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Nhung LUU

**Housing market, intergenerational transfers
and the macroeconomy**

Sous la direction de

M. Hippolyte d'Albis, Directeur de Recherche CNRS.

Membre du Jury

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(**rapporteure**).

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Résumé en français

Les répercussions de la hausse mondiale des prix du logement à partir des années 1980 sur l'économie ont attiré un intérêt croissant pour la recherche. Le logement représentant une part importante des dépenses des ménages ainsi que de la richesse totale, il est important de comprendre le rôle du logement dans l'inégalité des richesses. Cependant, en raison de la disponibilité des données, peu de choses ont été faites pour comprendre ce problème. La richesse des ménages s'accumulant à partir de deux sources principales: l'épargne et les transferts de capital, il est important de comprendre le lien qui existe entre ces sources et le marché du logement lorsque les prix de l'habitation changent. Ainsi, les deux premiers chapitres de cette thèse visent à mieux comprendre l'importance de la filière logement pour l'économie. Plus précisément, le premier chapitre attire l'attention sur la corrélation entre les prix du logement et la production. En introduisant différents chocs dans l'économie, nous pouvons analyser leurs effets sur les prix du logement et l'accumulation de capital, ainsi que sur le mécanisme de transmission de ces chocs. D'autre part, le deuxième chapitre porte sur la manière dont le transfert intergénérationnel rend l'inégalité persistante via le marché du logement. En considérant une économie avec deux actifs d'investissement différents: le logement et le capital, nous montrons qu'il existe un équilibre dans lequel le logement est plus rentable que le capital. En tant que famille riche, c'est-à-dire celle qui a hérité d'une richesse relativement plus grande, peut accéder au marché du logement alors que la famille pauvre ne le peut pas en raison de la contrainte d'emprunt, l'inégalité persiste à long terme.

En outre, le patrimoine et les transferts entre ménages sont les deux canaux de transmission essentiels qui relient la richesse de manière différente. Bien que l'on pense que les transferts entre ménages contribuent au développement du capital humain et donc des revenus du travail, l'héritage joue un rôle décisif dans l'accumulation de capital et de richesse. Les données du compte de transfert national en France nous ont montré qu'au cours des dernières décennies, la composition des transferts privés a sensiblement évolué: d'une part plus dominante des transferts entre ménages à une part plus dominante en matière de succession. Ce changement entraînant deux effets négatifs, il est donc intéressant de comprendre pourquoi et comment cela se produit, ainsi que son lien avec l'inégalité de la richesse. Ces questions sont abordées dans le troisième chapitre de ma thèse.

On constate que les prix réels du logement ont considérablement augmenté dans un certain nombre de pays à partir des années 1980. La figure (11) ci-dessous illustre les données relatives aux prix du logement dans de nombreux pays de l'OCDE. Il en ressort que l'indice des prix de l'immobilier a augmenté de 120% en moyenne sur la période 1980-2007, par exemple en France, au Royaume-Uni, en Espagne, en Finlande, aux États-Unis, etc. Bien que les prix du logement tendent à baisser après leur éclatement, ils restent à un niveau supérieur à celui de l'année précédant le boom. Ainsi, l'étude de la fluctuation des prix du logement sur l'économie a suscité un intérêt marqué ces dernières années. De plus, le logement représente la plus grande classe d'actifs du portefeuille des ménages (OCDE, 2018). Une explication de ce fait est que le logement, différent des autres types d'actifs, n'est pas seulement un investissement, mais également un bien de consommation fournissant un flux de services tels que le logement, la chaleur et le confort (King (2015)). Par conséquent, lors de l'étude de la décision de l'individu lors de l'achat d'une maison, il est important de prendre en compte le motif de devoir un propriétaire.

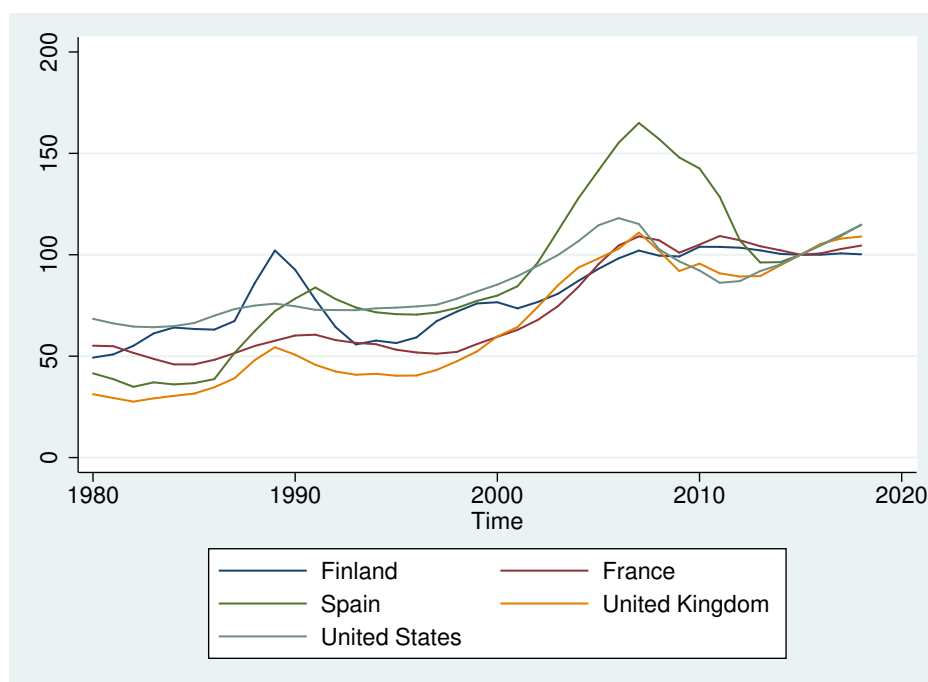


Figure 1: La dynamique de l'indice des prix réels du logement en France, en Espagne, en Finlande, au Royaume-Uni et aux États-Unis de 1980 à 2018.

Source: OECD

Dans le même temps, Piketty(2014), Capital au XXIe siècle, a attiré l'attention des universitaires et des lecteurs sur l'augmentation des inégalités. En particulier, à l'aide de données construites pour de nombreux pays tels que la Chine, l'Europe et les États-Unis, Piketty et ses collègues (voir, par exemple, Piketty(2014), Piketty et

Zucman(2014), Saez et Zucman(2016), Zucman(2019)) a montré que la concentration de la richesse a été signalée par une augmentation régulière remontant à la fin des années 1980. Les figures (2) et (3) ci-dessous montrent l'évolution de l'inégalité de la richesse en France, au Royaume-Uni et aux États-Unis. Comme on peut le constater, dans les trois pays, les inégalités affichent clairement une tendance à la hausse si l'on tient compte de la part de la richesse des 10% premiers ou des 1% premiers. On pourrait être intéressé de savoir si la hausse de la concentration de la richesse est due à la récente flambée des prix du logement . Si c'est le cas, on peut s'attendre à voir que certains groupes (les pauvres) sont systématiquement exclus du marché du logement. Connaître le mécanisme dans lequel l'inégalité de la richesse est générée et transmise d'une génération à l'autre est donc important pour l'implication politique.

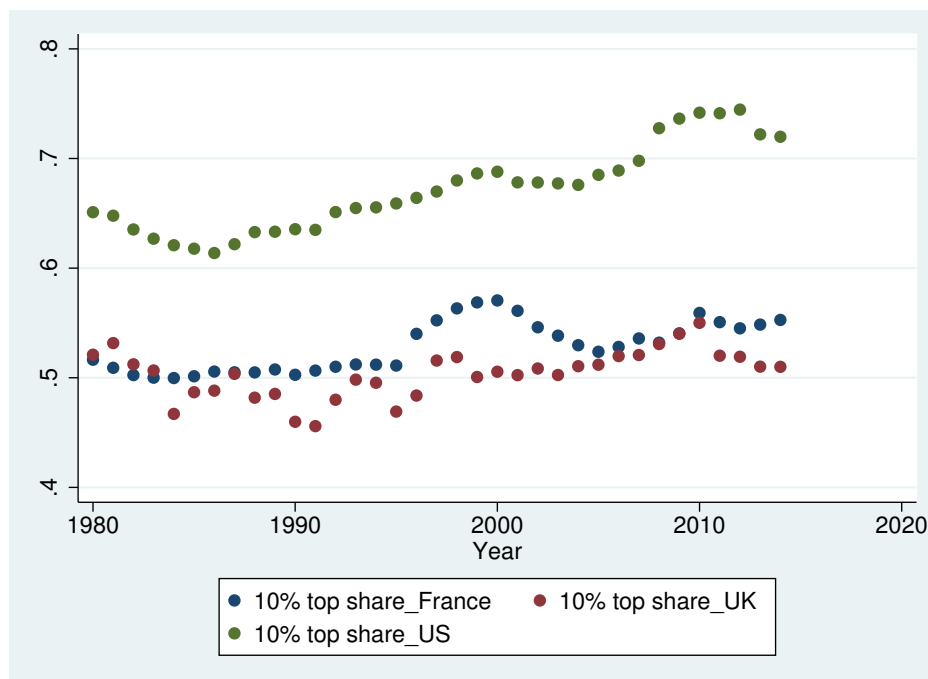


Figure 2: Evolution de la part de la richesse parmi les 10% les plus riches en France, au Royaume-Uni et aux États-Unis.

Source: WID

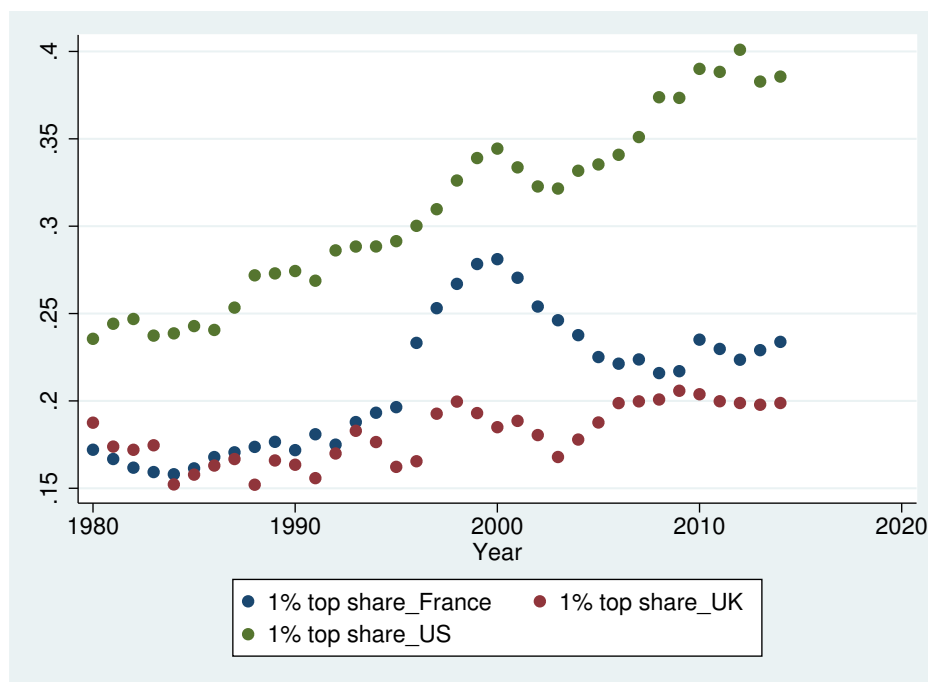


Figure 3: Evolution de la part de la richesse du top 1 % en France, au Royaume-Uni et aux États-Unis.

Source: WID

À partir de travaux préliminaires sur la richesse Guthrie (1963) ou Modigliani (1988), nous savons que l'épargne et les transferts, qui se présentent sous la forme de capital humain ou de transferts intergénérationnels, sont deux moyens essentiels pour que les ménages accumulent des richesses. L'étude du rôle du logement dans la disparité des richesses nécessite donc de comprendre ses relations avec l'accumulation de capital et les patrimoines.

L'importance du logement pour la consommation des ménages et l'épargne en capital a toujours été un sujet de recherche. Un certain nombre d'études ont attiré l'attention sur le bien-être des individus, car elles portent en particulier sur la relation entre les fluctuations des prix du logement et la consommation individuelle par effet de richesse et effet de substitution (voir, par exemple, Ludwig et Slok (2004), Iacoveillo (2004), Carroll et al. (2011) entre autres). Par exemple, Iacoveillo (2004) a suggéré que les ménages augmenteraient leur niveau de consommation si leur capacité d'emprunt était liée à leur richesse. En conséquence, la consommation globale augmentera en réponse à une augmentation du prix de l'immobilier. De plus, Buiter (2008) a ajouté qu'une modification des prix de l'immobilier n'aurait pas d'incidence pure sur la consommation si elle était due à une modification de la valeur fondamentale. Il a expliqué qu'une réduction de la valeur fondamentale des maisons rendrait les maisons moins chères. Ainsi, les propriétaires, c'est-à-dire ceux qui négocient sur

le marché du logement, seront les plus mal lotis, tandis que les locataires, c'est-à-dire ceux qui s'échangent sur le marché, seront mieux lotis. En conséquence, dans un contexte d'agent représentatif, la demande globale ne change pas car, dans un tel contexte, chaque locataire devient son propre locateur.

Cependant, de nombreuses études ont été réalisées sur les effets du logement sur la consommation des ménages et leur bien-être. Peu de choses ont été faites concernant le rôle du logement dans l'accumulation de capital et dans la croissance. Bien que le PIB et la consommation soient largement connus pour être fortement corrélés (Angelache (2011)). Cependant, il est nécessaire de comprendre les effets directs sur la décision d'épargne des ménages et la production économique via le mécanisme de logement. En outre, le logement et le capital étant les deux actifs courants des portefeuilles de ménages, il est important de comprendre si une augmentation du prix des logements peut entraîner une éviction ou une épargne en capital. Bien que ce sujet ait reçu peu d'attention dans la littérature.

Les recherches portant sur la question de savoir si les immobilisations et le logement sont substituables ou complémentaires fournissent des preuves mitigées. Les articles visant à évaluer la relation entre ces deux actifs sont, par exemple, Iacoviello (2005), Deaton and Laroque (2001), Kahn (2009) et Boris et Reichlin (2018). En particulier, Deaton and Laroque (2001) a étudié les impacts des terres sur le processus de croissance en utilisant un cadre de générations qui se chevauchent. En comparant les stocks de capital dans l'économie avec et sans terre, ils ont constaté que la présence de la terre abaisse le stock de capital à l'équilibre en entraînant la réaffectation de l'épargne du capital vers la terre. Le document suggère donc que les terres soient nationalisées et fournies à un loyer nul. Dans une approche différente, cite Iacoviello2005, utilisant un modèle de cycle économique monétaire avec une contrainte d'emprunt liée à la richesse du logement, a montré que les prix du logement correspondent à l'investissement en capital en réponse à un certain nombre de chocs. Cependant, le document a particulièrement attiré l'attention sur les effets d'un resserrement monétaire qui diminue les taux d'intérêt et déclenche une déflation par la dette. Selon les travaux, la déflation par la dette entraînera non seulement une baisse des prix de l'immobilier, mais également une démotivation des particuliers, ce qui entraînera une réduction de la production. Bien que le document ait également documenté les effets des chocs de production et des prix des actifs, la nature de ces chocs n'a pas été abordée. Dans le même esprit, Kahn (2008) a étudié le moteur de la récente flambée des prix de l'habitation et du PIB dans un certain nombre de pays; il a suggéré que cela était dû au changement de productivité. Utilisant un modèle de croissance complexe avec une spécification de

changement de régime de Markov pour la croissance de la productivité, le document a pu calibrer le comportement des prix de l'immobilier depuis les années 1960, y compris le récent ralentissement.

Par ailleurs, un long débat a été évoqué au cours duquel le débat sur la récente flambée des prix du logement était dû à un choc d'offre de logements ou à un choc de demande. Cependant, l'impact de ces chocs sur l'épargne en capital et la croissance reste encore ouvert. Du côté de l'offre, les études ont mis l'accent sur une pénurie de construction de logements. Par exemple, Glaeser et al. (2005) affirme que les prix de l'immobilier qui ont augmenté depuis 1980 reflètent en grande partie des restrictions artificielles de l'offre. De même, Gyourko et al. (2006) cite également de manière non radicale la construction de terrains, comparée à une demande de logements en hausse, en tant que facteur clé du phénomène qu'ils appellent les "villes de la superstar". Du côté de la demande, une tendance à la hausse des prix du logement serait due à la préférence pour les services de logement et/ou à la formation d'attentes. Par exemple, Huber et al. (2018) a exploré la pertinence des préférences pour les services de logement dans la bulle immobilière. À l'aide de données sur la consommation de logements, ils ont constaté que les pays caractérisés par une préférence pour le logement relativement plus faible verraient leurs prix du logement plus volatils de 1970 à 2014. De manière différente, Landvoigt (2017) a introduit un modèle de cycle de vie avec une demande de logements et des perspectives de rendement. Il a constaté que les convictions des ménages concernant les prix futurs des logements, motivées par un assouplissement de la contrainte d'emprunt, étaient à l'origine de la croissance des prix pendant le boom. Des résultats similaires peuvent être trouvés dans les travaux de Geanakoplos(2010) ou Piazzesi et Schneider(2009).

Selon une approche différente, la transmission intergénérationnelle jouerait un rôle clé dans la préservation des inégalités de richesse (voir, par exemple, Bewley (1977), Dynan et al. (2000), De Nardi (2004a), De Nardi and Yang (2016)). Selon ces spécialistes, les motifs de patrimoines, qu'ils soient volontaires, de luxe ou accidentels, pourraient être utilisés pour expliquer les comportements d'épargne des individus. Étant donné que les particuliers peuvent voir leur utilité augmentée lorsqu'ils laissent un héritage à leur progéniture et que le motif du patrimoine peut être beaucoup plus fort pour les ménages les plus riches, cela explique pourquoi les ménages riches épargnent non seulement à un taux beaucoup plus élevé, mais laissent également une richesse plus élevée. À mesure que ce comportement se répète d'une génération à l'autre, la répartition de la richesse devient persistante à long terme.

Bien que l'impact de la transition du patrimoine sur l'épargne et les inégalités ait été au centre des recherches récentes sur les inégalités, la question de savoir si et dans quelle mesure ce transfert affectera la répartition de la richesse par le biais du mécanisme de logement reste une question ouverte (Christophers (2018)). Dans le contexte de l'essor du logement, il peut être plus difficile pour les jeunes personnes de s'offrir leur première maison. Ainsi, le transfert des parents aux enfants peut être un moyen important de faciliter l'accès des jeunes individus aux ressources financières limitées à l'accès à la propriété. D'autre part, si les transferts monétaires, qui sont des vivres et des patrimoines, ont plus de chances d'être accordés par une famille aisée, ils peuvent potentiellement contribuer à l'exacerbation des inégalités existantes au fil du temps. Toutefois, en raison du manque de données de qualité sur la richesse, peu de travaux ont été consacrés à l'examen et à l'explication de l'importance du logement et des transferts face à la récente augmentation de la concentration de la richesse.

Le lien entre la transmission intergénérationnelle et la répartition de la richesse dans l'économie aux multiples actifs a été introduit dans les travaux pionniers de Matsuyama (2000) et de Gollier (2001). Dans leurs travaux, les actifs sont supposés fournir des taux de rendement hétérogènes. En présence de contraintes d'emprunt, certains groupes de personnes peuvent accéder à l'actif relativement plus rentable grâce à l'héritage d'une richesse relativement plus importante, tandis que les autres groupes, qui investissent relativement moins, sont exclus de ces marchés d'investissement. En conséquence, l'inégalité de la richesse persiste dans l'économie, dans laquelle les individus qui possèdent davantage de richesses héritées restent riches, tandis que ceux dont le transfert est moins héréditaire restent pauvres.

Cependant, les actifs modélisés dans ces travaux ont été supposés partager la même nature. La seule hétérogénéité de ces actifs est son retour sur investissement. Comme discuté précédemment, le logement se distingue d'autres types d'actifs d'investissement tels que les actions ou les obligations dans ses deux rôles. Le logement ne fournit pas seulement une opportunité d'accumuler et de stocker de la richesse, il génère également des services publics en consommant des services de logement. Par conséquent, si l'importance du logement est primordiale, il est nécessaire de modéliser le logement différemment par rapport aux autres actifs financiers afin de suivre la transmission de l'inégalité via le canal du logement.

Basés sur le cadre des générations imbriquées qui me permet de prendre en compte l'importance des transferts hérités, les trois articles de ma thèse visent à évaluer différents canaux menant à l'inégalité de la richesse. Les deux premiers chapitres por-

tent sur les impacts du logement sur les deux principaux canaux de création de richesse: l'accumulation de capital et la transmission intergénérationnelle. Le troisième chapitre accorde plus d'attention à la substitution entre différents types de transferts, c'est-à-dire aux transferts entre ménages et aux transferts entre générations (entre vivos et post mortem), et à son implication dans les inégalités. Dans chaque chapitre, un cadre théorique pratique est fourni pour traiter ces relations d'intérêt et pour améliorer notre compréhension du mécanisme économique à travers lequel les relations sont générées.

Chapitre 1: La relation entre les prix du logement et l'accumulation de capital, ainsi que la production, lors de la réalisation de différents chocs.

Comme indiqué ci-dessus, les recherches existantes sur la relation entre l'investissement en capital et les prix du logement ont donné des résultats mitigés. Alors que le stock de capital est déterminé de manière endogène par la décision d'épargne des individus, les prix du logement sont formés à partir du solde de la demande et de l'offre sur le marché du logement. Cela dit, une modification de l'accumulation de capital et des prix de l'immobilier est probablement la conséquence du changement d'un facteur commun de l'économie. Par conséquent, dans ce chapitre, nous étudierons leur rapprochement en présence de différents chocs d'intérêt.

De plus, étant donné que les prix du logement augmentent avec le temps, nous voudrions savoir comment cela affectera l'accumulation de capital ainsi que le PIB. Bien qu'un certain nombre d'articles aient examiné la relation entre les prix du logement et la consommation individuelle (voir, par exemple, Iacoviello (2005), Campbell and Cocco (2007)), les travaux qui visent directement le lien entre les prix de l'immobilier et la croissance sont toujours laissés ouverts. Notre objectif est de améliorer ce genre de connaissance.

Nous commençons par présenter notre exercice empirique visant à étudier le caractère cyclique de ces deux variables d'intérêt pour une sélection de pays de l'OCDE allant du premier trimestre de 1980 au quatrième trimestre de 2017. Le graphique (4) ci-dessous montre la cycle entre le prix d'immobilier et la croissance économique (PIB).

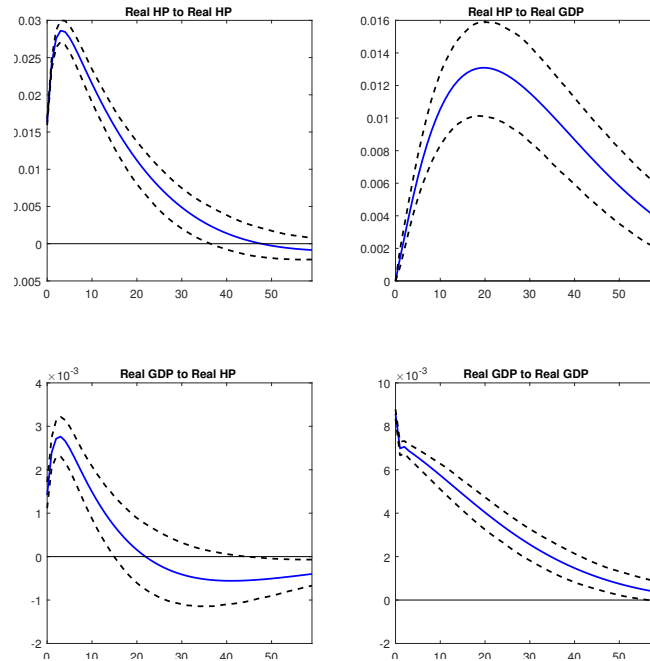


Figure 4: Panel VAR evidence, OECD countries

Notes: The solid line gives the estimated impulse responses. Dashed lines give the 90% confidence intervals generated by Monte Carlo with 5000 repetitions.

À l'aide du modèle d'autorégression à vecteur de panel, nous constatons que les prix des logements et la production sont co-fluctués en réponse aux chocs. Pour expliquer cette évidence, nous construisons un mécanisme de génération superposable qui nous permet d'identifier les facteurs qui déterminent l'essor du logement et/ou la production, tels qu'ils apparaissent dans les données. En incorporant les deux actifs capital et logement dans le modèle, nous expliquons le rapprochement comme conséquence de chocs différents. Justement, en introduisant un changement positif dans la technologie, nous constatons que cela entraîne à la fois une hausse des prix du logement et une accumulation de capital, à court et à long terme. De même, il a été démontré qu'une pénurie temporaire d'offre de logements était responsable de la hausse des deux variables d'intérêt, ce qui conforte l'hypothèse selon laquelle la récente bulle immobilière a été causée par une pénurie. Cependant, nous avons prouvé qu'un choc sur la demande de logements, en ce sens que les individus auraient une préférence plus grande pour les logements, ferait monter les prix des logements tout en évitant l'accumulation de capital. En conséquence, le PIB diminue en réponse à ce choc. Enfin, dans la dernière partie du chapitre, pour étudier les impacts de ces chocs sur la consommation individuelle, nous simulons l'économie à l'aide de paramètres calibrés. Étant donné que la consommation des jeunes et des personnes âgées réagit différemment à de tels chocs, connaître les conséquences ainsi que le mécanisme de transmission des chocs aiderait le planificateur social à définir une action politique appropriée en réponse au choc.

Chapitre 2: Impact du transfert intergénérationnel sur les disparités de richesse via le canal du logement.

Peu de travail a été fait sur l'importance du logement pour la disparité de la richesse due aux problèmes de disponibilité des données sur la richesse. Par exemple, Fuller and Regan (2019), qui a étudié l'inégalité de la richesse des pays de l'OCDE, a suggéré que la récente augmentation de la richesse était uniquement due à la hausse des prix de l'immobilier ainsi qu'à la variation des prix d'autres actifs financiers. En revanche, Bezrukovs (2013), utilisant les données de richesse des enquêtes auprès des ménages de 15 pays de la zone euro, a constaté que l'inégalité de la richesse dans le groupe des propriétaires est considérablement inférieure à celle du groupe des non propriétaires. Ainsi, la richesse du logement serait un atout pour compenser l'inégalité de la richesse par opposition à la richesse financière. Dans le même esprit, Kuhn and Grabka (2018) a utilisé les données du groupe socio-économique allemand et du PSM pour l'Allemagne et la Suisse et a mis en évidence une "forte corrélation négative entre les inégalités de richesse et les taux de propriété de plusieurs pays".

En outre, le fait de posséder une propriété peut générer un double revenu: revenus de location tirés de la location et valeur du logement tirée de la vente. Lorsque les prix de l'immobilier augmentent, la valeur de la maison d'un propriétaire ou d'un propriétaire augmente en conséquence. Si le logement n'est possédé que par certains groupes d'individus, la hausse des prix non seulement gêne les autres groupes pour accéder au marché du logement, mais augmente également la valeur nette de ceux qui sont redevables de biens. De plus, un gain de loyers augmente le revenu des propriétaires aux dépens du locataire car le coût du service de logement est maintenant plus élevé. Par conséquent, pour comprendre le rôle du logement dans l'inégalité, il faut à la fois étudier la contribution des prix de l'immobilier et des revenus locatifs dans l'accumulation de la richesse.

Bien que les prix du logement aient attiré une attention croissante sur les études sur l'inégalité de la richesse, les loyers des logements sont souvent ignorés dans la littérature courante. Très peu de documents ont examiné le rôle du marché locatif et des revenus locatifs dans la création de disparités de richesse. Par exemple, Bonnet et al (2014) ont fait valoir que les loyers, mais pas les prix de l'immobilier, devraient être utilisés pour prendre en compte l'inégalité de la richesse. Ils ont expliqué que "le capital immobilier produit un rendement réel pour les propriétaires, sous forme de loyer. Il génère également un rendement implicite en tant que loyer" économisé

“par le propriétaire occupant. Dans les deux cas, la valeur du capital immobilier doit être basée sur le loyer et non sur le loyer. sur l'évolution des prix du logement ”. De même, en utilisant un modèle de cycle de vie d'équilibre général avec un choix de mode d'occupation, Kindermann et Kohls(2018) ont trouvé une relation positive entre l'inégalité de la richesse et la taille du marché locatif. Ils ont fait valoir que lors de la comparaison entre les pays, quelle que soit leur taille relative, la majorité des locataires avait toujours peu de richesse. Par conséquent, en réduisant le nombre de locataires sur le marché, l'inégalité globale de l'économie diminuera en conséquence.

Dans ce chapitre, nous prêtons attention aux marchés de l'achat de logements et du loyer. Nous soutiendrons que ce ne sont pas seulement les prix du logement, mais aussi les loyers de logement qui contribuent à façonner les différences de richesse. Nous montrons ci-dessous l'évidence par rapport le lien entre le prix d'immobilier et l'inégalité autant que le loyer d'immobilier et l'inégalité qui support notre hypothèse.

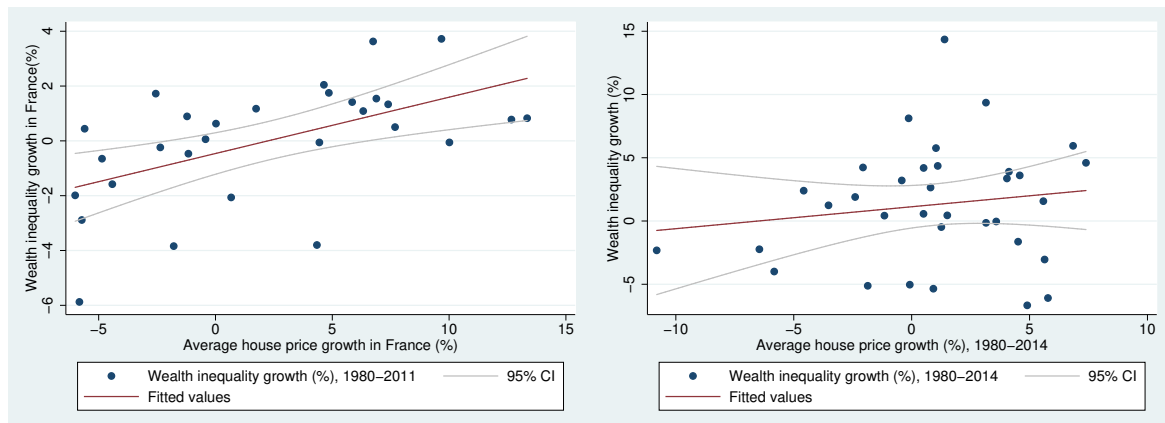


Figure 5: Le lien entre la croissance des prix du logement et la croissance de la richesse en France et aux États-Unis à partir de 1980.

Source: WID and OECD

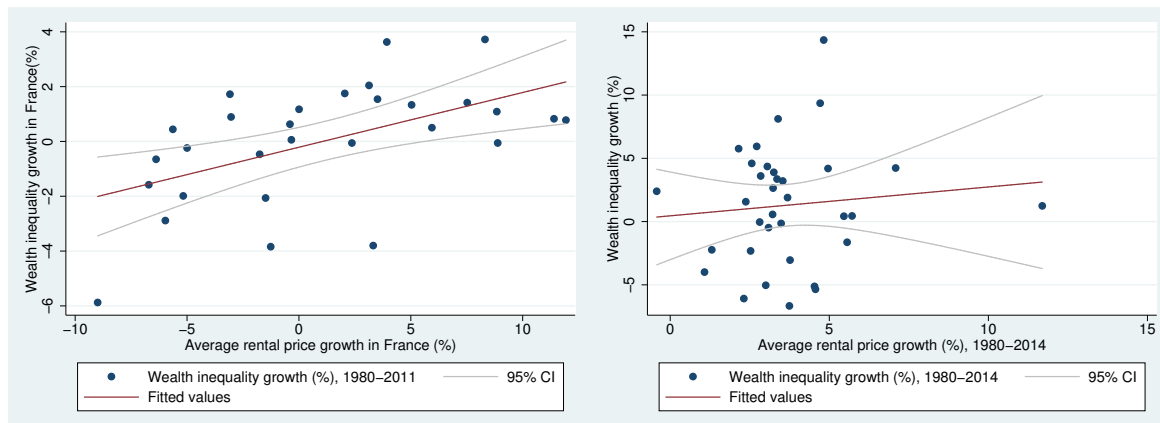


Figure 6: Le lien entre la croissance des prix de location et la croissance de la richesse en France et aux États-Unis de 1980 à 2011.

Source: WIID and OECD

Pour lier à la fois le rendement du logement et l'accumulation de la richesse, nous construisons un modèle de génération qui se chevauche avec des agents hétérogènes et des contraintes d'emprunt. Nous constatons qu'il existe une société dans laquelle les inégalités sont persistantes à long terme. En particulier, dans cet état stable, la riche dynastie devient à la fois emprunteur et propriétaire, tandis que la pauvre n'a d'autre choix que de devenir locataire. Deuxièmement, le modèle nous permet d'identifier le rôle différent des prix du logement et des revenus locatifs dans l'inégalité de la richesse à long terme. Considérant que différents chocs se sont produits dans l'économie, nous avons constaté qu'un assouplissement des limites d'emprunt qui augmente les prix de l'immobilier à l'équilibre tout en réduisant les loyers entraînerait une réduction des inégalités. D'autre part, lorsque nous introduisons une réforme des retraites qui augmente le taux de cotisation, cela entraîne une réduction de l'inégalité de la richesse via la baisse des prix du logement et des loyers. De telles conclusions suggèrent qu'en réagissant à ces chocs, les loyers, plutôt que les prix de l'immobilier, suivent mieux le mouvement de l'inégalité de la richesse.

Chapitre 3: Les implications de la substitution des transferts pour l'inégalité de la richesse.

La littérature existante est utilisée pour traiter séparément les impacts de la transmission du patrimoine et du capital humain dans la recherche sur les inégalités. S'il est vrai que ces transferts se font à différents moments de la vie d'une personne, où la plupart des patrimoines sont accordés à des personnes d'âge moyen, la transmission du capital humain sous forme de transfert entre ménages se fait au tout début de la vie, mais les recherches La richesse du cycle de vie doit également évaluer l'importance de

ces deux transferts. Cependant, peu d'efforts ont été faits pour capturer simultanément leurs impacts. Dans ce chapitre, nous introduisons le paramètre de génération se chevauchant sur 3 périodes qui nous permet de modéliser les transferts de patrimoines et les transferts entre ménages à différents moments de la vie. Ce faisant, nous pourrions évaluer dans quelle mesure la réception de ces différents transferts affecte le comportement en matière d'épargne des particuliers et son incidence sur l'accumulation de richesses.

Ce chapitre était motivé par les éléments de preuve que nous avons observés dans le compte de transfert national (NTA). Les nouvelles données de la NTA montrent que la composition des transferts privés entre générations en France a considérablement changé entre 1979 et 2011: d'une part dominante des transferts intra-ménages dans les années 1980 et 1990 à une part moins importante plus tard. années, ce qui a entraîné une diminution dans le temps du rapport entre les transferts entre ménages et les successions. Cela a conduit à une diminution du ratio de transferts entre ménages et d'héritage avec le temps. Il est de notre intérêt d'expliquer le phénomène et de déterminer le mécanisme qui conduit à la réduction de ce ratio.

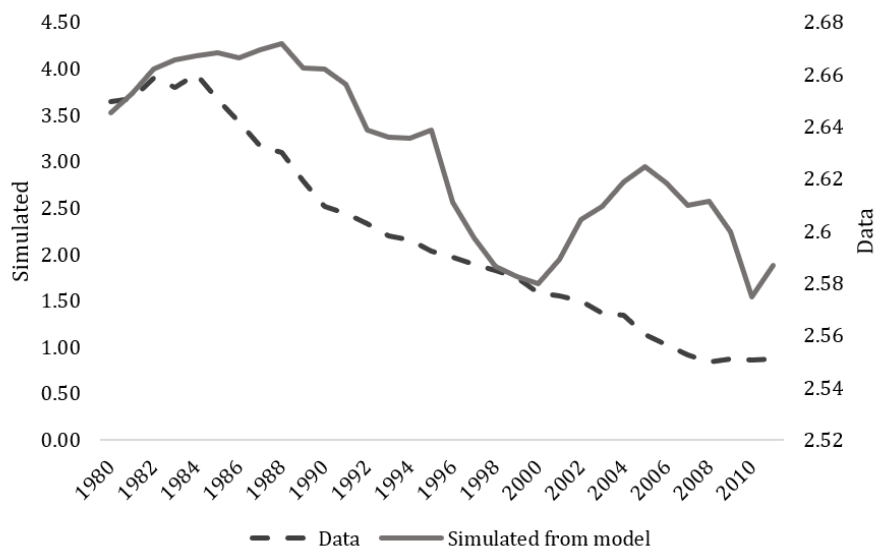
Dans ce chapitre, nous soutiendrons que ce phénomène peut s'expliquer par une différence dans les comportements d'épargne des individus dans le contexte d'inégalités croissantes. Pour générer une telle hétérogénéité, nous introduisons un modèle de générations qui se chevauchent qui inclut ces deux transferts privés. Nous permettons non seulement de léguer des préférences hétérogènes, mais également des différences dans le rôle joué par chacun de ces deux transferts. C'est la première fois que tous ces aspects sont intégrés en même temps, à notre connaissance. Conformément aux éléments de preuve, l'héritage est modélisé comme un revenu supplémentaire perçu à la fin de l'âge moyen, tandis que les transferts entre ménages soutiennent la consommation des enfants, modélisés à l'aide de la production de capital humain (une configuration conforme à Becker and Tomes (1986) , par exemple).

La contribution de notre travail est double. Premièrement, nous fournissons un cadre de génération superposable et traitable avec des transferts endogènes en termes de propriété et de succession. Le modèle nous permet de déduire la dynamique du ratio de transfert entre héritages entre ménages en fonction de l'inégalité de la richesse. Nous montrons que l'évolution de la composition, que nous avons observée à partir des données, peut être attribuée à une aggravation de l'inégalité de la richesse dans le cas de la France. En simulant l'évolution du ratio à l'aide de paramètres calibrés et de données d'inégalité de richesse en France, nous montrons que les données simulées

sont en mesure de bien reproduire la tendance globale observée dans les données. Deuxièmement, le modèle fournit des équilibres multiples avec des inégalités existant à long terme. Selon les paramètres choisis, la famille altruiste ou la famille égoïste peut être relativement plus riche. Les modèles avec l'altruisme à la Becker et Tom (1979) suggèrent généralement que la richesse dépend du degré d'altruisme (voir, par exemple, Becker, 1991) et que la dynastie la plus altruiste possède toute la richesse (Mayshar et Benninga (1996)). Différent de ces travaux, notre article peut définir une condition dans laquelle l'égoïste s'avère plus riche à long terme. De plus, en utilisant la calibration, nous avons montré qu'en renforçant le rôle du transfert intra-ménage dans l'accumulation de capital humain, il était possible de préserver les inégalités: la dynastie égoïste qui était née relativement plus pauvre que l'altruiste pouvait émerger du bas et devenir relativement plus riche. C'est un résultat inhabituel et intéressant que nous n'avons pas vu dans d'autres journaux.

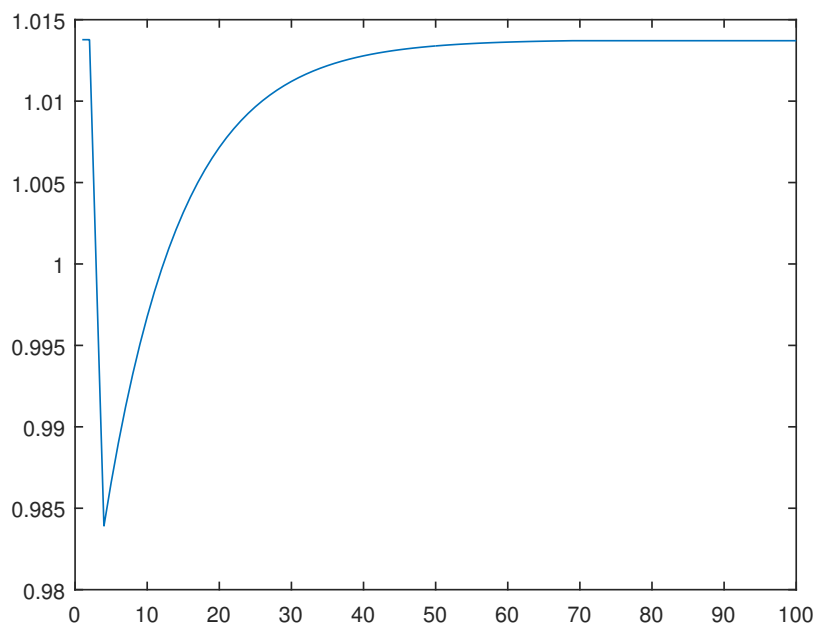
On montrés ci-dessous les graphics (7) et (8) qui illustrent les mains résultats de notre simulation.

Figure 7: Comparasion entre les rapports de transfert entre ménages générés par le modèle et dans les données



Source: Authors calculation, using the model and NTA (France)

Figure 8: Changement du régime d'inégalité: λ est passé de 0.9 à 0.99 pour 1 période.



Parameter choice: $\alpha = 0.2; \beta = 0.6, \theta = 0.4, \lambda = 0.5, \gamma = 0.2, p = 0.3.$

Conclusion générale

Il est essentiel de comprendre les effets de la hausse mondiale des prix du logement sur l'inégalité de la richesse, peu de choses ont été faites à ce sujet. Ma thèse vise à combler cette lacune.

On sait que l'épargne de capital est une des sources importantes de la concentration de la richesse. Ainsi, dans le premier chapitre, j'ai traité de l'interaction entre le logement et l'accumulation de capital et ai cherché à savoir si un gain de prix de l'habitation conduirait les particuliers à économiser plus ou moins.

Dans le deuxième chapitre, je me suis concentré sur la manière dont le transfert intergénérationnel rendrait l'inégalité persistante à long terme via le marché du logement. En considérant une économie avec deux actifs d'investissement différents: le logement et le capital, nous montrons qu'il existe un équilibre dans lequel le logement est plus rentable que le capital. En tant que famille riche, c'est-à-dire celle qui a hérité d'une richesse relativement plus grande, peut accéder au marché du logement alors que la famille pauvre ne le peut pas en raison de la contrainte d'emprunt, l'inégalité persiste à long terme.

Enfin, le patrimoine et le transfert entre ménages sont les deux canaux de transmission essentiels qui relient la richesse de manière différente. Bien que l'on pense que les transferts entre ménages contribuent au développement du capital humain et donc des revenus du travail, l'héritage joue un rôle décisif dans l'accumulation de capital et de richesse. Les données du compte de transfert national en France nous ont montré qu'au cours des dernières décennies, la composition des transferts privés a sensiblement évolué: d'une part plus dominante des transferts intra-ménages au début des années 1980 à une succession plus dominante au début des années 80s des années plus tard. Ce changement entraînant deux effets négatifs, il est donc intéressant de comprendre pourquoi et comment cela se produit, ainsi que son lien avec l'inégalité de la richesse. Ces questions sont abordées dans le troisième chapitre de ma thèse.

En étudiant les impacts du logement sur la richesse par différents canaux: accumulation de capital et transfert intergénérationnel, la thèse permet de mieux comprendre comment le logement devient le mécanisme par lequel la richesse est générée et transmise. Nous avons constaté que le fait que la richesse augmente en réponse à une modification des prix du logement et/ou des loyers dépend de la force motrice qui sous-tend ce changement. Ces conclusions seraient donc utiles dans le processus de décision politique. Si un gain de prix du logement va dans le même sens que la croissance économique en réponse à un choc, une intervention spectaculaire sur le marché du logement visant à réduire le prix du logement peut faire plus de mal que de bien.

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General Introduction

Real house prices are observed to increase substantially in a number of countries from 1980s. Figure (9) below illustrates housing price data in many OECD countries. It shows that the property price index has rose by 120% on average during the period 1980-2007, for example in France, United Kingdom, Spain, Finland, United States and so on. Although the housing prices tend to decrease after the housing burst, they still remain at higher level in comparison with the level in pre-boom year. Thus, studying on the fluctuation of housing prices on the economy has gained a prominent interest in the last few years. Additionally, housing represents a largest asset class in household portfolio (OECD,2018). One explanation for this fact is that housing, different from other types of assets, is not only an investment, but also a consumption good providing a flow of services such as shelter, warmth and comfort (King(2015)). Therefore, when studying the individual's decision in purchasing a house, it's important to take into account the motive of owning a house.

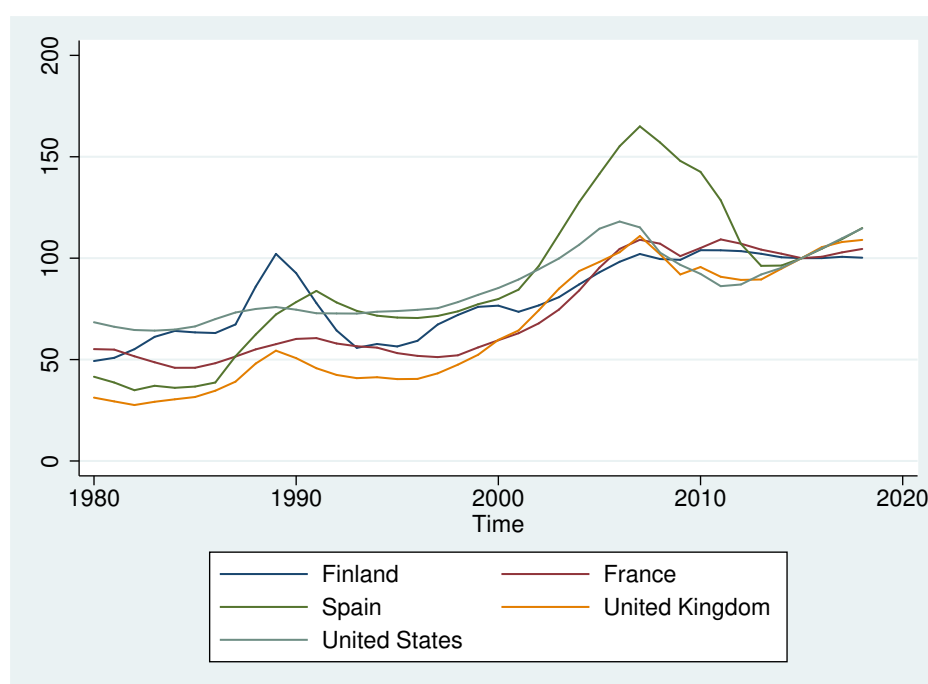


Figure 9: The dynamics of real housing price index in France, Spain, Finland, UK and US from 1980-2018.

Source: OECD

At the same time, Piketty (2014) Capital in the twenty-two century has drawn attention of both academia and public readers on the rise of inequality. Particularly, using data constructed for numerous countries such as China, Europe, and the United States, Piketty and his fellows (see, for example, Piketty's (2014), Piketty and Zucman

(2014), Saez and Zucman (2016), Zucman(2019)) showed that wealth concentration has been pointed to a steady rise stretching back to the end of the 1980s. Figure (10) and (11) below plot the pattern of wealth inequality in France, UK and US. As we can see, in the three countries, inequality displays clearly an increasing trend when considering either the wealth share of the top 10% or top 1%. One may be interested in knowing if the rise of wealth concentration is due to the recent housing price boom. If it is the case, one may expect to see that some (poor) groups are systematically excluded from the housing market. Knowing the mechanism in which wealth inequality is generated and transmitted from one generation to another is, thus, important for policy implication.

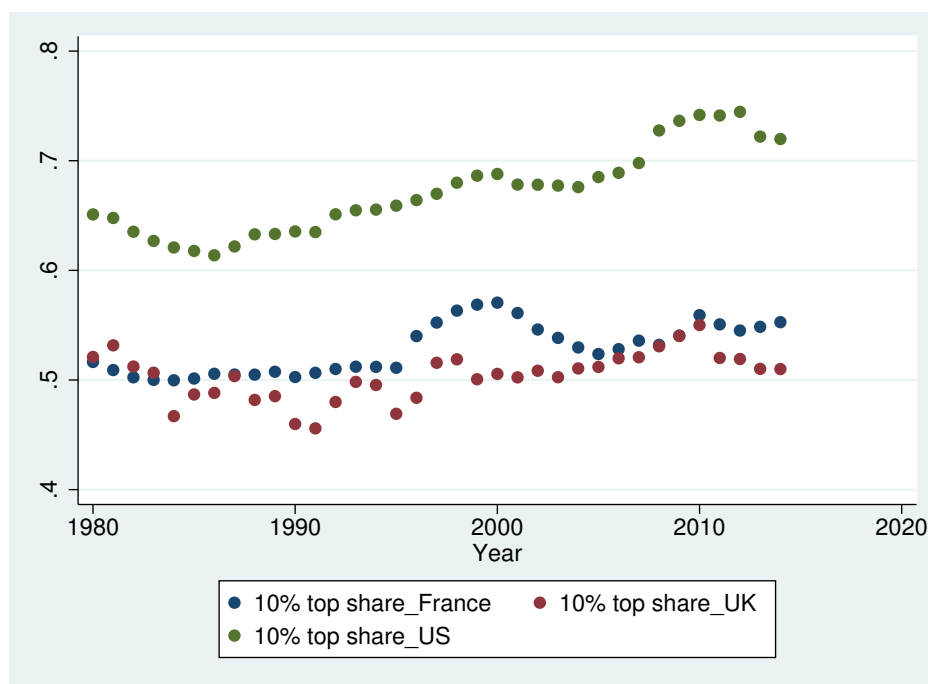


Figure 10: Evolution of wealth share of the top 10% in France, UK, and US.
 Source: WID

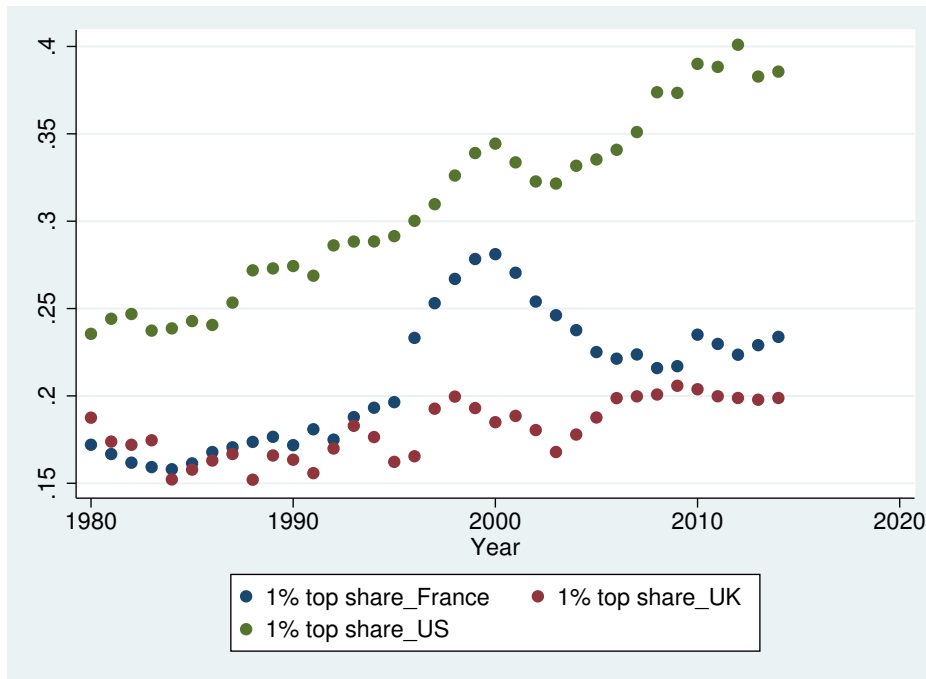


Figure 11: Evolution of wealth share of the top 1% in France, UK, and US.
Source: WID

Starting from early work on wealth Guthrie (1963) or Modigliani (1988), we know that savings and transfers, which are either in the form of human capital or inter-generational transfers, are two essential means for households to accumulate wealth. Studying the role of housing in shaping wealth disparity, therefore, requires understanding its relations with capital accumulation as well as bequest.

The importance of housing on household’s consumption and capital savings has always been of research interest. A number of studies has drawn attention to the welfare of individuals as they focus particularly on relationship between the fluctuations of housing prices and individual’s consumption through wealth effect and substitution effect (see, eg, Ludwig and Slok(2004), Iacoveillo(2004), Carroll et al. (2011) among others). For example, Iacoveillo(2004) suggested that households would increase their level of consumption if their capacity to borrow is tied to their wealth. As a result, aggregate consumption will increase in response to a gain in house price. Additionally, Buiter (2008) added that there’s no pure wealth impact on consumption from a change in property prices if it is due to a shift in fundamental value. He explained that a reduction in fundamental value of houses will make houses cheaper. Thus, the landlords, that is the one who trade down in the housing market will be worst off while the tenants, that is the ones who trade up in the market will be better off. As a result, in a representative agent setting, the aggregate demand doesn’t change due to, in such setting, each tenant becomes his/her own landlord.

Although, there has been numerous studies on the impacts of housing on household consumption and their welfare. Little has been done regarding the role of housing in capital accumulation as well as growth. Even though GDP and consumption are widely known to be highly correlated (Angelache (2011)). Yet, it's necessary to understand the direct effects on household's saving decision and economic production via housing mechanism. Moreover, as housing and capital are both assets commonly found in household portfolios, understanding if an increase in house prices may crowd out or crowd in savings in capital is an important task. Though, this topic has received little attention in literature.

Research examining if capital and housing assets are substitutable or complementary provides mixed evidence. Papers that aimed at assessing the relationship between these two assets are, for example, Iacoviello (2005), Deaton and Laroque (2001), Kahn (2009), and Boris and Reichlin(2018). Particularly, Deaton and Laroque (2001) studied the impacts of land on the growth process using an overlapping generations setting. By compares the capital stocks in the economy with and without land, they found that the presence of land lowers the equilibrium stock of capital by causing savings to be reallocated away from capital towards land. The paper suggested, thus, that land should be nationalized and provided at zero rent. In different approach, Iacoviello (2005), using a monetary business cycle model with borrowing constraint tied to housing wealth, showed that housing prices comove with capital investment in response to a number of shocks. However, the paper paid attention particularly on the effects from a monetary tightening that decreases interest rate and triggers debt-deflation. According to the work, the occurrence of debt-deflation will not only dampen house prices, but also demotivate individuals to invest, which leads to a reduction in production output. Although, the paper also documented the effects of output shocks and of asset price shocks, the nature of these shocks hasn't been addressed. In the same spirit, Kahn (2008) investigated the driving force of the recent upsurge of housing prices and GDP in a number of countries, he suggested that it was due to the productivity shift. Employing a complex growth model with Markov regime-switching specification for productivity growth, the paper was able to calibrate the behavior of housing prices since the 1960s, including the recent slowdown.

On the other hand, there has been raised a long debate in which discussing the recent upsurge of housing prices was due to a housing supply shock or a demand shock. However, how these shocks impact capital savings and growth is still left open. On the supply side, studies have focused on a shortage of housing construction. For exam-

ple, Glaeser et al. (2005) argues that property prices that increased since 1980 largely reflect artificial supply restrictions. Similarly, Gyourko et al. (2006) also cite inelastically land construction in comparison with a rising housing demand as a key driver of the phenomenon they call “superstar cities”. On the demand side, an upward shift in housing prices are argued to be driven by the preference for housing services and/or expectation formation. For example, Huber et al. (2018) explored the relevance of preferences for housing services in housing bubble. Using housing consumption data across countries, they found that countries that characterize with a relative weaker housing preference would experience a more volatile housing prices during 1970-2014. In a different manner, Landvoigt (2017) introduced a life-cycle model with housing demand and return expectation. He found that household beliefs about future house prices which was driven by a relaxation in borrowing constraint was responsible for driving price growth during the boom. Similar findings can be found in the work of Geanakoplos(2010) or Piazzesi and Schneider(2009).

From different approach, intergenerational transmission is claimed to play a key role in sustaining wealth inequality (see, for example, Bewley (1977), Dynan et al. (2000), De Nardi (2004a), De Nardi and Yang (2016)). According to these scholars, bequest motives, either in the voluntary, luxury-type or accidental form, are used as a possible explanation for individuals’ saving behaviors. As individuals might experience an increase in utility when they leave bequest to their offspring and the bequest motive may be much stronger for the richest households, it explains why wealthy households not only save at much higher rates but also leave a higher wealth than the rest. As this behavior is repeated from one generation to the next, the distribution of wealth becomes persistent in the long-run.

While the impact of transition of bequest on savings and inequality has been the focus of recent inequality research, whether and to what extent this transfer affects the distribution of wealth through housing mechanism remains an open question to debate (Christophers (2018)). In the context of housing boom, young individuals may find it more difficult to afford their first house. Thus, transfer given from parents to children may be an important means in facilitating young individuals who are financial-constrained to access to home ownership. On the other hand, if monetary transfers, which are inter-vivos and bequest, are more likely to be given by wealthy family, then it may potentially contribute to the exacerbation of existing inequality over time. However, due to the lack of good quality of data on wealth, little work has been done to examine and explain the importance of housing and transfers on the recent rise of wealth concentration.

The link between intergenerational transmission and the distribution of wealth in the economy with multiple assets has first introduced in the pioneering works of Matsuyama (2000) and Gollier (2001). In their works, assets are assumed to provide heterogeneous rates of returns. In the presence of borrowing constraint, some groups of individuals are able to access the relatively more profitable asset thanks to inheriting a relatively larger wealth while the other groups, who endow relatively less, are excluded from these investment markets. As a consequence, wealth inequality persists in the economy in which individuals who possess more inherited wealth remain rich while the ones with lower inherited transfer remain poor.

However, assets modelled in these works were assumed to share the same nature. The only heterogeneity among these assets is its investment return. As discussed previously, housing differs from other types of investment assets like stocks or bonds in its dual roles. Housing doesn't only provide an opportunity to accumulate and store wealth, it also generates utility through consuming housing service. Therefore, to highly the importance of housing, it's necessary to model housing differently in comparison with other financial assets in order to track the transmission of inequality via housing channel.

Based on the overlapping generation framework that allows me to take into account the importance of inherited transfers, the three papers in my dissertation aim at assessing different channels that leads to wealth inequality. The first two chapters focus on the impacts of housing on the two main channels of wealth generation: capital accumulation and intergenerational transmission. The third chapter pays more attention on the substitution between different types of transfer, i.e. intrahousehold versus intergenerational (inter-vivos and post-mortem) transfers, and its implication in inequality. In each chapter, a tractable theoretical framework is provided to address these relations of interest as well as to enhance our understanding on the economic mechanism through which the relations are generated.

Chapter 1: The relation between housing prices and capital accumulation as well as production output in the realization of different shocks.

As discussed above, existing research on the relation between capital investment and housing prices provided mixed findings. While capital stock is endogenously determined by the savings decision of individuals, housing prices are formed from the balance from the demand and supply in the housing market. With that being said, a

change in the capital accumulation and property prices are likely the consequence of a shift of a common factor in the economy. Therefore, in this chapter, we will study their comovement in the presence of different shocks of our interest.

Moreover, since housing prices are observed to increase over time, we are interested in knowing how it would affect capital accumulation as well as GDP. While there have been a number of papers investigating the relationship between housing prices and individual consumption (see, for example, Iacoviello (2005), Campbell and Cocco (2007)), work that targets directly the link between property prices and growth is still left open. Our purpose is to fulfil this gap.

We start by introducing our empirical exercise to study the cyclicity between these two variables of interest for a selection of OECD countries from Q1/1980 to Q4/2017. Using the panel vector autoregression model, we find that housing prices and production output co-move in response to shocks. To explain for this evidence, we construct a tractable overlapping generation mechanism that allows us to identify the factors that drives housing boom and/or production output seen in the data. By incorporating both two assets which are capital and housing in the model, we explain the comovement as a consequence of different shocks. Precisely, by introducing a positive shift in technology, we find that it drives up both housing prices and capital accumulation both in the short and long-run. Similarly, a temporary shortage on housing supply was shown to be responsible for shifting up the two variables of interest which supports the hypothesis that the recent housing bubble was caused by a shortage. However, we proved that a shock on the house demand in the sense that individuals have a greater preference for houses would drive up house prices while crowding out capital accumulation. As a result, GDP decreases in response to this shock. Finally, in the last part of the chapter, to study the impacts of these shocks on individual consumption, we simulate the economy using calibrated parameters. Since consumption of the young and the old respond differently to such shocks on consumption, knowing the consequences as well as the transmission mechanism of the shocks would help the social planner in defining an appropriate policy action in response to the shock.

Chapter 2: The impacts of intergenerational transfer on wealth disparity via housing channel.

Little work has been done on investigating the importance of housing on the disparity of wealth due to issues of data availability on wealth. For example, Fuller and Regan (2019) who studied wealth inequality of OECD countries suggested that the re-

cent rise in wealth is solely driven by rising house prices as well as price changes in other financial asset. On the other hand, Bezrukovs (2013), using household survey wealth data in 15 Eurozone, found that wealth inequality within the group of homeowners is considerably lower than within the group of non-homeowners. Thus, housing wealth is suggested to be an equalizing asset in wealth inequality as opposed to financial wealth. In the same spirit, Kuhn and Grabka (2018) using the German Socio-Economic Panel and SHP data for Germany and Switzerland and found a “strong negative correlation between wealth inequality and home-ownership rates across countries”.

Additionally, owning a property can provide dual earnings: rental incomes from lease and housing value from sale. When property prices increase, the house value of a home-owner or a landlord rises accordingly. If housing is possessed by only some groups of individuals, a gain in prices not only troubles the other groups to access to the housing market, but also increases the net worth of the ones who own properties. Moreover, a gain in rents increases the income of the landlords at the expense of the tenant as the cost of housing service is now higher. Therefore, understanding the role of housing in inequality requires both investigating the contribution of property prices and rental incomes in the accumulation of wealth.

Although, housing prices have drawn an increasing attention on the studies on wealth inequality, housing rents are often ignored in common literature. There have been quite few papers investigating the role of rental market as well as rental incomes in shaping wealth disparity. For example, Bonnet et al (2014) argued that rents, but not housing prices, should be used to account for wealth inequality. They explained that “housing capital produces a real return for landlords, from rent. It also produces an implicit return as rent “saved” by the owner-occupiers. On both accounts, the value of real estate capital must be based on rent and not on the evolution of housing prices”. Similarly, using a general equilibrium life-cycle model with housing tenure choice, Kindermann and Kohls(2018) found a positive relationship between wealth inequality and the size of rental market. They argued that when comparing among countries, no matter their relative size of the population, the majority of renters always has little wealth. Therefore, by reducing the number of renters in the market, the economy’s overall inequality will reduce, accordingly.

In this chapter, we pay attention on both housing purchase and rent markets. We will argue that it’s not only housing prices, but also housing rent that contribute in shaping wealth differences. To link both returns on housing to wealth accumulation, we construct an overlapping generation model with heterogeneous agents and borrow-

ing constraints. We find that there exists a society in which inequality is persistent in the long-run. Particularly, at this steady state, the rich dynasty becomes both capital borrower and home-owner while the poor one has no choice but becomes a renter. Secondly, the model allows us to identify different role of housing prices and rental income in wealth inequality in the long-run. Considering different shocks occurred in the economy, we found that a relaxation in borrowing limits that increases equilibrium house prices while reduces rents would drive a fall in inequality. On the other hand, when we introduce a pension reform that increases the contribution rate, it triggers a reduction in wealth inequality via the decrease in both housing prices and rents. Such findings suggest that, as responding to these shocks, rents, rather than house prices, follow better the movement of wealth inequality.

Chapter 3: The implications of transfers substitution for wealth inequality.

Existing literature used to address separately impacts from transmission of bequest and human capital in the research of inequality. While it is true that these transfers are made at different time of individual's life, in which most bequest is given to middle-age people, human capital transmission in the form of intrahousehold transfer is made at the early stage of one's life, research that investigates life-cycle wealth needs to evaluate equally the importance of these two transfers. However, little has been done to capture simultaneously their impacts. In this chapter, we introduce the 3-period overlapping generation setting which allows us to model the bequest and intrahousehold transfers at different timing of life. By doing that, we will be able to assess how receipt of these different transfers affects individual's saving behavior and its implication to the accumulation of wealth.

This chapter was motivated by the evidence that we observed in the National Transfer Account (NTA). New data from the NTA highlights that during the period 1979-2011, there has been a notable change in the composition of private transfers between generations in France: from a dominant share of intrahousehold transfers in the 1980s and 1990s to a less dominant one in later years, leading to a decrease in the ratio of intrahousehold transfers to inheritance over time. This has led to a decrease in the ratio of intrahousehold transfers-to-inheritance over time. It is of our interest to explain the phenomenon and determine the mechanism which leads to the reduction of this ratio.

In this chapter, we will argue that this phenomenon can be explained by a difference

in individuals' saving behaviors in the context of rising inequality. To generate such heterogeneity, we introduce an overlapping generations model that includes both of these private transfers. We not only allow for heterogeneous preferences to bequeath, but also differences in the role that each of these two transfers play. This is the first time that all of these aspects are incorporated at the same time, to our knowledge. In line with the evidence, inheritance is modeled as additional income received at the end of the middle-age time while while intrahousehold transfers support children's consumption, modeled through the production of human capital (a set-up in line with Becker and Tomes (1986), for instance).

The contribution of our work is twofold. First, we provide a tractable overlapping generation framework with endogenous transfers in terms of intrahousehold and inheritance. The model allows us to derive the dynamics of the intrahousehold transfer-to-inheritance ratio as a function wealth inequality. We show that the change in the composition, which we observed from the data, can be attributed to a rise in wealth inequality in the case of France. Simulating the evolution of the ratio using calibrated parameters and France wealth inequality data, we illustrate that the simulated data is able to replicate well the overall trend seen in the data. Secondly, the model provides multiple equilibria with inequality existing in the long-run. Depending on parameter choices, either the altruistic family or the egoistic family can be relatively richer. Models with altruism à la Becker and Tom(1979) usually suggest that wealth depends on the degree of altruism (See, for example, Becker, 1991) and the most altruistic dynasty owns all the wealth (Mayshar and Benninga (1996)). Different from these works, our paper can define a condition in which egoistic turns out to be richer in the long-run. Moreover, using calibration, we showed that by making the role of intrahousehold transfer more important in human capital accumulation, inequality can be reserved: the egoistic dynasty who was borne relatively poorer than the altruistic one can rise out from the bottom and becomes relatively richer. This is an uncommon and interesting result that we haven't seen in other papers.

Chapter 1

House price, Economic Growth, and Business Cycle¹

¹This chapter is co-written with Ekrame Boubtane and Dramane Coulibaly.

1. Introduction

The link between housing price appreciation, household behaviors and business cycle has gained much interest for decades, starting from the early 1990s when we observed in the US and many other countries a spectacular increase in house prices and then later followed by its sharp fall due to the housing bubble burst (Engelhardt (1996), Ortalo-Magne and Rady (2006), OECD (2015), Duca et al. (2011)). While most of the studies so far stressed their interest on the synchronization between housing price fluctuations and consumption (see, for example, Iacoviello (2005), Campbell and Cocco (2007), Carroll et al. (2011)), works targeting the relationship between house prices and economic output are still left open. Even though GDP and consumption are widely known to be highly correlated and many studies provoke that GDP and consumption co-move positively (Davis and Heathcote (2005), Angelache (2011)), research that directly investigates the link between house prices and production output should be called for to understand the driving force behind their movements.

Whether an increase in house prices crowds in or crowds out individual's savings remains controversial. The classic theory of rational bubbles addressed the capital crowding-out effect of bubbles. In detail, a gain in prices from an overheated housing market raises interest rate and crowds out productive investments such as commercial and industrial bank lending. As a consequence, the housing sector will experience excessive liquidity while investments go to non-housing sectors will be reduced (Tirole (1985), Bleck and Liu (2018), and Miao et al. (2015)). Moreover, according to the work of Farhi and Tirole (2011), this effect is even amplifying when liquidity is scarce and firms are financially constrained. Similarly, Miao et al. (2015) provided a model with credit frictions and productive externality where they proved that housing bubbles may drive resources away from more productive investment opportunities. In the same vein, the empirical work of Chakraborty et al. (2018) also suggested that housing price appreciations have negative spillover effects to the real economy.

In a different spirit, findings that support the positive effect of an increase in asset prices on productive real investment are also found in many works. They argued that the crowding-in effect between house prices and capital savings can be mainly explained by (but not limited to) the three following factors: the role of housing in borrowing constraint, its importance in generating wealth effects, and common causes. Although these explanations deserve credit, they are also somewhat problematic. For example, from the production's side, Holmstrom and Tirole (1997), Gan (2007) reached an agreement that since houses can be used as collateral, firms facing credit constraints are able to borrow and invest more because their owning collateral assets are worth more. At the same time, from individuals' side, Canner et al. (2002), Aoki et al. (2004)

claimed that when house prices rise, people would extract mortgage equity to finance further either consumption or non-housing investment, or to upgrade their housing stock. However, Cooper (2013) argued that individuals would look for mortgage extraction to finance further consumption only if they experience adverse income shocks. Additionally, Greenspan and Kennedy (2007) estimated the proportion of equity extracted that goes to non-housing investment and they found it around 6%, which is rather small. Similarly, Buiters (2008) claimed that there is no pure wealth effect on consumption from a shift in house prices if it is driven by a change in the fundamental value. They explained that as house price falls, individuals who “long” the asset are worse off, while the ones who “short” housing is better off. Hence, the net wealth effect on average is cancelled out. In the same spirit, Guerrieri and Iacoviello (2017) found that variation in housing prices has asymmetric effects on economic activity. In detail, using the DSGE model with Bayesian estimation method, they showed that expanding housing wealth had contributed little to consumption growth during the housing boom 2001-2006, while a housing collapse exacerbated the recession of 2007-2009. Based on such arguments, the influence from an appreciation of house price to capital savings rests ambiguous.

In alternative view, common causality are claimed to make housing price and GDP simultaneously co-move. For example, Mankiw and Weil (1989) suggested that the entry of Baby Boom generation were the fundamental cause of their pro-cyclicality, and predicted that house prices in the late 1990s would fall substantially at the moment that the Baby Bust generation would enter in their house-buying years. Specially, using However, as housing prices are observed to increase continuously over these years, it contradicts their finding. Moreover, Swan(1995) argued that the focus of a decline in house prices provided in Mankiw and Weil (1989) is based on a serious misinterpretation of their demand variable. He also added that the econometric work in Mankiw and Weil (1989) lacks credibility as they didn’t take into account cost factors and the supply of new houses. Martin (2005) renewed the argument for an important role for demographics. The recent work of Kahn (2009) brought again the interest in studying the co-movement between housing and final output. In this work, he argued that changing in labor productivity largely caused the movement of house prices in many OECD countries while credit market conditions just made it exacerbated² In the same spirit, Boris and Reichlin (2018) investigated the OECD data from 1970 to 2011 and identified two periods of high productivity: the first one occurred through the 1970s and the second one appeared after 2000. Their empirical work also confirms the pro-cyclicality between housing price and non-housing sector productivity within these

²This similar argument also found in the work of McCarthy and Peach (2005), but it was based on different method.

periods, i.e. real value added per hour worked, and adds that the causality is likely to go from productivity to housing sector. The two works mainly relied on the assumption that housing construction is less capital intensive than manufacturing while labor productivity in manufacturing is relatively more efficient than in construction. Moreover, when introducing a model with 2 sectors, manufacturing and construction ones, the wage equalization condition will require productivity to directly affect housing price. As a result, it's not surprising that technology shift will drive up housing cost and price.

Finally, if the relationship between house prices and economic fluctuation is pro-cycle while the saving rate is observed to be high in many countries like China, Korea or Taiwan, it might be interesting to know if there exists a “virtuous cycle” of housing and economics growth in which a gain in house price may drive individuals to save more which leads to higher economic growth and then, in return, drives up housing price. However, the work of Deaton and Laroque (2001) which developed an overlapping growth model in comparing the economy with and without land provides no support for this hypothesis.

In this paper, we reconsider the role of housing and its interaction with savings as well as economic growth. The aim of this work is to construct a tractable mechanism that can identify the factors that drives housing boom and/or production output as observed in the data as well as its impact on individual's welfare. In the same spirit with Deaton and Laroque (2001), we construct an overlapping generation economy in which each individual live for 2 periods and they can allocate their disposable income between buying a house or saving in the form of capital. Here, housing plays a dual role in which owner occupation provides both wealth and consumption services. In the setting, we omit the rental market for housing, for the fact that renters in general represent only a small fraction of the population. Similar to the setting of Deaton and Laroque (2001), we shut down the collateral constraint so individuals cannot borrow to acquire houses. Besides, we allow the existence of warm-glow bequest in our model. The idea of having bequest is to generate the inter-generational persistence of wealth so that any impacts of a shock on the current wealth today can transfer to their descendants' wealth tomorrow. Thus, it can account for the fact that even a temporary technology shock can lead to a persistent reaction of housing price via bequest mechanism³.

In this paper, we shut down the credit market channel. The first reason is that withdraw mortgage equity via collateral channel is only popularly exercised in the US and UK, but not so in other countries. And secondly, the presence of mortgage debt seems not change the main results of the overlapping generation model where housing

³See for example Piketty(2000, Chapter 8), Gomes and Mendicino (2015)

and capital savings are substitutes. Our intention is simply to understand what factors drive their co-movement and the pure impacts.

We find that the pro-cyclicality between house price and production output can be explained by either a positive technology shock or a negative shift on the house supply. On the contrary, we prove that house price acceleration will crowd out capital if there's a shock to the demand of housing, triggered by a shift in the housing preference. Therefore, it's less likely that the movements of the two variables that we observe in the data is driven by a change in house preference.

Regarding impacts of these shock on consumption, it's not surprising that the consumption of the two generations increase in response to a positive technology shift due to a higher wealth received by the young and the old individuals. Taking into account housing supply shock, we showed that a house supply shortage benefits the young individual at the expense of the old one. If we believe that the old is the rich one while the young represents the poor one, then the occurrence of this shock may help reduce the inequality of wealth in the society. Finally, a preference shift decreases the consumption of the youth while raises consumption of the old one. This shock may then involve in increasing inequality.

The paper is organized as follows. The second section presents our empirical evidence using panel VAR regression. The third section introduces the overlapping generation model. The fourth one studies the evolution path and the steady state solutions. Section fifth, sixth and seventh investigate the impacts of different shocks on price factors and production output while section eighth provides results from our numerical analysis. Section ninth concludes.

2. Data and Empirical Evidence

2.1. Data

In this empirical exercise, we will focus on the two variables of our interest which are GDP per capita and Housing Price Index. These two series are extracted from OECD national accounts and OECD house prices indicator databases. Our sample includes quarterly observations from 1990:1 to 2017:4 for a selection of OECD countries of which these two series are available during such period of time. They include 14 countries as follows: Australia, Canada, United States, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Spain, Switzerland and United Kingdom. Descriptive statistics of these two variables are provided in the table (1.2) in Appendix.

The house price indices is extracted from OECD house price indicator database. It is denoted by HPI and is index number that measure the prices of residential properties over time. Both new and existing dwellings are covered if available, independently of their final use and their previous owners. Only market prices are considered. They include the price of land on which residential buildings are located. For all these countries considered, the indices are computed by the ratio of seasonally adjusted nominal house prices to the seasonally adjusted consumers expenditure deflator in each country, from the OECD national account database.

Production output is measured by GDP per capita and is extracted from OECD national accounts. For comparative purpose, they are all measured in US \$ constant price using a specific PPPs in which the OECD reference year is 2010. They are also seasonally adjusted by using X-12 Reg-ARIMA method. Further details can be found in OECD report (2002).

2.2. Empirical strategy

The empirical analysis is based on a panel VAR model with the following specification:

$$Y_{it} = A_1 Y_{it-1} + \dots + A_p Y_{it-p} + u_i + \lambda_i t + \mu_t + \varepsilon_{it}, \quad i = 1, \dots, N \text{ and } t = 1, \dots, T \quad (1.1)$$

where $Y_{it} = (y_{it}^1, \dots, y_{it}^K)'$ is a $(K \times 1)$ vector of endogenous variables, the A_s for $s = 1, \dots, p$ are fixed $(K \times K)$ coefficient matrices, $u_i = (u_i^1, \dots, u_i^K)'$ is a fixed $(K \times 1)$ vector of region-fixed effects, $\lambda_i t$ stands for region-specific time trend, μ_t represents the common time-specific effect, and $\varepsilon_{it} = (\varepsilon_{it}^1, \dots, \varepsilon_{it}^K)'$ is the $(K \times 1)$ vector of residuals satisfying $E(\varepsilon_{it}) = 0$, $E(\varepsilon_{it}\varepsilon_{it}') = 0 \forall i$ and t and $E(\varepsilon_{it}\varepsilon_{j\tau}') = \Omega$ for $i \neq j$ or $t \neq \tau$.

We deal with possible heterogeneity in our panel data setting both by considering a somewhat homogeneous sample of OECD countries, and by including country-fixed effects (u_i) and country-specific time trends ($\lambda_i t$). We also account for cross-country contemporaneous interdependence by including year-specific effects (μ_t), as in ?.

Ordinary least-squares (OLS) estimation with fixed effects yield consistent estimates given the large time series dimension ($T = 136$) compared to the number of individuals ($N = 14$). All variables are de-trended with a country-specific quadratic trends. Panel unit root tests reject the null hypothesis of unit root, for all the de-trended variables.

We estimated a two-dimensional VAR model in which the vector of endogenous variables Y_{it} is:

$$Y_{it} = (HP_{it}, GDP_{it})',$$

where HP_{it} is the logarithm of property prices in country i and in year t and GDP_{it}

is the logarithm of GDP per capita ⁴;

The choice of the number of lags in the estimated models was made using AIC (Akaike information criterion) and BIC (Bayesian information criterion) tests. This leads us to select three lag.

After having estimated the VAR coefficients, we now aim to establish causal relationship between variables.

In VAR models, one can assess causality with Choleski decomposition. To this end, we order variables from the “least endogenous” to the “most endogenous”. In this decomposition, variables ordered first in the VAR system are allowed to have a contemporaneous impact on the other variables, while variables ordered later in the VAR system can affect those ordered first only with a lag. In other words, we make assumptions about the contemporary impacts of the shocks specifying which variables may be influenced in period t by a change in another variable in the same period t , while no restriction is placed on the variables for dates after t . Precisely, a structural shock, or innovation, on one variable can impact at time t this very variable and the other variables ordered afterwards, and from $t + 1$, all the variables of the system. As in Iacoviello (2005), we put property prices before GDP per capita because of nominal rigidities. Figure (1.1) below shows impulse responses to one standard deviation shock on housing prices and on GDP per capita while in the figure (1.2), we run the same regression using housing prices and Industrial Production Index instead. The 90% confidence intervals are generated by Monte Carlo with 5,000 repetitions.

⁴For robustness check, we will later run the regression using Industrial Production Index instead of GDP to measure Production output.

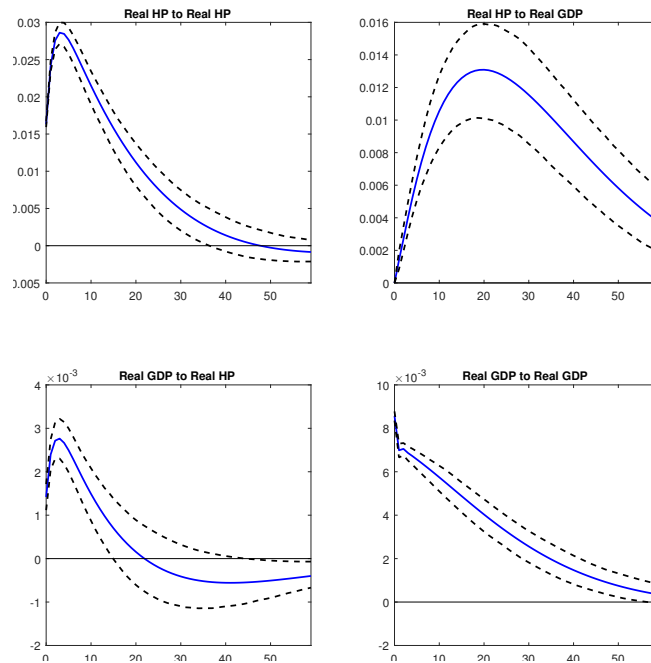


Figure 1.1: Panel VAR evidence, OECD countries

Notes: The solid line gives the estimated impulse responses. Dashed lines give the 90% confidence intervals generated by Monte Carlo with 5000 repetitions.

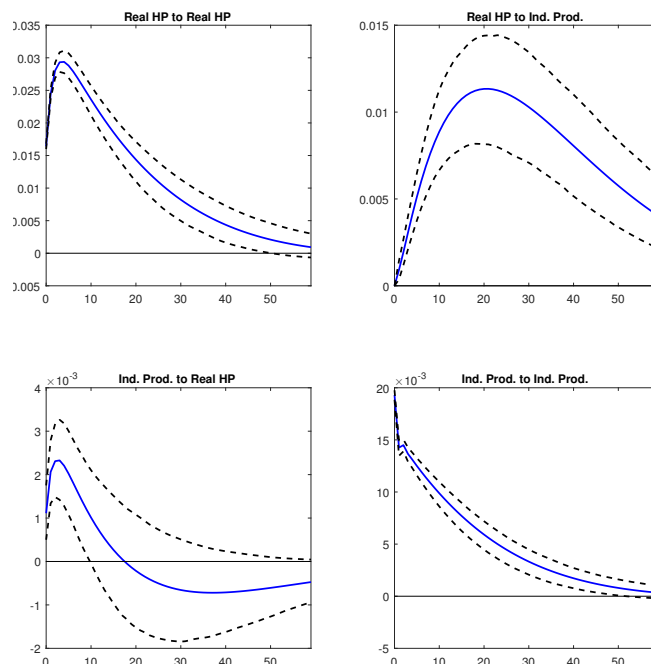


Figure 1.2: Panel VAR evidence, OECD countries

Notes: The solid line gives the estimated impulse responses. Dashed lines give the 90% confidence intervals generated by Monte Carlo with 5000 repetitions.

The results presented in both Figure (1.1) shows a positive comovement of housing prices and output. Adopting a row reading, we can see that when we introduce a shock

on housing sector in the sense that increases housing prices, real GDP per capita also increases significantly upon impact. The increase remains significant during almost 4 years after the housing price shock. Similarly, as displayed in the second row, following a shock in the output sector that increases GDP per capita, housing prices will rise significantly during the 5 years after the output shock. On the other hand, one may argue that the GDP measurement may endogenously take into account the housing wealth as well as its price. Therefore, to control for this issue, we re-estimate the model using industrial production index instead of GDP per capita to measure output. The corresponding impulse responses, reported in Figure (1.2), show the same response pattern which confirms that our results are robust. Moreover, in the two figures, one can notice that housing prices always respond immediately when the shocks occurs while production output reacts to a shock on housing sector with a lag.

In the rest of the paper, we'll construct and simulate a model that is consistent of these evidence and that can be helpful for political analysis. In the rest of the paper, we'll construct and simulate a model that is expected to replicate these evidence and that can be helpful for political analysis.

3. An overlapping generation model with two assets and bequest motive

3.1. Production

In the model, there is only one homogeneous good produced in the economy, which price is assumed to be numeraire. The good can be used in non-housing consumption as well as capital formation. Since non-housing consumption and capital are perfect substitutes, they share the same price.

The production technology is represented by a constant-return-to-scale Cobb-Douglas production function with Hicks-neutral technology A_t in the following form:

$$F(A_t, K_t, L_t) = A_t K_t^\alpha L_t^{1-\alpha}, \quad K_0 > 0 \text{ given} \quad (1.2)$$

where K_t is the gross capital in the economy, A_t is the total factor productivity, and L_t is the labor demand at time t .

In intensive form, the production function expressed in term of capital per capita is:

$$f(A_t, k_t) = \frac{F(A_t, K_t, L_t)}{L_t} = A_t k_t^\alpha \quad (1.3)$$

where $k_t = K_t/L_t$ is defined as the capital per capita and $f'(0) = \infty$ (Inada condition).

The representative firm maximizes its profit function under perfect competition.

Assuming that capital stock is fully depreciated at the end of each period, in equilibrium, factors are paid at their marginal products as follows:

$$R_t = f'(A_t, k_t) = \alpha A_t k_t^{\alpha-1} \quad (1.4)$$

$$w_t = f(A_t, k_t) - k_t f'(A_t, k_t) = (1 - \alpha) A_t k_t^\alpha \quad (1.5)$$

On the other hand, regarding the housing sector, depart from ? we assume that there is no production in housing sector. The housing stock per capita is exogenously given and fixed at h . Moreover, houses, unlike capital, are assumed to have no deterioration so that we have $h_t = h$ for every period t . This assumption will allow us to focus our entire attention on the demand side. In any case, we believe that the supply side is not the key to understand the substantial house price evolutions that we have observed in the data during the 1980-2007 period. Related to this, we abstract from housing growth. Similar setting can be found in the work of Eerola and Maatanen(2008), Arce and Lopez-Salido(2011),and Gary-Bobo and Nur(2015).

3.2. Consumers

To capture the relationship between housing prices and GDP, we develop a tractable overlapping generation economy that contains both capital and housing assets and that allows for intergenerational transfer. Similar with capital asset, here we assume that owning a house provides utility to individuals. Thus, individuals have incentive to invest in houses since they are playing a dual role: durable consumption good and investment good. This setting is common in housing modelling field and can be found in many papers, for example, Deaton and Laroque (2001), Arce and Lopez-Salido(2011), Gary-Bobo and Nur(2015), and eZhao(2015). Moreover, we add bequest in our model for technical purpose. As one can see from the empirical exercise, the response of both housing prices and GDP are quite persistent as it takes more than 50 quarters to converge to the steady state. According to the work of Boserup et al.(2014) and Gomes and Mendicino (2015), the inheritance mechanism allows the effects of shocks on wealth to transfer from today to tomorrow. Therefore, adding the variable bequest will help produce more persistent reactions of housing price which is consistent with our empirical findings.

We consider a society with overlapping generations. Each individual lives for two periods, so that there are always two generations alive at any given time. For simplicity, assume that there's no population growth. Thus, the size of the population is normalized to 1.

In the first period of life, individuals supply inelastically one unit of labor, earn labor income w_t and receive a bequest b_t from their ascendants. They then decide how much to consume when young c_t and how to divide their savings partly in capital s_{t+1} and housing z_{t+1} . When ones become old, they get retired, consume d_{t+1} out of their earnings from capital and housing wealth, and leave a bequest, b_{t+1} , to their descendants.

We assume utility takes the logarithmic form for reasons of tractability. The program of a young person born at time t is as follows:

$$U(c_t, d_{t+1}, z_{t+1}, b_{t+1}) = \ln(c_t) + \beta \ln(d_{t+1}) + \gamma \ln(z_{t+1}) + \rho \ln(b_{t+1}) \quad (1.6)$$

The budget constraint of a young individual in time t states:

$$c_t + s_{t+1} + p_t z_{t+1} = w_t + b_t \quad (1.7)$$

where p_t is the relative price of a unit of housing sold at time t and z_{t+1} captures the housing demand decided at time t .

At the period $t+1$ when the individuals become old, they take away their returns from savings and sell houses to finance consumption and to bequeath.

$$d_{t+1} + b_{t+1} = R_{t+1} s_{t+1} + p_{t+1} z_{t+1} \quad (1.8)$$

where R_{t+1} is the gross rate of return in capital at time $t+1$.

Concerning the initial old who is born in period -1 , he will decide how much they will consume and bequeath based upon their endowment of capital and housing stock. Their problem is following:

$$U(d_0, b_0) = \beta \ln(d_0) + \rho \ln(b_0) \quad (1.9)$$

$$s.t. \quad d_0 + b_0 = R_0 s_0 + p_0 z_0 \quad (1.10)$$

where R_0 and p_0 are chosen based upon the initial capital and housing endowment k_0 and h which are given at time 0 .

Given the initial capital stock, housing stock and the sequences of prices, k_0 , h , $\{R_t, p_t\}_{t=0}^{\infty}$, the first order conditions characterize the optimal decisions of any individuals born from at any time t are the following:

$$c_t = \frac{1}{\Gamma} (w_t + b_t) \quad (1.11)$$

where we define $\Gamma \equiv 1 + \beta + \gamma + \rho$.

$$d_{t+1} = \frac{\beta R_{t+1}}{\Gamma} (w_t + b_t) \quad (1.12)$$

$$z_{t+1} = \frac{\gamma}{\Gamma} \frac{w_t + b_t}{p_t - \frac{p_{t+1}}{R_{t+1}}} \quad (1.13)$$

$$b_{t+1} = \frac{\rho R_{t+1}}{\Gamma} w_t + \frac{\rho R_{t+1}}{\Gamma} b_t \quad (1.14)$$

Given (1.7) and (1.11), the total savings of individual i which is composed by capital and housing will be computed as:

$$s_{t+1} + p_t z_{t+1} = \frac{\Gamma - 1}{\Gamma} (w_t + b_t) \quad (1.15)$$

Due to the log utility function, it's no surprise that consumption, bequest and savings are proportional to total income. Moreover, since housing demand, captured by the variable z_{t+1} appears in the log utility function, it has to be positive. Therefore, from (1.13), one can pin down the condition on the returns of the two asset in equilibrium:

$$p_t - \frac{p_{t+1}}{R_{t+1}} > 0$$

which captures the cost of housing: the difference between the current cost and the discounted housing value. This result is reasonable as housing in our setting serves as consumption good. When one consumes house services, he has to pay a housing cost. Therefore, in equilibrium, this cost must be strictly positive, otherwise individuals will have infinite demand for housing. In a slightly different setting, Deaton and Laroque (2001) also presented in their work this condition on the user cost of housing.

Put differently, we obtain one condition on the gross rate of return on capital:

$$R_{t+1} > \frac{p_{t+1}}{p_t} \quad (1.16)$$

Secondly, one can prove that it's optimal if individuals decide not to save, i.e: $s_t = 0$. Indeed, from (1.14) that describe the dynamics of bequest, one can see that bequest will rise unbounded unless we impose the following condition for any time t :

$$\frac{\rho}{\Gamma} R_{t+1} < 1$$

where $\Gamma = 1 + \beta + \gamma + \rho$. Put it differently, we obtain another condition on the rate of return R_{t+1}

$$R_{t+1} < \frac{\Gamma}{\rho} \quad (1.17)$$

Since the capital rate of return is bounded by a finite number $\frac{\Gamma}{\rho}$, individual savings has to be finite and strictly positive, i.e. $s_{t+1} > 0$. Otherwise, the rate of return would go to infinity.

Solving the problem of the initial old agent given in (1.9) and (1.10) gives us:

$$d_0 = \frac{\beta}{\beta + \rho}(R_0 s_0 + p_0 z_0) \quad (1.18)$$

$$b_0 = \frac{\rho}{\beta + \rho}(R_0 s_0 + p_0 z_0) \quad (1.19)$$

4. Steady-state solutions and transitional dynamics

4.1. The inter-temporal equilibrium

Definition 1. *Given the initial capital stock per capita k_0 , housing stock per capita h , and a sequence of parameters $\{A_t\}_{t=0}^{\infty}$, an intertemporal equilibrium path is characterized by sequences of individual consumptions, capital and housing investment, bequest, $\{c_t, d_t, s_t, z_t, b_t\}_{t=0}^{\infty}$, and factor prices, $\{R_t, w_t, p_t\}_{t=0}^{\infty}$, such that:*

(i) for all $t > 0$, $\{c_t, d_t, s_t, z_t, b_t\}_{t=0}^{\infty}$ solve the problem of an agent born in period t given by (1.6)-(1.8)

(ii) (c_0, b_0) solve the problem of the initial old given by (1.9) and (1.10)

(iii) R_t, w_t satisfy the optimality conditions of firms given by (3.12) and (3.13)

(iv) housing, capital, labor and non-housing goods markets clear.

$$z_t = h \quad (1.20a)$$

$$s_{t+1} = k_{t+1} \quad (1.20b)$$

$$L_t = 1 \quad (1.20c)$$

$$f(A_t, k_t) = c_t + d_t + s_{t+1} \quad (1.20d)$$

Given the clearing conditions stated in (19a), (19b) and (19c), from (1.13) and (1.15), one can rewrite the functions define the dynamics of physical capital and housing as follows:

$$k_{t+1} = \frac{\beta + \rho}{\gamma} h p_t - \frac{\Gamma - 1}{\gamma} h \frac{p_{t+1}}{R_{t+1}} \quad (1.21)$$

$$\left(p_t - \frac{p_{t+1}}{R_{t+1}}\right) h = \frac{\gamma}{\Gamma} (w_t + b_t) \quad (1.22)$$

One can now construct the dynamic system of the whole economy. Notice that it

can be fully characterized by the dynamics of k_t and p_t . By replacing (18a) and (18b) into (1.15), (1.21) and (1.22), we obtain the dynamics of housing price and capital accumulation as follows:⁵

$$p_{t+1} = \alpha A_{t+1} k_{t+1}^{\alpha-1} p_t - \frac{\gamma\alpha}{h} \left(\frac{\rho + \beta(1 - \alpha)}{\Gamma\beta + \rho} A_t A_{t+1} k_t^\alpha k_{t+1}^{\alpha-1} - \frac{\rho}{\Gamma\beta + \rho} A_{t+1} k_{t+1}^\alpha \right) \quad (1.23)$$

$$k_{t+1} = \frac{(\Gamma - 1)(\rho + \beta(1 - \alpha))}{\Gamma(\beta + \rho)} A_t k_t^\alpha - \frac{\Gamma\beta + \rho}{\Gamma(\beta + \rho)} h p_t \quad (1.24)$$

Define a new variable $x_t \equiv \frac{p_t}{k_t^\alpha}$. We can see that x_t measures the ratio of house prices relative to income.

One can then reduce the dynamic system of (k_t, p_t) given in (2.25c) and (1.24) to the dynamics of solely x_t .

$$\frac{h(\Gamma\beta + \rho)}{A_{t+1}} x_{t+1} = \alpha\Gamma(\beta + \rho) \frac{\frac{h(\Gamma\beta + \rho)}{A_t} x_t - \gamma(\rho + \beta(1 - \alpha))}{(\Gamma - 1)(\rho + \beta(1 - \alpha)) - \frac{h(\Gamma\beta + \rho)}{A_t} x_t} + \gamma\alpha\rho \quad (1.25)$$

where recall that $\Gamma = 1 + \gamma + \beta + \rho$.

Notice, firstly, that (1.25) is defined if and only if for all t :

$$x_t \neq \frac{(\Gamma - 1)(\rho + \beta(1 - \alpha))A_t}{h(\Gamma\beta + \rho)}$$

Secondly, recall that x_t is constructed from a pre-determined variable k_t and a forward-looking variable p_t . Thus, x_t is a forward-looking variable.

4.2. The dynamic path of the economy

In what follows, we are going to provide the local analysis of the dynamic system. Recall that the dynamics of x_t is described in (1.25). To ease writing and facilitate our proof, we will rewrite the dynamics of x_t :

$$\begin{aligned} x_{t+1} &= g(x_t; A_{t+1}, A_t) \\ &\equiv \frac{\alpha\Gamma(\beta + \rho)A_{t+1}}{h(\Gamma\beta + \rho)} \left[\frac{\frac{h(\Gamma\beta + \rho)}{A_t} x_t - \gamma(\rho + \beta(1 - \alpha))}{(\Gamma - 1)(\rho + \beta(1 - \alpha)) - \frac{h(\Gamma\beta + \rho)}{A_t} x_t} + \gamma\alpha\rho \right] \end{aligned} \quad (1.26)$$

⁵Detailed proof is given in the section 1 of Appendix

Then, we are going to study the feature of the function $g(x_t; A_{t+1}, A_t)$.

Firstly, we observe that x_{t+1} receives negative values when x_t either tends to 0 or to $+\infty$. Indeed, one can show that:

$$\lim_{x_t \rightarrow 0} g(x_t; A_{t+1}, A_t) = -\frac{\alpha \gamma A_{t+1}}{(\Gamma - 1)h} < 0$$

$$\lim_{x_t \rightarrow +\infty} g(x_t; A_{t+1}, A_t) = \frac{\alpha A_{t+1}}{h(\Gamma\beta + \rho)} (\gamma\rho - \Gamma(\beta + \rho)) < 0^6$$

$$\lim_{x_t \rightarrow \frac{(\Gamma-1)(\rho+\beta(1-\alpha))A_t^-}{h(\Gamma\beta+\rho)}} g(x_t; A_{t+1}, A_t) = +\infty$$

$$\lim_{x_t \rightarrow \frac{(\Gamma-1)(\rho+\beta(1-\alpha))A_t^+}{h(\Gamma\beta+\rho)}} g(x_t; A_{t+1}, A_t) = -\infty$$

Secondly, from (1.26) one can prove that:

$$g'(x_t; A_{t+1}, A_t) = \frac{(\beta + \rho)^2 \alpha (\rho + \beta(1 - \alpha)) \Gamma}{(\Gamma\beta + \rho)^2 \left((\Gamma - 1)(\rho + \beta(1 - \alpha)) - \frac{h(\Gamma\beta + \rho)}{A_t} x_t \right)^2} \frac{A_{t+1}}{A_t} > 0$$

which means that x_t evolves monotonically over the following intervals:

$$\left(0, \frac{(\Gamma - 1)(\rho + \beta(1 - \alpha))A_t^-}{h(\Gamma\beta + \rho)} \right) \cup \left(\frac{(\Gamma - 1)(\rho + \beta(1 - \alpha))A_t^+}{h(\Gamma\beta + \rho)}, +\infty \right)$$

Thirdly, one can easily derive the second derivative of $g(x; A_{t+1}, A_t)$ as follows:

$$g''(x_t; A_{t+1}, A_t) = \frac{(\beta + \rho)^2 \alpha (\rho + \beta(1 - \alpha)) \Gamma}{(\Gamma\beta + \rho) \left((\Gamma - 1)(\rho + \beta(1 - \alpha)) - \frac{h(\Gamma\beta + \rho)}{A_t} x_t \right)^3} \frac{2hA_{t+1}}{A_t^2}$$

and show that:

(1) when $x_t \in \left(0, \frac{(\Gamma-1)(\rho+\beta(1-\alpha))A_t^-}{h(\Gamma\beta+\rho)} \right)$, then $g''(x_t; A_{t+1}, A_t) > 0$. Therefore, we can conclude that $g(x; A_{t+1}, A_t)$ is a convex function in such interval.

(2) when $x_t \in \left(\frac{(\Gamma-1)(\rho+\beta(1-\alpha))A_t^+}{h(\Gamma\beta+\rho)}, +\infty \right)$, then $g''(x_t; A_{t+1}, A_t) < 0$. Hence, $g(x; A_{t+1}, A_t)$ is a concave function in this interval.

⁶Notice that $\Gamma(\beta + \rho) > \gamma\rho$, therefore $\gamma\rho - \Gamma(\beta + \rho) < 0$.

Figure (1.3) below displays the global dynamics of x_t . It can be seen that the dynamics of x_t cuts the 45° line from below at one positive point x . Since the variable x_t is constructed by capital stock k_t and housing price p_t , a positive steady state value of x implies a positive long-run capital stock k and housing price p .

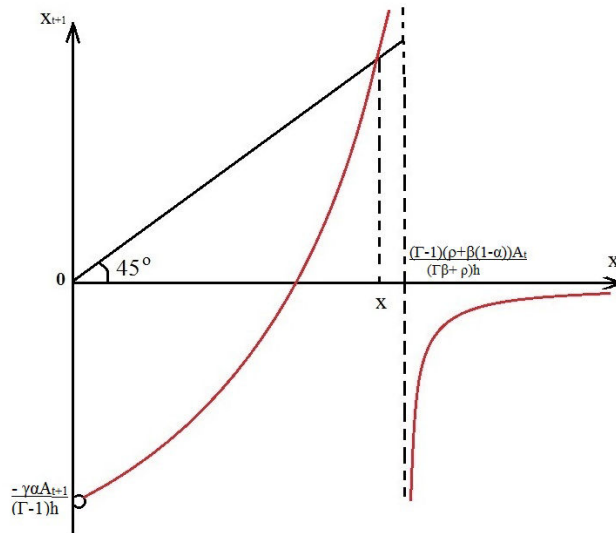


Figure 1.3: The global dynamic $g(x_t)$ in red lines.

Proposition 2. x is a locally unstable equilibrium point.

Proof. From the Figure (1.3), one can see that the function of x_t cuts the 45-degree line from below, the tangent at the point x is greater than 1. Therefore the equilibria x is locally unstable. \square

The locally unstable solution x requires that any divergence away from x would lead to explosive paths. Differently speaking, the system has to start at the steady state value x and remains there forever along the equilibrium path, i.e. $x_t = x \forall t$.

Recall that x_t captures the house price-to-income ratio. The finding hence suggests that, in the absence of shocks to the economy, this ratio remains constant over time. Equivalently, we expect that the prices of house and income should move in the same direction and at the same speed. This finding is consistent with the data. Figure (1.4) below plots the time-series price-to-income ratio in real term in US, France and OECD country average. What we see is that this ratio is rather stagnant from 1980s until the arrival of housing boom starting from 2000 which supports our finding.

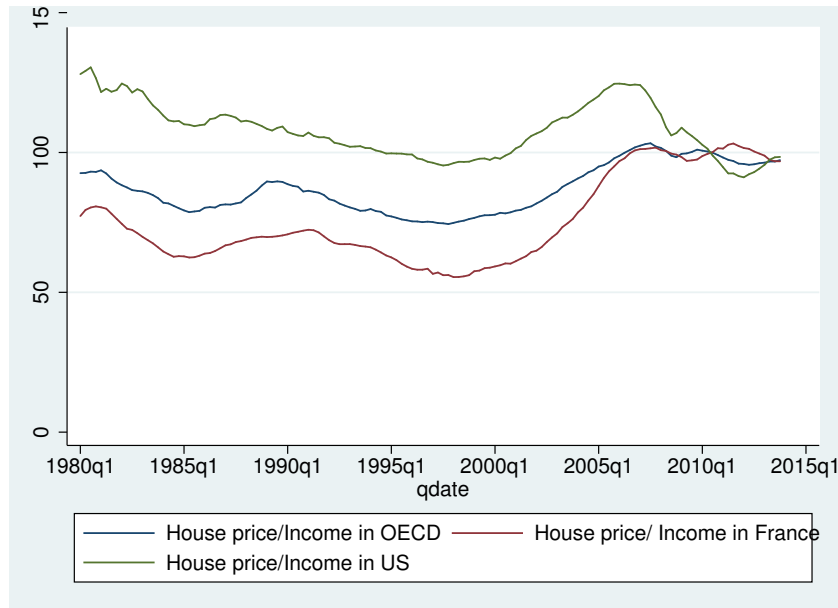


Figure 1.4: The evolution of price to income ratio in real term. Reference year= 2010
Source: *Federal Reserve Bank of Dallas*.

The result provided in the proposition 2 requires that for any time t , along the equilibrium path, we must have $x_t = x$, meaning that:

$$\frac{p_t}{k_t^\alpha} = \frac{p}{k^\alpha} \tag{1.27}$$

where p and k are the steady state values of house price and capital stock, respectively. Put differently, for all t

$$p_t = \frac{p}{k^\alpha} k_t^\alpha$$

Knowing that k_0 is initially given at time 0, one can then derive the initial price of housing p_0 . From (1.27), we can easily pin down its initial value:

$$p_0 = \frac{p}{k^\alpha} k_0^\alpha \tag{1.28}$$

We can now introduce the following proposition:

Proposition 3. *Housing prices and capital accumulation co-move along the equilibrium path.*

This comovement between the two variables are also found in many empirical studies. See for example, Iacoviello (2005), Muellbauer and Murphy (2008), Ferrara et al. (2009).

From (1.23) and (1.24) and recall that $\frac{p_t}{k_t^\alpha} = x_t$, we then can rewrite the dynamics of capital and housing prices as a function of the steady state value x_t as follows:

$$k_{t+1} = \left(\frac{(\Gamma - 1)(\rho + \beta(1 - \alpha))}{\Gamma(\beta + \rho)} A_t - \frac{\Gamma\beta + \rho}{\Gamma(\beta + \rho)} hx_t \right) k_t^\alpha \quad (1.29)$$

$$p_{t+1} = \left(\frac{(\Gamma - 1)(\rho + \beta(1 - \alpha))}{\Gamma(\beta + \rho)x_t} A_t - \frac{\Gamma\beta + \rho}{\Gamma(\beta + \rho)} h \right)^\alpha p_t^\alpha x_{t+1} \quad (1.30)$$

where x_{t+1} can be expressed as a function of x_t given in (1.26). We'll now study the dynamics of p_t and k_t in the short and long-run.

4.3. Steady State solutions

In what follows, we are going to study the long run economy. Notice that the system at steady state can be characterized as a function of x stated in eqn(1.25). Let $x_t = x$ and $A_t = A$. The number of equilibria in the model is then equal to the number of solutions of x when one solves $g(x) - x = 0$.

Having $g(x) - x = 0$ is equivalent with having:

$$\frac{(\Gamma\beta + \rho)}{\alpha\Gamma(\beta + \rho)A} hx - \frac{\frac{(\Gamma\beta + \rho)}{A} hx - \gamma(\rho + \beta(1 - \alpha))}{(\Gamma - 1)(\rho + \beta(1 - \alpha)) - \frac{(\Gamma\beta + \rho)}{A} hx} - \gamma\alpha\rho = 0 \quad (1.31)$$

From the dynamic analysis provided in the previous section, we can conclude that there exists a unique solution of x which is positive.

We can then derive the values of other variables according to x . Indeed, we can write the steady state values of other variables as a function of x as follows:

$$k^{1-\alpha} = \frac{(\Gamma - 1)(\rho + \beta(1 - \alpha))}{\Gamma(\beta + \rho)} A - \frac{\Gamma\beta + \rho}{\Gamma(\beta + \rho)} hx \quad (1.32)$$

$$p = xk^\alpha \quad (1.33)$$

$$R = \alpha Ak^{\alpha-1} \quad (1.34)$$

$$w = (1 - \alpha)Ak^\alpha \quad (1.35)$$

$$b = \frac{\rho R}{\Gamma - \rho R} w \quad (1.36)$$

$$c = \frac{1}{\Gamma - \rho R} w \quad (1.37)$$

$$d = \frac{\beta R}{\Gamma - \rho R} w \quad (1.38)$$

where $\Gamma = 1 + \beta + \theta + \rho$.

Since the steady state value of x is unique, the steady state value of $\{k, p, R, w, c, d, b\}$

are unique, accordingly. Therefore, we can now introduce the following proposition.

Proposition 4. *There exists unique equilibrium of the economy with positive capital stock and housing price.*

In what follows, we are going to study the impacts of different shocks on the equilibrium housing prices and production output. We are taking into account the following shifts: a shift in technology, a shift in per capita housing supply and a shift in housing preference.

5. Impact of a productivity shift A_t

Changes in technology has been recently cited as the driving force of the boom in house prices. For example, Kahn(2008) develops a calibrated growth model with land, housing services, and non-housing goods and shows that trend productivity growth is the key driver of the house price fluctuations over the past forty years, including the recent slowdown. In the same spirit but employing different setting that allows tractable analysis, we would like to know if a gain in technology is responsible for the comovement between house prices and GDP as well as the mechanism that passes this effect from production sector to housing sector.

5.1. Long term effects on output and house prices

Consider a permanent and positive shift in technology A_t . Putting (1.26) at steady state, one can derive the responsiveness of x when A shifts upward. Indeed,

$$\frac{\partial x}{\partial A} = \frac{x}{A} \tag{1.39}$$

Therefore, from (1.32) and (1.33), one can prove that:⁷

$$\frac{\partial p}{\partial A} > 0 \tag{1.40}$$

$$\frac{\partial k}{\partial A} > 0 \tag{1.41}$$

which will lead to:

$$\frac{\partial Y}{\partial A} > 0$$

which states that production output will also increase in response to the shock. We can now introduce the following proposition:

⁷Detailed proof is provided in the section 4 of Appendix

Proposition 5. *A positive and permanent shift in technology will increase the long-run production output and housing price.*

Eventually, a permanent gain in technology impacts on prices and capital accumulation via relaxation in the budget constraint. Through shifting up individual's income, he/she would have more resources to invest in both housing and capital. As a result, house prices and production output would rise. Similar papers that support the procyclicality of the two variables as a consequence of technology shift are Davis and Heathcote (2005), Kahn (2009), and Iacoviello (2005) among all others. For example, using a general equilibrium growth model with two sectors, Kahn (2009) suggested that the surge in home prices from the mid-1990s to 2007 was fueled by shifts in economic fundamentals.

5.2. Short-term effects on output and house prices

Let's take into account now a temporary and positive shift in technology at time t . Finding is stated in the following proposition.

Proposition 6. *A positive and transitory shift in technology will increase simultaneously housing prices and production output while capital accumulation is only driven up 1 period after the shock occurs.*

Proof. First, recall that capital stock k_t is pre-determined at time t . Thus, when the shock occurs at time t , k_t doesn't adjust immediately. We write:

$$\frac{\partial k_t}{\partial A_t} = 0$$

Secondly, note that output is a function of A_t and k_t . When A_t increases, output will be shifted immediately. The impact of a temporary shock on output will be:

$$\frac{\partial y_t}{\partial A_t} = k_t^\alpha > 0$$

Thirdly, recall that x_t is forward-looking variable. Therefore, when technology changes at time t , x_t adjusts immediately and accordingly. One can derive their link using (1.25):

$$\frac{\partial x_t}{\partial A_t} = \frac{x_t}{A_t} > 0 \tag{1.42}$$

Since x_t captures the house price to income ratio. Thus, the effect of a technology

shock on the housing price will be:⁸

$$\frac{\partial p_t}{\partial A_t} = k_t^\alpha \frac{\partial x_t}{\partial A_t} > 0$$

And from the dynamics of k_t stated in (1.24), one can easily prove that:

$$\frac{\partial k_{t+1}}{\partial A_t} > 0 \quad \square$$

Similarly with the results related to permanent positive technology shock, here we also find positive influences of a gain in short-run technology on both housing prices and GDP via income effect. As a gain in technology increases individual's income, they would have more disposable income to invest on houses and capital. As a result, property price will increase and capital accumulation rise.

6. Impact of a per capita housing stock shift h_t

Recall that the variable h_t measures housing stock per capita. Thus, a downward shift in h_t can be interpreted as either a negative change in aggregate housing stock or an increase in number of individuals in the market. A negative shift in aggregate stock can be triggered by natural disasters like earthquakes that causes great damage of houses. However, there's no evidence that such kind of disasters appeared globally in the period of time of our concern. On the other hand, an increasing house buyers is possibly the consequence of having either the booming generation entering the market during such period of time or/and the rise of immigration. There're quite few studies investigated the causes of per capita housing supply shortage, i.e. gap between housing supply relative to aggregate housing demand on the recent housing boom. For example, Glaeser et al (2005) argues that price increases since 1980 largely reflect artificial supply restrictions. Similarly, Gyourko et al (2006) also cite inelastically land construction in comparison with a rising housing demand as a key driver of the phenomenon they call "superstar cities". To see if our model can capture this evidence, we consider a negative shock on per capita housing stock h_t which means that housing supply is assume to fall short of the numbers of households. We are then interested in knowing whether this particular shock would be plausible for explaining the comovement between the prices and production output.

⁸Recall that p is not a pre-determined variable

6.1. Long term effects on capital accumulation and housing prices

Proposition 7. *An permanent drop in housing supply will be fully absorbed in housing market so that housing price immediately increases while capital accumulation and output aren't affected.*

Proof. For simplicity, assume that in this scenario, $A_t = A$ for all t . It can be seen from the equations (3.26), which are used to define the long-run value of x , that any adjustment in housing stock h will be fully offset by a change in x . Therefore, from (1.32), one can see that capital is independent of h . Put differently, any changes from housing supply has no impact on capital and output, i.e.

$$\frac{\partial k}{\partial h} = 0$$

$$\frac{\partial y}{\partial h} = 0$$

On the other hand, from (1.33), one can derive the reaction of price p when h changes:

$$\frac{\partial p}{\partial h} = -\frac{p}{h} < 0 \quad \square$$

Due to our log utility choice, housing wealth, which is measured by the product ph , is a constant proportion of total income⁹. As capital in the long-run is independent of the available stock of housing in the economy, any permanent adjustment housing stock doesn't have any impacts on capital and thus it has no effects on wealth. Moreover, as the shock is assumed to be deterministic, individuals are able to anticipate the change in housing supply and they adjust their demand in response to such change. As a result, a permanent shortage in housing supply would be absorbed by a shift upward in house prices.

6.2. Short term effects on capital accumulation and housing prices

Instead, we consider now an impact of a temporary change in housing stock. Assume that there occurs a short-term drop in the housing supply at time t so that the after-shock new stock is h'_t and h'_t is smaller than the initial housing stock h_t . We can interpret such shock as if there was a temporary shortage in stock in the sense that housing supply can't catch up a rise in demand. To study this effect, we assume that

⁹Indeed, by putting (1.22) at steady state, we obtain that $ph = \frac{1}{1-\rho R}w$.

before the shock occurrence, the economy is set in its steady state.

We state the following proposition.

Proposition 8. *Production output adjusts in the same direction with housing price in response to a negative temporary shock on house supply in which house price increases in the first place while output rises in the second place. After the shock, they fall back to their initial states.*

Proof. Let the housing supply h_t be time-variant and rewrite the (1.25) as:

$$\frac{h_t(\Gamma\beta + \rho)}{A}x_{t+1} = \alpha\Gamma(\beta + \rho)\frac{\frac{h_t(\Gamma\beta+\rho)}{A}x_t - \gamma(\rho + \beta(1 - \alpha))}{(\Gamma - 1)(\rho + \beta(1 - \alpha)) - \frac{h_t(\Gamma\beta+\rho)}{A}x_t} + \gamma\alpha\rho \quad (1.43)$$

The responsiveness of x_t to a temporary change in h_t can be derived as:

$$h_t\frac{\partial x_{t+1}}{\partial h_t} + x_{t+1} = \frac{(\beta + \rho)(\beta(1 - \alpha) + \rho)}{[(\Gamma - 1)(\rho + \beta(1 - \alpha)) - \frac{h_t(\Gamma\beta+\rho)}{A}x_t]^2}\left(\frac{\partial x_t}{\partial h_t}h_t + x_t\right) \quad (1.44)$$

Recall that x_t is locally unstable. Hence when the shock occurs at time t , x_t will jump immediately to the new value, maintain there and only jump back to the initial point at the moment that the shock is gone. It means that at the time $t + 1$, x_{t+1} doesn't change. Hence, we obtain that:

$$\frac{\partial x_{t+1}}{\partial h_t} = 0$$

From (1.44), it will lead to the following result:

$$\frac{\partial x_t}{\partial h_t}h_t + x_t = x_{t+1}\frac{[(\Gamma - 1)(\rho + \beta(1 - \alpha)) - \frac{h_t(\Gamma\beta+\rho)}{A}x_t]^2}{(\beta + \rho)(\beta(1 - \alpha) + \rho)} > 0$$

Equivalently, we can also say that:

$$\epsilon_{x_t, h_t} \equiv \frac{\partial x_t/x_t}{\partial h_t/h_t} > -1 \quad (1.45)$$

where ϵ_{x_t, h_t} captures the elasticity of substitution between price-to-income ratio and housing supply per capita. We found that this elasticity has to be greater than -1.

Since housing price is forward-looking variable, at the time t when the housing supply shock happens, the price will respond immediately to the shock. The effect of the shock to house price can be derived as:

$$\frac{\partial p_t}{\partial h_t} = \frac{\partial x_t}{\partial h_t} < 0$$

On the other hand, capital stock responds to the shock with one-period of delay. We

can now compute its responsiveness at time $t + 1$. From the dynamics of capital stock given by (1.29), we can derive the response of capital at time t and $t+1$ to a sudden reduction of housing supply at time t as follows:

$$\frac{\partial k_t}{\partial h_t} = 0 \tag{1.46a}$$

$$\frac{\partial k_{t+1}}{\partial h_t} = -x_t(1 + \epsilon_{x_t, h_t}) \tag{1.46b}$$

Taking the result stated in (1.45) and replacing it into $\frac{\partial k_{t+1}}{\partial h_t}$, we can now prove that:

$$\frac{\partial k_{t+1}}{\partial h_t} < 0$$

which states that capital stock also responds positively to a reduction in housing supply. \square

Let us now discuss the intuition of the finding. In our setting, there's no housing production and the stock is fixed. Therefore, a reduction in house supply eventually leads to a higher house price. We found that the absolute value of the supply elasticity of price is smaller than 1 which means that price adjusts inelastically and less proportionally to a shift in stock. As a result, one has to pay less for housing as the cost of housing that one has to purchase becomes eventually cheaper¹⁰. Since capital and housing are substitutable, a reduction in housing investment would give individuals more disposable income to invest to capital savings. Capital accumulation and GDP therefore increase in this case.

7. Impact of a housing preference shift

In this part, we will instead study impacts from a shift in housing preference. Work that supports the hypothesis that the recent housing boom is caused by changes in the housing preference are Kaplan et al.(2017), Piazzesi and Schneider(2016) among all others. For example, Kaplan et al.(2017) studied a model of the U.S. economy with multiple aggregate shocks. They found that stronger preference for housing services of households would be a potential driver of the recent housing boom and bust. Therefore, it's of our interest to study the impact of a shift in individual's housing desire on property prices as well as on production output.

¹⁰Due to the fact that $\frac{\partial p_t h_t}{\partial h_t} > 0$, $p_t h_t$ moves in the same direction with h_t . Thus when h_t decreases, $p_t h_t$ also diminishes accordingly.

7.1. Long term effects on economic output and housing prices

We are interested in knowing effects from a change in housing preference captured by parameter γ . In reality, a shift in one's desire for possessing a house can be interpreted as a demand shock on houses. This phenomenon is expected to be generated during the time where access to housing credit is easy. It's obvious that a higher γ leads to an increase in demand for houses.

Proposition 9. *A positive and permanent shock on housing preference induces an increase in housing price and crowds out the steady state capital stock via the budget constraint.*

This result¹¹ is consistent with the work of Deaton and Laroque (2001) about the behavior of capital when land is introduced into the model. However, in their work, they compare the two extreme economies in which one contains land while the other totally excludes land. As land and capital stock are substitutable, introducing land in the economy will reduce the amount of capital disposable for production. Therefore, a stronger desire to own land would make the capital stock to be smaller.

7.2. Short term effects on output and housing prices

Let γ_t be a time-variant variable and consider a positive and transitory shock on γ_t . Rewrite the (1.26) as a function of γ_t , we get:

$$x_{t+1} = \frac{\alpha\Gamma_t(\beta + \rho)A}{h(\Gamma_t\beta + \rho)} \left(\frac{\frac{h(\Gamma_t\beta + \rho)}{A}x_t - \gamma_t(\rho + \beta(1 - \alpha))}{(\Gamma_t - 1)(\rho + \beta(1 - \alpha)) - \frac{h(\Gamma_t\beta + \rho)}{A}x_t} + \gamma_t\alpha\rho \right) \quad (1.47)$$

where $\Gamma_t = 1 + \beta + \rho + \gamma_t$.

We use the same argument provided in the previous section in which x_t is argued to adjust immediately to any shocks at time t , maintain there and drop back to initial value when the shock is over. Put differently, we can say that the magnitude of response of x_t when the shock is temporary is equivalent with the one when shock is permanent. The only difference between the two cases is the length of reactions.

We then can write that:

$$\frac{\partial x_{t+1}}{\partial \gamma_t} = 0$$

Detailed proof is given in the appendix that states that:

$$\frac{\partial x_t}{\partial \gamma_t} > 0$$

¹¹Proof is given in the section 3 of Appendix.

which basically suggests that the house prices- to- income ratio increases in response to a stronger demand on housing.

Due to the fact that k_t is pre-determined, p_t will shift immediately at the time the shock happens. Therefore,

$$\frac{\partial p_t}{\partial \gamma_t} > 0$$

The intuition behind this result is simple. A gain in γ_t means a higher housing desirability. While housing stock is assumed to be fixed, a higher demand with a constant supply would lead to an immediate increase in housing price.

Finally, regarding the effects on capital accumulation, one can prove that a gain the desirability for houses will crowd out the capital stock. Hence,

$$\frac{\partial k_{t+1}}{\partial \gamma} < 0$$

$$\frac{\partial k_t}{\partial \gamma} = 0$$

Since capital stock is a pre-determined variable, it doesn't respond simultaneously at the time the shock happens, i.e. $\frac{\partial k_t}{\partial \gamma_t} = 0$.

Proposition 10. *The reaction of capital accumulation is opposite to housing price in response to a positive housing demand shock. While housing price increases, capital stock and also GDP reduce when the shock occurs. They back to their initial states once the shock is passed.*

8. Numerical analysis

In this section, we focus particularly on how individual consumptions change in response to shocks. the values of calibrated parameters are summarized in the table 1 below. The values of parameters for the simulation work are picked up from similar work provided in the Reference.

Table 1.1: Calibrated Parameters

Parameter	Value	Reference
Output elasticity of capital α	1/3	Ludwig and Vogel(2010), Cipriani(2014)
Discount factor β	0.67	De la croix(2001), Ludwig and Vogel(2010)
Housing preference γ	0.35	Gervais(1998), Zhao(2015)
Bequest preference ρ	0.4	Nishiyama,2002; Bellettini et al. (2017a),
Housing stock per capita h	1	
TFP A	1	Ludwig and Vogel(2010)

Recall that h measures the housing supply per capita. Thus, for simplicity, the variable h is normalized to 1. However, note that without loss of generality, this variable can be normalized to any strict positive values.

8.1. Dynamic system after a technology shock

Consider a shift upward in technology. According to Kahn (2008), boom in housing prices, which started in the late 1990s, occurred around the time many analysts determined that productivity growth believed to be trending up at an annual rate of nearly 3 percent. Therefore, we are interest in knowing how this kind of shock would affect the whole variables in the economy. We consider hereafter a permanent and temporary gain in technology at 3%. Figure (1.5) and (1.6) below summarize our finding, respectively.

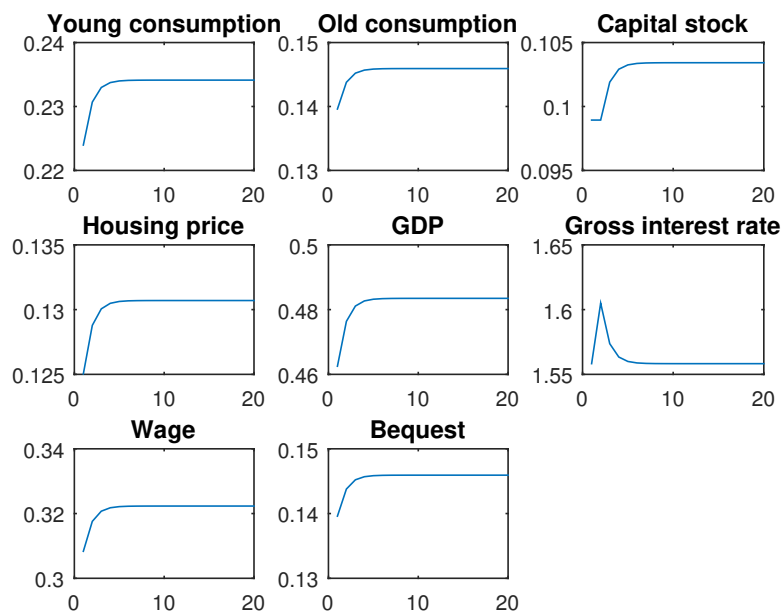


Figure 1.5: Dynamics after a permanent technology shock over time : A increases from 1 to 1.03

Note: The y-axis measures actual values of each variable

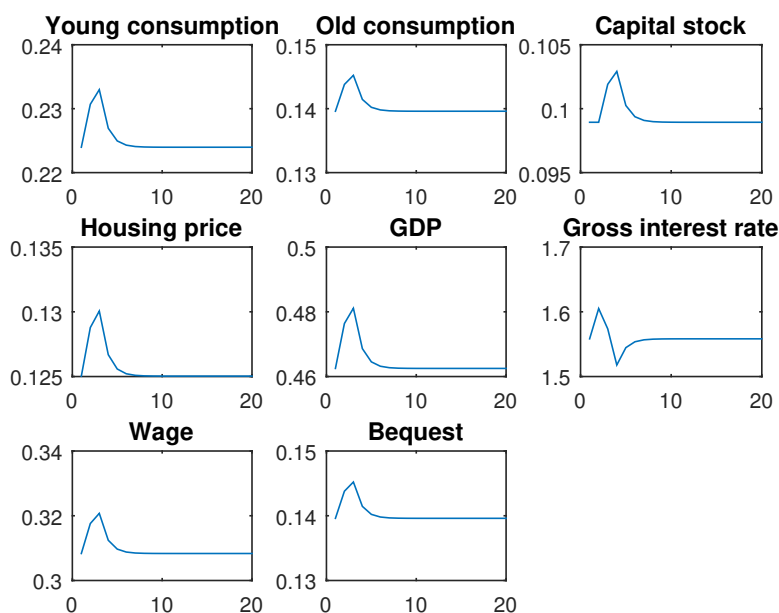


Figure 1.6: Dynamics after a temporary technology shock over time : A increases from 1 to 1.03 for 1 period.

Note: The y-axis measures actual values of each variable

As we can see, in response to such shocks, housing price, capital stock as well as GDP increase accordingly. Moreover, regarding impacts on consumption, we found that the occurrence of both temporary and permanent technology shocks benefit the young and the old's consumptions as they all increase accordingly. This positive effects are transferred through the income effect. Due to log utility setting, consumptions and savings are constant portion of total income. Therefore, a gain in technology makes the income of young households higher which, in turn, increases their housing and non-housing consumptions and their investments. Regarding effects on the old households, as their investments of increases in the first place, there's no surprise that the their total income when they get old increases. As a result, their consumption when they are old increases. Additionally, recall that in this setting we ignore population growth and assume that, in each period of time, there are equal fractions of young and old individuals. Thus, the aggregate consumption of the economy is simply the sum of their two assumptions. Since their both consumptions increase, the aggregate consumption rise accordingly. We observe in this scenario the comovement between consumption and house price in response to a technology shift. The procycle of these two variables has been studied in many papers such as Campbell and Cocco (2007) and Berger et al(2017).

8.2. Dynamic system after a negative housing supply shock

The response of all variables to a negative permanent shift in housing stock is summarized in the Figure (1.7) below. We assume that housing stock experiences a shortage of 5 %. As proved in the previous section, we observe similarly in this section that a permanent change in housing supply is fully absorbed by a shift in house price. Hence, consumptions do not respond to a permanent changes in housing stock.

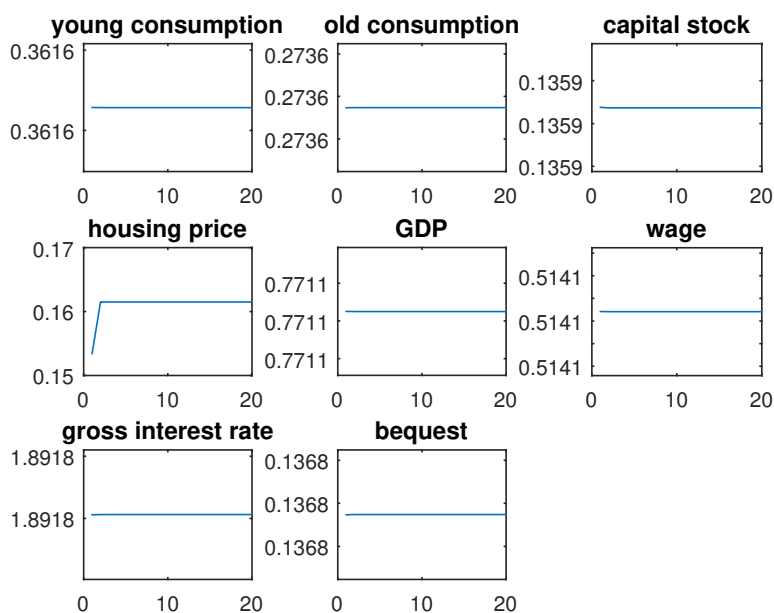


Figure 1.7: Transitional dynamics after a negative permanent housing supply shock : h decreases from 1 to 0.95.

Note: The y-axis measures actual values of each variable

Next, we present the simulation results concerning a negative and temporary shock to housing supply that lasts 1 period given by the figure (1.8) below.



Figure 1.8: Transitional dynamics after a negative temporary housing supply shock : h decreases from 1 to 0.95 for 2 periods.

Note: The y-axis measures actual values of each variable

One can see clearly that housing prices and GDP per capita co-move in response to a supply shock which confirms the evidence that a housing shock drives the pro-cycle of the two variables in the short-run. Concerning impacts on consumption, it's interesting that in this case young generations will be better off as their consumption increases at the cost of the old one. If we believe that the old generation is supposed to be the rich one in the society while the majority of poor individuals are young, this result would imply that a negative shock on supply would help reduce the consumption inequality between the rich and the poor.

8.3. Dynamic system after a positive housing demand shock

Due to the lack of data, in this section, we will assume that the preference for houses, which is captured by variable γ , slightly increases by 5%. We present in the following figure (1.9) and (1.10) the simulation results related to positive permanent and temporary shocks on housing preference, respectively.

One can observe that in the two cases the gains of house prices depress the accumulations of capital. It comes from the fact that in this setting the two factors are substitutes. Since individuals have a higher preference for housing, they would be willing to spend a larger fraction of income for housing at the expense of savings and consumption. As a result, capital savings and consumption would decrease in response to the shock.

Regarding impacts on consumption, the wealth of the old generations will increase due to a rise in housing price. On the other hand, as the young one has to pay at a higher price to own the same amount of houses, their disposable income available for consumption would reduce. When we compare the responsiveness of consumption between two generations, we found that, in both cases, the young generation (i.e. the poor) is worse off while the old generation (i.e. the rich) is better off. This result suggests that a shock on housing preference will dampen the inequality between the rich and the poor.

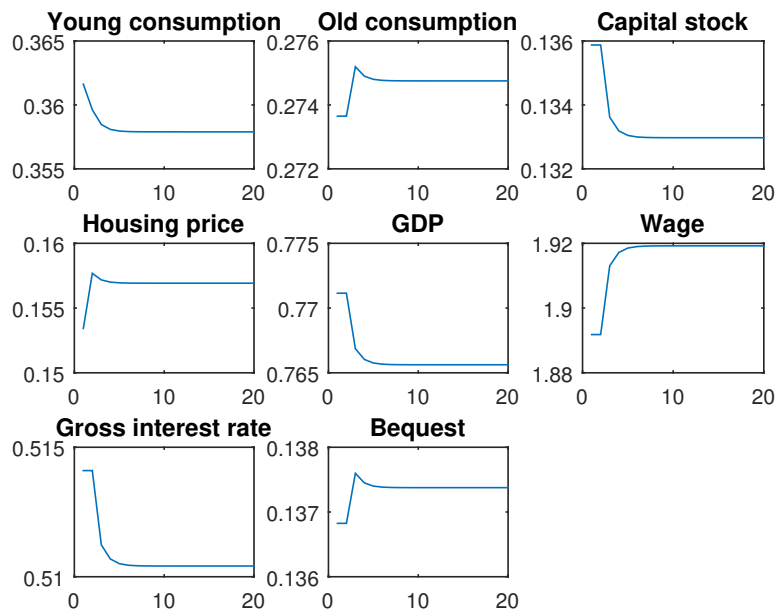


Figure 1.9: Transitional dynamics after a positive permanent housing preference shock : γ increases from 0.2 to 0.21

Note: The y-axis measures actual values of each variable

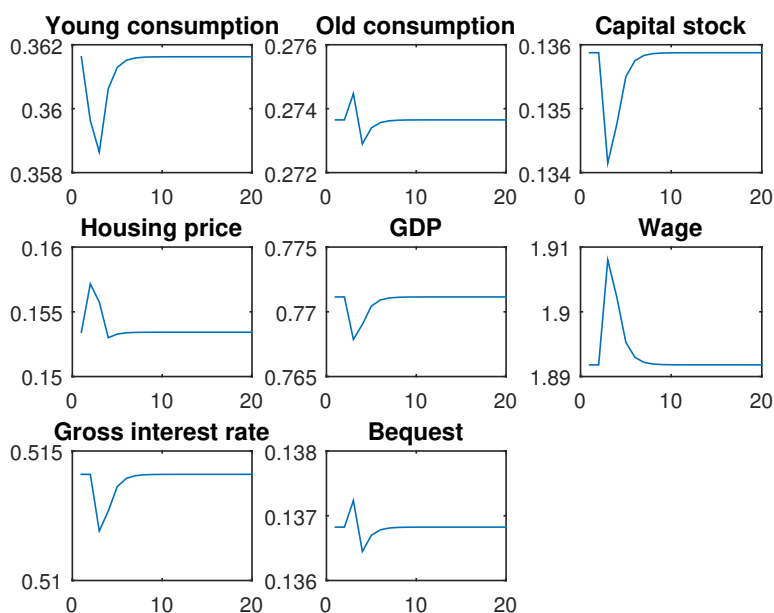


Figure 1.10: Transitional dynamics after a positive temporary housing preference shock : γ increases from 0.2 to 0.21 for 1 period.

Note: The y-axis measures actual values of each variable

9. Conclusion remarks

In this paper, we investigate the co-movement between housing price and economic output and we find that there exists a short-run virtuous cycle between them which is driven by either a temporary technology shift or a short-run shift in housing supply.

Particularly, GDP can fuel housing price appreciation via a technology shift which, in turn, leads to a further capital savings and higher economic growth on the next period. Similarly, via a shock on housing supply, its prices will respond immediately in the first place while capital and output only adjust accordingly at the second place.

On the other hand, the long-run pro-cyclicality between the two factors can only explained by long-term technology shock, which confirms the hypothesis that technology shift could have been one of the main factors that drove the recent housing price appreciation.

Regarding the impacts of these shocks on individual consumptions, our findings also suggest that a housing boom driven by a technology shock will benefit both the rich (the old) and the poor (the young) as their consumptions both raise up. Besides, a gain in house prices which is driven by the supply side may help reduce inequality by benefiting the poor at the expense of the rich while a rise in house prices triggered by the demand side may cause inequality to increase.

Knowing how different shocks impact differently on the aggregate economy would

be useful when one wants to construct fiscal policies targeting housing and capital that aims at boosting the economy while reducing the inequality distribution of wealth.

10. Appendices

A. The dynamics of capital and housing stock

The feasibility constraint of the economy at any time t is derived as follows:

$$f(k_t) = c_t^y + c_t^0 + s_{t+1}$$

Then replacing the eqn(14), (15) and (18) into the feasibility constraint, we have:

$$\frac{w_t + b_t}{\Gamma} + \frac{\beta}{\rho} b_t + k_{t+1} = f(k_t)$$

Hence, we can rewrite bequest as a function of the current and future capital stock:

$$b_t = \frac{\Gamma\rho}{\Gamma\beta + \rho} \left(\frac{\Gamma - 1}{\Gamma} w(k_t) + k_t R(k_t) - k_{t+1} \right) \quad (1.48)$$

where $R(k_t)$ and $w(k_t)$ are defined by eqn(8) and (9), respectively. Since

$$k_{t+1} = \frac{\Gamma - 1}{\Gamma} (w_t + b_t) - h_0 p_t$$

we can rewrite it using the eqn(42) and obtain the dynamics of capital:

$$k_{t+1} = \frac{E}{1 + M} A_t k_t^\alpha - \frac{h_0}{1 + M} p_t \quad (1.49)$$

where $E \equiv \frac{(\Gamma-1)(\rho+\beta(1-\alpha))}{\Gamma\beta+\rho}$ and $M \equiv \frac{(\Gamma-1)\rho}{\Gamma\beta+\rho}$. And also from eqn(16) we can define the evolutionary law of housing price:

$$p_{t+1} = \alpha A_{t+1} k_{t+1}^{\alpha-1} p_t - \frac{\gamma\alpha}{(\Gamma - 1)h_0} (E A_t k_t^\alpha - M k_{t+1}) A_{t+1} k_{t+1}^{\alpha-1} \quad (1.50)$$

Hence, we can reduce the dynamic system of three equations to the system of two which includes capital stock and housing price that can be characterized by eqn(43) and (44) above.

B. Prove the dynamics of x_t given in eqn(26)

From eqn(25), one can rewrite:

$$\frac{k_{t+1}}{k_t^\alpha} = \frac{E A_t}{M + 1} - \frac{h_0}{M + 1} \frac{p_t}{k_t^\alpha} \quad (1.51)$$

From eqn(24), one can derive:

$$\frac{p_{t+1}}{k_{t+1}^\alpha} = \alpha A_{t+1} \frac{k_t^\alpha}{k_{t+1}} \left(\frac{p_t}{k_t^\alpha} - \frac{\gamma EA_t}{(\Gamma - 1)h_0} \right) + \frac{\alpha \gamma M A_{t+1}}{(\Gamma - 1)h_0} \quad (1.52)$$

Defining $x_t = \frac{p_t}{k_t^\alpha}$ and replacing eqn(46) into (47) we obtain:

$$x_{t+1} = (M + 1)\alpha A_{t+1} \frac{x_t - \frac{\gamma EA_t}{h_0(\Gamma - 1)}}{EA_t - h_0 x_t} + \frac{\gamma \alpha M A_{t+1}}{h_0(\Gamma - 1)} \quad (1.53)$$

When $x_t \rightarrow 0$, i.e. $\frac{p_t}{k_t^\alpha} \rightarrow 0$ due to either $p_t \rightarrow 0$ or $k_t \rightarrow \infty$, then $x_{t+1} \rightarrow -\frac{\gamma \alpha}{(\Gamma - 1)h_0}$ which is strictly smaller than 0.

When $x_t \rightarrow \infty$, i.e. $\frac{p_t}{k_t^\alpha} \rightarrow \infty$, then $x_{t+1} \rightarrow -\frac{\alpha}{h_0}(1 + M(\frac{\beta + \rho}{\Gamma - 1})) (< 0)$.

Besides, when $x_t \rightarrow (\frac{E}{h_0})^-$, from eqn(26) we get $x_{t+1} \rightarrow +\infty$.

When $x_t \rightarrow (\frac{E}{h_0})^+$, from eqn(26) we get $x_{t+1} \rightarrow -\infty$.

C. Prove the response of x_t to γ_t

Given the function of $g(x)$ introduced in the section 7.2, we can derive:

$$\frac{\partial x_t}{\partial \gamma_t} = \frac{\partial x}{\partial \gamma} = -\frac{\partial g(x)/\partial \gamma}{\partial g(x)/\partial x} \quad (1.54)$$

- One can easily obtain:

$$\frac{\partial g(x)}{\partial x} = 2x + \alpha(M + 1) - \frac{\alpha \gamma M}{\Gamma - 1} - E \quad (1.55)$$

where again $E \equiv \frac{(\Gamma - 1)(\rho + \beta(1 - \alpha))}{\Gamma \beta + \rho}$ and $M \equiv \frac{(\Gamma - 1)\rho}{\Gamma \beta + \rho}$

Knowing that if x_1 and x_2 are the two solutions when $g(x) = 0$ then

$$\begin{cases} x_1 + x_2 = -(\alpha(M + 1) - \frac{\alpha \gamma M}{\Gamma - 1} - E) \\ x_1 x_2 < 0 \end{cases}$$

Assume that $x_1 > 0$ while $x_2 < 0$, then obviously $x_1 > -(\alpha(M + 1) - \frac{\alpha \gamma M}{\Gamma - 1} - E)$ and therefore

$$2x_1 > -(\alpha(M + 1) - \frac{\alpha \gamma M}{\Gamma - 1} - E)$$

Since we only take into account the positive value of x , we can then conclude that, from eqn(56), $\frac{\partial g(x)}{\partial x} > 0$

- On the other hand, one has:

$$\frac{\partial g(x)}{\partial \gamma} = x \left(\alpha \frac{\partial(M+1)}{\partial \gamma} - \alpha \frac{\partial \frac{\gamma M}{\Gamma-1}}{\partial \gamma} - \frac{\partial E}{\partial \gamma} \right) - \alpha \frac{\partial \frac{\gamma E}{\Gamma-1}}{\partial \gamma} \quad (1.56)$$

Equivalently, we will have:

$$\frac{\partial g(x)}{\partial \gamma} = x(\beta + \rho)(\beta(\alpha - 1) - \rho(\alpha\beta + 1)) - \alpha(\rho + \beta(1 - \alpha))(\rho + \beta(\Gamma - \gamma))$$

which is clearly negative. Therefore,

$$\frac{\partial x_t}{\partial \gamma_t} > 0$$

Since k_t is pre-determined variable, only p_t is adjusted at the time the shock happens.

Thus, $\frac{\partial p_t}{\partial \gamma_t} > 0$

Recall that $x_{t+1} = \frac{p_{t+1}}{k_{t+1}^\alpha}$, letting $\frac{x_{t+1}}{\gamma_t} = 0$ will give us:

$$\frac{\partial p_{t+1}}{\partial \gamma_t} = \alpha \frac{p_{t+1}}{k_{t+1}} \frac{\partial k_{t+1}}{\partial \gamma_t}$$

which suggests that the responses of both k_{t+1} and p_{t+1} to a temporary shock on housing preference move in the same direction. From the simulation part, we know that the two respond negatively to the shock.

D. Proof of the proposition 5

From (1.32), one can derive that:

$$\frac{\partial k}{\partial A} = \left(\frac{(\Gamma - 1)(\beta(1 - \alpha) + \rho)}{\Gamma(\beta + \rho)} - \frac{\Gamma\beta + \rho}{\Gamma(\beta + \rho)} h \frac{\partial x}{\partial A} \right) \frac{1}{(1 - \alpha)k^{-\alpha}}$$

Knowing that $\frac{\partial x}{\partial A} = \frac{x}{A}$, replacing in the formula before we obtain:

$$\frac{\partial k}{\partial A} = \left(\frac{(\Gamma - 1)(\beta(1 - \alpha) + \rho)}{\Gamma(\beta + \rho)} - \frac{\Gamma\beta + \rho}{\Gamma(\beta + \rho)} h \frac{x}{A} \right) \frac{1}{(1 - \alpha)k^{-\alpha}} = \frac{k^{1-\alpha}}{A} > 0$$

The effect of a technology shift on house price is then easily to compute. From (1.33) and knowing that $\frac{\partial x}{\partial A} > 0$ and $\frac{\partial k}{\partial A} > 0$, we obtain:

$$\frac{\partial p}{\partial A} = k^\alpha \frac{\partial x}{\partial A} + x \frac{k^\alpha}{\partial A} > 0$$

E. Descriptive statistics

Country	Housing price	GDP per capita
Australia	66.85	34495.95
Canada	68.90	35514.61
New Zealand	62.01	28596.96
United States	85.14	31891.11
Belgium	74.43	38223.24
Denmark	76.11	42564.43
France	76.39	31762.68
Germany	94.87	37656.29
Italy	86.21	34562.23
Norway	69.80	56456.51
Spain	76.11	30696.22
Sweden	71.01	38417.20
Switzerland	62.01	28596.96
United Kingdom	68.08	35232.04

Table 1.2: Data description at quarterly averages from Q1/1980 - Q4-2014
Source: OECD

Chapter 2

Asset prices and Inequality in a heterogeneous overlapping generation model

1. Introduction

There has been much evidence that wealth inequality had been constantly increasing across many advanced countries over recent decades. (See for example: Piketty and Saez(2003), Atkinson et al(2011), Saez and Zucman(2014)). At the same time, global house prices have also been observed to follow a clear, upward trajectory.(See Knoll et al.(2014), US Census(2015), IMF (2017)). Understanding whether the co-movement of these two trends are just coincidence or if there is a causal relation between the two receives less attention until recently, which is mainly due to the lacks of good quality of wealth data. In a well-known book of Thomas Piketty published in 2013, *Capital in the twenty-first century*, he emphasized the risk of an explosion of wealth inequality because capital accumulates faster than income. And this result is mainly based on the rise of housing capital via its prices. Similar findings can be found in the work of Rognlie(2014) and Weil (2015). In this paper, we are going to argue that there is an interaction between house prices and inequality. We display in the figure (2.1) below the relationship between the two factors in France and US. Note that in these figures wealth inequality is measured by the Gini wealth index obtained from the World Inequality Database (WID). Similar plots using different measurements of wealth inequality such as the wealth share of the top 1% and 10% can be found in the section A of Appendix. One can see that all these plots suggest a positive co-movement between house prices and inequality.



Figure 2.1: The link between housing price growth and the growth of wealth in France and US from 1980 onwards.

Source: WID and OECD

On the other hand, the link between housing rents and wealth inequality is rather weak, yet positive in the case of France as we can see in figure (2.2) below. It depicts the variations between rents and wealth inequality in the case of US and France from

1980 onwards. Therefore, their link is often ignored when it comes to study the role of housing in wealth inequality. However, with on average 35% the population in US who rent (US Census(2015)) and approximately 30% of the population of all 28 European countries over the past few years who are renters (Dan et Sanchez(2011)), income from lending houses should also generate uneven savings among individuals and may then result in a rising unequal wealth distribution. It is, therefore, important to take into account rental incomes when we want to investigate the role of housing in wealth inequality. In this paper, we will argue that it's not only house price but also rents that matter in determining wealth inequality in long-run.



Figure 2.2: The link between the growth of rental prices and the growth of wealth in France and US from 1980-2011.

Source: WIID and OECD

Literature studying the link between house prices and macroeconomic factors in the context of heterogeneous agents and incomplete financial market has been growing recently. For example, Kiyotaki et al(2011) developed a life-cycle model to study the interactions between housing prices, aggregate production, and household behavior over a life-time. They found that the procycle between house prices and production output can be explained by a change in productivity growth and a reduction in interest rate. As a result, homeowners will become richer at the expense of tenants. Although, they found that financial innovations, such as a loosening collateral constraint, have surprisingly little effects on house prices. In contrast to Kiyotaki et al(2011), Sommer et al(2013) investigated the equilibrium effect of fundamentals on house prices and rents. They instead found that financial liberalizations are mainly responsible for a dramatic hike in the US house price-rent ratio between 1995 and 2006. However, this paper didn't consider how it influences on wealth. With a different focus, Iacoveillo and Pavan(2013) introduced a general equilibrium model to explain for the procyclicality between house prices and mortgage debt. Using calibration, they can replicate the features of housing investment: "higher risk and lower downpayments can make

housing and debt more stable in response to small positive and negative shocks (as in the Great Moderation), but can make it more fragile in response to large negative shocks (as in the Great Recession)". However, in their setting, they did not endogenize housing prices, nor taking into account mortgage default. Similar findings can be found in the works of Nakajima(2005), Piazzesi and Schneider (2016) and so on.

On the other hand, little has been done to assess the impacts of rents and rental market on inequality. In fact, common literature often ignore the rental market. While it is true that renters are a minority in the total population, they, however, represent an overwhelming majority among the poorer and younger age groups. Campell and Cocco(2007) found that households who remain renters have lower income and consumption in comparison with homeowners. Kuhn and Grabka(2018) using the German Socio-Economic Panel and SHP data for Germany and Switzeland and found a "strong negative correlation between wealth inequality and home-ownership rates across countries". Bonnet et al (2014) argued that rents, but not housing prices, should be used to account for wealth inequality. They explained that "housing capital produces a real return for landlords, from rent. It also produces an implicit return as rent "saved" by the owner-occupiers. On both accounts, the value of real estate capital must be based on rent and not on the evolution of housing prices". They then recomputed the wealth dynamics and concluded that the findings of Piketty might not be true.

Our work in this paper will then stand in between the two trends. We will argue that it's not only house prices, but also the house rents that contribute to wealth in the long-run. The setting of our model will be closely related to the work of Matsuyama (2000), D'Albis and Iliopoulos(2013). While Matsuyama (2000) addressed the link between financial assets, credit market and wealth inequality, differently, in this paper, we will focus on housing asset and its link to wealth inequality. Depart from D'Albis and Iliopoulos(2013) in which they defined two dynasties that differ in discount rate, in initial wealth and in borrowing capacity, we will assume that the two dynasties are only different in their initial wealth.

To study the interaction between house prices, rents and wealth inequality, I set up an overlapping generation model of small open economy. Individuals are assumed to be altruistic and they differ only in their initial wealth endowment. Moreover, we introduce some non-convexities in housing market investment which are the borrowing limit and the minimum size friction. The combination of altruism, uneven distributed wealth endowment and non-convexity conditions cause the formation of class society. It turns out that there exists a steady state in which the poor dynasty has no choice but renting houses while the rich one can borrow to purchase houses which will then pay them a relatively higher return. Since returns on housing is maintained higher than on capital, wealth inequality becomes persistent in the long-run.

The contribution of the paper is twofold. First, we provide an analytical model that allows us to study the role of housing in wealth inequality. While many papers focus on the importance of housing on income inequality (see, for example, Määttä and Terviö(2014), Goda et al.(2016)), its contribution to the accumulation of wealth is still a missing piece in literature. Our aim is to fulfill this part. Moreover, in this setting, we assume that households share the same preferences and budget constraints. They only differ in their endowment wealth. Therefore, the model allows us to stress on the role of intergenerational transfers in inequality. Put differently, the long-run wealth discrepancies between households has to be driven by the position where they start.

Secondly, by considering impacts from different shocks, we can see that housing price and inequality don't necessarily move in the same direction as suggested in, for example, Arundel(2017) and Fuller et al.(2019). Particularly, as financial market liberalization is claimed to be the driving force of the housing boom starting from 1990s (Duca et al.(2010), Iacoveillo and Neri(2010)), we introduce such shock in our economy. We find that while housing prices increase due to the fact that households now find it easier to borrow to purchase houses, wealth inequality decreases. This finding is consistent with data. On the other hand, we found the pro-cycle between inequality and house rents in response to the shock, which suggests that rent may capture better movement of wealth gap.

Additionally, we consider impacts from a pension reform in which the contribution rate to the pension system is higher. A stylized version of life-cycle theory suggests that a generous social security benefits crowd out households' private saving (Alessi et al(2013)). However, little has been done to study the particular effects of a pension reform on housing returns as well as wealth. Cerny et al(2010) using a calibrated OLG model to match features of the UK economy and found that when there's a fall in the value of Pay-As-You-Go (PAYGO) pensions, the overall demand for housing is higher which will lead to a higher house price. Zhao(2015) introduced an OLG economy with homeowners and investors. He claimed that a liquidity shock can be generated by removing a pension system. His paper suggested that such shock would create a downward pressure on interest rate and trigger a possibility of having housing bubbles. However, his work doesn't pay attention on the effects on rents as well as on the wealth gap. On the other hand, PAYGO system is conventionally viewed as being inequality-reducing (Huggett and Ventura(1999), Liebman(2001), Krueger and Kubler(2002)). Using the OLG framework with heterogeneity in capital earnings, Fall(2007) suggested that the PAYGO system dampen wealth inequality by reducing interest rate. He argued that as the poor households are capital savers while the rich is borrower, a lower return on capital would make the rich better off at the expense of

the poor. In the same spirit, we are also interested in the impacts of pension reform on wealth inequality. By allowing heterogeneity in asset earnings, we, however, found that a higher contribution rate to a pension system would reduce wealth inequality via housing channel. Indeed, as disposable income is now lower, households would adjust the housing demand in a way that leads to a lower housing purchases as well as a lower house lending. As a result, the rich dynasties will experience a stronger negative income effect than the poor one. Wealth inequality will thus decrease. Additionally, our findings also suggest that in response to this shock, rents, rather than house prices, follow better the movement of inequality.

The paper is organized as follows. The second section presents introduces our overlapping generation model. The third one studies the evolution path of the economy while the fourth section provides the dynamics of wealth and the long-run solutions. Section fifth investigates the impacts of a loosening borrowing constraint and a change in pension contribution on individual's long-run wealth. Section sixth concludes our findings.

2. The model economy

The model structure In the same spirit with Matsuyama(2000) and D'Albis and Iliopoulos(2013) we consider a two-period overlapping generation (OLG) model with two different assets: houses and bonds; and two types of dynasties $i \in \{1, 2\}$. We assume that the two dynasties differ only in the initial amount of inheritance they receive at period 0, i.e. the beginning of the economy. We assume that dynasty 2 receives relatively higher initial endowment than dynasty 1.

Additionally, we consider a small and open economy where individuals can lend and borrow in the international capital market at a given interest rate.¹ Assume that the gross interest rate denoted by R and satisfies $R > 1$; otherwise, individuals have no incentive to save. Assuming also that the wage level (w) is exogenously given. Given our assumption, the interest rate and the wage are unaffected by the decisions made in the small open economy and, therefore, the net gap between domestic savings and investment is not necessarily equal to 0.

Moreover, note that in other related settings, for example, D'Albis and Iliopoulos (2013) and Zhao (2015), they assumed the existence of ex ante heterogeneity in the maximization problem of individuals. More specifically, to generate an equilibrium with different social classes, D'Albis and Iliopoulos (2013) assumed that the rich (i.e. patient) and the poor (i.e. impatient) experienced different borrowing constraints:

¹Note that in the closed economy setting, the interest rate is endogenously determined from the clearing condition on the capital market. Thus, in such setting, aggregate saving is always equal to aggregate investment.

only the impatient agent is assumed to face collateral constraint. In the same spirit, Zhao (2015) assumed that the rich dynasty differs from the poor one in the sense that they, as the investors in the society, don't value utility flows generated from housing service. Here in this work, we aim to provide a more general setting by relaxing this assumption. Particularly, we will assume that the wealthy-born and poor-born dynasties adopt identical utility and face the same budget constraints. Therefore, to compensate for this set-up, we need to assume that the economy is small open for technical reason. As we will see in the equilibrium part, there exists an equilibrium in which housing becomes more profitable than capital asset. As all types of dynasties desire to own houses while the poor one can't access the housing market due to borrowing limit condition, they will decide not to save either due to the fact that capital becomes unattractive asset. As a result, capital domestic supply turns out to be 0. In order to maintain the clearing housing market condition, one needs to assume that the rich dynasty can access to the international capital market so that they can borrow and buy houses.

Different from the capital market, we assume that there's no international mobility in housing market. Besides, housing asset is assumed to play a dual role in the economy: a consumption good as it provides housing services, and at the same time an investment good as it provides return. The first property is needed to make sure that individuals are incentive to buy houses. In fact, it resembles the modelling with fiat money in which they postulate that households gain utility from holding money. The second property captures the fact that individuals hold housing assets because they consider it as the store of value.

In each period t and for each dynasty i , the economy is populated by two generations, each living for two periods. After each period, the old individual of each type dies and is replaced by a new, young one, thus the total population remains constant over time.

At any period t , each young individual receives a constant income (w) and inherits (b_t^i) which is an inter-vivo gift transferred from the ancestor. In our setting, we assume that, at time 0, $b_0^2 > b_0^1$. Moreover, we assume that everyone has to pay a fixed contribution rate, τ , to the PAYGO pension system.

In the first period of life, they consume non-housing goods (c_t^i) and housing goods ($h_{R,t}^i$) which can be obtained either via purchasing or renting a house. They also decide whether to save or to borrow from the international capital market and whether to buy a house or just rent it. Individuals simultaneously face two different constraints when they decide to buy a house: a collateral borrowing constraint and a non-negativity constraint on the size of the house, as also introduced in the work of D'Albis and Iliopoulos(2013). The borrowing limit requires that individuals can not borrow more

than a certain fraction of their housing assets while the size constraint requires that the size of a house to be bought has to be greater or equal a minimum threshold. For simplicity, in this setting, we will assume that this threshold has to be strictly positive. Being owner of a house allows them to gain rental earnings (r_t) as part of the owner's income. On the other hand, if they can't meet these constraints, they will become a house renter.

In the last period of life, they receive a fixed amount of pension income ($w\tau$), sell the house, receive their saving return or pay back the debt. Then they decide how much to consume and how much to bequeath to their offspring (b_{t+1}^i).

The nature of housing For simplicity, we assume that the total stock of houses in the economy is always fixed at H , thus houses never deteriorate. This setting can be found in many papers such as Arce and Lopze-Salido(2011), Gary-Bobo and Nur(2015), Eerola and Maattanen(2018). This assumption will allow us to focus entirely on the housing demand side. In any case, we believe that supply side is not the key to understand the recent substantial house price evolution. Related from this, we will not take into account the growth in housing stock.

Individual's problem The generation born at time t of any type i , where $i \in \{1, 2\}$, who obtains utilities from non-housing consumption, housing service, and from giving a bequest will maximize the following:

$$\underset{c_t, h_{R,t}, d_{t+1}^i, b_{t+1}^i}{Max} \quad (1 - \beta) [(1 - \gamma) \ln c_t^i + \gamma \ln h_{R,t}^i] + \beta [(1 - \sigma) \ln d_{t+1}^i + \sigma \ln b_{t+1}^i]$$

with $0 < \beta < \frac{1}{2}$, $0 < \gamma < 1$, and $0 < \sigma < 1$. The parameter β , which captures time preference, must be in between 0 and 1/2 as otherwise individuals would value future utility more than present utility. Parameter σ captures the relative importance of bequest in comparison with old-age consumption and γ captures the degree of substitutability between housing and non-housing goods.

The budget constraints for an individual of type i are as follows:

$$c_t^i + s_t^i + r_t h_{R,t}^i + p_t h_{H,t}^i \leq w(1 - \tau) + b_t^i + r_t h_{H,t}^i \quad (2.1)$$

$$d_{t+1}^i + b_{t+1}^i \leq w\tau + p_{t+1} h_{H,t}^i + R s_t^i \quad (2.2)$$

$$h_{H,t}^i \geq 0 \quad (2.3)$$

$$s_t^i + (1 - \theta) p_t h_{H,t}^i \geq 0 \quad (2.4)$$

where r_t and p_t are the housing rents and prices, respectively; R is the exogenous gross return given by the international capital market; $h_{R,t}$ is the amount of houses that one

occupies to live in; and $h_{H,t}$ is the amount of houses that one possesses. Additionally, the parameter τ captures the contribution rate to the pension system and θ measures the down-payment rate, i.e. the portion of house values that one needs to save to be able to borrow.

We discuss now some important assumptions that we have made in this model. First, we assume that utility function is in the logarithmic form. The advantage of this form lies in their computation: it allows us to achieve tractable analysis.

Secondly, to generate long-run equilibrium with different social classes, one needs to introduce imperfect market conditions. Eventually, without imposing the two constraints stated in (2.3) and (2.4), we can't obtain the equilibrium of our interest. Note that the restriction defined in (3) has been introduced in previous works such as D'Albis et Iliopoulos(2013) and Zhao(2015). It states the feasibility constraint that housing demand can not be negative. More importantly, this constraint helps us distinguish house buyers, i.e: when $h_{H,t}^i = 0$, and house renters, i.e: when $h_{H,t}^i > 0$. The restriction (4) states the standard collateral borrowing limit, which is conventional in the set-up models on portfolio choice and asset prices (See for example Kiyotaki and Moore (1997), Gervais(2002), Iacoveillo(2005), Ortalo-Magne and Rady(2006)). This constraint states that at least a positive fraction, captured by θ , of the house purchases value must be financed with one own funds. Without this borrowing constraint, in the equilibrium where return on housing is higher than the world interest rate, individuals would borrow infinite sums at a lower rate and go long in housing to make profit. As a result, housing demand would be infinite and so would be house price. That being that, this borrowing constraint is necessary in obtain bounded solution.

Finally, the introduction of taxation is important for technical reasons. As we will see in the following section, the introduction of pension allows the consumption of the poor dynasty to be strictly positive. Additionally, if we shut down the contribution tax on the pension system, we will break the link between housing prices and rents in which rents now depend only on the rich household's wealth. As a result, without the taxation, any changes in property prices will have no influence on rents and our findings in this paper will no longer hold.

The Lagrangian for each dynasty's optimization problem is defined as:

$$\begin{aligned}
\mathcal{L}(s_t^i, h_{H,t}^i, b_t^i) = & (1 - \beta) [(1 - \gamma) \ln c_t^i + \gamma \ln h_{R,t}^i] + \beta [(1 - \sigma) \ln d_{t+1}^i + \sigma \ln b_{t+1}^i] \\
& + \mu^i [w(1 - \tau) + b_t^i + r_t h_{H,t}^i - c_t^i - s_t^i - r_t h_{R,t}^i - p_t h_{H,t}^i] \\
& + \varphi^i [w\tau + p_{t+1} h_{H,t}^i + R s_t^i - d_{t+1}^i - b_{t+1}^i] \\
& + \zeta^i h_{H,t}^i \\
& + \lambda^i [s_t^i + (1 - \theta) p_t h_{H,t}^i]
\end{aligned} \tag{2.5}$$

where μ^i and φ^i are the Lagrange multipliers associated to individual budget constraints (1) and (2) of each type of households, respectively; ζ^i is the Lagrange multiplier associated to the size restriction; and λ^i corresponds to the borrowing constraint.

The Kuhn-Tucker conditions for the maximization program of the generation are:

$$c_t^i : (1 - \gamma)(1 - \beta)/c_t^i = \mu^i \quad (2.6)$$

$$h_{R,t}^i : \gamma(1 - \beta)/h_{R,t}^i = r_t \mu^i \quad (2.7)$$

$$d_{t+1}^i : (1 - \sigma)\beta/d_{t+1}^i = \varphi^i \quad (2.8)$$

$$b_{t+1}^i : \frac{\sigma\beta}{b_{t+1}^i} = \varphi^i \quad (2.9)$$

$$s_t^i : \mu^i = \varphi^i R + \lambda^i \quad (2.10)$$

$$h_{H,t}^i : \mu^i(p_t - r_t) = \varphi^i p_{t+1} + \zeta^i + (1 - \theta)p_t \lambda^i \quad (2.11)$$

Moreover, complementary slackness conditions can be written as:

$$\zeta^i h_{H,t}^i = 0 \quad (2.12)$$

$$\lambda^i [s_t^i + (1 - \theta)p_t h_{H,t}^i] = 0 \quad (2.13)$$

Notice that depending on the different sets of binding conditions in equilibrium, the economy will reach different equilibria in the long-run.

3. Short-run equilibrium

Define a new variable \tilde{w}_t^i where is equal to the sum of disposable income and inherited wealth available at the beginning of period t , i.e. $\tilde{w}_t^i \equiv w(1 - \tau) + b_t^i$. Therefore, \tilde{w}_t^i captures the total wealth of dynasty i born at t . In period 0, we assume that $\tilde{w}_0^1 < \tilde{w}_0^2$. Note that \tilde{w}_t^i is a pre-determined variable.

Given $\{\tilde{w}_t^i\}_{i \in (1,2)}$, the wealth of dynasty i , the temporary equilibrium of the economy at time t is defined by:

1) The variables $\{c_t^i, d_t^i, b_t^i, h_{R,t}^i, h_{H,t}^i, s_t^i\}$ solve the optimal problem of each type of households.

2) The housing prices p_t , and housing rents r_t solve the following market clearing conditions.

$$\sum_i h_{R,t}^i = H \quad (2.14)$$

$$\sum_i h_{H,t}^i = H \quad (2.15)$$

where the clearing condition on the rental market given by (2.14) will define the equilibrium rental price whereas the balance in housing market given by (2.15) will provide us the equilibrium house price.

3) At any time t , we always have the balance in the sum of goods and capital market:

$$\sum_i c_t^i + \sum_i d_t^i + \sum_i s_t^i = 2w + R \sum_i s_{t-1}^i \quad (2.16)$$

4. Equilibrium solutions

In this section, we will first introduce the definition of household types according to their choices and then we will characterize in detail the two equilibria of the economy². We begin with the following definition.

Definition 11. *A household i is (i) a renter if $h_{R,t}^i \geq h_{H,t}^i$, (ii) a homeowner if $h_{R,t}^i < h_{H,t}^i$, (iii) capital saver if $s_t^i > 0$, and (iv) capital borrower if $s_t^i < 0$.*

Define the following wealth threshold:

$$\bar{w} \equiv \frac{(1 - \beta)(1 + \gamma \frac{1-(1-\theta)R}{R-1})}{\beta - \frac{1-(1-\theta)R}{R-1} \gamma (1 - \beta)}$$

Define also the threshold rate of return on capital R^* where:

$$R^* \equiv \frac{\beta + \gamma(1 - \beta)}{\beta + \gamma(1 - \beta)(1 - \theta)}$$

Our result can be summarized in the following two propositions. Detailed proof of these two propositions are provided in the section C1 of Appendix.

Proposition 12. *When interest rate is set high enough, $R > R^*$, we obtain the case where both dynasties have sufficient wealth, i.e. $\tilde{w}^i > \bar{w} \forall i$. In this case, two dynasties will buy houses. However, they are indifferent between being a homeowner and a renter.*

The proposition above deserves some discussions. Note that in this equilibrium, we obtain that:

$$R = \frac{p_{t+1}}{p_t - r_t}$$

Rewriting it gives us:

$$p_t - \frac{p_{t+1}}{R} = r_t$$

which basically states that the user cost of housing, which is captured by the difference between current housing price and the discounted value of future housing payoffs, is

²Proof for equilibrium existence is provided in the Appendix section A.

equal to the current rental price. As a result, individuals are indifferent between buying or renting a house. Thus, they won't exhaust their borrowing limit to buy houses³. Recall that wealth \tilde{w}_t^i is a predetermined variable. Thus, the dynamic system of the economy can be fully characterized by the following dynamics of household i 's wealth:

$$\tilde{w}_{t+1}^i = w(1 - \tau) + \sigma\beta R\left(\tilde{w}_t^i + \frac{w\tau}{R}\right) \quad (2.17)$$

Put differently,

$$\tilde{w}_{t+1}^i = w(1 - \tau) + \sigma\beta w\tau + \sigma\beta R\tilde{w}_t^i$$

We can see that the interest rate R measures the rate of return on their wealth. A higher R , therefore, implies a higher long-term level of wealth. Therefore, when R is too high, i.e. higher than a defined threshold \bar{R} , we obtain the case in which both dynasties are rich enough to access the housing market. In the opposite case where $R \leq R^*$, we procure the following equilibrium:

Proposition 13. *When the interest rate is small enough, i.e. $R \leq R^*$, we obtain the scenario where the poor dynasty (dynasty 1) has insufficient wealth, i.e. $\tilde{w}^1 < \bar{w}$. As a result, they will hold no savings and buy no housing assets. Thus, they become house-renters in the society. On the other hand, the rich one (dynasty 2) will be capital borrower and homeowner.*

Proof. Consider the case where for the dynasty 1:

$$\zeta^1 > 0$$

$$\lambda^1 > 0$$

From (2.12) and (2.13), we can derive the associated conditions on the two constraints of dynasty 1 as follows:

$$h_{H,t}^1 = 0$$

$$s_t^1 = 0$$

which states that along the equilibrium path the dynasty 1 doesn't possess house and doesn't save.

Due to the clearing condition on the housing market which requires that the total housing demand has equal to housing supply, it means that the rich dynasty will buy the whole housing stock, i.e:

$$h_{H,t}^2 = H > 0$$

³Recall that in this scenario, none of the borrowing constraints binds.

The Lagrange multipliers which corresponds to this condition will then be:

$$\zeta^2 = 0$$

And by elimination, the condition on the multiplier λ^2 of the rich has to be:

$$\lambda^2 > 0$$

These two conditions state that the rich dynasty will borrow at the international capital market to buy houses. \square

Details on this equilibrium be found in the C2 of Appendix. Note that in this scenario, we obtain that:

$$\frac{p_{t+1}}{p_t - r_t} > R$$

Put it differently:

$$r_t > p_t - \frac{p_{t+1}}{R} \tag{2.18}$$

which states that the rental price that one has to pay if he/she doesn't possess a house is strictly higher than the net cost from owning a house. Therefore, individuals now have desire to buy houses. However, only the rich one is able to borrow due to the borrowing constraint.

From the budget constraint equation of dynasty 2 given by the section C2 in Appendix, we define a new variable that measures the consumption discount rate of the dynasty 2:

$$\rho_{t+1} \equiv \frac{p_{t+1} - (1 - \theta)Rp_t}{\theta p_t - r_t}$$

Recall that in this scenario, the dynasty 2 borrows money in the world capital market to buy houses. Thus, their net rate of return from each unit of investment can be captured by the variable ρ_{t+1} . Clearly, this rate of return is higher than the rate of return from the capital measured by R .⁴

Finally, note that the dynamics of the whole economy can be characterized by the

⁴Having $\rho_{t+1} > R$ means that $\frac{p_{t+1}}{R} - (1 - \theta)p_t > \theta p_t - r_t$. Simplifying it gives us: $r_t > p_t - \frac{p_{t+1}}{R}$, which is proved in the (2.18)

wealth dynamics of the two households, prices and rents as follows:

$$\tilde{w}_{t+1}^1 = (1 - \tau)w + \sigma w \tau \quad (2.19a)$$

$$\tilde{w}_{t+1}^2 = (1 - \tau)w + \sigma \beta \rho_{t+1} \left(\tilde{w}_t^2 + \frac{w \tau}{\rho_{t+1}} \right) \quad (2.19b)$$

$$\rho_{t+1} = \frac{p_{t+1} - (1 - \theta) R p_t}{\theta p_t - r_t} \quad (2.19c)$$

$$r_t H = \gamma (1 - \beta) \left(\tilde{w}_t^2 + \frac{w \tau}{\rho_{t+1}} \right) + \gamma (w (1 - \tau) + \sigma w \tau) \quad (2.19d)$$

$$(\theta p_t - r_t) H = \beta \tilde{w}_t^2 - (1 - \beta) \frac{w \tau}{\rho_{t+1}} \quad (2.19e)$$

Solving for the FOC we obtain the following dynamics of consumption of the young and the old generation, housing services, and inheritance of dynasty 1 can be derived as follows:

$$c_t^1 = (1 - \gamma) (w (1 - \tau) + \sigma w \tau) \quad (2.20a)$$

$$r_t h_{R,t}^1 = \gamma (w (1 - \tau) + \sigma w \tau) \quad (2.20b)$$

$$d_{t+1}^1 = (1 - \sigma) w \tau \quad (2.20c)$$

$$b_{t+1}^1 = \sigma w \tau \quad (2.20d)$$

Notice that all variables of the dynasty 1 are independent of time. It is due to the fact that, in this scenario, the poor household doesn't either save or invest in housing. Thus, their income composes only the their wage w , which is assumed to be exogenous. With log utility setting, consumption on goods and on housing services of the young individual of dynasty 1 are fractions of their constant wage, captured by the parameters γ and σ . Similarly, the old individual's consumption and inheritance are proportional to their constant retired income.

Concerning the second dynasty, along the equilibrium path, their choices of housing and non-housing consumptions, bequests and savings in equilibrium are a function of wealth as follows:

$$c_t^2 = (1 - \gamma) (1 - \beta) \left(\tilde{w}_t^2 + \frac{w \tau}{\rho_{t+1}} \right) \quad (2.21a)$$

$$r_t h_{R,t}^2 = \gamma (1 - \beta) \left(\tilde{w}_t^2 + \frac{w \tau}{\rho_{t+1}} \right) \quad (2.21b)$$

$$d_{t+1}^2 = (1 - \sigma) \beta \rho_{t+1} \left(\tilde{w}_t^2 + \frac{w \tau}{\rho_{t+1}} \right) \quad (2.21c)$$

$$b_{t+1}^2 = \sigma \beta \rho_{t+1} \left(\tilde{w}_t^2 + \frac{w \tau}{\rho_{t+1}} \right) \quad (2.21d)$$

$$s_t = -(1 - \theta) p_t H \quad (2.21e)$$

Similarly, with log utility setting, consumption and inheritance of the dynasty 2 are proportional to their lifetime income. Clearly, housing earnings doesn't appear explicitly in the inheritance equation (2.21d). However, rewrite this equation (2.21d) as follows:

$$b_{t+1}^2 = \sigma\beta w\tau + \sigma\beta\rho_{t+1}\tilde{w}_t^2$$

Recall that ρ_{t+1} captures the net return on housing and ρ_{t+1} is a function of house prices and housing rental income described in (2.19c). We can see that ρ_{t+1} depends positively on rents r_t and house sale price at the time p_{t+1} while it depends negatively on p_t , which is the cost of buying a house at time t . Thus, a higher house earnings leads to a higher ρ_{t+1} which, in return, rises their total wealth.

Finally, note that the existence of multiple equilibria can be seen in a model that introduces multiple assets with borrowing constraints. For example, Matsuyama(2000) assumed that individuals in an economy face two opportunities to invest: capital savings which pays interest rate and "the project" which pays a relatively higher and exogenous return. In the context with borrowing constraint, he showed that there exists two equilibria: no-arbitrage condition on returns of the capital and the project and arbitrage condition in favor of the project. Similarly, we also obtained two scenarios. Particularly, in the first one, we found that returns on capital is equal to the net return on housing, i.e. $\frac{p_{t+1}}{p_t - r_t} = R$. In the second one, it's more profitable to borrow and invest on housing as net return on housing is higher than capital return, i.e. $\frac{p_{t+1}}{p_t - r_t} > R$.

5. Long-run Equilibrium and Wealth Inequality

Although the setting allows us to obtain multiple equilibria, we will restrict our attention to only the scenario provided in the proposition 3 in which the poor household becomes home-renter while the rich household becomes home-owner. We argue that this equilibrium is more relevant to the reality in which there are always a sufficient proportion of households who don't own any houses. While it's true that the number of individuals who rent houses decrease over time, this adjustment is still rather small. For example, Dan and Sanchez(2011) reported that, in the US, the average home-ownership rate increased from 63.9% in 1990 to 66.9% in 2010. It means that there are still one third of US residents who rent houses in 2010.

In this section, we are going to study the link between wealth inequality and asset prices in the long-run. We will note I a new variable to capture long-run wealth

inequality and I is defined as:

$$I \equiv \frac{\tilde{w}^2}{\tilde{w}^1}$$

which measures the gap between the steady-state wealth of the rich and poor households. Notice that $I \geq 1$ and that equality in the long-run is achieved when one obtains $I = 1$.

Consider now the scenario provided by the proposition 3. Recall that in this case the wealth of type 1 household is sufficiently low that he can't meet the borrowing limit, i.e.: $\tilde{w}^1 < \bar{w}$. Putting (2.19a) and (2.19b) at steady state we obtain the long-run wealths of the two households as follows:

$$\tilde{w}^1 = w(1 - \tau) + \sigma w \tau \tag{2.22}$$

$$\tilde{w}^2 = \frac{w(1 - \tau) + \sigma \beta w \tau}{1 - \sigma \beta \rho} \tag{2.23}$$

Recall that at steady state, the net return on wealth of the dynasty 2 can be computed as:

$$\rho = (1 - (1 - \theta)R) \frac{p}{\theta p - r}$$

Comparing the two steady state wealths, we obtain that⁵:

$$\tilde{w}^2 > \tilde{w}^1$$

Therefore, the long-run inequality is derived as:

$$I = \frac{\tilde{w}^2}{\tilde{w}^1} = \frac{w(1 - \tau) + \sigma \beta w \tau}{w(1 - \tau) + \sigma w \tau} \frac{1}{1 - \sigma \beta \rho} \tag{2.24}$$

which simply depends on ρ , the net return on wealth of dynasty 2. It is clear that $I > 1$. This result implies that wealth inequality is persistent in the long-run.

In what follows, we are going to characterize the model's steady state. Particularly, we will study if the model provides multiple steady states.

Recall that the economy can be fully characterized by the following system of equations which composes steady state values of property rents, prices and the discount

⁵Recall that the wealth of dynasty 2 computed from his budget constraint is $\tilde{w}^2 = w(1 - \tau) + \sigma(w\tau + pH(1 - (1 - \theta)R))$. Since $pH(1 - (1 - \theta)R) > 0$, it leads to: $\tilde{w}^2 > w(1 - \tau) + \sigma w \tau = \tilde{w}^1$

rate of the type-2 $\{p, r, \rho\}$:

$$\rho = (1 - (1 - \theta)R) \frac{p}{\theta p - r} \quad (2.25a)$$

$$rH = \gamma(1 - \beta) \left(\frac{w(1 - \tau) + \sigma\beta w\tau}{1 - \sigma\beta\rho} + \frac{w\tau}{\rho} \right) + \gamma(w(1 - \tau) + \sigma w\tau) \quad (2.25b)$$

$$(\theta p - r)H = \beta \frac{w(1 - \tau) + \sigma\beta w\tau}{1 - \sigma\beta\rho} - (1 - \beta) \frac{w\tau}{\rho} \quad (2.25c)$$

We will now prove that the equilibrium is unique. From (2.25a)-(2.25c), one can then reduce the system of 3 variables $\{r, p, \rho\}$ to a single equation of ρ as follows⁶:

$$\frac{\theta\rho}{1 - (1 - \theta)R} - \frac{\gamma\tilde{w}^1 + \gamma(1 - \beta) \left(\frac{w(1 - \tau) + \sigma\beta w\tau}{1 - \sigma\beta\rho} + \frac{w\tau}{\rho} \right)}{\beta \frac{w(1 - \tau) + \sigma\beta w\tau}{1 - \sigma\beta\rho} - (1 - \beta) \frac{w\tau}{\rho}} = 1 \quad (2.26)$$

where recall that $\tilde{w}^1 = w(1 - \tau) + \sigma w\tau$.

Lemma 14. *The steady state value of ρ is unique.*

Proof. To facilitate the proof, define the following sub-functions of (2.26) as follows:

$$\underbrace{\frac{\theta\rho}{1 - (1 - \theta)R}}_{\equiv \Pi(\rho)} - \underbrace{\frac{\gamma\tilde{w}^1 + \gamma(1 - \beta) \left(\frac{w(1 - \tau) + \sigma\beta w\tau}{1 - \sigma\beta\rho} + \frac{w\tau}{\rho} \right)}{\beta \frac{w(1 - \tau) + \sigma\beta w\tau}{1 - \sigma\beta\rho} - (1 - \beta) \frac{w\tau}{\rho}}}_{\equiv \Gamma(\rho)} = 1$$

To determine the number of ρ that solves the above equation, we are going to study the monotonicity of the LHS function measured by $\Pi(\rho) - \Gamma(\rho)$. Particularly, we're going to determine the sign of $\frac{\partial \Pi(\rho)}{\partial \rho} - \frac{\partial \Gamma(\rho)}{\partial \rho}$.

One can prove easily that $\Pi(\rho)$ is an increasing function of ρ , i.e:

$$\frac{\partial \Pi(\rho)}{\partial \rho} > 0$$

Moreover, if we rewrite $\Gamma(\rho)$ as follows:

$$\Gamma(\rho) = \frac{\frac{\gamma\tilde{w}^1}{w(1 - \tau) + \sigma\beta w\tau} (1 - \sigma\beta\rho) + \gamma(1 - \beta) \left(1 + \frac{w\tau}{w(1 - \tau) + \sigma\beta w\tau} \frac{1 - \sigma\beta\rho}{\rho} \right)}{\beta - (1 - \beta) \frac{w\tau}{w(1 - \tau) + \sigma\beta w\tau} \frac{1 - \sigma\beta\rho}{\rho}}$$

Clearly⁷,

$$\frac{\partial \Gamma(\rho)}{\partial \rho} < 0$$

⁶See Appendix section C5 for details

⁷See the section C, Appendix

It states that $\Gamma(\rho)$ is, therefore, a decreasing function of ρ .

As a result,

$$\frac{\partial \Pi(\rho)}{\partial \rho} - \frac{\partial \Gamma(\rho)}{\partial \rho} > 0$$

which states that $\Pi(\rho) - \Gamma(\rho)$ is an increasing function of ρ . Since its right-hand side is simply a constant, the solution of ρ derived from (2.26) is, therefore, unique. \square

Since the solution of the steady state values of ρ is unique. From (2.22) and (2.23), we can now conclude that the steady state wealth values of \tilde{w}^1 and \tilde{w}^2 are unique, accordingly. We introduce the following proposition.

Proposition 15. *In the equilibrium where $R < \bar{R}$ and $\tilde{w}^1 < \bar{w} < \tilde{w}^2$, wealth inequality is persistent in the long-run. The long-run wealth of the household type 2 will be:*

$$\tilde{w}^2 = \frac{w(1 - \tau) + \sigma\beta\tau w}{1 - \sigma\beta\rho}$$

while the type 1 will receive:

$$\tilde{w}^1 = w(1 - \tau) + \sigma w\tau$$

Moreover, $\tilde{w}^2 > \tilde{w}^1$.

The steady state wealths (\tilde{w}^1, \tilde{w}^2) are unique.

Before addressing the impacts of different shocks on prices, rents and inequality, it may be useful to study the sensitivity of inequality with respect to prices and rents. Recall that, from (2.24), steady state inequality is a function of the long-run net return rate of investment of the dynasty 2. Moreover, from (2.25a) one can see that ρ depends on house prices p and rents r . We can then write: $I = \mathcal{I}(\rho(p, r))$.

Therefore, to capture how wealth inequality changes relatively to a change in these market prices, we are going to compute the following partial derivatives:

$$\begin{aligned} \frac{\partial I}{\partial p} &= \frac{\partial \mathcal{I}}{\partial \rho} \frac{\partial \rho}{\partial p} \\ &= \underbrace{\frac{w(1 - \tau) + \sigma\beta w\tau}{w(1 - \tau) + \sigma w\tau}}_{>0} \underbrace{\frac{\sigma\beta}{(1 - \sigma\beta\rho)^2}}_{>0} \underbrace{\frac{-r}{(\theta p - r)^2}}_{<0} \end{aligned}$$

Therefore, $\frac{\partial I}{\partial p} < 0$.

$$\begin{aligned} \frac{\partial I}{\partial r} &= \frac{\partial \mathcal{I}}{\partial \rho} \frac{\partial \rho}{\partial r} \\ &= \underbrace{\frac{w(1-\tau) + \sigma\beta w\tau}{w(1-\tau) + \sigma w\tau}}_{>0} \underbrace{\frac{\sigma\beta}{(1-\sigma\beta\rho)^2}}_{>0} \underbrace{\frac{p}{(\theta p - r)^2}}_{>0} \end{aligned}$$

Thus, we can conclude that $\frac{\partial I}{\partial r} > 0$.

Let us now discuss the intuition behind these results. In this equilibrium, recall that only the rich household can invest in housing while the poor can only rent houses. Thus any gains from housing investment would widen the wealth gap between the rich and poor dynasties. As owning a house provides rental income, a higher rents would make the rich become richer at the expense of the poor. At the same time, a rise in prices would increase the user cost of housing and make housing less profitable. Hence, when all other things equal, a gain in rents increases wealth inequality while an increase in house price would reduce it.

7. Comparative Statics

In this section, we are going to study how a change in down-payment rate θ and contribution rate to the pension system τ may impact on the long-run wealth inequality and we'll show how the effects pass through the housing market.

7.1. Financial liberalization and wealth distribution

As financial liberalization is claimed to be the driving force of the housing boom starting from 1990s (Duca et al.(2010), Iacoveillo and Neri(2010)). It would be interesting to see how this shift may affect individual's wealth as well as wealth inequality.

A relaxation on the credit market friction can be captured by a reduction in the parameter θ . Table 1 below summaries the reactions of equilibrium wealth inequality, housing price, and rent in response to the shock. Note that the sign $+$ is interpreted that variables move in the same direction with θ , while the sign $-$ means that they move in opposite direction. Detailed proof is given in the section C4 of Appendix.

Table 2.1: Impact from an adjustment in borrowing constraint θ

Variables	θ
Inequality I	+
House price p	-
House rent r	+
Rate of return on wealth of dynasty 2ρ	+

One can read from the table that as the down-payment rate θ reduces, the long-run house price increases while rent decreases. Wealth inequality would be therefore decrease. Results provided in the table allows us to introduce the following proposition:

Proposition 16. *When the borrowing constraint is loosen, both equilibrium wealth inequality and rental prices will decrease while housing price will increase.*

In the next section, we are going to provide the simulation results to illustrate the long-run responsiveness of the wealth inequality and the asset returns when there is a reduction in down-payment rate and we are going to discuss the intuition of the findings.

Simulation

Consider the economy lies on a steady state at $t = 0$ and suppose that a negative shift on down-payment rate which are unanticipated and permanent takes place at time $t = 1$. Parameter choices are taken from a number of paper in similar literature. The table (3.1) below provides our parameter choices as well as some examples of papers which we took as reference.

Table 2.2: Calibrated Parameter Values

Parameter	Value	Reference
Discount factor β	0.4	De la Croix(2001), Chanda(2008)
Housing preference γ	0.35	Gervais(1998), Zhao(2015)
Bequest preference σ	0.3	Han and Mulligan(2001), Belettelli et al(2014)
Contribution rate τ	0.2	D'Autume(2003), Zhao(2015)
Down-payment rate θ	0.45	Briant(2010)
Interest rate R	1.05	Le Bayon and Madec(2014)
Housing stock H	1	See main text
Wage w	1	See main text

According to the statistical report of Briant(2010), along the period 1990-2001, in France a household had to pay up-front 45% value of their house to get a home loan. Thus, we will assume that the initial value of $\theta = 0.45$ before the credit market liberalization. Moreover, this report also states that the average of downpayment rate of the period 2001-2011, which is the known as the period experiencing borrowing relaxation, is about 0.35. Therefore, we will consider in this exercise a change of θ from 0.45 to 0.35.

Secondly, we will set interest term at $R = 1.05$ as following the report of Le Bayon and Madec(2014), the average mortgage interest rate in the case of France during the period 1990-2011 was approximately 5%.

Finally, recall that w and H are wage and housing supply measured in terms of per capita. Therefore, for the simulation work we will simply normalize them to 1.⁸ Figure (2.3) below report the responsiveness of variables to a relaxation in borrowing constraint.

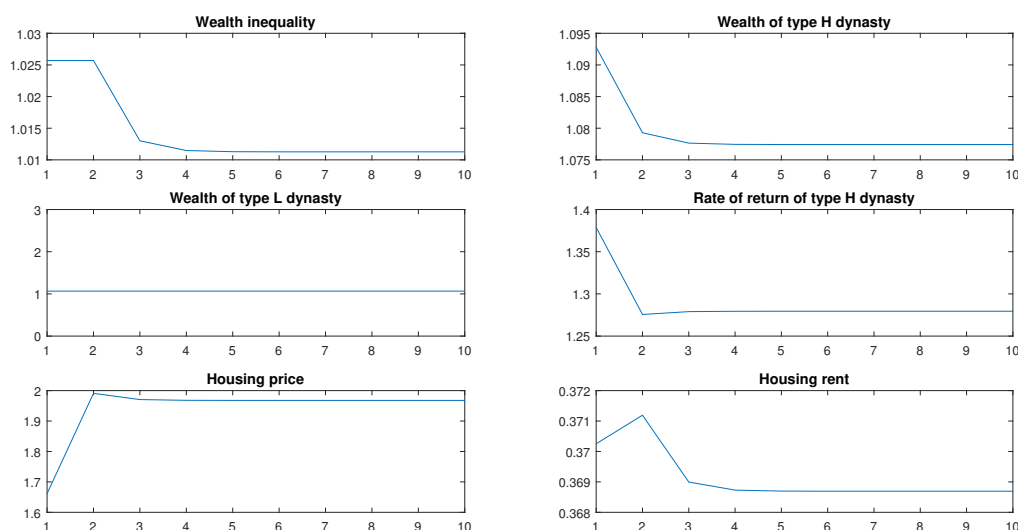


Figure 2.3: Response function of variables after a permanent shock on θ : θ decreases from 0.45 to 0.35.

Each period is equal to 30 years. Data is obtained from WID (France) and OECD.

Our simulation as well as analytic work confirm the co-movements of inequality and rents once the shock on the borrowing constraint happens. It suggests that in response to such shock rents rather than house prices should be used to trace the evolution of wealth inequality.

Let us now discuss the intuition of our findings. We found that a financial liberalization in which the down-payment rate becomes smaller leads to an increase of the

⁸Without loss of generality, these two variables can be normalized to any strictly positive levels.

equilibrium house price and a decrease of rental income.

Indeed, a lower down-payment rate makes borrowing become easier. Since homeownership is desirable by all individuals and houses now become more affordable, the demand of houses increases. Due to the house supply is fixed over time, house prices will rise in response to a shift in house demand. Turning to the rental market, a more accessibility of housing drives down the demand for renting a house. As a result, rental price will fall in a long-run.

Notice that the net return of the rich dynasty is captured by the rate of wealth return, ρ , which depends positively on rents and negatively on house prices. Since a relaxing borrowing constraint leads to a reduction in rents and increase in house prices, the net return on investment of dynasty 2, ρ , will clearly decrease. As a consequence, the equilibrium wealth of the rich dynasty will reduce.

Note also that the wealth of the poor household is not affected by a change in the downpayment rate as they are not qualified for mortgage loans. Therefore, wealth inequality, which is computed as the ratio between the wealth of rich and poor households, will fall in response to a financial liberalization. This result is supported by the data. As we show in the following figure (2.9) in Appendix, in the end of 1990s in which it was the period where relaxed or removed credit friction strongly took place, we observed that house prices increase while inequality decreases. This equalizing effect of housing boom during such period can also be found in the empirical work of Bastagli and Hills(2012), O'Farrell et al.(2016) and so on. For example, O'Farrell et al.(2016) documented the link between housing prices and inequality using OECD data. He found that a gain in house price reduces net wealth inequality in a number of OECD countries, including France, UK and Italy. He argues that the result comes from the fact that housing is mostly held by the middle-class households. given the sustained appreciation of house prices, the wealth of the middle income would lift up which will in return decline overall inequality.

7.2. Pension reform and wealth inequality

In this section, we are going to consider a shock from pension reform in the sense that individuals are required to contribute at a higher rate of τ . Little has been done to study the impacts of such shock on inequality via the housing mechanism. For example, Fall (2007) studied the impact of a shift on PAYGO system on inequality. He found that the effect of increasing the contribution rate to the pension system increases inequality by lowering the rate of return on capital savings. He explained that as interest rate is the lending rate of the poor households, they are worse off from the pension reform. On the contrary, the rich dynasty is better off as the reduction in the interest rate increases their net return on investment. In the same spirit, Zhao(2015)

used overlapping generation framework to study the impacts on house prices from a shock on the contribution rate in the PAYG pension system. He found that this particular shock will put pressure on interest rate. Whether this triggers a gain in house prices depends on the size of rental market friction as well as the borrowing capacity. As housing wealth and pension wealth are the two largest classes in household's balance sheet (Sierminska and Medgyesi (2013)), it's important to understand the impacts from a change in pension wealth on household's wealth accumulation through the housing market. Our work tries to provide an insight response to this question. A pension reform can be captured by an increase in the parameter τ . Table (2.3) below summarizes the reactions of equilibrium wealth inequality, housing price, rental income when the shock takes place. As stated above, the sign $+$ means that they move in the same direction whilst the sign $-$ refers to a negative relationship. We provide the proof of this part in the section C5 of the Appendix.

Table 2.3: Impact from an adjustment in contribution rate τ

Variables	τ
Inequality I	$-$
House price p	$-$
House rent r	$-$
Rate of return on wealth of dynasty 2 ρ	$-$

The table states that when the contribution rate to the pension system increases, the long-run house price would decrease as well as the house rent. As a result, wealth inequality would be reduced. The results provided in the table allows us to introduce the following proposition:

Proposition 17. *A reform that increases the contribution rate in the pension system drives down the equilibrium wealth inequality, housing price and rent.*

Simulation

In this section, we are simulating the long-run effects on the economy triggered from a positive shock on τ . We assume that the economy initially stays at steady state when $t = 0$ and the change which are unanticipated and permanent takes place at time $t = 1$. We will use the same parameter values as introduced in the previous section. Following the work of D'Autume(2003) and Zhao(2015) and we also set the initial payroll tax at 0.2. Due to the lack of data availability, we assume that it occurs a

very small upward shift in this rate, from 0.2 to 0.25. Since we are interested in the direction in which the variables respond rather than the magnitude of the response, we argue that the parameter choice would not change our results.

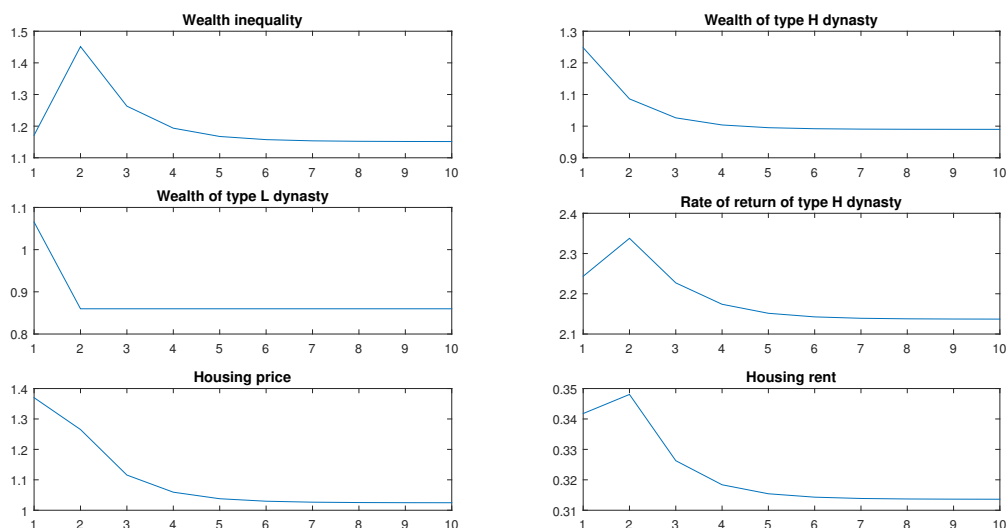


Figure 2.4: Response function of variables after a permanent shock on $\tau:\tau$ increases from 0.2 to 0.25.

Each period is equal to 30 years. Data is obtained from WID (France) and OECD.

In this scenario, we found that inequality, rents and house prices co-move once there is a pension reform. In details, we found that a higher contribution rate to the pension system leads to lower house prices and rents. Indeed, as seen from the budget constraint of the poor dynasty, a higher contribution rate reduces their income disposable. Since their non-housing and housing consumption are proportions of income, individuals from the poor household will reduce their rental demand for houses. As a result, rents will fall in long-run. On the other hand, a higher contribution rate also leads to a reduction in income of the rich household which will make them more difficult to borrow to buy house. Therefore, house demand will decrease which will decline house price.

Regarding the impacts on wealth, via the housing mechanism the pension reform makes the rich household much worse off than the poor one. It is the case because an increase in the contribution rate directly affects the poor via the reduction in their labor income. On the other hand, the income of rich will be detrimental by two adverse effects. Not only their labor income reduces in response to the pension reform, but also the rental incomes, which they obtain from lending their houses, also decreases. As a consequence, wealth inequality will fall in the long-run. The negative impact of a PAYG pension reform on housing prices can be found in the work of Zhao(2015) and

on wealth inequality can be found in, for example, Cubeddu(2000) and Fall(2003).

8. Conclusion remarks

The role of housing in the rise of wealth inequality has raised a lot attention in recent inequality research. However, the contribution of rental income in inequality has always been ignored. Our paper purpose is to fulfill this missing important piece.

First of all, by allowing housing tenure choice in the setting, we can create a society where the rich dynasty becomes capital borrower and home-owner while the poor one becomes capital saver and home renter. Wealth inequality is therefore persistent in the long-run.

Secondly, considering a relaxation in borrowing limits, we found that wealth inequality and rents reduce in response to such shock while house price increases. Regarding a shift in the contribution rate in the pension system, we found that the responses of inequality, rents and house price are all negative accordingly.

Lastly, our findings also suggest that in response to these shocks, rents, rather than house prices, follow better the movement of inequality.

9. Appendices

A. Link between inequality and housing prices and rents

Link between inequality and housing prices

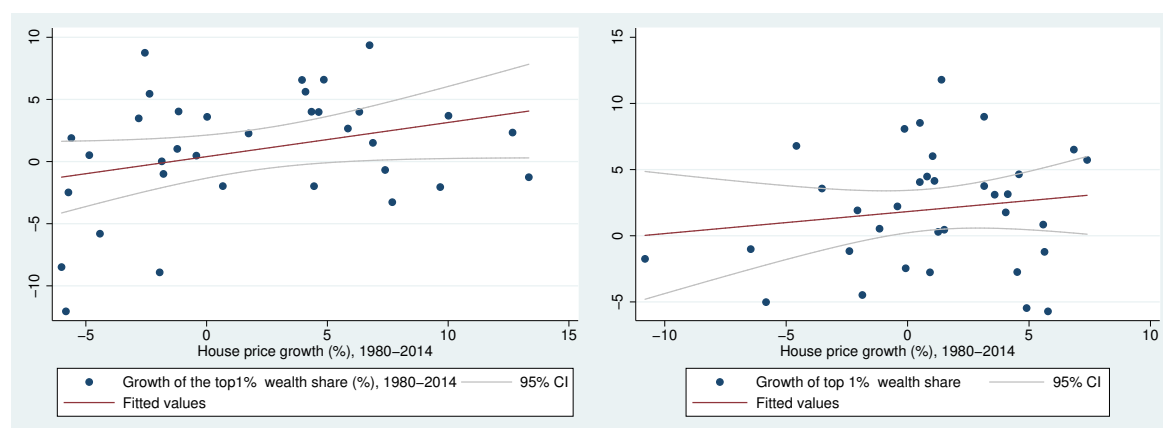


Figure 2.5: Correlation between housing prices and the wealth share of top 1% . The left-handed graph is for the case of France while the right-handed one reports the case of the US.

Source: WID and OECD

One can see that the relationship between changes in wealth share of top 1% and changes in house prices are positive in both France and the US. However, the level of significance is rather weak.

In what follows we present the relationship between house prices growth and the growth of top 10% wealth share in France and US. We notice that their correlations turn significant, which implies that the top 10% are significantly better off as house price increases.

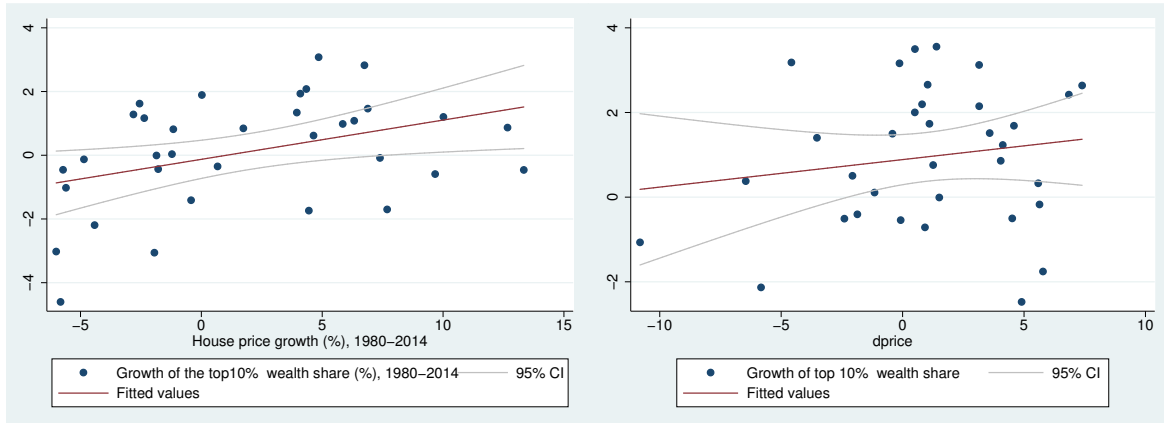


Figure 2.6: Correlation between housing rents and the wealth share of top 1% . The left-handed graph is for the case of France while the right-handed one reports the case of the US.

Source: WID and OECD

Link between inequality and housing rents

Hereafter are the relationship between changes in rents and changes in the wealth share of top 1% and of top 10%. We notice that these correlations in the case of France are also significant and positive, whereas in the case of US, they seems to be positive but insignificant. These plots suggest that, in the case of France, there's a positive correlation between house rents and wealth inequality.

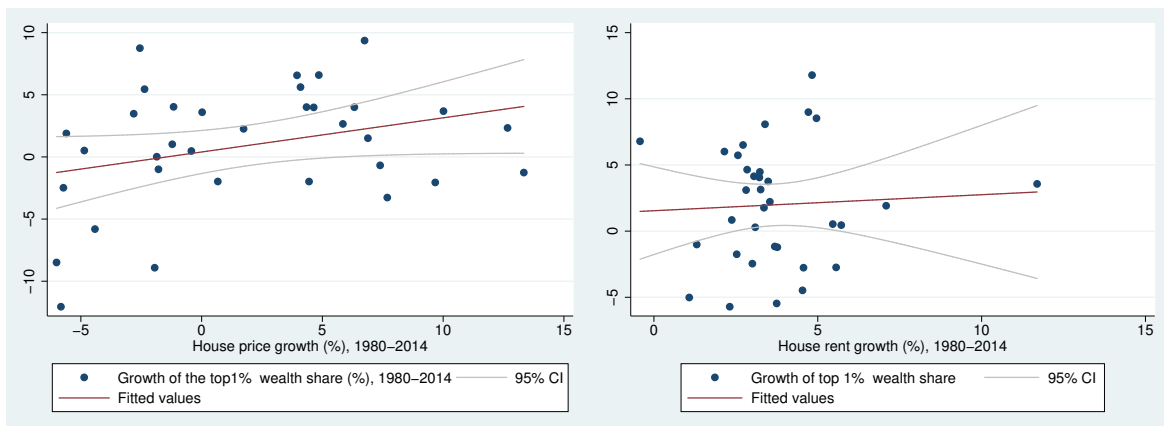


Figure 2.7: Correlation between housing rents and the wealth share of top 1% . The left-handed graph is for the case of France while the right-handed one reports the case of the US.

Source: WID and OECD

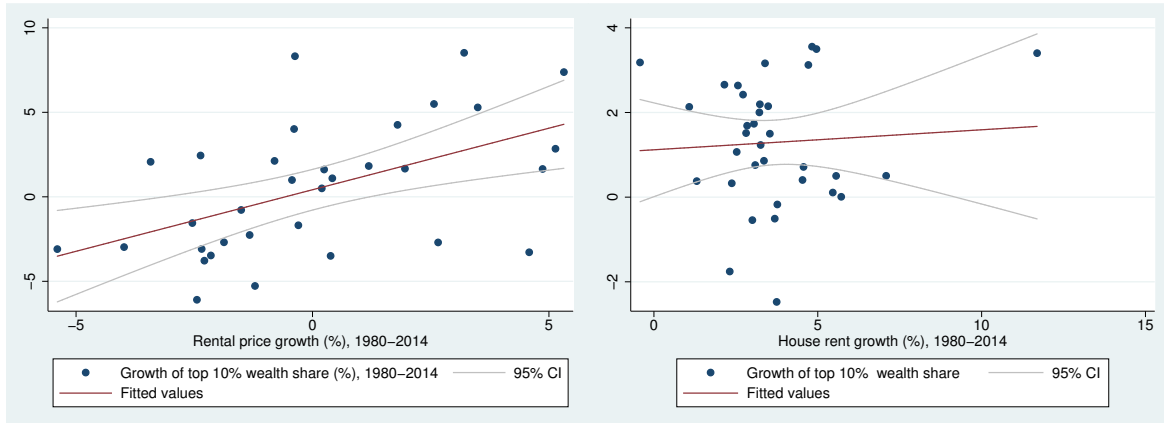


Figure 2.8: Correlation between housing rents and the wealth share of top 10% . The left-handed graph is for the case of France while the right-handed one reports the case of the US.

Source: WID and OECD

B. The evolution of Housing price and Wealth inequality in France

This figure plots the evolution of Housing price and Inequality captured by the top 10% wealth share in the case of France from 1980-2014. Data is taken from the WID and OECD.

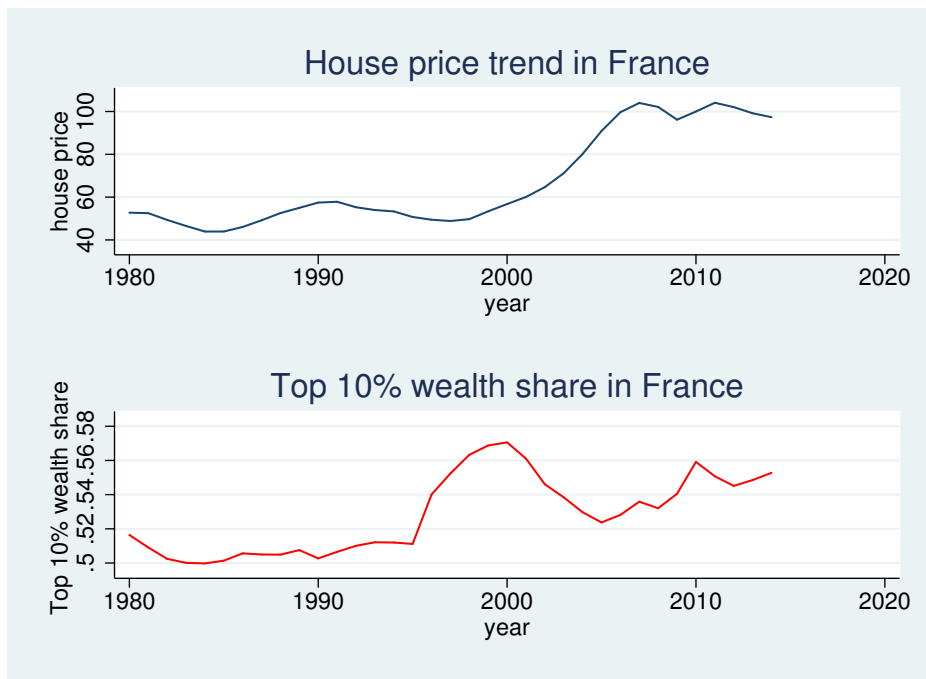


Figure 2.9: Source: WID and OECD

C. Solving for model's equilibrium

Recall the two following slackness conditions obtained from the non-negativity and

collateral constraints:

$$\zeta^i h_{H,t}^i = 0 \quad (2.27)$$

$$\lambda^i [s_t^i + (1 - \theta)p_t h_{H,t}^i] = 0 \quad (2.28)$$

1) Let us start by assuming that for both dynasties i :

$$\zeta^i > 0$$

$$\lambda^i > 0$$

From (2.27) and (2.28), these conditions imply that, for each dynasty i , the two constraints have to bind. It means that:

$$h_{H,t}^i = 0 \quad (2.29a)$$

$$s_t^i + (1 - \theta)p_t h_{H,t}^i = 0 \quad (2.29b)$$

However, this equilibrium cannot exist. Indeed, from (2.29a), we obtain that $\sum_i h_{H,t}^i = 0$. This result obviously contradicts the clearing condition in housing market, which requires that $\sum_i h_{H,t}^i = H$.

2) Assume now that for dynasty 1:

$$\zeta^1 > 0 \quad (2.30a)$$

$$\lambda^1 = 0 \quad (2.30b)$$

Therefore, (2.27) and (2.28) imply that only the non-negativity binds for dynasty 1, i.e:

$$h_{H,t}^1 = 0$$

$$s_t^1 > 0$$

Assume also that for dynasty 2:

$$\zeta^2 = 0 \quad (2.31a)$$

$$\lambda^2 > 0 \quad (2.31b)$$

Thus, only the borrowing constraint has to bind for dynasty 2. We write:

$$h_{H,t}^2 > 0$$

$$s_t^2 + (1 - \theta)p_t h_{H,t}^2 = 0$$

Recall that solving for FOC gives us:

$$\mu^i - \varphi^i R = \lambda^i \quad (2.32a)$$

$$\mu^i(p_t - r_t) - \varphi^i p_{t+1} = \zeta^i + (1 - \theta)p_t \lambda^i \quad (2.32b)$$

Then, replacing (2.30a) and (2.30b) into (2.32a) and (2.32b) above gives us the following relation between returns on asset prices:

$$R > \frac{p_{t+1}}{p_t - r_t} \quad (2.33)$$

On the other hand, by replacing (2.31a) and (2.31b) into (2.32a) and (2.32b), we obtain the opposite condition:

$$R < \frac{p_{t+1}}{p_t - r_t}$$

which clearly contradicts (2.33). Therefore, this equilibrium cannot exist.

3) Consider the opposite scenario of the case 3 presented above. In details, we assume for dynasty 1:

$$\zeta^1 = 0$$

$$\lambda^1 > 0$$

While for dynasty 2:

$$\zeta^2 > 0$$

$$\lambda^2 = 0$$

They lead to the result that only the non-negativity binds for dynasty 2 ($h_{H,t}^2 = 0$ and $s_t^2 > 0$) while uniquely the borrowing constraint binds for dynasty 1 ($h_{H,t}^1 > 0$ and $s_t^1 + (1 - \theta)p_t h_{H,t}^1 = 0$). Analogous argument will be used to prove that this equilibrium will not exist, either.

4) Assume the case where for dynasty 1:

$$\zeta^1 > 0$$

$$\lambda^1 = 0$$

which implies that they face uniquely non-negativity constraint, i.e. $h_{H,t}^1 = 0$ and $s_t^1 > 0$.

Then we obtain again the relations on asset prices stated in the case 2 in which:

$$R > \frac{p_{t+1}}{p_t - r_t}$$

On the other hand, assume that for dynasty 2:

$$\zeta^2 = 0$$

$$\lambda^2 = 0$$

which means that dynasty 2 is fully unconstrained. Therefore, we can write that $h_{H,t}^2 > 0$ and $s_t^2 + (1 - \theta)p_t h_{H,t}^2 > 0$.

Replacing these results into (2.32a) and (2.32b), one can derive that:

$$R = \frac{p_{t+1}}{p_t - r_t}$$

which contradicts the relation on asset returns obtained when solving for dynasty 1. This equilibrium, therefore, doesn't exist.

5) Analogous argument applies to the opposite of case 4:

$$\zeta^1 = 0 \quad \text{and} \quad \lambda^1 = 0$$

$$\zeta^2 > 0 \quad \text{and} \quad \lambda^2 = 0$$

Hence, in this scenario, dynasty 2 faces uniquely non-negativity constraint, i.e. $h_{H,t}^2 = 0$ and $s_t^2 > 0$, while dynasty 1 is fully unconstrained, i.e. $h_{H,t}^1 > 0$ and $s_t^1 + (1 - \theta)p_t h_{H,t}^1 > 0$.

6) Consider now the case where the Lagrangian multipliers of the rich dynasty⁹ are:

$$\zeta^2 > 0 \quad \text{and} \quad \lambda^2 > 0$$

It means that they have to face two binding constraints, i.e. $h_{H,t}^2 = 0$ and $s_t^2 = 0$. From (2.32a) and (2.32b) we can conclude that:

$$\frac{\mu^2}{\varphi^2} > R$$

⁹In this proof, we interchangeably call dynasty 2 as the rich one and dynasty 1 as the poor one. Recall that dynasty 2 is assumed to initially endow greater wealth than dynasty 1.

$$\frac{\mu^2}{\varphi^2} > \frac{p_{t+1}}{p_t - r_t}$$

From the FOC solved for dynasty 2, one can obtain his dynamics of consumption and wealth as follows:

$$c_t^2 = (1 - \gamma)(w(1 - \tau) + \sigma w\tau) \quad (2.34a)$$

$$\tilde{w}_t^2 = w(1 - \tau) + \sigma w\tau \quad (2.34b)$$

On the other hand, the multipliers of the poor dynasty are following:

$$\zeta^1 = 0 \quad \text{and} \quad \lambda^1 = 0$$

They imply that the dynasty 1 is now fully unconstrained, i.e: $h_{H,t}^1 = H > 0$ and $s_t^1 + (1 - \theta)Hp_t > 0$. It will then lead to:

$$R = \frac{p_{t+1}}{p_t - r_t}$$

The dynasty 1's wealth is then derived from:

$$\tilde{w}_{t+1}^1 = w(1 - \tau) + \sigma w\tau + \sigma(Hp_t + Rs_t^1)$$

where $Hp_t + Rs_t^1$ captures the net return on investment. This net return is required to be non-negative, i.e: $Hp_t + Rs_t^1 \geq 0 \forall t$, because otherwise, households don't have incentive to buy assets. As a result, we can conclude that:

$$\tilde{w}_t^2 \leq \tilde{w}_t^1 \quad \forall t$$

This result contradicts our assumption that, when $t = 0$, dynasty 1 endows less wealth than dynasty 2. Therefore, this equilibrium doesn't exist.

7) Similar argument would be used to reject the following scenario: For the rich household,

$$\zeta^2 > 0 \quad \text{and} \quad \lambda^2 > 0$$

For the poor household,

$$\zeta^1 = 0 \quad \text{and} \quad \lambda^1 > 0$$

They imply that the rich dynasty is fully constrained, i.e. $h_{H,t}^2 = 0$ and $s_t^2 = 0$, while the poor one is only borrowing constrained, i.e. $h_{H,t}^1 = H > 0$ and $s_t^1 + (1 - \theta)Hp_t = 0$.

Similarly, we will obtain the following dynamics of wealth of dynasty 2:

$$\tilde{w}_{t+1}^2 = w(1 - \tau) + \sigma w \tau$$

On the other hand, from the budget constraints of the old agent, the dynamics of wealth of dynasty 1 is computed as follows:

$$\tilde{w}_{t+1}^1 = w(1 - \tau) + \sigma w \tau + \sigma(p_{t+1} - (1 - \theta)Rp_t)H$$

We use the same argument that the aggregate return on investments has to be non-negative so that agents have incentive to invest on. Hence, we require $(p_{t+1} - (1 - \theta)Rp_t)H \geq 0$. As a result, we can conclude that

$$\tilde{w}_{t+1}^2 < \tilde{w}_{t+1}^1$$

which clearly contradicts to our assumption that $\tilde{w}_0^2 > \tilde{w}_0^1$. This scenario, therefore, doesn't exist.

8) Consider now the opposite of case 6:

For the rich household,

$$\zeta^2 = 0 \quad \text{and} \quad \lambda^2 = 0$$

For the poor household,

$$\zeta^1 > 0 \quad \text{and} \quad \lambda^1 > 0$$

Hence, the dynasty 1 now faces two binding constraints, i.e: $h_{H,t}^1 = 0$ and $s_t^1 = 0$, while the dynasty 2 are fully unconstrained, i.e: $h_{H,t}^2 = H$ and $s_t^2 + (1 - \theta)p_t H > 0$.

The associated slackness conditions then requires that, for dynasty 1, the both following conditions have to be satisfied:

$$\frac{\mu^1}{\varphi^1} > R$$

$$\frac{\mu^1}{\varphi^1} > \frac{p_{t+1}}{p_t - r_t}$$

Replacing the FOC equations on the condition above gives us:

$$\frac{(1 - \gamma)(1 - \beta) b_{t+1}^1}{\sigma \beta} \frac{1}{c_t^1} > R$$

Replacing back the dynamics of b_{t+1}^1 and c_t^1 , we obtain the following condition on the

interest rate R :

$$R < \frac{(1 - \beta)\tau}{\beta[1 - \tau(1 - \sigma)]} \quad (2.35)$$

Recall that in this scenario, the steady state wealth of dynasty 1 is:

$$\tilde{w}^1 = w(1 - \tau) + \sigma w \tau \quad (2.36)$$

On the other hand, the associated slackness conditions for dynasty 2 requires that:

$$\zeta^2 = 0$$

$$\lambda^2 = 0$$

which allows us to obtain the following non-arbitrage condition:

$$R = \frac{p_{t+1}}{p_t - r_t}$$

Similarly, the steady state wealth of dynasty 2 is computed as:

$$\tilde{w}^2 = \frac{w(1 - \tau) + \sigma\beta w \tau}{1 - \sigma\beta R} \quad (2.37)$$

Moreover, we argued that the net return on investment has to be always non-negative. As a result dynasty 2 will always be richer than dynasty 1. From (2.36) and (2.37), it follows that:

$$\frac{w(1 - \tau) + \sigma\beta w \tau}{1 - \sigma\beta R} > w(1 - \tau) + \sigma w \tau$$

which leads to the following condition on R :

$$R > \frac{(1 - \beta)\tau}{\beta[1 - \tau(1 - \sigma)]}$$

This condition obviously contradicts the condition stated in (2.35). Hence, this equilibrium doesn't exist.

Finally, there are two scenarios left that provide us equilibrium which are:

(i) $\zeta^i = 0$ and $\lambda^i = 0$ which means that both dynasties are fully unconstrained. For all i , we always have: $h_{H,t}^i > 0$ and $s_t^i + (1 - \theta)p_t h_{H,t}^i > 0$.

(ii) $\zeta^1 > 0$ and $\lambda^1 > 0$ while $\zeta^2 = 0$ and $\lambda^2 > 0$. As a result, the poor dynasty faces two binding constraints which are $h_{H,t}^1 = 0$ and $s_t^1 = 0$ while the rich dynasty is only borrowing constrained, i.e. $h_{H,t}^2 = H$ and $s_t^2 + (1 - \theta)p_t H = 0$.

D. List of Proofs

Proof of Proposition 2 and 3

To solve for equilibrium, we will consider different possible combinations of $\{\lambda^i, \zeta^i\}$.

Rewriting (10) and (11), we obtain:

$$\lambda^i = \mu^i - \varphi^i R \quad (2.38)$$

$$\zeta^i = \mu^i(\theta p_t - r_t) - \varphi^i(p_{t+1} - (1 - \theta)Rp_t) \quad (2.39)$$

Consider first the case where, for both dynasties, the non-negativity and collateral constraints are simultaneously unbinding, i.e: $\forall i$

$$h_{H,t}^i > 0$$

$$s_t^i + (1 - \theta)p_t h_{H,t}^i > 0$$

Hence, it follows that their Lagrange multipliers $\lambda^i = 0$ and $\zeta^i = 0$ for all i .

From (2.38) and (2.39), it leads to the following non-arbitrage condition between housing and capital returns:

$$\frac{p_{t+1}}{p_t - r_t} = R \quad (2.40)$$

From the FOC, the dynamics of consumptions, housing services, wealth and savings can be derived as follows:

$$c_t^i = (1 - \gamma)(1 - \beta)\left(\tilde{w}_t^i + \frac{w\tau}{R}\right) \quad (2.41a)$$

$$r_t h_{R,t}^i = \gamma(1 - \beta)\left(\tilde{w}_t^i + \frac{w\tau}{R}\right) \quad (2.41b)$$

$$d_{t+1}^i = (1 - \sigma)\beta R\left(\tilde{w}_t^i + \frac{w\tau}{R}\right) \quad (2.41c)$$

$$w_{t+1}^i = w(1 - \tau) + \sigma\beta R\left(\tilde{w}_t^i + \frac{w\tau}{R}\right) \quad (2.41d)$$

$$s_t^i = \beta\tilde{w}_t^i - (1 - \beta)\frac{w\tau}{R} - (p_t - r_t)h_{H,t}^i \quad (2.41e)$$

where recall that the household's wealth is $\tilde{w}_t^i = w(1 - \tau) + b_t^i$ and the equilibrium house rents and prices are computed as follows:

$$r_t H = \gamma(1 - \beta)\left(\sum_i \tilde{w}_t^i + \frac{2w\tau}{R}\right) \quad (2.42)$$

$$p_t H = \frac{R}{R - 1} r_t H \quad (2.43)$$

Since we focus only stationary steady state equilibria, from (2.41d), computing the

steady state wealths for each dynasty gives us:

$$\tilde{w}^1 = \tilde{w}^2 = \frac{w(1-\tau) + \sigma\beta w\tau}{1 - \sigma\beta R} \quad (2.44)$$

We will then define the existence condition for this particular equilibrium. Recall that this one occurs when all individuals have enough savings, i.e. $s_t^i > -(1-\theta)p_t h_{H,t}^i$. From (2.41e), it follows that:

$$\beta\tilde{w}_t^i > (\theta p_t - r_t)h_{H,t}^i + (1-\beta)\frac{w\tau}{R}$$

Replacing (2.42) and (2.43) into the inequation above gives us:

$$\tilde{w}_0^i > \underbrace{\frac{(1-\beta)(1 + \gamma\frac{1-(1-\theta)R}{R-1})}{\beta - \frac{1-(1-\theta)R}{R-1}\gamma(1-\beta)}}_{\equiv \bar{w}} \quad (2.45)$$

Notice that this condition exists if and only if

$$\beta > \frac{1 - (1-\theta)R}{R-1}\gamma(1-\beta)$$

Which leads to:

$$R > \underbrace{\frac{\beta + \gamma(1-\beta)}{\beta + \gamma(1-\beta)(1-\theta)}}_{\equiv R^*}$$

Rewrite the budget constraint of dynasties in the equilibrium provided in the Proposition 3

In this equilibrium, recall that the dynasty 1 doesn't save capital and doesn't buy house. The budget constraints of the young and old individuals are as follows:

$$c_t^1 + r_t h_{R,t}^1 = w(1-\tau) + b_t^1 \equiv \tilde{w}_t^1$$

$$d_{t+1}^1 + b_{t+1}^1 = w\tau$$

On the other hand, the budget constraint of the dynasty 2 is following:

$$c_t^2 + r_t h_{R,t}^2 = w(1-\tau) + b_t^2 - (\theta p_t - r_t)H \equiv \tilde{w}_t^2 - (\theta p_t - r_t)H \quad (2.46)$$

$$d_{t+1}^2 + b_{t+1}^2 = w\tau + (p_{t+1} - (1-\theta)Rp_t)H \quad (2.47)$$

For the dynasty 2, we already proved that their Lagrangian multipliers $\lambda^2 > 0$ and $\zeta^2 = 0$. Therefore, from (2.38) and (2.39), we can derive the following relation between

the two shadow prices μ^2 and φ^2 :

$$\frac{\mu^2}{\varphi^2} = \frac{p_{t+1} - (1 - \theta)Rp_t}{\theta p_t - r_t} \quad (2.48)$$

From FOC, we can rewrite (2.48) in terms of consumptions as follows:

$$\begin{aligned} \frac{d_{t+1}^2}{c_t^2} &= \frac{\beta(1 - \sigma)}{(1 - \beta)(1 - \gamma)} \underbrace{\frac{p_{t+1} - (1 - \theta)Rp_t}{\theta p_t - r_t}} \\ &\equiv \frac{\beta(1 - \sigma)}{(1 - \beta)(1 - \gamma)} \rho_{t+1} \end{aligned}$$

One can see that the new variable ρ_{t+1} where $\rho_{t+1} \equiv \frac{p_{t+1} - (1 - \theta)Rp_t}{\theta p_t - r_t}$ captures the discount rate of the dynasty 2.

From FOC, we also know that:

$$\begin{aligned} r_t h_{R,t}^2 &= \frac{\gamma}{1 - \gamma} c_t^2 \\ b_{t+1}^2 &= \frac{\sigma}{1 - \sigma} d_{t+1}^2 \end{aligned}$$

Replacing back into (2.46) and (2.47), one can find the dynamics of the young's consumption as follows:

$$c_t^2 = \frac{1}{(1 - \gamma)(1 - \beta)} \rho_{t+1} \left(\tilde{w}_t^2 + \frac{w\tau}{\rho_{t+1}} \right)$$

Therefore, we can say also that ρ_{t+1} measures the net return rate of dynasty 2.

Proof of Lemma 4

Recall that:

$$\gamma(\rho) = \frac{\gamma \tilde{w}^1 + \gamma(1 - \beta) \left(\frac{w(1 - \tau) + \sigma\beta w\tau}{1 - \sigma\beta\rho} + \frac{w\tau}{\rho} \right)}{\beta \frac{w(1 - \tau) + \sigma\beta w\tau}{1 - \sigma\beta\rho} - (1 - \beta) \frac{w\tau}{\rho}}$$

By dividing the numerator and denominator by $\frac{w(1 - \tau) + \sigma\beta w\tau}{1 - \sigma\beta\rho}$ we obtain:

$$\Gamma(\rho) = \frac{\frac{\gamma \tilde{w}^1}{w(1 - \tau) + \sigma\beta w\tau} (1 - \sigma\beta\rho) + \gamma(1 - \beta) \left(1 + \frac{w\tau}{w(1 - \tau) + \sigma\beta w\tau} \frac{1 - \sigma\beta\rho}{\rho} \right)}{\beta - (1 - \beta) \frac{w\tau}{w(1 - \tau) + \sigma\beta w\tau} \frac{1 - \sigma\beta\rho}{\rho}}$$

Equivalently,

$$\Gamma(\rho) = \underbrace{\frac{\gamma(1-\beta)}{\beta - (1-\beta)\frac{w\tau}{w(1-\tau)+\sigma\beta w\tau} \frac{1-\sigma\beta\rho}{\rho}}}_{\Gamma_1(\rho)} + \frac{\gamma\tilde{w}^1}{w(1-\tau) + \sigma\beta w\tau} \underbrace{\frac{1-\sigma\beta\rho}{\beta - (1-\beta)\frac{w\tau}{w(1-\tau)+\sigma\beta w\tau} \frac{1-\sigma\beta\rho}{\rho}}}_{\Gamma_2(\rho)} + \frac{\gamma(1-\beta)w\tau}{w(1-\tau) + \sigma\beta w\tau} \underbrace{\frac{1}{\beta\frac{\rho}{1-\sigma\beta\rho} - (1-\beta)\frac{w\tau}{w(1-\tau)+\sigma\beta w\tau}}}_{\Gamma_3(\rho)}$$

Notice that

$$\frac{\partial \frac{1-\sigma\beta\rho}{\rho}}{\partial \rho} < 0$$

Therefore $\frac{\partial \Gamma_1(\rho)}{\partial \rho} < 0$.

Similarly, one can prove that $\frac{\partial \Gamma_2(\rho)}{\partial \rho} < 0$ and $\frac{\partial \Gamma_3(\rho)}{\partial \rho} < 0$.

Hence, we conclude that

$$\frac{\partial \Gamma(\rho)}{\partial \rho} < 0$$

Proof of Proposition 6

To ease the writing, define $W \equiv w(1-\tau) + \theta\beta w\tau$.

Rewrite the equation (2.26) that defines the discount rate ρ as follows:

$$\underbrace{\frac{\theta\rho}{1 - (1-\theta)R}}_{\equiv \Pi(\rho;\theta)} - \underbrace{\frac{\gamma\tilde{w}_1 + \gamma\frac{W}{1-\sigma\beta\rho}}{\beta\frac{W}{1-\sigma\beta\rho} - (1-\beta)\frac{w\tau}{\rho}}}_{\equiv \Gamma(\rho;\theta)} + \gamma - 1 = 0 \quad (2.49)$$

Then taking the derivative with respect of θ from (2.49) gives us:

$$\frac{\partial \Pi(\rho;\theta)}{\partial \theta} - \frac{\partial \Gamma(\rho;\theta)}{\partial \rho} = 0 \quad (2.50)$$

where:

$$\frac{\partial \Pi(\rho;\theta)}{\partial \theta} = \frac{(1-R)\rho}{(1 - (1-\theta)R)^2} + \frac{\theta}{1 - (1-\theta)R} \frac{\partial \rho}{\partial \theta}$$

and

$$\frac{\partial \Gamma(\rho; \theta)}{\partial \theta} = \frac{\gamma}{\left(\beta \frac{W}{1-\sigma\beta\rho} - (1-\beta) \frac{w\tau}{\rho}\right)^2} \underbrace{\left[-\frac{(1-\beta)\beta\sigma w\tau + \sigma\beta^2 \tilde{w}_1 \rho}{\rho(1-\sigma\beta\rho)^2} W - (1-\beta) \frac{w\tau}{\rho^2} \left(\tilde{w}_1 + \frac{W}{1-\sigma\beta\rho}\right) \right]}_{\equiv M} \frac{\partial \rho}{\partial \theta}$$

Clearly, M is negative.

Replacing back into (2.50) we obtain:

$$\underbrace{\frac{(R-1)\rho}{(1-(1-\theta)R)^2}}_{>0} = \underbrace{\left[\frac{\theta}{1-(1-\theta)R} - \frac{\gamma}{\left(\beta \frac{W}{1-\sigma\beta\rho} + (1-\beta) \frac{w\tau}{\rho}\right)^2} M \right]}_{>0 \text{ due to } M < 0} \frac{\partial \rho}{\partial \theta}$$

Therefore, we can conclude that:

$$\frac{\partial \rho}{\partial \theta} > 0$$

which states that the asset return rate of the rich household will increase in response to a gain in down-payment rate.

From the system of equations (2.25a)-(2.25c), we can derive easily the effects on prices when the down-payment rate θ is relaxing:

$$\frac{\theta p - r}{\rho} \frac{\partial \rho}{\partial \theta} = -\frac{(1-R)p + Rr}{1-(1-\theta)R} + \frac{\partial r}{\partial \theta} - \frac{r}{p} \frac{\partial p}{\partial \theta} \quad (2.51)$$

$$\frac{\partial r}{\partial \theta} H = \gamma(1-\beta) \left(\frac{\sigma\beta W}{(1-\sigma\beta\rho)^2} - \frac{w\tau}{\rho^2} \right) \frac{\partial \rho}{\partial \theta} \quad (2.52)$$

Recall that $W = w(1-\tau) + \sigma\beta w\tau$.

$$(1-(1-\theta)R)H \frac{\partial p}{\partial \theta} + RpH = \frac{\beta W}{(1-\sigma\beta\rho)^2} \frac{\partial \rho}{\partial \theta} \quad (2.53)$$

Moreover, by adding up (2.25b) and (2.25c) we obtain an equation that defines the link between housing prices and rents:

$$\theta p H + \frac{1-\gamma}{\gamma} r H = \frac{W}{1-\sigma\beta\rho} + w(1-\tau) + \sigma w\tau$$

We can compute the derivation of house prices and rents as follows:

$$\frac{1-\gamma}{\gamma} H \frac{\partial r}{\partial \theta} + \theta H \frac{\partial p}{\partial \theta} + p H = \frac{\sigma\beta W}{(1-\sigma\beta\rho)^2} \frac{\partial \rho}{\partial \theta} \quad (2.54)$$

Replacing (2.53) into (2.51), we obtain the following function that allows us to compute the responsiveness of r when θ changes:

$$\frac{\partial r}{\partial \theta} = \underbrace{\frac{(1-R)p}{1-(1-\theta)R}}_{>0} + \underbrace{\left[\frac{\theta p - r}{\rho} + \frac{1}{p(1-(1-\theta)R)} \frac{\beta W r}{(1-\sigma\beta\rho)^2} \right]}_{>0} \frac{\partial \rho}{\partial \theta} \quad (2.55)$$

Since we already proved that $\frac{\partial \rho}{\partial \theta} > 0$, from (2.55) we can conclude that:

$$\frac{\partial r}{\partial \theta} > 0$$

which means that the rental prices responses positively to a tightening borrowing constraint.

Multiplying (2.54) by R and then subtracting (2.53) gives us a function that define the responsiveness of p to a financial liberalization as follows:

$$\underbrace{(R-1)}_{>0} \frac{\partial p}{\partial \theta} = - \underbrace{\frac{1-\gamma}{\gamma}}_{>0} R \underbrace{\frac{\partial r}{\partial \theta}}_{>0} - \underbrace{(1-\sigma\beta R)}_{>0} \frac{\beta W}{(1-\sigma\beta\rho)^2} \underbrace{\frac{\partial \rho}{\partial \theta}}_{>0}$$

As a result,

$$\frac{\partial p}{\partial \theta} < 0$$

It's easy to show that wealth inequality will increase in response to the shock. Recall that:

$$I = \frac{w(1-\tau) + \sigma\beta w\tau}{w(1-\tau) + \sigma w\tau} \frac{1}{1-\sigma\beta\rho}$$

Therefore, we have:

$$\frac{\partial I}{\partial \theta} = \frac{w(1-\tau) + \sigma\beta w\tau}{w(1-\tau) + \sigma w\tau} \frac{\sigma\beta}{(1-\sigma\beta\rho)^2} \frac{\partial \rho}{\partial \theta}$$

which states that the responsiveness of the inequality will move in the same direction with the one of ρ . Since we already proved that $\frac{\partial \rho}{\partial \theta} > 0$, therefore

$$\frac{\partial I}{\partial \theta} > 0$$

Wealth inequality will, thus, increase when borrowing constraint becomes tighter.

Proof of Proposition 7

From the system of equations (2.25a)-(2.25c), we can derive the impacts on prices when the contribution rate τ increases:

$$\frac{\theta p - r}{\rho} \frac{\partial \rho}{\partial \tau} = \frac{\partial r}{\partial \tau} - \frac{r}{p} \frac{\partial p}{\partial \tau} \quad (2.56)$$

$$(1 - (1 - \theta)R)H \frac{\partial p}{\partial \tau} = \frac{\beta W}{(1 - \sigma\beta\rho)^2} \frac{\partial \rho}{\partial \tau} - w \left(\frac{\beta(\rho - 1)}{1 - \sigma\beta\rho} + 1 \right) \quad (2.57)$$

$$\frac{1}{\gamma(1 - \beta)} H \frac{\partial r}{\partial \tau} = \left(\frac{\sigma\beta W}{(1 - \sigma\beta\rho)^2} - \frac{w\tau}{\rho^2} \right) \frac{\partial \rho}{\partial \tau} - w \left(\frac{\rho - 1}{\rho(1 - \sigma\beta\rho)} + \frac{1 - \sigma}{1 - \beta} \right) \quad (2.58)$$

where recall that $W \equiv w(1 - \tau) + \sigma\beta w\tau$.

Moreover, by adding up (2.25b) and (2.25c) we obtain an equation that defines the link between housing prices and rents:

$$\theta p H + \frac{1 - \gamma}{\gamma} r H = \frac{W}{1 - \sigma\beta\rho} + w(1 - \tau) + \sigma w \tau$$

We can compute the derivation of house prices and rents as follows:

$$\frac{1 - \gamma}{\gamma} H \frac{\partial r}{\partial \tau} + \theta H \frac{\partial p}{\partial \tau} = \frac{\sigma\beta W}{(1 - \sigma\beta\rho)^2} \frac{\partial \rho}{\partial \tau} - w \left(1 - \sigma + \frac{1 - \sigma\beta}{1 - \sigma\beta\rho} \right) \quad (2.59)$$

Replacing (2.57) and (2.58) into (2.59), we obtain the function that allows to compute the responsiveness of ρ once τ changes:

$$\begin{aligned} & \left[(1 - \gamma)(1 - \beta) \underbrace{\left(\frac{\sigma\beta W}{(1 - \sigma\beta\rho)^2} - \frac{w\tau}{\rho^2} \right)}_{\equiv X} + \underbrace{\left(\frac{\theta}{1 - (1 - \theta)R} - \sigma \right)}_{\equiv Z} \frac{\beta W}{(1 - \sigma\beta\rho)^2} \right] \frac{\partial \rho}{\partial \tau} = \\ & - \underbrace{\gamma w(1 - \rho)}_{>0} - \underbrace{\left(1 - (1 - \gamma)(1 - \beta) \right) \frac{w(1 - \sigma\beta)}{1 - \sigma\beta\rho}}_{>0} - \underbrace{\left(1 + \frac{\beta(\rho - 1)}{1 - \sigma\beta\rho} \right) \frac{\theta w}{1 - (1 - \theta)R}}_{>0} \end{aligned} \quad (2.60)$$

From the session above, we noticed that $X > 0$ ¹⁰. Thus, what is left is to prove that $Z > 0$.

Indeed, since $\frac{\theta}{1 - (1 - \theta)R} > 1$ due to $R > 1$ while the parameter $\rho < 1$, their subtraction captured by Z has to be positive, i.e:

$$Z > 0$$

¹⁰From (2.52), since $\frac{\partial r}{\partial \theta} > 0$ while $\frac{\partial \rho}{\partial \theta} > 0$, therefore, one can conclude that $X \equiv \frac{\sigma\beta W}{(1 - \sigma\beta\rho)^2} - \frac{w\tau}{\rho^2} > 0$

As a result, from (2.60) one can now conclude that

$$\frac{\partial \rho}{\partial \tau} < 0$$

From (2.57), one can see easily that:

$$\frac{\partial p}{\partial \tau} < 0$$

And from (2.58), we can state that:

$$\frac{\partial r}{\partial \tau} < 0$$

Finally, the impact from a change in the distribution rate on wealth inequality can be computed as follows:

$$\frac{\partial I}{\partial \tau} = - \underbrace{\frac{\sigma(1-\beta)}{(1-\sigma\beta\rho)(1-\tau+\sigma\tau)^2}}_{>0} + \underbrace{\frac{1-\tau+\sigma\beta\tau}{1-\tau+\sigma\tau}}_{>0} \underbrace{\frac{\sigma\beta}{(1-\sigma\beta\rho)^2}}_{>0} \frac{\partial \rho}{\partial \tau}$$

Since we already proved that $\frac{\partial \rho}{\partial \tau} < 0$, we can now conclude that:

$$\frac{\partial I}{\partial \tau} < 0$$

Proof of equation (2.26)

Rewrite (2.25a) as:

$$1 - \frac{r}{\theta p} = \frac{1 - (1 - \theta)R}{\theta \rho} \quad (2.61)$$

Dividing (2.25b) and (2.25c) gives us:

$$\frac{\theta p}{r} = \frac{(\beta + \gamma(1 - \beta)) \frac{w(1-\tau) + \sigma\beta w\tau}{1-\sigma\beta\rho} + (1 - \gamma)(1 - \beta) \frac{w\tau}{\rho} + \gamma\tilde{w}^1}{\gamma(1 - \beta) \left(\frac{w(1-\tau) + \sigma\beta w\tau}{1-\sigma\beta\rho} + \frac{w\tau}{\rho} \right) + \gamma\tilde{w}^1} \quad (2.62)$$

where recall that $\tilde{w}^1 = w(1 - \tau) + \sigma w\tau$.

Replacing (2.62) into (2.61) provides us the single equation of ρ :

$$\frac{1 - (1 - \theta)R}{\theta \rho} = \frac{\beta(w(1 - \tau) + \sigma\beta w\tau) \frac{1}{1-\sigma\beta\rho} - (1 - \beta) \frac{w\tau}{\rho}}{(\beta + \gamma(1 - \beta)) \frac{w(1-\tau) + \sigma\beta w\tau}{1-\sigma\beta\rho} - (1 - \gamma)(1 - \beta) \frac{w\tau}{\rho} + \gamma\tilde{w}^1}$$

Equivalently, it can be rewritten as:

$$\frac{\theta\rho}{1 - (1 - \theta)R} = 1 + \frac{\gamma\tilde{w}^1 + \gamma(1 - \beta)\left(\frac{w(1-\tau) + \sigma\beta w\tau}{1 - \sigma\beta\rho} + \frac{w\tau}{\rho}\right)}{\beta\frac{w(1-\tau) + \sigma\beta w\tau}{1 - \sigma\beta\rho} - (1 - \beta)\frac{w\tau}{\rho}}$$

To write with ease, we define $W \equiv w(1 - \tau) + \sigma\beta w\tau$ and replace back to equation above:

$$\frac{\theta\rho}{1 - (1 - \theta)R} = 1 + \frac{\gamma\tilde{w}^1 + \gamma(1 - \beta)\left(\frac{W}{1 - \sigma\beta\rho} + \frac{w\tau}{\rho}\right)}{\beta\frac{W}{1 - \sigma\beta\rho} - (1 - \beta)\frac{w\tau}{\rho}}$$

Simplifying it gives us:

$$\frac{\theta\rho}{1 - (1 - \theta)R} = 1 + \gamma\frac{\tilde{w}^1 + \frac{W}{1 - \sigma\beta\rho} + (1 - \beta)\frac{w\tau}{\rho} - \beta\frac{W}{1 - \sigma\beta\rho}}{\beta\frac{W}{1 - \sigma\beta\rho} - (1 - \beta)\frac{w\tau}{\rho}}$$

Equivalently, we have:

$$\frac{\theta\rho}{1 - (1 - \theta)R} = 1 - \gamma + \frac{\gamma\tilde{w}^1 + \gamma\frac{W}{1 - \sigma\beta\rho}}{\beta\frac{W}{1 - \sigma\beta\rho} - (1 - \beta)\frac{w\tau}{\rho}}$$

E. Justify that the Walras law doesn't apply in this setting

Recall that at any point in time t , there are always two living individuals: the old who was born at time $t - 1$ and the young who was born at time t . Their budget constraints at time t are following:

$$c_t^i + s_t^i + r_t h_{R,t}^i + p_t h_{H,t}^i \leq w(1 - \tau) + b_t^i + r_t h_{H,t}^i$$

$$d_t^i + b_t^i \leq w\tau + p_t h_{H,t-1}^i + R s_{t-1}^i$$

Recall that the economy composes of two different types of dynasties $i = 1, 2$. Therefore, we can derive the ...

$$\sum_i c_t^i + \sum_i d_t^i + \sum_i s_t^i + r_t \sum_i h_{R,t}^i + p_t \sum_i h_{H,t}^i = 2w + r_t \sum_i h_{H,t}^i + p_t \sum_i h_{H,t-1}^i + R \sum_i s_{t-1}^i$$

Notice that the rental and purchasing markets are required to clear. Hence, we write:

$$\sum_i h_{H,t}^i = \sum_i h_{H,t-1}^i = H$$

$$\sum_i h_{R,t}^i = H$$

Replacing back into the ... above and simplifying, we get:

$$\underbrace{\sum_i c_t^i + \sum_i d_t^i}_{\text{Total consumption at t}} + \sum_i s_t^i = 2w + R \sum_i s_{t-1}^i \quad (2.63)$$

As we can see, the equation (2.63) above composes the consumption goods and capital goods. As we assume that the economy is small and open, we doesn't require the capital market to clear, which means that:

$$\sum_i s_t^i \neq 0$$

Note that the economy contains 4 markets: consumption goods, capital, rental and housing. Since the capital market doesn't necessarily clear, the consumption good market doesn't have to clear, accordingly. Therefore, the Walras's law, which asserts that, general speaking, if demand is equal to supply in $n - 1$ markets then for the n -th market, its market-clearing conditions is satisfied automatically, doesn't hold in this setting.

Chapter 3

Intrahousehold transfer, inheritance and implications on inequality

⁰This chapter is co-written with Dalal Moosa.

1. Introduction

Research interest in private transfers between generations over the past few decades has focused largely on two types of financial transfers: (1) bequest, which are post-mortem, and (2) sizeable inter-vivo gifts, often made by living individuals to younger generations at later points in life, such as the transfer of real-estate ownership. Both of these transfers tend to be between households, meaning *interhousehold*, and are often lumped under the term “inheritance” - a term we also adopt in this paper.

Inheritance has been argued to play an important role in wealth accumulation. It has been estimated to contribute to about 80% of the stock of wealth in the U.S. in the 1970s (Kotlikoff and Summers (1981)),¹ and up to two-thirds of private wealth in France by 2010, with a rising share in other European countries such as Germany, Britain and Sweden post World War II (Piketty and Zucman (2015)).

We argue, however, that despite the importance of inheritance, there is a significant share of private intergenerational transfers that has received relatively less attention, which are *intrahousehold* transfers. These are transfers transmitted within the household, often from older generations (such as parents) to younger ones (such as children). They include, for instance, parental expenditure on children for food, for clothing, and even for shelter.

New data from the National Transfer Accounts (NTA) highlights the nontrivial size of these intrahousehold transfers in France over time, emphasizing the importance of looking at both inheritance and intrahousehold transfers when examining private intergenerational transfers. Thanks to the long series of inheritance data in France, the NTA database for the country uniquely combines these flows.

The results show an interesting trend. While the sum of these two transfers has remained relatively stable as a share of national income between 1979 and 2011, the composition has changed: from a dominant share of intrahousehold transfers in the 1980s and 1990s to a less dominant one in later years, leading to a decrease in the ratio of intrahousehold transfers to inheritance over time.

A change in fertility rates is unlikely to explain this trend for two reasons. Firstly, while total fertility rate (TFR) has decreased from the 1960s to today, it was relatively stable during the period of interest, at about 1.8-2.0 children per woman. Secondly, the data shows that there was a general increase of received intrahousehold transfers per capita to the age-groups that typically receive them, namely 0-19 year olds, which means that parents tended to increase transfers per child even when they had fewer children - a trend observed in the literature as well. However, despite the increase in per

¹This share was estimated at only 20% by Modigliani (1988) due to the different treatment of areas such as durable goods, children’s university fees and accrued interest on the stock of wealth (Gale and Scholz (1994))

capita transfers for this typical age-group recipient, inheritance increased much faster. This result remains the same even when we account for the rise in the population of those that typically make (and receive) inheritance, i.e. those that are 50+ years old. This implies that even when we account for a demographic change, the trends remain relatively the same and the ratio of intrahousehold transfer to inheritance increases over time (Refer to Figure (3.12) in Appendix (3)).

A change in inheritance tax is also unlikely to explain this trend. In fact, inheritance tax has increased significantly in France since the 1960s (Piketty (2011)), which would have theoretically discouraged inheritance rather than increased it.

In this paper, we argue that this phenomenon can be explained by a difference in individuals' saving behaviors in the context of rising inequality. To generate such heterogeneity, we introduce an overlapping generations model that includes both of these private transfers. We not only allow for heterogeneous preferences to bequeath, but also differences in the role that each of these two transfers play. This is the first time that all of these aspects are incorporated at the same time, to our knowledge. In line with the evidence, inheritance is modeled as additional income received during the middle of life, while intrahousehold transfers support children's consumption, modeled through the production of human capital (a set-up similar to Becker and Tomes (1986), for instance).

The impact of inheritance has been argued, from early on, to contribute to an increase in wealth inequality (Stamp (1926), Wedgwood (1929), Blinder (1976) and Harbury and Hitchens (1979)). Davies (1982) found that inheritance transferred from parents to children can have a "disequalizing" effect on current wealth, measured through an increase in the Gini coefficient for Canada. Gokhale et al. (2001) highlighted that bequest can propagate wealth inequality, even when accounting for a number of things such as skill heterogeneity, assortative mating and social security. Moreover, De Nardi (2004b) showed through a quantitative general equilibrium model for both the U.S. and Sweden that bequests help explain the emergence of large estates and wealth accumulation, and consequently the inequality seen at the time. Davies and Shorrocks (2000) argued that bequest is a crucial element in accurately measuring wealth distribution and inequality.

These findings can be explained by the fact that when inheritance is treated as a luxury good, individuals and households with higher lifetime incomes tend to have higher savings rates and consequently leave more wealth to their offspring, who, in turn, tend to do the same for the following generations. In the long-run, wealth becomes accumulated and transmitted across generations through bequest, leading to persistent inequality.

On the other hand, Becker and Tomes (1979) and Tomes (1981) argued that inheri-

tance can have an equalizing effect on wealth accumulation if it plays a “compensatory” role for children with lower incomes, meaning if parents give more inheritance to children with lower incomes. Moreover, Bossmann et al. (2007) showed that bequest can decrease wealth inequality when assuming stochastic individual income and when taking the coefficient of variation as a measure of inequality. Expected wealth and the variance of wealth increase with inheritance, with the former having a higher effect, consequently making wealth distribution more equal.

When accounting for other forms of intergenerational transfers, such as human capital-related transfers, as is in our model, the effect on inequality can be nontrivial. Ishikawa (1975), for instance, argued that accounting for intergenerational transfers for education and bequest can lead to strongly or weakly stratified societies in the long run, with the rich accumulating capital through inheritance and higher education, and the poor having little of either. Becker and Tomes (1986) argued that the degree of intergenerational mobility, and consequently the rise and fall of families, depends on endowments transmitted from parents and investment opportunities for each generation. Chanda (2008) argued that human capital investments can crowd out physical capital due to a rise in returns to education, as seen in the US, which can potentially reduce inequality.

The model introduced in this paper recognizes the role that human capital can play and its potential impact on inequality and allows for this channel in addition to inheritance. We show that heterogeneity in the preference to bequeath and differences in the purpose of these two intergenerational transfers can lead to increases in wealth inequality and a decrease in the ratio of intrahouseholds transfers to inheritance in the short run. We show that given these differences, long-run inequality exists, and that changes in the inheritance preference and the importance of intrahousehold transfers in human capital accumulation can lead to notable changes in long-run inequality.

The rest of the paper is structured as the following. Section 3 introduces the National Transfer Accounts (NTA) database and displays the main motivating facts for this research. Section 3 delves into the model set-up and its results regarding steady-state inequality and the dynamics of intrahousehold transfers-to-inheritance ratio, which help us explain what we see in the data. Section 3 further discusses these results and their assumptions. Finally, Section 3 concludes.

2. Motivating facts: Evidence from the National Transfer Accounts

The National Transfer Accounts (NTA), launched in the early 2000s, disaggregates economic flows seen in the System of National Accounts (SNA) into various ages. In

doing so, it allows for a better understanding of how people earn, consume, share and save their income across time, on a macroeconomic level. This can be useful in shedding more light on intergenerational (re)allocations and various transfers.

Based on lifecycle theory, the NTA quantifies the lifecycle deficit (or surplus) for each age, which is the difference between consumption and labor income. Children, for example, have lifecycle deficits because they do not earn any labor income, whereas working-age adults often have lifecycle surpluses. At each age, an individual can bridge (reallocate) this deficit (surplus) through three main channels: (1) public transfers, both cash and in-kind; (2) private transfers, within and between households (which exclude the inheritance flows introduced in the paper); and (3) asset-based reallocations such as financial income and savings (NTA Manual (2013)). This can be seen through the NTA Identity below:

$$C_{a,t} - L_{a,t} = \underbrace{T_{a,t}^{+public} - T_{a,t}^{-public}}_{\text{net public transfers}} + \underbrace{T_{a,t}^{+private} - T_{a,t}^{-private}}_{\text{net private transfers}} + \underbrace{A_{a,t}^+ - A_{a,t}^-}_{\text{net asset based reallocations}}$$

where a denotes age, t denotes time, $+$ stands for received and $-$ stands for given.

Public transfers in the NTA includes what individuals give in terms of taxes and what they receive in various subsidies and social security support, for instance. Private transfers include what is transmitted for purposes such education, healthcare, imputed rent² and others. Asset-based reallocations are largely intertemporal and can be divided into capital income and property income, both public and private³.

Due to the growing use of the NTA methodology across countries, the database can offer some unique cross-country perspectives on how people earn, consume, and reallocate their resources (d’Albis and Moosa (2015) and Lee and Mason (2011)). In France, for example, the NTA data show that the real per capita consumption profile by age has not changed drastically between 1979 and 2011, while the age profile of labor income shifted towards higher ages, in line with increasing years of education (d’Albis et al. (2015)).

However, more relevant to the purposes of this paper is the magnitude and trend of private transfers, which have often been difficult to quantify on an aggregate level. Private transfers in the NTA include two categories: (1) Intrahousehold transfers, i.e. those given and received within one household, which often mean a family; (2) Inter-

²Imputed rent is the use or consumption of owner-occupied housing. In the NTA it is calculated through the rental price of similar property at the time.

³While public capital income is quite negligible, public property income includes things such as public debt, sovereign wealth funds income and, in the special case of the U.S., student loan programs. Private capital income includes housing and consumer durables, while private property income includes consumer debt, land and subsoil minerals.

houshold transfers, i.e. between households, which include “regular” and “occasional” cash transfers, as well as “in-kind” transfers. For children, these transfers come to bridge the gap between their private consumption and their (lack of) labor income. The sweeping majority of this private consumption in France is in areas other than private education expenditures and private healthcare. Indeed, these transfers go to everything else to raise children, such as food, shelter, clothes (refer to Figure 3.16 in the Appendix).

The data show that “interhousehold” transfers as captured by the NTA are generally small, making up at most a tenth of the total intra- and interhousehold transfers during the period 1979-2011. For ease of reference, they are dropped from the following analysis, where we focus on intrahousehold transfers and inheritance flows.

The NTA France database was also able to provide age-profiles of bequest and inter-vivo gifts, referred to here as “inheritance”, utilizing estimates from a series of works on France by Thomas Piketty (Piketty (2011), Piketty et al. (2014) and Piketty (2014)). While they can also be categorized as “interhousehold”, they are not typically included in the NTA database for various reasons, including a lack of data. The availability of this data for France allows us to expand the analysis and incorporate more holistically all (monetary) intergenerational flows.

The trend and composition of these two types of private transfers show several important trends. The first is that the sum of these flows has remained a relatively stable share of Gross National Income in France, from about 25% in 1979 to a little over 22% in 2011, with a small dip in the late 1990s, as shown in Figure (3.1).

Figure 3.1: Received private transfers by type as a percentage of GNI

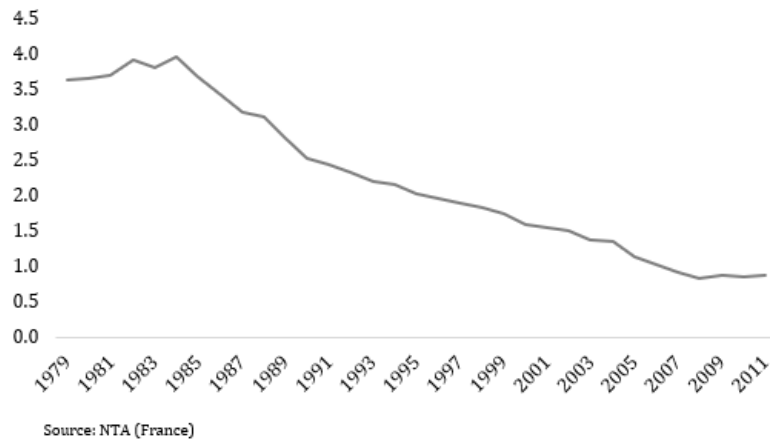


Source: NTA (France)

The second is that despite their relatively stable share, the composition of these transfers has changed over time, with a decrease in the share of intrahousehold transfers

in overall private transfers. Consequently, the ratio of intrahousehold transfers-to-inheritance has declined, from about 3.6 in 1979 to less than 1 in 2011 (Figure 3.2). This is a ratio of great interest to the purposes of this paper as it captures the change in the composition over time, which we can explain through the theoretical model.

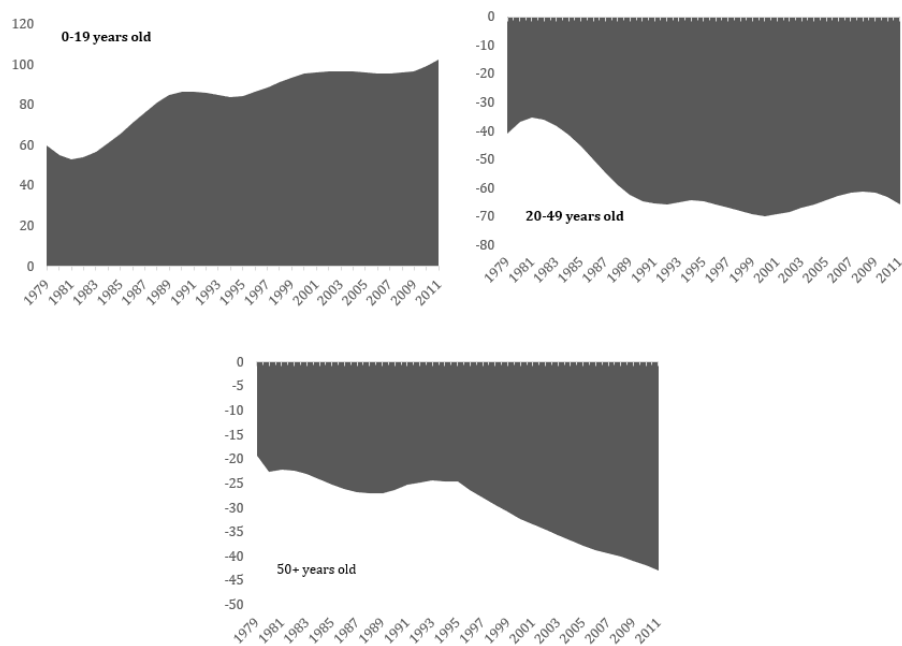
Figure 3.2: Ratio of intrahousehold transfers to inheritance



The NTA data also point to the directionality of intrahousehold transfers and inheritance. The working-age population, which is defined conservatively here as those 20-49 years old, is an important net giver of intrahousehold transfers, contributing at least two-thirds of transmitted intrahousehold transfers over the years. These transfers are received mostly by the young (0-19 years old), as shown in Figure (3.3).⁴

⁴If we define the working-age group as those 30-59 years old, or even 20-59 years old, for instance, the same conclusion holds.

Figure 3.3: Net intrahousehold transfers by broad age-group (billion real euros)

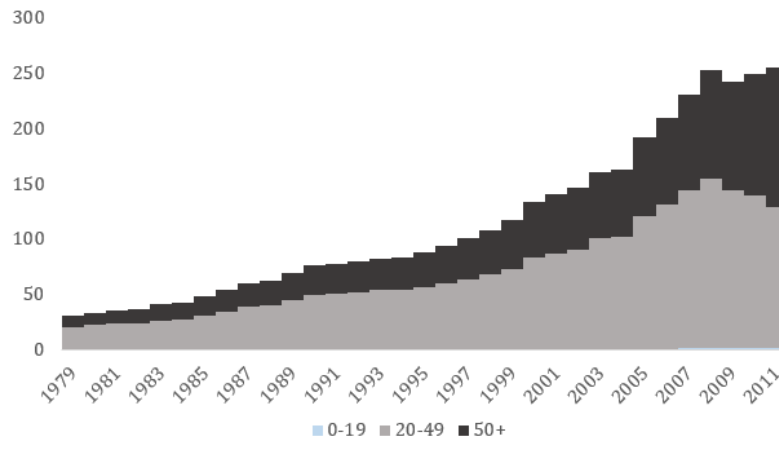


Source: NTA (France)

Inheritance, on the other hand, has been received over the years by both the “working-age” and those that are older, with the share of those that are older increasing over time, as shown in Figure (3.4). The NTA profiles also show that the highest value of received inheritance has not only increased over time, but has also been delayed over time. This means that individuals are now receiving more money at later ages. This is in line, of course, with rising life expectancy. The direction of these private transfers does not change when considering flows in per capita terms of each age-group (shown in Appendix 3).⁵

⁵Unfortunately, the data does not include the age-profile of those transmitting inheritance flows.

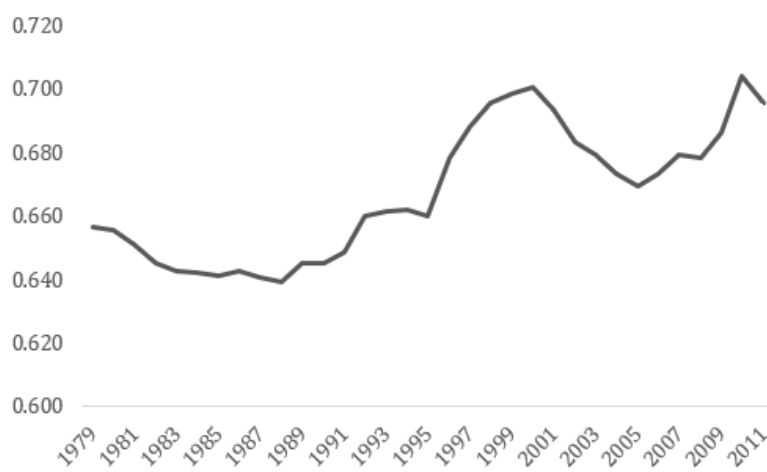
Figure 3.4: Aggregate received inheritance by broad age-group (billion real euros)



Source: NTA (France)

The final stylized fact that we make use of relates to the evolution of wealth inequality in France over time. Data from the World Inequality Database (WID) show that the Gini coefficient of wealth inequality has increased during the period that we consider in this paper, from 0.66 in 1979 to 0.7, with some variation in between, as shown in Figure (3.5). The share of wealth by the top decile and the top percentile of the population has increased. This is a fact that we exploit in the theoretical model in the following section, where we link private intergenerational transfers to wealth inequality.

Figure 3.5: Gini coefficient of wealth distribution in France



Source: World Inequality Database (WID)

3. The model set-up

We consider an economy of overlapping generations in discrete time, akin to Diamond (1965). Economic growth is endogenous and driven by human capital accumulation. Generations are linked to each other through altruism for various transfers, and they make up “dynasties.” Production, on the other hand, is made through a representative firm, which operates in perfect competition and produces with constant returns to scale. Below, we set-up the model and show its main conclusions.

The individual

Each individual in this model lives for three periods: childhood, adulthood and old-age. For ease of notation, we assume that children are born at time $(t - 1)$. However, decisions are taken by adults for their children and their old-age at time (t) .

There exists in each period two types of dynasties in the economy indexed by $i = \{1, 2\}$, which differ in their desire for bequest. Dynasty $i = 1$, which makes up a fixed proportion p of the population, where $p \in (0, 1)$, is egoistic and, therefore, does not make any bequest, whereas dynasty $i = 2$, with a proportion $1 - p$ of the population, has a desire to bequeath. This heterogeneity in the desire to bequeath is also found in the data, pioneered by Laitner and Juster (1996), who found this heterogeneity when examining a sample of pension holders in the U.S. Kopczuk and Lupton (2007), using a similar method but a different dataset, also found this heterogeneity, with about three-quarters of the elderly population having a bequest motive that caused them to reduce their consumption and transfer inheritance to the following generation.⁶

We assume, for further simplicity, no population growth, and thus population size is normalized to 1 over time.

Besides decisions pertaining to consumption and savings, individuals also make decisions regarding two kinds of intergenerational transfers. The first are transfers in the form of inheritance, denoted (b) , which are made at old age to the middle age-group (adulthood). The second are intrahousehold transfers made by adults for children, denoted (m) . These transfers are used solely for the development of children’s human capital, and they should be positively correlated with the child’s future labor income.

We assume, however, that the human capital of the individual does not only depend on these parental intrahousehold transfers, which are specific to dynasty i , but also on the average stock of human capital in society. This assumption is in line with the

⁶Based upon their suggestion, in the following simulation part, we will choose $p = 0.3$ which represents the proportion of the egoistic family in the society.

“external effect” argued originally by Lucas Jr (1988) and later integrated into models as in Tamura (1991) and Bovenberg and van Ewijk (1997). These models suggest that the average stock of human capital in society does not only affect the individual’s own human capital, but also by extension the productivity of all factors of production.⁷ It is also in line with the idea that individuals do not start with a clean slate, or zero human capital, when they are born, but that they are endowed from the beginning with a certain level of human capital that is best approximated by the average human capital in the economy (see, for example, Glomm and Ravikumar (1997) and de la Croix and Michel (2007)).

Therefore, the human capital, H , of an adult at time t of dynasty i can be expressed as a function of the previous period’s average human capital and the intrahousehold transfers received at childhood, as the following:

$$H_{i,t} = m_{i,t-1}^\lambda H_{t-1}^{1-\lambda}, \quad \lambda \in (0, 1) \quad (3.1)$$

where λ is the elasticity of human capital accumulation with respect to intrahousehold transfers - a crucial parameter in our model.

For every efficient unit of labor, an adult receives a wage w_t that entails a gross labor income of $w_t H_{i,t}$, as well as bequest from the older generation, $\beta_{i,t}$. From this inflow of income, he/she decides to consume $c_{i,t}$, to save $s_{i,t}$ and to invest in children $m_{i,t}$. At old age, the individual allocates from his/her capitalized savings, $R_{t+1} s_{i,t}$, how much to consume, $(d_{i,t+1})$, and how much to bequeath to the adult generation, $b_{i,t+1}$.

We denote V_t^i the utility of an adult of dynasty i and assume that it is a logarithmic function. The individual, therefore, maximizes the utility function, V_t^i , as the following:

$$\max_{c_{i,t}, m_{i,t}, d_{i,t+1}, b_{i,t+1}} \ln(c_{i,t}) + \theta \ln(m_{i,t}) + \beta \ln(d_{i,t+1}) + \beta \gamma_i \ln(b_{i,t+1}) \quad (3.2)$$

where θ captures the preference to give intrahousehold transfers, β captures time preference, and γ_i captures the intergenerational degree of altruism of dynasty i . We assume that $\gamma_1 = 0$ for the egoistic dynasty, and $\gamma_2 = \gamma \in (0, 1)$ for the altruistic dynasty. Note that this formulation expresses the inheritance motive as a “joy of giving,” as proposed by Yaari (1964). This is a common expression for altruism in the literature due to its tractability (Abel and Warshawsky (1987)).

⁷Note also that the average stock of human capital in society is found empirically to be significant and positive for productivities and incomes across countries (Benhabib and Spiegel (2005)).

The budget constraints for a type i individual are as follows:

$$c_{i,t} + s_{i,t} + m_{i,t} \leq w_t H_{i,t} + b_{i,t} \quad (3.3)$$

$$d_{i,t+1} + b_{i,t+1} \leq s_{i,t} R_{t+1} \quad (3.4)$$

where R_t and w_t are the rate of return on capital and wage per effective labor unit, respectively. We assume that at time 0, $s_{i,0}$ and $H_{i,0}$ are given.

From the first-order conditions we can derive optimal consumption, savings, intra-household transfers and inheritance for type i at time t as follows:

$$c_{i,t} = \frac{1}{1 + \theta + \beta(1 + \gamma_i)} (w_t H_{i,t} + b_{i,t}) \quad (3.5)$$

$$m_{i,t} = \frac{\theta}{1 + \theta + \beta(1 + \gamma_i)} (w_t H_{i,t} + b_{i,t}) \quad (3.6)$$

$$d_{i,t+1} = \frac{\beta R_{t+1}}{1 + \theta + \beta(1 + \gamma_i)} (w_t H_{i,t} + b_{i,t}) \quad (3.7)$$

$$b_{i,t+1} = \frac{\beta \gamma_i R_{t+1}}{1 + \theta + \beta(1 + \gamma_i)} (w_t H_{i,t} + b_{i,t}) \quad (3.8)$$

Given differences in the preference to bequeath, optimal inheritance for each dynasty can be more clearly expressed as the following:

$$b_{1,t+1} = 0 \quad (3.9a)$$

$$b_{2,t+1} = \frac{\beta \gamma R_{t+1}}{1 + \theta + \beta(1 + \gamma)} (w_t H_{2,t} + b_{2,t}) \quad (3.9b)$$

Note that given the logarithmic utility function, optimal inheritance and intra-household transfer will always be an interior solutions. This means that the set-up of the model does not allow for negative bequest and transfer.

The savings function for each dynasty can be derived as follows:

$$s_{1,t} = \frac{\beta}{1 + \beta + \theta} w_t H_{1,t} \quad (3.10)$$

$$s_{2,t} = \frac{\beta(1 + \gamma)}{1 + \theta + \beta(1 + \gamma)} (w_t H_{2,t} + b_{2,t}) \quad (3.11)$$

Due to our logarithm utility function, it is no surprise that the optimal level of our control variables is always proportional to the agent's wealth, $(w_t H_{i,t} + b_{i,t})$. Additionally, note that that:

$$\frac{\beta(1 + \gamma)}{1 + \theta + \beta(1 + \gamma)} > \frac{\beta}{1 + \beta + \theta}$$

which implies that the rate of savings of the altruistic household is always higher than that of the egoistic one.

Firms

Production in the economy, denoted by $F(K_t, H_t)$, occurs through a representative firm that operates according to constant returns to scale and uses two inputs: the aggregate stock of capital in the economy, K_t , and the aggregate stock of human capital, H_t , expressed as the following:

$$Y_t = K_t^\alpha H_t^{1-\alpha}$$

where α is the output elasticity of capital in the production function. To simplify the analysis, we assume that capital is fully depreciated after each period t .

We define new variables $\tilde{Y}_t \equiv \frac{Y_t}{H_t}$ and $\tilde{K}_t \equiv \frac{K_t}{H_t}$ the output and capital per effective worker, respectively. Then in intensive form, the production function can be written as $\tilde{Y}_t = \tilde{K}_t^\alpha$. In equilibrium, factors are paid their marginal products:

$$R_t = \alpha \tilde{K}_t^{\alpha-1} \quad (3.12)$$

$$w_t = (1 - \alpha) \tilde{K}_t^\alpha \quad (3.13)$$

4. Equilibrium in the economy

Given an initial capital for each dynasty $K_{i,0}$ and an initial human capital for each dynasty $H_{i,0}$, a competitive equilibrium for this economy implies a sequence of prices $\{R_t, w_t\}_{t=0}^\infty$ and quantities for dynasty- i variables $\{c_{i,t}, d_{i,t}, s_{i,t}, m_{i,t}, b_{i,t}, H_{i,t}\}_{t=0}^\infty$, together with aggregate variables $\{Y_t, H_t, K_t\}$, such that:

[(i)] Households behave optimally, given by Equations (3.5)-(3.11). Firms maximize their profit, given by Equations (3.12) and (3.13). All markets clear. The capital market clearing condition requires that the aggregate savings owned by members of both dynasties at time t are equal to the physical capital stock available at time $t + 1$:

$$K_{1,t} = s_{1,t-1} \quad (3.14a)$$

$$K_{2,t} = s_{2,t-1} \quad (3.14b)$$

$$K_t = pK_{1,t} + (1 - p)K_{2,t} = ps_{1,t-1} + (1 - p)s_{2,t-1} \quad (3.14c)$$

The human capital market clearing condition requires that the aggregate human capital owned by both dynasties equals the human capital stock of the economy at time t :

$$H_t = pH_{1,t} + (1-p)H_{2,t} = pm_{1,t-1}^\lambda H_{t-1}^{1-\lambda} + (1-p)m_{2,t-1}^\lambda H_{t-1}^{1-\lambda} \quad (3.15)$$

Finally, the goods market clearing condition, which results from the individual budget constraints, requires that what is produced is consumed, shared or saved in the economy, such that:

$$p(c_{1,t} + d_{1,t} + m_{1,t} + s_{1,t}) + (1-p)(c_{2,t} + d_{2,t} + m_{2,t} + s_{2,t}) = Y_t \quad (3.16)$$

From Equations (3.12) - (3.15), we can obtain the dynamic system which governs the equilibrium paths in the neighborhood of the steady state (K_1, K_2, H_1, H_2) . We can easily see that this is a four-dimensional dynamic system with four pre-determined variables:

$$\frac{1 + \theta + \beta}{\beta} K_{1,t+1} = w_t H_{1,t} \quad (3.17)$$

$$\frac{1 + \theta + \beta(1 + \gamma)}{\beta(1 + \gamma)} K_{2,t+1} - \frac{\gamma}{\gamma + 1} R_t K_{2,t} = w_t H_{2,t} \quad (3.18)$$

$$H_{1,t+1} = H_t^{1-\lambda} \left(\frac{\theta}{\beta} K_{1,t+1} \right)^\lambda \quad (3.19)$$

$$H_{2,t+1} = H_t^{1-\lambda} \left(\frac{\theta}{\beta(1 + \gamma)} K_{2,t+1} \right)^\lambda \quad (3.20)$$

where:

$$H_t = pH_{1,t} + (1-p)H_{2,t} \quad (3.21)$$

$$R_t = \alpha \left(\frac{pK_{1,t} + (1-p)K_{2,t}}{pH_{1,t} + (1-p)H_{2,t}} \right)^{\alpha-1} \quad (3.22)$$

$$w_t = (1 - \alpha) \left(\frac{pK_{1,t} + (1-p)K_{2,t}}{pH_{1,t} + (1-p)H_{2,t}} \right)^\alpha \quad (3.23)$$

5. Transitional dynamics and steady state

In order to examine the evolution of the economy in the short-term as well as its steady state in the long-term, we will transform in what follows all variables to the intensive form, i.e. per efficient unit of labor H_t . We will then denote the new variables with the symbol ($\tilde{\cdot}$) to mark this transformation.

We define a new variable $x_t \equiv \frac{\tilde{K}_{2,t}}{\tilde{K}_{1,t}}$ that captures the ratio of the capital intensity held by altruistic and egoistic dynasty, respectively. Note that $\tilde{K}_{2,t} \equiv \frac{K_{2,t}}{H_t}$ and $\tilde{K}_{1,t} \equiv \frac{K_{1,t}}{H_t}$. Since capital K is the only form of wealth in our model, we will be able to exploit

x_t in later sections to measure wealth inequality in the economy.

By dividing all variables of Equations (3.17)-(3.20) by the aggregate level of human capital, we can reduce the dynamic system of four dimensions to the dynamics of only x_t .⁸

In this setting, we are interested in two particular parameters, λ and γ , which capture the share of intrahousehold transfer in the production of human capital accumulation and the preference for inheritance, respectively. We, therefore, characterize the transitional dynamics of wealth inequality x_t as:

$$x_{t+1} = G(x_t; \lambda, \gamma) \quad (3.24)$$

where

$$G(x_t; \lambda, \gamma) \equiv \frac{(1 + \beta + \theta)(1 + \gamma)}{1 + \theta + \beta(1 + \gamma)} \left[\left(\frac{x_t}{1 + \gamma} \right)^\lambda + \frac{\gamma\alpha}{(1 + \gamma)(1 - \alpha)} x_t \frac{p + (1 - p)(x_t/(1 + \gamma))^\lambda}{p + (1 - p)x_t} \right]$$

and $x_0 = \frac{\tilde{K}_{2,0}}{\tilde{K}_{1,0}}$ is given at time 0.

Steady state solution

Let $x_{t+1} = x_t$ in Equation(3.24). Since the equilibrium is fully characterized by the dynamics of x , the number of steady states in the economy is equal to the number of solutions of x computed from (3.24). Letting $x = G(x; \lambda, \gamma)$ gives us:

$$x = \underbrace{\frac{(1 + \beta + \theta)(1 + \gamma)}{1 + \theta + \beta(1 + \gamma)} \left[\left(\frac{x}{1 + \gamma} \right)^\lambda + \frac{\gamma\alpha}{(1 + \gamma)(1 - \alpha)} \frac{p + (1 - p)(x/(1 + \gamma))^\lambda}{p + (1 - p)x} x \right]}_{\equiv G(x; \lambda, \gamma)} \quad (3.25)$$

Simplifying Equation (3.25) gives us:

$$\frac{1 + \theta + \beta(1 + \gamma)}{(1 + \beta + \theta)(1 + \gamma)} = \underbrace{\frac{x^{\lambda-1}}{(1 + \gamma)^\lambda} + \frac{\gamma\alpha}{(1 + \gamma)(1 - \alpha)} \frac{p + (1 - p)(x/(1 + \gamma))^\lambda}{p + (1 - p)x}}_{\equiv RHS(x)} \quad (3.26)$$

To determine the number of solutions of x , in what follows, we are going to study the shape of the function $RHS(x)$ given by (3.26). Clearly,

$$\frac{\partial RHS(x)}{\partial x} = \underbrace{\frac{x^{\lambda-2}}{(1 + \gamma)^\lambda}}_{>0} \underbrace{(\lambda - 1)}_{<0} + \underbrace{\frac{(1 - p)x^{\lambda-1}(p + (1 - p)x)}{(1 + \gamma)^\lambda(p + (1 - p)x)^2}}_{>0} \underbrace{(\lambda - 1)}_{<0} < 0$$

⁸Refer to Appendix 3 for more elaborate calculations.

Hence, $RHS(x)$ is a monotonically decreasing function of x .

Moreover, notice that

$$\lim_{x \rightarrow 0^+} RHS(x) = +\infty$$

Since the LHS function in (3.26) is independent of x . We can then conclude that there exists a unique positive steady state value denoted by x^* such that

$$\frac{1 + \theta + \beta(1 + \gamma)}{(1 + \beta + \theta)(1 + \gamma)} = RHS(x^*)$$

This allows us to make the following proposition.

3. Proposition 18. *The steady state value x^* is unique.*

This implies that, in steady state, capital in intensive form saved by each dynasty is positive. Since this x^* also serves as our wealth inequality measurement, this proposition also means that inequality exists in the long-run.

We are now in the position to study the key features of x^* . Recall that if $x^* > 1$, i.e. $\frac{\tilde{K}_2}{K_1} > 1$, we obtain a long-run wealth inequality that is driven by the altruistic dynasty. Alternatively, in the case $x^* < 1$, the long-run inequality is induced by the egoistic family. Thus, to study the value of x^* compared to 1, i.e. with complete equality, we compare the value of the function $G(1; \lambda, \gamma)$ to the value of 1 when $x = 1$. Note that $G(1; \lambda, \gamma) > 1$ infers that $x^* > 1$ while $G(1; \lambda, \gamma) < 1$ means that $x^* < 1$.

From (3.25), we can compute $G(1; \lambda, \gamma)$ as the following:

$$G(1; \lambda, \gamma) = \frac{(1 + \beta + \theta)(1 + \gamma)}{1 + \theta + \beta(1 + \gamma)} \left[\left(\frac{1}{1 + \gamma} \right)^\lambda + \frac{\gamma\alpha}{(1 + \gamma)(1 - \alpha)} (p + (1 - p) \left(\frac{1}{1 + \gamma} \right)^\lambda) \right] \quad (3.27)$$

Firstly, depending on our parameter choices of γ and λ , $G(1; \lambda, \gamma)$ can be greater or smaller than 1. Thus, we are going to determine the particular values of $\bar{\lambda}$ and $\bar{\gamma}$ such that $G(1; \bar{\lambda}, \bar{\gamma}) = 1$.

Secondly, notice that the function $G(1; \lambda, \gamma)$ is a decreasing function of λ . Solving $G(1; \bar{\lambda}, \bar{\gamma}) = 1$ gives us at most one solution of $\bar{\lambda}$. The solution $\bar{\lambda}$ lies in between the interval $(0, 1)$ if the following conditions are satisfied:

$$\lim_{\lambda \rightarrow 0} G(1; \lambda, \gamma) > 1$$

$$\lim_{\lambda \rightarrow 1} G(1; \lambda, \gamma) < 1$$

We can see that the first condition always holds since:

$$\lim_{\lambda \rightarrow 0} G(1; \lambda, \gamma) = \underbrace{\frac{(1 + \beta + \theta)(1 + \gamma)}{1 + \theta + \beta(1 + \gamma)}}_{>1} \underbrace{\left(1 + \frac{\gamma\alpha p}{(1 + \gamma)(1 - \alpha)} \right)}_{>1} > 1$$

The second condition holds if:

$$\lim_{\lambda \rightarrow 1} G(1; \lambda, \gamma) = \frac{1 + \beta + \theta}{1 + \theta + \beta(1 + \gamma)} \left(1 + \frac{\gamma\alpha}{1 - \alpha} \left(p + \frac{1 - p}{1 + \gamma} \right) \right) < 1$$

Simplifying it gives us the condition on γ compared to the critical value⁹ defined by $\bar{\gamma}$ as follows:

$$\gamma > \bar{\gamma} \equiv \frac{1 - p}{\frac{1 - \alpha}{\alpha} \frac{\beta}{1 + \beta + \theta} - p} - 1$$

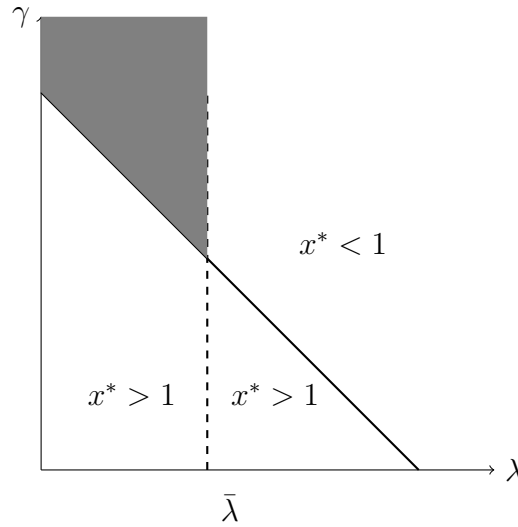
Therefore, we obtain the following scenarios:

- (1) If $\gamma < \bar{\gamma}$, then $\lambda < \bar{\lambda}$ for all $\lambda \in (0, 1)$ and $G(1; \lambda, \gamma) > 1$.
- (2) if $\gamma \geq \bar{\gamma}$, then we can determine the value of $G(1; \lambda, \gamma)$ conditional on λ :

$$G(1; \lambda, \gamma) = \begin{cases} \geq 1, & \text{if } \lambda \leq \bar{\lambda} \\ < 1, & \text{if } \lambda > \bar{\lambda} \end{cases}$$

Figure (3.6) summarizes graphically these conditions. Notice that $\bar{\lambda}$ is a decreasing function¹⁰ of γ .

Figure 3.6: Inequality conditional on choices of γ and λ



We then introduce the following proposition.

Proposition 19. *There exists a threshold of bequest preference, $\bar{\gamma}$, and of the share of intra-household transfers in human capital accumulation, $\bar{\lambda}$, where $G(1; \bar{\lambda}, \bar{\gamma}) = 1$. Under the condition that $\gamma > \bar{\gamma}$ and $\lambda > \bar{\lambda}$, long-run inequality exists such that the*

⁹Note that $\bar{\gamma}$ is independent of λ . The condition to have $\bar{\gamma} \in (0, 1)$ is $\frac{1+p}{2} < \frac{1-\alpha}{\alpha} \frac{\beta}{1+\beta+\theta} < 1$. Using conventional parameter choices such as $\alpha = 0.3$, $\beta = 0.6$, $\theta = 0.3$ and $p = 0.3$, we obtain $\bar{\gamma} = 0.6$, which is clearly between $(0, 1)$.

¹⁰Proof is given in the Appendix B.4

egoistic family accumulates more wealth than the altruistic family:

$$x^* < 1 \text{ and } \frac{\tilde{K}_2}{\tilde{K}_1} < 1$$

Otherwise, inequality in steady-state exists where the altruistic dynasty possesses more wealth than the egoistic dynasty:

$$x^* > 1 \text{ and } \frac{\tilde{K}_2}{\tilde{K}_1} > 1$$

This suggests that only if the degree of altruism (γ) is not too large, the altruistic dynasty would be able to accumulate higher long-run wealth than the egoistic dynasty, meaning $x^* > 1$. This is similar to the findings of Michel and Pestieau (2005), who argued that long-run wealth is only held by the most altruistic families.

Let us now discuss some intuitions of the findings. Understanding Proposition 19 necessitates understanding the effect of γ . From Equations (10), (11), (14a) and (14b) we can see that the steady state accumulated wealth of dynasty i , \tilde{K}_i , depends particularly on bequest preference γ , on inheritance given at time t and on their human capital accumulation as follows:

$$\begin{aligned} \tilde{K}_i &= \frac{\beta(1+\gamma)}{1+\theta+\beta(1+\gamma)}(w\tilde{H}_i + \tilde{b}_i) \\ &= \frac{\beta(1+\gamma)}{1+\theta+\beta(1+\gamma)}(w\tilde{m}_i^\lambda + \tilde{b}_i) \end{aligned}$$

To explain how these factors influence long-run capital for each dynasty, we can simplify this function to the following:

$$\tilde{K}_i = f(\gamma, \tilde{b}_i, \tilde{m}_i)$$

The impacts from a higher inheritance preference on \tilde{K}_i can be untangled as follows:

$$\frac{\partial \tilde{K}_i}{\partial \gamma} = \underbrace{\frac{\partial f}{\partial \gamma}}_{>0} + \underbrace{\frac{\partial f}{\partial \tilde{b}_i} \frac{\partial \tilde{b}_i}{\partial \gamma}}_{>0} + \underbrace{\frac{\partial f}{\partial \tilde{m}_i} \frac{\partial \tilde{m}_i}{\partial \gamma}}_{\substack{>0 \\ ?}}$$

Inheritance Effect *Income Effect*

One can see that an increase in the inheritance preference γ triggered two different effects which are called the inheritance effect and the income effect. Obviously, the inheritance effect in response to an increase in the degree of altruism has to be positive as people tend to leave more bequest for their offspring. We write: $\frac{\partial f}{\partial \tilde{b}_i} \frac{\partial \tilde{b}_i}{\partial \gamma} > 0$.

On the other hand, the income effect generated from changes in intrahousehold

transfer is still ambiguous. Recall that individuals divide their savings between capital and intra-household transfer. When the motive to bequeath is rather small, inheritance responds little to a change in γ . Therefore, one does not have to increase their savings and, hence, capital formation at the expense of intrahousehold transfer. In this case, we have: $\frac{\partial H_{i,t}}{\partial \gamma} > 0$.

However, when the intensity of altruism is sufficiently strong, then whether it leads to a strong reduction in income and, as a result, a fall in capital stock depends on how much intrahousehold transfer contributes to human capital production, captured by the parameter λ . If the role of these transfers is significant, i.e: λ is great enough, then a reduction in intrahousehold transfer translates in to a strong drop in human capital accumulation. In this case, the negative income effect will dominate the positive inheritance effect. Therefore, $\frac{\partial K_{i,t+1}}{\partial \gamma} < 0$ which means that the egoistic dynasty ends up accumulating more wealth in the long-run than altruistic one. If the role of these transfers is not significant, i.e. λ is small enough, the positive inheritance effect will dominate the negative income effect which leads to $\frac{\partial K_{i,t+1}}{\partial \gamma} > 0$. In this scenario, the altruistic dynasty will accumulate more capital in the long-run.

The transitional dynamics of the economy

In this part, we examine the stability of the steady state x^* that we found in the previous part. Noting the transitional dynamics of x_t , given by Equation (3.24), we can express the derivative of the function valued at the steady state value x^* and study its sign. We have:

$$G'(x^*) = \frac{(1 + \beta + \theta)(1 + \gamma)}{1 + \theta + \beta(1 + \gamma)} \left[\frac{\lambda(x^*)^{\lambda-1}}{(1 + \gamma)^\lambda} + \frac{\gamma\alpha}{(1 + \gamma)(1 - \alpha)} \frac{p^2 + \frac{p(1-p)(1+\lambda)}{(1+\gamma)^\lambda}(x^*)^\lambda + \frac{(1-p)^2\lambda}{(1+\gamma)^\lambda}(x^*)^{\lambda+1}}{(p + (1 - p)x^*)^2} \right] \quad (3.28)$$

One can easily see that:

$$G'(x^*) > 0$$

Moreover, one can also prove that:¹¹

$$G'(x^*) < 1$$

Therefore, one can conclude that x^* is an asymptotically stable equilibrium point.

On the other hand, from the difference equation of x_t given by Equation (3.24), we obtain:

$$\lim_{x_t \rightarrow 0^+} G(x_t) = 0 \quad \text{and} \quad \lim_{x_t \rightarrow 0^+} G'(x_t) = +\infty$$

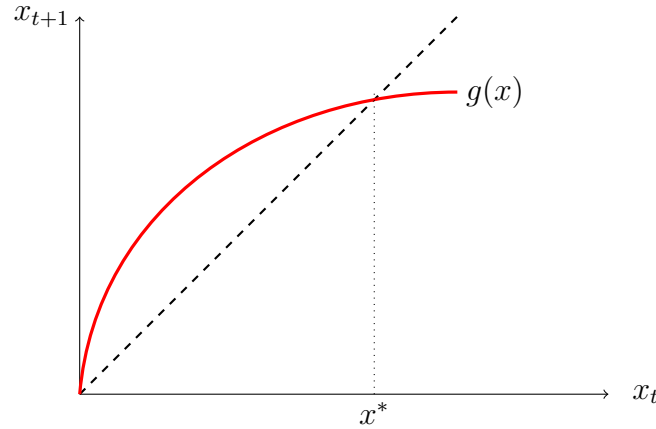
¹¹Refer to Appendix 3 for proof.

$$\lim_{x_t \rightarrow +\infty} G(x_t) = +\infty \quad \text{and} \quad \lim_{x_t \rightarrow +\infty} G'(x_t) = 0$$

These results allow us to draw the global dynamics of x_t , shown in Figure (3.7).

Proposition 20. *The steady state x^* is locally stable.*

Figure 3.7: Dynamics of wealth inequality, x_t



The balanced growth path

Along the balanced growth path, all variables in the economy grow at a constant rate. We first introduce the following proposition.

Proposition 21. *In the long-run, the economy follows a balanced growth path in which $\{c_{i,t}, d_{i,t}, m_{i,t}, b_{i,t}, H_{i,t}, K_{i,t}, K_t\}_{t=0}^{\infty}$ grow at the same rate as that of the aggregate human capital H_t , and $\{w_t, R_t\}$ remain constant.*

Proof of Proposition 21. In the long-run, all the endogenous variables, which are measured in efficient unit of labor, can be rewritten as a function of x^* . Indeed, from Equations (3.19), (3.20), and (3.21), we can derive:¹²

$$\tilde{H}_1 = \frac{1}{p + (1-p)\left(\frac{x^*}{1+\gamma}\right)^\lambda}$$

$$\tilde{H}_2 = \frac{\left(\frac{x^*}{1+\gamma}\right)^\lambda}{p + (1-p)\left(\frac{x^*}{1+\gamma}\right)^\lambda}$$

which are constant. Recall that, along the balanced growth path, $\tilde{H}_1 = \frac{H_{1,t+1}}{H_{t+1}}$ and $\tilde{H}_2 = \frac{H_{2,t+1}}{H_{t+1}}$. It means that $H_{1,t+1}$ and $H_{2,t+1}$ both grow at the same rate as the aggregate human capital H_{t+1} along the path.

¹²Detailed proof is given in B.4 in Appendix.

Moreover, from Equations (3.17) and (3.18), we can also derive \tilde{K}_1 , \tilde{K}_2 and \tilde{K} as a function of x^* as follows:

$$\tilde{K}_1 = \left(\frac{(1-\alpha)\beta^{1/(1-\lambda)}}{(1+\theta+\beta)\theta^{\lambda/(1-\lambda)}} \frac{(p+(1-p)x^*)^\alpha}{(p+(1-p)(\frac{x^*}{1+\gamma})^\lambda)^{1/(1-\lambda)}} \right)$$

$$\tilde{K}_2 = x^* \tilde{K}_1$$

$$\tilde{K} = p\tilde{K}_1 + (1-p)\tilde{K}_2 = (p+(1-p)x^*)\tilde{K}_1$$

which states that aggregate capitals $K_{1,t}$, $K_{2,t}$ and K_t move at the same rate as H_t .

Similarly, from Equations (3.5)-(3.8), we can obtain the steady state values of \tilde{c}_i , \tilde{d}_i , \tilde{m}_i , \tilde{b}_i . Obviously, these variables $\{c_{i,t}, d_{i,t}, b_{i,t}\}$ also grow at the same rate as H_t . \square

6. The intrahousehold transfers-inheritance ratio and wealth inequality

Since we are interested in the relative importance of intrahousehold transfers in comparison with inheritance over time, we construct their ratio denoted by $\frac{M_t}{B_t}$. In this section, we examine how the ratio evolves in the short- and long-run.

The evolution of $\frac{M_t}{B_t}$ in the long-run

Given Proposition (21), all the variables in the economy grow in the long-run at the same rate as that of the aggregate human capital, H_t . In other words, along the balanced growth path, the long-run inheritance and intrahousehold transfers, per efficient unit of labor, \tilde{b}_i and \tilde{m}_i respectively, are constant.

Since inheritance is only transmitted by the altruistic dynasty, we can express aggregate inheritance at time t as the following:

$$B_t = (1-p)b_{2,t} = (1-p)\tilde{b}_2 H_t \quad (3.29)$$

Similarly, aggregate intrahousehold transfers at time t can be expressed as the following:

$$M_t = pm_{1,t} + (1-p)m_{2,t} = p\tilde{m}_1 H_t + (1-p)\tilde{m}_2 H_t \quad (3.30)$$

Hence, the intrahousehold transfers to inheritance ratio in the long-run can be expressed as:

$$\frac{M_t}{B_t} = \frac{p\tilde{m}_1 H_t + (1-p)\tilde{m}_2 H_t}{(1-p)\tilde{b}_2 H_t} = \frac{p\tilde{m}_1 + (1-p)\tilde{m}_2}{(1-p)\tilde{b}_2} \quad (3.31)$$

which is a constant. This result suggests that in the long-run intrahousehold transfer

and inheritance should grow at the same rate determined by the rate of human capital change and should the ratio should be constant.

The evolution $\frac{M_t}{B_t}$ in the short-run

In this section, we examine the transitional dynamics of the ratio of our interest in the short-run. We can construct the ratio at time t in the short-run as the following:

$$\frac{M_t}{B_t} = \frac{pm_{1,t} + (1-p)m_{2,t}}{(1-p)b_{2,t}} = \frac{p\tilde{m}_{1,t} + (1-p)\tilde{m}_{2,t}}{(1-p)\tilde{b}_{2,t}} \quad (3.32)$$

From Equations (3.6), (3.8), (3.14), (3.17) and (3.18), we can re-express optimal intrahousehold transfers and inheritance as the following:

$$m_{i,t} = \frac{\theta}{\beta(1+\gamma_i)} K_{i,t+1}$$

$$b_{i,t} = \frac{\gamma_i}{1+\gamma_i} K_{i,t} R_t$$

We can then rewrite $\frac{M_t}{B_t}$ as a function of x_t , our wealth inequality measure, as follows:

$$\begin{aligned} \frac{M_t}{B_t} = & \frac{\theta}{1+\theta+\beta(1+\gamma)} + \frac{\theta p(1+\gamma)(1-\alpha)}{\alpha\gamma(1+\theta+\beta)(1-p)} \frac{\frac{p}{x_t} + 1 - p}{p + (1-p)\left(\frac{x_t}{1+\gamma}\right)^\lambda} \\ & + \frac{\theta(1+\gamma)(1-\alpha)}{\alpha\gamma(1+\theta+\beta(1+\gamma))} \frac{\frac{p}{x_t} + 1 - p}{p + (1-p)\left(\frac{x_t}{1+\gamma}\right)^\lambda} \left(\frac{x_t}{1+\gamma}\right)^\lambda \end{aligned} \quad (3.33)$$

Recall that this ratio is a forward-looking variable, whereas x_t , which is a function of capital, is a backward-looking one. Equation (3.33) allows to write the proposition below.

Proposition 22. *A higher wealth inequality at time t contributes to a lower intrahousehold transfers-to-inheritance ratio.*

Proof of Proposition 22. Recall that x_t is a predetermined variable at time t . To prove that $\frac{M_t}{B_t}$ is a decreasing function of x_t , we first simplify Equation (3.33) as follows:

$$\begin{aligned} \frac{M_t}{B_t} = & \frac{\theta}{1+\theta+\beta(1+\gamma)} \\ & + \frac{\theta(1-\alpha)(1+\gamma)}{\alpha\gamma(1+\theta+\beta(1+\gamma))} \frac{\frac{p}{x_t} + 1 - p}{p + (1-p)\left(\frac{x_t}{1+\gamma}\right)^\lambda} \left(\frac{p(1+\theta+\beta(1+\gamma))}{(1-p)(1+\theta+\beta)} + \left(\frac{x_t}{1+\gamma}\right)^\lambda \right) \end{aligned}$$

One can then derive $\frac{M_t}{B_t}$ with respect to x_t and obtain:

$$\begin{aligned} \frac{\partial M_t/B_t}{\partial x_t} = & \frac{1}{\underbrace{p + (1-p)(x_t/(1+\gamma))^\lambda}_{>0}} \left[- \underbrace{\left(Z + \left(\frac{x_t}{1+\gamma} \right)^\lambda \right) \frac{p}{x^2}}_{>0} \right. \\ & \left. - \lambda \underbrace{\left(\frac{p}{x_t} + 1 - p \right) \frac{x_t^{\lambda-1}}{(1+\gamma)^\lambda}}_{>0} \underbrace{\left(\left(Z + \left(\frac{x_t}{1+\gamma} \right)^\lambda \right) \frac{1-p}{p + (1-p)\left(\frac{x_t}{1+\gamma}\right)^\lambda} - 1 \right)}_{>0} \right] < 0 \end{aligned} \quad (3.34)$$

where $Z \equiv \frac{p(1+\theta+\beta(1+\gamma))}{(1-p)(1+\theta+\beta)}$.

Since the first three underbraced terms are positive, we provide the proof to show that the last term is also positive. Indeed, we want to show that:

$$\left(Z + \left(\frac{x_t}{1+\gamma} \right)^\lambda \right) \frac{1-p}{p + (1-p)\left(\frac{x_t}{1+\gamma}\right)^\lambda} > 1$$

which is true because it is equivalent to have:

$$\frac{p(1+\theta+\beta(1+\gamma))}{(1-p)(1+\theta+\beta)} + \left(\frac{x_t}{1+\gamma} \right)^\lambda > \frac{p}{1-p} + \left(\frac{x_t}{1+\gamma} \right)^\lambda$$

Then, it leads to have:

$$1 + \theta + \beta(1 + \gamma) > 1 + \theta + \beta$$

which is always true for all choices of parameters. Therefore, we conclude that $\frac{\partial M_t/B_t}{\partial x_t} < 0$. \square

The finding suggests that the ratio of our interest $\frac{M_t}{B_t}$ moves in the opposite direction with the variable x_t . As the ratio is observed decreasing over time, in fact since 1980s in France, our model implies that this can be explained can be explained by an increase in inequality x_t . Since we use x_t to capture wealth inequality in our model, the result would then imply that France were facing a rise in wealth gap during such period of time. This finding is also consistent with some of the findings in Piketty (2011) and Piketty and Zucman (2015), where wealth is seen to rise again since the late 1970s.

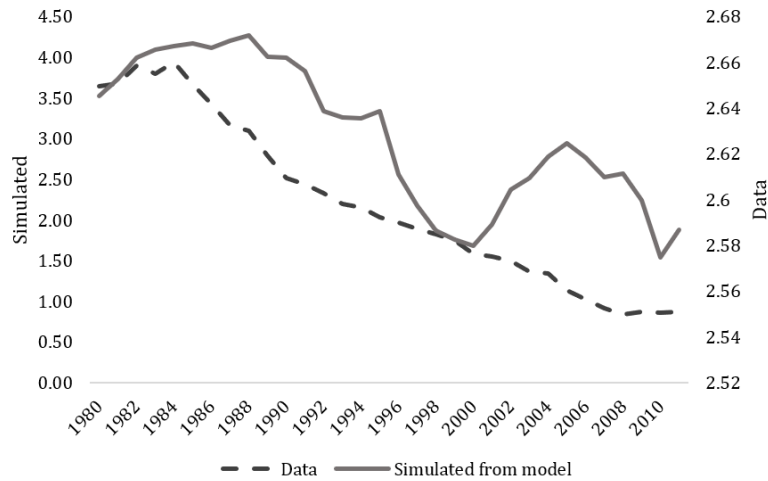
In what follows, we run a simple exercise to test how well the model matches the data by simulating the ratio of intrahousehold transfers-to-inheritance as computed through the equation (3.33). The parameter choices are taken from various papers in the literature, as shown in Table (3.1). Moreover, since our variable of interest, x_t , captures wealth inequality in the society, we take the Gini index of wealth as a close proxy, obtained from the World Inequality Database (WID).

Table 3.1: Calibrated Parameter Values

Parameter	Value	Reference
Capital elasticity of production α	0.3	Ludwig and Vogel (2010), Cipriani (2014)
Discount factor β	0.6	de la Croix (2001), Chanda (2008)
Intrahousehold transfer preference θ	0.5	Bellettini et al. (2017b)
Bequest preference γ	0.4	Han and Mulligan (2001), Bellettini et al. (2017b)
Proportion of egoistic families p	0.3	Kopczuk and Lupton (2007)
INTRAs transfer elasticity of output λ	0.5	de la Croix (2001)

Figure (3.8) below plots the intrahousehold-to-inheritance ratio obtained from the data and from our model. We can see that the model captures relatively well the overall decreasing trend of this ratio, albeit with wider variation. Interestingly, while the simulated ratio shows a significant upward kick in the ratio during the period of 2000-2007, which was the period of the housing boom and later bust, the data does not appear to capture this. This difference may come from the fact that while simulating the ratio, we assume that only wealth inequality changes while keeping other parameters constant.

Figure 3.8: The intrahousehold transfer -to- inheritance ratio in the model and in the data



Source: Authors calculation, using the model and NTA (France)

7. Comparative statics

In this part, we investigate the response of our wealth inequality indicator x_t when our key parameters λ and γ change.

Consider an economy in steady-state. Suppose that at time t , a temporary shock occurs through an increase in either λ or γ . Figures (3.9) and (3.10) below show the response function of x_t after introducing such shocks. We distinguish two scenarios: the one that starts at a steady state wealth greater than 1, i.e. $x^* > 1$, and another other that starts with $x^* < 1$. Before commenting on the results, it is worth noting that when $x_t > 1$, an increase in x_t means an increase in the gap between the two dynasties' wealth, and when $x_t < 1$ then an increase in x_t means a decreases in inequality.

Figure 3.9: Case 1: $x^* > 1$. Response function of wealth indicator x_t to a gain in inheritance preference (left graph) and to a gain in intrahousehold transfer share in human capital (right graph).

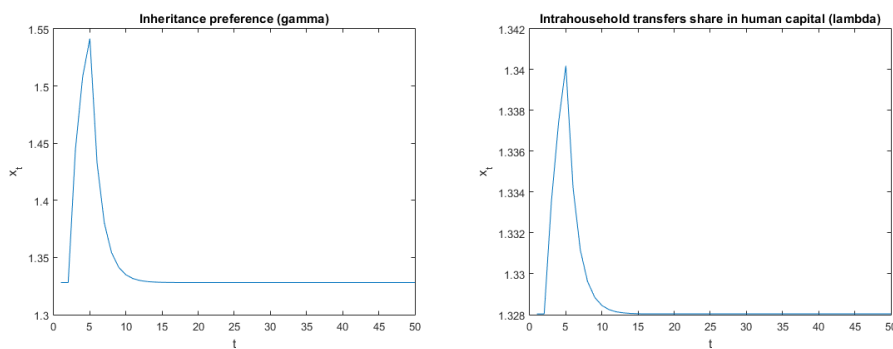
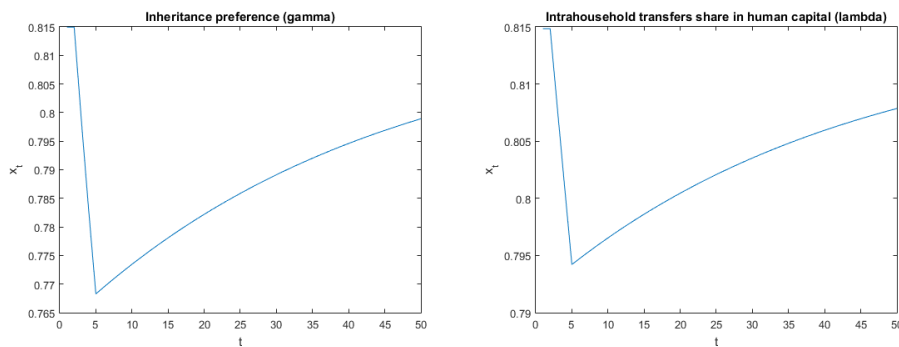


Figure 3.10: Case 2: $x^* < 1$. Response function of wealth indicator x_t to a gain in inheritance preference (left) and to a gain in intrahousehold transfer share in human capital (right).



The results show that an increase in the degree of altruism captured by γ leads to an increase in wealth inequality both when $x^* > 1$ and $x^* < 1$. A rise in γ at time t always increases the capital accumulation of the altruistic dynasty, which would widen the wealth gap between the two households if the altruistic family is already richer than the egoistic one. As a result, x_t increases in response to the shock in the case $x^* > 1$.

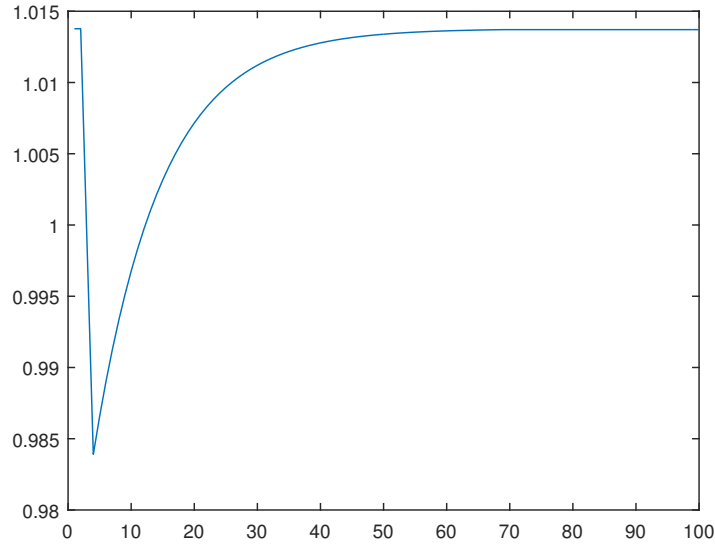
On the other hand, when $x^* < 1$ and thus γ is now above the threshold, further increases in the preference to bequeath decreases wealth accumulation of the altruistic dynasty through a larger decrease in their savings. This is the case when the preference to bequeath is already “too high.” The decrease in capital accumulation of the altruistic dynasty, and given no changes for the egoistic dynasty that does not bequeath, decreases x_t , and thus further increases inequality.

Similarly, an increase in the elasticity of intrahousehold transfers in human capital formation, λ , increases wealth inequality in both cases. Indeed, an increase in λ at time t generates a higher return from an additional unit of intrahousehold transfer. Thus, the richer household would invest more in their children by giving more intrahousehold transfer than the relatively poorer household. It would, in return, lead to a higher income at time $t+1$ for children in the rich family and, as a result, widen the inequality gap. In the case of $x^* > 1$, the altruistic household becomes richer than the egoistic one. However, in the opposite case where $x^* < 1$, the egoistic family becomes even richer than the altruistic one and inequality also widens.

Finally, we illustrate the importance of taking into account the role of λ in Figure (3.11) below. We show that an increase in λ above the threshold defined by $\bar{\lambda}$, when γ is already above its threshold $\bar{\gamma}$, can cause a temporary switch of the inequality regime, from $x > 1$, where the altruistic dynasty has more capital than the egoistic dynasty, to $x < 1$ where it has lower capital.

This is an uncommon result that we show from our model. We argue that if the shock is more persistent, there can be a drastic change in inequality over time. It highlights the important role that the efficiency parameter of intrahousehold transfers in human capital accumulation can play in changing inequality and which dynasty holds more capital.

Figure 3.11: Change in the inequality regime: λ is shifted from 0.9 to 0.99 for 1 period.



Parameter choice: $\alpha = 0.2; \beta = 0.6, \theta = 0.4, \lambda = 0.5, \gamma = 0.2, p = 0.3.$

8. Discussion

The theoretical results obtained here depend on the heterogeneity of agents beyond age, which is captured through the altruistic preference for inheritance, γ ,¹³ and the share of the population, p . Indeed, if we set $p = 0$ in Equation (3.33), then our ratio of interest becomes:

$$\frac{M_t}{B_t} = \frac{\theta}{1 + \theta + \beta(1 + \gamma)} + \frac{\theta(1 - \alpha)(1 + \gamma)}{\alpha\gamma}$$

which is independent of wealth inequality. Consequently, $\frac{\partial M_t/B_t}{\partial x_t} = 0$.

This result indicates that without the type of heterogeneity that drives savings differentials between dynasties, intrahousehold transfers and inheritance flows would grow at the same rate as that of the aggregate human capital, H_t , over time.

In fact, a representative agent model with endogenous growth, and assuming that the parameters we introduce in this model remain exogenous, would always lead to an expression of our ratio of interest that is constant over time. This is regardless of the timing of the flows. Heterogeneity plays an important role in explaining these aggregate observations in the data, given this class of growth models.

The results of our theoretical model also rely on the role of inheritance in wealth

¹³This is not contingent on one of the dynasties have zero inheritance preference. As long as $\gamma_1 < \gamma_2$, the results would hold - albeit with more complicated expressions.

accumulation - in the form of:

$$K_{t+1} = s_t + b_t$$

Indeed, a simple overlapping generations model where savings are the only form of asset accumulation would yield a ratio of intrahousehold transfers to inheritance that is also constant over time. This is so even if, for instance, savings is added to the utility function as a form of “capitalist spirit,” as framed by Carroll (1998). This highlights the importance of incorporating inheritance in wealth, or asset, accumulation. Such an assumption is not unreasonable given the fact that much of inheritance is in the form savings, transmitted to others after it accumulates a certain level of interest (think, for instance, of housing, land, or even company shares).

Still, a model that takes into account the role of inheritance in wealth accumulation but still ignores heterogeneity would not yield the results that we have here. In fact, we would end up with a long-run ratio of intrahouseholds transfers to inheritance that decreases with capital. This finding comes from the fact that as capital increases, its rate of return decreases, therefore decreasing the incentive to transmit inheritance - which itself can be expressed as a function of capital. Intrahousehold transfers would grow at a faster rate than inheritance, increasing the ratio over time.

A final remark concerns the potential role of demographic variables, had they been included in the model, such as survival probability. Indeed, the data shows that life expectancy in France has been increasing steadily since the 1950s. However, including a survival probability in the model, which would be in the form of ϕ in the utility function below:

$$\max_{c_{i,t}, m_{i,t}, d_{i,t+1}, b_{i,t+1}} \ln(c_{i,t}) + \theta \ln(m_{i,t}) + \phi \beta \ln(d_{i,t+1}) + \phi \beta \gamma_i \ln(b_{i,t+1})$$

would still yield a ratio that is constant over time. Without heterogeneity or inheritance in wealth accumulation, ϕ adds just another constant to this ratio. An increase in ϕ , should we show some comparative statics, for instance, increases the intrahousehold transfers to inheritance ratio because it decreases inheritance flows. As agents live longer, they would need to consume more out of their old-age income, and thus decrease what they bequeath.

Finally, we introduce a population growth rate to our model to examine the effects of this rate on inequality, as shown in Appendix 3. Assuming for simplicity that both dynasties grow at the same rate, an increase in the population growth rate, leads to a monotonic increase in inequality, regardless whether inequality is higher or lower than 1. Since the ratio of intrahousehold transfers to inheritance is a decreasing function of wealth inequality in the short-run, as shown in subsection 3, it means an increase in the population growth rate decreases the ratio in the short run. This is not

what one may expect. It comes from the fact that in OLG set-ups intergenerational transfers are given equally. An increase in the number of children means a lower intrahousehold transfer (and inheritance) per child. This, consequently means lower human capital per child. Since the altruistic dynasty accumulates capital also through inheritance, the egoistic dynasty is more affected, leading to a larger decrease in its capital accumulation compared to the altruistic one, thus further increasing inequality.

9. Concluding remarks

Data from the National Transfer Accounts in France show that the composition of private intergenerational transfers have changed in the period 1979-2011, from more intrahousehold transfers to more inheritance flows. This has occurred at a time when the sum of these private flows have remained at a relatively stable share of national income. Consequently, we observe a decrease in the intrahousehold transfers-to-inheritance ratio.

To explain this macro trend, we set up a three-period overlapping generations model with two dynasties that differ in their desire to bequeath; one being altruistic and bequeath to the following generation, and the other being egoistic and transferring only intrahousehold transfers to children. This difference captures not only empirical findings in the literature, but also allows us to link what we observe in the data to wealth inequality, which has long played an important role in understanding inheritance. In the model, intrahousehold transfers are made from the adult age-group to children, taking the form of an input to their human capital accumulation - which itself then serves as a multiplier for wages in the following period. Inheritance is made by the old age-group to the adult generation.

We show that in the long-run wealth inequality exists, defined as the ratio of accumulated capital of the altruistic dynasty to the egoistic one. However, which of the dynasties accumulates more capital depends on the inheritance preference and on the share of intrahousehold transfers in the production of human capital. The interplay between these two parameters is important. If both of these parameters are higher than a threshold that we define in the model, then the egoistic dynasty ends up accumulating more capital - the altruism parameter causes “too much” inheritance. In all other cases, however, the altruistic dynasty ends up accumulating more.

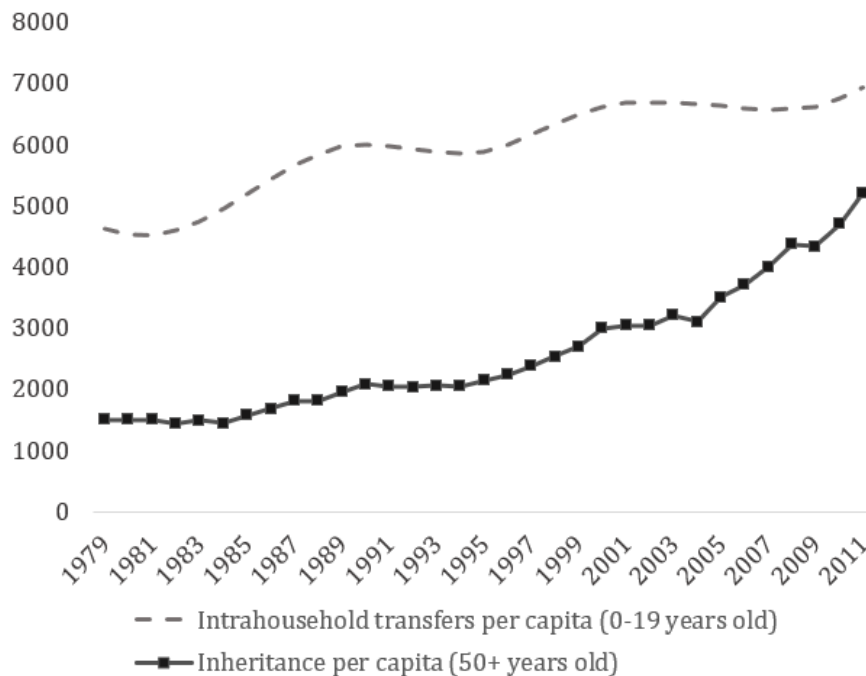
We also show that while the long-run ratio of intrahousehold transfers to inheritance is constant because all variables grow along the balanced growth path, the short-run ratio responds negatively to an increase in wealth inequality. We argue that a potential explanation for the case of France is that what we observe in the data is the process of moving from an initial wealth inequality that is lower than that of the

steady-state. However, we also show that changes in the share of intrahousehold transfers in capital accumulation can cause a change in short run inequality, from a regime where the altruistic dynasty accumulates more capital to one where it accumulates less capital. This is not a common finding in the literature.

10. Appendices

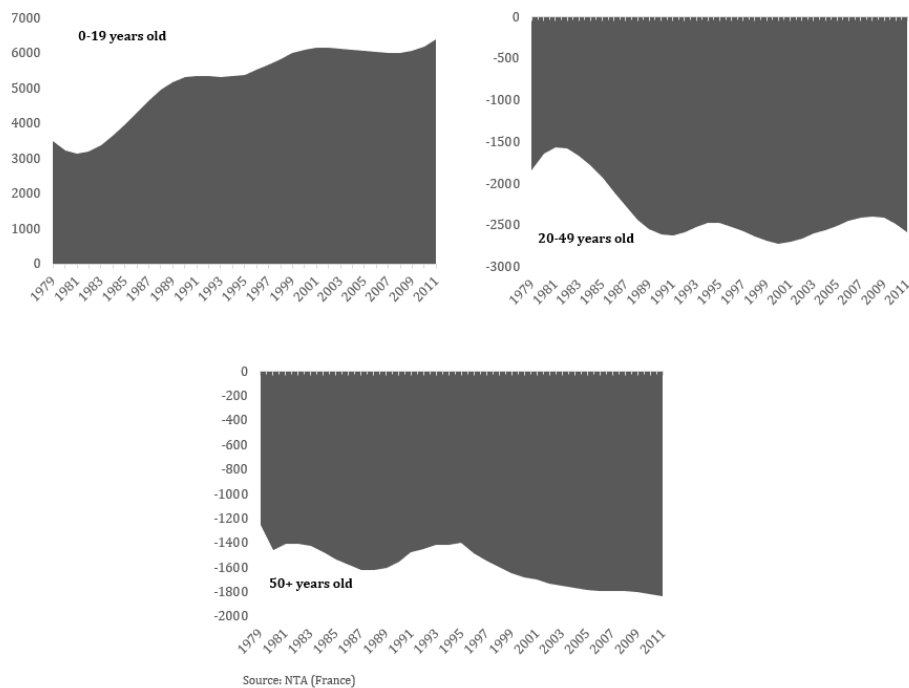
A. Further evidence from the NTA

Figure 3.12: Received intrahousehold transfers and inheritance per capita of typical recipient



Source: NTA (France)

Figure 3.13: Net per capita intrahousehold transfers by broad age-group (in real euros)



Note that these are smoothed profiles. We do not have smoothed age profiles for bequests and inter-vivo gifts.

Figure 3.14: Received inheritance in per age-group capita (in real euros)

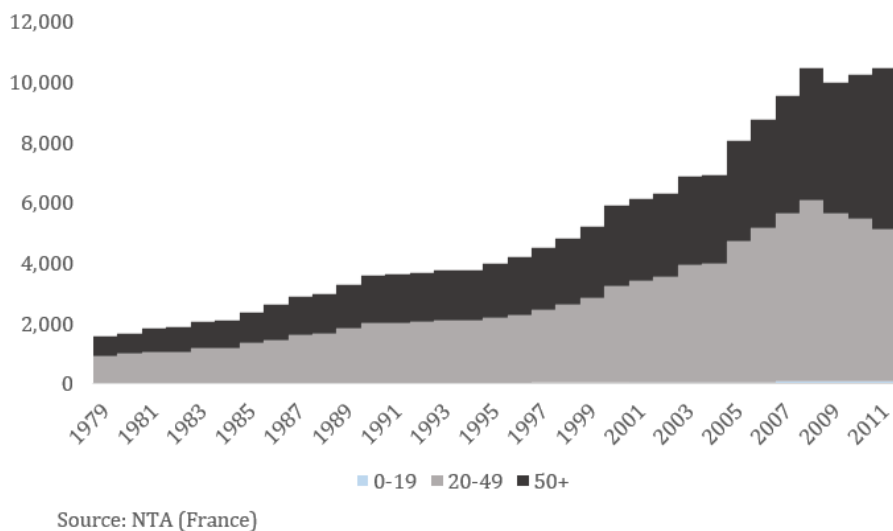


Figure 3.15: Age profile of received inheritance for selected years (in real euros)

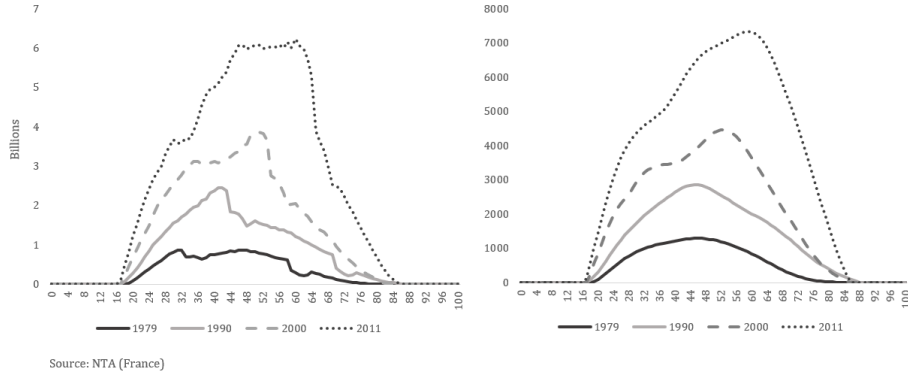
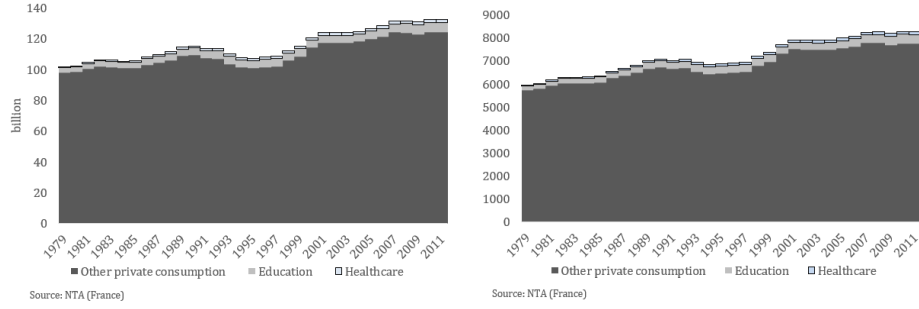


Figure 3.16: Aggregate and per capita private consumption by type for 0-19 year-olds (in real euros)



B. The model

Equilibrium in the economy

From Equations (3.17) - (3.20), we can reduce our system economy to a system of three variables, $\{\tilde{K}_{1,t}, \tilde{K}_{2,t}, \tilde{K}_t\}$, as follows:

$$\tilde{K}_{1,t+1} \left(p \left(\frac{\theta}{\beta} \tilde{K}_{1,t+1} \right)^\lambda + (1-p) \left(\frac{\theta}{\beta(1+\gamma)} \tilde{K}_{2,t+1} \right)^\lambda \right)^{1/1-\lambda} = Z_1 \frac{(1-\alpha) \tilde{K}_t^\alpha}{p + (1-p) \left(\frac{\tilde{K}_{2,t}}{(1+\gamma) \tilde{K}_{1,t}} \right)^\lambda} \quad (3.0.0.35)$$

$$\tilde{K}_{2,t+1} \left(p \left(\frac{\theta}{\beta} \tilde{K}_{1,t+1} \right)^\lambda + (1-p) \left(\frac{\theta}{\beta(1+\gamma)} \tilde{K}_{2,t+1} \right)^\lambda \right)^{1/1-\lambda} = Z_2 \left(\frac{(1-\alpha) \tilde{K}_t^\alpha}{p \left(\frac{(1+\gamma) \tilde{K}_{1,t}}{\tilde{K}_{2,t}} \right)^\lambda + 1 - p} + \frac{\gamma \alpha}{1+\gamma} \tilde{K}_t^{\alpha-1} \tilde{K}_{2,t} \right) \quad (3.0.0.36)$$

$$\tilde{K}_t = p \tilde{K}_{1,t} + (1-p) \tilde{K}_{2,t} \quad (3.0.0.37)$$

where $Z_1 \equiv \frac{\beta}{1+\theta+\beta}$ and $Z_2 \equiv \frac{\beta(1+\gamma)}{1+\theta+\beta(1+\gamma)}$.

Construct the variable x_t to measure wealth inequality

Define a new variable x_t that captures the wealth disparities between altruistic and egoistic families. We have

$$x_t \equiv \frac{\tilde{K}_{2,t}}{\tilde{K}_{1,t}}$$

Transforming (3.17) and (3.18) in intensive form, one can obtain:

$$\tilde{K}_{1,t+1} \frac{H_{t+1}}{H_t} = \frac{\beta}{1 + \theta + \beta} w_t \tilde{H}_{1,t} \quad (3.0.038)$$

$$\tilde{K}_{2,t+1} \frac{H_{t+1}}{H_t} = \frac{\beta(1 + \gamma)}{1 + \theta + \beta(1 + \gamma)} \left(w_t \tilde{H}_{2,t} + \frac{\gamma}{1 + \gamma} R_t \tilde{K}_{2,t} \right) \quad (3.0.039)$$

Dividing (3.0.039) by (3.0.038), we write:

$$\frac{\tilde{K}_{2,t+1}}{\tilde{K}_{1,t+1}} = \frac{(1 + \gamma)(1 + \theta + \beta)}{1 + \theta + \beta(1 + \gamma)} \left(\frac{\tilde{H}_{2,t}}{\tilde{H}_{1,t}} + \frac{\gamma}{1 + \gamma} \frac{R_t}{w_t} \frac{\tilde{K}_{2,t}}{\tilde{H}_{1,t}} \right) \quad (3.0.040)$$

On one hand, knowing that

$$\tilde{H}_{i,t+1} \frac{H_{t+1}}{H_t} = \tilde{m}_{i,t}^\lambda$$

$$\tilde{m}_{i,t} = \frac{H_{t+1}}{H_t} \frac{\theta}{\beta(1 + \gamma_i)} \tilde{K}_{i,t+1}$$

One can now construct the human capital accumulation ratio as a function of x_t :

$$\frac{\tilde{H}_{2,t}}{\tilde{H}_{1,t}} = \left(\frac{\tilde{K}_{2,t}}{(1 + \gamma) \tilde{K}_{1,t}} \right)^\lambda = \left(\frac{x_t}{1 + \gamma} \right)^\lambda \quad (3.0.041)$$

On the other hand, we know that

$$\frac{R_t}{w_t} \tilde{K}_{2,t} = \frac{\alpha}{1 - \alpha} \frac{\tilde{K}_{2,t}}{\tilde{K}_{1,t}}$$

Notice that $\tilde{K}_t = p\tilde{K}_{1,t} + (1 - p)\tilde{K}_{2,t}$, thus

$$\frac{R_t}{w_t} \tilde{K}_{2,t} = \frac{\alpha}{1 - \alpha p + (1 - p)x_t} x_t$$

Moreover, using the result provided in (3.0.041), we can rewrite $\tilde{H}_{1,t}$ as a function

of x_t as follows:

$$\begin{aligned}
\tilde{H}_{1,t} &= \frac{H_{1,t}}{H_t} \\
&= \frac{H_{1,t}}{pH_{1,t} + (1-p)H_{2,t}} \\
&= \frac{1}{p + (1-p)\frac{\tilde{H}_{2,t}}{\tilde{H}_{1,t}}} \\
&= \frac{1}{p + (1-p)\left(\frac{x_t}{1+\gamma}\right)^\lambda} \tag{3.0.0.42}
\end{aligned}$$

Replacing (3.0.0.41) and (3.0.0.42) into (3.0.0.40), we got:

$$x_{t+1} = \frac{(1+\gamma)(1+\theta+\beta)}{1+\theta+\beta(1+\gamma)} \left[\left(\frac{x_t}{1+\gamma}\right)^\lambda + \frac{\gamma\alpha}{(1+\gamma)(1-\alpha)} x_t \frac{p + (1-p)\left(\frac{x_t}{1+\gamma}\right)^\lambda}{p + (1-p)x_t} \right]$$

Transitional dynamic system in reduced form

One can also prove that:

$$\begin{aligned}
\frac{\lambda}{(1+\gamma)^\lambda} (x^*)^{\lambda-1} &< \frac{1}{(1+\gamma)^\lambda} (x^*)^{\lambda-1} \\
\frac{p^2 + \frac{p(1-p)(1+\lambda)}{(1+\gamma)^\lambda} (x^*)^\lambda + \frac{(1-p)^2\lambda}{(1+\gamma)^\lambda} (x^*)^{\lambda+1}}{(p + (1-p)x^*)^2} &< \frac{p + \frac{1-p}{(1+\gamma)^\lambda} (x^*)^\lambda}{p + (1-p)x^*}
\end{aligned}$$

Therefore, we can then prove that:

$$g'(x^*) < 1$$

Proof that $\bar{\lambda}$ is a decreasing function of γ

From (3.27), let $\lambda = \bar{\lambda}$. We already prove that there exists a unique solution of $\bar{\lambda}$ such that $G(1; \gamma, \bar{\lambda}) = 1$. Put differently, one is able to derive $\bar{\lambda}$ as a function of γ as :

$$\bar{\lambda} = f(\gamma)$$

Indeed, by letting $G(1; \gamma, \bar{\lambda}) = 1$ one has:

$$\frac{(1+\beta+\theta)(1+\gamma)}{1+\theta+\beta(1+\gamma)} \left[\left(\frac{1}{1+\gamma}\right)^{\bar{\lambda}} + \frac{\gamma\alpha}{(1+\gamma)(1-\alpha)} \left(p + \frac{1-p}{(1+\gamma)^{\bar{\lambda}}}\right) \right] = 1$$

Simplifying it gives us:

$$(1 + \gamma)^{\bar{\lambda}} = \frac{1 - \frac{\alpha\gamma p}{(1+\gamma)(1-\alpha)} + \frac{\beta\alpha}{1+\beta+\theta}}{1 + \frac{\gamma\alpha(1-p)}{(1+\gamma)(1-\alpha)}}$$

Taking logarithm the both sides of the equations gives us:

$$\bar{\lambda} = \underbrace{\frac{\ln\left(1 - \frac{\alpha\gamma p}{(1+\gamma)(1-\alpha)} + \frac{\beta\alpha}{1+\beta+\theta}\right) - \ln\left(1 + \frac{\gamma\alpha(1-p)}{(1+\gamma)(1-\alpha)}\right)}{\ln(1 + \gamma)}_{\equiv f(\gamma)}$$

Clearly, one can see that $f(\gamma)$ is a decreasing function of γ . Hence, $\bar{\lambda}$ is also a decreasing function of γ .

The model with population growth

To simplify the following analysis, note that we denote aggregate variables by capital letters, per capita variables by small letters, and per efficient capita (intensive form) by capital letters with tilde. For example, aggregate capital is denoted by K , capital saved by each individual denoted by $k = \frac{K}{L}$ and capital per efficient capita, or per efficient unit of labor, is denoted by $\tilde{K} = \frac{K}{hL}$.

The human capital, h , of an adult at time t of dynasty i can be expressed as a function of the previous period's average human capital and the intrahousehold transfers received at childhood, as the following:

$$h_{i,t} = m_{i,t-1}^{\lambda} h_{t-1}^{1-\lambda}, \quad \lambda \in (0, 1) \quad (3.0.043)$$

While the utility function does not change, as in Equation (3.2), the budget constraints of individual in dynasty i now become:

$$c_{i,t} + s_{i,t} + (1 + n)m_{i,t} \leq w_t h_{i,t} \quad (3.0.044)$$

$$d_{i,t+1} + (1 + n)b_{i,t+1} \leq (s_{i,t} + b_{i,t})R_{t+1} \quad (3.0.045)$$

The first-order conditions do not change, except for how we now express income, and where the population growth rate appears, to the following:

$$c_{i,t} = \frac{1}{1 + \theta + \beta(1 + \gamma_i)} (w_t h_{i,t} + b_{i,t}) \quad (3.0.046)$$

$$(1 + n)m_{i,t} = \frac{\theta}{1 + \theta + \beta(1 + \gamma_i)} (w_t h_{i,t} + b_{i,t}) \quad (3.0.047)$$

$$d_{i,t+1} = \frac{\beta R_{t+1}}{1 + \theta + \beta(1 + \gamma_i)}(w_t h_{i,t} + b_{i,t}) \quad (3.0.0.48)$$

$$(1 + n)b_{i,t+1} = \frac{\beta \gamma_i R_{t+1}}{1 + \theta + \beta(1 + \gamma_i)}(w_t h_{i,t} + b_{i,t}) \quad (3.0.0.49)$$

Consequently, given differences in the inheritance preference, we get the following more explicitly expressed equations:

$$b_{1,t+1} = 0 \quad (3.0.0.50a)$$

$$b_{2,t+1} = \frac{\beta \gamma R_{t+1}}{(1 + \theta + \beta(1 + \gamma))(1 + n)}(w_t h_{2,t} + b_{2,t}) \quad (3.0.0.50b)$$

The savings function for each dynasty can be derived as follows:

$$s_{1,t} = \frac{\beta}{1 + \beta + \theta} w_t h_{1,t} \quad (3.0.0.51)$$

$$s_{2,t} + b_{2,t} = \frac{\beta(1 + \gamma)}{1 + \theta + \beta(1 + \gamma)}(w_t h_{2,t} + b_{2,t}) \quad (3.0.0.52)$$

The production function of firm, as well as the rate of return on capital and wages now change to the following:

$$Y_t = K_t^\alpha (h_t L_t)^{1-\alpha} \quad (3.0.0.53)$$

$$R_t = \alpha \tilde{K}_t^{\alpha-1} \quad (3.0.0.53)$$

$$w_t = (1 - \alpha) \tilde{K}_t^\alpha \quad (3.0.0.54)$$

Equilibrium in the economy now can be expressed with the following market clearing conditions:

$$K_{1,t+1} = pN_{t+1}k_{1,t+1} = pN_{t-1}s_{1,t} = \frac{pN_{t+1}}{(1+n)^2}s_{1,t} \quad (3.0.0.55)$$

$$K_{2,t+1} = (1-p)N_{t+1}k_{2,t+1} = (1-p)N_{t-1}(s_{2,t} + b_{2,t}) = \frac{pN_{t+1}}{(1+n)^2}(s_{2,t} + b_{2,t}) \quad (3.0.0.56)$$

$$K_{t+1} = pN_{t+1}k_{1,t+1} + (1-p)N_{t+1}k_{2,t+1} = \frac{pN_{t+1}}{(1+n)^2}s_{1,t} + \frac{(1-p)N_{t+1}}{(1+n)^2}(s_{2,t} + b_{2,t}) \quad (3.0.0.57)$$

where it is important to note that the individual savings (and inheritance) is multiplied by the cohort at the time it is born, to remain consistent with the timing convention of the rest of the paper. Recall that $k_{i,t+1} = \frac{K_{i,t+1}}{N_{i,t+1}}$. Also note that Equation (3.0.0.55), for instance, gives the classic three-period market clearing condition $(1+n)^2 k_{1,t+1} = s_{1,t}$. This is the same for Equation (3.0.0.56). Also note that Equation (3.0.0.57) can be further simplified to look like the original market clearing condition $k_{t+1}(1+n)^2 = ps_{1,t} + (1-p)(s_{2,t} + b_{2,t})$.

The market clearing condition for labor remains the same. The human capital market clearing condition can be expressed as:

$$H_t = H_{1,t} + H_{2,t} = h_{1,t}pN_t + h_{2,t}(1-p)N_t \quad (3.0.058)$$

which can be further simplified to the original set-up as $h_t = ph_{1,t} + (1-p)h_{2,t}$. Finally, the good market clearing condition, which results from the individual budget constraints, requires that what is produced is consumed, shared or saved in the economy, such that:

$$p(c_{1,t} + d_{1,t} + (1+n)m_{1,t} + s_{1,t})N_t + (1-p)(c_{2,t} + d_{2,t} + (1+n)m_{2,t} + s_{2,t} + b_{2,t})N_{t-1} = Y_t \quad (3.0.059)$$

We can re-express the dynamics of our four predetermined variables as the following:

$$\frac{1 + \theta + \beta}{\beta}(1+n)^2k_{1,t+1} = w_t h_{1,t} \quad (3.0.060)$$

$$\frac{1 + \theta + \beta(1 + \gamma)}{\beta(1 + \gamma)}(1+n)^2k_{2,t+1} - \frac{\gamma(1+n)}{1 + \gamma}R_t k_{2,t} = w_t h_{2,t} \quad (3.0.061)$$

$$h_{1,t+1} = h_t^{1-\lambda} \left(\frac{\theta}{\beta}(1+n)k_{1,t+1} \right)^\lambda \quad (3.0.062)$$

$$h_{2,t+1} = h_t^{1-\lambda} \left(\frac{\theta}{\beta(1+\gamma)}(1+n)k_{2,t+1} \right)^\lambda \quad (3.0.063)$$

where:

$$h_t = ph_{1,t} + (1-p)h_{2,t} \quad (3.0.064)$$

$$R_t = \alpha \left(\frac{pk_{1,t} + (1-p)k_{2,t}}{ph_{1,t} + (1-p)h_{2,t}} \right)^{\alpha-1} \quad (3.0.065)$$

$$w_t = (1-\alpha) \left(\frac{pk_{1,t} + (1-p)k_{2,t}}{ph_{1,t} + (1-p)h_{2,t}} \right)^\alpha \quad (3.0.066)$$

We can re-write Equations (3.0.060) and (3.0.061) as the following:

$$\frac{1 + \theta + \beta}{\beta}(1+n)^2\tilde{K}_{1,t+1}\frac{h_{t+1}}{h_t} = w_t\tilde{H}_{1,t} \quad (3.0.067)$$

$$\frac{1 + \theta + \beta(1 + \gamma)}{\beta(1 + \gamma)}(1+n)^2\tilde{K}_{2,t+1}\frac{h_{t+1}}{h_t} = \frac{\gamma(1+n)}{1 + \gamma}R_t\tilde{K}_{2,t} + w_t\tilde{H}_{2,t} \quad (3.0.068)$$

where we make use of the following notation: $\tilde{K}_{1,t} = \frac{pk_{1,t}}{h_t}$, $\tilde{K}_{2,t} = \frac{(1-p)k_{2,t}}{h_t}$, $\tilde{K}_t = \tilde{K}_{1,t} + \tilde{K}_{2,t}$, $\tilde{H}_{1,t} = \frac{ph_{1,t}}{h_t}$ and $\tilde{H}_{2,t} = \frac{(1-p)h_{2,t}}{h_t}$. Dividing Equation (3.0.067) over (3.0.068)

gives us the following familiar expression:

$$\frac{1 + \theta + \beta(1 + \gamma)}{(1 + \theta + \beta)(1 + \gamma)} \frac{\tilde{K}_{2,t+1}}{\tilde{K}_{1,t+1}} = \frac{\gamma(1 + n)}{1 + \gamma} \frac{R_t}{w_t} \frac{\tilde{K}_{2,t}}{\tilde{H}_{1,t}} + \frac{\tilde{H}_{2,t}}{\tilde{H}_{1,t}} \quad (3.0.0.69)$$

which can be further simplified using our notation to be a function of inequality x , as the following:

$$\frac{1 + \theta + \beta(1 + \gamma)}{(1 + \theta + \beta)(1 + \gamma)} x_{t+1} = \frac{\gamma(1 + n)\alpha}{(1 + \gamma)(1 - \alpha)} \frac{x_t}{1 + x_t} \left(1 + \left(\frac{1 - p}{p} \right)^{1-\lambda} \left(\frac{x_t}{1 + \gamma} \right)^\lambda \right) + \left(\frac{1 - p}{p} \right)^{1-\lambda} \left(\frac{x_t}{1 + \gamma} \right)^\lambda \quad (3.0.0.70)$$

General conclusion

While it's essential to understand the impacts of the global rise of housing price on wealth inequality, little has been done on this subject. My thesis aims to fulfil this gap.

It's known that one important source that affects the concentration of wealth is capital savings. Thus, in the first chapter, I addressed the interaction of housing with capital accumulation and investigate if a gain in house prices would drive individuals to save more or less.

In the second chapter, I focused on how intergenerational transfer would make inequality persistent in long-run via housing market. By considering an economy with two different investment assets: housing and capital, we show that that there exists an equilibrium in which housing is more profitable than capital. As the rich family, i.e. the one who inherited relatively higher wealth, can access the housing market while the poor one can't due to the borrowing constraint, inequality maintains in the long run.

Finally, bequest and intrahousehold transfer are the two essential transmission channels that link to wealth in different way. While intrahousehold transfer is believed to contribute to the development of individual's human capital and hence labour incomes, inheritance plays a decisive role in one's accumulation of capital and wealth. Data from the national transfer account in France showed us that over the last decades, there has been a notable change in a composition of private transfers: from a more dominant share of intrahousehold transfers at the beginning of 1980s to a more dominant inheritance one at the later years. As this shift triggers two adverse effects, it's, therefore, interesting to understand why and how it happens as well as how it is related to wealth inequality. These questions are addressed in the third chapter of my thesis.

By studying the impacts of housing on wealth through different channels: capital

accumulation and intergenerational transfer, the thesis provides a better understanding on how housing becomes the mechanism through which wealth is generated and transmitted. We found that whether wealth increases in response to a shift in housing prices and/or housing rents is conditional on the driving force behinds this shift. Thus, these findings would be helpful in policy decision process. If a gain in housing price goes in the same direction with economic growth in response to a shock, a dramatic intervention on housing market in the way to reduce housing price may do more harm than good.

Bibliography

- Abel, A. B. and Warshawsky, M. (1987). Specification of the joy of giving: Insights from altruism. Technical report, National Bureau of Economic Research Working Paper No. 2154.
- Angelache, C. (2011). Analysis of the correlation between gdp and the final consumption. *Theoretical and Applied Economics*, 9:pp. 129–138.
- Aoki, K., Proudman, J., and Vlieghe, G. (2004). House prices, consumption, and monetary policy: a financial accelerator approach. *Journal of financial intermediation*, 13(4):414–435.
- Becker, G. S. and Tomes, N. (1979). An equilibrium theory of the distribution of income and intergenerational mobility. *Journal of political Economy*, 87(6):1153–1189.
- Becker, G. S. and Tomes, N. (1986). Human capital and the rise and fall of families. *Journal of labor economics*, 4(3, Part 2):S1–S39.
- Bellettini, G., Taddei, F., and Zanella, G. (2017a). Intergenerational altruism and house prices: evidence from bequest tax reforms in italy. *European Economic Review*, 92:1–12.
- Bellettini, G., Taddei, F., and Zanella, G. (2017b). Intergenerational altruism and house prices: evidence from bequest tax reforms in italy. *European Economic Review*, 92:1–12.
- Benhabib, J. and Spiegel, M. M. (2005). Human capital and technology diffusion. *Handbook of economic growth*, 1:935–966.
- Bewley, T. (1977). The permanent income hypothesis: A theoretical formulation. *Journal of Economic Theory*, 16(2):252–292.
- Bezrukovs, D. (2013). The role of housing in wealth inequality in eurozone countries. *Working Paper*.

- Bleck, A. and Liu, X. (2018). Credit expansion and credit misallocation. *Journal of Monetary Economics*, 94:27–40.
- Blinder, A. S. (1976). Inequality and mobility in the distribution of wealth. *Kyklos*, 29(4):607–638.
- Bossmann, M., Kleiber, C., and Wälde, K. (2007). Bequests, taxation and the distribution of wealth in a general equilibrium model. *Journal of Public Economics*, 91(7):1247–1271.
- Bovenberg, A. L. and van Ewijk, C. (1997). Progressive taxes, equity, and human capital accumulation in an endogenous growth model with overlapping generations. *Journal of Public Economics*, 64(2):153–179.
- Buiter, W. H. (2008). Housing wealth isn't wealth. Technical report, National Bureau of Economic Research.
- Campbell, J. Y. and Cocco, J. F. (2007). How do house prices affect consumption? evidence from micro data. *Journal of monetary Economics*, 54(3):591–621.
- Canner, G., Dynan, K., and Passmore, W. (2002). Mortgage refinancing in 2001 and early 2002. *Fed. Res. Bull.*, 88:469.
- Carroll, C. D. (1998). Why do the rich save so much? Technical report, National Bureau of Economic Research Working Paper No. 6549.
- Carroll, C. D., Otsuka, M., and Slacalek, J. (2011). How large are housing and financial wealth effects? a new approach. *Journal of Money, Credit and Banking*, 43(1):55–79.
- Chakraborty, I., Goldstein, I., and MacKinlay, A. (2018). Housing price booms and crowding-out effects in bank lending. *The Review of Financial Studies*, 31(7):2806–2853.
- Chanda, A. (2008). The rise in returns to education and the decline in household savings. *Journal of Economic Dynamics and Control*, 32(2):436–469.
- Christophers, B. (2018). Intergenerational inequality? labour, capital, and housing through the ages. *Working Paper*.
- Cipriani, G. P. (2014). Population aging and payg pensions in the olg model. *Journal of Population Economics*, 27(1):pp. 251–256.
- Cooper, D. (2013). House price fluctuations: the role of housing wealth as borrowing collateral. *Review of Economics and Statistics*, 95(4):1183–1197.

- d'Albis, H., Bonnet, C., Navaux, J., Pelletan, J., Toubon, H., and Wolff, F.-C. (2015). The lifecycle deficit in france, 1979-2005. *The Journal of the Economics of Ageing*, 5:79–85.
- Davies, J. B. (1982). The relative impact of inheritance and other factors on economic inequality. *The Quarterly Journal of Economics*, 97(3):471–498.
- Davies, J. B. and Shorrocks, A. F. (2000). The distribution of wealth. In *Handbook of income distribution*, volume 1, pages 605–675. Elsevier.
- Davis, M. A. and Heathcote, J. (2005). Housing and the business cycle. *International Economic Review*, 46(3):751–784.
- de la Croix, D. (2001). Growth dynamics and education spending: the role of inherited tastes and abilities. *European Economic Review*, 45(8):1415–1438.
- de la Croix, D. and Michel, P. (2007). Education and growth with endogenous debt constraint. *Economic Theory*, 33:pp. 509–530.
- De Nardi, M. (2004a). Wealth inequality and intergenerational links. *Review of Economic Studies*, 71(3):743–768.
- De Nardi, M. (2004b). Wealth inequality and intergenerational links. *The Review of Economic Studies*, 71(3):743–768.
- De Nardi, M. and Yang, F. (2016). Wealth inequality, family background, and estate taxation. *Journal of Monetary Economics*, 77:130–145.
- Deaton, A. and Laroque, G. (2001). Housing, land prices, and growth. *Journal of Economic Growth*, 6(2):87–105.
- Diamond, P. A. (1965). National debt in a neoclassical growth model. *The American Economic Review*, 55(5):1126–1150.
- Duca, J. V., Muellbauer, J., and Murphy, A. (2011). Shifting credit standards and the boom and bust in us house prices.
- Dynan, K. E., Skinner, J., and Zeldes, S. (2000). Do the rich save more? NBER Working Papers 7906, National Bureau of Economic Research, Inc.
- D'Albis, H. and Iliopoulos, E. (2013). Collateral constraints and rental markets. *Economics Letters*, 121(3):436–439.
- d'Albis, H. and Moosa, D. (2015). Generational economics and the national transfer accounts. *Journal of Demographic Economics*, 81(4):409–441.

- Engelhardt, G. V. (1996). House prices and home owner saving behavior. *Regional science and urban Economics*, 26(3-4):313–336.
- Fall, F. (2007). Pension reform, assets returns and wealth distribution. *Annales D'Économie Et De Statistique*, 85:81–96.
- Farhi, E. and Tirole, J. (2011). Bubbly liquidity. *The Review of Economic Studies*, 79(2):678–706.
- Ferrara, L., Vigna, O., et al. (2009). Cyclical relationships between gdp and housing market in france: Facts and factors at play. *Banque de France, Working Paper*, (268).
- Fuller, G., J. A. and Regan, A. (2019). Housing prices and wealth inequality in western europe. *West European Politics*, 0(0):1–24.
- Gale, W. G. and Scholz, J. K. (1994). Intergenerational transfers and the accumulation of wealth. *The Journal of Economic Perspectives*, 8(4):145–160.
- Gan, J. (2007). Collateral, debt capacity, and corporate investment: Evidence from a natural experiment. *Journal of Financial Economics*, 85(3):709–734.
- Glaeser, E. L., Gyourko, J., and Saks, R. E. (2005). Why have housing prices gone up? *American Economic Review*, 95(2):329–333.
- Glomm, G. and Ravikumar, B. (1997). Productive government expenditures and long-run growth. *Journal of Economic Dynamics and Control*, 21:pp. 183–204.
- Gokhale, J., Kotlikoff, L. J., Sefton, J., and Weale, M. (2001). Simulating the transmission of wealth inequality via bequests. *Journal of Public economics*, 79(1):93–128.
- Gollier, C. (2001). Wealth inequality and asset pricing. *The Review of Economic Studies*, 68(1):181–203.
- Gomes, S. and Mendicino, C. (2015). Housing market dynamics: Any new? *ECB working paper*.
- Greenspan, A. and Kennedy, J. (2007). Sources and uses of equity extracted from homes. federal reserve board finance and economics discussion series no. 2007-20.
- Guerrieri, L. and Iacoviello, M. (2017). Collateral constraints and macroeconomic asymmetries. *Journal of Monetary Economics*, 90:28–49.
- Guthrie, H. (1963). Intergeneration transfers of wealth and the theory of saving. *The Journal of Business*, 36(1).

- Gyourko, J., E., Mayer, C. J., and Sinai, T. M. (2006). Superstar cities? *NBER Working Paper*, w12355.
- Han, S. and Mulligan, C. B. (2001). Human capital, heterogeneity and estimated degrees of intergenerational mobility. *The Economic Journal*, 111(470):207–243.
- Harbury, C. and Hitchens, D. (1979). *Inheritance and Wealth Inequality in Britain*. Routledge Revivals. Routledge, Taylor and Francis Group.
- Holmstrom, B. and Tirole, J. (1997). Financial intermediation, loanable funds, and the real sector. *the Quarterly Journal of economics*, 112(3):663–691.
- Huber, S. J., Rott, C., and Giusti, G. (2018). Preference for housing services and house price bubble occurrence. evidence from a macro-experiment.
- Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American economic review*, 95(3):739–764.
- Ishikawa, T. (1975). Family structures and family values in the theory of income distribution. *Journal of Political Economy*, 83(5):987–1008.
- Kahn, J. (2009). Productivity swings and housing prices.
- Kahn, J. A. (2008). What drives housing prices?
- Kopczuk, W. and Lupton, J. P. (2007). To leave or not to leave: The distribution of bequest motives. *The Review of Economic Studies*, 74(1):207–235.
- Kotlikoff, L. J. and Summers, L. H. (1981). The role of intergenerational transfers in aggregate capital accumulation. *Journal of political economy*, 89(4):706–732.
- Kuhn, U. and Grabka, M. (2018). Homeownership and wealth in switzerland and germany. *Social Dynamics in Swiss Society*.
- Laitner, J. and Juster, F. T. (1996). New evidence on altruism: A study of taaa-cref retirees. *The American Economic Review*, pages 893–908.
- Landvoigt, T. (2017). Housing demand during the boom: The role of expectations and credit constraints. *The Review of Financial Studies*, 30(6):1865–1902.
- Lee, R. D. and Mason, A. (2011). *Population aging and the generational economy: A global perspective*. Edward Elgar Publishing.
- Lucas Jr, R. E. (1988). On the mechanics of economic development. *Journal of monetary economics*, 22(1):3–42.

- Ludwig, A. and Vogel, E. (2010). Mortality, fertility, education and capital accumulation in a simple olg economy. *Journal of Population Economics*, 23(2):703–735.
- Mankiw, N. G. and Weil, D. N. (1989). The baby boom, the baby bust, and the housing market. *Regional science and urban economics*, 19(2):235–258.
- Matsuyama, K. (2000). Endogenous inequality. *Review of Economic Studies*, 67(4):743–759.
- McCarthy, J. and Peach, R. W. (2005). Is there a bubble in the housing market now?
- Miao, J., Wang, P., and Zhou, J. (2015). Asset bubbles, collateral, and policy analysis. *Journal of Monetary Economics*, 76:S57–S70.
- Michel, P. and Pestieau, P. (2005). Fiscal policy with agents differing in altruism and ability. *economica*, 72(285):121–135.
- Modigliani, F. (1988). The role of intergenerational transfers and life cycle saving in the accumulation of wealth. *The Journal of Economic Perspectives*, 2(2):15–40.
- Muellbauer, J. and Murphy, A. (2008). Housing markets and the economy: the assessment. *Oxford review of economic policy*, pages 1–33.
- OECD (2015). *Prices : Analytical house price indicators*.
- Ortalo-Magne, F. and Rady, S. (2006). Housing market dynamics: On the contribution of income shocks and credit constraints. *The Review of Economic Studies*, 73(2):459–485.
- Piketty, T. (2011). On the long-run evolution of inheritance: France 1820–2050. *The Quarterly Journal of Economics*, 126(3):pp. 1071–1131.
- Piketty, T. (2014). *Capital in the twenty-first century*. Belknap Press of Harvard University Press Cambridge, MA.
- Piketty, T., Postel-Vinay, G., and Rosenthal, J.-L. (2014). Inherited vs self-made wealth: Theory & evidence from a rentier society (paris 1872–1927). *Explorations in economic history*, 51:21–40.
- Piketty, T. and Zucman, G. (2015). Wealth and inheritance in the long run. In *Handbook of income distribution*, volume 2, pages 1303–1368. Elsevier.
- Sierminska, E. and Medgyesi, M. (2013). The distribution of wealth between households. *Research note (European Commission, Brüssel 2013)*.

- Stamp, J. (1926). Inheritance as an economic factor. *Economic Journal*, pages 339–374.
- Tamura, R. (1991). Income convergence in an endogenous growth model. *Journal of Political Economy*, 99(3):522–540.
- Tirole, J. (1985). Asset bubbles and overlapping generations. *Econometrica: Journal of the Econometric Society*, pages 1499–1528.
- Tomes, N. (1981). The family, inheritance, and the intergenerational transmission of inequality. *Journal of Political Economy*, 89(5):928–958.
- Wedgwood, J. (1929). *The economics of inheritance*. Routledge, Taylor and Francis Group.
- Weil, D. (2015). Capital and wealth in the twenty-first century. *The American Economic Review*, 105.
- Yaari, M. E. (1964). On the consumer's lifetime allocation process. *International Economic Review*, 5(3):304–317.
- Zhao, B. (2015). Rational housing bubble. *Economic Theory*, 60(1):141–201.

Abstract

Housing market, intergenerational transfer, and the macroeconomy

Impacts of the global rise of housing price from 1980s on the economy has attracted a surging research interest. As housing represent a major share of household expenditure as well as total wealth, it's important to understand the role of housing on wealth inequality. Yet, due to data availability, little has been done to understand this issue. As wealth of households is accumulated from two main sources: capital savings and transfers, it's important to understand how these sources are linked to housing market when there's a shift in housing prices. Thus, the first two chapters of this thesis aim to a better understanding on the importance of housing channel on the economy. More specifically, the first chapter draws attention on the co-movement between housing prices and production output. By introducing different shocks to the economy, we can analyse their impacts on housing prices and capital accumulation as well as the mechanism in which these shocks are transmitted. On the other hand, the second chapter focuses on how intergenerational transfer makes inequality persistent via the housing market. By considering an economy with two different investment assets: housing and capital, we show that that there exists an equilibrium in which housing is more profitable than capital. As the rich family, i.e. the one who inherited relatively higher wealth, can access the housing market while the poor one can't due to the borrowing constraint, inequality maintains in the long run.

Furthermore, bequest and intrahousehold transfer are the two essential transmission channels that link to wealth in different way. While intrahousehold transfer is believed to contribute to the development of individual's human capital and hence labour incomes, inheritance plays a decisive role in one's accumulation of capital and wealth. Data from the national transfer account in France showed us that over the last decades, there has been a notable change in a composition of private transfers: from a more dominant share of intrahousehold transfers to a more dominant inheritance one. As this shift triggers two adverse effects, it's, therefore, interesting to understand why and how it happens as well as how it is related to wealth inequality. These questions are addressed in the third chapter of my thesis.

Keywords

Housing prices, Intergenetional Transfer, Inequality, Overlapping generation model

Marché d'immobilier, transferts entre générations, et la macroéconomie

Les répercussions de la hausse mondiale des prix du logement à partir des années 1980 sur l'économie ont attiré un intérêt croissant pour la recherche. Le logement représentant une part importante des dépenses des ménages ainsi que de la richesse totale, il est important de comprendre le rôle du logement dans l'inégalité des richesses. Cependant, en raison de la disponibilité des données, peu de choses ont été faites pour comprendre ce problème. La richesse des ménages s'accumulant à partir de deux sources principales: l'épargne et les transferts de capital, il est important de comprendre le lien qui existe entre ces sources et le marché du logement lorsque les prix de l'habitation changent. Ainsi, les deux premiers chapitres de cette thèse visent à mieux comprendre l'importance de la filière logement pour l'économie. Plus précisément, le premier chapitre attire l'attention sur la corrélation entre les prix du logement et la production. En introduisant différents chocs dans l'économie, nous pouvons analyser leurs effets sur les prix du logement et l'accumulation de capital, ainsi que sur le mécanisme de transmission de ces chocs. D'autre part, le deuxième chapitre porte sur la manière dont le transfert intergénérationnel rend l'inégalité persistante via le marché du logement. En considérant une économie avec deux actifs d'investissement différents: le logement et le capital, nous montrons qu'il existe un équilibre dans lequel le logement est plus rentable que le capital. En tant que famille riche, c'est-à-dire celle qui a hérité d'une richesse relativement plus grande, peut accéder au marché du logement alors que la famille pauvre ne le peut pas en raison de la contrainte d'emprunt, l'inégalité persiste à long terme.

En outre, le patrimoine et les transferts entre ménages sont les deux canaux de transmission essentiels qui relient la richesse de manière différente. Bien que l'on pense que les transferts entre ménages contribuent au développement du capital humain et donc des revenus du travail, l'héritage joue un rôle décisif dans l'accumulation de capital et de richesse. Les données du compte de transfert national en France nous ont montré qu'au cours des dernières décennies, la composition des transferts privés a sensiblement évolué: d'une part plus dominante des transferts entre ménages à une part plus dominante en matière de succession. Ce changement entraînant deux effets négatifs, il est donc intéressant de comprendre pourquoi et comment cela se produit, ainsi que son lien avec l'inégalité de la richesse. Ces questions sont abordées dans le troisième chapitre de ma thèse.

Mots-clés

Prix d'immobilier, Transfert intergénérationnel, Inégalité, Modèle à génération imbriquée

