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Modeling and observation applied to physiology-aware control for cycling

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THÈSE

pour obtenir le grade de

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UNIVERSITÉ GRENOBLE ALPES**

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Présentée par

Nadia Paola ROSERO IBARRA

Thèse dirigée par **John Jairo MARTINEZ MOLINA**

préparée au sein du **GIPSA-Lab**
dans l'**École Doctorale EEATS**

Modeling and observation applied to physiology-aware control for cycling

Thèse soutenue publiquement le **12 novembre 2018**,
devant le jury composé de :

M. Sorin OLARU

Professeur, CentraleSupélec, Rapporteur

M. Jimmy LAUBER

Professeur, Univ. de Valenciennes, Rapporteur

M. Nacim RAMDANI

Professeur, Univ. d'Orléans, Examineur

M. Christophe BERENGUER

Professeur, Grenoble INP, Président

M. John Jairo MARTINEZ MOLINA

Maître de conférences, HDR, Grenoble INP, Directeur de thèse

M. Matteo CORNO

Professeur associé, Politecnico di Milano, Invité

M. Johan CASSIRAME

Directeur Commercial, Societé Matsport, Invité



to Jacobo

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Notations

Mathematical notations

\mathbb{R}	Real values set
M^T	Transpose of matrix $M \in \mathbb{R}$
M^*	Conjugate of matrix $M \in \mathbb{C}$
$M = M^T$	Real symmetric matrix M
$M \succ (\succeq) 0$	M is symmetric and positive definite (semi definite) matrix
$M \prec (\preceq) 0$	M is symmetric and negative definite (semi definite) matrix
$\ \cdot\ _2$	Norm 2
$\ \cdot\ _\infty$	Norm ∞

Acronyms related to control theory

LMI	Linear Matrix Inequality
LPV	Linear Parameter Varying
LQR	Linear Quadratic Regulator
MPC	Model Predictive Control
PID	Proportional Integral Derivative
RPI	Robust Positive Invariant
SMC	Sliding Mode Control

Acronyms related to Physiology

AT	Anaerobic Threshold
ADP	Adenosine Diphosphate
ATP	Adenosine Triphosphate
bpm	beats per minute
CO_2	Carbon Dioxide
CP	Critical Power
EEG	Electroencephalogram
EPOC	Excess Post-Exercise Oxygen Consumption
H^+	Hydrogen Ion
HR	Heart Rate
HRR	Heart Rate Reserve
<i>La</i>	Lactate
LT	Lactate Threshold
MAOD	Maximal Accumulated Oxygen Deficit
MVC	Maximum Voluntary Contraction
NAD	Nicotinamide Adenine Dinucleotide
O_2	Oxygen
PCr	Phosphocreatine
RC	Respiratory Compensation
RER	Respiratory Exchange Ratio
RQ	Respiratory Quotient
RPE	Rating of Perceived Exertion
VO_{2max}	Maximal Oxygen uptake

Other acronyms

CVT	Continuously Varying Transmission
GPS	Global Positioning System
ICT	Incremental Cycling Test
PAS	Pedal Assist System

Modeling and observation applied to physiology-aware control for cycling

Abstract — Nowadays, most of the electrical assistances for bicycles are mainly based on measurements of mechanical variables. In this dissertation, cyclist and bicycle are considered as a whole energetic system in order to conceive physiology-aware control strategies. In this sense, the relationships between physiological indices and mechanical variables are studied. In particular, the gas exchange dynamics during cycling is modeled and analysed. The obtained model is discrete-time and linear parameter varying. Measured data of different cycling tests performed in GIPSA-Lab are used for its calibration. A methodology for parametric identification is developed as a solution of a sequence of non-linear unconstrained optimization problems. The gas exchange dynamical model allows the determination of excess carbon dioxide production, which is considered as an unsuitable physiological-state of the cyclist. Furthermore, a set-membership observer of carbon dioxide production is implemented using measurements of oxygen consumption and pedal power. It provides a deterministic interval which contains the real state. Finally, this thesis presents a method for finding sufficient conditions on the pedal power that guarantee admissible levels of carbon dioxide production.

Keywords: Physiological models, cycling, set-invariance approach, parametric identification, optimization.

GIPSA-Lab, 11 Rue des Mathématiques
Grenoble Campus, F-38402 Saint Martin d'Herès

Modélisation et observation pour la commande des variables physiologiques du cycliste

Résumé — De nos jours, la plupart des assistances électriques pour vélos sont principalement basées sur des mesures de variables mécaniques. Cette thèse se concentre sur l'analyse du cycliste et du vélo dans son ensemble, afin de concevoir une loi de commande qui prenne en compte des contraintes physiologiques. En ce sens, la relation entre les indices physiologiques et les variables mécaniques est abordée. En particulier, la dynamique d'échange de gaz pendant le pédalage est modélisée et analysée. Le modèle obtenu est linéaire à paramètres variables en temps discret. Les données mesurées de différents tests de cyclisme effectués au sein du GIPSA-Lab sont utilisées pour son calibration. La méthodologie d'identification paramétrique est développée comme solution à une séquence de problèmes d'optimisation non linéaire et non contraints. Le modèle dynamique d'échange de gaz fait référence à la détermination de la production excessive de dioxyde de carbone, qui est considérée comme un état physiologique indésirable du cycliste. En outre, un observateur de la production de dioxyde de carbone est mis en oeuvre en utilisant des mesures de la consommation d'oxygène et de la puissance de pédalage, ce qui fournit un intervalle déterministe contenant l'état réel. Enfin, une méthode est présentée pour trouver des conditions suffisantes sur la puissance de pédalage qui garantissent des niveaux admissibles de production de dioxyde de carbone.

Mots clés: modèle physiologique, cyclisme, ensembles invariants, identification paramétrique, optimisation.

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Main Introduction

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I.1 General context of the thesis

This thesis was partially supported by the Colombian government through a scholarship offered by COLCIENCIAS¹ and dedicated to developing doctoral studies abroad.

This thesis was developed between March 2015 and June 2018, within the SLR² research team at the Control Systems Department of GIPSA-Lab³, which is a joint research unit of CNRS⁴, Grenoble-INP⁵, and the *Université Grenoble Alpes*⁶. The major of the thesis is Automatic Control and Production Systems⁷, which belongs to the doctoral school EEATS⁸ of the *Université Grenoble Alpes*.

The following topics are part of the general framework of this thesis:

- Modeling of physiological systems oriented to control design processes.
- System identification and optimization for model parametrization using experimental data from different cycling scenarios.
- Use of optimal state estimation principles for reducing the number of measurements needed to assess the physiological state of cyclist.
- Use of Robust Positive Invariant Sets for computing the bounds of mechanical power required to avoid an inefficient cyclist physiological state.

¹ *Departamento Administrativo de Ciencia, Tecnología e Innovación* (Administrative Department of Science, Technology and Innovation), Colombia.

² *Systèmes Linéaires et Robustesse* (Linear Systems and Robustness)

³ *Grenoble Images Parole Signal Automatique* (Grenoble Images Speech Signal and Control Laboratory)

⁴ *Centre National de la Recherche Scientifique* (National Center for Scientific Research), France.

⁵ *Institut National Polytechnique de Grenoble* (Grenoble Institute of Technology)

⁶ *Université Grenoble Alpes*

⁷ *Automatique-Productique (AP)*

⁸ *Ecole Doctorale Electronique, Electrotechnique, Automatique et Traitement du Signal*

During the development of this project several multidisciplinary collaborations with other institutions were established. These institutions are:

- *Universidad de la Sabana*⁹. Bogotá, Colombia. Collaboration with Henry León for the analysis of human physiology and for the theoretical validation of the conceptual bio-energetic model.
- *Politecnico di Milano*¹⁰. Milan, Italy. Collaboration with Matteo Corno for the analysis of data from cycling scenarios and the parametrization of a gas exchange dynamical model.
- *Société Matsport*¹¹. Saint Ismier, France. Collaboration with Johan Cassirame for the physiological analysis of the gas exchange model and the use of professional medical equipment for cyclist measurements.

In addition, a student of the program of Bio-informatics Engineering of the *Université de Poitiers*¹² completed an internship in the analysis of ventilatory thresholds, and their relationships with mechanical variables.

I.2 Problem statement and motivation

The object of study of this thesis is the human-bicycle energetic system. The term bicycle in this document implies either of both, bicycles propelled only by human force or electric bicycles, used to describe electric-motor-powered bicycles. Then, assuming the human-bicycle system as a whole system, it is possible to distinguish three main components: human, bicycle, and electrical assistance (motor and battery pack). Human and battery are sources of energy. The human is capable of storing and transforming energy acquired as food and oxygen and then transform it into muscular movements which propel the pedals. In the electrical assistance case, a battery pack also stores energy and then transmits electricity to the motor, which generates bike movement. When these sources of energy interact with the bicycle, both power flows act in parallel upon the bicycle movement as shown in Fig.I.1.

The cyclist has an independent behavior. He could act like a controller, in the sense that he perceives bicycle speed through his senses and then he applies a force with his legs to the pedals in order to follow a speed reference, which is in his mind. The rate and amount of power produced by his legs are constrained by his physical, biomechanical and psychological characteristics.

On the other hand, the electrical motor acts according to a control law, which depends on the type of the electrical bicycle and the regulation of the country.

Regarding the simplest case, where the use of an electric bicycle is allowed even without pedaling, the rider can choose to rely on the motor completely, pedal and use the motor at the same time or pedal only (case of a conventional bicycle). For this kind of electrical bicycles,

⁹ *Universidad de la Sabana, Facultad de Medicina.*

¹⁰ *Politecnico di Milano, Dipartimento di Elettronica, Informazione e Bioingegneria.*

¹¹ *Société Matsport.*

¹² *Université de Poitiers, Faculté de Sciences Fondamentales et appliquées, Licence Génie Bio-Informatique.*

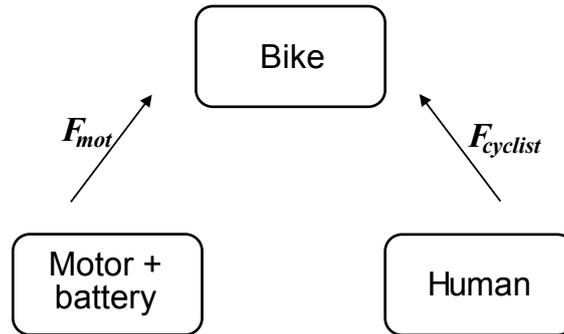


Figure I.1: Power flow in a human-bicycle system

the motor action typically relies on an algorithm that measures or estimates the bicycle speed with the motor speed and then calculates a rotational speed appropriate for following a speed reference assumed as a constant. In this case information exchange between components can be represented as the diagram in Fig.I.2.

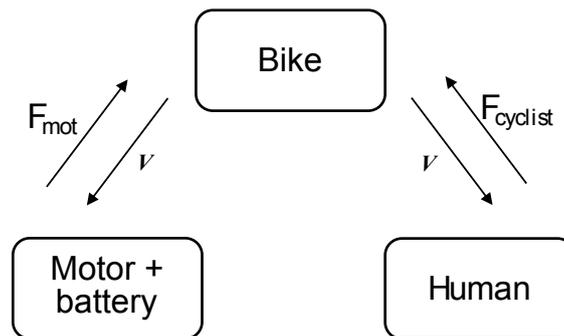


Figure I.2: Information flow in a human-bicycle system

Usually, the control law of the motor does not take into account the information concerning the human contribution to the system. In this regard, the information flow occurs between the sources of energy and the bike and not between the human and the motor controller.

Regarding pedelecs case, which have an electronic controller which cuts off the power supply to the motor when the rider is not pedaling or when a certain speed (usually 25 km/h) is reached, the information flow admits one of the following possibilities:

- Velocity and pedaling status (on/off): minimal information that must be measured or estimated for fulfilling pedelec regulation. It only includes a sensor of bicycle velocity for limiting speed, and detects if the rider is pedaling or not. This system does not include a control law for motor activation. It acts as a throttle. The cyclist selects the required level of assistance using a lever or a button.
- Cadence sensor: mainly consists of a magnet attached to the pedals and a sensor which picks up the movement of the magnet as the rider is pedaling. The motor power is

inversely proportional to the cadence.

- Torque sensor. measures the applied pedal torque and activates the motor power based on pedal pressure.

In the best scenario when all information above is used for conceiving a sophisticated control law, there is no coordination between both energy sources. This generates the following disadvantages:

- The sensation of comfort is not good enough. Since the control law of each source (human and electrical assistance) has different control objectives, sometimes the motor torque can act against the desires or needs of rider.
- The battery power delivery towards the bicycle is not consistent with cyclist power delivery. This is because the cyclist usually has more available energy at the beginning of a race and assistance torque is possibly not required at this time.
- In case of sport training, the assistance hinder physical improvement by limiting the required cyclist effort.

Lack of information exchange between human and control algorithm of electrical assistance is the main identified problem. Therefore, the present research focuses on the conception of an electrical assistance for bicycles adapted to the physiological needs of the cyclist. For this purpose, three main aspects are taken into account:

1. Modeling problems: physiological analysis of the cyclist is addressed from an engineering perspective aiming at obtaining a reliable model for a future physiology-aware control strategy for the electrical assistance.
2. Estimation problems: reducing the number of sensors used for evaluating the cyclist physiological state is an important issue in implementation. Here, the estimation of carbon dioxide production during cycling is tackled.
3. Control problems: analysis of energetic and gas exchange models of the system is addressed using a set-invariance approach, which in turn allows the proposition of a bounded power that fulfills physiological and mechanical objectives at the same time.

In a general context, topics discussed in this thesis could bring forward some positive and sustainable changes for the benefit of society in the following senses:

- Help people with physical disabilities providing them an electrical assistance adapted to their special needs.
- Increase the level of comfort experienced while riding a bike. The sensation of riding a bicycle without assistance must be achieved. The rider must neither notice the presence of electrical assistance nor feel it is acting against his desires. Consequently, he can feel himself stronger.
- Contribute to the promotion of bicycles as eco-friendly mean of transportation.
- Improve energetic efficiency of bicycles by only activating the electrical assistance when the cyclist really needs it.

I.3 Thesis structure

This thesis is organized in seven chapters:

Chapter I *Main Introduction*

The purpose of this chapter is to put into context the main topics related to this thesis. A recall of the main contributions is included as well as the structure of the dissertation and the main challenges considered.

Chapter II *State of the art of the human-bicycle system*

Provides a general framework about the main stages of the human-bicycle system and presents the main motivations for addressing this topic. It describes the basics of physiology, which discusses how the human body transforms chemical energy from food and oxygen into mechanical energy. Furthermore, it lists some important concepts and describes how related works have dealt with similar topics, mainly from physiological and engineering points of view.

Chapter III *Conceptual bio-energetic model of human-bicycle system*

A holistic approach of the human-bicycle system is presented in this chapter. It consists of a conceptual model based on physical laws that describe the dynamics of equivalent hydraulic and electro-mechanical systems. The model reproduces main dynamics of energy transformation during a bike ride at three different levels: metabolic, bio-mechanical and pure-mechanical level. The simulation results are presented and analyzed.

Chapter IV *Gas exchange model during cycling*

Explains a discrete-time linear parametric varying model, which is proposed to reproduce the gas exchange dynamics during cycling. The model relates physiological measurements as oxygen consumption and carbon dioxide production with a mechanical measurement, the power at pedal level. The methodology for parametric identification is developed as a solution of a sequence of non-linear unconstrained optimization problems. The experimental setup and data used for model identification and validation are respectively located and generated in GIPSA-Lab. The analysis and validation of experimental results are also presented for several subjects and scenarios.

Chapter V: *Robust estimation of carbon dioxide production*

Explains the design process and results about the observer of carbon dioxide production. It uses set-membership approach, which allows the estimation of states and moreover provides a deterministic computation for the error bounds. This contribution reduces the amount of sensors used for determining the physiological state of the cyclist. Simulation results together with measured data from tests in cyclo-ergometer are presented to illustrate the approach.

Chapter VI: *Control approach for physiology-aware assistance*

Presents a method for finding the maximal mechanical power that can be developed for a cyclist without excess of carbon dioxide production. Ellipsoidal invariant sets are used to characterize all possible trajectories of the system state in the presence of a bounded pedal power. The problem is solved as a geometrical optimization problem

involving an inequality constraint concerning measurements of produced carbon dioxide and consumed oxygen. The chapter also includes a numerical example and formulates the basis for control design of physiology-aware assistance.

Chapter VII: *Conclusions and Perspectives*

Aims to summarize this thesis contribution and to discuss further challenges arising from this work, in particular concerning the control strategies for physiology-aware electrical assistance for bicycles.

I.4 Main contributions

The contributions of this thesis can be summarized as follows:

- A conceptual bio-energetic model of the human-bicycle system. The model reproduces dynamical behavior of main variables involved during cycling. The modeling approach considers three main levels: i) physiological, ii) bio-mechanical and iii) pure-mechanical. The analysis of the model includes the meaning of the parameters, which are related to the physiological characteristics of the rider or mechanical characteristics of the bicycle. It also includes a pedagogical explanation about the dynamics of energy transformation processes during cycling. The model allows a better understanding of the bioenergetics of cycling performance. This contribution is presented in Chapter III.
- A discrete-time dynamical model of the gas exchange dynamics during cycling. The model reproduces dynamical behavior of oxygen consumption and carbon dioxide production using the measurement of power at pedal level as input. The prediction of carbon dioxide excess is also possible, which allows to know the physiological state of the cyclist. The difference between masses per unit time of oxygen and carbon dioxide is proposed for classifying data from any cycling scenario. The methodology for identification and validation of the model is done through a sequence of optimization problems, which makes it usable for obtaining a suitable model for any cyclist. The model is linear and has a varying parameter in the output matrix. This model can be used for enhancing the control design process of electrical assistance in bicycles. This contribution is presented in Chapter IV.
- A set-membership observer of the carbon dioxide production. The observer uses measurements of oxygen consumption and power at pedal level together with the discrete-time model of gas exchange dynamics proposed in Chapter IV. The set-membership approach provides a deterministic interval which contains the real state. Since the excess of carbon dioxide production is linked to the use of the anaerobic pathway, the observer can be applied to predict the physiological state of the cyclist by using a few number of sensors. This contribution is presented in Chapter V.
- A method for finding the sufficient conditions on the pedal power that guarantee admissible levels of carbon dioxide production. The invariant-sets approach allows the characterization of all possible trajectories of the system states subjects to a bounded input, the pedal power in this case. A geometrical optimization problem involving an

inequality constraint is solved to calculate an upper bound for the pedal power. It constitutes an advance in the search of a control law that includes physiological constraints related to gas measurements during cycling. This contribution is presented in Chapter VI.

Some results of the present thesis have already been published, and other ones are currently being edited by the authors for future submissions.

Journal papers

- J1** [Rosero 2019b] Nadia Rosero, John J. Martinez, Matteo Corno, Johan Cassirame. *Modeling and parameter identification of gas exchange dynamics during cycling* under review in Annual Reviews in Control
- J2** [Rosero 2019a] Nadia Rosero, John J. Martinez. *Estimation of carbon-dioxide production during cycling by using a set-membership observer approach* in preparation.

International conference papers

- C1** [Rosero 2017] Nadia Rosero, John J. Martinez, Henry Leon. *A bio-energetic model of cyclist for enhancing pedelec systems*, in preprints 20th IFAC World Congress, July 2017, Toulouse, France.
- C2** [Rosero 2018b] Nadia Rosero, John J. Martinez, Matteo Corno. *Modeling of gas exchange dynamics using cycle-ergometer tests*, in proceedings 9th Vienna International Conference on Mathematical Modelling (Mathmod), February 2018, Vienna, Austria.
- C3** [Rosero 2018a] Nadia Rosero, John J. Martinez. *An invariant-set approach for constraining gas exchange dynamics during cycling*, in proceedings 10th IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes (SAFEPROCESS), August 2018, Warsaw, Poland.

State of the art of the human-bicycle system

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II.1 Introduction

In this chapter is highlighted the relevance on bicycle research as a solution of current social and environmental problems. The analysis of bicycle as a human-powered vehicle offers a wide range of challenges for improvements that can be applied in other means of transportation. Therefore, main motivations of the thesis subject are listed in section II.2.

Then, the principles of physiology during exercise are explained in section II.3. A focus on gas exchange is given because of its importance in the next chapters. The human action in mechanical variables of bicycle movement is also analyzed.

Some models of human-bicycle system are briefly listed in section II.4. Despite the large amount of models present in the literature, here, control-oriented models and some others from physiology domain are described because of their relevance. Finally, a summary about related

bicycle research and physiological and mechanical measurements that have been applied for bicycle improvements is added in Section II.5.

The objective of this chapter is to provide a basic knowledge about the context of the thesis and glimpse of the position of this work in the scientific field.

II.2 Potential of human-powered transportation

The bicycle has recently acquired significant popularity since its usage to reduce oil-dependence. For this study, the understanding of cyclist and bicycle as a whole system facilitates for the comprehension of its variables and their dynamical behavior.

The analysis of human-bicycle system has been addressed from the perspective of different scientific fields. For physiologists, the comprehension of chemical reactions which transform nutriments and oxygen to movements at the muscular level is a notable point. For sport scientists, the main objective is to improve the mechanical performance during cycling [Abbiss 2005, Noakes 2000]. From mechanics and bio-mechanics, several works [Schwab 2013, Meijaard 2007] provide models for balance and mechanical control.

In the historical context, the beginning of bicycles is documented around 1815 [Limebeer 2006, Wilson 2004], when the german Baron Karl von Drais invented the running machine or draisine. The scenario of this invention was a difficult one: in 1815 the Indonesian volcano Tambora exploded, expelling the greatest known mass of dust in the atmosphere, thus making 1816 the year without summer in Central Europe and New England states. Starvation was widespread, and horses were killed for lack of fodder. The above circumstances and probably also the idea of skating without ice, made possibly one of the greatest inventions in transportation.

Nowadays, we are not facing the same problems of those days, however there is an increase in the research of human-powered transportation, mainly because it is seen as a good solution for actual problems in transportation such:

- Increasing fossil fuels cost
- Pollution produced by cars and motorcycles
- Space occupied by cars
- Traffic jams in big cities
- Health of population which do not practice any sport
- Deficiency in public transportation
- Oversized relation motor/weight in cars

Also, the bicycle industry has been seen as a good opportunity for obtaining economical income. Therefore, several well-known enterprises such as Audi, Volkswagen, Bosch, Mercedes Benz, Renault, Peugeot, among others, are actually investing in research, improving and invention of new commercial products in this field.

The field of assisted bicycles has evolved in the last years, giving rise to new vocabulary for classifying the types of bicycles today. According to [Parker 2004, Muetze 2007], some used terms are:

- *PAB*. General term for all Power assisted bicycles.
- *IC-PAB*. Early model PAB with a polluting two stroke internal combustion engine.
- *E-PAB*. Power assisted bicycle with a rechargeable battery and an electric motor with a maximum power output of 250 watts.
- *E-bike*. Electric bicycle with a motor rated at around 400 to 500 watts that does not need to be pedaled except on hills and against heavy winds.
- *Pedelec*. Bicycle with torque assistance through an electric motor, considerably lower than the pedaling effort. Motor input is only activated while pedaling and provide a maximum power of 250 W.
- *Intelligent passive bike*. It does not directly provide assisting traction power, but rather modifies the vehicle response according to the cyclist input. An example of such systems is automated gear shifting.

Since the introduction of the term pedelec (pedal electric cycle) in February 1999, there are a lot of people among manufacturers, dealers, scientists and everyday people, that are using this word for talking about vehicles that support the rider with electric power only while he is pedaling.

II.3 Physiological aspects during cycling

The analysis of human and bicycle as a whole system, implies necessarily the study of human physiology to understand what are the processes that take place within the body during exercise. This is a vast amount of knowledge to be covered. Thus, in this manuscript the most relevant information about main subsystems that intervene in energy transformation is listed. Also, the essentials of Physiology are briefly explained using basic literature [McArdle 2006, Kenney 2015, Hale 2005] about concepts related to this research.

II.3.1 Physiology definitions

Some important concepts of Physiology are defined in the following lines. They are used throughout the document and a basic knowledge of them provide a better understanding of the context of the thesis.

- Anaerobic threshold (AT). It is defined as the level of exercise intensity at which lactic acid builds up in the body faster than it can be cleared away. For this reason, it is also sometimes called the lactate threshold or lactate turnpoint (LT).

- Anaerobic capacity. It is defined as the maximal amount of adenosine triphosphate (ATP) that can be resynthesized by anaerobic metabolism; that is, mainly phosphocreatine (PCr) hydrolysis and glycolysis. The gold standard to determine anaerobic capacity has been the maximal accumulated oxygen deficit (MAOD).
- Excess Post-Exercise Oxygen Consumption (EPOC). Also known as oxygen debt, EPOC is the amount of oxygen required to restore your body to its normal, resting level of metabolic function (called homeostasis).
- Fatigue. Sensation of tiredness. During cycling it could be considered as the inability to maintain the required force output to continue pedaling. Main causes of fatigue are:
 - Decreased rate of energy delivery (ATP-PCr, anaerobic glycolysis and oxidative metabolism)
 - Accumulation of metabolic by products, such as lactate and Hydrogen ions H^+
 - Failure of the muscle fibers for the contractile mechanism
 - Alterations in neural control of muscle contraction
 Mechanisms of fatigue depend on the type and intensity of the exercise, the fiber type of the involved muscles, the subjects training status, and even his or her diet.
- Maximal Oxygen uptake (VO_{2max}). Is the highest value for oxygen uptake, which can be attained and measured during an incremental exercise protocol for a specific exercise mode. Attainment of VO_{2max} generally necessitates the use of large muscle groups over 5-15 min, so-called aerobic exercise. Hence, VO_{2max} is also called aerobic capacity.
- Maximal Power Output (W_{max}). W_{max} is determined during an incremental cycling test comprising at least 3-5 min stages with 25-50 W workload increments. The equation for calculating this power is:

$$W_{max} = W_{final} + \frac{t}{TW} \quad (\text{II.1})$$

Where W_{final} is the last completed stage, W is workload increment (W), t is the time spent in the final (uncompleted) stage, and T is the time (s) of the stage duration [Jeukendrup 2000].

- Ventilatory Threshold. During submaximal exercise, ventilation increases linearly with oxygen consumption and carbon dioxide production. When the exercise intensity increases, the frequency of breathing becomes more pronounced and minute ventilation ($\dot{V}E$) rises disproportionately to the increase in oxygen consumption. This disproportionate rise in breathing rate represents a state of ventilation that is no longer directly linked with oxygen demand at the cellular level and is generally termed the ventilatory threshold.

II.3.2 Physiological systems involved in cycling

Our body is a complex system composed of many subsystems: digestive, circulatory, respiratory, muscular, etc. When we are performing a physical activity, many of them act in a particular task. Several works have addressed the study of subsystems involved in exercise.

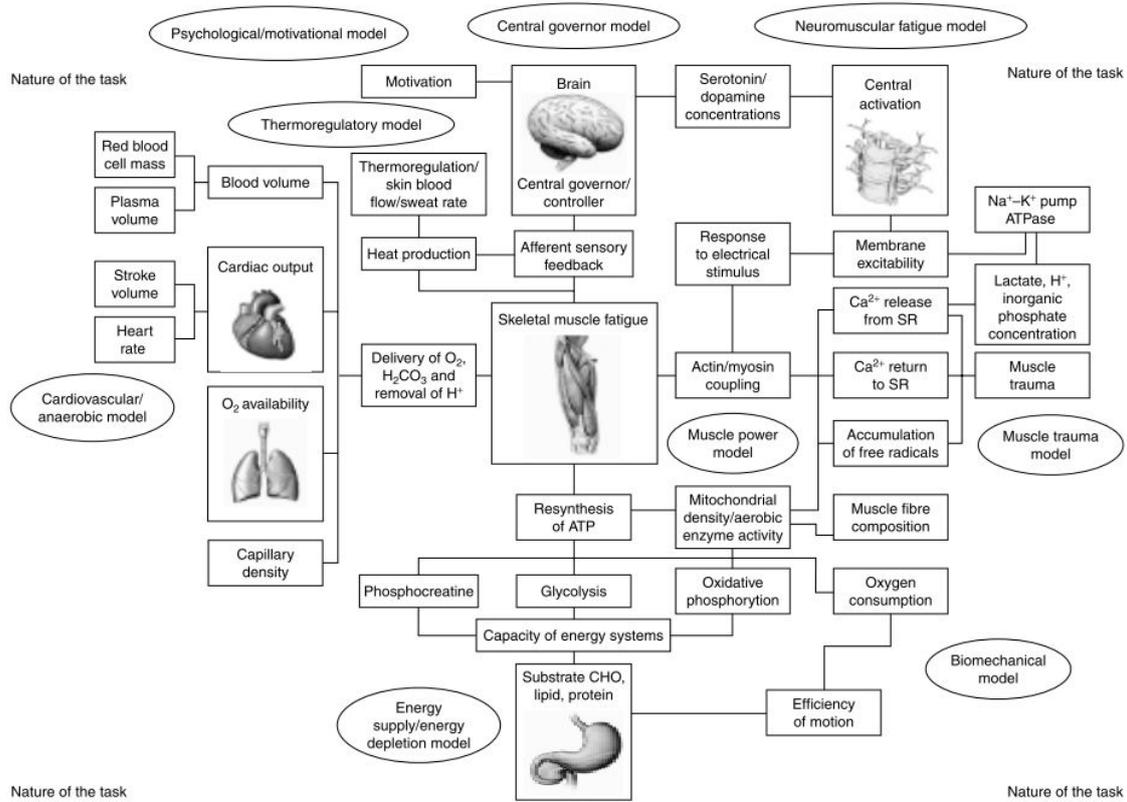


Figure II.1: Physiological systems involved on fatigue applied to prologued cycling, [Abbiss 2005]

In [Abbiss 2005], the cycling performance is explained in function of numerous physiological, biomechanical, environmental, mechanical and psychological factors.

Fig. II.1 shows a scheme of the main approaches used for analyzing the subsystems involved in cycling performance. These include: (i) cardiovascular/anaerobic; (ii) energy supply/energy depletion; (iii) neuro-muscular fatigue; (iv) muscle trauma; (v) bio-mechanical; (vi) thermo-regulatory; (vii) psychological/motivational; and (viii) central governor. In this document, only most relevant of them are described in the follows.

Cardiovascular/Anaerobic Model. During cycling dilatation of the blood vessels of active muscles increases the vascular area for blood flow. The alternate rhythmic contraction and relaxation of skeletal muscles forces blood through the vessels and returns it to the heart. Increased blood flow during moderate exercise increases systolic pressure in the first few minutes; diastolic pressure remains relatively unchanged.

The cardiovascular/anaerobic model of fatigue states that fatigue occurs during cycling when the heart is no longer able to supply oxygen and remove waste products to and from the working muscles.

Energy Supply/Energy Depletion Model The energy supply model proposes that fatigue during cycling is a direct consequence of a failure to supply sufficient ATP via the various metabolic pathways (phosphocreatine re-phosphorylation, glycolysis, lipolysis) to the

working muscles

Neuromuscular fatigue model This approach states that the functions involved in muscle excitation, recruitment and contraction are what limit exercise performance. Electromyography (EMG) is commonly used as a global indicator of these factors.

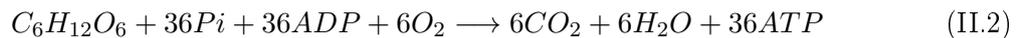
Other models for explaining fatigue during cycling as the bio-mechanical, the thermo-regulatory and the psychological constitute interesting subjects of research; however, they require the analysis of variables difficult to measure. These kind of approaches are beyond of the scope of this thesis. In this work, we focus on the analysis of the gas exchange measurements, which are well-known for being the most efficient way to assess the physiological state during the exercise [Maud 2006].

II.3.3 Gas exchange during cycling

The energy to perform a physical activity by humans, is obtained from a series of chemical reactions that transforms the nutrients from food into the energy currency of muscles which is the Adenosine Triphosphate (ATP).

There are mainly three ways to synthesize ATP: i) aerobic, ii) alactic anaerobic and iii) lactic anaerobic. The first is the most efficient way. However, as it requires an adequate supply of oxygen to the muscles, this pathway activates slowly. During the transient required to adapt the respiratory system, energy is produced through the alactic anaerobic pathway. This pathway use molecules of Phosphocreatine (PCr) and Adenosine Diphosphate (ADP) to synthesize ATP. For example, during moderate and prolonged exercise, energy comes mainly through the aerobic pathway, using firstly glycogen stored in active muscles and liver glycogen, which provides almost half of the energy requirement, secondly fat breakdown with minimal amounts from blood glucose.

Examples of aerobic ATP synthesis from glucose and fat can be represented by the reactions (II.2) and (II.3), respectively.



Assuming that the above chemical reactions come from the decomposition of substrates to produce ATP and carbon dioxide in presence or absence of oxygen, it is possible to simplify the ATP synthesis in a very general way as follows:

1) for aerobic reactions:



and 2) for anaerobic reactions:



where Ω represents other chemical substances produced during the reaction (including water), and where only the involved molecules are mentioned without specifying their number.

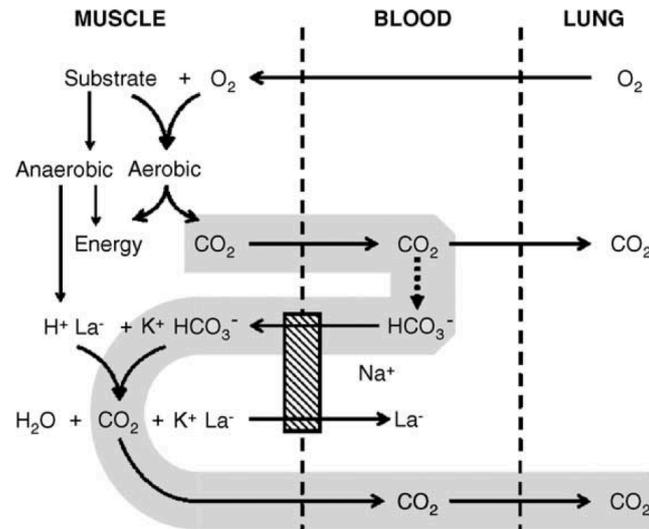
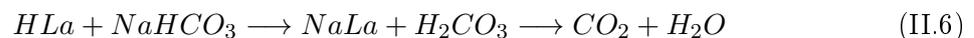


Figure II.2: Model for lactic acid buffering by bicarbonate and the control of hyperventilation above ventilatory threshold, [Wasserman 2005] redrawn by [Peronnet 2006]

Fig. II.2 shows a scheme of how the energy is obtained from the substrates. When a moderate exercise is performed, the energy is mainly obtained through aerobic reactions like (II.4) and carbon dioxide output is proportional to oxygen consumption.

When an exhaustive exercise is performed, anaerobic reactions are present, and in these reactions, carbon dioxide production has no correlation with oxygen consumption as in (II.5). Furthermore, lactate La and hydrogen ions H^+ are produced, which activates the lactate clearance mechanisms to maintain proper acid-base balance [McArdle 2006]. Then CO_2 production increases more than oxygen consumption, the sodium bicarbonate $NaHCO_3$ in the blood buffers or neutralizes the lactate generated during anaerobic metabolism in the following reaction



In the pulmonary capillaries, carbonic acid breaks down to its components carbon dioxide and water to allow carbon dioxide to readily exit through the lungs. Therefore, that buffering adds extra CO_2 to expired air above the quantity normally released during cellular energy metabolism. Relatively low CO_2 production occur after exhaustive exercise when carbon dioxide remains in body fluids to replenish bicarbonate that buffered the accumulating lactate.

II.4 Relevant models about human-bicycle system

When optimizing the performance of human-bike system is sought it is important to identify the inputs and outputs of the system and, if possible to have a model of the dynamical behavior. A lot of scientific efforts to understand and model the human behavior during exercise have been done from the physiological approach and recently from the engineering approach.

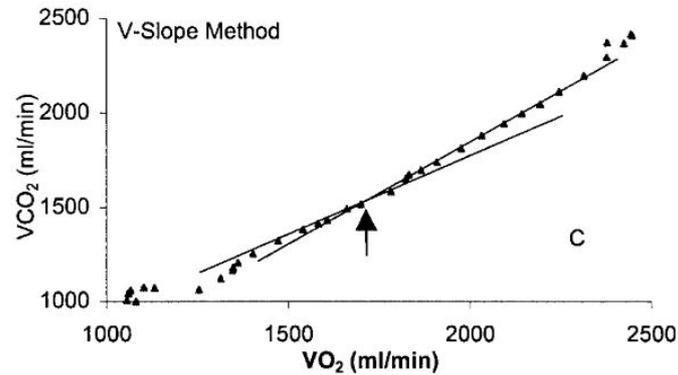


Figure II.3: $\dot{V}O_2$ vs. $\dot{V}CO_2$ used in the V-slope method for identifying the slope change, [Gaskill 2001]

In the following subsections some models, which were considered relevant during the development of the thesis are introduced. Their objectives, variables and hypothesis are briefly explained.

II.4.1 V-slope method

The relationship between gas exchange measurements and the onset of lactic acid production is known since 1923 mainly from [Hill 1923]. Later, some researchers used this relationship for finding a maximal safe power for exercising patients with cardiac disease [Wasserman 1973, Wasserman 1984]. This power is known as the anaerobic threshold, because it is supposed that it determines the activation of anaerobic pathway. In [Beaver 1986] the V-slope method is proposed to find the anaerobic threshold (AT).

Further works [Wasserman 1990, Shimizu 1991, Gaskill 2001] have validated and accepted this procedure and until our days it is used as a standard for AT calculation.

The V-slope method is performed during incremental tests. It uses the measured volume of oxygen consumed per time unit $\dot{V}O_2$ as an independent variable, because it is directly related with cyclist metabolism. The other measurements used in this method are the carbon dioxide production $\dot{V}CO_2$ and the Expired Volume $\dot{V}E$, both measured in l/m.

Three main characteristics are analyzed in the V-slope method, they are:

1. In the plot $\dot{V}CO_2$ vs. $\dot{V}O_2$ it is assumed that above the AT, CO_2 production increases more steeply. Then, the change in the slope is considered the AT. In this curve two lines from linear regression of $\dot{V}CO_2$ as a function of $\dot{V}O_2$ are proposed and the crosspoint between them is moved systematically until the mean square error is minimized.

Fig. II.3 shows the two lines and the crosspoint between them, known as the AT.

2. In the plot $\dot{V}E$ vs. $\dot{V}CO_2$ a zone of increased slope marks the start of respiratory compensation (RC) for metabolic acidosis. Before RC, $\dot{V}E$ is coupled to $\dot{V}CO_2$. After RC, $\dot{V}E$ rises more rapidly, which is known as hyperventilation.

3. In the plot $\dot{V}E$ vs. $\dot{V}O_2$ a three segment linear regression is proposed, where the first inflexion point determines the AT, which is difficult to detect. The second inflexion point corresponds to the RC and it is more evident.

The V-slope method does not give as a result mathematical equation of $\dot{V}CO_2 = f(\dot{V}O_2)$, because its objective is to find the point of slope change, which determines the AT. However, it uses a static linear model of the carbon dioxide production in function of oxygen consumption.

II.4.2 Critical power model

The basis of the critical power concept [Hill 1993] is that there is a hyperbolic relationship between power output and the time that the power output can be sustained. Theoretically, the power asymptote of the relationship, CP (critical power), can be sustained without fatigue; in fact, exhaustion occurs after about 30 to 60 minutes of exercise at CP.

A hydraulic model illustrates the main bio-energetic relationships during the procedure of energy transformation within the human body during exercise. The hydraulic representation of the model and some other proposed models in the same perspective are explained and analyzed in the review [Morton 2006]. This model is considered as a teaching and learning tool to give a macroscopic overview of the operation of the human bioenergetic system during exercise and recovery. Furthermore, it allows the formulation of dynamical equations about the use of pathways for ATP synthesis.

In Fig.II.4 is shown the most complete version of the Critical Power model. It includes three components which correspond to the available pathway for energy expenditure in human body. They are: The tank denoted with **O** represents the aerobic pathway, the middle tank with **AnA** stands for the alactic anaerobic pathway and the right tank **AnL** for the lactic anaerobic pathway.

In this model the liquid flow is analogous to the instantaneous power **P** and the integral in time corresponds to the total work **W** performed by the cyclist. **T** is a tap, which can be opened to allow a positive flow **P**.

The connecting tubes are: **R₁**, which has a fixed diameter with an upper bound for the flow at CP, **R₂** allows the flow from **AnL** to **AnA**, which is wide and can transport a big amount of fluid until **AnL** is empty. For its part, **R₃** is relatively so narrow and represents the repayment of oxygen debt when exercise is stopped, i.e. it is a very slow process.

Remark that tank **AnL**, except for the narrow tube **B**, has a limited capacity, which is called Anaerobic Work Capacity (AWC).

When a person begin to exercise, the energy is mainly provided by the **AnA** tank, but it empties quickly. Then, the **O** tank becomes the main source of energy and the most efficient one. The maximum output flow from **O** is CP. For greater powers than CP, the tank **AnL** is activated through the flow by **R₂**.

In order to estimate the parameters CP and AWC for any subject, it is necessary to complete at least two separate trials, cycling to exhaustion at two different but constant power outputs.

Differential equations of mass balance can be used for describing the dynamical behavior of the hydraulic system, but some other parameters like the transversal area of tanks or tubes are difficult to calibrate or are performed experimentally under strong assumptions. This is one of the shortcomings of this model.

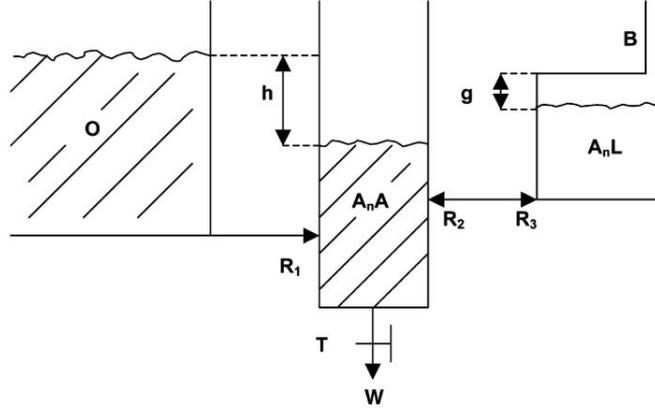


Figure II.4: Hydraulic representation of critical power model, [Morton 2006]

Some precisions about the vocabulary used in this model, especially the concepts of power and work, are commented in [Winter 2016].

The hydraulic model has been also used as a reference for other works. For example, in [Aftalion 2014], a dynamical model with a simplified version of two tanks (aerobic and anaerobic) is established for optimizing running strategies. The proposed dynamical equations for describing the system are:

$$\dot{x}_1 = x_2 \quad (\text{II.7})$$

$$\dot{x}_2 = f - \sigma(x_2) \quad (\text{II.8})$$

$$\dot{x}_3 = \theta(x_3) - fx_2 \quad (\text{II.9})$$

where x_1 is the height of depletion of the anaerobic container, x_2 is the instantaneous velocity, f is the propulsive force and $\sigma(x_2)$ is a drag function defined as $\sigma(x_2) = x_2/\tau$.

The state x_3 represents the anaerobic energy and $\theta(x_3)$ is a recreation function, which can be assumed as a positive constant.

Despite the fact that Critical Power model has been applied in several works, the difficulty to calibrate the parameters limits the viability of its use for real applications.

II.4.3 Control-oriented fatigue model

The fatigue is an index highly subject-dependent and unmeasurable in practice. Defining fatigue as reversible inability to maintain the required force output to continue muscular work, some authors have developed dynamical models of fatigue to compute a control law for optimal pacing or shift gear.

The works [Fayazi 2013, Wan 2014, Corno 2016] use an index called State of Fatigue (SoF), in order to overcome limitations of using a special equipment and the lack of mathematical models for others variables.

They take into account that depending on the cycling condition, the same amount of mechanical power can be delivered with more or less effort. In other words, the body accumulates

fatigue more or less rapidly depending on the cycling conditions. The model of SoF is described by:

$$SoF(t) = \frac{MVC - F_{max}(t)}{MVC - F_{th,iso}(t)} \quad (II.10)$$

where MVC is the Maximum Voluntary Contraction, F_{max} is the Instantaneous Maximum Force and $F_{th,iso}$ is the lowest level of steady-state force.

The value of MVC decreases due to muscular contraction velocity and fatigue, then the velocity sensitivity is taken into account by introducing the term MVC_ω . The combined effects of fatigue and resting can be modeled as a differential equation.

$$MVC_\omega = MVC \cdot \left(1 - \frac{\omega_{ped}(t)}{\omega_{max}}\right) \quad (II.11)$$

$$\frac{F_{max}(t)}{dt} = -k \cdot F_{max}(t) \frac{F_{cyclist}(t)}{MVC_\omega} + R \cdot (MVC_\omega - F_{max}(t)) \quad (II.12)$$

where ω_{ped} is the cyclist pedaling frequency, ω_{max} is the maximum pedaling frequency (190 rpm), k is the constant of fatigue dynamics and R the constant of resting dynamics.

II.4.4 Heart Rate models

Heart Rate is a physiological index easy to measure, therefore several works have tried to use this variable for the control design of electrical assistance for bicycles.

In the works [Meyer 2015a, Meyer 2015b] the objective is to maintain a HR reference for the rider. They use measurements of HR, torque and cadence to compute the motor power, while slope and headwind are considered as disturbances to reject. The HR model used in these works is as follows:

$$\dot{x}_1 = -a_1 x_1 + a_2 x_2 + a_6 u \quad (II.13)$$

$$\dot{x}_2 = -a_3 x_2 + \frac{a_4 x_1}{1 + \exp(-(x_1 - a_5))} \quad (II.14)$$

$$y = x_1 \quad (II.15)$$

The input u is the power (P_R) normalized by the estimated maximum power output of the cyclist, i.e.:

$$u = \frac{P_R}{P_{max}} \quad (II.16)$$

x_1 describes the immediate response of the cardiovascular system to workload, and is defined as the difference between the current HR and the HR at rest, normalized by the cyclist estimated maximum HR. It is:

$$x_1 = \frac{HR(t) - HR_{rest}}{HR_{max}} \quad (II.17)$$

x_2 describes the slower response of the HR for prolonged exercise due to the cardiovascular drift.

The parameters a_1 to a_6 in the model were estimated using ergometer experiments of one test subject. The subject conducted a total of four experiments on a cycling ergometer (LifeFitness, Brunswick Cooperation, USA). Each experiment consisted of three tasks where each task was divided into four phases: a resting phase, a warm-up phase, an exercise phase and a recovery. The experimental results of the fit between HR model and HR measurement to ergometer experiments are shown in Fig.II.5.

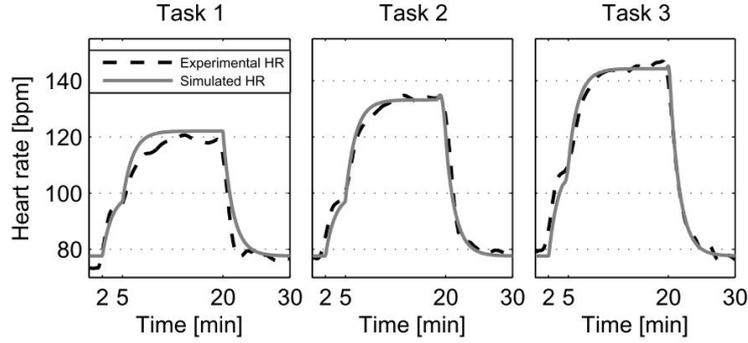


Figure II.5: Experimental results of HR model and HR measurement to ergometer experiments, [Meyer 2015a]

A HR model identified in treadmill scenarios have been used as a reference for control design in bicycles. The HR model corresponds to a Hammerstein model described in [Su 2006, Su 2007]. In the Hammerstein model the goal is to find a unique model in a homogeneous population, and different input/output data tests of all subjects are mixed and used together into a parameter optimization algorithm. The structure of Hammerstein model is shown in Fig. II.6. The same structure is also used in [Mohammad 2011, Mohammad 2013].

Other example of the use of HR model for control design of the assistance in bicycles is found in the works [Giani 2014, Corno 2015]. The model can be written as:

$$\dot{x} = \mathbf{A}x + \mathbf{B}\tilde{u} \quad (\text{II.18})$$

$$y = \mathbf{C}x + \mathbf{D}\tilde{u} \quad (\text{II.19})$$

where the input u represents the power, the model output y is a scaled version of the HR variations from the basal value. The model nonlinearity \mathcal{N} modify the input u . Then:

$$HR(t) = HR_{rest} + \gamma y(t) \quad (\text{II.20})$$

$$\tilde{u}(t) = \mathcal{N}(u(t)) \quad (\text{II.21})$$

The final transfer function of the model is:

$$G_{HR}(s) = G_{HR1}(s) + G_{HR2}(s) \quad (\text{II.22})$$

$$G_{HR}(s) = \frac{a_1}{(a_2s + 1)(a_3s + 1)}u_1 + \frac{a_4}{s}u_2 \quad (\text{II.23})$$

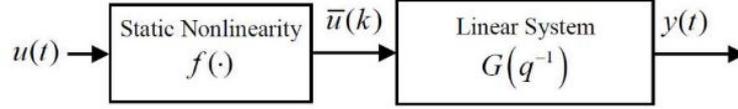


Figure II.6: Hammerstein model structure, [Mohammad 2013]

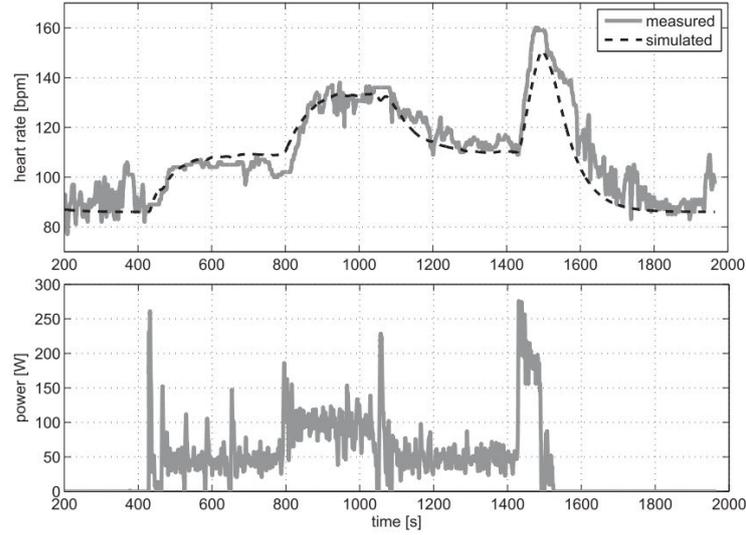


Figure II.7: HR model validation. Measured and simulated HR and pedaling power, [Corno 2015]

where $G_{HR1}(s)$ is the transfer function respect to power u_1 and G_{HR2} is a linear trend which represents the dynamics of the long term fatigue. Here, u_2 simply indicates whether the cyclist is pedaling or not.

The experimental results of the fit between HR model and HR measurement to ergometer experiments are shown in Fig.II.7.

In the work [Leitner 2014] it is identified a continuous time model for HR in function of breaking power of the ergometer and breathing volume. The proposed model has the following structure:

$$y(t) = \frac{a_1}{s^2 + b_1s + b_2}u_1(t) + \frac{a_2}{s^2 + b_3s + b_4}u_2(t) \quad (\text{II.24})$$

where the output $y(t)$ is the HR and the inputs to the model are $u_1(t)$ breaking power of the ergometer and $u_2(t)$ breathing volume. The fit of the model is shown in Fig. II.8. The model is only useful for aerobic scenarios.

In [Argha 2014, Argha 2016] the authors develop a non-model-based control strategy using proportional, integral and derivative (PID) controller/relay controller to regulate the HR to track a desired trajectory.

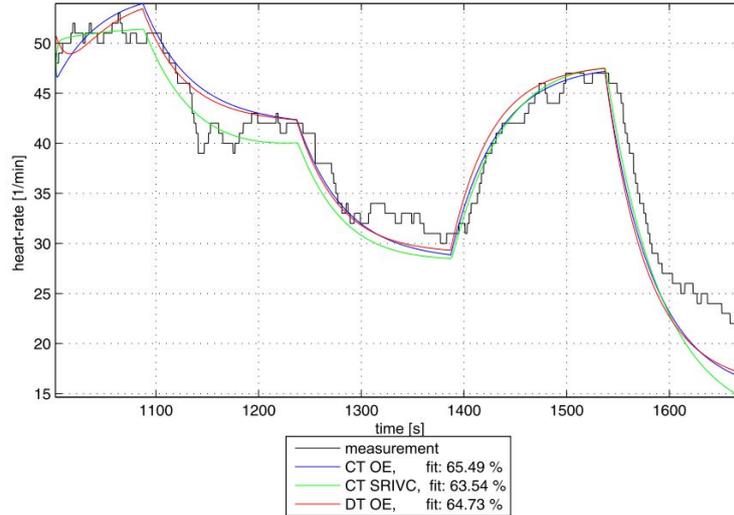


Figure II.8: Performance of the identified models on a validation data set (basal heart-rate of 55 bpm subtracted), [Leitner 2014].

II.5 Advances in bicycle research

Recent commercial bicycles are equipped with control systems to improve the rider comfort, optimize energy consumption, reduce the cyclist effort, reduce the commuting time, among others. This has been possible thanks to the instrumentation incorporated to measure variables, compute a control law and act in mechanical or electric components.

The instrumentation have increased the knowledge about mechanical systems as bicycles and has opened the possibility to new research topics. In the case of physiological variables, the fact that measurements are taken in the human body is a decisive factor for cyclist comfort and implementation of some solutions.

The opportunity to have an accurate evaluation of the physiological state of the cyclist determines the feasibility to apply a real-time control strategy. The used sensors, micro-controllers and actuators are concerned with the price of the proposed solutions, then have to be economically evaluated for the development of commercial products. A description of some of these instrumentation items is added below.

II.5.1 Cyclist physiological measurements

About security, health and comfort of the rider the following variables could be taken into account:

- **Heart Rate.** Consists in the number of contractions (beats) of the heart per minute (bpm). It varies according to the body needs, including the need to absorb oxygen and exhale carbon dioxide. Many factors can influence the variation of HR as physical exercise, sleep, anxiety, stress, illness and ingestion of drugs, even the presence of caffeine and nicotine in the body.

The main advantage of measuring the HR is its ease of measurement, because its value is usually equal or close to the pulse measured at any peripheral point. There is a wide range of products available into the market. They can be easily implemented in handlebars of the bike, in the chest, in the wrist or in the finger of the cyclist.

- **Gas exchange variables.** They constitutes mainly three measurements: oxygen consumption, carbon dioxide production and ventilation. Since the chemical reactions within the body for ATP synthesis have gases in the input and output of the equations, these variables are directly related with ATP production. In fact, in physiology and sport sciences the gas exchange variables are the most used for assessing the state of fatigue of the cyclist.

The ventilation can be estimated measuring the expansion of the chest. Another method, measures the percentage of oxygen in the blood with an spectrophotometric sensor and then calculates the oxygen intake. For measuring all the variables, it is necessary to use a mask with a flow sensor and chemical sensors for O_2 and CO_2 in order to calculate the real consumption or production of both gases with the input-output relation.

- **Ergonomic sensors.** The position of body rider in the bike is a key factor of health and comfort, a good position could avoid wound in muscles and joints. In road cycling, particularly when the velocity is high, the position of the cyclist affects significantly the aerodynamic behavior of the human-bicycle system.

Some important factors that can be taken into account are the high of the seat and inclination of back. The values of these factors depend on the body measures, for this reason it is necessary to know them before a calibration, for example, the right measure from the seat to the pedal center must be 0.885 the distance between groin and foot. The posture of the back can be calculated with a sensor in the helmet and other in the waist, later the angle between sensors and the distance of the head and the handset of the bike determines position of the back and inclination.

- **Electroencephalographic sensors.** Electroencephalogram (EEG) signals can be thought for detecting dangerous episodes in special patients as epileptic, somnolent and patients with another neurological diseases. Also, an EEG sensor can be installed in the helmet for calculating the state of stress of the cyclist.
- **Applied torque.** About this measurement, there exist a controversy for the difference between real torque produced by the cyclist and torque measured in the pedal axis. In fact, several bio-mechanical researches have addressed this topic having into account legs length, pedal angles, activated muscles among others.

When a controller is implemented in the bicycle, the torque measurement is usually applied to calculate the amount of assistance in a better way. In particular, it is useful for people that cannot apply the same torque with both legs; in this case the controller could estimate the appropriate help.

II.5.2 Bicycle instrumentation

The instrumentation of the human-bicycle system can define the possibilities of improve the comfort, safety, health and also allows a more sophisticated control. For example:

- **Sensors of torque, cadence and speed.** This instrumentation refers to how the assistance is applied. In general a torque sensor based assistance is better than the throttle one, because it determines the power of the motor based on how hard the rider is pedaling.

Cadence sensors can be found on most bicycles with pedal assist systems (PAS) available on the market, because torque sensors add considerably to the cost of an electric bike. For example, a torque sensor adding 200-300 USD to the retail cost of an electric bike, but the price of adding a cadence sensor to a bike is around 30 USD retail.

When this trio of sensors are used, torque, cadence and speed, the motor controller is able to have a complete picture of how the cyclist is riding so that it can give power in just the right amount at just the right time. It results in more intuitive operation, smooth performance, and outstanding efficiency.

- **Gyroscope.** It allows to measure the angle of slope in order to calculate the force due to slope, called F_{slope} . It can help to the controller to propose the activation of assistance motor in uphill or the simulation of resistance in downhill for decreasing the velocity or reducing the wear of brakes. Also, if the road is very rough, the controller can propose a change of route.
- **Bike gears.** There are many gearing systems, in order to offer a great variety of ways to front the scenarios of slope, flat road, training, velocity, etc. The most known brands in this field are Shimano and S-Ram, that sell internal systems with until 8 speeds and external ones until 21 speeds.

The role of gear shifting is important in the comfort of the rider, therefore the automatic shifting strategies can achieve an optimal trade-off between energy consumption and riding effort. A system called Continuously Variable Transmission (CVT) allows a continuous change of relation between rear wheel and crank set, achieving a fine regulation of the transmission. Also it offers the possibility of smoothly move the transmission along entire ratio range while in motion, under load, or stopped. [Giani 2013]

- **Global Positioning System.** A bicycle equipped with a Global Positioning System (GPS) uses satellite navigation technology to track speed and distance traveled. Also, it allows use maps for comparing routes, average speeds, elevations and some other calculations.

Another aspect is the safety, in case the bike was stolen, the GPS allows to know an accuracy position of the bike.

- **Detector of light.** A photocell or another sensor, only for turn on the lights when is needed. It can be seen as an additional item for the energy management of the bicycle electric components.

- **Proximity sensors.** For detecting near obstacles. They can be used in the controller for changes of speed, changes of route or simply for increasing awareness of the rider.
- **Charging battery system.** In downhills and even in flat road, the charging battery system must be turned on automatically or proposed to be turned on by the controller. Also it must be able to regulate currents and voltages.

Sometimes, when the state of battery is low, some capabilities of the assistance and controller are avoided in order to conserve energy for lights and most important functions.

- **Wireless communication.** It is extremely significant that the systems have some kind of communication with the controller, the user and in some cases between them. Actually, bluetooth is the most used, but also it is possible to use other ways. When the distance is short, wires are good, but it is better to avoid them.

II.5.3 Classification of related research works

The topic of physiology-aware control for pedelecs is increasing in the literature. The main related works are classified in this section according to i) the controlled variable, which refers to the physiological variable or set of variables the authors pretend to control; ii) the actuator, as the way they act to the bicycle or the rider for fulfilling the control objectives; and finally iii) the control technique, related to the approach they use for control design.

- **Controlled variable.** Most of the works analyzed during this research deal with Heart Rate (HR) control, mainly because of the ease of measurement of this variable. However, they recognize the limitations in the control regarding model uncertainties, intensity and kind of exercise. For example, most of the applications are only valid in cyclo-ergometers and not in road cycling; and for aerobic exercises, usually lower than 200 W of pedal power.

For instance, [Argha 2014, Argha 2016] focus on regulating the HR to a predefined reference trajectory, using different control strategies. The same case for [Leitner 2014], where the braking intensity of a bicycle ergometer is regulated based on information on the HR and breathing volume of the cyclist. The control system enables training at a desired HR by automatically adjusting the load under aerobic metabolic conditions.

In the articles [Mohammad 2011, Mohammad 2013] the objective is to automate and optimize physical activity for elderly non trained people. They also track a HR reference in a home trainer.

The works [Giani 2014, Corno 2015] address the design of a HR control system for a bicycle. They admit that the HR dynamics is subject to considerable uncertainties due to inter-subject differences, nonlinearities and other factors that are not directly influenced by the cycling physical effort. They compensate these withdraws in the control design.

In [Meyer 2015a, Meyer 2015b] the purpose is to maintain a desired HR level of the cyclist adjusting the motor assistance automatically. The control algorithm uses a set of variables: current HR, torque and cadence. Several test rides with only one subject at different intensity levels (maximum 150 W at pedal power) are performed to develop

and validate a nonlinear HR response model. In [Meyer 2018] the HR is also regulated, but also incorporates trip information to manage the motor assistance.

In [Fayazi 2013, Wan 2014] a phenomenological dynamic model for a rider fatigue is constructed and the model parameters are estimated using experimental data from road tests. For this case they also assume prior knowledge of the route elevation profile.

In [Corno 2016] a control-oriented analysis of three physiological features of the cyclist is done: i) equivalent cycling efficiency based on oxygen consumption, ii) the dynamic model of [Fayazi 2013, Wan 2014] for the state of fatigue (SoF), and iii) HR measurements. The analysis of the equivalent cycling efficiency and SoF dynamics, together with pedaling cadence measurement guide the design of the proposed charge-sustaining assistance algorithm.

Other application regarding physiological indices is MindRider [Strickland 2014], which consists in a bike helmet that reads rider brainwaves. The helmet also correlates the mental state of cyclist with their geographical routes, creating maps called by the inventor as psychogeography of the city.

- **Actuator.** Many innovative solutions about how to improve cyclist efficiency have been proposed. These include the use of an assistance motor as it is usual, but also other options as proposing a pacing strategy to the cyclist, changing automatically the gear or choosing a different route.

In most cases, commercial electric bicycles offer a motor assistance, that can be chosen through a simple interface of two or three buttons. The options of assistance are usually standard and boost, in which the feeling of assistance is not natural. The e-bike offers a resistance for negative slope, that is equal to apply breaks in downhill.

In [Fayazi 2013, Wan 2014] they do not use an electric bicycle. The solution suggests a pacing strategy that if followed can reduce total travel time.

In [Giani 2013, Corno 2015, Corno 2016], they work with a standard city bike equipped with roller-based Continuously Varying Transmission (CVT). The transmission system is embedded in the rear wheel hub: the freewheel is mounted on the gearbox input shaft, while the wheel hub constitutes the output shaft of the gearbox. The chain transfers the pedal torque from the front chain wheel to the rear freewheel, as in a standard bicycle.

Within the rear hub, a set of rotating and tilting balls is positioned between the input and output discs. Tilting the balls changes their contact diameters and varies the speed ratio of the discs, thus varying the transmission ratio. Fig.II.9 shows the CVT system used in these works.

Commercial electric bicycle with a front wheel hub motor is used in [Meyer 2015b, Meyer 2015a] for testing the controller design. The bicycle is modified to measure torque output and pedaling frequency of the rider and to control the motor torque. In [Meyer 2018] gear shifting and motor power are managed by the controller. A mid-mounted electric motor ($P_{max} = 250$ W) with internal transmission and CVT system (NuVinci N360, Fallbrook Technologies Inc.) are implemented in the bicycle.

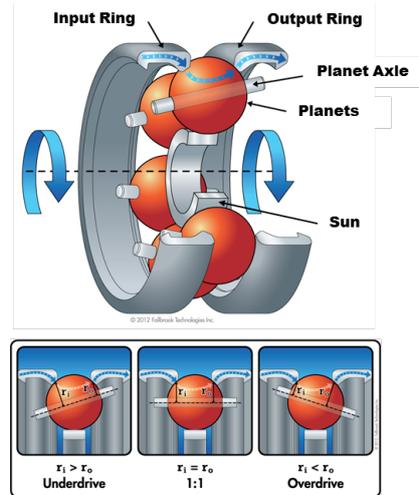


Figure II.9: Nu Vinci System of Continuously Varying Transmission [nuv 2018]

In the case of [Strickland 2014], an EEG sensor is installed in the helmet, which calculates the state of stress of the cyclist for helping him to choose the best route towards his destination.

- **Control algorithm.** It depends mainly on the expertise of the authors in a given technique. They include state machine, ruled-based algorithms, PID, LQR, SMC and other model-based control strategies.

For the case of control strategies implemented in cyclo-ergometer, [Argha 2014] implement a PID controller to regulate the HR to track a desired trajectory. In [Argha 2016] a model-based control strategy and a real-time damped parameter estimation scheme are combined. The controller is a time-varying integral sliding mode controller (SMC). In [Leitner 2014] a linear quadratic controller (LQR) is synthesized based on the identified model.

In [Mohammad 2011] they derive a control law based on Takagi-Sugeno fuzzy models. The discrete-time model representation allows them to deal with non quadratic Lyapunov functions. In order to derive a real-time control, an integral part is added and then they discuss stability according to exogenous inputs. Output feedback via an observer and stability of the complete loop is concluded. In [Mohammad 2013] output feedback control can be achieved via specific Lyapunov functions.

In [Fayazi 2013, Wan 2014], assuming prior knowledge of the route elevation profile, the optimal control problem is solved off-line using dynamic programming which generates a feedback strategy: Given measured bicycle velocity and the estimated rider state of fatigue, the solution suggests a pacing strategy for reducing total travel time.

In [Corno 2015] the HR model is the basis for the development of two controllers: A Proportional-Integral controller and a Second Order Sliding Mode Controller (SMC).

The control system helps the cyclist maintain a constant physical effort throughout the trip. With the proposed system the riders are capable of keeping their HR within 10 bpm from the desired one. A scheme of the proposed control-loop is shown in Fig.II.10.

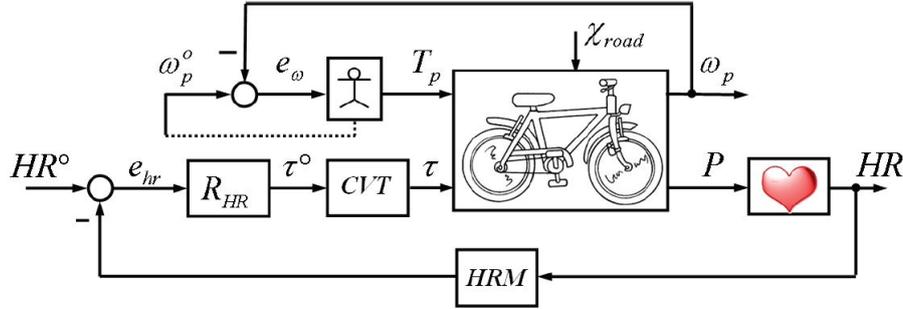


Figure II.10: Control loop HR controller, [Corno 2015].

In [Meyer 2015a] the controller consists of feedforward as well as feedback components: Whereas the HR is the feedback component, the system feedforward component is the rider estimated torque output at the desired HR level. Environmental changes like slope or headwind are modeled as disturbances that the controller has to reject.

In [Meyer 2018] the energy-management system is focused in i) an optimal control approach to regulate the HR of the cyclist and 2) trip information to manage the motor assistance. The system consists of a control stage and a planning stage. In the control stage, a model predictive controller (MPC) regulates the HR by changing the motor power and gear ratio to maintain a user-defined exertion while considering constraints. The planning stage processes a priori information about the user and the route to estimate the power demand during different sections of the trip and to calculate the optimal motor power for each section.

In [Strickland 2014] the information about mental state of the cyclist in real-time is sent to the MindRider app on the user phone. With the app, users can examine maps of their bike routes and potentially determine which streets are stressing them out. Fig.II.11 shows a scheme of this approach.

II.6 Conclusion

An overview about the impact of bicycle research and the evolution of human-powered transportation is briefly given in this chapter. The most of improvements about electrical assistances are based in bicycle mechanical variables, however, some recent researches are looking for fitting in the best way customer physiological requirements. Related works in physiology-aware control for bicycles are listed, to place this research as a work which analyses physiological factors and explore new approaches.

A presentation about basic human physiology gives a context about the concepts used in next chapters. Furthermore, the description of different types of models related with cyclist

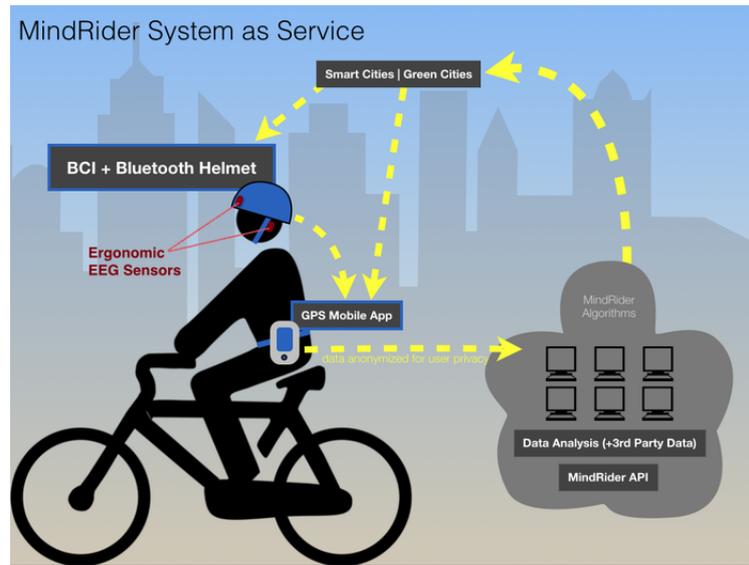


Figure II.11: MindRider System, [Strickland 2014]

physiological state found into the literature are reviewed in order to provide a background to compare the contributions of this research.

Some aspects about the implementation of control strategies are considered. It mainly consist in the instrumentation concerning sensors and actuators in the human-bicycle system, as well as the possibility to embed a control law. These factors define the feasibility of proposed solutions for commercial bicycles.

Finally, a classification of some related researches is done according three aspects i) physiological controlled variable, ii) actuator and iii) control technique, which gives a comprehensive overview of the thesis subject.

Conceptual bio-energetic model of human-bicycle system

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III.1 Introduction

In this chapter is presented a conceptual model of the human-bicycle system using a bio-energetic approach. Here, the human-bicycle system is seen as a set of subsystems that exchange energy among each other.

In order to design a more intelligent control system for pedelecs, it could be necessary to consider a model which is able to reproduce the behavior of the bio-energetic system of the cyclist; mainly, the energy transformation from chemical energy (food and oxygen) to mechanical movement in the pedal. Such a model has to be simple enough and energy-aware to be combined with previous electro-mechanical models that describe the electrical bike dynamics. In this way, it will be possible to incorporate the physiological state of the cyclist and/or new therapeutic objectives in future control solutions.

As presented in chapter II, most of the literature about models of human-bicycle system or human during cycling are static or based on steady-state points, while the interest of this thesis is to understand the dynamical behavior of human physiology. The proposed dynamical model is based on physical laws that describe hydraulic and electro-mechanical systems. Hence, the behavior of the system states respects the energy/mass conservation laws and allows the association of storage, transfer and dissipation of power in a more intuitive way.

In principle, this model is theoretical, but describes conceptually and pedagogically the system dynamical behavior at three different levels: metabolic, bio-mechanical and pure-mechanical level, where the latter corresponds to the interaction of cyclist with the bike dynamics. The differential equations formulated for each stage facilitates the simulation of practically any scenario.

This chapter has two main sections: description of the bio-energetic model, which is presented and discussed in Section III.2, and a simulated example of an Incremental Cycling Test scenario in Section III.3.

III.2 Conceptual bio-energetic model of human-bicycle system

An electro-hydraulic based model of the bio-energetic system of a cyclist is shown in Fig.III.1. The nomenclature is summarized in Table III.1. It contains three different interconnected sub-systems: i) a physiological stage represented as a hydraulic system, where mass balance is applied for every metabolic pathway to obtain the system equations, ii) a bio-mechanical stage, which acts as an electric engine in which electro-mechanical laws of motion are applied to obtain the dynamical equations for the transformation of physiological energy to mechanical energy, and iii) a pure mechanical stage which includes the dynamics of a bike by applying Newton's equations of motion.

According to the classification and descriptions given by [Abbiss 2005] and [Noakes 2000], it could be considered like an energy supply/energy depletion model.

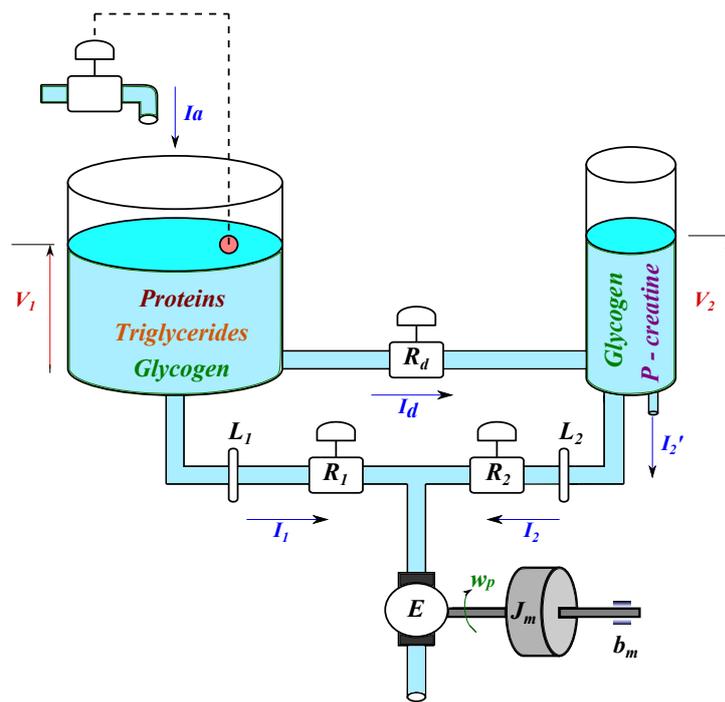


Figure III.1: Electro-hydraulic based model of the bio-energetic system of a cyclist.

The model is intended to describe the behavior of certain physiological variables accompanied by bio-mechanical and pure-mechanical dynamical interactions. Two metabolic pathways are considered i) an aerobic pathway and ii) an anaerobic pathway. The following additional issues were considered during the modeling process:

Table III.1: Nomenclature used in the conceptual bio-energetic model of human-bicycle system

Sym	Type	Meaning
I_a	Input	Amount of oxygen and nutrients transported by the cardiovascular system to the muscles
V_1	State	Vessel 1 level. Linked to the aerobic energy.
V_2	State	Vessel 2 level. Linked to the anaerobic energy.
I_1	State	Vessel 1 outflow. ATP from the aerobic pathway.
I_2	State	Vessel 2 outflow. ATP from the anaerobic pathway.
ω_p	State	Pedaling frequency. Measured at pedal level.
I_d	Variable	Flow between vessels 1 and 2.
I'_2	Variable	Losses when anaerobic pathway is used.
E	Variable	Counter-motive force in an electrical motor.
C_1	Parameter	Capacitance of vessel 1.
C_2	Parameter	Capacitance of vessel 2.
R_d	Parameter	Resistance of I_d flow. Regulates the recovery dynamics.
R_1	Parameter	Resistance of I_1 flow. Represents the voluntary desire to apply a force.
R_2	Parameter	Resistance of I_2 flow. Represents the involuntary and complementary action of the anaerobic pathway.
L_1	Parameter	Inductance at vessel 1 output. Models the dynamics of aerobic ATP synthesis.
L_2	Parameter	Inductance at vessel 2 output. Models the dynamics of anaerobic ATP synthesis.
K_e	Parameter	Counter-electromotive force. $E = K_e \cdot \omega_p$.
J_m	Parameter	Muscular inertia.
J_{eq}	Parameter	Equivalent inertia. It includes the cyclist, bike and wheels.
b_m	Parameter	Muscle viscous friction coefficient. It is useful to model isometric action.

- 1) The model has to describe the power contributions of every bio-energetic pathway, in a dynamical way.
- 2) The model has to reproduce the power conversion efficiency observed in every metabolic pathway.
- 3) The model has to be able to describe the energy consumption of muscles for both i) during isometric action (i.e. without muscular motion) and ii) during concentric/eccentric action (e.g. during pedaling).
- 4) The model has to reproduce the dependency of the Maximal Voluntary Contraction (MVC) with respect to the speed at which a muscle changes its length.
- 5) The model has to be simple, only retaining a few components to describe sources, storage, dissipation and conversion of energy at each stage of the model (i.e. for physiological and bio-mechanical stages).
- 6) The model has to allow the computation of a fatigue index that is compatible with the physiological energy storage process.

The stages of the model are:

III.2.1 The physiological stage

In a body, the cardiovascular system permits the blood to circulate and transport nutrients from and to the cells. For instance the Fick Principle relates the oxygen consumption with the cardiac output (heart rate HR by stroke volume SV) and the tissular oxygen extraction, this aspect is modeled as the input flow I_a . Here, I_a is controlled to regulate the oxygen supply and other necessary nutrients to produce energy in muscles. The stored energy takes the form of molecules of glycogen, triglycerides and proteins that are used for the aerobic system, and molecules of glycogen and phosphocreatine that are used for the anaerobic system. These stored molecules are modeled as the volume of the vessels, proportional to the levels V_1 and V_2 . Hydraulic models have been used previously for understanding the behavior of energy supply by different pathways during exercise, for example the ones cited by [Morton 2006].

Mass balance equations for the vessel 1 (used for the aerobic system) and the vessel 2 (used for the anaerobic system) model the storage process of nutrients and in particular the required time to fill and/or to empty these vessels. The equation for the aerobic system is:

$$C_1 \frac{dV_1}{dt} = I_a - I_1 - I_d \quad (\text{III.1})$$

and for the anaerobic system is:

$$C_2 \frac{dV_2}{dt} = I_d - I_2 - I'_2 \quad (\text{III.2})$$

The constants C_1 and C_2 represent the capacitance of vessel 1 and 2 respectively (i.e. the cross-sections of each vessel). The value of C_1 is linked to the endurance of the cyclist and/or the availability of muscle fibers type I, while C_2 is an image of the anaerobic capacity

of the cyclist and/or the availability of muscle fibers type II according to [Coyle 1992] and [Jeukendrup 2000].

The flow I_a is a function of the aerobic vessel level V_1 and a given metabolic reference value. It is assumed that this term regulates the level V_1 and it is proportional to the HR and the oxygen uptake $\dot{V}O_2$. For simplicity, this action is modeled by a saturated proportional control loop, as follows:

$$I_a = \begin{cases} -k_p(V_1 - V_1^{ref}) & \text{if } I_a \leq I_{max} \\ I_{max} & \text{otherwise} \end{cases} \quad (\text{III.3})$$

where the term V_1^{ref} could be obtained from the volume of available energy (in the form of proteins, lipids and glycogen) at the beginning of the workout and it will decrease during the exercise. Here the proposed variation of V_1^{ref} takes into account observations in endurance time by [Morton 2006] and his previous works. Then:

$$\dot{V}_1^{ref} = -\frac{1}{C_a} I_a \quad (\text{III.4})$$

where the initial condition $C_a V_1^{ref}(0)$ corresponds to the average amount of oxygen and nutrients that can be consumed above basal level in an endurance test. C_a is a constant parameter.

The maximum value of I_a will be I_{amax} which models the cardiovascular limit to provide oxygen and other nutrients to the aerobic system. This value is in fact an image of the maximum oxygen uptake $\dot{V}O_{2max}$. The flow I_d between the two vessels is described by the following equation:

$$I_d = \frac{V_1 - \alpha V_2}{R_d} \quad (\text{III.5})$$

with α being a constant parameter satisfying $0 \leq \alpha \leq 1$ and R_d being a resistance which limits the flow between vessels. It mainly models the molecular transfer between both storage systems. The parameter α can be used to finely model this exchange.

When an anaerobic effort is being carried out, the human body synthesizes Adenosine Triphosphate (ATP) from Phosphocreatine and Adenosine Diphosphate (ADP) or Anaerobic glycolysis; during this, it contracts an oxygen debt that will be paid off once the exercise is over. This phenomenon is known as Excess Post Exercise Oxygen Consumption (EPOC). The flows I_a and I_d reproduce this phenomenon. In particular, I_a increase during the effort to ensure that the level V_1 is high as possible and to recover the initial value after the workout.

In addition, the use of the anaerobic pathway produces an augmentation of lactate and acidosis caused by the hydrogen-ions H^+ product of anaerobic glycolysis, which saturate the clearance mechanisms as described in [Binzoni 2005] and [McArdle 2006]. In Fig. III.1, the flow I'_2 represents the fact that this pathway is less efficient than the aerobic one, generating losses in energy use, even if it is fast and synthesizes big amounts of ATP.

The human body cannot easily store ATP and it needs to be continuously created. Here, the synthesis of ATP molecules is modeled by two differential equations which relate the energy storage in the body considered as the source of power (proportional to V_1 and V_2), the dissipated energy required to synthesize ATP (I_1 and I_2), and the energy stored in the

muscles (proportional to L_1 and L_2). Thus, the dynamics of the flows I_1 and I_2 , describing the ATP synthesis dynamics, will be governed by the following equations:

$$L_1 \frac{dI_1}{dt} = V_1 - R_1 I_1 - E \quad (\text{III.6})$$

$$L_2 \frac{dI_2}{dt} = V_2 - R_2 I_2 - E \quad (\text{III.7})$$

where the constants L_1 and L_2 allow the characterization of the necessary time-response for generating ATP from the aerobic and anaerobic pathways, respectively. The synthesis of ATP molecules is related to the production of muscle force. Hence, the flows (or currents) I_1 and I_2 are images of the produced force from the aerobic and anaerobic pathways, respectively.

These dynamics are inspired from an electrical motor. As in electrical motors, the counter-motive force, denoted here by the symbol E , is proportional to the pedal angular speed ω_p (assuming that this speed is proportional to the speed of changing the muscle length).

The effort E pushes against the current $I_1 + I_2$ which induces it. It is given by

$$E = K_e \omega_p \quad (\text{III.8})$$

where K_e is a positive constant, which describes the power conversion from physiological energy to bio-mechanical one. Since the produced torque T_e is proportional to the sum of currents, i.e.

$$T_e = K_e (I_1 + I_2) \quad (\text{III.9})$$

Thus, the bio-mechanical power (without considering mechanical power losses and storage) can be calculated as

$$P_{bmec} = E (I_1 + I_2) = T_e \omega_p \quad (\text{III.10})$$

and the physiological power can be calculated as the sum of two contributions:

$$P_{phy} = (V_1 I_1) + (V_2 I_2) \quad (\text{III.11})$$

In the case of an isometric exercise, when muscle group moment becomes equal to resistance moment; ω_p and bio-mechanical power P_{bmec} will be zero even if the torque T_e is not zero. However, the physiological power P_{phy} will take a different value depending on the resistances R_1 and R_2 which, in turn depend on desired torques or forces.

When there is a constant applied force, the total force can be written as:

$$I_1 + I_2 = \left(\frac{V_1 - K_e \omega_p}{R_1} \right) + \left(\frac{V_2 - K_e \omega_p}{R_2} \right) \quad (\text{III.12})$$

Hence, the muscle fatigue appears in (III.12) as a consequence of the reduction of the levels V_1 and V_2 during an effort. The word "fatigue" is interpreted in this work as the inability to produce required/desired power, and "exhaustion" is interpreted as a condition acquired by accumulated fatigue.

According to [Kenney 2015], during long term exercises, fatigue coincides with a decreased concentration on muscle glycogen, no matter its rate of depletion. Here, the glycogen concentration is directly related to the levels of V_1 and V_2 . Therefore, two indices of fatigue are proposed:

$$SoF_1 = \frac{V_1(0) - V_1(t)}{V_1(0)} \quad (\text{III.13})$$

$$SoF_2 = \frac{V_2(0) - V_2(t)}{V_2(0)} \quad (\text{III.14})$$

Since both vessels levels represent available muscular substrates, the proposed indices are related with peripheral fatigue. The index SoF_2 could be the responsible of the sensation of fatigue due to metabolic acidosis and increase of lactate, while SoF_1 is more linked to glycogen depletion.

In addition to the previous exposed properties of the model, the chosen structure for modeling the human force generation reproduces the force-velocity relationship accepted by the physiology community. From (III.12), the variations of the torque or force (proportional to $I_1 + I_2$) with respect to the angular speed ω_p will be

$$\frac{\partial(I_1 + I_2)}{\partial\omega_p} = - \left(\frac{1}{R_1} + \frac{1}{R_2} \right) K_e \quad (\text{III.15})$$

which means that the force decreases as long as the speed increases, because the derivative is always negative. In other words, it will be harder to produce muscular force at high speeds. Hence, the proposed model seems to be well adapted to the observations presented by the physiology community.

III.2.2 The bio-mechanical stage

The role of resistances:

The resistance R_1 allows the inclusion of the "voluntary" desire of applying force through I_1 , and R_2 represents the "involuntary" but complementary role of the anaerobic pathway to apply I_2 . For instance, big values of resistances imply zero forces, and the smallest value of resistances provides the maximal forces that muscles can produce.

The maximum isometric force (i.e. a static force without movement of the muscles) that an individual can develop is called Maximum Voluntary Contraction (MVC). Here it can be computed using (III.12) for $\omega_p = 0$, given that currents are proportional to the cyclist force.

Suppose an individual desires to apply a given force which imposes a reference on I_1 , denoted I_1^{ref} . It is assumed that the desire to perform a force produces electrical signals coming from motor neurons. These electrical signals produce muscular actions. This voluntary control action is modeled as a proportional-integral control loop, that is

$$R_1 = K_p e_1 + K_i \int_0^t e_1 dt \quad (\text{III.16})$$

where e_1 stands for the force error signal $e_1 := I_1 - I_1^{ref}$. The constants K_p and K_i will depend on the capability of an individual to reproduce a desired force.

Concerning the resistance R_2 , it has to follow an internally generated reference which allows to perform more or less anaerobic flow according to the state of the aerobic pathway. Here this control action is modeled as follows:

$$R_2 = K_p e_2 + K_i \int_0^t e_2 dt \quad (\text{III.17})$$

where e_2 stands for the force error signal $e_2 := I_2 - I_2^{ref}$, with

$$I_2^{ref} = I_1^{ref} - I_1 = -e_1 \quad (\text{III.18})$$

This reference models the natural mechanism which intends to guarantee that the total desired force is assured by the sum of the aerobic and anaerobic contributions, i.e. $I_1 + I_2 = I_1^{ref}$ as much as possible. However, in practice the variables I_1 and I_2 do not perfectly track their references. The tracking error depends on the state of the aerobic and the anaerobic pathway. In particular, the state of the vessels levels V_1 and V_2 and the effort E .

The terms $-R_1 I_1$ and $-R_2 I_2$ in (III.6) and (III.7), respectively, have to be negative in order to model dissipative control actions (these actions do not generate power, they can only dissipate power).

Considering the same control gains K_p and K_i to simplify the model, thus, the time-response for each metabolic pathway is mainly assured by the choice of inductances L_1 and L_2 (for instance, the condition $L_1 > L_2$ models slower aerobic dynamics than anaerobic one).

In Fig.III.1, the electro-mechanical stage described by equations (III.6)-(III.7) can be used to model the interconnection between the physiological domain and the bio-mechanical one.

Recalling that the counter-motion force is $E = K_e \cdot \omega_p$, while the torque T_e is proportional to the sum of currents $I_1 + I_2$, this means that $T_e = K_e(I_1 + I_2)$. Newton second law for rotational motion can be used for obtaining the dynamics of the pedaling angular speed ω_p ; that is,

$$J_m \frac{d\omega_p}{dt} = T_e - b_m \omega_p - T_p \quad (\text{III.19})$$

where T_p is the torque at the pedal level, which is often measured in practice. The constants J_m and b_m model the muscular inertia and muscle viscous friction coefficient, respectively. Thus, during pedaling, the torque provided by the physiological stage, T_e , is converted into torque at the pedal level, T_p , but a fraction of this power is stored in the inertia and the rest is dissipated by friction.

III.2.3 The pure mechanical stage

A very simplified motion equation concerning the mechanical human-bicycle dynamics can be established as follows:

$$J_{eq} \frac{d\omega_p}{dt} = T_p - T_{load} \quad (\text{III.20})$$

where the inertia J_{eq} represents the equivalent inertia observed at the pedal level and it includes the bike, wheels and cyclist equivalent inertia. The load torque T_{load} includes all the dissipation terms which appears during bike motion. For instance, T_{load} includes mainly the aerodynamic losses, rolling resistance and a component of the gravity force due to slope.

Combining equations (III.19) and (III.20) and rearranging the torque produced by the cyclist at the physiological stage, we obtain:

$$(J_{eq} + J_m) \frac{d\omega_p}{dt} = T_e - b_m \omega_p - T_{load} \quad (\text{III.21})$$

which is more suitable for simulating the whole-body bio-energetic behavior because the torque produced at the physiological level T_e , explicitly appears into the motion equation.

Table III.2: Parameters used in the simulation example of the bio-energetic model of human-bicycle system

Parameter	Value	Units
C_1	1.23	m^2
C_2	0.41	m^2
R_d	2	$s \cdot m^{-2}$
L_1	0.92	H
L_2	0.0092	H
K_e	0.4612	$V \cdot s \cdot rad^{-1}$
b_m	0.1646	$N \cdot m \cdot s \cdot rad^{-1}$
$J_m + J_{eq}$	11.11	$kg \cdot m^2$
α	1	-
K_p	5000	-
K_i	50	-

III.3 A simulation example

Simulation data

The simulation has been performed to illustrate the dynamical behavior of the proposed model. The parameters have been chosen to fit available static and dynamical data from physiological tests. Even if the used parameters do not correspond to a particular individual, the results allow a pertinent evaluation of the proposed model. The parameters used are summarized in Table III.2.

Description of the presented scenario

A simulated scenario is proposed in order to illustrate the ability of the model to reproduce several physiological behaviors and phenomena during energy depletion and during the recovery process.

Fig. III.2 shows a scenario related to the incremental cycling test with resistance protocol, i.e. the resistance torque T_{load} is linearly increased during time, while the cyclist is required to maintain a constant pedaling frequency ω_p (freely chosen by him previously). The test is terminated when the cadence fell to more than 10 rpm below the chosen one for more than 10 s.

Fig. III.3 shows the main flows of the model. The cyclist starts from a basal level $V_1(0)$ and $I_a(0) = 1$ l/min at $t = 0$, which correspond to basal values. The load torque T_{load} increases as a ramp from 0 N.m to 50 N.m, as depicted in Fig. III.2. This produces a cyclist torque T_p , mostly provided by the aerobic pathway until around $t = 18$ min. At this time, the anaerobic threshold and the beginning of lactate accumulation could be considered. The contribution of aerobic pathway is saturated from this point until the end of the workout.

Since the beginning of the exercise, the flow I_a increases to provide the required oxygen and nutrients. A change in the slope is evident after the anaerobic threshold.

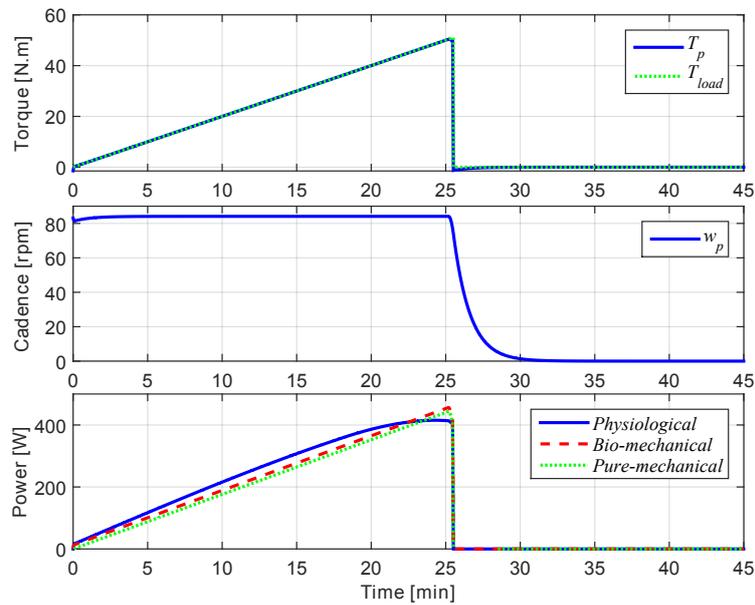


Figure III.2: Torque, cadence and power for the Incremental Cycling Test (ICT) scenario using the human-bicycle model

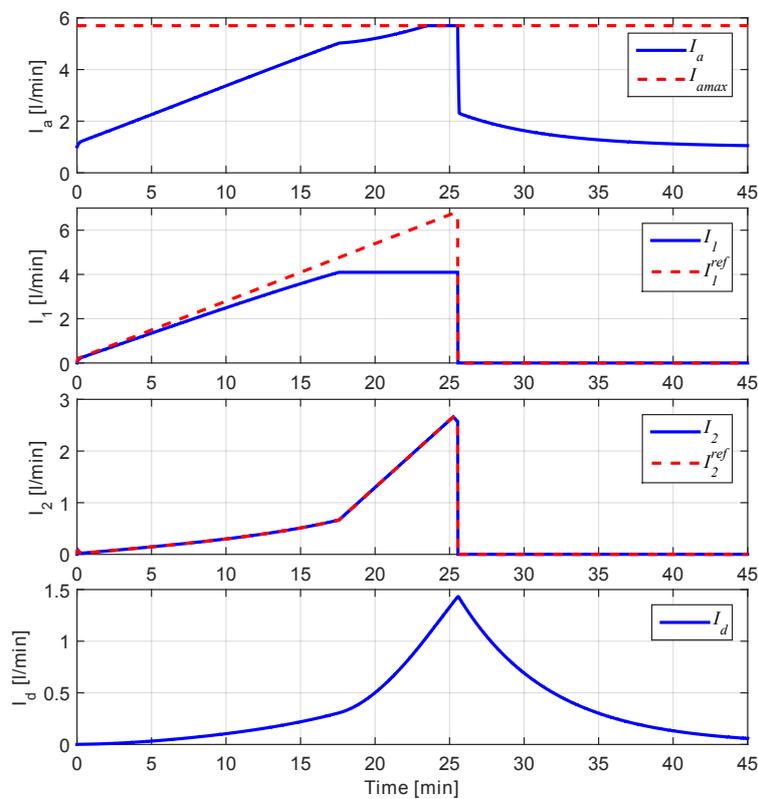


Figure III.3: Flows during simulation scenario.

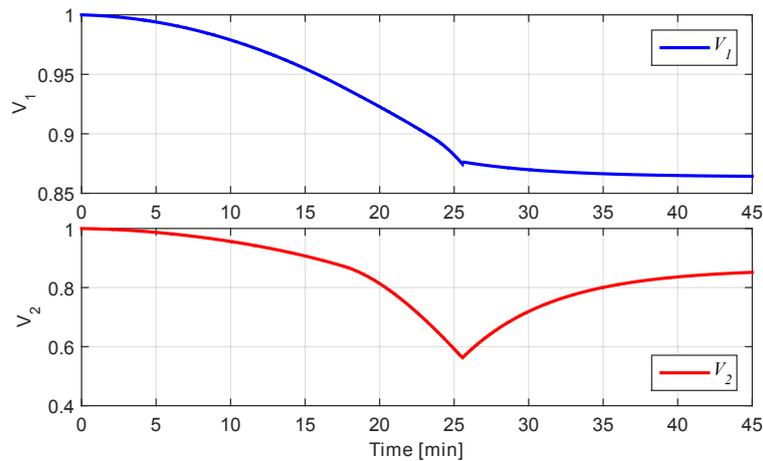


Figure III.4: Levels in vessels 1 and 2

Fig. III.4 shows how the levels V_1 and V_2 decrease during the increments in efforts. An important change in the anaerobic vessel level V_2 is observed especially in the last five minutes of the workout, producing an important level of fatigue. Remark also, that both metabolic pathways provide energy, noted by the decrements in vessels levels. However, the main source of energy comes from the aerobic pathway. The anaerobic pathway only intervenes when the aerobic system is not able to provide the desired effort. After an important effort, the cyclist feels fatigue sensation given by the low level in V_2 and he is not able to sustain the cadence, then the T_{load} is set to zero (see Fig. III.2) at $t = 27$ min.

The provided powers are also seen in Fig. III.2. The power rises in a proportional way to the load until its maximum value around $t = 26$ min, time at which the anaerobic system produces important quantities of ATP. Physiological power P_{phy} , bio-mechanical P_{bmec} and the mechanical one P_{mec} , which is equal to $T_p \cdot \omega_p$ were calculated. The energy losses during the transfer can be observed by the difference between the curves.

III.4 Conclusion

The bio-energetic approach of the presented model allows a comprehension of the interaction between subsystems of human-bicycle system. These subsystems are stages in the energy transformation process, which are: metabolic, bio-mechanical and pure-mechanical. The dynamical equations permit the simulation of dynamical behavior and reveal information about energy diminution in the cyclist.

The model structure incorporates the comprehension of internal control loops in the body, losses in muscles depending on the load or speed variation, and main relationships between physiological and mechanical variables. However, the variables and parameters used in the conceptual model are difficult to measure or even estimate.

This model constitutes a conceptual and theoretical effort in modeling dynamical relationships between physiological and mechanical variables. The graphical representation of the model allows an easy interpretation of the efficiency of the aerobic pathway, modeled as the

big vessel with a tap as source. If the intensity of exercise allows to keep the system working below the anaerobic threshold, i.e. below the saturation of I_1 ; the control mechanisms of the human body can ensure an optimal operation.

When the anaerobic pathway, represented as the smallest vessel, has to be the main source of energy, the system becomes inefficient. In this case, an external actuator in the mechanical part of the system can alleviate the output mechanical power and then return the system to its efficient mode.

In the next chapter, a dynamical model based on gas exchange variables is proposed because of its ease of measurement, which serves for parameter identification and for further use in control design for bicycle assistance.

Gas exchange model during cycling

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IV.1 Introduction

In this chapter, human physiology is addressed from an engineering perspective in order to interpret and model the human-bicycle system as an energetic system which transforms chemical energy into mechanical energy. In this sense, the energy/mass conservation law provides the idea that the energy from chemical reactions within the body is proportional to the mechanical power at the pedal level. Here, the relationship between the gas exchange measures during cycling and the mechanical power is described through a discrete-time dynamical model.

As it is expected, the chemical reactions within the body act like a black-box from the outside. It is extremely difficult to know exactly which substrates are reacting and which molecules are being produced, especially during road cycling. Therefore, in many cases, the evolution of fatigue during cycling is limited to the capability of measuring some markers (in blood or muscles) through the use of invasive or non invasive sensors or the capability of verifying these markers after the workout. At this point, the gas exchange measurements

constitute an image of the dynamical behavior of chemical reactions within the body and provide the easiest solution to investigate human physiology.

The development of a model that reproduces and predicts the behavior of gas exchange variables is explained throughout this chapter. The main focus is the relationship between consumed oxygen and produced carbon dioxide with power at the pedal level during cycling. The purpose of the model and its main features are described in Section IV.2. Basically, a discrete-time linear parametric varying model is proposed.

The parametric identification is developed as a solution of a sequence of non-linear unconstrained optimization problems described in Section IV.4. The structure of the model is explained in Section IV.3, which includes a variable parameter in the output matrix for the transition between two modes of the model, corresponding to a normal carbon dioxide production and a high carbon dioxide production. The experimental setup and the tests for obtaining the physiological data were performed in GIPSA-Lab. Several individuals of different conditions of training and age carried out the cycling tests. This is explained in Section IV.5 with the model results for two participants.

The model structure and the methodology for parametric identification constitute the main result of this work, because despite the values of parameters are subject-dependent, the calibration process can be done for any person and can be easily repeated when conditions change.

IV.2 Purpose of the model

The dynamical model of gas exchange dynamics with respect to the pedal power is intended not only for conceiving real-time applications including observation and control design, but also for system analysis. In particular, it is envisaged to use the proposed model for designing novel electrical assistance systems for bicycles.

As stated in [Casaburi 1989], the behavior of consumed oxygen and produced carbon dioxide over time reflects intramuscular biochemical events, transport of body fluids, delays and changes in gas stores. In view of these aspects, it seems unlikely that a static mathematical description would suffice to describe these dynamical responses. Consequently, a dynamical model described by differential equations seems to be more adapted. Assuming that measurements of consumed oxygen and produced carbon dioxide are available at every constant sample time, the discrete-time version of such dynamical models it is explored, thus obtaining a model of the form:

$$\mathbf{x}(k+1) = \mathbf{A}(\theta)\mathbf{x}(k) + \mathbf{B}(\theta)u(k) + \mathbf{B}(\theta)w_0 \quad (\text{IV.1})$$

where the system states at the time instant k are denoted as $\mathbf{x}(k) \in \mathbb{R}^3$ and their successors are denoted as $\mathbf{x}(k+1) \in \mathbb{R}^3$. These states concern the mass per unit time of consumed oxygen, denoted here as O_2 , produced carbon dioxide through aerobic pathway, denoted as CO_2 and excess carbon dioxide produced by anaerobic pathway, denoted as εCO_2 .

The matrices $\mathbf{A}(\theta) \in \mathbb{R}^{3 \times 3}$ and $\mathbf{B}(\theta) \in \mathbb{R}^{3 \times 1}$ are matrices which depend on a vector of constant parameters, denoted θ . These parameters have to be identified for each studied individual. The system inputs, which are responsible for future changes in the state evolution, are $u(k)$ and w , the pedal power and the consumed power at rest, respectively. In this work

it is considered that the input w_0 is a constant input to be identified. Also, it is assumed that these inputs are the image of the ATP production, O_2 consumption and CO_2 production.

In practice, the measurements of the mass per unit time of consumed oxygen and produced carbon dioxide as well as the measurement of the pedal power are available at every time-instant. However, it is not possible to explicitly distinguish the origin of the produced carbon dioxide. It can be produced from aerobic pathway and/or from anaerobic pathway. For this reason, the system output is modeled as a pair of sensors on the total mass per unit time of consumed oxygen and total mass per unit time of produced carbon dioxide. That is, $\mathbf{y}(k) \in \mathbb{R}^2$ described as

$$\mathbf{y}(k) = \mathbf{C}(\rho(k))\mathbf{x}(k) \quad (\text{IV.2})$$

where the matrix $\mathbf{C}(\rho(k)) \in \mathbb{R}^{2 \times 3}$ is intended for including the different additive contributions on the production of carbon dioxide. The varying parameter $\rho(k)$ allows us to include or exclude the anaerobic contribution of carbon dioxide into the measurements. Thus, the excess carbon dioxide is part of the output whenever a certain number of state conditions hold. Here, it is supposed that the varying parameter $\rho(k)$, which verifies $0 \leq \rho(k) \leq 1$, depends on the system vector $\mathbf{x}(k)$. A hyperbolic tangent function, which concerns one of the most commonly-used activation functions in Artificial Neural Network models, is used for modeling the relationship of $\rho(k)$ with respect to the state vector $\mathbf{x}(k)$.

Now, the model identification problem can be stated for finding the optimal vector θ , w_0 and the parameters \mathbf{z}_t and h of the hyperbolic tangent function, which describes the behavior of the varying parameter $\rho(k)$. These parameters must minimize, for instance, the cumulative sum of the norm-2 of the following prediction-error model:

$$\mathbf{e}(k) = \mathbf{y}(k) - \mathbf{y}^m(k) \quad (\text{IV.3})$$

for $k = \{1, \dots, N\}$, where $\mathbf{y}^m(k)$ denotes the available measured output data at the instant k , while $\mathbf{y}(k)$ concerns the obtained output signals by using the model (IV.1)-(IV.2) with measured input data $u(k)$. In this thesis, the term "residuals" indicates the vector whose elements are the prediction-errors (IV.3) for $k = \{1, \dots, N\}$.

IV.3 Novel index for anaerobic threshold identification

In a system, the following principle must be true: "*The mass that enters a system must, by conservation of mass, either leave the system or accumulate within the system*". Then, we decide to work with the mass per unit of time of O_2 and CO_2 , instead of using the volume as is usually done in the literature.

In this way, a novel index for identification of the anaerobic threshold is proposed. It is denoted $z(k)$. It concerns the difference between the mass of oxygen consumption and the mass of carbon dioxide production per unit time. It can be written in terms of $\dot{V}CO_2$ and $\dot{V}O_2$, measured at the instant k , as follows:

$$z(k) := \delta_1 \dot{V}O_2(k) - \delta_2 \dot{V}CO_2(k) \quad (\text{IV.4})$$

where the constants δ_1 and δ_2 correspond to the volumetric mass density in g/l (or equivalently kg/m³) of oxygen and carbon dioxide, respectively. The values used here are $\delta_1 = 1.429$

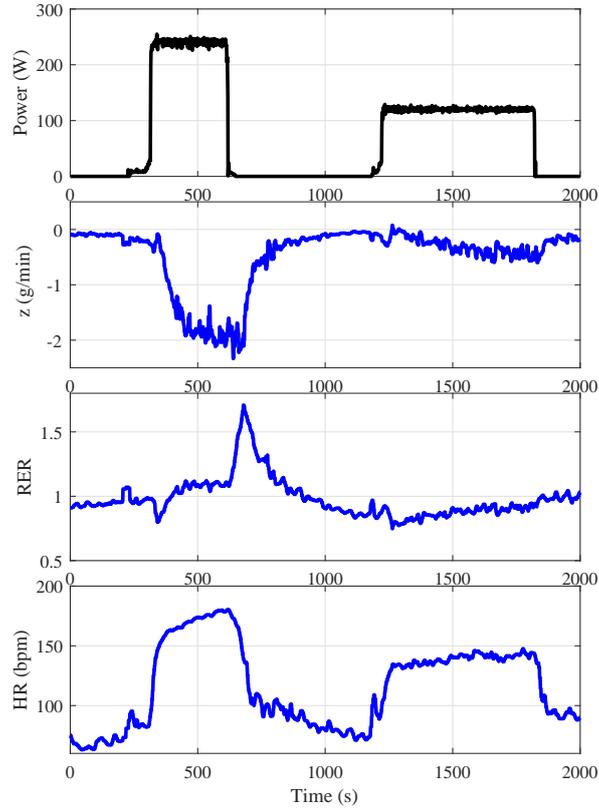


Figure IV.1: Example of evolution of pedal power, the proposed index $z(k)$, the RER index and the HR during high-intensity and moderate workouts.

g/l and $\delta_2 = 1.842$ g/l.

An example of the behavior of the index $z(k)$ for a given individual during a cycling test is shown in Fig. IV.1. Notice that the amplitude of the index $z(k)$ is small and around zero for moderate workout i.e. for pedal power smaller than 100 W, however it takes more negative values for high-intensity workout i.e. for pedal power near 200W. For comparison, the curves concerning the so-called Respiratory Exchange Ratio (RER) and the Heart Rate (HR) have been included in this figure.

The RER is the ratio between the amount of volumes of carbon dioxide output and oxygen consumption. That is:

$$RER = \frac{\dot{V}CO_2}{\dot{V}O_2} \quad (IV.5)$$

This index is usually employed as an approximation of the Respiratory Quotient (RQ) at cellular level and constitutes an indicator of the substrate (carbohydrate or fat), which is being metabolized. If the value of RER is greater than 1, it is associated to high-intensity exercise and it is often used as a criterion for determining the anaerobic threshold, as it is described in [Solberg 2005].

Concerning the HR, it is often used for estimating the amount of pedal power developed by the cyclist. However, this variable does not reach the same values for the same pedal powers and its steady-state values depend on the duration of the exercise. The HR has been largely studied in the past, for example in [Leitner 2014, Meyer 2015a, Corno 2015] and it is not into the scope of this thesis.

In this thesis, it is proposed to use the index (IV.4) instead of the RER or the HR values for estimating the anaerobic threshold. The main motivation of this choice concerns the fact that the index (IV.4) easily allows us to distinguish the level of the exercise (moderate or high-intensity workout). In addition, compared to the RER, the index (IV.4) seems to have a good signal-to-noise ratio. The latter could be explained by the fact that noises on signals are additive in (IV.4) while they are multiplicative in (IV.5). The proposed index $z(k)$ is quite similar to that proposed and analyzed in [Issekutz Jr 1961] for estimation of excess CO_2 production. In the sequel, this index is used for classifying the dynamical data into two modes: low carbon dioxide production (low εCO_2) or high carbon dioxide production (high εCO_2), as it is depicted in Fig.IV.2.

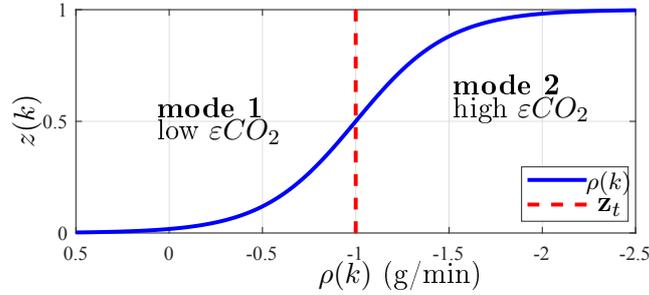


Figure IV.2: A graphical example of a hyperbolic tangent function. It models the progressive transition between two operation modes of the system output. Here the function follows (IV.11) with $z_t = -1$ g/min and $h = 0.5$.

IV.3.1 Structure of the model

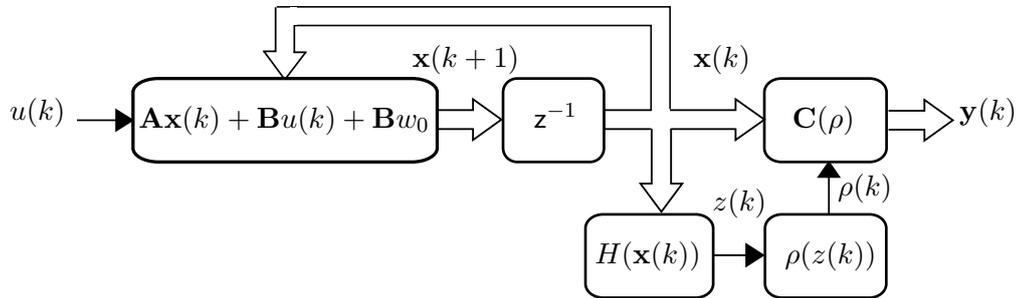


Figure IV.3: Equivalent block diagram of the proposed model.

An equivalent block diagram of the proposed model is depicted in Fig.IV.3. The mechanical power at pedal level is considered to be the input of the system, denoted $u(k)$; the state vector

denoted $\mathbf{x}(k)$ is formed of the mass per unit time of oxygen $x_1 = O_2$, aerobically produced carbon dioxide $x_2 = CO_2$, and excess of carbon dioxide, mainly produced by anaerobic pathway $x_3 = \varepsilon CO_2$. The vector $\mathbf{y}(k)$ is formed of mass per unit time of oxygen consumption and total carbon dioxide production, which can be calculated from $\dot{V}O_2$ and $\dot{V}CO_2$ measured in practice.

$$y_1(k) = x_1(k) \quad (\text{IV.6})$$

$$y_2(k) = x_2(k) + \rho(k)x_3(k) \quad (\text{IV.7})$$

The model provides the evolution of states at every time instant. The function $H(\mathbf{x}(k))$, permits the calculation of the index $z(k)$ in function of the states, it can be also calculated from measured outputs as stated in (IV.4) or from the model outputs as follows:

$$z(k) = y_1(k) - y_2(k) \quad (\text{IV.8})$$

The value of $z(k)$ determines in turn the value of the varying parameter $\rho(k)$ into the output matrix.

The system matrices are considered as follows:

$$\mathbf{A} = \begin{bmatrix} \theta_1 & \theta_2 & 0 \\ 0 & \theta_3 & 0 \\ 0 & \theta_5 & \theta_6 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} \theta_4 \\ \theta_4 \\ \theta_7 \end{bmatrix} \quad (\text{IV.9})$$

$$\mathbf{C}(\rho(k)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \rho(k) \end{bmatrix} \quad (\text{IV.10})$$

where θ_i for $i = \{1, \dots, 7\}$, stands for individual-dependent constant parameters.

The interest of the proposed matrices structure comes from the fact that it is considered that all the system states (mass per unit time of O_2 , CO_2 and εCO_2) are influenced by the pedal power and each of the states responds as a first order dynamical system. The latter is justified by the obtained data and by considering that the system involves storage and dissipation phenomena of such gases. Thus, the main diagonal of the matrix \mathbf{A} and the whole matrix \mathbf{B} consider these assumptions. In addition, in this work we propose the inclusion of two more parameters (θ_2 and θ_5 in \mathbf{A}) for modeling the possible interactions between CO_2 produced in an aerobic way and the values of O_2 and εCO_2 . These interactions can be justified from the chemical reactions (II.4) and (II.5).

The time-varying matrix $\mathbf{C}(\rho(k))$ models the behavior of the measured outputs (the total mass per unit of time of both O_2 and CO_2). The measured CO_2 concerns the sum of the aerobic and anaerobic contributions on CO_2 . The time-varying parameter $\rho(k)$, $0 \leq \rho(k) \leq 1$, conveniently reflects the assumption that a fraction of the excess CO_2 appears into the total measured CO_2 .

IV.3.2 Chosen anaerobic transition function

In this work, the following function is proposed for modeling the relationship between the parameter $\rho(k)$ and the system states:

$$\rho(k) := \rho(z(k)) = 0.5 + 0.5 \tanh\left(\frac{\mathbf{z}_t - z(k)}{h}\right) \quad (\text{IV.11})$$

where $z(k)$ represents the distance between oxygen consumption and carbon dioxide production at the instant k , as it is defined in (IV.4). The symbols \mathbf{z}_t and h represent constant parameters that can be identified from experimental data.

Thus, it is assumed that there exists a progressive transition of the observed εCO_2 on the total measured CO_2 . Hence, the value of \mathbf{z}_t can be seemed as an anaerobic threshold, allowing to define two operation modes, as it is depicted in Fig. IV.2. They are:

$$\begin{aligned} z(k) > \mathbf{z}_t & \text{ for } \mathbf{mode\ 1: low } \varepsilon CO_2 \\ z(k) \leq \mathbf{z}_t & \text{ for } \mathbf{mode\ 2: high } \varepsilon CO_2 \end{aligned} \quad (\text{IV.12})$$

Here, the names associated to these two modes have been chosen for simplicity. Recall that these two modes concerns anaerobic biochemical reactions for producing CO_2 . Thus, the parameter $\rho(k) := \rho(z(k))$ concerns the percentage of the excess of CO_2 , i.e. εCO_2 , contained into the total measured CO_2 .

IV.4 Methodology for parameter identification

IV.4.1 Optimization-based parameter identification

We consider the availability of several sequences of N uniform-sampled measurements for system input (pedal power) and outputs of mass per unit of time for O_2 and CO_2 calculated from measurements of oxygen consumption $\dot{V}O_2$ and carbon dioxide production $\dot{V}CO_2$ in different situations. In the sequel, the proposed model structure is used to find a vector of parameters \mathbf{p} that minimizes the output prediction-error. The identification problem is formulated as:

Find the vector of parameters $\mathbf{p} = [\theta, w_0, \mathbf{z}_t, h]^T$ which minimizes

$$J := \sum_{k=1}^N \|\mathbf{y}(k) - \mathbf{y}^m(k)\|_2 \quad (\text{IV.13})$$

subject to

$$\mathbf{x}(k+1) = \mathbf{A}(\theta)\mathbf{x}(k) + \mathbf{B}(\theta)u(k) + \mathbf{B}(\theta)w_0 \quad (\text{IV.14})$$

$$\mathbf{y}(k) = \mathbf{C}(\rho(k))\mathbf{x}(k) \quad (\text{IV.15})$$

$$\rho(k) = \rho(z(k)) \quad (\text{IV.16})$$

for $k = \{1, \dots, N\}$, $u(k)$ the measured input data, \mathbf{y}^m the measured output data and \mathbf{y} the model output, the function $\rho(z(k))$ defined in (IV.11), and $z(k)$ defined in (IV.4) and computed using measured output data.

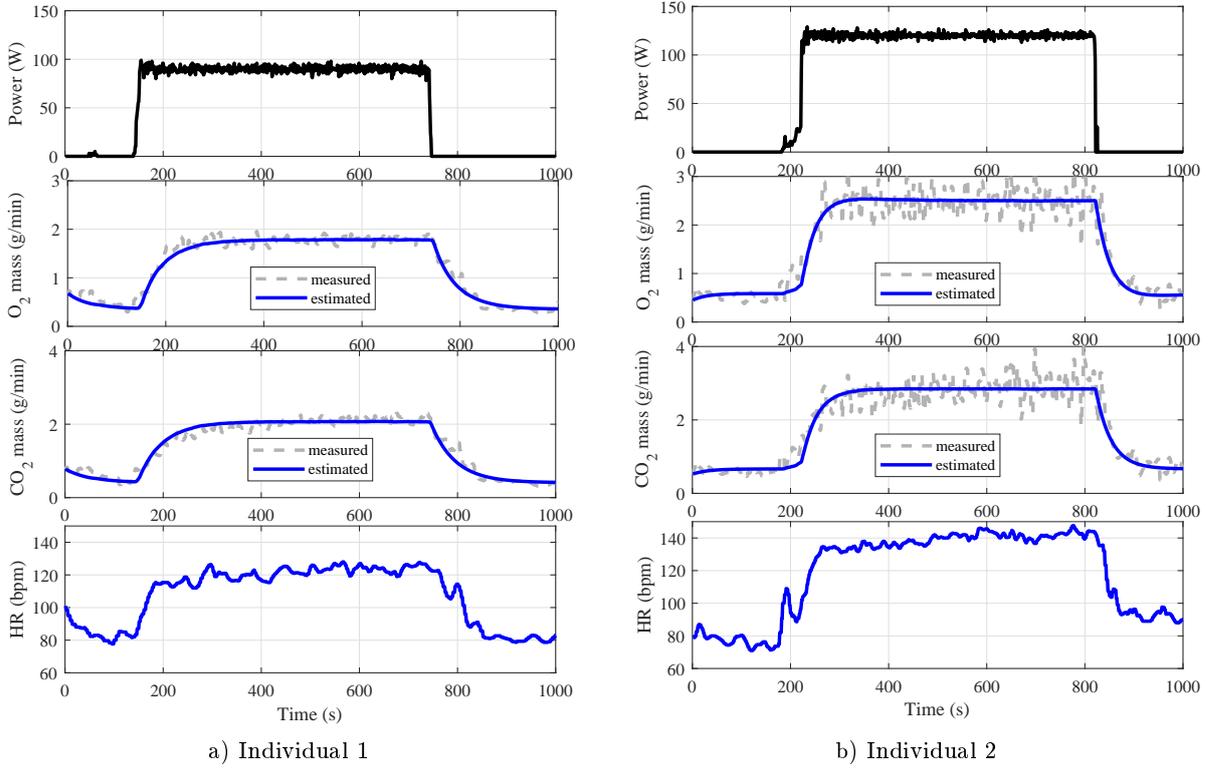


Figure IV.4: Scenario for parameter identification of the aerobic dynamics (fixed $\rho(k) = 0$).

Due to the nature of the problem and the fact that available data contain different modes, this optimization problem cannot be solved in one shot. Therefore, a methodology is proposed to solve the problem in three stages using pre-selected set of data. First, the parametric identification process of the aerobic dynamics, concerning the O_2 and CO_2 states will be dealt. Then, the parametric identification of the anaerobic dynamics, concerning the εCO_2 state. Finally, the parametric identification concerning the transition function $\rho(z(k))$ defined in (IV.11). In the following subsections, the proposed methodology is explained in more detail.

IV.4.2 Identification of the aerobic dynamics

The parametric identification of the aerobic dynamics requires a sequence of data where the pedal power corresponds to an aerobic exercise, i.e. moderate intensity. The identification process is carried out by considering only the parameters describing the aerobic dynamics, that is: “Find θ_i for $i = \{1, \dots, 4\}$ and w_0 ”.

In addition, we fix $\rho = 0$ because it is assumed that there is no excess of carbon dioxide production on the used data set.

The optimization-based identification problem is formulated as follows:

Find the vector of parameters $[\theta_1, \theta_2, \theta_3, \theta_4, w_0]^T$ which minimizes

$$J := \sum_{k=1}^N \|\mathbf{y}(k) - \mathbf{y}^m(k)\|_2$$

subject to

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}(\theta)\mathbf{x}(k) + \mathbf{B}(\theta)u(k) + \mathbf{B}(\theta)w_0 \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) \end{aligned}$$

with matrices $\mathbf{A}(\theta) = \mathbf{A}_1$, $\mathbf{B}(\theta) = \mathbf{B}_1$ and $\mathbf{C} = \mathbf{C}_1$, where the symbol * stands for "no-matter parameters". They are:

$$\mathbf{A}_1 = \begin{bmatrix} \theta_1 & \theta_2 & 0 \\ 0 & \theta_3 & 0 \\ 0 & * & * \end{bmatrix} \quad \mathbf{B}_1 = \begin{bmatrix} \theta_4 \\ \theta_4 \\ * \end{bmatrix} \quad \mathbf{C}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (\text{IV.17})$$

An example of the obtained model fit for a moderate exercise data-set is shown in Fig. IV.4. It shows the results for two individuals. Here, the level of the pedal power has been chosen according to the *a priori* anaerobic threshold of each individual.

IV.4.3 Identification of the anaerobic dynamics

The parametric identification of the anaerobic dynamics requires a sequence of data where the pedal power corresponds to an anaerobic exercise, i.e. a high intensity workout. The identification process is carried out by considering only the parameters describing the anaerobic dynamics; that is: "Find θ_i for $i = \{5, 6, 7\}$ ".

In addition, we fix $\rho = 1$ because it is assumed that the excess of carbon dioxide production on the used data set is considerable. The previous obtained aerobic parameters are also fixed.

The optimization-based identification problem is now formulated as follows:

Find the vector of parameters $[\theta_5, \theta_6, \theta_7]^T$ which minimizes

$$J := \sum_{k=1}^N \|\mathbf{y}(k) - \mathbf{y}^m(k)\|_2$$

subject to

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}(\theta)\mathbf{x}(k) + \mathbf{B}(\theta)u(k) + \mathbf{B}(\theta)w_0 \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) \end{aligned}$$

with matrices $\mathbf{A}(\theta)$ and $\mathbf{B}(\theta)$ defined in (IV.9) and $\mathbf{C} = \mathbf{C}_2$ with:

$$\mathbf{C}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad (\text{IV.18})$$

The pedal power, two measured outputs and estimated outputs using the anaerobic model are shown in Fig. IV.5. Notice that the aerobic dynamics, by considering $\rho = 0$ is not enough to reproduce the carbon dioxide production dynamics. However, in this scenario, for $\rho = 1$, the obtained model reproduces well the excess of carbon dioxide production.

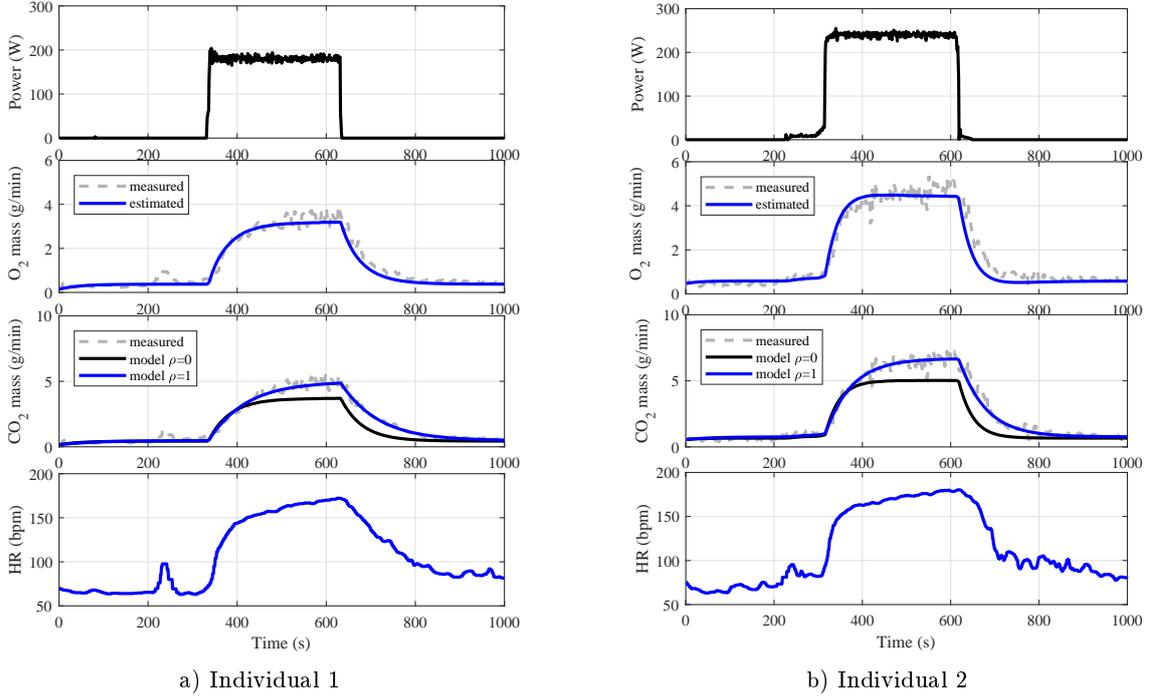


Figure IV.5: Scenario for parameter identification of the anaerobic dynamics (fixed $\rho(k) = 1$). For comparison, the expected aerobic dynamics is also depicted in this figure.

IV.4.4 Identification of transition function

Concerning the identification of the transition function parameters $\rho(z(k))$, given by (IV.11), a data sequence including moderate and high intensity exercise have to be used.

The parameter identification problem can be formulated as follows:

Find the vector of parameters $[z_t, h]$ which minimizes

$$J := \sum_{k=1}^N \|\mathbf{y}(k) - \mathbf{y}^m(k)\|_2$$

subject to

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}(\theta)\mathbf{x}(k) + \mathbf{B}(\theta)u(k) + \mathbf{B}(\theta)w_0 \\ \rho(k) &= \rho(z(k)) \\ \mathbf{y}(k) &= \mathbf{C}(\rho(k))\mathbf{x}(k) \end{aligned}$$

with matrices $\mathbf{A}(\theta)$ and $\mathbf{B}(\theta)$ defined in (IV.9) and using the previously obtained parameters, i.e. $[\theta, w_0]^T$. The matrix $\mathbf{C}(\rho(k))$ is defined in (IV.10) with $\rho(k)$ evolving as the function defined by (IV.11).

The different parameter identification problems, presented in sections IV.4.2, IV.4.3 and IV.4.4, can be solved as non-linear unconstrained optimization problems, by using for instance quasi-Newton methods. The results presented in this thesis have been obtained by using



Figure IV.6: System identification test

`fminsearch` solver proposed by *The MathWorks*. This solver uses the simplex search method presented in [Lagarias 1998], which is not guaranteed to converge to a local minimum.

Once these parameters have been obtained, a validation process can be launched by using different data-sets with different scenarios. In the following section, the model validation process is explained.

IV.4.5 Model validation

The model validation process can be performed by using different data-sets. The validation scenario should include aerobic and anaerobic sequences of exercises. As it is often practiced by engineers, the model can be validated by using, at least, the following validation tests: 1) the whiteness test, 2) the correlation test and 3) the goodness of fit. These tests are applied on the residuals acquired from the prediction-errors (IV.3), for N uniform-sampled data.

A numerical example is presented for illustrating the proposed parametric identification process and its corresponding model validation.

IV.5 Numerical results of gas exchange model

IV.5.1 Experimental setup

The used experimental setup is composed of:

- A mechanical cyclist trainer, Hammer (CycleOps, Madison, USA), which allows setting different cycling scenarios and provides measurements of pedal power and cadence.

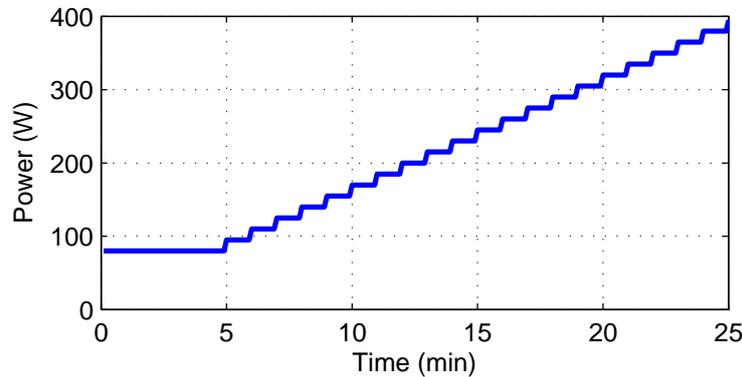


Figure IV.7: Typical profile of an Incremental Cycling Test (ICT) for men.

- A medical equipment to measure respiratory gases exchanges during exercise , Metalyzer 3b-R3 (Cortex Biophysik, Leipzig, Germany).
- A chest belt with Heart Rate sensor Polar H10 (Polar Electro Oy, Kempele, Finland), which transmits Heart Rate data via Bluetooth Low Energy.

IV.5.2 Description of the scenarios

Different data-sets are needed for the identification and further validation of the model. Four scenarios have been chosen for illustration of the proposed parametric identification method.

Incremental Cycling Test (ICT)

It consists in maintaining a specific cadence while the load is increased progressively, i.e. the load is increased 15W per minute. The test is finished when the cyclist is not able to maintain the power output (imposed by the mechanical cyclist trainer). An example of the power during an ICT is shown in Fig. IV.7.

During the ICT the *a priori* anaerobic threshold (AT) [Wasserman 2005] [Svedahl 2003] can be identified using the criterion of $RER > 1$. A power and an oxygen consumption can be associated to this moment.

Constant Power Aerobic Test:

It consists in maintaining a low specific power and constant cadence during pedaling. The power level has been chosen for performing a moderate exercise. The active part has a duration of 10 min and a recovery time of 5 min is also included.

Constant Power Anaerobic Test:

It consists in maintaining a high specific power and constant cadence during pedaling. The power level has been chosen for performing a high intensity workout (twice the power used

during the aerobic test). Here the test has a duration of $5min$. Compared to the previous scenario, the cyclist performs a pedaling with the same level of produced mechanical energy.

Combined Aerobic/Anaerobic Test:

It consists of a scenario which combines several Constant Power Aerobic Tests and Constant Power Anaerobic Tests. The scenario also concerns the periods of time after moderate and high intensity workouts. This scenario has been especially used for calibrating the transition function (IV.11).

IV.5.3 Example of obtained dynamical models

Two healthy young men have performed the previously described cycling tests to obtain gas exchange dynamical models according to the proposed methodology.

In order to establish the demanded power for the different proposed constant power tests, an ICT was realized at a freely chosen cadence (40 rpm and 50 rpm for individual 1 and 2, respectively). The ICT allows us to estimate an *a priori* Anaerobic Threshold (AT). This can be established by using the criterion $RER > 1$. Here, it is estimated that individual 1 has to perform more than 150W for reaching an anaerobic exercise, while individual 2 has to perform more than 200W for reaching the same situation. Thus, the moderate exercises have been performed at 60% of the estimated AT.

The identification of the aerobic dynamics has been performed by using the scenario called Constant Power Aerobic Test. The results are shown in Fig. IV.4. For both cases, the two system outputs, oxygen consumption and carbon dioxide production, present a good fit with the proposed model structure and identified parameters.

After obtaining the aerobic dynamics, the identification of the anaerobic dynamics can be started. This has been performed with data concerning high-intensity exercises, as described in Section 6.2.3. The results of the anaerobic identification are shown in Fig. IV.5.

Once the aerobic and anaerobic dynamics have been identified, the identification of the transition function parameters is done. That is, the parameters \mathbf{z}_t and h are identified by using the whole sequence of data, including moderate and high-intensity workout data.

The matrices that describe the gas exchange model for each individual are obtained. For individual 1:

$$\mathbf{A} = \begin{bmatrix} 0.943 & 0.029 & 0 \\ 0 & 0.981 & 0 \\ 0 & 0.009 & 0.994 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0.363 \\ 0.363 \\ 0.029 \end{bmatrix} \times 10^{-3} \quad (\text{IV.19})$$

and the parameters $w_0 = 18.288$, $\mathbf{z}_t = -0.692$ and $h = 0.487$.

For individual 2:

$$\mathbf{A} = \begin{bmatrix} 0.986 & -0.015 & 0 \\ 0 & 0.973 & 0 \\ 0 & 0.012 & 0.981 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0.523 \\ 0.523 \\ -0.139 \end{bmatrix} \times 10^{-3} \quad (\text{IV.20})$$

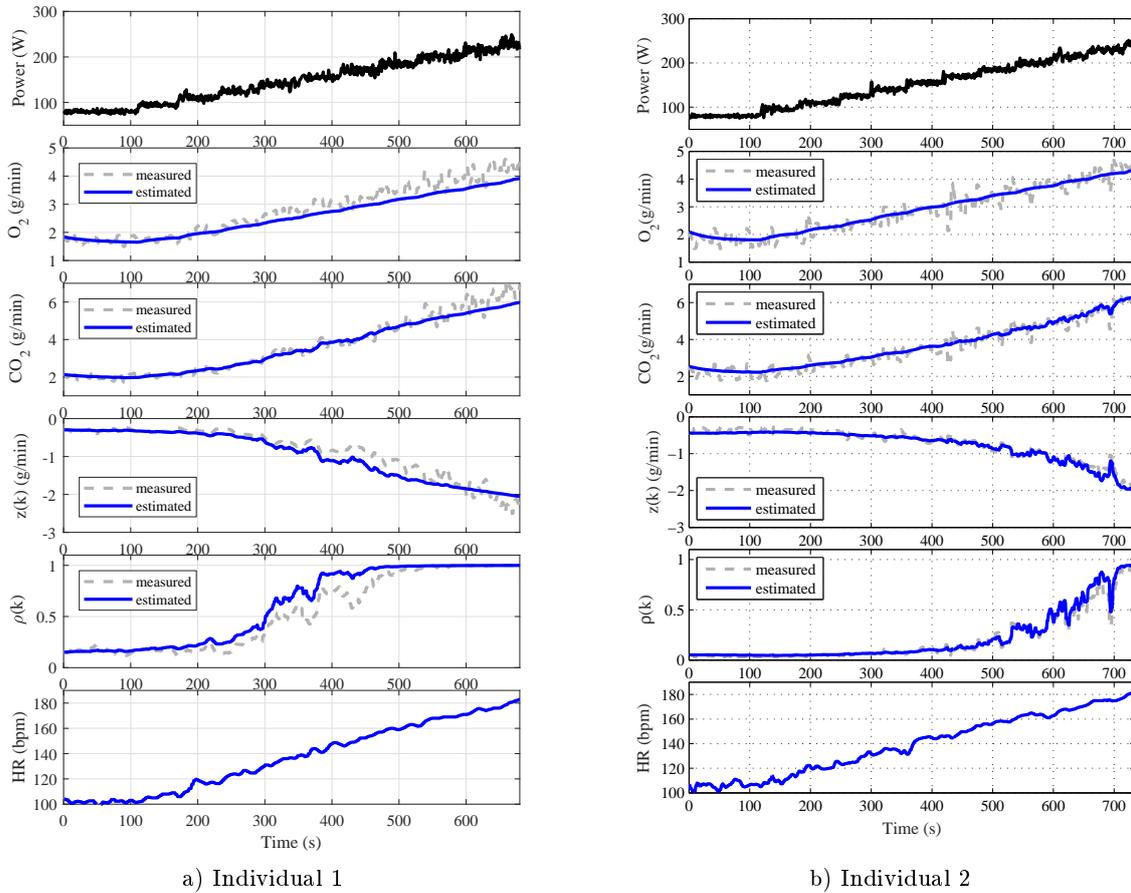


Figure IV.8: Validation of the obtained models for two individuals by using experimental data from an Incremental Cycling Test.

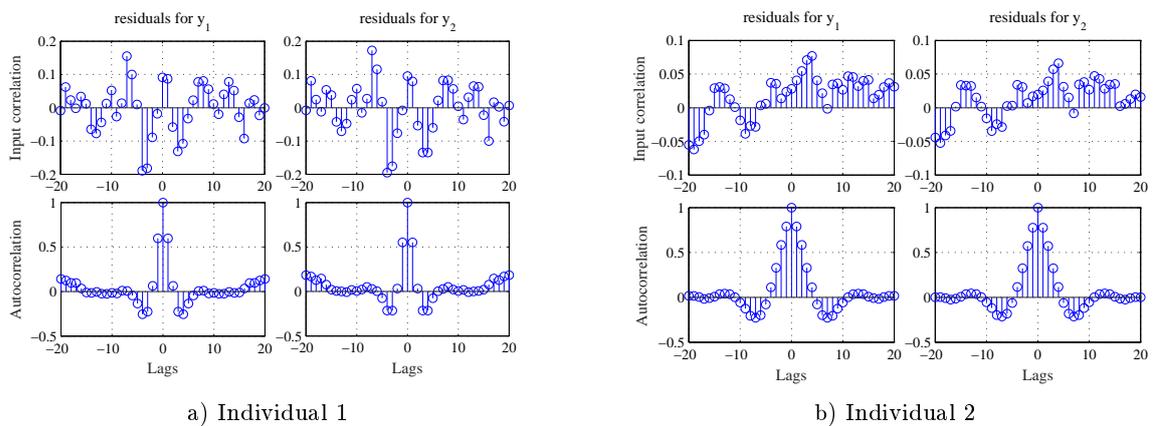


Figure IV.9: Validation of the obtained models for two individuals by using experimental data from an Incremental Cycling Test.

Table IV.1: Minimal and maximal values in g/min of measured mass per unit time of O_2 and CO_2 for two different individuals.

Measure	(g/min)	Individual 1	Individual 2
O_2	min	0.1437	0.1809
	max	5.4666	5.3524
CO_2	min	0.1625	0.2261
	max	7.0919	7.2626

Table IV.2: Prediction-error variance of gas exchange models for two individual and for different experimental scenarios. The ICT has been used for model validation. Outputs concern y_1 and y_2 , the mass per unit time of O_2 and CO_2 , respectively.

Scenario	Predicted output	Individual 1 variance	Individual 2 variance
Const. Power Aerobic Test	y_1	0.012	0.060
	y_2	0.020	0.089
Const. Power Anaerobic Test	y_1	0.041	0.123
	y_2	0.052	0.096
Combined Aero/Anaero Test	y_1	0.021	0.074
	y_2	0.031	0.084
Incremental Cycling Test	y_1	0.039	0.051
	y_2	0.068	0.066

and the parameters $w_0 = 32.03$, $\mathbf{z}_t = -1.218$ and $h = 0.532$.

The obtained models can be validated with a new sequence of data, different than that used for the identification process. Here, data from an ICT scenario is used for such validation.

The estimated outputs from the model and the used experimental data are shown in Fig. IV.8. Notice that the obtained model fitting deviation is small. Hence, the presented curves allow an easy estimation of the anaerobic thresholds (see for instance, the developed power and/or the HR at $\rho(k) = 0.5$).

Table IV.2 presents the obtained variances for the residuals concerning the system outputs for different scenarios. Notice that these variances are relatively small compared to the minimum and maximum values of such system outputs (see Table IV.1)

IV.6 Conclusion

In this chapter a model that relates physiological variables of the cyclist with pedal power is proposed. In this case, the gas exchange variables were chosen given their ease of measurement and their relation with internal chemical reactions.

A novel physiological index is used for the classification of two different modes of the model. The first mode is related to low levels of produced carbon dioxide and the second mode is related to high carbon dioxide production. Throughout the work, the analysis of gas exchange variables is based on masses instead of volume of gases as it is usual in physiology, thereby mass conservation laws are used to obtain a description of physiological phenomena.

A methodology for parameter identification and validation of this model is also presented. It is developed as a solution of a sequence of non-linear unconstrained optimization problems. Measured data of different cycling tests, that include moderate exercises, high-intensity exercises and Incremental Cycling Tests, all of them performed in GIPSA-Lab are used for model calibration.

The model structure and the methodology of parameter identification constitute an innovative approach for the analysis of physiological variables, in this case gas exchange measurements. Other models in the literature as the described ones in chapter II have usually nonlinear parts or are difficult to calibrate. Our model is simple, since it is linear except for the time-varying parameter ρ , into the output matrix, which models the transition between modes.

The experimental results allow considering that this kind of models can be used in simulation, prediction and analysis of consumed oxygen and produced carbon dioxide by using pedal power data.

Robust estimation of carbon dioxide production

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V.1 Introduction

The implementation of control laws often requires the knowledge of the system states to make decisions, thus it is convenient to have good estimations of states and hence reduce the quantity of sensors, especially for road cycling applications. In this aspect, observers are a useful tool.

This chapter presents a set-membership observer for the estimation of carbon dioxide production during cycling and moreover provides a deterministic computation for the error bounds. The observer consider the measurements of oxygen consumption and power at pedal level for the human-bicycle system defined by the model described in Chapter IV. The real process is assumed to be perturbed by unknown but bounded disturbances in measurements and model.

A brief explanation about the architecture and design of the observer are described in Sections V.2 and V.3. Then, the results of state estimation for a given individual are presented in Section V.4.

V.2 Architecture for estimation of carbon dioxide production

An architecture for the estimation of carbon dioxide production during cycling is proposed in this chapter, in order to take advantage of the useful physiological information that can be measured through sensors for the estimation of other variables. This approach can be applied to quantify in some way the current physiological state of the cyclist which is directly linked to the excess in carbon dioxide production.

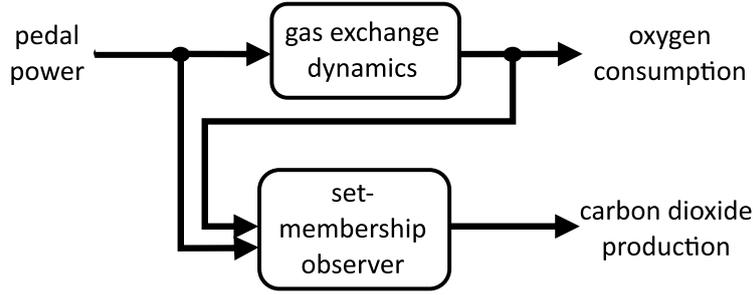


Figure V.1: Scheme of set-membership observer designed using the gas exchange model

The scheme for the implementation of the observer is depicted in Fig. V.1. The complete human-bicycle system acts like a system of energy transformation, where the power at pedal level could be understood as an image of internal chemical reactions, which involve oxygen consumption and carbon dioxide production.

The real system represented by the block “Gas exchange dynamics”, has a power input and two outputs, which can be all measured in practice. However, for observation purposes it is assumed that the input and only the output of oxygen consumption are the available measurements. The validated model of gas exchange dynamics presented in Chapter IV is utilized for the implementation of the observer.

The state estimation is performed by using a set-membership observer, which takes the power and oxygen consumption measurements and gives an estimation of the three states of the system. A non-linear output matrix, which depends on the estimated states is used for providing the carbon dioxide estimation.

The components of the proposed architecture are discussed in more detail in the next sections.

V.3 Design of a set-membership observer

In order to estimate the quantity of carbon dioxide production during cycling, the discrete-time system corresponding to the gas exchange dynamical model is:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}u(k) + \mathbf{B}w_0 + \mathbf{F}\mathbf{d}(k) \quad (\text{V.1})$$

$$y(k) = x_1(k) + G_1v(k) \quad (\text{V.2})$$

where only the availability of the first state x_1 , i.e. oxygen consumption measurement is considered as output $y(k) = x_1(k)$. The vector $\mathbf{d}(k)$ and the scalar $v(k)$ are unknown bounded process disturbances and unknown bounded measurement noise, respectively.

We propose to design a set-membership observer as the presented in [Loukkas 2017]. It has the form:

$$\hat{\mathbf{x}}(k+1) = (\mathbf{A} - \mathbf{LC})\mathbf{x}(k) + \mathbf{B}u(k) + \mathbf{B}w_0 + \mathbf{L}y(k) \quad (\text{V.3})$$

$$\mu(k+1) = (1 - \lambda)\mu(k) + \lambda \quad (\text{V.4})$$

$$\bar{\mathbf{x}}(k) = \hat{\mathbf{x}}(k) + \bar{\mathbf{e}}_\infty \mu^{1/2} \quad (\text{V.5})$$

$$\underline{\mathbf{x}}(k) = \hat{\mathbf{x}}(k) - \bar{\mathbf{e}}_\infty \mu^{1/2} \quad (\text{V.6})$$

where (V.3) concerns a punctual observer with a matrix \mathbf{L} computed using a suitable observer design method. Here the observer gain \mathbf{L} is computed using an H_∞ observer design approach in order to minimize the norm-2 of the estimation error with respect to the norm-2 of the disturbance. Equations (V.4)-(V.6) concern the estimation of the system state bounds. These bounds depend on the *a priori* known bounds of the system disturbances. In particular, the term $\bar{\mathbf{e}}_\infty$ stands for a constant column vector defined as follows:

$$\bar{\mathbf{e}}_\infty = \text{diag} \left(\left(\frac{\mathbf{P}}{\frac{1}{\lambda} \gamma^2 \bar{\mathbf{w}}^T \bar{\mathbf{w}}} \right)^{-1/2} \right) \quad (\text{V.7})$$

where \mathbf{P} stands for a positive definite matrix, γ and $\lambda \leq 1$ are positive scalars. The symbol $\bar{\mathbf{w}}$ stands for a vector with positive elements which bounds the system disturbances. That is, at every time-instant k , the disturbance satisfies $\mathbf{w}(k) \leq \bar{\mathbf{w}}$ (this inequality is interpreted as an elementwise inequality). Here, the system disturbances $\mathbf{w}(k) \in \mathbb{R}^m$ are defined as follows:

$$\mathbf{w}(k) := \begin{pmatrix} \mathbf{d}(k) \\ \mathbf{v}(k) \end{pmatrix} \quad (\text{V.8})$$

For a given observer gain matrix \mathbf{L} , the matrix \mathbf{P} and the positive scalars γ and λ are obtained by considering the estimation error dynamics:

$$\mathbf{e}(k+1) = \mathbf{A}_o \mathbf{e}(k) + \mathbf{E} \mathbf{w}(k) \quad (\text{V.9})$$

with $\mathbf{A}_o := (\mathbf{A} - \mathbf{LC})$ and $\mathbf{E} := [\mathbf{F} \quad -\mathbf{LG}]$, and by using the following well-known result [Boyd 1994]

Proposition V.1

(Bounded-real lemma) System (V.9) is stable if there exist a symmetric positive definite matrix $\mathbf{P} \succ 0$ and a positive scalar $\gamma > 0$ such that,

$$\begin{pmatrix} \mathbf{A}_o^T \mathbf{P} \mathbf{A}_o - \mathbf{P} + \mathbf{Q} & \mathbf{A}_o^T \mathbf{P} \mathbf{E} \\ \mathbf{E}^T \mathbf{P} \mathbf{A}_o & \mathbf{E}^T \mathbf{P} \mathbf{E} - \gamma^2 \mathbf{I}_m \end{pmatrix} \preceq 0 \quad (\text{V.10})$$

where $\mathbf{Q} \in \mathbb{R}^{n \times n}$ is a given (arbitrary) symmetric positive definite matrix and $\mathbf{I}_m \in \mathbb{R}^{m \times m}$ is an identity matrix. In addition, system (V.9), with output $\zeta(k) := \mathbf{Q}^{1/2} \mathbf{e}(k)$ and input $\mathbf{w}(k)$, has a Quadratic H_∞ performance equal to γ .

Here, the matrix \mathbf{Q} in (V.10) is computed as $\mathbf{Q} = \mathbf{V}^{-1}$, where the matrix \mathbf{V} denotes the steady-state estimation error covariance matrix, that can be obtained by solving the following Lyapunov equation, see for instance [Kofman 2012]:

$$\mathbf{A}_o \mathbf{V} \mathbf{A}_o^T - \mathbf{V} = -\mathbf{E} \mathbf{W} \mathbf{E}^T \quad (\text{V.11})$$

where $\mathbf{W} \in \mathbb{R}^{m \times m}$ concerns the covariance matrix of disturbances $\mathbf{w}(k)$.

The scalar λ is obtained as the minimum generalized eigenvalue of the pair (\mathbf{Q}, \mathbf{P}) .

Finally, in order to initialize the set-membership observer (V.3)-(V.6), it is supposed that the initial estimation error \mathbf{e}_0 is known and belongs, for instance, to a known ball, that is, $\mathbf{e}_0^T \mathbf{e}_0 \leq \delta^2$, for a given constant positive scalar δ . Then, as stated in [Loukkas 2017], the initial value of the scalar μ in (V.4) has to verify:

$$\mu_0 \geq \lambda_{\max}(\mathbf{P}) \delta^2 / \bar{c} \quad (\text{V.12})$$

with $\bar{c} = \frac{1}{\lambda} \gamma^2 \bar{\mathbf{w}}^T \bar{\mathbf{w}}$.

The complete observer design process is summarized in Algorithm 1. See [Martinez 2018] for more details.

Algorithm 1 Set-membership observer design.

Require: Matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{F}$ and \mathbf{Z} describing system (??), the observer gain \mathbf{L} obtained by performing an H_∞ observer synthesis and the covariance matrix of disturbances \mathbf{W} .

- 1: Compute $\mathbf{A}_o = \mathbf{A} - \mathbf{L}\mathbf{C}$.
 - 2: Compute $\mathbf{E} = [\mathbf{F} \quad -\mathbf{L}\mathbf{Z}]$
 - 3: Compute the covariance matrix \mathbf{V} using (V.11).
 - 4: Do $\mathbf{Q} = \mathbf{V}^{-1}$.
 - 5: Find a matrix \mathbf{P} and the minimum γ which satisfies the LMI (V.10).
 - 6: Compute λ as the minimum generalized eigenvalue of the pair (\mathbf{Q}, \mathbf{P}) .
 - 7: **Return** The observer parameters $\mathbf{L}, \mathbf{P}, \gamma$ and λ .
-

V.4 Data-based obtained results

The following matrices describing the gas exchange model are taken from the results presented in Chapter IV. Then:

$$\mathbf{A} = \begin{bmatrix} 0.944 & 0.029 & 0 \\ 0 & 0.981 & 0 \\ 0 & 0.001 & 0.994 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0.363 \\ 0.363 \\ 0.023 \end{bmatrix} \times 10^{-3} \quad (\text{V.13})$$

and the parameters $w_0 = 18.288$, $\mathbf{z}_t = -0.507$ and $h = 0.0452$.

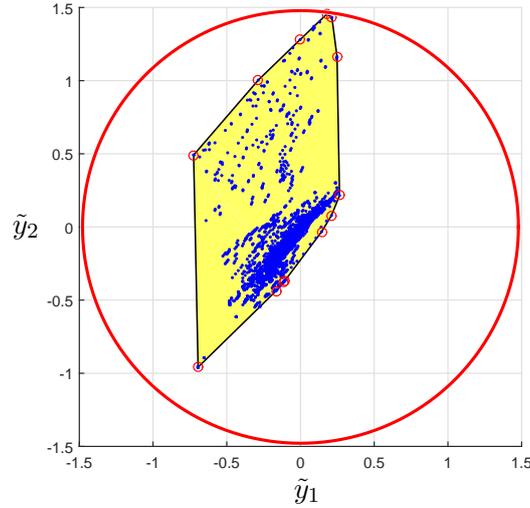


Figure V.2: Ball containing all the output errors from the identified system

The previous values are valid only for the subject of study, because they depend on the physical condition of the cyclist. However, model structure and matrix \mathbf{C}_t are the same regardless of the individual, as well as the proposed methodology for the observer design.

V.4.1 Estimation of the error values

The output error or measurement error $\mathbf{G}v(k)$ can be calculated in a deterministic way because of the availability of the measured data used for the identification and for further validation of the model.

The output error $\tilde{\mathbf{y}}$ is defined as the difference between the model output $\mathbf{y}(k)$ and the measured data $\mathbf{y}^m(k)$:

$$\tilde{\mathbf{y}}(k) = \mathbf{y}(k) - \mathbf{y}^m(k) \quad (\text{V.14})$$

Remember that for identification, two measured variables are used: oxygen consumption (y_1) and total carbon dioxide production (y_2), i.e. the vector $\mathbf{y} = [y_1, y_2]^T$. All the values of the error can be plotted in an error space $(\tilde{y}_1, \tilde{y}_2)$ as shown in Fig.V.2.

A polyhedron that contains all the error points is calculated. Then, a ball which contains the furthest vertex of the polyhedron, is also calculated. The radius of the ball is considered the maximal value of the output model. It is:

$$\text{radius} = 1.478$$

As shown in Fig.V.2, the ball constitutes a conservative estimation of the error, but assures the inclusion of all the measurement error. Thus for the output given by:

$$y_1(k) = \mathbf{C}_1 \mathbf{x}(k) + G_1 v(k) \quad (\text{V.15})$$

which is considered as the only output for the set-membership observer, we set $G_1 = 1.478$ and $v(k) = 1$.

For choosing the model error, a value of 30% of the off-set value $\mathbf{B}w_0$ is chosen, which corresponds to 1.993×10^{-3} for each state. As the values of disturbances must have appropriate dimensions, a vector $\mathbf{F} = 1.993 \times 10^{-3}[1, 1, 1]^T$ and $d(k) = 1$ are chosen.

V.4.2 Computation of observer gain matrix

According to the procedure suggested in [Loukkas 2017], in this example, firstly the LMI given by (V.10) is solved assuming \mathbf{Q} as an identity matrix, for obtaining initial values of \mathbf{P} and γ and a matrix \mathbf{L} as:

$$\mathbf{L} = \begin{bmatrix} 0.157 \\ 0.064 \\ 0.012 \end{bmatrix} \times 10^{-3} \quad (\text{V.16})$$

Afterwards, a new value of $\mathbf{Q} = \mathbf{V}^{-1}$ where \mathbf{V} obtained by solving (V.11) and $\mathbf{W} = \text{diag}([0.33, 0, 33])$ is provided as:

$$\mathbf{Q} = \begin{bmatrix} 6.37 & -5.93 & 1.63 \\ -5.93 & 5.58 & -1.73 \\ 1.63 & -1.73 & 2.01 \end{bmatrix} \times 10^6 \quad (\text{V.17})$$

Now, solving the LMI (V.10) with this matrix \mathbf{Q} , the appropriate values of \mathbf{P} and the minimum value of γ are obtained as:

$$\mathbf{P} = \begin{bmatrix} 2.26 & -2.00 & 0.22 \\ -2.00 & 1.92 & -0.54 \\ 0.22 & -0.54 & 2.97 \end{bmatrix} \times 10^8 \quad \gamma = 37.96 \quad (\text{V.18})$$

The obtained scalar λ was 2.612×10^{-3} , obtained as the minimum generalized eigenvalue of the pair (\mathbf{Q}, \mathbf{P}) .

All these observer data allow the computation of the following element-wise steady-state estimation-error bound:

$$\bar{\mathbf{e}}_\infty = \begin{bmatrix} 0.222 \\ 0.262 \\ 0.063 \end{bmatrix} \quad (\text{V.19})$$

For the simulation, the initial value of μ_0 is set to 2, which means that the initial value of the error is twice the error at steady-state value. This assumption assures the inclusion of initial conditions and therefore verifies equation (V.12).

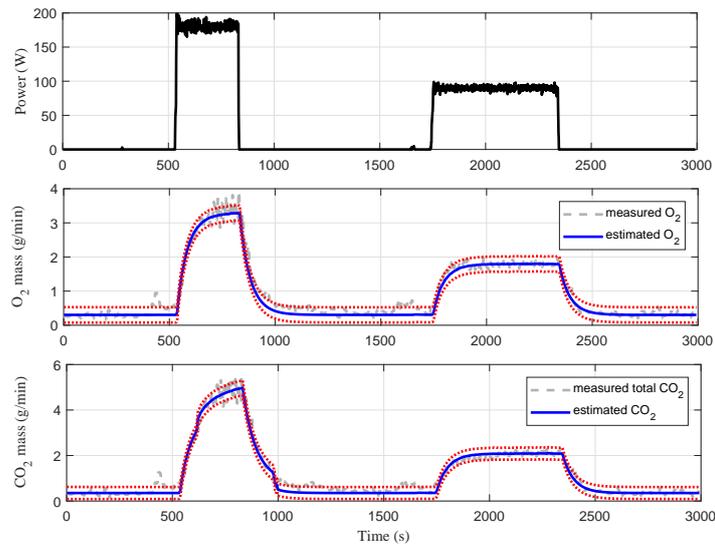


Figure V.3: Results of the set-membership observer using the gas exchange model

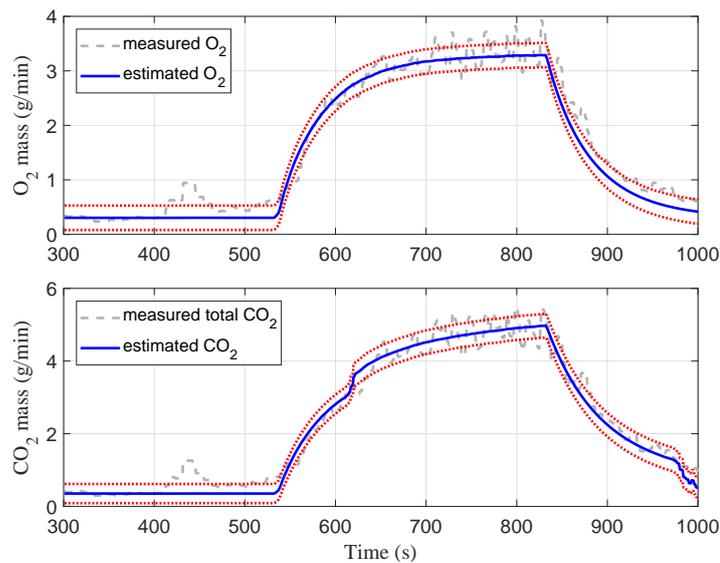


Figure V.4: Zoomed view of the outputs of Fig.V.3 between $t=300$ s and 1000 s. Data outside the bounds are noise with respect to the dynamical model.

V.4.3 Results of state estimation

The results of the estimation of carbon dioxide production are shown in Fig.V.3. The measured data are also included in the plot for comparing the dynamical behavior. This scenario contains two intensities of exercise, first a high intensity exercise, and after a moderate exercise.

The dotted lines correspond to the bounds obtained from the set-membership state ob-

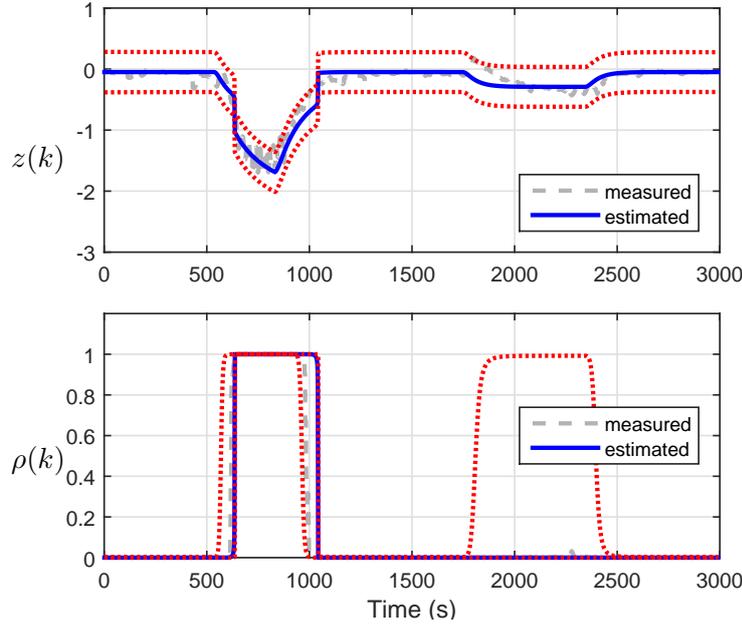


Figure V.5: $z(k)$ and $\rho(k)$ obtained from measured data and from the estimated states

server. Note that carbon dioxide production (CO_2 mass) is the sum of two states, thus its error bounds result from the operation of bounds for both states.

The values of parameters $z(k)$ and $\rho(k)$ can be seen in Fig.V.5. The estimated $z(k)$ is calculated using the values of the estimates of x_2 and x_3 , then the transition function of $\rho(k)$ is applied for obtaining the value of $\rho(k)$

$$\rho(k) := \rho(z(k)) = 0.5 + 0.5 \tanh\left(\frac{z_t - z(k)}{h}\right) \quad (\text{V.20})$$

For moderate exercise the bounds of $\rho(k)$ can include values corresponding to high intensity exercise, because of values near to $z(k) = 0.5$.

An advantage of this observer is that the bounds give a deterministic interval in which the existence of the states is certain. As it can be seen before the transients (around 500 s and 1600 s), there are some measured data outside the bounds, but they actually correspond to changes in body position and are not related to pedal power. The same situation occurs at the end of the first peak, but this is a known measurement noise introduced because the hyperventilation of the subject. A zoom between $t=300$ s and 1000 s is presented in Fig. V.4 to better see this phenomenon. In both cases, bounds are useful for filtering data, and for state estimation from the other side.

V.5 Conclusion

In this chapter a set-membership observer for the estimation of carbon dioxide production during cycling is presented. The results show a good estimation of measured and non-measured

states.

Furthermore, the proposed observer is able to provide, at every time-instant, accurate bounds of the estimated state. It is guaranteed that the state bounds include the actual state in presence of bounded disturbances and other physiological phenomena, which are not related with the pedaling power. Thus, the deterministic bounds provided by the observer constitute a useful tool for filtering data.

In the case of control design there are several control-strategies that includes switching. The existence of these bounds can be useful for avoiding over-switching in control laws based on the estimation of carbon dioxide production.

The proposed technique allows knowing carbon dioxide production using less sensors. Consequently, after a proper calibration of the model and the observer, it is possible to know the carbon dioxide production only measuring the oxygen consumption and pedal power. It could reduce the implementation cost of future solutions, for example avoiding the use of expensive chemical sensors of CO_2 .

On the other hand, since the carbon dioxide production is related with the fatigue of the cyclist, the possibility to know the value of this variable can be applied, together with other measured variables as HR and oxygen consumption, for a complete real-time assessment of the physiological state of the cyclist.

Control approach for physiology-aware assistance

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VI.1 Introduction

The identification of an adequate intensity of exercise that assures some safety constraints has been a research topic in Physiology since the 1960s. The objective of doing that, has been mainly neither not over-stressing patients suffering from cardiovascular disease nor not surpassing a limit on lactate blood concentration for maximizing endurance of athletes. In general, the goal is to improve the metabolic efficiency of a cyclist to generate power. This efficiency can be evaluated through the supervision of some markers in the gas exchange dynamics, which is one of the easiest solutions for studying the efficiency during cycling, because of its noninvasive characteristics.

In this chapter a method for finding the maximal mechanical power that can be developed for a cyclist without excess of carbon dioxide production is presented. The method is based on the discrete-time linear parameter-varying model of gas exchange dynamics presented in Chapter IV. The division of the output space depending on the value of the parameter z is described in Section VI.3 for introducing the topic. Some theory about the computation of ellipsoidal RPI sets is also included in Section VI.4.

A simple formulation of the problem statement is explained in Section VI.2 and then reformulated using ellipsoidal invariant set approach in Section VI.5. This approach characterizes

all possible trajectories of the system state in presence of a bounded pedal power. The latter allows solving the problem as a geometrical optimization problem involving an inequality constraint concerning the measurements of produced carbon dioxide and consumed oxygen. A numerical example concerning two scenarios, which cover moderate and high intensity exercises is also included in Section VI.6 .

VI.2 Problem statement

During cyclo-ergometer tests or road cycling, it is desirable to perform mechanical power in an efficient and/or safe way; for instance, to find a cadence for obtaining a suitable performance for athletes or to find an intensity of exercise that guarantees a healthy condition for people with cardiovascular or pulmonary diseases. Then, an interesting problem to solve can be formulated as follows: for a given individual, find the optimal intensity of exercise that satisfies a set of metabolic constraints.

The metabolic constraints can involve several criteria, for instance: medical, training or just personal criteria. Optimal exercise intensity can be useful for health assessment, evaluation of training programs, categorization of the perceived effort, improving exercise performance, avoidance of the development or progression of diseases. See [Svedahl 2003] and [Binder 2008].

Additionally, one of the main hindrances in the search of generalized solutions for exercise intensities is given by the fact that each subject has his own characteristics and limitations, which are highly stochastic and can change during time by age, training or health conditions, among others.

Concerning the constraints, for the purpose of this work, it was chosen to limit the excess CO_2 production of a given cyclist. Recall that limiting excess CO_2 could be useful for maintaining high levels of metabolic efficiency, and possibly decreasing the sensation of fatigue.

Considering the mechanisms for producing energy in the human body, see [McArdle 2006], it is accepted that the aerobic pathway is the most efficient and facilitates endurance.

The interest of this analysis is maximizing the intensity of exercise by avoiding an over-production of CO_2 . According to the model presented in Section 2, the difference between oxygen consumption and carbon dioxide output, called here $z(k)$, can be used for establishing a suitable metabolic region for performing any exercise, i.e. maintain the system dynamics in the aerobic mode.

The trajectories of the system states depend on the input $u(k)$ and are governed by the dynamical equation:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}u(k) + \mathbf{B}w_o \quad (\text{VI.1})$$

$$\mathbf{y}(k) = \mathbf{C}(\rho(k))\mathbf{x}(k) \quad (\text{VI.2})$$

Since the input $u(k)$ is produced by the cyclist, it cannot be controlled but only bounded in amplitude. Then the upper bound of $u(k)$, called here $\bar{\mathbf{u}}$, constitutes the only degree of freedom to act on system states trajectories.

The use of the set-invariance concept to establish the sufficient conditions for maintaining a given individual into an aerobic region during exercise is explored, therefore , the problem

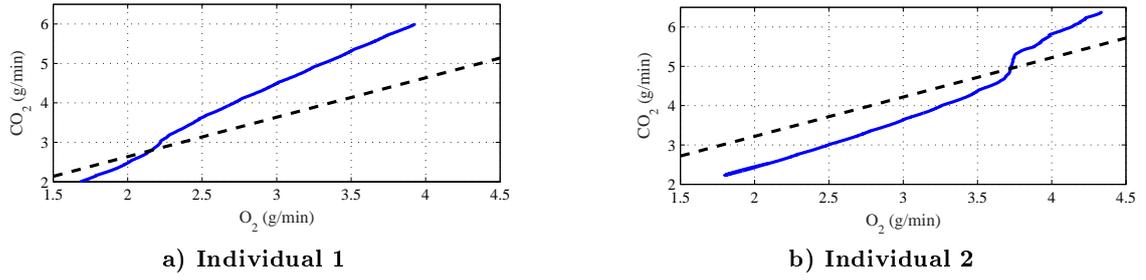


Figure VI.1: The output space divided by the line $z(k) = \mathbf{z}_t$ (dashed line) indicating the transition between mode 1 (low εCO_2) and mode 2 (high εCO_2). Solid lines corresponds to the measured data.

can be initially formulated as:

Problem VI.1

For the system (??), find the bound \bar{u} which defines the domain of $u(k)$, i.e. $0 \leq u(k) \leq \bar{u}$ such that for all time-instants $k \geq 0$, it is assured that $z(k) > \mathbf{z}_t$ (i.e. the states belong to the aerobic region).

The use of the invariant-set approach is proposed as a useful tool for addressing this problem for any cycling scenario.

VI.3 Regions in output space

The methodology for obtaining the gas exchange model consists in the identification of two models; one aerobic and other anaerobic. For intermediate scenarios, a transition is given through the varying parameter $\rho(k)$. The determination of the mode depends on the value of $z(k)$. Then, it is possible to establish two zones in the output space; first one for aerobic mode and second one for anaerobic mode, where excess of carbon dioxide is produced.

Fig. VI.1 shows the output space for an ICT scenario divided by the dashed line $\mathbf{z}_t - z(k) = 0$, which determines whether the data belong to aerobic (below the dashed lines) or anaerobic mode (above the dashed line). During the ICT, since the power is increasing linearly with the time, the crossing point between the two modes only occurs once. For other cycling scenarios when aerobic and anaerobic pathways are used alternately, it occurs many times and it is difficult to determine the mode which is being used.

In the V-slope method, a change in the slope of the curve $\dot{V}CO_2$ vs. $\dot{V}O_2$ is detected, resulting in a point in this plot, which determines the AT. The method uses linear regressions to establish a function of carbon dioxide production and oxygen consumption while power is increasing, i.e. a static model with an equation $\dot{V}CO_2 = f(\dot{V}O_2)$ for aerobic mode and other for anaerobic mode. The main disadvantage of this result is that these equations are only valid during ICT with the same protocol. Moreover, factors as noise can affect the belonging of the data to the lines.

The proposed dynamical model takes into account storage phenomena and relationships

between variables that are valid for any cycling scenario. The mode detection, in this case, is determined by a line in the output space, and not just by one point. Therefore, even if the data are very noisy, they have to belong to a set in the output space and not to a line equation, which makes easier the mode identification for any scenario.

From Fig.VI.1, it is possible to conclude that subjects have different levels of training. It can be seen from the final power achieved in ICT, the difference in AT and also the transition line between modes presented in Fig.VI.1. Despite this, the methodology and the model structure is valid for both cases.

VI.4 Computing ellipsoidal RPI sets

VI.4.1 The bounded-real lemma

The bounded-real lemma can be used to establish quadratic H_∞ performance of discrete-time linear systems, see for instance [Apkarian 1995]. Here, this result is used for obtaining invariant sets as proposed in [Martinez 2018]. The following proposition considers the discrete-time linear system in the form of (??), where the matrices $\mathbf{A} \in \mathbb{R}^{n \times n}$ and $\mathbf{B} \in \mathbb{R}^{n \times m}$ are real matrices.

Consider the following candidate Lyapunov function for the system (??):

$$V(\mathbf{x}(k)) = \mathbf{x}(k)^T \mathbf{P} \mathbf{x}(k) \geq 0 \quad (\text{VI.3})$$

Suppose there exist $\mathbf{P} \succ 0$ and a scalar $\gamma > 0$ verifying the following *dissipation* inequality:

$$V(\mathbf{x}(k+1)) - V(\mathbf{x}(k)) \leq -\mathbf{x}(k)^T \mathbf{Q} \mathbf{x}(k) + \gamma^2 u(k)^T u(k) \quad (\text{VI.4})$$

where $\mathbf{Q} \in \mathbb{R}^{n \times n}$ is a given (arbitrary) symmetric positive definite matrix.

Thus, the \mathcal{L}_2 gain of the input/output map is bounded by γ , which leads to the conclusion as stated in [Apkarian 1995], that the system has a Quadratic H_∞ performance equal to γ .

An equivalent Linear Matrix Inequalities representation of the Bounded-real lemma is presented in [Boyd 1994].

Proposition VI.1

The system (??) is stable if there exist a symmetric positive definite matrix $\mathbf{P} \succ 0$ and a positive scalar $\gamma > 0$ such that

$$\begin{pmatrix} -\mathbf{P} + \mathbf{Q} & 0 & \mathbf{A}^T \mathbf{P} \\ 0 & -\gamma^2 \mathbf{I}_m & \mathbf{B}^T \mathbf{P} \\ \mathbf{P} \mathbf{A} & \mathbf{P} \mathbf{B} & -\mathbf{P} \end{pmatrix} \preceq 0 \quad (\text{VI.5})$$

where $\mathbf{Q} \in \mathbb{R}^{n \times n}$ is a given (arbitrary) symmetric positive definite matrix and $\mathbf{I}_m \in \mathbb{R}^{m \times m}$ is an identity matrix. In addition, system (1), with output $\zeta(k) := \mathbf{Q}^{1/2} \mathbf{x}(k)$ and input $u(k)$, has a Quadratic H_∞ performance equal to γ .

VI.4.2 Ellipsoidal RPI sets

Let us introduce now a definition related to practical stability such as the robust positive invariant set, see for instance [Blanchini 1999].

Definition VI.1

(Robustly Positive Invariant set) The set $\Psi \in \mathbb{R}^n$ is a robustly positively invariant (RPI) set for the discrete-time linear system (??) with bounded input $u(k)$, if for any $\mathbf{x}(k) \in \Psi$, it holds that $\mathbf{x}(k+1) \in \Psi$ for all $k \geq 0$.

Now the following theorem, based on the results presented in [Martínez 2015], allows the computation of ellipsoidal RPI sets.

Theorem VI.1

Consider the system (??) with a bounded input $u(k)$ which verifies $0 \leq u(k) \leq \bar{u}$. If there exist a symmetric positive definite matrix \mathbf{P} and a scalar $\gamma > 0$ verifying condition (VI.5), for a given matrix \mathbf{Q} , then the following set is a Robust Positive Invariant set for system (??):

$$\Psi := \left\{ \mathbf{x} \in \mathbb{R}^n : \mathbf{x}^T \mathbf{P} \mathbf{x} \leq \frac{1}{\lambda} \gamma^2 \bar{\mathbf{u}}^T \bar{\mathbf{u}} \right\} \quad (\text{VI.6})$$

where, for a given non-zero vector \mathbf{x} , the scalar $\lambda > 0$ satisfies

$$\lambda \leq \frac{\mathbf{x}^T \mathbf{Q} \mathbf{x}}{\mathbf{x}^T \mathbf{P} \mathbf{x}} \leq 1 \quad (\text{VI.7})$$

Consider the generalized Rayleigh quotient (VI.7). Notice that the scalar λ satisfying this inequality can be obtained as the minimum generalized eigenvalue of the pair (\mathbf{Q}, \mathbf{P}) .

VI.5 Invariant-set approach formulation

Consider the ellipsoidal set (VI.6). It is built by considering a center at zero, i.e. by considering $w_o = 0$. However, in the system given by equation (VI.1) the additional known and constant input w_o shifts the equilibrium to \mathbf{x}_o , which corresponds to the so-called *basal level* calculated when the power input is equal to zero by the equation:

$$\mathbf{x}_o = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{B} w_o \quad (\text{VI.8})$$

Thus, the obtained RPI set has to be rewritten as follows:

$$\Psi_x := \left\{ \mathbf{x} \in \mathbb{R}^n : (\mathbf{x} - \mathbf{x}_o)^T \mathbf{P} (\mathbf{x} - \mathbf{x}_o) \leq \frac{1}{\lambda} \gamma^2 \bar{\mathbf{u}}^T \bar{\mathbf{u}} \right\} \quad (\text{VI.9})$$

which characterizes all possible state trajectories, for all $\mathbf{x} \in \Psi_x$, for any realization of the input $\mathbf{u}(k)$, $k \geq 0$, if this input is bounded by $\bar{\mathbf{u}}$.

Because the system output involves a matrix $\mathbf{C}(\rho)$, the projection of the RPI set (VI.9) into the output space is computed as follows:

$$\Psi_y = \mathbf{C}(\rho)\Psi_x \quad (\text{VI.10})$$

Recall that the measurable variables are the oxygen consumption y_1 and the total carbon dioxide output y_2 . The problem stated in the section VI.2 is then reformulated, using the RPI set, as the following optimization problem:

Problem VI.2

Find the maximum bound $\bar{\mathbf{u}}$ and ρ such that the set Ψ_y , defined by the equation (VI.10), belongs to the aerobic region \mathbb{C}_y , defined as:

$$\mathbb{C}_y := \{\mathbf{y} \in \mathbb{R}^2 : y_1 - y_2 \geq \mathbf{z}_t\} \quad (\text{VI.11})$$

allowing an individual to produce mechanical power by using mostly the aerobic pathway without producing excess CO_2 .

Thus, this problem is now a geometrical problem that can be solved by using a suitable optimization tool, for instance in this work we used CVX, a package for specifying and solving convex programs, see [Grant 2014].

VI.6 Numerical example of the control approach using invariant-sets

The presented approach is tested through the simulation of two cycling scenarios for showing the evolution of the outputs, oxygen consumption and carbon dioxide output, including or not the constraint \mathbb{C}_y stated in the problem formulation by the equation (VI.11).

The parameters for the invariant-set Ψ_x for the system ?? are $\gamma = 0.077$, $\lambda = 0.01$. The matrix \mathbf{P} is given by:

$$\mathbf{P} = \begin{bmatrix} 7.62 & -6.56 & 0.04 \\ -6.56 & 5.81 & -0.06 \\ 0.04 & -0.06 & 0.06 \end{bmatrix} \times 10^4 \quad (\text{VI.12})$$

The solution of the problem provides a maximal mechanical power $\bar{\mathbf{u}} = 150W$ and a maximal $\rho(k) = 0.15$.

For the following examples, a control is chosen such that assures for each instant k that $u(k)$ belongs to the domain $0 \leq u(k) \leq \bar{\mathbf{u}}$.

The results of the controlled scenarios are compared with the outputs for the same scenario without this control.

VI.6.1 Incremental Cycling Test

For this test the power is augmented 5W per minute during 30 minutes. Fig.VI.2 shows the input and outputs for this test without and with control. At the end of the test, there is an

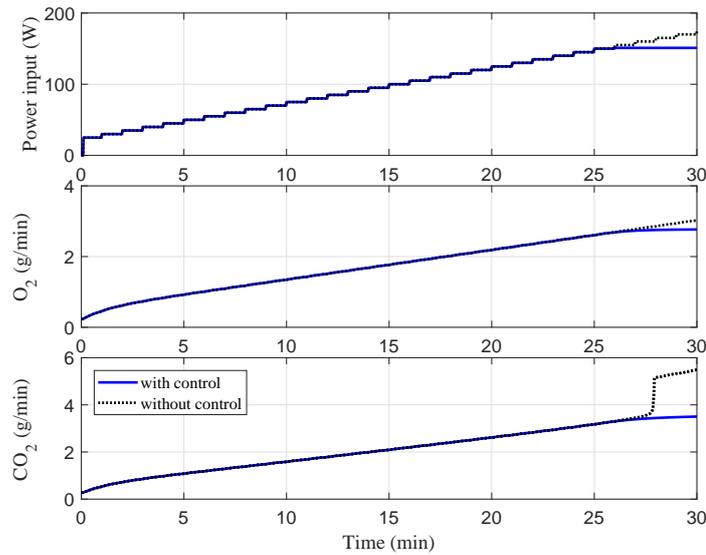


Figure VI.2: Power input $u(k)$ and outputs: oxygen consumption y_1 and produced carbon dioxide y_2 , for Incremental Cycling Test.

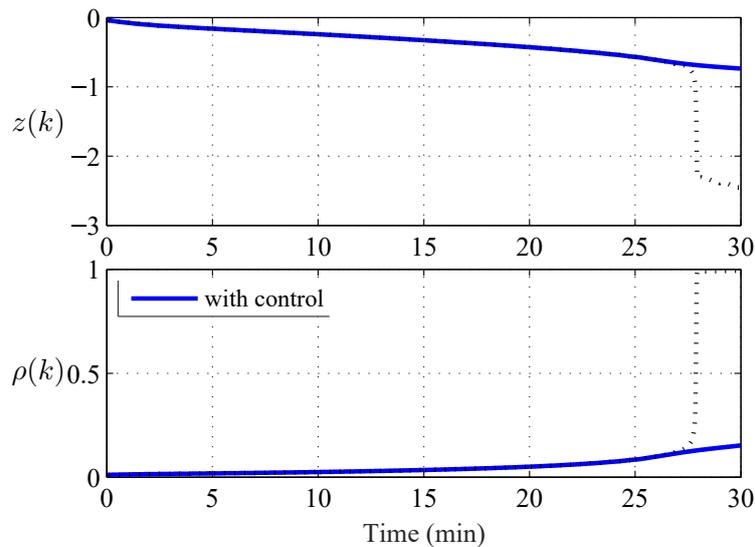


Figure VI.3: Indices $z(k)$ and $\rho(k)$, for Incremental Cycling Test.

overproduction of CO_2 that violates the constraint \mathcal{C}_y when the input is not constrained. The trajectories of states in the output space, the invariant-set Ψ_y and the constraint \mathcal{C}_y are shown in Fig.VI.4 for the scenario with unconstrained input and in Fig.VI.5 for the constrained case. The indices $z(k)$ and $\rho(k)$ can be seen in Fig.VI.3.

Notice that the ellipsoids of Fig.VI.4 and Fig.VI.5 include negative values, which are only theoretical since it is not possible to have negative values of these variables. The center of the ellipsoids corresponds to the basal values \mathbf{x}_0 for consumed O_2 and produced CO_2 .

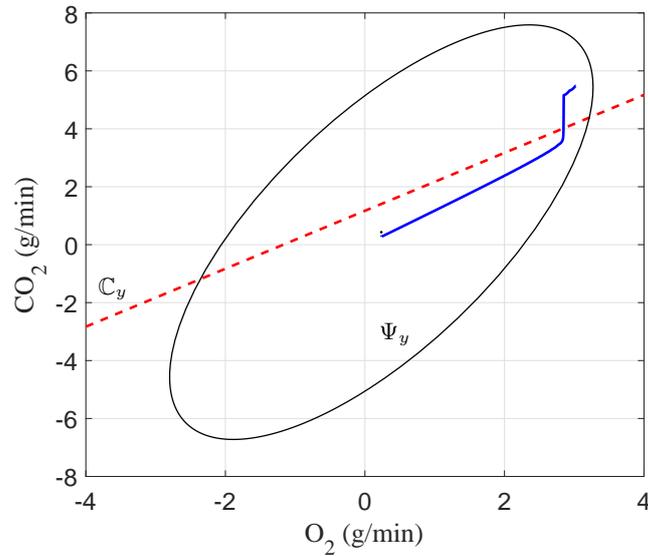


Figure VI.4: RPI set Ψ_y with unconstrained $u(k)$, which violate the constraint \mathbb{C}_y for Incremental Cycling Test

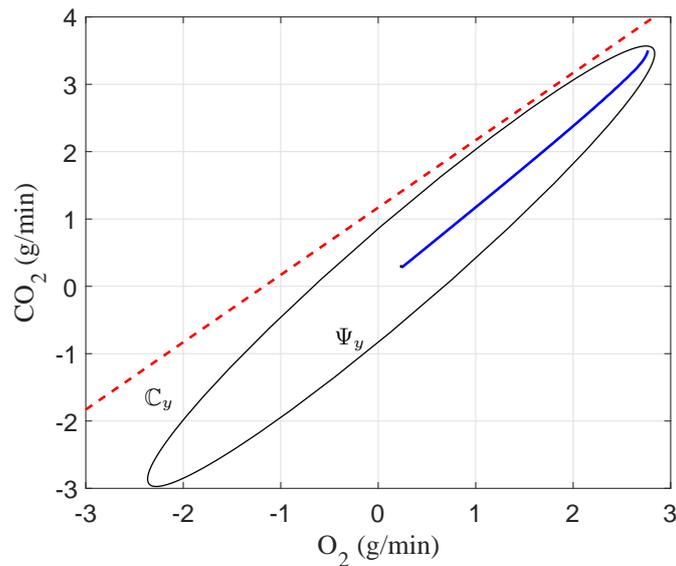


Figure VI.5: RPI set Ψ_y with constrained $u(k)$, $0 < u(k) \leq \bar{u}$. The states respect the constraint \mathbb{C}_y for Incremental Cycling Test.

VI.6.2 Flat road scenario

The Incremental Cycling test is a special case where the power increases linearly with the time, but usually during road scenarios the mechanical power depends on several factors like road conditions, aerodynamical forces, pedaling frequency, among others. Here, the measured power during a ride on a mainly flat road is taken as power input for the gas exchange model, allowing the comparison with a normal ride; i.e. unconstrained input and a constrained one.

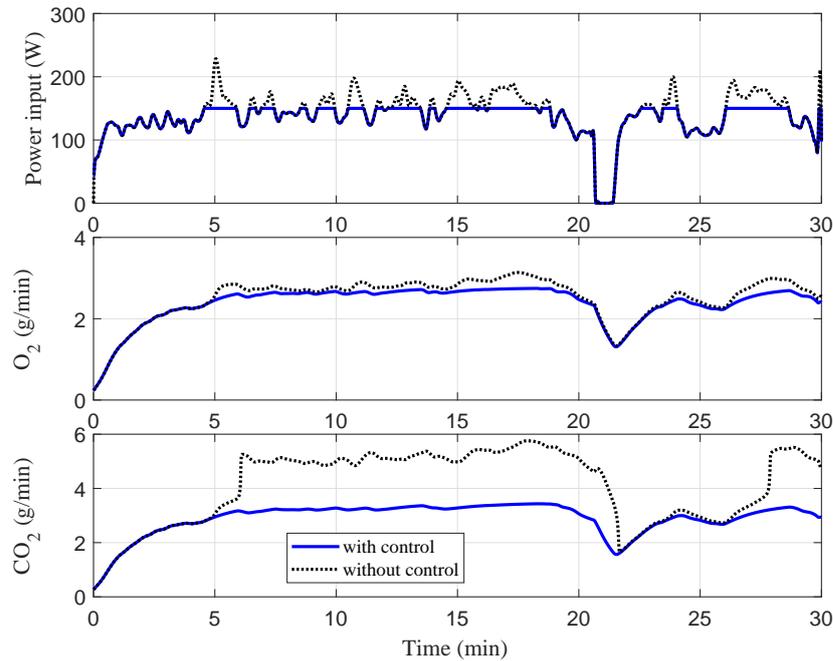


Figure VI.6: Power input $u(k)$ and outputs: oxygen consumption y_1 and produced carbon dioxide y_2 for flat road scenario

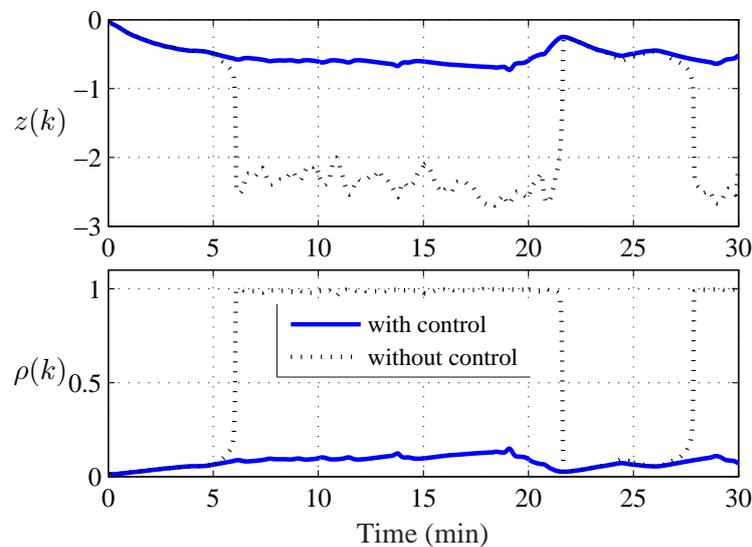


Figure VI.7: Indices $z(k)$ and $\rho(k)$ for flat road scenario

Fig.VI.6 and Fig.VI.7 show the values of the input, the indices and the evolution of the states O_2 and CO_2 during a flat road scenario. When the power is not controlled, an over-production of CO_2 is expected between minute 6 and 22 and also at the end of the workout. However, the power control permits to finish the scenario without touching the constraint.

The invariant-set representation Ψ_y in the output space is shown in Fig.VI.8 and Fig.VI.9.

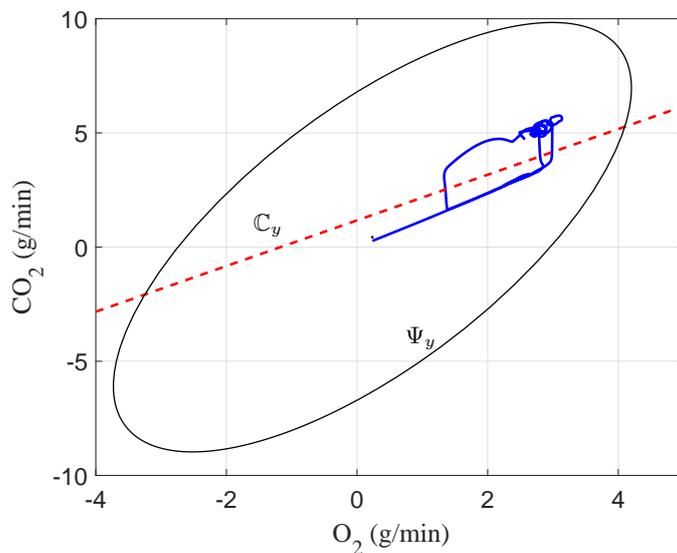


Figure VI.8: RPI set Ψ_y with unconstrained $u(k)$, which violates the constraint C_y for flat road scenario

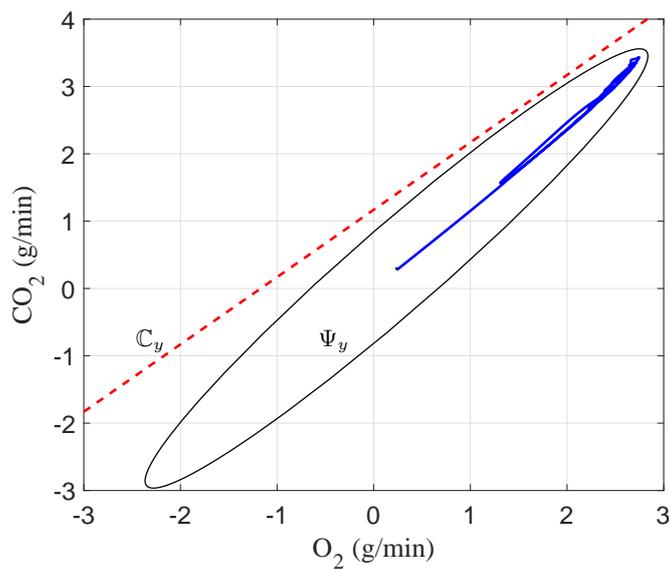


Figure VI.9: RPI set Ψ_y with constrained $u(k)$ for flat road scenario. The states belong to the aerobic region C_y .

The evolution of the states O_2 and CO_2 is different from the Invariant-set representation of Fig. VI.4 and Fig. VI.5, however the proposed method allows the characterization of all the state trajectories no matter the scenario.

VI.7 Conclusion

In this chapter, a method is proposed to find the maximal mechanical power, represented by \bar{u} , that can be developed by a cyclist without excess of carbon dioxide production. The analysis of the human-bicycle system using invariant-sets approach is proposed for constraining the outputs of the gas exchange dynamical model. In particular, the system outputs, y_1 oxygen consumption and y_2 carbon dioxide production, have to remain into a suitable "aerobic" region described by a metabolic constraint. The approach is applied to characterize all the possible trajectories of the system states in the presence of a bounded pedal power.

The used model and the constraint region depend on the studied individual. It could serve for the control design of electric assistances adapted to the particular needs of a cyclist.

The computation of Robustly Positively Invariant (RPI) sets parametrized by the possible maximal power at the pedal level is used as an analytical tool that can be applied for respecting metabolic constraints. The formulation of this problem in a geometrical representation permits to calculate an explicit solution that reduces computation times and facilitates the implementation.

Conclusions and Perspectives

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VII.1 General Conclusions

This thesis deals with modeling, observation and control for human-bicycle system dynamics. This work has been carried out for finding a suitable model for physiology-aware control strategies for bicycle electrical assistance. Therefore, a deep initial theoretical analysis of physiology from an engineering point of view has been addressed at the beginning of the work. Initially, a “conceptual” bio-energetic model allows the comprehension of the main physiology processes and simulation of cycling scenarios with dynamical responses similar to the ones described in physiology literature.

Then, taking into account the main relationships between physiological indices and mechanical variables, a gas exchange model is proposed, where oxygen consumption and carbon dioxide production are related with mechanical power at pedal level. This model calibrated with measured data from different cycling scenarios serves as a plant model for developing estimation methods of important physiological variables and for solving control issues using different techniques.

The contributions of this thesis are analyzed per chapter in the following lines.

Chapter III *Conceptual bio-energetic model of human-bicycle system*

The understanding of human physiology and bicycle mechanics as a whole dynamical system was the initial challenge of this thesis. The dynamical equations of the model presented in this chapter were obtained using a bio-energetic approach, mainly based on physiology literature, some steady-state data from cycling tests and discussions with sport science experts.

The presented model allows the analysis and simulation of energy diminution in the cyclist, which can be quantified using the fatigue indices SoF_1 and SoF_2 . However, the variables and parameters are difficult to measure or estimate. Then, the main purpose of this model is the comprehension of relationships between physiological and mechanical variables.

The simulation results shows the dynamical behavior of the human-bicycle system during a specific scenario. Afterwards, the effort is concentrated on modeling physiological and

mechanical relationships in terms of gas exchange variables and pedal power, because of the ease of measurement and calibration.

Chapter IV *Gas exchange model during cycling*

During exercise, there are several variables that can be measured for assessing the cyclist physiological state. Among which the gas exchange variables, i.e. oxygen consumption and carbon dioxide production provide an equilibrium between ease of measurement and correlation with metabolical processes. Other variables can be also used for the same purpose. The heart rate is often used in research works because it is easy to measure. However, a remarkable disadvantage of using the Heart Rate concerns the fact that it is highly influenced by other factors such as state of mood, neurological issues and general homeostasis of the human body. Invasive measurements such as blood lactate, glycogen concentration, or blood pressure are avoided for their difficult implementation in real-time applications.

The discrete-time LPV model proposed in this chapter reproduces the gas exchange dynamics of an individual during cycling. The model as well as the methodology of parameter identification are part of the novelty of this thesis, since to the best of the authors knowledge, other gas exchange models do not present a similar structure or approach for choosing parameters.

The model is linear and relatively simple, since the only time-varying parameter is in the output matrix and it is used to model the transition between two possible modes. One mode is related to low level of carbon dioxide production, while the second mode is related to high carbon dioxide production. In addition, this time-varying parameter has been written in terms of a novel physiological index by using a transition function, which significantly helps in the classification of those modes. Throughout the work, the analysis of gas exchange variables is based on masses instead of volume of gases as it is usual in Physiology, which allows obtain a description of physiological phenomena using mass conservation laws.

The experimental results show that the model can be used in simulation, prediction and analysis of consumed oxygen and produced carbon dioxide of a specific individual with an acceptable accuracy. Therefore, it can be used for control-design strategies adapted to the personal physiological needs.

Chapter V: *Robust estimation of carbon dioxide production*

An important indicator of cyclist fatigue is the overproduction of carbon dioxide. Therefore an estimator of this variable is useful for a real-time implementation, since it allows the reduction of installed sensors.

A set-membership observer for the estimation of carbon dioxide production during cycling is presented. The observer provides a good estimation of measured and non-measured states as well as deterministic bounds, which provide an interval where the real state surely evolves despite the presence of noise and other unknown physiological factors that affect the process.

The bounds provided by the observer are a useful tool to distinguish between the dynamics concerned to cycling and other physiological phenomena. Also, these bounds can

serve for avoiding over-switching in the implementation of control-strategies based on the estimation of carbon dioxide overproduction.

Chapter VI: *Control approach for physiology-aware assistance*

The limit between the use of aerobic and anaerobic pathway has been the subject of study of many physiology researches. However, a static value of power given by steady-state measurements is not enough to characterize such a physiological dynamical system involved during exercise. In this chapter, a method is proposed to find the maximal mechanical power \bar{u} that can be developed by a cyclist without excess of carbon dioxide production. Since carbon dioxide overproduction is related to the use of anaerobic pathway, i.e. the low efficiency mode of cyclist; the value of \bar{u} constitutes a certain bound to characterize the physiological state of the cyclist based on his particular dynamical model, which is useful for physiology-aware control strategies.

Ellipsoidal invariant sets approach is applied to characterize all the possible trajectories of the system states in the presence of a bounded pedal power. In particular, the system outputs, consumed O_2 and produced CO_2 are able to remain into a suitable "aerobic" region described by a metabolic constraint. The model and the constraint region depend on the studied individual.

In this case, RPI sets are presented as an analytical tool. The advantage of its geometric interpretation is that it allows an explicit calculation of the maximal input (pedal power) that can be applied for respecting a metabolic constraint.

VII.2 Perspectives

As the outcome of the presented work and the obtained results, the following perspectives (short or long-term) could be envisaged:

Modeling The two models presented in this thesis, the conceptual model and the gas exchange model are prone of improvements. However, a good option could be to concentrate the efforts on the refinement of the gas exchange model, since it is easily calibrated with measured data.

For the purposes of this thesis the input of the gas exchange model concerns the pedal power, but it is in turn the product of cadence and torque. The analysis of how affects different combinations of cadence and torque in the model fit is in the scope of further work.

By other hand, model results show a strong relationship between carbon dioxide production and pedal power. Although it can be concluded the same about the oxygen consumption and heart rate, these variables have been mainly studied during aerobic scenarios which limits the range of application in road cycling. Since the model allows differentiation between aerobically produced CO_2 and excess CO_2 , some energetic ratios could be calculated for the analysis of used metabolic pathways. The equations for these indices could be as follows,

For aerobic energy ratio:

$$\eta_{aer} = \frac{CO_2}{CO_2 + \varepsilon CO_2} \quad (\text{VII.1})$$

For anaerobic energy ratio:

$$\eta_{ana} = \frac{\varepsilon CO_2}{CO_2 + \varepsilon CO_2} \quad (\text{VII.2})$$

Other applications in the field of Medicine include to apply the model for diagnosis of cardiovascular or respiratory diseases. Respect to Sport Sciences, more validation in road cycling scenarios are required. The more data is obtained the more possibilities to assess changes in the model due to training and use of the model for design of adapted training programs for specific needs, since high level athletes to people with diseases.

Observation The proposed observation technique allows the estimation of carbon dioxide production using less sensors. It is proposed to evaluate more carefully the accuracy of results compared with measured data and then assess the diminution of cost in gas exchange equipment due the avoidance of carbon dioxide chemical sensors.

The bounds provided for the observer for measurable and non measurable data can be used for filtering data which is not concerned with the studied physiological phenomena. For example, considerable variations on carbon dioxide overproduction during recovery or noise due to increase in ventilation can be avoided.

The fact that there is used a recent formulated observation approach permits the exploration of theoretical issues as the inclusion of the measures in bounds calculation. The observed carbon dioxide production could be applied for the real-time assessing of the physiological state of the cyclist.

By other hand, since it is still necessary oxygen consumption measurement, the relationship of this variable with HR can be analyzed, for use a HR measure instead of oxygen consumption sensor and then provide an easier and cheaper implementation for commercial bicycles.

Control The main perspectives of this thesis are concerned with control design and implementation of model-based strategies for the advance on bicycle research.

Firstly, the gas exchange model constitutes a mathematical representation of physiological phenomena during cycling, which can be used in any control strategy for improving cyclist performance in the case of passive bicycles or for other control objectives in the case of electrical bicycles.

The existence of a bounded input $\bar{\mathbf{u}}$ treated in chapter VI can be implemented in a real-time application calculating an $\bar{\mathbf{u}}$ for a horizon of time. Also, it is interesting to explore the use of RPI sets approach for designing an event-switched control law as it is proposed in [Boisseau 2017].

Other application of invariant sets to this topic consists in the use of connected invariant-sets (with crossing points) as shown in Fig.VII.1. The center of each ellipsoid represents

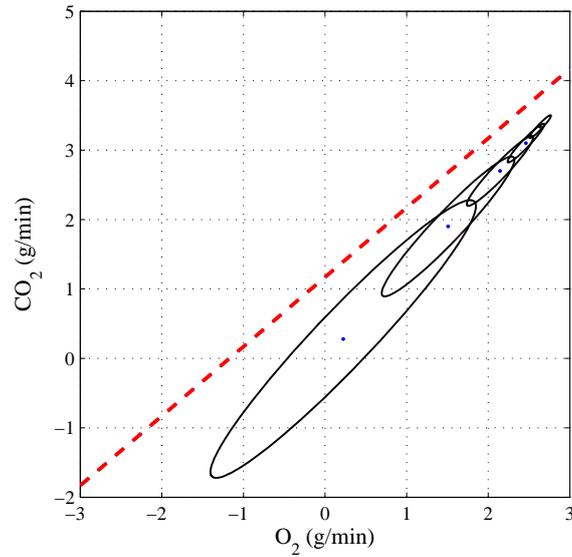


Figure VII.1: Connected invariant-sets

an operation point, for example, the basal metabolism. Once the cyclist has reached other known operation point, the invariant-set is calculated around it, hence the value of $\bar{\mathbf{u}}$ is particular for each case.

In general, the utilization of model-based control strategies for electrical bicycles seems adapted for the human-bicycle system. This research shows a focus on physiology-aware control based on gas exchange variables that can be used for adapting physiological objectives in control laws of electrical bicycles.

Another important aspect to be taken into account are the implementation issues for testing different control algorithms in an adequate benchmark. Hence, sensors, micro-controller and actuators must be carefully chosen with a trade-off between cost, lightness and computing power. The construction of this benchmark, able to be used in road cycling is already in process in GIPSA-Lab.

Résumé

Cette thèse a été partiellement soutenue par le gouvernement colombien par le biais d'une bourse d'études de COLCIENCIAS¹ et dédié au développement des études doctorales à l'étranger.

Cette thèse a été développée entre mars 2015 et juin 2018, au sein de l'équipe de recherche SLR² à GIPSA-Lab³, qui est une unité mixte de recherche du CNRS⁴, Grenoble-INP⁵, et l'Université Grenoble Alpes. La thèse s'inscrit dans la spécialité Automatique et Productique, de l'école doctorale EEATS⁶ de l'Université Grenoble Alpes.

Les sujets suivants font partie du cadre général de cette thèse :

- Modélisation de systèmes physiologiques orientés vers le contrôle des processus de conception.
- Identification et optimisation du système pour le paramétrage du modèle à l'aide de données obtenues à la suite de différents scénarios de tests sur vélo.
- Utilisation des principes d'estimation de l'état optimal pour réduire le nombre de mesures nécessaire pour évaluer l'état physiologique du cycliste.
- Utilisation d'ensembles invariants positifs robustes pour évaluer les limites de la puissance mécanique nécessaire pour éviter un état physiologique indésirable du cycliste .

Au cours de l'élaboration de ce projet, plusieurs collaborations multidisciplinaires avec d'autres instituts ont été mises en place. Ces institutions sont :

- Université de la Sabana. Bogotá, Colombie. Collaboration avec Henry León pour l'analyse de la physiologie humaine et pour la validation théorique du modèle bioénergétique conceptuel.
- Politecnico di Milano. Milan, Italie. Collaboration avec Matteo Corno pour l'analyse des données à partir de scénarios cycliques et de paramétrage d'un modèle dynamique d'échange de gaz.
- Société Matsport. Saint Ismier, France. Collaboration avec Johan Cassirame pour l'analyse physiologique du modèle d'échange gazeux et de l'utilisation d'équipements médicaux professionnels sur les mesures cyclistes.

¹ *Departamento Administrativo de Ciencia, Tecnología e Innovación*, Colombia.

² *Systèmes Linéaires et Robustesse*

³ *Grenoble Images Parole Signal Automatique*

⁴ *Centre National de la Recherche Scientifique*, France.

⁵ *Institut National Polytechnique de Grenoble*

⁶ *Ecole Doctorale Electronique, Electrotechnique, Automatique et Traitement du Signal*

De plus, un étudiant du programme de génie bio-informatique de l'Université de Poitiers a effectué un stage dans l'analyse des seuils ventilatoires et de leurs relations avec des variables mécaniques.

Motivation et objectifs

L'objet d'étude de cette thèse est le système énergétique du cycliste-vélo. Dans le présent document, le terme vélo désigne soit les vélos propulsés uniquement par la force humaine, soit les vélos électriques, utilisées pour décrire les vélos à moteur électrique. Puis, en supposant le système cycliste-vélo dans son ensemble, il est possible de distinguer trois composantes principales : l'homme, le vélo, et l'assistance électrique (c'est-à-dire le moteur et la batterie). L'homme et la batterie sont considérés comme sources d'énergie. L'être humain est capable de stocker et de transformer l'énergie acquise comme la nourriture et l'oxygène, puis les transformer en mouvements musculaires qui propulsent les pédales. Dans le cas d'une assistance électrique, un bloc batterie stocke également l'énergie et transmet ensuite l'électricité au moteur, qui génère le mouvement du vélo. Lorsque ces sources d'énergie interagissent avec le vélo, les deux flux d'énergie agissent en parallèle sur le mouvement du vélo, comme le montre la Fig.A.1.

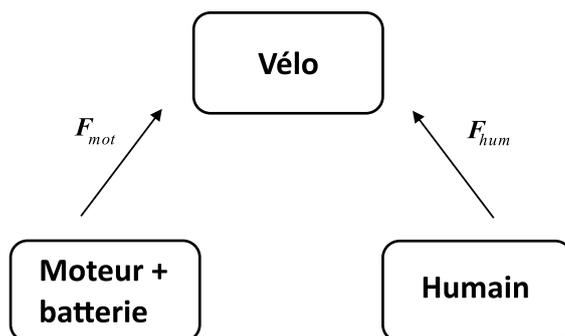


Figure A.1: Flux de puissance dans un système cycliste-vélo

Le cycliste a un comportement indépendant. Il pourrait agir comme un contrôleur, dans le sens où il perçoit la vitesse du vélo à travers ses sens et applique ensuite une force avec ses jambes aux pédales pour suivre son propre référence de vitesse. Le taux et la quantité de puissance produite par ses jambes est limitée par ses capacités physiques, biomécaniques et psychologiques.

D'autre part, le moteur électrique agit selon une loi de commande, qui dépend du type de vélo électrique et la réglementation du pays. En ce qui concerne le cas le plus simple, où l'utilisation d'un vélo électrique est autorisée même en l'absence de pédalage, le cycliste peut choisir de se fier entièrement au moteur ou de pédaler en utilisant en même temps l'assistance électrique ou non. Pour ce type de vélos électriques, l'action motrice repose généralement sur un algorithme qui mesure ou estime la vitesse du vélo avec la vitesse du moteur et calcule ensuite une vitesse de rotation appropriée pour suivre une vitesse de référence supposée comme constante.

Selon la description ci-dessus, l'échange d'information entre les composants est indiqué dans le Fig.A.2.

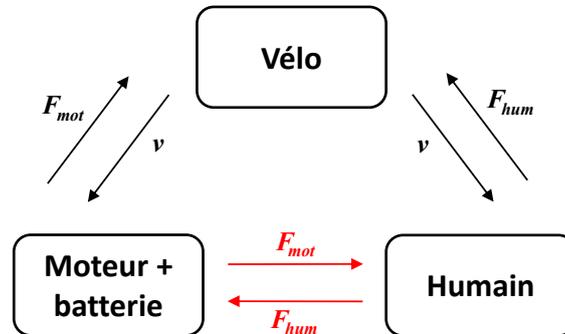


Figure A.2: Flux d'information dans un système cycliste-vélo

Habituellement, la loi de contrôle du moteur ne prend pas en compte les informations concernant la contribution humaine au système. À cet égard, la circulation de l'information se fait entre les sources d'énergie et le vélo et non entre l'homme et le contrôleur du moteur.

En ce qui concerne le cas des pedelecs, qui ont un contrôleur électronique qui coupe l'alimentation électrique du moteur lorsque le conducteur ne pédale pas ou quand une certaine vitesse (en général 25 km/h) est atteint, le flux d'informations admet l'une des possibilités suivantes :

- Vitesse et état du pédalage (on/off): informations minimales qui doivent être mesurées, ou estimé pour l'accomplissement de la régulation pedelec. Il ne comprend qu'un capteur de vitesse qui détecte si le conducteur pédale ou non. Ce système ne comprend pas une loi de contrôle pour l'activation du moteur. Il agit comme un accélérateur. Le cycliste sélectionne le niveau d'assistance qu'il désire à l'aide d'un levier ou d'un bouton.
- Capteur de cadence : composé principalement d'un aimant fixé aux pédales et d'un capteur qui capte le mouvement de l'aimant pendant que le cycliste pédale. La puissance du moteur est inversement proportionnelle à la cadence.
- Capteur de couple : mesure le couple appliqué aux pédales et active la puissance du moteur en fonction de la pression des pédales.

Dans le meilleur des cas, lorsque toutes les informations ci-dessus sont utilisées pour concevoir un contrôle sophistiqué il n'y a pas de coordination entre les deux sources d'énergie. Il présente les inconvénients suivants :

- La sensation de confort n'est pas suffisante. Puisque la loi de contrôle de chaque source (humaine et assistance électrique) a des objectifs de contrôle différents, le couple moteur peut agir contre les désirs du cycliste.
- La puissance délivrée vers le vélo n'est pas optimale. C'est parce que le cycliste a d'habitude plus d'énergie disponible en début de course et le couple d'assistance n'est pas nécessaire pour le moment.

- En cas d'entraînement sportif, l'aide entrave l'amélioration physique en limitant l'effort du cycliste.

Le manque d'échange d'informations entre l'homme et l'algorithme de contrôle de l'assistance électrique est le principal problème identifié. Par conséquent, la présente recherche se concentre sur la conception d'une assistance électrique pour vélos adaptée aux besoins physiologiques du cycliste. Pour cela trois aspects principaux sont pris en compte :

- Problèmes de modélisation : l'analyse physiologique du cycliste est réalisée depuis une perspective d'ingénierie, visant à obtenir un modèle fiable pour une future stratégie de contrôle physiologique pour l'assistance électrique.
- Problèmes d'estimation : réduire le nombre de capteurs utilisés pour évaluer l'état physiologique du cycliste est une question importante dans la mise en oeuvre. Ici, l'estimation de la production de dioxyde de carbone pendant le pédalage est abordée.
- Problèmes de contrôle : l'analyse des modèles énergétiques et d'échange gazeux du système est abordée en utilisant l'approche des ensembles invariants, ce qui permet de proposer une puissance limitée qui remplit à la fois des objectifs physiologiques et mécaniques.

Dans un contexte général, les sujets abordés dans cette thèse pourraient faire ressortir certains aspects positifs et durables au profit de la société dans les sens suivants :

- Aider les personnes handicapées physiques en leur fournissant une assistance électrique adaptée à leurs besoins particuliers.
- Augmentez le niveau de confort ressenti en faisant du vélo. La sensation de faire du vélo sans assistance doit être ressentie. Le cycliste ne doit pas non plus remarquer la présence de l'assistance électrique, ni qu'il agit à l'encontre de ses désirs. Par conséquent, il peut sentir plus fort.
- Contribuer à la promotion du vélo comme moyen de transport écologique.
- Améliorer l'efficacité énergétique des vélos en activant l'assistance électrique uniquement lorsque le cycliste en a vraiment besoin.

Résumé des contributions

Les contributions de cette thèse peuvent être résumées comme suit :

- Un modèle bioénergétique conceptuel du système cycliste-vélo. Le modèle reproduit le comportement dynamique des principales variables impliquées pendant le pédalage. L'approche de modélisation considère trois niveaux principaux : i) physiologique, ii) biomécanique et iii) purement mécanique. L'analyse du modèle inclut la signification des paramètres, qui sont liés aux caractéristiques physiologiques du cycliste ou caractéristiques mécaniques du vélo. Il comprend également une explication pédagogique sur la

dynamique des processus de transformation de l'énergie pendant le pédalage. Le modèle permet aussi de mieux comprendre la performance bioénergétique du cycliste. Cette contribution est présentée au chapitre III.

- Un modèle dynamique en temps discret de la dynamique des échanges gazeux pendant le pédalage. Le modèle reproduit le comportement dynamique de la consommation d'oxygène et de la production de dioxyde de carbone en utilisant la mesure de la puissance au niveau des pédales comme entrée. La prédiction de l'excès de dioxyde de carbone est également possible, ce qui permet de connaître l'état physiologique du cycliste. La différence entre les masses par unité de temps de l'oxygène et du dioxyde de carbone est proposée pour la classification à partir de n'importe quel scénario du cyclisme. La méthodologie d'identification et de validation du modèle se fait par le biais d'un problème d'optimisation, ce qui le rend utilisable pour l'obtention d'un modèle adapté à chaque cycliste. Ce modèle peut être utilisé pour améliorer la conception de la loi de commande pour l'assistance électrique des vélos. Cette contribution est présentée au chapitre IV.
- Un observateur en utilisant ensembles invariants de la production de dioxyde de carbone. L'observateur utilise des mesures de la consommation d'oxygène et de la puissance au niveau des pédales ainsi que le modèle dynamique en temps discret des échanges gazeux proposé au chapitre IV. L'approche des ensembles invariants fournit un intervalle déterministe qui contient l'état réel. Vu que l'excès de production du dioxyde de carbone est liée au dépassement du seuil anaérobie, l'observateur peut être appliquée pour prédire l'état physiologique du cycliste à l'aide de quelques capteurs. Cette contribution est présentée au chapitre V.
- Une méthode pour déterminer les conditions suffisantes sur la puissance de pédalage qui garantissent des niveaux admissibles de production de dioxyde de carbone. L'approche des ensembles invariants permet de caractériser toutes les trajectoires possibles des états du système soumis à une entrée limitée, la puissance de pédalage dans ce cas. Un problème d'optimisation géométrique impliquant une contrainte d'inégalité est résolu pour calculer une limite supérieure sur la puissance de pédalage. Cette méthode constitue une première étape dans la recherche d'une loi de contrôle incluant des contraintes physiologiques. Cette contribution est présentée au chapitre VI.

Conclusions générales

Cette thèse porte sur la modélisation, l'observation et la commande du système cycliste-vélo. Ce travail a été effectué afin de trouver un modèle approprié pour des stratégies de contrôle physiologique pour l'assistance électrique des vélos. Par conséquent, une analyse théorique initiale approfondie de la physiologie d'un point de vue technique a été abordée au début des travaux. Dans un premier temps, un modèle bioénergétique "conceptuel" permet la compréhension des principaux processus physiologiques et la simulation de scénarios du cyclisme avec des réponses dynamiques similaires à celles décrites dans la littérature en physiologie.

Ensuite, en tenant compte des principales relations entre les indices physiologiques et les variables mécaniques, un modèle d'échange gazeux est proposé, où la consommation d'oxygène

et la production de dioxyde de carbone sont liées à la puissance de pédalage. Ce modèle, calibré avec des données mesurées à partir de différents scénarios de cyclisme, sert de modèle pour l'élaboration de méthodes d'estimation des émissions importantes de variables physiologiques et pour résoudre les problèmes de contrôle en utilisant des différentes techniques. Les contributions de cette thèse sont analysées par chapitre dans les lignes suivantes.

Chapter III *Modèle bioénergétique conceptuel du système cycliste-vélo*

La compréhension de la physiologie humaine et de la mécanique du vélo en tant que système dynamique dans son ensemble était le défi initial de cette thèse. Les équations dynamiques du modèle présentées dans ce chapitre ont été obtenues à l'aide d'une approche bioénergétique, basée principalement sur la littérature en physiologie, certaines données en régime permanent issues d'essais cyclistes et de discussions avec des experts en sciences du sport.

Le modèle présenté permet l'analyse et la simulation de la diminution d'énergie chez le cycliste, qui peut être quantifiée en utilisant les indices de fatigue SoF_1 et SoF_2 . Cependant, les variables et les paramètres sont difficiles à mesurer ou à estimer. Ensuite, le but principal de ce modèle est la compréhension des relations entre les variables physiologiques et mécaniques.

Les résultats de la simulation montrent le comportement dynamique du système cycliste-vélo pendant un scénario spécifique. Ensuite, l'effort se concentre sur la modélisation des relations physiologiques et mécaniques en termes de variables d'échange de gaz et de puissance de pédalage, grâce à la facilité de mesure et d'étalonnage.

Chapter IV *Modèle d'échange de gaz pendant le pédalage*

Pendant l'exercice, plusieurs variables peuvent être mesurées pour évaluer l'état physiologique du cycliste. Parmi lesquelles les variables d'échange gazeux, c'est-à-dire la consommation d'oxygène et la production de dioxyde de carbone fournissent un équilibre entre la facilité de mesure et la corrélation avec les processus métaboliques. D'autres variables peuvent également être utilisées dans le même but. La fréquence cardiaque est souvent utilisée dans les travaux de recherche parce qu'elle est facile à mesurer. Cependant, un inconvénient remarquable de l'utilisation de la fréquence cardiaque est le fait qu'elle est fortement influencée par d'autres facteurs tels que l'état d'esprit, les problèmes neurologiques et l'homéostasie générale du corps humain. Les mesures invasives telles que le lactate sanguin, la concentration de glycogène ou la tension artérielle sont évitées en raison de leur mise en oeuvre difficile dans des applications en temps réel.

Le modèle LPV à temps discret proposé dans ce chapitre reproduit la dynamique d'échange gazeux d'un individu au cours d'un test du cyclisme. Le modèle ainsi que la méthodologie d'identification des paramètres font partie de la nouveauté de cette thèse, car au meilleur de la connaissance des auteurs, les autres modèles d'échange gazeux ne présentent pas une structure ou une approche similaire pour choisir les paramètres.

Le modèle est linéaire et relativement simple, puisque le seul paramètre variant dans le temps se trouve dans la matrice de sortie et il est utilisé pour modéliser la transition entre deux modes possibles. Le premier mode est lié à un faible niveau de production de dioxyde de carbone, tandis que le second mode est lié à une production élevée de dioxyde de

carbone. De plus, ce paramètre variable dans le temps a été écrit en fonction d'un nouvel indice physiologique à l'aide d'une fonction de transition, qui aide de manière significative à classer ces modes. Tout au long du travail, l'analyse des variables d'échange gazeux est basée sur les masses au lieu du volume des gaz comme c'est le cas en physiologie, ce qui permet d'obtenir une description des phénomènes physiologiques en utilisant les lois de conservation des masses.

Les résultats expérimentaux montrent que le modèle peut être utilisé dans la simulation, la prédiction et l'analyse de l'oxygène consommé et du dioxyde de carbone produit par un individu spécifique avec une précision acceptable. Il peut donc être utilisé pour des stratégies de conception de loi de commande adaptées aux besoins physiologiques personnels.

Chapter V: *Estimation robuste de la production de dioxyde de carbone*

Un indicateur important de la fatigue chez le cycliste est la surproduction de dioxyde de carbone. Un estimateur de cette variable est donc utile pour une implémentation en temps réel, puisqu'il permet de réduire le nombre de capteurs installés.

Un observateur qui utilise ensembles invariants est présenté pour l'estimation de la production de dioxyde de carbone pendant le pédalage. L'observateur fournit une bonne estimation des émissions mesurées et non mesurées, ainsi que des limites déterministes, qui fournissent un intervalle où l'état réel évolue sûrement malgré la présence de bruit et d'autres facteurs physiologiques inconnus qui affectent le processus. Les limites fournies par l'observateur sont utiles pour distinguer la dynamique concernée par le cyclisme des autres phénomènes physiologiques.

De plus, ces limites peuvent permettre d'éviter les sursauts dans la mise en oeuvre de stratégies de contrôle basées sur l'approche de l'estimation de la surproduction de dioxyde de carbone.

Chapter VI: *Analyse pour la commande des variables physiologiques du cycliste*

La limite entre l'utilisation des voies aérobie et anaérobie a fait l'objet de nombreuses recherches en physiologie. Cependant, une valeur statique de puissance donnée par des mesures en régime permanent n'est pas suffisante pour caractériser un tel système dynamique physiologique impliqué pendant l'exercice. Dans ce chapitre, une méthode est proposée pour déterminer la puissance mécanique maximale \bar{u} qui peut être développée par un cycliste sans excès de production de dioxyde de carbone. La surproduction de dioxyde de carbone étant liée à l'utilisation de la voie anaérobie, c'est-à-dire le mode de faible efficacité du cycliste, la valeur de \bar{u} constitue une certaine limite pour caractériser l'état physiologique du cycliste à partir de son modèle dynamique particulier, qui est utile pour des stratégies de contrôle sensibles à la physiologie.

L'approche des ensembles invariants ellipsoïdaux est appliquée pour caractériser toutes les trajectoires possibles des états du système pour une puissance de pédale limitée. En particulier, l' O_2 consommé et le CO_2 produit sont capables de rester dans un environnement aérobie approprié ou région décrite par une contrainte métabolique. Le modèle et la région de contrainte dépendent de l'individu étudié.

Dans ce cas, les ensembles RPI sont présentés comme un outil analytique. L'avantage de sa représentation géométrique est qu'elle permet un calcul explicite de l'entrée maximale (puissance de pédalage) pour respecter une contrainte métabolique.

Perspectives

A l'issue des travaux présentés et des résultats obtenus, les perspectives suivantes (à mi-parcours ou à long terme) pourraient être élaborées.

Modélisation Les deux modèles présentés dans cette thèse, le modèle conceptuel et le modèle de l'échange gazeux sont susceptibles d'être améliorés. Cependant, une bonne option pourrait être de concentrer les efforts sur le raffinement du modèle d'échange de gaz, puisqu'il est facilement étalonné à l'aide de données mesurées.

Pour les besoins de cette thèse, l'entrée du modèle d'échange de gaz concerne la puissance de pédalage, qui est quant à elle le produit de la cadence et du couple. L'analyse de la façon dont les effets sur les différentes combinaisons de cadence et de couple dans l'ajustement du modèle est envisagé comme travail futur.

D'autre part, les résultats du modèle montrent une forte relation entre la production de dioxyde de carbone et la puissance de pédalage. Bien que l'on puisse conclure la même chose sur la consommation d'oxygène et la fréquence cardiaque, ces variables ont été principalement étudiées lors de scénarios aérobies qui limitent le champ d'application dans le cyclisme sur route. Puisque le modèle permet de différencier le dioxyde de carbone produit sur voie aérobie de l'excès de dioxyde de carbone, certains rapports énergétiques pourraient être calculés pour l'analyse des voies métaboliques utilisées. Les équations pour ces indices pourraient être les suivantes,

Pour le rapport d'énergie aérobie :

$$\eta_{aer} = \frac{CO_2}{CO_2 + \varepsilon CO_2} \quad (A.1)$$

Pour le rapport d'énergie anaérobie :

$$\eta_{ana} = \frac{\varepsilon CO_2}{CO_2 + \varepsilon CO_2} \quad (A.2)$$

D'autres applications dans le domaine de la médecine comprennent l'application du modèle pour le diagnostic de la maladies cardiovasculaires ou respiratoires. Pour l'application du modèle aux sciences du sport, il faudrait reformuler plus de validation sur scénarios de route. Plus on obtient de données, plus on a de possibilités d'évaluer les changements apportés au modèle en raison de l'entraînement et de l'utilisation du modèle pour la conception d'un plan d'entraînement adaptée pour les besoins spécifiques, depuis les athlètes de haut niveau jusqu'aux personnes malades.

Observation La technique d'observation proposée permet d'estimer la production de dioxyde de carbone chez le cycliste, en utilisant moins de capteurs. Il est proposé d'évaluer plus soigneusement l'exactitude des données des résultats par rapport aux données mesurées et évaluer ensuite la diminution du coût de remplacer l'équipement de mesure en évitant l'utilisation de capteurs chimiques de dioxyde de carbone.

Les limites fournies pour l'observateur peuvent être utilisés pour filtrer les données qui ne sont pas concernées par les phénomènes physiologiques étudiés. Par exemple, la surproduction de dioxyde de carbone au cours de la récupération ou le bruit dû à l'augmentation de la ventilation peuvent être filtrés.

L'utilisation d'une approche d'observation récemment formulée permet l'exploration des questions théoriques comme l'inclusion des mesures dans le calcul des limites fournies pour l'observateur. L'estimation de la production de dioxyde de carbone pourrait être appliquée à l'évaluation en temps réel de l'état physiologique du cycliste.

D'autre part, puisqu'il est encore nécessaire de mesurer la consommation d'oxygène, la relation de cette variable avec HR peut être analysée, pour utiliser une mesure HR au lieu de mesurer la consommation d'oxygène, ce qui permettra une mise en oeuvre plus facile et moins chère pour les vélos commerciales.

Commande Les principales perspectives de cette thèse portent sur la conception et la mise en oeuvre de stratégies de contrôle basées en utilisant le modèle pour faire progresser la recherche sur la commande des variables physiologiques du cycliste.

Premièrement, le modèle d'échange de gaz constitue une représentation mathématique des phénomènes physiologiques pendant le pédalage, qui peut être utilisée dans plusieurs stratégies de contrôle pour améliorer les performances du cycliste dans le cas des vélos passives ou pour d'autres objectifs de contrôle dans le cas des vélos électriques. L'existence

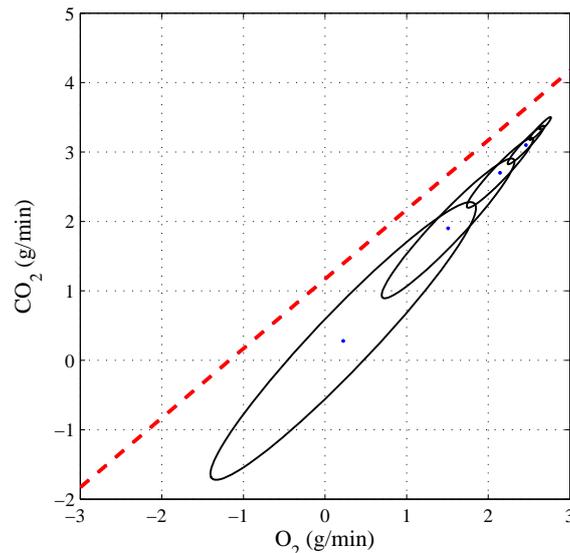


Figure A.3: Ensembles invariants connectés

d'une entrée de puissance de pédalage délimité \bar{u} traitée au chapitre VI peut être mise en oeuvre dans une application en temps réel calculant un \bar{u} pour un horizon temporel. De plus, il est intéressant d'explorer l'utilisation de l'approche des ensembles invariants pour la conception d'une loi de commande à commutation d'événements telle qu'elle est proposée dans [Boisseau 2017].

Une autre application des ensembles d'invariants à ce sujet consiste à utiliser des ensembles d'invariants connectés (avec points de croisement) comme le montre la Fig.A.3. Le centre de chaque ellipsoïde représente un point de fonctionnement, par exemple, le métabolisme de base. Une fois que le cycliste a atteint un autre point d'opération connu, l'invariant est calculé autour de lui, d'où la valeur de \bar{u} est particulière pour chaque cas.

En général, l'utilisation de stratégies de contrôle basées sur des modèles pour les vélos électriques semble adaptée au système cycliste-vélo. Cette recherche met l'accent sur un contrôle tenant compte de la physiologie basé sur des variables d'échange gazeux qui peuvent être utilisées pour l'adaptation des objectifs de contrôle en incluant les indices physiologiques.

Un autre aspect important à prendre en compte sont les problèmes de mise en oeuvre pour tester différents algorithmes de contrôle dans un benchmark adéquat. Par conséquent, les capteurs, les microcontrôleurs et les actionneurs doivent être choisis avec soin en tenant compte du coût, de la légèreté et de la puissance de calcul. La construction de ce banc d'essais, utilisable en cyclisme sur route, est déjà en cours au GIPSA-Lab.

Bibliography

- [Abbiss 2005] Chris R Abbiss et Paul B Laursen. *Models to explain fatigue during prolonged endurance cycling*. Sports Medicine, vol. 35, no. 10, pages 865–898, 2005. (cited on p. 10, 13 and 32.)
- [Aftalion 2014] Amandine Aftalion et J Frederic Bonnans. *Optimization of running strategies based on anaerobic energy and variations of velocity*. SIAM Journal on Applied Mathematics, vol. 74, no. 5, pages 1615–1636, 2014. (cited on p. 18.)
- [Apkarian 1995] Pierre Apkarian, Pascal Gahinet et Greg Becker. *Self-scheduled H_∞ control of linear parameter-varying systems: a design example*. Automatica, vol. 31, no. 9, pages 1251–1261, 1995. (cited on p. 72.)
- [Argha 2014] Ahmadreza Argha, Steven W Su, Sangwon Lee, Hung Nguyen et Branko G Celler. *On heart rate regulation in cycle-ergometer exercise*. In Engineering in Medicine and Biology Society (EMBC), 2014 36th Annual International Conference of the IEEE, pages 3390–3393. IEEE, 2014. (cited on p. 21, 25 and 27.)
- [Argha 2016] Ahmadreza Argha, Lin Ye, Steven W Su, Hung Nguyen et Branko G Celler. *Heart rate regulation during cycle-ergometer exercise using damped parameter estimation method*. In Engineering in Medicine and Biology Society (EMBC), 2016 IEEE 38th Annual International Conference of the, pages 2676–2679. IEEE, 2016. (cited on p. 21, 25 and 27.)
- [Beaver 1986] WILLIAM L Beaver, KARLMAN Wasserman et BRIAN J Whipp. *A new method for detecting anaerobic threshold by gas exchange*. Journal of applied physiology, vol. 60, no. 6, pages 2020–2027, 1986. (cited on p. 16.)
- [Binder 2008] Ronald K Binder, Manfred Wonisch, Ugo Corra, Alain Cohen-Solal, Luc Vanhees, Hugo Saner et Jean-Paul Schmid. *Methodological approach to the first and second lactate threshold in incremental cardiopulmonary exercise testing*. European journal of cardiovascular prevention & rehabilitation, vol. 15, no. 6, pages 726–734, 2008. (cited on p. 70.)
- [Binzoni 2005] Tiziano Binzoni. *Saturation of the Lactate Clearance Mechanisms Different from the Lactate Shuttle Determines the Anaerobic Threshold: Prediction from the Bioenergetic Model*. Journal of Physiological Anthropology and Applied Human Science, vol. 24, no. 2, pages 175–182, 2005. (cited on p. 35.)
- [Blanchini 1999] Franco Blanchini. *Set invariance in control*. Automatica, vol. 35, no. 11, pages 1747–1767, 1999. (cited on p. 73.)
- [Boisseau 2017] Bruno Boisseau, John Jairo Martinez, Thibaut Raharijaona, Sylvain Durand et Nicolas Marchand. *Event-switched control design with guaranteed performances*. International Journal of Robust and Nonlinear Control, vol. 27, no. 15, pages 2492–2509, 2017. (cited on p. 84 and 96.)

- [Boyd 1994] Stephen Boyd, Laurent El Ghaoui, Eric Feron et Venkataramanan Balakrishnan. Linear matrix inequalities in system and control theory, volume 15. Siam, 1994. (cited on p. 61 and 72.)
- [Casaburi 1989] Richard Casaburi, Thomas J Barstow, Terry Robinson et Karlman Wasserman. *Influence of work rate on ventilatory and gas exchange kinetics*. Journal of Applied Physiology, vol. 67, no. 2, pages 547–555, 1989. (cited on p. 44.)
- [Corno 2015] Matteo Corno, Paolo Giani, Mara Tanelli et Sergio Matteo Savaresi. *Human-in-the-Loop Bicycle Control via Active Heart Rate Regulation*. Control Systems Technology, IEEE Transactions on, vol. 23, no. 3, pages 1029–1040, 2015. (cited on p. 20, 21, 25, 26, 27, 28 and 47.)
- [Corno 2016] Matteo Corno, Daniele Berretta, Pierfrancesco Spagnol et Sergio M Savaresi. *Design, control, and validation of a charge-sustaining parallel hybrid bicycle*. IEEE Transactions on Control Systems Technology, vol. 24, no. 3, pages 817–829, 2016. (cited on p. 18 and 26.)
- [Coyle 1992] EF Coyle, LS Sidossis, JF Horowitz et JD Beltz. *Cycling efficiency is related to the percentage of type I muscle fibers*. Medicine and science in sports and exercise, vol. 24, no. 7, page 782, 1992. (cited on p. 35.)
- [Fayazi 2013] S Alireza Fayazi, Nianfeng Wan, Stephen Lucich, Ardalan Vahidi et Gregory Mocko. *Optimal pacing in a cycling time-trial considering cyclist’s fatigue dynamics*. In American Control Conference (ACC), 2013, pages 6442–6447. IEEE, 2013. (cited on p. 18, 26 and 27.)
- [Gaskill 2001] Steven E Gaskill, Brent C Ruby, Ava J Walker, Otto A Sanchez, Robert C Serfass et Arthur S Leon. *Validity and reliability of combining three methods to determine ventilatory threshold*. Medicine & Science in Sports & Exercise, vol. 33, no. 11, pages 1841–1848, 2001. (cited on p. 16.)
- [Giani 2013] P Giani, M Tanelli et SM Savaresi. *Identification and control of a continuously variable transmission for bicycles*. In IET Conference on Control and Automation 2013, pages 1–6, 2013. (cited on p. 24 and 26.)
- [Giani 2014] Paolo Giani, Matteo Corno, Mara Tanelli et Sergio M Savaresi. *Cyclist heart rate control via a continuously varying transmission*. Proceedings 19th IFAC World Congress, vol. 47, no. 3, pages 912–917, 2014. (cited on p. 20 and 25.)
- [Grant 2014] M. Grant et S. Boyd. *CVX: Matlab Software for Disciplined Convex Programming, version 2.1*, Mars 2014. (cited on p. 74.)
- [Hale 2005] Tudor Hale. Exercise physiology: a thematic approach, volume 5. John Wiley & Sons, 2005. (cited on p. 11.)
- [Hill 1923] AV Hill et Hartley Lupton. *Muscular exercise, lactic acid, and the supply and utilization of oxygen*. Quarterly Journal of Medicine, no. 62, pages 135–171, 1923. (cited on p. 16.)

- [Hill 1993] David W Hill. *The critical power concept*. Sports medicine, vol. 16, no. 4, pages 237–254, 1993. (cited on p. 17.)
- [Issekutz Jr 1961] B Issekutz Jr et K Rodahl. *Respiratory quotient during exercise*. Journal of Applied Physiology, vol. 16, no. 4, pages 606–610, 1961. (cited on p. 47.)
- [Jeukendrup 2000] Asker E Jeukendrup, Neil P Craig et John A Hawley. *The bioenergetics of world class cycling*. Journal of Science and Medicine in Sport, vol. 3, no. 4, pages 414–433, 2000. (cited on p. 12 and 35.)
- [Kenney 2015] W Larry Kenney, Jack Wilmore et David Costill. *Physiology of sport and exercise* 6th edition. Human kinetics, 2015. (cited on p. 11 and 36.)
- [Kofman 2012] Ernesto Kofman, José A De Doná et Maria M Seron. *Probabilistic set invariance and ultimate boundedness*. Automatica, vol. 48, no. 10, pages 2670–2676, 2012. (cited on p. 62.)
- [Lagarias 1998] J. C. Lagarias, J. A. Reeds, M. H. Wright et P. E. Wright. *Convergence Properties of the Nelder-Mead Simplex Method in Low Dimensions*. SIAM Journal of Optimization, vol. 9, no. 1, pages pp. 112–147, 1998. (cited on p. 53.)
- [Leitner 2014] Thomas Leitner, Harald Kirchsteiger, Hannes Trogmann et Luigi del Re. *Model based control of human heart rate on a bicycle ergometer*. In European Control Conference (ECC), 2014, pages 1516–1521. IEEE, 2014. (cited on p. 21, 22, 25, 27 and 47.)
- [Limebeer 2006] David JN Limebeer et Robin S Sharp. *Bicycles, motorcycles, and models*. IEEE control systems, vol. 26, no. 5, pages 34–61, 2006. (cited on p. 10.)
- [Loukkas 2017] Nassim Loukkas, John Jairo Martinez Molina et Nacim Meslem. *Set-Membership Observer Design Based on Ellipsoidal Invariant Sets*. In Proceedings of the IFAC World Congress, 2017. (cited on p. 60, 62 and 64.)
- [Martínez 2015] John J Martínez. *Minimal RPI sets computation for polytopic systems using the Bounded-real lemma and a new shrinking procedure*. IFAC-PapersOnLine, vol. 48, no. 26, pages 182–187, 2015. (cited on p. 73.)
- [Martinez 2018] J. Martinez, N. Loukkas et N. Meslem. *H-infinity set-membership observer design for discrete-time LPV systems*. Submitted to International Journal of Control, 2018. (cited on p. 62 and 72.)
- [Maud 2006] Peter J Maud et Carl Foster. *Physiological assessment of human fitness*. Human Kinetics, 2006. (cited on p. 14.)
- [McArdle 2006] William D McArdle, Frank I Katch et Victor L Katch. *Essentials of exercise physiology*. Lippincott Williams & Wilkins, 2006. (cited on p. 11, 15, 35 and 70.)
- [Meijaard 2007] Jaap P Meijaard, Jim M Papadopoulos, Andy Ruina et Arend L Schwab. *Linearized dynamics equations for the balance and steer of a bicycle: a benchmark and review*. In Proceedings of the Royal Society of London A: Mathematical, Physical and

- Engineering Sciences, volume 463, pages 1955–1982. The Royal Society, 2007. (cited on p. 10.)
- [Meyer 2015a] Daniel Meyer, Wenlong Zhang et Masayoshi Tomizuka. *Sliding Mode Control for Heart Rate Regulation of Electric Bicycle Riders*. In ASME 2015 Dynamic Systems and Control Conference, pages V002T27A003–V002T27A003. American Society of Mechanical Engineers, 2015. (cited on p. 19, 20, 25, 26, 28 and 47.)
- [Meyer 2015b] Daniel Meyer, Wenlong Zhang, Masayoshi Tomizuka et Veit Senner. *Heart rate regulation with different heart rate reference profiles for electric bicycle riders*. *Procedia Manufacturing*, vol. 3, pages 4213–4220, 2015. (cited on p. 19, 25 and 26.)
- [Meyer 2018] Daniel Meyer, Moritz Körber, Veit Senner et Masayoshi Tomizuka. *Regulating the Heart Rate of Human-Electric Hybrid Vehicle Riders Under Energy Consumption Constraints Using an Optimal Control Approach*. *IEEE Transactions on Control Systems Technology*, no. 99, pages 1–14, 2018. (cited on p. 26 and 28.)
- [Mohammad 2011] Sami Mohammad, Thierry Marie Guerra, Jean Marie Grobois et Bernard Hecquet. *Heart rate control during cycling exercise using Takagi-Sugeno models*. In 18th IFAC World Congress, Milano (Italy), pages 12783–12788, 2011. (cited on p. 20, 25 and 27.)
- [Mohammad 2013] Sami Mohammad, Thierry Marie Guerra, Bernard Hecquet et JM Grobois. *Heart Rate/Cycling Power Control System*. In 5th IFAC Symposium on System Structure and Control, Grenoble (France), pages 821–826. Elsevier, 2013. (cited on p. 20, 21, 25 and 27.)
- [Morton 2006] R Hugh Morton. *The critical power and related whole-body bioenergetic models*. *European journal of applied physiology*, vol. 96, no. 4, pages 339–354, 2006. (cited on p. 17, 18, 34 and 35.)
- [Muetze 2007] Annette Muetze et Ying C Tan. *Electric bicycles-A performance evaluation*. *IEEE Industry Applications Magazine*, vol. 13, no. 4, pages 12–21, 2007. (cited on p. 11.)
- [Noakes 2000] TD Noakes. *Physiological models to understand exercise fatigue and the adaptations that predict or enhance athletic performance*. *Scandinavian journal of medicine & science in sports*, vol. 10, no. 3, pages 123–145, 2000. (cited on p. 10 and 32.)
- [nuv 2018] *Nuvinci, Continuously Variable Planetary (CVP)*, 2005-2018. (cited on p. 27.)
- [Parker 2004] Alan Parker. *The electric powered assisted bicycle; a clean vehicle to reduce oil dependence and enhance the mobility of the elderly*. New Zealand Cycling Conference, Auckland, New Zealand, no. over 50, pages 1–21, 2004. (cited on p. 11.)
- [Peronnet 2006] Francois Peronnet et Bernard Aguilaniu. *Lactic acid buffering, nonmetabolic CO₂ and exercise hyperventilation: a critical reappraisal*. *Respiratory Physiology & Neurobiology*, vol. 150, no. 1, pages 4–18, 2006. (cited on p. 15.)

- [Rosero 2017] Nadia Rosero, John J Martinez et Henry Leon. *A bio-energetic model of cyclist for enhancing pedelec systems*. IFAC-PapersOnLine, vol. 50, no. 1, pages 4418–4423, 2017. (cited on p. 7.)
- [Rosero 2018a] Nadia Rosero et John J. Martinez. *An invariant-set approach for constraining gas exchange dynamics during cycling*. In Proceedings of 10th IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes, SAFEPROCESS 2018, volume 24, pages 1247–1256. Warsaw, Poland, 2018. (cited on p. 7.)
- [Rosero 2018b] Nadia Rosero, John J. Martinez et Matteo Corno. *Modeling of gas exchange dynamics using cycle-ergometer tests*. In Proceedings of 9th Mathmod conference, volume 24, pages 1247–1256. Vienna, 2018. (cited on p. 7.)
- [Rosero 2019a] Nadia Rosero et John J. Martinez. *Estimation of carbon-dioxide production during cycling by using a set-membership observer approach*. in preparation to be submitted to International Journal of Modelling, Identification and Control, 2019. (cited on p. 7.)
- [Rosero 2019b] Nadia Rosero, John J. Martinez, Matteo Corno et Johan Cassirame. *Modeling and parameter identification of gas exchange dynamics during cycling*. under review in Annual Reviews in Control, 2019. (cited on p. 7.)
- [Schwab 2013] Arend L Schwab et JP Meijaard. *A review on bicycle dynamics and rider control*. Vehicle System Dynamics, vol. 51, no. 7, pages 1059–1090, 2013. (cited on p. 10.)
- [Shimizu 1991] Masayoshi Shimizu, Jonathan Myers, Nancy Buchanan, Doug Walsh, Mark Kraemer, Paul McAuley et Victor F Froelicher. *The ventilatory threshold: method, protocol, and evaluator agreement*. American heart journal, vol. 122, no. 2, pages 509–516, 1991. (cited on p. 16.)
- [Solberg 2005] Geir Solberg, Bjørn Robstad, Ole Henning Skjønsberg et Fredrik Borchsenius. *Respiratory gas exchange indices for estimating the anaerobic threshold*. Journal of sports science & medicine, vol. 4, no. 1, page 29, 2005. (cited on p. 46.)
- [Strickland 2014] Eliza Strickland. *The bike helmet that reads your brainwaves*, 2014. (cited on p. 26, 27, 28 and 29.)
- [Su 2006] Steven W Su, Lu Wang, Branko G Celler, Andrey V Savkin et Ying Guo. *Modelling and control for heart rate regulation during treadmill exercise*. In Engineering in Medicine and Biology Society, 2006. EMBS'06. 28th Annual International Conference of the IEEE, pages 4299–4302. IEEE, 2006. (cited on p. 20.)
- [Su 2007] Steven W Su, Lu Wang, Branko G Celler, Andrey V Savkin et Ying Guo. *Identification and control for heart rate regulation during treadmill exercise*. Biomedical Engineering, IEEE Transactions on, vol. 54, no. 7, pages 1238–1246, 2007. (cited on p. 20.)

- [Svedahl 2003] Krista Svedahl et Brian R MacIntosh. *Anaerobic threshold: the concept and methods of measurement*. Canadian Journal of Applied Physiology, vol. 28, no. 2, pages 299–323, 2003. (cited on p. 54 and 70.)
- [Wan 2014] Nianfeng Wan, S Alireza Fayazi, Hamed Saeidi et Ardalan Vahidi. *Optimal power management of an electric bicycle based on terrain preview and considering human fatigue dynamics*. In American Control Conference (ACC), 2014, pages 3462–3467. IEEE, 2014. (cited on p. 18, 26 and 27.)
- [Wasserman 1973] Karlman Wasserman, Brian J Whipp, SN Koysl et WL Beaver. *Anaerobic threshold and respiratory gas exchange during exercise*. Journal of applied physiology, vol. 35, no. 2, pages 236–243, 1973. (cited on p. 16.)
- [Wasserman 1984] Karlman Wasserman. *The anaerobic threshold measurement to evaluate exercise performance*. American Review of Respiratory Disease, vol. 129, no. 2P2, pages S35–S40, 1984. (cited on p. 16.)
- [Wasserman 1990] Karlman Wasserman, William L Beaver et Brian J Whipp. *Gas exchange theory and the lactic acidosis (anaerobic) threshold*. Circulation, vol. 81, no. 1 Suppl, pages III4–30, 1990. (cited on p. 16.)
- [Wasserman 2005] Karlman Wasserman, James E Hansen, Darryl Y Sue, William W Stringer et Brian J Whipp. Principles of exercise testing and interpretation. Lippincott Williams & Wilkins, 2005. (cited on p. 15 and 54.)
- [Wilson 2004] David Gordon Wilson, Jim Papadopoulos et Frank Rowland Whitt. Bicycling science. MIT press, 2004. (cited on p. 10.)
- [Winter 2016] Edward M Winter, Grant Abt, FB Carl Brookes, John H Challis, Neil E Fowler, Duane V Knudson, Howard G Knuttgen, William J Kraemer, Andrew M Lane, Willem Van Mechelen et al. *Misuse of power and other mechanical terms in sport and exercise science research*. The Journal of Strength & Conditioning Research, vol. 30, no. 1, pages 292–300, 2016. (cited on p. 18.)

