool as it tries to capture period of turbulence. It is also based on a forward looking approach where we analyze economic risks that can hypothetical and not necessarily linked to past event. This particularity is important because we mainly observe that different crisis follow each others but they even reflect different economic crisis.

After the detailed presentation of stress-testing, we discuss the modeling hypothesis of stress testing and compare it with those of risk neutral pricing. We highlight the differences, particularly on the absence of probabilistic measure for stress-testing. We notice that non negligible differences can appear between a risk neutral anticipation and a (baseline) scenario based estimation. This difference would signify the presence of arbitrage opportunity but we can argue that the difference come from the difference of aims.

The “best estimation” can depend on its use. When market anticipation is adapted for pricing tradable derivatives because everyone uses this approach, using own anticipation is more adapted to management financial scale resources.

We also point that using scenario based projections for implied volatility and pricing with a risk-neutral model make the precision of the risk neutral model being useless. Risk neutral models are complex and give very precise results.

In the other hand, scenario based projection is very approximative and we are more interested by the magnitude than the exact number. Combining the two methods makes us using very precise tool to obtain and to use just a magnitude.

We end by taking a look at the results of the 2016 EBA stress-test. We notice that in this kind of exercise, we mostly analyze scale resources and not have a precise look on balance sheet elements. We can also notice the importance of stress-testing to benchmark different banks between each others.

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# Conclusion Générale

La thèse explore le sujet des stress-tests des valorisations théoriques de produits optionnels en se plaçant sous différents angles de vue. Néanmoins le cadre général reste celui de la gestion de bilans des institutions financières. C'est en effet là que l'étude des stress-tests de produits optionnels prend son importance comme outil de gestion stratégique du risque mais également de la performance. La thèse aborde ce vaste sujet en partant du niveau le plus granulaire pour arriver sur une vision grand angle des problématiques. La thèse est constituée de trois parties majeures qui sont les chapitres 2 à 4.

Après un chapitre introductif, le chapitre 2 nous permet de rentrer dans les détails de la valorisation de produits de taux et crédits présentant une option de remboursements anticipés. Ces produits sont très fréquents notamment aux passifs des institutions sous forme d'émission long terme. En effet, ce risque de pré-paiement n'est pas couvert et apporte un problème de liquidité dans le bilan de l'institution. Nous proposons notamment une méthode d'interpolation endogène à la modélisation des taux forward d'un Libor Market Model et la comparons à une méthode d'interpolation ex ante exogène linéaire sur les volatilités implicites de taux de swap. Nous observons que cette méthode permet d'obtenir de meilleurs résultats en terme de minimisation de la distance entre volatilité de taux de swap cotée et volatilité de taux de swap modélisée. Nous examinons également le cas d'un titre émis en une devise étrangère qui serait couvert par un swap de devise annulable à nominal croissant. Nous constatons notamment que ce produit est impacté par les trois marchés d'option du fait de la présence d'un spread de crédit libellé en devise locale. En effet, en l'absence de ce spread, la propriété "cross devise" du produit disparaît pour laisser place à une option en devise étrangère. Nous étudions la sensibilité de notre produit aux différents marchés d'option sous l'hypothèse de différentes courbes de spread. Nous constatons que le

marché des volatilité de swaptions en devise locale a un plus grand impact sur la valorisation par rapport au volatilité du taux de change.

Dans le chapitre 3, nous nous intéressons à la projection des volatilité implicites dans le cadre de stress-tests ou plus généralement lors d'étude d'impact par scénarios. Nous suggérons une méthode qui permet d'étendre les scénarios des variables de taux d'intérêt vers les volatilité implicites. Pour cela, nous sélectionnons certains points de la courbe comme variables explicatives alors que les variables expliquées sont représentées par un nombre limité de valeurs propres / facteurs (expliquants une grande partie de la variabilité de la donnée). Nos variables expliquées étant des variables cachées non directement observables, nous appliquons les filtres de Kalman afin d'obtenir une estimation moins bruitée. Dans notre cas, l'application du filtre de Kalman nous permet d'estimer les variables cachées par les valeurs prédites et non plus filtrées.

Nous comparons les résultats en appliquant lors d'un back-test nos différentes méthodes à des données historiques. Les résultats montrent que l'ACP et les modèles à facteurs donnent des résultats très proches si nous n'appliquons pas les filtres de Kalman, l'ACP donnant toutefois des résultats très légèrement plus performants. Ils réagissent par contre différemment à la méthode de nettoyage proposée. Nous remarquons notamment que lorsque le nombre d'observations dans l'ACP est faible, environ 300, la méthode dégrade énormément la qualité de la prévision. Ceci met en lumière les différences entre ces deux modèles qui a priori pouvaient sembler très proches. Après une étude des différentes configurations de modélisation, nous en déduisons que la méthode de nettoyage donne de bons résultats et améliore nos prévisions lorsque la période ne contient pas de changement de régime. Nous observons en effet qu'après chaque choc macroéconomique perturbant les relations entre variables, les résultats provenant du modèle avec la méthode du filtre de Kalman sont dégradés par rapport au modèle sans le filtre de Kalman. La méthode est efficace si nous avons assez de données dans notre ACP et que



ces données représentent un cycle macroéconomique sans choc. La dernière partie du chapitre applique la modèle de projection à la valorisation d'une swaption bermudienne.

Le dernier chapitre permet de contextualiser les chapitres précédents en définissant et illustrant les stress-tests dans le secteur financier. Nous détaillons notamment notre compréhension des hypothèses de modélisation qui sont derrière les stress-tests. Nous menons également une réflexion sur la compatibilité des stress de paramètres de marché et de la valorisation des produits structurés. La dernière partie du chapitre expose en détail les stress-tests réglementaires de 2016. Nous parcourons cet exercice de la conception des scénarios jusqu'aux résultats.

La thèse analyse sous différents angles les stress-tests comme outil de pilotage et gestion de risque structurel des institutions financières. Notre analyse part du niveau produit et va jusqu'à l'analyse de ces stress-tests au niveau microprudentiel. Un prolongement de la thèse consiste à étendre l'analyse au niveau macroprudentiel afin d'analyser les effets sur toute une économie. En effet, les bilans bancaires contiennent un niveau non négligeable de passif avec option de remboursement par anticipation; et c'est par ce biais que les projections de volatilité implicite impactent les indicateurs globaux de supervision macroprudentielle. Aujourd'hui, les notes méthodologiques de l'EBA préconisent des raccourcis simplificatrices. Les options à la main des contreparties sont supposées être exécutées à la première date d'option alors que les options à la main de la banque ne sont pas exécutées. Une analyse plus globale à l'échelle européenne apporterait une vision plus panoramique des impacts de projections des volatilités implicites permettant de valoriser les produits structurés et *de facto* de quantifier la liquidité au plus juste dans le scénario de stress.