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Design and pricing of new energy services in a competitive environment

Léonard von Niederhäusern

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Centrale Lille

Thèse

Présentée en vue d'obtenir le grade de

Docteur

En

Automatique, Génie Informatique, Traitement du Signal et Image

Par

Léonard von Niederhäusern

Doctorat délivré par Centrale Lille

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services en énergie dans un environnement
compétitif**

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Thèse préparée dans le Laboratoire CRISAL au sein de l'équipe INOCS à Inria Lille Nord-Europe, et dans le Laboratoire PROMES-CNRS à Perpignan Ecole doctorale SPI 072 (Lille, Artois, ULCO, UVHC, Centrale Lille)

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Presented to obtain the title of

PhD

In

Automation, Computer Engineering, Signal and Image Process

By

Léonard von Niederhäusern

PhD delivered by Centrale Lille

Design and pricing of new energy services in a competitive environment

Defended on April the 4th, 2019, in front of the jury:

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Doctoral school SPI 072 (Lille, Artois, ULCO, UVHC, Centrale Lille)

To my Impostor Syndrome (as a goodbye gift)

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Contents

1	Introduction	11
2	Literature review	17
2.1	Demand-Side Management and Demand Response	17
2.1.1	Smart Grid	20
2.2	Bilevel programming	22
2.2.1	Definitions and theory	22
2.2.2	Multi-leader-follower games	27
3	Single-Leader Single-Follower	33
3.1	Stochastic bilevel problem	34
3.1.1	Scenario tree method	34
3.1.2	Notations	35
3.1.3	Follower's problem	38
3.1.4	Leader's problem	39
3.2	Simplified problems and complexity	39
3.2.1	One device	40
3.2.2	Multiple devices	50
3.3	Numerical resolution of (SBPP)	53
3.3.1	One-level formulation of (SBPP)	53
3.3.2	A single scenario case	58
3.3.3	Several scenarios	72
3.4	Rolling horizons	78
3.4.1	Toy examples	81
3.4.2	Tests on large instances	83
3.5	Conclusion	85
4	Single-Leader Multi-Follower	87
4.1	Introduction	87
4.2	Problem formulation	90
4.2.1	Local agents	91
4.2.2	Aggregators	93
4.2.3	End users	94
4.2.4	Electricity supplier	94
4.3	From a trilevel model to a bilevel model	95

4.4	Reformulations of the bilevel program and alternative solution concepts	99
4.4.1	First order formulation of the bilevel model	99
4.4.2	Revisited optimistic approach	100
4.4.3	Semi-optimistic approach	106
4.5	Numerical results	107
4.5.1	Competitive case	107
4.5.2	Noncompetitive case	117
4.6	Conclusion	118
5	Multi-Leader Multi-Follower	121
5.1	Definition of the model	121
5.1.1	Local agents	122
5.1.2	Electricity suppliers	125
5.2	Solution methods	125
5.2.1	The method of Leyffer and Munson	127
5.2.2	Formulation as a GNEP	132
5.3	Conclusion	142
6	Conclusion	145
A	Appendix to Chapter 4	149
A.1	KKT conditions	149
A.2	Instances parameters	151
B	Appendix to Chapter 5	153
B.1	Price, profit and demand graphs	153
	Bibliography	167

Introduction

The title of this PhD thesis is *Design and pricing of new energy services in a competitive environment*. It is the result of three years of work within the INOCS team located at Inria Lille-Nord Europe, and the PROMES-CNRS laboratory in Perpignan. This thesis has been supported by the FMJH "Programme Gaspard Monge en Optimisation", and realized within a strong collaboration with EDF R&D.

The energy domain currently faces numerous changes. The growth of the global electricity consumption combined with the future scarcity of fossil fuels induce tremendous challenges. Many interests like ecological, economical, or political coexist and often collide. Whereas the old picture of electricity networks only included one-directional energy transports from large power plants to end consumers, the rise of distributed generation (DG) has been a game-changer. Indeed, distributed generation often means renewable energies that are scattered and irregular. These irregular energy supplies, combined with an increasing demand for energy from the customers, imply greater difficulties to ensure the supply-demand balance. Instead of installing costly generators that are able to cover any peak demand, consumers can help maintaining the balance by modifying their behavior. However, to know that such behavioral modifications are beneficial, a two-way communication has to take place between the consumers and the suppliers, leading to the birth of the smart grid paradigm (see Section 2.1.1). According to [Dept 2009], the smart grid is

"an automated, widely distributed energy delivery network, which will be characterized by a two-way flow of electricity and information and will be capable of monitoring everything from power plants to customer preferences to individual appliances. It incorporates into the grid the benefits of distributed computing and communications to deliver real-time information and enable the near-instantaneous balance of supply and demand at the device level."

Smart grids have numerous advantages: thanks to the installation of smart meters (e.g. *Linky* in France), they are observable at any moment. They allow for the easy addition or removal of distributed generation modules. They are able to detect the

failures occurring inside of them. Finally, they are more secure and more reliable. Smart grids are thus appropriate to minimize risks of brownouts or even blackouts that could be caused by the unpredictable supply of renewable energy into the grid.

The ability of the actors connected to the grid to communicate is crucial, and allows for the implementation of many interesting techniques. One of these techniques is *demand-side management* (DSM). DSM consists in shaping the electricity demand of end consumers to reach one or several of the next goals: to prevent blackouts by avoiding demands that cannot be satisfied, to make the best use of renewable energy, or, from a more economical point of view, to increase the benefits of the electricity producers in a competitive environment. In order to achieve DSM, there are six essential techniques, see Section 2.1. Among these, three of them concern medium or long-term strategies (strategic conservation, strategic load growth, and flexible load shape). According to [Kreith 2016], the three remaining DSM techniques are known as the *classic* forms of DSM: these are peak clipping, valley filling, and load shifting.

The concept of *demand response* (DR) is strongly linked to DSM. To ensure the supply-demand balance, the real demand has to be flexible, not only to reduce the peaks in consumption, but also to reduce the fluctuation of the demand. As defined in [Anjos 2017], "*the collection of approaches available to obtain this flexibility from the demand side of the balance is commonly referred to as DR*". Demand response concerns short-term decisions: for example, if an event causes the shutdown of a power plant, demand response mechanisms can be used to diminish the load and avoid a blackout. Such mechanisms often rely on pricing, the main categories being *time-of-use* (TOU) tariffs, *multi-TOU* tariffs, and *time-and-level-of-use* (TLOU) tariffs. In TOU pricing, the electricity tariffs depend on the time of consumption. In multi-TOU, several groups of customers get different TOU tariffs. In TLOU pricing, the prices depend not only on the time of consumption, but also on the energy quantities that are bought. Examples of TOU tariffs notably include *critical peak pricing* (CPP) and *variable peak pricing* (VPP). CPP tariffs involve a critical surcharge during time windows that can either be fixed in advance, or vary depending on the system needs. In the latter case, a notice is sent a few hours in advance to the customers so that they can react accordingly. For VPP tariffs, time windows are defined in advance, during which the prices may vary depending on the grid or market conditions. This thesis focuses on TOU pricing. Such pricing strategies already exist in France, with the *heures pleines/heures creuses* pricing (HPHC, in English peak/off-peak hours). This work intends to develop even more efficient pricing schemes, where one or several energy suppliers offer electricity prices to a set of end consumers.

New professions arose in the smart grid context. In particular, the so-called *aggregators* play an important role: they act as intermediaries between energy suppliers and consumers. On one hand, they offer financial rewards to consumers in exchange for control over all or part of their demand. On the other hand, they offer flexibility to the energy suppliers. The more clients are in contract with an aggregator, the more impact this aggregator has on the overall demand to the grid. Another feature of aggregators is their ability to act as energy suppliers, by renting production capacities of companies that they can activate at distance during given periods. However, this feature is not considered in this thesis.

The interaction between energy suppliers, aggregators, and end consumers is a hierarchical one: the suppliers offer prices, to which the aggregators and consumers react. Such interactions are known in economics as *Stackelberg games*. In mathematics, they constitute the core of *bilevel optimization*. In a bilevel problem, two actors, the *leader* and the *follower*, play a game: both actors have their own constraints and objective function, the leader takes decisions knowing that the follower is going to react optimally to them. Extensions of bilevel optimization include the introduction of multiple leaders and followers, which gives rise to the so-called *multi-leader-follower games* (MLFG). In a MLFG, the leaders play a game among themselves, like the followers do. Therefore, a solution to a MLFG takes the form of two Nash equilibria: one among the leaders and another one among the followers, the latter depending on the decisions of the leaders. For a set of leaders' decisions, there generally exist several possible Nash equilibria at the lower level. In a classical single-leader single-follower bilevel problem, this issue of potential multiple lower level solutions is usually considered through the optimistic-pessimistic dichotomy:

- In the *optimistic* case, the leader assumes that the follower will cooperate and select the most favorable solution to the leader among the set of optimal solutions.
- In the *pessimistic* case, the leader assumes that the follower will have an antagonistic behavior by selecting the solution that is least favorable for the leader. This case is often referred to as *risk-averse* instead of pessimistic.

In a single-leader multi-follower problem, this optimistic-pessimistic dichotomy can still be applied: there is only one leader, it is thus possible to determine which Nash equilibrium is favorable to him. However, in a multi-leader multi-follower setting, it is by far more difficult to discriminate the Nash equilibria: an equilibrium that is favorable to one leader might well be disadvantageous to another one. These issues have to be taken into account in the research of solutions, making MLFG intrinsically more complex than standard bilevel problems.

Bilevel optimization has been on the rise for the last thirty years and is becoming more and more popular for many kinds of applications, ranging from airline revenue management to human arm movement modeling, and from road network design to unit-commitment problems (see Section 2.2). Among these applications, pricing problems take up much space. In particular, the INOCS team at Inria Lille-Nord Europe gained expertise on bilevel pricing models for DSM inspired by industrial problems: [Afşar 2016b] and [Alekseeva 2018] both consider bilinear-bilinear bilevel pricing problems.

Building upon those two works, this thesis proposes various bilevel DSM problems in order to quantify the potential gains of one or several energy supplier in a competitive market, when pricing incentives are used to induce load shifts from the clients. The problems and results presented here constitute the premises of a promising research topic for the industrial partner.

The contributions of this thesis are of three kinds: model definitions, their theoretical analysis, and their numerical analysis. Three types of problems are considered:

1. Single-leader single-follower bilevel problems are studied in Chapter 3. The reference problem is an extension of the problems studied in [Afşar 2016a] and gets investigated from several angles. In particular, the follower is assumed to not only manage the usage schedule of his clients' appliances, but also a renewable energy source and storage capacities. The inherent uncertainty in the production of renewable energy leads to the definition of a stochastic bilevel problem based on a scenario tree method, which is solved with an exact method. Finally, a rolling horizon approach of the problem is proposed in order to solve larger size instances.
2. A single-leader multi-follower games with four kinds of actors is proposed in Chapter 4. At the upper level, an energy supplier sells electricity to actors of the lower level, who are either *local agents* or *aggregators*. A third level of optimization arises, as the aggregators are in contract with a set of *end users* who have to fulfill a demand that can be shifted. Theoretical results allow for a reformulation of the trilevel problem into a bilevel one. A characterization of the lower level shows that there is an optimistic response of the followers without energy exchanges among the aggregators and local agents, even though the model allows such exchanges. This result is used to drastically simplify the optimistic formulation of the problem, which leads to much faster and more reliable solutions (local optima of the optimization problem found by a commercial solver). Then we propose a second new approach where Nash

equilibria that are neither optimistic, nor pessimistic are selected, leading to pricing schemes that are more robust to responses of the followers. These newly defined equilibria are called *semi-optimistic*. Numerical tests assert the efficiency of the two solution methods resulting from the theoretical results.

3. Finally, multi-leader multi-follower problems are studied in Chapter 5. Such problems are by nature very difficult to handle, due to their complex structure. Building upon the results found for single-leader multi-follower games, a characterization of the Nash equilibria of the lower level is provided and used to make a selection among the Nash equilibria of the lower level that simplifies the MLFG. To solve this simplified MLFG, several methods introduced in [Leyffer 2010] are applied. However, they yield undesired equilibria of the upper level, in the sense that suppliers can offer prices that are below the costs of energy. Therefore, some conditions on the leader prices and on the followers' response are set to obtain a new problem that has nice properties: in particular, for any price profile of the leaders, there is a single-Nash equilibrium at the lower level. This approach leads to interesting numerical results.

Thesis structure

Chapter 2 is devoted to the literature review, divided into two topics: first, demand-side management and smart grids, second, bilevel programming. This last topic is itself divided into three parts: in a first step, the definitions and some basic results of classical bilevel programming (single-leader single-follower) are presented. Then, the case of multi-leader-follower games is addressed. Finally, an overview of applications of bilevel programming is provided.

Chapter 3 investigates a single-leader single-follower bilevel pricing problem for demand-side management. The follower has several specific features: storage capacities, distributed generation, and precise scheduling (each device being individually scheduled). A scenario tree approach as well as a rolling horizon method are presented.

In Chapter 4, a trilevel single-leader multi-follower problem is considered. Theoretical results allow for the transformation of the trilevel model into a bilevel one, whose optimistic version can be simplified with a characterization of the lower level. The chapter is actually comprised of the article "A Trilevel Model for Best Response in Energy Demand-Side Management", submitted after revision in December 2018.

Chapter 5 is devoted to a multi-leader-follower game logically following the problem studied in Chapter 4. Two solution methods are proposed, and useful

theoretical results are provided.

Finally, in Chapter 6, we summarize our contributions, draw conclusions and define some research prospects.

Literature review

2.1 Demand-Side Management and Demand Response

The transportation of electricity from the generation location to the place of consumption has always been a challenge, starting from the very beginning. Originally, electricity networks were conceived in a unidirectional way: the electricity was produced in large power plants (e.g. nuclear plants, which accounts for more than 70% of the total electricity production in France) and directly brought to the customers, in the aim to satisfy the demand at minimum cost. Such networks comprise three main parts (see Figure 2.1):

- The *transmission* network, directly connected to the large power plants, is able to transmit energy over long distances. To minimize the losses, the voltages applied to the transmission lines reach high values, between 200 kV and 800 kV [Garcia 2008].
- The *sub-transmission* network dispatches the energy to big consumers, such as heavy industries or railway companies, and to various substations that supply distribution networks. The voltages on sub-transmission networks range between 20 kV and 275 kV [Puret 1991].
- The *distribution* network connects the end-users (such as residential customers) to the aforementioned substations. In France, a single-phase connection has a voltage of 230 V, whereas a three-phase connection has a voltage of 400 V.

However, such networks have disadvantages. With the penetration of distributed generation (especially household photovoltaic (PV) panel installations) and their intermittence due to the unpredictability of the exact weather conditions, power injections into the network become more and more irregular [Barker 2000]. This can result in several issues: power outages, short-circuit current or islanding. Ensuring the supply-demand balance at any time is complex. To cope with the issue of supply-demand balance, two solutions coexist. Either the production can be adapted to

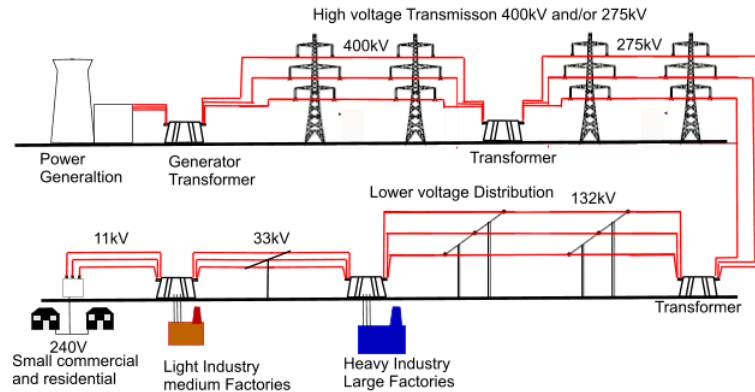


Figure 2.1: The structure of a traditional electricity network.

the demand, or the demand can be adapted to the production. This second option is referred to as *demand-side management* (DSM).

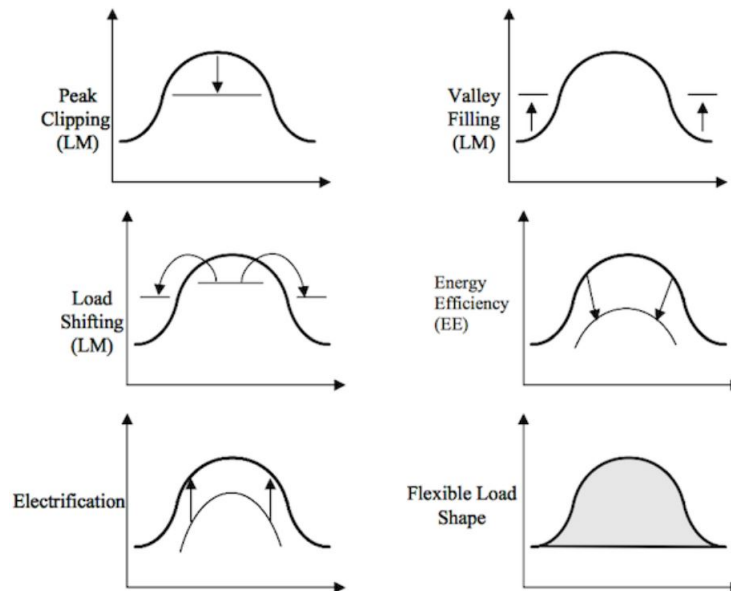
DSM is defined in [Gellings 1984] as

"the planning, implementation and monitoring of those utility activities designed to influence customer use of electricity in ways that will produce desired changes in the utility's load shape, i.e., changes in the time pattern and magnitude of a utility's load. Utility programs falling under the umbrella of demand-side management include: load management, new uses, strategic conservation, electrification, customer generation, and adjustments in market share."

DSM relies on six essential techniques [Kreith 2016], represented in Figure 2.2. The three first techniques are the classic forms of DSM, applied over short terms, whereas the latter techniques require an optimization over a longer term.

1. *Peak clipping* consists in a reduction of the peak loads, generally using direct load control.
2. *Valley filling* consists in building off-peak loads to smooth the demand curve. For example, energy can be stored during off-peak periods to ensure loads traditionally served by fossil fuels during peak periods.
3. *Load shifting* consists in moving loads from peak to off-peak periods. This can be achieved through rescheduling of flexible loads (delay or bring forward the usage of appliances), among other means.

4. *Strategic conservation* (also called *energy efficiency*, like in Figure 2.2) consists in decreasing the general energy consumption, for example by improving the energy efficiency of appliances.
5. *Strategic load growth*, at the contrary, corresponds to an increased, planned consumption of energy. For example, *electrification* describes the increased use of electricity to achieve tasks that previously required other energies. A good example is the breakthrough of electric vehicles in the automotive industry.
6. *Flexible load shape* relies on contracts between companies and customers, allowing customers to buy energy at a cheaper price in exchange of flexibility.



Source: Primer on Demand-Side Management, World Bank Document CRA No. D06090, 02/2005

Figure 2.2: The six techniques of DSM.

One of the methods to achieve DSM is *demand response* (DR), which typically consists in a system operator sending signals to customers. These signals contain information related to load shedding (also called *erasure*) or shifting, and can e.g. take the form of energy prices [Palensky 2011]. In [Han 2007], DR techniques are separated in two categories:

- *Incentive-based* DR: customers get payments or preferential prices during off-peak periods, in exchange for load reductions in peak periods. Incentive-

based DR includes several methods, which are *direct load control*, *interruptible/curtailable rates*, *emergency DR programs*, *capacity market programs*, and *demand bidding/buyback programs*.

- *Time-based rates* DR: customers receive price signals from the system operator, and react according to those signals. Again, time-based rates DR can be categorized as follows:
 - *Time-of-use* rates: the prices offered to the customers depend on the consumption time. Typically, high prices are offered during peak periods, whereas the energy is cheaper during off-peak periods. The prices can also vary seasonally.
 - *Critical peak pricing*: consists in setting very high prices during peak periods that are defined as critical by the supplier. Since this price raises are significant, the number of times the supplier is allowed to raise the tariffs during peak periods is often limited by contract.
 - *Real-time pricing*: in this case, the electricity rates may vary constantly over the day. The rates usually follow the fluctuations of the market prices.

Efficiently applying DR programs requires a lot of optimization. A tutorial on such approaches is [Anjos 2017]. This thesis considers only time-of-use pricing.

In order to be able to apply DSM/DR techniques, the suppliers must be able to communicate with the consumers. Such bidirectional information exchanges are made possible by the *smart grid*.

2.1.1 Smart Grid

In the last few years, three definitions of the smart grid were given by various authors:

- In [Singer 2010], the smart grid is

"a modern electric system. It uses sensors, monitoring, communications, automation and computers to improve the flexibility, security, reliability, efficiency, and safety of the electricity system."
- In [Dept 2009], the US Department of Energy describes the smart grid as

"an automated, widely distributed energy delivery network, which will be characterized by a two-way flow of electricity and information and will be capable of monitoring everything from power plants to

customer preferences to individual appliances. It incorporates into the grid the benefits of distributed computing and communications to deliver real-time information and enable the near-instantaneous balance of supply and demand at the device level".

- Finally, in [Canadian Electricity Association 2014],

"the smart grid is a suite of information-based applications made possible by increased automation of the electricity grid, as well as the underlying automation itself; this suite of technologies integrates the behavior and actions of all connected supplies and loads through dispersed communication capabilities to deliver sustainable, economic and secure power supplies."

Those three definitions are summarized in [Perles 2017] as follows:

"the smart grid is a concept of an autonomous electrical network able to adapt itself to client's needs in a secured, ecological and economical way. It enables bidirectional exchanges of electricity and information through lines."

The exchange of information resulting from the implementation of the smart grid paradigm is enabled by the installation of smart meters. In France, the smart meter *Linky* [Enedis 2018] is being deployed on a large scale. Every 10 minutes, Linky sends a data packet with relevant information about the actual energy consumption. Reading the meter does not need any physical presence anymore. Linky also allows the system operator to turn electricity on and off remotely. Finally, Linky is able to receive information about the electricity prices in real time. These features allow for the implementation of DR pricing techniques [Michiorri 2012].

The features of the smart grid gave rise to new professions. In particular, *aggregators* are bound to gain importance. Their role has been studied in [Gkatzikis 2013]: they act as intermediaries between an *electric utility operator* and *end users*. Each aggregator is in contract with his own set of end users. The operator wants to optimize the functioning of the grid by inducing load shifts. For this purpose, he offers a fraction of his profit to the aggregators, who themselves offer rewards to their end users. The end users finally implement the load shifts by maximizing their rewards and minimizing the inconvenience caused by the load shifts. This model is very similar to the one studied in Chapter 4. Other types of aggregators have been developed, especially in the aim of integrating electric vehicles (EVs) into the smart grid. In [Han 2010], the aggregator has contracts both with EV owners and a grid operator, which is the framework proposed in [Guille 2009].

Through an efficient management of the EVs batteries, the aggregator contributes to the regulation of the grid. The grid operator thus regularly sends regulation signals for the aggregator to know its needs. Finally, the same framework is slightly modified in [Wu 2010]: the aggregator is in charge for the intragrid management, that is the energy exchanges among the EVs. When an EV acts as a load and another one as a resource, the aggregator ensures a transmission from one to the other, avoiding unnecessary communication with the smart grid operator.

2.2 Bilevel programming

A bilevel problem is a decision process involving two decision-makers with a hierarchical structure. The situation described by a bilevel program is as follows: at the upper level, the *leader* makes decisions. Then, at the lower level, the *follower* reacts optimally to the leader's decisions. Both actors control their own decision variables in order to maximize their own objective function. Solving a bilevel program implies finding the optimal decisions of the leader, knowing that the follower is going to react in an optimal way. Bilevel programs were first introduced in an economic setting by von Stackelberg [von Stackelberg 1952], which explains why solutions of bilevel programs are called *Stackelberg equilibria*. The literature on bilevel optimization has expanded a lot since then, and reference monographs include [Bard 2010] and [Dempe 2002]. More recently, the survey [Dempe 2018] provides an introduction and an extensive list of publications in the field of bilevel optimization.

2.2.1 Definitions and theory

We choose to introduce the notations utilized in [Dempe 2018]. In a bilevel optimization problem, the leader and the follower aim to optimize their own objective function by deciding on their own set of variables. Let $x \in X \subseteq \mathbb{R}^m$ denote the variables of the leader and $y \in Y \subseteq \mathbb{R}^n$ the variables of the follower, with X and Y closed and nonempty. Then, for given decisions of the leader, the follower aims to optimize an objective function

$$\begin{aligned} f(x, \cdot) : \mathbb{R}^n &\rightarrow \mathbb{R} \\ y &\mapsto f(x, y), \end{aligned}$$

while satisfying a set of constraints $g_i(x, y) \leq 0$ for $i = 1, \dots, p$, where the g_i 's are functions

$$\begin{aligned} g_i(x, \cdot) : \mathbb{R}^n &\rightarrow \mathbb{R} \\ y &\mapsto g_i(x, y). \end{aligned}$$

The problem of the follower is then described by the parametric optimization problem

$$\min_y \{f(x, y) \mid g(x, y) \leq 0, y \in Y\}, \quad (2.1)$$

Next, define the optimal value function of Problem (2.1):

$$\varphi(x) := \min_y \{f(x, y) \mid g(x, y) \leq 0, y \in Y\}, \quad (2.2)$$

and its solution set mapping

$$\psi(x) := \{y \in Y \mid g(x, y) \leq 0, f(x, y) \leq \varphi(x)\}. \quad (2.3)$$

The function $\varphi : x \mapsto \varphi(x)$ returns the optimal value of the follower's objective for the leader's decision x , whereas $\psi(x)$ is the set of decisions of the follower that yield the optimal value $\varphi(x)$. Observe in particular that $\psi(x)$ is nonempty for any $x \in X$ such that $\varphi(x)$ exists. Next, define the graph of ψ :

$$G(\psi) := \{(x, y) \mid x \in X, y \in \psi(x)\} \subseteq \mathbb{R}^m \times \mathbb{R}^n. \quad (2.4)$$

If (x, y) belongs to $G(\psi)$, then y is an optimal response of the follower to the leader's decision x .

On the leader's side, the aim is to optimize the function

$$\begin{aligned} F : \mathbb{R}^m \times \mathbb{R}^n &\rightarrow \mathbb{R} \\ (x, y) &\mapsto F(x, y), \end{aligned}$$

under the constraints that y is an optimal response of the follower to x and that x satisfies a set of constraints $G_j : \mathbb{R}^m \rightarrow \mathbb{R}$, $j = 1, \dots, q$. The leader's problem is thus defined as

$$\text{"min"}_x \text{" } \{F(x, y) \mid G(x) \leq 0, (x, y) \in G(\psi), x \in X\}. \quad (2.5)$$

In other words, the leader optimizes $F(x, y)$ by deciding x , and knowing that y is a solution of the follower's problem. Due to their hierarchical relation, the leader's problem is called the *upper level* and the follower's problem the *lower level* of the bilevel problem.

Bilevel problems are not well-defined if there are several optimal solutions at the lower level for fixed leader decisions, thus the quotation marks around "min". To bypass this ambiguity, two main strategies have been proposed in the literature (see e.g. [Loridan 1996]):

1. In the *optimistic* formulation, the leader assumes that the follower will always select the solution in $\psi(x)$ that is most favorable to the leader. In that case, Problem (2.5) becomes

$$\min_x \{\varphi_o(x) \mid G(x) \leq 0, x \in X\}, \quad (2.6)$$

where

$$\varphi_o(x) := \min_y \{F(x, y) \mid G(x) \leq 0, x \in X, (x, y) \in G(\psi)\}.$$

This problem is generally written as follows:

$$\begin{aligned} & \min_x \min_y F(x, y) & (2.7) \\ \text{s.t.} & \begin{cases} x \in X \\ G(x) \leq 0 \\ y \in \operatorname{argmin} \{f(x, y) \mid g(x, y) \leq 0, y \in Y\}. \end{cases} \end{aligned}$$

Optimistic bilevel problems are also referred to as *strong* Stackelberg games.

2. At the opposite, the *pessimistic* formulation assumes that the follower is always going to choose the optimal reaction to x that is the least favorable to the leader. The pessimistic formulation considering the worst-case scenario is frequently referred to as *risk-averse* optimization, or *weak* Stackelberg game. In mathematical terms, the pessimistic formulation of Problem (2.5) is

$$\min_x \{\varphi_p(x) \mid G(x) \leq 0, x \in X\}, \quad (2.8)$$

where

$$\varphi_p(x) := \max_y \{F(x, y) \mid G(x) \leq 0, x \in X, (x, y) \in G(\psi)\}.$$

Pessimistic bilevel problems can also be written as

$$\begin{aligned} & \min_x \max_y F(x, y) & (2.9) \\ \text{s.t.} & \begin{cases} x \in X \\ G(x) \leq 0 \\ y \in \operatorname{argmin} \{f(x, y) \mid g(x, y) \leq 0, y \in Y\}. \end{cases} \end{aligned}$$

When a bilevel problem is solved, a solution for both the leader and the follower is computed, assuming that the follower is going to react in the predicted way.

However, there is no guarantee that the follower does in reality: the only certainty is that his answer will be optimal for himself. In [Alves 2016], the *rewarding* and *deceiving* solutions are introduced as alternative solutions of the bilevel problem. In the rewarding solution, the leader's decisions are obtained with the pessimistic formulation, but the solution chosen by the follower is the most favorable for the leader. On the other hand, in the deceiving solution, the leader's decisions are obtained with the optimistic formulation, but the follower chooses the solution that is the least favorable to the leader. The choice of the follower's reaction influences the leader's objective value. For the leader, the various solutions are ordered as follows:

$$\text{deceiving} \leq \text{pessimistic} \leq \text{rewarding} \leq \text{optimistic}.$$

The existence of solutions of bilevel problems has been thoroughly studied. First existence results are provided in [Lucchetti 1987]: sufficient and necessary conditions are given in the case where the follower's response set is a singleton for any leader's decision. In case of several possible responses, the pessimistic case is studied. In [Aboussoror 2002], the existence of solutions in the pessimistic case is tackled through successive approximations of the problem. Lastly, in [Lignola 1997], the case of optimistic bilevel problems is addressed, and conditions of minimal character are given to guarantee the lower semicontinuity of the upper level objective function.

From a computational point of view, bilevel problems are in general hard to solve. Even the simplest form of a bilevel problem, where all constraints and objectives are linear, is NP-hard, as it is showed in [Ben-Ayed 1990] or [Labbé 1998]. In general, the feasible region of the leader is nonconvex, and can even be disconnected or empty [Colson 2005].

When the follower's problem is convex for fixed decisions of the leader and a constraint qualification is satisfied (e.g. Slater's condition), the lower level problem can be replaced by its Karush-Kuhn-Tucker (KKT) conditions in the leader's problem. This gives rise to a *mathematical problem with equilibrium constraints* (MPEC) that can be implemented and solved numerically: this technique is widely used to solve bilevel problems. However, it has been shown in [Dempe 2012] that in the optimistic case, global and local optima of the bilevel problem and of the related MPEC do not always correspond. The conditions for global optima to correspond are usually satisfied, but the conditions for local optima to correspond are difficult to verify. This generates troubles, as most solvers guarantee local optimality of their solution, but not global optimality. The correspondence of optima in the pessimistic case has been considered in [Aussel 2019].

Despite their intrinsic complexity, bilevel problems are pervasive, because they

allow to model numerous real-life problems adequately. Bilevel programming originated from economic games [von Stackelberg 1952], but were soon used for military applications ([Bracken 1973], where the leader aims to find a minimum-cost weapons mix able to achieve a specified destruction of resources or weapons owned by the follower). The term *bilevel* itself was first mentioned in 1977 in an agricultural context, for the purpose of determining an optimal government policy [Candler 1977]. Other examples of problems that are adequately modeled with bilevel programming include e.g. facility location and production problems [Aardal 1996, Küçükaydin 2011], scheduling problems [Karlof 1996], gas cash-out problems [Dempe 2005, Kalashnikov 2010] or even human arm movement modelizations [Ulbrich 2012]. An extensive list of applications and references is given in [Dempe 2018].

This thesis focuses on pricing problems for demand-side management. Pricing problems constitute a class of problems that ideally fit in a bilevel framework: a leader aims to find an optimal pricing strategy, knowing that users/customers are going to adapt their behavior accordingly. In [Labbé 1998], a toll-setting problem defined on a multicommodity transportation network is studied, where the aim is to maximize the revenue of a taxation scheme. The followers solve shortest-path problems. Similar problems have been studied notably in [Brotcorne 2001] or in [Heilporn 2006]. In the latter, highways are the only roads that can be taxed.

Besides road networks, tariff schemes are also developed on telecommunication networks. In [Bouhtou 2007], the problem of determining a set of optimal tariffs on arcs of a network is addressed. In this setting, an actor only owns a subset of the arcs of the network. Like in road pricing problems, the followers are supposed to solve shortest-path problems. Another application of bilevel programming applied to telecommunication problems is considered in [Eytard 2017]: thanks to financial incentives, the aim is to avoid data congestion in a large mobile network, so that the quality of service is as high as possible.

In the energy domain, [Cervilla 2015] proposes a bilevel model where a regulator aims to ensure that his costs are covered, while customers intend to optimize their investments in DG, thus diminishing their energy costs. In [Vahid-Ghavidel 2018], DR aggregators procure two types of DR: reward-based DR and time-of-use. The obtained DR is sold into day-ahead and balancing markets. In [Zugno 2013], a retailer offers a price schedule to consumers that are going to minimize the total of their bill, and their inconvenience, which depends on the indoor temperature of a building. Some stochasticity is considered: the consumers' decisions depend on the weather forecast, i.e. the colder the outside temperature, the more energy will be bought for indoor heating. In [Alekseeva 2018], an electricity provider pro-

duces energy and has a duty to satisfy the demands of residential customers. The leader has some restrictions on his power generation capacities, which impact the pricing policy: the hourly demand of the customers is indeed bounded. The customers minimize their bill and their inconvenience, which is caused by the change of consumption schedule. This setting is very similar to that of [Afşar 2016b], which considers leaders and followers of the same type. The main difference between both articles lies in the consumers determining the schedule of each of their devices separately, giving rise to a very detailed problem. The main problem of Chapter 3 is an extension of the problem considered in [Afşar 2016b]: the follower is assumed to manage a production of renewable energy and storage capacities, which leads to the introduction of stochasticity into the problem.

A natural generalization of bilevel programming is multi-level programming. In this setting, there can be more than two levels of optimization, i.e. the constraints of the leader include solving another multi-level problem. Such problems are considered e.g. in [Migdalas 1998]. Examples of applications are scarce, but include in the energy domain [Ambrosius 2018] and [Grimm 2019], where multilevel problems are used to define price zones within the grid, [Kleinert 2019b] that extends the previous problem with network design, and the already mentioned [Gkatzikis 2013], where a system operator, aggregators and end users occupy the three levels of a trilevel problem. The latter article is strongly related to the problem studied in Chapter 4, which involves slightly modified aggregators. In our setting however, actors of the intermediate level have the possibility to exchange energy, which gives rise to a single-leader multi-follower game where a followers' response is a Nash equilibrium. Such problems are described in next section.

2.2.2 Multi-leader-follower games

Multi-leader-follower games (MLFG) constitute another generalization of bilevel programming. In this case, the leader and the follower are not necessarily unique. The leaders as well as the followers are usually assumed to be in a competition situation. Therefore, the upper and the lower level are modeled as games. The direct consequence is that the followers react with a (generalized) Nash equilibrium (GNE) to the leaders' decisions, and the optimal solution of the upper level is a GNE as well, though among the leaders. The following definition of GNE is taken from [Aussel 2018].

Let $P := \{1, \dots, p\}$ be a finite set of p players. Each player $\nu \in P$ controls variables $x_\nu \in \mathbb{R}^{n_\nu}$, also called *strategy*. The vector

$$x := (x_1, \dots, x_p) \in \mathbb{R}^n,$$

where $n = n_1 + \dots + n_p$, represents the *joint strategies* of all players. To denote the strategies of all players but $\nu \in P$, the notation $x_{-\nu}$ is commonly used. By abuse of notation, $x = (x_\nu, x_{-\nu})$. In a generalized Nash equilibrium problem (GNEP), the variable x_ν is constrained to belong to the domain $X_\nu(x_{-\nu})$, which is the set of feasible strategies, given the joint strategies of the other players. Furthermore, each player aims to minimize his objective function f_ν , which depends on the joint strategies of all players x . Given $x_{-\nu}$, the set

$$S_\nu(x_{-\nu}) := \operatorname{argmin}_{x_\nu} \{f_\nu(x_\nu, x_{-\nu}) \mid x_\nu \in X_\nu(x_{-\nu})\}$$

is the set of *best responses* of player $\nu \in P$ to the joint strategies $x_{-\nu}$ of his opponents.

Definition 2.2.1. A generalized Nash equilibrium problem (GNEP) consists in finding a joint strategy \bar{x} such that for each player $\nu \in P$, \bar{x}_ν is a best response to $\bar{x}_{-\nu}$, i.e. for all $\nu \in P$, $\bar{x}_\nu \in S_\nu(\bar{x}_{-\nu})$. Such a joint strategy \bar{x} is called an *equilibrium*, and by notation, $\bar{x} \in \text{GNEP}$.

If for all $\nu \in P$, the feasible domain for x_ν does not depend on $x_{-\nu}$, then it is only a *Nash equilibrium problem*. Constraints that comprise variables of several players are called *joint constraints* and therefore only appear in GNEPs.

In an MLFG, the situation is more complex than in a GNEP, which is just a specific case of MLFG. The players of an MLFG are separated in two categories: leaders ($i \in \mathcal{I}$), and followers ($j \in \mathcal{J}$). On one hand, the followers interact within a GNEP where each follower $j \in \mathcal{J}$ aims to solve the problem ($P_j(x, y_{-j})$):

$$\min_{y_j} \{f_j(x, y) \mid g_j(x, y) \leq 0, y_j \in Y_j(x, y_{-j})\} \quad (2.10)$$

where x and y represent the joint strategies of the leaders and the followers, respectively. The problems of the followers naturally depend on the variables of the leaders, thus the set of equilibria of the GNEP constituted by the problems ($P_j(x, y_{-j})$) $_{j \in \mathcal{J}}$ is denoted by $\text{GNEP}_{\mathcal{J}}(x)$.

On the other hand, the leaders also interact among themselves in a GNEP. The problem of leader i is denoted by ($P_i(x_{-i})$), and defined as

$$\begin{aligned} & \min_{x_i} F_i(x, y) \\ & \text{s.t.} \begin{cases} G_i(x, y) \leq 0 \\ x \in X_i(x_{-i}) \\ y \in \text{GNEP}_{\mathcal{J}}(x). \end{cases} \end{aligned} \quad (2.11)$$

The leaders' GNEP consists in finding a GNE for the problems $(P_i(x_{-i}))_{i \in \mathcal{I}}$. However, observe that this problem is ambiguous in its formulation, since $GNEP_{\mathcal{J}}(x)$ is in general no singleton. To cope with this issue in single-leader problems, an assumption is made about the followers' response: usually either the optimistic or the pessimistic formulation is chosen. When several leaders are involved in the problem, each one of them has to make an assumption about the followers' response, which might lead to contradictions. For example, assume that a follower j has to buy two items of a product from two leaders, i_1 and i_2 , who offer the same price. An optimistic solution for i_1 would be that j buys the two items from him, whereas an optimistic solution for i_2 would be the exact opposite. In this case, it might make sense to meet halfway and assume that j will opt for the last possible solution: buy one item from i_1 and one from i_2 .

To cope with these issues of leader assumptions, several methods have been defined. For example, in [Aussel 2018], a *multi-optimistic equilibrium* is defined. That is, each leader $i \in \mathcal{I}$ assumes a response y_i from the followers to the leaders' joint strategies $x = (x_i, x_{-i})$, and each couple (x_i, y_i) solves the problem

$$\begin{aligned} & \min_{x_i, y_i} F_i(x, y_i) \\ & \text{s.t.} \begin{cases} G_i(x, y_i) \leq 0 \\ x \in X_i(x_{-i}) \\ y_i \in GNEP_{\mathcal{J}}(x). \end{cases} \end{aligned} \quad (2.12)$$

This naturally defines a new GNEP among the leaders.

Several ways to solve a MLFG are suggested in [Leyffer 2010]. First, the followers' problems are replaced by their KKT conditions (see [Boyd 2004]). This induces the appearance of dual variables y^d , which are handled as if they were unique: if x is a joint strategy of the leaders, then (x_i, y, y^d) solves the problem $(P'_i(x_{-i}))$ (hereafter) for each $i \in \mathcal{I}$:

$$\begin{aligned} & \min_{x_i} F_i(x, y) \\ & \text{s.t.} \begin{cases} G_i(x, y) \leq 0 \\ x \in X_i(x_{-i}) \\ (y, y^d) \in KKT(GNEP_{\mathcal{J}}(x)), \end{cases} \end{aligned} \quad (2.13)$$

where $KKT(GNEP_{\mathcal{J}}(x))$ denotes the concatenation of the KKT conditions of all followers' problems.

The first solution method proposed in [Leyffer 2010] and applied in Chapter 5 consists in considering the concatenation of optimality conditions for the leaders' problems $(P'_i(x_{-i}))_{i \in \mathcal{I}}$. To reduce the number of solutions or the number of

complementarity constraints, appropriate objective functions can be used. Since optimality conditions are considered, many dual variables associated to the leaders' constraints appear. In the first method, it is assumed that each leader has a set of dual variables. However, some of the constraints are found in the optimization problems of all leaders: $(y, y^d) \in KKT(GNEP_{\mathcal{J}}(x))$. Therefore, it is sensible for all leaders to consider a single set of dual variables for those constraints. This results in a new formulation of the GNEP as a game with three kinds of players: the leaders $i \in \mathcal{I}$ who control their primal variables x_i , the followers who control their primal and dual variables (y, y^d) , and finally the markets, who control the dual variables associated to the KKT conditions of the followers. This idea gives rise to the *price-consistent formulations*. Note that equilibria for the price-consistent formulations are also equilibria of the first defined GNEP, but the converse does not hold.

In [Pang 2005], an example of a MLFG with no equilibrium is provided. To cope with the possible nonexistence of solutions of an MLFG, an alternative is proposed in [Kulkarni 2014]: the leaders share so-called *all equilibrium constraints*. Each leader makes an assumption y_i about the followers' response, but instead of leader i having to satisfy only the constraint $y_i \in GNEP_{\mathcal{J}}(x)$, he also has to satisfy the constraints $y_{i'} \in GNEP_{\mathcal{J}}(x)$ for all $i \neq i' \in \mathcal{I}$. Although the leaders' problems are more constrained in this new setting, new equilibria arise. Among others, it is proved that the example presented in [Pang 2005] has an equilibrium with the all equilibrium formulation.

On the applications' side, many papers focus on electricity markets. In [Aussel 2017a, Aussel 2017b], a multi-leader-common-follower problem is considered: energy producers act as leaders, whereas the system regulator is seen as the follower in a pay-as-bid electricity market. Similar problems were studied in [Allevi 2018, Aussel 2013, Fampa 2008] or in [Henrion 2012], where an equilibrium problem with equilibrium constraints (EPEC) is derived from an electricity spot market problem. In [Hesamzadeh 2014], the situation is reversed: the electricity producers are the followers, whereas the leader's role is fulfilled by a transmission planner. Different kinds of equilibria exist depending on the considered pricing type, as in [Lavigne 2000]: market equilibria obtained with an optimal pricing scheme obtained with a bilevel program are compared with traditional pricing methods, i.e. the price is either the marginal cost, or an affine function of the marginal cost. Finally, an interesting example of MLFG is provided in [Ramos 2016]: in an eco-industrial park, several companies aim to minimize their costs, whereas an authority wants to minimize the consumption of freshwater. Two cases are compared: in the first one, the authority is the leader and the companies are the follower, and in the

second one, the situation is reversed. Although both models are sensible, it turns out in the case study that the single-leader multi-follower model provides better results than the multi-leader single-follower one, in the sense that the sum of the companies' costs and the freshwater consumption are lower.

Single-Leader Single-Follower

This first research chapter focuses on a stochastic single-leader single-follower bilevel problem called (SBPP) (for stochastic bilevel pricing problem). (SBPP) involves two agents: at the upper level, an energy supplier aims to maximize his profit, and at the lower level, a *smart grid operator* (SGO) manages the consumption schedule of his clients' appliances.

At the upper level, the energy costs of the supplier depend on time, thus the supplier wants to sell more energy when the energy is cheap, and less when the energy is expensive. To induce those consumption changes, he offers time-dependent prices to the SGO, knowing that the SGO will shift his loads in an optimal way. It is assumed that the supplier acts in a competitive environment: a competitor also offers energy to the SGO, but the competitor's prices are assumed to be known in advance, making (SBPP) a best-response problem of the leader.

At the lower level, the SGO is related to a set of clients owning appliances to use during given time intervals. A *time window* is thus associated with each appliance. The SGO is in charge to ensure the powering of the appliances during their associated time window. For the clients, the ideal situation occurs when each appliance is powered at the beginning of the associated time window. If the SGO decides to schedule the appliance at another time within the time window, the client experiences *inconvenience*. The SGO aims therefore to minimize the *generalized cost* of his clients, which is the sum of the *billing cost* and the *inconvenience cost*. Furthermore, it is assumed that the SGO manages *distributed generation* (DG) capacities (e.g. solar panels) and a battery. To power his clients' appliances, the SGO has four possibilities: purchase energy from the leader, purchase energy from the competitor, consume energy produced by the DG, or consume energy stored in the battery. To store energy, the SGO can either purchase energy from the leader or the competitor, or use the energy produced by the DG.

The quantity of energy produced by solar panels (or possibly other renewable energy sources such as wind turbines) strongly depends on the weather conditions. As they are by nature unpredictable, a scenario tree approach is developed.

The problem (SBPP) is actually an extension of the problems proposed in [Afşar 2016b].

3.1 Stochastic bilevel problem

Before stating (SBPP), the scenario tree approach is introduced as a general method to consider stochasticity in an optimization problem.

3.1.1 Scenario tree method

Scenario tree approaches constitute the simplest way to take into account the uncertainty related to a situation in an optimization problem. Given a *time horizon* $\mathcal{H} = \{1, \dots, H\}$ divided into m *time periods* $TP_1 = \{1, \dots, t_2 - 1\}, \dots, TP_m = \{t_m, \dots, H\}$, a scenario tree is a set

$$\Sigma = \left\{ \left(\sigma^1, \dots, \sigma^m \right) \mid \sigma^i \in \{1, \dots, n_\Sigma\}, i = 1, \dots, m \right\},$$

where n_Σ is the number of base scenarios. The base scenarios are the elements $\sigma \in \Sigma$ such that $\sigma^i = \sigma^j$ for all $i, j = 1, \dots, m$, and are denoted by σ_i , with $i = 1, \dots, n_\Sigma$. Each base scenario corresponds to a set of parameters, which correspond to a sequence of H DG bounds in (SBPP) (denoted by $\lambda_{\max}^{h, \sigma}$, with $h \in \mathcal{H}$, see Subsection 3.1.2): σ_i corresponds to a base vector of bounds $\left(\lambda_{\max}^{1, \sigma_i}, \dots, \lambda_{\max}^{H, \sigma_i} \right)$, for $i = 1, \dots, n_\Sigma$. A scenario $\sigma \in \Sigma$ thus corresponds to a combination of the base vectors of bounds: if $\sigma^i = j$ for $i \in \{1, \dots, m\}$ and $j \in \{1, \dots, n_\Sigma\}$, the bounds associated to σ during the i th time period are the bounds of the j th base vector of bounds. Mathematically,

$$\sigma^i = j \Rightarrow \lambda_{\max}^{h, \sigma} = \lambda_{\max}^{h, \sigma_j} \quad \forall h \in TP_i.$$

To avoid confusion, recall that the time horizon is divided into time periods, and time periods are themselves divided into time slots, which are the smallest time units.

The elements of Σ can be seen as the leaves of a tree, as depicted in Figure 3.1 for $m = 3$, $n_\Sigma = 2$. Typically, $n_\Sigma = 2$ involves the presence of two basic scenarios: one with a large amount of sunshine and thus a large solar power production, and another one with a small amount of sunshine and thus a much smaller solar power production. During each of the three time periods, there can be a large or a small amount of sunshine. To determine the occurring probability of each scenario, it is assumed that at the end of each time period, there is a given probability to switch from the base scenario that just occurred to another base scenario. For example, if the probability to switch from one base scenario to the other is 0.5 at the end of each time period, each scenario of the tree depicted in Figure 3.1 has probability $0.5^3 = 0.125$ to occur.

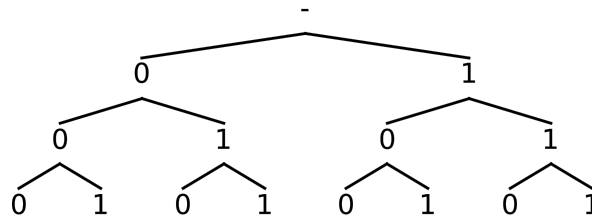


Figure 3.1: A scenario tree. Each leaf corresponds to a scenario, the branch to the left indicates low sunshine (0), the branch to the right indicates high sunshine (1). The sequence of the sunshine levels corresponding to a given scenario is determined by the sequence of its ancestors (starting from the root).

Introducing stochasticity with a scenario tree approach has some consequences on an optimization problem. First, a set of decision variables is associated to each scenario: they represent the decisions that must be taken if the associated scenario occurs. Then, the objective function becomes the expected value of the objective value for each scenario. Finally, nonanticipativity constraints are introduced: they ensure that the decisions are coherent. A stochastic model indeed represents a process over a whole time horizon. As long as two scenarios can be confounded as they occur, the decisions that are taken have to be the same. In the example depicted in Figure 3.1, consider the two scenarios $(0, 0, 0)$ and $(0, 0, 1)$. It is impossible to know which of the scenarios is happening during the two first time periods. Therefore, the variables associated to both scenarios have to take the same values during these two periods.

In the bilevel context of (SBPP), the situation is more complex. The stochasticity is introduced at the lower level. Therefore, both the leader and the follower optimize the expected value of their objective. Clearly, the SGO is directly impacted by the occurring scenario and has to modify his consumption schedule according to the DG. However, it is not acceptable for the leader to offer prices that differ according to the weather, thus there is a single vector of prices, not associated to a specific scenario, for the leader. Prices are thus *here-and-now* variables.

3.1.2 Notations

The parameters of the model are defined as follows:

- Time slots are denoted by $h \in \mathcal{H}$.

- The pair (n, a) denotes the *device* (also referred to as *appliance*) $a \in A_n$ of *client* $n \in N$.
- The devices are all preemptive, which means that their consumption can be adjusted (like air-conditioners or heaters). The device (n, a) is defined by three parameters: a time window containing successive time slots $T_{(n,a)} = \{T_{(n,a)}^{\text{first}}, \dots, T_{(n,a)}^{\text{last}}\}$ during which the device must receive $E_{(n,a)}$ energy units (kWh), with a maximum of $\beta_{(n,a)}^{\text{max}}$ energy units (usually kWh) per time slot.
- For each client n and device $a \in A_n$, the preferred time slot in the time window to start the corresponding device is the first one $T_{(n,a)}^{\text{first}}$. $C_{(n,a)}^h$ represents the inconvenience factor when (n, a) is not started at $T_{(n,a)}^{\text{first}}$. For the sake of consistency, the units of $C_{(n,a)}^h$ are €/kWh. As large delays induce large inconvenience, the following formula is considered:

$$C_{(n,a)}^h = \lambda_{(n,a)} \frac{h - T_{(n,a)}^{\text{first}}}{T_{(n,a)}^{\text{last}} - T_{(n,a)}^{\text{first}}}, \quad (3.1)$$

where $\lambda_{(n,a)}$ represents the heterogeneity of consumers with respect to the delay.

- There is a renewable energy production source, whose production at time $h \in \mathcal{H}$ in scenario $\sigma \in \Sigma$ is bounded by $\lambda_{\text{max}}^{h,\sigma}$.
- The storage capacities managed by the SGO are defined by the following parameters:
 - S^{min} and S^{max} are the lower and upper bound for the energy that can be stored.
 - Since storing energy implies losses, there are a charging coefficient $0 \leq \rho^c \leq 1$, and a discharging coefficient $0 \leq \rho^d \leq 1$.
- The energy cost function is denoted by K^h and depends on the time $h \in \mathcal{H}$ and the quantity of energy that must be supplied. For each $h \in \mathcal{H}$, K^h is a convex, monotonically increasing function. In this thesis, the energy is assumed to be bought in the spot market. For the sake of simplicity, we assume that K^h is a linear function in the market price, but K^h could be extended to an increasing piecewise linear function.
- Finally, the SGO can buy the energy from a competitor at a time-dependent price \bar{p}^h .

The decision variables of the leader and the follower are:

- The leader controls the energy prices p^h , which are not dependent on the scenarios.
- For time slot $h \in \mathcal{H}$ and scenario $\sigma \in \Sigma$, the follower defines the following variables:
 - $x_{(n,a)}^{h,\sigma}$ denotes the energy bought from the leader to be consumed by device (n, a) .
 - $\bar{x}_{(n,a)}^{h,\sigma}$ denotes the energy bought from the competitor to be consumed by device (n, a) .
 - $\lambda_{(n,a)}^{h,\sigma}$ denotes the energy that is consumed by device (n, a) coming from the DG.
 - $s_{(n,a)}^{h,\sigma}$ denotes the quantity of energy stored in the battery that is consumed by device (n, a) .
 - $S^{h,\sigma}$ denotes the quantity of energy stored in the battery at the beginning of time slot h .
 - $\lambda_s^{h,\sigma}$ denotes the energy coming from the DG that is stored into the battery.
 - $x_s^{h,\sigma}$ denotes the energy quantity bought from the leader to be stored into the battery.
 - $\bar{x}_s^{h,\sigma}$ denotes the energy quantity bought from the competitor to be stored into the battery.

In the rest of this section, bold variables indicate the aggregation of the variables that differ through their indices and exponents. For example, \mathbf{S} denotes the vectors of battery states $(S^{0,\sigma}, \dots, S^{H+1,\sigma})_{\sigma \in \Sigma}$, and \mathbf{x} is used to denote all variables of the form $x_{(n,a)}^{h,\sigma}$ or $x_s^{h,\sigma}$.

3.1.3 Follower's problem

For fixed leader's decisions, the optimization problem of the follower (P_f) can be formulated as:

$$\begin{aligned}
& \min_{\mathbf{x}, \bar{\mathbf{x}}, \boldsymbol{\lambda}, \mathbf{s}, \mathbf{S}} \sum_{\sigma \in \Sigma} P[\sigma] \cdot \left(\sum_{\substack{n \in N \\ a \in A_n \\ h \in T(n,a)}} \left(C_{(n,a)}^h \left(x_{(n,a)}^{h,\sigma} + \bar{x}_{(n,a)}^{h,\sigma} + \lambda_{(n,a)}^{h,\sigma} + s_{(n,a)}^{h,\sigma} \right) \right. \right. \\
& \quad \left. \left. + p^h x_{(n,a)}^{h,\sigma} + \bar{p}^h \bar{x}_{(n,a)}^{h,\sigma} \right) + \sum_{h \in \mathcal{H}} \left(p^h x_s^{h,\sigma} + \bar{p}^h \bar{x}_s^{h,\sigma} \right) \right) \\
& \text{s.t.} \left\{ \begin{array}{ll}
\sum_{h \in T(n,a)} \left(x_{(n,a)}^{h,\sigma} + \bar{x}_{(n,a)}^{h,\sigma} + \lambda_{(n,a)}^{h,\sigma} + s_{(n,a)}^{h,\sigma} \right) \geq E_{(n,a)} & \forall n \in N, a \in A_n, \sigma \in \Sigma \\
x_{(n,a)}^{h,\sigma} + \bar{x}_{(n,a)}^{h,\sigma} + \lambda_{(n,a)}^{h,\sigma} + s_{(n,a)}^{h,\sigma} \leq \beta_{(n,a)}^{\max} & \forall n \in N, a \in A_n, h \in T(n,a), \sigma \in \Sigma \\
\lambda_s^{h,\sigma} + \sum_{\substack{n \in N \\ a \in A_n}} \lambda_{(n,a)}^{h,\sigma} \leq \lambda_{\max}^{h,\sigma} & \forall h \in \mathcal{H}, \sigma \in \Sigma \\
\sum_{\substack{n \in N \\ a \in A_n}} s_{(n,a)}^{h,\sigma} \leq S^{h,\sigma} & \forall h \in \mathcal{H}, \sigma \in \Sigma \\
S^{0,\sigma} = S^{\text{start}} & \forall \sigma \in \Sigma \\
S^{h+1,\sigma} = \rho^d S^{h,\sigma} - \sum_{\substack{n \in N \\ a \in A_n}} s_{(n,a)}^{h,\sigma} \\
\quad + \rho^c \left(\lambda_s^{h,\sigma} + x_s^{h,\sigma} + \bar{x}_s^{h,\sigma} \right) & \forall h \in \mathcal{H}, \sigma \in \Sigma \\
S^{\min} \leq S^{h,\sigma} \leq S^{\max} & \forall h \in \mathcal{H}, \sigma \in \Sigma \\
\mathbf{x}, \bar{\mathbf{x}}, \boldsymbol{\lambda}, \mathbf{s} \geq 0, \\
x_{(n,a)}^{h,\sigma} = x_{(n,a)}^{h,\sigma'}, \quad \bar{x}_{(n,a)}^{h,\sigma} = \bar{x}_{(n,a)}^{h,\sigma'} & \forall h \in \mathcal{H}, n \in N, a \in A_n, (\sigma, \sigma') \in \text{Com}_h \\
\lambda_{(n,a)}^{h,\sigma} = \lambda_{(n,a)}^{h,\sigma'}, \quad s_{(n,a)}^{h,\sigma} = s_{(n,a)}^{h,\sigma'} & \\
\lambda_s^{h,\sigma} = \lambda_s^{h,\sigma'}, \quad S^{h,\sigma} = S^{h,\sigma'} & \forall h \in \mathcal{H}, (\sigma, \sigma') \in \text{Com}_h
\end{array} \right.
\end{aligned}$$

where

$$\text{Com}_h = \left\{ (\sigma, \sigma') \in \Sigma^2 \mid \sigma \prec \sigma', \lambda_{\max}^{h',\sigma} = \lambda_{\max}^{h',\sigma'} \quad \forall h' \in \mathcal{H}, h' \leq h \right\}$$

is the set of pairs of scenarios that are similar (at least) until $h \in \mathcal{H}$.

The follower's problem is a stochastic multi-stage linear problem. The objective function is the generalized cost of the consumer, composed of three parts: the

inconvenience cost, the cost of the energy that is purchased from the suppliers to power the devices, and the cost of the energy that is purchased to be stored. The first constraint ensures that each device receives enough energy during its associated time window. The second constraint ensures that the power limit of each device is not exceeded. The upper bound on the DG consumption is set by the third constraint. The fourth constraint asserts that the energy taken from the storage during time slot h does not exceed the available quantity of energy stored in the battery at the beginning of time h . The next three constraints describe the battery: the energy level in the battery is S^{start} at the beginning of the time horizon, the energy quantity in the battery depends on the energy taken from it and the energy stored in it, and the stored energy quantity is lower- and upper-bounded. Finally, after the necessary nonnegativity constraints, the last six constraints are the nonanticipativity constraints explained in Subsection 3.1.1.

To decrease the number of nonanticipativity constraints, we arbitrarily define a complete ordering of the scenarios, denoted by \prec . Only pairs of increasing scenarios are part of Com_h , which avoids considering for example both equations $\lambda_s^{h,\sigma} = \lambda_s^{h,\sigma'}$ and $\lambda_s^{h,\sigma'} = \lambda_s^{h,\sigma}$, which are the same for given $h \in \mathcal{H}$, $(\sigma, \sigma') \in \text{Com}_h$.

3.1.4 Leader's problem

The objective function of the leader consists in maximizing the expectation over all scenarios of his revenue, defined as the sum of the sales minus the energy costs. The sales are the sum of the energy sold to the SGO to power the appliances or to be stored, whereas the energy costs are described by the function K^h . This gives rise to Problem (P_l) :

$$\max_{\mathbf{p}} \sum_{\sigma \in \Sigma} P[\sigma] \cdot \left(\sum_{\substack{n \in N \\ a \in A_n \\ h \in T(n,a)}} p^h x_{(n,a)}^{h,\sigma} + \sum_{h \in \mathcal{H}} p^h x_s^{h,\sigma} - \sum_{h \in \mathcal{H}} K^h \left(x_s^{h,\sigma} + \sum_{\substack{n \in N, a \in A_n \\ \text{s.t. } h \in T(n,a)}} x_{(n,a)}^{h,\sigma} \right) \right)$$

$$\text{s.t. } (\mathbf{x}, \bar{\mathbf{x}}, \boldsymbol{\lambda}, \mathbf{s}, \mathbf{S}) \in \text{argmin} (P_f).$$

Note that lower and upper bounds on the prices are implicit. Negative prices would induce losses for the leader, whereas prices higher than the competitor's prices imply that the follower buys from the competitor.

3.2 Simplified problems and complexity

In this section, simplified versions of the problem (P_l) are defined and studied. In particular, they do not include DG or storage, nor do they consider multiple scenar-

ios. The idea is to study the theoretical computational complexity of the problem: the problem having a specific structure, the fact that bilinear bilevel problems are in general NP-hard does not guarantee the NP-hardness of (P_l) . The complexity may arise from two different features: the energy costs and/or the inconvenience costs. As for the rest of this chapter, the energy costs and inconvenience costs are assumed to be linear with respect to the energy that is produced, respectively consumed. However, the inconvenience factors can be arbitrarily chosen and do not depend on the delay.

3.2.1 One device

In this subsection, we assume that the follower wants to schedule the usage of a single device of a single client. We therefore omit the device's name in the model:

$$\begin{aligned} & \max_{\mathbf{p}} \sum_{h \in \mathcal{H}} (p^h x^h - K^h(x^h)) \\ & \left\{ \begin{array}{l} \min_{\mathbf{x}, \bar{\mathbf{x}}} \sum_{h \in \mathcal{H}} (p^h x^h + \bar{p}^h \bar{x}^h + C^h(x^h + \bar{x}^h)) \\ s.t. \left\{ \begin{array}{l} \sum_{h \in \mathcal{H}} (x^h + \bar{x}^h) \geq E \\ x^h + \bar{x}^h \leq \beta^{\max} \quad \forall h \in \mathcal{H} \\ x^h, \bar{x}^h \geq 0 \quad \forall h \in \mathcal{H}. \end{array} \right. \end{array} \right. \end{aligned} \quad (3.2)$$

3.2.1.1 No energy cost - no inconvenience cost

In this first case, we do not consider energy cost and inconvenience functions. It follows that:

- If the follower buys energy at time h , the price he pays will be equal to $\min\{x^h, \bar{x}^h\}$.
- As there is no inconvenience, the incentive for the follower to consume at a specific time slot is related to his billing cost, and thus to the price. Let $k = \lfloor E/\beta \rfloor$. The follower will consume β energy units during the k cheapest time slots, and $E - k\beta$ energy units during the $(k + 1)$ th cheapest slot.
- There are no production costs for the leader, thus maximizing the leader's objective requires to set the prices as high as possible.

It results that an optimal solution for the leader is defined as $p^h = \bar{p}^h$ for all $h \in \mathcal{H}$. This is naturally only true in the assumed optimistic setting.

3.2.1.2 Energy cost - no inconvenience cost

In this section, we assume that the cost function K^h is a linear function, with slope k^h . To get some intuition, consider the case of a device requiring three energy units during a time window of five time slots, with a maximal consumption of one energy unit per time slot (i.e. $E = 3$, $\beta^{max} = 1$, and $T = \mathcal{H} = \{1, \dots, 5\}$). Let $h : \mathbb{R}^5 \times \{1, \dots, 5\} \rightarrow \{1, \dots, 5\}$ be a map such that

$$i < j \Rightarrow \begin{cases} p^{h(\mathbf{p}, i)} < p^{h(\mathbf{p}, j)} \\ \text{or} \\ p^{h(\mathbf{p}, i)} = p^{h(\mathbf{p}, j)} \text{ and } k^{h(\mathbf{p}, i)} \leq k^{h(\mathbf{p}, j)}. \end{cases}$$

For a given price profile \mathbf{p} , the function $h(\mathbf{p}, \cdot) : \{1, \dots, 5\} \rightarrow \{1, \dots, 5\}$ gives an ordering of the time slots such that the prices are in increasing order, and in case of equality, the time slot with the smallest energy cost has a lower order. It follows directly that for a leader price profile \mathbf{p} , the follower will provide one energy unit to his appliance during time slots $h(\mathbf{p}, 1)$, $h(\mathbf{p}, 2)$, and $h(\mathbf{p}, 3)$. In particular, the optimal objective value of the follower is $\sum_{i=1}^3 p^{h(\mathbf{p}, i)}$, and the leader's profit is $\sum_{i=1}^3 (p^{h(\mathbf{p}, i)} - k^{h(\mathbf{p}, i)})$, since the profit per energy unit at time $h \in \mathcal{H}$ is $p^h - k^h$.

The competitor's prices and energy costs for each time slot are described in Figure 3.2. Without loss of generality, the competitor's prices are so that for $i \in \{1, \dots, 5\}$, $h(\bar{\mathbf{p}}, i) = i$, that is the competitor's prices are in increasing order. If the leader copies the prices of the competitor, the appliance is going to be used during the three first time slots, for a total price of 7.5.

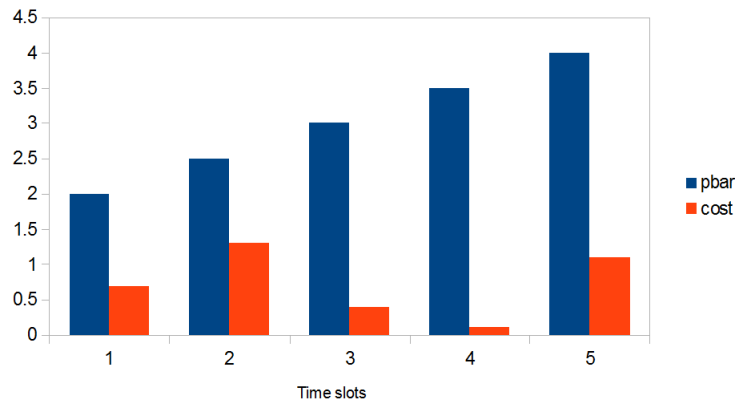


Figure 3.2: The competitor's prices, and the leader's energy costs.

To find the optimal price profile of the leader, we rely on inverse optimization

based on the shape of the follower's optimal response to the leader's prices. Assume first that the appliance is not necessarily powered during the first time slot. This means that the prices at time slots 2 to 5 are lower than or equal to the price offered at time slot 1. The optimal leader price profile \mathbf{p}_{-1} to achieve this is clearly $p_{-1}^h = \bar{p}^1$ for $h = 1, \dots, 5$, as in Figure 3.3.

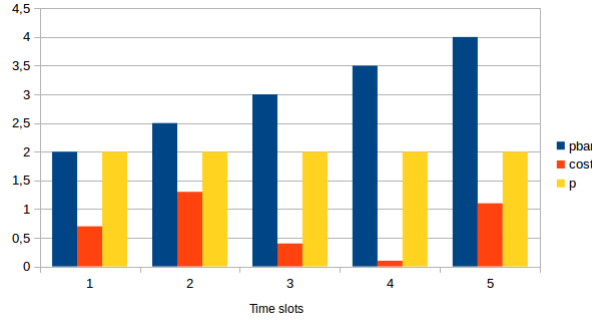


Figure 3.3: The competitor's prices, the leader's energy costs, and the leader's prices.

Since we assume an optimistic setting and the follower's reaction set is clearly no singleton, the follower will power his device during the time slots that are most advantageous for the leader, i.e. the slots 1, 3 and 4, since they are associated with the lowest energy costs. This induces a profit of 4.8 for the leader. However, this solution is not optimal for the leader. Indeed, let us assume that energy is necessarily consumed during the first time slot. Then, the price of the next time slots can be raised to increase the leader's profit. If the device is not necessarily powered during the second time slot, following the same reasoning, it results that the optimal leader price profile \mathbf{p}_{-2} to achieve such a follower's response is

$$p_{-2}^h = \begin{cases} \bar{p}^1 & \text{if } h = 1 \\ \bar{p}^2 & \text{if } h = 2, \dots, 5, \end{cases}$$

as depicted in Figure 3.4. The follower will then power his device during the time slots 1, 3 and 4, defining a profit of 5.8 for the leader.

Finally, we repeat the previous reasoning and assume that the device is powered during the two first time slots, but not during the third. As depicted in Figure 3.5, the corresponding price profile \mathbf{p}_{-3} is:

$$p_{-3}^h = \begin{cases} \bar{p}^1 & \text{if } h = 1 \\ \bar{p}^2 & \text{if } h = 2 \\ \bar{p}^3 & \text{if } h = 3, \dots, 5. \end{cases}$$

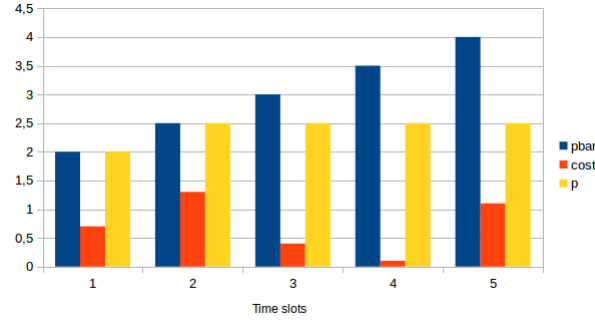


Figure 3.4: The competitor's prices, the leader's energy costs, and the leader's prices.

The follower's reaction to the price profile \mathbf{p}_{-3} consists in consuming the maximal energy quantity during time slots 1, 2 and 4, leading to a leader profit equal to 5.4.

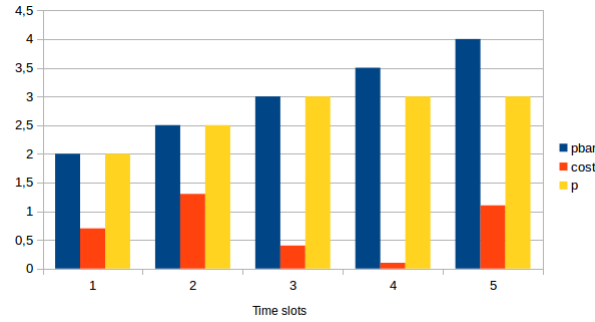


Figure 3.5: The competitor's prices, the leader's energy costs, and the leader's prices.

Observe that the three successive cases where energy is consumed or not necessarily can be represented as the leaves of a rooted tree, as in Figure 3.6. The i th level of the tree (i.e. nodes that have the distance i to the root) is associated with the i th time slot: the leaf to the left denoted by $-i$ represents the case where energy is not necessarily consumed at time i , whereas the node to the right denoted by i represents the case where energy is consumed at time i . The total number of leaves (and thus considered cases) is therefore $\lceil E/\beta^{max} \rceil$. These leaves define the set of the possible candidates for optimality. In the previous example, the optimal prices associated with each leaf $-i$ are the prices \mathbf{p}_{-i} for $i = 1, 2, 3$, whereas leaf 3 is associated with the prices $\mathbf{p}_3 = \bar{\mathbf{p}}$. Considering these four cases allows us to find the optimal leader price profile $\mathbf{p}_* = \mathbf{p}_{-2}$. By generalizing the previous reasoning,

it results that the optimal leader price profile for the simplified problem with energy costs belongs to the set

$$\left\{ (p^1, \dots, p^{|\mathcal{H}|}) \mid \begin{array}{ll} p^h = \bar{p}^h & \forall h \in \{1, \dots, i\} \\ p^h = \bar{p}^{i+1} & \forall h \in \{i+1, \dots, |\mathcal{H}|\} \end{array} \quad i = 0, \dots, |\mathcal{H}| \right\}. \quad (3.3)$$

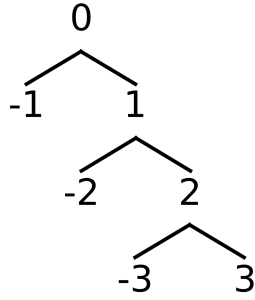


Figure 3.6: The tree of possible cases. Every leaf represents a case. The case where energy is not necessarily consumed at time h is denoted by $-h$, whereas h indicates the case where energy is necessarily consumed at time h .

In conclusion, to determine the optimal leader price profile, it is sufficient to compute the follower's reaction to each of the price profiles defined in (3.3), compute the corresponding leader's profit, and select the price profile that yields the best profit. This process is clearly running in polynomial time with respect to the number of time slots H .

3.2.1.3 No energy cost - inconvenience cost

In this subsection, we assume that the follower undergoes inconvenience depending on the time of usage of his device, but that the leader does not have energy costs. The inconvenience function C^h is linear with respect to the demand $x^h + \bar{x}^h$, with factor $c^h \geq 0$ at time $h \in \mathcal{H}$. As in the previous subsection, an example is provided before stating the general algorithm: the device needs three units of energy during a time window of five time slots, with a maximal consumption of one energy unit per time slot, as in the previous example. The parameters are given in Figure 3.7. Without loss of generality, the time slots are ordered in increasing order of values $\bar{p}^h + c^h$. The generalized cost for the follower to buy an energy unit at time $h \in \mathcal{H}$

is equal to $\min\{p^h, \bar{p}^h\} + c^h$. Therefore, if the leader sets the prices at the same level as the competitor for all time slots $h \in \mathcal{H}$, the follower will power his device during the three first time slots, defining a leader's profit equal to 7.5.

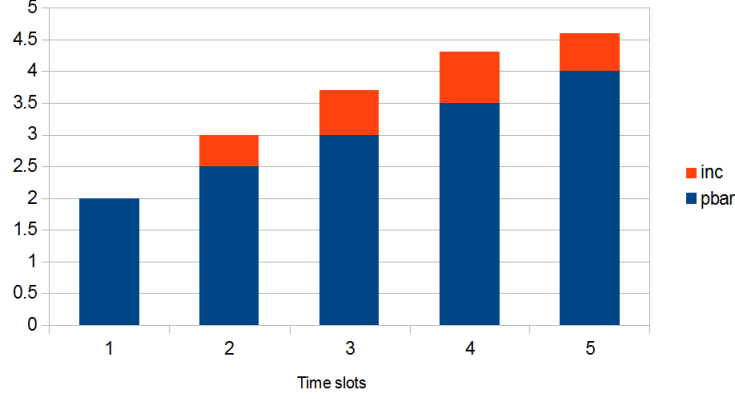


Figure 3.7: The competitor's prices and the inconvenience factors.

To find the optimal leader price profile, the solution algorithm proposed here is based on the scenario tree depicted in Figure 3.6. From the first to the k th time slot, where $k = \lceil E/\beta^{max} \rceil$, the follower's device can be powered or not. The case corresponding to leaf -1 is the case where the device is not necessarily powered during the first time slot. As a consequence, the generalized cost for the follower for any time slot is lower than or equal to the generalized cost of the first time slot, $\bar{p}^1 + c^1$. Therefore, the optimal price profile for this configuration is

$$p_{-1}^h = \bar{p}^1 + c^1 - c^h \quad \forall h \in \mathcal{H}.$$

The situation is depicted in Figure 3.8. Since we assume an optimistic setting, in case of equal generalized cost, the follower will power his device during the time slots generating a higher profit to the leader. In this case, they are the time slots with the smallest inconvenience factors, i.e. time slots 1, 2 and 5, yielding a profit equal to 4.9 for the leader.

If the device is powered during the first time slot, but not necessarily during the second one, the optimal leader price profile is given by

$$p_{-2}^h = \begin{cases} \bar{p}^1 & \text{if } h = 1 \\ \bar{p}^2 + c^2 - c^h & \text{otherwise.} \end{cases}$$

This price profile encourages the follower to consume energy during time slots 1,

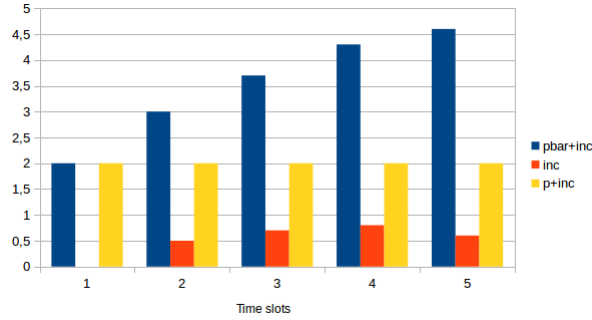


Figure 3.8: The competitor's prices plus the inconvenience factor, the inconvenience factor, and the leader's prices \mathbf{p}_{-1} plus the inconvenience factor.

2, and 5, like in the previous case, but the leader's profit increases to 6.9. This situation is depicted in Figure 3.9.

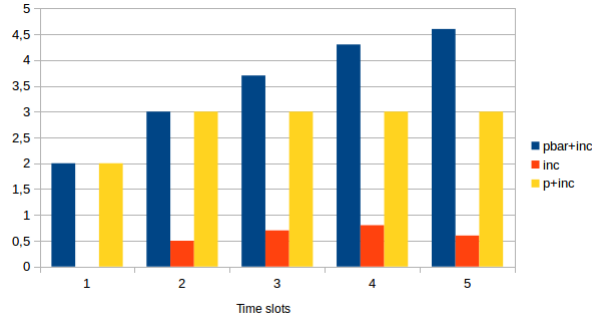


Figure 3.9: The competitor's prices plus the inconvenience factor, the inconvenience factor, and the leader's prices \mathbf{p}_{-2} plus the inconvenience factor.

Finally, let us assume that energy is consumed during the two first time slots, but not necessarily during the third one. The optimal price profile is then given by

$$p_{-3}^h = \begin{cases} \bar{p}^1 & \text{if } h = 1 \\ \bar{p}^2 & \text{if } h = 2 \\ \bar{p}^3 + c^3 - c^h & \text{otherwise,} \end{cases}$$

as represented in Figure 3.10. In this case, the follower powers his device during the time slots 1, 2 and 5 again, resulting in a leader's profit equal to 7.9. This situation corresponds to an optimal solution of the problem.

By generalizing this process, it results that the optimal leader price profile be-

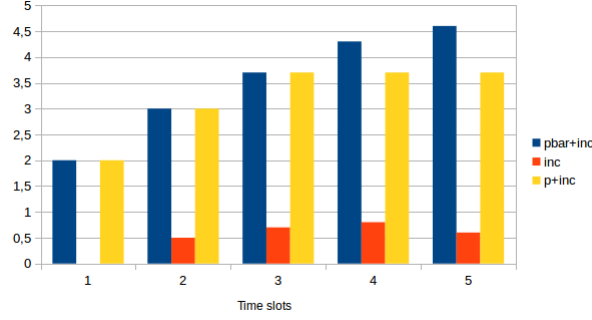


Figure 3.10: The competitor's prices plus the inconvenience factor, the inconvenience factor, and the leader's prices \mathbf{p}_{-3} plus the inconvenience factor.

longs to the set

$$\left\{ (p^1, \dots, p^{|H|}) \mid \begin{array}{ll} p^h = \bar{p}^h & \forall h \in \{1, \dots, i\} \\ p^h = \bar{p}^{i+1} + c^{i+1} - c^h & \forall h \in \{i+1, \dots, |H|\} \end{array} \quad i = 0, \dots, k \right\}, \quad (3.4)$$

where $k = \lceil E/\beta^{max} \rceil$, and the time slots are such that the values $\bar{p}^h + c^h$ are in increasing order, as assumed without loss of generality. In conclusion, we can argue that finding the optimal leader price profile takes polynomial time.

3.2.1.4 Energy cost - inconvenience cost

To conclude the study of problems with a single device, we consider that the leader faces a cost $k^h \geq 0$ for the energy he sells for all $h \in \mathcal{H}$, and the follower undergoes inconvenience, thus $c^h \geq 0$ for all $h \in \mathcal{H}$. Like in the two previous cases, we first consider an example, whose parameters are given in Figure 3.11. As in the previous case, it is assumed without loss of generality that the time slots are ordered such that the values $\bar{p}^h + c^h$ are increasing. Furthermore, if for some $h \in \mathcal{H}$, the equality $\bar{p}^h + c^h = \bar{p}^{h+1} + c^{h+1}$ holds, then $k^h \leq k^{h+1}$.

First, if the leader defines his prices according to the competitor's prices ($p^h = \bar{p}^h$ for all $h \in \mathcal{H}$), the follower will buy energy during the first three time slots, since they are the ones with the smallest generalized costs. The same tree as in the two previous cases is parsed (see Figure 3.6). Assume first that the device is not necessarily powered during the first time slot. The optimal price profile is:

$$p^h = \bar{p}^1 + c^1 - c^h \quad \forall h \in \mathcal{H}.$$

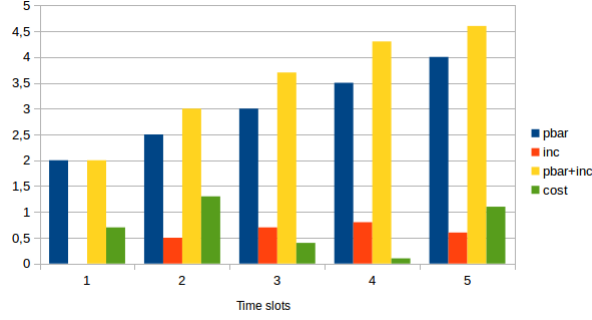


Figure 3.11: The competitor's prices, the inconvenience factor, their sum, and the energy cost.

The follower has thus access to energy at the same generalized cost (buying cost plus inconvenience) at all time slots. Therefore, he will power his device during the time slots 1, 3 and 5, since those are the time slots such that $p^h - k^h$ are the smallest, thus offering the highest revenue to the leader. Naturally, this results from the optimistic assumption. This first case is depicted in Figure 3.12, and results in a leader's profit equal to 3.3.

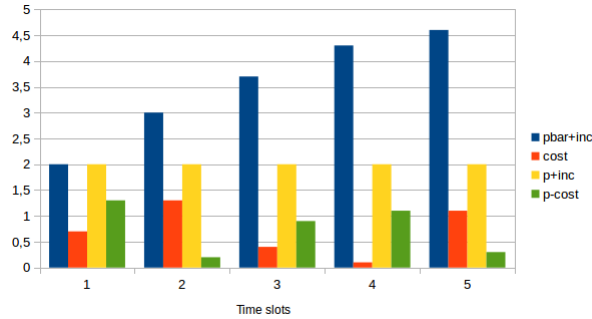


Figure 3.12: The competitor's prices plus the inconvenience factor, the energy cost, the leader's prices \mathbf{p}_{-1} plus the inconvenience factor, and the leader's profit per energy unit.

Second, let us assume that the follower does power his device during the first time slot, but not necessarily during the second one. The optimal leader price profile is

$$p_{-2}^h = \begin{cases} \bar{p}^1 & \text{if } h = 1 \\ \bar{p}^2 + c^2 - c^h & \text{otherwise,} \end{cases}$$

as represented in Figure 3.13. In this case, the follower powers his device during

the time slots 1, 3 and 5, which brings a profit of 5.3 to the leader.

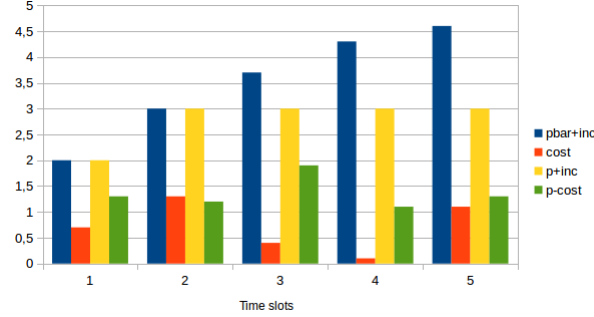


Figure 3.13: The competitor's prices plus the inconvenience factor, the energy cost, the leader's prices \mathbf{p}_{-2} plus the inconvenience factor, and the leader's profit per energy unit.

Finally, let us assume that the device is powered during the two first time slots, but not necessarily during the third one. The optimal leader price (see Figure 3.14) is

$$p_{-3}^h = \begin{cases} \bar{p}^1 & \text{if } h = 1 \\ \bar{p}^2 & \text{if } h = 2 \\ \bar{p}^3 + c^3 - c^h & \text{otherwise,} \end{cases}$$

which induces energy consumption by the follower at time slots 1, 2 and 3. The corresponding revenue for the leader is equal to 5.1.

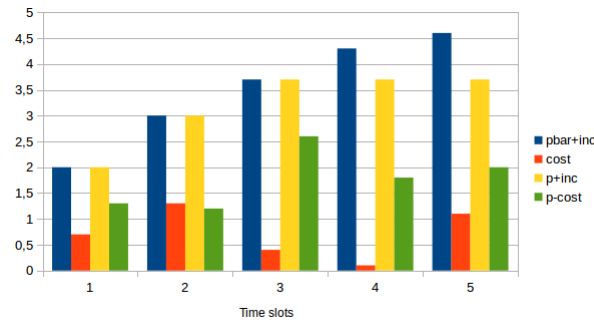


Figure 3.14: The competitor's prices plus the inconvenience factor, the energy cost, the leader's prices \mathbf{p}_{-3} plus the inconvenience factor and the leader's profit per energy unit.

As a conclusion for this example, the optimal price profile of the leader is \mathbf{p}_{-2} .

Let us remark that the price profiles \mathbf{p}_{-1} , \mathbf{p}_{-2} and \mathbf{p}_{-3} are the same as in Subsection 3.2.1.3. However, the follower's responses to those price profiles directly depend on the leader's energy costs. Hence, the optimal leader price profile belongs to the set defined in (3.4). The brute force algorithm consisting in solving the follower's problem for each of the potentially optimal price profiles and computing the corresponding leader revenue runs in polynomial time, with respect to the number of time slots.

It follows from these four studied cases that solving Problem 3.2 is easy: when there are no inconvenience costs for the follower, nor energy costs for the leader, the leader's optimal price profile is given by the competitor's prices. Otherwise, there is a brute force algorithm that runs in polynomial time with respect to the number of time slots.

3.2.2 Multiple devices

In this section, we consider the problem involving several devices:

$$\begin{aligned} \max_{\mathbf{p}} \quad & \sum_{h \in \mathcal{H}} \left(p^h \sum_{n \in N} \sum_{a \in A_n} x_{(n,a)}^h - K^h \left(\sum_{n \in N} \sum_{a \in A_n} x_{(n,a)}^h \right) \right) \\ \text{s.t.} \quad & \begin{cases} \min_{\mathbf{x}, \bar{\mathbf{x}}} \sum_{h \in \mathcal{H}} \sum_{n \in N} \sum_{a \in A_n} \left(p^h x_{(n,a)}^h + \bar{p}^h \bar{x}_{(n,a)}^h + C_{(n,a)}^h \left(x_{(n,a)}^h + \bar{x}_{(n,a)}^h \right) \right) \\ \sum_{h \in T_{(n,a)}} \left(x_{(n,a)}^h + \bar{x}_{(n,a)}^h \right) \geq E_{(n,a)} \quad \forall n \in N, a \in A_n \\ x_{(n,a)}^h + \bar{x}_{(n,a)}^h \leq \beta_{(n,a)}^{\max} \quad \forall h \in \mathcal{H}, n \in N, a \in A_n \\ x_{(n,a)}^h, \bar{x}_{(n,a)}^h \geq 0 \quad \forall h \in \mathcal{H}, n \in N, a \in A_n. \end{cases} \end{aligned} \quad (3.5)$$

The inconvenience costs and the energy costs are assumed to be linear functions: for $n \in N$ and $a \in A_n$, $C_{(n,a)}^h \left(x_{(n,a)}^h + \bar{x}_{(n,a)}^h \right) = c_{(n,a)}^h \cdot \left(x_{(n,a)}^h + \bar{x}_{(n,a)}^h \right)$, and $K^h(x) = k^h x$.

3.2.2.1 No energy cost - no inconvenience cost

If there are no inconvenience and no energy costs, the leader has no interest in inducing load shifts. Therefore, an optimal solution consists in mimicking the competitor's behavior and setting

$$p^h = \bar{p}^h \quad \forall h \in \mathcal{H}.$$

3.2.2.2 Energy cost - no inconvenience cost

In this section, we assume that the leader faces energy costs, but that the follower does not undergo any inconvenience. The parameters of Problem (3.5) are the com-

petitor's prices $\bar{\mathbf{p}}$, the energy costs factors k^h for all $h \in \mathcal{H}$, and the inconvenience factors $c_{(n,a)}^h$ for all $n \in N$, $a \in A_n$ and $h \in T_{(n,a)}$, the latter being all set to zero. Let us define

$$P = \left\{ \left(p^1, \dots, p^{|\mathcal{H}|} \right) \mid \forall h \in \mathcal{H} \exists h' \in \mathcal{H} : p^h = \bar{p}^{h'} \right\}.$$

It is easy to see that

$$|P| \leq |\mathcal{H}|^{|\mathcal{H}|}.$$

The size of P is thus polynomial with respect to the number of devices, even though it is superexponential with respect to the number of time slots. By proving that an optimal leader's price profile belongs to P , we conclude that for a fixed value of H , solving Problem (3.5) requires polynomial time with respect to the number of devices. The brute force algorithm consisting in testing each of the potentially optimal solutions in P indeed requires polynomial time, since testing one potential solution requires solving the follower's linear optimization problem, which can be done in polynomial time, with respect to the number of devices.

Proposition 3.2.1. *Given a price profile \mathbf{p} inducing an optimistic follower's response $(\mathbf{x}, \bar{\mathbf{x}})$, there is a price profile $\hat{\mathbf{p}} \in P$ such that:*

1. $\hat{p}^h \geq p^h$ for all $h \in \mathcal{H}$ such that $x^h > 0$,
2. $(\mathbf{x}, \bar{\mathbf{x}})$ belongs to the solution set of the follower's problem determined by $\hat{\mathbf{p}}$.

Proof. For all $h \in \mathcal{H}$, set

$$\hat{p}^h = \begin{cases} \max_{h' \in \mathcal{H}} \bar{p}^{h'} & \text{if } p^h > \max_{h' \in \mathcal{H}} \bar{p}^{h'} \\ \min\{\bar{p}^{h'} \mid h' \in \mathcal{H}, \bar{p}^{h'} \geq p^h\} & \text{otherwise.} \end{cases}$$

First, let us observe that if $p^h > \bar{p}^h$, the follower will not buy energy from the competitor at time $h \in \mathcal{H}$, therefore \hat{p} satisfies Condition 1.

Then, let us consider one of the devices $a \in A_n$, with $n \in N$. This device will be powered during the $k = \lceil E_{(n,a)} / \beta_{(n,a)}^{\max} \rceil$ time slots belonging to $T_{(n,a)}$ for which the energy is the cheapest. This means that the follower's response is uniquely determined by the order of the values $\min\{p^h, \bar{p}^h\}$. This order is preserved with $\hat{\mathbf{p}}$, since $p^h \leq p^{h'}$ for some $h, h' \in \mathcal{H}$ implies that $\hat{p}^h \leq \hat{p}^{h'}$. It follows directly that $(\mathbf{x}, \bar{\mathbf{x}})$ is an optimal follower's response to $\hat{\mathbf{p}}$. \square

Proposition 3.2.1 states that every follower's response $(\mathbf{x}, \bar{\mathbf{x}})$ that can be induced by a price profile \mathbf{p} can also be induced by a price profile $\hat{\mathbf{p}}$. Since $\hat{\mathbf{p}}$ satisfies Condition 1, the follower's response $(\mathbf{x}, \bar{\mathbf{x}})$ generates a higher profit for the leader

with $\hat{\mathbf{p}}$ than with \mathbf{p} . Furthermore, it is not guaranteed that $(\mathbf{x}, \bar{\mathbf{x}})$ is an optimistic response to $\hat{\mathbf{p}}$. However, an optimistic response to $\hat{\mathbf{p}}$ will yield at least the same profit for the leader as $(\mathbf{x}, \bar{\mathbf{x}})$. It directly follows from these remarks that for a given price profile \mathbf{p} , a higher profit is achieved by setting the prices to $\hat{\mathbf{p}}$. In conclusion, an optimal price profile always belongs to P . As a conclusion, for a fixed value of H , solving Problem (3.5) in the case where the follower does not undergo any inconvenience takes polynomial time with respect to the number of devices. It is not clear yet whether there exists a polynomial time algorithm with respect to the number of time slots as well.

3.2.2.3 No energy cost - inconvenience

In this section, we propose a conjecture, based on the same reasoning as in Section 3.2.2.2.

Definition 3.2.2. For all $h \in \mathcal{H}$, we define the set

$$P_{\text{opt}}^h = \{\bar{p}^{h'} + C_{(n,a)}^{h'} - C_{(n,a)}^h \mid h' \in \mathcal{H}, n \in N, a \in A_n\}.$$

The product of these sets is

$$P_{\text{opt}} = \prod_{h \in \mathcal{H}} P_{\text{opt}}^h.$$

Observe that for $h \in \mathcal{H}$, the size of P_{opt}^h is at most $H \cdot \sum_{n \in N} |A_n|$. Therefore, the size of P_{opt} is at most equal to $(H \cdot \sum_{n \in N} |A_n|)^H$, and thus of polynomial size with respect to the number of devices for a fixed value of H .

Conjecture 3.2.3. For any leader price profile \mathbf{p} inducing a follower's response $(\mathbf{x}_{(n,a)}, \bar{\mathbf{x}}_{(n,a)})_{n \in N, a \in A_n}$, there exists a price profile $\hat{\mathbf{p}} \in P_{\text{opt}}$ such that:

1. $\hat{p}^h \geq p^h$ for all $h \in \mathcal{H}$ such that there are $n \in N, a \in A_n$, with $x_{(n,a)}^h > 0$,
2. $(\mathbf{x}_{(n,a)}, \bar{\mathbf{x}}_{(n,a)})_{n \in N, a \in A_n}$ belongs to the solution set of the follower.

Conjecture 3.2.3 implies that for every price profile inducing a given followers' response, there is another price profile belonging to a finite set that brings a higher profit with the same followers' response, because the prices of the other price profile are higher than the ones in the initial price profile. Therefore, if Conjecture 3.2.3 is verified, then the brute force algorithm consisting in trying all the potentially optimal solutions in P_{opt} takes polynomial time with respect to the number of devices and yields an optimal solution for the leader, for the same reasons as in Subsection 3.2.2.2.

3.2.2.4 Energy cost - inconvenience

If Conjecture 3.2.3 holds, then the same conclusions can be derived if the energy costs are nonzero. That is, for a price profile \mathbf{p} inducing a follower's response $(\mathbf{x}_{(n,a)}, \bar{\mathbf{x}}_{(n,a)})_{n \in N, a \in A_n}$, there is a price profile $\hat{\mathbf{p}} \in P_{\text{opt}}$ such that $(\mathbf{x}_{(n,a)}, \bar{\mathbf{x}}_{(n,a)})_{n \in N, a \in A_n}$ belongs to the set of solutions of the follower's problem, and such that for all $h \in \mathcal{H}$, $p^h \leq \hat{p}^h$ if the demand is nonzero at this time. Thus, the leader price profile $\hat{\mathbf{p}}$ would generate a higher profit for the leader than \mathbf{p} , therefore there is an optimal price profile in P_{opt} , whose size is polynomial in the number of devices.

In conclusion, Problem 3.5 can possibly be solved in polynomial time with respect to the number of devices for a given number of time slots H . However, even if Conjecture 3.2.3 holds, the size of P_{opt} increases rapidly with H . The brute force algorithm requires solving $|P_{\text{opt}}|$ linear programs, which would require a long time. Thus for solving Problem 3.5, with multiple devices and inconvenience and/or energy costs, we surely need to design exact algorithms or heuristics based on the structure of the problem.

3.3 Numerical resolution of (SBPP)

In this section, we give a single-level mixed-integer formulation of (SBPP), and provide numerical results obtained through the resolution of the one-level reformulation using CPLEX [CPL 2014].

3.3.1 One-level formulation of (SBPP)

For fixed decisions of the leader, the follower's problem is linear. Therefore, it can be replaced by its optimality conditions (primal constraints, dual constraints, and the complementarity slackness constraints) in the leader's problem, giving rise to a mathematical program with complementarity constraints (MPCC), which can be linearized through the *big M* method, yielding a MIP. This method consists in replacing constraints of the following form

$$a \cdot b = 0$$

$$a, b \geq 0$$

by the constraints

$$a \leq M\vartheta$$

$$b \leq M(1 - \vartheta)$$

$$a, b \geq 0$$

$$\vartheta \in \{0, 1\},$$

which are equivalent for M sufficiently large. However, as shown in [Pineda 2018] and [Kleinert 2019a], determining the best value for M is hard. Despite this, the big M method is widely used in bilevel programming (as for instance in [Casorrán 2019] or [Garces 2009]), since MIPs can be solved efficiently using commercial solvers. For the numerical results, we proceed in an empirical way, systematically testing values for M until the solver is able to provide satisfactory results from the points of view of the computation time and the solution quality.

Let us denote a tuple of primal variables by

$$\varphi^p = (\mathbf{x}, \bar{\mathbf{x}}, \boldsymbol{\lambda}, \mathbf{s}, \mathbf{x}_s, \bar{\mathbf{x}}_s, \boldsymbol{\lambda}_s, \mathbf{S}).$$

This tuple belongs to *Primal* if it satisfies the primal constraints of (P_f) :

$$\begin{aligned} \sum_{h \in T(n,a)} \left(x_{(n,a)}^{h,\sigma} + \bar{x}_{(n,a)}^{h,\sigma} + \lambda_{(n,a)}^{h,\sigma} + s_{(n,a)}^{h,\sigma} \right) &\geq E_{(n,a)} & (dE_{(n,a)}^\sigma) & \quad \forall n \in N, a \in A_n, \sigma \in \Sigma \\ x_{(n,a)}^{h,\sigma} + \bar{x}_{(n,a)}^{h,\sigma} + \lambda_{(n,a)}^{h,\sigma} + s_{(n,a)}^{h,\sigma} &\leq \beta_{(n,a)}^{\max} & (d\beta_{(n,a)}^{\max,\sigma}) & \quad \forall n \in N, a \in A_n, h \in T(n,a), \sigma \in \Sigma \\ \lambda_s^{h,\sigma} + \sum_{\substack{n \in N \\ a \in A_n}} \lambda_{(n,a)}^{h,\sigma} &\leq \lambda_{\max}^{h,\sigma} & (d\lambda_{\max}^{h,\sigma}) & \quad \forall h \in \mathcal{H}, \sigma \in \Sigma \\ S^{0,\sigma} &= S^{\text{start}} & (dS_0^\sigma) & \quad \forall \sigma \in \Sigma \\ S^{h+1,\sigma} &= \rho^d S^{h,\sigma} - \sum_{\substack{n \in N \\ a \in A_n}} s_{(n,a)}^{h,\sigma} \\ &\quad + \rho^c (\lambda_s^{h,\sigma} + x_s^{h,\sigma} + \bar{x}_s^{h,\sigma}) & (dS^{h,\sigma}) & \quad \forall h \in \mathcal{H}, \sigma \in \Sigma \\ \sum_{\substack{n \in N \\ a \in A_n}} s_{(n,a)}^{h,\sigma} &\leq S^{h,\sigma} & (ds_{\max}^{h,\sigma}) & \quad \forall h \in \mathcal{H}, \sigma \in \Sigma \\ S^{\min} &\leq S^{h,\sigma} \leq S^{\max} & (dS_{\min}^{h,\sigma}, dS_{\max}^{h,\sigma}) & \quad \forall h \in \mathcal{H}, \sigma \in \Sigma \\ x_{(n,a)}^{h,\sigma} &= x_{(n,a)}^{h,\sigma'} & (dx_{(n,a)}^{\sigma,\sigma',h}) & \quad \forall h \in \mathcal{H}, n \in N, a \in A_n, (\sigma, \sigma') \in \text{Com}_h \\ \bar{x}_{(n,a)}^{h,\sigma} &= \bar{x}_{(n,a)}^{h,\sigma'} & (d\bar{x}_{(n,a)}^{\sigma,\sigma',h}) & \quad \forall h \in \mathcal{H}, n \in N, a \in A_n, (\sigma, \sigma') \in \text{Com}_h \\ \lambda_{(n,a)}^{h,\sigma} &= \lambda_{(n,a)}^{h,\sigma'} & (d\lambda_{(n,a)}^{\sigma,\sigma',h}) & \quad \forall h \in \mathcal{H}, n \in N, a \in A_n, (\sigma, \sigma') \in \text{Com}_h \\ s_{(n,a)}^{h,\sigma} &= s_{(n,a)}^{h,\sigma'} & (ds_{(n,a)}^{\sigma,\sigma',h}) & \quad \forall h \in \mathcal{H}, n \in N, a \in A_n, (\sigma, \sigma') \in \text{Com}_h \\ \lambda_s^{h,\sigma} &= \lambda_s^{h,\sigma'} & (d\lambda_s^{\sigma,\sigma',h}) & \quad \forall h \in \mathcal{H}, (\sigma, \sigma') \in \text{Com}_h \\ x_s^{h,\sigma} &= x_s^{h,\sigma'} & (dx_s^{\sigma,\sigma',h}) & \quad \forall h \in \mathcal{H}, (\sigma, \sigma') \in \text{Com}_h \\ \bar{x}_s^{h,\sigma} &= \bar{x}_s^{h,\sigma'} & (d\bar{x}_s^{\sigma,\sigma',h}) & \quad \forall h \in \mathcal{H}, (\sigma, \sigma') \in \text{Com}_h \\ S^{h,\sigma} &= S^{h,\sigma'} & (dS^{\sigma,\sigma',h}) & \quad \forall h \in \mathcal{H}, (\sigma, \sigma') \in \text{Com}_h \\ \mathbf{x}, \bar{\mathbf{x}}, \boldsymbol{\lambda}, \mathbf{s} \geq 0 & & (d\mathbf{x}_{(n,a)}^{h,\sigma}, d\bar{\mathbf{x}}_{(n,a)}^{h,\sigma}, dx_s^{h,\sigma}, d\bar{x}_s^{h,\sigma}, d\lambda_{(n,a)}^{h,\sigma}, d\lambda_s^{h,\sigma}, ds_{(n,a)}^{h,\sigma}) & \end{aligned}$$

The variables between parentheses are the dual variables associated to each of the constraints. Let us denote a tuple of dual variables by

$$\varphi^d = (d\mathbf{E}, d\boldsymbol{\beta}, d\boldsymbol{\lambda}_{\max}, d\mathbf{S}, d\mathbf{s}, d\mathbf{S}^{\min}, d\mathbf{S}^{\max}, d\mathbf{x}, d\bar{\mathbf{x}}, d\boldsymbol{\lambda}).$$

This tuple belongs to *Dual* if it satisfies the following constraints:

$$\begin{aligned}
dE_{(n,a)}^\sigma - d\beta_{(n,a)}^{\max,\sigma} + \sum_{\sigma' \in \text{Com}_h(\sigma)} \text{sgn}(\sigma, \sigma') dx_{(n,a)}^{\sigma, \sigma', h} &\leq P[\sigma] (p^h + C_{(n,a)}^h) & \forall n \in N, a \in A_n, h \in T_{(n,a)}, \sigma \in \Sigma \\
dE_{(n,a)}^\sigma - d\beta_{(n,a)}^{\max,\sigma} + \sum_{\sigma' \in \text{Com}_h(\sigma)} \text{sgn}(\sigma, \sigma') d\bar{x}_{(n,a)}^{\sigma, \sigma', h} &\leq P[\sigma] (\bar{p}^h + C_{(n,a)}^h) & \forall n \in N, a \in A_n, h \in T_{(n,a)}, \sigma \in \Sigma \\
dE_{(n,a)}^\sigma - d\beta_{(n,a)}^{\max,\sigma} - d\lambda_{\max}^{h,\sigma} + \sum_{\sigma' \in \text{Com}_h(\sigma)} \text{sgn}(\sigma, \sigma') d\lambda_{(n,a)}^{\sigma, \sigma', h} &\leq P[\sigma] C_{(n,a)}^h & \forall n \in N, a \in A_n, h \in T_{(n,a)}, \sigma \in \Sigma \\
dE_{(n,a)}^\sigma - d\beta_{(n,a)}^{\max,\sigma} + dS - ds_{(n,a)}^{h,\sigma} & & \\
+ \sum_{\sigma' \in \text{Com}_h(\sigma)} \text{sgn}(\sigma, \sigma') ds_{(n,a)}^{\sigma, \sigma', h} &\leq P[\sigma] C_{(n,a)}^h & \forall n \in N, a \in A_n, h \in T_{(n,a)}, \sigma \in \Sigma \\
-\rho^c dS^{h,\sigma} + \sum_{\sigma' \in \text{Com}_h(\sigma)} \text{sgn}(\sigma, \sigma') dx_s^{\sigma, \sigma', h} &\leq P[\sigma] p^h & \forall h \in \mathcal{H}, \sigma \in \Sigma \\
-\rho^c dS^{h,\sigma} + \sum_{\sigma' \in \text{Com}_h(\sigma)} \text{sgn}(\sigma, \sigma') d\bar{x}_s^{\sigma, \sigma', h} &\leq P[\sigma] \bar{p}^h & \forall h \in \mathcal{H}, \sigma \in \Sigma \\
-d\lambda_{\max}^{h,\sigma} - \rho^c dS^{h,\sigma} + \sum_{\sigma' \in \text{Com}_h(\sigma)} \text{sgn}(\sigma, \sigma') d\lambda_s^{\sigma, \sigma', h} &\leq 0 & \forall h \in \mathcal{H}, \sigma \in \Sigma \\
dS_0^\sigma - \rho^d dS^{h,\sigma} + ds_{(n,a)}^{h,\sigma} + \sum_{\sigma' \in \text{Com}_h(\sigma)} \text{sgn}(\sigma, \sigma') dS^{\sigma, \sigma', h} &\leq 0 & \forall \sigma \in \Sigma, h = 0 \\
dS^{h-1,\sigma} - \rho^d dS^{h,\sigma} + ds_{(n,a)}^{h,\sigma} + dS_{\min}^{h,\sigma} - dS_{\max}^{h,\sigma} & & \\
+ \sum_{\sigma' \in \text{Com}_h(\sigma)} \text{sgn}(\sigma, \sigma') dS^{\sigma, \sigma', h} &\leq 0 & \forall h \in \mathcal{H} \setminus \{0\}, \sigma \in \Sigma \\
dS^{h-1,\sigma} + dS_{\min}^{h,\sigma} - dS_{\max}^{h,\sigma} &\leq 0 & \forall \sigma \in \Sigma, h = |\mathcal{H}| + 1 \\
dE_{(n,a)}^\sigma, d\beta_{(n,a)}^{\max,\sigma}, d\lambda_{\max}^{h,\sigma}, ds_{(n,a)}^{h,\sigma}, dS_{\min}^{h,\sigma}, dS_{\max}^{h,\sigma} &\geq 0 & \forall \sigma \in \Sigma, h \in \mathcal{H}, n \in N, a \in A_n,
\end{aligned}$$

where

$$\text{Com}_h(\sigma) = \{\sigma' \in \Sigma \mid (\sigma, \sigma') \in \text{Com}_h \text{ or } (\sigma', \sigma) \in \text{Com}_h\},$$

and the sign function is defined as follows:

$$\text{sgn}(\sigma, \sigma') = \begin{cases} 1 & \text{if } \sigma \prec \sigma' \\ 0 & \text{if } \sigma = \sigma' \\ -1 & \text{if } \sigma \succ \sigma'. \end{cases}$$

Finally, let us denote by φ a pair (φ^p, φ^d) consisting of a tuple of primal variables φ^p and a tuple of dual variables φ^d . This tuple φ belongs to *Comp* if it satisfies the

following complementarity constraints:

$$\begin{aligned}
x_{(n,a)}^h \left(dE_{(n,a)}^\sigma - d\beta_{(n,a)}^{\max,\sigma} - P[\sigma] (p^h + C_{(n,a)}^h) \right) &= 0 & \forall n \in N, a \in A_n, h \in T_{(n,a)}, \sigma \in \Sigma \\
\bar{x}_{(n,a)}^h \left(dE_{(n,a)}^\sigma - d\beta_{(n,a)}^{\max,\sigma} - P[\sigma] (\bar{p}^h + C_{(n,a)}^h) \right) &= 0 & \forall n \in N, a \in A_n, h \in T_{(n,a)}, \sigma \in \Sigma \\
\lambda_{(n,a)}^h \left(dE_{(n,a)}^\sigma - d\beta_{(n,a)}^{\max,\sigma} - d\lambda_{\max}^{h,\sigma} - P[\sigma] C_{(n,a)}^h \right) &= 0 & \forall n \in N, a \in A_n, h \in T_{(n,a)}, \sigma \in \Sigma \\
s_{(n,a)}^h \left(dE_{(n,a)}^\sigma - d\beta_{(n,a)}^{\max,\sigma} + dS - ds_{\max}^{h,\sigma} - P[\sigma] C_{(n,a)}^h \right) &= 0 & \forall n \in N, a \in A_n, h \in T_{(n,a)}, \sigma \in \Sigma \\
x_s^h \left(-\rho^c dS^{h,\sigma} - P[\sigma] p^h \right) &= 0 & \forall h \in \mathcal{H}, \sigma \in \Sigma \\
\bar{x}_s^h \left(-\rho^c dS^{h,\sigma} - P[\sigma] \bar{p}^h \right) &= 0 & \forall h \in \mathcal{H}, \sigma \in \Sigma \\
\lambda_s^h \left(-d\lambda_{\max}^{h,\sigma} - \rho^c dS^{h,\sigma} \right) &= 0 & \forall h \in \mathcal{H}, \sigma \in \Sigma \\
S^h \left(dS_0^\sigma - \rho^d dS^{h,\sigma} + ds_{\max}^{h,\sigma} \right) &= 0 & \forall \sigma \in \Sigma, h = 0 \\
S^h \left(dS^{h-1,\sigma} - \rho^d dS^{h,\sigma} + ds_{\max}^{h,\sigma} + dS_{\min}^{h,\sigma} - dS_{\max}^{h,\sigma} \right) &= 0 & \forall h \in \mathcal{H} \setminus \{0\}, \sigma \in \Sigma \\
S^h \left(dS^{h-1,\sigma} + dS_{\min}^{h,\sigma} - dS_{\max}^{h,\sigma} \right) &= 0 & \forall \sigma \in \Sigma, h = |\mathcal{H}| + 1 \\
dE_{(n,a)}^\sigma \left(\sum_{h \in T_{(n,a)}} \left(x_{(n,a)}^{h,\sigma} + \bar{x}_{(n,a)}^{h,\sigma} + \lambda_{(n,a)}^{h,\sigma} + s_{(n,a)}^{h,\sigma} \right) - E_{(n,a)} \right) &= 0 & \forall n \in N, a \in A_n, \sigma \in \Sigma \\
d\beta_{(n,a)}^{\max,\sigma} \left(x_{(n,a)}^{h,\sigma} + \bar{x}_{(n,a)}^{h,\sigma} + \lambda_{(n,a)}^{h,\sigma} + s_{(n,a)}^{h,\sigma} - \beta_{(n,a)}^{\max} \right) &= 0 & \forall n \in N, a \in A_n, h \in T_{(n,a)}, \sigma \in \Sigma \\
d\lambda_{\max}^{h,\sigma} \left(\lambda_s^{h,\sigma} + \sum_{n \in N} \sum_{a \in A_n} \lambda_{(n,a)}^{h,\sigma} - \lambda_{\max}^{h,\sigma} \right) &= 0 & \forall h \in \mathcal{H}, \sigma \in \Sigma \\
ds_{\max}^{h,\sigma} \left(\sum_{n \in N} \sum_{a \in A_n} s_{(n,a)}^{h,\sigma} - S^{h,\sigma} \right) &= 0 & \forall h \in \mathcal{H}, \sigma \in \Sigma \\
dS_{\min}^{h,\sigma} \left(S^{\min} - S^{h,\sigma} \right) &= 0 & \forall h \in \mathcal{H}, \sigma \in \Sigma \\
dS_{\max}^{h,\sigma} \left(S^{h,\sigma} - S^{\max} \right) &= 0 & \forall h \in \mathcal{H}, \sigma \in \Sigma.
\end{aligned}$$

If $\varphi = (\varphi^p, \varphi^d)$ belongs to $(Primal \times Dual) \cap Comp$, then φ^p is primal optimal and φ^d is dual optimal. Therefore, the leader's problem is replaced by the single-level program (P_l^{MPCC})

$$\max_{\mathbf{p}, \varphi} \sum_{\sigma \in \Sigma} P[\sigma] \cdot \left(\sum_{\substack{n \in N \\ a \in A_n \\ h \in T_{(n,a)}}} p^h x_{(n,a)}^{h,\sigma} + \sum_{h \in \mathcal{H}} p^h x_s^{h,\sigma} - \sum_{h \in \mathcal{H}} K^h \left(x_s^{h,\sigma} + \sum_{\substack{n \in N, a \in A_n \\ \text{s.t. } h \in T_{(n,a)}}} x_{(n,a)}^{h,\sigma} \right) \right)$$

s.t. $\varphi \in (Primal \times Dual) \cap Comp$.

Problem (P_l^{MPCC}) is nonlinear due to the products of variables in the complementarity constraints and in the objective function. To get a mixed integer linear problem (MILP), we linearize those products. First, observe that the problem

$$\begin{aligned}
a \cdot b &= 0 \\
a, b &\geq 0
\end{aligned} \tag{3.6}$$

can be replaced by the problem

$$\begin{aligned}
a &\leq M \cdot \varepsilon \\
b &\leq M \cdot (1 - \varepsilon) \\
a, b &\geq 0, \\
\varepsilon &\in \{0, 1\}
\end{aligned} \tag{3.7}$$

with $M \in \mathbb{R}$ sufficiently large. The same process is applied to the complementarity constraints problem, and a binary variable as well as a *big* M are associated to each of the complementarity constraints. For example, we replace the complementarity constraint

$$dS_{\min}^{h,\sigma} (S^{\min} - S^{h,\sigma}) = 0$$

for some $h \in \mathcal{H}$ and some $\sigma \in \Sigma$ by the constraints

$$\begin{aligned}
dS_{\min}^{h,\sigma} &\leq M \cdot \varepsilon \\
S^{h,\sigma} - S^{\min} &\leq M \cdot (1 - \varepsilon) \\
dS_{\min}^{h,\sigma}, S^{h,\sigma} - S^{\min} &\geq 0 \\
\varepsilon &\in \{0, 1\}.
\end{aligned}$$

If $\varphi \in \Phi$ satisfies the modified version of the complementarity constraints, then $\varphi = (\varphi^p, \varphi^d)$ belongs to the set $Comp'$. Observe that

$$(Primal \times Dual) \cap Comp = (Primal \times Dual) \cap Comp'.$$

The leader's objective function is linearized with a strong duality result. If $\varphi = (\varphi^p, \varphi^d) \in \Phi$ belongs to $(Primal \times Dual) \cap Comp$, then φ^p is primal optimal and φ^d is dual optimal. By strong duality, the primal and dual objective functions take the same value, therefore

$$\sum_{\sigma \in \Sigma} P[\sigma] \cdot \left(\sum_{\substack{n \in N \\ a \in A_n \\ h \in T(n,a)}} \left(C_{(n,a)}^h (x_{(n,a)}^{h,\sigma} + \bar{x}_{(n,a)}^{h,\sigma} + \lambda_{(n,a)}^{h,\sigma} + s_{(n,a)}^{h,\sigma}) \right. \right. \\
\left. \left. + p^h x_{(n,a)}^{h,\sigma} + \bar{p}^h \bar{x}_{(n,a)}^{h,\sigma} \right) + \sum_{h \in \mathcal{H}} \left(p^h x_s^{h,\sigma} + \bar{p}^h \bar{x}_s^{h,\sigma} \right) \right)$$

is equal to

$$\sum_{\sigma \in \Sigma} \left(\sum_{\substack{n \in N \\ a \in A_n}} \left(E_{(n,a)} dE_{(n,a)}^\sigma - \sum_{h \in T_{(n,a)}} \beta_{(n,a)}^{\max} d\beta_{(n,a)}^{\max,\sigma} \right) + \sum_{h \in \mathcal{H}} \left(-\lambda_{\max}^h d\lambda_{\max}^{h,\sigma} + S^{\min} dS_{\min}^{h,\sigma} - S^{\max} dS_{\max}^{h,\sigma} \right) \right).$$

It follows from the equality that the leader's objective function can be rewritten as

$$F(\varphi) = \left\{ \begin{array}{l} \sum_{\sigma \in \Sigma} \left(\sum_{\substack{n \in N \\ a \in A_n}} \left(E_{(n,a)} dE_{(n,a)}^\sigma - \sum_{h \in T_{(n,a)}} \beta_{(n,a)}^{\max} d\beta_{(n,a)}^{\max,\sigma} \right) \right. \\ \left. + \sum_{h \in \mathcal{H}} \left(-\lambda_{\max}^h d\lambda_{\max}^{h,\sigma} + S^{\min} dS_{\min}^{h,\sigma} - S^{\max} dS_{\max}^{h,\sigma} \right) \right. \\ \left. - \left(\sum_{\substack{n \in N \\ a \in A_n \\ h \in T_{(n,a)}}} \left(\bar{p}^h \bar{x}_{(n,a)}^{h,\sigma} + C_{(n,a)}^h \left(x_{(n,a)}^{h,\sigma} + \bar{x}_{(n,a)}^{h,\sigma} + \lambda_{(n,a)}^{h,\sigma} + s_{(n,a)}^{h,\sigma} \right) \right) \right. \right. \\ \left. \left. + \sum_{h \in \mathcal{H}} \left(\bar{p}^h \bar{x}_s^{h,\sigma} + K^h \left(x_s^{h,\sigma} + \sum_{\substack{n \in N \\ a \in A_n}} x_{(n,a)}^{h,\sigma} \right) \right) \right) \right\}.$$

Finally, the reformulation of (SBPP) as a MIP is (P_l^{MIP})

$$\begin{array}{ll} \max_{\varphi} & F(\varphi) \\ \text{s.t.} & \varphi \in (\text{Primal} \times \text{Dual}) \cap \text{Comp}'. \end{array}$$

Problem (SBPP) can thus be solved to optimality by solving (P_l^{MIP}) .

3.3.2 A single scenario case

The numerical results for (SBPP) comprise two parts. In the first part, we perform sensitivity analyses for various parameters on large test instances considering a single scenario. In the next part, multiple scenarios are addressed.

All the numerical results presented in this chapter are obtained through the resolution of (P_l^{MIP}) with CPLEX 12.6 on a Linux virtual machine with 10GB RAM working on a computer equipped with an Intel i7-4600u processor at 2.10GHz.

3.3.2.1 Test instances

Numerical results with a single scenario are discussed on instances based on data retrieved from the industrial partner. The main characteristics of the base instance are as follows:

- Each time slot represents 30 minutes.
- The time horizon is composed of 336 time slots corresponding to half hours (thus a full week).
- The energy costs are based on the prices of electricity on the spot market during a fall week (see Figure 3.16)
- The consumption of the 120 devices approximates the electricity consumption due to the heating of a dozen households during a fall week (see Figure 3.15).
- The distributed generation scenario is based on the load factor profile for a fall week, with a generation capacity large enough to cover the electricity consumption during off-peak periods (see Figure 3.17).

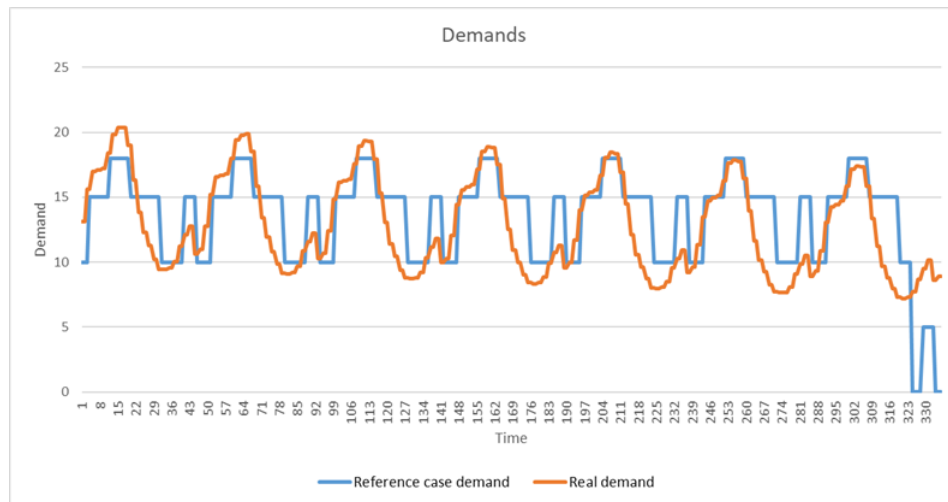


Figure 3.15: The energy consumption retrieved from industrial data, and implied by the devices if they are used during the first time slots of their associated time window.

The sensitivity analysis is based on the perturbation of the following parameters:

- DG scenarios: we consider instances with respectively 0, 0.5 and 1.5 times the generation considered in the base instance (see Figure 3.17).



Figure 3.16: The spot prices retrieved from industrial data, and the spot prices for the instance comprising high peak prices.

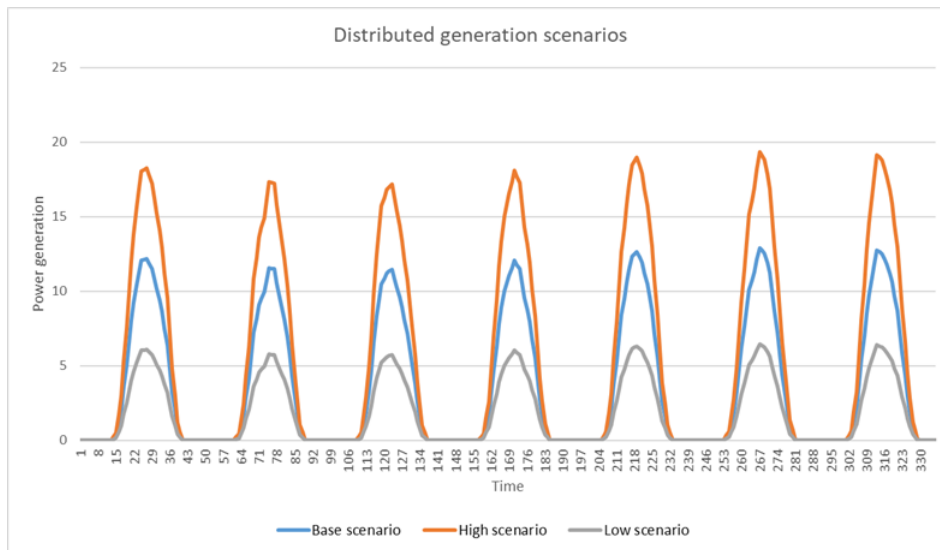


Figure 3.17: The distributed generations scenarios, with factors 0.5, 1 and 1.5.

- Inconvenience coefficients: three different instances where the devices have associated inconvenience factors equal to respectively 0, 0.5 and 1.5 times the inconvenience factor in the base instance.
- Energy costs: one instance where the energy costs are higher during peak periods (see Figure 3.16), compared to the base instance,
- Battery sizes: three instances with respectively a small, a large, and no storage

capacity.

- Time window sizes: one instance with larger time windows, one instance with narrower time windows.

This yields in total thirteen instances of (SBPP) to solve.

Note that only the instance with zero inconvenience is solved to optimality within the time limit of 1200 seconds. For the other ones, the optimality gaps range from 0.3% (instance with low inconvenience cost) to 1.9% (instance with larger time windows). The behavior of the solver CPLEX is illustrated in Figure 3.18 for the base instance. The solver is able to find a relatively good solution early in the process, but fails to close the optimality gap subsequently.

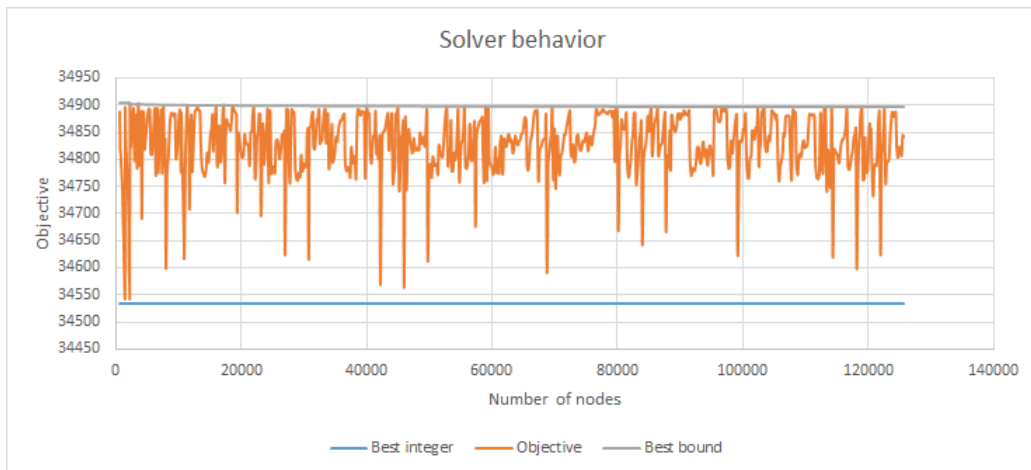


Figure 3.18: The solver’s behavior on the base instance.

The numbers of variables and constraints of the MIP do not only depend on the number of time slots and the number of devices, but also on the length of the time windows and their position in the time horizon. A time window located in the first time period will generate more nonanticipativity constraints than a time window located near the end of the time horizon. As an indication, the base instance involves 35530 constraints, 33745 continuous variables and 1157 binary variables, which are reduced to respectively 3356, 2290 and 1061 with CPLEX’ presolve process.

3.3.2.2 Sensitivity results with respect to the DG scenarios

The first parameter to be analyzed is the DG scenario. Four such scenarios are considered: 0, 0.5, 1, and 1.5 times the base DG scenario, as represented in Figure 3.17. All the energy coming from DG is either stored or directly consumed, as

can be seen in Figure 3.19 for the base scenario. This behavior results from the fact that DG is considered as free for the follower. It follows directly that if the distributed generation increases, the amount of energy bought from the grid will decrease accordingly. This affects the leader's profits (see Figure 3.20). As expected, the highest profit for the leader occurs when there is no distributed generation.

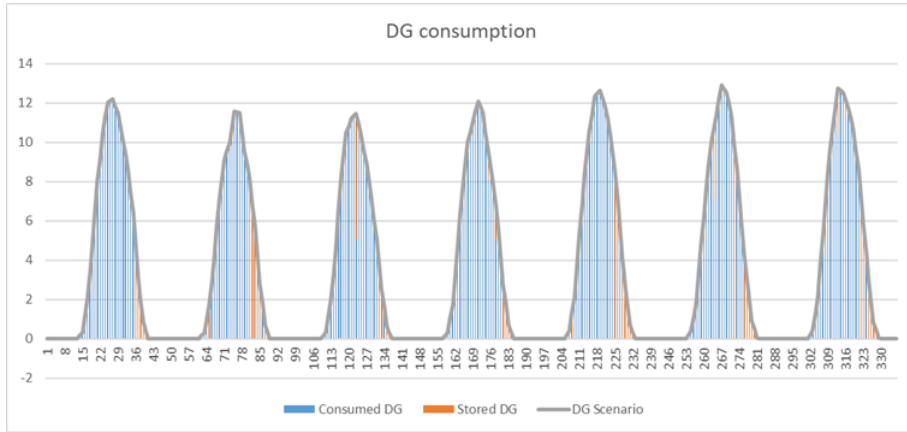


Figure 3.19: The DG consumption for the base instance.

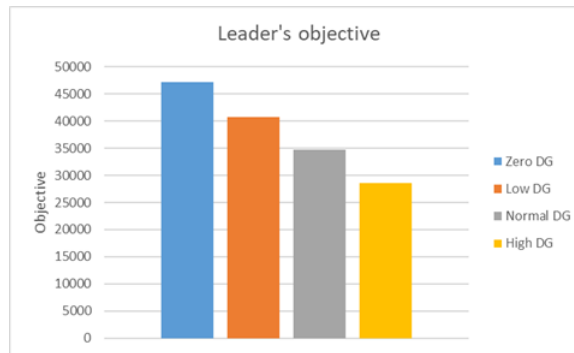


Figure 3.20: The leader's objective values for the instances with various DG scenarios.

As for the demand, the fact that the follower's problem is linear induces an "all or nothing" situation: if for time slots $h_1 < h_2$, the generalized cost of powering a device (n, a) during h_2 is smaller than during h_1 (i.e. $\min\{p^{h_1}, \bar{p}^{h_1}\} + C_{(n,a)}^{h_1} > \min\{p^{h_2}, \bar{p}^{h_2}\} + C_{(n,a)}^{h_2}$), then all the energy that would be consumed during h_1 in the reference case will be shifted to h_2 . This explains the huge variations in the follower's demand (e.g. from 0 during time slot 23 to 30 during the next time slot) for the whole horizon represented in the top graph of Figure 3.21. The bottom

graph of Figure 3.21 illustrates that power coming from the DG is totally used: smooth curves following the DG curve can be observed between time 16 and 29, especially on the curve illustrating the demand with a high DG scenario.

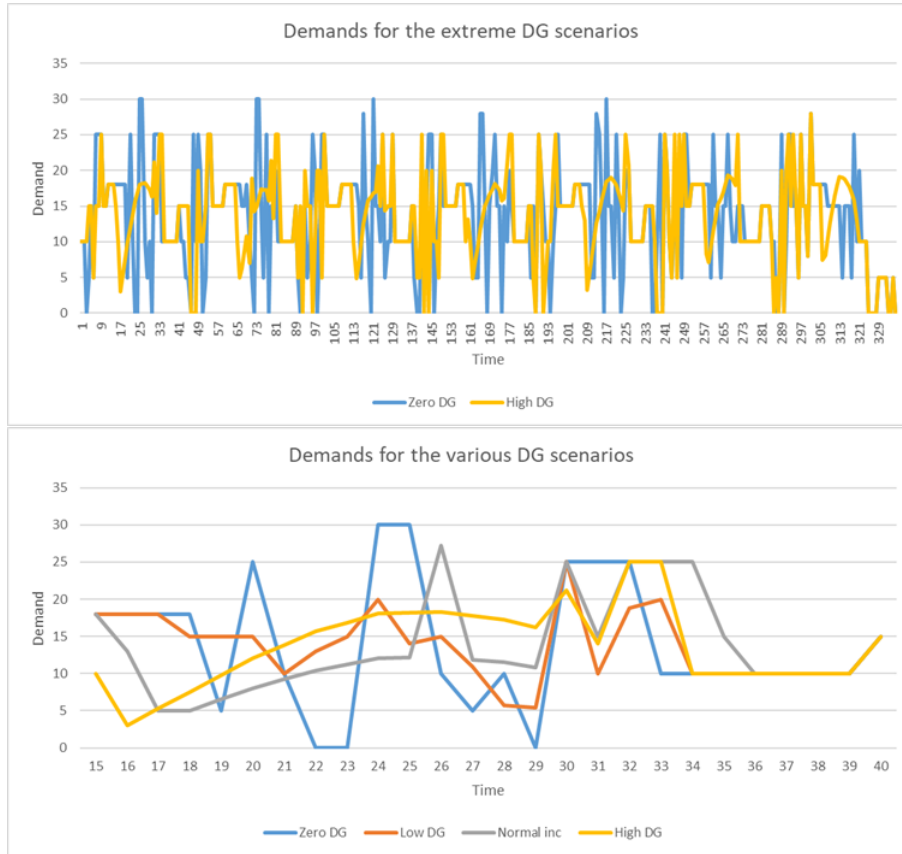


Figure 3.21: The follower's demands for the instances with extreme DG scenarios for the whole time horizon (top) and with all various DG scenarios for the time slots 15 to 40 (bottom).

3.3.2.3 Sensitivity results with respect to the inconvenience coefficients

Next, the influence of the inconvenience coefficients is studied. For each device, these coefficients increase linearly with the time. In this case study, the difference between the coefficients of two consecutive time slots is assumed to be the same for all devices. Four different values are tested for the difference: 0, meaning that the customers do not undergo inconvenience due to a potential load shift, 0.03125 (low inconvenience), 0.0625 (normal inconvenience), and 0.09375 (high inconvenience). Unsurprisingly, the leader's profit decreases as the inconvenience coefficients grow

(see Figure 3.22): if the consumers are more willing to shift their loads, less incentive is necessary to induce the same shift, and thus more profit is achievable for the leader.

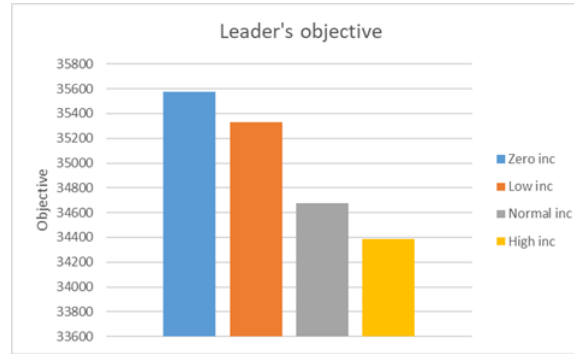


Figure 3.22: The leader's objective values for the instances with various inconvenience coefficients.

The latter fact can be easily observed in Figure 3.23. The optimal leader price profile when there is no inconvenience is constant at the level of the competitor's prices, because no incentive is required to induce a load shift. At the contrary, inducing load shifts for high inconvenience values is difficult. Prices resulting in a load shift despite a high inconvenience are clearly lower than prices achieving a similar load shift with a low inconvenience. This is conspicuous for example on time slots 33 or 273. For other time slots, the inconvenience is too high to induce shift: around time slot 8 or 249 for instance. The producer lowers his prices during these periods only for the instances with low inconvenience values.

Finally, let us observe that the prices tend to decrease linearly as the supplier induces delays. This can clearly be observed in the bottom graph of Figure 3.23: for example, the optimal prices for the instance with high inconvenience linearly decrease between time slots 11 and 19, with the noticeable exceptions constituted by time slots 13 and 18. The explanation lies in the optimistic assumption of the problem: when the follower gets his energy at the same generalized cost at two different time slots, he consumes during the time slot that is most advantageous for the leader. When the prices decrease linearly over a given period, they compensate the inconvenience costs that grow linearly with the delay, so that the generalized cost remains the same over the period.

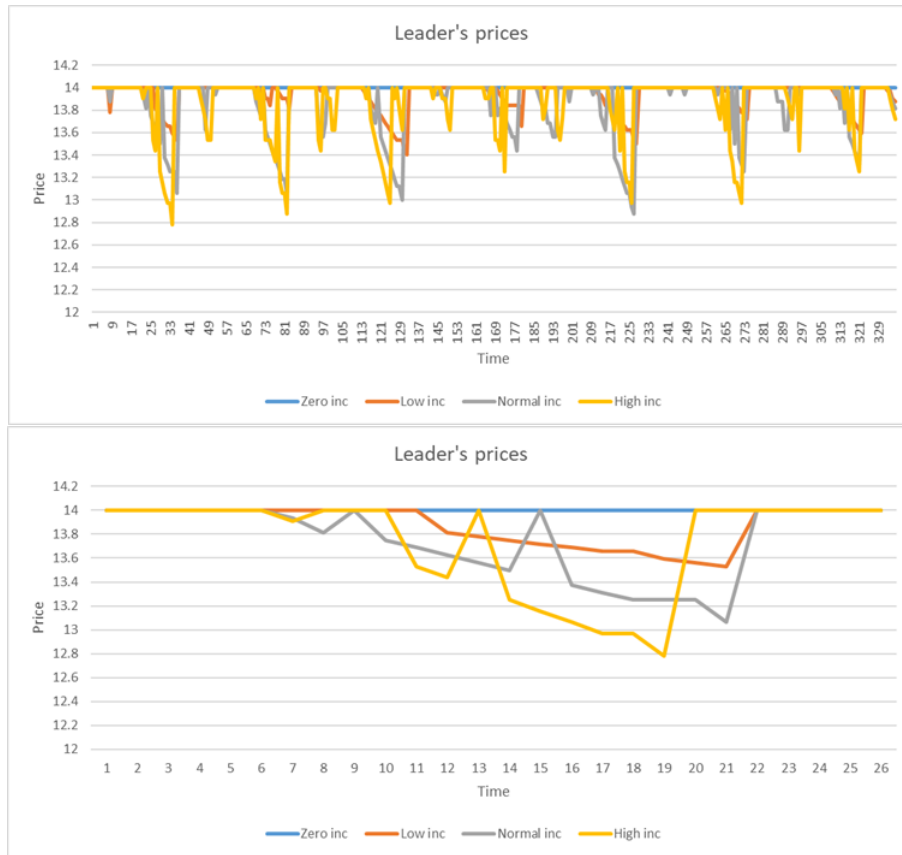


Figure 3.23: The leader price profiles for the instances with various inconvenience coefficients on the whole time horizon (top) and for the period between time slots 15 and 40 (bottom).

3.3.2.4 Sensitivity results with respect to the market prices

The spot market prices have a strong influence on the model. Indeed, they determine which time slots are the most profitable for the leader, or at the contrary which time slots are the least valuable. To study their influence, we consider an instance where the spot market prices (energy costs) are 20% higher during peak periods (e.g. periods where the market prices are the highest, see Figure 3.16). The incentive for the leader to induce a load shift from peak to off-peak periods is thus more substantial.

Like in the two previous examples, it is not surprising that the leader's optimal value is lower when the spot prices are higher, as it can be observed in Figure 3.24. Observe however that the difference is not as significant as when the DG scenario varies: whereas differences in the leader's optimal value can reach several

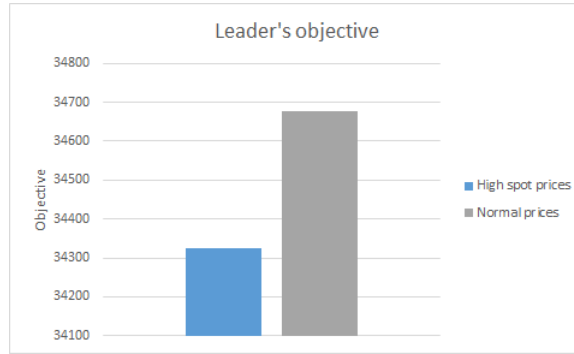


Figure 3.24: The leader's objective values for the instances with various spot market prices.

thousands, it is limited to 353 in the case of higher market prices. This value is in the same order of magnitude as the differences between the instance with various inconvenience coefficients (939 between the instance with low inconvenience and the instance with high inconvenience).

In Figure 3.25, observe that the demand curve differ for the high market prices instance and the base instance: the differences occur almost always during the peak periods, i.e. when the market prices in both considered instances are different. The higher need to shift load from peak to off-peak periods becomes clear while having a look at the optimal price profiles. The optimal prices for the high market prices instance are indeed in general lower than the optimal prices of the base instance, e.g. during time slots 25 and 170.

3.3.2.5 Sensitivity results with respect to the battery size

The size of the storage capacity owned by the follower has some influence as well. Clearly, storage capacities are filled when electricity prices are low, and the stored energy is consumed when prices are higher. One might think that a greater storage capacity represents an advantage for the follower. However, it turns out that a greater capacity is especially advantageous for the leader, as the leader's objective values indicate in Figure 3.26.

To illustrate this counter-intuitive fact, let us consider an example with a single device with $E = 2$, $\beta^{\max} = 1$, and associated time window $T = \{1, 2\}$. Moreover, let the renewable energy production be $\lambda_{max}^1 = 1$ and $\lambda_{max}^2 = 0$. The prices offered by the competitor for an energy unit is the same for both time slots: $\bar{p}^1 = \bar{p}^2 = 1$, whereas the energy costs differ: $c^1 = 0.6$ and $c^2 = 0.4$. The optimal price profile for the leader obviously consists in setting $p^1 = p^2 = 1$. The whole situation is

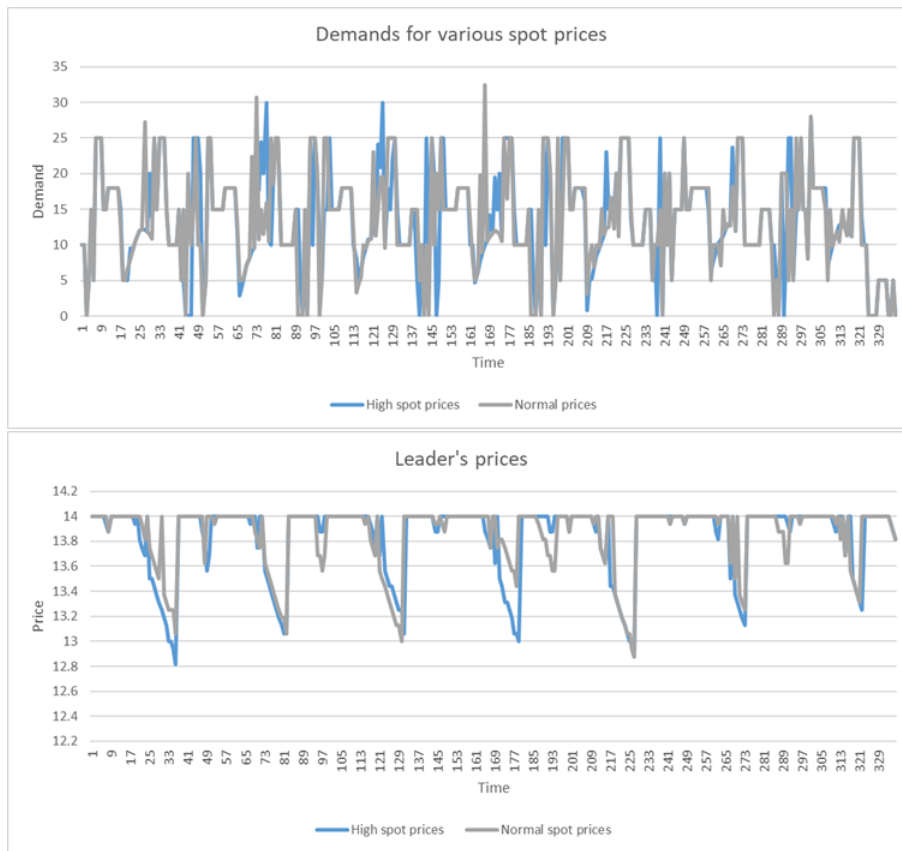


Figure 3.25: The follower’s demands and the leader price profiles for the instances with various spot market prices.

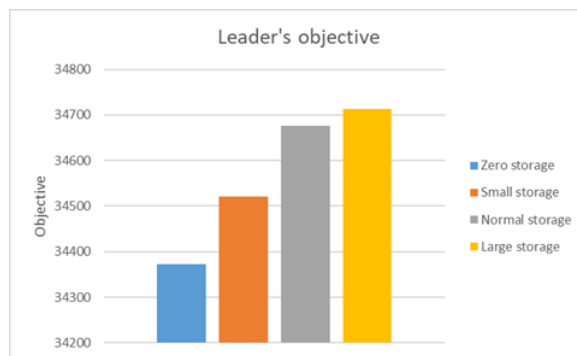


Figure 3.26: The leader’s objective values for the instances with various storage sizes.

summarized in Figure 3.27.

Let us consider two cases: in the first one, the capacity of the battery is equal

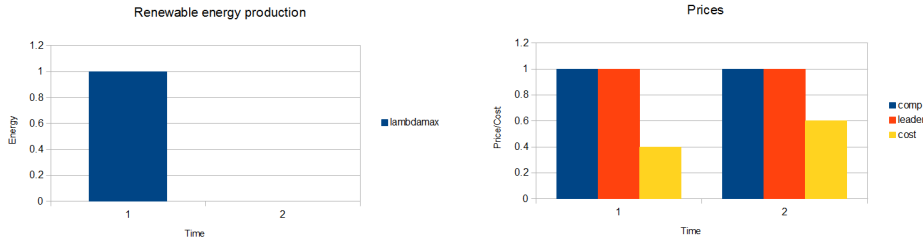


Figure 3.27: The set up of the example.

to zero, whereas in the second one, the battery can store one energy unit, with $\rho^c = 1$. It turns out that in the first case, all the renewable energy is consumed during the first time slot. The follower buys one energy unit from the leader during the second time slot at a price equal to 1, which generates a profit equal to 0.4 for the leader. In the second case, the renewable energy produced during the first time slot is stored into the battery and consumed during the second time slot. The follower then buys energy from the leader during the first time slot, when energy is cheaper on the market (remember that we are in an optimistic approach). The profit of the leader in the second case is 0.6. In conclusion, a larger storage capacity implying more freedom for the follower can imply a higher profit for the leader.

In Figure 3.28, the battery states are shown for the four considered storage sizes. The origin of the stored energy is indicated for the instance with the largest storage capacity. Observe that most of the stored energy is actually bought from the grid, instead of being taken from the DG. As for the demand curves, the "all or nothing" nature of linear programming is clearly visible, since the battery often oscillates between being full and being empty. These oscillations are particularly visible for the small battery instance during time slots 97 to 109 (Figure 3.28, bottom graph).

3.3.2.6 Sensitivity results with respect to the time windows sizes

Finally, the last parameters that have to be considered are the sizes of the time windows. Whereas the length of the time windows varies between 1.8 and 2.5 times the number of time slots needed to power each appliance in the base instance, these factors are in a range from 1.4 to 2 in the instance with narrower time windows, and between 2.14 and 3 in the instance with longer time windows. Figure 3.29 indicates that the normal instance brings the best results from the leader's point of view. However, the difference of profit for the leader is rather small (34677 with normal time windows, 34668 with narrow time windows and 34532 for longer time windows).

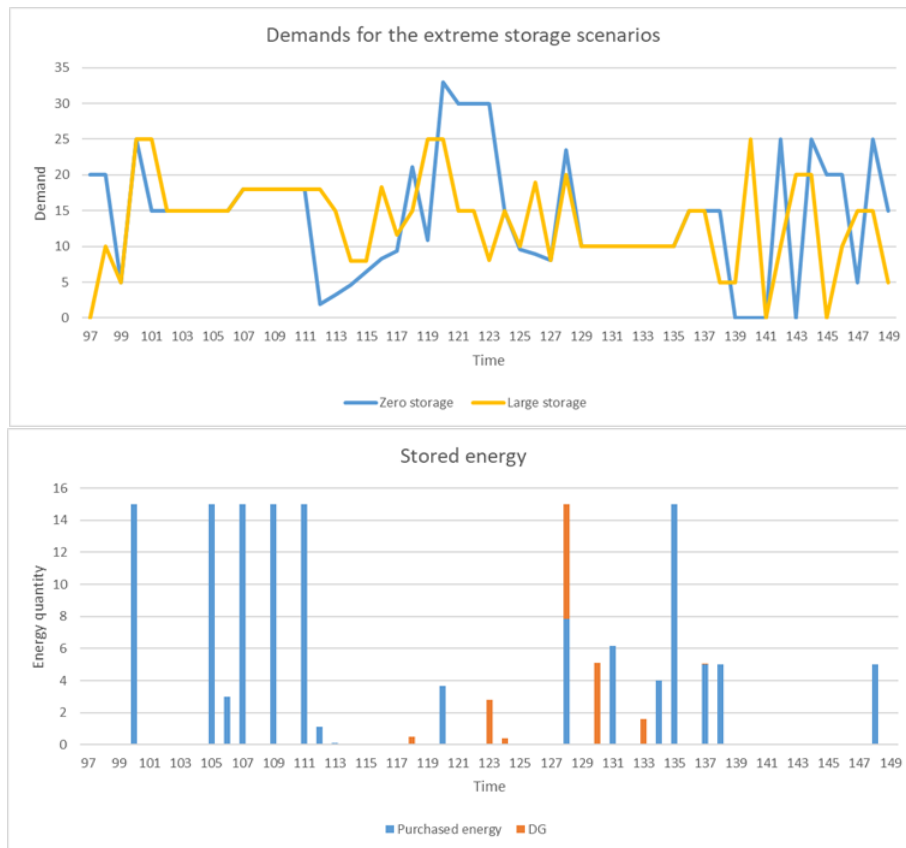


Figure 3.28: The battery states for the instances with various storage sizes and the origin of the stored energy for the large storage instance, for time slots 97 to 149.

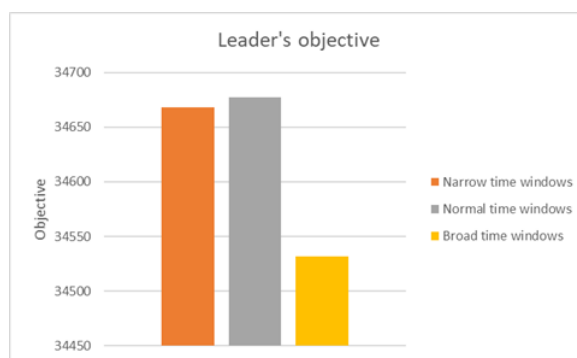


Figure 3.29: The leader's objective values for the instances with various time windows sizes.

3.3.2.7 Impact of the bilevel pricing model on the objective values

In this part, we consider the objective values of the leader and the follower. In order to quantify the impact of the bilevel model, we compare the so-called *reference case*, in opposition with the *optimized case*, which consists in the optimal solution of (SBPP) (found by the commercial solver). The reference case is defined as follows: first, the leader sets his prices at the same level as the competitor. Then the follower determines the schedule of the devices in the most convenient way: each device (n, a) is powered at its maximum power $\beta_{(n,a)}^{\max}$ during the first hours of the related time slots until $E_{(n,a)}$ is reached, then the device is stopped. To power the device, the follower makes DG his first choice, then energy from the battery, and finally energy from the leader. If at a given time slot, the DG exceeds the demand, the remaining energy is stored in the battery, unless the battery is full. The energy demand of the follower in the reference case is depicted in Figure 3.15.

The solutions of (SBPP) and the reference case are compared on the thirteen instances defined in Section 3.3.2.1. The objective values of the leader are given in Table 3.1. With the exception of the instance with high DG, the leader's profit is always higher in the optimized case than in the reference case. Even though the difference in profit is rather small (between 0.33% and 4.63%), it is not negligible. Unsurprisingly, the largest difference between the profits in the reference case and in the optimized case lies in the instance where the follower does not face inconvenience. As argued in Section 3.3.2.3, no incentive is required to make the follower react in the most favorable way for the leader. Therefore, 4.63% is the best possible increase for the leader's profit, in the case where the inconvenience factors vary. Concerning the instance with higher peak energy costs, the difference in profit is significant as well, with a difference equal to 3.26%. The energy cost differences indeed make load shifts more advantageous for the leader than in the instance with normal inconvenience. Finally, for the example with high DG resulting in a lower leader profit, the commercial solver has most probably found a local optimum that was not global: CPLEX indeed does not guarantee the global optimality of the solutions it returns, it might thus keep stuck on local optima.

On the follower's side, even though (SBPP) is not a zero-sum game, there is a clear relation between the follower's and the leader's objectives. The billing cost of the follower constitutes the revenues of the leader. Therefore, intuitively, the leader's optimized prices are going to induce a follower's reaction that will bring approximately the same objective value as in the reference case. Of course, the follower's optimal objective value in the reference case constitutes an upper bound for the follower's optimal value associated with any leader price profile, since the consumption schedule of the reference case is always feasible. The results for the

Instance	Ref. obj.	Opt. obj.	% diff.
Base	34172.68	34676.53	1.47
Zero bat.	34261.4	34372.9	0.33
Small bat.	34201.32	34521.54	0.94
Large bat.	34182.14	34714.16	1.56
Zero inc.	34172.68	35753.54	4.63
Low inc.	34172.68	35385.64	3.55
High inc.	34172.68	34389.21	0.63
Zero DG	46597.9	47101.38	1.08
Low DG	40390.54	40721.06	0.82
High DG	29084.13	28611.95	-1.62
Spot	33240.14	34323.98	3.26
Small TW	34172.68	34668.36	1.45
Large TW	34172.68	34531.8	1.05

Table 3.1: The leader's objective values in the reference case, in the optimized case, and the difference in percentage.

follower's optimal value are illustrated in Table 3.2. In the table, "bat." stands for battery, "inc." for inconvenience, "Spot" for the instance with higher market prices and "TW" for the instances where the sizes of the time windows vary. "BC ref" represents the billing cost of the follower in the reference case, "IC ref" the inconvenience cost in the reference case, "BC opt" the billing cost when the prices of the leader are optimized, and "IC opt" the inconvenience cost when the prices of the leader are optimized. Finally, % BC, % IC and % GC show the difference (in percentage) between the reference case and the optimized case of the billing cost, the inconvenience cost and the generalized cost respectively (e.g. % BC = $100 \cdot \text{BC opt} / \text{BC ref}$).

All follower's generalized costs are smaller in the optimized case than in the reference case, except for the low inconvenience instance, which is probably due to the commercial solver finding a local optimum instead of a global one. Moreover, the percentages are all very close to 100%, the furthest being the instance with large time windows (98.89%), which confirms the above-mentioned intuition. Addition-

Instance	BC ref	IC ref	BC opt	IC opt	% BC	% IC	% GC
Base	46573.2	525.13	46025.7	877.31	98.82	167.07	99.59
Zero bat.	46659.2	525.13	45964.7	856.52	98.51	163.11	99.23
Small bat.	46589.2	525.13	46246.8	737.52	99.27	140.45	99.72
Large bat.	46573.2	525.13	46379.4	646.92	99.58	123.19	99.84
Zero inc.	46573.2	0	46573.2	0	100	—	100
Low inc.	46573.2	262.56	46343.7	540.29	99.51	205.77	100.1
High inc.	46573.2	787.69	46052.1	1047.97	98.88	133.04	99.45
Zero DG	63882	525.13	63737.8	655.5	99.77	124.83	99.98
Low DG	55227.6	525.13	54784.7	712.29	99.2	135.64	99.54
High DG	39317.2	525.13	37755.5	786.96	96.03	149.86	96.74
Spot	46573.2	525.13	46002.4	905.36	98.77	172.41	99.6
Small TW	46573.2	525.13	46383.4	681.32	99.59	129.75	99.93
Large TW	46573.2	525.13	45543.8	1031.83	97.79	196.49	98.89

Table 3.2: The follower’s objectives.

ally, the billing cost is always smaller in the optimized case, but the inconvenience cost increases. This is due to the fact that in the reference case, the follower actually aims to minimize his inconvenience, since the leader’s prices in the reference case are the same for all time slots.

Note that in the reference case, the parameters influencing the billing cost are the size of the battery and the DG, whereas only the inconvenience coefficients induce a change in the inconvenience cost.

3.3.3 Several scenarios

For multiple scenarios, the instance of Section 3.3.2.1 is too large to be considered. Even with a small number of base scenarios (e.g. 3) and a small number of time periods (e.g. 7, one per day), the number of potential scenarios is significant: $3^7 = 2187$. As (SBPP) involves a set of variables per scenario and a lot of nonanticipativity constraints, the single-level formulation of (SBPP) leads to a large MIP that cannot be solved efficiently to optimality. Therefore, smaller instances are generated in Section 3.3.3.1, and the associated linear MIPs are solved using

CPLEX.

3.3.3.1 Instance generation

The test instances are created by a random instance generator. The input parameters of the generator are:

- The number of time slots H : either $H = 6$ or 12 ,
- The number of clients N : $N = 1$,
- The total number of appliances $|A_n|$: 5, 10 or 20,
- The number of time periods (thus the number of scenarios), \mathcal{H} being uniformly distributed among the time periods: 1, 2, 3 or 4 periods, which induce respectively 2, 4, 8 and 16 scenarios.

The output of the instance generator consists in the following data:

- For each scenario, the occurring probability $P[\sigma]$ is a random number between 1 and 100 that is normalized, so that the occurring probabilities of all scenarios sum up to 1,
- The competitor's prices \bar{p}^h are randomly chosen between 0.1 and 0.2 €/kWh,
- The energy costs k^h are randomly chosen between 40 and 70 €/MWh, which are typical values for electricity prices on the SPOT market,
- The first time slot of each time window $T_{(n,a)}^{\text{first}}$ is randomly chosen among \mathcal{H} , and $T_{(n,a)}^{\text{last}}$ is randomly chosen between $H - T_{(n,a)}^{\text{first}}$ and H ,
- The rated power of each appliance $\beta_{(n,a)}^{\text{max}}$ is randomly chosen between 500W and 3000W, covering a broad range of electrical devices, from air conditioning to washing machines,
- The amount of required energy ranges between $\beta_{(n,a)}^{\text{max}}$ and $(T_{(n,a)}^{\text{last}} - T_{(n,a)}^{\text{first}}) \cdot \beta_{(n,a)}^{\text{max}}$, in order to ensure that the problem is feasible,
- The two base DG scenarios are generated as follows: the production in the high scenario at each time slot is chosen between 0 and 2.31 kWh, whereas the production in the low scenario at each time slot is set between 0.2 and 1 time the production in the high scenario. The maximum energy production is based on the specifications of a typical photovoltaic panel, with a nominal capacity of 2.31 kWp (kiloWatt-peak),

- The storage capacity is not randomly determined. When all the other parameters have been fixed, nine instances are created with various storage capacities and starting states: the storage capacity S^{\max} can take the values 2.3, 4.6 or 9.2 kWh, and S^{start} can be $(k/2)S^{\max}$, with $k = 0, 1, 2$.

The randomly generated variables all follow a uniform distribution.

There are 3 possible numbers of appliances, 2 time horizon sizes, and 4 numbers of time periods, yielding 24 combinations. For each of those combinations, five instances are created. In conclusion, 1080 instances are created.

3.3.3.2 Bounds

In a stochastic optimization problem, the aim is to optimize the expected value over all scenarios of the objective function for a single scenario. This guarantees that the solution obtained with the stochastic optimization problem performs well in average, but not necessarily in all cases. Several values were developed to evaluate the quality of a solution obtained through a stochastic optimization process. The two most common values are the *value of the stochastic solution* (VSS) and the *expected value of perfect information* (EVPI). For formal definitions of these values, see [Birge 2011].

In single-level optimization, the EVPI measures the maximum amount a decision maker would be ready to pay in return for complete (and accurate) information about the future. It corresponds to the difference between two values, the so-called *wait-and-see* (WS) and *here-and-now* (STO) solutions. The here-and-now solution is the objective value of the considered stochastic problem (the Recourse Problem in [Birge 2011]), whereas the wait-and-see solution is the expected value of the optimal solution, namely the weighted sum of the optimal solution for each of the scenarios.

The VSS measures the performance of the stochastic approach compared to first-stage decisions made according to an average scenario. First, the average scenario, which is composed of the expected values of each of the random variables, is computed. Then, first stage decision variables are made in accordance with the average scenario: this gives the *expected value solution*. The second stage variables are then optimized for each scenario, and the expected value of their corresponding objective function yields the *expected result of using the expected value solution* (EEV). The difference between the here-and-now solution and the expected result of using the expected value solution is the VSS.

For a single-level convex maximization program, we have

$$EEV \leq STO \leq WS.$$

Bilevel optimization has a lot in common with stochastic optimization, more particularly with multi-stage decision processes. For (SBPP), the leader price profile can be seen as the first stage decision, whereas the second stage decision is the follower's response. The EVPI and the VSS are thus adapted to the bilevel context of (SBPP) as follows:

- To compute the EVPI, the wait-and-see solution must be determined. There are two ways to achieve this. The first one consists in solving $|\Sigma|$ versions of (SBPP) where the scenario tree is composed of a single scenario. This gives the optimal solution for each scenario, and it is enough to take the expected value of the objectives to find WS . In the other one, it is enough to solve (SBPP) with two modifications: first, one price profile is associated to each scenario, and second, all nonanticipativity constraints are removed: this directly yields WS . The difference between STO and WS is the EVPI.
- Four steps are necessary to compute the VSS:
 1. Compute the average scenario of DG production $\bar{\lambda}_{max}$: for each time slot $h \in \mathcal{H}$,

$$\bar{\lambda}_{max}^h = \frac{1}{|\Sigma|} \sum_{\sigma \in \Sigma} P[\sigma] \cdot \lambda_{max}^{h,\sigma}.$$
 2. Solve (SBPP) considering $\bar{\lambda}_{max}$ as the only possible scenario. The price profile \mathbf{p}^{avg} determined in this process is the expected value solution.
 3. Solve (SBPP) on the full scenario tree, with the prices fixed to \mathbf{p}^{avg} . The optimal value of this problem is EEV .

The VSS is the difference between STO and EEV .

Observe that the VSS is not absolutely well-defined here: the expected value problem could have several (first stage) optimal solutions, for which EEV could vary. As an example, consider a simplified instance of (SBPP), where there is no battery and no energy costs, a single device with $\beta^{\max} = 1$ has to be powered with one energy unit during the unique time slot of the horizon. Two scenarios are possible: either the sun is shining, which allows a high solar power production of two energy units, or it rains and no solar power can be produced. Both scenarios have one chance out of two to happen. The average scenario in this case is a production of one energy unit. Thus, the follower can power his device with renewable energy only, and the price offered by the leader does not matter, as long as it is positive. Assume that the competitor's price is 1€ per energy unit, which acts as an upper bound to the leader's price. Every $p \in [0, 1]$ is an acceptable expected value solutions.

However, it is clear that the VSS will vary depending on the chosen value for p . In the sunny scenario, the follower will always power his device with solar power, but in the rainy scenario, he has no choice but to buy energy from the leader (when $p \leq 1$) or from the competitor (when $p > 1$). Therefore, the expected value of using the expected value solution is clearly $P[\text{sunny}] \cdot 0 + P[\text{rainy}] \cdot p = 0.5 \cdot p$. It is also clear that in this example, the stochastic problem yields an optimal leader price of 1, and an objective value of 0.5. Therefore, the value of the stochastic solution can take any value in $[0, 0.5]$ depending on the choice of p .

The consequence of the reasoning above is that the concept of VSS is not uniquely defined, and depends on the choice of the first stage decisions on the average scenario. A better definition would imply taking the supremum of all possible values for the VSS, but computing such a bound might be annoyingly difficult.

3.3.3.3 Numerical results

Out of the 1080 considered instances, 911 are solved to optimality by CPLEX within the time limit of 1000 seconds. For the remaining 169 instances, the solver is able to find a feasible solution, but a gap is always remaining. The values for the gap range from 0% to 23% for all instances but three. For these three instances, the solver is unable to find a solution close to the optimum, as the leader's objective is negative at the end of the computation. The gap to optimality in those cases is about 238%.

Among the 169 instances that are not solved to optimality,

- 60 have a time horizon of 6 time slots, 109 of 12 time slots,
- 18 instances have 5 devices, 25 have 10 devices, and 126 have 20 devices,
- 35 instances have a scenario tree with two time periods, 49 with three, and 86 with four time periods.

Furthermore, among the 26 instances with a gap to optimality larger than 2%, only 2 involve a time horizon of 6 time slots, and both of them involve 20 devices and 4 time periods. There is therefore a clear and unsurprising tendency: the bigger the problem, the more difficulties the solver encounters. Note that the three instances for which the solver cannot find any good solution are the largest considered instances (12 time slots, 20 devices and 4 time periods in the scenario tree). Figure 3.30 shows clearly that a large number of instances need few time to be solved to optimality.

Concerning the VSS and the EVPI, let us first observe that for one time period and thus one scenario, the VSS and the EVPI are both equal to 0, since the wait-

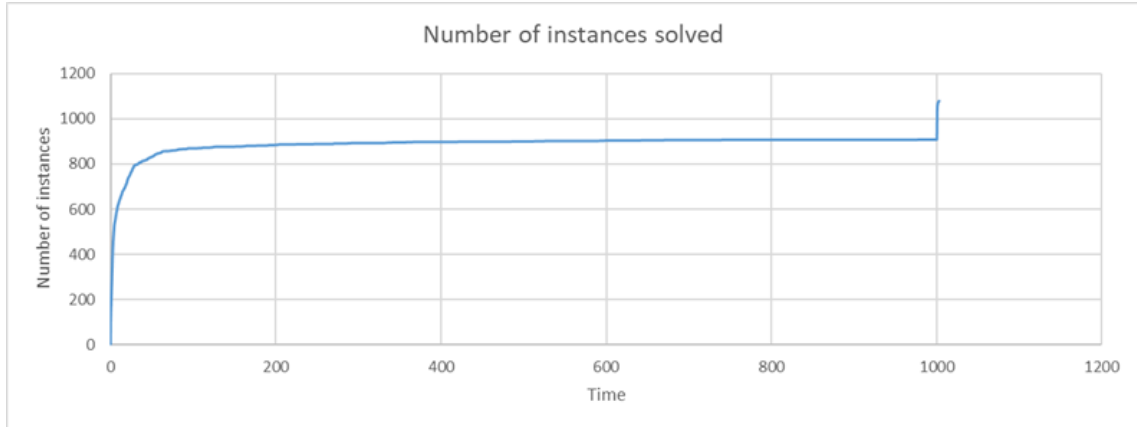


Figure 3.30: The number of solved instances per time.

and-see solution, the here-and-now solution and the expected value solution are identical. Therefore, we focus on the instances with several time periods, and thus several scenarios.

The values for the VSS range from 0 to 6.48. Considering the proportional difference in percent between the EEV and the here-and-now solution is more revealing: the improvement obtained by solving (SBPP) instead of considering the average scenario ranges from 0% to 1161%, which means that the expected benefit can be multiplied by 12 just by optimizing the leader's prices in a sensible way.

Finally, we expect the EVPI to be positive: being aware of the perfect information should allow the leader to optimize his prices in a better way. However, the values of the EVPI range from -0.35 to 0.81 . These values represent between -39.23% and 14.05% of the wait-and-see solution. Actually, in 596 out of the 810 instances with several scenarios, the EVPI is negative. It is positive for 93 instances, and equal to 0 for the 121 remaining instances. Three explanations are likely. First, bilevel problems are generally nonconvex, and thus do not necessarily meet the requirements for the EVPI to be positive. Then, the sometimes large MIP gaps can be a reasonable explanation. Finally, more intuitively, the perfect information is known not only by the leader, but also by the follower. This information can thus benefit the follower more than the leader.

In conclusion, the mixed performance of the commercial solver on relatively small instances is a strong argument for the further development of solution methods (exact algorithms or heuristics) based on the structure of the problem. Besides those computational considerations, the positive values of the VSS indicate that considering multiple scenarios is profitable for the leader and justify our bilevel

approach.

3.4 Rolling horizons

The previous section (in particular Subsection 3.3.3) outlines one particular drawback of the scenario tree-based approach: solving the MILP corresponding to an instance of (SBPP) is difficult and can be time-consuming. To avoid this difficulty, a rolling horizon method is applied. This method has been thoroughly studied in the field of stochastic optimization, see for example [Pironet 2014]. The idea behind rolling horizon methods is to solve many problems on short time periods in order to build a solution over the whole time horizon. The information obtained by solving the problem for a given time period is used in the definition of the problems for the subsequent time periods.

We first define the main parameters:

- The *time horizon* is the set $\mathcal{H} = \{0, 1, \dots, H\}$, where H can be infinite.
- The length of the rolling horizon l_{RH} is the number of time slots that will be considered for every computation of the rolling horizon method. Obviously, $l_{RH} \leq H$.
- The rolling horizon step s_{RH} . Every s_{RH} time slots, the data are actualized and the rolling horizon program is solved with respect to the new data set. There must be $s_{RH} \leq l_{RH}$.
- The length of the frozen horizon l_{FH} . In classical rolling horizon methods, this corresponds to the period of time for which decisions have already been made. More precisely, if a step of the computation is made at time $t \in \mathcal{H}$, then all the decisions for the time steps $t, \dots, t + l_{FH}$ are already fixed, since they were determined during the previous computations. In bilevel programming, the situation is slightly different: the decisions of the follower can be seen as recourses. In this case the leader has to fix its decisions in advance, whereas the follower chooses a schedule for its device but can adapt it at every step of the algorithm. So here, l_{FH} denotes the number of time periods for which the leader's prices are fixed. Therefore, the computation at time t consists in determining the leader's prices for the time slots $t + l_{FH} + 1, \dots, t + l_{RH}$. Of course, there must be $l_{RH} - l_{FH} \leq s_{RH}$.

See Figure 3.31 for a schematic view of the situation.

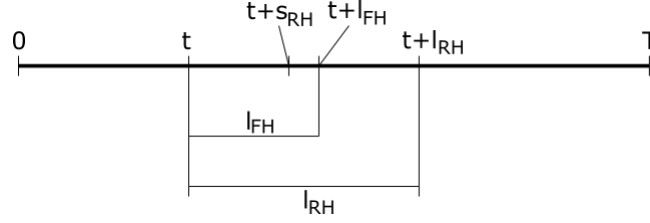


Figure 3.31: The time horizon with the various parameters of a rolling horizon method.

The instance comprising all the devices and all possible scenarios on the full time horizon \mathcal{H} is denoted by $P_{\mathcal{H}}$. The rolling horizon algorithm is given in Algorithm 1.

Algorithm 1 Rolling horizon algorithm

- 1: **procedure** RH($P_{\mathcal{H}}, l_{RH}, l_{FH}, s_{RH}$)
 - 2: $t \leftarrow 0$
 - 3: **repeat**
 - 4: Generate $P_{s_{RH}}^t$
 - 5: Solve $P_{s_{RH}}^t$
 - 6: Select a scenario σ
 - 7: Actualize data
 - 8: $t \leftarrow t + s_{RH}$
 - 9: **until** $t + l_{RH} \geq H$
 - 10: **return** data
 - 11: **end procedure**
-

In step 4, the subinstance $P_{s_{RH}}^t$ is generated. The time period considered in $P_{s_{RH}}^t$ is the time period $\{t, \dots, t + l_{RH}\}$. Therefore, the only devices in $P_{s_{RH}}^t$ are the devices (n, a) such that $T_{(n,a)} \cap \{t, \dots, t + l_{RH}\} \neq \emptyset$. Two cases may arise for a given device (n, a) :

- If $T_{(n,a)}^{\text{first}} < t$, then the device was already considered in the previous iterations of the algorithm. Hence it might have received some energy during the previous time slots: it results that the required energy has to be updated: the energy demand of (n, a) in $P_{s_{RH}}^t$ is denoted by $E_{(n,a)}^t$ and is worth $E_{(n,a)} - \sum_{h < t} (x_{(n,a)}^{h,\sigma} + \bar{x}_{(n,a)}^{h,\sigma} + s_{(n,a)}^{h,\sigma} + \lambda_{(n,a)}^{h,\sigma})$, with σ the scenario that has been selected at step 6.
- If $T_{(n,a)}^{\text{last}} > t + l_{RH}$, then $T_{(n,a)}$ partly belongs to the time period considered in $P_{s_{RH}}^t$. Several options are possible. Here, we choose to set

$E_{(n,a)}^t = \min \left\{ E_{(n,a)}, \left(t + l_{RH} - T_{(n,a)}^{\text{first}} \right) \beta_{(n,a)}^{\text{max}} \right\}$, meaning that the device (n, a) should consume as much energy as possible during the time period \mathcal{H}_t . Indeed, delaying the demand leads to inconvenience for the follower, thus powering the device as early as possible minimizes the inconvenience.

The instances $P_{\mathcal{H}}$ and $P_{s_{RH}}^t$ for all $t \in \mathcal{H}$ all involve a scenario tree. Here, we assume that the scenario tree of $P_{\mathcal{H}}$ is a complete tree with time periods of length s_{RH} . Therefore, one of the base scenarios is selected at every iteration of step 6: the scenario selected at time t is the scenario that comes to reality for the time slots $t, \dots, t + s_{RH} - 1$. However, considering a complete tree on \mathcal{H}_t can be difficult, even if \mathcal{H}_t is small, as shown in Section 3.3.3. Therefore, the scenario tree of $P_{s_{RH}}^t$ is assumed to only comprise the base scenarios. Furthermore, we assume that the probability to choose one of the scenarios only depends on the scenario that was selected at the previous iteration, as in a Markov process.

In step 5, the problem solved is the MIP defined in Section 3.3, to which we add constraints ensuring that the prices belonging to the frozen horizon are indeed the prices that have been determined in the previous iterations.

In step 7, the added information consist in the choice made for the scenario in step 6, the follower's decisions associated to the chosen scenario for the time slots t to $t + s_{RH} - 1$ and the leader's decision for the frozen horizon (except for the last iteration of the algorithm, where the leader's and follower's decisions are saved until the end of the horizon). At the end of the rolling horizon method, the data thus contains one scenario and the associated decisions for the whole time horizon \mathcal{H} .

To evaluate the quality of the results defined by the rolling horizon approach, we consider two cases that are obtained a posteriori: the reference case for the realized scenario (see Subsection 3.3.2.7) and the wait-and-see solution (see Subsection 3.3.3.2), which corresponds to the optimized case. The difference between the leader's performance with the rolling horizon and the leader's performance in the wait-and-see solution yields the *value of perfect information* (VPI) on this particular instance. Of course, the VPI can only be computed if the general instance is not too large. Indeed, even if there is only a single scenario, the number of devices and/or hours can cause many computational troubles. Computing the expected value of the VPI over all scenarios is intractable because of the number of scenarios in the scenario tree of $P_{\mathcal{H}}$.

3.4.1 Toy examples

Before considering numerical results over large instances, two toy examples are presented to illustrate the impact of the choice of some parameters.

- In the first example, a single device needs to be powered with one energy unit at time t (i.e. $E = 1$, $\beta^{\max} = 1$ and $TW = \{t\}$), with $t \geq 2$. There is a unique distributed generation scenario: one energy unit is produced at time 1, no energy is produced later on. Finally, there is a loss-less battery with capacity 1. The optimal solution for the follower consists in storing the energy that is produced during the first hour and to power his device with the stored energy, resulting in no energy sales of the leader. However, two cases are possible, depending on the chosen parameters:
 - In the first case, let us assume that l_{RH} is large enough, that is $l_{RH} \geq t - 1$. The device is thus part of the first subinstance P_{sRH}^1 considered by the rolling horizon method, which means that the follower knows that he will have to power a device in the future. Therefore, he stores the renewable energy produced during the first time slot, and the leader does not sell energy.
 - In the second case, let us consider that l_{RH} is too narrow, that is $l_{RH} < t - 1$. The follower does not know that he will have to power a device, since the device is not in the first subinstance considered by the rolling horizon method. Therefore, the follower has no incentive to store the renewable energy produced during the first time slot. As a result, the follower has to buy energy from either the leader or the competitor at time t (or possibly before to store it, as soon as the device enters the rolling horizon).

As a conclusion, it follows that the length of the rolling horizon plays a crucial role, not only for the leader but also for the follower, who could end up acting in a suboptimal way. It can furthermore be argued that the follower would automatically store as much of the produced renewable energy as possible: in that case, the rolling horizon method fails to adequately model the behavior of the follower.

- The second example considers again a single device, with $E = 1$, $\beta^{\max} = 1$, and $TW = \{1, \dots, 10\}$. There are no DG and no storage capacities. The energy costs and inconvenience coefficients, as well as the competitor's prices are illustrated in Figure 3.32. Clearly, the follower's answer consists in consuming one energy unit during a single time slot. Depending on the time

slot chosen by the follower, the leader's possible profit rank from 0.05 to 0.06 (with $p^h = \bar{p}^h - c^h$, so that all time slots have the same cost (energy price plus inconvenience) for the follower).

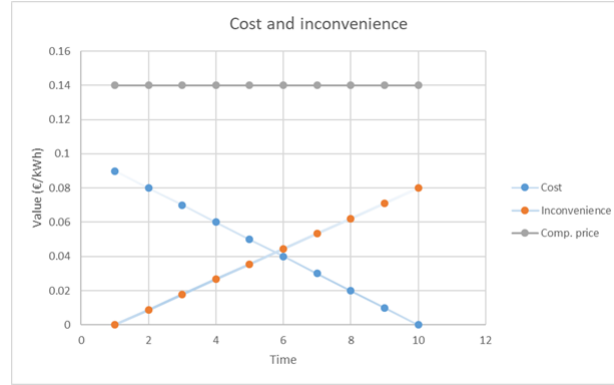


Figure 3.32: The energy costs, the inconvenience coefficients and the competitor's prices for the second toy example.

Here, the choices for l_{FH} , s_{RH} and l_{RH} are crucial. There are four possibilities:

- The reference case explained in Section 3.3.2.7 implies a consumption of one energy unit during the first time slot, with leader prices equal to the competitor's prices, which leads to a profit equal to 0.05 for the leader. The particular DG scenario allows the rolling horizon method to achieve this situation by setting $s_{RH} = 1$, $l_{FH} = 0$ and $l_{RH} = 1$.
- In the optimal case, the follower consumes one energy unit during time slot 10. The price paid for this energy unit is $p^{10} = 0.06$. This situation can be obtained with several "good" parameter sets (s_{RH}, l_{FH}, l_{RH}) , such as $(11, 0, 11)$, $(3, 2, 6)$, $(1, 1, 4)$, or $(1, 2, 4)$.
- Some parameter sets can be degenerate. For example, the parameter set $(3, 0, 3)$ leads the follower to consume one energy unit at time slot 2 at price $p^2 = 0.131$, which implies a profit equal to 0.051 for the leader. This case is degenerate, because each time slot is taken into account by a single iteration of the rolling horizon method.
- Finally, the last case occurs with the parameter set $(2, 0, 4)$ for example. The rolling horizon method with those parameters yields a consumption of one energy unit by the follower at time 10 at price $p^{10} = 0.104$. The induced profit for the leader amounts to 0.104, which is higher than the optimal case. This is due to the choice of the length of the frozen horizon: the leader announces prices to the follower at one iteration, but increases

them at the next iteration, since they are not part of the frozen horizon. Therefore, such cases should be avoided, since they put the leader in a position that is too favorable, and thus not acceptable for the follower.

It follows from this example that l_{FH} , s_{RH} and l_{RH} have to be chosen carefully. Typically, l_{RH} should be large enough to include as much information for the leader and for the follower as possible, but small enough for the subinstances to remain computationally tractable. The frozen horizon length l_{FH} has to be large enough: the follower has to know that the prices offered by the leader are trustworthy and are going to remain the same. Finally, a typical choice for s_{RH} is 1: new information is taken into account as soon as possible, which is supposed to improve the quality of the solution.

3.4.2 Tests on large instances

In this section, we conduct tests on the base instance presented in Section 3.3.2.1, and compare the results with the reference case and the optimized case. The rolling horizon step is set to $s_{RH} = 1$, the length of the frozen horizon to $l_{FH} = 6$, and the length of the rolling horizon to $l_{RH} = 12$. The scenario tree is a complete tree whose base scenarios are the three scenarios presented in Figure 3.17. In step 6 of Algorithm 1, the probability that the chosen scenario is the scenario selected at the previous iteration is 0.4. For either of the other scenarios, the probability is 0.3. For each iteration, the time limit of the solver is fixed to 150 seconds. The instance $P_{\mathcal{H}}$ comprising 336 time slots, the computation time required by CPLEX to get the result is large: between 23594 and 35511 seconds.

With these parameters, the rolling horizon method has been run five times. The results for the leader's objective value are given in Table 3.3, as well as the leader's objective values in the reference case and in the optimized case.

Considering the prices and demands obtained with the rolling horizon method compared with the prices and demands obtained with the *a posteriori* optimization, see Figure 3.33. The prices above 14 correspond to time slots where no energy is bought from the leader.

Except for the second run, the rolling horizon method always brings more profit to the leader than the optimal value that is computed *a posteriori*, even though the differences are small. This could be attractive for an energy supplier, but questions arise about the choice of the parameters. If the leader gets a profit higher than what is theoretically possible, it means that the follower buys energy at the wrong time and/or at the wrong price. This is probably due to the length of the frozen horizon $l_{FH} = 6$, which is small compared to the length of the rolling horizon $l_{RH} = 12$. At

Run	RH method	VPI	Ref case	Comp. time (s)
1	35092	34848	34348	35511
2	34452	34467	33918	23594
3	34048	34034	33603	23652
4	35207	35111	34750	24251
5	34530	34528	34053	24773

Table 3.3: The leader's objective values obtained with the rolling horizon method, with the optimized version on the realized scenario, and with the reference case method.

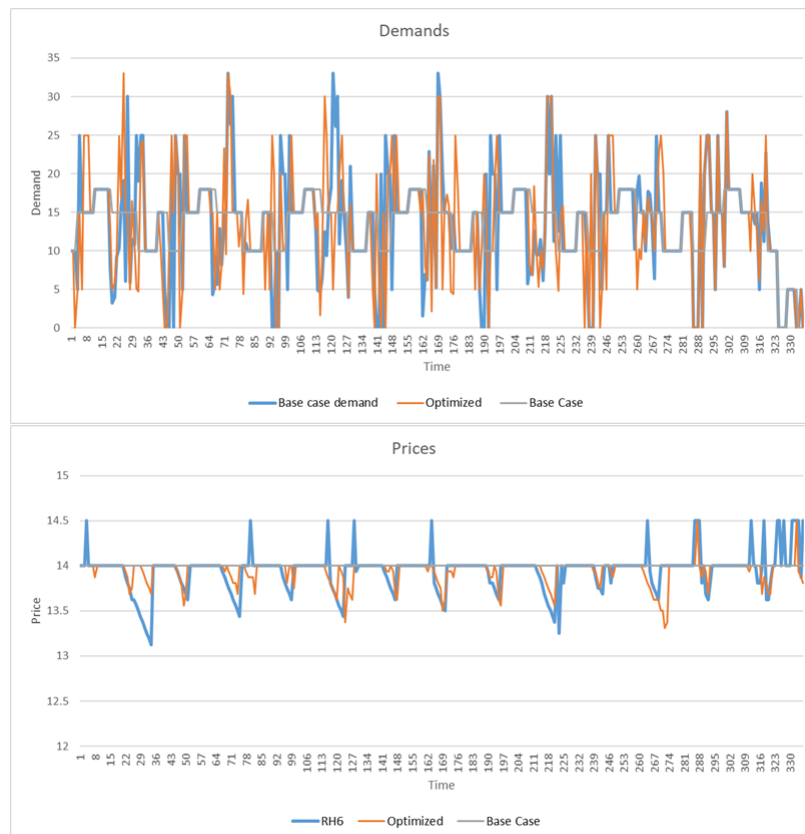


Figure 3.33: The prices and demands obtained with the first run of the rolling horizon method and with the VPI and reference case computations.

time t , the leader gives information about his prices for the next 12 time slots, for the last 5 time slots the leader can modify his prices later on. Whereas this is an

advantageous situation for the leader, the follower might not agree with the terms of the leader's pricing scheme. Nevertheless, the rolling horizon method yields results that are close to the optimal solution, which is a strong argument for the further development of such methods.

3.5 Conclusion

In this chapter, we define a single-leader single-follower bilevel model (SBPP) for a DSM pricing problem. At the lower level, the smart grid operator manages distributed generation capacities, e.g. solar panels. The production of solar energy depends on the weather and is therefore unpredictable. To handle this uncertainty, (SBPP) involves stochasticity in the form of a scenario tree.

To solve (SBPP), the follower's problem is replaced by its linearized KKT conditions in the leader's problem, giving rise to a MIP that can be solved with commercial solvers. However, numerical experiments show that even with a small scenario tree, the number of variables and constraints is large, making any resolution time-consuming. To bypass this issue, a rolling horizon method is applied to (SBPP). The method does not solve the problem (SBPP), but allows the supplier to determine his prices in real time, as one of the scenarios becomes real. In particular, we show that the parameters of the rolling horizon method have a large influence on the leader's result. Those parameters should therefore be chosen carefully.

Finally, simplified cases of (SBPP) are studied from a complexity point of view. We show that the case of a single device is easy to handle, but that several devices might make the problem complex. In particular, we prove that the optimal price profile in the case where the follower does not undergo inconvenience belongs to a discrete set of polynomial size with respect to the number of devices, but superexponential with respect to the number of time slots.

Single-Leader Multi-Follower

This chapter corresponds to the article "A Trilevel Model for Best Response in Energy Demand-Side Management", submitted in revised form on December 23rd, 2018, whose co-authors are Didier Aussel, Luce Brotcorne, and Sébastien Lepaul.

4.1 Introduction

Electricity production and consumption are one of the major challenges of today's society. Economical, ecological and political concerns are all at stake. In order to optimize the efficiency of the electricity distribution system, many approaches arose. One of them is demand-side management (DSM). The idea behind DSM is that instead of fitting the production to the demand, the demand can be adapted to the production. DSM has been an important topic for the last thirty years, see [Kreith 2016], and became more relevant with the introduction of the smart grid paradigm, see [Farhangi 2010, Fang 2012]. The ever growing communication among the actors of the grid indeed allows for a better management of the energy consumption.

Works on DSM include for example [Zhu 2012]: households possess appliances that need to receive energy with time and quantity constraints. The usage of the appliance is scheduled so as to minimize the load peak. A distributed DSM model is considered in [Ramchurn 2011]: a case study of the UK shows that efficient DSM could lead to smaller load peaks (-17%) and less carbon gas emissions (-6%). In [Arteconi 2012], the use of thermal energy storage for DSM is reviewed, whereas the integration of wind power generation is the object of [Moura 2010]. DSM can be achieved through many means, but the most used one consists in pricing policies. From a general point of view, [Murphy 2018] aims to quantify the effects of price regulation policies. In [Gottwalt 2011], the reaction of a household to time-based pricing is considered. Recent works include [Devine 2018], where a mixed complementarity problem is used to model electricity consumers offering load shedding on the market. In [Laur 2018], a three-stage stochastic flexibility problem is studied: in a market framework, a company wishes to purchase reserve capacities. From the point of view of the market, [Le Cadre 2018] is concerned with the DSO-TSO

interaction (respectively distribution and transmission system operator). Finally, [Tsitsiklis 2015] proposes a new pricing system, which is not based on the marginal cost, but aims to cover ancillary costs due to fluctuations in the demand.

In this work, we focus on load shifting, a technique of DSM that features a fixed overall demand. To achieve it, price incentives are determined. We thus model and solve the problem of an electricity supplier competing against other suppliers on the market to sell energy to various actors, in a context of demand-side management. More precisely, the energy supplier defines time-dependent prices for his energy, knowing that his clients will adapt by shifting their loads, either directly (local agents) or indirectly (aggregators). This sequential and hierarchical decision-making process can be adequately modeled as a trilevel problem, that is an extension of a bilevel problem.

Bilevel problems formalize the concept of Stackelberg games, and have received a great interest in the past thirty years, see e.g. [Dempe 2002, Bard 2010]. Let us recall that a bilevel problem is an optimization problem (called the *leader's problem*) in which at least one of the variables is constrained to be solution of another optimization problem (called the *follower's problem*). Applications of bilevel programming are numerous: toll pricing, network design problems, flight tickets pricing, see e.g. [Brotcorne 2008]. Bilevel programming is in general NP-hard, even in its simplest form where both objectives and all constraints are linear, see [Ben-Ayed 1990, Labbé 1998]. *Multilevel* optimization problems correspond to the more general case where more than two levels of optimization problems interlock. When there is more than one optimization problem at the leader level, the terminology used in the literature is Multi-Leader-Follower-Game (MLFG) if two additional conditions are met. First, the various leaders play in a noncooperation competition context, and second, each of the leaders' problems is a multilevel problem (see e.g. [Leyffer 2010]). The model considered in this work is a MLFG in which the leaders' problems are trilevel. Note that this structure of model has been rarely considered in the literature, due to its complexity - see for example [Migdalas 1998, Gkatzikis 2013].

Multilevel and MLFG are increasingly used in the energy domain, more precisely on pricing matters. For example, [Afşar 2016b] presents a bilevel energy pricing model in order to decrease the amplitude of the peak loads, whereas [Cervilla 2015] considers a longer time horizon and aims to make the energy system sustainable in a situation where the consumers optimize their investments in distributed generation. In [van Ackooij 2018], generation companies aiming to find the cheapest possible production schedule make contracts with micro-grids that can generate or store energy, and can thus help the generation companies to cope with production surplus

or lack of generation. In [Ambrosius 2018], multilevel problems are used to define price zones within the grid.

In [Gkatzikis 2013], the role of aggregators in the energy market is studied, and a trilevel model is proposed. In this model, a system operator aims to optimize his operating costs thanks to load shifts executed by end customers. Aggregators play an intermediary role between the system operator and the end customers. More precisely, the system operator offers a percentage of his benefit to each of the aggregators, and the aggregators offer pecuniary rewards to the end customers they are in contract with to induce a load shift. A heuristic to solve the trilevel model is defined. One limitation of the proposed model is that the cost function of the leader appears in the objective function of the aggregator, a situation that is not realistic for many trading situations since no aggregator is supposed to know the cost function of the supplier.

In this paper, we consider a similar interaction model, but first the cost function of the supplier only appears in his own problem, and second the actors of the intermediary level (ILAs) have the possibility to trade energy among themselves. Moreover we do not use heuristic techniques and our reformulation and resolution of the problem are theoretically thoroughly justified (see propositions 4.3.1, 4.4.1 and 4.4.3).

An important contribution of the paper is the definition of two new solution concepts that are adapted to this context of price management. Indeed, breaking with the classical optimistic approach usually made in case of nonuniqueness of the followers equilibrium solution, we define the so-called revisited optimistic approach and semi-optimistic approach that are based on some smart selection among the followers' equilibria, sharply taking into account the structure of the problem.

The paper is structured as follows: first, we present a *Trilevel Demand-Side Management model* (denoted by (TDSM)) and define the actors involved in the model in Section 4.2. Second, in Section 4.3, we formulate (TDSM) as a bilevel problem, thanks to an explicit resolution of the lowest level problems. In Section 4.4, we propose three different approaches to solve the bilevel version of (TDSM). First a classical method, consisting in replacing the lower level problems by their KKT conditions in the leader's problem leads in Subsection 4.4.1 to the so-called *Classical optimistic* approach. Second, in Subsection 4.4.2, we simplified the bilevel reformulation of the game by selecting a specific class of Generalized Nash equilibria (GNE) among the followers, providing thus the *revisited optimistic* approach. Third, in Subsection 4.4.3, we define a new class of GNE, called *semi-optimistic*, that simplifies a lot the formulation of the game and therefore its numerical treatment. Finally, in Section 4.5, we compare and comment the solutions computed by the

three approaches.

4.2 Problem formulation

Four kinds of actors are involved in the (TDSM) problem: electricity suppliers $s \in \mathcal{S}$, local agents $\ell \in \mathcal{L}$, aggregators $a \in \mathcal{A}$, and end users $i \in \mathcal{I}$. Interactions among actors of (TDSM) are described in Figure 4.1 and are of the following nature:

- being into a "best response approach", we assume that \mathcal{S} is composed of two suppliers, that is $\mathcal{S} = \{\hat{s}, \bar{s}\}$ where \bar{s} (the *competitor*) is a supplier with a fixed price strategy known by \hat{s} (the *leader*) that aims to determine his best strategy;
- for each period of time the suppliers propose electricity prices to the intermediate actors (aggregators and local agents);
- the intermediate actors can buy electricity from the suppliers or from other intermediate actors;
- each end user is in contract with only one aggregator and he can shift a part of his load in order to get some rewards from the aggregator.

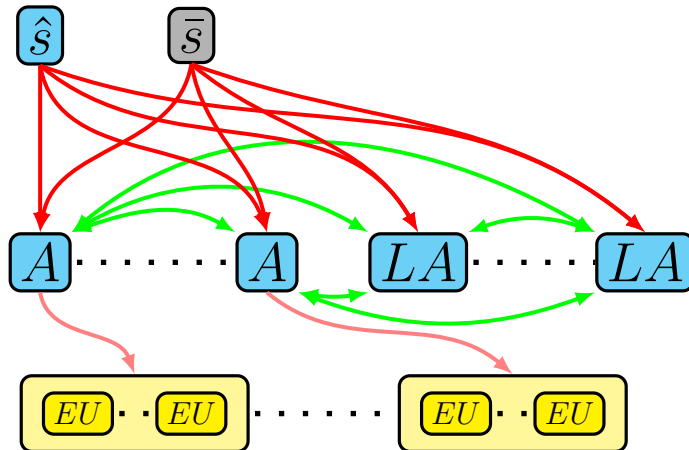


Figure 4.1: Structure of the model

Blue forms represent the various (dynamic) actors (the leader, the aggregators and the local agents), the grey supplier represents the static competitor, red links show unidirectional energy trades, green links designate bidirectional energy exchanges, and pink links indicate rewards for load shifting. Each aggregator is related to his own set of end users.

The variables and notations of the model are as follows:

- The set of the considered time slots is denoted by \mathcal{H} .
- Bold variables indicate vectors.
- $\mathbf{d}_x = (d_x^1, \dots, d_x^{|\mathcal{H}|})$ denotes the demand variables of $x \in \mathcal{I} \cup \mathcal{L}$.
- Variables p_{xy}^h denote the prices of an energy unit sold by x to y at time $h \in \mathcal{H}$.
- Variables e_{xy}^h represent the energy flow from y to x (bought by x from y) at time h .
- Variables r_{ai}^h denote the rewards given to the end user i by the aggregator a for an energy unit consumed at time $h \in \mathcal{H}$. The set of end users that are in contract with the aggregator a is denoted by \mathcal{I}_a .
- W_x indicates the overall demand that needs to be satisfied for $x \in \mathcal{I} \cup \mathcal{L}$.

In this paper we denote by *Intermediate Level Actors (ILAs)* the players of the second level (Aggregators and Local agents) in the trilevel structure.

4.2.1 Local agents

Local agents can buy energy from the suppliers or trade energy with the other ILAs. Each local agent $\ell \in \mathcal{L}$ has a fixed overall demand W_ℓ for the horizon and an *a priori* demand vector $(d_\ell^{h,0})_{h \in \mathcal{H}}$. This demand can be shifted, resulting in the *real* demand splitting/vector $(d_\ell^h)_{h \in \mathcal{H}}$. The energy bought can be used to satisfy the electricity demand or sold to the other ILAs. In case of shifting of their consumption, the local agents undergo an inconvenience cost that increases with the extent of the load shift. Local agents aim to minimize the sum of their electricity bill and the inconvenience due to the load shift. Formally, the local agent $\ell \in \mathcal{L}$ aims to solve the following problem:

$$(P_\ell) \quad \min_{\mathbf{e}_\ell, \mathbf{p}_\ell, \alpha_\ell} \sum_{h \in \mathcal{H}} \left(\sum_{s \in \mathcal{S}} p_{s\ell}^h e_{\ell s}^h + \sum_{a \in \mathcal{A}} (p_{a\ell}^h e_{\ell a}^h - p_{\ell a}^h e_{a\ell}^h) + \sum_{\ell' \neq \ell} (p_{\ell'\ell}^h e_{\ell\ell'}^h - p_{\ell\ell'}^h e_{\ell'\ell}^h) + V_\ell^h (d_\ell^h) \right)$$

$$s.t. \quad \begin{cases} \sum_{h \in \mathcal{H}} d_\ell^h = W_\ell \\ e_{\ell x}^h \geq 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \cup \mathcal{S}, \\ p_{\ell x}^h \leq \alpha_{\ell x}^h p_{sx}^h + (1 - \alpha_{\ell x}^h) p_{s\ell}^h & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \\ \alpha_{\ell x}^h \in [0, 1] & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S}, \end{cases}$$

where $V_\ell^h(d_\ell^h) = v_\ell^h (d_\ell^h - d_\ell^{h,0})^2$ is the inconvenience caused by the load shifting and $v_\ell^h > 0$ is a fixed inconvenience coefficient. Note that, for any h , the notation d_ℓ^h stands for the demand value and thus $d_\ell^h \stackrel{not.}{=} \sum_{s \in \mathcal{S}} e_{\ell s}^h + \sum_{a \in \mathcal{A}} (e_{\ell a}^h - e_{a \ell}^h) + \sum_{\ell' \neq \ell} (e_{\ell \ell'}^h - e_{\ell' \ell}^h)$ since no storage is considered in this model. The inconvenience function V_ℓ^h is the same as defined in [Gkatzikis 2013]. This function has nice mathematical properties, like convexity and differentiability, and it adequately models the real inconvenience that the consumers are undergoing. A small shift of the consumption will not represent a significant inconvenience, whereas an important shift will have strong repercussions on the consumer's comfort thanks to the square power.

The first constraint in problem (P_ℓ) ensures that ℓ 's cumulated demand remains constant, whereas the second constraint avoids negative energy exchanges. The third and fourth constraints result from a deeper observation. The second (intermediate) level of (TDSM) actually constitutes a noncooperative Nash game, where the players are the ILAs. Therefore, an optimal ILA's response to the leader's decisions is a Nash equilibrium. Since the decision variables of the ILA influence the feasible domains of the other ILAs, it is precisely a Generalized Nash Equilibrium (GNE). Let us recall that a GNE is reached whenever no player can unilaterally improve his situation by modifying his decision variables (see e.g. [Facchinei 2007]). Assume that an ILA x buys energy from another ILA y : the quantity is determined by x while the price is determined by y . Thus y can improve his revenue by unilaterally increasing the price, hence showing that without upper bounds on the prices, there cannot be energy exchanges among the ILAs in a GNE. But of course, if there is an upper bound to the prices, this upper bound is reached in any GNE, provided that there is a nonzero exchange between the affected ILAs.

For any $s \in \mathcal{S}$, a first obvious bound for p_{yx}^h (the price offered by y to x at time $h \in \mathcal{H}$) is p_{sx}^h , the price offered to x by s . Indeed, x would never buy energy from y if he can buy energy from s at a lower price. However, this bound can lead to undesired GNE. Indeed, let us assume that x buys energy from y : y has to furnish this energy, and thus can buy it from s and if p_{sx}^h is smaller than p_{sy}^h , then y ends up paying for the energy that x needs, even though this situation is a GNE.

As a result from the previous paragraph, if y sells energy to another ILA, he must sell it at least for p_{sy}^h to avoid losses. Therefore, we must have $p_{yx}^h \geq p_{sy}^h$. Since in a GNE, the upper bound to p_{yx}^h is reached, setting $p_{yx}^h \leq p_{sy}^h$ could make sense. However in this case, all the ILAs have access to energy at the lowest possible price, $\min_{x \in \mathcal{L} \cup \mathcal{A}} p_{sx}^h$. It is thus useless for the leader to offer different prices to different actors.

Under some competition assumptions on the market (further called *competitive*

case and *nondiscriminant exogenous pricing*), the forthcoming Proposition 4.4.1 shows that thanks to the third and fourth constraints of problem (P_ℓ) , if there is a nonzero exchange between two ILA x and y in a GNE of the intermediate level ($e_{yx}^h > 0$), then

$$p_{xy}^h = \max \left\{ \min \left\{ p_{sx}^h, p_{sx}^h \right\}, \min \left\{ p_{sy}^h, p_{sy}^h \right\} \right\}.$$

One can thus wonder why to use the artificial α_{xys}^h and third and fourth constraints instead of simply using the above formula for p_{xy}^h . The answer comes from the nonsmoothness of this formula and thus the computational difficulties that it could generate. This is why the approach based on the third and fourth constraints has been preferred.

4.2.2 Aggregators

Aggregators can buy energy from the suppliers, and trade energy with the other ILAs, in order to satisfy the demand of the end users they are in contract with. They offer rewards to their end users as incentives to shift their consumption. The aggregators aim to minimize the sum of the rewards offered to their end users and the cost of the electricity bought from the suppliers and the other ILAs. The optimization problem of the aggregator $a \in \mathcal{A}$ can be formally expressed as:

$$(P_a) \quad \min_{\mathbf{e}_a, \mathbf{r}_a, \mathbf{p}_a, \alpha_a} \min_{\mathbf{d}^a} \sum_{h \in \mathcal{H}} \left(\sum_{s \in \mathcal{S}} p_{sa}^h e_{as}^h + \sum_{\ell \in \mathcal{L}} (p_{\ell a}^h e_{a\ell}^h - p_{a\ell}^h e_{\ell a}^h) \right. \\ \left. + \sum_{a' \neq a} (p_{a'a}^h e_{aa'}^h - p_{aa'}^h e_{a'a}^h) + \sum_{i \in \mathcal{I}_a} r_{ai}^h d_i^h \right) \\ s.t. \quad \begin{cases} \sum_{s \in \mathcal{S}} e_{as}^h + \sum_{\ell \in \mathcal{L}} (e_{a\ell}^h - e_{\ell a}^h) + \sum_{a' \neq a} (e_{aa'}^h - e_{a'a}^h) = \sum_{i \in \mathcal{I}_a} d_i^h & \forall h \in \mathcal{H} \\ \mathbf{d}_i^a \in \text{argmax}(P_i) & \forall i \in \mathcal{I}_a \\ p_{ax}^h \leq \alpha_{ax}^h p_{sx}^h + (1 - \alpha_{ax}^h) p_{sa}^h & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \\ r_{ai}^h, e_{ax}^h \geq 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, i \in \mathcal{I}_a \\ \alpha_{ax}^h \in [0, 1] & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \end{cases}$$

where \mathbf{d}^a stands for the vector $\mathbf{d}^a = (\mathbf{d}_i^a)_{i \in \mathcal{I}_a}$, with $\mathbf{d}_i^a = (d_i^h)_{h \in \mathcal{H}}$, (P_i) is the end user i 's problem and \mathcal{I}_a denotes the set of end users who are in contract with aggregator a . It is here assumed that each end user is in contract with only one aggregator.

The first constraint ensures that the outcome of the energy exchanges equals the overall demand of the end users. The second constraint makes sure that the

demand of the end users is optimal. The utility of the third and last constraints has been discussed in Subsection 4.2.1.

4.2.3 End users

The end users are in contract with one of the aggregators that provides energy to them. They need to receive a given amount W_i of energy during the time horizon, and they receive rewards from the aggregator as an incentive to shift their energy consumption. However, shifting their consumption induces an inconvenience, which is similar to the inconvenience for the local agents. Therefore, the problem (P_i) of the end user $i \in \mathcal{I}$ can be expressed as:

$$(P_i) \quad \max_{\mathbf{d}_i} \sum_{h \in \mathcal{H}} r_{ai}^h d_i^h - V_i^h(d_i^h) \\ \text{s.t.} \quad \begin{cases} \sum_{h \in \mathcal{H}} d_i^h = W_i \\ d_i^h \geq 0 \quad \forall h \in \mathcal{H}, \end{cases}$$

where d_i^h denotes i 's demand at time h , W_i denotes i 's overall need in electricity and $V_i^h(d_i^h) = v_i^h(d_i^h - d_i^{h,0})^2$ is the inconvenience caused by the load shifting ($v_i^h > 0$ is fixed). As for the local agents, $(d_i^{h,0})_h$ stands for the *a priori* demand vector of end user i . Observe that (P_i) is convex.

4.2.4 Electricity supplier

The electricity supplier \hat{s} sells energy to aggregators and to local agents. Knowing the fixed price strategy $\mathbf{p}_{\bar{s}}$ of supplier \bar{s} , supplier \hat{s} aims to maximize his profit, which is the difference between the revenues resulting from sales and his production costs, described by the function $c_{\hat{s}}^h(\cdot)$. The electricity supplier influences the demand of the customers by setting hourly prices $p_{\hat{s}x}^h$, knowing that the clients will react optimally to these prices. Therefore, the problem of the supplier \hat{s} can be expressed as:

$$(P_{\hat{s}}) \quad \max_{\mathbf{p}_{\hat{s}}} \max_{\mathbf{e}, \mathbf{r}, \mathbf{p}_x, \boldsymbol{\alpha}} \sum_{h \in \mathcal{H}} \left(\sum_{a \in \mathcal{A}} p_{\hat{s}a}^h e_{a\hat{s}}^h + \sum_{\ell \in \mathcal{L}} p_{\hat{s}\ell}^h e_{\ell\hat{s}}^h - c_{\hat{s}}^h \left(\sum_{a \in \mathcal{A}} e_{a\hat{s}}^h + \sum_{\ell \in \mathcal{L}} e_{\ell\hat{s}}^h \right) \right) \\ \text{s.t.} \quad \begin{cases} p_{\hat{s}x}^h \geq 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \\ \{\mathbf{e}_{\ell}, \mathbf{p}_{\ell}, \boldsymbol{\alpha}_{\ell}\} \in \operatorname{argmax}(P_{\ell}) & \forall \ell \in \mathcal{L} \\ \{\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a, \mathbf{r}_a\} \in \operatorname{argmax}(P_a) & \forall a \in \mathcal{A}, \end{cases}$$

We assume here that the function $c_{\hat{s}}^h : t \mapsto c_{\hat{s}}^h(t)$ is increasing and convex for all $h \in \mathcal{H}$. According to the previous notations, (P_a) and (P_{ℓ}) respectively denote the

optimization problems of the aggregator $a \in \mathcal{A}$ and the local agent $\ell \in \mathcal{L}$.

In this paper, we will mainly consider the following two cases:

- *competitive case*: in this case, we assume that the leader is competitive compared to the competitor, which means that the leader can take over the whole demand and make profit with lower prices than the competitor. That is, for any $h \in \mathcal{H}$, the function $(d_x^h)_{x \in \mathcal{L} \cup \mathcal{A}} \mapsto \sum_{x \in \mathcal{L} \cup \mathcal{A}} p_{sx}^h d_x^h - c_s^h \left(\sum_{x \in \mathcal{L} \cup \mathcal{A}} d_x^h \right)$ is increasing with relation to each variable. Observe in particular that when the cost function of the leader is linear with factor k_s^h , the condition is fulfilled if for all $x \in \mathcal{L} \cup \mathcal{A}$, $p_{sx}^h \geq k_s^h$.
- *nondiscriminant exogenous pricing*: in this case, the competitor does not discriminate between the various consumers, thus for all $h \in \mathcal{H}$ and all $x, y \in \mathcal{L} \cup \mathcal{A}$, $p_{sx}^h = p_{sy}^h$.

Assuming that at least one of these cases is verified allows us to prove some interesting results (see propositions 4.4.1 and 4.4.3), which lead to efficient ways of solving the leader's problem (P_s) .

4.3 From a trilevel model to a bilevel model

Being a trilevel problem, (TDSM) is challenging to solve. Our goal is, deeply taking advantage of the special structure of the model, to rewrite this trilevel problem as a single-level optimization problem with complementarity constraints (MPCC). First, we give explicit formulas for the end users' problems and plug them into the aggregators' problems to obtain a bilevel reformulation (Subsection 4.3). To solve this bilevel problem, we then apply the classical method consisting in the replacement of the followers' problems by their KKT conditions in the leader's problem.

We next build an alternative bilevel formulation (P'_s) of the leader's problem and prove that any solution of the bilevel problem (P'_s) provides a solution of the initially considered trilevel problem (P_s) . For a given $a \in \mathcal{A}$, let us first rewrite the

aggregator a 's minimization problem (P_a) as

$$\begin{aligned} & \min_{\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a} \min_{\mathbf{r}_a, \mathbf{d}^a} \varphi_a(\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a) + \sum_{h \in \mathcal{H}} \sum_{i \in \mathcal{I}_a} r_{ai}^h d_i^h \\ & \text{s.t.} \left\{ \begin{array}{ll} \sum_{s \in \mathcal{S}} e_{as}^h + \sum_{\ell \in \mathcal{L}} (e_{a\ell}^h - e_{\ell a}^h) \\ \quad + \sum_{a' \neq a} (e_{aa'}^h - e_{a'a}^h) = \sum_{i \in \mathcal{I}_a} d_i^h & \forall h \in \mathcal{H} \\ \mathbf{d}_i^a \in \operatorname{argmax}(P_i) & \forall i \in \mathcal{I}_a \\ p_{ax}^h \leq \alpha_{ax}^h p_{sx}^h + (1 - \alpha_{ax}^h) p_{sa}^h & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \\ r_{ai}^h, e_{ax}^h \geq 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, i \in \mathcal{I}_a \\ \alpha_{ax}^h \in [0, 1] & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S}, \end{array} \right. \end{aligned}$$

setting

$$\varphi_a(\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a) = \sum_{h \in \mathcal{H}} \left(\sum_{s \in \mathcal{S}} p_{sa}^h e_{as}^h + \sum_{\ell \in \mathcal{L}} (p_{\ell a}^h e_{a\ell}^h - p_{a\ell}^h e_{\ell a}^h) + \sum_{a' \neq a} (p_{a'a}^h e_{aa'}^h - p_{aa'}^h e_{a'a}^h) \right)$$

and let us define the associated problem (P'_a)

$$\begin{aligned} (P'_a) \quad & \min_{\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a} \min_{\mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a} \varphi_a(\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a) + \sum_{h \in \mathcal{H}} \sum_{i \in \mathcal{I}_a} 2v_i^h d_i^h (d_i^h - d_i^{h,0}) + \sum_{i \in \mathcal{I}_a} W_i \hat{\lambda}_i \\ & \text{s.t.} \left\{ \begin{array}{ll} \sum_{s \in \mathcal{S}} e_{as}^h + \sum_{\ell \in \mathcal{L}} (e_{a\ell}^h - e_{\ell a}^h) \\ \quad + \sum_{a' \neq a} (e_{aa'}^h - e_{a'a}^h) = \sum_{i \in \mathcal{I}_a} d_i^h & \forall h \in \mathcal{H} \\ \sum_{h \in \mathcal{H}} d_i^h = W_i & \forall i \in \mathcal{I}_a \\ 2v_i^h (d_i^h - d_i^{h,0}) + \lambda_i \geq 0 & \forall h \in \mathcal{H}, \forall i \in \mathcal{I}_a \\ p_{ax}^h \leq \alpha_{ax}^h p_{sx}^h + (1 - \alpha_{ax}^h) p_{sa}^h & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \\ d_i^h, e_{ax}^h \geq 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, i \in \mathcal{I}_a \\ \alpha_{ax}^h \in [0, 1] & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S}. \end{array} \right. \end{aligned}$$

Proposition 4.3.1. *Let a belong to \mathcal{A} , and assume that for any $i \in \mathcal{I}_a$ and any $h \in \mathcal{H}$, the inconvenience coefficient v_i^h is positive. If $(\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a)$ is an optimal solution for (P'_a) then $(\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a, \mathbf{d}^a, \mathbf{r}_a)$ is an optimal solution for (P_a) , with $r_{ai}^h = 2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i$, for any $i \in \mathcal{I}_a$ and any $h \in \mathcal{H}$.*

Proof. Observe that for all $i \in \mathcal{I}_a$, the domain of (P_i) is nonempty, compact, and convex, and the objective function of (P_i) is strictly convex. Therefore, it has a unique solution \mathbf{d}_i^a . Furthermore, observe that Slater's condition is verified

for any \mathbf{r}_{ai} , since $W_i > 0$. It follows from Theorem 2.3 of [Dempe 2012] that if $(\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a, \mathbf{r}_a, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a, \boldsymbol{\mu}^a)$ is a global solution of (P_a^{MPCC}) :

$$\begin{aligned} & \min_{\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a, \mathbf{r}_a, \mathbf{d}^a, \boldsymbol{\mu}^a, \hat{\boldsymbol{\lambda}}^a} \varphi_a(\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a) + \sum_{h \in \mathcal{H}} \sum_{i \in \mathcal{I}_a} r_{ai}^h d_i^h \\ & \left. \begin{array}{l} \sum_{s \in \mathcal{S}} e_{as}^h + \sum_{\ell \in \mathcal{L}} (e_{a\ell}^h - e_{\ell a}^h) + \sum_{a' \neq a} (e_{aa'}^h - e_{a'a}^h) = \sum_{i \in \mathcal{I}_a} d_i^h \quad \forall h \in \mathcal{H} \\ \sum_{h \in \mathcal{H}} d_i^h = W_i \quad \forall i \in \mathcal{I}_a \\ \mu_i^h d_i^h = 0 \quad \forall h \in \mathcal{H} \\ -r_i^h + 2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i - \mu_i^h = 0 \quad \forall h \in \mathcal{H} \\ p_{ax}^h \leq \alpha_{ax}^h p_{sx}^h + (1 - \alpha_{ax}^h) p_{sa}^h \quad \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \\ d_i^h, e_{ax}^h, r_{ai}^h, \mu_i^h \geq 0 \quad \forall h \in \mathcal{H}, x \in \mathcal{S} \cup \mathcal{L} \cup \mathcal{A}, i \in \mathcal{I}_a \\ \alpha_{ax}^h \in [0, 1] \quad \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S}, \end{array} \right\} \text{s.t.} \end{aligned}$$

then $(\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a, \mathbf{r}_a, \mathbf{d}^a)$ is a global solution of (P_a) .

We immediately deduce that, for all $h \in \mathcal{H}$ and all $i \in \mathcal{I}_a$,

$$r_{ai}^h = 2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i - \mu_i^h,$$

and thus replacing r_{ai} in (P_a^{MPCC}) yields (P_a^{MPCC-r}) :

$$\begin{aligned} & \min_{\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a, \mathbf{d}^a, \boldsymbol{\mu}^a, \hat{\boldsymbol{\lambda}}^a} \varphi_a(\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a) + \sum_{h \in \mathcal{H}} \sum_{i \in \mathcal{I}_a} (2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i - \mu_i^h) d_i^h \\ & \left. \begin{array}{l} \sum_{s \in \mathcal{S}} e_{as}^h + \sum_{\ell \in \mathcal{L}} (e_{a\ell}^h - e_{\ell a}^h) + \sum_{a' \neq a} (e_{aa'}^h - e_{a'a}^h) = \sum_{i \in \mathcal{I}_a} d_i^h \quad \forall h \in \mathcal{H} \\ \sum_{h \in \mathcal{H}} d_i^h = W_i \quad \forall i \in \mathcal{I}_a \\ \mu_i^h d_i^h = 0 \quad \forall h \in \mathcal{H} \\ 2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i - \mu_i^h \geq 0 \quad \forall h \in \mathcal{H} \\ p_{ax}^h \leq \alpha_{ax}^h p_{sx}^h + (1 - \alpha_{ax}^h) p_{sa}^h \quad \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \\ d_i^h, e_{ax}^h, \mu_i^h \geq 0 \quad \forall h \in \mathcal{H}, x \in \mathcal{S} \cup \mathcal{L} \cup \mathcal{A}, i \in \mathcal{I}_a \\ \alpha_{ax}^h \in [0, 1] \quad \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S}. \end{array} \right\} \text{s.t.} \end{aligned}$$

Now, assume that we have an optimal solution $(\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a, \mathbf{d}^a, \boldsymbol{\mu}^a, \hat{\boldsymbol{\lambda}}^a)$ of the problem (P_a^{MPCC-r}) . Consider

$$(\mathbf{e}_a^* = \mathbf{e}_a, \mathbf{p}_a^* = \mathbf{p}_a, \boldsymbol{\alpha}_a^* = \boldsymbol{\alpha}_a, \mathbf{d}^{*a} = \mathbf{d}^a, \boldsymbol{\mu}^{*a} = 0, \hat{\boldsymbol{\lambda}}^{*a} = \hat{\boldsymbol{\lambda}}^a).$$

Obviously, $(\mathbf{e}_a^*, \mathbf{p}_a^*, \boldsymbol{\alpha}_a^*, \mathbf{d}^{*a}, \boldsymbol{\mu}^{*a}, \hat{\boldsymbol{\lambda}}^{*a})$ satisfies all the constraints of (P_a^{MPCC-r}) . Furthermore, observe that

$$\begin{aligned}
\sum_{h \in \mathcal{H}} \left(2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i - \mu_i^h \right) d_i^h &= \sum_{h \in \mathcal{H}: d_i^h \neq 0} \left(2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i - \mu_i^h \right) d_i^h \\
&= \sum_{h \in \mathcal{H}: d_i^h \neq 0} \left(2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i^* \right) d_i^h \\
&= \sum_{h \in \mathcal{H}} \left(2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i^* \right) d_i^h \\
&= \sum_{h \in \mathcal{H}} 2v_i^h d_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i^* W_i,
\end{aligned}$$

since $\mu_i^h > 0$ implies that $d_i^h = 0$, and $\sum_{h \in \mathcal{H}} d_i^h = W_i$. Therefore, $(\mathbf{e}_a^*, \mathbf{p}_a^*, \boldsymbol{\alpha}_a^*, \boldsymbol{\mu}^{*a}, \hat{\boldsymbol{\lambda}}^{*a})$ is an optimal solution of (P_a^{MPCC-r}) too, and we can restrain the search of optimal solutions to the sets of $(\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a, \boldsymbol{\mu}^a, \hat{\boldsymbol{\lambda}}^a)$ with $\boldsymbol{\mu}^a = 0$, which gives rise to (P'_a) . This concludes our proof. \square

Note also from the proof that the optimal reward (for a) to obtain a demand \mathbf{d}_i^a from the end user i at time h is

$$r_{ai}^h = 2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i \quad \forall i \in \mathcal{I}, h \in \mathcal{H},$$

with $\hat{\lambda}_i$ such that $\min_{h \in \mathcal{H}} r_{ai}^h = 0$.

Taking into account the formulation (P'_a) of the aggregators, we will now consider the following modified version problem of the leader, which is a bilevel problem, denoted by (P'_s) :

$$\begin{aligned}
(P'_s) \quad & \max_{\mathbf{p}_s} \max_{\mathbf{e}, \mathbf{p}_x, \boldsymbol{\alpha}, \mathbf{d}, \hat{\boldsymbol{\lambda}}} \sum_{h \in \mathcal{H}} \left(\sum_{a \in \mathcal{A}} p_{sa}^h e_{as}^h + \sum_{\ell \in \mathcal{L}} p_{s\ell}^h e_{\ell s}^h - c_s^h \left(\sum_{a \in \mathcal{A}} e_{as}^h + \sum_{\ell \in \mathcal{L}} e_{\ell s}^h \right) \right) \\
& s.t. \quad \begin{cases} p_{sx}^h \geq 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \\ \{\mathbf{e}_\ell, \mathbf{p}_\ell, \boldsymbol{\alpha}_\ell\} \in \operatorname{argmax} (P_\ell) & \forall \ell \in \mathcal{L} \\ \{\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a, \mathbf{d}_i, \hat{\boldsymbol{\lambda}}\} \in \operatorname{argmax} (P'_a) & \forall a \in \mathcal{A}, \end{cases}
\end{aligned}$$

and thus according to Proposition 4.3.1 from any solution $(\mathbf{p}_s, \mathbf{e}, \mathbf{p}_x, \boldsymbol{\alpha}, \mathbf{d}, \hat{\boldsymbol{\lambda}})$ of (P'_s) one obtains the solution $(\mathbf{p}_s, \mathbf{e}, \mathbf{p}_x, \boldsymbol{\alpha}, \mathbf{d}, \mathbf{r})$ of (P_s) with $r_{ai}^h = 2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i$, for any $i \in \mathcal{I}_a$ and any $h \in \mathcal{H}$.

4.4 Reformulations of the bilevel program and alternative solution concepts

As mentioned in Section 4.3, the resolution of the trilevel best response problem $(P_{\hat{s}})$ can be reduced to the computation of solutions of the single-leader multi-follower problem $(P'_{\hat{s}})$. Nevertheless, even if more simple from a structural point of view, single-leader multi-follower games are known to be very difficult to handle.

In this section, we present three different ways to transform/simplify $(P'_{\hat{s}})$ in order to be able to solve it. First, a somehow classical transformation of $(P'_{\hat{s}})$ into a mathematical program with complementarity constraints (MPCC) is developed in Subsection 4.4.1. Second, in subsections 4.4.2 and 4.4.3, two new approaches based on a technique of selection of the GNE at the ILAs' level are proposed to simplify $(P'_{\hat{s}})$.

4.4.1 First order formulation of the bilevel model

The classical method to solve a bilevel problem consists in replacing the followers' problems by their KKT conditions in the leader's problem, thus yielding an MPCC. Therefore, the bilevel reformulation of (TDSM) $(P'_{\hat{s}})$ becomes the single level MPCC $(P_{\hat{s}}^{cl})$:

$$(P_{\hat{s}}^{cl}) \quad \max_{\mathbf{p}_{\hat{s}}} \max_{\mathbf{e}, \mathbf{p}_x, \boldsymbol{\alpha}, \mathbf{d}, \hat{\boldsymbol{\lambda}}} \sum_{h \in \mathcal{H}} \left(\sum_{a \in \mathcal{A}} p_{\hat{s}a}^h e_{a\hat{s}}^h + \sum_{\ell \in \mathcal{L}} p_{\hat{s}\ell}^h e_{\ell\hat{s}}^h e_{\ell\hat{s}}^h - c_{\hat{s}}^h \left(\sum_{a \in \mathcal{A}} e_{a\hat{s}}^h + \sum_{\ell \in \mathcal{L}} e_{\ell\hat{s}}^h \right) \right)$$

$$s.t. \quad \begin{cases} p_{\hat{s}x}^h \geq 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \\ \{\mathbf{e}_{\ell}, \mathbf{p}_{\ell}, \boldsymbol{\alpha}_{\ell}\} \in \text{KKT}(P_{\ell}) & \forall \ell \in \mathcal{L} \\ \{\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a\} \in \text{KKT}(P'_a) & \forall a \in \mathcal{A}, \end{cases}$$

where $\text{KKT}(P_{\ell})$ and $\text{KKT}(P'_a)$ denote the solution sets of the KKT conditions. More precisely, $\text{KKT}(P_{\ell})$ is the set of tuples $(\mathbf{e}, \mathbf{p}, \boldsymbol{\alpha}, \boldsymbol{\mu}, \boldsymbol{\lambda})$ satisfying the primal feasibility, dual feasibility, stationarity and complementary slackness constraints related to (P_{ℓ}) . $\text{KKT}(P'_a)$ is the set of tuples $(\mathbf{e}, \mathbf{d}, \mathbf{p}, \boldsymbol{\alpha}, \boldsymbol{\mu}, \boldsymbol{\lambda})$ satisfying the KKT conditions of (P'_a) . All these constraints can be found in A.1.

It is now well known that, even if the lower level problems of (P_{ℓ}) and (P'_a) of the leader problems $(P'_{\hat{s}})$ are convex, a solution of the multi-leader-follower game composed of the problems $(P_{\hat{s}}^{cl})$ need not be a solution of the game composed with problems $(P'_{\hat{s}})$. Indeed as proved in [Dempe 2012, Aussel 2019] some qualification conditions are required but it is beyond the scope of this work to discuss such qualification conditions.

4.4.2 Revisited optimistic approach

We consider here the classical optimistic approach that was considered above in its bilevel expression (P'_s) but we show, using the specific characteristics of the (TDSM) problem, that at optimality (Nash equilibrium) we can actually select some special GNE at the intermediary level while maintaining the optimal values of the supplier. These special GNE get rid of the energy exchanges at the intermediary level.

Proposition 4.4.1. *Consider a leader price profile $\mathbf{p}_s = \{\mathbf{p}_{sx} \mid x \in \mathcal{L} \cup \mathcal{A}\}$, and*

$$S(\mathbf{p}) = \left\{ \left(\mathbf{e}_{xy}, \mathbf{e}_{xs}, \mathbf{p}_{xy}, \boldsymbol{\alpha}_{xy}, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a \right) \mid x, y \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S}, a \in \mathcal{A}, i \in \mathcal{I}_a \right\}$$

a GNE associated to \mathbf{p} . Assume that, for all $h \in \mathcal{H}$, at least one of the two following conditions holds:

$$(H1) \text{ For all } x \in \mathcal{L} \cup \mathcal{A}, p_{sx}^h \leq p_{sx}^h;$$

$$(H2) \text{ For all } x, y \in \mathcal{L} \cup \mathcal{A}, p_{sx}^h = p_{sy}^h.$$

Then for any $x, y \in \mathcal{L} \cup \mathcal{A}$ such that $e_{yx}^h > 0$ one has

$$p_{xy}^h = \max \left\{ \min \left\{ p_{sx}^h, p_{sx}^h \right\}, \min \left\{ p_{sy}^h, p_{sy}^h \right\} \right\}.$$

Let us first state the following technical lemma that will be useful to clarify the proof of Proposition 4.4.1.

Lemma 4.4.2. *Let a, b, c, d be real numbers, and define the two conditions*

1. $a \leq b$ and $c \leq d$;
2. $b = d$.

If at least one of these conditions is verified, then

$$\max \{ \min \{ a, b \}, \min \{ c, d \} \} = \min \{ \max \{ a, c \}, \max \{ b, d \} \}.$$

Proof of Lemma 4.4.2. Let us first assume that condition 1 holds true. In that case,

$$\max \{ \min \{ a, b \}, \min \{ c, d \} \} = \max \{ a, c \} = \min \{ \max \{ a, c \}, \max \{ b, d \} \},$$

since by the hypothesis, $\max \{ a, c \} \leq \max \{ b, d \}$. Now, assume that condition 2 is verified, hence $b = d$. If $a \leq b$ and $c \leq d$, then condition 1 is verified as well, and the lemma is verified. If condition 1 is not verified, then $a > b$ or $c > d$. Since both cases are symmetric, assume without loss of generality that $a > b$. In this case,

$$\max \{ \min \{ a, b \}, \min \{ c, d \} \} = \max \{ \min \{ a, b \}, \min \{ c, b \} \} = \max \{ b, \min \{ c, b \} \} = b.$$

On the other hand, $\min \{ \max \{ a, c \}, \max \{ b, d \} \} = \min \{ \max \{ a, c \}, b \} = b$, since it directly follows from the hypothesis that $\max \{ a, c \} > b$. \square

Proof of Proposition 4.4.1. Take $h \in \mathcal{H}$, and $x, y \in \mathcal{L} \cup \mathcal{A}$ such that $e_{yx}^h > 0$. First, observe that since $S(\mathbf{p})$ is a GNE, and since the energy bought from x by y is a decision of the ILA y , the price p_{xy}^h takes the highest possible value. This price is upper-bounded at the same time by $\alpha_{xy\hat{s}}^h p_{\hat{s}x}^h + (1 - \alpha_{xy\hat{s}}^h) p_{\hat{s}y}^h$ and $\alpha_{xy\bar{s}}^h p_{\bar{s}x}^h + (1 - \alpha_{xy\bar{s}}^h) p_{\bar{s}y}^h$. Observe that a judicious (i.e. 0 or 1) choice of $\alpha_{xy\hat{s}}^h$ and $\alpha_{xy\bar{s}}^h$ yields that those two upper bounds become $\max \{p_{\hat{s}x}^h, p_{\hat{s}y}^h\}$ and $\max \{p_{\bar{s}x}^h, p_{\bar{s}y}^h\}$, respectively. Therefore, if $e_{yx}^h > 0$, then

$$p_{xy}^h = \min \left\{ \max \{p_{\hat{s}x}^h, p_{\hat{s}y}^h\}, \max \{p_{\bar{s}x}^h, p_{\bar{s}y}^h\} \right\}.$$

Now, combining Lemma 4.4.2 with either hypothesis (H1) or (H2) directly implies that $p_{xy}^h = \max \left\{ \min \{p_{\hat{s}x}^h, p_{\bar{s}x}^h\}, \min \{p_{\hat{s}y}^h, p_{\bar{s}y}^h\} \right\}$. \square

Observe that the proof of Proposition 4.4.1 actually holds under the following hypothesis:

(H3) if $e_{xy}^h > 0$ for $h \in \mathcal{H}$ and $x, y \in \mathcal{L} \cup \mathcal{A}$, then at least one of the two conditions holds:

- $p_{\hat{s}x}^h \leq p_{fx}^h$ and $p_{\hat{s}y}^h \leq p_{sy}^h$ for all $s \neq \hat{s}$;
- $p_{\bar{s}x}^h = p_{fy}^h$ for all $s \neq \hat{s}$.

The hypothesis (H3) is clearly weaker than the hypothesis that for all $h \in \mathcal{H}$, (H1) or (H2) holds. However, its meaning is more difficult to interpret. If (H1) holds for $h \in \mathcal{H}$, then the leader \hat{s} is competitive at this time slot and offer cheaper prices than his competitors. If (H2) holds, then the competitors offer prices that are not buyer-dependent, and thus do not discriminate among the ILAs.

Let us recall from Subsection 4.2.4 that we defined two situations: the competitive case, and nondiscriminant exogenous pricing. Clearly, if any of these two situations is true, then the assumptions of Proposition 4.4.1 are fulfilled.

Proposition 4.4.1 is crucial in our characterization of GNEs at the intermediary level. The following proposition ensures that we can restrain the search of an optimal GNE in the set $GNE^o(\mathbf{p})$, which is the set of GNEs at the intermediary level where all energy exchanges among ILAs are equal to zero.

Proposition 4.4.3. *In a best response, optimistic context, we assume that:*

- $\mathbf{p} = \{p_{\hat{s}x} \mid x \in \mathcal{L} \cup \mathcal{A}\}$ is an optimal leader price profile;
- $S(\mathbf{p}) = \left\{ \left(\mathbf{e}_{xy}, \mathbf{e}_{xs}, \mathbf{p}_{xy}, \mathbf{d}^a, \hat{\lambda}^a \right) \mid x, y \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S}, a \in \mathcal{A} \right\}$ is a GNE among the ILAs with relation to \mathbf{p} that is optimal for the leader;
- for all $h \in \mathcal{H}$, (H1) or (H2) of Proposition 4.4.1 holds;

- for each $h \in \mathcal{H}$, the function $(\mathbf{e}_{x\hat{s}}^h)_{x \in \mathcal{L} \cup \mathcal{A}} \mapsto \sum_{x \in \mathcal{L} \cup \mathcal{A}} p_{\hat{s}x}^h e_{x\hat{s}}^h - c_{\hat{s}}^h \left(\sum_{x \in \mathcal{L} \cup \mathcal{A}} e_{x\hat{s}}^h \right)$ is increasing with relation to each variable;

then one can construct a leader price profile \mathbf{p}^* and a GNE $S^*(\mathbf{p}^*)$ such that $\mathbf{e}_{xy}^* = 0$ for all $x, y \in \mathcal{L} \cup \mathcal{A}$, and $z(S(\mathbf{p})) = z(S^*(\mathbf{p}^*))$, where z denotes the objective function of \hat{s} . That is, \mathbf{p}^* is an optimal price profile for the leader too.

Proof. Let h belong to \mathcal{H} , and for all $x \in \mathcal{L} \cup \mathcal{A}$, denote by $p_{x,\min}^h = \min \{p_{\hat{s}x}^h, p_{\bar{s}x}^h\}$ the lowest price offered to x by the suppliers.

Recall that from Proposition 4.4.1, if $e_{xy}^h > 0$, then $p_{yx}^h = \max \{p_{x,\min}^h, p_{y,\min}^h\}$. Furthermore, if $e_{xy}^h = 0$, then $p_{yx}^h \geq p_{x,\min}^h$, otherwise we would have a contradiction with $S(\mathbf{p})$ being a GNE, since x would be able to purchase cheaper energy from y than from the suppliers and thus unilaterally improve his situation by changing his energy source. Therefore, every actor $x \in \mathcal{L} \cup \mathcal{A}$ buys his energy at price $p_{x,\min}^h$, either from the supplier(s) offering this price, or from the other ILAs.

Claim:

For $x, y \in \mathcal{L} \cup \mathcal{A}$, if $p_{x,\min}^h < p_{y,\min}^h$, then $e_{xy}^h = 0$.

Proof:

Assume that $e_{xy}^h > 0$. It follows from Proposition 4.4.1 that $p_{yx}^h = p_{y,\min}^h > p_{x,\min}^h$, hence x has access to cheaper energy from at least one supplier than from y . It is therefore no optimal solution for x to buy energy from y and there is a contradiction with $S(\mathbf{p})$ being a GNE. Hence $e_{xy}^h = 0$ for all $x, y \in \mathcal{L} \cup \mathcal{A}$ such that $p_{x,\min}^h < p_{y,\min}^h$. Thus the claim is proved. ■

Claim:

If $S'(\mathbf{p})$ is a feasible solution for the followers satisfying

1. for all $x, y \in \mathcal{L} \cup \mathcal{A}$, if $p_{x,\min}^h \leq p_{y,\min}^h$, then $p_{xy}^h = p_{y,\min}^h$, and if $p_{x,\min}^h < p_{y,\min}^h$, then $p_{yx}^h > p_{x,\min}^h$,
2. for all $x \in \mathcal{L} \cup \mathcal{A}$, if $e_{xy}^h > 0$ for $y \in \mathcal{S} \cup \mathcal{L} \cup \mathcal{A}$, then $p_{yx}^h = p_{x,\min}^h$,
3. for all $\ell \in \mathcal{L}$,

$$d_\ell^h = \sum_{s \in \mathcal{S}} e_{\ell s}^h + \sum_{x \in \mathcal{L} \cup \mathcal{A}} (e_{\ell x}^h - e_{x\ell}^h) = \sum_{s \in \mathcal{S}} e_{\ell s}^h + \sum_{x \in \mathcal{L} \cup \mathcal{A}} (e_{\ell x}^h - e_{x\ell}^h) = d_\ell^h,$$

4. for all $i \in \mathcal{I}$, $d_i^h = d_i^h$ and $\hat{\lambda}_i^h = \hat{\lambda}_i$,

i.e. every local agent and every end user has the same demand in $S'(\mathbf{p})$ and in $S(\mathbf{p})$, then it is a GNE.

Proof:

By definition, $S'(\mathbf{p})$ is a GNE if no ILA $x \in \mathcal{L} \cup \mathcal{A}$ can unilaterally improve the value of his objective function.

The first condition ensures that $x \in \mathcal{L} \cup \mathcal{A}$ cannot buy cheaper energy from an ILA than from the cheapest supplier, and that y does not sell energy at a price too low to be profitable: otherwise, one has a contradiction with the fact that $S'(\mathbf{p})$ is a GNE. The second condition makes sure that x indeed buys the energy he needs at the lowest possible price. It is therefore not possible for x to improve his objective value by modifying his energy purchases while keeping the same consumption \mathbf{d}_x . On the other hand, the second condition makes sure that if x sells energy, he sells it at the maximum possible price. Hence x cannot improve his objective value by increasing his prices.

We thus deduce that the only way for x to strictly improve his objective value would imply a modification of his energy consumption \mathbf{d}_x . Let us assume that it is possible, with demand \mathbf{d}''_x . Since the energy sales of x are decision variables of the other ILAs and x buys all his energy at price $p_{x,\min}^h$, the difference in the revenue function of x is as follows:

- if x is a local agent,

$$\sum_{h \in \mathcal{H}} \left(p_{x,\min}^h (d_x^{''h} - d_x^{h}) + V_x^h (d_x^{''h}) - V_x^h (d_x^h) \right) < 0,$$

- if x is an aggregator,

$$\sum_{h \in \mathcal{H}} \left(p_{x,\min}^h (d_x^{''h} - d_x^h) + \sum_{i \in \mathcal{I}_x} (r_{xi}^{''h} d_i^{''h} - r_{ji}^h d_i^h) \right) < 0,$$

$$\text{with } r_{xi}^{''h} = 2v_i^h (d_i^{''h} - d_i^{h,0}) + \hat{\lambda}_i'' \text{ and } r_{xi}^h = 2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i'.$$

However in this case, x could also unilaterally strictly improve the value of his objective function in $S(\mathbf{p})$. If x is a local agent, it would be enough to set $\mathbf{d}_x = \mathbf{d}''_x$ and increase, respectively decrease, his energy purchases at time $h \in \mathcal{H}$ by $(d_x^{''h} - d_x^h)$. The difference in the revenue function of x is the same as with $S'(\mathbf{p})$, since $\mathbf{d}_x = \mathbf{d}'_x$. If x is an aggregator, observe from the proof of Proposition 4.3.1 that for a given demand of an end user \mathbf{d}_i^x , there is a unique optimal corresponding $\hat{\lambda}_i$. It follows that replacing \mathbf{d}_i^x and $\hat{\lambda}_i^x$ by $\mathbf{d}_i^{''x}$ and $\hat{\lambda}_i^{''x}$, and increasing/decreasing the energy purchases at time $h \in \mathcal{H}$ correspondingly finally yields the same revenue difference as previously, since $\mathbf{d}_x = \mathbf{d}'_x$ and thus $\hat{\lambda}_i = \hat{\lambda}_i'$. It follows from this that if x could unilaterally strictly improve the value of his objective function in $S'(\mathbf{p})$, so could he in $S(\mathbf{p})$, which is a contradiction with $S(\mathbf{p})$ being a GNE. Therefore, $S'(\mathbf{p})$ is a GNE and Claim 2 is proved.

■

Observe now that without loss of generality, $S(\mathbf{p})$ satisfies the conditions of Claim 4.4.2. Whereas conditions 3 and 4 are obviously verified, conditions 1 and 2 need not be valid. However, assume that condition 1 is not verified. Then there exist $x, y \in \mathcal{L} \cup \mathcal{A}$ with $p_{x,\min}^h \leq p_{y,\min}^h$ such that one of the two following cases holds true:

- $p_{xy}^h \neq p_{y,\min}^h$: it follows from the proof of Lemma 3 that $p_{xy}^h \leq p_{y,\min}^h$, thus here the inequality is strict. If $d_y^h + \sum_{x' \neq y} e_{x'y}^h > 0$, then $e_{x'y}^h > 0$ for x' offering the cheapest price to y , which is smaller than $p_{y,\min}^h$ by the assumption. This constitutes a contradiction with Lemma 3.

Now, if $d_y^h + \sum_{x' \neq y} e_{x'y}^h = 0$, then $e_{yx}^h = 0$, thus the value of p_{xy}^h can be increased up to $p_{y,\min}^h$ without modifying the value of the objective function of x . The value of the objective function of y is not modified either, and y does not get cheaper energy with the increased p_{xy}^h . Hence y cannot unilaterally strictly improve the value of his objective function with $p_{xy}^h = p_{y,\min}^h$, and $S(\mathbf{p})$ with $p_{xy}^h = p_{y,\min}^h$ is a GNE as well.

- $p_{yx}^h < p_{x,\min}^h$: this case is similar as the previous one, considering the two cases where $d_x^h + \sum_{x' \neq x} e_{x'x}^h > 0$ and $d_x^h + \sum_{x' \neq x} e_{x'x}^h > 0$. The same reasoning leads to the same conclusions.

As a result of this reasoning, we conclude that without loss of generality, $S(\mathbf{p})$ satisfies the conditions of Claim 4.4.2.

We say that there is a *cycle* in a followers' GNE $S(\mathbf{p})$ if there are ILA x_1, \dots, x_n such that $e_{x_i x_{i+1}}^h > 0$ for $i = 1, \dots, n$, with $x_{n+1} = x_1$.

Claim:

There exists a GNE $S'(\mathbf{p})$ such that there is no cycle.

Proof:

Assume there is a cycle (x_1, \dots, x_n) , and define $e = \min_{i=1, \dots, n} e_{x_i x_{i+1}}^h$. Observe that the first Claim implies that for any $i, j = 1, \dots, n$, $p_{x_i, \min}^h = p_{x_j, \min}^h$. Set $e_{x_i x_{i+1}}^h = e_{x_i x_{i+1}}^h - e$ for $i = 1, \dots, n$. The followers' answer that results $S'(\mathbf{p})$ clearly satisfies the conditions of Claim 2, therefore it is a GNE, concluding the proof of Claim 3.

■

Notice that, in the proof of Claim 4.4.2, no variable of the form e_{xs}^h with $x \in \mathcal{L} \cup \mathcal{A}$, $s \in \{\hat{s}, \bar{s}\}$ is different in $S'(\mathbf{p})$ and $S(\mathbf{p})$; therefore the leader's profit is constant and $S'(\mathbf{p})$ is a followers' answer that is optimal for the leader.

Claim 4.4.2 allows us to assume that $S(\mathbf{p}) = S(\mathbf{p}^*)$ has no cycle. Therefore, either there is at least one ILA $x \in \mathcal{L} \cup \mathcal{A}$ such that $0 < \sum_{y \in \mathcal{L} \cup \mathcal{A}} e_{yx}^h \leq e_{x\hat{s}}^h + e_{x\bar{s}}^h$, or

there are no energy exchanges among the ILAs in $S(\mathbf{p}^*)$. In the first case, choose y such that $e_{yx}^h > 0$. It follows from Claim 4.4.2 that $p_{y,\min}^h \geq p_{x,\min}^h$. Therefore, setting $e_{y\hat{s}}^h = e_{y\hat{s}}^h + e_{yx}^h$ and $e_{x\hat{s}}^h$ and $e_{x\bar{s}}^h$ such that $e_{x\hat{s}}^h + e_{x\bar{s}}^h = e_{x\hat{s}}^h + e_{x\bar{s}}^h - e_{yx}^h$ and $e_{x\hat{s}}^h \geq e_{x\hat{s}}^h - e_{yx}^h$ will yield a new followers' answer with $e_{yx}^h = 0$. Thanks to Claim 4.4.2, it is a GNE. Furthermore, the leader's profit cannot decrease in the process, because whenever an ILA buys less energy from \hat{s} , another ILA buys more energy (in greater or equal quantities) to a greater or equal price. Iterating this process will finally yield a GNE $S^*(\mathbf{p}^*)$ where $e_{xy}^h = 0$ for all $x, y \in \mathcal{L} \cup \mathcal{A}$, which concludes our proof. \square

It follows from Proposition 4.4.3 that a solution of (TDSM) can be obtained by simply solving the following revisited optimistic single-leader multi-follower problem ($P_{\hat{s}}^o$):

$$(P_{\hat{s}}^o) \quad \max_{\mathbf{p}_{\hat{s}x}} \max_{\mathbf{e}_x, \mathbf{d}^a, \hat{\lambda}^a} \sum_{h \in \mathcal{H}} \left(\sum_{x \in \mathcal{L} \cup \mathcal{A}} p_{\hat{s}x}^h e_{x\hat{s}}^h - c_{\hat{s}}^h \left(\sum_{x \in \mathcal{L} \cup \mathcal{A}} e_{x\hat{s}}^h \right) \right)$$

$$s.t. \quad \begin{cases} p_{\hat{s}x}^h \geq 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \\ (\mathbf{e}_{\ell\hat{s}}, \mathbf{e}_{\ell\bar{s}}) \in \operatorname{argmax} (P_{\ell}^{el}) & \forall \ell \in \mathcal{L} \\ (\mathbf{e}_{a\hat{s}}, \mathbf{e}_{a\bar{s}}, \mathbf{d}^a, \hat{\lambda}^a) \in \operatorname{argmax} (P_a^{el}) & \forall a \in \mathcal{A}, \end{cases}$$

where (P_x^{el}) denotes the optimization problem of $x \in \mathcal{L} \cup \mathcal{A}$ without energy exchanges (el stands for exchangeless), that is for $x \in \mathcal{L}$:

$$(P_x^{el}) \quad \min_{\mathbf{e}_x} \sum_{h \in \mathcal{H}} \left(p_{\hat{s}x}^h e_{x\hat{s}}^h + p_{\bar{s}x}^h e_{x\bar{s}}^h + V_k^h (e_{x\hat{s}}^h + e_{x\bar{s}}^h) \right)$$

$$s.t. \quad \begin{cases} \sum_{h \in \mathcal{H}} e_{x\hat{s}}^h + e_{x\bar{s}}^h = W_x \\ e_{x\hat{s}}^h, e_{x\bar{s}}^h \geq 0 & \forall h \in \mathcal{H}, \end{cases}$$

and for $x \in \mathcal{A}$, (P_x^{el}) is as follows:

$$(P_x^{el}) \quad \min_{\mathbf{e}_x, \mathbf{d}_i, \hat{\lambda}_i} \sum_{h \in \mathcal{H}} \left(p_{\hat{s}x}^h e_{x\hat{s}}^h + p_{\bar{s}x}^h e_{x\bar{s}}^h + \sum_{i \in \mathcal{I}_x} 2d_i^h v_i^h (d_i^h - d_i^{h,0}) \right) + \sum_{i \in \mathcal{I}_x} W_i \hat{\lambda}_i$$

$$s.t. \quad \begin{cases} \sum_{i \in \mathcal{I}_x} d_i^h = e_{x\hat{s}}^h + e_{x\bar{s}}^h & \forall h \in \mathcal{H} \\ \sum_{h \in \mathcal{H}} d_i^h = W_i & \forall i \in \mathcal{I}_x \\ d_i^h, e_{x\hat{s}}^h, e_{x\bar{s}}^h \geq 0 & \forall h \in \mathcal{H}, i \in \mathcal{I}_x \\ 2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i \geq 0 & \forall h \in \mathcal{H}, i \in \mathcal{I}_x. \end{cases}$$

4.4.3 Semi-optimistic approach

The developments of Section 4.3 and Subsection 4.4.2 show that the optimistic case of (TDSM) can be reformulated as a lighter bilevel problem, which can be efficiently solved by commercial solvers after a transformation into a MPCC. However, the optimistic approach is aptly named: if the follower has multiple solutions for a given decision of the leader, his decisions are assumed to be favorable to the leader, which might be too optimistic. At the other extreme, a possible pessimistic approach might be too pessimistic: if the follower has multiple solutions for a given decision of the leader, he will choose the least favorable for the leader. This is why we now consider a third case in between, which we call *semi-optimistic*. For the rest of this section, we will assume that we are in the competitive case, that is the leader can take over the whole electricity demand for cheaper prices than the competitor.

In the semi-optimistic case, we assume that the GNE among the ILAs will be of a specific type: at time $h \in \mathcal{H}$, the only ILA that buys energy from the leader \hat{s} is the ILA x that enjoys the lowest price, $\min_{x' \in \mathcal{A} \cup \mathcal{L}} p_{\hat{s}x'}^h \stackrel{not.}{=} p_{\min}^h$. The ILA y then buys the energy he needs from x at price $p_{\hat{s}y}^h$ (instead of \hat{s} , who offers the same price, in the optimistic case). Let us call this class of GNE $GNE^{so}(\mathbf{p})$. To motivate this choice, consider a simple case with a single time slot h and two local agents, x and y . Furthermore, assume that $p_{\hat{s}x}^h < p_{\hat{s}y}^h$, $W_x = 0$, $W_y = 1$. From the constraints of x , $p_{xy}^h \in [p_{\hat{s}x}^h, p_{\hat{s}y}^h]$. Observe that if $p_{xy}^h < p_{\hat{s}y}^h$, the optimal solution for y consists in setting $e_{yx}^h = 1$ and $e_{y\hat{s}}^h = 0$. However, if $p_{xy}^h = p_{\hat{s}y}^h$, the set of possible solutions for y is the set

$$\left\{ \left(e_{y\hat{s}}^h = \alpha, e_{yx}^h = 1 - \alpha \right) \mid \alpha \in [0, 1] \right\}.$$

With the classical optimistic approach, the decision taken by y would be $e_{y\hat{s}}^h = 1$ and $e_{yx}^h = 0$. The semi-optimistic approach implies that the function $e_{yx}^h(p_{xy}^h)$ that determines the optimal value of e_{yx}^h in (P_y) in function of p_{xy}^h is left-continuous in $p_{xy}^h = p_{\hat{s}y}^h$.

Another important notion to justify the semi-optimistic approach is the order in which the decisions are taken. Usually, bilevel models are used to model the response of a follower to decisions of a leader, which implies that the decisions of the leader are taken before the decisions of the follower. Consider the example given in the previous paragraph: if x sets his price p_{xy}^h to any value in $]p_{\hat{s}x}^h, p_{\hat{s}y}^h[$, he will get a strictly larger objective than with $p_{xy}^h = p_{\hat{s}y}^h$ in the optimistic approach. The semi-optimistic approach proposes a way to avoid this.

To solve (TDSM) with the semi-optimistic approach, remember that at time $h \in \mathcal{H}$, the leader \hat{s} sells energy at price p_{\min}^h , whereas each ILA x buys his energy at price p_{fx}^h . Thus we can determine an optimal profile of prices of the leader with

the following reformulation ($P_{\hat{s}}^{so}$):

$$(P_{\hat{s}}^{so}) \quad \max_{\mathbf{p}_{\min}, \mathbf{p}_{\hat{s}x}} \max_{\mathbf{e}_x, \mathbf{d}^a, \hat{\lambda}^a} \sum_{h \in \mathcal{H}} p_{\min}^h \left(\sum_{x \in \mathcal{A} \cup \mathcal{L}} e_{x\hat{s}}^h \right) - c_{\hat{s}}^h \left(\sum_{x \in \mathcal{A} \cup \mathcal{L}} e_{x\hat{s}}^h \right)$$

$$s.t. \quad \begin{cases} p_{\min}^h \leq p_{\hat{s}x}^h & \forall h \text{ in } \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \\ p_{\hat{s}x}^h \leq p_{sx}^h & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \\ (\mathbf{e}_{\ell\hat{s}}, \mathbf{e}_{\ell\bar{s}}) \in \operatorname{argmax} (P_{\ell}^{el}) & \forall \ell \in \mathcal{L} \\ (\mathbf{e}_{a\hat{s}}, \mathbf{e}_{a\bar{s}}, \mathbf{d}^a, \hat{\lambda}^a) \in \operatorname{argmax} (P_a^{el}) & \forall a \in \mathcal{A}, \end{cases}$$

where P_x^{el} is the same as in the previous subsection.

4.5 Numerical results

Based on the three different formulations of the best response problem of (TDSM) (the MPCC, the revisited optimistic and the semi-optimistic formulations), we provide in this section some numerical experiments to compare these approaches. The *competitive case* and the *noncompetitive case* are considered separately. The optimization problems resulting of the three reformulations lead to nonconvex problems with complementarity constraints and thus only local solutions will be obtained.

4.5.1 Competitive case

Spot market prices usually range in Europe from 0.01 to 0.08 € per kWh, whereas typical prices offered by a supplier to consumers start at 0.1 € per kWh. To get a more precise idea of what happens when this is not the case, see Subsection 4.5.2. This assumption allows us to get rid of the variables related to the competitors by setting the competitor's prices as upper bounds for the leader's prices. Therefore, we actually solve the following problems:

- With the classical approach, we solve ($P_{\hat{s}}^{c,KKT}$) (c stands for competitive):

$$(P_{\hat{s}}^{c,KKT}) \quad \max_{\mathbf{p}_{\hat{s}}} \max_{\mathbf{e}, \mathbf{p}_x, \boldsymbol{\alpha}, \mathbf{d}, \hat{\lambda}} \sum_{h \in \mathcal{H}} \left(\sum_{x \in \mathcal{L} \cup \mathcal{A}} p_{\hat{s}x}^h e_{x\hat{s}}^h - c_{\hat{s}}^h \left(\sum_{x \in \mathcal{L} \cup \mathcal{A}} e_{x\hat{s}}^h \right) \right)$$

$$s.t. \quad \begin{cases} 0 \leq p_{\hat{s}x}^h \leq p_{sx}^h & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \\ \{\mathbf{e}_{\ell}, \mathbf{p}_{\ell}, \boldsymbol{\alpha}_{\ell}\} \in \operatorname{KKT} (P_{\ell}^c) & \forall \ell \in \mathcal{L} \\ \{\mathbf{e}_a, \mathbf{p}_a, \boldsymbol{\alpha}_a, \mathbf{d}^a, \hat{\lambda}^a\} \in \operatorname{KKT} (P_a^c) & \forall a \in \mathcal{A}, \end{cases}$$

where $\operatorname{KKT} (P_{\ell}^c)$ and $\operatorname{KKT} (P_a^c)$ denote the solution sets of the KKT conditions respectively associated to local agents and aggregators. More precisely,

KKT (P_ℓ^c) is the set of tuples $(\mathbf{e}, \mathbf{p}, \boldsymbol{\alpha}, \boldsymbol{\mu}, \boldsymbol{\lambda})$ satisfying the primal feasibility, dual feasibility, stationarity and complementary slackness constraints related to problem (P_ℓ^c) , which is similar to (P_ℓ) , but with $\mathcal{S} = \{\hat{s}\}$. KKT (P_a^c) is the set of tuples $(\mathbf{e}, \mathbf{d}, \mathbf{p}, \boldsymbol{\alpha}, \boldsymbol{\mu}, \boldsymbol{\lambda})$ satisfying the KKT conditions of (P_a^c) , which is (P'_a) with $\mathcal{S} = \{\hat{s}\}$. The KKT conditions of (P_ℓ) and (P'_a) in the general case ($\mathcal{S} = \{\hat{s}, \bar{s}\}$) can be found in Subsection 4.3.

- With the optimistic reformulation, we solve $(P_{\hat{s}}^{o,KKT})$:

$$(P_{\hat{s}}^{o,KKT}) \quad \max_{\mathbf{p}^{\hat{s}x}} \max_{\mathbf{e}_x, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a, \boldsymbol{\mu}, \boldsymbol{\lambda}} \sum_{h \in \mathcal{H}} \left(\sum_{x \in \mathcal{L} \cup \mathcal{A}} p_{\hat{s}x}^h e_{x\hat{s}}^h - c_{\hat{s}}^h \left(\sum_{x \in \mathcal{L} \cup \mathcal{A}} e_{x\hat{s}}^h \right) \right)$$

$$s.t. \quad \begin{cases} 0 \leq p_{\hat{s}x}^h \leq p_{\bar{s}x}^h & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \\ (\mathbf{e}_\ell, \boldsymbol{\mu}_\ell, \boldsymbol{\lambda}_\ell) \in \text{KKT}(P_\ell^{el}) & \forall \ell \in \mathcal{L} \\ (\mathbf{e}_a, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a, \boldsymbol{\mu}_a, \boldsymbol{\lambda}_a) \in \text{KKT}(P_a^{el}) & \forall a \in \mathcal{A}, \end{cases}$$

where $\text{KKT}(P_x^{el})$ denotes the KKT conditions of the optimization program of $x \in \mathcal{L} \cup \mathcal{A}$ without energy exchanges. These KKT conditions can be found in A.1.

- For the semi-optimistic reformulation, we search solutions of $(P_{\hat{s}}^{so,KKT})$:

$$(P_{\hat{s}}^{so,KKT}) \quad \max_{\mathbf{p}^{\min}, \mathbf{p}^{\hat{s}x}} \max_{\mathbf{e}_x, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a, \boldsymbol{\mu}, \boldsymbol{\lambda}} \sum_{h \in \mathcal{H}} (p_{\min}^h - k_f^h) \left(\sum_{x \in \mathcal{A} \cup \mathcal{L}} d_x^h \right)$$

$$s.t. \quad \begin{cases} 0 \leq p_{\hat{s}x}^h \leq p_{\bar{s}x}^h & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \neq s' \in \mathcal{S} \\ p_{\min}^h \leq p_{\hat{s}x}^h & \forall x \in \mathcal{L} \cup \mathcal{A} \\ (\mathbf{e}_\ell, \boldsymbol{\mu}_\ell, \boldsymbol{\lambda}_\ell) \in \text{KKT}(P_\ell^{el}) & \forall \ell \in \mathcal{L} \\ (\mathbf{e}_a, \mathbf{d}^a, \hat{\boldsymbol{\lambda}}^a, \boldsymbol{\mu}_a, \boldsymbol{\lambda}_a) \in \text{KKT}(P_a^{el}) & \forall a \in \mathcal{A}, \end{cases}$$

where $\text{KKT}(P_x^{el})$ represents the ILA x 's program without exchanges. Observe that actually, the only differences between the optimistic and the semi-optimistic reformulations lie in the presence of p_{\min}^h and the adaptation of the leader's objective function.

We solved these MPCCs with the Knitro solver [Byrd 2006] included in the demo version of GAMS 24.9.2 [Gen 2017].

4.5.1.1 Classical optimistic versus revisited optimistic

The main interest of the revisited optimistic formulation lies in its low number of variables and constraints, compared to the classical optimistic formulation. Furthermore, the classical optimistic formulation allows degenerate cases: given tuples

$(\mathbf{e}_{x\cdot}, \mathbf{p}_{x\cdot}, \boldsymbol{\alpha}, (\mathbf{d}, \hat{\boldsymbol{\lambda}}), \boldsymbol{\mu}, \boldsymbol{\lambda}) \in KKT(P_x^{cl})$ for all ILAs, it is possible to build infinitely many tuples satisfying the same KKT conditions by setting $e_{xy}^{lh} = e_{xy}^h + c$, $e_{yx}^{lh} = e_{yx}^h + c$, $\mu_{pxy}^{lh} = \mu_{pxy}^h + c$ and $\mu_{pyx}^{lh} = \mu_{pyx}^h + c$ for every pair of ILAs (x, y) such that $p_{sx}^h = p_{sy}^h$, with $c > 0$. Observe that such new tuples $(\mathbf{e}'_{x\cdot}, \mathbf{p}_{x\cdot}, \boldsymbol{\alpha}, (\mathbf{d}, \hat{\boldsymbol{\lambda}}), \boldsymbol{\mu}', \boldsymbol{\lambda})$ imply the same energy consumption for the local agents and the end users, and the same energy sales for the suppliers. With the revisited optimistic formulation of the problem, there are no exchanges among the ILAs. Therefore, such degenerate cases are avoided. Note that it would be possible to allow unidirectional exchanges only by adding the following constraints:

$$e_{xy}^h \cdot e_{yx}^h = 0 \quad \forall x, y \in \mathcal{L} \cup \mathcal{A}.$$

However, this method not only increases the number of constraints, but also increases the number of necessary dual variables and complementarity constraints.

As proved in Subsection 4.4.2, the classical optimistic and the revisited optimistic formulations should yield the same optimal value for the objective function of the leader. Concerning the number of variables and constraints, a quick analysis of the resulting MPCCs gives, for the classical approach:

$$\begin{aligned} \#\{\text{variables}\} &= 2I + L + H(3I + 4A + 5L + 8A^2 + 16AL + 8L^2) \\ &= \Theta(H(I + A^2 + L^2)) \\ \#\{\text{constraints}\} &= 2I + L + H(4I + 4A + 5L + 10A^2 + 20AL + 10L^2) \\ &= \Theta(H(I + A^2 + L^2)), \end{aligned}$$

where $H = |\mathcal{H}|$, $I = |\mathcal{I}|$, $A = |\mathcal{A}|$, and $L = |\mathcal{L}|$. For the revisited optimistic formulation, these numbers reduce to:

$$\begin{aligned} \#\{\text{variables}\} &= 2I + L + H(3I + A + 3L) = \Theta(H(I + A + L)) \\ \#\{\text{constraints}\} &= 2I + L + H(4I + A + 3L) = \Theta(H(I + A + L)). \end{aligned}$$

Whereas for small instances (3 time slots, 2 local agents, 1 aggregator in contract with 2 end users), the difference between both methods might not be significant (282 variables, 342 constraints for the classical method versus 45 variables, 51 constraints for the revisited optimistic formulation), the classical method becomes intractable as soon as instances grow larger (24 time slots, 10 aggregators with 1 end user each, 10 local agents give 79710 variables and 99150 constraints, whereas the revisited optimistic formulation only needs 1710 variables and 1950 constraints).

These differences in terms of number of variables and constraints are experimentally confirmed in Table 4.1 summarizing some instances that have been tested.

This table also highlights the computational improvement (number of iterations and time) associated to the revisited formulation. The numerical experiments have been conducted on a laptop equipped with a processor Intel i7-4600u at 2.1 GHz and 16 GB of RAM.

Instance	#LA	Problem	#var	#cons	#iterations	time (s)
1	2	opt-classic	282	342		failed
1	2	opt-revis.	45	51	17048	3,789
2	1	opt-classic	146	176	31769	46,039
2	1	opt-revis.	35	41	23165	6,928
3	1	opt-classic	146	176	56357	61,371
3	1	opt-revis.	35	41	29487	5,164

Table 4.1: All the tests reported here involve three hours, one aggregator, and two end users. All the tests but one end up with a local optimum with a null gap. Each instance corresponds to a given set of parameters (production cost, competitor’s price, base demand, inconvenience coefficients). The parameters of the instances can be found in A.2.

The first tests below aim to study the sensitivity of the profit to the inconvenience coefficient. More precisely, we study the cases of one single local agent, and of one aggregator with a single end user. Since in both cases, there is only one actor, both approaches prove to be efficient and yield the same results, up to minor differences that can be explained by the margin of error of the commercial solver. The results of the computations can be found in Figures 4.2 and 4.3.

In both cases, we consider three time slots with base demand (4, 14, 14) kWh, which correspond to the demand of approximately ten households over one day (divided in three periods of 8 hours). The costs of energy for the leader were (0.02, 0.06, 0.05) € per kWh, and the competitor’s prices were assumed to be constant at 0.14 € per kWh. The inconvenience coefficients are supposed to be the same for the three time periods and vary between 10^{-6} and 0.1.

Several comments need to be highlighted. First, the energy consumption varies from the base demand to a complete shift of the whole demand on one time slot. The larger the inconvenience, the lesser the shift. When the price incentive is enough to result in a complete shift, the price curve rises towards the competitor’s price: when the inconvenience coefficient decreases, a smaller incentive is needed to

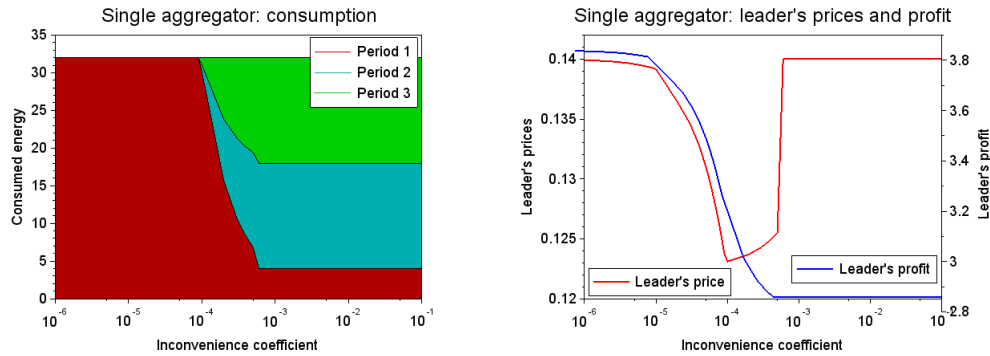


Figure 4.2: Situation with one aggregator

At left, the shift induced by the leader's prices. At right, the price offered by the leader during the first time period and the leader's profit.

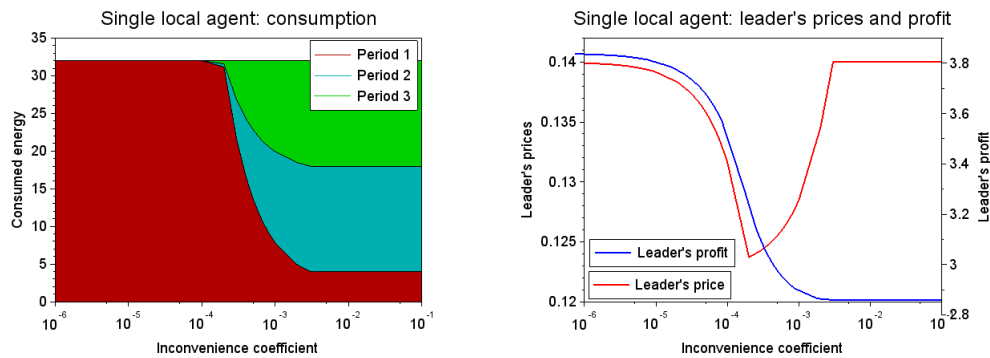


Figure 4.3: Situation with one local agent

At left, the shift induced by the leader's prices. At right, the price offered by the leader during the first time period and the leader's profit.

cause a similar shift. Then, the leader's profit increases as the inconvenience coefficients decrease. However, there is a noticeable difference between aggregators and local agents. In the aggregator case, there is a threshold: when the inconvenience coefficient reaches a certain value (around 0.0005 in this experiment), it becomes worth to incentivize load shifting with a large price difference, whereas the optimal price curve for local agents is smoother. Finally, observe that the actual interesting inconvenience coefficients are located in a narrow interval: between 10^{-4} and 10^{-2} . An inconvenience coefficient smaller than 10^{-4} will lead to higher billing costs for the consumer, whereas a coefficient larger than 10^{-2} will lead to an inability of the leader to imply load shifting with price incentives only.

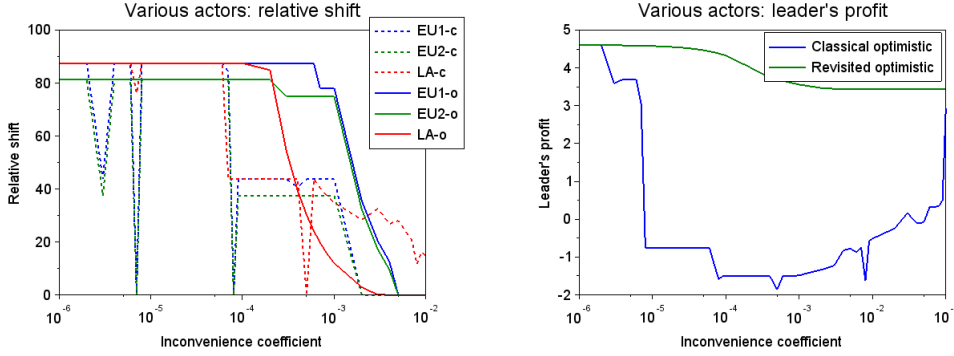


Figure 4.4: Situation with various actors, one local agent and one aggregator in contract with two end users

At left, the relative load shifting implied by the leader's prices for the various actors with the classical approach (c) and with the optimistic reformulation (o). At right, the profit obtained with the classical approach and with the optimistic reformulation.

After basic examples with only one ILA, we consider the case with one local agent and one aggregator, the latter in contract with two end users. The numerical results can be found in Figure 4.4. On the right hand side, the objective function of the leader is represented as a function of the inconvenience coefficients, assumed to be the same for all the actors. On the left hand side, the curves indicate the relative shift in percents, according to the following formula:

$$rs_x(\mathbf{v}_x) = \frac{\sum_{h \in \mathcal{H}} |d_x^h - d_x^{h,0}|}{2W_x},$$

where x stands for the local agent and the two end users. This value represents the percentage of energy that is consumed at another time slot than with the base demand.

These results were obtained with the multi-start option of Knitro. For each set of parameters, hundred starting points were automatically generated by the solver, and the best solution was returned. In all cases, the commercial solver returned a locally optimal solution. However, it is clear from Figure 4.4 that the local optima found with the classical method correspond to lower values of the objective function of the leader than the ones obtained with the local optima of the revisited optimistic formulation, and thus from the global optima of the problem. Hence the local optima found with the revisited optimistic reformulation are much better candidates for global optimality. This can be observed on the leader's profit graph:

in most cases, the leader's profit obtained with the classical method is far from the leader's profit obtained with the revisited optimistic formulation. The relative shifts obtained with the classical method give interesting informations as well. Whereas in this setting, the shift should be monotonously decreasing as the inconvenience coefficient increases, it is clearly not the case. Quite the opposite, the relative shift curves obtained with the classical method are chaotic compared to the relative shift curves obtained with the revisited optimistic formulation.

Finally note that the local optima of the MPCC obtained with the revisited optimistic formulation should have equivalent local optima (in the sense of same leader prices and consumers' demands) of the MPCC obtained with the classical method. However, the solver is unable to find these equivalent local optima. All in all, the formulation of the revisited optimistic case presented in Subsection 4.4.2 proves to be much more efficient: the MPCC have by far less variables and constraints, and the solver is able to find much better local optima. The difficulties to obtain meaningful results with the classical method build a strong argument for a thorough analysis of the lower level.

4.5.1.2 Revisited optimistic versus semi-optimistic

As observed in the previous subsection, the revisited optimistic formulation is computationally much better than the classical approach. Focusing on a subset of possible GNE at the lower level allows a great decrease in the number of necessary variables and constraints, which leads to better performances of the commercial solver, in terms of solving time and of quality of the local optima. We next discuss the advantages of the semi-optimistic formulation compared to the revisited optimistic one.

First, observe that the leader's objective value is always lower with the semi-optimistic approach than with the revisited optimistic one. Indeed, for any given price profile \mathbf{p} of the leader, the corresponding GNE in $GNE^o(\mathbf{p})$ yields a larger objective for the leader than the GNE in $GNE^{so}(\mathbf{p})$, since in the semi-optimistic case, at time $h \in \mathcal{H}$, all the energy is sold at the lowest offered price p_{\min}^h . This is not the case in the revisited optimistic setting, where the energy bought by each client $x \in \mathcal{L} \cup \mathcal{A}$ is sold by the leader at price $p_{sx}^h \geq p_{\min}^h$. Thus the interest of the semi-optimistic approach does not lie in its rough performance, but in its relative robustness towards the followers' reaction. The prices obtained with the semi-optimistic reformulation indeed ensure a lower bound for the leader's revenue under the mild assumption that the followers will buy energy from the leader instead of the competitors in the case of equal prices, whereas the prices obtained with the optimistic formulation only consider the leader's revenue in the most favorable case.

To have a meaningful comparison between the revisited optimistic and the semi-optimistic formulations, we need to compute the leader's revenue in the following configurations:

1. optimistic prices with the followers' response in $GNE^o(\mathbf{p})$, which yields a revenue z_o^o for the leader,
2. optimistic prices with the followers' response in $GNE^{so}(\mathbf{p})$, with revenue z_{so}^o for the leader,
3. semi-optimistic prices with the followers' response in $GNE^{so}(\mathbf{p})$, with revenue z_{so}^{so} for the leader.

The way z_o^o and z_{so}^{so} are computed is clear: it is enough to solve $(P_{\hat{s}}^{o,KKT})$ and $(P_{\hat{s}}^{so,KKT})$, respectively. For z_{so}^o however, there is more to do than just solve a MPCC. The prices and the energy consumption of the ILAs are the same as in the (revisited) optimistic case and are thus determined by solving $(P_{\hat{s}}^{o,KKT})$. Nevertheless, if we assume that the followers' GNE belongs to $GNE^{so}(\mathbf{p})$, then the ILA that gets the cheapest price at time $h \in \mathcal{H}$ from \hat{s} buys all the energy that is consumed at time h . Therefore, to compute z_{so}^o , we compute the following formula:

$$\sum_{x \in \mathcal{L} \cup \mathcal{A}} p_{\min}^h d_x^h - c_{\hat{s}}^h \left(\sum_{x \in \mathcal{L} \cup \mathcal{A}} d_x^h \right),$$

where $p_{\min}^h = \min_{x \in \mathcal{L} \cup \mathcal{A}} p_{\hat{s}x}^h$ is the minimum price offered to the ILAs at time h by the leader, and for all $x \in \mathcal{L} \cup \mathcal{A}$, $\mathbf{p}_{\hat{s}x}$ and \mathbf{d}_x are determined by solving $(P_{\hat{s}}^{o,KKT})$. This represents the revenue of the leader if the followers, as an answer to the prices $\mathbf{p}_{\hat{s}x}$, choose the GNE defined as follows: if \hat{x} is (one of) the ILA getting his electricity at price p_{\min}^h , then $e_{\hat{x}\hat{s}}^h = \sum_{x \in \mathcal{L} \cup \mathcal{A}} d_x^h$, $e_{x\hat{s}}^h = 0$ for $x \neq \hat{x}$, for $x, y \neq \hat{x}$, $e_{x\hat{x}}^h = d_x^h$ and $e_{xy}^h = 0$, and finally, $p_{xy}^h = \max\{p_{\hat{s}x}^h, p_{\hat{s}y}^h\}$. This is actually a worst-case scenario for the leader, under the assumption that no electricity is bought from the competitor. Nevertheless, it is a possible answer of the followers, since it is a GNE. Therefore, still under the assumption that no energy is bought from the competitor, GNE that are answers to the prices $\mathbf{p}_{\hat{s}}$ computed with $(P_{\hat{s}}^{o,KKT})$ can possibly take all values between z_o^o and z_{so}^o , whereas an answer to prices $\mathbf{p}_{\hat{s}}$ computed with $(P_{\hat{s}}^{so,KKT})$ will yield a revenue that is at least z_{so}^{so} . These values share a common ground idea with the *rewarding* and *deceiving* solutions defined in [Alves 2016].

Clearly,

$$z_o^o \geq z_{so}^{so} \geq z_{so}^o.$$

The question here is: how bad can the optimistic prices perform when the followers choose a GNE in $GNE^{so}(\mathbf{p})$, compared to the semi-optimistic prices? To prove the

interest of the semi-optimistic approach, we study a very simple case: three time slots and two local agents x and y with respective demands $(0, 10, 0)$ and $(0, 0, 10)$. Their inconvenience coefficients are as follows: $\mathbf{v}_x = (v, v, v)$ and $\mathbf{v}_y = (100, v, v)$, with v varying between 10^{-6} and 0.1 . Setting $v_y^1 = 100$ means that no energy should be consumed by y at the first time slot. The spot market prices are supposed to be $(0.01, 0.05, 0.08)$, so that the leader should incentivize its clients to consume energy as early as possible. This means that for the leader, an ideal consumption of x and y would be $(10, 0, 0)$ and $(0, 10, 0)$, respectively, considering that y should not consume during the first time slot. The competitor's prices are assumed to remain constant at $(0.14, 0.14, 0.14)$ as in the previous examples.

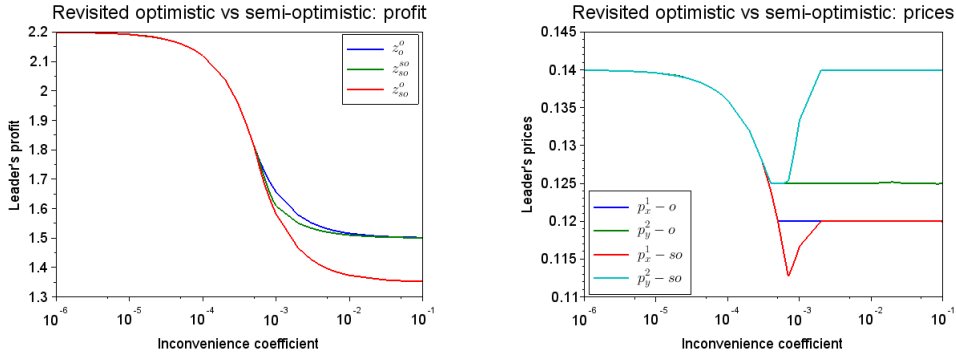


Figure 4.5: Comparison optimistic vs semi-optimistic reformulations

At left, the different profits: z_o^o , z_{so}^{so} and z_{so}^o . At right, the prices p_x^1 and p_y^2 with the optimistic (o) and with the semi-optimistic (so) reformulations.

It follows from the design of our example that we will have

$$p_{\hat{s}y}^1 = p_{\hat{s}x}^2 = p_{\hat{s}x}^3 = p_{\hat{s}y}^3 = 0.14.$$

Indeed, the leader does not want x or y to consume energy during these periods, therefore his price will take the highest possible value, that is $p_{\hat{s}x}^h = p_{\hat{s}y}^h = 0.14$ for all $h \in \mathcal{H}$. The prices that will directly influence the electricity consumption of x and y are thus the prices $p_{\hat{s}x}^1$ and $p_{\hat{s}y}^2$. In the revisited optimistic case, x and y buy their energy directly from the leader. In the semi-optimistic case, x will buy his energy from y during the second time slot, thus during this time slot, all the energy will be bought by y from the leader at price $p_{\hat{s}y}^2$. The results are presented in Figure 4.5. To the left, the leader's profit is represented in three cases: the blue curve depicts the result of the revisited optimistic formulation, the green curve the result of the semi-optimistic formulation, and finally the red curve illustrates the leader's profit when the leader's prices are computed with the optimistic reformulation, but the

GNE of the followers is in $GNE^{so}(\mathbf{p})$. The semi-optimistic formulation yields prices that are by far more robust, in the sense that for these prices, any followers' answer will yield at least the computed profit (provided that the followers favour the leader over the competitor in case of equal prices). By contrast, the prices computed with the (revisited) optimistic approach will yield the computed profit only if the followers' answer is the computed one (except for degenerate cases). An other possible followers' answer (in particular a semi-optimistic GNE) might provide less profit. Therefore, the prices computed with this formulation ensure a certain profit in a worst-case scenario. Furthermore, the difference between the profits obtained with the revisited optimistic z_o^o or the semi-optimistic formulations z_{so}^{so} are not very large, as it can be seen in Figure 4.5. Therefore, computing robust prices implies lower revenues in an optimal case (i.e. the followers' GNE is in $GNE^o(\mathbf{p})$), but ensures much better results in worst cases.

Let us end this subsection with a table summarizing some instances that have been tested. Note that in the tests presented above, the number of iterations and

Instance	#hours	#EU	#LA	Problem	#var	#cons	#iterations	time (s)
4	3	2	2	opt-revis.	45	51	17048	3,789
4	3	2	2	semi-opt.	48	60	22045	5,745
5	3	3	3	opt-revis.	66	75	29895	13,341
5	3	3	3	semi-opt.	69	87	48095	13,696
6	4	3	3	opt-revis.	85	97	17176	8,146
6	4	3	3	semi-opt.	89	113	14630	6,698
7	4	6	6	opt-revis.	166	190	20140	14,613
7	4	6	6	semi-opt.	170	218	28499	23,421
8	4	8	8	opt-revis.	220	252	31846	39,883
8	4	8	8	semi-opt.	224	288	41156	42,437

Table 4.2: All the tests reported here involve one aggregator. All the tests end up at a local optimum with a null gap. Each instance corresponds to a given set of parameters (production cost, competitor's price, base demand, inconvenience coefficients). The parameters of the instances can be found in A.2.

the computational time are always higher for the semi-optimistic approach than

for the revisited optimistic approach, except for instance 6. The behavior on this particular instance might be due to an usual structure of the instance, or more probably to the solving procedure used by Knitro.

4.5.2 Noncompetitive case

In the previous subsection, we assumed that the leader is competitive at any time, that is the competitor's price is larger than the production costs for any time slot. However, this highly depends on the studied data. In order to enlighten the shifting effect of the leader's pricing policy, we present a last example in which the leader is not always competitive. The data and results of the example are presented in Figure 4.6: we consider two time slots, and a single local agent with demand $(5, 5)$. Furthermore, the inconvenience coefficients $v^1 = v^2$ take three different values: 0.001, 0.002 and 0.003. These three values cause three very different situations: with 0.003, the inconvenience is too large to allow any shift and the optimal prices of the leader are similar to the competitor's prices. With 0.001, the inconvenience is small enough to cause, combined with the values of the optimal leader prices, a shift of the whole demand on the first time slot. The last case is intermediate: only a portion of the demand is shifted with the optimal leader prices.

First, observe that we are in an optimistic setting. The results presented here are therefore obtained with the revisited optimistic formulation. It is impossible for the leader to make any profit during the second time slot, because during this time slot, the leader's energy cost (factor of the linear cost function) is higher than the competitor's price (see top of Figure 4.6). Therefore, the optimal leader prices are greater than the competitor's prices during the second time slot, thus the energy that is consumed by the client at this time slot is bought from the competitor. The concern for the leader is to give an incentive to the client to shift his demand to the first time slot, where the leader can make some profit. In the base case, the profit achieved is $(0.07 - 0.01) * 5 = 3$: no optimization is done at all, the leader's prices are equal to the competitor's prices and thus the optimal follower's demand is the base demand. If the leader's price is too high, the follower will not shift his demand. If the leader's price is too low, the follower will shift his demand to a great extent, but the demand increase will not compensate the price decrease. The cases $v^h = 0.001$ and $v^h = 0.003$ are extreme: either the client is very shifting-averse, which leads to an optimal leader's price equal to the competitor's price and no shift, or the client undergoes a very low inconvenience with any shift, which leads to the leader offering a price small enough to induce a shift of the whole follower's demand to the first time slot. Finally, the case $v^h = 0.002$ is probably the most interesting: the client agrees to shift some of his demand, but not everything: exactly 12.5% of

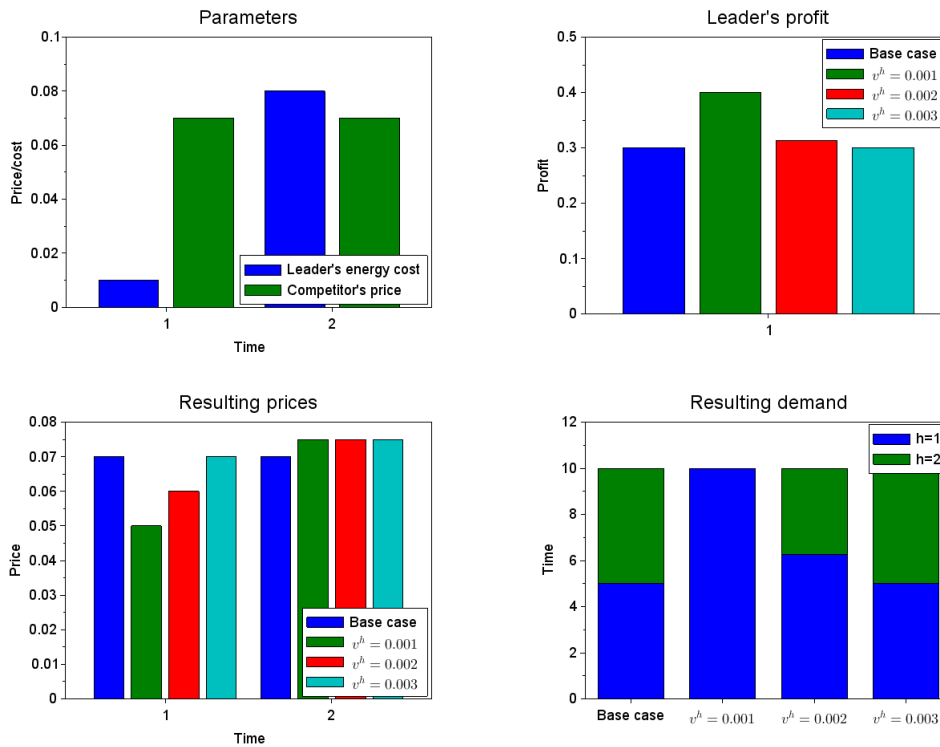


Figure 4.6: Noncompetitive case

On top left, the data for the example where the leader is not always competitive: the production cost and the competitor's prices. On top right, the resulting profit of the leader for the four various cases: no optimization (i.e. copying the competitor's prices), and optimization for the three possible values of v^h .

At bottom left, the optimal prices of the leader for the example where the leader is not always competitive. At bottom right, the follower's demand resulting of these prices.

his demand is shifted from the second to the first time slot. In this case, the leader's profit rises to 3.125, which represents an improvement of 4.16% of the leader's profit. This case can be considered as realistic. A shift of 12.5% of the demand does not seem too difficult to achieve for the client, and the leader's profit increases enough to be worth the effort of computing the optimal prices.

4.6 Conclusion

In this paper, we have defined a trilevel energy pricing model for Demand-Side Management. First, by explicit resolution of the lowest level, we reduced it to a

new bilevel problem. Besides the classical optimistic approach consisting in using first order reformulation of the lower level of this bilevel problem (the classical optimistic approach), we defined two new concepts of solutions of the problem. Both of those new approaches are based on a selection of special GNE of the lower level of the new model. The first one, called revisited optimistic, selects special GNE of the lower level of the new model while, theoretically, maintaining the optimal value of the leader. Actually our numerical experiments enlighten the fact that the revisited optimistic approach is clearly more stable than the classical approach, the latter providing often a lower optimal value of the supplier. Moreover the revisited formulation proved to be much more efficient than the classical method in computational terms (number of variables, constraints and thus solving time). Further analysis of the exchanges among the ILA allowed us to define the second new concept of solution, called semi-optimistic. The special kind of GNE considered at the lower level of the new bilevel model corresponds to an intermediate between optimistic and pessimistic concepts for the leader's problem. The numerical results obtained with the semi-optimistic approach have shown its robustness towards the followers' response to the leader's decisions while the optimal value for the supplier is only slightly impacted, thus furnishing a good compromise concept of solution. Note that the reformulations of the initial single-leader multi-follower game lead to nonconvex optimization problems with quite complex constraints. Thus the optimization tools that are used here can only reach local solutions (at best). The development of specific and adapted optimization techniques would be needed to go further and it is out of the scope of this work.

As a conclusion, this work provides strong arguments for a careful theoretical analysis of multilevel models for DSM pricing as well as numerical experiments.

Further works might imply various notions of equilibria at the lower level. Here, we chose to consider Nash equilibria. However, a more realistic approach might be to consider market equilibria in which an energy transaction is the result of a bargaining between the buyer and the seller, and not the sole fact that the buyer decides the quantity and the seller the price.

Multi-Leader Multi-Follower

In this chapter, the (TDSM) problem considered in Chapter 4 is reconsidered as a multi-leader multi-follower game. Instead of aiming to find the best response of an energy supplier to the decisions of other suppliers, we investigate here the situation in which all the suppliers are competing in a noncooperative way. Considering such a competition at the upper level makes the problem structure more complex. Therefore, to make the problem tractable, both from a theoretical and numerical point of view, the aggregators (and thus also their end users) are not considered here. This leads to a bilevel structure, with Nash-type competitions at the upper level among the suppliers and at the lower level among the local agents who also interact in a noncooperative way. Furthermore, the model of the local agents is slightly modified in order to be adapted to a situation of dynamic competition. This gives rise to a bilevel demand-side management problem denoted by (BDSM).

5.1 Definition of the model

Two kinds of actors are involved in the (BDSM) problem: electricity suppliers $s \in \mathcal{S}$ and local agents $\ell \in \mathcal{L}$. Interactions among actors of (BDSM) are described in Figure 5.1 and are of the following nature:

- For each period of time the suppliers propose electricity prices to the local agents,
- The local agents can buy electricity from the suppliers or from the other local agents.

Let us now define the variables and notations of the model:

- The set of the considered time slots is denoted by $\mathcal{H} = \{1, \dots, H\}$.
- Bold variables indicate vectors.
- $\mathbf{d}_\ell = (d_\ell^1, \dots, d_\ell^{|\mathcal{H}|})$ denotes the demand variables of the local agent $\ell \in \mathcal{L}$.
- Variables p_{xy}^h denote the prices of an energy unit sold by x to y at time $h \in \mathcal{H}$, the agent x being in $\mathcal{S} \cup \mathcal{L}$ while y is a local agent.

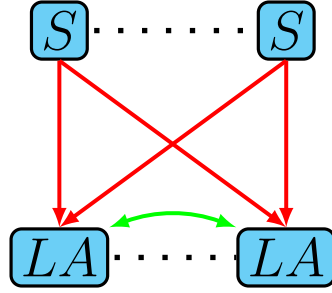


Figure 5.1: Blue forms represent the various actors (the suppliers and the local agents), red links show unidirectional energy trades, and green links designate bidirectional energy exchanges.

- Variables e_{xy}^h represent the energy flow from y to x (bought by x from y) at time h , the agent y being an element of $\mathcal{S} \cup \mathcal{L}$ and x belonging to \mathcal{L} .
- W_ℓ indicates the overall demand that needs to be satisfied for $\ell \in \mathcal{L}$.

5.1.1 Local agents

Local agents can not only buy energy from the suppliers, but also trade energy with the other local agents. Each local agent $\ell \in \mathcal{L}$ has a fixed overall demand W_ℓ for the horizon and an *a priori* demand splitting/vector $(d_\ell^{h,0})_h$. But this demand can be shifted, resulting in the *real* demand splitting/vector $(d_\ell^h)_h$. The bought energy is used either to satisfy the electricity demand or to sell a part to their peers. In case of shifting of their consumption, the local agents undergo an inconvenience increasing with the extent of the load shift. They aim to minimize the sum of their electricity bill and the inconvenience due to the load shift. Formally, the local agent $\ell \in \mathcal{L}$ aims to solve the following problem (P_ℓ):

$$(P_\ell) \quad \min_{e_\ell, p_\ell, \alpha_\ell} \sum_{h \in \mathcal{H}} \left(\sum_{s \in \mathcal{S}} p_{s\ell}^h e_{\ell s}^h + \sum_{\ell' \neq \ell} (p_{\ell'\ell}^h e_{\ell\ell'}^h - p_{\ell\ell'}^h e_{\ell'\ell}^h) + V_\ell^h(d_\ell^h) \right)$$

$$s.t. \quad \begin{cases} \sum_{h \in \mathcal{H}} d_\ell^h = W_\ell \\ e_{\ell x}^h \geq 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{S}, \\ p_{\ell\ell'}^h \leq \alpha_{\ell\ell' s s'}^h p_{s\ell}^h + (1 - \alpha_{\ell\ell' s s'}^h) p_{s'\ell'}^h & \forall \ell \neq \ell' \in \mathcal{L}, s, s' \in \mathcal{S} \\ \alpha_{\ell\ell' s s'}^h \in [0, 1] & \forall h \in \mathcal{H}, \ell \neq \ell' \in \mathcal{L}, s, s' \in \mathcal{S}, \end{cases}$$

where $V_\ell^h(d_\ell^h) = v_\ell^h (d_\ell^h - d_\ell^{h,0})^2$ is the inconvenience caused by the load shifting and $v_\ell^h > 0$ is a fixed inconvenience coefficient. Note that, for any h , the notation

d_ℓ^h stands for the demand value and thus $d_\ell^h \stackrel{\text{not.}}{=} \sum_{s \in \mathcal{S}} e_{\ell s}^h + \sum_{\ell' \neq \ell} (e_{\ell \ell'}^h - e_{\ell' \ell}^h)$. The inconvenience function V_ℓ^h is the same as defined in [Gkatzikis 2013]. This function has nice mathematical properties, like convexity and differentiability, and it adequately models the real inconvenience that the consumers are undergoing: a small shift of the consumption will not represent a significant inconvenience, whereas an important shift will have strong repercussions on the consumer's comfort thanks to the square power.

The first constraint in problem (P_ℓ) ensures that ℓ 's cumulated demand still cover, after shifting, the overall demand, whereas the second constraint avoids negative energy exchanges. The two other constraints result from a deeper observation for which explanations have been given in Chapter 4. The second level of (BDSM) actually constitutes a noncooperative Nash game, where the players are the local agents. Therefore, an optimal local agents' response to the leaders' decisions is a Nash equilibrium. Since the decision variables of the local agents influence the feasible domains of the other local agents, it is precisely a generalized Nash equilibrium (GNE). Let us recall that a GNE is reached whenever no player can unilaterally improve his situation by modifying his decision variables (see e.g. [Facchinei 2007]). The following proposition (analogous to Lemma 4.4.1) allows a proper characterization of the lower-level problem.

Proposition 5.1.1. *Given a leader price profile $\mathbf{p} = (\mathbf{p}_s)_{s \in \mathcal{S}}$, where \mathbf{p}_s denotes the price vector $\mathbf{p}_s = (p_{s\ell}^h)_{\substack{h \in \mathcal{H} \\ \ell \in \mathcal{L}}}$, and a lower-level GNE*

$$S(\mathbf{p}) = \left\{ e_{\ell x}^h, p_{\ell \ell'}^h, \alpha_{\ell \ell' s s'}^h \mid h \in \mathcal{H}, \ell, \ell' \in \mathcal{L}, s, s' \in \mathcal{S}, x \in \mathcal{S} \cup \mathcal{L} \setminus \{\ell\} \right\}.$$

If $e_{\ell \ell}^h > 0$ for $h \in \mathcal{H}$, $\ell, \ell' \in \mathcal{L}$, then

$$p_{\ell \ell'}^h = \max \left\{ \min_{s \in \mathcal{S}} p_{s\ell}^h, \min_{s \in \mathcal{S}} p_{s\ell'}^h \right\}.$$

Furthermore, if $e_{\ell \ell}^h = 0$ for $h \in \mathcal{H}$ and $\sum_{x \in \mathcal{S} \cup \mathcal{L} \setminus \{\ell\}} e_{\ell x}^h > 0$, $\ell, \ell' \in \mathcal{L}$, then

$$p_{\ell \ell'}^h \geq \min_{s \in \mathcal{S}} p_{s\ell'}^h.$$

Actually, the constraints on the prices are designed so that this proposition holds. What is the highest price that ℓ can offer to ℓ' , so that ℓ' buys from ℓ and not from one of the suppliers? Clearly, it is $\min_{s \in \mathcal{S}} p_{s\ell}^h$. However, setting constraints

$$p_{\ell \ell'}^h \leq p_{s\ell'}^h \quad \forall s \in \mathcal{S}$$

is not sufficient: if $\min_{s \in \mathcal{S}} p_{s\ell}^h > \min_{s \in \mathcal{S}} p_{s\ell'}^h$, ℓ would be forced to sell energy at a price that would be too low for him to make profits, as discussed in Section 4.2.1.

Therefore, one would like

$$p_{\ell\ell'}^h = \max \left\{ \min_{s \in \mathcal{S}} p_{s\ell}^h, \min_{s \in \mathcal{S}} p_{s\ell'}^h \right\}$$

to hold for all $\ell, \ell' \in \mathcal{L}$, $h \in \mathcal{H}$. To obtain this, the constraints

$$\begin{aligned} p_{\ell\ell'}^h &\leq \alpha_{\ell\ell'}^h p_{s\ell'}^h + (1 - \alpha_{\ell\ell'}^h) p_{s\ell}^h \quad \forall h \in \mathcal{H}, \ell' \in \mathcal{L}, s \in \mathcal{S} \\ \alpha_{\ell\ell'}^h &\in [0, 1] \quad \forall h \in \mathcal{H}, \ell \neq \ell' \in \mathcal{L} \end{aligned} \quad (5.1)$$

of local agent ℓ from Section 4.2.1 are replaced by the more numerous

$$\begin{aligned} p_{\ell\ell'}^h &\leq \alpha_{\ell\ell'ss'}^h p_{s\ell}^h + (1 - \alpha_{\ell\ell'ss'}^h) p_{s'\ell'}^h \quad \forall \ell \neq \ell' \in \mathcal{L}, s, s' \in \mathcal{S} \\ \alpha_{\ell\ell'ss'}^h &\in [0, 1] \quad \forall h \in \mathcal{H}, \ell \neq \ell' \in \mathcal{L}, s, s' \in \mathcal{S}. \end{aligned} \quad (5.2)$$

The reason for this is that the hypotheses (H1) or (H2) used in Chapter 4 are not necessary for Lemma 5.1.1 to hold with the new constraints. These two hypotheses indeed only make sense in a best response context, with only two suppliers: they are assumptions on the competitor's prices, and can therefore not be assumed in a dynamic competition context. Therefore, the new constraints on the prices have to be introduced.

Proof. First, assume that there are $h \in \mathcal{H}$ and $\ell, \ell' \in \mathcal{L}$ such that $e_{\ell'\ell}^h > 0$. The only constraints on this price are

$$p_{\ell\ell'}^h \leq \alpha_{\ell\ell'ss'}^h p_{s\ell}^h + (1 - \alpha_{\ell\ell'ss'}^h) p_{s'\ell'}^h \quad \forall \ell \neq \ell' \in \mathcal{L}, s, s' \in \mathcal{S}.$$

Denote by $s_{\min}, s'_{\min} \in \mathcal{S}$ the suppliers such that $p_{s_{\min}\ell}^h = \min_{s \in \mathcal{S}} p_{s\ell}^h$ and $p_{s'_{\min}\ell'}^h = \min_{s' \in \mathcal{S}} p_{s'\ell'}^h$. As an immediate consequence, one has

$$p_{\ell\ell'}^h \leq \alpha_{\ell\ell's_{\min}s'_{\min}}^h p_{s_{\min}\ell}^h + (1 - \alpha_{\ell\ell's_{\min}s'_{\min}}^h) p_{s'_{\min}\ell'}^h \leq \bar{p}$$

where $\bar{p} = \max \{ p_{s_{\min}\ell}^h, p_{s'_{\min}\ell'}^h \}$. Moreover, since $S(\mathbf{p})$ is a GNE and $e_{\ell'\ell}^h > 0$, the local agent ℓ chooses the highest possible price for $p_{\ell\ell'}^h$, and therefore the above inequality is an equality.

It directly follows from this observation that suitable values for the $\alpha_{\ell\ell'ss'}^h$ yield the highest possible value for $p_{\ell\ell'}^h$ of $\max \{ p_{s_{\min}\ell}^h, p_{s'_{\min}\ell'}^h \}$. In that case, $p_{\ell\ell'}^h$ has to take the highest possible value.

For the second part of the proof, assume by contradiction that there are $h \in \mathcal{H}$ and $\ell, \ell' \in \mathcal{L}$ such that $e_{\ell'\ell}^h = 0$, $\sum_{x \in S \cup \mathcal{L} \setminus \{\ell'\}} e_{\ell'x}^h > 0$ and $p_{\ell\ell'}^h < \min_{s \in \mathcal{S}} p_{s\ell'}^h$. Observe that the first part of this proposition implies that the energy that ℓ' buys is sold at price $\min_{s \in \mathcal{S}} p_{s\ell'}^h$. There is a contradiction with $S(\mathbf{p})$ being a GNE, since ℓ' could unilaterally modify his energy purchases, and buy from ℓ at price $p_{\ell\ell'}^h < \min_{s \in \mathcal{S}} p_{s\ell'}^h$, which would induce an improvement of the objective value of ℓ' . \square

5.1.2 Electricity suppliers

The electricity suppliers $s \in \mathcal{S}$ sell energy to the local agents. They aim to maximize their profit, which is the difference between the revenues resulting from sales and their production costs, described by the function $c_s^h(\cdot)$. The electricity supplier influences the demand of the customers by setting hourly prices p_{sx}^h , knowing that the clients will react optimally to these prices. Therefore, the problem of the supplier $s \in \mathcal{S}$ can be expressed as:

$$(P_s) \quad \max_{\mathbf{p}_s} \max_{\mathbf{e}, \mathbf{p}_x, \boldsymbol{\alpha}} \sum_{h \in \mathcal{H}} \left(\sum_{\ell \in \mathcal{L}} p_{s\ell}^h e_{\ell s}^h - c_s^h \left(\sum_{\ell \in \mathcal{L}} e_{\ell s}^h \right) \right)$$

$$s.t. \quad \begin{cases} p_{s\ell}^h \geq 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L} \\ \{\mathbf{e}_\ell, \mathbf{p}_\ell, \boldsymbol{\alpha}_\ell\} \in \operatorname{argmax}(P_\ell) & \forall \ell \in \mathcal{L} \end{cases}$$

All along the paper we assume that the energy cost functions of the various suppliers are all linear, with coefficient c_s^h for the cost function of supplier $s \in \mathcal{S}$ at time $h \in \mathcal{H}$.

5.2 Solution methods

In this section, several methods to solve the problem presented in Section 5.1 are presented. First, we show that the lower-level GNEP possibly admits several equilibria, and that a selection among those can be made. This is the object of Proposition 5.2.1.

Proposition 5.2.1. *Assume that $\mathbf{p} = \left\{ \left(p_{s\ell}^h \right)_{h \in \mathcal{H}, \ell \in \mathcal{L}} \mid s \in \mathcal{S} \right\}$ is a price profile for the upper level, that*

$$S'(\mathbf{p}) = \left\{ e_{\ell x}^h, p_{\ell\ell'}^h, \alpha_{\ell\ell' s s'}^h \mid h \in \mathcal{H}, \ell, \ell' \in \mathcal{L}, s, s' \in \mathcal{S}, x \in \mathcal{S} \cup \mathcal{L} \setminus \{\ell\} \right\} \quad (5.3)$$

is a GNE of the low level associated to \mathbf{p} and that

$$S(\mathbf{p}) = \left\{ e_{\ell x}^h, p_{\ell\ell'}^h, \alpha_{\ell\ell' s s'}^h \mid h \in \mathcal{H}, \ell, \ell' \in \mathcal{L}, s, s' \in \mathcal{S}, x \in \mathcal{S} \cup \mathcal{L} \setminus \{\ell\} \right\} \quad (5.4)$$

is a feasible solution for the followers (associated to \mathbf{p}) that satisfies

1. *for all $\ell, \ell' \in \mathcal{L}$, if $\min_{s \in \mathcal{S}} p_{s\ell}^h \leq \min_{s \in \mathcal{S}} p_{s\ell'}^h$, then $p_{\ell\ell'}^h = \min_{s \in \mathcal{S}} p_{s\ell'}^h$ and if $\min_{s \in \mathcal{S}} p_{s\ell}^h < \min_{s \in \mathcal{S}} p_{s\ell'}^h$, then $p_{\ell\ell'}^h > \min_{s \in \mathcal{S}} p_{s\ell}^h$,*
2. *for all $\ell \in \mathcal{L}$, if $e_{\ell x}^h > 0$ for $x \in \mathcal{S} \cup \mathcal{L} \setminus \{\ell\}$, then $p_{x\ell}^h = \min_{s \in \mathcal{S}} p_{s\ell}^h$,*

3. for all $\ell \in \mathcal{L}$,

$$\sum_{s \in \mathcal{S}} e_{\ell s}^h + \sum_{\ell' \neq \ell} (e_{\ell \ell'}^h - e_{\ell' \ell}^h) = \sum_{s \in \mathcal{S}} e_{\ell s}^{h'} + \sum_{\ell' \neq \ell} (e_{\ell \ell'}^{h'} - e_{\ell' \ell}^{h'}),$$

Then, $S(\mathbf{p})$ is a lower-level GNE associated to \mathbf{p} .

The proof of this proposition follows the same lines as the proof of Claim 2 in Lemma 4.4.3, with Proposition 5.1.1 replacing 4.4.1, and without aggregators. It is thus not repeated here.

Observe that in the above proposition, condition 3) simply expresses that $d_\ell^h = d_\ell^{h'}$, for any $\ell \in \mathcal{L}$. It follows that, as soon as conditions 1) and 2) are satisfied, the fact that the feasible solution $S(\mathbf{p})$ is actually a GNE does not depend on the values of the exchange vectors \mathbf{e} , but on the demand values $d_\ell^h = \sum_{s \in \mathcal{S}} e_{\ell s}^h + \sum_{\ell' \neq \ell} (e_{\ell \ell'}^h - e_{\ell' \ell}^h)$. As a consequence, given a GNE of the low level problem associated to a price profile \mathbf{p} , one can thus construct another GNE for the follower's level in which there are no exchanges between local agents. Indeed, starting from a GNE $S'(\mathbf{p})$ as defined in 5.3, for all $\ell \in \mathcal{L}$, $h \in \mathcal{H}$ set $e_{\ell s}^{h'}$ such that

$$\sum_{s \in \mathcal{S}} e_{\ell s}^h = \sum_{s \in \mathcal{S}} e_{\ell s}^{h'} + \sum_{\ell' \neq \ell} (e_{\ell \ell'}^{h'} - e_{\ell' \ell}^{h'}),$$

with $e_{\ell s}^h > 0$ only if $p_{s\ell}^h = \min_{s' \in \mathcal{S}} p_{s'\ell}^h$, and

$$e_{\ell \ell'}^h = 0 \quad \forall h \in \mathcal{H}, \ell \neq \ell' \in \mathcal{L}.$$

Furthermore, set the variables $\alpha_{\ell \ell' s s'}^h$ and $p_{\ell \ell'}^h$ for all $h \in \mathcal{H}$, $\ell \neq \ell' \in \mathcal{L}$, so that condition 1) is satisfied. Then according to Proposition 5.2.1, $S(\mathbf{p})$ as defined in 5.4 is a GNE. Therefore, it is possible to make a selection among the lower level GNEs and assume that the followers' response does not include any exchange among local agents, as in the revisited optimistic approach of Chapter 5.

This gives rise to simpler followers' problems where problem (P_ℓ) of local agent $\ell \in \mathcal{L}$ is replaced by

$$\begin{aligned} (P_\ell^{el}) \quad & \min_{\mathbf{e}_\ell} \sum_{h \in \mathcal{H}} \left(\sum_{s \in \mathcal{S}} p_{s\ell}^h e_{\ell s}^h + V_\ell^h \left(\sum_{s \in \mathcal{S}} e_{\ell s}^h \right) \right) \\ & s.t. \begin{cases} \sum_{h \in \mathcal{H}} \sum_{s \in \mathcal{S}} e_{\ell s}^h = W_\ell & (z_\ell) \\ e_{\ell s}^h \geq 0 & (z_{\ell s}^h) \quad \forall h \in \mathcal{H}, s \in \mathcal{S}. \end{cases} \end{aligned}$$

Here, the exponent "el" stands for "exchangeless". The variables between parentheses are the dual variables associated to each constraint.

The KKT conditions of this simpler problem are as follows:

$$\left\{ \begin{array}{l} p_{s\ell}^h + 2v_\ell^h \left(\sum_{s' \in \mathcal{S}} e_{\ell s'}^h - d_\ell^{h,0} \right) - z_\ell - z_{\ell s}^h = 0 \quad \forall h \in \mathcal{H}, s \in \mathcal{S} \\ \sum_{h \in \mathcal{H}} \sum_{s \in \mathcal{S}} e_{\ell s}^h = W_\ell \\ e_{\ell s}^h \geq 0 \quad \forall h \in \mathcal{H}, s \in \mathcal{S} \\ z_{\ell s}^h \geq 0 \quad \forall h \in \mathcal{H}, s \in \mathcal{S} \\ e_{\ell s}^h \cdot z_{\ell s}^h = 0 \quad \forall h \in \mathcal{H}, s \in \mathcal{S}. \end{array} \right.$$

The two next subsections aim to solve the MLFG implied by the leaders' problems and the followers' simplified problems.

5.2.1 The method of Leyffer and Munson

In order to solve the above problems, we will first consider the now classical method described by Leyffer and Musson in [Leyffer 2010].

Following the description of the KKT conditions of the simplified followers' problems, each leader's problem (P) can be rewritten as:

$$\begin{array}{l} \max_{\mathbf{p}, \mathbf{e}, \mathbf{z}} \sum_{h \in \mathcal{H}} \left(\sum_{\ell \in \mathcal{L}} (p_{s\ell}^h e_{\ell s}^h) - c_s^h \left(\sum_{\ell \in \mathcal{L}} e_{\ell s}^h \right) \right) \\ \text{s.t.} \left\{ \begin{array}{l} p_{s'\ell}^h + 2v_\ell^h \left(\sum_{s'' \in \mathcal{S}} e_{\ell s''}^h - d_\ell^{h,0} \right) \\ \quad - z_\ell - z_{\ell s'}^h = 0 \quad (\mu_{\ell s'}^h) \quad \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ \sum_{h \in \mathcal{H}} \sum_{s' \in \mathcal{S}} e_{\ell s'}^h = W_\ell \quad (\mu_\ell) \quad \forall \ell \in \mathcal{L} \\ e_{\ell s'}^h \geq 0 \quad (\lambda_{\ell s'}^h) \quad \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ z_{\ell s'}^h \geq 0 \quad (\psi_{\ell s'}^h) \quad \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ e_{\ell s'}^h \cdot z_{\ell s'}^h = 0 \quad (\sigma_{\ell s'}^h) \quad \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ p_{s\ell}^h \geq 0 \quad (\chi_{s\ell}^h) \quad \forall h \in \mathcal{H}, \ell \in \mathcal{L} \end{array} \right. \end{array}$$

The dual variables associated to each constraint are written between parentheses. Note that here, the optimistic formulation of the MPEC is used (see [Dempe 2002] for further informations on the optimistic approach).

An MPEC (P) is said to satisfy the MPEC-LICQ if (P) without the complementarity constraints satisfies the LICQ. Such a constraint qualification for the previous

problem is necessary for the resolution method to be theoretically justified. However, it is not clear whether the problem indeed satisfies the MPEC-LICQ, thus in the remaining of the section, we only assume it does.

The next step consists in computing the KKT conditions for each leader $s \in \mathcal{S}$, and then concatenate the KKT conditions of all leaders. The KKT conditions of $s \in \mathcal{S}$ are as follows:

$$\left\{ \begin{array}{ll} p_{s'\ell}^h + 2v_\ell^h \left(\sum_{s'' \in \mathcal{S}} e_{\ell s''}^h - d_\ell^{h,0} \right) - z_\ell - z_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ \sum_{h \in \mathcal{H}} \sum_{s' \in \mathcal{S}} e_{\ell s'}^h = W_\ell & \forall \ell \in \mathcal{L} \\ e_{\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ z_{\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ e_{\ell s'}^h \cdot z_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ p_{s\ell}^h \geq 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L} \\ e_{\ell s}^h + \chi_{s\ell}^h - \mu_{\ell s}^h = 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L} \\ \delta_s^{s'} \left(p_{s'\ell}^h - c_{s'}^h \right) - 2v_\ell^h \sum_{s'' \in \mathcal{S}} \mu_{\ell s''}^h \\ \quad - \mu_\ell + \lambda_{\ell s'}^h - \sigma_{\ell s'}^h \cdot z_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ \sum_{h \in \mathcal{H}} \sum_{s' \in \mathcal{S}} \mu_{\ell s'}^h = 0 & \forall \ell \in \mathcal{L} \\ \mu_{\ell s'}^h + \psi_{\ell s'}^h - \sigma_{\ell s'}^h \cdot e_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L}, s' \in \mathcal{S} \\ \chi_{s\ell}^h \geq 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L} \\ \lambda_{\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L}, s' \in \mathcal{S} \\ \psi_{\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L}, s' \in \mathcal{S} \\ \chi_{s\ell}^h \cdot p_{s\ell}^h = 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L} \\ \lambda_{\ell s'}^h \cdot e_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L}, s' \in \mathcal{S} \\ \psi_{\ell s'}^h \cdot z_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L}, s' \in \mathcal{S}, \end{array} \right.$$

with

$$\delta_s^{s'} = \begin{cases} 1 & \text{if } s = s' \\ 0 & \text{otherwise.} \end{cases}$$

The above system corresponds to problem (7) in [Leyffer 2010].

Following Leyffer and Munson's method, the primal and dual variables of the follower (that are respectively \mathbf{e} and \mathbf{z}) are the same for all leaders. However, in

their first formulation of the MLMF game, they assume that the dual variables corresponding to the constraints of the follower are specific to each leader. This means that each leader has a set of variables μ , λ , σ associated to him. Therefore, the concatenation of all KKT conditions of the leaders yields the following NCP (equivalent to (11) or (15) in [Leyffer 2010]):

$$\left\{ \begin{array}{ll} p_{s'\ell}^h + 2v_\ell^h \left(\sum_{s'' \in \mathcal{S}} e_{\ell s''}^h - d_\ell^{h,0} \right) - z_\ell - z_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ \sum_{h \in \mathcal{H}} \sum_{s \in \mathcal{S}} e_{\ell s}^h = W_\ell & \forall \ell \in \mathcal{L} \\ e_{\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ z_{\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ e_{\ell s'}^h \cdot z_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ p_{s\ell}^h \geq 0 & \forall h \in \mathcal{H}, s \in \mathcal{S}, \ell \in \mathcal{L} \\ e_{\ell s}^h + \chi_{s\ell}^h - \mu_{s\ell s}^h = 0 & \forall h \in \mathcal{H}, s \in \mathcal{S}, \ell \in \mathcal{L} \\ \delta_s^{s'} \left(p_{s'\ell}^h - c_{s'}^h \right) - 2v_\ell^h \sum_{s'' \in \mathcal{S}} \mu_{s\ell s''}^h \\ \quad - \mu_{s\ell}^h + \lambda_{s\ell s'}^h - \sigma_{s\ell s'}^h \cdot z_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, s, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ \sum_{h \in \mathcal{H}} \sum_{s' \in \mathcal{S}} \mu_{s\ell s'}^h = 0 & \forall s \in \mathcal{S}, \ell \in \mathcal{L} \\ \mu_{s\ell s'}^h + \psi_{s\ell s'}^h - \sigma_{s\ell s'}^h \cdot e_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L}, s, s' \in \mathcal{S} \\ \chi_{s\ell}^h \geq 0 & \forall h \in \mathcal{H}, s \in \mathcal{S}, \ell \in \mathcal{L} \\ \lambda_{s\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, s, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ \psi_{s\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, s, s' \in \mathcal{S}, \ell \in \mathcal{L} \\ \chi_{s\ell}^h \cdot p_{s\ell}^h = 0 & \forall h \in \mathcal{H}, s \in \mathcal{S}, \ell \in \mathcal{L} \\ \lambda_{s\ell s'}^h \cdot e_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L}, s, s' \in \mathcal{S} \\ \psi_{s\ell s'}^h \cdot z_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L}, s, s' \in \mathcal{S}. \end{array} \right.$$

The set of multipliers in this case is not unique though. Therefore, the problem

can be rewritten as:

$$\begin{aligned}
& \min_{\mathbf{p}, \mathbf{e}, \mathbf{z}, \lambda, \psi, \sigma, \chi, \mu} \sum_{s \in \mathcal{S}} \sum_{\ell \in \mathcal{L}} \sum_{s' \in \mathcal{S}} \sum_{h \in \mathcal{H}} \sigma_{s\ell s'}^h \\
& \text{s.t.} \left\{ \begin{array}{ll}
p_{s'\ell}^h + 2v_\ell^h \left(\sum_{s'' \in \mathcal{S}} e_{\ell s''}^h - d_\ell^{h,0} \right) - z_\ell - z_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\
\sum_{h \in \mathcal{H}} \sum_{s \in \mathcal{S}} e_{\ell s}^h = W_\ell & \forall \ell \in \mathcal{L} \\
e_{\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\
z_{\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\
e_{\ell s'}^h \cdot z_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\
p_{s\ell}^h \geq 0 & \forall h \in \mathcal{H}, s \in \mathcal{S}, \ell \in \mathcal{L} \\
e_{\ell s}^h + \chi_{s\ell}^h - \mu_{s\ell s}^h = 0 & \forall h \in \mathcal{H}, s \in \mathcal{S}, \ell \in \mathcal{L} \\
\delta_s^{s'} \left(p_{s'\ell}^h - c_{s'}^h \right) - 2v_\ell^h \sum_{s'' \in \mathcal{S}} \mu_{s\ell s''}^h \\
\quad - \mu_{s\ell}^h + \lambda_{s\ell s'}^h - \sigma_{s\ell s'}^h \cdot z_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, s, s' \in \mathcal{S}, \ell \in \mathcal{L} \\
\sum_{h \in \mathcal{H}} \sum_{s' \in \mathcal{S}} \mu_{s\ell s'}^h = 0 & \forall s \in \mathcal{S}, \ell \in \mathcal{L} \\
\mu_{s\ell s'}^h + \psi_{s\ell s'}^h - \sigma_{s\ell s'}^h \cdot e_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L}, s, s' \in \mathcal{S} \\
\chi_{s\ell}^h \geq 0 & \forall h \in \mathcal{H}, s \in \mathcal{S}, \ell \in \mathcal{L} \\
\lambda_{s\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, s, s' \in \mathcal{S}, \ell \in \mathcal{L} \\
\psi_{s\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, s, s' \in \mathcal{S}, \ell \in \mathcal{L} \\
\chi_{s\ell}^h \cdot p_{s\ell}^h = 0 & \forall h \in \mathcal{H}, s \in \mathcal{S}, \ell \in \mathcal{L} \\
\lambda_{s\ell s'}^h \cdot e_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L}, s, s' \in \mathcal{S} \\
\psi_{s\ell s'}^h \cdot z_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L}, s, s' \in \mathcal{S} \\
\sigma_{s\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L}, s, s' \in \mathcal{S}.
\end{array} \right.
\end{aligned}$$

This formulation corresponds to problem (16) in [Leyffer 2010]. Observe that σ is assumed to be nonnegative, because the associated equality

$$e_{\ell s'}^h \cdot z_{\ell s'}^h = 0 \quad \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L}$$

can actually be rewritten as the inequality

$$e_{\ell s'}^h \cdot z_{\ell s'}^h \leq 0 \quad \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L},$$

because $e_{\ell s'}^h$ and $z_{\ell s'}^h$ are both positive for all $h \in \mathcal{H}$, $s' \in \mathcal{S}$ and $\ell \in \mathcal{L}$.

Finally, the last reformulation is equivalent to Problem (17) in Leyffer and Munson, where all complementarity constraints are included in the objective function:

$$\begin{aligned}
& \min_{\mathbf{p}, \mathbf{e}, \mathbf{z}, \boldsymbol{\lambda}, \boldsymbol{\psi}, \boldsymbol{\sigma}, \boldsymbol{\chi}, \boldsymbol{\mu}} \sum_{s \in \mathcal{S}} \sum_{\ell \in \mathcal{L}} \sum_{h \in \mathcal{H}} \left(e_{\ell s}^h \cdot z_{\ell s}^h + \chi_{s\ell}^h \cdot p_{s\ell}^h + \sum_{s' \in \mathcal{S}} \left(\sigma_{s\ell s'}^h + \lambda_{s\ell s'}^h \cdot e_{\ell s'}^h + \psi_{s\ell s'}^h \cdot z_{\ell s'}^h \right) \right) \\
& \text{s.t.} \left\{ \begin{array}{ll}
p_{s'\ell}^h + 2v_{\ell}^h \left(\sum_{s'' \in \mathcal{S}} e_{\ell s''}^h - d_{\ell}^{h,0} \right) - z_{\ell} - z_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\
\sum_{h \in \mathcal{H}} \sum_{s \in \mathcal{S}} e_{\ell s}^h = W_{\ell} & \forall \ell \in \mathcal{L} \\
e_{\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\
z_{\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, s' \in \mathcal{S}, \ell \in \mathcal{L} \\
p_{s\ell}^h \geq 0 & \forall h \in \mathcal{H}, s \in \mathcal{S}, \ell \in \mathcal{L} \\
e_{\ell s}^h + \chi_{s\ell}^h - \mu_{s\ell s}^h = 0 & \forall h \in \mathcal{H}, s \in \mathcal{S}, \ell \in \mathcal{L} \\
\delta_s^{s'} \left(p_{s'\ell}^h - c_{s'}^h \right) - 2v_{\ell}^h \sum_{s'' \in \mathcal{S}} \mu_{s\ell s''}^h \\
\quad - \mu_{s\ell}^h + \lambda_{s\ell s'}^h - \sigma_{s\ell s'}^h \cdot z_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, s, s' \in \mathcal{S}, \ell \in \mathcal{L} \\
\sum_{h \in \mathcal{H}} \sum_{s' \in \mathcal{S}} \mu_{s\ell s'}^h = 0 & \forall s \in \mathcal{S}, \ell \in \mathcal{L} \\
\mu_{s\ell s'}^h + \psi_{s\ell s'}^h - \sigma_{s\ell s'}^h \cdot e_{\ell s'}^h = 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L}, s, s' \in \mathcal{S} \\
\chi_{s\ell}^h \geq 0 & \forall h \in \mathcal{H}, s \in \mathcal{S}, \ell \in \mathcal{L} \\
\lambda_{s\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, s, s' \in \mathcal{S}, \ell \in \mathcal{L} \\
\psi_{s\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, s, s' \in \mathcal{S}, \ell \in \mathcal{L} \\
\sigma_{s\ell s'}^h \geq 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L}, s, s' \in \mathcal{S}.
\end{array} \right.
\end{aligned}$$

This method (and variants) has been implemented in GAMS [Gen 2017], and solved with Artelys Knitro 10.3.0 [Byrd 2006]. However, it yields unexpected results, even for tiny instances. For example, consider a problem setting with a single time slot, a single follower and two suppliers s^1 and s^2 . The single follower has a demand of one unit, that he has to buy from either s^1 or s^2 . The energy costs are assumed to be linear with coefficient 0 for s^1 and 1 for s^2 .

In this example, the expected results are as follows:

$$p_{s^1} = p_{s^2} = 1 \quad \text{and} \quad e_{s^1} = 1, e_{s^2} = 0. \quad (5.5)$$

It seems unnatural that s^2 could offer prices lower than 1, because he would sell with a loss, making 1 a lower bound for both p_{s^1} and p_{s^2} . However, if the prices are greater than 1, the suppliers selling less energy can lower his price by $\varepsilon > 0$ in order to tap the follower's consumption.

On this example, all variants of Leyffer and Munson's method were tested: in all cases, Knitro yielded $p_{s^1} = p_{s^2}$ and $e_{s^1} = 1, e_{s^2} = 0$. However, the values for the prices ranged from 0.5057 to 0.6814. It turns out that all of them are equilibria: neither s^1 nor s^2 can unilaterally improve his objective function. On one hand, if s^1 lowers his price, he obviously diminishes his profit. If he increases his price, the follower buys his energy from s^2 , canceling s^1 's profit. On the other hand, if s^2 increases his price, nothing changes for him, but if he lowers it, the follower will purchase his energy from s^2 at a price that is lower than s^2 's production costs, leading to a deadweight loss.

Although mathematically correct, such equilibria are economically unrealistic, which is why other solution methods are looked for. The observations made on this tiny example motivate the rest of this chapter.

5.2.2 Formulation as a GNEP

Since the solving method presented in the previous subsection does not yield the anticipated results, other possibilities have to be investigated. Fortunately, the local agents are very similar to the end users of Chapter 4. Therefore, it is possible to find explicit formulas for their optimal demand. This leads to a natural attempt to get rid of the lower level and transform the problem (BDSM) into a single level GNEP (see 2.2.2). Although GNEPs are difficult to handle, they are less complex than MLFGs.

5.2.2.1 Theoretical preliminaries

In order to reformulate the problem (BDSM) as a GNEP, some assumptions are made:

- A1) First, the problem is going to be handled as in [Gkatzikis 2013], that is the demands of the followers are assumed to be strictly positive at every time slot. This assumption allows to get rid of some dual variables.
- A2) For each time slot, define the sets

$$\Pi_h = \left\{ s \in \mathcal{S} \mid c_s^h \leq c_{s'}^h \quad \forall s' \in \mathcal{S} \right\} \quad \text{and} \quad \Pi_s = \{ h \in \mathcal{H} \mid s \in \Pi_h \}.$$

At each time slot $h \in \mathcal{H}$, it is assumed that $|\Pi_h| = 1$ and its single element is denoted by s_h . Furthermore, it is assumed that only s_h sells energy at time h , and that his prices are in the interval $\left[c_{s_h}^h, \min_{s' \neq s} c_{s'}^h \right]$.

This second assumption is based on the observations made on the tiny example at the end of Section 5.2.1. On one time slot, this assumption leads to the expected

Nash equilibrium described in Formula 5.5. Whether making this assumption is a good idea for larger problems is debatable. However, it leads to interesting developments and allows for a resolution of the problem.

With assumption A1), the follower's problem becomes

$$(P_\ell^>) \quad \min_{\mathbf{d}_\ell} \sum_{h \in \mathcal{H}} \left(p_{s_h \ell}^h d_\ell^h + V_\ell^h(d_\ell^h) \right)$$

$$s.t. \quad \begin{cases} \sum_{h \in \mathcal{H}} d_\ell^h = W_\ell \\ d_\ell^h > 0 \quad \forall h \in \mathcal{H}. \end{cases}$$

It follows from the KKT conditions that

$$d_\ell^h = d_\ell^{h,0} - \frac{p_{s_h \ell}^h + \lambda}{2v_\ell^h}, \quad (5.6)$$

where

$$\lambda = - \sum_{h' \in \mathcal{H}} \frac{p_{s_{h'} \ell}^{h'}}{\alpha v_\ell^{h'}} \quad \text{and} \quad \alpha = \sum_{h' \in \mathcal{H}} \frac{1}{v_\ell^{h'}}.$$

After taking in account assumption A2) and replacing the demands in the followers' problems by their explicit formulas, the problem of leader $s \in \mathcal{S}$ becomes

$$(P'_s) \quad \max_{p_{s\ell}^h: h \in \Pi_s} \sum_{h \in \Pi_s} \left(\sum_{\ell \in \mathcal{L}} \left(p_{s\ell}^h - c_s^h \right) \left(d_\ell^{h,0} - \frac{p_{s\ell}^h - \sum_{h' \in \mathcal{H}} \frac{p_{s_{h'} \ell}^{h'}}{\alpha v_\ell^{h'}}}{2v_\ell^h} \right) \right)$$

$$s.t. \quad \begin{cases} p_{s\ell}^h \in [c_s^h, \min_{s' \neq s} c_{s'}^h] & \forall h \in \Pi_s, \ell \in \mathcal{L} \\ d_\ell^{h,0} - \frac{p_{s\ell}^h - \sum_{h' \in \mathcal{H}} \frac{p_{s_{h'} \ell}^{h'}}{\alpha v_\ell^{h'}}}{2v_\ell^h} > 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L}. \end{cases}$$

In general, this can only be done under given conditions, as discussed in [Dempe 2012]. To have a correspondence between the global solutions of (P'_s) and $(P_s^>)$ (i.e. (P_s) with the lower level problems replaced by $(P_\ell^>)$) is tricky due to the strict inequalities. However, with loose inequalities, Theorem 2.3 of [Dempe 2012] only requires the lower level problem to be convex and to satisfy Slater's CQ for any leader's decision. In particular, it is easy to see that (P_ℓ^{el}) does satisfy those conditions. Therefore, replacing (P_ℓ^{el}) by its KKT conditions in (P'_s) (i.e. (P_s) with (P_ℓ^{el}) at the lower level) is legitimate for the search of global optima of the supplier's problem.

The dual variables associated to the constraints are respectively $\mu_\ell^{\geq,h}$, $\mu_\ell^{\leq,h}$, and $\mu_\ell^h = 0$ (since the constraint is strict). The KKT conditions for supplier $s \in \mathcal{S}$ are thus as follows:

$$\left\{ \begin{array}{ll} p_{s\ell}^h \in [c_s^h, \min_{s' \neq s} c_{s'}^h] & \forall h \in \Pi_s, \ell \in \mathcal{L} \\ d_\ell^{h,0} - \frac{p_{s_h\ell}^h + \lambda}{2v_\ell^h} > 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L} \\ \mu_\ell^{\geq,h}, \mu_\ell^{\leq,h} \geq 0 & \forall h \in \Pi_s, \ell \in \mathcal{L} \\ \mu_\ell^{\geq,h} (p_{s\ell}^h - c_s^h) = 0 & \forall h \in \Pi_s, \ell \in \mathcal{L} \\ \mu_\ell^{\leq,h} (\min_{s' \neq s} c_{s'}^h - p_{s\ell}^h) = 0 & \forall h \in \Pi_s, \ell \in \mathcal{L} \\ d_\ell^{h,0} + \frac{1}{2\alpha v_\ell^h} \left(\sum_{h' \in \mathcal{H}} \frac{p_{s_{h'}\ell}^{h'}}{v_\ell^{h'}} + \sum_{h' \in \Pi_s} \frac{p_{s\ell}^{h'}}{v_\ell^{h'}} - \sum_{h' \in \Pi_s} \frac{c_{s'}^{h'}}{v_\ell^{h'}} \right) \\ - \frac{p_{s\ell}^h}{v_\ell^h} + \frac{c_s^h}{2v_\ell^h} + \mu_\ell^{\geq,h} - \mu_\ell^{\leq,h} = 0 & \forall h \in \Pi_s, \ell \in \mathcal{L} \end{array} \right.$$

Observe that, due to assumption A2), the dual variables $\mu_\ell^{\geq,h}$, $\mu_\ell^{\leq,h}$ can be assembled in a single free variable $\mu_\ell^h = \mu_\ell^{\geq,h} - \mu_\ell^{\leq,h}$. Furthermore, many terms of the stationarity constraint are constant and can be gathered in a single constant term C_ℓ^h . The KKT conditions then become

$$\left\{ \begin{array}{ll} p_{s\ell}^h \in [c_s^h, \min_{s' \neq s} c_{s'}^h] & \forall h \in \Pi_s, \ell \in \mathcal{L} \\ d_\ell^{h,0} - \frac{p_{s_h\ell}^h + \lambda}{2v_\ell^h} > 0 & \forall h \in \mathcal{H}, \ell \in \mathcal{L} \\ \mu_\ell^h (p_{s\ell}^h - c_s^h) \leq 0 & \forall h \in \Pi_s, \ell \in \mathcal{L} \\ \mu_\ell^h (\min_{s' \neq s} c_{s'}^h - p_{s\ell}^h) \geq 0 & \forall h \in \Pi_s, \ell \in \mathcal{L} \\ C_\ell^h - \frac{p_{s\ell}^h}{v_\ell^h} + \frac{1}{2\alpha v_\ell^h} \left(\sum_{h' \in \mathcal{H}} \frac{p_{s_{h'}\ell}^{h'}}{v_\ell^{h'}} + \sum_{h' \in \Pi_s} \frac{p_{s\ell}^{h'}}{v_\ell^{h'}} \right) + \mu_\ell^h = 0 & \forall h \in \Pi_s, \ell \in \mathcal{L} \end{array} \right.$$

Naturally, these replacements can only be made under certain conditions. In particular, if the parameters of the problem (that is, the inconvenience coefficient, the energy costs and the base demands) satisfy given constraints, then any possible price scheme of the leaders yields a strictly positive optimal demand of the followers.

Proposition 5.2.2. *Assume that for all $\ell \in \mathcal{L}$, the constant coefficients of the problem are linked by the following inequality*

$$\max_{h \in \mathcal{H}} \frac{\left(1 - \frac{1}{\alpha v_\ell^h}\right) \left(\min_{s' \neq s_h} c_{s'}^h\right) - \sum_{h' \neq h} \frac{c_{s_{h'}\ell}^{h'}}{\alpha v_\ell^{h'}}}{2v_\ell^h} < d_\ell^{h,0}. \quad (5.7)$$

Then for all $\mathbf{p} = (p_{sl}^h)_{h \in \mathcal{H}, s \in \mathcal{S}, \ell \in \mathcal{L}}$ such that

$$p_{s_h \ell}^h \in \left[c_{s_h}^h, \min_{s' \neq s_h} c_{s'}^h \right] \quad \forall h \in \mathcal{H}, \ell \in \mathcal{L}, \quad (5.8)$$

the optimal demand of each local agent is strictly positive:

$$d_\ell^h > 0 \quad \forall h \in \mathcal{H}, \ell \in \mathcal{L}.$$

It follows directly from this proposition that if the parameters of the problems (P_ℓ) satisfy Condition 5.7), the problems (P_s) and (P'_s) are equivalent. Actually, the constraints of (P'_s) forcing the demand to be strictly positive can even be left out.

Proof. Since, for any $\ell \in \mathcal{L}$, $\mathbf{d}_\ell = (d_\ell^h)_{h \in \mathcal{H}}$ is an optimal demand, for any $h \in \mathcal{H}$, the demand d_ℓ^h is given by Formula 5.6, and thus, combining with the fact that $p^h \in [c_{s_h}^h, \min_{s' \neq s_h} c_{s'}^h]$ and Assumption 5.7, one immediately has

$$\begin{aligned} d_\ell^h &= d_\ell^{h,0} - \frac{p^h - \sum_{h' \in \mathcal{H}} \frac{p^{h'}}{\alpha v_\ell^{h'}}}{2v_\ell^h} \\ &\geq d_\ell^{h,0} - \frac{\left(1 - \frac{1}{\alpha v_\ell^h}\right) (p^h) - \sum_{h' \neq h} \frac{p^{h'}}{\alpha v_\ell^{h'}}}{2v_\ell^h} \\ &> d_\ell^{h,0} - \frac{\left(1 - \frac{1}{\alpha v_\ell^h}\right) \left(\min_{s' \neq s_h} c_{s'}^h\right) - \sum_{h' \neq h} \frac{c_{s_{h'} \ell}^{h'}}{\alpha v_\ell^{h'}}}{2v_\ell^h} > 0 \end{aligned}$$

thus leading to the desired conclusion. \square

Denote by $(P'_{s\ell})$ the problem of the supplier $s \in \mathcal{S}$ with the single follower $\ell \in \mathcal{L}$:

$$(P'_{s\ell}) \quad \max_{p_{s\ell}^h: h \in \Pi_s} \sum_{h \in \Pi_s} (p_{s\ell}^h - c_s^h) \left(d_\ell^{h,0} - \frac{p_{s\ell}^h - \sum_{h' \in \mathcal{H}} \frac{p_{s_{h'} \ell}^{h'}}{\alpha v_\ell^{h'}}}{2v_\ell^h} \right)$$

$$s.t. \quad \begin{cases} p_{s\ell}^h \in [c_s^h, \min_{s' \neq s} c_{s'}^h] & \forall h \in \Pi_s \\ d_\ell^{h,0} - \frac{p_{s_{h\ell}}^h - \sum_{h' \in \mathcal{H}} \frac{p_{s_{h'} \ell}^{h'}}{\alpha v_\ell^{h'}}}{2v_\ell^h} > 0 & \forall h \in \mathcal{H}. \end{cases}$$

The following proposition shows that in the specific case considered in this section, it is enough to consider the behavior of a single follower, since the optimization problem (P'_s) is separable.

Proposition 5.2.3. *For each $\ell \in \mathcal{L}$, let $\mathbf{p}_\ell^* = (p_{s\ell}^{*,h})_{h \in \Pi_s}$ be an optimal solution of $(P'_{s\ell})$. Then the concatenation $\mathbf{p}^* = (p_{s\ell}^{*,h})_{h \in \Pi_s, \ell \in \mathcal{L}}$ of these solutions is an optimal solution of (P'_s) .*

Proof. Observe first that the constraints of (P'_s) are composed of the concatenation of the constraints of all $(P'_{s\ell})$. Therefore, \mathbf{p}^* satisfies the constraints of (P'_s) , since for each $\ell \in \mathcal{L}$, \mathbf{p}_ℓ^* satisfies the constraints of $(P'_{s\ell})$.

Now, consider the objective function of (P'_s) , and observe that the sums over $h \in \mathcal{H}$ and $\ell \in \mathcal{L}$ can be permuted:

$$\sum_{h \in \Pi_s} \left(\sum_{\ell \in \mathcal{L}} (p_{s\ell}^{*,h} - c_s^h) d_\ell^h(\mathbf{p}^*) \right) = \sum_{\ell \in \mathcal{L}} \left(\sum_{h \in \Pi_s} (p_{s\ell}^{*,h} - c_s^h) d_\ell^h(\mathbf{p}^*) \right),$$

where $d_\ell^h(\mathbf{p}^*)$ is defined by formula (5.6), and only depends on the prices offered to ℓ . By contradiction, assume that there is an optimal solution \mathbf{p} of (P'_s) such that

$$\sum_{\ell \in \mathcal{L}} \left(\sum_{h \in \Pi_s} (p_{s\ell}^h - c_s^h) d_\ell^h(\mathbf{p}) \right) > \sum_{\ell \in \mathcal{L}} \left(\sum_{h \in \Pi_s} (p_{s\ell}^{*,h} - c_s^h) d_\ell^h(\mathbf{p}_\ell^*) \right).$$

Then in particular, since $p_{s\ell}^h - c_s^h \geq 0$, there exists an $\ell \in \mathcal{L}$ such that

$$\sum_{h \in \Pi_s} (p_{s\ell}^h - c_s^h) d_\ell^h(\mathbf{p}) > \sum_{h \in \Pi_s} (p_{s\ell}^{*,h} - c_s^h) d_\ell^h(\mathbf{p}_\ell^*),$$

which is clearly a contradiction with the optimality of \mathbf{p}_ℓ^* in $(P'_{s\ell})$. \square

Observe that this proof relies in particular on the three assumptions made in this section:

1. The energy costs of the suppliers are linear;
2. No exchanges happen among the followers;
3. The leaders have the possibility to offer consumer-dependent prices.

Should any of these assumptions not be fulfilled, would Proposition 5.2.3 not hold.

5.2.2.2 Numerical results

To illustrate the effects of the GNEP reformulation of (BDSM), two examples are provided, each of them illustrating a peculiar behavior induced by the problem. The two examples share some common characteristics:

- There is only one local agent (denoted by ℓ): this follows directly from Proposition 5.2.3 that indicates that solving a problem with n followers is the same as solving n problems with one follower.
- The base demand is 1 at each hour:

$$d_\ell^{h,0} = 1 \quad \forall h \in \mathcal{H}.$$

- The sensitivity analysis is done on the inconvenience coefficients. In both cases, the inconvenience coefficients are the same at each time slot:

$$v_\ell^h = v_\ell^{h'} \quad \forall h, h' \in \mathcal{H}.$$

The sensitivity on the inconvenience coefficients is evaluated on 250 values: they range from 0.035 to 1.28 and are uniformly distributed.

- The parameters of the instances do not satisfy the condition stated in Proposition 5.2.2 for the smallest values of the inconvenience coefficients. However, for many of these values, the demands are positive at the optimum; thus those results are also presented here. For those to be theoretically valid, a uniqueness proof would be necessary, but this is a conjecture we make.
- Several cases with various numbers of suppliers are considered: one, two or three suppliers.

To solve the problem, the demo version of GAMS is used, with Artelys Knitro 10.3.0 as a solver. The computer on which computations have been done was equipped with a 2.10 GHz processor and 16 GB RAM. Solving times are not presented here, first because the separability of the problem means that the solving time increases linearly with the number of followers and second because each of the single follower problems gets solved in a really short time (less than one second).

In the first example, the time horizon is composed of three time slots: $\mathcal{H} = \{1, 2, 3\}$. The constraints on the prices are as follows:

$$p_{s_1}^1 \in [1, 2], \quad p_{s_2}^2 \in [1.5, 2.5] \quad \text{and} \quad p_{s_3}^3 \in [1, 2.5],$$

where $s_h \in \Pi_h$. These parameters satisfy the conditions of Proposition 5.2.2 when the inconvenience coefficients are larger than 0.875. However, the computations

yield that the optimal demands are strictly positive when those coefficients are in fact larger than 0.1. Four cases are studied: in the first one (case 1), a single supplier provides all the energy to the follower. In the second and third ones, there are two suppliers s^1 and s^2 , with $\Pi_{s^1} = \{1, 2\}$ and $\Pi_{s^2} = \{3\}$ in the case 2_1 and $\Pi_{s^1} = \{1\}$ and $\Pi_{s^2} = \{2, 3\}$ in the case 2_2. In the last case (case 3) there are three suppliers, each of them providing energy during a single time slot ($\Pi_{s^h} = \{h\}$).



Figure 5.2: Example 1: the case of a single supplier. The x -axes represent the inconvenience coefficients, the y -axes either the profit (€), the price (€/kWh) or the demand (kWh). The curves 1, 2, and 3 designate the corresponding time slot.

In Figure 5.2, the case of a single supplier is presented, with the results obtained for the total profit, the hourly prices and the corresponding optimal demands.

This case should yield similar results as the ones obtained in the previous chapter (Section 4.5) for a single local agent. However, there are several notable differences. First, the competitor's prices of the preceding chapter are here replaced by formal upper bounds on prices, and these bounds are not constant over the time horizon. This leads to an increasing profit as the inconvenience coefficients increase: as the coefficients increase, the follower's demand becomes closer to his base demand (1, 1, 1) as soon as the prices reach their upper bound. The demand decreases at the first time slot (the cheapest one) and increases at both other time slots. The profit per energy unit being larger at slot 3 than slot 1 and equal at slots 1 and 2, the profit logically increases.

A second difference lies in the computation process: since it is only assumed that the demand is positive at each time slot but not put into constraints, the prices are monotonously increasing: there is no decrease of the prices on the smallest inconvenience coefficients, as observed in Section 4.5.

Finally, observe that in this example, at equal price, the third slot is more advantageous than the second time slot for the supplier. Therefore, for the lowest inconvenience coefficients, the price offered by the supplier on the third time slot is smaller than the price offered on the second time slot, so that more energy is consumed during the third time slot. However, as soon as the price on p^3 is reached, the demand curves on slots 2 and 3 are the same, since the offered price is the same.

The results in cases 2_1 and 2_2 are shown in Figure 5.3. The "bumps" in the demand curves are caused by the prices reaching their upper bounds. As in case 1, the prices are all monotonously increasing, even though they are decided by different suppliers. The interesting fact here is that the difference between both cases is caused only by the allocation of the time slots to the suppliers. In case 2_1, s^2 can only make profit on the third time slot, thus he will offer prices that induce a large consumption during this time slot, whereas in case 2_2, s^2 can also make profit on the second time slot. At the contrary, s^1 can only make profit during the first time slot, which is why the demand during the first time slot is the highest in case 2_2 even for low values of the inconvenience coefficient. An expected consequence of this setting is that the supplier that sells energy during two time slots makes a higher profit than the other one.

Finally, the case 3 is perfect to observe what causes the "bumps" in the demand curves of each hour. The situation is represented in Figure 5.4. Since each supplier sells energy during a single time slot, they will all try to attract as much demand as possible on their slot. Since s^2 has to pay more for his electricity, he will be less eager to offer low prices to attract demand, thus his price is higher than the prices of s^1 and s^3 at times 1 and 3 respectively. Since s^1 and s^3 have the same production

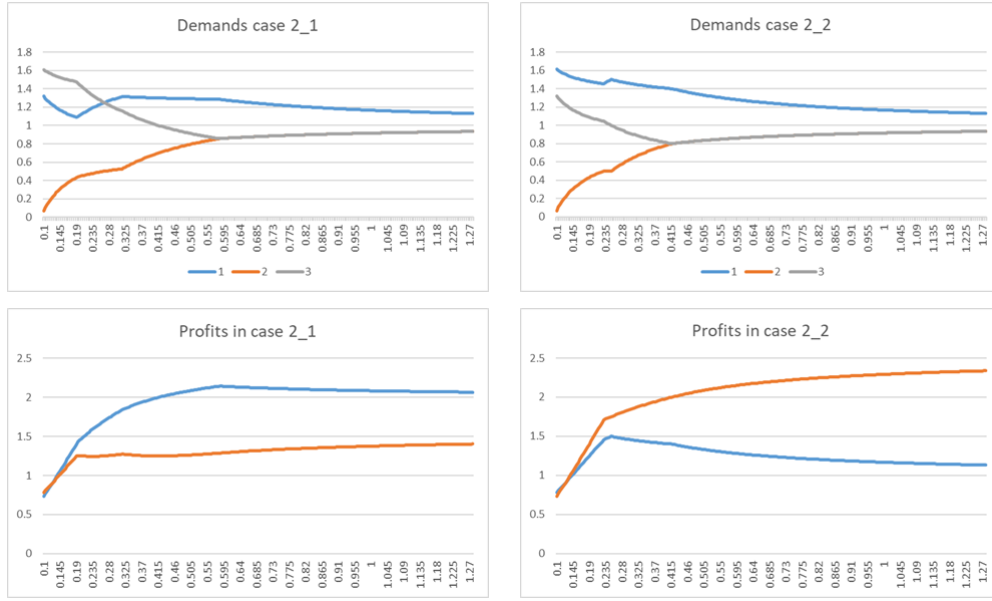


Figure 5.3: Example 1: the cases with two suppliers. The x -axes represent the inconvenience coefficients, the y -axes either the profit (€) or the demand (kWh). The curves 1, 2, and 3 correspond to the different time slots in the demand graphs, and the blue and orange curves represent the profits of s^1 and s^2 respectively in the profit charts.

cost, they do offer the same price, until $p_{s_1}^1$ reaches its upper bound around an inconvenience coefficient of 3. This causes a first "bump" in the demand curves. In particular, since the prices offered by s^2 and s^3 continue to increase with the inconvenience coefficient, the demand curve at time 1 increases suddenly. Then, $p_{s_2}^2$ reaches its upper bound around an inconvenience coefficient of 0.46, which causes a second obvious change in the slopes of the curves. Finally, $p_{s_3}^3$ reaches its upper bound as well. Since it is the same upper bound for $p_{s_2}^2$ and $p_{s_3}^3$, the demand curves for time 2 and 3 become the same.

In the second example, six time slots constitute the time horizon: $\mathcal{H} = \{1, \dots, 6\}$. The constraints on the prices are as follows:

$$\begin{aligned} p_{s_1}^1 &\in [1, 2], & p_{s_2}^2 &\in [1.5, 2.5] & p_{s_3}^3 &\in [1, 2.5] \\ p_{s_4}^4 &\in [0.5, 1.5], & p_{s_5}^5 &\in [0.25, 2] & p_{s_6}^6 &\in [0.75, 3] \end{aligned}$$

In this case, the conditions of Proposition 5.2.2 are met for inconvenience coefficients larger than 0.896, but the demands returned by the solver are strictly positive for inconvenience coefficients larger than 0.2 (and less depending on the number of suppliers). Again, four cases are studied: the case of a single supplier, two cases

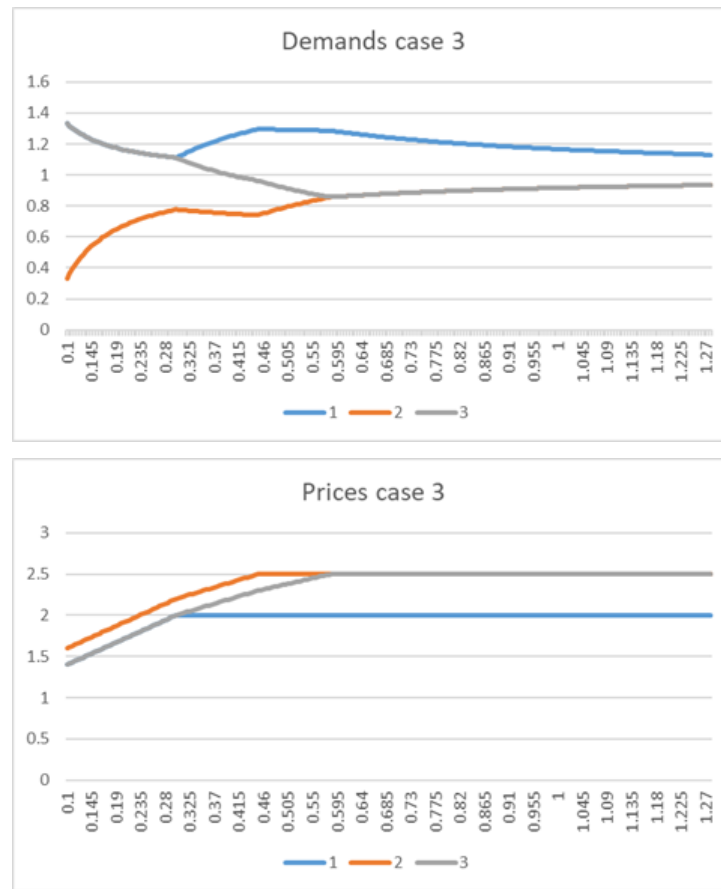


Figure 5.4: Example 1: the case of three suppliers. The x -axes represent the inconvenience coefficients, the y -axes either the profit (€), the price (€/kWh) or the demand (kWh). The curves 1, 2, and 3 designate the corresponding time slot.

with two and one case with three. The time slots are distributed as follows:

Case	Π_{s1}	Π_{s2}	Π_{s3}
1	$\{1, \dots, 6\}$		
2_1	$\{1, 2, 3\}$	$\{4, 5, 6\}$	
2_2	$\{1, 3, 5\}$	$\{2, 4, 6\}$	
3	$\{1, 4\}$	$\{2, 5\}$	$\{3, 6\}$

In case 1, the same behaviors than in the first example can be observed again. In cases 2_1 and 2_2, there is not much to add, compared to what has been observed in the first example. Since there are more time slots and different bounds, more "bumps" can be observed, but they have the same causes than in case 3 of example

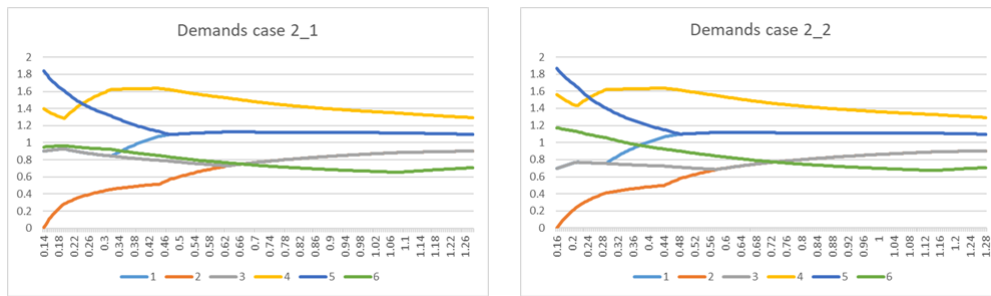


Figure 5.5: Example 2: the cases with two suppliers. The x -axes represent the inconvenience coefficients, the y -axes either the demand (kWh). The curves 1, 2, and 3 designate the corresponding time slot.

1. See Figure 5.5.

In case 3 however, there is a very interesting phenomenon, which is best seen in the lowest graph of Figure 5.6. Depending on the inconvenience coefficient, each of the suppliers can be the supplier making the most profit. For the lowest inconvenience coefficients, the suppliers having the lowest energy costs play their cards right: they can offer the cheapest prices and tap most of the demand, as can be seen in the price and demand charts. Then, $p_{s^1}^4$ reaches its upper bound, which turns out to be an advantage for inconvenience coefficients between 0.25 and 0.57. However, when $p_{s^1}^1$ reaches its upper bound too and the inconvenience coefficient continue to increase, these bounds are strong disadvantages, and s^1 becomes the supplier with the lowest profit, whereas s^2 has a period of grace for coefficients ranging from 0.57 to 1.05. Then, s^3 take the lead, as he can offer the highest prices and thus make the most profit per unit, while keeping a demand close to the base demand thanks to the unwillingness of the clients to shift their demand.

All the graphs (including cumulated demand) are found in Annex B.1.

5.3 Conclusion

In this chapter, we extended the problem of Chapter 4 to a MLFG, applied the resolution method of Leyffer and Munson, and finally developed a GNEP that is strongly linked to the original problem. The difficulties of handling a MLFG became apparent, notably with the unexpected (and undesired) equilibria found with the method of Leyffer and Munson. However, the problem (BDSM) is offering a wealth of development opportunities, due in particular to the nice structure of its lower problems.

Many prospects can be considered. First, the considered GNEP could be natu-

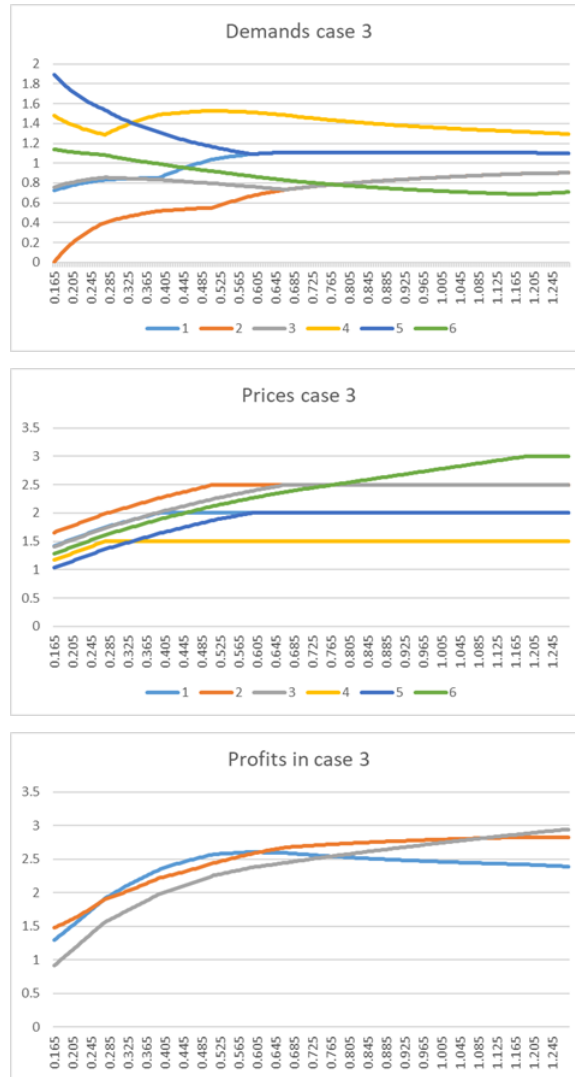


Figure 5.6: Example 2: the case of three suppliers. The x -axes represent the inconvenience coefficients, the y -axes either the profit (€), the price (€/kWh) or the demand (kWh). The curves 1, 2, and 3 designate the corresponding time slot in the demand and price charts, and the respective profit of s^1 , s^2 and s^3 in the profit chart.

rally extended: if n suppliers are competitive at time $h \in \mathcal{H}$ (i.e. the energy price offered by these n suppliers is greater than or equal to their respective production cost), the demand should be equally split among them. A natural extension of (BDSM) consists in considering nonlinear production costs, in order to capture a more realistic situation. As an alternative to the method developed by Leyffer and Munson, equilibria defined in [Kulkarni 2014] could be considered. However, the

problem we faced with Leyffer and Munson's method is rather that the obtained equilibria were undesired ones, not that they were nonexistent or impossible to obtain.

Another interesting prospect consists in trying to define semi-optimistic equilibria at the lower level (as they were defined in Chapter 4), and adapt the problem consequently to enable its resolution.

Conclusion

As the demand for energy grows and distributed generation (DG) expands, the energy domain has to face tremendous challenges. To be able to ensure the supply-demand balance at all time, several options are available. One of them consists in installing costly generators to cover peak demands, while another lies in controlling the demand: the latter being the aim of *demand-side management* (DSM).

This thesis focuses on one of the techniques to implement DSM, time-of-use pricing. Mathematically, bilevel optimization provides the perfect tools to model such situations. The problems considered throughout the thesis have the following characteristics in common:

- The considered DSM technique is load shifting: whereas the overall demand is constant, the consumption schedule can be modified.
- At the upper level, one or several energy suppliers sell energy at time-dependent prices, aiming to maximize their profit.
- At the lower level, one or several energy consumers adapt their schedule in an optimal way, minimizing their electricity bill and the inconvenience due to the load shifting.

Chapter 3 considers a single-leader single-follower problem. At the lower level, a smart grid operator (SGO) act as an intermediary agent between the supplier and the energy consumers. Each consumer transmits relevant information (power limit, needed energy, time window) about his appliances, whose usage get scheduled by the SGO. The overall energy demand thus results from the usage schedule of the customers' appliances. Furthermore, the SGO manages a DG source and storage capacities.

The production of renewable energy is unpredictable by nature, especially wind or solar power generation (as considered in the numerical results). Therefore, a stochastic model is developed, based on a scenario trees method. However, solving such problems is intractable, mainly because of their large size. Finally, a rolling horizon method is proposed to solve problems with long time horizons.

In the second research chapter, a trilevel single-leader multi-follower problem is studied. More precisely, an energy market model is presented where the lower level is constituted by so-called local agents and aggregators. Whereas the local agents consume directly the energy they buy, the aggregators transmit that energy to end users with whom they are in contract, giving rise to a third level of optimization. Furthermore, it is assumed that the aggregators and local agents can trade energy among themselves, the price being decided by the seller and the quantity by the buyer.

The particular energy exchange scheme of the intermediate level allows for many developments. First, an optimal reaction to the leader's prices consists in a Generalized Nash Equilibrium (GNE): a characterization of those GNE shows that in the optimistic case, one GNE does not include exchanges among the aggregators and local agents. A simplification of the problem directly follows from this result: the *revisited optimistic approach* proves to be much more efficient upon use of a commercial solver, both in terms of attained local optima and of solving times. However, energy exchanges can also be profitable to some followers. In particular, an interesting class of GNE of the intermediate level is the ground brick of the *semi-optimistic* approach, which provides results that are more robust for the leader in terms of followers' response. All these statements are supported by numerical experiments.

Finally, the last chapter of research focuses on a multi-leader multi-follower problem (multi-leader-follower game, MLFG). Knowing that single-leader single-follower problems are already difficult to solve, handling a MLFG requires even more care. Since more than one player compete at the upper level, an optimal solution of the upper level consists in a GNE. However, the assumptions that the leaders make about the followers' response might be conflicting. Besides applying without much success a resolution method developed by Leyffer and Munson, we take advantage of theoretical results to make strong assumptions and propose a new, simpler MLFG that can be efficiently solved. Numerical results on this new version of the problem show nice behaviors concerning the profits of the leaders or the load shifting induced by the prices offered by the leaders.

Prospects

The energy domain provides many great challenges, and will provide even more in the near future. In this thesis, our aim is to propose and solve new demand-side management problems. These models and solution methods can be extended as follows.

First, concerning the models, the energy costs of the leaders are supposed to be linear throughout the entire thesis. This is realistic when small amounts of energy are bought on the spot market, but not when the energy is directly produced by the energy suppliers. Using quadratic functions to represent the energy costs would be a first sensible improvement. On the other side of the problems, followers undergo inconvenience. In Chapter 3, the inconvenience functions are all linear, proportionally to the delay. This means that if the generalized cost (energy price + inconvenience coefficient) is lower for a time slot h_1 than for another time slot h_2 , the demand that would have been consumed during h_2 is shifted as much as possible to h_1 , potentially leaving h_2 without any demand. Introducing quadratic inconvenience functions could smooth the shifts: in the same situation as above, only a fraction of the demand would be shifted from h_2 to h_1 . Such inconvenience functions are considered in Chapters 4 and 5. They have very nice mathematical properties (strict convexity, differentiability), but are not related to the delays induced by the shifts: there is no difference if a load is shifted from the first time slot to the second one or to the last one. Designing an inconvenience function that combines both advantages is a challenge.

In Chapters 4 and 5, the exchanges among actors of the intermediate/lower level (ILA) are particular: it is assumed that the seller single-handedly decides the price and the buyer single-handedly decides the quantity, both decisions being taken at the same time. In reality, these exchanges result from negotiations among the concerned actors. An interesting modification of our problem thus consists in modeling such interactions.

Besides, none of the energy exchanges are bounded. However, the suppliers' capacities are not infinite, thus only a given amount of energy could be sold at each time slot. To model this, upper bounds can be set on exchanges, either at the upper or at the lower level.

Finally, it is assumed that ILAs can only sell the energy they buy from other actors. A possible extension of the model consists in assuming that the ILAs dispose of generation capacities and are thus able to sell the energy they produce to the other actors of the grid.

Another potential improvement lies in the introduction of other types of uncertainty into the models. Indeed, neither the energy costs, nor the competitor's prices, nor even the electricity demand are known in advance. Therefore, a broad range of new problems can be studied, as any of those parameters can be stochastically considered.

From a more theoretical point of view, it is still unsure whether the problems studied in Chapter 3 are NP-hard. Proving that they are or not would motivate the

search for heuristics or exact algorithms, respectively. The heuristic proposed in this work, i.e. the rolling horizon method, proves to be efficient to compute optimal prices for a realization of a scenario, but is not adapted to the resolution of the whole stochastic problem (SBPP).

To the best of our knowledge, stochastic bilevel optimization remains a rather open field. To design efficient solution methods remains a challenge. Furthermore, asserting the quality of a solution method is difficult, since classical bounds of stochastic programming as the EVPI or the VSS cannot be easily applied in a bilevel setting, as shown in Section 3.3.3.

In Chapter 5, MLFG are considered. Even though such games have been thoroughly studied, their complexity still makes them interesting. In particular, the classical optimistic/pessimistic dichotomy does not make sense in a multi-leader context. In this work, we suggest approaches that rely on GNE selections at the lower level, both in Chapters 4 and 5: smart selections can strongly simplify the resolution of MLFG. Such selection approaches should therefore be more thoroughly studied.

To conclude, this thesis answers a few questions, but opens many more.

Appendix to Chapter 4

A.1 KKT conditions

The KKT conditions of (P_ℓ) mentioned in Subsection 4.4.1 are as follows:

$$\left\{ \begin{array}{ll}
 \sum_{h \in \mathcal{H}} d_\ell^h = W_\ell & (\lambda_\ell) \\
 p_{lx}^h \leq \alpha_{lx}^h p_{sx}^h + (1 - \alpha_{lx}^h) p_{sl}^h & (\mu_{plx}^h) \quad \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \\
 e_{lx}^h \geq 0 & (\mu_{elx}^h) \quad \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \cup \mathcal{S}, \\
 \alpha_{lx}^h \in [0, 1] & (\mu_{\alpha lx}^{h, \min}, \mu_{\alpha lx}^{h, \max}) \quad \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \\
 \mu_{elx}^h \geq 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \cup \mathcal{S} \\
 \mu_{plx}^h, \mu_{\alpha lx}^{h, \min}, \mu_{\alpha lx}^{h, \max} \geq 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \\
 \mu_{plx}^h \left(p_{lx}^h + \sum_{s \in \mathcal{S}} (-\alpha_{lx}^h p_{sx}^h + (\alpha_{lx}^h - 1) p_{sl}^h) \right) = 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \\
 \mu_{elx}^h e_{lx}^h = 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \cup \mathcal{S} \\
 \mu_{\alpha lx}^{h, \min} \alpha_{lx}^h = 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \\
 \mu_{\alpha lx}^{h, \max} (1 - \alpha_{lx}^h) = 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \\
 p_{xl}^h + 2v_\ell^h (d_\ell^h - d_\ell^{h,0}) - \mu_{elx}^h + \lambda_\ell = 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \cup \mathcal{S} \\
 - \sum_{x \in \mathcal{L} \cup \mathcal{A}} e_{xl}^h + \mu_{plx}^h = 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \\
 (p_{sl}^h - p_{sx}^h) \mu_{plx}^h + \sum_{s \in \mathcal{S}} (-\mu_{\alpha lx}^{h, \min} + \mu_{\alpha lx}^{h, \max}) = 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}.
 \end{array} \right.$$

The variables between parentheses denote the dual variables associated to the primal constraints.

Next, the KKT conditions of (P'_a) , also mentioned in Subsection 4.4.1:

$$\left\{ \begin{array}{ll} \sum_{s \in \mathcal{S}} e_{as}^h + \sum_{\ell \in \mathcal{L}} (e_{a\ell}^h - e_{\ell a}^h) + \sum_{a' \neq a} (e_{aa'}^h - e_{a'a}^h) = \sum_{i \in \mathcal{I}_a} d_i^h & (\lambda_a) \quad \forall h \in \mathcal{H} \\ \sum_{h \in \mathcal{H}} d_i^h = W_i & (\lambda_i) \quad \forall i \in \mathcal{I}_a \\ p_{ax}^h \leq \alpha_{ax}^h p_{sx}^h + (1 - \alpha_{ax}^h) p_{sa}^h & (\mu_{pax}^h) \quad \forall h \in \mathcal{H}, s \in \mathcal{S}, x \in \mathcal{L} \cup \mathcal{A} \\ 2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i \geq 0 & (\mu_{ri}^h) \quad \forall h \in \mathcal{H}, i \in \mathcal{I}_a \\ d_i^h, e_{ax}^h \geq 0 & (\mu_{di}^h, \mu_{eax}^h) \quad \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \cup \mathcal{S}, i \in \mathcal{I}_a \\ \alpha_{ax}^h \in [0, 1] & (\mu_{\alpha ax}^{h,\min}, \mu_{\alpha ax}^{h,\max}) \quad \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \\ \mu_{eax}^h \geq 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \cup \mathcal{S} \\ \mu_{pax}^h, \mu_{\alpha ax}^{h,\min}, \mu_{\alpha ax}^{h,\max}, \mu_{ri}^h, \mu_{di}^h \geq 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S}, i \in \mathcal{I}_a \\ \mu_{pax}^h (p_{ax}^h - \alpha_{ax}^h p_{sx}^h + (\alpha_{ax}^h - 1) p_{sa}^h) = 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \\ \mu_{eax}^h e_{ax}^h = 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \cup \mathcal{S} \\ \mu_{\alpha ax}^{h,\min} \alpha_{ax}^h = 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \\ \mu_{\alpha ax}^{h,\max} (1 - \alpha_{ax}^h) = 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A}, s \in \mathcal{S} \\ \mu_{ri}^h (2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i) = 0 & \forall h \in \mathcal{H}, i \in \mathcal{I}_a \\ \mu_{di}^h d_i^h = 0 & \forall h \in \mathcal{H}, i \in \mathcal{I}_a \\ p_{xa}^h - \mu_{eax}^h + \lambda_a = 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \cup \mathcal{S} \\ - \sum_{x \in \mathcal{L} \cup \mathcal{A}} e_{xa}^h + \mu_{pax}^h = 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \\ (p_{sa}^h - p_{sx}^h) \mu_{pax}^h + \sum_{s \in \mathcal{S}} (-\mu_{\alpha ax}^{h,\min} + \mu_{\alpha ax}^{h,\max}) = 0 & \forall h \in \mathcal{H}, x \in \mathcal{L} \cup \mathcal{A} \\ 2v_i^h (2d_i^h - d_i^{h,0} - \mu_{ri}^h) + \hat{\lambda}_i - \lambda_a + \lambda_i - \mu_{di}^h = 0 & \forall h \in \mathcal{H}, i \in \mathcal{I}_a \\ d_i^h - \mu_{ri}^h = 0 & \forall h \in \mathcal{H}, i \in \mathcal{I}_a. \end{array} \right.$$

The KKT conditions of (P_x^{el}) are as follows: for $x = \ell \in \mathcal{L}$:

$$\left\{ \begin{array}{ll} \sum_{h \in \mathcal{H}} e_{\ell s}^h + e_{\ell \bar{s}}^h = W_\ell & (\lambda_\ell) \\ e_{\ell s}^h \geq 0 & (\mu_\ell^h) \quad \forall h \in \mathcal{H}, s \in \mathcal{S} \\ \mu_\ell^h \leq 0 & \forall h \in \mathcal{H}, s \in \mathcal{S} \\ \mu_\ell^h e_{\ell s}^h = 0 & \forall h \in \mathcal{H}, s \in \mathcal{S} \\ p_{s\ell}^h + 2v_\ell^h (d_\ell^h - d_\ell^{h,0}) + \lambda_\ell + \mu_\ell^h = 0 & \forall h \in \mathcal{H}, s \in \mathcal{S}, \end{array} \right.$$

and for $x = a \in \mathcal{A}$:

$$\left\{ \begin{array}{ll}
\sum_{i \in \mathcal{I}_a} d_i^h = e_{a\hat{s}}^h + e_{a\bar{s}}^h & (\lambda_a) \quad \forall h \in \mathcal{H} \\
\sum_{h \in \mathcal{H}} d_i^h = W_i & (\lambda_i) \quad \forall i \in \mathcal{I}_a \\
2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i \geq 0 & (\mu_{ri}^h) \quad \forall h \in \mathcal{H}, i \in \mathcal{I}_a \\
d_i^h, e_{as}^h \geq 0 & (\mu_{di}^h, \mu_{as}^h) \quad \forall h \in \mathcal{H}, i \in \mathcal{I}_a, s \in \mathcal{S} \\
\mu_{di}^h, \mu_{ri}^h, \mu_{as}^h \leq 0 & \quad \forall h \in \mathcal{H}, i \in \mathcal{I}_a, s \in \mathcal{S} \\
\mu_{di}^h d_i^h = 0 & \quad \forall h \in \mathcal{H}, i \in \mathcal{I}_a \\
\mu_{ri}^h (2v_i^h (d_i^h - d_i^{h,0}) + \hat{\lambda}_i) = 0 & \quad \forall h \in \mathcal{H}, i \in \mathcal{I}_a \\
\mu_{as}^h e_{as}^h = 0 & \quad \forall h \in \mathcal{H}, s \in \mathcal{S} \\
p_{sa}^h + 2v_i^h (2d_i^h - d_i^{h,0} + \mu_{ri}^h) + \mu_{di}^h + \lambda_i + \lambda_a = 0 & \quad \forall h \in \mathcal{H}, s \in \mathcal{S} \\
p_{sa}^h - \lambda_a + \mu_{as}^h = 0 & \quad \forall h \in \mathcal{H}, s \in \mathcal{S} \\
W_i + \sum_{h \in \mathcal{H}} \mu_{ri}^h = 0 & \quad \forall h \in \mathcal{H}, i \in \mathcal{I}_a.
\end{array} \right.$$

A.2 Instances parameters

This section of the appendix gathers the parameters of the instances used for the computing times in the subsections 4.5.1.1 and 4.5.1.2. First, the competitor's prices are all set to 0.14, and the inconvenience coefficients are assumed to be the same for all ILAs and all time slots: the chosen value is 0.0004.

Concerning the energy costs of the suppliers and the base demands of the local agents and end users, they are presented in the following table.

The instances are built by successive addition of local agents and end users. For example, Instance 4 involves the two first local agents ℓ_1 and ℓ_2 , and the two end users i_1 and i_2 , whereas the three local agents of Instance 5 are ℓ_1 , ℓ_2 and ℓ_3 , and its three end users are i_1 , i_2 , i_3 . Since these two instances only involve three hours, the parameters for $h = 4$ are ignored.

Parameter	$h = 1$	$h = 2$	$h = 3$	$h = 4$
$c_{\tilde{s}}^h$	0.02	0.06	0.06	0.03
$d_{\ell_1}^h$	4	14	14	9
$d_{\ell_2}^h$	4	14	14	8
$d_{\ell_3}^h$	5	12	13	5
$d_{\ell_4}^h$	9	11	15	3
$d_{\ell_5}^h$	3	8	7	9
$d_{\ell_6}^h$	2	20	19	5
$d_{\ell_7}^h$	4	14	14	8
$d_{\ell_8}^h$	5	12	13	5
$d_{\ell_9}^h$	9	11	15	3
$d_{i_1}^h$	0.4	1.4	1.4	0.9
$d_{i_2}^h$	0.4	1.4	1.4	0.55
$d_{i_3}^h$	0.4	1.4	1.2	0.8
$d_{i_4}^h$	0.3	1.5	1.6	0.5
$d_{i_5}^h$	0.7	2	1.5	1
$d_{i_6}^h$	0.2	0.9	0.8	0.3
$d_{i_7}^h$	0.4	1.4	1.2	0.8
$d_{i_8}^h$	0.3	1.5	1.6	0.5
$d_{i_9}^h$	0.7	2	1.5	1

Table A.1: The parameters of the test instances.

Appendix to Chapter 5

B.1 Price, profit and demand graphs

In this appendix, all graphs for the presented examples of Section 5.2.2.2 are shown.

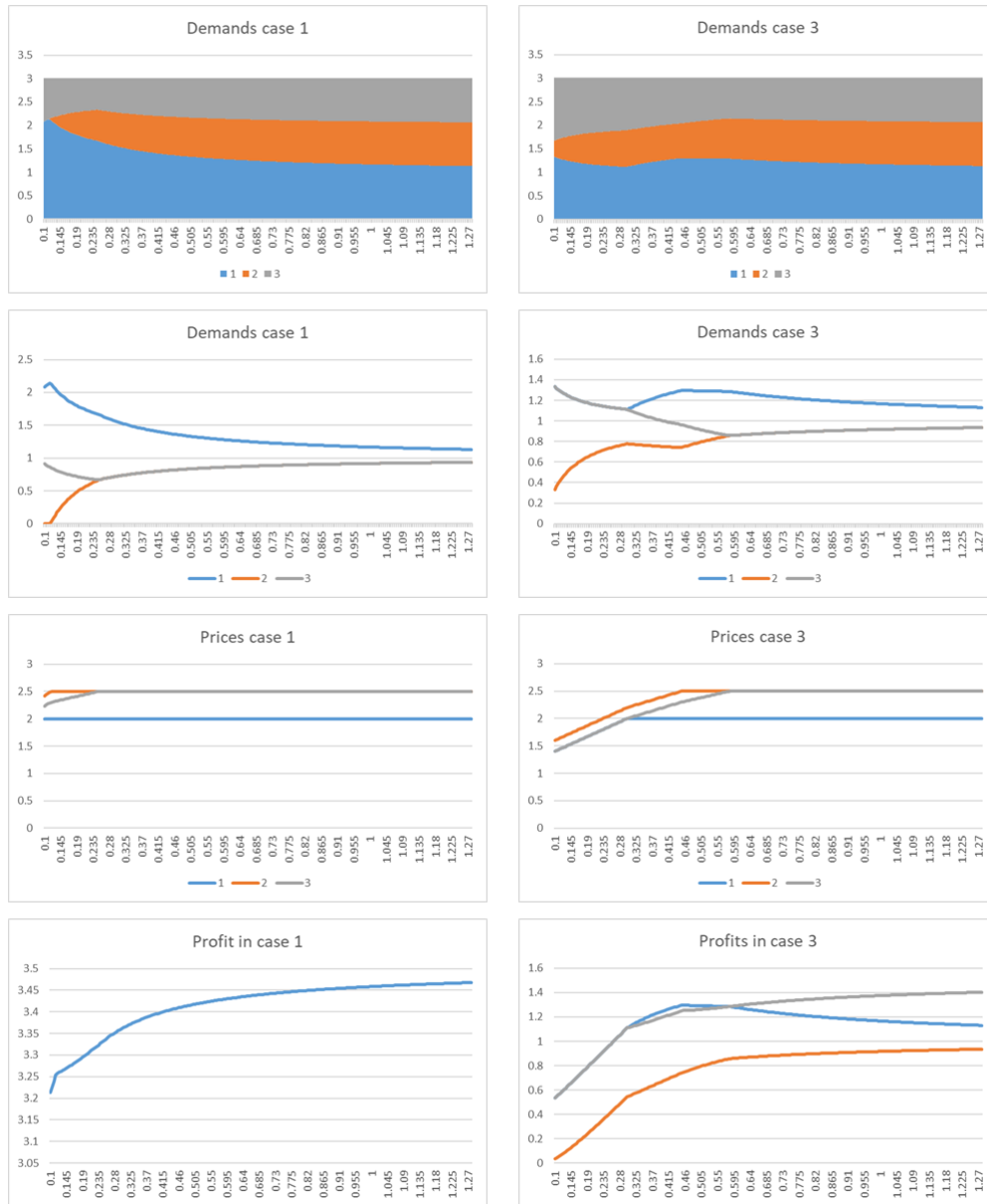


Figure B.1: Example 1: cases 1 and 3. The x -axes represent the inconvenience coefficients, the y -axes either the profit (€), the price (€/kWh) or the demand (kWh).

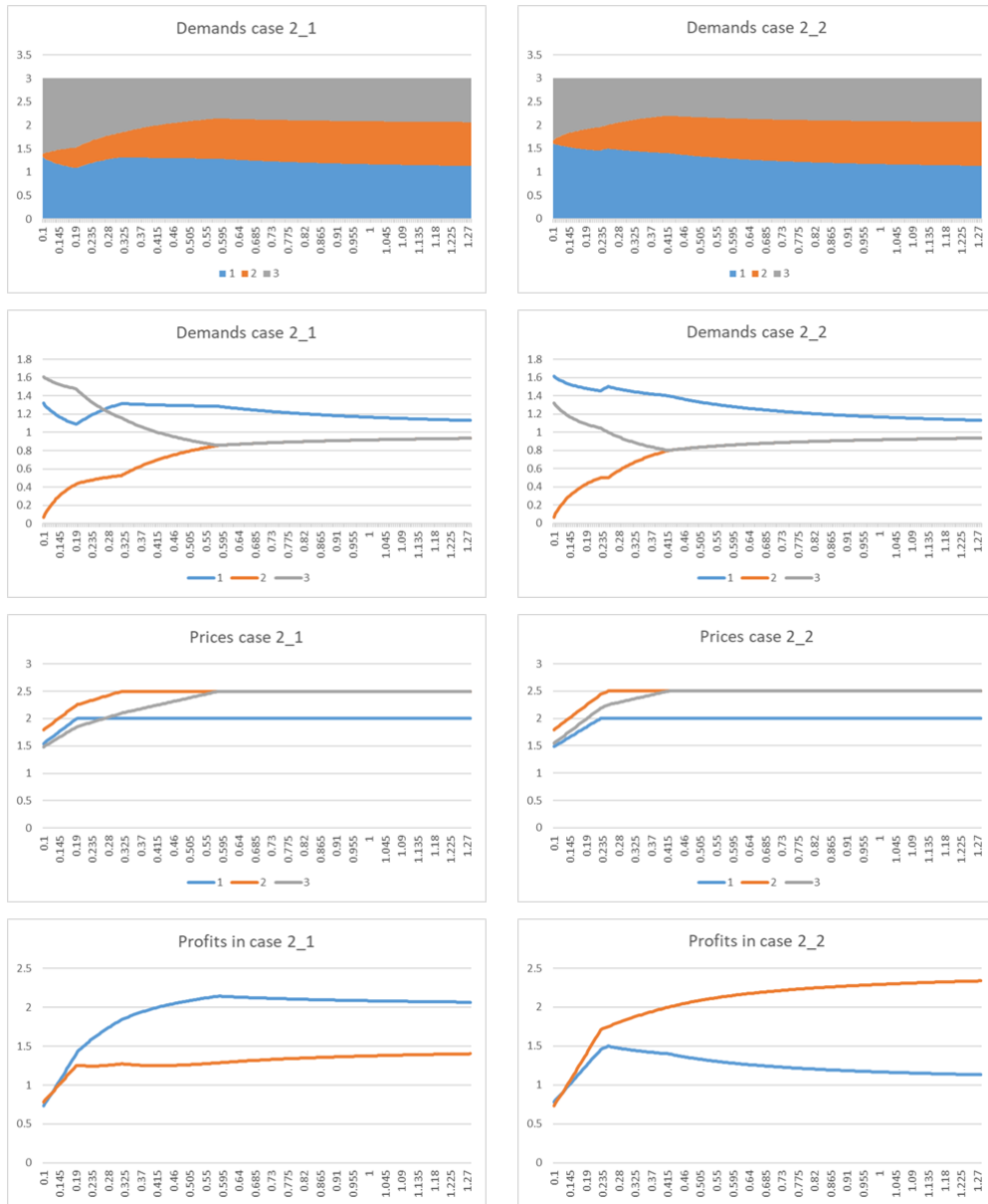


Figure B.2: Example 1: cases 2_1 and 2_2. The x -axes represent the inconvenience coefficients, the y -axes either the profit (€), the price ($\text{€}/\text{kWh}$) or the demand (kWh).

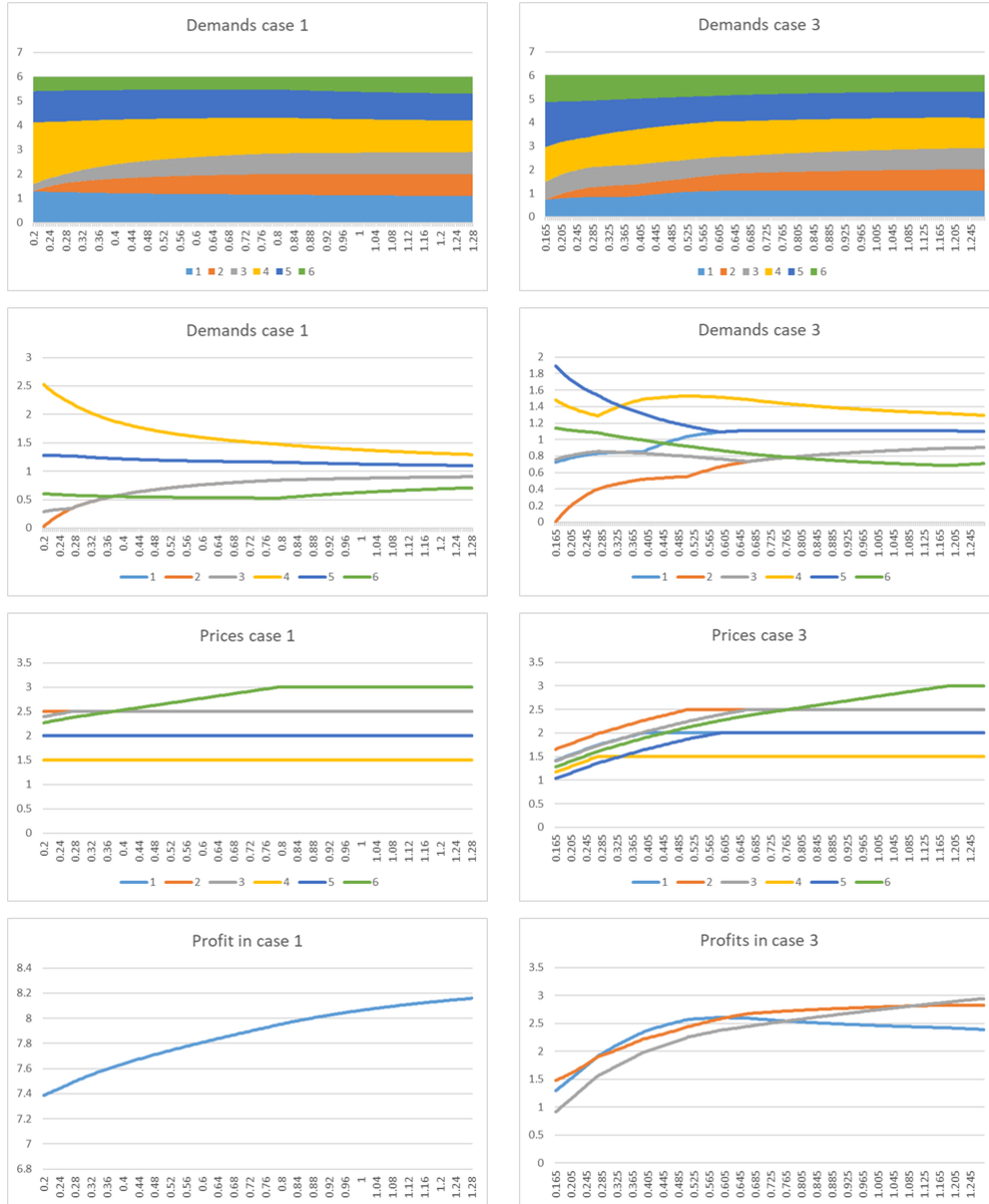


Figure B.3: Example 2: cases 1 and 3. The x -axes represent the inconvenience coefficients, the y -axes either the profit (€), the price (€/kWh) or the demand (kWh).

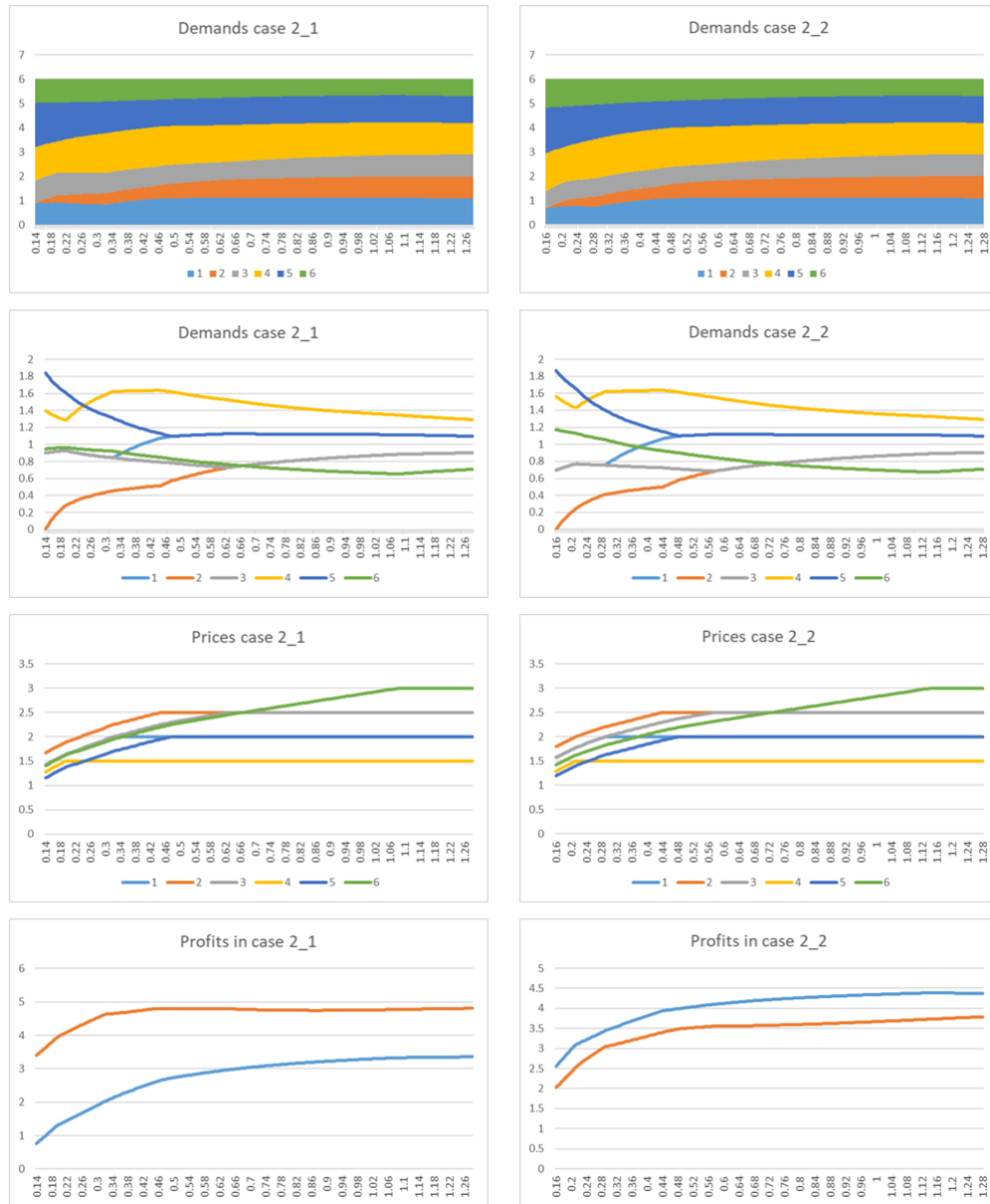


Figure B.4: Example 2: cases 2_1 and 2_2. The x -axes represent the inconvenience coefficients, the y -axes either the profit (€), the price (€/kWh) or the demand (kWh).

Résumé de la thèse

Cette thèse, intitulée "Conception et tarification de nouveaux services en énergie dans un environnement compétitif" est le fruit de trois années de travail au sein de l'équipe INOCS à Inria Lille-Nord Europe et du laboratoire PROMES-CNRS situé à Perpignan. Ce travail a été financé majoritairement par le Programme Gaspard Monge en Optimisation de la FMJH, et réalisé en collaboration avec EDF R&D.

Le domaine de l'énergie est actuellement confronté à de nombreux défis. L'augmentation de la consommation globale d'électricité, combinée à la future diminution des ressources en énergies fossiles, représente un énorme challenge. De plus, de multiples enjeux coexistent et se contredisent souvent, qu'ils soient économiques, politiques ou écologiques. Alors qu'historiquement, les réseaux électriques n'envisageait les transferts d'énergie que de manière mono-directionnelle, depuis de grandes centrales jusqu'aux consommateurs finaux, la démocratisation de la production décentralisée (*distributed generation*) a changé les règles du jeu. En effet, la production décentralisée repose principalement sur les énergies renouvelables telles que le solaire ou l'éolien, qui sont par nature irrégulières et imprévisibles. Cette production d'électricité irrégulière, en sus d'une demande croissante des consommateurs, induit de grandes difficultés à assurer l'équilibre offre-demande du réseau. Au lieu d'installer de coûteux moyens de production capables de couvrir n'importe quelle demande de pointe, les consommateurs peuvent aider à maintenir l'équilibre en modifiant leur comportement. En revanche, pour que de telles modifications de comportement soient efficaces, une communication bidirectionnelle entre consommateurs et producteurs doit être mise en place, ce qui a conduit à la naissance du paradigme du *smart grid*. Selon [Dept 2009], le smart grid se définit comme

*"an automated, widely distributed energy delivery network, which will be characterized by a two-way flow of electricity and information and will be capable of monitoring everything from power plants to customer preferences to individual appliances. It incorporates into the grid the benefits of distributed computing and communications to deliver real-time information and enable the near-instantaneous balance of supply and demand at the device level."*¹

¹Un réseau de distribution de l'énergie automatisé et largement distribué, qui sera caractérisé par un flux bidirectionnel d'électricité et d'informations, et qui sera capable de tout contrôler, des centrales de productions aux préférences des consommateurs et aux appareils seuls. Il incorpore au réseau les bénéfices des communications et du calcul distribué afin de livrer de l'information en temps réel, et il permet l'équilibre quasi-instantané de l'offre et de la demande au niveau des appareils électriques.

Les smart grids possèdent de nombreux avantages : grâce à l'installation de compteurs intelligents (tels *Linky* en France), ils sont observables à tout instant. Ils permettent l'intégration ou la suppression de modules de production décentralisée. Ils sont capables de détecter leurs pannes internes. Enfin, ils sont plus sûrs et plus fiables. Ainsi, les smart grids sont idoines pour minimiser les risques de baisses de tensions ou de black-outs pouvant être causés par l'injection imprévisible d'énergie renouvelable dans le réseau.

La capacité des acteurs du réseau à communiquer est cruciale et permet la mise en oeuvre de nombreuses techniques intéressantes. Une de ces techniques est la gestion active de la demande (*demand-side management*). La gestion active de la demande consiste à modeler la demande des consommateurs finaux, ce qui peut avoir plusieurs objectifs : empêcher des black-outs en évitant des demandes qui ne peuvent être satisfaites, optimiser l'usage des énergies renouvelables, ou, d'un point de vue plus économiques, augmenter les bénéfices d'un producteur d'électricité dans un environnement compétitif. Si certaines techniques de gestion de la demande s'utilisent à un niveau stratégique, et donc à long terme, d'autres peuvent être employées sur de courtes échelles de temps: il s'agit des techniques dites de *peak clipping* (réduction de la pointe), *valley filling* ("remplissage" des heures creuses) et de *load shifting* (déplacement de charge).

Le concept de réponse de la demande (*demand response*) est intimement lié à la gestion de la demande. Afin d'assurer l'équilibre offre-demande, la demande réelle doit être flexible, non seulement pour diminuer les pics de consommation, mais également pour réduire les fluctuations de la demande. Telle que définie dans [Anjos 2017], "*the collection of approaches available to obtain this flexibility from the demand side of the balance is commonly referred to as demand response.*"² La réponse de la demande concerne les décisions à court terme : par exemple, si un événement imprévu cause l'arrêt d'une centrale de production, des mécanismes de réponse de la demande peuvent être mis en oeuvre pour diminuer la demande totale et ainsi éviter un black-out. De tels mécanismes reposent généralement sur la tarification, dont les principales catégories sont les tarifs *time-of-use* (TOU), *multi-TOU* et *time-and-level-of-use* (TLOU). Les tarifs TOU dépendent de l'heure à laquelle l'électricité est consommée, les tarifs multi-TOU consistent à offrir des tarifs TOU différents selon les consommateurs, ceux-ci étant répartis en segments, et finalement, les tarifs TLOU dépendent non seulement de l'heure de consommation, mais également de la quantité d'énergie consommée. Parmi les tarifs TOU, plusieurs exemples peuvent être mentionnés, notamment le *critical peak pricing* (CPP) ou le

²L'ensemble des approches disponibles pour obtenir cette flexibilité de la demande s'appelle la réponse de la demande.

variable peak pricing (VPP). Dans le cas du CPP, les prix augmentent fortement lors de périodes de pointes, déterminées à l'avance ou en temps réel selon les besoins du système. Dans ce dernier cas, une alerte est envoyée quelques heures avant une augmentation significative du tarif pendant une période de pointe, afin que les consommateurs puissent réagir en conséquence. Les tarifs VPP quant à eux diffèrent du prix de base lors de certaines fenêtres de temps fixées à l'avance. Le niveau du tarif modifié dépend alors des conditions du système ou du marché.

Cette thèse vise à déterminer des tarifs TOU optimaux dans un modèle général où un ensemble de fournisseurs d'électricité proposent des tarifs à un ensemble de consommateurs, le tout étant considéré dans un contexte de load shifting, c'est-à-dire que la demande totale de chaque client reste constante sur l'horizon de temps considéré.

Différentes nouvelles professions sont apparues dans le contexte du smart grid. En particulier, les *agrégateurs* jouent un rôle important en tant qu'intermédiaires entre fournisseurs et consommateurs finaux. D'une part, ils offrent des avantages pécuniaires aux consommateurs en échange du contrôle de tout ou partie de leur demande. D'autre part, ils offrent de la flexibilité aux fournisseurs. Plus un agrégateur a de clients, plus l'impact potentiel dudit agrégateur sur la courbe de demande générale est important. De tels agrégateurs sont considérés dans le cadre de cette thèse.

L'interaction entre fournisseurs d'électricité, agrégateurs et consommateurs finaux est de nature hiérarchique : les fournisseurs proposent des prix auxquels les agrégateurs et consommateurs réagissent. De telles interactions sont connues dans le domaine de l'économie sous le nom de *jeux de Stackelberg*. En mathématiques, ceux-ci sont l'objet de *l'optimisation bi-niveau*. Dans un problème bi-niveau, deux acteurs, le *meneur* et le *suiveur*, jouent un jeu : chaque acteur subit ses propres contraintes et possède son propre objectif; le leader prend ses décisions en sachant que le suiveur va y réagir de manière optimale. Parmi les extensions possibles de l'optimisation bi-niveau, on retrouve des problèmes à plusieurs meneurs et/ou à plusieurs suiveurs, les *multi-leader-follower games* (MLFG). Dans un MLFG, les suiveurs jouent un jeu entre eux, tout comme les meneurs entre eux. Ainsi, une solution d'un MLFG prend la forme de deux équilibres de Nash : un équilibre parmi les meneurs, et un autre parmi les suiveurs qui dépend des décisions des meneurs. Pour un ensemble de décisions des meneurs, il existe généralement plusieurs équilibres potentiels au niveau inférieur. Dans un problème classique comprenant un seul meneur et un seul suiveur, le problème lié à de potentielles multiples solutions du niveau inférieur est traditionnellement considéré à travers le prisme de la dichotomie optimiste-pessimiste :

- Dans le cas *optimiste*, le meneur suppose que le suiveur va coopérer et choisir la solution la plus favorable au meneur parmi toutes les solutions (optimales) possibles.
- Dans le cas *pessimiste*, le meneur suppose que le suiveur va adopter un comportement antagoniste en choisissant la solution (optimale) la moins favorable au meneur. Cette approche est fortement liée à la notion d'aversion au risque.

Dans un problème à plusieurs suiveurs, mais un seul meneur, cette dichotomie reste sensée : puisqu'il n'y a qu'un seul meneur, il est possible de déterminer quel équilibre de Nash lui est le plus favorable. En revanche, dans un problème à plusieurs meneurs, il est bien plus compliqué de choisir parmi les équilibres possibles : un équilibre favorable à un meneur peut tout à fait être désavantageux pour un autre meneur. De tels considérations doivent être prises en compte lors de l'étude d'un MLFG, ce qui rend ces problèmes intrinsèquement plus compliqués que des problèmes bi-niveaux classiques.

L'optimisation bi-niveau a connu des avancées significatives lors des trente dernières années, et gagne continuellement en popularité grâce à sa faculté à modéliser de nombreuses situations de manière pertinente. Les applications potentielles vont des problèmes de gestion des recettes de lignes aériennes à la modélisation des mouvements du bras humain, et de problèmes de conception de réseaux autoroutiers à des problèmes de répartition des sources de production (unit commitment), par exemple. Parmi ces applications, les problèmes de tarification occupent une place importante. En particulier, l'équipe INOCS à Inria Lille-Nord Europe s'est forgé une expertise en problèmes de tarification bi-niveaux pour la gestion de la demande : de tels problèmes (bilinéaires-bilinéaires) sont étudiés dans [Afşar 2016b] et [Alekseeva 2018].

En se basant sur les deux travaux susnommés, cette thèse propose différents problèmes bi-niveaux de gestion de la demande visant à quantifier et optimiser les gains d'un fournisseur d'électricité dans un marché compétitif, où des incitations pécuniaires sont utilisées pour induire des déplacements de charges. Ces problèmes et les résultats relatifs constituent ainsi les prémices d'un champ de recherche prometteur pour le partenaire industriel.

Les contributions de la thèse sont de trois natures différentes : définition des modèles, leur analyse théorique et leur analyse numérique. Trois types de problèmes sont étudiés :

- Simple meneur, simple suiveur (Chapitre 3). Le problème de base de ce chapitre est une extension des problèmes étudiés dans [Afşar 2016b]. Le meneur est un fournisseur d'électricité visant à maximiser son profit,

i.e. la différence entre les revenus liés à la vente d'électricité et les coûts d'approvisionnements relatifs, qu'ils soient de production ou d'achat sur le marché spot. Le suiveur quant à lui est un opérateur de smart grid qui gère la consommation des appareils électriques de ses clients. Chaque appareil doit recevoir une quantité donnée d'énergie pendant une fenêtre temporelle d'utilisation, avec comme limite à la consommation la puissance nominale de chaque appareil. La première heure de la fenêtre de chaque appareil est supposée être l'heure préférée pour utiliser l'appareil en question, et tout retard cause un désagrément au client concerné. Ainsi, l'opérateur cherchera non seulement à minimiser les coûts d'achat de l'électricité au fournisseur, mais également à minimiser le désagrément subi par ses clients. En plus de la responsabilité de la programmation de l'usage des appareils de ses clients, l'opérateur de smart grid gère une production d'énergie renouvelable ainsi qu'une batterie, desquelles il peut puiser de l'énergie à fournir à ses clients.

Le problème est tout d'abord étudié d'un point de vue déterministe, où la quantité d'énergie renouvelable produite est supposée connue à l'avance, et une analyse de sensibilité est conduite sur différents paramètres du problème. Ensuite, de la stochasticité est introduite dans le problème, afin de tenir compte de l'imprévisibilité liée à l'énergie renouvelable: une approche par arbres de scénarios est proposée. En revanche, la taille par nature exponentielle des arbres de scénarios empêche toute résolution rapide et efficace du problème dès qu'un certain nombre d'heures est considéré, ainsi une méthode basée sur des horizons glissants est développée. Une telle méthode ne permet certes pas de résoudre le problème entier (en considérant tous les scénarios possibles), mais peut être appliquée dans un contexte réaliste où l'on observe quel scénario se réalise et recalcule une solution à intervalles réguliers. Enfin, une analyse de problèmes simplifiés donne quelques pistes quant à la complexité algorithmique du problème de base. En l'occurrence, une probable NP-complexité proviendrait essentiellement du nombre d'heures considérées, ainsi que du désagrément subi par les consommateurs.

- Simple meneur, multiples suiveurs (Chapitre 4). Dans ce chapitre, un modèle comprenant quatre types d'acteurs est proposé. Au niveau supérieur, un fournisseur vend de l'énergie aux acteurs du niveau inférieur qui peuvent être soit des *agrégateurs*, soit des *agents locaux*. Un troisième niveau d'optimisation apparaît, puisque les agrégateurs sont reliés, via contrats, à un ensemble de consommateurs finaux. Les rôles des différents acteurs se répartissent comme suit :

- Le fournisseur d'énergie offre des tarifs différenciés (selon le client et l'heure) aux agrégateurs et agents locaux. Son but est de maximiser son profit.
- Les agents locaux doivent satisfaire une demande qu'ils peuvent éventuellement déplacer. Chaque déplacement de charge occasionne du désagrément, ainsi les agents locaux cherchent à minimiser le coût de leurs achats en électricité plus le désagrément lié aux déplacements de charges.
- Les agrégateurs n'ont eux pas de contrôle direct sur une quelconque charge. En revanche, ils peuvent offrir des récompenses aux consommateurs finaux avec qui ils ont un contrat pour que ceux-ci déplacent leur charge. Ainsi, chaque agrégateur cherchera à minimiser le coût de ses achats, plus les récompenses offertes aux consommateurs finaux.
- Enfin, les consommateurs finaux minimisent leur désagrément tout en maximisant les récompenses offertes par leur agrégateur. Il est à noter toutefois qu'il ne s'agit pas ici d'optimisation multi-objectif : seule la différence du désagrément et des récompenses est optimisée.

En outre, les agrégateurs et agents locaux peuvent échanger de l'électricité entre eux. Dans ce cas, le vendeur décidera le prix, et l'acheteur décidera la quantité. De plus, ces acteurs peuvent se fournir en électricité auprès d'un fournisseur alternatif (le "compétiteur") au lieu de tout acheter au meneur. Par conséquent, le meneur cherche en fait une meilleure réponse aux prix du compétiteur, qui sont supposés connus à l'avance.

Afin de résoudre un tel modèle à trois niveaux d'optimisation, plusieurs résultats théoriques sont nécessaires. Tout d'abord, il s'agit de réduire le tout à un problème bi-niveau. Pour ceci, une méthode inspirée de [Gkatzikis 2013] est utilisée : les conditions d'optimalité des problèmes des consommateurs finaux permettent d'obtenir une formule explicite pour les demandes des consommateurs finaux en fonction des récompenses offertes, et inversement. En utilisant cette formule explicite, le problème tri-niveau est reformulé comme un problème bi-niveau.

A ce point, la méthode traditionnelle consistant à remplacer les problèmes des suiveurs par leurs conditions d'optimalité dans le problème du meneur est utilisée, mais s'avère peu efficace. La résolution par solveur commercial du problème d'optimisation avec contraintes d'équilibre (MPEC) ne retourne en effet que des points stationnaires du problème dont l'étude permet d'affirmer qu'ils ne sont en aucun cas des optimums globaux. Pour résoudre ce problème,

un premier résultat permet de déterminer le prix exact que chaque agrégateur ou agent local va payer pour son électricité, qu'il se fournisse chez le leader ou chez l'un de ses pairs. En se basant sur ce résultat, nous démontrons qu'il existe un équilibre de Nash optimiste au niveau inférieur dans lequel il n'y a aucun échange, ce qui permet, via une sélection des équilibres de Nash, de simplifier le problème de manière significative et d'enfin le résoudre efficacement à l'aide de solveurs commerciaux.

Finalement, une nouvelle sélection d'équilibres de Nash, dits *semi-optimistes*, permet de trouver des profils de prix du meneur plus robustes à la réponse des suiveurs que les profils trouvés grâce à la méthode optimiste. En effet, un problème intrinsèque à la méthode optimiste est qu'il n'y a aucun moyen de garantir que les suiveurs choisissent un équilibre "optimiste". Un équilibre optimiste est un équilibre réalisable, ni plus ni moins. En l'occurrence, les équilibres pessimistes peuvent s'avérer extrêmement mauvais pour le meneur et n'ont que peu de sens dans un contexte de meilleure réponse où les prix offerts par le compétiteur sont statiques, ce qui motive la définition des équilibres semi-optimistes susnommés. Ainsi, des prix trouvés grâce à la méthode semi-optimiste garantiront un certain profit pour une gamme entière d'équilibres réalisables.

Au cours du chapitre, des résultats numériques étayent nos résultats théoriques et prouvent donc la validité de nos approches originales.

- Multiples meneurs, multiples suiveurs (Chapitre 5). Dans ce dernier cas étudié, plusieurs fournisseurs d'électricité sont en concurrence. De tels problèmes sont par nature extrêmement difficiles à traiter, à cause de leur structure complexe. Le problème traité dans ce cas est inspiré du modèle tri-niveau étudié au chapitre précédent : au niveau supérieur, plusieurs fournisseurs rivalisent pour vendre de l'électricité à un ensemble d'agents locaux. La structure des agrégateurs étant plus complexe, ceux-ci sont abandonnés.

La première étape dans l'étude du problème concerné se base sur les résultats obtenus au chapitre précédent. En particulier, il existe un équilibre de Nash au niveau inférieur sans échanges entre les agents locaux. Ainsi, une première simplification du problème consiste à supposer que la réponse des suiveurs ne comportera en effet pas d'échanges, donnant lieu à un MLFG simplifié. Pour résoudre ce MLFG, plusieurs méthodes proposées dans [Leyffer 2010] sont utilisées. Malheureusement, les équilibres du niveau inférieur obtenus avec ces méthodes (grâce à un solveur commercial) ne sont pas les équilibres attendus: en particulier, certains fournisseurs peuvent proposer des prix qui

les feraient vendre à perte.

Afin de contourner cette difficulté, un problème d'équilibre de Nash généralisé (GNEP) est déterminé suite à quelques hypothèses relatives au problème bi-niveau. Grâce à ces hypothèses, la réponse des suiveurs aux prix des meneurs est unique. Bien qu'une solution du GNEP ne constitue pas forcément une solution du problème bi-niveau originellement considéré, il faut noter qu'une telle solution est réalisable, qu'il s'agisse des prix des meneurs ou de la réaction des suiveurs, qui est optimale. Cette approche permet de plus d'obtenir des résultats numériques très intéressants.

Pour résumer, cette thèse est consacrée à l'étude de trois problèmes bi-niveaux de gestion de la demande qui s'avèrent certes compliqués à traiter, mais modélisent de manière pertinente certaines situations. Les conclusions à tirer sont multiples. D'un point de vue pratique, les résultats numériques prouvent le bien-fondé de nos approches. Pour un fournisseur d'électricité, inciter ses clients à déplacer leur demande via des incitations pécuniaires permet une augmentation du profit, alors que les clients compensent les désagréments subis par des économies financières. D'un point de vue théorique, les résultats proposés ne peuvent en grande majorité qu'être appliqués aux problèmes étudiés ici, mais le fait qu'ils aident grandement à leur résolution constitue un argument fort pour une étude théorique approfondie d'un problème avant son éventuelle résolution. De plus, l'idée consistant à sélectionner une réponse particulière des suiveurs n'a été à notre connaissance que très peu étudiée, les approches optimiste et pessimiste se taillant la part du lion jusqu'ici. Or, ce travail prouve que de telles méthodes ont du sens, qu'il s'agisse d'efficacité computationnelle ou de robustesse des solutions vis-à-vis de la réponse des suiveurs.

Finalement, cette thèse clôt quelques questions, mais en ouvre beaucoup d'autres.

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Conception et tarification de nouveaux services en énergie dans un environnement compétitif

Abstract: L'objectif de cette thèse est de développer et étudier des modèles mathématiques d'échanges économiques, basés sur la flexibilité de la demande, entre fournisseurs et consommateurs d'électricité. D'une part, des fournisseurs d'électricité offrent des prix dépendant de l'heure de consommation. D'autre part, des consommateurs adaptent leur usage, minimisant leur facture et le désagrément lié aux changements de consommation induits. La structure de ces problèmes correspond à des problèmes d'optimisation bi-niveau.

Trois types de modèles sont étudiés. Tout d'abord, l'interaction entre un fournisseur et un opérateur de smart grid est modélisée par un problème à un seul meneur et un seul suiveur. Pour cette première approche, le niveau de détails du suiveur est particulièrement élevé, et inclut notamment une gestion stochastique de la production distribuée. La meilleure réponse d'un fournisseur dans un modèle à plusieurs meneurs et plusieurs suiveurs fait l'objet de la seconde partie de la thèse. Celle-ci intègre aussi la possibilité d'avoir des agrégateurs comme suiveurs. Deux nouvelles méthodes de résolution reposant sur la sélection d'équilibres de Nash entre suiveurs sont proposées. Enfin, dans une troisième et dernière partie, on se focalise sur la recherche d'équilibres non coopératifs pour ce modèle à plusieurs meneurs et plusieurs suiveurs.

Tous les problèmes abordés dans cette thèse le sont non seulement d'un point de vue théorique, mais également d'un point de vue numérique.

Mots-clefs: Gestion de la demande, tarification de l'électricité, optimisation bi-niveau, jeux à plusieurs meneurs/suiveurs

Design and pricing of new energy services in a competitive environment

Abstract: The objective of this thesis is to develop and study mathematical models of economical exchanges between energy suppliers and consumers, using demand-side management. On one hand, the suppliers offer time-of-use electricity prices. On the other hand, energy consumers decide on their energy demand schedule, minimizing their electricity bill and the inconvenience due to schedule changes. This problem structure gives rise to bilevel optimization problems.

Three kinds of models are studied. First, single-leader single-follower problems modeling the interaction between an energy supplier and a smart grid operator. In this first approach, the level of details is very high on the follower's side, and notably includes a stochastic treatment of distributed generation. Second, a multi-leader multi-follower problem is studied from the point of view of the best response of one of the suppliers. Aggregators are included in the lower level. Two new resolution methods based on a selection of Nash equilibriums at the lower level are proposed. In the third and final part, the focus is on the evaluation of noncooperative equilibriums for this multi-leader multi-follower problem.

All the problems have been studied both from a theoretical and numerical point of view.

Keywords: Demand-side management, electricity pricing, bilevel programming, multi-leader-follower games
