



Didactic Situations for the Development of Creative Mathematical Thinking: A study on Functions and Algorithms

Pedro Lealdino

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UNIVERSITÉ CLAUDE BERNARD - LYON I

DOCTORAL THESIS

Didactic Situations for the Development of Creative Mathematical Thinking: A study on Functions and Algorithms

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Defended Thursday November 29th, 2018,
in Lyon, in front of the jury

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- Marcelo BORBA
- Vincent BORRELLI
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- Nathalie SINCLAIR (reviewer)
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- Luc TROUCHE (invited)

*A thesis submitted in fulfillment of the requirements
for the degree of Doctor of Philosophy*

in the

Sciences, Société, Historicité, Éducation et Pratiques (S2HEP)
Institut de recherche sur l'enseignement des mathématiques de Lyon

May 22, 2019

Declaration of Authorship

I, Pedro LEALDINO FILHO, declare that this thesis titled, “Didactic Situations for the Development of Creative Mathematical Thinking: A study on Functions and Algorithms” and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed: _____

Date: _____

“Get out of my bed, Pedro!..., said my supervisor to his wife”

Christian MERCAT

UNIVERSITÉ CLAUDE BERNARD - LYON I

Abstract

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Doctor of Philosophy

Didactic Situations for the Development of Creative Mathematical Thinking: A study on Functions and Algorithms

by Pedro LEALDINO FILHO

Creativity is considered as a crucial skill for the contemporary world. The research described in this thesis had the Project MC Squared as the main context. Carried out between October 2013 and September 2016. The objective of the project was to develop a digital platform for the development of C-books for teaching mathematics in a way that develops Creative Mathematical Thinking both in the students and the authors.

This thesis, entitled: *Didactic Situations for the Development of Creative Mathematical Thinking* proposes an analysis of the design, development, implementation, and testing of digital and non-digital activities with the aim of improving and fostering Creative Mathematical Thinking having Functions and Algorithms as mathematical objects to analyze.

The following research questions raised from the problem:

- How to operationalize and revise existing definitions of Creative Mathematical Thinking?
- How can we assess the progress of a process involving Creative Mathematical Thinking?
- How the "Diamond of Creativity" model is an useful analytic tool to map the Creative Process path?

To answer such questions, the research followed a methodology based on an agile Design-Based Research. Four activities were cyclically developed. The first one, called: "*Function Hero*," is a digital game that uses body movements of the player to evaluate recognizability of functions. Three other activities called "*Binary Code*," "*FakeBinary Code*" and "*Op'Art*", aimed at the development of Computational Thinking.

The main constructs of this thesis are: (a) the "Diamond of Creativity" model to map the process of solving problems found in each activity, evaluating the process and the products derived from the students' work. (b) The digital game: "Function Hero".

To validate the research hypotheses, we collected data from each activity and analyzed them quantitatively and qualitatively. The results show that developed activities have awakened and engaged students into problem-solving and that the "Diamond of Creativity" model can help in identifying and labeling specific points in the creative process.

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List of Abbreviations

CMT	C reative M athematical T hinking
SRP	S tudy and R esearch P ath
ATD	A ntropological and T heory of D idactics
TDS	T heory of D idactic S ituations
CAT	C onsensual A ssessment T echnique
TTCT	T est of C reative T hinking
DME	D igital M athematical E nvironment
COI	C ommunity of I nterest
COP	C ommunity of P ractice

Dedicated to my Family

Chapter 1

Introduction

1.1 Background of the Study

I always appreciated video-games. I spent much time playing them since my childhood. When I started my license in teaching mathematics, during the internships that we have to do to get the diploma, I noticed something that seems to be common sense: Mathematics in schools are boring. If we ask anybody what they think about their mathematics courses, the majority of answers will be that they did not like that. Either because it was too hard, or because it was boring, or even because the teachers were weird.

Since I appreciate mathematics and did not find it annoying, I thought about ways to make it different, and I started asking myself about the use of games in mathematics courses. It was apparent, just observing my colleagues, that they would rather play video games than study. After finishing my license, I chose a Specialization course called “digital games for teaching mathematics.” Starting my research, I noticed that the majority of educational games do not cover much curriculum, and still, remain at the operational level of mathematics, like arithmetic, for instance.

My purpose to engage into this Specialization was, therefore, to learn how to develop digital games and lead to my master’s degree where I developed a small game called: “Simon Bile’s Adventures,” based on the competencies assessed by the Brazilian Institute INEP (Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira).

During my master’s degree, I met professor Christian MERCAT, on a research exchange on the teaching of mathematics for the engineers between the UTFPR (Universidade Tecnológica Federal do Paraná - Brazil) and the INSA (Institut national des sciences appliquées), in Lyon. I showed him my research, and we started to work on it together.

Another motivation aspect of mathematics is its use as a tool to solve problems. The cooperation with my advisor began with a study on math teaching for the engineers in the realm of the collaboration between UTFPR and INSA. Therefore when

I was proposed the opportunity to take part in two Tempus projects on that subject, I jumped in. I performed some research on the perception of mathematics which allowed me to understand that this perception is a social construct and that the basic needs for a student to be a successful engineering student are to change her perception on math and specifically on functions which are so very critical epistemologically and seen as the opposite of fun. Changing this view grew to be the challenge I wanted to address within this thesis.

I was also invited to participate in the MCSquared project. This project had the intention to design and develop an intelligent computational environment to support stakeholders from creative industries involved in the production of media content for educational purposes to engage in collective forms of creative thinking and working to co-design appropriate digital media. A primary focus of the project was on social creativity in the design of digital media intended to enhance Creativity in Mathematical Thinking (CMT). (Knygos, 2013; Essonnier, 2018; Daskolia, 2015; Daskolia, 2016)

In this project we developed the “c-book” (“c” for creative) infrastructure, extending e-book technologies to include various, dynamic widgets and an authorable data analytics engine. The members of four distinct Communities of Interest (including publishers, developers, researchers, school educators) worked together with the shared goal to creatively design c-book resources reflecting 21st-century pedagogy for CMT in schools and the workplace.

My collaboration intended to help in the development of the resources, focusing on the game based learning aspect, and also in developing the theoretical framework and operationalization of the concept: Creative Mathematical Thinking. Almost at the same time the MCSquared project started, another project called JEN.Lab - Learning with Digital Epistemic Games - took place in France. The focus of this project was the conception and the use of digital epistemic games in education and training. Again, the designers of the game were a team composed of teachers from a technical school in Saint-Nazaire. I, as a game designer, supported the first phases of the game development with teachers while the structure of the game also had the intention to enhance CMT in students. We understood during this practice that students from high school have a great interest in games and different methodologies, and engage themselves into learning activities when it is different from what they are used to, for instance, traditional courses.

Then, I was offered the opportunity to take part in two projects: Metamath and MathGeAr, in the Tempus framework. These projects aimed to address a broad spectrum of problems in math education for engineering programs in Russian, Georgian and Armenian universities. To solve these problems, the projects relied on a comprehensive approach including studying international best practices, analytical review,

and modernization of existing pedagogical approaches and math courses. The identification of functions as the main problematic topic in young engineering students lead me to realize that functional thinking was at stake and more generally the notion of algorithm, linking some input to an output in an algorithmic way. This is the reason why I decided to broaden my view on functions to encompass computational thinking and study the creativity therein.

1.1.1 Perception of Mathematics

The attitude towards mathematics is a long-standing strand of research and reliable measuring tools such as the seminal ATMI (Tapia and Marsh, 2004). However, our research identified axes which we find specific to engineering, primarily through its relationship with reality and the impact of the cultural differences.

Mathematics is considered as the foundation discipline for the entire spectrum of Science, Technology, Engineering, and Mathematics (STEM) curricula. Its weight in the curriculum is therefore high (Alpers et al., 2013). In Armenia, Georgia, and Russia, all university students pursuing this kind of curricula are obliged to take a three-semester standard course in Higher Mathematics. Special studies in Europe suggest that competencies gap in mathematics is a most typical reason for STEM students to drop out of study (Byrne and Flood, 2008; Araque, Roldán, and Salguero, 2009; Lassibille and Navarro Gómez, 2008; Heublein, Spangenberg, and Sommer, 2003).

Several research studies show that students' perceptions of mathematics and mathematics teaching have an impact on their academic performance of mathematics (Murtodi and Ngirande, 2014), positive attitude and perceptions toward the subject will encourage an individual to learn the subject matter better. In a broader sense, perceptions towards mathematics courses are as well essential to take into account in order to grasp the cultural differences between all the different institutions, from the students of course, of their teachers and the engineers themselves. The fact that culture does influence these beliefs while seeming obvious is not widely studied. Therefore this study fills a gap in the literature. Without assuming cultural determination, we do show significant differences between institutions.

1.1.2 Creative Mathematical Thinking within the MCSquared Project

As described in the proposal of MCSquared Project, Creative Mathematical Thinking (CMT) is a central aim of the European Union by being connected to personal and social empowerment for future citizens (Council of European Union, 2006). It is also considered as a highly valued asset in the industry (Noss et al., 2012) and as a prerequisite for meeting current and future economic challenges (Augustine, 2005).

CMT is seen as an individual and collective construction of mathematical meanings, norms and uses in novel and useful ways (Sawyer, 2014; Sawyer, 2004; Sternberg, 2003), which can be of relevance to a larger (academic, learning, professional, or other) community. Exploratory and expressive digital media are providing users with access to and potential for engagement with creative mathematical thinking in unprecedented ways (Hoyles and Noss, 2003; Healy and Kynigos, 2010). However, while we might expect that uses of digital media for CMT constitute a primary mission of schooling, education systems as well as industry routinely fail to rise to this challenge. On the contrary, restrictions to creativity enablement abound, mostly stemming from the priorities of a pre-digital industrial age emphasizing conformity and standardization in testing (Chevallard, 2015; Robinson, 2010). New designs are needed, new ways of thinking and learning about mathematics, mathematics education and the support of learners' engagement with creative mathematical thinking in collectives using dynamic digital media.

1.1.3 Game Based Learning approach in Mathematics Education

The reason for creating games in education, usually relies on motivation, aiming at creating alternative ways to engage students in performing activities which they might consider as boring otherwise, even though they are essential in their training.

Computer games can have a very favorable impact on a student's affective attachment to the Mathematics discipline (Kebritchi, 2008; Lealdino Filho, 2014). However, the ability to work with content is not always exploited with the same intensity as that of entertainment. Since a video game can be an original didactic medium, it is essential to systematize the construction of these programs and relate their design to didactical theories. The present work explores this issue, listing guidelines for the design of educational games, clarifying better the role of teachers, artists, and programmers in the work of implementing this type of software and describing a reference for the construction of educational video games seeking to combine fundamentals of software engineering and teaching and learning theories, with a particular focus on Mathematics teaching.

In the MCSquared research, my primary target was to look for affordances which could increase CMT (Lealdino Filho et al., 2017; El-Demerdash et al., 2016a). Initially thought to be complementary, the two approaches on knowledge on one hand and creativity on the other provoked a theoretical tension: the epistemic game intends to increase knowledge and the game that I was developing intended to increase Creativity in Mathematics. This point will be discussed later on in the corpus of the thesis.

1.1.4 Computational Thinking

The theme, Computational Thinking, emerged as an object of study in the thesis when we were choosing the treatments that would be used in Workshops and other interventions. The concern with this object of research was given by the appearance of content in the French curriculum and to be taught within the discipline of mathematics by mathematics teachers. On another hand, in collaboration with the Engineering Universities of Brazil, we observed that the disciplines of introduction to programming and algorithms had huge dropout and disapproval rates. Therefore the scope of functions, starting from functional thinking on real functions of a real variable, was broadened to encompass results of algorithms and algorithmic thinking in general.

1.2 Problem

Extensive research has been conducted on Creativity in Mathematics (Hadamard, 1996; Poincaré, 1913; Ervynck, 2002; Haylock, 1987; Rosa, 2004; Leikin and Lev, 2007; Sriraman, 2009a), Game Based Learning (Prensky, 2003; Dalla Vecchia, Maltempi, and Borba, 2015; Borba et al., 2017; Green and Kaufman, 2015; Gros, 2007; Lealdino Filho, 2014) and Teaching Functions (Vandebrouck and Leidwanger, 2016; Singer and Voica, 2015; Glazer, 2011; Runco et al., 1999). While these studies have described Creativity in Mathematics, there is little empirical research in putting together the subjects of technology, games, mathematics, inquiry-based learning, and creativity in the same scope, and the conditions for its efficacy in affording Creative Mathematical Thinking (CMT). The first reason is that there is no operational definition of Creativity in Mathematical Thinking. Some authors analyze the creativity of an individual or a group from their *products*, while others, analyze creativity as a *process*. Another reason can be thought of, such as the traditional setup of educational system, where not much room is devoted to activities and strategies focusing on creativity in mathematical concepts. However, creativity is a crucial element in the daily work of professional mathematicians.

The present study pursues to operationalize creative processes in mathematical thinking, identifying the affordances to integrate into artifacts, and setting up methodological tools to assess their efficacy in different classroom environments. In particular, we introduce the model of "The Diamond of Creativity" to observe creative manifestations during the problem-solving process and aim at validating it.

1.3 Significance of the Study

The significance of this study is to contribute a valuable theoretical and practical framework for successful implementation and development of educational digital games and didactical situations providing Creative Mathematical Thinking affordances. It is significant since creativity is an essential skill in 21st-century individuals (Greiff, Niepel, and Wistenberg, 2015; Pásztor, Molnár, and Csapó, 2015; Binkley et al., 2012), allowing them to solve problems in the real world. The results of this study may be utilized to develop, design, study, and improve digital games, game-based learning, and inquiry-based learning environments to enhance creativity, but as well, at a more basic level, for teachers and teacher trainers, to take into account, identify and foster creativity in their students.

1.4 Research Design

In this section, we clarify the research design because two main methodologies are presented in answering the research questions. More details are provided on 4. One methodology leads to the design of the activity used as a treatment in which students' behavior and production is assessed in its creative dimension (Function Hero), and the other research methodology leads to the development of the Diamond of Creativity (this model is described in Chapter 2), aiming to assess Creative Mathematical Thinking.

1.4.1 Research Questions

Digital Serious Games are used in education and most of them have, as a primary goal, teaching, training or simulating some "serious" content improving the knowledge of the user in some specific area. However, in this thesis, we are focused on the creative side of mathematics. Which, as I describe further in the text, does not necessarily have a direct link to learning. It means, becoming a creative person in the realm of mathematics may not have a direct learning side effect, this happens for example when the individual already has the target knowledge before being put in a situation fostering creativity.

Besides, other non-traditional activities can be inserted in a non-game context as motivational tools. Open-ended problems and magic, for example, can also be used to increase the student's motivation for pedagogical practice.

Therefore, our main research question is to know whether and how diverse didactic situations can enable the manifestation of observable behaviors through a descriptive model of the creative process, in the direction of mathematical creativity, having as mathematical object functions and algorithms.

From this general research context, several specific research questions arise:

Qualitative questions

1. How can we operationalize and revise existing definitions of Creative Mathematical Thinking?
2. How can we assess the progress of a process involving Creative Mathematical Thinking? We propose to objectify this development through an observing tool that we called the *Diamond of Creativity*.
3. How the "Diamond of Creativity" model is an useful analytic tool to map the Creative Process path?

Quantitative questions

1. How do students perceive mathematics?
2. To what extent can we validate a learning gain in function recognition tests after Function Hero activity?
3. What is the variability of the choreographies of the students in the Function Hero game?
4. What is the variability of solutions in the activities Binary Code and Fake Binary Code?

These research questions shaped the research in the following chronological format:

- Understand students' perceptions of mathematics in MetaMath and MathGear projects. The perception of the students from the partners' universities provided us with a foundation for developing a game with a mathematical object central in engineering courses: The concept of Functions.
- Develop a game with structural elements that enables the development of creativity. The criteria for the development of this game were built during the projects: MCSquared and Jen.Lab, with two prototypes: Space Math Ship (not discussed in this thesis) and Function Hero. The first and not definitive operational definition of creativity emerged in this process.
- Collect data on referring to Creative Mathematical Thinking from the games and activities that were developed. The game Function Hero served as a treatment to collect data from the participants' productions. An improvement in the theoretical framework on creativity has prompted us to study creativity not only in its state of "product" but also as a "process". In this context, Computational Thinking appeared to be essential and the Diamond of Creativity was put to the test in observing processes involving Computational Thinking.

- Evaluate Creative Mathematical Thinking quantitatively and qualitatively. This step was made possible using the Diamond of Creativity and the use of statistical techniques on graphs.

Details of this research design and its methodology are described in chapter 4.

1.5 Limitations

The study based on *Function Hero* contains two main objectives: (a) Develop a digital game which allows the enhancement of Creative Mathematical Thinking (b) Develop the theoretical framework to test the benefits and efficacy of the game. Thus, in this section, I present the limitations in three parts, one regarding limitations of the developed resource, one concerning experimentations, and the other regarding the theoretical limitations.

1.5.1 Artifact's limitations

1. The main limitation presented by the resource is the operating system in which it works. Using Microsoft SDK to be able to use the Kinect sensor tied the development to Windows users. Taking into account that educational environments are diverse, and most of the time, find for open source solutions, the game does not attend this need of school or whoever wants to use the game with Linux or MacOS for instance.
2. Another limitation is the fact that even trying to minimize the number of materials to put in practice this educational methodology, still is necessary to have at least a computer and the Kinect sensor to make it works. An optional to future research is to use the webcam instead of using the Kinect sensor since webcams are available in almost all computers and notebooks.
3. It needs a little effort from teachers or users to understand how the Kinect sensor gets the data and send to the Cinderella software. Thus, the user can decide and adapt the game for their own needs.

We are now working on this limitation, and we should have a new implementation shortly using the PoseNet library from the TensorFlow framework in JavaScript using CindyJS, a JavaScript rewrite of Cinderella at <http://bit.ly/FunctionHero>.

1.5.2 Experimentation limitations

1. The sample of schools was particular hence limited; therefore, results may not be generalized to all classroom settings.

2. Many variables outside the control of the researcher could impact the student manifestation of Creative Mathematical Thinking. These variables may include The role of the teacher in instrumentalizing the artifact since the researcher was always guiding the experimentation in the classroom. The gender of students, since the majority of the engineering courses, have boys as predominant gender, even knowing that there is no significant difference between gender's perception of Mathematics (Lealdino, Mercat, and El-Demerdash, 2016).

1.5.3 Theoretical limitations

In the chapter ??, in the topic: *Networking Strategies to develop the Model*, I show the principles of each theory used to build the final model. In this dissertation, I chose not to go deeper into the details that other finer theories can bring to the fore. The more prominent ones are the Documentational Approach and the Theory of Instrumental Orchestration.

The Documentational Approach (Gueudet and Trouche, 2008) insists on the slow appropriation of a pedagogical resource, in particular of a digital nature, by the teacher. A teacher is nurtured by different sources, and in order to use a particular resource as a valid teaching resource in the classroom, she has to appropriate the resource, make it her own and integrate it into a *system* of interrelated documents. This process is personal and depends on the technical, pedagogical and cultural backgrounds of the teacher. Such a fully appropriated resource then becomes a *document*. We have chosen not to study this appropriation and its documentation in the present thesis.

Once appropriated, a resource has to be used in the classroom. In the theory of Orchestration (Drijvers and Trouche, 2008), the teacher amounts to the *conductor* of a Jazz band, where every student, using the resources at hand as an instrument to tackle the situation, plays his part. I witnessed the way teachers began to appropriate and use my situations, and how students shared different roles in using them, one exploring new venues, another digging deeper in another direction, a third bringing into play alien resources such as a hand-held calculator, but I have chosen not to describe this role-play, erasing individuals behind the group and its production.

Being brought up to research in the theoretically very vivid city of Lyon, in the EducTICE team, I am fully aware of the theoretical limitations that these choices bring as a consequence. It is a matter of course that they would shed an interesting light on this work and I see this as avenues for further work.

1.6 Benefits of the Study

We believe the most significant benefit developed in this study was the digital educational game, Function Hero. We were able to use an incredible body data capture tool (Kinect) and integrate its functionality into a Dynamic Geometry Software (Cinderella), which has shaped a game adaptable to different age groups. We have also been able to test the game in real environments, different circumstances, and diverse scenarios such as, classrooms, science fairs, scientific exhibitions, and international congresses.

1.7 Summary

In this first chapter, we present an overview of the research, highlighting the background of the study, the problems to be solved, the significance of the study, the design of the research, its limitations and also its benefits. In the following chapters, we will present the theoretical foundations that supported our methodological choices to answer the research questions, the results and the discoveries made from the analysis of the collected data.

Chapter 2

Literature Review - The Diamond of Creativity

2.1 Introduction

This chapter presents the rationale for researching the relationships between the use of inquiry-based or game-based learning activities and the improvement of Creative Mathematical Thinking. We present a theoretical framework to support the assumptions, definitions, and methods used to validate the hypothesis and answer the research questions. The literature review did not strictly follow a systematic path. However, the development of the literature befallen mainly in correspondence with the MC Squared project and the publications linked to the project and this thesis.

Also, we describe how the diamond model of creativity was developed. We show its anatomy and the academic references used to develop the model. One of the hypothesis of the thesis shows that one can observe the creative process in problem-solving under the light of this model. Therefore, we explain how each stage of this process was elaborated and developed.

This chapter mainly addresses three aspects that support the applied methodology to answer the research questions:

- **Creative Mathematical Thinking Definition:** We seek in the literature whether there is any objective definition of Creative Mathematical Thinking to know how to develop this concept or skill in individuals or students.
- **Operationalization of Creative Mathematical Thinking:** We attempt to find examples of research already developed with the objective of developing Creative Mathematical Thinking, looking for clues in the development of methodological tools to assess it.
- **Improving Creative Mathematical Thinking:** To verify if there are researches that measure the gain of creativity in didactic situations, with different treatments.

Kaufman has been working on the subject of Creativity in the literature and says that the definition of Creativity is not too simple. (Kaufman, Cole, and Baer, 2009; Kaufman, Plucker, and Baer, 2008; Kaufman and Sternberg, 2010; Green and Kaufman, 2015) However, in this thesis, we need to define creativity, in particular, because the development of the methods to measure the progress of the individual on problem-solving relies on it. In the final years of MC Squared project, we come to a definition of Creativity as a process in which individuals insert themselves to create something new and somehow modify the space made up of innovative products. This model of Creativity is called "Diamond of Creativity", (see. Figure 2.1).

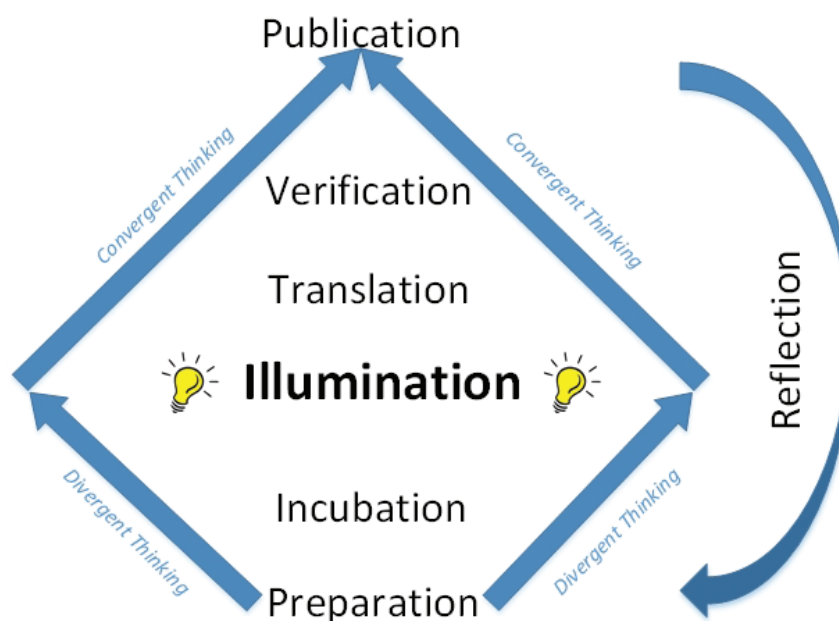


FIGURE 2.1: Diamond of Creativity

The Diamond of Creativity is a combination of principles which will be described in this chapter. The creative process is modeled as a path (see. Figure 2.1 with three distinct moments: Divergent phase, Illumination (AHA Moment), and Convergent Phase. This process happens recursively as the result of the engagement of an individual in solving a problem. This process is creative in the sense that it adds to the set of objects from a defined category, for instance the set of solutions to a problem, the set of answers, the set of concepts, etc., in an availability space defined by the community in which the individual belongs to.

One of the most challenging thing in the subject of creativity is the widely spread perception that it can be considered in a mystical, magical way, as a God-given gift. In many Mathematics Education events, such as CERME (Congress of the European Society of Research in Mathematics Education) and ICME (International Congress of Mathematical Education), for example, creativity is placed in the same working group of studies on high skills or talented students. In the same fashion, some people think that mathematics requires a special talent that can not be acquired. This thesis

refutes this argument, creativity and mathematics understanding can be the result of successful teaching.

2.2 Creativity

Kaufman, argues that perhaps the biggest reason why creativity can feel like an unwanted Windows upgrade is that many people do not know precisely what creativity is. At the beginning of the M C Squared project, we asked the participants of the committee of interest in France, what was their definition of Creativity and many definitions emerged: Think outside the box, unusual, novelty, artistic. Indeed these answers can be considered correct, however, what exactly does that mean?

The history of creativity, following Kaufman, has two "eras" so far - before 1950 and after 1950. Before 1950, there was insufficient research being conducted on creativity. Most of the early researchers who may have discussed or studied creativity focused on something else. Everything changed at the 1950 convention of the American Psychological Association. In his presidential address, Joy P. Guilford called for psychologists to increase their focus on creativity. He argued that creativity was an important topic and was not being studied or researched at the level it warranted.

Guilford examined the index of the Psychological Abstracts for each year since its origin. Of approximately 121,000 titles listed in the 23 years before 1950, only 186 were indexed as definitely bearing on the subject of creativity. Guilford galvanized the field and made it acceptable to study creativity (Guilford, 1950). A quick search on banks like Science Direct shows a result of 72,285 titles listed in research articles and book chapters after 1995.

Guilford (1950; 1967) placed creativity into a larger framework of intelligence in his Structure of Intellect model. See Fig. 2.2

With five operations, four contents, and six products, Guilford model had 120 different possible mental abilities. Guilford used a more psychometric approach to creativity, using Factor Analysis techniques to create the intelligence model.

As it is possible to see in the Figure (2.2), Guilford attempted to organize all of the human cognition along three dimensions. The first dimension called "operations" refers to the mental gymnastics needed for any task. These operations are:

- *Evaluation*: Evaluation is defined as a process of comparing a product of information with known information according to logical criteria, deciding criterion satisfaction.
- *Convergent Production*: Convergent production rather than divergent production is the dominant function when the input information is sufficient to determine a definite answer.

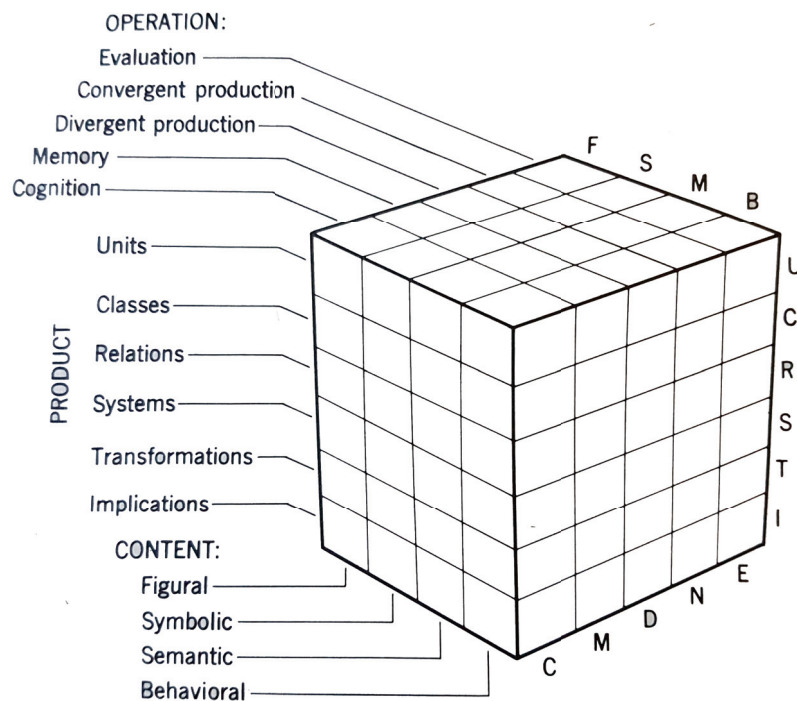


FIGURE 2.2: Structure of Intellect Model, by Guilford

- *Divergent Production*: Divergent production is a concept defined following a set of factors of intellectual ability that pertain primarily to information retrieval and their tests.
- *Memory*: The faculty of the mind by which information is encoded, stored, and retrieved.
- *Cognition*: Defined in the light of the kinds of tests needed to represent cognition factors, Guilford defines cognition as awareness, next discovery or re-discovery, or recognition of information in various forms; comprehension or understanding.

The second dimension was called "content," referred to the general subject area:

- *Figural*: Figural information has sensory character, like recognition in pictures or sounds for example. "Figural" skills are observed in image recognition for example.
- *Symbolic*: Symbolic information is in the form of tokens or signs, that are used to stand for something else.
- *Semantic*: It has more to do with the incorporated meaning of the word, but not with the word itself.

- *Behavioral*: For discussion purposes, Guilford says that behavioral content is defined as information, essentially nonverbal, involved in human interactions, where awareness of attention, perceptions, thoughts, desires, feelings, moods, emotions, intentions, and actions of other persons and ourselves is important.

Moreover, the last dimension, "product," represents the actual different forms of results of thinking in different kinds of subject matters:

- *Classes*: Classes can be somewhat operationally defined as recognized sets of items of information grouped under their common properties.
- *Relations*: It is a recognized connection between two items of information based upon variables or upon points of contact that apply to them. The term *variable*, according to Guilford, is included in the definition because many relations are of a quantitative or ordered sort, as described in such statements as "*A* is more *X* than *B*" or "*D* is less *X* than *B*."
- *Systems*: A system may be defined as an organized or structured aggregate of items of information, a complex of interrelated or interacting parts.
- *Transformations*: Transformations are changes of various kinds, of existing or known information in its attributes, meaning role, or use. The most common transformations in figural information include changes in sensory qualities and quantities, in location (movement), and an arrangement of parts.
- *Implications*: The definition of implication emphasizes expectancies, anticipations, and predictions, the fact that one item of information leads naturally to another. The implication, as a product, must be the connection, which is not a relation.

This componential study of creativity serves to describe creativity in general, in any field. In the field of mathematics, authors such Hadamard, 1996, Poincaré, 1913 e Cédric Villani, 2015, describe the emergence of ideas, theorems, theories and solutions as creative manifestations during the research process. Ervynck, 2002 hypothesize that the context for creativity is set by a preparatory stage in which mathematical procedures become interiorized through action before they can be the objects of mathematical thought. Preliminary to this may be an initial stage where the procedures might be used without even a full appreciation as to their theoretical status. We consider this creative "step" as incubation Wallas, 1926.

However, we realize that creativity, when described by these great mathematicians, is shown as the process of great discoveries, which drastically changed their investigations. Since our research is based on the hypothesis that anyone can be a creative person, especially in mathematics, the idea of a creative person should be attributed to any individual able to create something observable.

Amabile, 2012, argues that the first step to understanding the definition of creativity assumed in this thesis is to unwear the concept that creativity is something that happens only with geniuses or very talented people. Gardner, in his vast work on Creativity, defines creativity works as "the small subset" of works in a domain that ever deemed to be "highly novel, yet appropriate for the domain"; these works "actually cause a refashioning of the domain." Howard studied seven individuals: Sigmund Freud (1856-1939), Albert Einstein (1879-1955), Pablo Picasso (1881-1973), Igor Stravinsky (1882-1971), T.S. Elliot (1888-1965), Martha Graham (1894-1991), Mahatma Gandhi (1869-1948).

The study of Howards focused, in the words of Amabile, *squarely* on the creative individual defined as "a person who regularly solves problems, fashion products, or defines new questions in a domain in a way that is initially considered novel but that ultimately becomes accepted practice in a particular cultural setting." (p.35)

Following this reasoning, applying to the mathematical field, Creative people are the ones who create or had created things which changed the domain of mathematics. However, the MCSquared project was developed on the idea that Creative Mathematical Thinking is a vital construct or skill that every citizen from the 21st century must have: Like Amabile, we do not support the idea that Creativity is something exclusive to gifted or talented people. It is interesting to see that nowadays, in mathematics education conferences, the topic of creativity is very often related to giftedness in mathematics.

Howard's view is a "Big C" view of creativity, in the sense that every creative work is seen as both novel and valuable. These two values make a call to the idea that novelty, appropriateness, and value are possible to measure. However, in the following chapters, I am going to present the fact that these are only relative values to a particular context.

In contrast to Howard, Amabile proposes a "Little c" approach. Like Howard, she defines creativity as an appropriate novelty that is recognized as such by people knowledgeable in a domain. She tried to understand the social-environment forces that can dampen or enhance creativity. Both then, believe that the creative work changes the domain in which the product is inserted, however, the evaluation on how it is novel relies on the hands of the others. We can say then, that creativity depends on a social system in order to be evaluated and measured.

As our interest stands in creativity within mathematical thinking enhanced by technology, our social environments are educational contexts. It means that all the activities developed for this thesis and the MCSquared project, are intended to be used, mainly, by teachers in a *didactic situation*.

2.3 The Evolution of Creative Mathematical Thinking in the MCSquared Project

First we need to define two main categorizations: CoI (Community of Interests) and CoP (Community of Practices). Thanks to Essonnier, and her work on Social Creativity in the same project, we will use the definitions presented in her work.

- Community of Practice: According to Essonnier, the notion of Community of Practice (CoP) was introduced by Jean Lave and Etienne Wenger, as a group of person who shares an common interest, a job or profession (Lave, Wenger, and Wenger, 1991). She also says that, according to Wenger, 2011, a CoP can be characterized by three essential components: The domain, the community and the practice.
- Community of Interest: Essonnier cites (Fischer, 2001), who explains that a Community of Interest brings together stakeholders from different CoPs to solve a particular (design) problem of common concern. They can be thought of as "communities-of-communities (Brown & Duguid, 1991) or a community of representative communities". (p. 71)

The analysis and evaluation of the promotion of Mathematical Creativity and Creative Mathematical Thinking were one of the main tasks of the MCSquared project and also of this study. This assessment came in two distinct and complementary way. Firstly, the first level of analysis sought to identify the conceptions of mathematical creativity and creative mathematical thinking together with the members of the committee of interest of each country and how these conceptions impacted the considerations and definitions of c-books design criteria.

It is known that the MCSquared project has been developed for three years and for each year the methods of evaluations have evolved. In the following sections, we present the trajectory of evolution of the concepts of mathematical thinking in the three cycles of the project, ending with the construction of the diamond of creativity. These discussions are also present at the MCSquared Project Reports (Daskolia, 2015; 2016;) and also in the previous study of Essonnier, 2018 and El-Demerdash and Kortenkamp, 2010a.

The MC Squared project was divided into *work packages*. The WP2 was a work package described as an extended review on the theory and research on social creativity in the design of digital educational resources for CMT based on which we compose the working definitions and the measurement model of creativity to be used and describe the methodology to be followed. In WP2, the researchers from Université Claude Bernard were involved in the following tasks:

- Operational definitions and criteria for measuring social creativity in the design of digital educational resources for CMT.

- Developing an integrated theoretical framework on social creativity in the design of digital educational resources for CMT.
- A refined theoretical framework on social creativity in the design of digital educational resources for CMT.
- Guidelines and recommendations of setting up CoI for stimulating creative designs of educational resources for CMT.

The MC Squared project deals with the primary objective: improving Creative Mathematical Thinking in learners using the c-book resources developed by all communities inside the project. However, the concept of Creative Mathematical Thinking is being clarified from a theoretical perspective within the WP2 between researchers themselves.

Each CoI decided on ways to gather initial conceptions about CMT in different ways. They allowed building a common vocabulary about this concept among researchers. The outcomes of these discussions, in a concrete way, helped back the CoI members reflecting on the c-book unit CMT potential and drove their design process. One can think of a c-book unit as a digital interactive mathematical book that provides an environment to learn mathematics creatively. It allowed every CoI to define its criteria for CMT support in c-books and evaluate them on this Cycle 1 production.

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In the following sections, we present, the initial representations on CMT for the French COI, the evolution of these along the c-book design process and the results of the evaluation of the two c-book units the CoI has developed from the perspective of their potential for CMT.

Representations of Creative Mathematical Thinking

In this section we intend to answer for each CoI the following questions:

- What were the initial representations of the CoI members on CMT? How were they identified?
- Have they changed along the process of designing the c-books?
- How were these representations used as 'criteria' for evaluating the c-books? Through what process?

- What were the outcomes of the evaluation?

We describe the process of defining CMT in two ways:

- General: Where CMT is described as an outcome of discussions, meetings and literature review.
- Local: Where CMT is realized inside the c-book development.

The French and the Greek CoIs proposed a brainstorming and discussion on CMT issues. The Spanish CoI went even further and conducted activities aiming at reaching agreement on a definition on what mathematical creativity is, identifying topics and questions of interest and setting up sub-groups in charge of the design of a c-book unit around on of these topics. In the UK CoI, the ideas of first c-book units emerged also during the first face-to-face meeting of the CoI.

This preliminary question helped as a cheap teaser and warm-up to conduct a face-to-face brainstorming among the CoI members about CMT representations during first CoI meeting. The following issues came up from this brainstorming:

1. Situations of research, Open problems \Rightarrow means for developing creativity?
2. Classroom time, time for preparation, outside the classroom \Rightarrow conditions and constraints to take into account
3. Creative approach: Applying a different reasoning, different thought process
4. Teachers' epistemological foundations
5. Position of error in French Education
6. Early Education
7. Taking the initiative, at both teacher's and learner's side
8. Think differently, realize differently
9. Modelling

When trying to make sense of these issues, at the first glance divergent, we came up with the following organization putting forward three main aspects of the CoI representation of CMT:

- *Context fostering the development of the creative thinking:*
 1. Situation of research
 2. Open ended problems
 3. Modelling
- *Expression of the creative thinking:*

1. Creative approach: applying a different reasoning, a different 'train of thought', inquiry-based approach
 2. Taking initiatives on both teachers' and students' sides
 3. Think differently, produce differently
- *Conditions that favour or hinder the implementation of situations fostering creative thinking:*
 1. Classroom time, preparation time, outside the class
 2. Teachers' epistemological bases
 3. Early training
 4. Position of error in French Education

Organisation of CMT Representation using CoICode

When CoICode communication tool became available, the researchers asked the CoI members to start using it in order to summarize our views on CMT, previously expressed with paper and pens, sticky notes, Playmobil arrangements, billboards ... We didn't know at this moment exactly what to expect and whether CoICode was suitable for this task. Jana Trgalova and Nataly Essonnier had prepared textual summaries of these previous events and we tried to organize them as a graph in CoICode. Christian Mercat first tried to create nodes as buzz words popped out from the discussion, right during the discussion itself, but the resulting graph became rapidly "bushy" and unmanageable while the CoI members disagreed on different aspects of the connections that were made.

This first attempt (see Figure 2.3) was discarded and a more thoughtful try was endeavoured. After fiddling around, we decided to postpone this exercise and use a mind map tool that we mastered in order to create a concept map describing our conceptions of CMT (see Figure 2.4).

Then we went back to CoICode in order to encode the result with the desired format (see. Fig ??).

In order to develop a common consensus about what is CMT for our CoI, a discussion was held during a third CoI face-to-face meeting. The synthetic Figure 2.6 is a result of these discussions based on the initial brainstorming and on the representations of each group during the Creative workshop. Items related to the Approach, Effects, Conditions, Means and Context constitute criteria that will be used in the evaluation of the c-book units CMT potential.

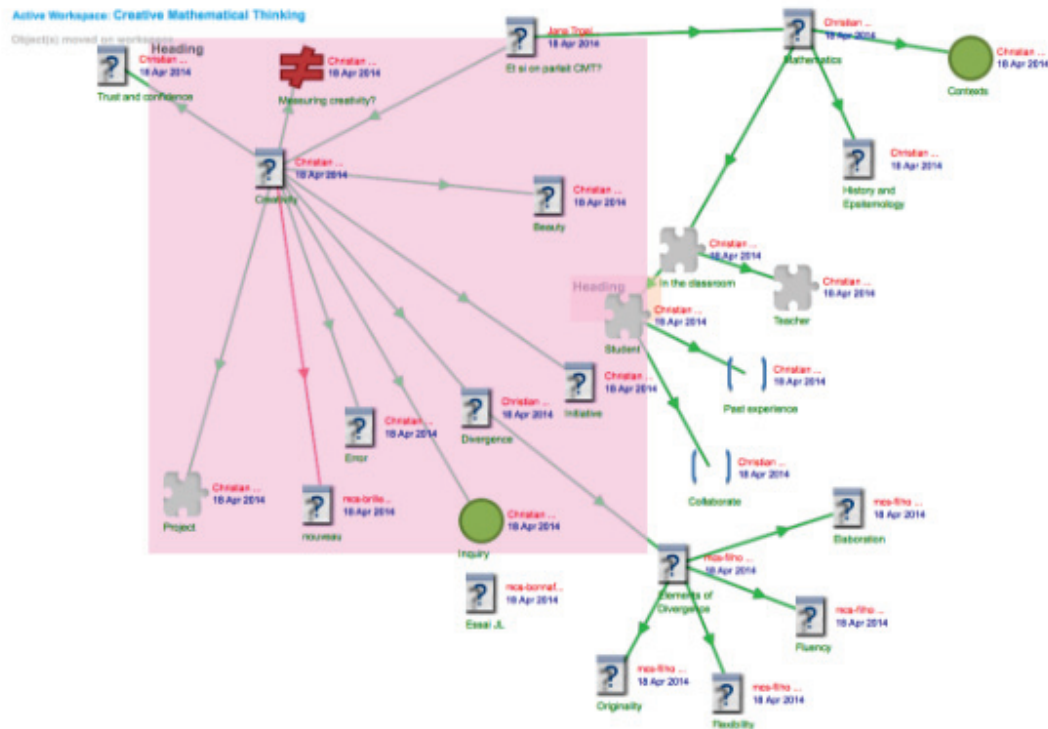


FIGURE 2.3: Resulting Graph

2.3.1 Second Cycle

In the following sections, we present, for each CoI, its evaluations of CMT affordances of c-book units, the evolution of their evaluation tools along cycle 2 and the results of these evaluations. The background theory on CMT is further addressed within the chapter ???. This evaluation will help the consortium to define common guidelines to design c-book units, based on converging and diverging point of view from the different CoIs.

In this section, we intend to answer for each CoI the following questions:

- How the CMT potential of c-book is being evaluated?
- Has it changed along the process of designing the c-books?
- What were the tools to evaluate the c-book CMT potential? How were they developed? What were the outcomes of the evaluations?

First, we recall here the different conceptualization of CMT during the first cycle. The French and the Greek CoIs proposed a brainstorming and a discussion on CMT issues among the CoI. The Spanish CoI went even further and conducted activities aiming at reaching agreement on a definition on what mathematical creativity is, identifying topics and questions of interest and setting up sub-groups in charge of the design of a c-book unit around one of these topics. In the UK CoI, the ideas of the first c-book units emerged also during the first face-to-face meeting of the CoI.

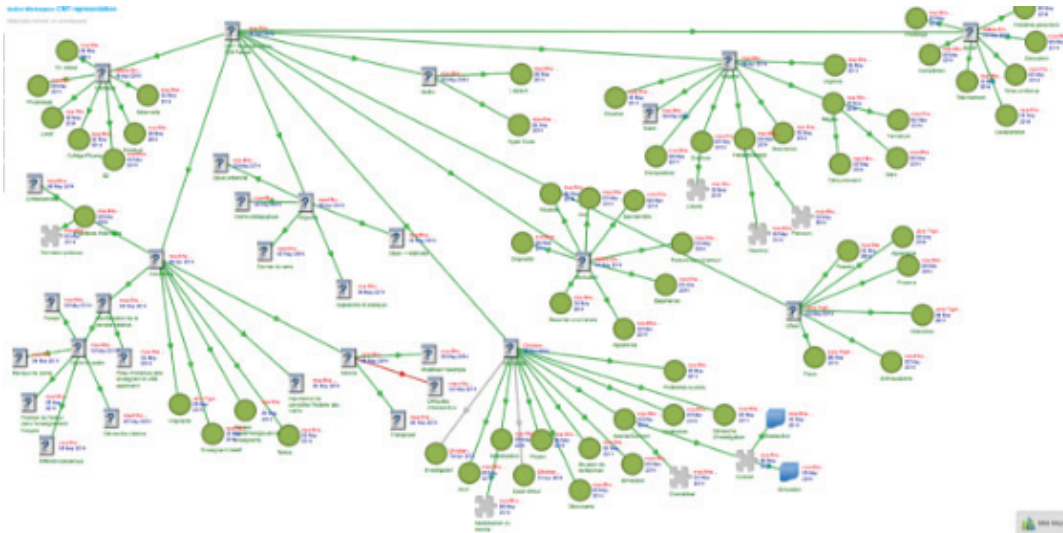


FIGURE 2.5: CMT Representation - French Community

MC2 CoI members who are involved in producing c-book units for CMT throughout the project. For identifying the CoI members' initial representations of CMT, questionnaires were designed and administered during the research teams face-to-face meetings with the CoIs. At a later stage, after the CoIs had completed with the production of their first c-book units, they were asked to become themselves evaluators of their productions, by thinking retrospectively on the criteria they used for designing their c-book productions in terms of CMT. This first after-task evaluation procedure aimed at collecting evidence from the part of the creators themselves on whether and how they had applied their initial representations as criteria for collectively designing c-book units in terms of CMT. The hypothesis to examine was that the CoI members' initial representations were important factors in shaping their design practice. This is also a question attached to the broader issue of the linkage between the teachers' conceptions of the nature of mathematics and their teaching (Olafson and Schraw, 2006)

Level 2: Self-evaluating and cross-evaluating the (perceived) CMT affordances of the c-book units productions - Production of a grid of analysis of CMT affordances for c-book units across CoIs.

The second level of CMT evaluation involves both initial designers as self-evaluators of their c-book units productions and members of a second CoI as external judges from the field in retrospectively evaluating and critically cross-examining their c-book unit productions in terms of the affordances they display in promoting CMT. During cycle 1, two units per CoI were completed and the criteria for their design were discussed. This resulted to an initial awareness of their CMT affordances. During cycle 2 the CoI members (from CoI A) will have to redesign one of their previous c-book productions and at the same time to evaluate and re-design the c-book production of another CoI (from CoI B). This is in accordance with the idea of involving

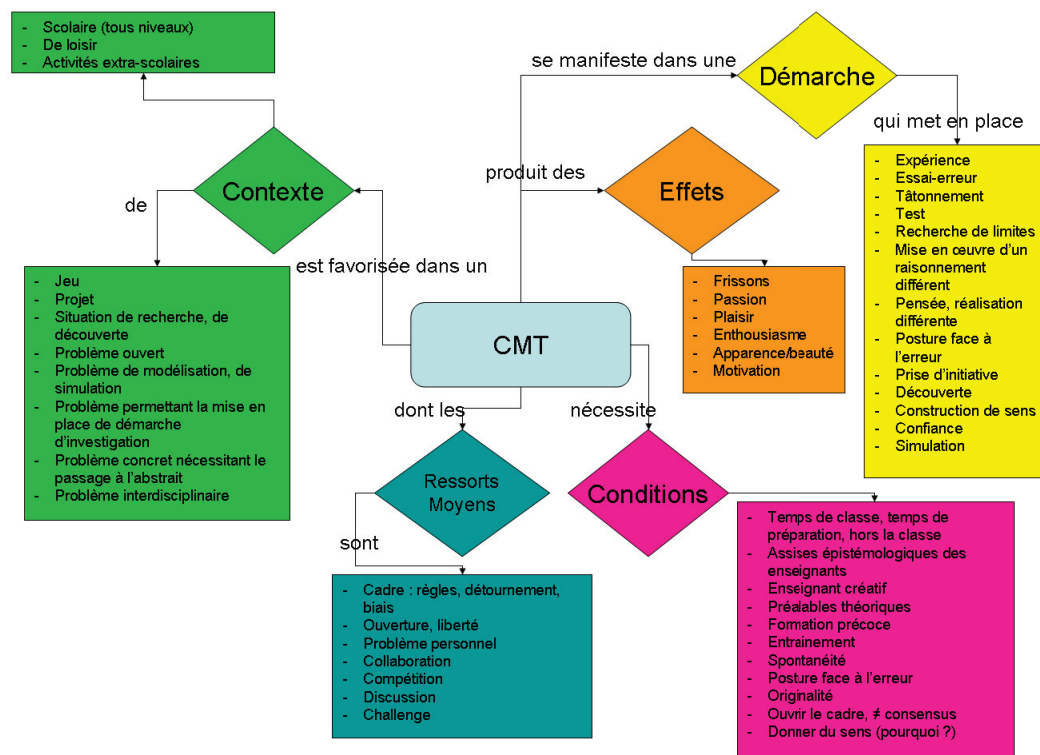


FIGURE 2.6: French Representation of CMT at the First Cycle

teachers into a quality assessment of existing resources (Trgalova, Soury-Lavergne and Jahn, 2011). This evaluation procedure will lead to a ‘common grid’ of commonly accepted CMT affordances of c-book units by combining and discussing evaluation forms from the four different CoIs. This grid will be further used as a “componential” measure of CMT for an in-context approach to CMT assessment. These CMT criteria (or “components”) will be elaborated within the realms of a collective (within the research groups and between researchers and CoI members) refinement process. Therefore, the construction of the grid will be used both a tool for facilitating shared understanding of CMT, thus playing the role of a ‘boundary object’, and as a tool to be used in a more contextualised, situated and holistic approach to CMT assessment rather than as a stand-alone tool for explicit criteria (O’Donovan, Price, and Rust, 2001).

Level 3: Assessing CMT manifestations in the students’ interaction in the use of c-book units.

Finally, the third level of CMT evaluation, conducted during Cycle 3, was based on the data obtained by conducting small-scale evaluation studies with students in real classroom conditions using the c-book units produced. The “componential” grid for CMT was used as the main instrument for identifying and rating evidence of CMT in the students’ processes and products. Since the c-book units are designed to promote

CMT in their prospective end-users (students), it is deemed necessary to engage students as evaluators of the creative potential of the c-book unit productions. There has been some important work focusing on defining what can stand as an evidence of CMT in students' activity (see for example the work of Leikin, Levav-Waynberg, and Guberman, 2011) that can be used as reference point, however in conjunction with our evolving understanding of CMT along the course of MC2 project and our on-going pursuit for more appropriate approaches to measuring mathematical creativity.

2.3.2 The c-book potential to improve Creative Mathematical Thinking

In this section, we provide the methodological approaches and the results of c-book's CMT affordances analysis. In order to evaluate the CMT affordances of c-books, each CoI developed its own tool which allows their c-book reviewers and evaluators to investigate the c-book and then, be guided on its analysis. The creation of these tools was necessary regarding the Level 2 of the CMT evaluation, as mentioned above. The criteria and indicators presented in the grid tool were thoroughly selected from existing literature and also discussed among CoIs. Considering the different set-ups of each CoI it is important to remark that each one has specific characteristics on how the evaluation tools were developed however, it's possible to see common indicators.

Methodological Approaches: Design of CMT Affordances evaluation tool

During the design and production of c-book units, the french CoI members kept in mind the initial intention to promote the Creative Mathematical Thinking on the learner's side. In this sense, the c-book unit were evaluated regarding CMT as a product itself and also for its capacity to promote the act of being mathematically creative when using it : CMT as a process.. It's important to remark that even evaluating the c-book as a product, does not exhaust the possibility to evaluate the unit as a promoter of mathematical creativity on both aspects: mathematical creativity as a product and mathematical creativity as a process.

The creation of the grid tool for evaluating the Creative Mathematical Thinking affordances raised from the necessity of knowing how the c-book unit could promote the development of CMT for the learner. The french CoI to analysed the c-book units through components and indicators that help us to establish the stages of performing mathematical creativity. It means that the CMT is considered as a process triggered and sustained by those factors, and Mathematical Creativity as a product or/and processes which can be concretely evaluated a-priori or a-posteriori analyzing the possible user interactions with the artefact.

The grid addresses four main axis:

- **General Information:** This section contains the mathematical fields to be covered by the c-book, if the c-book is intended to be used by users inside or outside classroom and with or without the orchestration by a tutor/teacher, and which kind of activities the c-book allows:
 1. Problems with many solutions
 2. Situation with many answers
 3. Open Problems
 4. Inquiry-based approaches
 5. Multidisciplinary
 6. Problem posing
- **Mathematical Creativity Indicators:** This section lists the most frequently used creativity indicators presented in literature review. The method to create the scale for measuring each indicator was the semantic differentiation, not a numerical value. In contrast to the Likert's scale, we don't take a position on the question to gain an agreement level, even because creativity is not a completely well shaped concept. What we did was to ask for answers without any pre-judgment compared to two different value extremes (– , ++). The indicators that we proposed were:
 1. Usefulness
 2. Originality
 3. Flexibility
 4. Fluency
 5. Elaboration

The French CoI focused on two main indicators: Flexibility and Fluency, and we asked how the evaluators perceive these indicators are present in the c-book. Then, we asked evaluators how they explicitly see Mathematical Creativity is embedded in the c-book, either as a process or as a product or both.

We always ask for comments and justifying answers for each question in order to get an idea of the level of comprehension regarding what is being asked.

- **Social Elements that foster creativity:** considering that social elements can foster creativity we embed social components to be evaluated with the same semantic differentiation approach. These elements are considered as supporters of creativity during the use of the c-book unit. They are:
 1. Discussion
 2. Collaboration

3. Challenge/Competition

- Affective elements that foster creativity: The affective side of using educational technologies has a great value in how the student appropriates the artefact and transform it as part of himself. For instance, with children, some educational videos use the visual appeal to take the attention of the spectator holding the interest in the artefact. In this sense, we believe that this kind of emotional link between an artefact and its user, supports and maintains an optimal state to be creative. The affective element evaluated in the c-books was Motivation.

Motivation can be triggered through two ways. Intrinsic motivation: When the person creates and hold by itself the desire to achieve something, for instance a piano player that trains a lot towards to be a professional and pleases herself. And on the other hand, extrinsic motivation, when the person is doing something motivated by an external factor like for instance, getting a diploma.

This evaluation grid was co-elaborated among french CoI and based on CMT representations and issues from Cycle 1. To finally agree on the items, the grid was tested on the UK c-book numbers for WP7 cross-evaluation work. Two groups of evaluator were formed, two forms were collected and then a synthesis was made collectively during a face-to-face CoI meeting and the final grid was realised.

When each c-book template was completed and its unit available inside DME, the CMT evaluation was realized by the designers and a reviewer filling an individual form and discussing it afterwards or filling collectively the form during a teleconference or face-to-face meeting.

For analyzing the collected data an histogram representation was provided in order to visualize the strengths and the weaknesses of the CMT affordances of the c-book produced during Cycle 2.

Inter-Coi Synthesis: Commonalities

At the all partner's work is possible to see some commonalities regarding how the evaluation tool was developed. Each CoI used the methodology which they elaborated for the c-book evaluation. Similarities between the CoI's are:

- The use of the indicators Originality, Flexibility, Fluency and Usefulness. French and UK CoI's used these indicators explicitly during their evaluation. Greece and Spain also based some axes of their evaluation on these, but the evaluation was made in the acts related to each criteria. For instance, Flexibility was evaluated as applying multiple strategies to solve problems.
- All CoI's agreed on the features of c-books which fosters creativity: open tasks, situation with many answers, inquiry, problem posing and multidisciplinary. This last (multidisciplinary) weights a lot in the design of all c-books, as each

CoI is composed by a granularity which takes individuals from diverse domains, is possible to remark that application of mathematical knowledge in diverse domain increases the sense of usefulness and appropriateness of the c-book.

- The affective element of c-books were taken account on all CoIs: maintaining motivation for the students by finding features that might enhance this: The aesthetic aspects, the sense of freedom, active engagement and playful disposition to use the mathematical knowledge.

Evaluation of CMT potential in c-book units: results and analysis

In this section, we give an example of the results and analysis for each CoI regarding their particular method of evaluation. The data presented here were collected from the c-book report delivered by each CoI and can be found in a complete way in the annex.

The CMT evaluation tool provided two results: quantitative and qualitative. In the next section we present the summarized results for one c-book produced during the cycle 2: *Algebraic Tetris*

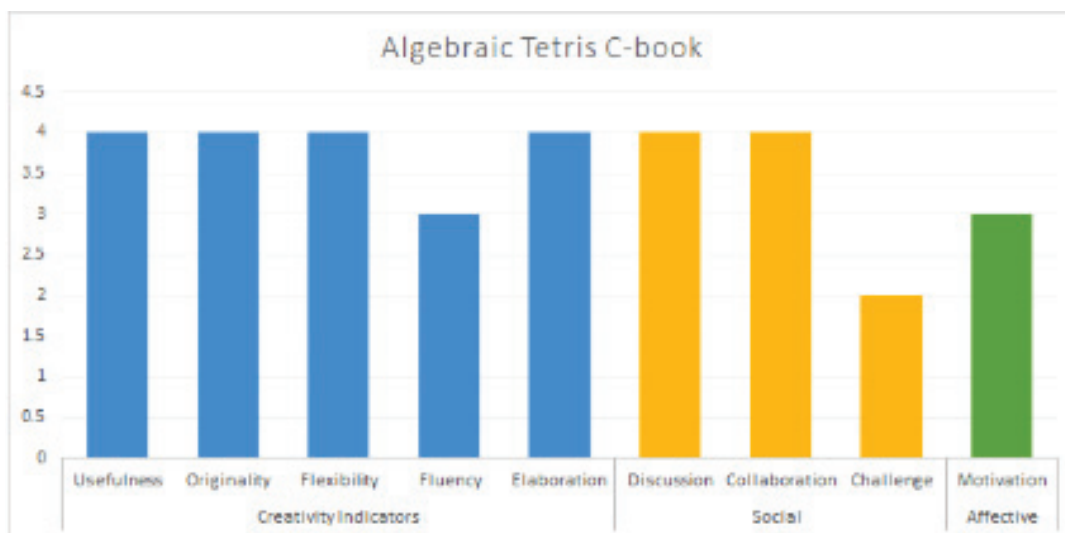


FIGURE 2.7: Algebraic Tetris c-book

As we can see on the figure 2.7, this c-book unit obtained a good evaluation from the researchers achieving the highest score for all indicators of creativity except Fluency. Regarding the social aspects, discussion and collaboration achieved the highest score and for the affective factors, it is possible to see that the c-book promotes motivation.

The only problem with the Fluency factor was that in the c-book unit, some activities are designed in a way to limit the number of answers asked to students. However, Fluency is rather related to the maximum possible amount of answers and solutions that a person can provide in a limited time. This aspect is easily changeable in the

c-book unit, such that a re-design was proposed in order to achieve a high level of Fluency potential as well.

Considering their evaluation, we can conclude that the c-book unit is potentially powerful, at least from the point of view of its designers, to improve the Creative Mathematical Thinking and also improve the competencies in mathematics (arithmetic and algebra).

2.3.3 Third Cycle

In this section, we provide the methodological approaches and the results of c-book's CMT affordances analysis for one c-book produced during the third cycle called *Math for Biology*. In order to evaluate the CMT affordances of c-books developed during the cycle 3, the CoIs used a common grid discussed during a workshop in Athens and finalized through a series of skype meetings between members of the four CoI. The development of this final form of the grid was based on three main aspects.

The first aspect includes 13 items aiming to evaluate the c-book unit affordances towards the development of CMT in users/students. These items relate the nature of the activities included in the c-book unit with the user's cognitive processes (fluency, flexibility, originality, elaboration). The second and third aspects are dealing with the social and affective aspects of the c-book unit that could enhance CMT for c-book users.

As for the first aspect, the responders were asked to evaluate the item in relation to each one of the four cognitive components of CMT in a scale from 1 (no affordance) up to 4 (strong affordance). There was additionally an extra option called N/A in case the affordance was not applicable for the specific item.

The evaluation of the CMT affordances of this c-book unit was done by three members of the French CoI. It was organized in three steps. Firstly, an e-mail was sent to each of the CoI members involved in the evaluation asking to have a look over the c-book unit and the grid. Secondly, a Skype meeting was organized by the main designer of the c-book to address evaluators needs for understanding and clarification. Thirdly, the evaluators evaluate the c-book affordances based on the grid using the Google form prepared for this purpose.

- *Cognitive Aspects of CMT:*

1. The c-book unit includes mathematical open problems or questions. (Silver, 1997)

2. The c-book unit includes devising and formulating problems by the students / users themselves or formulating new questions to extend the investigation of the initial problem (problem posing tasks). (Silver, 1997; Lev-Zamir and Leikin, 2011b)
3. The c-book unit includes calling for students' / users' constructions that call for mathematical thinking. (Silver, 1997; El-Demerdash and Kortenkamp, 2010a)
4. The c-book unit provides users with opportunities to establish connections between different knowledge areas and mathematics (interdisciplinary / cross-disciplinary / external connections). (Perry-Smith and Shalley, 2003).
5. The c-book unit provides users with opportunities to establish connections between different mathematical fields / concepts (internal connections). (Haylock, 1987; Haylock, 1997)
6. The c-book unit provides users with opportunities to establish connections between various representations of the mathematical concepts at stake (e.g. through a combination of widgets offering various representations). (Sriraman, Yaftian, and Lee, 2011)
7. The c-book unit stimulates/encourages users' exploratory activity and users' experimentation. (Silver, 1997; Leikin, 2009)
8. The c-book unit stimulates/encourages users to formulate and check their mathematical conjectures. (Haylock, 1987; Haylock, 1997).
9. The c-book unit stimulates/encourages users to search and find multiple solutions / multiple strategies to solve a mathematical problem. (Lev-Zamir and Leikin, 2011b; Skulimowski, 2011; Livne, Livne, and Wight, 2013)
10. The c-book unit stimulates/encourages to think about, reflect, summarize and evaluate the mathematical work already developed. (Lev-Zamir and Leikin, 2011b)
11. The c-book unit stimulates/encourages to generalize mathematical phenomena, going from concrete cases to general ones or to generalize real world phenomena through the use of mathematics. (Lev-Zamir and Leikin, 2011b)
12. The c-book unit includes non-standard problems calling for mathematical solutions. (Tan, 2007)
13. The c-book unit includes half-baked constructs (which are partially constructed or intentionally incorrect constructs) that call for intervention (Kynigos et al., 2014)

- *Social Aspect of CMT:*
 1. The c-book unit stimulates / encourages user’s collaboration / cooperation / interaction with other users. (Sriraman, 2009b)
 2. The c-book unit stimulates / encourages the students to develop their mathematical communicative skills. (Sriraman, 2009b)
 3. The c-book unit provides users with opportunities to stimulate competition. (Gross, 2018)
- *Affective Aspects of CMT:*
 1. The c-book unit actively promotes engagement by generating a perception of usefulness of mathematics, either in everyday life, or inside the mathematical context. (Shriki, 2010;
 2. The c-book unit actively promotes engagement by generating a feeling of pleasure / fun / challenge (narratives, game features, feeling of flow / immersion in the activities, etc.) (Bergen, 2009; Shriki, 2010; Keller, 2009)
 3. The c-book unit actively promotes engagement by generating a feeling of aesthetic pleasure from their contact with the mathematical concepts. (Bergen, 2009; Keller, 2009)

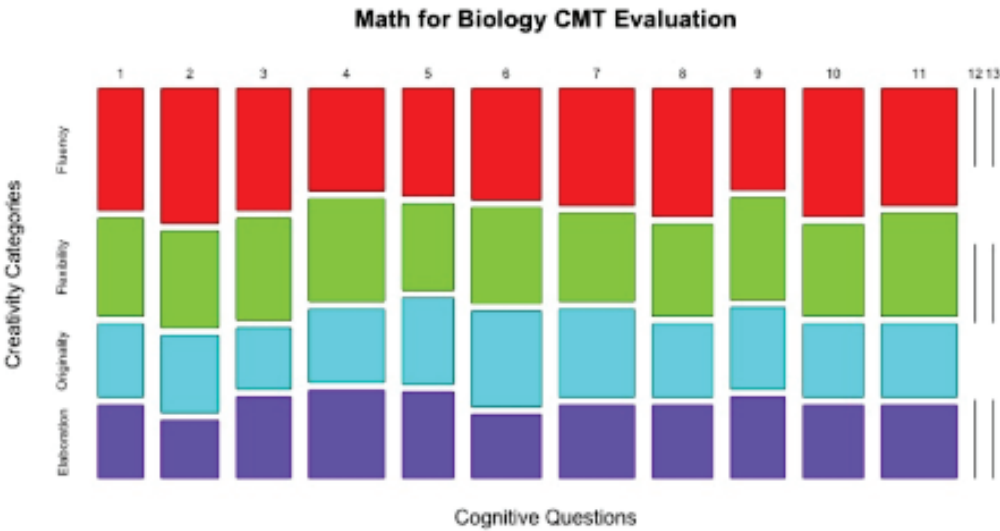


FIGURE 2.8: Cognitive aspects evaluation

The Figure 2.8 represents the evaluation of the cognitive aspects of CMT from the evaluator’s point of view. The height of the bars represents the distribution of the four components for each item (Fluency, Flexibility, Originality and Elaboration), while the thickness represents the mean between the four aspects for each question. From the evaluator’s point of view, only the items 12 and 13 achieved the value of N/A (no affordance). It means: 12. The c-book unit doesn’t include non-standard problems calling for mathematical solutions. 13. The c-book unit doesn’t include

half-baked constructs (which are partially constructed or intentionally incorrect constructs) that call for intervention.

Parameters	Fluency	Flexibility	Originality	Elaboration	Social	Affective
Mean	3.03	2.46	2.11	1.92	2.83	3.33

TABLE 2.1: Summary Mathematics for Biology - CMT Evaluation

To understand the table 2.1, we defined the follow statements of evaluation for the quantitative data:

- N/A means that the corresponding item is not present in the c-book (there is no such affordance)
- The value 1 means "no affordance", i.e., the corresponding item is present in the c-book unit but it does not foster the given dimension of the CMT;
- The value 2 means "weak affordance", i.e., the corresponding item is present in the c-book and might foster the given dimension of the CMT, but it is rather unlikely;
- The value 3 means "good affordance", i.e., the corresponding item is present in the c-book and might foster the given dimension of the CMT;
- The value 4 means "strong affordance", i.e., the corresponding item is present in the c-book and fosters the given dimension of the CMT.

From the scale defined to evaluate the c-book unit we got the following values for each component, as shown in the table 2.1: Fluency = 3.03, Flexibility = 2.46, Originality = 2.11 and Elaboration = 1.92. The highest value for this c-book in terms of cognitive aspects was Fluency in which your value achieved the rank of "good affordance". It means that, in general, the c-book facilitates the possibility of students in developing their ability to provide as many responses as possible or to come up with many strategies to solve a mathematical problem or challenge. Elaboration was the component with lower value.

	Fluency	Flexibility	Originality	Elaboration
Fluency	1.00	0.89	0.67	0.75
Flexibility	0.89	1.00	0.69	0.90
Originality	0.67	0.69	1.00	0.60
Elaboration	0.75	0.90	0.60	1.00

TABLE 2.2: Correlation Values - Mathematics for Biology

As for the correlation among the four cognitive components of CMT, the above table shows that there is strong correlation among some cognitive aspects. It means that, considering a significant value of $r > 0.70$, we may conclude that Fluency can be fostered in the same time as Flexibility and Elaboration. Flexibility might be fostered by Fluency and Elaboration. In the case of Originality, there isn't evidences

that supports the hypothesis that this component can be fostered by the other ones. Finally, there is evidence to accept that Elaboration might be fostered by Fluency and Flexibility.

To conclude, the c-book was well evaluated in terms of fluency and affective aspects. The c-book was based mainly in the connection between mathematics and biology. The c-book calls students to make relations with proportionality and golden ratio, emphasizing the role of mathematics and how it can be perceived in the nature.

Evaluators did not comment that much on their quantitative evaluation. However, they raised the need for orchestration to achieve the goals intended by the c-book unit.

In the previous sessions, we saw how the evaluation of creativity was performed and operationalized within the MCSquared project, within the French community, during the three cycles of the project. However, it was not useful to assess the student's productions in terms of Creativity. The evaluation methods shown before allow to pre-assess which component of Creativity the c-book can afford in terms of designers opinions.

The evaluation described above shows which structural elements should be present in an artifact in order to afford mathematical creativity. These criteria were taken into account for the development of the game *Function Hero*, as described on chapter 3.

2.4 Creativity and its connection with Learning

Kaufman, Plucker, and Baer proposes the "Four P" model to organize creativity research. This model distinguishes the creative *person*, from the creative *process*, leading to a creative *product* within a creative *environment*. We will use this model within an educational context focused on students, thus the creative person will be the student, the process will be the process in which a student engages him-herself in order to solve problems proposed by a teacher, the product will be the answers or productions from students, and the environment will be the educational context where the didactic situations occur. It is important to notice that Kaufman calls the *Environment* as *Press*, that is why the model is called "Four P."

2.4.1 The Creative Person - Student

As the title already says, studying the creative person may look at the individual characteristics of the creator. Amabile's componential model of creativity proposes that three variables are needed for creativity to occur: domain-relevant skills,

creativity-relevant skills, and task motivation. Domain-relevant skills include knowledge, technical skills, and specialized talents that individuals might possess that are important in particular domains, but not in others. In our case, if you are going to be a creative mathematician, according to this theory, you would need to know mathematics, but that mathematical knowledge might be of little use to someone who wants to be a creative painter. We see later, in the Op'Art section that they are not mutually exclusive! Indeed it can be useful for an artist, a painter, a dancer or a musician, to engage in Creative Computational Thinking.

Creativity-relevant skills are personal factors that are associated with creativity more generally, across many or all domains, such as tolerance for ambiguity, self-discipline, and a willingness to take appropriate risk. Self-discipline is necessary to solve challenging problems, especially when encountering fixations. (Reis, 2008).

As Kaufman, Plucker, and Baer says: If one focuses on the person as a possessor of such skills, the emphasis is on the person, but if one focuses on the underlying cognitive skill, then the emphasis is on the process itself rather than the person possessing it.

The third component in Amabile's model singles out one's motivation toward the task at hand. Intrinsic motivation - being driven by the enjoyment of a task - is more associated with creativity than extrinsic motivation, or being driven by external rewards such as money or praise. Unfortunately, intrinsic motivation is seldom encountered in the math classroom, and students often describe themselves as forced to study mathematics against their will (Middleton and Spanias, 1999).

This is the reason why, according to previous research, enriching the *milieu* with games, allows to increase intrinsic motivation for a task. In the case of the didactic situation, especially in mathematics, the motivation is driven by the milieu. Expressions of intrinsic motivation in mathematics classes are rare. That is why the development of the game became necessary because, in previous researches (Lealdino Filho, 2014), we realized that games and other didactic means could bring more motivation to the accomplishment of a task.

This is the reason why, in this thesis, the activities and didactic situations take a different form from the traditional classroom activities, changing the usual scenario in order to motivate the students to realize some tasks. In this setup, creativity is more accessible to spot through the engagement of the students in the proposed tasks.

In our case the creative individuals are the students, participating in the activities described in this thesis, namely *binary code trick*, *fake binary code*, *Op'Art* modeling and *Function Hero*. In this thesis, we will not study individual creativity and rather focus on the production and process of groups. In particular, we will not try to understand each student individually, neither assess their creativity through standardized

creativity tests, and we will only look at their evolution through the knowledge gain spectacles.

2.4.2 The Creative Process - Problem Solving

The creative process is the experience of being creative. As Kaufman, Plucker, and Baer says, a way of considering the creative process is found in the Geneplore Model (Finke, Ward, and Smith, 1992). The Geneplore model proposes that creative cognition cycles to gradually refine the initial pre-inventive structure. Exploring the pre-inventive structure, the initial idea is updated, and the cycle continues until a satisfactory result is reached. (Davis et al., 2011).

We are going to analyze this process of generation and exploration as divergent phase, convergent activities, explained in the next chapter, when we describe the Diamond of Creativity *per se*. We will see that this process can turn out to be recursive.

2.4.3 The Environment - Didactics of Mathematics

Amabile and Gryskiewicz, 1989 identify eight aspects of the working environment that stimulate creativity: adequate freedom, challenging work, appropriate resources, a supportive supervisor, diverse and communicative coworkers, recognition, a sense of cooperation, and an organization that supports creativity. They also list four aspects that restrain creativity: time pressure, too much evaluation, an emphasis on keeping the *status quo*, and too much organizational politics.

As said in Kaufman, Plucker, and Baer, the environment does not have to be a professional working environment; Indeed, we consider a school as an environment that should foster creativity. In line with these elements, the activities that we setup follow the guidelines mentioned above, in order to permit and develop Creative Mathematical Thinking in the students and more generally a creative attitude in the school. Some teachers and playground supervisors might think that students are creative enough when it comes to messing around. Our goal here is to focus their creativity in a positive direction concerning learning. We can perceive that in this setup, among some elements that impede creativity, the major one is a non-interactive traditional transmissive way of teaching, where students are not supposed to produce anything original instead reproduce what has been taught.

One theory that focuses on the relationship of a creator to the environment is the System Model proposed by Csikszentmihalyi, 1996. This model considers creativity to be a byproduct of the domain (i.e., mathematics), the field (the gatekeepers, such as editors and critics), and the person. In this model, these three elements work interactively.

In this case, we can understand the creative process happening in the following ways: (a) The individual as a system himself, where the products will change his/her domain of knowledge itself. (b) The individual within a community, such as classmates, when their response changes the domain in which all others are included and changes the domain of others. Moreover, finally, the universe as a system. I mean, if we have a new product that is inserted and modifies the domain of all individuals, universally, we usually can consider this kind of change or expansion as a Big C creativity event.

2.4.4 The Product - Answers and Solutions

The creative product - the things people make, the ideas they express, the responses they give. A big difference between product-focused assessment, such as the Consensual Assessment Technique (CAT), and process-focused assessments, such as the Tests of Creative Thinking (TTCT), is that products are typically domain-specific. A product might be a poem, a musical composition, or a mathematical proof (Kaufman, Plucker, and Baer, 2008).

In the case of the *Function Hero* activity, what is produced by the students is a choreography, a sequence of functions but we will analyze other types of products for other activities, an algorithm for *Binary code* and *Fake Binary Code*, a graphical sketch for *Op'Art*.

2.4.5 Creativity and Connectionism

Creativity is related to knowledge in the way that we know that something is new or innovative when we are not able to find in our memory something similar. Therefore, to assess students' creativity, we need a particular form of representation of what they already know. The basis of Artificial Intelligence can be used to represent a specific existing knowledge.

Martindale, 1995 argues that surprisingly, it turns out that behavioral, cognitive, and even psychoanalytic theories are substantially identical when expressed as a neural network theory. Even more surprisingly, this neural network theory is isomorphic to a set of existing connectionist models. A neural network theory tries to explain how neuronlike components could explain mental processes. Recently, Lakoff and Núñez, in *How The Embodied Mind Brings Mathematics Into being*, see the root of all logic in the embodiment of perceptions and movements into an evolving internal mental state of the world that allows predictions.

To build the knowledge base, we use a process called knowledge engineering. A knowledge engineer is someone who investigates a specific domain, learns which

concepts are essential in that domain, and creates a formal representation of objects and relationships in the domain.

Knowledge engineering projects vary widely in content, scope, and difficulty, but all of them include the following steps (Russell and Norvig, 2016):

- Identify the task: The knowledge engineer should outline the range of issues the knowledge base will support and the types of facts that will be available for each specific instance of the problem.
- Add relevant knowledge: The knowledge engineer must already be a domain expert or perhaps need to work with real experts to extract what they know - a process called *acquisition of knowledge*. At this stage, knowledge is not formally represented. The idea is to understand the scope of the knowledge base, determined by the task, and understand how the domain works.
- Define a vocabulary of predicates, functions, and constants: That is, convert the important domain-level concepts into logical-level names. It involves many engineering style questions of knowledge. Once the choices have been made, the result is a vocabulary known as domain *ontology*. The word ontology represents a specific theory about the nature of being or existing. Ontology determines the types of items that exist but does not determine their specific properties and interrelationships.
- Code knowledge of the domain: The knowledge engineer writes the axioms corresponding to all terms in the vocabulary. It fixes (as far as possible) the wording of the terms, allowing the expert to check the content.
- Encode a description of the specific instance of the problem: If the ontology is well elaborated, this step will be easy. It will involve the writing of simple atomic sentences on instances of concepts that are already part of the ontology.
- Query the inference procedure and get answers.
- Debugging the knowledge base.

In the article of (Leikin, Levav-Waynberg, and Guberman, 2011) she presents a method of measuring creativity based on authors such as (Ervynck, 2002); Krutetskii, WIRSZUP, and Kilpatrick, 1976; Polya, 2004b; Torrance, 1998). From a perspective of (Torrance, 1998), most of its main sources of knowledge, the reference language, the association current, and the use of basic and general knowledge. *Fluency* if time is unlimited, it refers to the process of solving a problem and switching between different solutions. If time is limited, (as in example of the number 36) fluency refers to the number of solutions created by the individual. *Flexibility* is related to work ideas producing a search solution and query the number of solutions by an individual. *Originality* for unique thinking and unique products. Leikin, 2009 proposed a model to evaluate

mathematical creativity using problems with multiple solutions (MST). Such a problem explicitly requires the student to solve it in different ways. Leikin, 2007 uses the concept of *Solution Space* to observe and examine the various aspects of a problem with multiple solutions.

Specialized Solutions Space contains both conventional and unconventional solutions. The *Individual Solutions Space*, which shows all solutions developed by an individual, is used as a tool to explore a student's creativity. The *Collective Solutions Space* includes all solutions produced by a group of students.

In the chapter 4, we explain in details the nodes and the nuclear components considered to develop the conceptual base and the solution space for each activity.

2.5 Computational and Mathematical Thinking

To better understand the approach in this work, we must define what an algorithm is. An algorithm can be defined as the translated process of solving a problem, using a finite amount of steps, in a way which is useful, non-ambiguous and organized, translated into instructions which are replicable and effectively carried out to solve problems of the same instances, by an individual, a computer, or an information-processing agent.

However, an algorithm by itself is not an *algorithmic thinking* in the same way a formula or a model in mathematics is not a *mathematical thinking*. We consider, in this case, that both are products from the creative process presented in this work.

Modeste, 2012 cites in his thesis Knuth, 1985, a mathematician who is one of the pioneers of informatics and author of the top-rated series of books *The Art of Computer Programming*. In his article *Algorithmic Thinking and Mathematical Thinking* Knuth, 1985, the role of algorithms in mathematical sciences is taken into the discussion and raise the question "What's the difference between algorithmic thinking and mathematical thinking"? In his words

Do most mathematicians have an essentially different thinking process from that of most computer scientists? Knuth, 1985

Modeste, 2012 argues that Knuth, 1985 understood *Algorithmic Thinking* as something distinct from mathematics. He was based on mathematical works and its proofs to analyze the *Mathematical Thinking* and compare it to the *Algorithmic Thinking*. He splits the mathematical thinking into nine *thinking modes* summarised at the Figure 2.9

1. Formula Manipulation
2. Representation of reality

	Formula manipulation	Representation of reality	Behavior of function values	Reduction to simpler problems	Dealing with infinity	Generalization	Abstract reasoning	Information structures	Algorithms
1 (Thomas)	**	**	**						
2 (Lavrent'ev/Nikol'skiĭ)	**		*		**				
3 (Kelley)	*					**	**		
4 (Euler)	**		**	*		**			*
5 (Zariski/Samuel)	*			*	**	*	**	**	
6 (Kleene)	*					**	**		*
7 (Knuth)	**	*		*					
8 (Pólya/Szegő)	**		**	**	**				
9 (Bishop)	**		**	**		*	**	**	*
"Algorithmic thinking"	*	**		**			**	**	**

FIGURE 2.9: The different mathematical thinking according to Knuth and their appearance in another works

3. Behavior of function values
4. Reduction to simpler problems
5. Dealing with infinity
6. Generalization
7. Abstract reasoning
8. Information structures
9. Algorithms

More recent work like Weintrop et al., 2016a proposed a definition of computational thinking for mathematics and science in the form of a taxonomy consisting of four main categories: data practices, modeling and simulation practices, computational problem-solving practices, and systems thinking practices, illustrated in the Figure 2.10.

Regarding Mathematical Thinking, authors like (Polya, 2004a) and (J. Mason, 2010) has provided stages through which solving a mathematical problem is likely to pass. Also, Schoenfeld, 1985 was concerned with problem-solving, organizing his work under four headings: the resources of mathematical knowledge and skills that the student brings to the task, the heuristic strategies that the student can use in solving problems, the monitoring, and control that the student exerts on the problem-solving process to guide it in productive directions, and the beliefs that the student holds about mathematics, which enables or disable problem-solving attempts.

J. Mason, 2010 also identified four fundamental processes, in two pairs, and showed how thinking mathematically very often proceeds by alternating between them:

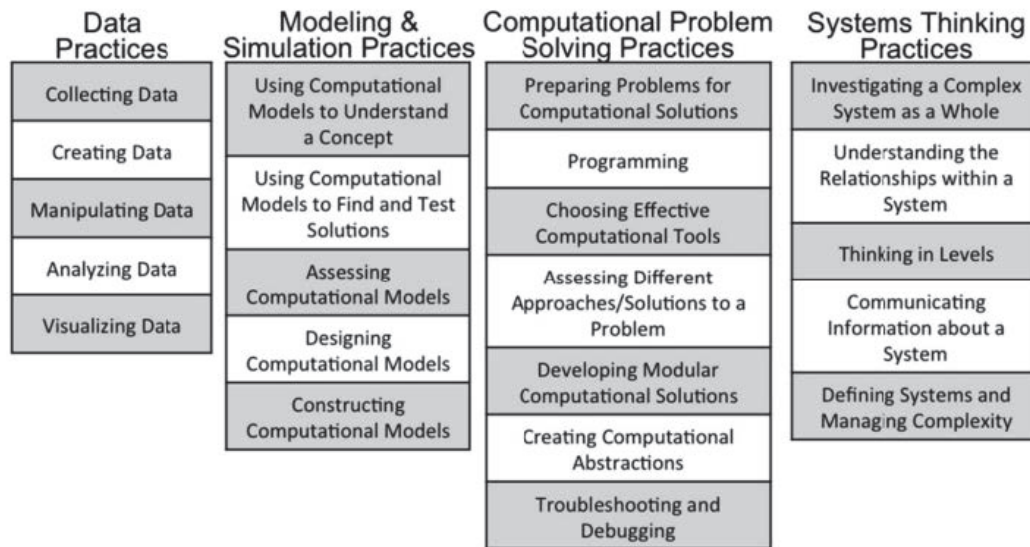


FIGURE 2.10: Computational Thinking in mathematics and science taxonomy

- specializing - trying special cases, looking at examples
- generalizing - looking for patterns and relationships
- conjecturing - predicting relationships and results
- convincing - finding and communicating reasons why something is true

In contribution Dreyfus, 1991 identifies two complementary fundamental processes:

- representation - generate an instance, specimen, example, image of a concept.
- abstraction - a constructive process, the building of mental structures from mathematical structures, i.e., from properties and relationships between mathematical objects.

Thus, we can perceive that both *Algorithmic Thinking* and *Mathematical Thinking* are processes intended to produce, often, the solution to a problem. The main difference is that the algorithm will always be translated into a set of steps to solve a problem, while in mathematics the product can be the solution, without the procedures to get there. In the next session, I will explain how this product can be measured as a creative product, elapsed by a creative process.

2.6 Building the Diamond of Creativity Model

In this session, we describe how the model *Diamond of Creativity* model was created. In the previous sessions, we saw how structural elements of a c-book could contribute to CMT enhancement in individuals, but we did not define how the CMT would be assessed.

There are several theories in Mathematics Education and especially on creativity in mathematics, it was difficult to have a consensus about what it is and how we should evaluate it. Then, Prediger, Bikner-Ahsbahs, and Arzarello contributes to the ongoing discussion on methods to deal with the diversity of theories in mathematics education research. Introducing and systematizing a collection of case studies using different strategies and methods for networking theoretical approaches that frame (qualitative) empirical research. The term 'networking strategies' is used to conceptualize those connecting strategies which aim at reducing the number of unconnected theoretical approaches while respecting their specificity.

Networking strategies are those connecting strategies that respect on the one hand the pluralism and/or modulate of autonomous theoretical approaches in the scientific discipline (Thomas et al., 2017). The figure 2.11 attempts to order those networking strategies in between the two extremes concerning the degree of integration, and it must be emphasized that it is not easy to specify their exact topology globally since the mutual degree of integration always depends on the concrete realizations and networking methodologies.

The networking strategies are structured in pairs of similar strategies for which gradual distinctions can be made: understanding and making understandable, comparing and contrasting, combining and coordinating, and integrating locally and synthesizing.

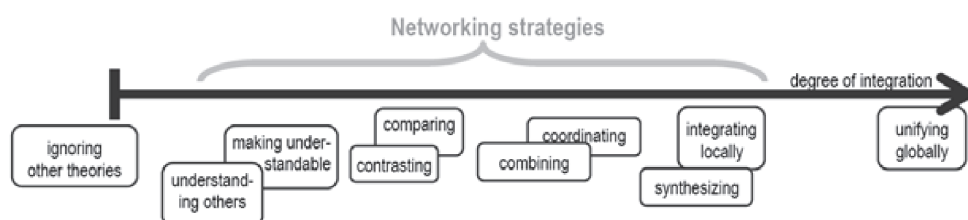


FIGURE 2.11: A landscape of strategies for connecting theoretical approaches

The main references that contributed to the development of the Diamond of Creativity were:

- *Divergent Thinking*: Based on Guilford' idea of divergent thinking where the components: Fluency, Elaboration, Flexibility, and Originality. We understand that in solving problems, an exploratory phase happens, a research quest for already existing knowledge. Also defined by Wallas on the Creative Process as Preparation, Incubation, Intimation.
- *C-K Theory*: It's an unified Design theory and reflects the assumption that Design can be modelled as the interplay between two interdependent spaces with different sctructures and logics: the space of concepts (C) and the space of

Knowledge (K). (Hatchuel and Weil, 2003a; Hatchuel and Weil, 2003b; Hatchuel, Le Masson, and Weil, 2017).

- *Bisociation*: A moment where ideas are spontaneously connected, enlightenment within the creative process (Koestler, 1964). Illumination from (Wallas, 1926). It's also called *AHA Moment*.
- *Convergent Thinking*: The opposite of divergent thinking, where ideas are translated, verified and published by the individual (Guilford, 1950). Verification (Wallas, 1926).
- *Theory of Didactic Situations*: All the situations proposed for the experiments are intended to make the student walk the path, that is, to expand the knowledge to solve the problem. That is why each situation is considered a didactic situation because it involves the artifacts, teacher and the workshop configuration as "milieu" to arrive at the answers expected by the teacher. (see Brousseau, 1997).
- *Didactic Engineering*: This theory gives us grounds to develop activities to be used within an educational environment. However, the development of activities had more to do with enhancing creativity than learning. Since we understand that learning something new is to insert a new concept or practice within the set of concepts and knowledge, it serves to define the characteristics of the milieu, the didactic variables, the interaction between knowledge and the student, the a priori ways to solve the problems and what opportunities are available for the student to be creative or to learn something. (Chevallard, 1982).
- *Study and Research Path*: It arises from Chevallard, from the Theory of Didactic Situations and Didactic Anthropology in what he calls the paradigm of questioning the world. For each question, there is a discovery, a path to be followed to arrive at an answer required by the teacher. (Chevallard, 2015; Chevallard, 2009; Barquero and Bosch, 2015).
- *Adjacent Possibles*: This study shows that innovations arise from the discovery and exploration of nuclear concepts close to each other, which can be achieved by exploring new problems. (Tria et al., 2014; Loreto et al., 2016).
- *Computational Thinking*: The taxonomy developed by Weintrop et al. gives us the necessary tags to identify the student's progress in developing the algorithms to solve the activities. (Weintrop et al., 2016a; Modeste, 2012)
- *Inquiry Based Learning*: Putting students in the role of a researcher, (Artigue and Blomhøj, 2013), inquiry is a multifaceted activity that involves making observations; posing questions; examining books and other sources of information to see what is already known; planing investigations; reviewing what is already known in light of experimental evidence; using tools to gather, analyze,

and interpret data; proposing answers, explanations and predictions; and communicating the results. (Council, 1996)

2.7 Summary

In this chapter, we have seen how creativity became a research topic and the problems that emerge from this discussion. We have seen that creativity can be seen from the perspective of different approaches, and yet it is not a manifestation exclusively for geniuses and talented people, but for any individual who engages in product creation and problem-solving. We define the variables that we observe in the creative process: Product and Process, based on the model of the four P's of Kaufman, Cole, and Baer.

Also, we described how Creative Mathematical Thinking was studied within the framework of the MCSquared project in its different cycles. Structural components for creating objects that enable the development of creativity have been defined to assist designers in creating their artifacts. At the beginning of the chapter, we highlighted in which tasks the research team of the Université Claude Bernard - Lyon were involved. These tasks addressed Social Creativity and the collaborative aspects of digital resource design. The factors that trigger or impede creativity were studied by Essonnier, 2018.

Based on the literature review on creativity (Guilford, 1950; Kaufman and Sternberg, 2010; Runco et al., 1999); mathematical creativity (Sriraman, 2009a; Leikin and Lev, 2007; Dreyfus, 1991; Haylock, 1987; Ervynck, 2002), and mathematical thinking (Tall, 1991; J. Mason, 2010; Knuth, 1985; Blinder, 2013), we understand CMT (*Creative Mathematical Thinking*) as a process (Wallas, 1926) combining three distinct and recurrent phases: (a) Divergent Thinking, (b) Illumination (Liljedahl, 2009; Liljedahl, 2013; Czarnocha and Prabhu, 2014; Palatnik and Koichu, 2014; Koestler, 1964), and (c) Convergent Thinking.

Since the actors observed in this thesis are not the resource developers, but the users, we needed a model to analyze and evaluate the data coming from the individuals who were submitted to the didactic activities. In the chapter 3, we describe how the Function Hero game was developed under the criteria established within the MCSquared project, and in the chapter 4, we describe the research methodology to answer the research questions related to individuals productions.

Chapter 3

Resource Design - *Function Hero*

3.1 Introduction

Several activities were used to answer the research questions, however, one, in particular, was developed and is unpublished: *Function Hero*. In this chapter, I describe the development of the tool, based on the theoretical framework of the development of activities that extend to creative mathematical thinking, as we can see in the Introduction. (Chapter 1).

In order to identify mechanisms and affordances that support the development of Creative Mathematical Thinking, this chapter analyses the design of the game *Function Hero*. This chapter hypothesizes that we can develop technologies that have the necessary criteria to improve and give the opportunity to Creative Mathematical Thinking.

3.2 What is a Game?

Games are present throughout the history of humankind, in various formats and situations: the term admits that can range from an utterly ludic end, associated with play, to serious situations such as word games and power. This trait of games is universal and crosses barriers between languages, as can be seen with the use in several contexts of the words play in English, or jouer in French, or still jogar in Portuguese, jugar in Spanish. There are two main approaches when using games in Mathematics Education: (a) By using the game as an artifact in which the mathematical content is inserted into the game to be somehow appropriated by the user. (b) By developing the game itself (Dalla Vecchia, Maltempi, and Borba, 2015; Borba et al., 2017), where the process of creating a game can hold modeling situations in which the designer must understand and apply mathematical models into the game structure. This thesis uses both methods combined, to play the game, the user must change it with mathematical knowledge. It is an adaptive game that is shaped by interaction with the user.

There are many definitions of games, attempts to classify and relate to definitions (Brougère, 1995). Finding a definition of games allows you to glimpse the richness of the subject and guide the process of conception. As a game developer, you have to keep in mind all the components that identify a game. We don't discuss the philosophical idea that a game is only a game if it is played because a chair, even if no one uses it, remains a chair. Given the polysemic character of the term 'game,' listing its components is one way to define it (Lealdino Filho, 2014). The classic "Homo Ludens" says that a game should be voluntary, have spatial and temporal boundaries, have rules, and goals (Huizinga, 2014).

Gathering some general characteristics, it can be said that a game is:

- Free and voluntary - players feel happy and attracted (Csikszentmihalyi, 1996). Any order or obligation immediately destroys its nature (Huizinga, 2014; Caillois, 2001). This component compromises what we mean by educational games since games of this type are applied in a non-voluntary way in classrooms, according to the characteristic, educational games would not be games. However, I understand that by using nature, the game remains a game, but what changes is engagement. Engagement cannot be forced, so it becomes free and voluntary. When a student engages in the game, he did so by his deliberation.
- Limited in space and time and repeatable - (Holopainen, 2011). It also has a space outside the real one, created specifically for the practice of the game. In educational games we break this barrier between the real and the fantastic.
- Uncertain - The result cannot be predicted, and the process can not be determined (Caillois, 2001). When we developed the game *Function Hero*, the software itself is a game, but the scenario we develop uses it as a tool to create a more massive game between teams of students.
- Immersive - absorbs all players' attention (Huizinga, 2014; Csikszentmihalyi, 1996; Kisielewicz, 2012)
- Regulated - It has a system of rules and laws valid during the activity (Caillois, 2001; Järvinen, 2008; Holopainen, 2011).
- Fictitious - There is a separation from the real world where a new space is created with different rules and possibilities (Caillois, 2001; Järvinen, 2008; Holopainen, 2011).

Among many different definitions and classifications, we retain a reasonably synthetic and adapted to software developers: a game must have one or more clear goals that direct attention, a set of rules and a system that demonstrates the progress of the player (feedback).

3.3 Mathematics, Games and Motivation

In discussing mathematics teaching, one of the recurring themes is the participation of students in the teaching process and the criticism of what is conventionally called *traditional teaching*. This denomination brings together characteristics such as the monologue class, the exhaustive repetition of exercises, the memorization of algorithms and the in-depth exploration of higher-order cognitive abilities such as logical reasoning and the use of heuristics. Although repetition and memorization are fundamental to the creation of expertise and show very positive results in the teaching of mathematics (McKenna, Hollingsworth, and Barnes, 2005), this discipline requires other cognitive abilities. Thus, alongside the acquisition of knowledge (multiplication tables, properties, and operations), mathematics also provides capabilities such as logical rationalization and heuristics for solving problems, which are perceived from the most basic levels:

do not rely solely on memorization; they do not employ only mechanical skills; they do not operate only on a 'concrete' level. They deal spontaneously and sometimes joyfully with mathematical ideas. (Ginsburg and Amit, 2008)

As a result, the active participation of the students is of particular importance. Getting attention and engaging the class is of paramount importance to improve the efficiency of the teaching process, increase comprehension and increase retention rates. Ironically, another fundamental feature of Mathematics - abstraction - stands as a barrier in this goal of motivation, in part by an inadequate approach in the classroom:

children might come to appreciate utility: how and why the mathematics is useful, a form of understanding that has been largely ignored in the literature and is typically given little prominence in schools. (Pratt and Noss, 2010)

Understanding aspects of human motivation are essential for designing classroom strategies and especially developing video game games for education. It can be classified as intrinsic or extrinsic (Vallerand et al., 1992). In the first case, the individual is moved to perform an activity because it gives him pleasure in some way. High-level athletes and entertainers are examples of people who volunteer to perform tasks. The extrinsic motivation is separated from the activity itself, which becomes an intermediary pass to the goal. An example of this is the student who is obliged to study because he wants to obtain a diploma.

In the development of games, it is crucial to build a compass in which the student, once he/she enters a state of flow (Csikszentmihalyi, 1996), does not leave this phase because it is considered the best state of mind to develop his creativity and solve

problems. When a game has phases or tasks to be solved, we can think of its linearity according to the Figures 3.1, 3.2 and 3.3:

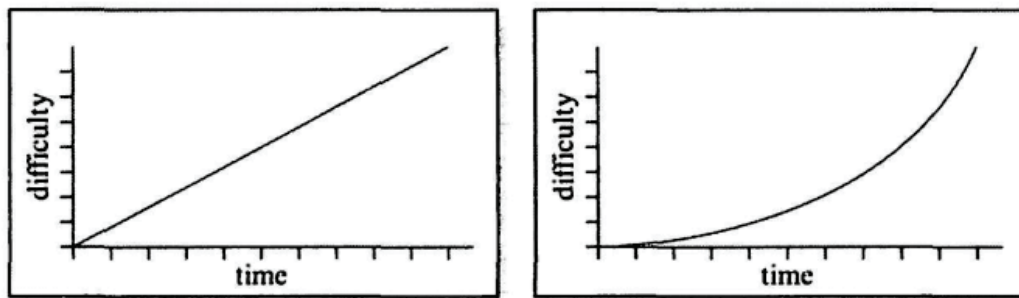


FIGURE 3.1: How should difficulty rise? - Rabin, 2010

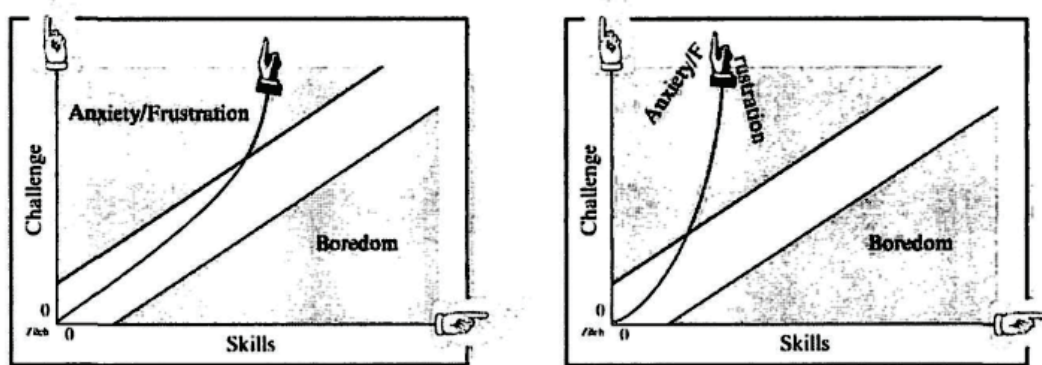


FIGURE 3.2: The player experience of the relentless rise in difficulty - Rabin, 2010

The figures 3.1, 3.2 and 3.3 show that when we develop a game, we must balance the difficulty with the player's abilities. If the skills are greater than the difficulty, the student will feel boredom while playing. If the difficulties are greater than the abilities, the feeling of anxiety and frustration may emerge. In this case, the motivation will be effectively the will of the player to win the tasks. An example of a game in which the difficulty is enormous is Dark Souls (FromSoftware, 2011).

The use of games and games in the classroom is a well-known subject so that video games should not be seen with anything new (Gee, 2003; Prensky, 2003). Computer games can act on both types of motivation, depending on how the content is treated within the program. In extrinsic motivation, a game can be applied as a reward for the student to achieve some goal set by the teacher, such as doing homework. To create the possibility of intrinsic motivation, the content being treated must be integrated into the game itself (Malone, 1981; Rieber, 1996). This goal is more comfortable to achieve. One way to explore it is to look for mapping between learning content and the elements that make up the video game scenario (Koscianski, 2010).

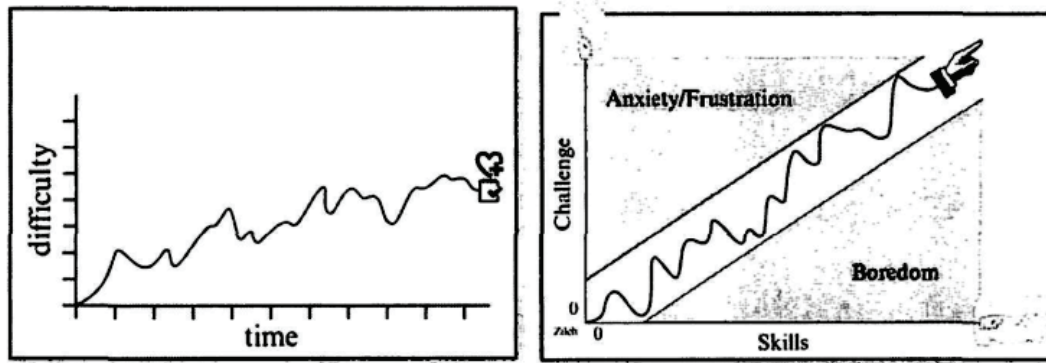


FIGURE 3.3: The ideal difficulty progression - Rabin, 2010

3.4 Learning Theories and Games

In order to fly over the triad teacher - content - student and place a computational game in it, we take three questions: a) consider how people learn; b) what elements must be present in an educational object so that learning becomes a reality; and c) what elements must be present in order for an educational object to be potentially a provider of the possibility of being creative.

The first question brings to the scene the various theories of learning. Each one of them offers a different perspective on the subject, but fortunately, all have common elements and therefore have non-negligible overlap between them (Schunk, 2012). Here we consider two general lines to draw relations with educational games, which are linked to constructivism and behaviorism.

Constructivist learning relies on the idea that individuals produce new knowledge through interaction with the environment and with other people, in addition to using prior knowledge (Bodner, 1986; Schunk, 2012). Some principles of constructivism consist in affirming that knowledge:

- is not taught by pure transmission, but is actively constructed
- can be obtained by experience or by deduction (there are variants of this idea according to the author researched);
- is symbolically constructed by the creation of representations of actions.

Some authors who work in the line of constructivism are Piaget, Vygotsky, Bruner, Papert. In general, in constructivism, everything the student experiences is compared or tested against his previous knowledge; new knowledge is assimilated with what the student already knows.

RPG (Role-Playing-Games) games have a similarity to this context because the user advances in the script in the sense that he understands how the game works and accumulates experience about his mechanics. In a pedagogical game, this translates into active experimentation (Kiili and Ketamo, 2007). The fact that the RPG scenario

naturally integrates texts makes this generation a little more propitious to deal with matters that involve detailed instructions or that need to be explained step by step, such as mathematical algorithms (Rosa, 2004).

Within the RPG category, the MMORPGs (Massive-Multiplayer-Online-RPG) are present. In them, thousands of users with varying levels of experience are mixed. It is part of the culture of communities that practice MMORPGs to teach the less experienced how to progress faster. It is a practical example of knowledge that is socially distributed and constructed. This type of learning, within a social interaction, was the target of the works of Vygotsky. Having even baptized his work, Vygotski left the door open for various denominations, such as 'socio-constructivism' (Prestes, 2010). A key concept in his work is the "Proximal Development Zone" ZPD - or more properly "imminent development zone," according to Prestes. The ZPD expresses the difference between the knowledge and skill that the individual already possesses and the potential capacity that he can only attain through interaction with another person.

Vygotsky argued that ZPD is an essential characteristic for learning, as well as the development of potential capacities, are only possible when the student is interacting with individuals in and in their environment (Vygotsky and Cole, 1978).

Going back to the gaming environment, while it is possible for a user to reach certain levels by himself, it is not uncommon that more difficult challenges are developed to be overcome with the help of other players. Creating a cyber-community is a strategy that is part of virtually every MMORPG, such as World of Warcraft (Entertainment, 2013). In these situations, the player will use the abilities and resources of other players present on the network at the moment they are interacting.

The second theory we examine here is the Behaviorist. It is found in the study of behaviors that can be observed and measured and considers the mind as a "black box" in the sense that the response to a stimulus can be observed quantitatively, ignoring the process of how thought happens (Bozarth, 1994). Some critical authors in the development of the behaviorist theory are Pavlov, Watson, Thorndike, and Skinner.

This theory also suggests that if positive actions reinforce a behavior, the subject tends to repeat the same behavior in the future (Watson, 2017). In this sense, learning is increasing the likelihood of a behavior happening based on past stimuli.

Several stimuli are used in video games to provoke emotions and fun in users (Freeman, 2004). Players can, for example, be driven by the desire for self-overcoming (Csikszentmihalyi, 1996). Registering and exposing scores on scorecards serves both to measure the player's performance and to act as a reinforcement of behavior (seeking to improve a record), or as a stimulus to competition. In addition to scoring the player can be rewarded by passing the "winning lives" phase or items used in subsequent phases. This idea is characterized as a meta-reward (Holopainen, 2011)

because the item received is not only an end in itself but a means to reach the next goal in the game that provides satisfaction to the user.

The escalation of difficulty serves to avoid monotony (Koster, 2013) and continue to challenge the skills of the player (Csikszentmihalyi, 1996). See figures 3.1, 3.3.

Behavioral logic is present in most video games. Educational games are no exception. Many math games employ some stimulus-reward activity. The typical scenario consists of a problem with multiple choice alternatives clothed in targets to be targeted, blocks to be transported, etc. The "stimulus scheme" - the rate at which each reward is given - is crucial to maintain motivation and lead to active learning. Planning and controlling this rate in a game is a non-trivial goal (Orvis, Horn, and Belanich, 2008).

This parameter has a direct influence on video game design. If the program offers few rewards and is difficult to advance, the user may be disappointed by the failure and unsure if the actions they are taking are correct or not. On the contrary, if many rewards are obtained quickly, and the gambler does not make a particular effort in the activity, it loses its purpose, and the tendency is to abandon it (Koster, 2013).

A multimedia resource - such as an educational video game - is a component of an instructional design that includes other items such as student readings, classroom explanations, exercises, etc. (Koscianski, 2010). Ideally, an educational game should not be designed in isolation, but as part of a broader context, influencing the design of other materials and being influenced by them.

3.5 Developing Games

The previous section presented two general conceptions of teaching that can be explored in the context of educational video games and how to insert elements that can promote the creativity of its users. In all cases, games can be explored directly in teaching or can be employed to flank the problem.

To illustrate it, in a more constructivist approach, the software can act as a promoter or facilitator of socialization. Network games can encourage users to exchange experiences and information, and the content itself is not a direct part of the actions within the program. Creating conditions for an individual to develop a step-by-step solution to a problem is not a simple goal in the classroom and can be even more challenging in a game. RPG style can be considered for this purpose, as it supports complex plots that can be exploited in different ways by each player. Of course, the design of such a game can be quite sophisticated.

In the case of the behaviorist approach, the situation arises differently. The instruction is given by course events and in which the amount of information presented is limited. To illustrate, in this genre an algorithm of multiplication or division can be

presented step by step, offering the player at every instant, options that correspond to a correct action or not. Traditionally, behavioral games summarize the learning experience to "hit" or "miss" within a somewhat limited mechanics.

The conception of an educational game, including the contents covered and the mechanics of it, is not a deterministic activity. As well as preparing an exciting and useful didactic sequence, the teacher requires experience, and a dose of creativity, the development of an educational game also involves a free design effort. Two subterfuges to assist this phase consist of defining mappings between operations to be taught and existing operations in games; and explore a range of options, such as different scenarios and genres (action, puzzle, RPG, etc.) before defining the final implementation (Koscianski, 2010).

Splitting a video game into a set of components can help organize the design phase. Among the various classifications of constituents of games, we can cite Jarvinen, 2009 and Holopainen, 2011, or the concise and practical approach given by Garris, Ahlers, and Driskell, 2002, containing six items: fantasy, norms and objectives, stimuli sensory, challenge, mystery, and control. Some complementary aspects are gameplay, conflict, challenge and interactivity (Rollings and Adams, 2003). These elements are summarized in figure 3.4

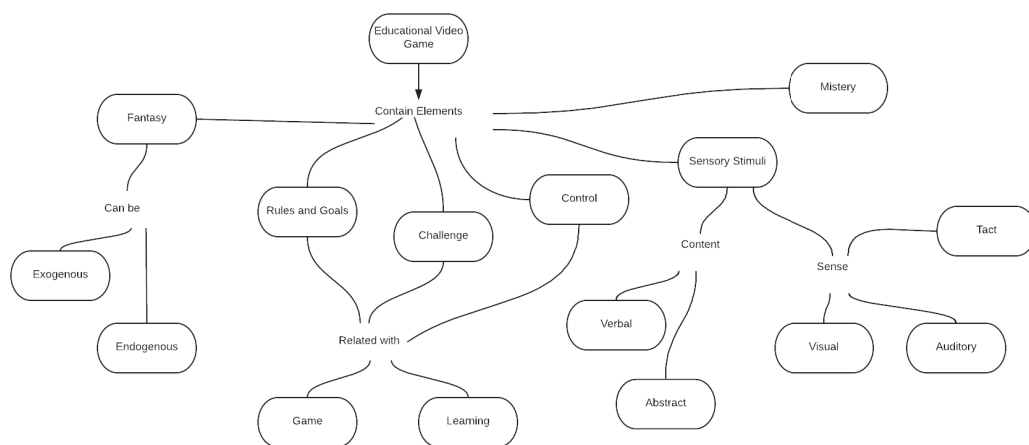


FIGURE 3.4: The classification of Garris for elements of games - Garris, Ahlers, and Driskell, 2002

The fantasy in a game can be of two types: exogenous and endogenous. In exogenous fantasy, game and content are independent; an extreme example would be to create an action game and introduce math problems as a condition for phase advancement. The second type, an endogenous fantasy, authentically merges the essence and game. The example here is a racing game, in which the user selects engines and fuels from yield information (price per ride) and price, to then build his vehicle and compete. RPG type joys are the most flexible for this purpose.

Standards and objectives demarcate mechanics and may have varied roles. In chess,

for example, rules occupy a central place in the activity. In action video games, the simple rules are preferred, leaving the action in the foreground (Koster, 2013). An educational game, as well as a lesson, must act so that the student concentrates on the content (Gagne, 1974).

Sensory stimuli, including graphics, texts and soundtracks and even tactile feedback, must be intentionally distributed in the scene to provoke the user's affective side (Feil and Scattergood, 2005). Drawing a parallel, in the textbooks of Brazil, the illustrations with techniques like watercolor or oil painting are rare, prevailing the simple trait, in the cartoon style. This strategy of low variety and low cost can be risky in the case of video games since users are accustomed to commercial titles whose budget can match that of a film. In addition to serving the aesthetic and emotional purposes, the sensory elements must be explored on the pedagogical side, since different combinations of texts, images, and sounds interfere with learning outcomes. Mathematics often employs graphical representations to illustrate functions, proportions, and measures. In a dyadic game, this can be easily combined with the presentation of formulas and explanatory texts, bringing a wealth of representations beneficial to the student. One researcher who has done much to study this aspect was Mayer, 2002.

The challenge component, already mentioned, must be measured in two aspects: the difficulty of progressing in the game, imposed by the mechanics of the same and the difficulty of didactic nature, of the contents being treated. The prototypes can help to perform a previous calibration of the software (Bethke, 2003), in search of a level of stress that maintains the interest of the player (Csikszentmihalyi, 1996).

One of the main attractions of video games is the novelty present behind each level (Koster, 2013). The mystery, or surprise, can create a positive tension that holds the player's interest. A simple solution is to introduce new characters and graphics at each stage without changing the rules. Varying the level of challenge also helps this goal. Another possibility would be to adjust the level of difficulty dynamically, depending on the behavior and performance of the player.

Finally, the control of the game can relate to two things. First, the user must feel in charge of the software, which must react precisely to their commands. This condition helps ensure immersion and increases the comfort of use (Koster, 2013). Second, depending on the teaching approach, one can give the student control over their advancement within the content (Garris, Ahlers, and Driskell, 2002). It can reduce the possibility of frustration and also act on the pleasure of winning phases of the game and pick correct answers to the teaching problems presented in it.

When creating a game within a team composed of teachers, designers, and programmers, it is essential to keep a record of ideas and decisions. It can be done using a document known as the Game Design Document. This document acts as a form,

guiding the creative team to think about all the elements of the game and the relationships between them (Bethke, 2003). Although the document does not deal with technical aspects such as programming languages and development tools, the more complete and detailed it is, the better the chances of implementation going well (Flynt and Salem, 2004). There is no single or conspicuous model of game project documents, especially since the purpose is precisely to help the creative work, but not to mold it. There are several models available, both in the literature and on the internet, that can be adapted according to what is agreed on by the project team.

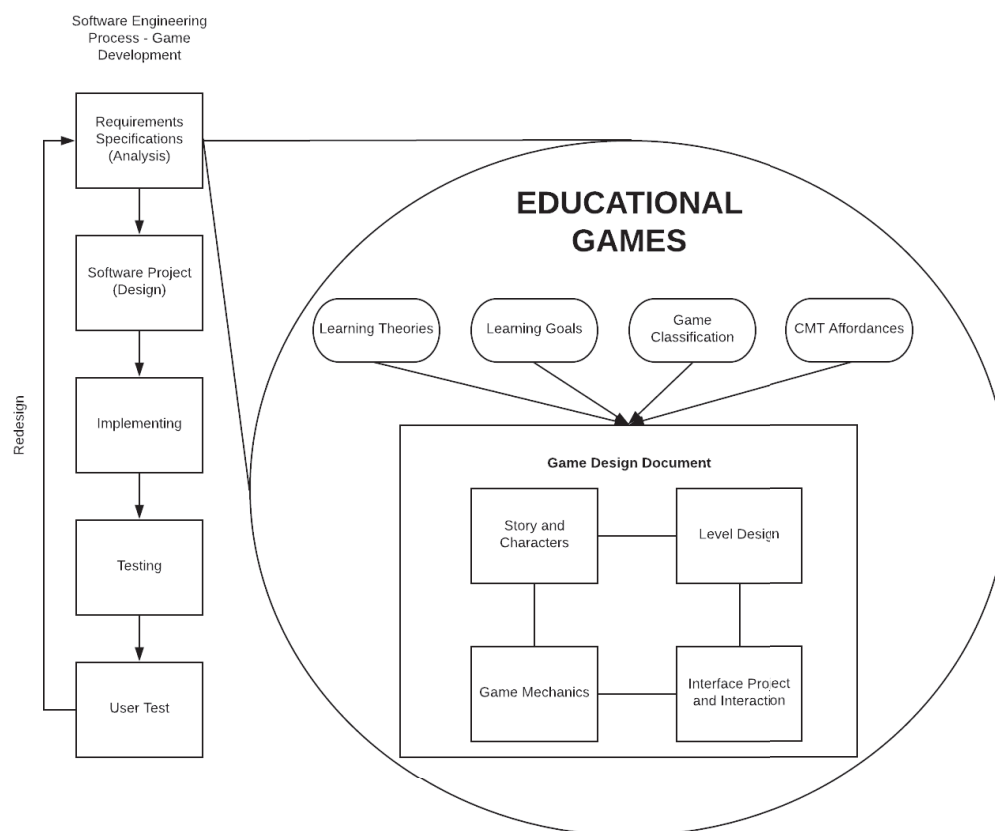


FIGURE 3.5: Software Development Process - Pressman, 2005

The creation of any program, including games, can be described in a series of steps studied in software engineering (Pressman, 2005), illustrated in figure 3.5. During the first stage, known as requirements analysis, the software must be clearly defined. To build an educational game, it is necessary to carry out a detailed survey of information with the people who can contribute to creating an efficient instrument; this can include teachers, pedagogues, graphic artists, and even students, who can evaluate initial designs, screens, and prototypes. The Game Design Document, presented in the previous section, is a central piece in the development of a game and can serve as a basis to harness the work of these different experts.

Although the activities of the figure 3.5 are listed in sequence, returns can occur. For example, in the project, it may be necessary to go back and review constraints due

to obstacles such as cost or time for implementation. The organization of this phase and team work is also the subject of software engineering (Pressman, 2005).

3.6 Game Development

Our intention in this session is not to teach or to show a tutorial on how to make a computer game. However, show what are the features present in the game *Function Hero* and relate them to the previous literature.

First of all, we emphasize that the development of a computer game is not a trivial task. Most of the educational games on the market are developed by teams that unite all the skills needed to build a good game. Usually, the roles present within a game development team are Programmers, Designers, Sound Engineers and Managers for the case of commercial games.

Several steps are taken until the end of a game. We will generally show those that were executed in the development of the game "*Function Hero*." We divide the process into three significant parts based on the conceptual stage, stage of elaboration and stage of refinement.

3.6.1 Conceptual Stage

The goal of developing an educational computer game reminds us to think in parallel of two fundamental pillars: fun and education. Still, as part of a game developed specifically for this research, creativity was a feature added to the development. Below we describe the concepts of the game, each paragraph deals with a fundamental characteristic of the game and therefore may seem disconnected.

The first step in achieving both goals is to set an initial idea of what the game will be like. Many ideas come from the experience we have had in various types of media - books, movies, games - and with that, we have mentally constructed a model to base the construction.

We had a longing for the MCSquared project to create applications that used body data for user interaction. Microsoft's Kinect sensor would make it possible to develop such activities as are present at the Museum of Mathematics in New York.

This sensor allows capturing 13 (thirteen) points of the body and their respective coordinates (x, y, z). Moreover, with that, we developed our first interactive object prototype where the user should move away or approach the sensor to make a point move according to a function described on the game screen.

The idea of *Function Hero* came up after I saw a meme on a social network, shown in Figure 3.6

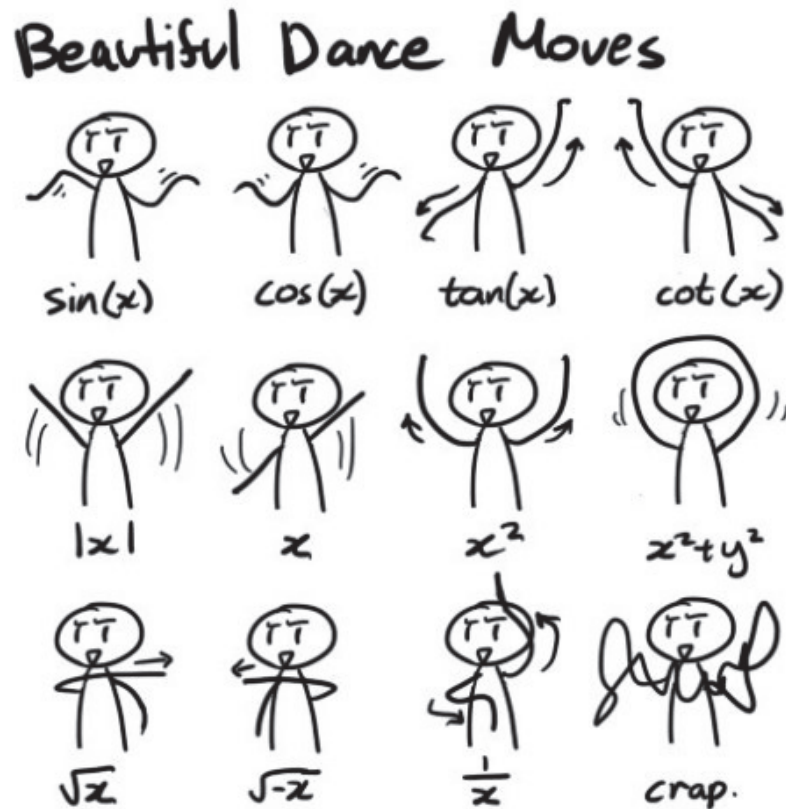


FIGURE 3.6: Math Dance Moves - "Internet Reproduction"

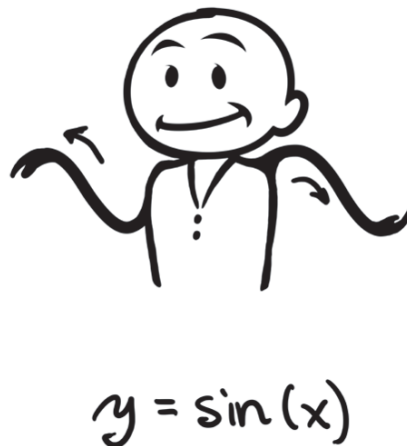
Then I had contact with the figures of *Michael Gralmann* in the Imaginary site with the figures of the gallery *Math Dance Moves* as in the Figure 3.7

Discussing with my supervisor on what we would develop as a game, at a luncheon at the university restaurant along with the idea of Just Dance and Guitar Hero games, we thought of a way to create a game in which students could dance to the algebraic expressions of functions and thus be born the idea of the game *Function Hero*.

The game shows the player a series of algebraic expressions that he or she must perform to earn points. The closer the arm points are to the function curve, the more points it gains.

The Educational Content

In order to provide different ways to perceive mathematical functions, we developed the game which is subject from this research, using the embodied motion as fundamental strategy. The use of kinesthetic approach in teaching and learning mathematics does not only promote students' understanding and visualization of mathematics contents, but also encourage them to be active learners (Tsai and Yen, 2013).

FIGURE 3.7: Movement - $\sin(x)$ - Matt, 2017

The contents of the kinesthetics digital materials includes a wide range of mathematical functions: Constant function, linear function, quadratic function, cubic function, absolute value function, fractional function, trigonometric functions, exponential function, logarithm function.

An achievement test is developed to test the impact of these kinesthetics digital materials in enhancing students' recognition and understanding of mathematical functions.

3.7 Elaboration Phase

To develop the kinesthetics digital materials employing Kinect sensor, some programming was required, using Unity 3D Development Kit, Microsoft Kinect (SDK) - Software Development Kit - and CindyScript the programming language associated with Cinderella - Dynamic Geometry Software (Kortenkamp and Richter-Gebert, 1999, Richter-Gebert and Kortenkamp, 2012), linked through a UDP (User Datagram Protocol) connexion. The software recognizes students' body gestures as input representations for mathematical function graphs.

As shown in the Figure 3.8, Cinderella software plots two function graphs, the red one corresponds to body gestures, and the green one corresponds to the goal graph which the student should perform to obtain the positive score during the game.

On the left side of the figure above we show Cinderella plotting the functions in accordance with what the player (right side) is performing in front of kinect sensor.

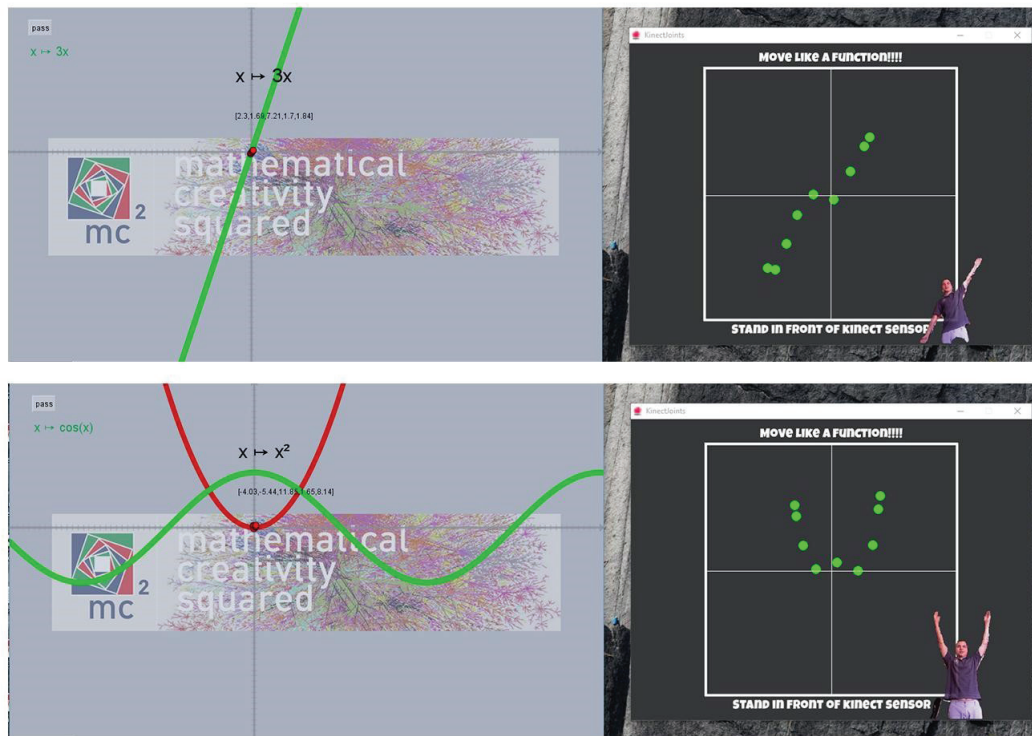


FIGURE 3.8: First version of Function Hero

3.7.1 Software Packages

Unity 3D Development Kit

Unity is a flexible and powerful development platform for creating multi-platform 3D and 2D games and interactive experiences. It's a complete ecosystem for anyone who aims to build a business on creating high-end content and connecting to their most loyal and enthusiastic players and customers.

However, even this resource had been developed to commercial purposes, we are using it to create educational content to teach mathematics. As unity allows having small teams and at the same time, achieve high quality results.

Kinect SDK

The Kinect SDK for Unity was developed by "RF Solutions" and provides a set of Kinect V1 examples that uses several major scripts, grouped in one set folder. It demonstrates how to use Kinect-controlled avatars, Kinect-detected gestures or other Kinect-related stuff in our own Unity projects.

Codenamed in development as Project Natal is a line of motion sensing input devices by Microsoft for Xbox 360 and Xbox One video game consoles and Windows PCs. Based around a webcam-style add-on peripheral, it enables users to control

and interact with their console/computer without the need for a game controller, through a natural user interface using gesture and spoken commands. On February 2011 Microsoft announced that it would release a non-commercial Kinect software development kit (SDK) for Windows. The SDK includes Windows 7 compatible PC drivers for Kinect device. It provides Kinect capabilities to developers to build applications with , C, or Visual Basic by using Microsoft Visual Studio and includes the following features:

- Raw sensor streams: Access to low-level streams from the depth sensor, color camera sensor, and four-element microphone array.
- Skeletal tracking: The capability to track the skeleton image of one or two people moving within Kinect's field of view for gesture-driven applications.
- Advanced audio capabilities: Audio processing capabilities include sophisticated acoustic noise suppression and echo cancellation, beam formation to identify the current sound source, and integration with Windows speech recognition API.
- Sample code and Documentation.

Cinderella

Cinderella is a program for doing geometry on a computer (Kortenkamp and Richter-Gebert, 1999; Richter-Gebert and Kortenkamp, 2012). It's the product of a sequence of three projects carried out between 1993 and 1998. It is based on various mathematical theories ranging from the great discoveries of geometers of the nineteenth century to newly developed methods that find their first applications in this program. The software can be downloaded at the link: <https://cinderella.de>

3.8 Refinement Phase

At this stage, new features are added to the game depending on the users' experience. Since the game "Function Hero" is intended to be easily usable without the need for an external sensor and independent of platform or operating system, the game's refinement phase is happening.

We intend to enhance the game so that it uses webcam computers and smartphones to capture body data, as well as a more user friendly and enjoyable user interaction design. Therefore, the game is still under development.

3.9 Summary

In this chapter we show how the game Function Hero was developed, presenting the theoretical bases to create a game of educational character and, hypothetically, to facilitate the improvement of the students in the recognition of functions by its algebraic form. In the next chapter we will discuss the Research Methodology which was developed to answer the research questions related to the creativity evaluation of the data from the Gameplay.

Chapter 4

Research Methodology

4.1 Introduction

The primary goal of this study was to answer the research questions related to the Diamond of Creativity, designing digital resources, didactic situations using mathematical games, student's achievements, creative mathematical thinking definition, operationalization of creativity in mathematics, and its evaluation, as stated in Chapter 1. The research, has a character qualitative and quantitative. Qualitative in the sense that, individuals were chosen and observed by the researcher. Afterward, the data collected were analyzed through the lenses of the theoretical approach called the *Diamond of Creativity* (see. Chapter 2). Quantitative in the sense that the productions from the individuals were examined under the lenses of Social Network Analysis, providing quantitative material to help to conclude. The methodology used to answer the research questions is presented in this chapter. The chapter is organized into (a) Selection of Participants, (b) Instrumentation, (c) Data Collection, and (d) Data Analysis.

4.2 Selection of Participants

Several countries participated in the study: France, Brazil, Russia, Georgia, Finland, Tunisia, Ireland, Colombia, Swiss, and Armenia. However, not all of them were observed to answer the research questions. Instead, they participated in activities in which we could improve the instruments and the experimental approach. Nevertheless, the contributions of each one enriched this research. As the study had several methodological steps, the countries of MetaMath and MathGeAr projects participated in a survey on the perception of mathematics in Engineering courses, providing the initial motivation on which mathematical content we would deal with and the first perceptions of mathematics from students and teachers' point of view. We can say that the sample was very heterogeneous because the situations in which we collected data, for the majority of the time, was not controlled as we detail in the

following sections. A brief description of the experiment participants can be seen in the Table 4.1.

<i>Country</i>	<i>Institution</i>	<i>Level</i>	<i>Participants</i>	<i>Activity</i>
Brazil	UEM	Post Graduate Students	15	Binary Code, Op'Art
	UTFPR - PG	Post Graduate Students	23	Binary Code, Op'Art
	UTFPR - GP	Undergraduate Students	5	Binary Code
	New Rome Faculty	Undergraduate Students	11	Binary Code
France	CFAI	Highschool	60	<i>Function Hero</i>
	IREM - Strasbourg	Researchers & in service teachers	25	Op'Art
	INSA - Lyon	Undergraduate Students	55	<i>Function Hero</i>
	IREM - Lyon	In Service Teachers	23	Binary Code
	College	HighSchool	43	<i>Function Hero</i>
	ICTMT	Researchers	14	Binary code
	Math C2+	Highschool	36	Binary code, Fake Binary
Tunisia	YFSF	Junior Highschool	40	Binary code
	YFSF	Engineer students	16	Binary code, Op'Art
Finland	TUT	Undergraduate Students	189	Perception of Mathematics
Russia	LETI	Undergraduate Students	x	Perception of Mathematics
	UNN	Undergraduate Students	35	Perception of Mathematics

	TSU	Undergraduate Students	27	Perception of Mathematics
	KNRTU-KAI	Undergraduate Students	23	Perception of Mathematics
	OMSU	Undergraduate Students	45	Perception of Mathematics
Georgia	UG	Undergraduate Students	53	Perception of Mathematics
	BSU	Undergraduate Students	24	Perception of Mathematics
	ATSU	Undergraduate Students	33	Perception of Mathematics
	GTU	Undergraduate Students	15	Perception of Mathematics
Armenia	NPUA	Undergraduate Students	46	Perception of Mathematics

TABLE 4.1: Participants Based on Country, Level and Activity

4.3 Sampling Procedure

Samples were selected in a non-random manner and for convenience. The institutions that participated in the experiments were volunteers in the participation and test of the instruments that were developed during this thesis. This sampling brought a granularity of the individuals, allowing us to observe some behavioral similarities in the resolution of problems that can be classified according to the age, the area of performance and the community which one belongs.

The activities described in the table 4.1 will be detailed in the next session.

4.4 Research Design

MetaMath and MathGeAr projects aimed to address a wide spectrum of the problem of math education in engineering programs of Russian, Georgian and Armenian universities. To solve these problems, the projects rely on a comprehensive approach including studying international best practices, analytical review and modernisation of existing pedagogical approaches and math courses.

The research questions related to the MetaMath and MathGeAr projects run away from the scope of this thesis in the sense of applicability of the technological tools developed. Since the central theme of this thesis is Creativity, the results of the modernization of engineering courses are not described in detail. However, the findings regarding the Perception of Mathematics are expressed here as motivation for the development of technological alternatives that may also influence the development of Creative Mathematical Thinking.

On the other hand, the project MC Squared sought from the beginning to develop digital tools to teach mathematics and also to promote the Creative Mathematical Thinking. For that reason, many theoretical gaps were open to being filled during the development of the project. Guiding us through the research questions:

Qualitative Questions

- RQ1: How can we operationalize Creative Mathematical Thinking?
- RQ2: How can we assess the progress of a process involving Creative Mathematical Thinking? We propose to objectify this development through an observing tool that we called the *Diamond of Creativity*, here described in Chapter 2.
- RQ3: How the "Diamond of Creativity" model is an useful analytic tool to map the Creative Process path?

Quantitative Questions

- RQ4: To what extent can we validate a learning gain in function recognition tests after Function Hero activity?
- RQ5: What is the variability of the choreographies of the students in the Function Hero game?

Each research question leads us to a research function (Plomp, 2013), which have as goal to contribute to the body of knowledge or a theory in the domain research. Combining the research questions into a chronological description of methodological events, our study was conducted to achieve different purposes and functions: (a) Developing didactic situations and artifacts that, a priori, can improve creative mathematical thinking (b) comparing the results obtained in the performance tests, (c) developing a model of analysis of the solutions for each situation (Diamond of Creativity), (d) compare the differences and similarities between the groups, (e) validate the developed model.

These purposes and functions can be accomplished by using different research methodologies, which refer generally to "principles, procedures, and practices that govern

research" (Marczyk, DeMatteo, and Festinger, 2005, p.22). However, Plomp suggested the following purposes that can be served by using the various research methodologies indicated (Alghamdi and Li, 2013):

- Survey: to describe, to compare, to evaluate;
- Case Studies: to explain, to compare;
- Experiments: to explain, to compare;
- Action research: to design/develop a solution to a practical problem;
- Ethnography: to describe, to explain;
- Correlational research: to describe, to compare;
- Evaluation research: to determine the effectiveness of a programme;
- Design research: to design/develop an intervention (such as programmes, teaching-learning strategies and materials, products and systems) with the aim to solve a complex educational problem to advance our knowledge about the characteristics of these interventions and the processes to design and develop them, or alternatively to design and develop educational interventions with the purpose to develop or validate theories (p. 17)

Therefore, to answer the research questions presented in this thesis, we use designs that can be classified as: (a) Design-based research, due to the development of activities to improve creative mathematical thinking. (b) Experimental, due to applications of situations in scenarios with different groups to compare their scores after the use of resources. At the same time, using participatory action research methodology of acting, observing and evaluating results, this study employed the mixed method approach to data collection, using both qualitative and quantitative methods, as defined by Johnson and Onwuegbuzie, 2004.

4.5 Instrumentation

Once we became interested in the role of activities for the enhancement of Creative Mathematical Thinking, such instruments needed to be developed and served as the treatment for the chosen samples. After constructing our theoretical framework and the definition of state of the art to measure Creative Mathematical Thinking (see. 2), we understand creativity as a construct that can be observed in four entities: Process, Product, Person, and Environment. In this thesis we work with the observations and manifestations of creativity in the aspects: Process and Product, using the methodological tools described in Chapter 2, it means, the Diamond of Creativity to observe the process, and the Statistical Analysis of Networks to observe the products and processes. For each of these instances, the instruments were:

- Creativity as Product: *Function Hero* - This is a Digital Educational Game where students have to move their body according to a sequence of functions, called a *choreography*. An optical system recognizes the player and attributes a score to the performance. The creative part is actually to *design* this choreography using algebraic expressions to describe the sequence of functions. The challenge is to design a choreography that will score high for their own team and that will be difficult to dance for other students. See fig.4.1



FIGURE 4.1: *Function Hero* - Screen Shot

- Creativity as Process:
 - *Binary Code* - The binary magic trick is based on a set of 5 cards with numbers on them. After having asked a participant to think about a secret number between 1 and 31 (like the date in the month of a birthday for example), the mathemagician shows the cards one by one and the participant selects all the cards that contain the number. Using the magic powers of mathematics, the mathemagician computes the chosen number.

3	7	23	19	14	26	10	27	6	5	20	14	14	10	15	31	17	23	19	25
17	9	11	31	3	30	15	18	28	23	15	13	11	27	29	13	27	30	29	18
5	27	21	13	31	6	19	23	31	29	21	22	26	28	9	12	22	21	26	24
1	25	29	15	2	7	11	22	4	30	7	12	8	30	25	24	16	28	31	20

FIGURE 4.2: Cards of Binary Code Trick

For instance, let's say that one chooses the cards, from left to right (see. Figure 4.2): [1, 3, 4]. Then, we can guess that the date of the birthday is 13.

- *Fake Binary Code* - A version of *Binary Code* which allows a person to guess a number following the commands given by another person.

65	5	35	21	27	24
22	42	72	66	60	63
62	2	32	26	20	23
16	76	46	52	54	57
25	45	75	61	67	64
51	31	1	15	13	10
56	36	6	12	14	17
11	71	41	55	53	50

TABLE 4.2: Fake Binary Code

The trick is to guess the number chosen from a viewer's commands. The individual must choose an available number in the table of numbers and say the numbers of the line in which that number is contained, lying the color of the number chosen. For example, if I choose the number 61, see 4.2 that is in the fifth line, my commands for the magician should be:

red red black black black red

TABLE 4.3: Command Example n 61 - Fake Binary Code

- *Op'Art Algorithms* - The Op'Art movement raised against figurative art in a rather minimalist approach. Along the line of the *Art by Telephone* exhibition Armajani, 1969, where artwork was performed by someone (or a computer) following instructions dictated over a phone, most productions can be described explicitly by a few sentences and yet the result is very compelling, often using anomalous motion optical illusions to create static but virtually vibrant compositions. Its main artist are Victor Vasarely, the G.R.A.V. artists, Julio Le Parc, Garcia Rossi, François Morellet, Francisco Sobrino, Stein, Jean-Pierre Vasarely (aka Yvaral), and Bridget Riley. Denise René gallery hosted and promoted these painters in Paris. Contemporary trends such as generative or algorithmic art movements can be linked to the Op'Art movement. For instance, see Figure 4.3.

Four activities were developed to be used as a treatment in the experiments. However, by limiting the depth of analysis, we bring the results of the situation: Function Hero and Binary Code. The others will be discussed in future work.

4.6 Data Collection

In this session we describe how the data collection for each activity was carried out, evidencing which research question we were able to answer with such data.

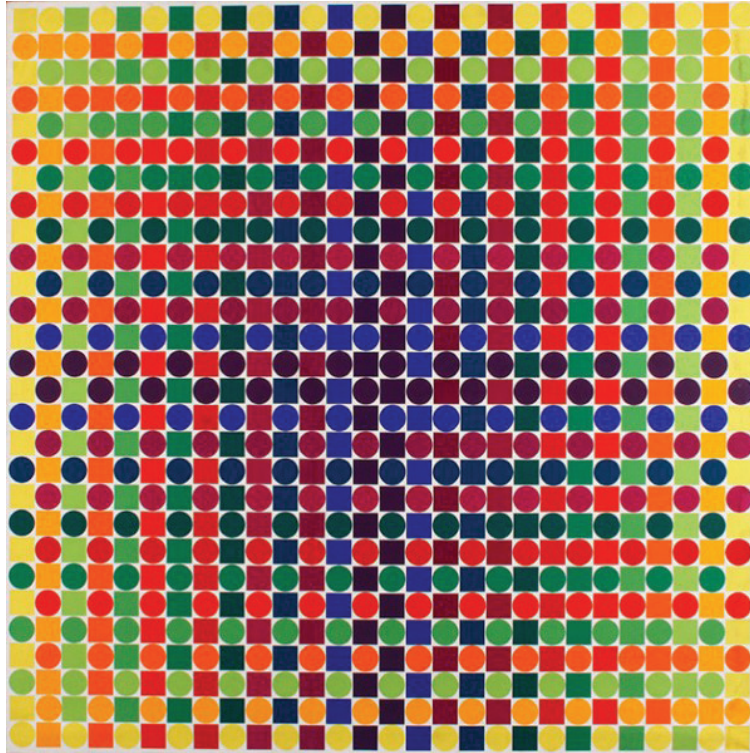


FIGURE 4.3: Variation on Ball by Julio Le Parc

4.6.1 Function Hero

We aimed at the development of Creative Mathematical Thinking, but also in improving the student's ability to recognize and understand the concept of functions presented in the game. Therefore, we developed two versions of performance tests for the situations involving the game *Function Hero*, described below.

Pre and Post Test - Version 1

We designed an achievement test that can be used in assessing students' performances before and after administering the prepared kinesthetics digital resources (Function Hero).

The table 4.4 shows the specification in terms of mathematical function types that have been covered in the digital resources with respect to recognition and understanding levels of mathematical achievement. The test consists of 40 items that are equally distributed among the two levels of achievement: recognition and understanding. 20 items are designed at each level in isomorphic pairs that was administered randomly as whether pretest or post-test.

The items type for the test are multi-choice questions of 4 choices to eliminate the guessing issues from the students' side. The test items were written in verbal and nonverbal, symbolic and graphical ways. The total score of this test is set to be (40)

marks. One point mark is given to each correct item response. (see Appendix A, Version 1).

Achievement Level Functions Type	a.i.	Recognition Items	a.i.	Understanding Items	Total	Perc
Constant Function	2	11, 12	2	23, 30	4	10%
Linear Function	2	9, 10	2	21, 24	4	10%
Quadratic Function	2	1, 15	2	27, 37	4	10%
Cubic Function	2	2, 16	2	28, 38	4	10%
Square Root Function	2	3, 6	2	32, 36	4	10 %
Absolute Value Function	2	17, 18	2	29, 35	4	10%
Fractional Function	2	7, 8	2	22, 31	4	10%
Trigonometric Function	2	19, 20	2	39, 40	4	10%
Exponential Function	2	4, 13	2	25, 34	4	10%
Logarithm Function	2	5, 14	2	26, 33	4	10%

TABLE 4.4: Specification of the Achievement Test - Version 1

Pre and Post Test - Version 2

To the second version of the experimentation with *Function Hero*, we designed an achievement test based on the study of Zachariades, Christou, and Papageorgiou, 2002 to collect information of students' understanding of function representation. This test consisted of two parts involving 26 tasks in total, split into four parts (see. Appendix A, Version 2). The first part included 9 relations and the students were asked to indicate whether or not the relations could describe one of more functions (see table 4.5).

The second part involved 11 graphs and students were asked to decide which of these graphs resulted from function of the form $y = f(x)$ (see picture 4.4).

The third part involved four body representations of graphs and students were asked to describe the dance movements in terms of algebraic expressions of functions (see pictures 4.5 and 4.6). The forth part involved one algebraic form of a

<i>Situation</i>	<i>Relations</i>
a	$x^2 + y^3 = 3$
b	$y = x^3 + x + 1$
c	$a^2 - b = 0$
d	$f(y) = e^y$
e	$x^4 = 3y$
f	$a = \sqrt{2}$
g	$f(x) = 3$
h	$y = x^2$
i	$s = 3t$

TABLE 4.5: Functions Represented by Symbolic Forms

function and students were asked to chose which graph represented that function (see Picture 4.7).

Game Data Logs

The game *Function Hero* has a data capture system, that is, the choreographies proposed by each team as we can see in the Figure 4.8.

Each choreography entered by the students allowed the development of the following Tables 4.6 and 4.7:

Team 1 Choreography - BTS		
Input	$f(x)$ Function (algebraic mode)	Function (computer mode)
1	$\log(x)$	$\log(x)$
2	$(\frac{4x}{5})(e^x)$	$((4*x)/5) * (\exp(x))$
3	$\frac{x^3}{5}$	$(x \wedge 3)/5$
4	x^2	x^2
5	$2x(\cos(x - 2))$	$2*x*(\cos(x-2))$
6	x	x

TABLE 4.6: Example of data from students - Team 1 BTS

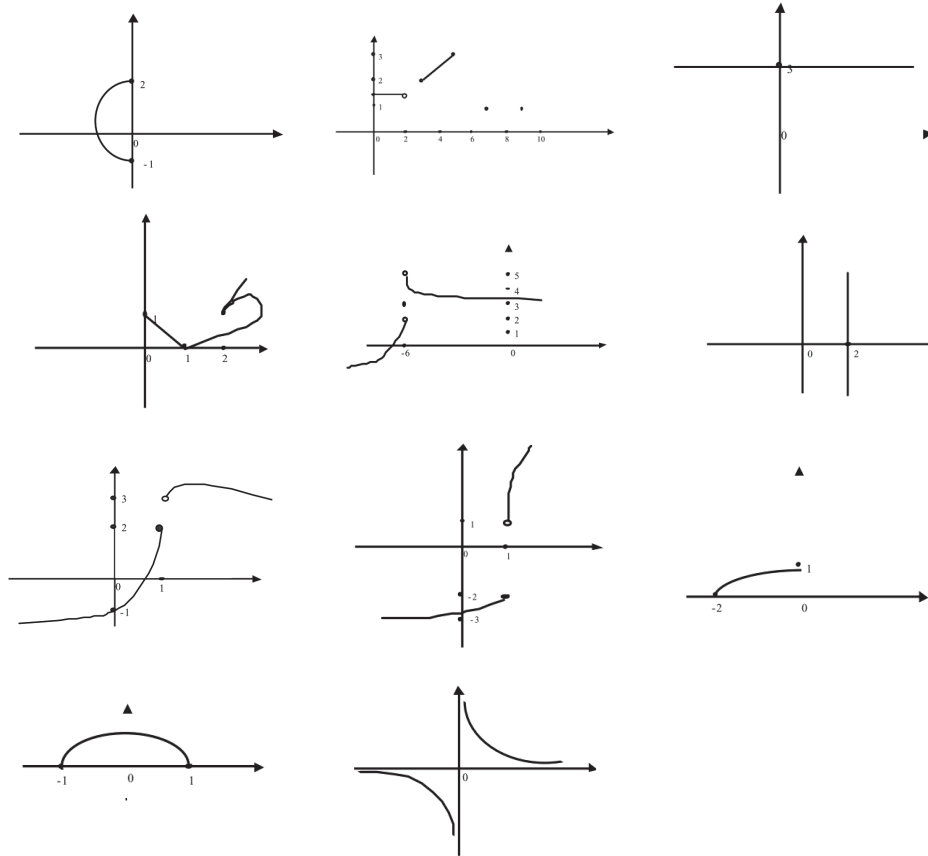


FIGURE 4.4: Functions Represented by Graphical Form - Pre-Post Test Version 2

4.6.2 Binary Code

To collect the data from participants of Binary Code Workshops. We asked them to summarize their resolution processes and, in the end, to develop a natural language algorithm to perform the tasks. The participants were asked to write down the solution of the following tasks:

1. Understand the trick and write an algorithm which performs it. It means, given a sequence of answers (Yes or No), one for each card, the algorithm returns the correct number chosen by someone.
2. Write an algorithm which, given a card rank between 1 and 5, return all the numbers on the corresponding card.
3. Write an algorithm which, given a number between 1 and 31, return the set of cards in which it is featured.

Students then developed their algorithms either in pseudocode version or implemented in some language they desired. The algorithms were collected to identify the development process. (see. Appendix G). Then, the data was coded into Q-A maps. (see. Appendix E).

4. If you could describe the following dance movements in terms of algebraic expression of functions. How would you do that?

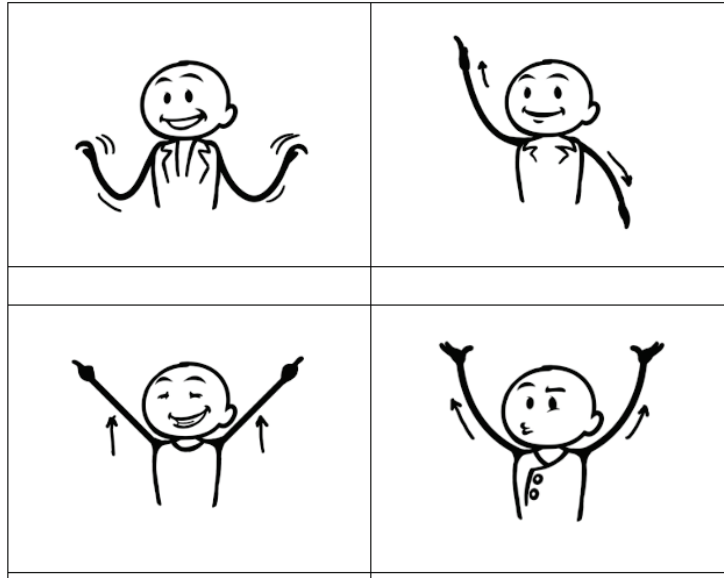


FIGURE 4.5: Function Movements - Pre Test

Team 2 Choreography - BTS		
Input	$f(x)$ Function (algebraic mode)	Function (computer mode)
1	x^2	x^2
2	x	x
3	$-x$	$-x$
4	$-x^2$	$-x^2$
5	e^x	$\exp(x)$
6	e^{-x}	$\exp(-x)$
7	$\log(x)$	$\log(x)$

TABLE 4.7: Example of data from students - Team 2 BTS

4.6.3 Fake Binary Code

To collect data from participants of Fake Binary Code activity .We asked them to try to summarize their resolution processes and, in the end, to develop a natural language algorithm to perform the tasks. The goals were:

- Understand the trick and write down an algorithm that performs it. It means, given a sequence of answers (Red or Black), the algorithm returns the correct number chosen by someone.

4.6.4 Op'Art

Experiments were short (less than four hours) and not with an audience of the target group of primary and secondary students, rather their teachers and teacher trainers,

4. If you could describe the following dance movements in terms of algebraic expression of functions. How would you do that?

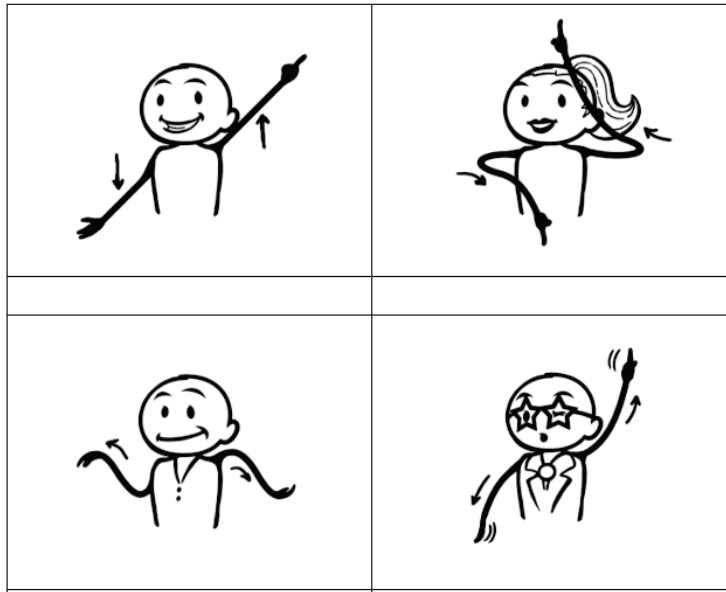


FIGURE 4.6: Function Movements - Post Test

but some conclusions can nevertheless be drawn, from the software produced and collected, as well as from oral exchanges.

4.7 Data Analysis

In this section we present the methods developed and the theoretical frames supporting the data analysis which help us to verify the following hypothesis related to research questions:

1. It's possible to model the process of creativity, allowing an observer to identify the critical episodes during a problem solving, inquiry based activity.
2. It's possible to model answers from the game Function Hero as Creative Products, allowing an observer to identify the cognitive components: Fluency, Flexibility, Originality and Elaboration on students' productions.
3. There was a significant difference in performance scores between before and after the use of the game.
4. The students were able to express creativity through the use of the game.
5. It is possible to highlight critical moments in the resolution of algorithms that have to do with the components of creativity's diamond.

As stated at the beginning of the data collection section, we consider mathematical creativity as a construct that combines Process, Person, Product, and Environment. Such contests are not individually determined but are part of a complete didactic

5. La représentation graphique de la fonction $f(x) = x^3$ est la figure ...

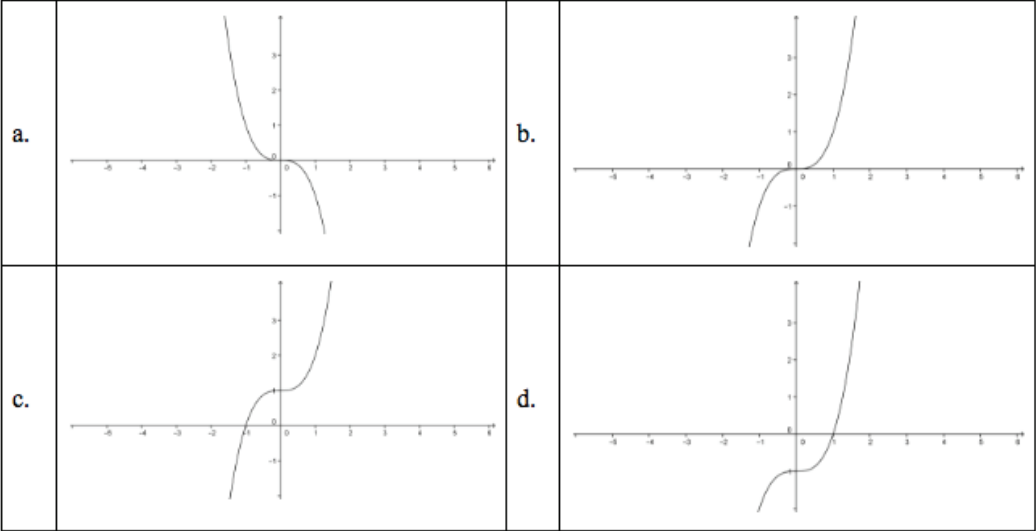


FIGURE 4.7: Function Test

$(1/2)*x^2, x, -x, (-1/2)*x^2, \sin(x), \cos(x), \text{abs}(x), \log(x)$

Entrez votre chorégraphie:

$(1/2)*x^2, x, -x, (-1/2)*x^2, \sin(x), \cos(x), \text{abs}(x), \log(x)$

OK

FIGURE 4.8: Function Hero Input

situation. In this thesis we observe and measure the Creativity as Process and Product. The notion of Study and Research Paths (SRP) (Chevallard, 2009) was adopted in this research as methodology to analyse the assemble of the components from the Inquiry based activities, and the the model of dynamics of novelties correlated via the adjacent possible from Tria et al., 2014 to analyze the productions from students. The model from Tria et al., 2014 predicts the statistical laws for the rate at which novelties happen (Heaps’ Law) and for the frequency distribution of the explored regions of the space (Zipf’s Law), as well the signatures of the correlation process by which one novelty sets the stage for another.

4.8 The Network Model: Process and Product

As seen on Chapter 2, several sources inspired the use of a graph system. By using these approach we were able to observe the collected data, labeling the concepts into a set of solutions and productions, in which the distinction between students’ productions could be addressed, helping us to answer the questions:

- Which are there types of nodes most used by students? - We will check the components: Fluency, Flexibility, Originality, and Elaboration. Related to the

RQ.4,5.

- What are the pieces of evidence, from the analysis of graphs, that allow us to conclude that the products and processes are more or less creative than others?

RQ.2,3

4.8.1 Function Hero

Knowing that the set of nodes represents all the possible "words" considered or belonging to a space of words, in the game *Function Hero*, we will use all the nodes written by all the students as the vocabulary already known. First, let's understand how the concepts were codified. The Table 4.8 shows how all the answers in the game *Function Hero* were encoded for an analysis.

id	function	type	name
n1	$\cot(x)$	function	cotangent
n2	$\tan(x)$	function	tangent
n3	x	function	x
n4	$\sin(x)$	function	sine
n5	$\cos(x)$	function	cosine
n6	$\log(x)$	function	logarithm
n7	$ x $	function	absolut
n8	e	function	exponential
n9	$\sec(x)$	function	secant
n10	$\operatorname{cosec}(x)$	function	cosecant
n11	a	constant	value
n12	$*$	operator	multiplication
n13	$-$	operator	minus
n14	$+$	operator	plus
n15	$/$	operator	divison
n16	\wedge	operator	power
n17	$\sqrt{\quad}$	function	square root
n18	1	value	value
n19	b	constant	constant
n20	p	expoent	value
n21	2	value	value
n22	3	value	value
n23	arcsin	function	arc-sine
n24	arccos	function	arc-cosine
n25	arctan	function	arc-tangent
n26	sinh	function	hyperbolic sine
n27	cosh	function	hyperbolic cosine
n28	arccosh	function	hyperbolic arc-cosine

n29	arcsenh	function	hyperbolic arc-sine
n30	pi	constant	constant
.n50	values	values	values

TABLE 4.8: Graph Nodes - *Function Hero*

We can then see in the figure 4.9, the graphic representation of all the nodes that constitute the students’ inputs during the course of the game *Function Hero*.

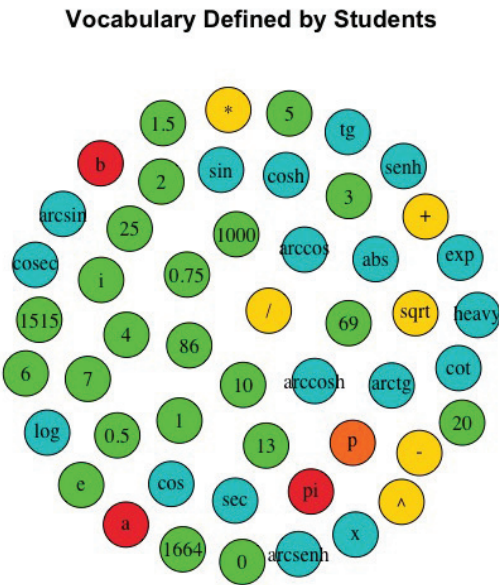


FIGURE 4.9: Vocabulary of Functions

Whenever a student develops a function and enters it in the game, we consider this as a joining of a number of nodes to represent the entire function. In this sense, for instance, if we consider a function of type $f(x) = ax + b$, the graph representing this function will be given by Figure 4.10:

4.8.2 Binary Code

In the case of Binary Code and other research problems. Conceptual space will not be formed in the same way as in *Function Hero*, but for already established knowledge as *semantic clusters of basic concepts*, which can form two types of results: Answers and Questions.

As we have seen in the methodology of Study and Research Path (Chevallard, 2009), for an initial problem, we have the desired answer (A^\heartsuit). Thus we will establish the ways to arrive at this answer as a dynamic, complex and recursive process, defined

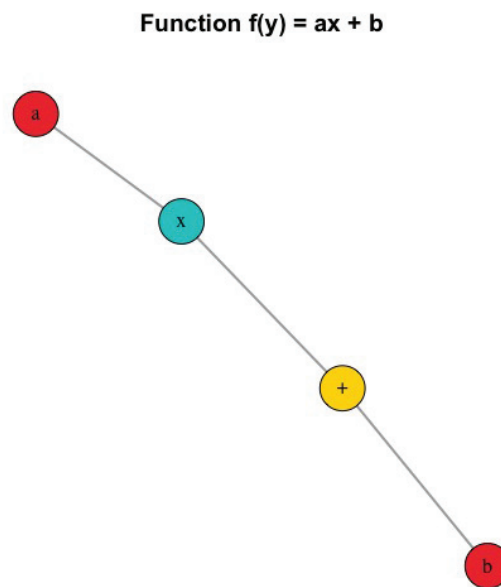


FIGURE 4.10: *Function Hero* Input - example $f(x) = ax + b$

by the Diamond of Creativity, explained in the chapter 2. In the figure 4.11, we see the scheme where the Diamond of Creativity is represented by the diamond between the initial question and the answer desired by the teacher.

As described by Jessen, 2017, the study and research process leads to a number of paths, detours and dead-ends in the the process of developing a coherent answer to the generation Q_0 (Bosch and Winsløw, 2016). The numbering of derived questions indicates the relation between the derived questions and the paths they belong to.

This diagram, from (Jessen, 2017. see. Picture 4.12), shows a tree with the questions generated by the initial question. Thus, the generation of new questions and answers may, slightly due to its complexity, be represented as follows in the Figure 4.13.

Usually, these questions arise from the students during the activity of research and study paths, self-generating according to the development of the task. However, to compare how creative the students are being, the problem was subdivided into questions that the teacher himself made. It creates a conceptual and knowledge space to "track" student development of the solution.

Each of these nodes, regarding Questions and Answers, helped us to develop a Q-A Map labeled following the taxonomy for Computational Thinking in Mathematics and Science, from Weintrop et al., defining the problem-solving process into four categories:

- Data practices

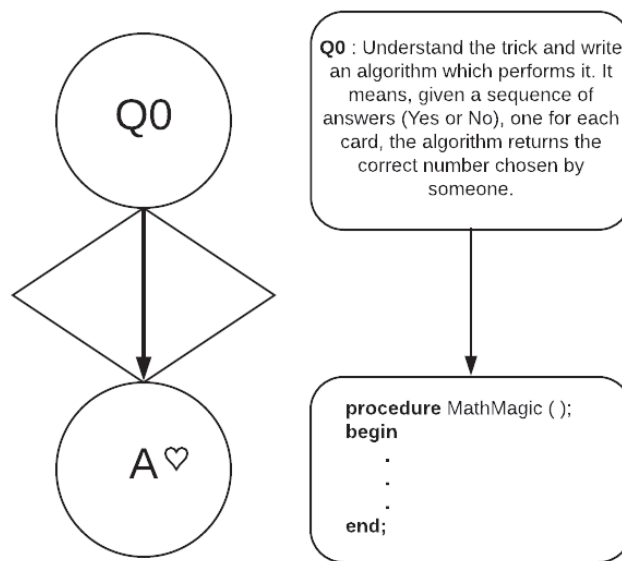


FIGURE 4.11: Simple Main Path - Binary Code

1. Collecting Data: Data are collected through observation and measurement. Computational tools play a key role in gathering and recording a variety of data across many different scientific and mathematical endeavors. However, in educational environments we do not always have computers at our disposal and therefore, in the magic Binary Code, the use of computers is not mandatory.

The Binary Code game data is displayed as five cards containing sixteen unordered numbers. Another necessary data for understanding the game is the answer that the individual proves saying yes or no.

2. Creating Data: In many cases, scientists and mathematicians use computational tools to generate data. This is the case when investigating phenomena that cannot be easily observed or measured or that are more theoretical in nature.

In the game Binary Code the data does not have to be created because they already are demonstrated in all the letters.

3. Manipulating Data: In mathematical and scientific fields, it is essential to manipulate data in order to *make meaning* of them. Computational tools make it possible to efficiently and reliably manipulate large and complex datasets. Data manipulation includes, sorting, filtering, cleaning, normalizing, and joining disparate datasets. These manipulations serve for both analysis and communication.

In Magical Binary Code, to understand the game, one can start by organizing the data of each card. In their original version, the numbers are

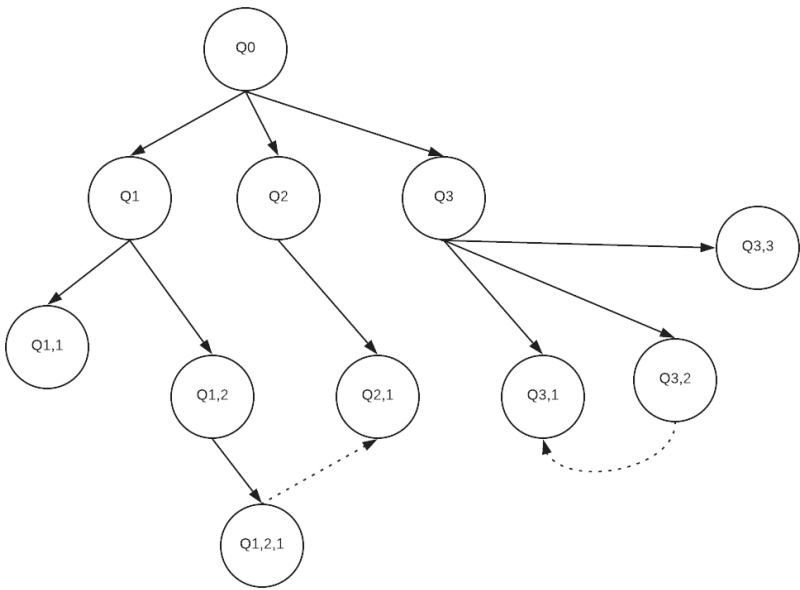


FIGURE 4.12: The tree diagram showing the derived questions during the Divergent Thinking and their internal relation

already organized in ascending order. To "give more power" to the magician when performing the trick, we decided to randomize the position of the numbers within the cards. It is up to the student to decide whether to organize them or not.

4. Analyzing Data: The true power of data lies in the information that can be gleaned from them through analysis. There are many strategies that can be employed when analyzing data for use in a scientific or mathematical context, including looking for patterns or anomalies, defining rules to categorize data, and identifying trends and correlations.

1	1	3	5	7	9	11	15	17	19	13	21	23	25	27	29	31
2	2	3	6	7	10	11	14	15	18	19	22	23	26	27	30	31
3	4	5	6	7	12	13	14	15	20	21	22	23	28	29	30	31
4	8	9	10	11	12	13	14	15	24	25	26	27	28	29	30	31
5	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31

In the Binary Code trick, one of the main characteristics that people find when they begin to analyze the cards is that the first card contains only odd numbers. After that, they soon go to the second card thinking that the same pattern can be applied, but soon find that the second card contains even and odd numbers.

Then, after analyzing the last letter (5), they realize that the sequence of numbers goes from 16 to 31. We will see later that students develop an algorithm to describe the input of the two cards.

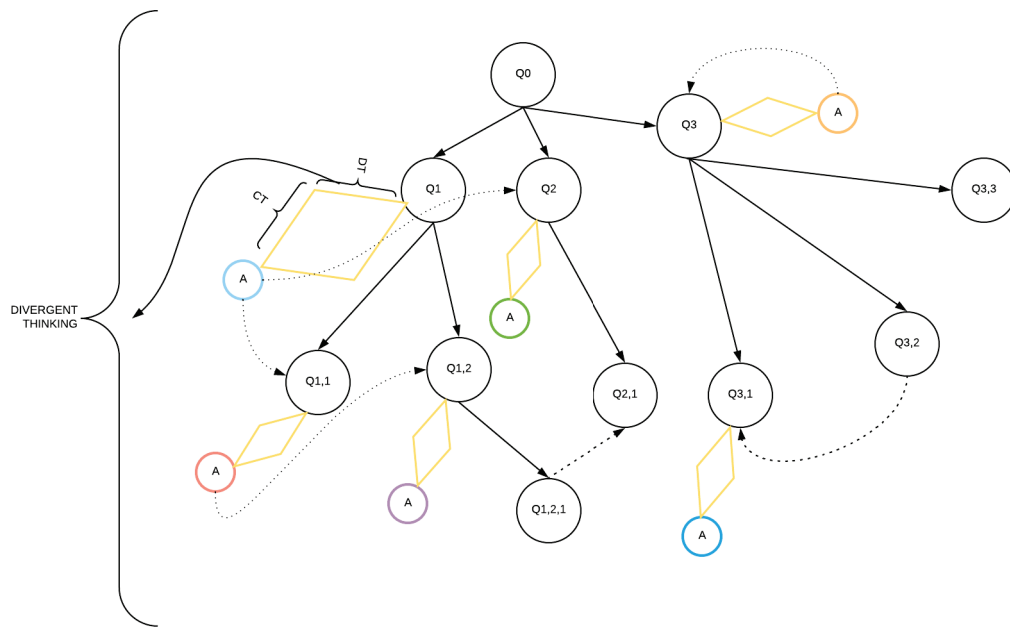


FIGURE 4.13: Divergent Thinking Diagram

5. Visualizing Data: Communicating results is an essential component of any knowledge-building endeavor, and computational tools can greatly facilitate that process.
- Modeling and simulation practices
 1. *Using Computational Models to Understand a Concept:* Computational models that demonstrate specific ideas or phenomena can serve as powerful learning tools. Students can use computational models to deepen their understanding of mathematical and scientific concepts, such as the interdependence within ecosystems, how objects move in a friction-less environment, and probabilistic distributions of random events.

However, in the trick of magic, students are asked to write and develop a computational model at different levels of understanding. Perform the trick, build the cards and give a number say the cards.

 2. *Using Computational Models to Find and Test Solutions:* Computational models can be also be use to test hypotheses and discover solutions to problems. They make it possible to test many different solutions quickly, easily, and inexpensively before committing to a specific approach.
 3. *Assessing Computational Models:* A key practice in using a computational model effectively is to understand how the model relates to the phenomenon being represented. This understanding is guided by a variety of questions including: Which aspects of the phenomenon have been faithfully modeled and which aspects have been simplified or ignored? In this case it

is the teacher who analyzes the models developed by the students elaborating these questions to understand the process that the students used to develop their algorithms.

4. *Designing Computational Models*: Part of taking advantage of computational power in the scientific disciplines is designing new models that can be run on a computational device. The process of designing a model is distinct from actually implementing it; designing a model involves making technological, methodological, and conceptual decisions.

In the case of Binary Code, the "design" happens by choosing the tools where the algorithm will be implemented. For example, we can develop the algorithm using scratch, or other programming languages. For this activity, we have decisions that are out of the understanding of the phenomenon to be implemented. For example, define how the cards will be presented to the user through images or a list of numbers.

5. *Constructing Computational Models*: An important practice in scientific and mathematical pursuits is the ability to create new or extend existing computational models. This requires being able to encode the model features in a way that a computer can interpret. Being able to implement modeling ideas is critical for advancing ideas beyond the work done by others and complements the previous practice of designing computational models.

Here we see the importance of mathematical thinking in resolving the binary code problem. The algorithm can be implemented in several ways, and yet the computer will be able to perform the trick or solve the problems proposed at the beginning of the activity. However, when we generalize the methods of solution, there will be a difference between the mathematical creativity present in the solution. For example, if the student realizes that the cards are organized in powers of two, he/she can write an algorithm easily.

- Computational problem solving practices

1. *Preparing Problems for Computational Solutions*: While some problems naturally lend themselves to computational solutions, more often, problems must be reframed so that existing computational tools - be they physical devices or software packages - can be utilized. Strategies for doing this include decomposing problems into subproblems, reframing new problems into known problems for which computational tools already exist, and simplifying complex problems so the mapping of problem features onto computational solutions is more accessible.

In the case of the Binary Code, and in the other problems, when we speak of divergent thinking and its recursion, it is the decomposition of the more

significant problem into smaller problems that can be solved with shorter or more elaborate diamond lines of creativity depending on the set of knowledge and concepts that the student has. To stimulate creativity, the problem should prompt the student to seek new solutions to these sub-problems that they do not yet know, increasing the flexibility and fluency to explore other problems of the same type, such as the Fake Binary Code.

2. *Programming*: The ability to encode instructions in such a way that a computer can execute them is a powerful skill for investigating and solving mathematical and scientific problems. Programs ranging from ten-line Python scripts to multimillion-line C++ libraries can be valuable for data collection and analysis, visualizing information, building and extending computational models, and interfacing with other existing computational tools.

In the Function Hero activity, encoding a function might not be straightforward for all students, such as using `sqrt` function instead of the $\sqrt{}$ symbol, using the caret sign $^$ for exponentiation, having to explicit the multiplication symbol $*$ or the linear way of coding a fraction for example, but there are as well impediments that are more mathematically meaningful such as entering a function (log, fractional powers such as $\sqrt{}$) whose definition domain does not contain the view $[-3, 3]$ used in the program. Then students tend to mistake the *intended* behaviour of the software, not showing any graph where the function is not defined, for a syntax error in their writing or a bug in the system.

In the Binary Code activity, students are instructed to develop their algorithms in a programming language that they want. Even the natural language serves, since the agent to execute the algorithm will be another person, who can understand the instructions passed to the user. At this stage, we are in the convergent phase of the diamond of creativity, because the process of finding the solution is now being translated into words that can be interpreted by another agent and verifying that the solution is correct.

3. *Choosing Effective Computational Tools*: Choosing an effective computational tool includes considering the functionality it provides, its scope and customizability, the type of data the tools expects and can produce, as well as questions that extend beyond the software itself, such as, whether or not there is an active user community that could assist with difficulties you might encounter.

In our case, for example, choosing a programming language is one of the choices students must make in order to develop their algorithms. Since we had different groups of samples with different degrees of knowledge in each tool to be used, we chose to leave this choice free. For some, we

got an implementation in Python, for others we got an implementation in Scratch. The second is widely used in educational scenarios because it is free and allows learning most of the basic programming concepts visually.

4. *Assessing Different Approaches/Solutions to a Problem:* When there are multiple approaches to solving a problem or multiple solutions to choose from, it is important to be able to assess the options and make an informed decision about which route to follow.

We know that different methods can achieve the same result, but we try to show precisely the different ways to achieve each result.

5. *Developing Modular Computational Solutions:* When working toward a specific scientific or mathematical outcome, there are often a number of steps or components involved in the process; these steps, in turn, can be broken down in a variety of ways that impact their ability to be easily reused, repurposed, and debugged. Elements of a solution can be large, complicated and uniquely designed for the problem at hand, or they can be small, modular, and reusable. Developing computational solutions in a modular, reusable way has many implications for both the immediate problem and future problems that may be encountered.

Again we see that these modular solutions may be responses to the subproblems encountered in exploring the problem. For example, in the case of Binary Code, a subproblem to be solved is to create the cards with the numbers to be shown to the user.

6. *Creating Computational Abstractions:* Creating an abstraction requires the ability to conceptualize and then represent an idea or a process in more general terms by foregrounding the important aspects of the idea while backgrounding less important features. The ability to create and use abstractions is used constantly across mathematical and scientific undertakings, be it creating computational abstractions when writing a program, generating visualizations of data to communicate an idea or finding, defining the scope or scale of a problem or creating models to further explore or understand a given phenomenon. Creating computational abstractions is essential for solving multiple problems that have structural similarity but differ in surface detail.

We see an example of this generalization when developing the algorithm to solve the Fake Binary Code trick. The decomposition in base two in the binary code trick is an abstraction that advances and improves the development of the algorithm. However, when we try to understand the structure of the Fake Binary Code, it may seem that the generalization structure is the same, but we end up falling into a mistake.

7. *Troubleshooting and Debugging*: Troubleshooting broadly refers to the process of figuring out why something is not working or behaving as expected. There are a number of strategies one can employ while troubleshooting a problem, including clearly identifying the issue, systematically testing the system to isolate the source of the error, and reproducing the problem so that potential solutions can be tested reliably.
- Systems thinking practices

1. *Investigating a Complex System as a Whole*: A system can be viewed as a single entity composed of many interrelated elements. Investigating a complex system as a whole relies on the ability to define and measure inputs and outputs of the system.

In this sense, when we ask the student to develop the Binary Code trick, we want him to write down all the processes involved in the creation, not only the algorithm but also the whole system that makes the user input "yes or no." We then have the inputs to the outputs and the process of processing those inputs to produce the expected result.

2. *Understanding the Relationship within a System*: Whereas some questions can best be answered by focusing on a system as a whole, other questions require understanding how the components within a system interact. Thus, it is important to be able to identify the different elements of a system and articulate the nature of their interactions.
3. *Thinking in Levels*: Systems can be understood and analyzed from different perspectives, ranging from a micro-level view that considers the smallest elements of the system to a macro-level view that considers the system as a whole.

Understanding the system is fundamental to its development. We will see in the results that the process can be understood at different levels. Sometimes such levels do not specify what the implementation should be. For example, if we use an object-oriented paradigm, where we have a class, and the class implements different methods such as organizing, doing, shuffling. If the understanding is low, we can say that the developer can only see and describe more general levels of actions.

4. *Communicating Information about a System*: A central challenge when investigating a system is figuring out how best to communicate what you have learned about it. Communicating information about a system often involves developing effective and accessible visualizations and infographics that highlight the most important aspects of what has been learned about the system in such a way that it can be understood by someone who does not know all the underlying details.

The very construction of the creativity diamond model is a sample of the communication of a complex system that needs a presentation such as Chapter 2, which presents in detail how the system works.

5. *Defining Systems and Managing Complexity*: Anything can be viewed as a system; the size and the membership of the system depends on where you define its boundaries.

In the sections below I present the same process as the items above, but with the answers and questions a priori made by the teacher before presenting the trick in schools and workshops.

Initial Question

- Q_0 - *Initial Question*: Understand the trick and write an algorithm which performs it. It means, given a sequence of answers (Yes or No), one for each card, the algorithm returns the correct number chosen by someone.

Data Practices

- *DP.Q1 - Collecting Data*: Which data can be collected from the trick?
 1. *AD1*: The answers from the participant of the trick
 2. *AD2*: The Data about the cards
- *DP.Q2 - Creating Data*: Do we need to generate more data than we have?
 1. *AD3*: No.
- *DP.Q3 - Manipulating Data*: How can we manipulate the data in order to make sense of it?
 1. *AD4*: It's possible to sort the numbers into a table

1	1	3	5	7	9	11	15	17	19	13	21	23	25	27	29	31
2	2	3	6	7	10	11	14	15	18	19	22	23	26	27	30	31
3	4	5	6	7	12	13	14	15	20	21	22	23	28	29	30	31
4	8	9	10	11	12	13	14	15	24	25	26	27	28	29	30	31
5	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31

2. *AD5*: A table with the cards and the numbers

<i>numbers from 1 - 31</i>	0	1	2	3	4
1	1	0	0	0	0
2	0	1	0	0	0
3	1	1	0	0	0
4	0	0	1	0	0
5	1	0	1	0	0
.
.
.
30	0	1	1	1	1
31	1	1	1	1	1

- **DP.Q4 - Analyzing Data:** Is there any conclusion to do about the observed data?

1. **AD6:** We can observe that in the first card we have only odd numbers.
2. **AD7:** We can observe that the last card has all numbers from 16 to 31.
3. **AD8:** We can observe that in each sequence of cards there are "holes" between blocks of consecutive numbers.
4. **AD9:** We can see that the first numbers of the sequence of each card increase as powers of 2.
5. **AD10:** We can observe that the presence or not, of the numbers of each card creates the binary decomposition of that number.
6. **AD11:** Each card has 16 numbers.
7. **AD12:** The range of possible numbers ranges from 1 to 31.

Modeling and Simulation Practices

- **MSP.Q1 - Using Computational Models to Understand a Concept:** Do you know any computational model to understand the phenomenon of the problem? If yes, which one?
- **MSP.Q2 - Using Computational Models to Find or Test Solutions:** Do you know any computational model to find or test the solutions you provided? If yes, which one?
- **MSP.Q3 - Assessing Computational Models:** How can you assess the computational model?
- **MSP.Q4 - Designing Computational Models:** Can I design a model to the problem?
 1. **AD13:** See figure 4.14

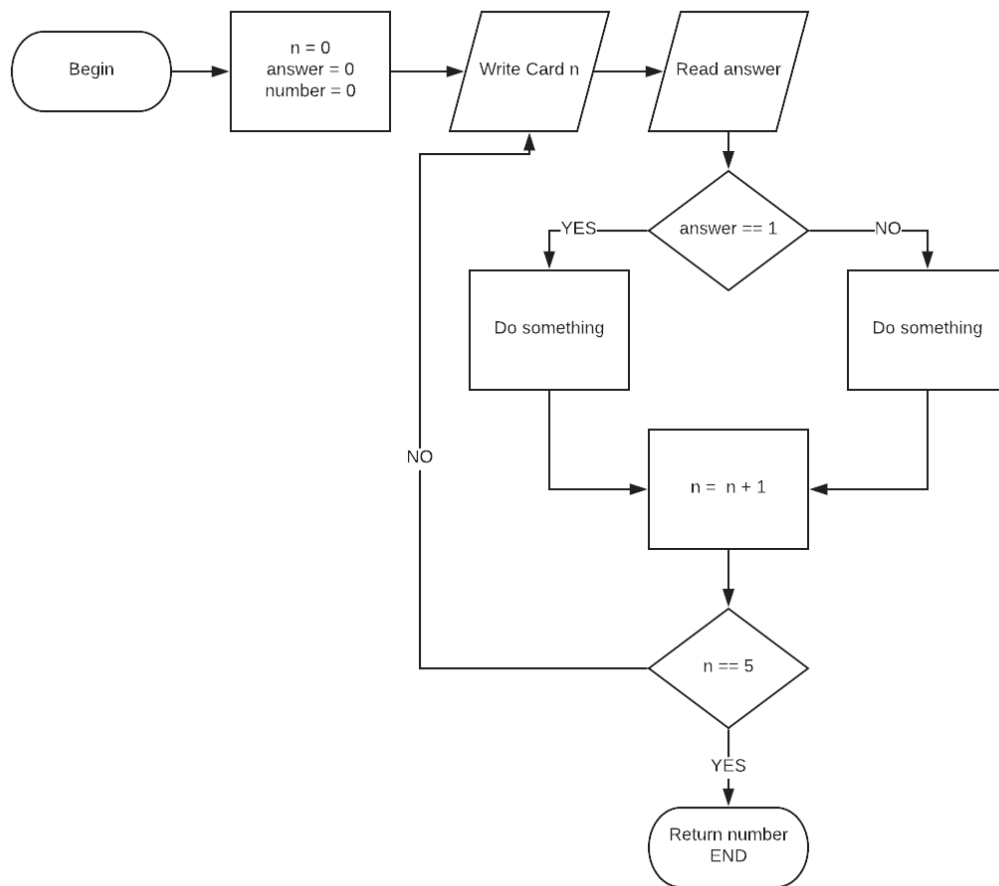


FIGURE 4.14: Binary Code - Simple Model

- **MSP.Q5 - Constructing Computational Models:** Can I construct a computational model?

Computational Problem Solving Practices

- **CPSP.Q1 - Preparing Problems for Computational Solutions:** Do we need to prepare the problem to be solved?
 1. **AD14:** We can write an algorithm in pseudocode before implementing it in a language.
- **CPSP.Q2 - Programming:** How can we start programming the solution?
 1. **AD15:** Defining user inputs.
 2. **AD16:** Creating the Calculation Routine.
 3. **AD17:** Showing the result on the screen.
- **CPSP.Q3 - Choosing Effective Computational Tools:** Which tools are we going to use to develop the algorithm?

1. *AD18*: Scratch
 2. *AD19*: Python
 3. *AD20*: Java
 4. *AD21*: Pseudocode
- *CPSP.Q4 - Assessing Different Approaches/Solutions to a Problem*: Can we think different approaches to the same problem?
 - *CPSP.Q5 - Developing Modular Computational Solutions*: Can we develop smaller pieces of code to help achieving the general solution?
1. *Q1*: How to show the cards to the user?
- (a) *AD22*:

```
import numpy as np

binTable = np.ones((31,5))

cards = np.ones((5,16))

for i in range(len(binTable)):
    tempDecimal = i + 1
    for j in range(5):
        binTable[i][j] = tempDecimal % 2
        tempDecimal = tempDecimal / 2

for i in range(5):
    control = 0
    for j in range(len(binTable)):
        if binTable[j][i] == 1:
            cards[i][control] = binTable[j][i] * (j
                + 1)
            control = control + 1

n = 0
for i in xrange(n, 32, i + 2 * n):
    print(i)
```

(b) *AD23*:

```
def card(n):
    '''The nth card'''
    n2 = 1 << n
    for i in range(16):
        print(i % n2 + ((1 + 2 * (i >> n)) * (2 **
            n)))
```

(c) AD24:

```
def card2(n):
    '''The nth card'''
    for i in range(32):
        if (i // (2 ** n)) % 2 == 1:
            print(i)
```

(d) AD25:

```
def card3(n):
    j=0
    while j<32:
        for i in range(n):
            print(0)
        for i in range(n):
            print(1)
        j += 2*n
```

- **CPSP.Q6 - Creating Computational Abstractions:** Can we develop a generalization to the problem?
 1. A25: Showing the cards in accordance with its index.
- **CPSP.Q7 - Troubleshooting and Debugging:** How can we verify whether the algorithm works?

Systems Thinking Practices

- **STP.Q1 - Investigating a Complex System as a Whole**
- **STP.Q2 - Understanding the Relationships within a System**
 1. AD27: Understand the role of binary numbers in other domains.
- **STP.Q3 - Thinking in Levels**
- **STP.Q4 - Communicating Information about a System:** How can we translate the solution into an entire system?

1. AD28:

```
#Funcao para mostrar a carta n
def card(n):
    n2 = 1 << n
    for i in range(16):
        print(i % n2 + ((1 + 2 * (i >> n)) * (2 ** n)))

#Interacao com Usuario
```

```

soma = 0
for i in range(5):
    card(i)
    resposta = int(input("is your number in the list
                        above? 1 = yes, 0 = no"))
    soma = soma + resposta * (2 ** i)

print("soma eh: ", soma)

```

2. **AD29** We make the sum of the power of two corresponding to the card requested by the user.
 3. **AD30** We make the sum of the numbers that are on the lower left side of each card.
- **STP.Q5 - Defining Systems and Managing Complexity**

The description above can be translated into a Q-A Map, which can be seen at the Figure 4.15

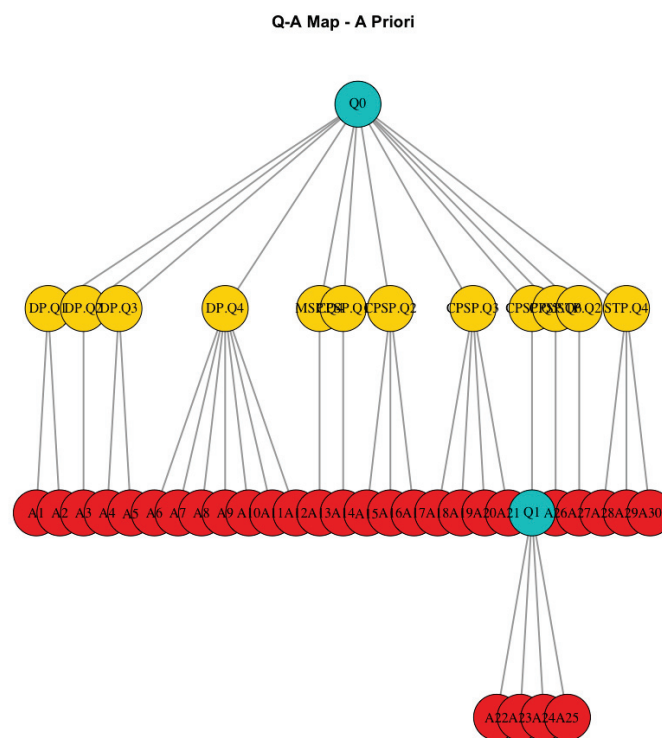


FIGURE 4.15: Q-A Map - A priori - Binary Code

The blue nodes are the trigger questions (Q_0 , Q_1). The yellow nodes are the questions pre-established by Weintrop et al.'s taxonomy. The red nodes are the answers given by the solver.

4.8.3 Fake Binary Code

The Fake Binary Code problem is an adaptation, or maybe we can consider as a variation of the Binary Code problem. The game consists in asking a person to choose a number from the grid of possible numbers and to repeat the sequence of numbers of that row by changing the color of the number selected. The magician then guesses which number was chosen.

The same methodology for analyzing the Binary Code trick was applied to the Fake Binary Code trick. The participants of the experiment should unravel the trick, and the path was analyzed as described in the sections before: A priori Q A map was created in order to assess the students creativity in the problem solution.

4.8.4 Op Art

In the Op Art activity we ask the students to analyze the following pieces of art and write down an algorithm to be performed by other students. Examples of these activities are the Figures 4.16a, 4.16b, 4.16c, and 4.16e.

4.9 Analysis of Creativity

As we mentioned in Chapter 2, Kaufman, 2016, says that one way to organize creativity research is the "Four P" model, which distinguishes the creative person, process, product, and press (i.e. environment). Also, we consider these components as part of a didactical system, where individuals are engaged into a problem-solving or inquiry activity.

To codify the problem-solving process to perform its quantitative analysis possible, we make use of the concept of *Study and Research Path* Chevallard, 2009. In any *Study and Research Paths* there is a generative question Q_0 which originates and inspires the process of study or problem solving. This question Q_0 , should allow the enunciation of another derivatives $(Q_i)_{1 \leq i \leq n}$. The different Mathematical Organizations are generated from every Q_i , as response to the questions that have aroused from their construction. The scheme that Chevallard, 2009 called *developed herbatian* $[S(X; Y; Q) \rightarrow \{R_1^\diamond, R_2^\diamond, \dots, R_n^\diamond, O_{n+1}, \dots, O_m\}] \hookrightarrow R^\heartsuit$, enables to interpret the following: the RSP must be organized around a generative question (Q_0), must produce the constitution of a didactic means $M = \{R_1^\diamond, R_2^\diamond, \dots, R_n^\diamond, O_{n+1}, \dots, O_m\}$, where R_i^\diamond correspond to the ready-made answers, which have not been originated in the same class; and the works O_j , the available theories and Mathematical Organizations, from which it is a possible to deconstruct the R^\diamond . From the ready-made

answers and those which are generated in the didactic medium; the elaboration, validation and institutionalization of a response R^\heartsuit as result of the process of study are expected to be obtained.

More specifically Chevallard, 2009, explains that the didactic system can be defined as:

$$S(X; Y; Q)$$

, where around a question Q , are present: X is a collective (a class, a team of students, a team of researchers, a journalist, etc.) and Y a team (usually reduced: sometimes it may even be a null set) of help in studying or solving the problem (teachers, tutors, research directors, etc.).

Chevallard, 2009 evokes the scheme saying:

A teacher y gives a class X a job to do, say a math problem. This problem can be observed as a question Q , which each student $x \in X$ must give a R_x answer, his solution to the problem, which we can denote as R_x^\diamond , because this response is "punctured" by the student x . We can imagine that the class then works to produce, under the direction of y , its response R^\heartsuit from the answers $R_x^\diamond, x \in X$.

4.9.1 Person

In this thesis we don't observe the individual characteristics. At first, the only individual variable we tried to measure was the ability to recognize functions, for the Function Hero game, to then correlate with the points of creativity obtained during the game. Making use of the Herbatian scheme, the person is represented by $x \in X$.

4.9.2 Product

The creative product - the things people make, the ideas they express, the responses they give. This is one of the focuses of the thesis. According to Kaufman, 2016, most creativity tests tend to focus on a tangible product (such as a poem, drawing, or answers to open questions or problems). We consider as products: the functions of the choreography of the game *Function Hero* and the algorithms developed for the activities *Binary Code*, *Fake Binary Code* and *Op' Art*.

4.9.3 Process

To analyze the creative process, we will use the Creativity Diamond, described in Chapter 2, to label, in the procedures developed by students, each step of the process. To recap: We understand the creative process as a combination of divergent

and convergent thinking in problem solving activities.

This conceptual framework describes the creative process when someone attempts to solve mathematical problems. As seen before on the example of Julio le Parc, creating a figure is a creative process and the figure itself is a creative product. In this generative abstract art version, notice that the algorithm to produce the figure must be seen as a creative product as well.

Guilford emphasized the distinction between convergent and divergent thinking. He introduced the model of "Divergent Thinking" as the main ingredient of creativity. Drawing on his model Guilford, the generation of new ideas shows the cognitive elements called fluency, flexibility, originality, and elaboration in mathematics that are defined as follows: El-Demerdash et al.:

- **Fluency:** means the individual's ability to pose or come up with many mathematical ideas or configurations related to a mathematical problem or situation in a short time.
- **Flexibility:** refers to the individual's ability to vary the approach or suggest a variety of different methods toward a mathematical problem or situation.
- **Originality:** means the individual's ability to try novel or unique approaches toward a mathematical problem or situation.
- **Elaboration:** is the individual's ability to redefine a single mathematical problem or situation to create others, going deeper, giving elaborated solutions, by changing one or more aspects by substituting, combining, adapting, altering, expanding, eliminating, rearranging, or reversing and then speculation on how this single change would have a ripple effect on other aspects of the problem or the situation at hand.

To illustrate the usage of the Guilford's approach in a product's evaluation of creativity. Consider we have an activity which asks students to write down all the possible ways that he/she knows to get the number 36 in a limited time. (see. ??).

However, these stages are related only with the Divergent Thinking stage, where the individuals work in combination with conscious and unconscious processing generating what we call the fractal of the creative process. It means that the creative process not only happens in a cyclic movement, but also has recursive moments. Deciding to give a name to a variable is a little c creative moment, compared to a capital C creative moment like "defining Fuchsian groups" by Henri Poincaré for example.

At the climax of the divergent thinking process, the *Illumination* is an inflection point which allows the creative process to converge, leading to the actual production of something (an object, a story, a formal proof, a software...) outside the creators' brain, that can be shared with others and confronted with their point of views. Ideas

can then be externalized and translated into things, i.e. this phase we call convergent thinking.

4.9.4 Environment

Amabile, 1983 has done many studies that consider the importance for creativity of intrinsic motivation, or being driven by a passion for the activity. Amabile and Gryskiewicz, 1989 identify eight aspects of the work environment that stimulate creativity: adequate freedom, challenging work, appropriate resources, a supportive supervisor, diverse and communicative coworkers, recognition, a sense of cooperation, and an organization that supports creativity. They also list four aspects that restrain creativity: time pressure, too much evaluation, and emphasis on keeping the status quo, and too much organizational politics.

In this study we did not evaluate the environment due to the complexity and quantity of variables involved. We could not separate which factors in the environment can influence student quantitative outcomes.

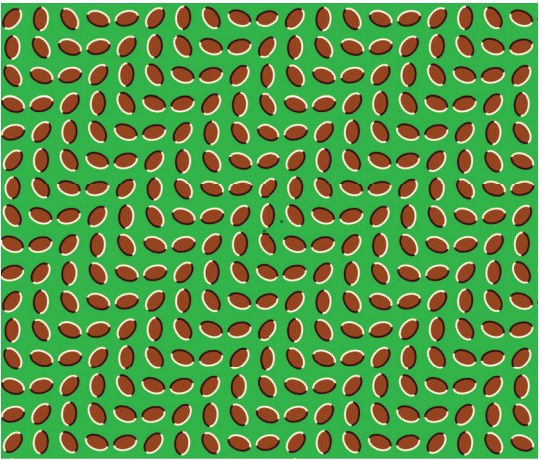
4.10 Summary

In this chapter, we explain how the research methodology was delineated and chosen to answer the research questions established in the first chapters of the thesis. We describe how the research participants were selected, and which institutions and projects are included. We showed how the procedure for sample selection was performed, taking into account the individuals' willingness to participate in the sessions and workshops. We describe the choice of research design according to the research functions related to the questions. Also, we describe the instruments used for the interventions, data collection, and analysis, detailing the activities: Function Hero, Binary Code, Fake Binary Code, and Op'Art.

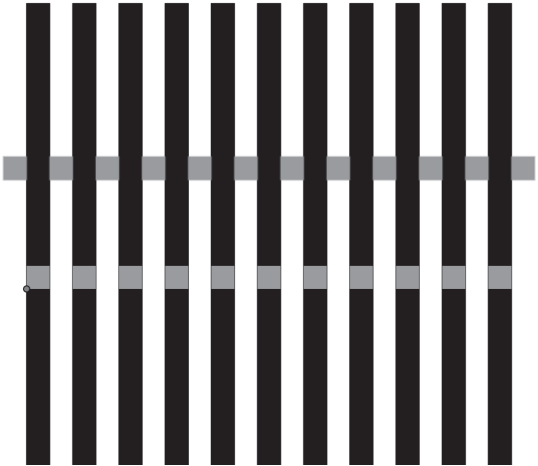
In the next chapter, we show the results obtained from the situations developed.



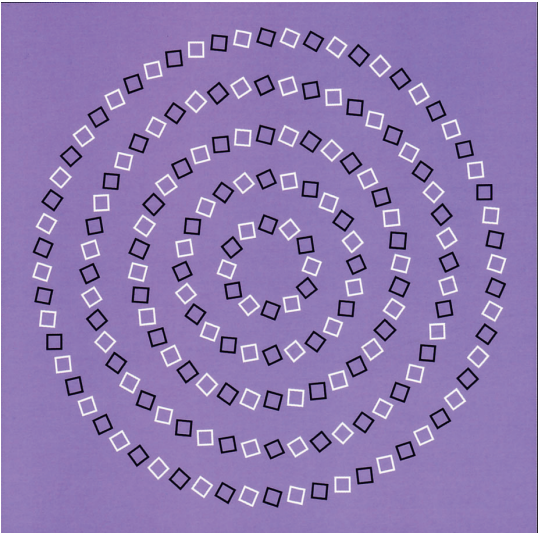
(A) Bridget Riley - Blaze Study



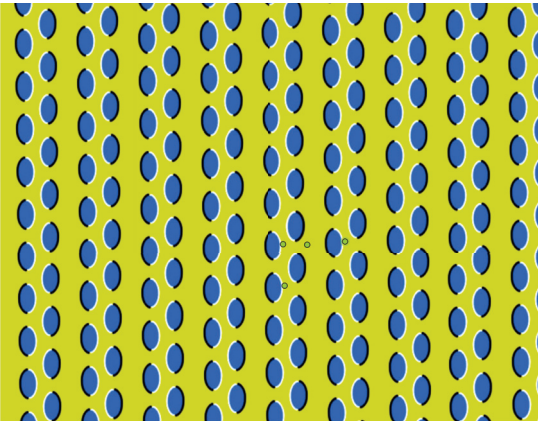
(B) Coffee Bean



(C) Gris Noir



(D) Psycho



(E) Blue Bean (inspired by Akiyoshi Kitaoka)

FIGURE 4.16: Op Art Activities

Id	Ans 1	Ans 2	Ans 3	Ans 4	Ans 5	Ans 6	Evaluation
Student 1	35 + 1	1 + 35	12 + 24	13 + 23	21 + 15	5 + 31	*
Student 2	5 + 31	7 + 29	13 + 23	17 + 19			*
Student 3	35 + 1	37 - 1	6 × 6	$\frac{15}{25}$			**
Student 4	23 + 13	45 - 11	6 ²	$\frac{150}{25}$			**
Student 5	Thirty Six	XXXVI	2 ² + 4 ² + 4 ²				***
Student 6	2 × (7 + 11)	(36 × 0) - (2 × ((5 × 2) + $\frac{16}{2}$))	2 ² × 3 ²				****
Student 7	00100100						*****

TABLE 4.9: Activity Example to afford Creativity

- * Highest fluency (most responses). Operator or Prime numbers used as fluency indicator.
- ** Highest flexibility (most type of responses). Operators used to achieve the result.
- *** Highest Originality.
- **** Highest Elaboration.
- ***** Usefulness, in other domains like computer science.

Chapter 5

Results

5.1 Introduction

In this chapter we present the results of the pre and post test of *Function Hero* experimentation, Binary Code, Fake Binary Code and Op'Art activities, the statistical analysis result of testing the study hypothesis, the results of the quantitative and qualitative analysis of the results from each activity. Also, we present the interpretation of these results in the light of the literature review and the purposes of the study.

Prior knowledge and creativity skills in students are important (Ahmar et al., 2017; Oxman, 1990), however in this thesis, we concentrate on other aspects, namely the progress of the process and the final products designed during the different activities.

5.2 *Function Hero*

Here we would like to mention our observation of students behaviour during the experimentation in terms of students' involvement during the intervention, students pleasure, students uses of the digital materials, and student's originality of the choreography.

According to the teachers' feedback after the intervention, the students tested during this experimentation are in a technical school. They are not particularly interested in maths, but the playfulness of the activity engaged them in the action, even very weak students, usually very passive, tried to play and they all enjoyed the game. These students have generally difficulties with abstraction, they are not used to create mental representations. The challenge has stimulated their curiosity, they explored new representations and they were happy to discover their algebraic expressions, according to one of their teachers. Moreover, they were not that interested in the post test because they had their break just one or two minutes before this test, they were hurry to go out. Then, they confessed that they did not actually care for

their answers, hence the results of the first post test did not bring anything to the fore.

The students were overall very motivated by the experimentation but not at all by the post-test, which was performed once the prizes were distributed. This was clearly a mistake because they were eager to adjourn for recess and did not pay attention to the version 1 of the post-test. The data were therefore not usable as it was, forcing us to retake the test. (experiment version 2).

The two tests that were carried out with the students were to test the hypotheses:

- H_0 : The didactic situation using the Function Hero game does not have any effect on the average accuracy in recognizing functions in their algebraic and graphical forms.
- H_1 : The average accuracy in recognition of functions in their forms: algebraic and graphical, is different in the pretest of the post-test.

5.2.1 Pre and Post Test - Version 1

As said before, the quantitative data collected through the pre and post test doesn't allow us to make conclusions about the question of recognizing functions. It's possible to perceive that the means changed negatively. However, the reason for that is not related to the data but on how it was collected. Because of this, the statistical tests could not be performed effectively.

5.2.2 Pre and Post Test - Version 2

In the second version of the experiment, the classes were observed with the help of the teacher regent. We would like to observe whether the game *Function Hero* would influence in any way the performance of the students in recognizing functions from their algebraic form. To measure it, we use the same surveying instrument developed by Zachariades, Christou, and Papageorgiou, 2002 . We can see the results of the two tests in the table 5.1

TABLE 5.1: Descriptive Statistics - Function Hero

Test	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Pre Test	41	9.37	2.55	5.00	7.00	11.00	14
Post Test	41	10.37	2.47	6.00	9.00	12.00	16

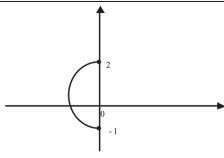
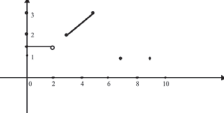
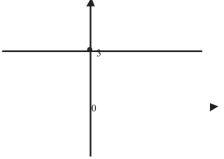
For each situation of the table 5.2, the subjects were asked to indicate whether the symbolic representation corresponded or not to a function. The table 5.2 shows that situations $f(y) = e^y$ and $f(x) = 3$ were the easiest symbolic functions identified

by the students ($\bar{X}_{f(y)=e^y} = 0.95$, $SD = 0.21$; $\bar{X}_{f(x)=3} = 0.85$, $SD = 0.35$), while situations $x^2 + y^3 = 3$, and $a^2 - b = 0$ were the hardest for students to determine whether the relation was a function or not ($\bar{X}_a = 0.09$, $SD = 0.30$; $\bar{X}_c = 0.04$, $SD = 0.21$). The situation c was not correctly answered by the majority of students while half of the students correctly identified the situation h , which is equivalent to the situation c .

		Pre Test		Post Test	
Situation	Relations	Mean	Std. Deviation	Mean	Std. Deviation
a	$x^2 + y^3 = 3$	0.0975	0.3004	0.1951	0.4012
b	$y = x^3 + x + 1$	0.5121	0.5060	0.4878	0.5060
c	$a^2 - b = 0$	0.0487	0.2180	0.0731	0.2636
d	$f(y) = e^y$	0.9512	0.2180	0.9756	0.1561
e	$x^4 = 3y$	0.1463	0.3578	0.2195	0.4190
f	$a = \sqrt{2}$	0.1219	0.3312	0.3414	0.4800
g	$f(x) = 3$	0.8536	0.3578	0.8536	0.3578
h	$y = x^2$	0.5365	0.5048	0.5609	0.5024
i	$s = 3t$	0.2195	0.4190	0.3414	0.4800

TABLE 5.2: The Difficulty Level of the Functions Represented by Symbolic Forms

The table 5.3 shows the difficulty level of the tasks given in graphical forms. The graph depicted in situation t was correctly identified as a function by 97% of the students ($\bar{X} = 0.97$, $SD = 0.15$). Situation o was the hardest task for students since only 19% of them answered it correctly in the pre-test, and 7% in the post-test. The situation r had a great change between two tests ($\bar{X}_{pre-test} = 0.39$, $SD = 9.49$, $\bar{X}_{post-test} = 0.60$, $SD = 0.49$).

		Pre Test		Post Test	
Situation	Graphs presented to students	Mean	Std. Deviation	Mean	Std. Deviation
j		0.4878	0.5060	0.6341	0.4876
k		0.1463	0.3578	0.1951	0.4012
l		0.5365	0.5048	0.6097	0.4938

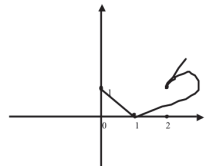
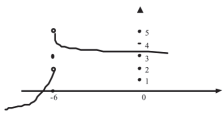
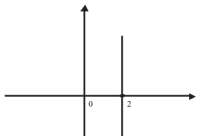
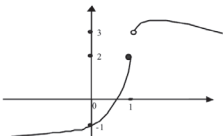
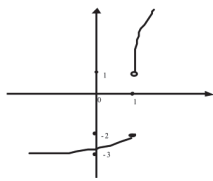
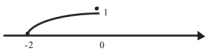
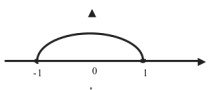
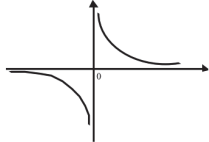
m		0.7560	0.4347	0.8536	0.3578
n		0.2682	0.4485	0.2195	0.4190
o		0.1951	0.4012	0.0731	0.2636
p		0.3170	0.4711	0.2195	0.4190
q		0.7804	0.4190	0.7317	0.4485
r		0.3902	0.4938	0.6097	0.4938
s		0.5121	0.5060	0.5365	0.5048
t		0.9756	0.1561	0.9756	0.1561

TABLE 5.3: The Difficulty Level of the Functions Represented by Graphical Forms

To verify if there was a performance difference between the two tests, in the same samples, we performed a paired T-test. The maximum score that could be obtained in the tests was 23. The figure 5.1 shows the difference between the post-test and pre-test scores. Points below the blue line indicate observations where Pre-Test is greater than Post-Test, that is where $(\text{Post-Test} - \text{Pre-Test})$ is negative.

Plot 5.2 of paired samples from a paired t-test. Circles below or to the right of the blue one-to-one line indicate observations with a higher value for Pre-Test than for

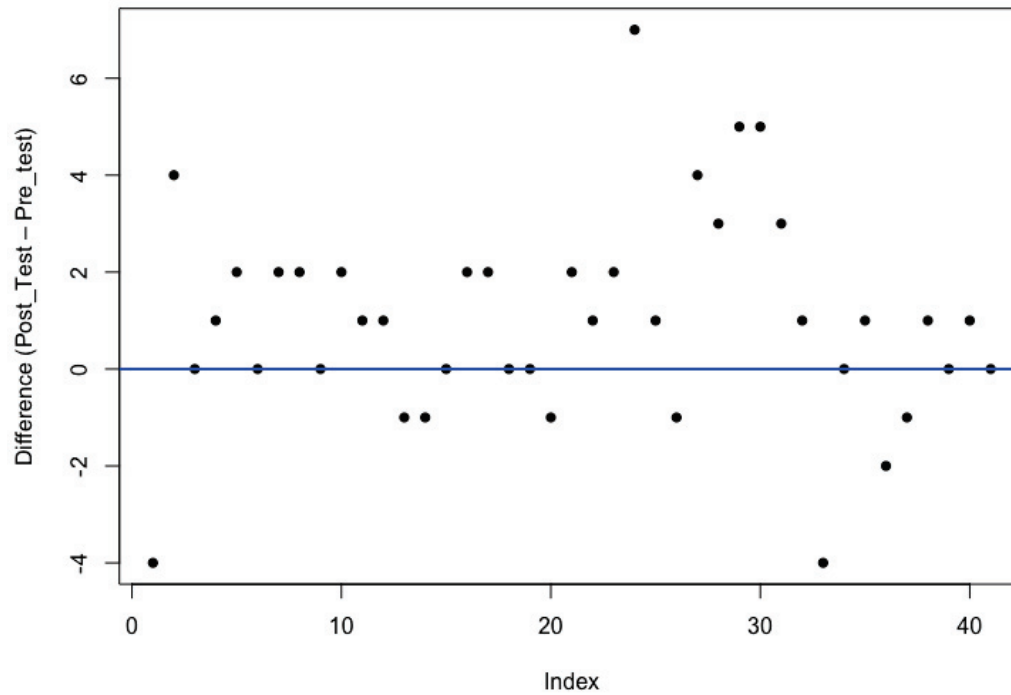


FIGURE 5.1: Difference Post-Test - Pre-Test

Post-Test.

Histogram of differences of two populations from a paired t-test (Figure 5.3). Distribution of differences should be approximately normal. Bins with negative values indicate observations with a higher value for Pre-Test than for Post-Test.

Recalling the question we wanted to answer: *Does the didactic situation using the Function Hero have any effect on the average accuracy in recognizing functions in their algebraic and graphical forms?*. To answer this question One-Sample t test was performed on the Pre-Test and Post-Test data to analyze the mean of the learning gain among both tests. There was significant difference between the mean of correctness on Pre-test and Post-test, $t(40) = 2.94$, $p = .005$. Table ?? illustrates the results of one-sample t test for each assessment.

Test	Results
1 Paired t-test:	$t(40) = 2.94$, $p = .005$, $d = 0.65$

Considering a p -value ($p = 0.05$), see Table ??, it's possible to reject the null hypothesis which says that there was no significant difference between the two measurements, Pre-test and Post-test.

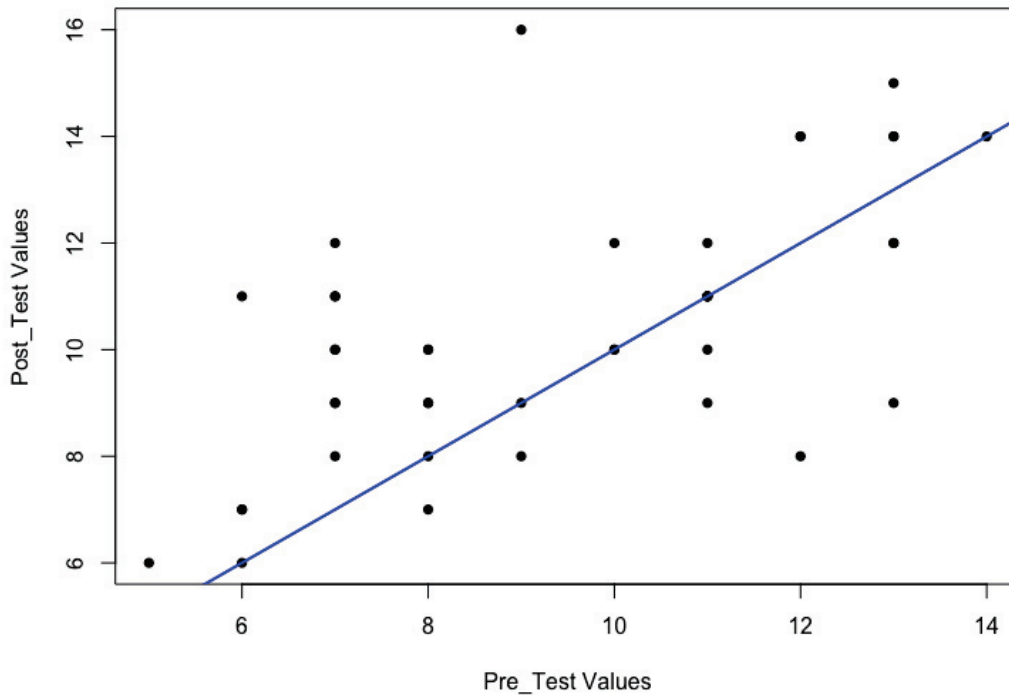


FIGURE 5.2: Pre Test Values vs Post Test Values

5.2.3 Network Analysis - Function Hero

As the fundamental elements of networks graphs are their vertices and edges, Kolaczyk and Csárdi, 2014 reinforce that there are a number of network characterizations centered upon these. In this subsection, I present the results of the network analysis of the game *Function Hero*, trying to elucidate the components of creativity related with the quantitative results.

Our procedure, following the Kolaczyk and Csárdi, 2014 approach, is broken down according to (i) those characterizations based upon vertex degrees, and (ii) those seeking to capture some more general notion of the *importance* of a vertex - typically referred to as vertex centrality measures.

In total there were 20 teams participating in the experiments and competitions with the game *Function Hero*.

We can see a very large difference in the number of vertices in the table 5.4. That's because the INSA students participated in the competition which had several moments.

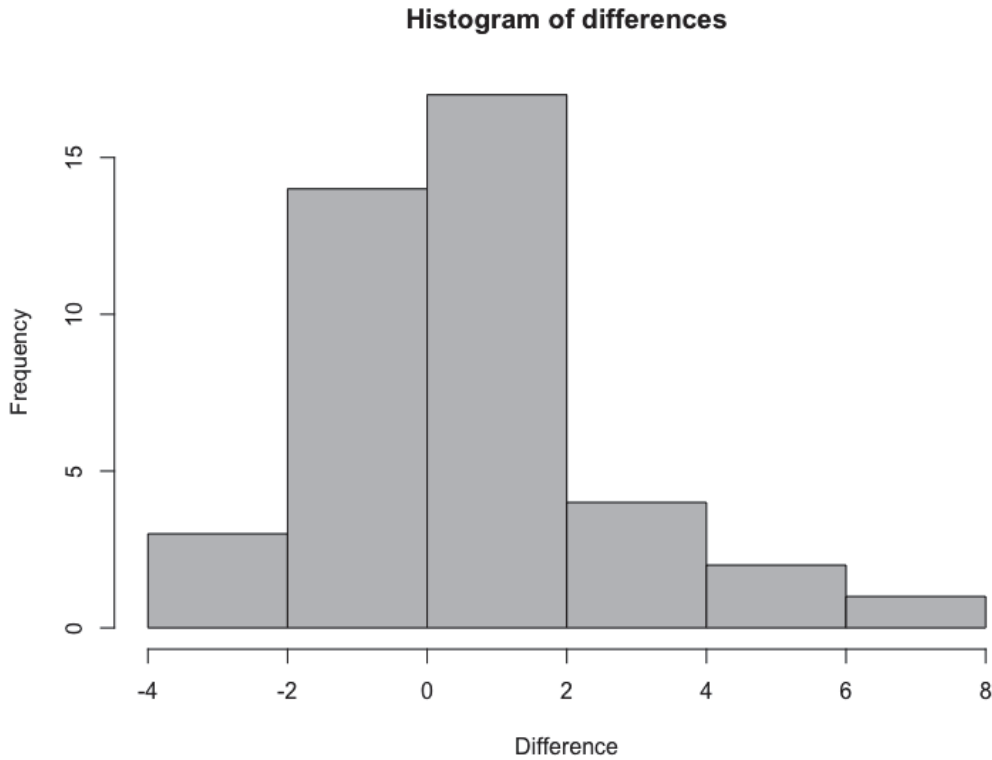


FIGURE 5.3: Histogram of differences Post-Test - Pre-Test

Vertex Degree

The degree d_v of a vertex v , in a network graph $G = (V, E)$, counts the number of edges in E incident upon v . Given the networks, from the Function Hero game, G , define f_d to be the fraction of vertices $v \in V$ with degree $d_v = d$. The collection $\{f_d\}_{d \geq 0}$ is called *degree distribution* of G . Then we can create the degree distribution for each sample as shown in the following figures.

In the figure 5.4 we can see that the highest concentration of connections is at the node x with $d_x = 9$, while the less used function is \cos , with $d_{\cos} = 1$, the second function less used is \exp , with $d_{\exp} = 2$, according to the table 5.5.

Now we give a look on how another group of students developed its choreography, as we can see in the Fig. 5.5

In the figure 5.5 we can see that the highest concentration of connection is at node $/$ with $d_{/} = 12$, and $power$ with $d_{power} = 12$, while that the nodes with the lowest concentration of connections were 6 , $\sqrt{}_t$, $-e$ \exp com $d_{6,\sqrt{}_t,-e,\exp} = 1$, according to the table 5.6.

With the graphs from the Figures 5.6, we measure the cognitive components of Creative Mathematical Thinking, i.e.: Fluency, Flexibility, Originality and Elaboration in a quantitative way, considering the following aspects for each component:

Sample	Institution	Vertex Count	Edges Count
Team 1	BTS	8	17
Team 2	BTS	7	13
Team 3	BTS	11	25
Team 4	BTS	7	13
Team 5	BTS	8	15
Team 6	BTS	8	10
Team 7	BTS	13	22
Team 8	BTS	11	18
Team 9	BTS	12	32
Team 10	BTS	9	15
Team 11	BTS	15	27
Team 12	BTS	15	31
Team 13	BTS	16	31
Team 1	INSA	14	224
Team 2	INSA	23	553
Team 3	INSA	29	177
Team 4	INSA	17	98
Team 5	INSA	19	176
Team 6	INSA	15	46
Team 7	INSA	16	74

TABLE 5.4: Description of Graph Networks of the Samples

node id	label	type	degree	normalized
n3	x	function	9	1.000
n16	$power$	operator	8	0.8888
n8	exp	function	2	0.2222
n13	$-$	operator	2	0.2222
n21	2	value	2	0.2222
n22	3	value	2	0.2222
n5	cos	function	1	0.1111

TABLE 5.5: BTS Team 1 - Choreography Vertex Degree

- Fluency: The highest relative frequency of degrees of a node of the type other than x .
- Flexibility: The number of function types used for the same choreography.
- Originality: The lowest relative frequency of degrees of a node value.
- Elaboration: The greatest amount of edges of a step in the choreography.

Our question then, from these two sets of data, is: *which of the two teams was the most creative? I.E., Which of the two products is the most innovative?*

Considering the components Creativity, we can say that the second choreography, see Fig. 5.6 right, is the most creative in terms of:

- Fluency: The INSA team uses the functions \sin and \cos two times (degree = 2) while the BTS team uses \cos one time (degree = 1).

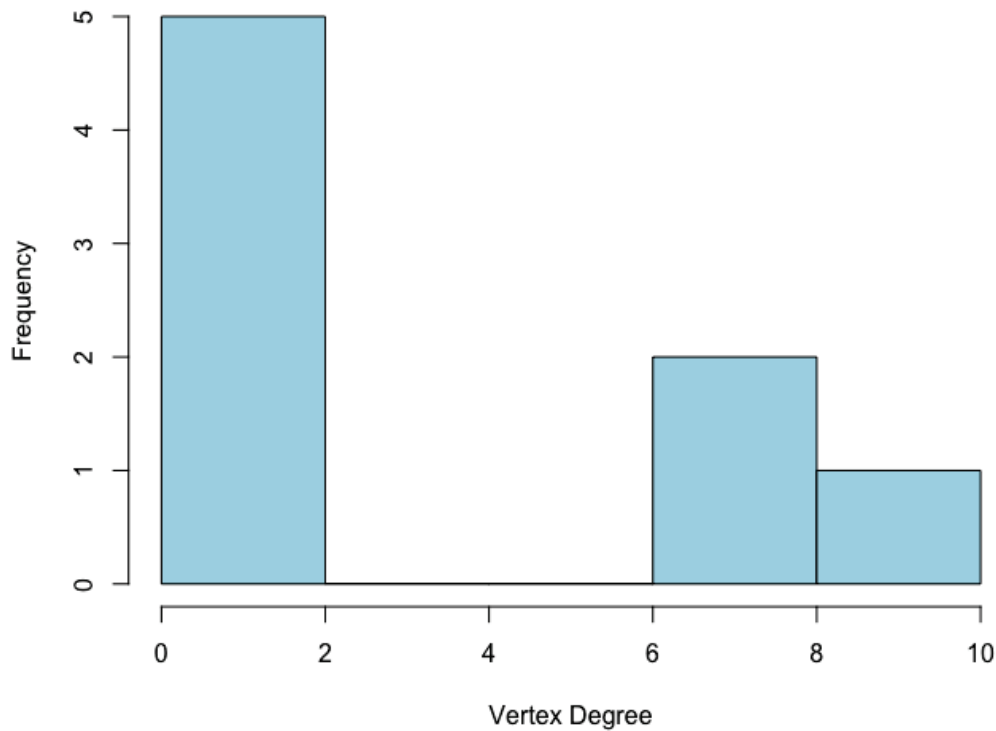


FIGURE 5.4: Vertex Degree - Histogram - BTS - Team 1

- Flexibility: The INSA team used six types of functions (*sin*, *cos*, *abs*, *exp*, *x*, *sqrt*) while BTS uses three types (*cos*, *exp*, *x*).
- Originality: As Originality must be compared, INSA used more possible nodes to develop their choreographies. INSA team used 15 nodes while BTS used 8, all the nodes in INSA choreography was used by the BTS team.
- Elaboration: As the Elaboration is measured by the edges of the network, INSA elaborated more since the amount of Edges(INSA) = 33, while Edges(BTS) = 17.

5.3 Binary Code

Very different Study Research Paths shown by different produced graphs, building on different knowledge and different creativity skills that were not assessed. Using the taxonomy of Weintrop et al., 2016a, seen at the figure 2.10 and Chevallard, 2009, we created labels to code the solution process, they are: (1) Data Practices, (2) Modeling and Simulation Practices, (3) Computational Problem Solving Practices, (4) System Thinking Practices. To remember the trick, the cards are as follows in the Figure 5.7:

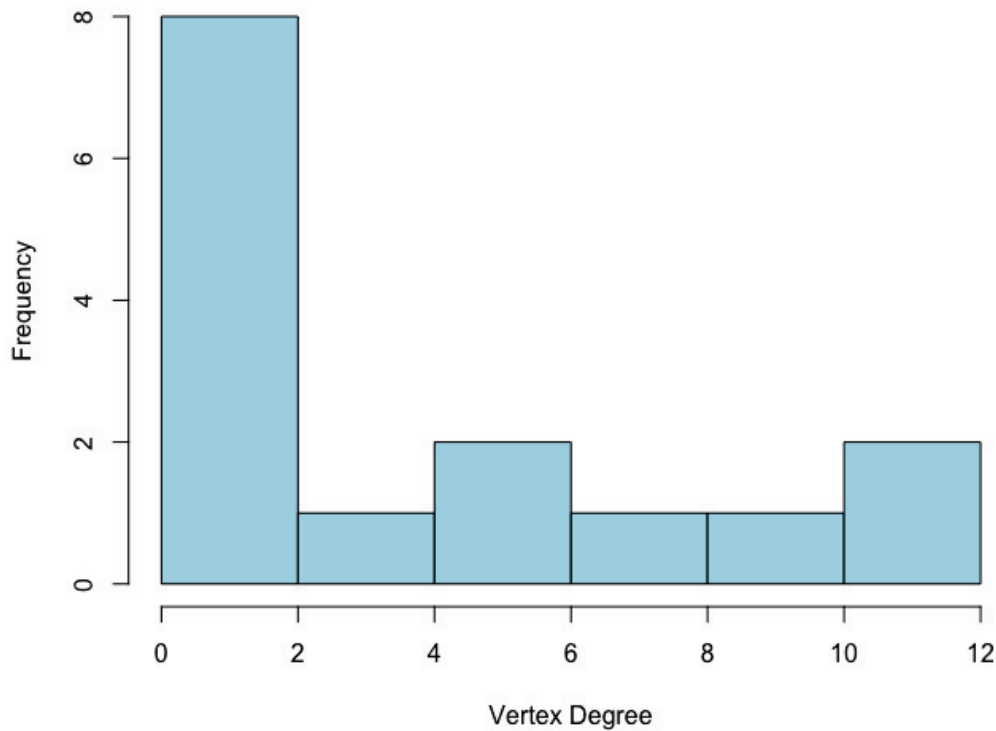


FIGURE 5.5: Vertex Degree - Histogram - INSA - Team 1

5.3.1 Network Analysis - Binary Code

As Kolaczyk and Csárdi, 2014 says, many questions that might be asked about a vertex in a network graph essentially seek to understand its 'importance' in the network. Measures of *centrality* are designed to quantify such notions of 'importance' and thereby facilitate the answering of such questions.

We can see in the Fig. 5.8 that we have a degree distribution in which more than 40 nodes have 0 or 1 connections, while 5 nodes have 14 or more. To get easier to see the network, the Fig. 5.9 shows the questions and answers split into categories. The blue nodes are questions raised by the problem or the students, these questions are not part of the set of a priori nodes. The yellow nodes correspond to the Weintrop et al., 2016a taxonomy of computational thinking. The red nodes correspond to the answers given by the solver.

The figure 5.9 shows all the nodes from all the samples, i.e. (Priori, Teacher, Students). As we are studying the process of the solution and trying to observe some steps of Diamond's Creativity, we take the student's network from the graph shown in the figure 5.9.

node id	label	type	degree	normalized
n15	/	operator	12	1.000
n16	<i>power</i>	operator	12	1.000
n3	x	function	10	0.8333
n12	*	operator	8	0.6666
n18	1	value	5	0.4166
n21	2	value	5	0.4166
n14	+	operator	3	0.2500
n4	\sin	function	2	0.1666
n5	\cos	function	2	0.1666
n22	3	value	2	0.1666
n7	abs	function	1	0.0833
n8	\exp	function	1	0.0833
n13	-	operator	1	0.0833
n17	$\sqrt{}$	function	1	0.0833
n35	6	value	1	0.0833

TABLE 5.6: INSA Team 1 - Choreography Vertex Degree

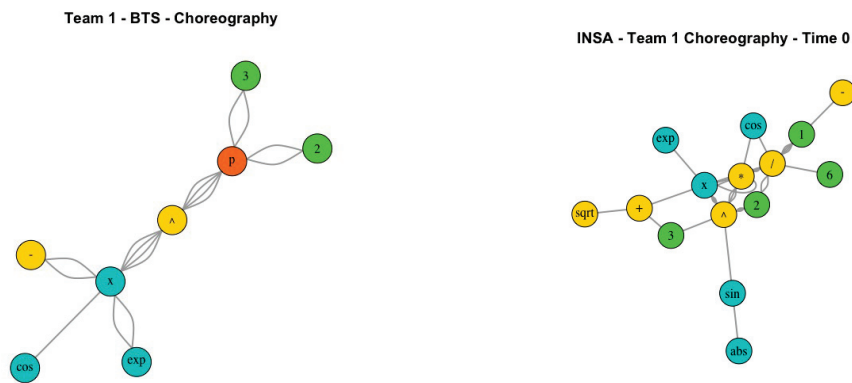


FIGURE 5.6: Choreographies BTS left and INSA right

From this graph (see. Picture 5.9) we would like to know, quantitatively, if there is a node more important than others. There are different network measures which can help us to analyze a network. We use four of them: (a) Degree (b) Closeness (c) Betweenness and (d) Eigenvalues.

Taking into account the *Degree Centrality* we make evident the most important node from it's connection amount. In the figure 5.11 we have the node n_{21} , which can be translated as a Question of the taxonomy: *STP.Q4: How can we translate the solution into an entire system?*. The second node more important is the n_4 , which can be translated as a Question of the taxonomy: *DP.Q4: Is there any conclusion to do about the observed data?*. The answer with more connection is the n_{24} , which can be translated as: *AD2: The Data from the cards*.

Taking into account the *Betweenness Centrality* we make evident the most important

3	7	23	19	14	26	10	27	6	5	20	14	14	10	15	31	17	23	19	25
17	9	11	31	3	30	15	18	28	23	15	13	11	27	29	13	27	30	29	18
5	27	21	13	31	6	19	23	31	29	21	22	26	28	9	12	22	21	26	24
1	25	29	15	2	7	11	22	4	30	7	12	8	30	25	24	16	28	31	20

FIGURE 5.7: Cards of Binary Trick

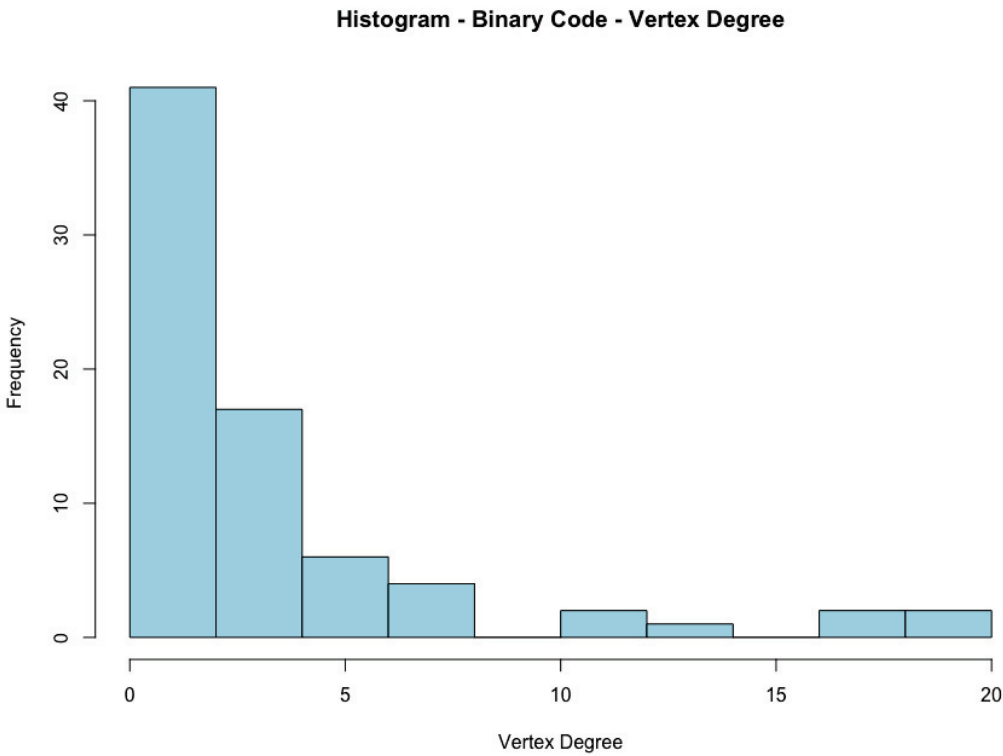


FIGURE 5.8: Histogram - Vertex Degree - Binary Code

node in terms of how it connects other ones. In the figure 5.12, we have the node $n4$, which can be translated as a Question of the taxonomy: *DP.Q4: Is there any conclusion to do about the observed data?*.

5.4 Fake Binary Code

In the same approach as the situation *Binary Code* the solutions from students were coded and transformed into network graphs in order to understand the topology of these solutions in terms of Creativity.

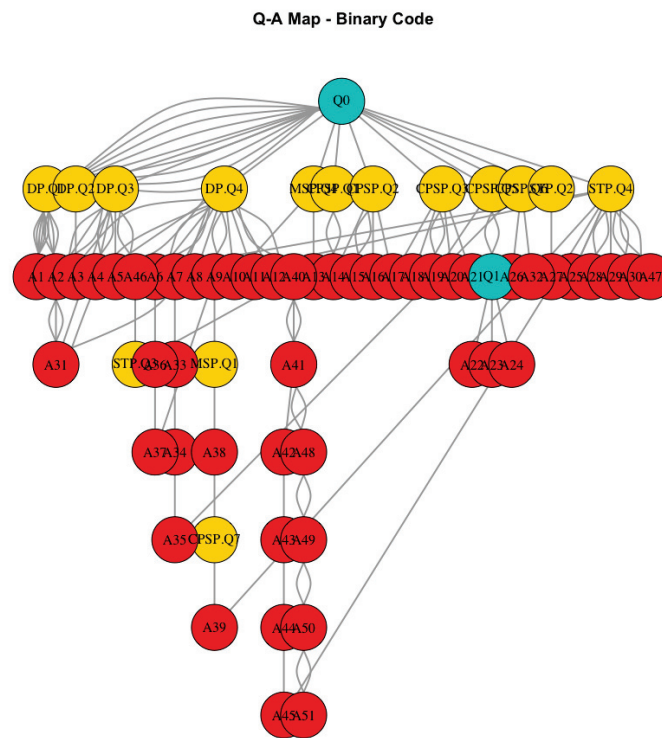


FIGURE 5.9: Q-A Map of Binary Code Trick

5.4.1 Network Analysis - Fake Binary Code

From the graph shown in the figure 5.13 we see a tree structure beginning at the $Q0$ node, which represents the initial question of the problem. As we can see there are some different levels of resolution if we take into account the yellow nodes, representing the questions from the initial framework.

Taking into account the *Degree Centrality* we make evident the most important node from it's connection amount. In the figure 5.14 we have the node $n4$, which can be translated as a Question of the taxonomy: *DP.Q4: Is there any conclusion to make about the observed data?.* The second node more important is the $n37$, which can be translated as an Answer of the taxonomy: *A15: Putting The lines side by side and comparing the Binary Decomposition with the number..*

Taking into account the *Betweenness Centrality* we make evident the most important node in terms of how it connects the other nodes. In the figure 5.15 we have again the node $n4$, which can be translated as a Question of the taxonomy: *DP.Q4: Is there any conclusion to make about the observed data?.* The second node more important is the $n7$, which can be translated as a Question of the taxonomy: *MSP.Q2: Do you know any computational model to find or test the solutions you provided?.*

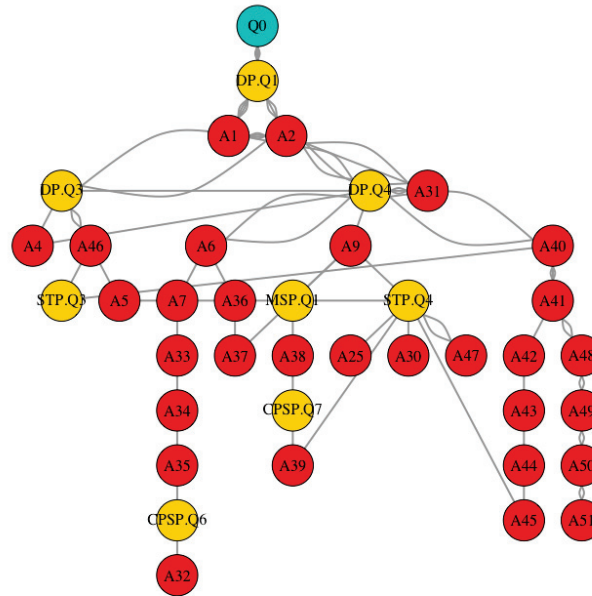


FIGURE 5.10: Q-A Map of Binary Code Trick - Students

5.5 Op Art

In this section we present the results of a study we have done (Mercat and Lealdino, 2017), having as goal to observe the creativity on algorithms development to describe pieces of art.

Experiments were short (less than 4 hours) and not with an audience of the target group of primary and secondary students, rather engineering students, teachers and teacher trainers, but some conclusions can nevertheless be drawn, from the software produced and collected and from the oral exchanges which are only summarized here. First, although technology is lurking behind the scene and does promote advanced creative mathematical thinking, a basic unplugged version is doing the job of fostering creative mathematical thinking and might be preferable as an introduction: Paper printed versions were preferred even though the electronic versions allowed for electronic color picking or precise measurements. This is the sign of the conceptual nature of the pieces: something begs to be understood in these pictures and to be faithful to the actual artwork is not the challenge. We present here Julio Le Parc artworks Ball, Composition, Sectors, the Railway illusion, the Coffee Bean illusion which are examples of what was done on other pieces, climbing from confusion to subjective descriptions and finally to formalized algorithm.

The exercises led to more and more precise terms and algorithms when the protocol

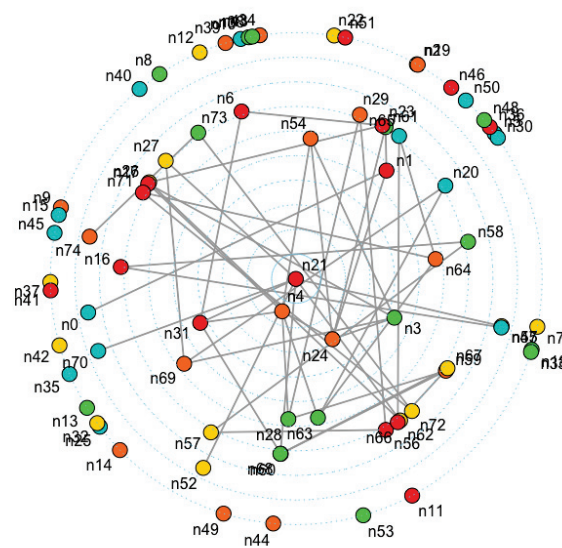


FIGURE 5.11: Binary Code - Students - Vertex Centrality - Degree

was followed, but many took the instructions as a student's guide to be followed by the students and not by them, refusing the homology of the situation, avoiding the several steps scaffolding.

In the Railway illusion (Fig. 5.16 left), the illustrated implementation uses the spreadsheet containing geometric formulae, using translation and symmetry. It has been scaffolded on an example, not related to the Railway but using the geometric spreadsheet, that has to be modified and generalized in order to reach the final goal of reproducing the visual effect. Meanwhile, the users got the crucial addition, with respect to a static reproduction, of an interactive exploration of the space of configurations (density, size and angle of lines and segments). Technically, each individual segment can be identified in the spreadsheet, which is not the case of the Coffee Bean illusion (Fig. 5.16 right) implementation, which was based on a rather sophisticated double loop of a translated and rotated base image. There are only four objects in this mesmerizing figure, two points, corners of an image, and this large list, it is a complex and abstract "one-liner" solution. In a more "concrete" way, using the spreadsheet for explicitly unfolding a loop at each step, being able to understand the relationships between the objects, is a good pedagogical idea in building the notion of loop and sequence (Haspekian and Bruillard, 2007) that can be scaffolded on 1-loop and then 2-loop examples leading to this Coffee Bean type of nested loops associated with a Cartesian product configuration space.

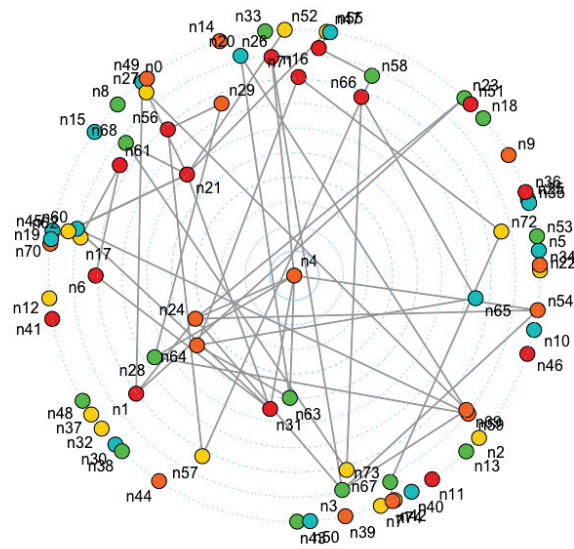


FIGURE 5.12: Binary Code - Students - Vertex Centrality - Betweenness

In the Julio Le Parc Sector artwork (Fig. 5.17) for an implementation in Scratch), the construct drawing an angular arc of a given angle being already defined, the user is supposed to use it in two nested loops in order to draw grey and white sectors. Their angles increase linearly in each line y and each column x , and at each point (x,y) add to a flat angle, therefore can be modeled by $\pi/2(1 \pm xy)$.

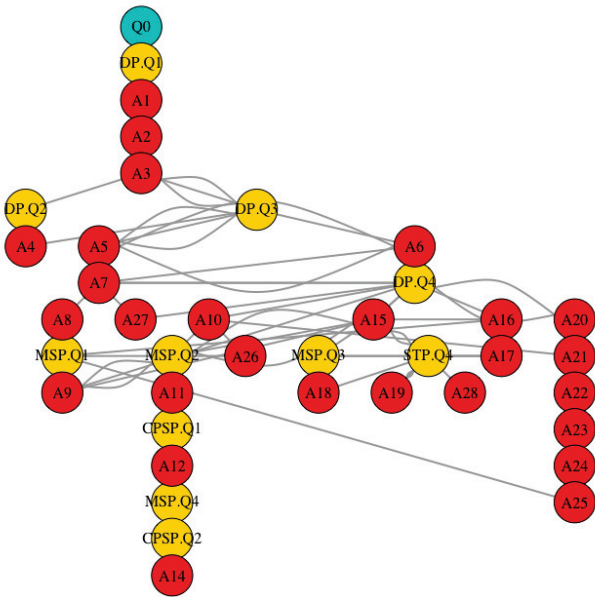


FIGURE 5.13: Q-A Map - Fake Binary Code

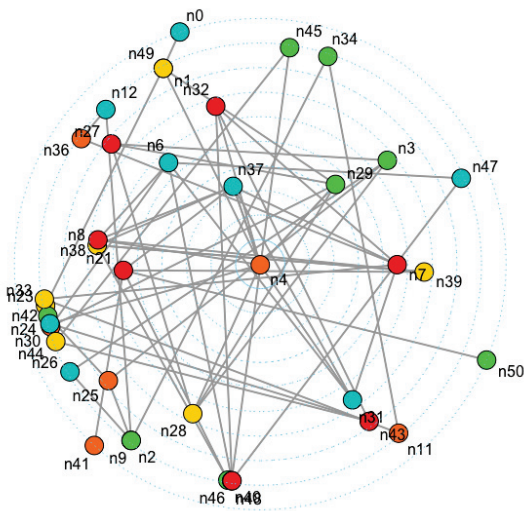


FIGURE 5.14: Fake Binary Code - Students - Vertex Centrality - Degree

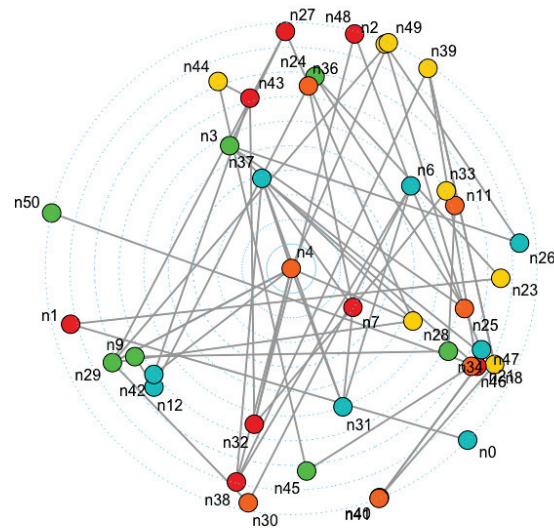


FIGURE 5.15: Fake Binary Code - Students - Vertex Centrality - Betweenness

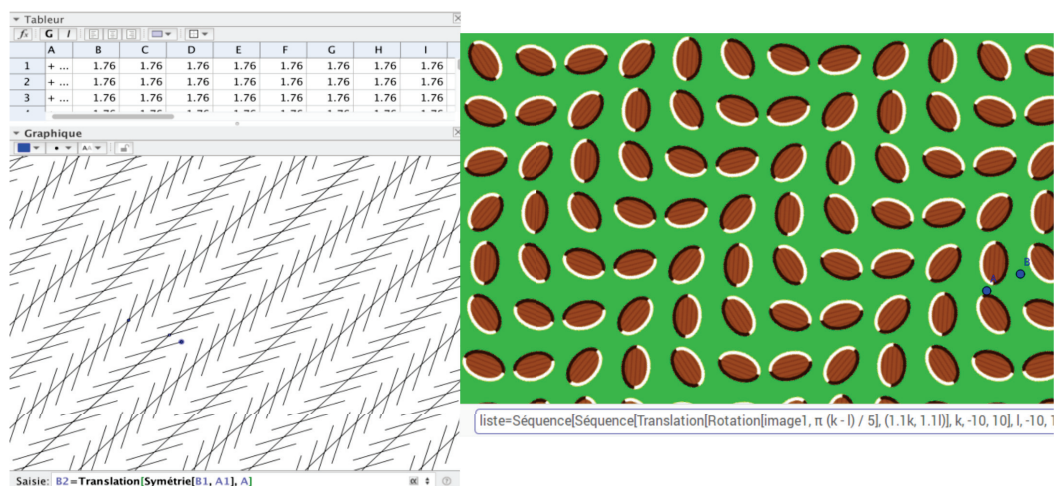


FIGURE 5.16: The Railway respectively Coffee Bean illusions based on a scaffolding of the use of the geometric spreadsheet (each cell contains a segment) resp. of sequences (loops) in Geogebra producing a large list.

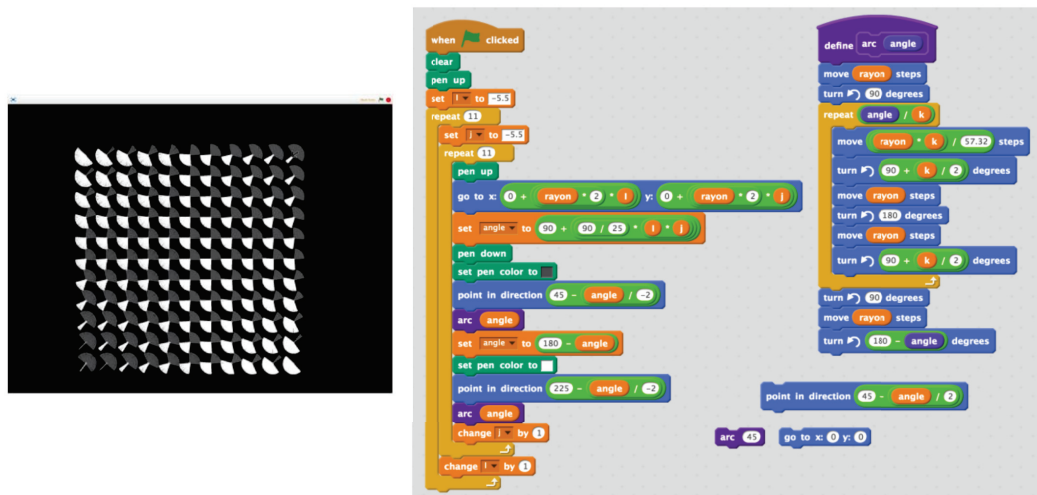


FIGURE 5.17: Given the angle construct, Julio Parc's Sector can be modeled with two nested loops.

Chapter 6

Discussion and Recommendations

6.1 Introduction

In the preceding chapter, the presentation and analysis of data have been reported. The Chapter 6 consists of a summary of the study, discussion of the finding, implications for practice, recommendations for further research, and conclusions. The purpose of the latter sections is to expand upon the concepts that were studied in an effort to provide a further understanding of Creative Mathematical Thinking and its development making use of different didactic situations. Finally, a synthesizing statement is offered to capture the substance and scope of what has been attempted in this research.

6.2 Summary of the Study

This chapter begins with a summary of the purpose and structure of this research and is followed by the most relevant discoveries related to the promotion of creative mathematical thinking from didactic situations based on games and problem-solving. Conclusions on the findings of this study are discussed about the definition, operationalization, and evaluation of Creative Mathematical Thinking. Finally, the implications on the practice of teachers or people who want to use them, and recommendations for future work are presented and discussed.

The purpose of this study was to develop an analytical tool for Creative Mathematical Thinking, analyzing the problem-solving process of computational and mathematical modeling, exemplified in particular on the analysis of so-called choreographies for a video game called Function Hero and algorithm production. We also had a purpose of defining structural bases of design for didactic situations and digital artifacts, that can contribute and scaffold the development of Creativity.

Creative Mathematical Thinking is a concept related to essential and vital skills for individuals of the 21st century. We took part in further developing it during the term of research projects that involved the development and use of technologies in math

courses in different phases and levels of education (MC Squared, MetaMath, Math-GeAr). The methodological tool we use here was created for this study and tested in situations involving teachers, educators, researchers, and students, engaged in the participation of workshops on Creativity in Mathematics, Modeling and Computational Thinking.

Participants were asked to perform different tasks for each particular didactic situation. In the game, Function Hero, the participants had to create choreography steps, in forms of function graphs, for other teams to mimic them. In activities, Binary Code and Fake Binary Code, participants had to develop an algorithm to perform a magical trick. In the Op'Art activity, participants had to develop an algorithm for another participant to execute, intending to reproduce a given original work of art. Besides, in the Function Hero activity, participants responded to a function recognition test before and after the activity so that we could verify that the game also contributed to the recognition performance of functions in their graphical and algebraic forms.

The study included, from all the scope of this thesis, a total of $n = 1410$ subjects (958 from MathGeAr and MetaMath projects, coming from Russia, Finland, France, Armenia and Georgia. 54 from Tunisia, 256 from France and 140 from Brazil), selected by convenience, they were students or participants in the workshops taught in these countries, or students of the Universities belonging to the European projects. However, for the didactical situations in which we analyzed creativity, only a few solutions made by the participants were analyzed, since the data collection didn't allow us to get all the algorithms produced by the students. This study included the research questions:

Qualitative

1. RQ1: How can we operationalize Creative Mathematical Thinking?
2. RQ2: How can we assess the progress of a process involving Creative Mathematical Thinking? We propose to objectify this development through an observing tool that we called the *Diamond of Creativity*, here described in Chapter 2.
3. RQ3: How the "Diamond of Creativity" model is an useful analytic tool to map the Creative Process path?

To answer the three questions to the qualitative phase of the study, data were obtained from the algorithms developed during the activities and the observation of the experiments with the game Function Hero. The data were categorized, coded, and transformed into Network Graphs. The qualitative research addressed the complexity of the data using the Diamond of Creativity, validating the quantitative research and its conclusions.

Quantitative

1. RQ4: To what extent can we validate a learning gain in function recognition tests after Function Hero activity?
2. RQ5: What is the variability of the choreographies of the students in the Function Hero game?

Question four was answered quantitatively from the data obtained from participant scores on the pre-test and post-test on function recognitions. The question was answered using the results from a *t* test for dependent means comparing means of the same group over two measurements, before and after playing the game Function Hero.

Question five was answered quantitatively from the data obtained from participants choreographies on the Function Hero game, codified as a graph. The question was answered adapting the global score of social creativity (Essonner, 2018) comparing the components of divergent thinking (Guilford, 1950) in terms of *Fluency, Flexibility, Elaboration and Originality*, based on the topology of the network graphs produced from the datalog of the game.

6.3 Discussion and Findings

Previous researchers (El-Demerdash and Kortenkamp, 2010b; Essonner, 2018; Lev-Zamir and Leikin, 2011a; Sriraman, 2009a; Eryvynck, 2002; Haylock, 1987) studied how to assess creativity in a mathematics education context, whether using software, or developing artifacts to scaffold creativity, whether developing theoretical approaches to understand the phenomena of being creative in mathematics. The goal of our study was to conceive games and activities to afford Creative Mathematical Thinking. Also, to conceive a methodology to observe and assess Creative Mathematical Thinking in two forms: Process and Product, on inquiry-based and game-based learning activities. This section discusses the implications of the findings for each of the five research questions.

6.3.1 Research Question One: How can we operationalize Creative Mathematical Thinking?

The revision of the existent literature on Creative Mathematical Thinking was established within the context of the MC Squared project. A team of researchers participant of the project helped to develop the main theoretical framework to assess creativity in a social context when creating the c-book units.

To this thesis in particular, we make use of the network strategies (Prediger, Bikner-Ahsbahs, and Arzarello, 2008) to create the Diamond of Creativity. This model of

the creative process, as said Essonnier, 2018, allows us to distinguish the cycle of creating something into the main episodes, which can be illustrated in Fig. 2.1:

- **Divergent Thinking:** where the individuals start looking for already known knowledge in order to solve sub-problems related to the big one. In this phase, also the incubation happens if the individual doesn't achieve the solution to the problem to be solved.
- **Aha Moment:** where the individual have the illumination on how to solve the problem. The time of the incubation phase is not an observable variable in this thesis.
- **Convergent Thinking:** when the individuals translate their ideas into a common language, shared with a community, which will validate the production, in our case it could be a teacher, to publish the creation and insert this new idea into the knowledge space to be reused in other situations.

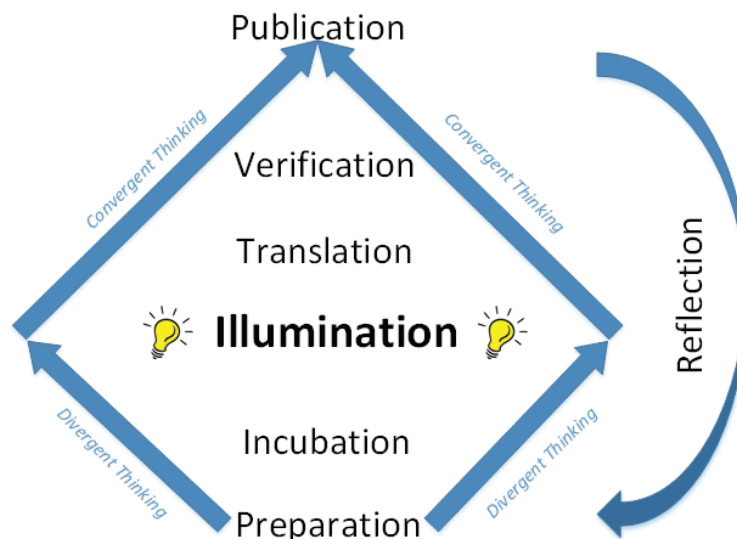


FIGURE 6.1: Diamond of Creativity

6.3.2 Research Question Two: How can we assess the progress of a process involving Creative Mathematical Thinking?

Considering Creative Mathematical Thinking as a process in which new products are created to change a conceptual space dependent on the community level in which the individual belongs, after the person is involved in a problem solving or design activity, we can classify and to categorize the user's attitudes by creating an initial product space (i.e. Questions, Answers, Concepts) and to use the Creativity Diamond to elucidate the events of a new product creation.

For instance, in the Binary Code and Fake Binary Code activities, the solution spaces were transformed into graphs and observing their form and the centrality measures

of each node, we identify at what stage of problem-solving the individual encounters. On the other hand, in the Function Hero activity, the solution space was defined as all the functions and choreographies which students created to play the game.

Using statistical analysis techniques of network graphs, we can identify the importance of each node, representing possible steps and answers in solving a problem, measuring the frequency of its use and how it can become an essential episode in solving a problem. With the descriptive statistics of a network, we can give value to the components: Fluency, Flexibility, Elaboration, and Originality.

6.3.3 Research Question Three: How the "Diamond of Creativity" model is an useful analytic tool to map the Creative Process path?

This question can be answered qualitatively from the analysis of the solutions carried out by the students. By decomposing the solutions into nuclear structures, with each node signifying a practice within the computational thinking taxonomy, we were able to identify the stages of resolution within the framework of the Diamond of Creativity.

In some cases, such as when students describe: "After thinking hard," or "after testing for many cases," we have been able to evidence practices of mathematical and computational thinking within a structure that first diverges, starting with the initial question and then converge, ending in a product that can be measured and compared against others.

Recalling the example of the modeling of a variation on Ball by Julio Le Parc (Fig. 6.2), we can describe the different levels of descriptions that went into different generic stages, following the diamond of creativity model (See Fig. 2.1) and the cycle of modeling: "I don't understand the question, can you please repeat, I don't see anything" (preparation), "I see a large square with a yellow cross in it" (elaboration on the preparation - collecting data), "there are colours, disks and squares" (identifications of the relevant variables - describing the data), "the disks are on the squares, they are both periodically coloured and arranged in a table" (identification of preliminary relations between the variables), "the colours of the disks are always the same on a diagonal, likewise the colours of the squares along another diagonal" (elaboration on the identification of relations), some silent or vocal incubation, until the Ahah! Moment (Koestler, 1964) where the relationship between the variables colour/shape/position springs to mind : "the colour of a disk depends on the sum of its coordinates, the colour of a square on their difference" and the user can now try to translate, explain this crucial relationship and implement this algorithm using whether coloured paper clips or a software of her choice. In some cases, amounting to 11 collective productions on diverse proposals, students went as far as programming these algorithms (see Fig. 5.16, 5.17).

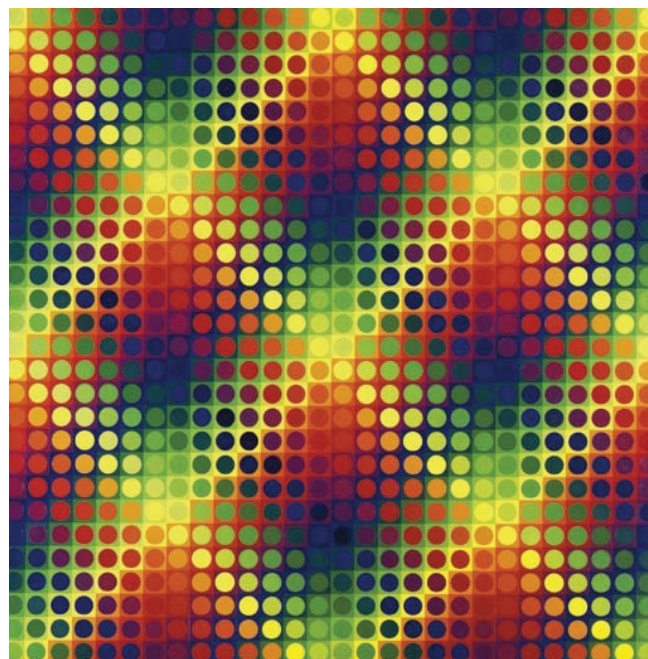


FIGURE 6.2: Variation on Ball by Julio le Parc

6.3.4 Research Question Four: To what extent can we validate a learning gain in function recognition tests after Function Hero activity?

The findings from this first research question showed that there is a significant difference between the means of success in the tests of recognition of functions before the practice and after the practice of the game. This finding is related to the participants of the BTS group since the other groups that participated in the game did not perform the tests to measure the score.

However, the majority of students who participated in the interventions with the game Function Hero engaged in the activity and participated in creating and testing their choreography. Given this, it is essential to have alternatives so that these creative potentialities are put into practice to motivate them to use the knowledge they already have and also to increase it.

6.3.5 Research Question Five: What is the variability of the choreographies of the students in the Function Hero game?

The students began to use their hand-held calculator in order to graph functions. The most bizarre were tried, for example $\sin(x^2)$. But actually trying to get a high score on these deterred them from using them in the choreography. They fell back to simpler functions and whereas the two groups were subjected to different warm-up functions (circular for one, polynomial for the other), some of their findings were common. During the practice, they nicknamed some of the functions, for example

“sinus, that’s the Egyptian” for the sine function because they had to turn sideways in order to bend their arms in the right concavity, or “happy” for x^2 , or “sad” for $-x^2$.

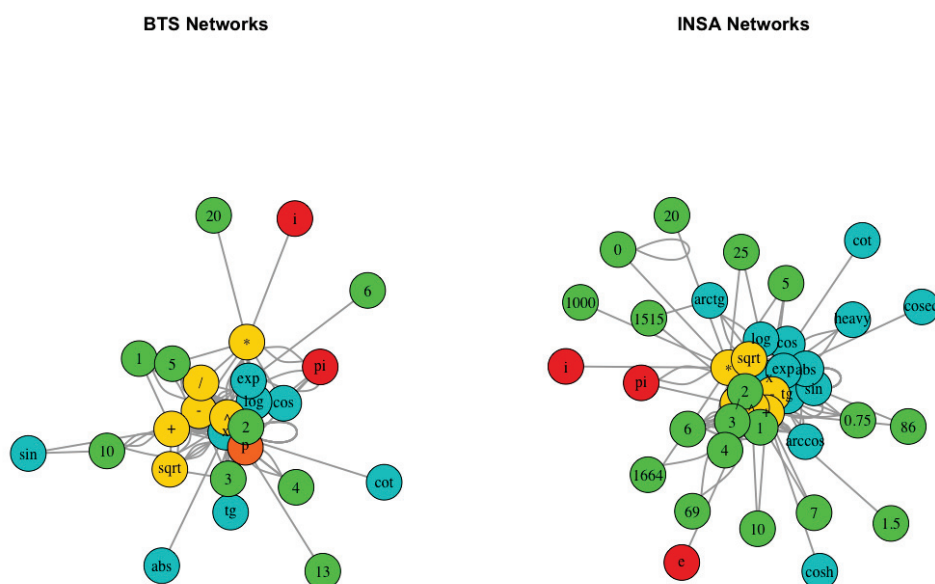


FIGURE 6.3: Graph Networks from samples of the Function Hero treatment

Observing the figure 6.3 from the students’ answers of the BTS and INSA samples, we can verify a greater distribution in the use of the elements that are part of the vocabulary of functions in the network of the right. However, once we measure the productions in terms of Fluency, Flexibility, Elaboration and Originality, we can compare them in a more precise way.

In Figure 6.4, the graphs show the centrality of each node of the function vocabularies. In the first column, we show the degrees of each node, in which we can compare the fluency of each team by the frequency of use of each element. We observed that the BTS team is more fluent in using the elements: n3: x , n22: 3, n30: π , n31: 13, n32: 10, n33: 5.

However, this does not mean that the INSA team is more fluent in the other nodes. It would be interesting if we could see a difference between nodes that are classified as functions, for example: n4: $\sin(x)$, n5: $\cos(x)$, n6: $\log(x)$, but after the normalization of graphs, these nodes have the same degree distribution.

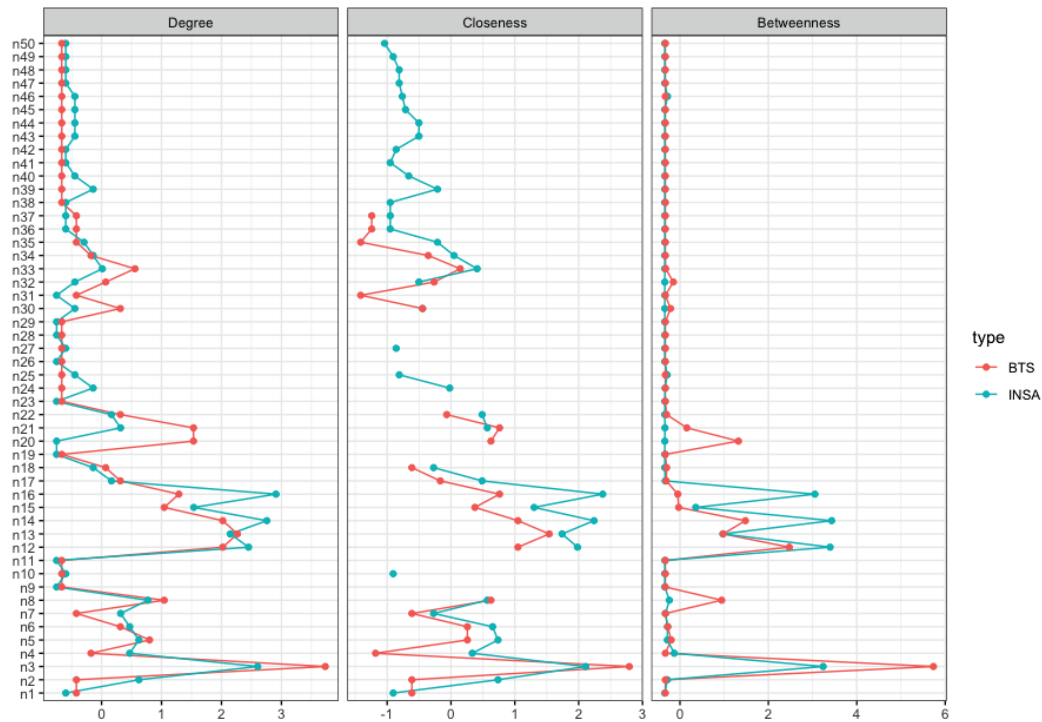


FIGURE 6.4: Networks Centralities - BTS and INSA

6.4 Implications for Practice

The new curriculum format where the disciplines of computer science are coming in several countries. In France, this is already a reality since 2016 when they implemented the teaching of algorithms within the mathematics discipline. In Brazil, the steps are still timid. However, events, such as the Brazilian Congress of Informatics in Education, are bringing the discussion of the teaching of computational thinking to the fore and allow the discussion of exciting topics, offering workshops, lectures, courses and contributes, such as articles and magazines.

Like Creativity, the development of skills in computer science is also of importance to the citizens of this new century, and from the following centuries as well. Therefore, this research sought to contribute to the advancement of the treatment of mathematics teaching, as well as the teaching of computational thinking, in order to provide methodological tools to analyze the results of didactic situations in which students engage in a research proposal.

The findings of my research also provide teachers with evidence that using alternative methodologies such as games can evolve student knowledge. Moreover, the methodological tool can assist them in observing the paths taken to solve problems, showing which are the essential practices for solving a problem. Thus, teachers and educators can point the events during the inquiry activity.

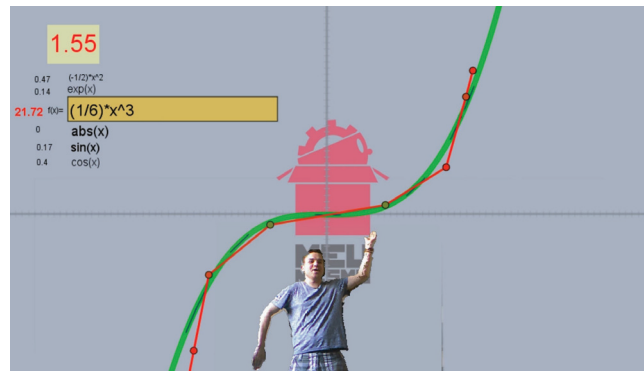


FIGURE 6.5: Function Hero - Example

Brousseau, 2008 and his followers advocate a teaching approach that plays with the didactic variables for the building of knowledge. Since a didactic situation occurs when: "one of the subjects demonstrates the intention to modify the knowledge system of the other (the means of decision, the vocabulary, the forms of argumentation, the cultural references)".

On the developer side of the artifacts, the choices regarding its structure can be classified as didactic variables, since changing certain aspects of the resource, new problem solving strategies can emerge. It is the role of the designer to think about the possible strategies to move from one level in the game to another, to ensure that the activity is both challenging and interesting.

6.4.1 Didactic Variables at Function Hero

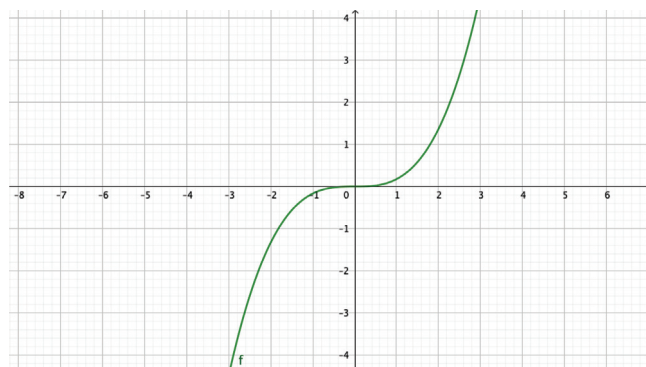
The didactic variables in the Function Hero game appear in two distinct moments, in their development as software and in their implementation as an artifact within a didactic situation in the classroom. As we saw in chapter 2, we can define a didactic situation as the "means" used by the teacher so that the student experiences the necessary experiences to develop skills and abilities, making learning meaningful.

When we developed the game, Function Hero, we firstly aimed to improve the ability to recognize functions graph by its algebraic form, keeping in mind the use of body data capture technologies: Kinect, within a game category software. Nevertheless, another purpose was to measure students' creativity in gameplay. So we decided that the game would not be static, like most electronic games, and the game could be modified according to the interests of teachers and students.

Let's remember how the game works:

In the yellow rectangle at the Picture 6.5, there is a function in its algebraic form: $\frac{x^3}{6}$

It's graphical representation is shown on the Picture 6.6

FIGURE 6.6: $\frac{x^3}{6}$

The goal of the player is to look at the algebraic form within the yellow rectangle within a given time, 5 seconds, and perform with his body the graph of the function. In the figure 6.5 of the game, we see the answer curve in green. However, this curve does not appear when the player is playing before a time interval, also predefined.

The faster the player performs the function graph with his body, the higher the score on his score. After 5 seconds, another function appears within the rectangle, the player must change the movement according to the new function, and so on until the cycle ends. Thus, we can define our didactic variables within the two approaches:

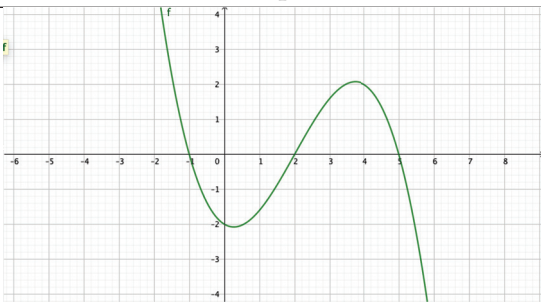
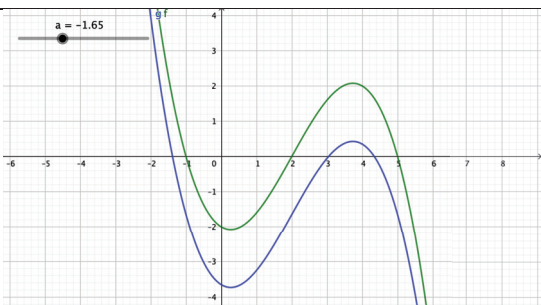
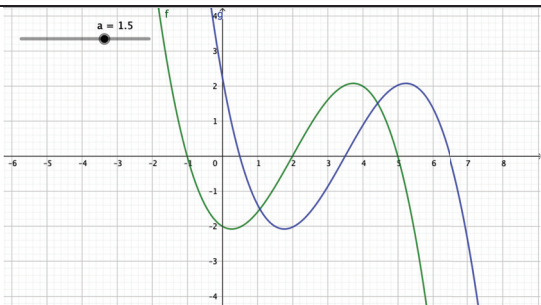
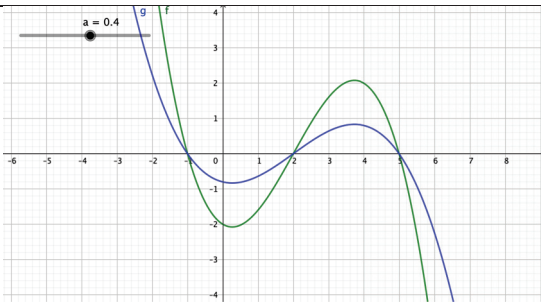
Activity	Goal	Didactical Variable
Function Hero (Artifact)	Recognizing graphs of functions from its algebraic expressions	Types of Functions
Dance like a Function	Creating Choreographies to other teams	Type of Functions and Transformation of Functions

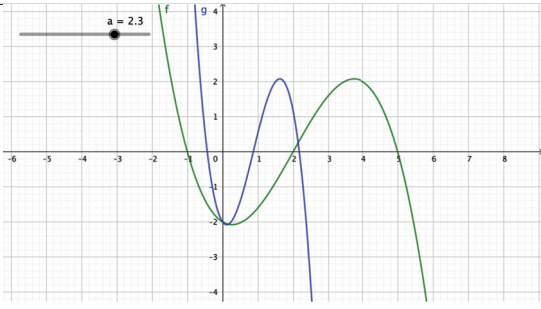
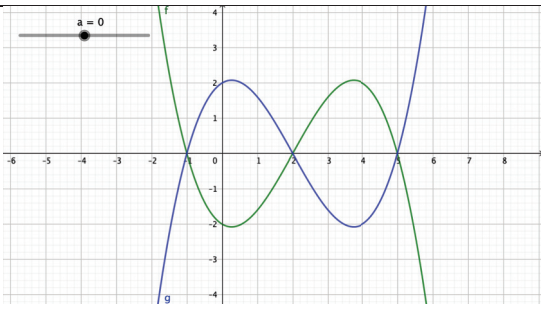
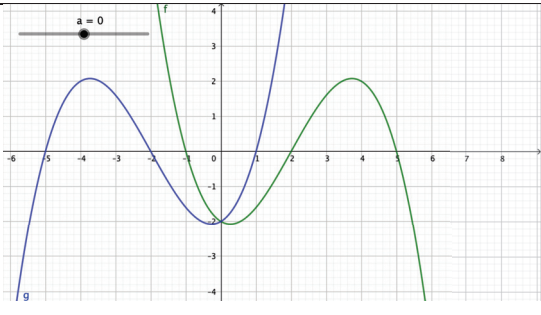
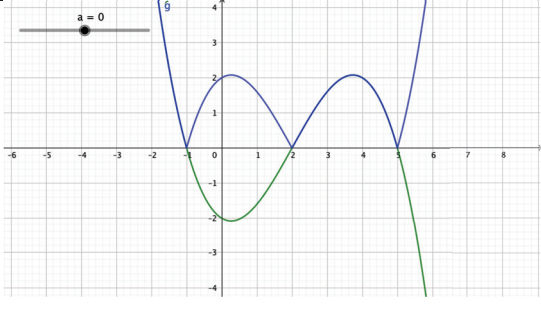
TABLE 6.1: Didactical Variables of Function Hero Game

For the "base" game, where the functions that players were supposed to perform were pre-established, several functions were e: Constant function, Even functions, Odd functions, First-degree related or polynomial function, Linear function, Increasing function, Decreasing function, Quadratic or polynomial function of the second degree, Modular function, Exponential function, Logarithmic function, Trigonometric functions, Root function.

For the battle of functions, the objective was to understand how the students transform functions that already knew, in order to develop their choreographies. When we study the transformations that can occur in the graph of a function, we aim to foster the perception that the knowledge of the graph of a function quite simple, allows us to discover the graph of other functions, which being of the same type,

result from the one of these transformations. Let us use the following function as an example: $f(x) = -0.2(x + 1)(x - 5)(x - 2)$, see the table 6.2.

Function	Graph	Transformation
$y = f(x)$		The original function was drawn on all graphs with the green color and was obtained from the following function: $f(x) = -0.2(x + 1)(x - 5)(x - 2)$
$y = f(x) - a$		Translating a in the y-axis direction
$y = f(x + a)$		Translating a in the x-axis direction
$y = a \times f(x)$		Expansion or contraction according to the factor a in the y-axis direction

$y = f(a \times x)$	 <p>The graph shows a green curve labeled 'f' and a blue curve labeled 'g'. The blue curve is a horizontal compression of the green curve. A slider at the top left is set to 'a = 2.3'.</p>	Expansion or contraction according to the factor a in the x-axis direction
$y = -f(x)$	 <p>The graph shows a green curve labeled 'f' and a blue curve labeled 'g'. The blue curve is a vertical reflection of the green curve across the x-axis. A slider at the top left is set to 'a = 0'.</p>	Reflection with respect to the x-axis
$y = f(-x)$	 <p>The graph shows a green curve labeled 'f' and a blue curve labeled 'g'. The blue curve is a horizontal reflection of the green curve across the y-axis. A slider at the top left is set to 'a = 0'.</p>	Reflection with respect to the y-axis
$y = f(x) $	 <p>The graph shows a green curve labeled 'f' and a blue curve labeled 'g'. The blue curve is the absolute value of the green curve, reflecting the negative parts of the green curve above the x-axis. A slider at the top left is set to 'a = 0'.</p>	The ordinate points are positive or zero and the remainder are obtained from the negative ordinate points by a reflection of the x-axis.

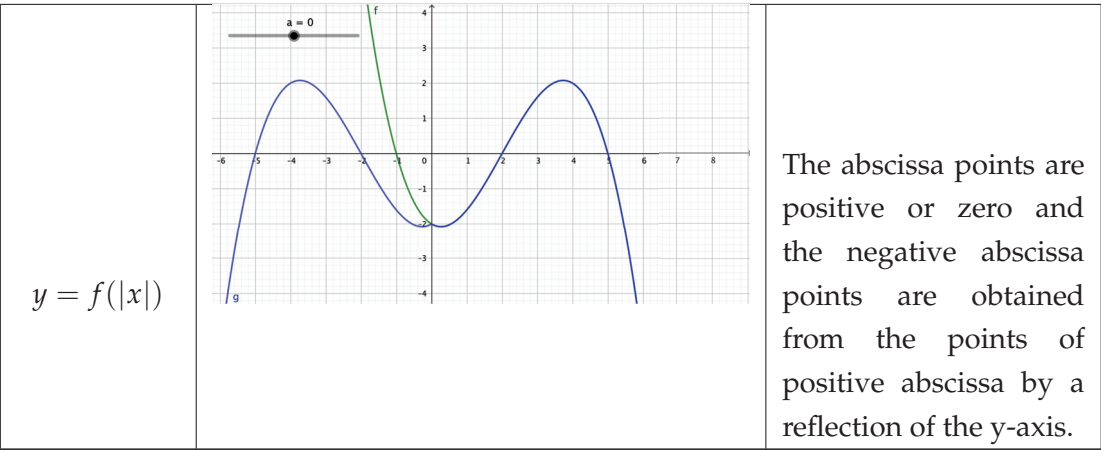


TABLE 6.2: Transformation of Functions

We then realized that the students used such transformations as a strategy to create choreographies for competing teams, for example, hiding the appearance of the graph of the function on the screen, translating a in the y-axis direction.

6.4.2 Didactic Variables at Binary Code

Recalling the Binary Code trick (see. Fig 6.7), we gave the students the main tasks:

- Develop an algorithm that performs the trick.

3	7	23	19	14	26	10	27	6	5	20	14	14	10	15	31	17	23	19	25
17	9	11	31	3	30	15	18	28	23	15	13	11	27	29	13	27	30	29	18
5	27	21	13	31	6	19	23	31	29	21	22	26	28	9	12	22	21	26	24
1	25	29	15	2	7	11	22	4	30	7	12	8	30	25	24	16	28	31	20

FIGURE 6.7: Binary Code - Cards

The first didactic variable defined for this trick is the order of the elements of the cards. The unsorted numbers induce the viewer a notion of lack of pattern between the cards. The *a priori* strategy is that the viewer sort the numbers in each card to understand the relation between the numbers in each one.

Another variable is the insertion of the powers of 2 in the lower left corner of the cards. It is not uncommon for people to discover how the trick is done using only this information. By summing these powers, in the cards in which the chosen number appears, we obtain the result that is the number itself. Generally, in shorter situations such as workshops and lecture presentations, this pattern is identified by most people.

The creative manifestations of the students can occur and can be perceived when we ask them to translate their reasoning to solve a problem. It also allows us to verify the difficulties of reasoning, giving a more precise diagnosis on how to advance it.

In several situations, putting myself now as an observer teacher, I realize that students can not understand the initial question of the problem. When we ask, in the Binary Code trick, for example, how does the trick work? Answers like: "You have memorized all the cards," or "You look the number in my eyes," and also "It has something to do with prime numbers," are frequent when I perform the workshops presenting the trick. At all ages and academic levels, the practice of analyzing data, organizing them, and making conclusions about them are essential factors in solving the problem.

Another important finding was that prior knowledge could serve both as a helper and as a blocker to new related problems. In the Fake Binary Code trick, students who had performed the Binary Code trick easier took much longer to unravel the Fake Binary Code trick. While the students who "learned" about binary decomposition in the first trick, did better in the second one.

This research may also be useful for researchers in digital game development for teaching as it provides a rationale for the elements of design, didactic variables and production of artifacts to achieve desired learning outcomes.

6.5 Recommendations for Further Research

We began this thesis with the idea of developing a methodology of analysis of creative mathematical thinking, this methodology could be quantitative or qualitative, but it should allow in some way, to measure objectively and operationally, the creative process of students, in producing solutions to different didactic situations. This goal led us to develop a methodology, theoretically called the diamond of creativity, which considers creativity as a process of creation that begins with a divergent state, an inflection point that is the AHA Moment (or Illumination), and then, a convergent process, leading to the concretization of mental activities, translated into some symbolic type, understandable by the community.

During the research, several questions arose but could not be answered within the scope of this thesis. We consider four components for the evaluation of creativity: Person, Product, Environment, and Process. Here, we evaluate product and process. For example, in the game Function Hero, we realized that the choreographies of the first institution were less creative than the second. Understanding the causes of this difference in creativity could be interesting to know what are the personal and environmental factors that can affect creativity.

Another question is whether the coding of problems in networks is useful in the classroom when the number of students is high. Improving the process of encoding the mapping of questions that students make during the problem-solving process would serve to expedite the analysis of the graphs produced from these resolutions.

Also, a more detailed and explanatory mathematical work should be carried out around the topology of the graphs developed from the students' productions. Works such as Gera et al., 2018, in which he develops a similarity test to know whether partial information from a network is similar to the ground truth. Such a metric could be used to verify how similar a network of one institution or individual is similar to that of another institution.

I also propose as future research the development of an analytical tool to integrate into games like Function Hero and Space Math Ship, to create an automatic and adaptive feedback system according to the performance of its users.

Regarding the perception of mathematics, we suggest further investigation avenues: the first one is to study deeper the influence of engineering students' perception on mathematics performance for each institution. The second one is the elaboration of questionnaires targeting engineers in order to study the perception and actual usage of mathematics by professional and whether its use allow them to be more creative in their jobs. Because the link between students and engineers goes through teachers, we need to study as well the perception of teachers themselves.

6.6 Conclusions

The findings of this study contributed the work of previous researchers in the areas of: Educational Digital Games Design, Creativity in Mathematical and Computational Thinking, and Inquiry-Based Learning. This investigation revealed that students engage in game-based activities and can increase their knowledge from their use. We also found that the "Diamond of Creativity" is a useful tool for observing creative manifestations of people engaged in problem-solving in mathematics and computer science, specifically in algorithms design, as well as participating in educational games like the Function Hero. Still, we realize that the process of solving a problem can be broken down into elemental and identified parts to facilitate the intervention of teachers in the development of students' reasoning.

This thesis has enriched my eyes on how Creativity can be studied and observed at various levels of knowledge. It manifests itself not only in great geniuses but in all people engaged in solving a problem. I was able to witness manifestations of intrinsic motivation in students who solved math problems, and who played a math game. This study made me believe that we can vulgarize this discipline and make it accessible to all individuals in a captivating and effective way because, in the end,

they will improve their understanding and knowledge about the concepts involved in each activity.

Appendix A

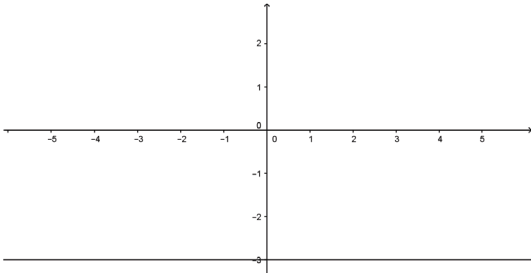
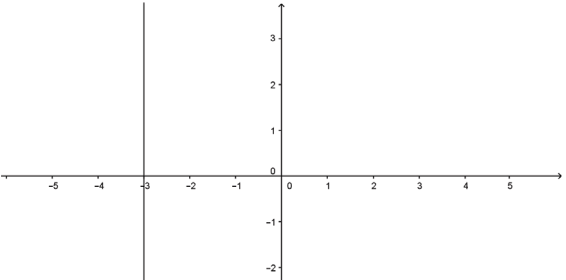
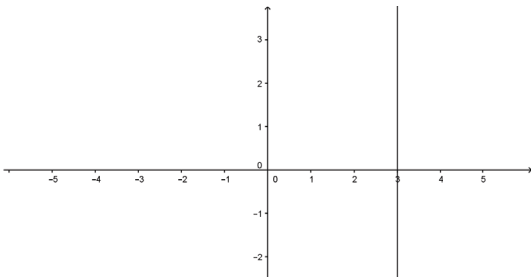
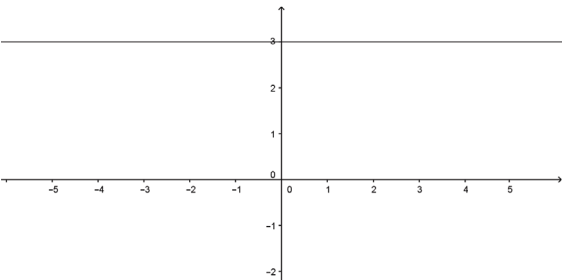
Pre and Post Tests - Version 1

Univeristé Claude Bernard - Lyon I
Function Hero - Pre_Test

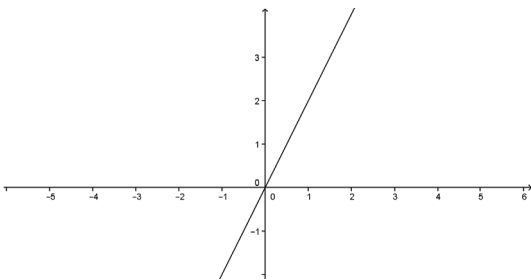
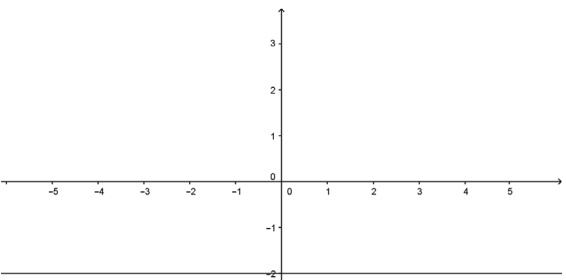
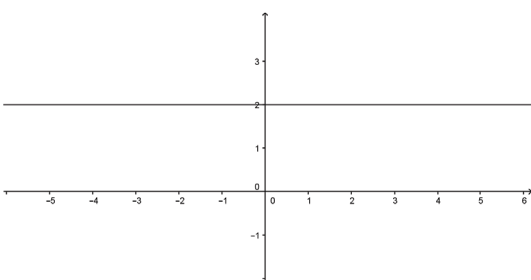
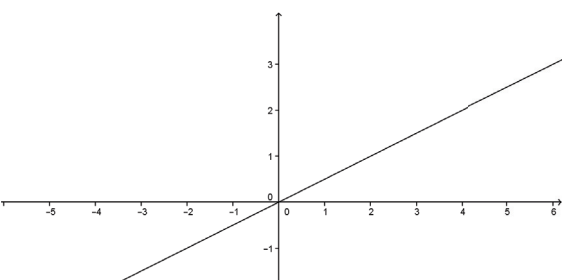
Nom: _____

Choisis la réponse correcte

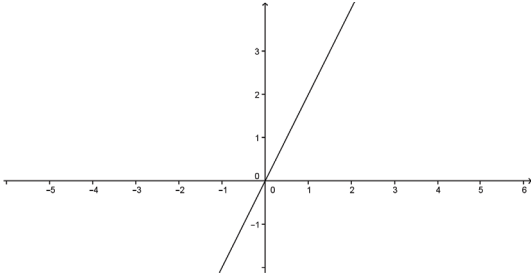
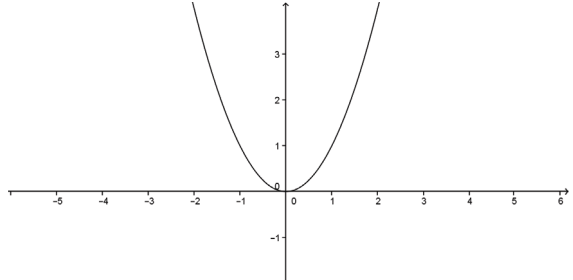
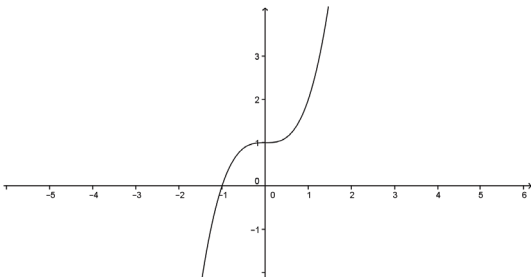
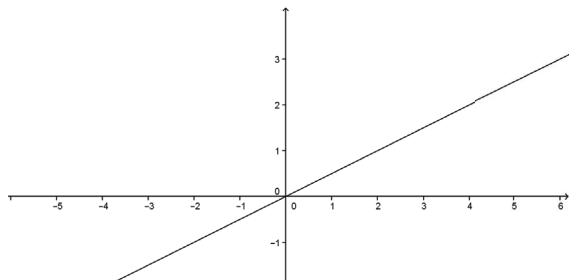
1. La représentation graphique de la fonction $f(x) = 3$ est la figure ...

a.		b.	
c.		d.	

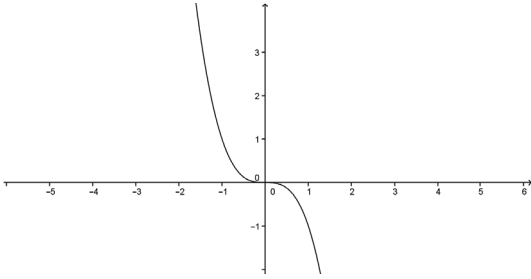
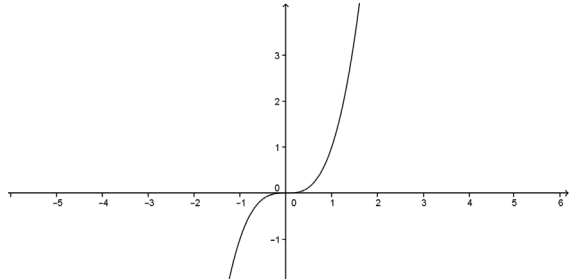
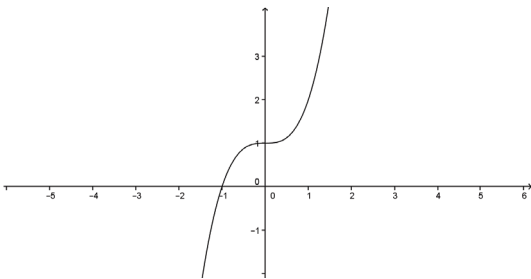
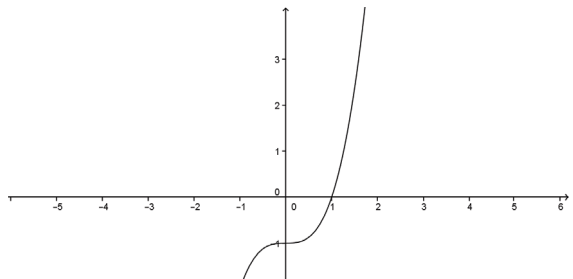
2. La représentation graphique de la fonction $f(x) = 2x$ est la figure ...

a.		b.	
c.		d.	

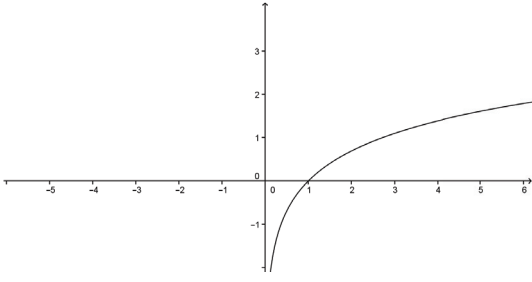
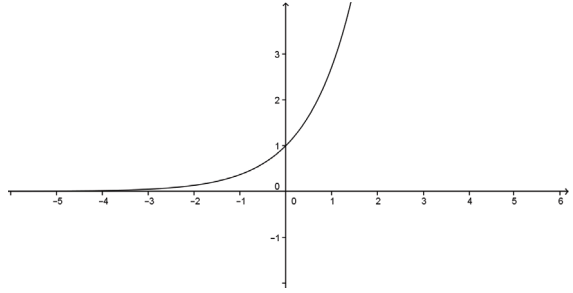
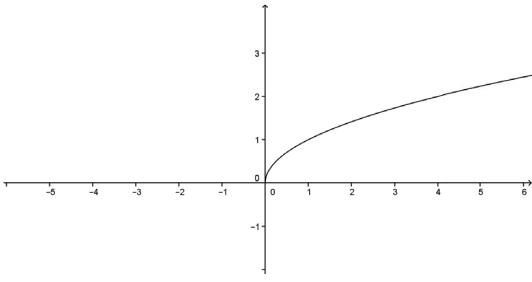
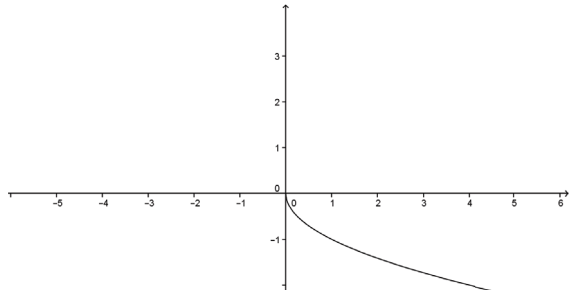
3. La représentation graphique de la fonction $f(x) = x^2$ est la figure ...

a.		b.	
c.		d.	

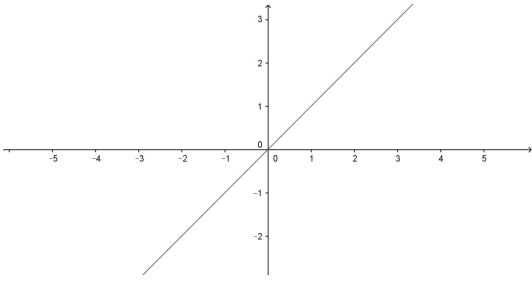
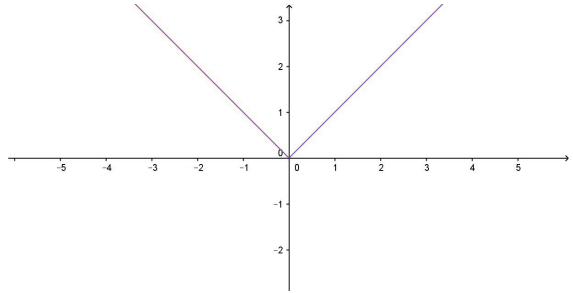
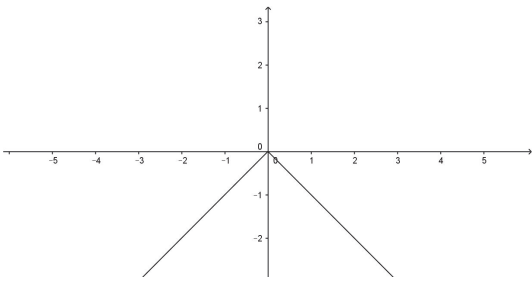
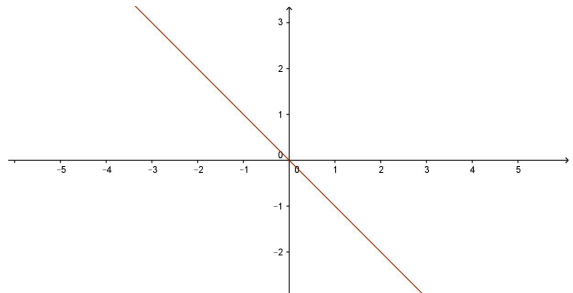
4. La représentation graphique de la fonction $f(x) = x^3$ est la figure ...

a.		b.	
c.		d.	

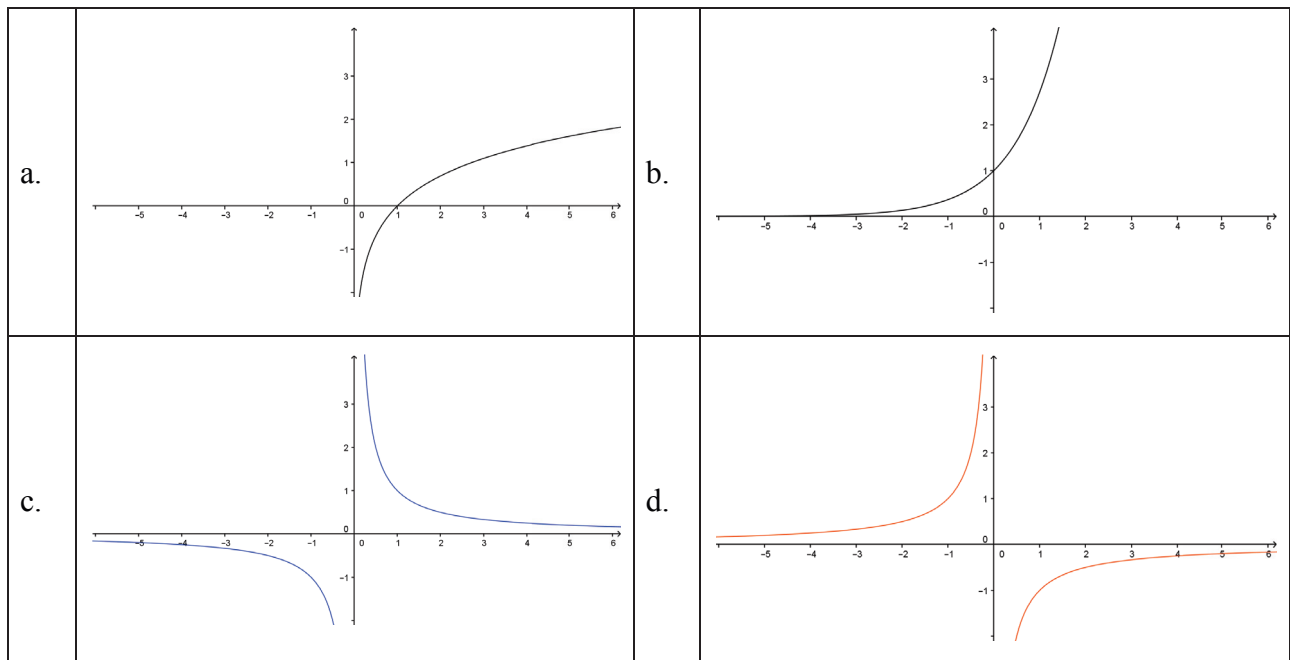
5. La représentation graphique de la fonction $f(x) = \sqrt{x}$ est la figure ...

a.		b.	
c.		d.	

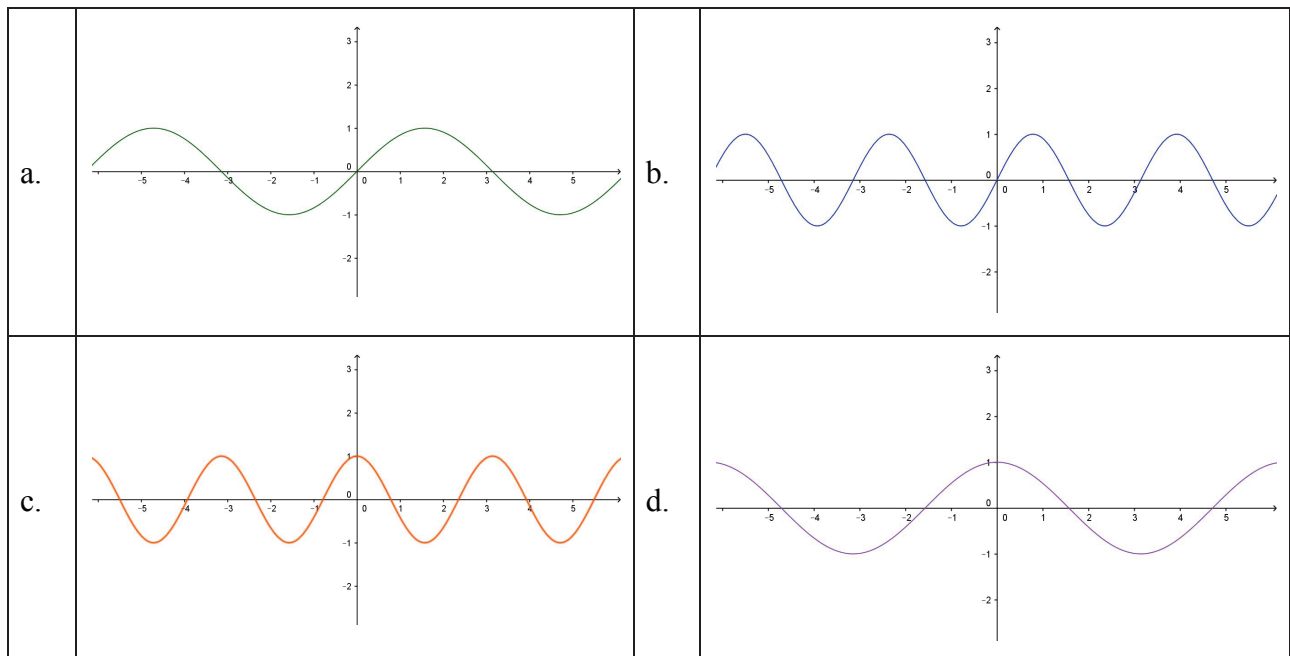
6. La représentation graphique de la fonction $f(x) = |x|$ est la figure ...

a.		b.	
c.		d.	

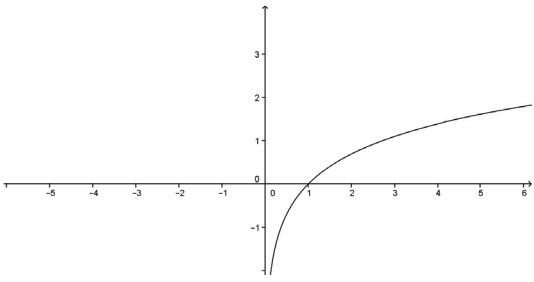
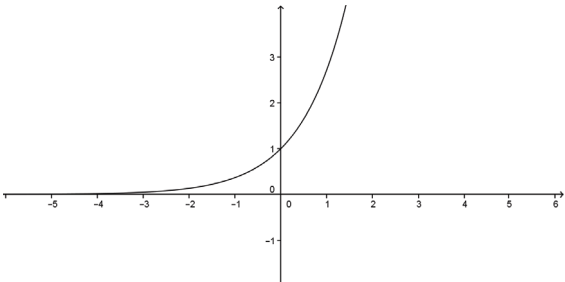
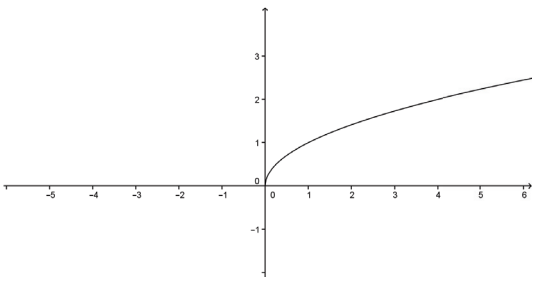
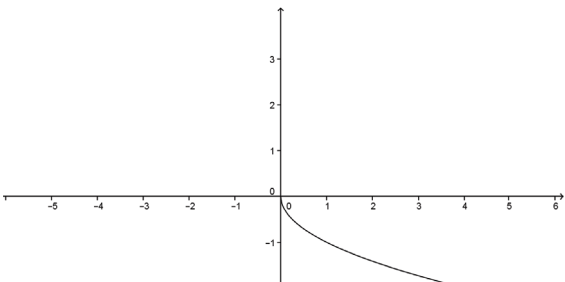
7. La représentation graphique de la fonction $f(x) = \frac{1}{x}$ est la figure ...



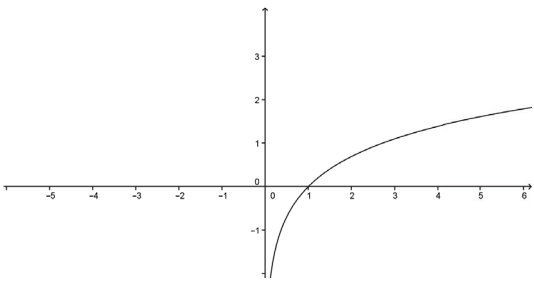
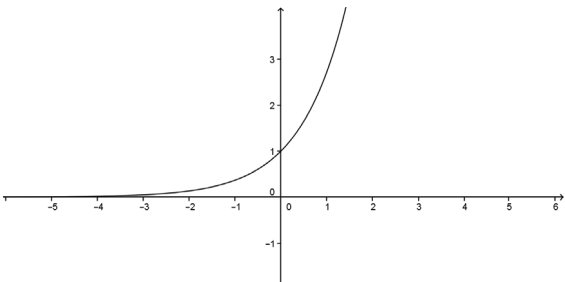
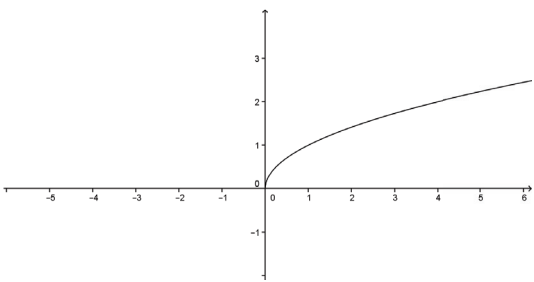
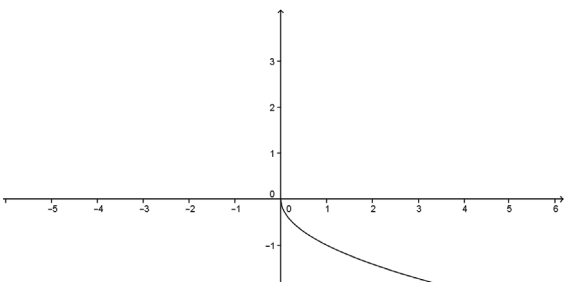
8. La représentation graphique de la fonction $f(x) = \sin(x)$ est la figure ...



9. La représentation graphique de la fonction $f(x) = e^x$ est la figure ...

a.		b.	
c.		d.	

10. La représentation graphique de la fonction $f(x) = \ln(x)$ est la figure ...

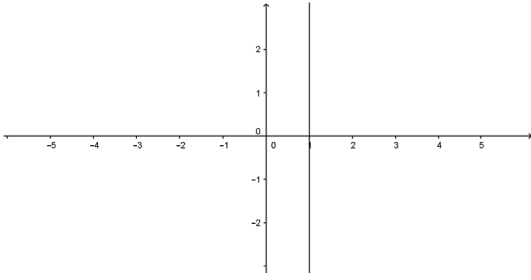
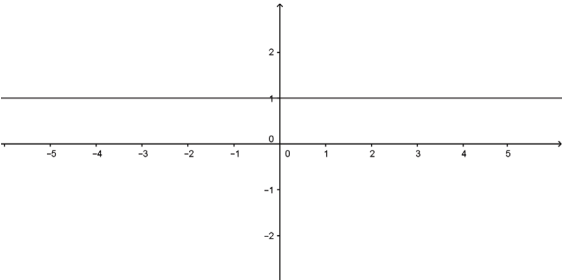
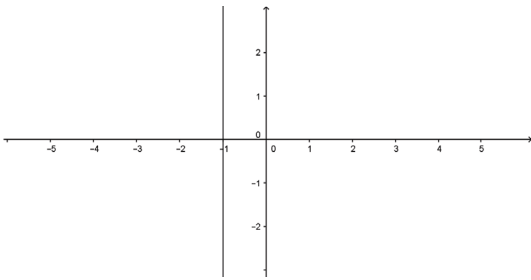
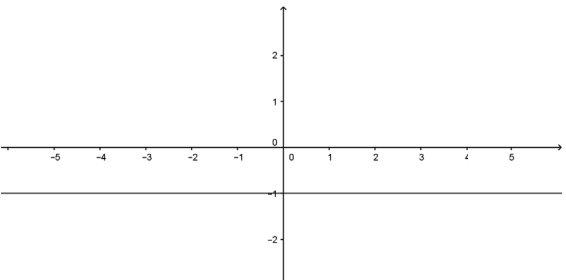
a.		b.	
c.		d.	

Université Claude Bernard - Lyon I
Function Hero - Post_Test

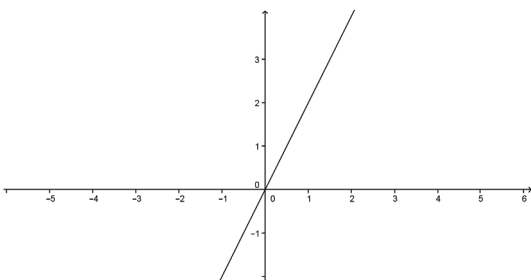
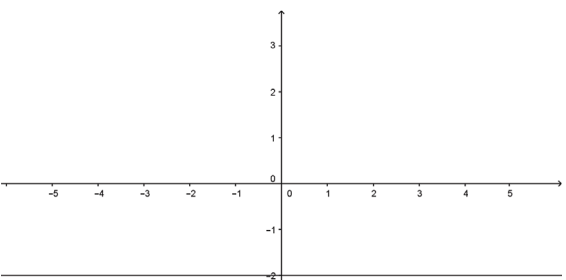
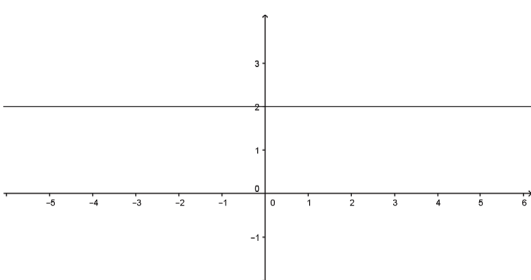
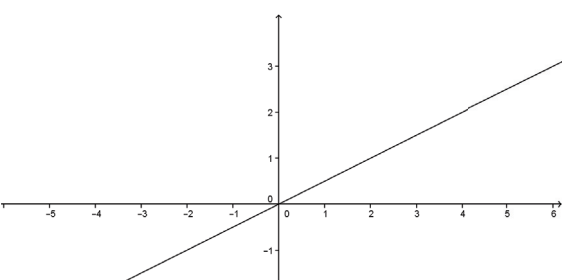
Nom: _____

Choisis la réponse correcte

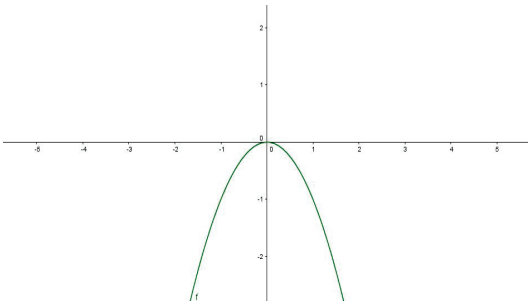
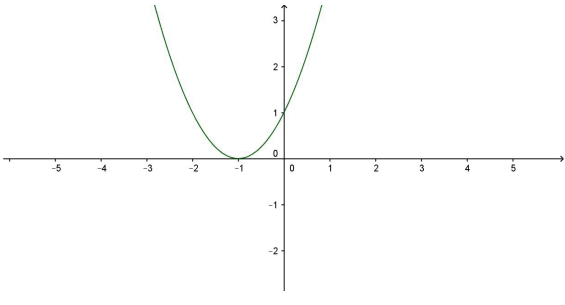
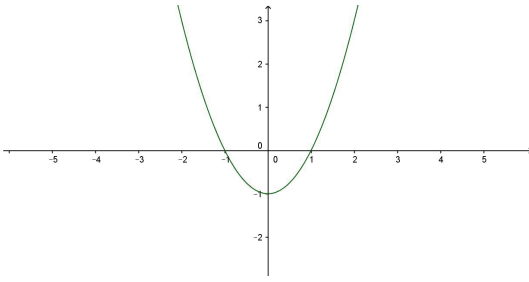
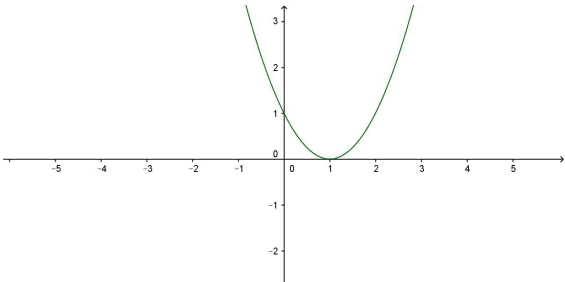
1. La représentation graphique de la fonction $f(x) = -1$ est la figure ...

a.		b.	
c.		d.	

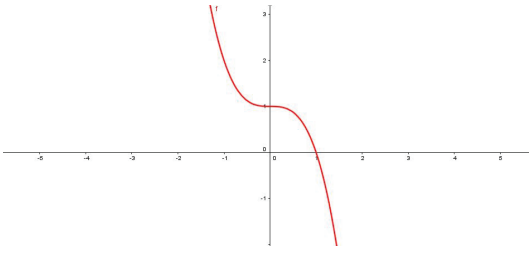
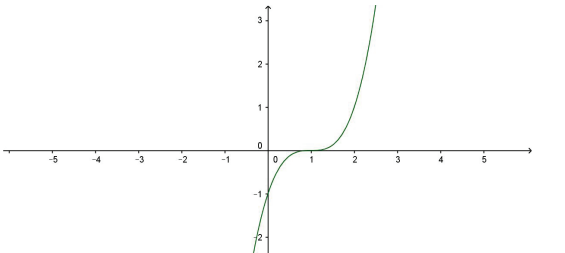
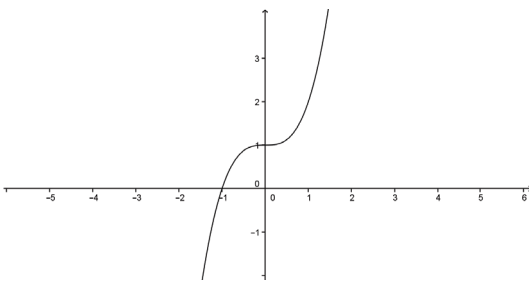
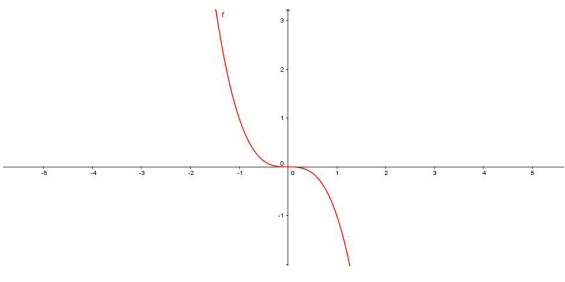
2. La représentation graphique de la fonction $f(x) = \frac{1}{2}x$ est la figure ...

a.		b.	
c.		d.	

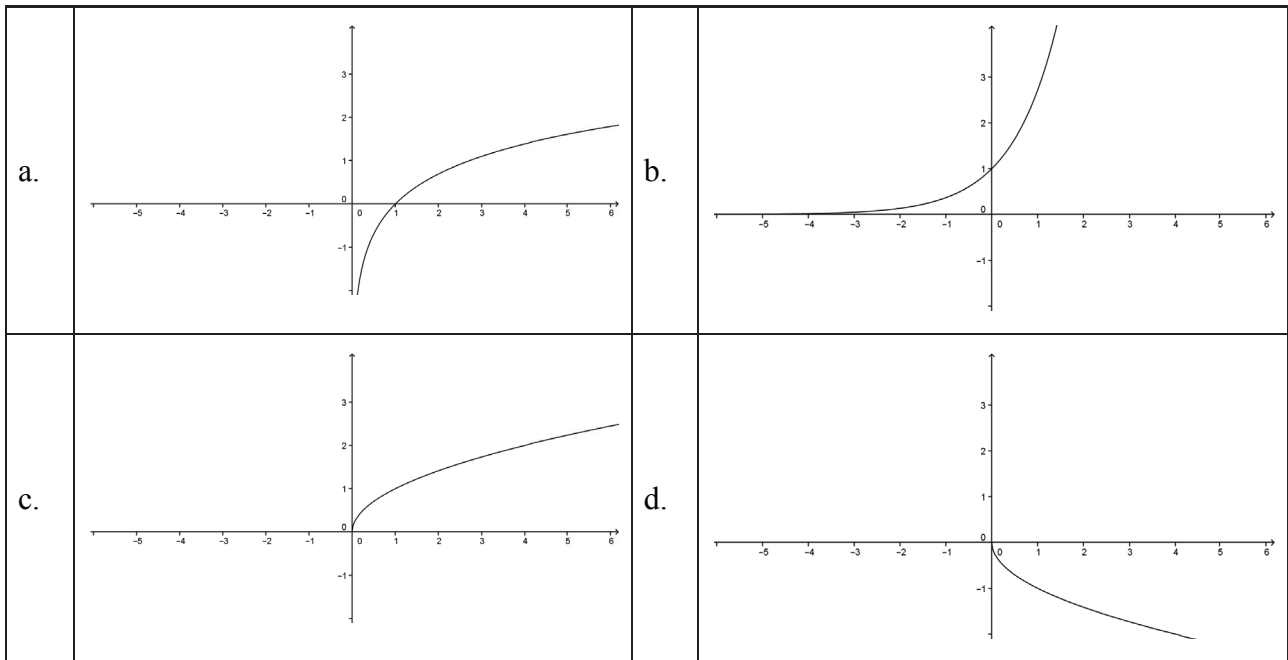
3. La représentation graphique de la fonction $f(x) = -x^2$ est la figure ...

a.		b.	
c.		d.	

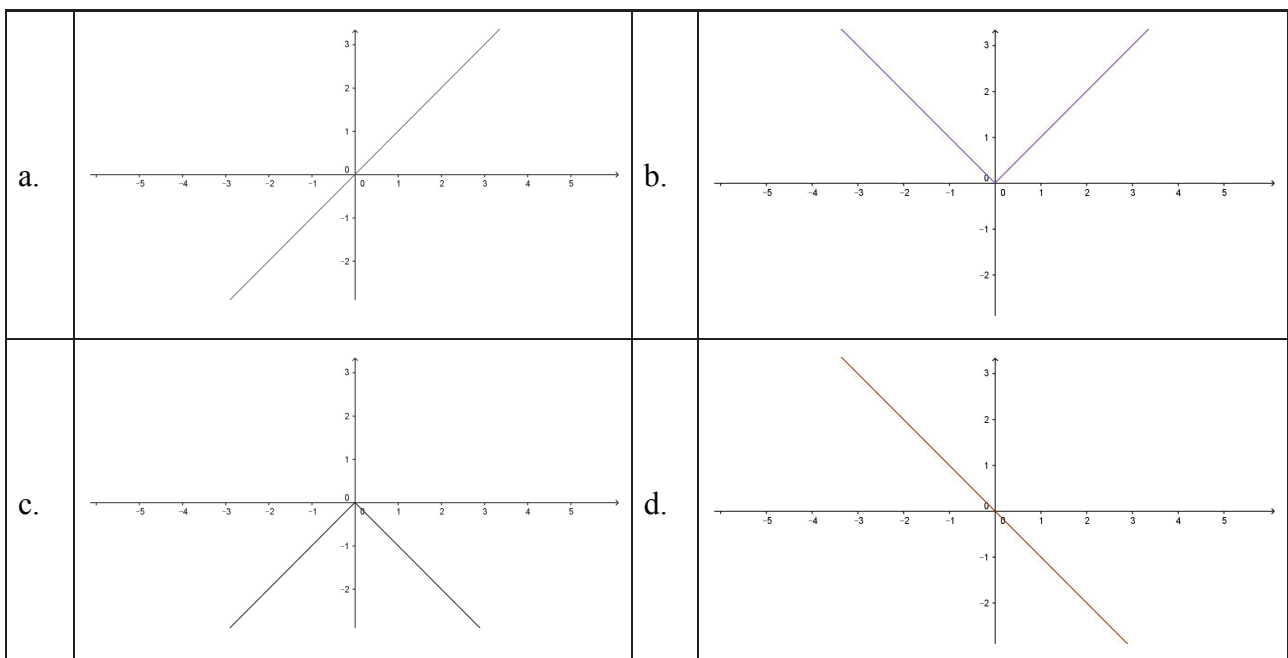
4. La représentation graphique de la fonction $f(x) = -x^3$ est la figure ...

a.		b.	
c.		d.	

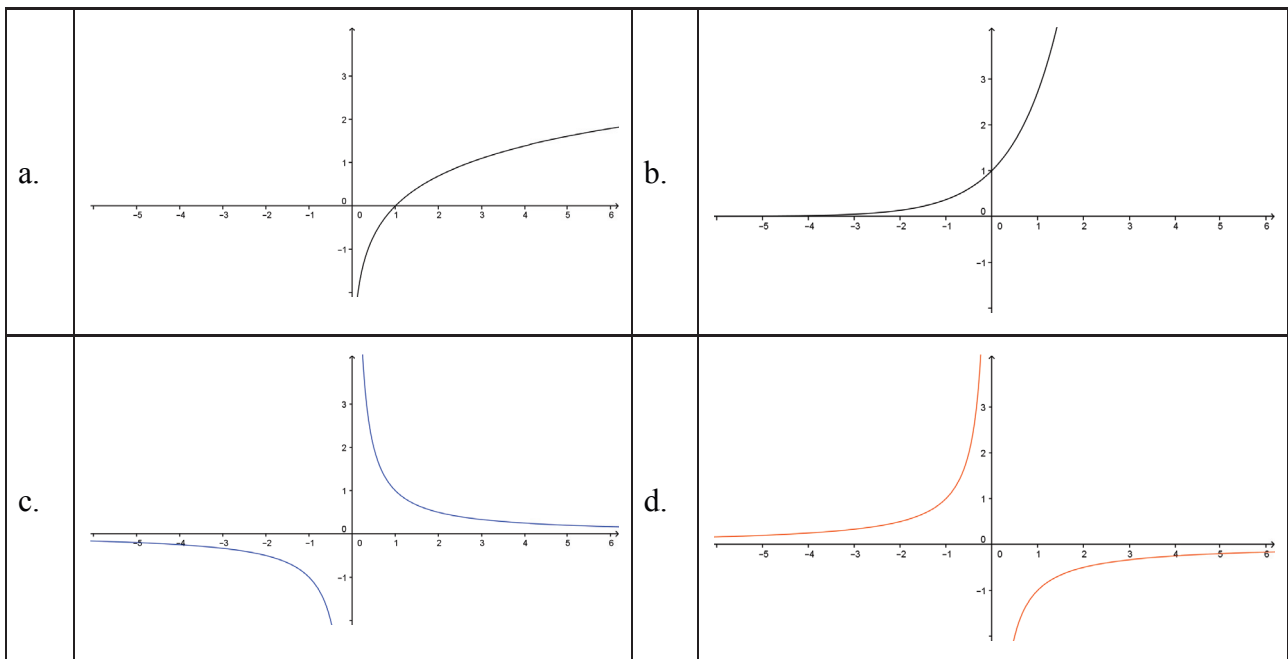
5. La représentation graphique de la fonction $f(x) = -\sqrt{x}$ est la figure ...



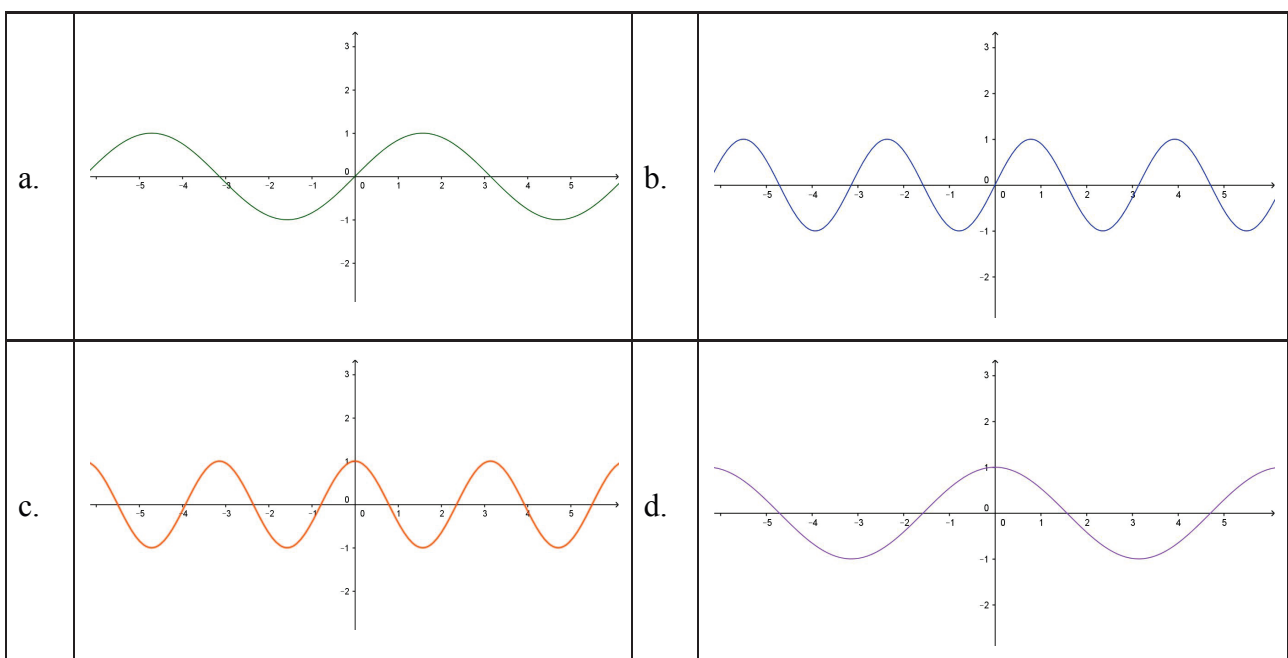
6. La représentation graphique de la fonction $f(x) = -|x|$ est la figure ...



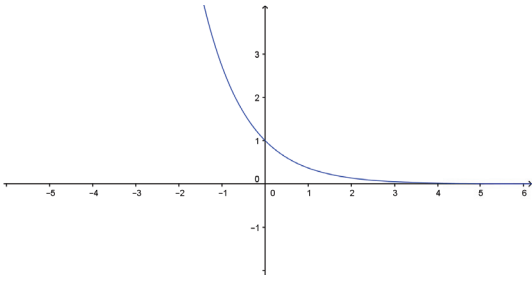
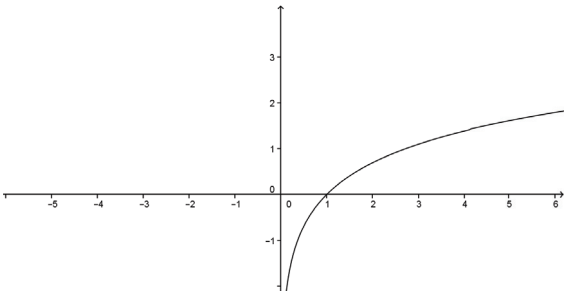
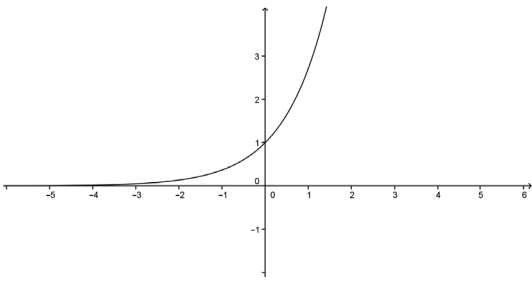
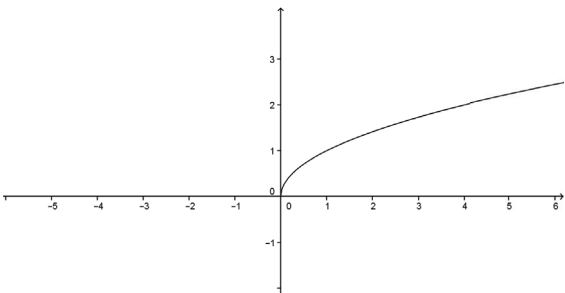
7. La représentation graphique de la fonction $f(x) = -\frac{1}{x}$ est la figure ...



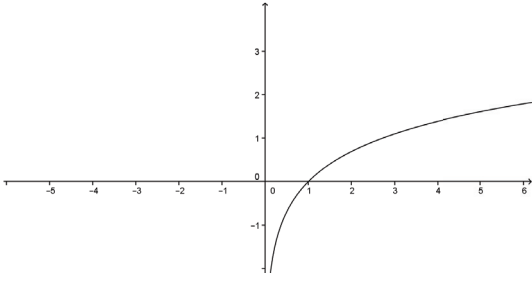
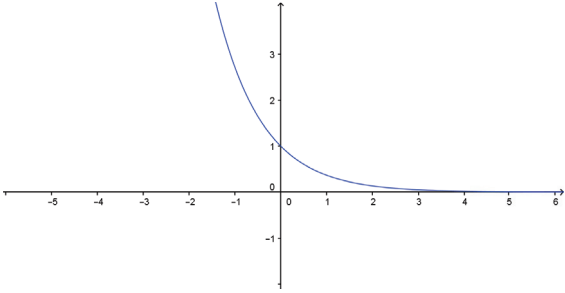
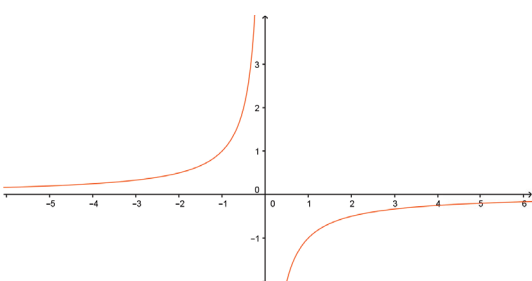
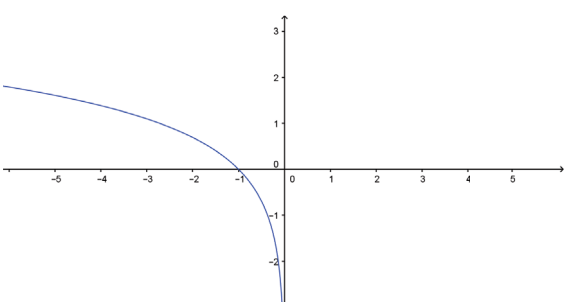
8. La représentation graphique de la fonction $f(x) = \cos(x)$ est la figure ...



9. La représentation graphique de la fonction $f(x) = e^{-x}$ est la figure ...

a.		b.	
c.		d.	

10. La représentation graphique de la fonction $f(x) = \ln(-x)$ est la figure ...

a.		b.	
c.		d.	

Appendix B

Pre and Post Test - Version 2

UNIVERSITÉ CLAUDE BERNARD - LYON I
Pre Test

Nom: _____

Instructions:

First of all, thank you for participating on this study. It will lead us to develop and improve better technologies to teach mathematics and make the courses more attractive. So, I gently ask you to answer the following questions very seriously. It's an anonymous test, so, don't hesitate in answering wrong if you don't know the correct one.

Some more rules:

- You're not allowed to use your calculator.
- This test is individual, don't try copy your colleagues' answers.
- Have fun.

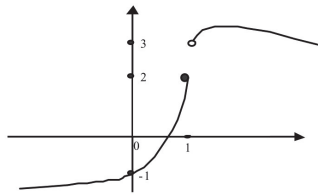
1. Check the following expressions which represents a function:

a) $x^2 + y^3 = 3$	b) $y = x^3 + x + 1$	c) $a^2 - b = 0$
d) $f(y) = e^y$	e) $x^4 = 3y$	f) $a = \sqrt{2}$
g) $f(x) = 3$	h) $y = x^2$	i) $s = 3t$

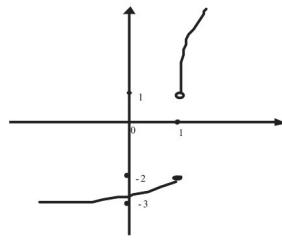
2. Check the following graphs which represents a function:

A) 	B) 	C)
D) 	E) 	F)

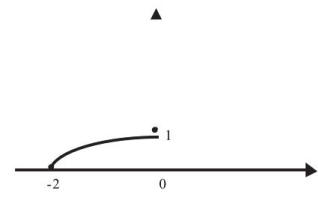
G)



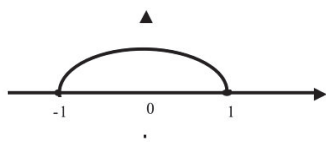
H)



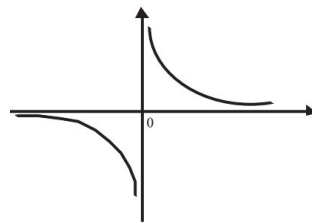
I)







K)



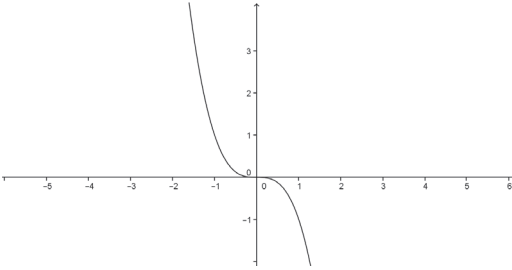
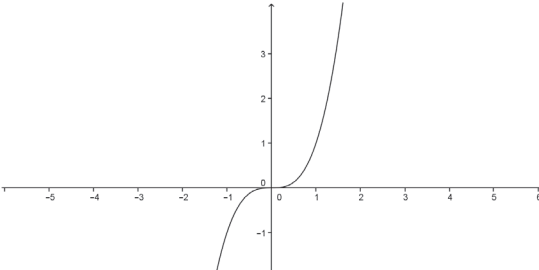
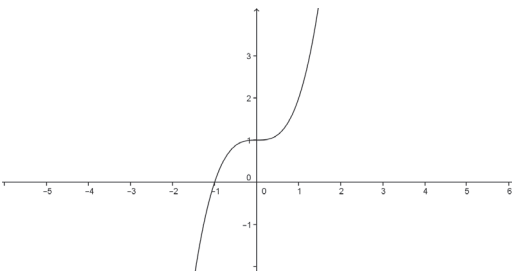
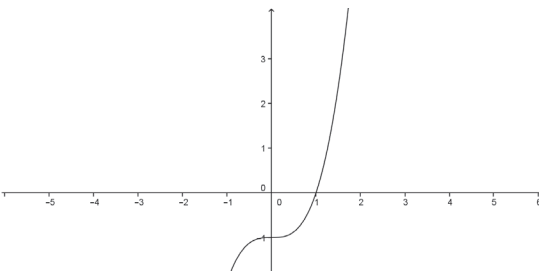
L)



4. If you could describe the following dance movements in terms of algebraic expression of functions. How would you do that?

5. La représentation graphique de la fonction $f(x) = x^3$ est la figure ...

a.		b.	
c.		d.	

UNIVERSITÉ CLAUDE BERNARD - LYON I
Post Test

Nom: _____

Instructions:

First of all, thank you for participating on this study. It will lead us to develop and improve better technologies to teach mathematics and make the courses more attractive. So, I gently ask you to answer the following questions very seriously. It's an anonymous test, so, don't hesitate in answering wrong if you don't know the correct one.

Some more rules:

- You're not allowed to use your calculator.
- This test is individual, don't try copy your colleagues' answers.
- Have fun.

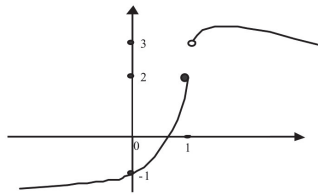
1. Check the following expressions which represents a function:

a) $x^2 + y^3 = 3$	b) $y = x^3 + x + 1$	c) $a^2 - b = 0$
d) $f(y) = e^y$	e) $x^4 = 3y$	f) $a = \sqrt{2}$
g) $f(x) = 3$	h) $y = x^2$	i) $s = 3t$

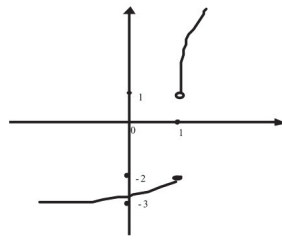
2. Check the following graphs which represents a function:

A) 	B) 	C)
D) 	E) 	F)

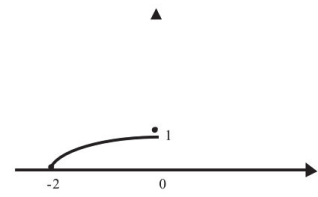
G)



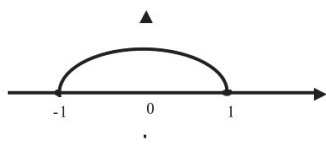
H)



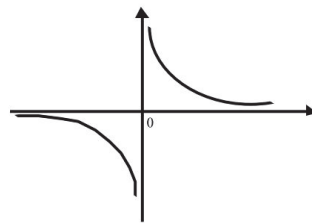
I)



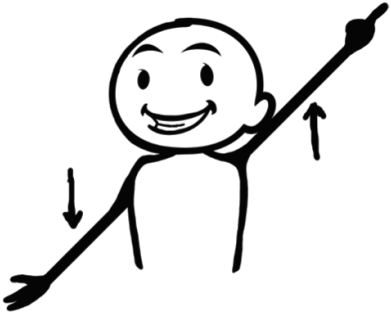

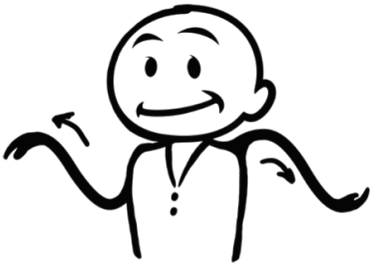

K)



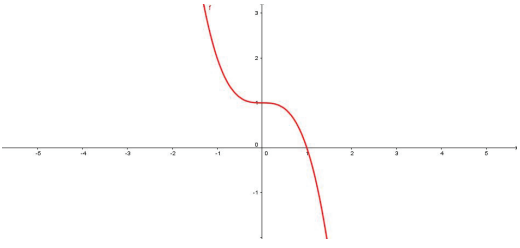
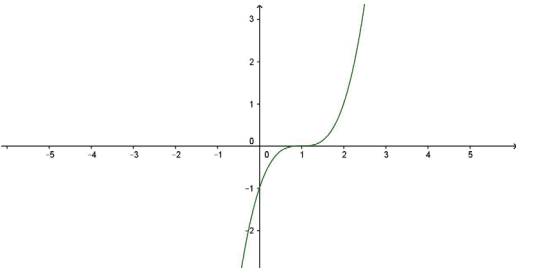
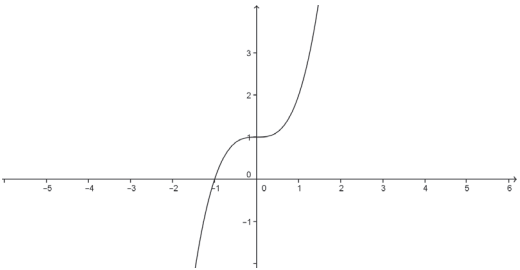
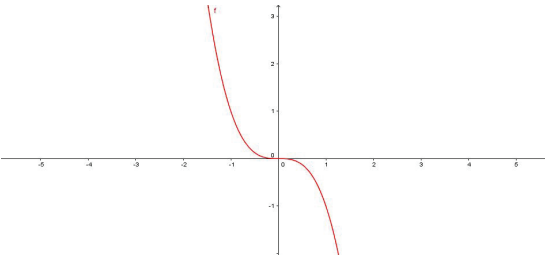
L)



4. If you could describe the following dance movements in terms of algebraic expression of functions. How would you do that?

5. La représentation graphique de la fonction $f(x) = -x^3$ est la figure ...

a.		b.	
c.		d.	

Appendix C

BTS Choreographies

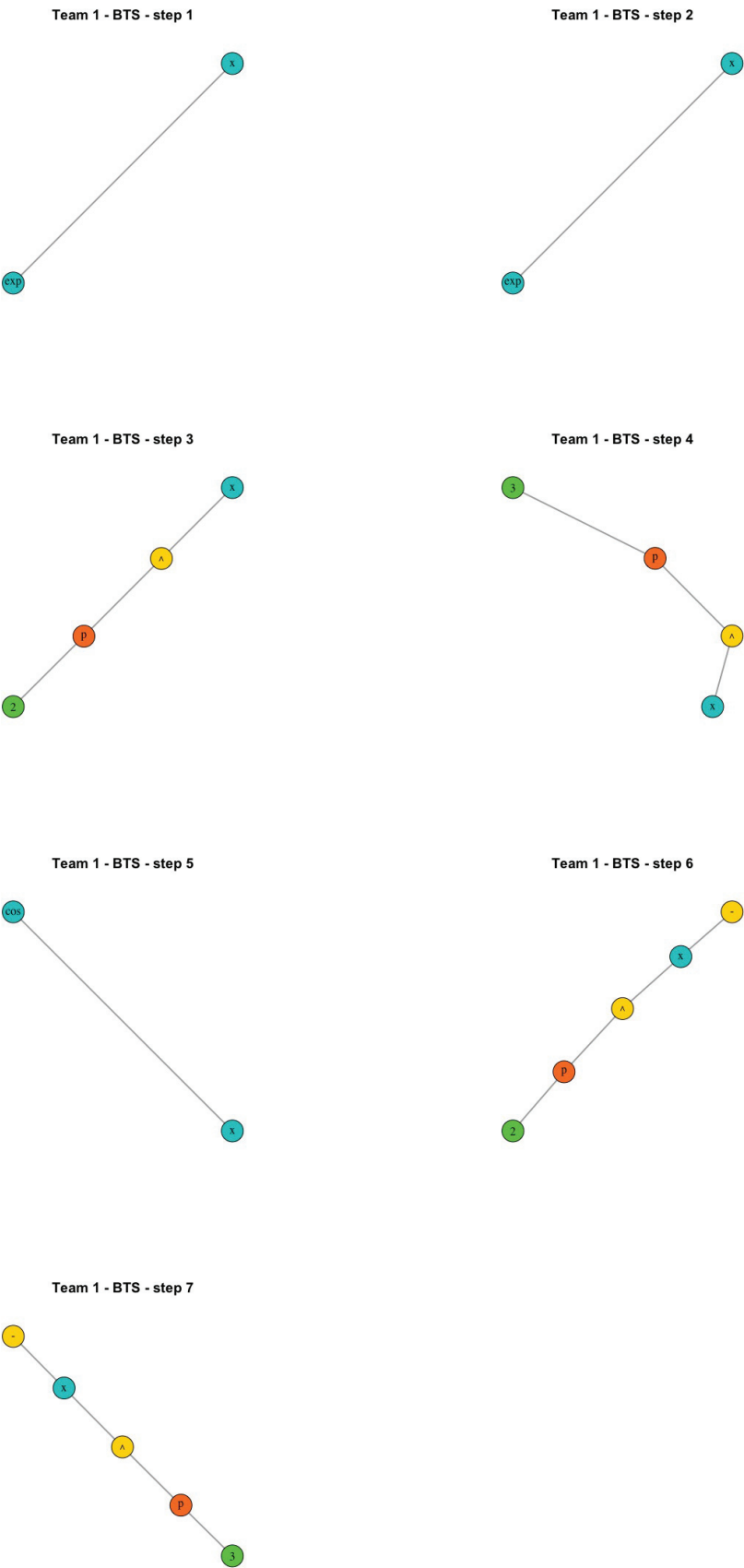
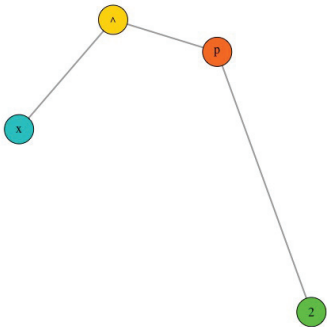


FIGURE C.1: Steps of Team 1 - Choreography

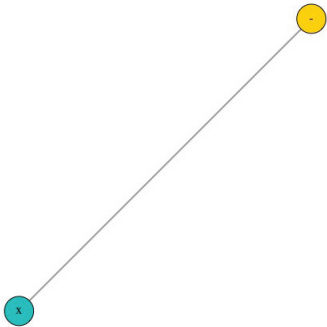
Team 2 - BTS - step 1



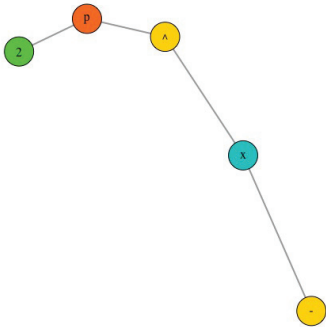
Team 2 - BTS - step 2



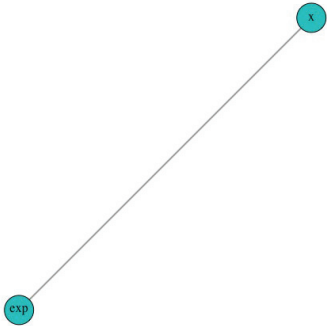
Team 2 - BTS - step 3



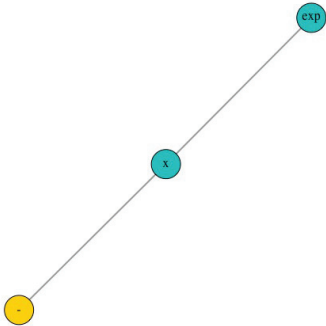
Team 2 - BTS - step 4



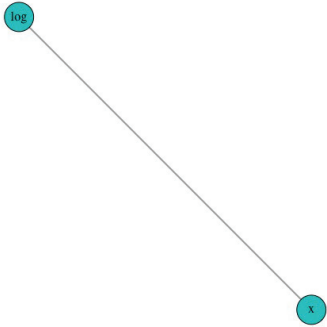
Team 2 - BTS - step 5



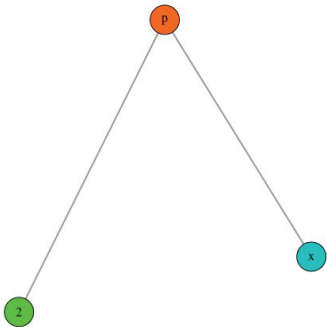
Team 2 - BTS - step 6



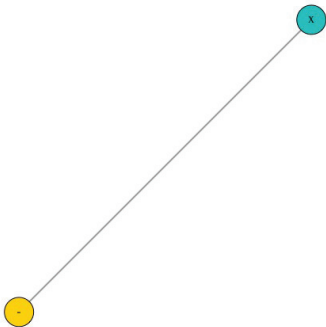
Team 2 - BTS - step 7



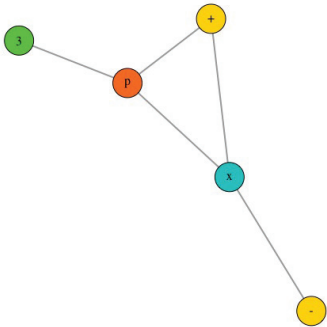
Team 3 - BTS - step 1



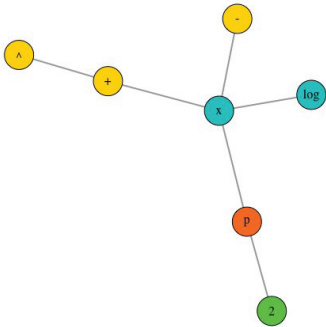
Team 3 - BTS - step 2



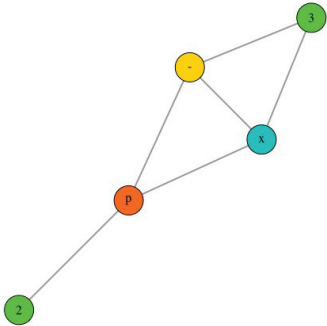
Team 3 - BTS - step 3



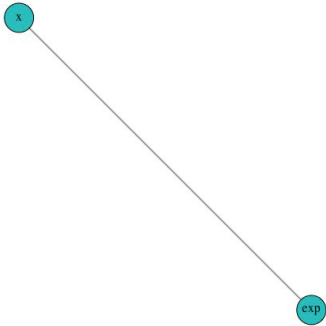
Team 3 - BTS - step 4



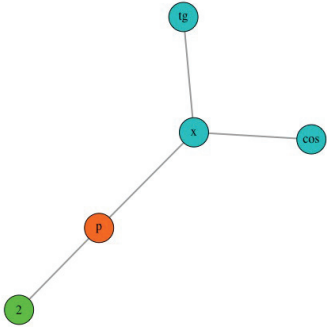
Team 3 - BTS - step 5



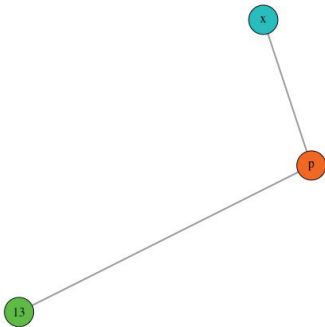
Team 3 - BTS - step 6



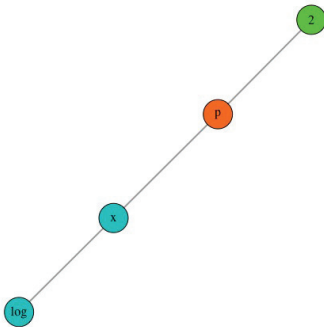
Team 3 - BTS - step 7



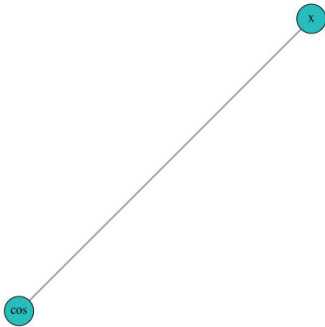
Team 4 - BTS - step 1



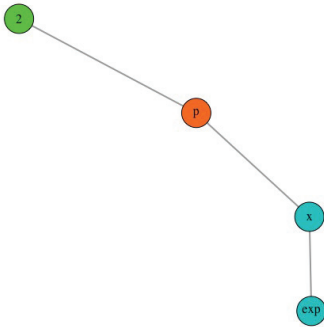
Team 4 - BTS - step 2



Team 4 - BTS - step 3



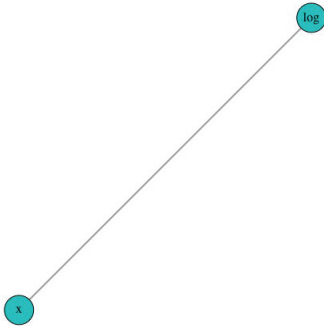
Team 4 - BTS - step 4



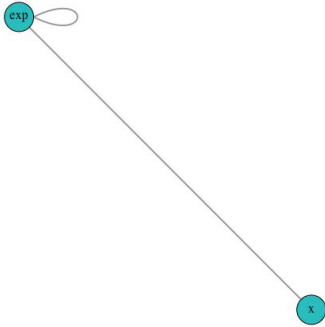
Team 4 - BTS - step 5



Team 4 - BTS - step 6



Team 4 - BTS - step 7



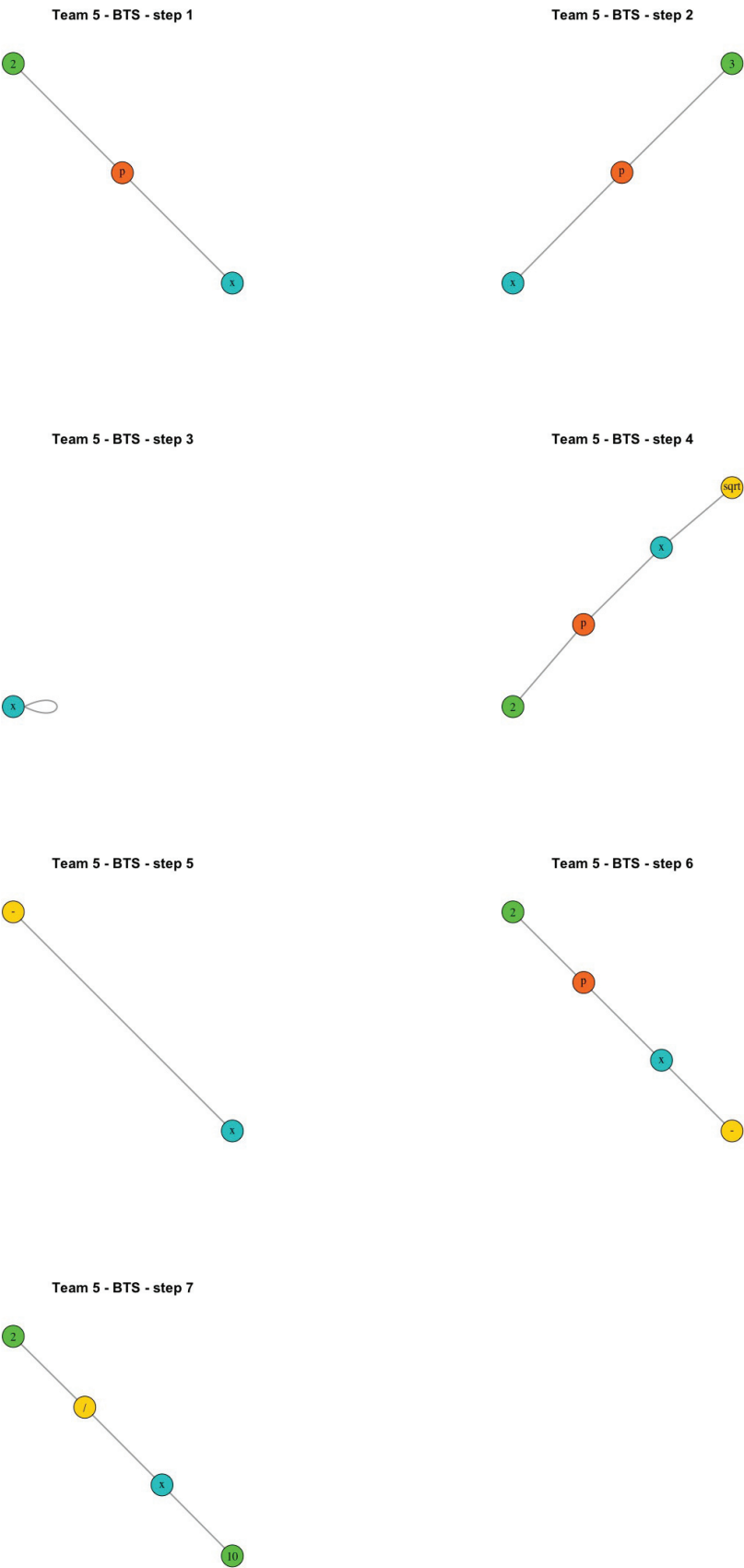


FIGURE C.5: Steps of Team 5 - Chorepography

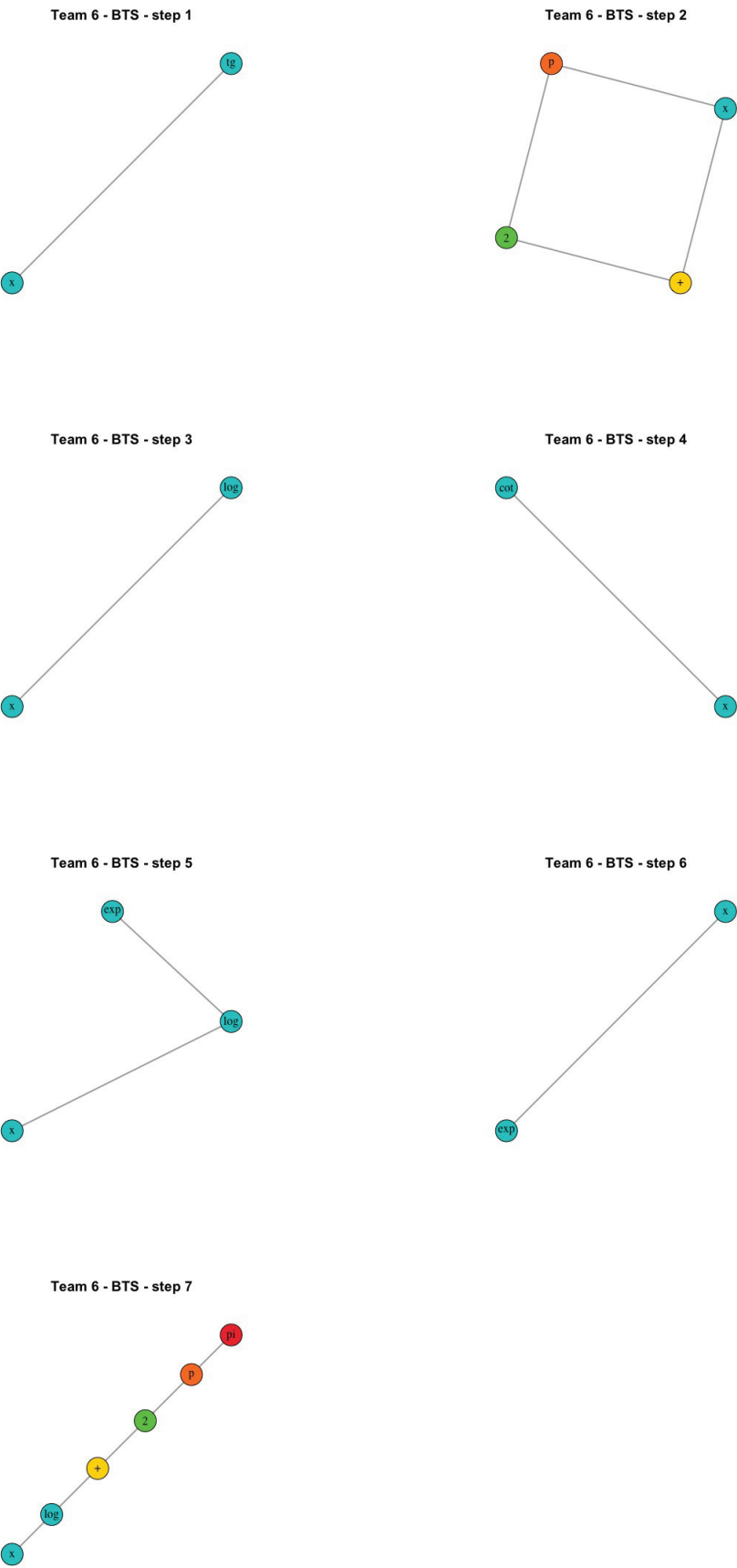


FIGURE C.6: Steps of Team 6 - Chorepography

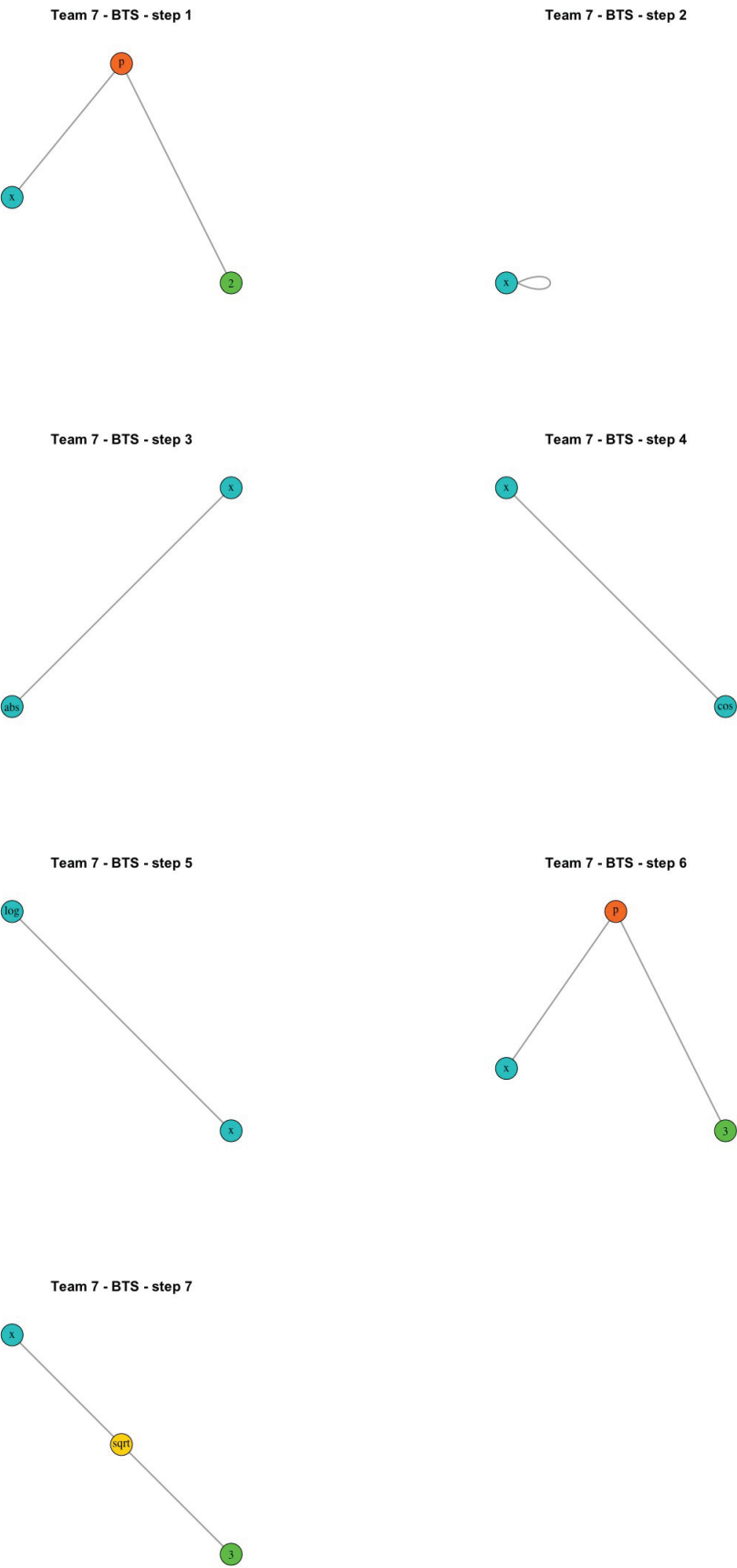


FIGURE C.7: Steps of Team 7 - Chorepography

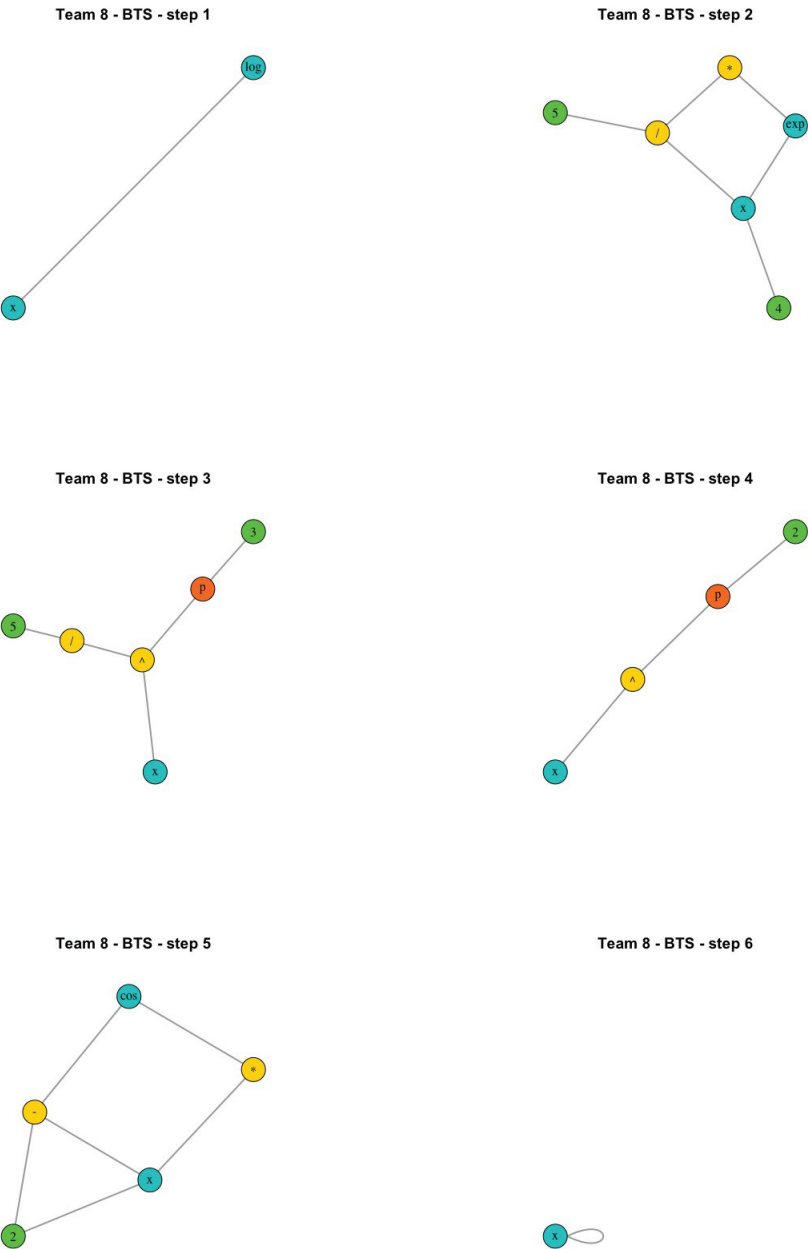


FIGURE C.8: Steps of Team 8 - Chorepography

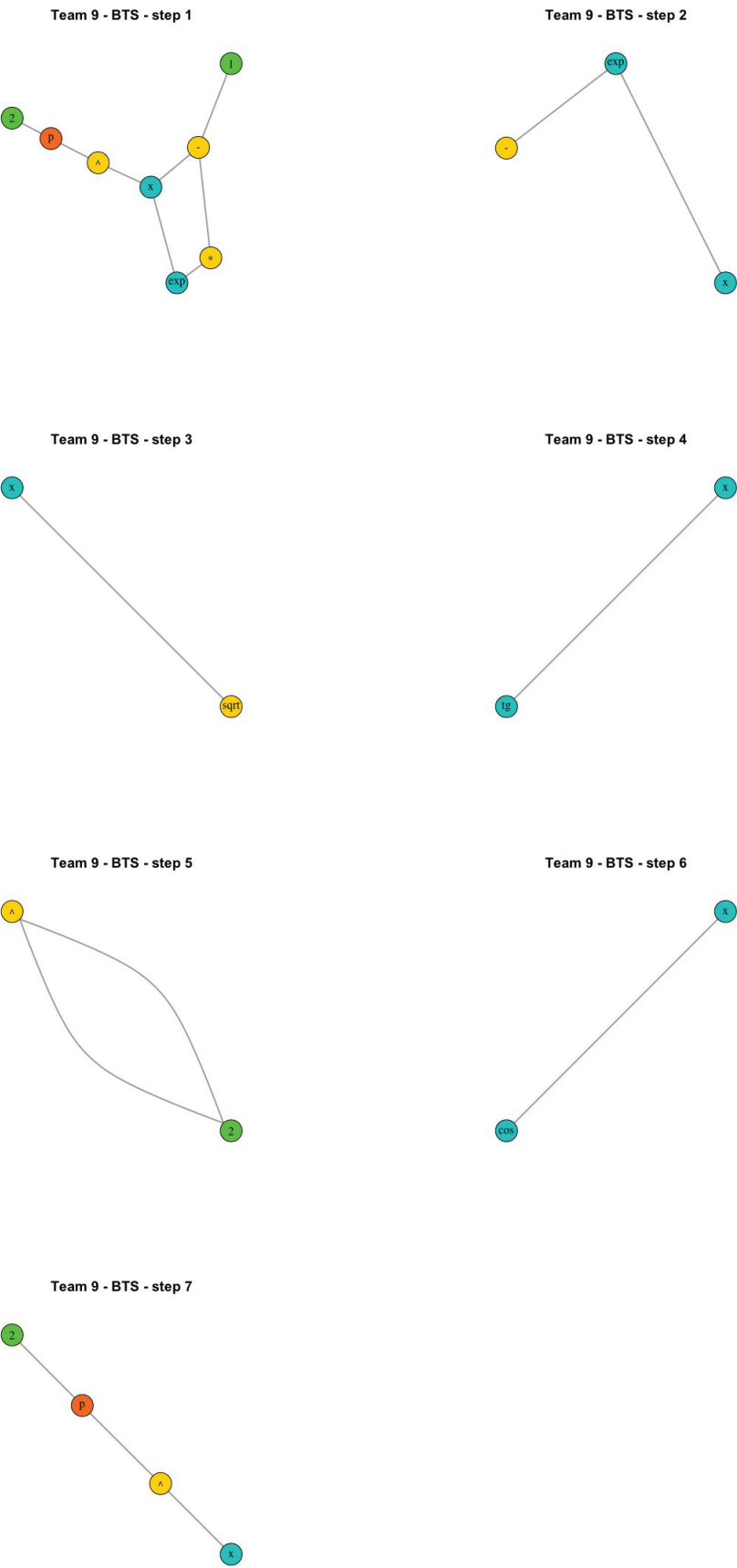


FIGURE C.9: Steps of Team 9 - Chorepography

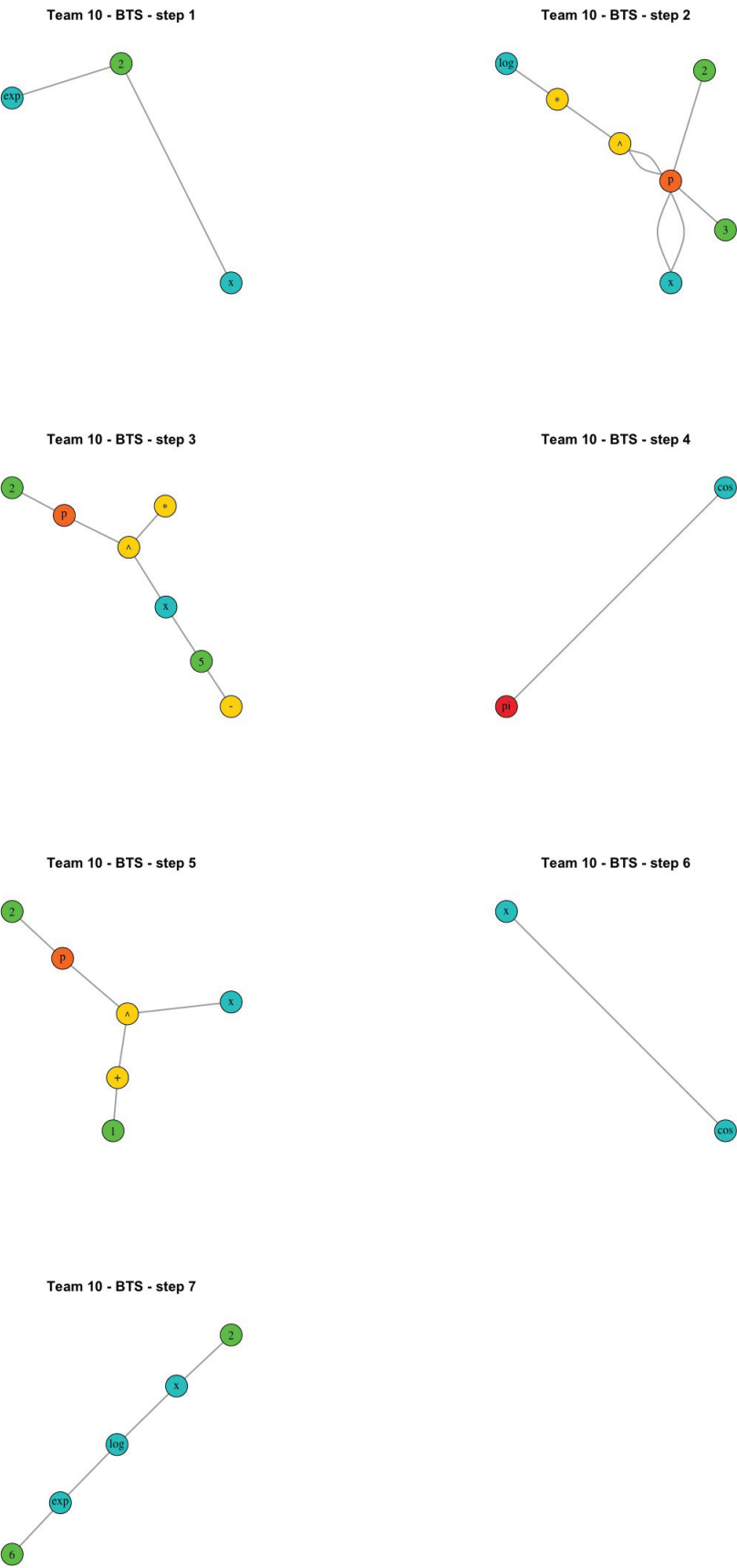


FIGURE C.10: Steps of Team 10 - Chorepography

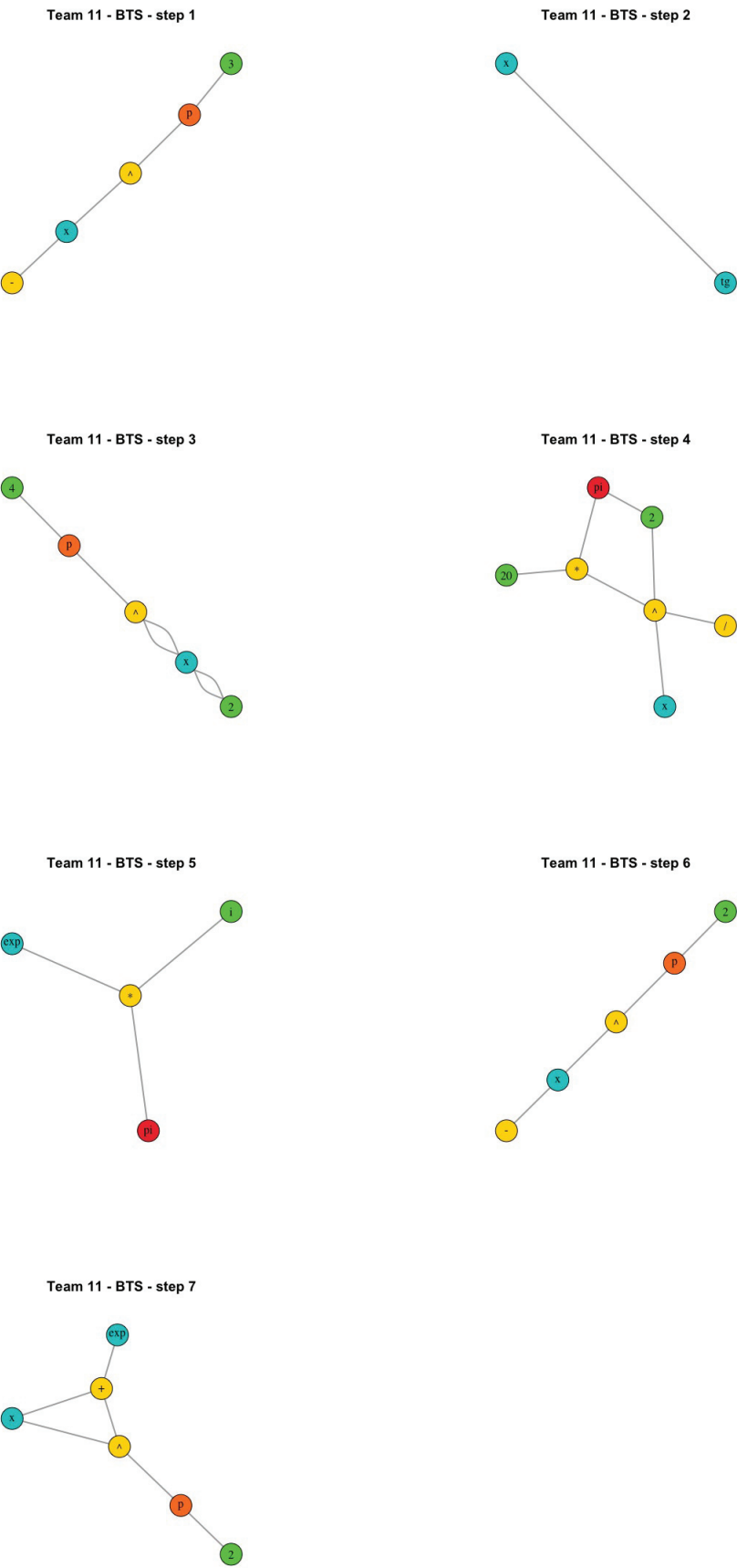


FIGURE C.11: Steps of Team 11 - Chorepography

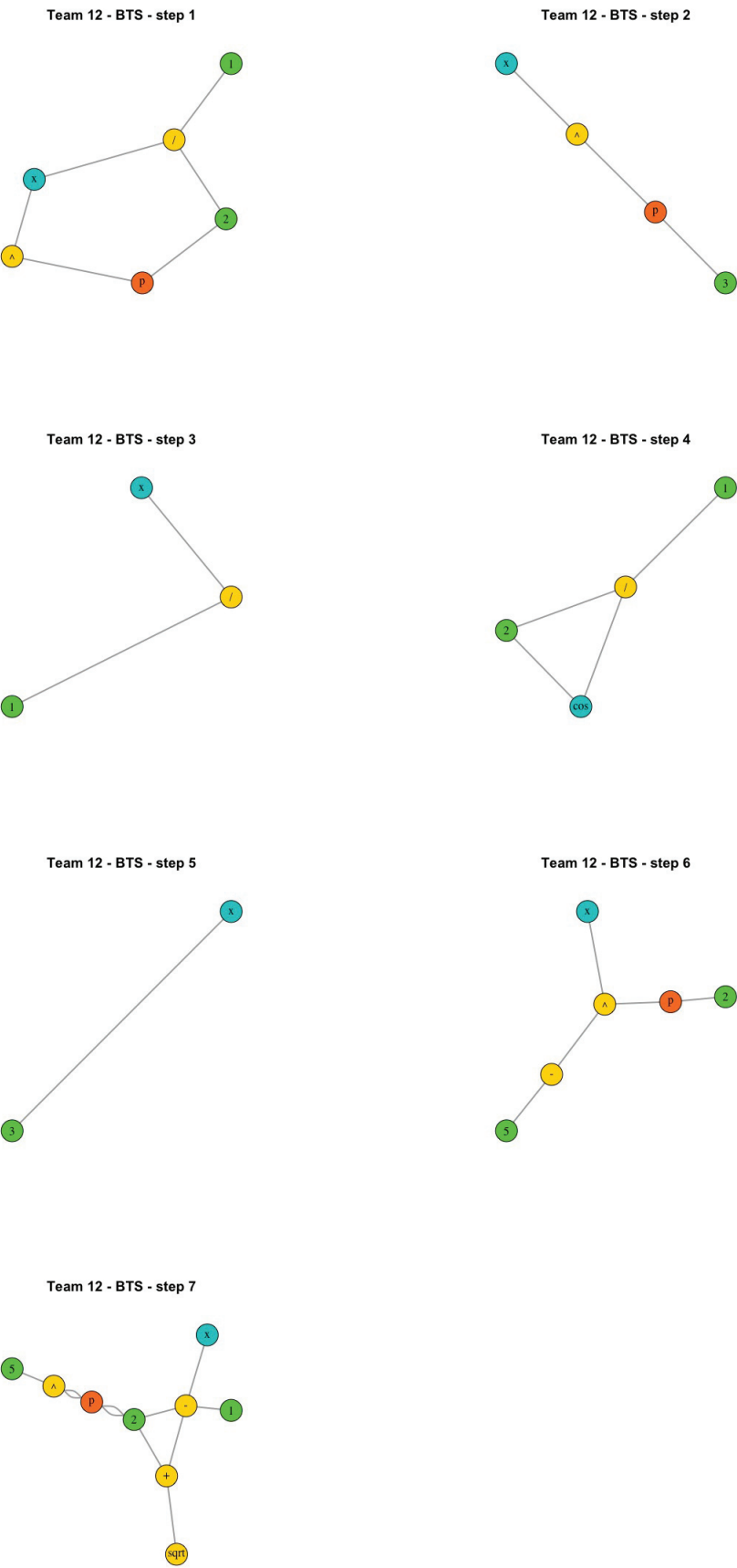


FIGURE C.12: Steps of Team 12 - Choreopography

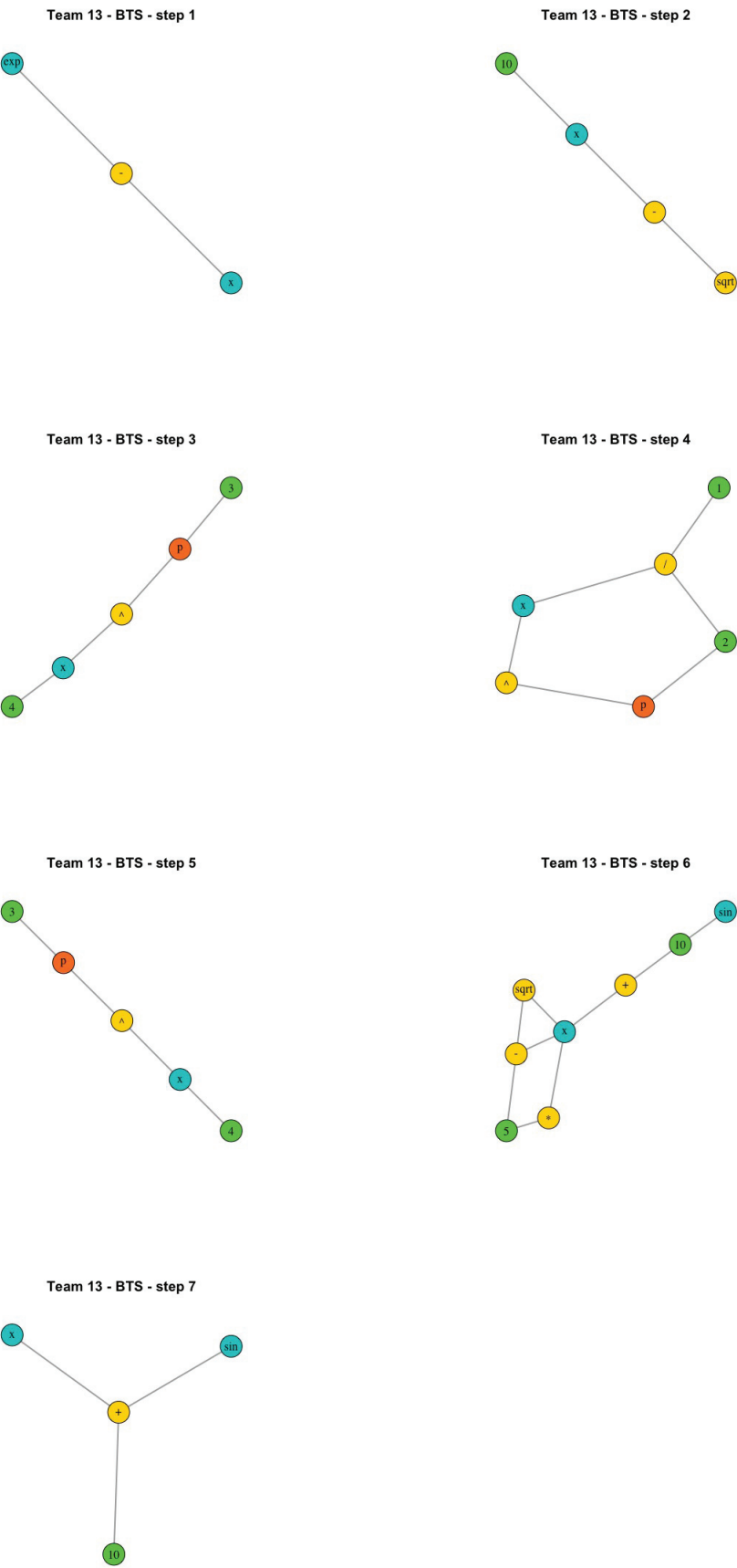


FIGURE C.13: Steps of Team 13 - Chorepography

Appendix D

INSA Choreographies

Esse é o C

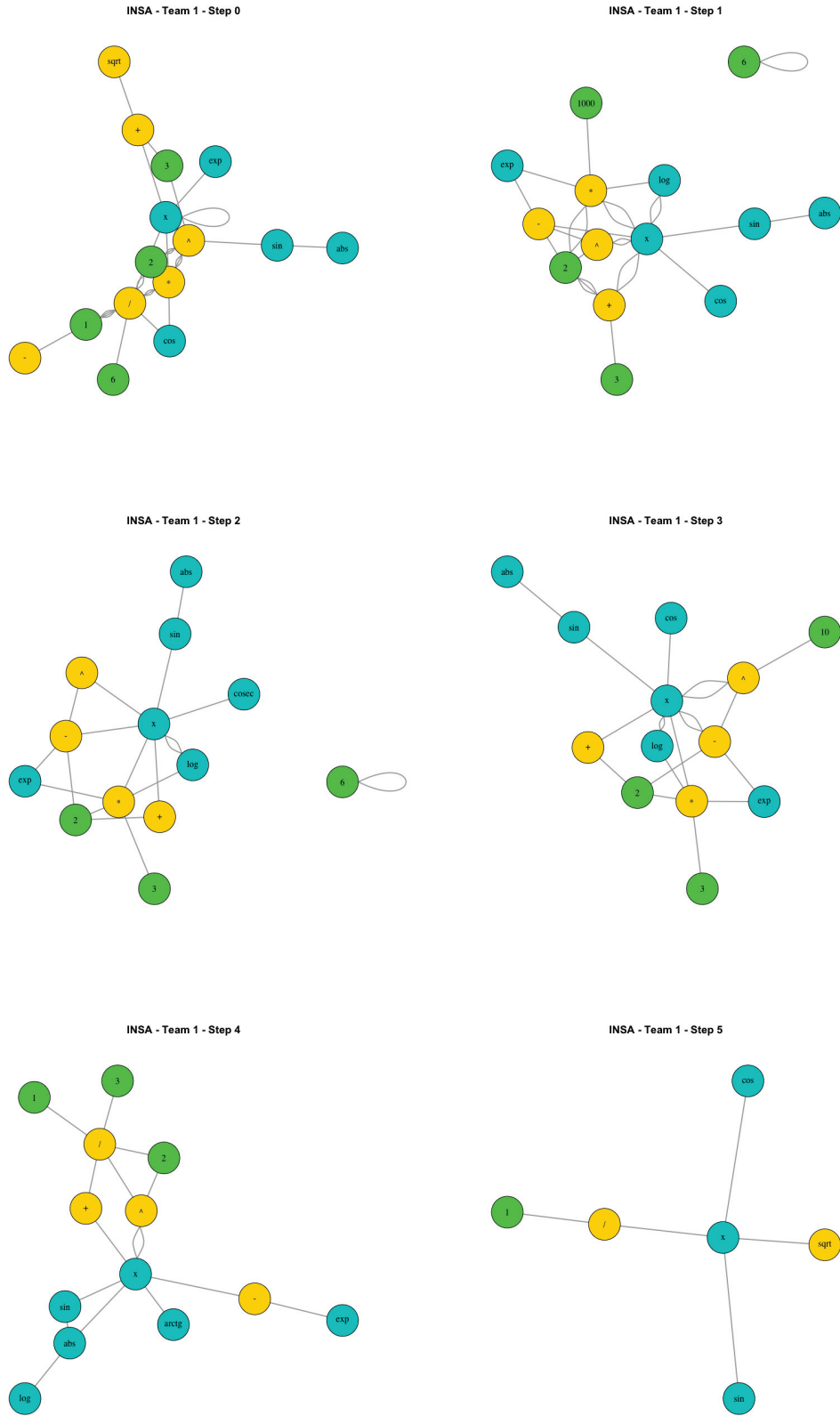


FIGURE D.1: Steps of Team 1 - Choreography - INSA

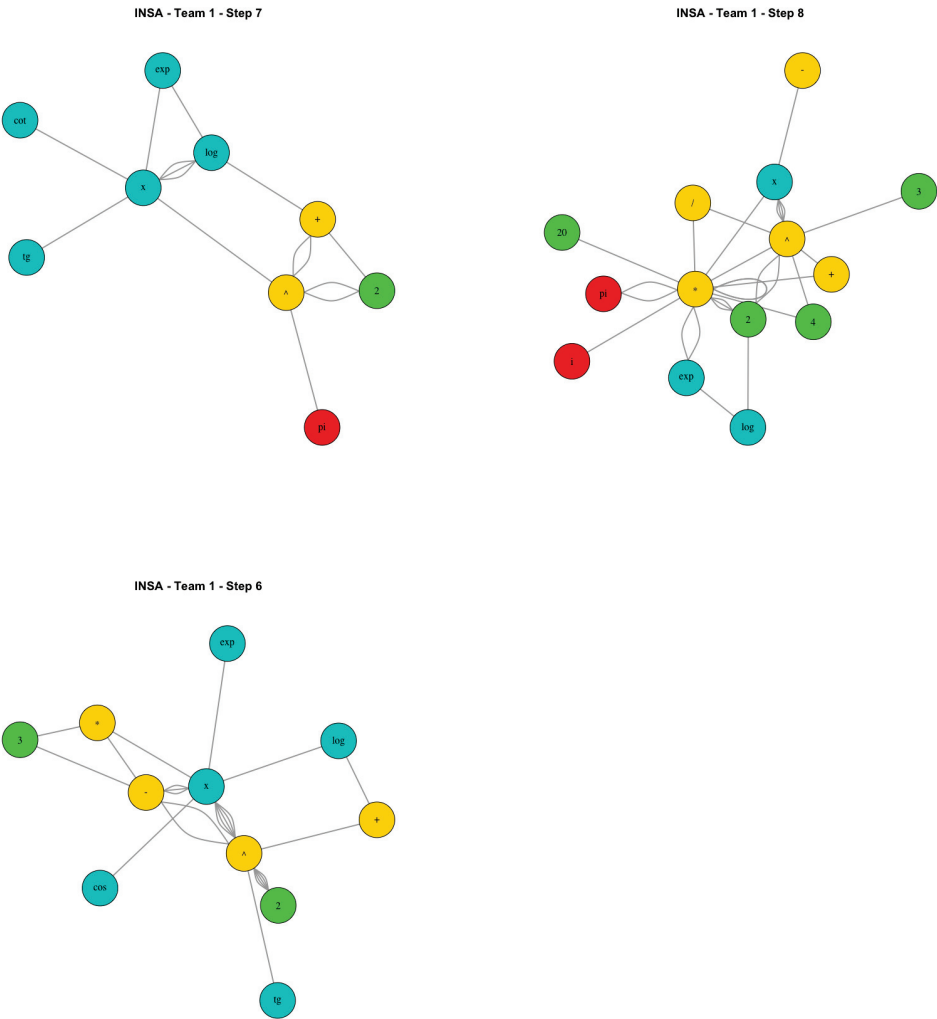


FIGURE D.2: Steps of Team 1 - Choreography - INSA

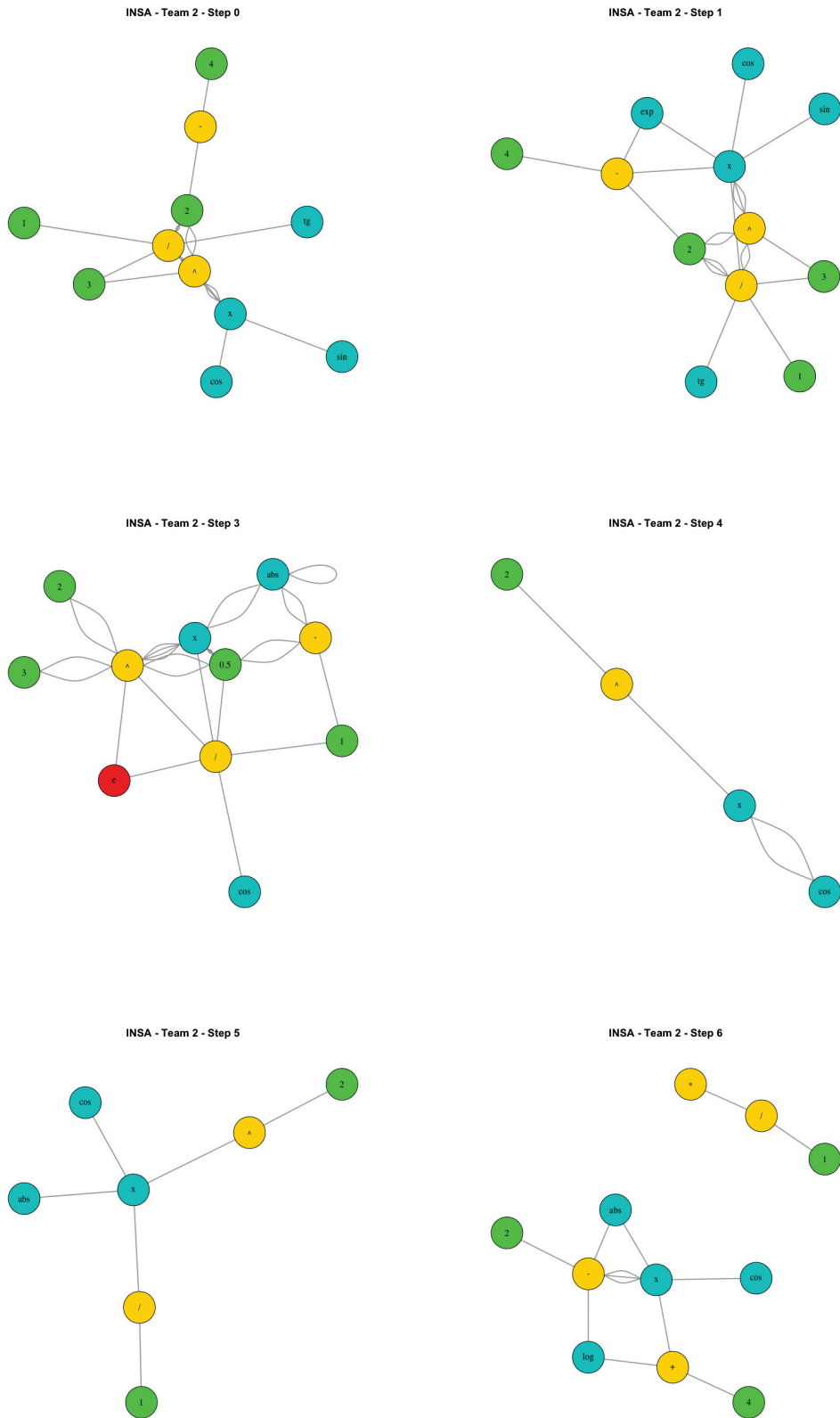


FIGURE D.3: Steps of Team 2 - Choreography - INSA

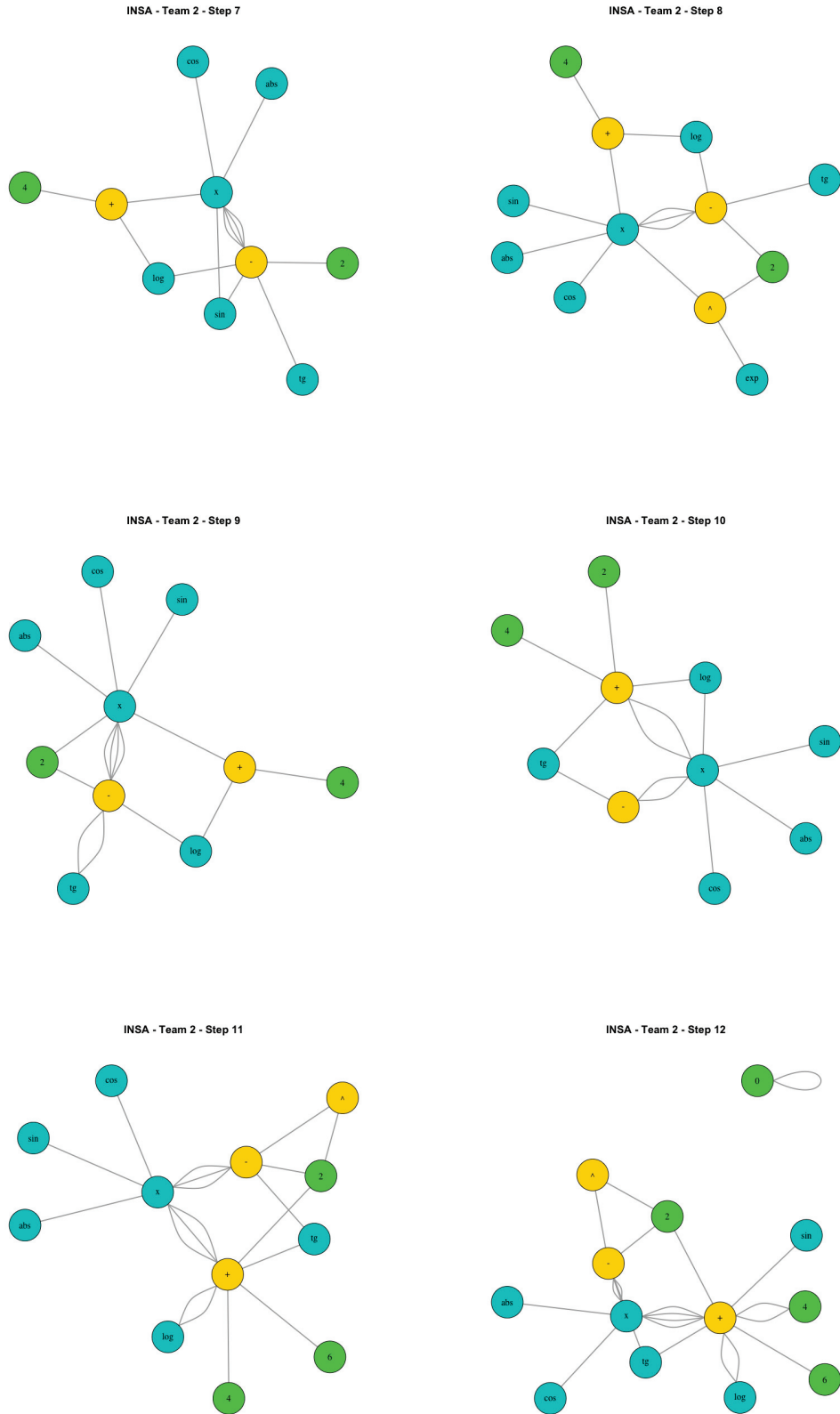


FIGURE D.4: Steps of Team 2 - Choreography - INSA

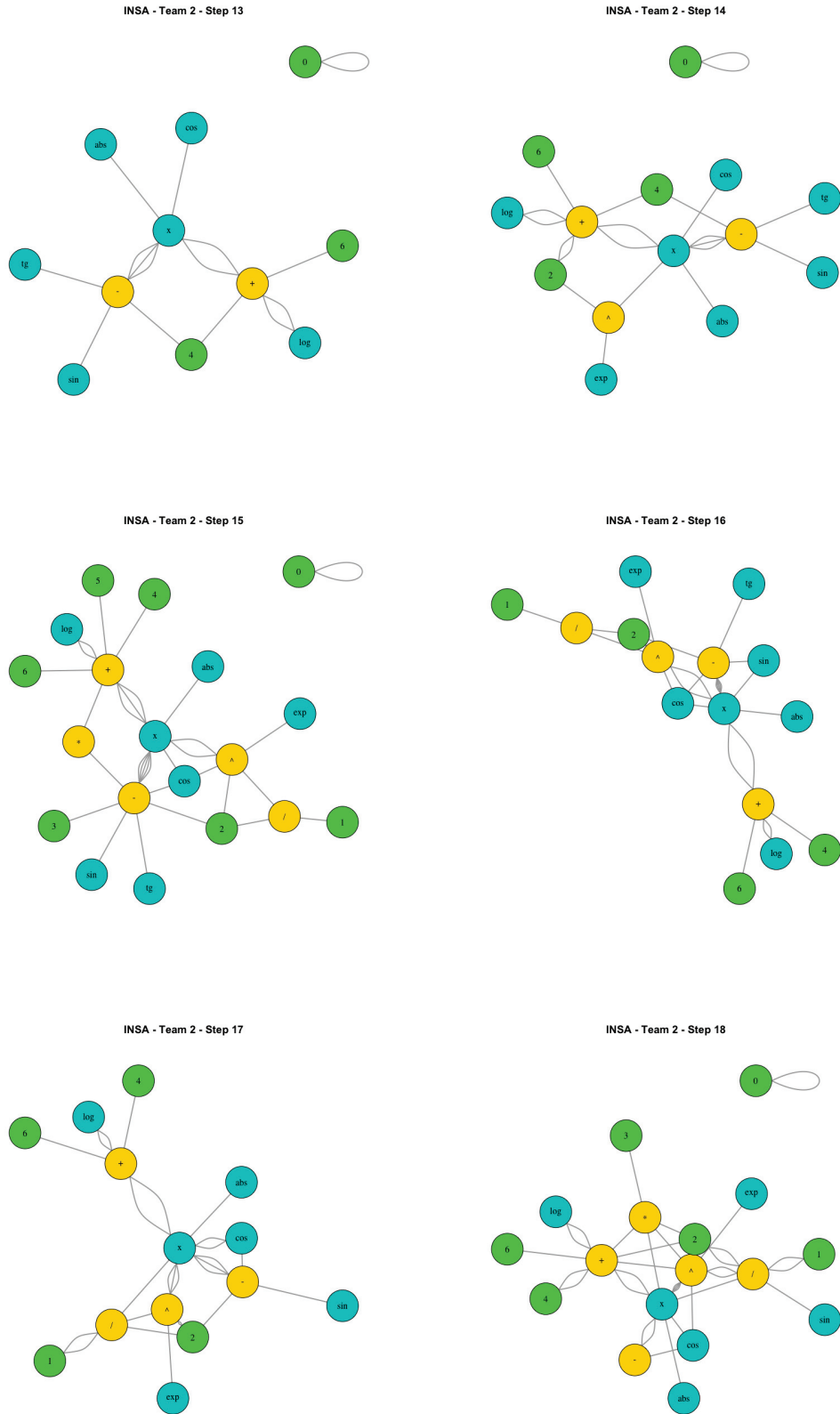


FIGURE D.5: Steps of Team 2 - Choreography - INSA

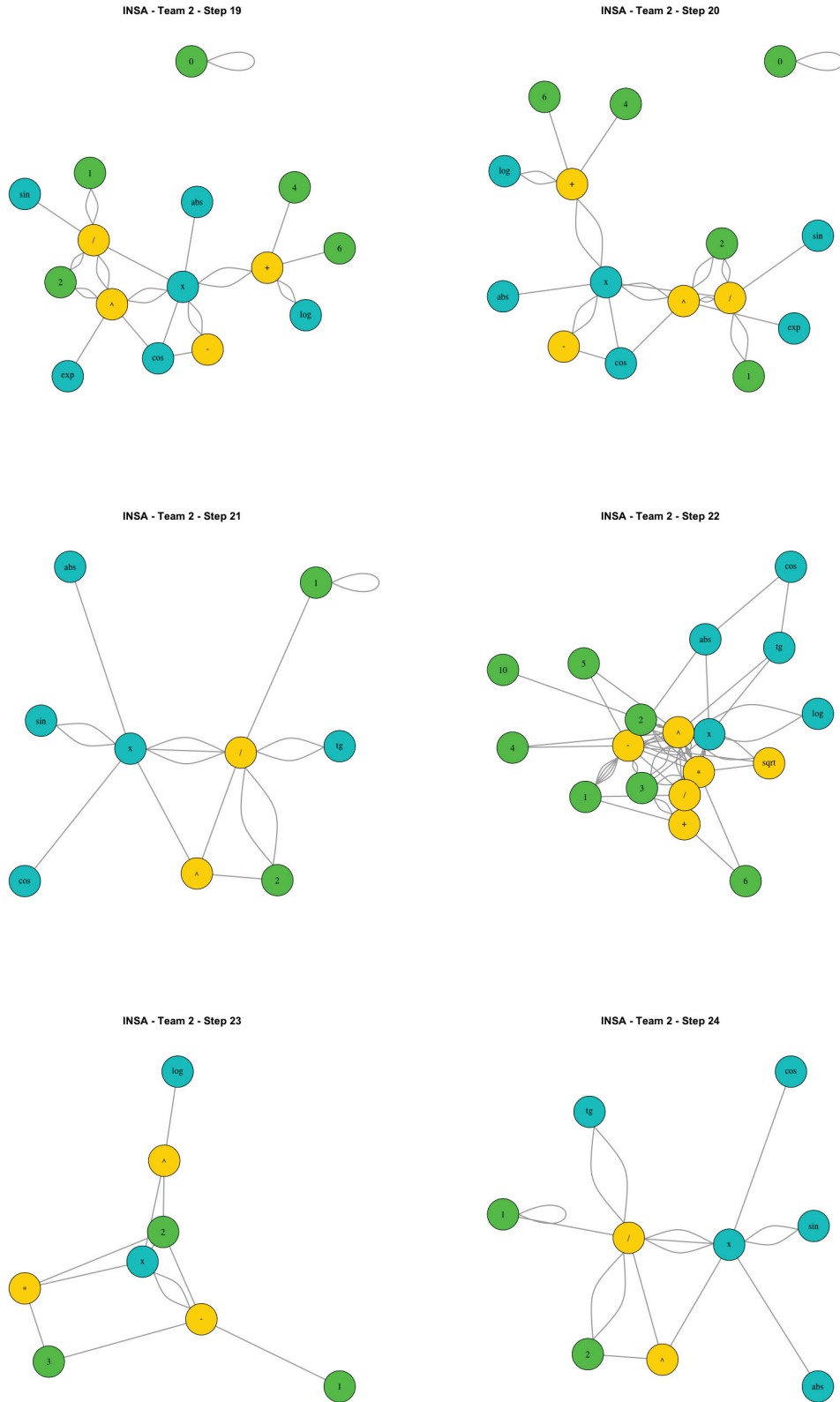


FIGURE D.6: Steps of Team 2 - Choreography - INSA

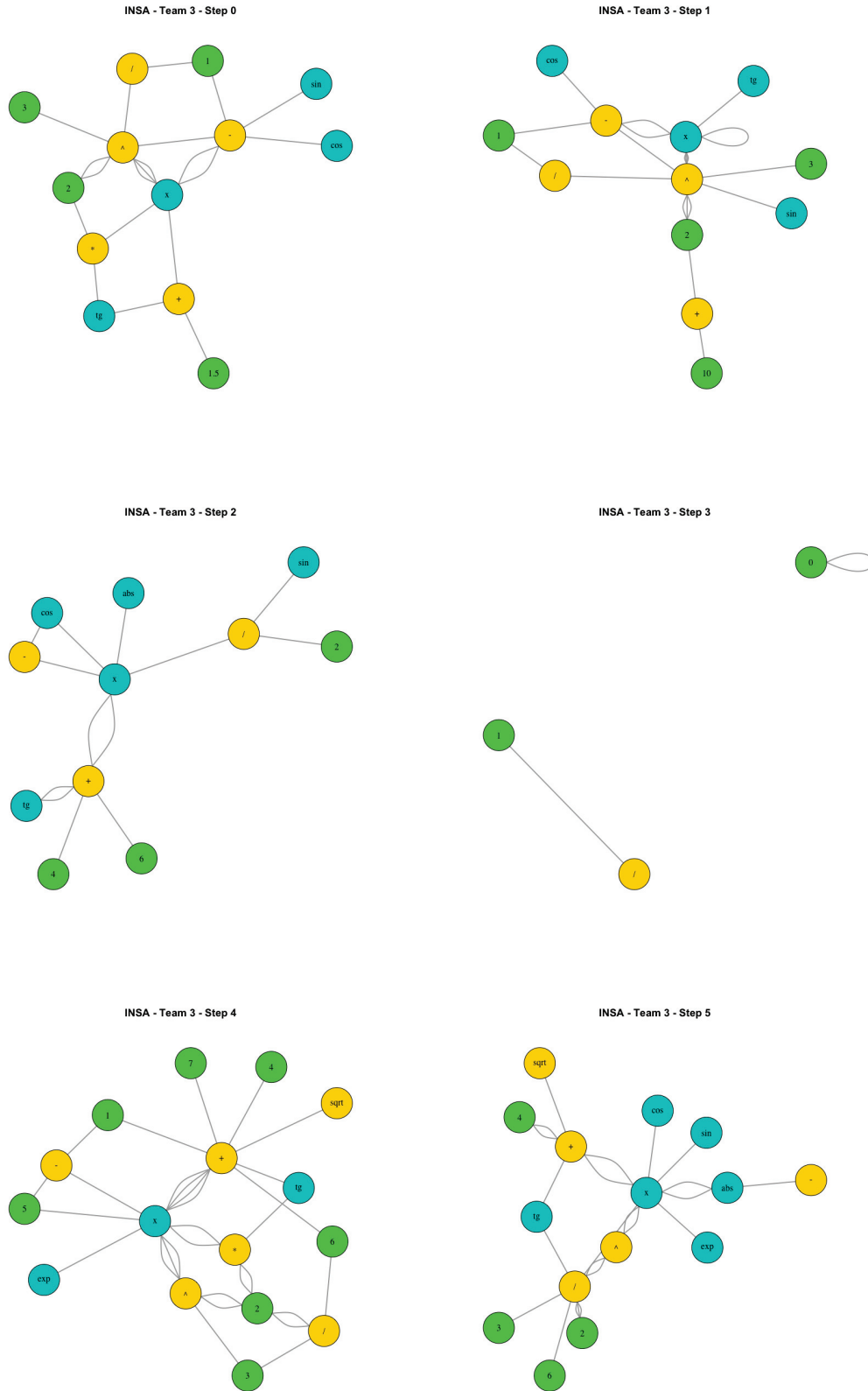


FIGURE D.7: Steps of Team 2 - Choreography - INSA

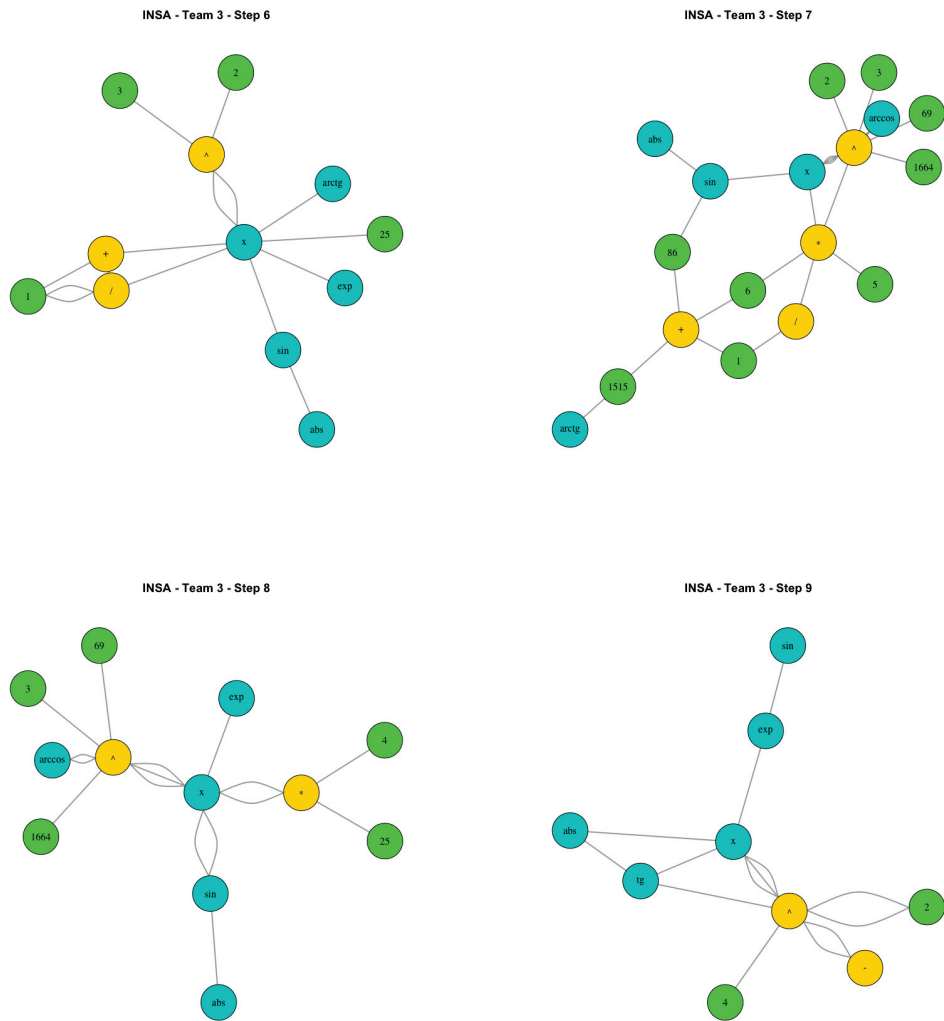


FIGURE D.8: Steps of Team 3 - Choreography - INSA

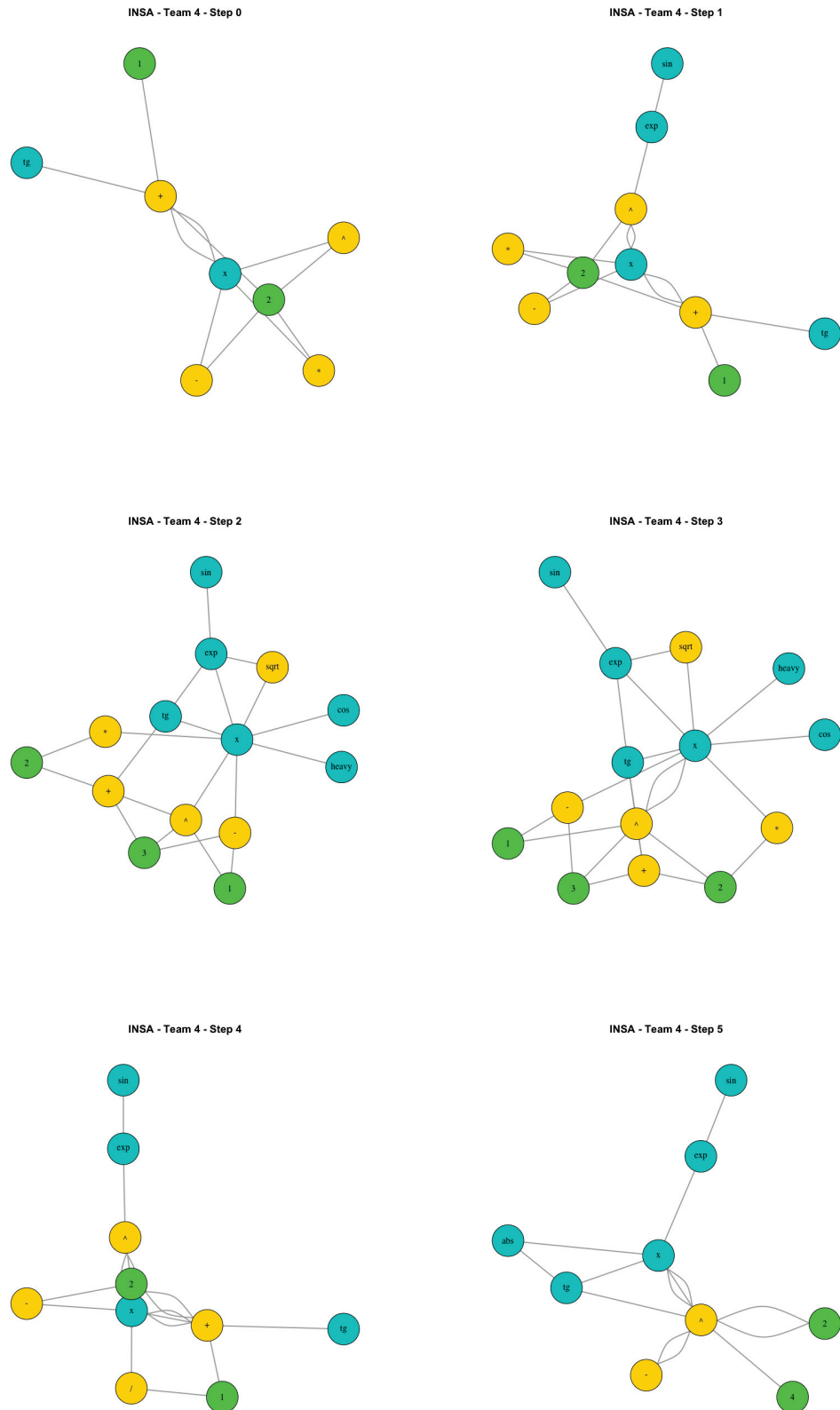


FIGURE D.9: Steps of Team 2 - Choreography - INSA



FIGURE D.10: Steps of Team 2 - Choreography - INSA

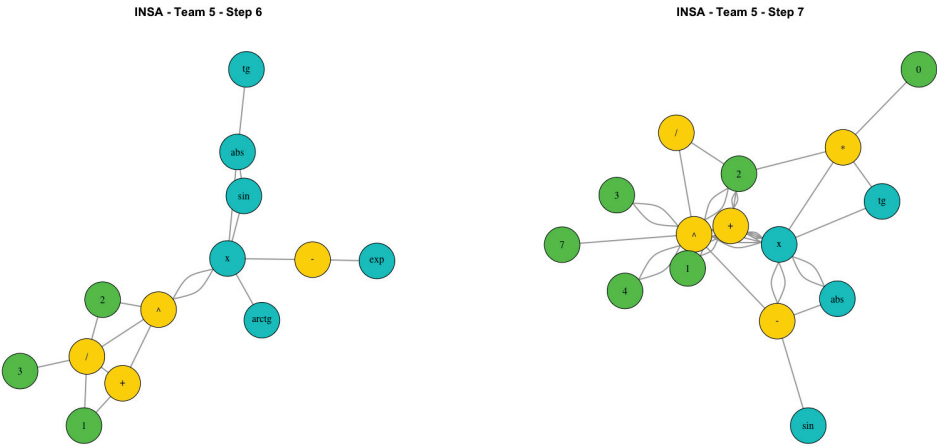


FIGURE D.11: Steps of Team 2 - Choreography - INSA

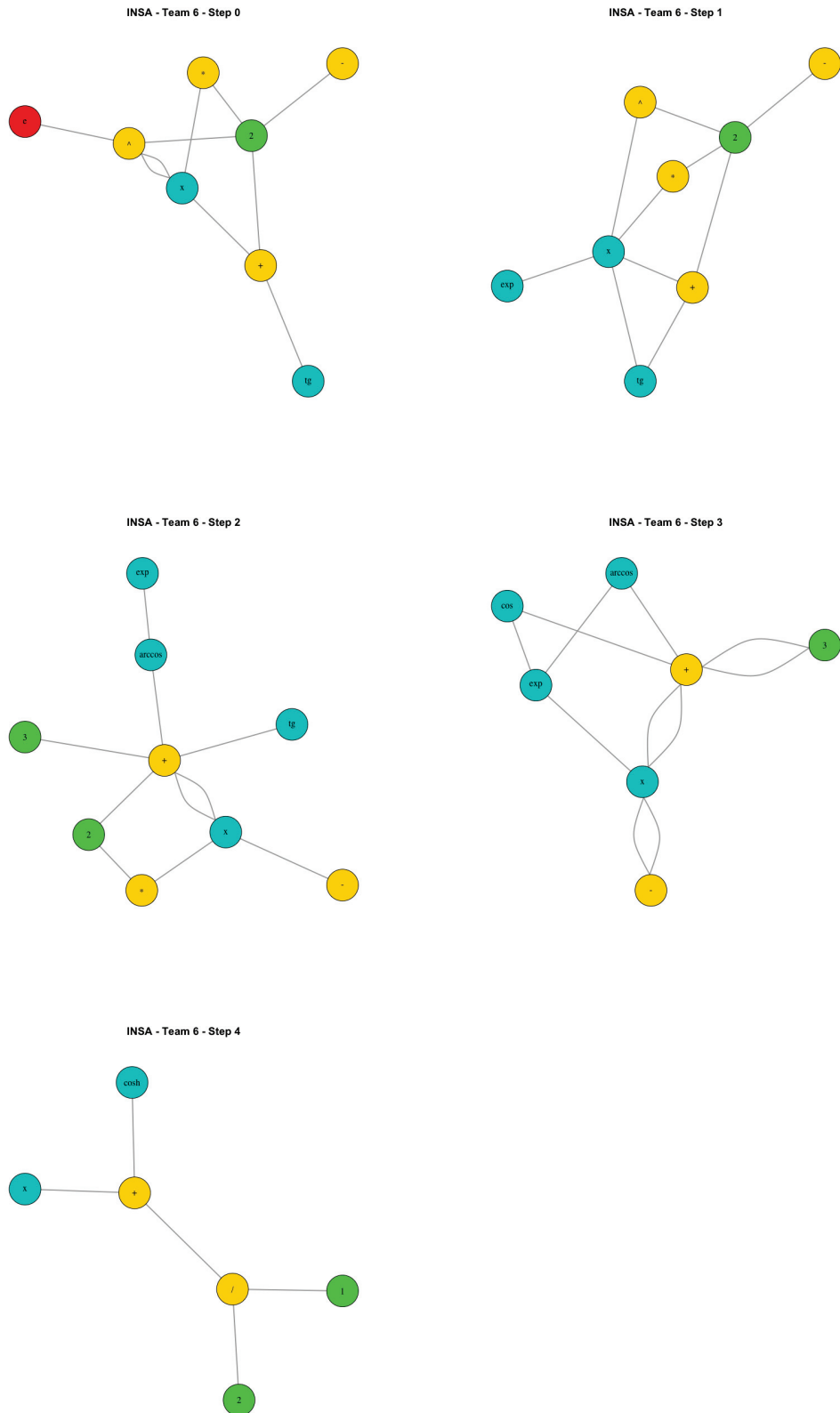


FIGURE D.12: Steps of Team 2 - Choreography - INSA

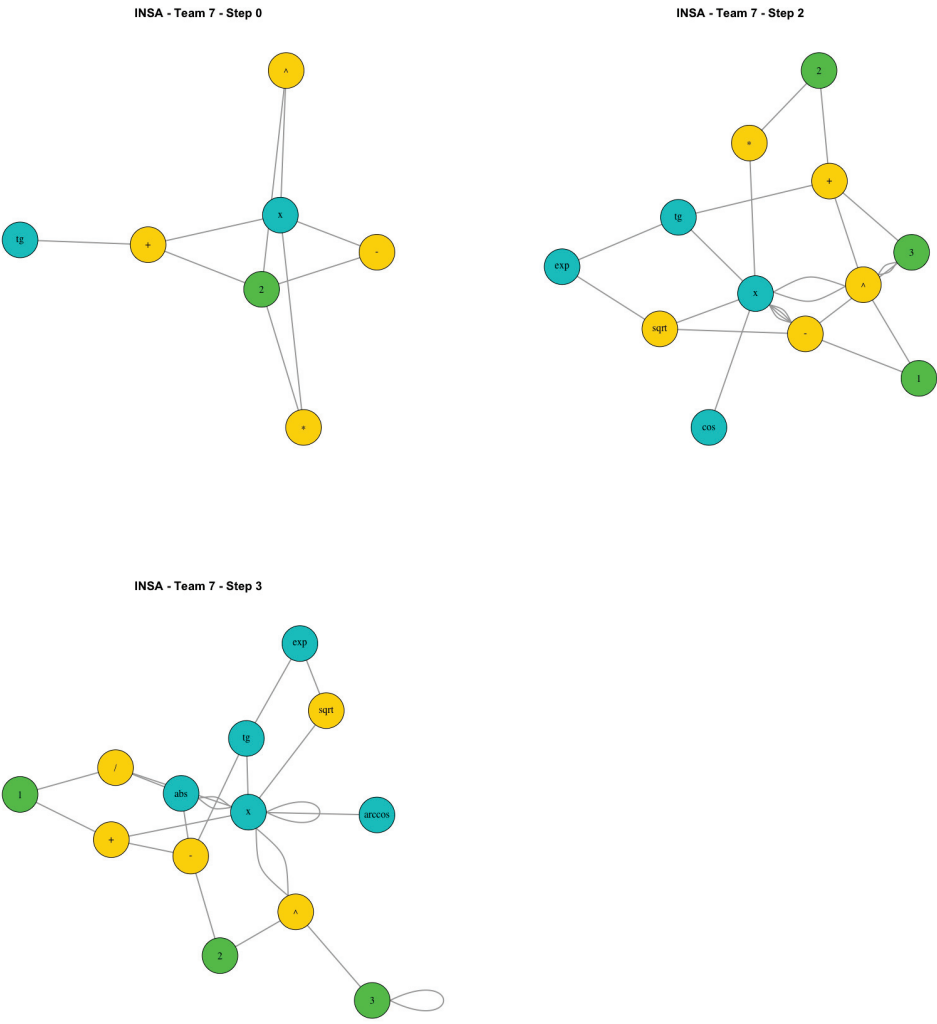


FIGURE D.13: Steps of Team 2 - Choreography - INSA

Appendix E

Binary Code: Q-A Map Solutions

E.1 Sample 01

E.1.1 Initial Question - Sample 01

Q_0 : Understand the trick and write an algorithm which performs it. It means, given a sequence of answers (Yes or No), one for each card, the algorithm returns the correct number chosen by someone.

E.1.2 Data Practices

- *DP.Q1 - Collecting Data*: Which data can be collected?
 1. *AD2*: The Data about the cards
 - *AD31*: Each card separately.
- *DP.Q4 - Analyzing Data*: Is there any conclusion to make about the observed data?
 1. *AD6*: Podemos observar que na primeira carta somente temos os numeros impares.
 2. *AD7*: Podemos observar que a última carta possui numeros de 16 a 31 em uma sequencia crescente.
 3. (*AD33*): O numero um está somente na primeira carta
 4. *AD34*: Se dizemos sim todas as vezes é o número 31 porque ele aparece em todas as cartas.
 5. *AD35*: A carta que começa pelo 17 contém todos os números 10 a 20.
 6. *AD8*: Podemos observar que em cada sequencia de cartas existem "bura-cos" entre os números.

E.1.3 Computational Problem Solving Practices

- *CPSP.Q6 - Creating Computational Abstractions*: Can we develop a generalization to the problem?
 1. *AD32*: Se o numero estiver na primeira carata ele será necessariamente impar.

E.2 Sample 02

E.2.1 Initial Question - Sample 02

Q_0 : Understand the trick and write an algorithm which performs it. It means, given a sequence of answers (Yes or No), one for each card, the algorithm returns the correct number chosen by someone.

E.2.2 Data Practices

- *DP.Q1 - Collecting Data*: Which data can be collected?
 1. *AD2*: The Data about the cards
 - *AD31*: Each card separately.
- *DP.Q4 - Analyzing Data*: Is there any conclusion to make about the observed data?
 1. *AD6*: Podemos observar que na primeira carta somente temos os numeros impares.
 2. *AD36*: Todas as outras cartas tem a mesma quantidade de números pares e ímpares.
 3. *AD37*: Os numeros 1, 2, 4, 8 e 16 aparecem somente uma vez.
 - *AD9*: Podemos observar que os primeiros números da sequência de cada carta aumentam em uma potência de 2.

E.2.3 Modeling Simulation Practices

- *MSP.Q1 - Using Computational Models to Understand a Concept*: Do you know any computational model to understand the phenonema of the problem? If yes, which one?
 1. *AD38*: Binary Decomposition.

E.2.4 Computational Problem Solving Practices

- *CPSP.Q7 - Troubleshooting and Debugging*: How can we verify whether the algorithm works?
 1. *AD39*: Verifying some examples.

E.2.5 System Thinking Practices

- *STP.Q4 - Communicating Information about a System*: How can we translate the solution into an entire system?
 1. *AD30* Realizamos a soma dos números que estão no lado inferior esquerdo de cada tabela.

E.3 Sample 03

E.3.1 Initial Question - Sample 03

Q0: Understand the trick and write an algorithm which performs it. It means, given a sequence of answers (Yes or No), one for each card, the algorithm returns the correct number chosen by someone.

E.3.2 Data Practices

- *DP.Q1 - Collecting Data*: Which data can be collected?
 1. *AD2*: The Data about the cards
 - *AD31*: Each card separately.
- *DP.Q3 - Manipulating Data*: How can we manipulate the data in order to make sense of it?
 1. *AD46*: Traduzimos as respostas sim por 1 e as respostas não por 0

E.3.3 System Thinking Practices

- *STP.Q3 - Thinking in Levels*
 1. *AD40*: Describe the cards separately
 - *AD41 - Card 01*:
 - * Sim: O número é ímpar

- * Não: O número é par
- AD42 - Card 02:
 - * Sim: O número faz parte da lista a partir de $2 + 1 + 3...$
 - * Nao: O número faz parte da lista a partir de $1 + 1 + 3...$
- AD43 - Card 03:
 - * Sim: O número faz parte da lista a partir de $4 + 1 + 1 + 5...$
 - * Nao: O número faz parte da lista a partir de $1 + 1 + 1 + 5...$
- AD44 - Card 04:
 - * Sim: O numero faz parte da lista a partir de $8 + 1 + 1 + 1 + ... + 9...$
 - * Nao: O numero faz parte da lista a partir de $1 + 1 + 1 + 1 + ... + 7...$
- AD45 - Card 05:
 - * Sim: numero ≥ 16
 - * Nao: numero < 16
- 1. **STP.Q4 - Communicating Information about a System:** How can we translate the solution into an entire system?
 - (a) AD47: Coletamos as respostas sim e não do participante, mudamos a ordem do numero binário e convertemos a decimal.

E.4 Sample 04

E.4.1 Initial Question - Sample 04

- **Q₀ - Initial Question:** Understand the trick and write an algorithm which performs it. It means, given a sequence of answers (Yes or No), one for each card, the algorithm returns the correct number chosen by someone.

E.4.2 Data Practices

- **DP.Q1 - Collecting Data:** Which data can be collected?
 1. AD1: The answers from the participant of the trick
 2. AD2: The Data about the cards
- **DP.Q3 - Manipulating Data:** How can we manipulate the data in order to make sense of it?

1. *AD46*: Traduzimos as respostas sim por 1 e as respostas não por 0
2. *AD5*: A table with the cards and the numbers

<i>numbers from 1 - 31</i>	0	1	2	3	4
1	1	0	0	0	0
2	0	1	0	0	0
3	1	1	0	0	0
4	0	0	1	0	0
5	1	0	1	0	0
.
.
.
30	0	1	1	1	1
31	1	1	1	1	1

E.4.3 System Thinking Practices

- *STP.Q4 - Communicating Information about a System*: How can we translate the solution into an entire system?
 1. *AD47*: Coletamos as respostas sim e não do participante, mudamos a ordem do numero binário e convertemos a decimal.

E.5 Sample 05

E.5.1 Initial Question - Sample 05

Q0: Understand the trick and write an algorithm which performs it. It means, given a sequence of answers (Yes or No), one for each card, the algorithm returns the correct number chosen by someone.

E.5.2 Data Practices

- *DP.Q1 - Collecting Data*: Which data can be collected?
 1. *AD1*: The answers from the participant of the trick
 2. *AD2*: The Data about the cards
- *DP.Q4 - Analyzing Data*: Is there any conclusion to make about the observed data?
 1. *AD40*: Describe the cards separately

- *AD41 - Card 01:*
 - * Sim: O número é ímpar
 - * Não: O número é par
- *AD48 - Card 02:*
 - * É composta apenas de numeros com 2^1 no sistema binário e por isso possui o numero 1 nessa coluna.
- *AD49 - Card 03:*
 - * É composta apenas de numeros com 2^2 no sistema binário e por isso possui o numero 1 nessa coluna.
- *AD50 - Card 04:*
 - * É composta apenas de numeros com 2^3 no sistema binário e por isso possui o numero 1 nessa coluna.
- *AD51 - Card 05:*
 - * É composta apenas de números com 2^4 no sistema binário e por isso possui o numero 1 nessa coluna.

E.6 Sample 06

E.6.1 Initial Question - Sample 06

Q_0 : Understand the trick and write an algorithm which performs it. It means, given a sequence of answers (Yes or No), one for each card, the algorithm returns the correct number chosen by someone.

E.6.2 Data Practices

- *DP.Q1 - Collecting Data:* Which data can be collected?
 1. *AD1:* The answers from the participant of the trick
 2. *AD2:* The Data about the cards
- *DP.Q4 - Analyzing Data:* Is there any conclusion to make about the observed data?
 1. *AD40:* Describe the cards separately
 - *AD41 - Card 01:*
 - * Sim: O número é ímpar

- * Não: O número é par
- AD48 - Card 02:
 - * É composta apenas de numeros com 2^1 no sistema binário e por isso possui o numero 1 nessa coluna.
- AD49 - Card 03:
 - * É composta apenas de numeros com 2^2 no sistema binário e por isso possui o numero 1 nessa coluna.
- AD50 - Card 04:
 - * É composta apenas de numeros com 2^3 no sistema binário e por isso possui o numero 1 nessa coluna.
- AD51 - Card 05:
 - * É composta apenas de números com 2^4 no sistema binário e por isso possui o numero 1 nessa coluna.

E.7 Sample 07

E.7.1 Initial Question - Sample 07

- **Q_0 - Initial Question:** Understand the trick and write an algorithm which performs it. It means, given a sequence of answers (Yes or No), one for each card, the algorithm returns the correct number chosen by someone.

E.7.2 Data Practices

- **DP.Q1 - Collecting Data:** Which data can be collected?
 1. AD1: The answers from the participant of the trick
 2. AD2: The Data about the cards
- **DP.Q3 - Manipulating Data:** How can we manipulate the data in order to make sense of it?
 1. AD4: It's possible to sort the numbers into a table

1	1	3	5	7	9	11	15	17	19	13	21	23	25	27	29	31
2	2	3	6	7	10	11	14	15	18	19	22	23	26	27	30	31
3	4	5	6	7	12	13	14	15	20	21	22	23	28	29	30	31
4	8	9	10	11	12	13	14	15	24	25	26	27	28	29	30	31
5	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31

- **DP.Q4 - Analyzing Data:** Is there any conclusion to make about the observed data?
 1. **AD9:** Podemos observar que os primeiros números da sequência de cada carta aumentam em uma potência de 2.

E.7.3 System Thinking Practices

- **STP.Q4 - Communicating Information about a System:** How can we translate the solution into an entire system?
 1. **AD30** Realizamos a soma dos números que estão no lado inferior esquerdo de cada tabela.

Appendix F

Fake Binary Code: Q-A Map Solutions

F.1 Teacher Q-A Map

F.1.1 Initial Question - Fake Binary Code - Teacher

Q_0 :

F.1.2 Data Practices

- **DP.Q1 - Collecting Data:** Which data can be collected?
 1. AD1: The answers from the participant
 2. AD2: The numbers of the trick
 3. AD3: The colors of the numbers
- **DP.Q2 - Creating Data:** Do we need to generate more data than we have?
 1. AD4: No
- **DP.Q3 - Manipulating Data:** How can we manipulate the data in order to make sense of it?
 1. AD5: Sorting the numbers into a table
 2. AD6: Converting the colors into a binary code.
- **DP.Q4 - Analyzing Data:** Is there any conclusion to make about the observed data?
 1. AD7: If the last number is odd, all the others are even.
 2. AD8: If the last number is even, all the others are odd.
 3. AD10: The Binary Decomposition doesn't work for all the numbers.

4. AD15: If the last number is red, the number is even.
5. AD16: If the last number is black, the number is odd.

F.1.3 Modeling Simulation Practices

- **MSP.Q1 - Using Computational Models to Understand a Concept:** Do you know any computational model to understand the phenonema of the problem? If yes, which one?
 1. AD9: Yes, the binary decomposition of the number could give me a hint.
- **MSP.Q2 - Using Computational Models to Find or Test Solutions:** Do you know any computational model to find or test the solutions you provided? If yes, which one?
 1. AD11: With the Binary Code the numbers could be at maximum 64, however there are number greater than 64.
 2. AD15: Putting The lines side by side and comparing the Binary Decomposition with the number, I revealed the pattern. (Illumination).
- **MSP.Q3 - Assessing Computational Models:** How can we assess the computational model?
 1. AD17: Testing for some numbers.
- **MSP.Q4 - Designing Computational Models:** Can we design a model to the problem?
 1. AD13:

F.1.4 Computational Problem Solving Practices

- **CPSP.Q1 - Preparing Problems for Computational Solutions:** Do we need to prepare the problem to be solved?
 1. AD12: Sorting all the lines
- **CPSP.Q2 - Programming:** How can we start programming the solution?
 1. AD14: Considering red as 1 and black as 0
- **CPSP.Q6 - Creating Computational Abstractions:** Can we develop a generalization to the problem?
 1. AD16: The numbers to multiply by the code of red and black are: 40, 20, 10, 4, 2, 1.

F.1.5 System Thinking Practices

- **STP.Q4 - Communicating Information about a System:** How can we translate the solution into an entire system?

1. AD19: Get the sequence of red and blacks from left to right, consider the values 1 to red and 0 to black, and perform the following:

$$value_0 * 40 + value_1 * 20 + value_2 * 10 + value_3 * 4 + value_4 * 2 + value_5 * 1 = result$$

F.1.6 Initial Question - Fake Binary Code - Sample 1

Q₀:

F.1.7 Data Practices

- **DP.Q1 - Collecting Data:** Which data can be collected?
 1. AD1: The answers from the participant
 2. AD2: The numbers of the trick
 3. AD3: The colors of the numbers
- **DP.Q3 - Manipulating Data:** How can we manipulate the data in order to make sense of it?
 1. AD5: Sorting the numbers into a table
 2. AD6: Converting the colors into a binary code.
- **DP.Q4 - Analyzing Data:** Is there any conclusion to make about the observed data?
 1. AD20: For the 1, there was only at the first column red, then the first column is 1.
 2. AD21: For the 2, there was only the second column red, then the second column is 2.
 3. AD22: For the 5, we have the first column which values 1 so the 3rd column is 4.
 4. AD23: For the 10, There was only the 4th column which is red, so it's 10.
 5. AD24: For the 20, There was only the 5th column which is red, so it's 20.
 6. AD25: For the 40, There was only the 6th column which is red, so it's 40.

F.1.8 Modeling Simulation Practices

- *MSP.Q1 - Using Computational Models to Understand a Concept:* Do you know any computational model to understand the phenomenon of the problem? If yes, which one?
 1. AD9: Yes, The Binary Code.
- *MSP.Q2 - Using Computational Models to Find or Test Solutions:* Do you know any computational model to find or test the solutions you provided? If yes, which one?
 1. AD15: Putting The lines side by side and comparing the Binary Decomposition with the number.

F.1.9 System Thinking Practices

- *STP.Q4 - Communicating Information about a System:* How can we translate the solution into an entire system?
 1. AD19: Get the sequence of red and blacks from left to right, consider the values 1 to red and 0 to black, and perform the following:

$$value_0 * 40 + value_1 * 20 + value_2 * 10 + value_3 * 4 + value_4 * 2 + value_5 * 1 = result$$

F.1.10 Initial Question - Fake Binary Code - Sample 2

Q₀:

F.1.11 Data Practices

- *DP.Q1 - Collecting Data:* Which data can be collected?
 1. AD1: The answers from the participant
 2. AD2: The numbers of the trick
 3. AD3: The colors of the numbers
- *DP.Q3 - Manipulating Data:* How can we manipulate the data in order to make sense of it?
 1. AD6: Converting the colors into a binary code.
- *DP.Q4 - Analyzing Data:* Is there any conclusion to make about the observed data?

1. AD20: For the 1, there was only at the first column red, then the first column is 1.
2. AD21: For the 2, there was only the second column red, then the second column is 2.
3. AD22: For the 5, we have the first column which values 1 so the 3rd column is 4.
4. AD10: The Binary Decomposition doesn't work for all the numbers.
5. AD26: Each column has an independent value from the others.

F.1.12 Modeling Simulation Practices

- *MSP.Q1 - Using Computational Models to Understand a Concept:* Do you know any computational model to understand the phenomenon of the problem? If yes, which one?
 1. AD9: Yes, The Binary Code.
- *MSP.Q2 - Using Computational Models to Find or Test Solutions:* Do you know any computational model to find or test the solutions you provided? If yes, which one?
 1. AD15: Putting The lines side by side and comparing the Binary Decomposition with the number.

F.1.13 System Thinking Practices

- *STP.Q4 - Communicating Information about a System:* How can we translate the solution into an entire system?
 1. AD19: Get the sequence of red and blacks from left to right, consider the values 1 to red and 0 to black, and perform the following:

$$value_0 * 40 + value_1 * 20 + value_2 * 10 + value_3 * 4 + value_4 * 2 + value_5 * 1 = result$$

F.1.14 Initial Question - Fake Binary Code - Sample 3

Q₀:

F.1.15 Data Practices

- *DP.Q1 - Collecting Data:* Which data can be collected?

1. AD1: The answers from the participant
 2. AD2: The numbers of the trick
 3. AD3: The colors of the numbers
- **DP.Q3 - Manipulating Data:** How can we manipulate the data in order to make sense of it?
 1. AD5: Sorting the numbers into a table
 2. AD6: Converting the colors into a binary code.
 3. AD27: Splitting the sequence in two parts with three codes. Dizaines and unities.
 - **DP.Q4 - Analyzing Data:** Is there any conclusion to make about the observed data?
 1. AD15: If the last number is red, the number is even.
 2. AD16: If the last number is black, the number is odd.

F.1.16 Modeling Simulation Practices

- **MSP.Q1 - Using Computational Models to Understand a Concept:** Do you know any computational model to understand the phenonema of the problem? If yes, which one?
 1. AD9: Yes, the binary decomposition of the number could give me a hint.
- **MSP.Q2 - Using Computational Models to Find or Test Solutions:** Do you know any computational model to find or test the solutions you provided? If yes, which one?
 1. AD15: Putting The lines side by side and comparing the Binary Decomposition with the number.
- **MSP.Q3 - Assessing Computational Models:** How can we assess the computational model?
 1. AD17: Testing for some numbers.

F.1.17 System Thinking Practices

- **STP.Q4 - Communicating Information about a System:** How can we translate the solution into an entire system?

1. AD28: Get the sequence of red and blacks from left to right, consider the values 1 to red and 0 to black, and perform the following:

$$(value_0 * 4 + value_1 * 2 + value_2 * 1) * 10 + (value_3 * 4 + value_4 * 2 + value_5 * 1) = result$$

Appendix G

Binary Code - Data

110 00

$1 \times 2^2 + 4 \times 2^0$

111

16

$2^4 2^3 2^2 2^1 2^0$
10110

1

2^4	2^3	2^2	2^1	2^0
1	1	0	1	
1	1	1	1	

~~16 + 8 + 2 + 1~~

13 = 1

11100
1011

2^4	2^3	2^2	2^1	2^0

1100

11011

- 1 ... 31
- 2 ... 34
- 4 ... 31
- 8 ... 31
- 16 ... 31

8

$7 = 11100$

00010

10101

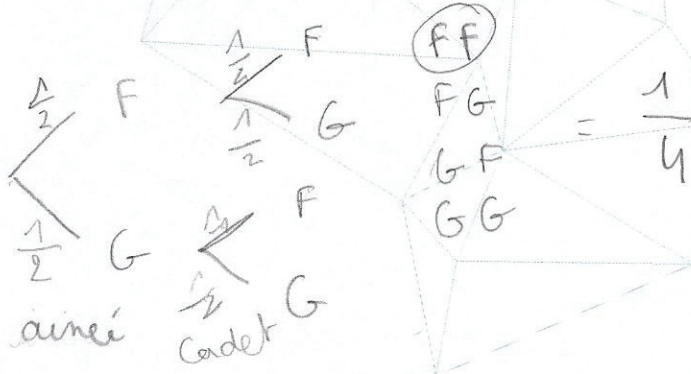
5

10010

"Les sophie"

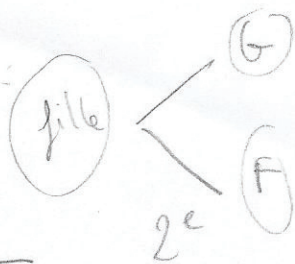
MathC2+

On connaît un couple qui a 2 enfants.
Quelle est la proba pour que ce soient 2 filles ?



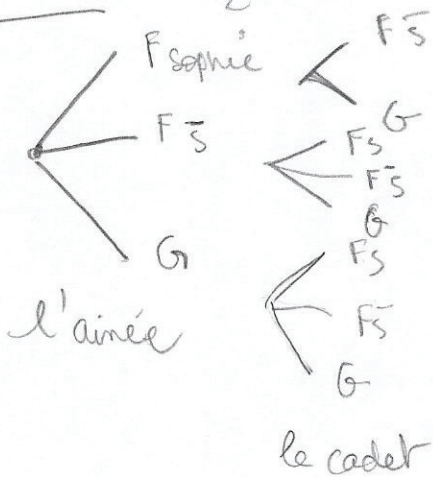
	F	G	
F	25%	25%	50%
G	25%	25%	50%

$P(2f) = \frac{1}{4}$ $P(1f+1g) = \frac{1}{2}$ $P(2g) = \frac{1}{4}$



$\frac{1}{2} = g$
 $\frac{1}{2} = f$

$P_{\text{au-1}^{\text{er}}} (2f) = \frac{1}{3}$



$P_{FS} (2 \text{ filles})$

	FS	F \bar{S}	G	
FS	0	$\frac{1}{600}$	$\frac{1}{600}$	$\frac{1}{300}$
F \bar{S}	$\frac{1}{600}$	$\frac{148}{6000}$	$\frac{149}{6000}$	$\frac{149}{3000}$
G	$\frac{1}{600}$	$\frac{149}{6000}$	$\frac{1}{4}$	$\frac{1}{2}$
	$\frac{1}{300}$	$\frac{149}{3000}$	$\frac{1}{2}$	

27 35 20 2 21 34

BALLESTEROS
Chloe'

17 15 24 23 25 27 28 29

La première carte n'a que des nombres impairs.
↳ si le nombre est dedans, il sera forcément impair.

1 + 2

Une carte ai les nombres sont tous plus grands que 16

+ 16

Le 1 n'est que sur une seule carte.

Si on dit tout oui, le nombre est le 31 car il est sur toutes les cartes.

La carte commençant par 17 contient tout les nombres de 16 à 20.

Une carte avec des suites de 4.



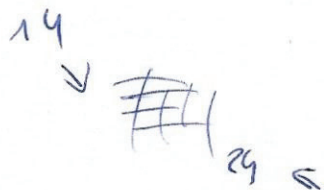
si impair = table du 3

0 0 1 0 1

1 + 2 + 4

0 0 1 1 1

5 → 1 +



2 suite

8 → 15
24 → 31

16 + 4 = 1

+ 8 unique

1 0 1 1 0 0 0

6 →

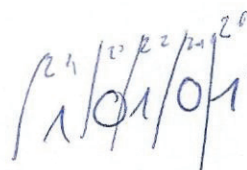
12

suite de 4 : 4 → 7

20 → 23

12 → 15

28 → 31



4 3 1 1 0

1 0 1 0 0

MathC2+

atelier mathématiques mercredi 27 juin

- 1 grille tous les nombres impairs
- les autres ont un nombre égal de nombre pairs et impairs
- le n° 1 et 2, 4, 8, 16 n'apparaissent qu'une fois
- et ne sont pas sur la même grille. → puissance de 2.
- une grille [16 ; 31]

16

binaire.

16	8	4	2	1
2	2	2	2	2
0	1	1	0	0

$$8 + 4 = 12$$

$$1 \quad 0 \quad 1 \quad 0 \quad 1$$

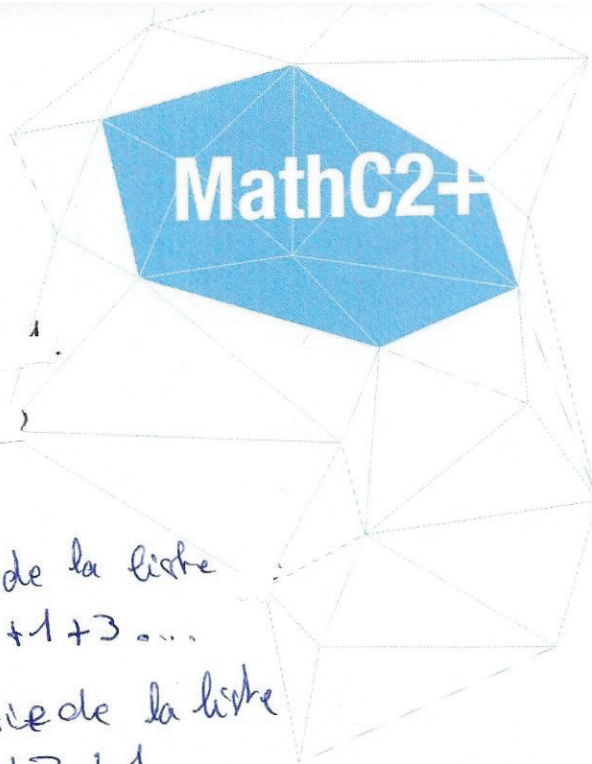
$$16 + 4 + 1 = 21$$

$$1 \quad 1 \quad 1 \quad 0 \quad 1$$

$$16 + 8 + 4 + 1 = 29$$

$$1 + 2 + 4 + 8 + 16 + \dots$$

quand oui → ajoute nb en bas gauche.
si non → rien.



Grille 1

oui \rightarrow nb impair

non \rightarrow nb pair

Grille 2

oui \rightarrow nb fait partie de la liste

à partir de 2 puis $+1+3 \dots$

non \rightarrow nb fait partie de la liste

à partir de 1 puis $+3+1 \dots$

Grille 3

oui \rightarrow nb fait partie de la liste à partir de 4 ~~+~~ puis $+7+1+1$

non \rightarrow ⁺⁵ nb fait partie de la liste à partir de 1 puis $+1+1+5$

Grille 5

oui \rightarrow nb ≥ 16

non \rightarrow nb < 16

Grille 4

oui \rightarrow nb fait partie de la liste à partir de 8 puis $+1+1+1+1+1$
 $+1+1+9 \dots$

non \rightarrow nb fait partie de la liste à partir de 1 puis $+1+1+1$
 $+1+1+1+1+7 \dots$

on écrit les chiffres de 1 à 31 à l'envers (2: 0100)

On traduit les oui par des 1 et des non par des 0 (oui, non,

oui, oui, non \rightarrow 10110 \rightarrow 01101 \rightarrow 13)

on traduit

ou le mot
à l'envers

on traduit
du binaire
au décimal



g²

2-3-6-7-10-11-14-15-18-19-22-23-26-27-30-31

1 145 89

g³

4-5-6-7-12-13-14-15-20-21-22-23-28-29-30-31

6891011

-2-3

g⁵

8-9-10-11-12-13-14-15-24-25-26-27-28-29-30-31

8+1 11+1 11+1 11+1

+1+7

+7

Pour chaque nombre, code binaire de 5 chiffres: s'il y est, on met 1 dans la première, deuxième, troisième, quatrième ou cinquième position ~~suivant~~ le numéro du tableau numérotés de 1 à 5 sinon on y met un zéro

1:	1 0 0 0 0	U	00001
2:	0 1 0 0 0	U	00010
3:	1 1 0 0 0	U	00011
4:	0 0 1 0 0	U	00100
5:	1 0 1 0 0	U	00101
6:	0 1 1 0 0	U	00110
7:	1 1 1 0 0	U	00111
8:	0 0 0 1 0	U	01001
9:	1 0 0 1 0	U	01010
10:	0 1 0 1 0	U	01011
11:	1 1 0 1 0	U	01100
12:	0 0 1 1 0	U	01101
13:	1 0 1 1 0	U	01110
14:	0 1 1 1 0	U	01111
15:	1 1 1 1 0	U	10000
16:	0 0 0 0 1	U	10001
17:	1 0 0 0 1	U	10010
18:	0 1 0 0 1	U	10011
19:	1 1 0 0 1	U	10100
20:	0 0 1 0 1	U	10101
21:	1 0 1 0 1	U	10110
22:	0 1 1 0 1	U	10111
23:	1 1 1 0 1	U	11000
24:	0 0 0 1 1	U	11001
25:	1 0 0 1 1	U	11010
26:	0 1 0 1 1	U	11011
27:	1 1 0 1 1	U	11100
28:	0 0 1 1 1	U	11101
29:	1 0 1 1 1	U	11110
30:	0 1 1 1 1	U	11111
31:	1 1 1 1 1	U	

Grille 1: Si on choisit un chiffre de la grille, ie est forcément impair car il n'y a que des ~~chiffres~~ ^{nombre} impairs et donc si on ne choisit pas un chiffre de cette grille alors le ^{nombre} ~~chiffre~~ est pair.

Donc la grille 1 permet de savoir si le ^{nombre} ~~chiffre~~ choisi est pair ou impair. La grille 1 n'est composée que de nombre 2^0 en binaire donc elle possède 1 dans cette colonne.

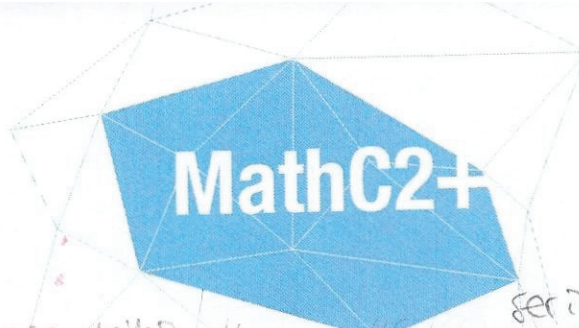
Grille 2: La grille 2 est composée exclusivement de ~~chiffres~~ ^{nombre} avec 2^1 dans le système binaire donc elle possède 1 dans cette colonne.

Grille 3: Cette grille est composée de nombre ~~de nombre~~ ^{nombre} avec 2^2 dans le système binaire, donc elle possède 1 dans cette colonne.

Grille 4: Cette grille est composée de nombre avec 2^3 dans le système binaire.

Grille 5: Cette grille est composée de nombre avec 2^4 dans le système binaire. donc elle possède 1 dans cette colonne.

- Grille 1: elle donne l'indice des 2^0 dans le système binaire, les nombres dessus sont tous les impairs entre 1 et 31 (l'ensemble de définition). le nombre choisi est donc forcément impair s'il s'y trouve.
- Grille 2: elle donne l'indice des 2^1 dans le système binaire, les nombres qui s'y trouvent possèdent un 1 dans la colonne des 2^1 .
- Grille 3: elle donne l'indice des 2^2 dans le système binaire, les nombres qui s'y trouvent possèdent un 1 dans la colonne des 2^2 .
- Grille 4: elle donne l'indice des 2^3 dans le système binaire, les nombres qui s'y trouvent possèdent un 1 dans la colonne des 2^3 .
- Grille 5: elle donne l'indice des 2^4 dans le système binaire, les nombres qui s'y trouvent possèdent un 1 dans la colonne des 2^4 .



① impaire ou paire

3	7	23	19
17	9	11	31
5	27	21	13
1	25	29	15

plus petit

pair = table 2
impair = table 3

14	26	6	27
3	30	15	18
31	6	19	23
2	7	11	22

plus petit numero 2 non

serie des
8 9 10 11 12 13 14 15
24 25 26 27 28 29 30 31

14	10	15	31
11	27	29	13
26	28	9	12
8	30	35	24

plus petit numero 8

17	23	19	25
27	30	29	18
22	21	26	24
16	28	31	20

16 à 31 en dessous de 16

4 5 6 7
12 13 14 15
20 21 22 23
29 29 30 31

6	5	20	14
28	23	15	13
31	29	21	22
4	30	7	12

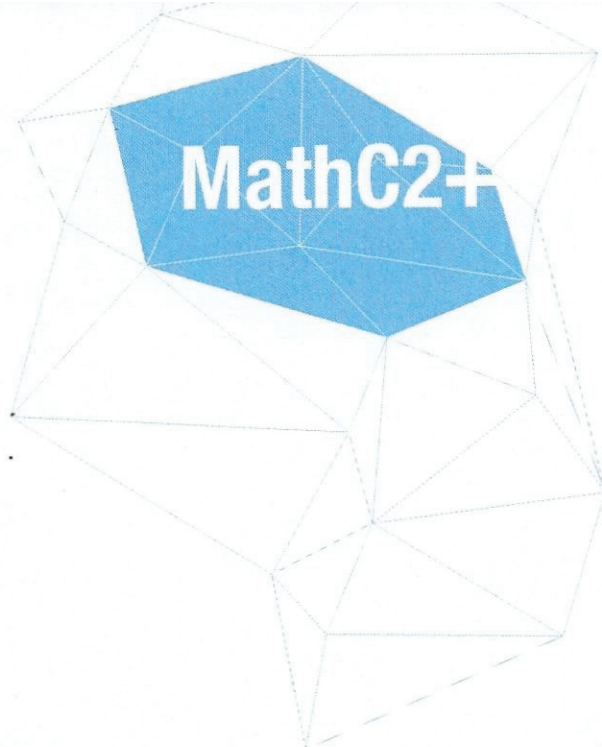
plus petit

16 8 9 2 0
24 23 22 21 20
0 0 1 0 1

ou additionner les derniers
nombres ⇒ si oui

- 1 001
- 2 010
- 3 011
- 4 100
- 5 101
- 6 110
- 7 111
- 8 1000
- 9
- 10
- 11
- 12
- 13
- 14
- 15
- 16
- 17
- 18

Dany-



La première carte montre tous les nombres impairs de l'intervalle $[1; 31]$. Si la personne dit "oui", le nombre est impair. Sinon, il est pair.

La deuxième carte comporte autant de nombres pairs qu'impairs. Les trois suivantes présentent la même caractéristique.

La troisième carte comprend ^{nombre} quatre suites de 4 chiffres consécutifs : "4; 5; 6; 7" / "12; 13; 14; 15" / "20; 21; 22; 23" / "28; 29; 30; 31"

→ la quatrième comprend une suite manquante à la troisième : "8; 9; 10; 11"
même schéma pour la 5^e : "16; 17; 18; 19"

On note que la cinquième carte montre les 16 derniers nombres de l'intervalle $[1; 31]$

→ Sur la deuxième carte, chaque
lui est adjacent.



chiffre
nombre

a un autre nombre, qui

Modèle

- Dans la première grille, tous les nombres sont impairs
Donc si le nombre y est, c'est qu'il est impair mais si x n'y est pas, x peut être impair.
- Si x est de la première grille mais pas dans la deuxième, il ne peut pas être 27; 3; 15; 19; 23; 7 et ~~21~~; 31
Donc x peut être ^{ou} 1; 5; 9; 13; 17; 21; 25; 29 ~~ou 31~~.
- Dans la deuxième grille, il y a un nombre pair sur deux et pareil pour les nombres impairs
- Dans la troisième grille, il y a plusieurs sortes de 4 nombres et entre chaque sorte 4 nombres ne sont pas présents
- 4ème grille: 2 sortes de 8 nb (de 8 à 15 et de 24 à 31)
- 5ème: il y a tous les nb compris entre 16 et 31!

TEST:

~~1; 2; 5; 7; 9; 11; 13; 15; 17; 19; 21; 23; 25; 27; 29; 31~~
~~2; 4; 6; 8; 10; 12; 14; 16; 18; 20; 22; 24; 26; 28; 30~~

~~2~~ ~~4~~ 6 ~~8~~ ~~10~~ ~~12~~ ~~14~~ ~~16~~ ~~18~~ ~~20~~ 22 ~~24~~ ~~26~~ ~~28~~ 30

MathC2+

1^{er} feuille que impair donc si oui impair
si non pair

2^{em} feuille ~~autre~~ paire impaire

2^{em} feuille nombre pair sur ~~deux~~ idm impaire

3^{em} feuille ~~pas~~ nbr pas écrit 4 nbr écrit

4^{em} feuille nbr pas écrit 8 nbr écrit

5^{em} feuille tout nbr plus grand que 16

3 5 7 9 11 13 15 17 19 21 23 25 27 29 31
~~4~~ ~~6~~ ~~8~~ ~~10~~ ~~12~~ ~~14~~ ~~16~~ ~~18~~ ~~20~~ ~~22~~ ~~24~~ ~~26~~ ~~28~~ ~~30~~
 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

Act. with 10000 Leonardo 11110

Stages 16 nombres

1-31

1ère case:

1-3-5-7-9-11-13-15-17-19-21-23-25-27-29-31

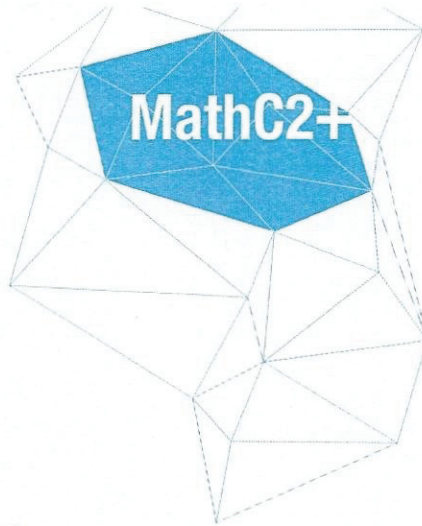
2ème:

2-3-6-7-11-14-15-18-19-22-23-26-27-30-31

3ème: 4-5-6-7-12-13-14-15-21-22-23-28-29-30-31

4ème: 8-9-10-11-12-13-14-15-24-25-26-27-28-29-30-31

5ème: 16-17-18-19-20-21-22-23-24-25-26-27-28-29-30-31



Un nombre peut apparaître au moins 2 fois.
 Certaines cases ont plus de nombres pairs que de nombres impairs ^{et inversement} ~~et inversement~~
 Certaines cases ont ~~des~~ plus de nombres dizaines que de nombres ^à unités.
 Certains nombres apparaissent plusieurs fois de suite. 123

$$\begin{array}{cccccc} 0 & & 1 & & 2 & & 3 & & 4 & & 5 \\ 0 < 2^4 & + & 0 < 2^3 & + & 0 < 2^2 & + & 0 < 2^1 & + & 0 < 2^0 \\ 1 & & 1 & & 1 & & \frac{3}{4} & & \frac{2}{1} \end{array}$$

- 1 0 ou 1
- 2 0 ou 1
- 3 0 ou 1
- 4 0 ou 1
- 5 0 ou 1

Ahmed-Emin ~~Robin~~ Robin

1: |

1^{er} carré: si le nombre est pair, il est impair
sinon, pair.

2: | 0

3: | 1

4: | 0 0

5: | 0 1

6: | 1 0

7: | 1 1

8: | 0 0 0

9: | 0 0 1

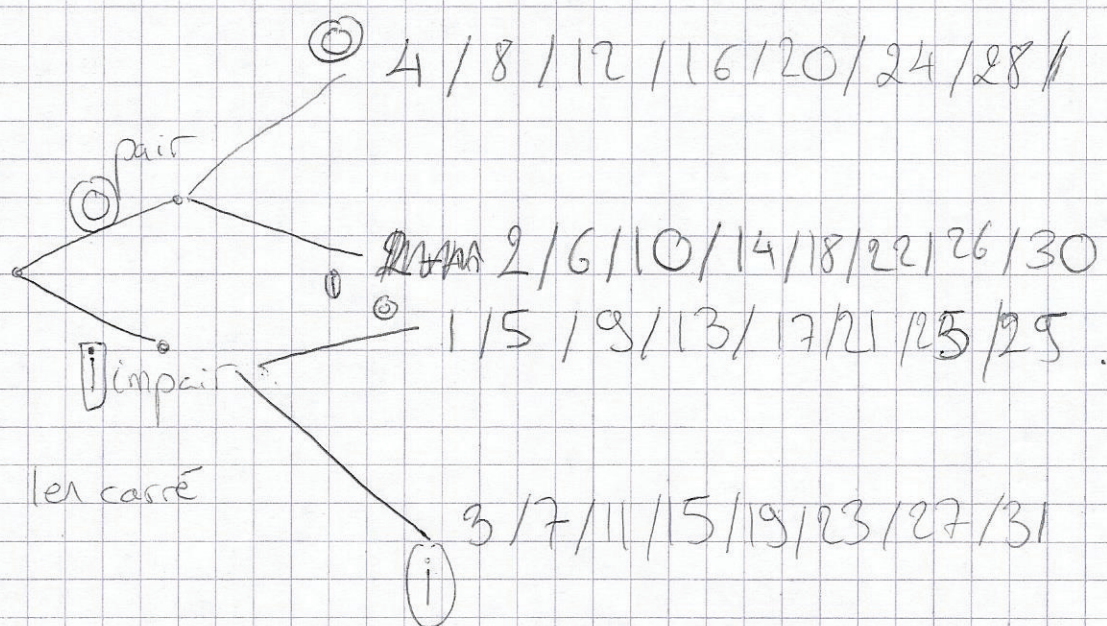
10: | 0 1 0

2^{ème} carré: 2/3; 6/7; 10/11; 14/15; 18/19; 22/23
26/27; 30/31

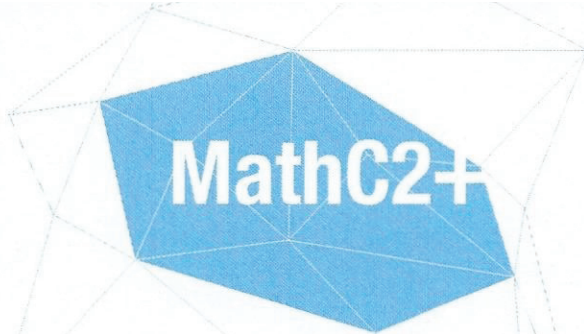
3^{ème} carré: 4, 5, 6, 7 / 12, 13, 14, 15 / 20, 21, 22, 23
28, 29, 30, 31.

4^{ème} carré: 8, 9, 10, 11, 12, 13, 14, 15 / 24, 25, 26, 27, 28, 29
30, 31.

5^{ème} carré: 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,
28, 29, 30, 31.



THEOIS
Touza
[1; 31]



TOUZNA

Dans un tableau ① il y a que des chiffres impaires.
Si il est de ce tableau le nombre est impair aussi non il est pair.

Dans le tableau ② on prend 4 on saute 2, 3.

Dans le ③ on saute 4 chiffre on prend le chiffres

Dans le ④ on vas de 8 en 8 à partir de 8

Dans le ⑤ c'est tous les nombre ≥ 16 [16; 20]

$$2^6 = 16 \quad 2^3 = 8 \quad 2^2 = 4 \quad 2^1 = 2 \quad 2^0 = 1$$

present = 1 absent = 0

$$\begin{cases} 27 = 26 + 1 \\ 27 = 16 + 8 + 0 + 2 + 1 \end{cases}$$

1	1	0	1	1
16	8	0	2	1

$$16 + 8 + 0 + 2 + 1 = 27$$

Il faut compter au fur et à mesure les puissances de 2 en fonction de la réponse. Si Oui calculer (additionner la dernière case à gauche) Non ne pas calculer

MathC2+

- 1 * Trouver le nombre par élimination :
- une feuille avec les nombres impairs par réduire de 16 chiffres l'intervalle
 - une autre feuille de 2 en 2
 - la troisième feuille de 3 en 3
 - une quatrième feuille de 8 en 8
 - la dernière avec tout les nombre de l'intervalle à partir de 7

2 * Avec l'écriture binaire quand on juxtapose les chiffres ça forme le code binaire du nombre choisi

Feuille 1 = 2^0

2 = 2^1

3 = 2^2

4 = 2^3

5 = 2^4

=> code binaire

$$\begin{array}{r|l} 0 & 1 \\ 8 & 4 + 2 + 1 \\ \hline 17 & 0 \end{array}$$

calcul mental

ajouter les chiffres en bas à gauche

= out
= in

ycle
ycle
ycle

side 3

side

$\{4, 8, 12, 16, 20, 24, 28\}$

(27L)
Pair

$\{2, 6, 10, 14, 18, 22, 26, 30\}$

$\{1, 5, 9, 13, 17, 21, 25, 29\}$

$\{3, 7, 11, 15, 19, 23, 27, 31\}$

$\{4, 12, 20, 28\}$

$\{2, 10, 18, 26\}$

$\{6, 14, 22, 30\}$

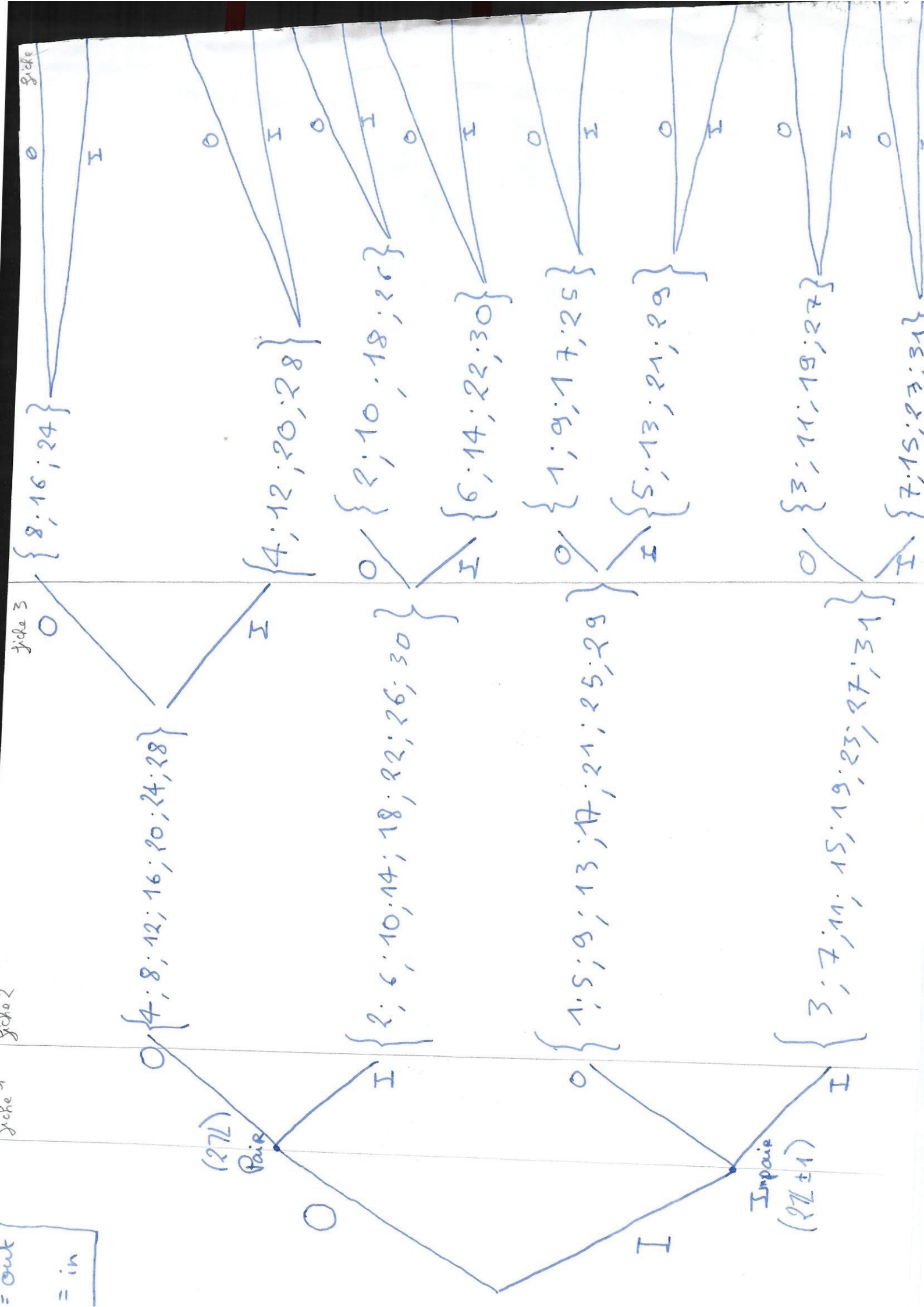
$\{1, 9, 17, 25\}$

$\{5, 13, 21, 29\}$

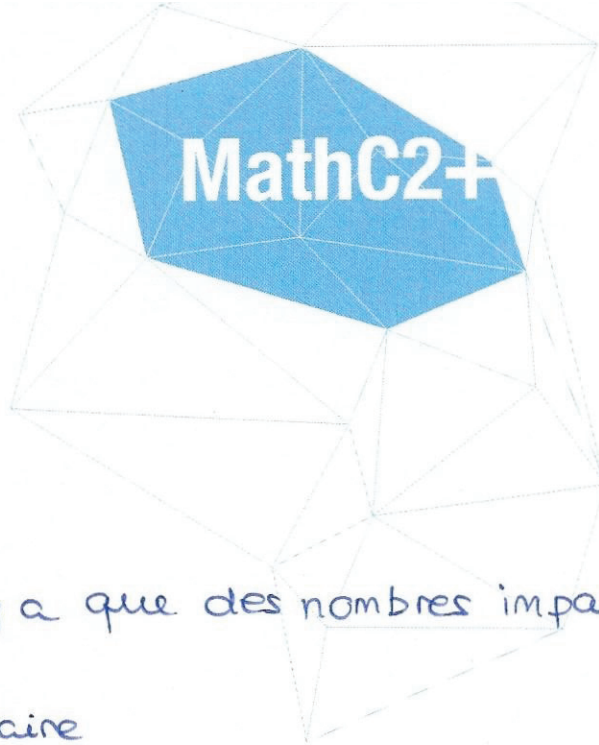
$\{3, 11, 19, 27\}$

$\{7, 15, 23, 31\}$

$\{8, 16, 24\}$



Océane
Lou



$$\begin{array}{r} 01010 \\ 11010 \\ 168020 \\ \hline 26 \end{array}$$

$$\begin{array}{r} 01010 \\ 11010 \\ 168020 \\ \hline 26 \end{array}$$

- Dans la 1^e, il y a que des nombres impairs

* On utilise le binaire

- quand le nombre y est pas on met 0

" " " y est on met 1

→ on convertit le nombre binaire obtenu en décimal.

[A] 10001
 $1 \times 2^4 = 16$
 $0 \times 2^3 = 0$
 $0 \times 2^2 = 0$
 $0 \times 2^1 = 0$
 $1 \times 2^0 = 1$

[25] 11001
 $1 \times 16 = 16$
 $1 \times 8 = 8$
 $0 \times 4 = 0$
 $0 \times 2 = 0$
 $1 \times 1 = 1$
 $16 + 8 + 1 = 25$

[9] 01001
 $0 \times 16 = 0$
 $1 \times 8 = 8$
 $0 \times 4 = 0$
 $0 \times 2 = 0$
 $1 \times 1 = 1$
 $8 + 1 = 9$

01100
 $0 \times 16 = 0$
 $1 \times 8 = 8$
 $1 \times 4 = 4$
 $0 \times 2 = 0$
 $0 \times 1 = 0$
 $8 + 4 = 12$

10101
 $1 \times 16 = 16$
 $0 \times 8 = 0$
 $1 \times 4 = 4$
 $0 \times 2 = 0$
 $1 \times 1 = 1$
 $16 + 4 + 1 = 21$

[21] 10101
 $1 \times 16 = 16$
 $0 \times 8 = 0$
 $1 \times 4 = 4$
 $0 \times 2 = 0$
 $1 \times 1 = 1$
 $16 + 4 + 1 = 21$

00110
 $0 \times 16 = 0$
 $0 \times 8 = 0$
 $1 \times 4 = 4$
 $1 \times 2 = 2$
 $0 \times 1 = 0$
 $4 + 2 = 6$

11101
 $1 \times 16 = 16$
 $1 \times 8 = 8$
 $1 \times 4 = 4$
 $0 \times 2 = 0$
 $1 \times 1 = 1$
 $16 + 8 + 4 + 1 = 29$

11110

quand il y a 16, faire + 16

~~10000~~
 16
 8
 4
 2
 1



8
 4
 2
 1

8
 4
 2
 1

Apolline

MathC2+

5 cartes - 16 cases

Carte 1: seulement des
nombres impairs

carte 2: nombres pairs
+ nombres impairs
déjà présents
dans la carte 1

carte 3: idem que carte 2
mais pas \hat{m} nombre
que en carte 2

carte 4: idem que carte 2
mais pas \hat{m} nombres

carte 5: idem que carte 2
mais pas \hat{m} nombres

donc si nb choisi dans carte 1 alors il est forcément impair
- 16 possibilités en \odot
et inversement, pair si pas dans carte 1

Nous utilisons le système binaire

Si le nombre est présent sur une carte on met 1, sinon 0, puis la forme du
nombre en question

10010 10010

faux, il faut
écrire dans l'autre
sens du fait de l'ordre
des feuilles

On peut aussi additionner les chiffres en base 2 gauche
correspondant aux cases dans lesquelles le nombre choisi
est présent et on obtient le nb en question.

01001

$$9 + 1 = 9$$

10001

$$16 + 1 = 17$$



MINISTÈRE
DE L'ÉDUCATION
NATIONALE

Rachel CHOLLE

Dans chaque cané on retire des intervalles : se range de 16 à 31
+ carte de nombres impaire.

Étape à venir donc l'ordre des fiches est importante : fiche (des impaire?)

(F₁) \Rightarrow Marque $\begin{matrix} \swarrow \\ 2 \end{matrix}$ 4 6 8 10 12 14 16 18 20 22 24 26 28 30 $\begin{matrix} \nearrow \\ (31) \end{matrix}$

(F₂) \Rightarrow Marque $\begin{matrix} \swarrow \\ 16 \end{matrix}$ 17 18 19 20 21 22 23 $\begin{matrix} \nearrow \\ 24 \end{matrix}$ (31)

(F₃) \Rightarrow M : $\begin{matrix} \swarrow \\ 2 \end{matrix}$ 4 6 8 16 17 14 16 18 20 22 24 26 28 30 $\begin{matrix} \nearrow \\ (31) \end{matrix}$

(F₄) \Rightarrow M : $\begin{matrix} \swarrow \\ 8 \end{matrix}$ 9 10 11 16 17 18 19 24 25 26 27 $\begin{matrix} \nearrow \\ (31) \end{matrix}$

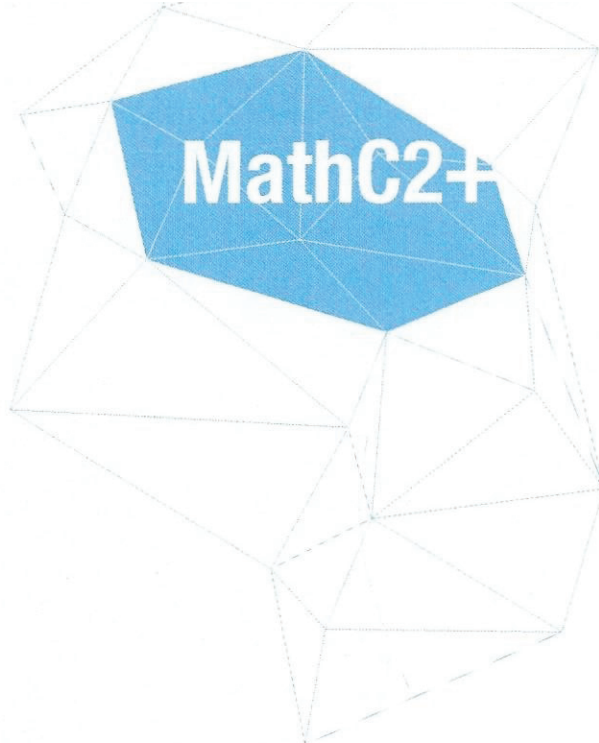
(F₅) \Rightarrow M : $\begin{matrix} \swarrow \\ -4 \end{matrix}$ 5 - 8 9 - 12 3 - 14 17 - 20 21 - 24 25 - 28 29 $\begin{matrix} \nearrow \\ (31) \end{matrix}$

(F₆) \Rightarrow M $\begin{matrix} \swarrow \\ 1 \end{matrix}$ à 15

$$17 \Rightarrow 2^4 \times 1 \quad 2^3 \times 0 \quad 2^2 \times 0 \quad 2^1 \times 0 \quad 2^0 \times 1 = 10001$$

F₁ \Rightarrow

Létizia



$$\begin{array}{r} 11011 \\ 168021 = 27 \end{array}$$

16 cases

Nb entre 1 et 31

carte 1: Que des nb impairs.

carte 2: même nb impairs que carte 1 + nb pairs

carte 3: même configuration que carte 2 mais

carte 4: Nb impairs qui se trouvent dans les nbs de la carte 1

carte 5: //

nbs impairs: 1-3-5-7-9-11-13-15-17-19-21-23-25-27-29-31

⇒ si le nb n'est pas dans la 1^{re} carte alors il est pair et inversement.

nbs pairs: 2-4-6-8-10-12-14-16-18-20-22-24-26-28-30

carte 2: 2-6-10-14-18-22-26-30

carte 3: 4-6-12-14-20-22-28-30

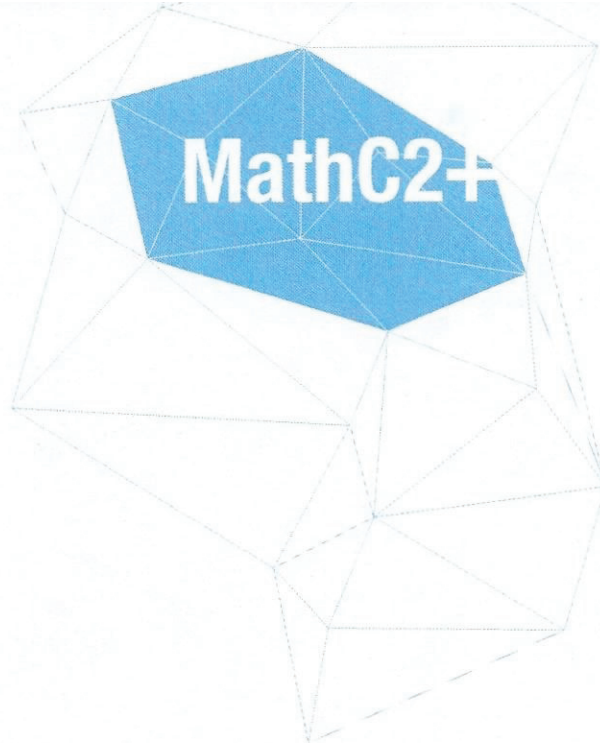
carte 4: 8-10-12-14-24-26-28-30

carte 5: 16-18-20-22-24-26-28-30

Système binaire: 1 = oui 0 = non

$$\begin{array}{l} N-O-O-O-O = 30 \\ N-O-O-O-N = 14 \\ N-O-O-N-O = 22 \\ N-O-N-O-N = 10 \\ N-O-N-O-O = 26 \\ N-O-N-N-N = 2 \\ N-N-O-N-N = 4 \\ N-N-N-O-N = 8 \\ N-N-N-N-O = 16 \\ N-O-O-N-N = 6 \end{array}$$

Imane.



2^4	2^3	2^2	2^1	2^0
0	1	1	0	1

$$8 + 4 + 0 + 1$$

16 possibilités
[1; 31]

1^{ère} carte : tous les nombres impairs

2^{ème} carte : tous les nombres > 16 .

3^{ème} carte : $[24; 31] \cup [8; 14]$

$2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$
1 0 0 0 0

On additionne les chiffres en bas à gauche quand la réponse est oui.

Jenna -

MathC2+

Chaque

16 possibilités de chaque carte.

(il y a 31 possibilités en tout.)

- on a à la fois des nombres pairs et impairs de chaque carte.

- 1^{ère} carte = que des nombres impairs.

$$3 = 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1 \\ = 011$$

$$5 = 2^2 \times 1 + 2^1 \times 0 + 2^0 \times 1 \\ = 101$$

$$7 = 2^2 \times 1 + 2^1 \times 1 + 2^0 \times 1 \\ = 111$$

$$9 = 2^2 \times 2 + \\ =$$

- ds le système binaire, tous les nombres impairs se finissent par un 1.

- les nombres se suivent dans chaque carte.

- regarder l'étendue. fiche 1 = 1 à 31

fiche 1 : tous les nombres impairs.

fiche 2 : tous les nombres se suivent de 16 à 31.

fiche 3 : il manque 2 nombres à chaque fois.

fiche 4 : il manque 4 nombres, on commence à 4.

fiche 5 :

(ils font apparaitre un 1 au système binaire dans 2^0)

fiche 1 : 1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31

fiche 5 : 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
(ils font apparaitre un 1 au système binaire dans 2^4)

fiche 2 : 2 3 6 7 10 11 14 15 18 19 22 23 26 27 30 31
(pair + impair) (ils font apparaitre un 1 au système binaire ds 2^1)

fiche 3 : 4 5 6 7 12 13 14 15 20 21 22 23 28 29 30 31
(ils font apparaitre un 1 au système binaire dans 2^2)

fiche 4 : 8 9 10 11 12 13 14 15 24 25 26 27 28 29 30 31
(ils font apparaitre un 1 au système binaire dans 2^3)

$$18 = 2^2 \times$$

ordre des cartes à peu près,

...	2^4	2^3	2^2	2^1	2^0
	1	0	0	1	0

17

10001

1 ... 31

2 ... 31

4 ... 31

8 ... 31

16 ... 31

2 3 5 7 ... 31

2 3 6 7 ... 31

4 5 6 7 ... 31

17

f1 oui nonne impair
f2 non
f3 non
f4 non
f5 oui



X axis/
columns

X X X X X X
Y Y Y Y Y Y
Y Y Y Y Y Y

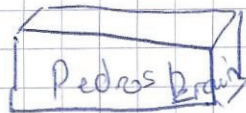
X @ = even

Figure 10
X axis/
columns

3

B/R.

NNNNNN



X axis/
columns
Y axis/
row.

X X X X X X
Y Y Y Y Y X

Y = odd
X = even

N = 1 or 0

1 0 1 1 1 0
32 16 8 4 2 1
1 1 0 1 0 1
0 0 1 0 1 0

1 R
2 R
3 R
6 R
11 R
12 R
14
15 B
16 B
17 R
17 B
20 R
21 R
22 R
23 B
24 R
25 R
26 B
27 B

20
21
22
23
24
25
26
27

31
32
35
36

41
42
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60
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62
63
64
65
66
67

71
72
75
76

90
93
95
96
98
99
99
99
99
99

61 67 64
62 65 60 66
69

71 75 76 72

(2)

	132	16	8	4	2	1		5
	1	2	4	8	16	32		
0	1	0	1	0	1	1	= 21	R = 1
1	1	0	0	1	0	0	= 50	B = 0
0	1	0	0	1	0	0	= 18	
1	0	1	1	1	0	0	= 46	
1	1	0	1	0	1	1	= 53	
0	0	1	0	0	0	1	= 9	
0	0	1	1	1	1	0	= 14	
1	0	1	0	0	0	1	= 41	40
1	0							3

1	1	1	1	1	0
0	0	0	0	0	1
0	0	0	0	0	1
0	0	0	0	0	1
1	1	1	1	1	0
1	1	1	1	1	0
0	0	0	0	0	1
1	1	1	1	1	0

(13)

1-31

8

1	2	3	4	5
---	---	---	---	---

(13)

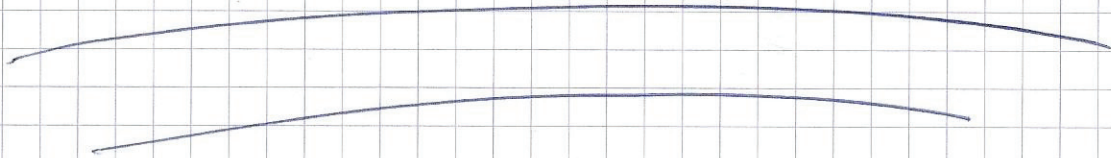
1	0	1	1	0
---	---	---	--------------	---

Yes - 1

(1)

1	0	0	0	0
---	---	---	---	---

No - 0



35

1
 2
 3
 4
 5
 6
 10
 11
 12
 13
 14
 15
 16
 17
 20
 21
 22
 23
 24
 25
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 27
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 60
 61
 62
 63
 64
 65
 66
 67
 71
 72
 75
 76

32 16 8 4 2 1

0 0 0 0 0 1
 0 0 0 0 1 0
 0 0 0 1 0 1
 6 0 0 1 1 0
 0 0 1 0 0 0
 0 0 1 0 1 1
 0 1 0 0 1 0

B B B B B R
 B B B B R B
 B B B R B R
 B B B R R B
 B B B R B B
 B B R B R R
 B B R B R B
 B B R B B B
 B B R B B R
 B

6

1 3
 2 2
 32 10 4 2 1
 10 4 2 1

10 4 2 1
 R R R R R

10 10 4 2 1
 R R R B R
 1 1 0 1

1 0 0 0 0 0

34 20 10 4 2 1
 0 0

20 10 4 2 1

R B R R B R
 1 0 1 1 0 1

20 10 4 2 1
 1 0 0 0 1 0
 0 1 1 0 1 0

32

1 1 1 1 1 1

①

6

8

B	R	B	R	B	R
R	R	B	B	R	B
B	R	B	B	R	B
R	B	R	R	R	B
R	R	B	R	B	R
B	B	R	B	B	R
B	B	R	R	R	B
R	B	R	B	B	R

B = 1
R = 0

32	16	8	4	2	1	
1	0	1	0	1	0	= 41
0	0	1	1	0	1	= 13
1	0	1	1	0	1	= 45
0	1	0	0	0	1	= 17
0	0	1	0	1	0	= 10
1	1	0	1	1	0	= 54
1	1	0	0	0	1	= 49
0	1	0	1	1	0	= 26

Notée

5 cases

16 possibilités par carte

31 nb possibles.

$$13 = 01101$$

$$31 = 11111$$

tableau eau



$$\text{Tab 1} = 2^0$$

$$\text{Tab 2} = 2^1$$

$$\text{Tab 3} = 2^2$$

$$\text{Tab 4} = 2^3$$

$$\text{Tab 5} = 2^4$$

$$1 = 00001$$

$$2 = 00010$$

$$3 = 00011$$

$$4 = 00100$$

$$5 = 00101$$

$$6 = 00110$$

$$7 = 00111$$

$$8 = 01000$$

$$9 = 01001$$

$$10 = 01010$$

$$11 = 01011$$

$$12 = 01100$$

$$13 = 01101$$

$$14 = 01110$$

$$15 = 01111$$

$$16 = 10000$$

$$17 = 10001$$

$$18 = 10010$$

$$19 = 10011$$

$$20 = 10100$$

$$21 = 10101$$

$$22 = 10110$$

$$23 = 10111$$

$$24 = 11000$$

$$25 = 11001$$

$$26 = 11010$$

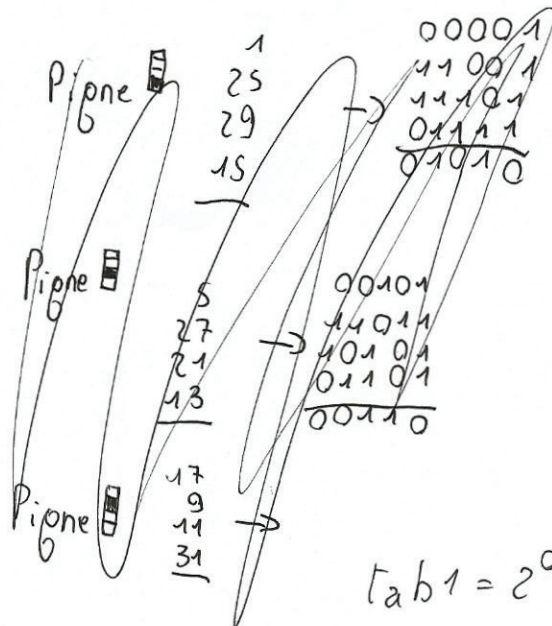
$$27 = 11011$$

$$28 = 11100$$

$$29 = 11101$$

$$30 = 11110$$

$$31 = 11111$$



$$\text{Tab 1} = 2^0 = 00001$$

tous nb ayant ds binaire
ce 1 y sont

$$\text{Tab 2} = 2^1 = 00010$$

tous nb ayant ds binaire 1
y sont

Pour trouver le nombre choisi, on retrouve son nb binaire.

Si il se trouve ds le tab 1, alors sa compo est 1

sinon, c'est 0.

À la fin, on trouve le nb binaire et on convertit

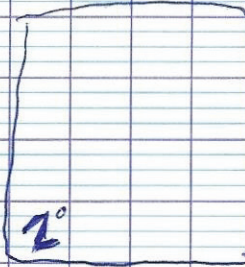
$$\text{ex : } 01101 = 0 \times 16 + 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 = 8 + 4 + 1 = 13$$

Plus rapide, on additionne les puissances de 2 des tableaux où sont se
trouve le nombre et on l'obtient ainsi.

Roman PACCARD

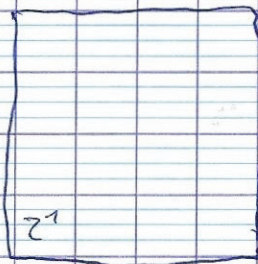
16 possibilités par carte

1^{re} carte :
Que des impairs
↳ donc on sait si
c'est pair ou pas



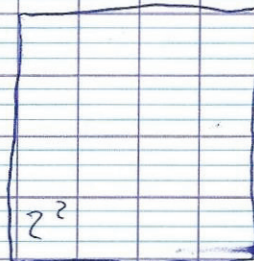
Si le ~~chiffre~~ nombre
est dedans, on met 1
à la fin. (car en binaire
ils se finissent tous par 1)

2^e carte



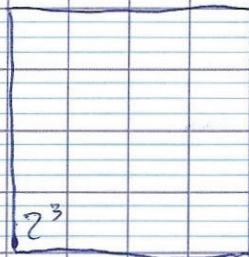
~~Et~~ Ensuite on continue
en mettant un 0 ou
un 1 devant l'ancien
chiffre

3^e carte



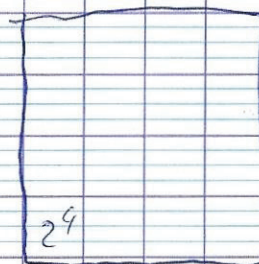
Où
Si il est dans une
case, on prend le
chiffre dans le coin
(1, 2, 4, 8 ou 16)

4^e carte



et ensuite on les
additionne et on
obtient le chiffre qu'on
cherche.

5^e carte
Tout les chiffres ici
ont en binaire au
moins 5 caractères



Le 1, 2, 4, 8 et 16 ne sont que sur une grille
Si on dit oui partout, alors c'est 31

En binaire:

1 \rightarrow 1

2 \rightarrow 10

3 \rightarrow 11

4 \rightarrow 100

5 \rightarrow 101

6 \rightarrow 110

7 \rightarrow 111

8 \rightarrow 1000

9 \rightarrow 1001

10 \rightarrow 1010

11 \rightarrow 1011

12 \rightarrow 1100

13 \rightarrow 1101

14 \rightarrow 1110

15 \rightarrow 1111

16 \rightarrow 10000

17 \rightarrow 10001

18 \rightarrow 10010

19 \rightarrow 10011

20 \rightarrow 10100

21 \rightarrow 10101

22 \rightarrow 10110

23 \rightarrow 10111

24 \rightarrow 11000

25 \rightarrow 11001

26 \rightarrow 11010

27 \rightarrow 11011

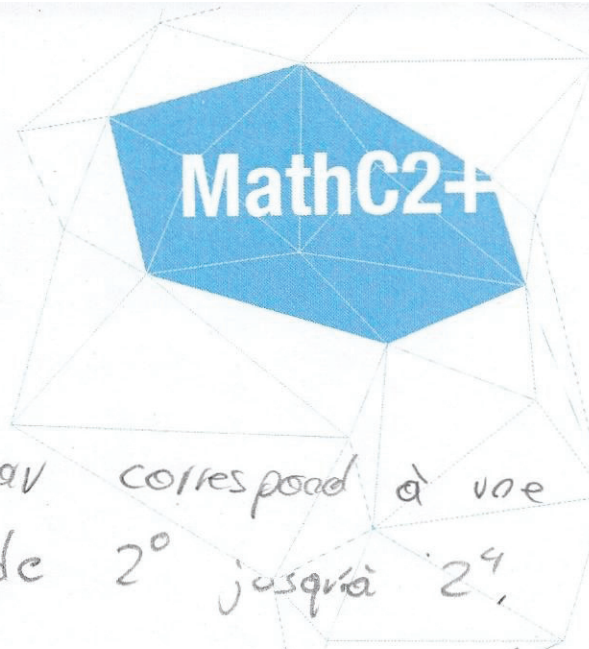
28 \rightarrow 11100

29 \rightarrow 11101

30 \rightarrow 11110

31 \rightarrow 11111

Maxence
Alexandra



chaque tableau correspond à une puissance de 2
en partant de 2^0 jusqu'à 2^4 .

Ainsi si le nombre est présent, alors cela vaut
1 en binaire pour la puissance de 2 correspondante

On obtient :

oui	oui	non	oui	oui	non
1	1	0	1	1	0

donc $2^0 + 2^2 + 2^3$

donc $1 + 4 + 8$

donc 13

Cela permet de trouver le chiffre choisi

BELGAÏDI

Nil

MathC2+

= je pense que c'est relié au chiffre binaire

Atividade

1º Passo: Defina em quais imagens tem o número da data de aniversário, a imagem que não tiver elimine todos os números.

2º Passo: Pegue as 3 imagens que tem a data, e retire os números iguais 11, 22, 31, 15, 5, 21, 29.

3º Passo: O número que estiver nas 3 imagens é o resultado.

Lucas Barros



Determine o valor
 se tiver na tabela manter
 se não estiver descartar a tabela
 fazer isso para todas as tabelas
 mostra o numero que se repete nas tabelas restantes

Walter Lopes da Silva



Ailton Ruiz

Atividade - Definição de Processos / Algoritmo.

01. Apresente a imagem

02. Se houver a resposta, armazene-a, senão descarte-a.

03. Prosiga para o próximo quadro

04. Repita o passo 1, 2, 3

05. Verifique se há resposta em todos os quadros.

Atividade - Definição de processos / Algoritmos

- 1- Observar os números
- 2- Analisar a quantidade de números iguais
- 3- Destacar números que não se repetem nas tabelas seguintes
- 4- Pegar os números que se repetem e observar suas posições
- 5- Destacar os números que estão nas mesmas células das tabelas seguintes
- 6- Pegar o único número que se repete nas tabelas seguintes e que estejam em células diferentes.

12
21
31/58
70
91

122

- 1- Observar os números que se repetem nas tabelas
- 2- Destacar os números iguais que mais se repetem
- 3- Observar se nas tabelas que não possuem números iguais se tem outro número que somando irá dar o valor dos números iguais com o das tabelas

70

73

79

Arthur Vinícius

1- Observar os números

58

70

91

- 0 Apresente a imagem
- 1 Analize a resposta
- 2 Se sim marque os números que compõem o quadrado analisado. Se não desconte-os
- 3 Prossiga ao próximo quadrado
- 4 Repita o passo 1, 2 e 3 até cobrir as imagens
- 5 Verifique o número que está em todos os quadrados em que a resposta foi sim.

ATIVIDADE - DEFINIÇÃO DE PROCESSO/ALGORITMOS

1 - PASSO: ANALISAR AS IMAGENS MOSTRADAS

2 - PASSO: VER QUE AS IMAGENS EM COMUM TEM

3 - PASSO: OLHAR NAS TABELAS ESCOLHIDAS QUAL O NUMERAL REPETIDOR
E [ELIMINAR AS OUTRAS TABELA COM OS SEUS DEMAIS NUMERAIS]

4 - PASSO: COMPARAR AS TABELAS E ACHAR O NUMERAL INCOMUM
QUE APARECEM NAS TABELAS ESCOLHIDAS

5 - PASSO: DEFINIR O NUMERAL REPETIDOR, O QUAL SERIA A RESPOSTA

Lucas Eduardo C. Luz e Lho.

ADS 2018.1

Heitor França ADS

1- Passos: o Passos finais estão em SIM e
quais estão em Não

2- Passos: eliminar os que estão ~~em~~ no
Não

3- Passos: eliminar todos que não se repetem
nos 3 SIM

~~4- Passos: eliminar os que estão em ambos~~

Aluno: Leanny Heitor

curso: Ciência da computação "nemha"

passos para descobrir a data de nascimento com 5 tabelas.

1º. Encontre as tabelas com a possível data.

2º compare com as não possíveis e elimine as datas iguais.

4º compare com as tabelas possíveis novamente.

5º Encontre a data que se repete.

6º Esta data é o resultado.

05.03.18

ATIVIDADE - DEFINIÇÃO DE PROCESSOS/ALGORITMOS

ADS - *João Filipe*

- 1:P - PERGUNTAR AO USUÁRIO, SE A DATA ESTÁ NA IMAGEM.
- 2:P - SE SIM, ARMAZENAR DADOS DA IMAGEM.
- 3:P - SE NÃO, DESCARTAR E IR PARA A PRÓXIMA IMAGEM.
- 4:P - ~~(RECEB)~~ RECEBER, APENAS OS NÚMEROS IGUAIS NAS IMAGENS ARMAZENADAS.
- 5:P - APRESENTAR O VALOR FINAL.

05/03/18

Definição de processos/algoritmos

Atividade

Luon Raphael
Curso: ADS

Passo 1 = a mostrar as 5 imagens

Passo 2 = Perguntar se a data do aniversário do participante está em qual imagem e marcar se sim (estar a data) ou não (se não estar), em todas as imagens.

Passo 3 = Comparar ~~os~~ os números de cada imagem com sim ou não.

Passo 4 = E ~~os~~ ^{os} ~~imagens~~ números que tem em não são descartados e os números iguais em sim ou não são também descartados

Passo 5 = Comparar se nas imagens sim o número se repete em todas as imagens sim.

Passo 6 = o número que repetir em todas as imagens sim é o número do resultado procurado.

05 / 03 / 18

- 1º passo: Definir em quais imagens, entre as mostradas, está presente o dia do aniversário.
- 2º passo: ~~Comparar~~ Comparar os números das imagens em que está presente o dia do aniversário com as que não está presente.
- 3º passo: Excluir os números em comum entre as imagens em que está presente o dia e as que não está presente o dia.
- 4º passo: Comparar as imagens em que o número do dia está presente.
- 5º passo: Excluir os números que não estão comuns as imagens em que o número do dia está presente.
- 6º passo: O número que restar, for comum as imagens, é o número do dia do aniversário.

FACULDADE NOVA ROMA

ALUNO: GUILHERME SILVEIRA CONTINHO - COMP 1-M

Atividade -> Definição de processos

Daniel Souza

3 7 23 19	14 26 10 27	6 5 20 14
17 9 11 31	3 30 15 18	28 23 15 13
5 27 21 13	31 6 19 23	31 29 21 22
1 25 29 15	2 7 11 22	4 30 7 12
S	N	N

14 10 15 31	17 23 19 25	
11 27 29 13	27 30 29 18	R = 25
26 28 9 12	22 21 26 24	
8 30 25 24	16 28 31 20	
S	S	

- 1-| Descarta os m: dentro dos quadrados que não tem
- 2-| Corroia os m: de SIM que ja tem nos do Não
- 3-| Descarta os mesmos
- 4-| Analisa o que sobrou
- 5-| Confere se o m: está presente nos 3 que tem SIM



~~- Mostra-se uma cartela com 36 números~~

Aluna: Karen Samara

- Identifica-se um número em uma cartela com 36 números, a resposta podendo ser sim ou não.

~~- Repete-se~~ - A resposta foi sim.

- Repete-se o mesmo processo mais ^{duas} ~~uma~~ vezes, a resposta sendo não.

- Repete-se o mesmo processo mais duas ~~vezes~~ vezes, a resposta sendo sim.

- Descartar os números das cartelas onde a resposta foi não.

- Considerar os números das cartelas onde a resposta foi sim.

- Verificar números em comum, onde a resposta foi sim.

- Chegar ao resultado final, número 25

COMECEI COLOCANDO OS NÚMEROS EM ORDEM CRESCENTE E EM SEUS DETERMINADOS GRUPOS.

I 1 - 3 - 5 - 7 - 9 - 11 - 13 - 15 - 17 - 19 - 21 - ~~23~~ - ~~25~~ - ~~27~~ - 29 - 31

II 2 - ~~3~~ - 6 - ~~7~~ - 10 - 11 - ~~14~~ - ~~15~~ - 18 - 19 - 22 - 23 - 26 - ~~27~~ - 30 - 31

III 4 - 5 - 6 - 7 - 12 - 13 - 14 - 15 - 20 - 21 - 22 - 23 - 28 - 29 - 30 - 31

IV 8 - 9 - 10 - 11 - 12 - 13 - 14 - 15 - 24 - 25 - 26 - 27 - ~~28~~ - 29 - 30 - 31

V 16 - 17 - 18 - 19 - 20 - 21 - 22 - 23 - 24 - 25 - ~~26~~ - 27 - 28 - 29 - 30 - 31

APÓS PENSAR EM UM NÚMERO, VERIFIQUEI SE ESTAVAM CONTIDOS NO GRUPO

I E **II**, E DEPOIS ELIMINEI OS ^{QUE} NÃO ERAM COMUM EM AMBOS OS GRUPOS.

OS NÚMEROS QUE SOBRAREM, SERAM PARES OU ÍMPARES. ENTÃO ELIMINO AS

DIFERENTES NO GRUPO **III**. NO GRUPO **IV**, DESCARTO OS QUE SOBRAREM QUE

NÃO PERTENCEM. E NO GRUPO **V**, ELIMINO OS NÚMEROS MENORES QUE 16

E OS QUE ESTAVAM NOS GRUPOS QUE O NÚMERO NÃO PERTENCIA, OU O

INVERSO.

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