



HAL
open science

Gestion des risques des taux d'intérêt et de liquidité dans le secteur bancaire libanais

Rudy Daccache

► **To cite this version:**

Rudy Daccache. Gestion des risques des taux d'intérêt et de liquidité dans le secteur bancaire libanais. Business administration. Université Claude Bernard - Lyon I, 2014. English. NNT : 2014LYO10100 . tel-02381225

HAL Id: tel-02381225

<https://theses.hal.science/tel-02381225>

Submitted on 26 Nov 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

École Doctorale Sciences Économiques et de Gestion

THÈSE

Présentée et soutenue publiquement le 26 juin 2014

pour l'obtention du

Doctorat de l'Université Claude Bernard Lyon I

En Sciences de Gestion

par

Rudy DACCACHE

Interest Rate and Liquidity Risk Management for Lebanese Commercial Banks

Composition du jury

Rapporteurs: Pr. Jean-Paul LAURENT
Pr. Duc Khuong NGUYEN

Examineurs: Pr. Stéphane LOISEL
Dr. Rami EL HADDAD
Mr. Mazen HAMDOUN
Pr. Christian ROBERT (directeur de thèse)

Acknowledgement

“The
astronomer
may speak to you of
his understanding of space,
but he cannot give you his understanding.
The musician may sing to you of the rhythm which
is in all space, but he cannot give you the ear which arrests
the rhythm nor the voice that echoes it. And he who is versed in the science
of numbers can tell of the regions of weight and measure, but he cannot conduct you
thither. For the vision
of one man lends not
its wings to another man.”

“Gibran Khalil Gibran”

I like to think of learning similar to mountain climbing, for it allows me to visualize the responsibilities of the student and the teacher. A teacher is like a guide who knows a particular mountain well because he/she has been climbing that mountain for a long time. Such a guide can inform the climbers about the best routes to the top and can warn them about the dangers that may lie ahead. But a guide, no matter how skilled, can't do the climbing for you. You must carry your own gear and supplies, and you must do your own climbing.

I would like to express my sincere gratitude to my supervisor Christian Robert for helping me climbing to the top of my mountain. His guidance and comments were instrumental to my research and writing of this thesis. His mentorship was paramount in providing a well rounded experience consistent my long-term career goals. For everything you've done for me, “Christian, I thank you.”

I would like to thank Mr. Adel Satel, Group Chief Risk Officer at Bank Audi-Lebanon, who gave me the opportunity to undertake my PhD at ISFA, Claude Bernard University-Lyon.

My sincere thanks go to my colleagues at Bank Audi : Mazen, Chadi, Patrick, Remy, Abdallah and Karim, for their insight throughout this thesis.

Rami and Toni heads of the Mathematics Department at Saint Joseph University-Beirut, thank you for your ongoing support since 2008, and thanks to all my colleagues at Bank Audi risk management department and at faculty of science-Saint Joseph University.

My colleagues at ISFA research laboratory, Samy, Abdou, Sawssen, Manel, Elena, Yahia, Julien A., Julien V., Julien T., Ibrahima, Huy, Andres, Anisa, Reda, Khalil, Alexandre, Ines,... the list is endless... have all extended their support in a very special way, and I gained a lot from them, through their personal and scholarly interactions, their

suggestions at various points... Thank you...

I would like to acknowledge my other friends for their moral support and motivation, which drives me to give my best. Larry, Rasha, Bassem, Emil, Pierre, Youssef... list is endless... thanks to one and all. My special gratitude to Elias, Karim and my cousin David for being with me in thicks and thins of life, I find myself lucky to have friends like them in my life.

Last but not the least, I dedicate this thesis to my family for their constant support and unconditional love. I love you all dearly. I owe a lot to my parents, who encouraged and helped me at every stage of my personal and academic life, and longed to see this achievement come true. I thank my uncle Pierre and his family for their wonderful help all these years. I am very much indebted to my brother and his wife who supported me in every possible way to see the completion of this work, and not to forget my new born nephew Angela, "I see the joy in your smile".

It has been a tough climb, as Barry Finlay said: "Every mountain top is within reach if you just keep climbing." Cheers to new mountain's tops that I will never stop chasing.

Abstract

The aim of this thesis is to provide Bank Audi with econometric tools for sake of a more robust risk management. Lebanese businesses today are faced with greater challenges than ever before, both economical and political, and there is a question about the future of the middle east region after the Syrian civil war. Thus, Lebanese commercial banks face greater complications in the management of interest rate and liquidity risk.

The first part of this thesis discusses interest rate risk management and measurement in the Lebanese market. First, we seek to build the Lebanese term structure. This market is known by its illiquidity, yields for a given maturity make a large jump with a small impact on other yields even if close to this maturity. Therefore, we face challenges in calibrating existing yield curve models. For this matter, we get historical prices of bonds issued by the Lebanese government, and denominated in Local currency and in US dollar. A new estimation method has been added to Nelson Siegel and Svensson model, we call it “Correlation Constraint Approach”. Model parameters can be interpreted from economical perspective which will be helpful in forecasting yield curve movements based on economist’s opinion.

On the second hand, traditional customer deposits are the main funding source of Lebanese commercial banks (80-85% of liabilities). Although they are contractually short term (mainly one month) paying fixed interest rates, these deposits are historically known to be a stable source of funding and therefore exhibit a sticky behavior to changes in market interest rates. We develop an error correction model showing a long-run equilibrium between Libor and Lebanese banking sector average rate offered on USD deposits. Results make it possible to determine the behavioral duration (repricing date) of customer deposits when market interest rates fluctuate. Therefore, the behavioral duration of liabilities will be higher than the contractual one which will lower the duration gap between assets and liabilities and thus the negative impact of positive interest rate shocks.

After understanding interest risk profile of customers’ deposits, we start the second part by determining their behavioral liquidation maturity. We get Bank Audi’s historical deposits outstanding balances filtered into the following categories: currency, account typology and residency of depositor. We develop an error correction model for each filter. Results show relationship between deposits behaviors, the coincident indicator and spreads between offered rates in the Lebanese market. The model will lead to assess behavioral liquidation maturity to deposits and understand their liquidity risk profile. This will be helpful for the funding liquidity risk management at Bank Audi.

Large financial institutions are supposed to hold large positions of given assets. The last topic is related to market liquidity risk management. We suppose an investor holds a large position of a given asset. Then at time 0, a severe shock causes a large depreciation of the asset value and makes the investor decides to liquidate the portfolio as soon as possible with limited losses. Stock returns are modeled by GARCH process which has tail behaviors after large variation at time 0. Trading on liquid and illiquid markets, we provide the trader with best exit trading strategy maximizing his utility function, finally we incorporate into the model an expert opinion which will help the investor in taking the decision.

Résumé

L'objectif de cette thèse est de fournir à la Banque Audi des outils économétriques et appliqués pour une gestion des risques plus efficace et plus robuste. Les banques libanaises sont aujourd'hui confrontées à des défis plus importants que jamais: l'avenir de la région Moyen-Orient repose sur les conséquences de la guerre civile syrienne. Dans ce contexte, la gestion des taux d'intérêt et de la liquidité s'avère de plus en plus compliquée pour les banques commerciales.

En premier lieu, le risque de taux d'intérêt sur le marché libanais sera étudié. Ce marché est connu pour son manque de liquidité et le problème de calibrage des modèles de taux est difficile. Afin de résoudre ce problème, nous utilisons les prix historiques des obligations émises par le gouvernement libanais et libellées en monnaie locale et en dollars américains. Nous considérons des modèles de Nelson-Siegel et Svensson et contraignons le niveau de corrélation des facteurs pour stabiliser l'estimation des paramètres de ces modèles. La méthode conduit à des résultats qui s'interprètent très facilement d'un point de vue économique et peuvent être utilisés pour la prévision des variations de la courbe des taux en se basant sur une analyse économique prospective.

En second lieu, la problématique des dépôts des clients traditionnels sera étudiée. Ces derniers sont reconnus comme étant la source principale de financement des banques commerciales libanaises (80-85% du passif). Bien qu'ils soient contractuellement des dépôts à court terme (principalement un mois) versant des taux d'intérêt fixes, ces dépôts sont assimilés à une source de financement stable possédant un comportement proche des taux d'intérêt du marché. Nous développons un modèle à correction d'erreur représentant un équilibre à long terme entre le Libor et le taux moyen du secteur bancaire libanais offert sur les dépôts en dollars américains. Les résultats permettent de déterminer une date de réévaluation des dépôts clientèles en cas de fluctuation des taux d'intérêt. Une nouvelle durée du passif tenant compte des comportements des clients a été mise en place. Elle sera par construction plus élevée que la durée contractuelle. En cas de hausse des taux d'intérêt, une baisse de l'écart entre la durée des actifs et des passifs sera alors observée menant à la diminution de l'impact négatif de la hausse.

Après avoir étudié le profil de risque des taux des dépôts clientèles, nous commençons la deuxième partie de la thèse par la détermination de l'échéancier des retraits. Nous segmentons les données historiques des données sur les dépôts clientèles selon: la monnaie, le type de dépôt et la résidence du déposant. Pour chaque filtre, un modèle à correction d'erreur est développé. Les résultats montrent la relation entre les dépôts clientèles, un indicateur relatif du niveau économique et les écarts entre les taux offerts sur le marché libanais. Ainsi, le modèle permettra d'évaluer le comportement des retraits des dépôts clientèles et de comprendre leur profil de risque de liquidité.

Les grandes institutions financières détiennent des positions importantes en actifs financiers. La dernière partie de la thèse discute de la gestion du risque de liquidité de marché en cas de session forcée de ces actifs. Nous supposons qu'un investisseur détient une position importante d'un actif donné, à $t = 0$, un choc sévère provoque une forte dépréciation de la valeur de l'actif et par conséquent, force l'investisseur à opter pour la liquidation du portefeuille dès que possible en limitant ses pertes. Les rendements des actions sont modélisés par des processus de type GARCH qui sont adaptés pour décrire

des comportements extrêmes suite à une grande variation de l'actif au temps initial. Suivant que le marché est liquide ou illiquide, nous proposons une stratégie optimale à l'investisseur qui maximise sa fonction d'utilité. Enfin, nous intégrons dans le modèle un avis d'expert pour optimiser la prise d'une décision.

Keywords

Interest rate risk, Liquidity risk, Nelson-Siegel, Nelson-Siegel-Svensson, Liquidation Strategies, Market impact, Time Series Analysis.

Mots Clés

Risque des taux d'intérêt, Risque de liquidité, Nelson-Siegel, Nelson-Siegel-Svensson, Stratégies de liquidation, l'impact du marché, analyse des séries temporelles.

Discipline

Sciences de gestion

Intitulé et adresse de l'U.F.R. ou du laboratoire:

Université Lyon 1, ISFA, 50 Avenue Tony Garnier, 69366 Lyon Cedex 7

Contents

Introduction	12
I Interest Rate Risk Management and Measurement in the Lebanese Market	18
1 Introduction to Interest Rate Risk Management	20
1.1 Introduction	20
1.2 Interest Rate Risk Management	21
1.2.1 Definition of interest rate risk	21
1.2.2 The management of Net Interest Income	22
1.2.3 Duration	22
1.2.4 Convexity	23
1.2.5 Principal Component Analysis	23
1.3 Zero-Coupon Yield Curves	25
1.3.1 Theoretical Background	26
1.3.2 Zero-coupon yield curves in central banks	33
1.4 The Repricing Gap Model	33
1.5 Conclusion	34
Appendices	34
1.A-Fixed Income Portfolio Example	35
1.B- Yield Curve Models in Central Banks	36
Bibliography of the chapter	37
2 Lebanese Yield Curve	38
2.1 Introduction	38
2.2 Data	40
2.3 Estimation Methods	41
2.4 Application	43
2.4.1 Nelson-Siegel Applied on the USD Yield Curve	45
2.4.2 Nelson-Siegel Applied on the LBP Yield Curve	47
2.4.3 Nelson-Siegel-Svensson Applied on the USD Yield Curve	50
2.4.4 Nelson-Siegel-Svensson Applied on the LBP Yield Curve	52
2.4.5 Conclusion	53

2.5	Lebanese Yield Curves Movements	55
2.6	Economical Interpretation	57
2.7	Conclusion	61
Appendices		61
2.A-	Principal Component Analysis Tables	62
Bibliography of the chapter		64
3	Pass-Through of Interest Rate Shocks to Lebanese Deposit Rates	66
3.1	Introduction	66
3.2	Data	67
3.2.1	Banking Sector Average Rate (BSAR)	68
3.2.2	London InterBank Offered Rate (LIBOR)	68
3.2.3	Descriptive statistics of the two variables	69
3.3	Error Correction model	71
3.3.1	Definition	71
3.3.2	Model Results	72
3.4	Shock Analysis	74
3.5	Application	75
3.6	Conclusion	77
Appendices		77
3.A-	Vector Autoregression and Error Correction Model	78
3.B-	Cointegration and ECM Tools	89
Bibliography of the chapter		93
II Liquidity Risk Measurement and Management in Banks		96
4	Liquidity Risk in Banks	98
4.1	Introduction	98
4.2	Liquidity Operations	99
4.3	Sources of liquidity	102
4.3.1	Sources of Asset Liquidity	102
4.3.2	Sources of Funding Liquidity	103
4.3.3	Sources of Off-Balance Sheet Liquidity	103
4.3.4	Amalgamating Liquidity assets	104
4.4	Liquidity Problems	104
4.4.1	Funding Liquidity Risk	104
4.4.2	Asset Liquidity Risk	106
4.5	Liquidity Risk in Basel Standards	107
4.5.1	Introduction	107
4.5.2	Principles for sound liquidity risk management and supervision . . .	108
4.5.3	Regulatory standards	111

4.6	Conclusion	113
Appendices		113
4.A	Illustrative template for the LCR	114
4.B	Components of Available Stable Funding and Associated ASF Factors . . .	119
4.C	Detailed Composition of Asset Categories and Associated RSF Factors . . .	120
4.D	Composition of Off-balance Sheet Categories and Associated RSF Factors .	122
4.E	Composition of Off-balance Sheet Categories and Associated RSF Factors .	123
Bibliography of the Chapter		125
5	Behavioral Maturity of Non-Maturing Deposits	126
5.1	Introduction	126
5.2	Data	128
5.2.1	Account typology: Demand or Term Deposit	128
5.2.2	Depositor Residency Indicator	129
5.2.3	Coincident Indicator	129
5.2.4	Bank Audi Deposits Distribution	130
5.3	Market Analysis	131
5.3.1	Deposit Rates	131
5.3.2	Total Deposits	132
5.3.3	Demand Deposits	133
5.3.4	Term Deposits	134
5.4	Maturity Adjustment	135
5.4.1	Contractual Maturity	135
5.4.2	Econometric Model	136
5.4.3	Cointegration Model	136
5.4.4	Behavioral Maturity	140
5.5	Results	142
5.6	Conclusion	143
Appendices		143
5.A	Fit Results of Historical Bank Run Scenarios	144
Bibliography of the Chapter		146
6	Exit Trading Strategies in stressed markets using Expert Opinion	150
6.1	Introduction	150
6.2	Basic Model	152
6.2.1	Modeling fat tails of Stock Returns	153
6.2.2	Fitting to ARCH(1) process	154
6.2.3	The case of a GARCH(1,1) process	158
6.3	Trading Strategies in Liquid Markets	162
6.3.1	Selection of the proper trading strategy	162
6.3.2	ARCH(1) Process	164
6.3.3	GARCH(1,1)	165

6.3.4	Application to financial markets	167
6.4	Trading Strategies in illiquid markets	172
6.4.1	Market Impact	172
6.4.2	Optimization Problem	174
6.4.3	Temporary market impact	176
6.4.4	Permanent Market Impact	179
6.4.5	Incorporating Expert Opinion into the model	181
6.5	Conclusion	183
Appendices		183
6.A-	Modeling Stock Returns	184
6.B-	Proof of Tail and Spectral Process Theorems	187
6.C-	Proof of the martingale $(\delta_i)_{i \in \mathbb{N}}$	189
6.D-	Proof of the square integrable martingale $(\omega_i)_{i \in \mathbb{N}}$	191
6.E-	Generating Randoms of GARCH processes with volatility tail process . . .	192
Bibliography of the Chapter		193
Conclusion		196
Bibliography		200

Introduction

Financing Lebanese government depends mainly on commercial bank deposits growth. With the high government debt largely held by domestic commercial banks, their ongoing funding, mainly from deposits, is an important key to ensure the stability of the Lebanese financing activity.

We represent, in Table 1, the consolidated Balance Sheet of Lebanese commercial banks published by the Lebanese Central Bank as end of September 2013. Figures show the 82% non-financial customers deposits occupation of total assets therefore understanding the risk profile of these deposits is an important issue for Lebanese commercial banks. On the other side, 23% of total assets are constituted of claims on public sector whereas claims on resident private sector stand only on 25% of total assets. This explains the high liquidity availability in Lebanese Banks since domestic sovereign debts are classified as high quality liquid assets in the liquidity coverage ratio.

Table 1: Commercial Banks Consolidated Balance Sheet (In Billions of LBP) as Sep 2013

ASSETS		LIABILITIES	
RESERVES	81'679.1 (34%)	TOTAL RESIDENT PRIVATE SECTOR DEPOSITS	158'732 (66%)
VAULT CASH	592.3 (0%)	DEMAND DEPOSITS IN LBP OF RESIDENT PRIVATE SECTOR	4'332.7 (2%)
DEPOSITS WITH CENTRAL BANK	81'086.8 (34%)	TIME AND SAVING DEPOSITS IN LBP OF RESIDENT PRIVATE SECTOR	58'777.6 (24%)
CLAIMS ON RESIDENT PRIVATE SECTOR	61'138.6 (25%)	RESIDENT PRIVATE SECTOR DEPOSITS IN FC	95'621.3 (40%)
CLAIMS ON RESIDENT PRIVATE SECTOR IN LBP	16'112.4 (7%)	PUBLIC SECTOR DEPOSITS	4'313.1 (2%)
CLAIMS ON RESIDENT PRIVATE SECTOR IN FC	45'026.2 (19%)	DEPOSITS OF NON RESIDENT	39'291.8 (16%)
CLAIMS ON PUBLIC SECTOR	54'515.4 (23%)	DEPOSITS OF NON RESIDENT FINANCIAL SECTOR	7'357.2 (3%)
FOREIGN ASSETS	36'318.0 (15%)	OTHER FINANCIAL LIABILITIES	428.8 (0%)
FIXED ASSETS	5'815.1 (2%)	CAPITAL ACCOUNTS	21'216.7 (9%)
UNCLASSIFIED ASSETS	612.5 (0%)	UNCLASSIFIED LIABILITIES	8'739.3 (4%)
TOTAL BALANCE SHEET	240'079 (100%)	TOTAL BALANCE SHEET	240'079 (100%)

Consequently, two main components of Lebanese banks are the fixed income portfolio and customer deposits. For this matter, this thesis focuses on interest rate and liquidity risk management.

Interest rate risk is one of the most important risk in managing fixed income portfolio and the whole balance sheet of a bank. In the first part, we aim to study dynamics of interest rates in the Lebanese market. First, we study the yield curve obtained from observed rates of bonds issued by the Lebanese Government in Local Currency and in US dollar, the objective of this study is to use a more robust yield curve for the shareholders' equity sensitivity. Second, we identify the impact of Market interest rates shocks on US dollar deposit rate offered by Lebanese Banks in order to understand the behavior of deposit rates in response to LIBOR, such analysis guides us to obtaining the behavioral repricing date of customers' deposits when analyzing also the shareholders' equity sensitivity.

Firstly, we seek to build the Lebanese term-structure of interest rates based on

bonds issued by the Lebanese Government in local currency and US dollar. This model can be used for risk management and plays an important role in pricing fixed-income securities, interest rate derivatives, as well as other financial assets. Since zero coupon rates are not observable for a range of maturities, an estimation methodology is required to derive the zero coupon yield curves from observable data.

After reviewing articles related to the topic and seeing models used by central banks, we decide to implement Nelson Siegel Model and its extension proposed by Svensson. Many papers highlight difficulties found in estimating model parameters specifically in Svensson model due to the number of coefficients to be estimated relative to the limited set of observable yields on the secondary market.

For this issue, we collect historical prices of Lebanese government bonds from January 2009 until January 2013, since earlier data is not available on Bloomberg platform. Then we apply the bootstrapping method to obtain exact zero yields.

Turning to the two models, we apply various parameters estimation methods and we define a new optimization approach called "Correlation Constraint Approach" in which a new constraint has been added to maintain a small correlation between the loading factors. Nevertheless, this approach yields more stable parameters that change in response to any remarkable regional political or economical situation change.

During the last four years, there have been many changes in the economical and political situations. These events are reflected in our results by a regime switching analysis applied on obtained parameters. Therefore, any change in the situation will be helpful in forecasting the Lebanese term-structure. Moreover, an economic expert opinion can be added to the model for prediction.

Secondly, we aim to estimate the behavioral duration of customers' deposits since these traditional customer deposits are the main funding source of Lebanese commercial banks (80-85% of liabilities). Although they are contractually short term (mainly one month) paying fixed interest rates, these deposits are historically known to be a stable source of funding and therefore exhibit a sticky behavior to changes in market interest rates.

This sticky behavior gives Lebanese banks a high bargaining power to control the pass-through of market shocks and therefore control their sensitivity to interest rates. Consequently, we will model the behavioral versus contractual repricing of customer deposits. In other words, we measure the real impact of interest rate shocks on the bank's profits and economic value of shareholders' equity.

For this matter, we get interest rate data from 2003 until the end of 2011 since earlier data is no longer relevant to the actual state of the banking sector and the Lebanese economy in general. We use LIBOR as a proxy for market interest rates and the Banking Sector Average Rate (BSAR) as a proxy for interest rates paid by Lebanese banks on

customer deposits.

After reviewing the literature concerning this subject and applying different econometric models, we come to the conclusion that an Error Correction Model (ECM) is the most appropriate to reach the results we are aiming for.

Within this ECM framework, we are able to identify the short and long run pass-through rates in response to interest rate shocks. In addition, we formulate an impulse response function that measures the speed, in months, it would take a shock to be transmitted to rates paid on deposits.

Results show that 7% of a market shock will be passed-through immediately while the final pass-through rate is 33%. These results would make it possible to determine the behavioral duration (repricing date) of customer deposits when market interest rates fluctuate.

A positive shock in interest rates, which is the standard stress test used to measure banks' sensitivity to interest rates, will usually yield a negative impact as assets' duration is higher than liabilities' duration.

When considering the results of our model, the behavioral duration of liabilities will be higher than the contractual one which will lower the duration gap between assets and liabilities and thus the negative impact of positive interest rate shocks.

After understanding the interest rate risk profile of main components of Lebanese commercial banks' balance sheet, we go forward in the study by analyzing their liquidity risk side.

Liquidity is the lifeblood of every commercial and sovereign entity, in fact, we define it as the availability of cash or equivalent resources, and it allows expected and unexpected obligations to be met when needed so that daily business affairs can proceed uninterrupted. However, when there is absence of sufficient cash resources, the probability of encountering more severe financial distress increases. Liquidity is therefore a fundamental element of financial management, and must be considered and managed with care.

As we have mentioned above that customers' deposits constitute 80-90% of Lebanese banks' liabilities, understanding their liquidity risk profile is a key issue for Lebanese banks. Moreover, in terms of liquidity risk, considering contractual maturity of these deposits leads to a large liquidity gap for short-term analysis. First, we seek to build a robust econometric model to capture the behavioral maturity of non-maturing deposits at Bank Audi.

For this issue, we gather historical monthly outstanding balances of total customers' deposits from January 2006 to June 2013, then we subdivide these deposits into different classes depending on account typology, currency, residency indicator,... Results show the

absence of any severe historical bank run scenario at Bank Audi despite regional political problems from 2006 till today including the war of 2006 between Israel and Lebanon and the current Syrian civil war. However, we see during these periods large depositors convert their deposits into other currencies without withdrawing them.

Moreover, Lebanese banks prefer not to take FX positions in order to avoid currency risk. Therefore, they manage their asset-liability per currency, for this reason, we will neglect currency conversion when observing negative trend in deposit category behavior.

After discussion with members of Asset Liability Committee (ALCO) , we decide to build an econometric model that explains a relationship between deposits behaviors, interest rates and macroeconomic factors. We conclude that the coincident indicator, considered as a proxy indicator of the Lebanese economy situation, has an important impact on the behavior, nevertheless it can determine the level of deposits' growth. Nonetheless, the asset liability management requires adding Bank Audi's policy to the equation, we added the spread between Bank Audi rates and other Lebanese banks as an exogenous variable. Results show it as an important factor for predicting residents depositors accounts since these clients don't depend only on the economic situation but it can convert or transfer their money in order to get a higher interest.

After building the model, the equation shows that deposits will always grow up if we don't have an important change in the exogenous variables. For this reason, we suppose that at time 0 there is negative jumps in exogenous variables and we can generate the outflows in response to this scenario. And we build their behavioral maturity based on this scenario.

Since the frequency of data used in the model above is monthly, obtained behavioral maturities are also monthly. However, we also need to study short-term liabilities resilience. To be more conservative, for the maturities less than one month, we get the daily data of deposits behaviors and we select the worst historical withdraw scenario for each filter. Then, we suppose behavioral maturities are the same as the generated outflows fitted by Matlab Gaussian function with an adjusted R^2 greater than 99%.

Finally, we propose to ALCO to manage its liquidity by considering these behavioral maturities in normal business conditions, and the contractual maturities in stressed market conditions.

Another important asset on Lebanese commercial banks' balance sheet is investments in Beirut stock exchange which is considered as an illiquid market. Lots of banks hold large positions in a given security, which requires a high level of management since a financial turmoil may cause a large decline in the value of the portfolio.

We suppose that an investor holds a large portfolio of a given asset and wishes to liquidate it following a severe shock in the market. The objective is to develop an ap-

proach to estimate the best exit trading strategy in stressed markets (liquid or illiquid). In the first analysis, we suppose that the asset is traded on a liquid market so there is no market liquidity problem whereas in the second analysis we consider the case of illiquid markets where the investor should trade while taking into consideration the timing risk as well as the market liquidity risk.

First, we suppose that asset daily changes can be modeled by regularly varying process; we select the best trading strategy which maximizes investor's utility function. Simulations results show the existence of model risk in a way that the best trading strategy depends largely on the selected process and not on the asset. In other words, we observe the best trading strategy of an ARCH process isn't the same for a GARCH process having the same tail behavior.

In the first application, we suppose that the asset is traded on a liquid market so there is no market liquidity premium on large orders. We gather daily stock prices of CAC 40 and other liquid assets. We fit these variables by an ARCH or GARCH process then we run simulations in order to get best liquidation trading strategy. Results show that the trader should liquidate his portfolio the soonest possible since there is no market liquidity premium.

Turning to illiquid markets, we introduce a market impact function that depends on the size of the order, its estimation is based on large orders historical during crisis. Using Bloomberg tools, we get daily stock returns of most traded stocks on Beirut stock exchange then we refer to the order book to get market premiums applied on executable orders during stressed periods. After fitting stock returns to GARCH process and market premiums to a predefined function, we run Monte Carlo simulations to get results of different trading strategies then we plot the expected return of each exit trading strategy as function of its volatility. This plot will help the investor to select the exit trading strategy that maximizes his utility function.

Moreover, adding an expert opinion to this model in these conditions is fundamental for the investor, for this matter, we predefine a trend of stock price after the shock and we suppose that this trend is provided by the investor. The volatility is being estimated by ARCH or GARCH process, we can run Monte Carlo Simulation to get the plot between expected return and the volatility, and the investor can choose his proper liquidation trading strategy.

Two fundamentals results from this study are the fire sale in the case of liquid market and the failure of obtaining the same exit trading strategy for ARCH and GARCH process when trading on illiquid market. Therefore the analysis shows a model selection risk since liquidation strategies depend on model selection even if they have the same tail behavior.

The rest of the thesis is organized as follows. Chapter I provides an introduction to interest rate risk management and the theoretical background to understand interest

rate dynamics in Lebanon. Chapter II discusses a new approach for the construction of term structures of interest rates, which can be applied to the Lebanese market. Chapter III presents a relationship between LIBOR and USD deposits rates offered by Lebanese banks and the resulting behavioral duration of customers' deposits. Chapter IV provides an introduction to liquidity risk management in banks. Chapter V looks at trends in Bank Audi customers' deposits to build their behavioral maturity. Chapter VI discusses optimal liquidation of large positions in stressed markets. The last chapter concludes.

Part I

Interest Rate Risk Management and Measurement in the Lebanese Market

Chapter 1

Introduction to Interest Rate Risk Management

1.1 Introduction

Modeling and forecasting interest rate dynamics is of great importance in many areas of finance such as derivatives pricing, asset allocation and debt restructuring. Not surprisingly, a vast amount of literature is devoted to research in this part of academia in order to find optimal methods to estimate and forecast the term-structure of interest rates.

In this part, we concentrate on the Lebanese Market since this work is helpful for Bank Audi-Beirut in managing market risks using advanced econometric models. The literature of existing models is explained in the introduction of each chapter.

Before outlining the developed parts, we provide a historical review for the Lebanese Economy between 1950 and 1995 to show how Lebanon is trying to enter global markets by maintaining a fixed exchange rate between the domestic currency and the US dollar. (cf. Corm (2005))

Lebanon has always been known as the Switzerland of the Middle-East mainly due to its first class banking services benefiting from a banking secrecy law that was passed in 1956. Beirut, its capital, is regarded as Paris of the East with all the touristic facilities it offers.

Until 1950, the Lebanese economy relied essentially on the exportation of agricultural products. Unfortunately, in 1975, the Lebanese civil war erupted and lasted until 1990 destroying the infrastructure and paralyzing the economic activity. Lebanon started losing gradually its status as “Switzerland of the Middle-East”. Moreover, after the civil war, economic problems persisted and led to an actual public debt near 56 Billion USD and representing 137% of the country’s GDP.

During the civil war, the worst period for the Lebanese economy was between 1982 and 1988. The Israeli invasion caused a chaos in the country. LBP deposits in Lebanese

Banks decreased, in value, from 12 Billion USD in 1982 to 3.5 Billion USD in 1988 due to a depreciation of the Lebanese pound that went from 4.5 pounds per US dollar to 500 pounds before stabilizing at a level between 800 to 1000 pounds. The country, emerging from fifteen years of violence, had a damaged infrastructure and 600'000 people displaced which affected the productive capacity of the economy. In addition, the Israeli army occupied a large part of Southern Lebanon until 2000. Reconstruction of the infrastructure costs around 3.5 Billion USD. Thus, the country was facing a difficult situation and the deterioration of the currency continued until it reached 2800 pounds per US dollar in summer 1992.

Since 1992, Lebanon is trying to regain its post war status and enter global markets. Between 1995 and 1996, the Lebanese pound was pegged against the US dollar at an average fixed rate of 1507.5 USD/LBP. The fixed exchange rate was the first step of the Lebanese Central Bank strategy to regain local and international investors confidence. This led to a monetary policy in Lebanon highly influenced by the US monetary policy.

The rest of the part is organized as follows. In this chapter, we give a general introduction to interest rate risk management in Banks as well as an overview of the different models employed in building the term structure of zero yields and the used models in top world central banks. Section 1.3 presents the repricing gap concept used in Chapter 3. In Chapter 2, we develop a new estimation method called "Correlation Constraint approach" which can be applied on Nelson-Siegel Model and its extension proposed by Svensson, also we estimate the Lebanese Government term-structure of zero yields denominated in LBP and USD currency then we study the evolution of the parameters in order to identify interest rate dynamics which is explained in the end of the chapter by an eco-political interpretation. Chapter 3 presents an econometric model interpreting a cointegration relationship between Libor and US deposits rates in Lebanese Banks, a model that gives the behavioral duration of customers' deposits in order to have a more robust measure of shareholders' equity sensitivity to interest rate shocks.

1.2 Interest Rate Risk Management

1.2.1 Definition of interest rate risk

Interest rate risk is the exposure of a bank's financial condition to adverse movements in interest rates. Banks accept this risk since it's the main source of profitability and shareholder value. Fluctuations in interest rates cause changes in the net interest income of banks also in other interest-sensitive income and operating expenses. Nevertheless, the underlying value of the bank's assets, liabilities and off-balance-sheet instruments can be affected by changes in interest rates since the present value of future cash flows change when interest rates change. Before setting out how to measure this risk, we provide a brief introduction to the sources of interest rate risk. (cf. BIS (2004))

Sources of Interest Rate Risk:

Repricing Risk. The repricing risk arises from timing differences in the maturity for fixed-rate instruments and repricing for floating-rate instruments. For instance, the income of a loan with a variable rate will increase when rates rise and decrease when rates fall, whereas if the loan is funded with fixed rate deposits, the bank's interest margin will fluctuate.

Yield Curve Risk. The yield curve risk originates from the spread between short-term and long-term interest rates. For instance, the underlying economic value of a long position in 10-year government bonds hedged by a short position in 5-year government notes could decline sharply if the yield curve steepens, even if the position is hedged against parallel movements in the yield curve.

Basis Risk. The basis risk arises from imperfect correlation in the adjustment of the rates earned and paid on different instruments with otherwise similar repricing characteristics. For example, a plan of funding of a one-year loan that reprices monthly based on 1 month US LIBOR rate with a one-year deposit that reprices monthly based on 1 Month US Treasury Bills, exposes the bank to the risk that the spread between the two index rates may change unexpectedly.

Option Risk. The option risk originates from the options embedded in many bank assets, liabilities, and off-balance sheet portfolios. In other words, mortgage loans present significant option risk due to prepayment speeds that change dramatically when interest rates rise and fall. For example, borrowers will refinance and repay their loans and leave the bank with uninvested cash when interest rates have declined. Alternately, rising interest rates cause mortgage borrowers to repay slower, leaving the bank with more loans based on prior, lower interest rates.

In the following, we provide a brief summary for modeling the interest rate risk.

1.2.2 The management of Net Interest Income

The net interest income of a bank is defined as the excess of interest received over interest paid (cf. Hull (2012)). The asset-liability management unit takes the charge of ensuring that net interest income remains roughly through time. One way of doing this job is to ensure that the maturities of the assets on which interest is earned and the maturities of the liabilities on which interest is paid are matched. Referring to the liquidity preference theory, long-term rates should be higher than short-term rates.

1.2.3 Duration

Duration is a widely used measure of a portfolio's exposure to yield curve movements. Suppose y is a bond's yield and B is its market price. The duration D of the bond is given by

$$\frac{\Delta B}{B} = -D\Delta y$$

where Δy is a small change in the bond's yield and ΔB is the corresponding change in its price. Duration measures the sensitivity of percentage changes in the bond's price to changes in its yield. Using calculus notation, we can write

$$D = -\frac{1}{B} \frac{dB}{dy}$$

We consider a bond that provides cash flows c_1, c_2, \dots, c_n at times t_1, t_2, \dots, t_n . Then the price of the bond is defined by the following formula:

$$B = \sum_{i=1}^n c_i \exp(-yt_i)$$

From this, it follows that

$$D = \sum_{i=1}^n t_i \left(\frac{c_i \exp(-yt_i)}{B} \right)$$

Thus, the duration is a weighted average of the times when payments are made, with the weight applied to time t_i being equal to the proportion of the bond's total present value provided by the cash flow at time t_i .

1.2.4 Convexity

The duration relationship measures exposure to small changes in yields. We introduce a factor known as convexity which improves the relationship between bond prices and yields. The convexity for a bond is:

$$C = \frac{1}{B} \frac{d^2 B}{dy^2} = \frac{\sum_{i=1}^n c_i t_i^2 \exp(-yt_i)}{B}$$

Therefore the second-order approximation to change in the bond price is:

$$\frac{\Delta B}{B} = -D\Delta y + \frac{1}{2}C(\Delta y)^2$$

We illustrate the importance of the convexity in Appendix 1.A.

1.2.5 Principal Component Analysis

Principal Component Analysis is a statistical technique in which the original variables are replaced by a smaller number of artificial variables that preserve as much as possible of the variability of the original variables. In other words, PCA reduces the number of the variables and detects structural relationships between variables. We implement this method to build the structure between different yields used to build the yield curve. We consider weekly changes in France Eurobond Yields to identify the behavior of the yield curve and factor loadings from France Eurobond Yields are represented in Table 1.1.

Table 1.1: Factor Loadings for EUR France Eurobond Yields

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10
3M	-0.28	0.49	-0.24	-0.20	0.26	-0.10	0.31	-0.10	0.01	-0.26
6M	-0.29	0.49	-0.21	-0.16	0.01	-0.11	0.11	-0.04	0.22	0.22
1Y	-0.30	0.39	-0.10	0.23	-0.30	0.23	-0.61	0.29	-0.28	0.07
2Y	-0.30	0.09	0.34	0.71	-0.17	0.11	0.28	-0.33	0.18	-0.09
3Y	-0.31	-0.03	0.47	0.08	0.48	-0.41	0.02	0.28	-0.42	0.15
4Y	-0.28	-0.11	0.31	-0.18	0.16	0.09	-0.29	0.29	0.72	-0.16
5Y	-0.28	-0.11	0.22	-0.36	0.02	0.25	-0.16	-0.56	-0.11	0.36
6Y	-0.28	-0.15	0.11	-0.26	-0.08	0.24	0.04	-0.16	-0.15	-0.08
7Y	-0.27	-0.18	0.00	-0.16	-0.19	0.24	0.25	0.23	-0.20	-0.51
8Y	-0.26	-0.20	-0.09	-0.09	-0.26	-0.02	0.22	0.23	-0.13	-0.04
9Y	-0.25	-0.22	-0.19	-0.03	-0.32	-0.28	0.14	0.05	0.04	0.35
10Y	-0.23	-0.25	-0.20	0.06	-0.21	-0.39	-0.03	0.04	0.20	0.17
15Y	-0.19	-0.22	-0.31	0.11	0.18	-0.31	-0.42	-0.39	-0.09	-0.45
20Y	-0.15	-0.21	-0.33	0.22	0.30	0.14	-0.02	-0.03	0.05	0.04
30Y	-0.13	-0.20	-0.34	0.20	0.42	0.46	0.14	0.18	0.00	0.29

We present the first ten factors describing the rate moves. The first factor, shown in the column labeled PC1, corresponds to a roughly parallel shift in the yield curve. When we have one unit of that factor, the three-month rate increases 0.28, the 6-month rate 0.29 and so on. The second factor is shown in column labeled PC2. It corresponds to a twist or change of slope of the yield curve. Rates between 3 months and 2 years move in one direction; rates between 3 years and 30 years move in the other direction. The third factor corresponds to a “bowing” of the yield curve. Rates at the short end and long end of the yield curve move in one direction; rates in the middle move in other direction.

The importance of each factor is measured by the standard deviation of its fac-

Table 1.2: Standard Deviation of factor scores

	PC1	PC2	PC3	PC4
Standard Deviation	1.729593	0.858561	0.440685	0.191791
Proportion of Variance	74.31%	18.31%	4.82%	0.91%
Cumulative Proportion	74.31%	92.62%	97.45%	98.36%

tor score. We represent standard deviations of the factor scores in Table 1.2. The table shows that the first factor accounts for 74.31% of the variance and the most three important factors accounts for 97.45% of the variance.

We illustrate the importance of the PCA by an example. We suppose that an investor hold a large portfolio and we represent the exposures to interest rate moves in Table 1.3, a 1 basis point change in the one-year rate causes the portfolio to increase by

Table 1.3: Change of portfolio value for a 1 basis point rate move

Rate	1Y	2Y	3Y	4Y	5Y
Change	15	9	-10	-17	1

\$15 million; a 1 basis point change in the two-years rate causes it to increase by \$9 million dollars; and so on. We use the first two factors to model rate moves:

FACTOR 1

$$0.3 \times 15 + 0.3 \times 9 + 0.31 \times -10 + 0.28 \times -17 + 0.28 \times 1 = -0.52$$

FACTOR 2

$$-0.39 \times 15 - 0.09 \times 9 + 0.03 \times -10 + 0.11 \times -17 + 0.11 \times 1 = -8.73$$

Results show that the exposure of the portfolio to the second shift is about 17 times greater than the exposure to the first shift. However, the first shift is about four times as important in terms of the extent to which it occurs.

1.3 Zero-Coupon Yield Curves

The relationship between the yields of default-free zero coupon bonds and their length to maturity is defined as the term structure of interest rates and is shown pictorially in the yield curve. This relation can be used for risk management and has an important role in pricing fixed-income securities and interest rate derivatives, as well as other financial assets. Because of its numerous uses, an accurate estimate of the term structure has constituted a major question in the empirical literature in economics and finance.

There are two distinct approaches to estimate the term structure of interest rates: the equilibrium models and the statistical techniques. Examples of the first approach include Vasicek (1977), Dothan (1978), Brennan and Schwartz (1979), Cox and Ingersoll Ross (1985) and Duffie and Kan (1996). These models are formalized by defining state variables characterizing the state of the economy which are driven by these random processes and are related in some way to the prices of bonds. It then uses no-arbitrage arguments to infer the dynamics of term structure. In contrast to equilibrium models, the statistical techniques focusing on obtaining a continuing yield curve from cross-sectional coupon bond data based on curve fitting techniques are able to describe a richer variety of yield patterns in reality. The most known models employed in the second approach are Nelson-Siegel (1987) and its extension by Svensson (1994).

Most central banks use either the Nelson-Siegel or the extended version suggested by Svensson (see e.g. Appendix 1.B). Exceptions are Canada, Japan, Sweden, the United Kingdom, and the United States of America which all apply variants of the smoothing splines method. They use government bonds in the estimations since they carry no default risk.

1.3.1 Theoretical Background

The spot interest rate of a given maturity m is defined as the yield on a pure discount bond of that maturity. The spot rates are discount rates determining the present value of a unit payment at a given time in the future. Spot rates considered as a function of maturity are referred to as the term-structure of interest rates. (cf. Nawalkha *et al.*(2005))

The value of a coupon bond is simply the present value of the stream of future cash flows it provides. That is,

$$B = \sum_{m=1}^T \frac{CF_m}{(1 + R(m))^m}$$

where T denotes the maturity of the bond, CF_m is the cash flow paid by the bond at time $m = 1, \dots, T$ and $R(m)$ is referred to as the spot interest rate for maturity m years.

Our objective is to estimate zero-coupon rates, or forward rates, or discount functions from a set of coupon bond prices. Generally this requires fitting a parsimonious functional form that is flexible in capturing stylized facts regarding the shape of the term structure. A good term structure estimation method should satisfy the following requirements:

- The method ensures a suitable fitting of the data.
- The estimated zero-coupon rates and the forward rates remain positive over the entire maturity spectrum.
- The estimated discount functions, and the term structures of zero coupon rates and forward rates are continuous and smooth.
- The method allows asymptotic shapes for the term structures of zero-coupon rates and forward rates at the long end of the maturity spectrum.

In the following, we focus on five commonly used term structure estimation methods given as the bootstrapping method, the cubic-spline, Merrill Lynch exponential spline, the Nelson-Siegel and the Nelson-Siegel-Svensson method.

Bootstrapping Method

The bootstrapping method consists of iteratively extracting zero-coupon yields using a sequence of increasing maturity coupon bond prices. This method requires the existence of at least one bond that matures at each bootstrapping date. To illustrate this method, we consider N bonds maturing at dates t_1, \dots, t_N , and let $C_{i,t}$ be the total cash flow payments of the i th bond on the date t . The discount rates are obtained by solving the following system of equations:

$$\begin{pmatrix} P(t_1) \\ P(t_2) \\ \vdots \\ P(t_N) \end{pmatrix} = \begin{pmatrix} C_{1,t_1} & C_{1,t_2} & \cdots & C_{1,t_N} \\ C_{2,t_1} & C_{2,t_2} & \cdots & C_{2,t_N} \\ \vdots & \vdots & \vdots & \vdots \\ C_{N,t_1} & C_{N,t_2} & \cdots & C_{N,t_N} \end{pmatrix} \begin{pmatrix} d(t_1) \\ d(t_2) \\ \vdots \\ d(t_N) \end{pmatrix}$$

The zero-coupon rates are obtained from the corresponding discount functions by the following equation:

$$r(t) = \frac{-\ln(d(t))}{t}$$

Then, to build the yield curve, we estimate zero-coupon rates using a simple linear interpolation, and thus the whole term structure of zero-coupon rates is obtained.

The bootstrapping method has two main limitations. First, since this method does not perform optimization, it computes zero-coupon rates that exactly fit the bond prices thus it contains idiosyncratic errors due to lack of liquidity, bid-ask spreads, specific tax effects, and so on. Second, the bootstrapping method requires ad-hoc adjustments when the number of bonds are not the same as the bootstrapping maturities, and when cash flows of different bonds don't fall on the same bootstrapping dates.

Cubic-Spline Method

We build a relationship between the observed price of a coupon bond maturing at time t_m , and the term structure of discount functions. The price of this bond can be expressed as:

$$P(t_m) = \sum_{i=1}^m C_i \times d(t_i) + \epsilon$$

where C_i is the total cash flow from the bond at time t_i . The cubic-spline method addresses the first issue by dividing the term structure in many segments using a series of points that are called knot points. Different functions of the same class (polynomial, exponential, etc...) are then used to fit the term structure over these segments. These functions are limited to be continuous and smooth around each knot point to ensure the continuity and smoothness of the fitted curves, using spline methods. To illustrate the method, we consider a set of N bonds with maturities of t_1, t_2, \dots, t_N years. The range of maturities is divided into $n - 2$ intervals defined by $n - 1$ knot points T_1, T_2, \dots, T_{n-1} , where $T_1 = 0$ and $T_{n-1} = t_N$. Therefore the cubic polynomial spline of the discount function $d(t)$ is defined by the following equation:

$$d(t) = 1 + \sum_{i=1}^n \alpha_i g_i(t)$$

where $g_1(t), g_2(t), \dots, g_n(t)$ are the n basis cubic functions and $\alpha_1, \alpha_2, \dots, \alpha_n$ are unknown parameters that must be estimated.

Since the discount factor for time 0 is 1 by definition, we have:

$$g_i(0) = 0, \forall i \in \{1, 2, \dots, n\}$$

To ensure the continuity and smoothness at the knot points, the polynomial functions defined over adjacent intervals (T_{i-1}, T_i) and (T_i, T_{i+1}) must have a common value also equal first and second derivatives at T_i . These constraints lead to the following definitions for the set of basis functions:

Case 1: $i < n$

$$g_i(t) = \begin{cases} 0 & t < T_{i-1} \\ \frac{(t-T_{i-1})^3}{6(T_i-T_{i-1})} & T_{i-1} \leq t \leq T_i \\ \frac{(T_i-T_{i-1})^2}{6} + \frac{(T_i-T_{i-1})(t-T_i)}{2} + \frac{(t-T_i)^2}{2} - \frac{(t-T_i)^3}{6(T_{i+1}-T_i)} & T_i \leq t \leq T_{i+1} \\ (T_{i+1} - T_{i-1}) \left(\frac{2T_{i+1}-T_i-T_{i-1}}{6} + \frac{t-T_{i+1}}{2} \right) & t \geq T_{i+1} \end{cases}$$

Case 2: $i = n$

$$g_i(t) = t$$

By replacing the discount rate by its expected value in the equation of the price of a bond, we obtain the following:

$$P(t_m) = \sum_{j=1}^m C_j \left(1 + \sum_{i=1}^n \alpha_i g_i(t_i) \right) + \epsilon$$

Assuming that ϵ is the white noise, we can estimate the unknown parameters $\alpha_1, \alpha_2, \dots, \alpha_n$ using ordinary least squares regression.

Merrill Lynch Exponential Spline

The Merrill Lynch Exponential Spline is used to model the discount function, $d(m)$, as a linear combination of exponential basis functions. The discount function is given as

$$d(m) = \sum_{k=1}^N \beta_k \exp(-k\alpha m)$$

The β_k are unknown parameters for $k = 1, \dots, N$ that must be estimated. The parameter α , while also unknown, can be interpreted as the long-term instantaneous forward rate. The larger the number of basis functions used N , the more accurate the fit that is realized.

Given the above theoretical form for the discount functions, now we have to compute the theoretical bond prices. The theoretical price of any bond is simply the sum of the discounted values of its component cash flows, including principal and interest payments. We can express it as follows:

$$\hat{P}_i = \sum_{j=1}^{m_i} (C_{i,j} \times d(\tau_{i,j}))$$

where m_i denotes the number of cash flows generated by the i th bond, $C_{i,j}$ is the specific cash flow associated with time $\tau_{i,j}$ and d is the appropriate discount factor.

The final step in deriving the discount function is to estimate the parameters β_1, \dots, β_N . We assume that the pricing errors $\hat{P}_i - P_i$ are normally distributed with

a zero mean and a variance that is proportional to $1/w_i$, where w_i is the weight assigned to bond i . We next need to find the parameters that maximizes the log-likelihood function:

$$\mathcal{L}(\beta_1, \dots, \beta_N) = - \sum_{i=1}^N w_i (\hat{P}_i - P_i)^2$$

We still have to estimate the non linear parameter α , and there are two options for dealing with it. First, as stated earlier, the value of α can be interpreted as the long-term instantaneous forward rate. As such, we can utilize economic theory and estimate the parameter directly, rather than treat it as an unknown. Second we can use numerical optimization techniques to solve for the value of α that minimizes the residual pricing error.

Nelson-Siegel Model

The Nelson-Siegel Model uses a function form of the forward rate curve that allows it to take a number of shapes. The instantaneous forward rate at maturity m is given by the solution to a second-order differential equation with real and equal roots. The function form is:

$$f(m) = \beta_1 + \beta_2 \exp\left(\frac{-m}{\tau}\right) + \beta_3 \left[\left(\frac{m}{\tau}\right) \exp\left(\frac{-m}{\tau}\right) \right]$$

β_1 , β_2 and β_3 denote respectively the long-term value of the interest rate, the slope and curvature parameter. The time τ represents the scale parameter that measures the rate at which the short-term and medium term components decay to zero. For example, small values of τ result in rapid decay in the predictor variables and therefore will be suitable for curvature at low maturities. Large values of τ produce slow decay in the predictor variables and will be suitable for curvature over long maturities. The spot interest rate for maturity m can be derived by integrating the previous equation from zero to m and dividing by m . The resulting function can be expressed as follows:

$$R(m) = \beta_1 + \beta_2 \left(\frac{\tau}{m}\right) \left[1 - \exp\left(\frac{-m}{\tau}\right) \right] + \beta_3 \left(\frac{\tau}{m}\right) \left[1 - \exp\left(\frac{-m}{\tau}\right) \left(\frac{m}{\tau} + 1\right) \right]$$

Diebold and Li (2006) reformulated the original Nelson-Siegel expression by setting $\lambda = \frac{1}{\tau}$:

$$R(m) = \beta_1 + \beta_2 \frac{1 - \exp(-\lambda m)}{\lambda m} + \beta_3 \left(\frac{1 - \exp(-\lambda m)}{\lambda m} - \exp(-\lambda m) \right)$$

The advantage of the representation is that it provides economic interpretations of the parameters β_1 , β_2 and β_3 .

Long-term value of the interest rate The loading on β_1 is equal to 1, a constant that doesn't depend on the maturity. Moreover, loadings on other parameters decay to zero when the maturity m tends towards infinity, therefore β_1 can be regarded as a level factor as well as the long-term value of the interest rate. Another interpretation that can be

applied on Nelson Siegel Model is that the three loading factors have the same behavior of the first three component vectors obtained from the principal component analysis applied on yield changes explained in the first chapter. For this issue, in the remaining part of this section, we will plot each loading factor against the correspondent component vector obtained from the PCA applied on French Yield Curve. Figure 1.1 plots these two factors.

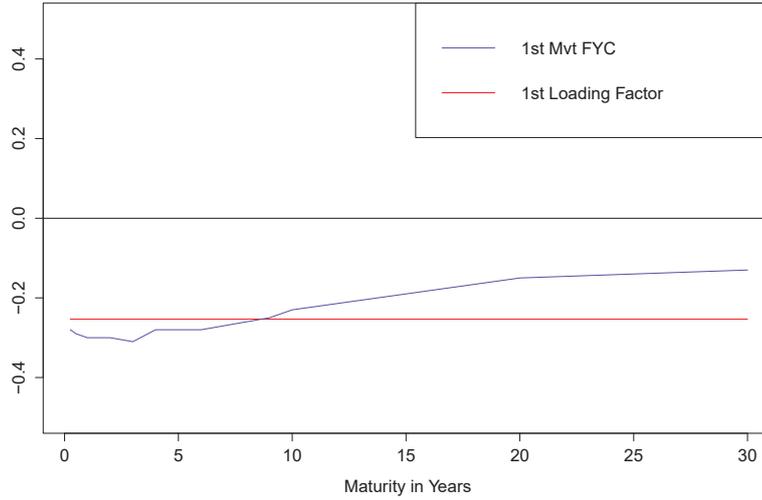


Figure 1.1: β_1 Loading Factor against First Component Factor (France Yield Curve)

Short-term Yields We denote the loading on the second parameter β_2 by $h_1(m)$ which can be written as follows:

$$h_1(m) = \frac{1 - \exp(-\lambda m)}{\lambda m}$$

This function is unity for $m = 0$ and exponentially decays to zero as m grows, hence β_{2t} has an important impact on short term yields and this can be depicted at the short end of the yield curve. Figure 1.2 plots this loading factor against the second component factor obtained from PCA applied on the French yield curve.

Mid-term Yields We denote the loading on the second parameter β_3 by $h_2(m)$ which can be written as follows:

$$h_2(m) = \frac{1 - \exp(-\lambda m)}{\lambda m} - \exp(-\lambda m)$$

This function is equal to 0 for $m = 0$ and exponentially increases to its maximum 0.298 for $m = -1.793/\lambda$ then decays to zero as m grows to $+\infty$, hence β_3 has an important impact on mid-term yields and this can be noticed at the end of the yield curve. Figure 1.3 represents the plot of the loading function $h_2(m)$ against the third component factor.

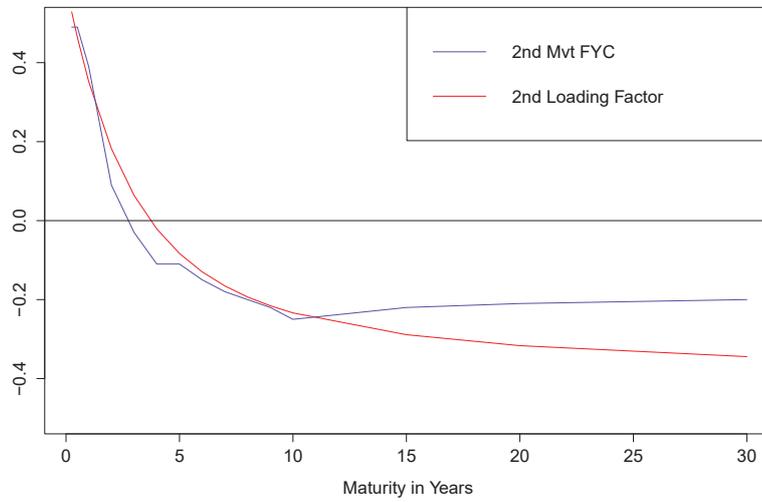


Figure 1.2: β_2 Loading Factor against Second Component Factor(France Yield Curve)

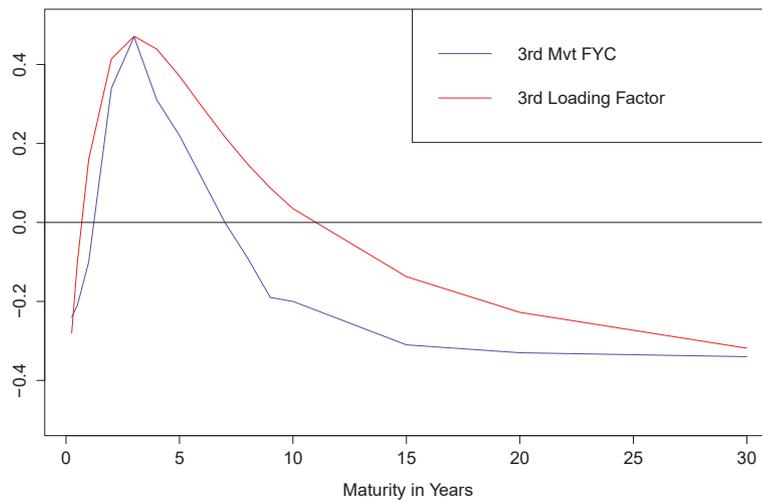


Figure 1.3: β_3 Loading Factor against third Component Factor(France Yield Curve)

Summary With longer time to maturity the spot rate curve approaches β_1 . To avoid negative interest rates β_1 must be positive. If m gets small the limiting value for $R(m)$ is $(\beta_1 + \beta_2)$. Thus, also the sum $(\beta_1 + \beta_2)$ is required to be positive. The parameter λ is bounded to positive values that guarantee convergence to the long term value β_1 . In Table 1.3.1, we provide a brief summary of Nelson-Siegel yield curve for all possible parameters constellations with $\lambda > 0$.

Shape of the Spot Rate	β_1	β_2	β_3	Condition
Increasing	+	-	+	$ \beta_2 \geq \beta_3 $
Increasing	+	-	-	$ \beta_2 \geq \beta_3 $
Decreasing	+	+	-	$ \beta_2 \geq \beta_3 $
Decreasing	+	+	+	$ \beta_2 \geq \beta_3 $
Hump, above β_1	+	+	+	$ \beta_2 < \beta_3 $
Hump, crosses β_1	+	-	+	$ \beta_2 < \beta_3 $
Trough, below β_1	+	-	-	$ \beta_2 < \beta_3 $
Trough, crosses β_1	+	+	-	$ \beta_2 < \beta_3 $

Table 1.4: The term structure shapes resulting from all possible parameters constellations.

Nelson-Siegel-Svensson Model

To increase the flexibility and improve the fitting performance, Svensson (1994) extends Nelson-Siegel's instantaneous forward rate function by adding a fourth term, a second hump-shape (or U-shape), $\beta_3 \frac{m}{\tau_2} \exp\left(\frac{-m}{\tau_2}\right)$, with two additional parameters, β_3 and τ_2 . The function is then set as:

$$f(m) = \beta_1 + \beta_2 \exp\left(\frac{-m}{\tau_1}\right) + \beta_3 \left[\left(\frac{m}{\tau_1}\right) \exp\left(\frac{-m}{\tau_1}\right) \right] + \beta_4 \left[\left(\frac{m}{\tau_2}\right) \exp\left(\frac{-m}{\tau_2}\right) \right]$$

where the unknown parameters $\beta_1, \beta_2, \beta_3$ and τ_1 have the same economic interpretation as the Nelson-Siegel model, and the two additional parameters, β_4 and τ_2 have the same meaning as β_3 and τ_1 . The spot rate is derived by integrating the forward rate and is given by:

$$\begin{aligned} R(m) &= \beta_1 + \beta_2 \left(\frac{\tau_1}{m}\right) \left[1 - \exp\left(\frac{-m}{\tau_1}\right) \right] \\ &+ \beta_3 \left(\frac{\tau_1}{m}\right) \left[1 - \exp\left(\frac{-m}{\tau_1}\right) \left(\frac{m}{\tau_1} + 1\right) \right] \\ &+ \beta_4 \left(\frac{\tau_2}{m}\right) \left[1 - \exp\left(\frac{-m}{\tau_2}\right) \left(\frac{m}{\tau_2} + 1\right) \right] \end{aligned}$$

As the reformulation of Diebold and Li (2006) for the original Nelson-Siegel expression, we reformulate the Nelson-Siegel Svensson model.

$$\begin{aligned} R(m) &= \beta_1 + \beta_2 \frac{1 - \exp(-\lambda_1 m)}{\lambda_1 m} \\ &+ \beta_3 \left(\frac{1 - \exp(-\lambda_1 m)}{\lambda_1 m} - \exp(-\lambda_1 m) \right) \\ &+ \beta_4 \left(\frac{1 - \exp(-\lambda_2 m)}{\lambda_2 m} - \exp(-\lambda_2 m) \right) \end{aligned}$$

Also, parameters have the same economic interpretation of the Nelson-Siegel-Svensson unless the two additional parameters β_4 and λ_2 can be interpreted as the second yield

curve shape employed to long-term yields behavior. We can not state that Svensson model is better than Nelson-Siegel model since he adds another shape to the curve and we will see in the following that some central banks prefer to use Nelson-Siegel model and not Svensson model.

1.3.2 Zero-coupon yield curves in central banks

In this section, we provide brief notes on approaches employed by central banks to build the yield curves reported to Basel Committee. (cf. BIS (2005))

National Bank of Belgium estimates the yield curve using the Nelson-Siegel-Svensson model (1994). The estimation of the parameters is based on minimizing the sum of squared bond price errors weighted by the following function:

$$\frac{1 + \text{yield to maturity}}{\text{duration} \times \text{price}}$$

The Bank of Canada employs the Merrill Lynch Exponential Spline model. The model is based on a functional form for the discount function $d(t)$, as

$$d(t) = \sum_{i=1}^9 a_k \exp(-kbt)$$

where a_k ($k = 1, \dots, 9$) and b are the parameters to be estimated. The estimation method is to minimize the sum of squared bond price errors weighted by the inverse of the duration of each bond.

The Bank of Finland estimates the daily term structure of interest rates using the methods developed by Nelson-Siegel (1987). The estimation is based on the minimization of the yield errors.

In France, the yield curve is determined by the original Nelson-Siegel function and the augmented function as proposed by Svensson. The parameters are obtained by minimizing the weighted sum of the square of the errors on the prices, the weights are the interest rate sensitivity factors of prices. First, the Nelson-Siegel model is being fitted to the data. Second, they introduce the two parameters that are specific to Svensson model to the fitted equation and the new parameters are estimated by the same method used to the first four parameters. The selection between the basic and the augmented Nelson-Siegel functions is based on the Fisher test (at the 5% significance level).

Appendix 1.B provides a summary table for these employed yield curve models with their estimation methods in different central banks.

1.4 The Repricing Gap Model

The repricing gap is certainly the best known and most widely used in Interest Rate Risk Management. It is defined as the bank's exposure to interest rate risk derives from the fact that interest-earning assets and interest-bearing liabilities show differing sensitivities to changes in market rates. (cf. Resti and Sironi (2007))

To cope with interest rate risk arising from fluctuations in markets, risk managers look at the income Gap Analysis. We measure the sensitivity of the bank's interest income to changes in interest rates:

$$\text{GAP} = \text{RSA} - \text{RSL}$$

where RSA is rate sensitive short-term assets and RSL is rate sensitive short-term liabilities. The effect on bank income is given by:

$$\Delta I = \text{GAP} \times \Delta i$$

where Δi denotes the change in interest rates. According to the circular number 250 published by Banking Control Commission of Lebanon, we suppose that all interest rates will increase by 200 bps.

Measuring interest risk with this repricing gap technique, as it is common among banks, involves several problems. Limitations of this approach are mainly due to:

- Assuming uniform changes of interest rates of assets and liabilities, and of rates for different maturities.
- Measuring repricing gaps arising from on-demand assets and liabilities since those instruments don't have a fixed maturity date.

A briefly proposed solution is outlined in the following three steps.

- Identify a reference rate, such as USD 1M Libor rate (for Lebanese Commercial Banks).
- Estimate interest rates sensitivities of assets and liabilities with respect to Libor rate changes.
- Calculate an adjusted gap that can be used to estimate the actual change that the bank's economic value would undergo in response to Libor rate fluctuations.

In Chapter 3, we study the impact of Libor rate on deposits rate in order to estimate their effective duration sensitivity to market rate changes. The method can be fine-tuned even further to deal with on-demand instruments which have no automatic indexing mechanism. For this matter, we will estimate the structure of average delays in rate adjustments with respect to the point in time when a market rate change occurs.

Having built the relationship, we will use a behavioral maturity adjustment concept in Chapter 3 where customers' deposits will be repriced when deposit rates change in response to market rates and we will call this approach "Behavioral duration of customers' deposits."

1.5 Conclusion

In this chapter, we provided an introduction to interest rate risk management in order to show the importance of this risk in banks' management. Also, we presented the theory behind yield curve models being used by central banks, as well as an introduction to the repricing gap model.

Appendix 1.A - Fixed Income Portfolio Example

We consider a two-year 10% with a face value of \$100. Suppose that the yield on the bond is 12% with continuous compounding and coupon payments of \$2.5 are made every three months. The calculation of the duration and Convexity is represented in the table 1.5.

The table shows that the duration is equal to 1.833 whereas the convexity is equal to

Table 1.5: Calculation of Duration and Convexity

Time	Cash flow	Present Value	Weight	Duration	Convexity
0.25	\$ 2.50	\$ 2.4261	0.025	0.006	0.000
0.50	\$ 2.50	\$ 2.3544	0.024	0.012	0.003
0.75	\$ 2.50	\$ 2.2848	0.024	0.018	0.010
1.00	\$ 2.50	\$ 2.2173	0.023	0.023	0.023
1.25	\$ 2.50	\$ 2.1518	0.022	0.028	0.044
1.50	\$ 2.50	\$ 2.0882	0.022	0.033	0.073
1.75	\$ 2.50	\$ 2.0265	0.021	0.037	0.113
2.00	\$ 102.50	\$ 80.6294	0.838	1.677	6.707
Total	\$ 120.00	\$ 96.1784	1.000	1.833	6.973

6.973. So we have the following equation:

$$\Delta B = -96.1784 \times 1.833 \times \Delta y + \frac{1}{2} \times 6.973 \times 96.1784 \times (\Delta y)^2$$

We apply this formula to our example and we outlines results in the table 1.6. The results

Table 1.6: Impact of interest rate change on Bond Price

	Real Change	1st order	2nd order
0.10%	-0.176	-0.176	-0.176
1.00%	-1.746	-1.763	-1.729

show that it is more likely to introduce the convexity in case of high changes in interest rates.

Appendix 1.B- Yield Curve Models in Central Banks

Table 1.7: Yield Curve Models in Central Banks

Central Bank	Estimation Method	Minimised Error	Availability(BIS)
Belgium	Svensson or Nelson-Siegel	Weighted Prices	up to 16 years
Canada	Merrill Lynch Exponential Spline	Weighted Prices	up to 30 years
Finland	Nelson-Siegel	Weighted Prices	up to 12 years
France	Svensson or Nelson-Siegel	Weighted Prices	up to 10 years
Germany	Svensson	Yields	up to 10 years
Italy	Nelson-Siegel	Weighted Prices	up to 30 years
Japan	Smoothing Splines	Prices	up to 10 years
Norway	Svensson	Yields	up to 10 years
Spain	Svensson	Weighted Prices	up to 10 years
Sweden	Smoothing Splines and Svensson	Yields	up to 10 years
Switzerland	Svensson	Yields	up to 30 years
UK	VRP	Yields	up to 30 years
USA	Smoothing Splines	Weighted Prices or Prices	up to 10 years

Bibliography

- [1] BIS PAPERS (2004). Principles for the management and supervision of interest rate risk. *BIS Working Paper*.
- [2] BIS PAPERS (2005). Zero-coupon yield curves: technical documentation. *BIS Working Paper*.
- [3] BRENNAN M., SCHWARTZ E. (1979). A continuous time approach to the pricing of bonds. *Journal of Banking and Finance, Volume 3, Issue 2*, 133-155.
- [4] CORM G. (2005). *L'économie libanaise: risques et perspectives*. www.georgescorm.com.
- [5] COX J., INGERSOLL J., ROSS S.(1985). *A theory of the term structure of interest rates. Econometria, Volume 53, Issue 2*, 285-408.
- [6] DOTHAN L. (1978). On the term structure of interest rates. *Journal of Financial Economics, Volume 6, Issue 1*, 58-69.
- [7] DUFFIE D., KAN R. (1996). A yield-factor of interest rates. *Mathematical Finance, Volume 6, No 4*, 379-406.
- [8] HULL J. (2012). *Risk management and financial institutions*. Wiley Finance.
- [9] NAWALKHA S., SOTO G., BELIAEVA N. (2005). *Interest rate risk modeling*. Wiley Finance.
- [10] NELSON C., SIEGEL A. (1987). Parsimonious modeling of yield curves. *Journal of Business, Volume 60, Issue 4*, 473-489.
- [11] RESTI A., SIRONI A. (2007). *Risk management and shareholders' value in banking*. Wiley Finance.
- [12] SVENSSON L. (1994). Estimating and interpreting forward interest rates: Sweden 1992-1994. *NBER Working Paper No 4871*.
- [13] VASICEK O. (1977). An equilibrium characterization of the term structure. *Journal of Financial Economics, Volume 5*, 177-188.

Chapter 2

Lebanese Yield Curve

2.1 Introduction

The model of Nelson and Siegel (1987) and its extension by Svensson (1994) are used by 75% of central banks as a model for the term structure of interest rates (cf. Gimenoa and Nave (2009), BIS (2005)). Many academic studies show difficulties when estimating the parameters. For instance, a small change in a yield may cause a large change in the parameters due to the colinearity between the loading factors.

Duffee (2002), and Ang and Piazzesi (2003) document that beating the random walk in terms of forecasting is indeed difficult, in particular when using vector autoregressive and standard affine models. Diebold and Li (2006) and Christensen *et al.* (2009) show that dynamic Nelson-Siegel-style factor models forecast particularly well. Even more bright results are obtained with models that incorporate macroeconomic factors. Ang and Piazzesi (2003) and Mönch (2008) improve forecasts for U.S. Treasury yields using affine models which have been augmented to include principal component-based macro factors. Hordahl *et al.* (2006) makes improvements in German zero-coupon bond yields predictability using inflation and industrial production. Ludvigson and Ng (2009) report improvements in forecasting yield curves by adding macro factors containing predictive information that doesn't exist in forward rates and yield spreads. Diebold and Li (2006), de Pooter (2007), Gilli *et al.* (2010) propose a new methodology for calibrating the Nelson-Siegel-Svensson model by working on the colinearity between loadings on factors. Annaert *et al.* (2012) introduces a ridge regression in calibrating Nelson Siegel model.

The closest market to the Lebanese Market is the Turkish sector, several studies discuss the Turkish yield curve, for instance, Alper *et al.* (2004) performs factor analysis on Turkish yield curves estimated by McCulloch and Nelson-Siegel methods, Kaya (2010) includes macroeconomic factors to Nelson-Siegel model in order to show a relationship between parameters and these exogenous factors.

The only available yield curve for the Lebanese Sector is the Bloomberg Valuation (BVAL) curve, these curves are individually constructed using comparable bonds from the same industry, credit rating category and currency, and are further refined by:

- Outlier detection: bonds that breach certain yield-to-maturity thresholds are excluded from curve construction. Outlier detection prevents instability associated with technical trading factors.
- BVAL score: only bonds with a BVAL score of 6 or higher are used in curve construction. (Bonds with transparent prices)
- Bond count requirements: curves are constructed when the following parameters are met:
 - At least 15 bonds across the term structure
 - At least 5 bonds with maturities between 5 and 10 years
 - At least 5 bonds with maturities beyond 10 years
- Minimum bond maturity: only bonds with a minimum of 2 months to maturity are used in curve construction.
- Bond structures:
 - Retail MTN: exclude bonds with an embedded survivor put option or are of retail size.
 - Embedded structures: exclude bonds with call/put options, convertibles and sinking funds.
 - Coupon types: exclude inflation linked, floating rate and structured notes.

This method is confidential and can be purchased from Bloomberg services. But in the following, we will build the Lebanese term structure using only Lebanese government bonds with their market prices.

This chapter discusses existing estimation methods of Nelson Siegel and Nelson Siegel Svensson models. We show the failure of these estimation methods in giving results that provide an economic interpretation for parameters when applied on the secondary Lebanese Market. In fact, in this illiquid market, yields for a given maturity could make a large jump with a small impact on other yields even if close to the maturity however these changes will cause changes in Nelson Siegel Svensson parameters. For example, a change in the yield of one month may cause a large variation in the long-run parameter.

Thus, a new estimation method is required for this term structure of interest rates and we will call it “Constraint Correlation approach” in which we work on the linear correlation between the loading factors. This approach will provide us results with an economic interpretation of the parameters. Nevertheless a economist’s opinion will be helpful for yield curve forecasting.

This chapter is organized as follows. Section 2.2 presents data used for this research. In Section 2.3, we provide the existing estimation methods as well as the constraint correlation approach. Section 2.4 shows the application of the estimation methods of Nelson

Siegel and Svensson models and discusses the results focusing on the stability and the interpretation of the parameters. Section 2.5 shows the application of the PCA on the fitted yields in order to compare the component factors to the loading factors. An eco-political interpretation of parameters behavior is given in Section 2.6 and finally the conclusion in Section 2.7.

2.2 Data

At Bank Audi, or in general at Lebanese Banks, in order to calculate the economic value of different financial products in LBP or USD currency, we build two zero-coupon yield curves. The data used for estimating zero coupon yield curves are divided into two parts, the first considers the Lebanese Pound Yield Curve which is built from bonds issued by the Lebanese Government and denominated in the local currency whereas the second is the US dollar yield curve which is estimated from Eurobonds issued by the Lebanese Government denominated in US dollar for long-term yields, and from the US treasury yields and the Lebanese Credit Default Swap (CDS) points for short term yields. Lebanese Government Bonds denominated in Local currency are divided into two categories:

- Treasury Bills: Zero coupon bonds that mature within one year or less from their time of issuance.
- Treasury Notes: Bonds issued with maturities of one, three, five, seven, 10 years.

We obtain the yield of LBP bonds traded on the secondary market for all maturities. These bonds are known for their illiquidity and their yields might be stale for a while before experiencing some jumps.

For Eurobonds, the market is more liquid especially for newly issued bonds. We get the yields from different available maturities from the secondary market. But, sometimes, we don't have maturing bonds in the short term, therefore we use US treasury yields to which we add the Lebanese Credit Default Swap points as an approximation usually used for the first year of the yield curve.

We have shown how we obtain observed rates for zero coupon bonds and coupon bonds from the secondary market. Therefore we apply the bootstrapping method to obtain observed zero rates for LBP yield curve and USD yield curve. Then we apply linear interpolation to obtain zero rates for all maturities to build the yield curve. The aim of the following section is to find a method more robust than the linear interpolation to build the yield curve from the observed zero rates obtained from the bootstrapping method.

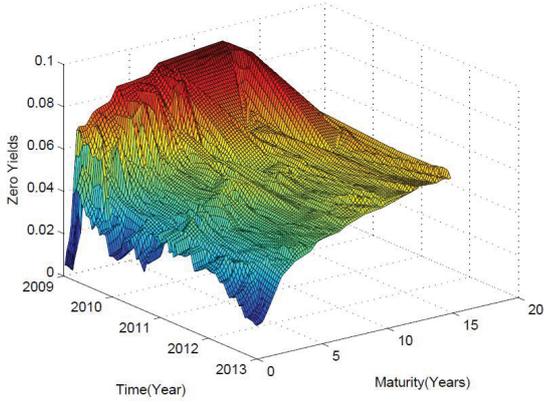


Figure 2.1: Historical US Dollar Yield Curve (2009-2013).

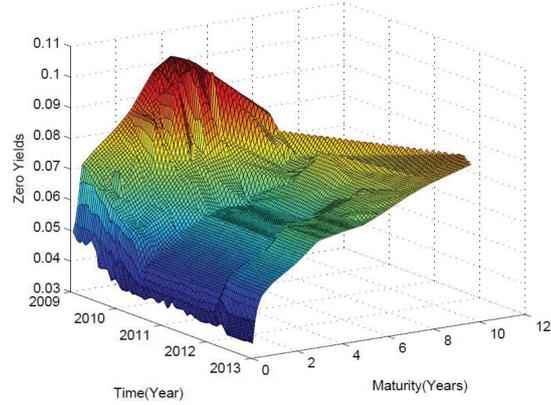


Figure 2.2: Historical Lebanese Pound Yield Curve (2009-2013).

Figures 2.1 and 2.2 represent historical zero yields for USD bonds and LBP bonds respectively. The two surfaces show the absence of any shock on both of the yield curves and the rates decreased slowly during the period of study. Therefore, the long-run parameter β_1 should not have large variations during this period.

2.3 Estimation Methods

We look into the two main models “Nelson-Siegel” model and its extension proposed by Svensson (1994). We have to estimate four linear parameters $\beta_1, \beta_2, \beta_3$ and β_4 , and two non-linear parameters λ_1 and λ_2 . For n observed yields with n different maturities, we have n equations. The simplest way to estimate the parameters is to fix λ_1 and λ_2 , and then estimate the β -values using ordinary least square method or maximum likelihood method. But this method will generate unacceptable errors distribution.

More generally, we can estimate the parameters of the models by minimizing the difference between the model rates \hat{r} and observed rates r thus the optimisation problem is as follows:

$$\begin{aligned} & \underset{\beta, \lambda}{\text{minimize}} && \sum (r - \hat{r})^2 \\ & \text{subject to} && \beta_1 > 0, \beta_1 + \beta_2 > 0 \end{aligned}$$

The two constraints enable the parameters to have a clear economic interpretation, $\beta_1 > 0$ confirms the non-negativity of the interest rate long-run and $\beta_1 + \beta_2 > 0$ show that the overnight zero rate should be positive. However, when using this estimation method, we find a lot of jumps in the behavior of the parameters due to the colinearity between

the loadings factors:

$$\begin{cases} h_1(m) = \frac{1 - \exp(-\lambda_1 m)}{\lambda_1 m} \\ h_2(m) = \left(\frac{1 - \exp(-\lambda_1 m)}{\lambda_1 m} - \exp(-\lambda_1 m) \right) \\ h_3(m) = \left(\frac{1 - \exp(-\lambda_2 m)}{\lambda_2 m} - \exp(-\lambda_2 m) \right) \end{cases}$$

When analyzing the factor loadings, we conclude that a high correlation exists between them for many values of λ_1 and λ_2 . Figure 2.3 shows the correlation between $h_1(m)$ and $h_2(m)$, Figure 2.4 represents the surface of the function

$$\mathcal{R} = \sqrt{\frac{1\rho(h_1(m), h_2(m))^2 + \rho(h_1(m), h_3(m))^2 + \rho(h_2(m), h_3(m))^2}{3}}$$

where ρ denotes the linear correlation.

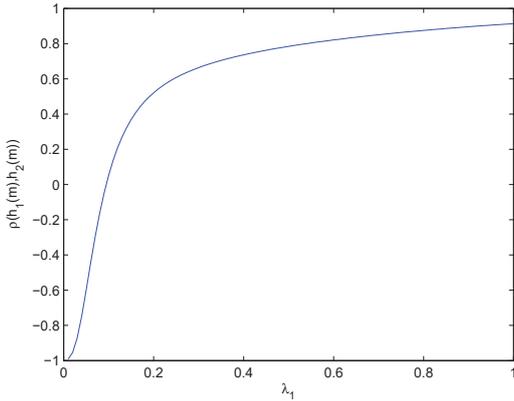


Figure 2.3: Nelson-Siegel Loadings Correlation

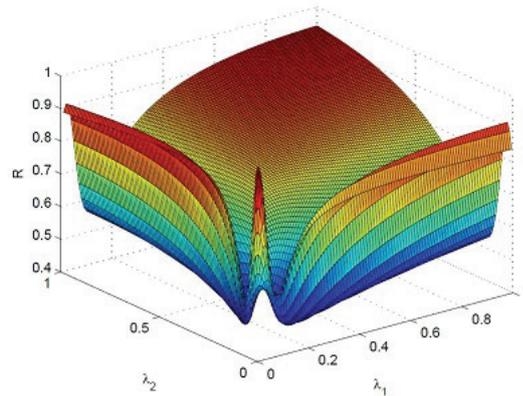


Figure 2.4: Svensson Loadings Correlation

We see that correlation is -1 for λ close to zero, rapidly grows to 1 as λ grows. The correlations are computed for maturities up to 10 years, so we agree that the correlation between the first two loading factors is negligible for a small interval of λ_1 . Also, for the case of Nelson-Siegel-Svensson model, the euclidean measure of the vector of correlations $(\rho(h_1, h_2), \rho(h_1, h_3), \rho(h_2, h_3))$ is small in value for a small range of (λ_1, λ_2) as observed in Figure 2.4. If we only want to obtain an approximation of the current yield curve, we can disregard the colinearity since many different parameter values give similar fits. But in this work, we are interested in giving an economic interpretation to the estimated parameters. For this issue, we add an additional constraint to the optimization problem in order to limit the correlation and obtain more robust parameters for interpretation. In other words, the new constraint adds bounds to the correlation between the two first loading factors for Nelson-Siegel Model and to the euclidean norm of the vector of correlations

$(\rho(h_1, h_2), \rho(h_1, h_3), \rho(h_2, h_3))$ for Nelson-Siegel-Svensson model.

Nelson Siegel Model We tend to minimize the sum of squared errors between the observed and the estimated yields with the given constraints:

- Non-negativity of the long run parameter $\lim_{m \rightarrow +\infty} r(m) = \beta_1$;
- Non-negativity of the “overnight” rate $\hat{r}(0) = \beta_1 + \beta_2$;
- The correlation between the two loadings factors is bounded to 0.2, to assure the linear independence between the endogenous variables.

$$\begin{aligned} & \underset{\beta_1, \beta_2, \beta_3, \lambda_1}{\text{minimize}} && \sum (r - \hat{r})^2 \\ & \text{subject to} && \beta_1 > 0, \beta_1 + \beta_2 > 0 \\ & && |\rho(h_1, h_2)| \leq 0.2 \end{aligned}$$

Nelson Siegel Svensson Model The same constraints stated previously apply to Nelson Siegel Svensson Model, we have added an additional constraint related to the Euclidean norm of the loadings factors correlations.

The fourth constraint have been upper bounded by $0.8/\sqrt{3}$, since by checking Figure 2.4 we notice that $\mathcal{R} \leq 0.8/\sqrt{3}$ for 1% of λ_1 and λ_2 values. In this way, we have selected the 1% smallest values of \mathcal{R} .

$$\begin{aligned} & \underset{\beta_1, \beta_2, \beta_3, \beta_4, \lambda_1, \lambda_2}{\text{minimize}} && \sum (r - \hat{r})^2 \\ & \text{subject to} && \beta_1 > 0, \beta_1 + \beta_2 > 0 \\ & && \mathcal{R} = \sqrt{\frac{\rho(h_1, h_2)^2 + \rho(h_1, h_3)^2 + \rho(h_2, h_3)^2}{3}} \leq \frac{0.8}{\sqrt{3}} \end{aligned}$$

We call this estimation method: “Correlation Constraint Approach”.

2.4 Application

As mentioned above, bonds issued by the Lebanese Government are divided into two types zero coupon bonds and coupon bonds. Thus, we apply the bootstrapping method to calculate the observed zero coupon yields. We use weekly zero rates for maturities 1 day to 15 years from January 2009 until end of January 2013. For the Nelson Siegel Model, we apply the following methods:

R Package Solution: We use the functions included in the package “Yield Curve”.

Correlation Zero Method: We fix λ_1 which set the correlation between loading factors to zero then we estimate the linear parameters using ordinary least square method.

Ordinary Least Square Method: We use this method to compare it to the function of the “Yield Curve” package.

Correlation Constraint Approach We use the approach explained in Section 2.3.

For the Nelson-Siegel-Svensson model, we used the R package `Solution` and correlation constraint approach. Before interpreting the results, we cite the required criterion for the model validation. During our research, we find a lot of papers highlighting difficulties in calibrating these two models due to the limited observed points as we have seen for a given date, the difference between the number of observed points on the market and the number of parameters to be estimated is very small when applying Nelson-Siegel-Svensson model, thus the parameter are very sensitive to variations if the loading factors are linearly dependent.

To study the residuals as a time series, we draw the histogram of available maturities for all the dates in Figures 2.5 and 2.6.

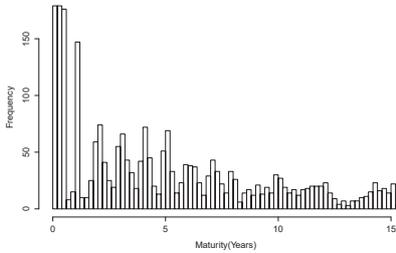


Figure 2.5: Histogram of Maturities for USD Bonds

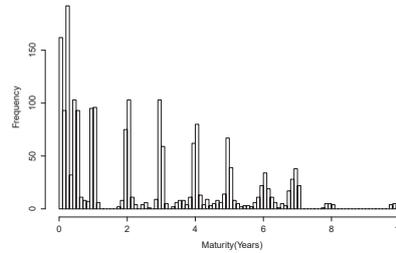


Figure 2.6: Histogram of Maturities for LBP Bonds

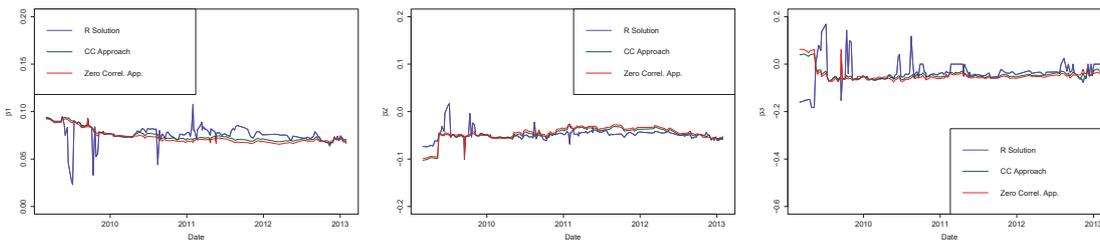
Therefore, we divide yield errors into intervals limited by the following maturities: 0, 1 Month, 3 Months, 90 Months, 1 Year, 2 Years, 3 Years, 4 Years, 5 Years, 6 Years, 7 Years, 8 Years, 9 Years and 10 Years. For each interval, we take yield errors as a time series from the start date until the last date. For a date, if there exit two errors in the same interval we take the mean error. Then we study the distribution of theses errors to see if the model is more robust for short term yields or long term yields.

Table 2.1: Distribution of the maturities

	USD	LBP		USD	LBP
]0M-1M]	179	115]7Y-8Y]	145	33
]1M-3M]	179	237]8Y-9Y]	75	4
]3M-9M]	175	230]9Y-10Y]	97	13
]9M-1Y]	24	214]10Y-11Y]	89	7
]1Y-2Y]	251	187]11Y-12Y]	95	0
]2Y-3Y]	214	241]12Y-13Y]	57	0
]3Y-4Y]	201	165]13Y-14Y]	38	0
]4Y-5Y]	201	209]14Y-15Y]	86	0
]5Y-6Y]	178	101]15Y-16Y]	25	0
]6Y-7Y]	139	162			

Table 2.1 represents the distribution of these maturities in figures. For the USD yields, the maturities are nearly uniformly distributed for the first eight years excluding maturities around 1 year and this mismatch is due to the combination of Eurobond yields and US treasury yields plus Credit Default Swap, thus we will find a lot of observed yields having maturities between 9 months and two years. Also, yields having maturities greater than 9 years are nearly uniformly distributed but with a smaller frequency. On the other hand, for the LBP yields, the maturities are equally distributed for those less or equal to 7 years, but on September 2012, the Lebanese Government started to issue LBP bonds having a maturity greater than 7 years for the time in history and this is why we observe low frequency for the long term maturities.

2.4.1 Nelson-Siegel Applied on the USD Yield Curve

Figure 2.7: Nelson-Siegel Linear Parameters Behavior ($\beta_1, \beta_2, \beta_3$) for US Dollar Yield Curve

In Figures 2.7 and 2.8, we show the evolution of the parameter values for all the dates. The parameter β_1 reveals that the long-term interest rate jumps several times when applying the R package solution. But when we refer to the historical yield curves (Figure 2.1), we see that during these two years there is no shocks neither in the short-term yields nor in the long-term yields, therefore, the estimated parameters β_1 and β_2 should vary in a limited interval fixing the long-run of interest rates and the overnight

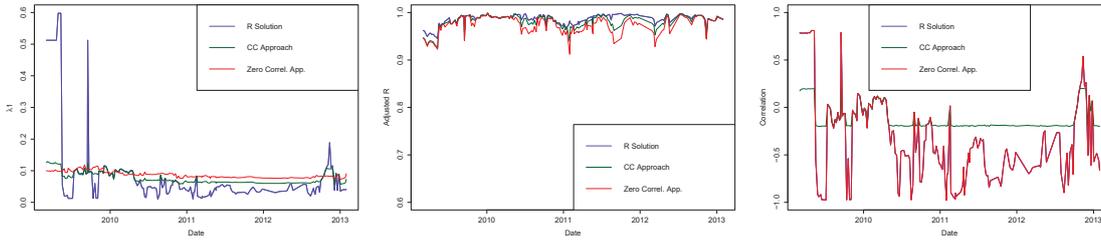


Figure 2.8: λ_1 , Adjusted R^2 , and Correlation between the two loadings factors for US Dollar Yield Curve

rate. Therefore, adding the colinearity constraint has made the parameters more robust and can be interpreted economically. The same concept applies for the third linear parameter β_3 .

In addition, the correlation between the two loading factors seen in Figure 2.8 shows the three curves related to R solution package, Null correlation approach and correlation constraint approach. In this work, the null correlation approach has fixed a λ that tends to minimize the absolute correlation to zero; hence the zero constant red line in the figure. Comparing the two other methods, it is obvious that the interval of the green line curve has less boundaries than the one in blue line, which is of added value to the constraint correlation approach.

On the other hand the adjusted R figures in the center, calculating the fitting performance of the three different methods using:

$$R^2 = 1 - \frac{\sum_{i=1}^n (r_i - \hat{r}_i^2)/(n - k)}{\sum_{i=1}^n (r_i - \bar{r}_i^2)/(n - 1)}$$

clearly shows the good fit of all three methods.

Figures 2.9 and 2.10 represent the errors in a three-dimensional plot. Figure 2.11 shows the time series of generated errors of all four categories, we mark the attention that on the long-run (maturity of 10 years), null correlation method fits very poorly. The QQ-plots generated in Figure 2.12 shows the normality of the errors for R package solution and correlation constraint approach. Whereas, Figure 2.13 represents the auto-correlation function of the errors showing the existence of the auto-correlation in all methods that can be explained by the rigidity of the Nelson-Siegel function.

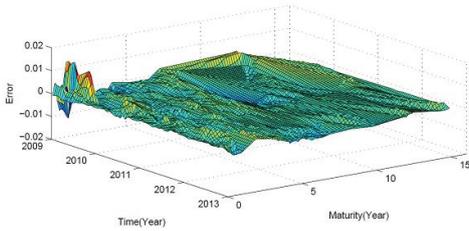


Figure 2.9: USD Nelson-Siegel Errors (R solution)

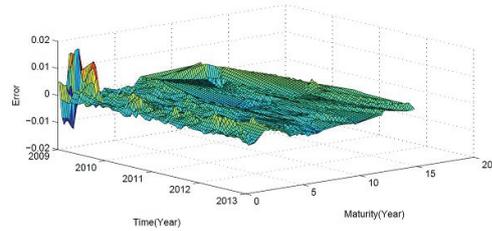


Figure 2.10: USD Nelson-Siegel Errors (Correlation Constraint Approach)

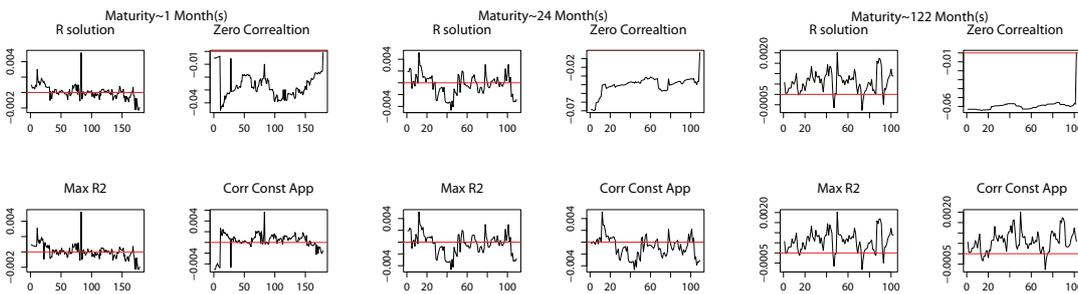


Figure 2.11: Time Series Errors of Nelson Siegel Model Applied on USD Curve

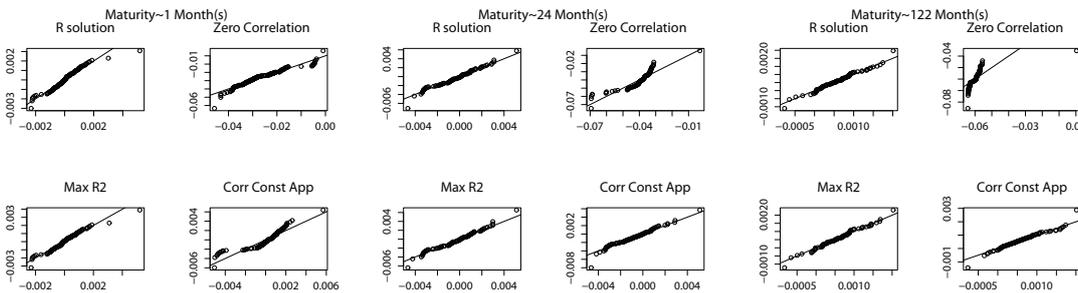


Figure 2.12: QQplot Errors of Nelson Siegel Model Applied on USD Curve

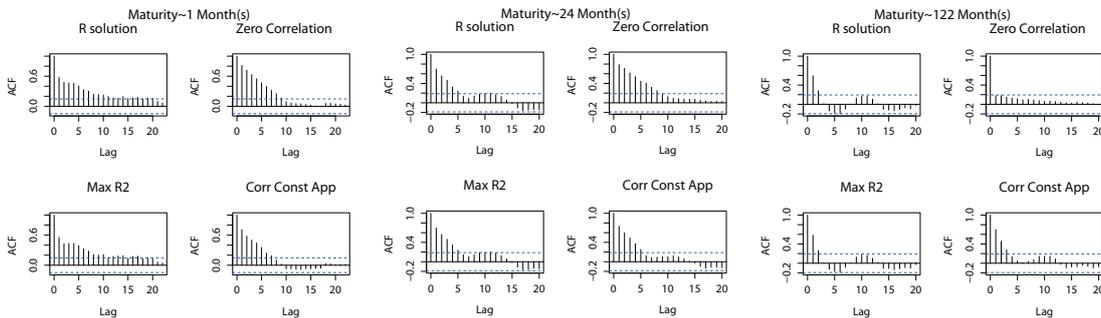


Figure 2.13: Auto Correlation Errors of Nelson Siegel Model Applied on USD Curve

2.4.2 Nelson-Siegel Applied on the LBP Yield Curve

In Figures 2.14 and 2.15, the time series of estimated parameters are also plotted for all the dates. As we have shown for the USD yield curve, the long-run parameter β_1

jumps several times due to small changes in yields when using the R-package solution. According to the historical Lebanese Pound Yields, the estimated parameters β_1 and β_2 should have small variations since the Lebanese Bonds are illiquid and there is absence of any shock in their prices. Therefore, adding the colinearity constraint is essential to maintain the variation of the parameters.

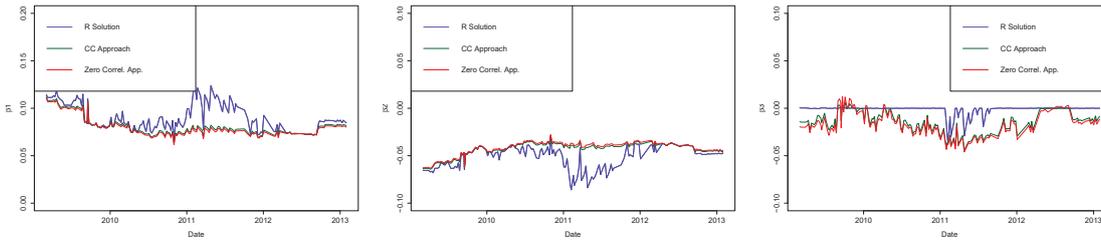


Figure 2.14: Nelson-Siegel Linear Parameters Behaviour ($\beta_1, \beta_2, \beta_3$) for US Dollar Yield Curve

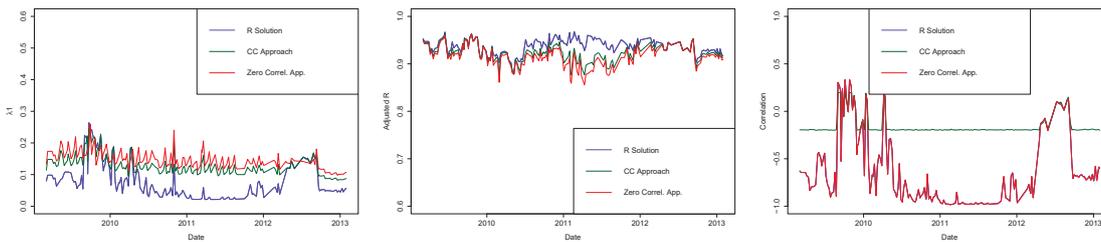


Figure 2.15: λ_1 , Adjusted R^2 , and Correlation between the two loadings factors for US Dollar Yield Curve

Similarly to the previous case, the correlation between the two loading factors is represented in Figure 2.15 the three curves related to R solution package, null correlation approach and constraint correlation approach solution. The null-correlation approach is observed in the zero constant red line in the figure. Comparing the two other approaches, it's obvious that the interval of the green line curve has less boundaries then the one in blue line as in the previous case. Once again, the constraint correlation approach outperforms the other proposed methods.

Turning to the adjusted R figure in the center, we observe that the fitting performance isn't as well as we are seeking, therefore adding Svensson model parameters may achieve the objective.

Figures 2.16 and 2.17 plot the errors in a three-dimensional space. The first figure 2.18 shows the time series of generated errors of all four categories, we mark the attention that on the long-run (maturity of 10 years), null-correlation approach fits very poorly. The QQ-plots generated in figure 2.19 shows the normality of the errors for R

package solution and accepted correlation approach. The figure 2.20 represents the auto-correlation function of the errors also showing the existence of the auto-correlation in all methods.

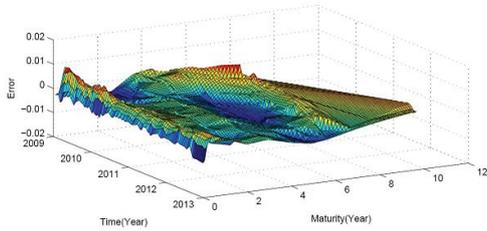


Figure 2.16: LBP Nelson-Siegel Errors (R solution)

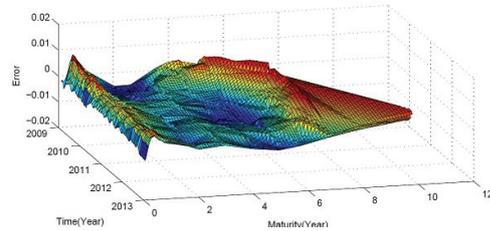


Figure 2.17: LBP Nelson-Siegel Errors (Acceptable Correlation solution)

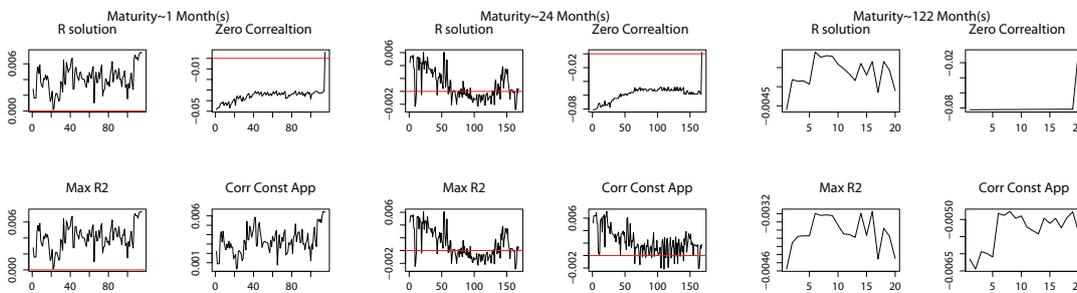


Figure 2.18: Time Series Errors of Nelson Siegel Model Applied on LBP Curve

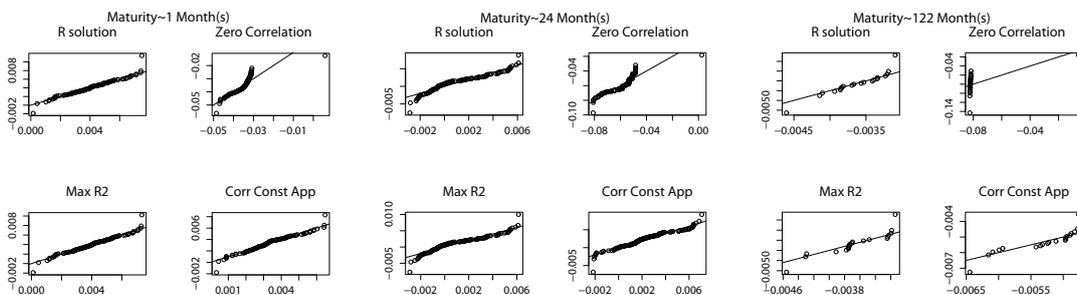


Figure 2.19: QQplot Errors of Nelson Siegel Model Applied on LBP Curve

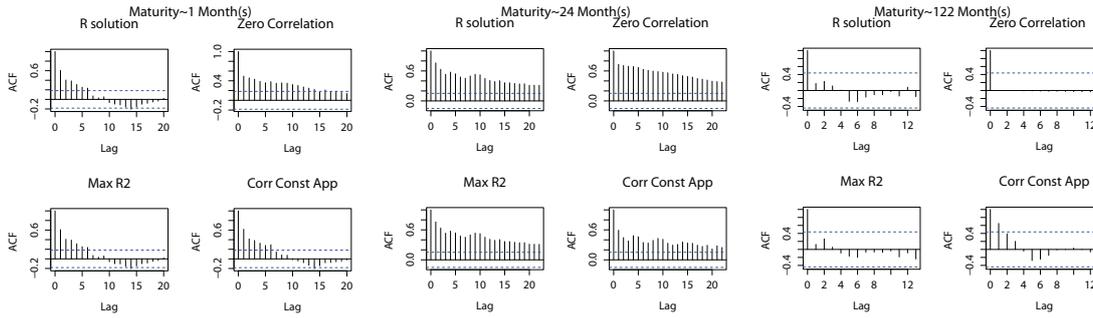


Figure 2.20: Auto Correlation Errors of Nelson Siegel Model Applied on LBP Curve

2.4.3 Nelson-Siegel-Svensson Applied on the USD Yield Curve

The behaviors of the parameter values are represented in Figures 2.21 and 2.22. We have shown that in the Nelson-Siegel model there exists several jumps in the long-run parameter due to the collinearity of the two loading factors; whereas in the Nelson-Siegel-Svensson, we have three loading factors and the last two are very similar. Hence, applying R solution will lead to observe lots of jumps in the time series of β_1 due to small changes in yields. We reject the absence of these jumps by referring to the historical Lebanese Pound Yields which shows small changes in yields and by the collinearity hypothesis for the ordinary least square method. Nevertheless, we observe a large set of dates where the long-run is negative or greater than 0.12 when neglecting the correlation criteria. The same analysis is done for β_2 to maintain the long-run of interest rates and the overnight rate. Thus, in order to obtain more robust parameters we should add the collinearity constraint.

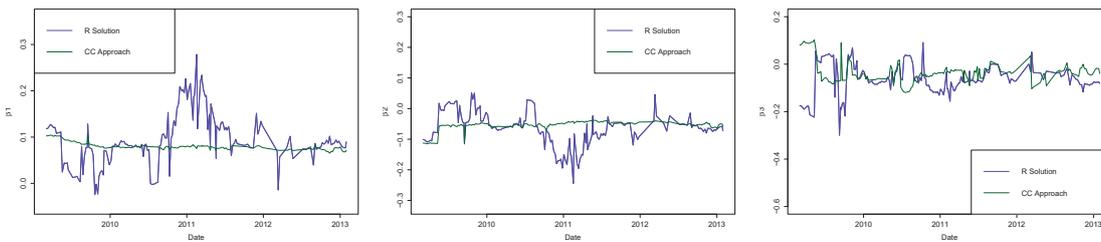


Figure 2.21: Nelson-Siegel-Svensson Linear Parameters Behaviour ($\beta_1, \beta_2, \beta_3$) for US Dollar Yield Curve

Concerning the fitting performance, we observe that the adjusted R^2 is higher than the Nelson-Siegel model for the both approaches. Therefore, for the Lebanese Government Eurobonds, it is better to apply the Svensson model. Figures 2.23 and 2.24 show the errors in a three-dimensional plot where errors seems to be small as a consequence of the high adjusted R^2 . Figures 2.25, 2.26 and 2.27 show the time series of generated errors of the two fitting models, we mark the attention that on the mid and long-run maturities, the two methods have residuals almost normally distributed. But, for the short-term yields, the residuals of the acceptable correlation approach are far from the normal distribution

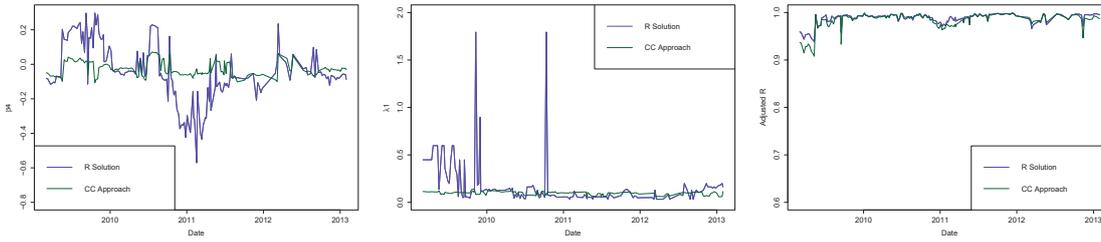


Figure 2.22: β_4 , λ_1 , Adjusted R^2 for US yield curve

and it can be explained by the mismatch between the yields used for the short-term and long-term bonds. This mismatch will be discussed in details in the conclusion of this section. The mismatch illustrated by Svensson adds a positive value to be admitted. Also, an auto-correlation exists in all residuals deduced from the two fitting approaches.

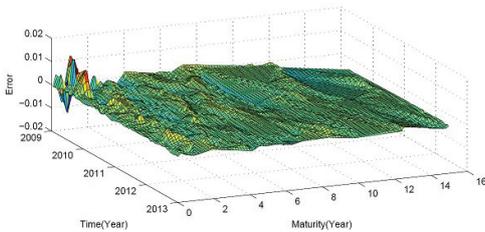


Figure 2.23: LBP Nelson-Siegel-Svensson Errors (R solution)

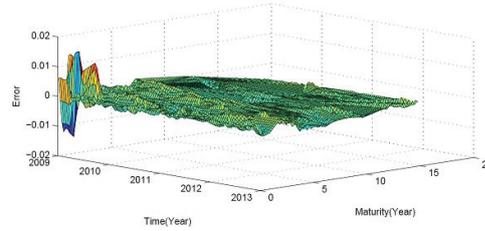


Figure 2.24: LBP Nelson-Siegel-Svensson Errors (Acceptable Correlation solution)

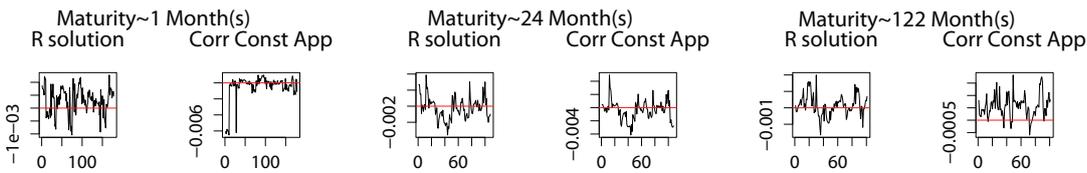


Figure 2.25: Time Series Errors of Nelson-Siegel-Svensson Model Applied on LBP Curve

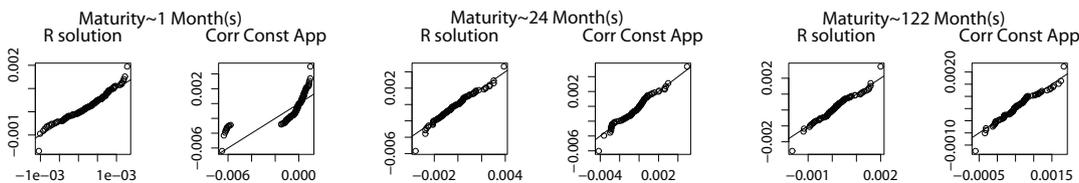


Figure 2.26: QQplot Errors of Nelson-Siegel-Svensson Model Applied on LBP Curve

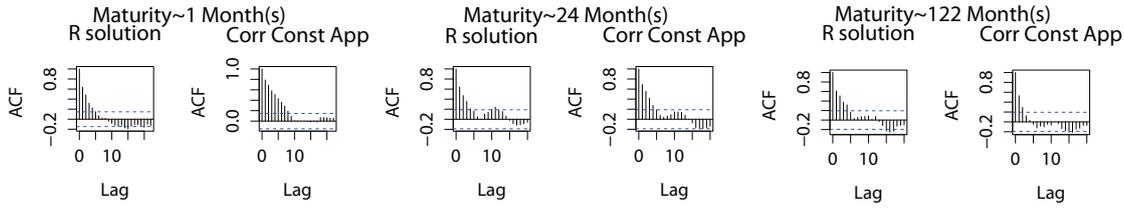


Figure 2.27: Auto Correlation Errors of Nelson-Siegel-Svensson Model Applied on LBP Curve

2.4.4 Nelson-Siegel-Svensson Applied on the LBP Yield Curve

In Figures 2.28 and 2.29, we draw the evolution of the parameter values also for all the dates. The same analysis done on the USD yield curve Nelson-Siegel-Svensson method is applied for this case. β_1 and $\beta_1 + \beta_2$ must have small variations for all the dates.

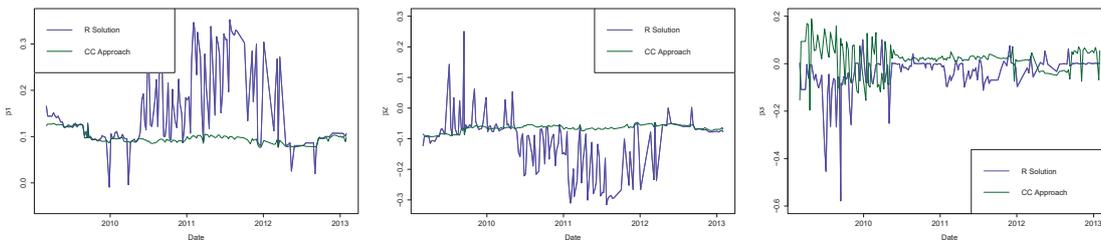


Figure 2.28: Nelson-Siegel-Svensson Linear Parameters Behaviour ($\beta_1, \beta_2, \beta_3$) for LBP Yield Curve

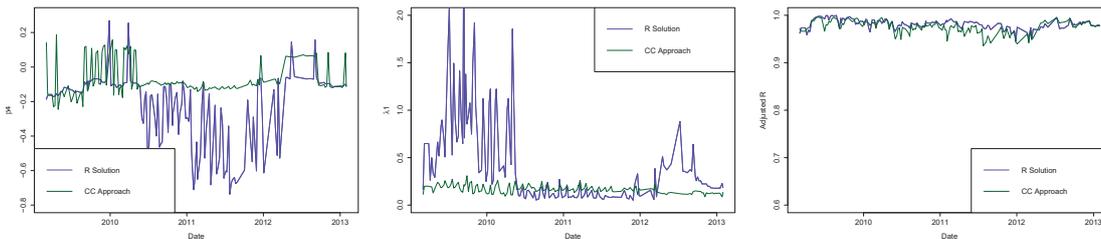


Figure 2.29: $\beta_4, \lambda_1, \text{Adjusted } R^2$ for LBP yield curve

The same interpretation for the adjusted R^2 which is higher in this case than the case of Nelson-Siegel model. Figures 2.30 and 2.31 show the small absolute value of errors and the good fit done by the Svensson model. Moreover, the analysis of residuals can be done by interpreting results outlined in the figures 2.32, 2.33 and 2.34. In fact, residuals are normally distributed for the two methods which also adds value to Svensson model but the autocorrelation problem isn't resolved yet.

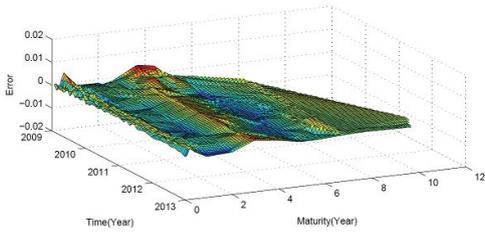


Figure 2.30: LBP Nelson-Siegel-Svensson Errors (R solution)

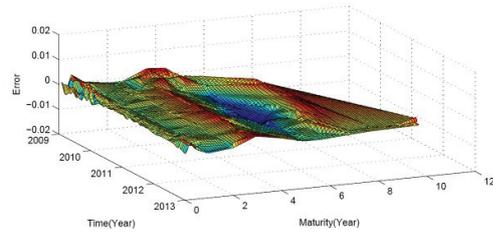


Figure 2.31: LBP Nelson-Siegel-Svensson Errors (Acceptable Correlation solution)

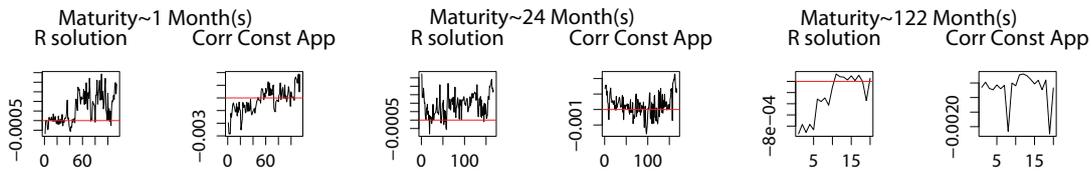


Figure 2.32: Time Series Errors of Nelson-Siegel-Svensson Model Applied on LBP Curve

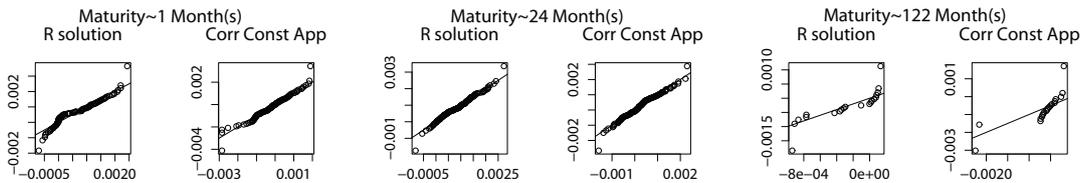


Figure 2.33: QQplot Errors of Nelson-Siegel-Svensson Model Applied on LBP Curve

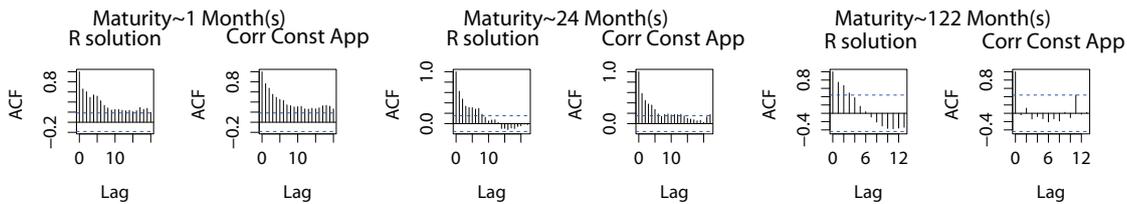


Figure 2.34: Auto Correlation Errors of Nelson-Siegel-Svensson Model Applied on LBP Curve

2.4.5 Conclusion

The Lebanese term-structure of interest rates is here to stay an important concept in pricing all fixed income securities when observing the evolutions of the term-structure of interest rates. An almost over fit estimation can be helpful in building investment strategies, forecasting future interest rates and managing interest rate risk.

To sum up, we have shown in this section that the most proper model is the Nelson-

Siegel-Svensson since the Lebanese secondary market is not liquid enough to give one shape for the yield curve as in Nelson-Siegel. Our results show that Svensson model almost over fit the observed values. On the other hand, concerning the estimation methods, we show that the best method is to minimize yield errors but while adding a collinearity constraint to maintain variations of the parameters which should reflect the real fluctuations of the yields.

Therefore, in the following, we analyze the fitted yield curve by Nelson-Siegel-Svensson model with the correlation constraint approach as an estimation method for the parameters. Table 2.2 represents the fitting performance for all the maturities.

Table 2.2: Fitting Performance for different maturities

	LBP	USD		LBP	USD
1 Month	94.45%	96.75%	5 Years	99.27%	98.07%
3 Months	93.84%	98.76%	6 Years	98.86%	98.29%
6 Months	97.87%	78.81%	7 Years	99.79%	98.99%
1 Year	99.30%	86.60%	8 Years	99.99%	98.87%
2 Years	99.32%	96.82%	9 Years		93.57%
3 Years	97.05%	97.20%	10 Years	99.96%	96.16%
4 Years	99.43%	98.76%	>10 Years		99.87%

The results add a value to Svensson model to be employed as an estimator for the historical yields in order to study the historical evolution of yields with fixed maturities not greater than 10 years.

For the LBP yield curve, as we have mentioned the absence for the bonds having maturities less than seven years for the period before september 2012 when the Lebanese government issued bonds having maturities greater than 7 years and denominated in LBP for the time in history, we would like to do further studies on the sensitivity of the yield curve. For this matter, we estimated the parameters based on bonds maturing less than 7 years then we predict zero yields having maturity greater than 7 years. Figure 3.35 shows the impact of the issuing new bonds on the adjusted R^2 where the blue line denotes the old fitting performance indicator and the red line the new one. Therefore, extending the yield curve is not a good indicator for the yields of bonds issued with a maturity greater than the largest available bond maturity in the market. Of course, issuing new bonds with new maturities will produce a movement in the yield curve because this effect is due to a change in the monetary strategy of the government.

We didn't do the same for the USD yield curve since short-term yields, US treasury plus CDS, hold 50% of observed yields possessing maturities less than 7 years. So we can't do the same study due to liquidity mismatch. But, we have eliminated all the yields having maturities less than or equal to one year and we keep yields observed on the secondary Lebanese market and we show the fitting performance without adding the

calculated yields to the model. Figure 3.36 shows the almost over fit of the mid and long term yields.

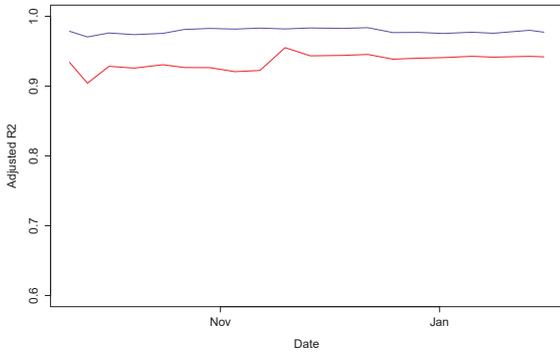


Figure 2.35: Black: Adjusted R^2 of the fitted LBP yield curve; Red: Adjusted R^2 of the fitted LBP yield curve using yields less than 7 years

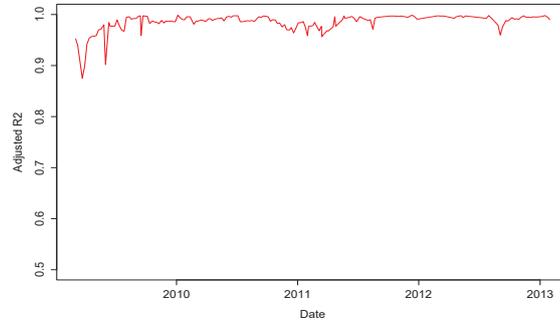


Figure 2.36: Adjusted R^2 of the USD yield curve fitted only on observed Eurobonds yields on the secondary Lebanese Market

2.5 Lebanese Yield Curves Movements

To adjust the movement of the Lebanese Yield Curve, we apply the principal component analysis as we have done for the Euro France Eurobond Yields in Section 1.1.5.

For the USD yield curve, we build the historical yield curves using Nelson-Siegel-Svensson model with correlation constraint approach. Then we calculate the first difference yields and we apply the principal component analysis. The results are outlined in Table 2.3 (Appendix 2.A). A brief summary of the USD yield curve is outlined hereafter:

Movement 1: Twist or change in the slope of the yield curve (Probability=57.6%)

Movement 2: Parallel Shift (Probability=37.2%)

Movement 3: Bowing in the yield curve (Probability=3.6%)

Generally, first loading factors should be similar to the first loading factors of Nelson-Siegel Svensson model. But in this case, the first loading factor with 57.6% as a proportion of the variance isn't a parallel shift which should have the same behavior as of the first loading factor. This can be explained by the mismatch between the liquidity of the Lebanese Eurobonds and the US treasury bonds since this yield curve is built from these two variables. Moreover, US treasury yields are more volatile than Lebanese Eurobonds and can behave differently due to many reasons, hence this mismatch can be observed in studying yield curve movements when applying principal component analysis.

To resolve this problem, in Figure 2.37, we eliminated the first component factor from the curve in order to compare the next four component factors to the four Svensson loading factors. Curves confirm both loading factors and component factors having the same behavior after eliminating the first one. Nevertheless, we eliminated the yields having maturities less than 1 year which are extracted from US treasury yields and the credit default spread, we fit the Svensson yield curve to the available observed yields on the Lebanese Market then we estimate short term yields by extending the estimated curve to the short term yields. And we apply the principal component analysis and the results are published in Table 2.5 (Appendix 2.A) which show the absence of a parallel shifting in the first component factor and we can conclude that adding US treasury plus CDS isn't the best solution to construct the term-structure of USD zero yields but is better than eliminating them and extending the curve for the short term yields.

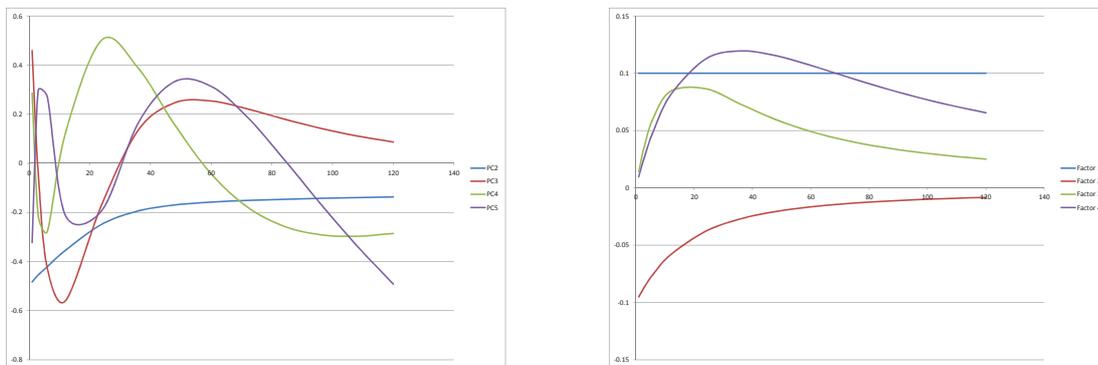


Figure 2.37: Component Factors (USD Yield Curve) compared to Nelson-Siegel-Svensson loading factors

Turning to the Lebanese Yield Curve, the output obtained from the principal component analysis is in Table 2.4 (Appendix 2.A) and hereafter outlines a brief summary of the results:

Movement 1: Parallel Shift (Probability=88.5%)

Movement 2: Twist or change in the slope of the yield curve (Probability=7.6%)

Movement 3: Bowing in the yield curve (Probability=1.7%)

The first three principal components explain almost all of the variance of interest rate changes. The first factor accounts for 88.5% of the total variance, while the second and third factors accounts for 7.6% and 1.3%, respectively. In sum the first three principal components explain 98% percent of the variability of the data, which indicate that these factors are sufficient for describing the changes in the term structures. We represent them in the figure 2.38 with a comparison to the loading factors of Nelson-Siegel-Svensson model.

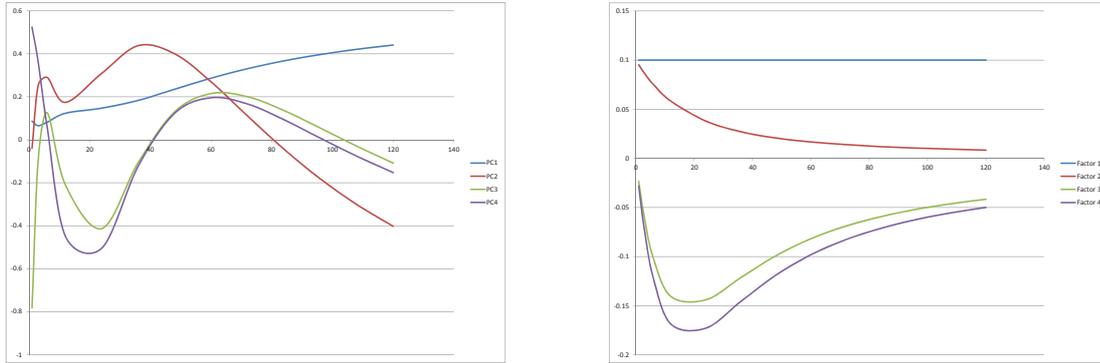


Figure 2.38: Component Factors (LBP Yield Curve) compared to Nelson-Siegel-Svensson loading factors

The first component basically represents a parallel change in LBP yield curve, and this is why we usually call β_1 as the level or the height factor. The second principal component represents a change in the steepness, and is referred to as the slope factor as β_2 . The third principal component is called the curvature curve the same interpretation of β_3 , since it basically affects the curvature of the Yield curve by inducing a butterfly shift.

2.6 Economical Interpretation

Both methods, “Ordinary Least Squared errors” used by R packages and “Correlation constraint approach”, give a good fit of observable yields. However, a good approximation to yield curve should not only fit well in sample but also forecast well out-of sample (cf. Diebold and Li (2006)). For this issue, we studied the stationarity of Svensson parameters obtained from the correlation constraint approach by using Augmented Dickey Fuller test which shows that they are integrated of order 0 or 1, moreover Johansen Cointegration test confirms the absence of any cointegration relationship between parameters integrated of order 1. Therefore, these time series can be forecasted using autoregressive integrated moving average models. But when observing time series, we conclude that a regime switching model performs better in this case; moreover the evolution of the parameters can be economically interpreted. It is also known that the Lebanese economy depends largely on the political situation since it is considered as a touristic country for people coming from the Arabian Gulf.

Unfortunately, the historical data is not sufficient to build econometric models for prediction, so we give a brief economical interpretation of the evolution of parameters. Figure 2.39 shows the existence of five different regimes and Figure 2.40 plots their relative yield curves.

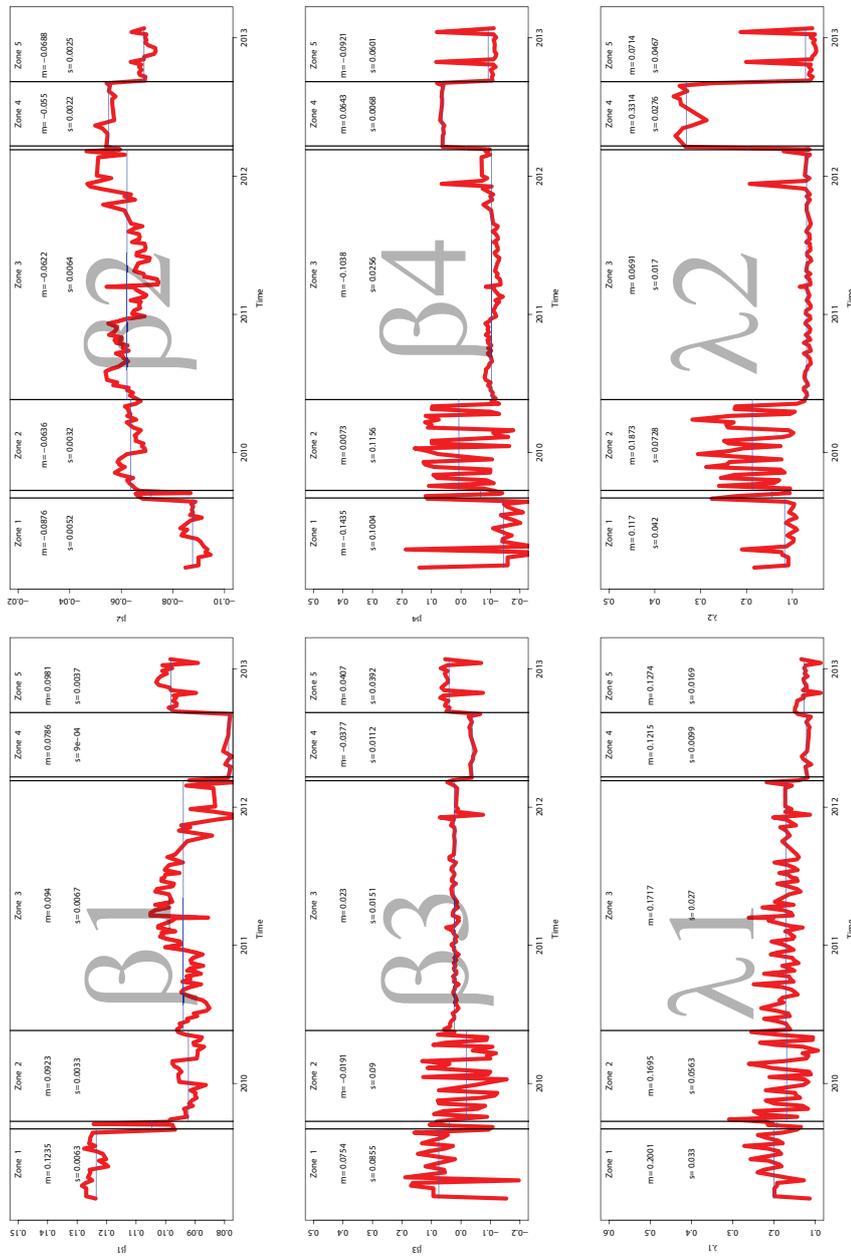


Figure 2.39: Estimated Svensson Parameters using Constraint Correlation Approach divided into 5 regimes.

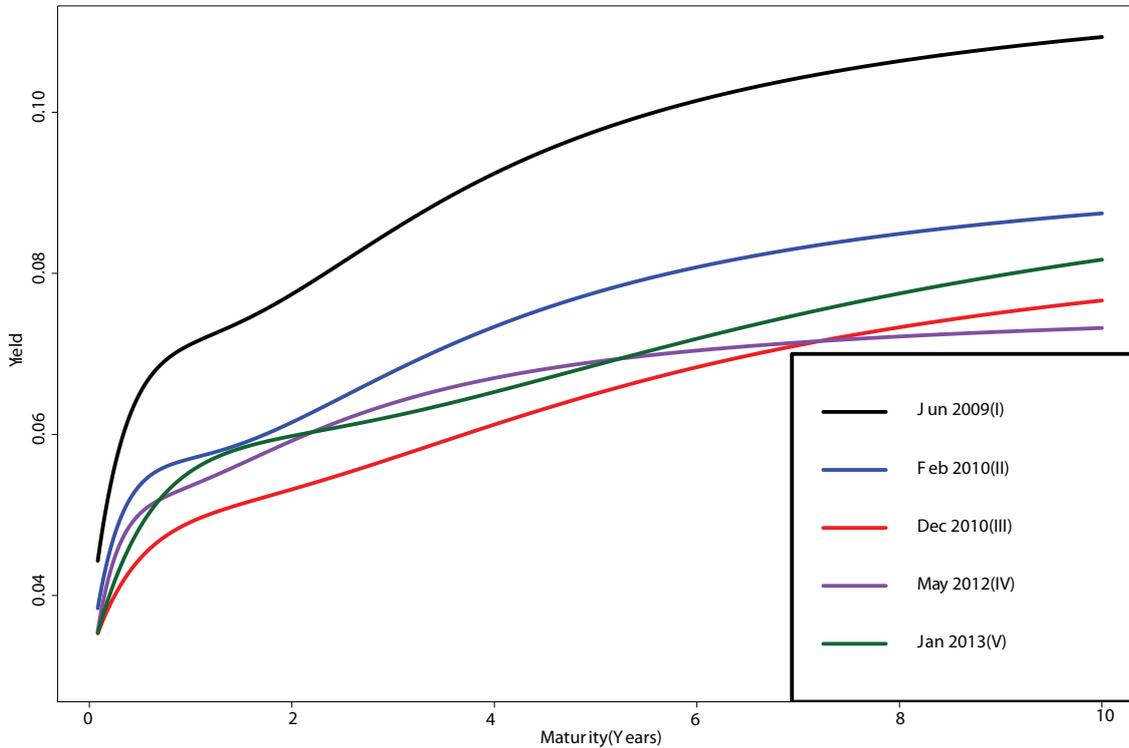


Figure 2.40: Lebanese Pound Fitted Yield Curve on 5 dates of different regimes.

The first phase is between January 2009 and September 2010, we observe high yields for all maturities during this period because Lebanon was emerging from four years of violence and political problems. It started by the assassination of the former prime minister Rafik Hariri and the cedar revolution causing the withdrawal of Syrian troops from Lebanon in 2005. The political assassinations didn't end in 2005; a series of bombs attacks rocked Lebanon during the next two years. On 12th of July 2006, Israel declared the war on Lebanon. This conflict killed 1200 Lebanese citizens and severely damaged Lebanese civil infrastructure. Nevertheless, in 2007, there was a conflict between Fatah al-Islam, an Islamist militant organization, and the Lebanese Army; and several terrorist bombings took place in and around Lebanon's capital, Beirut. The war ended in September 2007. In 2008, after a conflict with Future movement militiamen loyal to the government, Hezbollah took control of all the capital. It ended with a deal between Lebanese leaders on May 21, 2008 to terminate the 18 month political dispute that exploded into fighting and nearly drove the country to a new civil war.

In the second and third phase, the yield curve varied by a negative parallel shift which shows amelioration in the Lebanese economical situation and we observe a higher volatility in the second phase which can be called a transformation phase from first regime to the third. In fact, after the elections of 2009, the Lebanese government was constituted of all Lebanese political movements. Moreover, in 2008, there was the global financial crisis which led to a rise of liquidity in the Lebanese banking system and of the reserves

at the central bank. Thus, banks had excess liquidity with restricted investments opportunities incurred unnecessary higher costs of funds which led to a decline in interest rates independently of international markets. The Lebanese economy continued to demonstrate remarkable resilience to the global financial crisis, as signs of improvement in domestic conditions gave a boost to real economy activity with raising its real GDP growth estimated to 7.0% for 2009. We also mention the growth of passengers at the airport (26.3%) and the evolution of real estate prices which increased 200% during 2 years.

On the 15th of March 2011, a popular uprising started against the Baathist Regime of Bachar El Assad in Syria. This uprising was peaceful at the beginning, but it was slowly transformed into a military confrontation between the newly formed Free Syrian Army (FSA) and the pro Assad (pro governmental) Syrian Arab Army (SAA) in the first quarter of 2012. Taking into consideration that Lebanon shares with Syria a common border that extends from the north east to the south west of the country, making Syria the obligatory passage from Lebanon to the Arab Gulf countries, this conflict affected negatively the Lebanese economical situation: the Arab tourists were not able to reach Lebanon through land, and the commercial transitions between Lebanon and the rest of middle-east region was slowed down. Moreover, the Syrian civil war spilled over into Lebanon. Fighting has mainly occurred between Lebanese opponents and supporters of the Syrian government, the spillover effect has resulted in violent unrest and kidnappings of foreign citizens across Lebanon. Islamists across northern Lebanon have arranged sit-ins and blocked roads to protest against the arrest of a Lebanese Islamist linked to the Syrian uprising. The conflict escalated from previous sectarian clashes between pro-Syrian government Alawites and anti-Syrian government Sunni militants in Tripoli in June 2011 and February 2012. Sit-ins were also held in south Lebanon by anti-Hezbollah Salafists, which heightened tensions further. It evolved into a conflict between the anti-Syrian March 14 Alliance and the pro-Syrian March 8 Alliance. The Saudi backed March 14 alliance supporting the Syrian rebels, whereas the Iranian backed March 8 alliance supporting the Syrian government. This caused a slow down in the tourism in Lebanon. Adding to this, Lebanese banks' operations in neighboring Syria have lost a combined of \$400 million during the first 20-month-old conflict there¹. This turmoil can be observed in the transformation from phase 3 to phase 4 when mid term yields shifted up slowly.

At the beginning of the fourth quarter of 2012, the finance ministry issued 10 years treasury bills in Lebanese denominated currency with 8.25% interest for the first time in the history of Lebanon. The subscription in the 10 years bonds was just one of series of steps taken by the Central Bank to raise the interest to more than 8 percent. This guided most Lebanese banks to refrain from subscribing to the short and medium-term T-bills because the yields on them were too small and therefore not very appealing to investors and lenders. This description is explained in the fifth phase after issuing the new government bonds.

¹Source: Reuters (21 November 2012)

2.7 Conclusion

The model of the term structure applied in this chapter allowed us to build the LBP Yield Curve and US Yield Curve based on bonds issued by the Lebanese Government. For the USD yield curve, the term structure is built from two different types of Bonds, for the short-term we used US treasury bonds plus Lebanese Credit Default spread. This mismatch was illustrated in the principal component analysis when the most probable yield curve movement was not a parallel shift. To solve this problem, we tried to estimate the yield curve based only on observed Eurobonds yields issued by the Lebanese Government on the secondary market but the results weren't satisfactory, therefore adding US treasury yields for the short-term is a temporary solution and shouldn't be permanent.

For the Lebanese Yield Curve, we showed that "Correlation Constraint Approach" is a good solution for in-sample data. The Augmented-Dickey Fuller test confirms that obtained parameters are integrated of order 0 or 1, however when observing the time series we conclude that a regime switching may exist in parameters due to exogenous factors. We interpreted the evolution of the parameters by a political-economical analysis. Nevertheless, due to the insufficient number of observed yields, we can't apply time series models for the forecasting. An alternate solution is to take an economic expert opinion for the upcoming economical situation to predict the future possible movement of the yield curve. Today, the upcoming situation can't be expected to become unambiguous before the formation of the new Lebanese government and the elections of a new parliament without waiting for the end of the current Syrian Civil war.

Appendix 2.A - Principal Component Analysis Tables

Table 2.3: Factor Loadings for USD Zero Yields

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10
1M	-0.52	-0.48	0.46	0.29	-0.32	-0.22	-0.15	-0.12	-0.08	-0.03
3M	-0.33	-0.46	-0.04	-0.21	0.30	0.36	0.39	0.40	0.30	0.11
6M	-0.10	-0.42	-0.43	-0.28	0.27	0.10	-0.21	-0.46	-0.43	-0.17
1Y	0.17	-0.35	-0.56	0.12	-0.21	-0.44	-0.25	0.21	0.37	0.18
2Y	0.32	-0.25	-0.17	0.50	-0.19	0.12	0.50	0.14	-0.35	-0.30
3Y	0.31	-0.19	0.14	0.39	0.16	0.35	-0.06	-0.41	0.16	0.49
4Y	0.27	-0.17	0.25	0.16	0.33	0.11	-0.37	0.06	0.28	-0.31
5Y	0.25	-0.16	0.25	-0.04	0.31	-0.19	-0.21	0.33	-0.16	-0.27
6Y	0.23	-0.15	0.22	-0.18	0.19	-0.33	0.10	0.16	-0.31	0.27
7Y	0.22	-0.15	0.18	-0.26	0.02	-0.30	0.29	-0.15	-0.05	0.32
8Y	0.22	-0.14	0.14	-0.29	-0.16	-0.12	0.27	-0.31	0.27	-0.14
9Y	0.21	-0.14	0.11	-0.30	-0.34	0.14	0.04	-0.16	0.27	-0.41
10Y	0.21	-0.14	0.09	-0.29	-0.49	0.45	-0.34	0.31	-0.27	0.26
% of Variance	0.576	0.371	0.036	0.013	0.003	0.001	0.000	0.000	0.000	0.000
Cumulative %	0.576	0.948	0.984	0.997	0.999	1.000	1.000	1.000	1.000	1.000

Table 2.4: Factor Loadings for LBP Zero Yields

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10
1M	0.09	-0.04	-0.78	0.52	0.09	0.25	-0.12	0.13	-0.06	0.02
3M	0.07	0.25	-0.10	0.36	-0.49	-0.44	0.37	-0.41	0.21	-0.06
6M	0.08	0.29	0.13	0.06	-0.66	0.04	-0.31	0.51	-0.30	0.09
1Y	0.12	0.17	-0.21	-0.45	-0.34	0.62	-0.03	-0.34	0.29	-0.10
2Y	0.15	0.31	-0.41	-0.50	0.13	-0.25	0.42	0.13	-0.38	0.21
3Y	0.18	0.44	-0.11	-0.12	0.25	-0.32	-0.35	0.22	0.41	-0.45
4Y	0.24	0.40	0.13	0.12	0.23	0.00	-0.34	-0.29	0.06	0.48
5Y	0.29	0.27	0.22	0.20	0.18	0.20	0.03	-0.25	-0.36	0.05
6Y	0.33	0.11	0.20	0.17	0.11	0.23	0.28	0.06	-0.21	-0.41
7Y	0.37	-0.04	0.14	0.09	0.04	0.15	0.31	0.27	0.17	-0.16
8Y	0.40	-0.18	0.05	0.01	-0.02	0.02	0.17	0.25	0.34	0.30
9Y	0.42	-0.30	-0.03	-0.08	-0.08	-0.13	-0.07	0.03	0.17	0.34
10Y	0.44	-0.40	-0.11	-0.15	-0.13	-0.27	-0.35	-0.31	-0.34	-0.31
% of Variance	0.885	0.076	0.017	0.012	0.008	0.002	0.000	0.000	0.000	0.000
Cumulative %	0.885	0.961	0.978	0.991	0.998	1.000	1.000	1.000	1.000	1.000

Table 2.5: Factor Loadings for USD Zero Yields Basing on Observed Yields on Lebanese Market

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10
1M	0.78	-0.42	0.37	-0.25	-0.09	-0.02	0.04	-0.03	-0.02	-0.01
3M	0.54	0.20	-0.43	0.52	0.26	0.13	-0.25	0.20	0.13	0.06
6M	0.30	0.55	-0.27	-0.15	-0.22	-0.19	0.42	-0.40	-0.27	-0.14
1Y	0.09	0.56	0.16	-0.54	-0.03	0.04	-0.24	0.40	0.32	0.19
2Y	0.01	0.24	0.35	-0.07	0.57	0.28	-0.27	-0.24	-0.36	-0.36
3Y	0.01	0.12	0.32	0.17	0.43	-0.08	0.38	-0.23	0.19	0.55
4Y	0.01	0.11	0.27	0.23	0.14	-0.36	0.32	0.23	0.27	-0.23
5Y	0.01	0.12	0.24	0.23	-0.08	-0.42	-0.05	0.29	-0.12	-0.35
6Y	0.01	0.12	0.23	0.22	-0.22	-0.30	-0.31	0.03	-0.32	0.14
7Y	0.01	0.12	0.21	0.21	-0.29	-0.07	-0.33	-0.26	-0.09	0.34
8Y	0.01	0.11	0.21	0.20	-0.30	0.18	-0.15	-0.34	0.31	-0.01
9Y	0.01	0.11	0.20	0.20	-0.27	0.40	0.13	-0.10	0.38	-0.40
10Y	0.01	0.11	0.20	0.19	-0.21	0.51	0.36	0.45	-0.45	0.20
% of Variance	0.991	0.007	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Cumulative %	0.991	0.998	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Bibliography

- [1] ALPER C., AKADEMIR A., KAZIMOV K. (2004). Estimating yield curves in Turkey: Factor Analysis Approach. *www.researchgate.com*.
- [2] ANNAERT J., CLAES A., DE CEUSTER M., ZHANG H. (2012). Estimating the yield curve using the Nelson-Siegel model: a ridge regression approach. *International Review of Economics & Finance, Forthcoming*.
- [3] ANG A., PIAZZESI M. (2003). A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary Economics, Volume 50*, 745-787.
- [4] BIS PAPERS (2005). Zero-coupon yield curves: technical documentation. *BIS Working Paper*.
- [5] CHRISTENSEN J. H. E., DIEBOLD F. X., RUDEBUSCH G. D. (2009). The affine arbitrage-free class of Nelson-Siegel term structure models. *Working paper, University of Pennsylvania and Federal Reserve Bank of San Francisco*.
- [6] DIEBOLD F., LI C. (2006). Forecasting the term structure of government bond yields. *Journal of Econometrics, Volume 130*, 337-364.
- [7] DE POOTER M. (2007). Examining the Nelson-Siegel class of term structure models. *Tinbergen Institute Discussion Paper*., TI 2007-043/4
- [8] DUFFEE G. R. (2002). Term premia and interest rate forecasts in affine models. *Journal of Finance, Volume 5*, 405-443.
- [9] GIMENO R., NAVE J. (2009). A genetic algorithm estimation of the term structure of interest rates. *Computational Statistics & Data Analysis, Volume 53*, 2236-2250.
- [10] HORDAHL P., TRISTANI O., VESTIN D. (2006). A joint econometric model of macroeconomic and term-structure dynamics. *Journal of Econometrics, Volume 131*, 405-444.
- [11] KAYA H. (2010). Term structure of interest rate and macroeconomic variables: the Turkish case. *International Journal of Economics and Finance Studies*, ISSN: 1309-8055(online).
- [12] LUDVIGSON S. C., NG S. (2009). Macro factors in bond risk premia. *forthcoming in Review of Financial Studies*.

- [13] MÖNCH, E. (2008). Forecasting the yield curve in a data-rich environment: A No-Arbitrage factor-augmented VAR approach. *Journal of Econometrics, Volume 146*, 24-43.
- [14] NELSON C., SIEGEL A. (1987). Parsimonious modeling of yield curves. *Journal of Business, Volume 60, Issue 4*, 473-489.
- [15] SVENSSON L. (1994). Estimating and interpreting forward interest rates: Sweden 1992-1994. *NBER Working Paper No 4871*.

Chapter 3

Pass-Through of Interest Rate Shocks to Lebanese Deposit Rates

3.1 Introduction

The objective of this chapter is to identify the impact of market interest rate changes on interest rates paid by Lebanese banks on term and saving deposits. During the two past decades, many papers show that changes in official and/or money market rates are not fully reflected in short-term bank deposit rates (cf. BIS (1994), Cottarelli and Kourelis (1994), and Borio and Fritz (1995)). Moreover, several researches examine the issue of an asymmetric interest rate pass-through, in other words, the response of bank deposit rates to changes in official and/or money market rates depends in some cases on whether official and/or money market rates are rising or falling (cf. Borio and Fritz (1995), and Mojon (2000)) or bank deposit rates are above or below cointegration levels (cf. Hofmann (2000)). Shin *et al.* (2014) develops a cointegrating nonlinear autoregressive distributed lag (NARDL) model in which short- and long-run nonlinearities are introduced via positive and negative partial sum decompositions of the explanatory variables. Hammoudeh *et al.* (2014) employs NARDL model to examine the pass-through of changes in crude oil prices, natural gas prices, coal prices and electricity prices to the CO₂ emission allowance prices in order to simultaneously test the short- and long-run nonlinearities through the positive and negative partial sum decompositions of the predetermined explanatory variables. Scholnick (1996) examines the asymmetric interest rate pass-through process in Malaysia and Singapore. Cross-country studies by Kleimeier and Sander (2000), Donnay and Degryse (2001), Toolsema *et al.* (2001), Bondt (2002), and Sorensen and Werner (2006) confirm these findings.

However, in our case, figures show a symmetric interest rate pass-through process. For this purpose, we employ an error-correction model to identify the adjustment of interest rates on USD term deposits in Lebanon following changes in market interest rates. The main advantage of this empirical method is that it allows us to capture both short run and long run domestic deposit interest rate dynamics. In addition, we determine a pass-through rate and the time lag in months that will take a market interest rate shock to be absorbed by the Lebanese Banking System. Our results show that an interest rate

shock in global markets is partially passed through over a period of time to USD term and saving deposits interest rates.

Results shows that 6.5% of a market shock will be passed-through immediately while the final pass-through rate is 33%. Profiting from this result, Lebanese banks have the possibility to determine the behavioral duration (repricing date) of customer deposits when market interest rates fluctuate.

A positive shock in interest rates, which is the standard stress test used to measure banks' sensitivity to interest rates, will usually yield a negative impact as assets' duration is higher than liabilities' duration.

When considering behavioral duration of customers' deposits, the duration of liabilities will be higher than the contractual one which will lower the duration gap between assets and liabilities and thus the negative impact of positive interest rate shocks.

The rest of the chapter is organized as follows. Section 3.2 provides the required data for the econometric model. In Section 3.3, we give the econometric equation explaining the relationship between Libor and USD Deposits Rates. The shock analysis is outlined in Section 3.4 and its application in Section 3.5. At the end of this chapter, in Appendices 3.A and 3.B, we present theoretical backgrounds being used in this model¹.

3.2 Data

For our model, we selected data from 2003 until 2011 as we consider data prior to 2003 not relevant to the actual relation we have between USD LIBOR and BDL published deposit rates. The turning point was the Paris Club II conference that was held in November 2002. After the conference, the mood in Lebanon was optimistic which allowed the Central Bank to cut interest rates (lower spread between interest on FC deposits and international market interest rates) while FX reserves were rising (which also helped decrease the rates on deposits). Disruptions on the political level occurred between 2003 and 2011 but had no major long term effect on deposit interest rates. A rise of bank liquidity in the banking system after mid 2008 and of reserves at the central bank coincided with drops in international benchmark rates (most importantly, the Fed Funds target) to face the crisis and its global economic impact. The lower interest rate environment internationally left bank liquidity (excess over lending) with restricted investment opportunities while incurring a relatively high cost of funds. The booming economy at the time (GDP growth in the 8% region) resulting from political stability paved the way for liquidity to be poured into the system also due to a favorable rate spread and a preference of Lebanese expects to take familiar Lebanese Risk vs. unknown risks they would face abroad as bankruptcies and Government support for Banks repeated abroad (home bias).

¹For more information concerning the theory, please refer to Banerjee *et al.* (1993), Hamilton (1995), Letkepoh (2003)

Consequently, this environment allowed Lebanese banks to lower interest offered on deposits during the last months of 2009 benefiting from a reduction in the risk premium required by investors to place their funds in Lebanon. Before explaining this observation in a quantitative way, we introduce the two variables used in the model.

3.2.1 Banking Sector Average Rate (BSAR)

The main variable of the model is the weighted average of term deposits interest rate in Lebanon. The historical time series data is collected by the Lebanese Central Bank and is available on its official website ². With respect to circular BDL18 issued by Lebanese Central Bank in 1992, each Lebanese Bank should report monthly the below table.

Table 3.1: BSAR calculation

	Monthly Average	Weight
Interest Rate on Term Deposits		
Interest Rate on Current Accounts		
Interest Rate on Saving Deposits		
Accounts Maturity		
1 Month		
3 Months		
6 Months		
1 year and above		
TOTAL		100%

Then, the central bank will calculate the weighted average using the following formula:

$$\text{BSAR} = \frac{\sum_i (\text{Customers' Deposits})_i \times (\text{Interest Rate})_i}{\sum_i (\text{Customers' Deposits})_i}$$

3.2.2 London InterBank Offered Rate (LIBOR)

LIBOR is a weighted average interest rate at which a selection of banks are prepared to lend one another unsecured funds on the London money market. The selection of the banks is made every year by the British Bankers' Association with assistance from the Foreign Exchange and Money Markets Committee. The calculation of LIBOR is in charge of Thomson Reuters which collects the rates from all panel Banks then eliminates the highest and lowest 25% and calculates the weighted average of the mid values.

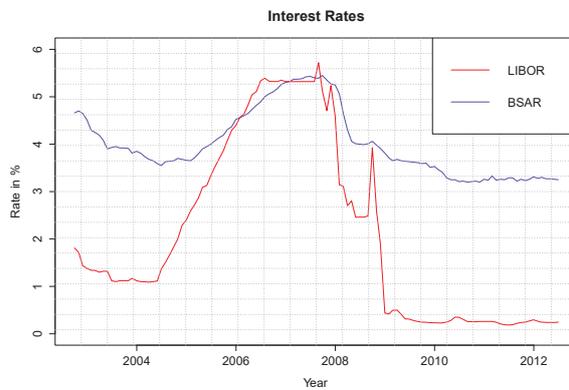
We chose LIBOR as a proxy of market interest rates as it is known as the most important benchmark in the world for short-term interest rates. In addition to the fact that Banks use LIBOR as the reference rate when pricing loans, savings and mortgages.

²Official website of the Lebanese Central Bank: www.bdl.gov.lb

3.2.3 Descriptive statistics of the two variables

Before we start identifying an econometric causality relationship between LIBOR and BSAR, we show in figure 3.1 the historical trend of both interest rate series since 2003. We observe a partial pass-through in addition to a lagged response of BSAR to fluctuations in LIBOR. Although the two time series are non stationary, still we can study their interdependency using first differences if we can identify a cointegration relationship between them.

Figure 3.1: BSAR versus LIBOR



We represent first differences in Figure 3.2 and its histogram in Figure 3.3. We observe that both variables behave in a similar fashion though LIBOR experiences higher volatility than BSAR. In addition, it's worth mentioning that large changes in LIBOR have a high impact on BSAR.

Figure 3.2: First differences of the two rates

Figure 3.3: Histogram of first differences

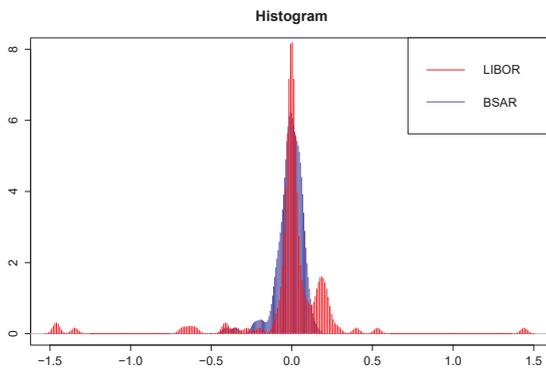
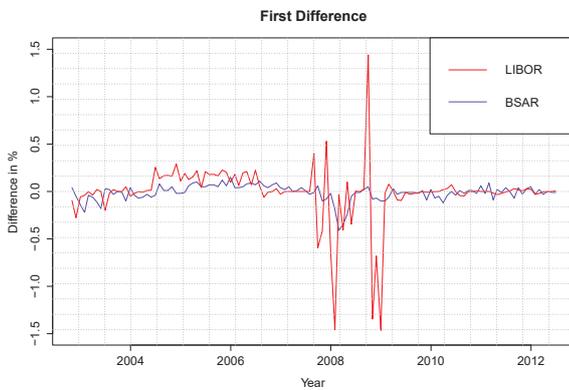
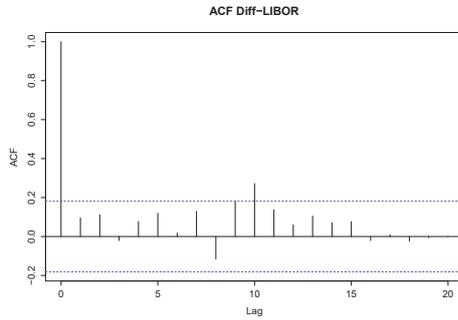
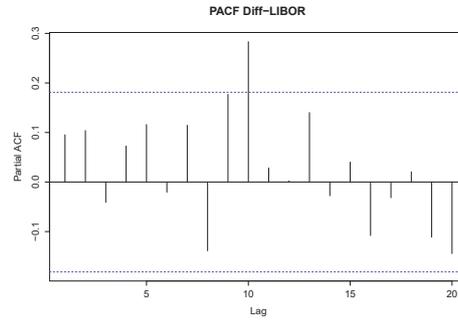
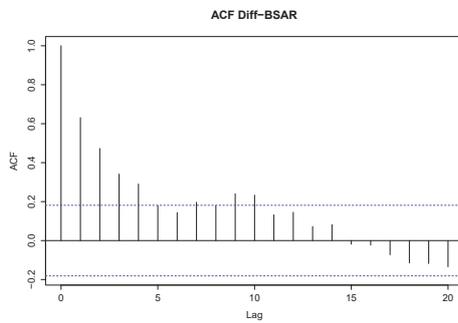
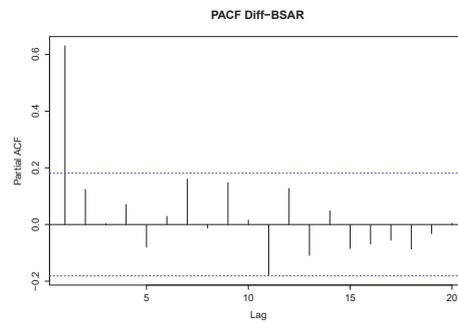


Figure 3.4: ACF of Δ LIBORFigure 3.5: PACF of Δ LIBORFigure 3.6: ACF of Δ BSARFigure 3.7: PACF of Δ BSAR

Figures 3.4, 3.5, 3.6 and 3.7 plot respectively autocorrelation and partial autocorrelation functions of first differences of Libor and BSAR. Then, we employ Augmented-Dickey Fuller test³ to assess the stationarity of the two variables. Results show stationary in first differences. However, working with first differences would only help determining short-run. Given that the aim of this chapter is to model the long-run as well as the short-run relationships, we will need instead to employ an error correction model to investigate interdependency between LIBOR and BSAR.

Table 3.2: Descriptive statistic of first differences

	DBSAR	Dlibor
Mean	-0.01	-0.01
STD	0.08	0.31
VaR _{2.5%}	-0.22	-0.75
Median	-0.01	0.00
VaR _{97.5%}	0.10	0.30
ADF(P-value)	0.05	<0.01

³The results are represented Table 3.2.

3.3 Error Correction model

3.3.1 Definition

The error-correction mechanism is a transformation of a general linear model incorporating past values of both the explained variable (interest on term deposits in Lebanon) and the explanatory variables (LIBOR).

An advantage of the error-correction model is that it provides explicitly the short term deviations from long-run equilibrium in the estimated equation. This ECM will allow us to assess the following:

- Short term reaction of interest rates in response to a shock
- The long run equilibrium coefficients
- The speed of adjustment towards the long run equilibrium after a shock

This turns out to be very useful for the purpose of our study as we are modeling the elasticity and the time it takes for an interest rate shock in the market to be grasped and priced by the Lebanese banking sector on customer deposits.

This will allow us to assess the effective impact of an interest rate shock on the Bank's economic value of shareholders' equity. Since customer deposits constitute the major part of our liabilities, a reduction and delay of an interest rate shock on these deposits in response to an shock in the market will reduce our interest rate risk (reduce Shareholders Economic value sensitivity to interest rate shocks).

The transformation of the auto-regressive process to an error correction model is as follows (cf. Hendry (1995))

We consider the autoregressive distributed lag model with a lag equal to 1:

$$\text{BSAR}_t = \alpha_0 + \alpha_1 \text{BSAR}_{t-1} + \beta_0 \text{LIBOR}_t + \beta_1 \text{LIBOR}_{t-1} + \epsilon_t$$

where $\forall t \geq 0$, $\epsilon_t \hookrightarrow \mathcal{N}(0, \sigma^2)$ and $|\alpha_1| < 1$. We suppose that we have a static equilibrium, noting that $(\text{BSAR}_t, \text{LIBOR}_t)$ are jointly stationary, therefore we have the following long-run values: $\forall t \geq 0$, $\mathbf{E}(\text{BSAR}_t) = \text{BSAR}^*$ and $\mathbf{E}(\text{LIBOR}_t) = \text{LIBOR}^*$. Since $\mathbf{E}(\epsilon_t) = 0$, we can write:

$$\text{BSAR}^* = \frac{\alpha_0 + (\beta_0 + \beta_1)\text{LIBOR}^*}{1 - \alpha_1} = k_0 + k_1 \text{LIBOR}^*$$

Hence, $k_1 = \frac{\beta_0 + \beta_1}{1 - \alpha_1}$ is the long-run multiplier of term deposits interest rate with respect to libor. By a simple calculation, we obtain the equation of the vector error correction model:

$$\Delta \text{BSAR}_t = \alpha_0 + (\alpha_1 - 1)(\text{BSAR}_{t-1} - k_1 \text{LIBOR}_{t-1}) + \beta_0 \Delta \text{LIBOR}_t + \epsilon_t$$

where the coefficient α_2 determines the short-term pass-through, β_2 the final pass-through and $(1 - \alpha_2/\beta_1)$ the speed (in months) at which market interest rate shocks are passed to Term Deposits.

3.3.2 Model Results

We got the following VEC Model:

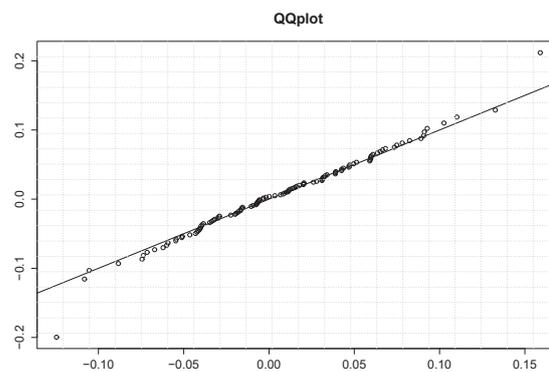
$$\Delta\text{BSAR}_t = 0.4469 + 0.06478\Delta\text{LIBOR}_t - 0.13798(\text{BSAR}_{t-1} - 0.3329\text{LIBOR}_{t-1})$$

	Estimate	Std. Error	t value	Pr(> t)
LIBOR	0.3329	0.0146	22.76	0.0000
(Intercept)	0.4469	0.0709	6.30	0.0000
ΔLIBOR	0.0648	0.0205	3.15	0.0021
Cointegration Relationship	-0.1380	0.0213	-6.48	0.0000

Applying Student test, the P-values of all coefficients are less than 0.5% so the parameters are significant with an adjusted R^2 equal to 33.44%. Thus, we confirm the existence of a long-run equilibrium between the rates and a short-term pass-through after a shock in LIBOR of 6.48% whereas the final pass-through stands at 33.29%. The mean speed of adjustment is around 7 months.

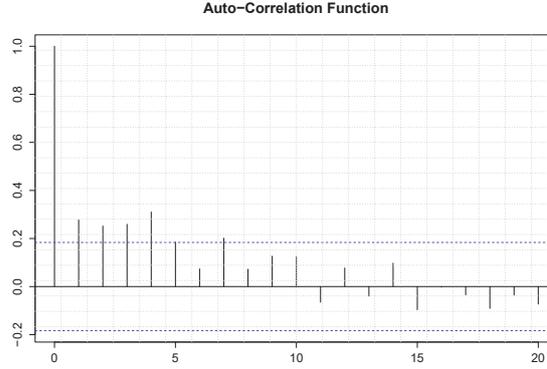
While analysing residuals of the model, we spotted two outliers that occurred in LIBOR during the financial crisis of 2008. The Kolmogorov-Smirnov test confirmed the normality of residuals while it was rejected by the Jarque-Bera test before removing these two outliers. Once we remove these two data points, the Jarque-Bera test couldn't reject the normality hypothesis anymore. Figure 3.8 represents the QQ-plot of residuals and shows the normality.

Figure 3.8: QQplot of Residuals



In order to validate the error-correction equation, we still have to show that residuals process is a white noise process. Figure 2 represents the auto-correlation function that shows a low correlation in lags. Also, Augmented-Dickey-Fuller test confirms stationarity with two lags. Thus, the equation can be employed to measure the impact of shocks in LIBOR on BSAR.

Figure 3.9: Auto-Correlation Function of Residuals



The most important step in the study is to draw the impulse response function; we calculate the impulse response function of the following error correction equation:

$$\begin{aligned} \Delta \text{BSAR}_t &= \alpha_1 + \alpha_2 \Delta \text{LIBOR}_t - \beta_1 (\text{BSAR}_{t-1} - \beta_2 \text{LIBOR}_{t-1}) \\ \Leftrightarrow \text{BSAR}_t &= \alpha_1 + (1 - \beta_1) \text{BSAR}_{t-1} + \alpha_2 \text{LIBOR}_t + (\beta_1 \beta_2 - \alpha_2) \text{LIBOR}_{t-1} \end{aligned}$$

We suppose that the shocked interest rate is indicated by the symbol " ' ". If at time t LIBOR will increase ΔI in value, we have the following:

Time t :

$$\begin{cases} \text{LIBOR}'_t - \text{LIBOR}_t = \Delta I \\ \text{BSAR}'_t - \text{BSAR}_t = \alpha_2 \Delta I \end{cases}$$

Time $t+1$:

$$\begin{aligned} \text{BSAR}'_{t+1} &= \alpha_1 + (1 - \beta_1) \text{BSAR}'_t + \alpha_2 \text{LIBOR}_{t+1} + (\beta_1 \beta_2 - \alpha_2) \text{LIBOR}'_t \\ \Leftrightarrow \text{BSAR}'_{t+1} - \text{BSAR}_{t+1} &= (1 - \beta_1) \alpha_2 \Delta I + (\beta_1 \beta_2 - \alpha_2) \Delta I \end{aligned}$$

Time $t+2$:

$$\begin{aligned} \text{BSAR}'_{t+2} &= \alpha_1 + (1 - \beta_1) \text{BSAR}'_{t+1} + \alpha_2 \text{LIBOR}_{t+2} + (\beta_1 \beta_2 - \alpha_2) \text{LIBOR}_{t+1} \\ \Leftrightarrow \text{BSAR}'_{t+2} - \text{BSAR}_{t+2} &= (1 - \beta_1) (\text{BSAR}'_{t+1} - \text{BSAR}_{t+1}) \end{aligned}$$

We denote $u_t = \text{BSAR}'_t - \text{BSAR}_t$ by the instantaneous impact of LIBOR on BSAR rate at time t and if LIBOR realizes a shock of ΔI at time 0 thus we have the following:

$$\begin{cases} u_0 = \alpha_2 \Delta I \\ u_1 = (1 - \beta_1) \alpha_2 \Delta I + (\beta_1 \beta_2 - \alpha_2) \Delta I \\ u_{t+1} = (1 - \beta_1) u_t \end{cases}$$

Therefore the cumulative impulse response function is calculated through the following formula:

$$F(t) = \sum_{i=0}^t u_i \quad \forall i \in \mathbb{N}$$

Then, the above formula provides the impulse response function plot in Figure 3.10, that shows existence of a positive impact that decreases in value reaching 1 basis point after 12 months. On the other hand, Figure 3.11 shows the convergence of BSAR to its long-run equilibrium.

Figure 3.10: Impulse Response Function

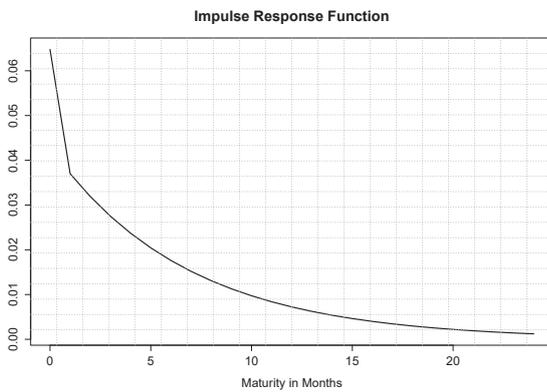
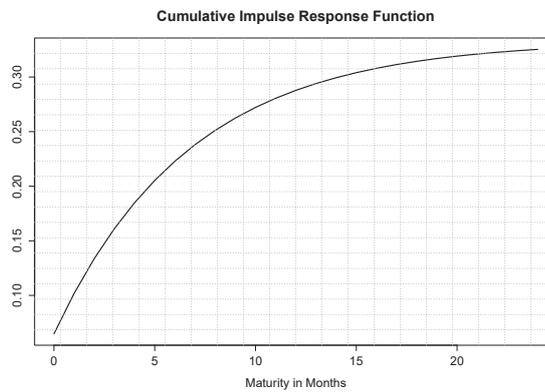


Figure 3.11: Cumulative IRF



3.4 Shock Analysis

We represent in Table 3.3 the speed of adjustment. Results show that 86.44% of the long-run effect will be grasped by the Lebanese Banking sector in the first year. We study the impact of a 200 bps positive jump in LIBOR on BSAR and we obtain the following:

Short-Run: $0.0647 \times 200\text{bps} = 12.95\text{bps}$

Impact after one year: $0.8644 \times 0.3329 \times 200\text{bps} = 57.56\text{bps}$

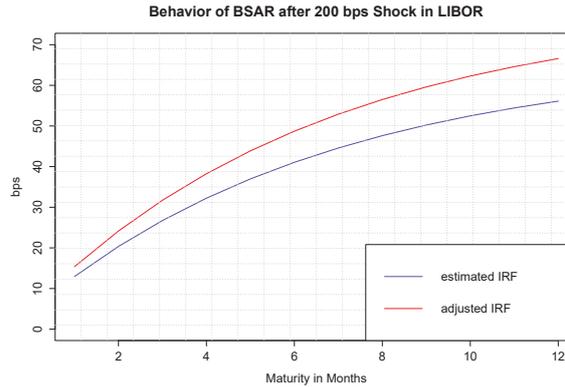
Long-Run: $0.3329 \times 200\text{bps} = 66.58\text{bps}$

Table 3.3: Distribution of the shock

Maturity	% of the long-run	Maturity	% of the long-run
3 Months	48.41%	15 Months	91.31%
6 Months	66.95%	18 Months	94.44%
9 Months	78.83%	21 Months	96.44%
12 Months	86.44%	24 Months	97.72%

To be more conservative, we assume that the shock will be totally absorbed in 12 months by the Lebanese Banking sector deposit rates. Therefore, we shift the cumulative impulse response function (divide all the number by 0.8644) to obtain the following curve which will be used to model the impact of a shock in LIBOR on the repricing of USD customers' deposits.

Figure 3.12: Behavior of BSAR in response of 200 bps changes in LIBOR



3.5 Application

As previously mentioned, we concentrate on a hypothetical Lebanese bank's customer deposits which constitute the majority (80%-90%) of the bank's liabilities. The biggest portion of these deposits has a residual contractual maturity below or equal to 1 month. Though this is the simplest assumption for deposits' repricing, it doesn't however reflect their behavioral repricing profile.

In this section, we formulate a methodology for rebucketing deposits in the bank's repricing gap (a report typically used by banks to give a picture of their repricing interest rate risk) according to their behavioral repricing that we got from our model.

As mentioned previously, our model estimates a total pass-through of 33% from which 86% within a year time.

To be more conservative, we will assume the following:

- We suppose that the long-run impact is totally passed through within 1 year then we construct the cumulative impulse response function which converges to 100% instead of 86% (as detailed earlier).
- The remaining 67% will be passed through at the end of the first year. We will use the impulse response function we developed earlier, to determine the percentage of deposits that are repriced at each point in time.

For example, if the impulse response function shows that 7% of the shock is passed through the first month, therefore 7% of deposits will be repriced in 1 month. The remaining 93% are shifted to the 2nd month where we redo this procedure until we reach the 1 year point where all the remaining deposits are assumed to reprice.

This will yield us with the behavioral repricing of deposits which is further in time than their contractual repricing. Therefore, the behavioral duration of our liabilities will be higher than the contractual duration. This will lower the duration gap between assets and liabilities and as a result the bank's sensitivity to interest rate risk will decrease. We

Table 3.4: Model Results

Maturity	1 Month	3 Months	6 Months	12 Months
1	7.69%	-	-	-
2	4.39%	-	-	-
3	3.78%	18.04%	-	-
4	3.26%	3.71%	-	-
5	2.81%	3.20%	-	-
6	2.42%	2.76%	31.21%	-
7	2.09%	2.38%	2.68%	-
8	1.80%	2.05%	2.31%	-
9	1.55%	1.77%	1.99%	-
10	1.34%	1.52%	1.71%	-
11	1.15%	1.31%	1.48%	-
12	67.70%	63.27%	58.63%	100.00%

illustrate an application of the model in the below example summarized in Table 3.5.

Table 3.5: Maturity Adjustment

Maturity	1 Month	3 Months	6 Months	1 Year
Customer Deposits	\$ 1,890,000	\$ 450,000	\$ 120,000	\$ 40,000
	Present Value		Present Value	
Contractual	\$ 2,487,491	Adjusted	\$ 2,429,173	
Contractual +100bps	\$ 2,483,847	Adjusted +100bps	\$ 2,410,105	
Contractual +200bps	\$ 2,480,217	Adjusted +200bps	\$ 2,391,228	
Contractual +400bps	\$ 2,472,997	Adjusted +400bps	\$ 2,354,035	

We suppose a bank holds 2,500,000 USD as customers' deposits distributed as represented in Table 3.5. The present value of deposits without any adjustment is equal to USD 2,487,491 whereas the adjusted present value declines to USD 2,429,173. The change in value of liabilities after a shock of 200 bps increased from -7,274 to -37,946. In the table, we see also results getting from 100 and 400 bps shock. This shows that liabilities sensitivity is higher and will compensate the decrease in value of assets after a positive

shock in interest rates.

3.6 Conclusion

Our model's intent is to identify the behavioral duration (repricing date) of customer deposits when market interest rates fluctuate. This alters the sensitivity of the economic value of shareholders' equity to interest rate shocks, a measure of interest rate risk used to assess capital requirements for banks.

Our econometric model shows the existence of a partial pass-through with a significant time lag of domestic USD deposit interest rates after an interest rate shock in global market benchmark interest rates. The resulting behavioral deposit repricing profile is shifted in time beyond the contractual one (i.e. it becomes longer). As a result, the interest rate duration gap between assets and liabilities decreases, implying lower interest rate repricing risk.

Appendix 3.A - Vector Autoregression and Error Correction Model

We provide the applied econometric model in Chapter 3. Multivariate time series methods are widely used by lot of financial analysts. The econometricians have focused on the time series analysis which could help in answering economic and financial questions. This appendix develops the theoretical background of two models widely used in studying linear dependencies between variables: Vector Autoregression Model (VAR) and Vector Error Correction Model (VECM). First section outlines the statistical tools used to verified the required hypothesis for applying one of the models, whereas in the second section we provide a brief description of the two models.

Statistical Background

Stationarity

A stationary variable is a stochastic process that doesn't change when shifted in time or space. The parameters such as the mean and the variance, if it exists, also don't change over time or position.

Unit Root Process A linear stochastic process has a unit root if 1 is a root of the process's characteristic equation. In other words, we consider a discrete time stochastic process y_t and suppose that it can be written as an auto-regressive process of order p :

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \dots + a_p y_{t-p} + \epsilon_t$$

where ϵ_t is a white noise process. If $m = 1$ is a root of the characteristic equation,

$$m^p - a_1 m^{p-1} - a_2 m^{p-2} - \dots - a_p = 0,$$

then the stochastic process has a unit root. The properties and characteristics of unit-root processes are:

- Shocks to a unit root process have permanent effects which do not decay as they would if the process were stationary.
- A unit root process has a variance that depends on t , and diverges to infinity.
- If it is known that a series has a unit root, the series can be differenced to render it stationary.

Stationary Process: A stochastic process is stationary if their first and second moments are time invariant. In other words, a stochastic process x_t is stationary if:

- $E(X_t) = \mu, \forall t.$
- $cov(X_t, X_{t+j}) = \gamma_j, \forall t, j$

We have said that the variance of a unit process depends on time t therefore a unit root process is non stationary.

Proof: A unit root process has a variance that depends on time

We consider discrete time stochastic process x_t and suppose that it can be written as an autoregressive process of order 1:

$$x_t = a_1 x_{t-1} + \epsilon_t.$$

The process has a unit root when $a_1 = 1$ since the characteristic equation is $m - a_1 = 0$. Therefore, we have:

$$y_t = y_{t-1} + \epsilon_t.$$

By repeated substitution, we can write $y_t = y_0 + \sum_{j=1}^t \epsilon_j$. Then the variance of y_t is given by:

$$\text{VAR}(y_t) = \sum_{j=1}^t \sigma^2 = t\sigma^2.$$

The variance depends on t since $\text{VAR}(y_1) = \sigma^2$, while $\text{VAR}(y_2) = 2\sigma^2$.

Proof: If a process has a unit root then the process will be non stationary

We consider X_t as a process having a unit root and we assume that it can be written as an autoregressive process of order p .

$$X_t = \sum_{i=1}^p a_i X_{t-i} + \epsilon_t$$

where ϵ_t is a white noise. After rearranging the terms we obtain the following equation:

$$\sum_{i=0}^P a_i X_{t-i} = \epsilon_t.$$

We denote by B the backward operator, hence we have $BX_t = X_{t-1}$ then:

$$\left(\sum_{i=0}^p a_i B^i \right) X_t = \epsilon_t.$$

As X_t is a unit process, the characteristic polynomial can be divided by $(B - I)$ where I is the identity matrix. Then we have the following equation:

$$(B - I)BX_t = \epsilon_t$$

If $(B - I)B$ is invertible then $X_t = [(B - I)B]^{-1}\epsilon_t$ is stationary. We proof that the last proposition is incorrect by contradiction.

We suppose that $(B - I)^{-1}$ exist then $BX_t = (B - I)^{-1}\epsilon_t$ is stationary, on the other hand we have:

$$\begin{aligned} (B - I)BX_t &= \epsilon_t \\ BBX_t - BX_t &= \epsilon_t \\ BX_t &= BX_{t-1} - \epsilon_t. \end{aligned}$$

We calculate recursively and we obtain:

$$BX_t = B^{t-1}\epsilon_1 - B^{t-2}\epsilon_2 - \dots - B\epsilon_{t-1} - \epsilon_t.$$

Then the variance of BX_t depends on time t which means that BX_t is non stationary.

Unit Root Tests: In order to test the stationarity or non-stationarity of a time series, we use the unit root theory which has been developed by Dickey and Fuller. In our work, we select the Augmented Dickey-Fuller test more for convenience than any other reason. The augmented Dickey-Fuller test constructs a model with higher order lag and tests the significance of the parameter estimates using non-standard t-test. The model used for this test is:

$$\Delta x_t = \alpha_1 x_{t-1} + \beta_1 \Delta x_{t-1} + \beta_2 \Delta x_{t-2} + \dots + \beta_{p-1} \Delta x_{t-p+1} + \epsilon_t$$

where the t-test checks significance of the α_1 term.

Student's T-test: We consider the following regression equation:

$$Y_t = \alpha + \beta X_t + \epsilon_t$$

for $t = 1, \dots, n$. Once we have estimated the coefficients α and β , we aim to test if the slope β is equal to some specified value β_0 (often taken to be 0, in which case the hypothesis is that x and y are unrelated).

Let:

- $\hat{\alpha}$ and $\hat{\beta}$ be the least-squares estimators.
- $SE_{\hat{\alpha}}$ and $SE_{\hat{\beta}}$ be the standard errors of least-squares estimators.

We have:

$$SE_{\hat{\beta}} = \frac{\sqrt{\frac{1}{n-2} \sum_{i=1}^n (Y_i - \hat{y}_i)^2}}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}}$$

then

$$t = \frac{\hat{\beta} - \beta_0}{SE_{\hat{\beta}}}.$$

has a T-distribution with $n - 2$ degrees of freedom if the null hypothesis is true.

Order of Integration: A time series is integrated of order 0 if the series is stationary and we note $X_t \sim I(0)$. A time series is integrated of order P if:

$$(1 - L)^P X_t \sim I(0)$$

where L is the lag operator and we note $X_t \sim I(P)$. In other words, a series is said to be integrated of order 1 if although it is itself non stationary, the changes in this series form a stationary. It is said to be integrated of order 2 if, although the changes are non stationary, the changes in changes form a stationary series. Generally, if the series must be differenced k times to achieve stationarity, then the series is integrated of order k .

Granger Causality Test: One of the key questions that can be addressed with vector autoregression is how useful some variables are for forecasting others. This section discusses Granger causality test which is an econometric tool to study the causality. The idea of Granger causality is a pretty simple one:

"A variable X Granger-causes Y if Y can be better predicted using the histories of both X and Y than it can use the history of Y alone."

To implement this test, we assume a particular autoregressive lag length p and estimated by OLS method:

$$x_t = c_1 + a_1x_{t-1} + a_2x_{t-2} + \dots + a_px_{t-p} + b_1y_{t-1} + b_2y_{t-2} + \dots + b_py_{t-p} + u_t.$$

We then conduct an F test of the null hypothesis:

$$H_0 : b_1 = b_2 = \dots = b_p = 0.$$

We calculate the sum of squared residuals and we obtain:

$$RSS_1 = \sum_{t=1}^T \hat{u}_t^2.$$

And we estimate the coefficients of the univariate regression for x_t :

$$x_t = c_1 + c_1x_{t-1} + c_2x_{t-2} + \dots + c_px_{t-p} + \epsilon_t.$$

Then we calculate the sum squared residuals of the univariate model:

$$RSS_0 = \sum_{t=1}^T \hat{\epsilon}_t^2.$$

The objective is to compare RSS_1 with RSS_0 . For this purpose, we calculate the following ratio:

$$S_1 = \frac{\frac{RSS_0 - RSS_1}{p}}{\frac{RSS_1}{T - 2p - 1}}.$$

If S_1 is greater than the 5% critical value for an $F(p, T - 2p - 1)$ distribution, then we reject the null hypothesis that y does not Granger-cause x .

Lag Length Criteria: To estimate the vector autoregression model, we have to select the most appropriate number of lagged terms in the model. For this reason, we use statistical tools to estimate the lag order. In this section we discuss three information criteria: Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Hannan & Quinn (HQ). All these criterias aim at minimizing the residual sum of squares and impose a penalty for including an increasingly large number of regressors (lag values).

Akaike Information Criterion: We consider a stationary time series $X_t, t = 1, 2, \dots, N$, the Akaike Information criteria consist of minimizing the function:

$$AIC(p) = N \log \sigma_p^2 + 2p \text{ for } p \in \{0, 1, 2, \dots, m\}$$

where σ_p^2 is the estimated residual variance for a fitted $AR(p)$ model, N is the number of observations and m is a pre-determined upper autoregressive order.

The proper model is which has the smallest AIC. The advantage of AIC is that the criterion can be used for in-sample data and out-of sample data forecasting performance of a model.

Bayesian Information Criteria: The Bayesian Information Criterion is obtained by replacing the non-negative penalty function 2 by $\log N$. Hence, we have:

$$BIC(p) = N \log \sigma_p^2 + p \log N \text{ for } p \in \{0, 1, 2, \dots, m\}$$

where σ_p^2 is obtained as above and the appropriate model is obtained by selecting the order p which minimizes the BIC(p). The BIC is useful in case of a wide range of statistical problems i.e. when the sample size grows to infinity, the probability of choosing the right model converges to unity. As AIC, the BIC can be used to compare in-sample or out-of-sample forecasting performance of a model.

Hannan-Quinn Criterion: It is obtained by replacing the non-negative penalty function by $C \log C \log N$. Thus we have:

$$HQ(p) = N \log \sigma_p^2 + pC \log C \log N \text{ for } p \in \{0, 1, 2, \dots, m\}$$

where C is a constant such $C > 2$. We use this order selection in case of a small sample size.

Jarque-Bera Test The Jarque-Bera test is a goodness-of-fit test of whether sample data have the skewness and kurtosis matching a normal distribution. The test Jarque-Bera is defined as:

$$JB = \frac{n}{6} \left(S^2 + \frac{1}{4}(K - 3)^2 \right)$$

where S is the sample skewness of the variable:

$$S = E \left[\left(\frac{X - \mu}{\sigma} \right)^3 \right] = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^3}{\left(\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right)^{3/2}}$$

and K is the sample kurtosis:

$$K = E \left[\left(\frac{X - \mu}{\sigma} \right)^4 \right] = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^4}{\left(\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right)^2}$$

If the data come from a normal distribution, the Jarque-Bera statistic asymptotically has a chi-square distribution with two degrees of freedom.

Cointegration

Definition: If two or more series are individually integrated but some linear combination of them has a lower order of integration, then the series are said to be co-integrated. In other words, we define the time series vector $X_t = (X_{1t}, X_{2t}, \dots, X_{nt})'$. We say that the components of the vector X_t are cointegrated of order (d, b) , which is denoted $X_t \sim CI(d, b)$ if:

- $X_{1t}, X_{2t}, \dots, X_{nt}$ are integrated of order d .
- There exists a non-null vector β such that the linear combination:

$$\beta X_t = \beta_1 X_{1t} + \beta_2 X_{2t} + \dots + \beta_n X_{nt}$$

is integrated of order $(d - b)$, where b is positive and β is called the cointegrating vector.

βX_t can be called as the error correction term or the long run equilibrium error. We give some remarks about the cointegration:

- $(\beta_1, \beta_2, \dots, \beta_n)$ is a cointegrating vector, then for all λ different than 0 we have $\lambda\beta$ as a cointegrating vector.
- A prior condition for the presence of a cointegrating relationship is that all variables should be integrated of the same order.
- If the vector X_t has n components, there may be as many as $(n - 1)$ linearly independent cointegrating vectors. For example, if $n=2$ then there can be at most one independent cointegrating vector.

Engle-Granger Methodology: This method is as a basic concept to understand the cointegration between the variables. The first step is to test each series apart for their order of integration. In this case, we can use the augmented Dickey-Fuller test to check for the presence of a unit-root. If the individual time series are integrated of different orders than we can conclude that they are not cointegrated. A cointegrating relationship may be only present between variables integrated of the same order. And we advance to second step of the methodology. The second step is to estimate a linear regression between the time series and to test if the residuals are stationary. For example, if x_t and y_t are both integrated of order one than the linear regression will have the following form:

$$y_t = \beta_0 + \beta_1 x_t + \epsilon_t.$$

We apply the augmented Dickey-Fuller test to determine the stationary of the residual series. If ϵ_t is stationary therefore $\epsilon_t = y_t - \beta_1 x_t - \beta_0$ is stationary. Then we can say that x_t and y_t are cointegrated of order $(1,1)$ and we call the equation as the long-run equilibrium.

Johansen Cointegration Test: The Johansen test is a procedure for testing cointegration of several I(1) time series. The advantage of this test is that it permits to estimate more than one cointegrating relationship. So this test is more applicable than the Engle-Granger test which is based on the augmented Dickey-Fuller test. We will now explain the Johansen Approach in the following procedures:

We consider the univariate case:

$$y_t = a_1 y_{t-1} + \epsilon_t$$

or

$$\Delta y_t = (a_1 - 1)y_{t-1} + \epsilon_t$$

If $(a_1 - 1) = 0$ then it can be concluded that y_t has a unit root and is a non stationary process. If $(a_1 - 1) \neq 0$ the y_t is a stationary process. We generalize the previous in a multivariate model. We denote x_t by a vector of n variables, we assume that x_t can be written as a vector autoregressive of order p .

$$x_t = A_1 x_{t-1} + A_2 x_{t-2} + \dots + A_p x_{t-p} + u_t$$

Reparameterizing the VAR model and we obtain the following equation:

$$\Delta x_t = -\Pi x_{t-1} + \sum_{i=1}^{p-1} \Phi_i \Delta x_{t-i} + \epsilon_t \quad (1)$$

where $\Pi = (I - \sum_{i=1}^p A_i)$ and $\Phi_i = -(\sum_{j=i+1}^p A_j)$.

Each variable appearing in (1) is integrated of order 0 either because of first-differencing or to taking linear combinations of variables, which are stationary.

The Johansen approach is based on the relationship between the rank of a matrix and its characteristic roots. We have that the number of non-null characteristic roots of a matrix is equal to the rank of the matrix. As we have that each variable appearing in (1) is stationary, we can say that the number of cointegration vectors is equal to the rank of Π .

We distinguish three cases:

- Rank $(\Pi) = 0$: There are no cointegrating variables, all rows are linearly dependent, and the system is non stationary. We apply the first difference on all the variables and we apply the VAR model on first difference:

$$\Delta y_t = \sum_{i=1}^{n-1} \Phi_i \Delta y_{t-i} + \epsilon_t$$

- Rank $(\Pi) = n$ (number of the variables): All rows are linearly independent; therefore there are n linearly independent cointegration equations. We can conclude that all variables are integrated of order 0 and the system is stationary.
- Rank $(\Pi) = r < n$: The system is non stationary but there are r cointegrating relations among the variables (r rows are linearly independent, thus r linearly independent combinations of y_{it} are stationary). The cointegrating vectors are determined by $\Pi = \alpha\beta'$ where:

- α is a $n \times r$ matrix which measures the average speed of convergence towards Long-run equilibrium.
- β is a $n \times r$ matrix of parameters determining the cointegrating vectors $\beta' y_{t-1}$ is the long run equilibrium error.

We explain the methodology of Johansen in estimating the number of cointegrating vectors:

Once we have estimated the parameters of the matrix Π and get the associated eigen values, we classify in order the eigen values as $\lambda_1 > \lambda_2 > \dots > \lambda_k$ and we determine the number of characteristic roots that are significantly different than zero.

For this issue, Johansen proposes two different likelihood ratio tests:

The trace test:

$$J_{trace} = -T \sum_{i=r+1}^n \ln(1 - \lambda_i),$$

The maximum eigen value test:

$$J_{max} = -T \ln(1 - \lambda_{r+1})$$

where T is the sample size. The Trace test tests the null hypothesis of r cointegrating vectors against the alternative hypothesis of n cointegrating vectors. The maximum eigen-value value test tests the null hypothesis of r cointegrating vectors against the alternative hypothesis of $r + 1$ cointegrating vectors. These test statistics should be compared to a chi-square distribution.

VAR Model

Definition: The VAR is a statistical model used to capture the evolution and the inter-dependencies between multiple time series. All the variables in a VAR model are treated symmetrically; each variable has an equation explaining its evolution based on its own lags and the lags of all the other variables. The variables are collected in a $k \times 1$ vector y_t which has the i^{th} element $y_{(i,t)}$ the time t observation of y_i . Hereafter outlines the VAR(p) model:

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + \epsilon_t$$

where c is a $k \times 1$ vector of constants, A_i is a $k \times k$ matrix (for every $i = 1, \dots, p$) and ϵ_t is a $k \times 1$ vector of error terms satisfying:

- $E(\epsilon_t) = 0$.
- $E(\epsilon_t \epsilon_t') = \Omega$.
- $E(\epsilon_t \epsilon_{t-k}') = 0$.

ϵ_t is called the white noise process.

Order of Integration: All variables in the equation should have the same order of integration. We distinguish three cases:

- All variables are integrated of order 0: The VAR model is stable.
- All variables are integrated of order d with $d > 0$:
 - The variables are cointegrated : The error correction term has to be included in the VAR model. And the VAR model becomes a Vector Error Correction Model.
 - The Variables are not cointegrated: We have to differ d times the variables and we apply the VAR model on the difference.

Stability of the VAR model: One important characteristic of a VAR(p) process is its stability. This means that it generates stationary time series with time invariant means, variances and covariance structure, given sufficient starting values. We can check this by evaluating the characteristic polynomial:

$$\det(I_k - A_1z - \dots - A_pz^p) \neq 0 \text{ for } |z| \leq 1.$$

If the solution of the above equation has a root for $z = 1$, then either some or all variables in the VAR(p)-process are integrated of order one. It might be the case that cointegration between the variables does exist. In this case, we are required to apply the Vector Error Correction model which is developed in the next section.

Impulse Response Function: A primary tool for the VAR analysis is the impulse response function which simulates the effects of a shock to one variable in the system on the conditional forecast of another variable. The mathematical expression of the impulse response function is defined in this paragraph.

We begin by rewriting the VAR(p)-model as a VAR(1)-model. We consider a time series vector $y_t = (y_{1t}, y_{2t}, \dots, y_{nt})'$ and we suppose that y_t can be written as a VAR(p):

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \epsilon_t$$

$$\begin{bmatrix} y_t \\ y_{t-1} \\ \vdots \\ \vdots \\ y_{t-p+1} \end{bmatrix} = \begin{bmatrix} c \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_1 & \phi_2 & \dots & \phi_p \\ I_n & 0 & \dots & 0 \\ 0 & I_n & \dots & 0 \\ \vdots & \dots & \ddots & \vdots \\ 0 & \dots & I_n & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ \vdots \\ \vdots \\ y_{t-p} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}$$

where I_n is the identity matrix of size n. We set:

$$\xi_t = \begin{bmatrix} y_t \\ y_{t-1} \\ \vdots \\ \vdots \\ y_{t-p+1} \end{bmatrix}; \gamma_t = \begin{bmatrix} c \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}; F = \begin{bmatrix} \phi_1 & \phi_2 & \dots & \phi_p \\ I_n & 0 & \dots & 0 \\ 0 & I_n & \dots & 0 \\ \vdots & \dots & \ddots & \vdots \\ 0 & \dots & I_n & 0 \end{bmatrix}; \nu_t = \begin{bmatrix} \epsilon_t \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}$$

And we obtain:

$$\xi_t = \gamma + F\xi_{t-1} + \nu_t.$$

Therefore, we have transformed a VAR(p) into a VAR(1) model. And now we are interested in determining ξ_{t+s} in function of ξ_t in order to study the response of ξ_{t+s} to an irregular movement in ξ_t .

$$\xi_{t+1} = \gamma + F\xi_t + \nu_{t+1}.$$

$$\xi_{t+1} = (I_n + F)\gamma + F^2\xi_{t-1} + \nu_{t+1} + F\nu_t.$$

By a simple calculation, we conclude the following:

$$\xi_{t+s} = (I_n + F + F^2 + \dots + F^{s-1})\gamma + F^s\xi_t + \nu_{t+s} + F\nu_{t+s-1} + F^2\nu_{t+s-2} + \dots + F^{s-1}\nu_{t+1}.$$

Hence, $\frac{\partial \xi_{t+s}}{\partial \xi_t'} = F^s$ is the response of ξ_{t+s} to a shock in ξ_t .

Let $\Psi_s = F^s$, therefore we have the following equation:

$$y_{t+s} = c_s + \epsilon_{t+s} + \Psi_1\epsilon_{t+s-1} + \Psi_2\epsilon_{t+s-2} + \dots + \Psi_{s-1}\epsilon_{t+1} + F_{12}^{(s)}y_{t-1} + F_{13}^{(s)}y_{t-2} + \dots + F_{1p}^{(s)}y_{t-p+1}$$

where

$$c_s = (I_n + \Psi_1 + \Psi_2 + \dots + \Psi_{s-1})c.$$

Thus the response of y_{t+s} to a shock in y_t is equal to:

$$\frac{\partial y_{t+s}}{\partial y_t'} = \frac{\partial y_{t+s}}{\partial \epsilon_t'} = \Psi_s.$$

The plot of Ψ_s as a function of $s = 1, 2, \dots$ is called the nonorthogonalized impulse-response function. And the (i, j) element of Ψ_s identifies the consequences of one-unit increase in $y_{j,t}$ for the value of $y_{i,t+s}$.

VEC Model

Definition: The vector error correction model is a dynamic econometric tool that provides us the short term deviations from long-run equilibrium. We have shown in the previous section how to derive the vector error correction model from the vector auto regression model:

$$\Delta x_t = -\Pi x_{t-1} + \sum_{i=1}^{p-1} \Phi_i \Delta x_{t-i} + \epsilon_t$$

where x_t is a vector of n univariate time series, $\Pi = (I - \sum_{i=1}^p A_i)$ and $\Phi_i = -(\sum_{j=i+1}^p A_j)$. To have a vector error correction model, the variables should be cointegrated hence the rank of Π should be equal to $r < n$. We can conclude that Π can be written as the product of two matrix as mentioned above ($\Pi = \alpha\beta'$). Then the equation becomes:

$$\Delta x_t = -\alpha\beta'x_{t-1} + \sum_{i=1}^{p-1} \Phi_i \Delta x_{t-i} + \epsilon_t$$

Or

$$\Delta x_t = -\alpha w_{t-1} + \sum_{i=1}^{p-1} \Phi_i \Delta x_{t-i} + \epsilon_t \text{ where } w_t = \beta' x_t$$

We can see clearly that w_t is a process integrated of order 0. Thus, it represents the linear combination of the I(1) elements of x_t . The VECM imposes $n - r$ unit roots in the VAR by including first differences of all of the variables and r linear combinations of levels of the variables. The levels of x_t are introduced in a special way that all the variables in the regression are integrated of order 0.

Interpretation of the coefficients

We reintroduce the previous equation:

$$\Delta x_t = -\alpha \beta' x_{t-1} + \sum_{i=1}^{p-1} \Phi_i \Delta x_{t-i} + \epsilon_t$$

And we have the following properties:

- The coefficients of β show the long run equilibrium relationships between levels of variables.
- The coefficients of α show the amount of changes in the variables to bring the system back to equilibrium.
- The coefficients of the Φ_i show the short run changes occurring due to previous changes in variables.

Further interpretations related to VAR and VECM model are represented in Appendix 3.B.

Appendix 3.B - Cointegration and ECM Tools

If the vector X_t has two components therefore the number of linearly independent cointegrating vectors will be at most equal to one. We consider the simple vector auto-regression (VAR) model:

$$\begin{cases} y_t = a_{11}y_{t-1} + a_{12}z_{t-1} + \epsilon_{yt} \\ z_t = a_{21}y_{t-1} + a_{22}z_{t-1} + \epsilon_{zt} \end{cases} .$$

Applying the lag operator on y_{t-1} and z_{t-1} , and rearranging we get:

$$\begin{cases} y_t = a_{11}Ly_t + a_{12}Lz_t + \epsilon_{yt} \\ z_t = a_{21}Ly_t + a_{22}Lz_t + \epsilon_{zt} \end{cases}$$

$$\begin{cases} (1 - a_{11}L)y_t - a_{12}Lz_t = \epsilon_{yt} \\ -a_{21}Ly_t + (1 - a_{12}L)z_t = \epsilon_{zt} \end{cases} .$$

Applying Cramer's rule on the system and we obtain:

$$\begin{cases} y_t = \frac{(1 - a_{22}L)\epsilon_{yt} + a_{12}L\epsilon_{zt}}{(1 - a_{11}L)(1 - a_{22}L) - a_{12}a_{21}L^2} \\ z_t = \frac{a_{21}L\epsilon_{yt} + (1 - a_{11}L)\epsilon_{zt}}{(1 - a_{11}L)(1 - a_{22}L) - a_{12}a_{21}L^2} \end{cases} .$$

The equations have the same inverse characteristic equation:

$$(1 - a_{11}L)(1 - a_{22}L) - a_{12}a_{21}L^2 = 0 \text{ and } \lambda = 1/L.$$

Then this implies that:

$$\lambda^2 - (a_{11} + a_{22})\lambda + (a_{11}a_{22} - a_{12}a_{21}) = 0.$$

Whereby the characteristic roots λ_1 and λ_2 determine the time paths of both variables. We will see in which case we have the two variables cointegrated of order (1,1).

- If (λ_1, λ_2) lie inside the unit circle then stable solutions for the series y_t and z_t exists and the variables are stationary. An important implication of this is that they cannot be cointegrated of order (1, 1).
- If either root lies outside the unit circle then we call that the solutions are explosive, we have

$$y_t = \frac{(1 - a_{22}L)\epsilon_{yt} + a_{12}L\epsilon_{zt}}{(1 - a_{11}L)(1 - a_{22}L) - a_{12}a_{21}L^2} = \frac{(1 - a_{22}L)\epsilon_{yt} + a_{12}L\epsilon_{zt}}{(1 - \lambda_1L)(1 - \lambda_2L)}$$

since λ_1 and λ_2 are the roots of the characteristic equation. If (λ_1, λ_2) lie inside the unit circle then the solution is stable, however if either root lies outside the unit circle then the system is explosive. Neither variable is difference stationary which implies that the variables cannot be cointegrated of order (1, 1).

- If $a_{12} = a_{21} = 0$, then

$$\begin{cases} y_t = a_{11}y_{t-1} + \epsilon_{yt} \\ z_t = a_{22}z_{t-1} + \epsilon_{zt} \end{cases}.$$

And the solution is trivial.

- If $a_{11} = a_{22} = 1$, then the two variables have unit roots. Therefore $\lambda_1 = \lambda_2 = 1$ and

$$y_t = y_{t-1} + a_{12}z_{t-1} + \epsilon_{yt}.$$

This implies the two variables cannot be cointegrated of order $(1, 1)$.

We have shown that the two variables cannot be cointegrated of order $(1, 1)$ if the two roots lies outside or inside the unit circle or if the two roots are equal to one. Thus for y_t and z_t to be cointegrated of order $(1, 1)$, it is necessary for one characteristic root to be unity and the other less than unity in absolute value.

For example, if $|\lambda| = 1$ then

$$y_t = \frac{(1 - a_{22}L)\epsilon_{yt} + a_{12}L\epsilon_{zt}}{(1 - L)(1 - \lambda_2L)}$$

Multiplying the right-hand side and the left-hand side by $(1 - L)$:

$$(1 - L)y_t = \frac{(1 - a_{22}L)\epsilon_{yt} + a_{12}L\epsilon_{zt}}{1 - \lambda_2L}$$

Or

$$\begin{aligned} (1 - L)y_t &= y_t - Ly_t \\ &= y_t - y_{t-1} \\ &= \Delta y_t \end{aligned}$$

Then

$$\Delta y_t = \frac{(1 - a_{22}L)\epsilon_{yt} + a_{12}L\epsilon_{zt}}{1 - \lambda_2L}$$

is stationary if $|\lambda_2| < 1$. We apply these conditions on the coefficients:

If $|\lambda_1| = 1$ then:

$$1 - (a_{11} + a_{22}) + a_{11}a_{22} - a_{12}a_{21} = 0 \text{ (refer to the characteristic equation)}$$

$$\text{Hence, } a_{11} = \frac{1 - a_{22} - a_{12}a_{21}}{1 - a_{22}}.$$

And if $|\lambda_2| < 1$, then $a_{11}a_{22} - a_{12}a_{21} < 1$

Transformation of a simple VAR model to VECM: We consider a simple VAR model:

$$\begin{cases} y_t = a_{11}y_{t-1} + a_{12}z_{t-1} + \epsilon_{yt} \\ z_t = a_{21}y_{t-1} + a_{22}z_{t-1} + \epsilon_{zt} \end{cases}.$$

By a simple calculation, we obtain:

$$\begin{pmatrix} \Delta y_t \\ \Delta z_t \end{pmatrix} = \begin{pmatrix} a_{11} - 1 & a_{12} \\ a_{21} & a_{22} - 1 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{yt} \\ \epsilon_{zt} \end{pmatrix}.$$

We have

$$a_{11} - 1 = \frac{-a_{12}a_{21}}{1 - a_{22}}.$$

Therefore

$$\begin{cases} \Delta y_t = \frac{-a_{12}a_{21}}{1-a_{22}}y_{t-1} + a_{12}z_{t-1} + \epsilon_{yt} \\ \Delta z_t = a_{21}y_{t-1} + (1 - a_{22})z_{t-1} + \epsilon_{zt} \end{cases}.$$

If $a_{12}a_{21} \neq 0$ then we can build the cointegrating vector with respect to either variable. In this case we build with respect to y_{t-1} . And we obtain

$$\begin{cases} \Delta y_t = \alpha_y(y_{t-1} - \beta z_{t-1}) + \epsilon_{yt} \\ \Delta z_t = \alpha_z(y_{t-1} - \beta z_{t-1}) + \epsilon_{zt} \end{cases}$$

where

$$\begin{cases} \alpha_y = \frac{-a_{12}a_{21}}{1-a_{22}} \\ \alpha_z = a_{21} \\ \beta = \frac{1-a_{22}}{a_{21}} \end{cases}$$

We say that α_y and α_z determine the speed of adjustment to bring the system back to equilibrium and β show the long-run equilibrium between y_t and z_t .

We introduce the lag of Δy_t and Δz_t , and we obtain the general form of the vector error correction model:

$$\begin{cases} \Delta y_t = \alpha_y(y_{t-1} - \beta z_{t-1}) + c_{11}\Delta y_{t-1} + c_{12}\Delta z_{t-1} + \epsilon_{yt} \\ \Delta z_t = \alpha_z(y_{t-1} - \beta z_{t-1}) + c_{21}\Delta y_{t-1} + c_{22}\Delta z_{t-1} + \epsilon_{zt} \end{cases}.$$

Johansen Procedure: We denote by y_t a vector of k variables and we assume that y_t can be written as a vector autoregressive of order p .

$$y_t = A_1y_{t-1} + A_2y_{t-2} + \dots + A_p y_{t-p} + \epsilon_t$$

where A_i is a $k \times k$ matrix (for every $i = 1, \dots, p$) and ϵ_t is a $k \times 1$ white noise process containing the error terms. We reparametrize the VAR:

- Add and subtract $A_p y_{t-(p-1)}$ from right-hand side

$$y_t = A_1y_{t-1} + A_2y_{t-2} + \dots + A_{p-1}y_{t-(p-1)} + (A_p - A_p)y_{t-(p-1)} + A_p y_{t-p} + \epsilon_t$$

$$y_t = A_1y_{t-1} + A_2y_{t-2} + \dots + A_{p-2}y_{t-(p-2)} + (A_{p-1} + A_p)y_{t-(p-1)} - A_p \Delta y_{t-(p-1)} + \epsilon_t$$

- Add and subtract $(A_{p-1} + A_p)y_{t-(p-1)}$ from right-hand side

$$y_t = A_1y_{t-1} + A_2y_{t-2} + \dots + A_{p-3}y_{t-(p-3)} + (A_{p-2} + A_{p-1} + A_p)y_{t-(p-2)} - (A_{p-1} + A_p)\Delta y_{t-(p-2)} - A_p\Delta y_{t-(p-1)} + \epsilon_t.$$

- By recurrence, we obtain:

$$y_t = (A_1 + \dots + A_p)y_{t-1} - (A_2 + \dots + A_p)\Delta y_{t-1} - (A_3 + \dots + A_p)\Delta y_{t-2} - \dots - A_p\Delta y_{t-p} + \epsilon_t.$$

- We subtract y_{t-1} from left-hand side and right-hand side and we have

$$\Delta y_t = -(I - A_1 - \dots - A_p)y_{t-1} - (A_2 + \dots + A_p)\Delta y_{t-1} - (A_3 + \dots + A_p)\Delta y_{t-2} - \dots - A_p\Delta y_{t-p} + \epsilon_t.$$

- Finally we conclude the generalized equation:

$$\Delta y_t = -\Pi y_{t-1} + \sum_{i=1}^{p-1} \Phi_i \Delta y_{t-i} + \epsilon_t$$

where $\Pi = (I - \sum_{i=1}^n A_i)$ and $\Phi_i = -(\sum_{j=i+1}^n A_j)$.

If N variables can be written in N linearly independent stationary equations, then the N variables are stationary. Let $y_t = (y_{1t}, \dots, y_{Nt})'$ be a vector of N variables. There exist N linearly independent combinations of the components of y_t such that the combinations are stationary. We can conclude that there exist a $N \times N$ matrix A of rank equal to N and a stationary process $\epsilon_t = (\epsilon_{1t}, \dots, \epsilon_{Nt})'$ such that:

$$Ay_t = \epsilon_t.$$

As the rank of A is equal to N then A^{-1} exists and we have:

$$y_t = A^{-1}\epsilon_t.$$

So all the components of y_t can be written as a combination of stationary processes. Hence all the components of y_t are integrated of order 0.

Bibliography

- [1] BANERJEE A., DOLADO J., GALBRAITH J., HENDRY D. (1993). *Co-integration, error correction, and the econometric analysis of non-stationary data*. Oxford University Press.
- [2] BIS PAPERS (1994). National differences in interest rate transmission. *BIS Working Paper*.
- [3] BONDT G. (2002). Retail bank interest rate pass-Through: new evidence at the euro area level. *European Central Bank Working Paper No 136*.
- [4] BORIO C.E.V., FRITZ W. (1995). The response of short-term bank lending rates to policy rates: a cross-country perspective. *BIS, Financial Structure and the Monetary Transmission Policy Transmission Mechanism, March, Basel*, 106-153.
- [5] COTTARELLI C., KOURELIS A. (1994). Financial structure, bank lending rates, and the transmission mechanism of monetary policy. *IMF Staff Papers, Volume 41, Issue 4*, 587-623.
- [6] DONNAY M. AND DEGRYSE H. (2001). Bank lending rate pass-through and differences in the transmission of a single EMU monetary policy. *Discussion Paper, Center for Economic Studies, K.U. Leuven*.
- [7] HAMILTON J. (1994). *Time series analysis*. Princeton University Press.
- [8] HAMMOUDEH S., LAHIANI A., NGUYEN D.K., SOUSA R.M. (2014). Asymmetric and nonlinear pass-through of energy prices to CO2 emission allowance prices. *NIPE Working Papers 05/2014, NIPE - Universidade do Minho*.
- [9] HENDRY D. (1995). *Dynamic econometrics*. Oxford University Press.
- [10] HOFMANN B. (2000). Non-linear adjustment of loan rates to money market rates: some evidence for the euro area. *Paper Presented at Internal ECB Seminar, 18 October*.
- [11] KLEIMEIER S., H. SANDER (2000). Asymmetric adjustment of commercial bank interest rates in the euro area: implications for monetary policy. *Paper Presented at the Conference Financial Structure, Bank Behavior and Monetary Policy in the EMU, Groningen 5-6 October*.

- [12] LETKEPOHL H. (2003). *New introduction to multiple time series*, Springer.
- [13] MOJON B. (2000). Financial structure and the interest rate channel of ECB monetary policy. *European Central Bank Working Paper Series No 40*.
- [14] SCHOLNICK B. (1996). Asymmetric adjustment of commercial bank interest rates: evidence from Malaysia and Singapore. *Journal of International Money and Finance, Volume 15*, 485-496.
- [15] SHIN Y., YU B., GREENWOOD-NIMMO M. (2014). Modelling asymmetric cointegration and dynamic multipliers in an ARDL framework. *Horrace W.C., Sickles R.C. (Eds.), Festschrift in Honor of Peter Schmidt. Springer Science & Business Media, New York(NY)*.
- [16] SORENSEN C., WERNER T. (2006). Bank interest rate pass-through in the euro area, a cross country comparison. *European Central Bank Working Paper Series No 580*.
- [17] TOOLSEMA L.A., STURM J.-E., DE HAAN J. (2001). Convergence of monetary transmission in EMU: new evidence. *CESifo Working Paper*, 465.

Other References

- BLOOMBERG: *www.bloomberg.com*.
- LEBANESE CENTRAL BANK: *www.bdl.gov.lb*.

Part II

Liquidity Risk Measurement and Management in Banks

Chapter 4

Liquidity Risk in Banks

4.1 Introduction

Liquidity is the level to which an asset or security can be bought or sold in the market without affecting the asset's price. In other words, liquidity is defined by the availability of cash or equivalents. We fix the liquidity risk as the risk of loss arising from an absence of cash or equivalents, or more specifically, the risk of loss arising from inability to obtain funding at economically reasonable levels, or sell or pledge an asset at carrying prices, in order to cover an expected or unexpected obligation.

We have two types of liquidity risk:

- Funding liquidity risk
- Asset liquidity risk

Funding liquidity is defined as the availability of unsecured liabilities that can be drawn to create cash, including short-term and long-term debt facilities. Hence, the funding liquidity risk is the risk of loss stemming from an inability to access unsecured funding sources at an economically reasonable cost in order to meet obligations.

Asset liquidity focuses on the availability of assets, such as marketable securities, inventories, receivables, and plant and equipment, which can be sold or pledged to generate cash. Asset liquidity risk is the risk of loss arising from an inability to convert assets into cash at carrying value in order to meet obligations.

In banks, we join together the two risks to produce an incremental degree of risk, which we term joint together to produce an incremental degree of risk, which we term joint asset/funding liquidity - the risk of loss that happens when funding cannot be accessed and assets cannot be converted into cash in order to meet obligations.

Several banks having jointly asset/funding liquidity problem may guide to a systemic liquidity problem. It is exactly a global or regional crisis because of its potential to destabilize the financial system. We call it systemic liquidity risk.

We can consequently consider liquidity risk in the context of internal and external forces. Figure 4.1 presents a general taxonomy of risks in order to show the existence of liquidity risk with other risks.

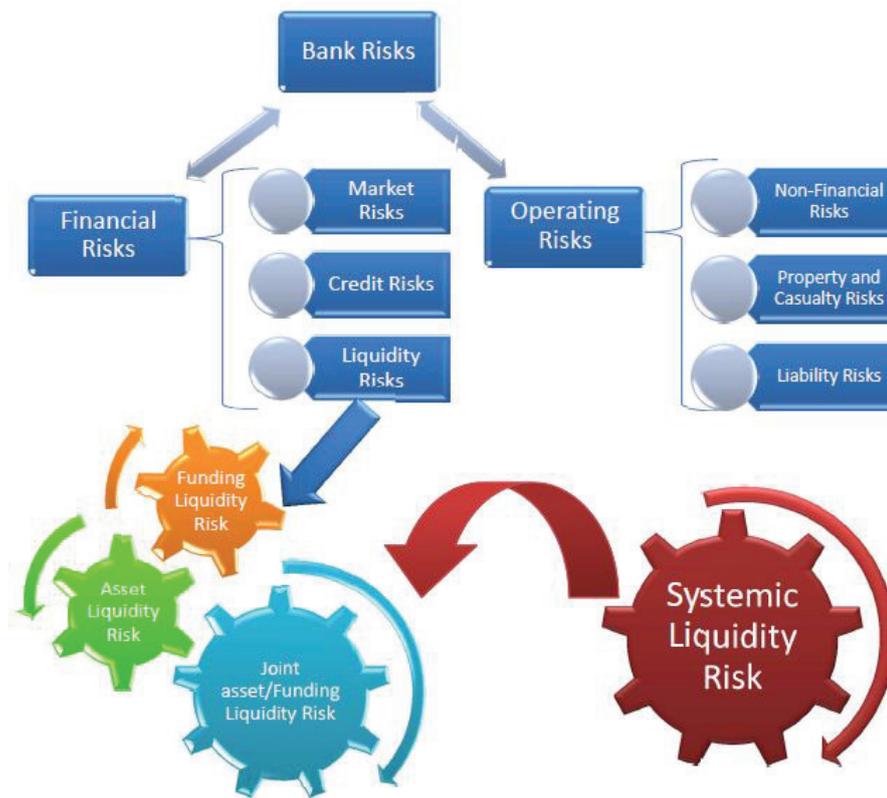


Figure 4.1: A general taxonomy of risks

Many references discuss the liquidity risk management in general. This synthesis is deduced mainly from Banks (2005), McNeil *et al.* (2006) and Hull (2012). Section 4.2 discusses liquidity operations in large companies. Section 4.3 outlines sources of liquidity. Section 4.4 represents liquidity problems in two parts funding liquidity risk and asset liquidity risk. Section 4.5 shows regulatory requirement for liquidity risk management in banks (cf. BIS (2008) and (2010)).

4.2 Liquidity Operations

Liquidity management is therefore a dynamic process due to liquidity changes in cycles. At any time, the liquidity position can change from adequate to inadequate due

to: time, market movements, cash flow cycles,...

Since liquidity changes continuously, it is helpful to consider cycles of cash inflows, outflows, and requirements. Firms obviously do not have the same demand for liquidity at all points in time - either seasonally or structural - so a proper plant must be developed. We illustrate these points into the following example (cf. Banks (2005)):

We consider three companies A, B and C where:

Company A operates in a very foreseeable corporate environment, plans ahead for seasonal variations in cash flow, and keeps a liquidity buffer to cope with unexpected payments. A's demands for incremental liquidity under any scenario are likely to be negligible.

Company B operates in a seasonal business with cash inflows and outflows occurring at regular intervals during the cycle. When B is in the "cash flow positive" portion of the cycle, it does not need extra liquidity and can accumulate a buffer for unexpected payments, or for the cash outflow part of the cycle. When B moves into the "cash flow negative" portion it needs access to liquidity. If B has planned ahead it might have accumulated enough of a buffer to meet requirements, otherwise it will have to turn to alternative sources, intensifying liquidity pressures.

Company C is periodically impacted by unexpected payments, and has not established a buffer to cover emergencies. C always requires external liquidity and thus suffers from more serious liquidity pressures.

The primary point to punctuate is that companies, regardless of industry, face different liquidity profiles and needs over time, and must deal with a range of internal and external forces when creating a management process. The illustration of the discussion is represented in the figure 4.2.

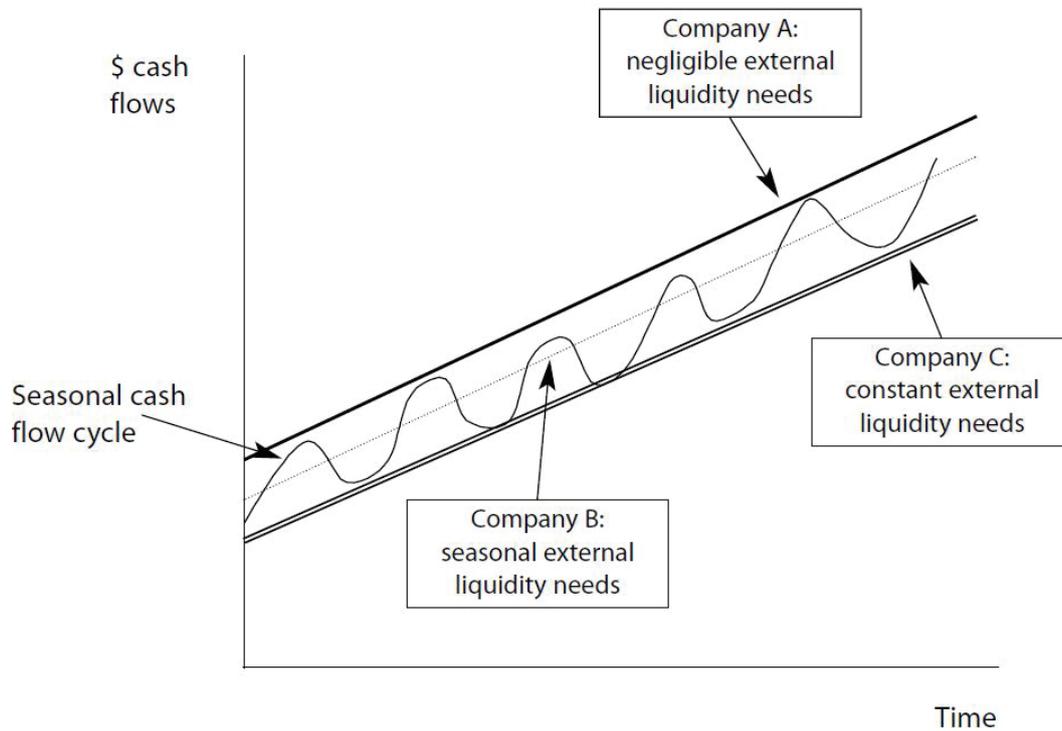


Figure 4.2: Corporate liquidity needs

Eventually, in order to cover the liquidity risk, the company should be in a situation where the following conditions exist:

- The firm can hold all assets and liabilities until maturity.
- Assets and liabilities are properly matched; liabilities, in particular, cannot be presented for early repayment.
- No unexpected payments arise (or those that arise can be met with an extra buffer of liquidity).
- Accounts are not marked-to-market.
- Assets are not subject to default, and yield a defined value at maturity.

In order to have greater control of its liquidity profile, the company should create a plan based on the techniques cited below:

Matching cash flows: laddering cash inflows/outflows assets and liabilities as closely as possible in order to keep the funding gap tight. This may involve lengthening liabilities in order to avoid instances of simultaneous redemption.

Diversifying assets and liabilities: ensuring that portfolios of assets and liabilities are diversified across maturities, markets, sectors, and providers in order to reduce over-reliance on any single source.

Creating a liquidity warehouse: developing a portfolio of securities that can easily be pledged as collateral or sold in order to raise new funds.

Developing committed funding sources: obtaining bank facilities that are truly committed, and that will not be withdrawn under any circumstances.

Purchasing liquidity options: using financial resources to acquire contracts that will provide cash injection when needed.

Liquidity risk management is no longer only a process driven by internal goals but after the 2008 crisis, regulatory requirement has been added to ensure the stability of the liquidity of each bank. The aim of regulators is to avoid the financial system from a global liquidity problem and to preserve the stability of the market in the sake of reinforcing the confidence to investors and creditors. Affairs of regulators are thus concerned with:

- maintaining systemic stability (particularly in industries that supply liquidity to others, intermediate in the origination and allocation of capital funds, or create liabilities that are due and payable to others)
- creating investor, depositor, and lender confidence (which can help strengthen capital flows and participation in government securities markets, including benchmarks used to price other capital instruments and derivative contracts)
- ensuring conditions are conducive to monetary policy activities (including open market operations, which rely heavily on active trading in government securities and repurchase agreements).

4.3 Sources of liquidity

As liquidity risks and challenges can emerge from an asset and funding perspective, it is useful to start by analyzing sources of liquidity that firms from a broad range of industries can access in support of their operations. This supplies an understanding of how liquidity structure must be managed to ensure availability of cash resources when needed. In this section, we present an overview on sources of liquidity found in the asset and liability accounts of the balance sheet, as well as those that exist off-balance sheet.

4.3.1 Sources of Asset Liquidity

Liquid assets

- Cash and marketable securities: A ready source of liquidity, either through outright sale or pledge of unencumbered securities for cash.
- Receivables: A ready source of liquidity, either through outright sale (factoring) or pledge of unencumbered receivables for cash.
- Inventories: An acceptable source of liquidity, either through outright sale or pledge of unencumbered inventories; most effective for standard, durable inventories.

Fixed and intangibles

- Fixed assets: A possible source of liquidity, primarily through pledge of unencumbered plant and equipment for cash.
- Intangibles: Not a source of liquidity.

4.3.2 Sources of Funding Liquidity

Short-term funding markets

The short-term funding markets sources are ready sources of liquidity, but ones that are more complex to manage and can be withdrawn or cancelled very rapidly. The correspondent sources are cited below:

- Commercial paper
- Euro commercial paper
- Short-term bank facilities
- Payables
- Deposits and repurchase agreements

Medium-/long-term funding

The medium-/long-term funding source are ready sources of liquidity that provide a greater degree of funding stability; secured facilities remove some balance sheet flexibility. The sources are cited below:

- Medium-term notes and Euronote facilities
- Funding agreements and GICs
- Long-term bonds
- Loans

4.3.3 Sources of Off-Balance Sheet Liquidity

- Securitization: An acceptable source of liquidity, primarily through transfer of securities or receivables to a conduit in exchange for cash.
- Contingent financing: A good source of liquidity, to be drawn down as needed.
- Leases: A good source of liquidity, releasing cash to be used to meet other obligations.
- Derivatives: A limited source of liquidity, primarily through off-market, synthetic, or leveraged structures that provide upfront cash or relieve funding requirements.

4.3.4 Amalgamating Liquidity assets

To minimize costs and avoid any possibility of disruption, companies often develop plans on how to access their joined sources of liquidity. Assuming normal market conditions, a firm is likely to use most, or all, of the liquidity mechanisms that it can access in a timely, and economically rational, manner. A typical "rank ordering" might therefore be as follow:

- Rollover of existing facilities.
- Draw-down of bank lines or contingent funding sources.
- Pledge of unencumbered assets for loans.
- Sale of liquid assets from the liquidity warehouse, in order of marketability.
- Securitization of assets.
- Sale of additional illiquid assets, including fixed assets and entire business units.

4.4 Liquidity Problems

4.4.1 Funding Liquidity Risk

Funding liquidity risk is the risk arising from inability to obtain unsecured funding for an economic reason when needed. It can be become from exogenous or endogenous factors. If a company has an internal funding liquidity problem, it will a have a weak credit rating or it should increase the capital. It is very important to note that funding liquidity risk management depends largely on the market liquidity. In liquid markets, companies can roll over existing financing arrangements or liquidate positions without losses to meet obligation needs. However, illiquid markets will absorb a firm's funding requirement and firm's economic value can change in a point of time. In illiquid markets, holding large positions even if high quality assets (i.e. Lebanese Eurobonds) could not be categorized as high quality liquid assets because may the holder will be forced to liquidate a portion of this position in many small trades with more transaction and liquidity costs in order to pay short-term liabilities.

Unexpected demand for cash is at the heart of funding liquidity risk. The first source is called unpredictable cash flows, any company faces at any time unexpected expenses such as a firm might misjudge the size of its daily business requirements or might be presented unexpected payment demands from suppliers. Bank run scenario is one of banks challenges in managing funding liquidity risk. Unexpected cash demand can arise also from unfavorable legal or regulatory actions, for instance, a firm may have penalties to buy when a court system might find product defects or environmental damages caused by this company. An internal mismanagement may let employee to make a fraud, hence the mismanagement can be considered as a source of cash demand. Reputational risk can be also added to funding liquidity risk sources.

When a company find liquidity difficulties due to one of the factors cited above, its management will turn to a funding program in order to address the crisis. The company may face rollover problems, lack of market access to a reputational risk. Funding liquidity problem may lead investors to withdraw their commitments and will have an impact on other firms' liquidity (i.e. the case of Lehman Brothers Bank in 2008). Therefore, funding liquidity risk can rise to the level of financial distress.

Figure 4.3 summarizes the above discussion.

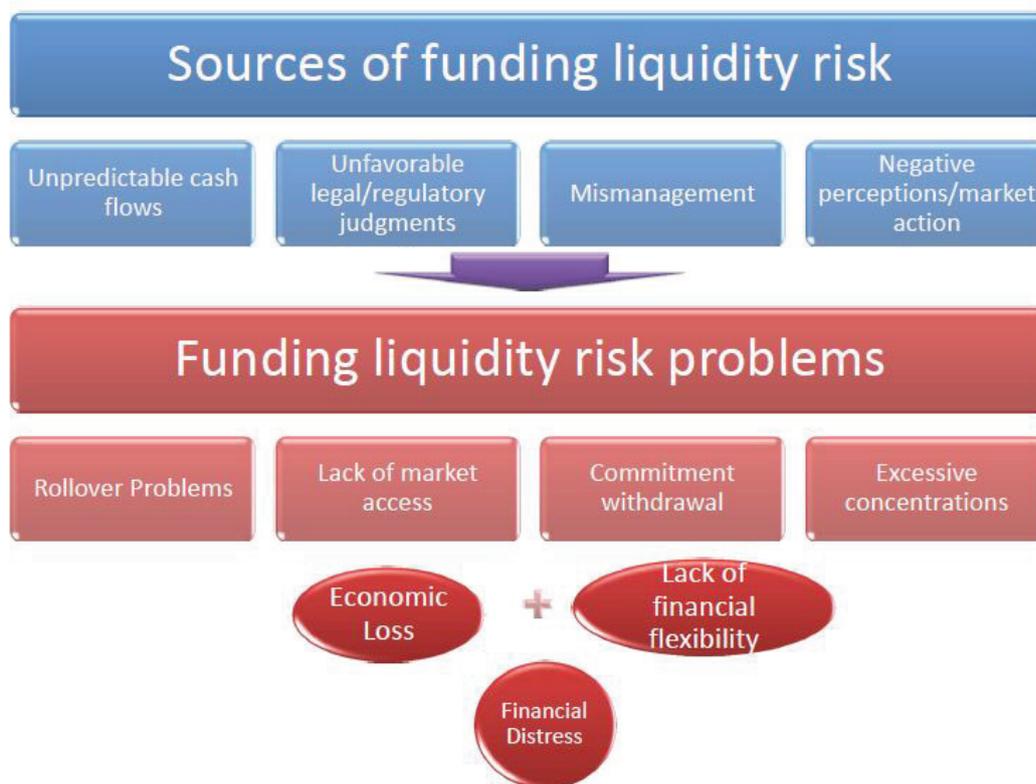


Figure 4.3: Funding Liquidity Risk

4.4.2 Asset Liquidity Risk

Asset liquidity risk is also known as market liquidity risk since the process relates to market price of the asset. A company, that manages robustly operating cash flows and funding sources and can hold all assets to their maturities, will not face asset liquidity problems. The risk is injected when unexpected expenses appear on the balance sheet, existing cash flows are inadequate and funding sources prove insufficient, unpredictable or too expensive, therefore assets prices and their holding periods will become uncertain. The same case of banks when they need to liquidate positions in order to provide liquidity to depositors when needed.

Asset liquidity risk can arise endogenously from funding liquidity risk. However, a company can manage its asset liquidity in a prudent manner, but still faces problems related to external effects and actions. This can be expressed in terms of marketability, concentration, and misvaluation.

Lack of asset marketability is also a problem for asset liquidity risk. A firm holding assets that cannot be readily liquidated or transferred, injects structural illiquidity into its business leading to large losses for surviving. A firm may propose to borrow against its assets instead of selling them, in this case, it will fail to control over its balance sheet. Asset liquidity risk can arise from excessive concentrations and vice-versa. Moreover, asset liquidity problem may lead to misvaluation of illiquid assets therefore misvaluation of firms' economic value. Collateral taken to secure transactions can also be impacted by asset liquidity problems.

Both asset and funding liquidity risks are strongly correlated and led to financial turmoils. Figure 4.4 summarizes the asset liquidity risk.

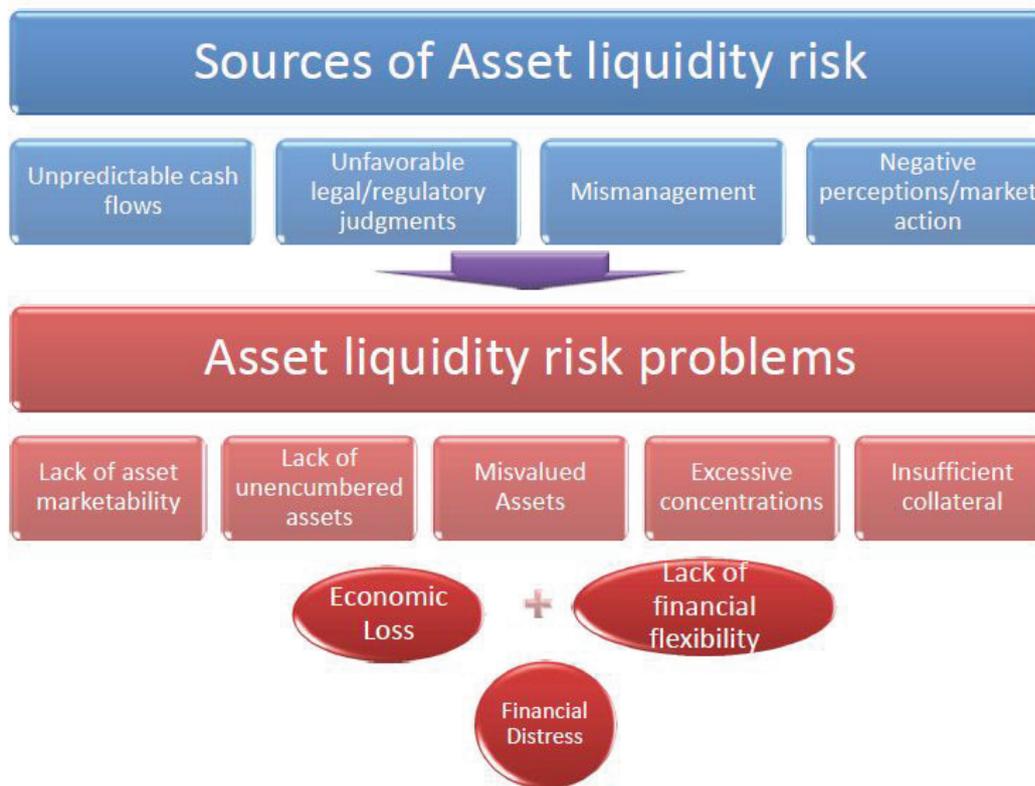


Figure 4.4: Asset Liquidity Risk

4.5 Liquidity Risk in Basel Standards

4.5.1 Introduction

The liquidity is defined as the ability of a bank to fund increases in assets and meet obligations as they come due, without incurring unacceptable losses. The objective of the Basel III standards for the liquidity risk is to improve the banking sector's ability to absorb shocks arising from financial and economic stress, whatever the source, thus reducing the risk of spillover from financial sector to the real economy. The banks become inherently vulnerable to liquidity risk when there's transformation of short-term deposits into long term loans. In addition, if the liquidity risk management is effective we can ensure the bank's ability to meet cash flow obligation. The last financial crisis shows us the importance of the liquidity risk where a lot of banks had ignored this risk and became bankrupt or still experienced difficulties. And, we have show how quickly liquidity can evaporate in the case of rapid reversal in market conditions. The banking system came under serve stress, which necessitated central bank to support both the functioning of

money markets and, in some cases, individual institutions.

4.5.2 Principles for sound liquidity risk management and supervision

Learning from the turmoil, the supervisors should be responsible for some key areas cited as follow:

- The importance of establishing a liquidity risk tolerance
- The maintenance of an adequate level of liquidity, including through a cushion of liquid assets
- The necessity of allocating liquidity costs, benefits and risks to all significant business activities
- The identification and measurement of the full range of liquidity risks, including contingent liquidity risks
- The design and use of severe stress test scenarios
- The need for a robust and operational contingency funding plan
- The management of intra-day liquidity risk and collateral
- Public disclosure in promoting market discipline

This guidance is arranged around seventeen principles for managing and supervising liquidity risk. And these principles are divided into five categories:

Fundamental principle for the management and supervision of liquidity risk

Principle 1

A bank is responsible for the sound management of liquidity risk. A bank should establish a robust liquidity risk management framework that ensures it maintains sufficient liquidity, including a cushion of unencumbered, high quality liquid assets, to withstand a range of stress events, including those involving the loss or impairment of both unsecured and secured funding sources. Supervisors should assess the adequacy of both a bank's liquidity risk management framework and its liquidity position and should take prompt action if a bank is deficient in either area in order to protect depositors and to limit potential damage to the financial system.

Governance of liquidity risk management

Principle 2

A bank should clearly articulate a liquidity risk tolerance that is appropriate for its business strategy and its role in the financial system.

Principle 3

Senior management should develop a strategy, policies and practices to manage liquidity risk in accordance with the risk tolerance and to ensure that the bank maintains sufficient liquidity. Senior management should continuously review information on the bank's liquidity developments and report to the board of directors on a regular basis. A bank's board of directors should review and approve the strategy; policies and practices related to the management of liquidity at least annually and ensure that senior management manages liquidity risk effectively.

Principle 4

A bank should incorporate liquidity costs, benefits and risks in the internal pricing, performance measurement and new product approval process for all significant business activities (both on- and off-balance sheet), thereby aligning the risk-taking incentives of individual business lines with the liquidity risk exposures their activities create for the bank as a whole.

Measurement and management of liquidity risk

Principle 5

A bank should have a sound process for identifying, measuring, monitoring and controlling liquidity risk. This process should include a robust framework for comprehensively projecting cash flows arising from assets, liabilities and off-balance sheet items over an appropriate set of time horizons.

Principle 6

A bank should actively monitor and control liquidity risk exposures and funding needs within and across legal entities, business lines and currencies, taking into account legal, regulatory and operational limitations to the transferability of liquidity.

Principle 7

A bank should establish a funding strategy that provides effective diversification in the sources and tenor of funding. It should maintain an ongoing presence in its chosen funding markets and strong relationships with funds providers to promote effective diversification of funding sources. A bank should regularly gauge its capacity to raise funds quickly from each source. It should identify the main factors that affect its ability to raise funds and monitor those factors closely to ensure that estimates of fund raising capacity remain valid.

Principle 8

A bank should actively manage its intra-day liquidity positions and risks to

meet payment and settlement obligations on a timely basis under both normal and stressed conditions and thus contribute to the smooth functioning of payment and settlement systems.

Principle 9

A bank should actively manage its collateral positions, differentiating between encumbered and unencumbered assets. A bank should monitor the legal entity and physical location where collateral is held and how it may be mobilized in a timely manner.

Principle 10

A bank should conduct stress tests on a regular basis for a variety of short-term and protracted institution-specific and market-wide stress scenarios (individually and in combination) to identify sources of potential liquidity strain and to ensure that current exposures remain in accordance with a bank's established liquidity risk tolerance. A bank should use stress test outcomes to adjust its liquidity risk management strategies, policies, and positions and to develop effective contingency plans.

Principle 11

A bank should have a formal contingency funding plan (CFP) that clearly sets out the strategies for addressing liquidity shortfalls in emergency situations. A CFP should outline policies to manage a range of stress environments, establish clear lines of responsibility, include clear invocation and escalation procedures and be regularly tested and updated to ensure that it is operationally robust.

Principle 12

A bank should maintain a cushion of unencumbered, high quality liquid assets to be held as insurance against a range of liquidity stress scenarios, including those that involve the loss or impairment of unsecured and typically available secured funding sources. There should be no legal, regulatory or operational impediment to using these assets to obtain funding.

Public disclosure**Principle 13**

A bank should publicly disclose information on a regular basis that enables market participants to make an informed judgment about the soundness of its liquidity risk management framework and liquidity position.

The role of supervisors**Principle 14**

Supervisors should regularly perform a comprehensive assessment of a bank's overall liquidity risk management framework and liquidity position to determine whether they deliver an adequate level of resilience to liquidity stress given the bank's role in the financial system.

Principle 15

Supervisors should supplement their regular assessments of a bank's liquidity risk management framework and liquidity position by monitoring a combination of internal reports, prudential reports and market information.

Principle 16

Supervisors should intervene to require effective and timely remedial action by a bank to address deficiencies in its liquidity risk management processes or liquidity position.

Principle 17

Supervisors should communicate with other supervisors and public authorities, such as central banks, both within and across national borders, to facilitate effective cooperation regarding the supervision and oversight of liquidity risk management. Communication should occur regularly during normal times, with the nature and frequency of the information sharing increasing as appropriate during times of stress.

4.5.3 Regulatory standards

Liquidity coverage ratio

In order to advance short-term resilience of a bank's liquidity risk profile by ensuring that it has sufficient high-quality liquid assets to survive a significant stress scenario lasting for one month, the committee developed the liquidity coverage ratio.

$$\text{LCR} = \frac{\text{Stock of high quality liquid assets}}{\text{Total net cash outflows over the next 30 calendar days}} \geq 100\%$$

The scenario for this standard entails a combined idiosyncratic and market-wide shock that would result in:

- The run-off of a proportion of retail deposits
- A partial loss of unsecured wholesale funding capacity
- A partial loss of secured, short-term financing with certain collateral and counterparties
- Additional contractual outflows that would arise from a downgrade in the bank's public credit rating by up to and including three notches, including collateral posting requirements
- Increases in market volatilities that impact the quality of collateral or potential future exposure of derivative positions and thus require larger collateral haircuts or additional collateral, or lead to other liquidity needs
- Unscheduled draws on committed but unused credit and liquidity facilities that the bank has provided to its clients

- The potential need for the bank to buy back debt or honor non-contractual obligations in the interest of mitigating reputational risk

The calculus of the LCR is defined in Appendix 4.A.

Net stable funding ratio

Furthermore, the committee developed the net stable funding ratio so as to promote resilience over a longer time horizon by creating additional incentives for banks to fund their activities with more stable sources for funding on an ongoing basis and to provide a sustainable maturity structure of assets and liabilities. Especially it is structured to ensure that long term assets are funded with at least a minimum amount of stable liabilities in relation to their liquidity risk profiles.

$$\text{NSFR} = \frac{\text{Available amount of stable funding}}{\text{Required amount of stable funding}} > 100\%$$

With this standard, the bank encounters, and investors and customers become aware of:

- A significant decline in profitability or solvency arising from heightened credit risk, market risk or operational risk and/or other risk exposures
- A potential downgrade in a debt, counterparty credit or deposit rating by any nationally recognized credit rating organization
- A material event that calls into question the reputation or credit quality of the institution

The calculus of the NSFR is defined Appendices 4.B, 4.C, 4.D and 4.E.

Monitoring tools

The application of the standards is important but not sufficient, so the committee proposes the following monitoring tools:

Contractual maturity mismatch

The contractual maturity mismatch profile identifies the gaps between the contractual inflows and outflows of liquidity for defined time bands. These maturity gaps indicate how much liquidity a bank would potentially need to raise in each of these time bands if all outflows occurred at the earliest possible date. This metric provides insight into the extent to which the bank relies on maturity transformation under its current contracts.

Concentric of funding

This metric is meant to identify those sources of wholesale funding that are of such significance that withdrawal of this funding could trigger liquidity problems. The metric thus encourages the diversification of funding sources recommended in the Committee's Sound Principles.

Available unencumbered assets

This metric provides supervisors with data on the quantity and key characteristics, including currency denomination and location, of banks' available unencumbered assets. These assets have the potential to be used as collateral to raise additional secured funding in secondary markets and/or are eligible at central banks and as such may potentially be additional sources of liquidity for the bank.

LCR by significant currency

While the standards are required to be met in one single currency, in order to better capture potential currency mismatches, banks and supervisors should also monitor the LCR in significant currencies. This will allow the bank and the supervisor to track potential currency mismatch issues that could arise.

Market-related monitoring tools

High frequency market data with little or no time lag can be used as early warning indicators in monitoring potential liquidity difficulties at banks.

4.6 Conclusion

In this introduction chapter, we provide a general overview on Liquidity risks in large companies and financial institutions and in the last section we show what the regulatory requirements for banks in managing this risk.

We discuss two related topics in the two following chapters, Chapter 5 discusses the maturity adjustment of non-maturing deposits at Bank Audi and this subject concerns the funding liquidity risk because knowing short-term deposits outflows will lead the bank to manage the liquidity in a robust manner.

In Chapter 6, we suppose a large company holds a large position of an asset and decides to liquidate the portfolio in a stressed market. Clearly, this chapter will be a part of the asset liquidity risk management.

Appendix 4.A - Illustrative template for the LCR

Item	Factor (to be multiplied against total amount)	Total amount	With factor applied
Stock of high-liquidity assets			
A. Level 1 assets:			
Cash	100%		
Qualifying marketable securities from sovereigns, central banks, public sector entities, and multilateral development banks	100%		
Qualifying central bank reserves	100%		
Domestic Sovereign or central bank debt in domestic currency	100%		
B. Level 2 assets:			
Sovereign, central bank, and PSE assets qualifying for 20% risk weighting	85%		
Qualifying corporate bonds rated AA- or higher	85%		
Qualifying corporate bonds rated AA- or higher	85%		
Calculation of 40% cap of liquid assets	Maximum 2/3 of adjusted Level 1 assets that would exist after an unwind of all secured funding transactions		
Total value of stock of highly liquid assets			

Cash Outflows			
A. Retail Outflows			
Demand deposit and qualifying term deposits with residual maturity or notice period within 30 days			
- stable deposits	Minimum 5%(additional categories to be determined by jurisdiction)		
- less stable retail deposits	Minimum 10%(additional categories to be determined by jurisdiction)		
Term deposit with residual maturity greater than 30 days with a withdrawal with a significant penalty, or no legal right to withdraw	0%(or higher rate to be determined by jurisdiction)		
B. Unsecured wholesale funding:			
Funding from:			
Stable small business customers	Minimum 5%(additional categories to be determined by jurisdiction)		
Less stable small business customers	Minimum 10%(additional categories to be determined by jurisdiction)		
Legal entities with relationships	25% of deposits needed for operational purposes		
Portion of corporate deposits with operational relationships covered by deposit insurance - same treatment as for retail demand deposits			

Cooperative banks in an institutional network	25% of the qualifying deposits with the centralised institution		
Non-financial corporates, sovereigns, central banks and PSEs	75%		
Other legal entity customers	100%		
C. Secured funding:			
Secured funding transactions backed by Level 1 assets, with any counterparty	0%		
Secured funding transactions backed by Level 2 assets, with any counterparty	15%		
Secured funding transactions backed by assets that are not eligible for the stock of highly liquid assets, with domestic sovereigns, domestic central banks, or domestic public sector entities as a counterparty	25%		
All other secured funding transactions	100%		
D. Additional requirements:			
Liabilities related to derivative collateral calls related to a downgrade of up to 3-notches	100% of collateral that would be required to cover the contracts in case of up to a 3-notch downgrade		
Market valuation changes on derivatives transactions	Treatment determined by supervisors in each jurisdiction		
Valuation changes on posted collateral securing derivative transactions that is comprised of non-Level 1 assets	20%		

ABCP, SIVs, Conduits, etc:			
Liabilities from maturing ABCP, SIVs, SPVs, etc	100% of maturing amounts and 100% of returnable assets		
Asset Backed Securities (including covered bonds)	100% of maturing amounts		
Currently undrawn portion of committed credit and liquidity facilities to:			
- retail and small business clients	5% of outstanding credit and liquidity lines		
- non-financial corporates, sovereigns and central banks, and PSEs; credit facilities	10% of outstanding credit lines		
- non-financial corporates, sovereigns and central banks, and PSEs; liquidity facilities	100% of outstanding liquidity lines		
- other legal entity customers, credit and liquidity facilities	100% of outstanding credit and liquidity lines		
Other contingent funding liabilities (such as guarantees, letters of credit, revocable credit and liquidity facilities, derivative valuations, etc)	Treatment determined by supervisors in each jurisdiction		
Any additional contractual outflows	100%		
Net derivative payables	100%		
Any other contractual cash outflows	100%		
Total cash outflows			
Cash Inflows			
Reverse repos and securities borrowing, with the following as collateral:			
- Level 1 assets	0%		
- Level 2 assets	15%		
- All other assets	100%		
Credit or liquidity facilities	0%		

Operational deposits held at other financial institutions - Deposits held at centralized institution of a network of cooperative banks	0% 0% of the qualifying deposits with the centralised institution		
Other inflows by counterparty:			
- Amounts receivable from retail counterparties	50%		
- Amounts receivable from non-financial wholesale counterparties, from transactions other than those listed in the inflow categories above.	50%		
- Amounts receivable from financial institutions, from transactions other than those listed in the inflow categories above.	100%		
Net derivative receivables	100%		
Other contractual cash inflows	Treatment determined by supervisors in each jurisdiction		
Total inflows			
Total net cash outflows = Total cash outflows minus min [total cash inflows, 75% of gross outflows]			
LCR (= Total value of stock of highquality liquid assets / Net cash outflows)			

Appendix 4.B - Components of Available Stable Funding and Associated ASF Factors

ASF Factor	Components of ASF Category
100%	<p>The total amount of capital, including both Tier 1 and Tier 2 as defined in existing global capital standards issued by the Committee.</p> <p>The total amount of any preferred stock not included in Tier 2 that has an effective remaining maturity of one year or greater taking into account any explicit or embedded options that would reduce the expected maturity to less than one year.</p> <p>The total amount of secured and unsecured borrowings and liabilities (including term deposits) with effective remaining maturities of one year or greater excluding any instruments with explicit or embedded options that would reduce the expected maturity to less than one year. Such options include those exercisable at the investors discretion within the one-year horizon.</p>
90%	"Stable" non-maturity (demand) deposits and/or term deposits with residual maturities of less than one year provided by retail customers and small business customers.
80%	"Less stable" non-maturity (demand) deposits and/or term deposits with residual maturities of less than one year provided by retail and small business customers.
50%	Unsecured wholesale funding, non-maturity deposits and/or term deposits with a residual maturity of less than one year, provided by non-financial corporates, sovereigns, central banks, multilateral development banks and PSEs.
0%	All other liabilities and equity categories not included in the above categories.

Appendix 4.C - Detailed Composition of Asset Categories and Associated RSF Factors

Components of RSF Category	RSF Factor
<p>Cash immediately available to meet obligations, not currently encumbered as collateral and not held for planned use (as contingent collateral, salary payments, or for other reasons)</p> <p>Unencumbered short-term unsecured instruments and transactions with outstanding maturities of less than one year</p> <p>Unencumbered securities with stated remaining maturities of less than one year with no embedded options that would increase the expected maturity to more than one year</p> <p>Unencumbered securities held where the institution has an offsetting reverse repurchase transaction when the security on each transaction has the same unique identifier (eg ISIN number or CUSIP)</p> <p>Unencumbered loans to financial entities with effective remaining maturities of less than one year that are not renewable and for which the lender has an irrevocable right to call</p>	0%
<p>Unencumbered marketable securities with residual maturities of one year or greater representing claims on or claims guaranteed by sovereigns, central banks, BIS, IMF, EC, non-central government PSEs) or multilateral development banks that are assigned a 0% risk-weight under the Basel II standardised approach, provided that active repo or sale-markets exist for these securities</p>	5%
<p>Unencumbered corporate bonds or covered bonds rated AA- or higher with residual maturities of one year or greater satisfying all of the conditions for Level 2 assets in the LCR.</p> <p>Unencumbered marketable securities with residual maturities of one year or greater representing claims on or claims guaranteed by sovereigns, central banks, non-central government PSEs that are assigned a 20% risk-weight under the Basel II standardised approach, provided that they meet all of the conditions for Level 2 assets in the LCR</p>	20%

<p>Unencumbered gold</p> <p>Unencumbered equity securities, not issued by financial institutions or their affiliates, listed on a recognised exchange and included in a large cap market index</p> <p>Unencumbered corporate bonds and covered bonds that satisfy all of the following conditions:</p> <ul style="list-style-type: none"> - Central bank eligibility for intraday liquidity needs and overnight liquidity shortages in relevant jurisdictions - Not issued by financial institutions or their affiliates (except in the case of covered bonds) - Not issued by the respective firm itself or its affiliates - Low credit risk: assets have a credit assessment by a recognised ECAI of A+ to A-, or do not have a credit assessment by a recognised ECAI and are internally rated as having a PD corresponding to a credit assessment of A+ to A- - Traded in large, deep and active markets characterised by a low level of concentration <p>Unencumbered loans to non-financial corporate clients, sovereigns, central banks, and PSEs having a remaining maturity of less than one year</p>	50%
<p>Unencumbered residential mortgages of any maturity that would qualify for the 35% or lower risk weight under Basel II Standardised Approach for credit risk</p> <p>Other unencumbered loans, excluding loans to financial institutions, with a remaining maturity of one year or greater, that would qualify for the 35% or lower risk weight under Basel II Standardised Approach for credit risk</p>	65%
<p>Unencumbered loans to retail customers (ie natural persons) and small business customers (as defined in the LCR) having a remaining maturity of less than one year (other than those that qualify for the 65%</p> <p>All other assets not included in the above categories</p>	100%

Appendix 4.D - Composition of Off-balance Sheet Categories and Associated RSF Factors

RSF Category	RSF Factor
Conditionally revocable and irrevocable credit and liquidity facilities to any client	5% of the currently undrawn portion
<p data-bbox="177 633 804 712">Other contingent funding obligations, including products and instruments such as:</p> <p data-bbox="177 745 804 824">Unconditionally revocable "uncommitted" credit and liquidity facilities;</p> <p data-bbox="177 824 804 857">Guarantees;</p> <p data-bbox="177 857 804 891">Letters of credit;</p> <p data-bbox="177 891 804 925">Other trade finance instruments; and</p> <p data-bbox="177 925 804 958">Non-contractual obligations such as:</p> <ul style="list-style-type: none"> <li data-bbox="177 969 804 1115">- Potential requests for debt repurchases of the bank's own debt or that of related conduits, securities investment vehicles and other such financing facilities; <li data-bbox="177 1126 804 1272">- Structured products where customers anticipate ready marketability, such as adjustable rate notes and variable rate demand notes (VRDNs); and <li data-bbox="177 1283 804 1449">- Managed funds that are marketed with the objective of maintaining a stable value such as money market mutual funds or other types of stable value collective investment funds etc. 	National supervisors can specify the RSF factors based on their national circumstances.

Appendix 4.E - Composition of Off-balance Sheet Categories and Associated RSF Factors

Available Stable Funding (Sources)		Required Stable Funding(Uses)	
Item	Availability Factor	Item	Required Factor
<ul style="list-style-type: none"> - Tier 1&2 Capital Instruments - Other preferred shares and capital instruments in excess of Tier 2 allowable amount having an effective maturity of one year or greater - Other liabilities with an effective maturity of one year or greater 	100%	<ul style="list-style-type: none"> - Cash - Shortterm unsecured actively-traded instruments(<1yr); Securities with exactly offsetting reverse repo; Securities with remaining maturity < 1 yr - Non-renewable loans to financials with remaining maturity < 1yr 	0%
<ul style="list-style-type: none"> - Stable deposits of retail and small business customers(non-maturity or residual maturity <1yr) 	90%	<ul style="list-style-type: none"> - Debt issued or guaranteed by sovereigns, central banks, BIS, IMF, EC, non-central government, multilateral development banks with a 0% risk weight under Basel II standardized approach 	5%
<ul style="list-style-type: none"> - Less stable deposits of retail and small business customers (non-maturity or residual maturity <1yr) 	80%	<ul style="list-style-type: none"> - Unencumbered non-financial senior unsecured corporate bonds rated at least AA-, and debt that is issued by sovereigns, central banks, and PSEs with a risk-weighting of 20%; maturity \geq1yr 	20%

- Wholesale funding provided by non-financial corporate customers, sovereign central banks, multilateral development banks and PSEs (non-maturity or residual maturity < 1yr)	50%	-Unencumbered listed equity securities or non-financial senior unsecured corporate bonds (or covered bonds) rated from A+ to A-, maturity \geq 1 yr ; Gold ; Loans to non-financial corporate clients, sovereigns, central banks, and PSEs with a maturity < 1 yr	50%
- All other liabilities and equity not included above	0%	- Unencumbered residential mortgages of any maturity and other unencumbered loans, excluding loans to financial institutions with a remaining maturity of one year or greater that would qualify for the 35% or lower risk weight under Basel II standardised approach for credit risk	65%
		- Other loans to retail clients and small businesses having a maturity < 1 yr	85%
		- All other assets	100%
		Off Balance Sheet Exposures	
		- Undrawn amount of committed credit and liquidity facilities	5%
		- Other contingent funding obligations	National Supervisory Discretion

Bibliography

- [1] BANKS E. (2005). *Liquidity risk-managing asset and funding risk*. Palgrave Macmillan.
- [2] BIS PAPERS (2008). Principles for sound liquidity risk management and supervision. *BIS Working Paper*.
- [3] BIS PAPERS (2010). Basel III: International framework for liquidity risk measurement, standards and monitoring.
- [4] HULL J. (2012). *Risk management and financial institutions*. Wiley Finance.
- [5] MCNEIL A., FREY R., EMBRECHTS P. (2006). *Quantitative risk management: concepts, techniques, and tools*. Princeton Series in Finance.

Chapter 5

CONFIDENTIAL

Behavioral Maturity of Non-Maturing Deposits

5.1 Introduction

Lebanese Banks' customers deposits constitute the majority (80%-90%) of the bank's liabilities and main part of these deposits has a contractual maturity below or equal to one month. The main reason banks have a liquidity problem is that the amount of deposits is subject to constant, and sometimes unpredictable change. Consequently any development that affects the stability of deposits directly involves the liquidity of Banks. Therefore, the asset and liability management of banks depends on an accurate understanding of the liquidity risk and interest rate risk profile of these deposits.

Chapter 3 employs an error-correction framework to identify the impact of market interest rate shocks on offered USD term deposits rates by Lebanese banks. This model allows to build behavioral repricing maturities for customers' deposits. Therefore we can estimate the real sensitivity of shareholders' equity to interest rate shocks.

After understanding the interest rate risk profile of these deposits, we have to study their liquidity risk profile. If a bank could be sure that the volume of deposits would not vary, the maintenance of liquidity would call for nothing more than providing money for working purposes. The distribution of maturities of earning assets could be determined solely on the basis of safety and earnings. Nevertheless, if deposits, even though not constant in amount, were to change in a predictable manner, the problem of liquidity would be far simpler than at present. The maturities of banks' assets could then be arranged according to foreseeable needs; the determination of banks' liabilities would approach the nature of an exact science and there would be little excuse for having either too many or too few liquid assets.

There is a lot of literature with different topics related to depositor withdrawal and the study differs largely from a country to others. For instance, D'Amato *et al.* (1997) studies the case of Argentine banking sector, Flannery(1998) works on US banks supervision based on market information, Gonzalez-Hermosillo (1999) analyzes the con-

tribution of microeconomic and macroeconomic factors in five recent episodes of banking system problems in US Southwest (1986-1992), Northeast (1991-1992), California (1992-1993), Mexico (1994-1995) and Colombia (1982-1987), Barajas and Steiner (2000) works on depositors behaviors in the Colombian market, Schumacher(2000) studies bank runs scenario in Argentine (Tequila shock), Calomiris and Powell (2001) discusses sources of historical banking panics in US, Budnevich and Franken (2003) studies the case of Chilean Banks, Ghosh and Das(2003) the case of Indian banks, McCandless *et al.* (2003) shows the cause of Argentine bank run scenarios, Sironi (2003) shows the existence of market discipline evidence for European banks, and Opiela (2006) discusses the case of Thai banks.

Another topic related to depositor behavior is the bank run where large depositors withdraw their deposits from a bank at the same time and either demand cash or transfer those funds into a safer institution or other securities because they believe the bank will become insolvent due to internal or macroeconomic factors. There is an important literature on these events with panics unrelated to bank-specific factors. Calomiris and Gorton (1991), Calomiris and Schweikart (1991), Kaufman (1994), Saunders and Wilson (1996), Calomiris and Mason (1997), Kelly and O Grada (2000), and O Grada and White (2002) discuss historical bank run scenarios.

The above papers request a large database related to depositors or important historical bank run scenarios. Selvaggio (1996), O'Brien (2000), de Jong and Wielhouver (2001), Dewachter *et al.* (2006) use autoregressive processes in the formulation of deposits outstanding balances equations. Moreover, O'Brien (2000) includes also client rates and macroeconomic factors. Florentina and Schurle (2010) build relationships between client rates, market rates and deposits outstanding balances with autoregressive process for the prediction of customers' deposits.

Adam *et al.* (2009) assumes demand deposits evolution to be related to both interest rates and some exogenous factor which can be interpreted as business risk or model risk when working on hedging interest rate Margins on demand deposits. This work studies how macroeconomic factors, offered interest rates and competitors offered rates can impact the depositor's behavior at Bank Audi. The essence of this model is to build behavioral maturity for non-maturing deposits. Results show that interest rates impact only resident depositors whereas macro economic factors impact both resident and non-resident depositors. This macroeconomic factor is exogenous to the banking system therefore a change in the economic situation will hit all Lebanese banks simultaneously. On the other hand, since Bank Audi's rates are lower than other Lebanese banks' rates, the spread between rates will be an exogenous factor for Bank Audi's clients deposits.

This chapter is organized as follows: Section 5.2 presents the data with descriptive analysis. Section 5.3 shows comparison between Bank Audi and other Lebanese banks. Section 5.3 discusses the methodology of adjusting behavioral maturities. Results are presented and interpreted in Section 5.4. In the last section we conclude and we briefly discuss future possible extensions.

Bibliography

- [1] ADAM A., AND HOUKARI M., LAURENT J.P. (2009). Hedging Interest Rate Margins on Demand Deposits. *Available at SSRN: <http://ssrn.com/abstract=1361660> or <http://dx.doi.org/10.2139/ssrn.1361660>.*
- [2] BARAJAS A., STEINER B. (2000). Depositor behavior and market discipline in Colombia. *International Monetary Fund Seminar Series, Volume 51*, 1-39.
- [3] BUDNEVICH C., FRANKEN H. (2003). Disciplina de mercado en la conducta de los depositantes y rol de las agencias clasificadoras de riesgo: el caso de Chile. *Economía Chilena, Volume 6*, 5-70.
- [4] CALOMIRIS C., GORTON G. (1991). The origins of banking and panics: models, facts, and bank regulation. *In Financial Markets and Financial Crises, edited by Robert G. Hubbard, Chicago: University of Chicago Press*, 109-174.
- [5] CALOMIRIS C., MASON J. (1997). Contagion and bank failures during the great depression: the June 1932 Chicago banking panic. *American Economic Review, Volume 87*, 863-883.
- [6] CALOMIRIS C., ANDREW P. (2001). Can emerging market bank regulators establish credible discipline? The case of argentina, 1992-1999. *in Prudential Supervision: What Works and What Doesn't, University of Chicago Press*.
- [7] CALOMIRIS C., SCHWEIKART L. (1991). The panic of 1857: origins, transmission, and containment. *Journal of Economic History, Volume 51*, 807-834.
- [8] D'AMATO L., GRUBISIC E., POWELL A. (1997). Contagion, bank fundamentals or macroeconomic shock? An empirical analysis of the Argentine 1995 banking problems. *Working Paper No. 2, Central Bank of Argentina*.
- [9] DE JONG F., WIELHOUWER J. (2001). The valuation and hedging of variable rate savings accounts. Amsterdam : Tinbergen Institute, Discussion paper.
- [10] DEWACHTER H., LYRIO M., MAES K. (2006). A multi-factor model for the valuation and risk management of demand deposits. *National Bank of Belgium, Working Papers*.
- [11] FLANNERY M.J. (1998), Using market information in prudential bank supervision: A review of the U.S. empirical evidence. *Journal of Money, Credit, and Banking, Volume 30*, 273-305.

-
- [12] GONZALEZ-HERMOSILLO B. (1999). Determinants of ex-ante banking system distress: A macro-micro empirical exploration of some recent episodes. *Working Paper No. 33, International Monetary Fund*.
- [13] GHOSH S., DAS A. (2003). Market discipline in the Indian banking sector: An empirical exploration. *NSE Research Initiative, NSE, Mumbai*.
- [14] KAUFMA G. (2003). Bank contagion: A review of the theory and evidence. *Journal of Financial Services Research, Volume 8*, 123-150.
- [15] KELLY M., GRADA C.O.(2000). Market Contagion: Evidence from the panics of 1854 and 1857. *American Economic Review, Volume 90*, 1110-24.
- [16] MCCANDLESS G., GABRIELLI M., ROUILLET M. (2003). Determining the causes of bank runs in Argentina during the crisis of 2001. *Revista de Análisis Económico, Volume 18*, 87-102.
- [17] O'BRIEN J. (2000). Estimating the value and interest rate risk of interest-bearing transactions deposits. *Federal Reserve System*.
- [18] GRADA O. C., WHITE E. (2002). Who panics during panics? Evidence from a nineteenth century savings bank. *NBER Working Paper No. 8856*.
- [19] OPIELA T.P. (2006), A test of the effect of systemic risk on deposit market discipline at Thai banks. *Mimeo, DePaul University*.
- [20] PARASCHIV F. AND SCHURLE M. (2010). Modeling client rate and volumes of non-maturing accounts. *Institute for Operations Research and Computational Finance, University of St. Gallen*.
- [21] SAUNDERS A., WILSON B. (2006). Contagious bank runs: evidence from the 1929-1933 period. *Journal of Financial Intermediation, Volume 5*, 409-423.
- [22] SCHUMACHER L. (2000). Bank runs and currency run in a system without a safety net: Argentina and the Tequila shock. *Journal of Monetary Economics, Volume 46*, 257-277.
- [23] SELVAGGIO R. (1996). Using the OAS methodology to value and hedge commercial bank retail demand deposit premiums. *Commercial Bank Retail Demand Deposit Premiums, Chapter 12*, 363-373.
- [24] SIRONI A. (2003). Testing for market discipline in the European banking industry: evidence from subordinated debt issues. *Journal of Money, Credit and Banking, 35*, 443-72.
- [25] TAKEMERA T., TAKASHI K. (2010 a). Modeling individuals' deposit-withdrawal behaviors after receiving information on financial turmoil. *Research in Business and Economics Journal*

- [26] TAKEMERA T., TAKASHI K. (2010 b). A micro data analysis on individual's deposit-withdrawal behavior. *Discussion Paper Series*, No 93.
- [27] TAKEMERA T., TAKASHI K., KOBAYASHI T. (2011). The empirical study of Japanese deposit withdrawal behaviors on unstable financial environment. *American Journal of Economics and Business Administration*, Volume 3, Issue 2, 288-295.
- [28] TAKEMERA T., TAKASHI K., KOBAYASHI T. (2012), Modeling deposit-withdrawal behavior with risk tolerance *Journal of Management Policy and Practice*, Volume 13, Issue 5, 116-127.

Chapter 6

Exit Trading Strategies in stressed markets using Expert Opinion

6.1 Introduction

The credit crisis that started in the middle of 2007 emphasized the importance of liquidity risk for both financial institutions and their regulators. Investors lost confidence in banks and retired their deposits. Nevertheless, banks found that many instruments for which there had previously been a liquid market could only be sold at fire-sale prices during the crisis.

We should distinguish solvency from liquidity. The solvency is defined as the ability of the bank to cover liabilities from its assets whereas the liquidity is the ratio liquid assets to total assets. We can find a bank with 90% as solvency ratio but 5% as liquid ratio therefore this bank is exposed to the liquidity risk, if its clients decide to retire their deposits, the bank will be forced to liquidate illiquid assets with high transaction costs due to the market impact and liquidity crisis.

Market Liquidity is defined as an asset's ability to be sold without causing a significant movement in the price and with minimum loss of value. In this chapter, we study the optimal trading strategies in stressed markets where an investor finds difficulties in selling a large block of single security. Therefore our analysis focuses on large institutional investors like banks, insurance companies or pension funds. Moreover, Acquiring or unwinding large positions in one security may incur significant costs which directly influences the return on the investment. For instance, an investor can execute small market orders with or without little effect on prices and he usually pays one-half of the bid-ask spread for the opportunity to immediately trade. On the other hand, large market orders are more difficult to execute than smaller ones as they run through order book until they are completely filled. Nevertheless, an investor has a positive initial investment in an asset, after seeing a shock in prices he decides to close this position within a trading window of short period. For this purpose, he can submit market orders basically continuously throughout this period. By that means the trader has to balance different issues. Obviously, the investor can liquidate the portfolio all at once immediately after the opening

of the stock exchange. However this strategy will presumably have a substantial price impact and consequently lead to high liquidity costs. Thus, the liquidation of a large portfolio should be executed with smaller packages.

Regardless the reason of the demand for liquidity, banks may be forced to liquidate their assets equivalently raise money quickly. Referring to Sheilfer-Vishny (1992), Allen and Gale (1995), assets would have to be sold at fire sale prices. But the fire sale strategy could increase the liquidity problem of the bank since asset values may be depressed in a sense the bank becomes insolvent.

For this matter, we review some literature related to trading strategies in illiquid markets. This kind of researches discusses a lot of types and forms of the transaction cost: linear or non-linear, permanent or temporal price impact. Also, optimal trading strategies are formulated in discrete-time or continuous time.

Chan and Lakonishok (1995) studies the price impact of the entire sequence of trades executed by large investment management firms. Bertsimas and Lo (1998) derives dynamic optimal trading strategies that minimize the expected cost of trading a large block of equity over a fixed time horizon. He and Mamaysky (2005) analyzes the optimal policy for a risk averse agent who wants to liquidate a large block of a security in the presence of market impact and transaction costs basing on the standard Merton portfolio problem. Ly *et al.* (2007) seeks to select an optimal portfolio of one risk-free and on risky asset subject to liquidity risk and price impact. Gatheral (2008) demonstrates the relationship between the shape of market impact function describing the average response of the market price to traded quantity and the function that describes the decay of market impact. Brigo and Nordio (2010) estimates liquidity adjusted risk measures with a stochastic time horizon. Bayraktar and Ludkovski (2011) studies optimal trade execution strategies with Poisson process order flow with a finite liquidation horizon.

Almegren and Chris (2001) works on portfolio transactions with the aim of minimizing a combination of volatility risk and transaction costs arising from permanent and temporary market impact. Almegren (2003) determines optimal trading strategies for liquidation of a large single-asset portfolio with the same aim of the previous paper. Schied and Schoneborn (2008) uses the same problem of Almegren (2003) and analyzes the sensitivities of the value function and the optimal strategy with respect to the various model parameters. Kharroubi and Pham (2009) uses a continuous-time modeling framework to obtain optimal portfolio liquidation strategy over a finite horizon in a limit order book with bid-ask spread and temporary market price impact penalizing speedy execution trades. Marzo *et al* (2011) go forward with the optimal trade execution strategies for a large portfolio of single stocks proposed by Almegren (2003) with adding the option of trading in the opening period and the optimal trading strategy will depend on trading activity in the initial period.

Moreover, Engle (1982), Bollerslev (1986), Embrechts *et al.* (1997), Xekalaki *et al.* (2010) demonstrate the modeling of stock returns by ARCH and GARCH processes.

On the other hand, modeling stock returns and its fat tail are one of the important ingredients in our model. We fit stock returns to regular varying processes in order to forecast stock prices. These types of processes were studied by Basrak *et al.* (2002), Posedel (2005), Basrak and Johan (2008), Meinguet (2010), Laurini and Tawn (2012).

We fit stock returns to ARCH or GARCH process then we determine their tail behaviors after a big change at time 0. Then we try to find the best exit trading strategy that maximizes the utility function of the trader. Our results show that trading strategies depends largely on the selection between ARCH and GARCH process.

The rest of the chapter is as follows. Section 6.2 outlines the basic model with required theories. Section 6.3 discusses the case of liquidating in liquid market. Section 6.4 provides best exit trading strategies in illiquid markets as well as it incorporates an expert opinion into the model. Section 6.5 concludes.

6.2 Basic Model

We suppose an investor holding a block of X units of a security (Stock, Future Contracts or Units of Foreign Currency) that he plans to liquidate before time T . We postulate that at terminal date T all unsold units are immediately disposed off as one large trade through the traditional limit order book. Thus, there is always one more matching order arriving at time T . The variable T denotes the actual time needed to sell completely the initial position X . We divide the period from $t_0 = 0$ to time T into N intervals of $\tau = T/N$. The trading strategy is defined by the list x_0, \dots, x_N where x_k denotes the number of shares held by the investor at time t_k . We may equivalently specify a strategy by the trade list n_0, \dots, n_N , where $n_k = x_k - x_{k+1}$ denotes the number of units that the investor will sell at time t_k . We define S_k by the value of one security. We define

$$Y_k = \ln \left(\frac{S_k}{S_{k-1}} \right)$$

by the stock return on time t_k . Therefore, at time t_k , the investor will sell n_k share(s) and he will earn

$$n_k S_k = n_k S_0 \exp(Y_1 + Y_2 + \dots + Y_k)$$

in cash. We denote by M_k the amount of cash held by the investor at time t_k . Therefore, at date T , the investor will have the following amount of cash at maturity.

$$\begin{aligned} M_N &= n_0 S_0 + n_1 S_1 + \dots + n_N S_N \\ &= n_0 S_0 + \sum_{i=1}^{i=N} n_i S_i \\ &= n_0 S_0 + \sum_{i=1}^{i=N} n_i S_0 \exp(Y_1 + Y_2 + \dots + Y_i) \end{aligned}$$

Thus, the objective of the problem is to find an optimal trading strategy maximizing the utility¹ of obtained cash at the horizon time T .

Moreover in our work, the liquidation of the portfolio is done in a stressed market. For this matter, we will suppose that the investor is liquidating the portfolio after a severe shock at time $t = 0$ and we have to add a condition to the distribution of M_N . Therefore the expected resulted cash flow from liquidation becomes:

$$E(M_N | Y_0 < -w) = E(n_0 S_0 + \sum_{i=1}^{i=N} n_i S_0 \exp(Y_1 + Y_2 + \dots + Y_i) | Y_0 < -w)$$

where w denotes the threshold. By adding the above condition, stock returns will behave extremely.

6.2.1 Modeling fat tails of Stock Returns

To be able to model extreme stock returns, we have to add an assumption that the studied process of stock returns should have regular variations. And we take the definition from Basrak *et al.* (2002):

A d -dimensional random vector X has regular variations with index $\alpha \geq 0$ if there exists a sequence (a_n) and a random vector Θ with values in \mathbb{S}^{d-1} almost sure, where \mathbb{S}^{d-1} denotes the unit sphere in \mathbb{R}^d with respect to the norm $|\cdot|$, such that for all $t > 0$,

$$n\mathcal{P}(|X| > ta_n, X/|X| \in \cdot) \xrightarrow{v} t^{-\alpha} \mathcal{P}(\Theta \in \cdot), \text{ as } n \rightarrow \infty$$

As the investor plans to liquidate a large portfolio of a single asset after a severe shock in the market, we have to use the theorems cited in Meinguet 2010 where he outlines the tail behavior of regular varying time series².

Tail Process Theorem *Let $(Y_t)_{t \in \mathbb{Z}}$ be a stationary process in the set of all real numbers \mathbb{R} . The following are equivalent:*

(i) $(Y_t)_{t \in \mathbb{Z}}$ is jointly regularly varying with index $\alpha > 0$.

(ii) There exists a process $(F_n)_{n \in \mathbb{N}}$ in \mathbb{R} , called forward tail process, with $\mathbf{P}(F_0 < -f) = f^{-\alpha}$ for $f \geq 1$ and, as $y \rightarrow +\infty$,

$$\mathcal{L}((Y_n/y)_{n \in \mathbb{N}} | Y_0 \leq -y) \xrightarrow{d} \mathcal{L}((F_n)_{n \in \mathbb{N}})$$

(iii) There exists a process $(F_t)_{t \in \mathbb{Z}}$ in \mathbb{R} , called tail process, with $\mathbf{P}(F_0 < -f) = f^{-\alpha}$ for $f \geq 1$ and, as $y \rightarrow +\infty$,

$$\mathcal{L}((Y_t/y)_{t \in \mathbb{Z}} | Y_0 \leq -y) \xrightarrow{d} \mathcal{L}((F_t)_{t \in \mathbb{Z}})$$

¹We use logarithm utility function (see Appendix 6.C)

²Proof of theorems in Appendix 6.B

Spectral Process Theorem Let $(Y_t)_{t \in \mathbb{Z}}$ be a stationary process in the set of all real numbers \mathbb{R} . We assume that $\mathbf{P}(Y_0 < \cdot)$ is regular varying with index α . The following are equivalent:

(i) $(Y_t)_{t \in \mathbb{Z}}$ is jointly regularly varying with index $\alpha > 0$.

(ii) There exists a process $(S_n)_{n \in \mathbb{N}}$ in \mathbb{R} , called forward spectral process, such that, as $y \rightarrow +\infty$,

$$\mathcal{L}((Y_n/\|Y_0\|)_{n \in \mathbb{N}} \mid Y_0 \leq -y) \xrightarrow{d} \mathcal{L}((S_n)_{n \in \mathbb{N}})$$

(iii) There exists a process $(S)_{t \in \mathbb{Z}}$ in \mathbb{R} , called spectral process, such that, as $y \rightarrow +\infty$,

$$\mathcal{L}((Y_t/\|Y_0\|)_{t \in \mathbb{Z}} \mid Y_0 \leq y) \xrightarrow{d} \mathcal{L}((S_t)_{t \in \mathbb{Z}})$$

In this case, the tail process $(F_t)_{t \in \mathbb{Z}}$ of $(Y_t)_{t \in \mathbb{Z}}$ is given by $(F_t)_{t \in \mathbb{Z}} \stackrel{d}{=} (FS_t)_{t \in \mathbb{Z}}$ where F is independent of $(S_t)_{t \in \mathbb{Z}}$ and is Pareto(α)-distributed.

6.2.2 Fitting to ARCH(1) process

In this section, we study the tail behavior of the resulted cash flows at the horizon time under the assumption that the daily stock returns is modeled by an ARCH(1) process. We outline its properties clarifying the reason of modelling a financial data series by an ARCH(1) process:

- No correlation in the data.
- Volatility varies in time.
- Data are heavy-tailed.
- High correlation of the squares and absolute values of the data.
- High threshold exceedances appear in clusters.

Definition

We base on the model of Engle (1982) and we suppose that Y_k is an ARCH process therefore Y_k can be estimated using the following recursive system:

$$\begin{cases} Y_k = \sigma_k Z_k \\ \sigma_k^2 = \alpha_0 + \alpha_1 Y_{k-1}^2 \end{cases}$$

where Z_k is a sequence of iid innovations such that $E(Z_k) = 0$ and $V(Z_k) = 1$ for all $k > 0$, and $\alpha_0, \alpha_1 > 0$. By a simple calculation we have $Y_k^2 = \underbrace{\alpha_0 Z_k^2}_{b_k} + \underbrace{\alpha_1 Z_k^2}_{a_k} Y_{k-1}^2$ and

These can be rewritten as:

$$\underbrace{\begin{pmatrix} Y_k^2 \\ \sigma_k^2 \end{pmatrix}}_{\Gamma_k} = \underbrace{\begin{pmatrix} \alpha_1 Z_k^2 & 0 \\ \alpha_1 & 0 \end{pmatrix}}_{A_k} \underbrace{\begin{pmatrix} Y_{k-1}^2 \\ \sigma_{k-1}^2 \end{pmatrix}}_{\Gamma_{k-1}} + \underbrace{\begin{pmatrix} \alpha_0 Z_k^2 \\ \alpha_0 \end{pmatrix}}_{B_k}$$

Therefore we have written the ARCH(1) model as a multivariate stochastic recurrence equation:

$$\Gamma_k = A_k \Gamma_{k-1} + B_k$$

We return to the first representation of ARCH(1) process and we develop the equations and we obtain the following:

$$\begin{aligned} Y_k^2 &= \sigma_k^2 Z_k^2 \\ &\vdots \\ Y_k^2 &= \alpha_0 \sum_{j=0}^{k-1} \alpha_1^j \left(\prod_{i=0}^j Z_{k-i}^2 \right) + \alpha_1^k \left(\prod_{j=0}^{k-1} Z_{k-j}^2 \right) Y_0^2 \end{aligned}$$

Then we conclude the expression of σ_k^2 by writing $\sigma_k^2 = Y_k^2 / Z_k^2$ and we rewrite the solution in a matrix form:

$$\underbrace{\begin{pmatrix} Y_k^2 \\ \sigma_k^2 \end{pmatrix}}_{\Gamma_k} = \underbrace{\begin{pmatrix} \alpha_1^k \left(\prod_{i=0}^{k-1} Z_{k-i}^2 \right) & 0 \\ \alpha_1^k \left(\prod_{i=1}^{k-1} Z_{k-i}^2 \right) & 0 \end{pmatrix}}_{\mathcal{A}_k} \underbrace{\begin{pmatrix} Y_0^2 \\ \sigma_0^2 \end{pmatrix}}_{\Gamma_0} + \underbrace{\begin{pmatrix} \alpha_0 \sum_{j=0}^{k-1} \alpha_1^j \left(\prod_{i=0}^j Z_{k-i}^2 \right) \\ \alpha_0 \left(1 + \sum_{j=1}^{k-1} \alpha_1^j \left(\prod_{i=1}^j Z_{k-i}^2 \right) \right) \end{pmatrix}}_{\mathcal{B}_k}$$

Thus we have simplified the system of equation in a stochastic recurrence equation:

$$\Gamma_k = \mathcal{A}_k \Gamma_0 + \mathcal{B}_k$$

where $\mathcal{A}_k = \prod_{i=1}^k A_i$ and $\mathcal{B}_k = \sum_{i=0}^{k-1} \left(\prod_{j=0}^i (A_{k-j}) B_{k-(i+1)} \right) + B_k$. In the following, we will distinguish two cases study. In the first part, we assume that the innovations are independent and identically $N(0, 1)$ distributed whereas in the second part the innovations are independent and identically Student's t-distributed.

Normal distributed Innovations

To apply theorems of the convergence, we use the theorem cited in Basrak *et al.* (2002). A stationary ARCH Process is a regular varying process. So, we begin by putting the conditions of the stationarity on the parameters (cf. Embrecht *et al.* (1997)):

- $\alpha_0 \geq 0$.
- $0 \leq \alpha_1 < 2e^\gamma$ where $\gamma \approx 0.5772$ is Euler's constant.

To study the tail behavior of an ARCH Process, we will define a new function h . For a standard normal random variable Z and $\alpha_1 \in]0, 2e^\gamma[$, we define for all $u \geq 0$:

$$\begin{aligned} h(u) &= E(\alpha_1 Z^2)^u \\ &= \frac{(2\alpha_1)^u}{\sqrt{\pi}} \Gamma\left(u + \frac{1}{2}\right) \end{aligned}$$

The function h is strictly convex in u , and there exists a unique solution $\kappa = \kappa(\alpha_1) > 0$ to the equation $h(u) = 1$. The value $\kappa = \kappa(\alpha_1)$ is extremely significant for the extremal behavior of the ARCH(1) process. It cannot be written in an explicit form but it is estimated using mathematical simulations and the results can be found in the following table.

α_1	0.1	0.3	0.5	0.7	0.9	1.0	1.5	2.0	2.5	3.0	3.5
κ	13.24	4.18	2.37	1.59	1.15	1.0	.54	.31	.17	.075	.007

Theorem (The tail behavior of an ARCH(1) process) *Let (Y_t) be a stationary ARCH(1) process with parameters $\alpha_0 > 0$ and $\alpha_1 \in]0, 2e^\gamma[$, where γ is Euler's constant. Let $\kappa > 0$ be the unique positive solution of the equation $h(u) = 1$. Then*

$$\mathbf{P}(Y > y) \sim \frac{c}{2} y^{-2\kappa}, x \rightarrow +\infty$$

where

$$c = \frac{E [((\alpha_0 + \alpha_1 Y^2)^\kappa - (\alpha_1 Y^2)^\kappa) Z^{2\kappa}]}{\kappa E [(\alpha_1 Z^2)^\kappa \ln(\alpha_1 Z^2)]} \in]0, +\infty[$$

for a standard normal random variable Z , independent of Y .

We conclude that a stationary ARCH(1) process is jointly regular varying with index $\alpha = 2\kappa > 0$. Thus we can apply the two theorems cited in Meinguet (2010). We use the absolute value as a norm: $\|Y_t\| = |Y_t|$. We have Y_t a stationary ARCH(1) process thus Y_t is jointly regular varying with index $\alpha = 2\kappa$. We conclude that:

- There exists a process $(W_k)_{k \in \mathbb{N}}$, called tail process, with $\mathbf{P}(W_0 < -w) = w^{-\alpha}$ for $w \geq 1$ and, as $w \rightarrow +\infty$,

$$\mathcal{L}((Y_k/w)_{k \in \mathbb{N}} \mid Y_0 \leq -w) \xrightarrow{d} \mathcal{L}((W_k)_{k \in \mathbb{N}}).$$

- And there exists a process $(\theta)_{k \in \mathbb{N}}$, called spectral process, such that, as $w \rightarrow +\infty$,

$$\mathcal{L}((Y_k/\|Y_0\|)_{k \in \mathbb{N}} \mid Y_0 \leq -w) \xrightarrow{d} \mathcal{L}((\theta_k)_{k \in \mathbb{N}}).$$

where $(W_k)_{k \in \mathbb{N}} \stackrel{d}{=} (W\theta_k)_{k \in \mathbb{N}}$ and Y is independent of $(\theta_k)_{k \in \mathbb{N}}$ and is Pareto(α)-distributed.

We apply the theorem cited above for $k = 0$ and we have, as $w \rightarrow +\infty$,

$$\mathcal{L}((Y_0/w) \mid Y_0 \leq -w) \xrightarrow{d} \mathcal{L}((W_0)).$$

where $\mathbf{P}(W_0 \leq -w) = w^{-\alpha}$ for $w \geq 1$. Then, conditionally on $Y_0 \leq -w$, as $w \rightarrow +\infty$,

$$\begin{aligned} Y_1/w &= \sqrt{Y_1^2/w^2} \times \text{sign}(Z_1) \\ &= \sqrt{(b_1 + a_1 Y_0^2)/w^2} \times \text{sign}(Z_1) \\ &= \sqrt{b_1/w^2 + a_1 (Y_0/w)^2} \times \text{sign}(Z_1) \\ &\xrightarrow{d} \sqrt{a_1 W_0^2} \times \text{sign}(Z_1) \\ &= \alpha_1^{\frac{1}{2}} Z_1 W_0 \\ &= W_1 \end{aligned}$$

By doing the same calculation, we obtain

$$\begin{aligned}
Y_2/w &= \sqrt{b_2/w^2 + a_2(Y_1/w)^2} \times \text{sign}(Z_2) \quad \xrightarrow{d} \quad \sqrt{a_2 W_1^2} \times \text{sign}(Z_2) = W_2 \\
Y_3/w &= \sqrt{b_3/w^2 + a_3(Y_2/w)^2} \times \text{sign}(Z_3) \quad \xrightarrow{d} \quad \sqrt{a_3 W_2^2} \times \text{sign}(Z_3) = W_3 \\
&\dots \\
Y_k/w &= \sqrt{b_k/w^2 + a_k(Y_{k-1}/w)^2} \times \text{sign}(Z_k) \quad \xrightarrow{d} \quad \sqrt{a_k W_{k-1}^2} \times \text{sign}(Z_k) = W_k
\end{aligned}$$

Hence the tail process $(W_k)_{k \in \mathbb{N}}$ of $(Y_k)_{k \in \mathbb{N}}$ has the property

$$\begin{aligned}
W_k &= |W_0| \text{sign}(Z_k) \sqrt{a_1 a_2 \cdots a_k} \\
&= |W_0| \underbrace{\alpha_1^{\frac{k}{2}} Z_k \prod_{i=1}^{k-1} |Z_i|}_{c_k}
\end{aligned}$$

and the spectral process $(\theta_k)_{k \in \mathbb{N}}$ of $(Y_k)_{k \in \mathbb{N}}$ satisfies:

$$\theta_k = |\theta_0| \underbrace{\alpha_1^{\frac{k}{2}} Z_k \prod_{i=1}^{k-1} |Z_i|}_{c_k}$$

Tail behavior of M_N

After determining the tail behavior of Y_k , we aim to study the tail behavior of cash flows at time $t_N = T$:

$$M_N = n_0 S_0 + \sum_{i=1}^N n_i S_0 e^{Y_1 + Y_2 + \cdots + Y_i}$$

As the investor plans to liquidate the portfolio after an extreme shock in the market, we will study the tail behavior conditionally on $Y_0 < -w$. Once the distribution of $(e^{Y_1 + Y_2 + \cdots + Y_i} \mid Y_0 < -w)$ has been determined for all $i = 1, \dots, N$, we can study the tail behavior of M_N for each trading strategy. By a simple calculation, we obtain, $w \rightarrow +\infty$,

$$\begin{aligned}
\mathcal{L}(e^{Y_1 + Y_2 + \cdots + Y_i} \mid Y_0 < -w) &= \mathcal{L}(e^{|Y_0|(Y_1/|Y_0| + Y_2/|Y_0| + \cdots + Y_i/|Y_0|)} \mid Y_0 < -w) \\
&\xrightarrow{d} \mathcal{L}(e^{|Y_0|(\theta_1 + \theta_2 + \cdots + \theta_i)}) \\
&= \mathcal{L}(e^{|Y_0|(c_1|\theta_0| + c_2|\theta_0| + \cdots + c_i|\theta_0|)}) \\
&= \mathcal{L}(e^{|Y_0|\theta_0(c_1 + c_2 + \cdots + c_i)})
\end{aligned}$$

Up through this point, we have provided a thorough investigation and detailed analysis of the tail behavior of the stock returns. The investor is facing a challenge to liquidate his portfolio after the shock, thus we have to maximize $E(M_N)$ conditionally to $Y_0 = -w$. As we are working in extreme cases, we have to study the tail behavior of M_n

conditionally to $Y_0 < -w$ or $Y_0 = -w$. Thus, we will the convergence theorem mentioned in the previous section and we have, as $w \rightarrow +\infty$,

$$\mathcal{L} \left(e^{Y_0+Y_1+\dots+Y_i} | Y_0 < -w \right) \xrightarrow{d} \mathcal{L} \left(e^{Y_0+|Y_0|\theta_0(c_1+c_2+\dots+c_i)} \right)$$

In order to stay in the problematic, we should have that all the trading strategies will cause a loss for the investor, in other words, we explain it mathematically:

$$\begin{aligned} Y_0 + |Y_0|\theta_0(c_1 + c_2 + \dots + c_i) &< 0 \quad \forall i = 1 \dots N \\ -1 + \theta_0(c_1 + c_2 + \dots + c_i) &< 0 \quad \forall i = 1 \dots N \end{aligned}$$

As a first step of the study, we analyse the process $\delta_i = \sum_{k=1}^i c_k$ where $c_k = \alpha^{k/2} Z_k \prod_{j=1}^{k-1} |Z_j|$ because it is the main process which depends on time. The process $(\delta_i)_{i \in \mathbb{N}}$ is a martingale for all $\alpha_1 \in [0; \frac{\pi}{2}]^3$.

6.2.3 The case of a GARCH(1,1) process

It is well known that using a single model for the financial time series is insufficient. Under the same conditions and hypothesis, we will apply the GARCH(1,1) model in order to determine the tail behavior of M_N .

Definition of the process

Let (Z_k) be sequence of i.i.d. random variables such that $E(Z_k) = 0$ and $V(Z_k) = 1$, if (X_t) is a GARCH(1,1) process then $(X_t)_{t \in \mathbb{N}}$ is given by the equations:

$$\begin{cases} Y_k = \sigma_k Z_k \\ \sigma_k^2 = \alpha_0 + \alpha_1 Y_{k-1}^2 + \beta_1 \sigma_{k-1}^2 \end{cases}$$

where $\alpha_0, \alpha_1, \beta_1 > 0$, These can be rewritten as:

$$\underbrace{\begin{pmatrix} Y_k^2 \\ \sigma_k^2 \end{pmatrix}}_{\Gamma_k} = \underbrace{\begin{pmatrix} \alpha_1 Z_k^2 & \beta_1 Z_k^2 \\ \alpha_1 & \beta_1 \end{pmatrix}}_{A_k} \underbrace{\begin{pmatrix} Y_{k-1}^2 \\ \sigma_{k-1}^2 \end{pmatrix}}_{\Gamma_{k-1}} + \underbrace{\begin{pmatrix} \alpha_0^2 Z_k^2 \\ \alpha_0 \end{pmatrix}}_{B_k}$$

Therefore we have written the GARCH(1,1) model as a multivariate stochastic recurrence equation:

$$\Gamma_k = A_k \Gamma_{k-1} + B_k$$

As in the previous case, the stationarity of the process is a required condition for the convergence. For this reason, we refer to Nelson (1991) and the sufficient condition for the stationarity is given by:

$$E \log(\alpha_1 Z^2 + \beta_1) < 0 \text{ and } \alpha_0 > 0$$

³Proof in Appendix 6.C

We note that a stationary GARCH(1,1) is a regularly varying process with index $\alpha = 2\kappa$ where κ is the unique solution of the equation:

$$E[(\alpha_1 Z^2 + \beta_1)^\kappa] = 1$$

This equation can be solved for κ by numerical and/or simulation methods for fixed values of α_1 and β_1 from the stationarity region of a GARCH(1,1) process and assuming a concrete density for Z .

Tail Process and Spectral Process

After determining the κ of a GARCH(1,1) process, we apply the two theorems that we have applied in the case of ARCH(1) process. And we use the same norm of the previous case: $\|Y_t\| = |Y_t|$. However, we have Y_t a stationary GARCH(1,1) process thus Y_t is jointly regular varying with index $\alpha = 2\kappa$. We conclude that:

- There exists a process $(W_k)_{k \in \mathbb{N}}$, called tail process, with $\mathbf{P}(W_0 < -w) = w^{-\alpha}$ for $w \geq 1$ and, as $w \rightarrow +\infty$,

$$\mathcal{L}((Y_k/w)_{k \in \mathbb{N}} \mid Y_0 \leq -w) \xrightarrow{d} \mathcal{L}((W_k)_{k \in \mathbb{N}})$$

- And there exists a process $(\theta)_{k \in \mathbb{N}}$, called spectral process, such that, as $w \rightarrow +\infty$,

$$\mathcal{L}((Y_k/\|Y_0\|)_{k \in \mathbb{N}} \mid Y_0 \leq -w) \xrightarrow{d} \mathcal{L}((\theta_k)_{k \in \mathbb{N}})$$

where $(W_k)_{k \in \mathbb{N}} \stackrel{d}{=} (W\theta_k)_{k \in \mathbb{N}}$ and Y is independent of $(\theta_k)_{k \in \mathbb{N}}$ and is Pareto(α)-distributed.

We apply the theorem cited above for $k = 0$ and we will have, as $w \rightarrow +\infty$,

$$\mathcal{L}((Y_0/w) \mid Y_0 \leq -w) \xrightarrow{d} \mathcal{L}((W_0))$$

where $\mathbf{P}(W_0 \leq -w) = w^{-\alpha}$ for $w \geq 1$.

On the other hand, we have:

$$\Gamma_k = A_k \cdots A_1 \Gamma_0 + \sum_{i=1}^{k-1} A_k \cdots A_{i+1} B_i + B_k$$

Then, conditionally on $Y_0 \leq -w$, as $w \rightarrow +\infty$,

$$\begin{aligned} Y_k/w &= \sqrt{Y_k^2/w^2} \times \text{sign}(Z_k) \\ &= \sqrt{(A_k \cdots A_1)_{(1,1)} Y_0^2/w^2 + (A_k \cdots A_1)_{(1,2)} \sigma_0^2/w^2 + \left(\sum_{i=1}^{k-1} A_k \cdots A_{i+1} B_i + B_k\right)_{(1,1)}/w^2} \times \text{sign}(Z_k) \\ &\xrightarrow{d} \sqrt{(A_k \cdots A_1)_{(1,1)} W_0^2 + (A_k \cdots A_1)_{(1,2)} U_0^2} \times \text{sign}(Z_k) \end{aligned}$$

where $U_0 = \lim \sigma_0^2/w^2$ is tail process of the volatility. Hence the tail process $(W_k)_{k \in \mathbb{N}}$ of $(Y_k)_{k \in \mathbb{N}}$ has the property

$$W_k = \sqrt{(A_k \cdots A_1)_{(1,1)} W_0^2 + (A_k \cdots A_1)_{(1,2)} U_0^2} \times \text{sign}(Z_k)$$

and the spectral process $(\theta_k)_{k \in \mathbb{N}}$ of $(Y_k)_{k \in \mathbb{N}}$ satisfies:

$$\theta_k = \sqrt{(A_k \cdots A_1)_{(1,1)} \theta_0^2 + (A_k \cdots A_1)_{(1,2)} \Omega_0^2} \times \text{sign}(Z_k)$$

where Ω_0 is the spectral process of σ_0 . We will distinguish our case study into two cases, the first case is that the volatility doesn't have a tail process and we will have $U_0 = \Omega_0 = 0$ whereas in the second case we will study the tail behavior of the volatility, in this case we will see if a shock in the returns will cause an irregular variation in the volatility.

Volatility without tail process

We denote by d_k :

$$\begin{aligned} d_k &= \sqrt{(A_k \cdots A_1)_{(1,1)} \times \text{sign}(Z_k)} \\ &= \sqrt{\alpha_1^k Z_k^2 Z_{k-1}^2 \cdots Z_1^2 + \sum_{i=1}^{k-2} a^{k-i} b^i Z_k^2 S_{k-1, k-(i+1)} + ab^{k-1} Z_k^2} \times \text{sign}(Z_k) \end{aligned}$$

where $S_{i,t}$ is the sum product of i -combinations of $\{Z_1^2, Z_2^2, \dots, Z_t^2\}$, for instance:

$$S_{2,3} = Z_1^2 Z_2^2 + Z_1^2 Z_3^2 + Z_2^2 Z_3^2$$

. Then we will have that the tail process $(W_k)_{k \in \mathbb{N}}$ of $(Y_k)_{k \in \mathbb{N}}$ has the property

$$W_k = d_k * |W_0|$$

and the spectral process $(\theta_k)_{k \in \mathbb{N}}$ of $(Y_k)_{k \in \mathbb{N}}$ satisfies:

$$\theta_k = d_k * |\theta_0|$$

After analysing the extreme value of Y_k , we have to study the tail behavior of the cash flows at time $t_N = T$:

$$M_N = n_0 S_0 + \sum_{i=1}^N n_i S_0 e^{Y_1 + Y_2 + \cdots + Y_i}$$

Using the same hypothesis mentioned in the previous case, we will obtain, $w \rightarrow +\infty$,

$$\mathcal{L}(e^{Y_1 + Y_2 + \cdots + Y_i} | Y_0 < -w) \xrightarrow{d} \mathcal{L}(e^{Y_0 \theta_0 (d_1 + d_2 + \cdots + d_i)})$$

As we have discussed in the case of the ARCH(1) process. We study the tail behavior of M_N conditionally to $Y_0 < -w$ or $Y_0 = -w$. Thus, we apply the convergence theorem:

$$\mathcal{L}(e^{Y_0 + Y_1 + \cdots + Y_i} | Y_0 < -w) \xrightarrow{d} \mathcal{L}(e^{Y_0 + |Y_0| \theta_0 (d_1 + d_2 + \cdots + d_i)})$$

And we have the following condition:

$$\begin{aligned} Y_0 + |Y_0|\theta_0(d_1 + d_2 + \dots + d_i) &< 0 \quad \forall i = 1 \dots N \\ -1 + \theta_0(d_1 + d_2 + \dots + d_i) &< 0 \quad \forall i = 1 \dots N \end{aligned}$$

We begin by studying the process $\omega_i = \sum_{k=1}^i d_k$ where $d_k = \sqrt{(A_k \dots A_1)_{(1,1)}} \times \text{sign}(Z_k)$. In fact the process $(\omega_i)_{i \in \mathbb{N}}$ is a square integrable martingale for all $\alpha_1, \beta_1 \in]0, 1[$ such that $\alpha_1 + \beta_1 < 1^4$

Volatility with tail process

In the previous case, we have neglected the effect of the volatility on the extreme returns. In the following, we aim to determine the joint tail behavior of $\Gamma_0 = (Y_0^2, \sigma_0^2)$. First we evaluate the regular variation κ and then derive the joint and marginal tail behaviors of Γ_0 . We denote by $\widehat{\Gamma}_0 = (W_0^2, U_0^2)$ the distribution of the tail process of Γ_0 , then, as $u \rightarrow +\infty$:

$$\Gamma_0/u | Y_0^2 > u \xrightarrow{d} \widehat{\Gamma}_0$$

Then, the joint distribution of $\widehat{\Gamma}_0$ (cf. Laurini and Tawn (2007)):

$$f_{\widehat{\Gamma}_0}(w, u) = \frac{\kappa}{E(|Z|^{2\kappa})w^{1/2}u^{\kappa+3/2}} f_Z(\sqrt{w/u}) \text{ for } w \geq 1, u \geq 0$$

As we cannot factorize $f_{\widehat{\Gamma}_0}$, then W_0 and U_0 are dependent, with marginal distributions given by:

$$F_{W_0^2}(w) = 1 - w^{-\kappa}, \quad w \geq 1$$

and, with $1_{[\cdot]}$ denoting the indicator function,

$$F_{U_0^2}(u) = 1 - \frac{E(|Z|^{2\kappa} 1_{[|Z| \leq u^{-1/2}]}) + 2u^{-\kappa}(1 - F_Z(u^{-1/2}))}{E(|Z|^{2\kappa})}, \quad u \geq 0$$

We denote by $(R, T) =^d (W_0^2, W_0/U_0)$ then the joint density function of (R, T) for $r \geq 1$ and $t \geq 0$ is:

$$f_{R,T}(r, t) = \underbrace{\frac{\kappa}{r^{\kappa+1}}}_{f_R(r)} \times \underbrace{\frac{2t^{2\kappa} f_Z(t)}{E(|Z|^{2\kappa})}}_{f_T(t)}$$

Thus R and T are independent variables with density functions $f_R(r)$ and $f_W(w)$ respectively. Consequently, $\widehat{\Gamma}_0$ can be simulated via simulation of independent R and W and setting $\widehat{\Gamma}_0 = (R, R/T^2)$.⁵

⁴Proof in Appendix 6.D

⁵Algorithm of random generation is outlined in Appendix 6.E

6.3 Trading Strategies in Liquid Markets

In this section, we concentrate on liquid markets and we provide a full analysis of different trading strategies that can be executed. The aim of the study to maximize the utility function of the investor. Utility is an economic term used to measure the total satisfaction received from consuming a good or service. We can determine theoretically by a measured function, thus the investor will strive to maximize this utility function, and in this analysis we use the logarithm utility function since it can transform products into sums:

$$u(w) = \ln(w)$$

where w is the wealth. It has the following properties:

Non-satiation We have $u(w) = \frac{d(\ln(w))}{dw} = \frac{1}{w} > 0$ for all $w > 0$. The increasing function means that each investor will be more satisfied more he earns money whereas $u(w) \neq 0$ means that the investor never becomes completely satisfied and will always prefer more to less.

Risk Aversion We take the case of the beggar and the millionaire, we observe that as wealth increases, each additional amount has a lower perceived value. We have $u(w) = \frac{d^2(\ln(w))}{dw^2} = -\frac{1}{w^2} < 0$ then each additional unit of wealth increases utility by a smaller amount. Another consequence of decreasing marginal utility is risk aversion and the level of risk aversion is measured by the way of ARA (*absolute risk aversion*):

$$R_a(w) = -\frac{u''(w)}{u'(w)} = \frac{1}{w}$$

it measures risk aversion to a loss in absolute terms.

DARA *Decreasing Absolute Risk Aversion* We have $u(w) = \frac{d^3(\ln(w))}{dw^3} = \frac{2}{-w^3} < 0$ and $R_a(w) = -\frac{1}{w^2} < 0$ therefore the investor is classified in the DARA classes which it means that as wealth increases he will hold more pounds in risky assets. And it is a necessary condition for the positivity of the bid-selling spread.

We help the investor how to select a proper exit trading strategy in order to maximize the logarithm utility function of the cash value obtained at the horizon time.

6.3.1 Selection of the proper trading strategy

The optimal trading strategy by investor is which maximizes the utility of his wealth at the horizon time T . In the following, we provide an optimization technique to determine this optimal exit trading strategy in absence of the market impact. We suppose that the investor plans to liquidate X blocks of a stock. Therefore, we have the following optimization problem:

$$\begin{aligned} & \underset{n_0, \dots, n_N}{\text{Maximize}} && E(\ln M_N | Y_0 < -w) \\ & \text{Subject to} && n_0 + n_1 + \dots + n_N = X \end{aligned}$$

where w is the selected threshold which indicates the existence of an extreme event. Given $Y_0 < -w$ we have:

$$\begin{aligned}
M_N &= \sum_{i=0}^N n_i S_{-1} e^{Y_0 + \dots + Y_i} \\
&= \sum_{i=0}^N n_i S_{-1} e^{|Y_0| \left(-1 + \frac{Y_1}{|Y_0|} + \dots + \frac{Y_i}{|Y_0|} \right)} \\
&\sim \sum_{i=0}^N n_i S_{-1} e^{|Y_0| (-1 + c_1 |\theta_0| + \dots + c_i |\theta_0|)} \\
&= \sum_{i=0}^N n_i S_{-1} e^{|Y_0| \delta_i}
\end{aligned}$$

We classify in a decreasing order the time series $(\delta_i)_{i \in \mathbb{N}}$ such that:

$$\delta_{(0)} \geq \delta_{(1)} \geq \dots \geq \delta_{(N)}$$

Therefore, the limited development of M_N of order 1 is equal to:

$$\begin{aligned}
M_N &\approx n_{(0)} S_{-1} e^{|Y_0| \delta_{(0)}} (1 + o(1)) \\
\Leftrightarrow \ln M_N &\approx \ln n_{(0)} + \ln S_{-1} + |Y_0| \delta_{(0)} + o(1) \\
\Leftrightarrow E(\ln M_N) &\approx E(\ln n_{(0)}) + \ln S_{-1} + |Y_0| E(\delta_{(0)})
\end{aligned}$$

We denote γ_i by the distribution of the order of the maximum $\delta_{(0)}$ such that:

$$\gamma_i = \mathbb{P}(\delta_1 = \delta_{(0)}) \quad \forall i \in \{0, 1, \dots, N\}$$

Then, we can calculate easily the expected value of the Neperian Logarithm of $n_{(0)}$:

$$E(\ln n_{(0)}) = \sum_{i=0}^N \gamma_i \ln n_i$$

And we have the following equivalence between the two optimization problems:

$$\begin{aligned}
&\underset{n_0, \dots, n_N}{\text{Maximize}} && E(\ln M_N | Y_0 < -w) \\
&\text{Subject to} && n_0 + n_1 + \dots + n_N = X \\
&&& \approx \\
&\underset{n_0, \dots, n_N}{\text{Maximize}} && E(\ln n_0) \\
&\text{Subject to} && n_0 + n_1 + \dots + n_N = X
\end{aligned}$$

We introduce the Lagrange multiplier λ and we have the following Lagrange function:

$$\begin{aligned}\Delta(n_0, n_1, \dots, n_N, \lambda) &= E(\ln n_{(0)}) + \lambda(n_0 + n_1 + \dots + n_N - X) \\ &= \sum_{i=0}^N \gamma_i \ln n_i + \lambda \left(\sum_{i=0}^N n_i - X \right)\end{aligned}$$

Setting the gradient $\nabla_{n_0, n_1, \dots, n_N, \lambda} \Delta(n_0, n_1, \dots, n_N, \lambda) = 0$ yields the following, for all $i \in \{0, 1, \dots, N\}$,

$$\frac{\partial \Delta}{\partial n_i} = \frac{\gamma_i}{n_i} - \lambda = 0$$

On the other hand we have $\sum_{i=0}^N \gamma_i = 1$ and $\sum_{i=0}^N n_i = X$, therefore the solution of the optimization problem is given by the following system:

$$\begin{cases} \lambda = \frac{1}{X} \\ n_i = \gamma_i X \end{cases} \text{ For all } i \in \{0, 1, \dots, N\}$$

6.3.2 ARCH(1) Process

Optimal Trading Strategies Comparison

We study the tail behavior of the daily returns for several values of α_1 then we select at which time the price has the greater value. We obtain the distribution of the index of the maximum and the results of the case of ARCH(1) process are presented in Table 6.1.

We suppose that we have modeled the daily returns of a security and we obtained that

Table 6.1: Optimal trading strategies basing on ARCH(1) process

Maturity	$\alpha_1 = 0.05$	$\alpha_1 = 0.1$	$\alpha_1 = 0.2$	$\alpha_1 = 0.3$	$\alpha_1 = 0.4$	$\alpha_1 = 0.6$	$\alpha_1 = 0.7$	$\alpha_1 = 0.9$
0	49.37%	49.78%	48.13%	46.71%	44.13%	40.85%	38.63%	35.90%
1	25.40%	24.99%	23.78%	22.52%	22.31%	20.60%	19.67%	18.53%
2	12.24%	12.17%	12.37%	11.92%	12.42%	11.98%	11.60%	11.65%
3	6.70%	6.35%	7.23%	7.18%	7.47%	8.41%	8.64%	8.31%
4	3.16%	3.40%	3.81%	5.01%	4.80%	5.59%	5.55%	6.23%
5	1.61%	1.79%	1.90%	2.56%	3.28%	3.60%	4.51%	4.36%
6	0.76%	0.64%	1.36%	1.65%	2.13%	2.59%	3.19%	3.81%
7	0.46%	0.40%	0.66%	1.09%	1.42%	2.07%	2.05%	2.42%
8	0.10%	0.20%	0.34%	0.39%	0.72%	1.22%	1.59%	1.85%
9	0.11%	0.08%	0.19%	0.36%	0.44%	0.86%	1.27%	1.62%
10	0.03%	0.07%	0.12%	0.21%	0.32%	0.61%	0.90%	1.28%

α_1 is equal to 0.3, therefore the investors will sell immediately 46.71% of the portfolio whereas he will sell 22.52% at time t_1 and he will continue exit trading strategy using the table cited above. We recall that the objective of the problematic is to reduce the loss caused by the severe shock, thus the question may be asked by the trader is about

the existence of trading strategy allowing to catch up the loss and give a profit to the placement despite the severe shock. To answer this question, we counted the number of trading strategies giving a positive result and results are presented in Table 6.2.

The results confirm the proposition that the probability of catch up for risky assets is

Table 6.2: Possibility of obtaining a gain trading strategy basing on ARCH(1) model

α_1	0.05	0.1	0.2	0.3	0.4	0.6	0.7	0.9
Probability of catch up	0.0%	0.5%	3.7%	8.0%	11.4%	18.2%	21.9%	27.4%

higher than for the stable assets. In other words, we define it mathematically, the first coefficient α_1 of the ARCH(1) process is the main determinant of the trading strategy also defines the risk level of the asset. For instance, an investor holds a large block of an asset fitted to an ARCH(1) process with $\alpha_1 = 0.3$ and after a given shock, the catch up of the losses according to the investor will be equal to 8%.

6.3.3 GARCH(1,1)

Volatility without tail process

As we have estimated the optimal exit trading strategy for the case of a ARCH(1) model, by the same method we determine the optimal exit trading strategy when the process is modelled by a GARCH(1,1) process. In fact, in order to be able to compare the results we should have the same extremal index κ . For instance, we have fixed $\kappa = 4.18$ and we calculate the different parameters of GARCH(1,1) model having the same κ , then we run simulations and present them in Tables 6.3 and 6.4.

Table 6.3: Optimal trading strategies basing on GARCH(1,1) process with the same κ .

Maturity	$\alpha_1=0.2$	$\alpha_1=0.225$	$\alpha_1=0.25$	$\alpha_1=0.275$	$\alpha_1=0.3$
	$\beta_1=0.56$	$\beta_1=0.46$	$\beta_1=0.328$	$\beta_1=0.175$	$\beta_1=0$
0	25.40%	31.80%	33.60%	45.60%	47.10%
1	14.60%	14.70%	16.80%	20.80%	21.60%
2	8.80%	9.60%	9.60%	10.30%	11.40%
3	8.50%	8.40%	7.40%	6.60%	7.80%
4	6.20%	6.70%	7.60%	4.80%	5.00%
5	4.50%	4.60%	5.50%	4.10%	2.70%
6	3.00%	3.90%	4.70%	2.80%	1.80%
7	2.80%	2.70%	2.90%	1.70%	1.00%
8	3.10%	2.40%	2.40%	0.80%	0.60%
9	2.20%	2.40%	2.60%	0.70%	0.30%
10	2.60%	2.10%	1.60%	0.70%	0.40%

Table 6.4: Optimal trading strategies basing on GARCH(1,1) process with the fixed $\alpha_1 + \beta_1$.

	$\alpha_1=0.2$ $\beta_1=0.56$	$\alpha_1=0.225$ $\beta_1=0.46$	$\alpha_1=0.25$ $\beta_1=0.328$	$\alpha_1=0.275$ $\beta_1=0.175$	$\alpha_1=0.3$ $\beta_1=0$
Probability of catch up	0.114	0.102	0.068	0.039	0.016

We conclude from the two tables cited above that the main determinant of the risk level of a security is the sum of the first two coefficients of the GARCH(1,1) model. The asset the less risky is the one having the smallest $\alpha_1 + \beta_1$. In the same way, we estimate the optimal exit trading strategy for several GARCH(1,1) processes under the same constraint $\alpha_1 + \beta_1 = \text{constant}$ and results are presented in Tables 6.5 and 6.6.

Results are too close and we deduce the main determinant of the optimal trading

Table 6.5: Probability of catch up

Maturity	$\alpha_1=0.05$ $\beta_1=0.25$	$\alpha_1=0.1$ $\beta_1=0.2$	$\alpha_1=0.15$ $\beta_1=0.15$	$\alpha_1=0.2$ $\beta_1=0.1$	$\alpha_1=0.25$ $\beta_1=0.05$
0	44.70%	47.10%	45.30%	45.50%	45.40%
1	22.70%	22.20%	23.80%	22.60%	22.60%
2	11.70%	14.00%	10.00%	12.80%	12.00%
3	6.00%	5.30%	8.70%	6.80%	6.90%
4	4.90%	4.20%	4.20%	5.40%	4.60%
5	3.10%	2.90%	3.00%	2.60%	2.90%
6	2.60%	1.60%	1.60%	2.20%	2.50%
7	2.00%	0.90%	0.90%	1.10%	1.10%
8	0.50%	0.30%	1.1%	0.3%	0.70%
9	0.50%	0.50%	0.50%	0.2%	0.50%
10	0.60%	0.20%	0.30%	0.00%	0.40%

Table 6.6: Probability of cacth up

	$\alpha_1=0.05$ $\beta_1=0.25$	$\alpha_1=0.1$ $\beta_1=0.2$	$\alpha_1=0.15$ $\beta_1=0.15$	$\alpha_1=0.2$ $\beta_1=0.1$	$\alpha_1=0.25$ $\beta_1=0.05$
Probability of Catch up	0.00	0.02	0.03	0.10	0.12

strategy is the sum of the first two coefficients of GARCH(1,1) model. We can say that two securities, modeled by GARCH(1,1) processes having the sum of the two first coefficients of the model equal to 0.3, should be traded by the same trading strategies.

Volatility with tail process

We simulate with fixed $Y_0 = -10\%$ and several values of σ_0 and using the same coefficients of GARCH(1,1) process. In our example, we take $\alpha_1 = 0.1$ and $\beta_1 = 0.2$ and

the results are shown in Tables 6.7 and 6.8.

Table 6.7: Optimal trading strategies basing on GARCH processes with different volatilities

Maturity	$\sigma_0 = 0.01$	$\sigma_0 = 0.05$	$\sigma_0 = 0.1$	$\sigma_0 = 0.2$	$\sigma_0 = 0.4$
0	46.49%	45.85%	45.75%	46.39%	46.43%
1	22.70%	22.76%	23.29%	23.09%	23.67%
2	12.13%	11.88%	12.13%	11.19%	11.31%
3	6.73%	6.50%	6.11%	6.53%	6.48%
4	4.14%	4.49%	4.48%	4.32%	4.29%
5	2.76%	3.10%	2.96%	2.94%	2.92%
6	1.67%	2.01%	1.84%	1.96%	1.88%
7	1.25%	1.11%	1.29%	1.16%	1.17%
8	0.81%	0.82%	0.78%	0.74%	0.70%
9	0.48%	0.46%	0.36%	0.59%	0.42%
10	0.28%	0.35%	0.37%	0.36%	0.26%

Table 6.8: Probability of Catch up

Maturity	$\sigma_0 = 0.01$	$\sigma_0 = 0.05$	$\sigma_0 = 0.1$	$\sigma_0 = 0.2$	$\sigma_0 = 0.4$
Probability of Catch up	0.02	0.04	0.09	0.21	0.33

We conclude that the distribution of the maximum of the the index is independent of the volatility of the stock. Whereas the probability of the catch is related to volatility of the stock.

6.3.4 Application to financial markets

In this section, we apply our theory to financial markets and we collect data from liquid markets since the first theory is applied without the market impact. In the first part, we outline the algorithm to determine the optimal trading strategy, whereas in the second market test the accuracy of the model on the real data.

Algorithm

Here we support the optimisation problem by presenting a dynamic programming solution. We download historical prices of a security and we calculate the frequent returns with the same time interval using the following formula:

$$Y_t = \ln \left(\frac{S_t}{S_{t-1}} \right)$$

where Y_t denotes the stock return at time t and S_t the price of the security at time t . We choose the specific model ARCH(1) or GARCH(1,1) with Normal or Student innovations.

Then we estimate the parameters of the model using maximum likelihood method. The optimal trading is a rule for determining n_0, n_1, \dots, n_N at time $t_0 = 0$ in order to maximise $E(\ln(M_N))$. However we fix the initial security return Y_0 which should be a large number i.e. -10%, -20% or -30%. We forecast the stock return basing on the asymptotic behavior:

$$\begin{cases} Y_t = \sigma_t Z_t \\ \text{ARCH} & : \sigma_t^2 = \alpha_1 Y_{t-1}^2 \\ \text{GARCH} & : \sigma_t^2 = \alpha_1 Y_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \end{cases}$$

where α_1 and β_1 are positive, we calculate the stock price at time t ,

$$S_t = S_{t-1} \exp(Y_t)$$

And we find at which time S_t has the greater value. We repeat this step 10000 times and we obtain the distribution of the maximum index of the stock prices, therefore we arrived to determine the optimal trading strategy. In the following, we consider three types of trading strategies:

Fire Sale We sell the stock at the first opportunity which is considered in this paper as the first maturity after the shock.

Linear Sale The investor sell the asset linearly over all the period.

Optimal Trading Strategy The liquidation of the portfolio is determined by the optimal trading strategy resulted from ARCH or GARCH process.

We simulate the stock price basing on ARCH(1) process or GARCH(1,1) process:

$$\begin{cases} \text{ARCH} & : \sigma_t^2 = \alpha_1 Y_{t-1}^2 \\ \text{GARCH} & : \sigma_t^2 = \alpha_1 Y_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \\ Y_t = \sigma_t Z_t \\ S_t = S_{t-1} \exp(Y_t) \end{cases}$$

Thus we have the distribution of the horizon time cash value of each trading strategy. Referring to the theoretical part, the optimal trading strategy should maximize $E(\ln M_N)$. Since the optimization problem is solved after a limited development of order 1, we repeat the simulation 10000 times and we see how much the optimal trading strategy maximizes $\ln(M_N)$.

Application and empirical analysis

Data

Turning to the application, we download the daily historical prices of our liquid security CAC40. Using R programming environment, we fit a GARCH(1,1) and ARCH(1) time series model to the data by computing the maximum-likelihood estimates of the conditionally normal model and we obtain the following results:

$$\begin{cases} \text{ARCH} & : \sigma_t^2 = 0.2402Y_{t-1}^2 + 0.00016 \\ \text{GARCH} & : \sigma_t^2 = 0.0865Y_{t-1}^2 + 0.8985\sigma_{t-1}^2 + 0.000003 \\ Y_t = \sigma_t Z_t \\ S_t = S_{t-1} \exp(Y_t) \end{cases}$$

	Estimate	Std. Error	t value	Pr(> t)
ARCH				
(Intercept)	1.58e-04	2.48e-06	63.44	<2e-16
Y_{t-1}^2	2.40e-01	1.40e-02	17.14	<2e-16
GARCH				
(Intercept)	3.05e-06	3.36e-07	9.10	<2e-16
Y_{t-1}^2	8.65e-02	5.73e-03	15.11	<2e-16
σ_{t-1}^2	8.99e-01	6.50e-03	138.33	<2e-16

To study model errors, we draw the quantile-quantile plot between the residuals and the Normal distribution in Figures 6.1 and 6.2 which show the abnormality of errors distribution. For this issue, we regress innovations to a Student distribution such that its

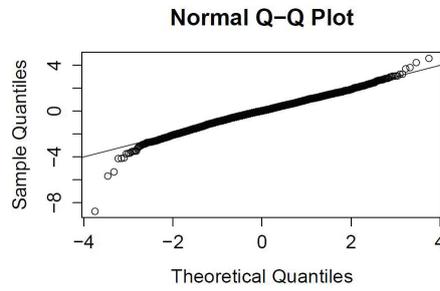
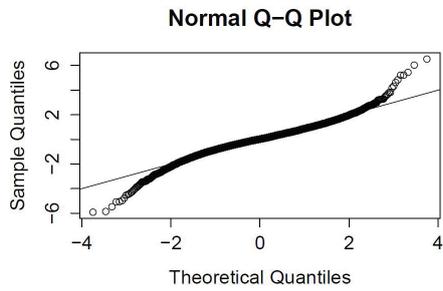


Figure 6.1: QQplot Residuals vs Normal Distribution (ARCH)

Figure 6.2: QQplot Residuals vs Normal Distribution (GARCH)

degree freedom is obtained from the Hill estimator then we plot the regression in Figures 6.3 and 6.4. And the results shows that it is better to use Student innovations in the following study.

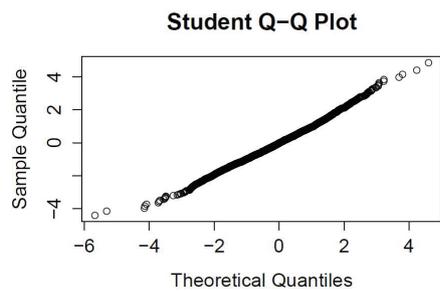
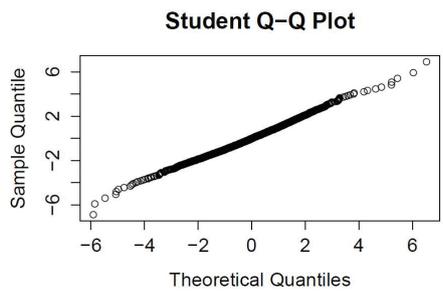


Figure 6.3: QQplot Residuals vs Student Distribution (ARCH)

Figure 6.4: QQplot Residuals vs Student Distribution (GARCH)

Optimal trading strategies

After estimating the parameters, we found that the residuals can be modeled better by a Student distribution with a degree of freedom determined by the Hill estimator. We

run the algorithm outlined in the previous section and we determine the optimal trading strategy. We process the problem with an initial shock value $Y_0 = -10\%$. The optimal trading strategies are represented in the following table:

Table 6.9: Optimal Exit Trading Strategy for CAC40

Maturity	ARCH	GARCH	Maturity	ARCH	GARCH
1	47.52%	10.19%	21	0.00%	1.60%
2	23.88%	5.16%	22	0.00%	1.59%
3	12.53%	3.91%	23	0.00%	1.61%
4	6.96%	3.29%	24	0.00%	1.57%
5	3.95%	2.88%	25	0.00%	1.57%
6	2.21%	2.63%	26	0.00%	1.59%
7	1.27%	2.40%	27	0.00%	1.59%
8	0.72%	2.27%	28	0.00%	1.64%
9	0.40%	2.15%	29	0.00%	1.69%
10	0.24%	2.05%	30	0.00%	1.71%
11	0.14%	1.96%	31	0.00%	1.75%
12	0.07%	1.92%	32	0.00%	1.79%
13	0.04%	1.86%	33	0.00%	1.91%
14	0.02%	1.79%	34	0.00%	2.00%
15	0.01%	1.75%	35	0.00%	2.13%
16	0.01%	1.73%	36	0.00%	2.33%
17	0.00%	1.67%	37	0.00%	2.57%
18	0.00%	1.65%	38	0.00%	3.05%
19	0.00%	1.63%	39	0.00%	3.98%
20	0.00%	1.63%	40	0.00%	7.82%

Distribution of the Utility function after liquidation

We compare the optimal trading strategy to simple liquidation strategies. For this matter, we suppose that the investor is facing three possible trading strategies with a fixed time horizon:

- Linear liquidation of the asset
- Aggressive sale of the asset
- Optimal trading strategies

Unfortunately, when plotting the results in Figures 6.5 and 6.6 in order to show the distribution of $\ln(M_N)$ at maturity, results show that the optimal trading strategy isn't the best solution despite it maximizes the expected return at maturity. An explanation of this conclusion is provided in the following.

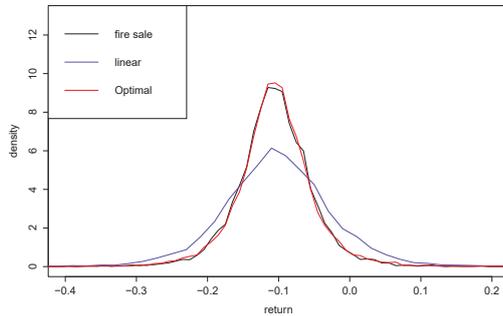


Figure 6.5: ARCH

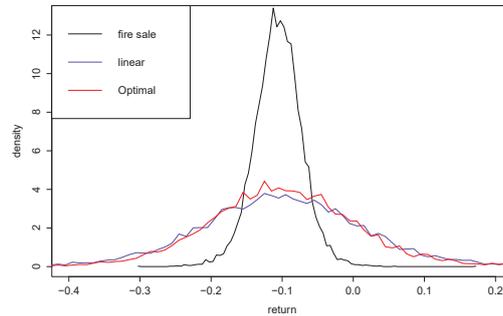


Figure 6.6: GARCH

Empirical Analysis and Application in the real situation

As the coefficient of GARCH is significant, we study the fitted GARCH model and the same analysis can be done when applying ARCH processes.

The main determinant of the trading strategy are the expected volatility of the return as well as the stock price after the shock. For this matter, we plot in Figure 6.7 the tail behavior of the volatility after the shock whereas Figure 6.8 represent the Stock price behavior after a shock of 10%.

In Figure 6.8 the volatility started from the mean of historical daily standard deviations, after the shock, the volatility make a jump and it's normal that in the first day of a crisis, we have incertitude in forecasting stock returns due to large demands and supplies may exist on the order book. Then we draw the distribution of the future trajectories of the volatility, we show that on average it returns to the mean of the historical volatility with a speed of adjustment that depends on GARCH parameters.

On the other hand, we draw the distribution of future stock prices for the upcoming 40 days and it is obvious that the volatility of the stock price will raise with the time, for instance, forecasting of a stock price for one maturity is less volatile than projecting for two maturities since it depends on the estimated value on the first maturity and so on. Concerning the mean of the stock price increases but not significantly due to the decrease of the volatility of stock return with the time and the common used distribution for the simulations.

In order to determine the best trading strategy, we run an algorithm that takes around 8 hours to provide results. The program consists of generating all the possible trading strategies for a given a maturity (40 days in our example) then we generate stock returns and we can obtain the distribution of cash flows obtained from any trading strategy at maturity. Then we calculate the expected logarithm of the cash flow of each maturity and we see if there exists a convergence to a trading strategy that maximizes the utility function \ln but results show its absence and it depends largely on random generation. However, we observe a convergence for a trading strategy that minimizes that the stan-

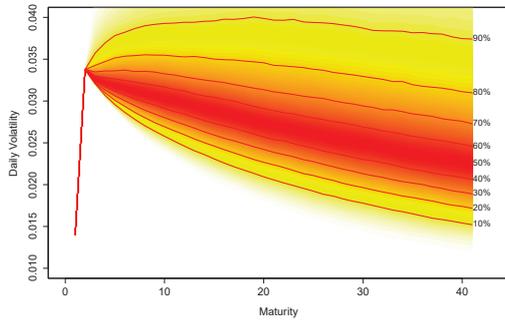


Figure 6.7: Tail Behavior of volatility

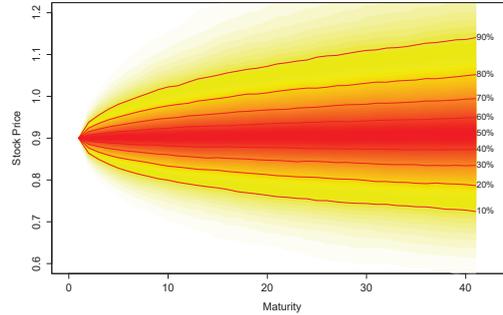


Figure 6.8: Stock Price Behavior after Shock

dard deviation of the cash flow obtained as well as of its logarithm. And this trading strategy show that the investor should fire sale his whole portfolio in absence of market liquidity costs.

6.4 Trading Strategies in illiquid markets

6.4.1 Market Impact

Definition

The market impact is defined as the difference between the transaction cost and what the market price would have been in the absence of the transaction. The market impact is significant in illiquid markets because of difficulties found in trading large blocks. Nevertheless, for large investors, the market impact is a key consideration that needs to be considered before any decision to move money within or between financial markets. The market impact can't be easily measure since we can not observe the occurrence and non-occurrence of the transaction. For this matter, we refer to the order book and we look at quoted prices just prior to the transactions. The market impact can be divided into a permanent component associated with the information, and a temporary component arising from the liquidity demands made by execution in a short time.

Mathematical definition

We suppose that an investor decides at time t to make an sell order in stock of size q . If the current price is S_t , and the time lag from the last order is τ , the transaction price of a security will be equal to:

$$S_t \times f(q, \tau)$$

where f is the temporary price impact function from $[0, +\infty[\times [0, T]$ into $[0, 1]$. We assume that the function satisfies the following properties:

- (i) f is continuous on $[0, +\infty[\times [0, T]$.

- (ii) f is C^1 on $[0, +\infty[\times [0, T]$.
- (iii) $f(0, \tau) = 1$ for all $\tau \in [0, T]$, it means that no trade incurs no impact on the market price.

We denote by the multiplier k_b such that $k_b \times S_t$ reflects the bid price. Therefore, the usual form of the temporary price impact and transaction cost function f is: (cf. Kharroubi (2009))

$$f(q, \tau) = k_b e^{-\lambda \left| \frac{q}{\tau} \right|^\beta}$$

Graphical analysis

Before applying to financial markets, we provide a description of parameters of the function. First, we suppose that the investor can trade at each instant, therefore the time lag between the trades will be equal to one at each instant and $\tau = 1$. When collecting data from Bloomberg professional platform, we measure the market impact by calculating the following ratio:

$$\text{Market Impact} = \frac{\text{Purchase Price}}{\text{Market Price}}$$

And the simple form of the market impact function will be as the following:

$$f(x) = \exp(-\lambda x^\beta)$$

Where λ and β are the non-negative parameters to be estimated, x denotes the percentage of the portfolio to be sold at time. In absence of market impact, we have $f(x) = 1$ for all $x \in [0, 1]$. On the other hand, if the trader decides to sell all the portfolio at time, thus the market impact function will have the lowest value $\exp(-\lambda)$ therefore λ denotes the highest market impact can be occurred. Whereas β is an index for the market liquidity. To put it more simply, we draw the market impact function for the same but with different market liquidity indicators.

We conclude from the figure that an asset is more sensitive to the size of the order when the market liquidity index β is small. For instance, we consider that an investor plans to liquidate two large portfolios and the two assets have the same reaction if he plans to liquidate them immediately with $\lambda = 0.05$. On the contrary, each asset have his proper liquidity index β , we suppose that the Security A has β equal to 0.25 whereas the Security B has β equal to 4. Referring to the figure, we conclude that the asset A is more liquid than the asset B since that the more impact function of the asset A is always above the function of the asset B.

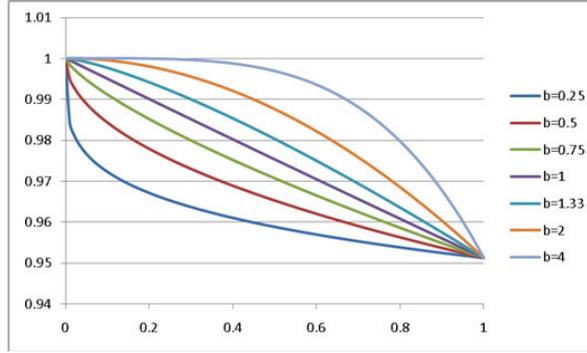


Figure 6.9: Market Impact function

6.4.2 Optimization Problem

We introduce the market impact to our model and we will have:

$$\begin{aligned}
 M_N &= n_0 S_0 e^{-\lambda \left| \frac{n_0}{\tau} \right|^\beta} + n_1 S_1 e^{-\lambda \left| \frac{n_1}{\tau} \right|^\beta} + \dots + n_N S_N e^{-\lambda \left| \frac{n_N}{\tau} \right|^\beta} \\
 &= \sum_{i=0}^N n_i S_i e^{-\lambda \left| \frac{n_i}{\tau} \right|^\beta} \\
 &= \sum_{i=0}^N n_i S_{-1} e^{Y_0 + Y_1 + Y_2 + \dots + Y_i} e^{-\lambda \left| \frac{n_i}{\tau} \right|^\beta}
 \end{aligned}$$

where we denote the time lag from the last order executed by τ . Therefore, the resolution of the optimisation problem isn't simple as in the first case:

$$\begin{aligned}
 &\text{Maximize}_{n_0, \dots, n_N} \quad \sum_{i=0}^N n_i e^{y_0 + \dots + y_i} e^{-\lambda \left| \frac{n_i}{\tau_i} \right|^\beta} \\
 &\text{Subject to} \quad n_0 + n_1 + \dots + n_N = X
 \end{aligned}$$

In our case, we have $\tau_i = 1$ for all $i \in \{0, \dots, T\}$. And we have the following optimization problem:

$$\begin{aligned}
 &\text{Maximize}_{n_0, \dots, n_N} \quad E(\ln M_N | Y_0 < -w) \\
 &\text{Subject to} \quad n_0 + n_1 + \dots + n_N = X
 \end{aligned}$$

where w is the selected threshold which indicates the existence of an extreme event. Given $Y_0 < -w$ we have:

$$\begin{aligned}
M_N &= \sum_{i=0}^N n_i S_{-1} e^{Y_0 + \dots + Y_i} e^{-\lambda n_i^\beta} \\
&= \sum_{i=0}^N n_i S_{-1} e^{|Y_0| \left(-1 + \frac{Y_1}{|Y_0|} + \dots + \frac{Y_i}{|Y_0|} \right)} e^{-\lambda n_i^\beta} \\
&\sim \sum_{i=0}^N n_i S_{-1} e^{|Y_0| (-1 + c_1 |\theta_0| + \dots + c_i |\theta_0|)} e^{-\lambda n_i^\beta} \\
&= \sum_{i=0}^N n_i S_{-1} e^{|Y_0| \delta_i} e^{-\lambda n_i^\beta}
\end{aligned}$$

We classify in a decreasing order the time series $(\delta_i)_{i \in \mathbb{N}}$ such that:

$$\delta_{(0)} \geq \delta_{(1)} \geq \dots \geq \delta_{(N)}$$

Therefore, the limited development of M_N of order 1 is equal to:

$$\begin{aligned}
M_N &\approx n_{(0)} S_{-1} e^{|Y_0| \delta_{(0)}} e^{-\lambda n_{(0)}^\beta} (1 + o(1)) \\
\Leftrightarrow \ln M_N &\approx \ln n_{(0)} - \lambda n_{(0)}^\beta + \ln S_{-1} + |Y_0| \delta_{(0)} + o(1) \\
\Leftrightarrow E(\ln M_N) &\approx E(\ln n_{(0)} - \lambda n_{(0)}^\beta) + \ln S_{-1} + |Y_0| E(\delta_{(0)})
\end{aligned}$$

We denote the distribution of the order of the maximum $\delta_{(0)}$ by γ_i such that:

$$\gamma_i = \mathbb{P}(\delta_1 = \delta_{(i)}) \quad \forall i \in \{0, 1, \dots, N\}$$

Then, we can calculate easily the expected value of the Neperian Logarithm of $n_{(0)}$:

$$E(\ln n_{(0)} - \lambda n_{(0)}^\beta) = \sum_{i=0}^N \gamma_i (\ln n_i - \lambda n_i^\beta)$$

And we have the following equivalence between the two optimization problems:

$$\begin{aligned}
&\underset{n_0, \dots, n_N}{\text{Maximize}} && E(\ln M_N | Y_0 < -w) \\
&\text{Subject to} && n_0 + n_1 + \dots + n_N = X \\
&&& \Leftrightarrow \\
&\underset{n_0, \dots, n_N}{\text{Maximize}} && E(\ln n_0 - \lambda n_{(0)}^\beta) \\
&\text{Subject to} && n_0 + n_1 + \dots + n_N = X
\end{aligned}$$

We introduce the Lagrange multiplier Λ and we have the following Lagrange function:

$$\begin{aligned}\Delta(n_0, n_1, \dots, n_N, \Lambda) &= E(\ln n_{(0)} - \lambda n_{(0)}^\beta) + \Lambda (n_0 + n_1 + \dots + n_N - X) \\ &= \sum_{i=0}^N \gamma_i (\ln n_i - \lambda n_{(0)}^\beta) + \Lambda \left(\sum_{i=0}^N n_i - X \right)\end{aligned}$$

Setting the gradient $\nabla_{n_0, n_1, \dots, n_N, \Lambda} \Delta(n_0, n_1, \dots, n_N, \lambda) = 0$ yields the following, for all $i \in \{0, 1, \dots, N\}$,

$$\frac{\partial \Delta}{\partial n_i} = \frac{\gamma_i}{n_i} - \lambda \beta n_i^{\beta-1} - \Lambda = 0$$

We solve this system numerically and we obtain optimal exit trading strategies in illiquid markets.

6.4.3 Temporary market impact

In liquid markets, the best exit trading strategy is to liquidate immediately the entire portfolio since that close values of the cash obtained on maturity in absence of market impact. In the following, we implement the market impact function to the model and we see what are effects of the market liquidity on the optimal exit trading strategy and how much the expected utility of the cash obtained from the optimal exit trading strategy is far from others when applying linear or aggressive liquidation strategies.

We outline the algorithm of the resolution of the problematic in order to determine the optimal exit trading strategy based on theory, then we apply the model on most liquid assets traded on Beirut Stock Exchange:

- Solidere A: Lebanese joint-stock company in charge of planning and redeveloping Beirut Central District. ‘A’, amounting to 100M shares made up the contribution in kind to the paid-in capital of Solidere. ‘A’ shareholders were given the option to recuperate their properties in exchange of surrendering said shares back into the Company.
- Solidere B: Same legal and financial rights and obligations as Solidere A. ‘B’, amounting to 65M shares made up the cash contribution.

Algorithm

As in the previous section, we outline the resolution of the optimisation problem by a dynamic programming solution. We begin by downloading the historical returns of the stocks and we determine the optimal exit trading strategy without the market impact as defined in the previous section.

Nevertheless, we have to introduce the market impact function to the model. For this reason, we begin by downloading the daily order book of each stock and we select the large orders having a high impact on stock returns. Therefore, we have a table of

two columns such that in the first we have the sizes of the large executed orders whereas the second outlines the impact of each large transaction on the stock returns. To have a proper estimation of the market impact, we take from the investor the number of the assets to be liquidated then we select from the table all the executed transactions with sizes less or equal of the total number of the assets to be liquidated then we fit the data to the market impact function $f(x) = \exp(\lambda * x^\beta)$ and we obtain the estimation of the parameters λ and β .

In order to determine the optimal exit trading strategy in illiquid markets, we solve the system explained in the last optimisation problem,

$$\frac{\gamma_i}{n_i} - \lambda \beta n_i^{\beta-1} - \Lambda = 0, \forall i = 0 \dots T$$

where Λ denotes the Lagrange multiplier. We solve this system by iterations and we arrive to the optimal liquidation strategy. Then we do the same procedure of the previous case in order to compare the different trading strategies.

Fitting to ARCH/GARCH process

We fit the data to GARCH/ARCH Models and we represent the results in the table 6.10

Also we regress residuals to Student distribution and we will apply GARCH model

Table 6.10: Parameters Estimation

	α_0 (STD)	α_1 (STD)	β_1 (STD)
SOL A ARCH	3.00E-04 (6.09e-06)	0.468 (2.30e-02)	-
SOLA GARCH	1.04E-04 (5.27e-06)	0.362 (1.50e-02)	0.499 (1.74e-02)
SOLB ARCH	4.00E-04 (7.16e-06)	0.359 (2.18e-02)	-
SOLB GARCH	3.00E-05 (1.73e-06)	0.188 (8.01e-03)	0.781 (5.724e-03)

since all the parameters are significant.

Market Impact Function

In this section, we estimate parameters of the market impact function. For this reason, we take a look over the daily order book of Beirut stock exchange to find orders that made an impact on the stock returns. After finding these orders, we fit them to the market impact function and we outline the results in the table 6.11.

Exit Trading Strategy

As we have mentioned that the expected stock price stays in a stable region for a short period since we base on ARCH/GARCH processes. But when we introduce the market impact, the return on investment depends largely on the trading strategy.

Table 6.11: Market Impact Function Parameters

	Qunatity	Impact	X	λ	β
Solidere A	13000	10%	1	10%	0.897712
	1000	1%	0.076923		
Solidere B	10000	10%	1	10%	0.788855
	1300	2%	0.13		

In the following, we suppose that an investor holds 10'000 shares of Solidere B and he decided to liquidate them after shock of 10%. The same study can be done when considering other illiquid assets. In fact, as the investor liquidate quickly as he avoids the time risk but with high market impacts. For this issue, we suppose that the best trading strategy is to liquidate fast as he can for a given market impact. In Figure 6.10, we present the best trading strategy to be executed when the investor is ready to support a high market impact equal to 8% where the investor should liquidate 90.63% on the first opening trading day and 9.37% on the second, and so on for Figures 6.11, 6.12 and 6.13 representing best trading strategies with the respective market impacts 5%, 4% and 3%.

We observe the period of the exit trading strategy depends largely on the selected market impact to be supported by the investor.

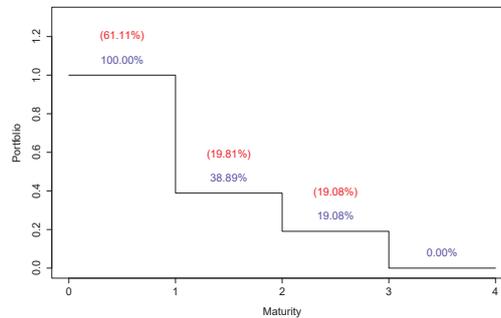
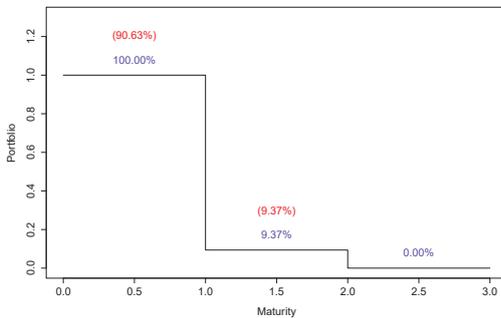


Figure 6.10: Best Exit Trading Strategy with a market impact equal to 8% Figure 6.11: Best Exit Trading Strategy with a market impact equal to 5%

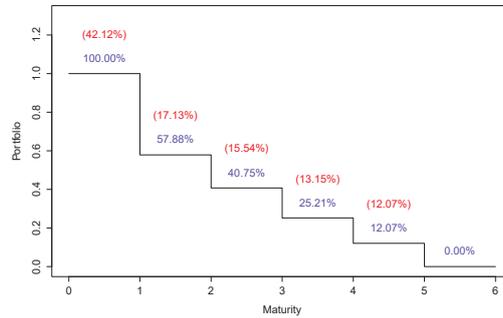
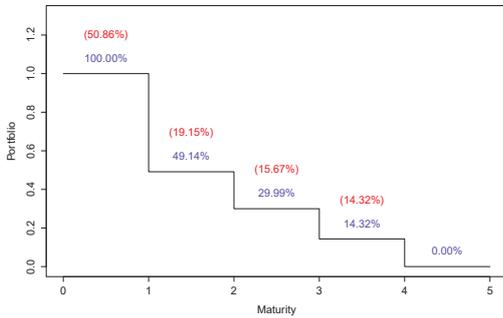


Figure 6.12: Best Exit Trading Strategy with a market impact equal to 4% Figure 6.13: Best Exit Trading Strategy with a market impact equal to 3%

We have linked the best exit trading strategy to the expected return at maturity basing on the market impact function. In order to finalize the study, we should introduce the volatility of the cash flow letting the investor to select the best strategy depending on his utility function. Figure 6.14 plots the volatility in percentage in function of the market impact. The figure shows that the volatility and the timing risk vary in a opposite sense of the market impact. Therefore, the investor can now select the best trading strategy basing on the given information in this section.

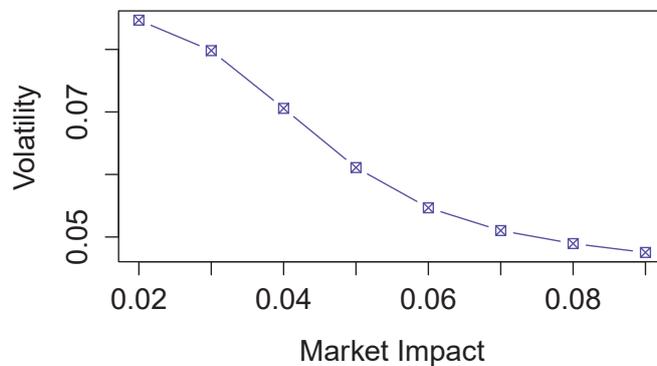


Figure 6.14: The volatility of the best exit trading strategy in function of the market impact

6.4.4 Permanent Market Impact

In the previous case, we supposed that the investor is facing a temporary market impact due to large orders on an illiquid market. But now, we suppose that the investor is market leader for a given asset and he holds a large portfolio of this asset in such way that

the market price of the asset makes a reaction immediately and permanently depending on trading strategy executed by the investor. Therefore, the money obtained at maturity from the liquidation strategy is as follows:

$$M_N = \sum_{i=1}^N [n_i \times S_i \times \prod_{k=1}^i f(n_k)]$$

Where n_i denotes the trading strategy and f the market impact function.

Linear Sale Strategy

First to show what is the difference between these two market impacts, we plot in Figures 6.15 and 6.16 the market impact of linear trading strategies for different maturities. We observe that with the temporary liquidity cost, the investor can minimize the market impact to zero if he accepts to face the volatility risk. We can say that in this case the trading is done in a stressed illiquid market where the investor is forced to pay transaction costs in order to liquidate in a way that the asset price is not impacted by these transactions. On the second hand, when the investor is a market leader, we observe that the market is at the minimum when the trade is done on the first 4 dates due to the impact of these orders on the stock price behavior.

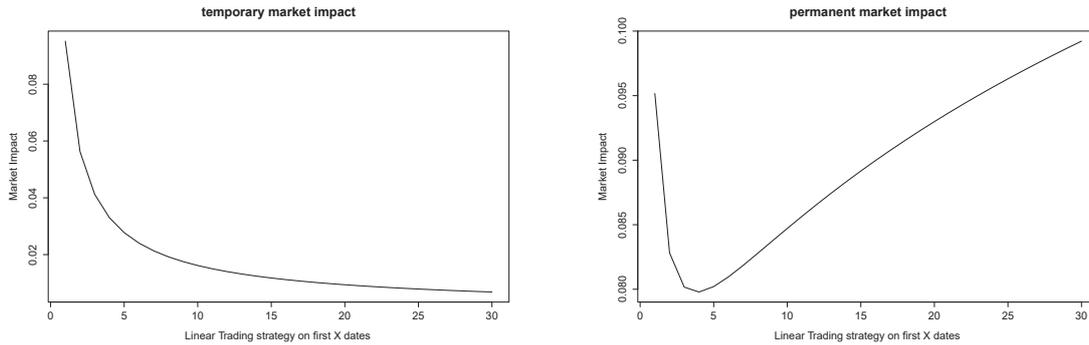


Figure 6.15: Temporary Market impact of Figure 6.16: Permanent Market impact of linear trading strategies for different maturities

Application

As we have mentioned before, the investor should liquidate the whole portfolio as soon as possible with taking into consideration the market impact due to the liquidity problem.

In order to minimize the market impact, we should minimize the following function for a given horizon time:

$$\begin{aligned} & \text{Minimize}_{n_0, \dots, n_N} \sum_{i=1}^N [n_i \times \prod_{k=1}^i f(n_k)] \\ & \text{Subject to } n_0, n_1, \dots, n_N > 0 \end{aligned}$$

Basing on simulations, we plot in Figures 6.17, 6.18, 6.19 and 6.20 the best trading strategy to be executed in order to minimize the market impact with a given horizon time.

We conclude from figures that the market impact can not be minimized to less

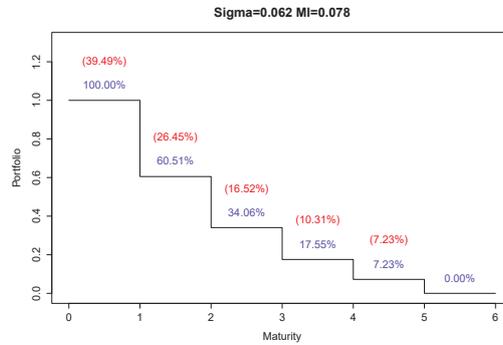
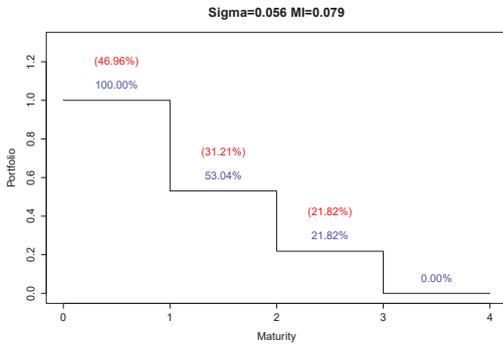


Figure 6.17: Best Exit Trading Strategy within 3 days. Figure 6.18: Best Exit Trading Strategy within 5 days.

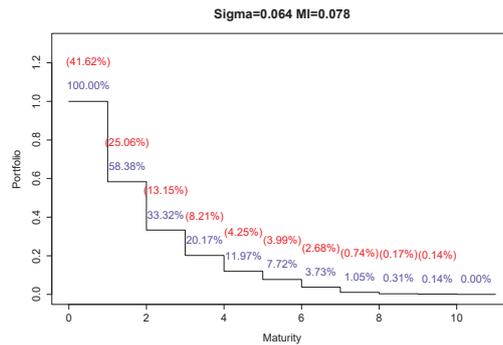
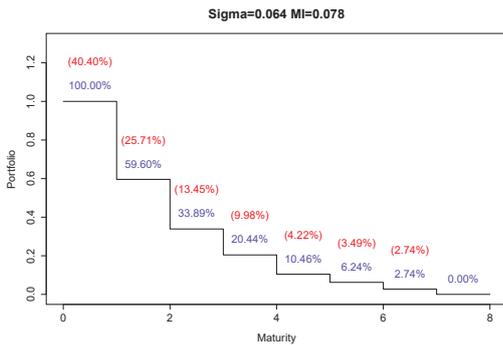


Figure 6.19: Best Exit Trading Strategy within 7 days. Figure 6.20: Best Exit Trading Strategy within 10 days.

than 7.8% even if we increase the horizon time of the trading strategy. Also, we can do a simple comparison, in the case of temporary market impact, the investor can liquidate the portfolio in two days with a market impact equal to 8% but when the liquidity cost is considered permanent, the investor should liquidate within 3 days or more to obtain a market impact equal to 8%. As we have done in the previous section, Figure 6.21 plots the volatility of the cash obtained from liquidation in function of the market impact and we let the investor decides which trading strategy to decide depending on his utility function and risk aversion.

6.4.5 Incorporating Expert Opinion into the model

So far we have analyzed the case where the investor possesses no information to predict the stock price while being able to forecast the volatility. However, a more practical

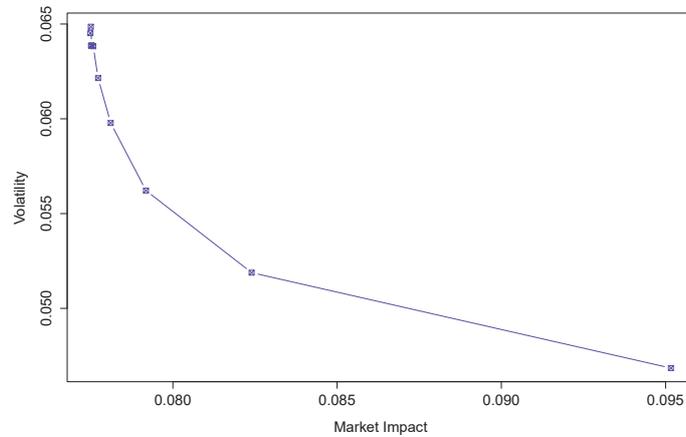


Figure 6.21: The volatility of the best exit trading strategy in function of the market impact

approach is to incorporate expert opinion into the analysis thereby allowing the prediction of non-zero expected return while maintaining the same estimation methodology of the volatility.

The approach consists of adding the expertise of practitioners as the mean of the randomly generated number while the forecasting of the volatility remains the same.

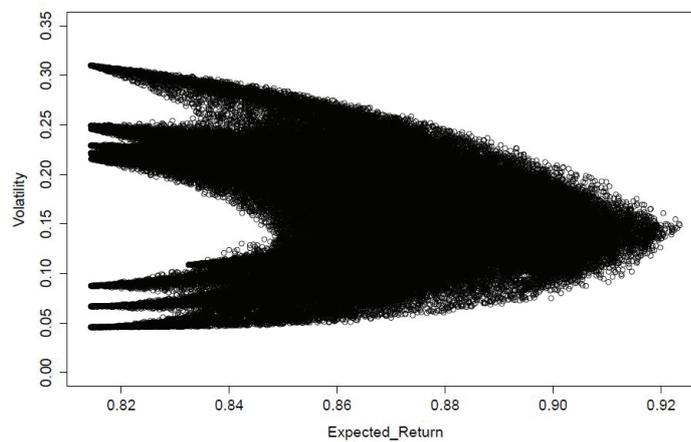


Figure 6.22: The volatility of the best exit trading strategy in function of the expected return at maturity

Figure 6.22 shows the volatility of the obtained cash flow in function of the expected return. Therefore the investor will choose the best couple (Expected Return , Volatility) basing on his utility function in order to take the best applied trading strategy.

6.5 Conclusion

In this chapter, we consider an investor holding a large portfolio of one security, and after a severe shock he decides to liquidate the entire portfolio. Using data from Bloomberg and basing on ARCH/GARCH models, our objective is to determine the best exit trading strategy to be executed by the investor with optimizing trading losses and trader's utility function.

In the first part, the trade is executed on liquid markets and the optimal trading strategy is independent of the size of the portfolio due to the absence of the market impact in such case. We suggest the investor to fire his whole portfolio at time 0 to avoid timing risk.

Conversely, in the second part, the main aim is to put emphasis on the importance of the market impact in illiquid markets. Therefore, the optimization problem is resolved while accounting for the market impact. Results here indicate a significant spread between expected returns of the different trading strategies, and highlight the effect of market liquidity that should be taken into consideration in the pre-trade analysis. Unfortunately, we conclude that trading strategies depend largely on the selection between ARCH and GARCH process.

The fundamental idea of this study resides in finding the ability to assess the losses arising from an exit strategy in absence of a liquid market and hence computing an optimal proportion of our portfolio to be sold in each trading day with the purpose of trying to minimize the damage while avoiding the timing risk. For this matter, we plot results in a figure showing a relationship between the volatility of the trading strategy and its expected return and the investor will select his best trading strategy basing on these two indicators.

Moreover, we go forward with the study by incorporating an expert opinion in the model and we also present results in a scatter plot for the selection of the best trading strategy by the investor.

The weakness of the analysis is that trading strategies differ largely when considering ARCH or GARCH process for the same asset. Therefore this will be discussed in future studies in the sense of model risk.

Appendix 6.A - Modeling Stock Returns⁶

The problem of modeling financial series is known by its complexity. This complexity is not only due to the availability of very large data sets, to the importance of the frequency of observations (second, minute, hour, day, etc...) or to the variety of the series in use (stocks, exchange rates, interest rates, etc...). It is mainly due to the existence of statistical regularities which are common to a large number of financial series and are difficult to reproduce artificially using stochastic models.

In our model, we considered the log return $Y_k = \ln(S_k/S_{k-1})$ as the main variable which is very close to the series of relative price variations $r_k = (S_k - S_{k-1})/S_{k-1}$ since $Y_k = \ln(1 + r_k)$. Hereafter outlines the properties of the stock returns and we used CAC 40 returns to illustrate the properties.

1. *Nonstationarity of stock returns.* The daily returns have a random walk without intercept and oscillate around zero. The oscillations doesn't have a steady magnitude but it varies in times. However the recent extreme volatility of prices was induced by the financial crisis of 2008.

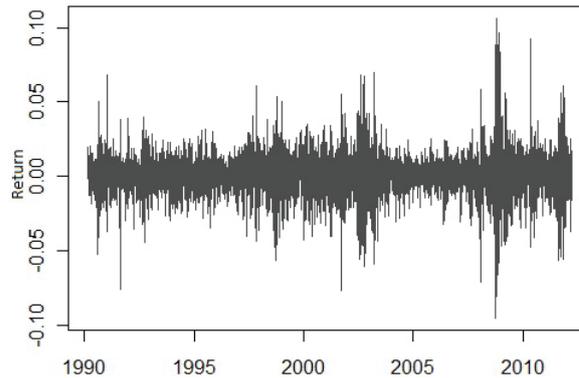


Figure 6.23: CAC 40 returns (March 2, 1990 to March 27, 2012); September 2001, fall of the Twin Towers; October 2008, financial crisis

2. *Absence of autocorrelation for the price variations.* The series of price variations generally displays small autocorrelations, which it means that the variable is close to the white noise. This is illustrated in Figure 2.
3. *Autocorrelations of the squared price returns.* Squared returns (Y_t^2) or absolute returns ($|Y_t|$) are generally strongly autocorrelated. This property show the assumption of the white noise is not strong.
4. *Volatility clustering.* Large absolute returns ($|Y_t|$) tend to appear in clusters. High volatility periods are followed by low volatility subperiods. These subperiods are recurrent but do not appear in a periodic way (see Figure 4).

⁶For more details, please refer to Emprechts *et al.* (1997), Xekalaki (2010)

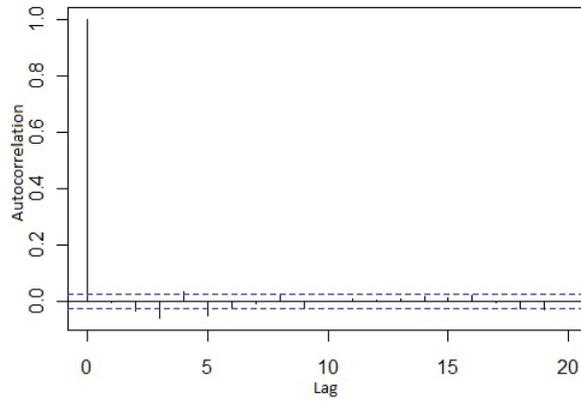


Figure 6.24: Autocorrelation of CAC 40 returns

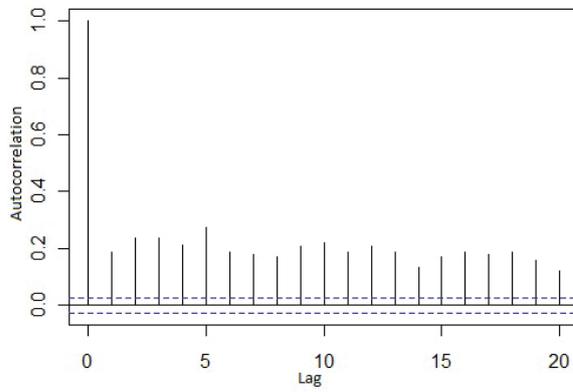


Figure 6.25: Autocorrelation of CAC 40 squared returns

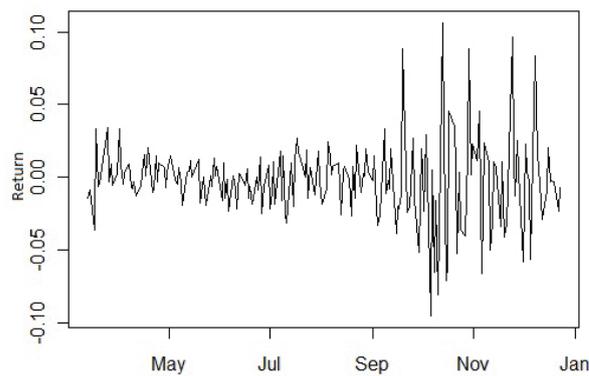


Figure 6.26: Returns of CAC 40 from March 13, 2008 to December 23, 2008

5. *Fat-tailed distribution.* When the empirical distribution of daily returns is drawn, one can generally observe that it does not resemble a Gaussian distribution. But

monthly returns have a distribution that is much closer to normal than daily returns. Figure 5 compares the density of the CAC 40 daily returns with a Gaussian density. The peak around zero appears clearly, but the thickness of the fat tails is more difficult to visualize.

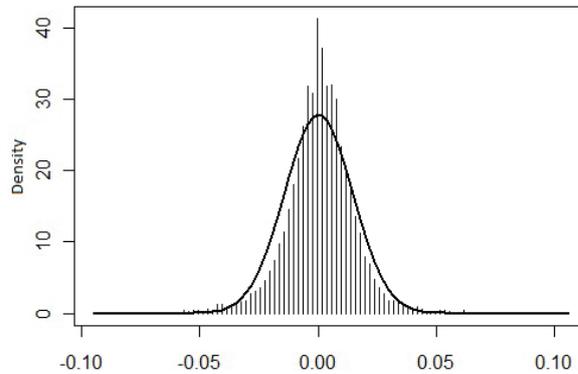


Figure 6.27: CAC 40 returns density (Column lines) and density of a Gaussian with mean and variance equal to the sample mean and variance of returns (solid line)

6. *Leverage effects and Seasonality.* If we analyse the impact of past values on the volatility, we will observe that there is asymmetry of the impact of past positive and negative values on the current volatility. Negative returns tend to increase volatility by a larger amount than positive returns of the same magnitude. For this reason, in our case, we will work on negative returns with high magnitude. Nevertheless, working with intraday series will show the presence of the seasonal effect. Following a period of market closure, volatility tends to increase, reflecting the information cumulated during the break.

The properties cited below show the difficulty of financial series modeling. Any statistical model for daily returns must be able to capture the main stylized facts described in the previous properties. The main propriety will be discussed in this paper is the conditional heterosedasticity because it illustrates mainly the fact that large absolute returns tend to be followed by large by large absolute returns. For this reason, we will in this paper with the autoregressive conditional heteroskedasticity models (Engle 1982) and generalized autoregressive conditional heteroskedasticity (Bollerslev 1986).

Appendix 6.B - Proof of Tail and Spectral Process Theorems

(ii) implies (i)

Let $(Y_t)_{t \in \mathbb{Z}}$ be a stationary time series in \mathbb{R} that admits a forward tail process $(F_t)_{t \geq 0}$ for which the law of $\|F_0\|$ is non-degenerate at 1. Then there exists $\alpha > 0$ such that $\|Y_0\|$ is regularly varying with index α and $\|F_0\| \sim \text{Pareto}(\alpha)$.

Proof. The definition of forward tail process compounded with the continuous function $g((y_t)_{t \geq 0}) = y_0$ yields the weak convergence

$$\mathcal{L}\left(\frac{Y_0}{y} \mid Y_0 < -y\right) \rightarrow \mathcal{L}(F_0), \quad \text{asy } y \rightarrow \infty$$

On the one hand, since $\mathcal{L}(F_0)$ possesses at most countably many atoms, it follows the Portmanteau Lemma that, for almost every $f \geq 1$,

$$P(\|Y_0\| > yf \mid Y_0 < -y) \rightarrow P(\|F_0\| > f)$$

as $y \rightarrow \infty$. On the other hand, since $f \geq 1$,

$$P(\|Y_0\| > yf \mid Y_0 < -y) = \frac{P(\|Y_0\| > yf)}{P(Y_0 < -y)}$$

As $P(\|Y_0\| > yf)/P(Y_0 < -y)$ possesses a limit almost everywhere as $x \rightarrow \infty$, it must be $f^{-\alpha}$ for some $\alpha \in \mathbb{R}$ and every $f \geq 1$ (cf. Bingham *et al.* (1987), Theorem 1.4.1). So $P(F_0 < -f) = f^{-\alpha}$ for $f \geq 1$. According to the fact that $\mathcal{L}(F_0)$ is a probability distribution, we have $\alpha > 0$ and $P(F_0 < -f) = 1$ for $f < 1$. Since $\|F_0\|$ is nondegenerate in 1, we obtain $\alpha > 0$.

(i) implies (iii)

If the series $(Y_t)_{t \in \mathbb{Z}}$ is regularly varying with index $\alpha > 0$, then $(Y_t)_{t \in \mathbb{Z}}$ has a tail process with $\|F_0\|$ Pareto(α)-distributed.

Proof. We define the following two sets of measures:

$$M_b(X) := \{\mu \text{ is a measure on } \mathcal{B}_X \mid \mu(X) < +\infty\}$$

$$M_0(\mathbb{R}) := \{\mu \in M_b(\mathbb{R} \setminus \{0\}) \mid \forall r > 0, \mu(\|X\| > r) < +\infty\}$$

Let $s, t \geq 0$. By definition, the random object $Y = (Y_{-s}, \dots, Y_t)$ in \mathbb{R}^{t+s+1} is regularly varying with index $\alpha > 0$, so there exists $\mu_{s,t} \in M_0(\mathbb{R}^{t+s+1})$ such that

$$\frac{1}{V(y)} \mathcal{P}(Y \in y \cdot) \xrightarrow{M_0} \mu_{s,t}(\cdot) \quad \text{as } y \rightarrow +\infty$$

where $V(y) = P(Y_0 < -y) \in RV_{-\alpha}$. Set $\nu_{s,t}$ the restriction of $\mu_{s,t}$ to

$$\{(y_{-s}, \dots, y_t) \in \mathbb{R}^{t+s+1} \mid \|y_0\| > 1\}.$$

By construction $\nu_{s,t}$ is a probability measure.

Let (F_{-s}, \dots, F_t) be a random function having $\nu_{s,t}$ as law and let f be a bounded continuous function on \mathbb{R}^{t+s+1} . Since $\mu_{s,t}(\{\|y_0\| = 1\}) = 0$ and $0 \notin \{\|y_0\| \geq 1\}$, we apply the lemma 2.5.20 in Meinguet (2010) and we obtain:

$$E\left(f\left(\frac{Y_{-s}}{y}, \dots, \frac{Y_t}{x}\right) \mid Y_0 < -y\right) \rightarrow \int f d\mu_{s,t} \llcorner \{\|x_0\| > 1\} = E(f(F_{-s}, \dots, F_t))$$

as $y \rightarrow +\infty$, which establishes the weak convergence. Referring to the Daniell-Kolmogorov existence theorem (cf. Pollard 2002, Theorem 53), there exists a stochastic process $(F_t)_{t \in \mathbb{Z}}$ in \mathbb{R} such that, for every $s, t \in \mathbb{Z}$, the distribution of (F_{-s}, \dots, F_t) is $\nu_{s,t}$.

Since the law F_0 satisfies $\nu_{0,0}(\lambda A) = \lambda^{-\alpha} \nu_{0,0}(A), \forall \lambda > 0$ and $\nu_{0,0}(\{\|y_0\| \geq 1\}) = 1$, we have $\mathcal{P}(F_0 < -f) = f^{-\alpha}$.

(iii) implies (ii) evident

Then we introduce, for $t \geq 0$, the random element

$$S_t = \frac{F_t}{\|F_0\|}$$

so that $(S_t)_{t \geq 0}$ is, by definition, the forward spectral process of $(Y_t)_{t \in \mathbb{Z}}$.

Appendix 6.C - Proof of the martingale $(\delta_i)_{i \in \mathbb{N}}$

The process $(\delta_i)_{i \in \mathbb{N}}$ is a martingale for all $\alpha_1 \in [0; \frac{\pi}{2}[$ since:

- $E(|\delta_i|) < +\infty$, proof:

$$\begin{aligned}
 E(|\delta_i|) &= E\left(\left|\sum_{k=1}^i c_k\right|\right) \\
 &\leq E\left(\sum_{k=1}^i |c_k|\right) \\
 &= \sum_{k=1}^i E(|c_k|) \\
 &= \sum_{k=1}^i \alpha_1^{k/2} E(|Z_1|)^k \\
 &= \sum_{k=1}^i \left(\frac{2\alpha_1^{1/2}}{\sqrt{2\pi}}\right)^k
 \end{aligned}$$

Therefore $E(|\delta_i|)$ is upper bounded by a sum of geometric sequence which converges if:

$$\begin{aligned}
 \frac{2\alpha_1^{1/2}}{\sqrt{2\pi}} &< 1 \\
 \frac{4\alpha_1}{2\pi} &< 1 \\
 \alpha_1 &< \frac{\pi}{2} \approx 1.57
 \end{aligned}$$

- $E(\delta_{i+1}|Z_0, Z_1, \dots, Z_i) = \delta_i$, proof:

$$\begin{aligned}
 E(\delta_{i+1}|Z_0, Z_1, \dots, Z_i) &= \sum_{k=1}^i E(c_k|Z_0, \dots, Z_i) + E(c_{i+1}|Z_0, \dots, Z_i) \\
 &= \sum_{k=1}^i c_k + E(Z_{i+1})E(|c_i||Z_0, \dots, Z_i) \\
 &= \delta_i
 \end{aligned}$$

Nevertheless, we have:

$$\begin{aligned}
E(\delta_i^2) &= E\left(\left(\sum_{k=1}^i c_k\right)^2\right) \\
&= \sum_{k=1}^i E(c_k)^2 \\
&= \sum_{k=1}^i E(\alpha_1^k \prod_{j=1}^k Z_j^2) \\
&= \sum_{k=1}^i [E(\alpha_1 Z_1^2)]^k \\
&= \sum_{k=1}^i (h(1))^k < +\infty \text{ if } h(1) < 1 \Leftrightarrow \alpha_1 < 1
\end{aligned}$$

Therefore $\sup_i E(\delta_i^2) < +\infty$ for all $\alpha_1 < 1$. And $(\delta_i)_{i \in \mathbb{N}}$ is a square integrable martingale and it converges almost surely to a finite process δ . Up to this point, we have shown that $(\delta_i)_{i \in \mathbb{N}}$ is a bounded process.

Appendix 6.D - Proof of the square integrable martingale $(\omega_i)_{i \in \mathbb{N}}$

The process $(\omega_i)_{i \in \mathbb{N}}$ is a square integrable martingale for all $\alpha_1, \beta_1 \in]0, 1[$ such that $\alpha_1 + \beta_1 < 1$ since:

•

$$\begin{aligned}
 E(\omega_i^2) &= E\left(\left(\sum_{k=1}^i d_k\right)^2\right) \\
 &= \sum_{k=1}^i E(d_k^2) \\
 &= \sum_{k=1}^i \left(\alpha_1^k [E(Z_k^2)]^k + \sum_{j=1}^{k-2} \binom{k-1}{j+1} \alpha_1^{k-j} \beta_1^j [E(Z_k^2)]^{k-j} + \alpha_1 \beta_1^{k-1} E(Z_k^2) \right) \\
 &= \sum_{k=1}^i \left(\alpha_1 E(Z_k^2) \left[\alpha_1^{k-1} [E(Z_k^2)]^{k-1} + \sum_{j=1}^{k-2} \binom{k-1}{j+1} \alpha_1^{k-(j+1)} \beta_1^j [E(Z_k^2)]^{k-(j+1)} + \beta_1^{k-1} \right] \right) \\
 &= \sum_{k=1}^i \left(\alpha_1 E(Z_k^2) (\alpha_1 E(Z_k^2) + \beta_1)^{k-1} \right)
 \end{aligned}$$

On the other hand, we have $E(Z_k^2) = 1$ therefore $\sup_i E(\omega_i^2) < +\infty$ if $\alpha_1 + \beta_1 < 1$.

• $E(w_{i+1} | Z_0, Z_1, \dots, Z_i) = w_i$ since $E(d_{i+1} | Z_0, \dots, Z_i) = 0$

Therefore w_i is a bounded process and converges almost surely to a finite process w .

Appendix 6.E - Generating Randoms of GARCH processes with volatility tail process

Generating the random R We begin by calculating the distribution function, for $r \geq 1$, we have:

$$\begin{aligned} F_R(r) &= \int_1^r \frac{\kappa}{t^{\kappa+1}} dt \\ &= [-t^{-\kappa}]_1^r \\ &= 1 - r^{-\kappa} \end{aligned}$$

We denote by \mathcal{U} by a standard random variable. We introduce this variable in order to facilitate the random generations using R program. However, we have:

$$\begin{aligned} \mathcal{U} &= 1 - r^{-\kappa} \\ \iff r^{-\kappa} &= 1 - \mathcal{U} \\ \iff r^{-\kappa} &= \mathcal{U} \\ \iff r &= \mathcal{U}^{-1/\kappa} \end{aligned}$$

Generating the random T By the same method, we begin by determining the distribution function of T . For $t \geq 0$, we have:

$$F_T(t) = \int_0^t \frac{2u^{2\kappa} f_Z(u)}{E(|Z|^{2\kappa})} du \quad (6.1)$$

$$= \frac{\sqrt{\pi}}{2^\kappa \Gamma(\kappa + \frac{1}{2})} \times G(t, \kappa) \quad (6.2)$$

Where,

$$\begin{aligned} G(t, k) &= 2^k \left(\frac{2^{3/4-1/2k} t^{2k+1} (t^2)^{-1/2k-1/4} e^{-1/4t^2} \mathbf{M}(1/2k+1/4, 1/2k+3/4, 1/2t^2)}{(k+1/2)(3+2k)} \right. \\ &\quad \left. + \frac{2^{3/4-1/2k} t^{2k-1} (t^2)^{-1/2k-1/4} e^{-1/4t^2} \mathbf{M}(1/2k+5/4, 1/2k+3/4, 1/2t^2)}{k+1/2} \right) \frac{1}{\sqrt{\pi}} \end{aligned}$$

where M represents the whittaker M-function.

Bibliography

- [1] ALLEN F., GALE D. (1995). Financial innovation and risk sharing. *The Review of Financial Studies*, Volume 8, Issue 4, 1238-1243.
- [2] ALMGREN R., CHRISS N. (2001). Optimal execution of portfolio transactions. *Journal of Risk*, Volume 3, 5-39.
- [3] ALMGREN R. (2003). Optimal execution with nonlinear impact functions and trading-enhanced risk. *Applied Mathematical Finance*, Volume 10, 1-18.
- [4] BASRAK B., DAVIS R., MIKOSCH T. (2002). Regular variation of GARCH processes. *Stochastic Processes and their Applications*, Volume 99, Issue 1, 95-115.
- [5] BASRAK B., JOHAN S. (2008). Regularly varying multivariate time series. *Stochastic Processes and their Applications*, Volume 119, Issue 4, 1055-1080.
- [6] BAYRAKTAR E., LUDKOVSKI M. (2011). Optimal trade execution in illiquid markets. *Mathematical Finance*, Volume 21, Issue 4, 681-701.
- [7] BERTSIMAS D., LO A. (1998). Optimal control of liquidation cost, *Journal of Financial Markets*, 1-50.
- [8] BINGHAM N. H., GLODIE C. M., TEUGELS J. L. (1987). Regular variation. *Encyclopedia of Mathematics and its Applications*, Cambridge University Press, Cambridge.
- [9] BOLLERSLEV T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, Volume 31, 307-327.
- [10] BRIGO D., NORDIO C. (2010). Liquidity-adjusted market risk measures with stochastic holding period.
- [11] CHAN L.K.C., LAKONISHOK J. (1995). The behavior of stock prices around institutional trades. *Journal of Finance*, Volume 50, 1147-1174.
- [12] EMBRECHTS P., KLUPPELBERG C., MIKOSCH T. (1997). *Modeling extremal events*. Springer 1997.
- [13] ENGLE R. (1982). Autoregression conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, Volume 50, Issue 4, 987-1007.

-
- [14] GATHERAL J. (2008). No-dynamic-arbitrage and market impact. *Mathematics in Finance Working Paper Series, WP2008-6*.
- [15] HE H., MAMAYSKY H. (2005). Dynamic trading policies with price impact. *Journal of Economic Dynamics and Control, Volume 29*, 891-930.
- [16] KHARROUBI I., PHAM H. (2009). Optimal portfolio liquidation with execution cost and risk. *SIAM Journal on Financial Mathematics, Volume 1, Issue 1*, 897-931.
- [17] LAURIN F., TAWN F. (2007). The extremal index for GARCH(1,1) processes. *Extremes, Volume 15, Issue 4*, 511-529.
- [18] LY VATH V., MNIF M., H. PHAM (2007). A model of optimal portfolio selection under liquidity risk and price impact. *Finance Stoch, Volume 11*, 51-90.
- [19] MARZO M., RITELLI D., ZAGAGLIA P. (2011). Optimal trading execution with non-linear market impact: An alternative Solution Method. *Quaderni - Working Paper DSE 797*.
- [20] MEINGUET T. (2010). Heavy tailed functional time series. *Ph.D. Thesis, Louvain-la-Neuve*.
- [21] NELSON D. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica, Volume 59, Issue 2*, 347-370.
- [22] POLLARD D. (2002). A user's guide to measure theoretic probability. *Cambridge Series in Statistical and Probabilistic Mathematics, Volume 8*, ISBN 0-521-80242-3; 0-521-00298-3.
- [23] POSEDEL P. (2005). Properties and estimation of GARCH(1,1) model. *Metodoloski zvezki 2,2*, 243-257.
- [24] SCHIED A., SCHONEBORN T. (2008). Risk aversion and the dynamics of optimal liquidation strategies in illiquid markets. *Finance Stoch 13*, 181-204.
- [25] SHLEIFER A., VISHNY R. (1992). Liquidation values and debt capacity: A market equilibrium approach. *Journal of Finance, Volume 47, Issue 4*, 1343-1366.
- [26] XEKALAKI E., DEGIANNAKIS S. (2010). *ARCH models for financial applications*. Wiley.

Conclusion

This thesis provided Bank Audi Risk Management with new econometric tools. We worked on the Lebanese market for many reasons: the main activity of Bank Audi is in Lebanon, the lack of papers focusing on this market, the lack of data on Lebanese Banks, the lack of papers combining and comparing Lebanese banks to banks in developed countries, the big difference between this country and other countries.

Using Lebanese market and Bank Audi data, this thesis answered the following research questions:

- Is there a methodology for building the Lebanese term structure that can be economically interpreted?
- Can we find a relationship between market rates and USD deposits rates offered by Lebanese banks?
- As large banks develop behavioral maturities for customers' deposits in order to study the funding liquidity risk, what is the behavioral maturity of customers' deposits at Bank Audi?
- As a trader holding a large position of a given asset, how do you liquidate your portfolio after a severe shock in asset value and in stressed market conditions?

We answered these questions in two parts, the first is related to interest rate risk management whereas the second to liquidity risk management.

The first part starts with an introduction to interest rate risk management showing the weight of this risk in banks' management. Nevertheless, we provided an introduction to yield curve models being used by central banks as well as an introduction to the repricing gap model.

In the second chapter of this part, we used Nelson-Siegel and Nelson-Siegel-Svensson model in building the Lebanese term structure based on bonds issued by the Lebanese government denominated in USD and LBP. Using secondary market prices for bonds, we developed a new approach called "Correlation Constrained Approach". The USD yield is built from two different types of bonds, for the short-term we used US treasury bonds plus Lebanese Credit Default spread since short term Lebanese eurobonds are illiquid and not traded on the secondary market, for long term we used eurobonds having maturities greater than one year. Clearly, we found a liquidity mismatch between short and long term yields since US treasury bonds are more liquid and volatile than Lebanese eurobonds, it is illustrated in the principal component analysis when the most probable yield curve movement is not a parallel shift. To solve this problem, we tried to estimate the yield curve based only on observed Eurobonds yields issued by the Lebanese Government on the secondary market but results were not satisfactory, therefore adding US treasury yields for the short-term is a temporary solution and shouldn't be permanent.

For the Lebanese Yield Curve, we showed the importance and the essence of the "Correlation Constraint Approach" in interpreting model coefficients. Augmented-Dickey Fuller test confirmed that estimated parameters are integrated of order 0 or 1, however when observing the time series we concluded that a regime switching may exist in parameters due to exogenous factors. We interpreted parameters evolution by a political-economical analysis. For the forecasting we proposed to take an economic expert opinion for the upcoming economical situation to predict the future possible movement of the yield curve because the data was limited for the prediction. Today, the upcoming situation can't be expected before the formation of the new Lebanese government and the elections of a new parliament without waiting for the end of the current Syrian Civil war.

The intent of Chapter 3 is to identify the behavioral duration (repricing date) of customer deposits when market interest rates fluctuate. This alters the sensitivity of the economic value of shareholders' equity to interest rate shocks, a measure of interest rate risk used to assess capital requirements for banks.

For this matter, we built an error correction model showing the existence of a partial pass-through with a significant time lag of domestic USD deposit interest rates after an interest rate shock in global market benchmark interest rates. The behavioral duration of deposits is adjusted based on the response of USD deposit interest rates to Libor changes. The resulting behavioral deposit repricing profile is shifted in time beyond the contractual one (i.e. it becomes longer). As a result, the interest rate duration gap between assets and liabilities decreases, implying lower interest rate repricing risk.

The second part discussed two topics related to liquidity risk management.

The first chapter presented a general overview on Liquidity risks in large companies and financial institutions and showed the regulatory requirements for banks in managing this risk.

The first liquidity research topic is related to funding liquidity risk management and discussed in Chapter 5. We aimed to build behavioral liquidity maturity of non-maturing deposits at Bank Audi since knowing short-term deposits outflows will help the bank in managing the liquidity in a robust manner.

Using cointegration relationships, we showed a relationship between Bank Audi deposits behaviors, interest rates spread and coincident indicator. As the objective is to select a best practice model to obtain behavioral maturities for non maturing deposits.

Having the relationships, we made a severe shock on exogenous factors in order to forecast deposits outflows which will guide to obtain behavioral maturity of customers' deposits. To be more conservative, the bank will take these results to manage the liquidity in normal business situation and, in time of extreme crisis, the best solution is to work with contractual perspectives in order to estimate the survival horizon.

With the result of this chapter, the bank can manage its liquidity with a more robust study based on econometric model and not on simple assumptions.

The last chapter discussed an issue related to asset liquidity risk. We supposed an investor holding a large portfolio of one security, and after a severe shock he decides to liquidate the entire portfolio.

Using data from Bloomberg and ARCH/GARCH models, we tried to find the best exit trading strategy to be executed by the investor in order to minimize the losses and maximizes the utility function.

First trade is supposed to be executed on liquid markets and results showed that the optimal trading strategy is independent of the size of the portfolio due to the absence of the market impact in such case. We concluded that the expected returns of various trading strategies are too close and this is due to the symmetric random generated by ARCH/GARCH processes.

For the second trade, we aimed to put emphasis on the importance of the market impact in illiquid markets. Therefore, the optimization problem is solved while accounting in this case for the market impact. Results indicated a significant spread between the expected returns of the different trading strategies, which highlights the effect of market liquidity that should be taken into consideration in the pre-trade analysis.

The fundamental idea of this study resided in finding the ability to assess the losses arising from an exit strategy in absence of a liquid market and hence computing an optimal proportion of our portfolio to be sold in each trading day with the purpose of trying to minimize the damage while avoiding the timing risk. For this matter, we showed results presenting a relationship between the volatility of the trading strategy and its expected return and the investor will select his best trading strategy based on these two indicators.

Moreover, we went forward with the study by incorporating an expert opinion in the model and we also presented results in a scatter plot for the selection of the best trading strategy by the investor.

The weakness of the analysis is that trading strategies differs largely when considering ARCH or GARCH process for the same asset. Therefore this will be discussed in future studies from a model risk perspective.

Finally, we can enumerate many contributions of this thesis. In contrary to existing papers focusing only on application in advanced countries, the ongoing work is to be concentrated on Lebanese market. Nevertheless, this thesis employs simple models with limited data for the sake of obtaining best practice models for risk management. As proposals, forthcoming studies can:

- Find yield curve models for illiquid markets;

- Enlarge the study of behavioral duration of customer's deposits by working on different deposit rates in different currencies;
- Build depositor rating scale similar to the credit scoring scale after getting depositors' data;
- Discuss the model risk in selecting between ARCH or GARCH process for estimating extreme risk measures.

Thesis Bibliography

- [1] ADAM A., AND HOUKARI M., LAURENT J.P. (2009). Hedging Interest Rate Margins on Demand Deposits. *Available at SSRN: <http://ssrn.com/abstract=1361660> or <http://dx.doi.org/10.2139/ssrn.1361660>.*
- [2] ALLEN F., GALE D. (1995). Financial innovation and risk sharing. *The Review of Financial Studies, Volume 8, Issue 4*, 1238-1243.
- [3] ALMGREN R., CHRISS N. (2001). Optimal execution of portfolio transactions. *Journal of Risk, Volume 3*, 5-39.
- [4] ALMGREN R. (2003). Optimal execution with nonlinear impact functions and trading-enhanced risk. *Applied Mathematical Finance, Volume 10*, 1-18.
- [5] ALPER C., AKADEMIR A., KAZIMOV K. (2004). Estimating yield curves in Turkey: Factor Analysis Approach. *www.researchgate.com*.
- [6] ANG A., PIAZZESI M. (2003). A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary Economics, Volume 50*, 745-787.
- [7] ANNAERT J., CLAES A., DE CEUSTER M., ZHANG H. (2012). Estimating the yield curve using the Nelson-Siegel model: a ridge regression approach. *International Review of Economics & Finance, Forthcoming*.
- [8] BANERJEE A., DOLADO J., GALBRAITH J., HENDRY D. (1993). *Co-integration, error correction, and the econometric analysis of non-stationary data*. Oxford University Press.
- [9] BANKS E. (2005). *Liquidity risk-managing asset and funding risk*. Palgrave Macmillan.
- [10] BARAJAS A., STEINER B. (2000). Depositor behavior and market discipline in Colombia. *International Monetary Fund Seminar Series, Volume 51*, 1-39.
- [11] BASRAK B., DAVIS R., MIKOSCH T. (2002). Regular variation of GARCH processes. *Stochastic Processes and their Applications, Volume 99, Issue 1*, 95-115.
- [12] BASRAK B., JOHAN S. (2008). Regularly varying multivariate time series. *Stochastic Processes and their Applications, Volume 119, Issue 4*, 1055-1080.

-
- [13] BAYRAKTAR E., LUDKOVSKI M. (2011). Optimal trade execution in illiquid markets. *Mathematical Finance, Volume 21, Issue 4*, 681-701.
- [14] BERTSIMAS D., LO A. (1998). Optimal control of liquidation cost, *Journal of Financial Markets*, 1-50.
- [15] BINGHAM N. H., GLODIE C. M., TEUGELS J. L. (1987). Regular variation. *Encyclopedia of Mathematics and its Applications, Cambridge University Press, Cambridge*.
- [16] BIS PAPERS (1994). National differences in interest rate transmission. *BIS Working Paper*.
- [17] BIS PAPERS (2004). Principles for the management and supervision of interest rate risk. *BIS Working Paper*.
- [18] BIS PAPERS (2005). Zero-coupon yield curves: technical documentation. *BIS Working Paper*.
- [19] BIS PAPERS (2008). Principles for sound liquidity risk management and supervision. *BIS Working Paper*.
- [20] BIS PAPERS (2010). Basel III: International framework for liquidity risk measurement, standards and monitoring.
- [21] BOLLERSLEV T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics, Volume 31*, 307-327.
- [22] BONDT G. (2002). Retail bank interest rate pass-Through: new evidence at the euro area level. *European Central Bank Working Paper No 136*.
- [23] BORIO C.E.V., FRITZ W. (1995). The response of short-term bank lending rates to policy rates: a cross-country perspective. *BIS, Financial Structure and the Monetary Transmission Policy Transmission Mechanism, March, Basel*, 106-153.
- [24] BRENNAN M., SCHWARTZ E. (1979). A continuous time approach to the pricing of bonds. *Journal of Banking and Finance, Volume 3, Issue 2*, 133-155.
- [25] BRIGO D., NORDIO C. (2010). Liquidity-adjusted market risk measures with stochastic holding period.
- [26] BUDNEVICH C., FRANKEN H. (2003). Disciplina de mercado en la conducta de los depositantes y rol de las agencias clasificadoras de riesgo: el caso de Chile. *Economía Chilena, Volume 6*, 5-70.
- [27] CALOMIRIS C., SCHWEIKART L. (1991). The panic of 1857: origins, transmission, and containment. *Journal of Economic History, Volume 51*, 807-834.
- [28] CALOMIRIS C., GORTON G. (1991). The origins of banking and panics: models, facts, and bank regulation. In *Financial Markets and Financial Crises*, edited by Robert G. Hubbard, Chicago: University of Chicago Press, 109-174.

- [29] CALOMIRIS C., MASON J. (1997). Contagion and bank failures during the great depression: the June 1932 Chicago banking panic. *American Economic Review, Volume 87*, 863-883.
- [30] CALOMIRIS C., ANDREW P. (2001). Can emerging market bank regulators establish credible discipline? The case of argentina, 1992-1999. in *Prudential Supervision: What Works and What Doesn't*, University of Chicago Press.
- [31] CHAN L.K.C., LAKONISHOK J. (1995). The behavior of stock prices around institutional trades. *Journal of Finance, Volume 50*, 1147-1174.
- [32] CHRISTENSEN J. H. E., DIEBOLD F. X., RUDEBUSCH G. D. (2009). The affine arbitrage-free class of Nelson-Siegel term structure models. *Working paper, University of Pennsylvania and Federal Reserve Bank of San Francisco*.
- [33] CORM G. (2005). *L'économie libanaise: risques et perspectives*. www.georgescorm.com.
- [34] COTTARELLI C., KOURELIS A. (1994). Financial structure, bank lending rates, and the transmission mechanism of monetary policy. *IMF Staff Papers, Volume 41, Issue 4*, 587-623.
- [35] COX J., INGERSOLL J., ROSS S. (1985). *A theory of the term structure of interest rates*. *Econometrica, Volume 53, Issue 2*, 285-408.
- [36] D'AMATO L., GRUBISIC E., POWELL A. (1997). Contagion, bank fundamentals or macroeconomic shock? An empirical analysis of the Argentine 1995 banking problems. *Working Paper No. 2, Central Bank of Argentina*.
- [37] DE JONG F., WIELHOUWER J. (2001). The valuation and hedging of variable rate savings accounts. Amsterdam : Tinbergen Institute, Discussion paper.
- [38] DE POOTER M. (2007). Examining the Nelson-Siegel class of term structure models. *Tinbergen Institute Discussion Paper*., TI 2007-043/4
- [39] DEWACHTER H., LYRIO M., MAES K. (2006). A multi-factor model for the valuation and risk management of demand deposits. National Bank of Belgium, Working Papers.
- [40] DIEBOLD F., LI C. (2006). Forecasting the term structure of government bond yields. *Journal of Econometrics, Volume 130*, 337-364.
- [41] DONNAY M. AND DEGRYSE H. (2001). Bank lending rate pass-through and differences in the transmission of a single EMU monetary policy. *Discussion Paper, Center for Economic Studies, K.U. Leuven*.
- [42] DOTHAN L. (1978). On the term structure of interest rates. *Journal of Financial Economics, Volume 6, Issue 1*, 58-69.

- [43] DUFFEE G. R. (2002). Term premia and interest rate forecasts in affine models. *Journal of Finance*, Volume 5, 405-443.
- [44] DUFFIE D., KAN R. (1996). A yield-factor of interest rates. *Mathematical Finance*, Volume 6, No 4, 379-406.
- [45] EMBRECHTS P., KLUPPELBERG C., MIKOSCH T. (1997). *Modeling extremal events*. Springer 1997.
- [46] ENGLE R. (1982). Autoregression conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, Volume 50, Issue 4, 987-1007.
- [47] FLANNERY M.J. (1998), Using market information in prudential bank supervision: A review of the U.S. empirical evidence. *Journal of Money, Credit, and Banking*, Volume 30, 273-305.
- [48] GATHERAL J. (2008). No-dynamic-arbitrage and market impact. *Mathematics in Finance Working Paper Series*, WP2008-6.
- [49] GHOSH S., DAS A. (2003). Market discipline in the Indian banking sector: An empirical exploration. *NSE Research Initiative*, NSE, Mumbai.
- [50] GIMENO R., NAVE J. (2009). A genetic algorithm estimation of the term structure of interest rates. *Computational Statistics & Data Analysis*, Volume 53, 2236-2250.
- [51] GONZALEZ-HERMOSILLO B. (1999). Determinants of ex-ante banking system distress: A macro-micro empirical exploration of some recent episodes. *Working Paper No. 33*, International Monetary Fund.
- [52] HAMILTON J. (1994). *Time series analysis*. Princeton University Press.
- [53] HAMMOUDEH S., LAHIANI A., NGUYEN D.K., SOUSA R.M. (2014). Asymmetric and nonlinear pass-through of energy prices to CO2 emission allowance prices. *NIPE Working Papers 05/2014*, NIPE - Universidade do Minho.
- [54] HE H., MAMAYSKY H. (2005). Dynamic trading policies with price impact. *Journal of Economic Dynamics and Control*, Volume 29, 891-930.
- [55] HENDRY D. (1995). *Dynamic econometrics*. Oxford University Press.
- [56] HOFMANN B. (2000). Non-linear adjustment of loan rates to money market rates: some evidence for the euro area. *Paper Presented at Internal ECB Seminar, 18 October*.
- [57] HORDAHL P., TRISTANI O., VESTIN D. (2006). A joint econometric model of macroeconomic and term-structure dynamics. *Journal of Econometrics*, Volume 131, 405-444.
- [58] HULL J. (2012). *Risk management and financial institutions*. Wiley Finance.

- [59] KAUFMA G. (2003). Bank contagion: A review of the theory and evidence. *Journal of Financial Services Research, Volume 8*, 123-150.
- [60] KAYA H. (2010). Term structure of interest rate and macroeconomic variables: the Turkish case. *International Journal of Economics and Finance Studies*, ISSN: 1309-8055(online).
- [61] KELLY M., GRADA C.O.(2000). Market Contagion: Evidence from the panics of 1854 and 1857. *American Economic Review, Volume 90*, 1110-24.
- [62] KHARROUBI I., PHAM H. (2009). Optimal portfolio liquidation with execution cost and risk. *SIAM Journal on Financial Mathematics, Volume 1, Issue 1*, 897-931.
- [63] KLEIMEIER S., H. SANDER (2000). Asymmetric adjustment of commercial bank interest rates in the euro area: implications for monetary policy. *Paper Presented at the Conference Financial Structure, Bank Behavior and Monetary Policy in the EMU, Groningen 5-6 October*.
- [64] LAURIN F., TAWN F. (2007). The extremal index for GARCH(1,1) processes. *Extremes, Volume 15, Issue 4*, 511-529.
- [65] LUDVIGSON S. C., NG S. (2009). Macro factors in bond risk premia. *forthcoming in Review of Financial Studies*.
- [66] LETKEPOHL H. (2003). *New introduction to multiple time series*, Springer.
- [67] LY VATH V., MNIF M., H. PHAM (2007). A model of optimal portfolio selection under liquidity risk and price impact. *Finance Stoch, Volume 11*, 51-90.
- [68] MARZO M., RITELLI D., ZAGAGLIA P. (2011). Optimal trading execution with non-linear market impact: An alternative Solution Method. *Quaderni - Working Paper DSE 797*.
- [69] MCCANDLESS G., GABRIELLI M., ROUILLET M. (2003). Determining the causes of bank runs in Argentina during the crisis of 2001. *Revista de Análisis Económico, Volume 18*, 87-102.
- [70] MCNEIL A., FREY R., EMBRECHTS P. (2006). *Quantitative risk management: concepts, techniques, and tools*. Princeton Series in Finance.
- [71] MEINGUET T. (2010). Heavy tailed functional time series. *Ph.D. Thesis, Louvain-la-Neuve*.
- [72] MOJON B. (2000). Financial structure and the interest rate channel of ECB monetary policy. *European Central Bank Working Paper Series No 40*.
- [73] MÖNCH, E. (2008). Forecasting the yield curve in a data-rich environment: A No-Arbitrage factor-augmented VAR approach. *Journal of Econometrics, Volume 146*, 24-43.

- [74] NAWALKHA S., SOTO G., BELIAEVA N. (2005). *Interest rate risk modeling*. Wiley Finance.
- [75] NELSON C., SIEGEL A. (1987). Parsimonious modeling of yield curves. *Journal of Business*, Volume 60, Issue 4, 473-489.
- [76] NELSON D. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, Volume 59, Issue 2, 347-370.
- [77] O'BRIEN J. (2000). Estimating the value and interest rate risk of interest-bearing transactions deposits. Federal Reserve System.
- [78] GRADA O. C., WHITE E. (2002). Who panics during panics? Evidence from a nineteenth century savings bank. *NBER Working Paper No. 8856*.
- [79] OPIELA T.P. (2006), A test of the effect of systemic risk on deposit market discipline at Thai banks. *Mimeo, DePaul University*.
- [80] PARASCHIV F. AND SCHURLE M. (2010). Modeling client rate and volumes of non-maturing accounts. *Institute for Operations Research and Computational Finance, University of St. Gallen*.
- [81] POLLARD D. (2002). A user's guide to measure theoretic probability. *Cambridge Series in Statistical and Probabilistic Mathematics, Volume 8*, ISBN 0-521-80242-3; 0-521-00298-3.
- [82] POSEDEL P. (2005). Properties and estimation of GARCH(1,1) model. *Metodoloski zvezki 2,2*, 243-257.
- [83] RESTI A., SIRONI A. (2007). *Risk management and shareholders' value in banking*. Wiley Finance.
- [84] SAUNDERS A., WILSON B. (2006). Contagious bank runs: evidence from the 1929-1933 period. *Journal of Financial Intermediation*, Volume 5, 409-423.
- [85] SCHIED A., SCHONEBORN T. (2008). Risk aversion and the dynamics of optimal liquidation strategies in illiquid markets. *Finance Stoch 13*, 181-204.
- [86] SCHOLNICK B. (1996). Asymmetric adjustment of commercial bank interest rates: evidence from Malaysia and Singapore. *Journal of International Money and Finance*, Volume 15, 485-496.
- [87] SCHUMACHER L. (2000). Bank runs and currency run in a system without a safety net: Argentina and the Tequila shock. *Journal of Monetary Economics*, Volume 46, 257-277.
- [88] SELVAGGIO R. (1996). Using the OAS methodology to value and hedge commercial bank retail demand deposit premiums. *Commercial Bank Retail Demand Deposit Premiums*, Chapter 12, 363-373.

- [89] SHIN Y., YU B., GREENWOOD-NIMMO M. (2014). Modelling asymmetric cointegration and dynamic multipliers in an ARDL framework. *Horrace W.C., Sickles R.C. (Eds.), Festschrift in Honor of Peter Schmidt. Springer Science & Business Media, New York(NY).*
- [90] SHLEIFER A., VISHNY R. (1992). Liquidation values and debt capacity: A market equilibrium approach. *Journal of Finance, Volume 47, Issue 4*, 1343-1366.
- [91] SIRONI A. (2003). Testing for market discipline in the European banking industry: evidence from subordinated debt issues. *Journal of Money, Credit and Banking, 35*, 443-72.
- [92] SORENSEN C., WERNER T. (2006). Bank interest rate pass-through in the euro area, a cross country comparison. *European Central Bank Working Paper Series No 580.*
- [93] SVENSSON L. (1994). Estimating and interpreting forward interest rates: Sweden 1992-1994. *NBER Working Paper No 4871.*
- [94] TAKEMERA T., TAKASHI K. (2010 a). Modeling individuals' deposit-withdrawal behaviors after receiving information on financial turmoil. *Research in Business and Economics Journal*
- [95] TAKEMERA T., TAKASHI K. (2010 b). A micro data analysis on individual's deposit-withdrawal behavior. *Discussion Paper Series, No 93.*
- [96] TAKEMERA T., TAKASHI K., KOBAYASHI T. (2011). The empirical study of Japanese deposit withdrawal behaviors on unstable financial environment. *American Journal of Economics and Business Administration, Volume 3, Issue 2*, 288-295.
- [97] TAKEMERA T., TAKASHI K., KOBAYASHI T. (2012), Modeling deposit-withdrawal behavior with risk tolerance *Journal of Management Policy and Practice, Volume 13, Issue 5*, 116-127.
- [98] TOOLSEMA L.A., STURM J.-E., DE HAAN J. (2001). Convergence of monetary transmission in EMU: new evidence. *CESifo Working Paper*, 465.
- [99] VASICEK O. (1977). An equilibrium characterization of the term structure. *Journal of Financial Economics, Volume 5*, 177-188.
- [100] XEKALAKI E., DEGIANNAKIS S. (2010). *ARCH models for financial applications.* Wiley.

Other References

- BLOOMBERG: *www.bloomberg.com.*
- LEBANESE CENTRAL BANK: *www.bdl.gov.lb.*

List of Tables

1	Commercial Banks Consolidated Balance Sheet (In Billions of LBP) as Sep 2013	12
1.1	Factor Loadings for EUR France Eurobond Yields	24
1.2	Standard Deviation of factor scores	24
1.3	Change of portfolio value for a 1 basis point rate move	25
1.4	The term structure shapes resulting from all possible parameters constellations.	32
1.5	Calculation of Duration and Convexity	35
1.6	Impact of interest rate change on Bond Price	35
1.7	Yield Curve Models in Central Banks	36
2.1	Distribution of the maturities	45
2.2	Fitting Performance for different maturities	54
2.3	Factor Loadings for USD Zero Yields	62
2.4	Factor Loadings for LBP Zero Yields	62
2.5	Factor Loadings for USD Zero Yields Basing on Observed Yields on Lebanese Market	63
3.1	BSAR calculation	68
3.2	Descriptive statistic of first differences	70
3.3	Distribution of the shock	74
3.4	Model Results	76
3.5	Maturity Adjustment	76
5.1	Contractual Maturity of USD Deposits (in millions of USD)	135
5.2	Contractual Maturity of LBP Deposits (in millions of USD)	136
5.3	Behavioral Maturity of USD Deposits (in millions of USD)	141
5.4	Behavioral Maturity of LBP Deposits (in millions of USD)	141
5.5	Model Results of USD Deposits (in millions of USD)	142
5.6	Model Results of LBP Deposits (in millions of USD)	142
6.1	Optimal trading strategies basing on ARCH(1) process	164
6.2	Possibility of obtaining a gain trading strategy basing on ARCH(1) model .	165
6.3	Optimal trading strategies basing on GARCH(1,1) process with the same κ .	165
6.4	Optimal trading strategies basing on GARCH(1,1) process with the fixed $\alpha_1 + \beta_1$	166

6.5	Probability of catch up	166
6.6	Probability of catch up	166
6.7	Optimal trading strategies basing on GARCH processes with different volatilities	167
6.8	Probability of Catch up	167
6.9	Optimal Exit Trading Strategy for CAC40	170
6.10	Parameters Estimation	177
6.11	Market Impact Function Parameters	178

List of Figures

1.1	β_1 Loading Factor against First Component Factor (France Yield Curve)	30
1.2	β_2 Loading Factor against Second Component Factor(France Yield Curve)	31
1.3	β_3 Loading Factor against third Component Factor(France Yield Curve)	31
2.1	Historical US Dollar Yield Curve (2009-2013).	41
2.2	Historical Lebanese Pound Yield Curve (2009-2013).	41
2.3	Nelson-Siegel Loadings Correlation	42
2.4	Svensson Loadings Correlation	42
2.5	Histogram of Maturities for USD Bonds	44
2.6	Histogram of Maturities for LBP Bonds	44
2.7	Nelson-Siegel Linear Parameters Behavior ($\beta_1, \beta_2, \beta_3$) for US Dollar Yield Curve	45
2.8	λ_1 , Adjusted R^2 , and Correlation between the two loadings factors for US Dollar Yield Curve	46
2.9	USD Nelson-Siegel Errors (R solution)	47
2.10	USD Nelson-Siegel Errors (Correlation Constraint Approach)	47
2.11	Time Series Errors of Nelson Siegel Model Applied on USD Curve	47
2.12	QQplot Errors of Nelson Siegel Model Applied on USD Curve	47
2.13	Auto Correlation Errors of Nelson Siegel Model Applied on USD Curve	47
2.14	Nelson-Siegel Linear Parameters Behaviour ($\beta_1, \beta_2, \beta_3$) for US Dollar Yield Curve	48
2.15	λ_1 , Adjusted R^2 , and Correlation between the two loadings factors for US Dollar Yield Curve	48
2.16	LBP Nelson-Siegel Errors (R solution)	49
2.17	LBP Nelson-Siegel Errors (Acceptable Correlation solution)	49
2.18	Time Series Errors of Nelson Siegel Model Applied on LBP Curve	49
2.19	QQplot Errors of Nelson Siegel Model Applied on LBP Curve	49
2.20	Auto Correlation Errors of Nelson Siegel Model Applied on LBP Curve	50
2.21	Nelson-Siegel-Svensson Linear Parameters Behaviour ($\beta_1, \beta_2, \beta_3$) for US Dollar Yield Curve	50
2.22	β_4 , λ_1 , Adjusted R^2 for US yield curve	51
2.23	LBP Nelson-Siegel-Svensson Errors (R solution)	51
2.24	LBP Nelson-Siegel-Svensson Errors (Acceptable Correlation solution)	51
2.25	Time Series Errors of Nelson-Siegel-Svensson Model Applied on LBP Curve	51
2.26	QQplot Errors of Nelson-Siegel-Svensson Model Applied on LBP Curve	51

2.27	Auto Correlation Errors of Nelson-Siegel-Svensson Model Applied on LBP Curve	52
2.28	Nelson-Siegel-Svensson Linear Parameters Behaviour $(\beta_1, \beta_2, \beta_3)$ for LBP Yield Curve	52
2.29	β_4, λ_1 , Adjusted R^2 for LBP yield curve	52
2.30	LBP Nelson-Siegel-Svensson Errors (R solution)	53
2.31	LBP Nelson-Siegel-Svensson Errors (Acceptable Correlation solution)	53
2.32	Time Series Errors of Nelson-Siegel-Svensson Model Applied on LBP Curve	53
2.33	QQplot Errors of Nelson-Siegel-Svensson Model Applied on LBP Curve	53
2.34	Auto Correlation Errors of Nelson-Siegel-Svensson Model Applied on LBP Curve	53
2.35	Black: Adjusted R^2 of the fitted LBP yield curve; Red: Adjusted R^2 of the fitted LBP yield curve using yields less than 7 years	55
2.36	Adjusted R^2 of the USD yield curve fitted only on observed Eurobonds yields on the secondary Lebanese Market	55
2.37	Component Factors (USD Yield Curve) compared to Nelson-Siegel-Svensson loading factors	56
2.38	Component Factors (LBP Yield Curve) compared to Nelson-Siegel-Svensson loading factors	57
2.39	Estimated Svensson Parameters using Constraint Correlation Approach divided into 5 regimes.	58
2.40	Lebanese Pound Fitted Yield Curve on 5 dates of different regimes.	59
3.1	BSAR versus LIBOR	69
3.2	First differences of the two rates	69
3.3	Histogram of first differences	69
3.4	ACF of Δ LIBOR	70
3.5	PACF of Δ LIBOR	70
3.6	ACF of Δ BSAR	70
3.7	PACF of Δ BSAR	70
3.8	QQplot of Residuals	72
3.9	Auto-Correlation Function of Residuals	73
3.10	Impulse Response Function	74
3.11	Cumulative IRF	74
3.12	Behavior of BSAR in response of 200 bps changes in LIBOR	75
4.1	A general taxonomy of risks	99
4.2	Corporate liquidity needs	101
4.3	Funding Liquidity Risk	105
4.4	Asset Liquidity Risk	107
5.1	Bank Audi Term(Left) and Demand (Deposits) from 2006 to 2013 (in/converted to USD)	128
5.2	Bank Audi Total Deposits from 2006 to 2013(in/converted to USD): Residents (Left) and Non Residents(Right)	129
5.3	Coincident Indicator (Source: www.bdl.gov.lb)	129

5.4	Bank Audi Total Deposits Distribution	130
5.5	Bank Audi Deposits Distribution: Term (Left), Demand (Right)	131
5.6	Offered Interest Rates on Term Deposits	132
5.7	Total Deposits: Bank Audi vs Other Lebanese Banks	132
5.8	Demand Deposits: Bank Audi vs Other Lebanese Banks	133
5.9	Term Deposits: Bank Audi vs Other Lebanese Banks	134
5.10	Historical Scenario 1: USD resident deposits percentage of withdrawal in function of time(days)	139
5.11	Historical Scenario 2: USD non resident deposits percentage of withdrawal in function of time(days)	139
5.12	Historical Scenario 3: LBP resident deposits percentage of withdrawal in function of time(days)	140
6.1	QQplot Residuals vs Normal Distribution (ARCH)	169
6.2	QQplot Residuals vs Normal Distribution (GARCH)	169
6.3	QQplot Residuals vs Student Distribution (ARCH)	169
6.4	QQplot Residuals vs Student Distribution (GARCH)	169
6.5	ARCH	171
6.6	GARCH	171
6.7	Tail Behavior of volatility	172
6.8	Stock Price Behavior after Shock	172
6.9	Market Impact function	174
6.10	Best Exit Trading Strategy with a market impact equal to 8%	178
6.11	Best Exit Trading Strategy with a market impact equal to 5%	178
6.12	Best Exit Trading Strategy with a market impact equal to 4%	179
6.13	Best Exit Trading Strategy with a market impact equal to 3%	179
6.14	The volatility of the best exit trading strategy in function of the market impact	179
6.15	Temporary Market impact of linear trading strategies for different maturities	180
6.16	Permanent Market impact of linear trading strategies for different maturities	180
6.17	Best Exit Trading Strategy within 3 days.	181
6.18	Best Exit Trading Strategy within 5 days.	181
6.19	Best Exit Trading Strategy within 7 days.	181
6.20	Best Exit Trading Strategy within 10 days.	181
6.21	The volatility of the best exit trading strategy in function of the market impact	182
6.22	The volatility of the best exit trading strategy in function of the expected return at maturity	182
6.23	CAC 40 returns (March 2, 1990 to March 27, 2012); September 2001, fall of the Twin Towers; October 2008, financial crisis	184
6.24	Autocorrelation of CAC 40 returns	185
6.25	Autocorrelation of CAC 40 squared returns	185
6.26	Returns of CAC 40 from March 13, 2008 to December 23, 2008	185

6.27 CAC 40 returns density (Column lines) and density of a Gaussian with mean and variance equal to the sample mean and variance of returns (solid line)	186
---	-----