PARIS-SACLAY

# Contribution to the sizing of the modular multilevel converter 

Thèse de doctorat de l'Université Paris-Saclay
préparée à l'Université Paris-Sud
Ecole doctorale $n^{\circ} 575$ Electrical, optical, bio-physics and engineering (EOBE)
Spécialité de doctorat : Génie Electrique

Thèse présentée et soutenue à Gif-sur-Yvette, le 25/09/2019, par

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#### Abstract

The modular multilevel converter is a suitable solution for high voltage direct current grids thanks to its modularity, low switching frequency and quasi-sinusoidal AC voltage. However, due to its topology, its mathematical model is quite complex and is therefore often simplified at the design stage. In particular, the arm equivalent resistance R, the arm inductance L and the circulating current are often neglected. But experimental results obtained with our 1-ph 6-level full-bridge MMC prototype showed that these hypotheses are not always acceptable. In this context, the goal of this thesis is to study the impact of accounting for R , L and the circulating current on the module capacitor voltage and on the operating area of the converter.

First, we extended the commonly used integral based model and we clarified the hypotheses behind it. Among others, expressions for the circulating and dc currents have been developed and compared with the one that can be found in the literature. It allowed us to analyze the module capacitor voltage ripple as a function of $R$ and $L$, without circulating current only.

Second, to overcome the limitations of the integral based model, we proposed to use a steady state time invariant $\Delta \Sigma$ MMC model in $d q 0$ frame. Only few hypotheses are required to obtain this model, but a numerical evaluation is required. It allowed us to analyze the module capacitor average voltage and the module capacitor voltage ripple as a function of R and L , with and without circulating current.

Third, using the steady state time invariant model, we developed a detailed PQ diagram of the MMC. In addition to the conventional AC current limit, DC current limit and modulation index limit, we added several internal limits: IGBT current, arm rms current and module capacitor voltage and current ripple. The results have been confirmed by numerical simulation using a detailed Matlab Simulink SimPowerSystems model.

The results presented in this thesis could be used to optimize the sizing of the components of the MMC considering its operating area, and to assess the impact of different parameters on the MMC performance.


## Résumé

## Introduction

Pour faire face aux défis tant du côté de la génération et que de celui de la consommation, le réseau de transport d'énergie électrique évolue. En parallèle du réseau à courant alternatif haute tension (HVAC), des liaisons à courant continu haute tension (HVDC) sont mises en place. De nos jours, la transmission HVDC est déjà utilisée dans plus de 150 installations à travers le monde. Les raisons de ce déploiement rapide sont multiples :

- des coûts réduits pour le transport de l'électricité sur de longues distances par rapport à la technologie HVAC
- la possibilité de transporter de l'énergie électrique par câbles HVDC souterrains ou sous-marins, au-delà de la limite technique des 50 km traditionnellement associée à la technologie HVAC
- l'augmentation des besoins en termes d'interconnexion électrique entre les pays, en particulier dans le contexte du développement du marché de l'électricité
- la possibilité de connecter des réseaux électriques AC asynchrones.

Parmi les convertisseurs AC/DC, le convertisseur modulaire multiniveau (MMC) est une solution adaptée aux réseaux HVDC. En comparaison avec le convertisseur VSC à deux niveaux, le MMC possède plusieurs avantages :

- basse fréquence de commutation,
- basse distorsion harmonique totale (THD),
- topologie modulaire qui peut évoluer pour s'adapter aux hautes puissances,
- fiabilité inhérente.


## Difficultés

## Courant de circulation

Le courant de circulation influence le courant nominal des composants du MMC. C'est donc l'un des facteurs à prendre en compte lors de la conception du convertisseur. Mais la relation entre les éléments passifs du bras du MMC (résistance du bras et inductance du bras) et le courant de circulation n'a pas été étudiée en détails dans la littérature.

## Tension des condensateurs

L'énergie stockée dans le MMC détermine la valeur des condensateurs des modules. Cela a un impact direct sur la taille, le poids et le coût du convertisseur. Or l'influence des éléments passifs du bras du MMC sur la tension des condensateurs est souvent négligée dans la littérature.

## Objectif de thèse

Compte tenu des difficultés soulignées, il est nécessaire de clarifier l'influence de la résistance et de l'inductance du bras sur le courant de circulation et sur la tension des condensateurs des modules (moyenne et ondulation). C'est l'objectif principal de cette thèse.

Avec une meilleure connaissance de ces deux grandeurs internes, on peut alors mieux définir la zone d'opération du convertisseur. Pour ce faire, nous proposons de compléter le diagramme PQ du MMC avec plusieurs limites internes.

## Modélisation du MMC

Dans un premier temps, nous avons redéveloppé trois modèles du MMC. Inspirée de la littérature, la dérivation est complétée et clarifiée. Une attention particulière a été portée au formalisme et à l'identification des hypothèses requises. Le convertisseur a d'abord été modélisé en fonction des grandeurs de bras dans le repère abc (modèle $U L a b c$ ). Puis ce modèle a été transformé afin de faire intervenir les grandeurs des côtés AC et DC du convertisseur (modèle $\Delta \Sigma a b c$ ). Finalement, deux transformées de Park ont été utilisées afin d'obtenir le modèle du convertisseur dans le repère $d q 0$ (modèle $\Delta \Sigma d q 0$ ).

## Modèle intégral

Obtenu à partir du modèle $\Delta \Sigma a b c$, nous avons proposé une extension du « modèle intégral» proposé dans la littérature. Cette extension permet de prendre en compte les éléments passifs du bras du convertisseur. Elle nous a permis de dériver des expressions analytiques plus générales de certaines grandeurs internes du MMC : courant du réseau DC , courant de circulation, tension du condensateur du module. A l'aide de cette dernière expression, nous avons analysé l'ondulation de la tension du module en fonction de la résistance et de l'inductance du bras. Nous avons souligné qu'un choix judicieux de l'inductance est nécessaire afin de respecter les limites de la tension du condensateur du module. Comme l'ondulation de tension est maximale lorsque le transfert de puissance active est faible et la génération/consommation de puissance réactive est importante, la sélection des éléments passifs est particulièrement importante pour un MMC utilisé comme compensateur statique d'énergie réactive (STATCOM). Cependant, de nos jours, de nombreux MMC sont conçus pour transférer de la puissance active (et une puissance réactive relativement faible) [1]. Dans ce cas, l'ondulation de la tension du condensateur est faible et le courant de bras devient la grandeur dimensionnante pour la capacitance. L'utilisation de l'ondulation de tension maximale comme critère de conception amènerait à un condensateur surdimensionné. Du point de vue énergétique, plusieurs auteurs ont étudié le MMC comme un dispositif stockant l'énergie [44, 45, 46]. Cette étude est également bénéfique dans la mesure où elle élabore le comportement énergétique instantané en tenant compte de la résistance et de l'inductance de bras du MMC. Le modèle intégral permet d'obtenir des expressions analytiques, cependant il est basé sur des équations dans le domaine temporel et fait appel à de nombreuses hypothèses.

## Modèle invariant dans le temps en régime permanent (SSTI)

Obtenu à partir du modèle $\Delta \Sigma d q 0$, nous avons proposé une nouvelle dérivation du < modèle invariant dans le temps en régime permanent » (SSTI) proposé par [75, 76]. Ce
modèle nous a permis d'évaluer les grandeurs internes du convertisseur sans faire appel aux hypothèses requises pour développer le modèle intégral. En particulier, il est possible de prendre en compte l'impact du courant de circulation. Par contre, une procédure numérique est nécessaire.

Dans la littérature, de nombreuses simplifications ont été envisagées en raison de la complexité de la méthode à base intégrale. Habituellement, ces simplifications considèrent que l'inductance du bras et la résistance du bras sont négligées. A côté de cela, le courant circulant est souvent supprimé. De plus, le choix de l'inductance peut provoquer la résonance du courant circulant. Le courant de circulation faisant partie du courant du bras, la tension du condensateur est également fonction du courant de circulation. Par conséquent, le choix de l'inductance affecte clairement la tension du condensateur et donc l'énergie instantanée du condensateur.

## Tension moyenne du condensateur du module

Dans la littérature, deux modèles ont été proposés afin d'évaluer la tension moyenne du condensateur du module. Le premier modèle néglige les éléments passifs ainsi que le courant de circulation : la tension moyenne des condensateurs est constante. Le deuxième modèle inclut la résistance du bras mais néglige son inductance : la tension moyenne des condensateurs varie linéairement en fonction de la puissance. Le modèle SSTI nous a permis d'évaluer la tension moyenne du condensateur du module en fonction du point de fonctionnement du convertisseur (et de son contrôle) pour différentes valeurs de la résistance et de l'inductance de bras. La tension moyenne du condensateur diminue avec l'augmentation de la puissance mais aussi avec l'augmentation des valeurs des éléments passifs du bras. Les résultats ont été comparés avec deux modèles précédents.

## Ondulation de tension du condensateur du module

Le modèle SSTI nous a également permis d'étudier l'ondulation de tension du condensateur du module en fonction des éléments passifs de bras. Lorsque les éléments passifs sont négligés, les résultats du modèle SSTI concordent avec la littérature. Lorsque les éléments passifs sont pris en compte, nous avons mis en évidence que (i) l'ondulation augmente lorsque l'inductance augmente et (ii) l'ondulation en fonction de la puissance active est asymétrique lorsque la résistance augmente.

## Diagrammes PQ

Le diagramme $\mathrm{PQ} \ll$ conventionnel $\gg$ du MMC comporte 3 limites : la limite en courant AC , la limite en courant DC et la limite en modulation. Les deux premières limites sont indépendantes du modèle du MMC. Pour obtenir la dernière limite, un modèle qui réduit le MMC à une source de tension en série avec un circuit RL est souvent utilisé. Ce modèle nécessite des hypothèses rarement justifiées, notamment une résistance du bras nulle et une tension interne idéale [74, 73, 77]. Nous avons proposé un diagramme PQ du MMC plus précis et plus complet. Pour cela, nous avons utilisé le modèle SSTI. D'une part, cela nous a permis de déterminer une limite en modulation qui s'affranchisse des hypothèses précédentes. D'autre part, cela nous a permis de tracer d'autres limites : la limite en courant de l'IGBT, la limite en courant de bras, la limite en ondulation de tension de condensateur et la limite en courant de condensateur. L'intersection des zones intérieures aux limites précédentes définit la zone d'opération du convertisseur. Nous avons effectué des simulations avec un modèle détaillé implémenté à l'aide de la toolbox Matlab/Simulink

SimPowerSystems. Les résultats de simulation sont en accord avec les résultats théoriques. Pour les points d'opération à l'intérieur de zone d'opération, les variables examinées sont à l'intérieur des limites. Dès que le point d'opération sort de la zone d'opération, les variables examinées dépassent les limites.

## Conclusion

Le convertisseur multiniveau modulaire est une solution appropriée pour les réseaux HVDC en raison de ses nombreux avantages par rapport aux convertisseurs de source de tension à 2 niveaux. Considérant la nécessité de mieux comprendre le comportement de ce convertisseur afin de le dimensionner correctement, l'objectif de cette thèse était de clarifier l'influence de la résistance équivalente du bras et de l'inductance du bras sur le courant de circulation et sur la tension du condensateur du module. Pour ce faire, nous avons étendu plusieurs modèles de la littérature, les avons comparés et nous avons proposé de visualiser certains résultats à l'aide d'un diagramme PQ. Les modèles et les résultats présentés ici pourraient être utiles aux fabricants de convertisseurs. En effet, une meilleure compréhension de l'impact des éléments passifs des bras du MMC sur les grandeurs internes (courant de circulation et tension des condensateurs) permet d'améliorer le dimensionnement du MMC. Le diagramme PQ du MMC qui a été proposé ici pourrait servir aux fabricants de convertisseur et aux gestionnaires de réseaux électriques. En phase de conception, il permet de vérifier que le composant sélectionné permet au convertisseur de couvrir la zone d'opération spécifiée par le cahier des charges. En phase d'opération, ce diagramme permet de définir de manière précise une zone d'opération du convertisseur qui offre une marge de sécurité suffisante, tout en prenant en compte l'évolution des différents paramètres du convertisseur.

Tout d'abord, une démonstration complète du modèle MMC $\Delta \Sigma$ dans les repère $a b c$ et $d q 0$ a été élaborée. Les différentes hypothèses requises pour dériver ces modèles ont été clarifiées.

Deuxièmement, à partir du modèle MMC $\Delta \Sigma$ dans le repère $a b c$, un modèle basé sur l'intégral (IB) étendue a été obtenu. Cela nous a permis de trouver une expression de courant de circulation généralisée. Cette expression a permis de quantifier la réduction de l'amplitude du courant circulant à mesure que l'inductance du bras augmentait. De plus, une expression généralisée du courant réseau continu a été dérivée. Cela nous a permis d'expliquer pourquoi le courant du réseau continu est toujours différent de zéro, même pour un transfert de puissance réactive pure. Et une expression généralisée d'ondulation de tension de condensateur de module a été développée. En comparaison avec l'expression de la littérature, il permet d'étudier l'influence des éléments du bras passif sur la tension du condensateur. Mais la méthode à base intégrale présente plusieurs limitations.

Troisièmement, à partir du modèle MMC $\Delta \Sigma$ dans $d q 0$ frame, le modèle invariant dans le temps en régime permanent (SSTI) d'un convertisseur multiniveau modulaire a été obtenu. Cela nous a permis, pour la première fois, d'estimer la tension moyenne du condensateur du module en fonction du point de fonctionnement. Par exemple, nous avons observé que la tension moyenne du condensateur du module diminuait plus rapidement avec la puissance pour les valeurs élevées des éléments passifs du bras. De plus, le SSTI nous a permis d'étudier l'ondulation de la tension du condensateur du module. Nous avons remarqué un comportement asymétrique de l'ondulation de tension en fonction du transfert de puissance active. Nous avons déterminé que cette asymétrie est due à la résistance équivalente du bras. De plus, nous avons étudié l'ondulation de tension en fonction de la valeur d'inductance du bras et trouvé le comportement de résonance attendu.

Quatrièmement, le diagramme PQ de la console MMC a été construit à l'aide du modèle SSTI. Cette nouvelle méthode de construction nous a permis de nous débarrasser des simplifications habituellement apportées pour tracer le diagramme PQ de la MMC. En particulier, il s'est avéré possible de montrer l'influence des éléments du bras passif sur la limite d'indice de modulation. Nous avons conclu que la zone d'utilisation du MMC diminue lorsque l'inductance du bras augmente. En outre, nous avons représenté certaines des limites internes de la MMC dans le diagramme PQ : limite de courant IGBT, limite de courant RMS du bras, ondulation de la tension du condensateur du module et ondulation du courant du condensateur du module. Ce diagramme PQ est utile car il permet d'expliquer les comportements obtenus avec des modèles de simulation détaillés.

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## Chapter 1

## Introduction

### 1.1 The HVDC systems of today

To face the new challenges in generation and consumption, the electrical grid is evolving to include both high voltage alternating current (HVAC) and high voltage direct current (HVDC) systems $[1,2,3$, $4,77,6,13]$. Nowadays, HVDC transmission has been used in over 150 installations worldwide [6]. There are four principal motivations for using HVDC over HVAC grids [77]:

1. Costs: Fig. 1.1 shows a comparison of the cost between HVDC and HVAC transmission systems. The capital investment is higher for HVDC because of the converter price. Over a break-even distance, the overall cost becomes lower for a DC system than for an AC system [6]. The breakeven distance for submarine cables ${ }^{1}$ is 50 km . The break-even distance for overhead lines ${ }^{2}$ is 600 to 800 km .
2. Underground cable: Up until now, overhead transmission lines were often used because of their cost advantage benefits when compared to underground cables. Today, because of the political, social and environmental pressure [58, 59], it is more and more difficult to build new overhead lines. As a result, the usage of underground cables is constantly increasing. ${ }^{34}$ Besides, when compared to HVAC overhead lines, HVDC grids need 2 to 3 times smaller right of way for the same rating.
3. Power demand: The power demand is constantly rising. ${ }^{5}$ The existing HVAC grid needs reinforcement in order to increase the amounts of power transfer over long distances [80, pg. 4.].
4. Asynchronous grid interconnection: An HVDC grid can interconnect two AC systems having different frequencies. We witness different cases around the world. ${ }^{6}$


Figure 1.1 - HVDC and HVAC cost comparison (adapted from [6, 77])
HVDC grids can be categorized into two groups:

1. LCC-based HVDC: This is the oldest HVDC technology. It consists of Line Commutated Converters (LCC) using thyristors. They need big filters and a strong grid in order to operate. Nowadays they are present in the HVDC grids with high voltages and high power ranges.
2. VSC-based HVDC: Contrary to the LCC, the VSC can achieve a relatively fast power reversal (no DC voltage reversal), it can control the reactive power on the AC side and it has a "black start" capability (necessary for offshore applications). This type of HVDC technology features either 2level Voltage Source Converters (VSC) using IGBTs, or Modular Multilevel Converters (MMC) using IGBTs.
[^0]
### 1.2 Modular multilevel converter

In 2003, the modular multilevel converter was proposed by Lesnicar et al. [70].

### 1.2.1 Advantages and drawbacks

The MMC is a suitable solution for several applications because of its benefits compared to the 2-level VSC [5, 77, 6, 80],

- low switching frequency and thus lower switching losses [60],
- low total harmonic distortion (THD) on the AC side implying a reduced footprint of the filters [80],
- modular design enabling scalability to higher voltage and power ratings [6],
- inherent reliability (in case of a module failure, the desired output voltage can still be obtained because of the module redundancy) [61, 6].

Nevertheless, the MMC has some disadvantages in comparison to conventional converters. These include,

- complex mathematical model and control system since the MMC features a high number of modules [75, 76]
- capacitor voltage balancing is challenging because of the requirement of fast simultaneous control [6],
- higher conduction losses because of the high number of switches [60],
- difficult choice of passive elements [23, 53],
- higher number of switches [6].


### 1.2.2 Overview and conventions

A Modular Multilevel Converter (MMC) is an AC-DC converter (Fig. 1.2). A detailed view of a 3 -phase MMC is shown in Fig. 1.3. Each phase (leg) of the converter has two arms. Each arm has N modules connected in series with an arm inductance.


Figure 1.2 - Overview of a 3-phase MMC and conventions

### 1.2.3 Principle of operation

It is assumed that each module is a half-bridge module $(\mathrm{HBm})$ ie. a module composed of two IGBT switches with antiparallel diodes and a capacitor (Fig. 1.4). The half-bridge modes of operation are shown in the Fig. 1.5. At any time, exactly $N$ modules are $\mathrm{ON}\left(v_{\bmod }=v_{c a p}\right)$ and N modules are OFF $\left(v_{\text {mod }}=0\right)$ in each phase of the MMC. With the Nearest Level Control (see subsection A.1.3), the AC phase-to-ground voltage has $N+1$ levels $[6,80,77,75,76]$.


Figure 1.3 - Detailed view of a 3-phase MMC and conventions

### 1.3 MMC prototype

### 1.3.1 Prototype overview

A laboratory scale 1-phase 6-level MMC prototype has been developed at the GeePs laboratory, Figs. 1.6 and 1.7. This was a joint project between the laboratory GeePs at CentraleSupélec, France and the laboratory GPEC at Universidade Federal do Cearà, Brazil [69]. Details about the prototype conception and the implementation of the control are given in Appendix A. 2 and A.3. The prototype parameters are summarized in Table 1.1.

Table 1.1 - MMC experimental parameters

| Parameter | Variable | Value |
| :---: | :---: | :---: |
| DC voltage | $v_{d c}$ | 150 V |
| DC bus capacitance | $C_{b}$ | $4400 \mu \mathrm{~F}$ |
| Number of modules per arm | $N$ | 5 |
| Module capacitance | $C$ | $2240 \mu \mathrm{~F}$ |
| Commutation frequency | $f_{s w}$ | 100 kHz |
| Arm inductor resistance | $R$ | $0.1 \Omega$ |
| Arm coil inductance | $L$ | 30 mH |
| Load resistance | $R_{o}$ | $6 \Omega$ |



Figure 1.4 - Topology of modules: Half-bridge module (left), Full-bridge module (right)


Figure 1.5 - Half-bridge module operation

### 1.3.2 Experimental results

In this section, the steady-state open-loop (see Appendix A.3) experimental results are presented. We compare the results with the simulation, also with the open-loop control (see Appendix A.1).

The DC side of the converter is connected to a DC source $v_{d c}$ via DC bus capacitors $C_{b}$. The AC side of the converter is connected to a resistive load $R_{0}$. The upper and lower arm voltages, $v_{u}$ and $v_{l}$ are controlled to form a staircase quasi-sinusoidal voltage (see sections A.1.3 and A.1.4). This creates a quasi-sinusoidal voltage $v_{s}$. With the open loop control the load current $i^{\Delta}$ is not controlled and it depends on the load.

The experiment is conducted for three values of $L$ at the active AC power of 300 W .


Figure 1.6 - Electrical circuit and connection of the 1-phase MMC prototype


Figure 1.7 - Photo of the 1-phase prototype


Figure 1.8 - Upper and lower arm voltages and module capacitor voltages experimental (blue) and simulation results (red)

## Results

The Fig. 1.8 shows the upper arm voltage $v_{u}$, the lower arm voltages $v_{l}$, the module capacitor voltage $v_{\text {cap }}$, the AC grid current $i^{\Delta}$, the DC grid current $i_{d c}$, upper and lower arm current $i_{u}$ and $i_{l}$, and the circulation current $i_{2}^{\Sigma}$, for different values of $L$.

The voltages $v_{u}$ and $v_{l}$ have six levels, an amplitude slightly lower than $v_{d c}$ and a frequency of 50 Hz . This shows that the NLC and CVB algorithms work properly. The voltage $v_{c a p}$ has a mean value lower than $v_{d c} / N$ : for $L=18 \mathrm{mH}$ it is 24 V , for $L=30 \mathrm{mH}$ it is 23 V and for $L=42 \mathrm{mH}$ it is 22.5 V . The voltage ripple decreases slightly when $L$ increases (especially visible from the simulation results). For $L=18 \mathrm{mH}$ it is $13.3 \%$, for $L=30 \mathrm{mH}$ it is $10 \%$ and for $L=42 \mathrm{mH}$ it is $8.3 \%$ of $v_{d c} / N$.

The currents $i^{\Delta}, i_{u}$ and $i_{l}$ are quasi-sinusoidal at 50 Hz . Let us remind that in this case the converter operates in the open-loop, and therefore $i^{\Delta}$ is not controlled. It depends only on the arm voltages and on the load. The sinusoidal waveform shows that the arm inductance play their role of smoothing correctly.

The circulating current $i_{2}^{\Sigma}$ has a zero mean value and a frequency of 100 Hz . Its amplitude decreases when $L$ increases ( 0.5 A for $L=18 \mathrm{mH}, 0.3 \mathrm{~A}$ for $L=30 \mathrm{mH}$ and 0.1 A for $L=42 \mathrm{mH}$ ).

### 1.4 Issues

### 1.4.1 Impact of the arm passive elements on the circulating current

The circulating current influences the current rating of the components of the MMC. It is one of the factors to consider at the design stage of the converter. As observed in Fig. 1.8, the arm inductance can play a role in reducing the circulating current. But only few authors actually studied the relationship between the arm passive elements (the arm resistance and the arm inductance) and the circulating current [23].

### 1.4.2 Impact of the arm passive elements on the module capacitor average voltage

The rated energy stored in the module capacitors of the MMC is the primary factor of the design of the module capacitor [49]. It directly impacts the size, the weight and the cost of the converter. As observed in Fig. 1.8, the module capacitor average voltage can vary with the value of the arm passive elements. But only few authors pointed that out [44].

### 1.4.3 Impact of the arm passive elements on the module capacitor voltage ripple

The module capacitor voltage ripple is a key point to consider when selecting the module capacitor. Even though this has been studied by many groups [49, 20, 50, 51], the influence of the arm passive elements was often neglected in the analysis. But as observed in Fig. 1.8, the arm passive elements have a clear impact on the module capacitor voltage ripple. Therefore, it should be further discussed.

### 1.5 Aim of the thesis

Considering the issues underlined in section 1.4, it is necessary to clarify the influence of the arm resistance and arm inductance on the module capacitor voltage (average and ripple), and on the circulating current. This is the primary objective of this thesis.

With a better knowledge of the module capacitor voltage and of the circulating current, one can then define the safe operating area of the converter. To do so, we propose to complete the conventional PQ diagram of the MMC [74, 73, 77, 6, 72] with several internal limits.

### 1.6 Thesis content

This thesis is organized as follows :

1. In Chapter 2 we develop the UL and $\Delta \Sigma$ mathematical models for an MMC in $a b c$ and $d q 0$ frame.
2. In Chapter 3 we extend the integral-based model of the MMC. It allows us to derive more general analytical expression for the circulating current, the DC grid current and the module capacitor voltage ripple. The limitations of the integral-based model are underlined.
3. In Chapter 4 we introduce the steady state time invariant MMC $\Delta \Sigma$ model in $d q 0$ frame. This model enables us to study the MMC with as few simplifications as possible. We study the evolution of the module capacitor average voltage and voltage ripple as a function of the passive arm elements.
4. In Chapter 5 we build the PQ diagram of the MMC. We elaborate some of the internal converter limits and we examine the influence of the passive arm elements on the operating area of the converter. The results are discussed through detailed MMC model simulations.

## Chapter 2

## Modeling of the Modular Multilevel Converter

Objectives: Definition of the conventions used in this thesis. Derivation of the MMC UL model in $a b c$ frame, the MMC $\Delta \Sigma$ model in $a b c$ frame and the MMC $\Delta \Sigma$ model in $d q 0$ frame. Motivation: Lack of precision of the derivation of the MMC equations that can be found in the literature.

Contributions: Rigorous derivation of the equations describing the MMC behavior in $a b c$ and $d q 0$ frames. In particular, clarification of the hypotheses required to derive these models.

### 2.1 Modeling of the MMC

The conventions adopted in this thesis are summarized in Fig. 1.2 and 1.3. In the following, we derive the UL MMC model in $a b c$ frame, the $\Delta \Sigma$ MMC model in $a b c$ frame and the $\Delta \Sigma$ MMC model in $d q 0$ frame. The reader can jump to section 2.2 for a summary of the three models.

### 2.1.1 Mathematical model of a 1-phase MMC

Here we develop the 1-phase MMC equations. The phase index $\{a, b, c\}$ is removed for the sake of clarity.

### 2.1.1.1 Kirchoff's laws

Applying the Kirchhoff's voltage law,

$$
\begin{align*}
& v_{s}(t)=\frac{v_{d c}(t)}{2}-v_{u}(t)-R i_{u}(t)-L \frac{d i_{u}(t)}{d t}  \tag{2.1}\\
& v_{s}(t)=-\frac{v_{d c}(t)}{2}+v_{l}(t)+R i_{l}(t)+L \frac{d i_{l}(t)}{d t} \tag{2.2}
\end{align*}
$$

where $v_{d c}$ is the DC voltage, $L$ is the arm inductance, $R$ is the equivalent arm resistance ${ }^{1}$, $v_{s}$ is the AC phase-to-ground voltage, $v_{u}$ and $v_{l}$ are the upper and lower stack voltages, and $i_{u}$ and $i_{l}$ are the upper and lower arm currents respectively. Throughout this document we assume that the DC link voltage is constant,

$$
\begin{equation*}
v_{d c}(t) \stackrel{\text { hyp }}{=} \text { const. } \tag{2.3}
\end{equation*}
$$

Adding Eqs. (2.1)-(2.2) the governing equation for AC grid the phase current $i^{\Delta}$ is obtained,

$$
\begin{gather*}
v_{s}(t)=\frac{v_{l}(t)-v_{u}(t)}{2}-\frac{R}{2} i^{\Delta}(t)-\frac{L}{2} \frac{d}{d t} i^{\Delta}(t)  \tag{2.4}\\
i^{\Delta}(t) \triangleq i_{u}(t)-i_{l}(t) \tag{2.5}
\end{gather*}
$$

Subtracting Eqs. (2.1)-(2.2), the governing equation for the differential current $i^{\Sigma}$ is obtained,

$$
\begin{gather*}
v_{d c}=v_{u}(t)+v_{l}(t)+2 R i^{\Sigma}(t)+2 L \frac{d i^{\Sigma}(t)}{d t}  \tag{2.6}\\
i^{\Sigma}(t) \triangleq \frac{i_{u}(t)+i_{l}(t)}{2} \tag{2.7}
\end{gather*}
$$

Eqs. (2.4) and (2.6) always hold. From Eqs. (2.5) and (2.7) we can express the upper and the lower currents as,

$$
\begin{align*}
& i_{u}(t)=\frac{i^{\Delta}(t)}{2}+i^{\Sigma}(t)  \tag{2.8}\\
& i_{l}(t)=-\frac{i^{\Delta}(t)}{2}+i^{\Sigma}(t) \tag{2.9}
\end{align*}
$$

[^1]
### 2.1.1.2 Modulation index

Let us assume [6, 76, 75],

$$
\begin{align*}
& v_{m o d, u}(t) \stackrel{\text { hyp }}{=} m_{u}(t) v_{c a p, u}(t)  \tag{2.10}\\
& v_{\text {mod }, l}(t) \stackrel{\text { hyp }}{=} m_{l}(t) v_{c a p, l}(t) \tag{2.11}
\end{align*}
$$

where $v_{m o d, u}$ and $v_{m o d, l}$ are the upper and lower module voltages respectively and $v_{c a p, u}$ and $v_{c a p, l}$ are the upper and lower instantaneous module capacitor voltages respectively (both shown in the Fig. 1.5).

Supposing that,

$$
\begin{align*}
& i_{\text {mod }, u}(t) v_{\text {mod }, u}(t) \stackrel{\text { hyp }}{=} i_{\text {cap }, u}(t) v_{\text {cap }, u}(t)  \tag{2.12}\\
& i_{\text {mod }, l}(t) v_{\text {mod }, l}(t) \stackrel{\text { hyp }}{=} i_{\text {cap }, l}(t) v_{\text {cap }, l}(t) \tag{2.13}
\end{align*}
$$

where $i_{\text {mod }}$ and $i_{\text {cap }}$ are the module and capacitor current respectively. Input Eqs. (2.10)-(2.11),

$$
\begin{gather*}
i_{\text {cap }, u}(t)=m_{u}(t) i_{\text {mod }, u}(t)  \tag{2.14}\\
i_{\text {cap }, l}(t)=m_{l}(t) i_{\text {mod }, l}(t) \tag{2.15}
\end{gather*}
$$

Since the module currents are the arm currents, $i_{\text {mod }, u}=i_{u}$ and $i_{\text {mod }, l}=i_{l}$ we obtain,

$$
\begin{gather*}
i_{c a p, u}(t)=m_{u}(t) i_{u}(t)  \tag{2.16}\\
i_{c a p, l}(t)=m_{l}(t) i_{l}(t) \tag{2.17}
\end{gather*}
$$

Let us assume that $m_{u}$ and $m_{l}$ are of the form [6, 76, 75],

$$
\begin{align*}
& m_{u}(t) \stackrel{\text { hyp }}{=} \frac{1}{2}-\frac{m^{\Delta}(t)}{2}  \tag{2.18}\\
& m_{l}(t) \stackrel{\text { hyp }}{=} \frac{1}{2}+\frac{m^{\Delta}(t)}{2}  \tag{2.19}\\
& m^{\Delta}(t)=m_{l}(t)-m_{u}(t) \tag{2.20}
\end{align*}
$$

The upper and lower arm voltages, $v_{u}$ and $v_{l}$, are equal to the sum of all the module voltages of upper and lower arm respectively. So, if the module number is indexed with $i$, we obtain,

$$
\begin{align*}
& v_{u}(t)=\sum_{i=1}^{N} v_{m o d, u, i}(t)  \tag{2.21}\\
& v_{l}(t)=\sum_{i=1}^{N} v_{m o d, l, i}(t) \tag{2.22}
\end{align*}
$$

Let us assume that all the instantaneous module capacitor voltages of one arm are equal, $v_{c a p, u, 1} \stackrel{h y p}{\underline{\underline{p}}}$ $v_{c a p, u, 2} \stackrel{h y p}{=} \ldots \stackrel{h y p}{=} v_{c a p, u, N}$ and $v_{c a p, l, 1} \stackrel{h y p}{=} v_{c a p, l, 2} \stackrel{h y p}{\underline{=}} \ldots \stackrel{\text { hyp }}{=} v_{c a p, l, N}$. Then, inserting Eqs. (2.21)-(2.22) into Eqs. (2.10)-(2.11),

$$
\begin{gather*}
v_{u}(t)=m_{u}(t) N v_{c a p, u}(t)  \tag{2.23}\\
v_{l}(t)=m_{l}(t) N v_{c a p, l}(t) \tag{2.24}
\end{gather*}
$$

### 2.1.1.3 Maximal arm voltages

Let the "maximal arm voltages" $v_{u, C}$ and $v_{l, C}$ be defined as,

$$
\begin{align*}
& v_{u, C}(t) \triangleq N v_{c a p, u}(t)  \tag{2.25}\\
& v_{l, C}(t) \triangleq N v_{c a p, l}(t) \tag{2.26}
\end{align*}
$$

Then the arm voltages in Eqs. (2.23)-(2.24) can be rewritten as,

$$
\begin{gather*}
v_{u}(t)=m_{u}(t) v_{u, C}(t)  \tag{2.27}\\
v_{l}(t)=m_{l}(t) v_{l, C}(t) \tag{2.28}
\end{gather*}
$$

Since all the capacitor voltages in one arm are considered equal, the series connection of the modules in each arm can be replaced by a circuit-based average model [ $6,80,76,77]$, corresponding to the average arm model (AAM), as indicated in Fig. 1.3 in the lower arm of the phase $c$. In this case, each arm can be modeled as a modulated voltage source interfacing the arm inductor and a controlled voltage source interfacing the capacitor $C_{a r m}$,

$$
\begin{equation*}
C_{a r m} \triangleq \frac{C}{N} \tag{2.29}
\end{equation*}
$$

The differential equations describing the maximal arm voltages then are $[6,75,76]$,

$$
\begin{align*}
& \frac{C_{a r m}}{m_{u}(t)} \frac{d v_{u, C}(t)}{d t}=i_{u}(t)  \tag{2.30}\\
& \frac{C_{a r m}}{m_{l}(t)} \frac{d v_{l, C}(t)}{d t}=i_{l}(t) \tag{2.31}
\end{align*}
$$

Replacing the $i_{u}$ and $i_{l}$ using the Eqs.(2.8)-(2.9),

$$
\begin{gather*}
C_{a r m} \frac{d v_{u, C}(t)}{d t}=m_{u}(t)\left(\frac{i^{\Delta}(t)}{2}+i^{\Sigma}(t)\right)  \tag{2.32}\\
C_{a r m} \frac{d v_{l, C}(t)}{d t}=m_{l}(t)\left(-\frac{i^{\Delta}(t)}{2}+i^{\Sigma}(t)\right) \tag{2.33}
\end{gather*}
$$

### 2.1.2 Mathematical model of a 3-phase MMC

For a 3-phase system, in matrix notation, let's define,

$$
\boldsymbol{\alpha}_{x y z}(t) \triangleq\left[\begin{array}{c}
\alpha_{x}(t)  \tag{2.34}\\
\alpha_{y}(t) \\
\alpha_{z}(t)
\end{array}\right], \quad\{x y z\} \in\{a b c, d q 0\}
$$

### 2.1.2.1 MMC UL model in $a b c$ frame

Eq. (2.1) in 3-phase $a b c$ frame is,

$$
\boldsymbol{v}_{s, a b c}(t)=\frac{1}{2}\left[\begin{array}{l}
v_{d c}  \tag{2.35}\\
v_{d c} \\
v_{d c}
\end{array}\right]-\boldsymbol{v}_{u, a b c}(t)-R \boldsymbol{i}_{u, a b c}(t)-L \frac{d \boldsymbol{i}_{u, a b c}(t)}{d t}
$$

Eq. (2.2) in 3-phase $a b c$ frame is,

$$
\boldsymbol{v}_{s}(t)=-\frac{1}{2}\left[\begin{array}{l}
v_{d c}  \tag{2.36}\\
v_{d c} \\
v_{d c}
\end{array}\right]+\boldsymbol{v}_{l, a b c}(t)+R \boldsymbol{i}_{l, a b c}(t)+L \frac{d \boldsymbol{i}_{l, a b c}(t)}{d t}
$$

From Eqs. (2.30)-(2.31) the 3-phase maximal arm voltage equations in $a b c$ frame are,

$$
\begin{align*}
C_{a r m} \frac{d \boldsymbol{v}_{u, C, a b c}(t)}{d t} & =\boldsymbol{m}_{u, a b c}(t) \odot \boldsymbol{i}_{u, a b c}  \tag{2.37}\\
C_{a r m} \frac{d \boldsymbol{v}_{l, C, a b c}(t)}{d t} & =\boldsymbol{m}_{l, a b c}(t) \odot \boldsymbol{i}_{l, a b c} \tag{2.38}
\end{align*}
$$

where the operator $\odot$ represents the Hadamard's product (element wise multiplication, Appendix B). From Eq.(2.27)-(2.28), the upper and lower arm voltage equations in $a b c$ frame are,

$$
\begin{gather*}
\boldsymbol{v}_{u, a b c}(t)=\boldsymbol{m}_{u, a b c}(t) \odot \boldsymbol{v}_{u, C, a b c}(t)  \tag{2.39}\\
\boldsymbol{v}_{l, a b c}(t)=\boldsymbol{m}_{l, a b c}(t) \odot \boldsymbol{v}_{l, C, a b c}(t) \tag{2.40}
\end{gather*}
$$

The 6 equations (2.35)-(2.40) describe the average arm model of MMC in $a b c$ frame and hold as long as hypotheses in Eqs. (2.10)-(2.11) and (2.18)-(2.19) are verified.


Figure 2.1 - MMC UL model in $a b c$ frame

### 2.1.2.2 MMC $\Delta \Sigma$ model in $a b c$ frame

The analysis of the MMC with arm (UL) quantities is not an easy task. Therefore it has been proposed by $[75,76,80]$ to use $\Delta \Sigma$ quantities. Under this representation it is possible to derive 12 states and 6 control variables for an average model of a 3-phase MMC. In particular, our demonstration adopts the conventions proposed by Bergna et al. [75, 76].

Let us define $\boldsymbol{v}_{m, a b c}^{\Delta}, \boldsymbol{v}_{m, a b c}^{\Sigma}, \boldsymbol{i}_{a b c}^{\Delta}, \boldsymbol{i}_{a b c}^{\Sigma}, \boldsymbol{v}_{c, a b c}^{\Delta}$ and $\boldsymbol{v}_{c, a b c}^{\Sigma}$ by,

$$
\left[\begin{array}{c}
\boldsymbol{v}_{m, a b c}^{\Delta}(t)  \tag{2.41}\\
\boldsymbol{v}_{m, a b c}^{\Sigma}(t) \\
\boldsymbol{i}_{a b c}^{\Delta}(t) \\
\boldsymbol{i}_{a b c}^{\Sigma}(t) \\
\boldsymbol{v}_{\boldsymbol{c}, a b c}^{\Delta}(t) \\
\boldsymbol{v}_{c, a b c}^{\Sigma}(t)
\end{array}\right] \triangleq\left[\begin{array}{cccccc}
\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\
0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2}
\end{array}\right]\left[\begin{array}{c}
\boldsymbol{v}_{l, a b c}(t) \\
\boldsymbol{v}_{u, a b c}(t) \\
\boldsymbol{i}_{l, a b c}(t) \\
\boldsymbol{i}_{u, a b c}(t) \\
\boldsymbol{v}_{l, C, a b c}(t) \\
\boldsymbol{v}_{u, C, a b c}(t)
\end{array}\right]
$$

Therefore, the inverse transformation is,

$$
\left[\begin{array}{c}
\boldsymbol{v}_{l, a b c}(t)  \tag{2.42}\\
\boldsymbol{v}_{u, a b c}(t) \\
\boldsymbol{i}_{l, a b c}(t) \\
\boldsymbol{i}_{u, a b c}(t) \\
\boldsymbol{v}_{l, C, a b c}(t) \\
\boldsymbol{v}_{u, C, a b c}(t)
\end{array}\right]=\left[\begin{array}{cccccc}
1 & 1 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{2} & 1 & 0 & 0 \\
0 & 0 & \frac{1}{2} & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & -1 & 1
\end{array}\right]\left[\begin{array}{c}
\boldsymbol{v}_{m, a b c}^{\Delta}(t) \\
\boldsymbol{v}_{m, a b c}^{\Delta}(t) \\
\boldsymbol{i}_{a b c}^{\Delta}(t) \\
\boldsymbol{i}_{a b c}^{\Sigma}(t) \\
\boldsymbol{v}_{c, a b c}^{\Sigma}(t) \\
\boldsymbol{v}_{c, a b c}^{\Sigma}(t)
\end{array}\right]
$$

Additionally,

$$
\left[\begin{array}{l}
\boldsymbol{m}_{a b b}^{\Delta}(t)  \tag{2.43}\\
\boldsymbol{m}_{a b c}^{\Sigma}(t)
\end{array}\right] \triangleq\left[\begin{array}{cc}
1 & -1 \\
1 & 1
\end{array}\right]\left[\begin{array}{l}
\boldsymbol{m}_{l, a b c}(t) \\
\boldsymbol{m}_{u, a b c}(t)
\end{array}\right]
$$

Therefore,

$$
\left[\begin{array}{l}
\boldsymbol{m}_{l, a b c}(t)  \tag{2.44}\\
\boldsymbol{m}_{u, a b c}(t)
\end{array}\right]=\frac{1}{2}\left[\begin{array}{cc}
1 & 1 \\
-1 & 1
\end{array}\right]\left[\begin{array}{l}
\boldsymbol{m}_{a b c}^{\Delta}(t) \\
\boldsymbol{m}_{a b c}^{\Sigma}(t)
\end{array}\right]
$$

AE1 ${ }^{2}$ : $\quad$ Subtracting the Eqs. (2.39) and (2.40),

$$
\begin{equation*}
\boldsymbol{v}_{l, a b c}(t)-\boldsymbol{v}_{u, a b c}(t)=\boldsymbol{m}_{l, a b c}(t) \odot \boldsymbol{v}_{l, C, a b c}(t)-\boldsymbol{m}_{u, a b c}(t) \odot \boldsymbol{v}_{u, C, a b c}(t) \tag{2.45}
\end{equation*}
$$

Apply the transformation in Eq. (2.42),

$$
\begin{align*}
\boldsymbol{v}_{m, a b c}^{\Delta}(t)+\boldsymbol{v}_{m, a b c}^{\Sigma}(t)-\left(-\boldsymbol{v}_{m, a b c}^{\Delta}(t)+\boldsymbol{v}_{m, a b c}^{\Sigma}(t)\right) & =\frac{1}{2}\left(\boldsymbol{m}_{a b c}^{\Delta}(t)+\boldsymbol{m}_{a b c}^{\Sigma}(t)\right) \odot\left(\boldsymbol{v}_{c, a b c}^{\Delta}(t)+\boldsymbol{v}_{c, a b c}^{\Sigma}(t)\right) \\
- & \frac{1}{2}\left(-\boldsymbol{m}_{a b c}^{\Delta}(t)+\boldsymbol{m}_{a b c}^{\Sigma}(t)\right) \odot\left(-\boldsymbol{v}_{c, a b c}^{\Delta}(t)+\boldsymbol{v}_{c, a b c}^{\Sigma}(t)\right) \tag{2.46}
\end{align*}
$$

Simplify using the Theorem 2 from Appendix B,

$$
\begin{equation*}
\boldsymbol{v}_{m, a b c}^{\Delta}(t)=\frac{1}{2}\left(\boldsymbol{m}_{a b c}^{\Delta}(t) \odot \boldsymbol{v}_{c, a b c}^{\Sigma}(t)+\boldsymbol{m}_{a b c}^{\Sigma}(t) \odot \boldsymbol{v}_{c, a b c}^{\Delta}(t)\right) \tag{2.47}
\end{equation*}
$$

AE2: Summing the Eqs. (2.39) and (2.40),

$$
\begin{equation*}
\boldsymbol{v}_{u, a b c}(t)+\boldsymbol{v}_{l, a b c}(t)=\boldsymbol{m}_{u, a b c}(t) \odot \boldsymbol{v}_{u, C, a b c}(t)+\boldsymbol{m}_{l, a b c}(t) \odot \boldsymbol{v}_{l, C, a b c}(t) \tag{2.48}
\end{equation*}
$$

Apply the transformation in Eq. (2.42),

$$
\begin{align*}
\boldsymbol{v}_{m, a b c}^{\Delta}(t)+\boldsymbol{v}_{m, a b c}^{\Sigma}(t)-\boldsymbol{v}_{m, a b c}^{\Delta}(t)+\boldsymbol{v}_{m, a b c}^{\Sigma}(t) & =\frac{1}{2}\left(\boldsymbol{m}_{a b c}^{\Delta}(t)+\boldsymbol{m}_{a b c}^{\Sigma}(t)\right) \odot\left(\boldsymbol{v}_{c, a b c}^{\Delta}(t)+\boldsymbol{v}_{c, a b c}^{\Sigma}(t)\right) \\
+ & \frac{1}{2}\left(-\boldsymbol{m}_{a b c}^{\Delta}(t)+\boldsymbol{m}_{a b c}^{\Sigma}(t)\right) \odot\left(-\boldsymbol{v}_{c, a b c}^{\Delta}(t)+\boldsymbol{v}_{c, a b c}^{\Sigma}(t)\right) \tag{2.49}
\end{align*}
$$

Simplify using the Theorem 2 from Appendix B,

$$
\begin{equation*}
\boldsymbol{v}_{m, a b c}^{\Sigma}(t)=\frac{1}{2}\left(\boldsymbol{m}_{a b c}^{\Sigma}(t) \odot \boldsymbol{v}_{c, a b c}^{\Sigma}(t)+\boldsymbol{m}_{a b c}^{\Delta}(t) \odot \boldsymbol{v}_{c, a b c}^{\Delta}(t)\right) \tag{2.50}
\end{equation*}
$$

ODE1 ${ }^{3}$ : Adding Eqs. (2.35)-(2.36) the 3-phase grid current equation in $a b c$ frame is,

$$
\begin{equation*}
\frac{\boldsymbol{v}_{l, a b c}-\boldsymbol{v}_{u, a b c}}{2}-\boldsymbol{v}_{s, a b c}(t)=\frac{R}{2}\left(\boldsymbol{i}_{u, a b c}(t)-\boldsymbol{i}_{l, a b c}(t)\right)+\frac{L}{2} \frac{d}{d t}\left(\boldsymbol{i}_{u, a b c}(t)-\boldsymbol{i}_{l, a b c}(t)\right) \tag{2.51}
\end{equation*}
$$

Apply the Eq. (2.42) to Eq. (2.51),

$$
\begin{equation*}
\frac{\boldsymbol{v}_{m, a b c}^{\Delta}(t)+\boldsymbol{v}_{m, a b c}^{\Sigma}(t)-\left(-\boldsymbol{v}_{m, a b c}^{\Delta}(t)+\boldsymbol{v}_{m, a b c}^{\Sigma}(t)\right)}{2}-\boldsymbol{v}_{s, a b c}(t)=\frac{R}{2} \boldsymbol{i}_{a b c}^{\Delta}(t)+\frac{L}{2} \frac{d}{d t} \boldsymbol{i}_{a b c}^{\Delta}(t) \tag{2.52}
\end{equation*}
$$

and simplify,

$$
\begin{equation*}
\frac{L}{2} \frac{d \boldsymbol{i}_{a b c}^{\Delta}(t)}{d t}=\boldsymbol{v}_{m, a b c}^{\Delta}(t)-\boldsymbol{v}_{s, a b c}(t)-\frac{R}{2} \boldsymbol{i}_{a b c}^{\Delta}(t) \tag{2.53}
\end{equation*}
$$

[^2]ODE2: Subtracting Eqs. (2.35)-(2.36) the 3-phase grid differential equation in $a b c$ frame is,

$$
\left[\begin{array}{l}
v_{d c}  \tag{2.54}\\
v_{d c} \\
v_{d c}
\end{array}\right]=\boldsymbol{v}_{u, a b c}(t)+\boldsymbol{v}_{l, a b c}(t)+R\left(\boldsymbol{i}_{u, a b c}(t)+\boldsymbol{i}_{l, a b c}(t)\right)+L \frac{d}{d t}\left(\boldsymbol{i}_{u, a b c}(t)+\boldsymbol{i}_{l, a b c}(t)\right)
$$

Apply the transformation in Eq. (2.42) to Eq. (2.54),

$$
\left[\begin{array}{l}
v_{d c}  \tag{2.55}\\
v_{d c} \\
v_{d c}
\end{array}\right]=\boldsymbol{v}_{m, a b c}^{\Delta}(t)+\boldsymbol{v}_{m, a b c}^{\Sigma}(t)-\boldsymbol{v}_{m, a b c}^{\Delta}(t)+\boldsymbol{v}_{m, a b c}^{\Sigma}(t)+2 R \boldsymbol{i}_{a b c}^{\Sigma}(t)+2 L \frac{d}{d t} \boldsymbol{i}_{a b c}^{\Sigma}(t)
$$

and simplify,

$$
L \frac{d \boldsymbol{i}_{a b c}^{\Sigma}(t)}{d t}=\frac{1}{2}\left[\begin{array}{l}
v_{d c}  \tag{2.56}\\
v_{d c} \\
v_{d c}
\end{array}\right]-\boldsymbol{v}_{m, a b c}^{\Sigma}(t)-R \boldsymbol{i}_{a b c}^{\Sigma}(t)
$$

ODE3: Subtract the Eqs. (2.37)-(2.38),

$$
\begin{equation*}
C_{a r m} \frac{d \boldsymbol{v}_{u, C, a b c}(t)}{d t}-C_{a r m} \frac{d \boldsymbol{v}_{l, C, a b c}(t)}{d t}=\boldsymbol{m}_{u, a b c}(t) \odot \boldsymbol{i}_{u, a b c}-\boldsymbol{m}_{l, a b c}(t) \odot \boldsymbol{i}_{l, a b c} \tag{2.57}
\end{equation*}
$$

Apply the transformation in Eq. (2.42) and (2.44),

$$
\begin{array}{r}
C_{a r m} \frac{d\left(-\boldsymbol{v}_{c, a b c}^{\Delta}(t)+\boldsymbol{v}_{c, a b c}^{\Sigma}(t)-\boldsymbol{v}_{c, a b c}^{\Delta}(t)-\boldsymbol{v}_{c, a b c}^{\Sigma}(t)\right)}{d t}=\frac{1}{2}\left(-\boldsymbol{m}_{a b c}^{\Delta}(t)+\boldsymbol{m}_{a b c}^{\Sigma}(t)\right) \odot\left(\frac{1}{2} \boldsymbol{i}_{a b c}^{\Delta}(t)+\boldsymbol{i}_{a b c}^{\Sigma}(t)\right)- \\
\frac{1}{2}\left(\boldsymbol{m}_{a b c}^{\Delta}(t)+\boldsymbol{m}_{a b c}^{\Sigma}(t)\right) \odot\left(-\frac{1}{2} \boldsymbol{i}_{a b c}^{\Delta}(t)+\boldsymbol{i}_{a b c}^{\Sigma}(t)\right) \tag{2.58}
\end{array}
$$

Simplify using Theorem 2 from Appendix B,

$$
\begin{equation*}
2 C_{a r m} \frac{d \boldsymbol{v}_{c, a b c}^{\Delta}(t)}{d t}=-\boldsymbol{m}_{a b c}^{\Sigma}(t) \odot \frac{\boldsymbol{i}_{a b c}^{\Delta}(t)}{2}+\boldsymbol{m}_{a b c}^{\Delta}(t) \odot \boldsymbol{i}_{a b c}^{\Sigma}(t) \tag{2.59}
\end{equation*}
$$

ODE4: Sum the Eqs. (2.37)-(2.38),

$$
\begin{equation*}
C_{a r m} \frac{d \boldsymbol{v}_{u, C, a b c}(t)}{d t}+C_{a r m} \frac{d \boldsymbol{v}_{l, C, a b c}(t)}{d t}=\boldsymbol{m}_{u, a b c}(t) \odot \boldsymbol{i}_{u, a b c}+\boldsymbol{m}_{l, a b c}(t) \odot \boldsymbol{i}_{l, a b c} \tag{2.60}
\end{equation*}
$$

Apply the transformation in Eq. (2.42) and (2.44),

$$
\begin{array}{r}
C_{a r m} \frac{d\left(-\boldsymbol{v}_{c, a b c}^{\Delta}(t)+\boldsymbol{v}_{c, a b c}^{\Sigma}(t)+\boldsymbol{v}_{c, a b c}^{\Delta}(t)+\boldsymbol{v}_{c, a b c}^{\Sigma}(t)\right)}{d t}=\frac{1}{2}\left(-\boldsymbol{m}_{a b c}^{\Delta}(t)+\boldsymbol{m}_{a b c}^{\Sigma}(t)\right) \odot\left(\frac{1}{2} \boldsymbol{i}_{a b c}^{\Delta}(t)+\boldsymbol{i}_{a b c}^{\Sigma}(t)\right)+ \\
\frac{1}{2}\left(\boldsymbol{m}_{a b c}^{\Delta}(t)+\boldsymbol{m}_{a b c}^{\Sigma}(t)\right) \odot\left(-\frac{1}{2} \boldsymbol{i}_{a b c}^{\Delta}(t)+\boldsymbol{i}_{a b c}^{\Sigma}(t)\right) \tag{2.61}
\end{array}
$$

Simplify using Theorem 2 from Appendix B,

$$
\begin{equation*}
2 C_{a r m} \frac{d \boldsymbol{v}_{c, a b c}^{\Sigma}(t)}{d t}=-\boldsymbol{m}_{a b c}^{\Delta}(t) \odot \frac{\boldsymbol{i}_{a b c}^{\Delta}(t)}{2}+\boldsymbol{m}_{a b c}^{\Sigma}(t) \odot \boldsymbol{i}_{a b c}^{\Sigma}(t) \tag{2.62}
\end{equation*}
$$

The 6 equations $(2.47),(2.50),(2.53),(2.56),(2.59)$ and (2.62) describe the average arm $\Delta \Sigma$ model of MMC in $a b c$ frame and hold as long as hypotheses in Eqs. (2.10)-(2.11) and (2.18)-(2.19) are verified.

Equivalent schemes The MMC can be studied using the equivalent schemes. The Eq. (2.53) is represented in the Fig. 2.3. The Eq. (2.56) is represented in the Fig. 2.4.


Figure 2.2 - MMC $\Delta \Sigma$ model in $a b c$ frame


Figure 2.3 - Equivalent circuit from Eq. (2.53)


Figure 2.4 - Equivalent circuit from Eq. (2.56)

### 2.1.2.3 MMC $\Delta \Sigma$ model in $d q 0$ frame

Here we pass from the $a b c$ to the $d q 0$ frame. This enables us to represent the AC waveforms in the form of DC signals [16]. Simplified calculations can then be carried out on these DC quantities.
The used Park transformation, $\boldsymbol{P}_{n \omega}$ (see Appendix C), is [16],

$$
\begin{equation*}
\boldsymbol{f}_{d q 0}=\boldsymbol{P}_{n \omega}(\theta) \boldsymbol{f}_{a b c} \tag{2.63}
\end{equation*}
$$

where $n=1$ for $\Delta$ variables and $n=-2$ for $\Sigma$ variables (see Chapter 4). The inverse transformation is,

$$
\begin{equation*}
\boldsymbol{f}_{a b c}=\boldsymbol{P}_{n \omega}^{-1}(\theta) \boldsymbol{f}_{d q 0} \tag{2.64}
\end{equation*}
$$

AE1: Let us apply the Eq. (2.64) to Eq. (2.47),

$$
\begin{equation*}
\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{v}_{m, d q 0}^{\Delta}(t)\right)=\frac{1}{2}\left(\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{m}_{d q 0}^{\Delta}(t)\right) \odot\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)\right)+\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{m}_{d q 0}^{\Sigma}(t)\right) \odot\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{v}_{c, d q 0}^{\Delta}(t)\right)\right) \tag{2.65}
\end{equation*}
$$

Multiply the left and the right side by $\boldsymbol{P}_{\omega}$,

$$
\begin{equation*}
\boldsymbol{P}_{\omega}\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{v}_{m, d q 0}^{\Delta}(t)\right)=\frac{1}{2} \boldsymbol{P}_{\omega}\left(\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{m}_{d q 0}^{\Delta}(t)\right) \odot\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)\right)+\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{m}_{d q 0}^{\Sigma}(t)\right) \odot\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{v}_{c, d q 0}^{\Delta}(t)\right)\right) \tag{2.66}
\end{equation*}
$$

Simplify,

$$
\begin{gather*}
\boldsymbol{v}_{m, d q 0}^{\Delta}(t)=\frac{1}{2} \boldsymbol{P}_{\omega}\left[\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{m}_{d q 0}^{\Delta}(t)\right) \odot\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)\right)\right]+\frac{1}{2} \boldsymbol{P}_{\omega}\left[\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{m}_{d q 0}^{\Sigma}(t)\right) \odot\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{v}_{c, d q 0}^{\Delta}(t)\right)\right]  \tag{2.67}\\
\boldsymbol{v}_{m, d q 0}^{\Delta}(t)=\left[\boldsymbol{M}_{A 1 a}(t)^{\Delta}\right] \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)+\left[\boldsymbol{M}_{A 1 b}(t)^{\Sigma}\right] \boldsymbol{v}_{c, d q 0}^{\Delta}(t) \tag{2.68}
\end{gather*}
$$

where

$$
\begin{gather*}
{\left[\boldsymbol{M}_{A 1 a}^{\Delta}(t)\right]=\frac{1}{4}\left[\begin{array}{ccc}
2 m_{0}^{\Delta} \lambda_{1}-m_{q}^{\Delta} & 2 m_{0}^{\Delta} \lambda_{2}-m_{d}^{\Delta} & 2 m_{d}^{\Delta} \\
-m_{d}^{\Delta}-2 m_{0}^{\Delta} \lambda_{2} & m_{q}^{\Delta}+2 m_{0}^{\Delta} \lambda_{1} & 2 m_{q}^{\Delta} \\
m_{d}^{\Delta} \lambda_{1}-m_{q}^{\Delta} \lambda_{2} & m_{q}^{\Delta} \lambda_{1}+m_{d}^{\Delta} \lambda_{2} & 2 m_{0}^{\Delta}
\end{array}\right]}  \tag{2.69}\\
{\left[\boldsymbol{M}_{A 1 b}^{\Sigma}(t)\right]=-\frac{1}{4}\left[\begin{array}{ccc}
m_{q}^{\Sigma}-2 m_{0}^{\Sigma} & m_{d}^{\Sigma} & -2 m_{d}^{\Sigma} \lambda_{1}-2 m_{q}^{\Sigma} \lambda_{2} \\
m_{d}^{\Sigma} & -2 m_{0}^{\Sigma}-m_{q}^{\Sigma} & 2 m_{d}^{\Sigma} \lambda_{2}-2 m_{q}^{\Sigma} \lambda_{1} \\
-m_{d}^{\Sigma} \lambda_{1}-m_{q}^{\Sigma} \lambda_{2} & m_{d}^{\Sigma} \lambda_{2}-m_{q}^{\Sigma} \lambda_{1} & -2 m_{0}^{\Sigma}
\end{array}\right]}  \tag{2.70}\\
\lambda_{1}(t)=\cos (3 \omega t) \tag{2.71}
\end{gather*} ; \quad \lambda_{2}(t)=\sin (3 \omega t) .
$$

AE2: Let us apply the Eq. (2.64) to Eq. (2.50),

$$
\begin{equation*}
\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{v}_{m, d q 0}^{\Sigma}(t)\right)=\frac{1}{2}\left(\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{m}_{d q 0}^{\Sigma}(t)\right) \odot\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)\right)+\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{m}_{d q 0}^{\Delta}(t)\right) \odot\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{v}_{c, d q 0}^{\Delta}(t)\right)\right) \tag{2.72}
\end{equation*}
$$

Multiply the left and the right side by $\boldsymbol{P}_{-2 \omega}$,

$$
\begin{equation*}
\boldsymbol{P}_{-2 \omega}\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{v}_{m, d q 0}^{\Sigma}(t)\right)=\frac{1}{2} \boldsymbol{P}_{-2 \omega}\left(\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{m}_{d q 0}^{\Sigma}(t)\right) \odot\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)\right)+\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{m}_{d q 0}^{\Delta}(t)\right) \odot\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{v}_{c, d q 0}^{\Delta}(t)\right)\right) \tag{2.73}
\end{equation*}
$$

Simplify,

$$
\begin{equation*}
\boldsymbol{v}_{m, d q 0}^{\Sigma}(t)=\frac{1}{2} \boldsymbol{P}_{-2 \omega}\left[\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{m}_{d q 0}^{\Sigma}(t)\right) \odot\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)\right)\right]+\frac{1}{2} \boldsymbol{P}_{-2 \omega}\left[\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{m}_{d q 0}^{\Delta}(t)\right) \odot\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{v}_{c, d q 0}^{\Delta}(t)\right)\right] \tag{2.74}
\end{equation*}
$$

$$
\begin{equation*}
\boldsymbol{v}_{m, d q 0}^{\Sigma}(t)=\left[\boldsymbol{M}_{A 2 a}(t)^{\Sigma}\right] \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)+\left[\boldsymbol{M}_{A 2 b}(t)^{\Delta}\right] \boldsymbol{v}_{c, d q 0}^{\Delta}(t) \tag{2.75}
\end{equation*}
$$

where,

$$
\begin{gather*}
{\left[\boldsymbol{M}_{A 2 a}^{\Sigma}(t)\right]=\frac{1}{4}\left[\begin{array}{ccc}
2 m_{0}^{\Sigma}-m_{q}^{\Sigma} \chi_{1}+m_{d}^{\Sigma} \chi_{2} & -m_{d}^{\Sigma} \chi_{1}-m_{q}^{\Sigma} \chi_{2} & 2 m_{d}^{\Sigma} \\
-m_{d}^{\Sigma} \chi_{1}-m_{q}^{\Sigma} \chi_{2} & 2 m_{0}^{\Sigma}+m_{q}^{\Sigma} \chi_{1}-m_{d}^{\Sigma} \chi_{2} & 2 m_{q}^{\Sigma} \\
m_{d}^{\Sigma} & m_{q}^{\Sigma} & 2 m_{0}^{\Sigma}
\end{array}\right]}  \tag{2.76}\\
{\left[\boldsymbol{M}_{A 2 b}^{\Delta}(t)\right]=\frac{1}{4}\left[\begin{array}{ccc}
2 m_{0}^{\Delta} \lambda_{1}-m_{q}^{\Delta} & -m_{d}^{\Delta}-2 m_{0}^{\Delta} \lambda_{2} & 2 m_{d}^{\Delta} \lambda_{1}-2 m_{q}^{\Delta} \lambda_{2} \\
2 m_{0}^{\Delta} \lambda_{2}-m_{d}^{\Delta} & m_{q}^{\Delta}+2 m_{0}^{\Delta} \lambda_{1} & 2 m_{q}^{\Delta} \lambda_{1}+2 m_{d}^{\Delta} \lambda_{2} \\
m_{d}^{\Delta} & m_{q}^{\Delta} & 2 m_{0}^{\Delta}
\end{array}\right]}  \tag{2.77}\\
\chi_{1}(t)=\cos (6 \omega t) \tag{2.78}
\end{gather*} \quad ; \quad \chi_{2}(t)=\sin (6 \omega t) .
$$

ODE1: Let us apply the Eq. (2.64) to Eq. (2.53),

$$
\begin{equation*}
\frac{L}{2} \frac{d\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{i}_{d q 0}^{\Delta}(t)\right)}{d t}=\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{v}_{m, d q 0}^{\Delta}(t)\right)-\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{v}_{s, d q 0}(t)\right)-\frac{R}{2}\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{i}_{d q 0}^{\Delta}(t)\right) \tag{2.79}
\end{equation*}
$$

Multiply the left and the right side by $\boldsymbol{P}_{\omega}$ and develop the product,

$$
\begin{equation*}
\frac{L}{2} \boldsymbol{P}_{\omega}\left(\frac{d \boldsymbol{P}_{\omega}^{-1}}{d t} \boldsymbol{i}_{d q 0}^{\Delta}(t)+\boldsymbol{P}_{\omega}^{-1} \frac{d \boldsymbol{i}_{d q 0}^{\Delta}(t)}{d t}\right)=\boldsymbol{v}_{m, d q 0}^{\Delta}(t)-\boldsymbol{v}_{s, d q 0}(t)-\frac{R}{2} \boldsymbol{i}_{d q 0}^{\Delta}(t) \tag{2.80}
\end{equation*}
$$

Simplify using Eq. (C.6),

$$
\begin{equation*}
\frac{L}{2} \frac{d \boldsymbol{i}_{d q 0}^{\Delta}(t)}{d t}=\boldsymbol{v}_{m, d q 0}^{\Delta}(t)-\boldsymbol{v}_{s, d q 0}(t)-\frac{R}{2} \boldsymbol{i}_{d q 0}^{\Delta}(t)-\frac{L}{2} \boldsymbol{J}_{\omega} \boldsymbol{i}_{d q 0}^{\Delta}(t) \tag{2.81}
\end{equation*}
$$

where $\boldsymbol{J}_{\omega}$ is defined in Eq. (C.6).
ODE2: Let us apply the Eq. (2.64) to Eq. (2.56),

$$
L \frac{d\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{i}_{d q 0}^{\Sigma}(t)\right)}{d t}=\left[\begin{array}{c}
\frac{v_{d c}}{2}  \tag{2.82}\\
\frac{v_{d c}}{2} \\
\frac{v_{d c}}{2}
\end{array}\right]-\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{v}_{m, d q 0}^{\Sigma}(t)\right)-R\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{i}_{d q 0}^{\Sigma}(t)\right)
$$

Multiply the left and the right side by $\boldsymbol{P}_{-2 \omega}$ and develop the product,

$$
L \boldsymbol{P}_{-2 \omega}\left(\frac{d \boldsymbol{P}_{-2 \omega}^{-1}}{d t} \boldsymbol{i}_{d q 0}^{\Sigma}(t)+\boldsymbol{P}_{-2 \omega}^{-1} \frac{d \boldsymbol{i}_{d q 0}^{\Sigma}(t)}{d t}\right)=\boldsymbol{P}_{-2 \omega}\left[\begin{array}{c}
\frac{v_{d c}}{2}  \tag{2.83}\\
\frac{v_{d c}}{2} \\
\frac{v_{d c}}{2}
\end{array}\right]-\boldsymbol{P}_{-2 \omega}\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{v}_{m, d q 0}^{\Sigma}(t)\right)-R \boldsymbol{P}_{-2 \omega}\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{i}_{d q 0}^{\Sigma}(t)\right)
$$

Simplify using the Eq. (C.6),

$$
L \frac{d \boldsymbol{i}_{d q 0}^{\Sigma}(t)}{d t}=\left[\begin{array}{c}
0  \tag{2.84}\\
0 \\
\frac{v_{d c}}{2}
\end{array}\right]-\boldsymbol{v}_{m, d q 0}^{\Sigma}(t)-R \boldsymbol{i}_{d q 0}^{\Sigma}(t)-L \boldsymbol{J}_{-2 \omega} \boldsymbol{i}_{d q 0}^{\Sigma}(t)
$$

ODE3: Let us apply the Eq. (2.64) to Eq. (2.59),

$$
\begin{equation*}
2 C_{a r m} \frac{d\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{v}_{c, d q 0}^{\Delta}(t)\right)}{d t}=-\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{m}_{d q 0}^{\Sigma}(t)\right) \odot \frac{\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{i}_{d q 0}^{\Delta}(t)\right)}{2}+\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{m}_{d q 0}^{\Delta}(t)\right) \odot\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{i}_{d q 0}^{\Sigma}(t)\right) \tag{2.85}
\end{equation*}
$$

Multiply the left and the right side by $\boldsymbol{P}_{\omega}$ and develop the product,

$$
\begin{align*}
2 C_{a r m} \boldsymbol{P}_{\omega}\left(\frac{d \boldsymbol{P}_{\omega}^{-1}}{d t} \boldsymbol{v}_{c, d q 0}^{\Delta}(t)+\boldsymbol{P}_{\omega}^{-1} \frac{d \boldsymbol{v}_{c, d q 0}^{\Delta}(t)}{d t}\right)= & -\boldsymbol{P}_{\omega}\left[\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{m}_{d q 0}^{\Sigma}(t)\right) \odot \frac{\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{i}_{d q 0}^{\Delta}\right)(t)}{2}\right] \\
& +\boldsymbol{P}_{\omega}\left[\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{m}_{d q 0}^{\Delta}(t)\right) \odot\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{i}_{d q 0}^{\Sigma}(t)\right)\right] \tag{2.86}
\end{align*}
$$

Simplify using the Eq. (C.6),

$$
\begin{align*}
2 C_{a r m} \frac{d \boldsymbol{v}_{c, d q 0}^{\Delta}(t)}{d t}= & -\boldsymbol{P}_{\omega}\left[\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{m}_{d q 0}^{\Sigma}(t)\right) \odot \frac{\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{i}_{d q 0}^{\Delta}(t)\right)}{2}\right] \\
& +\boldsymbol{P}_{\omega}\left[\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{m}_{d q 0}^{\Delta}(t)\right) \odot\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{i}_{d q 0}^{\Sigma}(t)\right)\right]-2 C_{a r m} \boldsymbol{J}_{\omega} \boldsymbol{v}_{c, d q 0}^{\Delta}(t)  \tag{2.87}\\
\frac{d \boldsymbol{v}_{c, d q 0}^{\Delta}(t)}{d t}= & \frac{1}{2 C_{a r m}}\left(\left[\boldsymbol{M}_{3 a}^{\Sigma}(t)\right] \boldsymbol{i}_{d q 0}^{\Delta}(t)+\left[\boldsymbol{M}_{3 b}^{\Delta}(t)\right] \boldsymbol{i}_{d q 0}^{\Sigma}(t)\right)-\boldsymbol{J}_{\omega} \boldsymbol{v}_{c, d q 0}^{\Delta}(t) \tag{2.88}
\end{align*}
$$

where,

$$
\begin{gather*}
{\left[\boldsymbol{M}_{3 a}^{\Sigma}(t)\right]=-\frac{1}{4}\left[\begin{array}{ccc}
2 m_{0}^{\Sigma}-m_{q}^{\Sigma} & -m_{d}^{\Sigma} & 2 m_{d}^{\Sigma} \lambda_{1}+2 m_{q}^{\Sigma} \lambda_{2} \\
-m_{d}^{\Sigma} & 2 m_{0}^{\Sigma}+m_{q}^{\Sigma} & 2 m_{q}^{\Sigma} \lambda_{1}-2 m_{d}^{\Sigma} \lambda_{2} \\
m_{d}^{\Sigma} \lambda_{1}+m_{q}^{\Sigma} \lambda_{2} & m_{q}^{\Sigma} \lambda_{1}-m_{d}^{\Sigma} \lambda_{2} & 2 m_{0}^{\Sigma}
\end{array}\right]}  \tag{2.89}\\
{\left[\boldsymbol{M}_{3 b}^{\Delta}(t)\right]=\frac{1}{2}\left[\begin{array}{ccc}
2 m_{0}^{\Delta} \lambda_{1}-m_{q}^{\Delta} & 2 m_{0}^{\Delta} \lambda_{2}-m_{d}^{\Delta} & 2 m_{d}^{\Delta} \\
-m_{d}^{\Delta}-2 m_{0}^{\Delta} \lambda_{2} & m_{q}^{\Delta}+2 m_{0}^{\Delta} \lambda_{1} & 2 m_{q}^{\Delta} \\
m_{d}^{\Delta} \lambda_{1}-m_{q}^{\Delta} \lambda_{2} & m_{q}^{\Delta} \lambda_{1}+m_{d}^{\Delta} \lambda_{2} & 2 m_{0}^{\Delta}
\end{array}\right]} \tag{2.90}
\end{gather*}
$$

ODE4: Let us apply the Eq. (2.64) to Eq. (2.62),

$$
\begin{equation*}
2 C_{a r m} \frac{d\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)\right)}{d t}=-\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{m}_{d q 0}^{\Delta}(t)\right) \odot \frac{\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{i}_{d q 0}^{\Delta}(t)\right)}{2}+\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{m}_{d q 0}^{\Sigma}(t)\right) \odot\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{i}_{d q 0}^{\Sigma}(t)\right) \tag{2.91}
\end{equation*}
$$

Multiply the left and the right side by $\boldsymbol{P}_{-2 \omega}$ and develop the product,

$$
\begin{align*}
2 C_{a r m} \boldsymbol{P}_{-2 \omega}\left(\frac{d \boldsymbol{P}_{-2 \omega}^{-1}}{d t} \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)+\boldsymbol{P}_{-2 \omega}^{-1} \frac{d \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)}{d t}\right)= & -\boldsymbol{P}_{-2 \omega}\left[\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{m}_{d q 0}^{\Delta}(t)\right) \odot \frac{\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{i}_{d q 0}^{\Delta}(t)\right)}{2}\right] \\
& +\boldsymbol{P}_{-2 \omega}\left[\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{m}_{d q 0}^{\Sigma}(t)\right) \odot\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{i}_{d q 0}^{\Sigma}(t)\right)\right] \tag{2.92}
\end{align*}
$$

Simplify using the Eq. (C.6),

$$
\begin{align*}
2 C_{a r m} \frac{d \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)}{d t}= & -\boldsymbol{P}_{-2 \omega}\left[\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{m}_{d q 0}^{\Delta}(t)\right) \odot \frac{\left(\boldsymbol{P}_{\omega}^{-1} \boldsymbol{i}_{d q 0}^{\Delta}(t)\right)}{2}\right] \\
& +\boldsymbol{P}_{-2 \omega}\left[\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{m}_{d q 0}^{\Sigma}(t)\right) \odot\left(\boldsymbol{P}_{-2 \omega}^{-1} \boldsymbol{i}_{d q 0}^{\Sigma}(t)\right)\right]  \tag{2.93}\\
& -2 C_{a r m} \boldsymbol{J}_{-2 \omega} \boldsymbol{v}_{c, d q 0}^{\Sigma}(t) \\
\frac{d \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)}{d t}= & \frac{1}{2 C_{a r m}}\left(\left[\boldsymbol{M}_{4 a}^{\Delta}(t)\right] \boldsymbol{i}_{d q 0}^{\Delta}(t)+\left[\boldsymbol{M}_{4 b}^{\Sigma}(t)\right] \boldsymbol{i}_{d q 0}^{\Sigma}(t)\right)-\boldsymbol{J}_{-2 \omega} \boldsymbol{v}_{c, d q 0}^{\Sigma}(t) \tag{2.94}
\end{align*}
$$

where,

$$
\begin{align*}
& {\left[\boldsymbol{M}_{4 a}^{\Delta}(t)\right]=-\frac{1}{4}\left[\begin{array}{ccc}
2 m_{0}^{\Delta} \lambda_{1}-m_{q}^{\Delta} & -m_{d}^{\Delta}-2 m_{0}^{\Delta} \lambda_{2} & 2 m_{d}^{\Delta} \lambda_{1}-2 m_{q}^{\Delta} \lambda_{2} \\
2 m_{0}^{\Delta} \lambda_{2}-m_{d}^{\Delta} & m_{q}^{\Delta}+2 m_{0}^{\Delta} \lambda_{1} & 2 m_{q}^{\Delta} \lambda_{1}+2 m_{d}^{\Delta} \lambda_{2} \\
m_{d}^{\Delta} & m_{q}^{\Delta} & 2 m_{0}^{\Delta}
\end{array}\right]}  \tag{2.95}\\
& {\left[\boldsymbol{M}_{4 b}^{\Sigma}(t)\right]=\frac{1}{2}\left[\begin{array}{ccc}
2 m_{0}^{\Sigma}-m_{q}^{\Sigma} \chi_{1}+m_{d}^{\Sigma} \chi_{2} & -m_{d}^{\Sigma} \chi_{1}-m_{q}^{\Sigma} \chi_{2} & 2 m_{d}^{\Sigma} \\
-m_{d}^{\Sigma} \chi_{1}-m_{q}^{\Sigma} \chi_{2} & 2 m_{0}^{\Sigma}+m_{q}^{\Sigma} \chi_{1}-m_{d}^{\Sigma} \chi_{2} & 2 m_{q}^{\Sigma} \\
m_{d}^{\Sigma} & m_{q}^{\Sigma} & 2 m_{0}^{\Sigma}
\end{array}\right]} \tag{2.96}
\end{align*}
$$

The $\Delta \Sigma$ equations in $d q 0$ frame are summarized in Fig. 2.5. The 6 equations (2.68), (2.75), (2.81), (2.84), (2.88) and (2.94) describe the average arm $\Delta \Sigma$ model of MMC in $d q 0$ frame and hold as long as hypotheses in Eqs. (2.10)-(2.11) and (2.18)-(2.19) are verified.


Figure 2.5 - MMC $\Delta \Sigma$ model in $d q 0$ frame

### 2.1.3 AC grid power equations

The AC grid instantaneous power $p$ is defined as,

$$
\begin{equation*}
p(t) \triangleq \boldsymbol{v}_{s a b c}^{T} \boldsymbol{i}_{a b c}^{\Delta} \tag{2.97}
\end{equation*}
$$

By definition,

$$
\begin{equation*}
\bar{S}(t) \triangleq P_{s}(t)+j Q_{s}(t) \tag{2.98}
\end{equation*}
$$

where $\bar{S}(t)$ is the AC grid complex apparent power, and $P_{s}(t)$ and $Q_{s}(t)$ are the AC grid active and reactive power. Let us define the voltage and current space vectors (phasors) as explained in C.2,

$$
\begin{gather*}
\overline{v_{s d q}}(t) \triangleq v_{s q}(t)-j v_{s d}(t)  \tag{2.99}\\
\overline{i_{d q}^{\Delta}}(t) \triangleq i_{q}^{\Delta}(t)-j i_{d}^{\Delta}(t) \tag{2.100}
\end{gather*}
$$

The apparent power equation is derived as [46], [9, p. 977], [80, pg. 184],

$$
\begin{equation*}
\bar{S}(t)=\frac{3}{2} \overline{v_{s d q}}(t) \overline{i_{d q}^{\Delta}}(t)^{*} \tag{2.101}
\end{equation*}
$$

where superscript * is the complex conjugation. Input Eqs. (2.99)-(2.100),

$$
\begin{equation*}
\bar{S}(t)=\frac{3}{2}\left(v_{s q}(t)-j v_{s d}(t)\right)\left(i_{q}^{\Delta}(t)+j i_{d}^{\Delta}(t)\right) \tag{2.102}
\end{equation*}
$$

Identifying the real and imaginary part in Eq. (2.98),

$$
\begin{align*}
P_{s}(t) & =\frac{3}{2}\left(v_{s d}(t) i_{d}^{\Delta}(t)+v_{s q}(t) i_{q}^{\Delta}(t)\right)  \tag{2.103}\\
Q_{s}(t) & =\frac{3}{2}\left(-v_{s d}(t) i_{q}^{\Delta}(t)+v_{s q}(t) i_{d}^{\Delta}(t)\right) \tag{2.104}
\end{align*}
$$

In the grid voltage-oriented $d q$ reference frame, $v_{s d}(t)$ is zero. The AC grid active and reactive powers become,

$$
\begin{align*}
& P_{s}(t)=\frac{3}{2} v_{s q}(t) i_{q}^{\Delta}(t)  \tag{2.105}\\
& Q_{s}(t)=\frac{3}{2} v_{s q}(t) i_{d}^{\Delta}(t) \tag{2.106}
\end{align*}
$$

### 2.1.4 DC grid power equation

The DC grid (active) power $P_{d c}$ is by definition,

$$
\begin{equation*}
P_{d c} \triangleq v_{d c} i_{d c} \tag{2.107}
\end{equation*}
$$

where $i_{d c}$ is the DC grid current.

### 2.2 Summary

In this chapter we developed the MMC UL model in $a b c$ and MMC $\Delta \Sigma$ models in $a b c$ and $d q 0$ frame. For the sake of clarity, the three models are summarized here.

## MMC UL model in $a b c$ frame

$$
\begin{gather*}
\boldsymbol{v}_{u, a b c}(t)=\boldsymbol{m}_{u, a b c}(t) \odot \boldsymbol{v}_{u, C, a b c}(t)  \tag{2.108a}\\
\boldsymbol{v}_{l, a b c}(t)=\boldsymbol{m}_{l, a b c}(t) \odot \boldsymbol{v}_{l, C, a b c}(t)  \tag{2.108b}\\
\boldsymbol{v}_{s, a b c}(t)=\frac{1}{2}\left[\begin{array}{c}
v_{d c} \\
v_{d c} \\
v_{d c}
\end{array}\right]-\boldsymbol{v}_{u, a b c}(t)-R \boldsymbol{i}_{u, a b c}(t)-L \frac{d \boldsymbol{i}_{u, a b c}(t)}{d t}  \tag{2.108c}\\
\boldsymbol{v}_{s, a b c}(t)=-\frac{1}{2}\left[\begin{array}{l}
v_{d c} \\
v_{d c} \\
v_{d c}
\end{array}\right]+\boldsymbol{v}_{l, a b c}(t)+R \boldsymbol{i}_{l, a b c}(t)+L \frac{d \boldsymbol{i}_{l, a b c}(t)}{d t}  \tag{2.108d}\\
C_{a r m} \frac{d \boldsymbol{v}_{u, C, a b c}(t)}{d t}=\boldsymbol{m}_{u, a b c}(t) \odot \boldsymbol{i}_{u, a b c}  \tag{2.108e}\\
C_{a r m} \frac{d \boldsymbol{v}_{l, C, a b c}(t)}{d t}=\boldsymbol{m}_{l, a b c}(t) \odot \boldsymbol{i}_{l, a b c} \tag{2.108f}
\end{gather*}
$$

MMC $\Delta \Sigma$ model in $a b c$ frame

$$
\begin{gather*}
\boldsymbol{v}_{m, a b c}^{\Delta}(t)=\frac{1}{2}\left(\boldsymbol{m}_{a b c}^{\Delta}(t) \odot \boldsymbol{v}_{c, a b c}^{\Sigma}(t)+\boldsymbol{m}_{a b c}^{\Sigma}(t) \odot \boldsymbol{v}_{c, a b c}^{\Delta}(t)\right)  \tag{2.109a}\\
\boldsymbol{v}_{m, a b c}^{\Sigma}(t)=\frac{1}{2}\left(\boldsymbol{m}_{a b c}^{\Sigma}(t) \odot \boldsymbol{v}_{c, a b c}^{\Sigma}(t)+\boldsymbol{m}_{a b c}^{\Delta}(t) \odot \boldsymbol{v}_{c, a b c}^{\Delta}(t)\right)  \tag{2.109b}\\
\frac{L}{2} \frac{d \boldsymbol{i}_{a b c}^{\Delta}(t)}{d t}=\boldsymbol{v}_{m, a b c}^{\Delta}(t)-\boldsymbol{v}_{s, a b c}(t)-\frac{R}{2} \boldsymbol{i}_{a b c}^{\Delta}(t)  \tag{2.109c}\\
L \frac{d \boldsymbol{i}_{a b c}^{\Sigma}(t)}{d t}=\frac{1}{2}\left[\begin{array}{c}
v_{d c} \\
v_{d c} \\
v_{d c}
\end{array}\right]-\boldsymbol{v}_{m, a b c}^{\Sigma}(t)-R \boldsymbol{i}_{a b c}^{\Sigma}(t)  \tag{2.109d}\\
2 C_{a r m} \frac{d \boldsymbol{v}_{c, a b c}^{\Delta}(t)}{d t}=-\boldsymbol{m}_{a b c}^{\Sigma}(t) \odot \frac{\boldsymbol{i}_{a b c}^{\Delta}(t)}{2}+\boldsymbol{m}_{a b c}^{\Delta}(t) \odot \boldsymbol{i}_{a b c}^{\Sigma}(t)  \tag{2.109e}\\
2 C_{a r m} \frac{d \boldsymbol{v}_{c, a b c}^{\Sigma}(t)}{d t}=-\boldsymbol{m}_{a b c}^{\Delta}(t) \odot \frac{\boldsymbol{i}_{a b c}^{\Delta}(t)}{2}+\boldsymbol{m}_{a b c}^{\Sigma}(t) \odot \boldsymbol{i}_{a b c}^{\Sigma}(t) \tag{2.109f}
\end{gather*}
$$

MMC $\Delta \Sigma$ model in $d q 0$ frame

$$
\begin{gather*}
\boldsymbol{v}_{m, d q 0}^{\Delta}(t)=\left[\boldsymbol{M}_{A 1 a}^{\Delta}(t)\right] \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)+\left[\boldsymbol{M}_{A 1 b}^{\Sigma}(t)\right] \boldsymbol{v}_{c, d q 0}^{\Delta}(t)  \tag{2.110a}\\
\boldsymbol{v}_{m, d q 0}^{\Sigma}(t)=\left[\boldsymbol{M}_{A 2 a}^{\Sigma}(t)\right] \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)+\left[\boldsymbol{M}_{A 2 b}^{\Delta}(t)\right] \boldsymbol{v}_{c, d q 0}^{\Delta}(t)  \tag{2.110b}\\
\frac{L}{2} \frac{d \boldsymbol{i}_{d q 0}^{\Delta}(t)}{d t}=\boldsymbol{v}_{m, d q 0}^{\Delta}(t)-\boldsymbol{v}_{s, d q 0}(t)-\frac{R}{2} \boldsymbol{i}_{d q 0}^{\Delta}(t)-\frac{L}{2} \boldsymbol{J}_{\omega} \boldsymbol{i}_{d q 0}^{\Delta}(t)  \tag{2.110c}\\
L \frac{d \boldsymbol{i}_{d q 0}^{\Sigma}(t)}{d t}=\left[\begin{array}{c}
0 \\
0 \\
\frac{v_{d c}}{2}
\end{array}\right]-\boldsymbol{v}_{m, d q 0}^{\Sigma}(t)-R \boldsymbol{i}_{d q 0}^{\Sigma}(t)-L \boldsymbol{J}_{-2 \omega} \boldsymbol{i}_{d q 0}^{\Sigma}(t)  \tag{2.110d}\\
\frac{d \boldsymbol{v}_{c, d q 0}^{\Delta}(t)}{d t}=  \tag{2.110e}\\
\frac{1}{2 C_{a r m}}\left(\left[\boldsymbol{M}_{3 a}^{\Sigma}(t)\right] \boldsymbol{i}_{d q 0}^{\Delta}(t)+\left[\boldsymbol{M}_{3 b}^{\Delta}(t)\right] \boldsymbol{i}_{d q 0}^{\Sigma}(t)\right)-\boldsymbol{J}_{\omega} \boldsymbol{v}_{c, d q 0}^{\Delta}(t)  \tag{2.110f}\\
\frac{d t}{\Sigma}= \\
\frac{1}{2 C_{a r m}}\left(\left[\boldsymbol{M}_{4 a}^{\Delta}(t)\right] \boldsymbol{i}_{d q 0}^{\Delta}(t)+\left[\boldsymbol{M}_{4 b}^{\Sigma}(t)\right] \boldsymbol{i}_{d q 0}^{\Sigma}(t)\right)-\boldsymbol{J}_{-2 \omega} \boldsymbol{v}_{c, d q 0}^{\Sigma}(t)
\end{gather*}
$$

## Chapter 3

## Integral-based model

Objectives: Derivation of the integral-based model that can be found in the literature using a unique framework and clarification of the hypotheses behind it. Extension of the model to include more details, such as arm equivalent resistance and arm inductance.

Motivations: Need for a more general expression of the circulating current that takes the passive arm elements into account (see section 1.4.1). Need for a more general expression of the module capacitor voltage ripple that takes the passive arm elements into account (see section 1.4.3).

Contributions: Complete derivation of the integral-based model starting from the $\Delta \Sigma$ MMC model in $a b c$ frame. Development of a generalized circulating current expression. Development of a generalized DC grid current expression. Development of a generalized module capacitor voltage ripple expression. Comparison of the obtained expressions to the state of the art. Study of the influence of the arm equivalent resistance and arm inductance on the module capacitor voltage ripple.

### 3.1 Introduction

## Circulating current

The analytical expression of the circulating current is necessary in order to predict the frequencies, amplitudes and phase angles of the current harmonic components within the converter. In addition, the understanding of the circulating current helps in the design stage of the converter since it affects the current and voltage ratings of its components. Further, this expression helps designing the circulating current suppression controllers. The assumptions and derivation of the analytical expression of the circulating current are adapted from [79].

## Module capacitor voltage ripple

Here we use the integral approach starting from the $\Delta \Sigma$ MMC model in $a b c$ frame to derive the energy variation for the arm. This will allow us to obtain the module capacitor voltage ripple.

In the literature, many simplifications have been considered due to the complexity of the integral-based method. Usually, these simplifications consider that the arm inductance and arm resistance are neglected. Next to that, the circulating current is often suppressed. Furthermore, it has been shown in Eq. (3.68) that the choice of inductance can provoke the resonance of the circulating current. Since the circulating current is part of the arm current, the capacitor voltage is also the function of the circulating current. Therefore, the choice of inductance clearly affects the capacitor voltage and therefore the instantaneous energy of the capacitor.

For example, Merlin et al. [20] used the integral-based method in order to calculate the instantaneous arm energy supposing that $L=0, R=0$, and $i_{2}^{\Sigma}=0$. They pushed forward this study to calculate the voltage ripple. This study showed that the instantaneous energy is directly proportional to the converters apparent power and inversely proportional to the grid frequency. Further they demonstrated that the instantaneous arm energy is non-linear function of grid current amplitude and phase. They concluded that the biggest module capacitor voltage ripple occurs when the MMC transfers only the reactive power. However, all of these conclusions are valid only for the adopted hypotheses.

Ilves et al. [49] have addressed the subject of energy storage requirements of the MMC using the integral-based model. They explicitly neglected the circulating current $\left(i_{2}^{\Sigma}=0\right)$. However, it was unclear if the impact of the arm coil was considered. Since the authors use [49, Eq. 7, pg. 79], we conclude that they considered $L=0$ and $R=0$. The authors have studied the impact of the modulation index on the instantaneous energy and have reached similar results to [20]. They concluded that the instantaneous arm energy depends on the power transfer and that the highest arm energy ripple occurs for the reactive power transfer.

Tang et al. [50] studied the capacitor voltage ripple with the goal of selecting the appropriate capacitor value. The authors considered that the circulating current is suppressed, but propose a way of including the arm inductance by adapting the modulation index using [50, Eq. 51]. The authors concluded as well that the maximal voltage ripple occurs for the pure reactive power transfer and that an increase of the modulation index increases the voltage ripple. However, even though the inductance is considered, capacitor average voltage is still considered constant. One can see that this is not true, using Eq. (3.45), where the modulation index affects the capacitor average voltage.

Kim et al. [51] employed the integral-based model for the analytical calculation of the instantaneous arm energy with the goal of extending the PQ capability of the MMC during a module failure. In these situations the MMC should be able to deliver the demanded power and the circulating current could make this possible at the cost of increased power loss. The authors consider $L=0$ and $R=0$ but as they make the hypothesis that the circulating current can help in increasing the power capability of the converter, therefore $i_{2}^{\Sigma} \neq 0$. The authors concluded that the circulating current increases the area of operation of the MMC and provide the demanded power.

### 3.2 Hypotheses and definitions

In this chapter we adopt the following hypotheses.

Module voltages Let us remind the assumptions from Eqs. (2.10)-(2.11),

$$
\begin{equation*}
v_{m o d, u}(t) \stackrel{\text { hyp }}{=} m_{u}(t) v_{c a p, u}(t) \tag{3.1}
\end{equation*}
$$

$$
\begin{equation*}
v_{m o d, l}(t) \stackrel{\text { hyp }}{=} m_{l}(t) v_{c a p, l}(t) \tag{3.2}
\end{equation*}
$$

where $v_{c a p}$, is defined as,

$$
\begin{equation*}
v_{c a p}(t) \triangleq V_{c a p}+\tilde{v}_{c a p}(t) \tag{3.3}
\end{equation*}
$$

where $V_{c a p}$ is the module capacitor average voltage and $\tilde{v}_{c a p}$ is module capacitor the voltage fluctuation.
Current and voltage waveforms Throughout this document, the angle of the phase $a$ of the grid voltage $v_{s}$ is taken as a reference,

$$
\begin{equation*}
v_{s}(t) \stackrel{h y p}{=} V_{s} \cos (\omega t) \tag{3.4}
\end{equation*}
$$

where $V_{s}$ is the grid voltage amplitude. Let us assume the grid current $i^{\Delta}$,

$$
\begin{equation*}
i^{\Delta}(t) \stackrel{h y p}{=} I^{\Delta} \cos \left(\omega t+\varphi_{i}^{\Delta}\right) \tag{3.5}
\end{equation*}
$$

where $I^{\Delta}$ is the amplitude and $\varphi_{i}^{\Delta}$ is the phase shift of the grid current. Therefore, as a result of hypotheses in Eqs. (3.4)-(3.5), the power factor at the Point of Common Coupling (PCC) is $\cos \left(\varphi_{i}^{\Delta}\right)$. If $n$ is the harmonic order, we can assume the form of each harmonic of the current $i^{\Sigma}$,

$$
\begin{gather*}
i^{\Sigma}=I_{0}^{\Sigma}+\sum_{1}^{\infty} i_{n}^{\Sigma}(t)  \tag{3.6}\\
i_{n}^{\Sigma}(t) \stackrel{h y p}{=} I_{n}^{\Sigma} \cos \left(n \omega t+\varphi_{n}^{\Sigma}\right) \tag{3.7}
\end{gather*}
$$

where $I_{n}^{\Sigma}$ is the amplitude and $\varphi_{n}^{\Sigma}$ is the phase shift of the current $i_{n}^{\Sigma}$. Similarly, we assume the form of $v_{m}^{\Sigma}$,

$$
\begin{align*}
& v_{m}^{\Sigma}(t) \stackrel{h y}{\underline{h y p}} V_{0}^{\Sigma}+\sum_{n=1}^{\infty} v_{n}^{\Sigma}(t)  \tag{3.8}\\
& v_{n}^{\Sigma}(t) \stackrel{h \underline{y} p}{=} V_{n}^{\Sigma} \cos \left(n \omega t+\psi_{n}^{\Sigma}\right) \tag{3.9}
\end{align*}
$$

where $V_{0}$ is the constant term of $v_{m}^{\Sigma}$ and $v_{n}^{\Sigma}$ is the n-th harmonic voltage with the amplitude $V_{n}^{\Sigma}$ and phase shift $\psi_{n}^{\Sigma}$. Let us assume the waveform of the voltage $v_{m}^{\Delta}$ as well,

$$
\begin{equation*}
v_{m}^{\Delta}(t) \stackrel{h y p}{=} V_{m}^{\Delta} \cos \left(\omega t+\varphi_{t}^{\Delta}\right) \tag{3.10}
\end{equation*}
$$

where $V_{m}^{\Delta}$ is the amplitude and $\varphi_{t}^{\Delta}$ is the phase shift of $v_{m}^{\Delta}$.
Modulation index waveform Let us assume the modulation index waveform as,

$$
\begin{equation*}
m^{\Delta}(t) \stackrel{h y p}{=} M^{\Delta} \cos \left(\omega t+\varphi_{m}^{\Delta}\right) \tag{3.11}
\end{equation*}
$$

where $M^{\Delta}$ and $\varphi_{m}^{\Delta}$ are the modulation index amplitude and phase, respectively.
Complex transformation definition Let us define the transformation from time to complex domain as,

$$
\begin{equation*}
f(t)=F \cos \left(\omega t+\theta_{f}\right) \stackrel{\text { def }}{\Longrightarrow} \bar{f}(t)=F e^{j\left(\omega t+\theta_{f}\right)} \tag{3.12}
\end{equation*}
$$

Now the current $i_{n}^{\Sigma}$ and voltage $v_{n}^{\Sigma}$ in Eqs. (3.7) and (3.9) in the complex domain are,

$$
\begin{align*}
& \overline{i_{n}^{\Sigma}}(t)=I_{n}^{\Sigma} e^{j\left(n \omega t+\varphi_{n}^{\Sigma}\right)}  \tag{3.13}\\
& \overline{v_{n}^{\Sigma}}(t)=V_{n}^{\Sigma} e^{j\left(n \omega t+\psi_{n}^{\Sigma}\right)} \tag{3.14}
\end{align*}
$$

### 3.3 Arm currents

Here we develop the waveform of upper and lower arm current. From Fig. 1.3,

$$
\begin{equation*}
\sum_{k=a, b, c} i_{u, k}(t)=i_{d c} \tag{3.15}
\end{equation*}
$$

Insert Eq. (2.8),

$$
\begin{equation*}
\sum_{k=a, b, c}\left(\frac{i_{k}^{\Delta}(t)}{2}+i_{k}^{\Sigma}(t)\right)=i_{d c} \tag{3.16}
\end{equation*}
$$

Since the AC grid is assumed symmetrical, $\sum_{k=a, b, c} i_{k}^{\Delta}=0$, the Eq. (3.16) becomes,

$$
\begin{equation*}
\sum_{k=a, b, c} i_{k}^{\Sigma}(t)=i_{d c} \tag{3.17}
\end{equation*}
$$

Input Eq. (3.6) into Eq. (3.17),

$$
\begin{equation*}
\sum_{k=a, b, c}\left(I_{0, k}^{\Sigma}+\sum_{0}^{n} i_{n, k}^{\Sigma}(t)\right)=i_{d c} \tag{3.18}
\end{equation*}
$$

Taking the mean of both sides,

$$
\begin{equation*}
\sum_{k=a, b, c} I_{0, k}^{\Sigma}=i_{d c} \tag{3.19}
\end{equation*}
$$

Since the system is balanced, the current distributes equally across the three phases of the converter,

$$
\begin{equation*}
3 I_{0, k}^{\Sigma}=i_{d c} \tag{3.20}
\end{equation*}
$$

Therefore, the Eq. (3.6) is equal to,

$$
\begin{equation*}
i_{k}^{\Sigma}(t)=\frac{i_{d c}}{3}+\sum_{1}^{n} i_{n, k}^{\Sigma}(t) \tag{3.21}
\end{equation*}
$$

The currents $\sum_{1}^{n} i_{n, k}^{\Sigma}$ are the circulating currents, see section 3.4. Finally, insert Eq. (3.21) into Eqs. (2.8) and (2.9),

$$
\begin{align*}
& i_{u, k}(t)=\frac{i_{d c}}{3}+\frac{i_{k}^{\Delta}(t)}{2}+\sum_{1}^{n} i_{n, k}^{\Sigma}(t)  \tag{3.22}\\
& i_{l, k}(t)=\frac{i_{d c}}{3}-\frac{i_{k}^{\Delta}(t)}{2}+\sum_{1}^{n} i_{n, k}^{\Sigma}(t) \tag{3.23}
\end{align*}
$$

Thus, the analytical expressions of the arm currents are derived.

### 3.4 Equivalent arm capacitor $C_{\text {arm }}$ charge

### 3.4.1 Development

Let us summarize the $\Delta \Sigma$ model in $a b c$ frame for the phase $a$ (Eq. (2.109)). Omitting the phase index they become,

$$
\begin{gather*}
v_{m}^{\Delta}(t)=\frac{1}{2}\left(m^{\Delta}(t) v_{c}^{\Sigma}(t)+m^{\Sigma}(t) v_{c}^{\Delta}(t)\right)  \tag{3.24}\\
v_{m}^{\Sigma}(t)=\frac{1}{2}\left[m^{\Sigma}(t) v_{c}^{\Sigma}(t)+m^{\Delta}(t) v_{c}^{\Delta}(t)\right]  \tag{3.25}\\
L \frac{d i^{\Sigma}(t)}{d t}=\frac{1}{2} v_{d c}-v^{\Sigma}(t)-R i^{\Sigma}(t) \tag{3.26}
\end{gather*}
$$

$$
\begin{align*}
\frac{L}{2} \frac{d i^{\Delta}(t)}{d t} & =v_{m}^{\Delta}(t)-v_{s}(t)-\frac{R}{2} i^{\Delta}(t)  \tag{3.27}\\
2 C_{a r m} \frac{d v_{c}^{\Delta}(t)}{d t} & =-m^{\Sigma}(t) \frac{i^{\Delta}(t)}{2}+m^{\Delta}(t) i^{\Sigma}(t)  \tag{3.28}\\
2 C_{a r m} \frac{d v_{c}^{\Sigma}(t)}{d t} & =-m^{\Delta}(t) \frac{i^{\Delta}(t)}{2}+m^{\Sigma}(t) i^{\Sigma}(t) \tag{3.29}
\end{align*}
$$

The calculations here are based on Eqs.(3.25), (3.28) and (3.29) Let us integrate the Eqs. (3.28) and (3.29),

$$
\begin{align*}
& v_{c}^{\Delta}(t)=\int_{t_{0}}^{t} \frac{1}{2 C_{a r m}}\left(-m^{\Sigma}(\tau) \frac{i^{\Delta}(\tau)}{2}+m^{\Delta}(\tau) i^{\Sigma}(\tau)\right) d \tau+v_{c}^{\Delta}\left(t_{0}\right)  \tag{3.30}\\
& v_{c}^{\Sigma}(t)=\int_{t_{0}}^{t} \frac{1}{2 C_{a r m}}\left(-m^{\Delta}(\tau) \frac{i^{\Delta}(\tau)}{2}+m^{\Sigma}(\tau) i^{\Sigma}(\tau)\right) d \tau+v_{c}^{\Sigma}\left(t_{0}\right) \tag{3.31}
\end{align*}
$$

Let us insert Eqs. (3.30) and (3.31) into Eq. (3.25),

$$
\begin{align*}
v_{m}^{\Sigma}(t)= & \frac{1}{2}\left[m^{\Sigma}(t)\left(\int_{t_{0}}^{t} \frac{1}{2 C_{a r m}}\left(-m^{\Delta}(\tau) \frac{i^{\Delta}(\tau)}{2}+m^{\Sigma}(\tau) i^{\Sigma}(\tau)\right) d \tau+v_{c}^{\Sigma}\left(t_{0}\right)\right)\right.  \tag{3.32}\\
& \left.+m^{\Delta}(t)\left(\int_{t_{0}}^{t} \frac{1}{2 C_{a r m}}\left(-m^{\Sigma}(\tau) \frac{i^{\Delta}(\tau)}{2}+m^{\Delta}(\tau) i^{\Sigma}(\tau)\right) d \tau+v_{c}^{\Delta}\left(t_{0}\right)\right)\right]
\end{align*}
$$

According to the hypotheses in Eqs. (2.18) and (2.19) and the definition of $m^{\Sigma}$ in Eq. (2.43), $m^{\Sigma}$ is always equal to one (when the circulating current is not suppressed), so,

$$
\begin{align*}
v_{m}^{\Sigma}(t)= & \frac{1}{2}\left[\left(\int_{t_{0}}^{t} \frac{1}{2 C_{a r m}}\left(-m^{\Delta}(\tau) \frac{i^{\Delta}(\tau)}{2}+i^{\Sigma}(\tau)\right) d \tau+v_{c}^{\Sigma}\left(t_{0}\right)\right)\right.  \tag{3.33}\\
& \left.+m^{\Delta}(t)\left(\int_{t_{0}}^{t} \frac{1}{2 C_{a r m}}\left(-\frac{i^{\Delta}(\tau)}{2}+m^{\Delta}(\tau) i^{\Sigma}(\tau)\right) d \tau+v_{c}^{\Delta}\left(t_{0}\right)\right)\right]
\end{align*}
$$

Let us multiply both sides by $2 \frac{C}{N}$,

$$
\begin{align*}
2 \frac{C}{N} v_{m}^{\Sigma}(t)= & \int_{t_{0}}^{t}\left(-m^{\Delta}(\tau) \frac{i^{\Delta}(\tau)}{2}+i^{\Sigma}(\tau)\right) d \tau+2 \frac{C}{N} v_{c}^{\Sigma}\left(t_{0}\right) \\
& +m^{\Delta}(t)\left(\int_{t_{0}}^{t}\left(-\frac{i^{\Delta}(\tau)}{2}+m^{\Delta}(\tau) i^{\Sigma}(\tau)\right) d \tau+2 \frac{C}{N} v_{c}^{\Delta}\left(t_{0}\right)\right) \tag{3.34}
\end{align*}
$$

Insert Eq. (3.8),

$$
\begin{align*}
2 \frac{C\left(V_{0}^{\Sigma}+\sum_{n=1}^{\infty} v_{n}^{\Sigma}(t)\right)}{N}= & \int_{t_{0}}^{t}\left(-m^{\Delta}(\tau) \frac{i^{\Delta}(\tau)}{2}+i^{\Sigma}(\tau)\right) d \tau+2 \frac{C}{N} v_{c}^{\Sigma}\left(t_{0}\right)  \tag{3.35}\\
& +m^{\Delta}(t)\left(\int_{t_{0}}^{t}\left(-\frac{i^{\Delta}(\tau)}{2}+m^{\Delta}(\tau) i^{\Sigma}(\tau)\right) d \tau+2 \frac{C}{N} v_{c}^{\Delta}\left(t_{0}\right)\right)
\end{align*}
$$

Since all the modules are identical and instantaneous capacitor voltage balancing is assumed, the voltages $v_{c}^{\Sigma}\left(t_{0}\right)$ and $v_{c}^{\Delta}\left(t_{0}\right)$ are equal to $\frac{N}{2}\left(v_{u, c a p}\left(t_{0}\right)+v_{l, c a p}\left(t_{0}\right)\right)$ and $\frac{N}{2}\left(v_{l, c a p}\left(t_{0}\right)-v_{u, c a p}\left(t_{0}\right)\right)$ respectively,

$$
\begin{align*}
2 \frac{C\left(V_{0}^{\Sigma}+\sum_{n=1}^{\infty} v_{n}^{\Sigma}(t)\right)}{N}= & \int_{t_{0}}^{t}\left(-m^{\Delta}(\tau) \frac{i^{\Delta}(\tau)}{2}+i^{\Sigma}(\tau)\right) d \tau+C\left(v_{u, c a p}\left(t_{0}\right)+v_{l, c a p}\left(t_{0}\right)\right) \\
& +m^{\Delta}(t)\left(\int_{t_{0}}^{t}\left(-\frac{i^{\Delta}(\tau)}{2}+m^{\Delta}(\tau) i^{\Sigma}(\tau)\right) d \tau+C\left(v_{l, c a p}\left(t_{0}\right)-v_{u, c a p}\left(t_{0}\right)\right)\right) \tag{3.36}
\end{align*}
$$

Replace the currents $i^{\Delta}$ and $i^{\Sigma}$ using the Eq. (2.41),

$$
\begin{align*}
2 \frac{C\left(V_{0}^{\Sigma}+\sum_{n=1}^{\infty} v_{n}^{\Sigma}(t)\right)}{N}= & \int_{t_{0}}^{t}\left(-m^{\Delta}(\tau) \frac{i_{u}(\tau)-i_{l}(\tau)}{2}+\frac{i_{u}(\tau)+i_{l}(\tau)}{2}\right) d \tau \\
& +C\left(v_{u, c a p}\left(t_{0}\right)+v_{l, c a p}\left(t_{0}\right)\right) \\
& +m^{\Delta}(t)\left(\int_{t_{0}}^{t}\left(-\frac{i_{u}(\tau)-i_{l}(\tau)}{2}+m^{\Delta}(\tau) \frac{i_{u}(\tau)+i_{l}(\tau)}{2}\right) d \tau\right.  \tag{3.37}\\
& \left.+C\left(v_{l, c a p}\left(t_{0}\right)-v_{u, c a p}\left(t_{0}\right)\right)\right)
\end{align*}
$$

Insert Eqs. (3.22)-(3.23) and simplify,

$$
\begin{align*}
\frac{2 C\left(V_{0}^{\Sigma}+\sum_{n=1}^{\infty} v_{n}^{\Sigma}(t)\right)}{N}= & \int_{t_{0}}^{t}\left(\frac{i_{d c}}{3}-\frac{i^{\Delta}(\tau)}{2} m^{\Delta}(\tau)+\sum_{n=1}^{\infty} i_{n}^{\Sigma}(\tau)\right) d t- \\
& -m^{\Delta}(t) \int_{t_{0}}^{t}\left(-\frac{i_{d c}}{3} m^{\Delta}(t)+\frac{i^{\Delta}(\tau)}{2}-m(\tau) \sum_{n=1}^{\infty} i_{n}^{\Sigma}(\tau)\right) d t  \tag{3.38}\\
& +C\left(v_{c a p, u}\left(t_{0}\right)+v_{c a p, l}\left(t_{0}\right)\right) \\
& -m^{\Delta}(t) C\left(v_{c a p, u}\left(t_{0}\right)-v_{c a p, l}\left(t_{0}\right)\right)
\end{align*}
$$

Using Eqs. (2.20),(3.5) and (3.7), Eq. (3.38) can be developed as,

$$
\begin{equation*}
\frac{2 C\left(V_{0}^{\Sigma}+\sum_{n=1}^{\infty} v_{n}^{\Sigma}(t)\right)}{N}=Q_{0}+Q_{1}+Q_{2}+Q_{3}+Q_{4}+Q_{5}+Q_{6} \tag{3.39}
\end{equation*}
$$

where,

$$
\begin{align*}
& Q_{0}=C\left(v_{c a p, u}\left(t_{0}\right)+v_{c a p, l}\left(t_{0}\right)\right)-m^{\Delta}(t) C\left(v_{c a p, u}\left(t_{0}\right)-v_{c a p, l}\left(t_{0}\right)\right)  \tag{3.40a}\\
& Q_{1}=\int_{t_{0}}^{t} \frac{i_{d c}}{3} d \tau  \tag{3.40b}\\
& Q_{2}=\int_{t_{0}}^{t} \sum_{n=1}^{\infty} i_{n}^{\Sigma} d \tau=\int_{t_{0}}^{t} \sum_{n=1}^{\infty} I_{n}^{\Sigma} \cos \left(n \omega \tau+\varphi_{n}^{\Sigma}\right) d \tau  \tag{3.40c}\\
& Q_{3}=-\int_{t_{0}}^{t} \frac{I^{\Delta}}{2} M^{\Delta} \cos \left(\omega \tau+\varphi_{i}^{\Delta}\right) \cos \left(\omega \tau+\varphi_{m}^{\Delta}\right) d \tau  \tag{3.40d}\\
& Q_{4}=-M^{\Delta} \cos \left(\omega t+\varphi_{m}^{\Delta}\right) \int_{t_{0}}^{t} \frac{I^{\Delta}}{2} \cos \left(\omega \tau+\varphi_{i}^{\Delta}\right) d \tau  \tag{3.40e}\\
& Q_{5}=M^{\Delta} \cos \left(\omega t+\varphi_{m}^{\Delta}\right) \int_{t_{0}}^{t} \frac{i_{d c}}{3} M^{\Delta} \cos \left(\omega \tau+\varphi_{m}^{\Delta}\right) d \tau  \tag{3.40f}\\
& Q_{6}=M^{\Delta} \cos \left(\omega t+\varphi_{m}^{\Delta}\right) \int_{t_{0}}^{t} M^{\Delta} \cos \left(\omega \tau+\varphi_{m}^{\Delta}\right) \sum_{n=1}^{\infty} I_{n}^{\Sigma} \cos \left(n \omega \tau+\varphi_{n}^{\Sigma}\right) d \tau \tag{3.40~g}
\end{align*}
$$

The integrations yield,

$$
\begin{align*}
Q_{0} & =C\left(v_{c a p, u}\left(t_{0}\right)+v_{c a p, l}\left(t_{0}\right)\right)-m^{\Delta}(t) C\left(v_{c a p, u}\left(t_{0}\right)-v_{c a p, l}\left(t_{0}\right)\right)  \tag{3.41a}\\
Q_{1} & =\frac{i_{d c}}{3}\left(t-t_{0}\right)  \tag{3.41b}\\
Q_{2} & =\sum_{n=1}^{\infty} \frac{I_{n}}{n \omega}\left(\sin \left(n \omega t+\varphi_{n}\right)-\sin \left(n \omega t_{0}+\varphi_{n}\right)\right)  \tag{3.41c}\\
Q_{3} & =-\frac{I^{\Delta}}{4} M^{\Delta} \cos \left(\varphi_{m}^{\Delta}-\varphi_{i}^{\Delta}\right)\left(t-t_{0}\right)-\frac{I^{\Delta}}{8 \omega} M^{\Delta}\left(\sin \left(2 \omega t+\varphi_{i}^{\Delta}+\varphi_{m}^{\Delta}\right)\right.  \tag{3.41d}\\
& \left.-\sin \left(2 \omega t_{0}+\varphi_{i}^{\Delta}+\varphi_{m}^{\Delta}\right)\right) \\
Q_{4} & =-M^{\Delta} \frac{I^{\Delta}}{4 w}\left(\sin \left(2 \omega t+\varphi_{i}^{\Delta}+\varphi_{m}^{\Delta}\right)-\sin \left(\varphi_{m}^{\Delta}-\varphi_{i}^{\Delta}\right)-\sin \left(\omega t+\omega t_{0}+\varphi_{i}^{\Delta}+\varphi_{m}^{\Delta}\right)\right.  \tag{3.41e}\\
& \left.+\sin \left(\omega t+\varphi_{m}^{\Delta}-\omega t_{0}-\varphi_{i}^{\Delta}\right)\right) \\
Q_{5} & =\left(M^{\Delta}\right)^{2} \frac{i_{d c}}{6 w}\left(\sin \left(2 \omega t+2 \varphi_{m}^{\Delta}\right)-\sin \left(\omega t+\omega t_{0}+2 \varphi_{m}^{\Delta}\right)+\sin \left(\omega t-\omega t_{0}\right)\right)  \tag{3.41f}\\
Q_{6} & =\left(M^{\Delta}\right)^{2} \sum_{n=1}^{\infty} \frac{I_{n}}{4} \frac{1}{\omega(n+1)}\left(\sin \left(\omega t(n+2) 2 \varphi_{m}^{\Delta}+\varphi_{n}\right)+\sin \left(n \omega t+\varphi_{n}\right)\right.  \tag{3.41~g}\\
& \left.-\sin \left(\omega t+\omega t_{0}(n+1)+2 \varphi_{m}^{\Delta}+\varphi_{n}\right)+\sin \left(\omega t-\omega t_{0}(n+1)-\varphi_{n}\right)\right) \\
& +\left(M^{\Delta}\right)^{2} \cos \left(\omega t+\varphi_{m}^{\Delta}\right) \frac{I_{1}}{2} \cos \left(\varphi_{m}^{\Delta}-\varphi_{n}\right)\left(t-t_{0}\right) \\
& \frac{\left(M^{\Delta}\right)^{2}}{4 w} \sum_{n=2}^{\infty} \frac{I_{n}}{n-1}\left(\sin \left(n \omega t+\varphi_{m}^{\Delta}\right)-\sin \left(\omega t(2-n)+\varphi_{m}^{\Delta}-\varphi_{n}\right)\right) \\
& -\frac{\left(M^{\Delta}\right)^{2}}{4 w} \sum_{n=2}^{\infty}\left(\sin \left(\omega t+(n-1) \omega t_{0}+\varphi_{n}\right)-\sin \left(\omega t-(n-1) \omega t_{0}+2 \varphi_{m}^{\Delta}-\varphi_{n}\right)\right)
\end{align*}
$$

We choose $t_{0}$ such as $v_{\text {cap }}\left(t_{0}\right)=V_{c a p}$, therefore,

$$
\begin{align*}
Q_{0} & =2 C V_{c a p}  \tag{3.42a}\\
Q_{1} & =\frac{i_{d c}}{3} t  \tag{3.42b}\\
Q_{2} & =\sum_{n=1}^{\infty} \frac{I_{n}}{n w}\left(\sin \left(n \omega t+\varphi_{n}\right)-\sin \left(\varphi_{n}\right)\right)  \tag{3.42c}\\
Q_{3} & =-\frac{I^{\Delta}}{4} M^{\Delta} \cos \left(\varphi_{i}^{\Delta}\right) t-\frac{I^{\Delta}}{8 w} M^{\Delta} \sin \left(2 \omega t+\varphi_{i}^{\Delta}\right)+\frac{I^{\Delta}}{8 w} M^{\Delta} \sin \left(\varphi_{i}^{\Delta}+\varphi_{m}^{\Delta}\right)  \tag{3.42d}\\
Q_{4} & =M^{\Delta} \frac{I^{\Delta}}{4 \omega}\left(\sin \left(2 \omega t+\varphi_{i}^{\Delta}\right)-\sin \left(\varphi_{m}^{\Delta}-\varphi_{i}^{\Delta}\right)-\sin \left(\omega t+\varphi_{i}^{\Delta}+\varphi_{m}^{\Delta}\right)+\sin \left(\omega t+\varphi_{m}^{\Delta}-\varphi_{i}^{\Delta}\right)\right)  \tag{3.42e}\\
Q_{5} & =-\left(M^{\Delta}\right)^{2} \frac{i_{d c}}{6 w}\left(\sin \left(2 \omega t+2 \varphi_{m}^{\Delta}\right)-\sin \left(\omega t+2 \varphi_{m}^{\Delta}\right)+\sin (\omega t)\right)  \tag{3.42f}\\
Q_{6} & =\left(M^{\Delta}\right)^{2} \sum_{n=1}^{\infty} \frac{I_{n}}{4} \frac{1}{\omega(n+1)}\left(\sin \left(\omega t(n+2) 2 \varphi_{m}^{\Delta}+\varphi_{n}\right)+\sin \left(n \omega t+\varphi_{n}\right)\right.  \tag{3.42~g}\\
& \left.-\sin \left(\omega t+2 \varphi_{m}^{\Delta}+\varphi_{n}\right)+\sin \left(\omega t-\varphi_{n}\right)\right) \\
& +\left(M^{\Delta}\right)^{2} \cos \left(\omega t+\varphi_{m}^{\Delta}\right) \frac{I_{1}}{2} \cos \left(\varphi_{m}^{\Delta}-\varphi_{n}\right) t \\
& +\frac{\left(M^{\Delta}\right)^{2}}{4 w} \sum_{n=2}^{\infty} \frac{I_{n}}{n-1}\left(\sin \left(n \omega t+\varphi_{m}^{\Delta}\right)-\sin \left(\omega t(2-n)+\varphi_{m}^{\Delta}-\varphi_{n}\right)\right) \\
& -\frac{\left(M^{\Delta}\right)^{2}}{4 w} \sum_{n=2}^{\infty} \frac{I_{n}}{4 \omega(n-1)}\left(\sin \left(\omega t+\varphi_{n}\right)-\sin \left(\omega t+2 \varphi_{m}^{\Delta}-\varphi_{n}\right)\right)
\end{align*}
$$

In the following, we will use the principle of superposition to analyze each harmonic of Eq. (3.39) separately. We focus on constant, linear, fundamental harmonic and second harmonic terms. With the aim of expressing the circulating current $i_{2}^{\Sigma}$, we observe in particular the second harmonic influence, claimed to be the most important one [79]. For this scenario, the circuit in the Fig. 3.1 is formed, valid for $n>0$. Given that the modules are ideal and identical according to the adopted assumptions, the sum of their $v_{m o d}$ voltages is equal to $v_{n}^{\Sigma}$.


Figure 3.1 - MMC equivalent circuit for n-th harmonic [79]

### 3.4.2 Constant terms

Extract the constant terms from Eq. (3.39),

$$
\begin{align*}
\frac{2 C V_{0}^{\Sigma}}{N}= & 2 C V_{c a p}-\sum_{n=1}^{\infty} \frac{I_{n}^{\Sigma}}{n \omega} \sin \varphi_{n}+M^{\Delta} \frac{I^{\Delta}}{8 \omega} \sin \left(\varphi_{m}^{\Delta}+\varphi_{i}^{\Delta}\right)  \tag{3.43}\\
& +M^{\Delta} \frac{I^{\Delta}}{4 \omega} \sin \left(\varphi_{m}^{\Delta}-\varphi_{i}^{\Delta}\right)-\frac{\left(M^{\Delta}\right)^{2} I_{2}^{\Sigma}}{4 \omega} \sin \left(2 \varphi_{m}^{\Delta}-\varphi_{2}^{\Sigma}\right) \\
V_{0}^{\Sigma}= & N V_{c a p}-\frac{N}{2 C} \sum_{n=1}^{\infty} \frac{I_{n}^{\Sigma}}{n \omega} \sin \varphi_{n}+\frac{N}{2 C} M^{\Delta} \frac{I^{\Delta}}{8 \omega} \sin \left(\varphi_{m}^{\Delta}+\varphi_{i}^{\Delta}\right)  \tag{3.44}\\
& +\frac{N}{2 C} M^{\Delta} \frac{I^{\Delta}}{4 \omega} \sin \left(\varphi_{m}^{\Delta}-\varphi_{i}^{\Delta}\right)-\frac{N}{2 C} \frac{\left(M^{\Delta}\right)^{2} I_{2}^{\Sigma}}{4 \omega} \sin \left(2 \varphi_{m}^{\Delta}-\varphi_{2}^{\Sigma}\right)
\end{align*}
$$

After rearranging, we are able to express the average module voltage as,

$$
\begin{align*}
V_{c a p}= & \frac{V_{0}^{\Sigma}}{N}+\frac{1}{2 C} \sum_{n=1}^{\infty} \frac{I_{n}^{\Sigma}}{n \omega} \sin \varphi_{n}-\frac{1}{2 C} M^{\Delta} \frac{I^{\Delta}}{8 \omega} \sin \left(\varphi_{m}^{\Delta}+\varphi_{i}^{\Delta}\right)  \tag{3.45}\\
& -\frac{1}{2 C} M^{\Delta} \frac{I^{\Delta}}{4 \omega} \sin \left(\varphi_{m}^{\Delta}-\varphi_{i}^{\Delta}\right)+\frac{1}{2 C} \frac{\left(M^{\Delta}\right)^{2} I_{2}^{\Sigma}}{4 \omega} \sin \left(2 \varphi_{m}^{\Delta}-\varphi_{2}^{\Sigma}\right)
\end{align*}
$$

The mathematical expression in Eq. (3.45) has not been reported in the literature. It describes the behavior of the module capacitor average voltage but does not account for arm passive elements directly.

### 3.4.3 Linear terms

Extract terms that increase linearly with time in Eqs. (3.39),

$$
\begin{equation*}
\left[\frac{i_{d c}}{3}-\frac{I^{\Delta}}{4} M^{\Delta} \cos \left(\varphi_{m}^{\Delta}-\varphi_{i}^{\Delta}\right)+\frac{\left(M^{\Delta}\right)^{2} I_{1}^{\Sigma}}{2} \cos \left(\omega t+\varphi_{m}^{\Delta}\right) \cos \left(\varphi_{m}^{\Delta}-\varphi_{1}^{\Sigma}\right)\right] t=0, \quad \forall t \tag{3.46}
\end{equation*}
$$

The third term in the parentheses is sinusoidal. It is zero only if $I_{1}^{\Sigma}=0$. Then,

$$
\begin{equation*}
i_{d c}=3 \frac{I^{\Delta}}{4} M^{\Delta} \cos \left(\varphi_{m}^{\Delta}-\varphi_{i}^{\Delta}\right) \tag{3.47}
\end{equation*}
$$

The Eq. (3.47) shows that DC current increases linearly with the increase of the AC grid current amplitude. On the other hand, we can see that the DC current is not directly proportional to the power factor $\cos \left(\varphi_{i}^{\Delta}\right)$, due to the modulation index phase shift $\varphi_{m}^{\Delta}$. If $\varphi_{m}^{\Delta}=0$ is assumed, this leads to,

$$
\begin{equation*}
i_{d c}=3 \frac{I^{\Delta}}{4} M^{\Delta} \cos \left(\varphi_{i}^{\Delta}\right) \tag{3.48}
\end{equation*}
$$

which is the same as the expressions found in many articles calculated using the balance of power between the AC and the DC side (neglecting the $\varphi_{m}^{\Delta}$ ). $[20,23,62,63,64]$. The expression proposed here is more generic as it takes into account $\varphi_{m}^{\Delta}$.

To the best of our knowledge, this is the only mathematical demonstration that shows that $I_{1}^{\Sigma}=0$.

### 3.4.4 Fundamental harmonic terms

Extract the fundamental harmonic terms from Eq. (3.39),

$$
\begin{align*}
\frac{2 C v_{1}^{\Sigma}(t)}{N}= & \frac{I_{1}^{\Sigma}}{\omega} \sin \left(\omega t+\varphi_{1}^{\Sigma}\right)-M^{\Delta} \frac{I^{\Delta}}{4 \omega} \sin \left(\omega t+\varphi_{i}^{\Delta}+\varphi_{m}^{\Delta}\right)+M^{\Delta} \frac{I^{\Delta}}{4 \omega} \sin \left(\omega t+\varphi_{m}^{\Delta}-\varphi_{i}^{\Delta}\right) \\
& +\left(M^{\Delta}\right)^{2} \frac{i_{d c}}{6 \omega} \sin \left(\omega t+2 \varphi_{m}^{\Delta}\right)-\left(M^{\Delta}\right)^{2} \frac{i_{d c}}{6 w} \sin (\omega t)+\left(M^{\Delta}\right)^{2} \frac{I_{1}^{\Sigma}}{8 \omega} \sin \left(\omega t+\varphi_{1}^{\Sigma}\right) \\
& -\left(M^{\Delta}\right)^{2} \sum_{n=1}^{\infty} \frac{I_{n}^{\Sigma}}{4} \frac{1}{\omega(n+1)} \sin \left(\omega t+2 \varphi_{m}^{\Delta}+\varphi_{n}^{\Sigma}\right)+\left(M^{\Delta}\right)^{2} \sum_{n=1}^{\infty} \frac{I_{n}^{\Sigma}}{4} \frac{1}{\omega(n+1)} \sin \left(\omega t-\varphi_{n}^{\Sigma}\right) \\
& +\frac{\left(M^{\Delta}\right)^{2}}{4 \omega} \frac{I_{3}^{\Sigma}}{2} \sin \left(\omega t-\varphi_{m}^{\Delta}+\varphi_{n}^{\Sigma}\right) \\
& -\frac{\left(M^{\Delta}\right)^{2}}{4 \omega} \sum_{n=2}^{\infty} \frac{I_{n}^{\Sigma}}{4 \omega(n-1)}\left(\sin \left(\omega t+\varphi_{n}^{\Sigma}\right)-\sin \left(\omega t+2 \varphi_{m}^{\Delta}-\varphi_{n}^{\Sigma}\right)\right) \tag{3.49}
\end{align*}
$$

The Eq. (3.49) describes the relationship between $v_{1}^{\Sigma}$ and modulation index and various currents in the MMC circuit. It does not account for arm passive elements directly. Due to its complexity, it seems that its applicability is limited. This is probably the reason why this equation has not been reported in the literature.

### 3.4.5 Second harmonic terms

Extract the terms at $2 \omega$ from Eq. (3.39),

$$
\begin{align*}
\frac{2 C v_{2}^{\Sigma}(t)}{N} & =\frac{I_{2}^{\Sigma}}{2 \omega} \sin \left(2 \omega t+\varphi_{2}^{\Sigma}\right)-\frac{I^{\Delta}}{8 \omega} M^{\Delta} \sin \left(2 \omega t+\varphi_{i}^{\Delta}+\varphi_{m}^{\Delta}\right)-M^{\Delta} \frac{I^{\Delta}}{4 \omega} \sin \left(2 \omega t+\varphi_{i}^{\Delta}+\varphi_{m}^{\Delta}\right) \\
& +\left(M^{\Delta}\right)^{2} \frac{i_{d c}}{6 \omega} \sin \left(2 \omega t+2 \varphi_{m}^{\Delta}\right)+\frac{\left(M^{\Delta}\right)^{2}}{4} \frac{I_{2}^{\Sigma}}{\omega} \sin \left(2 \omega t+\varphi_{2}^{\Sigma}\right)  \tag{3.50}\\
& +\frac{\left(M^{\Delta}\right)^{2}}{4} \frac{I_{4}^{\Sigma}}{3 \omega} \sin \left(2 \omega t-2 \varphi_{m}^{\Delta}-\varphi_{4}^{\Sigma}\right)+\frac{\left(M^{\Delta}\right)^{2}}{4} \frac{I_{2}^{\Sigma}}{3 \omega} \sin \left(2 \omega t+\varphi_{2}^{\Sigma}\right)
\end{align*}
$$

Ilves et al. [79] consider $I_{4}^{\Sigma} \ll I_{2}^{\Sigma}$. Adopting this, Eq. (3.50) simplifies to,

$$
\begin{align*}
\frac{2 C v_{2}^{\Sigma}(t)}{N}= & -\frac{3 M^{\Delta} I^{\Delta}}{8 \omega} \sin \left(2 \omega t+\varphi_{m}^{\Delta}+\varphi_{i}^{\Delta}\right) \\
& +\left(\frac{I_{2}^{\Sigma}}{2 \omega}+\frac{\left(M^{\Delta}\right)^{2} I_{2}^{\Sigma}}{4 \omega}+\frac{\left(M^{\Delta}\right)^{2} I_{2}^{\Sigma}}{12 \omega}\right) \sin \left(2 \omega t+\varphi_{2}^{\Sigma}\right)  \tag{3.51}\\
& +\frac{\left(M^{\Delta}\right)^{2} i_{d c}}{6 \omega} \sin \left(2 \omega t+2 \varphi_{m}^{\Delta}\right)
\end{align*}
$$

Since $\sin (x)=\cos \left(x-\frac{\pi}{2}\right)$,

$$
\begin{align*}
\frac{2 C v_{2}^{\Sigma}(t)}{N}= & -\frac{3 M^{\Delta} I^{\Delta}}{8 \omega} \cos \left(2 \omega t+\varphi_{m}^{\Delta}+\varphi_{i}^{\Delta}-\frac{\pi}{2}\right) \\
& +\left(\frac{I_{2}^{\Sigma}}{2 \omega}+\frac{\left(M^{\Delta}\right)^{2} I_{2}^{\Sigma}}{4 \omega}+\frac{\left(M^{\Delta}\right)^{2} I_{2}^{\Sigma}}{12 \omega}\right) \cos \left(2 \omega t+\varphi_{2}^{\Sigma}-\frac{\pi}{2}\right)+  \tag{3.52}\\
& \left(M^{\Delta}\right)^{2} \frac{2}{6 \omega} \cos \left(2 \omega t+2 \varphi_{m}^{\Delta}-\frac{\pi}{2}\right)
\end{align*}
$$

According to the superposition theorem, it is possible to separate the converter circuit into several circuits of the same frequency. The sum of the upper and lower arm impedance is $\overline{Z_{n}}=2(R+j n \omega L)$. From Fig. 3.1, the voltage drop across the leg impedance then is,

$$
\begin{equation*}
\overline{v_{n}^{\Sigma}}(t)=-\overline{Z_{n}} \overline{i_{n}^{\Sigma}}(t)=-2(R+j n w L) I_{n}^{\Sigma} e^{j\left(n \omega t+\varphi_{n}^{\Sigma}\right)} \tag{3.53}
\end{equation*}
$$

which for $n=2$ becomes,

$$
\begin{equation*}
\overline{v_{2}^{\Sigma}}(t)=-\overline{Z_{2}} \overline{i_{2}^{\Sigma}}(t)=-2(R+j 2 w L) I_{2}^{\Sigma} e^{j\left(2 \omega t+\varphi_{2}^{\Sigma}\right)} \tag{3.54}
\end{equation*}
$$

Transform Eq. (3.52) to complex domain according to Eq. (3.12),

$$
\begin{align*}
\frac{2 C \overline{v_{2}^{\Sigma}}(t)}{N}= & -\frac{3 M^{\Delta} I^{\Delta}}{8 \omega} e^{j\left(2 \omega t+\varphi_{m}^{\Delta}+\varphi_{i}^{\Delta}-\frac{\pi}{2}\right)} \\
& +\frac{3+2\left(M^{\Delta}\right)^{2}}{6 w} I_{2}^{\Sigma} e^{j\left(2 \omega t+\varphi_{2}^{\Sigma}-\frac{\pi}{2}\right)}+  \tag{3.55}\\
& \left(M^{\Delta}\right)^{2} \frac{i_{d c}}{6 \omega} e^{j\left(2 \omega t+2 \varphi_{m}^{\Delta}-\frac{\pi}{2}\right)}
\end{align*}
$$

Insert Eq. (3.54),

$$
\begin{align*}
-\frac{2 C}{N} 2(R+j 2 \omega L) I_{2} e^{j\left(2 \omega t+\varphi_{2}\right)}= & -\frac{3 M^{\Delta} I^{\Delta}}{8 \omega} e^{j\left(2 \omega t+\varphi_{m}^{\Delta}+\varphi_{i}^{\Delta}-\frac{\pi}{2}\right)} \\
& +\frac{3+2\left(M^{\Delta}\right)^{2}}{6 w} I_{2}^{\Sigma} e^{j\left(2 \omega t+\varphi_{2}^{\Sigma}-\frac{\pi}{2}\right)}  \tag{3.56}\\
& +\left(M^{\Delta}\right)^{2} \frac{i_{d c}}{6 \omega} e^{j\left(2 \omega t+2 \varphi_{m}^{\Delta}-\frac{\pi}{2}\right)}
\end{align*}
$$

Simplify,

$$
\begin{align*}
-\frac{2 C}{N} 2(R+j 2 \omega L) I_{2}^{\Sigma} e^{j\left(2 \omega t+\varphi_{2}\right)}= & j \frac{3 M^{\Delta} I^{\Delta}}{8 \omega} e^{j\left(2 \omega t+\varphi_{m}^{\Delta}+\varphi_{i}^{\Delta}\right)}-j \frac{3+2\left(M^{\Delta}\right)^{2}}{6 w} I_{2}^{\Sigma} e^{j\left(2 \omega t+\varphi_{2}^{\Sigma}\right)}  \tag{3.57}\\
& -j\left(M^{\Delta}\right)^{2} \frac{i_{d c}}{6 \omega} e^{j\left(2 \omega t+2 \varphi_{m}^{\Delta}\right)} \\
\left(-\frac{2 C}{N} 2(R+j 2 \omega L)+j \frac{3+2\left(M^{\Delta}\right)^{2}}{6 \omega}\right) I_{2}^{\Sigma} e^{j\left(2 \omega t+\varphi_{2}^{\Sigma}\right)}= & j \frac{3 M^{\Delta} I^{\Delta}}{8 \omega} e^{j\left(2 \omega t+\varphi_{m}^{\Delta}+\varphi_{i}^{\Delta}\right)}  \tag{3.58}\\
& -j\left(M^{\Delta}\right)^{2} \frac{i_{d c}}{6 \omega} e^{j\left(2 \omega t+2 \varphi_{m}^{\Delta}\right)} \\
I_{2}^{\Sigma} e^{j\left(2 \omega t+\varphi_{2}^{\Sigma}\right)}= & j \frac{\frac{3 M^{\Delta} I^{\Delta}}{8 \omega} e^{j\left(\varphi_{m}^{\Delta}+\varphi_{i}^{\Delta}\right)}-\left(M^{\Delta}\right)^{2} \frac{i_{d c}}{6 \omega} e^{j\left(2 \varphi_{m}^{\Delta}\right)}}{-\frac{2 C}{N} 2(R+j 2 \omega L)+j \frac{3+2\left(M^{\Delta}\right)^{2}}{6 \omega}} e^{j 2 \omega t} \tag{3.59}
\end{align*}
$$

Finally,

$$
\begin{equation*}
i_{2}^{\Sigma}(t)=\Re e\left\{I_{2}^{\Sigma} e^{j\left(2 \omega t+\varphi_{2}^{\Sigma}\right)}\right\}=\Re e\left\{j \frac{\frac{3 M^{\Delta} I^{\Delta}}{8 \omega} e^{j\left(\varphi_{m}^{\Delta}+\varphi_{i}^{\Delta}\right)}-\left(M^{\Delta}\right)^{2} \frac{i_{d c}}{6 \omega} e^{j\left(2 \varphi_{m}^{\Delta}\right)}}{-\frac{2 C}{N} 2(R+j 2 \omega L)+j \frac{3+2\left(M^{\Delta}\right)^{2}}{6 \omega}} e^{j 2 \omega t}\right\} \tag{3.60}
\end{equation*}
$$

The current $i_{2}^{\Sigma}$ is the circulating current. The expression in Eq. (3.60) is valid if the harmonics higher than $n=2$ are neglected. Ilves et al. used the same method but the calculation was not detailed [79]. The authors consider that $\varphi_{m}^{\Delta}=0$ which is not necessarily true. Using this hypothesis, with our notations, we obtain the expression identical to the one in [79, Eq. (143)],

$$
\begin{equation*}
i_{2}^{\Sigma}(t)=\Re e\left\{j \frac{\frac{3 M^{\Delta} I^{\Delta}}{8 \omega} e^{j\left(\varphi_{i}^{\Delta}\right)}-\left(M^{\Delta}\right)^{2} \frac{i_{d c}}{6 \omega}}{-\frac{2 C}{N} 2(R+j 2 \omega L)+j \frac{3+2\left(M^{\Delta}\right)^{2}}{6 \omega}} e^{j 2 \omega t}\right\} \tag{3.61}
\end{equation*}
$$

This makes the Eq. (3.60) more generic as it accounts for $\varphi_{m}^{\Delta}$.
Finally, from Eqs. (3.22) and (3.23), the arm currents can be expressed as,

$$
\begin{align*}
& i_{u}(t)=\frac{i_{d c}}{3}+\frac{i^{\Delta}(t)}{2}+i_{2}^{\Sigma}(t)  \tag{3.62}\\
& i_{l}(t)=\frac{i_{d c}}{3}-\frac{i^{\Delta}(t)}{2}+i_{2}^{\Sigma}(t) \tag{3.63}
\end{align*}
$$

### 3.4.6 Resonance of the circulating current

From Eq. (3.60),

$$
\begin{equation*}
I_{2}^{\Sigma}=\left|j \frac{\frac{3 M^{\Delta} I^{\Delta}}{8 \omega} e^{j\left(\varphi_{m}^{\Delta}+\varphi_{i}^{\Delta}\right)}-\left(M^{\Delta}\right)^{2} \frac{i_{d c}}{6 \omega} e^{j\left(2 \varphi_{m}^{\Delta}\right)}}{-\frac{2 C}{N} 2(R+j 2 \omega L)+j \frac{3+2\left(M^{\Delta}\right)^{2}}{6 \omega}}\right| \tag{3.64}
\end{equation*}
$$

The resonance of the circulating current occurs at $\omega=\omega_{\text {res }}$ when the amplitude of the circulating current $I_{2}^{\Sigma}$ is maximal. This happens when the denominator of Eq. (3.64) is minimal,

$$
\begin{gather*}
\min \left|-\frac{2 C}{N} 2(R+j 2 w L)+j \frac{3+2\left(M^{\Delta}\right)^{2}}{6 w}\right| \Longleftrightarrow  \tag{3.65}\\
\Longleftrightarrow\left\{\begin{array}{c}
\Longleftrightarrow \min \left(\Re e\left\{-\frac{2 C}{N} 2\left(R+j 2 \omega_{\text {res }} L\right)+j \frac{3+2\left(M^{\Delta}\right)^{2}}{6 \omega_{\text {res }}}\right\}\right)=\min \left(\frac{4 C R}{N}\right) \\
\min \left(\Im m\left\{-\frac{2 C}{N} 2\left(R+j 2 \omega_{\text {res }} L\right)+j \frac{3+2\left(M^{\Delta}\right)^{2}}{6 \omega_{\text {res }}}\right\}\right)=\min \left(\frac{-8 C \omega_{\text {res }} L}{N}+\frac{3+2\left(M^{\Delta}\right)^{2}}{6 \omega_{\text {res }}}\right)
\end{array}\right. \tag{3.66}
\end{gather*}
$$

The first equation states that the resistance has to be minimal. Equating the second equation to zero,

$$
\begin{gather*}
\frac{-8 C \omega_{\text {res }} L}{N}+\frac{3+2\left(M^{\Delta}\right)^{2}}{6 \omega_{\text {res }}}=0  \tag{3.67}\\
\omega_{\text {res }}^{2}=\frac{N}{L C} \frac{3+2\left(M^{\Delta}\right)^{2}}{48} \tag{3.68}
\end{gather*}
$$

Supposing that MMC operates with $M^{\Delta} \approx 1[6,47]$, we obtain,

$$
\begin{equation*}
\omega_{r e s}^{2}=\frac{N}{L C} \frac{5}{48} \tag{3.69}
\end{equation*}
$$

With our notations, this is identical to the expressions proposed by Ilves et al. [79, Eq. (120)] and Yue et al. [47, Eq. (26)].

Conclusion We developed the method proposed by Ilves et al. [79]. It enabled us to obtain Eq. (3.47), Eq. (3.60) and Eq. (3.68) that can be commonly found in the literature (but often obtained with hypothesis $\varphi_{m}^{\Delta}=0^{1}$ ). The equations obtained here are more generic as they take the effect of $\varphi_{m}^{\Delta}$ into account. Further in the thesis, it will be shown that $\varphi_{m}^{\Delta}$ has a direct influence on the voltage drop across the arm coil.

### 3.5 Instantaneous arm energy and module capacitor voltage ripple

### 3.5.1 Introduction

Here we develop the integral-based model proposed in [20]. We study three possible cases:

- Case I: $L=0, R=0$, and $i_{2}^{\Sigma}=0$ (analogue to [20], the case most often found in the literature)
- Case II: $L=0, R=0$, and $i_{2}^{\Sigma} \neq 0$ (this case can potentially show the effect of circulating current without passive arm elements)
- Case III: $L \neq 0, R \neq 0$, and $i_{2}^{\Sigma}=0$ (realistic test-case scenario)

Furthermore, we study the arm resistance and inductance influence on the voltage ripple in the Case III.

[^3]
### 3.5.2 Hypotheses

The integral-based model for determining the instantaneous arm energy and the module capacitor voltage ripple is based on the Eqs. (3.24) and (3.27).

In this section we add the following hypothesis,

$$
\begin{equation*}
v_{u, C} \stackrel{h y p}{=} v_{l, C} \stackrel{h y p}{=} v_{d c} \tag{3.70}
\end{equation*}
$$

which as a consequence has,

$$
\begin{gather*}
v_{c}^{\Delta}=0  \tag{3.71}\\
v_{c}^{\Sigma}=v_{d c} \tag{3.72}
\end{gather*}
$$

Input Eq. (3.72) into Eq. (3.24),

$$
\begin{equation*}
v_{m}^{\Delta}(t)=m^{\Delta}(t) \frac{v_{d c}}{2} \tag{3.73}
\end{equation*}
$$

Input Eqs. (3.10) and (3.11) into Eq. (3.73),

$$
\begin{gather*}
V_{m}^{\Delta}=M^{\Delta} \frac{v_{d c}}{2}  \tag{3.74}\\
\varphi_{t}^{\Delta}=\varphi_{m}^{\Delta} \tag{3.75}
\end{gather*}
$$

### 3.5.3 Upper arm voltage

The upper arm voltage can now be expressed from Eqs. (2.27) and (3.70),

$$
\begin{equation*}
v_{u}(t)=m_{u}(t) v_{u, C}=m_{u}(t) v_{d c} \tag{3.76}
\end{equation*}
$$

Input Eq. (2.44)

$$
\begin{equation*}
v_{u}(t)=\frac{1-m^{\Delta}(t)}{2} v_{d c} \tag{3.77}
\end{equation*}
$$

Input Eq. (3.73),

$$
\begin{equation*}
v_{u}(t)=\frac{v_{d c}}{2}-v_{m}^{\Delta}(t) \tag{3.78}
\end{equation*}
$$

### 3.5.4 Upper arm current

Let us remind the upper arm current from Eq. (3.62),

$$
\begin{equation*}
i_{u}(t)=\frac{i_{d c}}{3}+\frac{i^{\Delta}(t)}{2}+i_{2}^{\Sigma}(t) \tag{3.79}
\end{equation*}
$$

### 3.5.5 Fresnel's diagram

The Fresnel's diagram is a phasor diagram used to show the phase relationships between the sine waves of the same frequency. The phasor length corresponds to the amplitude of the wave it represents, and the angle between the phasors is the same angle of the phase difference between the sine waves. To easily represent the amplitudes and phases, we transform the sine waves to the corresponding complex variables as described in Eq. (3.12). In complex domain, Eq. (3.27) is,

$$
\begin{equation*}
\overline{V_{m}^{\Delta}}=\frac{R}{2} \bar{I}^{\Delta}+j \omega \frac{L}{2} \bar{I}^{\Delta}+\overline{V_{s}} \tag{3.80}
\end{equation*}
$$

The corresponding Fresnel's diagram is shown in the Fig. 3.2.
From the Fig. 3.2, by projection on the real and imaginary axes, we obtain,

$$
\begin{equation*}
V_{m}^{\Delta} \cos \left(\varphi_{t}^{\Delta}\right)=V_{s}+\frac{R}{2} I^{\Delta} \cos \left(\varphi_{i}^{\Delta}\right)+\omega \frac{L}{2} I^{\Delta} \sin \left(\varphi_{i}^{\Delta}\right) \tag{3.81}
\end{equation*}
$$



Figure 3.2 - Fresnel's diagram of Eq. (3.80)

$$
\begin{equation*}
V_{m}^{\Delta} \sin \left(\varphi_{t}^{\Delta}\right)=-\frac{R}{2} I^{\Delta} \sin \left(\varphi_{i}^{\Delta}\right)+\omega \frac{L}{2} I^{\Delta} \cos \left(\varphi_{i}^{\Delta}\right) \tag{3.82}
\end{equation*}
$$

Squaring each equation and adding them,

$$
\begin{equation*}
V_{m}^{\Delta}=\sqrt{V_{s}^{2}+R V_{s} I^{\Delta} \cos \left(\varphi_{i}^{\Delta}\right)+\omega L V_{s} I^{\Delta} \sin \left(\varphi_{i}^{\Delta}\right)+\left(\frac{R}{2} I^{\Delta}\right)^{2}+\left(\omega \frac{L}{2} I^{\Delta}\right)^{2}} \tag{3.83}
\end{equation*}
$$

Dividing Eqs. (3.81) by (3.82),

$$
\begin{equation*}
\varphi_{t}^{\Delta}=\arctan \left(\frac{-\frac{R}{2} I^{\Delta} \sin \left(\varphi_{i}^{\Delta}\right)+\omega \frac{L}{2} I^{\Delta} \cos \left(\varphi_{i}^{\Delta}\right)}{V_{s}+\frac{R}{2} I^{\Delta} \cos \left(\varphi_{i}^{\Delta}\right)+\omega \frac{L}{2} I^{\Delta} \sin \left(\varphi_{i}^{\Delta}\right)}\right) \tag{3.84}
\end{equation*}
$$

### 3.5.6 Instantaneous arm power and energy

The instantaneous power of the upper arm is,

$$
\begin{equation*}
p_{u}(t)=v_{u}(t) i_{u}(t) \tag{3.85}
\end{equation*}
$$

The instantaneous energy of the upper arm is obtained through the integration of the instantaneous power,

$$
\begin{equation*}
E_{u}(t)=\int_{0}^{t} p_{u}(t) d t \tag{3.86}
\end{equation*}
$$

The instantaneous arm energy consists of a DC component, $E_{u 0}$, and an AC component $\tilde{E}_{u}$,

$$
\begin{equation*}
E_{u}(t)=E_{u 0}+\tilde{E}_{u}(t) \tag{3.87}
\end{equation*}
$$

### 3.5.7 Module capacitor voltage ripple

The energy stored in a module capacitor $E_{c}$ is defined as,

$$
\begin{equation*}
E_{c}(t) \triangleq \frac{C}{2} v_{c a p}^{2}(t) \tag{3.88}
\end{equation*}
$$

In this case, the instantaneous arm energy $E_{u}$ is,

$$
\begin{equation*}
E_{u}(t)=N \frac{C}{2} v_{c a p}^{2}(t) \tag{3.89}
\end{equation*}
$$

and the arm energy ripple then is,

$$
\begin{equation*}
\Delta E_{u}=\max _{t}\left(E_{u}(t)\right)-\min _{t}\left(E_{u}(t)\right) \tag{3.90}
\end{equation*}
$$

Replacing Eq. (3.3) in Eq. (3.89), the constant part of the instantaneous arm energy is,

$$
\begin{equation*}
E_{u 0}=N \frac{C}{2} V_{c a p}^{2} \tag{3.91}
\end{equation*}
$$

and the variable part is,

$$
\begin{equation*}
\tilde{E}_{u}(t)=N C V_{c a p} \tilde{v}_{c a p}(t)+N \frac{C}{2} \tilde{v}_{c a p}^{2}(t) \tag{3.92}
\end{equation*}
$$

On the other hand, input Eq. (3.3) in Eq. (3.89) and express $\tilde{v}_{\text {cap }}$,

$$
\begin{equation*}
\tilde{v}_{c a p}(t)=\sqrt{\frac{2 E_{u}(t)}{C N}}-V_{c a p} \tag{3.93}
\end{equation*}
$$

therefore from Eq. (3.91),

$$
\begin{equation*}
\tilde{v}_{c a p}(t)=\sqrt{\frac{2 E_{u}(t)}{C N}}-\sqrt{\frac{2 E_{u 0}}{C N}} \tag{3.94}
\end{equation*}
$$

The capacitor voltage ripple $\Delta v_{c a p}$ is,

$$
\begin{equation*}
\Delta v_{c a p}=\max _{t}\left(\tilde{v}_{c a p}(t)\right)-\min _{t}\left(\tilde{v}_{c a p}(t)\right) \tag{3.95}
\end{equation*}
$$

## Procedure

To calculate the voltage ripple, the procedure is the following,

1. Express the upper arm current and voltage $i_{u}$ and $v_{u}$ using Eqs. (3.79) and (3.78).
2. Calculate the upper arm power $p_{u}$ using Eq. (3.85).
3. Calculate the upper arm instantaneous energy $E_{u}$ using Eq. (3.86).
4. Calculate the voltage fluctuation $\tilde{v}_{\text {cap }}$ using Eqs. (3.93) and (3.94).
5. Calculate the voltage ripple $\Delta v_{c a p}$ from Eq. (3.95).
6. Calculate the arm energy ripple $\Delta E_{u}$ from Eq. (3.90).

In this section, the numerical application is carried out for the MMC having the parameters of Table 5.3. This corresponds to our prototype (see section 1.3).

## Study cases

We look for the upper arm current $i_{u}$ and upper arm voltage $v_{u}$ (step 1 from sec. 3.5.7). Here we study three cases.
3.5.7.1 Case I: $L=0, R=0, i_{2}^{\Sigma}=0$

From Eq. (2.4), $v_{s}=v_{m}^{\Delta}$ since the passive components are neglected. Using Eq. (3.73),

$$
\begin{equation*}
v_{s}(t)=m^{\Delta}(t) \frac{v_{d c}}{2} \tag{3.96}
\end{equation*}
$$

Then, from Eqs. (3.4) and (3.11) we conclude,

$$
\begin{gather*}
V_{s}=M^{\Delta} \frac{v_{d c}}{2}  \tag{3.97}\\
M^{\Delta}=\frac{2 V_{s}}{v_{d c}} \tag{3.98}
\end{gather*}
$$

and,

$$
\begin{equation*}
\varphi_{m}^{\Delta}=0 \tag{3.99}
\end{equation*}
$$

Replacing Eq. (3.98) into Eq. (3.47), the DC current becomes,

$$
\begin{equation*}
i_{d c}=3 \frac{I^{\Delta}}{2} \frac{V_{s}}{v_{d c}} \cos \left(\varphi_{i}^{\Delta}\right) \tag{3.100}
\end{equation*}
$$

Replacing Eq. (3.100) and (3.5) into Eq. (3.62),

$$
\begin{equation*}
i_{u}(t)=\frac{I^{\Delta}}{2} \frac{V_{s}}{v_{d c}} \cos \left(\varphi_{i}^{\Delta}\right)+\frac{I^{\Delta}}{2} \cos \left(\omega t+\varphi_{i}^{\Delta}\right) \tag{3.101}
\end{equation*}
$$

From Eq. (3.78), the upper arm voltage is,

$$
\begin{gather*}
v_{u}(t)=\frac{v_{d c}}{2}-v_{s}(t)  \tag{3.102}\\
v_{u}(t)=\frac{v_{d c}}{2}-V_{s} \cos (\omega t) \tag{3.103}
\end{gather*}
$$

Replace Eqs. (3.101) and (3.103) into Eq. (3.85),

$$
\begin{equation*}
p_{u}^{I}(t)=\left(\frac{v_{d c}}{2}-V_{s} \cos (\omega t)\right)\left(\frac{I^{\Delta}}{2} \frac{V_{s}}{v_{d c}} \cos \left(\varphi_{i}^{\Delta}\right)+\frac{I^{\Delta}}{2} \cos \left(\omega t+\varphi_{i}^{\Delta}\right)\right) \tag{3.104}
\end{equation*}
$$

Using Eq. (3.86),

$$
\begin{gather*}
E_{u}^{I}(t)=E_{u 0}^{I}+E_{u 1}^{I}+E_{u 2}^{I}  \tag{3.105}\\
E_{u 0}^{I}=\frac{-S v_{d c} \sin \left(\varphi_{i}^{\Delta}\right)}{6 V_{s} \omega}+\frac{S \sin \left(\varphi_{i}^{\Delta}\right)}{12 \omega}+E_{u 0}  \tag{3.106}\\
E_{u 1}^{I}(t)=\frac{S v_{d c} \sin \left(\omega t+\varphi_{i}^{\Delta}\right)}{6 V_{s} \omega}-\frac{S V_{s}\left(\sin \left(\omega t+\varphi_{i}^{\Delta}\right)+\sin \left(\omega t-\varphi_{i}^{\Delta}\right)\right)}{6 v_{d c} \omega}  \tag{3.107}\\
E_{u 2}^{I}(t)=-\frac{S \sin \left(2 \omega t+\varphi_{i}^{\Delta}\right)}{12 \omega} \tag{3.108}
\end{gather*}
$$

where $E_{u 0}^{I}, E_{u 1}^{I}$ and $E_{u 2}^{I}$ are the constant, fundamental and second harmonic component respectively. The energy fluctuation for the case I is the function of $S$ and $\varphi_{i}^{\Delta}$. It shown in the Fig. 3.3a.
3.5.7.2 Case II: $L=0, R=0, i_{2}^{\Sigma} \neq 0$

From Eq. (2.4), $v_{s}=v_{m}^{\Delta}$. Using Eq. (3.73),

$$
\begin{equation*}
v_{s}(t)=m^{\Delta}(t) \frac{v_{d c}}{2} \tag{3.109}
\end{equation*}
$$

Then, from Eqs. (3.4) and (3.11) we conclude,

$$
\begin{gather*}
V_{s}=M^{\Delta} \frac{v_{d c}}{2}  \tag{3.110}\\
M^{\Delta}=\frac{2 V_{s}}{v_{d c}} \tag{3.111}
\end{gather*}
$$

and,

$$
\begin{equation*}
\varphi_{m}^{\Delta}=0 \tag{3.112}
\end{equation*}
$$

Replacing Eq. (3.111) into Eq. (3.47), the dc current becomes,

$$
\begin{equation*}
i_{d c}=3 \frac{I^{\Delta}}{2} \frac{V_{s}}{v_{d c}} \cos \left(\varphi_{i}^{\Delta}\right) \tag{3.113}
\end{equation*}
$$

Replacing Eq. (3.111) and (3.112) into Eq. (3.60),

$$
\begin{equation*}
i_{2}^{\Sigma}(t)=\Re e\left\{j \frac{\frac{3 \frac{2 V_{s}}{v_{d c}} I^{\Delta}}{8 \omega} e^{j\left(\varphi_{i}^{\Delta}\right)}-\left(\frac{2 V_{s}}{v_{d c}}\right)^{2} \frac{i_{d c}}{6 \omega}}{j \frac{3+2\left(\frac{2 V_{d c}}{v_{d c}}\right)^{2}}{6 \omega}} e^{j 2 \omega t}\right\} \tag{3.114}
\end{equation*}
$$

Replacing Eq. (3.113) and (3.5) into Eq. (3.62),

$$
\begin{equation*}
i_{u}(t)=\frac{I^{\Delta}}{2} \frac{V_{s}}{v_{d c}} \cos \left(\varphi_{i}^{\Delta}\right)+\frac{I^{\Delta}}{2} \cos \left(\omega t+\varphi_{i}^{\Delta}\right)+\Re e\left\{j \frac{\frac{3 \frac{2 V_{s}}{v_{d c}}}{8 \omega} e^{j\left(\varphi_{i}\right)}-\left(\frac{2 V_{s}}{v_{d c}}\right)^{2} \frac{i_{d c}}{6 \omega}}{j \frac{3+2\left(\frac{2 V_{v s}}{v_{d c}}\right)^{2}}{6 \omega}} e^{j 2 \omega t}\right\} \tag{3.115}
\end{equation*}
$$

From Eq. (3.78), the upper arm voltage is,

$$
\begin{gather*}
v_{u}(t)=\frac{v_{d c}}{2}-v_{s}(t)  \tag{3.116}\\
v_{u}(t)=\frac{v_{d c}}{2}-V_{s} \cos (\omega t) \tag{3.117}
\end{gather*}
$$

Replace Eqs. (3.115) and (3.117) into Eq. (3.85),

$$
\begin{align*}
p_{u}(t)= & \left(\frac{v_{d c}}{2}-V_{s} \cos (\omega t)\right)\left(\frac{I^{\Delta}}{2} \frac{V_{s}}{v_{d c}} \cos \left(\varphi_{i}^{\Delta}\right)+\frac{I^{\Delta}}{2} \cos \left(\omega t+\varphi_{i}^{\Delta}\right)\right. \\
& \left.+\Re e\left\{j \frac{\frac{3 \frac{2 V_{s}}{v_{d c}} I^{\Delta}}{8 \omega} e^{j\left(\varphi_{i}^{\Delta}\right)}-\left(\frac{2 V_{s}}{v_{d c}}\right)^{2} \frac{i_{d c}}{6 \omega}}{j \frac{3+2\left(\frac{2 v_{s}}{v_{d c}}\right)^{2}}{6 \omega}} e^{j 2 \omega t}\right\}\right) \tag{3.118}
\end{align*}
$$

Using Eq. (3.86),

$$
\begin{align*}
& E_{u}^{I I}(t)=E_{u 0}^{I I}+E_{u 1}^{I I}+E_{u 2}^{I I}+E_{u 3}^{I I} \\
& E_{u 0}^{I I}=\frac{-I^{\Delta}}{8 v_{d c}{ }^{2} \omega\left(4 V_{s}+3 v_{d c}\right)}\left(6 v_{d c}{ }^{4} \sin \left(\varphi_{i}^{\Delta}\right)+5 V_{s} v_{d c}{ }^{3} \sin \left(\varphi_{i}^{\Delta}\right)-28 V_{s}{ }^{2} v_{d c}{ }^{2} \sin \left(\varphi_{i}^{\Delta}\right)\right. \\
& \left.+9 V_{s} v_{d c}{ }^{3} \sin \left(\varphi_{i}^{\Delta}\right)\right)+E_{u 0} \\
& E_{u 1}^{I I}(t)=\frac{-I^{\Delta}}{8 v_{d c}{ }^{2} \omega\left(4 V_{s}+3 v_{d c}\right)}\left(-6 v_{d c}{ }^{4} \sin \left(\varphi_{i}^{\Delta}+\omega t\right)-8 V_{s} v_{d c}{ }^{3} \sin \left(\varphi_{i}^{\Delta}+\omega t\right)-48 V_{s}{ }^{4} \sin (\omega t) \cos \left(\varphi_{i}^{\Delta}\right)\right. \\
& +16 V_{s}{ }^{3} v_{d c} \sin (\omega t) \cos \left(\varphi_{i}^{\Delta}\right)+48 V_{s}{ }^{2} v_{d c}{ }^{2} \sin (\omega t) \cos \left(\varphi_{i}^{\Delta}\right)+8 V_{s}^{4} 3 \sin (\omega t) \cos \left(\varphi_{i}^{\Delta}\right) \\
& \left.+6 V_{s}{ }^{2} v_{d c}{ }^{2} \sin \left(\varphi_{i}^{\Delta}\right) 3 \cos (\omega t)-6 V_{s}{ }^{2} v_{d c}{ }^{2} 3 \sin (\omega t) \cos \left(\varphi_{i}^{\Delta}\right)\right) \\
& E_{u 2}^{I I}(t)=\frac{-I^{\Delta}}{8 v_{d c}{ }^{2} \omega\left(4 V_{s}+3 v_{d c}\right)}\left(+3 V_{s} v_{d c}{ }^{3} \sin \left(\varphi_{i}^{\Delta}+2 \omega t\right)+4 V_{s}{ }^{2} v_{d c}{ }^{2} \sin \left(\varphi_{i}^{\Delta}+2 \omega t\right)\right. \\
& \left.-9 V_{s} v_{d c}{ }^{3} \sin (2 \omega t) \cos \left(\varphi_{i}^{\Delta}\right)+12 V_{s}^{3} v_{d c} \sin (2 \omega t) \cos \left(\varphi_{i}^{\Delta}\right)-9 V_{s} v_{d c}{ }^{3} \sin \left(\varphi_{i}^{\Delta}\right) \cos (2 \omega t)\right)  \tag{3.122}\\
& E_{u 3}^{I I}(t)=\frac{-I^{\Delta}}{8 v_{d c}{ }^{2} \omega\left(4 V_{s}+3 v_{d c}\right)}\left(-8 V_{s}^{4} \sin (3 \omega t) \cos \left(\varphi_{i}^{\Delta}\right)+6 V_{s}{ }^{2} v_{d c}{ }^{2} \sin \left(\varphi_{i}^{\Delta}\right) \cos (3 \omega t)\right.  \tag{3.123}\\
& \left.+6 V_{s}^{2} v_{d c}{ }^{2} \sin (3 \omega t) \cos \left(\varphi_{i}^{\Delta}\right)\right)
\end{align*}
$$

where $E_{u 0}^{I I}, E_{u 1}^{I I}, E_{u 2}^{I I}$ and $E_{u 3}^{I I}$ are the constant, fundamental, second and third harmonic component respectively. The energy fluctuation for the case II is shown in the Fig. 3.3b.
3.5.7.3 Case III: $L \neq 0, R \neq 0, i_{2}^{\Sigma}=0$

In this case, we can calculate the amplitude and the phase of the voltage $v_{m}$ as shown in Eqs. (3.83) and (3.84). Using the hypothesis in Eq. (3.73) and the definition of the modulation index in Eq. (3.11),

$$
\begin{gather*}
V_{m}^{\Delta}=M^{\Delta} \frac{v_{d c}}{2}  \tag{3.124}\\
M^{\Delta}=\frac{2 V_{m}^{\Delta}}{v_{d c}} \tag{3.125}
\end{gather*}
$$

and,

$$
\begin{equation*}
\varphi_{t}^{\Delta}=\varphi_{m}^{\Delta} \tag{3.126}
\end{equation*}
$$

Replacing Eqs. (3.125) into Eq. (3.126) into Eq. (3.47),

$$
\begin{equation*}
i_{d c}=3 \frac{I^{\Delta}}{2} \frac{V_{m}}{v_{d c}} \cos \left(\varphi_{t}^{\Delta}-\varphi_{i}^{\Delta}\right) \tag{3.127}
\end{equation*}
$$

Replacing Eq. (3.127) and (3.5) into Eq. (3.62),

$$
\begin{equation*}
i_{u}(t)=\frac{I^{\Delta}}{2} \frac{V_{m}}{v_{d c}} \cos \left(\varphi_{t}^{\Delta}-\varphi_{i}^{\Delta}\right)+\frac{I^{\Delta}}{2} \cos \left(\omega t+\varphi_{i}^{\Delta}\right) \tag{3.128}
\end{equation*}
$$

From Eq. (3.78), the upper arm voltage is,

$$
\begin{gather*}
v_{u}(t)=\frac{v_{d c}}{2}-v_{m}^{\Delta}(t)  \tag{3.129}\\
v_{u}(t)=\frac{v_{d c}}{2}-V_{m}^{\Delta}\left(\omega t+\varphi_{t}^{\Delta}\right) \tag{3.130}
\end{gather*}
$$

In this case in the first approximation (developed in detail in section 4.5.1) the voltage $V_{c a p}$ is [44],

$$
\begin{equation*}
V_{c a p}=\frac{v_{d c}}{N}-2 R \frac{i_{d c}}{3 N} \tag{3.131}
\end{equation*}
$$

Replace Eqs. (3.128) and (3.130) into Eq. (3.85),

$$
\begin{equation*}
p_{u}^{I I I}(t)=\left(\frac{v_{d c}}{2}-V_{m}^{\Delta} \cos \left(\omega t+\varphi_{t}^{\Delta}\right)\right)\left(\frac{I^{\Delta}}{2} \frac{V_{m}^{\Delta}}{v_{d c}} \cos \left(\varphi_{t}^{\Delta}-\varphi_{i}^{\Delta}\right)+\frac{I^{\Delta}}{2} \cos \left(\omega t+\varphi_{i}^{\Delta}\right)\right) \tag{3.132}
\end{equation*}
$$

Using Eq. (3.86),

$$
\begin{gather*}
E_{u}^{I I I}(t)=E_{u 0}^{I I I}+E_{u 1}^{I I I}+E_{u 2}^{I I I}  \tag{3.133}\\
E_{u 0}^{I I I}=\frac{I^{\Delta} V_{m}^{\Delta} \sin \left(\varphi_{i}^{\Delta}+\varphi_{t}^{\Delta}\right)}{8 \omega}+\frac{\left(V_{m}^{\Delta}\right)^{2} \cos \left(\varphi_{i}^{\Delta}-\varphi_{t}^{\Delta}\right) \sin \left(\varphi_{t}^{\Delta}\right)}{2 v_{d c} \omega}-\frac{I^{\Delta} v_{d c} \sin \left(\varphi_{i}^{\Delta}\right)}{4 \omega}+E_{u 0}  \tag{3.134}\\
E_{u 1}^{I I I}(t)=\frac{-I^{\Delta}\left(V_{m}^{\Delta}\right)^{2} \cos \left(\varphi_{i}^{\Delta}-\varphi_{t}^{\Delta}\right) \sin \left(\omega t+\varphi_{t}^{\Delta}\right)}{2 v_{d c} \omega}+\frac{I^{\Delta} v_{d c} \sin \left(\omega t+\varphi_{i}^{\Delta}\right)}{4 \omega}  \tag{3.135}\\
E_{u 2}^{I I I}(t)=\frac{-I^{\Delta} V_{m}^{\Delta} \sin \left(2 \omega t+\varphi_{i}^{\Delta}+\varphi_{t}^{\Delta}\right)}{8 \omega} \tag{3.136}
\end{gather*}
$$

where $E_{u 0}^{I I I}, E_{u 1}^{I I I}$ and $E_{u 2}^{I I I}$ are the constant, fundamental and second harmonic component respectively. The energy fluctuation for the case III is shown in the Fig. 3.3c. We underline here that if $L$ and $R$ are neglected, we obtain the instantaneous arm energy of the first case.

### 3.5.7.4 Results

Follow the steps 4,5 and 6 from sec. 3.5.7 to calculate the instantaneous arm energy and module capacitor voltage ripple. The three studied cases are traced in the Fig. 3.3d.

We can see that if the passive elements in the arm are neglected, the module voltage ripple is symmetrical about the y -axis and about the x -axis. The maximal voltage ripple value occurs for the pure reactive power transfer.

Inspecting the case III (yellow line), we can see that the passive elements influence the voltage ripple shape. Firstly, the voltage ripple loses the symmetry indicating that the ripple will have different values for the same amount of power transfered in the two directions. This asymmetry is also reflected on the instantaneous arm energies, seen in Fig. 3.3c. The maximal value of the module voltage ripple does not occur for pure reactive power anymore, but is dislocated.

Therefore, here we can conclude that the passive elements do influence the shape of the voltage ripple. As we have seen, if they are neglected, it can lead to improper estimation of the voltage ripple. Furthermore, the voltage ripple is stated as one of the most important criteria for the capacitor selection, therefore its proper estimation is of best interest for the manufacturers.


Figure 3.3 - Instantaneous energy and voltage ripple for case I, case II and case III

### 3.5.7.5 Module capacitor voltage ripple as a function of arm inductance

To further investigate the effect of the arm coil resistance and inductance on the energy ripple and voltage ripple, we consider case III $\left(R, L, i_{2}^{\Sigma}=0\right)$.


Figure 3.4 - Arm inductance influence on capacitor voltage ripple


Figure 3.5 - Arm resistance influence on capacitor voltage ripple
In Fig. 3.4 we plotted the capacitor voltage ripple for four different inductance values and $R=1 \Omega$. The biggest capacitor voltage ripple occurs for $L=1 \mathrm{mH}$. Furthermore, the maximum does not occur for pure reactive power transfer ${ }^{2}$. The influence of the inductance would increase if we would be able to study the situation where $i_{2}^{\Sigma} \neq 0$ but this is not possible to do since the equivalent scheme in Fig. 2.3 would no longer be valid. The influence would be more emphasized because of the resonance with the capacitance at the frequency of the second harmonic, Eq. (3.68).

In Fig. 3.5 we plotted the capacitor voltage ripple for four different resistance values and $L=10 \mathrm{mH}$. The voltage ripple increases with the increase of the arm resistance. This implies that, if lower voltage ripple is desired, the resistance should be lower. From the efficiency point of view, the resistance should be as low as possible as well.

### 3.6 Discussions

In this chapter, we used the integral-based model to derive classical formulas for DC grid current, circulating current and module capacitor voltage ripple. A special attention was paid to the formalism and the identification of the hypotheses that are required. Then we studied the module voltage ripple calculation using the integral-based model in function of the arm equivalent resistance and inductance.

[^4]We have developed the generic expression for the DC grid current $i_{d c}$, Eq. (3.47). It shows that $i_{d c}$ is a function of $\varphi_{m}^{\Delta}$. This means that even for pure reactive power $\left(\varphi_{i}^{\Delta}= \pm 90^{\circ}\right)$, a DC grid current can exist due to modulation index phase shift. We have also developed the generic expression for the circulating current $i_{2}^{\Sigma}$, Eq. (3.60). This current is a function of $\varphi_{m}^{\Delta}$ as well.

We showed that the inductance affects the voltage ripple. We saw that the choice of $L$ for a given $C$ changes the ripple of the capacitor voltage. Therefore, a proper dimensioning has to be employed in order to respect the limits of the module capacitor voltage. As the maximal voltage ripple occurs for mainly the reactive power transfer (or not far from it in terms of the power factor), this is especially important for the STATCOM MMCs. In this case, the voltage ripple would be paramount for the capacitance and inductance selection. However, nowadays, many MMCs are designed to transfer active power (and relatively low reactive power) [6]. In this situation, as the voltage ripple for the active power is lower than for the reactive power, the most important variable for the capacitance and inductance selection would be the arm current. If one was to design the capacitor solely according to the maximal voltage ripple (occuring for the pure reactive power), the capacitor would be oversized.

From the energy point of view, several authors study the MMC as a device that stocks energy [49, $50,51]$. This study is beneficial in this case as well, as it elaborates the instantaneous energy behavior with arm inductance and resistance taken into account. Therefore, it contributes to the dimensioning of the MMC from the energy storage point of view.

Nevertheless, the integral-based model for module capacitor voltage ripple is predicated on time domain equations with the hypothesis in Eq. (3.70) which for a consequence has Eq. (3.73). In Chapter 2 we have adopted an MMC model from [75] represented through a system of 6 equations that eliminates this hypothesis, section 2.1.2.2.

In the following, to solve above problems, we propose an alternative method to the integral-based model in order to derive the module capacitor voltage ripple.

## Chapter 4

## Steady-state time invariant model

Objectives: Derivation of the steady-state time invariant model. Study of the module capacitor average voltage and voltage ripple as a function of the passive arm elements.

Motivations: Need for a model with a limited number of assumptions for sizing purpose. Lack of discussion on the module capacitor average voltage. Lack of discussion on the influence of the passive arm elements on the module capacitor voltage ripple.

Contributions: Clarification of the derivation of the steady-state time invariant model from $[75,76]$. Application of this model to estimate the module capacitor average voltage and voltage ripple. Comparison of the results to the state of the art and to simulations. Study of the influence of the arm equivalent resistance and arm inductance on the module capacitor voltage (average and ripple).

### 4.1 Introduction

In this chapter, we develop the steady-state time invariant (SSTI) $\Delta \Sigma$ MMC model in $d q 0$ frame, with as few simplifications as possible. This model allows us access to internal variables of the converter.

At this point we are able to study the module capacitor voltage from two aspects,

- module capacitor average voltage: In the literature, this voltage is often considered independent of the operating point $[20,40,41,42,43]$. We elaborate the influence of the operating point on $V_{c a p}$
- module capacitor voltage ripple: We elaborate the impact of the arm resistance and inductance on $\Delta v_{c a p}$.


### 4.2 Hypotheses and definitions

Neglect harmonics: Remind the matrices in Eqs. (2.69), (2.70), (2.76), (2.77), (2.89), (2.90), (2.95), (2.96) where the parameters $\lambda$ and $\chi$ are defined in Eqs. (2.71) and (2.78), respectively. Let us neglect the $3^{\text {rd }}$ and $6^{\text {th }}$ harmonics in these matrices, or in other words, let us suppose,

$$
\begin{equation*}
\lambda_{1} \stackrel{\text { hyp }}{=} 0 \quad ; \quad \lambda_{2} \stackrel{\text { hyp }}{=} 0 \quad ; \quad \chi_{1} \stackrel{\text { hyp }}{=} 0 \quad ; \quad \chi_{2} \stackrel{\text { hyp }}{=} 0 \tag{4.1}
\end{equation*}
$$

The matrices then become,

$$
\begin{gather*}
{\left[\boldsymbol{M}_{A 1 a}^{\Delta *}\right]=\frac{1}{4}\left[\begin{array}{ccc}
-m_{q}^{\Delta} & -m_{d}^{\Delta} & 2 m_{d}^{\Delta} \\
-m_{d}^{\Delta} & m_{q}^{\Delta} & 2 m_{q}^{\Delta} \\
0 & 0 & 2 m_{0}^{\Delta}
\end{array}\right]}  \tag{4.2}\\
{\left[\boldsymbol{M}_{A 1 b}^{\Sigma *}\right]=-\frac{1}{4}\left[\begin{array}{ccc}
m_{q}^{\Sigma}-2 m_{0}^{\Sigma} & m_{d}^{\Sigma} & 0 \\
m_{d}^{\Sigma} & -2 m_{0}^{\Sigma}-m_{q}^{\Sigma} & 0 \\
0 & 0 & -2 m_{0}^{\Sigma}
\end{array}\right]}  \tag{4.3}\\
{\left[\boldsymbol{M}_{A 2 a}^{\Sigma *}\right]=\frac{1}{4}\left[\begin{array}{ccc}
2 m_{0}^{\Sigma} & 0 & 2 m_{d}^{\Sigma} \\
0 & 2 m_{0}^{\Sigma} & 2 m_{q}^{\Sigma} \\
m_{d}^{\Sigma} & m_{q}^{\Sigma} & 2 m_{0}^{\Sigma}
\end{array}\right]}  \tag{4.4}\\
{\left[\boldsymbol{M}_{A 2 b}^{\Delta *}\right]=}  \tag{4.5}\\
\left.\boldsymbol{M}_{3 a}^{\Sigma *}\right]=-\frac{1}{4}\left[\begin{array}{ccc}
-m_{q}^{\Delta} & -m_{d}^{\Delta} & 0 \\
-m_{d}^{\Delta} & m_{q}^{\Delta} & 0 \\
m_{d}^{\Delta} & m_{q}^{\Delta} & 2 m_{0}^{\Delta}
\end{array}\right]  \tag{4.6}\\
{\left[\begin{array}{lll}
2 m_{0}^{\Sigma}-m_{q}^{\Sigma} & -m_{d}^{\Sigma} \\
-m_{d}^{\Sigma} & 2 m_{0}^{\Sigma}+m_{q}^{\Sigma} & 0 \\
0 & 0 & 2 m_{0}^{\Sigma}
\end{array}\right]}  \tag{4.7}\\
{\left[\boldsymbol{M}_{3 b}^{\Delta *}\right]=\frac{1}{2}\left[\begin{array}{ccc}
-m_{q}^{\Delta} & -m_{d}^{\Delta} & 2 m_{d}^{\Delta} \\
-m_{d}^{\Delta} & m_{q}^{\Delta} & 2 m_{q}^{\Delta} \\
0 & 0 & 2 m_{0}^{\Delta}
\end{array}\right]}  \tag{4.8}\\
{\left[\boldsymbol{M}_{4 a}^{\Delta *}\right]=-\frac{1}{4}\left[\begin{array}{ccc}
-m_{q}^{\Delta} & -m_{d}^{\Delta} & 0 \\
-m_{d}^{\Delta} & m_{q}^{\Delta} & 0 \\
m_{d}^{\Delta} & m_{q}^{\Delta} & 2 m_{0}^{\Delta}
\end{array}\right]}  \tag{4.9}\\
{\left[\boldsymbol{M}_{4 b}^{\Sigma *}\right]=\frac{1}{2}\left[\begin{array}{ccc}
2 m_{0}^{\Sigma} & 0 & 2 m_{d}^{\Sigma} \\
0 & 2 m_{0}^{\Sigma} & 2 m_{q}^{\Sigma} \\
m_{d}^{\Sigma} & m_{q}^{\Sigma} & 2 m_{0}^{\Sigma}
\end{array}\right]}
\end{gather*}
$$

where the superscript * is related to the matrices where parameters $\lambda$ and $\chi$ are zero. Note that with this hypotheses, the matrices become independent of time and therefore ( $t$ ) has been removed.

Form of $\Delta \Sigma$ variables: Let us take the hypotheses on the form of $\Delta \Sigma$ variables in $a b c$ frame. These assumptions have been proposed by Bergna et al. in [75, 76], and are in agreement with some results of the integral based model as will be shown case by case.
$\boldsymbol{m}_{a b c}^{\Delta}$ oscillates at $\omega$ and $\boldsymbol{m}_{a b c}^{\Sigma}$ has a dc component and a possible $-2 \omega$ oscillation.

$$
\begin{gather*}
\boldsymbol{m}_{a b c}^{\Delta}(t) \stackrel{\text { hyp }}{=}\left[\begin{array}{c}
M^{\Delta} \cos \left(\omega t+\varphi_{m}^{\Delta}\right) \\
M^{\Delta} \cos \left(\omega t+\varphi_{m}^{\Delta}-\frac{2 \pi}{3}\right) \\
M^{\Delta} \cos \left(\omega t+\varphi_{m}^{\Delta}-\frac{4 \pi}{3}\right)
\end{array}\right]  \tag{4.10}\\
\boldsymbol{m}_{a b c}^{\Sigma}(t) \stackrel{\text { hyp }}{=}\left[\begin{array}{c}
M_{0}^{\Sigma}+M^{\Sigma} \cos \left(-2 \omega t+\varphi_{m}^{\Sigma}\right) \\
M_{0}^{\Sigma}+M^{\Sigma} \cos \left(-2 \omega t+\varphi_{m}^{\Sigma}-\frac{2 \pi}{3}\right) \\
M_{0}^{\Sigma}+M^{\Sigma} \cos \left(-2 \omega t+\varphi_{m}^{\Sigma}-\frac{4 \pi}{3}\right)
\end{array}\right] \tag{4.11}
\end{gather*}
$$

This is in agreement with integral-based model, Eqs. (2.18)-(2.19) and (3.11).
Under normal operating conditions the grid current $i_{a b c}^{\Delta}$ of the MMC should contain only fundamental harmonic $\omega$. In addition, the circulating current consists of a DC component in addition to second harmonic at $-2 \omega$.

$$
\begin{gather*}
\boldsymbol{i}_{a b c}^{\Delta}(t) \stackrel{\text { hyp }}{=}\left[\begin{array}{c}
I^{\Delta} \cos \left(\omega t+\varphi_{i}^{\Delta}\right) \\
I^{\Delta} \cos \left(\omega t+\varphi_{i}^{\Delta}-\frac{2 \pi}{3}\right) \\
I^{\Delta} \cos \left(\omega t+\varphi_{i}^{\Delta}-\frac{4 \pi}{3}\right)
\end{array}\right]  \tag{4.12}\\
\boldsymbol{i}_{a b c}^{\Sigma}(t) \stackrel{\text { hyp }}{=}\left[\begin{array}{c}
I_{0}^{\Sigma}+I^{\Sigma} \cos \left(-2 \omega t+\varphi_{i}^{\Sigma}\right) \\
I_{0}^{\Sigma}+I^{\Sigma} \cos \left(-2 \omega t+\varphi_{i}^{\Sigma}-\frac{2 \pi}{3}\right) \\
I_{0}^{\Sigma}+I^{\Sigma} \cos \left(-2 \omega t+\varphi_{i}^{\Sigma}-\frac{4 \pi}{3}\right)
\end{array}\right] \tag{4.13}
\end{gather*}
$$

where $I_{0}^{\Sigma}$ corresponds to the arm DC current discussed earlier, $I_{0}^{\Sigma}=\frac{i_{d c}}{3} . I^{\Sigma}$ is the circulating current amplitude, and $\varphi_{i}^{\Sigma}$ is the circulating current phase shift. This is in agreement with Eqs. (3.5) and (3.7).

For $\boldsymbol{v}_{c, a b c}^{\Delta}$ we assume that the right-hand side of Eq. (2.59) will be of fundamental harmonic $\omega$. For $\boldsymbol{v}_{c, a b c}^{\Sigma}$ the right-hand side of Eq. (2.62) we assume that it consists of a DC component and a second harmonic $-2 \omega$.

$$
\begin{gather*}
\boldsymbol{v}_{c, a b c}^{\Delta}(t) \stackrel{\text { hyp }}{=}\left[\begin{array}{c}
V_{c}^{\Delta} \cos \left(\omega t+\varphi_{v c}^{\Delta}\right) \\
V_{c}^{\Delta} \cos \left(\omega t+\varphi_{v c}^{\Delta}-\frac{2 \pi}{3}\right) \\
V_{c}^{\Delta} \cos \left(\omega t+\varphi_{v c}^{\Delta}-\frac{4 \pi}{3}\right)
\end{array}\right]  \tag{4.14}\\
\boldsymbol{v}_{c, a b c}^{\Sigma}(t) \stackrel{\text { hyp }}{=}\left[\begin{array}{c}
V_{c 0}^{\Sigma}+V_{c}^{\Sigma} \cos \left(-2 \omega t+\varphi_{v c}^{\Sigma}\right) \\
V_{c 0}^{\Sigma}+V_{c}^{\Sigma} \cos \left(-2 \omega t+\varphi_{v c}^{\Sigma}-\frac{2 \pi}{3}\right) \\
V_{c 0}^{\Sigma}+V_{c}^{\Sigma} \cos \left(-2 \omega t+\varphi_{v c}^{\Sigma}-\frac{4 \pi}{3}\right)
\end{array}\right] \tag{4.15}
\end{gather*}
$$

where $V_{c}^{\Delta}$ is the amplitude of $v_{c}^{\Delta}$ and $\varphi_{v c}^{\Delta}$ is its phase shift. Similarly, $V_{c}^{\Sigma}$ is the amplitude of the AC component of $v_{c}^{\Sigma}$ and $\varphi_{v c}^{\Sigma}$ is its phase shift. $V_{c 0}^{\Sigma}$ is the DC component of $v_{c}^{\Sigma}$.

Note that here we do not take the hypothesis on the form of $v_{m}^{\Delta \Sigma}$ voltage. This is the main advantage of the SSTI model over the integral-based one ${ }^{1}$.

We remind the hypothesis taken on the AC grid voltage as well,

$$
\boldsymbol{v}_{s, a b c}(t) \stackrel{\operatorname{hyp}}{=}\left[\begin{array}{c}
V_{s} \cos (\omega t)  \tag{4.16}\\
V_{s} \cos \left(\omega t-\frac{2 \pi}{3}\right) \\
V_{s} \cos \left(\omega t-\frac{4 \pi}{3}\right)
\end{array}\right]
$$

We use the Park transformation with $n=1$ for $\Delta$ variables, whereas for $\Sigma$ variables we use $n=-2$. When transformed to $d q 0$ frame, these hypotheses become,

$$
\begin{gather*}
\boldsymbol{m}_{d q 0}^{\Delta}=\left[\begin{array}{c}
-M^{\Delta} \sin \left(\varphi_{m}^{\Delta}\right) \\
M^{\Delta} \cos \left(\varphi_{m}^{\Delta}\right) \\
0
\end{array}\right]  \tag{4.17}\\
\boldsymbol{m}_{d q 0}^{\Sigma}=\left[\begin{array}{c}
-M^{\Sigma} \sin \left(\varphi_{m}^{\Sigma}\right) \\
M^{\Sigma} \cos \left(\varphi_{m}^{\Sigma}\right) \\
M_{0}^{\Sigma}
\end{array}\right] \tag{4.18}
\end{gather*}
$$

[^5]\[

$$
\begin{gather*}
\boldsymbol{i}_{d q 0}^{\Delta}=\left[\begin{array}{c}
-I^{\Delta} \sin \left(\varphi_{i}^{\Delta}\right) \\
I^{\Delta} \cos \left(\varphi_{i}^{\Delta}\right) \\
0
\end{array}\right]  \tag{4.19}\\
\boldsymbol{i}_{d q 0}^{\Sigma}=\left[\begin{array}{c}
-I^{\Sigma} \sin \left(\varphi_{i}^{\Sigma}\right) \\
I^{\Sigma} \cos \left(\varphi_{i}^{\Sigma}\right) \\
I_{0}^{\Sigma}
\end{array}\right]  \tag{4.20}\\
\boldsymbol{v}_{c, d q 0}^{\Delta}=\left[\begin{array}{c}
-V_{c}^{\Delta} \sin \left(\varphi_{v c}^{\Delta}\right) \\
V_{c}^{\Delta} \cos \left(\varphi_{v c}^{\Delta}\right) \\
0
\end{array}\right]  \tag{4.21}\\
\boldsymbol{v}_{c, d q 0}^{\Sigma}=\left[\begin{array}{c}
-V_{c}^{\Sigma} \sin \left(\varphi_{v c}^{\Sigma}\right) \\
V_{c}^{\Sigma} \cos \left(\varphi_{v c}^{\Sigma}\right) \\
V_{c 0}^{\Sigma}
\end{array}\right]  \tag{4.22}\\
\boldsymbol{v}_{s, d q 0}=\left[\begin{array}{c}
0 \\
V_{s} \\
0
\end{array}\right] \tag{4.23}
\end{gather*}
$$
\]

Note that the above variables are independent of time and therefore $(t)$ has been removed.

### 4.3 SSTI MMC $\Delta \Sigma$ model in $d q 0$ frame

Using the hypotheses from the section 4.2 we can simplify the equations of the MMC $\Delta \Sigma$ model in $d q 0$ frame (Eq. (2.110)).

AE1: Replacing $\left[\boldsymbol{M}_{A 1 a}^{\Delta}\right]$ and $\left[\boldsymbol{M}_{A 1 b}^{\Sigma}\right]$ with Eqs. (4.2) and (4.3) in Eq. (2.68) and adopting Eqs. (4.21) and (4.22),

$$
\begin{equation*}
\boldsymbol{v}_{m, d q 0}^{\Delta}=\left[\boldsymbol{M}_{A 1 a}^{\Delta *}\right] \boldsymbol{v}_{c, d q 0}^{\Sigma}+\left[\boldsymbol{M}_{A 1 b}^{\Sigma *}\right] \boldsymbol{v}_{c, d q 0}^{\Delta} \tag{4.24}
\end{equation*}
$$

AE2: Replacing $\left[\boldsymbol{M}_{A 2 a}^{\Sigma}\right]$ and $\left[\boldsymbol{M}_{A 2 b}^{\Delta}\right]$ with Eqs. (4.4) and (4.5) in Eq. (2.75) and adopting Eqs. (4.21) and (4.22),

$$
\begin{equation*}
\boldsymbol{v}_{m, d q 0}^{\Sigma}=\left[\boldsymbol{M}_{A 2 a}^{\Sigma *}\right] \boldsymbol{v}_{c, d q 0}^{\Sigma}+\left[\boldsymbol{M}_{A 2 b}^{\Delta *}\right] \boldsymbol{v}_{c, d q 0}^{\Delta} \tag{4.25}
\end{equation*}
$$

ODE1: Using the Eqs. (4.19) and (4.23) in Eq. (2.81),

$$
\begin{equation*}
0=\boldsymbol{v}_{m, d q 0}^{\Delta}-\boldsymbol{v}_{s, d q 0}-\frac{R}{2} \boldsymbol{i}_{d q 0}^{\Delta}-\frac{L}{2} \boldsymbol{J}_{\omega} \boldsymbol{i}_{d q 0}^{\Delta} \tag{4.26}
\end{equation*}
$$

ODE2: Using the Eqs. (4.20) and (4.23) in Eq. (2.84),

$$
0=\left[\begin{array}{c}
0  \tag{4.27}\\
0 \\
\frac{v_{d c}}{2}
\end{array}\right]-\boldsymbol{v}_{m, d q 0}^{\Sigma}-R \boldsymbol{i}_{d q 0}^{\Sigma}-L \boldsymbol{J}_{-2 \omega} \boldsymbol{i}_{d q 0}^{\Sigma}
$$

ODE3: Using the Eqs. (4.21), (4.19), (4.20) in Eq. (2.88) and replacing $\left[\boldsymbol{M}_{3 a}^{\Sigma}\right]$ and $\left[\boldsymbol{M}_{3 b}^{\Delta}\right]$ with Eqs. (4.6) and (4.7),

$$
\begin{equation*}
0=\frac{1}{2 C_{a r m}}\left(\left[\boldsymbol{M}_{3 a}^{\Sigma *}\right] \boldsymbol{i}_{d q 0}^{\Delta}+\left[\boldsymbol{M}_{3 b}^{\Delta *}\right] \boldsymbol{i}_{d q 0}^{\Sigma}\right)-\boldsymbol{J}_{\omega} \boldsymbol{v}_{c, d q 0}^{\Delta} \tag{4.28}
\end{equation*}
$$

ODE4: Using the Eqs. (4.22), (4.19), (4.20) in Eq. (2.94) and replacing $\left[\boldsymbol{M}_{4 a}^{\Delta}\right]$ and $\left[\boldsymbol{M}_{4 b}^{\Sigma}\right]$ with Eqs. (4.8) and (4.9),

$$
\begin{equation*}
0=\frac{1}{2 C_{a r m}}\left(\left[\boldsymbol{M}_{4 a}^{\Delta *}\right] \boldsymbol{i}_{d q 0}^{\Delta}+\left[\boldsymbol{M}_{4 b}^{\Sigma *}\right] \boldsymbol{i}_{d q 0}^{\Sigma}\right)-\boldsymbol{J}_{-2 \omega} \boldsymbol{v}_{c, d q 0}^{\Sigma} \tag{4.29}
\end{equation*}
$$

The matrix $\boldsymbol{J}$ is defined in Appendix C. The MMC DS model in $d q 0$ frame become steady-state time invariant with the hypotheses from section 4.2 , therefore the $(t)$ is omitted. The 6 equations (4.24)-(4.29) hold as long as the hypotheses from the section 4.2 are verified.


Figure 4.1 - MMC steady-state time-invariant model
The system of these 6 equations can be rewritten as,

$$
\begin{equation*}
\boldsymbol{A} \boldsymbol{x}_{d q 0}^{\Delta \Sigma}=\boldsymbol{B} \tag{4.30}
\end{equation*}
$$

The vector $\boldsymbol{x}_{d q 0}^{\Delta \Sigma}$ is,

$$
\boldsymbol{x}_{d q 0}^{\Delta \Sigma} \triangleq\left[\begin{array}{lllll}
\left(\boldsymbol{v}_{m d q 0}^{\Delta}\right)^{T} & \left(\boldsymbol{v}_{m d q 0}^{\Sigma}\right)^{T} & \left(\boldsymbol{i}_{d q 0}^{\Delta}\right)^{T} & \left(\boldsymbol{v}_{d q 0}^{\Sigma}\right)^{T} & \left(\boldsymbol{v}_{c d q 0}^{\Delta}\right)^{T}
\end{array}\left(\begin{array}{l}
\left.\boldsymbol{v}_{c d q 0}^{\Sigma}\right)^{T} \tag{4.31}
\end{array}\right]^{T}\right.
$$

The matrix $\boldsymbol{A}$ is,

$$
\left.\boldsymbol{A}=\left[\begin{array}{cc}
{\left[\boldsymbol{A}_{1}\right.}  \tag{4.32}\\
{\left[\boldsymbol{A}_{4}\right.} \\
\hdashline \boldsymbol{A}_{7}
\end{array}\right] \quad\left[\begin{array}{c}
\boldsymbol{A}_{2} \\
\boldsymbol{A}_{5} \\
\hdashline \boldsymbol{A}_{8}
\end{array}\right] \quad\left[\begin{array}{c}
\boldsymbol{A}_{3} \\
\boldsymbol{A}_{6} \\
\vdots \\
\boldsymbol{A}_{9}
\end{array}\right]\right]
$$

where,

$$
\left.\begin{array}{c}
{\left[\boldsymbol{A}_{1}\right]=}
\end{array} \begin{array}{llllll}
1 & 0 & 0 & 0 & 0 & 0  \tag{4.33}\\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right] .
$$

$$
\begin{align*}
& {\left[\boldsymbol{A}_{4}\right]=\left[\begin{array}{cccccc}
-1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]}  \tag{4.36}\\
& {\left[\boldsymbol{A}_{5}\right]=\left[\begin{array}{cccccc}
\frac{R}{2} & -\omega \frac{L}{2} & 0 & 0 & 0 & 0 \\
\omega \frac{L}{2} & \frac{R}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{R}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & R & 2 \omega L & 0 \\
0 & 0 & 0 & -2 \omega L & R & 0 \\
0 & 0 & 0 & 0 & 0 & R
\end{array}\right]}  \tag{4.37}\\
& {\left[\boldsymbol{A}_{6}\right]=[0]_{6 \times 6}}  \tag{4.38}\\
& {\left[\boldsymbol{A}_{7}\right]=[0]_{6 \times 6}}  \tag{4.39}\\
& {\left[\boldsymbol{A}_{8}\right]=\left[\begin{array}{cccccc}
\frac{m_{0}^{\Sigma}}{2}-\frac{m_{a}^{\Sigma}}{4} & -\frac{m_{d}^{\Sigma}}{4} & 0 & \frac{m_{a}^{\Delta}}{2} & \frac{m_{d}^{\Delta}}{d} & -m_{d}^{\Delta} \\
-\frac{m_{d}^{\Sigma}}{4} & \frac{m_{0}^{\Sigma}}{2}+\frac{m_{q}^{\Sigma}}{4} & 0 & \frac{m_{a}^{\Delta}}{2} & -\frac{m_{q}^{\Delta}}{2} & -m_{q}^{\Delta} \\
0 & 0 & \frac{m_{0}^{\Sigma}}{2} & 0 & 0 & m_{0}^{\Delta} \\
-\frac{m_{a}^{\Delta}}{4_{\Delta}} & -\frac{m_{\Delta}^{\Delta}}{4} & 0 & -m_{0}^{\Sigma} & 0 & -m_{d}^{\Sigma} \\
-\frac{m_{a}^{\Delta}}{4} & 0 & 0 & -m_{0}^{\Sigma} & -m_{q}^{\Sigma} \\
\frac{m_{\Delta}^{d}}{4} & \frac{m_{a}^{\Delta}}{4} & \frac{m_{0}^{\Delta}}{2} & -\frac{m_{d}^{\Sigma}}{2} & -\frac{m_{a}^{2}}{2} & -m_{0}^{\Sigma}
\end{array}\right]}  \tag{4.40}\\
& {\left[\boldsymbol{A}_{9}\right]=\left[\begin{array}{cccccc}
0 & -2 C_{\text {arm }} \omega & 0 & 0 & 0 & 0 \\
2 C_{\text {arm }} \omega & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 C_{\text {arm }} \omega & 0 \\
0 & 0 & 0 & -4 C_{\text {arm }} \omega & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]} \tag{4.41}
\end{align*}
$$

We can see that the matrix $\boldsymbol{A}$ consists of control variables $m^{\Delta}$ and $m^{\Sigma}$ and passive elements $R, L$ and $C_{\text {arm }}$. The matrix $\boldsymbol{B}$ is,

$$
\boldsymbol{B}=\left[\begin{array}{lllll}
{[0]_{1 \times 7}} & -V_{s} & {[0]_{1 \times 3}} & \frac{v_{d d}}{2} & {[0]_{1 \times 6}} \tag{4.42}
\end{array}\right]^{T}
$$

The matrix $\boldsymbol{B}$ consists of AC grid voltage amplitude and the DC grid voltage. These voltages are imposed, therefore this matrix is always known.

Inverting the system in Eq. (4.30), we can obtain,

$$
\begin{equation*}
\boldsymbol{x}_{d q 0}^{\Delta \Sigma}=\boldsymbol{A}^{-1} \boldsymbol{B} \tag{4.43}
\end{equation*}
$$

which enables the calculation of all variables within the vector $\boldsymbol{x}_{d q 0}^{\Delta \Sigma}$. For this, the determinant of the matrix $\boldsymbol{A}$ has to be different from 0 .

### 4.4 Evaluation of the SSTI model

The SSTI must be evaluated numerically. This is performed as follows,

1. Set hte operating point: $M^{\Delta}$ varies from 0 to 1.4 and $\varphi_{i}^{\Delta}$ varies from $-\pi$ to $\pi$.
2. Express $\boldsymbol{m}_{d q 0}^{\Delta}$ from $M^{\Delta}$ and $\varphi_{m}^{\Delta}$ using Eq. (4.10).
3. Set $\boldsymbol{m}_{d q 0}^{\Sigma}$ intervals: $m_{d}^{\Sigma}$ and $m_{q}^{\Sigma}$ both vary from -1 to 1 and $m_{0}^{\Sigma}=1^{2}$.
4. Input $\boldsymbol{m}_{d q 0}^{\Delta}$ and $\boldsymbol{m}_{d q 0}^{\Sigma}$ into $\boldsymbol{A}$ in Eq. (4.30).
5. Invert Eq. (4.30) to obtain the vector $\boldsymbol{x}_{d q 0}^{\Delta \Sigma}$.

### 4.5 Capacitor voltage

In this section, we use the SSTI model to determine the module capacitor average voltage and the module capacitor voltage ripple. Here, we focus on the upper arm of the phase $a$ so the phase index is omitted. A similar calculation can be done for the lower arm and other phases as well. From Eq. (2.42) and hypotheses in Eqs. (4.14) and (4.15),

$$
\begin{equation*}
v_{u, C}(t)=V_{c 0}^{\Sigma}-V_{c}^{\Delta} \cos \left(\omega t+\varphi_{v c}^{\Delta}\right)+V_{c}^{\Sigma} \cos \left(-2 \omega t+\varphi_{v c}^{\Sigma}\right) \tag{4.44}
\end{equation*}
$$

From Eq. (2.25),

$$
\begin{equation*}
v_{u, c a p}(t)=\frac{1}{N}\left(V_{c 0}^{\Sigma}-V_{c}^{\Delta} \cos \left(\omega t+\varphi_{v c}^{\Delta}\right)+V_{c}^{\Sigma} \cos \left(-2 \omega t+\varphi_{v c}^{\Sigma}\right)\right) \tag{4.45}
\end{equation*}
$$

The amplitudes $V_{c}^{\Delta}$ and $V_{c}^{\Sigma}$ are determined as,

$$
\begin{align*}
& V_{c}^{\Delta}=\sqrt{\left(v_{c d}^{\Delta}\right)^{2}+\left(v_{c q}^{\Delta}\right)^{2}}  \tag{4.46}\\
& V_{c}^{\Sigma}=\sqrt{\left(v_{c d}^{\Sigma}\right)^{2}+\left(v_{c q}^{\Sigma}\right)^{2}} \tag{4.47}
\end{align*}
$$

The phases $\varphi_{v c}^{\Delta}$ and $\varphi_{v c}^{\Sigma}$ are calculated using Eq. (C.12),

$$
\begin{align*}
& \varphi_{v c}^{\Delta}=\arctan \left(\frac{-v_{c d}^{\Delta}}{v_{c q}^{\Delta}}\right)  \tag{4.48}\\
& \varphi_{v c}^{\Sigma}=\arctan \left(\frac{-v_{c d}^{\Sigma}}{v_{c q}^{\Sigma}}\right) \tag{4.49}
\end{align*}
$$

By solving the SSTI model, $v_{c d}^{\Delta}, v_{c q}^{\Delta}, v_{c d}^{\Sigma}$ and $v_{c q}^{\Sigma}$ are determined. One can then determine the $v_{c a p}$ using the Eq. (4.45).
In the following, we focus on the module capacitor average voltage $V_{c a p}$ and on the module capacitor voltage ripple $\Delta v_{c a p}$, Then, for a given operating point, the voltage ripple is defined as,

$$
\begin{equation*}
\Delta v_{c a p} \triangleq \max _{t}\left(v_{c a p}(t)\right)-\min _{t}\left(v_{c a p}(t)\right) \tag{4.50}
\end{equation*}
$$

Following Eq. (3.3) the voltage $V_{c 0}^{\Sigma}$ is the average arm voltage ${ }^{3}$,

$$
\begin{equation*}
V_{c a p}=\frac{V_{c 0}^{\Sigma}}{N} \tag{4.51}
\end{equation*}
$$

### 4.5.1 Module capacitor average voltage

In the literature, two equations have been used to calculate the module capacitor average voltage.

[^6]
### 4.5.1.1 Case I: $R=0, L=0, i_{2}^{\Sigma}=0$

To simplify, the module capacitor average voltage is often considered independent of the operating point and equal to,

$$
\begin{equation*}
V_{c a p}=\frac{v_{d c}}{N} \tag{4.52}
\end{equation*}
$$

In the Fig. 4.2, this is shown using the dotted line. The Eq. (4.52) has been used in [20, 40, 41, 42, 43].

### 4.5.1.2 Case Ia: $R \neq 0, L=0, i_{2}^{\Sigma} \neq 0$

The average module capacitor voltage is susceptible to variation with power. Christe [44], developed an equation based on the integral method. This equation describes the linear dependence between the average module capacitor voltage and the active power transfer. However, this equation is only valid if the arm inductance is very low (or neglected). From Eq. (2.56), for phase $a$ with omitted index, we obtain,

$$
\begin{equation*}
L \frac{d i^{\Sigma}(t)}{d t}=\frac{v_{d c}}{2}-v_{m}^{\Sigma}(t)-R i^{\Sigma}(t) \tag{4.53}
\end{equation*}
$$

Replace $v_{m}^{\Sigma}$ using Eq. (2.41),

$$
\begin{equation*}
L \frac{d i^{\Sigma}(t)}{d t}=\frac{v_{d c}}{2}-\frac{v_{u}(t)+v_{l}(t)}{2}-R i^{\Sigma}(t) \tag{4.54}
\end{equation*}
$$

Replace $v_{u}$ and $v_{l}$ from Eqs. (2.23) and (2.24),

$$
\begin{equation*}
L \frac{d i^{\Sigma}(t)}{d t}=\frac{v_{d c}}{2}-\frac{m_{u}(t) N v_{c a p, u}(t)+m_{l}(t) N v_{c a p, l}(t)}{2}-R i^{\Sigma}(t) \tag{4.55}
\end{equation*}
$$

Under hypothesis $v_{c a p, u}(t) \stackrel{h y p}{=} v_{c a p, l}(t) \stackrel{h y p}{=} V_{c a p}$, and using Eqs. (2.18) and (2.19) we obtain,

$$
\begin{equation*}
L \frac{d i^{\Sigma}(t)}{d t}=\frac{v_{d c}}{2}-N V_{c a p}-R i^{\Sigma}(t) \tag{4.56}
\end{equation*}
$$

Replace $i^{\Sigma}$ with Eq. (4.13)

$$
\begin{equation*}
L \frac{d\left(I_{0}^{\Sigma}+I^{\Sigma} \cos \left(-2 \omega t+\varphi_{i}^{\Sigma}\right)\right)}{d t}=\frac{v_{d c}}{2}-N V_{c a p}-R\left(I_{0}^{\Sigma}+I^{\Sigma} \cos \left(-2 \omega t+\varphi_{i}^{\Sigma}\right)\right) \tag{4.57}
\end{equation*}
$$

Take the mean of Eq. (4.57),

$$
\begin{equation*}
0=\frac{v_{d c}}{2}-N V_{c a p}-R I_{0}^{\Sigma} \tag{4.58}
\end{equation*}
$$

The current $I_{0}^{\Sigma}$ corresponds to the dc phase current, $\frac{i_{d c}}{3}$, derived from Eq. (3.47). Therefore,

$$
\begin{equation*}
V_{c a p}=\frac{v_{d c}}{N}-2 R \frac{i_{d c}}{3 N} \tag{4.59}
\end{equation*}
$$

This equation includes the impact of the arm equivalent resistance. It is based on the fact that the dc phase current $\frac{i_{d c}}{3}$ creates the dc voltage drop across the arm resistance. However, the impact of the arm inductance cannot be seen. The voltage obtained this way is represented with dashed lines in the Fig. 4.2.
4.5.1.3 Case IV: $R \neq 0, L \neq 0, i_{2}^{\Sigma} \neq 0$

The module capacitor average voltage is calculated using the SSTI model in Eq. (4.51). It derives directly by inverting Eq. (4.30).


Figure 4.2 - Module average voltage as a function of active power $P_{s}$ and $Q_{s}=0$.

### 4.5.1.4 Results

In the Fig. 4.2a, we can observe the module capacitor average voltage for $R=0 \Omega$ and $R=1 \Omega$ and for several arm inductance values. We look for this voltage in the $P_{s} \in[-2000 \mathrm{~W}, 2000 \mathrm{~W}]$ interval while $Q_{s}=0$.

In idealized case (for $R=0$ and $L=0$ ), Fig. 4.2a, the three results correlate. If the resistance is considered, the voltage drop is linear with power. The SSTI model gives identical results as the model from Christe [44].

Increasing the inductance leads to a discrepancy of the SSTI model results in comparison to the two previous simplifications. For higher inductances ie. 15 mH , Fig. 4.2 h , the SSTI model is able to predict effects that could not be seen with classical formulas. In fact, the module capacitor average voltage obtained through SSTI decreases more and more with power increase. This leads to conclusion that, arm inductance contributes to module capacitor average voltage drop. This can lead to smaller operating areas of the converter (studied further in this document).

To quantify the discrepancy between the models let us consider Fig. 4.2f. At $P_{s}=1500 \mathrm{~W}$ the ideal model (Eq. (4.52)) gives 30 V . On the other hand, the model from Christe (Eq. (4.59)) gives 28.5 V . Finally, the SSTI model results with a voltage slightly lower than 28 V . This represents a $7 \%$ difference from the ideal model in Eq. (4.52). Therefore, we underline here that the inductance does affect the module capacitor average voltage. In Fig. 4.2f, a simulation for the detailed MMC model (see section 5.3) has been performed confirming the theoretical analysis.

To our best knowledge, apart from Christe [44], this phenomenon has not been reported in the literature.

### 4.5.2 Module capacitor voltage ripple

In this section we use the SSTI model from section 4.3 to calculate $\Delta v_{c a p}$ for the following case studies.

- Case I: $R=0, L=0, i_{2}^{\Sigma}=0$
- Case II: $R=0, L=0, i_{2}^{\Sigma} \neq 0$
- Case IV: $R \neq 0, L \neq 0, i_{2}^{\Sigma} \neq 0$
4.5.2.1 Case I: $R=0, L=0, i_{2}^{\Sigma}=0$

To study Case I, let us adopt the hypotheses from Merlin et al. [20] model. In this case, the SSTI model cannot be applied directly since the matrix $\boldsymbol{A}$ in Eq. (4.30) is not invertible. The derivation for Case I is detailed in Appendix D.

In the Fig. 4.3, the module capacitor voltage ripple calculated using the integral based (IB) model is compared to SSTI model. The result obtained using the integral based method is the one obtained by Merlin et al. in [20]. We observe that the result obtained using SSTI model is exactly the same. The voltage ripple is symmetrical about the x - and y -axis in the Fig. 4.3b, and the maximal ripple is attained for the pure reactive power transfer.


Figure 4.3 - Module capacitor voltage ripple for Case I: $R=0, L=0, i_{2}^{\Sigma}=0$ for SSTI and IB models

## Arm energy ripple $\Delta E_{u}$

Here, we calculate the arm energy ripple in two ways.
Option 1 - equidistant module capacitor voltage ripple In [20] the arm energy ripple has, $"[\ldots]$ been defined such that the maximum voltage deviation of the cells (modules) has a value opposite to their minimum voltage deviation, thus centred around the nominal cell (module) voltage value."
Adopting this hypothesis we can express from Eq. (3.89),

$$
\begin{align*}
& N \frac{C}{2}\left(V_{c a p}+\frac{\Delta v_{c a p}^{*}}{2}\right)^{2}=E_{u 0}+\Delta E_{u, \max }  \tag{4.60}\\
& N \frac{C}{2}\left(V_{\text {cap }}-\frac{\Delta v_{c a p}^{*}}{2}\right)^{2}=E_{u 0}+\Delta E_{u, \min } \tag{4.61}
\end{align*}
$$

where the $\Delta v_{c a p}^{*}$ is the equidistant voltage deviation from $V_{c a p}$, and $\Delta E_{u, \text { max }}$ and $\Delta E_{u, \text { min }}$ are the maximal and minimal arm energy voltage deviation (also equidistant from $E_{u 0}$ ). Subtracting these two equations,

$$
\begin{equation*}
\Delta E_{u}=N C V_{c a p} \Delta v_{c a p}^{*} \tag{4.62}
\end{equation*}
$$

The $\Delta E_{u}$ calculated this way using Eq. (4.50) is shown in Fig. 4.4.


Figure 4.4 - Arm energy ripple for $R=0, L=0, i_{2}^{\Sigma}=0$, adapted from [20]
The IB and SSTI models give the same result if an equidistant module capacitor voltage ripple is assumed.
Option 2 - no hypothesis on module capacitor voltage ripple Input Eq. (4.45) in Eq. (3.89)

$$
\begin{gather*}
E_{u}=N \frac{C}{2}\left(\frac{1}{N}\left(V_{c 0}^{\Sigma}-V_{c}^{\Delta} \cos \left(\omega t+\varphi_{v c}^{\Delta}\right)+V_{c}^{\Sigma} \cos \left(-2 \omega t+\varphi_{v c}^{\Sigma}\right)\right)^{2}\right.  \tag{4.63}\\
E_{u}=\frac{C}{2 N}\left(V_{c 0}^{\Sigma}-V_{c}^{\Delta} \cos \left(\omega t+\varphi_{v c}^{\Delta}\right)+V_{c}^{\Sigma} \cos \left(-2 \omega t+\varphi_{v c}^{\Sigma}\right)\right)^{2}  \tag{4.64}\\
E_{u}=\frac{C}{4 N}\left[V_{c}^{\Delta^{2}} \cos \left(2 \varphi_{c}^{\Delta}+2 \omega t\right)+V_{c}^{\Sigma^{2}} \cos \left(2 \varphi_{c}^{\Sigma}-4 \omega t\right)+V_{c}^{\Delta^{2}}+V_{c}^{\Sigma^{2}}+2 V_{c 0}^{\Sigma^{2}}+\right. \\
4 V_{c}^{\Sigma} V_{c 0}^{\Sigma} \cos \left(\varphi_{c}^{\Sigma}-2 \omega t\right)-2 V_{c}^{\Delta} V_{c}^{\Sigma} \cos \left(\varphi_{c}^{\Delta}+\varphi_{c}^{\Sigma}-\omega t\right)-4 V_{c}^{\Delta} V_{c 0}^{\Sigma} \cos \left(\varphi_{c}^{\Delta}+\omega t\right)  \tag{4.65}\\
\left.-2 V_{c}^{\Delta} V_{c}^{\Sigma} \cos \left(\varphi_{c}^{\Delta}-\varphi_{c}^{\Sigma}+3 \omega t\right)\right]
\end{gather*}
$$



Figure 4.5 - Arm energy ripple for $R=0, L=0, i_{2}^{\Sigma}=0$ for the SSTI and IB models

The arm energy ripple is calculated using Eq. (3.90), Fig. 4.5.
The IB and SSTI models do not give the same result when no hypothesis on module capacitor voltage ripple is taken.

Discussion The model proposed by Merlin et al. has been compared to the SSTI model, using the same hypotheses. We observed a correlation between the two for $\Delta v_{\text {cap }}$.

The issue that appeared here is a definition of the voltage ripple. The IB model takes a hypothesis of an equidistant voltage ripple. This reflected on the arm energy ripple in particular. Since the SSTI model does not need this hypothesis, the energy ripple we obtained is not symmetrical around $\varphi_{i}^{\Delta}=0$ which provided a more precise result.

In addition, the case study analyzed here $\left(R=0, L=0\right.$ and $\left.i_{2}^{\Sigma}=0\right)$ is not a reflection of a realistic MMC. To obtain more precise results, we will analyze the impact of the passive elements on the module capacitor voltage ripple using the SSTI model (see section 4.3) in the following section.

### 4.5.2.2 Case II: $R=0, L=0, i_{2}^{\Sigma} \neq 0$

Let us study the case II using the SSTI model. Remind that this case was studied in 3.5.7.2 using the integral-based (IB) model.

In Fig. 4.6 we can see the results from both SSTI and IB models. There is a difference between the two due to hypotheses taken in the IB model (Eq. (3.70) which as a consequence has Eq. (3.73)). Nevertheless a similar tendency is observed. The highest $\Delta v_{c a p}$ occurs for pure positive reactive power. A slight asymmetry about the x -axis is observed.


Figure $4.6-\Delta v_{c a p}$ for Case II: SSTI model (black line), IB model (red line)

### 4.5.2.3 Case IV: $R \neq 0, L \neq 0, i_{2}^{\Sigma} \neq 0$

The SSTI model allows us to study the Case IV, which was not possible with the integral method. In Fig. 4.8, we can see the comparison of the SSTI model and the simulation of an detailed MMC model (see section 5.3). The two results are in good agreement.

The $\Delta v_{c a p}$ diagram is asymmetrical. This implies that $\Delta v_{c a p}$ will be different for the same power transfer in the two directions. Let us take the active power transfer as an example: $\Delta v_{c a p}$ has lower value for the positive active power transfer than for the case of negative active power transfer. A similar conclusion can be drawn for the reactive power.

By inspection, we observe that the asymmetry is introduced by the arm equivalent resistance, rather than arm inductance.


Figure $4.7-\Delta v_{c a p}$ for Case IV and for $R=1 \Omega, L=5 \mathrm{mH}$


Figure $4.8-\Delta v_{c a p}$ for Case IV and $R=1 \Omega$ and $L=10 \mathrm{mH}$ : SSTI model (black), simulation result (red)

### 4.5.2.4 Module capacitor voltage as a function of arm inductance

In the following, we calculate the maximal voltage ripple as a function of the arm resistance and inductance.

The Fig. 4.10 shows the case where resistance is neglected. We can identify three regions:

- small inductance region (here from 0 to 4 mH ) where the maximal voltage ripple increases with the inductance increase.
- resonant-like behavior (here occurring for 4 mH ) where the voltage ripple becomes abnormally high. This is the consequence of neglecting the arm resistance.


Figure $4.9-\Delta v_{c a p}$ for Case IV and $R=1 \Omega, L=15 \mathrm{mH}$

- big inductance region (here from 4 to 15 mH ) where the maximal voltage ripple decreases with the inductance increase

Inspecting the Fig. 4.10b, we can see that the lowest maximal voltage ripple is obtained for $L=0$. This indicates that, from the $\Delta v_{\text {cap }}$ perspective, a low inductance is desirable. Before selecting the value of the arm coil inductance, we remind that there are other criteria that have to be considered [23]. In Fig. 4.11 we observe the $\Delta v_{c a p}$ when the resistance is not neglected. We can see that the resonant-like behavior has a much lower value indicating that the resistance increase limits the voltage ripple. But in the low inductance region, $\Delta v_{c a p}$ is larger than for $R=0$, whereas in the big inductance region, the $\Delta v_{c a p}$ stays at the same level.


Figure $4.10-\Delta v_{\text {cap }}$ for $R=0 \Omega$ as a function of $L$ (case IIb)


Figure $4.11-\Delta v_{c a p}$ for $R=1 \Omega$ as a function of $L$ (case IV)


Figure $4.12-\Delta v_{c a p}$ for $R=2 \Omega$ as a function of $L$ (case IV)

### 4.6 Discussions

In this chapter we developed the steady-state time invariant MMC $\Delta \Sigma$ model in $d q 0$ frame. The equations are summarized here,

$$
\begin{gather*}
\boldsymbol{v}_{m, d q 0}^{\Delta}=\left[\boldsymbol{M}_{A 1 a}^{\Delta *}\right] \boldsymbol{v}_{c, d q 0}^{\Sigma}+\left[\boldsymbol{M}_{A 1 b}^{\Sigma *}\right] \boldsymbol{v}_{c, d q 0}^{\Delta}  \tag{4.66a}\\
\boldsymbol{v}_{m, d q 0}^{\Sigma}=\left[\boldsymbol{M}_{A 2 a}^{\Sigma *}\right] \boldsymbol{v}_{c, d q 0}^{\Sigma}+\left[\boldsymbol{M}_{A 2 b}^{\Delta *}\right] \boldsymbol{v}_{c, d q 0}^{\Delta}  \tag{4.66b}\\
0=\boldsymbol{v}_{m, d q 0}^{\Delta}-\boldsymbol{v}_{s, d q 0}-\frac{R}{2} \boldsymbol{i}_{d q 0}^{\Delta}-\frac{L}{2} \boldsymbol{J}_{\omega} \boldsymbol{i}_{d q 0}^{\Delta}  \tag{4.66c}\\
0=\left[\begin{array}{c}
0 \\
0 \\
\frac{v_{d c}}{2}
\end{array}\right]-\boldsymbol{v}_{m, d q 0}^{\Sigma}-R \boldsymbol{i}_{d q 0}^{\Sigma}-L \boldsymbol{J}_{-2 \omega} \boldsymbol{i}_{d q 0}^{\Sigma}  \tag{4.66d}\\
0=\frac{1}{2 C_{a r m}}\left(\left[\boldsymbol{M}_{3 a}^{\Sigma *}\right] \boldsymbol{i}_{d q 0}^{\Delta}+\left[\boldsymbol{M}_{3 b}^{\Delta *}\right] \boldsymbol{i}_{d q 0}^{\Sigma}\right)-\boldsymbol{J}_{\omega} \boldsymbol{v}_{c, d q 0}^{\Delta}  \tag{4.66e}\\
0=\frac{1}{2 C_{a r m}}\left(\left[\boldsymbol{M}_{4 a}^{\Delta *}\right] \boldsymbol{i}_{d q 0}^{\Delta}+\left[\boldsymbol{M}_{4 b}^{\Sigma *}\right] \boldsymbol{i}_{d q 0}^{\Sigma}\right)-\boldsymbol{J}_{-2 \omega} \boldsymbol{v}_{c, d q 0}^{\Sigma} \tag{4.66f}
\end{gather*}
$$

In section 4.5.1, we dealt with the module capacitor average voltage $V_{c a p}$. Three models were compared: Case I - $R=0, L=0$ and $i_{2}^{\Sigma}=0$, Case Ia - $R \neq 0, L=0$ and $i_{2}^{\Sigma} \neq 0$, and Case IV - $R \neq 0, L \neq 0$ and $i_{2}^{\Sigma} \neq 0$. We demonstrated that this voltage depends on the arm resistance, arm inductance and the
power transfer. During a power transfer, the bigger the inductance, the bigger is the voltage drop for the same operating point. A simulation was performed, showing the validity of the result obtained by the SSTI model.

In section 4.5.2, we dealt with the module capacitor voltage ripple $\Delta v_{\text {cap }}$. Several cases were studied: Case I - $R=0, L=0$ and $i_{2}^{\Sigma}=0$, Case II - $R=0, L=0$ and $i_{2}^{\Sigma} \neq 0$, and Case IV - $R \neq 0, L \neq 0$ and $i_{2}^{\Sigma} \neq 0$. The Case IV could not be studied for the IB model due to its complexity. The SSTI model result was presented along with the simulation result. The two are in good agreement. The asymmetry of the $\Delta v_{c a p}$ was observed: the same power transfer in different directions results in a different $\Delta v_{c a p}$.

## Chapter 5

## PQ diagrams

Objectives: Construction of the PQ diagram of the MMC including the "internal limits". Discussion of the results through simulations for various operating points and arm inductances.

Motivations: Lack of discussion on PQ diagrams for the modular multilevel converters. Need for information on the "inner" converter limits and their influence on the operating area of the converter.

Contributions: Use of the steady-state time invariant model to construct a PQ diagram. Representation of the internal limits of the MMC within the PQ diagram (IGBT current, arm RMS current, module capacitor voltage ripple, module capacitor current ripple).

### 5.1 Introduction

The converter PQ diagram is one of the technical indices during the converter design stage. It allows us to check if the components of the converter are qualified to cover the required power. Since all the components need to satisfy all the constraints, it is essential to study the operating areas of the converter.

The PQ diagram of the MMC is commonly obtained from a simplified model that reduces the MMC to a voltage source in series with a RL circuit $[74,73,77]$. Such model, denominated "conventional model", takes hypotheses that are rarely justified, including a zero equivalent arm resistance and a hypothesis on modulated voltage (3.73). It corresponds to Eq. (3.27) in the IB model. This conventional model includes only 3 limits.

In an attempt to go further than the conventional model, Jovcic et al. constructed the PQ diagram of the MMC using a phasor model [6]. Considering a weak AC system, they underlined that a voltage drop at the PCC would seriously affect the power transfer capability of the MMC, and that this should be considered from the design stage. They concluded that the PQ diagram resembles to the one of a 2-level voltage source converter (VSC).

Another approach, proposed by Kim et al. [72], is to use the IB model to derive the equations of the MMC. They defined the PQ capability of the MMC delimited by the apparent power and the imposed module capacitor voltage ripple. This allowed them to study the operating range of the MMC with and without the circulating current suppression. They demonstrated that the operating area (OA) can be extended when suppressing the circulating current.

In above references, the influence of the MMC arm resistance and inductance on the PQ diagram has not been studied. To do so, we use the MMC SSTI model in $d q 0$ frame. This allows us to discuss the impact of the arm resistance and inductance on the PQ diagram.

### 5.2 PQ diagram development

### 5.2.1 Conventional model

Firstly, we elaborate the conventional model that will be used for comparisons. The conventional model for constructing the PQ diagram takes only one equation into account, Eq. (3.27), with several hypotheses $[74,73,77]$.

For this let us define two new study cases.

- Case IIa - $R=0, L \neq 0, i_{2}^{\Sigma}=0$ where the waveform of $v_{m}^{\Delta}$ is assumed (Eq. (3.73)). This corresponds to the IB model.
- Case IIb - $R=0, L \neq 0, i_{2}^{\Sigma}=0$ where the waveform of $v_{m}^{\Delta}$ is not assumed. This corresponds to the SSTI model.
Using the conventional model we can derive the following equations concerning the power transfer.

Limit on modulation index For the half-bridge MMC, the maximal modulation index amplitude is $M_{\max }^{\Delta}=1[6]$. Let us remind the Fig. 3.2 and Eqs. (3.81) and (3.82). Since $R=0$, they simplify to,

$$
\begin{gather*}
V_{m}^{\Delta} \cos \left(\varphi_{t}^{\Delta}\right)=V_{s}+\omega \frac{L}{2} I^{\Delta} \sin \left(\varphi_{i}^{\Delta}\right)  \tag{5.1}\\
V_{m}^{\Delta} \sin \left(\varphi_{t}^{\Delta}\right)=\omega \frac{L}{2} I^{\Delta} \cos \left(\varphi_{i}^{\Delta}\right) \tag{5.2}
\end{gather*}
$$

Multiply the equations with $V_{s}$,

$$
\begin{gather*}
V_{s} V_{m}^{\Delta} \cos \left(\varphi_{t}^{\Delta}\right)=V_{s}^{2}+V_{s} \omega \frac{L}{2} I^{\Delta} \sin \left(\varphi_{i}^{\Delta}\right)  \tag{5.3}\\
V_{s} V_{m}^{\Delta} \sin \left(\varphi_{t}^{\Delta}\right)=V_{s} \omega \frac{L}{2} I^{\Delta} \cos \left(\varphi_{i}^{\Delta}\right) \tag{5.4}
\end{gather*}
$$

The complex apparent power is $S=\frac{3}{2} \bar{V}_{s} \bar{I}^{\Delta *}$. Transforming the hypotheses in Eqs. (3.5) and (3.4) using the Eq. (3.12), we obtain $S=\frac{3}{2} V_{s} I^{\Delta} e^{-j \varphi_{i}}$. Since $P_{s}$ and $Q_{s}$ have been defined as a real and imaginary part of $\bar{S}$ in Eq. (2.98), we obtain

$$
\begin{align*}
P_{s} & =\frac{3}{2} V_{s} I^{\Delta} \cos \left(\varphi_{i}^{\Delta}\right)  \tag{5.5}\\
Q_{s} & =\frac{3}{2} V_{s} I^{\Delta} \sin \left(\varphi_{i}^{\Delta}\right) \tag{5.6}
\end{align*}
$$

Insert Eqs. (5.5)-(5.6) into Eqs. (5.3)-(5.4) and simplify,

$$
\begin{gather*}
Q_{s}=3 \frac{V_{s} V_{m}^{\Delta} \cos \left(\varphi_{t}^{\Delta}\right)-V_{s}^{2}}{\omega L}  \tag{5.7}\\
P_{s}=3 \frac{V_{s} V_{m}^{\Delta} \sin \left(\varphi_{t}^{\Delta}\right)}{\omega L} \tag{5.8}
\end{gather*}
$$

Remind that,

$$
\begin{equation*}
\sin ^{2}\left(\varphi_{t}^{\Delta}\right)+\cos ^{2}\left(\varphi_{t}^{\Delta}\right)=1 \tag{5.9}
\end{equation*}
$$

Isolating $\sin \left(\varphi_{t}^{\Delta}\right)$ and $\cos \left(\varphi_{t}^{\Delta}\right)$ from Eqs. (5.7) and (5.8) and replacing them into Eq. (5.9),

$$
\begin{equation*}
P_{s}^{2}+\left(Q_{s}+\frac{3 V_{s}^{2}}{\omega L}\right)^{2}=\left(\frac{3 V_{s} V_{m}^{\Delta}}{\omega L}\right)^{2} \tag{5.10}
\end{equation*}
$$

This is the equation of a circle with the center at $\left(0,-\frac{3 V_{s}^{2}}{\omega L}\right)$ and a radius of $\frac{3 V_{s} V_{m}^{\Delta}}{\omega L}$. The voltage amplitude $V_{m}^{\Delta}$ is directly related to $M^{\Delta}$, Eq. (3.124). Therefore, for $M_{\max }^{\Delta}=1$, we obtain $V_{m, \max }^{\Delta}=\frac{v_{d c}}{2}$.

Limit on AC grid current The AC grid current limit is expressed as,

$$
\begin{equation*}
P_{s}^{2}+Q_{s}^{2}=S^{2}=\left(\frac{3}{2} V_{s} I^{\Delta}\right)^{2} \tag{5.11}
\end{equation*}
$$

The center of this circle is at $(0,0)$ and its radius is $\frac{3}{2} V_{s} I^{\Delta}$. The AC grid limit is imposed by the ratings of the AC grid cables. It is independent of the MMC model. We look at the active and the reactive power when the converter is operated along the AC grid current limit. It is commonly expressed as in Eq. (5.11), where $I^{\Delta}=I_{\text {max }}^{\Delta}$ and $I_{\text {max }}^{\Delta}$ is the AC grid current amplitude limit.

DC power: The DC grid current is limited by the DC cable current capacity. From Eq. (2.107),

$$
\begin{equation*}
P_{d c, l i m i t}=v_{d c} i_{d c, \max } \tag{5.12}
\end{equation*}
$$

where $i_{d c, \text { max }}$ is the DC cable maximal current and the $P_{d c, l i m i t}$ is the corresponding power. This limit is thus represented by two vertical lines (for positive and negative active power).

### 5.2.2 SSTI model

The MMC behavior is modeled through six equations, Eqs. (4.24)-(4.29). The active and reactive power are calculated by inserting Eq. (4.23) into Eqs. (2.105) and (2.106),

$$
\begin{align*}
P_{s} & =\frac{3}{2} V_{s} i_{q}^{\Delta}  \tag{5.13}\\
Q_{s} & =\frac{3}{2} V_{s} i_{d}^{\Delta} \tag{5.14}
\end{align*}
$$

### 5.2.3 Some converter limitations

With the SSTI model, we have access to the internal variables of the MMC. This allows us to derive supplementary limits.

Limit on AC grid current: The AC grid limitation is derived using the procedure in section 4.4. When $\left(I_{\text {max }}^{\Delta}\right)^{2}=\left(i_{d}^{\Delta}\right)^{2}+\left(i_{q}^{\Delta}\right)^{2}$ the corresponding powers are calculated (Eqs. (5.13) and (5.14)).

Limit on modulation index To determine the limit on modulation index, the SSTI model is evaluated as in section 4.4. When $M^{\Delta}=1$, use Eqs. (5.13)-(5.14) to obtain $P_{s}$ and $Q_{s}$.

Limit on module capacitor voltage ripple Following the procedure in section 4.4, the limit on the module capacitor voltage ripple using SSTI model can then be elaborated as in the Eq. (4.45) and (4.50). When $\Delta v_{c a p}=\Delta v_{c a p, \text { max }}$, use Eqs. (5.13)-(5.14) to obtain corresponding $P_{s}$ and $Q_{s}$.

Limit on the IGBT current The IGBT current limitation is represented through the rms arm current. From Eq. (2.42), using the hypotheses in Eqs. (4.12) and (4.13) the upper arm current is equal to,

$$
\begin{equation*}
i_{u}(t)=I_{0}^{\Sigma}+\frac{I^{\Delta}}{2} \cos \left(\omega t+\varphi_{i}^{\Delta}\right)+I^{\Sigma} \cos \left(-2 \omega t+\varphi_{i}^{\Sigma}\right) \tag{5.15}
\end{equation*}
$$

where $I_{0}^{\Sigma}$ is calculated directly from the Eq. (4.30) and,

$$
\begin{gather*}
I^{\Delta}=\sqrt{i_{d}^{\Delta 2}+i_{q}^{\Delta 2}}  \tag{5.16}\\
I^{\Sigma}=\sqrt{i_{d}^{\Sigma 2}+i_{q}^{\Sigma 2}}  \tag{5.17}\\
\varphi_{i}^{\Delta}=\arctan \left(\frac{-i_{d}^{\Delta}}{i_{q}^{\Delta}}\right)  \tag{5.18}\\
\varphi_{i}^{\Sigma}=\arctan \left(\frac{-i_{d}^{\Sigma}}{i_{q}^{\Sigma}}\right) \tag{5.19}
\end{gather*}
$$

The rms arm current is then,

$$
\begin{equation*}
i_{u, R M S}=\sqrt{\frac{1}{T} \int_{0}^{T} i_{u}^{2} d t} \tag{5.20}
\end{equation*}
$$

To evaluate this limitation, follow the procedure in section 4.4. Then, use Eq. (5.20) to calculate $i_{u, R M S}$. When $i_{u, R M S}=i_{u, R M S, \max }$, use Eqs. (5.13)-(5.14) to obtain corresponding $P_{s}$ and $Q_{s}$.

Limit on arm rms current Passing through the arm resistance, the arm current causes Joule's losses. In order to keep these losses on a reasonable limit, the upper arm current rms value should be limited at a certain level. The limitation can then be expressed as,

$$
\begin{equation*}
P_{R}=R\left(i_{u, R M S}\right)^{2} \tag{5.21}
\end{equation*}
$$

where $P_{R}$ is the power loss on one equivalent arm resistance. Therefore, to limit this loss, the $i_{u, R M S}$ should be limited. To evaluate this limitation, follow the procedure in section 4.4. Then, use Eq. (5.20) to calculate $i_{u, R M S}$. When $i_{u, R M S}=i_{u, R M S, \text { max }}$, use Eqs. (5.13)-(5.14) to obtain corresponding $P_{s}$ and $Q_{s}$.

Limit on module capacitor current ripple The capacitor current ripple is the rms value of the current passing through the capacitor. The excessive values of this current can lead to fast heating and degradation of the capacitors [67, 68]. Therefore, the capacitor current should be kept within the limit detailed in the capacitor datasheet.
From Eq. (2.43) and hypothesis in Eq. (4.10),

$$
\begin{equation*}
m_{u}(t)=\frac{1}{2}\left(1-M^{\Delta} \cos \left(\omega t+\varphi_{m}^{\Delta}\right)\right) \tag{5.22}
\end{equation*}
$$

Insert Eqs. (5.15) and (5.22) into Eq. (2.16),

$$
\begin{equation*}
i_{c a p}(t)=\frac{1}{2}\left(1-M^{\Delta} \cos \left(\omega t+\varphi_{m}^{\Delta}\right)\right)\left(I_{0}^{\Sigma}+\frac{I^{\Delta}}{2} \cos \left(\omega t+\varphi_{i}^{\Delta}\right)+I^{\Sigma} \cos \left(-2 \omega t+\varphi_{i}^{\Sigma}\right)\right) \tag{5.23}
\end{equation*}
$$

The rms capacitor current is then,

$$
\begin{equation*}
i_{c a p, R M S}=\sqrt{\frac{1}{T} \int_{0}^{T} i_{c a p}^{2} d t} \tag{5.24}
\end{equation*}
$$

To evaluate this limitation, follow the procedure in section 4.4. Then, use Eq. (5.24) to calculate $i_{c a p, R M S}$. When $i_{c a p, R M S}=i_{c a p, R M S, \text { max }}$, use Eqs. (5.13)-(5.14) to obtain corresponding $P_{s}$ and $Q_{s}$.

Table 5.1 - Summary of the limits

| Limit | Conventional model | SSTI model |
| :---: | :---: | :---: |
| AC grid current | Eq. (5.11) | Eqs. (5.13) and (5.14) |
| DC grid current | Eq. (5.12) | Eq. (5.12) |
| RMS arm current | NA* | Eq. (5.21) |
| Modulation index | Eq. (5.10) | Eqs. (5.13) and (5.14) |
| Module capacitor voltage ripple | NA | Eq. (4.45) and (4.50) |
| Module capacitor current limit | NA | Eqs. (5.23) and (5.24) |
| IGBT current limit | NA | Eqs. (5.15) and (5.20) |
| * Not Available |  |  |

The procedure in section 4.4 allows us to calculate all operating points. Then, we compare the calculated limits to their maximal admissible values in table 5.2, and plot them for their respective operating points.

Table 5.2 - MMC parameters

| Parameter | Variable | Value |
| :---: | :---: | :---: |
| Module capacitance | $C$ | $2240 \mu \mathrm{~F}$ |
| Number of modules per arm | $N$ | 5 |
| Angular frequency | $\omega$ | 50 Hz |
| AC grid voltage amplitude | $V_{s}$ | 60 V |
| DC grid voltage | $v_{d c}$ | 150 V |
| Modulation index limit | $M_{\bmod }^{\Delta}$ | 1 |
| AC grid current limit amplitude | $I_{\max }^{\Delta}$ | $32 \sqrt{2} \mathrm{~A}$ |
| DC grid current limit amplitude | $i_{d c, \text { max }}$ | 32 A |
| Module capacitor voltage ripple | $\Delta v_{\text {cap, max }}$ | $60 \% V_{0}^{\Sigma}$ |
| IGBT rms current limit | $i_{u, R M S, \max }$ | 40 A |
| Arm rms current limit | $i_{u, R M S, \max }$ | 10 A |
| Capacitor current ripple | $i_{\text {cap, } R M S, \max }$ | 1.8 A |
| Circulating current amplitude tolerance | $\varepsilon$ | 0.1 A |

### 5.2.4 Comparison of conventional PQ diagram and proposed PQ diagram

In this section, we obtain the PQ diagram including the limit on AC grid current, the limit on DC grid current and the limit on modulation index. The limit on modulation index is obtained by both model.

The use of the SSTI model allows us to get rid of the hypotheses relative to the conventional model (section 5.2.1) one by one. The SSTI is evaluated according to the procedure in section 4.4. Note that for the cases without the circulating current (cases IIa, IIb and III), $\boldsymbol{m}_{d q 0}^{\Sigma}$ is found by trial and error so that $I^{\Sigma} \leq \varepsilon$.

### 5.2.4.1 Case IIa: $R=0, L \neq 0, i_{2}^{\Sigma}=0$ and assumed $v_{m}^{\Delta}$

This case corresponds to $R=0, L \neq 0, i_{2}^{\Sigma}=0$ where the waveform of $v_{m}^{\Delta}$ is assumed (Eq. (3.73)). This is shown in the Fig. 5.1a. The modulation index limit obtained with the conventional model and
the one obtained with the SSTI model overlap. This tends to validate the SSTI model. Several features are worth underlining. First, the PQ diagram is symmetrical about the $y$-axis. This indicates that the maximal active power is the same for both transfer directions. For this case study, it can be reached only if there is a negative reactive power. Second, the PQ diagram is asymmetrical about the x-axis. This indicates that the maximal reactive power is different for the two transfer directions. The limit on the modulation index restricts the positive reactive power. The limit on the AC grid current restricts the negative reactive power. The maximal positive and negative reactive powers are both obtained for $P_{s}=0$.

### 5.2.4.2 Case IIb: $R=0, L \neq 0, i_{2}^{\Sigma}=0$

This case corresponds to $R=0, L \neq 0, i_{2}^{\Sigma}=0$ where the waveform of $v_{m}^{\Delta}$ is not assumed. The obtained result is shown in the Fig. 5.1b. The operating area of the converter is increased in comparison to the conventional model. The symmetry about the y - axis and the assymetry about the x -axis are still valid here. The maximal active power transfer is also obtained only if there is negative reactive power, but in this case the reactive power is lower.

### 5.2.4.3 Case III: $R \neq 0, L \neq 0, i_{2}^{\Sigma}=0$

This case corresponds to $R \neq 0, L \neq 0, i_{2}^{\Sigma}=0$ (section 3.5.7.3). The result is shown in the Fig. 5.1c. The PQ diagram is completely asymmetrical indicating different active and reactive power transfer capabilities in both directions. The positive active power is now limited by the modulation index limitation whereas the negative active power is limited by the AC grid current limitation. The positive reactive power in this case is higher than in the previous case. In addition, it occurs for the negative active power transfer.

### 5.2.4.4 Case IV: $R \neq 0, L \neq 0, i_{2}^{\Sigma} \neq 0$

This case corresponds to $R \neq 0, L \neq 0, i_{2}^{\Sigma} \neq 0$ (section 4.5.2.3) In other words, the Fig. 5.1d shows the case where the circulating current is no longer neglected. The positive reactive power capability of the converter increases but there are no other significant changes within the PQ plane compared to the previous one.

### 5.2.5 Discussion

The DC grid current limitation is represented with the dashed vertical lines in Fig. 5.1. This limitation is independent of the MMC model. For this case study, the limit on the DC grid current does not reduce the OA of the converter.

The SSTI model enables us to study the PQ diagrams of the MMC without the hypotheses traditionally associated to its construction. The inclusion of the arm resistance changes the PQ diagram in the comparison to the conventional one. In this study, the positive active power is lower than the one obtained in the conventional model, but the negative one is higher. This is important because the PQ diagram gives the information on the MMC components design as well as the MMC power capabilities needed by the TSOs.

### 5.2.6 Passive arm elements influence on PQ diagrams

In this section, we use only the SSTI model. Our goal is to study the impact of the arm equivalent resistance and inductance on the PQ diagram. First we consider a MMC without circulating current suppression $\left(I^{\Sigma} \neq 0\right)$. The Fig. 5.2 shows the PQ diagrams for various arm resistances and inductances. As concluded earlier, the arm resistance inclusion in the calculation enables us to show the asymmetry of the PQ diagram as indicated in Fig. 5.2a. An increase of the resistance decreases the operating area of the MMC. This is, in addition of the efficiency, another reason for, the arm resistance to be as low as possible. In the Fig. 5.2 b we see that a diminution of the inductance increases the operating area of the converter. Therefore, a low inductances is desirable from the point of view of the operating area. However, let us not forget that the inductance design is based on several criteria that should be fulfilled as well [23]. Second, we consider a MMC with circulating current suppression ( $I^{\Sigma}=0$ ). Figs. 5.2 and 5.2 d show the influence of the resistance and inductance. We can see that the circulating current suppression has bigger effects on the operating area decrease for lower arm parameter values. This is expected because the higher arm


Figure 5.1 - Comparison of the conventional (+ markers) and SSTI models (o markers)
parameter values are, the lower the circulating current is (see section 5.3). Therefore, the suppression effect is better seen for the lower arm parameters.

### 5.2.7 PQ diagrams with internal constraints

The SSTI model is evaluated according to procedure in section 4.4. Then, we compare the calculated limits to their maximal admissible values in table 5.2 , and plot them for their respective operating points. All the figures correspond to Case IV (section 4.5.2.3).

In this section we inspect the constraints from section 5.2.3. Firstly, let us remind the PQ diagram for the AC grid current, DC grid current and modulation index limits for the Case IV, Fig. 5.3. As discussed, the diagram is not symmetrical and the power transfer is limited either by the modulation index either by the AC grid current limit.

Let us add the IGBT current limit, Fig. 5.4a. The rms arm current is limited to 40 A which is considered as the IGBT maximal current in steady-state. This limit is far from the ones studied up until now and therefore does not affect the operating area. It is worthy to underline here that the violet line does not form a closed surface. This is because, the operating point on the IGBT current limit cannot be obtained with the input parameters $\boldsymbol{m}_{d q 0}^{\Delta \Sigma}$ in the range specified in the evaluation process in section 4.4. And, it would not make sense to go beyond that evaluation range (for example a modulation index amplitude of 1.5 is not of interest. )

This is because the variable sweep is not high enough for this. Going beyond the modulation index amplitude of 1.5 is not of interested (nor theoretical nor in diagram determination).

Let us now add the module capacitor voltage ripple limit (Eqs. (4.45) and (4.50)). The red line in


Figure 5.2 - PQ diagrams for different values of arm resistance and inductance. (a) and (c) show the diagrams for $R=0 \Omega$ (blue squares), $R=1 \Omega$ (black squares) and $R=2 \Omega$ (red squares) for $L=10 \mathrm{mH}$ and with and without circulating current respectively. (b) and (d) show the diagrams for $L=8 \mathrm{mH}$ (blue diamonds), $L=10 \mathrm{mH}$ (black diamonds) and $L=12 \mathrm{mH}$ (red diamonds) for $R=1 \Omega$ and with and without circulating current respectively.


Figure 5.3 - PQ diagram with AC grid current (dotted line), DC grid current (dashed line) and modulation index limits (blue line)

Fig. 5.5 shows the module capacitor voltage ripple set to $60 \%$ of the module capacitor average voltage ${ }^{1}$.

1. The voltage ripple limit is usually taken equal to $10 \%$ of the module capacitor average voltage. But for our laboratory-


Figure 5.4 - PQ diagram with AC grid current (dotted line), DC grid current (dashed line), modulation index limit (blue line) and IGBT current limits (purple line)

The operating area is limited by the voltage ripple limit.


Figure 5.5 - PQ diagram with AC grid current (dotted line), DC grid current (dashed line), modulation index (blue line), IGBT current(purple line) and module capacitor voltage ripple limit (red line)

Let us add the capacitor current limit. In our prototype the module capacitor consists of four capacitors connected in parallel (see section A.2). Each capacitor "sees" a current equal to $i_{\text {cap }} / 4$, whose rms value is limited to 1.8 A . In Fig. 5.6 the limit is shown with green dots. This limit is not as restraining as the module capacitor voltage ripple one However, let us underline that this limitation would be different if other type of capacitor connections are considered. Imagine only one capacitor in the module (and not a parallel connection). With the same current limit, the green line would form a much lower surface, since the capacitor current would not be divided by the parallel branches.

Let us now add the arm current limit (yellow line in Fig. 5.7). This limit is of elliptical form. It slightly reduces the available operating area. The arm current limit taken here is 10 A . This limit is more restraining than the one associated with the IGBT current limit.

[^7]

Figure 5.6 - PQ diagram with AC grid current (dotted line), DC grid current (dashed line), modulation index (blue line), IGBT current (purple line), module capacitor voltage (red line) and module capacitor current ripple limits (green line)


Figure 5.7 - PQ diagram with AC grid current (dotted line), DC grid current (dashed line), modulation index (blue line), IGBT current (purple line), module capacitor voltage (red line), module capacitor current ripple limits (green line) and arm rms current limits (yellow line)

### 5.3 Simulations

### 5.3.1 Test case

Simulations have been performed with Matlab/Simulink SimPowerSystems using a detailed MMC model. To the contrary of the arm average model, the detailed model consists of IGBTs and their antiparallel diodes. It possesses the advantage of reproducing the behavior of the power electronic components and their losses and allowing the use of different module topologies. The simulation parameters are summarized in Table 5.3.

The detailed model overview is shown in Figs. 1.2 and 1.3. The AC side of the converter is connected to a strong AC grid. The DC side of the converter is connected to a strong DC grid.

A cascaded closed-loop control in the $d q$ reference frame is adopted. The inner loop controls the d- and q-axis current while the outer loop controls the active and reactive power. The PI controllers are tuned according to the symmetrical optimum method (see section A.1.2.4.2). The converter is synchronized to the grid using a phase locked loop (PLL). The control scheme includes NLC and CVB (see sections A.1.3 and A.1.4). No circulating current suppression controller has been implemented in order to underline the influence of the passive elements on the circulating current.

In all the simulations, the reference apparent power is $S^{*}=1500 \mathrm{VA}$. The power factor angle varies from $-\pi$ to $\pi$ with a step of $\frac{\pi}{4}$. Each operating point is simulated for four inductances: $L=5 \mathrm{mH}$,
$L=10 \mathrm{mH}, L=15 \mathrm{mH}$ and $L=20 \mathrm{mH}$. All the simulations in this section correspond to Case IV (see chapter 4).

Table 5.3-MMC parameters for the detailed model

| Parameter | Variable | Value |
| :---: | :---: | :---: |
| AC grid line-to-ground voltage amplitude | $V_{s}$ | 60 V |
| AC grid frequency | $f$ | 50 Hz |
| DC grid voltage | $v_{d c}$ | 150 V |
| Apparent power | $S$ | 1500 VA |
| Switching frequency | $f_{s w}$ | 2000 Hz |
| Number of modules per arm | $N$ | 5 |
| Arm equivalent resistance | $R$ | $1 \Omega$ |
| Module capacitance | $C$ | $2240 \mu \mathrm{~F}$ |
| Arm inductance | $L$ | variable |

### 5.3.2 Simulation results

The steady-state operating point of the MMC for various $L$ is plotted in the PQ plane Fig. 5.8. In the Figs. 5.8 a and 5.8 b , the measured power for all operating points attained the reference value. However, in Fig. 5.8 c the measured power for $\varphi_{i}^{\Delta}=\frac{\pi}{4}$ and $\varphi_{i}^{\Delta}=\frac{\pi}{2}$ did not reach the reference. In Fig. 5.8 d this is even more explicit.

In section 5.2.6, we concluded that an increase of the arm inductance can reduce the operating area by modifying the modulation index limit. This is what is happening in Figs. Fig. 5.8c and Fig. 5.8d.

To prove this, we analyze four operating points:

1. OP1: $P_{s}^{*}=-1500 \mathrm{~W}$ and $Q_{s}^{*}=0 \operatorname{VAr}$
2. OP2: $P_{s}^{*}=0 \mathrm{~W}$ and $Q_{s}^{*}=-1500 \mathrm{VAr}$
3. OP3: $P_{s}^{*}=1500 \mathrm{~W}$ and $Q_{s}^{*}=0 \mathrm{VAr}$
4. OP4: $P_{s}^{*}=0 \mathrm{~W}$ and $Q_{s}^{*}=1500 \mathrm{VAr}$

## $\mathrm{OP} 1: P_{s}^{*}=-1500 \mathrm{~W}$ and $Q_{s}^{*}=0 \mathrm{VAr}$

The results are shown in Fig. 5.9. The active and reactive power references are reached for all $L$. This is in agreement with Fig. 5.8.

The DC grid current $i_{d c}$ is the same for the first three cases. For the fourth case we see a ripple of 6 A. The AC grid current $i_{a b c}^{\Delta}$ reaches the amplitude of $I^{\Delta}=16.5 \mathrm{~A}$ for all $L$.

The circulating current $i_{2}^{\Sigma}$ reduces with the inductance increase (except for $L=20 \mathrm{mH}$ ). The same circulating current amplitude for the first three $L$ is calculated using Eq. (3.60): 2.8, 1.2 and 0.8 A .

The average capacitor voltage $V_{c a p}$ decreases with the inductance increase (green line for $v_{c a p}$ diagrams). For the inductances $L=5 \mathrm{mH}, L=10 \mathrm{mH}$ and $L=15 \mathrm{mH}$, the simulation result yields 31, 30.6 and 30 V . This same results were calculated using the SSTI in section 4.5.1. The voltage ripple $\Delta v_{c a p}$ for the same three inductances are $12.5,11$ and 11 V . The same voltage ripples were obtained in section 4.5.2.

The capacitor current ripple for all $L$ is lower than the limit in Table 5.2.

## OP2: $P_{s}^{*}=0 \mathrm{~W}$ and $Q_{s}^{*}=-1500 \mathrm{VAr}$

The results are shown in Fig. 5.10. The active and reactive power references are reached for all $L$. This is in agreement with Fig. 5.8.

The DC grid current $i_{d c}$ is the same for all $L$ (except for $L=20 \mathrm{mH}$ ) and is different from zero. This is in agreement with the dc grid current generic expression in Eq. (3.47). The reason for this is that $\varphi_{m}^{D}$ elta $\neq 0$. The AC grid current $i_{a b c}^{\Delta}$ reaches the amplitude of $I^{\Delta}=16.5 \mathrm{~A}$ for all $L$.

The circulating current $i_{2}^{\Sigma}$ reduces with the inductance increase. The same circulating current amplitude is calculated using Eq. (3.60): 4.3, 1.3, 0.5 and 0.3 A .


Figure 5.8 - PQ diagrams for various $L$. The cross markers are the power references, the circle markers are the measured power, and the black line is the modulation index limit.

The average capacitor voltage $V_{c a p}$ decreases with the inductance increase (green line for $v_{c a p}$ diagrams). The simulation results are $32.7,31.7,31.2$ and 30.2 V . The voltage ripple $\Delta v_{\text {cap }}$ decreases with inductance increase as well: $15,12.8,12,11.5 \mathrm{~V}$. The same voltage ripples were obtained in section 4.5.2.

The capacitor current ripple for all $L$ is lower than the limit in Table 5.2.
OP3: $P_{s}^{*}=1500 \mathrm{~W}$ and $Q_{s}^{*}=0 \mathrm{VAr}$
The results are shown in Fig. 5.11. The active and reactive power references are reached for all $L$. This is in agreement with Fig. 5.8.

The DC grid current $i_{d c}$ is the same for all $L$. The AC grid current $i_{a b c}^{\Delta}$ reaches the amplitude of $I^{\Delta}=16.5 \mathrm{~A}$ for all $L$.

The circulating current $i_{2}^{\Sigma}$ reduces with the inductance increase. The same circulating current amplitude is calculated using Eq. (3.60): 3.6, 1.5, 1.6 and 0.85 A.

The average capacitor voltage $V_{c a p}$ decreases with the inductance increase (green line for $v_{c a p}$ diagrams). The simulation results are $28.7,27.7,27$ and 26.4 V . This is in agreement with Figs. 4.7-4.7. The voltage ripple $\Delta v_{c a p}$ decreases with inductance increase as well: $12,10,9$ and 8 V . The same voltage ripples were obtained in section 4.5.2.

The capacitor current ripple for all $L$ is lower than the limit in Table 5.2.

## OP4: $P_{s}^{*}=0 \mathrm{~W}$ and $Q_{s}^{*}=1500 \mathrm{VAr}$

The results are shown in Fig. 5.12. The active and reactive power references are reached for $L=5 \mathrm{mH}$ and $L=10 \mathrm{mH}$. For $L=15 \mathrm{mH}$ and $L=20 \mathrm{mH}$, the power reference is not reached. The OP for these
two inductances is therefore out of the operating area of the converter. In other words, the modulation index amplitude is reached ( $M^{\Delta}$ in Fig. 5.12 is higher than 1 for these two inductances). This is in agreement with Fig. 5.8.

The DC grid current $i_{d c}$ is the same for $L=5 \mathrm{mH}$ and $L=10 \mathrm{mH}$. We observe the same behavior as for OP2. The AC grid current $i_{a b c}^{\Delta}$ reaches the amplitude of $I^{\Delta}=16.5 \mathrm{~A}$ for $L=5 \mathrm{mH}$ and $L=10 \mathrm{mH}$.

The circulating current $i_{2}^{\Sigma}$ reduces with the inductance increase for $L=5 \mathrm{mH}$ and $L=10 \mathrm{mH}$. The same circulating current amplitude is calculated using Eq. (3.60): 4.1 and 2.3 A .

The average capacitor voltage $V_{c a p}$ decreases with the inductance increase (green line for $v_{c a p}$ diagrams) for $L=5 \mathrm{mH}$ and $L=10 \mathrm{mH}$. The simulation results are 27 and 26.4 V . The voltage ripple $\Delta v_{\text {cap }}$ decreases with inductance increase as well: 14.5 and 13.8 V . The same voltage ripples were obtained in section 4.5.2.

The capacitor current ripple for $L=5 \mathrm{mH}$ and $L=10 \mathrm{mH}$ is lower than the limit in Table 5.2.

### 5.4 Discussion

We developed the PQ diagram of an MMC using the SSTI model. We elaborated both "external" limits and "internal" limits. For the case study in this thesis, the operating area is delimited by the limit on the modulation index, the limit on the module capacitor voltage ripple and the limit on the arm current. We underline here that this study is case-dependent and that a different case study might result in drastically different results. Note that such a PQ diagram has not, to our knowledge, been reported in the literature.

We remind that, for a half-bridge MMC, the modulation index limit cannot be exceeded. The other limits could be exceeded, but only temporarily and with the risk of excessive losses and/or high voltage ripple.

The detailed MMC model simulations were performed for various operating points. The simulation results have confirmed the theoretical tendencies anticipated by the steady state time invariant model.


Figure 5.9 - Simulation results for OP1 $\left(P_{s}^{*}=-1500 \mathrm{~W}\right.$ and $\left.Q_{s}^{*}=0 \mathrm{VAr}\right)$


Figure 5.10 - Simulation results for OP2 $\left(P_{s}^{*}=0 \mathrm{~W}\right.$ and $\left.Q_{s}^{*}=-1500 \mathrm{VAr}\right)$


Figure 5.11 - Simulation results for OP3 $\left(P_{s}^{*}=1500 \mathrm{~W}\right.$ and $\left.Q_{s}^{*}=0 \mathrm{VAr}\right)$


Figure 5.12 - Simulation results for OP4 $\left(P_{s}^{*}=0 \mathrm{~W}\right.$ and $\left.Q_{s}^{*}=1500 \mathrm{VAr}\right)$

## Chapter 6

## Conclusions

The modular multilevel converter has been proposed as a suitable solution for the HVDC grids due to its many advantages over 2-level voltage source converters. Considering the need to better understand the behavior of this converter in order to size it correctly, the goal of this thesis was to clarify the influence of the arm equivalent resistance and arm inductance on the module capacitor voltage and on the circulating current. To do so, we extended several models from the literature, compared them and proposed to visualize some results using a PQ diagram.

First, a complete demonstration of the $\Delta \Sigma$ MMC model in $a b c$ and $d q 0$ frames was elaborated. The various hypotheses required to derive these models were clarified.

Second, starting from the $\Delta \Sigma$ MMC model in $a b c$ frame, an extended integral-based (IB) model was obtained. It enabled us to find a generalized circulating current expression. This expression made it possible to quantify the reduction of the circulating current amplitude as the arm inductance increases. Moreover, a generalized DC grid current expression was derived. It allowed us to explain why the DC grid current is always different from zero, even for a pure reactive power transfer. And a generalized module capacitor voltage ripple expression was developed. In comparison to the expression of the literature, it permits to study the influence of the passive arm elements on the capacitor voltage. But the integral-based method has several limitations were underlined.

Third, starting from the $\Delta \Sigma$ MMC model in $d q 0$ frame, the steady-state time invariant (SSTI) model of a modular multilevel converter was obtained. It allowed us, for the first time, to estimate the module capacitor average voltage as a function of the operating point. For example, we observed that the module capacitor average voltage dropped faster with power for high values of the arm passive elements. In addition, the SSTI allowed us to study the module capacitor voltage ripple. We noticed an asymmetrical behavior of the voltage ripple as a function of the active power transfer. We determined that this asymmetry is due to the arm equivalent resistance. Furthermore, we studied the voltage ripple as a function of the arm inductance value and found the expected resonance behavior.

Fourth, the PQ diagram of the MMC was built using the SSTI model. This new method of construction allowed us to get rid of the simplifications that are usually made to plot the PQ diagram of the MMC. In particular, it proved possible to show the influence of the passive arm elements on the modulation index limit. We concluded that the operating area of the MMC reduces when the arm inductance increases. In addition, we represented some of the internal limits of the MMC within the PQ diagram: IGBT current limit, arm RMS current limit, module capacitor voltage ripple and module capacitor current ripple. Such PQ diagram is useful, as it permits to explain behaviors obtained with detailed simulation models.

This research could be useful to converter manufacturers since the presented results could be used at the design stage to optimize the sizing of the components of the MMC considering its operating area, and to asses the impact of different parameters on the MMC performance.

## Appendices

## Appendix A

## Converter control and prototype control implementation

## A. 1 Control of the Modular Multilevel Converter

## A.1.1 Plant



Figure A. 1 - MMC control plants and controllers

## A.1.1.1 Decoupling control

Apply the transformation in Eq. (2.42) to Eq. (2.81) and neglect the zero sequence component,

$$
\frac{d}{d t}\left[\begin{array}{l}
i_{q}^{\Delta}(t)  \tag{A.1}\\
i_{d}^{\Delta}(t)
\end{array}\right]=\left[\begin{array}{cc}
-\frac{R}{L} & -\omega \\
\omega & -\frac{R}{L}
\end{array}\right]\left[\begin{array}{l}
i_{q}^{\Delta}(t) \\
i_{d}^{\Delta}(t)
\end{array}\right]+\frac{2}{L}\left[\begin{array}{l}
v_{m q}^{\Delta}(t)-v_{s q}(t) \\
v_{m d}^{\Delta}(t)-v_{s d}(t)
\end{array}\right]
$$

Note the cross-coupling terms $-\omega i_{d}^{\Delta}(t)$ and $\omega i_{q}^{\Delta}(t)$ between the two axis. A decoupling can be obtained by defining $\Delta v_{q}(t)$ and $\Delta v_{d}(t)$ as $[14,15]$,

$$
\begin{align*}
& \Delta v_{q}(t) \triangleq v_{m q}^{\Delta}(t)-v_{s q}(t)-\omega \frac{L}{2} i_{d}^{\Delta}(t)  \tag{A.2}\\
& \Delta v_{d}(t) \triangleq v_{m d}^{\Delta}(t)-v_{s d}(t)+\omega \frac{L}{2} i_{q}^{\Delta}(t) \tag{A.3}
\end{align*}
$$

Inserting Eqs.(A.2) and (A.3) into Eq.(A.1),

$$
\frac{d}{d t}\left[\begin{array}{l}
i_{q}^{\Delta}(t)  \tag{A.4}\\
i_{d}^{\Delta}(t)
\end{array}\right]=\left[\begin{array}{cc}
-\frac{R}{L} & 0 \\
0 & -\frac{R}{L}
\end{array}\right]\left[\begin{array}{l}
i_{q}^{\Delta}(t) \\
i_{d}^{\Delta}(t)
\end{array}\right]+\frac{2}{L}\left[\begin{array}{c}
\Delta v_{q}(t) \\
\Delta v_{d}(t)
\end{array}\right]
$$

Then $i_{q}^{\Delta}$ and $i_{d}^{\Delta}$ respond to $\Delta v_{q}$ and $\Delta v_{d}$ respectively, through a simple first-order transfer function $F_{r}(s)$,

$$
\begin{equation*}
F_{r}(s)=\frac{i_{q}^{\Delta}}{\Delta v_{q}}=\frac{i_{d}^{\Delta}}{\Delta v_{d}}=\frac{2}{R} \frac{1}{1+\tau_{r} s} \tag{A.5}
\end{equation*}
$$

where $\tau_{r}=L / R$.

## A.1.1.2 DC-link equations

According to $[14,15], v_{d c}$ responds to $i_{d}$ through a simple first-order transfer function $G_{v}(s)$,

$$
\begin{equation*}
G_{v}(s)=\frac{v_{d c}}{i_{d}^{\Delta}}=\frac{3}{4 C_{d c}} \frac{1}{s} \tag{A.6}
\end{equation*}
$$

where $C_{d c}$ is defined as $C_{d c}=\frac{6 C}{N}$, [48, Eq. (2.34)], [65].
From a control point of view, the MMC is taken into account by means of a first order transfer function $G_{d}(s)[11,12]$,

$$
\begin{equation*}
G_{d}(s)=\frac{v_{m d}^{\Delta}}{v_{m d}^{\Delta *}}=\frac{v_{m q}^{\Delta}}{v_{m q}^{\Delta *}}=\frac{1}{1+T_{a} s} \tag{A.7}
\end{equation*}
$$

where $v_{m d}^{\Delta}$ is the input control voltage, and $v_{m d}^{\Delta *}$ is the output voltage. $T_{a}$ is the time delay due to the computation and generation time. To simplify, we take $T_{a}=1 / f_{C V B}$ where $f_{C V B}$ is the frequency of Capacitor Voltage Balancing control, see section A.1.4.

## A.1.2 Controller

## A.1.2.1 Controller design

The inner controller design is done in accordance with $[6,14,15]$ and Fig. 1.3.
A vector-control approach done in the grid voltage-oriented dq reference frame is adopted. Following Eqs.(2.105) and (2.106), the d-axis current contributes to the instantaneous active power and the q-axis current to the instantaneous reactive power. The controller has a cascaded structure. A fast inner current loop controls the converter d- and q-axis currents. An slower outer loop controls the DC-link voltage to its reference value by the d-axis current, and the reactive power by the q-axis current.

A decoupled control is obtained by defining the feedback loops and controller $G_{P I_{i}}(s)$ as,

$$
\begin{align*}
& v_{m q}^{\Delta *}=G_{P I_{i}}(s)\left(i_{q}^{\Delta *}-i_{q}^{\Delta}\right)+\omega \frac{L}{2} i_{d}^{\Delta}+v_{s q}  \tag{A.8}\\
& v_{m d}^{\Delta *}=G_{P I_{i}}(s)\left(i_{d}^{\Delta *}-i_{d}^{\Delta}\right)-\omega \frac{L}{2} i_{q}^{\Delta}+v_{s d} \tag{A.9}
\end{align*}
$$

The inner open loop transfer function is,

$$
\begin{equation*}
H_{O L, r}(s)=G_{d}(s) F_{r}(s)=\frac{\frac{2}{R}}{\left(1+T_{a} s\right)\left(1+\tau_{r} s\right)} \tag{A.10}
\end{equation*}
$$

For the design of the outer controller, the grid-side inner closed loop transfer function is approximated by a first order transfer function $[17,14,15]$,

$$
\left\{\begin{array}{l}
H_{e q, r}(s)=\frac{i_{q}^{\Delta}}{i_{q}^{\Delta *}}=\frac{i_{d}^{\Delta}}{i_{d}^{\Delta *}}=\frac{1}{1+T_{e q, r} s}  \tag{A.11}\\
T_{e q, r}=t_{s, 10} / 2.3
\end{array}\right.
$$

where $t_{s, 10}$ is the $10 \%$ settling time of the inner closed loop transfer function. The control is obtained by defining the feedback loops and controller $G_{P I_{v}}(s)$ as,

$$
\begin{equation*}
i_{d}^{*}=G_{P I_{v}}(s)\left(v_{d c}^{*}-v_{d c}\right) \tag{A.12}
\end{equation*}
$$

The grid-side d-axis outer open loop transfer function is,

$$
\begin{equation*}
H_{O L, r}(s)=G_{v}(s) H_{e q, r}(s)=\frac{\frac{3}{4 C_{d c}}}{\left(1+T_{e q, r} s\right) s} \tag{A.13}
\end{equation*}
$$

Table A. 1 - Controller tuning

|  | $\omega_{c}$ | $a$ | $K_{p}$ | $T_{i}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | $\left[\mathrm{rad} . \mathrm{s}^{-1}\right]$ | - | $[\mathrm{si}]$ | $[\mathrm{s}]$ |
| $G_{P I_{i}}$ | $\frac{2 \pi f_{C V B}}{20}$ | $\frac{1}{\omega_{c} T_{a}}$ | $\frac{\tau_{r} R_{r}}{a T_{a}}$ | $a^{2} T_{a}$ |
| $G_{P I_{v}}$ | - | 4 | $\frac{4 C_{d c}}{3} \frac{T_{e}}{a T_{e q, r}}$ | $a^{2} T_{e q, r}$ |

## A.1.2.2 $V_{d c}-Q$ control mode

In this mode of operation, the MMC maintains a constant DC side voltage and supply the desired reactive power at the AC side.

## A.1.2.3 $P-Q$ control mode

In this control mode, the active and reactive power transfer is controlled.
The power references are determined following the Eq. (2.105) and (2.106). The inner loop is similar to $v_{d c}-Q$ control mode.

## A.1.2.4 PI controller tuning

The PI controllers are tuned according to [14, 15].
A.1.2.4.1 Symmetrical optimum method Consider an open loop transfer function of the form,

$$
\begin{equation*}
H_{O L}(s)=\frac{K}{\left(1+T_{\alpha} s\right) s} \tag{A.14}
\end{equation*}
$$

The controller tuning can be done using the symmetrical optimum method [14, 15]. The method has the advantage of maximizing the phase margin and therefore the system can withstand more delay. Besides it has good disturbance rejection compared to modulus optimum method and it is possible to specify the open loop crossover frequency $[17,18]$. The parameters of the PI controller,

$$
\begin{equation*}
G_{P I}(s)=K_{p} \frac{1+T_{i} s}{T_{i} s} \tag{A.15}
\end{equation*}
$$

can be found with [18],

$$
\begin{equation*}
\left\{a, T_{i}, K_{p}\right\}=\left\{\frac{1}{\omega_{c} T_{\alpha}}, a^{2} T_{\alpha}, \frac{1}{a T_{\alpha} K}\right\} \tag{A.16}
\end{equation*}
$$

where $\omega_{c}$ is the open loop crossover frequency, and $a$ is a design parameter in the range $\{2,4\}$. A higher value of $a$ leads to better damping but slower response.

Consider an open loop transfer function of the form,

$$
\begin{equation*}
H_{O L}(s)=\frac{K}{\left(1+T_{\alpha} s\right)\left(1+T_{\beta} s\right)} \tag{A.17}
\end{equation*}
$$

with $T_{\beta} \gg T_{\alpha}$. The controller tuning can be done using the symmetrical optimum method around crossover frequency by approximating $\frac{1}{1+T_{\beta} s} \approx \frac{1}{T_{\beta} s}[17]$.
A.1.2.4.2 Controller tuning goals As the inner and outer open loop transfer functions of the system are all of the form of Eqs.(A.14) or (A.17), all the controllers can be tuned using the symmetrical optimum method. The tuning goals are:
(i) inner loop closed loop bandwidth one order of magnitude smaller than the frequency of the CVB.
(ii) slow outer loops in comparison to the inner loops.

To achieve (i), we use Eq.(A.16) to calculate the minimal value of $a$ that meets the bandwidth requirement for the inner loops. To achieve (ii) we set $a=4$ leading to the slowest response. Analytical formulas for the systematic tuning of the PI controllers are summarized in Table A.1.


Figure A. 2 - MMC modulation techniques

## A.1.3 Nearest Level Control (NLC)

The different MMC modulation techniques are summarized in Fig. A.2. We adopt here the Nearest Level Control (NLC). The three phases are controlled separately. The idea is to transform $m_{u}(t)$ and $m_{l}(t)$ into the corresponding staircase waveforms, $\delta_{u}(t)$ and $\delta_{l}(t)$, following the procedure in Fig. A.3. The output of the NLC is later used by the Capacitor Voltage Balancing algorithm. The basic NLC is shown in the Fig. A.3.
The staircase signals are obtained as,

$$
\begin{align*}
\delta_{u}^{*}(t) & =\operatorname{round}\left(N m_{u}(t)\right)  \tag{A.18}\\
\delta_{l}^{*}(t) & =\operatorname{round}\left(N m_{l}(t)\right) \tag{A.19}
\end{align*}
$$

The meaning of the staircase signal can be seen in section A.1.4.


Figure A. 3 - Basic principle of Nearest Level Control


Figure A. 4 - NLC example for $\mathrm{N}=5$ : (a) sine wave before the NLC, (b) sine wave multiplied by the number of the modules and (c) staircase signal output $\delta$

## A.1.4 Capacitor Voltage Balancing

For a given number of modules to be ON in an arm, there are different combinations. The role of the CVB is to select the best combination based on the capacitor voltages and arm current direction. The other modules are OFF. Note that if the $i_{u}\left(i_{l}\right)$ is positive, the ON state of the modules corresponds to
charging of the module capacitor whereas if the $i_{u}\left(i_{l}\right)$ is negative, the ON state will lead to discharging of the module capacitor, according to Fig. 1.5.


Figure A. 5 - Capacitor Voltage Balancing algorithm for upper arm


Figure A. 6 - Detailed MMC control scheme

## A. 2 Description of the prototype modules

The modules of the prototype are full-bridge modules. Nevertheless, in this thesis, they are always modeled/controlled as half-bridge modules.
The module consists of:

- one IGBT H-bridge pack module SKiiP 16GH066V1 from Semikron [35]. Its nominal collector current is $I_{C, n o m}=50 \mathrm{~A}$ and maximal collector-emitter voltage is $V_{C E S}=600 \mathrm{~V}$.
- two IGBT drivers $2 \mathrm{SC} 0108 \mathrm{~T} 2 \mathrm{H} 0-17$ from Power Integrations. This is a two-channel driver with isolated DC/DC converter, short-circuit protection, soft shut down, etc. Each channel is electrically isolated from the primary side and the other channel [36]. Each driver has two status output (SO) signals, signalizing the proper operation of the driver. If any problem is detected, the driver changes the status outputs from 1 to 0 .
- one main module board with protection and acquisition circuits (LEM voltage sensor LV25-P [37]),
- one capacitor board with module capacitors and a choke capacitor and,
- one heatsink.


Figure A. 7 - Photo of the one module of the prototype

## Module concept

The driver boards of the module comprise the IGBT gate driver with its interface circuits. Each driver controls one half-bridge of the IGBT module. The primary side interface circuit adapts the signals received from the FPGA. The secondary driver side has two identical channels. Each of them has an interface circuit that adapts the controls signals for the IGBT gates. The secondary side driver channels are controlled dependently (the output channels are complementary). The dead time is generated automatically by the driver in this case. This is called "half-bridge mode".

The module contains four capacitor connected in parallel, forming the "module capacitor". The voltage sensor provides the capacitor voltage measurement $v_{c a p}$ and this value is dispatched towards the arm FPGA.

The main module board contains the circuits for signalization and protection.

Signalization The set of four LEDs give information on the converter operation. The red LED indicates IGBT module overheat, the yellow LED signals IGBT overvoltage, while the blue and green LEDs give information on the supply voltage state.

If the module overheats, the red LED turns off and the overheat signal $O H$ drops to zero. Similarly, in the case of overvoltage the overvoltage signal $O V$ drops to zero and the yellow LED turns off. The blue and green LEDs turn off if the supply voltage is lower than the value specified in driver datasheet. This facilitates the problem diagnostics.

Protection In addition to the protection included in the drivers, two additional protection have been added to the module: an overheat protection using the IGBT pack temperature sensor and an overvoltage protection using the module capacitor voltage sensor. For the overheat the voltage of the internal IGBT resistance related to its temperature is compared to a constant voltage. If a certain value is attained, the OH signal drops to zero signifying the overheat. For the overvoltage, the capacitor voltage is compared to a fixed value. If the capacitor voltage exceeds this value, the $O V$ signal drops to zero.

The signals $S O 1, S O 2, S O 3, S O 4, O H$ and $O V$ are dispatched towards arm FPGA through the module signal connector. These signals belong to error signals $e$.

## A. 3 Prototype control description

The control is implemented in two levels, Fig. A.8:

- "Low level" control mainly in charge of CVB,
- "High level" control mainly in charge of NLC.


Figure A. 8 - Prototype control overview

The control is employed via the DE0-Nano FPGA Board, [38], Fig. A.9. This controller has two external GPIO headers and is supplied via 5 V USB mini-AB port. It implements Cyclone IV device. The FPGA is programmed by the Quartus Prime software from Intel [39].

## A.3.1 "Low level" control

The operation of the arm FPGA is illustrated in the Fig. A.10. The arm FPGA has the following tasks:

- Verify the module status (verification of the module errors such as IGBT overheat, IGBT overvoltage, driver status),
- Treatment of the arm current measurements,
- Treatment of the module capacitor voltage,
- Module capacitor voltage sorting
- Control signal generation and
- Dispatch of the generated signals towards the respective modules
- Reception of the $\delta_{u}$ or $\delta_{l}$ signals from the master FPGA

The arm operation is as follows. Firstly, the FPGA reads the outputs of the modules. Each output contains the capacitor voltage $v_{c a p}$ and error signals $e$. Next to that the FPGA reads the arm current


Figure A. 9 - Arm FPGA controller [38]
measurement provided by the current sensor. The current measurement is put through a gain/offset circuit to assimilate signal to the FPGA characteristics. Secondly, these measurements are converted to digital values using a 12 -bit ADC integrated in the FPGA. The variables $v_{c a p}[k]$ and $i_{u(l)}[k]$ are obtained.

Thirdly, the CVB is executed. The CVB reads the number of modules to be inserted $\delta$ provided by the master FPGA. According to the $i_{u(l)}[k]$ direction and $\delta$, the code decides on the exact modules to be inserted at the given moment. The CVB code compares the five capacitor voltages and provides an output as explained in A.1.4.

Finally, The FPGA outputs five signals $I N$ that are dispatched to the module drivers. Note that in the end it is the driver who generates the IGBT gate signals, not the FPGA.

The error signals are put through the error verification block. The arm FPGA reads the state of the overheat, overvoltage and driver status outputs. If the errors are not detected, the enable signal is sent to CVB block, enabling its execution. As a measure of safety, the error signals are also sent to master FPGA. If any of errors are detected, the CVB execution is stopped and all of the IGBTs are blocked.


Figure A. 10 - Arm FPGA overview [69]

## A.3.2 "High level" control

The operation of the master FPGA is illustrated in the Fig. A.11. It has the following tasks:

- Modulation wave generation
- Implementation of the round function
- Creation of two signals ( $\delta_{u}$ and $\delta_{l}$ )
- Dispatch of these two signals towards the respective arm FPGAs.

The master FPGA operation is as follows. Firstly, the sinusoidal modulation index wave is generated using a lookup table in 12-bits. Secondly, the round function is implemented. The output of the round block is the staircase signal of 6 levels, $\delta_{l}$. The signal $\delta_{u}$ is generated as $\delta_{u}=1-\delta_{l}$. This ensures a $180^{\circ}$ phase shift and that $\delta_{u}+\delta_{l}=5$ at all times. These two signals are then dispatched towards the arm FPGAs.

The master FPGA also receives the enable signal from the arm FPGAs. If any errors is met, the dispatching of $\delta_{u}$ and $\delta_{l}$ signals are stopped immediately.


Figure A. 11 - Master FPGA overview [69]

## Appendix B

## Hadamard's product

Definition Let $A$ and $B$ be $m \times n$ matrices. The Hadamard product of $A$ and $B$ is defined by,

$$
\begin{equation*}
[A \odot B]_{i j} \triangleq[A]_{i j}[B]_{i j}, \quad i \in[1, m] \wedge j \in[1, n] . \tag{B.1}
\end{equation*}
$$

The Hadamard product is simply element-wise multiplication. Note that matrices A and B need to be the same size, but not necessarily square. In order to avoid confusion with classical matrix multiplication, we use $\odot$ for the Hadamard product.

Theorem 1 Let $A$ and $B$ be $m \times n$ matrices. Then,

$$
\begin{equation*}
A \odot B=B \odot A \tag{B.2}
\end{equation*}
$$

Theorem 2 Suppose $\alpha \in \mathbb{C}$, and $A, B$ and $C$ are $m x$ matrices. Then,

$$
\begin{equation*}
C \odot(A+B)=C \odot A+C \odot B \tag{B.3}
\end{equation*}
$$

Furthermore,

$$
\begin{equation*}
\alpha(A \odot B)=(\alpha A) \odot B=A \odot(\alpha B) . \tag{B.4}
\end{equation*}
$$

For further information on Hadamard product and its properties, refer to [45].

## Appendix C

## Park transformation

## C. 1 Definition

The used Park transformation, $\boldsymbol{P}_{n \omega}$, is [16],

$$
\begin{gather*}
\boldsymbol{f}_{d q 0}=\boldsymbol{P}_{n \omega}(\theta) \boldsymbol{f}_{a b c}  \tag{C.1}\\
\boldsymbol{P}_{n \omega}(\theta)=\frac{2}{3}\left[\begin{array}{ccc}
\sin (\theta) & \sin \left(\theta-\frac{2 \pi}{3}\right) & \sin \left(\theta+\frac{2 \pi}{3}\right) \\
\cos (\theta) & \cos \left(\theta-\frac{2 \pi}{3}\right) & \cos \left(\theta+\frac{2 \pi}{3}\right) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{array}\right]  \tag{C.2}\\
n \omega=\frac{d \theta}{d t} \tag{C.3}
\end{gather*}
$$

where $n \in \mathbb{Z}$. The inverse transformation is,

$$
\begin{gather*}
\boldsymbol{f}_{a b c}=\boldsymbol{P}_{n \omega}^{-1}(\theta) \boldsymbol{f}_{d q 0}  \tag{C.4}\\
\boldsymbol{P}_{n \omega}^{-1}(\theta)=\left[\begin{array}{ccc}
\sin (\theta) & \cos (\theta) & 1 \\
\sin \left(\theta-\frac{2 \pi}{3}\right) & \cos \left(\theta-\frac{2 \pi}{3}\right) & 1 \\
\sin \left(\theta+\frac{2 \pi}{3}\right) & \cos \left(\theta+\frac{2 \pi}{3}\right) & 1
\end{array}\right] \tag{C.5}
\end{gather*}
$$

Here, the q -axis is ahead of the d -axis and the angle $\theta$ is the angle between the phase $a$ and d-axis, Fig.C.1.


Figure C. 1 - Park transformation

[^8]\[

$$
\begin{align*}
& \boldsymbol{J}_{n \omega}=\boldsymbol{P}_{n \omega} \frac{d \boldsymbol{P}_{n \omega}^{-1}}{d t}=\left[\begin{array}{ccc}
0 & -n \omega & 0 \\
n \omega & 0 & 0 \\
0 & 0 & 0
\end{array}\right]  \tag{C.6}\\
& \left(\boldsymbol{P}_{n \omega}^{-1} \boldsymbol{f}\right)^{T} \boldsymbol{P}_{n \omega}^{-1}=\left(\left[\begin{array}{ccc}
\frac{3}{2} & 0 & 0 \\
0 & \frac{3}{2} & 0 \\
0 & 0 & 3
\end{array}\right] \boldsymbol{f}\right)^{T} \tag{C.7}
\end{align*}
$$
\]

where $\boldsymbol{f}$ is a matrix of 3 x 1 dimension.

## C. 2 Space vector

Let us assume that $f_{a b c}$ is a 3 -phase system,

$$
\boldsymbol{f}_{a b c}=\left[\begin{array}{l}
f_{a}  \tag{C.8}\\
f_{b} \\
f_{c}
\end{array}\right]=\left[\begin{array}{c}
F \cos \left(\omega t+\varphi_{f}\right) \\
F \cos \left(\omega t+\varphi_{f}-\frac{2 \pi}{3}\right) \\
F \cos \left(\omega t+\varphi_{f}-\frac{4 \pi}{3}\right)
\end{array}\right]
$$

Applying the Park transformation on $\boldsymbol{f}_{a b c}$,

$$
\begin{gather*}
\boldsymbol{f}_{d q 0}=\left[\begin{array}{c}
f_{d} \\
f_{q} \\
f_{0}
\end{array}\right]=\boldsymbol{P}_{\omega}\left[\begin{array}{c}
F \cos \left(\omega t+\varphi_{f}\right) \\
F \cos \left(\omega t+\varphi_{f}-\frac{2 \pi}{3}\right) \\
F \cos \left(\omega t+\varphi_{f}-\frac{4 \pi}{3}\right)
\end{array}\right]  \tag{C.9}\\
{\left[\begin{array}{c}
f_{d} \\
f_{q} \\
f_{0}
\end{array}\right]=\left[\begin{array}{c}
-F \sin \left(\varphi_{f}\right) \\
F \cos \left(\varphi_{f}\right) \\
0
\end{array}\right]} \tag{C.10}
\end{gather*}
$$

The space vector is defined as,

$$
\begin{equation*}
\bar{f} \triangleq F e^{j \varphi_{f}} \tag{C.11}
\end{equation*}
$$

Using the Euler identity and the Eq. (C.10), we develop the following expression,

$$
\begin{align*}
\bar{f} & =F e^{j\left(\varphi_{f}\right)} \\
& =F\left(\cos \varphi_{f}+j \sin \varphi_{f}\right)  \tag{C.12}\\
& =f_{q}-j f_{d}
\end{align*}
$$

## Appendix D

## SSTI MMC $\Delta \Sigma$ model adapted for Case I

In the Case I, we have the following hypotheses [20],

$$
\begin{gather*}
R \stackrel{h y p}{=} 0  \tag{D.1}\\
L \stackrel{h y p}{=} 0  \tag{D.2}\\
\boldsymbol{v}_{m, a b c}^{\Delta} \stackrel{h y p}{=} \boldsymbol{m}_{a b c}^{\Delta} \frac{v_{d c}}{2}  \tag{D.3}\\
\boldsymbol{v}_{m, a b c}^{\Sigma} \stackrel{h y p}{=} \boldsymbol{m}_{a b c}^{\Sigma} \frac{v_{d c}}{2}  \tag{D.4}\\
\boldsymbol{i}_{a b c}^{\Sigma} \stackrel{h y p}{=}\left[\begin{array}{c}
I_{02}^{\Sigma} \\
I_{02}^{\Sigma} \\
I_{02}^{\Sigma}
\end{array}\right] \tag{D.5}
\end{gather*}
$$

Apply the Park transformation,

$$
\begin{align*}
\boldsymbol{v}_{m, d q 0}^{\Delta} & =\boldsymbol{m}_{d q 0}^{\Delta} \frac{v_{d c}}{2}  \tag{D.6}\\
\boldsymbol{v}_{m, d q 0}^{\Sigma} & =\boldsymbol{m}_{d q 0}^{\Sigma} \frac{v_{d c}}{2}  \tag{D.7}\\
\boldsymbol{i}_{d q 0}^{\Sigma} & =\left[\begin{array}{c}
0 \\
0 \\
I_{02}^{\Sigma}
\end{array}\right] \tag{D.8}
\end{align*}
$$

With these hypotheses, the SSTI model cannot be applied directly since the matrix $\boldsymbol{A}$ in Eq. (4.30) is not invertible. Therefore we need to go back to the MMC $\Delta \Sigma$ model in $d q 0$ frame (section 4.3) and insert these hypotheses into Eqs. (4.24)-(4.29),

$$
\begin{align*}
& \boldsymbol{v}_{m, d q 0}^{\Delta}=\boldsymbol{m}_{d q 0}^{\Delta} \frac{v_{d c}}{2}  \tag{D.9}\\
& \boldsymbol{v}_{m, d q 0}^{\Sigma}=\boldsymbol{m}_{d q 0}^{\Sigma} \frac{v_{d c}}{2}  \tag{D.10}\\
& 0=\boldsymbol{v}_{m, d q 0}^{\Delta}-\boldsymbol{v}_{s, d q 0}  \tag{D.11}\\
& 0=\left[\begin{array}{c}
0 \\
0 \\
\frac{v_{d c}}{2}
\end{array}\right]-\boldsymbol{v}_{m, d q 0}^{\Sigma} \tag{D.12}
\end{align*}
$$

$$
\begin{align*}
0 & =\frac{1}{2 C_{a r m}}\left(\left[\boldsymbol{M}_{3 a}^{\Sigma *}\right] \boldsymbol{i}_{d q 0}^{\Delta}+\left[\boldsymbol{M}_{3 b}^{\Delta *}\right] \boldsymbol{i}_{d q 0}^{\Sigma}\right)-\boldsymbol{J}_{\omega} \boldsymbol{v}_{c, d q 0}^{\Delta}  \tag{D.13}\\
0 & =\frac{1}{2 C_{a r m}}\left(\left[\boldsymbol{M}_{4 a}^{\Delta *}\right] \boldsymbol{i}_{d q 0}^{\Delta}+\left[\boldsymbol{M}_{4 b}^{\Sigma *}\right] \boldsymbol{i}_{d q 0}^{\Sigma}\right)-\boldsymbol{J}_{-2 \omega} \boldsymbol{v}_{c, d q 0}^{\Sigma} \tag{D.14}
\end{align*}
$$

From Eqs. (D.11) and (D.12) the voltages $\boldsymbol{v}_{m, d q 0}^{\Delta}$ and $\boldsymbol{v}_{m, d q 0}^{\Sigma}$ are determined. Therefore, from Eqs. (D.9) and (D.10) the modulating indexes $\boldsymbol{m}_{d q 0}^{\Delta}$ and $\boldsymbol{m}_{d q 0}^{\Sigma}$ are also determined. They are,

$$
\begin{gather*}
\boldsymbol{m}_{d q 0}^{\Delta}=\left[\begin{array}{c}
0 \\
\frac{2 V_{s}}{v_{d c}} \\
0
\end{array}\right]  \tag{D.15}\\
\boldsymbol{m}_{d q 0}^{\Sigma}=\left[\begin{array}{l}
0 \\
0 \\
1
\end{array}\right] \tag{D.16}
\end{gather*}
$$

And,

$$
\begin{gather*}
{\left[\boldsymbol{M}_{3 a}^{\Sigma *}\right]=-\frac{1}{2}\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right]}  \tag{D.17}\\
{\left[\boldsymbol{M}_{3 b}^{\Delta *}\right]=\left[\begin{array}{ccc}
-\frac{V_{s}}{v_{d c}} & 0 & 0 \\
0 & \frac{V_{s}}{v_{c c}} & \frac{2 V_{s}}{v_{d c}} \\
0 & 0 & 0
\end{array}\right]}  \tag{D.18}\\
{\left[\boldsymbol{M}_{4 a}^{\Delta *}\right]=-\frac{1}{2}\left[\begin{array}{ccc}
-\frac{V_{s}}{v_{d c}} & 0 & 0 \\
0 & \frac{V_{s}}{v_{d c}} & 0 \\
0 & \frac{V_{s}}{v_{d c}} & 0
\end{array}\right]}  \tag{D.19}\\
{\left[\boldsymbol{M}_{4 b}^{\Sigma *}\right]=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right]} \tag{D.20}
\end{gather*}
$$

Inspecting the Eqs. (D.13) and (D.14), the unknowns are $\boldsymbol{v}_{c, d q 0}^{\Delta}$ and $\boldsymbol{v}_{c, d q 0}^{\Sigma}$. Therefore,

$$
\begin{gather*}
\boldsymbol{J}_{\omega} \boldsymbol{v}_{c, d q 0}^{\Delta}=\frac{1}{2 C_{a r m}}\left(\left[\boldsymbol{M}_{3 a}^{\Sigma *}\right] \boldsymbol{i}_{d q 0}^{\Delta}+\left[\boldsymbol{M}_{3 b}^{\Delta *}\right] \boldsymbol{i}_{d q 0}^{\Sigma}\right)  \tag{D.21}\\
\boldsymbol{J}_{-2 \omega} \boldsymbol{v}_{c, d q 0}^{\Sigma}=\frac{1}{2 C_{a r m}}\left(\left[\boldsymbol{M}_{4 a}^{\Delta *}\right] \boldsymbol{i}_{d q 0}^{\Delta}+\left[\boldsymbol{M}_{4 b}^{\Sigma *}\right] \boldsymbol{i}_{d q 0}^{\Sigma}\right) \tag{D.22}
\end{gather*}
$$

Once $\boldsymbol{v}_{c, d q 0}^{\Delta}$ and $\boldsymbol{v}_{c, d q 0}^{\Sigma}$ are obtained the capacitor voltage can be calculated as explained in the section 4.5.


Figure D. 1 - Simplified MMC steady-state time-invariant model

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Titre : Contribution au dimensionnement du convertisseur modulaire multiniveau Mots clés : MMC, tension de condensateurs, diagramme PQ, prototype

Résumé : Le convertisseur multiniveau modulaire (MMC) est une solution appropriée pour les réseaux HVDC grâce à sa modularité, sa faible fréquence de commutation et sa tension alternative quasisinusoïdale. En raison de sa topologie, son modèle mathématique est assez complexe et est donc souvent simplifié au stade de la conception. En particulier, la résistance équivalente au bras $R$, l'inductance du bras L et le courant circulant sont souvent négligés. Toutefois, les résultats expérimentaux obtenus avec notre prototype monophasé de MMC à pont complet à six niveaux ont montré que ces hypothèses ne sont pas toujours acceptables. Dans ce contexte, l'objectif de cette thèse est d'étudier l'impact de $R$, $L$ et du courant de circulaiton sur la tension du condensateur du module et sur la zone de fonctionnement du MMC.
Premièrement, nous avons étendu le modèle basé sur les intégrales communément utilisé et nous avons clarifié les hypothèses sur lesquelles il repose. Entre autres, des expressions pour les courants de circulation et courant DC ont été développées et comparées à celles que l'on trouve dans la littérature. Cela nous a permis d'analyser l'ondulation de la tension du condensateur du module en fonction de $R$ et $L$, sans courant de circulation.

Deuxièmement, pour surmonter les limites du modèle basé sur l'intégrale, nous avons proposé d'utiliser un modèle MMC invariant dans le temps en régime permanent dans le système dq0. Quelques hypothèses seulement sont nécessaires pour obtenir ce modèle, mais une évaluation numérique est requise. Cela nous a permis d'analyser la tension moyenne du condensateur du module et l'ondulation de tension du condensateur du module en fonction de $R$ et $L$, avec et sans courant de circulation.
Troisièmement, en utilisant le modèle invariant dans le temps en régime permanent, nous avons développé un diagramme PQ détaillé du MMC. Outre la limite de courant AC, la limite de courant DC et la limite d'indice de modulation classiques, nous avons ajouté plusieurs limites internes: courant de l'IGBT, courant efficace des bras et ondulation du courant et de la tension du condensateur du module. Les résultats ont été confirmés par simulation numérique à l'aide d'un modèle détaillé Matlab Simulink SimPowerSystems.
Les résultats présentés dans cette thèse pourraient être utilisés pour optimiser le dimensionnement des composants de la MMC en fonction de sa zone d'exploitation et pour évaluer l'impact de différents paramètres sur les performances du MMC.

Title : Contribution to the sizing of the modular multilevel converter Keywords : MMC, capacitor voltage, PQ diagrams, prototype


#### Abstract

The modular multilevel converter is a suitable solution for HVDC grids thanks to its modularity, low switching frequency and quasi-sinusoidal AC voltage. However, due to its topology, its mathematical model is quite complex and is therefore often simplified at the design stage. In particular, the arm equivalent resistance $R$, the arm inductance $L$ and the circulating current are often neglected. But experimental results obtained with our 1-ph 6-level full-bridge MMC prototype showed that these hypotheses are not always acceptable. In this context, the goal of this thesis is to study the impact of accounting for $R, L$ and the circulating current on the module capacitor voltage and on the operating area of the converter. First, we extended the commonly used integral based model and we clarified the hypotheses behind it. Among others, expressions for the circulating and dc currents have been developed and compared with the one that can be found in the literature. It allowed us to analyze the module capacitor voltage ripple as a function of $R$ and $L$, without circulating current only.


Second, to overcome the limitations of the integral based model, we proposed to use a steady state time invariant $\Delta \Sigma$ MMC model in dq0 frame. Only few hypotheses are required to obtain this model, but a numerical evaluation is required. It allowed us to analyze the module capacitor average voltage and the module capacitor voltage ripple as a function of $R$ and $L$, with and without circulating current.
Third, using the steady state time invariant model, we developed a detailed PQ diagram of the MMC. In addition to the conventional AC current limit, DC current limit and modulation index limit, we added several internal limits: IGBT current, arm rms current and module capacitor voltage and current ripple. The results have been confirmed by numerical simulation using a detailed Matlab Simulink SimPowerSystems model.
The results presented in this thesis could be used to optimize the sizing of the components of the MMC considering its operating area, and to assess the impact of different parameters on the MMC performance.

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[^0]:    1. Example: NorNed HVDC connection between Norway and Netherlands.
    2. Example: Pacific DC intertie ( 1360 km ).
    3. Example: Transiting large amount of power via undersea or underground cables, e.g. the 2000 MW interconnection IFA2000 [55] between France and the United Kingdom under the English Channel.
    4. Example: Increasing the capacity of an existing AC transmission with an HVDC system using underground cables, e.g. the HVDC transmission between France and Spain using MMCs (INELFE [56])
    5. Example: Delivery of electrical bulk power over long distances, e.g. the 3000 MW connection of the hydroelectric power station "Three Gorges Dam" to Changzhou, China [57]
    6. Examples: (i) Itapu Power Station built due to frequency difference between Brazil $(60 \mathrm{~Hz})$ and Paraguay ( 50 Hz$)$, (ii) GCC interconnection between UAE ( 50 Hz ) and Saudi Arabia ( 60 Hz ), (iii) HVDC Hokkaidō-Honshū link in Japan, etc.
[^1]:    1. The arm coil resistance, the inter-module connections and the conduction resistance of the switches are modeled as a series equivalent arm resistance $R$
[^2]:    2. Analytical Equation.
    3. Ordinary Differential Equation.
[^3]:    1. The hypothesis $\varphi_{m}^{\Delta}=0$ is very often taken in the literature. This is valid for the cases where the arm inductance is sufficiently low so that the voltage drop across the coil can be neglected. As the main objective of this thesis is to derive the impact of the arm parameters on various elements of the MMC, the calculations had to be done in a more generic manner.
[^4]:    2. The study is case dependent. Using different parameters may drastically change the result.
[^5]:    1. The integral-based model has to take the hypothesis on the form of $v_{m}^{\Delta \Sigma}$ voltage (Eqs. (3.10) and (3.72)) in order to derive the equations. The reason for this is that the Eqs. (2.68) and (2.75) from the $\Delta \Sigma$ model are not considered in their proper form.
[^6]:    2. This is a direct consequence of $m_{u}$ and $m_{l}$ definition
    3. Obtained directly from the SSTI model
[^7]:    scale prototype, this constraint would seriously limit the OA of the converter.

[^8]:    Note that,

