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Mathieu Boullot

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## **THÈSE**

Pour l'obtention du titre de Docteur en sciences économiques

Présentée et soutenue publiquement le

15 Mars 2019 par

**Mathieu Boullot**

## **Essays on Asset Bubbles and Secular Stagnation**

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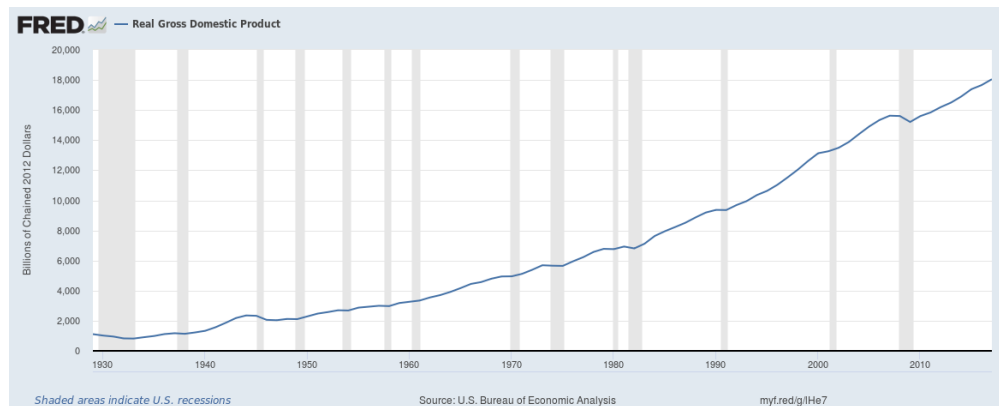
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# Introduction

The graphic below plots the real gross domestic product (RGDP) of the United States since 1929. Strikingly, over the long run, even the financial crises of 1929 and 2008 are almost indistinguishable.



**Figure 1:** US RGDP, Billions of Chained 2012 Dollars, from 1929 to 2017. Source: Bureau of Economic Analysis

Yet, to quote the famous words of John Maynard Keynes, "in the long run, we're all dead". Even if it was a minor event from a (very) long run perspective, the Great Recession of 2008 was a painful experience for dozens of millions of people.

Over the course of two years, the US unemployment rate nearly doubled, and it remained elevated for almost ten years. Even now, the growth rate of nominal wages is still low and the employment-population ratio didn't recover: some discouraged workers decided to exit the labor force during the crisis, but didn't re-enter the labor market after the crisis.

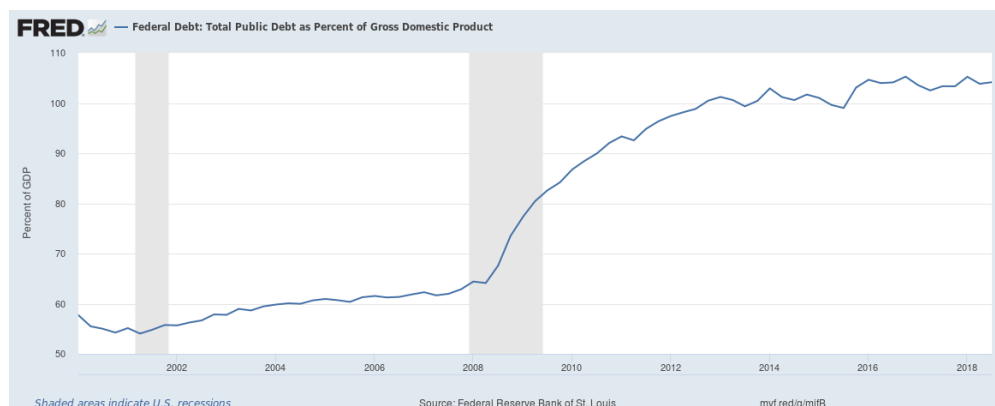




**Figure 2:** US Unemployment Rate: 20 years and over, %, from 2000 to 2019. Source: Bureau of Labor Statistics

As of today, the US economy appears quite strong. However, the next crisis might already be looming on the horizon: according to the NBER <sup>1</sup>, the average duration between two troughs is 70 months in the US since 1945. The last expansion began in June 2009: most than 100 months have passed since the last trough.

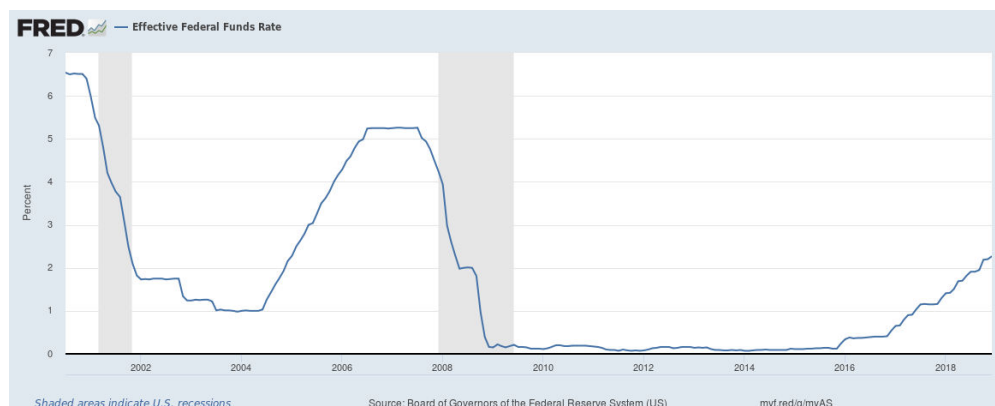
And the US economy is unprepared to yet another recession. The Great Recession lasted for almost 10 years despite large-scale programs aimed at supporting aggregate demand (i.e zero nominal interest rates, quantitative easing, stimulative fiscal policy ect.). But, since public debt has dramatically increased during the crisis, but it wasn't reduced during the expansion, the US fiscal capacity is strongly impaired.



**Figure 3:** US Public Debt / GDP ratio, %, from 2000 to 2019. Source: Saint Louis FED

Furthermore, both the federal funds rate (the nominal interest rate) as well as the real interest rate remain close to zero: there isn't much room for conventional monetary policy either.

<sup>1</sup>Data are available at: <https://www.nber.org/cycles.html>



**Figure 4:** Federal Funds Rate, %, from 2000 to 2019. Source: Board of Governors of the FED system

Thus, if another crisis of the same caliber were to hit the US during the next decade, the situation could quickly become out of control. Therefore, it is more crucial than ever to understand the roots of the 2008 crisis, and hence to determine how to prevent such crises to happen again.

According to [Summers \(2013\)](#) and [Krugman \(2013\)](#), there are two key notions to understand the 2008 crisis: secular stagnation and asset bubbles. I'll briefly develop those notions, and then provide a narrative of this "bubble-secular stagnation" theory of the 2008 crisis.

The price of an asset includes a bubble when it exceeds the fundamental value of this asset, which is equal to the sum of discounted flows of future dividends. Historical examples include the South Sea bubble (England, 18th century), the Tulip Mania bubble (Netherlands, 17th century), or the recent Dot Com bubble (US, 2001). Usually, those bubbly episodes involve speculative behaviors: investors start to accumulate the bubbly asset because they expect to realize high capital gains. Seemingly unrelated, the secular stagnation hypothesis traces back to [Hansen \(1939\)](#). Although Keynesian, demand-led, recessions are usually thought of as temporary, i.e the typical recession in the US lasts around five years, [Hansen \(1939\)](#) speculated that, absent adequate aggregate demand management policies, demand-led depressions could be very persistent or even permanent.

The narrative of the secular stagnation - asset bubble theory goes as follows. For decades, aggregate demand in the US has been on a downward trend because of divers structural changes <sup>2</sup>. However, the mortgage-backed securities (MBS) bubble kept aggregate demand from falling "too low": the central bank still had some leverage over the economy. But this bubble imploded shortly after Lehman Brothers went bankrupt in 2008: investors realized that those MBS weren't as safe

<sup>2</sup>E.g lower rates of productivity and population growth, an increasing demand for USD-denominated assets by emerging economies, sky-rocketing income and wealth inequalities, a higher share of intangible capital vs physical capital etc.

as they sought, but rather exposed to the housing market that had just crashed a few months ago. This led them to re-evaluate a large fraction of those assets as worthless. This sudden and permanent shock to the supply of assets drastically reduced aggregate demand. It couldn't be fully offset by the FED because of the binding Zero Lower Bound (ZLB). Thus, the US economy entered a period of secular stagnation. Among others, [Caballero et al. \(2008\)](#) provide some stylized facts consistent with this interpretation of the events.

This thesis consists in three chapters, each analyzing a particular aspect of the asset bubble - secular stagnation theory. The first paper, "Asset Bubbles and the Income Distribution", which is based on my Master's thesis, focuses on the emergence of asset bubbles. More specifically, it analyzes from a theoretical point of view whether a high concentration at the top of the income distribution promotes or prevents the emergence of asset bubbles. I show that a high level of inequality promotes the emergence of asset bubbles whenever asset bubbles are illiquid and/or financial markets are arbitrage-free; a contrario, a low level of inequality promotes the emergence of asset bubbles when those bubbles are liquid and liquid assets pay a premium under illiquid assets.

The second paper, "Secular Stagnation, Liquidity Trap and Asset Bubbles", deals more directly with the [Summers \(2013\)](#)-[Krugman \(2013\)](#) hypothesis: it analysis under which circumstances asset bubbles are expansionary in the long run in a New Keynesian model that includes capital. I show that secular stagnation is a necessary, but not sufficient, condition for asset bubbles to be expansionary. Indeed, asset bubbles raise a trade-off between a positive demand-side effect vs a negative supply-side effect. The demand-side effect dominates if and only if the bubble-less economy suffers from a strong enough shortage of aggregate demand.

The third paper, "Secular Stagnation or Secular Boom" is more technical: it shows that "standard" New Keynesian models make puzzling predictions when aggregate demand is chronically deficient – they predict a secular boom, and seeks to understand how those models must be adjusted to analyze secular stagnation. I emphasize the crucial role of the long run elasticities of asset demand and supply with respect to the output gap in general equilibrium: if the former is greater than the latter, a persistent shortage of aggregate demand generates a secular stagnation; if the difference is negative, it generates a secular boom. I also connect the failure to meet this condition to other puzzling predictions of the New Keynesian model.

Each paper can be read independently of the others. A substantial summary in french can be found near the end of this manuscript.

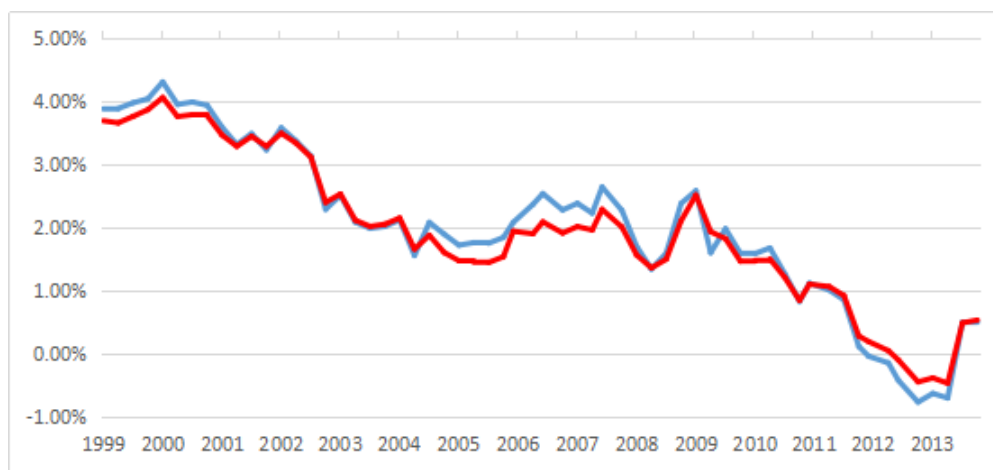
# Chapter I

## Asset Bubbles and the Income Distribution

### 1 Introduction

Three stylized facts have become increasingly important in the macroeconomic discourse since the financial crisis of 2008:

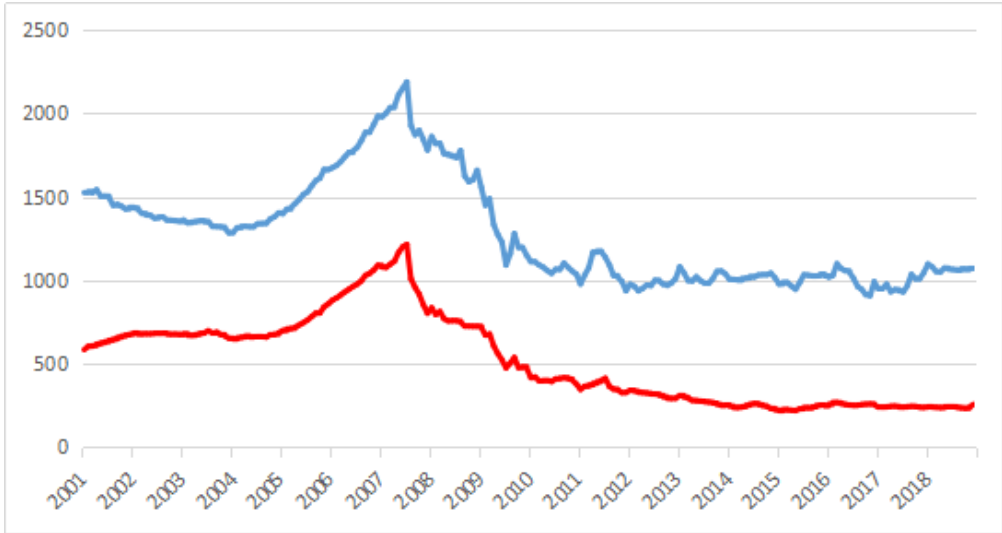
*Fact 1:* The world real interest rate has been on a downward trend for decades <sup>1</sup> and it is currently close to, or below, zero in a lot of (most?) developed economies.



**Figure 1:** World real interest rate in % (blue) and US real interest rate in % (red), from 1999 to 2013. Source: King & Low (2014)

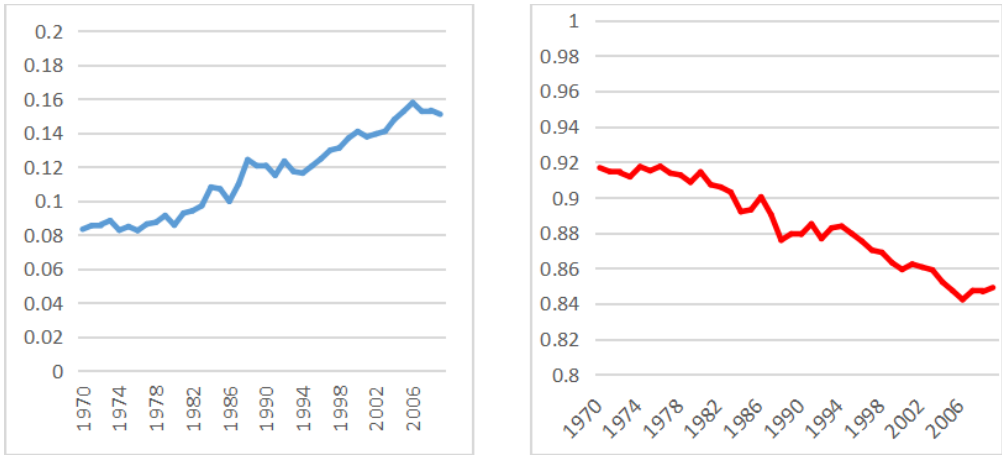
<sup>1</sup> See [IMF \(2014\)](#) and especially chapter 3.

*Fact 2:* Financial bubbles have been popping and bursting here and there, and their macroeconomic consequences are dramatic. Japan experienced a lost decade since the burst of a stock and housing market bubble in the 90's. The bust of the mortgage-backed securities (MBS) bubble in the US in 2008 led to a worldwide collapse.



**Figure 2:** Outstanding Commercial Paper in the US in Trillions of Dollars, Total (blue) and Asset-Backed (red), from 2001 to 2018. Source: FRED

*Fact 3:* Income and wealth inequality have sky-rocketed in developed economies since the 70's. By providing a huge volume of new data, [Piketty \(2014\)](#), among others, brought awareness of those 'internal imbalances'.



**Figure 3:** Post-tax income shares of the top 1% (blue) vs the bottom 99 % (red) in the US from 1970 to 2009. Source: WID

Facts 1 and 2 are crucial to understand the Great Recession of 2008 in the US. Indeed, prominent authors such as [Summers \(2013\)](#) and [Krugman \(2013\)](#) have advanced the idea that the burst of a financial bubble was "the" shock that initiated the crisis. Furthermore, ultra-low real interest rates prevented the major central banks to respond adequately. At first glance, fact 3 doesn't seem to belong to this list: it doesn't appear especially relevant for a macro-economist interested in business cycles and financial crisis. However, since 2008, several economists <sup>2</sup> have informally concluded that the huge inequality shock observed in developed economics was an important driver behind facts 1 and 2.

This "conventional" story is usually framed as follows: in the data, we observe that wealthy households have higher savings rates than poor households. Thus, a higher concentration at the top of the income distribution (fact 1) should raise the aggregate demand for assets. If the aggregate asset supply isn't affected by this inequality shock, the equilibrium interest rate must fall in order to clear the financial markets (fact 2). As the interest rate turns very low, agents have incentives to invest in, or create, asset bubbles (fact 3). According to this logic, fact 3 implies fact 1, and fact 1 implies fact 2: the inequality shock promotes the emergence of financial bubbles.

The implication between facts 1 and 2 is well-known since the seminal work of [Samuelson \(1958\)](#) and [Tirole \(1985\)](#). Both papers have proven that "dynamic inefficiency" – i.e  $R < g$  where  $R$  is the interest rate and  $g$  the growth rate of output, both measured in steady state – is a necessary and sufficient condition for the existence of rational asset bubbles. Intuitively, if the economy cannot produce enough assets, too much savings are chasing too few stores of value: there's a shortage of assets ([Caballero, 2006](#)), and financial bubbles become attractive to investors, even through they're inherently unproductive. But fact 3 has been barely included in the literature on rational bubbles: this paper attempts to fill this gap. Clearly, if one accepts the idea that asset bubbles can be major drivers of business cycles, then it is crucial to better understand how to prevent those bubbles in the first place, that is, to understand which circumstances promote the emergence of asset bubbles.

Although very intuitive, the "conventional" story makes several assumptions, more or less implicitly. In this paper, I relax two of those assumptions: rather than being perfect substitutes to each others, some assets are more liquid than others – I'll use the following definition of liquidity: liquid assets can be traded by all agents whereas illiquid assets cannot be traded by some agents; and wealthier households hold a much higher fraction of illiquid assets in their portfolio <sup>3</sup>. With

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<sup>2</sup>Examples include: [Stiglitz \(2009\)](#), [Bardhan \(2009\)](#) or [Fitoussi and Saraceno \(2010\)](#).

<sup>3</sup>Recent studies find empirical support for the hypothesis that the equity premium is, at least partially, a participation or liquidity premium ([Krishnamurthy and Vissing-Jorgensen, 2012](#)). Currently, participation in the US

regard to the conventional story, this adds a *composition* effect to inequality shocks. Indeed, an inequality shock now affects both the average savings rate but *also* the average desired fraction of liquid assets in agents' portfolio. Taking this composition effect into account turns the inequality-interest rate-asset bubbles nexus upside down: if asset bubbles are liquid, a high concentration of income at the top of the distribution doesn't promote the emergence of financial bubbles, but a low concentration does.

The model I'll present has heterogeneous agents and imperfect financial markets that limit arbitrage between liquid vs illiquid assets. When financial frictions are non-binding, financial markets are functional, i.e arbitrage-free. As liquid and illiquid assets are perfect substitutes to each others, an inequality shock only has a *level* effect: by redistributing income from agents who have a low savings rate to agents who have a high savings rate, it raises asset demand, lowers the interest rate, and, if the interest rate falls enough, it makes rational bubbles possible. Thus, the model is able to reproduce the conventional story.

However, when financial frictions are binding, financial markets are dysfunctional and not arbitrage-free. Instead, liquid assets pay a liquidity premium under illiquid assets: the interest rate is lower than the rate of return on illiquid assets; this liquidity premium is the relative price of illiquid vs liquid assets. An inequality shock has dramatically different implications for the interest rate. Indeed, it redistributes income toward agents who have a higher savings rate, but also hold much less liquid assets in their portfolio. Hence, an inequality shock still raises asset demand and decreases the rate of return on illiquid assets. But, it simultaneously reduces the demand for liquid assets relative to illiquid assets, implying a fall in the liquidity premium, and therefore a higher interest rate *despite* a lower rate of return on illiquid assets. Consequently, the higher interest rate prevents the emergence of liquid rational bubbles; the lower rate of return on illiquid assets promotes the emergence of illiquid rational bubbles *despite* a higher interest rate. Thus, the model also underlines that the conventional story isn't robust: binding financial frictions together with another layer of heterogeneity (portfolio choices) turn the results upside down.

I build a standard two-periods OLG model with competitive markets, a single consumption good, three factors of production: capital, skilled and unskilled labor, two liquid assets: bonds and bubbles, one illiquid asset: equity-capital, and two types of agents: investors and workers. Inequality are to be understood as differences in permanent income in a class society: there isn't

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stock market is estimated at around 40%; and large, wealth- or income-dependent, portfolio differences are well documented.

any income risk nor social mobility. Instead, workers and investors differ at birth along three dimensions: (i) investors supply skilled labor whereas workers supply unskilled labor – skilled labor is relatively scarcer, and therefore better remunerated; (ii) investors are more patient than workers; (iii) they face different investment opportunities: workers hold liquid assets only, whereas investors can take advantage of any arbitrage opportunity between liquid and illiquid assets. Agents within one class are homogenous, there's a fixed mass of each type.

Those assumptions are very tractable ways to capture differences in portfolio choices and savings rates that would arise in a fully-fledged model that includes (ii): idiosyncratic shocks to labor endowment as well as (iii): non-linear costs of re-balancing portfolios. Those classes should be thought of as the "99%", the workers, vs the "1%", the investors, or any other arbitrary segmentation of the population such that there are fewer investors who differ from the rest of the population because (i) they earn more, (ii) they have a higher savings rate and (iii) they have a much higher fraction of illiquid assets in their portfolio. The income distribution is calibrated by a parameter in the production function that determines how the aggregate labor income is distributed between skilled vs unskilled labor, i.e between investors vs workers. If the workers receive a high share of the aggregate labor income, I'll say that the society is relatively equal: agents in the bottom of the distribution collectively earn a large fraction of aggregate income. An "inequality shock" is an exogenous redistribution of income from the workers to the investors – it is micro-founded by a shock to the production function that changes the optimal input mix of the firm, which starts to demand more skilled labor and less unskilled labor.

All agents face other financial frictions: they cannot short-sale the asset bubble, and their supply of bonds is limited by a borrowing constraint. In equilibrium, equity-capital is the only asset in positive net supply: all savings must be channeled to the capital market. Investors act as financial intermediaries between workers and the firm: they sell liquid bonds to the workers and accumulate illiquid equity issued by the firm. The equilibrium asset demand equals the sum of all agents' savings; the equilibrium demand for liquid assets equals workers' aggregate savings. Quite intuitively, a shock to the income distribution affects both: if the society becomes more unequal, asset demand rises because investors have a higher savings rate, but the demand for liquid assets shrinks because investors directly accumulate equity – they don't use financial intermediation. As the shock doesn't affect the equilibrium supply of assets, it simultaneously raises the rate of return on liquid assets (the interest rate) and reduces the rate of return on illiquid assets (the marginal product of capital).



If asset bubbles are liquid, a high level of inequality prevents the emergence of rational asset bubbles; if asset bubbles are illiquid, or all assets are perfect substitutes to each others, a high level of inequality promotes the emergence of rational asset bubbles. Whether the "conventional" story is right (asset bubbles arise because of rising inequality) or wrong (asset bubbles arise despite rising inequality) depends on financial markets imperfections and the type of asset bubble under consideration. With regard to the impact of an inequality shock on the size of the equilibrium asset bubble (that is, conditional on existence), we observe the same phenomenon: an inequality shock inflates already-existing financial bubbles when financial markets are functional or asset bubbles illiquid, but deflates those bubbles when financial markets are dysfunctional and asset bubbles liquid.

Of course, the way the model includes heterogeneity, inequality and financial frictions is very crude: the framework developed in this paper is too simplistic to offer a definitive conclusion about the inequality-interest rate-asset bubbles nexus. However, it allows to capture this additional effect on asset demand: total asset demand vs the demand for a particular class of asset. It is entirely conceivable, and even quite intuitive, that a higher level of inequality in the distribution of wealth or income leads to *both* a higher demand for assets in general and a lower demand for specific asset class (here, liquid).

One way to understand the sub-prime crisis is through the lens of the housing market (an arguably illiquid asset). Another is through the lens of the mortgage-backed securities (MBS) market (an arguably liquid asset). This model is rather coherent with the latter: as the demand for liquid assets has steadily increased over the last 30 years in the US, banks had high incentives to issue as much liquid assets as possible. To do so, they began to lend a lot in order to issue mortgage-backed liquid assets: according to this interpretation, the high housing prices pre-2008 were a by-product of the MBS bubble (as banks started to lend much more for housing-purposes, housing demand rose a lot more than the supply, hence leading to rising prices). This model doesn't provide any explanation of how to engineer an asset bubble, but provides a narrative that questions whether asset bubbles arise because of or despite an increasing concentration of income and wealth at the top of the distribution.

Nevertheless, the predictions of the model are at odds with the data along some dimensions. In particular, if inequality is rising and financial markets imperfect, the model predicts a higher interest rate and a lower marginal product of capital. In the data, we rather observe a more or less stable marginal product of capital, and a falling interest rate. But the model doesn't include other

well-documented macroeconomic trends, including rising mark-ups, changes in productivity and population growths, the savings glut etc. Anyway, this paper makes a simple theoretical point: the "conventional" story is very fragile; it is perfectly conceivable that higher income and wealth inequality were a "stabilizing" force from a macroeconomic point of view.

**Related literature** Since [Samuelson \(1958\)](#) and [Tirole \(1985\)](#), it is well-known that asset bubbles are possible if and only if the real interest rate is lower than the growth rate of output in steady state. More recently, [Martin and Ventura \(2012\)](#) and [Farhi and Tirole \(2012\)](#) have introduced financial frictions in the literature on rational bubbles. This allows to disconnect the interest rate from the rate of return on other assets: asset bubbles become possible even if the marginal product of capital (the rate of return on illiquid assets) is above the growth rate of output – the relevant interest rate in the  $R < g$  condition becomes the highest rate of return the most constrained agent can reach.

I build on those papers, and in particular on [Martin and Ventura \(2016\)](#) who also develop an OLG model with financial frictions, heterogeneous agents and asset bubbles. They show that, if agents have linear preferences, illiquid bubbles can raise investment and output. Indeed, by providing the investors (they call them entrepreneurs) with higher collateral, illiquid asset bubbles raise the interest rate, which induces the savers to fully postpone their consumption into old-age, i.e. it generates a savings glut. Although the basic structure of my model is quite similar to their, my focus is different. I don't try to provide a theory as to why asset bubbles might be expansionary, but rather how the income distribution affects the existence condition as well as the equilibrium size of those bubbles. To do that, I introduce two types of labor associated to the two types of agents. This allows me to calibrate the income distribution and study what's going on in the model when one makes this distribution vary – the income distribution in [Martin and Ventura \(2016\)](#) is very simple: young investors receive nothing and optimally decide not to consume; young workers receive the entire labor income and may decide to consume (if the interest rate is low) or to fully postpone consumption (if the interest rate is high). I also introduce liquid asset bubbles as well as differences in discount rates.

[Graczyk and Phan \(2018\)](#) also analyze the effects of within-cohort inequality on asset bubbles using an OLG model with heterogeneous agents. In their endowment economy, all agents have the same preferences and face the same set of investment opportunities. They find that an inequality shock promotes the emergence of asset bubbles. Indeed, as in their model, all agents receive the

same old-age endowment but different young-age endowment, poor agents want to borrow whereas wealthy agents want to save, both for a consumption-smoothing motive. This is another way of micro-founding the conventional story. In my paper, inequality interacts with heterogeneous investment opportunities and preferences. If financial frictions are non-binding, I recover their result (although through a completely different mechanism); but, if financial frictions are binding, an inequality shock prevents the emergence of liquid asset bubbles whereas it promotes the emergence of illiquid asset bubbles.

Using a three-periods OLG model with financial frictions, [Raurich and Seegmuller \(2017\)](#) show that the distribution of income by age-group, i.e young, middle-age and old, is a crucial determinant of whether asset bubbles are possible or not, as well as whether they're productive or unproductive. While they focus on financial intermediation between agents of different cohorts and the a-synchronicity between investment opportunities and income along the life-cycle, I rather focus on income inequality, heterogeneous investment opportunities and financial inter-mediation within one cohort. Both approaches are complementary.

Closely related are [Ikeda and Phan \(2015\)](#) and [Bengui and Phan \(2018\)](#). Both papers introduce financial frictions in OLG models. They distinguish between safe and risky bubbles, and study how various structural attributes of the financial markets, such as the degree of pledge-ability, the possibility of default or limited liability, promote the existence of one type of bubbles or the other. I introduce a different set of financial frictions, inequality, and I rather distinguish between liquid and illiquid bubbles. Again, both approaches are complementary.

Finally, there's a growing literature on secular stagnation that seeks to explain the downward trend in the interest rate over the last decades in developed economies. Examples include the three-periods OLG model of [Eggertsson et al. \(2017\)](#) and the Bewley-like incomplete markets' model of [Auclert and Rognlie \(2018\)](#). A general conclusion of this literature is that inequality was an important, although probably not the most important, factor. But those papers typically abstract from the distinction between liquid vs illiquid assets and portfolio differences, or don't consider the possibility of dysfunctional financial markets, which are key elements of this paper. Indeed, how the liquidity premium adjusts to inequality shocks is the driving force that generates positive co-movements between the interest rate the aggregate demand for assets in my model.

Section 2 introduces the model and derives the system of equations that characterizes an equilibrium; section 3 studies how inequality affects the long run interest rate when asset bubbles

aren't valued; section 4 introduces liquid asset bubbles; section 5 introduces illiquid asset bubbles; section 6 provides results related to equilibrium determinacy, the proof of which can be found in A.

## 2 Basic environment and equilibrium

The basic setup is an OLG model with a single final good, three factors of production: capital, skilled and unskilled labor, and three assets: a real bond, equity-capital and asset bubbles. Time is discrete,  $t \in \{0, 1, \dots\}$ , and the horizon infinite. Agents have perfect foresight; all markets are competitive. In period 0, old agents are endowed with  $I_{-1} > 0$  units of equity and  $H = 1$  units of bubbles.

### 2.1 Production

A firm uses capital,  $K_t$ , and labor – skilled,  $N_t^i$ , and unskilled,  $N_t^w$ , to produce the final good according to a Cobb-Douglas technology:

$$Y_t = K_t^\alpha \left( N_t^{i\gamma} N_t^{w1-\gamma} \right)^{1-\alpha}, \quad \alpha, \gamma \in (0, 1) \quad (1)$$

Capital takes one period to build, and it fully depreciates during the production process. In period  $t$ , the firm issues  $K_{t+1}$  units of equity in order to build capital in  $t + 1$ . Firm's factor demand functions are:

$$\frac{W_t^i N_t^i}{Y_t} = \gamma(1 - \alpha), \quad \frac{W_t^w N_t^w}{Y_t} = (1 - \gamma)(1 - \alpha), \quad \frac{Q_t K_t}{Y_t} = \alpha \quad (2)$$

$Q_t$  is the rate of dividends on equity and  $W_t^j$  is the wage rate for type- $j$  labor,  $j \in \{i, w\}$ , respectively equal to the marginal products of capital and type- $j$  labor in equilibrium.

Here, a higher  $\gamma$  raises firm's demand for skilled labor but decreases the demand for unskilled labor. Thus, given the labor supply, the wage rate for skilled labor increases whereas that for unskilled labor decreases, implying corresponding changes in type- $j$  labor share in aggregate income.

### 2.2 Households

At the beginning of period  $t$ , a new generation of agents appears. Those agents live two periods, young and old. A generation is made up of investors and workers. Those agents differ along three dimensions: (i) productivity / labor type; (ii) discount factor; (iii) investment opportunities. (i)

is a proxy for differences in inherited skills or human capital whereas (ii) and (iii) are proxies for idiosyncratic shocks to labor endowment as well as non-linear costs of adjusting asset holdings. To simplify the analysis, I focus on a class society à la Matsuyama (2000) and neglect within-class heterogeneity. Those classes should be thought of as the "99%", the workers, vs the "1%", the investors, or any other arbitrary segmentation of the population such that there are fewer investors who differ from the rest of the population because (i) they earn more, (ii) they have a higher savings rate and (iii) they have a much higher fraction of illiquid assets in their portfolio. Although very simplistic, this framework is complex enough to capture and question the conventional intuition.

Each generation consists of a mass  $\lambda \in (0, 1)$  of investors and a mass  $1 - \lambda$  of workers. Agents within one class are homogenous; superscript  $j = i$  will denote investors, while superscript  $j = w$  will denote workers. An agent of generation  $t$  and type  $j$  derives utility from young-age consumption,  $C_{1t}^j$ , as well as old-age consumption,  $C_{2+t1}^j$ ,

$$\mathcal{U}_t^j \equiv \log C_{1t}^j + \beta^j \log C_{2+t1}^j, \quad \beta^j \in (0, 1)$$

Investors are relatively more patient than workers,  $\beta^i > \beta^w$ : this first layer of heterogeneity captures in a simple way the fact that wealthy households save more than non-wealthy households, which is necessary to reproduce the conventional intuition.

A young agent inelastically supplies one unit of labor of type  $j$ , consumes and accumulates three types of assets: real bonds,  $L_t^j$ , issued by other young agents; bubbles,  $H_t^j$ , sold by older agents; and equity,  $I_t^j$ , issued by the firm. His per-period budget constraint is:

$$C_{1t}^j + L_t^j + B_t H_t^j + I_t^j = W_t^j$$

Income-wise, the only difference between a young worker and a young investor is the type of labor each supplies. Under assumption 1 below, skilled labor is both more demanded and less supplied than unskilled labor: in equilibrium, young investors will earn a higher wage rate than young workers.

**Assumption 1**  $\gamma > \lambda$

The within-cohort income distribution is calibrated by two parameters:  $\gamma$  governs the distribution of labor income between classes;  $\lambda$  governs the distribution of population between classes. However, as all policy functions will be linear in income,  $\lambda$  won't affect the aggregate variables which will

depend on  $\gamma$  only. Thus, I'll call an exogenous positive variation of skilled labor share in aggregate income, i.e in investors' aggregate income share,  $d \log \gamma > 0$ , an inequality shock.

In his second period of life, the retiree collects returns on his asset holdings, sells his stock of bubbles and consumes his entire income,

$$C_{2t+1}^j = R_t L_t^j + B_{t+1} H_t^j + Q_{t+1} I_t^j$$

Here,  $R_t$  is the real interest rate.

There are three forms of financial frictions in this economy. First, agents cannot short-sale the asset bubble,

$$H_t^j \geq 0$$

Second, agents' portfolio must satisfy a borrowing constraint,

$$L_t^j \geq -\rho \frac{Q_{t+1}}{R_t} I_t^j - \rho \frac{B_{t+1}}{R_t} H_t^j, \quad \rho \in (0, 1)$$

The supply of bonds,  $-L_t^j$ , is bounded from above by the future discounted returns the agent is able to pledge. Although agents cannot borrow using the asset bubble because they cannot commit to repay, holding asset bubbles allow them to pledge higher returns (although not the full returns, because of the commitment problem). Here,  $L_t^j$  can be thought off as a form of backed credit whereas  $H_t^j$  is a form of un-backed credit.

Third, the equity market is segmented,

$$I_t^w = 0 \tag{3}$$

Workers cannot hold any equity. Thus, the entire capital stock of the economy belongs to the investors, who act as financial intermediaries between workers and the firm in equilibrium.

This last assumption is too simplistic since middle-class agents own stocks and housing. But richer agents invest a much larger fraction of their wealth in illiquid, high-returns, assets. Furthermore, those agents own and manage the largest banks and firms, which issue liquid assets and accumulate illiquid assets.

As both types of agents have free access to the markets for bubbles and bonds, I'll refer to those assets as liquid, or liquidity – respectively bubbly and "fundamental"; a contrario, as the worker cannot buy any equity, the capital market is illiquid.

**Lemma 1** *In equilibrium, the following non-arbitrage conditions must be satisfied:*

$$Q_{t+1} \geq R_t, \quad B_t = \frac{B_{t+1}}{R_t} \quad \text{and} \quad B_t \geq 0 \quad (4)$$

Because all agents can trade in liquid assets, bubbles and bonds must offer the same rate of return in equilibrium ( $B_t = B_{t+1} = 0$  if bubbles aren't traded). But the various financial frictions place a limit to arbitrage between liquid and illiquid assets<sup>4</sup>. Consequently, the equity must be at least as attractive as liquid assets to investors,  $Q_{t+1} \geq R_t$ , but it can offer higher returns in equilibrium.

Thanks to logarithmic preferences, both types save a constant fraction of their labor income,

$$I_t^j + L_t^j + B_t H_t^j = s^j W^j \quad (5)$$

$$\text{Where} \quad s^j \equiv \frac{\beta^j}{1 + \beta^j}$$

Since workers don't have access to the equity market, (3), and the two types of liquid assets offer the same rate of return, (4), workers' portfolio is indeterminate in partial equilibrium.

Investors, however, are capable of taking advantage of arbitrage opportunities between liquid and illiquid assets. If both types of assets offer the same rate of return,  $Q_{t+1} = R_t$ , investors' portfolio is also indeterminate in partial equilibrium. But, if liquid assets pay a liquidity premium under illiquid assets,  $Q_{t+1} > R_t$ , there's an arbitrage opportunity: investors can borrow from other households at a low interest rate, lend all raised funds to the firm at a higher rate of return, and hence make free money out of it. As they implement this strategy, they issue liquid assets until they hit the borrowing constraint, and invest all funds on the equity market. Thus, if  $Q_{t+1} > R_t$ , investors' liquid asset demand functions can be written as:

$$L_t^i = -\bar{L}_t, \quad H_t^i = 0 \quad (6)$$

$$\text{Where} \quad \bar{L}_t \equiv \frac{\rho^{\frac{Q_{t+1}}{R_t}}}{1 - \rho^{\frac{Q_{t+1}}{R_t}}} s^i W_t^i$$

Given (6), we can use (5) to determine the equity demand function.

Here,  $\bar{L}_t$  is the maximal supply of liquid assets consistent with the borrowing constraint. Through the maximal leverage (debt-savings ratio), i.e.  $\frac{\rho^{\frac{Q_{t+1}}{R_t}}}{1 - \rho^{\frac{Q_{t+1}}{R_t}}}$ , it is endogenously determined by

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<sup>4</sup>Short-sales and borrowing constraints bound the supply of liquid assets from above: investors cannot issue as much liquidity as they wish to; market segmentation bounds the demand from below: workers buy liquid assets even if they are dominated by illiquid assets.

the interest rate spread between liquid and illiquid assets,  $\frac{Q_{t+1}}{R_t}$ . A higher spread allows investors to pledge higher discounted returns, and therefore to issue more debt and invest more.

It is perhaps surprising that "poor" agents lend to "wealthy" agents. But both types save because they anticipate retirement. Here, agents aren't temporarily poor or wealthy, they remain so permanently. Thus, the income distribution doesn't affect incentives to save/borrow for a consumption-smoothing motive: inequality in this model refers to differences in permanent income, rather than temporary fluctuations around a given average. But wealthy agents have the opportunity to borrow at a low cost to invest in high-return assets: they act as financial intermediaries between the workers and the firm.

The income distribution,  $\gamma$ , affects both the level of asset demand – agents have different savings rate – as well as its composition – agents make different portfolio choices. In partial equilibrium, a higher level of inequality, i.e  $d \log \gamma > 0$ , tends to affect both the level and composition of asset demand. Indeed, on the one hand, the typical investor has a higher savings rate than the typical worker: the aggregate demand for assets rises; on the other hand, the typical worker holds only liquid assets whereas the typical investor wants to sell liquid assets: the demand for liquid assets falls.

### 2.3 Equilibrium

In the rest of the paper,  $b_t \equiv \frac{B_t}{(1-\alpha)Y_t}$  will denote the bubble-labor income ratio.

**Market for bubbles** In equilibrium, young agents must buy the entire stock of bubbly asset sold by older agents:  $\lambda H_t^i + (1-\lambda)H_t^w = 1$ ; and a standard non-arbitrage condition must be satisfied:

$$b_{t+1} = R_t \left( \frac{K_{t+1}}{K_t} \right)^{-\alpha} b_t \quad (7)$$

I made use of (1) and (2) to re-write (4) in terms of bubble-labor income ratio and capital stock.

**Labor markets** Equalizing labor demand, (2), to labor supply for each type,

$$W_t^i = \frac{\gamma}{\lambda}(1-\alpha)Y_t, \quad W_t^w = \frac{1-\gamma}{1-\lambda}(1-\alpha)$$

Under assumption 1, an investor receives a higher labor income than a worker. Note also that the distribution of labor income between classes is fixed: on aggregate, investors receive a share  $\gamma$  of



the aggregate labor income. Since all policy functions are linear in income, there's a representative worker as well as a representative investor. Given the income distribution between classes,  $\lambda$  determines the income level of each individual, which is neutral on aggregate.

**Equity market** From (2), the rate of dividends equals the marginal product of capital. Using (1) to re-write in terms of capital stock:

$$Q_{t+1} = \alpha K_{t+1}^{\alpha-1} \quad (8)$$

Firm's supply of equity only depends on the rate of dividends: it isn't affected by asset bubbles or inequality. To compute the economy's stock of capital, we can use the liquidity market clearing condition,  $\lambda L_t^i + (1 - \lambda)L_t^w = 0$ , as well as the savings function, (5), to determine investors' equilibrium equity demand and impose market clearing,  $I_t = K_{t+1}$ :

$$K_{t+1} = \left( s^i \gamma + s^w (1 - \gamma) - b_t \right) (1 - \alpha) K_t^\alpha \quad (9)$$

And in period 0,  $K_0 = I_{-1}$ .

From an aggregate point of view, capital is the only asset in positive net supply: in general equilibrium, investors' equity demand equals the aggregate asset demand, which is the sum of all agents' savings (including old agents dis-savings when bubbles are valued). Indeed, investors collectively issue just enough liquid assets to meet workers' demand,  $-\lambda L_t^i = (1 - \lambda)L_t^w$ , and use those borrowed funds together with their own savings to accumulate equity.

Thanks to logarithmic preferences, the aggregate asset demand is fully inelastic with respect to the interest rate. The aggregate savings rate:  $s_t \equiv (s^i \gamma + s^w (1 - \gamma) - b_t)(1 - \alpha)$ , is affected both by the distribution of income between classes ( $\gamma$ ) and generations ( $\alpha, b$ ). Indeed, young investors have a lower marginal propensity to consume (MPC) than young workers, who them-selves have a lower MPC than old households. Hence, any redistribution of income between those agents affects the aggregate asset demand and hence the economy's stock of capital: I'll speak of the *level* effect.

$$d \log K_{t+1} = \frac{(s^i - s^w) \gamma}{s^i \gamma + s^w (1 - \gamma) - b_t} d \log \gamma - \frac{b_t}{s^i \gamma + s^w (1 - \gamma) - b_t} d \log b_t$$

In particular, as  $s^i > s^w$ , an inequality shock has a positive level effect: it increases asset demand and hence the equilibrium stock of capital; a bigger asset bubbles, that redistributes income from

asset buyers (young agents) to asset sellers (old agents), has a negative level effect: it decreases asset demand and the equilibrium stock of capital.

**Debt market** Since the economy is closed, liquidity is in zero net supply:  $\lambda L_t^i + (1 - \lambda)L_t^w = 0$ . As workers' demand for liquid assets is fully inelastic, investors' supply must adjust in general equilibrium to ensure that all savings are channeled to the capital market. To determine the interest rate consistent with this condition, it is necessary to distinguish between the two cases depicted in lemma 1. Let  $\bar{L}_t^c \equiv \frac{\rho}{1-\rho} s^i (1 - \alpha) K_t^\alpha$  be the maximal supply of liquidity of an individual investor when the liquidity premium is nil, i.e  $Q_{t+1} = R_t$ .

As long as workers' aggregate demand for liquid assets remains limited, that is, as long as:  $(1 - \lambda)L_t^w \leq \lambda \bar{L}_t^c$ , financial markets are functional, meaning arbitrage-free. The supply of liquidity by the investors is infinitely elastic:  $Q_{t+1} = R_t$ , and financial constraints are irrelevant. I'll refer to this regime as to either "functional financial markets", or the "high-liquidity", *h*-, regime. But, as the demand for liquidity rises relative to the supply, liquidity becomes scarce:  $(1 - \lambda)L_t^w > \lambda \bar{L}_t^c$ , and financial markets turn dysfunctional, not arbitrage-free. Indeed, if all assets offer the same rate of return, investors aren't able to issue enough debt to meet the demand. Instead, to clear the financial markets, the interest rate must fall under the rental rate of capital:  $Q_{t+1} > R_t$ , until:  $(1 - \lambda)L_t^w = \lambda \bar{L}_t^c$ . This liquidity premium,  $\frac{Q_{t+1}}{R_t} > 1$ , raises investors' maximal leverage, allowing them to borrow more. I'll refer to this as to either "dysfunctional financial markets", or the "low-liquidity", *l*-, regime.

To summarize, we can write the liquidity market clearing condition as:

$$R_t = \min \left\{ \frac{\rho (s^i \gamma + s^w (1 - \gamma) - b_t)}{s^w (1 - \gamma) - b_t}, 1 \right\} Q_{t+1} \quad (10)$$

Given the rate of return on equity, the liquidity market clearing condition determines the interest rate spread. When liquidity is abundant, the economy is in regime *h*: the interest rate spread is nil, and the interest rate is determined by the aggregate asset supply and demand. However, when liquidity is scarce, the economy is in regime *l*: through the liquidity premium, the interest rate is also shaped by the supply and demand for liquid assets. Liquidity is scarce when the supply of liquid assets – the pledged fraction of equity-investment,  $\rho (s^i \gamma + s^w (1 - \gamma) - b_t) (1 - \alpha)$ , is lower than the demand for liquid assets,  $(s^w (1 - \gamma) - b_t) (1 - \alpha)$ , both measured as shares of output; this condition is equivalent to  $(1 - \lambda)L_t^w > \lambda \bar{L}_t^c$ .

Those two regimes have drastically different implications for the interest rate, at least given the rate of return on equity. Indeed, in regime  $l$ , any shock that impacts the supply or demand for liquid assets affects the interest rate because it changes the relative price of liquid vs illiquid assets, i.e it affects the liquidity premium, even if the shock leaves the aggregate asset supply and demand constant. This is the *composition* effect. In regime  $h$ , this composition effect vanishes: all assets are perfect substitutes to each other.

Through the composition effect, the liquidity premium in regime  $l$  is decreasing as the asset bubble grows bigger and also as the income distribution gets more unequal.

$$d \log \frac{Q_{t+1}}{R_t} = - \left( \frac{(s^i - s^w)\gamma}{s^i\gamma + s^w(1 - \gamma) - b_t} + \frac{s^w\gamma}{s^w(1 - \gamma) - b_t} \right) d \log \gamma \\ - \left( \frac{b_t}{s^w(1 - \gamma) - b_t} - \frac{b_t}{s^i\gamma + s^w(1 - \gamma) - b_t} \right) d \log b_t$$

Indeed, a lower workers' share in income reduces workers' demand for liquid assets, and it simultaneously raises the supply of liquid assets because investors accumulate more equity and therefore have higher returns to pledge. Through similar mechanisms, a bigger asset bubble reduces both the supply and demand for liquid assets, but the latter is relatively more affected – the second term between brackets is clearly positive.

**Equilibrium** Given  $K_0 > 0$ , an equilibrium is a sequence  $\{b_t, Q_{t+1}, K_{t+1}, R_t\}$  such that (7), (8), (9) and (10) are satisfied,  $K_{t+1} \geq 0$  and  $b_t \geq 0$ , for all  $t \geq 0$ .

### 3 Income distribution and the interest rate

Let's first assume that bubbles aren't valued in the long run. There's always a unique "fundamental" steady state:

$$b = 0 \\ R^* = \min \left\{ \frac{\rho (s^i\gamma + s^w(1 - \gamma))}{s^w(1 - \gamma)}, 1 \right\} Q^* \\ Q^* = \frac{\alpha}{(s^i\gamma + s^w(1 - \gamma)) (1 - \alpha)} \\ K^* = \left( s^i\gamma + s^w(1 - \gamma) \right)^{\frac{1}{1-\alpha}} (1 - \alpha)^{\frac{1}{1-\alpha}}$$

Investors act as financial intermediaries: they borrow from the workers in order to lend to the firm. Because agents have logarithmic preferences and prices are flexible <sup>5</sup>, financial frictions are irrelevant to the steady state stock of capital and output: it is "as if" all agents had access to the equity market. However, the distribution of income between classes is important if agents have heterogeneous preferences. Indeed, as workers are less patient than investors, an inequality shock that redistributes income from the workers to the investors, captured by a higher  $\gamma$ , raises the aggregate savings rate and the long run stock of capital.

**Proposition 1** *Assume that  $b = 0$ . In steady state:*

1. *Financial markets are dysfunctional if and only if  $\gamma < \gamma^c$ .*
2. *If financial markets are functional, an inequality shock reduces the interest rate as well as the marginal product of capital:  $\frac{d \log R^*}{d \log \gamma} = \frac{d \log Q^*}{d \log \gamma} < 0$ .*
3. *If financial markets are dysfunctional, an inequality shock increases the interest rate but reduces the marginal product of capital:  $\frac{d \log Q^*}{d \log \gamma} < 0 < \frac{d \log R^*}{d \log \gamma}$ .*

Where  $\gamma^c \equiv \frac{s^w(1-\rho)}{s^w(1-\rho)+s^i\rho} \in (0, 1)$ .

But, as proposition 1 illustrates, financial frictions greatly matter for the inequality - interest rate nexus. While the conventional intuition – that a redistribution from agents whose savings rate is low to agents whose savings rate is high lowers the interest rate – is verified when financial markets are functional, it is turned upside down when financial markets are dysfunctional. Instead, a redistribution of this kind leads to both a higher interest rate *and* a lower marginal product of capital in the long run. In this model, inequality affects asset demand through both a *level* effect – on which the conventional intuition is based – but also a *composition* effect: the demand for liquid assets may well rise while the aggregate demand for assets falls.

The intuition goes as follows. Although the typical worker cannot directly accumulate capital, investors act as financial intermediaries and allocate both their and workers' savings on the capital market. If financial markets are functional, i.e if  $(1 - \lambda)L^w \leq \lambda\bar{L}^c$ , liquidity isn't particularly scarce: investors' supply is infinitely elastic. Hence, the interest rate is the price of all assets, and it is shaped by the aggregate asset supply and demand: there isn't any composition effect. A redistribution of income from the workers to the investors raises the aggregate savings rate, i.e it

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<sup>5</sup>Implying: (i) agents' savings rates don't depend on the interest rate; (ii) the interest rate adjusts to clear the financial markets, i.e production is supply-determined.

raises asset demand. As it leaves firm's supply unaffected, the interest rate must fall to clear the financial markets:

$$d \log R^* = d \log Q^*$$

$$d \log Q^* = -\frac{(s^i - s^w)\gamma}{s^i\gamma + s^w(1 - \gamma)} d \log \gamma$$

However, if the financial markets are dysfunctional, liquidity is scarce and pays a liquidity premium under the rate of dividends: without this liquidity premium, investors aren't able to pledge enough returns to meet the demand for liquid assets by the workers. Any transfer from the workers to the investors simultaneously raises the aggregate asset demand – the aggregate savings rate increases, raises the supply of liquid assets – that is proportional to investors' pledged returns – and decreases the demand for liquid assets – workers' income, and hence liquidity demand, shrinks. This leads to a lower marginal product of capital, a lower liquidity premium and a higher interest rate:

$$d \log R^* = \left( \frac{(s^i - s^w)\gamma}{s^i\gamma + s^w(1 - \gamma)} + 1 \right) d \log \gamma + d \log Q^*$$

$$d \log Q^* = -\frac{(s^i - s^w)\gamma}{s^i\gamma + s^w(1 - \gamma)} d \log \gamma$$

Crucially, whether financial markets are functional or dysfunctional is also determined by the income distribution. Given financial markets frictions, that is, given  $\rho$ , more unequal societies, that is, with a higher  $\gamma$ , are less likely to have dysfunctional financial markets. Indeed, if those agents who cannot access all financial markets receive a low aggregate income share, the demand for liquid assets will be low. Thus, even through the supply of liquid assets is severely limited, those financial frictions are irrelevant.

## 4 Asset bubbles, the interest rate and inequality

Given the supply, a strong demand for a specific class of assets puts upward pressures on its price. In particular, if there's either a very high demand for liquid assets – *l*-regime – or a very high demand for assets in general – *h*-regime, the rate of return on those assets falls below the growth rate of output. To remedy this dynamic inefficiency, private agents have one great tool: asset bubbles. If they succeed in coordinating their expectations <sup>6</sup>, they can create rational asset

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<sup>6</sup>This paper, as well as virtually the entire literature on rational asset bubbles, doesn't provide any explanation for this dynamic process of expectations-formation and coordination.

bubbles. Those are assets that don't bring any dividends nor utility, i.e they aren't productive, but may nevertheless be valued as a way to store wealth. Even if they are in zero net supply on aggregate (old households sell to young households), they improve consumption-smoothing by redistributing income between cohorts, which reduces the aggregate asset demand.

$$b = \max \left\{ b^l \equiv s^w(1 - \gamma) - \rho \frac{\alpha}{1 - \alpha}, b^h \equiv s^i \gamma + s^w(1 - \gamma) - \frac{\alpha}{1 - \alpha} \right\}, \quad b > 0$$

$$R^b = 1$$

$$Q = \min \left\{ Q^l \equiv \frac{\alpha}{s^i \gamma(1 - \alpha) + \rho \alpha} R^b, Q^h \equiv R^b \right\}$$

$$k = \min \left\{ k^l \equiv \left( s^i \gamma(1 - \alpha) + \rho \alpha \right)^{\frac{1}{1 - \alpha}}, k^h \equiv (\alpha)^{\frac{1}{1 - \alpha}} \right\}$$

As there's always a fundamental steady state, if a bubbly steady state also exists, the economy has two steady states. Those aren't necessarily in the same regime. Two types of bubbly steady states can arise, although they don't co-exist. In one case, financial markets are functional, all agents value the bubbles. This is the old fashion dynamic inefficiency of e.g [Tirole \(1985\)](#), which is symptomatic of a lack of assets in general; in the other, financial markets are dysfunctional, the workers value the bubbles whereas the investors don't. This is the "new" form of dynamic inefficiency introduced by [Martin and Ventura \(2012\)](#), which is symptomatic of a lack of liquid assets (or a particular class of assets, more generally).

A bubbly steady state of type  $j \in \{l, h\}$  exists under two conditions: first, the "fundamental" interest rate must be lower than the growth rate of output, that is, either  $R^* < \min\{1, Q^*\}$  or  $R^* = Q^* < 1$ ; second, the "bubbly" economy needs to be in regime  $j$ , that is, either  $R^b < Q^b$  or  $R^b = Q^b$ . Both conditions can be expressed in terms of parameter  $\gamma$  that calibrates the income distribution.

**Proposition 2** *1. There's a bubbly steady state with functional financial markets if and only if  $\gamma > \max\{\gamma^h, \gamma^{c-b}\}$ . An inequality shock doesn't affect the interest rate nor the marginal product of capital, but the bubble grows bigger:  $\frac{d \log R^b}{d \log \gamma} = \frac{d \log Q^h}{d \log \gamma} = 0 < \frac{d \log b^h}{d \log \gamma}$ .*

*2. There's a bubbly steady state with dysfunctional financial markets if and only if  $\gamma < \min\{\gamma^l, \gamma^{c-b}\}$ . An inequality shock doesn't affect the interest rate, but it reduces the marginal product of capital and the asset bubble:  $\frac{d \log R^b}{d \log \gamma} = 0 > \frac{d \log Q^l}{d \log \gamma}, \frac{d \log b^l}{d \log \gamma}$ .*

Where  $\gamma^h \equiv \frac{\alpha - s^w}{s^i - s^w}$ ,  $\gamma^{c-b} \equiv \frac{1 - \rho}{s^i} \frac{\alpha}{1 - \alpha}$ , and  $\gamma^l \equiv 1 - \frac{\rho}{s^w} \frac{\alpha}{1 - \alpha}$ .

As proposition (2) illustrates, the way inequality affects asset bubbles is conditional on whether financial markets are functional or not. Rational asset bubbles are possible when the economy suffers from a strong lack of assets, implying a form of dynamic inefficiency. Those bubbles reduce asset demand until the interest rate is equalized to the growth rate of the economy. Thus, a lower "bubble-less" interest rate implies a larger asset bubble in steady state. But, as discussed in the previous section, the correlation between the interest rate and inequality, and therefore asset bubbles and inequality, depends on the regime the economy is in.

There's a bubbly steady state with functional financial markets when the workers receive a small share of output,  $\gamma > \max\{\gamma^h, \gamma^{c-b}\}$ . In regime  $h$ , the liquidity premium is nil: the asset bubble of type  $h$  equalizes both the interest rate and the marginal product of capital to the growth rate of output. It occurs in economies that suffer from a lack of assets in general: investors receive a large share of output such that (i) the rental rate of capital is lower than the growth rate of the economy,  $\gamma > \gamma^h$ ; (ii) the liquidity premium is nil when there's a bubble,  $\gamma > \gamma^{c-b}$ . Both conditions are equivalent to a lower bound on the asset price bubble: (i) it must be traded at a positive price; (ii) the price must be high enough to equalize the rate of return on liquid vs illiquid assets.

This type of bubble is valued by all agents and completely determines the stock of capital as a function of the capital share and the growth rate of output. Thus, it "isolates" the stock of capital from any fluctuations in the distribution of income that are "absorbed" by the asset bubble. In particular, very unequal societies are characterized by a high fundamental savings rate, a low fundamental marginal product of capital and hence a large asset bubble.

$$\begin{aligned} d \log R^b &= 0 \\ d \log Q^h &= 0 \\ d \log b^h &= \frac{(s^i - s^w)\gamma}{s^i\gamma + s^w(1 - \gamma) - \frac{\alpha}{1-\alpha}} d \log \gamma \end{aligned}$$

A contrario, there's a bubbly steady state with dysfunctional financial markets when the workers receive a large share of output,  $\gamma < \min\{\gamma^l, \gamma^{c-b}\}$ . In this other regime, the asset bubble equalizes the interest rate to the growth rate of output, but it remains below the marginal product of capital. A bubble of type  $l$  is possible in economies that suffer from a lack of liquid assets: the investors receive a small share of output such that (i) the interest rate is lower than the growth rate of the economy,  $\gamma < \gamma^l$ ; (ii) the liquidity premium remains positive when there's a bubble,  $\gamma < \gamma^{c-b}$ . Here, the first condition is, again, a lower bound on the asset price bubble: (i) it must be traded

at a positive price; but the second is an upper bound: (ii) the price must be low enough such that it doesn't equalize the rate of return on liquid vs illiquid assets.

This type of bubble is valued by the workers only. As workers' share decreases, they demand less liquid assets which deflates the steady state bubble. But this bubble affects the interest rate through two channels: the rental rate of capital and the liquidity premium. Thus, this bubble doesn't fully "isolate" the stock of capital from fluctuations in the income distribution. Rather, as workers' share in income falls, there's a simultaneous adjustment in both the liquidity premium, that increases, and the rental rate of capital, that decreases:

$$\begin{aligned} d \log R^b &= 0 \\ d \log Q^l &= -\frac{s^i \gamma (1 - \alpha)}{s^i \gamma (1 - \alpha) + \rho \alpha} d \log \gamma \\ d \log b^l &= -\frac{s^w \gamma}{s^w (1 - \gamma) - \rho \frac{\alpha}{1 - \alpha}} d \log \gamma \end{aligned}$$

Note that, while a bubble of type  $h$  can emerge even if the fundamental steady state is of type  $l$ , a bubble of type  $l$  is possible only if the fundamental steady state is of type  $l$ :  $\min\{\gamma^l, \gamma^{c-b}\} < \gamma^c$ . Indeed, as an asset bubble decreases the liquidity premium, the "fundamental" liquidity premium must be large for a bubbly steady state of type  $l$  to exist.

The correlation between asset bubbles and inequality is turned upside down when financial markets are dysfunctional. This is quite natural given the results in the previous section: a rational bubble exists if and only if the interest rate is lower than the growth rate of output, and it grows just large enough to equalize  $R$  to  $g$  ( $= 1$  here). Since the correlation between the level of inequality and the interest rate changes of sign when financial markets become dysfunctional, the same change is observed when we consider the correlation between the level inequality and asset bubbles.

In this section, I implicitly considered that asset bubbles are liquid. However, if asset bubbles are illiquid, we recover the conventional story: an inequality shock promotes the emergence of assets bubble, *but despite a higher interest rate*.

## 5 Illiquid asset bubble

Until now, the asset bubble was treated as a liquid asset similar to debt. In this extension, asset bubbles are illiquid:

$$H_t^w = 0, \quad H_t^i \geq 0 \tag{11}$$



This condition replaces the other no-short-sales constraint: only investors can hold the bubbly asset. This is the only modification with respect to the basic model. As now the marginal asset bubble holder is an investor, condition (4) is replaced by:

$$Q_{t+1} \geq R_t, \quad B_t = \frac{B_{t+1}}{Q_{t+1}}, \quad B_t \geq 0$$

Riding the bubble must provide the same rate of return as holding the equity.

Savings are still given by (5) for both types; (3) and (11) allow us to determine workers' portfolio. If  $Q_{t+1} = R_t$ , investors' portfolio is fully indeterminate in partial equilibrium. However, if  $Q_{t+1} > R_t$ , investors' liquid asset demand functions can be written as:

$$L_t^i + B_t H_t^i = -\frac{\rho \frac{Q_{t+1}}{R_t}}{1 - \rho \frac{Q_{t+1}}{R_t}} s^i W_t^i, \quad H_t^i \geq 0$$

Given (6), this equation determines investors' equity demand.

As on aggregate, agents still have the same savings rate, and firm's program hasn't changed, (8) and (9) remain valid. But (7) is replaced by:

$$b_{t+1} = Q_{t+1} \left( \frac{K_{t+1}}{K_t} \right)^{-\alpha} b_t \tag{12}$$

And the liquidity market clearing condition, (10), is replaced by:

$$R_t = \min \left\{ \frac{\rho (s^i \gamma + s^w (1 - \gamma))}{s^w (1 - \gamma)}, 1 \right\} Q_{t+1} \tag{13}$$

Asset bubbles don't affect the liquidity premium. Indeed, they don't lower the demand for liquid assets by the workers: bubbles aren't liquid, and therefore workers' demand is nil; and they don't decrease the supply by the investors: the lower pledged returns on equity investment are fully compensated by the higher pledged returns on riding the bubble.

Given  $K_0 > 0$ , an equilibrium is a sequence  $\{b_t, Q_{t+1}, K_{t+1}, R_t\}$  such that (12), (8), (9) and (13) are satisfied,  $K_{t+1} \geq 0$  and  $b_t \geq 0$ , for all  $t \geq 0$ .

Of course, this modification doesn't affect the fundamental steady state, which is similar to that of section 3. But the bubbly steady states differ. Indeed, there's a unique bubbly steady state, which has the same bubble and capital stock as that of type  $h$ , section 4, but exists under the condition:  $\gamma > \gamma^h$ , i.e the usual dynamic inefficiency condition:  $Q^* < 1$ . The other difference is the

liquidity premium:

$$R^b = \min \left\{ \frac{\rho (s^i \gamma + s^w (1 - \gamma))}{s^w (1 - \gamma)}, 1 \right\} Q^b$$

The dynamic of capital and asset prices are also similar to that of regime  $h$ , section 4.

Thus, illiquid asset bubbles satisfy the conventional intuition. There's however, an important addendum when financial markets are dysfunctional, i.e  $\gamma < \gamma^c$ : an inequality shock favors the emergence of financial bubbles *despite a higher interest rate*. Indeed, the existence of illiquid bubbles is conditional on the rate of return on illiquid assets, which doesn't equal the interest rate. If financial markets are dysfunctional, an inequality shock lowers the rate of return on illiquid assets but raises that on liquid assets.

## 6 Dynamic and stability

The entire dynamic can be described by a 2-dimensional system with one backward-looking variable, the stock of capital, and one forward-looking, the asset price bubble:

$$K_{t+1} = \left( (s^i \gamma + s^w (1 - \gamma) - b_t) (1 - \alpha) \right) K_t^\alpha$$

$$b_{t+1} = \begin{cases} \frac{\rho \alpha}{\rho \alpha + (b^l - b_t)(1 - \alpha)} & \text{if } b_t < b^l + \frac{\rho}{1 - \rho} (b^l - b^h) \\ \frac{\alpha}{\alpha + (b^h - b_t)(1 - \alpha)} & \text{otherwise} \end{cases}$$

Given  $\{b_t\}$ , the dynamic of the stock of capital is stable: there are equilibria as long as the asset price bubble doesn't explode as a share of output. The analysis of the difference equation corresponding to the asset bubble is relegated to appendix A.

**Proposition 3** *1. If the fundamental steady state doesn't co-exist with a bubbly steady state, there's a unique equilibrium:  $b_t = 0$  for all  $t \geq 0$ .*

*2. If the fundamental steady state co-exists with a bubbly steady state of type  $j \in \{l, h\}$ ,*

- i. There's a continuum of equilibria, indexed by  $b_0 \in [0, b^j)$ , that converges to the fundamental steady state;*
- ii. As well as a unique equilibrium that converges to the bubbly steady state:  $b_t = b^j$  for all  $t \geq 0$ .*

The fundamental steady state is globally determinate if and only if it is unique; if it co-exists with a bubbly steady state, it is globally indeterminate but the bubbly steady state is globally

determinate. The presence of two distinct regimes doesn't change this conclusion, which is usual in the literature, and traces back to [Tirole \(1985\)](#).

## 7 Conclusion

I have provided a simple, analytical, model to study how inequality promotes or prevents the emergence of (rational) financial bubbles. An inequality shock promotes the emergence of financial bubbles if and only if financial markets aren't too imperfect, *or* the bubble is illiquid.

This model is, of course, way too simplistic to offer a definitive conclusion on this matter. But it underlines the very importance of financial frictions to understand how inequality shapes the rates of return on particular asset classes and therefore promotes or prevents the emergence of asset bubbles of particular types.

## A Bubbly dynamic

The dynamic of the bubble-labor income ratio is described by the following non-linear equation:

$$b_{t+1} = \begin{cases} \frac{\rho\alpha}{\rho\alpha + (b^l - b_t)(1-\alpha)} & \text{if } b_t < b^l + \frac{\rho}{1-\rho}(b^l - b^h) \\ \frac{\alpha}{\alpha + (b^h - b_t)(1-\alpha)} & \text{otherwise} \end{cases}$$

In a given regime  $j \in \{l, h\}$ , the dynamic is unstable: the bubble explodes as a share of output, i.e.  $\lim_{t \rightarrow \infty} b_t \rightarrow \infty$ , if  $b_t > b^j$ ; it implodes,  $\lim_{t \rightarrow \infty} b_t = 0$ , if  $b_t < b^j$ ; and it has a unique steady state,  $b_t = b^j$ . A path that leads to an explosion cannot be part of an equilibrium because it would violate either the bubble market clearing condition, or young agents' budget constraint. A path that leads to an implosion converges to the fundamental steady state.

It is useful to distinguish three cases: (i) there isn't any bubbly steady state; (ii) there's a bubbly steady state of type  $l$ ; (iii) there's a bubbly steady state of type  $h$ . Remember that there's always a unique fundamental steady state.

(i) There isn't any bubbly steady state if and only if  $\max\{b^l, b^h\} < 0$ . Consequently, if  $b_0 > 0$ , then  $b_0 > \max\{b^l, b^h\}$ : all path such that  $b_t > 0$  lead to an explosion. Hence, there's a unique equilibrium, and it is bubble-less:  $b_t = 0$  for all  $t \geq 0$ .

(ii) There's a bubbly steady state of type  $l$  if and only if  $b^l > \max\{0, b^h\}$ . As  $b^l + \frac{\rho}{1-\rho}(b^l - b^h) >$

$b^l > b^h$ , a path that starts in, or switches to, the  $h$ -regime leads to an explosion: this rules out all path such that  $b_0 > b^l$ . Furthermore, since  $b^l + \frac{\rho}{1-\rho}(b^l - b^h) > 0$  is equivalent to  $\gamma < \gamma^c$ , the fundamental steady state is of type  $l$ . Hence, there's a unique path that converges to the bubbly steady state,  $b_t = b^l$  for all  $t \geq 0$ ; and a continuum of paths,  $b_0 \in [0, b^l)$ , that converge to the fundamental steady state.

- (iii) There's a bubbly steady state of type  $h$  if and only if  $b^h > \max\{0, b^l\}$ . Here, we have:  $b^l + \frac{\rho}{1-\rho}(b^l - b^h) < b^l < b^h$ . There are two sub-possibilities. (a) The fundamental steady state is of type  $h$ , that is,  $\gamma > \gamma^c$ . As  $b^l + \frac{\rho}{1-\rho}(b^l - b^h) < 0$ , the economy is always in the  $h$ -regime. All paths such that  $b_0 \in [0, b^h)$  lead to an implosion; all paths such that  $b_0 > b^h$  lead to an explosion; and there's a unique path such that  $b_t = b^h$  for all  $t \geq 0$ . (b) The fundamental steady state is in regime  $l$ , that is,  $\gamma < \gamma^c$ . Again, if  $b_t > b^h$ , the economy starts in the  $h$ -regime, remains there, and the bubble explodes. If  $b_t < b^l + \frac{\rho}{1-\rho}(b^l - b^h)$ , the economy starts in the  $l$ -regime, remains there, and the bubble implodes. If  $b_t \in (b^l + \frac{\rho}{1-\rho}(b^l - b^h), b^h)$ , the economy starts in regime  $h$ , but the bubble begins to implode; there's a switch to the  $l$ -regime as soon as  $b_t < b^l + \frac{\rho}{1-\rho}(b^l - b^h)$ , and the bubble continues to implode. Hence, again, there's a unique path that converges to the bubbly steady state,  $b_t = b^h$  for all  $t \geq 0$ ; and a continuum of paths,  $b_0 \in [0, b^h)$ , that converge to the fundamental steady state.

## Chapter II

# Secular Stagnation, Liquidity Trap and Asset Bubbles

### 1 Introduction

Some, if not most, of the major financial crises were initiated by boom-burst cycles in asset prices. But the crisis that started in 2008 in the US stands out, for at least two main reasons. First, the burst wasn't preceded by a boom. Instead, from the beginning of the 90's until 2008, the US economy experienced a prolonged episode of low volatility, modest GDP growth and inflation. In retrospect, this "great moderation" despite a large financial bubble is difficult to rationalize. Second, the crisis seems to have had a very persistent, or even permanent, effect on the US economy. More than 10 years after the crash, even through unemployment recovered, labor market participation remains weak, capital accumulation below the pre-2008 trend and the nominal interest rate close to zero percent.

To explain the dynamic of the US economy pre- and post-2008, [Summers \(2013\)](#) and [Krugman \(2013\)](#) have advanced a simple, yet powerful, explanation: the "bubble-secular stagnation theory". The narrative goes as follows. For decades, aggregate demand in the US has been on a downward trend because of various structural changes – lower rates of productivity and population growth, an increasing demand for USD-denominated assets by emerging economies etc. However, the mortgage-backed securities bubble kept aggregate demand from falling "too low", i.e the Zero Lower Bound (ZLB) wasn't binding. The burst of this bubble in 2008 was a large, permanent, negative shock to aggregate demand. Because it soon hit the ZLB, the FED wasn't able to fully offset the effect of

this shock: the US economy entered a period of secular stagnation, that is, a permanent depression due to a shortage of aggregate demand. Among others, [Caballero et al. \(2008\)](#) provide some stylized facts consistent with this interpretation of the events.

In this paper, I formalize [Summers \(2013\)](#) and [Krugman \(2013\)](#)'s intuition using an OLG version of the standard New Keynesian (NK) model, augmented with capital and asset bubbles. I prove their intuition correct, although with a slight refinement: secular stagnation is a necessary, but not sufficient, condition for asset bubbles to be expansionary. Indeed, whereas asset bubbles always push the economy out of secular stagnation, this doesn't necessarily translate into a higher level of output. Instead, even though asset bubbles allow the economy to produce at full capacity, asset bubbles also drastically reduce this capacity under some calibrations: a supply-side stagnation is substituted for a demand-side stagnation. In particular, asset bubbles are expansionary when the economy suffers from a "strong" secular stagnation, in the sense that aggregate demand is severely deficient; when there's only a "mild" secular stagnation, asset bubbles are contractionary.

The model has three key ingredients: nominal rigidities, the OLG structure and a particular – but intuitive – distribution of factorial income between young and old agents<sup>1</sup>. There's a unique final good that can be either consumed or transformed into capital. It is produced by a competitive sector that aggregates a continuum of intermediate varieties. Each variety is produced by a particular intermediate firm out of capital and labor using a Cobb-Douglas technology. Those firms are in monopolistic competition and face a Rotemberg-style pricing friction. Households live for two periods. When young, they supply labor, consume and purchase three types of assets: bubbles, nominal bonds and equity-capital. When old, they collect payoffs on their asset holdings, sell their bubbles, receive profits distributed by the firms and consume their entire income. Labor and financial markets are competitive.

As in the basic NK model, given the stock of capital, there's a unique natural/potential/supply-determined level of output<sup>2</sup>, and a corresponding natural interest rate that equalizes aggregate demand to the natural level of output. In a flexible-price economy, if young households have a strong desire to save, the natural interest rate falls in order to bring down the cost of capital and stimulate investment by the firms. This restores equilibrium in the financial and goods markets. But in a

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<sup>1</sup>The last two ingredients are necessary to introduce (i) a wedge between the interest rate and agents' discount factor as well as (ii) pro-cyclical income risk. Other assumptions about financial markets and the distribution of income would produce similar results, conditional on (i) and (ii).

<sup>2</sup>The natural/potential level of X refers to its value when prices are fully flexible, *conditional on a path for capital and asset prices*. The X gap refers to the deviation of X from its natural level; the potential X gap refers to the deviation of the natural level of X when prices are sticky from the natural level when prices are flexible, *conditional on a path for asset prices*.

sticky-price economy, the interest rate is under central bank's control and doesn't need to match its flexible-price counterpart. In normal times, the inflation-targeting central bank implements a nominal interest rate consistent with a zero interest rate gap and therefore a zero output gap.

This, however, might not be possible because of the ZLB. If asset demand (aggregate savings) is large relative to asset supply (aggregate investment), the natural interest rate is so low that the central bank hits the ZLB. In the liquidity trap, any shock that reduces the natural interest rate translates fully into an interest rate gap: given the natural level of output, aggregate savings exceed aggregate investment. Facing a low demand, firms reduce their production, that is, they ask for less capital and labor. In the short run, i.e. given the stock of capital, equilibrium hours fall relative to the natural level: there's a negative output gap. In the longer run, firms also invest less such that the sticky-price economy accumulates less capital than the flexible-price economy: there's a negative potential output gap. Importantly, this liquidity trap can last for an arbitrary number of periods, or even permanently: this is the secular stagnation.

But this depression isn't incurable. Instead, the decentralized economy has one great tool to deal with a shortage of aggregate demand: asset bubbles. Indeed, under the very weak condition that the growth rate of output is higher than the ZLB minus the central bank's inflation target – which I'll call the Real Lower Bound (RLB), an economy in secular stagnation is dynamically inefficient. As [Tirole \(1985\)](#) has shown, dynamic inefficiency is a necessary and sufficient condition for the emergence of rational asset bubbles: while a dynamically efficient economy has a unique steady state which is bubble-less, a dynamically inefficient economy has two steady states, only one of which is bubble-less. In an OLG model, an asset bubble redistributes income from the asset buyers – the young households – to the asset sellers – the old households. Since financial markets are incomplete, the marginal propensity to consume out of current income of the former exceeds that of the latter. Thus, an asset bubble reduces the aggregate savings rate and sustains aggregate consumption in partial equilibrium.

Given the interest rate, a bubble shock, i.e. a transition from a bubble-less to a bubbly equilibrium, therefore permanently raises aggregate demand. Hence, it stimulates the equilibrium hours and capital stock as long as the economy remains in the trap: both the output gap and potential output gap increase. If aggregate demand increases a lot after the shock, the economy may even escape the liquidity trap. As the central bank recovers some leverage over the economy, it starts to offset the *marginal* effect of the shock on aggregate demand in order to fight inflationary pressures. Thus, as soon as the ZLB becomes non-binding, the output and potential output gaps both stabilize

at zero – it is as if prices were flexible at the micro-level, but potential output decreases as capital is gradually crowded-out by the higher interest rate – with respect to the bubble-less flexible-price equilibrium. As a consequence, the effect of the initial bubble shock on output become ambiguous: the output and potential output gaps have increased, but the natural level of output has decreased. If the shock doesn't trigger a large monetary response, i.e it doesn't push the economy "too far away" from the ZLB, the positive demand-side effect (higher output and potential output gaps) is greater than the negative supply-side effect (lower potential output); the converse is true if the shock triggers a large monetary response.

In general equilibrium, the size of the asset bubble in steady state, and therefore the strength of the shock, is endogenous. Indeed, the equilibrium bubble equalizes the natural interest rate to the growth rate of the economy. Assuming that this growth rate is above the RLB, asset bubbles always push the economy out of the liquidity trap. As long as the economy remains in the trap, it is Keynesian: the strength of the positive demand-side effect can be proxied by the gap between the bubble-less natural interest rate and the RLB; but as soon as the economy exits the trap, it turns Neo-classical: the strength of the negative supply-side effect can be proxied by the gap between the RLB and the growth rate of output. Thus, asset bubbles are expansionary in economies that suffer from a strongly deficient aggregate demand – a large RLB-natural interest rate gap, where the growth rate of output is relatively close to the RLB. In such an environment, the burst of an asset bubble pushes the economy in the liquidity trap, possibly forever: asset bubbles prevent or delay a depression due to a shortage of aggregate demand.

Although the preceding analysis was mainly concerned with the steady states of the model, I also provide a dynamic analysis. In particular, I show that the model always has equilibria; the bubble-less steady state is locally determinate if and only if the bubbly steady state doesn't exist; if both steady states co-exist, the output gap is locally determinate around both, but the asset bubble is locally determinate only around the bubbly steady state. I use this local indeterminacy to model the bubble-secular stagnation theory in another way: I introduce a stochastic bubble that follows a two-states Markov process. In the short run, that is, as long as the bubble doesn't burst, it sustains investment and output: the economy is out of the liquidity trap and gradually accumulates capital. But as soon as the bubble bursts, the economy falls in the liquidity trap. The output gap undershoots its long run value during the burst phase because the economy suddenly dis-accumulates capital in excess with respect to its steady state stock. Then, after one period, the economy converges back to a secular stagnation steady state.



**Related literature** Since [Summers \(2013\)](#)'s speech at the IMF, there's a fast-growing literature on secular stagnation. An early contribution is [Eggertsson et al. \(2017\)](#), who showed that a three-periods OLG-NK model with sticky wages and financial frictions exhibits a secular stagnation steady state under some calibrations. They provided various examples of shocks that persistently depress aggregate demand – e.g a deleveraging shock or population or productivity slowdowns – as well as several escape policies – e.g a higher inflation target or higher government spending. [Garrec and Touze \(2016\)](#) complemented this analysis by showing that a government consumption shock raises a trade-off in a version of the model that includes capital: if the shock is too small, the economy remains in secular stagnation; if the shock is too large, it reduces output by crowding-out capital. [Michau \(2018\)](#) provided another way of modeling secular stagnation that doesn't rely on an OLG structure, but rather on a utility function that includes wealth as an argument in a representative agent economy. He further characterized the peculiar properties of the secular stagnation steady state – the paradoxes of thrift, flexibility and toil, performed a well-fare analysis and also computed various tax schemes that implement the optimal allocation.

I contribute to this literature by studying another way out of secular stagnation: asset bubbles. Indeed, I show that asset bubbles can prevent or delay a secular stagnation. One important difference with respect to other types of demand shocks, e.g a government consumption shock, is that asset bubbles are endogenous: the strength of the shock in the long run is determined by the fundamentals of the economy; furthermore, as this shock is driven by asset prices expectations, which are prone to sudden changes, it provides an explanation as of how the crisis started. Finally, my two-periods OLG model with sticky prices is considerably simpler, and much closer to the textbook NK model than the three-periods OLG models with sticky wages and financial frictions of [Eggertsson et al. \(2017\)](#) and [Garrec and Touze \(2016\)](#). This makes it easier to compare the model to the textbook NK model and understand which deviations are responsible for the possibility of secular stagnation.

The first paper which included asset bubbles in OLG-NK models was [Kocherlakota \(2013\)](#). He showed that a fall in the price of land – due to either an imploding bubble or lower fundamentals – permanently depresses aggregate demand when monetary policy is unresponsive and nominal wages rigid, i.e a secular stagnation (although he didn't claim the label). In an OLG-NK model that includes financial frictions and heterogeneous agents, [Caballero and Farhi \(2018\)](#) showed that secular stagnation can arise because of a safety trap rather than a liquidity trap if some agents are

infinitely risk-averse; they also showed that safe asset bubbles raise aggregate demand and output in such an environment.

But those papers don't include capital, i.e they assume that the stock of capital is fixed. While this approximation has some validity in the short run, it is clearly violated in the long run. This omission shuts off an important propagation channel through which demand shocks affect the supply-side of the economy in the long run. Without capital, asset bubbles are either expansionary (in the trap), or they don't affect the real allocation (out of the trap). With capital, I show that they raise a new trade-off: a positive demand-side effect (higher output and potential output gaps) vs a negative supply-side effect (lower potential output). Importantly, whether asset bubbles are expansionary or not depends on the fundamentals of the economy.

[Gali \(2014\)](#) studied how the presence of asset bubbles affect the conduct of the optimal monetary policy in an OLG-NK model. Using the indeterminacy of asset prices around a fundamental but dynamically inefficient steady state, [Gali \(2017\)](#) showed that asset bubbles provide a theory of endogenous demand shocks; he further studied how the existence of asset bubbles modify the usual Taylor principle. My analysis is complementary to his. Indeed, I answer the opposite question: how does the conduct of monetary policy affects the macroeconomic effects of asset bubbles? To do that, I use a model that includes capital and the ZLB – both features he abstracts from but which are crucial to my story.

In this regard, my analysis is closer to [Hanson and Phan \(2017\)](#). They introduced sticky wages in an OLG model with heterogeneously productive investors and financial frictions. They showed that asset bubble creation shocks can stimulate investment and output, but possibly at the cost of a large recession post-burst. Our analyses are complementary. I reproduce their results, but through a different mechanism and with one notable difference: the pre-burst economy isn't necessarily booming. Rather, it produces at full capacity as long as the bubble doesn't burst, undershoots during the burst phase, and then remains permanently depressed. Furthermore, in my model, an asset bubble stimulates investment by raising aggregate consumption; in their model, bubble creation shocks raise the marginal efficiency of investment.

Since the seminal papers by [Martin and Ventura \(2012\)](#) and [Farhi and Tirole \(2012\)](#), several authors showed that asset bubbles can stimulate investment and output in flexible-price economies with financial frictions<sup>3</sup>. In those papers, asset bubbles redistribute income between heterogeneous

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<sup>3</sup>A limited list includes: [Kocherlakota \(2009\)](#), [Shi and Suen \(2014\)](#), [Miao and Wang \(2015a\)](#), [Miao and Wang \(2015b\)](#), [Hirano et al. \(2015\)](#). See [Miao \(2014\)](#) for a good introduction to that literature.

agents, and those transfers raise investment and output because either of: (i) the savings rate, (ii) the labor supply, (iii) the marginal efficiency of investment, or (vi) TFP, goes up. In a sticky-price economy, as they raise the natural level of output, (i), (ii) and (iv) would qualify as negative demand shocks: my paper relies on opposite mechanisms; (iii) would be amplified by the presence of nominal rigidities, but I abstract from this channel to keep the paper as simple as possible.

Finally, following [Krugman \(1998\)](#) and [Eggertsson and Woodford \(2003\)](#), there's a large literature on liquidity traps in NK models <sup>4</sup>. While this literature typically relies on an exogenous shock on a structural parameter <sup>5</sup> to provoke a temporary fall in the natural interest rate, here, a non-fundamental shock to asset prices expectations drives the economy into a permanent liquidity trap.

The paper is organized as follows. Section 2 exposes the basic model. Section 3 derives the general equilibrium system. Section 4 studies the macroeconomic effects of asset bubbles in the long run. Section 5 studies the short run dynamic. All proofs for sections 4 and 5 are respectively in appendices A and B.

## 2 Basic environment

The basic setup is an overlapping generations model. Time is discrete,  $t \in \{0, 1, \dots\}$ , and the horizon infinite. There's a single final good; two factors of production, capital and labor; and three assets: nominal bonds issued by the government, equity-capital issued by the firms, and asset bubbles. In period 0, old agents are endowed with  $H = 1$  units of bubbles and  $I_{-1} > 0$  units of equity-capital. Agents have perfect foresight in the non-linear model.

### 2.1 Households

Each generation consists of a mass one of homogeneous households who live for two periods, young and old. At birth, an individual of generation  $t$  is endowed with the following utility function:

$$\mathcal{U}_t \equiv \log \left( C_{1t} - A_t \frac{N_t^{1+\varphi}}{1+\varphi} \right) + \beta \log C_{2t+1}, \quad \varphi > 0, \quad \beta \in (0, 1) \quad (1)$$

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<sup>4</sup>A limited list includes: [Lorenzoni and Guerrieri \(2011\)](#), [Christiano et al. \(2011\)](#), [Eggertsson and Krugman \(2012\)](#), [Correia et al. \(2013\)](#), [Korinek and Simsek \(2016\)](#)

<sup>5</sup>Exceptions include [Schmitt-Grohe and Uribe \(2012\)](#), who consider a shock to inflation expectations.

Here,  $C_{1t}$  and  $C_{2t+1}$  respectively denote young- and old-age consumptions;  $N_t$  is the labor supply;  $A_t = g^t$  is the exogenous productivity of labor,  $g \geq 1$ , which I include in the utility function to make it consistent with the existence of a balanced growth path when  $g \neq 1$ . As GHH preferences rule out any wealth effect on the flexible-price labor supply, they allow to focus on the sticky-price labor demand.

A young individual consumes, supplies labor and chooses a portfolio of assets: nominal bonds,  $L_t$  issued by the government (expressed in real terms); bubbles,  $H_t$ ; and equity issued by the firms,  $I_t$ . His per-period budget constraint (expressed in real terms) is:

$$C_{1t} + L_t + B_t H_t + I_t = W_t N_t - T_t \quad (2)$$

Here,  $W_t$  is the real wage,  $B_t$  is the price of the bubbly asset and  $T_t$  is a lump-sum tax.

During his second period of life, the retiree sets up a firm – described below – and collects the corresponding profits,  $D_{t+1}$ , receives payoffs on his bond and equity holdings, sells his stock of bubbly asset and consumes his entire income,

$$C_{2t+1} = D_{t+1} + R_t L_t + B_{t+1} H_t + Q_{t+1} I_t \quad (3)$$

Here,  $Q_{t+1}$  is the rate of dividends – the marginal product of capital in equilibrium – and  $R_t$  is the (gross real) interest rate. Capital, and therefore the equity that funded it, fully depreciates within one period; each firm disappears with its owner.

Each household maximizes (1) under (2), (3), taking prices and his profit income as given. Thanks to the GHH preferences, labor supply only depends on the productivity-adjusted wage rate:

$$N_t = \left( \frac{W_t}{A_t} \right)^{\frac{1}{\varphi}} \quad (4)$$

In equilibrium, the following non-arbitrage conditions must hold true for all  $t \geq 0$ :

$$R_t = Q_{t+1}, \quad B_t \leq R_t B_{t+1}, \quad B_t \geq 0 \quad (5)$$

From households' point of view, all assets are perfect substitutes for each other. Thus, assuming that there's a positive supply of bonds and equity, the first equality ensures that they offer the same rate of return. The next two inequalities are complementary slackness conditions: either asset bubbles offer the same rate of return as the other assets,  $B_t = R_t B_{t+1}$ , or they aren't traded,

$B_t = 0$ . Taking this into account, we can solve household's program for the consumption function:

$$C_{1t} = \frac{1}{1+\beta} \left( W_t N_t - T_t + \frac{D_{t+1}}{R_t} \right) + \frac{\beta}{1+\beta} \frac{W_t N_t}{1+\varphi}, \quad C_{2t+1} = \beta R_t C_{1t} - \beta R_t \frac{W_t N_t}{1+\varphi} \quad (6)$$

As he's smoothing consumption across his life-time, each young household saves a constant fraction  $\frac{\beta}{1+\beta} \in (0, 1)$  of his wealth, which equals the sum of discounted flows of non-financial income: net labor income,  $W_t N_t - T_t$ , plus discounted profit income,  $\frac{D_{t+1}}{R_t}$ . The complementarity between young-age consumption and labor introduced by GHH preferences raises young-age consumption further by a fraction  $\frac{\beta}{1+\beta} \frac{1}{1+\varphi} \in (0, 1)$  of household's labor income.

## 2.2 Firms and nominal rigidities

The final good good,  $Y_t$ , is produced by a competitive sector that aggregates a continuum of intermediate varieties,  $Y_t(\omega)$ ,  $\omega \in [0, 1]$ , according to a Dixit-Stiglitz technology:  $Y_t = \left( \int Y_t(\omega)^{\frac{\iota-1}{\iota}} d\omega \right)^{\frac{\iota}{\iota-1}}$ ,  $\iota > 1$ . Solving the problem of the final good producer, the demand for each individual variety is:

$$Y_t(\omega) = \left( \frac{P_t(\omega)}{P_t} \right)^{-\iota} Y_t \quad (7)$$

Where  $P_t(\omega)$  is the price of the variety  $\omega$ , and  $P_t = \left( \int P_t(\omega)^{1-\iota} d\omega \right)^{\frac{1}{1-\iota}}$  is the aggregate price level.

Intermediate goods are supplied by a continuum of monopolists, each producing a particular variety  $\omega$  out of labor and capital according to a Cobb-Douglas technology:

$$Y_t(\omega) = K_t(\omega)^\alpha (A_t N_t(\omega))^{1-\alpha} \quad (8)$$

As it takes time to build capital, they issue equity one period before starting to produce,  $I_{t-1}(\omega) \equiv K_t(\omega)$ . Both the labor and equity markets are competitive. Solving the dual problem of a given monopolist leads to the (real) marginal cost of production:

$$mc_t = \left( \frac{Q_t}{\alpha} \right)^\alpha \left( \frac{W_t}{1-\alpha} \right)^{1-\alpha}$$

As well as factor demands:

$$\frac{K_t(\omega) Q_t}{Y_t(\omega)} = \alpha mc_t, \quad \frac{N_t(\omega) W_t}{Y_t(\omega)} = (1-\alpha) mc_t \quad (9)$$

Each firm belongs to a particular old household and disappears when he dies, i.e it has a planning horizon of one period only. Thus, a monopolist  $\omega$  maximizes its flow of real profits,

$$D_t(\omega) = \left( \frac{P_t(\omega)}{P_t} - (1 - \tau)mc_t \right) Y_t(\omega) - AC_t(\omega)$$

Under the demand constraint, (7). Here,  $\tau = \frac{1}{\iota}$  is a production subsidy that corrects the static distortion due to monopoly pricing – this is without consequences for the results but simplifies the analysis, and  $AC_t(\omega)$  is a quadratic quadratic cost of adjusting nominal prices,

$$AC_t(\omega) = \frac{\Phi}{2} \left( \frac{P_t(\omega)}{\Pi^n P_{t-1}} - 1 \right)^2 \frac{Y_t}{\pi_t}, \quad \Phi \geq 0$$

Where  $\pi_t \equiv \frac{P_t}{\Pi^n P_{t-1}}$  is the (gross) inflation rate in deviation from a reference level  $\Pi_n \geq 1$  – the inflation rate henceforth.

This cost is measured in final output and increases with the size of the nominal adjustment relative to a reference price,  $\Pi^n P_{t-1}$  where  $\Pi_n \geq 1$  is the central bank's inflation target and  $P_{t-1}$  is last period's price level. This cost can be interpreted as follows: because firms have well-anchored expectations, they use a simple indexation rule; if they decide to deviate from this rule, they must re-optimize their price plan, which is costly. I normalize this adjustment cost by the current "inflation gap",  $\pi_t$ , to avoid dealing with the downward-sloping segment of the AS curve <sup>6</sup>.

Solving the firm's problem yields the usual optimal pricing condition:

$$\left( \frac{P_t(\omega)}{P_t} - mc_t \right) Y_t(\omega) + \frac{\Phi}{\iota - 1} \frac{P_t(\omega)}{\Pi^n P_{t-1}} \left( \frac{P_t(\omega)}{\Pi^n P_{t-1}} - 1 \right) \frac{Y_t}{\pi_t} = 0 \quad (10)$$

Finally, distributed profits are net of subsidies but include adjustment costs,

$$D_t = \int (D_t(\omega) + AC_t(\omega) - \tau mc_t) d\omega$$

Discussing the implications of the non-linear vs log-linear solutions of the New Keynesian model with Rotemberg or Calvo pricing at the ZLB, [Eggertsson and Singh \(2016\)](#) uncover that the two solutions diverge with Rotemberg pricing as the recession becomes severe. Indeed, adjustment costs explode as a share of output in the non-linear model. I by-pass this issue the way they recommend,

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<sup>6</sup>Without this normalization, the AS curve is a quadratic polynomial with two solutions:  $\pi_t = \frac{1}{2} \pm \frac{1}{2} \Delta(x_t)$  where the function  $\Delta(\cdot)$  is strictly increasing. If  $\pi_t \approx 1$  – as is the case in the basic NK model, the economy is on the upward sloping segment; however, there's usually another solution with  $\pi_t$  way below 1.

that is, I consider adjustment costs as "psychological".

### 2.3 Monetary and fiscal policies

Following the New Keynesian tradition, the central bank sets the nominal interest rate according to a Taylor rule. The (gross real) interest rate on nominal bonds is then:

$$R_t \equiv \frac{r_t}{\Pi_{t+1}} = \frac{\max\{R_t^n \pi_t^\phi, \underline{R}\}}{\pi_{t+1}}, \quad \underline{R} \leq 1 \leq g, \quad \phi \geq 0 \quad (11)$$

Here,  $r_t$  is the (gross) nominal interest rate,  $\underline{R} \equiv \frac{\underline{r}}{\Pi^n}$  is the real lower bound (RLB) constraint, that is, the lowest interest rate the inflation-targeting central bank can deliver when the zero lower bound constraint (ZLB) on the nominal interest rate,  $\underline{r}$ , is binding without violating its inflation mandate;  $R_t^n$  is the natural interest rate – see below – which the central bank takes as exogenous.

The government borrows by issuing nominal bonds and taxes young households in a lump-sum fashion. Its budget constraint is given by:

$$R_{t-1} L_{t-1}^g = L_t^g + T_t$$

I'll consider that the supply of nominal bonds is strictly positive (thus justifying (5)), but infinitesimal,  $L_t^g \rightarrow 0$  for all  $t \geq 0$  – such that  $T_t \rightarrow 0$ .

## 3 General equilibrium

Let smaller case letters denote productivity-adjusted variables,  $e_t \equiv \frac{E_t}{A_t}$ ; the only exception is the asset bubble that is normalized by young households' savings out of labor income net of utility cost,  $b_t \equiv \frac{B_t}{\frac{\beta}{1+\beta} \frac{\varphi}{1+\varphi} W_t N_t}$ . Given a sequence of asset prices and capital,  $\{k_t, b_t\}$ , I define the natural allocation, denoted by a superscript  $n$ , as the sequence of outputs and interest rates,  $\{y_t^n, R_t^n\}$ , that would obtain if prices were fully flexible,  $\Phi = 0$ , in every period  $t \geq 0$ .

### 3.1 Labor market and the AS curve

From (10), were firms free to adjust their price,  $\Phi = 0$ , they'd equalize it to the nominal marginal cost,

$$mc_t^n = 1$$

Given the stock of capital, the labor market equilibrium,  $N_t = N_t(\omega)$ , (4) and (9), together with the production function, (8), would determine the natural level of output:

$$y_t^n = (1 - \alpha)^{\frac{1}{\psi}} k_t^\chi \tag{12}$$

$$\text{Where } \psi \equiv \frac{\alpha + \varphi}{1 - \alpha} \quad \text{and} \quad \chi \equiv \alpha \frac{1 + \psi}{\psi} \in (0, 1)$$

In the presence of nominal rigidities, however, the economy is likely to deviate from the natural allocation. Using the last two equations, we can write the labor market equilibrium condition as:

$$mc_t = x_t^\psi, \quad x_t \equiv \frac{y_t}{y_t^n}$$

The output gap,  $x_t$ , is a measure of the static – for a given stock of capital – output gain (or loss) due to nominal rigidities, i.e the labor market slack,  $x_t = (\frac{N_t}{N_t^n})^{1-\alpha}$  where  $N_t^n$  is the natural level of hours. Given aggregate demand,  $y_t$ , the flexible-price firm continuously adjusts its price to implement the optimal mark-up – here equal to 1. The sticky-price firm, however, must balance the marginal benefit versus the marginal cost of price adjustment. In general, it optimally decides to partially adjust both its price and production. This logic is illustrated by the aggregate supply (AS) curve, derived from (10) and  $mc_t = x_t^\psi$ :

$$\pi_t = 1 + \lambda(x_t^\psi - 1) \tag{13}$$

$$\text{Where } \lambda \equiv \left(\frac{\Phi}{\iota - 1}\right)^{-1}$$

The slope of the AS curve, which measures the degree of price stickiness, determines the distribution of adjustment in price versus production. As nominal adjustment costs become infinitely large,  $\lambda = 0$ , the AS curve becomes flat,  $\pi_t = 1$ : there is a full production response; a contrario, as adjustment cost become infinitesimal,  $\lambda \rightarrow \infty$ , the AS curve becomes vertical: there's a full price response. In the intermediate case, a deficient aggregate demand leads to both a fall in the inflation rate relative to the reference level,  $\pi_t < 1$ , and a fall in production relative to the natural level,  $x_t < 1$ .

**Assumption 1**  $\lambda < 1$ .

Under this assumption, inflation cannot fall below a strictly positive lower floor.

To produce less, the firms collectively cut their demand for capital and labor. This, in turn,



has distributive consequences. Plugging  $mc_t = x_t^\psi$  in (9):

$$\frac{Q_t k_t}{y_t} = \alpha x_t^\psi, \quad \frac{w_t N_t}{y_t} = (1 - \alpha) x_t^\psi, \quad \frac{d_t}{y_t} = 1 - x_t^\psi \quad (14)$$

The output gap acts as a wedge between factor prices and their marginal product, hence redistributing income between labor/capital and profits. During a recession, the binding demand constraint bids down the value of capital and labor for the firms, putting downward pressures on factor prices. Given the stock of capital, households supply less labor at a lower wage, (4).

### 3.2 Financial markets: IS curve, capital accumulation and asset pricing

The asset price bubble must satisfy the non-arbitrage condition, (5):

$$b_t = \frac{g}{R_t} \frac{y_{t+1}^n}{y_t^n} \frac{x_{t+1}^{1+\psi}}{x_t^{1+\psi}} b_{t+1}, \quad b_t \in [0, 1] \quad (15)$$

In equilibrium, households' demand for bonds must equal the government's supply,  $L_t = L_t^g$ , which tends to zero,  $L_t^g \rightarrow 0$ . Using this as well as the budget constraint, (3), and imposing equilibrium in the market for equity in period  $t - 1$ ,  $I_{t-1} = K_t$ , we can compute old-age consumption:

$$C_{2t} = D_t + Q_t K_t + B_t$$

Old agents are hand-to-mouth: they consume their entire income, which is made of non-financial profit income and financial, bubble plus dividends, income. Summing up with (6), aggregate consumption,  $C_t \equiv C_{1t} + C_{2t}$  is equal to:

$$C_t = \frac{1}{1 + \beta} \left( W_t N_t + \frac{D_{t+1}}{R_t} \right) + \frac{\beta}{1 + \beta} \frac{W_t N_t}{1 + \varphi} + D_t + Q_t K_t + B_t$$

Using factor shares, (14), and re-arranging, we can derive the asset demand function:

$$i_t = y_t - c_t = \frac{\beta}{1 + \beta} \frac{\varphi}{1 + \varphi} (1 - \alpha) (1 - b_t) x_t^\psi y_t - \frac{g}{1 + \beta} \frac{(1 - x_{t+1}^\psi) y_{t+1}}{R_t}$$

Where  $y_t = x_t y_t^n$  for all  $t \geq 0$ .

On aggregate, bubbles are in zero net supply because young households buy those bubbles from the old ones. Thus, I'll refer to  $i_t$  as to aggregate savings, asset demand or equity demand

interchangeably.

As the model is non-Ricardian, asset demand is determined both by the growth rate of aggregate income,  $\frac{y_{t+1}}{y_t}$ , as well as its distribution,  $x_t^\psi, x_{t+1}^\psi$ . During a recession,  $x_t < 1$ , labor income falls more than output but profit income falls less: the income share of young agents shrinks and that of old agents rises. As the latter's marginal propensity to consume out of current income is greater than the former's, asset demand is pro-cyclical,  $\frac{di_t}{dx_t} = 1 + \psi > 1$  – given expectations and asset prices. By the same logic, expectations of a recession,  $x_{t+1} < 1$ , raise young households' expected wealth and thereby consumption, but doesn't affect old agents' behavior: asset demand contracts. In short, and to follow the terminology introduced by [Werning \(2015\)](#), income risk is pro-cyclical. The magnitude of those redistributive effects is calibrated by the elasticity of the marginal cost with respect to the output gap: if  $\psi$  is very high, the distribution of non-financial income varies a lot along the cycle and asset demand is strongly pro-cyclical.

Through similar mechanisms, asset bubbles reduce asset demand in partial equilibrium. Indeed, higher asset prices redistribute financial income from the young households – asset buyers – to the old ones – asset sellers. The consumption of the former isn't affected – their life-time wealth hasn't changed – but the latter consume more. This is the usual "crowding-out" on aggregate savings identified by [Tirole \(1985\)](#).

From (14) and (5), the supply of financial assets by the firms is:

$$gk_{t+1} = \alpha g \frac{x_{t+1}^\psi y_{t+1}}{R_t} \tag{16}$$

As with  $i_t$ , I'll refer to  $gk_{t+1}$  as to equity supply, asset supply or aggregate investment interchangeably.

Given expectations, asset supply is a-cyclical; but expectations of a crisis depress investment and asset supply because firms anticipate a lower demand as well as a higher marginal cost of production.

In equilibrium, asset demand must equal asset supply,  $i_t = gk_{t+1}$ . This condition leads to the IS curve:

$$x_t = \left( \frac{R_t^n}{R_t} \frac{1 - \eta x_{t+1}^\psi}{(1 - \eta)x_{t+1}^\psi} \right)^{\frac{1}{1+\psi}} x_{t+1} \tag{17}$$

Where  $\eta \equiv 1 - \alpha(1 + \beta)$

Where  $R_t^n$  is the natural interest rate, that, conditional on a path for potential capital and asset prices,  $\{k_t, b_t\}$ , would obtain if prices fully flexible in periods  $t$  and  $t + 1$ ,

$$R_t^n = \frac{\alpha g}{\frac{\beta}{1+\beta} \frac{\varphi}{1+\varphi} (1-\alpha)(1-b_t)} \frac{y_{t+1}^n}{y_t^n} \quad (18)$$

Where  $y_t^n$  is given by (12).

An excess demand for assets,  $i_t - gk_{t+1} > 0$ , translates into an excess supply of goods:  $c_t + gk_{t+1} < y_t$ . Facing a low demand, firms cut their production and price until aggregate supply equals aggregate demand. Assuming (for now) that the central bank keeps the interest rate constant and expectations are unaffected, the output gap stabilizes at a level where young households' labor income is sufficiently low to balance the supply and demand for assets. The adjustment goes through both a lower wage rate and lower hours worked. Thus, given the interest rate and expectations, the model reproduces Keynes (1936)'s insight: if aggregate savings exceed aggregate investment, output drops to clear the financial markets.

Taking logs and total-differentiating the IS curve,

$$d \log x_t = \left( 1 - \frac{\psi}{(1+\psi)(1-\eta x_{t+1}^\psi)} \right) d \log x_{t+1} + \frac{1}{1+\psi} (d \log R_t^n - d \log R_t)$$

The output gap is increasing in the "interest rate gap",  $\frac{R_t^n}{R_t}$ : a higher interest rate raises both the cost of capital for the firms and young households' wealth. Hence, asset supply contracts while asset demand expands. But, given asset prices, monetary policy is less effective than in the representative agent (RA) version of the model. Indeed, the additional redistributive effect due to pro-cyclical income risk makes asset demand more responsive to the output gap and therefore reduces the equilibrium adjustment in output following a monetary shock – or any other type of demand shocks affecting the economy through  $R_t^n$ .

The output gap also depends on expectations over the future output gap, but, again, less than in the RA version. If income risk is very (very) pro-cyclical, the elasticity of the output gap with respect to the expected output gap may even turn negative – young households' expected wealth rises a lot if they anticipate a crisis, hence they demand much more assets. In the rest of the paper, I'll make the following assumption:

**Assumption 2**  $\frac{\psi}{(1+\psi)(1-\eta)} \in (0, 1)$ , i.e.  $\alpha \in \left( \frac{\varphi}{\beta+\varphi(1+\beta)}, \frac{1}{1+\beta} \right)$ .

The upper bound on  $\frac{\psi}{(1+\psi)(1-\eta)}$  (lower bound on  $\alpha$ ) ensures that  $\frac{d \log x_t}{d \log x_{t+1}} > 0$ : everything else

equal, expectations of a recession in period  $t + 1$  depress aggregate demand and output in period  $t$ . Given the upper bound, the lower bound (upper bound on  $\alpha$ ) makes the IS curve discounted,  $\frac{d \log x_t}{d \log x_{t+1}} < 1$ : because young-age income is more output-elastic than old-age income, households are less responsive to news about future shocks than they're about current shocks. Both restrictions are economically meaningful; furthermore, as a standard calibration sets  $\alpha \approx \frac{1}{3}$  and  $\beta < 1$ , the second isn't particularly strong.

### 3.3 Equilibrium

Given  $k_0 > 0$ , an equilibrium is a sequence  $\{x_t, \pi_t, R_t, b_t, k_{t+1}, R_t^n, y_t^n\}$  that satisfies for all  $t \geq 0$ : the AS curve, (13), the IS curve, (17), the Taylor rule, (11), the asset pricing equation, (15), the demand for capital, (16), and the natural interest rate and output are respectively given by (18) and (12);  $k_{t+1} \geq 0, b_t \geq 0$ .

I summarize below all relevant equations:

$$\begin{aligned}\pi_t &= 1 + \lambda(x_t^\psi - 1) \\ x_t &= \left( \frac{R_t^n}{R_t} \frac{1 - \eta x_{t+1}^\psi}{(1 - \eta)x_{t+1}^\psi} \right)^{\frac{1}{1+\psi}} x_{t+1} \\ R_t &= \frac{\max\{R_t^n \pi_t^\phi, \mathbf{R}\}}{\pi_{t+1}}\end{aligned}$$

Given a natural allocation  $\{y_t^n, R_t^n\}$ , this 3-equations NK model determines the inflation rate, the output gap and the interest rate. In turn, the natural allocation is defined conditionally on  $\{k_t, b_t\}$ ,

$$\begin{aligned}R_t^n &= \frac{\alpha g}{\frac{\beta}{1+\beta} \frac{\varphi}{1+\varphi} (1 - \alpha)(1 - b_t)} \frac{y_{t+1}^n}{y_t^n} \\ y_t^n &= (1 - \alpha)^{\frac{1}{\psi}} k_t^\chi\end{aligned}$$

And those satisfy the laws of motion:

$$\begin{aligned}b_t &= \frac{g}{R_t} \frac{y_{t+1}^n}{y_t^n} \frac{x_{t+1}}{x_t} b_{t+1}, \quad b_t \in [0, 1] \\ k_{t+1} &= \alpha \frac{x_{t+1}^{1+\psi} y_{t+1}^n}{R_t}\end{aligned}$$

In the rest of the paper,  $\hat{e}_t \equiv \frac{e_t - e}{e}$  will denote the log-deviation of  $e_t$  from its steady state value;

the only exception is  $\hat{b}_t \equiv b_t - b$ . If I drop the time index,  $\hat{e} \equiv d \log e$  will denote the log-deviation from the previous steady state – and  $\hat{b} \equiv db$ . I'll also introduce three composite parameters:

$$\Omega_0^\phi \equiv 1 + \psi + \phi\kappa, \quad \Omega_1^\phi \equiv \frac{\psi}{1 - \eta x^\psi} + (\phi - 1)\kappa, \quad \Omega_p^\phi \equiv (1 - p)\Omega_0^\phi + p\Omega_1^\phi$$

Here,  $\kappa \equiv \psi \frac{\lambda x^\psi}{1 + \lambda(x^\psi - 1)} \geq 0$  is the elasticity of inflation with respect to the output gap and  $\Omega_p^\phi$  is the general equilibrium (inverse) elasticity of the output gap with respect to a demand shock of persistence  $p$  in general equilibrium when the central bank is  $\phi$ -active.

**Lemma 1** *Under assumptions 1, 2,  $\Omega_p^\phi > 0$  and  $\frac{d\Omega_p^\phi}{dp} < 0$  for all  $\phi \geq 0$  and  $p \in [0, 1]$ .*

This elasticity falls in the long run but remains strictly positive,  $\Omega_0^\phi > \Omega_1^\phi > 0$ , even if the central bank isn't active – in the standard NK model without capital nor cyclical income risk,  $\Omega_1^0 < 0$ .

Before going on with the analysis of the steady states of the model, let's solve a semantic issue that can be confusing. I'll say that the level of output is efficient if and only if  $x_t = 1$ ; and a steady state is efficient if and only if  $x = 1$ . This definition implicitly neglects the usual "dynamic inefficiency" which arises whenever  $R < g$  in steady state: I focus on aggregate demand and the output gap; efficient doesn't mean Pareto-efficient, but rather "production-efficient" in the sense that, given a path for asset prices and capital, the sticky-price economy reproduces the real allocation of its flexible-price counterpart.

## 4 Secular stagnation, liquidity trap and asset bubbles in the long run

We'll study steady states in three steps: first, the 3-equations NK model given the natural interest rate; second, asset bubbles and the natural interest rate; third, capital. But before doing that, we'll analyze a simple experiment that will shed light on the long run mechanics of the model.

All proofs are in appendix A.

### 4.1 Long run response to a bubble shock: a primer

Consider that the economy is hit by a permanent bubbly shock,  $\hat{b} > 0$ , and converges to a new steady state <sup>7</sup>. In partial equilibrium, this shock redistributes financial income toward old agents.

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<sup>7</sup>We can roughly interpret this experiment as a transition from a bubble-less to a bubbly steady state. Note that, although I take it as given here,  $\hat{b}$  is endogenous in general equilibrium.

Because young households' marginal propensity to consume out of financial income is equal to zero, but old agents' equals one, asset demand shifts down, i.e the natural interest rate rises:

$$\hat{r}^n = \frac{\hat{b}}{1-b}$$

Given this natural interest rate shock, we can use the 3-equations NK model,

$$\begin{aligned}\hat{\pi} &= \kappa \hat{x} \\ \hat{x} &= \frac{\hat{r}^n - \hat{r}}{1 - \eta x^\psi} \\ \hat{r} &= \max\{-r^n + \hat{r}^n + \phi \hat{\pi}, -\underline{r}\} - \hat{\pi}\end{aligned}$$

To determine the long run output gap multiplier:

$$\hat{x} = \begin{cases} \frac{r^n}{\Omega_1^\phi} & \text{if } \hat{r}^n > \frac{\Omega_1^0}{\Omega_1^\phi} r^n - \underline{r} \\ \frac{\hat{r}^n + \underline{r}}{\Omega_1^0} & \text{otherwise} \end{cases}$$

Here,  $r^n \equiv \log \frac{R\pi}{R^n \pi^\phi} \geq 0$  and  $\underline{r} \equiv \log \frac{R\pi}{\underline{R}} \geq 0$  where  $R\pi$  is the pre-shock steady state nominal interest rate.

The general equilibrium effect on the output gap crucially depends on monetary policy. Given aggregate income, the shock generates a disequilibrium on the financial and goods markets: there's an excess supply of assets and a corresponding excess demand for goods. Facing a high demand, firms raise the inflation rate and produce more: asset supply expands and asset demand contracts until equilibrium is restored. In the liquidity trap, i.e binding ZLB *after* the shock, this is it: the economy adjusts to a new steady state with a lower interest rate, higher output gap and inflation rate. Out of the trap, however, the central bank increases the nominal interest rate in order to fight inflationary pressures. It does so until the interest rate equals its natural value. This, in turn, attenuate the initial expansionary effects of the bubbly shock. There are two different cases, depending on the degree to which the central bank was constrained by the ZLB *before* the shock. If the economy was already out of the trap,  $r^n = 0 < \underline{r}$ , there's a complete monetary offset:  $\hat{x} = 0$ ; if the economy used to be in the trap,  $r^n > 0 = \underline{r}$ , the monetary offset is only partial:  $\frac{\hat{r}^n}{\Omega_1^0} > \hat{x} = \frac{r^n}{\Omega_1^\phi} > 0$ . Intuitively, the central bank fully offsets the *marginal* expansionary effect as soon as the economy escapes the liquidity trap, but not necessarily the *total* effect.

How the shock affects capital depends on the relative strength of this crowding-in effect – that goes through the output gap – and the usual crowding-out effect – that goes through the interest rate,

$$\hat{k} = \frac{(1 + \psi)\hat{x} - \hat{r}}{1 - \chi}$$

On the one hand, firms need more capital to meet the higher demand for goods; on the other hand, the cost of capital rises with the interest rate. Using our previous results, the long run capital multiplier is:

$$\hat{k} = \begin{cases} \frac{\Omega_0^1 + \Omega_1^0}{1 - \chi} \frac{r^n}{\Omega_1^\phi} - \frac{\hat{r}^n}{1 - \chi} & \text{if } \hat{r}^n > \frac{\Omega_1^0}{\Omega_1^\phi} r^n - \underline{r} \\ \frac{\Omega_0^1}{1 - \chi} \frac{\hat{r}^n + \underline{r}}{\Omega_1^0} & \text{otherwise} \end{cases}$$

In the liquidity trap, a positive demand shock crowds capital in. This is a *long* run version of Keynes (1936)'s paradox of thrift<sup>8</sup>: as households want to save less in partial equilibrium, they end up saving more in general equilibrium. Indeed, the shock raises aggregate demand and, through expected inflation, also lowers the cost of capital. Out of the trap, the capital multiplier is clearly negative if the ZLB wasn't binding pre-shock: the central bank has increased its nominal interest rate, but inflation and the output gap haven't changed. This reproduces Tirole (1985)'s seminal analysis: bubbles lower aggregate savings and thereby capital. In the intermediate case with a binding ZLB pre-shock but not post-shock, the net effect is ambiguous. Capital is crowded in as long as the ZLB is binding, but starts to be crowded out as soon as it isn't. As the former effect depends on the degree to which the central was constrained pre-shock,  $r^n$ , while the latter depends the size of the shock,  $\hat{r}^n$ , the capital multiplier is positive if and only if the ZLB was "strongly" binding pre-shock, i.e  $r^n$  is high, and the shock isn't too large, i.e  $\hat{r}^n$  is low.

Thus, a necessary condition for expansionary bubbles in the long run is a binding ZLB pre-shock: demand shocks must raise the output gap; but the effect on the stock of capital and potential output is ambiguous. Quite intuitively, and as we'll develop in the next subsection, if the central bank isn't completely passive, this condition is satisfied only in a situation of secular stagnation: the long run natural interest rate is so low that it lies below the real lower bound,  $\frac{\underline{R}}{R^n} < 1$ .

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<sup>8</sup>Eggertsson (2011), Eggertsson and Krugman (2012), among numerous other papers, have proven that NK models reproduce this paradox in the *short* run.

## 4.2 Secular stagnation in the 3-equations NK model

Let us turn back to studying the various possible steady states. Given the natural interest rate,  $R^n$ , we can solve the usual 3-equations NK model – provided by the IS curve, (17), the AS curve, (13), and the Taylor rule, (11) – for the output gap, inflation and interest rates. Following Eggertsson and Krugman (2012) and Caballero and Farhi (2018), it is convenient to plug the Taylor rule in the IS curve, and use the  $AS - AD$  representation of the model in the plane  $(x^\psi, \pi)$ ,

$$\begin{aligned} \pi &= 1 + \lambda(x^\psi - 1) \\ \frac{1 - \eta x^\psi}{(1 - \eta)x^\psi} &= \max\left\{\pi^{\phi-1}, \frac{\underline{R}}{R^n} \pi^{-1}\right\} \end{aligned}$$

The AS curve can be interpreted as the labor market equilibrium condition and the AD curve as the financial markets equilibrium condition. The AS curve is increasing in the plane, but the slope of the AD curve depends on the monetary regime. Under assumption 1, the LHS of the second equation is increasing in the output gap: the AD curve is decreasing if and only if the ZLB isn't binding and  $\phi > 1$ . Under assumptions 1 and 2, it is possible to show that, if the AD curve is increasing, its slope is greater than that of the AS curve.

**Proposition 1** *Given the natural interest rate,  $R^n$ , the  $AS - AD$  system has a unique solution.*

1. *If the economy isn't in secular stagnation,  $\frac{\underline{R}}{R^n} \leq 1$ , the ZLB isn't binding, inflation is on target and output equals the natural level:  $\pi^n = x^n = 1$ .*
2. *In secular stagnation,  $\frac{\underline{R}}{R^n} > 1$ , the ZLB is binding, inflation is below the target and output below the natural level:  $\pi^s \leq 1$  and  $x^s \leq x^{rp} < 1$ . A higher "RLB-interest rate gap",  $d \frac{\underline{R}}{R^n} > 0$ , or more flexible prices,  $d\lambda > 0$ , result in lower steady state inflation rate and output gap.*

Where  $x^{rp} \equiv \left( (1 - \eta) \frac{\underline{R}}{R^n} + \eta \right)^{-\frac{1}{\psi}}$  is the steady state output gap in the rigid-price limit,  $\lambda \rightarrow 0$ .

As long as the ZLB isn't binding, the central bank has some leverage over the economy and therefore manages to hit its inflation target in the long run; the "divine coincidence" in sticky-price models implies that the economy produces at full potential (1.). But the central bank cannot deliver an interest rate lower than  $\underline{R}$  without violating either the ZLB or its inflation mandate. Thus, if for whatever reason (inequality, savings glut, liquid or safe asset shortage etc.) the natural interest rate turns negative and lies below the RLB, this steady state ceases to exist.



Instead, in secular stagnation,  $\frac{R}{R^*} > 1$ , the ZLB is binding, monetary policy becomes impotent and the economy enters the liquidity trap (2.). In the trap, the ratio of the RLB to the natural interest rate,  $\frac{R}{R^*}$ , is a proxy for the disequilibrium in the financial and goods markets when production is efficient. If  $x = 1$ ,  $\pi = 1$  and  $\frac{R}{R^*} > 1$ , there's an excess demand for assets and an excess supply of goods. The sticky-price firms optimally respond by cutting the inflation rate and the level of production. If it wasn't for the ZLB, the central bank would lower the policy rate until inflation is back on target. This, in turn, would stimulate investment and restore equilibrium in the financial and goods markets consistent with the natural level of output. But in secular stagnation, the central bank cannot further cut the nominal interest rate. Rather, output freely falls until the demand for financial assets equals the supply. This alternative equilibrating mechanism is costly: production falls below the efficient level.

The severity of the depression in the trap depends on the initial disequilibrium, and therefore the necessary output adjustment. As the RLB-natural interest rate gap widens, the output gap must grow larger (in absolute value). The upper bound on the output gap in secular stagnation is an implication of the paradox of flexibility (e.g. [Kocherlakota \(2016\)](#) or [Bhattarai et al. \(2014\)](#)): in the liquidity trap, stickier prices are stabilizing,  $\frac{dx^s}{d\lambda} < 0$ . Indeed, given an initial RLB - interest rate gap,  $\frac{R}{R^*} > 1$ , as output falls below the natural level to clear the financial markets, inflation also drops below the reference level. This raises the interest rate gap even more,  $\frac{R}{R^*}\pi^{-1}$ , implying a further fall in output, which further reduces inflation etc. Large adjustment costs make inflation much less responsive to the output gap and limit this endogenous amplification mechanism.

Finally, note that this proposition doesn't assume that the central bank follows some kind of Taylor principle, i.e.  $\phi > \bar{\phi}$  where  $\bar{\phi} > 0$  ( $= 1$  in the standard model). Under assumptions 1 and 2, it is valid for any  $\phi \geq 0$ : this is a direct consequence of income risk being pro-cyclical.

### 4.3 Bubbles and the natural interest rate

Equation (15) has two solutions: the bubble-less,  $b = 0$ , and the bubbly,  $R = g$ . If  $b = 0$ , using (17):

$$R^n = R^* = \frac{\alpha g}{\frac{\beta}{1+\beta} \frac{\varphi}{1+\varphi} (1 - \alpha)}$$

The natural interest rate is increasing in the supply of assets as a share of output,  $\alpha g$ , but it is decreasing in the demand,  $\frac{\beta}{1+\beta} \frac{\varphi}{1+\varphi} (1 - \alpha)$  (both measured when output is efficient). In particular, a lower capital intensity,  $d\alpha < 0$ , implies a higher demand for financial assets – young households'

labor income rises – and a lower supply – firms’ optimal capital-labor ratio shrinks. Thus, as  $\alpha$  becomes lower and lower, so does the natural interest rate.

But a shortage of assets isn’t an incurable disease. Instead, the decentralized economy has one great tool to remedy a deficient relative asset supply: asset bubbles. Those bubbles implement inter-generational transfers from young agents to old agents: consumption-smoothing is improved and aggregate consumption is increased in partial equilibrium.

**Assumption 3**  $\frac{R^*}{g} > \mathcal{R}^b \equiv \frac{(1-\eta) \left(1 + \frac{R^* - 1}{\lambda}\right)}{1-\eta \left(1 + \frac{R^* - 1}{\lambda}\right)}$

This assumption ensures that there’s a unique bubbly steady state. If it isn’t verified, then the economy has two bubbly steady states: the non-ZLB studied in the main text, plus a ZLB steady state. Both share the same interest rate, but the latter has a lower level of output. This ZLB bubbly steady state has "weird" properties – demand shocks lower the output gap – and it is indeterminate. Ruling it out is neutral with respect to all other results.

From (15), if  $b > 0$ , then  $R = g$ . As by assumption  $g \geq \underline{R}$ , the ZLB isn’t binding and  $x = 1$ . Thus, using (18), the long run asset price bubble must be equal to:

$$b = 1 - \frac{R^*}{g}$$

That is, the economy has a bubbly steady state,  $b \in (0, 1)$ , if and only if the bubble-less steady state is dynamically inefficient,  $\frac{g}{R^*} > 1$  – a conclusion already reached by [Tirole \(1985\)](#) and numerous other papers. The equilibrium asset bubble equalizes the interest rate to the growth rate of output. Hence, the bubble gets larger as the gap between  $g$  and  $R^*$  increases.

**Proposition 2** Let  $\alpha^b \equiv \frac{\beta\varphi}{\beta\varphi + (1+\beta)(1+\varphi)} \in (0, 1)$  and  $\alpha^s \equiv \frac{\beta\varphi}{\beta\varphi + \frac{g}{\underline{R}}(1+\beta)(1+\varphi)} \in (0, \alpha^b]$ .

1. The economy has at least one steady state, and at most two.
2. There’s always a unique bubble-less steady state,  $R^n = R^*$  and  $b = 0$ . It is dynamically inefficient,  $\frac{g}{R^*} > 1$ , if and only if  $\alpha < \alpha^b$ ; it is in secular stagnation,  $\frac{R}{R^*} > 1$ , if and only if  $\alpha < \alpha^s$ .
3. There’s a bubbly steady,  $R^n = g$  and  $b = 1 - \frac{R^*}{g}$ , if and only if the bubble-less steady state is dynamically inefficient.

Propositions (1) and (2) jointly imply that in secular stagnation, the economy has two steady states: the bubble-less steady state in the liquidity trap; and a bubbly version of the efficient steady state which isn't in the liquidity trap. Thus, in an environment characterized by ultra-low interest rates, asset bubbles may not be harmful but rather helpful. At the micro-level, those bubbles allow households to better smooth consumption; at the macro-level, they raise aggregate demand and allow the central bank to recover some leverage over the economy.

#### 4.4 Capital, the ZLB and the paradox of thrift

But since the ZLB isn't binding if the steady state is bubbly, bubbles may crowd capital out with respect to the bubble-less steady state. Indeed, as  $R^n = g \geq \max\{R^*, \underline{R}\}$ , the interest rate in a bubbly steady state is higher than the interest rate in the corresponding non-bubbly steady state. Thus, even through asset bubbles may well raise aggregate demand, labor demand and the equilibrium labor in steady state, they may simultaneously reduce the equilibrium stock of capital.

To compute the stock of capital in a particular steady state, we can use the asset supply function, (16), as well as our previous results. If the steady state is efficient,  $x = 1$  and  $R = R^*$ ,

$$k^* = \left( \frac{\frac{\beta}{1+\beta} \frac{\varphi}{1+\varphi} (1-\alpha)^{\frac{1+\psi}{\psi}}}{g} \right)^{\frac{1}{1-x}}$$

Note that  $\frac{dk^*}{d\beta} > 0 > \frac{dk^*}{dg}$ : as agents become more patient, or expect a lower long run output growth, asset demand rises. The interest rate being flexible, it adjusts downward to clear the financial markets: asset supply is crowded in.

Things work out very differently in secular stagnation. As the output gap is bounded from above by the output gap in the rigid price limit,  $x^s \leq x^{rp}$ , and the stock of capital is strictly increasing in the output gap,  $\frac{dk^s}{dx^s} > 0$ , the stock of capital in secular stagnation is bounded from above by the stock of capital in the rigid price limit,  $k^s \leq k^{rp}$ , which is easy to compute. Indeed, plugging  $x = x^{rp}$  and  $R = \underline{R} > R^*$  in (16),

$$k^{rp} = \left( \frac{\alpha(1-\alpha)^{\frac{1}{\psi}}}{\underline{R} \left( \frac{\underline{R}}{g} \beta \frac{\varphi}{1+\varphi} (1-\alpha) + 1 - \alpha(1+\beta) \right)^{\frac{1+\psi}{\psi}}} \right)^{\frac{1}{1-x}}$$

Because the paradox of thrift holds true in the liquidity trap, the stock of capital falls as households

demand more assets,  $\frac{dk^{rp}}{d\beta} < 0 < \frac{dk^{rp}}{dg}$ . Furthermore, in secular stagnation, the economy accumulates less capital,  $k^{rp} < k^*$ . Here,  $k^*$  can be interpreted as the stock of capital in a fictive economy that shares the same primitives as that in secular stagnation, except that the ZLB isn't a constraint in the efficient economy, i.e  $\underline{R} = 0$  – the ZLB isn't a determinant of  $k^*$ .

This divergence between  $k^*$  and  $k^{rp}$  implies that the long run *measured* level of potential output falls, even through the "true" level of potential output,  $k^*$ , doesn't. This hysteresis isn't necessarily permanent but rather conditional on monetary policy. If the central bank decides to raise its inflation target or implement negative nominal interest rate, i.e  $d\underline{R} < 0$ , the "potential output gap",  $(\frac{k^{rp}}{k^*})^\alpha$ , will shrink. The total loss in output due to the demand shortfall is therefore:  $\frac{y^{rp}}{y^*} = x^{rp}(\frac{k^{rp}}{k^*})^\alpha$ . Thus, a focus on the output gap may lead policy-makers to under-estimate the economy' slack in secular stagnation.

If the economy is bubbly,  $x^b = 1$  and  $R = g$ ,

$$k^b = \left( \frac{\alpha(1-\alpha)^{\frac{1}{\psi}}}{g} \right)^{\frac{1}{1-\alpha}}$$

The stock of capital doesn't depend on agents' discount factor at all. Instead, the asset price bubble absorbs any demand shock such that the interest rate remains at  $R = g$ . It is immediate to verify that, if the bubbly steady state exists, bubbles crowd out capital with respect to the efficient steady state. But, as discussed in the first sub-section, this doesn't imply a crowding out with respect to the secular stagnation steady state. Indeed, asset bubbles lower the savings rate, but raise the output gap: the net effect on aggregate savings is ambiguous.

**Proposition 3** *Assume that  $g \geq \underline{R} > R^*$ . Let  $k^j$  be the steady state stock of capital where  $j = b$  stands for bubbly,  $j = s$  secular stagnation and  $j = *$  efficient and bubble-less – the stock of capital in a fictive economy with the same primitives, except that the ZLB is removed:  $\underline{R} = 0$ .*

1. *If the economy is either in secular stagnation or bubbly, the stock capital falls with respect to the efficient steady state:  $k^* > \max\{k^s, k^b\}$ .*
2. *If the RLB - natural interest rate gap is above a given threshold,  $\frac{R}{R^*} > \mathcal{R}^k$ , or equivalently if  $\alpha < \alpha^k$ , the bubbly economy accumulates more capital than the economy in secular stagnation:  $k^b > k^s$ .*

Where  $\mathcal{R}^k \equiv \frac{(\frac{g}{\underline{R}})^{\frac{\psi}{1+\psi}} - \eta}{1-\eta}$  and  $\alpha^k \equiv \frac{\beta\psi - \frac{g}{\underline{R}}(1+\varphi) \left( (\frac{g}{\underline{R}})^{\frac{\psi}{1+\psi}} - 1 \right)}{\beta\varphi + \frac{g}{\underline{R}}(1+\beta)(1+\varphi)} \leq \alpha^s$ ; in the particular case:  $g = \underline{R}$ ,

$\mathcal{R}^k = 1$  and  $\alpha^k = \alpha^s = \alpha^b$ .

Proposition 3 basically re-states the results we derived in the first sub-section.

The effect of asset bubbles on investment, and therefore output, in general equilibrium is ambiguous: on the one hand, the higher aggregate demand increases the output gap and stimulates investment; on the other hand, the higher interest rate rises the cost of capital and therefore decreases investment.

With respect to the efficient steady state, bubbles always reduce investment and output. Indeed, the bubble-induced higher aggregate demand leads to inflationary pressures that the central fights by increasing its policy rate. Thus, the bubbly steady state features a higher interest rate but the same levels of inflation and output gap as the efficient one. A higher cost of capital translates into lower capital demand by the firms: [Tirole \(1985\)](#)'s crowding out.

In secular stagnation, however, asset bubbles also exert a positive effect on aggregate demand, and therefore investment: [Keynes \(1936\)](#)'s paradox of thrift. The Keynesian effect increases with the gap between the RLB and the natural interest rate,  $\frac{\mathbf{R}}{\mathbf{R}^*}$ , whereas the crowding out effect increases with the gap between the growth rate of output and the RLB,  $\frac{g}{\mathbf{R}}$ . Thus, if the latter is sufficiently low with respect to the former, asset bubbles are expansionary. In particular, in the limit  $\frac{g}{\mathbf{R}} = 1$ , the steady state interest rate is the same in secular stagnation and in the bubbly economy such that bubbles are always expansionary. As  $\frac{g}{\mathbf{R}}$  rises, a lower  $\alpha$  is necessary for expansionary bubbles, that is, the output gap in secular stagnation must be larger (in absolute value).

Finally, note that, since  $\frac{y^s}{y^b} = x^s \left(\frac{k^s}{k^b}\right)^\alpha$ , there's an intermediate case that I didn't discuss here: the bubbly steady state has a higher level of output than the secular stagnation steady state despite a lower stock of capital, i.e there's a positive effect on the equilibrium labor that overcomes the negative effect on the equilibrium stock of capital. It arises whenever the RLB-interest rate gap is relatively high relative to the RLB-growth rate of output gap, but not too high.

## 5 Short run dynamic and "bubble theory" of the 2008-crisis

In this section, I relax the assumption of perfect foresight in favor of rational expectations. Around a given steady state, the log-linearized equilibrium system is:

$$\hat{\pi}_t = \kappa \hat{x}_t \tag{19}$$

$$\hat{x}_t = \left(1 - \frac{\psi}{(1 + \psi)(1 - \eta x^\psi)}\right) E_t \hat{x}_{t+1} + \frac{1}{1 + \psi} (\hat{r}_t^n - \hat{r}_t) \tag{20}$$

$$\hat{r}_t = \max\{\hat{r}_t^n + \phi \hat{\pi}_t - r^n, -\underline{r}\} - E_t \hat{\pi}_{t+1} \tag{21}$$

$$\hat{b}_t = \frac{g}{R} E_t \hat{b}_{t+1} + b \chi (\hat{k}_{t+1} - \hat{k}_t) + b(1 + \psi)(E_t \hat{x}_{t+1} - x_t) - b \hat{r}_t \tag{22}$$

$$\hat{r}_t^n = \frac{1}{1 - b} \hat{b}_t + \chi (\hat{k}_{t+1} - \hat{k}_t) \tag{23}$$

$$\hat{k}_{t+1} = \frac{(1 + \psi) E_t \hat{x}_{t+1} - \hat{r}_t}{1 - \chi} \tag{24}$$

The first two sub-sections study the local dynamic given a monetary regime: the economy remains close to the considered steady state such that it is either always out of, or always in, the liquidity trap. The third considers a simple experiment that formalizes the "bubble - secular stagnation" theory: the asset bubble follows a Markov process with two states. In the long run (second state), the bubble has imploded and the economy is in secular stagnation. In the short run (first state), the economy is booming – relative to the secular stagnation steady state – thanks to temporarily high asset prices.

All proofs are in appendix B.

### 5.1 Dynamic out of the liquidity trap

Let's start with the dynamic around a no-liquidity trap steady state:  $x = \pi = 1$ ,  $R = R^n$ , which can be either bubbly,  $R^n = g$  and  $b = 1 - \frac{R^*}{g} > 0$ , or fundamental,  $R = R^*$  and  $b = 0$ .

As the ZLB never binds in (21),

$$\hat{r}_t = \hat{r}_t^n + \phi \kappa \hat{x}_t - \kappa E_t \hat{x}_{t+1} \tag{25}$$

I used the AS curve, (19), to express the inflation rate as a function of the output gap.

Plugging in (20), this rule implies that output gap fluctuations are purely expectations-driven:

$$\hat{x}_t = \left(1 - \frac{\Omega_1^\phi}{\Omega_0^\phi}\right) \mathbb{E}_t \hat{x}_{t+1} \quad (26)$$

Since income risk is pro-cyclical, future news are discounted in the IS curve, (20), and therefore also in (26):  $\frac{\Omega_1^\phi}{\Omega_0^\phi} \in (0, 1)$  for all  $\phi \geq 0$ . Thus, there's a unique non-explosive solution:  $\hat{x}_t = 0$  for all  $t \geq 0$ .

Given a sequence of output gaps,  $\{\hat{x}_t\}$ , the sequence of asset prices,  $\{\hat{b}_t\}$ , is determined by the policy rule, (25), and the non-arbitrage condition, (22),

$$\hat{b}_t = b(1 - b) \left(\Omega_0^1 \mathbb{E}_t \hat{x}_{t+1} - \Omega_0^\phi \hat{x}_t\right) + (1 - b) \frac{g}{R^n} \mathbb{E}_t \hat{b}_{t+1} \quad (27)$$

As Galí (2017) noted in a different but related model, if liquidity isn't valued in steady state,  $b = 0$  and  $R^n = R^*$ , short run fluctuations in asset prices are completely orthogonal to the fundamentals of the economy, including the interest rate; endogenous fluctuations are possible if and only if the bubbly steady state exists, that is, if and only if:  $\frac{R^*}{g} < 1$ . A contrario, if liquidity is valued,  $b = 1 - \frac{R^*}{g}$  and  $R^n = g$ , endogenous fluctuations aren't possible, but asset prices are influenced by the stance of monetary policy.

Given  $\hat{k}_0$  and  $\{\hat{x}_t, \hat{b}_t\}$ , we can use (24) and (25) to compute the sequence of capital stocks  $\{\hat{k}_{t+1}\}$ :

$$\hat{k}_{t+1} = \Omega_0^1 \mathbb{E}_t \hat{x}_{t+1} - \phi \kappa \hat{x}_t + \chi \hat{k}_t - \frac{1}{1 - b} \hat{b}_t \quad (28)$$

As  $\chi \in (0, 1)$ , the dynamic is stable.

**Proposition 4** *Out of the liquidity trap, the dynamic of capital is stable; the dynamic of the output gap is determinate; the dynamic of the bubble is determinate if and only if the considered steady state isn't both bubble-less and dynamically inefficient.*

## 5.2 Dynamic in the liquidity trap

Now, let's study the dynamic around a liquidity trap steady state, that is, a steady state in secular stagnation:  $x = x^s < 1$ ,  $\pi = \pi^s < 1$ ,  $R = \frac{R}{\pi^s} < R^n$  and  $b = 0$ .

The ZLB always binds in (21),

$$\hat{r}_t = -\kappa \mathbb{E}_t \hat{x}_{t+1} \quad (29)$$

Since a secular stagnation steady must be bubble-less, asset prices are likely to be subject to endogenous shocks,

$$\hat{b}_t = \frac{g}{R^s} E_t \hat{b}_{t+1} \quad (30)$$

Indeed, depending on the calibration, dynamic efficiency,  $R^s > g$ , can require an implausibly large rate of deflation.

Given  $\hat{k}_0$  and output gap expectations,  $\{E_t \hat{x}_{t+1}\}$ , we can substitute (29) in (24) to determine  $\{\hat{k}_{t+1}\}$ :

$$\hat{k}_{t+1} = \frac{\Omega_0^1 E_t \hat{x}_{t+1}}{1 - \chi} \quad (31)$$

In the trap, the binding ZLB breaks the link between the current and future stocks of capital (that goes through the natural interest rate). Because of the time-to-build constraint, investment decisions are based on anticipations of future production rather than realized production: investment in period  $t$  solely depends on expectations over the output gap in period  $t + 1$ . Plugging (29) and (31) in (20) if  $t > 0$ ,

$$\hat{x}_t = \left(1 - \frac{\Omega_1^0}{\Omega_0^0}\right) E_t \hat{x}_{t+1} + \frac{\chi \frac{\Omega_0^1}{1-\chi}}{\Omega_0^0} (E_t \hat{x}_{t+1} - E_{t-1} \hat{x}_t) + \frac{1}{\Omega_0^0} \hat{b}_t$$

Taking expectations conditional on the information set of period  $t - 1$ ,

$$E_{t-1} \hat{x}_t = \left(1 - \frac{\Omega_1^0}{\Omega_0^0 + \chi \frac{\Omega_0^1}{1-\chi}}\right) E_{t-1} \hat{x}_{t+1} + \frac{1}{\Omega_0^0 + \chi \frac{\Omega_0^1}{1-\chi}} E_{t-1} \hat{b}_t$$

Since households use (30) to forecast asset prices, they must either expect that:  $E_{t-1} \hat{b}_{t+j} = (\frac{R^s}{g})^j E_{t-1} \hat{b}_t$  when  $\frac{R^s}{g} < 1$ , or  $E_{t-1} \hat{b}_{t+j} = \hat{b}_{t+j} = 0$  for all  $j \geq 0$  otherwise. Furthermore, as the IS curve, (20), is discounted and the output gap raises investment,  $\Omega_0^1 > \Omega_0^0 > \Omega_1^0 > 0$ , the previous equation has a unique non-explosive solution for all  $t \geq 1$ :

$$E_{t-1} \hat{x}_t = \frac{1}{\Omega_p^0 + (1-p)\chi \frac{\Omega_0^1}{1-\chi}} E_{t-1} \hat{b}_t \quad (32)$$

Where  $E_{t-1} \hat{b}_t = 0$  if endogenous fluctuations in asset prices aren't possible and  $p \equiv \frac{R^s}{g}$  is the coefficient of auto-correlation of asset prices in (30). If asset prices are indeterminate, we can treat the sequence  $\{\hat{b}_t\}$  as demand shocks affecting the economy. Those shocks, which are endogenous, follow an AR(1) process given by (30). Agents correctly anticipate that asset bubbles raise aggregate



demand, and, because prices are sticky and the ZLB binding, a higher demand translates into a higher output gap. With respect to (27), the term  $(1-p)\chi\frac{\Omega_0^1}{1-\chi} > 0$  captures the additional effect of the output gap on the natural interest rate that goes through the growth rate of potential output when the ZLB is binding.

Plugging (32) in the initial equation describing the dynamic of the output gap if  $t > 0$ :

$$\hat{x}_t = \frac{\Omega_0^0 + (1-p)\chi\frac{\Omega_0^1}{1-\chi}}{\Omega_p^0 + (1-p)\chi\frac{\Omega_0^1}{1-\chi}} \frac{\hat{b}_t}{\Omega_0^0} + \frac{p\chi\frac{\Omega_0^1}{1-\chi}}{\Omega_p^0 + (1-p)\chi\frac{\Omega_0^1}{1-\chi}} \frac{\hat{b}_t - \hat{b}_{t-1}}{\Omega_0^0} \quad (33)$$

In the liquidity trap, given expectations, the interest rate is fixed. Thus, as the natural interest rate varies, so does the interest rate gap,  $\hat{r}_t^n - \hat{r}_t$ , and hence aggregate demand and the output gap. In general equilibrium, asset bubbles affect the natural interest rate through two channels, (23). First, there's the direct, redistributive, channel: by implementing transfers from low-MPC households to high-MPC households, asset bubbles increase aggregate consumption. Second, there's the indirect, potential output growth, channel: asset bubbles affect investment decisions by the firms and therefore the growth rate of potential output. A high growth rate of asset prices implies a high growth rate of potential output, which increases aggregate consumption for a consumption-smoothing motive.

Finally, it will be useful for the following analysis to determine the output gap conditional on the stock of capital in period  $t$  if households expect the ZLB to bind forever from period  $t$  on,

$$\hat{x}_t = \frac{\Omega_0^0 + \chi\frac{\Omega_0^1}{1-\chi}}{\Omega_p^0 + (1-p)\chi\frac{\Omega_0^1}{1-\chi}} \frac{\hat{b}_t}{\Omega_0^0} - \frac{\chi}{\Omega_0^0} \hat{k}_t \quad (34)$$

If the economy was in the liquidity trap in period  $t-1$ ,  $k_t$  is given by (31) such that (33) and (34) coincide. But this isn't the case in period 0, or, more generally, if the economy wasn't in the liquidity trap in period  $t-1$ . Instead, as the economy enters the trap, capital takes one period to adjust. During this transition, the output gap can be lower than its steady state value – if the initial stock of capital is high such that households expect potential output to fall – or above – the other way around.

To summarize the results in this sub-section,

**Proposition 5** *Around a liquidity trap steady state, there isn't any dynamic of capital; the dynamic of the output gap is determinate; the dynamic of the bubble is determinate if and only if the*

considered steady state is dynamically efficient.

In this model, the dynamic of the output gap is always determinate, even if the central bank doesn't target inflation. As income risk is pro-cyclical, the income of young households is more elastic with respect to the output gap than the income of old households. This, in turn, greatly reduces the importance of future news relative to current news on aggregate demand.

### 5.3 Stochastic bubble: great moderation and secular stagnation

According to the "bubble-secular stagnation" theory of the post-2008 depression, i.e. Summers (2013) and Krugman (2013), the US great moderation, starting from the 90's until 2008, didn't arise despite, but rather thanks to, a large financial bubble. As we'll see in this section, the model confirms this intuition. To simplify the exposition, I'll take the rigid price limit,  $\kappa = 0$ , implying that  $\Omega_p^\phi = \Omega_p^0$  for all  $p, \phi$  – similar results can be derived for the general case  $\kappa \geq 0$ , but the computations are much more involved.

I consider a secular stagnation steady state which is dynamically inefficient and I assume that  $\{\hat{b}_t\}$  follows a two-states Markov chain:  $\hat{b}_t \in \{0, \hat{b}^h \in (0, 1)\}$ . The transition probabilities are:

$$E_t\{\hat{b}_{t+1} = 0 | \hat{b}_t = 0\} = 1, \quad E_t\{\hat{b}_{t+1} = 0 | \hat{b}_t = \hat{b}^h\} = 1 - p$$

If the economy starts period  $t$  in state  $h$ , asset prices are temporarily high,  $\hat{b}_t = \hat{b}^h \in (0, 1)$ . In period  $t + 1$ , this bubble bursts,  $\hat{b}_{t+1} = 0$ , with a probability  $1 - p$ ; it keeps going,  $\hat{b}_{t+1} = \hat{b}^h$ , with a probability  $p$ . Once the bubble has imploded, it doesn't recover: if  $\hat{b}_{t+1} = 0$ , then  $\hat{b}_{t+T} = 0$  for all  $T \geq 1$ . I'll refer to  $h$  as to the short run; the other state is the long run.

As the model features capital, the natural interest rate is endogenous, (23). This, combined with the non-linearity introduced by the ZLB, (21), and the forward-looking nature of the output gap, implies that the model can generate complicated dynamics with endogenous – deterministic or stochastic – regime-switching. A full characterization of all possible equilibria is beyond the scope of this paper. To focus purely on bubbles, monetary policy and secular stagnation, I'll select equilibria that satisfy three restrictions: first, they're non-explosive; second, the economy enters the liquidity trap as soon as the bubble bursts, or remains in the liquidity trap during the burst period; third, there aren't any endogenous regime-switches, i.e in state  $h$  the ZLB is either always or never binding. Given the Markovian structure of the shock process as well as local uniqueness given monetary regime, those restrictions are natural – although the second and third aren't necessarily

satisfied for arbitrary initial conditions. The results of this section are intuitive and generally hold true along other types of equilibria, but the analysis is much more involved.

To construct such equilibria, we'll work backward. If the bubble implodes in period  $t$ , it doesn't recover:  $b_s = 0$  for all  $s \geq t$ . Thus, under the second restriction, the economy enters the liquidity trap in period  $t$  with a stock of capital  $\hat{k}_t$  and remains in secular stagnation forever. As the stock of capital takes one period to adjust, the first restriction implies that the economy is in steady state from period  $t + 1$  on:  $\hat{x}_{s+1} = \hat{k}_{s+2} = 0$  for all  $s \geq t$ . I'll denote by a superscript  $l$  the burst period. Using (31), (34) and (23):

$$\hat{k}_{t+1}^l = 0, \quad \hat{r}_t^{n,l} = -\chi \hat{k}_t^h, \quad \hat{x}_t^l = -\frac{\chi}{\Omega_0^0} \hat{k}_t^h \quad (35)$$

Here,  $\hat{k}_t^j$  denotes the stock of capital conditional on being in state  $j \in \{h, l\}$  in period  $t - 1$ . We'll ensure later on that the ZLB is binding in (21), i.e.  $-\chi \hat{k}_t^h < r^n$ .

If the bubble raised the stock of capital,  $\hat{k}_t^h > 0$ , the natural interest rate undershoots its long run value during the burst period, which depresses aggregate demand and thereby the output gap. Indeed, the economy suddenly dis-accumulates capital in excess to the steady state value: the magnitude of the recession is proportional to the short run investment boom generated by the asset bubble <sup>9</sup>.

Having determined post-burst expectations, let's study the short run dynamic. Conditional on being in state  $h$  until period  $t - 1$  included, asset prices are constant:  $\hat{b}_s = \hat{b}^h$  for all  $s \leq t - 1$ . Under the third restriction, the economy is either always in the trap, or never. I'll start by the liquidity trap, and then the no-liquidity trap.

**Liquidity trap in the short run.** If it is in the liquidity trap despite the asset bubble, the output gap in the short run is given by (33):

$$\hat{x}_t^h = x_z^h \equiv \frac{\Omega_0^0 + (1-p)\chi \frac{\Omega_0^0}{1-\chi} \hat{b}^h}{\Omega_p^0 + (1-p)\chi \frac{\Omega_0^0}{1-\chi} \Omega_0^0} \quad (36)$$

The output gap rises with respect to the long run value. But this shouldn't be interpreted as a sign of a booming economy. Instead, the ZLB is still binding and the output gap strictly positive, but asset bubbles mitigate the chronic shortage of aggregate demand that the economy suffers from.

<sup>9</sup>This trade-off was also identified by [Hanson and Phan \(2017\)](#) in a different but related model. However, in [Hanson and Phan \(2017\)](#), the economy converges back to the efficient steady state.

Using (31) and (32), we can compute the stock of capital:

$$\hat{k}_{t+1}^h = k_z^h \equiv \frac{p \frac{\Omega_0^0}{1-\chi} \hat{b}^h}{\Omega_p^0 + (1-p)\chi \frac{\Omega_0^0}{1-\chi}} \quad (37)$$

As monetary offset is fully neutralized, the paradox of thrift holds true: a higher output gap translates into a higher level of investment and therefore a larger stock of capital.

Finally, we're left to verifying the two selection restrictions. Using (23), (35), (36) and (37), the ZLB isn't binding in (21) in state  $h$ , but is in state  $l$ , if and only if the following condition is satisfied:

$$\max \left\{ \hat{b}^h + \chi \max \{ \hat{k}_z^h - \hat{k}_0, 0 \}, -\chi \hat{k}_0 \right\} < r^n$$

Assuming that  $\hat{k}_0 \in [0, \hat{k}^h]$ ,  $b^h < \frac{\Omega_p^0 + (1-p)\chi \frac{\Omega_0^0}{1-\chi}}{\Omega_p^0 + \chi \frac{\Omega_0^0}{1-\chi}} r^n$  is a sufficient condition.

**No-liquidity trap in the short run.** Now, let's consider that the economy escapes the liquidity trap in the short run. It is slightly more complicated because, since the economy isn't in the same monetary regime in the short and long runs, we cannot iterate (26) forward. Assume that the economy is in state  $h$  in period  $t$ . Given the Markov structure of the shock, rational expectations satisfy:  $E_t \hat{x}_{t+1} = p E_t \hat{x}_{t+1}^h + (1-p)x_{t+1}^l$ . Plugging in (26):

$$\hat{x}_t^h = \left( 1 - \frac{\Omega_1^0}{\Omega_0^0} \right) \left( p E_t \hat{x}_{t+1}^h + (1-p)x_{t+1}^l \right) + \frac{r^n}{\Omega_0^0}$$

Where  $x_{t+1}^l$  is given by (35). Using (28), the path of capital solves:

$$\hat{k}_{t+1}^h = \Omega_0^0 \left( p E_t \hat{x}_{t+1}^h + (1-p)\hat{x}_{t+1}^l \right) + \chi \hat{k}_t^h - \hat{b}^h + r^n$$

We have a two-dimensional dynamic system in  $\{\hat{x}_t^h, \hat{k}_{t+1}^h\}$ . I show in appendix B that under assumptions 1 and 2, this system has a unique non-explosive solution,

$$\hat{x}_t^h = \hat{x}^h - v(\hat{k}_t - \hat{k}^h), \quad \hat{k}_{t+1}^h = \hat{k}^h + \delta(\hat{k}_t - \hat{k}^h) \quad (38)$$

Where  $\delta \in (0, 1)$  and  $v > 0$  are composite parameters that don't depend on  $\hat{b}^h$  nor on  $r^n$ , and  $\hat{x}^h$ ,

$\hat{k}^h$  are the stationary values of the output gap and stock of capital in state  $h$ ,

$$\hat{x}^h \equiv \frac{r^n}{\Omega_p^0} - \left(1 - \frac{\Omega_1^0}{\Omega_0^0}\right) \frac{(1-p)\chi\Omega_0^0}{\Omega_p^0} \hat{k}^h, \quad \hat{k}^h \equiv \frac{\left(p\Omega_0^0 + \Omega_p^0\right) \frac{r^n}{\Omega_p^0} - \hat{b}^h}{1 - \chi + \frac{(1-p)\chi\Omega_0^0}{\Omega_p^0}}$$

It is possible to show that  $\hat{x}^h > 0$  and  $\frac{d\hat{x}^h}{dr^n} > 0$ .

As in the long run, the output gap and the stock of capital are increasing in the steady state interest rate gap,  $r^n$ , and the stock of capital is decreasing in the size of the asset bubble,  $\hat{b}^h$ . If the bubble isn't too large, i.e

$$\hat{b}^h < \frac{p\Omega_0^0 + \Omega_p^0}{\Omega_p^0} r^n$$

The stock of capital rises with respect to the secular stagnation steady state,  $\hat{k}^h > 0$ . If the shock isn't permanent,  $p < 1$ , bubbles also affect the output gap through a novel and counter-intuitive mechanism. By partially crowding-out the short run stock of capital, asset bubbles increase the output gap. Indeed, households anticipate a severer crisis during the burst period if the economy accumulates a lot of capital in the short run – in excess to the long run level. This, in turn, depresses aggregate demand.

Finally, we're left to verifying the two selection restrictions. Using (23), (35) and (38), the ZLB isn't binding in (21) in state  $h$ , but is in state  $l$ , if and only if the following condition is satisfied for all  $t \geq 0$ :

$$-\chi\hat{k}^h + \chi\delta^t (\hat{k}^h - \hat{k}_0) < r^n < \hat{b}^h + \chi\delta^t(1 - \delta) (\hat{k}^h - \hat{k}_0)$$

Assuming that  $\hat{k}_0 \in [0, \hat{k}^h]$ , a sufficient condition for the existence of an equilibrium with a positive capital multiplier out of the liquidity trap in the short run is:  $\hat{b}^h > r^n$ .

It is therefore possible to construct equilibria that formalize Summers (2013) and Krugman (2013)'s "bubble-based" theory of the 2008 crisis: financial bubbles stimulate aggregate demand and investment in the short run. In particular, if the economy escapes the liquidity trap as long as the bubble doesn't burst, investment and output gradually increase over time. They suddenly and sharply fall when the bubble bursts. Furthermore, measured potential output also falls, although with a lag, because capital adjusts slowly. But the boom that predates the burst is relative to a secular stagnation steady state: this boom pushes the economy closer to the "true" level of potential output, which is higher than the post-crisis "measured" level. Thus, during a bubbly

episode, the economy doesn't necessarily consume and invest too much. Rather, this bubbly episode may allow the economy to reach its full potential.

## 6 Conclusion

The model provides a coherent interpretation of the sub-prime crisis: since the demand for US-originated assets gradually increased over the last few decades, it is quite possible that, prior to the 2008 crisis, the bubble-less natural interest rate was negative and way below the real lower bound. But, to paraphrase [Krugman \(2013\)](#), the housing and mortgage-backed securities bubble masked this striking fact and sustained aggregate demand. *Instead of causing the crisis, financial bubbles delayed the crisis.* The burst of the bubble in 2008 was a large, *permanent* negative shock on the supply of liquidity. Consequently, neither the length of the resulting depression, nor the ultra-low interest rates observed post-2008, should be surprising. Furthermore, looking at the stock of capital post-2008, one may tend to under-estimate potential output.

The predictions of the model are consistent with the broad trends of GDP, investment, interest and inflation rates during and after two of the recent crises triggered by the burst of financial bubbles, the US in 2008 and Japan in the 90's, as well as during the decade that predated the crash. In particular, the model can explain why: (i) a large financial bubble did not generate an economic boom; and (ii) the burst depressed labor, investment and output; (iii) the economy didn't enter a deflationary spiral at the ZLB.

One obvious limitation of the present model, which it shares with the rest of the literature on rational asset price bubbles, is that it cannot explain how bubbles emerge and crash. It would also be interesting to extend the model to a monetary union and check whether asset bubbles in peripheral countries of the Euro-zone can explain the divergence with the core both before and after 2008. I leave these questions for future research.

## A Long run: the bubble-less 3-equations NK model

We are looking for  $x^\psi \geq 0$  and  $\pi \geq 0$  that jointly solve the  $AS - AD$  system:

$$\begin{aligned} \pi &= 1 + \lambda(x^\psi - 1) \\ \frac{1 - \eta x^\psi}{(1 - \eta)x^\psi} &= \max \left\{ \pi^{\phi-1}, \frac{\mathbf{R}}{R^n} \pi^{-1} \right\} \end{aligned}$$

Note that any solution must satisfy:  $x^\psi \in (0, \eta^{-1})$ . Let  $x^z(\frac{\mathbf{R}}{R^*}) \equiv \left(1 + \frac{(\frac{\mathbf{R}}{R^*})^{\frac{1}{\phi}} - 1}{\lambda}\right)^{\frac{1}{\psi}}$ ;  $x^z(1) = 1$ ; if  $x^z(\frac{\mathbf{R}}{R^*}) > 0$ , then  $\frac{d \log x^z(\frac{\mathbf{R}}{R^*})}{d \log \frac{\mathbf{R}}{R^*}} = \frac{1}{\phi\kappa}$ . The ZLB binds if and only if  $x < x^z(\frac{\mathbf{R}}{R^*})$ . If  $\frac{\mathbf{R}}{R^*} < (1 - \lambda)^\phi$ , then  $x^z(\frac{\mathbf{R}}{R^*}) < 0$ : the ZLB never binds.

**Non-ZLB steady state** First, assume that  $x < x^z$  in steady state – a condition that we’ll check later on. The steady state output gap solves:

$$\mathcal{A}(x) \equiv \frac{(1 - \eta)x^\psi}{1 - \eta x^\psi} \left(1 + \lambda(x^\psi - 1)\right)^{\phi - 1} = 1$$

Here,  $\mathcal{A}(x) > 0$  for all  $x \in (0, \eta^{-1})$ ;  $\frac{d \log \mathcal{A}(x)}{d \log x} = \Omega_1^\phi$ . As  $\Omega_1^\phi > 0$ , the function  $\mathcal{A}(\cdot)$  is monotonically increasing on  $(0, \eta^{-1})$  and the equation  $\mathcal{A}(x) = 1$  has a unique solution. Since  $\mathcal{A}(1) = 1$ , the efficient solution:  $x = \pi = 1$  is our unique candidate. It satisfies the non-ZLB requirement if and only if the economy isn’t in secular stagnation,  $\frac{\mathbf{R}}{R^*} \leq 1$ .

**ZLB steady state** Second, assume that  $x < x^z$  in steady state. For future references, I’ll let  $\mathcal{R}(x) \equiv \left(1 + \lambda(x^\psi - 1)\right) \frac{1 - \eta x^\psi}{(1 - \eta)x^\psi}$ , where  $\mathcal{R}(x) > 0$  for all  $x \geq 0$ ,  $\mathcal{R}(1) = 1$  and  $\frac{d \log \mathcal{R}(x)}{d \log x} = -\Omega_1^0 < 0$ ;  $\mathcal{A}^z(x^z(\frac{\mathbf{R}}{R^*})) \equiv \mathcal{A}(\frac{\mathbf{R}}{R^*})$  where  $\mathcal{A}(\frac{\mathbf{R}}{R^*}) > 0$  for all  $\frac{\mathbf{R}}{R^*} > (1 - \lambda)^\phi$ ,  $\mathcal{A}(1) = 1$  and  $\frac{d \log \mathcal{A}(\frac{\mathbf{R}}{R^*})}{d \log \frac{\mathbf{R}}{R^*}} = \frac{\Omega_1^\phi}{\phi\kappa} > 0$ .

The output gap solves:

$$\mathcal{A}^z(x) \equiv \frac{\frac{\mathbf{R}}{R^*}}{1 - \eta x^\psi} \left(1 + \lambda(x^\psi - 1)\right)^{-1} = 1$$

Here,  $\mathcal{A}(x) > 0$  for all  $x \in (0, \eta^{-1})$ ;  $\frac{d \log \mathcal{A}(x)}{d \log x} = \Omega_1^0$ . As  $\Omega_1^0 > 0$ , the function  $\mathcal{A}(\cdot)$  is monotonically increasing on  $(0, \eta^{-1})$  and the equation  $\mathcal{A}(x) = 1$  has a unique solution. Since  $\lim_{x \rightarrow 0} \mathcal{A}^z(x) = 0 < \lim_{x \rightarrow \eta^{-1}} \mathcal{A}^z(x) = \infty$ , the equation  $\mathcal{A}^z(x) = 1$  has a unique solution. It is a steady state if and only if it satisfies the ZLB requirement:  $x < x_z$ , or  $\mathcal{A}(\frac{\mathbf{R}}{R^*}) > 1$ . Thus, there’s a ZLB steady state if and only if  $\frac{\mathbf{R}}{R^*} \geq 1$ . Finally, since  $\mathcal{A}(1) = \frac{\mathbf{R}}{R^*}$  and  $\frac{d \log \mathcal{A}(x)}{d \log x} > 0$ ,  $x < 1$  in steady state if and only if  $\frac{\mathbf{R}}{R^*} > 1$ .

## B Short run: the bubble-secular stagnation theory

If out of the liquidity trap, the short run dynamic system can be re-written as:

$$\hat{x}_t^h = p\left(1 - \frac{\Omega_1^0}{\Omega_0^0}\right)E_t\hat{x}_{t+1}^h - p\left(1 - \frac{\Omega_1^0}{\Omega_0^0}\right)\frac{(1-p)\chi}{\Omega_0^0}\hat{k}_t^h + \frac{r^n}{\Omega_0^0} \quad (\text{B-1})$$

$$\hat{k}_{t+1}^h = p\Omega_0^0 E_t\hat{x}_{t+1}^h + p\chi\hat{k}_t^h - b^h + \hat{r}^n \quad (\text{B-2})$$

The characteristic polynomial of this system is:  $P(z) = z^2 - p\left(1 - \frac{\Omega_1^0}{\Omega_0^0} + \chi\right)z + p\left(1 - \frac{\Omega_1^0}{\Omega_0^0}\right)\chi$ . As  $p\left(1 - \frac{\Omega_1^0}{\Omega_0^0}\right)\chi \in (0, 1)$ , a necessary and sufficient condition for both roots to be inside the unit circle is  $P(-1)P(1) > 0$ , or:

$$\left(1 - p\left(1 - \frac{\Omega_1^0}{\Omega_0^0}\right)\right)(1 - \chi) + (1 - p)\chi > 0 \quad (\text{B-3})$$

$$\left(1 + p\left(1 - \frac{\Omega_1^0}{\Omega_0^0}\right)\right)(1 + \chi) - (1 - p)\chi > 0 \quad (\text{B-4})$$

Both conditions are clearly satisfied under assumptions 1 and 2. Thus, there's an equilibrium, and it is locally unique.

To solve for the equilibrium, we can guess that the solution takes the form:

$$\hat{x}_t^h = -v\hat{k}_t^h + \zeta, \quad \hat{k}_{t+1}^h = \delta\hat{k}_t^h + \xi$$

Plugging this guess in (B-1) and (B-2) and collecting terms,

$$v = \left(1 - \frac{\Omega_1^0}{\Omega_0^0}\right)\frac{\frac{(1-p)\chi}{\Omega_0^0}}{1 - p\left(1 - \frac{\Omega_1^0}{\Omega_0^0}\right)\delta}, \quad \delta = p\left(\Omega_0^0\lambda\delta + \chi\right)$$

Using those two equations,  $\delta$  is solution of:  $G(\delta) \equiv (\delta - \chi)\left(1 - p\left(1 - \frac{\Omega_1^0}{\Omega_0^0}\right)\delta\right) + (1 - p)\chi = 0$ . (B-3) and (B-4) imply that  $G(-1) < 0 < G(1)$ ; furthermore,  $G'(\delta) > 0$  if and only if  $\delta < \frac{1+\chi}{2p\left(1 - \frac{\Omega_1^0}{\Omega_0^0}\right)}$  and

$G(0) = -p\chi$ . Thus, there's a unique  $\delta \in (0, 1)$  that solves  $G(\delta) = 0$ .



From (B-1) and (B-2), in the long run, the economy converges to:

$$\hat{x}^h \equiv \frac{r^n}{\Omega_p^0} - \left(1 - \frac{\Omega_1^0}{\Omega_0^0}\right) \frac{(1-p)\chi\Omega_0^0}{\Omega_p^0} \hat{k}^h$$

$$\hat{k}^h \equiv \frac{\left(p\Omega_0^0 + \Omega_p^0\right) \frac{r^n}{\Omega_p^0} - \hat{b}^h}{1 - \chi + \frac{(1-p)\chi\Omega_0^0}{\Omega_p^0}}$$

The remaining parameters are therefore:

$$\zeta = \hat{x}^h + v\hat{k}^h, \quad \xi = (1 - \delta)\hat{k}^h$$

# Chapter III

## Secular stagnation or secular boom?

### 1 Introduction

Motivated by the long-lasting crisis of 1929 in the US, [Hansen \(1939\)](#) introduced the concept of secular stagnation: he predicted that the US had entered a very persistent, or even permanent, depression due to a shortage of aggregate demand. Shortly after Hansen's address to the AEA, the second world war and the economic boom that followed disproved the secular stagnation hypothesis. Almost 80 years later, [Summers \(2013\)](#) brought this idea back to life in order to explain the dynamic of the US economy pre- and post-2008.

However, to quote [Eggertsson et al. \(2017\)](#), "secular stagnation does not emerge naturally from the current vintage of models in use in the literature". The New Keynesian (NK) model has been built to study short run fluctuations around a given trend. While the textbook model <sup>1</sup> captures in an elegant way the Keynesian intuition that short run output fluctuations arise because of shocks to aggregate demand (a negative shock lowers output, a positive shock raises output), it cannot, or should not, be used to study secular stagnation.

Indeed, secular stagnation involves very persistent or even permanent demand shocks. A characteristic that is very problematic for the NK model: unless subtle assumptions are made, the NK model isn't Keynesian at all in the medium-to-long run. Rather, it turns fully "Neo-Fisherian" as shocks become persistent enough: positive demand shocks lower output, negative demand shocks raise output. Instead of a secular stagnation, the model predicts a secular boom when aggregate demand is persistently deficient (if the economy converges).

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<sup>1</sup>That is, the 3-equations model that features complete financial markets, no liquidity, ect. More generally, I call textbook 3-equations NK models with a-cyclical inequality/income risk/liquidity and standard preferences.

It is also well-known that the basic NK model makes other puzzling predictions. A non-exhaustive list includes the following points: when monetary policy follows a Taylor rule that includes the ZLB, local determinacy implies global indeterminacy (Benhabib et al., 2001); expected future demand shocks have way too strong effects on current output (Del Negro et al., 2015); the Neo-Fisherian paradox: nominal interest rate cuts can be deflationary (Cochrane, 2014); and more generally, the laws of NK economics seem to drastically change when the ZLB binds and shocks become sufficiently persistent, e.g the secular boom.

Through an illustrative model (a TANK with cyclical inequality and cyclical liquidity) I provide a coherent framework (that heavily builds on Auclert and Rognlie (2018) and Auclert et al. (2018), reviewed below) to understand the puzzling predictions of NK models, and to analyze and compute the effects of demand shocks (temporary, persistent, or permanent) in analytically tractable NK models. I represent the equilibrium of the model in terms of asset demand and supply curves, and I underline the crucial importance of the elasticities of asset demand and asset supply with respect to the output gap in general equilibrium (the GE elasticities). The sign(s) of those elasticities at various time horizons, at the ZLB and in normal times, determine(s) whether the model makes puzzling and/or indeterminate predictions, the secular boom included, or conventional and determinate predictions.

For example, to a macro-economist interested in studying secular stagnation, the two following properties are desirable: (i): there's a globally unique steady state equilibrium; (ii): if aggregate demand is chronically deficient, then there's a secular stagnation rather than a secular boom. I show that the model without long run liquidity predicts (i) and (ii) if and only if the long run GE elasticity of asset demand is strictly positive both at the ZLB and in normal times.

Of course, this condition may fail on multiple fronts: if the elasticity changes of sign when the ZLB binds, then the economy hasn't any steady state when aggregate demand is chronically deficient, and it has multiple steady states otherwise (one in the liquidity trap, the other out of the trap); if both elasticities are negative, the economy has a unique steady state equilibrium, but, if aggregate demand is chronically deficient, there's a secular boom at the ZLB rather than a secular stagnation.

Those results can be generalized to shorter time horizons as well, i.e non-permanent demand shocks. For example, if the shock process follows a two-states Markov chain – state  $l$  is the steady state and it is absorbing; in the short run, the economy is in state  $h$ ; the transition probability from  $h$  to  $l$  is  $p \in [0, 1)$  – then the exact same proposition applies if we replace the long run GE

elasticity by that at time horizon (persistence)  $p$ , and steady state by stationary state.

In the log-linear model, the persistence- $p$  GE elasticity of asset demand can be computed as a linear combination of the short run (persistence zero) and the long run (steady state) elasticities:

$$\Omega_p^\phi = (1 - p)\Omega_0^\phi + p\Omega_1^\phi$$

Here,  $\phi$  indexes the coefficient on inflation in the Taylor rule:  $\phi = p$  corresponds to a constant real interest rate;  $\phi = 0$  to a constant nominal interest rate.

The textbook NK model (with both a-cyclical inequality and liquidity) has a strictly positive short run GE elasticity:  $\Omega_0^\phi > 0$  for all  $\phi \geq 0$ ; however, the long run GE elasticity is strictly positive if and only if the central bank follows the Taylor principle:  $\Omega_1^\phi > 0$  if and only if  $\phi > 1$  (and  $\Omega_1^1 = 0$ ). Thus, as shocks become more and more persistent, the persistence- $p$  GE elasticity in the liquidity trap slowly and continuously decreases, until it turns negative: mildly negative demand shocks induce two stationary equilibria (one in the trap, the other out of the trap); strongly negative demand shocks induce the absence of any stationary equilibrium.

Those problematic predictions can be traced back to the sign of the output gap response to the shock in GE. Along a stationary equilibrium, and abstracting from sunspots, the GE response of output  $\hat{x}_t$  to a given demand shock  $e_t$  of persistence  $p$  can be computed as follows <sup>2</sup> (up to a first-order approximation):

$$\hat{x}_t = -\frac{d\hat{a}_t}{de_t} e_t$$

Where  $\frac{d\hat{a}_t}{de_t}$  is the partial equilibrium (PE) elasticity of asset demand to the shock.

Here, I'll assume that the GE elasticity is strictly positive in normal times; the mechanics when the elasticity is negative can be deduced by symmetry. When the central bank targets inflation, a strongly negative shock reduces the output gap until the ZLB binds. If the GE elasticity doesn't change of sign at the ZLB, the output gap continues to fall until asset demand has decreased enough to clear the financial markets. If, however, the GE elasticity changes of sign at the ZLB, the negative demand shock suddenly generates a large output boom that pushes the economy out of the trap. But, out of the trap, the multiplier changes of sign again: this cannot be an equilibrium, and therefore all equilibria are non-stationary. Furthermore, if the shock isn't strongly negative, there are multiple equilibria if and only if the GE elasticity changes of sign – I refer to sub-sections

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<sup>2</sup>Auclert and Rognlie (2018) derive such a formula for the long run output multipliers, i.e  $p = 1$ , in a fully-fledged HANK model under a constant-R policy, i.e  $\phi = p$ . I generalize this formula, although in much simpler setting.

3.2 (global multiplicity) and 3.5 (local multiplicity) for a detailed discussion.

Similar, although more complicated, conditions that involve the GE elasticities of asset demand and asset supply can be derived in a model with an endogenous asset supply. In particular, a necessary condition to rule problematic predictions is the difference between the GE elasticities of asset demand and asset supply be strictly positive. Thus, in the extended model, a strictly positive GE elasticity of asset demand is neither a necessary nor a sufficient condition anymore.

I introduce liquidity in a peculiar way: I focus on asset bubbles. As bubbly and fundamental (bubble-less) steady states co-exist, it provides a natural framework to compare the dynamic response of the economy to various shocks depending on whether there's a positive long run supply of liquidity or not. However, it places strong restrictions on the GE elasticity of asset supply: it tends to infinity as shocks become permanent, and it turns negative whenever the ZLB binds. Consequently, my "full" model fails to predict (i) and (ii) when shocks are very persistent. This underlines the very importance of how the long run is treated in NK models: subtle differences in the assumptions lead to large differences in outcomes, both qualitatively and quantitatively.

Nevertheless, the extended model allows me to analyze the reciprocal interactions between asset bubbles and monetary policy. A usual and intuitive conclusion in the literature is that, since monetary policy cannot affect the natural interest rate, it doesn't affect the existence condition for assets bubbles. I show that this conclusion will generally prove wrong in TANK and HANK models. Indeed, in NK models with cyclical inequality, if prices remain sticky even in the long run, monetary policy can affect the steady state level of asset demand. Depending on the sign of the long run GE elasticity of asset demand, a low nominal interest rate can promote or prevent the emergence of asset bubbles of the rational kind.

In the first part of the paper, I build on [Werning \(2015\)](#) and [Bilbiie \(2019\)](#). I introduce cyclical inequality and incomplete markets in the textbook NK model. I consider a very simple structure à la [Woodford \(1990\)](#): there are two states, characterized by different income levels, and two representative agents continuously switch from one state to the other in a deterministic fashion at the end of every period. The income distribution between those two states is endogenous: the "saver", i.e the agent in state  $s$ , receives a higher share of aggregate income than the "borrower", the agent in state  $b$ , but this income share varies along the cycle. Saver's income share is a function of the output gap: if this function is increasing, inequality is pro-cyclical; if it is constant, inequality is a-cyclical; and if it is decreasing, inequality is counter-cyclical. Because of financial frictions, the agent in state  $b$  is liquidity-constrained: income shocks aren't insurable. This triggers

a precautionary savings motive that has the same "cyclicality" as inequality <sup>3</sup>. If inequality is strongly pro-cyclical, the strongly pro-cyclical precautionary savings motive makes the long run GE elasticity of asset demand strictly positive, even in the liquidity trap. This solves the problems discussed above.

In the second part of the paper, I introduce an endogenous supply of liquidity: rational asset bubbles. I show that it can restore all puzzles, i.e restore negative multipliers if shocks are very persistent, despite a long run positive GE elasticity. Intuitively, now, the relevant statistics to determine whether the economy is Keynesian or Neo-Fisherian in the long run includes both the long run GE elasticities of asset demand and asset supply. To have a truly Keynesian NK model, the long run GE elasticity of asset demand must be greater than that of asset supply: a lower (higher) level of output must decrease (increase) asset demand more than asset supply. As I discussed above, my model doesn't satisfy this criterion because of the particular type of liquidity I consider.

Finally, I compare the multipliers of the model depending on whether inequality is cyclical and/or liquidity is cyclical. In isolation, both pro-cyclical inequality or counter-cyclical liquidity tend to reduce the multipliers with respect to the textbook model (both a-cyclical). However, the predictions of the model that jointly assumes cyclical inequality and liquidity aren't as sharp. Indeed, endogenous liquidity interacts with the endogenous precautionary savings motive that is generated by cyclical inequality: as the saver accumulates a buffer stock of assets, the precautionary savings motive is reduced. Quite counter-intuitively, a pro-cyclical supply of liquidity can be stabilizing whereas a counter-cyclical supply of liquidity can be destabilizing, depending on the cyclical nature of inequality. But, under relatively weak assumptions, the full model predicts lower multipliers than the textbook model if inequality is pro-cyclical and liquidity counter-cyclical.

Although I use a particular set of micro-foundations, the analysis in terms of GE elasticities, and the associated no-puzzles conditions, can be imported in virtually any tractable NK model. At the micro-level, those models differ from one another: the determinants of the GE elasticities will change from one model to the other. However, those models share the same aggregate representation and therefore all results in terms of GE elasticities should hold true.

**Related literature** As there's a growing literature trying to quantify the macroeconomic effects of various shocks in models that include a realistic distribution of income and wealth <sup>4</sup>

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<sup>3</sup>Here, the term "cyclical precautionary savings motive" is a short-cut to summarize how incomplete markets and cyclical inequality affect households' asset demand. I refer to [Bilbiie \(2019\)](#) for a detailed analysis of this mechanism.

<sup>4</sup>Recent examples include: [McKay and Reis \(2016\)](#), [McKay et al. \(2016\)](#), [Kaplan et al. \(2016\)](#) or [Auclert and](#)

there's a growing literature that seeks to better understand the insight of those models in simpler frameworks. Indeed, HANK models are difficult to solve numerically, cannot be solved analytically, and include many channels that are absent from the basic model.

[Werning \(2015\)](#) is an early example<sup>5</sup>: he's shown that *either* pro-cyclical income risk *or* counter-cyclical liquidity allow to solve the forward guidance puzzle in a NK model with incomplete markets. I extend his analysis to other types of shocks, other paradoxes, including the possibility of secular stagnation rather than a secular boom, and I include *both* cyclical inequality and cyclical liquidity: I show that those two key mechanisms may undermine each other in GE.

The most closely related papers to mine are [Auclert and Rognlie \(2018\)](#) and [Auclert et al. \(2018\)](#). This paper can be seen as a extension of their results, but in a simplified version of their model. That is, because I restrict my-self to a (much) less general class of models than they do, I can extend their results along several dimensions within this particular class of models. [Auclert et al. \(2018\)](#) introduced a new set of moments, the Inter-temporal Marginal Propensities to Consume (IMPC), and showed how the multipliers of demand shocks in HANK models can be analytically computed using the IMPC-matrix (assuming that the central bank keeps the interest rate constant). Furthermore, they showed that macroeconomists can discriminate across models by comparing the theoretical IMPC to to the measured IMPC. [Auclert and Rognlie \(2018\)](#) built a HANK model to analyze the macroeconomic effects of various types of inequality shocks, both in the short and long run. They showed that, when the central bank is unresponsive, a particular type of inequality shock has a large effect on employment in the long run. They also provided some analytical results: secular stagnation is possible only if the long run GE elasticity of asset demand is higher than that of asset supply; the long run multipliers to a permanent shock can be computed using the long run GE elasticities of asset supply and demand.

From a methodological point of view, I heavily borrow from them: as [Auclert and Rognlie \(2018\)](#), I focus on the GE elasticities of asset demand and supply (which can be used to compute the IMKC), and as [Auclert et al. \(2018\)](#), I show that the GE elasticities can be used to compute the GE multipliers in a simple way – in a much simpler model. Nevertheless, I extend their results, although in much more restrictive class of models. First, I provide two sufficient statistics to compute the GE multiplier in analytically tractable NK models: the difference between the short run GE elasticities of asset demand and supply as well as the difference between the long run

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[Rognlie \(2018\)](#).

<sup>5</sup>Other recent papers include that analyze income risk in NK models with incomplete markets include: [Ravn and Sterk \(2017\)](#), [Sterk and Ravn \(2017\)](#), [Acharya and Dogra \(2018\)](#).

elasticities. Second, I provide a unified treatment of multipliers in the liquidity trap or when the central bank follows a Taylor rule. Third, I study the predictions of the model when the sign of those elasticities is horizon-dependent or/and regime-dependent: along the way, I provide an explanation for some puzzling predictions of the NK model.

This relates my paper to [Bilbiie \(2019\)](#) and [Bilbiie \(2018a\)](#). [Bilbiie \(2019\)](#) provided a Keynesian cross representation for NK models. He used this representation to compute the multipliers of demand shocks, and performed a decomposition between direct vs indirect effects, in various NK models, including an analytical HANK model with incomplete markets and cyclical inequality. Finally, he proved that a calibrated version of his HANK can approximately reproduce the predictions of the richer quantitative HANKs. Using the HANK model built in [Bilbiie \(2019\)](#), [Bilbiie \(2018a\)](#) showed that various problems of the NK model (such as determinacy under a peg or the forward guidance puzzle) are cured if inequality is sufficiently pro-cyclical, but at the cost of lower demand shocks multipliers. He then provided two solutions out of this trade-off: Wicksellian interest rate rules or counter-cyclical enough income risk. Finally, he computed the optimal monetary policy, both in and out of the liquidity trap.

My model without liquidity is a simplified version of his HANK – a TANK – that nevertheless captures the main mechanism described in [Bilbiie \(2019\)](#) (the inter-temporal Keynesian cross); but, as it is simpler, I can also analyze a version of the model that includes liquidity. I build on his analysis and reproduce some of his results, although from a different perspective: I emphasize the sign(s) of the GE elasticities of asset supply and demand at various time-horizon, in the liquidity and in normal times. This provides a general framework to understand whether and why analytical NK models make puzzling predictions. Furthermore, while he focuses on temporary shocks and optimal policies, I mostly focus on very persistent or permanent shocks and the possibility of secular stagnation. Our analyzes are complementary.

There's a growing literature on secular stagnation in NK models. Using a OLG model, [Kocherlakota \(2013\)](#) showed that a fall in the price of land generates a secular stagnation when monetary policy is unresponsive and nominal wages rigid. In an OLG-NK model that includes financial frictions and heterogeneous agents, [Caballero and Farhi \(2018\)](#) showed that secular stagnation can arise because of a safety trap rather than a liquidity trap if some agents are infinitely risk-averse. [Eggertsson et al. \(2017\)](#) showed that a three-periods OLG-NK model with sticky wages and financial frictions exhibits a secular stagnation steady state under some calibrations. [Michau \(2018\)](#) provided another way of modeling secular stagnation that doesn't rely on an OLG structure, but



rather on a utility function that includes wealth as an argument in a representative agent economy.

It is rather difficult to understand what are the key assumptions that make secular stagnation in those papers possible, and what are the key mechanisms. Indeed, from one paper to the other, nominal rigidities, financial frictions, the demographic structure, preferences, etc. all vary. Furthermore, subtle differences matter: while the three-periods OLG model of [Eggertsson et al. \(2017\)](#) is capable of generating secular stagnation as a steady state with sticky *wages*, the two-periods OLG model of [Boullot \(2017\)](#) wouldn't. Instead, [Boullot \(2017\)](#) needs sticky *prices* to get a "conventional" secular stagnation rather than a secular boom. Here, I provide a coherent framework to understand what's going on in those models: they all make assumptions that lead to a long run GE elasticity of asset demand that is higher than the long run GE elasticity of asset supply.

The same remarks apply to [Michaillat and Saez \(2018\)](#), who showed that a NK model that includes wealth in the utility function can solve most puzzles of the textbook model, as well as generate secular stagnation as a steady state. My analysis reveals that, although [Bilbiie \(2018a\)](#) and [Michaillat and Saez \(2018\)](#)'s papers have very different micro-foundations, they've the same macro-representation: their results are driven by the strictly positive long run GE elasticity of asset demand in the trap. However, in general, those results aren't robust to the inclusion of liquidity in the model.

Since [Martin and Ventura \(2012\)](#) and [Farhi and Tirole \(2012\)](#) have shown that asset bubbles can be expansionary in OLG models that include financial frictions, a growing literature has studied asset bubbles-driven business cycles <sup>6</sup>. Even more recently, this literature has merged with the NK literature (e.g [Kocherlakota \(2013\)](#), [Gali \(2014\)](#), [Gali \(2017\)](#), [Caballero and Farhi \(2018\)](#), [Boullot \(2017\)](#) and [Wang et al. \(2017\)](#)). I make two contributions to this literature. First, I show that the usual conclusion that monetary policy cannot affect the conditions under which asset bubbles exist <sup>7</sup> will generally prove wrong in TANK and HANK models. Instead, when inequality is cyclical and prices sticky, there's a multiplicity of bubbly steady states if the central bank follows a Taylor rule: monetary policy, through the "choice" of an interest rate floor, i.e the "level" of the ZLB, can influence the size of the steady state bubble along on of them, and even eliminate it; alternatively, if monetary policy is modeled as setting an interest rate peg, then there's a unique bubbly steady state, and its existence as well as the size of the steady state bubble are both conditional on the

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<sup>6</sup>[Miao \(2014\)](#) provides an introduction to, as well as a review of, this literature.

<sup>7</sup>One exception is [Wang et al. \(2017\)](#) where the inflation target of the central bank affects the existence condition, but through a completely different mechanism: their model doesn't include the ZLB nor cyclical inequality, but rather a very particular kind of financial friction that makes inflation non-neutral even in the long run.

level of the nominal interest rate peg. Second, I study how assets bubbles interact with cyclical inequality to influence the dynamic response of the economy to demand shocks as well as the determinacy conditions.

Finally, there's a large literature on liquidity traps in NK models, which can be roughly divided into two camps: fundamentals-driven liquidity traps vs expectations-driven liquidity traps – to borrow the terminology of [Bilbiie \(2018b\)](#). The former literature started with the seminal work of [Krugman \(1998\)](#) and [Eggertsson and Woodford \(2003\)](#): an exogenous shock to the fundamentals of the economy reduces aggregate demand and pushes the economy into the liquidity trap<sup>8</sup>; the latter started with [Benhabib et al. \(2001\)](#): the textbook NK model has two equilibria, one of which is a liquidity trap. My analysis of the liquidity trap is close to that of [Bilbiie \(2018b\)](#), who analyzed those two types of liquidity traps in the textbook NK model. In particular, he showed that the latter type arises when there is "enough shock persistence and endogenous inter-temporal amplification of future news"; and he noticed that multipliers of usual fiscal and monetary policies change of sign when liquidity trap are expectations-driven rather than fundamentals-driven. Furthermore, he computed the optimal monetary and fiscal policies in the Neo-Fisherian case.

I provide an alternative, general, simple condition to determine which type of liquidity trap is possible in analytically tractable NK models: without long run liquidity, if the GE elasticity of asset demand doesn't change of sign at the ZLB, then liquidity traps are fundamentals-driven; if it does, then liquidity traps are expectations-driven. I also provide similar, although more complicated, conditions for a model that includes long run liquidity. This leads me to conclude that all multipliers change of sign when the economy enters an expectations-driven liquidity trap. I also study permanent liquidity traps driven by large demand shocks: secular stagnation vs secular boom, which he abstracts from. Our analyzes are complementary.

Section 2 introduces the basic environment and equilibrium; section 3 analyzes the model without long run liquidity; section 4 analyzes the model with long run liquidity. The proofs for sections 3 and 4 are in appendices A and B.

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<sup>8</sup>A limited list includes: [Lorenzoni and Guerrieri \(2011\)](#), [Christiano et al. \(2011\)](#), [Eggertsson and Krugman \(2012\)](#), [Correia et al. \(2013\)](#), [Korinek and Simsek \(2016\)](#).

## 2 Basic environment and equilibrium

The basic setup is a two-agents NK model (TANK). There is a single consumption good and two assets, a nominal bond in zero net supply and asset bubbles. Time is discrete,  $t \in \{0, 1, \dots\}$ , and the horizon infinite. In period 0, agents are collectively endowed with  $H = 1$  units of bubbles. In the non-linear model, agents have perfect foresight; I relax this assumption in favor of rational expectations in the log-linear version.

### 2.1 Firms, nominal rigidities and the AS curve

The supply side of the economy plays a very minor role in this paper: several configurations lead to the same results and intuitions. Hence, I simplify it as much as possible. With respect to the textbook NK model, I make three assumptions. First, I follow [Werning \(2015\)](#) and by-pass the labor market, that is, I avoid micro-founding the labor supply, production function and the corresponding distribution of income between agents. Second, I assume that the natural level of output is fixed. Third, I consider a particular version of Rotemberg's model that makes pricing decisions static and eliminates the downward-sloping solution.

The final good,  $Y_t$ , is produced by a competitive sector that aggregates a continuum of intermediate varieties,  $Y_t(\omega)$ ,  $\omega \in [0, 1]$ , according to the Dixit-Sitglitz technology,  $Y_t = \left( \int Y_t(\omega)^{\frac{\iota-1}{\iota}} d\omega \right)^{\frac{\iota}{\iota-1}}$ ,  $\iota > 1$ . As well-known, this leads to the following demand function:

$$Y_t(\omega) = \left( \frac{P_t(\omega)}{P_t} \right)^{-\iota} Y_t$$

Where  $P_t(\omega)$  is the price of variety  $\omega$ , and  $P_t = \left( \int P_t(\omega)^{1-\iota} d\omega \right)^{\frac{1}{1-\iota}}$  is the aggregate price level.

Intermediate firms are in monopolistic competition. Each has access to a CRS technology to produce a particular variety at a (real) marginal cost  $mc_t$ ,

$$mc_t = y_t^\psi, \quad \psi > 0$$

Here,  $y_t \equiv \frac{Y_t}{A_t}$  is the productivity-adjusted level of output, and  $A_t = g^t, g \geq 1$ , is a productivity index.

Firm  $\omega$ 's flow of profits in period  $t$  is:

$$D_t(\omega) = \left( \frac{P_t(\omega)}{P_t} - mc_t \right) Y_t(\omega) - AC_t(\omega)$$

Where  $AC_t(\omega)$  is a cost of adjusting nominal prices.

Each monopolist maximizes the sum of discounted flows of profits under the demand constraint. Absent any form of nominal rigidities,  $AC_t(\omega) = 0$ , each firm sets  $P_t(\omega) = \frac{\iota}{\iota-1} P_t mc_t$ . Imposing symmetry across firms,  $P_t(\omega) = P_t$ , the equilibrium marginal cost is fixed at:  $mc_t = \frac{\iota-1}{\iota}$ , such that output is constant over time and it is equal to:

$$y_t = y^n \equiv \left( \frac{\iota-1}{\iota} \right)^{\frac{1}{\psi}}$$

Here,  $y^n$  is the natural/potential/supply-determined level of output. However, in this economy, firms face a quadratic cost of adjusting nominal prices, which is measured in final output:

$$AC_t(\omega) = \frac{\Phi}{2} \left( \frac{P_t(\omega)}{\Pi^n P_{t-1}} - 1 \right)^2 \frac{Y_t}{\pi_t}, \quad \Phi \geq 0$$

$$\text{Where } \pi_t \equiv \frac{P_t}{\Pi^n P_{t-1}}$$

I make two adjustments with respect to the usual version of (Rotemberg, 1982)'s model found in the NK literature. First, the cost of adjusting nominal prices depends on the size of the nominal adjustment relative to a reference price,  $\Pi^n P_{t-1}$ , rather than relative to firm's past price,  $P_{t-1}(\omega)$ : the pricing decision is static. One way to think of this is that firms use a rule of thumb to set their price, which delivers the optimal outcome as long as inflation equals the target. They may decide to revise their plan and deviate from this rule, but the resulting optimization process is "psychologically" costly. Consistent with this interpretation, adjustment costs are rebated lump-sum to households – the good market clearing condition is thereby:  $Y_t = C_t$  where  $C_t$  denotes aggregate consumption. Second, I normalize the adjustment cost by the current inflation gap,  $\pi_t$ , to suppress the downward-sloping segment of the AS curve <sup>9</sup>.

Solving firm's program yields the optimal pricing condition:

$$\left( \frac{P_t(\omega)}{P_t} - \frac{\iota}{\iota-1} mc_t \right) Y_t(\omega) + \frac{\Phi}{\iota-1} \frac{P_t(\omega)}{\Pi^n P_{t-1}} \left( \frac{P_t(\omega)}{\Pi^n P_{t-1}} - 1 \right) \frac{Y_t}{\pi_t} = 0$$

<sup>9</sup>Without this normalization, the AS curve is a quadratic polynomial with two solutions:  $\pi_t = \frac{1}{2} \pm \frac{1}{2} \Delta(x_t)$  where the function  $\Delta(\cdot)$  is strictly increasing. If one requires that  $\pi_t \approx 1$  – as is the case in the basic NK model, one implicitly picks the upward-sloping solution:  $\frac{d\pi_t}{dx_t} > 0$ ; however, there's another solution at  $\pi_t \approx 0$  and  $\frac{d\pi_t}{dx_t} < 0$ .

Imposing symmetry across firms,  $P_t(\omega) = P_t$ , we can derive the aggregate supply (AS) curve:

$$\pi_t = 1 + \lambda(x_t^\psi - 1) \tag{1}$$

$$\text{Where } \lambda \equiv \left(\frac{\Phi}{\iota - 1}\right)^{-1} \quad \text{and} \quad x_t \equiv \frac{y_t}{y^n}$$

Here,  $\lambda$  is a measure of price stickiness and  $x_t$  is the output gap. Prices are fully rigid when  $\lambda = 0$ : firms never deviate from the target; prices are fully flexible when  $\lambda \rightarrow \infty$ : output is supply-determined.

Firms optimally deviate from central bank's target if and only if aggregate demand differs from the potential level of output. Up to a first-order approximation, this AS curve is similar to the usual NK Philips curve if firms have a limited planing horizon.

**Assumption 1**  $1 > \lambda$ .

This assumption bounds the elasticity of inflation with respect to the output gap from below. It isn't necessary, but it will simplify the steady state analysis.

As the natural level of output is fixed, all fluctuations in aggregate income are due to fluctuations in the output gap. Thus, I'll refer to variations in aggregate income, output or the output gap interchangeably.

## 2.2 Households, incomplete markets and the IS curve

The demand side is the crucial part of the model. Again, I'll try to avoid too specific micro-foundations in order to clarify the underlying mechanics. When one wishes to study analytically (rather than numerically) NK models with incomplete markets, there's a trade-off between a realistic structure of financial markets and shocks, i.e a continuum of agents and states as well as occasionally binding constraints, and a positive supply of liquidity in equilibrium: if a model includes both, the entire distribution of asset holdings is a state variable, which makes the model completely intractable. While most papers take the first option, I pick the second that allows to study the interplay between liquidity, inequality, and incomplete markets <sup>10</sup>. I study a particular form of liquidity, bubbly liquidity: as bubbly and fundamental (bubble-less) steady states co-exist, it provides a natural framework to compare the dynamic response of the economy

<sup>10</sup>Exceptions include [Bilbiie and Ragot \(2017\)](#) and [Nistico \(2016\)](#) who analyze optimal monetary policy in a framework with limited heterogeneity and a positive supply of liquid assets, respectively money and stocks. Both papers abstract from cyclical inequality, the ZLB etc. Some papers study OLG models without idiosyncratic shocks, e.g [Gali \(2017\)](#), which is intuitively equivalent to the second option. The same remarks apply.

to various shocks depending on whether there's a positive long run supply of liquidity or not. Furthermore, how asset bubbles interact with the conduct of monetary policy is still an open question.

There are two representative agents,  $\{1, 2\}$ , and two states,  $i \in \{s, b\}$ , corresponding to two different income levels:  $Y_t^s > Y_t^b > 0$ . As in Woodford (1990), agents switch from one state to the other in a deterministic fashion at the end of every period, and they each start in a different state. If in period  $t$ , 1 is in state  $s$  and 2 is in state  $b$ , in period  $t + 1$ , 1 will be in state  $b$  and 2 in state  $s$ ; and so on. Knowing an agent's type at a given point in time is therefore a sufficient information to determine his identity: I'll index variables by state; the agent in state  $s$  in period  $t$  is the "saver", while the agent in state  $b$  is the "borrower".

Each agent is endowed with the following discounted utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^i{}^{1-\sigma}}{1-\sigma}$$

$i = s$  during even periods and  $i = b$  during odd periods for agent 1; the reverse is true for agent 2.

An agent of type  $i$  consumes,  $C_t^i$ , receives an exogenous (from his viewpoint) flow of income  $Y_t^i$  and chooses asset holdings – nominal bonds,  $L_t^i$  (expressed in real terms) and bubbles,  $H_t^i$ . His per-period budget constraint (expressed in real terms) is:

$$C_t^i + L_t^i + B_t H_t^i = Y_t^i + R_{t-1} L_{t-1}^j + B_t H_{t-1}^j, \quad i, j \in \{s, b\}, \quad j \neq i$$

Here,  $R_t$  is the (gross real) interest rate on the nominal bonds and  $B_t$  is the price of the bubbly asset.

I introduce heterogeneity and an endogenous distribution of income in a very simple way: the agent in state  $s$  receives a share  $\gamma_t$  of aggregate income:  $Y_t^s = \gamma_t Y_t$ , and the agent in state  $b$  receives what's left:  $Y_t^b = (1 - \gamma_t) Y_t$ ; importantly, saver's income share is endogenous as it is a function of the output gap,  $\gamma_t = \gamma(x_t)$ .

**Assumption 2** *Let  $x^{max} > 1 > x^{min} \geq 0$ ;  $\gamma(x)$  is continuously differentiable on  $[x^{min}, x^{max}]$ ; if  $x < x^{min}$ , then  $\gamma(x) = \gamma(x^{min}) \equiv \gamma^{min}$ ; if  $x > x^{max}$ , then  $\gamma(x) = \gamma(x^{max}) \equiv \gamma^{max}$ ;  $\gamma(1) \equiv \gamma^n \in (\frac{1}{2}, 1)$ ; and either:*

$$i. \theta \equiv \frac{d \log \gamma(x)}{d \log x} > 0 \text{ for all } x \in (x^{min}, x^{max}) \text{ and } 1 > \gamma^{max} > \gamma^{min} > \frac{1}{2}.$$

ii.  $\theta < 0$  for all  $x \in (x^{min}, x^{max})$  and  $\frac{1}{2} < \gamma^{max} < \gamma^{min} < 1$ .

In the rest of the paper, I'll implicitly assume that  $x_t \in (x^{min}, x^{max})$  for all  $t \geq 0$ .

The upper and lower bounds on  $\gamma(x)$  ensure that each individual receives a strictly positive income, but saver's is higher than borrower's:  $\frac{Y_t^s}{Y_t} > \frac{Y_t^b}{Y_t} > 0$ . When the economy produces the efficient level, the saver receives a flow of income equals to:  $\gamma^n y^n A_t$  and the borrower:  $(1 - \gamma^n) y^n A_t$ . However, the level and distribution of aggregate income vary along the cycle:

$$\frac{d \log Y_t^s}{d \log x_t} = 1 + \theta, \quad \frac{d \log Y_t^b}{d \log x_t} = 1 - \frac{\gamma}{1 - \gamma} \theta$$

Here,  $\theta$  is the elasticity of saver's income share with respect to the output gap. If  $\theta = 0$ , saver's income share is a-cyclical: agents in both states are equally affected by the output gap; if  $\theta > 0$ , saver's income share is pro-cyclical: saver's income rises more than output during expansions but shrinks more than output during contractions; if  $\theta < 0$ , saver's income share is counter-cyclical: saver's income rises and shrinks less than output. As state  $s$  is the "good", high-income, state and  $b$  the "bad", low-income, state, I'll say that (income) inequality is pro-cyclical if and only if  $\theta > 0$ ; a-cyclical if and only if  $\theta = 0$ ; counter-cyclical if and only if  $\theta < 0$ .

**Assumption 3**  $\theta \in (-1, \frac{1-\gamma}{\gamma})$

If inequality is either very strongly pro-cyclical,  $\theta > \frac{1-\gamma}{\gamma}$ , or very strongly counter-cyclical:  $\theta < -1$ , income in one state is negatively correlated with the output gap. Although it may well be verified in the data that some agents' income level rises during crisis and shrinks during booms, this doesn't make much sense in a model with two representative agents switching from one state to the other. Under assumption 3 above, the income *level* in both states rises with the output gap, even through the income *share* in one state increases while that in the other decreases <sup>11</sup>.

Since this paper doesn't seek to explain the distribution of income, but rather to study the macroeconomic implications of various distributions, I take the short road and avoid any kind of micro-foundations. But one should think of the function  $\gamma(x)$  as capturing various factors, including for example an unequal distribution of factor income (labor, capital, profit), a non-linear tax system, other forms of liquidity omitted from the model, etc. In particular, as [Werning \(2015\)](#) noticed, such an endogenous distribution naturally arises through the distribution of labor vs profit income in OLG-NK models.

<sup>11</sup> [Bilbiie \(2008\)](#) analyzes monetary policy in a NK model with two agents – who permanently remain in a given state – in which this assumption might be violated. He shows this gives rise to an "inverted" aggregate demand logic.

With complete financial markets, idiosyncratic income fluctuations are completely smoothed away. But in this economy, financial markets are dysfunctional: agents cannot issue any assets, that is, they cannot borrow from each other:

$$L_t^i \geq 0, \quad H_t^i \geq 0$$

Income shocks aren't insurable: this limits consumption-smoothing/risk-sharing in equilibrium and may therefore trigger a precautionary savings motive. However, as in period 0, there's a positive supply of bubbles,  $H_t = 1$ , distributed to the  $b$ -agent, some consumption-smoothing/risk-sharing remains possible if bubbles are traded in equilibrium. Indeed, the agent presently in the good state,  $s$ , can accumulate a buffer stock of assets that he'll liquidate in the bad state,  $b$ .

Each household maximizes his utility under the budget and financial constraints, taking the prices and his non-financial income as given. The Euler equations for bonds and bubbles are:

$$\begin{aligned} \left( C_t^{i-\sigma} - \beta \mathbf{E}_t \left\{ C_{t+1}^{j-\sigma} R_t \right\} \right) L_t^i &= 0, & C_t^{i-\sigma} &\geq \beta \mathbf{E}_t \left\{ C_{t+1}^{j-\sigma} R_t \right\}, & L_t^i &\geq 0 \\ \left( C_t^{i-\sigma} B_t - \beta \mathbf{E}_t \left\{ C_{t+1}^{j-\sigma} B_{t+1} \right\} \right) H_t^i &= 0, & C_t^{i-\sigma} B_t &\geq \beta \mathbf{E}_t \left\{ C_{t+1}^{j-\sigma} B_{t+1} \right\}, & H_t^i &\geq 0 \end{aligned}$$

Here,  $i \in \{s, b\}$  indexes the current state and  $j \in \{s, b\}$ ,  $j \neq i$  indexes the future state.

Consumption and asset holdings must also satisfy a transversality condition:

$$\lim_{T \rightarrow \infty} \beta^T \mathbf{E}_0 \left( \frac{C_T^j}{C_0^i} \right)^{-\sigma} \left( L_T^j + B_T H_T^j \right) \leq 0 \quad (2)$$

To study the non-linear model, I abstract from uncertainty: agents have perfect foresight. I re-introduce uncertainty in the log-linear model. Smaller-case letters denote potential output-adjusted variables,  $e_t \equiv \frac{E_t}{y^n A_t}$ , with the exceptions of saver's asset demand,  $a_t \equiv \frac{c_t^s - \gamma_t x_t}{\gamma_t x_t}$ , and the asset bubble,  $b_t \equiv \frac{B_t}{\gamma_t x_t y^n A_t}$ , which are normalized by saver's income.

Given the unequal distribution of aggregate income, in financial autarky, the agent in state  $s$  expects his consumption to fall between  $t$  and  $t + 1$ ; and the reverse is true of the agent in state  $b$ . Hence, in order to smooth consumption, the former wishes to save and the latter to borrow: I consider equilibria along which the saver accumulates assets – the financial frictions are non-binding<sup>12</sup> – whereas the borrower gets rid of his – the financial frictions are binding.

<sup>12</sup>As McKay et al. (2017), I consider the limiting case when the financial constraint allows to issue a positive,  $L_t^i \geq -\epsilon$ , but infinitesimal quantity of bonds,  $\epsilon \rightarrow 0^+$ .



Under this scenario, the borrower lives hand-to-mouth, that is, he consumes his entire income plus the proceed of his asset sales, if any:

$$c_t^b = (1 - \gamma_t)x_t + \gamma_t x_t b_t$$

Since he was a saver during the preceding period, he enters the low-income state owning the entire stock of bubbles,  $H_{t-1}^s = 1$ ; and because it isn't possible to borrow, it is also impossible to save in equilibrium,  $L_{t-1}^s = 0$ . Thus, the budget constraint implies the consumption function above. Higher asset prices raise agent  $b$ 's financial income and therefore consumption one-for-one.

As he used to be a borrower, the current saver enters the high-income state without any assets:  $H_{t-1}^b = L_{t-1}^b = 0$ ; and as he expects to be liquidity-constrained during the next period, he has a limited planning horizon. Iterating the budget constraint over  $t$  and  $t + 1$ :

$$c_t^s = \frac{1}{1 + \beta^{\frac{1}{\sigma}} R_t^{\frac{1}{\sigma} - 1}} \left( \gamma_t x_t + \frac{g}{R_t} (1 - \gamma_{t+1}) x_{t+1} \right)$$

Unlike the hand-to-mouth borrower, the consumption-smoothing saver isn't only sensitive to his current income, but also to his expected income and the interest rate. His marginal propensity to consume (MPC) out of wealth,  $(1 + \beta^{\frac{1}{\sigma}} R_t^{\frac{1}{\sigma} - 1})^{-1}$ , is strictly lower than one. Although he accumulates assets, asset prices don't affect his behavior. Indeed, using the Euler equations, the asset bubble must satisfy a standard no-arbitrage condition:

$$b_t = \frac{\gamma_{t+1} x_{t+1}}{\gamma_t x_t} \frac{g}{R_t} b_{t+1}, \quad b_t \geq 0 \quad (3)$$

Asset prices fluctuations redistribute saver's financial income between  $t$  and  $t + 1$ , but don't affect his level of wealth and therefore don't affect his consumption. His asset demand, measured as a share of his income, can be expressed as:

$$a_t = \frac{1}{1 + \beta^{\frac{1}{\sigma}} R_t^{\frac{1}{\sigma} - 1}} \left( \beta^{\frac{1}{\sigma}} R_t^{\frac{1}{\sigma} - 1} - \frac{g}{R_t} \frac{x_{t+1}}{x_t} \frac{1 - \gamma_{t+1}}{\gamma_t} \right) \quad (4)$$

The last term,  $\frac{1 - \gamma_{t+1}}{\gamma_t}$ , is the contribution of incomplete markets and cyclical inequality. As the agent faces idiosyncratic income shocks, his expected income growth doesn't match the growth rate of aggregate output, it is smaller:  $\frac{1 - \gamma_{t+1}}{\gamma_t} < 1$ . This introduces a precautionary savings motive: the *level* of asset demand is higher than that of the representative agent (RA) – I say that the

economy has a RA when  $\gamma_t = \frac{1}{2}$  and  $b_t = 0$  for all  $t \geq 0$ . However, if inequality is a-cyclical,  $\theta = 0$ , incomplete markets don't affect the *elasticity* of asset demand with respect to the output gap: the strength of the precautionary savings motive doesn't vary along the cycle.

But, if inequality is pro-cyclical,  $\theta > 0$ , the precautionary savings motive is also pro-cyclical: precautionary savings increase during expansions and decrease during contractions. Indeed, if  $\theta > 0$ , saver's current income rises more than output: he has stronger incentives to accumulate assets when output is high but lower incentives to accumulate assets when output is low. This makes asset demand more elastic with respect to the output gap. And the reverse is true when inequality is counter-cyclical,  $\theta < 0$ : the precautionary savings motive becomes counter-cyclical, which makes asset demand less elastic with respect to the output gap.

In equilibrium, saver's asset demand,  $a_t$ , must be equal to borrower's supply,  $b_t$  – both measured in terms of saver's income:

$$a_t = b_t \quad (5)$$

Although the borrower sells a fixed quantity of assets, the value of those assets is endogenously determined in equilibrium by the no-arbitrage condition. Thus, while I'll refer to (4) as to "saver's asset demand" or "asset demand", I'll refer to (3) as to "asset supply" or "borrower's asset supply". Of course, in this economy, the *net* aggregate asset demand is equal to zero in equilibrium:  $x_t - c_t = 0$ .

Finally, we must verify that agent  $b$  doesn't purchase any assets in equilibrium:

$$b_t \in [0, 1] \quad (6)$$

And bubbles cannot be traded at a negative price.

### 2.3 Monetary policy

The central bank sets the nominal interest rate according to a Taylor rule,

$$R_t \equiv \frac{\tilde{R}_t}{\Pi_{t+1}} = \frac{\max \{ R^n \pi_t^\phi, \underline{R} \}}{\pi_{t+1}}, \quad \underline{R} \leq 1 \leq g, \quad \phi \geq 0 \quad (7)$$

Here,  $\tilde{R}_t$  is the nominal interest rate – in the rest of the paper, I'll focus exclusively on the real interest rate,  $R_t$ ;  $R^n$  is the steady state natural interest rate – that is consistent with a zero output gap in steady state, i.e  $x = 1$  only if  $R = R^n$  – which is exogenous from the central bank's viewpoint;  $\underline{R} \equiv \frac{\tilde{R}}{\Pi^n}$  is the lower bound on the real interest rate (RLB): the lowest interest rate central bank

can deliver at the zero lower bound,  $\tilde{\mathbf{R}}$ , without violating its inflation target,  $\Pi^n \geq 1$ .

I'll say that the central bank is *active* when it follows the Taylor principle, i.e it sets  $\phi$  sufficiently high to determine inflation and output – but the Taylor principle isn't equivalent to  $\phi > 1$  as in the textbook model.

## 2.4 Equilibrium

An equilibrium is a sequence  $\{a_t, b_t, x_t, \pi_t, R_t\}$  such that (1), (2), (3), (4), (5), (6), (7) are satisfied for all  $t \geq 0$ ,  $\gamma_t = \gamma(x_t)$  and  $x_t \geq 0$ .

I summarize below all equations characterizing the dynamic of the economy,

$$\begin{aligned} a_t &= \frac{1}{1 + \beta^{\frac{1}{\sigma}} R_t^{\frac{1}{\sigma}-1}} \left( \beta^{\frac{1}{\sigma}} R_t^{\frac{1}{\sigma}-1} - \frac{g}{R_t} \frac{x_{t+1}}{x_t} \frac{1 - \gamma_{t+1}}{\gamma_t} \right) \\ b_t &= \frac{\gamma_{t+1} x_{t+1}}{\gamma_t x_t} \frac{g}{R_t} b_{t+1}, \quad b_t \in [0, 1) \\ a_t &= b_t \\ R_t &= \frac{\max \{ R^n \pi_t^\phi, \mathbf{R} \}}{\pi_{t+1}} \\ \pi_t &= 1 + \lambda(x_t^\psi - 1) \end{aligned}$$

Although this version of the AS curve is static, it is innocuous here: qualitatively similar results can be derived with the standard, dynamic, NKPC. In particular, up to a constant, all results related to the long run aren't affected. The important deviation from the basic model is the IS curve that obtains if we let  $a_t = b_t$  in the first equation. Here, through  $b_t$ , asset supply is endogenous; and as financial markets are incomplete and income inequality cyclical, there's an endogenous, cyclical, precautionary savings motive: this affects how asset demand responds to fluctuations in the output gap. It nests the textbook IS curve as a special case when  $b_t = 0$  and  $\gamma_t = \frac{1}{2}$  for all  $t \geq 0$ .

In the rest of the paper,  $\hat{e}_t \equiv \frac{E_t - E}{E}$  will denote log-deviation from steady state; the only exceptions are  $\hat{a}_t \equiv a_t - a$  and  $\hat{b}_t \equiv b_t - b$ ;  $\epsilon_t \equiv \frac{\beta_t - \beta}{\beta}$  will denote a discount rate shock and  $\zeta_t$  a monetary shock. If I drop the time index,  $\hat{e} \equiv d \log E$  will denote log-deviation from the previous steady state;  $\hat{a} \equiv da$  and  $\hat{b} \equiv db$ .

I'll sometimes use the abbreviations "PE" and "GE" respectively for partial equilibrium, i.e given asset prices and the output gap, and general equilibrium. When I'll refer to asset demand, asset

supply, or asset prices, I'll omit "as a share of saver's income", but it is an implicit normalization.

### 3 Zero long run liquidity

The first sub-section computes the long run output response to a permanent demand shock; the second studies the possible steady states, including secular stagnation; the third briefly discusses well-fare; the fourth and fifth analyze the short run dynamic – respectively determinacy and endogenous shocks to asset supply vs exogenous shocks to asset demand.

All proofs for this section can be found in appendix A.

#### 3.1 Long run response to a demand shock: Keynes vs the Neo-Fisherians

Imagine that households suddenly, and permanently, become more patient:  $\epsilon > 0$  and/or the central bank systematically over-estimates the natural interest rate:  $\zeta > 0$ . Taking convergence to a new steady state as granted, how are inflation and output affected in the long run? The answer, it turns out, might be quite counter-intuitive.

Since liquidity isn't valued, the supply of asset available to the saver is nil:  $b = 0$ . Thus, the financial markets clearing condition,  $a_t = b_t$ , implies that  $a = 0$  in steady state. Taking a log-linear approximation of saver's asset demand, (4), around a steady state with  $a = b = 0$ ,

$$\hat{a}_t = m(1 + \theta)\hat{x}_t - m\left(1 - \frac{\gamma}{1 - \gamma}\theta\right)E_t\hat{x}_{t+1} + \frac{m}{\sigma}(\epsilon_t + \hat{r}_t) \quad (8)$$

$$\text{Where } m \equiv \frac{\beta^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}}{1 + \beta^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}} \in (0, 1)$$

Here,  $m$  is saver's marginal propensity to save around a no-liquidity steady state.

In partial equilibrium, a discount rate shock lowers saver's desired consumption: asset demand dries up. Absent financial bubbles, the borrower consumes his entire income: the supply of assets is fixed and equals zero. Thus, the shock creates a disequilibrium in the financial and goods markets that must be reabsorbed in general equilibrium. Given asset supply, there are two potential equilibrating mechanisms in the financial markets: the interest rate and saver's income growth. Because of the substitution effect, saver's asset demand is increasing in the interest rate; and asset demand decreases with saver's expected income growth because of a consumption-smoothing motive – this second channel is reinforced or weakened by the precautionary savings motive discussed above.

I'll refer to those equilibrating mechanisms so as to the "interest rate channel" and the "income

growth channel"; I'll say that a channel/effect is positive (negative) if, given the other effect, it implies that asset demand is pro-cyclical, i.e the income growth effect is positive if and only if asset demand rises with the output gap when the interest rate is kept constant.

In a sticky-price economy, the interest rate channel is partially under central bank's control. Log-linearizing (7):

$$\hat{r}_t = \max\{r^n + \phi\hat{\pi}_t, \underline{r}\} - E_t\hat{\pi}_{t+1} + \zeta_t \quad (9)$$

$$\text{Where } r^n \equiv -\log \frac{R\pi}{R^n\pi\phi} \leq 0, \quad \underline{r} \equiv -\log \frac{R\pi}{R} \leq 0$$

Here,  $\phi = 0$  if the ZLB binds; the constants  $r^n$  and  $\underline{r}$  are the log-deviations of the nominal interest rate from the previous steady state: if the economy remains in the same monetary regime pre- and post-shock,  $r^n = 0 < \underline{r}$  – the ZLB doesn't bind – or  $r^n < 0 = \underline{r}$  – the ZLB binds; I made use of the AS curve, (1), to express the inflation rate as a function of the output gap:

$$\hat{\pi}_t = \kappa\hat{x}_t$$

$$\text{Where } \kappa \equiv \psi \frac{\lambda x^\psi}{1 + \lambda(x^\psi - 1)} \in (0, \psi)$$

The central bank keeps the interest rate constant as long as inflation isn't affected <sup>13</sup>:

$$\hat{r} = (\phi - 1)\kappa\hat{x} + \zeta$$

As the elasticity of asset demand with respect to the interest rate is fixed at:  $\frac{m}{\sigma} > 0$ , the sign as well as the magnitude of the interest rate channel is determined by the elasticity of the interest rate with respect to the output gap. If  $\phi > 1$ , the central bank implements a higher interest rate when inflation goes up. Since firms set higher prices when the output gap is high, the interest rate effect is positive: asset demand is increasing in the interest rate, which is it-self increasing in the output gap. If  $\phi < 1$ , however, the central bank lets the interest rate fall when inflation goes up: the interest rate effect becomes negative.

Now, let's turn to the income growth channel. While the interest rate channel is very standard, and at work in virtually all NK models, both at short and long time horizons, the textbook NK

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<sup>13</sup>I implicitly assume that the central bank fails to recognize the shock as permanent: it doesn't adjust its policy rule to accommodate the higher steady state natural interest rate. If it does, shocks don't affect the output gap if the ZLB doesn't bind.

model doesn't include an income growth effect in the long run. Indeed, the income growth channel is determined by the elasticity of asset demand with respect to saver's income growth, which is fixed at:  $m > 0$ , and that of saver's income growth with respect to the output gap. In turn, the output gap influences saver's income growth through both the growth rate of aggregate income,  $g \frac{x_{t+1}}{x_t}$ , as well as its distribution,  $\frac{1-\gamma_{t+1}}{\gamma_t}$ :

$$d \log \left\{ g \frac{x_{t+1}}{x_t} \frac{1-\gamma_{t+1}}{\gamma_t} \right\} = \left( 1 - \frac{\gamma}{1-\gamma} \theta \right) \hat{x}_{t+1} - (1+\theta) \hat{x}_t$$

But, in the long run, the growth rate of aggregate output is exogenous <sup>14</sup>. Hence, the long run income growth channel is entirely driven by the income distribution, that is, by the precautionary savings motive. Consequently, the income growth channel is muted in steady state if income inequality is a-cyclical:

$$\hat{a} = m \frac{\theta}{1-\gamma} \hat{x} + \frac{m}{\sigma} (\epsilon + \hat{r})$$

When inequality is cyclical,  $\theta \neq 0$ , the income distribution between the two states is endogenous. When inequality is pro-cyclical,  $\theta > 0$ , saver's steady state income growth increases with the output gap: the long run precautionary savings motive is pro-cyclical, and it is therefore high when the output gap is high. Thus, given the interest rate, the long run asset demand is pro-cyclical: it rises with the output gap because there's a positive income growth effect. And the reverse is true when inequality is counter-cyclical,  $\theta < 0$ : the long run precautionary savings motive is counter-cyclical, and this, given the interest rate, makes asset demand counter-cyclical: the income growth effect is negative.

Plugging (9) in (8), the long run asset demand curve can be expressed as:

$$\hat{a} = \Omega_1^\phi \hat{x} + \frac{m}{\sigma} (\epsilon + \zeta)$$

Where  $\Omega_1^\phi \equiv m \frac{\theta}{1-\gamma} + \frac{m}{\sigma} (\phi - 1) \kappa$

Here,  $\Omega_1^\phi$  is the long run elasticity of asset demand with respect to the output gap in general equilibrium when the central bank is  $\phi$ -active (the long run GE elasticity of asset demand). It is the sum of the interest rate effect,  $\frac{m}{\sigma} (\phi - 1) \kappa$  – keeping the distribution of income constant – and the income growth effect,  $m \frac{\theta}{1-\gamma}$  – keeping the interest rate constant.

<sup>14</sup>See [Benigno and Fornaro \(2017\)](#) for secular stagnation in a NK model with an endogenous long run aggregate income growth, but an exogenous distribution.

As bubbles aren't valued, borrower's asset supply is nil:

$$\hat{b} = 0$$

Without long run liquidity, the long run asset supply curve is fully inelastic with respect to the output gap, that is, it is horizontal in the frame  $(\hat{x}, \hat{b})$ ; the long run asset demand curve is increasing in the frame if and only if  $\Omega_1^\phi > 0$ ; it is otherwise decreasing. Equalizing asset supply and demand,  $\hat{a} = \hat{b}$ , the long run output response to a demand shock in general equilibrium is given by:

$$\hat{x} = -\frac{m}{\Omega_1^\phi}(\epsilon + \zeta) \quad (10)$$

As [Auclert and Rognlie \(2018\)](#) have shown in a different but related model <sup>15</sup>, the output gap response to a given shock  $e$  can be computed using the partial equilibrium elasticities of asset supply and demand to the shock – I'll speak of PE elasticities, as well as the general equilibrium elasticities with respect to the output gap – I'll speak of GE elasticities. Here, asset supply is exogenous; as asset demand's PE elasticity is positive, the sign of the output response is determined by asset demand's GE elasticity:

$$\hat{x} = -\frac{d\hat{a}}{\frac{d\hat{a}}{d\hat{x}}}e \quad (11)$$

I show below that this formula is valid at any time-horizon  $p$  if we replace the long run elasticities by the persistence- $p$  elasticities.

The standard NK model with a-cyclical inequality,  $\theta = 0$ , is fully driven by the interest rate channel in the long run:  $\Omega_1^\phi = \frac{m}{\sigma}(\phi - 1)\kappa$ . Following a negative demand shock that lowers aggregate consumption, equilibrium requires a fall in the interest rate to equalize asset demand back to zero. As long as the central bank is active,  $\phi > 1$ , this isn't particularly worrisome: the GE elasticity is positive, such that the model predicts a negative long run response of inflation and output. Indeed, the shock lowers aggregate consumption. Since prices are sticky, firms produce less and reduce their price. The resulting fall in the inflation rate induces the inflation-targeting central bank to cut its policy rate more than one-for-one with inflation. And finally, the lower interest rate restores equilibrium in the financial and goods markets.

But, if one wishes to study secular stagnation and the liquidity trap, i.e  $\phi = 0$ , troubles arise. Indeed, as the interest rate effect changes of sign, so does the GE elasticity of asset demand: a lower

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<sup>15</sup>They apply the formula below to a different kind of shocks – permanent shocks to stochastic process that drives the income distribution in their heterogeneous agents, Bewley-like, model – but the idea is the same.

interest rate requires a higher inflation rate and hence a higher output gap. Thus, if the economy converges in the long run, negative demand shocks raise inflation and output. This "Neo-Fisherian" paradox (Cochrane (2017), Schmitt-Grohe and Uribe (2017)) has proven exceedingly difficult to rationalize in terms of micro-economic behaviors, and also contradicts a large body of empirical studies.

To recover "conventional" multipliers, even in a permanent liquidity trap, one needs to make the NK model "truly" Keynesian in the long run. In the trap, the model predicts a long run negative response of the output gap if and only if the long run GE elasticity of asset demand is positive. Hence, the NK model needs a strongly positive long run income growth effect that dominates the interest rate effect in the liquidity trap: as firms produce less and set a lower inflation rate, asset demand must fall *despite* a higher interest rate. In this particular model, this is possible if and only if inequality is strongly pro-cyclical:  $\Omega_1^0 > 0 \Leftrightarrow \theta > \frac{1-\gamma}{\sigma}\kappa$ , which implies a large pro-cyclical precautionary savings motive.

Note, however, that the mere existence of a positive long run income growth effect can worsen the paradox if it isn't strong enough to make the GE elasticity of asset demand positive in the liquidity trap. Indeed, if  $\theta \in (0, \frac{1-\gamma}{\sigma}\kappa)$ , the pro-cyclical nature of asset demand at a given interest rate tends to amplify the output boom following a negative demand shock: as the GE elasticity of asset demand remains negative, but it is slightly lower (in absolute value) than when  $\theta = 0$ , a larger output response is necessary to clear the financial markets. Intuitively, given the interest rate, as output rises, asset demand also rises because of the stronger precautionary savings motive: this amplifies the initial disequilibrium between asset demand and asset supply. A contrario, counter-cyclical inequality,  $\theta < 0$ , cannot solve the Neo-Fisherian paradox because it implies a negative long run income growth effect, but it can mitigate the paradox.

The preceding analysis is true for any type of demand shock, including monetary or fiscal shocks: a lower interest rate peg or a lower government consumption depress inflation and output in the long run if and only if the long run GE elasticity of asset demand is strictly positive.

### 3.2 Steady states: secular stagnation or secular boom?

The lacking long run income growth channel has very strong implications for the NK model, in particular if one wants to study secular stagnation. As one can guess from the long run response to a demand shock, the textbook NK model predicts a secular boom rather than a secular stagnation



when aggregate demand is permanently depressed <sup>16</sup>. To recover a conventional secular stagnation rather than a secular boom, the NK model must be adjusted to include a strongly positive long run income growth effect.

A steady state is a solution of:

$$a = \frac{1}{1 + \beta^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}} \left( \beta^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1} - \frac{g}{R} \frac{1-\gamma}{\gamma} \right) = 0$$

Where the interest rate is given by:

$$R = \max \left\{ R^* \left( 1 + \lambda(x^\psi - 1) \right)^{\phi-1}, \underline{R} \left( 1 + \lambda(x^\psi - 1) \right)^{-1} \right\}$$

$$\text{Where } R^* \equiv \beta^{-1} \left( g \frac{1-\gamma^n}{\gamma^n} \right)^\sigma$$

In this model,  $R^*$  is the long run natural interest rate, consistent with financial markets clearing when production is efficient,  $x = x^n = 1$ . Because financial markets are incomplete, it generally doesn't match the natural interest rate of the textbook NK model – the two are equal if and only if  $\gamma^n = \frac{1}{2}$ . As I show in appendix C, the transversality condition (2) in the TANK model is nevertheless satisfied as long as the frictionless interest rate,  $R^{ra}$ , is above the growth rate of output in the long run – the same condition as in the textbook model.

**Assumption 4**  $R^{ra} \equiv \beta^{-1} g^\sigma > g$ .

Thus, it doesn't place any restriction on  $R^*$ , which can be below the growth rate of output,  $R^* < g$ , or even below the RLB,  $R^* < \underline{R}$  <sup>17</sup>. I'll say that aggregate demand is permanently depressed when the natural interest rate is below the RLB:  $\frac{\underline{R}}{R^*} > 1$ .

Slightly anticipating, in the TANK model without long run liquidity, the central bank is active if and only if  $\Omega_1^\phi > 0$  – see proposition 3 below.

**Proposition 1** *Assume that the supply of liquidity is nil,  $b = 0$ , and inequality isn't strongly procyclical,  $\Omega_1^0 < 0$ . Then:*

1. *If the economy isn't in secular stagnation,  $\frac{R}{R^*} \leq 1$ , it has at least one, and at most two, steady state(s).*

<sup>16</sup>If there's a steady state equilibrium. This conclusion is valid even if one doesn't interpret secular stagnation as a steady state, but rather as a very persistent, but not permanent, state. See the last sub-section, 3.5.

<sup>17</sup>The intuition is that agents continuously switch from a state with a high consumption growth,  $b$  state, to a state with a low consumption growth,  $s$ : on average, their stochastic discount factor is equal to that of the RA.

- i. There's a unique no-ZLB steady state; inflation is on target, output is efficient,  $x = \pi = 1$ .
  - ii. It co-exists with a ZLB steady state iff the central bank is active,  $\Omega_1^\phi > 0$ , and the RLB-interest rate gap isn't too low,  $\frac{R}{R^*} \in (\mathcal{R}_c^{\min}, 1)$ ; inflation and output are depressed,  $x, \pi < 1$ .
2. In secular stagnation,  $\frac{R}{R^*} > 1$ , there's at most one steady state, and the ZLB binds. It exists iff the central bank is passive,  $\Omega_1^\phi < 0$ , and the RLB-interest rate gap isn't too high,  $\frac{R}{R^*} \in (1, \mathcal{R}_c^{\max})$ ; inflation and output are booming,  $\pi, x > 1$ .

The conditions  $\Omega_1^0 < 0$  and either  $\Omega_1^\phi > 0$  or  $\Omega_1^\phi < 0$  are true for all  $x \in (x^{\min}, x^{\max})$ .

This proposition offers sufficient, but not necessary, conditions on  $\Omega_1^0$  and  $\Omega_1^\phi$ ; the lower/upper bounds  $\mathcal{R}_c^j, j \in \{\min, \max\}$  ensure that the steady state is located in the cyclical inequality region,  $x \in (x^{\min}, x^{\max})$ . As  $x^{\min}, x^{\max}$  can be set quite arbitrarily, this isn't a strong restriction.

Points 1.i and 1.ii aren't new to the NK literature: when aggregate demand isn't depressed, there's an efficient steady state; furthermore, if the central bank is active, there's also an expectations-driven liquidity trap steady state. However, the possible nonexistence of type 1. steady states and the secular boom steady state of point 2. have been overlooked. Indeed, assumption 4 rules out a permanent shortage of aggregate demand in the textbook model; but this isn't true in this model, or more generally in RANK or HANK models.

Given the mechanics of the textbook NK model, this secular boom is intuitive. Indeed, as the long run GE elasticity of asset demand is negative when  $\phi = 0$ , a higher level of output is necessary to reduce asset demand. There's shortage of aggregate demand when agents want to save way too much relative to the assets the economy can produce. Assuming that  $\theta = 0$ , a very low interest rate is necessary to clear the financial markets, which, because of an unadapted inflation target and the ZLB, the central bank cannot deliver. Given the RLB, a steady state exists if and only if the central bank is willing to let inflation rise without raising even more the nominal interest rate, i.e. if and only if it is passive. But a higher inflation rate requires a boom in output in order to provide firms with incentives to set higher prices. If  $\theta \neq 0, \theta < \frac{1-\gamma}{\sigma} \kappa$ , the cyclical nature of asset demand tends to amplify ( $\theta > 0$ ) or mitigate ( $\theta < 0$ ) the necessary output boom.

**Proposition 2** *Assume that the supply of liquidity is nil,  $b = 0$ , and inequality is strongly procyclical,  $\Omega_1^0 > 0$ . Then:*

- 1. *If the economy isn't in secular stagnation,  $\frac{R}{R^*} \leq 1$ , it has at least one, and at most two, steady state(s).*

- i. There's a unique no-ZLB steady state; inflation is on target, output is efficient,  $x = \pi = 1$ .*
  - ii. It co-exists with a ZLB steady state iff the central bank isn't active,  $\Omega_1^\phi < 0$ , and the RLB-interest rate gap isn't too low,  $\frac{R}{R^*} \in (\mathcal{R}_p^{min}, 1)$ ; inflation and output are booming,  $x, \pi > 1$ .*
2. In secular stagnation,  $\frac{R}{R^*} > 1$ , there's at most one steady state, and the ZLB binds. It exists iff the central bank is active,  $\Omega_1^\phi > 0$ , and the RLB-interest rate gap isn't too high,  $\frac{R}{R^*} \in (1, \mathcal{R}_p^{max})$ ; inflation and output are depressed,  $\pi, x < 1$ .

The conditions  $\Omega_1^0 > 0$  and either  $\Omega_1^\phi > 0$  or  $\Omega_1^\phi < 0$  are true for all  $x \in (x^{min}, x^{max})$ .

Note that  $\Omega_1^\phi < 0 < \Omega_1^0$  implies that the central bank puts a negative weight on inflation, i.e  $\phi < 0$ . I include this possibility to cover all cases, but in the rest of the paper, I'll stick to  $\phi \geq 0$ .

A necessary condition for the existence of a "conventional" secular stagnation steady state is the existence of a region where the model is Keynesian rather than Neo-Fisherian: the GE elasticity of asset demand has to be positive<sup>18</sup>. Intuitively, there's a strong shortage of aggregate demand when households want to save way too much relative to the assets the economy can produce. As the GE elasticity of asset demand is positive, this surplus of savings is re-absorbed in general equilibrium through an endogenous fall in output. Output and inflation are depressed because the Neo-Fisherian paradox doesn't materialize.

More generally, propositions 1 and 2 jointly develop a general framework to analyze the steady states of a NK model. There are four possibilities to consider, depending on whether the two following conditions are satisfied: (a) the long run GE elasticity of asset demand is strictly positive out of the liquidity trap; (b) the sign of the long run GE elasticity of asset demand doesn't change in the liquidity trap.

When (b) isn't satisfied, the model has multiple steady states. To understand this result, let's assume that (a) is satisfied, and the economy starts in the efficient steady state but agents expect output to fall permanently. Under (a), if agents don't expect the ZLB to bind, this shock reduces asset demand in PE; it doesn't affect asset supply. Thus, in GE, output must adjust up to clear the financial markets. As this violates agents' initial expectations, this type of sunspots isn't possible. If agents expect the ZLB to bind, they must anticipate a lower nominal interest rate with respect to the efficient steady state. Thus, asset demand is affected by two shocks in PE: the initial shock

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<sup>18</sup>It is possible to show that, if prices become fully rigid below a certain level, i.e  $\pi = \max\{1 + \lambda(x^\psi - 1), \pi^{min}\}$ , where  $\pi^{min} > 0$  is a constant, then there's a conventional secular stagnation steady state when  $\frac{R}{R^*} > 1$ , and it is the unique steady state: one doesn't need to assume that  $\Omega_1^0 > 0$  for all levels of output.

to agents' output expectations as well as a monetary shock. Under (a) and (b), both shocks reduce asset demand and therefore imply an output boom in GE. Again, this is ruled out. But, under (a) and non-(b), the first shock raises asset demand whereas the second reduces it. If the latter effect dominates, i.e if agents don't expect output to fall "too much": the endogenous shock isn't "too large", asset demand decreases in PE. Hence, a slightly lower level of output is necessary to restore equilibrium in the financial markets: agents' initial expectations are validated.

When (b) isn't satisfied, the model also cannot handle large negative demand shocks (to either the fundamentals or expectations), i.e it cannot generate a secular stagnation steady state nor a secular boom. Again, let's assume that (a) is satisfied, and the economy starts in the efficient steady state. A large negative demand shock greatly increases asset demand in PE. Under (a), output falls in GE until the ZLB binds. However, because the shock is large, it isn't sufficient yet: asset demand must decrease further. Under (a) and (b), output continues to fall until the GE effect on asset demand perfectly offsets the PE effect of the shock. Under (a) and non-(b), however, a large negative demand shock implies a large output boom in the liquidity trap. But, as this boom pushes the economy out of the trap, this cannot be an equilibrium because the economy is back at square one.

Finally, when (a) isn't satisfied, the predictions of the model are symmetric but of opposite sign. Under non-(a) and non-(b), there's a secular boom rather than a secular stagnation when aggregate demand is permanently depressed; under non-(a) and (b), a liquidity trap steady state is driven by expectations of a permanent output boom rather than expectations of a permanent fall in output.

I generalize those results to temporary shocks in sub-section 3.5 below.

### 3.3 Steady states: well-fare

As both agents continuously switch from one state to the other, we can take the sum of the discounted utility over two periods in steady state (saver + borrower) as a well-fare index:

$$\mathcal{W}(x, b) \equiv \frac{(\gamma(1-b)x)^{1-\sigma} + \beta(g(1-\gamma(1-b)x))^{1-\sigma}}{1-\sigma} \quad (12)$$

Evaluating the well-fare function in steady state when liquidity isn't valued:

$$\frac{d\mathcal{W}(x, 0)}{dx} = (\gamma x)^{-\sigma} \left\{ \gamma(1+\theta) + \frac{(1-\gamma)g}{R} \left(1 - \frac{\gamma}{1-\gamma}\theta\right) \right\}$$

Under assumption 3, given asset prices, well-fare is strictly increasing in the output gap. Hence, the efficient steady state Pareto-dominates the bad-expectations-driven liquidity steady state and the secular stagnation steady state<sup>19</sup>. But it is Pareto-dominated by the good-expectations-driven liquidity trap steady state and the secular boom.

### 3.4 Determinacy

I consider the log-linear dynamic around a steady state without liquidity,  $b = 0$  and  $R^n = R^*$ . I'll assume that the economy remains in the vicinity of this steady state such that  $x_t \in (x^{min}, x^{max})$  for all  $t \geq 0$ , and the ZLB remains either binding or non-binding forever:  $r^n = \underline{r} = 0$ .

Taking a log-linear approximation of  $a_t$  around a steady state with  $a = 0$ , (8), and plugging in the log-linear Taylor rule, (9):

$$\hat{a}_t = \Omega_0^\phi \hat{x}_t - (\Omega_0^\phi - \Omega_1^\phi) E_t \hat{x}_{t+1} + \frac{m}{\sigma} (\epsilon_t + \zeta_t) \quad (13)$$

Where  $\Omega_0^\phi \equiv m(1 + \theta) + \frac{m}{\sigma} \phi \kappa$

Here,  $\Omega_0^\phi$  is the short run GE elasticity of asset demand when the central bank is  $\phi$ -active. Since a fully temporary shock doesn't affect expectations, under assumption 3, both the short run interest rate and income growth effects are positive and reinforced with respect to their long run counterparts:  $\Omega_0^\phi > \max\{\Omega_1^\phi, 0\}$  for all  $\phi \geq 0$ . The elasticity of asset demand with respect to future income equals the difference between the short run and long run GE elasticities: asset demand is more sensitive to current income than to future income if and only if the long run GE elasticity is strictly positive.

Although liquidity isn't valued in steady state, asset prices can fluctuate in the short run. Log-linearizing (3),

$$\hat{b}_t = \frac{g}{R} E_t \hat{b}_{t+1}$$

Around a bubble-less steady state, the dynamic of the asset price bubble doesn't depend on the fundamentals of the economy. However, endogenous fluctuations are possible if and only if the bubbly steady state exists,  $\frac{R}{g} < 1$ . Hence, we can treat the sequence of asset prices  $\{\hat{b}_t\}$  as endogenous shocks affecting the economy: as Galí (2017) noticed, this provides a theory of demand

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<sup>19</sup>When assumption 3 isn't verified, well-fare might be decreasing in the output gap. Indeed, the output gap affects both the distribution and level of consumption. If a lower output gap redistributes income in such a way that consumption-smoothing is improved, well-fare may rise despite a lower aggregate consumption.

shocks.

Indeed, imposing equilibrium in the financial markets,  $\hat{a}_t = \hat{b}_t$  and re-arranging:

$$\hat{x}_t = \left(1 - \frac{\Omega_1^\phi}{\Omega_0^\phi}\right) E_t \hat{x}_{t+1} + \frac{\hat{b}_t - \frac{m}{\sigma}(\epsilon_t + \zeta_t)}{\Omega_0^\phi}$$

Given expectations, higher asset prices inflate borrower's asset supply. As the short run GE elasticity of asset demand is strictly positive:  $\Omega_0^\phi > 0$ , this shock tends to raise the output gap.

Given a sequence of shocks and asset prices  $\{\epsilon_t, \zeta_t, \hat{b}_t\}$  that satisfy their respective laws of motion, and assuming that the economy remains in the vicinity of the considered steady state, the entire dynamic is summarized by the equation above. It has a unique solution  $\{x_t\}$  if and only if  $\frac{\Omega_1^\phi}{\Omega_0^\phi} \in (0, 2)$ : (i) the GE elasticity mustn't change of sign in the short vs long run; (ii) the short run elasticity must be twice as large as the long run's, in absolute value. Under assumption 3, the short run elasticity is positive, and (i) implies (ii). Thus, determinacy requires a strictly positive long run GE elasticity:  $\Omega_1^\phi > 0$ .

**Proposition 3** *The asset bubble is determinate if and only if  $\frac{R}{g} > 1$ . The output gap is determinate if and only if  $\Omega_1^\phi > 0$ , which is equivalent to:*

$$\phi > \phi^* \equiv 1 - \frac{\sigma \frac{\theta}{1-\gamma}}{\kappa}$$

Here,  $\phi^* < 0$  if and only if  $\theta > \frac{1-\gamma}{\sigma} \kappa$ ;  $\phi^* < 1$  if and only if  $\theta > 0$ .

Local determinacy around a steady state is equivalent to the economy being Keynesian along this particular steady state. The condition  $\phi > \phi^*$  is a modified Taylor principle that nests the usual Taylor principle:  $\phi > 1$ , as a special case when inequality isn't cyclical:  $\theta = 0$ . As long as inequality is pro-cyclical, the Taylor principle is weakened; determinacy is even possible in the liquidity trap,  $\phi = 0$ , if inequality is strongly pro-cyclical. A contrario, when inequality is counter-cyclical, the Taylor principle must be strengthened.

The key is how inequality and incomplete markets affect the long run GE elasticity of asset demand. From (13), when inequality is pro-cyclical, the pro-cyclical long run precautionary savings motive makes asset less sensitive to the future level of output than to the current level of output: the IS curve becomes discounted. The reverse is true whenever inequality is counter-cyclical: the long run income growth effect is negative, the IS curve becomes compounded.

Those conclusions were also reached by numerous recent papers in different-but-related models,

e.g, [Sterk and Ravn \(2017\)](#), [Bilbiie \(2018a\)](#), [Acharya and Dogra \(2018\)](#), although those papers don't relate this result to the long run GE elasticity of asset demand. In particular, [Bilbiie \(2018a\)](#) provides a similar condition in terms of the elasticity of aggregate demand to news. This elasticity to news can be computed using the short and long run GE elasticities of asset demand:  $v^\phi = \frac{\Omega_0^\phi - \Omega_1^\phi}{\Omega_0^\phi}$ . Under the intuitive assumption that  $\Omega_0^\phi > 0$  for all  $\phi \geq 0$ , this condition is equivalent to  $\Omega_1^\phi > 0$ : this provides another simple condition to test determinacy in the data.

### 3.5 Multipliers and puzzles in/out of the liquidity trap

I consider a bubble-less steady state around which the asset price bubble is indeterminate,  $R^* < g$ . Following the ZLB literature starting with [Eggertsson and Woodford \(2003\)](#), I'll assume that all shocks follow a two-states Markov chain,  $\xi_t \in \{\xi^h, 0\}$ : if  $\xi_t = \xi^h$ , then  $\epsilon_t = \epsilon^h$ ,  $\zeta_t = \zeta^h$  and  $\hat{b}_t = \hat{b}^h \in (0, 1)$ ; if  $\xi_t = 0$ ,  $\epsilon_t = \zeta_t = \hat{b}_t = 0$ . The economy starts in state  $h$  and converges back to steady state with a probability  $p = \frac{R}{g} \in (0, 1)$  at the beginning of each period; state  $l$  is absorbing.

Given the Markovian structure of the shocks, I'll focus on Minimum State Variable (MSV) solutions: the endogenous variables depend on the minimum number of state variables. Here, a MSV solution requires that output only depends on the shocks:  $\hat{x}_t^h = \hat{x}^h$  and  $x_t^l = 0$  for all  $t \geq 0$  – if  $\Omega_1^\phi > 0$  and  $\Omega_p^0 > 0$ , this MSV solution is also the unique equilibrium. Thus, state  $l$  corresponds to the steady state.

In state  $h$ , saver's asset demand can be expressed as:

$$\hat{a}^h = \Omega_p^\phi \hat{x}^h + \frac{m}{\sigma} (\epsilon^h + \zeta^h + r)$$

$$\text{Where } \Omega_p^\phi \equiv (1 - p)\Omega_0^\phi + p\Omega_1^\phi$$

Here,  $\Omega_p^\phi$  is the persistence- $p$  GE elasticity of asset demand when the central bank is  $\phi$ -active; the  $p$ -elasticity is a linear combination of the short run and long run elasticities. Thus, as shock become more and more persistent, the long run GE elasticity of asset demand becomes increasingly important.

**Lemma 1**  $\frac{d\Omega_p^\phi}{dp} < 0$  and  $\frac{d\Omega_p^\phi}{d\phi} > 0$  for all  $p \in [0, 1]$  and  $\phi \geq 0$ .

In partial equilibrium, the discount rate and monetary shocks induce the saver to postpone consumption: both shocks raise saver's asset demand. However, those shocks don't affect the consumption of the financially-constrained borrower: they leave asset supply unaffected. Higher asset

prices don't affect saver's asset demand because he's smoothing consumption across periods, but induce the borrower to consume more: asset supply increases,  $\hat{b}_t = \hat{b}^h$ .

Equalizing asset demand and supply,  $\hat{a}^h = \hat{b}^h$ , we can compute the equilibrium output gap:

$$\hat{x}^h = \frac{\hat{b}^h - \frac{m}{\sigma}(\epsilon^h + \zeta^h + r)}{\Omega_p^\phi}$$

Where  $r = \underline{r}$  and  $\phi = 0$  if the ZLB binds;  $r = r^n$  otherwise.

From an aggregate point of view, a bubbly shock is similar to a discount rate or a monetary shock: the micro-level implications are different, but the macro-level impact on aggregate demand and the output gap is the same. Endogenous shocks to asset prices therefore provide an interesting way to explain financial without relying on some exogenous shock to a structural parameter <sup>20</sup>.

The ZLB binds in state  $h$  if and only if  $\hat{x}^h < \frac{\underline{r} - r^n}{\phi\kappa}$ . If we let

$$\xi_t \equiv -\hat{b}_t + \frac{m}{\sigma}(\epsilon_t + \zeta_t), \quad \xi^z \equiv \frac{\Omega_p^0 r^n - \Omega_p^\phi \underline{r}}{\phi\kappa}$$

We can re-write the no-ZLB or ZLB conditions as follows:

$$\text{no-ZLB: } \frac{\xi^h}{\Omega_p^\phi} < \frac{\xi^z}{\Omega_p^\phi}, \quad \text{ZLB: } \frac{\xi^h}{\Omega_p^\phi} > \frac{\xi^z}{\Omega_p^0}$$

Here,  $\xi_t$  is the "aggregate" demand shock:  $\xi_t > 0$  is a negative shock: the PE effect on asset demand is higher than the PE effect on asset supply;  $\xi_t < 0$  is a positive shock.

**Proposition 4 1.** *If the shock isn't too large,  $\xi^h < \xi^z$ ,*

- i. There's a unique MSV solution iff  $\Omega_p^0 > 0$ , the ZLB doesn't bind and  $\frac{d\hat{x}^h}{d\xi^h} < 0$ .*
  - ii. There are two MSV solutions iff  $\Omega_p^\phi > 0 > \Omega_p^0$ , the ZLB doesn't bind and  $\frac{d\hat{x}^h}{d\xi^h} < 0$ ; or the ZLB binds and  $\frac{d\hat{x}^h}{d\xi^h} > 0$ .*
2. *If the shock is large,  $\xi^h > \xi^z$ ,*

- i. There's a unique MSV solution iff either: (a)  $\Omega_p^0 > 0$ , the ZLB binds and  $\frac{d\hat{x}^h}{d\xi^h} < 0$ ; or (b)  $\Omega_p^\phi < 0$ , the ZLB binds and  $\frac{d\hat{x}^h}{d\xi^h} > 0$ .*
- ii. There is no MSV solution iff  $\Omega_p^\phi > 0 > \Omega_p^0$ .*

<sup>20</sup>Bouillot (2017) formalizes the "asset bubble - secular stagnation" theory of the sub-prime crisis advanced by Summers (2013) and Krugman (2013).



This proposition generalizes the results of propositions 1 and 2 to temporary shocks – although I assumed here that  $\phi \geq 0$ , which rules out one possibility:  $\Omega_p^\phi < 0 < \Omega_p^0$ .

To rule out a multiplicity or in-existence of stationary equilibria, the persistence- $p$  GE elasticity must not change of sign when the economy enters the liquidity trap. Furthermore, to get conventional multipliers, the long run GE elasticity must be strictly positive. The intuitions are similar to those developed at the end of section 3.2.

By virtue of lemma 1, a sufficient condition to rule out all puzzling predictions is a strictly positive long run GE elasticity of asset demand:  $\Omega_1^0 > 0$ ; a necessary condition is:  $\Omega_p^0 > 0$ ; both imply that the MSV is unique, and it is further the unique equilibrium of the model. The determinacy condition,  $\Omega_p^0 > 0$ , can be written as:

$$\phi_p^* \equiv p - \sigma \frac{(1-p)(1+\theta) + p \frac{\theta}{1-\gamma}}{\kappa} < 0$$

As  $\frac{d\phi_p^*}{dp} > 0$  and  $\phi_1^* = \phi^*$ , determinacy is possible with Markov shocks even if it isn't possible when  $p = 1$ : counter-cyclical inequality doesn't necessarily leads to indeterminacy at the ZLB if shocks aren't too persistent.

Finally, when inequality is pro-cyclical and the economy Keynesian, the output response to a demand shock is muted with respect to the textbook model. Since pro-cyclical inequality raises the GE elasticity of asset demand,  $\frac{d\Omega_p^\phi}{d\theta} > 0$ , a lower adjustment in output is necessary to clear the financial markets following a demand shock. As Bilbiie (2018a) puts it, there's a trade-off between puzzles and amplifications.

To conclude this section on the model without long run liquidity, as Bilbiie (2018a) underlined, there's a single parameter that drives the all puzzling predictions of the NK model. In this paper, I provide an explanation of this phenomenon as well as an interpretation for this parameter. The model makes conventional and intuitive predictions if and only if the long run GE elasticity of asset demand is strictly positive in normal times as well as in the liquidity trap. If this condition isn't met, multiplicity of equilibria and puzzles are guaranteed as shocks become more and more persistent.

Cyclical liquidity is a particular way to "cure" the NK model; in particular, the results of Michailat and Saez (2018) and Michau (2018) show that utility functions that include wealth as an argument are also capable of making the long run GE elasticity of asset demand strictly positive

in the trap. And there surely are a lot of other micro-foundations that would do the trick.

I'll show in the next section that, including an endogenous supply of liquidity can restore all puzzles despite a positive long run GE elasticity of asset demand: when asset supply is endogenous, both the long run GE elasticities of asset demand and supply become important.

## 4 Positive long run liquidity

The first sub-section computes the long run output response to a permanent demand shock; the second studies the possible bubbly steady states; the third briefly discusses well-fare; the fourth and fifth analyze the short run dynamic – respectively determinacy and liquidity as an automatic (de-)stabilizer.

Given a shock  $e_t$ , I'll say that liquidity

- Is counter-cyclical if and only if  $\frac{d\hat{x}_t}{de_t} \frac{d\hat{b}_t}{de_t} < 0$ .
- Acts as an automatic stabilizer if and only if:  $|\frac{d\hat{x}_t}{de_t}|_{b>0} < |\frac{d\hat{x}_t}{de_t}|_{b=0}$ .

Liquidity is counter-cyclical if and only if it rises as a share of saver's income when output falls – or the other way around; liquidity acts as an automatic stabilizer if and only if the output response in general equilibrium is dampened with respect to the no-liquidity equilibrium, implicitly taking the primitives of the economy as given (the only difference being  $b = 0$  or  $b > 0$ ).

All proofs for this section can be found in appendix B.

### 4.1 Long run response to a demand shock

Let's re-do the analysis of the long run impact of a permanent demand shock when liquidity is valued. Taking a log-linear approximation of  $a_t$  around a steady state with  $a = b > 0$ ,

$$\hat{a}_t = m^b(1 + \theta)\hat{x}_t - m^b(1 - \frac{\gamma}{1 - \gamma}\theta)E_t\hat{x}_{t+1} + \frac{m^b(1 + \chi)}{\sigma}\epsilon_t + \frac{m^b(1 + (1 - \sigma)\chi)}{\sigma}\hat{r}_t \quad (14)$$

$$\text{Where } m^b \equiv \frac{\beta^{\frac{1}{\sigma}}g^{\frac{1}{\sigma}-1}}{1 + \beta^{\frac{1}{\sigma}}g^{\frac{1}{\sigma}-1}} - b \in (0, m), \quad \chi \equiv b\frac{1 - m^b - b}{m^b} > 0$$

It is possible to show that  $m^b = (1 - b)(1 - \gamma)$  and  $\chi = \frac{b\gamma}{1 - \gamma}$ .

When liquidity is valued, the saver accumulates assets that he'll liquidate as soon as he becomes a borrower: some consumption-smoothing/risk-sharing is allowed in steady state. Thus, borrower's consumption rises above his non-financial income:  $\frac{c^b}{(1 - \gamma)x} = 1 + \chi$ . Since the saver expects to be a

borrower during the next period, this lowers his marginal propensity to save,  $m^b < m$ , inflates the substitution effect of interest rate or discount rate shocks,  $+\chi$ , and also generates a new income effect that decreases saver's sensitivity to the interest rate,  $-\sigma\chi$ .

**Assumption 5**  $\frac{1}{\sigma} \geq 1 - \left(\frac{R^*}{g}\right)^{\frac{1}{\sigma}}$ .

Assumption 5 is equivalent to  $1 + (1 - \sigma)\chi \geq 0$ : the partial equilibrium elasticity of asset demand with respect to the interest rate is positive. This assumption can be relaxed to:  $1 + (1 - \sigma)\chi \geq -\frac{b}{2m^b}$ , hence allowing a negative elasticity, without affecting the results.

Plugging in the log-linear Taylor rule, the steady state asset demand is:

$$\hat{a} = \Upsilon_1^\phi \hat{x} + \frac{m^b(1 + \chi)}{\sigma} \epsilon + \frac{m^b(1 + (1 - \sigma)\chi)}{\sigma} \zeta$$

$$\text{Where } \Upsilon_1^\phi \equiv m^b \frac{\theta}{1 - \gamma} + \frac{m^b(1 + (1 - \sigma)\chi)}{\sigma} (\phi - 1)\kappa$$

$\Upsilon_1^\phi$  is the long run GE elasticity of asset demand when the central bank is  $\phi$ -active. It includes a new term with respect to  $\Omega_1^\phi$  that accounts for the higher substitution and income effects of the interest rate.

From (3), if liquidity is valued in the long run, asset bubbles must offer the same rate of return as the bond,  $R = g$ :

$$\hat{b} = -b \lim_{p \rightarrow 1} \frac{\hat{r}}{1 - p}$$

As the shock doesn't affect  $g$ , if  $\hat{r} > \hat{g} = 0$ , the bond offer a higher rate of return: the bubble implodes,  $\hat{b} \rightarrow -\infty$  and its price converges to zero,  $b = 0$ ; if  $\hat{r} < \hat{g} = 0$ , the bond offers a lower rate of return: the bubble explodes,  $\hat{b} \rightarrow +\infty$ , but as it violates saver's budget constraint, it cannot be part of an equilibrium. Therefore, if liquidity remains valued in the long run, the interest rate must remain constant:  $\hat{r} = 0$ .

Using the Taylor rule, (9), the condition  $\hat{r} = 0$  is equivalent to:

$$\hat{x} = -\frac{1}{(\phi - 1)\kappa} \zeta \tag{15}$$

Borrower's asset supply is infinitely elastic – it is vertical in the frame  $(\hat{x}, \hat{b})$ .

Given the output gap, asset prices adjust to clear the financial markets,  $\hat{a} = \hat{b}$ :

$$\hat{b} = \frac{m^b(1 + \chi)}{\sigma} \epsilon - \frac{m^b \frac{\theta}{1 - \gamma}}{(\phi - 1)\kappa} \zeta \tag{16}$$

Endogenous liquidity adds another equilibrating mechanism in the financial markets. Given saver's asset demand, if there's a disequilibrium in the financial markets, it can be reabsorbed by fluctuations in asset prices: asset supply adjusts to asset demand. Alternatively, given saver's consumption, asset prices adjust such that borrower's consumption clears the goods market. Quite intuitively, endogenous liquidity interacts with the precautionary savings motive: as households can accumulate assets, they need less self-insurance.

$$\hat{x} = -\frac{\frac{d\hat{a}}{de} - \frac{d\hat{b}}{de}}{\frac{d\hat{a}}{d\hat{x}} - \frac{d\hat{b}}{d\hat{x}}}e, \quad \hat{b} = \frac{d\hat{a}}{d\hat{x}}\hat{x} + \frac{d\hat{a}}{de}e \quad (17)$$

(17) is a generalized version of (11) when asset supply is endogenous. With asset bubbles, in the long run, both the PE and GE elasticities of asset supply tend to infinity,  $\frac{d\hat{b}}{d\hat{x}} = -b \lim_{p \rightarrow 1} \frac{\phi - p}{1 - p}$  and  $\frac{d\hat{b}}{d\hat{c}} = -b \lim_{p \rightarrow 1} \frac{1}{1 - p}$ , whereas those of asset demand are finite. Thus, the equilibrium output gap doesn't depend on asset demand; given the equilibrium output gap, assets prices adjust to asset demand.

**Proposition 5** *Assume that  $\phi > 1$  and  $\min\{\Upsilon_1^\phi, \Omega_1^\phi\} > 0 > \Omega_1^0$ . In the long run,*

1. *Against discount rate shocks, assets prices*

- i. *Are counter-cyclical.*
- ii. *Act as automatic stabilizers.*

2. *Against monetary shocks, asset prices*

- i. *Are counter-cyclical if and only if  $\theta < 0$ .*
- ii. *Act as automatic stabilizers if and only if  $\theta < 0$ .*

The conditions  $\phi > 1$  and  $\min\{\Omega_1^\phi, \Upsilon_1^\phi\} > 0 > \Omega_1^0$  imply that both steady states under consideration are determinate when the ZLB doesn't bind – see propositions 3 and 7, but indeterminate otherwise: the long run output response has the same sign around both.

In the NK model, cycles are generated by shocks that lead to fluctuations of asset demand relative to asset supply in partial equilibrium. According to (17), the output gap is determined by the ratio of the PE to the GE elasticities. Consequently, liquidity acts as an automatic stabilizer in general equilibrium if and only if it attenuates the PE effect relative to the GE response.

If a given shock affects asset demand but not asset supply in PE, the equilibrium output gap equals zero; the equilibrium asset prices adjust so as to fully "absorb" the PE effect on asset demand:

asset prices are counter-cyclical and stabilizing. To understand this result, let's analyze the GE response step by step. In PE, the shock raises asset demand: given asset prices, the equilibrium output gap and interest rate fall on impact to clear the financial markets <sup>21</sup>:

$$\hat{x}_{|\hat{b}=0} = -\frac{\frac{m^b(1+\chi)}{\sigma}}{\Upsilon_1^\phi} \epsilon, \quad \hat{r}_{|\hat{b}=0} = -(\phi - 1) \frac{\frac{m^b(1+\chi)}{\sigma}}{\Upsilon_1^\phi} \epsilon$$

But, as the central bank has cut the interest rate to fight the deflationary pressures, the interest rate is below the growth rate of output:  $\hat{r} < \hat{g} = 0$ . Hence, asset prices immediately jump such that the no-arbitrage condition is satisfied. Since the PE effect of the shock on asset supply is nil,  $\hat{r} = (\phi - 1)\hat{x}$ , the no-arbitrage condition is satisfied,  $\hat{r} = 0$ , if and only if the equilibrium output gap doesn't change:  $\hat{x} = 0$ . Thus, asset prices increase until the PE effect on asset demand is fully offset, (16). This "irrelevance" of demand shocks can be generalized to other shocks and other forms of liquidity, as long as: (i) the long PE and GE elasticities of asset supply tends to infinity; and (ii) the shock doesn't affect asset supply in PE. Against discount rate shocks, aggregate consumption doesn't change, but it is redistributed: in the new steady state, saver's consumption has fallen while borrower's has increased in the same proportions due to higher asset prices. At the *macro*-level, endogenous liquidity fully insures the economy against fluctuations in asset demand.

If a given shock affects both asset demand and asset supply in PE, the equilibrium output gap is determined so as to fully offset the PE effect of the shock on the interest rate; given the equilibrium output gap, asset prices adjust to clear the financial markets. Liquidity is stabilizing and asset prices counter-cyclical if and only if inequality is also counter-cyclical; liquidity is otherwise destabilizing and asset prices pro-cyclical. Again, let's analyze the GE response step by step. In PE, the shock raises asset demand: given asset prices, the equilibrium output gap fall on impact to clear the financial markets:

$$\hat{x}_{|\hat{b}=0} = -\frac{\frac{m^b(1+(1-\sigma)\chi)}{\sigma}}{\Upsilon_1^\phi} \zeta, \quad \hat{r}_{|\hat{b}=0} = \frac{m \Omega_1^1}{\Upsilon_1^\phi} \zeta$$

As the shock also affects the interest rate, whether it increases or decreases depend on the cyclical nature of inequality. If  $\theta = 0$ ,  $\Omega_1^1 = 0$ , asset demand is a-cyclical: given asset supply, equilibrium in the financial markets requires that the interest rate returns to its pre-shock value,  $\hat{r}_{|\hat{b}=0} = 0$ . If  $\theta > 0$ , however,  $\Omega_1^1 > 0$ , asset demand is pro-cyclical: the interest rate response remains limited:  $\hat{r}_{|\hat{b}=0} > 0$ . To satisfy the no-arbitrage condition, asset prices fall until the interest rate returns to

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<sup>21</sup>I am implicitly assuming that the long run GE elasticity of asset demand is positive:  $\Upsilon_1^\phi > 0$ . If it isn't, the intuition is the same, although output and the interest rate initially rise, and then fall as asset prices adjust up.

its pre-shock value:  $\hat{r} = 0$ , i.e until the equilibrium output gap is given by (15) and asset prices by (16). Asset prices have fallen: they're pro-cyclical and therefore amplify the PE effect of the shock in general equilibrium. And the reverse is true when inequality is counter-cyclical: the interest rate falls,  $\hat{r}|_{\hat{b}=0} < 0$ , such that asset prices are counter-cyclical and dampen the PE effect of the shock: asset prices increase when asset demand is high. Liquidity doesn't provide any insurance at the macro-level against shocks to asset supply: it absorbs only the PE effect on asset demand, but not the PE effect on asset supply.

This may sound paradoxical: whereas if liquidity isn't valued, pro-cyclical inequality is stabilizing, it might be destabilizing when liquidity is valued. However, that interpretation is only partially correct. If liquidity is endogenous and inequality pro-cyclical, the monetary multiplier is greater than without liquidity, but it is exactly the same as if liquidity wasn't valued and inequality was a-cyclical. Thus, the correct interpretation is rather that liquidity shuts down the income growth channel in the long run: as there's a positive supply of liquidity, households don't need to self-insure, i.e the precautionary savings motive vanishes in the long run – or, at least, fluctuations in the precautionary savings motive vanish.

If the dynamic is indeterminate, i.e if  $\phi < 1$  and  $\Omega_1^\phi < 0$ , another form of the Neo-Fisherian paradox materializes that is difficult to rationalize in terms of micro-economic behaviors: the equilibrium level of output rises when there's a positive monetary shock. Indeed, the condition  $\hat{r} = 0$  requires an interest rate cut. As the central bank doesn't implement a positive correlation between inflation and the interest rate, this implies a boom in output that raises inflation expectations.

Adding asset bubbles in the NK model with cyclical inequality makes it behave quite similarly to the textbook model. Indeed, in the textbook model, the long run asset demand curve is infinitely elastic with respect to the interest rate; here, the long run asset supply curve is infinitely elastic: both specifications imply that  $R = R^n$ , i.e  $\hat{r} = 0$ , in the long run. This isn't a general result but rather a limiting case: if, for example, the asset under consideration pays dividends, and those dividends are correlated with the output gap, then the asset supply curve is no longer infinitely elastic.

## 4.2 Bubbly steady states: existence.

A bubbly steady state is a solution of <sup>22</sup>:

$$a = \frac{1}{1 + \beta^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}} \left( \beta^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1} - \frac{g}{R} \frac{1-\gamma}{\gamma} \right) = b, \quad b > 0$$

Where the output gap solves:

$$g = \max \left\{ g \left( 1 + \lambda(x^\psi - 1) \right)^{\phi-1}, \underline{R} \left( 1 + \lambda(x^\psi - 1) \right)^{-1} \right\}$$

Here, the natural interest rate is equal to the growth rate of output,  $R^n = g$ ; I plugged the conditions  $R = g$  and  $a = b$  in saver's asset demand, (4), and the Taylor rule, (7).

The condition  $R = g$ , the long run asset supply, is isomorphic to the long run NK IS curve when the natural interest rate of the textbook model equals  $g$ . The sole difference is the interpretation of this condition: in the textbook NK model, the long run asset demand curve is infinitely elastic with respect to the interest rate; in this model, while the long run asset demand curve isn't, the long run asset supply curve is infinitely elastic.

Thus, as in the textbook NK model, the last equation has either: no solutions if and only if  $\phi < 1$ ; or two solutions if and only if  $\phi > 1$ , the efficient,  $x = x^n = 1$ , and the liquidity trap,  $x = x^b \equiv \left( 1 + \frac{\underline{R}-1}{\lambda} \right)^{\frac{1}{\psi}} < 1$ . This multiplicity is similar to that of the textbook NK model with a-cyclical inequality: those two steady states share the same interest rate, which equals the natural interest rate, but they've different inflation rates and output gaps. Furthermore, asset bubbles prevent any shortage of aggregate demand under the weak condition that the growth rate of the economy is higher than the RLB:  $g > \underline{R}$  – which I assumed.

Given the output gap, the IS curve determines the steady state bubble:

$$b = \frac{1 - \left( \ell(x) \frac{R^*}{g} \right)^{\frac{1}{\sigma}}}{1 + \left( \frac{R^a}{g} \right)^{\frac{1}{\sigma}}}$$

Where  $\ell(x) \equiv \left( \frac{\gamma^n}{1 - \gamma^n} \frac{1 - \gamma}{\gamma} \right)^\sigma$

And the equilibrium bubble must satisfy:  $b \in [0, 1]$ , i.e.  $\frac{R^*}{g} \ell(x) < 1$ . This last restriction sometimes rules out indeterminacy: the steady level of output, and hence asset demand, isn't necessarily

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<sup>22</sup>We can verify that under assumption 4, bubbles don't violate the transversality condition, (2) – see appendix C.

compatible with a positive supply of assets when inequality is cyclical. Here,  $\ell(x)$  can be understood as a liquidity premium relative to the efficient steady state. This liquidity premium is decreasing in the output gap,  $\frac{d \log \ell(x)}{d \log x} = -\sigma \frac{\theta}{1-\gamma} < 0$ , if and only if inequality is pro-cyclical. When  $\theta > 0$ , saver's asset demand raises with the output gap. Thus, a higher output gap leads to a higher asset demand. To be consistent with  $R = g$ , asset prices must rise correspondingly to satisfy the higher asset demand.

**Proposition 6** *Assume that the growth rate of output is higher than the RLB,  $\frac{R}{g} \leq 1$ .*

1. *The economy has at most two bubbly steady states, one ZLB and one no-ZLB.*
2. *There's a no-ZLB bubbly steady state if and only if  $\frac{R^*}{g} < 1$ . Inflation is on target and output is efficient,  $\pi = x = 1$ .*
3. *There's a ZLB bubbly steady state if and only if  $\ell(x^b) \frac{R^*}{g} < 1$  and the central bank is active,  $\phi > 1$ . Inflation and output are depressed,  $\pi, x = x^b < 1$ .*

Abstracting from either the ZLB, sticky prices or cyclical inequality – such that, in steady state, either  $x = 1$  and  $\ell(1) = 1$ , or  $\ell(x) = 1$  for all  $x$  – we recover the standard results of [Tirole \(1985\)](#): there's a unique value of  $b$  consistent with  $R^b = g$  in the long run, and  $b > 0$  if and only if the bubble-less efficient steady state is dynamically inefficient,  $R^* < g$ .

But in the present version of the NK model, the bubble-less interest rate is a function of the output gap,  $R = \ell(x)R^*$ . Thus, both conditions  $R < g = R^b$  are conditional on a given level of output, which differs from one monetary regime to the other. In particular, if inequality is counter-cyclical, the economy features a bigger asset bubble at the ZLB, and rational bubbles are possible even if the bubble-less equilibrium is dynamically efficient – in the sense that  $R^* > g > \ell(x^b)R^*$ : the lower level of output (relative to the efficient steady state) raises saver's asset demand. A contrario, if inequality is pro-cyclical, the economy features a smaller asset bubble at the ZLB, and dynamic inefficiency is a necessary but not sufficient condition for the existence of rational bubbles: the lower level of output decreases saver's asset demand.

This stand in stark contrast with other monetary models with asset bubbles (e.g [Gali \(2017\)](#), [Asriyan et al. \(2015\)](#)) which conclude that monetary policy doesn't affect the existence condition for bubbles<sup>23</sup>. Here, through  $\underline{R}$ , the central bank does determine the long run level of output at the

<sup>23</sup>One exception is [Wang et al. \(2017\)](#) where the inflation target of the central bank affects the existence condition, but through a completely different mechanism: their model doesn't include the ZLB nor cyclical inequality, but rather a very particular kind of financial friction that makes inflation non-neutral even in the long run.



ZLB, and therefore whether asset demand is high or low. If there's a sufficiently high asset demand in steady state, asset bubbles become possible. As the level of output is lower when  $\underline{R}$  is high, a high  $\underline{R}$  promotes the emergence of asset bubbles when inequality is counter-cyclical, but it prevents the emergence of asset bubbles when income is pro-cyclical. Thus, if inequality is pro-cyclical, quite counter-intuitively, by committing to a high RLB, the central bank can prevent the existence of the "bad" bubbly steady state.

Finally, at the ZLB, the central bank sets an interest rate peg equal to  $\underline{R}$ . Thus, if one forgets Taylor rules but rather assumes that the central bank sets  $R = \underline{R}$  in the long run, there's a at most one bubbly steady state, and the central bank can determine whether it exists or not.

### 4.3 Bubbly steady states: well-fare

The well-fare function is still given by (12). Evaluating well-fare in steady state when liquidity is valued, one can show that the steady state bubble maximizes well-fare given the output gap:

$$\frac{\mathcal{W}(x, b)}{db} = 0$$

Asset bubbles allow agents to better smooth consumption: asset bubbles partially "cure" a form of dynamic inefficiency which arises because of the binding financial frictions. Thus, asset bubbles allow full-insurance against idiosyncratic income shocks in the long run, which explains the disappearance of the precautionary savings motive. Indeed, whatever the output gap, the equilibrium bubble sets saver's consumption share at:

$$\gamma(1 - b) = \frac{\left(\frac{R^r a}{g}\right)^{\frac{1}{\sigma}}}{1 + \left(\frac{R^r a}{g}\right)^{\frac{1}{\sigma}}}$$

This implies that if we take a "fundamental" steady state  $x \in (0, 1]$  and consider a bubbly steady state  $(x, b)$ , then:  $\mathcal{W}(x, b) > \mathcal{W}(x, 0)$  for all  $x$ . In particular, the bubbly version of the efficient steady state dominates the fundamental version:  $\mathcal{W}(1, b) > \mathcal{W}(1, 0)$ .

It is also possible to compare the two types of bubbly steady states. The well-fare function of a bubbly steady state can be written as a function of  $x$  only:

$$\mathcal{W}^b(x) \equiv \left(\frac{\left(\frac{R^r a}{g}\right)^{\frac{1}{\sigma}}}{1 + \left(\frac{R^r a}{g}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma} + \beta \left(g \frac{\left(\frac{R^r a}{g}\right)^{\frac{1}{\sigma}}}{1 + \left(\frac{R^r a}{g}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma} x^{1-\sigma}$$

Here, the output gap doesn't affect the distribution of consumption – because the bubble adjusts – but only the level of consumption. Clearly, a higher output gap delivers a higher consumption level and hence a higher well-fare: the efficient bubbly steady state Pareto-dominates the ZLB bubbly steady state. Hence, the bubbly efficient steady state, if it exists, Pareto-dominates all other steady states if we abstract from the peculiar secular boom and good-expectations-driven liquidity trap.

#### 4.4 Determinacy and asset bubbles

I consider the log-linear dynamic around a steady state with liquidity,  $b > 0$  and  $R = R^n = g$ . I'll assume that the economy remains in the vicinity of this steady state such that  $x_t \in (x^{min}, x^{max})$  for all  $t \geq 0$ , and the ZLB remains either binding or non-binding forever:  $r^n = \underline{r} = 0$ .

Taking a log-linear approximation of  $a_t$  around a steady state with  $a = 0$ , (11), and plugging in the log-linear Taylor rule, (9):

$$\hat{a}_t = \Upsilon_0^\phi \hat{x}_t - (\Upsilon_0^\phi - \Upsilon_1^\phi) \text{E}_t \hat{x}_{t+1} + \frac{m^b(1+\chi)}{\sigma} \epsilon_t + \frac{m^b(1+(1-\sigma)\chi)}{\sigma} \zeta_t \quad (18)$$

Where  $\Upsilon_0^\phi \equiv m^b(1+\theta) + \frac{m^b(1+(1-\sigma)\chi)}{\sigma} \phi \kappa$

Here,  $\Upsilon_0^\phi$  is the short run GE elasticity of asset demand when the central bank is  $\phi$ -active. Under assumption 3 and 5:  $\Upsilon_0^\phi > \max\{\Upsilon_1^\phi, 0\}$  for all  $\phi \geq 0$ .

Log-linearizing (3) and plugging in the log-linear Taylor rule:

$$\hat{b}_t = -\Gamma_0^\phi \hat{x}_t + (\Gamma_0^\phi - \Gamma_1^\phi) \text{E}_t \hat{x}_{t+1} + \text{E}_t \hat{b}_{t+1} - b(\zeta_t + r) \quad (19)$$

Where  $\Gamma_p^\phi \equiv b(1-p)(1+\theta) + b(\phi-p)\kappa$

Here,  $-\Gamma_p^\phi$  is the GE elasticity of asset supply at time horizon  $p$ , *keeping asset prices expectations constant*: the GE elasticity including endogenous asset prices expectations is equal to  $-\frac{\Gamma_p^\phi}{1-p}$ , which tends to infinity as shocks become permanent. Under assumption 3, it is possible to show that:  $\Gamma_0^\phi > \max\{\Gamma_1^\phi, 0\}$  for all  $\phi \geq 0$ .

Around a bubbly steady state:  $R^b = g$ , this equation isn't discounted. The asset price bubble depends on the fundamentals of the economy through the interest rate: given expectations, an interest rate cut tends to inflate asset prices – to satisfy the no-arbitrage condition. Output also enters the pricing equation because the asset bubble is measured as share of saver's income.

Given a sequence of shocks  $\{\epsilon_t, \zeta_t\}$  that satisfy their respective laws of motion, and assuming

that the economy remains in the vicinity of the considered steady state, the entire dynamic is summarized by a 2-dimensional system:

$$\begin{aligned}\hat{x}_t &= \left(1 - \frac{\Upsilon_1^\phi + \Gamma_1^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi}\right) \mathbb{E}_t \hat{x}_{t+1} + \frac{1}{\Upsilon_0^\phi + \Gamma_0^\phi} \mathbb{E}_t \hat{b}_{t+1} - \frac{m^b(1+\chi)}{\Upsilon_0^\phi + \Gamma_0^\phi} \epsilon_t - \frac{m^b(1+(1-\sigma)\chi) + b}{\Upsilon_0^\phi + \Gamma_0^\phi} (\zeta_t + r) \\ \hat{b}_t &= \left(\frac{\Upsilon_1^\phi + \Gamma_1^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} \Gamma_0^\phi - \Gamma_1^\phi\right) \mathbb{E}_t \hat{x}_{t+1} + \left(1 - \frac{\Gamma_0^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi}\right) \mathbb{E}_t \hat{b}_{t+1} + \frac{m^b(1+\chi) \Gamma_0^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} \epsilon_t + \left(\frac{m^b(1+(1-\sigma)\chi) + b}{\Upsilon_0^\phi + \Gamma_0^\phi} \Gamma_0^\phi - b\right) (\zeta_t + r)\end{aligned}$$

I imposed equilibrium in the financial markets,  $\hat{a}_t = \hat{b}_t$  and re-arranged.

**Proposition 7** *The asset bubble is determinate if and only if  $\Gamma_1^\phi = b(\phi - 1)\kappa > 0$ . The output gap is determinate if and only if  $\Upsilon_1^\phi + \Gamma_0^\phi > 0$ . Both conditions are equivalent to:*

$$\phi > \phi^b \equiv \max\left\{1, 1 - \frac{m^b \frac{\theta}{1-\gamma} + b(1 + \theta + \kappa)}{\left(\frac{m^b(1+(1-\sigma)\chi)}{\sigma} + b\right)\kappa}\right\}$$

Furthermore,  $\phi^b < \phi^*$  if and only if  $\theta < 0$ .

Around a bubbly steady state, the asset bubble isn't discounted: to avoid endogenous fluctuations of asset prices, the central bank must follow the usual Taylor principle:  $\phi > 1$  <sup>24</sup>.

The modified, no-liquidity, Taylor principle can be weakened when inequality is counter-cyclical, but it must be strengthened if inequality when pro-cyclical. As we've discussed in section 4.1, the asset bubble neutralizes the income growth affect in the long run. If  $\theta > 0$ , this effect was stabilizing: asset bubbles make determinacy more difficult to reach; if  $\theta < 0$ , asset bubbles neutralize a destabilizing force in the long run and therefore make determinacy easier to reach.

#### 4.5 Liquidity as an automatic (de-)stabilizer?

I'll study two types of demand shocks: monetary and discount rate shocks. While they're observationally equivalent if  $b = 0$ , they've different implications around a steady state with a positive supply of liquidity. Both shocks follow the same kind of Markov chains as in the previous section, and I focus on MSV solutions.

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<sup>24</sup>This isn't particularly surprising nor interesting because it is clearly not robust to other specifications for asset prices: a rational bubble has a unit root whenever the interest rate isn't responsive to economic conditions. This wouldn't be the case if one includes dividends or bubble creation or taxes etc.

In the short run, saver's asset demand and borrower's asset supply can be expressed as:

$$\begin{aligned}\hat{a}^h &= \Upsilon_p^\phi \hat{x}^h + \frac{m^b(1+\chi)}{\sigma} \epsilon^h + \frac{m^b(1+(1-\sigma)\chi)}{\sigma} (\zeta^h + r) \\ \hat{b}^h &= -\frac{\Gamma_p^\phi}{1-p} \hat{x}^h - \frac{b}{1-p} (\zeta^h + r)\end{aligned}$$

In partial equilibrium, both shocks raise saver's asset demand; the discount rate shock doesn't affect the interest rate, hence it doesn't affect borrower's supply; the monetary shock, however, increases the rate of return of the bond relative to the bubble and therefore reduces asset prices.

**Lemma 2**  $\frac{d\Upsilon_p^\phi}{dp}, \frac{d\Gamma_p^\phi}{dp} < 0$  and  $\frac{d\Upsilon_p^\phi}{d\phi}, \frac{d\Gamma_p^\phi}{d\phi} > 0$  for all  $p \in [0, 1]$  and  $\phi \geq 0$ .

Imposing equilibrium in the financial markets,  $\hat{b}^h = \hat{a}^h$ , we can solve for the time-invariant output gap and asset prices:

$$\begin{aligned}\hat{x}^{h,b} &= -\frac{\frac{m^b(1+\chi)}{\sigma} \epsilon^h - \frac{m^b(1+(1-\sigma)\chi)}{\sigma} + \frac{b}{1-p}}{\Upsilon_p^\phi + \frac{\Gamma_p^\phi}{1-p}} (\zeta^h + r) \\ \hat{b}^h &= \frac{\frac{m^b(1+\chi)}{\sigma} \Gamma_p^\phi}{\Upsilon_p^\phi + \frac{\Gamma_p^\phi}{1-p}} \epsilon^h + \left( \frac{\frac{m^b(1+(1-\sigma)\chi)}{\sigma} + \frac{b}{1-p}}{\Upsilon_p^\phi + \frac{\Gamma_p^\phi}{1-p}} \frac{\Gamma_p^\phi}{1-p} - \frac{b}{1-p} \right) (\zeta^h + r)\end{aligned}$$

Where  $r = \underline{r}$  and  $\phi = 0$  if the ZLB binds,  $r = r^n = 0$  otherwise.

The output multiplier of both shocks is negative if and only if the difference between the GE elasticities of asset demand and supply is positive:  $\Upsilon_p^\phi + \frac{\Gamma_p^\phi}{1-p} > 0$  – where  $\phi = 0$  if the ZLB binds. As a discount shock doesn't affect the no-arbitrage condition directly, the asset bubble is counter-cyclical if and only if its GE elasticity is negative:  $-\frac{\Gamma_p^\phi}{1-p} < 0$ ; against monetary shocks, the asset bubble is counter-cyclical if and only if the general equilibrium response,  $-\frac{\Gamma_p^\phi}{1-p} \hat{x}^h$  is greater than the PE effect,  $-\frac{b}{1-p}$ .

If we let:

$$\xi_t^b \equiv \frac{m^b(1+\chi)}{\sigma} \epsilon_t + \left( \frac{m^b(1+(1-\sigma)\chi)}{\sigma} + \frac{b}{1-p} \right) \zeta_t, \quad \xi^{z,b} \equiv \frac{(\Upsilon_p^0 + \frac{\Gamma_p^0}{1-p})r^n - (\Upsilon_p^\phi + \frac{\Gamma_p^\phi}{1-p})\underline{r}}{\phi\kappa}$$

We can re-write the no-ZLB or ZLB conditions as follows:

$$\text{no-ZLB: } \frac{\xi^{h,b}}{\Upsilon_p^\phi + \frac{\Gamma_p^\phi}{1-p}} < \frac{\xi^{z,b}}{\Upsilon_p^\phi + \frac{\Gamma_p^\phi}{1-p}}, \quad \text{ZLB: } \frac{\xi^{h,b}}{\Upsilon_p^0 + \frac{\Gamma_p^0}{1-p}} > \frac{\xi^{z,b}}{\Upsilon_p^0 + \frac{\Gamma_p^0}{1-p}}$$

If we replace  $\xi^h$  and  $\xi^z$  by  $\xi^{h,b}$  and  $\xi^{z,b}$ , as well as  $\Omega_p$  by  $\Upsilon_p^0 + \frac{\Gamma_p^0}{1-p}$ , we recover the exact same results as in sub-section 3.5.

**Proposition 8** 1. *If the shock isn't too large,  $\xi^{h,b} < \xi^{z,b}$ ,*

- i. There's a unique MSV solution iff  $\Upsilon_p^0 + \frac{\Gamma_p^0}{1-p} > 0$ , the ZLB doesn't bind and  $\frac{d\hat{x}^{h,b}}{d\xi^{h,b}} < 0$ .*
- ii. There are two MSV solutions iff  $\Upsilon_p^\phi + \frac{\Gamma_p^\phi}{1-p} > 0 > \Upsilon_p^0 + \frac{\Gamma_p^0}{1-p}$ , the ZLB doesn't bind and  $\frac{d\hat{x}^{h,b}}{d\xi^{h,b}} < 0$ ; or the ZLB binds and  $\frac{d\hat{x}^{h,b}}{d\xi^{h,b}} > 0$ .*

2. *If the shock is large,  $\xi^{h,b} > \xi^{z,b}$ ,*

- i. There's a unique MSV solution iff either: (a)  $\Upsilon_p^0 + \frac{\Gamma_p^0}{1-p} > 0$ , the ZLB binds and  $\frac{d\hat{x}^{h,b}}{d\xi^{h,b}} < 0$ ; or (b)  $\Upsilon_p^\phi + \frac{\Gamma_p^\phi}{1-p} < 0$ , the ZLB binds and  $\frac{d\hat{x}^{h,b}}{d\xi^{h,b}} > 0$ .*
- ii. There is no MSV solution iff  $\Upsilon_p^0 + \frac{\Gamma_p^0}{1-p} < 0$ .*

There are three main differences with respect to sub-section 3.5.

First, the GE elasticity of asset demand is no longer a sufficient statistics to determine whether the economy is Keynesian or Neo-Fisherian. Going back to (17), it is clear that, even if the GE elasticity of asset demand is negative, but that of asset supply is also negative and even more so, the economy remains Keynesian. Intuitively, the economy is Keynesian whenever a higher output gap tends to increase asset demand more than asset supply: the relative variation of asset demand with respect to asset supply is important, not the absolute variation of each.

Second, the condition  $\Upsilon_p^0 + \frac{\Gamma_p^0}{1-p} > 0$  cannot be true when  $p \rightarrow 1$ . Indeed, as shocks become more and more persistent, the GE elasticity of asset supply becomes increasingly important relative to that of asset demand. Thus, asset bubbles rule out the possibility of secular stagnation, even if it is understood as a persistent state rather than a permanent. This isn't problematic if shocks don't affect the interest rate in PE, but it has troubling implications if they do. Indeed, looking at the expressions for  $\xi_t^b$  and  $\xi^{z,b}$ , it is clear that: (i) as discount rate shocks become persistent, there's a strong adjustment in asset prices that insures the economy against the shocks such that the ZLB never binds; (ii) large and persistent monetary shocks lead to a complete breakdown of the economy: there isn't any stationary equilibrium.

Third, in general, the condition  $\Upsilon_p^0 + \frac{\Gamma_p^0}{1-p} > 0$  is only necessary, but not sufficient, for determinacy – see appendix B; the determinacy condition:  $\min\{(1-p)\Upsilon_0^0 + p\Upsilon_p^0 + \Gamma_0^0, (1-p)\Upsilon_p^0 + \Gamma_p^0\} > 0$ , can

be written as:

$$\phi_p^b \equiv \max \left\{ p^2 - \frac{p^2 m^b \frac{\theta}{1-\gamma} + (1-p^2)m^b(1+\theta) + b(1+\theta+p^2\kappa)}{\left(\frac{m^b(1+(1-\sigma)\chi)}{\sigma} + b\right)\kappa}, p - \frac{pm^b \frac{\theta}{1-\gamma} + (1-p)(1+\theta)(m^b + \frac{b}{1-p})}{\left(\frac{m^b(1+(1-\sigma)\chi)}{\sigma} + \frac{b}{1-p}\right)\kappa} \right\} < 0$$

Here,  $\frac{d\phi_p^b}{dp} > 0$  and  $\phi_1^b = \phi^b$ : this condition is verified if the ZLB doesn't bind around a steady state that satisfies the condition in proposition 7.

The "fully-fledged" NK model that includes *both* cyclical inequality and liquidity behaves in a similar manner as the textbook model, including the same paradoxes. Intuitively, while in the textbook NK model, puzzling properties arise because the long run GE asset demand can be negative, puzzling properties arise in the fully-fledged model because the long run elasticity of asset supply becomes increasingly large relative to that of asset demand *and* it can become positive.

Now, let's compare the impact of demand shocks on output depending on whether liquidity is valued or not. I consider two steady states, both efficient, but one is bubbly:  $b > 0$ , and the other is fundamental:  $b = 0$ . Those two economies share the same primitives, except that  $R = R^n = R^*$  and  $b = 0$  in one,  $R = R^n = g$  and  $b > 0$  in the other. As  $R$  doesn't enter the formula for the multipliers, this difference is innocuous – we may even assume that those steady states don't share the same  $R^*$ , i.e  $R^* > g$  in the fundamental,  $R^* < g$  in the bubbly:  $\beta$  doesn't enter the multipliers either. The monetary regime is the same in both economies; furthermore,  $\phi > \max\{\phi^*, \phi^b\}$  and  $\max\{\phi_p^*, \phi_p^b\} < 0$ : both steady states are locally determinate, and there's a unique MSV solution.

**Proposition 9** *Assume that  $\phi > \max\{\phi^b, \phi^*\}$  and  $\max\{\phi_p^b, \phi_p^*\} < 0$ .*

1. *If the shocks are discount rate's, the asset bubble*

- i. *Is counter-cyclical if and only if:  $\phi > \phi_p^{\epsilon,cc} \equiv p - \frac{(1-p)(1+\theta)}{\kappa}$ .*
- ii. *Acts as an automatic stabilizer if and only if:  $\phi > \phi_p^{\epsilon,as} \equiv \phi^{\epsilon,cc} + \chi \frac{m^b}{\frac{b}{1-p} - \chi m^b} \frac{p \frac{\theta}{1-\gamma}}{\kappa}$ .*

2. *If the shocks are monetary, the asset bubble*

- i. *Is counter-cyclical if and only if:  $0 > \phi_p^{\zeta,cc} \equiv -\frac{1-\sigma}{\sigma}(1+\chi)(1-p)(1+\theta) + p \frac{\theta}{1-\gamma}$ .*
- ii. *Acts as an automatic stabilizer if only if:  $0 > \phi_p^{\zeta,as} \equiv \phi_p^{\zeta,cc} + \chi \left(1 + \frac{m^b(1+\chi)}{\frac{b}{1-p} - \chi m^b}\right) p \frac{\theta}{1-\gamma}$ .*

Where  $\phi = 0$  if the ZLB binds in equilibrium;  $\frac{b}{1-p} - \chi m^b > 0$ .

Under assumption 3 and 5, as  $\max\{\phi_p^{\epsilon,cc}, \phi_p^{\epsilon,as}\} < p$  for all  $p \in [0, 1]$ , 1.i and 1.ii are implied by  $\phi > \max\{\phi^b, \phi^*\}$  when the ZLB doesn't bind; when the ZLB binds, 1.i and 1.ii are implied by  $\max\{\phi_p^*, \phi_p^b\} < 0$  if 2.i and 2.ii are respectively satisfied.

If *either* inequality is a-cyclical:  $\theta = 0$ , shocks fully temporary:  $p = 0$ , or fully permanent:  $p = 1$ , 1.i and 1.ii are equivalent to each other, and 2.i and 2.ii are also equivalent to each other: the bubble acts as an automatic stabilizer if and only if it is counter-cyclical. But those equivalences break down when inequality becomes cyclical *and* shocks somewhat persistent but not permanent. Instead, when inequality is counter-cyclical, counter-cyclical asset prices is a sufficient, but not necessary, condition for liquidity to be stabilizing the economy; when inequality is pro-cyclical, counter-cyclical asset prices is a necessary, but not sufficient, condition liquidity to be stabilizing.

Conditional on determinacy, a crisis in the NK model is the endogenous response of the economy to shocks that increase asset demand more than asset supply in PE. Quite intuitively, if the difference between asset demand and asset supply is strongly pro-cyclical, the GE output response to shocks will be very limited: a small output adjustment up is sufficient to provoke a large rise in asset demand over asset supply. Including a positive long run supply of liquidity affects the PE and GE elasticities of both asset demand and supply. But, if asset demand is a-cyclical given the interest rate,  $\theta = 0$ , *and* asset supply is a-cyclical in general equilibrium,  $\phi = \phi^{e,cc}$  where  $e$  is the shock under consideration, liquidity is neutral: the higher/lower PE elasticities are perfectly offset by the higher/lower PE elasticities. We also recover this neutrality, whatever  $\theta$ , if shocks are fully temporary:  $p = 0$ .

To understand deviations from this intuitive neutrality benchmark when  $\theta \neq 0$  and  $p \in (0, 1)$ , remember that, in the presence of liquidity, the long run income growth effect is muted: as they're fully insured against the endogenous component of idiosyncratic income shocks, households don't have an endogenous precautionary savings motive. If either  $p = 0$  or  $\theta = 0$ , the long run income growth effect is neutralized anyway, with or without endogenous liquidity. However, it kicks in in the economy with exogenous liquidity when  $p\theta \neq 0$ . If  $\theta > 0$ , liquidity tends to neutralize a stabilizing force: without liquidity, asset demand would be pro-cyclical. Thus, liquidity is stabilizing if and only if it is counter-cyclical enough to make up for this loss. A contrario, if  $\theta < 0$ , liquidity neutralizes a destabilizing force. Thus, it can be stabilizing despite being pro-cyclical.

There's one important difference between shocks that affects both asset demand and supply in PE, and those that only affect asset demand: whether the monetary regime interacts with asset prices or not. If shocks don't affect the interest rate in PE, a very aggressive monetary policy makes

asset prices counter-cyclical and stabilizing; however, if the central bank isn't aggressive enough, it makes asset prices pro-cyclical and destabilizing. If the shocks also affect the interest rate in PE, the central bank cannot affect the cyclical nature of asset prices, nor their stabilizing properties: although monetary policy affects the no-liquidity / liquidity multipliers, it doesn't affect the difference between those multipliers; furthermore, monetary policy affects the magnitude of asset prices adjustment in GE, but not the sign.

Those results are related to [Werning \(2015\)](#)'s, who's shown in a different-but-related model that monetary shocks are dampened when *either* liquidity is counter-cyclical,  $b > 0 = \theta$ ,  $\sigma < 1$ , *or* income risk is pro-cyclical but liquidity a-cyclical,  $\theta > 0 = b$ . The present model reproduces his results, and also extends them by considering *both* cyclical liquidity and inequality as well as other types of shocks. While it may have been intuitive that, if each assumption (counter-cyclical liquidity, pro-cyclical inequality) has a stabilizing effect in isolation, they'll complement each other in general equilibrium, it isn't the case. Instead, they undermine each other. Indeed, if the substitution effect dominates the income effect,  $\sigma < 1$ , and inequality is counter-cyclical,  $\theta < 0$ , all conditions are satisfied: conditional on determinacy, asset prices are counter-cyclical and stabilizing against both types of shocks. But if  $\sigma < 1$  and  $\theta > 0$ , it depends on the calibration: several configurations are possible, including endogenous liquidity as a destabilizing factor.

Nevertheless, if we compare the multipliers of the full model with cyclical inequality and liquidity with respect to the textbook model,  $\theta = b = 0$ , we find that the fully-fledged model predicts a lower GE response than the textbook model under weak conditions.

**Proposition 10** *Assume that  $\phi > \max\{\phi^b, \phi_{\theta=0}^*\}$  and  $\max\{\phi_p^b, \phi_{p,\theta=0}^*\} < 0$ .*

1. *If the shocks are discount rate's, asset prices and cyclical inequality act as automatic stabilizers if and only if:  $\phi > \phi_p^{\epsilon,as-b} \equiv p - \frac{-m^b(1-p)(1+\chi)+(1-p)(1+\theta)(m^b+\frac{b}{1-p})+m^b p \frac{\theta}{1-\gamma}}{(\frac{b}{1-p}-m^b\chi)\kappa}$ .*
2. *If the shocks are monetary, asset prices and cyclical inequality act as automatic stabilizers if and only if:  $0 > \phi_p^{\zeta,as-b} \equiv -(1-\sigma)(1-p) - \theta \left(1-p + p \frac{\frac{m^b}{1-\gamma}}{\frac{b}{1-p}-\chi m^b}\right)$ .*

Where  $\phi = 0$  if the ZLB binds;  $\frac{b}{1-p} - \chi m^b > 0$ .

When the ZLB doesn't bind, the response of the full model against discount rate shocks is limited with respect to that of textbook model as 1. is implied by  $\phi > \phi^b$ ; at the ZLB,  $\theta > 0$  is a sufficient condition for:  $\phi_{p,\theta=0}^* > \phi_p^{\epsilon,as-b}$  – although it isn't necessary. Furthermore, 1.ii is satisfied if  $\sigma < 1$  and  $\theta > 0$ .



Although the assumptions of cyclical liquidity and cyclical inequality undermine each other in general equilibrium, it remains partial: the multipliers are still lower than in the textbook model, but to a limited extent. This may be important to explain the multipliers in HANK models that include fiscal policy and heterogeneous agents.

Adding endogenous liquidity might restore all puzzles of the textbook model despite a positive long run GE elasticity of asset demand; alternatively, it could solve all puzzles despite a negative long run GE elasticity – although bubbly liquidity clearly cannot. Indeed, the relevant parameter to determine whether the economy is Keynesian, and thus "conventional", or Neo-Fisherian, and therefore "puzzling", is now the difference between the long run GE elasticities of asset demand and supply.

The way I have included liquidity here is very peculiar: rational bubbles imply a long run elasticity of asset supply that tends to infinity, and becomes positive whenever the ZLB binds. Nevertheless, it underlines the very importance of long run liquidity in NK models, and how subtle differences in the long run elasticities have large implications, in particular if one wants to study persistent shocks.

## A Model without long run liquidity: steady state

A steady state is a solution of:

$$a = \frac{1}{1 + \beta^{\frac{1}{\sigma}} R^{\frac{1}{\sigma} - 1}} \left( \beta^{\frac{1}{\sigma}} R^{\frac{1}{\sigma} - 1} - \frac{g}{R} \frac{1 - \gamma}{\gamma} \right) = 0$$

Such that  $x \in (x^{min}, x^{max})$ ,  $x \geq 0$ , and the interest rate is given by:

$$R = \max \left\{ R^* \left( 1 + \lambda(x^\psi - 1) \right)^{\phi - 1}, \underline{R} \left( 1 + \lambda(x^\psi - 1) \right)^{-1} \right\}$$

$$\text{Where } R^* \equiv \beta^{-1} \left( g \frac{1 - \gamma^n}{\gamma^n} \right)^\sigma$$

Let  $x^z(\frac{R}{R^*}) \equiv \left( 1 + \frac{(\frac{R}{R^*})^{\frac{1}{\phi} - 1}}{\lambda} \right)^{\frac{1}{\psi}}$ ;  $x^z(1) = 1$ ; if  $x^z(\frac{R}{R^*}) > 0$ , then  $\frac{d \log x^z(\frac{R}{R^*})}{d \log \frac{R}{R^*}} = \frac{1}{\phi \kappa}$ . The ZLB binds if and only if  $x < x^z(\frac{R}{R^*})$ . If  $\frac{R}{R^*} < (1 - \lambda)^\phi$ , then  $x^z(\frac{R}{R^*}) < 0$ : the ZLB never binds.

**no-ZLB steady state** First, assume that  $x < x^z$  in steady state – a condition that

we'll check later on. The steady state output gap solves:

$$A(x) \equiv \left(1 + \lambda(x^\psi - 1)\right)^{\frac{\phi-1}{\sigma}} \ell(x)^{-\frac{1}{\sigma}} = 1$$

$$\text{Where } \ell(x) \equiv \left(\frac{1-\gamma}{\gamma} \frac{\gamma^n}{1-\gamma^n}\right)^\sigma$$

I plugged the no-ZLB Taylor rule in the asset demand curve and re-arranged.

As  $\hat{A}(x) = \Omega_1^\phi \hat{x}$ , this equation has at most one solution  $x \in (x^{min}, x^{max})$  if, for all  $x \in (x^{min}, x^{max})$ , either  $\Omega_1^\phi < 0$  or  $\Omega_1^\phi > 1$ . As  $A(1) = 1$ ,  $x = 1$  is our unique candidate. It satisfies the no-ZLB requirement if and only if the economy isn't in secular stagnation,  $\frac{\mathbf{R}}{\mathbf{R}^*} \leq 1$ .

**ZLB steady state** Second, assume that  $x > x^z$  in steady state. The steady state output gap solves:

$$A^z(x) \equiv \left(\frac{\mathbf{R}}{\mathbf{R}^*}\right)^{\frac{1}{\sigma}} \left(1 + \lambda(x^\psi - 1)\right)^{-\frac{1}{\sigma}} \ell(x)^{-\frac{1}{\sigma}} = 1$$

As  $\hat{A}^z(x) = \Omega_1^0 \hat{x}$ , this equation has at most one solution  $x \in (x^{min}, x^{max})$  if, for all  $x \in (x^{min}, x^{max})$ , either  $\Omega_1^0 < 0$  or  $\Omega_1^0 > 0$ . For future references, I'll let  $\mathcal{R}(x) \equiv \left(1 + \lambda(x^\psi - 1)\right) \ell(x)$ , where  $\mathcal{R}(x) > 0$  for all  $x \geq 0$ ,  $\mathcal{R}(1) = 1$  and  $d \log \mathcal{R}(x) = -\sigma \Omega_1^0 \hat{x}$ ;  $A^z(x^z(\frac{\mathbf{R}}{\mathbf{R}^*})) \equiv \mathcal{A}(\frac{\mathbf{R}}{\mathbf{R}^*})$  where  $\mathcal{A}(\frac{\mathbf{R}}{\mathbf{R}^*}) > 0$  for all  $\frac{\mathbf{R}}{\mathbf{R}^*} > (1 - \lambda)^\phi$ ,  $\mathcal{A}(1) = 1$  and  $d \log \mathcal{A}(\frac{\mathbf{R}}{\mathbf{R}^*}) = \frac{\Omega_1^\phi}{\phi \kappa} d \log \frac{\mathbf{R}}{\mathbf{R}^*}$ .

First, assume that  $\Omega_1^0 < 0$  for all  $x \in (x^{min}, x^{max})$ . There's a solution if and only if  $A^z(x^{min}) > 1 > A^z(x^{max})$ , which is equivalent to:  $\frac{\mathbf{R}}{\mathbf{R}^*} \in (\mathcal{R}(x^{min}), \mathcal{R}(x^{max}))$ . It is a steady state if and only if it satisfies the ZLB requirement:  $x < x_z$ , or  $\mathcal{A}(\frac{\mathbf{R}}{\mathbf{R}^*}) < 1$ . Thus, if  $\Omega_1^\phi > 0 > \Omega_1^0$ , there's a ZLB steady state if and only if  $\frac{\mathbf{R}}{\mathbf{R}^*} \in (\mathcal{R}_c^{min}, 1)$  where  $\mathcal{R}_c^{min} \equiv \max\{(1 - \lambda)^\phi, \mathcal{R}(x^{min})\}$ ; if  $\Omega_1^\phi, \Omega_1^0 < 0$ , there's a ZLB steady state if and only if  $\frac{\mathbf{R}}{\mathbf{R}^*} \in (1, \mathcal{R}_c^{max})$  where  $\mathcal{R}_c^{max} \equiv \mathcal{R}(x^{max})$ . Finally, since  $A(1) = \frac{\mathbf{R}}{\mathbf{R}^*}$  and  $\frac{d\hat{A}}{d\hat{x}} < 0$ ,  $x < 1$  in steady state if and only if  $\frac{\mathbf{R}}{\mathbf{R}^*} < 1$ .

Second, assume that  $\Omega_1^0 > 0$  for all  $x \in (x^{min}, x^{max})$ . There's a solution if and only if  $A^z(x^{min}) < 1 < A^z(x^{max})$ , which is equivalent to:  $\frac{\mathbf{R}}{\mathbf{R}^*} \in (\mathcal{R}(x^{max}), \mathcal{R}(x^{min}))$ . It is a steady state if and only if it satisfies the ZLB requirement:  $x < x_z$ , or  $\mathcal{A}(\frac{\mathbf{R}}{\mathbf{R}^*}) > 1$ . Thus, if  $\Omega_1^\phi, \Omega_1^0 > 0$ , there's a ZLB steady state if and only if  $\frac{\mathbf{R}}{\mathbf{R}^*} \in (1, \mathcal{R}_p^{max})$  where  $\mathcal{R}_p^{max} \equiv \mathcal{R}(x^{min})$ ; if  $\Omega_1^\phi < 0 < \Omega_1^0$ , there's a ZLB steady state if and only if  $\frac{\mathbf{R}}{\mathbf{R}^*} \in (\mathcal{R}_p^{min}, 1)$  where  $\mathcal{R}_p^{min} \equiv \max\{(1 - \lambda)^\phi, \mathcal{R}(x^{max})\}$ . Finally,

since  $A(1) = \frac{R}{R^*}$  and  $\frac{d\hat{A}}{dx} > 0$ ,  $x < 1$  in steady state if and only if  $\frac{R}{R^*} > 1$ .

All conditions are sufficient for the existence of a unique steady state; they aren't necessary for existence, nor uniqueness. What's, however, clearly needed for the existence of a steady state in secular stagnation is a segment where  $\max\{\Omega_1^\phi, \Omega_1^0\} > 0$ .

## B Model with long run liquidity.

### B.1 Determinacy

We have a 2-dimensional dynamic system:

$$\begin{aligned}\hat{x}_t &= \left(1 - \frac{\Upsilon_1^\phi + \Gamma_1^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi}\right) E_t \hat{x}_{t+1} + \frac{1}{\Upsilon_0^\phi + \Gamma_0^\phi} E_t \hat{b}_{t+1} - \frac{m^b(1+\chi)}{\Upsilon_0^\phi + \Gamma_0^\phi} \epsilon_t - \frac{m^b(1+(1-\sigma)\chi) + b}{\Upsilon_0^\phi + \Gamma_0^\phi} (\zeta_t + r) \\ \hat{b}_t &= \left(\frac{\Upsilon_1^\phi + \Gamma_1^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} \Gamma_0^\phi - \Gamma_1^\phi\right) E_t \hat{x}_{t+1} + \frac{\Upsilon_0^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} E_t \hat{b}_{t+1} + \frac{m^b(1+\chi)\Gamma_0^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} \epsilon_t + \frac{m^b(1+(1-\sigma)\chi)\Gamma_0^\phi - b\Upsilon_0^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} (\zeta_t + r)\end{aligned}$$

The characteristic polynomial is:  $P(z) = z^2 - \text{tr } z + \det$  where:

$$\begin{aligned}\det &= 1 - \frac{\Upsilon_1^\phi + \Gamma_0^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} \\ \text{tr} &= 1 + \det - \frac{\Gamma_1^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi}\end{aligned}$$

The two roots are solutions of  $P(z) = 0$ . Since both variables are non-predetermined, the model is locally determinate if and only if the two roots are inside the unit circle. Necessary and sufficient conditions for determinacy are:  $\det \in (-1, 1)$ ,  $1 - \text{tr} + \det > 0$  and  $1 + \text{tr} + \det > 0$ <sup>25</sup>, which lead to the following conditions:  $2 - \frac{\Upsilon_1^\phi + \Gamma_0^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} - \frac{1}{2} \frac{\Gamma_1^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} > 0$ ,  $\frac{\Upsilon_1^\phi + \Gamma_0^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} > 0$  and  $\frac{\Gamma_1^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} > 0$ . Under assumption 3 and 5,  $\Upsilon_0^\phi \geq \max\{\Upsilon_1^\phi, 0\}$  and  $\Gamma_0^\phi > \max\{\Gamma_1^\phi, 0\}$  for all  $\phi \geq 0$ . Thus, the determinacy conditions are:

$$\begin{aligned}\Upsilon_1^\phi + \Gamma_0^\phi > 0 &\Leftrightarrow \phi > 1 - \frac{m^b \frac{\theta}{1-\gamma} + b(1 + \theta + \kappa)}{\left(\frac{m^b(1+(1-\sigma)\chi)}{\sigma} + b\right)\kappa} \\ \Gamma_1^\phi > 0 &\Leftrightarrow \phi > 1\end{aligned}$$

<sup>25</sup>Strictly speaking, we need:  $(1 - \text{tr} + \det)(1 + \text{tr} + \det) > 0$ . However, if  $1 - \text{tr} + \det < 0$ , the conditions  $1 + \text{tr} + \det < 0$  and  $\det \in (-1, 1)$  are mutually exclusive.

Let  $\phi^b \equiv \max\{1, 1 - \frac{m^b \frac{\theta}{1-\gamma} + b(1+\theta+\kappa)}{(\frac{m^b(1+(1-\sigma)\chi)}{\sigma} + b)\kappa}\}$ . If we note that  $\frac{m^b}{1-\gamma} + b = 1$ , then  $\phi^b < 1$  if and only if  $\theta < -b(1 + \kappa)$ . Since  $1 - \frac{m^b \frac{\theta}{1-\gamma} + b(1+\theta+\kappa)}{(\frac{m^b(1+(1-\sigma)\chi)}{\sigma} + b)\kappa} > \phi^*$  if and only if:  $\theta > \frac{1+\kappa}{\frac{\gamma(1-b)-(1-\gamma)}{1-\gamma} + \sigma\chi} > 0$ , if  $\theta < -b(1 + \kappa)$ , then  $\phi^b < \phi^*$ . If  $\theta > -b(1 + \kappa)$ ,  $\phi^b = 1$ . Since  $\phi^* > 1$  if and only if  $\theta < 0$ ,  $\phi^b < \phi^*$  if and only if  $\theta < 0$ .

## B.2 Markov shocks: determinacy

Here, I assume that the considered steady state is determinate, i.e it satisfies the condition in proposition 7. The dynamic system is:

$$\begin{aligned}\hat{x}_t^h &= \left(1 - \frac{\Upsilon_1^\phi + \Gamma_1^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi}\right) p E_t \hat{x}_{t+1}^h + \frac{1}{\Upsilon_0^\phi + \Gamma_0^\phi} p E_t \hat{b}_{t+1}^h - \frac{m^b(1+\chi)}{\Upsilon_0^\phi + \Gamma_0^\phi} \epsilon^h - \frac{m^b \frac{1+(1-\sigma)\chi}{\sigma} + b}{\Upsilon_0^\phi + \Gamma_0^\phi} (\zeta^h + r) \\ \hat{b}_t^h &= \frac{\Upsilon_1^\phi \Gamma_0^\phi - \Upsilon_0^\phi \Gamma_1^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} p E_t \hat{x}_{t+1}^h + \frac{\Upsilon_0^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} p E_t \hat{b}_{t+1}^h + \frac{m^b(1+\chi) \Gamma_0^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} \epsilon^h + \frac{m^b \frac{1+(1-\sigma)\chi}{\sigma} \Gamma_0^\phi - b \Upsilon_0^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} (\zeta^h + r)\end{aligned}$$

The characteristic polynomial is:  $P(z) = z^2 - \text{tr } z + \det$  where:

$$\begin{aligned}\det &= p^2 \left(1 - \frac{\Upsilon_1^\phi + \Gamma_0^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi}\right) = p \left(1 - \frac{\Upsilon_p^\phi + \Gamma_0^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi}\right) \\ \text{tr} &= p \left(1 + \frac{\det}{p^2} - \frac{\Gamma_1^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi}\right)\end{aligned}$$

Necessary and sufficient conditions for determinacy are:  $\det \in (-1, 1)$ ,  $1 - \text{tr} + \det > 0$  and  $1 + \text{tr} + \det > 0$ , which lead to:  $p \left(1 - \frac{\Upsilon_p^\phi + \Gamma_0^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi}\right) \in (-1, 1)$ ,  $\frac{(1-p)\Upsilon_p^\phi + \Gamma_p^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} > 0$  and  $2 + 2p \left(1 - \frac{\Upsilon_p^\phi + \Gamma_0^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi}\right) - \frac{(1-p)\Upsilon_p^\phi + \Gamma_p^\phi}{\Upsilon_0^\phi + \Gamma_0^\phi} > 0$ . Using lemma 2,  $\Upsilon_0^\phi + \Gamma_0^\phi > \max\{0, \Upsilon_p^\phi + \Gamma_p^\phi\}$  for all  $\phi \geq 0$  and  $p \in [0, 1]$ . Thus, necessary and sufficient conditions are:

$$\begin{aligned}(1-p)\Upsilon_0^\phi + p\Upsilon_p^\phi + \Gamma_0^\phi > 0 &\Leftrightarrow \phi > p^2 - \frac{p^2 m^b \frac{\theta}{1-\gamma} + (1-p^2)m^b(1+\theta) + b(1+\theta+p^2\kappa)}{(\frac{m^b(1+(1-\sigma)\chi)}{\sigma} + b)\kappa} \\ (1-p)\Upsilon_p^\phi + \Gamma_p^\phi > 0 &\Leftrightarrow \phi > p - \frac{p m^b \frac{\theta}{1-\gamma} + (1-p)m^b(1+\theta) + b(1+\theta)}{\left(\frac{m^b(1+(1-\sigma)\chi)}{\sigma} + \frac{b}{1-p}\right)\kappa}\end{aligned}$$

Which we can write as:

$$\phi_p^b \equiv \max \left\{ p^2 - \frac{p^2 m^b \frac{\theta}{1-\gamma} + (1-p^2)m^b(1+\theta) + b(1+\theta+p^2\kappa)}{(\frac{m^b(1+(1-\sigma)\chi)}{\sigma} + b)\kappa}, p - \frac{p m^b \frac{\theta}{1-\gamma} + (1-p)(1+\theta)(m^b + \frac{b}{1-p})}{\left(\frac{m^b(1+(1-\sigma)\chi)}{\sigma} + \frac{b}{1-p}\right)\kappa} \right\} < 0$$

## C A note on bubbles and transversality condition

Absent any shocks, along the bubbly saddle path,  $b_t = b$  and  $x_t = 1$  for all  $t \geq 0$ ; as in equilibrium,  $L_t^j = 0$  for all  $t \geq 0$ , the transversality condition, (2), is equivalent to:

$$\lim_{T \rightarrow \infty} \left( \frac{g}{R^{ra}} \right)^T \left( \frac{c_T^j}{c_0^j} \right)^{-\sigma} H_t^j b = 0$$

Using the consumption functions,

$$\left( \frac{c_T^j}{c_0^j} \right)^{-\sigma} = \left( \frac{\gamma(1-b)}{1-\gamma(1-b)} \right)^{-\sigma} \left( \frac{1-\gamma(1-b)}{\gamma(1-b)} \right)^{-\sigma} \dots = O^{i,j} \quad (\text{C-1})$$

Where  $O^{s,s} = O^{b,b} = 1$ ,  $O^{s,b} = \frac{1}{O^{b,s}} = \left( \frac{1-\gamma(1-b)}{\gamma(1-b)} \right)^{-\sigma}$ .

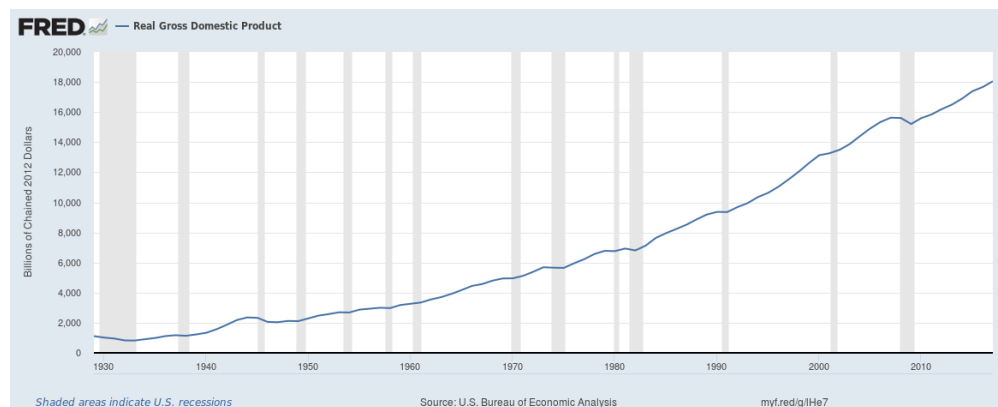
Thus, the transversality condition,

$$\lim_{T \rightarrow \infty} \left( \frac{g}{R^{ra}} \right)^T O^{i,j} H_t^j b = 0$$

Is satisfied if either  $R^{ra} > g$  or  $b = 0$ . Since  $b > 0$  if and only if  $R^f < g$  – here,  $R^f$  stands for the steady state interest rate in the bubble-less economy – bubbles aren't possible in a RA economy; however, if  $R^* < g < R^{ra}$ , bubbles are possible when financial frictions are binding.

# Résumé

Le graphique ci-dessous présente le Produit Intérieur Brut Réel (PIBR) des Etats-Unis (EU) depuis 1929. Il est frappant de constater qu'à long terme, même les crises économiques majeures de 1929 et 2008 sont à peine discernables.



**Figure 1:** PIBR EU, Milliards de Dollars de base 2012, de 1929 à 2017. Source: Bureau of Economic Analysis

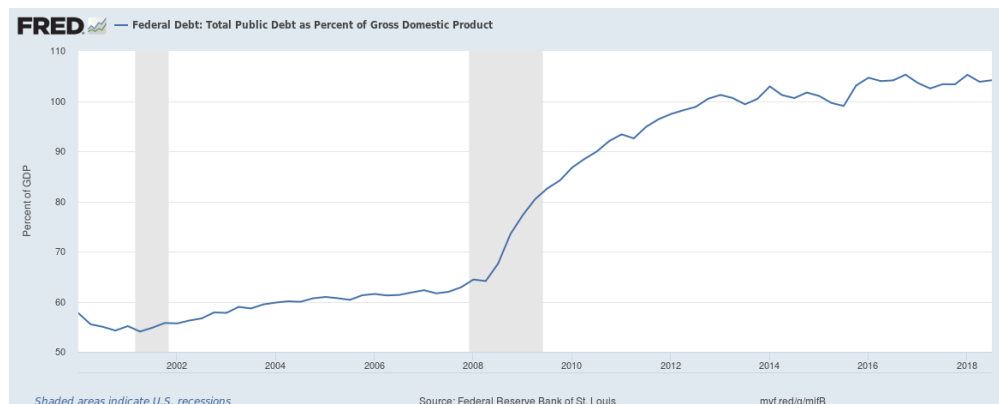
Et pourtant, pour citer le célèbre macro-économiste John Maynard Keynes, "à long terme, nous sommes tous morts". Bien qu'un événement finalement peu important selon une perspective de très long terme, la crise financière de 2008 a été une longue et pénible expérience pour des millions de personnes à travers le monde. En l'espace de deux ans, le taux de chômage aux Etats-Unis a plus que doublé, et est resté élevé durant près de dix ans. De plus, le taux de croissance des salaires nominaux est toujours faible, et le ratio emploi-population en dessous de son niveau pré-crise: il semble probable que des travailleurs découragés aient quitté le marché du travail durant la crise et n'y soient pas ré-entrés.



**Figure 2:** Taux de chômage aux EU: 20 ans et plus, %, de 2000 à 2019. Source: Bureau of Labor Statistics

Aujourd'hui, l'économie américaine est vigoureuse. Cependant, la prochaine crise pourrait déjà se profiler à l'horizon: selon le NBER <sup>26</sup>, la durée moyenne entre deux creux (le point le plus bas du cycle économique) est de 70 mois aux EU depuis 1945. La reprise de la croissance datant de Juin 2009, plus de 100 mois se sont écoulés depuis le dernier creux.

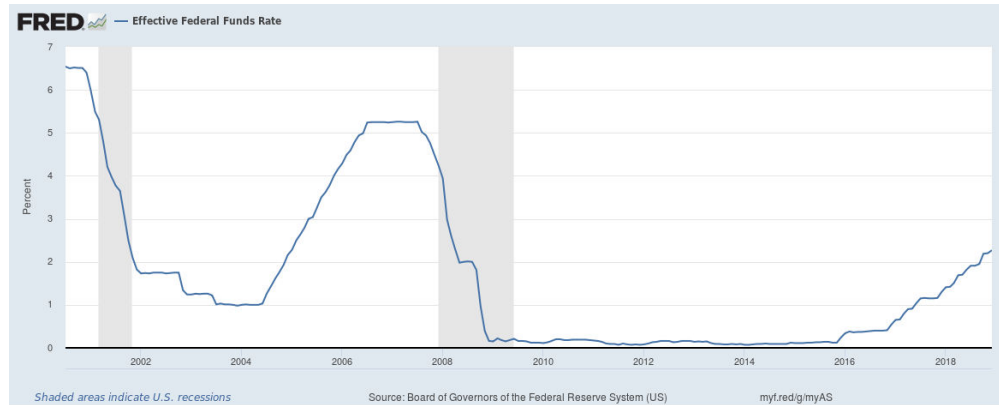
Et l'économie américaine apparaît bien mal préparée à la prochaine recession. La crise de 2008 a duré presque une décennie, et ce malgré d'ambitieux programmes de soutien à la demande agrégée (la politique de taux zéro par la FED, l'assouplissement quantitatif, une politique fiscale accommodante ect.). La dette publique ayant fortement augmenté, non seulement durant la crise, mais également durant la reprise, la capacité fiscale américaine est fortement réduite.



**Figure 3:** Ratio Dette Publique / PIB aux EU, %, de 2000 à 2019. Source: Saint Louis FED

Qui plus est, le taux d'intérêt nominal fixé par la FED est toujours très proche de zéro, de même que le taux d'intérêt réel, laissant peu d'espace à la politique monétaire.

<sup>26</sup>Les données se trouvent à l'adresse suivante: <https://www.nber.org/cycles.html>



**Figure 4:** Taux Nominal fixé par la FED, %, de 2000 à 2019. Source: Board of Governors of the FED system

Les politiques monétaires et fiscales étant considérablement affaiblies, si l'économie américaine venait à essuyer une nouvelle crise de même calibre que celle de 2008 durant la prochaine décennie, la situation pourrait rapidement devenir totalement hors de contrôle. Dès lors, il est absolument crucial de comprendre les origines de la crise de 2008, et pouvoir ainsi mettre en place des politiques adaptées pour prévenir ce type de crise et espérer éviter un scénario catastrophe.

Selon [Summers \(2013\)](#) et [Krugman \(2013\)](#), il y a deux notions clés pour comprendre la Grande Récession: la stagnation séculaire et les bulles d'actifs. Avant de développer la théorie "bulle – stagnation séculaire", il apparaît nécessaire d'en définir brièvement les termes.

Suivant la définition courante, le prix d'un actif inclut une bulle lorsqu'il excède la valeur fondamentale de cet actif, elle-même calculable comme la somme des flux de dividendes futurs actualisés. Certaines bulles d'actifs sont entrées dans l'histoire: la bulle de la compagnie des mers du sud en Angleterre au 18ème siècle; la bulle de la tulipe aux Pays-Bas au 17ème siècle, ou, plus récemment, la bulle des nouvelles technologies aux Etats-Unis en 2001. Ces bulles d'actifs sont typiquement liées à des comportements spéculatifs: les investisseurs achètent ces actifs car ils anticipent de forts gains en capital. A priori sans rapport, la notion de stagnation séculaire a été introduite par [Hansen \(1939\)](#). La plupart des économistes, que ce soit à l'époque ou encore aujourd'hui, considèrent qu'une récession Keynesienne, c'est-à-dire provoquée par un déficit de demande agrégée, est de nature temporaire – en moyenne, une crise économique dure cinq ans aux Etats-Unis. Cependant, selon [Hansen \(1939\)](#), si des politiques de soutien à la demande agrégée ne sont pas mises en place, une telle récession pourrait se transformer en dépression très persistente, voire même permanente.

La théorie de la bulle-stagnation séculaire offre une grille d'analyse des événements récents



aux Etats-Unis. Selon cette théorie, divers changements structurels <sup>27</sup> ont considérablement, et durablement, diminué la demande agrégée aux Etats-Unis depuis des décennies. Cependant, une importante bulle spéculative, portant sur des actifs adossés au marché hypothécaire américain, a soutenu la demande agrégée, prévenant ainsi une trop forte chute malgré les tendances sus-mentionnées. Cette bulle a imploré lorsque Lehman Brothers a déclaré faillite en 2008: les investisseurs ont soudainement réalisé que ces actifs adossés au marché hypothécaire n'étaient pas sans risque comme ils l'avaient cru, mais au contraire très exposés au marché immobilier, lequel venait de s'effondrer. Cette prise de conscience a induit les investisseurs à ré-évaluer une large portion de ces actifs comme sans valeur aucune. Ce choc soudain et permanent touchant l'offre d'actifs a très fortement réduit la demande agrégée. Malheureusement pour l'économie américaine, la FED n'a pu en annuler totalement les effets à cause du taux d'intérêt plancher (le ZLB): les Etats-Unis sont alors entrés dans une période de stagnation séculaire. Parmi d'autres, Caballero et al. (2008) présentent des faits stylisés qui soutiennent cette interprétation de la crise.

Cette thèse comporte trois chapitres, chacun analysant un aspect particulier de la théorie bulle – stagnation séculaire. Le premier chapitre, "Asset Bubbles and the Income Distribution", qui est une extension de mon mémoire de Master 2, se concentre sur la formation de bulles financières (d'un point de vue théorique): la hausse documentée des inégalités de revenus dans les pays développés a-t-elle contribué à l'émergence de bulles spéculatives? Le second papier, "Secular Stagnation, Liquidity Trap and Asset Bubbles", étudie plus directement la théorie bulle – stagnation séculaire: j'analyse les circonstances sous lesquelles une bulle d'actif stimule l'investissement et l'emploi dans un modèle Nouveau Keynesien (NK) incluant le capital. Le troisième papier, "Secular Stagnation or Secular Boom?", est plus technique: j'y démontre que les modèles NK font des prédictions paradoxales lorsque la demande agrégée est chroniquement déficiente – un boom séculaire plutôt qu'une stagnation séculaire, et analyse comment ajuster ces modèles pour qu'ils deviennent viables dans l'environnement actuel.

## Asset Bubbles and the Income Distribution

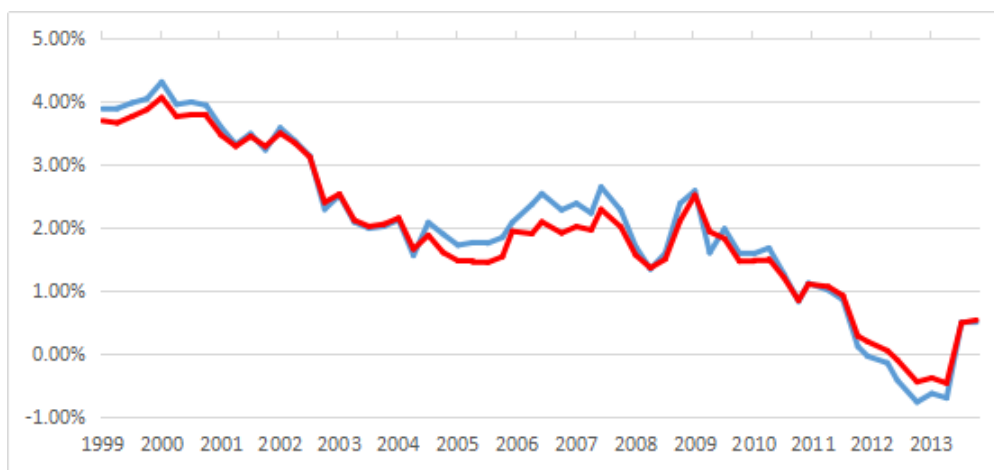
Trois faits stylisés ont dominé les discussions macroéconomiques depuis le début de la crise de 2008:

*Fait 1:* On observe un déclin du taux d'intérêt réel sur toute la période 1970–2008, que ce soit au

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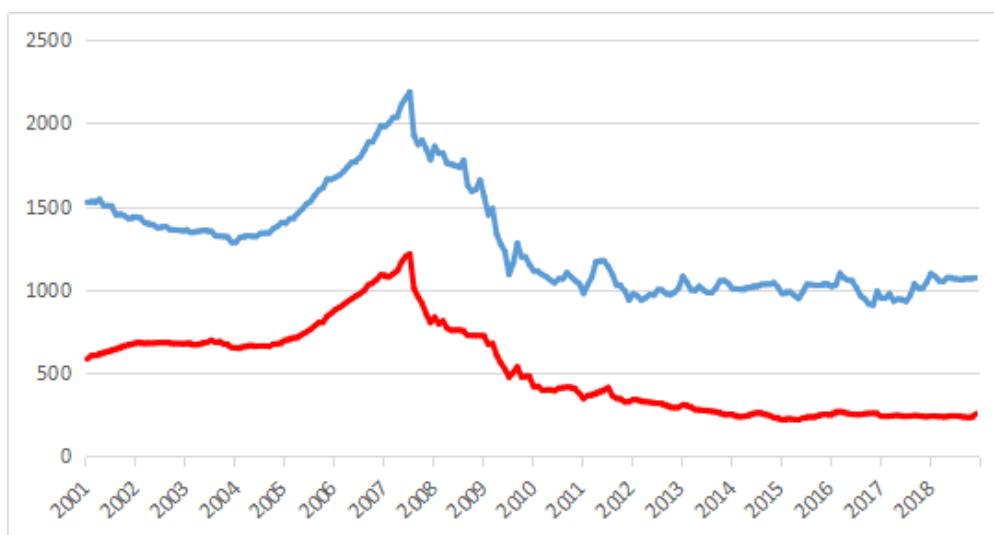
<sup>27</sup>Par exemple une baisse des taux de croissance de la productivité et de la population, une demande toujours croissante d'actifs en dollars de la part des économies émergentes, une hausse des inégalités, ect.

niveau mondial ou encore aux Etats-Unis <sup>28</sup>. Le taux d'intérêt réel est aujourd'hui proche de zéro, voire en dessous de zéro, dans la plupart des économies développées.



**Figure 5:** Taux d'Intérêt Réel Mondial en % (bleu) et Taux d'Intérêt Réel EU en % (rouge), de 1999 à 2013. Source: King & Low (2014)

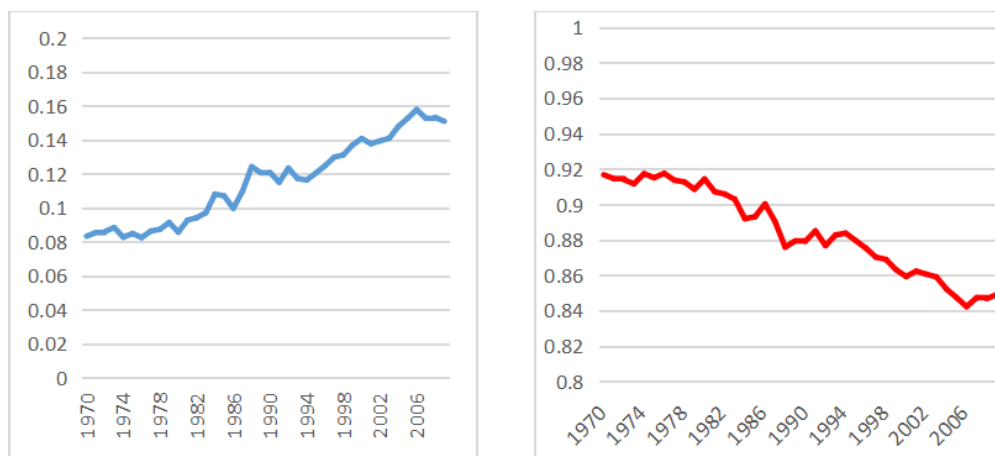
*Fait 2:* Des bulles ont explosé puis implosé en divers endroits du globe, avec des conséquences dramatiques: le malaise japonais (ou stagnation séculaire) peut être attribué à l'implosion d'une bulle d'actifs sur les marchés immobiliers et d'actions; le crash de la bulle aux Etats-Unis en 2008 a plongé les économies développées dans une dépression persistente.



**Figure 6:** Billets de Trésorerie en Circulation aux EU en Milliards de Dollars, Total (bleu) et Adossés à des actifs (rouge), de 2001 à 2018. Source: Federal Reserve Board

<sup>28</sup>Voir IMF (2014), en particulier le chapitre 3.

*Fait 3:* On observe dans les pays développés une concentration croissante des richesses et revenus (Piketty, 2014). En particulier, la part du PIB versée au top 1% a presque doublé de 1970 à 2009.



**Figure 7:** Part dans le PIB, après taxes du top 1 1% (bleu) vs le bas 99 % (rouge) aux EU, de 1970 à 2009. Source: WID

Les faits 1 and 2 sont cruciaux pour comprendre la Grande Récession de 2008. En effet, des économistes influents tels que Summers (2013) et Krugman (2013) ont avancé l'idée que l'implosion de la bulle en 2008 était "le" choc ayant initié la crise. De plus, les taux d'intérêts réels extrêmement faibles ont empêché les principales banques centrales de répondre adéquatement à ce choc. A première vue, le fait 3 ne semble pas devoir appartenir à cette liste: il n'apparaît pas particulièrement important pour un macroéconomiste intéressé par les cycles et la crise financière. Cependant, quelques mois après l'implosion de la bulle sur le marché des actifs adossés au marché hypothécaire (les actifs MBS), divers économistes<sup>29</sup> ont attribué l'origine des faits 1 et 2 au fait 3, qui serait donc le principal "moteur" de l'émergence de la bulle puis de la stagnation séculaire.

Cette théorie "conventionnelle" – bien que non formalisée – se présente généralement comme suit. Dans les données, on constate que les ménages les plus riches choisissent des taux d'épargne plus élevés que les ménages les plus pauvres. Une concentration plus importante des richesses et revenus (fait 3) auraient donc tendance à augmenter la demande (aggrégée) d'actifs. Si l'offre d'actifs n'est pas affectée par ce choc, le taux d'intérêt réel devrait alors baisser pour équilibrer le marché des actifs (fait 1). Or, lorsque le taux d'intérêt chute sous un certain seuil, les investisseurs ont des incitations à créer ou investir dans des bulles d'actifs (fait 2). Selon cette logique, le fait 3 implique le fait 1, et le fait 1 implique le fait 2: le choc d'inégalités promeut l'émergence de bulles d'actifs.

<sup>29</sup>Une liste partielle inclut: Stiglitz (2009), Bardhan (2009) et Fitoussi and Saraceno (2010).

D'un point de vue théorique, l'implication du fait 1 au fait 2 est bien connue depuis les travaux de [Samuelson \(1958\)](#) et [Tirole \(1985\)](#). Ces papiers ont montré que l'inefficience dynamique, c'est-à-dire  $R < g$  où  $R$  est le taux d'intérêt et  $g$  le taux de croissance du PIB, tous deux mesurés à l'état stationnaire, est une condition nécessaire et suffisante à la formation de bulles d'actifs rationnelles. Intuitivement, si l'économie ne peut produire suffisamment d'actifs pour satisfaire la demande, les épargnants font face à une pénurie d'actifs ([Caballero, 2006](#)), ce qui rend les bulles d'actifs désirables, bien qu'elles soient intrinsèquement improductives. En revanche, la littérature académique étudiant la formation de bulles d'actifs n'a pas réellement pris en compte la distribution des revenus dans la population à un instant donné: ce papier est une tentative pour y remédier. Si l'on accepte l'idée que les bulles d'actifs peuvent être un important facteur d'explication des cycles, il est alors crucial de mieux comprendre les facteurs favorisant l'émergence desdites bulles et donc comment les prévenir.

Bien qu'intuitive, la théorie conventionnelle repose néanmoins sur des hypothèses très fortes, plus ou moins implicitement. Dans ce papier, je relâche deux de ces hypothèses: tous les actifs ne sont pas également liquides, mais certains actifs sont plus liquides que d'autres – j'utilise la définition suivante de la liquidité: un actif liquide peut être acheté ou vendu par tous les agents, alors qu'un actif illiquide ne peut l'être que par un sous-ensemble d'agents; les agents les plus riches détiennent également une fraction beaucoup plus importantes d'actifs illiquides dans leur portefeuille d'actifs que la population générale<sup>30</sup>. Par rapport à la théorie conventionnelle, ces deux hypothèses ajoutent un effet de *composition* à un choc d'inégalités. En effet, un choc d'inégalités affecte maintenant non seulement le taux d'épargne moyen, mais aussi la répartition désirée moyenne de l'épargne entre actifs liquides et non-liquides. La prise en compte de cet effet de composition modifie drastiquement la corrélation entre les inégalités et le taux d'intérêt, et donc entre les inégalités et les bulles d'actifs: si ces bulles sont liquides, une importante concentration au plus haut de la distribution des revenus ne promeut pas l'émergence de bulles, mais une faible concentration si.

Le modèle utilisé inclut des agents hétérogènes et des frictions financières qui limitent l'arbitrage entre les actifs liquides et non-liquides. Lorsque les frictions financières ne sont pas saturées, les marchés des actifs sont fonctionnels, c'est-à-dire qu'il n'y a pas d'opportunité d'arbitrage. Puisque les actifs liquides et illiquides sont alors parfaitement substituables, un choc d'inégalités a un effet

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<sup>30</sup>De nombreuses études récentes confirment que la prime de risque est, au moins partiellement, attribuable à des différentiels de liquidité ([Krishnamurthy and Vissing-Jorgensen, 2012](#)). On estime qu'environ 40% seulement de la population américaine est active sur les marchés financiers non liquides (directement ou indirectement); et d'importantes différences de portefeuilles selon le niveau de richesse sont documentées.

de *niveau* uniquement: en redistribuant le PIB d'agents à faible taux d'épargne vers des agents à fort taux d'épargne, ce choc augmente la demande d'actifs, diminue le taux d'intérêt, et si l'effet est suffisamment fort, rend les bulles d'actifs désirables. Le modèle est donc capable de reproduire la théorie conventionnelle.

Cependant, lorsque les frictions financières sont saturées, les marchés financiers sont dysfonctionnels, c'est-à-dire non exempts d'opportunités d'arbitrage. En effet, les actifs liquides paient alors une prime de liquidité: le taux d'intérêt est inférieur au taux de rendement des actifs non liquides; cette prime de liquidité est le prix relatif des actifs non liquides par rapport aux actifs liquides. Un choc d'inégalités a dès lors des implications radicalement différentes pour le taux d'intérêt. En effet, il redistribue le PIB vers des agents qui choisissent non seulement un taux d'épargne plus élevé, mais qui détiennent également des actifs beaucoup moins liquides dans leur portefeuille. Par conséquent, un choc d'inégalités augmente la demande d'actifs et diminue le taux de rendement des actifs non liquides; mais, simultanément, il réduit la demande d'actifs liquides par rapport aux actifs non liquides, ce qui implique une baisse de la prime de liquidité et donc un taux d'intérêt plus élevé *malgré* un taux de rendement plus faible des actifs non liquides. Le taux d'intérêt plus élevé empêche l'émergence de bulles rationnelles liquides; le taux de rendement faible des actifs illiquides favorise l'émergence de bulles rationnelles illiquides *malgré* un taux d'intérêt plus élevé. Ainsi, le modèle souligne également que la théorie conventionnelle n'est pas robuste: des frictions financières saturées associées à un autre niveau d'hétérogénéité (choix de portefeuille) bouleversent les résultats.

J'utilise un modèle OLG standard à deux périodes avec des marchés concurrentiels, un seul bien de consommation, trois facteurs de production: capital, travail qualifié et non qualifié, deux actifs liquides: des obligations et des bulles, un actif illiquide: des actions (du capital), et deux types d'agents: investisseurs et travailleurs. Les inégalités doivent être comprises comme une différence de revenu permanent dans une société de classe: il n'y a pas de risque de revenu ni de mobilité sociale. Les travailleurs et les investisseurs diffèrent à la naissance selon trois dimensions: (i) les investisseurs fournissent une main-d'oeuvre qualifiée alors que les travailleurs fournissent une main-d'oeuvre non qualifiée – et la main-d'oeuvre qualifiée est relativement plus rare, et donc relativement mieux rémunérée; (ii) les investisseurs sont plus patients que les travailleurs; (iii) ils font face à différentes opportunités d'investissement: les travailleurs ne détiennent que des actifs liquides, tandis que les investisseurs peuvent tirer parti de toute opportunité d'arbitrage entre actifs liquides et non liquides. Les agents d'une même classe sont homogènes, il y a une masse fixe de

chaque type.

Ces hypothèses permettent de saisir très simplement les différences de choix de portefeuille et taux d'épargne que l'on observerait dans un modèle plus structuré incluant (*i*): des différences de revenu permanent; (*ii*): des chocs idiosyncrasiques portant sur la dotation en travail, ainsi que (*iii*): des coûts non-linéaires d'ajustement de portefeuille. Ces classes doivent être considérées comme les "99%", les travailleurs, vs les "1%", les investisseurs, ou toute autre segmentation arbitraire de la population, de sorte qu'il y ait moins d'investisseurs qui diffèrent du reste de la population parce que (*i*): ils reçoivent un flux de revenu plus élevé en moyenne; (*ii*) ils choisissent un taux d'épargne plus élevé et (*iii*): ils détiennent une fraction beaucoup plus élevée d'actifs non liquides dans leur portefeuille. La répartition du revenu est calibrée par un paramètre de la fonction de production qui détermine la répartition du revenu agrégé du travail entre travail qualifié et non qualifié, c'est-à-dire entre les investisseurs et les travailleurs. Si les travailleurs reçoivent une part élevée du revenu du travail total, je dirai que la société est relativement égalitaire: les agents situés au bas de la distribution reçoivent collectivement une fraction importante du revenu agrégé. Un "choc d'inégalités" est une redistribution exogène en faveur des investisseurs – il est micro-fondé par un choc sur la fonction de production qui modifie la composition optimale des intrants de l'entreprise, qui demande alors plus de main-d'oeuvre qualifiée et moins de main-d'oeuvre non qualifiée.

Tous les agents sont confrontés à d'autres frictions financières: ils ne peuvent pas vendre à découvert la bulle d'actifs et leur offre d'obligations est limitée par une contrainte d'emprunt. A l'équilibre, les actions (le capital) représentent le seul actif dont l'offre nette est positive: toutes l'épargne doit être dirigée vers le marché des capitaux. Les investisseurs font office d'intermédiaires financiers entre les travailleurs et l'entreprise: ils vendent des obligations liquides aux travailleurs et accumulent les actions non liquides émises par l'entreprise. La demande d'actifs est égale à la somme de l'épargne de tous les agents; la demande d'actifs liquides est égale à l'épargne totale des travailleurs. Tout à fait intuitivement, un choc sur la distribution des revenus affecte les deux: si la société devient plus inégalitaire, la demande d'actifs augmente parce que les investisseurs ont un taux d'épargne plus élevé, mais la demande pour les actifs liquides se contracte car les investisseurs accumulent directement des actions – ils n'ont pas besoin d'intermédiation financière. Le choc n'affectant pas l'offre d'actifs, il augmente simultanément le taux de rendement des actifs liquides (le taux d'intérêt) et réduit le taux de rendement des actifs non liquides (le produit marginal du capital).

Si les bulles d'actifs sont liquides, un niveau élevé d'inégalités empêche l'apparition de bulles

d'actifs rationnelles; si les bulles d'actifs sont illiquides ou si tous les actifs sont des substituts parfaits les uns des autres, un niveau élevé d'inégalités favorise l'émergence de bulles d'actifs rationnelles. Que la théorie conventionnelle soit juste (les bulles d'actifs résultent d'inégalités croissantes) ou erronée (les bulles d'actifs surgissent en dépit d'inégalités croissantes) dépend des imperfections des marchés financiers et du type de bulles d'actifs considéré. En ce qui concerne l'impact d'un choc d'inégalités sur la taille de la bulle d'actifs à l'équilibre (conditionnellement à l'existence d'une bulle), on observe le même phénomène: un choc d'inégalités gonfle des bulles financières déjà existantes lorsque les marchés financiers sont fonctionnels ou les bulles d'actifs illiquides, mais dégonfle ces bulles lorsque les marchés financiers sont dysfonctionnels et que les actifs sont liquides.

Bien entendu, le modèle développé dans le présent article est trop simpliste pour offrir une conclusion définitive sur le lien entre les bulles d'actifs, les inégalités et le taux d'intérêt. Cependant, il permet de capturer cet effet supplémentaire sur la demande d'actifs: la demande totale d'actifs par rapport à la demande d'une classe d'actifs particulière. Il est tout à fait concevable, et même assez intuitif, qu'une plus grande inégalité dans la répartition de la richesse ou des revenus entraîne à la fois une augmentation de la demande d'actifs en général et une baisse de la demande pour des classes d'actifs spécifiques (ici liquides).

L'une des manières d'aborder la crise de 2008 consiste à prendre pour cadre le marché de l'immobilier résidentiel (un actif illiquide). Une autre est de prendre pour cadre le marché des titres adossés à des créances hypothécaires (un actif liquide). Ce modèle est plutôt cohérent avec la deuxième option: puisque la demande d'actifs liquides a fortement augmenté aux Etats-Unis au cours des 30 dernières années, les banques étaient fortement incitées à émettre de plus en plus d'actifs liquides. Pour ce faire, elles ont fortement augmenté leurs crédits, notamment hypothécaires, pour pouvoir émettre des actifs liquides adossés à des créances hypothécaires: selon cette interprétation, les prix élevés de l'immobilier observés avant 2008 étaient un sous-produit de la bulle des MBS. Ce modèle ne fournit aucune explication sur la manière de concevoir une bulle d'actifs, mais pose la question suivante: est-ce que des bulles d'actifs surviennent en raison d'une concentration croissante des revenus et richesses, ou bien malgré une concentration croissante?

Néanmoins, les prédictions du modèle sont en contradiction avec certaines observations macroéconomiques. En particulier, lorsque les inégalités augmentent et que les marchés financiers sont imparfaits, le modèle prédit un taux d'intérêt plus élevé et un produit marginal du capital plus faible. Dans les données, nous observons plutôt un produit marginal du capital plus ou moins stable et un taux d'intérêt en baisse. Mais le modèle n'inclut pas d'autres tendances

macroéconomiques bien documentées, comme par exemple des taux de marges à la hausse, la baisse des taux de croissance de la productivité et de la population, une surabondance de l'épargne au niveau mondial, etc. En tout état de cause, cet article présente un argument théorique simple: la théorie conventionnelle est très fragile; il est parfaitement concevable que l'accroissement des inégalités de revenus et de richesse soit une force "stabilisatrice" du point de vue macroéconomique.

**Revue de la littérature** Depuis [Samuelson \(1958\)](#) et [Tirole \(1985\)](#), il est bien connu que des bulles d'actifs rationnelles sont possibles si et seulement si le taux d'intérêt réel est inférieur au taux de croissance de la production en régime permanent. Plus récemment, [Martin and Ventura \(2012\)](#) et [Farhi and Tirole \(2012\)](#) ont introduit des frictions financières dans la littérature sur les bulles rationnelles, qui permettent de déconnecter le taux d'intérêt du taux de rendement des autres actifs: les bulles d'actifs deviennent possibles même si le produit marginal du capital (le taux de rendement des actifs non liquides) est supérieur au taux de croissance de la production.

Mon modèle se base sur ces travaux, et en particulier sur [Martin and Ventura \(2016\)](#), qui ont également développé un modèle OLG avec des frictions financières, des agents hétérogènes et des bulles d'actifs. Ils montrent que, si les agents ont des préférences linéaires, des bulles illiquides peuvent augmenter l'investissement et la production. En effet, en fournissant aux investisseurs (ils les appellent des entrepreneurs) des actifs collatéraux plus élevées, des bulles d'actifs illiquides augmentent le taux d'intérêt, ce qui incite les épargnants à reporter complètement leur consommation à un âge avancé, c'est-à-dire qu'elles provoquent une surabondance d'épargne et donc un fort investissement. Bien que la structure de base de mon modèle soit assez similaire à la leur, mon objectif est différent. Je n'essaie pas d'expliquer par quel biais les bulles d'actifs stimulent l'investissement et la production, mais plutôt comment la distribution des revenus affecte la condition d'existence ainsi que la taille d'équilibre de ces bulles. Pour ce faire, j'introduis deux types de facteur travail associés aux deux types d'agents. Cela me permet de calibrer la distribution des revenus et d'étudier ce qui se passe dans le modèle lorsque l'on fait varier cette distribution – la répartition des revenus dans [Martin and Ventura \(2016\)](#) est très simple: les jeunes investisseurs ne reçoivent rien et décident de manière optimale de ne pas consommer; les jeunes travailleurs perçoivent l'intégralité du revenu du travail et peuvent décider de consommer (si le taux d'intérêt est bas) ou de différer totalement leur consommation (si le taux d'intérêt est élevé). J'introduis aussi des bulles d'actifs liquides ainsi que des différences de taux d'actualisation temporelle.



[Graczyk and Phan \(2018\)](#) analysent également les effets des inégalités de revenus au sein d'une cohorte sur les bulles d'actifs en utilisant un modèle OLG avec des agents hétérogènes. Dans leur économie de dotation, tous les agents ont les mêmes préférences et ont accès au même ensemble d'opportunités d'investissement. Ils constatent qu'un choc d'inégalités favorise l'émergence de bulles d'actifs. En effet, puisque dans leur modèle tous les agents reçoivent la même dotation, mais les personnes âgées des dotations différentes, les agents pauvres veulent emprunter tandis que les agents riches veulent épargner - pour lisser leur consommation. C'est une autre manière de micro-fonder la théorie conventionnelle. Dans mon modèle, les inégalités interagissent avec des opportunités d'investissement et des préférences hétérogènes. Si les frictions financières ne sont pas saturées, je retrouve leur résultat (bien que par un mécanisme complètement différent); mais, si les frictions financières sont saturées, un choc d'inégalités empêche l'émergence de bulles d'actifs liquides alors qu'il favorise l'émergence de bulles d'actifs non liquides.

En utilisant un modèle OLG à trois périodes avec des frictions financières, [Raurich and Seegmuller \(2017\)](#) montrent que la répartition des revenus par groupe d'âge, c'est-à-dire entre les jeunes, les personnes d'âge moyen et les personnes âgées, est un facteur déterminant pour savoir si des bulles d'actifs rationnelles peuvent émerger, et si elles sont productives ou non-productives. Alors qu'ils se concentrent sur l'intermédiation financière entre des agents de différentes cohortes et l'asynchronicité entre opportunités d'investissement et revenus tout au long du cycle de vie, je me concentre plutôt sur l'inégalité des revenus, les opportunités d'investissement hétérogènes et l'intermédiation financière au sein d'une cohorte. Les deux approches sont complémentaires.

[Ikeda and Phan \(2015\)](#) et [Bengui and Phan \(2018\)](#) introduisent des frictions financières dans les modèles OLG. Ils distinguent les bulles sûres des bulles risquées et étudient comment divers attributs structurels des marchés financiers, tels que la possibilité de déclarer faillite ou la responsabilité limitée, favorisent l'existence d'un type de bulle ou de l'autre. J'introduis un ensemble différent de frictions financières, des inégalités et je fais plutôt la distinction entre les bulles liquides et non liquides. Encore une fois, les deux approches sont complémentaires.

Enfin, il existe une littérature de plus en plus importante sur la stagnation séculaire qui cherche à expliquer la tendance à la baisse du taux d'intérêt au cours des dernières décennies dans les économies développées. Deux exemples représentatifs sont le modèle OLG à trois périodes de [Eggertsson et al. \(2017\)](#) et le modèle de marchés incomplets à la Bewley de [Auclert and Rognlie \(2018\)](#). Une conclusion générale de cette littérature est que la hausse des inégalités de revenus est un facteur important, bien que probablement pas le plus important, pour expliquer la baisse

des taux. Mais ces papiers font abstraction de la distinction entre actifs liquides et non liquides, ainsi que des différences de portefeuille, ou encore ne tiennent pas compte de la possibilité de dysfonctionnements des marchés financiers, qui sont des éléments clefs du présent papier. En effet, l'ajustement de la prime de liquidité à un choc d'inégalités constitue la force motrice qui génère des co-mouvements positifs entre le taux d'intérêt et la demande d'actifs dans mon modèle.

## Secular Stagnation, Liquidity Trap and Asset Bubbles

Certaines, sinon la plupart, des crises financières majeures ont été déclenchées par des cycles d'explosion-implosion des prix des actifs. Mais la crise de 2008 aux Etats-Unis se démarque, pour au moins deux raisons principales. Premièrement, la récession n'a pas été précédée par un boom économique. En effet, du début des années 90 jusque 2008, l'économie américaine a connu un épisode prolongé de faible volatilité et modestes taux d'inflation et de croissance du PIB. Rétrospectivement, cette "grande modération" malgré une importante bulle financière est difficile à rationaliser. Deuxièmement, la crise semble avoir eu un effet très persistant, voire permanent, sur l'économie américaine. Plus de 10 ans après la crise, et même si le taux de chômage est retourné à son niveau "naturel", la participation au marché du travail reste faible, l'accumulation de capital en dessous de la tendance pré-crise et le taux d'intérêt nominal proche de zéro.

Pour expliquer la dynamique de l'économie américaine avant et après 2008, [Summers \(2013\)](#) et [Krugman \(2013\)](#) ont avancé une théorie simple mais puissante, qui peut s'exposer comme suit. Depuis des décennies, la demande agrégée aux Etats-Unis a eu tendance à baisser en raison de divers changements structurels – moindres taux de croissance de la productivité et de la population, augmentation de la demande d'actifs libellés en dollars par les économies émergentes, ect. Cependant, la bulle des titres adossés à des créances hypothécaires a empêché la demande agrégée de "trop" chûter, c'est-à-dire que la contrainte représentée par le taux d'intérêt plancher (le ZLB) n'était pas saturée. Mais dès que cette bulle a implosé, la FED a heurté le ZLB et l'économie américaine est entrée dans une période de stagnation séculaire: une dépression permanente due à une pénurie de demande agrégée. Entre autres, [Caballero et al. \(2008\)](#) fournissent des faits stylisés compatibles avec cette interprétation des événements.

Dans cet article, je formalise l'intuition de [Summers \(2013\)](#) et [Krugman \(2013\)](#) à l'aide d'une version OLG du modèle New Keynesian (NK) standard, augmenté de bulles d'actifs et de capital. Je prouve que leur intuition est correcte, quoique avec un léger raffinement: la stagnation séculaire

est une condition nécessaire, mais non suffisante, pour que les bulles d'actifs stimulent le PIB. En effet, alors que les bulles d'actifs éloignent systématiquement l'économie de la stagnation séculaire, cela ne se traduit pas nécessairement par un niveau de production supérieur. Des bulles d'actifs permettent à l'économie de produire à pleine capacité, tout en réduisant considérablement cette capacité sous certaines calibrations: une stagnation de l'offre se substitue à une stagnation de la demande. En particulier, les bulles d'actifs sont expansionnistes si et seulement si l'économie souffre d'une stagnation séculaire "forte", en ce sens que la demande agrégée est gravement déficiente; s'il n'y a qu'une stagnation séculaire "légère", les bulles d'actifs réduisent le PIB.

Le modèle comporte trois ingrédients principaux: des rigidités nominales, la structure en OLG et une distribution particulière – mais intuitive – du revenu des facteurs entre les agents jeunes et les agents âgés <sup>31</sup>. Il y a un unique bien final qui peut être consommé ou transformé en capital. Ce bien est produit par un secteur concurrentiel qui agrège un continuum de variétés intermédiaires. Chaque variété est produite par une entreprise intermédiaire à partir de capital et de travail suivant une technologie Cobb-Douglas. Ces entreprises sont en concurrence monopolistique et font face à des frictions dans l'ajustement des prix nominaux à la Rotemberg. Les ménages vivent deux périodes. Jeunes, ils fournissent du travail, consomment et épargnent via trois actifs: les bulles, des obligations nominales et des actions/du capital. Quand ils sont âgés, ils perçoivent les rendements de leurs actifs, vendent leur stock de bulles, reçoivent les profits distribués par les entreprises et consomment la totalité de leur revenu. Les marchés du travail ainsi que les marchés financiers sont concurrentiels.

Comme dans le modèle NK de base, étant donné le stock de capital, il existe un niveau de production naturel / potentiel / déterminé par l'offre <sup>32</sup>, et un taux d'intérêt naturel qui permet d'égaliser la demande agrégée au niveau naturel de production. Dans une économie à prix flexibles, si les jeunes ménages souhaitent beaucoup épargner, le taux d'intérêt naturel diminue afin de réduire le coût du capital et de stimuler l'investissement par les entreprises. Ceci rétablit l'équilibre sur les marchés financiers et le marché des biens. Mais dans une économie à prix visqueux, le taux d'intérêt est sous le contrôle de la banque centrale, et ne correspond donc pas nécessairement au taux naturel. En temps normal, la banque centrale qui cible l'inflation implémente un taux d'intérêt

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<sup>31</sup>Les deux derniers ingrédients sont nécessaires pour introduire (*i*): un écart entre le taux d'intérêt et le facteur d'actualisation ainsi que (*ii*): un risque de revenu procyclique. D'autres hypothèses sur les marchés financiers et la répartition du revenu produiraient des résultats similaires, sous réserve de (*i*) et de (*ii*).

<sup>32</sup>Le niveau naturel / potentiel de  $X$  fait référence à sa valeur lorsque les prix sont totalement flexibles, *étant donnée une séquence de capital et bulle*. L'écart de  $X$  fait référence à la déviation de  $X$  par rapport à son niveau naturel; l'écart de  $X$  potentiel correspond à l'écart du niveau naturel de  $X$  lorsque les prix sont visqueux par rapport au niveau naturel lorsque les prix sont flexibles, *étant donnée une séquence de bulle*.

nominal compatible avec un écart de taux d'intérêt nul et donc un écart de production nul.

Cependant, cela peut être rendu impossible par le ZLB. Si la demande d'actifs (l'épargne agrégée) est importante par rapport à l'offre d'actifs (l'investissement agrégé), le taux d'intérêt naturel est si bas que la banque centrale heurte le ZLB. Dans la trappe à liquidité, tout choc réduisant le taux d'intérêt naturel se traduit par un écart de taux d'intérêt: compte tenu du niveau naturel de production, l'épargne agrégée excède l'investissement agrégé. Faisant face à une faible demande, les entreprises réduisent leur production, c'est-à-dire qu'elles demandent moins de capital et de travail. A court terme, i.e étant donné le stock de capital, le travail d'équilibre diminue par rapport au niveau naturel: il existe un écart de production négatif; à plus long terme, les entreprises investissent moins et l'économie à prix visqueux accumule moins de capital que l'économie à prix flexibles: il existe un écart de production potentielle négatif. Il est important de noter que cette trappe à liquidité peut durer un nombre arbitraire de périodes, voire même de manière permanente: c'est la stagnation séculaire.

Mais cette dépression n'est pas incurable: l'économie décentralisée dispose d'un outil formidable pour faire face à une pénurie de demande agrégée: les bulles d'actifs. En effet, sous l'hypothèse – relativement faible – que le taux de croissance de la production est supérieur au ZLB moins la cible d'inflation de la banque centrale - que j'appellerai la limite inférieure réelle (RLB), une économie en stagnation séculaire est dynamiquement inefficace. Comme [Tirole \(1985\)](#) l'a démontré, l'inefficience dynamique est une condition nécessaire et suffisante pour l'émergence de bulles d'actifs rationnelles: si une économie dynamiquement efficiente a un régime permanent unique sans bulle, une économie dynamiquement inefficace a deux régimes permanents, dont l'un est sans bulle mais l'autre inclut une bulle. Dans un modèle OLG, une bulle d'actifs redistribue les revenus des acheteurs d'actifs - les jeunes ménages - aux vendeurs d'actifs - les ménages âgés. Les marchés financiers étant incomplets, la propension marginale à consommer des revenus courants des premiers est supérieure à celle des personnes âgées. Ainsi, une bulle d'actifs réduit le taux d'épargne agrégé et soutient la consommation agrégée.

Etant donné le taux d'intérêt, un choc de bulle, c'est-à-dire le passage d'un équilibre sans bulle à un équilibre avec bulle, augmente donc de manière permanente la demande agrégée. Par conséquent, il stimule le travail ainsi que le stock de capital à l'équilibre tant que l'économie reste dans la trappe à liquidité: l'écart de production et l'écart de production potentielle augmentent. Si la demande agrégée augmente fortement après le choc, l'économie peut même sortir de la trappe à liquidité. Au fur et à mesure que la banque centrale recouvre son pouvoir sur l'économie, elle

commence à compenser l'effet "marginal" du choc sur la demande agrégée afin de lutter contre les pressions inflationnistes. Ainsi, dès que le ZLB devient non contraignant, les écarts de production et de production potentielle se stabilisent à zéro – comme si les prix étaient flexibles, mais la production potentielle diminue à mesure que l'investissement est progressivement découragé par le taux d'intérêt plus élevé - par rapport à l'équilibre à prix flexibles sans bulle. En conséquence, l'effet initial du choc de bulle sur la production devient ambigu: les écarts de production et production potentielle ont augmenté, mais le niveau naturel de la production a diminué. Si le choc ne déclenche pas une réaction monétaire importante, c'est-à-dire si l'économie ne s'éloigne pas "trop" du ZLB, l'effet positif sur la demande (accroissement des écarts de production et production potentielle) est supérieur à l'effet négatif sur l'offre (production potentielle plus faible); l'inverse est vrai si le choc déclenche une réaction monétaire importante.

En équilibre général, la taille de la bulle d'actifs en régime permanent, et donc la force du choc, est endogène. En effet, la bulle d'équilibre égalise le taux d'intérêt naturel au taux de croissance de l'économie. En supposant que ce taux de croissance soit supérieur au RLB, les bulles d'actifs conduisent toujours l'économie hors de la trappe à liquidité. Tant que l'économie demeure dans la trappe, elle est Keynésienne: la force de l'effet positif lié à la demande peut être déterminée par l'écart entre le taux d'intérêt naturel sans bulle et le RLB; mais dès que l'économie sort de la trappe à liquidité, elle devient Néo-classique: la force de l'effet négatif sur l'offre peut être déterminée par l'écart entre le RLB et le taux de croissance de la production. Ainsi, les bulles d'actifs sont expansionnistes dans les économies qui souffrent d'une demande agrégée fortement déficiente, et où le taux de croissance de la production est relativement proche du RLB. Dans un tel environnement, l'éclatement d'une bulle d'actifs plonge l'économie dans la trappe à liquidité, potentiellement pour toujours: les bulles d'actifs évitent ou retardent une dépression due à une pénurie de demande agrégée.

Bien que l'analyse précédente ait principalement concerné les régimes permanents du modèle, je propose également une analyse dynamique. En particulier, je montre que le modèle a toujours des équilibres; la dynamique autour du régime sans bulle est déterminée si et seulement si le régime avec bulle n'existe pas; si les deux régimes coexistent, l'écart de production est déterminé autour de l'un comme de l'autre, mais la bulle d'actifs n'est déterminée qu'autour du régime avec bulle. J'utilise cette indétermination pour modéliser la théorie de la bulle-stagnation séculaire d'une autre manière: j'introduis une bulle stochastique qui suit un processus de Markov à deux états. A court terme, c'est-à-dire tant que la bulle n'éclate pas, elle soutient l'investissement

et la production: l'économie échappe à la trappe à liquidité et accumule progressivement du capital. Mais dès que la bulle éclate, l'économie tombe dans la trappe à liquidité. L'écart de production est inférieur à sa valeur de long terme au cours de la phase d'éclatement car l'économie désaccumule soudainement l'excédent de capital par rapport au stock en régime permanent. Puis, après une période, l'économie retrouve son régime permanent de stagnation séculaire.

**Revue de la littérature** Depuis le discours de [Summers \(2013\)](#) au FMI, la littérature sur la stagnation séculaire croît rapidement. L'une des premières contributions est [Eggertsson et al. \(2017\)](#), qui ont montré qu'un modèle OLG-NK à trois périodes avec des salaires visqueux et des frictions financières présente un régime permanent de stagnation séculaire sous certains étalonnages. Ils ont fourni divers exemples de chocs qui dépriment de manière persistante la demande agrégée - par exemple un choc de désendettement ou un ralentissement de la croissance de la population ou de la productivité - ainsi que diverses politiques permettant d'échapper à la stagnation séculaire - par exemple un objectif d'inflation plus élevé ou des dépenses publiques plus élevées. [Garrec and Touze \(2016\)](#) ont complété cette analyse en montrant qu'un choc de consommation du gouvernement soulève un arbitrage dans une version du modèle qui inclut le capital: si le choc est trop faible, l'économie demeure en stagnation séculaire; s'il est trop important, il réduit la production en évincant le capital. [Michau \(2018\)](#) a fourni un autre moyen de modéliser la stagnation séculaire, qui ne repose pas sur une structure OLG, mais plutôt sur des préférences pour la richesse dans une économie avec un agent représentatif. Il a ensuite décrit les propriétés particulières du régime permanent de stagnation séculaire - les paradoxes de l'épargne, de la flexibilité et du travail, effectué une analyse de bien-être et proposé diverses politiques fiscales permettant d'implémenter l'allocation optimale.

Je contribue à cette littérature en étudiant un autre moyen de sortir de la stagnation séculaire: les bulles d'actifs. En effet, je montre que les bulles d'actifs peuvent empêcher ou retarder une stagnation séculaire. Une différence importante par rapport aux autres types de chocs de demande, par exemple un choc de consommation du gouvernement, réside dans le fait que les bulles d'actifs sont endogènes: la force du choc à long terme est déterminée par les fondamentaux de l'économie; de plus, comme ce choc est généré par les anticipations des agents, qui sont sujettes à des changements soudains, il fournit une explication à l'origine de la crise. Enfin, mon modèle OLG à deux périodes avec des prix visqueux est beaucoup plus simple et beaucoup plus proche du modèle NK classique que les modèles OLG à trois périodes avec des salaires visqueux et des frictions financières de

[Eggertsson et al. \(2017\)](#) et de [Garrec and Touze \(2016\)](#). Cela facilite la comparaison avec le modèle NK usuel et permet de comprendre quelles déviations sont responsables de la possibilité d'une stagnation séculaire.

Le premier article qui incluait des bulles d'actifs dans les modèles OLG-NK est [Kocherlakota \(2013\)](#). Il a montré qu'une baisse du prix de la terre - due à l'implosion d'une bulle ou à une baisse des fondamentaux - déprime de manière permanente la demande agrégée lorsque la politique monétaire ne réagit pas et que les salaires nominaux sont rigides, c'est-à-dire génère une stagnation séculaire (bien qu'il n'ait pas revendiqué l'étiquette). Dans un modèle OLG-NK qui inclut des frictions financières et des agents hétérogènes, [Caballero and Farhi \(2018\)](#) ont montré que la stagnation séculaire peut résulter d'une trappe à sûreté plutôt que d'une trappe à liquidité si certains agents sont infiniment averses au risque; ils ont également montré que des bulles d'actifs non risquées augmentaient la demande et la production agrégées dans un tel environnement.

Mais ces papiers n'incluent pas le capital, c'est-à-dire qu'ils supposent que le stock de capital est fixe. Bien que cette approximation ait une certaine validité à court terme, elle est clairement violée à long terme. Cette omission néglige un important canal de propagation par lequel les chocs de demande affectent l'offre à long terme. Sans capital, les bulles d'actifs sont expansionnistes (dans la trappe) ou n'affectent pas l'allocation réelle (hors de la trappe). Incluant le capital, je montre que les bulles soulèvent un nouveau compromis: un effet positif sur la demande (augmentation des écarts de production et production potentielle) contre un effet négatif sur l'offre (réduction de la production potentielle). Selon les fondamentaux de l'économie, des bulles d'actifs peuvent alors stimuler ou diminuer la production.

[Gali \(2014\)](#) a étudié l'incidence de la présence de bulles d'actifs sur la conduite de la politique monétaire optimale dans un modèle OLG-NK. [Gali \(2017\)](#), en utilisant l'indétermination des prix des actifs autour d'un régime permanent sans bulles mais dynamiquement inefficace, a montré que les bulles d'actifs fournissent une théorie des chocs de demande endogènes; il a ensuite étudié dans quelle mesure l'existence de bulles d'actifs modifiait le principe de Taylor habituel. Mon analyse est complémentaire à la sienne. En effet, je réponds à la question opposée: comment la conduite de la politique monétaire affecte-t-elle les effets macroéconomiques des bulles d'actifs? Pour ce faire, j'utilise un modèle qui inclut le capital et le ZLB – deux caractéristiques dont il fait abstraction, mais qui sont essentielles à mon histoire.

A cet égard, mon analyse est plus proche de [Hanson and Phan \(2017\)](#). Ils ont introduit des salaires visqueux dans un modèle OLG avec des investisseurs hétérogènes qui diffèrent selon leur

productivité, ainsi que des frictions financières. Ils ont montré que des chocs de création de bulles d'actifs peuvent stimuler l'investissement et la production, mais au prix d'une récession importante post-éclatement. Nos analyses sont complémentaires. Je reproduis leurs résultats, mais à travers un mécanisme différent et avec une différence notable: l'économie avant l'éclatement de la crise n'est pas nécessairement en plein essor. Au contraire, l'économie produit à pleine capacité tant que la bulle n'éclate pas, connaît une grave crise lors de la phase d'éclatement, puis reste définitivement déprimée. De plus, dans mon modèle, une bulle d'actifs stimule l'investissement en augmentant la consommation agrégée; dans leur modèle, les chocs de création de bulles augmentent l'efficience marginale de l'investissement.

Depuis les articles phares de [Martin and Ventura \(2012\)](#) et de [Farhi and Tirole \(2012\)](#), divers auteurs ont montré que les bulles d'actifs peuvent stimuler l'investissement et la production dans des économies à prix flexibles avec des frictions financières.<sup>33</sup> Dans ces articles, les bulles d'actifs redistribuent le revenu entre agents hétérogènes, et ces transferts accroissent l'investissement et la production car: (i) le taux d'épargne, (ii) l'offre de travail, (iii) l'efficience marginale de l'investissement, ou (vi) la PTF, augmente. Dans une économie à prix visqueux, puisqu'ils augmentent le niveau naturel de production, (i), (ii) et (iv) constitueraient des chocs de demande négatifs: mon article repose sur des mécanismes opposés; (iii) serait amplifié par la présence de rigidités nominales, mais j'en fais ici abstraction pour garder le modèle aussi simple que possible.

Enfin, depuis [Krugman \(1998\)](#) et [Eggertsson and Woodford \(2003\)](#), il existe une littérature abondante sur la trappe à liquidité dans les modèles NK.<sup>34</sup> Alors que cette littérature repose généralement sur un choc exogène portant sur un paramètre structurel pour plonger l'économie dans une trappe à liquidité temporaire<sup>35</sup>, je montre qu'un choc non-fondamental portant sur les anticipations de prix des actifs peut conduire l'économie dans la trappe à liquidité pour une durée indéterminée.

## Secular Stagnation or Secular Boom?

Motivé par la longue crise américaine de 1929, [Hansen \(1939\)](#) a introduit le concept de stagnation séculaire: il a prédit que les Etats-Unis étaient entrés dans une dépression très persistante, voire

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<sup>33</sup>Une liste restreinte comprend: [Kocherlakota \(2009\)](#), [Shi and Suen \(2014\)](#), [Miao and Wang \(2015a\)](#), [Miao and Wang \(2015b\)](#), [Hirano et al. \(2015\)](#). Voir [Miao \(2014\)](#) pour une bonne introduction à cette littérature.

<sup>34</sup>Une liste restreinte comprend: [Lorenzoni and Guerrieri \(2011\)](#), [Christiano et al. \(2011\)](#), [Eggertsson and Krugman \(2012\)](#), [Correia et al. \(2013\)](#), [Korinek and Simsek \(2016\)](#)

<sup>35</sup>Les exceptions incluent [Schmitt-Grohe and Uribe \(2012\)](#), qui considèrent un choc sur les anticipations d'inflation.



permanente, due à une pénurie de demande agrégée. Peu de temps après le discours de Hansen devant l'AEA, la seconde guerre mondiale et le boom économique qui a suivi ont réfuté l'hypothèse de la stagnation séculaire. Près de 80 ans plus tard, [Summers \(2013\)](#) a redonné vie à cette idée afin d'expliquer la dynamique de l'économie américaine avant et après 2008.

Cependant, pour citer [Eggertsson et al. \(2017\)](#), "la stagnation séculaire n'émerge pas naturellement des modèles utilisés actuellement dans la littérature". Le modèle Nouveau Keynesien (NK) a été conçu pour étudier des fluctuations à court terme autour d'une tendance donnée. Bien que le modèle classique <sup>36</sup> illustre avec élégance l'intuition Keynésienne selon laquelle les fluctuations de la production à court terme résultent de chocs sur la demande agrégée (un choc négatif diminue la production, un choc positif augmente la production), il ne peut pas, ou ne devrait pas, être utilisé pour étudier la stagnation séculaire.

En effet, la stagnation séculaire implique des chocs de demande très persistants, voire permanents. Une caractéristique très problématique pour le modèle NK: à moins que des hypothèses subtiles ne soient ajoutées au modèle classique, le modèle NK n'est pas du tout Keynésien à moyen et à long terme. Au lieu de cela, il devient entièrement "Néo-Fisherien" lorsque que les chocs deviennent suffisamment persistants: des chocs de demande positifs provoquent une baisse de la production, des chocs de demande négatifs augmentent la production. Au lieu d'une stagnation séculaire, le modèle prédit un boom séculaire lorsque la demande agrégée est chroniquement déficiente (si l'économie converge).

Il est également bien connu que le modèle NK classique fait d'autres prédictions surprenantes. Une liste non exhaustive comprend les points suivants: lorsque la politique monétaire suit une règle de Taylor qui inclut le ZLB, la détermination locale implique l'indétermination globale ([Benhabib et al., 2001](#)); des chocs de demande futurs anticipés ont des effets beaucoup trop importants sur la production actuelle ([Del Negro et al., 2015](#)); le paradoxe Néo-Fisherien: une réduction du taux d'intérêt nominal peut être déflationniste ([Cochrane, 2014](#)); et plus généralement, les lois de la macroéconomie NK semblent changer radicalement lorsque le ZLB est saturé et que les chocs deviennent suffisamment persistants, par exemple le boom séculaire.

A travers un modèle illustratif (un TANK avec inégalités cycliques et liquidité cyclique), je fournis un cadre cohérent (qui s'appuie fortement sur [Auclert and Rognlie \(2018\)](#) et [Auclert et al. \(2018\)](#), décrits ci-dessous) pour comprendre les prédictions étonnantes du modèle NK, et analyser et

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<sup>36</sup>C'est-à-dire le modèle à 3 équations avec des marchés financiers complets, pas de liquidité, etc. En général, j'appelle modèles classiques les modèles NK à 3 équations, avec inégalités a-cycliques, risque de revenu a-cyclique et liquidité a-cyclique, et préférences usuelles.

calculer les effets de chocs de demande (temporaires, persistants ou permanents) dans des modèles NK résolubles sans recourir à des simulation numériques. Je représente l'équilibre du modèle en termes de courbes d'offre et de demande d'actifs et souligne l'importance cruciale des élasticités de la demande d'actifs et de l'offre d'actifs par rapport à l'écart de production en équilibre général (les élasticités GE). Le(s) signe(s) de ces élasticités à divers horizons temporels, au ZLB et en temps normal, détermine(nt) si le modèle fait des prédictions déroutantes et / ou indéterminées, y compris le boom séculaire, ou des prédictions classiques et déterminées.

Par exemple, pour un macro-économiste intéressé par l'étude de la stagnation séculaire, les deux propriétés suivantes sont souhaitables: (i): il existe un unique équilibre en régime permanent; (ii): si la demande agrégée est chroniquement déficiente, il y a stagnation séculaire plutôt qu'un boom séculaire. Je montre que le modèle sans liquidité à long terme prédit (i) et (ii) si et seulement si l'élasticité GE à long terme de la demande d'actifs est strictement positive à la fois au ZLB et en temps normal.

Bien entendu, cette condition peut échouer sur plusieurs fronts: si l'élasticité change de signe lorsque le ZLB est saturé, l'économie n'a pas de régime permanent lorsque la demande agrégée est chroniquement déficiente, mais elle a plusieurs états d'équilibre en régime permanent (un dans la trappe à liquidité, l'autre non); si les deux élasticités sont négatives, l'économie présente un unique équilibre en régime permanent, mais, si la demande agrégée est chroniquement déficiente, elle subit un boom séculaire au ZLB plutôt qu'une stagnation séculaire.

Ces résultats peuvent être généralisés à des horizons temporels plus courts, à savoir des chocs de demande non permanents. Par exemple, si le processus de choc suit une chaîne de Markov à deux états – l'état  $l$  est l'état stationnaire et il est absorbant; à court terme, l'économie est dans l'état  $h$ ; la probabilité de transition de  $h$  à  $l$  est  $p \in [0, 1)$  – alors la même proposition s'applique si nous remplaçons l'élasticité GE à long terme par celle de l'horizon temporel (persistance)  $p$ , et régime permanent par équilibre stationnaire.

Dans le modèle log-linéaire, l'élasticité GE de persistance  $p$  de la demande d'actifs peut être calculée comme une combinaison linéaire des élasticités à court terme (zéro persistance) et à long terme (régime permanent):

$$\Omega_p^\phi = (1 - p)\Omega_0^\phi + p\Omega_1^\phi$$

Ici,  $\phi$  indexe le coefficient d'inflation dans la règle de Taylor:  $\phi = p$  correspond à un taux d'intérêt réel constant;  $\phi = 0$  à un taux d'intérêt nominal constant.

Le modèle NK classique (avec à la fois inégalités et liquidité a-cycliques) présente une élasticité en équilibre général (GE) à court terme strictement positive:  $\Omega_0^\phi > 0$  pour tout  $\phi \geq 0$ ; toutefois, l'élasticité GE à long terme est strictement positive si et seulement si la banque centrale suit le principe de Taylor:  $\Omega_1^\phi > 0$  si et seulement si  $\phi > 1$  (et  $\Omega_1^1 = 0$ ). Ainsi, alors que les chocs deviennent de plus en plus persistants, l'élasticité diminue progressivement jusqu'à devenir négative: des chocs de demande légèrement négatifs induisent deux équilibres stationnaires (l'un dans la trappe, l'autre en dehors); des chocs de demande fortement négatifs induisent l'absence de tout équilibre stationnaire.

Ces prédictions problématiques peuvent être reliées au signe de la réponse de la production à un choc en GE. Le long d'un équilibre stationnaire, et en faisant abstraction de "tâches solaires", la réponse GE de la production  $\hat{x}_t$  à un choc de demande donné  $e_t$  de persistance  $p$  peut être calculée comme suit: <sup>37</sup> (à une approximation du premier ordre près):

$$\hat{x}_t = -\frac{d\hat{a}_t}{de_t} e_t$$

Où  $\frac{d\hat{a}_t}{de_t}$  est l'élasticité à l'équilibre partiel (PE) de la demande d'actifs par rapport au choc.

Dans l'explication qui suit, je vais supposer l'élasticité GE strictement positive en temps normal; la mécanique lorsque l'élasticité est négative peut être déduite par symétrie. Lorsque la banque centrale cible l'inflation, un choc fortement négatif réduit l'écart de production jusqu'à ce que le ZLB devienne satûré. Si l'élasticité GE ne change pas de signe au ZLB, l'écart de production continue à se réduire jusqu'à ce que la demande d'actifs ait suffisamment diminué pour équilibrer les marchés financiers. Si, toutefois, l'élasticité GE change de signe au ZLB, le choc de demande négatif génère soudainement un important boom de la production qui éloigne l'économie de la trappe. Mais, hors de la trappe, le multiplicateur change à nouveau de signe: cela ne peut pas être un équilibre, et par conséquent tous les équilibres sont non-stationnaires. De plus, si le choc n'est pas fortement négatif, il y a des équilibres multiples si et seulement si l'élasticité GE change de signe – l'intuition est discutée dans le texte principal.

Des conditions similaires, bien que plus complexes, impliquant les élasticités GE de la demande d'actifs et de l'offre d'actifs, peuvent être dérivées dans le modèle avec une offre d'actifs endogène. En particulier, une condition nécessaire pour régler les prédictions problématiques est que la dif-

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<sup>37</sup> [Auclert and Rognlie \(2018\)](#) dérivent une telle formule pour les multiplicateurs de la production à long terme, c'est-à-dire  $p = 1$ , dans un modèle HANK sous une politique de taux d'intérêt constant, c'est-à-dire  $\phi = p$ . Je généralise, bien que dans un cadre beaucoup plus simple.

férence entre les élasticités GE de la demande d'actifs et l'offre d'actifs soit strictement positive. Ainsi, dans le modèle avec liquidité, une élasticité GE strictement positive de la demande d'actifs n'est plus une condition nécessaire ni suffisante.

J'introduis une offre de liquidité d'une manière particulière: je me concentre sur les bulles d'actifs. Etant donné que les régimes permanents avec et sans bulles coexistent, cela fournit un cadre naturel pour comparer la réponse dynamique de l'économie à divers chocs selon qu'il existe ou non une offre de liquidité positive à long terme. Cependant, cela impose de fortes restrictions sur l'élasticité GE de l'offre d'actifs: elle tend vers l'infini lorsque les chocs deviennent permanents, et devient négative au ZLB. Par conséquent, mon modèle "complet" ne permet pas de prédire (i) et (ii) lorsque les chocs sont très persistants. Cela souligne l'importance du traitement du long terme dans les modèles NK: des différences subtiles dans les hypothèses conduisent à de grandes différences dans les résultats, à la fois qualitativement et quantitativement.

Néanmoins, le modèle avec liquidité me permet d'analyser les interactions réciproques entre les bulles d'actifs et la politique monétaire. Une conclusion habituelle et intuitive dans la littérature est que, puisque la politique monétaire ne peut pas affecter le taux d'intérêt naturel, elle n'affecte pas la condition d'existence des bulles d'actifs. Je montre que cette conclusion se révélera généralement fautive dans les modèles TANK et HANK. En effet, dans les modèles NK avec inégalités cycliques, si les prix restent rigides même à long terme, la politique monétaire peut affecter le niveau de la demande d'actifs en régime permanent. Selon le signe de l'élasticité GE à long terme de la demande d'actifs, un taux d'intérêt nominal bas peut favoriser ou empêcher l'émergence de bulles d'actifs de type rationnel.

Dans la première partie de l'article, je me base sur l'analyse séminale de [Werning \(2015\)](#). J'introduis des inégalités cycliques et des marchés incomplets dans le modèle NK classique. Je considère une structure très simple à la [Woodford \(1990\)](#): il existe deux états, caractérisés par des niveaux de revenus différents, et deux agents représentatifs basculant continuellement d'un état à l'autre à la fin de chaque période. La répartition des revenus entre ces deux états est endogène: l'épargnant, c'est-à-dire l'agent dans l'état  $s$ , reçoit une part plus grande du revenu agrégé que l'emprunteur, l'agent dans l'état  $b$ , mais cette part de revenu varie le long du cycle. La part du revenu de l'épargnant est fonction de l'écart de production: si cette fonction est croissante, les inégalités sont pro-cycliques; si elle est constante, les inégalités sont a-cycliques; et si elle est décroissante, les inégalités sont contra-cycliques. En raison des frictions financières, l'agent dans l'état  $b$  ne peut emprunter: les chocs de revenus ne sont pas assurables. Cela génère un motif

d'épargne de précaution qui a la même "cyclicité" que les inégalités. Si les inégalités sont fortement pro-cycliques, l'épargne de précaution est fortement pro-cyclique, ce qui rend l'élasticité GE à long terme de la demande d'actifs strictement positive, même dans la trappe de la liquidité, ce qui résout alors les problèmes discutés ci-dessus.

Dans la deuxième partie du papier, je présente un modèle avec une offre de liquidités endogène: des bulles d'actifs rationnelles. Je montre que la liquidité endogène peut restaurer tous les problèmes du modèle NK, c'est-à-dire rétablir des multiplicateurs négatifs si les chocs sont très persistants, et ce malgré une élasticité GE positive de la demande d'actifs à long terme. Intuitivement, les statistiques suffisantes permettant de déterminer si l'économie est Keynésienne ou Néo-Fisherienne à long terme incluent les élasticités GE à long terme de la demande et de l'offre d'actifs. Pour avoir un modèle NK véritablement Keynésien, l'élasticité GE à long terme de la demande d'actifs doit être supérieure à celle de l'offre d'actifs: un niveau de production inférieur (plus élevé) doit diminuer (augmenter) davantage la demande d'actifs que l'offre d'actifs. Comme indiqué plus haut, mon modèle ne satisfait pas à ce critère en raison du type particulier de liquidité que je considère.

Enfin, je compare les multiplicateurs du modèle selon que les inégalités et / ou la liquidité sont cycliques. Pris isolément, les inégalités pro-cycliques ou la liquidité contra-cyclique ont tendance à réduire les multiplicateurs par rapport au modèle classique (tous deux non cycliques). Cependant, les prédictions du modèle qui suppose conjointement des inégalités cycliques et liquidité cyclique ne sont pas aussi claires. En effet, la liquidité endogène interagit avec le motif d'épargne de précaution endogène généré par les inégalité cycliques: lorsque l'épargnant accumule un stock d'actifs, le motif d'épargne de précaution est réduit. De manière tout à fait contre-intuitive, une offre de liquidité pro-cyclique peut constituer une force stabilisatrice, tandis qu'une offre de liquidité contra-cyclique peut constituer une force déstabilisatrice, selon de la nature cyclique des inégalités. Mais, sous des hypothèses relativement faibles, le modèle complet prédit des multiplicateurs plus faibles que le modèle classique si les inégalités sont pro-cycliques et la liquidité contra-cyclique.

Bien que j'utilise un ensemble particulier de micro-fondations, l'analyse en termes d'élasticités GE et les conditions associées peuvent être importées dans virtuellement tous les modèles NK analysables sans recourir à des simulations numériques. Au niveau micro, ces modèles diffèrent les uns des autres: les déterminants des élasticités GE changeront d'un modèle à l'autre. Cependant, ces modèles partagent la même représentation agrégée et, par conséquent tous les résultats en termes d'élasticités GE devraient être vérifiés.

**Revue de la littérature** Puisqu'il existe une littérature de plus en plus large qui tente de quantifier les effets macroéconomiques de différents chocs dans des modèles incluant une distribution réaliste des revenus et richesses <sup>38</sup>, il existe une littérature de plus en plus large qui cherche à mieux comprendre les prédictions de ces modèles dans des cadres plus simples. En effet, les modèles HANK sont difficiles à résoudre numériquement, ne peuvent pas être résolus analytiquement et incluent de nombreux canaux absents du modèle classique.

[Werning \(2015\)](#) est un exemple précoce. <sup>39</sup>: il a montré que *soit* le risque de revenu pro-cyclique *ou* la liquidité contra-cyclique permettait de résoudre le problème de la "forward guidance" dans un modèle NK avec des marchés incomplets. J'étends son analyse à d'autres types de chocs, à d'autres paradoxes, y compris la possibilité d'une stagnation séculaire plutôt que d'un boom séculaire, et j'inclus des inégalités cycliques et de la liquidité cyclique: je montre que ces deux mécanismes clés peuvent s'annuler mutuellement en GE.

Les articles les plus étroitement liés aux miens sont [Auclert and Rognlie \(2018\)](#) et [Auclert et al. \(2018\)](#). Cet article peut être vu comme une généralisation de leurs résultats, mais dans une version simplifiée de leur modèle. En effet, étant donné que je me limite à une classe de modèles (beaucoup) moins générale qu'ils ne le font, je peux étendre leurs résultats sur plusieurs dimensions au sein de cette classe de modèles. [Auclert et al. \(2018\)](#) ont introduit un nouvel ensemble de moments, les propensions marginales intertemporelles à consommer (IMPC), et ont montré que les multiplicateurs de chocs de demande dans les modèles HANK peuvent être calculés analytiquement à l'aide de la matrice IMPC (en supposant un taux d'intérêt constant). En outre, ils ont montré que les macroéconomistes peuvent discriminer les modèles en comparant les IMPC théoriques aux IMPC mesurés. [Auclert and Rognlie \(2018\)](#) ont créé un modèle HANK pour analyser les effets macroéconomiques de divers types de chocs d'inégalités, à court et à long terme. Ils ont montré que, lorsque la banque centrale ne réagit pas, un type particulier de choc d'inégalités a un effet important sur l'emploi à long terme. Ils ont également fourni des résultats analytiques: la stagnation séculaire n'est possible que si l'élasticité de la demande d'actifs est à long terme supérieure à celle de l'offre d'actifs; les multiplicateurs à long terme d'un choc permanent peuvent être calculés à l'aide des élasticités GE à long terme de l'offre et de la demande d'actifs.

D'un point de vue méthodologique, je leur emprunte beaucoup: comme [Auclert and Rognlie](#)

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<sup>38</sup>Des exemples récents incluent: [McKay and Reis \(2016\)](#), [McKay et al. \(2016\)](#), [Kaplan et al. \(2016\)](#) ou [Auclert and Rognlie \(2018\)](#).

<sup>39</sup>Parmis les articles récents analysant le risque de revenu dans les modèles NK avec des marchés incomplets, citons: [Ravn and Sterk \(2017\)](#), [Sterk and Ravn \(2017\)](#), [Acharya and Dogra \(2018\)](#).

(2018), je me concentre sur les élasticités GE de la demande et de l'offre d'actifs, et comme [Auclert et al. \(2018\)](#), je montre que les élasticités GE peuvent être utilisées pour calculer les multiplicateurs GE de manière simple - dans un modèle beaucoup plus simple. Néanmoins, j'étends leurs résultats, bien que dans une classe de modèles beaucoup plus restrictive. Premièrement, je fournis deux statistiques suffisantes pour calculer le multiplicateur GE dans des modèles NK analysables: la différence entre les élasticités GE à court terme de la demande et de l'offre d'actifs ainsi que la différence entre les élasticités à long terme. Deuxièmement, je propose un traitement unifié des multiplicateurs dans la trappe à liquidité ou lorsque la banque centrale suit une règle de Taylor. Troisièmement, j'étudie les prédictions du modèle lorsque le signe de ces élasticités dépend de l'horizon temporel ou / et du régime monétaire: en cours de route, je fournis une explication pour certaines prédictions surprenantes des modèles NK.

Ceci relie mon article à [Bilbiie \(2019\)](#) et à [Bilbiie \(2018a\)](#). [Bilbiie \(2019\)](#) a fourni une représentation des modèles NK en termes de fonction de consommation Keynesienne. Il a utilisé cette représentation pour calculer les multiplicateurs de chocs de demande, et effectué une décomposition entre effets directs et indirects, dans divers modèles NK, notamment un modèle analytique HANK avec des marchés incomplets et des inégalités cycliques. Il a également prouvé qu'une version calibrée de son HANK peut reproduire approximativement les prédictions des HANK quantitatifs. En utilisant le modèle HANK développé dans [Bilbiie \(2019\)](#), [Bilbiie \(2018a\)](#) a montré que divers problèmes du modèle NK (tels que la détermination sous un ancrage nominal ou la "forward guidance") sont résolus si les inégalités sont suffisamment pro-cycliques, mais au prix d'une baisse des multiplicateurs. Il a ensuite proposé deux solutions permettant de palier cet arbitrage: des règles de taux d'intérêt Wickselliennes ou un risque de revenu suffisamment contra-cyclique. Enfin, il a calculé la politique monétaire optimale, à la fois dans et hors de la trappe à liquidité.

Mon modèle sans liquidité est une version simplifiée de son HANK - un TANK - qui capture néanmoins le mécanisme principal décrit dans [Bilbiie \(2019\)](#) (la croix keynésienne intertemporelle); mais, puisque mon modèle est plus simple, je peux analyser une version du modèle qui inclut la liquidité. Je me base sur son analyse, et reproduis certains de ses résultats, mais selon une perspective différente: j'insiste sur le ou les signes des élasticités GE de l'offre et de la demande d'actifs à divers horizons, dans la trappe à liquidité et en temps normal. Ceci fournit un cadre général pour comprendre si et pourquoi les modèles analytiques NK font des prédictions surprenantes. De plus, s'il se concentre sur des chocs temporaires et des politiques optimales, je me concentre principalement sur des chocs très persistants ou permanents et sur la possibilité d'une stagnation séculaire.

Nos analyses sont complémentaires.

Il existe une littérature croissante sur la stagnation séculaire dans les modèles NK. En utilisant un modèle OLG, [Kocherlakota \(2013\)](#) a montré qu'une baisse du prix de la terre engendre une stagnation séculaire lorsque la politique monétaire ne répond pas et que les salaires nominaux sont rigides. Dans un modèle OLG-NK qui inclut des frictions financières et des agents hétérogènes, [Caballero and Farhi \(2018\)](#) ont montré qu'une stagnation séculaire peut survenir à cause d'une trappe à sûreté plutôt que d'une trappe à liquidité si certains agents sont infiniment averses au risque. [Eggertsson et al. \(2017\)](#) ont montré qu'un modèle OLG-NK à trois périodes, incluant des salaires visqueux et des frictions financières, présente un régime permanent de stagnation séculaire sous certaines calibrations. [Michau \(2018\)](#) a fourni un autre moyen de modéliser la stagnation séculaire, qui ne repose pas sur une structure OLG, mais plutôt sur des préférences incluant la richesse dans une économie avec un agent représentatif.

Il est assez difficile de comprendre quelles sont les hypothèses clefs qui rendent possible la stagnation séculaire dans ces articles, et quels sont les mécanismes clefs. En effet, d'un papier à l'autre, rigidités nominales, frictions financières, structure démographique, préférences, etc. varient. De plus, des différences subtiles sont importantes: alors que le modèle OLG à trois périodes de [Eggertsson et al. \(2017\)](#) est capable de générer une stagnation séculaire avec des *salaires* visqueux, le modèle OLG à deux périodes de [Bouillot \(2017\)](#) ne le pourrait. [Bouillot \(2017\)](#) a plutôt besoin de *prix* visqueux pour obtenir une stagnation séculaire "conventionnelle" plutôt qu'un boom séculaire. Je propose ici un cadre cohérent pour comprendre ce qui se passe dans ces modèles: ils font tous des hypothèses qui conduisent à une élasticité GE à long terme de la demande d'actifs supérieure à l'élasticité GE à long terme de l'offre d'actifs.

Les mêmes remarques s'appliquent à [Michaillat and Saez \(2018\)](#), qui ont montré qu'un modèle NK qui inclut la richesse dans la fonction d'utilité peut résoudre la plupart des problèmes du modèle manuel et générer une stagnation séculaire en tant que régime permanent. Mon analyse révèle que, bien que les papiers de [Bilbiie \(2018a\)](#) et de [Michaillat and Saez \(2018\)](#) aient des micro-fondements très différents, leurs résultats sont liés à l'élasticité GE de la demande d'actifs à long terme qui est strictement positive. Cependant, ces résultats ne sont pas robustes pas à l'inclusion de la liquidité dans le modèle.

Depuis que [Martin and Ventura \(2012\)](#) et [Farhi and Tirole \(2012\)](#) ont montré que les bulles d'actifs peuvent être expansionnistes dans les modèles OLG qui incluent des frictions financières, une littérature de plus en plus large a examiné les cycles économiques induits par les bulles d'actifs



<sup>40</sup>. Encore plus récemment, cette littérature a fusionné avec la littérature NK (par exemple, [Kocherlakota \(2013\)](#), [Gali \(2014\)](#), [Gali \(2017\)](#)). Je fais deux contributions à cette littérature. Premièrement, je contredis la conclusion habituelle selon laquelle la politique monétaire ne peut pas influencer sur les conditions sous lesquelles existent des bulles d'actifs. <sup>41</sup> Je montre que, lorsque les inégalités sont cycliques et que les prix sont visqueux, il existe deux régime permanents avec bulles si la banque centrale suit une règle de Taylor: la politique monétaire, par le "choix" d'un taux plancher, c'est-à-dire le "niveau" du ZLB, peut influencer la taille de la bulle dans l'un de ces régimes permanents et même l'éliminer. Si la politique monétaire est modélisée comme établissant une ancrage de taux d'intérêt nominal, il existe alors un régime permanent unique avec bulle, et son existence ainsi que la taille de la bulle sont toutes deux conditionnées au niveau du taux d'intérêt nominal. Deuxièmement, j'étudie comment les bulles d'actifs interagissent avec les inégalités cycliques pour influencer sur la réponse dynamique de l'économie à des chocs de demande.

Enfin, il existe de nombreux articles sur les trappes à liquidité dans les modèles NK, qui peuvent être divisés en deux camps: les trappes à liquidité fondamentales et les trappes à liquidité liées aux anticipations - pour reprendre la terminologie de [Bilbiie \(2018b\)](#). La littérature fondamentale a débuté avec les travaux phares de [Krugman \(1998\)](#) et de [Eggertsson and Woodford \(2003\)](#): un choc exogène sur les fondamentaux de l'économie réduit la demande agrégée et pousse l'économie dans la trappe à liquidité <sup>42</sup>; la littérature anticipations a débuté avec [Benhabib et al. \(2001\)](#): le modèle NK classique a deux équilibres, dont l'un est une trappe à liquidité. Mon analyse de la trappe à liquidité est proche de celle de [Bilbiie \(2018b\)](#), qui a analysé ces deux types de trappe à liquidité dans le modèle NK classique. En particulier, il a montré que les trappes liées aux anticipations surviennent lorsqu'il y a "suffisamment de persistance des choc et d'amplification intertemporelle endogène des informations futures"; et il a remarqué que les multiplicateurs des politiques budgétaires et monétaires habituelles changent de signe lorsque la trappe à liquidité est due à un changement dans les anticipations plutôt que dans les fondamentaux. En outre, il a calculé les politiques monétaire et fiscale optimales dans le cas Néo-Fisherien.

Je propose une condition simple et alternative pour déterminer quel type de trappe à liquidité est possible: sans liquidité à long terme, si l'élasticité GE de la demande d'actifs ne change pas

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<sup>40</sup>[Miao \(2014\)](#) propose une introduction à, ainsi qu'une revue de, cette littérature.

<sup>41</sup>Une exception est la [Wang et al. \(2017\)](#) où la cible d'inflation de la banque centrale a une incidence sur la condition d'existence, mais par un mécanisme complètement différent: leur modèle n'inclut ni le ZLB ni les inégalités cycliques, mais plutôt un type de friction financière très particulier qui rend l'inflation non neutre, même à long terme.

<sup>42</sup>Une liste restreinte comprend: [Lorenzoni and Guerrieri \(2011\)](#), [Christiano et al. \(2011\)](#), [Eggertsson and Krugman \(2012\)](#), [Correia et al. \(2013\)](#), [Korinek and Simsek \(2016\)](#).

de signe au ZLB, les trappe à liquidité sont liées aux fondamentaux; s'il y a changement de signe, les trappe à liquidité sont liées aux anticipations. Je propose également des conditions similaires, bien que plus compliquées, pour un modèle incluant de la liquidité à long terme. Ceci m'amène à conclure que tous les multiplicateurs changent de signe lorsque l'économie entre dans une trappe à liquidité liée aux anticipations. J'étudie également les trappes permanentes générées par des chocs importants: stagnation séculaire ou boom séculaire, dont il fait abstraction. Nos analyses sont complémentaires.

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# Essays on Asset Bubbles and Secular Stagnation

The first chapter questions the conventional intuition that a high concentration of income at the top of the distribution should promote the emergence of rational asset bubbles. I use an OLG model with financial frictions and heterogeneous agents that differ in terms of savings rate, portfolio choices and skills. I show that a high concentration at the top promotes the emergence of asset bubbles if and only if those asset bubbles are illiquid or financial markets are arbitrage-free. Instead, if asset bubbles are liquid and liquid assets pay a premium under illiquid assets, a low concentration promotes the emergence of asset bubbles. The second chapter studies the circumstances under which asset bubbles are expansionary in an OLG-New Keynesian that includes capital. I show that secular stagnation is a necessary but not sufficient condition. Indeed, asset bubbles stimulate investment, consumption and output if and only if there's a strong shortage of aggregate demand. Finally, the third paper shows that "standard" New Keynesian models make puzzling predictions when aggregate demand is chronically deficient – they predict a secular boom, and seeks to understand how those models must be adjusted to analyze secular stagnation. I emphasize the crucial role of the long run elasticities of asset demand and supply with respect to the output gap in general equilibrium; and I also connect the secular boom to other puzzling predictions of the New Keynesian model.

**Keywords:** Asset Bubbles, Secular Stagnation, New Keynesian, Macroeconomics

# Essais sur les Bulles d'Actifs et la Stagnation Séculaire

Le premier chapitre questionne l'intuition conventionnelle selon laquelle une forte concentration au plus haut de la distribution des revenus devrait favoriser l'émergence de bulles d'actifs rationnelles. J'utilise un modèle OLG avec des frictions financières et des agents hétérogènes qui diffèrent en termes de taux d'épargne, portefeuilles d'actifs et talents. Je montre qu'une forte concentration promeut l'émergence de bulles si et seulement si ces bulles sont illiquides ou si tous les actifs offrent les mêmes rendements. À l'inverse, lorsque les bulles sont liquides et les actifs liquides paient une prime de liquidité, une faible concentration promeut l'émergence de bulles. Le deuxième papier étudie les conditions sous lesquelles une bulle d'actif augmente le PIB dans un modèle OLG-Nouveau Keynesien incluant le capital. Je montre que la stagnation séculaire est une condition nécessaire mais non suffisante. En effet, les bulles ne stimulent le PIB que si la demande agrégée est très fortement déficiente. Le troisième papier démontre que les modèles Nouveaux Keynesiens font des prédictions paradoxales lorsque la demande agrégée est chroniquement déficiente – un boom séculaire plutôt qu'une stagnation séculaire, et analyse comment ajuster ces modèles pour qu'ils deviennent viables dans l'environnement actuel. Je souligne l'importance cruciale des élasticités de l'offre et de la demande d'actifs par rapport au PIB à long terme; j'effectue également une connexion entre le boom séculaire et d'autres prévisions paradoxales du modèle NK.

**Mots-clés:** Bulles d'Actifs, Stagnation Séculaire, Nouveau Keynesien, Macroéconomie