# Atmospheric turbulence effects mitigation for a ground to geostationary satellite optical link: impact on the ground terminal architecture 

Adrien-Richard Camboulives

## To cite this version:

Adrien-Richard Camboulives. Atmospheric turbulence effects mitigation for a ground to geostationary satellite optical link: impact on the ground terminal architecture. Optics / Photonic. Université Paris Saclay (COmUE), 2017. English. NNT: 2017SACLS564 . tel-02185130

HAL Id: tel-02185130
https://theses.hal.science/tel-02185130
Submitted on 16 Jul 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
universite PARIS-SACLAY

NNT : 2017SACLS564

# Compensation des effets de la turbulence atmosphérique sur un lien optique montant sol-satellite géostationnaire : impact sur l'architecture du terminal sol 

Thèse de doctorat de l'Université Paris-Saclay préparée à l'Université Paris-Sud

École doctorale $n^{\circ} 572$ Ondes et matières (EDOM) Spécialité de doctorat: optique et photonique

Thèse présentée et soutenue à Palaiseau, le 13 Décembre 2017, par

## Adrien-Richard CAMBOULIVES

Composition du Jury :
Yves JAOUEN Président du jury \&
Professeur, Télécom Paris
Aniceto BELMONTE
Professeur, Technical University of Catalonia, BarcelonaTech
Michel TALLON
Professeur, Observatoire Astronomique de Lyon
Jérome LACAN
Professeur, ISAE
Géraldine ARTAUD
Ingénieur de recherche, CNES
Vincent MICHAU
Professeur, ONERA
Rapporteur
Rapporteur
Examinateur
Examinateur

Marie-Thérèse VELLUET
Ingénieur de recherche, ONERA Invitée
Laurent SAINT-ANTONIN
Ingénieur de recherche, OneWeb
Invité
Examinateur
Directeur de thèse


# Mitigation of Turbulence effects on optical GRound to Geostationary satellite link: Impact on ground terminal Architecture 

## A THESIS PRESENTED BY <br> Adrien-Richard Camboulives

AND PREPARED AT
Institut de Recherche Technologique Saint Exupéry \&
Onera - Office National d'Études et de Recherches Aérospatiales

FOR THE DEGREE OF
Doctor of Sciences
OF
Université Paris Saclay
École Doctorale ED572 - Ondes et matières (EDOM)
Specialty: optics and photonics

Directed and supervised by
Vincent Michau
Marie-Thérèse Velluet
Laurent Saint-Antonin
\& Sylvain Poulenard

Defended publicly on December $13^{\text {th }}, 2017$
JURY COMPOSED BY:

| Yves Jaouen | Telecom Paris | President and <br> Rapporteur |
| :--- | :--- | :--- |
| Aniceto Belmonte | BarcelonaTech | Rapporteur |
| Michel Tallon | Obs. Astr. De Lyon | Examiner |
| Jérome Lacan | ISAE | Examiner |
| Géraldine Artaud | CNES | Examiner |
| Vincent Michau | Onera | Director |
| Marie-Thérèse Velluet | Onera | Guest |
| Laurent SAint-Antonin | OneWeb | Guest |

## Acknowledgments

"Hakuna Matata"<br>-Timon and Pumbaa, The Lion King

I would like, in these few lines, to express my gratitude to all of those who have contributed to help me achieve the conclusion of this thesis. I had the chance to work in two laboratories: the High Angular Resolution unit of Onera Châtillon and the ALBS project team of the IRT Saint Exupéry, in Toulouse.

I would like to start by thanking my thesis supervisor, Vincent Michau, who has provided me, throughout my thesis, with insightful suggestions and helped me develop hindsight on my work. I am grateful for the time you took from your busy schedule, especially at the end of my thesis, to help me refine my work.

At Onera, I thank Marie-Thérèse Velluet, for her ceaseless help and support throughout all these three years. I have always appreciated our discussions on trying to understand the results from the simulations and on our respective interpretations.

I would also like to thank Jean-Marc Conan and Clélia Robert, for the interesting discussions we had on my work, as well as Nicolas Vedrenne, for his help when I used TURANDOT. I warmly thank all of the members of the High Angular Resolution unit of Onera for welcoming me among them.

I have a particular thought for the other thesis students within the unit, in particular those who were in the same year as me and have become close friends: Sebastien, Joel, Kassem and Lucien. I always enjoyed the time we spent during breaks or outside of Onera, or even when discussing our respective works. I wish the best to all the thesis students who are still on the path to reach their Ph.D.

From my experience at IRT Saint-Exupéry, I thank Laurent Saint-Antonin, for his support throughout my thesis. Working with you has been one of the major reasons that made my thesis so enjoyable.

I also thank Sylvain Poulenard, for his help during the whole duration of the thesis as well as for welcoming me in his home three months, one of the first times I went to Toulouse. You have always provided me with interesting comments and suggestions on my work and you have helped me place my work in the broader context of optical telecommunications.

I also thank the whole ALBS project team for the great atmosphere there was when working in Toulouse. I also thank my two thesis companions, Karim and Jean-Alain. I
wish you all the best, Jean-Alain, for your defense and Karim, for your final year.
I am deeply grateful to all the members of my defense jury - Yves Jaouen, Aniceto Belmonte, Michel Tallon, Géraldine Artaud and Jerome Lacan - for having done me the honor of their presence and for their interest in my work. I appreciated the discussions we had on the different topics of this thesis.

I would also like to thank all of my friends who have supported me during this adventure.

Finally, of course, I want to warmly thank my parents who have supported me tremendously throughout my studies and who have done everything to help me achieve my thesis as well as Alice who has always been present and has managed to support me in all stages of the thesis.

Thanks to all and everyone of you!

## Contents

I Context ..... 3
1 Optical communications for high throughput satellite feeder links ..... 5
1.1 Geostationary satellite system for broadband internet ..... 6
1.1.1 Space, ground and user segments ..... 6
1.1.2 Feeder link ..... 6
1.1.3 Limitations to the use of radio frequencies ..... 6
1.1.4 Advantages and challenges of free-space optical communications ..... 7
1.1.5 History of optical communications ..... 8
1.2 Optical communication device architecture: main technological drivers ..... 12
1.2.1 Wavelength ..... 12
1.2.2 Optical receiver - direct detection ..... 13
1.2.3 Modulation - NRZ OOK ..... 13
1.2.4 Detection architecture ..... 13
1.3 Link Budget ..... 14
1.3.1 NRZ-OOK: power detection threshold - $P_{R}$ ..... 15
1.4 Conclusion ..... 19
2 Optical Beam Propagation in Turbulent Media ..... 21
2.1 Atmospheric turbulence ..... 22
2.1.1 Physical phenomenon ..... 22
2.1.2 Outer scale, inner scale and inertial range ..... 23
2.1.3 Structure function and spectral density of the fluctuations of the refractive index ..... 23
2.1.4 $\quad C_{n}^{2}$ vertical profiles ..... 24
2.2 Optical Propagation in Turbulent Media ..... 25
2.2.1 Propagation equations of an electromagnetic wave ..... 25
2.2.2 Spatial beam coherence ..... 26
2.2.3 The Rytov approximation: an analytical solution to Helmholtz equa- tion ..... 27
2.2.4 Statistical properties of the irradiance of the propagated beam in the Rytov regime ..... 27
2.2.5 Power spectral densities in weak perturbations for plane wave ..... 28
2.3 Modal analysis of turbulent phase ..... 28
2.3.1 Zernike polynomials ..... 28
2.4 Atmospheric turbulence effects on Gaussian beams ..... 29
2.4.1 Gaussian beam properties ..... 29
2.4.2 Irradiance fluctuations and scintillation ..... 30
2.4.3 Fade statistics ..... 38
2.5 Propagation simulation with phase screens ..... 39
2.5.1 Principle of phase screen propagation ..... 39
2.5.2 Temporal effects simulation ..... 40
2.5.3 TURANDOT ..... 41
2.6 Conclusion ..... 41
3 Mitigation techniques for link budget improvement ..... 43
3.1 Diversity techniques ..... 43
3.1.1 Wavelength diversity ..... 44
3.1.2 Temporal diversity ..... 44
3.1.3 Multiple emitters ..... 44
3.2 Adaptive optics pre-compensation ..... 45
3.2.1 Reciprocity principle ..... 45
3.2.2 Correlation between downlink and uplink ..... 47
3.2.3 Study of the correlation with altitude ..... 49
3.2.4 Residual Zernike coefficients estimation ..... 49
3.2.5 Residual scintillation ..... 52
3.3 Conclusion ..... 53
4 Thesis objectives ..... 55
II Irradiance fluctuations modeling for ground-to-satellite geostation- ary satellite optical links ..... 57
5 Validity of existing models ..... 59
5.1 Comparison of literature models with TURANDOT ..... 59
5.2 Taking into account the tracking ..... 61
5.3 Conclusion ..... 63
6 Irradiance fluctuations modeling for ground-to-satellite geostationary satel- lite optical links ..... 67
6.1 A multi-layer model - the WPLOT model ..... 68
6.1.1 Taking into account the beam truncation by the telescope ..... 72
6.1.2 Conclusion ..... 73
6.2 Discussion and validity of the WPLOT model ..... 73
6.2.1 Criteria for validation ..... 73
6.2.2 Propagation channel used for validation ..... 74
6.2.3 Without tracking ..... 74
6.2.4 With tracking ..... 83
6.3 Conclusion ..... 94
III WPLOT model application for the sizing of an Optical Ground Station ..... 95
7 Optical ground station sizing ..... 97
7.1 Standard parameters ..... 98
7.1.1 Channel parameters ..... 98
7.1.2 Optical ground station parameters ..... 99
7.1.3 Link budget parameters ..... 100
7.2 Maximum acceptable residual tilt ..... 100
7.3 Sizing using the WPLOT model ..... 102
7.3.1 Introduction to architecture sizing ..... 103
7.4 Sensitivity study ..... 111
7.4.1 Anisoplanatism induced by the point-ahead angle ..... 111
7.4.2 Delay before correction ..... 111
7.4.3 Sensitivity of the sizing to atmospheric turbulence conditions ..... 114
7.5 Conclusion on the sizing of the optical ground station ..... 118
8 Model application to error correcting codes and interleaving sizing : time series creation ..... 121
8.1 Obtaining irradiance time series ..... 121
8.2 Fade statistics description ..... 122
8.3 WPLOT to test error correcting codes and interleaving ..... 125
8.4 Conclusion ..... 125
A Detection architecture noise estimation ..... 131
B Zernike polynomials ..... 135
C Chassat's correlation functions ..... 139
D Scintillation index formulas ..... 147
E Résumé en Français ..... 149
F Publications list ..... 151

## List of Figures

1.1 Description of the architecture for a geostationary satellite for broadband internet.[4] ..... 7
1.2 Lunar Lasercomm ground terminal in White Sands (NASA) ..... 9
1.3 Schematic diagram of an optical feeder link ..... 12
1.4 Atmospheric transmission through the atmosphere as a function of the wave- length.[16] ..... 12
1.5 Illustration of NRZ OOK modulation. ..... 13
1.6 Presentation of the direct detection architecture for NRZ-OOK modulation. ..... 14
1.7 Presentation of the bit errors sources. The detection threshold of the in- tensity $I_{D}$ isn't necessarily placed in the intersection of the two probability density distributions. $P(1 \mid 0)$ corresponds to a false alarm case and $P(0 \mid 1)$ corresponds to a no-detection case. ..... 16
1.8 Presentation of the benefits of interleaving. In this example, the signal sent is comprised of packets of 4 bits. Without interleaving, 3 of the 4 bits of a packet are lost and the error correcting codes will not be able to recover the information. With interleaving, 3 packets have lost only one bit and the error correcting codes will be able to recover the information. ..... 19
2.1 Theory of energy cascades and the dispersion of energy in smaller turbulent eddies ..... 22
2.2 Power spectral density of the refractive index according to the modified von Kármán spectrum, for different values of outer and inner scales. The spectrum is normalized by the refractive index structure parameter $C_{n}^{2}$. ..... 24
2.3 Gaussian beam amplitude profile convention. ..... 30
2.4 Propagation of a convergent Gaussian beam (curvature radius $>0$ at $\mathrm{z}=0$ ) in vacuum ..... 31
2.5 Effects of beam wander on irradiance fluctuations[35] ..... 32
2.6 Illustration of the beam at the satellite's altitude, taking into account the effects of beam wander and beam spread.[36] ..... 32
2.7 Instantaneous beam profile in the receiver plane as a function of the ratio $\frac{w_{0}}{r_{0}}$. ..... 33
2.8 Phase screens propagation principle.[36] ..... 40
3.1 Illustration of the reciprocity principle in the simplified case of plane waves. ..... 46
3.2 Envisioned system presentation. WFS means Wave-Front Sensor. ..... 46
3.3 Point-ahead angle origin. $c$ is the speed of light ..... 47
3.4 Envisioned system presentation with point-ahead angle. WFS means Wave- Front Sensor. ..... 48
3.5 Point-ahead angle and the difference in atmospheric turbulence seen by the downlink and the uplink ..... 49
3.6 Presentation of monostatic configuration. ..... 50
3.7 Tilt variances resulting from each layer of the discretized atmosphere with and without tracking. ..... 52
5.1 Probability density function comparison between TURANDOT, a tilt-only model and the tilt model with on-axis scintillation model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case. ..... 61
5.2 Cumulative density function comparison for $P\left(I<I_{T}\right)<0.2$ between TU- RANDOT, a tilt-only model and the tilt model with on-axis scintillation model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case. ..... 62
5.3 Probability density function comparison between TURANDOT and the Dios model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case. ..... 63
5.4 Cumulative density function comparison between TURANDOT and the Dios model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case. ..... 64
5.5 Probability density function comparison between TURANDOT, a beam wander with on-axis scintillation model and the beam wander only model for 2000 draws, using a Hufnagel-Valley 5/7 atmospheric profile in a tracking case. ..... 65
5.6 Cumulative density function comparison for $P\left(I<I_{T}\right)<0.2$ between TU- RANDOT, a tilt-only model and the tilt model with on-axis scintillation model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile in a tracking case. ..... 66
6.1 Random irradiance draws obtained with the $2 \mathrm{~L}-\mathrm{WO}$ model and the LOT model. ..... 68
6.2 Comparison between random irradiance draws obtained with the $2 \mathrm{~L}-\mathrm{WO}$ model and the LOT model with tilt as the only considered aberration. ..... 69
6.3 Comparison between random irradiance draws obtained with the $2 \mathrm{~L}-\mathrm{WO}$ model and the LOT model with defocus as the only considered aberration. ..... 69
6.4 Comparison between random irradiance draws obtained with the $2 \mathrm{~L}-\mathrm{WO}$ model and the LOT model with astigmatism as the only considered aberration. 70
6.5 Probability Density Functions of irradiance statistics between LOT solution,2L-WO simulation and a 10L-WO simulation when considering only defocus,for an emitted waist size of 8 cm .70
6.6 Cumulative Density Functions of irradiance statistics between LOT solution, $2 \mathrm{~L}-\mathrm{WO}$ simulation and a $10 \mathrm{~L}-\mathrm{WO}$ simulation when considering only defocus, for an emitted waist size of 8 cm . ..... 70
6.7 Comparison between random irradiance draws obtained with the $2 \mathrm{~L}-\mathrm{WO}$ model and the WPLOT model with defocus as the only considered aberration. 71
6.8 Comparison between random irradiance draws obtained with the $2 \mathrm{~L}-\mathrm{WO}$ model and the WPLOT model. ..... 72
6.9 Cumulative density functions of irradiance fluctuations obtained with the 2L-WO simulation taking into account the truncation and the WPLOT model with 2 layers taking into account, or not the truncation. In this case, $D_{T_{X}}=2^{3 / 2} w_{0}$, which is a value usually found in the literature[42]. ..... 73
6.10 Mean irradiance as a function of the emitted waist. Comparison between the TURANDOT simulation and the WPLOT model for 2000 draws, with a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case. ..... 75
6.11 Difference in dB between the estimation of the mean irradiance as a function of the waist with the TURANDOT simulation and the WPLOT model, for a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case. ..... 75
6.12 Probability density function comparison between TURANDOT and the WPLOT model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case. ..... 76
6.13 Cumulative density function comparison between TURANDOT and the WPLOT model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case. ..... 77
6.14 Comparison of the estimation of the irradiance threshold $5 \%$ of the normal- ized CDF as a function of the waist size at emission between TURANDOT and the WPLOT model, with a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case. ..... 77
6.15 Comparison of the estimation of the losses due to turbulence at $5 \%$ of the normalized CDF as a function of the waist size at emission between TU- RANDOT and the WPLOT model, with a Hufnagel-Valley 5/7 atmospheric profile in a no tracking case. ..... 78
6.16 Temporal power spectral density comparison between TURANDOT and the WPLOT model obtained from 4 s times series with a 2 kHz sampling, using a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case. For a waist size of 5 cm , TURANDOT returned a numerical error probably due to sampling. We have not been able to correct this error yet. ..... 79
6.17 Mean irradiance as a function of the emitted waist. Comparison between the TURANDOT simulation and the WPLOT model for 2000 draws, with stronger atmospheric conditions in a no tracking case. ..... 80
6.18 Difference in dB between the estimation of the mean irradiance as a function of the waist with the TURANDOT simulation and the WPLOT model. ..... 80
6.19 Probability density function comparison between TURANDOT and theWPLOT model for 2000 draws, using the stronger atmospheric profile ina no tracking case.81
6.20 Cumulative density function comparison between TURANDOT and the WPLOT model for 2000 draws, using the stronger atmospheric profile in a no tracking case. ..... 82
6.21 Comparison of the estimation of the irradiance threshold $5 \%$ of the normal- ized CDF as a function of the waist size at emission between TURANDOT and the WPLOT model, with the stronger atmospheric profile in a no track- ing case ..... 82
6.22 Estimation of the losses due to turbulence at $5 \%$ of the normalized CDF as a function of the waist size at emission, using the stronger atmospheric profile in a no tracking case. ..... 83
6.23 Temporal power spectral density comparison between TURANDOT and the WPLOT model obtained from 4 s times series with a 2 kHz sampling, using a stronger atmospheric profile in a no tracking case. For a waist size of 5 cm , TURANDOT returned a numerical error probably due to sampling. We have not been able to correct this error yet. ..... 84
6.24 Mean irradiance as a function of the emitted waist. Comparison between the TURANDOT simulation and the WPLOT model for 2000 draws, with a Hufnagel-Valley 5/7 atmospheric profile and tracking. ..... 85
6.25 Difference in dB between the estimation of the mean irradiance as a function of the waist with the TURANDOT simulation and the WPLOT model. ..... 85
6.26 Probability density function comparison between TURANDOT and the WPLOT model for 2000 draws, using a Hufnagel-Valley 5/7 atmospheric profile and tracking. ..... 86
6.27 Cumulative density function comparison between TURANDOT and the WPLOT model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile and tracking. ..... 87
6.28 Comparison of the estimation of the irradiance threshold $5 \%$ of the normal- ized CDF as a function of the waist size at emission between TURANDOT and the WPLOT model, with a Hufnagel-Valley $5 / 7$ atmospheric profile and tilt tracking ..... 87
6.29 Comparison of the estimation of the losses due to turbulence at $5 \%$ of the normalized CDF as a function of the waist size at emission between TU- RANDOT and the WPLOT model, with a Hufnagel-Valley 5/7 atmospheric profile and tilt tracking. ..... 88
6.30 Temporal power spectral density comparison between TURANDOT and the WPLOT model calculated from a 1 s times series with a 2 kHz sampling, using a Hufnagel-Valley $5 / 7$ atmospheric profile and tracking. ..... 88
6.31 Mean irradiance as a function of the emitted waist. Comparison between the TURANDOT simulation and the WPLOT model for 2000 draws, with stronger conditions and tracking. ..... 89
6.32 Difference in dB between the estimation of the mean irradiance as a function of the waist with the TURANDOT simulation and the WPLOT model, with stronger conditions and tracking. ..... 89
6.33 Probability density function comparison between TURANDOT and the WPLOT model for 2000 draws, using the stronger atmospheric profile and tracking. ..... 90
6.34 Cumulative density function comparison between TURANDOT and the WPLOT model for 2000 draws, using the stronger atmospheric profile and tracking. ..... 91
6.35 Comparison of the estimation of the irradiance threshold $5 \%$ of the normal- ized CDF as a function of the waist size at emission between TURANDOT and the WPLOT model, using the stronger atmospheric profile and tracking. ..... 92
6.36 Estimation of the losses due to turbulence at $5 \%$ of the normalized CDF as a function of the waist size at emission, using the stronger atmospheric profile and tracking. ..... 92
6.37 Temporal power spectral density comparison between TURANDOT and the WPLOT model calculated from a 1 s times series with a 2 kHz sampling, using a stronger atmospheric profile and tracking. ..... 93
7.1 Link distance and elevation angle as a function of the latitude. ..... 99
7.2 Description of the Gaussian beam parameters ..... 100
7.3 Maximum allowable variance for the tip/tilt coefficients over uncompensated tilt variances ratio as a function of the waist size at emission, taking into account a telescope truncature of diameter $D_{T_{X}}=2^{\frac{3}{2}} w_{0}$ in order to compare with the results in the rest of the study. ..... 102
7.4 Evolution of the optical link budget as a function of the waist in a no tracking case. ..... 104
7.5 Evolution of the optical link budget as a function of the waist with perfect tracking. ..... 104
7.6 Residual tilt variance compared to the initial uncompensated tilt as a func- tion of the wave-front pupil diameter to waist size ratio, considering $w_{0}=8 \mathrm{~cm} .106$
7.7 Evolution of the link budget as a function of the wave-front pupil diameter to waist size ratio, considering $w_{0}=8 \mathrm{~cm}$. ..... 106
7.8 Examples of bi-static configurations. ..... 107
7.9 Residual tilt variance compared to the initial uncompensated tilt as a func- tion of the waist size in the case of an off-axis configuration. ..... 108
7.10 Evolution of the link budget as a function of the waist size in the case of an off-axis configuration. ..... 108
7.11 Residual tilt variance compared to the initial uncompensated tilt for a 8 cm waist size at emission as a function of the $D_{T T S}$ to $D_{T_{X}}$ ratio. ..... 109
7.12 Link budget for a 8 cm waist size at emission as a function of the $D_{T T S}$ to $D_{T_{X}}$ ratio. ..... 109
7.13 Effect of the emitter truncature on the optical link budget for different waist sizes without atmospheric turbulence. ..... 110
7.14 Effect of the emitter truncature on the optical link budget for different waist sizes with atmospheric turbulence. ..... 110
7.15 Residual tilt variance over the uncompensated tilt variance as a function of the beam waist size at emission when taking into account the point-ahead angle in the tracking. ..... 112
7.16 Evolution of the optical link budget as a function of the emitted waist sizes, either considering a point-ahead angle or not. ..... 112
7.17 Residual tilt variance compared to the initial uncompensated tilt as a func- tion of the delay between the wavefront distortion measurement and the correction for waists of 5,8 and 12 cm . ..... 113
7.18 Link budget evolution as a function of the delay between the wavefront distortion measurement and the correction for waists of 5,8 and 12 cm . ..... 113
7.19 Comparison of the link budget evolution as a function of the emitted waist in a case with tracking considering a ground turbulence strength $C_{n}^{2}(0)=$ $1.7 \times 10^{-14} \mathrm{~m}^{-\frac{2}{3}}, C_{n}^{2}(0)=5.4 \times 10^{-14} \mathrm{~m}^{-\frac{2}{3}}$ and $C_{n}^{2}(0)=1 \times 10^{-13} \mathrm{~m}^{-\frac{2}{3}}$. ..... 114
7.20 Considered atmospheric profiles for study of the impact of the high altitude layers on the link budget. ..... 116
7.21 Comparison of the link budget evolution as a function of the emitted waist in a case with tracking considering the atmospheric profiles presented in Figure 7.20 . ..... 117
7.22 Optical link budgets for different outer scales. ..... 118
8.1 Irradiance time series over 1 s with a 2000 Hz sampling using the final ar- chitecture from Table 7.5 in Chapter 7 for a Hufnagel-Valley atmospheric profile with $C_{n}^{2}(0)=5.4 \times 10^{-14} \mathrm{~m}^{-2 / 3}$ and an elevation $\phi_{\text {elev }}=40^{\circ}$. ..... 123
8.2 Fade duration occurrence and exceedance probability plots for the final ar- chitecture from Table 7.5 in Chapter 7 for a Hufnagel-Valley atmospheric profile with $C_{n}^{2}(0)=5.4 \times 10^{-14} \mathrm{~m}^{-2 / 3}$ and an elevation $\phi_{\text {elev }}=40^{\circ}$ in a tracking case. ..... 123
8.3 BER time series over 1 s with a 2000 Hz sampling using the final architecture from Table 7.5 in Chapter 7 for a Hufnagel-Valley atmospheric profile with $C_{n}^{2}(0)=5.4 \times 10^{-14} \mathrm{~m}^{-2 / 3}$ and an elevation $\phi_{\text {elev }}=40^{\circ}$. ..... 124
8.4 BER curve for the detection architecture presented in Chapter 1 and Ap- pendix A. ..... 124
8.5 Correlation of the 45 first Zernike coefficients for the stronger atmospheric profile and the architecture from Table 8.1. ..... 129
8.6 Correlation presented as a function of the radial degree for the stronger atmospheric profile and the architecture from Table 8.1. ..... 129
A. 1 Effects of each component of the detection architecture on the temporal power spectral densities of the signal and ASE fields leading to the power spectral densities of the currents SxASE and ASExASE ..... 132
B. 1 The first 15 Zernike polynomials, ordered vertically by radial degree and horizontally by azimuthal degree. ..... 136
C. 1 Definition of $d_{1 \rightarrow 2}(h)$, the distance between the centers of the two considered beams at an altitude $h[27]$. ..... 141
C. 2 Zernike coefficients variance for a wavefront perturbed by a Kolmogorov Turbulence (infinite outer scale)[27].
C. 3 Dependence of $\sigma_{j j}\left(0,1, \frac{D}{2 L_{0}}\right)$ for the polynomials $j$ with a radial degree $n \leq 4$ in function of $D / L_{0}[27]$. . . . . . . . . . . . . . . . . . . . . . . . . . . 145

## List of Tables

1.1 Past optical link demonstrations between ground and space. ..... 11
1.2 Average photon/bit required depending on optical receiver design (values obtained from [17]). ..... 13
1.3 Contributors to the optical link budget. ..... 20
3.1 Parameters for the study ..... 50
3.2 Table describing the impact of each layer on the overall residual tilt after compensation ..... 51
7.1 Contributors to the optical link budget. ..... 101
7.2 Parameters of the sensitivity study. ..... 103
7.3 Table of the isoplanatic angle and Fried parameter obtained with each at- mospheric profile ..... 115
7.4 Table describing the impact of each layer on the residual tilt after compen- sation considering the atmospheric profiles presented in Figure 7.20. ..... 117
7.5 Final architecture characteristics. ..... 119
8.1 Final architecture characteristics. ..... 128
B. 1 The first 15 Zernike polynomials, their Noll index, radial degree, azimuthal degree and classical name ..... 137
C. 1 Definition of $K_{j_{1} j_{2}}^{1}$ as a function of $j_{1}$ and $j_{2}$. ..... 140
C. 2 Definition of $K_{j_{1} j_{2}}^{2}$ as a function of $j_{1}$ and $j_{2} .\left(s_{j_{1} j_{2}}=\operatorname{sign}\left(m_{1}-m_{2}\right)\right.$ if $m_{1}-m_{2}$ is odd, else $s_{j_{1} j_{2}}=1$.) ..... 141
E. 1 Caractéristiques de l'architecture retenue. ..... 150

## List of symbols and abbreviations

$\chi \quad$ Log-amplitude of the optical field
$\lambda \quad$ Wavelength
$\nu_{p} \quad$ Carrier frequency
$\phi \quad$ Phase of the optical field
$\phi_{\text {elev }}$ Elevation angle $\phi_{\text {elev }}=90^{\circ}-\zeta$
$\Re \quad$ Responsiveness of the PIN photodiode
$\theta_{\text {IPA }}$ Isoplanatic angle
$\theta_{I P A}$ Isoplanatic angle
$\theta_{P A A}$ Point-ahead angle
$\xi \quad$ Angular offset due to the ground station
$B_{E} \quad$ Bandwith of the electrical filter
$B_{O}$ Bandwith of the optical filter
$C_{n}^{2} \quad$ Index-of-refraction structure parameter
$D_{n}(\vec{\rho})$ Refractive index structure function
$D_{R_{X}}$ Diameter of the receiving telescope
$D_{T_{X}} \quad$ Diameter of the transmitting telescope
$D_{\text {throughput }}$ Targeted data throughput
$G \quad$ Gain of the Erbium-Doped Fiber Amplifier
$G_{E} \quad$ Emission gain
$G_{R} \quad$ Receiver gain
$h \quad$ Planck constant
$I_{E} \quad$ Electrical current after detection
$I_{R} \quad$ Irradiance threshold
$k_{0} \quad$ Wave number
$L \quad$ Propagation distance
$L_{0} \quad$ Outer scale
$l_{0} \quad$ Inner scale
$L_{a b} \quad$ Absorption and scattering loss
$L_{\text {clouds }}$ Transmission through thin clouds
$L_{\text {margin }}$ System margin
$L_{\text {Others }}$ Losses in link budget other than turbulence losses
$N F \quad$ Noise Factor of the Erbium-Doped Fiber Amplifier
$P_{E} \quad$ Optical emitted power
$P_{R} \quad$ Optical received power
$q$ Electrical charge
$R(z)$ Curvature radius of the beam at position $z$
$T_{R_{X}}$ Transmission loss inside optical terminal in $R_{X}$
$T_{T_{X}} \quad$ Transmission loss inside optical terminal in $T_{X}$
$w(z)$ Radius of the beam at $1 / \mathrm{e}$ in amplitude at position $z$
$w_{0} \quad$ Waist size of the beam at emission
$W_{n} \quad$ Spectral density of the refractive index fluctuations
$Z_{R} \quad$ Rayleigh distance of the Gaussian beam
ALBS Accès Large Bande pour Satellite
AO Adaptive Optics
ASE Amplified Spontaneous Emission
BER Bit Error Rate
EDFA Erbium-Doped Fiber Amplifier
NRZ OOK Non Return to Zero On-Off-Keying

OGS Optical Ground Station
PSD Power Spectral Density
RF Radio Frequency

## Introduction

In order to face the ever growing needs for higher data rates and because of the saturation of the radio frequency ( RF ) bands, a multiplex of wavelengths around the $1.55 \mu \mathrm{~m}$ spectral band is being investigated for the next generation of high throughput satellite system aiming at a 1 Terabits/s capacity.

In addition to cloud coverage, one of the major limitations for the use of optical wavelengths is atmospheric turbulence, which induces local fluctuations of the refractive index which deteriorate the optical quality of a laser beam during its propagation within the atmosphere and lead to fluctuations of the detected telecommunication signal. In the case of an uplink (a link going from the ground to a geostationary satellite), the optical beam is deflected (a phenomenon called 'beam wandering') and distorted ('beam spreading') by atmospheric turbulence. These effects lead to irradiance fluctuations which can result, in the worst case, in a loss of detected signal. These effects also have an important temporal variability and therefore increase the error rate as well as prevent the system from working at the desired capacity. Propagation of ground phase perturbations induced tracking errors which cannot be compensated at satellite level. The solution is therefore to compensate these effects at emission. For this, different systems are envisioned such as compensating the phase with an adaptive optics system or using a diversity of emitters.

Adaptive optics (AO) systems for correcting the effects of atmospheric turbulence were first envisioned for astronomy in 1953 [1]. The idea is to improve the performance of optical systems by reducing the effect of wave front distortions. The phase perturbations are measured with a wave front sensor and compensated with a deformable mirror. This leads to better quality images with bigger telescopes. At the time, adaptive optics systems were limited due to the requirements in computational power. However, since the 1990s, adaptive optics systems equip the world's leading telescopes. In the case of satellites for telecommunications, using AO systems should permit to concentrate and stabilize the energy on the satellite's detection pupil, thus reducing the error rates. This idea was first introduced in the 1970s by Fried [2], relying on the reciprocity of propagation through atmosphere. In the case of a ground to satellite propagation, such an approach makes sense because the atmospheric perturbations are close to the ground station telescope, mostly modifying the phase of the electromagnetic field and can therefore be compensated by using a deformable mirror. In such a system, a beam coming from the satellite will be used to measure and estimate the perturbations that need to be applied to the emitted
wavefront. Because of point-ahead angle between the downlink and the uplink (due to the Earth's rotation), of the optical ground station (OGS) architecture and of the delay between the measurement and the correction, the turbulence effects experienced by the downlink and the uplink are slightly different, leading to partial compensation only.

The knowledge, characterization and modeling of the effects of turbulence, and their correction, on an optical link are essential for the optimization of a ground station. My thesis focused on this subject, in the particular context of a grounf to satellite feeder link, capable of providing the targeted capacity ( 1 Terabit/s by 2025). To achieve this goal, most of the effort will be focused on developing a model describing the effects of turbulence on the optical uplink in the presence of adaptive optics pre-compensation.

In Part I of this thesis, the aim is to introduce optical satellite communications. In Chapter 1, I will introduce the link budget performance criteria which we will use throughout this manuscript. It gives an estimation of the received power as a function of the propagation channel losses and emitted power, and estimates the link's performance. Within the link budget, I will focus on the losses due to turbulence as a function of the emitting architecture. In order to estimate the losses due to turbulence, I will need a model which describes accurately irradiance fluctuations, particularly in the lower irradiances. In Chapter 2, I am going to describe irradiance fluctuations using results from the literature in order to comprehend how turbulence affects irradiance fluctuations. Chapter 3 will be focused on presenting turbulence effects mitigation techniques, such as adaptive optics.

In Part II, I will present the simplified model for describing irradiance fluctuations that we have developed. In Chapter 5, we will compare the models described in Chapter 2 with TURANDOT (a wave optics simulation tool developed by ONERA also presented in Chapter 2) and highlight that the presented models are insufficient to model irradiance fluctuations, particularly in a tilt tracked case. I will present in Chapter 6 a model found in the literature [3] which we have improved. I will compare this new model to TURANDOT in order to validate its range of validity but also demonstrate its limits.

Finally, the objective of Part III is to do a system sizing using the link budget from Chapter 1 and the irradiance fluctuations model from Chapter 6 . The objective will be to evaluate the impact of the optical ground station parameters and of the propagation channel on the link budget and converge with an optical ground station capable of fulfilling the link budget. Finally, in Chapter 8, I will present the temporal statistics of irradiance times series obtained with the model and their impact on the error correcting codes performance in turbulence.

## Part I

## Context

## Optical communications for high throughput satellite feeder links

After a brief introduction of the history, context and actual state of the art of optical freespace communications, we will focus on satellite optical communications, more specifically on ground-to-space links.

The second part of this chapter will present the link parameters (wavelength, modulation scheme, etc.). We will briefly present the detection architecture which will result in a performance criteria that will be used throughout the manuscript. Finally, we will introduce the link budget.

## Contents

### 1.1 Geostationary satellite system for broadband internet <br> 6

1.1.1 Space, ground and user segments ..... 6
1.1.2 Feeder link ..... 6
1.1.3 Limitations to the use of radio frequencies ..... 6
1.1.4 Advantages and challenges of free-space optical communications ..... 7
1.1.5 History of optical communications ..... 8
1.2 Optical communication device architecture: main technolog- ical drivers ..... 12
1.2.1 Wavelength ..... 12
1.2.2 Optical receiver - direct detection ..... 13
1.2.3 Modulation - NRZ OOK ..... 13
1.2.4 Detection architecture ..... 13
1.3 Link Budget ..... 14
1.3.1 NRZ-OOK: power detection threshold - $P_{R}$ ..... 15
1.4 Conclusion ..... 19

### 1.1 Geostationary satellite system for broadband internet

In order to benefit from global links without the limitations from heavy and vulnerable terrestrial structures, satellites appeared to be a promising solution. In 1962, Telstar 1 was the first true communications satellite. Equipped with a receiver and a very powerful transmitter, it transmitted live in the United States the first television shows broadcasted in Europe. The next year, NASA sent the first satellite on the geostationary orbit, Syncom 1 , with a capacity equivalent to one television channel or 50 phone calls at the same time. In 1965, the launch of the first telecommunications geostationary satellite, Intelsat 1 , marked the true beginning of the telecommunications by satellites era which allowed a total coverage of the globe.

### 1.1.1 Space, ground and user segments

The architecture of geostationary satellite systems for broadband can be divided in three segments: the space segment, the ground segment and the user segment. Figure 1.1 provides an illustration of the different segments.

The ground segment consists in one or several gateways, which comprise of large antenna dishes, modems and the Internet service provider equipment. The gateway is at the interface between the Internet and the satellite link.

The user segment uses small antenna dishes and a modem to interface between the satellite system and the end-user's network.

Finally, the space segment comprises of two parts: the platform and the payload. For geostationary communications satellites, the payload is usually transparent, meaning that it will only amplify the received signal before transmitting it to the following segment. The signal is therefore modified without any impact on the satellite during its lifetime. The platform defines the mass, consumption and energy dissipation of the satellite.

### 1.1.2 Feeder link

The feeder link transmits the data between the ground segment and the spatial segment and the links between the space segment and the user segment are called user links. Both links are bidirectional. The links going from the ground to the satellite are called uplinks while those going in the other direction are called downlinks.

### 1.1.3 Limitations to the use of radio frequencies

The next generation of high throughput satellite system is expected to reach 1 Terabits/s, in order to meet the data rate requirements for mobile telephony and Internet.

Nowadays, there are two principal limitations to the use of radio frequencies (RF). The first is the saturation of the RF bands. Many satellites use the Ka band today, which has a bandwidth of 3.5 GHz for both the downlink and the uplink around their central frequencies respectively at 20 GHz and 30 GHz . The second limitation is the need for a higher capacity, which the RF can't deliver.

In order to deal with these limitations, the satellite industry pushes towards the higher frequencies, as the available bandwidth increases with the frequency. The Q/V band (which

### 1.1 Geostationary satellite system for broadband internet



Figure 1.1: Description of the architecture for a geostationary satellite for broadband internet.[4]
has a bandwidth of 5 GHz at each polarization for both the downlink and the uplink around their central frequencies respectively at 40 GHz and 50 GHz ) is currently in development. Using this band will liberate the Ka band which may then be fully allocated to the user links. However, using this frequency band would lead to using over 30 gateways, because of diversity techniques to mitigate the rain attenuation, in order to satisfy an availability over $99.9 \%$.

Optical links permit to deal with the problem of the saturation of the RF bands as well as significantly increase the bandwidth, with a central frequency around 200 THz .

### 1.1.4 Advantages and challenges of free-space optical communications

The laser beam has a low divergence (typically around 10 to $100 \mu \mathrm{rad}$ ) in contrast to RF waves for which the divergence is at least 1000 times bigger. The flux budget is thus more favorable with a gain of the order of 60 dB reducing the source's required power. By its directivity, the beam is difficult to intercept and thus allows inherently secured communications. Interference problems between channels are also resolved and the spectrum regulation becomes unnecessary. The mass, space and electrical power required are much lower as well as the overall cost of the system. Antennas embedded on satellite have dimensions of a few tens of centimeters and emit the order of the watt for the optical links; they are rather of the order of the meter and several tens of watt for RF links.

However, the low divergence of the beam presents a technological challenge, as it requires a very precise pointing system of the beam towards the receiver. The pointing may be disturbed by atmospheric turbulence close to the ground. Another technological difficulty is the stabilization of the optical axes of the pointing systems at emission and at reception due to possible mechanical drifts or vibrations.

Chapter 1. Optical communications for high throughput satellite feeder links

Another limitation is the absorption by clouds at optical wavelengths. This means that a feeder link can not function if there is a cloud above the ground station. The solution is to use a diversity of ground stations so that at least one station is not under a cloud. This will not be treated in this thesis.

### 1.1.5 History of optical communications

## Brief history of optical communications

The first real communication network appeared at the end of the French Revolution: in 1794, Chappe's optical telegraph connected Paris to Lille via a network of semaphores conveying step by step coded messages. Its effective propagation speed was $35 \mathrm{~km} / \mathrm{h}$. However, optics were quickly abandoned at the benefit of electronics whose mastery progressed more rapidly.

In 1880, Graham Bell developed his photophone [5]. It worked using light modulation induced by the deformation of a mirror under the effect of voice and a selenium receiver whose resistance varied according to the light intensity received, which enabled information recovery.

It was the arrival of lasers, in 1960, that revived interest in optical transmissions by establishing an optical transmission link in direct line of sight. But they were limited by their lifespans, their bulk and their insufficient luminous power.

In the 1980s, the arrival of semiconductor lasers made it possible to meet these limitations thanks to a longer lifespan, reduced bulk and high efficiency. Thanks to the directivity and their high power, they allowed to work with high signal-to-noise ratios.

It was in the 1990s that fiber lasers appeared, bringing technological maturity and a response to increasing bandwidth demand. They had very low attenuation which allowed the creation of high-speed long-range line such as submarine cables, therefore in guided propagation. The large variations in transmission induced by atmospheric conditions (precipitation, aerosols, etc.) in free space propagation as well as the need for high availability for the targeted applications initially led to the development of short-range systems.

## Ground-to-space optical links

For the time being, optical free-space telecommunications through space are still at the stage of demonstration.

Initial searches for ground-to-space link applications started in the late 1970s. In 1992, the Galileo probe received the first optical link while being 6 million km away from Earth[6]. In 1995, the first bidirectional link was made between the Japanese geostationary ETS-VI satellite and the ground during the Ground/Orbiter Lasercomm Demonstration (GOLD) program $[7,8]$.

In 2001, a two-way link was made between a LEO satellite (SPOT-4) and a GEO satellite (Artemis) as part of the SILEX program[9]. The link delivered 50 Mbps . The system was made by Astrium (now Airbus Defence \& Space) and was embedded on both satellites. It used a GaAlAs laser diode, emitting 60 mW at 800 nm , an avalanche photodiode for detection as well as a tracking system comprising of a tilting mirror and CCD matrices. It

### 1.1 Geostationary satellite system for broadband internet

used a "pulse position modulation" (PPM) (cf Section 1.2.3). The Artemis satellite was also used to make two-way links with the ground from November 2001[10].

In 2006, the LOLA experiment was a two-way link between the GEO satellite Artemis and an airplane. The uplink had a 2 Mbps capacity modulated by a BPPM while the downlink had a 50 Mbps with an OOK modulation.

In 2008, the first bi-directional optical telecommunications link between the LEO satellite OICETS/Kirari and a ground station in Tokyo (uplink with a 2 Mbps capacity and BPPM while downlink had a 50 Mbps OOK modulation) was achieved. The connection was maintained during 6 minutes during the satellite's passage in clear weather. The achieved bit error rates were $10^{-7}$ on the uplink and $10^{-4}$ on the downlink.

In 2011, a bi-directional broadband link was achieved between the LEO satellite NFIRE from NASA to an OGS in Tenerife (part of the TERRASAR project [11]). This link was achieved by the European Space Agency (ESA), the German space agency (DLR) and TESAT (German company specialized in satellite telecommunications). It had a 5.6 Gbps capacity using a BPSK modulation with a homodyne detection. It used a YAG laser source (at a $1,06 \mu \mathrm{~m}$ wavelength) which delivered 0.7 W . During this demonstration, there were long periods of error less telecommunications intersected with peaks of error due to the scintillation of the laser beam in the turbulent atmosphere.

In 2014, the GEO satellite Alphasat[12] was sent into orbit with a laser terminal developed by the TESAT and provided by the DLR, with a capacitu of the order of 1.8 Gbps and a ground station in Tenerife. It used a laser source at $1.06 \mu \mathrm{~m}$ with a BPSK modulation.

Also in 2014, a broadband link between the ground and the Moon was achieved with the Lunar Laser Communication Demonstrator (LLCD)[13].Three stations were used : the OGS in Tenerife, the Optical Comm Telescope Lab of the JPL at Table Mountain and a specific terminal at White Sands with 4 emitters with a 15 cm diameter and 4 receivers with a 40 cm diameter (cf. Figure 1.2). It used a PPM with wavelengths around $1.5 \mu \mathrm{~m}$ and achieved a 622 Mbps capacity on the downlink and a 20 Mbps on the uplink.

The SOTA (SmallOpticalTrAnsponder) terminal, part of the SOCRATES (Space Optical Communications Research Advanced Technology Satellite) has been used to establish link with an optical ground station at the Observatoire de la Côte d'Azur (OCA) in France [14]. This was a project financed by CNES (the french space agency) with the participation of Airbus Defence \& Space, Thales Alenia Space and ONERA.

Finally, Opals (Optical PAyload Lasercomm Science) on the International Space Station (ISS) has been used for a downlink transmission at $50 \mathrm{Mbps}[15]$, with the aim to prove the feasibility of satellite to ground links.

Throughout these examples, we can see that the ques-


Figure 1.2: Lunar Lasercomm ground terminal in White Sands (NASA). tion of optical links, and particularly of uplinks, is gaining in maturity. All these demonstrations characteristics are summarized in Table 1.1. These results show that, while the first demonstration of a ground to satellite link dates from 1995, there has recently
been an acceleration in the field with multiple demonstrations in 2014. We can observe that the wavelength has increased over the demonstrations, as it started around a $0.8 \mu \mathrm{~m}$ wavelength whereas the most recent experiments all used a $1.55 \mu \mathrm{~m}$ wavelength. Most demonstrations used a NRZ OOK modulation or PPM modulation, even though Europeans also seem to be interested in the BPSK modulation. Finally, adaptive optics are beginning to be tested in the very recent demonstrations on downlinks whereas beam diversity is still most commonly used for uplinks. There has not yet been any demonstrations of ground to satellite uplinks using adaptive optics. This is the next step toward the use of bidirectional optical feeder links.
Table 1.1: Past optical link demonstrations between ground and space.

| Name of program | GOLD | SILEX | LUCE | TERRASAR | ALPHASAT | LLCD | OPALS | SOCRATES |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | 1995 | 2001 | 2008 | 2011 | 2014 | 2014 | 2014 | 2014 |
| Region | USA/Japan | Europe | Japan | Europe/USA | Europe | USA | USA | Japan |
| Application | Ground to GEO satellite link | Intersatellite communication | Ground to LEO satellite | $\begin{aligned} & \text { Ground to } \\ & \text { LEO } \end{aligned}$ | $\begin{aligned} & \text { Ground to } \\ & \text { GEO } \end{aligned}$ | Ground-to- <br> Moon | ISS to ground | LEO to ground |
| Link direction | Bi- <br> directional | Bi- <br> directional | Bi- <br> directional | Bi- <br> directional | Bi- <br> directional | Bi- <br> directional | Downlink | Downlink |
| $\begin{array}{ll} \text { Link } & \text { range } \\ (\mathrm{km}) & \end{array}$ | 38.000 | 38.000 | 2.640 | 45.000 | 38.000 | 385.000 | 700 | 2.640 |
| Uplink wavelength | 515 | 830 | 847 | 1064 | 1064 | 1550 | / | / |
| Downlink wavelength | 830 | 830 | 815 | 1064 | 1064 | 1550 | 1550 | 1550 |
| Data rate up(Mbps) | 1 | 50 | 2 | 5600 | 1800 | 622 | $/$ | / |
| Data rate down(Mbps) | 1 | 50 | 50 | 5600 | 1800 | 20 | 50 | 50 |
| Modulation up | 2-PPM | 2-PPM | 2-PPM | BPSK | BPSK | PPM | / | $/$ |
| Modulation down | 2-PPM | NRZ-OOK | NRZ-OOK | BPSK | BPSK | PPM | NRZ-OOK | NRZ-OOK |
| Detection | Direct | Direct | Direct | Coherent | Coherent | Direct | Direct | Direct |
| Atmospheric mitigation | Spatial and temporal diversity | / | / | / | / | Diversity \& tracking | Adaptive <br> optics (receiver) | Adaptive optics (receiver) |

Chapter 1. Optical communications for high throughput satellite feeder links

### 1.2 Optical communication device architecture: main technological drivers

The optical feeder link may be decomposed in multiple parts. On the ground segment, there is the laser source, the modulator (a Mach-Zender interferometer for example) as well as the optical architecture of the emitting telescope, which may comprise adaptive optics. On the space segment, there is the receiving telescope and the detection architecture. A simple schematic diagram of an optical feeder link is presented in Figure 1.3.


Figure 1.3: Schematic diagram of an optical feeder link.

### 1.2.1 Wavelength

Because of the absorption at certain wavelengths by molecules present in the atmosphere, there is a limited number of windows of emission.


Figure 1.4: Atmospheric transmission through the atmosphere as a function of the wavelength.[16]

A longer wavelength leads to improved transmission through the atmosphere and a weaker degradation due to atmospheric turbulence. From Figure 1.4, three wavelengths have been envisioned: $0.8,1.06$ and $1.55 \mu \mathrm{~m}$.

To make a telecommunications transmission, it is necessary to have the optical and electronic equipment. The optical components were developed for fiber-based terrestrial networks at $1.55 \mu \mathrm{~m}$ in the 90 s with the Erbium-Doped Fiber Amplifier (EDFA). The limits of electronics are day by day pushed back with smaller electronic circuits, dedicated and operating at frequencies far superior to the ten Gbps.

### 1.2.2 Optical receiver - direct detection

Two detection means can be envisaged:

- Direct detection, which is only sensitive to the intensity of the received field.
- Coherent detection, which detects the phase and the intensity of the received signal, which interferes with a local oscillator.

Coherent detection has the advantage of being more sensitive than direct detection by 3 dB in the signal-to-noise ratio and therefore of having a lower photon / bit ratio. However, since the invention of EDFA, the gap between the two solutions has narrowed as can be seen in Table 1.2. Moreover, coherent detection is more complex and therefore costly to implement, as it needs a local oscillator controlled by a phase locked loop. We will consider only a direct detection receiver in this thesis.

Table 1.2: Average photon/bit required depending on optical receiver design (values obtained from [17]).

| Direct detection | Direct detection with EDFA | Coherent Detection |
| ---: | ---: | ---: |
| 500 | 30 | 18 |

### 1.2.3 Modulation - NRZ OOK



Figure 1.5: Illustration of NRZ OOK modulation.
Due to the choice of the direct detection, only a modulation of the amplitude field is possible. We will consider a Non Return to Zero On-Off-Keying (NRZ OOK) modulation, which is robust and simple, presented in Figure 1.5.

### 1.2.4 Detection architecture

The architecture used to detect an OOK signal is presented in Figure 1.6. In this Figure, $P_{R}$ is the received optical power. $G$ is the gain of the EDFA and $N F$ is its noise factor. $B_{O}$ and $B_{E}$ are, respectively, the bandwidths of the optical and electrical filters. $\Re$ is the responsiveness of the photodiode (in ampere per watt) and $I_{E}$ is the electrical current after detection.

After propagation through the atmosphere, the optical signal $P_{R}$ is received by the EDFA, which will amplify it. It will then be detected by a PIN photodiode. The EDFA

Chapter 1. Optical communications for high throughput satellite feeder links


Figure 1.6: Presentation of the direct detection architecture for NRZ-OOK modulation.
will be a source of significant noise due to the spontaneous emission that takes place within it. Indeed, this spontaneous emission will be amplified in the same way as the signal received (this is called the amplified spontaneous emission or ASE ). To limit the impact of noise during detection, an optical bandpass filter is added between the EDFA and the photodiode as well as an electrical lowpass filter after the photodiode. This system leads to noise variance of the current fluctuations $\sigma^{2}$ [17]:

$$
\begin{equation*}
\sigma^{2}=\sigma_{s h o t}^{2}+\sigma_{S \times A S E}^{2}+\sigma_{A S E \times A S E}^{2}+\sigma_{T}^{2}, \tag{1.1}
\end{equation*}
$$

where $\sigma_{T}^{2}$ is the variance resulting from the thermal noise and $\sigma_{S}^{2}$ is the variance from the shot noise. The shot noise has a component resulting from the detected signal and one component resulting from the ASE of the EDFA. It is defined:

$$
\begin{equation*}
\sigma_{\text {shot }}^{2}=2 q \Re\left(G P_{R}+P_{A S E}\right) B_{E}=\sigma_{\text {shot }, \text { signal }}^{2}+\sigma_{\text {shot }, A S E}^{2} \tag{1.2}
\end{equation*}
$$

where $q$ is the electric charge.
$\sigma_{S \times A S E}^{2}$ and $\sigma_{A S E \times A S E}^{2}$ are the variances resulting of the beating noise of the signal with the spontaneous emission and, respectively, of the spontaneous emission with itself (cf. Appendix A). They are equal to[17]:

$$
\begin{gather*}
\sigma_{S \times A S E}^{2}=4 \Re^{2} G P_{R} S_{A S E} B_{E},  \tag{1.3}\\
\sigma_{A S E \times A S E}^{2}=2 \Re^{2} S_{A S E}^{2} B_{E}\left(2 B_{O}-B_{E}\right), \tag{1.4}
\end{gather*}
$$

where $S_{A S E}=n_{s p}(G-1) . h . \nu_{c}$ is the spectrum of the amplified spontaneous emission and $n_{s p}$ is the spontaneous emission factor. Modeling the detector noise sources will allow to identify the minimum required power to achieve the targeted capacity.

### 1.3 Link Budget

The link budget (or flux budget) is a tool that gives an indication of the overall performance of the link. It estimates the received optical power by the satellite $P_{R}$ as a function of the

### 1.3 Link Budget

emitted optical power $P_{E}$, taking into account the losses in the propagation channel:

$$
\begin{equation*}
P_{R}=G_{R} T_{R X} \underbrace{L_{\text {margin }} L_{\text {clouds }} L_{a b}}_{L_{O T H E R S}} L_{F S} L_{T U R B} T_{T_{X}} G_{E} P_{E} . \tag{1.5}
\end{equation*}
$$

The gains $G_{R}$ and $G_{E}$ are usually used to model the link in the case of RF communications. They are dimensionless quantities. The initial assumption is that the transmitters (and receivers) are isotropic and the gains permit to take into account the directivity of the transmission (and reception). They can easily be adapted to model the propagation of a Gaussian beam and are determined from [18]. The gain at emission $G_{E}$ is:

$$
\begin{equation*}
G_{E}\left(\xi, D_{T_{X}}\right)=\left(\frac{\pi D_{T_{X}}}{\lambda}\right)^{2} \times 2 \alpha_{0}^{2}\left|\int_{0}^{1} e^{-\alpha_{0}^{2}} J_{0}\left(\frac{k D_{T_{X}}}{2} \sin (\xi) u^{\frac{1}{2}}\right) d u\right|^{2} \tag{1.6}
\end{equation*}
$$

where $D_{T_{X}}$ is the diameter of the transmitting telescope, $\lambda$ is the wavelength, $\alpha_{0}=\frac{D_{T_{X}}}{2 w_{0}}$ (with $w_{0}$ the waist size of the beam at emission) and $\xi$ is an angular offset due to the ground station. We will not consider any obscuration, because covering the Gaussian beam at its peak would reduce the transmitted power excessively. The gain on reception without obscuration is equal to[19]:

$$
\begin{equation*}
G_{R}=\left(\frac{\pi D_{R_{X}}}{\lambda}\right)^{2} \tag{1.7}
\end{equation*}
$$

where $D_{R_{X}}$ is the receiving telescope diameter. It assumes that the beam arriving on the satellite's telescope is a plane wave. Its expression is simpler than $G_{E}$ because we assume that the satellite terminal perfectly points towards the ground. In the case of an unperturbed propagation, the received power is given by:

$$
\begin{equation*}
P_{R}=G_{R} L_{F S} G_{E} P_{E} \tag{1.8}
\end{equation*}
$$

where $L_{F S}=\left(\frac{\lambda}{4 \pi L}\right)^{2}$, in which $L$ is the propagation distance, are the free-space loss that take into account the power attenuation due to diffraction during the propagation.
$L_{\text {OTHERS }}$ corresponds to other losses on the link budget: losses due to transmission through emission and reception optics, taking into account fiber injection losses, ( $T_{T_{X}}$ and $T_{R_{X}}$ ), atmospheric absorption ( $L_{a b}$ ) and transmission through thin clouds ( $L_{\text {clouds }}$ ). We also take a margin $L_{\text {margin }}$. As the gains $G_{E}$ and $G_{R}$, they are dimensionless quantities. The contributors on the link budget are summarized in Table 1.3.

### 1.3.1 NRZ-OOK: power detection threshold - $P_{R}$

Because of the detection architecture's inherent current noises and the small envisioned received power at satellite level (magnitude of a few nW), errors will occur in the detection, which means that some of the transmitted signal will be lost.

When the sent bit is 1 , the resulting current variance is ${ }^{1}$ :

$$
\begin{equation*}
\sigma_{1}^{2}=\sigma_{s h o t}^{2}+\sigma_{S \times A S E}^{2}+\sigma_{A S E \times A S E}^{2} \tag{1.9}
\end{equation*}
$$

Chapter 1. Optical communications for high throughput satellite feeder links


Figure 1.7: Presentation of the bit errors sources. The detection threshold of the intensity $I_{D}$ isn't necessarily placed in the intersection of the two probability density distributions. $P(1 \mid 0)$ corresponds to a false alarm case and $P(0 \mid 1)$ corresponds to a no-detection case.
and when the sent bit is 0 , the resulting current variance becomes:

$$
\begin{equation*}
\sigma_{0}^{2}=\sigma_{s h o t, A S E}^{2}+\sigma_{A S E \times A S E}^{*} \tag{1.10}
\end{equation*}
$$

Assuming the current fluctuations are Gaussian, Figure 1.7 presents bit errors sources. The probability of a no-detected bit is:

$$
\begin{equation*}
P(0 \mid 1)=\frac{1}{\sigma_{1} \sqrt{2 \pi}} \int_{-\infty}^{I_{D}} \exp \left(-\frac{\left(I-I_{1}\right)^{2}}{2 \sigma_{1}^{2}}\right) d I=\frac{1}{2} \operatorname{erfc}\left(\frac{I_{1}-I_{D}}{\sigma_{1} \sqrt{2}}\right) \tag{1.11}
\end{equation*}
$$

and the probability of a false alarm is:

$$
\begin{equation*}
P(1 \mid 0)=\frac{1}{\sigma_{0} \sqrt{2 \pi}} \int_{I_{D}}^{\infty} \exp \left(-\frac{\left(I-I_{0}\right)^{2}}{2 \sigma_{0}^{2}}\right) d I=\frac{1}{2} \operatorname{erfc}\left(\frac{I_{D}-I_{0}}{\sigma_{0} \sqrt{2}}\right) \tag{1.12}
\end{equation*}
$$

where $I_{0}$ is the mean intensity after detection when a 0 is sent, $I_{1}$ is the mean intensity after detection when a 1 is sent and $I_{D}$ is the detection threshold for the intensity. $I_{0}, I_{1}$ and $I_{D}$ are all given in Amperes. The complementary error function is defined:

$$
\begin{equation*}
\operatorname{erfc}(x)=\frac{2}{\sqrt{\pi}} \int_{x}^{\infty} \exp \left(-y^{2}\right) d y \tag{1.13}
\end{equation*}
$$

The performance criteria that we will use is the Bit Error Rate (BER), which is the ratio between the incorrect detected bits (errors) on the number of transmitted bits:

$$
\begin{equation*}
B E R=\frac{\text { Number of errors }}{\text { Number of transmitted bits }} . \tag{1.14}
\end{equation*}
$$

[^0]page 16

### 1.3 Link Budget

It is equal to:

$$
\begin{equation*}
B E R=p(0) P(1 \mid 0)+p(1) P(0 \mid 1), \tag{1.15}
\end{equation*}
$$

where $p(0)$ and $p(1)$ are respectively the probabilities to obtain a bit equal to 0 or 1 . As $p(0)=p(1)=1 / 2$, the BER is equal to:

$$
\begin{equation*}
B E R=\frac{1}{2}[P(0 \mid 1)+P(1 \mid 0)]=\frac{1}{4}\left[\operatorname{erfc}\left(\frac{I_{1}-I_{D}}{\sigma_{1} \sqrt{2}}\right)+\operatorname{erfc}\left(\frac{I_{D}-I_{0}}{\sigma_{0} \sqrt{2}}\right)\right] . \tag{1.16}
\end{equation*}
$$

Usually, $I_{D}$ is chosen so as to minimize the $\operatorname{BER}\left(\frac{d(B E R)}{d I_{D}}=0\right)$. It can be shown that a minimum is obtained for[17]:

$$
\begin{equation*}
I_{D}=\frac{\sigma_{0} I_{1}+\sigma_{1} I_{0}}{\sigma_{0}+\sigma_{1}} . \tag{1.17}
\end{equation*}
$$

Using this detection threshold leads to the BER:

$$
\begin{equation*}
B E R=\frac{1}{2} \operatorname{erfc}\left(\frac{Q}{\sqrt{2}}\right) \approx \frac{\exp \left(-\frac{Q^{2}}{2}\right)}{Q \sqrt{2 \pi}}, \tag{1.18}
\end{equation*}
$$

with:

$$
\begin{equation*}
Q=\frac{I_{1}-I_{0}}{\sigma_{1}+\sigma_{0}} \tag{1.19}
\end{equation*}
$$

To simplify, we will assume that the currents $i_{S \times A S E}$ and $i_{A S E \times A S E}$ (resulting from the spontaneous emission beating with the signal and with itself) are negligible compared to the signal current $i_{\text {sig }}$. This means that $I_{1}$ will correspond to the current of the signal:

$$
\begin{equation*}
I_{1}=i_{s i g}=\Re G P_{R}, \tag{1.20}
\end{equation*}
$$

where $P_{R}=\left|E_{R}\right|^{2}$ corresponds to the power of the signal when the bit sent is 1. And $I_{0}$ will be negligible compared to $I_{1}$.

Assuming that the gain G of the amplifier is high and that the optical filter is chosen so as to limit the $A S E \times A S E$ beat noise, then only the beat noise $S \times A S E$ remains and dominates the other noises (shot noise in particular). As a result, the electrical signal-tonoise ratio is[17]:

$$
\begin{equation*}
S N R=\frac{\langle I\rangle^{2}}{\sigma^{2}}=\frac{\left(\Re \cdot G \cdot P_{R}\right)^{2}}{4 \cdot \Re^{2} \cdot G \cdot P_{R} \cdot S_{A S E} \cdot B_{E}}=\frac{G \cdot P_{R}}{4 \cdot S_{A S E} \cdot B_{E}} \approx \frac{P_{R}}{4 \cdot n_{s p} \cdot h \cdot \nu_{c} \cdot B_{E}}=Q^{2}, \tag{1.21}
\end{equation*}
$$

where $h$ is the Planck constant and $\nu_{c}$ is the carrier frequency ( $\nu_{c}=193.1 \times 10^{12} \mathrm{~Hz}$ for $\lambda=1.55 \mu \mathrm{~m}$ ).

Numerical application The objective we set on the BER is to have $B E R=10^{-3}$ before using the forward error correcting codes, which leads to:

$$
\begin{equation*}
Q>3.0902 . \tag{1.22}
\end{equation*}
$$

Assuming the amplifier is ideal, $n_{s p}=1$. The targeted data throughput is $D_{\text {throughput }}=$ $10 \times 10^{9} \mathrm{bits} / \mathrm{s}$ (the targeted capacity of 1 Tbps from a multiplexing at different wavelengths) and the bandwidth of the electric filter is $B_{E}=0.8 \times D_{\text {throughput }}$.

Chapter 1. Optical communications for high throughput satellite feeder links

Equation (1.21) using the result from Equation (1.22) leads to the result on the minimum power needed in order to have a $\operatorname{BER}$ lower than $10^{-3}$ :

$$
\begin{equation*}
P_{R} \geq-44 \mathrm{dBm} \approx 40 \mathrm{nW} \tag{1.23}
\end{equation*}
$$

This leads to approximately 30 photons per bit for a 10 Gbps capacity channel. If we do not neglect the other sources of noise, we obtain $P_{R} \geq-43 \mathrm{dBm}$.

Interleaving and error correcting codes The error correction codes enable to go from $B E R=10^{-3}$ to $B E R=10^{-9}$. We will not describe precisely the working of error correction codes, as it is not the objective of this thesis. The idea is that bits of information are grouped into packages and that if one of the bits in the package is detected as an error, the overall code is still capable of extracting the right information from the package.

Atmospheric turbulence is a random phenomenon and thus implies that detected irradiance fluctuations are random as well. There can be times during which, despite our best efforts, the detected power becomes lower than the necessary threshold power $P_{R}$. These moments are called fades. During these moments, the BER of the uncorrected signal becomes higher than $10^{-3}$, which means that the targeted performance can not be met. It is at these times that the interleaving will come into play to distribute the losses on all the data and allow a better performance of the error correcting codes. An example is given in Figure 1.8, in which the error correcting codes are assumed to be able to recover the information of 1 bit lost within a 4 bit package. We observe that, without interleaving, the error correcting codes will not be able to recover the 3 bits contained in the package under the fade whereas with interleaving, forward error correcting codes are able to recover the each bit of data lost in the 3 packages.

In this thesis, we will assume that if the total of the fade durations (during which the detected power falls below the required power threshold) represent less than $5 \%$ of the time, the interleaving coupled with the error correcting codes will mitigate the losses and make it possible to obtain a BER equal to $10^{-9}$. To estimate the performance of interleaving, there is a need for temporal simulations which will permit to estimation fade durations. Indeed, longer fade durations lead to bigger buffers used for interleaving and therefore to additional complexity.
$L_{T U R B}$ estimation The loss term $L_{T U R B}$ which is integrated in the link budget is:

$$
\begin{equation*}
L_{T U R B}=10 \log _{10}\left(\frac{I_{R}}{I_{0}}\right) \tag{1.24}
\end{equation*}
$$

where $I_{0}$ is the irradiance (in $\mathrm{W} / \mathrm{m}^{2}$ ) in the satellite plane in a no turbulence case:

$$
\begin{equation*}
P_{0}=I_{0} \times \pi\left(\frac{D_{R_{X}}}{2}\right)^{2}=G_{R} L_{O T H E R S} G_{E} P_{E} \tag{1.25}
\end{equation*}
$$

And the irradiance threshold $I_{R}$ is defined by estimating the probability $P\left(I>I_{R}\right)=0.95$, where $I$ is the instantaneous detected irradiance.

### 1.4 Conclusion



Figure 1.8: Presentation of the benefits of interleaving. In this example, the signal sent is comprised of packets of 4 bits. Without interleaving, 3 of the 4 bits of a packet are lost and the error correcting codes will not be able to recover the information. With interleaving, 3 packets have lost only one bit and the error correcting codes will be able to recover the information.

### 1.4 Conclusion

In this first chapter, we have started by presenting the context and the recent demonstrations of free-space optical telecommunications satellites. This technology seems to be very promising but limitations regarding clouds and atmospheric turbulence have to be dealt with. In particular, there has yet to have demonstrations of optical uplinks using adaptive optics. The impact of atmospheric turbulence and its mitigation on a ground-to-space link will be the subject of this thesis.

We have presented in the second part of this chapter a performance budget taking into account all the assumptions on the emitted power, modulation, wavelength, propagation channel losses, etc. Presenting the state of the art in direct detection architectures and the BER as our performance criteria, we have identified the necessary optical power in order achieve the targeted capacity ( 10 Gbps per channel). The performance budget is summarized in Table 1.3.

The impact of the losses due to turbulence on the link budget are characterized by the $L_{T U R B}$ parameter. Moreover, using irradiance fluctuations time series will permit to test error correcting codes and interleaving.

There is now a need for accurate models describing irradiance fluctuations resulting from atmospheric turbulence. This will be the aim of the following chapter.

Chapter 1. Optical communications for high throughput satellite feeder links

Table 1.3: Contributors to the optical link budget.

| Contributors | Description |
| :---: | :--- |
| $P_{E}$ | Optical power booster amplifier |
| $T_{T_{X}}$ | Transmission loss inside optical terminal in $T_{X}$ |
| $G_{E}$ | Emitter gain with pointing error |
| $L_{F S}$ | Free-space loss |
| $L_{a b}$ | Absorption and scattering loss |
| $L_{T U R B}$ | Atmospheric turbulence loss |
| $L_{\text {clouds }}$ | Cloud loss |
| $G_{R}$ | Receiver gain |
| $T_{R_{X}}$ | Transmission and injection loss inside optical terminal in $R_{X}$ |
| $L_{\text {margin }}$ | System margin |
| $P_{R}$ | Injected optical power |

## Optical Beam Propagation in Turbulent Media

In the previous chapter, we described the the overall link and targeted performance. We also arrived at the conclusion that a description of the irradiance fluctuations resulting from atmospheric turbulence was necessary. In this chapter, the objective is to introduce the necessary mathematical formalism and results on the irradiance fluctuations from propagation of optical beams through turbulent media.

We will first start by describing atmospheric turbulence. Then, we will present the equations of an electromagnetic beam propagation through turbulent media, and results in the case of "weak" perturbations. We will focus on the specific case of a Gaussian beam propagation from the ground to a satellite and present results from the literature describing the irradiance in different atmospheric regimes. We will also use this chapter to present the Zernike polynomials as a way to describe the phase perturbations of the beam. Finally, we will present the principle of wave optics using Fresnel propagation.

## Contents

2.1 Atmospheric turbulence ..... 22
2.1.1 Physical phenomenon ..... 22
2.1.2 Outer scale, inner scale and inertial range ..... 23
2.1.3 Structure function and spectral density of the fluctuations of the refractive index ..... 23
2.1.4 $C_{n}^{2}$ vertical profiles ..... 24
2.2 Optical Propagation in Turbulent Media ..... 25
2.2.1 Propagation equations of an electromagnetic wave ..... 25
2.2.2 Spatial beam coherence ..... 26
2.2.3 The Rytov approximation: an analytical solution to Helmholtz equation ..... 27
2.2.4 Statistical properties of the irradiance of the propagated beam in the Rytov regime ..... 27
2.2.5 Power spectral densities in weak perturbations for plane wave ..... 28
2.3 Modal analysis of turbulent phase ..... 28
2.3.1 Zernike polynomials ..... 28
2.4 Atmospheric turbulence effects on Gaussian beams ..... 29
2.4.1 Gaussian beam properties ..... 29
2.4.2 Irradiance fluctuations and scintillation ..... 30
2.4.3 Fade statistics ..... 38
2.5 Propagation simulation with phase screens ..... 39
2.5.1 Principle of phase screen propagation ..... 39
2.5.2 Temporal effects simulation ..... 40
2.5.3 TURANDOT ..... 41
2.6 Conclusion ..... 41

### 2.1 Atmospheric turbulence

### 2.1.1 Physical phenomenon

Earth's atmosphere is a mixture of gas subject to solar radiation as well as radiative transfer from the ground. The shearing due to the wind produces turbulent air movements. A flow is turbulent when it becomes chaotic, particularly when eddies appear. This situation leads to the swirling of the physical quantities that characterize the fluid, such as temperature, speed, refractive index and so forth. The complexity of this phenomena has led scientists to adopt a phenomenological approach of turbulence and try to describe it statistically.

The first theoretical works of importance related to the statistic description of atmospheric turbulence were those of Kolmogorov [20]. He developed a theory in which he predicted that some initial energy was injected within large eddies and that this kinetic energy was transmitted to smaller eddies until the energy is dissipated by friction. It is called the theory of energy cascades. One way to understand this theory is through Figure 2.1.


Figure 2.1: Theory of energy cascades and the dispersion of energy in smaller turbulent eddies.

### 2.1.2 Outer scale, inner scale and inertial range

In Figure 2.1, we see a range appearing between the spatial frequencies $1 / L_{0}$ and $1 / l_{0}$ : this is the inertial range. It defines the range of spatial frequencies in which the atmospheric turbulence can be statistically modeled. The lengths $l_{0}$ and $L_{0}$ associated to these frequencies are respectively called the inner and outer scales.

The inner scale $l_{0}(\vec{r}, t)$, where $\vec{r}$ is a position in a tridimensional space and $t$ is time, is an estimation of the smallest distances over which fluctuations in the atmosphere are correlated. Below this length, we consider that the kinetic energy is dissipated in heat because of friction. The length $l_{0}$ can go from a few millimeters near the ground to a few centimeters at the tropopause (atmospheric layer at a 10 km altitude).

The outer scale $L_{0}(\vec{r}, t)$ on the other hand is an estimation of the largest distances over which fluctuations in the atmosphere are correlated. It results from macroscopic phenomena such as the movement of the atmospheric layers, wind or meteorological perturbations. When working in the optical range, the outer scale is not precisely known. It may range anywhere from 1 to 100 meters, with great spatial and temporal variability.
N.B.: In literature, it is also possible to find scales defined as $L_{0}^{\prime}=L_{0} / 2 \pi$ which considers that the scales are $2 \pi$ times smaller to describe the same effects. It is therefore important to verify the definition of the scale to prevent differences due to the $2 \pi$ factor.

### 2.1.3 Structure function and spectral density of the fluctuations of the refractive index

In Optics and to characterize the atmospheric turbulence, we study the fluctuations of the refractive index. Within the inertial range, we will study the correlation between the refractive indexes $n(\vec{r})$ and $n(\vec{r}+\vec{\rho})$, where $\vec{\rho}$ is a distance in a tridimensional space and $l_{0}<|\vec{\rho}|<L_{0}$. We consider that the turbulent regime is both temporally and spatially stationary. Kolmogorov and Obukhov have shown that the variance of the difference in refractive indexes between two points of space, also called structure function, is given by [20, 21]:

$$
\begin{equation*}
\left.D_{n}(\vec{\rho})=\langle | n(\vec{r})-\left.n(\vec{r}+\vec{\rho})\right|^{2}\right\rangle=C_{n}^{2} \rho^{\frac{2}{3}}, \tag{2.1}
\end{equation*}
$$

where $\langle$.$\rangle represents the statistical average over time. D_{n}(\vec{\rho})$ is the refractive index structure function. $C_{n}^{2}$, the index-of-refraction structure parameter, represents a measure of the magnitude of the fluctuations in the refractive index and is expressed in $m^{-\frac{2}{3}}$.

Another way to statistically characterize the refractive index fluctuations is to consider the power spectral density. It has been shown that the spectral density of the refractive index fluctuations may be expressed as [22, 21]:

$$
\begin{equation*}
W_{n}(\vec{f})=0.033(2 \pi)^{-\frac{2}{3}} C_{n}^{2} f^{-\frac{11}{3}} . \tag{2.2}
\end{equation*}
$$

The spectrum defined in Equation (2.2) is known as the Kolmogorov spectrum. This expression is only valid within the inertial range. In order to take into account these parameters, the spectrum was modified into the modified von Kármán spectrum presented in Equation (2.3) [23]:

$$
\begin{equation*}
W_{n}(\vec{f})=0.033(2 \pi)^{-\frac{2}{3}} C_{n}^{2}\left(f^{2}+\frac{1}{L_{0}^{2}}\right)^{-\frac{11}{6}} e^{-\left(\frac{2 \pi l_{0}}{5.9 f}\right)^{2}}, \quad 0 \leq f<\infty . \tag{2.3}
\end{equation*}
$$

Figure 2.2 shows the modified von Kármán spectrum for different values of inner and outer scales.


Figure 2.2: Power spectral density of the refractive index according to the modified von Kármán spectrum, for different values of outer and inner scales. The spectrum is normalized by the refractive index structure parameter $C_{n}^{2}$.

Applying the Wiener-Kinchine theorem, the covariance function $B_{n}$ of a $n$ quantity is:

$$
\begin{equation*}
B_{n}(\vec{\rho})=\langle n(\vec{r}) n(\vec{r}+\vec{\rho})\rangle=\int d \vec{f} W_{n}(\vec{f}) e^{-2 i \pi f \cdot \vec{\rho}} . \tag{2.4}
\end{equation*}
$$

Note that $B_{n}(0)$ is infinite when considering the Kolmogorov spectrum. The structure function defined in Equation (2.1) can then be rewritten as dependent on the covariance function:

$$
\begin{equation*}
D_{n}(\vec{\rho})=2\left[B_{n}(0)-B_{n}(\vec{\rho})\right] \tag{2.5}
\end{equation*}
$$

### 2.1.4 $C_{n}^{2}$ vertical profiles

Throughout this thesis, we will use the Hufnagel-Valley profile as vertical $C_{n}^{2}$ profile[24]. It is defined by:

$$
\begin{equation*}
C_{n}^{2}(h)=0.00594\left(\frac{v}{27}\right)^{2}\left(10^{-5} h\right)^{10} e^{-\frac{h}{1000}}+2.7 \times 10^{-16} e^{-\frac{h}{1500}}+C_{g} e^{-\frac{h}{100}} \tag{2.6}
\end{equation*}
$$

Typical values for parameters $v$ and $C_{g}$ are: $C_{g}=1.7 \times 10^{-14} \mathrm{~m}^{-\frac{2}{3}}$ and $v=21 \mathrm{~m} / \mathrm{s}$, which results in a $r_{0}$ calculated at zenith equal to 19 cm for a $1.55 \mu \mathrm{~m}$ wavelength. This particular profile is refered to the H-V 5/7 profile. It corresponds to a night profile.

### 2.2 Optical Propagation in Turbulent Media

### 2.2.1 Propagation equations of an electromagnetic wave

While propagating through a turbulent medium, an electromagnetic wave at a visible or infrared wavelength will fluctuate in amplitude and phase because of the refractive index variations. The description of these fluctuations relies on the analytic resolution of the Helmholtz propagation equation, which can't be analytically resolved unless the weak atmospheric turbulence assumption is made.

## Helmholtz equation

The propagation of an electromagnetic beam in a random dielectric medium is governed by Maxwell's laws. Considering the propagation of a narrow-band, in which the signal frequency is significantly higher than the frequency of the refractive index fluctuations, the electric field $\vec{E}(\vec{r})$ obeys the Helmholtz propagation equation [23]:

$$
\begin{equation*}
\nabla^{2} \vec{E}(\vec{r})+k_{0}^{2} n^{2}(\vec{r}) \vec{E}(\vec{r})+2 \nabla(\vec{E}(\vec{r}) \cdot \nabla(\ln (n(\vec{r}))))=0 \tag{2.7}
\end{equation*}
$$

where $k_{0}=2 \pi / \lambda$ is the wave number of the beam propagating through vacuum and $\nabla^{2}=\Delta=\frac{\partial}{\partial x^{2}}+\frac{\partial}{\partial y^{2}}+\frac{\partial}{\partial z^{2}}$ denotes the Laplacian. It can be shown that the last term in Equation (2.7), which represents coupling produced between the different polarization components of the electric field, is negligible if $\lambda$ is lower than the inner scale $l_{0}$, which is the case for a wavelength of $1.55 \mu \mathrm{~m}$ compared to the typical inner scale size of a millimiter [25]. Equation (2.7) can therefore be simplified:

$$
\begin{equation*}
\nabla^{2} \vec{E}(\vec{r})+k_{0}^{2} n^{2}(\vec{r}) \vec{E}(\vec{r})=0 \tag{2.8}
\end{equation*}
$$

## Paraxial approximation

We consider that the beam is propagating along the $\overrightarrow{e_{z}}$ axis. The amplitude of the scalar field $E(\vec{r})$ can be written ${ }^{1}$ :

$$
\begin{equation*}
E(\vec{r})=\Psi(\vec{r}) e^{i k z}, \tag{2.9}
\end{equation*}
$$

where $\Psi(\vec{r})$ is the complex amplitude of the wave. By injecting Equation (2.9) in Equation (2.8), we obtain an equation on $\Psi$ :

$$
\begin{equation*}
\nabla^{2} \Psi(\vec{r})+2 i\langle n\rangle k_{0} \frac{\partial \Psi(\vec{r})}{\partial z}+k_{0}\left(n^{2}-\langle n\rangle^{2}\right) \Psi(\vec{r})=0 . \tag{2.10}
\end{equation*}
$$

According to the paraxial approximation, the scale of the spatial fluctuations of $\Psi$ is small compared to the wavelength. We can therefore consider:

$$
\begin{equation*}
\left|\frac{\partial^{2} \Psi}{\partial z^{2}}\right| \ll\left|k \frac{\partial \Psi}{\partial z}\right| . \tag{2.11}
\end{equation*}
$$

We can also approximate the refractive index. Indeed, the refractive index can be written $n=\langle n\rangle+N$, where $N$ are the fluctuations of the refractive index around the mean value.

[^1]The square of the refractive index while considering that $N \ll 1$ gives, according to Taylor: $n^{2} \approx\langle n\rangle^{2}+2\langle n\rangle N$. Then Equation (2.10) becomes the paraxial equation:

$$
\begin{equation*}
\frac{\partial^{2} \Psi(\vec{r})}{\partial x^{2}}+\frac{\partial^{2} \Psi(\vec{r})}{\partial y^{2}}+2 i\langle n\rangle k_{0} \frac{\partial \Psi(\vec{r})}{\partial z}+2 k_{0}\langle n\rangle N \Psi(\vec{r})=0 . \tag{2.12}
\end{equation*}
$$

The solution of this equation in a homogeneous media $(N=0)$ is given:

$$
\begin{equation*}
\Psi(x, y, z+d)=\Psi(x, y, z) * \frac{e^{i k d}}{i \lambda d} e^{i k\left(\frac{x^{2}+y^{2}}{2 d}\right)}=\Psi(x, y, z) * F_{d}(x, y) . \tag{2.13}
\end{equation*}
$$

$F_{d}(x, y)$ is called the Fresnel propagator over a distance $d$ and $*$ means a convolution. This equation gives the field after propagation in vacuum over a distance $d(\Psi((x, y, z+d))$ knowing the initial field of the beam $\Psi((x, y, z)$. This solution will be particularly important later (in Section 2.5.1)when we explain the use of phase screens to simulate the propagation of an optical beam within a turbulent media.

### 2.2.2 Spatial beam coherence

At this point in our resolution of the propagation, we will take a small pause and focus on the notion of spatial coherence of a propagated in a turbulent media. The Fried parameter $r_{0}$ is defined from the wave structure function, in the case of a Kolmogorov spectrum[26, 27]:

$$
\begin{equation*}
D_{\phi, z}(\vec{r})=2.91 k_{0}^{2} C_{n}^{2}(z) d z r^{\frac{5}{3}} \tag{2.14}
\end{equation*}
$$

and rewritten:

$$
\begin{equation*}
D_{\phi, z}(\vec{r})=6.88\left(\frac{r}{r_{0}}\right)^{\frac{5}{3}} \tag{2.15}
\end{equation*}
$$

with

$$
\begin{equation*}
r_{0}=\left[0,423 k_{0}^{2} \int_{\text {Path }} d z C_{n}^{2}(z)\right]^{-\frac{3}{5}} \tag{2.16}
\end{equation*}
$$

The spatial coherence function $B_{\Psi}$ of the complex field $\Psi$ quantifies the loss in spatial coherence and is defined by:

$$
\begin{equation*}
B_{\Psi}=\left\langle\Psi(\vec{r}) \Psi^{*}(\vec{r}+\vec{\rho})\right\rangle \tag{2.17}
\end{equation*}
$$

Yura has shown that [28]:

$$
\begin{equation*}
B_{\Psi}=e^{-\left(\frac{\rho}{\rho_{0}}\right)^{\frac{5}{3}}} \tag{2.18}
\end{equation*}
$$

where $\rho=|\rho|$ and:

$$
\begin{equation*}
\rho_{0}=\left[1.46 k_{0}^{2} \int_{\text {Path }} d z C_{n}^{2}(z)\right]^{-\frac{3}{5}} \tag{2.19}
\end{equation*}
$$

$\rho_{0}$ is called the coherence length of the field. It is the length between two points in the field for which the spatial correlation is equal to $1 / e$.

Both $r_{0}$ and $\rho_{0}$ are linked by the relation.

$$
\begin{equation*}
r_{0} \approx 2,11 \rho_{0} \tag{2.20}
\end{equation*}
$$

### 2.2.3 The Rytov approximation: an analytical solution to Helmholtz equation

For the analytic resolution of the Helmholtz propagation equation, simplifying assumptions are necessary. The weak atmospheric turbulence assumption allows for a perturbative type resolution which has been developed using two different methods: the Rytov approximation[23] and the Born approximation. The Rytov approximation considers that the perturbation is multiplicative to the unperturbed field while the Born approximation [29] considers it as additive. In scientific literature, the Rytov approximation is the most commonly used one as it gives closer results to the experimental data and is therefore the one we will explain hereafter.

The complex amplitude can be written $\Psi=\exp (\psi)$. When substituting this field in the propagation Equation (2.8), one obtains:

$$
\begin{equation*}
\nabla^{2} \psi+(\nabla \psi)^{2}+k_{0}^{2} n^{2}=0 \tag{2.21}
\end{equation*}
$$

$\psi$ can be developed

$$
\begin{equation*}
\psi=\psi_{0}+\psi_{1}+\psi_{2}+\ldots \tag{2.22}
\end{equation*}
$$

where $\Psi_{0}(\vec{r})=e^{\psi_{0}(\vec{r})}$ is a complex amplitude, solution of Equation (2.10) in an homogenous medium with refractive index $\langle n\rangle$. In the Rytov method, only the first order is taken into account and the higher orders are neglected, which means that we consider: $\psi=\psi_{0}+\psi_{1}$. To simplify, we consider $n \approx 1+N$, i.e. the mean value of $n$ is equal to 1 . At first order, $n^{2} \approx 1+2 N$.

The unperturbed field is solution of the propagation in vacuum:

$$
\begin{equation*}
\nabla^{2}\left(\psi_{0}\right)+\left(\nabla\left(\psi_{0}\right)\right)^{2}+k_{0}^{2} n^{2}=0 \tag{2.23}
\end{equation*}
$$

Equation (2.21) leads to

$$
\begin{equation*}
\nabla^{2}\left(\psi_{1}\right)+\nabla\left(\psi_{1}\right)\left(\nabla\left(\psi_{1}\right)+2 \nabla\left(\psi_{0}\right)\right)+2 N k_{0}^{2}=0 . \tag{2.24}
\end{equation*}
$$

In the weak turbulence assumption, the Rytov approximation assumes $\left|\nabla \psi_{1}\right| \ll\left|\nabla \psi_{0}\right|$. Equation (2.24) becomes:

$$
\begin{equation*}
\nabla^{2}\left(\psi_{1}\right)+2 \nabla\left(\psi_{1}\right) \nabla\left(\psi_{0}\right)+2 N k_{0}^{2}=0 \tag{2.25}
\end{equation*}
$$

which has a solution [30]:

$$
\begin{equation*}
\psi_{1}(\vec{r})=\frac{k^{2}}{2 \pi \Psi_{0}(\vec{r})} \int d \overrightarrow{r^{\prime}} N\left(\overrightarrow{r^{\prime}}\right) \Psi_{0}\left(\overrightarrow{r^{\prime}}\right) \frac{e^{i k\left|\vec{r}-\overrightarrow{r^{\prime}}\right|}}{\left|\vec{r}-\overrightarrow{r^{\prime}}\right|}, \tag{2.26}
\end{equation*}
$$

where $\Psi_{0}(\vec{r})=\exp \left(\psi_{0}\right)$.

### 2.2.4 Statistical properties of the irradiance of the propagated beam in the Rytov regime

$\psi_{1}$ can be decomposed $\psi_{1}=\chi+i \phi$, where $\chi$ is the $\log$-amplitude and $\phi$ is the phase of the field $\psi_{1}$.

Empirically, it has been shown that the Rytov approximation was valid as long as the log-amplitude variance $\sigma_{\chi}^{2}$ was inferior to 0.3 [30]. Indeed, with the Rytov approximation, the variance of the log-amplitude, $\sigma_{\chi}^{2}$, or of the irradiance, $\sigma_{I}^{2}$, are supposed to increase without limits but experimentation shows a saturation [31].

In the case of weak atmospheric turbulence, $\chi($ and $\phi)$ is the sum of random independent Gaussian variables. Therefore, the probability density of $\chi$ is a centered Gaussian function. The irradiance $I$ is equal to : $I=\Psi \Psi^{*}=I_{0} e^{2 \chi}$. As $\chi$ is a normally distributed random variable, the probability density of $I$ is log-normal:

$$
\begin{equation*}
P(I)=\frac{1}{\sqrt{2 \pi} I \sigma_{\chi}} \exp \left[-\frac{(\ln (I)-\langle\chi\rangle)^{2}}{2 \sigma_{\chi}^{2}}\right], I>0 \tag{2.27}
\end{equation*}
$$

A very useful variable is the scintillation index, or normalized variance of the intensity. It is linked to the log-amplitude variance [30]:

$$
\begin{equation*}
\sigma_{I}^{2}=\frac{\left\langle I^{2}\right\rangle}{\langle I\rangle^{2}}-1=\exp \left[4 \sigma_{\chi}^{2}\right]-1 \tag{2.28}
\end{equation*}
$$

In weak turbulence, considering that $\sigma_{\chi}^{2}$ is small enough, it can be simplified:

$$
\begin{equation*}
\sigma_{I}^{2}=4 \sigma_{\chi}^{2} \tag{2.29}
\end{equation*}
$$

### 2.2.5 Power spectral densities in weak perturbations for plane wave

The power spectral densities of $\chi$ and $\phi$ in weak perturbations can be deduced from Equation (2.26):

$$
\begin{align*}
& W_{\chi}(\vec{f})=k_{0}^{2} \int_{0}^{L} W_{n}(\vec{f}, z) \sin ^{2}\left(\pi z \lambda f^{2}\right) d z  \tag{2.30}\\
& W_{\phi}(\vec{f})=k_{0}^{2} \int_{0}^{L} W_{n}(\vec{f}, z) \cos ^{2}\left(\pi z \lambda f^{2}\right) d z \tag{2.31}
\end{align*}
$$

Where $L$ is the propagation distance and $W_{n}(\vec{f}, z)$ is the index fluctuations spectrum at the distance $z$ given in Equation (2.3). We have presented the results for a plane wave but results can also be obtained in spherical and even Gaussian waves[32].

### 2.3 Modal analysis of turbulent phase

Within the Rytov approximation, turbulence effects on phase perturbation can be described using a modal description, such as Zernike polynomials, when experienced on a circular aperture.

### 2.3.1 Zernike polynomials

The Zernike polynomials are a sequence of polynomials that are orthonormal on the unit disk. They are described in Appendix B, according to Noll's definition[33].

### 2.4 Atmospheric turbulence effects on Gaussian beams

## Statistical properties

The turbulence phase $\phi(\vec{r})$ can be expressed with the Zernike polynomials:

$$
\begin{equation*}
\phi(r)=\sum_{(i=1)}^{\infty} a_{i} Z_{i}(r) . \tag{2.32}
\end{equation*}
$$

Noll provides the results of the variances for the coefficients of each Zernike coefficients, for a Kolmogorov spectrum. The total phase variance (without piston) is equal to

$$
\begin{equation*}
\sigma_{\phi}^{2}=\sum_{i=2}^{\infty}\left\langle a_{i}^{2}\right\rangle=1.03\left(\frac{D}{r_{0}}\right)^{\frac{5}{3}} . \tag{2.33}
\end{equation*}
$$

Because, we prefer using a Von Karman spectrum in order to take into account the effects of the outer scale $L_{0}$, we will use Chassat's results presented in Appendix C in Equations (C.18), (C.19) and (C.20).

## Temporal properties

Conan has shown that for a plane wave experienced on a circular aperture, the temporal power spectral densities of the Zernike coefficient $a_{i}$ are given by[34]:

$$
\begin{equation*}
W_{a_{i}}(\nu)=\int_{0}^{L} \int_{-\infty}^{\infty} \frac{1}{V(z)}\left|M_{a_{i}}\left(\frac{\nu}{V(z)}, f_{y}\right)\right|^{2} W_{\phi}\left(\frac{\nu}{V(z)}, f_{y}\right) d z d f_{y}, \tag{2.34}
\end{equation*}
$$

where $W_{\phi}$ is the phase spatial power spectrum and $\left|\tilde{M}\left(a_{i}\right)\right|^{2}$, the Fourier transform of the Zernike polynomial:

$$
\left|\tilde{M}_{a_{i}}(\vec{f})\right|=\sqrt{n+1} \frac{2\left|J_{n+1}\left(\pi d_{0} f\right)\right|}{\pi d_{0} f} \times\left\{\begin{array}{lc}
\sqrt{2}|\cos (m \theta)| & \text { for } m \neq 0 \& j \text { even }  \tag{2.35}\\
\sqrt{2}|\sin (m \theta)| & \text { for } m \neq 0 \& j \text { odd } \\
1 & \text { for } m=0
\end{array}\right.
$$

### 2.4 Atmospheric turbulence effects on Gaussian beams

### 2.4.1 Gaussian beam properties

The fundamental mode Gaussian laser beam, equivalent to a $T E M_{00}$ electromagnetic wave, propagating in vacuum along the $\vec{z}$ direction is:

$$
\begin{equation*}
E_{0}(r, z)=\frac{w_{0}}{w(z)} \exp \left(-\frac{\rho^{2}}{w(z)^{2}}\right) \exp \left(-\frac{i k_{0} \rho^{2}}{2 R(z)}\right) \times \exp \left(i k z-i \tan ^{-1}\left(\frac{z}{Z_{R}}\right)\right) \tag{2.36}
\end{equation*}
$$

$Z_{R}=\frac{\pi w_{0}^{2}}{\lambda}$ is called the Rayleigh distance, $w(z)$ is the radius of the beam at $1 / \mathrm{e}$ in amplitude:

$$
\begin{equation*}
w(z)=\sqrt{w_{0}^{2}\left(1+\left(\frac{z-z_{0}}{Z_{R}}\right)^{2}\right)} \tag{2.37}
\end{equation*}
$$

and $R(z)$ is the curvature radius:

$$
\begin{equation*}
R(z)=\left(z-z_{0}\right)\left(1+\left(\frac{Z_{R}}{z-z_{0}}\right)^{2}\right) . \tag{2.38}
\end{equation*}
$$

$w_{0}$, its minimal radius size is called the waist size. At $z=z_{0}$, the wavefront is plane. $w\left(Z_{R}\right)=\sqrt{2} w_{0}$ for $z_{0}=0$. We will consider that the beam is collimated, i.e. close to a plane wave, for $z$ between $z_{0}$ and $Z_{R}$. For very long propagation distances $\left(z \gg Z_{R}\right)$, the approximation $w(z) \simeq \theta_{0} z$ with the angle $\theta_{0}=\frac{\lambda}{\pi w_{0}}$ is used. This is illustrated in Figure 2.4.

Figure 2.3 shows the convention used in this document. This convention also corresponds to a reduction by a factor of $1 / \mathrm{e}$ from their maximum values of the amplitude distributions (which leads to a value of $1 / e^{2}$ for irradiance distributions). ${ }^{2}$

## Amplitude



Figure 2.3: Gaussian beam amplitude profile convention.

### 2.4.2 Irradiance fluctuations and scintillation

## Statistical properties

Atmospheric turbulence introduces variations of the refractive index along the propagation path and these variations have a significant impact on the detected irradiance. We will focus particularly on the effects observed on a ground-to-satellite path. Depending on the

[^2]
### 2.4 Atmospheric turbulence effects on Gaussian beams



Figure 2.4: Propagation of a convergent Gaussian beam (curvature radius $>0$ at $\mathrm{z}=0$ ) in vacuum.
size of the turbulent eddies and on the beam radius, the effects on the beam will differ. Two major effects can be observed.

One of the effects is called beam spreading. It corresponds to a widening of the beam size larger than natural diffraction due to turbulent eddies being smaller than the beam. It doesn't have an effect on the beam's direction. However, it corresponds to an energy spreading. The diffraction due to the smaller eddies also leads to interference and therefore irradiance fluctuations and scintillation.

The other effect consists in the beam deviating from its original direction. This effect is called "Beam Wander". It results from turbulent eddies larger than the beam radius, which will refract the beam, thus deviating it. This can add important irradiance variations on the optical link and increases the BER. Beam wander becomes significant for very long propagation paths, as the one between the ground and a geostationary satellite. This phenomenon is illustrated in Figure 2.5

In this manuscript, the term "scintillation" will be used to describe all irradiance fluctuations including those resulting from beam wander.

Following Fante, we introduce 3 variables to describe these effects[30]. The first one is the short-term beam radius $r_{s}$. It corresponds to the radius of the broadened laser spot (due to the small eddies) observed with a very short exposure. The beam is deflected by a distance $r_{c}$. It is a measure of the beam displacement at satellite altitude due to beam wander effects (from the large eddies). The last variable is the long-term beam radius $r_{L}$. It is a measure of the beam radius with an exposure much longer than the time intervals $\Delta t=D /\left|V_{\perp}\right|$ between two deflections of the beam, where $D$ is the beam diameter and $V_{\perp}$ is the transverse flow velocity of the turbulent eddies. Figure 2.6 illustrates the physical meaning of these three parameters. Fante has shown that these variables are linked by the formula [30]:

$$
\begin{equation*}
\left\langle r_{L}^{2}\right\rangle=\left\langle r_{s}^{2}\right\rangle+\left\langle r_{c}^{2}\right\rangle . \tag{2.39}
\end{equation*}
$$

Chapter 2. Optical Beam Propagation in Turbulent Media


Figure 2.5: Effects of beam wander on irradiance fluctuations[35].


Figure 2.6: Illustration of the beam at the satellite's altitude, taking into account the effects of beam wander and beam spread.[36]

### 2.4 Atmospheric turbulence effects on Gaussian beams

Fante has given analytical expressions for the mean square of these variables. $\left\langle r_{L}^{2}\right\rangle$ and $\left\langle r_{s}^{2}\right\rangle$ are defined as the radii at which the long and short-term averaged intensity distributions are reduced by a factor of $e^{-2}$. We will first present the results for $\left\langle r_{L}^{2}\right\rangle^{3}$ :

$$
\begin{equation*}
\left\langle r_{L}^{2}\right\rangle=\left(\frac{L \lambda}{\pi w_{0}}\right)^{2}+w_{0}^{2}+8.8 *\left(\frac{L \lambda}{\pi r_{0}}\right)^{2} \tag{2.40}
\end{equation*}
$$

The first two terms represent the beam propagating through vacuum. The term on the right takes into account all of the turbulence and is the term describing both the beam wander and the beam spreading.

The beam wander is given by:

$$
\begin{equation*}
\left\langle r_{c}^{2}\right\rangle=\frac{18.2 L^{2}}{k_{0}^{2} r_{0}^{\frac{5}{3}} w_{0}^{\frac{1}{3}}} \tag{2.41}
\end{equation*}
$$

The short-term beam spread $\left\langle r_{s}^{2}\right\rangle$ is given by subtracting $\left\langle r_{c}^{2}\right\rangle$ from $\left\langle r_{L}^{2}\right\rangle$. The long-term beam divergence takes into account beam wander whereas the short-term beam divergence doesn't. Depending on the size of the waist at the emission, the effects of atmospheric turbulence on the beam will differ and lead to three distinct irradiance fluctuations regimes. This is illustrated in Figure 2.7 for a ground to satellite propagation.


Figure 2.7: Instantaneous beam profile in the receiver plane as a function of the ratio $\frac{w_{0}}{r_{0}}$.
When the waist size at emission is much lower than the coherence diameter described by the Fried parameter $r_{0}$, the effects of atmospheric turbulence are limited. Usually, the Rayleigh distance is small and the beam diffracts rapidly. At satellite level, the beam size is therefore very large, which means that beam wander is negligible compared to the size of the beam. In this case, the irradiance fluctuations can usually be modeled by using the Rytov approximation. Using a very diffracted beam means that much of the power is lost at satellite level. Due to the limited powers of the envisioned laser sources, there is therefore a need to focus the energy on the satellite by increasing the waist size at emission.

When increasing the waist size at emission, the beam divergence is reduced and beam pointing becomes a key issue when $w_{0} \approx r_{0}$, i.e. the major impact of atmospheric turbulence on the beam becomes beam wander. This regime is particularly interesting because

[^3]beam wander may be compensated using a tilt tracking mirror, which immediately leads to important mitigation of irradiance fluctuations. Moreover, in this regime, the beam maintains its coherence and the impact of higher orders is negligible compared to beam wander.

When the waist size at emission becomes significantly larger than $r_{0}$, we see in Equation (2.40) that the beam spread is primarily due to the Fried parameter $r_{0}$. Moreover, Equation (2.41) shows that the larger the waist size, the lower the impact of beam wander. In this regime, beam coherence is lost and beam breakup occurs. The beam is consists of several hot "spots" and the center of the beam is difficult to position. Beam wander does not have any significance in this case. Mitigation of atmospheric effects becomes more difficult.

## Weak perturbation case - $w_{0} \ll r_{0}$

The first studies that dealt with scintillation in Gaussian laser beams are those of Schmeltzer [37] and Ishimaru [38]. In this thesis, we will use the expressions derived later by Andrews [39], presented in Appendix D.

The scintillation index for a laser beam at a certain distance L may be divided into two terms: the first is the on axis scintillation, measured on the optical axis, and the second takes into account the dependence of the scintillation to the distance from the optical axis. This gives ${ }^{4}$ :

$$
\begin{equation*}
\sigma_{I}^{2}(r, L)=\sigma_{I}^{2}(0, L)+\sigma_{I, r}^{2}(r, L) . \tag{2.42}
\end{equation*}
$$

We have seen that in the Rytov approximation, the irradiance fluctuations due to the scintillation phenomenon follows a log-normal probability density function, given in Equation (2.27). The log-amplitude is therefore a normal variable, and these relations are approximately satisfied:

$$
\begin{array}{r}
\langle\chi\rangle=-\sigma_{\chi}^{2} \\
\sigma_{\chi}^{2}=\frac{1}{4} \ln \left(1+\sigma_{I}^{2}(0, L)\right) . \tag{2.44}
\end{array}
$$

We can therefore obtain an irradiance PDF in the weak perturbation case.

## Beam wander regime - $w_{0} \approx r_{0}$

When the beam size at emission increases, discrepancies appear between the scintillation theory developed from the Rytov perturbation method and numerical wave optics simulations. For these beams, the scintillation is dominated by turbulence-induced beam wander at the target.

Many studies argue that the beam wander phenomenon has to be added to the Rytov phenomenon [40, 41]. Considering the log-amplitude fluctuations $I_{S}$ and beam wander $I_{B W}$, the normalized on axis instantaneous irradiance is given by:

$$
\begin{equation*}
I(0,0, L)=I_{S} \times I_{B W} . \tag{2.45}
\end{equation*}
$$

[^4]
### 2.4 Atmospheric turbulence effects on Gaussian beams

There are two approaches in the literature describing how to take into account the scintillation with beam wander. The first considers that only the on-axis scintillation $\sigma_{I}^{2}(0, L)$ should be taken into account[42]:

$$
\begin{equation*}
I(0,0, L)=\exp [2 \chi(0,0)] \exp \left[-2 \frac{\delta x^{2}+\delta y^{2}}{w_{S T}^{2}(L)}\right] \tag{2.46}
\end{equation*}
$$

where $(\delta x, \delta y)$ are the position of the centroid of the beam resulting from beam wander, $w_{S T}^{2}(L)$ is the short term beam waist $\left(w_{S T}^{2}(L)=\left\langle r_{s}^{2}\right\rangle\right)$. The decorrelation between $I_{S}$ and $I_{B W}$ leads to the following scintillation index:

$$
\begin{equation*}
\sigma_{I}^{2}=\sigma_{I, S}^{2}(0,0, L)+\sigma_{I, B W}^{2}(L), \tag{2.47}
\end{equation*}
$$

with $\sigma_{I, S}^{2}$ is given from Equation (D.1), and $\sigma_{I, B W}^{2}(L)$ is equal to [43]:

$$
\begin{equation*}
\sigma_{I, B W}^{2}(L)=\frac{4 \alpha^{2}}{1+4 \alpha}, \tag{2.48}
\end{equation*}
$$

where $\alpha=\left\langle r_{c}^{2}\right\rangle / w_{S T}^{2}(L)$. The probability density function of the irradiance fluctuations from beam wander is a modified beta distribution [44, 3]:

$$
\begin{equation*}
P_{I_{B w}}(I)=\frac{1}{2 \alpha}(I)^{\frac{1}{2 \alpha}-1} . \tag{2.49}
\end{equation*}
$$

Kiasaleh proposes analytical expressions of both the PDF and CDF of the irradiance $I$ taking into account both scintillation and beam wander[44].

The second method, proposed by Dios, considers that the log-amplitude scintillation is determined as a function of beam wander, as it shifts the Gaussian beam in the satellite plane[40]. The normalized on-axis instantaneous irradiance is given by:

$$
\begin{equation*}
I(0,0, L)=\exp [2 \chi(\delta x, \delta y)] \exp \left[-2 \frac{\delta x^{2}+\delta y^{2}}{w_{S T}^{2}(L)}\right] . \tag{2.50}
\end{equation*}
$$

Compared to the case without beam wander, this means that $\chi$ now has to be estimated at ( $\delta x, \delta y$ ). This means that Equation (2.44) becomes:

$$
\begin{equation*}
\sigma_{\chi}^{2}=\frac{1}{4} \ln \left(1+\sigma_{I, G b}^{2}(\delta x, \delta y, L)\right), \tag{2.51}
\end{equation*}
$$

in which $\sigma_{I, G b}^{2}(\delta x, \delta y, L)$ is given by:

$$
\begin{equation*}
\sigma_{I, G b}^{2}=\left(\sigma_{I}^{2}(0, L)+\sigma_{I, r}^{2}(\delta x, \delta y, L)\right)\langle I(\delta x, \delta y, L)\rangle^{2}, \tag{2.52}
\end{equation*}
$$

where $\langle I(\delta x, \delta y, L)\rangle^{2}=\exp \left[-2 \frac{\delta x^{2}+\delta y^{2}}{w_{L T}^{2}(L)}\right]$. This means that a log-amplitude value is obtained as a function of the centroid displacement $(\delta x, \delta y)$.

In the regime where beam wander is predominant, the results of the Rytov approximation are sometimes neglected because of their poor fit to experimental data.

The Low-Order of Turbulence solution Another approach is to consider that the irradiance fluctuations result from beam wander as well as beam deformations, while neglecting scintillation from the Rytov approximation. Baker proposes to describe the irradiance fluctuations by the deformations of a Gaussian beam propagating through the atmosphere only using the first and second order Zernike polynomials: tip/tilt, defocus and astigmatism[3]. This model is called the Low-Order of Turbulence model. In the case of a ground to satellite propagation, the turbulence is in the near field while the satellite is in the far field. According to Baker, this means that the turbulence can be integrated in a single phase screen placed at the transmitter prior to propagation. The phase screen at the transmitter has the phase value:

$$
\begin{equation*}
\phi_{L O T}(x, y)=k_{0}\left[\theta_{x} x+\theta_{y} y+\Delta \kappa \frac{1}{2}\left(\left(x^{2}+y^{2}\right)^{\frac{1}{2}} w_{0}^{2}\right)+c_{5} \frac{1}{2}\left(x^{2}-y^{2}\right)+c_{6} x y\right], \tag{2.53}
\end{equation*}
$$

where $\theta_{\{x, y\}}$ corresponds to the angular tip/tilt in radians. $\Delta \kappa$ corresponds to the defocus curvature and $c_{\{5,6\}}$ to the astigmatism curvature (given in $m^{-1}$ ).

Baker has shown that the resulting normalized irradiance of a Gaussian beam propagating through atmospheric turbulence up to a satellite at a distance $L$ with the phase screen $\phi_{\text {LOT }}$ placed at the transmitter is expressed[3]:

$$
\begin{align*}
I(x, y, L)=\frac{2}{\pi} \frac{1}{w_{x} w_{y}} \times \exp ( & \left.-\frac{2}{w_{x}^{2}}\left[\left(x-\delta_{x}\right) \cos (\omega)+\left(y-\delta_{y}\right) \sin (\omega)\right]^{2}\right) \\
& \times \exp \left(-\frac{2}{w_{y}^{2}}\left[\left(x-\delta_{x}\right) \sin (\omega)-\left(y-\delta_{y}\right) \cos (\omega)\right]^{2}\right), \tag{2.54}
\end{align*}
$$

where the beam wander, partial beam radii, and astigmatism parameters in Equation (2.54) are given by, respectively:

$$
\begin{gather*}
\delta_{\{x, y\}}=L \theta_{\{x, y\}}  \tag{2.55}\\
w_{\{x, y\}}=w_{0} \sqrt{\left[1+L\left(\Delta \kappa \pm \sqrt{c_{5}^{2}+c_{6}^{2}}\right)\right]^{2}+\left(\frac{2 L}{k w_{0}^{2}}\right)^{2}}  \tag{2.56}\\
\omega=\frac{1}{2} \arg \left(c_{5}+j c_{6}\right) \tag{2.57}
\end{gather*}
$$

In Equation (2.56), the first term under the root square expresses the beam broadening induced by defocus and astigmatism and the second term is the broadening induced by diffraction.

Using Noll's phase variance[33], the model omits the following phase variance:

$$
\begin{equation*}
\sigma_{\delta \phi}(j>6)=0.065\left(\frac{d_{0}}{r_{0}}\right)^{\frac{5}{3}} \tag{2.58}
\end{equation*}
$$

where $d_{0}=2 w_{0}$ and $j$ is the number of the considered Zernike polynomial (in our case, all the polynomials of higher order than astigmatism). We will assume that statistical equivalence between the LOT model and a case where the whole phase is taken into account

### 2.4 Atmospheric turbulence effects on Gaussian beams

is achieved when there is less than 0.1 wave squared difference (within a circle of radius $w_{0}$ ) between the two:

$$
\begin{equation*}
\sigma_{\delta \phi}(j>6) \leq\left(\frac{2 \pi}{10}\right)^{2} \tag{2.59}
\end{equation*}
$$

This adds a constraint on the maximum waist size of the beam:

$$
\begin{equation*}
w_{0} \leq 1.5 r_{0} . \tag{2.60}
\end{equation*}
$$

In order to determine the range of validity, Baker [3] proposes two dimensionless parameters to define the region of interest in which turbulence is located in the near-field while the satellite is in the far-field. These parameters are:

$$
\begin{equation*}
N_{L}=\frac{\pi w_{0}^{2}}{\lambda L} \tag{2.61}
\end{equation*}
$$

and

$$
\begin{equation*}
N_{\tau}=\frac{\pi w_{0}^{2}}{\lambda z_{\tau}} \tag{2.62}
\end{equation*}
$$

with $z_{\tau}$ defined by:

$$
\begin{equation*}
z_{\tau}=\frac{\int_{0}^{L} z C_{n}^{2}(z) d z}{\int_{0}^{L} C_{n}^{2}(z) d z} \tag{2.63}
\end{equation*}
$$

$z_{\tau}$ gives an estimation of the centroid of the turbulence strength on the path. The parameters $N_{L}$ and $N_{\tau}$ correspond to Fresnel numbers of the initial beam observed at, respectively, distance $L$ and $z_{\tau}$. They compare the Rayleigh range $Z_{R}$ of the beam to, respectively, the distances $L$ and $z_{\tau}$. According to Baker, the bounds of this region are given by two conditions. The satellite being in the far field leads to a hypothesis on $N_{L}$ :

$$
\begin{equation*}
N_{L}<1 \tag{2.64}
\end{equation*}
$$

Assuming the turbulence is located near the emitter leads to the resulting hypothesis:

$$
\begin{equation*}
N_{\tau}^{2} \gg 1 \tag{2.65}
\end{equation*}
$$

These hypotheses impose a minimum waist for the validity of the LOT model. Because the satellite is so far away, the condition $N_{L}<1$ will always be true. This means that it is the condition $N_{\tau}^{2} \gg 1$ that will set the minimum waist.

## Strong perturbations regime - $w_{0} \gg r_{0}$

Habash have proposed a distribution called the Gamma-Gamma distribution to describe the irradiance fluctuations for strong turbulence [45]. In this manuscript, we will usually consider that strong turbulence correspond to cases where the emitted waist size $w_{0}$ is larger than the Fried parameter $r_{0}$, which therefore corresponds to a loss of spatial coherence of the beam during its propagation through the atmosphere. The model assumes that the irradiance fluctuations resulting from small eddies (diffractive phenomenon) are modulated multiplicatively by the irradiance fluctuations resulting from large eddies (refractive
phenomenon). Mathematically put: $I=I_{x} I_{y}$ where $I_{x}$ and $I_{y}$ represent the irradiance fluctuations after propagation resulting from small and large eddies ${ }^{5}$, respectively. $I_{x}$ and $I_{y}$ are assumed statistically independent and described by a Gamma distribution. I therefore has a Gamma-Gamma probability density function:

$$
\begin{equation*}
p(I)=\frac{2(\alpha \beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha) \Gamma(\beta)} I^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}(2 \sqrt{\alpha \beta I}), I>0, \tag{2.66}
\end{equation*}
$$

where $K_{p}$ is a modified Bessel function of $p$ order. $\alpha$ and $\beta$ represent the number of small and large eddies. $\Gamma$ represents the "Euler gamma" function defined by:

$$
\begin{equation*}
\Gamma(s)=\int_{0}^{\infty} d x e^{-x} x^{s-1} \tag{2.67}
\end{equation*}
$$

The parameters $\alpha$ and $\beta$ are defined by[46]:

$$
\begin{array}{r}
\frac{1}{\alpha}=34.29\left(\frac{\Lambda L}{k_{0} r_{0}^{2}}\right)^{\frac{5}{6}}\left(\frac{\sigma_{p e}}{w(L)}\right)^{2}+\exp \left[\frac{0.49 \sigma_{B u}^{2}}{\left(1+(1+\Theta) 0.56 \sigma_{B u}^{\frac{12}{5}}\right)^{\frac{7}{6}}}\right]-1, \\
\frac{1}{\beta}=\exp \left[\frac{0.51 \sigma_{B u}^{2}}{\left(1+0.69 \sigma_{B u}^{\frac{12}{5}}\right)^{\frac{5}{6}}}\right]-1, \tag{2.69}
\end{array}
$$

where $\sigma_{B u}^{2}=\sigma_{I}^{2}(0, L)$, as defined in Equation (D.1) and

$$
\begin{equation*}
\sigma_{p e}^{2}=\left\langle r_{c}^{2}\right\rangle\left[1-\left(\frac{\pi^{2} w_{0}^{2} / r_{0}^{2}}{1+\pi^{2} w_{0}^{2} / r_{0}^{2}}\right)^{\frac{1}{6}}\right] . \tag{2.70}
\end{equation*}
$$

This distribution is very interesting because its behavior in weak turbulence are close to the log-normal distribution and fits well with the experimental results when the turbulence's strength increases, as it becomes closer to a negative exponential which is the assumed probability density function for strong turbulence. However, it results from an empiric fitting and not from any analytic description and its domain of validity remains to be confirmed. In order to be able to model the irradiance fluctuations more physically with less assumptions, wave optics simulations have to be considered.

### 2.4.3 Fade statistics

There are not any analytical models in the literature that provide irradiance time series. The usual approach in the literature to describe irradiance statistics is through the expected number of fades per second (for example in [47]), which is equal to the number of negative crossings below a specified threshold level, and the mean fade time, which represents the

[^5]average time during which the irradiance stays below the prescribed threshold level of irradiance. The mean fade time is calculated by taking the ratio of the probability of fade and the expected number of fades per second [48]. These parameters have been obtained for Log-normal and Gamma-Gamma irradiance PDFs (summarized in [47]), for cases where $w_{0} \ll r_{0}$ and $w_{0} \gg r_{0}$.

Temporal power spectral density of beam wander In the literature, the temporal power spectral density for the case $w_{0} \approx r_{0}$ is treated using only irradiance fluctuations resulting from beam wander. Yura [49] provides level crossing statistics in such as case. A comprehensive estimate of the single-axis wander angle temporal autocorrelation function and the corresponding power spectrum is used to develop analytic expressions for the mean angular level crossing rate and the mean duration of such crossings. However, the higher orders are not considered. This means that when we will consider tip/tilt precompensation, we will not be able to model the residual irradiance fluctuations.

### 2.5 Propagation simulation with phase screens

The Helmholtz equation does not have a solution in the general case and the Rytov approximation only permits to obtain results for weak turbulence. The Markov approximation yields rigorous yet intractable results for a turbulence domain. Numerical simulation of wave propagation enables tho overcoming of the analytical models which are limited to weak perturbations [50]. Using a Monte-Carlo approach permits to estimate the statistical properties of the propagated field through strong perturbations.

### 2.5.1 Principle of phase screen propagation

Numerical wave optics simulations rely on the separation of the propagation and turbulence effects on phase. It consists in discretizing the turbulent volume in multiple turbulent layers. These layers are simulated using thin phase screens. Between the phase screens, the propagation occurs in the vacuum. The thickness of the layers (or the distance between the phase screens) has to be small enough so that the diffraction effects within the layer can be neglected and big enough so that two successive phase screens are decorrelated.

Within the paraxial approximation, the propagation in vacuum of a field $\Psi(x, y, z)$ can be described with Equation (2.13)[50]. Figure 2.8 presents the principle of using phase screens to model propagation through a turbulent volume. Each turbulent layer is assimilated to a phase screen that adds the perturbation to the field. Mathematically, the propagation is modeled:

$$
\begin{align*}
\Psi(x, y, z) & =A_{0}(x, y, z) \cdot e^{i \phi_{0}(x, y, z)} \\
\Psi^{\prime}(x, y, z) & =\Psi(x, y, z) \cdot e^{i \phi_{1}(x, y, z)} \\
\Psi(x, y, z+d) & =\Psi^{\prime}(x, y, z) * F_{d}  \tag{2.71}\\
\Psi^{\prime}(x, y, z+d) & =\Psi(x, y, z+d) \cdot e^{i \phi_{2}(x, y, z+d)} \\
\Psi(x, y, z+2 d) & =\Psi^{\prime}(x, y, z+d) * F_{d}
\end{align*}
$$

The propagated field is the result of the succession of phase perturbations introduced by the phase screens and of phase and amplitude variations due to the propagation of the


Figure 2.8: Phase screens propagation principle.[36]
phase errors between the screens (diffraction). This method (called wave optics) is used by ONERA in their code TURANDOT (TUrbulence simulator for SpAce-grouND Optical Telecommunication[35]).

### 2.5.2 Temporal effects simulation

The time scales during which the turbulence moves in front of the beam $\Delta t=D /\left|V_{\perp}\right|$, where $D$ is the beam diameter and $V_{\perp}$ is the transverse wind speed, are considered to be sufficiently small in comparison to the temporal evolution of turbulence to satisfy Taylor's hypothesis, also known as the hypothesis of "frozen turbulence". This hypothesis assumes that the only modification of the turbulent layers between two instants is a translation whose amplitude depends on $V_{\perp}$. The refractive index at time $t$ can therefore be calculated by : $n \approx n\left(\vec{r}-V_{\perp}(\vec{r}) t\right)$. The modeling of the temporal evolution will therefore be done by shifting the phase screens orthogonally to the direction of propagation.

## Wind profile

Throughout this thesis, the wind will be modeled by a Bufton wind profile[24], described in Equation (2.72), in which the atmospheric layers move with a $5 \mathrm{~m} / \mathrm{s}$ speed at ground level and with a $30 \mathrm{~m} / \mathrm{s}$ speed at an altitude around 12.5 km :

$$
\begin{equation*}
V(h)=S_{G}+S_{P} e^{-\left(\frac{h-H_{P}}{W_{P}}\right)^{2}} \tag{2.72}
\end{equation*}
$$

in which we have taken: $S_{G}=5 \mathrm{~m} / \mathrm{s}, S_{P}=25 \mathrm{~m} / \mathrm{s}, H_{P}=12448 \mathrm{~m}$ and $W_{P}=4800 \mathrm{~m}$.
In Part II, we will only present the case in which the wind moves in the $x$ direction for all layers. Extension to the wind moving in multiple direction is straightforward ([27, 34]).

### 2.5.3 TURANDOT

The French aerospace laboratory ONERA has developed for more than a decade a computer code, called PILOT (Propagation and Imaging, Laser and Optics, through Turbulence), based on the split step method previously to simulate the propagation of an optical beam through random media. TURANDOT is an automatically parametrized version of PILOT. It relaxes the issues of spatial and temporal sampling of the physical phenomena, using dimensioning tools to optimize the computation time while ensuring reliable simulation results. It has been validated in a downlink case with the experimental data from Optical Inter-orbit Communications Engineering Test Satellite [51]. For the uplink, statistical spatial characteristics of the beam irradiance profile were compared to expected beam profile [30].

TURANDOT is particularly useful for studying situations which cannot be described satisfactorily with analytic methods. It is able to consider all type of wave-forms (spherical, plane or truncated Gaussian beams) and deal with high turbulence strength. It is also able to simulate temporal evolution of the perturbations. The latest version of TURANDOT is able to produce end-to-end simulations with tilt tracking using the information coming from the downlink to pre-compensate the uplink and taking into account point-ahead angle and aperture mismatch. While we will consider TURANDOT as our reference, it is important to note that its results rely on a Monte-Carlo approach, meaning that the precision of its results will depend on the number of considered draws.

However, while TURANDOT is capable of giving very precise results, it is very computationally intensive. This means that the study of a single case can require up to a day of simulations. During our sizing study, many cases will have to be considered, as we will be studying different waist sizes, for different telescope truncations and different ground architectures (cf. Chapter 7)... This means that TURANDOT is too slow to be used for the sizing study. This why we need to use analytic models. These models' validity range are usually much shorter but permit to obtain results much faster. The results obtained with these models can than be validated for selected operating points using TURANDOT.

### 2.6 Conclusion

In this chapter, we have presented the different mathematical descriptions of irradiance fluctuations resulting from atmospheric turbulence, the analytic descriptions as well as wave optics simulations. We will consider in this manuscript that the wave optics simulations as the reference. However, they have very long simulation times which means that they can not be used to do a sizing. This is why we are also interested in analytic descriptions which generally have smaller ranges of validity. The validity of these analytic descriptions compared to wave optics simulations (i.e. TURANDOT) will be presented in Chapter 5.

Chapter 2. Optical Beam Propagation in Turbulent Media

## Chapter 3

## Mitigation techniques for link budget improvement

After having presented the effects of atmospheric turbulence on irradiance fluctuations in the previous chapter, we will present different mitigation techniques for improving the link budget. There are two major mitigation techniques. The first is using diversity, which corresponds to an averaging of the irradiance fluctuations. The second is adaptive optics, in which a servo system modifies in real time the emitted wavefront in order to make it recover an unperturbed waveform when reaching the satellite.

## Contents

3.1 Diversity techniques . . . . . . . . . . . . . . . . . . . . . . . . . . 43
3.1.1 Wavelength diversity . . . . . . . . . . . . . . . . . . . . . . . . . 44
3.1.2 Temporal diversity . . . . . . . . . . . . . . . . . . . . . . . . . . 44
3.1.3 Multiple emitters . . . . . . . . . . . . . . . . . . . . . . . . . . . 44
3.2 Adaptive optics pre-compensation . . . . . . . . . . . . . . . . . 45
3.2.1 Reciprocity principle . . . . . . . . . . . . . . . . . . . . . . . . . 45
3.2.2 Correlation between downlink and uplink . . . . . . . . . . . . . 47
3.2.3 Study of the correlation with altitude . . . . . . . . . . . . . . . 49
3.2.4 Residual Zernike coefficients estimation . . . . . . . . . . . . . . 49
3.2.5 Residual scintillation . . . . . . . . . . . . . . . . . . . . . . . . . 52
3.3 Conclusion . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 53

### 3.1 Diversity techniques

The idea behind beam diversity is to average the effects of atmospheric turbulence on several uncorrelated and statistically independent beams. Mathematically, the laser beam intensity distribution of a beam $(i)$ is described as:

$$
\begin{equation*}
I_{i}(x, y)=s(x, u)+n_{i}(x, y), \tag{3.1}
\end{equation*}
$$

Chapter 3. Mitigation techniques for link budget improvement
where $s(x, y)$ is the signal component without the noise component and $n_{i}(x, y)$ is the noise component due to turbulence. At the receiver, the effects of all the beams will sum up to average the effects of turbulence [52]:

$$
\begin{equation*}
I(x, y)=s(x, u)+\frac{1}{N} \sum_{i=1}^{N} n_{i}(x, y) \tag{3.2}
\end{equation*}
$$

where $N$ is the number of beams used. This means that the scintillation $\sigma_{I}^{2}$ by $N$. Methods relying on diversity necessitate a synchronization process between each beam, in terms of data transmission. This solution is therefore not optimal for a telecommunication application.

### 3.1.1 Wavelength diversity

Wavelength diversity is the idea of simultaneously using multiple beams having different wavelengths and each carrying the same information. Unfortunately, the dependence of the atmospheric turbulence to the wavelength is weak and so the different beams are only weakly decorrelated. Therefore, only a small reduction of the scintillation index is expected with this method (around $10 \%$ according to Kiasaleh [53]). The reduction is improved with the increase in difference between the wavelengths but the optics are harder to make. Wavelength diversity will therefore not be considered to mitigate irradiance fluctuations effects.

Moreover, in practice, a multiplexing of wavelengths around $1.55 \mu \mathrm{~m}$ will be used to achieve the targeted capacity. Each link associated with each wavelength targeting a 10 Gbps capacity. The weak dependence of atmospheric turbulence to wavelength will be considered as negligible.

### 3.1.2 Temporal diversity

The coherence time of the link channel (i.e. the atmosphere) is long compared to the time necessary to emit a bit ( 1 millisecond compared to 1 nanosecond). The idea is therefore to emit the data multiple times relying on the temporal decorrelation of the atmosphere. The correlation time is around 10 ms . Moreover, the bigger the aperture, the longer the correlation time. A long coherence time implies having big buffers. This solution is therefore not applicable as it generates system complexity.

### 3.1.3 Multiple emitters

Finally, the most interesting idea for reducing the atmospheric turbulence influence is to use an emitter diversity. Emitter diversity is based on a spatial decorrelation of the effects of turbulence, which means using different beams which do not go through the same optical paths. Each laser beam should be incoherent with one another in order to reduce any chance of interference at the receiver plane and thus reduce speckle effects and the scintillation index. Moreover, in order for a very good spatial decorrelation, each beam should be separated by a distance superior than $\rho_{0}$. This is the technique used in 2014 by

NASA for the Lunar LaserComm Demonstration [13]. Liu proposes an estimation of the scintillation index in case the decorrelation between each beam isn't perfect [54, 55]:

$$
\begin{equation*}
\sigma_{I, n}^{2}=\frac{1}{n}(1-\eta) \sigma_{I}^{2}(0)+\eta \sigma_{I}^{2}(0) \tag{3.3}
\end{equation*}
$$

where $\eta$ is the average spatial correlation between the beams.
Between 2012 and 2013, transmitter diversity has been tested with two measurement campaigns between the ESA Optical Ground Station (OGS) on Canary Islands and the geostationary satellite ARTEMIS [56]. The main scope of those campaigns was to analyze transmitter diversity mitigation (using 2 beams) effect on the uplink scintillation. In these campaigns, downlink tracking was used to compensate beam wander but due to the fact that the point-ahead angle is larger than the isoplanatic and tilt-anisoplanatic angle, the compensation was not perfect, resulting in fades deeper than 20dB. They showed that with transmitter diversity, the fades become shorter in time.

The problem with using diversity techniques is the necessary synchronization at the receiver. This is due to the fact that, since the beams do not go through the same atmospheric paths, they each arrive with a slightly different phase. From a telecommunications point of view, this mitigation technique leads there to complexity at reception and in terms of coding.

### 3.2 Adaptive optics pre-compensation

### 3.2.1 Reciprocity principle

The idea of pre-compensating the beam results from Fermat's principle which states that the path taken between two points by a ray of light is the one that can be traversed in the least time and particularly independently of the direction of the beam. Using this principle and thinking in terms of geometrical optics with plane waves, if one adds the conjugate of the downlink's phase to the uplink's at emission and if the uplink propagates through the medium along the same path as the downlink, a plane wavefront at the output of the medium should be obtained. This is illustrated in Figure 3.1. Being able to control the wave front's shape at the atmospheric output means that it is easier to predict the wavefront in the receiver plane.

Shapiro [57] and Fried\&Yura [2] have been the first to propose the use of reciprocity for beam pre-compensation. The envisioned technique is adaptive optics (AO) in which a servo system modifies in real time the emitted wavefront (with a deformable mirror) in order to make it recover a plane waveform when reaching the satellite. To do so, the beam coming from the satellite will be used to estimate the perturbations that need to be applied to the emitted wavefront. This is illustrated in Figure 3.2.

In the case of a ground to satellite feeder link: two effects prevent us from strictly applying the reciprocity principle. The first results from the point-ahead angle (developed in Section 3.2.2) which induce that the downlink and the uplink do not propagate through exactly the same atmospheric turbulence. The second is the fact that the uplink as a Gaussian amplitude while the downlink is homogeneous in the receiver pupil.

Chapter 3. Mitigation techniques for link budget improvement


Figure 3.1: Illustration of the reciprocity principle in the simplified case of plane waves.


Figure 3.2: Envisioned system presentation. WFS means Wave-Front Sensor.

### 3.2 Adaptive optics pre-compensation

### 3.2.2 Correlation between downlink and uplink

In practice, the downlink and the uplink do not go through the atmosphere on the exact same path. It will therefore be essential to characterize the correlation between the uplink and the downlink. It will be a parameter to describe what pre-compensation will be able to achieve.The correlation between the Zernike coefficients of the downlink and the uplink can be estimated using Chassat's correlation functions presented in Appendix C.

## Point-ahead angle

As we work with geostationary satellites, they are always directly above the same location on Earth. The point-ahead angle is the result of the Earth's rotation during the beam's propagation from the satellite to the ground and then from the ground to the satellite because the time taken by the beam to travel such distances is no longer negligible (magnitude of 2.4 ms ). This is illustrated in Figure 3.3.


Figure 3.3: Point-ahead angle origin. $c$ is the speed of light.
The point-ahead angle can be calculated from [58]:

$$
\begin{equation*}
\theta_{P A A} \approx 2 \frac{V_{s a t, \perp}-V_{O G S, \perp}}{c}, \tag{3.4}
\end{equation*}
$$

Chapter 3. Mitigation techniques for link budget improvement
where $c$ is the speed of light, $V_{s a t, \perp}$ and $V_{O G S, \perp}$ are the satellite and OGS speeds transverse to the link. For a GEO satellite, we have:

$$
\begin{equation*}
V_{s a t, \perp}=\omega_{E a r t h}\left(R_{E}+H_{G E O}\right), \tag{3.5}
\end{equation*}
$$

with $\omega_{\text {Earth }}=2 \pi / 86400 \mathrm{~s}=73 \mu \mathrm{rad} / \mathrm{s}, R_{E}=6400 \mathrm{~km}$ and $H_{G E O}=36,000 \mathrm{~km}$. When the OGS sees the satellite at zenith (OGS is then on equator), we have $V_{O G S, \perp}=\omega_{\text {Earth }} R_{E}$. For the transverse OGS speed, we can then consider the range:

$$
\begin{equation*}
0<V_{O G S, \perp} \leq \omega_{E a r t h} R_{E} \tag{3.6}
\end{equation*}
$$

Finally, we obtain:

$$
\begin{equation*}
2 \frac{\omega_{E a r t h} H_{G E O}}{c} \leq \theta_{P A A}<2 \frac{\omega_{E a r t h}\left(R_{E}+H_{G E O}\right)}{c} \tag{3.7}
\end{equation*}
$$

or:

$$
\begin{equation*}
17.5 \mu r a d \leq \theta_{P A A}<20.6 \mu r a d \tag{3.8}
\end{equation*}
$$

In this document, we will take $\theta_{P A A}=18.5 \mu \mathrm{rad}$, usually considered for European latitudes. The resulting envisioned system is illustrated in Figure 3.4.


Figure 3.4: Envisioned system presentation with point-ahead angle. WFS means WaveFront Sensor.

### 3.2 Adaptive optics pre-compensation

## Anisoplanatism

The notion of point-ahead angle leads to introduce the notion of isoplanatism. Isoplanatism is the angular zone over which optical phase distortion due to turbulence is nearly the same (the criteria used is the to have phase structure function $D_{\phi}(\theta, L)=1 \mathrm{rad}^{2}$ ). As defined by Fried, the isoplanatic angle $\theta_{I P A}$ is [59]:

$$
\begin{equation*}
\theta_{I P A}=\left[2.91 k^{2} \sec ^{\frac{8}{3}}(\zeta) \int_{0}^{L} C_{n}^{2}(h) h^{\frac{5}{3}} d h\right]^{-\frac{3}{5}} \tag{3.9}
\end{equation*}
$$

Using a classical Hufnagel-Valley $5 / 7$ profile with $\phi_{\text {elev }}=0$ (described in Chapter 2.1.4), the isoplanatic angle is estimated at $13 \mu \mathrm{rad}$ for $\lambda=1.55 \mu \mathrm{~m}$. Therefore, the point-ahead angle being bigger than the isoplanatic angle means that the downlink and the uplink do not see exactly the same atmospheric turbulence and an adaptive optics correction will be imperfect. This is illustrated in Figure 3.5.


Figure 3.5: Point-ahead angle and the difference in atmospheric turbulence seen by the downlink and the uplink.

### 3.2.3 Study of the correlation with altitude

The aim of this Section is to help comprehend how anisoplanatism from the point-ahead angle will deteriorate the correction quality and to underscore the origin of the residual tilt after pre-compensation.

### 3.2.4 Residual Zernike coefficients estimation

The variance of the residual Zernike coefficient after precompensation $\sigma_{\epsilon}^{2}$ is estimated for each layer and is given by:

$$
\begin{equation*}
\sigma_{\epsilon}^{2}=\left\langle\left(a_{j, U}-a_{j, D}\right)^{2}\right\rangle . \tag{3.10}
\end{equation*}
$$

where $a_{j, U}$ is the coefficient associated with the $j^{\text {th }}$ Zernike polynomial estimated on the uplink $a_{j, D}$ is the coefficient associated with the $j^{\text {th }}$ Zernike polynomial estimated on the downlink (cf. Figure 3.6). This leads to

$$
\begin{equation*}
\sigma_{\epsilon}^{2}=\left\langle a_{j, U}^{2}\right\rangle+\left\langle a_{j, U}^{2}\right\rangle-2\left\langle a_{j, U} a_{j, D}\right\rangle . \tag{3.11}
\end{equation*}
$$

Chapter 3. Mitigation techniques for link budget improvement
$\left\langle a_{j, U} a_{j, D}\right\rangle$ is the covariance between the $j^{\text {th }}$ Zernike coefficients of the uplink and the downlink.

## Propagation geometry analysis

Atmospheric turbulence is usually very strong at ground level, and decreases rapidly with altitude. However, assuming an architecture for which the only source of error on the tracking comes from the point-ahead angle, it is intuitive that the residual errors will result from atmospheric turbulence in altitude. To prove this, we will discretize the atmosphere into multiple layers (arbitrarily chose at 10). We will present the results on the estimation of the residual tilt and study the impact of each layer on the strength of the residual tilt.


Figure 3.6: Presentation of monostatic configuration.

Table 3.1: Parameters for the study

| Wave length $\lambda$ | $1.55 \mu \mathrm{~m}$ |
| :---: | :---: |
| Waist | 8 cm |
| Pointing telescope diameter | 16 cm |
| Tracking telescope diameter | 16 cm |
| Point-ahead angle | $18.5 \mu \mathrm{rad}$ |
| Delay before correction | 0.004 s |
| Configuration | Merged pupils |
| Elevation angle | $90^{\circ}$ |
| Atmospheric profile | Hufnagel-Valley $5 / 7$ |
| Wind profile | Bufton |

Fusco [60] introduces the notion of equivalent layer (EL) in order to discretize the atmospheric profile for good approximation of the angular correlations between the Zernike polynomials coefficients. The equivalent layers are defined as follows by their respective altitude $h_{i}$ and turbulence strength $C_{n}^{2}\left(h_{i}\right)$ :

$$
\begin{equation*}
h_{i}=\left(\frac{\int_{h_{\min }}^{h_{\max }} C_{n}^{2}(h) h^{\frac{5}{3}} d h}{\int_{h_{\min }}^{h_{\max }} C_{n}^{2}(h) d h}\right)^{\frac{3}{5}} \tag{3.12}
\end{equation*}
$$

Where $h_{\min }$ and $h_{\max }$ are the lower and outer limit of the discretized area of the atmospheric profile. The equivalent strength of the turbulence is given by:

$$
\begin{equation*}
\delta h_{i} C_{n}^{2}\left(h_{i}\right)=\int_{h_{\min }}^{h_{m} a x} C_{n}^{2}(h) d h \tag{3.13}
\end{equation*}
$$

The equivalent wind speed is obtained with the same method:

$$
\begin{equation*}
V_{i}=\left(\frac{\int_{h_{\min }}^{h_{\max }} C_{n}^{2}(h) V(h)^{\frac{5}{3}} d h}{\int_{h_{\min }}^{h_{\max }} C_{n}^{2}(h) d h}\right)^{\frac{3}{5}} \tag{3.14}
\end{equation*}
$$

Table 3.2: Table describing the impact of each layer on the overall residual tilt after compensation

| Layer num- <br> ber | Layer alti- <br> tude (m) | $C_{n}^{2}\left(m^{-\frac{2}{3}}\right)$ | Residual tilt <br> variance at <br> each layer <br> $\left[\mathrm{rad}^{2}\right]$ | Residual <br> tilt/total <br> tilt $[\%]$ | Distance <br> between the <br> two beams <br> $(\mathrm{m})$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 326.24 | $9.99 \mathrm{e}-16$ | $1.10 \mathrm{e}-03$ | 4.5 | 0.01 |
| 2 | 2824.04 | $3.95 \mathrm{e}-17$ | $2.76 \mathrm{e}-03$ | 11.26 | 0.05 |
| 3 | 4911.04 | $1.30 \mathrm{e}-17$ | $2.07 \mathrm{e}-03$ | 8.43 | 0.09 |
| 4 | 7073.87 | $1.20 \mathrm{e}-17$ | $2.74 \mathrm{e}-03$ | 11.2 | 0.13 |
| 5 | 9037.98 | $1.59 \mathrm{e}-17$ | $4.25 \mathrm{e}-03$ | 17.34 | 0.17 |
| 6 | 10978.27 | $1.56 \mathrm{e}-17$ | $4.54 \mathrm{e}-03$ | 18.54 | 0.2 |
| 7 | 12932.06 | $1.12 \mathrm{e}-17$ | $3.48 \mathrm{e}-03$ | 14.22 | 0.24 |
| 8 | 14897.57 | $6.42 \mathrm{E}-18$ | $2.08 \mathrm{e}-03$ | 8.5 | 0.28 |
| 9 | 16871.26 | $3.07 \mathrm{E}-18$ | $1.03 \mathrm{e}-03$ | 4.21 | 0.31 |
| 10 | 18850.64 | $1.28 \mathrm{E}-18$ | $4.41 \mathrm{e}-04$ | 1.8 | 0.35 |

In Table 3.2, we present the results of the estimation of the residual tilt after tracking for each layer for a 10 layer discretization of the atmospheric profile. We observe that most of the residual tilt results from the atmospheric layers around a 10 km altitude. The residual tilt results from both the quality of the correction and the strength of the turbulence within the considered layer. With our assumptions, the quality of the correction depends primarily from the distance between the two beams $d_{1 \rightarrow 2}$ and on the strength of each atmospheric layer. We observe that for a height of 9 km , the distance between the two beams is equal to 17 cm , which is already bigger than twice the radius of each beam (for simplicity, we consider a plane wave). This means that the beams no longer overlap and the decorrelation is already very important. We observe that in the layers above 9 km , the residual tilt is stronger than the initial uncorrected tilt. This is verified in Figure 3.7, in which we plot the tilt variances with and without tracking.

It will be interesting to study the evolution of these results as a function of the atmospheric profile. In particular, for a day atmospheric profile with much stronger turbulence near the ground.


Figure 3.7: Tilt variances resulting from each layer of the discretized atmosphere with and without tracking.

### 3.2.5 Residual scintillation

Tyson [61, 62] uses Sasiela's formalism [63] to take into account adaptive optics in the scintillation variance. In a perfect correction case, the residual phase spectrum is filtered by the Zernike polynomials spectrum. This means that the residual phase spectrum is, in the case of a plane wave:

$$
\begin{equation*}
W_{\phi}=k^{2} d z W_{n}(\vec{f})\left[1-\sum_{i=2}^{N}\left|\tilde{M}_{a_{j}}(\vec{f})\right|^{2}\right] \tag{3.15}
\end{equation*}
$$

with $\left|\tilde{M}_{a_{j}}(\vec{f})\right|$ defined in Equation (2.35). In terms of Zernike mode correction, the mismatch due to point-ahead angle translates into a performance reduction that can be approximated by the following expression [61]:

$$
\begin{equation*}
N_{e f f} \approx\left\lfloor N \exp \left[-\left(\frac{\theta_{P A A}}{\theta_{I P A}}\right)^{\frac{5}{3}}\right]\right\rfloor, \tag{3.16}
\end{equation*}
$$

where $\lfloor$.$\rfloor is the floor operator,$ theta $_{P A A}$ is the point-ahead angle, theta $a_{I P A}$ is the isoplanatic angle, $N$ is the number of Zernike modes that the AO system is trying to correct, and $N_{\text {eff }}$ is the effective number of Zernike modes that AO system can actually correct because of the mismatch between the downlink and uplink. After correction of the first $N_{\text {eff }}$ Zernike modes, the residual phase variance is reduced [33].

Regarding the possible scintillation index improvement when applying pre-distortion AO in an optical GEO uplink, Tyson [61] proposes the relation:

$$
\begin{equation*}
\sigma_{I, N_{e f f}}^{2}=\frac{\sigma_{I}^{2}}{N_{e f f}}, \tag{3.17}
\end{equation*}
$$

### 3.3 Conclusion

where $\sigma_{I, N_{e f f}}^{2}$ is the scintillation index after correction of $N_{\text {eff }}$ Zernike modes, and $\sigma_{I}^{2}$ is the scintillation index without any adaptive optics mode correction.

The AO modeling is based on simplified error terms that are not sufficient for a precise system analysis. However, this way of taking into account adaptive optics, while imperfect for modeling irradiance fluctuation of an AO pre-compensated uplink is still the one usually used (for example in [64]).

### 3.3 Conclusion

In conclusion, we have presented in this chapter the two major techniques for mitigating atmospheric turbulence irradiance fluctuations : adaptive optics and diversity.

We have briefly presented the idea behind diversity mitigation. However, diversity, while being efficient in terms of irradiance mitigation, requires that the data of each beam is synchronized at reception. It therefore adds complexity. For this reason, we will focus the majority of our efforts on the second mitigation technique, adaptive optics.

We have seen the adaptive optics was limited due to the fact that the downlink and the uplink did not follow the exact same paths through atmospheric turbulence (because of the point-ahead angle particularly). We have also presented the effects of anisoplanatism due to the point-ahead angle on the residual tilt.

Chapter 3. Mitigation techniques for link budget improvement

## com 4

## Thesis objectives

To achieve optical uplinks between the ground and geostationary satellites, enough power must be detected by the satellite terminal. This is the link budget. Through clear skies conditions, the major effect which reduces the link budget is atmopsheric turbulence. Its impact depends on the optical architecture of the optical ground station (i.e. size of the beam at emission) as well as turbulence conditions. Most of the time, the link budget is not fulfilled and pre-compensation must be applied. The knowledge, characterization and modeling of the effects of turbulence, and their correction, on an optical link are essential for the optimization of a ground station. My thesis focused on this subject, in the particular context of a ground to satellite feeder link, capable of providing the targeted capacity (1 Terabit/s by 2025).

This work was done within the context of the ALBS (Accès Large Bande pour Satellite - Broadband Satellite Access) project within the IRT Antoine de Saint Exupéry. The purpose of the ALBS project is to mature technologies that increase the efficiency of satellite transmissions or increase systems' capacity.

In order to optimize the OGS, a sensitivity study of its main parameters is performed ans requires a lot of simulations. The parameters of interest are, on the one hand, the parameters of the OGS (i.e. the waist size at emission, the telescope truncation, and the adaptive optics architecture) and, on the other hand, the turbulence strength of the propagation channel to study the sensitivity of the optimization. The criteria of performance is the minimum detectable irradiance $I_{R}$ defined at $5 \%$ of the CDF. It will lead to whether or not the link budget is fulfilled.

The TURANDOT simulation, available at ONERA, models the propagation of a laser beam through atmospheric turbulence accurately. It could be used to determine the above criteria. However, it requires a lot of computations and is therefore not adapted for performing an important number of simulations.

I have developed a fast model derived from the LOT model proposed by Baker and have adapted it to my needs with a sufficient accuracy. I have validated this model by comparing it with TURANDOT in a few cases on the criteria of interest. It takes into account the first orders of aberration. It also takes into account tilt tracking, taking into account the error induced by point-ahead and compensation delay. It is called the WPLOT model.

The main activities in this thesis have been:

- Development of a model for fast estimation of the impact of atmospheric turbulence on irradiance fluctuations (Chapter 6.1).
- Validation by comparing it with TURANDOT (Chapter 6.2).
- Sensitivity study leading to an OGS sizing (Chapter 7).
- Study of the temporal fluctuations and their impact on error correction codes (Chapter 8).


## Part II

## Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links

## Validity of existing models

The objective of this chapter is to present a discussion on the literature models presented in Chapter 2 . We will consider the advantages of each model, their capacity to be adapted in order to model residual irradiance fluctuations after using mitigation techniques (adaptive optics in particular, tilt tracking at the very least).

We are going to focus on the models for the case in which $w_{0} \approx r_{0}$. We will see in Part III that for $w_{0} \ll r_{0}$, the link budget can't be fulfilled and that considering cases where $w_{0} \ll r_{0}$ may not be necessary. This will also allow to compare the results from the literature models to our chosen model presented in Chapter 6 and compared with TURANDOT in Chapter 6.2.

The literature models we will consider are

- the model which takes into account beam wander with decorrelated on-axis only Rytov scintillation presented in Equation (2.46) of Chapter 2, which we will refer to as the "Beam wander with on-axis scintillation" model.
- the model presented by Dios\&al.[40] in Equation (2.50), which we will refer to as the "Dios model".

For this study, we will compare the probability density functions obtained with the literature models and TURANDOT. We will also focus on the lower irradiances, i.e. the CDFs around the $5 \%$ threshold, in order to estimate the errors on the loss term $L_{T U R B}$ obtained with the analytic models compared to TURANDOT.

## Contents

5.1 Comparison of literature models with TURANDOT . . . . . . 59
5.2 Taking into account the tracking . . . . . . . . . . . . . . . . . . 61
5.3 Conclusion . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 63

### 5.1 Comparison of literature models with TURANDOT

The comparison between the literature models and TURANDOT will be done for a HufnagelValley 5/7 atmospheric profile, described in Section 2.1.4. The elevation angle is taken
equal to $\phi_{\text {elev }}=40^{\circ}$, making the effective $r_{0}$ equal to 15 cm at a $1.55 \mu \mathrm{~m}$ wavelength. The outer scale $L_{0}$ is fixed by the wave optics simulations: we have taken the outer scale to be equal to TURANDOT's phase screens' sizes, which gives $L_{0}=2.5 \mathrm{~m}$.

## Beam wander with on-axis scintillation

The comparison of the probability density functions obtained with TURANDOT and with the beam wander with on-axis scintillation model are presented in Figure 5.1. In this Figure, we have also added a case considering a perfect Gaussian beam moving because of beam wander. We will refer to it as the beam wander only case.

We observe that the the beam wander with on-axis scintillation model allows for a better fit of the irradiance fluctuations for a waist size of 5 cm compared to beam wander only case. However, for the other considered waists, the differences between the beam wander with on-axis scintillation model and the beam wander only case are negligible.

The PDFs' shapes are close to TURANDOT's results as well as the estimation of the mean irradiance and irradiance variance. Particularly, for all the considered wait sizes, the lower irradiances are nearly perfectly superimposed. In Figure 5.2, the CDFs comparison between TURANDOT with the beam wander with on-axis scintillation model and a beam wander only case shows that the lowest irradiances obtained with TURANDOT can be well modeled with beam wander only. Indeed, in this case the error obtained for estimating the turbulence term $L_{T U R B}$ is low for all the considered cases and reaches a maximum of approximately 1 dB for $w_{0}=14 \mathrm{~cm}$.

## Dios Model

The comparison of the probability density functions obtained with TURANDOT and Dios' model is presented in Figure 5.3 for different waist sizes. We observe that, for a 5 cm waist size, the mean irradiances around which are centered the PDFs is higher with the Dios model than with TURANDOT. However, the respective mean irradiances decrease much faster with the Dios model than with TURANDOT. Moreover, the PDFs obtained with the Dios model are much wider than those obtained with TURANDOT, leading to irradiance variance being much higher. This signifies that irradiance fluctuations resulting from the Dios model are overestimated. In their article, Dios\&al.[40] question whether the beam wander effect is actually considered in the Rytov method. These results seem to show that taking into account both the irradiance fluctuations from the off-axis scintillation (from Equation (D.3)) and beam wander effects seem to overestimate the actual irradiance fluctuations from our reference TURANDOT.

The study of the comparison of the CDFs between TURANDOT and the Dios model in Figure 5.4 corroborates the fact that Dios' model overestimates irradiance fluctuations. Particularly, we have focused on lower parts of the CDFs $\left(P\left(I<I_{T}\right)=0.2\right)$. These results show that using the Dios model can lead to an error of up to $2-3 \mathrm{~dB}$ on the turbulence loss term $L_{T U R B}$.
5.2 Taking into account the tracking


Figure 5.1: Probability density function comparison between TURANDOT, a tilt-only model and the tilt model with on-axis scintillation model for 2000 draws, using a HufnagelValley 5/7 atmospheric profile in a no tracking case.

## Conclusion

The presented results show that both the considered models do not give perfect irradiance fluctuations estimation compared to TURANDOT. The Dios model clearly overestimates the irradiance fluctuations while the beam wander with on-axis scintillation model slightly underestimates them. However, the lower irradiances are well modeled by beam wander.

From this study, we conclude that the Dios model can't be used in our considered cases to model irradiance fluctuations of uplinks. However, it would probably give better results for much smaller waist sizes (cf [40]). We have also shown that considering only beam wander allows for a good estimation of the irradiance fluctuations in the lower irradiances.

### 5.2 Taking into account the tracking

Works describing irradiance fluctuations and taking into account tilt tracking exist in the literature. Basu in [65] proposes a description of the residual irradiance description in a case with tracking (taking into account many parameters such as point-ahead angle, aperture


Figure 5.2: Cumulative density function comparison for $P\left(I<I_{T}\right)<0.2$ between TURANDOT, a tilt-only model and the tilt model with on-axis scintillation model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case.
mismatch and delay from the servo system). He provides information on the probability of fade, mean time of fade and mean number of fade over one second, by using the beam wander with on-axis scintillation technique. However, he considers that the beam wander is modeled by a negative exponential instead of the modified beta distribution usually used in the literature [44, 3].

We are going to compare the results of the beam wander with on axis scintillation model with tilt tracking taken into account to TURANDOT. The tracking is considered perfect with only the point-ahead angle as a decorrelation source. We present the PDFs for waist sizes at emission of $5,8,11$ and 14 cm in a case with tracking for the same atmospheric profile as previously in Figure 5.5 . For a 5 cm waist size at emission, taking into account the on-axis scintillation permits to significantly improve the resemblance of the PDF obtained with the residual beam wander with on-axis scintillation compared to the PDF obtained with TURANDOT. We see in this case that the error on the estimation of the turbulence loss term $L_{T U R B}$ at $5 \%$ of the CDF is of the order of 1 dB . For the bigger waist sizes, the impact of scintillation becomes negligible and on top of that, the errors on the estimation of the turbulence loss term $L_{T U R B}$ become of the order of 3 to 4 dB (cf.

### 5.3 Conclusion



Figure 5.3: Probability density function comparison between TURANDOT and the Dios model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case.

Figures $5.5 \mathrm{~b}, 5.5 \mathrm{c}$ and 5.5 d ). These results show that the residual beam wander is not sufficient to irradiance fluctuations when tilt tracking is considered, even when taking into account the on-axis scintillation.

### 5.3 Conclusion

In this chapter, we have compared irradiance fluctuations obtained with TURANDOT to the analytic descriptions found in the literature in a regime in which $w_{0} \approx r_{0}$. The two models from the literature we have considered are the beam wander with on-axis scintillation model presented in Chapter 2 and the model presented by Dios\&al.[40] in Equation (2.50). They both rely on the Rytov approximation coupled with beam wander.

We have shown that the Dios model does not give an accurate description of the effective irradiance fluctuations as it overestimates them, even in a case without tracking. We have also shown that the model considering a Gaussian beam displaced by beam wander perfectly modeled the lower irradiances in a no tracking case. On the other hand,


Figure 5.4: Cumulative density function comparison between TURANDOT and the Dios model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case.
we have seen that, when taking into account tilt tracking, the beam wander with on-axis scintillation did not model the residual irradiance fluctuations and greatly underestimated them.

From these results, we can confirm that, in a case with no tilt tracking, the lower irradiances $(<20 \%)$ are perfectly modeled by a Gaussian beam being displaced by beam wander. We are in a region in which the literature often states that Rytov does not give an accurate description of the irradiance fluctuations. This is confirmed with the presented results, particularly in a case with tilt tracking. This means that the residual irradiance fluctuations are the results of other phenomenons as, for example, the Gaussian beam deformations presented in the LOT solution (cf Chapter 2). This method will be the subject of the following chapter.

### 5.3 Conclusion



Figure 5.5: Probability density function comparison between TURANDOT, a beam wander with on-axis scintillation model and the beam wander only model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile in a tracking case.


Figure 5.6: Cumulative density function comparison for $P\left(I<I_{T}\right)<0.2$ between TURANDOT, a tilt-only model and the tilt model with on-axis scintillation model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile in a tracking case.

## Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links

In the previous chapter, we have presented a comparison of models from the literature with TURANDOT. We have observed that these models do not give an accurate description of irradiance fluctuations. In this chapter, we are going to improve the Low-Order of Turbulence (LOT) model, which was first introduced by Baker [3] and presented in Chapter 2 , by taking into account effects of the propagation. The final model will be called WPLOT (With Propagation Low-Order of Turbulence) model.

In the first part of this Chapter, we will present the LOT model's limitations regarding propagation through multiple layers and explain how we have come to develop the WPLOT model. In the second part of this Chapter, we will compare and validate the WPLOT model with TURANDOT's results for different atmospheric turbulence conditions.

## Contents

6.1 A multi-layer model - the WPLOT model ..... 68
6.1.1 Taking into account the beam truncation by the telescope ..... 72
6.1.2 Conclusion ..... 73
6.2 Discussion and validity of the WPLOT model ..... 73
6.2.1 Criteria for validation ..... 73
6.2.2 Propagation channel used for validation ..... 74
6.2.3 Without tracking ..... 74
6.2.4 With tracking ..... 83
6.3 Conclusion ..... 94

Chapter 6. Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links

### 6.1 A multi-layer model - the WPLOT model

The LOT solution assumes that the turbulence can be integrated in a single phase screen placed at the transmitter prior to propagation. This assumes that the effects of the propagation through multiple atmospheric layers are negligible inside the turbulence volume. In order to validate this assumption, we have compared the detected irradiance results obtained with the LOT solution with results obtained after wave optics propagation through phase screens evenly distributed along the propagation path. The phase screens are constructed as a linear combination of tip/tilt, defocus and astigmatism. They are the same phase screens as in the LOT solution $\phi_{L O T}$, i.e.:

$$
\begin{equation*}
\phi_{i}=\phi_{L O T}(x, y)=k_{0}\left[\theta_{x} x+\theta_{y} y+\Delta \kappa \frac{1}{2}\left(\left(x^{2}+y^{2}\right)_{\frac{1}{2}} w_{0}^{2}\right)+c_{5} \frac{1}{2}\left(x^{2}-y^{2}\right)+c_{6} x y\right], \tag{6.1}
\end{equation*}
$$

The difference is in the estimation of the Zernike coefficients: in the LOT solution, they are estimated over the whole turbulence volume whereas, here, they are estimated for each atmospheric layer. To illustrate the effects of the propagation through multiple phase screens, we will consider only a two layer atmosphere. This will be referred to as the 2L-WO simulation (for 2-Layer Wave Optics simulation). The objective of the 2L-WO simulation is to highlight the effects of the propagation through multiple atmospheric layers using the hypothesis of the LOT model that tip/tilt, defocus and astigmatism are sufficient to describe the beam deformations resulting from atmospheric turbulence in the D1 region. Using only two layers is sufficient to make these effects appear.

In this study, we will use the Hufnagel-Valley $5 / 7$ atmospheric profile with a 8 cm waist size at emission. In the case of the 2L-WO simulation, we will consider one ground layer and on layer at a 10 km altitude. This leads to $N_{L}=3.6 \times 10^{-4}$ and $N_{\tau}=9.4$, well within the D1 region of validity of the LOT solution.


Figure 6.1: Random irradiance draws obtained with the 2L-WO model and the LOT model.
We observe, in Figure 6.1, a good correlation of the results even though some differences appear. Studying the effects of each optical aberration separately, we can see that for

### 6.1 A multi-layer model - the WPLOT model

tip/tilt and astigmatism, the effects of propagation through multiple layers are negligible on the irradiance estimation, as can be seen in Figure 6.2 and 6.4. However, when considering only the defocus aberration, the effects of taking into account the propagation through multiple phase screens are clearly visible as can be seen in Figure 6.3. An irradiance threshold appears for the LOT model compared to the 2L-W0 model. This irradiance threshold results from diffraction and is equal to (in the case of a normalized irradiance):

$$
\begin{equation*}
I_{\text {threshold }}(0, L)=\frac{2}{\pi w(L)^{2}} \approx 2 \pi\left(\frac{w_{0}}{\lambda L}\right)^{2} . \tag{6.2}
\end{equation*}
$$



Figure 6.2: Comparison between random irradiance draws obtained with the 2L-WO model and the LOT model with tilt as the only considered aberration.


Figure 6.3: Comparison between random irradiance draws obtained with the 2L-WO model and the LOT model with defocus as the only considered aberration.

Figure 6.5 compares the Probability Density Functions (PDF) of irradiance in cases where only defocus is applied for 1,2 or 10 layers. It shows that using only one layer greatly reduces the dynamic of irradiance fluctuations: the PDF for only one layer is

Chapter 6. Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links


Figure 6.4: Comparison between random irradiance draws obtained with the $2 \mathrm{~L}-\mathrm{WO}$ model and the LOT model with astigmatism as the only considered aberration.
narrower than for multiple layers. However, the differences between 2 or 10 layers are not significant. On the other hand, Figure 6.6 shows that the impact of taking into account multiple layers does not significantly impact the lower irradiances.

This means that for more realistic irradiance fluctuations modeling (for example, better modeling of mean irradiance), at least two layers should be used.


Figure 6.5: Probability Density Functions of irradiance statistics between LOT solution, 2L-WO simulation and a $10 \mathrm{~L}-\mathrm{WO}$ simulation when considering only defocus, for an emitted waist size of 8 cm .


Figure 6.6: Cumulative Density Functions of irradiance statistics between LOT solution, 2L-WO simulation and a $10 \mathrm{~L}-\mathrm{WO}$ simulation when considering only defocus, for an emitted waist size of 8 cm .

In order to take into account the propagation through multiple phase screens, we propose to use the ABCD matrix propagation method [66], where the ABCD matrix is obtained from:

$$
\left(\begin{array}{cc}
A & B  \tag{6.3}\\
C & D
\end{array}\right)=\left(\begin{array}{cc}
1 & L_{p r o p} \\
0 & 1
\end{array}\right) \prod_{i}^{N}\left(\begin{array}{cc}
1 & 0 \\
\Delta \kappa_{i} & 1
\end{array}\right)\left(\begin{array}{cc}
1 & \Delta z_{i} \\
0 & 1
\end{array}\right)
$$

### 6.1 A multi-layer model - the WPLOT model

where $\Delta \kappa_{i}$ is the curvature (in meters ${ }^{-1}$ ) resulting from defocus at the $i^{\text {th }}$ layer, $\Delta z_{i}$ is the distance between two successive phase screens $i$ and $i+1$ and $N$ is the total number of considered layers. $L_{\text {prop }}$ is the remaining distance to the satellite: $L_{\text {prop }}=L-\sum_{i}^{N} \Delta z_{i}$, where $L$ is the distance between the ground and the satellite ( $L=36000 \mathrm{~km}$ ). Usually $\sum_{i}^{N} \Delta z_{i}$ is of the order of 20 km . This permits to obtain the same irradiance estimation as wave optics propagation when only defocus is considered, as can be seen in Figure 6.7.


Figure 6.7: Comparison between random irradiance draws obtained with the 2L-WO model and the WPLOT model with defocus as the only considered aberration.

Considering the following initial complex radius of curvatures:

$$
\begin{gather*}
\frac{1}{q_{x 0}}=\left(c_{5}^{2}+c_{6}^{2}\right)^{\frac{1}{2}}+\frac{\lambda i}{\pi w_{0}^{2}},  \tag{6.4}\\
\frac{1}{q_{y 0}}=-\left(c_{5}^{2}+c_{6}^{2}\right)^{\frac{1}{2}}+\frac{\lambda i}{\pi w_{0}^{2}} . \tag{6.5}
\end{gather*}
$$

Using the ABCD matrix, the complex radius curvatures after propagation are ${ }^{1}$ :

$$
\begin{equation*}
q_{x}=\frac{A q_{x 0}+B}{C q_{x 0}+D} . \tag{6.6}
\end{equation*}
$$

This leads to a modified estimation of the beam radii $w_{x}$ and $w_{y}$ compared to Baker's approach[3]:

$$
\begin{equation*}
w_{(x)}=\sqrt{\frac{\lambda}{\pi \Im\left(1 / q_{(x)}\right)}}, \tag{6.7}
\end{equation*}
$$

where $\Im$ means the imaginary part. These beam radii are implemented in the LOT solution presented in Equation (2.54). This permits to obtain the same irradiance estimation as wave optics propagation through multiple screens when only defocus is considered. We call this model WPLOT (for With Propagation Low-Order of Turbulence).

A comparison between the WPLOT model and the 2L-WO simulation is given in Figure 6.8. We observe that the higher irradiances are much better modeled. For lower

[^6]Chapter 6. Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links


Figure 6.8: Comparison between random irradiance draws obtained with the 2L-WO model and the WPLOT model.
irradiances, the results were already convincing. It is because these lower irradiances are principally due to beam wandering. The remaining errors are due to neglecting the effects of propagation through multiple phase screens for astigmatism. This approach taking into account the propagation through multiple phase screens does not significantly change the estimation of the irradiance threshold at $5 \%$ of the CDF in the case without tracking.

### 6.1.1 Taking into account the beam truncation by the telescope

The model needs to take into account the parameters of the optical ground station. In particular, one of those parameters is the truncation of the infinite Gaussian beam by the telescope. The truncation will have two effects. The first one is that it will reduce the transmitted power by the telescope [18]. The second one is that it will add a diffraction effect in the far-field.

The effects of the diffraction of a Gaussian beam by a circular aperture leads to a convolution in the case of propagation without turbulence, whose result is not easy to use. Yura has proposed an effective beam diameter at $1 / e^{2}$ in irradiance at the emission when the beam is weakly diffracted by a circular aperture. It is estimated in the case of a propagation without turbulence, but Yura has assumed that it can be applied to a case with turbulence. This effective beam diameter is given by[67]:

$$
\begin{equation*}
D_{e f f}=d_{0}\left[\tanh \left(D_{T_{X}}^{2} / 2 d_{0}^{2}\right)\right]^{\frac{1}{2}} \tag{6.8}
\end{equation*}
$$

This effective beam diameter yields accurate results well as long as $d_{0}<D_{T_{X}}$. It is valid only within the main central lobe of the beam ${ }^{2}$. The encircled power distributions are valid to better than approximately $2 \%$.

The effects of taking into account the truncation of the beam by the telescope in the WPLOT model can be observed in the CDFs in Figure 6.9. We observe that taking into account the truncation does not have a significant impact on the lower irradiances.

[^7]However, it greatly reduces the higher irradiances. This has the effect of improving the estimation of the mean irradiance and therefore improving the physical representativeness of the WPLOT model.


Figure 6.9: Cumulative density functions of irradiance fluctuations obtained with the 2LWO simulation taking into account the truncation and the WPLOT model with 2 layers taking into account, or not the truncation. In this case, $D_{T_{X}}=2^{3 / 2} w_{0}$, which is a value usually found in the literature[42].

### 6.1.2 Conclusion

In this chapter, we have presented a model which is equivalent to using a wave-optics propagation in which the phase screens are obtained from tip/tilt, defocus and astigmatism. This model also takes into account the effects resulting from the truncation of the beam with the telescope. In the next section, we will compare the WPLOT model's results with TURANDOT simulations in order to validate the model's irradiance description.

### 6.2 Discussion and validity of the WPLOT model

The objective of this section is to compare the results from the WPLOT model with TURANDOT model, dedicated to ground to space optical communications, which we will consider as our reference. In the WPLOT model, we will consider two atmospheric layers : one at ground level and one at a 10 km altitude. From this comparison, we will be able to estimate the performance of the WPLOT model by assessing the errors between the model and TURANDOT.

### 6.2.1 Criteria for validation

The criteria we will consider to validate the WPLOT model are: the mean irradiance, the probability density functions, the cumulative density functions, the losses $L_{T U R B}$ at $5 \%$ of the cumulative density function, and the temporal power spectral densities. Looking

Chapter 6. Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links
at the mean irradiance and the probability density functions enable to validate the representativity of the irradiance fluctuations obtained with the WPLOT model. From the perspective of the link budget, however, the main parameter of interest is the irradiance at $5 \%$ of the cumulative density functions $I_{R}$, which leads to losses due to turbulence term in the link budget: $L_{T U R B}=10 \log _{10}\left(I_{R} / I_{0}\right)$, where $I_{0}$ is the irradiance in a case of propagation without turbulence ${ }^{3}$. Validating the CDFs, especially in the lower irradiances, and the temporal power spectral densities will confirm that the WPLOT model will enable the study of the fade statistics.

If the errors on the estimation of $L_{\text {turb }}$ are less than 1 dB , the results are satisfying. If the errors are between 1 and 3 dB , a margin will have to be taking into account in the link budget. Finally, if the errors are higher than 3 dB , the model's results can not be used.

### 6.2.2 Propagation channel used for validation

## $C_{n}^{2}$ vertical profiles

We will consider two atmopsheric profiles. The first case we will consider is the common Hufnagel-Valley $5 / 7$, described in Section 2.1.4. The elevation angle will be considered equal to $40^{\circ}$, making the effective $r_{0}$ equal to 15 cm . The outer scale $L_{0}$ is fixed by the wave optics simulations: we have taken the outer scale to be equal to TURANDOT's phase screens' sizes, which gives $L_{0}=2.5 \mathrm{~m}$. This case corresponds to a low turbulence day.

The second case we will consider is for stronger turbulence conditions at ground level. The atmospheric profile is still the Hufnagel-Valley but the ground level is $C_{g}=5.4 \times 10^{-14} \mathrm{~m}^{-\frac{2}{3}}$ and $v=21 \mathrm{~m} / \mathrm{s}$, which results in a $r_{0}$ calculated at zenith equal to 11 cm for a $1.55 \mu \mathrm{~m}$ wavelength. At an elevation angle of $40^{\circ}$, the effective $r_{0}$ is equal to 8 cm . In mid latitudes near ground level, the ground $C_{n}^{2}(0)<5.4 \times 10^{-14} 60 \%$ of the time [68]. This case corresponds to a strong turbulence day. For this atmospheric profile, the sampling forced TURANDOT to use larger phase screens, inducing an outer scale $L_{0}=5 \mathrm{~m}^{4}$.

### 6.2.3 Without tracking

We start the validation by comparing the results from the WPLOT model and TURANDOT in a no tracking case. A no tracking case is interesting because it takes into account beam wander effects resulting from tilt which, according to Noll's results [33], represent around $90 \%$ of the turbulent phase variance. We have also seen in Chapter 5 that beam wander provided a good estimation of the irradiance fluctuations.

## Hufnagel-Valley 5/7

The condition from Equation (2.65) imposes a minimum waist size of 8 cm and the condition from Equation (2.60) imposes a maximum waist size of 22.5 cm . Despite this validity range, we will present results for lower waists ( 5 cm ) because we can assume that the lower

[^8]irradiances fluctuations are solely due to strong beam wander, even though scintillation will have an impact on the irradiance fluctuations closer to the mean irradiance.

The maximum diameter envisioned for the emitting telescope is 40 cm . Taking a telescope diameter to waist size ratio of $2^{\frac{3}{2}}$ (a usual value in the literature[42] which optimizes the transmitted energy) leads to a maximum waist size of 14 cm in this study.


Figure 6.10: Mean irradiance as a function of the emitted waist. Comparison between the TURANDOT simulation and the WPLOT model for 2000 draws, with a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case.


Figure 6.11: Difference in dB between the estimation of the mean irradiance as a function of the waist with the TURANDOT simulation and the WPLOT model, for a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case.

Mean irradiance The mean irradiances obtained with TURANDOT and the WPLOT model are compared in Figure 6.10. The WPLOT model almost perfectly estimates the mean irradiances for the considered waists, as Figure 6.11 shows that the difference between TURANDOT and the WPLOT model is less than 0.5 dB .

Probability Density Functions The PDFs for waists of $5,8,11$ and 14 cm are given in Figure 6.12. The PDFs obtained with the WPLOT model are very close to TURANDOT. This confirms the physical representativeness of the irradiance fluctuations obtained with the WPLOT model in this particular case.

In all the Figures, the normalized irradiance, expressed in dB , is defined : $10 \log _{10}\left(I / I_{0}\right)$.
Cumulative Density Functions The CDFs for waists of 5, 8, 11 and 14 cm are given in Figure 6.12. These CDFs focus on the lower irradiances. There is a very good fit for waists of 5,8 and 11 cm , as the differences are below 1 dB . For a 14 cm waist, a gap slightly larger than 1 dB starts to appear. We can therefore consider that the WPLOT model is valid when considering this particular case.

Losses due to turbulence estimation We start by estimating the irradiance threshold at $5 \%$ of the cumulative density function. It is given in Figure 6.14. We observe that an optimum appears for a waist size at emission of 8 cm .

Chapter 6. Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links


Figure 6.12: Probability density function comparison between TURANDOT and the WPLOT model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case.
6.2 Discussion and validity of the WPLOT model


Figure 6.13: Cumulative density function comparison between TURANDOT and the WPLOT model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case.


Figure 6.14: Comparison of the estimation of the irradiance threshold $5 \%$ of the normalized CDF as a function of the waist size at emission between TURANDOT and the WPLOT model, with a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case.

Chapter 6. Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links

Obtaining the threshold leads to the estimation of $L_{T U R B}$ with TURANDOT and the WPLOT model in Figure 6.15. The fit between the two is overall very good, even though gaps bigger than 1 dB appear for waist sizes bigger than 13 cm . In the Hufnagel-Valley $5 / 7$ atmopsheric profile, is precise within the 1 dB range.


Figure 6.15: Comparison of the estimation of the losses due to turbulence at $5 \%$ of the normalized CDF as a function of the waist size at emission between TURANDOT and the WPLOT model, with a Hufnagel-Valley $5 / 7$ atmospheric profile in a no tracking case.

Temporal Power Spectral Density The temporal power spectral densities (PSD) of the irradiance fluctuations are compared in Figure 6.16. They are estimated from 4s time series with a 2 kHz sampling. The fit between the results from TURANDOT and the WPLOT model are overall very good, as there is a good estimation of the cutoff frequency and asymptotic power laws ${ }^{5}$.

## Stronger turbulence profile

In the stronger atmospheric conditions, Equation (2.65) imposes a minimum waist size of 4 cm and the condition from Equation (2.60) imposes a maximum waist size of 12 cm . We have considered waists slightly outside the validity range of the model, in order to determine the behavior of the model even outside the validity range.

Mean irradiance The mean irradiances obtained with TURANDOT and the WPLOT model are compared in Figure 6.17. The mean irradiance obtained with the WPLOT model as a function of the waist size at emission increases much faster than with TURANDOT. For a 14 cm waist at emission, the difference between the two is of the order of 2 dB . The difference is less than 1 dB for waist sizes lower than 10 cm . These results induce that the long-term beam radius at satellite level is larger with TURANDOT than it is with the WPLOT model.

[^9]

Figure 6.16: Temporal power spectral density comparison between TURANDOT and the WPLOT model obtained from 4 s times series with a 2 kHz sampling, using a HufnagelValley $5 / 7$ atmospheric profile in a no tracking case. For a waist size of 5 cm , TURANDOT returned a numerical error probably due to sampling.

Chapter 6. Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links


Figure 6.17: Mean irradiance as a function of the emitted waist. Comparison between the TURANDOT simulation and the WPLOT model for 2000 draws, with stronger atmospheric conditions in a no tracking case.


Figure 6.18: Difference in dB between the estimation of the mean irradiance as a function of the waist with the TURANDOT simulation and the WPLOT model.

Probability Density Functions The PDFs for waists of 5, 8, 11 and 14 cm are given in Figure 6.19. We observe satisfying results for waists sizes of 5 and 8 cm . However, major differences appear for the larger waist sizes. We assume that these differences result from the higher Zernike orders which are not considered in the WPLOT model and whose effects are no longer negligible.

Cumulative Density Functions The results for the CDFs are presented in Figure 6.20. The results from Figures 6.20c and 6.20 d seem to indicate that, assuming tilt is well modeled, the short-term waist sizes are larger with TURANDOT than with the WPLOT model.

Losses due to turbulence estimation We start by comparing the irradiance threshold at $5 \%$ of the cumulative density function. It is given in Figure 6.21. We observe that the threshold $I_{R}$ obtained with the WPLOT model decreases while TURANDOT's estimation seems to reach minimum and then stays approximately constant.

We compare the estimation of $L_{T U R B}$ with TURANDOT and the WPLOT model in Figure 6.22. The error between the two is of the order of 1 dB for waist sizes at emission lower than 8 cm . However, for larger waist sizes, we observe that TURANDOT's results change slopes and decrease more slowly. It is interesting to notice that the Fried parameter $r_{0}$ is equal to 8 cm in the considered profile for stronger conditions. This means that the beam diameter at emission becomes significantly bigger than the atmospheric turbulence coherence length. This means that speckles will start to appear and that the short term beam spread will be larger than what the WPLOT model takes into account. Higher orders should be taken into account in this case. In a case without tracking, the WPLOT model can not be used for waist sizes larger than 8 cm for the stronger atmospheric turbulence conditions.


Figure 6.19: Probability density function comparison between TURANDOT and the WPLOT model for 2000 draws, using the stronger atmospheric profile in a no tracking case.

Chapter 6. Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links


Figure 6.20: Cumulative density function comparison between TURANDOT and the WPLOT model for 2000 draws, using the stronger atmospheric profile in a no tracking case.


Figure 6.21: Comparison of the estimation of the irradiance threshold $5 \%$ of the normalized CDF as a function of the waist size at emission between TURANDOT and the WPLOT model, with the stronger atmospheric profile in a no tracking case.


Figure 6.22: Estimation of the losses due to turbulence at $5 \%$ of the normalized CDF as a function of the waist size at emission, using the stronger atmospheric profile in a no tracking case.

Temporal Power Spectral Density The temporal PSDs are compared in Figure 6.23. The fit between the results from TURANDOT and the WPLOT model are good for a 8 cm waist size. However, for a 14 cm waist size, a gap appears in the higher frequencies, probably due to the fact that some speckle effects resulting from higher order Zernike polynomials are not taken into account.

### 6.2.4 With tracking

We now consider cases with tracking taken into account. Tracking is considered perfect. We consider that the apertures of the uplink and the downlink are merged and aligned and there is no delay before the pre-compensation. Only the point-ahead angle $\theta_{P A A}=18.5 \mu \mathrm{~m}$ is taken into account.

## Hufnagel-Valley 5/7

Mean irradiance The mean irradiances obtained with TURANDOT and the WPLOT model are compared in Figure 6.24. Even though Figure 6.24 shows an increasing error on the estimation of the mean irradiances as the waist size increases, Figure 6.25 shows that the difference between TURANDOT and the WPLOT model is less than 1 dB .

Probability Density Functions The PDFs for waists of 5, 8, 11 and 14 cm with a Hufnagel-Valley $5 / 7$ atmospheric profile and tilt tracking taken into account are given in Figure 6.26. There are also some differences between the results from TURANDOT and the WPLOT model. These differences are due to the fact that we neglect the impact of the higher orders of Zernike polynomials on the irradiance fluctuations.

Cumulative Density Functions The CDFs for waists of 5, 8, 11 and 14 cm with tracking taken into account are given in Figure 6.27. From these CDFs, we observe that

Chapter 6. Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links


Figure 6.23: Temporal power spectral density comparison between TURANDOT and the WPLOT model obtained from 4 s times series with a 2 kHz sampling, using a stronger atmospheric profile in a no tracking case. For a waist size of 5 cm , TURANDOT returned a numerical error probably due to sampling.


Figure 6.24: Mean irradiance as a function of the emitted waist. Comparison between the TURANDOT simulation and the WPLOT model for 2000 draws, with a Hufnagel-Valley 5/7 atmospheric profile and tracking.


Figure 6.25: Difference in dB between the estimation of the mean irradiance as a function of the waist with the TURANDOT simulation and the WPLOT model.
the differences between TURANDOT and the WPLOT model are of the order of 1 dB . This means that the modeling with the WPLOT model is valid.

Losses due to turbulence estimation The estimation of the irradiance threshold at $5 \%$ of the cumulative density function is given in Figure 6.28. We observe that the irradiance threshold increases with the waist size. Due to the model's validity range, we can not observe an optimum that appears for the bigger waists. We also observe a slight overestimation of the threshold $I_{R}$.

We can then obtain the estimation of $L_{T U R B}$ with TURANDOT and the WPLOT model in Figure 6.29. We observe that there is less a 1 dB difference between the results from TURANDOT and the WPLOT model, which validates the model's precision in the studied range.

Temporal Power Spectral Density The temporal PSDs are compared in Figure 6.30 .

The fit between the results from TURANDOT and the WPLOT model are very good overall, as there is a good estimation of the cutoff frequency and asymptotic power laws. For a 5 cm waist size at emission, we see a small error appear around the 100 Hz frequency. We attribute this error to the fact that a 5 cm waist is at the limit of the WPLOT model validity range and that, for lower waists, the on-axis scintillation from the Rytov perturbation method (cf. Figure 5.5a and Equation 2.46) is not negligible compared to beam wander and beam deformations in terms of impact on the irradiance fluctuations.

Chapter 6. Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links


Figure 6.26: Probability density function comparison between TURANDOT and the WPLOT model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile and tracking.


Figure 6.27: Cumulative density function comparison between TURANDOT and the WPLOT model for 2000 draws, using a Hufnagel-Valley $5 / 7$ atmospheric profile and tracking.


Figure 6.28: Comparison of the estimation of the irradiance threshold $5 \%$ of the normalized CDF as a function of the waist size at emission between TURANDOT and the WPLOT model, with a Hufnagel-Valley $5 / 7$ atmospheric profile and tilt tracking.

Chapter 6. Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links


Figure 6.29: Comparison of the estimation of the losses due to turbulence at $5 \%$ of the normalized CDF as a function of the waist size at emission between TURANDOT and the WPLOT model, with a Hufnagel-Valley $5 / 7$ atmospheric profile and tilt tracking.


Figure 6.30: Temporal power spectral density comparison between TURANDOT and the WPLOT model calculated from a 1 s times series with a 2 kHz sampling, using a HufnagelValley $5 / 7$ atmospheric profile and tracking.

## Stronger atmospheric profile

We now compare the results from the WPLOT model and TURANDOT for stronger atmospheric conditions, with tracking.


Figure 6.31: Mean irradiance as a function of the emitted waist. Comparison between the TURANDOT simulation and the WPLOT model for 2000 draws, with stronger conditions and tracking.


Figure 6.32: Difference in dB between the estimation of the mean irradiance as a function of the waist with the TURANDOT simulation and the WPLOT model, with stronger conditions and tracking.

Mean irradiance The mean irradiances obtained with TURANDOT and the WPLOT model are compared in Figure 6.31. The mean irradiance obtained with the WPLOT model as a function of the waist size at emission increases much faster than with TURANDOT. For a 14 cm waist at emission, the difference between the two is of the order of 2.5 dB . As in the no tracking case, this can be interpreted as the long-term beam waist at satellite level increasing much faster for TURANDOT than in the WPLOT model. Because tilt is well tracked, these results are due to the beam spreading from the higher order Zernike polynomials.

Probability Density Functions Figure 6.33d presents a comparison of the PDFs for waists sizes of $5,8,11$ and 14 cm in the case of stronger atmospheric conditions and tracking. We indeed observe that the mean irradiance is higher for the WPLOT model than for TURANDOT. However, the variance of the irradiance fluctuations seems to be close for waists sizes of 8 and 11 cm .

Cumulative Density Functions Figure 6.34d presents a comparison of the CDFs for waists sizes of $5,8,11$ and 14 cm in the case of stronger atmospheric conditions and tracking. The gap between TURANDOT and the WPLOT model increases with the waist size. Except for a 5 cm (which could be improved by taking into account irradiance fluctuations from the Rytov perturbation method), the curves seem to have the same shape with only an offset as difference. This offset increases with the waist size.

Chapter 6. Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links


Figure 6.33: Probability density function comparison between TURANDOT and the WPLOT model for 2000 draws, using the stronger atmospheric profile and tracking.


Figure 6.34: Cumulative density function comparison between TURANDOT and the WPLOT model for 2000 draws, using the stronger atmospheric profile and tracking.

Chapter 6. Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links

Losses due to turbulence estimation The estimation of the irradiance threshold at $5 \%$ of the cumulative density function in the case of stronger atmospheric conditions and tilt tracking is given in Figure 6.35. We observe that the irradiance threshold increases with the waist size. Due to the model's validity range, we can not observe an optimum that appears for the bigger waists. We also observe an overestimation of the threshold $I_{R}$.


Figure 6.35: Comparison of the estimation of the irradiance threshold $5 \%$ of the normalized CDF as a function of the waist size at emission between TURANDOT and the WPLOT model, using the stronger atmospheric profile and tracking.


Figure 6.36: Estimation of the losses due to turbulence at $5 \%$ of the normalized CDF as a function of the waist size at emission, using the stronger atmospheric profile and tracking.

The estimation of $L_{\text {turb }}$ at $5 \%$ of the normalized CDFs is given in given in Figure 6.36. The error between TURANDOT and the WPLOT is of the order of $1.5-2 \mathrm{~dB}$ for waist sizes $w_{0}<1.5 r_{0}$. The underestimation is of the order of 3 dB for waist sizes bigger than $1.5 r_{0}$, outside the assumed validity range of the WPLOT model. Because of these errors on the estimation, we will have to take a margin when using the WPLOT model to do a sizing in Part III.


Figure 6.37: Temporal power spectral density comparison between TURANDOT and the WPLOT model calculated from a 1 s times series with a 2 kHz sampling, using a stronger atmospheric profile and tracking.

Chapter 6. Irradiance fluctuations modeling for ground-to-satellite geostationary satellite optical links

Temporal Power Spectral Density The temporal PSDs in the case of stronger atmospheric conditions and tracking are compared in Figure 6.37. The fit between the results from TURANDOT and the WPLOT model are good. For the 5 cm waist size at emission, there is again a gap around the 100 Hz frequency. We attribute it to the scintillation from the Rytov Perturbation method for the same reason as for the HufnagelValley $5 / 7$ profile case.

### 6.3 Conclusion

We have compared the results of the WPLOT model with results from TURANDOT for two cases: one modeling classic daytime atmospheric conditions (the Hufnagel-Valley 5/7) and one modeling stronger atmospheric conditions at ground level.

We have observed very good fit between the WPLOT model's and TURANDOT's results in the case of the Hufnagel-Valley $5 / 7$ atmospheric profile without tracking. When we considered tracking, the results were still good as the mean error for estimating $L_{\text {turb }}$ was of the order of 1 dB . However, we start to notice irradiance fluctuations resulting from the higher Zernike polynomials order, which are not taken into account in the WPLOT model.

In the case of stronger atmospheric conditions, the results between TURANDOT and the WPLOT model were not as good. We attribute these differences to the higher orders whose effects are no longer negligible. In the case without tracking in the stronger turbulence conditions, the results incite us to modify the range of validity of the model from $w_{0}<1.5 r_{0}$ to $w_{0}<r_{0}$. Moreover, the gaps between the LOT and TURANDOT's estimations incite us to consider a 3 dB margin in the link budget.

## Part III

## WPLOT model application for the sizing of an Optical Ground Station



## Optical ground station sizing

The objective of this chapter is to present an application of the WPLOT model to do a sensitivity study resulting in the sizing of an optical ground station. The approach is as follows: firstly, the characteristics of the ground station, such as the waist size at emission, the beam truncation and architecture configuration, are chosen for representative atmospheric conditions. Subsequently, the performances of the ground station are estimated by varying the atmospheric conditions, such as the ground layer strength, the $C_{n}^{2}$ profile and the outer scale.

The first part of this Chapter describes the parameters for the study, i.e. the channel, atmospheric conditions, ground station and link budget parameters. The second part of this Chapter consists in the sensitivity study.

## Contents

7.1 Standard parameters ..... 98
7.1.1 Channel parameters ..... 98
7.1.2 Optical ground station parameters ..... 99
7.1.3 Link budget parameters ..... 100
7.2 Maximum acceptable residual tilt ..... 100
7.3 Sizing using the WPLOT model ..... 102
7.3.1 Introduction to architecture sizing ..... 103
7.4 Sensitivity study ..... 111
7.4.1 Anisoplanatism induced by the point-ahead angle ..... 111
7.4.2 Delay before correction ..... 111
7.4.3 Sensitivity of the sizing to atmospheric turbulence conditions ..... 114
7.5 Conclusion on the sizing of the optical ground station ..... 118

### 7.1 Standard parameters

### 7.1.1 Channel parameters

## Geometry

Because the envisioned optical ground stations (OGS) will be located in Europe, we need to consider the fact that the propagation distance from the OGS to the satellite will not be exactly 36000 km but a slightly longer distance due to the latitudes of the OGS. This is illustrated in Figure 7.1. $L$ is the overall link distance, $R_{E}=6378 \mathrm{~km}$ is the Earth's mean radius, $H_{\text {sat }}=35786 \mathrm{~km}$ is the geostationary satellite's altitude, $\alpha$ is the latitude, $\phi_{\text {elev }}$ is the elevation angle and $\psi_{z e n}$ is the zenith angle. It can be shown that $L$ and $\phi_{\text {elev }}$ (and therefore $\psi_{z e n}$ as well) can be expressed as a function of $\alpha$. The results are:

$$
\begin{equation*}
L=H_{s a t} \sqrt{1+2\left(\frac{R_{E}\left(R_{E}+H_{s a t}\right)}{H_{s a t}^{2}}\right)(1-\cos (\alpha))} \tag{7.1}
\end{equation*}
$$

and

$$
\begin{equation*}
\phi_{\text {elev }}=\arctan \left(\frac{\cos (\alpha)-\left(\frac{R_{E}}{R_{E}+H_{\text {sat }}}\right)}{\sin (\alpha)}\right) . \tag{7.2}
\end{equation*}
$$

In this approach, we have assumed that the OGS and the satellite were at the same longitude (i.e. same plane), but these results can easily be generalized [69]. Using these equations, it can be shown that at latitude $\alpha=43^{\circ}$, the zenith angle is $\phi_{\text {elev }}=40^{\circ}$. This leads to the link distance $L=37750 \mathrm{~km}$.

## Turbulence

The station characteristics are optimized by considering daytime conditions, i.e. the stronger turbulence profile presented in Section 6.2.2. Section 6.2 has shown that the WPLOT model leads to an underestimation of around 1 to 2 dB of the losses due to the turbulence at $5 \%$ of the normalized CDF, in a case with tracking.

It is important to note that these conditions are not a worst case scenario, as cases where $C_{n}^{2}(0)=1 \times 10^{-13} m^{-\frac{2}{3}}$ can occur, for which $r_{0}=6 \mathrm{~cm}$. However, in mid latitudes near ground level, the ground $C_{n}^{2}(0)<5.4 \times 10^{-14} 60 \%$ of the time [68]. The considered atmospheric profile is therefore representative of a majority of cases.

In these atmospheric conditions, the Fried parameter is equal to 8 cm . We will consider an outer scale of 5 m . The bigger the outer scale, the stronger the tip/tilt variances. However, its effect on the higher Zernike orders is negligible, as can be seen in Figure C. 3 (in Appendix C). The inner scale, on the other hand, has a small influence in the case of Rytov scintillation, which is negligible in the considered regime where $w_{0} \approx r_{0}$. The inner scale will therefore be neglected.

The considered wind profile is still the Bufton profile described in Section 2.5.2. We will consider that the wind is in the same direction as the point-ahead angle (i.e. worst case scenario).


Figure 7.1: Link distance and elevation angle as a function of the latitude.

### 7.1.2 Optical ground station parameters

The foreseen optical ground station will comprise a Gaussian laser beam of waist size $w_{0}$ (and of diameter $d_{0}=2 w_{0}$ ) emitted through a pupil of diameter $D_{T_{X}}$, which truncates the beam. Figure 7.2 describes the different parameters of the emitted beam.

The emitted Gaussian beam is considered collimated, i.e. that the position of the waist $w_{0}$ is in the exit pupil of the emitter. Obstructions were not considered because adding one will modify the shape of the beam propagating through the atmosphere [70] as well as significantly reduce the transmitted power of the Gaussian beam.

The dimension of the beam waist $w_{0}$ is the first parameter that will be studied. A small waist at the emission ( $w_{0} \ll r_{0}$ ) is less susceptible to the atmospheric turbulence effects but, due to the limited power of lasers envisioned for feeder links, the beam divergence has to be dramatically reduced, leading to larger waist sizes, making the beam pointing a key issue. A trade-off will therefore have to be achieved, using a pointing error correction system, necessary to achieve the targeted power detected by the satellite.

Another parameter regarding the optical ground station that needs to be optimized is the diameter of the emission telescope $D_{T_{X}}$. The constraints of realizing an actual optical ground station impose a maximum size for the emission telescope. The maximum diameter $D_{T_{X}}$ considered for the telescope $T_{X}$ is 40 cm .

Finally, the optical ground station will comprise an emitting telescope $T_{X}$ and a receiving telescope $R_{X}$ of diameter $D_{R_{X}}$. We assume that the phase measurement on the downlink will be made using a wavefront sensor of pupil diameter $D_{T T S}$ which is part of $R_{X} . R_{X}$ and $T_{X}$ may or may not be merged, depending on whether the possible stray light problems can be delt with (the idea is to use two different circular polarization between the downlink which receives a power of a few nW and the uplink which emits a power of


Figure 7.2: Description of the Gaussian beam parameters

50 W ). Unless stated otherwise, we will consider that the pupil of the wavefront sensor is merged with the emitted beam.

Throughout the study, we assume that the measurement of the Zernike coefficients on the downwards propagating beam by the wavefront sensor is perfect and that the correction is perfectly applied by the pointing mirror to the upwards propagating beam.

### 7.1.3 Link budget parameters

We will assume that 50 W in continuous power will be emitted by the ground station, i.e.:

$$
\begin{equation*}
P_{E}=50 \mathrm{~W}=17 \mathrm{dBW} . \tag{7.3}
\end{equation*}
$$

We assume also that the receiving pupil on the satellite has a diameter $D_{R_{X}}=20 \mathrm{~cm}$. This leads to $G_{R}=112.2 \mathrm{~dB}$.

The other contributors taken into account in the link budget in $L_{O T H E R S}$ are summarized with their numerical values for this study in Table 7.1. In order to take into account the differences between the WPLOT model and TURANDOT (our reference) observed in Chapter 6.2 , we will add in the link budget a system margin loss term equal to $L_{\text {margin }}=-3 \mathrm{~dB}$.

### 7.2 Maximum acceptable residual tilt

We present in this section a rough estimation of the maximum acceptable tilt after precompensation. For simplification, in this Section we are going to consider two main parameters: the waist size at emission and residual tilt as the only aberration. With these parameters, we are able to evaluate the link budget.

Table 7.1: Contributors to the optical link budget.

| Contributors | Description | Value in link budget |
| :---: | :--- | :--- |
| $P_{E}$ | Optical power booster amplifier | $50 \mathrm{~W}=17 \mathrm{dBW}$ |
| $T_{T_{X}}$ | Transmission loss inside optical <br> terminal in $T_{X}$ | -3 dB (Independent of the size of <br> the telescope $)$ |
| $G_{E}$ | Emitter gain with pointing error | Equation $(1.6)$ |
| $L_{F S}$ | Free-space loss | $10 \log _{10}\left(\left(\frac{\lambda}{4 \pi L}\right)^{2}\right)=-290 \mathrm{~dB}$ |
| $L_{a b}$ | Absorption and scattering loss | $10 \log _{10}\left(\exp \left(\frac{\log \left(T_{z e n}\right)}{\sin \left(E_{\text {deg }}\right)}\right)\right), \quad$ with <br> $T_{z e n}=0.9$ the atmospheric trans- <br> mission and $E_{\text {deg }}$ the elevation an- <br> gle $[71]$ |
| $L_{T U R B}$ | Atmospheric turbulence loss | Thesis objective |
| $L_{c l o u d s}$ | Cloud loss | -5 dB |
| $G_{R}$ | Receiver gain | 112.2 dB, for $D_{R_{X}}=20 \mathrm{~cm}$ |
| $T_{R_{X}}$ | Transmission and injection loss in- <br> side optical terminal in $R_{X}$ | -5 dB |
| $L_{\text {margin }}$ | System margin | -3 dB |
| $P_{R}$ | Injected optical power | $P_{R}>-43 \mathrm{dBm}$ |

The on axis irradiance is:

$$
\begin{equation*}
I(x, y, L)=\frac{2}{\pi} \frac{1}{w(L)^{2}} \exp \left[-2 \frac{(x-\delta x)^{2}+(y-\delta y)^{2}}{w(L)^{2}}\right] \tag{7.4}
\end{equation*}
$$

where $w$ is the beam radius at a distance $L$ and $\vec{\delta}=(\delta x, \delta y)$ the displacement of the beam due to tilt. The on-axis irradiance probability distribution is the modified beta distribution[3]:

$$
\begin{equation*}
P_{I}(I, 0)=\frac{1}{2 \alpha I_{0}}\left(\frac{I}{I_{0}}\right)^{\frac{1}{2 \alpha}-1} \tag{7.5}
\end{equation*}
$$

where $I_{0}=\frac{2}{\pi w^{2}}$ and $\alpha=\frac{\langle\vec{\delta}\rangle}{\left.w^{2}\right\rangle}$. The cumulative distribution function at a threshold irradiance $I_{S}$ is therefore:

$$
\begin{equation*}
C D F_{I}\left(I \leq I_{S}\right)=\left(\frac{I_{S}}{I_{0}}\right)^{\frac{1}{2 \alpha}} \tag{7.6}
\end{equation*}
$$

From the link budget in Table 7.1, we can determine the maximum acceptable turbulence loss $L_{T U R B, m}\left(w_{0}\right)=104.3-G_{E}$, in which $G_{E}$ is the emitter gain defined in Equation (1.6). In our simplified case, $G_{E}=10 \log _{10}\left(\frac{8 \pi^{2} w_{0}^{2}}{\lambda^{2}}\right)$. Using $L_{T U R B}$ 's definition ( $L_{T U R B}=$ $\left.10 \log _{10}\left(\frac{I_{R}}{I_{0}}\right)\right)$ leads to

$$
\begin{equation*}
I_{R}=I_{0} \times 10^{\frac{L_{T U R B}}{10}} . \tag{7.7}
\end{equation*}
$$

Assuming $C D F_{I}\left(I \leq I_{R}\right)=0.05$ and inserting Equation (7.7) in Equation (7.6) leads to:

$$
\begin{equation*}
\alpha=\frac{L_{T U R B, m}\left(w_{0}\right)}{20} \frac{\log (10)}{\log (0.05)} . \tag{7.8}
\end{equation*}
$$

Therefore,

$$
\begin{equation*}
\left\langle\vec{\delta}^{2}\right\rangle=w^{2} \frac{L_{T U R B, m}\left(w_{0}\right)}{20} \frac{\log (10)}{\log (0.05)} \tag{7.9}
\end{equation*}
$$

This result therefore gives the maximum acceptable displacement resulting from as a function of the waist size at emission and wavelength in a simplified case with no truncature from the telescope and no beam deformation resulting from turbulence. It leads to the maximum Zernike coefficients variance, assuming $w(L) \approx \frac{L \lambda}{\pi w_{0}}$ :

$$
\begin{equation*}
\sigma_{a_{2}, \max }^{2}+\sigma_{a_{3}, \max }^{2}=\frac{L_{T U R B, m}\left(w_{0}\right)}{20} \frac{\log (10)}{\log (0.05)} . \tag{7.10}
\end{equation*}
$$

In uncompensated cases, $\sigma_{a_{2}}^{2}=\sigma_{a_{3}}^{2}$. However, in tracked cases, the anisoplanatism effects mean that the correlation for tip and tilt are not equal. Therefore, we will introduce the variable $\sigma_{T T}^{2}=\sigma_{a_{2}}^{2}+\sigma_{a_{3}}^{2}$. Figure 7.3 illustrates the maximum allowable variance for the tip/tilt coefficients, calculated using Equation (7.10), over uncompensated tilt variances ratio as a function of the waist size at emission.

The WPLOT model is much more precise than this rough approach. However, in the case of a sizing considering the quality of tilt tracking, this analytical approach gives a first estimation of whether or not the link budget is fulfilled.


Figure 7.3: Maximum allowable variance for the tip/tilt coefficients over uncompensated tilt variances ratio as a function of the waist size at emission, taking into account a telescope truncature of diameter $D_{T_{X}}=2^{\frac{3}{2}} w_{0}$ in order to compare with the results in the rest of the study.

### 7.3 Sizing using the WPLOT model

In the sizing, we will want to study the impact of many parameters. These parameters and their range of study are presented in Table 7.2.

### 7.3 Sizing using the WPLOT model

Table 7.2: Parameters of the sensitivity study.

| Parameters | Range of study |
| :---: | :---: |
| Sensibility to the OGS parameters |  |
| Waist $w_{0}$ | 4 to 12 cm |
| Telescope diameter $D_{T_{X}}$ | $D_{T_{X}}=2 w_{0}$ to $D_{T_{X}}=5 w_{0}$ |
| PAA | $P A A=0 \mu \mathrm{rad}$ or $P A A=18.5 \mu \mathrm{rad}$ |
| Architecture configuration |  |
| Monostatic configuration | $D_{T T S}=d_{0} / 4$ to $D_{T T S}=2 d_{0}$ |
| Bistatic configuration | - off-axis configuration <br> - annular configuration |
| Sensitivity to the propagation channel |  |
| Ground layer | $\begin{aligned} & C_{n}^{2}(0)=1.7 \times 10^{-14} \mathrm{~m}^{-2 / 3}, C_{n}^{2}(0)=5.4 \times 10^{-14} \mathrm{~m}^{-2 / 3} \text { and } \\ & C_{n}^{2}(0)=1 \times 10^{-13} \mathrm{~m}^{-2 / 3} \end{aligned}$ |
| Altitude layers | Turbulence layers strengths increased at 5 km and 10 km altitudes |
| Outer scale | $L_{0}$ from 1 m to 100 m |

### 7.3.1 Introduction to architecture sizing

## No pointing correction

Figure 7.4 shows the results of the link budget as a function of the emitted waist size in a no tracking case. Due to the model's validity regime, the waist sizes range from 4 to 12 cm . The effects of the truncature by the telescope and of the station's vibrations are not considered for now. The link budget is never fulfilled, as there is, in the best case for $w_{0}=4 \mathrm{~cm}$, a 10 dB gap between the link budget and the detection threshold. An optimum appears for the lower waist sizes. This optimum stays below the detection threshold as the detected power is limited by the beam divergence for lower waist sizes. This therefore indicates that adding tilt correction is necessary. In the rest of the study, tilt tracking will always be considered.

## Perfect tilt tracking

Figure 7.5 presents the evolution of the optical link budget as a function of the emitted waist size with perfect tracking, i.e. we do not into account decorrelation from the pointahead angle, aperture mismatch or delay between measurement and pre-compensation. The effects of the truncature by the telescope and of the station's vibrations are not considered for now.

A 4.5 cm waist size appears to be the minimum waist size that fulfills the link budget. Increasing the waist size at emission improves the link budget. This is logical because increasing the waist size at emission focuses the energy in the far field. However, the gains on the link budget are not as important between two successive relatively big waist sizes

Chapter 7. Optical ground station sizing


Figure 7.4: Evolution of the optical link budget as a function of the waist in a no tracking case.


Figure 7.5: Evolution of the optical link budget as a function of the waist with perfect tracking.
(i.e. $>10 \mathrm{~cm}$ ). There is a 1 dB difference on the link budget between $w_{0}=8 \mathrm{~cm}$ and $w_{0}=12 \mathrm{~cm}$ while there is almost a 3 dB difference between $w_{0}=4 \mathrm{~cm}$ and $w_{0}=8 \mathrm{~cm}$.

## Choice of the architecture for tilt tracking

There are two considered cases; mono-static and bi-static configurations. Mono-static configurations refer to cases where the emitting pupil and Tilt Tracking Sensor (TTS) pupil are merged while bi-static configurations refer to cases where the pupils are separated. The most intuitive configuration is the mono-static where the measurement of the phase aberration on the downlink is done on the same pupil as the one considered for the emitting beam (of diameter $d_{0}=2 w_{0}$ ). However, such configurations might be difficult to implement due to possible stray light problems and the receiving and emitting pupils may have to be separated, i.e. bi-static configurations may have to be envisioned. In this subsection we will evaluate the performances of these configurations. We will assume in this study that the measurement on the downlink is directly applied to pre-compensate the uplink.

Mono-static configuration In the case of a mono-static configuration, we will consider the possibility that the TTS pupil does not have a diameter equal to $d_{0}$. However, it is still assumed that their respective centers are merged. The results on the link budget of considering TTS pupil diameters in the range $0.5 w_{0} \leq D_{T T S} \leq 4 w_{0}$ are presented in Figure 7.7 , for a waist size at emission $w_{0}=8 \mathrm{~cm}$. These results show that the optimal solution is obtained for $D_{T T S}=2 w_{0}$. The link budget is fulfilled for the range $1.1 w_{0}<D_{T T S}<3 w_{0}$. Moreover, there is less than 1 dB loss from the maximum for the range $1.3 w_{0}<D_{T T S}<$ $2.7 w_{0}$.

Figure 7.6 shows the results on the evolution of the residual tilt to initial tilt variance ratio as a function of the wave-front pupil diameter to waist size ratio, for a waist size at emission $w_{0}=8 \mathrm{~cm}$. Particularly, we observe the range of $D_{T T S}$ for which the link budget is fulfilled corresponds to residual tilt being less than $22 \%$ of the initial tilt. This value is slightly less than the result given in Figure 7.3 which allowed for a residual tilt up to $26 \%$ of the initial tilt. This is because we take into account the effects of defocus and astigmatism in the WPLOT model whereas they were neglected in Section 7.2.

Bi-static configurations Two cases of bi-static configurations will be envisioned for our study : off-axis and annular configurations. They are recalled in Figure 7.8.

Off-axis configuration Off-axis configurations are presented in Figure 7.8a and Figure 7.8b. They correspond to cases where the emitting pupil and the TTS pupil are next to each other but do not overlap. Figure 7.10 presents the results of the link budget assuming that $D_{T T S}=2 w_{0}$ and the distance between the center of the beam and the center of the TTS is $d=w_{0}+2^{\frac{1}{2}} w 0$ (as we consider that the diameter of the emitting telescope is $D_{T_{X}}=2^{\frac{3}{2}} w_{0}$ ). This corresponds in the case described in Figure 7.8a. This result shows that an off-axis configuration does not permit to fulfill the link budget. This can be easily justified by looking at the residual tilt variance compared to the initial uncompensated tilt in Figure 7.9. We observe that the residual tilt is actually stronger than the uncompen-


Figure 7.6: Residual tilt variance compared to the initial uncompensated tilt as a function of the wave-front pupil diameter to waist size ratio, considering $w_{0}=8 \mathrm{~cm}$.


Figure 7.7: Evolution of the link budget as a function of the wave-front pupil diameter to waist size ratio, considering $w_{0}=8 \mathrm{~cm}$.

### 7.3 Sizing using the WPLOT model



Figure 7.8: Examples of bi-static configurations.
sated tilt. This means that the tracking does not compensate at all the tilt and even adds errors.

In the case described in Figure 7.8b, the idea was to try to use the off-axis configuration and the PAA to improve the correlation between the uplink and the downlink in the higher layers. In this configuration, the tip/tilt correlation is slightly improved but still does not correct the tilt and still results in a stronger residual tilt than uncompensated tilt. This is due to the fact that the ground layers are much stronger than altitude layers.

Finally, increasing the size of the wavefront sensor pupil diameter $D_{T T S}$ increases the residual tilt and therefore does not improve the performance of the system.

Annular configuration The annular configuration is presented in Figure 7.8c. It corresponds to a case where the wavefront sensor has an annular pupil around the emitting telescope. Greenwood [72] gives a result for tilt estimation on an annulus:

$$
\begin{equation*}
a_{2, \text { annulus }}=\frac{\pi D}{\left(A_{D}^{2}-A_{d}^{2}\right)}\left[A_{D}^{2} \cdot \frac{1}{\pi D} \cdot a_{2, D}-A_{d}^{2} \cdot \frac{1}{\pi d} \cdot a_{2, d}\right] . \tag{7.11}
\end{equation*}
$$

This result is interesting because it means that we will be able to express tilt on an annulus as a function of tilts estimated on plain disks of diameters $D$ and $d$ (cf. Figure 7.8c). Even though some work has been done to define a polynomial base close to Zernike polynomials on an annulus [73], these polynomials are no longer valid with Chassat's correlation functions. Therefore, we will only be able to consider tilt tracking when studying the annular

Chapter 7. Optical ground station sizing


Figure 7.9: Residual tilt variance compared to the initial uncompensated tilt as a function of the waist size in the case of an off-axis configuration.


Figure 7.10: Evolution of the link budget as a function of the waist size in the case of an off-axis configuration.

### 7.3 Sizing using the WPLOT model

configuration. The correlation between the annulus tilt and tilt on the pupil of diameter $d$ is

$$
\begin{equation*}
\left\langle a_{2, \text { annulus }}\right\rangle=\frac{\pi D}{\left(A_{D}^{2}-A_{d}^{2}\right)}\left[A_{D}^{2} \cdot \frac{1}{\pi D} \cdot\left\langle a_{2, D} a_{2, d}\right\rangle-A_{d}^{2} \cdot \frac{1}{\pi d} \cdot\left\langle a_{2, d}^{2}\right\rangle\right] . \tag{7.12}
\end{equation*}
$$

In Figure 7.12 is illustrated the link budget for a 8 cm waist size at emission as a function of the $D_{T T S}$ to $D_{T_{X}}$ ratio in Figure 7.8c. The annular configuration does not permit to fulfill the link budget. Moreover, the link budget decreases very fast. The tilt improvement for $D_{T T S} \leq 1.6 D_{T_{X}}$ showed in Figure 7.11 is insufficient. The link budget is closed to being fulfilled for $D_{T T S} \approx D_{T_{X}}$. However, such a system is practically impossible because not enough light would reach the wave-front sensor.


Figure 7.11: Residual tilt variance compared to the initial uncompensated tilt for a 8 cm waist size at emission as a function of the $D_{T T S}$ to $D_{T_{X}}$ ratio.


Figure 7.12: Link budget for a 8 cm waist size at emission as a function of the $D_{T T S}$ to $D_{T_{X}}$ ratio.

In conclusion, neither of the considered bi-static configurations permit to fulfill the link budget. This means that a mono-static configuration is mandatory. Moreover, the
most effective mono-static configuration is for $D_{T T S}=2 w_{0}$, which will be the considered configuration in the rest of the study.

## Effects of the beam truncation

We study here the effects of beam truncation. First, in Figure 7.13, the effects of beam truncature by the telescope on the link budget are shown in a case without turbulence. These results highlight the fact that a telescope diameter $D_{T_{X}}=2 w_{0}$ induces an almost 4 dB loss on the link budget compared to cases where the telescope diameter is significantly larger than the waist size (i.e. $D_{T_{X}}=5 w_{0}$ in Figure 7.13). Moreover, as could intuitively be suspected, an asymptote appears. Indeed there is only approximately a 0.5 to 1 dB difference between $D_{T_{X}}=3 w_{0}$ and $D_{T_{X}}=5 w_{0}$, for the different considered waist sizes. This justifies the value usually taken in the literature: $D_{T_{X}}=2^{\frac{3}{2}} w_{0} \approx 2.8 w_{0}$ [42].


Figure 7.13: Effect of the emitter truncature on the optical link budget for different waist sizes without atmospheric turbulence.


Figure 7.14: Effect of the emitter truncature on the optical link budget for different waist sizes with atmospheric turbulence.

When we add atmospheric turbulence while considering perfect tracking, the results are different, as can be seen in Figure 7.14. It appears that there is an optimum truncature for
each considered waist size. Moreover, the difference between $D_{T_{X}}=2 w_{0}$ and $D_{T_{X}}=5 w_{0}$ is much smaller as there is only a 2 dB difference for a 4 cm waist size and for a 12 cm waist size. $D_{T_{X}}=2 w_{0}$ even gives a slightly better results on the link budget than $D_{T_{X}}=5 w_{0}$ for a 12 cm waist size. We attribute this to the fact that, while a smaller truncature decreases the mean irradiance (as can be seen in Figure 7.13), the fact that is leads to a bigger beam radius in the far field which decreases the impact of atmospheric turbulence on irradiance fluctuations.

Despite the fact that there is an optimum diameter for each waist, we will consider $D_{T_{X}}=2^{\frac{3}{2}} w_{0}$ in the rest of the sizing.

## Initial sizing conclusion

From this initial sizing study, we can conclude that tilt tracking is mandatory and enables to fulfill the link budget. The configuration architecture needs to be mono-static with the tip/tilt measured over a pupil $D_{T T S}=2 w_{0}$ and bi-static configurations may not be envisioned as they do not permit to fulfill the link budget. Finally, we have chosen a reasonable beam truncation $D_{T_{X}}=2^{3 / 2} w_{0}$ which does not significantly reduce the performance compared to a no truncation case.

### 7.4 Sensitivity study

### 7.4.1 Anisoplanatism induced by the point-ahead angle

We now add point-ahead angle in the simulation. This means that tip/tilt precompensation is no longer perfect but residual tip/tilt will appear due to the anisoplanatism effects from the point-ahead angle. Figure 7.15 presents the residual tilt in the presence of point-ahead angle. The residual tilt is much lower than our reference in Figure 7.3. Figure 7.16 shows the results on the link budget. The impact of point-head angle on the link budget is small, as it is less than 0.5 dB . In the rest of the study, the point-ahead angle will always be taken into account in the tracking.

### 7.4.2 Delay before correction

There will be a delay between the measurement of the wavefront distortion on the downlink and the application of the precompensation on the uplink. Figure 7.18 shows the evolution of the link budget as a function of the delay between the wavefront distortion measurement and the correction for waists of 5,8 and 12 cm and Figure 7.17 shows the evolution of the residual tilt variance compared to the initial uncompensated tilt. These Figures show that the delay between the wavefront measurement and the correction should be of the order of the millisecond.

While the limitations on the quality of the correction due to the point-ahead angle will always be present, the limitations due to the delay can easily be dealt with by increasing the bandwidth of the correction loop. In the case of a 8 cm waist size at emission with a 4 ms delay, the tilt correlation obtained from Chassat's functions is equal to 0.86 . This leads to a residual tilt variance of $0.03 \mathrm{rad}^{-2}$, which then leads to a standard deviation of the displacement in the satellite plane of 38 m . If the delay before the correction is zero,


Figure 7.15: Residual tilt variance over the uncompensated tilt variance as a function of the beam waist size at emission when taking into account the point-ahead angle in the tracking.


Figure 7.16: Evolution of the optical link budget as a function of the emitted waist sizes, either considering a point-ahead angle or not.


Figure 7.17: Residual tilt variance compared to the initial uncompensated tilt as a function of the delay between the wavefront distortion measurement and the correction for waists of 5,8 and 12 cm .


Figure 7.18: Link budget evolution as a function of the delay between the wavefront distortion measurement and the correction for waists of 5,8 and 12 cm .
then the tilt correlation becomes 0.9 , the residual tilt variance becomes $0.02 \mathrm{rad}^{-2}$ and the standard deviation of the displacement is 33 m . Compared to the beam radius in the satellite plane, equal to 254 m , this 5 m displacement error is negligible, which means that there is no need for a lower delay time.

### 7.4.3 Sensitivity of the sizing to atmospheric turbulence conditions

We have seen how the OGS architecture parameters impact the link budget. In this section, we study how propagation channel variations impact the link budget. Tip/tilt tracking with a $18.5 \mu \mathrm{rad}$ point-ahead angle is taken into account. The OGS is in a monostatic configuration $\left(D_{T T S}=d_{0}\right)$ and a 4 ms delay is considered.

## Ground layer impact

We start by studying the influence of the turbulence strength near the ground, which can greatly fluctuate depending on the time of the day, on the season or on the localization of the station. We compare the conditions from the Hufnagel-Valley $5 / 7$ profile, in which $C_{n}^{2}(0)=1.7 \times 10^{-14} \mathrm{~m}^{-\frac{2}{3}}$, from the stronger atmospheric profile presented in Section 6.2.2, in which $C_{n}^{2}(0)=5.4 \times 10^{-14} \mathrm{~m}^{-\frac{2}{3}}$, and from an even stronger atmospheric profile, in which $C_{n}^{2}(0)=5.4 \times 10^{-14} \mathrm{~m}^{-\frac{2}{3}}$. The results are presented in Figure 7.19:


Figure 7.19: Comparison of the link budget evolution as a function of the emitted waist in a case with tracking considering a ground turbulence strength $C_{n}^{2}(0)=1.7 \times 10^{-14} \mathrm{~m}^{-\frac{2}{3}}$, $C_{n}^{2}(0)=5.4 \times 10^{-14} \mathrm{~m}^{-\frac{2}{3}}$ and $C_{n}^{2}(0)=1 \times 10^{-13} \mathrm{~m}^{-\frac{2}{3}}$.

The impact of the ground layer on the system link budget is quite important. Considering an elevation angle $\phi_{\text {elev }}=40^{\circ}$, the Fried parameter is equal to $r_{0}=15 \mathrm{~cm}$ for $C_{n}^{2}(0)=1.7 \times 10^{-14} \mathrm{~m}^{-\frac{2}{3}}$, to $r_{0}=8 \mathrm{~cm}$ for $C_{n}^{2}(0)=5.4 \times 10^{-14} \mathrm{~m}^{-\frac{2}{3}}$ and to $r_{0}=6 \mathrm{~cm}$ for $C_{n}^{2}(0)=1 \times 10^{-13} \mathrm{~m}^{-\frac{2}{3}}$. For the strongest atmospheric profile, we are therefore outside the validity range of WPLOT for waist sizes $w_{0}>9 \mathrm{~cm}$. In terms of link budget, the gap is of the order of 2 to 3 dB between the weakest and strongest ground turbulent layer. In order to fulfill the link budget, even in the strongest cast, a waist size bigger than 7 cm must be considered, leading to a telescope diameter $D_{T_{X}}=20 \mathrm{~cm}$.

### 7.4 Sensitivity study

## Altitude layers impact

We are going to consider different atmospheric profiles, adding stronger turbulence around the 5 km and 10 km altitude layers to the reference profile (Figure 7.20 a ). These profiles are presented in Figure 7.20. In Table 7.3, we present the impact of taking into account more turbulent layers at high altitude on the isoplanatic angle and the Fried parameter.

| Atmospheric profile | IPA $(\mu \mathrm{rad})$ | $r_{0}(\mathrm{~cm})$ |
| :--- | :---: | :---: |
| Hufnagel-Valley profile with $C_{n}^{2}(0)=5.4 \times 10^{-14} \mathrm{~m}^{2 / 3}($ Figure <br> $7.20 \mathrm{a})$ | 13 | 8.3 |
| Peak at $C_{n}^{2}(10 \mathrm{~km})=1 \times 10^{-16} \mathrm{~m}^{2 / 3}($ Figure 7.20 b$)$ | 10 | 8.2 |
| Peak at $C_{n}^{2}(5 \mathrm{~km})=1 \times 10^{-16} \mathrm{~m}^{2 / 3}($ Figure 7.20 c$)$ | 12 | 8.2 |
| Peak at $C_{n}^{2}(5 \mathrm{~km})=3 \times 10^{-16} \mathrm{~m}^{2 / 3}($ Figure 7.20 d$)$ | 10 | 8 |
| Peaks at $C_{n}^{2}(5 \mathrm{~km})=1 \times 10^{-16} \mathrm{~m}^{2 / 3}$ and $C_{n}^{2}(10 \mathrm{~km})=1 \times$ <br> $10^{-16} \mathrm{~m}^{2 / 3}($ Figure 7.20 e$)$ | 9.5 | 8.1 |

Table 7.3: Table of the isoplanatic angle and Fried parameter obtained with each atmospheric profile.

The link budgets are obtained for each of these atmospheric profiles and are presented in Figure 7.21. We observe from these results that the loss of performance is more strongly correlated with the additional turbulence strength (i.e.the lower Fried parameters). In that regard, the worst result is obtained for the atmospheric profile with the higher peak at a 5 km altitude with a loss of around 1 dB on the link budget.

Comparing the results for the atmospheric profile with a peak at $C_{n}^{2}(5 \mathrm{~km})=1 \times$ $10^{-16} \mathrm{~m}^{2 / 3}$ with the atmospheric profile with a peak at $C_{n}^{2}(10 \mathrm{~km})=1 \times 10^{-16} \mathrm{~m}^{2 / 3}$ (both resulting with the same $r_{0}$ ), we can observe that the influence of the impact of the height of the stronger atmospheric layer is negligible, as there is a maximum difference of 0.1 dB between the two. This means that the correlation at the 5 km altitude is slightly stronger than at the 10 km altitude, but this effect is not significant.

Table 7.4 gives an estimation of the strongest layers (following the same approach as in Chapter 3.2.4). It shows that the relative impact of each added layer is quite important on the residual tilt. It also shows how the added layers impact the total residual tilt variance, which is what translates into the link budget.

## Outer scale impact

Finally, the effects of the outer scale on the optical link budget are evaluated. Without any correction, the outer scale plays a very important role on the pointing errors and a larger outer scale induces bigger pointing errors.

However, after correction, we see that the influence of the outer scale is very small, as can be seen in Figure 7.22. For small outer scales such as 1 m , there is a small improvement on the uplink optical budget. However, there is almost no difference for outer scales bigger than 3 m . The criteria that can be used is the $d_{0} / L_{0}$ ratio. The outer scale has no influence when it is inferior to 0.1 .

Chapter 7. Optical ground station sizing


(e) $C_{n}^{2}(5 \mathrm{~km})=1 \times 10^{-16} \mathrm{~m}^{2 / 3}$ and $C_{n}^{2}(10 \mathrm{~km})=$ $1 \times 10^{-16} \mathrm{~m}^{2 / 3}$

Figure 7.20: Considered atmospheric profiles for study of the impact of the high altitude layers on the link budget.

### 7.4 Sensitivity study



Figure 7.21: Comparison of the link budget evolution as a function of the emitted waist in a case with tracking considering the atmospheric profiles presented in Figure 7.20.

Table 7.4: Table describing the impact of each layer on the residual tilt after compensation considering the atmospheric profiles presented in Figure 7.20.

| Residual tilt for each layer / Total residual tilt [\%] |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Layer <br> number | Layer <br> altitude <br> $[\mathrm{km}]$ | Figure <br> 7.20 a | Figure <br> 7.20 b | Figure <br> 7.20 c | Figure <br> 7.20 d | Figure <br> 7.20 e |
| 1 | 152 | 2,49 | 1,57 | 1,84 | 1,15 | 1,28 |
| 2 | 1463 | 4,28 | 2,70 | 3,15 | 1,97 | 2,20 |
| 3 | 2456 | 5,70 | 3,60 | 4,20 | 2,63 | 2,93 |
| 4 | 3455 | 5,24 | 3,30 | 4,27 | 3,27 | 2,99 |
| 5 | 4465 | 4,26 | 2,69 | 14,63 | 25,49 | 10,23 |
| 6 | 5493 | 3,92 | 2,47 | 16,59 | 29,84 | 11,59 |
| 7 | 6516 | 4,68 | 2,95 | 4,20 | 3,70 | 2,93 |
| 8 | 7520 | 6,24 | 3,93 | 4,59 | 2,87 | 3,21 |
| 9 | 8514 | 7,90 | 5,78 | 5,82 | 3,64 | 4,71 |
| 10 | 9506 | 9,04 | 23,04 | 6,66 | 4,16 | 18,79 |
| 11 | 10498 | 9,34 | 23,80 | 6,87 | 4,30 | 19,41 |
| 12 | 11491 | 8,80 | 6,43 | 6,48 | 4,05 | 5,24 |
| 13 | 12485 | 7,67 | 4,84 | 5,64 | 3,53 | 3,95 |
| 14 | 13480 | 6,25 | 3,94 | 4,60 | 2,88 | 3,21 |
| 15 | 14476 | 4,80 | 3,03 | 3,54 | 2,21 | 2,47 |
| 16 | 15472 | 3,52 | 2,22 | 2,59 | 1,62 | 1,81 |
| 17 | 16469 | 2,46 | 1,55 | 1,81 | 1,13 | 1,27 |
| 18 | 17466 | 1,66 | 1,05 | 1,22 | 0,76 | 0,85 |
| 19 | 18463 | 1,08 | 0,68 | 0,80 | 0,50 | 0,56 |
| 20 | 19461 | 0,68 | 0,43 | 0,50 | 0,31 | 0,35 |
|  |  |  |  |  |  |  |
| Total residual tilt variance $\left[\mathrm{rad}^{2}\right]$ | 0,0250 | 0,0382 | 0.0332 | 0.0508 | 0.0470 |  |
|  |  |  |  |  |  |  |



Figure 7.22: Optical link budgets for different outer scales.

### 7.5 Conclusion on the sizing of the optical ground station

Using the WPLOT model for obtaining irradiance as a function of the optical ground station architecture and propagation channel, we have been able to identify important results for the optimization of a ground to space telescope. Tip/tilt correction using a fine pointing mirror has been confirmed as mandatory in order to reach the necessary powers for a functioning system. The telescope's truncature ratio induces some loss on the optical link budgets but with a sufficiently big telescope $\left(D_{T_{X}}=2^{3 / 2} w_{0}\right.$ seems to provide a good trade-off between compactness and performance), the losses become negligible. With good tip/tilt correction, increasing the waist at the emission leads to better performance as long as the beam isn't too deformed by atmospheric turbulence.

The most significant result is that the tip/tilt correction is not efficient enough with bi-statics optical ground station architecture (annular and off-axis configurations). On the other hand, tip/tilt correction with a monostatic configuration brings a gain compatible with the considered laser power. This is very important because it will have great impact on the OGS architecture. Using the same pupil for both emission and reception can lead to difficulties due to stray light problems (even though the emitted beam will not have the same polarization as the received one).

Turbulence profiles with stronger turbulence layers in altitude result in an increase of the BER and impossibility to reach the desired capacity due to anisoplanatism from the point-ahead angle. These effects lead to larger waist sizes to fulfill the link budget. Finally, after correction, we've shown that the outer scale does not have any impact on our system performance.

The sizing's optimization leads to a Gaussian beam of waist size equal to 8 cm . The telescope diameter will have a diameter equal to $D_{T_{X}}=2^{\frac{3}{2}} w_{0}=22.6 \mathrm{~cm}$. Only tilt tracking will be considered in a mono-static configuration. This is summarized in Table 7.5.

| Waist size | 8 cm |
| :---: | :--- |
| Telescope diameter | $D_{T_{X}}=22.6 \mathrm{~cm}\left(2^{\frac{3}{2}} w_{0}\right)$ |
| Architecture configuration | Mono-static configuration: tip/tilt measured over <br>  <br>  <br> $D_{T T S}=16 \mathrm{~cm}\left(2 w_{0}\right)$ |
| Delay before correction | 4 ms |
| Link Budget - $P_{R} @ 5 \% \mathrm{CDF}$ | $-40.7 \mathrm{dBm}>-43 \mathrm{dBm}$ for the stronger atmospheric pro- <br>  <br> file with $C_{n}^{2}(0)=5.4 \times 10^{-14} \mathrm{~m}^{-\frac{2}{3}}$. |

Table 7.5: Final architecture characteristics.

Chapter 7. Optical ground station sizing

## Model application to error correcting codes and interleaving sizing : time series creation

The objective of this Chapter is to provide a study of the temporal irradiance fluctuations in a tracking case. In the second part of this Chapter, we will present with an example the major advantage of using the WPLOT comparing to other works (such as those from Basu[65] for example) in that its ability to create time series permits to test the performance of interleaving coupled with error correcting codes.

## Contents

### 8.1 Obtaining irradiance time series <br> 121

8.2 Fade statistics description . . . . . . . . . . . . . . . . . . . . . . 122
8.3 WPLOT to test error correcting codes and interleaving . . . . 125
8.4 Conclusion . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 125

### 8.1 Obtaining irradiance time series

The WPLOT model relies on estimating Zernike coefficient time series, which can be obtained from Eq.8.1:

$$
\begin{equation*}
a_{i}(t)=\int \sqrt{W_{a_{i}}(f)} e^{i \phi_{n}} e^{2 i \pi f_{n} . t} d f_{n}, \tag{8.1}
\end{equation*}
$$

where $\phi_{n}$ is the phase randomly attributed to the spectral component at the frequency $f_{n}$ and $W_{a_{i}}(f)$ is the temporal PSD of the $a_{i}$ coefficient, estimated in Equation (2.34). If we wan to consider a tracking, we need to estimate the residual tilt temporal Power Spectral Density. The following phase-related quantity can be used[74]:

$$
\begin{equation*}
G_{j}=[\phi(r+d, t)-\phi(r, t)] * Z_{j}(r) \tag{8.2}
\end{equation*}
$$

Chapter 8. Model application to error correcting codes and interleaving sizing : time series creation

If the distance $\vec{d}$ between the two beams is along the $\vec{x}$ axis, and if the diameter of the beams are the same, the spectrum of $G_{j}$ is:

$$
\begin{equation*}
\left|M_{G_{j}}(f, h)\right|^{2}=4 \sin ^{2}\left(\pi d \frac{\nu}{V(h)}\right) \times\left|M_{a_{j}}(f)\right|^{2} \tag{8.3}
\end{equation*}
$$

$\left|M_{a_{j}}(f)\right|^{2}$ is obtained from Chapter 2.3.1. Time series of the residual tilt coefficient may then be obtained.

Once the irradiance time series are estimated, they can directly serve as inputs in the WPLOT model.

### 8.2 Fade statistics description

For describing the fade duration the ITU gives the following two measures based on the CDF of the fade duration (see [75, 76]):

1. Fade duration occurrence probability, that is, the probability of occurrence of fades of duration $d_{F}$, defined as the time interval between two crossings (down and up) of a threshold level, longer than a given duration threshold $D$ :

$$
\begin{equation*}
P\left(d_{F}>D\right)=\frac{N\left(d_{F} \mid d_{F}>D\right)}{N\left(d_{F}\right)} \tag{8.4}
\end{equation*}
$$

where $N(x)$ denotes the number of occurrences of $x$.
2. Cumulative fade duration exceedance probability, that is, the probability that, if a fade occurs, it has a duration $d_{F}>D$ :

$$
\begin{equation*}
F\left(d_{F}>D\right)=\frac{\sum_{i}\left(d_{F, i} \mid d_{F, i}>D\right)}{\sum_{i} d_{F, i}} \tag{8.5}
\end{equation*}
$$

The two given measures can be used to describe the distribution of the fade durations, but they do not describe the occurrence of the fades. This is described by the probability of fade.

Using the WPLOT model, we can obtain time series for the final architecture from Table 7.5 for a Hufnagel-Valley atmospheric profile with $C_{n}^{2}(0)=5.4 \times 10^{-14} \mathrm{~m}^{-2 / 3}$ and an elevation angle $\phi_{\text {elev }}=40^{\circ}$. A time series is presente in Figure 8.1 in which we can observe the improvement of taking into account the tip/tilt tracking. Using very long time series, i.e. having access to a high number of fade occurrences, we can obtain the fade statistics results presented in Figure 8.2 in a tracking case.

From the irradiance time series obtained in Figure 8.1, the evolution of the BER is obtained. The BER is estimated for each sample over a duration of $500 \mu \mathrm{~s}$, assuming that the irradiance is constant over this time period. The BER as a function of time for the same irradiance time series as Figure 8.1 is presented in Figure 8.3. We observe that, as intended, periods during which the received power is below -43 dBm correspond to a BER higher than $10^{-3}$. This is obtained from the BER curve in Figure 8.4 ${ }^{1}$.

[^10]

Figure 8.1: Irradiance time series over 1 s with a 2000 Hz sampling using the final architecture from Table 7.5 in Chapter 7 for a Hufnagel-Valley atmospheric profile with $C_{n}^{2}(0)=5.4 \times 10^{-14} \mathrm{~m}^{-2 / 3}$ and an elevation $\phi_{\text {elev }}=40^{\circ}$.


Figure 8.2: Fade duration occurrence and exceedance probability plots for the final architecture from Table 7.5 in Chapter 7 for a Hufnagel-Valley atmospheric profile with $C_{n}^{2}(0)=5.4 \times 10^{-14} \mathrm{~m}^{-2 / 3}$ and an elevation $\phi_{\text {elev }}=40^{\circ}$ in a tracking case.

Chapter 8. Model application to error correcting codes and interleaving sizing : time series creation


Figure 8.3: BER time series over 1 s with a 2000 Hz sampling using the final architecture from Table 7.5 in Chapter 7 for a Hufnagel-Valley atmospheric profile with $C_{n}^{2}(0)=5.4 \times$ $10^{-14} \mathrm{~m}^{-2 / 3}$ and an elevation $\phi_{\text {elev }}=40^{\circ}$.


Figure 8.4: BER curve for the detection architecture presented in Chapter 1 and Appendix A.

### 8.3 WPLOT to test error correcting codes and interleaving

At the satellite receiver side, the electrical signal $y_{k}$ after photo detection at the input of the receiver decoder is:

$$
\begin{equation*}
y_{k}(n)=h(n) \times x_{k}(n)+n_{k}(n) \tag{8.6}
\end{equation*}
$$

Where:

- $h$ is the channel power attenuation
- $x$ is the mean electrical power function of the emitted bit k and the optical link budget
- $n$ is a random value representing the noise power. It is function of the emitted bit k , the channel power attenuation, the optical transmission chain.

As the free space optical channel is a slow fading channel (compared to optical symbols duration), interleaving is interesting to spread over time the bad channel conditions.

End-to-end simulations have been performed for a 10Gbit/s optical uplink between the Earth and a geostationary satellite. The ground emitter is based on a 22 cm diameter with on-axis tracking $\left(D_{T T S}=2 w_{0}\right)$ and a 50 W optical amplifier. The satellite receiver is 20 cm large with a pre-amplified optical receiver and NRZ-OOK demodulator.

First results have highlighted the required trade-off between physical layer channel code ratio and the interleaver size (and latency associated). With physical layer channel code ratio of 0.5 , limited interleaver size of $100-500 \mathrm{Mbit}$ seems to be accessible. Greater interleaver will imply greater latency that could be detrimental to the service.

### 8.4 Conclusion

The WPLOT model is able to provide time series of irradiance fluctuations. This means that we are able to describe the irradiance fluctuations through the fade duration occurrence probability and the cumulative fade duration exceedance probability. Finally, end-to-end simulations have been performed for a $10 \mathrm{Gbit} / \mathrm{s}$ optical uplink between the Earth and a geostationary satellite, and have permitted to obtain primary results on the sizing of interleaving and error correcting codes. The most suitable error correcting code (interleaver included) schemes will be the next hot topic of the optical communication community. WPLOT should be an interesting model for such an activity.

Chapter 8. Model application to error correcting codes and interleaving sizing : time series creation

## Conclusion

## Summary

The work presented in this manuscript has focused on the atmospheric turbulence effects and mitigation techniques on ground to space optical links performance.

In the Chapter 1 of this thesis, after a brief presentation of past optical links demonstrations, I have presented our description of the optical link through the link budget. This description has led to the definition of the loss term relating to the impact of atmospheric turbulence $L_{T U R B}$. In particular, I have shown that in order to determine this loss term, a precise knowledge of the irradiance fluctuations, especially in the lowest percents of the cumulative density functions (CDF), is required. The link budget enables the sizing of an optical ground station architecture. However, in order to test the performances of forward error correcting codes, I need to obtain time series of irradiance fluctuations.

In Chapter 2, I presented the state of the art of optical propagation through turbulent media. I distinguished three regimes: the weak turbulence regime in which irradiance fluctuations are modeled from scintillation resulting from the Rytov approximation, the beam wander regime in which beam deflection from the optical axis due to beam wander accounts for the majority of irradiance fluctuations and finally the strong fluctuations regime in which the beam losses its coherence and breaks into multiple speckles. I also introduced the Zernike polynomials to describe the phase perturbations. Finally, I presented wave optics simulations (such as TURANDOT), which allow for precise estimation of irradiance fluctuations at the expanse of computation time and which I have considered as our reference for estimating the precision of analytic models. In Chapter 3, I introduced the different envisioned mitigation techniques, such as diversity techniques and adaptive optics, and how they are modeled in the literature.

The objective of Chapter 5 was to compare the irradiance fluctuations obtained with TURANDOT and with the analytic models proposed in the literature and listed in Chapter 2. I have observed that the analytic models presented did not allow for a good estimation of atmospheric turbulence particularly on the lower irradiances when tip/tilt tracking is considered.

This has led us to consider, in Chapter 6, another approach which assumes that the irradiance fluctuations resulted from beam wandering as well as from beam deformations such as defocus and astigmatism, as suggested by Baker [3]. Particularly, I have improved

Chapter 8. Model application to error correcting codes and interleaving sizing : time series creation
the modeling of the impact of defocus by taking into account the propagation through multiple atmospheric layers and not using only a single layer at ground level. Additionally, in order to be able to model an optical ground station, I have presented how to model the impact of the telescope truncation on the Gaussian beam. I have named this model the With Propagation Low Order of Turbulence model (WPLOT).

I have compared the irradiance fluctuations obtained with the WPLOT model with those obtained with TURANDOT. For a weak turbulence case, modeled by an HufnagelValley $5 / 7 C_{n}^{2}$ profile, the precision of the WPLOT model were convincing as there was less than 1 dB difference for the estimation of the turbulence losses on the link budget. For a stronger atmospheric profile near ground, the error was of the order of 1 to 2 dB in the case with tracking, which is sufficient even though the model limitations started to appear. In particular, the effects of beam spreading are not well taken into account, despite our modeling of defocus, which leads to the mean irradiances obtained with the WPLOT model being usually higher than with TURANDOT.

The WPLOT model has enabled us to do a sizing of an optical ground station architecture (OGS) in Chapter 7. In particular, I have been able to study the impact of numerous parameters of the OGS on the link budget (such as the waist, telescope truncation, pointahead angle, delay between measure and correction application, aperture mismatch,etc) as well as the impact of the propagation channel (impact of a stronger ground $C_{n}^{2}$, stronger atmospheric layers at different altitudes and outer scale). This has led to the definition of an optimal OGS providing a trade-off between size/complexity and performance. The final architecture is reminded in Table 8.1.

| Waist size | 8 cm |
| :---: | :--- |
| Telescope diameter | $D_{T_{X}}=2^{\frac{3}{2}} w_{0}=22.6 \mathrm{~cm}$ |
| Architecture configuration | Mono-static configuration: TTS pupil merged with beam <br> with $D_{T T S}=d_{0}$ |
| Delay before correction | $\leq 4 \mathrm{~ms}$ |

Table 8.1: Final architecture characteristics.

Finally, tests on the performance of interleaving coupled with forward error correcting codes have been performed in Chapter 8 in order to estimate the capacity achievable with the chosen architecture.

## Perspectives

The WPLOT model provides a description of irradiance fluctuations resulting from beam propagation through atmospheric turbulence. This description can be either statistical, through the PDFs and CDFs, as well as temporal, by providing irradiance time series.

However, the model relies on assumptions which lead to an underestimation of the impact of atmospheric turbulence effects on irradiance fluctuations. In particular, the WPLOT model only relies on tilt, defocus and astigmatism (i.e. the 5 first Zernike polynomials, neglecting piston). Because tilt accounts for approximately $90 \%$ of the phase variance[33], the irradiance fluctuations are usually well modeled in a case without tilt

### 8.4 Conclusion

tracking (cf Section 6.2.3) for $w_{0}<r_{0}$. However, I have observed an underestimation of irradiance fluctuations in a tracked case. It can be assumed that this is the result of neglecting the higher orders of Zernike polynomials which will have an impact on beam deformations at satellite level.

Another limitation of the WPLOT model is that, while it has showed that tilt tracking was necessary but also sufficient in order to obtain the targeted performance in the considered cases, it does not allow the estimation of gains of using adaptive optics and correcting the higher orders of Zernike polynomials. Correcting higher orders of Zernike polynomials can be envisioned to further reduce irradiance fluctuations and their impact on the sizing of error correcting codes and interleaving. Moreover, in the stronger atmospheric turbulence conditions profile I have considered, the ground $C_{n}^{2}(0)=5.4 \times 10^{-14} \mathrm{~m}^{-2 / 3}$ is at $60 \%$ of the CDF of the ground $C_{n}^{2}(0)$ (i.e. $\mathrm{P}\left(C_{n}^{2}(0)<5.4 \times 10^{-14} \mathrm{~m}^{-2 / 3}\right)=0.6$ ) [68]. The objective for the availability of the feeder link is $99.9 \%$. For possible stronger ground $C_{n}^{2}(0)$, adaptive optics can be envisioned to reduce irradiance fluctuations. Indeed, using the chosen architecture after the sizing presented in Table 8.1 with a point-ahead angle of $18.5 \mu \mathrm{rad}$ and the stronger ground atmopsheric profile presented in Section 6.2 .2 (leading to $r_{0}=8 \mathrm{~cm}$ ) with a Bufton wind profile, Chassat's correlation functions from Equation (C.10) in Appendix C show that the Zernike polynomials with a radial degree $n \leq 6$ can be precompensated in order to mitigate the impact of irradiance fluctuations in Figure 8.6 (adaptive optics lead to an improvement as long as the correlation of the considered Zernike polynomials is above $50 \%$ ), except for the $16^{t h}$ Zernike polynomial (cf. Figure 8.5). The study of the impact of adaptive optics correction will have to be done with other tools than the WPLOT model, such as TURANDOT or through lab demonstrations (this will be the objective of the FEEDELIO project done at ONERA for the European Space Agency).


Figure 8.5: Correlation of the 45 first Zernike coefficients for the stronger atmospheric profile and the architecture from Table 8.1.


Figure 8.6: Correlation presented as a function of the radial degree for the stronger atmospheric profile and the architecture from Table 8.1.

In terms of sizing the optical ground station, I have considered that the wave front sensor perfectly measured the wave front arriving from the downlink and that the deformable mirror perfectly pre-compensated the uplink. However, because of the very low powers of

Chapter 8. Model application to error correcting codes and interleaving sizing : time series creation
the received downlink (of the order of a few nW ), there can be errors on the measurement of the wave front sensor. Moreover, because we have showed that a mono-static configuration was mandatory, there can be problems due to stray light despite the fact that downlink and the uplink will have opposite circular polarizations.

Finally, finding the most suitable error correcting codes (interleaving included) will be the next hot topic in the optical communication community. In order to size and test these, the IRT Saint Exupéry has developed an optical link communication testbed named ELLA. It emulates the propagation through atmospheric turbulence channel by using a variable attenuation emulator. The irradiance time series obtained with the WPLOT model will be used to pilot the variable attenuator. Later, the objective will be to substitute the variable attenuator with a true free-space optical testbed also being developed at the IRT. This testbed will be using a Digital Micromirror Device (DMD) to code the effects of atmospheric turbulence. Representativeness with respects to GEO feeder links will be key, taking into account the effects of anisoplanatism from the point-ahead angle, compared to other existing lab and terrain demonstrations[77, 78].

## Appendix

## Detection architecture noise estimation

The following results are a summary of [79].
The electric current is given by

$$
\begin{equation*}
I_{E}=\Re\left(\left|\sqrt{G} E_{s i g}+E_{A S E, c p}\right|^{2}+\left|E_{A S E, o p}\right|^{2}\right)+i_{s}+i_{T}, \tag{A.1}
\end{equation*}
$$

where $E_{\text {sig }}$ is the field of the detected optical signal, $E_{A S E}$ is the field due to the ASE (Amplified Spontaneous Emission).cp indicates that the ASE field is polarized in the same direction as the signal field and op means that the polarization is orthogonal. $i_{s}$ and $i_{T}$ are the current fluctuations induced by the shot noise and the thermal noise of the photodiode.

The shot and thermal noise are neglected for the moment. Equation (A.1) becomes:

$$
\begin{align*}
I_{E}=\Re\left(G\left|E_{s i g}\right|^{2} \pm 2 \sqrt{G}\left|E_{s i g}\right|\left|E_{A S E, c p}\right|+\left|E_{A S E, c p}\right|^{2}\right. & \left.+\left|E_{A S E, o p}\right|^{2}\right) \\
& =i_{s i g}+i_{S \times A S E}+i_{A S E \times A S E} \tag{A.2}
\end{align*}
$$

in which,

$$
\begin{equation*}
i_{s i g}=\Re G\left|E_{s i g}\right|^{2} \tag{A.3}
\end{equation*}
$$

is the current resulting from the optical signal, and

$$
\begin{equation*}
i_{S \times A S E}=2 \Re \sqrt{G}\left|E_{s i g}\right|\left|E_{A S E, c p}\right|, \tag{A.4}
\end{equation*}
$$

and

$$
\begin{equation*}
i_{A S E \times A S E}=\Re\left(\left|E_{A S E, c p}\right|^{2}+\left|E_{A S E, o p}\right|^{2}\right)=2 \Re\left|E_{A S E, c p}\right|^{2}=2 \Re\left|E_{A S E, o p}\right|^{2} \tag{A.5}
\end{equation*}
$$

The photodetector being essentially a square-law detector of electric fields, the received signal field will mix, leading to a beating phenomenon, with the components of the ASE noise field that are polarized in the same detection as the signal, this is $i_{S \times A S E}$ (signal-cross-ASE-noise). The various components in the ASE noise field will also mix with themselves, giving $i_{A S E \times A S E}$ (ASE-cross-ASE-noise). The powers $\left|E_{A S E, o p}\right|^{2}$ and $\left|E_{A S E, c p}\right|^{2}$ are given using the power spectral density of the ASE $S_{A S E}$.

$$
\begin{equation*}
\left|E_{A S E, c p}\right|^{2}=\left|E_{A S E, o p}\right|^{2}=S_{A S E} B_{O} \tag{A.6}
\end{equation*}
$$

$S_{A S E}$ is considered constant (as it is assumed to be a white noise) and depends on the gain and the spontaneous emission factor $n_{s p}$ :

$$
\begin{equation*}
S_{A S E}=n_{s p}(G-1) \cdot h \cdot \nu_{c} \tag{A.7}
\end{equation*}
$$

Where $h$ is the Planck constant and $\nu_{c}$ is the carrier frequency $\left(\nu_{c}=193.1 T H z\right.$ for a $1.55 \mu \mathrm{~m}$ wavelength). $n_{s p}$ is the spontaneous emission factor defined by

$$
\begin{equation*}
n_{s p}=\frac{N_{2}}{N_{2}-N_{1}} \tag{A.8}
\end{equation*}
$$

Where $N_{1}$ and $N_{2}$ are the atomic populations at the low and excited states, respectively. The spontaneous emission factor is related to the amplifier's noise figure:

$$
\begin{equation*}
N F=10 \log _{10}\left[\frac{2 n_{s p}(G-1)}{G}+\frac{1}{G}\right] \approx 10 \log _{10}\left[2 n_{s p}\right] \quad \text { for large gain } . \tag{A.9}
\end{equation*}
$$

By showing the DSPs of the beats SxASE and ASExASE, i.e., passing through to the frequency domain, they result in convolutions, as can be seen in the Figure A.1, showing the evolution of the two frequency components of the signal and the noise of the preamplified optical receiver. The convolutions occur at the level of the photodetector.


Figure A.1: Effects of each component of the detection architecture on the temporal power spectral densities of the signal and ASE fields leading to the power spectral densities of the currents SxASE and ASExASE

The power spectral densities of the currents SxASE and ASExASE are

$$
P S D_{i_{S \times A S E}}(f)=4 \Re^{2} G P_{R} S_{A S E} \begin{cases}1 & \text { if } f \leq B_{O} / 2  \tag{A.10}\\ 0 & \text { if } f>B_{O} / 2\end{cases}
$$

$$
P S D_{i_{A S E \times A S E}}(f)=4 \Re^{2} S_{A S E}^{2} \begin{cases}B_{O}-f & \text { if } f \leq B_{O} / 2  \tag{A.11}\\ 0 & \text { if } f>B_{O} / 2\end{cases}
$$

These results are given with the frequency $f>0$. These above spectral densities are considered just before the electrical low-pass filter. By integrating on the passband of the electric filter and assuming $B_{E}<B_{0}$ (which is generally the case), we obtain:

$$
\begin{gather*}
\sigma_{S \times A S E}^{2}=4 \Re^{2} G P_{R} S_{A S E} B_{E}  \tag{A.12}\\
\sigma_{A S E \times A S E}^{2}=2 \Re^{2} S_{A S E}^{2} B_{E}\left(2 B_{O}-B_{E}\right) \tag{A.13}
\end{gather*}
$$

\section*{| Appendix |
| :--- |}

## Zernike polynomials

Zernike polynomials [80] define an orthonormal basis on the unit circle and are generally used to describe the wavefront in terms of optical aberrations. In the coordinate system $(r, \theta)$, these polynomials are the result of the product of functions of $\rho$ with functions of $\theta$ :

$$
Z_{n}^{m}(r, \theta)= \begin{cases}\sqrt{\left(2 \frac{n+1}{1+\delta_{m, 0}}\right)} \cdot R_{n}^{m} \cdot(r) \cos (m \theta) & \text { if } m \geq 0  \tag{B.1}\\ \sqrt{2 \cdot(n+1)} \cdot R_{n}^{|m|}(r) \sin (m \theta) & \text { if } m<0\end{cases}
$$

Where $\sqrt{2 \frac{n+1}{1+\delta_{m, 0}}}$ and $\sqrt{2(n+1)}$ are normalization constants and the polynomials $R_{n}^{m}(r), n$ is an integer and $m \in Z$, are given by:

$$
\begin{equation*}
R_{n}^{m}(r)=\sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^{k}(n-k)!}{k!\left(\frac{n+m}{2}-k\right)!\left(\frac{n-m}{2}-k\right)!} r^{n-2 k} \text {, with } n \geq|m| \text { and } n-|m| \text { even } \tag{B.2}
\end{equation*}
$$

$n$ is called the radial degree. $m$ is called the azimuthal degree and takes values going from $-n$ to $n$ verifying that $(n-m)$ is even. Figure and Table show and describe the first 15 Zernike Polynomials.


Figure B.1: The first 15 Zernike polynomials, ordered vertically by radial degree and horizontally by azimuthal degree.

Table B.1: The first 15 Zernike polynomials, their Noll index, radial degree, azimuthal degree and classical name.

| Noll in$\operatorname{dex}(j)$ | Radial degree ( $n$ ) | Azimutha degree ( $m$ ) | $1 \quad Z_{j}$ | Classical name |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 1 | Piston |
| 2 | 1 | 1 | $2 \rho \cos \theta$ | Tip (X-Tilt) |
| 3 | 1 | -1 | $2 \rho \sin \theta$ | Tilt (Y-Tilt) |
| 4 | 2 | 0 | $\sqrt{3}\left(2 \rho^{2}-1\right)$ | Defocus |
| 5 | 2 | -2 | $\sqrt{6} \rho^{2} \sin 2 \theta$ | Oblique astigmatism |
| 6 | 2 | 2 | $\sqrt{6} \rho^{2} \cos 2 \theta$ | Vertical astigmatism |
| 7 | 3 | -1 | $\sqrt{8}\left(3 \rho^{3}-2 \rho\right) \sin \theta$ | Vertical coma |
| 8 | 3 | 1 | $\sqrt{8}\left(3 \rho^{3}-2 \rho\right) \cos \theta$ | Horizontal coma |
| 9 | 3 | -3 | $\sqrt{8} \rho^{3} \sin 3 \theta$ | Vertical trefoil |
| 10 | 3 | 3 | $\sqrt{8} \rho^{3} \cos 3 \theta$ | Oblique trefoil |
| 11 | 4 | 0 | $\sqrt{5}\left(6 \rho^{4}-6 \rho^{2}+1\right)$ | Primary spherical |
| 12 | 4 | 2 | $\sqrt{10}\left(4 \rho^{4}-2 \rho^{2}\right) \cos 2 \theta$ | Vertical secondary astigmatism |
| 13 | 4 | -2 | $\sqrt{10}\left(4 \rho^{4}-2 \rho^{2}\right) \sin 2 \theta$ | Oblique secondary astigmatism |
| 14 | 4 | 4 | $\sqrt{10} \rho^{4} \cos 4 \theta$ | Vertical quadrafoil |
| 15 | 4 | -4 | $\sqrt{10} \rho^{4} \sin 4 \theta$ | Oblique quadrafoil |

## Appendix

## Chassat's correlation functions

One of the most interesting works to characterize correlation between two beams has been made by F. Chassat during his $\mathrm{PhD}[27]$. Chassat's idea was to calculate the covariance functions between the coefficients of Zernike polynomials describing two different wavefronts:

$$
\begin{equation*}
\phi^{(i)}\left(R_{i} \rho\right)=\sum_{j=2}^{\infty} a_{j}^{(i)} Z_{j}(\rho), i=1,2 \tag{C.1}
\end{equation*}
$$

Where $i$ describes the considered beam, $a_{j}$ is the coefficient associated to the Zernike polynomial $Z_{j}$. Each wavefront (from beam (1) and (2)) is therefore described by all the coefficients $\left(a_{j_{1}}^{(1)}\right)_{j_{1}=2, \infty}$ and $\left(a_{j_{2}}^{(2)}\right)_{j_{2}=2, \infty} . R_{i}$ describes the radius of the $i$ beam at the ground. For two of these coefficients (not belonging to the decomposition of the same wavefront), their covariance will be mathematically noted $C_{j_{1} j_{2}}$ and defined as:

$$
\begin{equation*}
C_{j_{1} j_{2}}=\left\langle\left(a_{j_{1}}^{(1)}-\left\langle a_{j_{1}}^{(1)}\right\rangle\right)\left(a_{j_{2}}-\left\langle a_{j_{2}}^{(2)}\right\rangle\right)\right\rangle \tag{C.2}
\end{equation*}
$$

The functions $\phi^{(i)}(R \rho)$ represent the differences to an unperturbed wavefront and thus their means is equal to zero because the turbulence is supposed stationary. Therefore, we can deduce that $\left\langle a_{j}^{(i)}\right\rangle=0$ and thus:

$$
\begin{equation*}
C_{j_{1} j_{2}}=\left\langle a_{j_{1}}^{(1)} \cdot a_{j_{2}}^{(2)}\right\rangle \tag{C.3}
\end{equation*}
$$

Chassat has shown that the covariance between the Zernike coefficients between two beams $C_{j_{1} j_{2}}$ is equal to:

$$
\begin{align*}
& C_{j_{1} j_{2}}\left(d_{1 \rightarrow 2}^{\prime}(h), R_{1}(h), R_{2}(h), C_{n}(h), L_{0}(h)\right)= \\
&  \tag{C.4}\\
& \left(\frac{D_{1}}{r_{0}^{(1)}}\right)^{\frac{5}{3}} \frac{\int_{0}^{L} d h C_{n}^{2}(h) R_{1}^{\frac{5}{3}} \sigma_{j_{1} j_{2}}\left(\frac{d_{1 \rightarrow 2}^{\prime}(h)}{R_{1}(h)}, \omega_{2,1}(h), \frac{R_{1}(h)}{L_{0}(h)}\right)}{\int_{0}^{L} d h C_{n}^{2}(h) R_{1}^{\frac{5}{3}}(h)}
\end{align*}
$$

Where:

$$
\begin{gather*}
d_{1 \rightarrow 2}^{\prime}(h)=d_{1 \rightarrow 2}(h)+v(h) \tau_{1 \rightarrow 2}  \tag{C.5}\\
\omega_{2,1}(h)=\frac{R_{2}(h)}{R_{1}(h)}  \tag{C.6}\\
\sigma_{j_{1} j_{2}}(\xi, \zeta, \Lambda)=3.895(-1)^{\left(n_{1}+n_{2}-m_{1}-m_{2}\right) / 2} \sqrt{\left(n_{1}+1\right)\left(n_{2}+1\right)} \\
\times\left[\frac{K_{j_{1} j_{2}}^{1}(\Theta)}{\zeta} \int_{0}^{\infty} d x \cdot x^{-\frac{14}{3}} J_{n_{1}+1}(x) J_{n_{2}+1}(\zeta x) J_{m_{1}+m_{2}}(\xi x)\left(1+\left(\frac{\Lambda}{x}\right)^{2}\right)^{\frac{-11}{6}}\right. \\
\left.+\frac{K_{j_{1} j_{2}}^{2}(\Theta)}{\zeta} \int_{0}^{\infty} d x \cdot x^{\frac{-14}{3}} J_{n_{1}+1}(x) J_{n_{2}+1}(\zeta x) J_{m_{1}+m_{2}}(\xi x)\left(1+\left(\frac{\Lambda}{x}\right)^{2}\right)^{\frac{-11}{6}}\right]  \tag{C.7}\\
D_{1}=2 * R_{1}(0) \tag{C.8}
\end{gather*}
$$

Where $n_{i}$ and $m_{i}$ are respectively the radial degree and the azimuthal frequency. $J_{k}$ is the Bessel function of the $k^{t h}$ order. $\Theta$ is the angle defined in each of pupils $P^{(i)}$ between the line resulting from the intersection of the pupil and the plane defined by the two optical axes and the axis taken as origin of $\theta$ in the coordinate system $(\rho, \theta)$ defined for the Zernike polynomials (usually, it is possible to have $\Theta=0$ ). The term $K_{j_{1} j_{2}}^{1}$ and $K_{j_{1} j_{2}}^{2}$ are given by Table C. 1 and Table C.2:

Table C.1: Definition of $K_{j_{1} j_{2}}^{1}$ as a function of $j_{1}$ and $j_{2}$.

| $K_{j_{1} j_{2}}^{1}$ |  | $m_{1}=0$ | $m_{1} \neq 0$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $j_{1}$ even | $j_{1}$ odd |
| $m_{2}=0$ |  |  | 1 | $\sqrt{2} \cos m_{1} \Theta$ | $\sqrt{2} \sin m_{1} \Theta$ |
| $m_{2} \neq 0$ | $j_{2}$ even | $\begin{aligned} & (-1)^{m_{2}} \sqrt{2} \\ & \times \cos m_{2} \Theta \end{aligned}$ | $\begin{gathered} (-1)^{m_{2}} \times \\ \cos \left(m_{1}+m_{2}\right) \Theta \end{gathered}$ | $\begin{gathered} (-1)^{m_{2}} \times \\ \sin \left(m_{1}+m_{2}\right) \Theta \end{gathered}$ |
|  | $j_{2}$ odd | $\begin{aligned} & (-1)^{m_{2}} \sqrt{2} \\ & \times \sin m_{2} \Theta \end{aligned}$ | $\begin{gathered} (-1)^{m_{2}} \times \\ \sin \left(m_{1}+m_{2}\right) \Theta \end{gathered}$ | $\begin{gathered} (-1)^{m_{2}} \times \\ \cos \left(m_{1}+m_{2}\right) \Theta \end{gathered}$ |

To compute the covariance $C_{j_{1} j_{2}}$, it is usually discretized along the propagation path and the function $\sigma_{j_{1} j_{2}}$ is computed at each considered altitude. The value $d_{1 \rightarrow 2}(h)$ is defined in Figure 3.4:

Table C.2: Definition of $K_{j_{1} j_{2}}^{2}$ as a function of $j_{1}$ and $j_{2}$. $\left(s_{j_{1} j_{2}}=\operatorname{sign}\left(m_{1}-m_{2}\right)\right.$ if $m_{1}-m_{2}$ is odd, else $s_{j_{1} j_{2}}=1$.)

| $K^{2} K_{j_{1} j_{2}}^{2}$ | $m_{1}=0$ | $m_{1} \neq 0$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $j_{1}$ even | $j_{1}$ odd |  |
| $m_{2}=0$ |  | 0 | 0 | 0 |
| $m_{2} \neq 0$ | $j_{2}$ even | 0 | $s_{j_{1} j_{2}} \cos \left(\left(m_{1}-m_{2}\right) \Theta\right)$ | $s_{j_{1} j_{2}} \sin \left(\left(m_{1}-m_{2}\right) \Theta\right)$ |
|  | $j_{2}$ odd | 0 | $-s_{j_{1} j_{2}} \sin \left(\left(m_{1}-m_{2}\right) \Theta\right)$ | $s_{j_{1} j_{2}} \cos \left(\left(m_{1}-m_{2}\right) \Theta\right)$ |



Figure C.1: Definition of $d_{1 \rightarrow 2}(h)$, the distance between the centers of the two considered beams at an altitude $h[27]$.

Finally, $v(h)$ is the wind profile and $\tau_{1 \rightarrow 2}$ is the time between when the instant the downlink crosses the layer and the instant the uplink crosses the layer back.

The ponderation functions $\sigma_{j_{1} j_{2}}(\xi, \zeta, \Lambda)$ are key to compute the covariance functions. The parameters are equal to:

$$
\begin{equation*}
\xi=\frac{d_{1 \rightarrow 2}^{\prime}(h)}{R_{1}}, \quad \zeta=\omega_{2,1}(h)=\frac{R_{2}(h)}{R_{1}(h)} \quad \text { and } \quad \Lambda=\frac{R_{1}(h)}{L_{0}(h)} \tag{C.9}
\end{equation*}
$$

$\xi$ represents the spatial deviation between the two beam paths, $\zeta$ the geometrical differences between the two beams and $\Lambda$ the effects of the outer scale.

Sensibility to the atmospheric profile As seen in Equation (C.4), the turbulence profile is normalized. This means that the influence of each layer in the computation of $C_{j_{1} j_{2}}$ doesn't depend on the absolute value $C_{n}^{2}(h)$ but on its relative value compared to the value of the other layers. Therefore, it depends solely on the atmospheric profile. The intensity of the total turbulence intervenes in the factor $\left(r_{0}^{(1)}\right)^{-\frac{5}{3}}$ and affects evenly the covariance values. Moreover, this ponderation is independent of the point-ahead angle. This leads Chassat to introduce the correlation function $\Gamma_{j_{1} j_{2}}$ which is independent from $r_{0}$ and defined by:

$$
\begin{align*}
& \Gamma_{j_{1} j_{2}}\left(d_{1 \rightarrow 2}^{\prime}(h), R_{1}(h), R_{2}(h),\right. \\
& \left.C_{n}(h), L_{0}(h)\right)=  \tag{C.10}\\
& \\
& \qquad \frac{C_{j_{1} j_{2}}\left(d_{1 \rightarrow 2}^{\prime}(h), R_{1}(h), R_{2}(h), C_{n}(h), L_{0}(h)\right)}{C_{j_{1} j_{2}}\left(0, R_{1}(h), R_{2}(h)=R_{1}(h), C_{n}(h), L_{0}(h)\right)}
\end{align*}
$$

Which can therefore be written:

$$
\begin{align*}
& \Gamma_{j_{1} j_{2}}\left(d_{1 \rightarrow 2}^{\prime}(h), R_{1}(h), R_{2}(h), C_{n}(h), L_{0}(h)\right)= \\
& \frac{\int_{0}^{L} d h C_{n}^{2}(h) R_{1}^{\frac{5}{3}} \sigma_{j_{1} j_{2}}\left(\frac{d_{1 \rightarrow 2}^{\prime}(h)}{R_{1}(h)}, \omega_{2,1}(h), \frac{R_{1}(h)}{L_{0}(h)}\right)}{\int_{0}^{L} d h C_{n}^{2}(h) R_{1}^{\frac{5}{3}} \sigma_{j_{1} j_{2}}\left(0,1, \frac{R_{1}(h)}{L_{0}(h)}\right)} \tag{C.11}
\end{align*}
$$

Equations (C.10) and (C.11) are in the case of merged apertures of same diameter at ground level. In the case of separate apertures, the correlation function is:

$$
\begin{align*}
& \Gamma_{j_{1} j_{2}}\left(d_{1 \rightarrow 2}^{\prime}(h), R_{1}(h), R_{2}(h), C_{n}(h), L_{0}(h)\right)= \\
& \frac{C_{j_{1} j_{2}}\left(d_{1 \rightarrow 2}^{\prime}(h), R_{1}(h), R_{2}(h), C_{n}(h), L_{0}(h)\right)}{\sqrt{C_{j_{1} j_{2}}\left(0, R_{1}(h), R_{1}(h), C_{n}(h), L_{0}(h)\right)} \times \sqrt{C_{j_{1} j_{2}}\left(0, R_{2}(h), R_{2}(h), C_{n}(h), L_{0}(h)\right)}} \tag{C.12}
\end{align*}
$$

Sensibility to Zernike modes Chassat computes the variance of the Zernike coefficients in the case of a single aperture and no point-ahead angle by taking $\xi=0$ and $\zeta=1$. From Equation (C.7) and taking into account that $J_{k \neq 0}(0)=0$, it is possible to deduce that the covariance will be different from 0 only for coefficients for which $m_{1}-m_{2}=0$. Knowing that $J_{0}(0)=1$, it is possible to obtain:

$$
\begin{align*}
& \sigma_{j_{1} j_{2}}(0,1, \Lambda)=3,895(-1)^{\left(n_{1}+n_{2}-2 m\right) / 2} \sqrt{\left(n_{1}+1\right)\left(n_{2}+1\right)} \\
& \times\left(\int_{0}^{\infty} d x \cdot x^{\frac{-14}{3}} J_{n_{1}+1}(x) J_{n_{2}+1}(x)\left(1+\left(\frac{\Lambda}{x}\right)^{2}\right)^{\frac{-11}{6}}\right) \tag{C.13}
\end{align*}
$$

The outer scale will not be taken into account and will be considered infinite. Thus, $\Lambda=0$ and:

$$
\begin{align*}
\sigma_{j_{1} j_{2}}(0,1,0)=2,256(-1)^{\left(n_{1}+n_{2}-2 m\right) / 2} & \sqrt{\left(n_{1}+1\right)\left(n_{2}+1\right)} \\
\times & \Gamma\left[\begin{array}{l}
\frac{n_{1}+n_{2}}{2}-\frac{5}{6} \\
\frac{n_{1}+n_{2}}{2}+\frac{23}{6}, \frac{n_{2}-n_{1}}{2}+\frac{17}{6}, \frac{n_{1}-n_{2}}{2}+\frac{17}{6}
\end{array}\right] \tag{C.14}
\end{align*}
$$

And for the same coefficient $j$ :

$$
\sigma_{j j}(0,1,0)=0.76(n+1) \times \Gamma\left[\begin{array}{l}
n-\frac{5}{6}  \tag{C.15}\\
n+\frac{23}{6}
\end{array}\right]
$$

Where $\Gamma$ here represents the "Euler gamma" function and the notation used is equivalent to:

$$
\Gamma\left[\begin{array}{l}
x_{1}, x_{2}, \ldots, x_{m}  \tag{C.16}\\
y_{1}, y_{2}, \ldots, y_{n}
\end{array}\right]=\frac{\Gamma\left(x_{1}\right) \times \Gamma\left(x_{2}\right) \times \ldots \times \Gamma\left(x_{m}\right)}{\Gamma\left(y_{1}\right) \times \Gamma\left(y_{2}\right) \times \ldots \times \Gamma\left(y_{n}\right)}
$$

In Figure C. 2 are plotted the Zernike coefficients variance. It shows that the lower Zernike coefficients have a much bigger variance and are therefore more interesting to compensate.


Figure C.2: Zernike coefficients variance for a wavefront perturbed by a Kolmogorov Turbulence (infinite outer scale)[27].

Sensibility to the parameters of atmospheric turbulence (inner scale, outer scale) We will now take into account the effect of the outer scale on the Zernike coefficients variances. To do this, Chassat stays in the case of a single aperture and no point-ahead angle. The general result is:

$$
\begin{align*}
& \sigma_{j j}\left(0,1, \frac{D}{2 L_{0}}\right)=1.168(n+1) \\
& \times\left(\sum_{p=0}^{\infty} \frac{(-1)^{p}}{p!}\left(\frac{D}{2 L_{0}}\right)^{2 p+2 n-\frac{5}{3}} \Gamma\left[\begin{array}{l}
p+n+\frac{3}{2},-p-n+\frac{5}{6}, p+n+1 \\
p+2 n+3, p+n+2
\end{array}\right]\right. \\
&+\sum_{p=0}^{\infty} \frac{(-1)^{p}}{p!}\left(\frac{D}{2 L_{0}}\right)^{2 p} \Gamma\left[\begin{array}{ll}
-p+n-\frac{5}{6}, & p+\frac{7}{3}, \\
p+n+\frac{23}{6}, & p+\frac{17}{6}
\end{array}\right] \tag{C.17}
\end{align*}
$$

Next are approximations:

- For Zernike polynomials with a radial degree $n=1$ (tip/tilt):

$$
\begin{align*}
\sigma_{j j}\left(0,1, \frac{D}{2 L_{0}}\right)= & 0.451 \\
& \times\left(1-0,77\left(\frac{D}{L_{0}}\right)^{\frac{1}{3}}+0,09\left(\frac{D}{L_{0}}\right)^{2}-0.054\left(\frac{D}{L_{0}}\right)^{\frac{7}{3}}\right) \tag{C.18}
\end{align*}
$$

- For Zernike polynomials with a radial degree $n=2$ :

$$
\begin{equation*}
\sigma_{j j}\left(0,1, \frac{D}{2 L_{0}}\right)=2.34 \times 10^{-2}\left(1-0,39\left(\frac{D}{L_{0}}\right)^{2}+0.27\left(\frac{D}{L_{0}}\right)^{\frac{7}{3}}\right) \tag{C.19}
\end{equation*}
$$

- For Zernike polynomials with a radial degree $n \geq 3$ :

$$
\begin{align*}
\sigma_{j j}\left(0,1, \frac{D}{2 L_{0}}\right)=0.756 & (n+1) \\
& \times \Gamma\left[\begin{array}{l}
n-\frac{5}{6} \\
n+\frac{23}{6}
\end{array}\right]\left(1-\frac{0,38}{\left(n-\frac{11}{6}\right)\left(n+\frac{23}{6}\right)}\left(\frac{D}{L_{0}}\right)^{2}\right) \tag{C.20}
\end{align*}
$$

Figure C. 3 presents the dependence of $\sigma_{j j}\left(0,1, \frac{D}{2 L_{0}}\right)$ for the polynomials $j$ with a radial degree $n \leq 4$ in function of $D / L_{0}$. When decreasing the outer scale $L_{0}$, the variances of the Zernike coefficient also decrease. This decrease is particularly important for the tip/tilt. This therefore means that a smaller outer scale is equivalent to a lower tip/tilt variance and thus reduces the impact of beam wander.


Figure C.3: Dependence of $\sigma_{j j}\left(0,1, \frac{D}{2 L_{0}}\right)$ for the polynomials $j$ with a radial degree $n \leq 4$ in function of $D / L_{0}[27]$.

## Appendix

## Scintillation index formulas

We present here the expressions derived by Andrews [39]. The expression for the on-axis scintillation is:

$$
\begin{align*}
\sigma_{I}^{2}(0, L)=8 \pi^{2} k_{0}^{2} \int_{0}^{L} \int_{0}^{\infty} & \kappa W_{n}(z, \kappa) \exp \left[-\frac{\Lambda L \kappa^{2}}{k_{0}}\left(\frac{L-z}{L}\right)^{2}\right] \\
& \times\left(1-\cos \left(\frac{L \kappa^{2}}{k_{0}}\left(\frac{L-z}{L}\right)\left[\Theta+\frac{(1-\Theta) z}{L}\right]\right)\right) d \kappa d z \tag{D.1}
\end{align*}
$$

where $\Lambda$ and $\Theta$ are parameters of the laser beam:

$$
\begin{equation*}
\Lambda=\frac{2 L}{k_{0} w^{2}(L)}, \Theta=\left[1+\left(\frac{L}{z_{R}}\right)^{2}\right]^{-1} \tag{D.2}
\end{equation*}
$$

assuming that the beam is initially collimated.
The scintillation term that depends on the distance from the optical axis in the horizontal plane of the satellite, $r$, is:

$$
\begin{align*}
\sigma_{I, r}^{2}(r, L)=8 \pi^{2} k_{0}^{2} \int_{0}^{L} \int_{0}^{\infty} \kappa W_{n}(z, \kappa) \exp & {\left[-\frac{\Lambda L \kappa^{2}}{k_{0}}\left(\frac{L-z}{L}\right)^{2}\right] } \\
& \times\left(I_{0}\left[2 \Lambda \kappa r\left(\frac{L-z}{L}\right)\right]-1\right) d \kappa d z \tag{D.3}
\end{align*}
$$

where $I_{0}$ is, here, a modified Bessel function of the first class and zero order.

## ${ }_{\text {Appendix }} \square$

## Résumé en Français

Les travaux présentés dans ce manuscrit ont porté sur la compensation des effets de la turbulence atmosphérique sur un lien allant du sol vers un satellite géostationnaire.

Dans le Chapitre 1, après une brève présentation des démonstrations de liens optiques réalisées, j'ai présenté une description du lien optique à partir du bilan de liaison. Le critère de performance de la liaison est la puissance minimale détectable $95 \%$ du temps. Cette description m'a amené à définir le terme de pertes dû à la turbulence atmosphérique $L_{T U R B}$. Pour déterminer ce terme, une connaissance précise de des basses puissances est nécessaire, en particulier dans les bas pourcentages de la fonction de répartition (ou CDF, pour Cumulative Density Function). Le bilan de liaison sera utile pour réaliser le dimensionnement d'une station sol. De plus, pour être capables de tester les performances des codes correcteurs d'erreurs, l'obtention de séries temporelles se montre indispensable.

Dans le Chapitre 2, j'ai présenté l'état de l'art de la propagation d'un faisceau optique à travers un milieu turbulent. J'ai distingué trois régimes: celui de la faible turbulence où les fluctuations de puissances détectées sont modélisées à partir de la scintillation résultant de l'approximation de Rytov, le régime dit «beam wander » où les déviations du faisceau de l'axe optique à cause de la réfraction du faisceau dans la turbulence entrainent la majeur partie des fluctuations de la puissance détectée et enfin, le régime de forte turbulence où le faisceau perd sa cohérence spatiale et se casse en de multiples speckles. J'y ai également introduit les polynômes de Zernike pour décrire les perturbations de phase. Enfin, j'ai présenté la méthode de propagation de Fresnel, utilisée dans des logiciels de simulation tels que TURANDOT (développé par l'ONERA). Cette méthode permet une estimation précise des fluctuations de la puissance détectée au prix de longs temps de calculs. J'ai considéré cette méthode comme la référence lorsque j'ai voulu valider les modèles analytiques. Dans le Chapitre 3 , j'ai introduit les différents moyens de compensation envisagés, comme les techniques de diversité et l'optique adaptative.

L'objectif du Chapitre 5 a été de comparer les fluctuations de puissance obtenues avec TURANDOT et le modèle analytiques existants dans la littérature présentés dans le Chapitre 2. Cela m'a permis de montrer que ces modèles analytiques n'estimaient pas les effets de la turbulence atmosphérique sur la puissance détectée avec une précision suffisante, particulièrement dans des cas avec prise en compte de la pré-compensation du tip/tilt.

Cela nous a conduit à considérer, au Chapitre 6, à utiliser une méthode qui prend en compte à la fois des déviations du faisceau ainsi que des déformations du faisceau sous l'effet du defocus et de l'astigmatisme, comme suggérer par Baker[3]. J'ai amélioré l'estimation de l'impact du terme de defocus en prenant en compte la propagation à travers de multiples couches et non pas seulement une seule couche au sol. J'ai également pris en compte les effets liés à la troncature du faisceau par le télescope de la station sol. J'ai nommé ce modèle WPLOT (pour With Propagation Low Order of Turbulence).

Ce modèle a été validé en le comparant avec TURANDOT. Dans un cas de turbulence jour faible, modélisé par un profil de $C_{n}^{2}$ Hufnagel-Valley $5 / 7$, nous avons observé moins de 1 dB d'écart pour l'estimation de la puissance minimale détectable $95 \%$ du temps, que ce soit avec ou sans prise en compte de la pré-compensation du tip/tilt. Pour un profil atmosphérique avec une couche au sol plus forte, l'erreur était de l'ordre de 1 à 2 dB dans le cas avec pré-compensation du tip/tilt, ce qui suffit malgré le fait que les limitations du modèle commencent à apparaitre. Les effets de l'étalement de la tâche, malgré notre prise en compte du defocus, sont sous-estimés, ce qui signifie que les puissances moyennes obtenues avec WPLOT sont souvent supérieures à celles obtenues avec TURANDOT.

WPLOT nous a permis de réaliser une étude de dimensionnement de l'architecture dans le Chapitre 7. Cela m'a permis d'étudier l'impact de nombreux paramètres de la station sol optique sur le bilan de liaison (comme le waist du faisceau, la troncature du faisceau, la configuration de la compensation, etc) ainsi que l'impact du canal de propagation (en fonction de la force du $C_{n}^{2}$ au sol, des couches plus fortes en altitude et de l'échelle externe). Cela nous a conduit à la définition de l'architecture d'une station sol optimale issue d'un compromis entre la taille/complexité et la performance. L'architecture finale retenue est présentée dans le Tableau E.1.

| Taille du waist | $w_{0}=8 \mathrm{~cm}$ |
| :---: | :--- |
| Diamètre du télescope | $D_{T_{X}}=2^{\frac{3}{2}} w_{0}=22.6 \mathrm{~cm}$ |
| Configuration de la correction | Configuration mono-statique : la pupille de l'analyseur <br> de front d'onde TTS est confondue $D_{T T S}=2 w_{0}$ |
| Délai avant la correction | $\leq 4 \mathrm{~ms}$ |

Table E.1: Caractéristiques de l'architecture retenue.
Enfin, des séries temporelles ont étés obtenues pour cette configuration dans le Chapitre 8, ce qui a permis de faire des tests de performances des codes correcteurs d'erreur couplés à un entrelaceur. Cette thématique sera le prochain sujet d'intérêt dans la communauté des télécommunications optiques. Le modèle WPLOT devrait être un outil très intéressant dans ce cadre.

## Appendix

## Publications list

## Peer-reviewed article

A-R. Camboulives, M-T. Velluet, S. Poulenard, L. Saint-Antonin and V. Michau, "Statistical and temporal irradiance fluctuations modeling for ground-to-geostationary satellite optical link," Appl. Opt. 57, 709-721 (2018)

## Conference proceedings

A-R. Camboulives, M-T. Velluet, S. Poulenard, L. Saint-Antonin, V. Michau, "Optical ground station optimization for future optical geostationary satellite feeder uplinks", Proc. SPIE 10096, Free-Space Laser Communication and Atmospheric Propagation XXIX, 1009608 (24 February 2017)

# Statistical and temporal irradiance fluctuations modeling for a ground-to-geostationary satellite optical link 

A.-R. Camboulives, ${ }^{1,2, *}$ (1) M.-T. Velluet, ${ }^{2}$ S. Poulenard, ${ }^{3}$ L. Saint-Antonin, ${ }^{1,3}$ and V. Michau ${ }^{2}$<br>${ }^{1}$ IRT Saint-Exupery, 118 Route de Narbonne, Toulouse, France<br>${ }^{2}$ ONERA-DOTA, 29 Avenue de la Division Leclerc, Châtillon, France<br>${ }^{3}$ Airbus Defence and Space, 31 Rue des Cosmonautes, Toulouse, France<br>*Corresponding author: adrien-richard.camboulives@irt-saintexupery.com

Received 18 September 2017; revised 19 December 2017; accepted 19 December 2017; posted 20 December 2017 (Doc. ID 305508); published 25 January 2018


#### Abstract

An optical communication link performance between the ground and a geostationary satellite can be impaired by scintillation, beam wandering, and beam spreading due to its propagation through atmospheric turbulence. These effects on the link performance can be mitigated by tracking and error correction codes coupled with interleaving. Precise numerical tools capable of describing the irradiance fluctuations statistically and of creating an irradiance time series are needed to characterize the benefits of these techniques and optimize them. The wave optics propagation methods have proven their capability of modeling the effects of atmospheric turbulence on a beam, but these are known to be computationally intensive. We present an analytical-numerical model which provides good results on the probability density functions of irradiance fluctuations as well as a time series with an important saving of time and computational resources. © 2018 Optical Society of America


OCIS codes: (010.1080) Active or adaptive optics; (010.1300) Atmospheric propagation; (010.3310) Laser beam transmission; (060.2605) Free-space optical communication.
https://doi.org/10.1364/AO.57.000709

## 1. INTRODUCTION

An optical link based on a multiplex of wavelengths around the $1.55-\mu \mathrm{m}$ spectral band is foreseen to be a valuable alternative to the conventional radio-frequencies feeder links for nextgeneration broadband geostationary satellites, targeting a capacity of around 1 Tbps . In addition to cloud obstruction, one of the major limitations to optical links is the presence of atmospheric turbulence during the first 20 km of the propagation. In this paper, we will focus on the ground-to-satellite link, or uplink.

During its propagation from a ground station to a geostationary satellite, the optical beam is deflected (beam wandering) and possibly distorted (beam spreading) by atmospheric turbulence. It induces strong fluctuations of the detected telecommunication signal, thus increasing the bit error rate (BER). To correct these effects, the beam characteristics need to be modified at the emission (pre-compensation). The envisaged technique is adaptive optics ( AO ) in which a servo system modifies in real time the emitted wavefront in order to make it recover a plane waveform when reaching the satellite, using the reciprocity principle as envisioned by Fried and Yura [1]. To do so, the beam coming from the satellite will be used to estimate the perturbations that need to be applied to the emitted wavefront.

However, there is a point-ahead angle between the downlink and the uplink in order for the uplink to intercept the moving satellite at the position where it will be when the pulse arrives. This means that the turbulence effects experienced by the downlink and the uplink are slightly different, leading to only partial compensation. Other effects also have an impact on the quality of the compensation, such as the optical ground station architecture through the beam size and aperture mismatch.

Because of the turbulence-induced irradiance fluctuations (the power detected by the satellite's terminal), the signal modulation detection and the link budget are disturbed. To evaluate the degradation, we need to estimate the probability density function and time series of the received signal. Wave optics simulations (in this paper, we will use TURANDOT [2]) permit us to obtain very precise results for the irradiance fluctuations statistics as well as time series. However, they require a lot of computational power due to the requirements on the phase screens' sampling and sizing in the case of a time series [3]. This leads to long-duration simulations. Wave optics simulations are therefore not practical for realizing sensitivity studies with many parameters of the optical ground station and propagation channel or a long time series. Hence, there
is a need to obtain similar results to wave optics simulations while requiring less computational power.

There have been many studies aiming to give a thorough description of the irradiance fluctuations on a ground-tosatellite propagation. These studies' objectives are usually to present analytical expressions for the scintillation index (the normalized irradiance variance), the irradiance probability density functions, and even for the probability of fades ([4,5]). These studies usually rely on the Rytov perturbation method (RPM) [6]. However, discrepancies appear for ground-to-space paths that are usually attributed to beam wander $[7,8]$. Moreover, the temporal aspects of the irradiance fluctuations are usually described by the number of fades per second and the mean fade time. With these analytical approaches, it is not possible to create irradiance series and test the performance of error correcting codes. In order to obtain irradiance time series, we have relied on the existing work regarding the impact of the temporal effects of atmospheric turbulence on beam propagation ([9]).

The objective of this paper is to present a model that performs time series of irradiance fluctuations as detected by the satellite, with and without the tracking system. In order to do so, we will base our work on an existing model from the literature that gives statistical results of irradiance fluctuations in the case of a beam propagating from the ground to a geostationary satellite [10]. This model is called the low order of turbulence (LOT) model. We have compared the LOT model's results (cumulative density functions and temporal power spectral density functions) with wave optics simulations to discuss its validity. We have demonstrated that the LOT model is not as accurate as we need due to its lack of modeling the propagation of a beam through multiple phase screens. We propose a method to improve and expand it in order to take into account the tracking and obtain time series of irradiance fluctuations. The final model is called the WPLOT model (with propagation low order of turbulence model). All the models to which we will refer throughout the paper are described in Appendix A.

The WPLOT model will depend on the parameters of the optical ground station and of the propagation channel and will take into account the inherent errors within tracking. It should permit us to optimize many parameters of an optical ground station architecture. The parameters required to evaluate the turbulence effect on the link performance are introduced in Section 2. In Section 3, the developed model is described. Results obtained with this model are compared with those obtained with wave optics simulations in Section 4. Finally, in Section 5, we present results when tracking is taken into account.

## 2. MODEL REQUIREMENTS

Thanks to the optical link budget, an estimation of the general performance of the system can be assessed by estimating the received power detected by the satellite. On the other hand, knowledge of the temporal irradiance fluctuations will permit us to study the viability of envisioned error correction codes coupled with interleaving.

## A. Optical Link Budget

The optical link budget gives an estimation of the received power $P_{R}$ as a function of the emitted power $P_{E}$, taking
into account all losses during the beam propagation: $P_{R}=$ $G_{R} L_{\text {TURB }} L_{\text {OTHERS }} G_{E} P_{E}$. $L_{\text {TURB }}$ are the losses induced by the irradiance fluctuations due to turbulence. $L_{\text {OTHERS }}$ are those induced by other contributors such as transmission losses through the emitter and receiver, atmospheric absorption and scattering losses, cloud margin, fiber injection, and the free-space losses. $G_{E}$ and $G_{R}$ are the gains at the emission and reception, respectively, taking into account the diameters of the telescopes at the emission and on the satellite (and the waist size at the emission).

Atmospheric turbulence is a random phenomenon and thus implies that detected irradiance fluctuations are random as well. A statistical approach is therefore considered, focusing on one quantity: the irradiance threshold $I_{T}$ defined by estimating the probability $P\left(I>I_{T}\right)=0.95$, where $I$ is the instantaneous detected irradiance. Finding $I_{T}$ provides the loss term $L_{\text {TURB }}$ due to atmospheric turbulence at a $5 \%$ probability of the cumulative distribution function. The other losses ( $L_{\text {OTHERS }}, L_{F S}$ ) in the optical link budget are set and static. The gains $G_{E}$ and $G_{R}$ can be easily determined [11].

In order to find the irradiance threshold $I_{T}$, we need to have access to the irradiance cumulative density function and thus to the irradiance probability function.

## B. Temporal Fluctuations

The irradiance threshold is defined at a 5\% probability of the cumulative distribution function. This means that the instantaneous irradiance will spend $5 \%$ of the time below the threshold. During these periods, interleaving (coupled with the error correction codes) will permit us to mitigate the losses and significantly improve the BER. In order to optimize the codes, there is a need for realistic irradiance time series in order to estimate the probability density functions of the fading times, the number of fadings per second, etc. These statistics will be obtained with long time series of irradiance.

In conclusion, the model we aim to develop must be able to provide not only the cumulative density functions of the irradiance fluctuations but also be able to generate irradiance time series.

## 3. MODEL DESCRIPTION

We will consider a ground-to-satellite optical link that corresponds to a weak turbulence regime, for which the scintillation index $\left(\sigma_{I}^{2}=\frac{\left\langle I^{2}\right\rangle-\left\langle\left. I\right|^{2}\right.}{\langle I\rangle^{2}}\right)$ is lower than 1. Baker describes different regions of distinct behavior for Gaussian beam on-axis weak scintillation [10]. In one region, named $D_{1}$, the Rytov perturbation method (RPM) fails to give an accurate description of the beam-wave scintillation [12]. Baker proposes to bound this region using Eq. (B4) with the metrics $N_{L}$ and $N_{\tau}$ defined, respectively, in Eqs. (B1) and (B2) in Appendix B. These bounds imply that the majority of the turbulence is in the near field of the beam while the satellite is in the far field. Due to the distance between the ground and the geostationary satellite ( $36,000 \mathrm{~km}$ ), the bound on $N_{L}$ is never predominant over the bound on $N_{\tau}$, which therefore bounds the minimum waist size $w_{0}$ at the emission as a function of the turbulence strength distribution along the path.

## A. Low-Order Turbulence Solution

Baker proposes to describe the irradiance fluctuations by estimating the deformations of a Gaussian beam propagating through the atmosphere with the first- and second-order Zernike polynomials $[13,14]$ : tip/tilt, defocus, and astigmatism. This is the LOT solution. The LOT's major results are recalled in Appendix B. This solution is equivalent to integrating the refractive index fluctuations present along the path into a single phase screen placed at the transmitter (Fig. 1).

## 1. Effects of Propagation and LOT Solution

The LOT solution assumes that the turbulence can be integrated in a single phase screen placed at the transmitter prior to propagation. This assumes that the effects of the propagation through multiple atmospheric layers are negligible inside the turbulence volume. In order to validate this assumption, we have compared in Fig. 2 the detected irradiance results obtained with the LOT solution with results obtained after Fresnel propagation through phase screens evenly distributed along the propagation path. The phase screens are constructed as a linear combination of tip, tilt, defocus, and astigmatism. The Zernike coefficients are obtained from the Noll variances in Eqs. (C1) and (C2) for the atmospheric parameters


Fig. 1. Presentation of the LOT model. This model is equivalent to a single phase screen resulting from tilt, defocus, and astigmatism placed in the emission plane of the beam.


Fig. 2. Random draws of irradiance statistics between LOT model (one layer) and the 2L-WO model for an emitted waist of 8 cm . In this case, $N_{L}=3.6 \times 10^{-4}$ and $N_{\text {tau }}=9.4$, well within the $D_{1}$ region.
presented in Section 4. For this study, we will consider a Gaussian beam with a waist size $w_{0}=8 \mathrm{~cm}$. This leads to the metrics $N_{L}=3.6 \times 10^{-4}<1$ and $N_{\text {tau }}=9.4\left(N_{\text {tau }}^{2} \gg 1\right)$ being well within the $D_{1}$ region.

To illustrate the effects of the propagation through multiple phase screens, we will consider only a two-layer atmosphere. The first layer is placed at the transmitter while the second layer is at a $10-\mathrm{km}$ altitude. This will be called the two-layer wave optics (2L-WO) simulation. The objective of the $2 \mathrm{~L}-\mathrm{WO}$ simulation is to highlight the effects of the propagation through multiple atmospheric layers using the hypothesis of the LOT model that tip/tilt, defocus, and astigmatism are sufficient to describe the beam deformations resulting from atmospheric turbulence in the $D_{1}$ region. Using only two layers is sufficient to make these effects appear.

In order to keep with the different used models in this paper, Table 1 summarizes them with their specificities in Appendix A.

We observe, in Fig. 2, a good correlation of the results even though some differences appear. Studying the effects of each optical aberration separately, we can see that for tip/tilt and astigmatism there is a near-perfect fit whether one or more layers are used. However, differences appear for the defocus aberration, as can be seen in Fig. 3.

An irradiance threshold appears for the LOT model compared to the 2L-WO simulation. This is because the focusing effects are limited by diffraction when using only one phase screen. The volume effects from propagating through multiple phase screens lead to a more focused beam than when using only one phase screen.

Figure 4 compares the probability density functions (PDF) of irradiance for cases where only defocus is applied at 1,2 , or 10 layers. It shows that using only one layer greatly reduces the dynamic of irradiance fluctuations: the PDF for only one layer is narrower than for multiple layers. However, the differences between 2 or 10 layers are not significant. This means that at least two atmospheric layers should be used.

In order to take into account the propagation through multiple phase screens, we propose to use the $A B C D$ matrix propagation method [4], where the $A B C D$ matrix is obtained from

$$
\left(\begin{array}{ll}
A & B  \tag{1}\\
C & D
\end{array}\right)=\left(\begin{array}{cc}
1 & L_{\text {prop }} \\
0 & 1
\end{array}\right) \prod_{i}^{N}\left(\begin{array}{cc}
1 & 0 \\
\Delta \kappa_{i} & 1
\end{array}\right)\left(\begin{array}{cc}
1 & \Delta z_{i} \\
0 & 1
\end{array}\right)
$$

where $\Delta \kappa_{i}$ is the curvature (in meters ${ }^{-1}$ ) resulting from defocus at the $i^{\text {th }}$ layer, $\Delta z_{i}$ is the distance between two successive phase screens $i$ and $i+1$, and $N$ is the total number of considered layers. $L_{\text {prop }}$ is the remaining distance to the satellite: $L_{\text {prop }}=L-\sum_{i}^{N} \Delta z_{i}$, where $L$ is the distance between the ground and the satellite ( $L=36,000 \mathrm{~km}$ ). Usually $\sum_{i}^{N} \Delta z_{i}$ is of the order of 20 km .

Consider the following initial complex radius of curvatures:

$$
\begin{align*}
& \frac{1}{q_{x 0}}=\left(c_{5}^{2}+c_{6}^{2}\right)^{\frac{1}{2}}+\frac{\lambda i}{\pi w_{0}^{2}}  \tag{2}\\
& \frac{1}{q_{y 0}}=-\left(c_{5}^{2}+c_{6}^{2}\right)^{\frac{1}{2}}+\frac{\lambda i}{\pi w_{0}^{2}} \tag{3}
\end{align*}
$$

Table 1. Description of the Models

| Model | Phase Screens Distribution | Propagation Method |
| :--- | :---: | :---: |
| LOT | One phase screen obtained from the linear combination of tip/tilt, <br> defocus, and astigmatism. It is placed at transmitter. <br> Multiple phase screens obtained from the linear combination <br> of tip/tilt, defocus, and astigmatism. They are distributed | Analytic propagation of Gaussian beam <br> $\left(w_{x}\right.$ and $w_{y}$ estimation). |
| WPLOT | Analytic propagation of defocus leading to |  |
|  | another estimation of $w_{x}$ and $w_{y}$. |  |

Fresnel propagation between the phase screens and up to the satellite.
Fresnel propagation between the phase screens and up to the satellite.


Fig. 3. Random draws of irradiance statistics between LOT solution and $2 \mathrm{~L}-\mathrm{WO}$ model when considering only defocus for an emitted waist of 8 cm . In this case, $N_{L}=3.6 \times 10^{-4}$ and $N_{\text {tau }}=9.4$, well within the $D_{1}$ region.
where $w_{0}$ is the waist size at emission, $\lambda$ is the wavelength, and $c_{5,6}$ are the curvatures resulting from astigmatism [10].

Using the $A B C D$ matrix, the complex radius curvatures after propagation through multiple phase screens are

$$
\begin{equation*}
q_{x}=\frac{A q_{x 0}+B}{C q_{x 0}+D} \tag{4}
\end{equation*}
$$

We present the results in the $x$ direction, but the method is exactly the same in the $y$ direction. This leads to a modified estimation of the beam radii $w_{x}$ and $w_{y}$ compared to Baker's approach [10],


Fig. 4. PDF of irradiance statistics between LOT solution, $2 \mathrm{~L}-\mathrm{WO}$ model, and a 10L-WO model when considering only defocus for an emitted waist of 8 cm .

$$
\begin{equation*}
w_{(x)}=\sqrt{\frac{\lambda}{\pi \mathfrak{S}\left(1 / q_{(x)}\right)}} \tag{5}
\end{equation*}
$$

where $\mathfrak{J}$ means the imaginary part. These beam radii are implemented in the LOT solution presented in Eq. (B5). This permits us to obtain the same irradiance estimation as wave optics propagation through multiple screens when only defocus is considered. We call this model WPLOT (for with propagation low order of turbulence).

A comparison between the WPLOT model and the 2L-WO simulation is given in Fig. 5. The WPLOT model considers the same two layers as the $2 \mathrm{~L}-\mathrm{WO}$ simulation. We observe that the higher irradiances are much better modeled. For lower irradiances-which are the most interesting ones-the results were already convincing. It is because these lower irradiances are principally due to beam wandering. The remaining errors are due to neglecting the effects of propagation through multiple phase screens for astigmatism. For tilt, the contributions of each layer can be summed while keeping a perfect fit in the far field regardless of the number of atmospheric layers. This approach taking into account the propagation through multiple phase screens does not significantly change the estimation of the irradiance threshold at $5 \%$ of the cumulative density functions (CDFs) in the case without tracking. However, it will significantly improve the validity of the results when tracking is taken into account.

## 2. LOT Applicability

The LOT solution does not take into account Zernike polynomials higher than astigmatism. Using Noll's results for phase variance [15] for a plane wave, the phase variance corresponding to higher orders is


Fig. 5. Random draws of irradiance statistics between WPLOT model (for two layers) and the 2L-WO model for an emitted waist of 8 cm .

$$
\begin{equation*}
\sigma_{\delta \phi}(j>6)=0.065\left(\frac{d_{0}}{r_{0}}\right)^{\frac{5}{3}}, \tag{6}
\end{equation*}
$$

where $r_{0}$ is the Fried parameter [16], $d_{0}=2 w_{0}$, and $j$ is the number of the considered Zernike polynomial (in our case, all the polynomials of higher order than astigmatism). Statistical equivalence between the model and a case where the whole phase is taken into account is achieved when there is less than 0.1 wave squared difference (within a circle of radius $w_{0}$ ) between the two [10]:

$$
\begin{equation*}
\sigma_{\delta \phi}(j>6) \leq\left(\frac{2 \pi}{10}\right)^{2} \tag{7}
\end{equation*}
$$

This adds a constraint on the maximum waist size of the beam:

$$
\begin{equation*}
w_{0} \leq 1.5 r_{0} . \tag{8}
\end{equation*}
$$

This condition ensures that the higher-order $(j>6)$ Zernike coefficients' impact on the beam are negligible.

## B. Telescope Truncation-Far-Field Angle

The model takes into account the parameters of the optical ground station. In particular, one of those parameters is the truncation of the infinite Gaussian beam by the telescope. The truncation will have two effects. The first one is that it will reduce the transmitted power by the telescope [11]. The second one is that it will add a diffraction effect in the far field. The effects of the diffraction of a Gaussian beam by a circular aperture usually lead to a convolution whose result is not easy to use. Because we consider telescope sizes that will always be larger than the Gaussian beam diameter at emission $\left(2 \times w_{0}\right)$, it is usually considered that the effects of the diffraction of the beam by the aperture will only slightly modify the shape of the beam, and, in particular, it will become wider while keeping its Gaussian shape. We can use the results from Belland and Crenn [17], acquired using energy conservation, to obtain a simple expression of the size of the diffracted waist by the telescope in the case of a propagation without atmospheric turbulence:

$$
\begin{equation*}
w_{L}=\frac{\lambda L}{\pi w_{0}} \frac{\sqrt{1-\exp \left(-\frac{2 R_{\mathrm{Tel}}^{2}}{w_{0}^{2}}\right)}}{1-\exp \left(-\frac{R_{\mathrm{Tel}}^{2}}{w_{0}^{2}}\right)} \tag{9}
\end{equation*}
$$

where $R_{\mathrm{Tel}}$ is the radius of the telescope. To take into account the truncation effect of the emitter on the beam size, the farfield diffraction expression in Eq. (B7) is replaced by the waist expression of Eq. (9). This leads to a modified expression of the waist in the satellite plane, in the case of Baker's LOT approach,

$$
\left.\begin{array}{rl}
w_{\{x, y\}}= & w_{0} \times\left[\left[1+L\left(\Delta \kappa \pm \sqrt{c_{5}^{2}+c_{6}^{2}}\right)\right]^{2}\right. \\
& +\left[\frac{\lambda L}{\pi w_{0}} \frac{\sqrt{1-\exp \left(-\frac{2 R_{T d}^{2}}{w_{0}^{2}}\right)}}{1-\exp \left(-\frac{R_{\mathrm{Td}}^{2}}{w_{0}^{2}}\right)}\right] \tag{10}
\end{array}\right]^{\frac{1}{2}} .
$$

This result can lead to an alternate effective beam waist $D_{\text {eff }}$ at $1 / e^{2}$ in irradiance at the emission in order to take into account the effects of the diffraction of the Gaussian beam by a circular aperture. In this paper, we will use Yura's result [18] for the effective beam diameter at $1 / e^{2}$ in irradiance at the emission, which is given by

$$
\begin{equation*}
D_{\mathrm{eff}}=d_{0}\left[\tanh \left(D_{\mathrm{Tel}}^{2} / 2 d_{0}^{2}\right)\right]^{\frac{1}{2}} \tag{11}
\end{equation*}
$$

This effective beam diameter yields accurate results as long as $d_{0}<D_{\text {Tel }}$ (where $d_{0}=2 w_{0}$ ). The encircled power distributions are valid to better than approximately $2 \%$.

However, the strength of the Zernike polynomials' coefficients still has to be estimated for a beam of diameter size $d_{0}$ at the emission. Both the methodologies from Yura and Belland et al. yield similar results. In the rest of the paper, we will consider only the methodology from Yura, which is simpler to implement.

The improvement of taking into account the truncation of the beam by the telescope in the WPLOT model can be observed in Fig. 6. The results are compared to the result from a $2 \mathrm{~L}-\mathrm{WO}$ simulation taking into account a truncation.

Taking into account the truncation of the Gaussian beam reduces the maximum irradiance that can be reached (at the center of the Gaussian beam) while increasing the width of the beam. This means that at the extremities of the Gaussian beam, when beam wander shifts the Gaussian by a distance equivalent to the beam size, the detected irradiance with a truncated Gaussian beam will be slightly higher than in a no-truncation case. The results show that assuming that the shape of the beam is still Gaussian is relevant.

## C. Presentation of the Model Process

There are two aspects to the process of the WPLOT model: the estimation of the Zernike coefficients and the propagation through multiple phase screens. We have shown in Section 3.A. 1 that propagation through multiple phase screens essentially impacts defocus, and we have proposed a solution to model these effects. For tilt and astigmatism, the turbulence can be integrated and placed at the transmitter prior to propagation.

In a no-tracking case, the random draws for tilt and astigmatism can be obtained using the variances given in


Fig. 6. Random draws of irradiance statistics between WPLOT solution taking into account, or not, the truncation of the Gaussian beam by the telescope compared with $2 \mathrm{~L}-\mathrm{WO}$ simulation taking into account a truncation.

Eqs. (C1) and (C2) in Appendix C. These variances are estimated over the whole turbulence volume and assume the beam can be modeled as a plane wave. This results from our assumption that the majority of the turbulence is located close to the emitter, where the beam is collimated. If tracking is taken into account, then the estimation of the correlation has to be done over the whole turbulence volume, using multiple layers, because the decorrelation between the downlink and the uplink is usually a function of the altitude. Chassat's correlation functions [19] provide an analytic estimation of the correlation between two beams calculated over the whole turbulence volume and can permit us to estimate the tilt contribution of each layer. These contribution can than be integrated and placed at the transmitter prior to propagation.

For defocus, Zernike polynomials have to be estimated for each atmospheric layer. The turbulence strength division between these layers is made to keep the same overall phase variance. In Eq. (C2), this implies a change in the estimation of the Fried parameter $r_{0}$, which is now estimated for turbulence volume corresponding to each layer. In our analysis, we will assume that the beam is collimated during its propagation through atmospheric turbulence. This means that we will consider the initial beam diameter $d_{0}$ for each atmospheric layer.

We assume that the Zernike coefficients are Gaussian random variables with zero mean [15] and that they are decorrelated [20].

Times series of each Zernike coefficient can be obtained from their respective temporal PSDs. Their PSDs are wellknown in the literature, especially in the case of plane waves [9] when using Taylor's hypothesis of frozen turbulence, and are given in Eq. (C3). The time series of the different Zernike coefficients can be obtained from Eq. (12):

$$
\begin{equation*}
a_{i}(t)=\int \sqrt{W_{a_{j}}\left(f_{n}\right)} e^{i \phi_{n}} e^{2 i \pi f_{n} \cdot t} \mathrm{~d} f_{n}, \tag{12}
\end{equation*}
$$

where $\phi_{n}$ is the phase attributed to the spectral component at the frequency $f_{n}$, and $W_{a_{j}}(f)$ is the temporal PSD of the $a_{j}$ coefficient.

The tilt Zernike coefficients $a_{\{2,3\}}$ are transformed into an angular deviation $\theta_{\{x, y\}}$ using Eq. (C5), the defocus Zernike coefficient $a_{4}$ into the defocus curvature $\Delta \kappa$ using Eq. (C6), and the Zernike astigmatism coefficients $a_{\{5,6\}}$ into the astigmatism curvatures $c_{\{5,6\}}$ using Eq. (C7). The final step is to insert the obtained Zernike coefficients described in Appendix B with the waist solution from Eq. (10).

## 4. MODEL VALIDATION

We have validated our model by comparing its results to wave optics simulations (TURANDOT model [2], dedicated to ground-to-space optical communications). TURANDOT models the propagation of a beam through a realistic atmospheric turbulence. The phase screens are obtained from a von Kármán spectrum and the sampling of the atmospheric turbulence is done using 24 atmospheric layers nonuniformly distributed with their locations optimized. For the WPLOT model, we are going to consider the same atmospheric profile as TURANDOT, i.e., 24 layers with the same distribution. Because we focus on the lower irradiances, we need an
important number of samples in order to obtain a sufficiently precise estimation of the lower percentages of the irradiance CDFs. However, with TURANDOT, we are limited by the computation time. In our study, we have taken 2000 samples for each case considered with TURANDOT, and this led to a computation time of approximately 5 h each time. On the other hand, with the WPLOT model, we are able to consider a very large number of draws (chosen arbitrarily at 250,000 samples) in approximately 1 min .

## A. Turbulence Conditions

The vertical $C_{n}^{2}$ profile used is the Hufnagel-Valley profile [21], defined by

$$
\begin{align*}
C_{n}^{2}(h)= & 0.00594\left(\frac{v}{27}\right)^{2}\left(10^{-5} h\right)^{10} e^{-\frac{b}{1000}} \\
& +2.7 \times 10^{-16} e^{-\frac{b}{1500}}+C_{g} e^{-\frac{b}{100}} . \tag{13}
\end{align*}
$$

In this paper, we consider $C_{g}=1.7 \times 10^{-14} \mathrm{~m}^{-\frac{2}{3}}$ and $v=21 \mathrm{~m} / \mathrm{s}$, which results in a $r_{0}$ calculated at zenith equal to 19 cm for a $1.55-\mu \mathrm{m}$ wavelength. The elevation angle will be considered equal to $45^{\circ}$, making the effective $r_{0}$ equal to 15 cm . The outer scale $L_{0}$ is fixed by the wave optics simulations; we have taken the outer scale to be equal to TURANDOT's phase screens' sizes, which gives $L_{0}=2.5 \mathrm{~m}$ in our case.

The temporal effects are modeled using Taylor's hypothesis of frozen turbulence and a Bufton wind profile [21], described in Eq. (14), in which the atmospheric layers move with a $5 \mathrm{~m} / \mathrm{s}$ speed at ground level and with a $30 \mathrm{~m} / \mathrm{s}$ speed at an altitude around 12.5 km ,

$$
\begin{equation*}
V(h)=S_{G}+S_{P} e^{-\left(\frac{b-H_{p}}{W_{P}}\right)^{2}}, \tag{14}
\end{equation*}
$$

in which we have taken $S_{G}=5 \mathrm{~m} / \mathrm{s}, S_{P}=25 \mathrm{~m} / \mathrm{s}$, $H_{P}=12448 \mathrm{~m}$, and $W_{P}=4800 \mathrm{~m}$.

We will consider only the case in which the wind moves in the $x$ direction for all layers. Extension to the wind moving in multiple direction is straightforward ( $[9,19]$ ).

Using these atmospheric conditions and Baker's conditions on $N_{L}$ and $N_{\text {tau }}$ presented in Eq. (B4) to define the $D_{1}$ region lead to a minimum waist size of 4.9 cm [and a maximum waist size of 4.2 m , but our assumptions on phase variance already impose the maximum bound $w_{0} \leq 1.5 r_{0}=22.5 \mathrm{~cm}$ presented in Eq. (8)].

## B. Average Irradiance and Scintillation Index

In this example, we will consider the telescope diameter $D_{T_{X}}=2^{\frac{3}{2}} w_{0}$, which is a value often proposed in the literature [22]. It optimizes the losses due to beam truncation and beam diffraction. We have compared the mean detected on-axis irradiance (Fig. 7) and on-axis scintillation index $\sigma_{I}^{2}$ (Fig. 8) obtained with wave optics simulations and our model as a function of the waist size at emission.

There is good agreement between both TURANDOT and WPLOT model, which tends to validate the WPLOT model.

## C. Cumulative Density Function

We also compare the Cumulative Density Functions (CDF) obtained with TURANDOT simulations and WPLOT model.


Fig. 7. Mean detected on-axis irradiance as a function of the emitted waist size. The mean on-axis irradiance has been calculated over 2000 samples for the TURANDOT simulations and over 250,000 samples for the WPLOT model for waist sizes ranging from 5 to 14 cm .


Fig. 8. On-axis scintillation index as a function of the emitted waist size. The on-axis scintillation index has been calculated for over 2000 samples for the TURANDOT simulations and over 250,000 samples for the WPLOT model for waist sizes ranging from 5 to 14 cm .

Some examples are given in Figs. 9-12 for waist sizes of 5, 8, 11 , and 14 cm .

There are some differences between our model and the results coming from TURANDOT. However, there is a good fit for the lower irradiances, which are the interesting parameter for our study. The threshold at $5 \%$ of the CDF for different waist sizes at the emission is given in Fig. 13.

We observe a very good fit for the smaller waists, but a gap starts to appear for waist sizes bigger than 9 cm . The error on the estimation of the threshold starts to be bigger than 1 dB for waists bigger than 12 cm . There can be multiple reasons for this gap. It might mean that the impact of higher-order phase effects, such as spherical aberration or coma for example, which are not taken into account, are no longer negligible. This would mean that the condition from Eq. (8) may not be sufficiently strong.
N.B.: The envisioned diameters for the emitting telescopes will probably not be bigger than 40 cm . Therefore, a waist size of 14 cm is the maximum that needs to be considered in a case where the emitting telescope diameter $D_{T_{X}}$ is equal to $2^{\frac{3}{2}} w_{0}$.

## D. Temporal Power Spectral Density

We have also compared the on-axis irradiance temporal PSD obtained with TURANDOT and the WPLOT model. These temporal PSDs are obtained from time series estimated


Fig. 9. CDF of the detected on-axis irradiance comparison between the TURANDOT simulation and the WPLOT model for a waist size of 5 cm . The CDFs have been calculated over 2000 samples for the TURANDOT simulation and over 250,000 samples for the WPLOT model.


Fig. 10. CDF of the detected on-axis irradiance comparison between the TURANDOT simulation and the WPLOT model for a waist size of 8 cm . The CDFs have been calculated over 2000 samples for the TURANDOT simulation and over 250,000 samples for the WPLOT model.
over a duration of 4 s with a 2500 Hz sampling. Some examples are given in Figs. 14 and 15 for waist sizes of 8 and 14 cm .

Both figures show good agreement between the WPLOT model and TURANDOT. We observe a floor on the temporal PSD obtained with TURANDOT that resembles aliasing


Fig. 11. CDF of the detected on-axis irradiance comparison between the TURANDOT simulation and the WPLOT model for a waist size of 11 cm . The CDFs have been calculated for over 2000 samples for the TURANDOT simulation and over 250,000 samples for the WPLOT model.


Fig. 12. CDF of the detected on-axis irradiance comparison between the TURANDOT simulation and the WPLOT model for a waist size of 14 cm . The CDFs have been calculated for over 2000 samples for the TURANDOT simulation and over 250,000 samples for the WPLOT model.


Fig. 13. Comparison of the estimation of the threshold at $5 \%$ of the CDF between the TURANDOT simulations and the WPLOT model for different waist sizes at the emission. The bars correspond to a $1-\mathrm{dB}$ margin from TURANDOT's results.
effects. It would mean that the phase screen sampling of TURANDOT is insufficient. However, since these effects appear after the cutoff frequency, we have not looked further into this subject. As the WPLOT model is much faster than TURANDOT, it is able to create much longer time series in reasonable time. For example, the 4 -s time series with 2500 Hz samplings considered were obtained in a few seconds with the WPLOT model, whereas it took around a day with


Fig. 14. Temporal power spectral density of on-axis irradiance comparison for a waist size of 8 cm calculated for an irradiance time series of 4 s with $2500-\mathrm{Hz}$ sampling.


Fig. 15. Temporal power spectral density of on-axis irradiance comparison for a waist size of 14 cm calculated for an irradiance time series of 4 s with $2500-\mathrm{Hz}$ sampling.

TURANDOT. Because we have shown that we have a precise estimation of the irradiance threshold at $5 \%$ of the irradiance CDF (cf. Fig. 13), the fact that we can obtain very long time series means that the WPLOT model can be used to study the fade statistics.

## E. Justification of Model Particularities

## 1. Impact of Taking into Account the Beam Truncation

Most models found in the literature do not take into account the impact of the beam truncation $([5,8,22])$. However, we show that its influence is not always negligible, and it is therefore an important parameter to take into account if the objective is to optimize the architecture of an optical ground station. In Fig. 16, we use the scintillation index as an indicator of the influence of the beam truncation as a function of the emitting waist size. The beam truncation is $D_{T_{X}}=2^{\frac{3}{2}} w_{0}$.

We therefore observe that if the beam truncation is not taken into account, there is an overestimation of the scintillation index. This is because the waist size at satellite level is smaller. Therefore, beam wander induces more important irradiance fluctuations.
2. Comparison of the WPLOT Model to the Rytov Perturbation Method
We justify here our choice of Baker's model [10] as the base for our model by comparing it with other usual models in the literature where the defocus and the astigmatism are not taken into account and rely for the higher orders on the Rytov approximation ([5,8,22]). In Fig. 17, we compare the


Fig. 16. Impact of not taking into account the beam truncation on the on-axis scintillation index for waist sizes ranging from 5 to 14 cm and estimated over 250,000 samples.


Fig. 17. Impact of not taking into account the defocus and astigmatism on the on-axis scintillation index and estimated over 250,000 samples.
scintillation results from the LOT model to results obtained from Dios's methodology [8]. In order to take into account the truncation, we have taken the waists obtained from Yura [see Eq. (11)].

We observe that results from Dios slightly overestimate the scintillation index compared to TURANDOT and the WPLOT model. This proves that the effects of defocus and astigmatism cannot be neglected. Moreover, using the Rytov perturbation method does not permit us to compensate for the effects of defocus and astigmatism on irradiance fluctuations.

## 5. MODEL APPLICATION TO TRACKING

Taking into account tracking within the model is pretty straightforward using the literature $([9,19])$. The important results are presented in Appendix D.

It is now possible to compare the statistical results as well as the time series when tracking is added. We present in this section the results obtained for a specific example in which the point-ahead angle is considered equal to $18.5 \mu \mathrm{rad}$. The delay between the measurement on the downlink and the pre-compensation of the uplink is considered equal to 4 ms . Therefore, tracking will not be perfect. The pupils of downlink and uplink are merged at ground level. The atmospheric parameters are the same as in Section 4.

While the limitations on the quality of the correction due to the point-ahead angle will always be present, the limitations due to the delay can easily be dealt with by increasing the bandwidth of the correction loop. In the considered case, the tilt correlation obtained from Chassat's functions is equal to 0.86 for a waist size of 8 cm . This leads to a residual tilt variance of $0.03 \mathrm{rad}^{-2}$, which then leads to a standard deviation of the displacement in the satellite plane of 38 m . If the delay before the correction is zero, then the tilt correlation becomes 0.9 , the residual tilt variance becomes $0.02 \mathrm{rad}^{-2}$, and the standard deviation of the displacement is 33 m . Compared to the beam radius in the satellite plane, equal to 254 m , this 5 -m displacement error is negligible, which means that there is no need for a lower delay time.

## A. Validation

We have compared the results obtained with the WPLOT model and TURANDOT when taking into account tracking.


Fig. 18. Comparison of the estimation of the threshold at $5 \%$ of the CDF between the TURANDOT simulation and the WPLOT model for different waist sizes at the emission. The bars correspond to $1-\mathrm{dB}$ margin from TURANDOT's results.

Figure 18 shows the estimation of the threshold at $5 \%$ of the CDF between TURANDOT and the WPLOT model for different waist sizes at emission. This shows that the estimation of the threshold is with a precision lower than 1 dB . Figure 19 shows the temporal PSDs of irradiance fluctuations for the specific case of an $8-\mathrm{cm}$ waist size at emission obtained with TURANDOT and the WPLOT model. We observe that there is a good match.


Fig. 19. Comparison of the on-axis irradiance temporal PSD between the TURANDOT simulation and the WPLOT model with tracking taken into account for NA $8-\mathrm{cm}$ waist size at emission calculated for an irradiance time series of 4 s with 10,000 samples.


Fig. 20. Comparison of the mean detected on-axis irradiance as a function of the emitted waist size calculated over 250,000 samples with and without tracking using the WPLOT model with two layers.


Fig. 21. Comparison of the on-axis scintillation index as a function of the emitted waist size calculated over 250,000 samples with and without tracking using the WPLOT model with two layers.

## B. Improvement on Link Budget

We have compared the mean detected irradiance (Fig. 20) and scintillation index $\sigma_{I}^{2}$ (Fig. 21) with and without tracking as a function of the waist size at emission using the WPLOT model. In this section, we will consider two layers.

The results are consistent with what one would expect: tracking the tip/tilt significantly reduces the scintillation and increases the mean irradiance. These effects are stronger for the larger waist sizes at the emission than for the lower waist sizes.

We can also observe the evolution of time series with and without tracking in Fig. 22. With tracking, we observe the


Fig. 22. 2-s on-axis irradiance time series with a $2500-\mathrm{Hz}$ sampling for a waist size of 8 cm obtained with the WPLOT model with and without tracking.


Fig. 23. $\quad L_{\text {TURB }}$ estimation with and without tracking for different emitted waist sizes obtained using the WPLOT model using 250,000 samples.
disappearance of the very low irradiance fadings. This means that, if we keep the irradiance threshold at $5 \%$ of the CDF, the fading times are going to be shorter but more numerous. It also shows the significant increase of the irradiance threshold at 5\% of the cumulative density function and therefore the important decrease of the losses due to turbulence $L_{\mathrm{TURB}}$, as can be seen in Fig. 23.

## 6. CONCLUSION

We have presented that which performs time series of the irradiance fluctuations induced by the atmospheric turbulence on the axis in the satellite plane in the case of uplink. Moreover, this model is directly linked to the optical ground station characteristics and turbulence profile. Compared to wave optics simulation (TURANDOT), the computation time is considerably reduced (approximately 5 h to obtain 2000 samples using TURANDOT while the WPLOT model provides the results for 250,000 samples in less than a minute). This allows sensitivity studies and longer time series generation.

Because we focus on a ground-to-satellite propagation case, we have been able to make justified hypotheses that significantly simplify our model by restricting its range of validity to telescope diameters between 10 cm and 40 cm . This model has been validated by comparing it to wave optics simulations.

The model takes into account the different parameters of the optical ground station architecture. So we are able to evaluate the optical link budget as a function of them and optimize the performance of the ground station. Besides, different errors can be considered for the correction (point-ahead, angle, servo-loop delay, and even pupil misalignment between the measuring pupil of the downlink and the emitting pupil of the uplink) as well as effects due to the finite telescope diameter on the infinite Gaussian beam. These effects are not usually taken into account in the literature $[8,22]$.

Contrary to many works in the literature ([4,5,23]), which focus on the mean fade duration and mean number of fades per second, our model can create time series of the irradiance fluctuations. This approach will be useful to work on the contributions of interleaving and on the error correction codes. Despite some discrepancies that appear for the greater irradiances, the time series obtained with the WPLOT model enable a precise study of the fades as they occur in the lower irradiances, which are well modeled. The WPLOT model therefore provides the possibility to estimate the fade duration occurrence probability and the cumulative fade duration exceedance probability as defined in [24] from the International Telecommunication Union (ITU) recommendations [25].

Some work has already been done on the optimization of an optical ground station using this model [26]. The time series provided will be useful for future work, notably the optimization of interleaving and error correction codes in the presence of turbulence.

## APPENDIX A: DESCRIPTION OF THE MODELS

Throughout this paper, we use multiple models, which are presented in Table 1 for improved clarity.

## APPENDIX B: LOT IRRADIANCE MODEL PRESENTATION

## 1. Hypotheses and Range of Validity

In order to determine the range of validity, Baker [10] proposes two dimensionless parameters to define the region of interest in which beam wander becomes predominant. These parameters are

$$
\begin{align*}
& N_{L}=\frac{\pi w_{0}^{2}}{\lambda L}  \tag{B1}\\
& N_{\tau}=\frac{\pi w_{0}^{2}}{\lambda z_{\tau}} \tag{B2}
\end{align*}
$$

with $z_{\tau}$ defined by

$$
\begin{equation*}
z_{\tau}=\frac{\int_{0}^{L} z C_{n}^{2}(z) \mathrm{d} z}{\int_{0}^{L} C_{n}^{2}(z) \mathrm{d} z} \tag{B3}
\end{equation*}
$$

$w_{0}$ is the waist radius at $1 / e$ of the amplitude, $\lambda$ is the wavelength, $L$ is the distance between the ground and the satellite, and $C_{n}^{2}$ is the refractive index structure constant corresponding to the variance of the refractive index between two points separated between $1 \mathrm{~m} . z_{\tau}$ gives an estimation of the centroid of the turbulence strength on the path. The shorter it is, the stronger is the turbulence near the emitter. The parameters $N_{L}$ and $N_{\tau}$ correspond to Fresnel numbers of the initial beam observed at, respectively, distances $L$ and $z_{\tau}$. They compare the Rayleigh range of the beam to, respectively, the distances $L$ and $z_{\tau}$. According to Baker, the bounds of this region are given by

$$
\begin{equation*}
N_{L}<1 \quad \text { and } \quad N_{\tau}^{2} \gg 1 \tag{B4}
\end{equation*}
$$

## 2. Low-Order Turbulence Solution

Baker [10] proposes a solution to describe the irradiance fluctuations relying on an approach that describes the deformations of a Gaussian beam propagating through the atmosphere as dependent on the first- and second-order Zernike polynomials [13,14]: tip/tilt, defocus, and astigmatism. He has shown that the resulting irradiance of a Gaussian beam, of wavelength $\lambda$ (and wavenumber $k=\frac{2 \pi}{\lambda}$ ) and of waist size $w_{0}$ (at $1 / e^{2}$ of the Gaussian irradiance) at the emission, propagating through atmospheric turbulence up to a satellite at a distance $L$, is expressed:

$$
\begin{align*}
I(x, y, L)= & \frac{2}{\pi} \frac{1}{w_{x} w_{y}} \\
& \times \exp \left(-\frac{2}{w_{x}^{2}}\left[\left(x-\delta_{x}\right) \cos (\omega)+\left(y-\delta_{y}\right) \sin (\omega)\right]^{2}\right) \\
& \times \exp \left(-\frac{2}{w_{y}^{2}}\left[\left(x-\delta_{x}\right) \sin (\omega)-\left(y-\delta_{y}\right) \cos (\omega)\right]^{2}\right), \tag{B5}
\end{align*}
$$

where the beam wander, partial beam radii, and astigmatism parameters in Eq. (B5) are given by, respectively,

$$
\begin{equation*}
\delta_{\{x, y\}}=L \theta_{\{x, y\}} \tag{B6}
\end{equation*}
$$

$$
\begin{equation*}
w_{\{x, y\}}=w_{0} \sqrt{\left[1+L\left(\Delta \kappa \pm \sqrt{c_{5}^{2}+c_{6}^{2}}\right)\right]^{2}+\left(\frac{2 L}{k w_{0}^{2}}\right)^{2}} \tag{B7}
\end{equation*}
$$

and

$$
\begin{equation*}
\omega=\frac{1}{2} \arg \left(c_{5}+j c_{6}\right) . \tag{B8}
\end{equation*}
$$

$\theta_{\{x, y\}}$ corresponds to the angular tip/tilt in radians. $\Delta \kappa$ corresponds to the defocus curvature, and $c_{\{5,6\}}$ to the astigmatism curvature (given in $\mathrm{m}^{-1}$ ).

In Eq. (B7), the first term under the root square expresses the broadening of the beam induced by defocus and astigmatism, and the second term is the broadening induced by diffraction.

## APPENDIX C: ZERNIKE COEFFICIENTS

## 1. Variances

The variances of the Zernike coefficients are very well known in the literature [19] in the case of a von Kármán spectrum for atmospheric turbulence. The variance for tip and tilt, of radial order $n=1$, is

$$
\begin{aligned}
\sigma_{a_{2,3}}^{2} & =0.451\left(\frac{d_{0}}{r_{0}}\right)^{\frac{5}{3}} \\
& \times\left[1-0.77\left(\frac{2 \pi d_{0}}{L_{0}}\right)^{\frac{1}{3}}+0.09\left(\frac{2 \pi d_{0}}{L_{0}}\right)^{2}-0.054\left(\frac{2 \pi d_{0}}{L_{0}}\right)^{\frac{7}{3}}\right]
\end{aligned}
$$

(C1)
where $L_{0}$ is the outer scale. The defocus and astigmatism Zernike coefficients variances, of radial order $n=2$, are

$$
\begin{align*}
\sigma_{a_{4,5,6}}^{2}= & 2.34 \times 10^{-2}\left(\frac{d_{0}}{r_{0}}\right)^{\frac{5}{3}} \\
& \times\left[1-0.39\left(\frac{2 \pi d_{0}}{L_{0}}\right)^{2}+0.27\left(\frac{2 \pi d_{0}}{L_{0}}\right)^{\frac{7}{3}}\right] \tag{C2}
\end{align*}
$$

N.B.: In the case of the WPLOT model, $\sigma_{a_{4}}^{2}$ has to be estimated for each layer. This means that the Fried parameter $r_{0}$ in Eq. (C2) has to be calculated between the bounds of each atmospheric layer.

## 2. Power Spectral Densities

The PSDs for each Zernike coefficient are [9]

$$
\begin{align*}
W_{a_{j}}(\nu)= & \int_{0}^{L} \int_{-\infty}^{\infty} \frac{1}{V(z)}\left|\tilde{M}_{a_{j}}\left(\frac{\nu}{V(z)}, f_{y}\right)\right|^{2} \\
& \times W_{\phi}\left(\frac{\nu}{V(z)}, f_{y}\right) \mathrm{d} z \mathrm{~d} f_{y} \tag{C3}
\end{align*}
$$

where $W_{\phi}$ is the phase spatial power spectrum and $\left|\tilde{M}_{a_{j}}(\vec{f})\right|$ is the Fourier transform of the Zernike polynomial,

$$
\begin{align*}
\left|\tilde{M}_{a_{j}}(\vec{f})\right|= & \sqrt{n+1} \frac{2\left|J_{n+1}\left(\pi d_{0} f\right)\right|}{\pi d_{0} f} \\
& \times\left\{\begin{array}{cc}
\sqrt{2}|\cos (m \theta)| & \text { for } m \neq 0 \text { and } j \text { even } \\
\sqrt{2}|\sin (m \theta)| & \text { for } m \neq 0 \text { and } j \text { odd } \\
1 & \text { for } m=0
\end{array}\right. \tag{C4}
\end{align*}
$$

where $\vec{f}=\left(f_{x}, f_{y}\right)$ when the frequency vector is described in Cartesian coordinates or $\vec{f}=(f, \theta)$ when it is described in
polar coordinates, $n$ is the radial degree, and $m$ is the azimuthal frequency of the $j^{\text {th }}$ Zernike polynomial. These results are for a plane wave.

## 3. From Zernike Coefficients to Phase Coefficients

Once the Zernike coefficients are obtained, they can easily be converted into the corresponding phase coefficients in order to insert them in the model.

The angular deviation due to tip/tilt is

$$
\begin{equation*}
\theta_{x, y}=\left(\frac{2 \lambda}{\pi d_{0}}\right) a_{2,3} \tag{C5}
\end{equation*}
$$

while the defocus curvature is

$$
\begin{equation*}
\Delta \kappa=\left(\frac{8 \sqrt{3}}{\pi}\right)\left(\frac{\lambda}{d_{0}^{2}}\right) a_{4} \tag{C6}
\end{equation*}
$$

and the astigmatism curvature is

$$
\begin{equation*}
c_{5,6}=\left(\frac{4 \sqrt{6}}{\pi}\right)\left(\frac{\lambda}{d_{0}^{2}}\right) a_{5,6} \tag{C7}
\end{equation*}
$$

## APPENDIX D: TAKING INTO ACCOUNT TRACKING

The idea is to inject the opposite of the Zernike tip/tilt coefficients $\left(a_{2} / a_{3}\right)$ measured on the downlink to pre-compensate the uplink. The pointing error at the exit of the atmosphere is

$$
\begin{equation*}
a_{i, \mathrm{res}}(t)=a_{i, U}(t)-a_{i, D}(t) \tag{D1}
\end{equation*}
$$

where $a_{i, \text { res }}, a_{i, U}(t)$, and $a_{i, D}(t)$ are the residual tip/tilt error ( $i=2,3$ ), the tip/tilt on the uplink without pre-compensation and the tip/tilt measured on the downlink, respectively.

## 1. Reciprocity Principle and Implications

The idea of compensation relies on the reciprocity of light propagation. However, a difficulty is that while the downlink is a plane wave, the uplink is a Gaussian beam. It has been shown $([27,28])$ that the upward propagation of a Gaussian beam can be modeled by the downward propagation of a plane wave. This result simplifies the estimation of the correlation between the downlink and the uplink. It justifies our use of the Zernike polynomials compared to the Laguerre polynomials that are used in the case of a Gaussian beam. It is also consistent with the assumption that the beam is collimated throughout its propagation in the atmosphere.

## 2. Statistical Approach

The variance of the residual Zernike coefficient is

$$
\begin{equation*}
\sigma_{a_{i, \mathrm{res}}}^{2}=\left\langle\left(a_{i, U}-a_{i, D}\right)^{2}\right\rangle \tag{D2}
\end{equation*}
$$

This leads to

$$
\begin{equation*}
\sigma_{a_{i, \mathrm{res}}}^{2}=\left\langle a_{i, U}^{2}\right\rangle+\left\langle a_{i, D}^{2}\right\rangle-2\left\langle a_{i, U} a_{i, D}\right\rangle, \tag{D3}
\end{equation*}
$$

where $\left\langle a_{i, U} a_{i, D}\right\rangle$ is the covariance between the $i^{\text {th }}$ Zernike coefficients of the uplink and the downlink. An estimation is given in Chassat's thesis [19].

## 3. Power Spectral Densities after Tracking Correction

In the case of tracking, the results are obtained from the differential phase. Assuming the on-ground pupil is the same diameter, we obtain the phase-related quantity $G$ [9],

$$
\begin{equation*}
G(\vec{r})=[\phi(\vec{r}+\vec{D}, t)-\phi(\vec{r}, t)] * Z_{j}(\vec{r}), \tag{D4}
\end{equation*}
$$

where $Z_{j}(\vec{r})$ is the $j^{\text {th }}$ Zernike polynomial defined on a pupil of diameter $d_{0}$.

If we consider that $\vec{D}$ is oriented along the $x$ axis, the spectrum from Eq. (C3) becomes

$$
\begin{equation*}
\left|\tilde{M}_{G}(\vec{f}, h)\right|^{2}=4 \sin ^{2}\left(\pi D \frac{\nu}{V(h)}\right) \times\left|\tilde{M}_{a_{j}}(\vec{f})\right|^{2} \tag{D5}
\end{equation*}
$$

where $\left|\tilde{M}_{a_{j}}(\vec{f})\right|$ is obtained from Eq. (C4).
Acknowledgment. This paper is part of the work done in the context of a Ph.D. thesis, which was co-financed by two institutions: IRT Saint-Exupery and ONERA. The authors thank Jean-Marc Conan and Nicolas Védrenne for many helpful discussions. They also thank the anonymous reviewers for their help in improving the clarity of this paper.

## REFERENCES

1. D. L. Fried and H. T. Yura, "Telescope-performance reciprocity for propagation in a turbulent medium," J. Opt. Soc. Am. 62, 600-602 (1972).
2. N. Védrenne, J. Conan, M. Velluet, M. Sechaud, M. Toyoshima, H. Takenaka, A. Guérin, and F. Lacoste, "Turbulence effects on bi-directional ground-to-satellite laser communication systems," in International Conference on Space Optical Systems and Applications (2012).
3. J. M. Martin and S. M. Flatté, "Intensity images and statistics from numerical simulation of wave propagation in 3-D random media," Appl. Opt. 27, 2111-2126 (1988).
4. L. C. Andrews and R. L. Phillips, Laser Beam Propagation Through Random Media (SPIE, 2005), Vol. 1.
5. S. Basu, D. Voelz, and D. K. Borah, "Fade statistics of a ground-to-satellite optical link in the presence of lead-ahead and aperture mismatch," Appl. Opt. 48, 1274-1287 (2009).
6. L. C. Andrews, R. L. Phillips, and P. T. Yu, "Optical scintillations and fade statistics for a satellite-communication system," Appl. Opt. 34, 7742-7751 (1995).
7. G. J. Baker and R. S. Benson, "Gaussian-beam weak scintillation on ground-to-space paths: compact descriptions and Rytov-method applicability," Opt. Eng. 44, 106002 (2005).
8. F. Dios, J. A. Rubio, A. Rodríguez, and A. Comerón, "Scintillation and beam-wander analysis in an optical ground station-satellite uplink," Appl. Opt. 43, 3866-3873 (2004).
9. J.-M. Conan, G. Rousset, and P.-Y. Madec, "Wave-front temporal spectra in high-resolution imaging through turbulence," J. Opt. Soc. Am. A 12, 1559-1570 (1995).
10. G. J. Baker, "Gaussian beam weak scintillation: low-order turbulence effects and applicability of the Rytov method," J. Opt. Soc. Am. A 23, 395-417 (2006).
11. B. J. Klein and J. J. Degnan, "Optical antenna gain. 1: transmitting antennas," Appl. Opt. 13, 2134-2141 (1974).
12. M. I. Charnotskii, "Asymptotic analysis of finite-beam scintillations in a turbulent medium," Waves Random Media 4, 243-273 (1994).
13. F. Zernike, "Diffraction theory of the knife-edge test and its improved form, the phase-contrast method," Mon. Not. R. Astron. Soc. 94, 377-384 (1934).
14. M. Born and E. Wolf, Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light (Elsevier, 1975).
15. R. J. Noll, "Zernike polynomials and atmospheric turbulence," J. Opt. Soc. Am. 66, 207-211 (1976).
16. D. L. Fried, "Optical resolution through a randomly inhomogeneous medium for very long and very short exposures," J. Opt. Soc. Am. 56, 1372-1379 (1966).
17. P. Belland and J. P. Crenn, "Changes in the characteristics of a Gaussian beam weakly diffracted by a circular aperture," Appl. Opt. 21, 522-527 (1982).
18. H. T. Yura, "Optimum truncation of a Gaussian beam for propagation through atmospheric turbulence," Appl. Opt. 34, 2774-2779 (1995).
19. F. Chassat, "Propagation optique à travers la turbulence atmosphérique: étude modale de l'anisoplanétisme et application à I'optique adaptative," Physique, Université Paris-Sud (1992).
20. N. A. Roddier, "Atmospheric wavefront simulation using Zernike polynomials," Opt. Eng. 29, 1174-1180 (1990).
21. J. Hardy, Adaptive Optics for Astronomical Telescopes, Oxford Series in Optical and Imaging Sciences (Oxford University, 1998).
22. L. C. Andrews, R. L. Phillips, R. J. Sasiela, and R. R. Parenti, "Strehl ratio and scintillation theory for uplink Gaussian-beam waves: beam wander effects," Opt. Eng. 45, 076001 (2006).
23. H. T. Yura and R. A. Fields, "Level crossing statistics for optical beam wander in a turbulent atmosphere with applications to ground-to-space laser communications," Appl. Opt. 50, 2875-2885 (2011).
24. B. Epple, "Simplified channel model for simulation of free-space optical communications," J. Opt. Commun. Netw. 2, 293-304 (2010).
25. "Prediction method of fade dynamics on earth-space paths," Technical Report (International Telecommunication Union (ITU), Recommendation ITU-R, 2005), pp. 1623-1631.
26. A.-R. Camboulives, M.-T. Velluet, S. Poulenard, L. Saint-Antonin, and V. Michau, "Optical ground station optimization for future optical geostationary satellite feeder uplinks," Proc. SPIE 10096, 1009608 (2017).
27. J. H. Shapiro and A. L. Puryear, "Reciprocity-enhanced optical communication through atmospheric turbulence-Part I: reciprocity proofs and far-field power transfer optimization," J. Opt. Commun. Netw. 4, 947-954 (2012).
28. C. Robert, J.-M. Conan, and P. Wolf, "Impact of turbulence on highprecision ground-satellite frequency transfer with two-way coherent optical links," Phys. Rev. A 93, 033860 (2016).

## Bibliography

[1] Horace W Babcock. "The possibility of compensating astronomical seeing". In: Publications of the Astronomical Society of the Pacific 65.386 (1953), pp. 229-236.
[2] D. L. Fried and H. T. Yura. "Telescope-Performance Reciprocity for Propagation in a Turbulent Medium". In: J. Opt. Soc. Am. 62.4 (Apr. 1972), pp. 600-602. Doi: 10.1364/JOSA.62.000600. URL: http://www.osapublishing.org/abstract.cfm? URI=josa-62-4-600.
[3] Gary J Baker. "Gaussian beam weak scintillation: low-order turbulence effects and applicability of the Rytov method". In: JOSA A 23.2 (2006), pp. 395-417.
[4] Gérard Maral and Michel Bousquet. Satellite communications systems: systems, techniques and technology. John Wiley \& Sons, 2011.
[5] Alexander Graham Bell. "On the production and reproduction of sound by light". In: American journal of science 118 (1880), pp. 305-324.
[6] Keith E Wilson, James R Lesh, and Tsun-Yee Yan. "GOPEX: a laser uplink to the Galileo spacecraft on its way to Jupiter". In: (1993).
[7] KE Wilson. "An overview of the GOLD experiment between the ETS-6 satellite and the table mountain facility". In: (1996).
[8] Keith E Wilson. "Overview of the ground-to-orbit lasercom demonstration". In: (1997).
[9] GD Fletcher, TR Hicks, and Bernard Laurent. "The SILEX optical interorbit link experiment". In: Electronics \& communication engineering journal 3.6 (1991), pp. 273279.
[10] Angel Alonso, Marcos Reyes, and Zoran Sodnik. "Performance of satellite-to-ground communications link between ARTEMIS and the Optical Ground Station". In: Proc. of SPIE Vol. Vol. 5572. 2004, p. 373.
[11] Stefan Seel et al. "Space to ground bidirectional optical communication link at 5.6 Gbps and EDRS connectivity outlook". In: Aerospace Conference, 2011 IEEE. IEEE. 2011, pp. 1-7.
[12] Knut Böhmer et al. "Laser communication terminals for the European data relay system". In: Proc. SPIE. Vol. 8246. 2012, p. 82460D.
[13] Don M Boroson et al. "The lunar laser communications demonstration (LLCD)". In: Space Mission Challenges for Information Technology, 2009. SMC-IT 2009. Third IEEE International Conference on. IEEE. 2009, pp. 23-28.
[14] C Petit et al. "Adaptive optics results with SOTA". In: Space Optical Systems and Applications (ICSOS), 2015 IEEE International Conference on. IEEE. 2015, pp. 1-7.
[15] Bogdan V Oaida et al. "OPALS: an optical communications technology demonstration from the International Space Station". In: Aerospace Conference, 2013 IEEE. IEEE. 2013, pp. 1-20.
[16] Florian Moll and Markus Knapek. "Wavelength selection criteria and link availability due to cloud coverage statistics and attenuation affecting satellite, aerial, and downlink scenarios". In: Proceedings of SPIE-Volume 6709 (2007), pp. 670616-1.
[17] Govind P Agrawal. Fiber-optic communication systems. Vol. 222. John Wiley \& Sons, 2012.
[18] Bernard J. Klein and John J. Degnan. "Optical Antenna Gain. 1: Transmitting Antennas". In: Appl. Opt. 13.9 (Sept. 1974), pp. 2134-2141. DOI: 10. 1364/AO. 13. 002134. URL: http://ao.osa.org/abstract.cfm?URI=ao-13-9-2134.
[19] John J Degnan and Bernard J Klein. "Optical antenna gain. 2: Receiving antennas". In: Applied optics 13.10 (1974), pp. 2397-2401.
[20] Andrei N Kolmogorov. "The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers". In: Dokl. Akad. Nauk SSSR. Vol. 30. 4. JSTOR. 1941, pp. 301-305.
[21] AM Obukhov. Structure of the temperature field in turbulent flow. Tech. rep. DTIC Document, 1968.
[22] AM Yaglom. "On the local structure of a temperature field in a turbulent flow". In: Dokl. Akad. Nauk SSSR. Vol. 69. 6. 1949, p. 743.
[23] Valerian Ilich Tatarski. Wave propagation in a turbulent medium. Courier Dover Publications, 1967.
[24] J.W. Hardy. Adaptive Optics for Astronomical Telescopes. Oxford series in optical and imaging sciences. Oxford University Press, 1998. ISBN: 9780195090192. URL: https://books.google.com.pe/books?id=-0aAWyckS\\_8C.
[25] J Strohbehn and S Clifford. "Polarization and angle-of-arrival fluctuations for a plane wave propagated through a turbulent medium". In: IEEE Transactions on Antennas and Propagation 15.3 (1967), pp. 416-421.
[26] David L Fried. "Optical resolution through a randomly inhomogeneous medium for very long and very short exposures". In: JOSA 56.10 (1966), pp. 1372-1379.
[27] François Chassat. "Propagation optique à travers la turbulence atmosphérique. Etude modale de l'anisoplanétisme et application à l'optique adaptative". PhD thesis. 1992.
[28] Hal T Yura. "Physical model for strong optical-amplitude fluctuations in a turbulent medium". In: JOSA 64.1 (1974), pp. 59-67.
[29] SF Clifford. "The classical theory of wave propagation in a turbulent medium". In: Laser beam propagation in the atmosphere. Springer, 1978, pp. 9-43.
[30] Ronald L Fante. "Electromagnetic beam propagation in turbulent media". In: Proceedings of the IEEE 63.12 (1975), pp. 1669-1692.
[31] SF Clifford, GR Ochs, and RS Lawrence. "Saturation of optical scintillation by strong turbulence". In: JOSA 64.2 (1974), pp. 148-154.
[32] Richard J Sasiela. Electromagnetic wave propagation in turbulence: evaluation and application of Mellin transforms. Vol. 18. Springer Science \& Business Media, 2012.
[33] Robert J Noll. "Zernike polynomials and atmospheric turbulence". In: JOSA 66.3 (1976), pp. 207-211.
[34] Jean-Marc Conan, Gérard Rousset, and Pierre-Yves Madec. "Wave-front temporal spectra in high-resolution imaging through turbulence". In: J. Opt. Soc. Am. A 12.7 (July 1995), pp. 1559-1570. DOI: 10.1364/JOSAA.12.001559. URL: http://josaa. osa.org/abstract.cfm?URI=josaa-12-7-1559.
[35] Nicolas Védrenne et al. "Turbulence effects on bi-directional ground-to-satellite laser communication systems". In: Proc. International Conference on Space Optical Systems and Applications. 2012.
[36] Noah Schwartz. "Précompensation des effets de la turbulence par optique adaptative: application aux liaisons optiques en espace libre". PhD thesis. Université Nice Sophia Antipolis, 2009.
[37] Robert A Schmeltzer. "Means, variances, and covariances for laser beam propagation through a random medium". In: Quarterly of Applied Mathematics 24.4 (1967), pp. 339-354.
[38] Akira Ishimaru. "Fluctuations of a beam wave propagating through a locally homogeneous medium". In: Radio Science 4.4 (1969), pp. 295-305.
[39] L. C. Andrews, R. L. Phillips, and P. T. Yu. "Optical scintillations and fade statistics for a satellite-communication system". In: Appl. Opt. 34.33 (Nov. 1995), pp. 77427751. DOI: 10.1364/A0.34.007742. URL: http://ao.osa.org/abstract.cfm?URI= ao-34-33-7742.
[40] Federico Dios et al. "Scintillation and beam-wander analysis in an optical ground station-satellite uplink". In: Appl. Opt. 43.19 (July 2004), pp. 3866-3873. DOi: 10. 1364/A0.43.003866. URL: http://ao.osa.org/abstract.cfm?URI=ao-43-193866.
[41] Gary J Baker and Robert S Benson. "Gaussian beam scintillation on ground-tospace paths: the importance of beam wander". In: Optical Science and Technology, the SPIE 49th Annual Meeting. International Society for Optics and Photonics. 2004, pp. 225-235.
[42] Larry C Andrews et al. "Strehl ratio and scintillation theory for uplink Gaussianbeam waves: beam wander effects". In: Optical Engineering 45.7 (2006), pp. 076001076001.
[43] Paul J. Titterton. "Power Reduction and Fluctuations Caused by Narrow Laser Beam Motion in the Far Field". In: Appl. Opt. 12.2 (Feb. 1973), pp. 423-425. Doi: 10.1364/ AO.12.000423. URL: http://ao.osa.org/abstract.cfm?URI=ao-12-2-423.
[44] Kamran Kiasaleh. "On the probability density function of signal intensity in freespace optical communications systems impaired by pointing jitter and turbulence". In: Optical Engineering 33.11 (1994), pp. 3748-3757.
[45] MA Al-Habash, Larry C Andrews, and Ronald L Phillips. "Mathematical model for the irradiance probability density function of a laser beam propagating through turbulent media". In: Optical Engineering 40.8 (2001), pp. 1554-1562.
[46] Larry C Andrews et al. "PDF models for uplink to space in the presence of beam wander". In: Proc. SPIE. Vol. 6551. 2007, pp. 655109-1.
[47] Frida Strömqvist Vetelino, Cynthia Young, and Larry Andrews. "Fade statistics and aperture averaging for Gaussian beam waves in moderate-to-strong turbulence". In: Applied optics 46.18 (2007), pp. 3780-3789.
[48] Larry C Andrews, Ronald L Phillips, and Cynthia Y Hopen. Laser beam scintillation with applications. Vol. 99. SPIE press, 2001.
[49] Harold T. Yura and Renny A. Fields. "Level crossing statistics for optical beam wander in a turbulent atmosphere with applications to ground-to-space laser communications". In: Appl. Opt. 50.18 (June 2011), pp. 2875-2885. DOI: 10.1364/A0.50.002875. URL: http://ao.osa.org/abstract.cfm?URI=ao-50-18-2875.
[50] JM Martin and Stanley M Flatté. "Intensity images and statistics from numerical simulation of wave propagation in 3-D random media". In: Applied Optics 27.11 (1988), pp. 2111-2126.
[51] Marc Sechaud et al. "High resolution imaging through atmospheric turbulence: Link between anisoplanatism and intensity fluctuations". In: PROC SPIE INT SOC OPT ENG. Vol. 3866. 1999, pp. 100-109.
[52] Tomoko Nakayama et al. "Application of Compact Optical Duplicate System as a Multi-beam Generation Device for Satellite-Ground Laser Communications". In: Proceedings of ICSOS, P-9 (2014).
[53] Kamran Kiasaleh. "On the scintillation index of a multiwavelength Gaussian beam in a turbulent free-space optical communications channel". In: JOSA A 23.3 (2006), pp. 557-566.
[54] Weihui Liu and Jian Wu. "Influence of space between atmospheric channels and beams' number on scintillation". In: Chinese Optics Letters 1.11 (2003), pp. 621623.
[55] Wu-ming Wu et al. "Scintillation analysis for multiple uplink Gaussian beams in the presence of beam wander". In: Selected Proceedings of the Photoelectronic Technology Committee Conferences held July-December 2013. International Society for Optics and Photonics. 2014, pp. 914223-914223.
[56] Ramon Mata Calvo et al. "Transmitter diversity verification on ARTEMIS geostationary satellite". In: SPIE LASE. International Society for Optics and Photonics. 2014, pp. 897104-897104.
[57] Jeffrey H Shapiro. "Reciprocity of the turbulent atmosphere". In: JOSA 61.4 (1971), pp. 492-495.
[58] H. Bischl et al. ORELIA - Feasibility Assessment of Optical Technologies \& Techniques for Reliable High Capacity Feeder Links. Tech. rep. European Space Agency, 2010.
[59] David L Fried. "Anisoplanatism in adaptive optics". In: JOSA 72.1 (1982), pp. 52-61.
[60] Thierry Fusco. "Correction partielle et anisoplanetisme en Optique Adaptative: traitements a posteriori et Optique Adaptative Multiconjuguée". PhD thesis. Nice, 2000.
[61] Robert K Tyson. "Adaptive optics and ground-to-space laser communications". In: Applied optics 35.19 (1996), pp. 3640-3646.
[62] Robert K. Tyson. "Bit-error rate for free-space adaptive optics laser communications". In: J. Opt. Soc. Am. A 19.4 (Apr. 2002), pp. 753-758. DOI: $10.1364 / \mathrm{JOSAA} .19$. 000753. URL: http://josaa.osa.org/abstract.cfm?URI=josaa-19-4-753.
[63] Richard J Sasiela. A unified approach to electromagnetic wave propagation in turbulence and the evaluation of multiparameter integrals. Tech. rep. MASSACHUSETTS INST OF TECH LEXINGTON LINCOLN LAB, 1988.
[64] Svilen Dimitrov et al. "Digital modulation and coding for satellite optical feeder links with pre-distortion adaptive optics". In: International Journal of Satellite Communications and Networking 34.5 (2016), pp. 625-644.
[65] Santasri Basu, David Voelz, and Deva K Borah. "Fade statistics of a ground-tosatellite optical link in the presence of lead-ahead and aperture mismatch". In: Applied optics 48.7 (2009), pp. 1274-1287.
[66] Larry C Andrews and Ronald L Phillips. Laser beam propagation through random media. Vol. 1. SPIE press Bellingham, WA, 2005.
[67] HT Yura. "Optimum truncation of a Gaussian beam for propagation through atmospheric turbulence". In: Applied optics 34.15 (1995), pp. 2774-2779.
[68] Karin Weiss-Wrana and Leslie S Balfour. "Statistical analysis of measurements of atmospheric turbulence in different climates". In: International Symposium on Remote Sensing. International Society for Optics and Photonics. 2002, pp. 93-101.
[69] Tomás Soler and David W Eisemann. "Determination of look angles to geostationary communication satellites". In: Journal of surveying engineering 120.3 (1994), pp. 115127.
[70] Virendra N Mahajan. "Uniform versus Gaussian beams: a comparison of the effects of diffraction, obscuration, and aberrations". In: JOSA A 3.4 (1986), pp. 470-485.
[71] Hamid Hemmati. Deep space optical communications. Vol. 11. John Wiley \& Sons, 2006.
[72] Darryl P Greenwood. "Tracking turbulence-induced tilt errors with shared and adjacent apertures". In: JOSA 67.3 (1977), pp. 282-290.
[73] Virendra N. Mahajan. "Zernike annular polynomials for imaging systems with annular pupils". In: J. Opt. Soc. Am. 71.1 (Jan. 1981), pp. 75-85. DOI: 10.1364/JOSA. 71. 000075. URL: http://www.osapublishing.org/abstract.cfm?URI=josa-71-1-75.
[74] Jean-Marc Conan, Gérard Rousset, and Pierre-Yves Madec. "Wave-front temporal spectra in high-resolution imaging through turbulence". In: J. Opt. Soc. Am. A 12.7 (July 1995), pp. 1559-1570. DOI: 10.1364/JOSAA.12.001559. URL: http://josaa. osa.org/abstract.cfm?URI=josaa-12-7-1559.
[75] Bernhard Epple. "Simplified channel model for simulation of free-space optical communications". In: Journal of Optical Communications and Networking 2.5 (2010), pp. 293-304.
[76] Prediction method of fade dynamics on Earth-space paths. Tech. rep. International Telecommunication Union (ITU), Recommendation ITU-R P.1623-1, 2005.
[77] Nina Leonhard et al. "Real-time adaptive optics testbed to investigate point-ahead angle in pre-compensation of Earth-to-GEO optical communication". In: Optics express 24.12 (2016), pp. 13157-13172.
[78] Aoife Brady et al. "Experimental validation of phase-only pre-compensation over 494 m free-space propagation". In: Optics Letters 42.14 (2017), pp. 2679-2682.
[79] Stephen B Alexander. Optical communication receiver design. Vol. 37. SPIE Press, 1997.
[80] F Zernike. "Diffraction theory of the knife-edge test and its improved form, the phasecontrast method". In: Monthly Notices of the Royal Astronomical Society 94 (1934), pp. 377-384.

Titre : Compensation des effets de la turbulence atmosphérique sur un lien optique montant solsatellite géostationnaire : impact sur l'architecture du terminal sol

Mots clés : optique adaptative ; propagation ; communications optiques en espace libre


#### Abstract

Résumé : Un lien optique basé sur un multiplex de longueurs d'onde autour de $1,55 \mu \mathrm{~m}$ est une alternative intéressante pour pallier la saturation des bandes radio-fréquences classiquement utilisées et pour répondre aux besoins de liens haut débit par satellite géostationnaire de la prochaine génération de télécommunication. Compte-tenu de la puissance limitée des lasers envisagés, la divergence du faisceau doit être considérablement réduite. Par conséquent, le pointage du faisceau devient un paramètre critique. Au cours de sa propagation entre la station sol et un satellite géostationnaire, le faisceau optique est dévié et éventuellement déformé par la turbulence atmosphérique. Cela induit de fortes fluctuations du signal de télécommunication détecté, réduisant le débit disponible. Un miroir de basculement est utilisé pour pré-compenser la déviation mesurée à partir d'un faisceau provenant du satellite. Du fait de l'angle de pointage en avant entre la liaison des-


cendante et la liaison montante, les effets de turbulence subis par les deux faisceaux sont légèrement différents, ce qui induit une erreur dans la correction.
Le critère de performance de la liaison est l'intensité minimale détectable $95 \%$ du temps. Un modèle rapide, nommé WPLOT, prenant en compte les erreurs de pointage et leur évolution temporelle, est proposé pour évaluer cette intensité minimale en fonction des paramètres de la station sol et de la qualité de la correction. Les résultats obtenus avec ce modèle sont comparés avec ceux obtenus par un modèle physique mais plus couteux en temps de calcul; le code TURANDOT. Grâce à ce modèle, une étude de sensibilité a été réalisée et a permis de proposer un dimensionnement de la station sol. Ce modèle permet également de générer des séries temporelles afin d'optimiser les codes de correction d'erreur et optimiser le débit (1Terabit/s d'ici 2025).

Title : Mitigation of turbulence effects on ground-to-geostationary link: impact on ground terminal architecture

Keywords : adaptive optics; propagation; optical free-space communications


#### Abstract

An optical link based on a multiplex of wavelengths at $1.55 \mu \mathrm{~m}$ is foreseen to be a valuable alternative to the conventional radiofrequencies for the feeder link of the nextgeneration of high throughput geostationary satellite. Considering the limited power of lasers envisioned for feeder links, the beam divergence has to be dramatically reduced. Consequently, the beam pointing becomes a key issue. During its propagation between the ground station and a geostationary satellite, the optical beam is deflected and possibly distorted by atmospheric turbulence. It induces strong fluctuations of the detected telecom signal, thus reducing the capacity. A steering mirror using a measurement from a beam coming from the satellite is used to pre-compensate the deflection. Because of the point-ahead angle between the downlink and the


uplink, the turbulence effects experienced by both beams are slightly different, inducing an error in the correction. The performance criteria is the minimum detectable irradiance $95 \%$ of the time. A fast model, named WPLOT, taking into account pointing errors and their temporal evolution, is proposed to evaluate the minimum irradiance as a function of the ground station parameters and quality of the correction. The model's results are compared to those obtained with a more physical but requiring more computation power: TURANDOT. A sensitivity study has been realized and led to a sizing of a ground station. The model also enables the generation of time series in order to optimize the forward error correction codes in order to be compliant with the targeted capacity (1Terabit/s by 2025).


[^0]:    ${ }^{1}$ We have neglected the thermal noise.

[^1]:    ${ }^{1}$ We will not consider a spherical wave here.

[^2]:    ${ }^{2}$ Another convention, for example Fante[30], defines the radius at $1 / \mathrm{e}$ of the irradiance distribution. The radius size value using this convention is smaller than the radius size value in the case defined above by a factor $\sqrt{2}$ in amplitude and 2 in intensity. It is therefore key to verify which convention is being used when studying beam radii.

[^3]:    ${ }^{3}$ The results are slightly modified compared to the initial formula from [30] due to the difference in waist definition

[^4]:    ${ }^{4}$ It is important to keep in mind that the scintillation index is normalized. Moreover, in this subsection, the irradiances are normalized, i.e. $\langle I\rangle=1$.

[^5]:    ${ }^{5}$ Note that this is the same assumption as considering that beam wander results from refraction through large eddies while beam spread results from small eddies.

[^6]:    ${ }^{1}$ We present the results in the $x$ direction but the method is exactly the same in the $y$ direction.

[^7]:    ${ }^{2}$ The other lobes resulting from the circular aperture are therefore neglected.

[^8]:    ${ }^{3}$ All the irradiances presented in this Chapter are normalized. This means that $\iint I_{0} \exp \left(-r^{2} / w^{2}\right) r d r d \theta=1$.
    ${ }^{4}$ I have verified that the size of the phase screens acted as an outer scale in TURANDOT by comparing a tilt estimation obtained from measuring the beam displacement in TURANDOT and Chassat's analytical expressions for a von Kármán spectrum. The results fit well but are not presented in this manuscript.

[^9]:    ${ }^{5}$ We observe on TURANDOT's PSDs a spectrum folding which we attribute to the sampling of the phase screens.

[^10]:    ${ }^{1}$ Figure 8.3 and Figure 8.4 have been obtained thanks to Karim Elayoubi's work with the VPI simulator.

