

Asset Return Determinants: risk Factors, Asymmetry and Horizon consideration

Abdoulaye Ndiaye Dieye

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Université de Lyon

Université Lumière Lyon 2

THÈSE

présentée pour obtenir le titre de DOCTEUR en Sciences de Gestion

ASSET RETURN DETERMINANTS: RISK FACTORS, ASYMMETRY AND HORIZON CONSIDERATION

Abdoulaye Ndiaye DIEYE

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Résumé

Cette thèse aborde la question du choix des déterminants ou des sources d'explication du rendement moyen des actifs. La question des déterminants du rendement moyen d'un actif est consubstantielle à la littérature sur l'évaluation des actifs, comme le souligne le nombre d'études qui ont été générées par cette question de recherche. Toutefois, cette question n'est pas réglée et demeure en suspens dans la littérature financière. La présente étude complète la littérature existante en proposant des arguments- structurés autour de 3 essais- sur les facteurs de risque, l'asymétrie de la distribution des rendements et l'horizon d'investissement comme déterminants du rendement moyen des actifs.

Dans le premier essai nous considérons l'analyse de la composition sectorielle des dix portefeuilles de Fama et French classés par taille en ayant recours à la procédure de sélection LASSO pour démontrer que l'effet de taille peut être considéré comme étant partiellement le fait de certains secteurs industriels jugés statistiquement pertinents pour expliquer spécifiquement les portefeuilles constitués d'entreprises de petites (grandes) tailles. Ainsi autour des années 2000 les entreprises du secteur des nouvelles technologies suffisent à expliquer le comportement des portefeuilles constitués d'entreprises de petites tailles. Nous étudions également les implications empiriques de notre hypothèse sectorielle sur les modèles d'évaluation prenant spécifiquement en compte cet effet. Au niveau cross-sectionnel, nous observons des rentabilités anormales significativement supérieures pour les portefeuilles de tailles construits à l'aide du LASSO par rapport aux portefeuilles de référence de Fama et French ainsi qu'un impact significatif sur les facteurs de taille (SMB) et de qualité (QMJ) proposés pour prendre en compte cet effet.

Nous conduisons dans le deuxième essai une étude sur la relation entre le marché et les principaux autres facteurs de risque en considérant une approche non-linéaire de la relation entre le marché avec ces facteurs dans le but d'en réduire le nombre. Ainsi, nous mettons en évidence l'existence d'une transformation non-linéaire du facteur de marché qui conduit à distinguer, sans *a priori* et de manière statistiquement optimale, les rendements de marché positifs des rendements négatifs. En d'autres termes les facteurs de risque considérés peuvent être en partie pris en compte de façon non-linéaire par le facteur de marché. Nous explorons également dans quelle mesure le lien entre le marché et certains facteurs de risque peut s'avérer profitable en termes de stratégies d'investissements. Nous montrons qu'une exposition différenciée aux bêtas des rendements de marché positifs et (surtout) négatifs procure une rentabilité supérieure (et anormale) à celle du marché. En outre la performance d'un investissement basé sur ces stratégies s'avère particulièrement significative. Dans le troisième essai nous revenons sur la question de la diversification temporelle en analysant l'impact de l'horizon sur les propriétés des distribution de rendements composés – par opposition aux rendements simples. Dans un premier temps, nous généralisons l'expression analytique des moments d'ordre supérieurs de la distribution des rendements obtenue par Arditti and Levy (1975). Nous constatons qu'à mesure que nous allongeons l'horizon de placement, l'asymétrie de la distribution des rendements devient de plus en plus positive, ce qui implique, pour un investisseur de long terme, la nécessité d'adapter sa stratégie d'investissement. Nous montrons par ailleurs que l'effet de composition est le principal raison expliquant la forme des distributions de rendement d'actifs à long terme. A la lumière de ces considérations, nous examinons ensuite les questions d'allocation d'actifs et de mesure de performance à long terme pour proposer une explication pouvant réconcilier les points de vue apparemment opposés sur la question exprimés dans la littérature. Nos résultats apportent un nouvel éclairage à ces divergences d'opinions quant aux stratégies d'investissement à suivre sur le long terme.

Abstract

This thesis addresses the question of choosing determinants or sources of explanations of asset returns. The question of the determinants of an asset's average return is consubstantial with the asset pricing literature as underlined by the number of studies that has been generated by this research question. However the subject is not resolved and remains an issue in the asset pricing literature. The present study adds to the extant literature by proposing arguments– structured around 3 essays– on the pricing factors, the asymmetry in the distribution of asset returns and the investment horizon as determinants of asset returns.

In the first essay we analyze the composition of the ten Fama-French portfolios sorted by size in terms of industry sectors through the LASSO selection procedure to demonstrate that the size effect can be considered as partly due to specific industrial sectors that are considered statistically relevant to explain portfolios of small (large) size firms. We also study the empirical implications of our sector-based hypothesis on asset pricing models that specifically take the size effect into account. Hence we report significantly higher abnormal returns for the size portfolios constructed using the LASSO compared to the Fama-French benchmark portfolios as well as a significant impact on the size (SMB) and quality (QMJ) factors proposed to take this effect into account.

In the second essay we conduct a study of the relationship between the market and other standard risk factors considering, in particular, a non-linear relation between the market and these factors in order to reduce their number. Thus, we highlight the existence of a non-linear transform of the market factor that leads to a distinction, without *a priori* and in a statistically optimal way, between upside and downside market returns. In other words, the considered risk factors may be partially taken into account in a non-linear way by the market factor. We also explore the extent to which the relation between the market and certain risk factors can be profitable in terms of investment strategies. We show that a differentiate exposure to upside and (especially) downside market betas leads to high (and abnormal) returns. Additionally, investments based on these strategies prove to be particularly profitable.

In the second essay we revisit the issue of time diversification and analyze the impact of horizon on the properties of the distribution of compound returns – as opposed to simple returns. First, we generalize the analytical expression of the higher order moments of the return distributions obtained by Arditti and Levy (1975). We notice that the skewness of the distribution of returns becomes increasingly positive as we extend the investment horizon. For a long-term investor, it implies the need to adapt her investment strategy. We also show that the composition

effect is the main reason for the shape of long-term return distributions. In light of these considerations, we then examine the questions of long-term asset allocation and performance measurement to propose an explanation that can reconcile the apparently dissenting views expressed in the literature on the issue. Our results shed new light on these divergences of opinion regarding long-term investment strategies.

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Introduction Générale

Introduction

La détermination des facteurs explicatifs du rendement des actifs financiers est une question centrale en finance tant par son intérêt théorique évident que du point de vue pratique. Elle est, en effet, nécessaire aux investisseurs pour prendre des décisions basées sur des anticipations fondées sur des éléments théoriquement pertinents. Cependant, comme le souligne Merton (1980), il est particulièrement difficile d'estimer le rendement espéré d'un actif. La question des déterminants du rendement moyen d'un actif n'est pas réglée et demeure une question en suspens dans la littérature financière. De nombreux modèles ont été proposés, sans qu'un consensus n'ait encore émergé. Presque tous les modèles existants sont basés sur l'approche positive-normative et peuvent être regroupés selon ce que Cochrane (2009) appelle des théories absolue (approche positive) ou relative (approche normative). Toutefois Cochrane (2011) regrette que les modèles initialement pensés pour l'Evaluation deviennent de plus en plus des modèles de *Rentabilisation* des actifs. En effet, il défend qu'initialement les modèles se voulaient *évaluer* le prix d'un actif et par conséquent en déduire une *rentabilité* qui est devenue l'objet principal de ces modèles avec la recherche incessante de nouveaux facteurs explicatif (générant des rendements anormaux).

Les travaux présentés dans cette thèse portent sur les déterminants du rendement des actifs abordés sous l'angle des facteurs de risque, de l'asymétrie de la distribution des rendements et de l'horizon d'investissement. A cet égard, ils contribuent à la littérature sur la "Rentabilisation" des actifs. Ces déterminants constituent l'articulation autour de laquelle se développe notre sujet de recherche. Les raisons qui nous ont poussées à choisir les facteurs de risque (*pricing factors*), l'asymétrie de la distribution des rendements et l'horizon d'investissement comme déterminants ou comme sources d'explications des rentabilités sont multiples et nous essaierons d'en présenter les plus pertinentes dans ce qui suit. Le choix de ce thème de recherche émane d'un contexte économique et financier, et fait suite aux enseignements théoriques et empiriques de l'évaluation d'actifs et la gestion de portefeuille qui restent des champs de recherche très dynamiques.

Ainsi notre travail se veut proposer des arguments sur des déterminants des rendements d'actifs tels que l'effet de taille en particulier, les facteurs de risque de façon plus générale, l'asymétrie et l'horizon en traitant les problématiques de recherche suivantes :

- Banz (1981) a fait valoir qu'à long terme, les petites entreprises présentaient des rendements supérieurs à ceux des grandes entreprises et que, par ailleurs, les modèles existants n'expliquaient pas les rentabilités excessives des entreprises de petite taille. Plusieurs hypothèses, sur lesquelles nous reviendrons en détail plus loin, ont été avancées pour expliquer ce phénomène. Contribuant à cette littérature, nous étudions le lien (s'il existe) entre secteurs industriels et effet de taille et, plus largement, les implications d'un tel impact sectoriel pour l'effet de taille sur certains modèles d'évaluation classiques.
- L'effet de taille est une anomalie parmi d'autres et celles-ci sont prises en compte à travers de facteurs de risque dans les modèles d'évaluations d'actifs. Toutefois, on en recense aujourd'hui une multitude (Harvey et al., 2016) ce qui implique une certaine redondance. Harvey and Siddique (2000), de même que Chung et al. (2006) entre autres, suggèrent que le facteur de taille pourrait n'être qu'une façon de capturer l'asymétrie des distribution de rendements. En d'autres termes cet effet pourrait être en lien avec la dimension asymétrique observée dans la dynamique des rendements d'actifs. Fama and French (2015), quant à eux, parlent de redondance entre certains des cinq facteurs qu'ils ont introduit dans la dernière version de leur célèbre modèle. Ainsi il semble intéressant de reconsidérer dans un premier temps le lien entre le marché et certains facteurs de risque pour ensuite étudier les conséquences de ce lien sur l'évaluation des rendements d'actifs.
- La plupart des études portent sur le comportement des rendements de court terme alors même que des effets séculaires (*e.g.* bulles spéculatives) semblent avoir un impact significatif, si ce n'est déterminant, sur le mouvement des prix des actifs. Ce constat rend, selon Cochrane (2011), l'étude des rendements à long terme particulièrement intéressante, voire nécessaire. Il est donc pertinent de considérer l'effet de l'horizon sur les propriétés de la dynamique des rendements d'actifs et, en particulier, le comportement des moments d'ordre supérieur, parmi lesquels l'asymétrie et l'aplatissement, sur le long terme et les implications qui en résultent sur la gestion de portefeuille, la mesure de performance et l'allocation d'actifs.

Les trois essais présentés dans cette thèse apportent des éléments de réponses à ces problématiques. Ainsi, le premier essai établit un lien entre l'effet de taille et une explication sectorielle. Différentes explications ont été avancées dans la littérature pour expliquer cet effet de taille, incluant les biais de survivance conduisant à une surestimation de la performance des petites entreprises (Kothari et al., 1995), les effets de liquidité (Amihud, 2002) ou les explications comportementales (Shefrin and Statman, 1994). Toutefois ce sujet reste d'actualité avec des résurgences récentes tel l'article d'Asness et al. (2018) qui défend une prime de taille beaucoup plus forte, plus stable et robuste dans le temps si les petites entreprises de faible qualité (*junk*) sont contrôlées.

Nous nous proposons, à travers le premier essai, d'examiner si l'effet de taille est le fait de quelques secteurs industriels spécifiques. Pour ce faire nous recourons au Least Absolute Shrinkage and Selection Operator (LASSO) développée par Tibshirani (1996) qui permet, sur la base d'une régression linéaire pénalisée, la sélection des variables explicatives pertinentes en exerçant des contraintes sur les coefficients de régression. Cette approche connait un certain succès en finance empirique notamment avec Feng et al. (2019) qui y recourent pour étudier le lien cross-sectionnel des facteurs de risque dans l'explication des rendements. Toutefois notre essai constitue le premier, à notre connaissance, à utiliser cette approche pour explorer le lien entre l'effet de taille et les caractéristiques sectorielles des entreprises. En effet en se basant sur les critères de classifications en termes de taille et de secteurs d'activité, des relations significatives peuvent être mises en évidence. Les résultats obtenus donnent aussi une idée de l'évolution de cet effet en fonction des secteurs industriels. Il est montré que certains secteurs présentent un lien plus étroit que d'autres avec l'effet de taille et en particulier, pendant la bulle internet, les industries de nouvelles technologies constituent celles qui sont les plus significatives pour expliquer l'effet de taille. Ce résultat a un impact dans la détermination des rendements d'actifs comme cela sera illustré à travers la partie présentant notre contribution.

Toutefois, l'effet de taille ne constitue qu'une anomalie parmi d'autres et la proposition de facteurs de risque pour expliquer les caractéristiques spécifiques des rendements de tel ou tel groupe de titres a conduit à la prolifération de facteurs avec un certain nombre de conséquences pour la validité des modèles. En effet, d'une part Kan and Robotti (2009) considèrent que trop de facteurs dans un modèle peut avoir des conséquences sur la validité de celui-ci et d'autre part Harvey et al. (2016) répertorient plus de 301 anomalies (facteurs) dans la littérature et concluent à un nécessaire réexamen de leur pertinence.

Ainsi, le deuxième essai de cette thèse a pour objectif d'explorer le lien entre le marché – facteur initial de référence absolue – et certains de ces facteurs. Pour ce faire, nous considérons la méthode Alternating Conditionnal Expectation (ACE) développée par Breiman and Friedman (1985), qui consiste à mettre en évidence, par une transformation non-linéaire, le lien optimal entre une variable dépendante et des variables prédictives. Nous reprenons les facteurs considérés dans les modèles classiques d'évaluation des actifs financiers à savoir le modèle d'équilibre des actifs financiers (MEDAF), le modèle à trois (cinq) facteurs de Fama and French (1993, 2015) et le modèles à quatre facteurs de Carhart (1997). L'idée est de trouver la transformation optimale qui donne une meilleure relation entre d'une part un facteur de risque donné et d'autre part le marché. Cela fait écho à la remarque de redondance entre certains facteurs évoquée par Fama and French (2015) et à l'idée selon laquelle la linéarité supposée par ces modèles ne serait pas suffisante pour rendre compte de la réalité de la relation entre rentabilité attendue et facteurs de risques (Rubinstein, 1973; Jurczenko and Maillet, 2006; Gu et al., 2018). L'étude de la relation entre les facteurs dans le but de réduire leur nombre n'est pas nouvelle (Klein and Chow, 2013; Baker and Wurgler, 2006). Cependant notre approche est la première à appliquer la méthode ACE pour répondre à ce type de considérations. Ainsi l'ACE propose une transformation non-linéaire du facteur de marché qui conduit à distinguer, sans a priori et de manière statistiquement optimale, les rendements de marché positifs des rendements négatifs à l'instar de ce que Pettengill et al. (1995) avaient proposé de manière *ad hoc*. Dans la lignée des travaux de Ang et al. (2006) et Lettau et al. (2014) nous considérons des modèles traitant séparément les rendements de marché positifs et négatifs pour élaborer des stratégies d'investissement au rendement anormal.

Enfin, dans le troisième essai de cette thèse, nous revenons sur la question largement débattue de l'effet de la diversification temporelle. En considérant les rendements composés – par opposition aux rendements simples – nous constatons qu'à mesure que nous allongeons l'horizon de placement, l'asymétrie de la distribution des rendements devient de plus en plus positive, ce qui implique, pour un investisseur de long horizon, la nécessité d'adapter sa stratégie d'investissement. Cet essai fait suite aux articles de Bessembinder (2018) et Fama and French (2018b) qui ont récemment jeté un nouvel éclairage sur les propriétés des distributions de rendements à long terme et suscité un regain d'intérêt pour ce sujet. En effet ils ont montré que les moments d'ordres supérieurs, et particulièrement l'asymétrie et l'aplatissement, étaient touchés par les effets de l'horizon. De plus, nous avons analysé les effets de l'horizon sur les mesures de performance ainsi que sur l'allocation d'actif. Nos résultats apportent un nouvel éclairage à la divergence de points de vue exprimés dans la littérature quant aux stratégies d'investissement à suivre sur le long terme.

La suite de cette introduction comprend une section qui définit le cadre théorique permettant de positionner les travaux réalisés par rapport à la littérature existante. Les méthodologies mobilisées ainsi que les principales contributions de cette thèse aux enseignements théoriques et empiriques en matière d'évaluation d'actifs et de gestion de portefeuille sont ensuite présentées.

1 Cadre théorique de la thèse et positionnement des travaux

Dans cette section, nous commençons par contextualiser le positionnement de notre étude en présentant le cadre théorique sur lequel s'appuie la formulation de nos problématiques. Cette partie couvre les aspects théoriques et empiriques des approches dites absolue (approche positive) et relative (approche normative) de l'évaluation d'actifs traduites par les modèles d'équilibre, dont le MEDAF, et les modèles d'évaluation par arbitrage. Afin d'améliorer la compréhension de l'objet de nos travaux, les modèles d'évaluation des actifs fondés sur le risque de pertes (*downside risk*) ainsi que les modèles basés sur les moments d'ordres supérieurs sont aussi abordés.

1.1 Cadre théorique

L'étude des modèles d'évaluation d'actifs a toujours été d'une grande importance de par leur utilisation dans de nombreux domaines d'application, notamment l'estimation des rentabilités, le calcul du coût du capital, l'évaluation des performances ou encore le choix de portefeuilles. Tout commence avec l'introduction du modèle d'évaluation des actifs financiers, le MEDAF, qui a été proposé indépendamment par Sharpe (1964), Lintner (1965) et Mossin (1966). Sharpe (1964) soutient qu'avant le MEDAF, il n'existait aucune théorie qui reliait le prix d'un actif aux préférences des investisseurs, ni aucune caractéristique des actifs qui détermine leurs prix. L'absence d'une telle théorie rendait difficile l'établissement de la relation entre le prix et le risque d'un actif. Il a donc proposé une théorie de prix d'équilibre. Il s'agit du modèle le plus connu et utilisé, qui se veut répondre à des questions structurelles et décrire l'économie dans le but d'expliquer pourquoi les prix des actifs sont ce qu'ils sont. Cochrane (2009) considère cette perspective de la théorie de l'évaluation des actifs comme absolue dans le sens où ce type de modèles vise à expliquer ce que devraient être réellement les rendements des actifs par opposition aux modèles relatifs d'évaluation des actifs qui tentent d'être proches des observations empiriques. Le MEDAF traduit le caractère moyenne-variance efficace du portefeuille de marché à l'équilibre et se traduit, techniquement, par la relation :

$$\mathbf{E}[R_i] = R_f + \beta_{i,R_M} \cdot \mathbf{E}[R_M - R_f] , \qquad (1.1)$$

où R_i et R_M sont respectivement les rendements d'un *i* actif donné et du portefeuille de marché. Le paramètre β_{i,R_M} , appelé bêta, est la sensibilité de l'actif au marché; R_f est le taux sans risque.

Dans le MEDAF, l'excès de rentabilité espéré d'un titre est donc une fonction linéaire croissante du "bêta" qui selon ce modèle constitue l'unique source de risque pour expliquer les rentabilités. En effet, selon ce modèle, seul le risque non diversifiable – associé au portefeuille de marché – est rémunéré. Le MEDAF apparait comme un modèle assez simple, quoique raisonnable en proposant une détermination des prix d'équilibre des actifs sous plusieurs hypothèses :

- Il n'y a pas de coûts de transactions ou de taxes ; la vente à découvert ou l'achat d'un titre n'a aucune incidence sur son prix ; les investisseurs sont averses au risque et rationnels.
- Tous les investisseurs ont le même horizon d'investissement; les investisseurs contrôlent le risque de leur portefeuille par la diversification; le marché est entièrement libre et tous les actifs peuvent y être échangés.
- Les investisseurs peuvent emprunter et prêter des montants illimités au taux sans risque; toutes les informations sur le marché sont disponibles dans les même conditions pour tous les investisseurs; la concurrence sur les marchés est parfaite et non faussée; tous les actifs financiers sont infiniment divisibles.

Cependant, un vaste pan de la littérature suggère que les résultats empiriques sur la validité du MEDAF sont pour la plupart peu concluants. On peut citer, entre autres, Black et al. (1972) qui ont notamment étudié la validité empirique du modèle et leurs résultats suggèrent l'existence d'un écart systématique, statistiquement significatif, par rapport au MEDAF. Sharpe and Cooper (1972) se sont opposés aux conclusions de Black et al. (1972) et ont fait état de constatations à l'appui du ME-DAF de même que Fama and MacBeth (1973) dont les travaux apportent un soutien empirique important au MEDAF. Cependant Shanken (1985) a observé que la précision des paramètres estimés dans la méthodologie de Fama and MacBeth (1973) est surévaluée. MacKinlay and Richardson (1991a) ont montré que la Méthode des Moments Généralisée (MMG) introduite par Hansen (1982) fournit un cadre unifié pour le test de modèles d'évaluation avec des hypothèses plus réalistes.

D'autres critiques, plus fondamentales, adressées au MEDAF portent sur l'efficience du marché, notamment avec Roll (1977), qui remet en cause le choix du portefeuille de marché. Cette critique est confirmée dans le cadre des travaux de Ferguson and Shockley (2003) qui démontrent que le portefeuille de marché doit contenir tous les actifs financiers échangeables dans l'économie.

De nombreuses tentatives visant à modifier ou à assouplir certaines des hypothèses du MEDAF ont donné lieu à des variations de ce modèle. Parmi les extensions les plus importantes, citons le modèle zéro-beta développé par Black (1972) qui se montre plus robuste vis-à-vis des tests empiriques et a eu une influence sur l'adoption généralisée du MEDAF. L'hypothèse interdisant la possibilité de prêts et d'emprunts illimités à un taux sans risque est relâchée dans ce modèle. Le MEDAF inter-temporel (ICAPM) proposé par Merton (1973) constitue un autre exemple important. Son caractère dynamique permet l'inclusion de facteurs autres que le marché, tels des facteurs macroéconomiques et des facteurs propres aux entreprises.

D'autres critiques ont porté sur la condition d'équilibre jugée trop forte. Ainsi l'hypothèse d'équilibre présente dans le MEDAF, et les alternatives précédemment citées, a été remplacée par une simple hypothèse de non-arbitrage, condition nécessaire (mais non suffisante) à l'équilibre. A cet égard, Ross (1976) propose l'utilisation de portefeuilles d'arbitrage qui aboutissent au Modèle d'Évaluation par Arbitrage (MEA). Le MEA et les modèles multifactoriels sont alors apparus. Ce modèle est sous-tendu par la relation factorielle

$$R_i = \alpha_i + \sum_j \beta_{i,j} \cdot F_j + \varepsilon_i , \qquad (1.2)$$

où $\beta_{i,j}$ est la sensibilité du titre (ou portefeuille) de rendement R_i au facteur F_j et ε_i représente le risque spécifique de ce titre.

Le MEA porte principalement sur l'évaluation relative des actifs en tenant compte d'un ensemble de facteurs pour décrire le rendement relatif attendu des actifs. Le MEA suggère qu'il existe d'autres facteurs que le portefeuille de marché qui affectent le risque systématique. Ce modèle est plus souple et permet aux chercheurs d'étendre le MEDAF en y ajoutant des facteurs de risque autres que le marché. Chaque facteur peut être considéré comme ajoutant un coefficient bêta spécifique à une prime de risque spécifique. Cochrane (2011) comptabilise plus d'une centaine de facteurs proposés afin d'expliquer le rendement des actifs au cours des deux dernières décennies et constate que, de plus en plus, les chercheurs se consacrent à en trouver de nouveaux. En effet comme le MEA ne précise pas les facteurs de risque à considérer, le modèle s'ouvre à de nombreuses sources possibles de risque et de nombreuses recherches ont été menées afin d'identifier les facteurs potentiels. Récemment, Cooper et al. (2019) – et Bai and Ng (2002) avant eux – ont proposé un protocole pour identifier les facteurs de risque réels. Ils estiment les facteurs communs qui résument l'information à partir de l'ensemble des rendements boursiers et considèrent qu'une dizaine de facteurs communs sont statistiquement significatifs.

Alors que le MEA s'appuie sur le principe de non-arbitrage, un autre volet de

l'évaluation des actifs repose simplement sur l'ajout, de manière *ad hoc*, de facteurs systématiques (variables d'états), afin d'appréhender la part des rendements laissée inexpliquée – les *anomalies* – par les approches précédentes : c'est l'approche multifactorielle, dont l'exemple le plus célèbre est le modèle à trois facteurs de Fama and French (1992, 1993). Il ne s'appuie sur aucune théorie particulière mais se veut corriger certaines anomalies, dont l'effet de taille, non pris en compte par le MEDAF. En effet, les conclusions de Banz (1981) contredisent les conclusions du MEDAF selon lesquelles seul le risque systématique a un pouvoir explicatif des rendements attendus et ont souligné qu'à long terme, les petites entreprises obtenaient des rendements supérieurs à ceux des grandes entreprises et que par ailleurs le MEDAF n'expliquait pas les rentabilités excessives des entreprises de petite taille.

1.2 L'effet de taille : enseignements théoriques et empiriques

Récemment on observe un regain d'intérêts pour l'explication de cet effet de taille. Ainsi Asness et al. (2018) ont réexaminé certaines critiques émises contre l'effet de taille et montrent que celles-ci portent essentiellement sur la performance volatile des petites entreprises dites de faible qualité ou *junk*. Ils expliquent que si ces entreprises sont contrôlées à travers le facteur *Quality minus Junk* (QMJ) qu'ils proposent, une prime de taille beaucoup plus forte, plus stable et robuste dans le temps, y compris pendant les périodes où l'effet de taille semble disparaitre est alors obtenue. Ils montrent aussi que l'effet de taille n'est pas lié à la liquidité ni concentré sur les *très* petites entreprises et est robuste aux effets calendaires (Keim, 1983) et au choix de la mesure de taille (non fondée sur les valeurs de marché).

Ainsi le premier essai de cette thèse s'intéresse à cette problématique. Il faut dire que cette anomalie a fait l'objet d'une abondante littérature. En effet plusieurs hypothèses/critiques ont été avancées pour expliquer ce phénomène parmi lesquelles Chan and Chen (1988) qui considèrent que si le bêta est défini comme une variable explicative robuste, l'effet taille peut provenir de la forte corrélation systématiquement constatée entre les variables taille et bêta. Il est alors délicat de distinguer l'effet taille de l'effet bêta sur les rentabilités. *A contrario*, pour Chan and Chen (1991), un véritable effet taille peut exister et refléter la sensibilité des entreprises de grandes tailles aux fluctuations économiques.

Pour éviter des approches biaisées par la forte corrélation existante entre le bêta et la taille, Fama and French (1992) suggèrent de construire des portefeuilles par taille et simultanément par bêta reprenant ainsi la méthodologie développée par Fama and MacBeth (1973). Leur étude met alors en évidence la supériorité d'un modèle fondé sur la variable taille mesurée par la capitalisation boursière pour expliquer les rentabilités des actions.

Cependant, Berk (1995) conteste l'intérêt de rechercher une relation entre la taille de l'entreprise et les rentabilités observées selon la méthodologie de Fama and French. La taille mesurée par la capitalisation boursière est implicitement dépendante du cours de l'action et donc du risque de l'entreprise. Il stipule que la relation entre les variables associées à des anomalies et la rentabilité espérée n'est pas due aux caractéristiques opérationnelles de l'entreprise mesurées par ces variables (les re-

venus ou la taille de l'entreprise). Cette relation exprime plutôt une prime de risque théorique contenue dans les caractéristiques de "marché" de ces variables. Pour lui, il est par conséquent presque tautologique de rechercher une relation entre les rentabilités et la capitalisation boursière et plus généralement entre les rentabilités et des variables liées au cours des titres. La relation négative entre la capitalisation et les rentabilités ne possède pas de réel pouvoir explicatif mais présente un intérêt uniquement "confirmatoire". Berk (1995, 1997) montrent que des mesures de taille non dépendantes du risque ne présentent aucune corrélation avec les rentabilités ou la partie des rentabilités non expliquée par le MEDAF.

Pour Amihud and Mendelson (1986) une corrélation entre la liquidité et la valeur de marché peut expliquer les écarts de rentabilités constatées selon la taille de l'entreprise. Ainsi l'effet taille n'apparait donc pas comme une anomalie du MEDAF mais comme la confirmation d'une relation entre la valeur de marché, le taux d'actualisation des dividendes (une évaluation du risque), la liquidité et les *cashflows* futurs et que ces éléments limitent sérieusement la portée des résultats sur l'effet de taille défendu par Fama and French (1993, 1996).

Horowitz et al. (2000) étudient la persistance de l'effet de taille sur la période 1963-1981, trouvent une différence en termes de rentabilité annualisée de 13% entre portefeuilles et petites et grandes capitalisations tandis que cette différence devient négative et égale à -2% après 1982. Ils en concluent que l'effet taille a disparu et que ce dernier ne doit pas être considéré comme facteur de risque. Suite au constat des études sur la disparition de l'effet taille, van Dijk (2011) démontre que cette disparition est tributaire des chocs inattendus qui affectent la profitabilité des petites et grandes capitalisations. Cet auteur trouve que les petites firmes subissent des chocs négatifs importants qui affectent leur profitabilité à partir de 1980 tandis que les grandes firmes subissent des chocs positifs.

Pour résumer, de nombreuses études ont été menées pour expliquer (et constater la réalité de) l'effet de taille, néanmoins les explications ne font pas l'unanimité et sont souvent elles-même sujettes à débat. Par conséquent aucun consensus sur la raison sous-jacente de cette anomalie ne semble être trouvé. Nous contribuons ainsi au débat en cours en analysant la composition sectorielle des portefeuilles classés par taille. A notre connaissance, aucun article n'a encore établi un lien entre la prime de taille et une explication sectorielle. Pour combler cette lacune, dans le premier essai, nous testons si l'effet de taille est le fait de secteurs industriels spécifiques. En effet avec l'augmentation des firmes dans les bases de données utilisées pour étudier cet effet on constate un problème de concentration entre les déciles de taille qui n'existait pas à l'origine de cette anomalie. Par ailleurs nous étudions les implications empiriques de notre hypothèse sectorielle sur les modèles prenant spécifiquement en compte l'effet de taille au moyen, notamment, des facteurs SMB de Fama and French et QMJ d'Asness et al..

1.3 Repenser le lien marché - facteurs de risque

Il s'avère que, comme l'effet de taille, d'autres anomalies sont aussi observées dans la littérature résultant des critiques adressées au MEDAF. Basu (1977) démontre qu'en

classant les actifs selon le ratio valeur comptable sur valeur de marché, les rentabilités des actions de faible ratio (*value*) affichent un rendement moyen supérieur aux actions de fort ratio (*growth*); c'est ce que l'on appelle effet ratio valeur comptable sur valeur de marché (*value effect*) pris en compte par le facteur HML (High minus Low) introduit par Fama and French (1993). Jegadeesh and Titman (1993), entre autres, ont constaté qu'il était possible de dégager un rendement excédentaire en achetant les titres les plus performants sur les douze derniers mois et en vendant les titres les moins performants sur la même période. C'est l'effet *Momentum* associé au facteur WML (Winners minus Loosers), différence entre les rentabilités des titres gagnants et celles de titres perdants. Fama and French (2015) préconisent la prise en compte des critères de profitabilité et d'investissement des firmes. Par conséquent ils introduisent les facteurs RWM (Robust minus Weak) et CMA (Conservative minus Agressive) qui prennent en compte, respectivement, la différence de rentabilité entre les firmes à faibles et fortes marges opérationnelles ainsi que la rentabilité entre les firmes qui investissent d'avantage et celles qui investissent moins.

Au total, Harvey et al. (2016) ont recensé 316 anomalies pour lesquelles des facteurs potentiels sont proposés dans la littérature. Parmi ceux-ci on peut citer les plus connus dont le modèle à trois (cinq) facteurs de Fama and French (1993, 2015) intégrant en plus du facteur de marché les facteurs de taille et *value* (les facteurs RWM et CMA) ou le modèle de Carhart (1997) qui ajoute le facteur Momentum au modèle à trois facteurs de Fama and French. Ce sont des modèles *ad hoc* qui n'ont de légitimité qu'à travers leur réussite empirique.

Plusieurs hypothèses ont été avancées pour expliquer ces anomalies, parmi lesquelles la dimension asymétrique des distributions de rendements. Selon celles-ci, les facteurs précédents ne seraient que des succédanés permettant de rendre compte de l'impact de l'asymétrie des distributions de rendement – et autres non-linéarités – sur la valeur des actifs. De fait, il est établit que les investisseurs considèrent de façon distinctes leurs gains et leurs pertes. Roy (1952) a été le premier à plaider en faveur d'une mesure du risque fondée sur le concept de risque de pertes. C'est la logique qui sous-tend le *Roy's Safety First* Ratio. Markowitz (1959) lui-même propose, en lieu et place de la variance, une mesure du risque fondée sur le risque de pertes, la semi-variance, qu'il juge plus adaptée mais plus compliquée à utiliser opérationnellement. Ang et al. (2006) soutiennent que les entreprises dont le bêta mesure le risque de pertes systématique affichent des rendements moyens plus élevés qu'avec les bêta classiques. Lettau et al. (2014) constatent une prime de risque plus importante et significative pour les actifs sur devises en considérant le risque de pertes basé essentiellement sur la semi-variance proposée par Ang et al. (2006).

Depuis Arditti (1967), et même avant, le caractère non-normal de la distribution des rendements est accepté avec la nécessité de prendre en compte l'asymétrie. Rubinstein (1973) est parmi les premiers à proposer un modèle d'évaluation incluant l'asymétrie. Kraus and Litzenberger (1976) suivent Rubinstein (1973) et proposent le MEDAF étendu qui considère aussi l'asymétrie systématique pour confirmer la théorie soutenant la prise en compte par les investisseurs des moments d'ordres plus élevés que la variance. Harvey and Siddique (2000) proposent un modèle qui incorpore l'asymétrie conditionnelle. Ils soutiennent que l'asymétrie systématique devrait être rémunérée. Leur modèle leur permet d'expliquer en partie la dimension crosssectionnelle des rendements. Dittmar (2002) va plus loin en ajoutant l'aplatissement (kurtosis) en plus de l'asymétrie. Ses résultats suggèrent que la qualité de prévision des rendements s'améliorent lorsque l'asymétrie et l'aplatissement sont inclus dans le modèle tout comme Christie-David and Chaudhry (2001) qui plaident en faveur du MEDAF à quatre moments.

On trouve dans la littérature de nombreux articles proposant une explication des facteurs de risque par la biais de l'asymétrie des rendements. Ainsi Klein and Chow (2013) concluent que la Value (HML) et/ou le Momentum (WML) jouent un faible rôle dans la volatilité des actions, Ang et al. (2006) soutiennent que l'effet Momentum lui-même peut être lié à l'asymétrie et qu'une partie peut être expliquée comme la compensation requise pour l'exposition au risque élevé de pertes. Ils ont indiqué aussi que la prime en lien avec le risque de pertes était différente de la prime de risque associée à l'asymétrie. Harvey and Siddique (2000) voient également les facteurs SMB and HML comme une approximation de la coasymétrie tout comme Chung et al. (2006) qui défendent que la prise en compte de moments d'ordre supérieur a pour effet de réduire l'importance des facteurs de Fama-French. Hung (2007) montre que les effets de Momentum et de taille sont attribuables au risque de co-mouvements systématiques d'ordre supérieur.

Un autre versant de la littérature, enfin, porte sur le lien entre les facteurs de risque. On peut citer l'explication de l'effet de taille (SMB) par la liquidité avec Amihud and Mendelson (1986) ou Fama and French (2015) qui évoque une redondance possible entre les facteurs de leur modèle à cinq facteur notamment les facteurs HML et CMA.

Il semble donc pertinent de reconsidérer l'existence d'un lien entre les facteurs et même entre les modèles proposés. D'autant plus, comme le soutiennent Fama and French (2018a), que le choix entre des modèles concurrents est un défi ouvert vu la pléthore de facteurs qui pourraient être inclus dans un modèle. En effet il n'existe toujours aucun consensus sur le nombre ou la nature des facteurs et les relations entre facteurs de risque et leur impact sur le rendement cross-sectionnel des actifs. Ainsi dans notre deuxième essai, nous conduisons une étude de la relation entre le marché et les autres facteurs de risque en considérant, notamment, une approche non-linéaire de la relation du marché avec les autres facteurs de risque dans le but d'en réduire le nombre. Nous explorons également dans quelle mesure le lien entre le marché et certains facteurs de risque peut s'avérer profitable à travers des stratégies d'investissements.

1.4 Considérations sur l'horizon temporel

Les études sur les déterminants du rendement des actifs, particulièrement quand il s'agit de prendre en compte les effets des moments d'ordre supérieur, portent principalement sur le court terme. Cependant Cochrane (2011) considère les horizons à long terme plus intéressants, en ce qu'ils lient plus étroitement les grandes tendances du mouvement des prix à la volatilité sur le long terme. Bessembinder (2018) et Fama and French (2018b) ont récemment jeté un nouvel éclairage sur les propriétés des distributions de rendement à long terme et suscité un regain d'intérêt pour ce sujet. En effet ils ont montré que les moments d'ordre supérieur et particulièrement l'asymétrie et l'aplatissement étaient affectés par les effets de l'horizon en considérant la règle de la composition pour le rendement à long terme. Les travaux fondateurs de Merton (1973) ont fourni un cadre général pour comprendre les choix de portefeuille des investisseurs à long terme lorsque des opportunités d'investissement varient au fil du temps. Toutefois, jusqu'à récemment, les travaux empiriques sur le choix de portefeuille à long terme ont pris beaucoup de retard par rapport à la littérature théorique. Une des raisons de la lenteur du développement de ce champ d'étude a été la difficulté de résoudre le modèle inter-temporel de Merton. En outre, l'impact de l'horizon sur l'évaluation de la performance et l'allocation d'actifs a conduit à des conclusions et interprétations divergentes. Thorley (1995) et Hansson and Persson (2000) sont parmi ceux qui plaident pour une participation accrue au marché lorsque l'horizon de placement augmente alors que les opposants à cette théorie, comme Gunthorpe and Levy (1994) et Bodie (1995), préconisent plutôt une diminution de la part d'actifs risqués dans l'allocation. Ces derniers s'appuient principalement sur l'argument de Merton and Samuelson (1974) contre l'utilisation fallacieuse du théorème de la limite centrale pour justifier la diversification du risque alors que les premiers rejettent cet argument sur la base du bon sens et de l'indifférence quant à l'horizon de placement dans la théorie d'utilité espérée.

La même divergence peut également être constatée lorsqu'il s'agit d'examiner la mesure de performance des actifs. En effet, Zakamouline and Koekebakker (2009) soutiennent que l'une des mesures de performance les plus utilisées, à savoir le ratio de Sharpe, est une fonction croissante de l'horizon alors que van Binsbergen and Koijen (2017) et Madan and Schoutens (2018) affirment le contraire.

Il semble donc intéressant, à la lumière de ces considérations, d'examiner les questions d'allocation d'actifs et de mesure de performance sous l'angle de l'horizon temporel d'autant plus que Pastor and Stambaugh (2012b) ont conclu que la variance présente une tendance à la hausse lorsque l'horizon d'investissement considéré augmente, ce qui montre l'incidence de l'horizon sur la mesure de performance. Plus récemment, Bessembinder (2018) et Fama and French (2018b) montrent, en considérant la règle de composition pour les rendements d'actifs de long terme, que l'asymétrie devient de plus en plus positive avec l'horizon.

Ainsi, dans le troisième essai de cette thèse, nous mesurons l'effet de l'horizon sur les propriétés des distribution de rendements d'actifs. L'impact de l'horizon sur les mesures de performance ainsi que sur l'allocation d'actifs est aussi reconsidéré selon la préconisation de Bessembinder (2018) qui évoque la nécessité de "réévaluer les méthodes standards d'évaluation de la performance de la gestion des investissements". On examine si une explication, se basant sur l'impact de l'horizon sur l'asymétrie et l'aplatissement, pourrait être fournie pour réconcilier les visions apparemment opposées sur l'horizon à considérer.

Pour résumer, dans toute cette section nous nous sommes attelés à positionner les travaux réalisés dans cette thèse par rapport à la littérature existante pour ainsi montrer l'intérêt des problématiques dont nous semble-t-il le traitement apportera des contributions tant théoriques que pratiques. Nous synthétisons graphiquement



sur la Figure 1.1 l'articulation entre les différentes dimensions considérées.

FIGURE 1.1 : Articulation des Déterminants.

2 Méthodologie

Sur le plan méthodologique, nos travaux s'inscrivent très largement dans le cadre proposé par Fama and MacBeth (1973). Bien que déjà ancien et parfois critiqué (Shanken, 1985, parmi bien d'autres), ce cadre demeure aujourd'hui encore la référence dans notre champ d'étude.

La méthodologie de Fama and MacBeth repose sur deux piliers : la constitution de portefeuilles homogènes et une régression linéaire en deux étapes du rendement des portefeuilles préalablement constitués contre, premièrement, les facteurs de risques dont on cherche à évaluer la pertinence (régression longitudinale) puis, deuxièmement, contre les sensibilités aux facteurs – les $b\hat{e}tas$ – qui viennent d'être estimées (régression en coupe transversale) afin de déterminer les primes de risque associées à chacun de ces facteurs ainsi que leur significativité statistique.

Outre la détermination de la significativité *statistique* de tel ou tel facteur – et donc de l'anomalie qui lui est généralement associée – il est également important d'établir la significativité *économique* de cette anomalie. Pour cela, avant même de procéder à la régression en deux étapes proposée par Fama and MacBeth, il est important d'étudier l'écart de rendements entre les portefeuilles initialement constitués. Un écart significatif atteste d'une anomalie par rapport au MEDAF. Cette approche est largement, si ce n'est systématique, utilisée pour illustrée l'existence de l'anomalie étudiée (e.g. Fama and French, 1993; Jegadeesh and Titman, 1993; Frazzini and Pedersen, 2014; Asness et al., 2018). Il convient cependant de souligner que la significativité économique d'une anomalie ne se traduit pas toujours par la significativité statistique du facteur de risque qui peut lui être associé. Une telle anomalie n'en demeure pas moins intéressante d'un point de vue pratique dans la mesure où elle est de nature à capturer un rendement anormal.

Par ailleurs, la faible stationnarité des séries financières nous a conduit à appliquer la méthodologie de Fama and MacBeth sur des fenêtres glissantes. Cette approche est classique pour la constitution des portefeuilles homogènes qui doivent être rebalancés régulièrement, mais beaucoup moins pour la phase de régression en deux étapes et de tests de significativité qui s'ensuit. Cette approche nous est cependant apparue nécessaire dans la mesure où les portefeuilles, bien qu'homogènes, ne présentent pas des caractéristiques parfaitement stables dans le temps.

Conduire la phase de régressions, et de tests associés, sur des fenêtres glissantes suppose de tester des hypothèses multiples (une pour chaque fenêtre glissante) et requière une méthodologie adaptée. En effet, réaliser un test d'hypothèse (unique) au seuil de significativité de 5%, par exemple, signifie que l'on accepte de rejeter à tord l'hypothèse nulle une fois sur vingt (*i.e.* 5% pour temps). On comprend aisément que répéter le test plusieurs fois (cas d'hypothèses multiples) conduira à un taux de rejet bien supérieur à celui attendu. Il convient donc de corriger le seuil significativité du test pour tenir compte de la présence d'hypothèses multiples. Bonferroni (1936) fut le premier à proposer une telle correction, mais celle-ci est connue pour être trop sévère et nous avons préféré, chaque fois que nécessaire, avoir recours à la procédure de Benjamini and Yekutieli (2001) qui permet de déterminer une correction beaucoup plus fine et qui reste valide en cas d'hypothèses multiples mutuellement dépendantes, ce qui est notre cas dans la mesure où les fenêtres glissantes présentent un recouvrement important.

Enfin, l'originalité de nos travaux repose sur les méthodes de constitution des portefeuilles que nous avons retenues. Dans la littérature, les portefeuilles sont généralement constitués sur la base de critères (taille, profitabilité, qualité...) choisis *a priori* et "aisément" observables. Nous avons retenu une approche différente cher-

chant à tirer parti de certains développements récents en matière de traitement des données et d'apprentissage statistique (communément appelé *Intelligence Artificielle*).

Les méthodes statistiques standards, dont l'objectif est d'extraire l'information utile d'un ensemble de données, deviennent rapidement inefficaces dès lors que le nombre de variables considérées augmente. Dans le cas de la régression linéaire ordinaire, par exemple, la seule minimisation de la somme des carrés des résidus devient rapidement inappropriée lorsque le nombre de variables explicatives est important. Elle conduit, en effet, à des estimateurs très instables qui ne permettent plus de déterminer quelles variables sont véritablement significatives et donc d'opérer la sélection des variables. Plusieurs techniques statistiques ont été proposées pour remédier à ce problème, parmi lesquelles les méthodes relevant de l'apprentissage statistique qui constitue une classe d'outils permettant un meilleur traitement des données de grandes dimensions.

L'apprentissage statistique a pour objectif d'expliquer une variable dépendante (ou réponse) par un ensemble de variables indépendantes (ou prédicteurs). Dans la littérature, il existe plusieurs approches, linéaires et non-linéaires, paramétriques et non-paramétriques, pour estimer la fonction liant la variable dépendante aux variables indépendantes. Dans notre premier essai, nous avons recours au *Least Absolute Shrinkage and Selection Operator* (LASSO) développé par Tibshirani (1996) pour palier les limites de la régression linéaire en grande dimension. Le LASSO est une technique de régularisation qui cherche à minimiser la somme des carrés des résidus en respectant une contrainte de type ℓ_1 sur les coefficients. Cette méthode permet simultanément la sélection de variables et l'estimation des coefficients de régression. Le LASSO nous permet de sélectionner les secteurs industriels permettant d'expliquer la performance des portefeuilles de taille de Fama and French (1993).

Dans un esprit différent du LASSO, les modèles additifs constituent des méthodes de régressions non-linéaires bien connues pour trouver la relation optimale entre variables dépendante et indépendantes avec des données de grandes dimensions. L'objectif de ces modèles est de maximiser la qualité de prévision de la variable dépendante en opérant une transformation non-linéaire des variables dépendantes. En d'autres termes, au lieu d'estimer des paramètres simples (comme les coefficients de la régression dans une régression linéaire multiple), on cherche à déterminer la fonction qui permet de lier les variables indépendantes à la variable dépendante de manière optimale. Dans cette famille d'approches, l'*Alternating Conditionnal Expectation* (ACE) développé par Breiman and Friedman (1985), permet de définir la meilleure relation entre des variables indépendantes et une variable dépendante de manière non-paramétrique.

Nous avons recours à cette méthode dans notre deuxième essai. Elle nous permet d'étudier le lien entre le facteur de marché et certains autres facteurs de risque comme ceux proposés par Fama and French ou Carhart. Ainsi, pour chaque facteur de risque considéré, nous cherchons à établir la transformation optimale du facteur de marché que rend compte le mieux possible (au sens de l'erreur quadratique moyenne) de l'information commune avec le facteur de risque en question.

3 Contributions et implications de la recherche

Les recherches menées dans cette thèse nous semble de nature à présenter plusieurs contributions à la littérature existante sur l'évaluation des actifs et à générer des implications pratiques pour les investisseurs.

Le premier essai démontre que l'effet de taille peut être considéré comme partiellement le fait de certains secteurs industriels. Ce résultat émane des secteurs sélectionnés par le LASSO et jugés pertinents pour expliquer spécifiquement les portefeuilles constitués d'entreprises de petites tailles, d'une part, et de grandes tailles, d'autre part. Notre approche permet de mettre en exergue, par exemple, qu'autour des années 2000 les entreprises du secteur des nouvelles technologies suffisent à expliquer l'effet de taille. Il est aussi intéressant de noter que certains secteurs industriels sont sélectionnées autant pour expliquer le rendement des petites que les grandes entreprises. Ce résultat implique que l'effet de taille est probablement concentré sur un nombre limité de secteurs industriels. Nous montrons également que la prise en compte des portefeuilles de petite (grande) taille formés spécifiquement par ces industries impacte, au niveau cross-sectionnel, le facteur de taille SMB et a une influence certaine sur le facteur QMJ d'Asness et al.. En effet si l'on compare la différence de rentabilités anormales obtenues en considérant tous les facteurs des modèles de Fama-French et de Carhart ainsi que le facteur QMJ, on trouve des rentabilités anormales significativement supérieures pour les portefeuilles de tailles construits à l'aide du LASSO par rapport aux portefeuilles de référence de Fama-French. Celles-ci sont plus importantes, avec notre approche, sur la période complète de notre étude tout comme sur les périodes définies par Asness et al. (2018) (en particulier le Golden Age, de juillet 1957 à décembre 1979, durant lequel l'effet de taille est considéré comme le plus marqué). Par ailleurs alors que, classiquement, les entreprises de grande taille résistent mieux aux récessions, si l'on se focalise sur les périodes de récession ou d'expansion telles qu'elles sont définies par le NBER. la prime de taille est plus stable avec les portefeuilles formés avec notre approche. Ainsi ces résultats apportent un éclairage nouveau sur les déterminants de l'effet de taille et contribuent à une meilleure compréhension de cette anomalie.

Au fil des travaux réalisés dans le deuxième essai, nous avons pu mettre en lumière le lien entre le facteur de marché et certains facteurs de risque et les implications en termes de stratégies d'investissement. Cette étude fournit de nouveaux éléments sur la relation optimale définie comme asymétrique (non-linéaire) entre les facteurs de risque et surtout met en exergue la transformation, à travers l'approche ACE, du facteur de marché qui conduit à distinguer, sans *a priori* et de manière statistiquement optimale, les rendements de marché positifs des rendements négatifs. Ainsi nos résultats montrent que les facteurs de risque considérés notamment dans les modèles de Fama and French (1993, 2015) ou Carhart (1997), peuvent être en partie pris en compte de façon non-linéaire par le facteur de marché. Ces résultats font échos à l'argument selon lequel il y a une dissymétrie entre les rendements positifs et négatifs largement répandu dans la littérature. Ainsi à l'image de Frazzini and Pedersen (2014), avec leur facteur "*bet against beta*" (BAB) proposé pour profiter de rentabilité anormale entre les portefeuilles classés suivant leur bêta, nous classons aussi les bêta (et semi-bêta comme dans Post et al. 2009) suivant les régressions conduites sur les parties négatives et positives du facteur de marché. Il est montré que l'exposition aux rendements positifs, d'une part, et (surtout) négatifs, d'autre part, du marché conduit à une rentabilité supérieure (et donc anormale) à celle du marché. En effet l'évolution dans le temps d'un investissement basé sur ces stratégies se montre très performant. Certaines spécificités ont été aussi découvertes au fil de notre étude et nous ont semblé pertinentes à rapporter notamment notre stratégie de classement suivant les bêta négatifs (positifs) délivrent des rentabilités anormales plus importantes en comparaison de la stratégie BAB défendue par Frazzini and Pedersen (2014). Ces résultats sont d'autant plus marqués que l'on considère les portefeuilles au préalable classés suivant la taille ou le Momentum.

Le dernier essai s'intéresse plutôt aux comportements de long terme des moments d'ordre supérieur et leurs implications en termes de mesure de performance et d'allocation d'actifs. Les travaux menés nous permettent d'aboutir à différents résultats. La première contribution est l'explication de la positivité, sur le long terme, de l'asymétrie des distributions de rendement par l'effet de composition ainsi qu'une confirmation de l'effet de l'horizon temporel sur le risque relatif des actifs financiers. Nous montrons que la composition joue un rôle important dans les caractéristique de la dynamique à long terme des rentabilités. En effet nous démontrons, théoriquement, que la principale raison de l'asymétrie positive des rendements à long terme, observée empiriquement par Bessembinder (2018) et Fama and French (2018b), est le fait de la composition et que l'asymétrie à court terme n'est qu'un effet de second ordre. En d'autres termes l'effet de la composition est le principal moteur qui explique ce qui peut être considéré comme un fait stylisé à savoir la positivité de l'asymétrie des distributions de rendements à long terme. Ces résultats ont des conséquences importantes sur l'évaluation des performances et les décisions d'investissement, avec la nécessité de "réévaluer les méthodes standard d'évaluation des performances de gestion des investissements" (Bessembinder, 2018). Ainsi le deuxième apport de ce travail est de permettre une meilleure compréhension des divergences de points de vue sur le comportement temporel des mesures de performance ainsi que des arguments en faveur d'une plus ou moins forte détention d'actifs risqués sur le long ou court terme. En effet, la part des actifs risqués croit ou décroit en fonction de la prise en compte ou non de ces moment d'ordre supérieur dont l'importance augmente avec l'horizon d'investissement. Il semble que l'analyse de l'impact des moments sur la performance à long terme des actifs risqués tout comme le lien entre le comportement des moments sur le long terme et le problème de l'allocation optimale des actifs permettent de concilier des conclusions divergentes sur les avantages de la diversification temporelle.

Le travail mené dans cette thèse comporte plusieurs implications pratiques. A travers l'étude menée dans le premier essai, nous avons discuté de l'intérêt d'investir dans les petites entreprises de secteurs spécifiques. Investissement qui, par ailleurs, serait une solution potentielle qui pourrait profiter à tous. En effet la Commission européenne souhaite améliorer le financement de ce segment d'entreprise, notamment par le biais des marchés financiers, pour favoriser leur croissance et leur développement. Elle est ainsi en ligne avec la volonté du Conseil européen, qui a

par exemple agréé le 31 mai 2011 les conclusions sur le Small business act, où il souligne "le besoin d'aider les PME à avoir un accès renforcé aux marchés financiers et accueille favorablement l'intention de la Commission d'inclure des mesures visant les PME dans les révisions à venir de la règlementation des services financiers". En outre, selon un rapport de Morningstar (2014), l'investissement dans les entreprises de petites capitalisation s'est révélé plus bénéfique et plus rémunérateur aussi bien dans les portefeuilles des particuliers que des investisseurs institutionnels. Et toujours dans le même rapport il est stipulé que cette sur-performance est d'autant plus importante que l'horizon est lointain et cet investissement constitue une source de financement qui fait défaut à ce segment du marché financier. Ainsi la possibilité d'obtenir une meilleure performance sur un horizon de long terme constitue une problématique importante. D'une part compte tenu du vieillissement de la population et l'organisation du système de retraite et d'autre part les nouvelles règles prudentielles en l'occurrence Solvency II pour l'assurance et Bâle III pour la finance, la rentabilité et l'allocation de portefeuille des investisseurs ayant un horizon long constituent des sujets qui prennent de plus en plus d'importance. Cette dimension de l'horizon est abordé avec notre troisième essai.

Par conséquent les contributions présentées dans ce travail constitueraient à notre sens une aide pour le régulateur dans la mesure où elles apportent des pistes de réflexion sur ces questions.

4 Plan de la thèse

Le Chapitre 1 de la thèse présente une rapide synthèse de la littérature relative aux modèles d'évaluation des actifs en passant d'un cadre d'équilibre avec les modèles afférents à un cadre plus général s'appuyant sur l'approche par actualisation (*Stochastic Discount Factor*) mais aussi en discutant l'approche par arbitrage ainsi que les approches multifactorielles tout en mettant en lumière le lien avec nos travaux. On y fait l'état des lieux de la littérature avec les critiques adressées aux modèles existants notamment la non prise en compte de certaines caractéristiques des rendements traduites par des anomalies.

Dans le Chapitre 2, nous reconsidérons une de ces anomalies, en l'occurrence l'effet de taille. Pour ce faire, à l'aide du LASSO de Tibshirani (1996), nous testons l'hypothèse d'une contribution sectorielle à la prime de taille. Nous étudions la composition des portefeuilles de petite et grandes taille selon les secteurs industriels et testons également si des industries spécifiques sont déterminantes dans l'explication de la prime de taille. Par ailleurs notre approche donne une nouvelle compréhension sur l'effet de taille qui reste un sujet très discuté en finance empirique.

Nous considérons, dans le Chapitre 3, d'autres anomalies et tentons d'établir une relation entre le facteur de marché et les facteurs de risque proposés pour prendre en compte ces anomalies. Ainsi nous utilisons l'approche ACE développée par Breiman and Friedman (1985) et montrons l'existence de relations non-linéaires entre le facteur de marché et les facteurs de risque qui conduisent à des stratégies d'investissement au rendement anormal.

Dans le Chapitre 4, nous passons brièvement en revue la littérature sur la mesure

de performance. Ceci nous a mené à considérer la généralisation, dans le Chapitre 5, de l'expression analytique des moments d'ordre supérieur tels que l'asymétrie et l'aplatissement de la distribution rendements composés en fonction de l'horizon d'investissement. Nous y analysons également l'impact de la composition sur la mesure de performance et l'allocation d'actifs.

La Conclusion générale – rédigée en anglais et en français tout comme la présente introduction – est l'occasion de synthétiser les enseignements de nos travaux, d'en examiner les limites et de présenter les implications de nos résultats en soulignant des pistes de recherches futures.

Il est aussi à préciser que l'ensemble des références bibliographiques auxquelles nous nous sommes référés tout au long de cette thèse est regroupé à la fin. Toutefois, chacun des trois essais comporte une section bibliographique qui lui est propre ainsi que des annexes présentant des éléments additionnels et, le cas échéant, les preuves détaillées de certains résultats.

General introduction

Introduction

The determination of the factors that explain the expected asset is a central issue in finance, both in terms of its obvious theoretical interest and from a practical point of view. It is indeed necessary for investors to make decisions based on expectations supported by theoretically relevant elements. However, as pointed out by Merton (1980), it is particularly difficult to estimate the expected return on an asset. The issue of the determinants of an asset's expected return is not resolved and remains an open question in the financial literature. Many models have been proposed, but no consensus has yet emerged. Almost all existing models are based on the positive-normative approach and can be grouped according to what Cochrane (2009) calls absolute (positive approach) or relative (normative approach) theories. However, Cochrane (2011) regrets that the models initially designed for Valuation are increasingly becoming asset *Return* models. Indeed, he argues that the models were initially intended to evaluate the price of an asset and thus to deduce a return. However the main focus of these models has become a search of abnormal return with the constant search for new explanatory factors.

The research presented in this thesis focuses on the determinants of asset returns from the perspective of the risk factors, the asymmetry of the distribution of returns and the investment horizon. Hence our research project is structured around these determinants. There are many reasons to focus on the risk factors, the asymmetry of the distribution of returns and the investment horizon as determinants or sources of explanations of expected returns, and we will try to present the most relevant ones in the following. The choice of this research theme stems from an economic and financial context, and follows the theoretical and empirical lessons of asset pricing and portfolio management, which remain very dynamic fields of research.

Thus our work aims to propose arguments on the determinants of asset returns such as the pricing factors, the asymmetry in the distribution of asset returns and the investment horizon by addressing the following research questions :

• Banz (1981) argued that in the long run, small (capitalization) firms outperformed the big ones and consider that the existing models did not explain the abnormal return delivered by these small firms. Several hypotheses, which will be discussed below, have been provided to explain this considered anomaly coined Size effect. Contributing to this literature, we study the relation (if their is any) between the firm sector and the size effect and also consider the implications of such a sectoral impact on pricing models.

- The size effect is an anomaly among others and these ones are taken into account in asset pricing models by risk factors. However Harvey et al. (2016) argue that it exists a plethora factors which might imply a redundancy between these pricing factors. Chung et al. (2006), among others, suggest that the size factor, for example, may only be a way to capture the asymmetry in asset return distributions. In other words, the size effect could be related to the asymmetric dimension observed in the asset returns dynamics. Fama and French (2015) also point out a potential redundancy between factors introduced in the latest version of their famous model. Thus, it seems interesting to reconsider the relation between the market and other risk factors and then study the implications of this relation for asset pricing models.
- While most studies in the asset pricing literature focus on short-term returns Cochrane (2011) argue that the long-term horizons are the most interesting because they tie the return predictability to volatility, "bubbles," and the nature of price movements. It is therefore interesting to consider the horizon effect on the properties of asset return distributions and particularly the higher moments in the long term and the resulting implications for portfolio management, performance measurement and asset allocation.

The three essays presented in this thesis give some answers to these mentioned research questions. Thus, the first essay suggests that some specific industries matter for characterizing the size effect. Many hypothesis have been provided to explain the size effect, including survival biases with an overestimation of small firms performance (Kothari et al., 1995), liquidity effects (Amihud, 2002) or behavioral explanations (Shefrin and Statman, 1994). However, this subject still remains examined with among other Asness et al. (2018) who argue a much higher, more stable and robust premium over time if small, low quality companies (junk) are controlled. Hence we propose to test whether the size effect is the result of some specific industries. To this aim, we use the Least Absolute Shrinkage and Selection Operator (LASSO) approach developed by Tibshirani (1996) which allows, on the basis of a penalized linear regression, the selection of relevant explanatory variables by imposing constraints on the regression parameters. This approach has been used by Feng et al. (2019) to study the cross sectional relation of risk factors in explaining returns. However, to our knowledge, this approach has never been used before to select the preeminent industries that characterize size firms. The results suggest that some specific industries matter for characterizing the small (big) size portfolio. It has been shown that some industries are more related to the size effect than others. Hence during the Internet bubble, New Technology (NT) industries are identified to mostly explain the size effect. These results have an impact on the determination of asset returns as it will be illustrated in the section presenting our contribution.

The size effect is an anomaly among others and many risk factors have been proposed to explain these anomalies. However, an increasing number of proposed risk factors is observed. On the one hand, Kan and Robotti (2009) consider that too many factors in a model can impact the model validity and on the other hand Harvey et al. (2016) identify more than 301 anomalies (factors) in the literature and conclude that their relevance should be examined.

Thus, our second essay aims to investigate the relation between the market and some considered risk factors. Hence, we consider the Alternating Conditional Expectation (ACE) method developed by Breiman and Friedman (1985) which consists of an estimation of the optimal transformations for both risk factors and the market in regression and correlation analysis. Therefore we use the risk factors considered in traditional asset pricing models, namely, the three (five) factor model of Fama and French (1993, 2015) and the four-factor model of Carhart (1997). The idea is to find the optimal transformation that gives a best relation between the market and a given risk factor. This follows the remark of redundancy between certain risk factors mentioned by Fama and French (2015) and the idea that the linearity assumed by the above mentioned models would not be sufficient to reflect the reality of the relation between expected return and risk factors (Rubinstein, 1973; Jurczenko and Maillet, 2006; Gu et al., 2018). The study of the relationship between factors in order to reduce their number is not new (Klein and Chow, 2013; Baker and Wurgler, 2006). However, our approach is the first to apply the ACE method to address these types of considerations. Thus, the ACE proposes a non-linear transformation of the market factor that leads to an *a priori* and statistically optimal distinction between positive and negative market returns, as proposed by Pettengill et al. (1995) on an ad hoc basis. In line with Ang et al. (2006) and Lettau et al. (2014), we also consider models that separately account for the positive and negative market returns to develop investment strategies.

In our third essay, we reconsider the question of horizon effect on asset return. This essay follows the articles of Fama and French (2018b) and Bessembinder (2018) who recently shed new light on the properties of long-term return distributions and generated renewed interest in the subject. In fact they showed that higher order moments, and particularly the skewness (asymmetry) and kurtosis (peakedness or flatness) were affected by the effects of the horizon. Considering the compounding rule we show that as we extend the investment horizon, the asymmetry in the distribution of returns becomes increasingly positive, which implies, for a long-term investor, the need to adapt his investment strategy. In addition, we analyzed the horizon effects on performance measures and asset allocation. Our results shed new light on long-term investment strategies divergent views expressed in the literature.

This general introduction includes a section that presents the theoretical framework of our work relative to the existing literature. A section in which we present our adopted methodology is also provided, as well as a section summarizing the main theoretical and empirical contributions of this thesis to asset pricing and portfolio management literature.

1 Theoretical framework and position of the thesis

In this section, we begin by presenting the theoretical framework on which the formulation of our problems relies on. This section covers the theoretical and empirical aspects of the absolute (positive) approach, including the CAPM as well as the relative (normative) approach with arbitrage pricing and multi factorial models. To improve the understanding of the purpose of our work, downside risk asset pricing models as well as models based on higher order moments are also discussed.

1.1 Theoretical framework

The study of asset pricing models has always been of great importance since these models are used in many areas of application, including asset return estimation, cost of capital determination, performance evaluation and portfolio selection. The introduction of the capital asset pricing model, the CAPM, by Sharpe (1964), Lintner (1965) and Mossin (1966) independently characterizes the beginning. In fact Sharpe (1964) argues that prior to the CAPM, there was no theory that linked the price of an asset to investor preferences, nor any characteristic of the assets that determined their prices. The absence of such a theory made it difficult to establish the relationship between the price and risk of an asset. He therefore proposed a theory of equilibrium price. It is the most well known and used model, which aims to answer structural questions and describe the economy in order to explain why asset prices are what they are. Cochrane (2009) considers this perspective of asset pricing theory to be absolute in the sense that this type of models aims to explain what asset returns should really be as opposed to relative asset pricing models that attempt to be close to empirical observations. The CAPM reflects the mean-variance efficient character of the market portfolio at equilibrium and is defined by the relation :

$$E(R_i) = r_f + \beta_{i,R_M} \cdot E(R_M - r_f),$$

where R_i and R_M denote the return of a given asset *i* and the return of a hypothetical market portfolio which consists of all (value-weighted) assets. r_f is the free rate return. The scaling factor β_{i,R_M} is the covariance between the asset returns and the market returns normalized by the variance of the market returns.¹

In the CAPM, the expected excess return of an asset is therefore an increasing linear function of beta, which according to this model is the only source of risk to explain asset return. In this model, only the non-diversifiable risk associated with the market portfolio is remunerated. The CAPM appears to be a simple model, but admits several assumptions:

- All investors are single period risk-averse agents who maximize the expected utility of their terminal wealth and can choose among portfolios solely on the basis of their mean return and variance.
- There are no taxes nor transactions costs, all investors have homogeneous views regarding the parameters of the joint probability distribution of all security returns.
- All investors can borrow and lend at a given risk-free rate of interest.

¹The systematic risk of the asset, $\beta_{i,R_M} = \frac{Cov(R_i,R_M)}{Var(R_M)}$, is the sole source of risk in the model that cannot be diversified away by the investor.

However, a large literature suggests that the empirical results on the validity of the CAPM are mostly inconclusive. Black et al. (1972), for example, studied the empirical validity of the model and their results suggest that there is a systematic, statistically significant deviation from the CAPM. Sharpe and Cooper (1972) opposed the conclusions of Black et al. (1972) and reported findings in support of the CAPM as well as Fama and MacBeth (1973) who provide significant empirical support for the CAPM. However, Shanken (1985) observed that the accuracy of the parameters estimated in the Fama and MacBeth (1973) methodology is overestimated. MacKinlay and Richardson (1991a) show that the Generalized Moment Method (GMM) introduced by Hansen (1982) provides an unified framework for testing pricing models with more realistic assumptions.

Ferguson and Shockley (2003) show that the market portfolio should account for all traded financial assets to be efficient reinforcing the Roll (1977) critic on the market portfolio efficiency assumed by the CAPM.

Many attempts to modify or relax some assumptions of the CAPM have resulted in variations in this model. The zero-beta model developed by Black (1972) is among the most important extension and is more robust to empirical tests and has had an impact on the widespread adoption of the CAPM. The inter-temporal CAPM proposed by Merton (1973) is another important extension and allows for the inclusion of risk factors, such as macroeconomic and business-specific factors.

The CAPM equilibrium condition has also been considered as a too strong assumption. Thus the equilibrium assumption has been replaced by a non-arbitrage hypothesis which is a sufficient (but not necessary) condition for equilibrium. In this regard Ross (1976) proposes the use of arbitrage portfolios leading to the Arbitrage Pricing Theory (APT). The APT and multi factorial models then appeared. This model is defined by :

$$R_i = \alpha_i + \sum_j \beta_{i,j} \cdot F_j + \varepsilon_i \,,$$

with $\beta_{i,k}$ is the sensitivity of asset *i* to the factor F_k .

The APT takes into account a set of factors that describe the expected asset returns. This model is more flexible and allows to extend the CAPM by adding risk factors. Each factor can be considered as adding a specific beta coefficient to a specific risk premium. Cochrane (2011) reports more than 100 proposed factors over the past two decades and finds that new ones are still proposed. Since the APT does not specify the risk factors to be considered many possible sources of risk exist and identify potential factors remains challenging. Recently, Cooper et al. (2019) – and Bai and Ng (2002) before – proposed a protocol to identify relevant risk factors. They estimate the common factors that summarize information from all stock market returns and consider that there are ten common factors that are statistically relevant.

While the APT is based on the principle of non-arbitrage, another aspect of asset pricing is simply based on the addition, on an *ad hoc* basis, of systematic factors, in order to understand the proportion of returns left unexplained by the previous approaches: this is the multi factorial approach. On of the most famous example
is the three-factor model by Fama and French. It is not based on any particular theory but aims to account for among other the Size effect. In fact, the Banz (1981) conclusion pointed out that in the long term, small (capitalization) firms outperform big firms what is not accounted for by the CAPM.

1.2 The size effect: Theoretical and empirical evidence

We notice a recent renewed interest in the literature in explaining the size effect. Asness et al. (2018) have re-examined some criticisms addressed to the size effect and show that these ones are mainly due to the volatile performance of small (capitalization) firms particularly the low quality (*junk*) ones. They argue that if these firms are controlled through the *Quality minus Junk* (QMJ) factor they propose, a much higher, more stable and robust size premium over time, even during periods when the size effect seems to disappear, is then obtained. They also show that size effect is not related to liquidity or concentrated on very small firms and is also robust to calendar effects (Keim, 1983) and to the choice of size measure (not based on market prices).

Thus in our first essay we consider the size effect investigation which remains a debated subject with an abundant literature. In fact, several hypotheses/critics have been given to explain this anomaly, including Chan and Chen (1988) who consider that the size effect might be explained by the strong correlation systematically observed between the size and the market beta. It is then difficult to distinguish the size effect on profitability from the beta one. For Chan and Chen (1991), in contrast, a true size effect can exist and reflects the sensitivity of big firms to economic fluctuations.

Fama and French (1992) suggest to sort portfolios by size and simultaneously by beta. To this aim, they use the two-step methodology developed by Fama and MacBeth (1973) to avoid the strong correlation between the market beta and the size that might bias the results. Their study then highlights the superiority of a model based on the size variable measured by the market capitalization to explain equity returns.

However, Berk (1995) argues against the Fama and French methodology to measure the firm size. He considers that the size measured by the market capitalization is implicitly dependent on the asset price and therefore on the firm's risk. It states that the relationship between the variables associated with considered firms anomalies and the expected return is not due to the firm characteristics measured by these variables (size for example). Rather, this relationship expresses a theoretical risk premium contained in the "market" characteristics of these variables. For him, it is therefore almost tautological to look for a relationship between returns and the market capitalization and more generally between returns and firm characteristics related to the price of securities. The negative relationship between capitalization and profitability does not have any real explanatory power but is of purely confirmatory interest. Berk (1995, 1997) show that non-risk-based size measures do not correlate with profitability or the part of profitability not explained by the CAPM.

For Amihud and Mendelson (1986), a correlation between liquidity and the market

may explain the abnormal return observed for the size. Thus, the size does not appear to be an anomaly of the CAPM but rather a confirmation of a relationship between the market, the dividend discount rate (a risk assessment), the liquidity and the cash flows. Thus accounting for these elements seriously limit the importance of the results on the size effect defended by Fama and French (1993, 1996). Horowitz et al. (2000) study the persistence of the size effect over the period 1963-1981 and find a difference in terms of annualized return of 13% while this difference becomes negative and equal to -2% after 1982. They conclude that the size has disappeared and should not be considered as a risk factor. Following the findings of studies on the disappearance of effect size, van Dijk (2011) shows that this disappearance is dependent on unexpected shocks that affect the return of small and large caps. He finds that small firms face on significant negative shocks that affect their return from 1980 while for big firms the shocks are rather positive.

To sum up, many studies have been carried out to explain (and determine the reality of) the size effect. However in spite of the given explanations, this subject is still debated. Therefore, no consensus on the underlying reason for this anomaly seems to be found. We contribute to the ongoing debate by considering the hypothesis of a link between the industry sorted and the Fama and French size sorted portfolios. To our knowledge, no research has yet established a link between the size premium and a firm composition explanation. To fill this gap, in our first essay, we test whether the size is the result of specific industries. In fact with the increase firm number in the databases used to study this effect, a concentration problem – that did not exist at the origin of this anomaly – in the size deciles can be underlined due to the fact that the prominently listed firms in AMEX and NASDAQ are the concentrated ones in the lower decile portfolios. We also consider the empirical implications of our result on existing pricing models with the impact on risk factors such the SMB factors of Fama and French and QMJ of Asness et al. proposed to account for the size effect.

1.3 Rethinking the market - risk factors relation

It appears that, like the size, many anomalies are also observed in the literature resulting on critics addressed to the CAPM. Basu (1977) shows that by ranking assets according to the book to market value ratio, the returns of low ratio stocks (value) exhibit an average return higher than high ratio stocks (growth); this is called the book to market ratio (value effect) taken into account by the HML factor (High minus Low). Jegadeesh and Titman (1993), among others, found that an abnormal excess return could be achieved by buying the best performing stocks over the past 12 months and selling the worst performing stocks over the same period. This is the momentum effect associated with the WML factor (Winners minus Loosers), the difference between the returns of winning stocks and those of losing stocks.

Overall, Harvey et al. (2016) identified 316 anomalies for which potential factors are proposed in the literature. Among these the Fama and French (1993) threefactor model (five) including the size and value factors (in addition to RWM and CMA factors) with the market factor, and the Carhart (1997) model (adding the Momentum factor to the Fama and French's three-factor model) are well known. They are *ad hoc* models and are legitimated only through their empirical success.

Several hypotheses have been given to explain these anomalies, including the asymmetric dimension of return distributions. In fact, it is shown that investors asymmetrically consider losses and gains. Roy (1952) was the first to advocate for risk measurement based on the concept of downside risk. This is the logic behind Roy's Safety First Ratio. Markowitz (1959) proposed, instead of variance, a risk measure based on the downside risk, the semi-variance, which he considered more appropriate but more complicated to use operationally. Ang et al. (2006) argue that firms with a downside measured beta deliver higher average returns than conventional betas. Lettau et al. (2014) also find a higher and more significant risk premium by considering the downside risk based essentially on the semi-variance measure proposed by Ang et al. (2006) in the case of currency assets.

Since Arditti (1967), and even before, asset return distribution has been shown to be non normal, thus underlining the necessity to take into account the asymmetry. Rubinstein (1973) is among the first to propose an asset pricing model including the skewness. Kraus and Litzenberger (1976) follow Rubinstein (1973) and derive and test the extended CAPM model that includes systematic conditional skewness which is known as the 3-Moment CAPM. Harvey and Siddique (2000) also propose a model that incorporates conditional skewness and argue that systematic skewness should be rewarded. Furthermore their model allows them to explain in part the crosssection of asset returns. Dittmar (2002) went further with fourth-order moments and argued that, in addition to skewness, kurtosis has to be priced. He improved the CAPM by adding terms for skewness and kurtosis, and found that the tested multifactor models lose their significance in explaining the cross-section of stock returns. Along this line, Christie-David and Chaudhry (2001) provide empirical evidence in favor of the four-moment CAPM. They argue that investors with non-increasing risk aversion will prefer an investment with positive co-skewness.

There are many studies in the literature that suggest an explanation of risk factors through the return asymmetry. Klein and Chow (2013) conclude that Value (HML) and/or Momentum (WML) play a small role in equity volatility, Ang et al. (2006) argue that Momentum effect itself can be related to asymmetry and explained as the compensation required for exposure to the high downside risk. They also indicated that the premium for the downside risk was different from the risk premium associated with the return asymmetry. Harvey and Siddique (2000) also argue that the Fama and French SMB and HML factors can be seen as an approximation of asymmetry. Chung et al. (2006) argue that taking into account higher moments reduces the importance of the Fama and French factors. Hung (2007) shows that Momentum and size effects are attributable to the risk of higher order systematic co-moment.

However another trend of the literature focuses on the relationship between risk factors. Amihud and Mendelson (1986) link the size effect (SMB) to liquidity while Fama and French (2015) suggest a possible redundancy between the factors of their five-factor model, particularly between the HML and CMA factors. It therefore seems appropriate to reconsider the potential existence of a relation between the

risk factors and even though between the proposed models. Moreover, as Fama and French (2018a) argue, given the plethora of factors that could be included in a model, the choice between competing models remains a challenge. In addition there is still no consensus on the number or nature of factors and the relation between risk factors and their impact on the cross section of asset return and investment strategies. Thus, in our second essay, we investigate the relation between the market and other risk factors in order to reduce their number. We also explore to which extent the relation between the market and risk factors can be profitable through investment strategies.

1.4 Horizon consideration

Many studies on the determinants of asset returns, particularly when considering higher order moments, often focus on the short term consideration. However Cochrane (2011) argue for long-term horizons to predict return. Bessembinder (2018) and Fama and French (2018b) investigate the properties of long-term return distributions by considering the compounding rule for long-term return. They show that higher order moments and especially the skewness and kurtosis were affected by the effects of the horizon. Merton (1973) seminal work provided a general framework for understanding long-term investors' portfolio choices when investment opportunities vary over time. However, until recently, their is little empirical work on long term portfolio selection compared to the significant theoretical literature. One of the reasons for the slow development of this field was the difficulty to solve Merton's inter-temporal model. In addition, the long-term impact on performance assessment as well as asset allocation has led to divergent conclusions and interpretations. Thorley (1995) and Hansson and Persson (2000) are among those who argue for increased market participation when the investment horizon increases, while opponents of this theory such as Gunthorpe and Levy (1994) and Bodie (1995) advocate a decrease in the share of risky assets in the allocation. The latter rely mainly on the argument of Merton and Samuelson (1974) against the misleading use of the central limit theorem to justify the diversification of risk, while the former reject this argument on the basis of common sense and the indifference to the investment horizon.

The same divergence can be also observed when it comes to examine the asset return performance measurement. In fact, Zakamouline and Koekebakker (2009) argue that one of the most widely used performance measure, namely the Sharpe ratio, is an increasing function of the horizon while van Binsbergen and Koijen (2017) and Madan and Schoutens (2018) argue the opposite.

It therefore seems interesting, in light of these considerations, to examine asset allocation and performance measurement issues from a horizon perspective, especially since Pastor and Stambaugh (2012b) concluded that the variance tends to increase when the considered investment horizon increases. Bessembinder (2018) and Fama and French (2018b) also show that the investment horizon impacts the skewness and kurtosis of the return distribution.

Hence, in the third essay, we theoretically investigate the horizon effect on the

properties of asset return distributions. The impact of the horizon on performance measures and asset allocation is also considered consistent with Bessembinder (2018) recommendation, which refers to the need to "reassess standard methods for evaluating investment management performance". We examine if an explanation, based on the impact of the horizon on the skewness and kurtosis of the return distribution, could be provided to reconcile the apparently opposing views on the horizon to be considered.

To sum up, in this section we position our research relative to the existing literature in order to show both the theoretical and practical interest of our work. We graphically summarize in Figure 1.1 the articulation between the different considered dimensions.

2 Methodology

From a methodological point of view, our work is in line with the framework proposed by Fama and MacBeth (1973). This framework remains the reference in the asset pricing field despite many criticisms (Shanken, 1985, among many other).

The Fama and MacBeth methodology is based on the formation of homogeneous portfolios and a two-step linear regression of the return on these portfolios. The first step regression is conducted on the risk factors whose relevance is being assessed (longitudinal regression) and the second one on the sensitivities to the factors – betas – that have just been estimated (cross-sectional regression) to determine the risk premia associated with each of these factors and their statistical significance.

The statistical significance of a particular factor – and therefore the generally associated anomaly – is as much important as the economical significance of the anomaly. Hence, even before proceeding with the two-step regression proposed by Fama and MacBeth, it is important to study the difference in returns between the portfolios initially created. A significant deviation indicates an anomaly with respect to the CAPM. This approach is widely, if not systematically, used to illustrate the existence of the studied anomaly (e.g. Fama and French, 1993; Jegadeesh and Titman, 1993; Frazzini and Pedersen, 2014; Asness et al., 2018). However, it should be noticed that the economical significance of an anomaly does not always translate into the statistical significance of the associated risk factor. Nevertheless, such an anomaly remains interesting from a practical point of view since it captures an abnormal return.

In addition, due to the low stationarity of the financial time series we apply the Fama and MacBeth methodology with rolling windows procedure. This approach is classic for building homogeneous portfolios that are regularly rebalanced. However for the two-step regression phase and the subsequent significance tests this rolling windows approach is not common. In our case this approach was necessary because the portfolios, although homogeneous, do not have perfectly stable characteristics over time.

Performing a (unique) hypothesis test at the 5% significance threshold, for example, means accepting to wrongly reject the null hypothesis once out of twenty (*i.e.* 5% of the time). It is easy to understand that repeating the test several times



Figure 1.1: Asset return determinants relation .

(multiple hypotheses) will lead to a rejection rate much higher than expected. It is therefore appropriate to adjust the significance threshold of the test to take into account the presence of multiple hypotheses. Bonferroni (1936) was the first to propose such a correction, but it is known to be too conservative. Thus we use the procedure of Benjamini and Yekutieli (2001) which allows us to determine a more accurate correction in case of multiple mutually dependent hypotheses. The Benjamini and Yekutieli (2001) adjustment method, based on the procedure that controls the False discovery rate (FDR), seems to be well adapted for the test of the overall significance of the regression parameters.

Finally, the originality of our work is based on the portfolio construction methods that we choose. In the literature, portfolios are generally formed on the basis of criteria (size, profitability, quality, etc.) *a priori* chosen and "easily" observed. We have adopted a different approach seeking to take advantage of some recent developments in data processing and statistical learning theory (commonly known as *Artificial Intelligence*).

Standard statistical methods, which aim to extract useful information from a data set, quickly become ineffective as the number of considered variables increases. In the case of ordinary linear regression, for example, the minimization of the sum of the squared residuals quickly becomes inappropriate when the number of explanatory variables is large. It leads to very unstable estimators that no longer allow to know the variables that are truly significant and therefore to select them. Several statistical techniques have been proposed to address this problem, including statistical learning methods known to better process large data set.

The objective of a given statistical learning tool is to explain a dependent variable (or response) by a set of independent variables (or predictors). In the literature, there are several approaches, linear and non-linear, parametric and non-parametric, for estimating the function linking the dependent variable to the independent variables. In our first essay, we use the *Least Absolute Shrinkage and Selection Operator* (LASSO) developed by Tibshirani (1996) to overcome the limitations of linear regression in large dimensions. The LASSO is a regularization technique that seeks to minimize the sum of squared residuals by respecting a constraint of type ℓ_1 on the coefficients. This method allows the simultaneous selection of variables and estimation of regression coefficients. The LASSO allows us to select the industrial sectors that explain the performance of Fama and French size portfolios.

In a different spirit, additive models are well-known non-linear regression methods for finding the optimal relationship between dependent and independent variables with large data. The objective of these models is to maximize the predictive power of the dependent variable by performing a non-linear transformation of the dependent variables. In other words, instead of estimating simple parameters (such as regression coefficients in multiple linear regression), we try to determine the function that allows independent variables to be optimally linked to the dependent variable. In this family of approaches, the *Alternating Conditional Expectation* (ACE) developed by Breiman and Friedman (1985), allows to define the best relationship between independent variables and a dependent variable non-parametrically.

We use this method in our second essay. It allows us to study the relation between

the market factor and risk factors such as those proposed by Fama and French and Carhart. Thus, for each considered risk factor, we seek to establish the optimal transformation of the market factor that best reflects (in the sense of the mean squared error) the common information between the market and this risk factor.

3 Contributions and implications of the research

Several contributions emanate from this work and are of interest for many reasons. In fact each essay of this thesis highlights various contributions to the existing asset pricing literature and generates practical implications for investors.

The first essay shows that the size effect can be considered as partially due to some specific firms. In fact our results suggest that some specific industries matter for characterizing the small and the big capitalization firms. Overall, our results indicate that not all industries and, in fact, only a few matter to explain size portfolio returns over time and particularly the small and big cap portfolios. Around the 2000s, for example, internet related firms are identified and selected to explain the return on the small size portfolio. We also show that the specific industries selected by the Lasso are relevant, as much as the SMB or QMJ in explaining or/and accounting for the size effect. Particularly if we focus on periods of recession or expansion as defined by NBER, our Lasso small portfolio resists better than the Fama and French one during recessions. Thus, these results shed new light on the determinants of size effect and contribute to a better understanding of this anomaly.

We investigate in our second essay the relation between the market factor and some other risk factors and the implications in terms of investment strategies. Our results show that the risk factors considered in the Fama and French (1993, 2015) or Carhart (1997) models can be partially taken into account non-linearly by the market factor. In fact we provide new information on the optimal relation –between the market and some risk factors – that is defined as asymmetric (non-linear). Overall, we underline, through the ACE approach, the transformation of the market factor leads to an *a priori* and statistically optimal distinction between positive and negative market returns. These results are consistent with the argument that there is a widespread asymmetry between positive and negative returns in the literature. Frazzini and Pedersen (2014) proposed their bet against beta (BAB) factor to account for abnormal return between market beta sorted portfolios. In this line we also form beta sorted and downside (upside) beta sorted portfolios (as in Post et al. 2009). We show that the portfolios based on the downside market betas have consistently delivered positive returns. Particularly we show that our strategy based on the downside beta sorted portfolio delivers a higher abnormal returns – compared to the beta sorted portfolio in Frazzini and Pedersen (2014) – when the size or Momentum portfolios are considered. In sum, our results underline a real investment opportunity when accounting for the difference of sensitivities to the market upside and downside moves.

Our last essay focuses on the long-term behaviors of higher-order moments and their implications for performance measurement and asset allocation. The first contribution allows to make more precise the fact that the compounding effect is the main driving force that explains the increasingly positive asymmetry of the return distributions in the long-term. This positivity in the long term can be considered as a stylized fact. In fact we demonstrate that the main reason for the increasingly positive asymmetry of long-term returns, empirically observed by Fama and French (2018b) and Bessembinder (2018), is the compounding rule and that short-term asymmetry is only a second order. These results have important implications for performance assessment and investment decisions. Thus, the second contribution of this work is to reconcile the divergent views on the horizon impact on performance measurement and optimal asset allocation. In fact we relate the behavior of the long-term higher order moments to the problem of the performance measure and optimal asset allocation to show that theses ones are impacted in the long-term by higher moments accountancy.

Through the study carried out in the first essay, the importance of investing in small businesses is underlined. This is consistent with Morningstar (2014), in which the investment in small-capitalization firms has be proven to be more profitable. In addition the European Commission wishes to improve the financing of this segment, in particular through financial markets, to promote their growth and development. Hence we give arguments for the investment on small business since this investment would also be a potential solution that could benefit everyone.

On the one hand, given the ageing of the population and the organization of the pension system, and on the other hand, the new prudential rules, Solvency II for insurance and Basel III for finance, portfolio allocation for investors with a long horizon are increasingly important issues. Therefore the possibility of obtaining better performance over a long-term horizon is an important issue. Hence the horizon dimension is addressed in our third essay and some contributions are provided.

Consequently, the contributions presented in this work would, in our opinion, could be very useful to the regulator since some answers to the debated questions are provided.

4 Organization of the thesis

Chapter 1 of the thesis identifies and reviews the literature on asset pricing models, from an equilibrium framework to a more general framework based on the discount factor approach (*Stochastic Discount Factor* : SDF). Are also discussed the arbitrage and multi factorial approaches. In Chapter 2, we reconsider the study of size effect by considering the Lasso procedure introduced by Tibshirani (1996). We hypothesize that the firm composition can explain the size effect. Hence we study the composition of small (big) size portfolio by industry and also test whether these specific industries are relevant for explaining the size premium. Our approach gives new insight on the size effect which remains an empirical pricing debated subject. In Chapter 3, we consider some other anomalies – then risk factors – and try to establish the relation between the market and risk factors. To this aim we use the ACE approach developed by Breiman and Friedman (1985). Our results show the existence of non-linear relation between the market and the tested risk factors that lead to profitable investment opportunities. In Chapter 4, we briefly review the literature on performance measurement. Hence we consider the generalization, in Chapter 5, of the analytical expression of higher order moments of the compound return distribution. We also analyze the impact of horizon on performance measurement and asset allocation. The Conclusion – written in English and French, as well as this introduction – provides the opportunity to synthesize the results of our work, examine its limitations and present some implications for future research.

It should also be noticed that all the referred bibliography is grouped at the end. However, each of the three essays has its own bibliography and appendices containing additional results and when it is needed detailed proofs.

Chapter 1 Asset pricing literature review

Introduction

In this chapter we review the most important asset-pricing models encountered in the literature. Many different approaches have been introduced, going from equilibrium models to the stochastic discount factor framework and the no-arbitrage principle. The goal of this chapter is not to be exhaustive. We choose to review selected models in order to give as much an intuitive presentation of the central theories as possible while covering a large part of the asset pricing literature. In fact there is a huge literature available on the theoretical developments and the empirical evidence on their applicability in real world markets. Then we simply cannot explain every single models developed in the field of asset pricing. Hence in the scope of this chapter we choose to present some relevant examples to understand the theories that drive the pricing models and what are their relations.

We provide a review of the literature of the most important equilibrium models, starting with the well-known Capital Asset Pricing Model (CAPM). We also present related frameworks in which attempts have been made to take into account important empirical patterns. The downside risk based approach as well as the higher-order moment models have been showed to be of importance in the asset pricing literature since these models have been proposed to account for the CAPM empirical anomalies. Alternative frameworks, such as the Arbitrage Pricing Theory (APT) and the Multi-Factor models, also constitute an important part of this trend of literature. We also introduce the pricing kernel or Stochastic Discount Factor (SDF) framework which encompasses all the asset pricing models as showed by Cochrane (2009). We will use the terms pricing kernel and SDF interchangeably.

The first section of this chapter covers theoretical and empirical aspects of the equilibrium asset pricing approaches. The review of the literature presents the equilibrium models, starting with the CAPM and its related models. The CAPM is discussed on the basis of the critical literature dealing with the assumptions, the methodologies and anomalies. In the second section the downside risk based asset pricing models are presented to enlarge the understanding of our subject of investigation and particularly to account for the specific impact of negative returns. The third section develops the higher-moment models to account for some empirical

observations not taken into consideration by the CAPM. In the fourth section, the Arbitrage approach of asset pricing is presented together with some related factor models. The fifth part deals with the Stochastic Discount Factor framework from which almost all pricing models can be derived. We summarize all these literature in the sixth section to underline the relations between the models. The final section gives a summary and provides some comments based on the previous sections to describe the main argument or at least an important part of the thesis defended in the present study.

1 Equilibrium pricing models

Equilibrium models are designed to answer structural questions and describe the economy in order to explain why asset prices are what they are. Cochrane (2009) considers this perspective of the asset pricing theory as *absolute* in the sense that models aim to explain what should really be the asset prices as opposed to *relative* asset pricing models which try to be close to the empirical observations. In the following sections, we present the best-known equilibrium model, the CAPM, and also some related models.

The Capital Asset Pricing Model: A reference model

Among the equilibrium models, the most famous and (so far) most widely used model in asset pricing is the Capital Asset Pricing Model introduced by Sharpe (1964), Lintner (1965) and Mossin (1966). Sharpe (1964) argues that before the CAPM there was no theory that relates the price of an asset to the investors' preferences, neither were given asset attributes, that determine the asset prices. The absence of such a theory made it difficult to establish the relationship between the price and the risk of an asset. Hence Sharpe (1964) proposed an equilibrium pricing theory where it is assumed that (1) all investors are single period risk-averse agents who maximize the expected utility of their terminal wealth and can choose among portfolios solely on the basis of their mean return and variance, (2) there are no taxes nor transactions costs, (3) all investors have homogeneous views regarding the parameters of the joint probability distribution of all security returns, and (4) all investors can borrow and lend at a given risk-free rate of interest. As a consequence, the optimal portfolio is the same for all investors and, by virtue of the equilibrium, the market portfolio is mean-variance efficient.

The consequence of this results is the existence of a linear relation between the expected excess return and the systematic risk of an asset captured by the coefficient beta. This relation can be expressed as follows

$$\mathbf{E}[R_i] = R_f + \beta_{i,R_M} \cdot \mathbf{E}[R_M - R_f] , \qquad (1.1)$$

where R_i and R_M denote the return of a given asset *i* and the return of a hypothetical market portfolio which consists of all (value-weighted) assets. R_f is the risk-free rate return. The scaling factor β_{i,R_M} is the covariance between the asset returns and the

market returns normalized by the variance of the market returns.¹

The CAPM remains today a benchmark model in asset pricing, largely due to its simple and straightforward interpretation. It has also become one of the most widely used theoretical models to assess, among other things, the performance of individual assets as well as investment funds, portfolio diversification and for investment valuation.

However an important literature suggests that the empirical findings on the validity of the CAPM are mostly inconclusive. Among others, Black et al. (1972) investigated the empirical validity of the CAPM by time series regression. Under the null hypothesis that the CAPM holds, the intercept α_i in the regression equation

$$R_{i,t} - R_f = \alpha_i + \beta_{i,R_M} \cdot (R_{M,t} - R_f) + \varepsilon_{i,t}, \qquad (1.2)$$

should equal zero (as usual, $\varepsilon_{i,t}$ is the residual, assumed uncorrelated with $R_{M,t}$). Black et al. (1972) used monthly returns of stocks listed on the New York Stock Exchange (NYSE) over the time period from 1926 to 1966. Their findings suggested a misspricing with the intercept statistically significantly different from zero. In addition they observed that the intercept was time varying. They found that the intercept was negatively related to the beta: when the beta was greater than one the intercept was negative and when the beta was less than one the intercept was positive.

Sharpe and Cooper (1972) also tested the empirical validity of the CAPM for the period from 1931 to 1967 for stocks listed on the NYSE. They argued against Black et al.'s findings by reporting evidence of the stability over time of systematic risk for most of the studied stocks. Fama and MacBeth (1973) provided substantial empirical support for the CAPM. They developed the two pass cross-sectional regression method to examine whether the relation between the expected returns and the betas are linear. Betas are estimated using time series regression in the first pass and the relation between expected returns and betas are estimated using a second pass cross-sectional regression. They investigated stocks listed on the NYSE between January 1926 and June 1968 and reported findings that supported the CAPM. In details, they formed portfolios of individual stocks to test the empirical validity of the CAPM. They sorted individual securities to form these portfolios to overcome the problem of reduced beta range and statistical power caused by use of portfolio betas in the test of CAPM. They developed a testing methodology that employed a month-by-month cross-sectional regression to overcome the problem of auto-correlation in residuals from a single cross-sectional regression. On this basis, they reported a positive trade-off between risk and return and found that beta was the only relevant measure of risk for investors in asset pricing in comparison with risk measures like the standard deviation, the idiosyncratic volatility or the downside volatility.

Shanken (1985) observed that the precision of the parameters estimated in the Fama and MacBeth methodology is overstated. Indeed, the use of estimated betas

¹The systematic risk of the asset, $\beta_{i,R_M} = \frac{\text{Cov}(R_i,R_M)}{\text{Var}(R_M)}$, is the sole source of risk in the model that cannot be diversified away by the investor.

in the second pass introduces the classic errors-in-variables problem. Pasquariello (1999) revisited the Fama and MacBeth methodology utilizing the same data set and analyzed it with a modified econometric model. His findings suggest that although the relationship between risk and return on average was positive, non linear and non-beta measures of risk were priced by investors. Pettengill et al. (1995) refined the cross-sectional testing methodology of Fama and MacBeth by controlling for positive and negative realized risk premia through a dummy variable. They argued that the CAPM predicts positive risk premia. However, realized market risk premia over different periods are sometimes negative and sometimes positive. The usual cross-sectional methodology to empirically test the CAPM amounts to average all the cross-sectional time periods disregarding market ups and downs. They found evidence for a significant systematic relation between beta and expected returns whenever the market is upturns or downturns. If this relation between beta and returns is positive (negative) then the market risk premium is positive (negative) whereas in the traditional static CAPM setting, the expected market risk premium is a non negative number.

Gibbons (1982) showed that the classical maximum likelihood method can be used to estimate and test linear beta pricing models when stock returns are i.i.d and jointly normal. Ordinary least square (OLS) or General Least Square (GLS) or Weighted Least Square (WLS) have also been used in many empirically works in asset pricing (see Black et al., 1972; Fama and MacBeth, 1973; Shanken, 1992; Hodrick and Zhang, 2001). The Generalized Method of Moments (GMM), developed by Hansen (1982), has made econometric analysis of stock returns possible under more realistic assumptions regarding the nature of the stochastic processes governing the time varying nature of economic variables. In fact it provides an econometric framework that allows conditional heteroskedasticity and time dependence. Moreover, the GMM approach offers the important advantage to provide a unified framework for the testing of conditional linear beta pricing models as showed by MacKinlay and Richardson (1991b) who extend OLS statistic to account for non-normality and develop asset pricing tests using the GMM.

The CAPM is built upon the assumption that the market portfolio is meanvariance efficient. In fact, Roll (1977) criticized the CAPM tests on the basis that the market portfolio is not observable, so that they amount to test whether the *proxy* market portfolio is mean-variance efficient. Stambaugh (1982) investigated the empirical validity of the market Sharpe-Lintner-Mossin model and concluded that tests of the CAPM were not dependent upon the choice of proxy portfolio. So he could not report empirical evidence in support of the classic Sharpe-Lintner-Mossin CAPM.

Gibbons and Ferson (1985) relaxed the assumption of constant risk premium in the presence of an unobservable market portfolio (accounting for Roll's Critique) and investigated the empirical validity of the CAPM on the Dow Jones companies over the sample period from 1962 to 1980. Their findings were supportive of the CAPM with time varying risk premium. With a reverse-engineering approach Levy and Roll (2010), further expanded by Ni et al. (2011), argue that the market portfolio may after all be mean-variance efficient. They show that the difference between the values obtained with the sample mean returns and standard deviations of a portfolio that satisfies the mean-variance efficiency constraints and the theoretical values obtained from the CAPM, was lying within the statistical bound which made the market proxy not rejectable to explain asset returns.

Among the failure of the CAPM, its inability to explain some stylized facts in the cross-section of asset returns is largely reviewed in the asset pricing literature. In fact the findings of Banz (1981), among others, contradicted the CAPM which argues that only systematic risk has explanatory power for the cross-section of expected stock returns. Banz (1981) argued that over long time periods, small firms earn higher returns than big firms. This is known as the *size effect*, and size is measured as the natural logarithm of the firm's market capitalization.

Fama and French (1992) investigated the cross-section of expected stock returns and risks and noticed that stocks with low (high) betas were systematically under-(over-) priced. Their findings refuted the propositions entailed in the CAPM and failed to provide any empirical support for beta to explain the cross-section of stock returns. Among the most prominent new patterns in cross-sectional risk premia are a number of investment and profitability based anomalies. An investment anomaly can be broadly classified as a pattern in which stocks of firms that invest more exhibit lower average returns than the stocks of firms that invest less. The profitabilitybased anomalies refer to the evidence indicating that more profitable firms earn higher average returns than less profitable firms (Fama and French, 2015).

Among the assumptions of the CAPM, the possibility of unlimited lending and borrowing at the same risk-free rate is questionable. Black (1972) develop a model (the Zero-beta CAPM) in which this assumption is relaxed. This CAPM version was more robust against empirical testing and was influential in the widespread adoption of the CAPM. In fact Stambaugh (1982), for example, found support for this Black's version of the CAPM while reporting no support for the Sharpe-Lintner-Mossin version.

Other existing models also result in modifications and relaxations of the CAPM main assumptions. Some of the prominent extensions include the inter-temporal CAPM (ICAPM) proposed by Merton (1973). This model allows for multi-period and time varying investment opportunities. Then, the market portfolio is not the sole risk factor to be accounted for at the equilibrium. Additional macroeconomic and firm-specific factors also come into play.

Lucas (1978) and Breeden (1979) proposed the Consumption maximization based model (CCAPM). More recently Jagannathan and Wang (2007) reconsider this model to show that the CCAPM explains the cross-section of stock returns almost as well as the Fama and French model without substituting it. Cochrane (1991b) considers the Production as the variable to maximize before testing the Investment maximization Based CAPM (Cochrane, 1996). Mayers (1974) proposed an equilibrium framework in which some assets are allowed to be non marketable with Non-Marketable Human Capital while the inflation CAPM is introduced by Friend et al. (1976).

The literature review in this section suggests that the empirical findings on the validity of the CAPM are mostly inconclusive and the alternative equilibrium models

also face theoretical and empirical failures. The lack of empirical support for the CAPM has been mostly attributed to its too stringent assumptions and to the weakness of the empirical testing methodology.

Due to the empirical failure of the CAPM, alternative frameworks have been proposed. Markowitz (1959) himself recognized that investors care about downside risk and that asset returns may not be normally distributed. Therefore, he argued for the use of a downside risk based measure in making portfolio decisions. He suggested that downside risk can be measured in two ways: a measure of semivariance calculated from below mean deviations and a measure of semi-variance calculated from below target return deviations. He proved that when asset returns are non-normally distributed downside risk measures provide a better account of risk than the variance. The next section introduces this literature.

2 Downside-risk based models

The Semi-variance has been introduced for a better consideration of risk. In fact it is important to acknowledge the role of downside risks and account for the co-movements of assets with the market during downturns to understand the cross-section of stocks returns.²

Roy (1952) was the first who argued in favor of a risk measure that was based on the concept of downside risk. He pointed out that investors would have rational inclination to first look for ensuring the safety of their main investment and a minimum acceptable return, which can be the risk-free rate of return for instance. He argued that investors would favor investment with the lowest probability of achieving a return below the target. This is the logic behind the Roy's *Safety First Ratio*.

Hogan and Warren (1974) and Bawa and Lindenberg (1977) proposed a model based on this downside risk measure, namely the semi-variance. In fact, they underline the inappropriateness of the CAPM beta as a good measure of systematic risk. They choose the risk-free rate as threshold return, which allows them to distinguish losses from gains and define their downside beta.³ These models are developed in the Lower Partial Moment framework and allow them to take into account the asymmetric treatment of risk by specifying downside and upside betas. They account for both systematic risk measures as captured by the CAPM beta and by the downside risk betas. They report that these later perform at least as well as the CAPM. Jahankhani (1976) investigated the comparative ability of the mean-variance (MV-CAPM) and the mean-semi-variance (MS-CAPM) frameworks and concluded that this latter did not perform any different than the MV-CAPM. However, these findings have been thought to be biased as they were based on a short sample period. Price et al. (1982a) compared regular and downside beta effects on US stock returns

²The semi-variance measures the average squared downward deviation from a return threshold $(R_f, \text{ for instance})$ that separates losses from gains: $SV = \mathbb{E}\left[\left.\left(R - R_f\right)^2\right| R \le R_f\right]$.

³The downside beta is defined by Bawa and Lindenberg (1977) as the ratio between the second lower partial co-moment between the asset return R_i and the market return R_M and the second lower partial moment of the market: $\beta = \frac{\mathbb{E}[(R_f - R_M)(R_f - R_i)|R_M \leq R_F]}{\mathbb{E}[(R_f - R_M)^2|R_M \leq R_F]}$.

and showed that there is a systematic difference between these betas leading to underestimate (overestimate) the risk for low-beta (high beta) stocks.

Harlow and Rao (1989), among others, extended the earlier work of Hogan and Warren (1974) and Bawa and Lindenberg (1977) and generalized the Lower Partial Moment (LPM) framework by introducing an Asymmetric response model (ARM). They defined a semi-variance measure with a given threshold return to distinguish losses and gains.⁴ They argued that the use of the risk-free rate as investor's target rate of return is more a technical assumption than an assumption rooted in the economic theory. To overcome this deficiency they proposed the generalized Mean-LPM model which enables them to accommodate any investor's target rate of return. They proved that their model was consistent with utility functions exhibiting aversion to risk and preference for skewness. They provide empirical evidence in support of the generalized Mean-LPM model and pointed out that downside risk was more related to deviations below mean equity market return than the risk free rate.

Estrada (2002, 2004) also used a semi-deviation for measuring the total downside risk and introduced the Downside-CAPM model to explain the cross-section of Emerging Markets asset returns. This semi-deviation measure, instead of the classic variance, is used to define a beta which replaces the CAPM beta.⁵ Estrada (2002) empirically compared the MV-CAPM and the MS-CAPM on a sample of equity returns of both developed and emerging markets. The findings from the correlation analysis revealed that Downside-CAPM beta had the highest correlation with average returns and significantly outperformed the MV-CAPM. Furthermore he ranks the betas obtained with the CAPM and Downside-CAPM for both developed and emerging markets to form portfolios and shows that the difference between the average returns of low and high beta portfolios was larger for Downside-CAPM betas than for CAPM beta in these markets.

Estrada and Serra (2003) argued that practitioners are more interested in economic significance than statistical significance of return spreads between a high risk portfolio and a low risk portfolio. Following Fama and MacBeth methodology and the GMM estimation procedure, they analyzed different models classified into traditional, factor and downside risk families. Their findings suggest that the value effect (over-performance of high book-to-market stocks over lower ones) was sensitive to the sample period and outliers. However, they concluded that the relation between the risk variables and the cross-section of stock returns was weak. They explained this failure of the models to be the result of cross-sectional variations from country to country. However, the findings from their economic analysis revealed that global

⁴Harlow and Rao (1989) define the semi-variance below threshold k as $SV = E\left[\left(R-k\right)^2 | R \leq k\right]$ and introduce a downside beta as the regression coefficient of X in the regression model: $R_i = \alpha_i + \beta_{i,ARM}X + \gamma_i Z + \varepsilon_i$, where $X = R_M \cdot 1_{R_M \leq 0} + E\left[R_M | R_M > 0\right] \cdot 1_{R_M > 0}$ and $Z = R_M \cdot 1_{R_M > 0} - E\left[R_M | R_M > 0\right] \cdot 1_{R_M > 0}$ with R_M the market return and $1_{R_M \leq 0}$ is a downturn-market dummy (see Post et al., 2009, for further details).

⁵Estrada (2002, 2004) defines his downside beta as the ratio between the co-semivariance (or downside co-variance) of the asset return R_i with the market R_M return and the market semi-variance: $\beta = \frac{\mathrm{E}[\min(R_M, 0) \cdot \min(R_i, 0)]}{\mathrm{E}[\min(R_M, 0)^2]}$.

downside betas significantly explained the cross-section of stock returns. They show that a portfolio of high global downside beta stocks outperformed a portfolio of low global downside beta stocks both re-balanced every five years by an average 10% per year over a period of twenty years.

Post and van Vliet (2006) compared both unconditional and conditional MV-CAPM and MS-CAPM. Their findings reject the conventional MV-CAPM and strongly support the MS-CAPM. In particular the relation between downside risk beta and expected return is very good in bad times. They further report that downside risk betas have explanatory power for size portfolios as well as the momentum effect but share no relation with distress risk. Post and van Vliet (2006) also compared the CAPM and multi-factor pricing models based on Fama-French three and four factors and found that empirical support for the multi-factor models was sensitive to the sample period. They concluded that this evidence supports the data snooping hypothesis or factors like transaction costs, liquidity features of small stocks.

Ang et al. (2006) argued that firms with high downside betas experience high average returns. They report that risk premia captured by downside betas are different from the risk premium associated with the co-skewness.⁶ They point out that the downside betas do measure risk conditional on market downturns. However, the co-skewness measures unconditional relationship of stock returns and extreme market downside returns. They further report that, beside extremely volatile stocks, past downside betas have predictive ability for future downside market returns. They showed that the cross-section of stock returns reflects a premium for downside risk.

Post et al. (2009), starting from the semi-variance framework of Hogan and Warren (1974) and Bawa and Lindenberg (1977), propose a semi-variance beta. They show that the Asymmetric Response Model (ARM) of Harlow and Rao (1989) does not directly assess the co-variation between a bad market state and the downside risk premium because it is not a pure measure of downside risk. They also argue that the downside co-variance model (DC) of Ang et al. (2006) is based on a conditional measure of variance rather than a second lower partial moment. Then the implied pricing kernel of an equilibrium model that reconciles the mean return and the downside covariance beta would not always be positive and decreasing and may be inconsistent with the investor's non-satiation and risk aversion hypothesis. They also compare the spread between higher and lower portfolio returns among those sorted with downside beta criteria (theirs, ARM and DC). The spread obtained with their semi-variance beta is the largest one compared to the other beta rankings. They concluded that their model explains better the cross-section of US stock returns in comparison to others.

More recently Lettau et al. (2014) introduced a downside risk model (DR-CAPM) essentially based on the Ang et al. (2006) downside beta measure. They apply it to currency returns and find a significant risk premium related to downside risks.

To sum up, the literature review on downside risk models underlines the importance for investors to control for downside risk. However as much as the CAPM

⁶Downside beta is defined by Ang et al. (2006) as the standardized covariance between asset and market returns: $\beta = \frac{\text{Cov}(R_M, R_i | R_M \leq 0)}{\text{Var}(R_M | R_M \leq 0)}$.

fails to account for downside risk some others return features are not accounted for by this alternative approach. The empirical evidence on the linear risk-return relationship implied by the CAPM has been found to be very weak and in addition one of the necessary assumptions of the Sharpe-Lintner-Mossin CAPM is that asset returns are normally distributed. In fact, Arditti (1967), among others, provided empirical evidence showing that asset returns were non-normally distributed. To account for the non-normality in asset returns, the pricing models like the CAPM was extended to include higher order moments. These higher order moments included the skewness and kurtosis. Skewness measures the asymmetry around the mean of a distribution. Negative skewness suggests asymmetry towards negative values in the tail of a distribution while positive skewness suggests asymmetry towards positive values. Arditti (1967) argued that investors' risk aversion is negatively related with investors' wealth and hence causes investors' preference for positive skewness. A distribution may also be more or less peaked or flat relative to a normal distribution. Kurtosis is a measure of this relative peakedness or flatness of a distribution. Whereas skewness differentiates extreme values in one versus the other tail, kurtosis measures extreme values in both tails. Distributions with large (low) kurtosis exhibit tail data exceeding (that are generally less extreme than) the tails of the normal distribution. Hence risk averse investors prefer assets with lower kurtosis as well as the prefers assets with lower variance. The next section presents some relevant pieces of the asset pricing literature on the higher order moments framework.

3 Higher moments framework

Rubinstein (1973) argues that all the moments of the distribution of returns are significant for investors given that their utility function is not quadratic and asset returns are non-normally distributed. Jurczenko and Maillet (2006) argue that the worldwide success of derivatives, both as investments and risk management tool, active portfolio management and the existence of hedge funds provide evidence that investors are sensitive to moments of order higher than two, namely the mean and variance considered in the CAPM.

Due to the empirical failure of the CAPM, alternative models incorporating higher order (co-) moments have emerged. On the one hand, the CAPM is based on the first two moments of the return distributions while, on the other hand, Arditti (1967) argued that a systematic part of their skewness and kurtosis is non diversifiable. The systematic skewness (resp. kurtosis) risk is evaluated in the same way as the CAPM beta with the co-skewness (resp. co-kurtosis) instead of the covariance between the asset returns and the market returns. Samuelson (1970) demonstrated that, assuming that both absolute risk and trading interval are small, the meanvariance-skewness analysis framework results in a relatively better approximation to the expected utility than the approximation based on quadratic utility, that is the mean-variance framework.

Following this trend of literature, Rubinstein (1973) is among the first to propose an asset pricing model including the skewness. Kraus and Litzenberger (1976) follow Rubinstein (1973) and derive and test an extended CAPM that includes moments up to systematic co-skewness which is known as the 3-Moment CAPM. Their estimated coefficients for both the beta risk premium and the co-skewness risk premium were statistically significant and carried the appropriate sign to confirm the theory backing higher-moment CAPM. The coefficient for the co-skewness risk premium was negative and significant characterizing the investor's preference for positive coskewness meaning that investors want to increase their wealth.

Lim (1989) confirmed the earlier findings of Kraus and Litzenberger (1976) by providing empirical evidence. He divided his sample into ten sub-sample periods each consisting of five years. His findings suggest that investors favored co-skewness given a positive skewness of the market returns. However, investors do not prefer co-skewness given a negative skewness of the market returns. Harvey and Siddique (2000) also show that stocks with negative skewness are not desired by investors and then stocks with low (high) co-skewness tend to have higher (lower) average returns. They propose a model that incorporates conditional skewness and argue that systematic skewness should be rewarded. Their model allows them to explain in part the cross-section of asset returns and supports the idea that the Fama and French and Carhart factors, particularly SMB and WML, are proxies for co-skewness.

Dittmar (2002) went further with fourth-order moments and argued that, in addition to skewness, kurtosis might be priced. He improves the CAPM by adding terms for skewness and kurtosis, and confirms that Fama and French and Carhart factors lose their significance in explaining the cross-section of stock returns. His results show that the forecasting power improves when the skewness and kurtosis are included in a model. Along this line, Christie-David and Chaudhry (2001) provide empirical evidence in favor of the four-moment CAPM. They argue that investors with non-increasing risk aversion will prefer an investment with positive co-skewness given that the market might have positive skewness and investors' preference tilts towards investments that have small co-kurtosis. Chung et al. (2006) also found substantial evidence in support for the higher-moment CAPM. They reported that the inclusion of higher-moments results in reducing the Fama-French factors' significance. Hence they termed the Fama-French factors proxy for the higher systematic co-moments.

Nevertheless, some researchers as Post et al. (2009) claim that models based on cubic utility functions as those of Harvey and Siddique (2000) or Dittmar (2002) are not sufficiently flexible to predict strong enough downside risk aversion and lower preference for upside potential at the same time. In fact they show that for investors with high preference for skewness, the models based on cubic utility functions result on a large reduction upside potential with a small reduction in downside risk.

Agren (2006) extended the cumulative prospect theory (CPT) of Tversky and Kahneman (1992) by relating it with skewness and kurtosis. He assumed returns to be normal inverse Gaussian (NIG) distributed to accommodate skewness and kurtosis in addition of the mean and variance. The findings of the study revealed that investors' optimal portfolio allocation considerably changes when higher-moments were considered and hence portfolio allocations were not mean-variance efficient.

Hung (2007) investigated the empirical validity of the higher moment CAPM. He investigated the hypothesis that the four-moment CAPM should adequately explain the cross-section of stock returns given that portfolio returns were meanvariance-skewness-kurtosis efficient. To test this hypothesis he formed momentum and size portfolios of international stocks invested in the U.S. equity market. He also conducted out-of-sample empirical tests of the model on a recent U.S. data set and international stocks. His findings suggest that the four-moment CAPM outperforms the three-moment CAPM. The findings also reveal that the higher-moment CAPM is relevant for explaining the time series of returns on portfolios of winner and small size stocks. Furthermore this model was found to have the smallest prediction error among all the competing forecasting models (among which the CAPM).

Mitton and Vorkink (2007) as well as Barberis and Huang (2008) introduced models in which negative correlation between expected returns and idiosyncratic skewness is accounted for. They argue that, in order to preserve a positive asymmetry, skewness seeking investors choose to under-diversify their portfolio as it is observed in real life. Boyer et al. (2010) also find consistency between theory and empirical observations about this negative correlation between expected returns and idiosyncratic skewness.

Chabi-Yo et al. (2014) extend the mean-variance two-fund separation theorem to a three-fund separation theorem with skewness portfolio as the additional fund.⁷ They analyze the pricing of skewness risk for non myopic agent in an incomplete market and exhibit missing factors which justify the account of the pentosis, the fifth order moment, in pricing models.

More recently Langlois (2019) extends the CAPM in terms of a Generalized Asymmetric (GA)-CAPM in which he proposes a setup where the distribution of returns allows to incorporate systematic and idiosyncratic asymmetry components to account for return asymmetries. In equilibrium he obtains a three fund separation theorem in which systematic and idiosyncratic return asymmetries are taken into account. He proposes factors that capture these systematic and idiosyncratic asymmetries to explain the cross-section of stock returns.

Dahlquist et al. (2017) argue that Langlois (2019) does not consider investor's preferences and show that return asymmetries have only a marginal effect on the portfolio choice of investors with standard expected utility preferences and then some investors have non-standard preferences. They propose a model that accounts for both investors preferences and return asymmetries.

The importance to account for the higher moments in order to better describe some patterns of expected returns is well established. However an important strand of the asset pricing literature still questions the reason of the failure of market beta to explain cross-sectional variations in stock returns. Ross (1976) proposed using a simple non-arbitrage argument to weaken the equilibrium assumption underlying the CAPM. The Arbitrage Pricing Theory has emerged and multi-factor models have become widespread. These models are more versatile and allow the researcher to extend the CAPM by adding risk factors other than the market. The next section presents some related literature.

⁷Tobin (1958) argued that investors should only allocate their wealth between the free risk asset and a risky portfolio that maximizes the Sharpe ratio.

4 Arbitrage Pricing Theory and multi-factor models

4.1 Arbitrage Pricing Theory

In the wording of Cochrane (2009), the Arbitrage Pricing Theory (APT) is mainly about relative asset pricing: given a set of factors, what should be the *relative* expected returns of the assets in the economy. The APT suggests that there are factors, different from the market, which affect systematic risk. Since the APT does not specify these risk factors, the model opens up to many possible sources of risk and a lot of research has been conducted in order to identify the potential factors. To some extend, the APT can be seen as a fishing license to researchers who dedicate to find new risk factors.

Furthermore while the CAPM assumes that asset returns are normally distributed, the APT does not rely on any hypothesis on the nature of the distributions. The APT does not include any assumptions on individuals' utility functions either, but simply assumes that individuals are risk averse. This simplification of the assumptions allows the model to be validated empirically. The APT assumes that the return of an asset is generated by a multiple factors model. Each factor can be viewed as adding a specific beta coefficient and a specific risk premium. The return generating process is assumed to be given by:

$$R_i = \alpha_i + \sum_k \beta_{i,k} \cdot F_k + \varepsilon_i \,, \tag{4.1}$$

with $\beta_{i,k}$ is the sensitivity of asset *i* in relation to the factor F_k .

The model assumes that markets are efficient and that the factor structure of the asset returns is common knowledge. The number of assets is assumed to be very large compared with the number of factors. Arbitrage reasoning then allows to end up with the following relationship:

$$\mathbf{E}[R_i] - R_f = \sum_k \beta_{i,k} \cdot \lambda_k \,, \tag{4.2}$$

where λ_k is the risk premium of factor k. It is also the excess expected return of a portfolio with a sensitivity to factor k equal to 1 and a sensitivity to the other factors equal to zero, namely the excess expected return of a portfolio replicating the factor k.

This relationship explains the average asset return as a function of the exposure to the different risk factors and the market compensation for those factors. This formula shows that the CAPM is a specific case of the APT, as long as we assume that the equilibrium holds. We then simply need the market portfolio as the sole factor. The APT allows us to use several factors to explain the returns, which provides it an advantage over the CAPM. The market portfolio no longer has any particular role. It is simply one factor among many.

The risk factors are often represented as returns of factor replicating portfolios, macroeconomic factors or firm-specific factors. Since the theory is of no guidance regarding the number and the nature of the factors, this model has been the object of numerous empirical validation tests to identify the most significant factors. Chen et al. (1986) managed to identify a set of macroeconomic variables, which in their view, explained expected returns. They are: the growth rate of industrial production, the market risk premium, the spread between long-term and short-term interest rates, the spread between high-grade and low-grade bonds, and inflation. While it does not claim to have found the "true" set of factors for asset pricing, this article has since motivated a series of research (see Shanken and Weinstein, 2006, for instance). Recently Cooper et al. (2019) describe the broad cross-section of average stock returns based on the APT principles. They estimate the common factors that summarize the information from the broad cross-section of stock returns and consider that there are ten common factors that are statistically significant. They also claim to outperform the empirical horse-races in the literature when it comes to determine asset returns. The literature points out the necessity to rely on the use of three to five factors (Roll and Ross, 1980).

Although the APT seems to be an extremely appealing model, the absence of agreement among practitioners and academics regarding risk factors has limited its applications (Ameer, 2007). The lack of consensus on the number and the type of factors that should be included in a model remains one of the main shortcomings of the APT even if there are statistical procedures for consistently determining the number of factors from the observed data (Bai and Ng, 2002, among others). Additionally, the APT does not provide an economic explanation for the risk premium associated with each original source of systematic risk (the factors).

Among the existing factor models, some of them are proposed on an *ad hoc* basis, simply adding more systematic factors (state variables), to explain anomalies that cannot be taken into account by the CAPM. The most famous of these models is probably the Fama and French three factor model which accounts for the Size effect (Small firms outperform big firms on average) and the Value effect (value firms outperform growth firm). It is typical of a popular approach that amounts to design risk factor replicating portfolios such as the size factor with the SMB portfolio, the book-to-market ratio factor with the HML portfolio, the momentum factor with the WML portfolio, the liquidity factor with the IML portfolio. Despite their sometimes weak foundations, workhorse factor models are important for both academic research and investment management practice. We present some of them in the next subsection.

4.2 Factor models

4.2.1 Fama-French models

Banz (1981) shows that small stocks (lower market capitalization) tend to have higher average returns. The findings of Banz contradicted the CAPM which states that only systematic risk has explanatory power for the cross-section of expected returns. De Bondt and Thaler (1985) conclude that stocks with high book-to-market ratios have higher average returns (see also Fama and French, 1992, and later).

Fama and French (1992) investigated the cross-section of stock returns. Their findings contradicted the propositions entailed in the CAPM. They further extended their sample period. Even then they failed to provide any empirical support for the

CAPM beta to explain the cross-section of stock returns. However, they reported the size of the firm, its book-to-market equity ratio, its price-earning ratio and its debt-to-equity ratio as statistically significant at explaining the cross-section of stock returns.

All these observed features lead Fama and French (1993) to add the size (SMB) and the value versus growth (HML) to the market factor and introduce the most prominent multi-factor model, namely the Fama-French three Factor model (FF3F). In their view, small firms and stocks with high book-to-market (B/M) ratio tend to do better than the overall market. In fact they found that size and value factors when added on top of the overall market factor greatly improves the explanatory power of the CAPM on stock returns. The size factor captures the expected additional risk premium for holding small stocks as compared to large stocks while the value factor captures the expected additional risk premium for investing in firms with high B/M ratios as compared with firms with low B/M ratios.

However the size and value factors do not have strong theoretical foundations, even if Fama and French (1993) claim that their premia are related to firms' probability of distress. Later, Fama and French (1996) found that the FF3F is also able to explain the strong patterns in returns observed when portfolios are formed on earnings-toprice ratio, cash flow-to-price ratio, and sales growth. However the FF3F was found to leave many other important anomalies unexplained.

Daniel and Titman (1997) further investigated the claim of Fama and French (1992) that size and value factors have explanatory power for the cross-section of stock returns and that the risk premium for each of these firm characteristics arise because both size and value are surrogates for market-like risk factors which are not diversifiable. They failed to provide any support, however, in favor of this claim. They observe that the characteristics of the firm rather than the covariance structure have explanatory ability for the cross-section of stock returns.

More recently, Fama and French (2015) extend their three factor model (FF3F) in a five factor model (FF5F) with Profitability (RMW) and Investment (CMA) as additional factors. A restricted version of (FF5) that excludes HML is also proposed and becomes the four factor model (FF4F). They claim that this model is able to account for the observed investment anomalies among others.

Fama and French (2015) intend to study the interactions of CMA and RMW with their original three-factor model. Actually, they find that their value factor (HML) seems to become redundant for describing asset returns in the sense that RMW, and especially, CMA, seem to capture all the risk dimensions of HML: when adding CMA and RMW in their asset pricing model, the factor loading of HML is no longer significant. In fact, firms whose stocks have a high book-to-market ratio tend to invest less and also to be less profitable. Alternatively, firms whose stocks have a low book-to-market ratio tend to invest aggressively and be profitable. The factor HML thus interacts with CMA and RMW, this interaction being so high in their sample that it seems to render HML insignificant in their model. However, as mentioned by Fama and French (2015), the redundancy of HML may be attributable to their sample and other studies must be conducted to be more conclusive. Racicot and Théoret (2016) test the FF5F with a model that lies on CMA and RMW factors

on a sample of hedge fund strategies and find difficult to establish that most of the impact of HML is absorbed by the CMA.

4.2.2 Other factor models

Jegadeesh and Titman (1993) show that U.S. stocks keep doing well if it is has been the case in the past and this fact is known as the *Momentum effect* The stocks with good performance in the last months tend to sustain high returns in the short term and stocks with poor performance tend to have low returns in the near future. They study the strategies for buying shares of the first group, the winners, and selling shares of the second group, the losers, and test whether this strategy generates significant positive returns. The return differential between the winner portfolio and the loser portfolio comprises the momentum factor.

Based on Jegadeesh and Titman (1993), Carhart (1997) adds the momentum factor to the FF3F model, thus developing the four factor model and shows that it captures part of the variation in stock returns that is not explained by the size and value factors. Carhart (1997) four factor model, as much as FF3F, have been shown to capture a large part of the variation of stock returns in the US as well as a wide range of developed and emerging markets around the world. Nonetheless, due to the short-lived nature of the momentum effect, Fama and French (2004) argued that it is largely irrelevant for the estimation of the cost of equity.

Still in attempt to explain some of the CAPM anomalies the four-factor model of Pastor and Stambaugh (2003b) includes a stock liquidity factor. They consider an extension of the FF3F model in which the liquidity factor is added to price this return pattern. It is a four factor model including the liquidity which is defined as the degree to which an asset can be bought or sold in the market without affecting the asset prices. It is a well-known model to account for some of the CAPM anomalies and it performs well in explaining the cross-section of stock returns.

Malevergne et al. (2009) underlined the importance of an additional risk factor in a time series regression model, the Zipf factor. This factor accounts for the heavy tailed distribution of firm sizes. Without this factor, the diversification of idiosyncratic risks that underlies the derivation of the APT breaks down. They show that this Zipf factor is relevant and with the market portfolio form a twofactor model that performs equally well as, and sometimes even better than, the FF3F model to explain the cross-section of asset returns.

As we saw, the APT and the factor models testify of the amount of existing factors to account for the CAPM anomalies. Harvey et al. (2016) even review 316 anomalies proposed as potential factors in asset-pricing models and this illustrates the existence of redundancy between these factors.

Many alternative approaches have been proposed in asset pricing literature with the same purpose to determine the price of a given asset. Cochrane (2009) synthesizes all the asset pricing theory by what he calls the central asset pricing formula. He argues that most of the existing pricing theory results on the manipulations of this central formula. Hence he develops a general framework to reconcile or nest all these approaches. It is the Stochastic Discount Factor (SDF) or Kernel Pricing approach. We present it in the next section.

5 General framework : SDF

The pricing kernel or stochastic discount factor (SDF) is a key component of any asset pricing model (Cochrane, 2009). A kernel is a commonly used mathematical term to represent an operator. The term Stochastic Discount Factor (SDF) extends concepts from economics and finance to include adjustments for risk. There is a close connection between the terms pricing kernel and SDF which are often used interchangeably. It summarizes investor preferences for payoffs over different states of the world and gives a complete description of asset prices, expected returns, and risk premia.

In this framework it is shown that the price of an asset at time t, p_t , equals the expected discounted payoff, x_{t+1} , where m_{t+1} stands for the discount factor:

$$p_t = \mathcal{E}_t \left[m_{t+1} x_{t+1} \right] \,. \tag{5.1}$$

This approach allows to generalize all existing asset pricing models in a same framework and we get (Cochrane, 2009):

$$E_t [m_{t+1} \cdot r_{t+1}] = 0, \qquad (5.2)$$

where r_{t+1} stands for an asset or portfolio excess return over the risk-free rate.

The existence of a unique positive discount factor m_{t+1} is always granted in a complete market in which the law of one price and the no-arbitrage condition hold.⁸ Cochrane (2009) shows that the above mentioned models only differ in how the SDF is defined allowing to express all these models in the SDF framework.⁹ Harrison and Kreps (1979), as well as Hansen and Jagannathan (1991) among others, considered this framework to characterize investor's inter-temporal marginal rate of substitution.

Cochrane (1996) is one of the most important articles in this field of research, where the SDF methodology is explained in detail. Harvey and Siddique (2000) and Dittmar (2002) respectively consider a pricing kernel expressed in terms of a quadratic and a cubic function of the market returns in order to investigate the cross-section of asset returns.¹⁰ They both find that models based on nonlinear pricing kernels can significantly improve the linear pricing kernel in explaining the asset returns cross-sectionally. In option markets, Bakshi et al. (2010) use a model free approach to estimate the shape of the pricing kernel. They find that a U-shaped pricing kernel can account for the average returns of contingent claims. Chabi-Yo (2012) consider a stochastic discount factor approach by including additional terms to allow the recovery of pricing kernels that depend not only on the volatility of the

⁸Arrow and Debreu (1954) define a complete market as a market in which all relevant assets are priced appropriately and there is no way for one's gains to outpace market gains without taking on more risk.

⁹We have, for example, in the case of the CAPM: $m_t = a + b \cdot R_{M,t}$ and for the Fama-French three factor model: $m_t = a + b \cdot R_{M,t} + c \cdot SMB_t + d \cdot HML_t$.

¹⁰The kernel pricing considered by Harvey and Siddique (2000) reads $m_t = a + b \cdot R_{M,t} + c \cdot R_{M,t}^2$ while the kernel pricing chosen by Dittmar (2002) is given by $m_t = a + b \cdot R_{M,t} + c \cdot R_{M,t}^2 + d \cdot R_{M,t}^3$.

market return, but more importantly on the skewness and kurtosis of the market returns while Post and van Vliet (2006) consider this framework to account for downside risks effects.¹¹

As the pricing kernel is unknown, it is usually approximated. To test SDF based models, empirical researchers compare the performance of different asset pricing models by use, for example, of GMM methods (Dittmar, 2002; Cochrane, 2009) and of the Hansen and Jaganathan (HJ)-distance (Kan and Robotti, 2012).¹² The GMM and the HJ-distance metric are interesting in so far as they can be used whether or not the pricing model is linear in a set of systematic risk factors.

Despite its convenience, the SDF framework suffers from shortcomings. In fact, the pricing kernel is directly related to the representative agent's utility function. As a consequence, based on economic theory, the SDF should be a decreasing function of the state variables since the marginal utility is decreasing function of these state variables. However, Jackwerth (2000) and Rosenberg and Engle (2002) among others, find that the pricing kernel is not an overall decreasing function. In fact, they observe a locally increasing pricing kernel, implying a locally increasing marginal utility and a convex utility function which contradict the standard assumption that investors satisfy the non-satiation property and are risk averse. So, empirical findings suggest that the SDF is a non-monotonic function of underlying asset return and is time varying (Bakshi et al., 2010).

Post and van Vliet (2004) also argue that despite the standard asset pricing theory prevents arbitrage opportunities, a linear pricing kernel can not guarantee that the pricing kernel follows the non-arbitrage constraint (non negative). Thus the specification of an asset's return in a linear factor model itself restricts the specification of the pricing kernel making difficult the search for a good pricing kernel in a linear framework. In fact, linear pricing kernels assume constant volatility, which is problematic when it comes to evaluating, for example, option data. (Heston and Nandi, 2000; Christoffersen et al., 2013).

Because there is considerable debate among researchers over the state variables that enter into the pricing kernel, pricing kernel projections are interesting insofar as they can be estimated without specifying these variables. In fact the SDF is a state-dependent function that discounts payoffs using time and risk preferences and generally, it can depend on many (possibly unknown) state variables. Hence some researchers are interested in projecting the pricing kernel onto the payoffs of a traded asset and as discussed in Cochrane (2009), this projected pricing kernel has exactly the same pricing implications as the original pricing kernel. However this projected pricing kernel is not necessarily identical to the original one and among the admissible SDFs, there exists only one that is a function of available payoffs. To find it, several methods have been considered to infer it directly by using the fundamental asset pricing equation (Rosenberg and Engle, 2002) or indirectly by estimating first the physical and risk-neutral densities and then obtain the pricing kernel in a second step (Jackwerth, 2000) such a kernel from observed (option) market prices.

¹¹The downside SDF in Post and van Vliet (2006) reads $m_t = a + b \cdot R_{M,t} \cdot 1_{(R_{M,t} < 0)}$.

¹²Hansen and Jaganathan (1997) proposed a scalar measure of model misspecification, the HJdistance, which is based on the GMM.

Furthermore in a complete market, the pricing kernel is uniquely determined by the price of the traded assets. However, when the market is incomplete, the pricing kernel is not unique. How to select the optimal pricing kernel is a difficult issue as state it by Cochrane (2009) "choosing an optimal pricing kernel is equivalent to choosing optimal risk premia".

6 Summary and comments

The Chapter gave a review of the theoretical and empirical literature on the traditional CAPM and the related equilibrium models. Downside risk as well as highermoment based models and alternative frameworks as the APT and Multi-Factor models have also been reviewed with the aim to survey the asset-pricing models and position our research topic in relation to the literature. Finally this chapter provided an overview of the derivation of each of the asset pricing models under consideration. As outlined by Cochrane (2009), the asset pricing literature exhibits two main strands defined, on the one hand, as absolute and, on the other hand, as relative. The latter are part of the factor models or arbitrage approaches, while the asset-pricing equilibrium models such as the CAPM are part of the absolute trend. Cochrane (2009) also showed that the main pricing models developed with the absolute as well as the relative approaches could be summarized in an unique framework: the SDF. In the diagram 6.1, we summarize the asset pricing literature, moving from the initial equilibrium model to the discount factor framework. The diagram establishes the relation between existing approaches and allows us to give a global vision of the literature on asset pricing.

Our review of literature reveals that empirical evidence on the CAPM is not strong and mostly rejects the theory. Researchers have attributed the empirical failure of the CAPM to misspecifications of the model and limitations of the empirical testing methodology (Roll, 1977). The most important issues include the use of market proxies, biases in both time series and cross-sectional testing methodologies.¹³ However Elton et al. (2014) argue that despite changes in the assumptions, many conclusions of the original CAPM remain the same. This shows that the simple form of the CAPM is amazingly robust and might explain why academicians, researchers and professionals have particular liking for this model despite its weak empirical support.

We also review the downside risk based asset pricing models and outlined substantial theoretical and empirical evidence. It has been argued, in different articles (see Estrada, 2004, 2002), that the Mean-Semi-Variance framework is superior to the Mean-Variance framework and provide empirical support in favor of this model.

The literature review further suggests that the introduction of higher-moment models allows to account for the non normality of the asset returns distribution and empirical evidence on the higher-moment models is quite promising (Harvey

¹³Particularly the use of OLS as an estimation technique in the Fama-MacBeth cross-sectional two-step methodology has been criticized as it fails to account for auto-correlation and heteroskedasticity. The review of previous studies further suggests GMM to be an improved estimation technique in the Fama-MacBeth cross-sectional two-step methodology.



Figure 6.1: Asset Pricing literature Summary

and Siddique, 2000, among others). Particularly skewness should be priced in stock markets (Harvey and Siddique, 2000). Even though the empirical support for the higher-moment models is encouraging, there is no definitive evidence against the shortcomings they share with the CAPM, namely the size, value and momentum effects.

Furthermore the most important competitors of the CAPM revealed by the literature review are the factors models. However, as stated by Cochrane (2011), we notice "a zoo of new factors". Hundreds of factors have been proposed to explain asset returns during the past decades. Harvey et al. (2016) even identify 316 anomalies proposed as potential factors in asset-pricing models, and they notice that there are others that do not make their list. Fama and French (2018a) argue that given the plethora of factors that might be included in a model, choosing among competing models is an open challenge and there is no consensus on the number of factors or the type of factors in literature. Moreover, the relationships between all these risk factors and their impact on the cross-section of asset returns and investment strategies still let unexplained many pricing anomalies.

The present study investigates the link between downside risk, risk factors and higher-order moments. As reported by Kan and Robotti (2012), unnecessary factors have a large impact on the acceptance or the rejection of existing asset pricing models. They argue that, for the common factor models, the standard errors of the factors risk-premia are affected by the presence of an irrelevant factor. A reason might also be the commonalities between risk factors. In fact, Harvey and Siddique (2000) consider Fama-French factors as proxies for co-skewness and further demonstrated that this latter also explains the momentum effect (Jegadeesh and Titman, 1993). Klein and Chow (2013) apply orthogonal transform to other risk factors to specify the role of each of them. They conclude that some risk factors as Value (HML) or Momentum (MOM) play small role in explaining the stock return volatility.¹⁴ Ang et al. (2006) claimed that their downside risk effect is different from the impact of the co-skewness in Harvey and Siddique (2000). They argue that Momentum effect itself can be related to co-skewness and a fraction of it can be explained as the compensation required for the exposure to high downside risk (see also Post and van Vliet, 2006). Chung et al. (2006) suggest that the size and book-to-market factors of Fama and French (1996), may be proxies for higher-order systematic comoments of returns. They show that, adding a set of systematic co-moments (but not standard moments) reduces the explanatory power of the Fama-French factors to insignificance in almost every tested cases. In addition, Hung (2007) shows that both momentum and size effects are attributable to higher order systematic co-moments. Fama and French (2015) themselves call upon possible redundancy among factors.

Thus we reconsider the study of the relation between the market and the other risk factors by expressing several questions still in debate in the asset pricing literature :

- What are the links between risk factors?
- To which dimensions of the cross-section of stock returns are these factors

¹⁴They compare different approaches; among others, the Principal Component Analysis (PCA), the Gram-Schmidt (GS) process and the Schweinleir-Wigner/Lowdin symmetric procedure (SWL).

more correlated?

- Is the market the only factor necessary to properly describe the broad crosssection of stock returns?
- To what extent (and how) can we improve the current multi-factor models proposed in the literature in order to achieve a better description of large-scale cross-sectional risk premia?

In the next chapter, we will focus on the size effect to contribute to the ongoing debate on its reality and relevance by considering industry-specific effects. In fact, the size effect is gaining renewed interest with the recent work of Asness et al. (2018), among others, who argue to overcome the main criticisms levied against this anomaly. Then, in the following Chapter, with a more general consideration, we investigate the relations between the market factor and the main factors proposed in the literature to account for anomalies such the size effect and others.

Chapter 2

And if the size premium was driven by specific industries?

Résumé

Motivé par le récent regain d'intérêt pour l'effet taille dans la littérature financière, nous considérons l'analyse de la composition sectorielle des dix portefeuilles de Fama et French classés par taille. En ayant recours à la procédure de sélection LASSO, nous démontrons que l'effet de taille peut être considéré comme étant partiellement le fait de certains secteurs industriels jugés statistiquement pertinents pour expliquer spécifiquement les portefeuilles constitués d'entreprises de petites (grandes) tailles. Ainsi autour des années 2000 les entreprises du secteur des nouvelles technologies suffisent à expliquer le comportement des portefeuilles constitués d'entreprises de petites tailles. Nous étudions également les implications empiriques de notre hypothèse sectorielle sur les modèles d'évaluation prenant spécifiquement en compte cet effet. Au niveau cross-sectionnel, nous observons des rentabilités anormales significativement supérieures pour les portefeuilles de tailles construits à l'aide du LASSO par rapport aux portefeuilles de référence de Fama et French ainsi qu'un impact significatif sur les facteurs de taille (SMB) et de qualité (QMJ) proposés pour prendre en compte cet effet.

And if the size premium was driven by specific industries?

Abstract

Motivated by the recent surge of interest for the size effect in the financial literature, we analyze the composition of the ten Fama-French portfolios sorted by size in terms of industry sectors. Relying on the LASSO selection procedure, we demonstrate that the size effect can be considered as partly due to specific industrial sectors that are considered statistically relevant to explain portfolios of small (large) size firms. For example, around the years 2000s, the New Technologies sector was sufficient to explain the behavior of portfolios of small firms. We also study the empirical implications of our sector-based hypothesis on asset pricing models that specifically take the size effect into account. At the cross-sectional level, we report significantly higher abnormal returns for the size portfolios constructed using the LASSO compared to the Fama-French benchmark portfolios as well as a significant impact on the size (SMB) and quality (QMJ) factors proposed to take this effect into account.

Introduction

Sharpe (1964), Lintner (1965) and Mossin (1966) separately developed a pricing model, the CAPM, that supports a linear relation between the risk premium of any asset and the risk premium of the market portfolio. However, for the purpose of determining the expected return of small firms, this model fails to fit the empirical observation which is puzzling and has been coined the *size effect*. Banz (1981) and Reinganum (1981) were among the first to report that small firms earn higher returns than large firms on average. To explain this seemingly anomaly two trends of literature have opposed: on the one hand the tenants of a second source of priced risk (Fama and French, 1992, 1993) and on the other hand the supporters of a more fundamental conceptual challenge to the market efficiency (Roll, 1977). Yet, despite many papers dealing with the size anomaly, there remains much debate about the origins and the very existence of the size effect.

In this Chapter we contribute to this ongoing debate by questioning the composition of the Fama-French size-sorted portfolios with a particular attention on small and big firm portfolios. To this aim (a) we first hypothesize that some specific booming industries (high tech for example) explain the size effect (observed during the 2000's) and (b) we then test the resulting asset pricing implications of this hypothesis. It is established that the size premium mostly originates from microcap stocks and when the size effect was discovered, the size anomaly was only due to firms with small capitalization in the sense that they were small high-potential
and high-risk firms that captured a large premium. However, when taking into account the AMEX and NASDAQ listed firms in addition to the firms listed on the NYSE, the question of size portfolios composition seems to be legitimated.¹ In fact the prominently listed firms in AMEX and NASDAQ are the concentrated ones in the lower decile portfolios. Thus the initial rationale of limiting the influence of micro-caps and small stocks is no longer observed since these small (big) firms now represent 35 (5) % of the total size firm while it used to be 10%. Hence the study of the industry composition of size deciles with particular attention to the small and big ones might allow to make more precise the size effect explanation and to a larger extent allow to a better accountancy of the size effect.

On the one hand a large number of papers, among which Dichev (1998), Chan et al. (2000), Horowitz et al. (2000) and Amihud (2002), suggest the disappearance of the size effect since the early 1980s by reporting that small firms do not outperform big firms during the 1980s and 1990s. Alquist et al. (2018) also find that the returns of the size factor are far less stable, less persistent, and less robust compared to other exiting pricing factors such as the the HML which account for the value effect. Regarding the universe of US stocks, Cattaneo et al. (2018) find that the size anomaly is represented by a monotonically decreasing and convex relationship between returns and size, is highly significant, and is robust to different sub-periods including the period from 1980 to 2015. However they conclude that the size anomaly is not robust in sub-samples which exclude "smaller" small firms (*i.e.*, considering only the firms listed on the NYSE).

On the other hand, many authors argue that the size effect is still and has been relevant even during the period 1980-1990. Hou and Van Dijk (2019) show that although the size effect has disappeared from *ex post* realized returns after the early 1980s, there remains a robust size effect in *ex ante* expected returns. They also suggest at least three hypothesis to explain why the size effect disappeared: the absence of relation between size and expected returns, the non-systematic difference in cash flow shocks between small firms and big firms, and a genuine shift in the relation between size and expected returns in the studied samples. Asness et al. (2018) recently argue that after controlling for the quality of a firm, that they define as "a characteristic of an asset that, all else equal, commands a higher price", a significant size premium emerges with more stability over time and also explains the interactions between size and other return-related firm characteristics.

Along the line of several recent papers we use the *Least Absolute Shrinkage and Selection Operator* (Lasso), introduced by Tibshirani (1996), to select the preeminent industries that characterize small cap firms and, to a larger extent, the big caps also. The Lasso, as a machine learning tool for variable selection in high-dimensional settings, is becoming increasingly popular in finance with, among others, Feng et al. (2019) or Freyberger et al. (2018) who use it to analyze cross-sectional returns.

We use the monthly returns on 30 Industry and 10 Size portfolios from the Fama-French library over the sample period from January 1926 to November 2018. These

¹Stocks listed on the NYSE enters the Center for Research in Securities Prices (CRSP) database as of January 1926, stocks listed on the AMEX as July 1962 and stocks listed on the NASDAQ as of January 1973 (Center for Research in Security Prices, 2018)

data include all stocks listed on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX) and the NASDAQ. In addition to the full sample period (from July 1926 to November 2018), we also consider the three sub-periods defined by Asness et al. (2018) as the *Golden Age* (from July 1957 to December 1979) which coincides with the time when the size effect was the most pronounced, the *Embarrassment* (from January 1980 to December 1999) when it seems to have disappeared immediately after its discovery, and the *Resurrection* (from January 2000 to December 2012) when the size effect appears to revive. The NBER recession and expansion periods are also taken into consideration.

The results suggest that some specific industries matter for characterizing the small and the big capitalization firms. Overall, our results indicate that not all industries and, in fact, only a few matter to explain size portfolio returns over time and particularly the small and big cap portfolios. Around the 2000s, for example, internet related firms are identified and selected to explain the return on the small size portfolio.

We also focus on the interactions between the size effect and other variables that explain cross-sectional returns such as value, momentum and quality by estimating the risk-adjusted spread using factor analysis in order to control for the effects of other risk factors. To this aim we construct two portfolios, $L_{-}IND_{Small}$ and $L_{-}IND_{Big}$ (also noted $L_{-}IND_{Size}$ for short), with a methodology that follows Fama and French (1993) and Frazzini and Pedersen (2014). $L_{-}IND_{Small}$ (resp. $L_{-}IND_{Big}$) is an equally-weighted portfolio made of the industries selected by the Lasso to explain Fama-French small (resp. big) firm size portfolio over the last five years. We rebalance the two portfolios every three months. Given these two portfolios, we built the portfolio $L_{-}IND_{S-B}$ that goes long on $L_{-}IND_{Small}$ and short on $L_{-}IND_{Big}$.

On the basis of the Fama and French small and big size portfolios, our results confirm that small firm size portfolios outperform the portfolios of big firms during the *Golden Age* while other periods show mixed evidence. The raw spread between the portfolios $L_{-}IND_{Small}$ and $L_{-}IND_{Big}$ follows the same pattern. The risk-adjusted spreads provide results consistent with the models in which the factor QMJ of Asness et al. (2019) is accounted for. We obtain a significant spread for both the Fama and French size portfolios and ours. Moreover the spread obtained with the Lasso selected industries is the highest when the full period is considered. Thus the spread depends on the way the small and big size portfolios are obtained. Preliminary results suggest that during recession and expansion periods, the spread is more stable with our approach compared to the one with Fama and French portfolios. Particularly our Lasso small portfolio resists better than the Fama and French one during recessions.

To further test the relation between our $L_{-}IND_{Size}$ portfolios and other factors, we consider the cross-sectional regressions with the Fama and French and Carhart models. To this aim, we apply the Fama and MacBeth (1973) cross-sectional procedure to the 10 Size portfolios and add our factor $L_{-}IND_{S-B}$ to the competing models. Very interestingly, accounting for $L_{-}IND_{S-B}$ leads to a systematic reduction of the intercept (the alpha) in the cross-sectional regression (second step regression). More important, we find that the intercept becomes statistically insignificant during the Golden Age for most asset pricing specifications that encompasses the factor $L_{-}IND_{S-B}$ while this intercept is significant absent this factor. In addition we notice that $L_{-}IND_{S-B}$ is associated with a significant risk premium, even in the presence of the factors SMB or QMJ. Hence our results show that the specific industries selected by the Lasso are relevant, as much as the SMB or QMJ in explaining or/and accounting for the size effect.

The rest of the Chapter is organized as follows. In Section 1, we begin with a brief discussion of our motivation to investigate the size effect and the related literature. Section 2 first presents our hypothesis to link the size effect to specific industries and then the resulting analysis. We define our methodological approach and investigate the empirical pricing implications in Section 3. In Section 4, we present the crosssectional implications. The Appendix 2.A details the Lasso procedure while the Appendices 2.B, 2.C and 2.D provide additional results and robustness check.

1 Motivation and background

1.1 Preliminary consideration

This section provides motivation for our study of the exposure of the Fama-French (FF hereafter) small and big size portfolios to industry factors. The portfolio of small (resp. big) firms consists of the smallest (resp. largest) firms in terms of market capitalization and results from the ranking of firms in 10 deciles of size. Thus, the portfolio of small firms consists of firms in the first decile of the size distribution while the portfolio of large firms consists of firms in the last decile. The rationale for portfolio ranking is to determine whether the expected returns on assets are associated with certain firm characteristics. A natural and popular way to uncover the existence of such relations is to sort stocks by firm characteristics, form portfolios according to the sorted characteristic, and then compare differences in average returns across the portfolios. This methodological approach has found wide popularity in the empirical finance literature and is applied to study the size effect. However as mentioned by Beedles (1992) the vast majority of small firms that financial research has defined as "small", in relative rather than absolute value, are not particularly small. For instance, Reinganum (1981) studied both AMEX and NYSE firms while many others exclude the AMEX, which is considered as dominated by "small" companies (at least by Big Board standards).

Then, it is important to better characterize the composition of the size-sorted portfolios. In particular, the available equity data represents a highly unbalanced panel over the sample period usually considered in the literature. The sample size over time of both the total CRSP universe and then those firms who are listed on the NYSE, AMEX and NASDAQ is varying. At the beginning of the sample, in 1927, the CRSP universe includes approximately 500 firms, increases to nearly 8000 firms in the late 1990s, and currently comprises about 4000 firms. In Figure 1.1, we focus on the FF-Small and Big size portfolios and depict the evolution of the number of stocks in each of these portfolios (red line) as well as the percentage of the total number

of stocks in the CRSP they represent (blue line). As defined by Fama and French, the composition of the size-sorted portfolios is determined by the breakpoints of the size distribution of the NYSE listed companies. This choice explains the presence of two sharp jumps in the number of firms that occur in 1962 and 1972 and represent the addition to sample of firms listed on the AMEX and NASDAQ respectively. The maximum number of listed firms is reached in 1997. The number of small cap firms was about 80 in 1927, before it rises to a high of approximately 2760, and is currently slightly below 1200. The Figure 1.1 also underlines many interesting points on size portfolios composition evolution over time. In fact, as we can notice with the blue line, until 1962 the Small as Big size portfolios actually represented 10% of total number of firms, in accordance with the definition of the breakpoints of the size distribution of firms listed on the NYSE. Then after 1962 with the addition of firms listed on the AMEX, the small caps represented 35% of total number of firms while the big caps decreased from 10% to 6%. The introduction of NASDAQ firms even amplified this trend after 1972 with a Big size portfolio only representing 2% of the sample of firms and a Small size portfolio made of 60% of the firms of the sample. Today, these two portfolios are stabilized at 38% of the sample size for the Small caps and 5% for the Big caps, which is still far from the initial definition based on the deciles of the size distribution.

The initial rationale for maintaining the choice of the breakpoints of the size distribution of NYSE-listed firms, even after the introduction of the stocks listed on AMEX and NASDAQ in the sample, was to limit the influence of micro-caps and small stocks and therefore the variations in the composition of lower decile portfolios. However, this choice as led to a huge concentration of firms in lower decile portfolios. It is confirmed in Table 2.1 which presents the evolution of the Gini coefficient which is close to 0 before 1963 with only NYSE listed firms.² This concentration can be considered as an issue, all the more so that these firms concentrated in the lower decile portfolios are prominently listed on the AMEX and NASDAQ. Given the industry unbalance in the firms listed on the three major US stock markets (Goyal et al., 2008), the concentration of NASDAQ firms in the Small size portfolio can lead to a significant difference in the industry composition of the FF size-sorted portfolios, which could be responsible for a part of the size premium.

In Table 2.1, we summarize some features of the 10 FF-size portfolios. The size effect namely the higher return of small caps with respect to big caps is observed all along the considered period but with a varying level. We can also notice the evolution of both the firm numbers and capitalization over time.

1.2 Related literature

In this subsection we revisit some notable papers in the literature about the size anomaly and introduce the empirical relevance to consider the study the composition of the FF size-sorted portfolios. In fact prior research on the size effect has

 $^{^{2}}$ The Gini coefficient measures the degree of inequality in a given distribution with a value of 0 for a perfect equality and 1 maximal inequality among values.

Decile	Firm Number	Firm Number (%)	Firm Cap (M\$)	Mean Return (%)	Standard Deviation (%)							
-		Bet	fore 1963 (NYS	SE)	()							
1 -	81	9.74	3	1.77	13.63							
2	83	9.98	6	1.49	11.37							
3	83	9.98	9	1.36	10.17							
4	83	9.98	14	1.35	9.34							
5	83	9.98	21	1.20	8.77							
6	83	9.98	30	1.31	8.56							
7	84	10.10	45	1.13	7.94							
8	84	10.10	75	1.10	7.51							
9	84	10.10	145	1.04	7.23							
10	84	10.10	685	0.93	6.09							
SMB				0.20	3.38							
Gini	0.0005											
-	Between 1963-1973 (NYSE/AMEX)											
1 -	764	36.56	12	1.00	6.95							
2	225	10.77	33	0.68	6.11							
3	169	8.09	50	0.87	5.93							
4	150	7.18	71	0.83	5.68							
5	140	6.70	101	0.84	5.11							
6	134	6.41	143	0.71	5.01							
7	130	6.22	216	0.67	4.66							
8	128	6.12	356	0.69	4.35							
9	125	5.98	617	0.57	3.90							
10	125	5.98	2 806	0.64	3.26							
SMB				0.16	3.08							
Gini	0.32											
-	Be	tween 1973-19	997 (NYSE/AN	/IEX/NASD	AQ)							
1 -	2757	52.88	17	1.32	5.89							
2	628	12.04	67	1.36	5.93							
3	407	7.81	120	1.44	5.83							
4	313	6.00	194	1.43	5.66							
5	255	4.89	306	1.47	5.48							
:												

Table 2.1: 10 Size Structure

The table reports the summary statistics of the FF 10 size-sorted portfolios and the SMB factor for the period from July 1926 to November 2018. We present the mean (%) and Standard deviation (%) for SMB and each size portfolio. In addition, for these latter we give the number of firms they comprise and their capitalization (in Million \$). The Gini parameter is also given to illustrate the level of firm concentration in each portfolio.

Decile	Firm Firm I		Firm Cap	Mean	Standard
	Number	Number	(M\$)	\mathbf{Return}	Deviation
		(%)		(%)	(%)
-	Be	tween 1973-19	997 (NYSE/AN	MEX/NASD.	AQ)
:					
6	206	3.95	477	1.38	5.23
7	182	3.49	747	1.38	5.22
8	168	3.22	1 221	1.29	5.02
9	152	2.92	$2 \ 209$	1.24	4.73
10	146	2.80	8 749	1.12	4.38
SMB				0.24	2.74
Gini	0.55				
-		After 1997(NYSE/AMEX	/NASDAQ)	
1	2011	44.55	89	0.97	6.27
2	584	12.94	357	0.99	6.69
3	393	8.71	651	1.00	6.10
4	311	6.89	$1 \ 028$	0.89	5.83
5	253	5.60	1548	0.90	5.72
6	220	4.87	2 296	0.90	5.22
7	201	4.45	$3\ 451$	0.93	5.12
8	192	4.25	5 898	0.96	5.12
9	180	3.99	11 877	0.86	4.54
10	169	3.74	60 565	0.66	4.26
SMB				0.20	3.38
Gini	0.46				
-		Full Period	(NYSE/AMEX	(NASDAQ)	
1	1297	43.77	27	1.38	9.88
2	356	12.01	106	1.25	8.67
3	248	8.37	191	1.24	7.91
4	203	6.85	302	1.21	7.39
5	174	5.87	457	1.16	7.00
6	153	5.16	683	1.17	6.74
7	142	4.79	1 035	1.10	6.38
8	136	4.59	1 752	1.07	6.10
9	129	4.35	$3\ 451$	1.00	5.76
10	125	4.22	16 885	0.89	5.02
SMB				0.21	3.19
Gini	0.44				

Table 2.1: 10 Size Structure (C	Continued
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The table reports the summary statistics of the FF 10 size-sorted portfolios and the SMB factor for the period from July 1926 to November 2018. We present the mean (%) and Standard deviation (%) for SMB and each size portfolio. In addition, for these latter we give the number of firms they comprise and their capitalization (in Million). The Gini parameter is also given to illustrate the level of firm concentration in each portfolio.



Figure 1.1: Small and Big Size Firm number

The figure depicts the evolution of the number of stocks in the FF-Small and Big size portfolios (red line) as well as their percentage of the total number of stocks (blue line) listed on the NYSE, AMEX and NASDAQ for the period from July 1926 to November 2018.

produced mixed evidence. There is still much debate on the theoretical and empirical justifications for including a size premium in cost of equity estimates. Prior evidence does not yet lead to consensus conclusion.

Recently we notice that the studies on the size effect regained interest in the asset pricing literature (van Dijk, 2011; Alquist et al., 2018; Asness et al., 2018, among others) with no consensus for an explanation. van Dijk (2011) even state the need for more empirical research to examine the robustness of the size effect on US and international equity markets.

In the trend of literature on the size premium, we can find a set of studies arguing that firm size is a proxy for underlying risk characteristics, *i.e.*, that small cap and large cap firms' share prices respond differently to changes in risk factors. For example, studies claim that the size effect disappears if one adequately controls for beta risk, measuring betas using annual (rather than daily or monthly) returns (Handa et al., 1989), over longer periods (Chan and Chen, 1988), or using a broader market index including debt and equity claims (Ferguson and Shockley, 2003).

Similarly, Chan et al. (1985) show that part of the size premium disappears after controlling for observable macroeconomic risk factors. Furthermore, Chan and Chen (1991) provide evidence that is consistent with the idea that very small firms with low production efficiency and high leverage drive the size effect. As Jegadeesh (1992) points out, however, if firm size measures and observable risk characteristics are closely correlated, empirically disentangling the effects of size and risk on returns becomes a difficult task. Indeed, examining size portfolios that have (almost) equal betas, he shows that the size effect cannot be explained by beta risk. Given that small cap shares typically have lower liquidity and higher transaction costs than large cap shares, some studies test whether the small versus large share price return difference reflects investors' compensation for liquidity risk and/or transaction costs. Stoll and Whaley (1983) find that the size effect disappears after controlling for differences in transaction costs between small and large company shares.

However, Schultz (1983) shows that this finding is not robust to a change in the sample composition. Peek (2016) indicate that illiquidity (risk) is positively associated with returns (see also Datar et al., 1998). Nonetheless, some of these studies explicitly show that firm size remains significant in explaining returns also after controlling for liquidity (risk), suggesting that liquidity differences do not fully explain the size effect (van Dijk, 2011). So liquidity is only a partial explanation for the size effect (Asness et al., 2018). Lo and MacKinlay (1990) argue that the observed association between market value and realized returns may be an empirical irregularity uncovered through data snooping, as opposed to a theoretically robust finding. This risk arises because many studies examine overlapping samples of share price returns and are motivated by prior empirical findings rather than theory. To counter this argument some studies analyze the stability of the size effect over time (Horowitz et al., 2000) or the existence of the size effect on non-US stock exchanges (Bagella et al., 2000; Beedles, 1992; Chan and Chen, 1991), producing mixed evidence. A limitation of these studies could be, however, that a focus on shorter time periods or smaller non-US samples reduces statistical power and may produce unreliable results. Furthermore, if risk differences between small cap and large cap firms drive the size effect, it is plausible that large cap firms' share returns exceed small cap firms' share returns in some time periods or sub-samples (van Dijk, 2011).

These observations motivate us to re-examine the size effect on a recent sample. In fact early studies following Banz (1981) confirm the existence of an average size premium effect and document some of its peculiarities. For example, Brown et al. (1983) show that during their sample period (1967-1979) the effect is linear if the log of market value is used but is not stable over time (when investigating the shape of the relation between excess return and size). Further, Keim (1983) and Lamoureux and Sanger (1989) find that a substantial proportion of the size premium is earned in January. During the three decades following Banz (1981), researchers have challenged the existence of the size effect using various arguments.

Recently Asness et al. (2018) examine seven empirical challenges that have been hurled at the size effect – that it is weak overall, has not worked out of sample and varies significantly through time, only works for extremes, only works in January, only works for market-price based measures of size, is subsumed by illiquidity, and is weak internationally – and systematically dismantle each one by controlling for firms' quality. They show that previous evidence on the variability of the size effect is largely due to the volatile performance of small, low quality "junk" firms. They also argue that when these junk firms are controlled for, a much stronger and more stable size premium emerges. It is robust across time, including those periods where the size effect seems to fail; monotonic in size and not concentrated in the extremes; robust across months of the year; robust across non-market price based measures of size; not subsumed by illiquidity premia. Alquist et al. (2018) also examine many claims about the size effect and aim to clarify some of the misunderstanding surrounding it by performing simple tests using publicly available data. In the line of this trend of literature we want to contribute on this ongoing debate by questioning the composition of the size-sorted portfolios with particular attention on the small and big ones.

Furthermore, the industry portfolios are acknowledged to be useful for robustness test of empirical results. In fact as recently done by Asness et al. (2018) for confirming the role of the QMJ factor in resurrecting the size premium or Feng et al. (2019) and Freyberger et al. (2018) in explaining the cross-sectional impact of firm characteristics on returns, we rely on the 30 Industry portfolios available on Kenneth French' website to analyze the composition of the size-sorted portfolios.

To make more precise the rationale for our attempt to disentangle the size premium from any industry-specific factors, let us take the example of the internet related firms. In the 2000's ("internet bubble") the investment on internet related firms, mainly traded on the NASDAQ, exhibited abnormal returns. These tech startups were, at their beginning, small high-potential high-risk firms that captured a large premium. So, if as argued by many papers, as Asness et al. (2018), the size premium is resurrected after the 2000's, then the Internet related firms might explain this resurrection. A similar argument could be also given for the 70's and the 80's with Electronic bubble. Hence we consider that explicitly accounting for specific booming industries might allow us to make more precise the size effect explanation. The next section defines our main hypothesis to link industry composition and size effect.

2 Framework of the study

2.1 Methodology

To examine the reason of the persistence or not over time of the size effect, we hypothesize the existence of a relation between the Fama-French size-sorted portfolios and selected industry portfolios. We test if the size effect is due to a (small) set of specific industries composing the small and big cap portfolios. Hence to test the relation between small/large cap stock returns and industry portfolio returns, we consider the following multiple regression based on equation (2.1) hereafter, that relates small/big cap returns, namely R_t^{Big}/R_t^{Small} , to the returns obtained from a set of individual industry portfolios, $R_{i,t}$, where the 30 industry portfolios are indexed by $i = 1, \ldots, 30$:

$$R_t^{Big/Small} = \alpha^{Big/Small} + \sum_{i=1}^{30} \beta_i^{Big/Small} \cdot R_{i,t} + \varepsilon_t \,. \tag{2.1}$$

We cannot rely on the standard Ordinary Least Squared (OLS) regression since we want to select only the most important explanatory variables in order to identify the relevant industries (see Chinco et al., 2019). To address this challenge, and in the line with several recent papers, we use a machine learning tool called the *least absolute*

shrinkage and selection operator (Lasso). The Lasso introduced by Tibshirani (1996) is a powerful tool for variable selection in high-dimensional settings (Zhang et al., 2008; Bickel et al., 2009; Meinshausen et al., 2009). It avoids over-fitting problems by the introduction of a penalty function that removes all but the strongest explanatory variables.

Machine learning tools are becoming increasingly popular in finance. Feng et al. (2019), Freyberger et al. (2018), and Kozak et al. (2019) among others rely on the Lasso to analyze cross-sectional returns, while Chinco et al. (2019) analyze cross-firm return predictability at the one-minute horizon. Han et al. (2018) and Diebold and Shin (2019) employ the Lasso to select the individual forecasts to include in a combination forecast in an effort to refine the prediction of cross-sectional returns. They show that the individual forecasts selected by the Lasso provide insight into the relevance of individual firm characteristics over time and justify the real contribution of these procedures in asset pricing.³

Hence we apply the Lasso (detailed in Appendix 2.A) to explore the relation between small/big cap stock returns and industry portfolio returns. We can then analyze the relevance of certain industries to explain small and big size portfolio returns over time. To this aim we consider a rolling period of 60 months with lag of 3 months between two successive periods.⁴ For each rolling window, the Lasso estimate of equation (2.1) solves the following problem in the case of the large cap portfolio (for instance):

$$\arg\min_{\alpha,\beta\in\mathbb{R}} \left(\frac{1}{2} \sum_{t=1}^{T} \left(R_t^{Big} - \alpha^{Big} - \sum_{i=1}^{30} \beta_i^{Big} \cdot R_{i,t} \right)^2 + \lambda \sum_{i=1}^{30} \left| \beta_i^{Big} \right| \right).$$
(2.2)

The parameter λ is a regularization parameter. When $\lambda = 0$, equation (2.2) reduces to the familiar OLS objective function. The presence of λ in equation (2.2) shrinks the slope estimates (the β_i s) to zero and then tends to reduce the number of explanatory variables. As a consequence, the Lasso performs variable selection.

Many papers have been concerned with the choice of the penalty parameter λ required for the implementation of the Lasso estimator. As a result, several methods to choose λ have been developed and theoretically justified. For instance the Akaike Information Criterion can be used to select the optimal λ as done by Han et al. (2018). However, in practice, researchers often rely upon cross-validation to estimate λ (see Ghosh, 2012; Zou et al., 2007; Chatterjee and Jafarov, 2015; Hastie et al., 2015; Chinco et al., 2019). We choose to follow this approach which amounts to partition the sample of data into complementary subsets, perform the estimation on one subset and validate on the other in order to derive the most accurate estimates

³More general approaches, based on neural networks for instance, are also very promising. Chen et al. (2019) propose to estimate a non-linear asset pricing model with neural networks. They estimate the stochastic discount factor that explains all asset returns from the conditional moment constraints implied by no-arbitrage. They argue that their model outperforms outof-sample all other benchmark approaches in terms of Sharpe ratio, explained variation and pricing errors.

⁴Many papers also underline the relevance of the rolling window approach (see Lee and Chen, 1982, among others).

in terms of model prediction error.

2.2 Results of the Lasso selection

By examining the individual industries selected by the Lasso in equation (2.2), we can get a sense of how the number and the kind of relevant industry factors evolve over time. The top left panels of Figures 2.1-2.2 show a heatmap that represents the selected industry portfolios over time (the darker the color the larger the value of the corresponding β_i). The evolution of the number of selected industries over time is plotted on the bottom left panels. The top right panels represent the fraction of time a given industry is selected over the total sample period.

The results suggest that a limited number of specific industries matter for explaining the small and the big size portfolio returns. In addition, these industries evolve over time. For the full sample period, between 1926 to 2018, the Lasso identifies an average number of explanatory industries equal to 5 (resp. 11) for the small (resp. big) size portfolio. The number of selected industries is rather stable over time. We get an average number of significant industries equal to 6 (resp. 10) during the *Golden Age*, while it is equal to 3 (resp. 11) for the *Embarrassment* period and 7 (resp. 13) in the so-called *Resurrection* era. We also notice that the number of relevant industries is consistently smaller – and much smaller – for the small size portfolio compared with the portfolio of big firms. Besides, unsurprisingly some industries are relevant only to explain small caps, *e.g.* Apparel (Clths), whereas others only matters for big firms, *e.g.* Chemicals (Chems) or for both, *e.g.* Banking, Insurance, Real Estate, Trading (Fin).

Among the selected industries, certain are more persistent over time than others. A limited number of industries are pertinent at each point in time. Particularly we notice that there is much more variability among the selected industries to explain the small caps. For example, the Printing and Publishing (Books) industry is only selected in the *Resurrection* era (Figure 2.B.1 in Appendix 2.B). In comparison we can underline more stability in terms of persistence over time concerning the selected firms for the large caps with, for example, the Petroleum and Natural Gas (Oil) industry is relevant for all considered periods (Figures 2.B.2, 2.B.4 and 2.B.6 in Appendix 2.B).

It is also interesting to notice on top right panels of Figures 2.1-2.2 that the fraction of time an industry is selected varies. The Game Industry, in the 1970-80's (the *Golden Age*), and the Business equipment (BusEq) during the 2000's (The *Resurrection*), are two examples of particularly relevant industries to explain the returns of small capitalization stocks. It is worth noting that these industries where booming at that time and thus experienced large realized returns which corroborates our initial hypothesis. Overall, our results indicate that a limited number of industries matter to explain the returns of small and big cap stocks over time.



Figure 2.1: Lasso of Small Size on 30 Industry Portfolios in Full period.

The top left panel depicts a heatmap that represents the industry portfolios selected by the Lasso to explain the returns of the small size portfolio for the full sample period from July 1926 to October 2018 (the darker the color the larger the value of the corresponding β_i). The evolution of the number of selected industries over time is plotted on the bottom left panel. The top right panel represents the fraction of time a given industry is selected over the total sample period.

Figure 2.2: Lasso of Big Size on 30 Industry Portfolios in Full period.



Big Size

The top left panel depicts a heatmap that represents the industry portfolios selected by the Lasso to explain the returns of the large size portfolio for the full sample period from July 1926 to October 2018 (the darker the color the larger the value of the corresponding β_i). The evolution of the number of selected industries over time is plotted on the bottom left panel. The top right panel represents the fraction of time a given industry is selected over the total sample period.

3 Empirical implications

3.1 Lasso consideration

On the basis of the industries selected by the Lasso to explain the Fama-French small and big size portfolios, we form two equally-weighted portfolios, respectively named L_IND_{Small} and L_IND_{Big} . The construction of these portfolios follows Fama and French (1993), Frazzini and Pedersen (2014) or Asness et al. (2018). Considering the month of July of year t as the beginning of our testing period, we select the relevant industries to explain the return of the FF small and big size portfolios with the Lasso over the time period from July of year t - 5 to June of year t (60 months). We assign the average return of these industries to a portfolio – either L_IND_{Small} or L_IND_{Big} – which is kept during the three following months, namely July, August and September. We repeat this procedure on the following rolling windows (of 60 months) with three months lag and so on. According as we apply our selection procedure to the small or big caps, we form L_IND_{Small} and L_IND_{Big} portfolios. We also consider the long-short portfolio L_IND_{S-B} that goes long with L_IND_{Small} and short with L_IND_{Big} .

Table 2.2 reports the correlations between the returns on the Market portfolio, the SMB, HML, MOM, QMJ factors, and the returns on our three portfolios $L_{-}IND_{Small}, L_{-}IND_{Big}$ and $L_{-}IND_{S-B}$ over the full sample period from July 1957 to October 2018.⁵ We see that the returns on these three portfolios are strongly positively correlated. They also are positively correlated with the Market with, unsurprisingly, an (almost) perfect correlation between $L_{-}IND_{Big}$ and the market (correlation coefficient equal to 0.97). A bit more surprising is the large correlation between $L_{-}IND_{Small}$ and the market (0.87).

Since the long-short portfolio $L_{-}IND_{S-B}$ is, by construction, rather close to the factor SMB, it is sensible to compare their respective correlations with the other factors (*i.e.* Market, HML, MOM and QMJ). To the noticeable exception of the momentum, the correlations between $L_{-}IND_{S-B}$ and the others factors remain very close to the values observed for the correlation between SMB and the others factors which are all significant except the one with MOM. We also observe an insignificant correlation between the HML and QMJ.

3.2 Size premium measurement

The spread between small and big size portfolio returns captures the risk reward (or premium) for holding small with respect to big stock portfolios. It is a signature of the size anomaly in so for as no reward should be expected according to the CAPM. One way to measure the size risk premium for a given market is to compute the difference of the average returns during a given time period between small and big

⁵Our sample period starts from July 1957 due to the availability of the data for Asness et al. (2019) QMJ factor.

	MKT	SMB	HML	MON	ON?	LINDS-B	L-IND Small
SMB	0.29						
HML	-0.25	-0.19					
MOM	-0.14	-0.01^{\dagger}	-0.20				
QMJ	-0.51	-0.47	-0.06^{\dagger}	0.28			
$L_{-}IND_{S-B}$	0.41	0.45	-0.16	-0.30	-0.49		
$L_{-}IND_{Small}$	0.89	0.39	-0.22	-0.29	-0.57	0.76	
$L_{-}IND_{Big}$	0.97	0.23	-0.20	-0.19	-0.45	0.35	0.87

 Table 2.2: Correlation between Factors

The table reports the correlation between the Market portfolio, the SMB, HML, MOM, QMJ factors, and the portfolios $L - IND_{Small}$, $L - IND_{Big}$ and $L - IND_{S-B}$ over the full sample period from July 1957 to October 2018. All the reported correlations are statistically significant at the 5% level except those with [†] which are not.

stock portfolios. It is called the raw spread and is given by:

$$Raw_Spread = \overline{R_{Small}} - \overline{R_{Big}}, \qquad (3.1)$$

where $\overline{R_{Small}}$ and $\overline{R_{Big}}$ represent the average return during a given period of time of small and big stock portfolios respectively.

Many studies in the literature rely on equation (3.1) to compute the *ex post* size premium for a given market during a considered time period (van Dijk, 2011). However, using average raw returns to compute size risk premium is not accurate since raw returns result from the interaction of other common risk factors — such as the Market, HML or WML risk effects — which do not impact small and big portfolio raw returns in the same way. It is then important to control for other risk factors in order to isolate the pure size effect. For this reason, we use factor analysis in order to control for the effects of other risk factors on raw portfolio returns and thus measure the risk-adjusted size risk premia. Based on the factors proposed by Fama and French, Carhart and Asness et al., we perform the following regression

$$R_t^{Size} = \alpha^{Size} + \beta^{Size} \cdot Factors_t + \varepsilon_t , \qquad (3.2)$$

where R_t^{Size} stands for the returns on $L_{-IND_{Small}}$, $L_{-IND_{Big}}$ or $L_{-IND_{S-B}}$ at date t and Factors is a vector that contains the return on the Market portfolio, and different subsets of the following factors: SMB, HML, MOM and QMJ. The subset depends on the asset pricing model under consideration. In the following, we consider four alternative models: (I) HML and MOM; (II) SMB, HML, MOM; (III) HML, MOM and QMJ; and (IV) SMB, HML, MOM and QMJ. The parameter vector β^{Size} refers to the vector of the factor loadings. The idea behind the relation expressed by equation 3.2 is to determine the existence of abnormal returns by regressing the returns of the portfolios $L_{-}IND_{Size}$ against the factors. Then we investigate the impact of the size premium measured by :

$$Risk_Adjusted_Spread = \alpha^{Small} - \alpha^{Big}.$$
(3.3)

3.3 Size premium analysis

We present some preliminary statistical observations on the spreads between small and big size portfolio returns. Table 2.3 reports the average returns of small and big stock portfolios, as well as the resulting raw spread given by equation (3.1). These statistics are estimated on the basis of the whole sample data, that is over the time period from July 1957 to November 2018, but also on the three sub-periods defined by Asness et al. (2018) as the *Golden Age*, from July 1957 to December 1979, the *Embarrassment*, from January 1980 to December 1999 and the *Resurrection* from January 2000 to December 2012.

Table 2.3 reports the raw spreads obtained with equation (3.1) for the Fama and French small and big size portfolios (R_{Small}, R_{Big}) and for the portfolios obtained with the Lasso $(L_{IND_{Small}}, L_{IND_{Big}})$. For the Fama and French portfolios the results supporting the idea that the spread between small and large size portfolio returns is important in the US stock market are not confirmed over the whole sample period but only for the *Golden Age* with a significant positive spread. In comparison, the spread observed with the small and big size portfolios obtained with the Lasso, namely $L_{IND_{Small}}$ and $L_{IND_{Biq}}$, follows the same trend but with lower levels. In fact, for the *Golden Age* for example, the spread goes from 0.68% (with the Fama-French portfolios) to 0.38% (with the Lasso portfolios). It clearly illustrates the fact that a small number of selected industries captures a large part of the size premium during this time period. However the spread becomes insignificantly negative (-0.38%) during the *Embarrassment* period. Hence as argued among other Alguist et al. (2018) the size spread seems to be not stable over time. This instability is also confirmed when it comes to consider the *ex post* US business cycle expansion and contraction periods reported by the National Bureau of Economic Research (NBER). The results (reported in Table 2.9 in Appendix 2.C) show that the size premium also depends on the business cycle. For the FF small and big size portfolios, we notice a positive and significant spread during expansion periods (0.35%) with a t-stat of 1.96) while it becomes insignificant, during recession periods (-0.06%) with a t-stat of -0.14). The Lasso portfolios exhibit a similar behavior over the business cycle.

Working with raw returns may alter the results regarding the magnitude of the size effect for a given period. Thus we account for the traditional factors that explain the cross-section of asset returns, as described by the pricing equation (3.2), to estimate the risk-adjusted spread between the small and big size portfolio returns over the same time periods as for the raw spreads. Tables 2.4 and 2.5 report the risk-adjusted spreads based on (3.3). These Tables summarize the results obtained

		Fama-French			Lasso				
	R_{Small}	Raw spread	R_{Big}	R_{Small}	Raw spread	R_{Big}			
eriod	1.15	0.30	0.85	0.86	-0.10	0.96			
Total P	(3.45)	(1.73)	(3.19)	(3.81)	(-0.80)	(6.15)			
Golden Age	1.29 (3.24)	0.68 (2.32)	0.61 (2.57)	1.07 (3.29)	0.38 (2.60)	0.69 (2.76)			
Embarrassment	1.10 (3.13)	-0.38 (-1.47)	1.50 (5.41)	1.20 (3.42)	-0.28 (-1.63)	1.48 (5.40)			
Resurrection	0.97 (1.73)	0.83 (1.94)	0.14 (0.39)	-0.10 (-0.15)	-0.66 (-1.63)	0.56 (1.45)			

Table 2.3: Lasso selected Industries Raw Size Premium

The table reports the mean returns (R_{Size}) of the small and big size portfolios and their raw spreads (as defined by equation 3.1) over the full sample period, from July 1957 to October 2018, and during the Golden Age, from July 1957 to December 1979, the Embarrassment from January 1980 to December 1999 and the Resurrection from January 2000 to December 2012 (Asness et al., 2018). All the figures are expressed in percentage and the t-stats of the estimates are given below within parenthesis. for both the FF and Lasso portfolio returns regressed against the Market portfolio, and different combinations of the factors SMB, HML, MOM and QMJ.

First we notice that for all considered models as well as for all tested periods, the portfolio $L_{-}IND_{Big}$ exhibits a positive and significant abnormal return (α_{Big}). We observe the same result for the FF big size portfolio except for with the models (I) and (III) during the *Resurrection* period when the α_{Big} exhibits insignificant returns of 0.03% (*t*-stat of 0.44) and -0.08% (-1.16). It is also interesting to notice that the abnormal return is always higher with the Lasso portfolio except for the model (III) which accounts for the factor QMJ during the full sample period with $\alpha_{Big} = 0.86\%$ for the FF portfolio compared to $\alpha_{Big} = 0.37\%$ for $L_{-}IND_{Big}$. Hence, with these results, we can underline that the impact of QMJ is different depending on the considered portfolio construction scheme. Asness et al. (2019) introduced the QMJ as a quality factor; it is thus reasonable that it tempers the impact of $L_{-}IND_{Big}$ since this latter also accounts for this feature.

When we consider the results for $L_{-IND_{Small}}$ and the FF small size portfolio, we can notice that they both exhibit significantly positive α_{Small} for the full sample period as well as the *Golden Age* with abnormal returns respectively equal to 0.80% (with a *t*-statistic of 6.57) and 1.16% (with a *t*-statistic of 5.08) with the model (III). The FF small size portfolio also exhibits a significant abnormal return, except for the model (I) during the *Embarrassment* and model (II) for the *Resurrection* periods during which $L_{-IND_{Small}}$ performs poorly.

Overall as summarized in Tables 2.4 and 2.5, the risk-adjusted size premium is larger with the Lasso formed portfolios compared to the one obtained with the FF small and big size portfolios during the full sample period. In fact considering the model (III), premia of 0.24% (with a t-statistic of 2.23) and 0.15% (with a t-statistic of 2.27) are respectively exhibited by the Lasso and FF portfolios. However, during the *Golden Age*, the trend is reversed with the FF portfolios exhibiting a larger risk-adjusted spread of 0.93% (with a t-statistic of 3.59) while it is 0.42% (with a t-statistic of 3.19) for the Lasso portfolio. In addition the spreads decrease with respect to their counterparts reported in Table 2.3 during the *Golden Age*. Thus the spread becomes smaller after adjusting for common risk factors. However the results are mixed for the Lasso portfolio when it comes to consider the *Embarrassment* and the *Resurrection* periods. During these two sub-periods, the risk-adjusted spreads obtained with the Fama and French portfolios remain significant while they are not for the Lasso-formed portfolio.

We also notice that, both FF and Lasso portfolios, exhibit a similar behavior through the business cycle (reported on Table 2.10 in Appendix 2.C). As a result, investors seeking size premia would optimize their returns on investment after controlling for regime shifts. In fact this is consistent with the literature which argues that small firms risk and expected return are more strongly affected by the business cycle than large firms. Large firms expected return also displays state dependencies but to a lesser extent compared to small firms (see Cooley and Quadrini, 2006; Perez-Quiros and Timmermann, 2000).

Our results go against the argument that during recessions, investors prefer large over small stocks, which may explain the bad conditional performance of small stock

			Lasso		F	ama-Frenc	h
		α_{Small}	R_A_S	α_{Big}	α_{Small}	R_A_S	α_{Big}
	т	0.42	-0.03	0.45	0.51	0.06	0.45
pq	1	(4.07)	(-0.29)	(12.85)	(3.41)	(0.84)	(3.53)
ric	П	0.37	-0.09	0.46	0.32	0.04	0.28
Pe	11	(3.81)	(-0.92)	(13.63)	(4.76)	(0.58)	(6.49)
al	TTT	0.61	0.24	0.37	1.01	0.15	0.86
ot	111	(5.90)	(2.23)	(10.85)	(7.58)	(2.27)	(7.39)
	W	0.48	0.11	0.39	0.45	0.12	0.33
	1 V	(4.76)	(0.86)	(11.39)	(6.53)	(1.69)	(7.39)
	т	0.61	0.19	0.42	0.80	0.46	0.34
e	1	(4.69)	(1.31)	(10.98)	(3.22)	(1.59)	(6.40)
Ag	ΤT	0.42	-0.02	0.44	0.36	-0.06	0.42
ü	11	(5.11)	(-0.23)	(11.74)	(4.13)	(-0.76)	(14.58)
lde	TTT	0.80	0.42	0.38	1.16	0.93	0.23
0	111	(6.57)	(3.19)	(10.20)	(5.08)	(3.59)	(5.33)
J	W	0.44	0.05	0.39	0.32	-0.04	0.36
	ΙV	(5.20)	(0.53)	(10.38)	(3.56)	(-0.47)	(13.17)

Table 2.4: Risk-adjusted size premium (1)

The table reports the abnormal returns (α_{Size}) of the small and big size portfolios and their risk-adjusted spreads (R_A_S) over the full sample period, from July 1957 to October 2018, and during the Golden Age, from July 1957 to December 1979 (Asness et al., 2018). All the figures are expressed in percentage and the t-stat of the estimates are given below within parenthesis. Model (I) accounts for the Market, HML and MOM; Model (II) accounts for the Market, SMB, HML and MOM; Model (III) replaces SMB by QMJ; Model (IV) accounts for all the factors.

portfolio in the recession since both small as big stocks exhibit a positive spread. Thus our lasso characterization of the size effect may improve our understanding on size risk premia behavior. Generally speaking, our results confirm that the size effect crucially depends on how the small and big size portfolios are defined, consistent with the divergent conclusions of the literature.

4 Cross-section considerations

We test the cross-sectional implications of our previous results by considering a model defined by equation 4.1 that accounts for the above factors and portfolio in the cross-section :

$$R_{Size,t} - R_f = \alpha + \beta \cdot (R_{M,t} - R_f) + \gamma \cdot Factor_t + \varepsilon_t, \qquad (4.1)$$

			Lasso			F	ama-Frenc	eh
		α_{Small}	R_A_S	α_{Big}	-	α_{Small}	R_A_S	α_{Big}
د.	т	0.41	-0.25	0.66		0.36	-0.24	0.60
eni	1	(2.57)	(-1.41)	(10.92)		(1.49)	(-0.86)	(8.75)
sm	ΤT	0.39	-0.26	0.67		0.30	-0.32	0.62
ass	11	(2.71)	(-1.83)	(11.81)		(2.42)	(-2.64)	(15.82)
arr	TTT	0.48	-0.03	0.51		1.28	0.92	0.36
lba	111	(2.72)	(-0.15)	(8.17)		(5.77)	(3.49)	(5.44)
En	IV	0.17	-0.40	0.57		0.60	0.05	0.55
		(1.07)	(-2.37)	(9.10)		(4.53)	(0.38)	(12.69)
					-			
	т	-0.02	-0.42	0.40	-	0.85	0.82	0.03
nd	1	(-0.07)	(-1.53)	(3.83)		(2.40)	(1.97)	(0.44)
tic	ΤT	-0.02	-0.44	0.42		0.28	0.12	0.16
rec	11	(-0.07)	(-1.56)	(3.96)		(1.52)	(0.66)	(4.33)
un	TTT	0.02	-0.30	0.32		1.47	1.55	-0.08
est	111	(0.08)	(-1.07)	(3.12)		(5.16)	(4.71)	(-1.16)
Ĥ	W	0.07	-0.22	0.29		0.6	0.46	0.14
	IV	(0.21)	(-0.78)	(2.73)		(3.30)	(2.61)	(3.53)

Table 2.5: Risk-adjusted size premium (2)

The table reports the abnormal returns (α_{Size}) of the small and big size portfolios and their risk-adjusted spreads (R_A_S) during the Embarrassment and the Resurrection periods, respectively from January 1980 to December 1999 and from January 2000 to December 2012 (Asness et al., 2018). All the figures are expressed in percentage and the t-stat of the estimates are given below within parenthesis. Model (I) accounts for the Market, HML and MOM; Model (II) accounts for the Market, SMB, HML and MOM; Model (III) replaces SMB by QMJ; Model (IV) accounts for all the factors. where $R_{Size,t}$ and R_f stand for the returns on the 10 FF size-sorted portfolios and the risk-free rate. $Factor_t$ is a vector of returns on the SMB, HML, MOM, QMJ and $L_{-}IND_{S-B}$ portfolio at date t. The parameter γ refers to the vector of corresponding factor loadings.

Equation 4.1 nests the CAPM, the Fama and French 3-factor (FF3) and the Carhart 4-factor (CARH4) models. Asness et al. (2018) argue that controlling for QMJ, the size premium is substantially stronger. In this line we include the $L_{-IND_{S-B}}$. The idea is to study the influence/control of $L_{-IND_{S-B}}$ over the size effect in the cross-section. Hence we consider the Fama and MacBeth procedure for the CAPM, FF3 and CARH4 models augmented with the $L_{-IND_{S-B}}$ and QMJ factors separately or together. We report the results of the cross-sectional regressions over the full sample period as well as for the *Golden Age* and the *Resurrection* periods in Tables 2.6, 2.7 and 2.8.

As argued in the literature, the CAPM fails to account for the size effect (Fama and French, 1993) as evidenced by the significant value of the model intercept (t-stat of 3.09) as well as the modest adjusted- R^2 of 47%. Consistent with the literature, the FF3 and CARH4 models provide a better account of of the size effect as showed by the increase of the R^2 , respectively, of 88% and 97%. However the intercept remains significant (they even become grater compared to the CAPM for both of these models meaning that a part of the size premium remains unexplained. Consistent with Asness et al. (2019) results, we notice that the inclusion of the QMJ factor reduces the significance level of the intercept with a decrease of the t-stat from 3.94 to 2.84. However the inclusion of QMJ also impacts the SMB premium. In fact we observe a decrease of both the premium value (from 0.52% to 0.49%) and significance level (from 3.35 to 2.14) for the SMB in the FF3 model. The same observations can also be made for the CARH4.

The inclusion of the Lasso portfolio $L_{-}IND_{S-B}$ exhibit interesting results. We can first notice that for all considered models the inclusion of the $L_{-}IND_{S-B}$ have a positive impact which manifests by the reduction of the intercept (Table 2.6) which, for example in the CAPM, decreases from 1.54% (with a *t*-stat of 3.1) to 1.31% (with a *t*-stat of 1.39). The intercept of the CAPM is only reduced to insignificance by the inclusion of the SMB and QMJ factors and our $L_{-}IND_{S-B}$ portfolios. This result underlines the link between the size effect and these factors. As previously mentioned, the FF3 and CARH4 models exhibit a significant positive intercept. Hence, in Table 2.6, it is interesting to notice the extent to which the $L_{-}IND_{S-B}$ portfolio allows to explain the size premium when included in these models compared to the QMJ.

In Table 2.8 we report the results for the Golden Age and the Resurrection periods. We notice a consistency in terms of intercept reduction with the inclusion of $L_{-}IND_{S-B}$ in the CAPM, FF3 and CARH4 models. In addition, for the Golden Age, all considered models exhibit an increase of their adjusted- R^2 resulting from the addition of $L_{-}IND_{S-B}$ to the set of factors. Above all, for this period, significant premia associated to our $L_{-}IND_{S-B}$ are respectively observed with -0.5%(with a t-stat of -2.48) for FF3 and -0.5% (with a t-stat of -2.22) for CARH4. To sum up, these results are consistent with the hypothesis that some specific industries command a significant part of the size premium.

We also report a series of additional results in the Table 2.7 by considering regressions in which the SMB is replaced by the $L_{-}IND_{S-B}$ in the FF3 and CARH4 models. In fact we want to test if the $L_{-}IND_{S-B}$ provides a better account of the size premium. For both models (FF3 and CARH4) the intercept is lower with the $L_{-}IND_{S-B}$ than the SMB and the $L_{-}IND_{S-B}$ risk premia are always significant. The results are getting even better with the CARH4 in which a significant the risk premium (1.61%) is associated with the $L_{-}IND_{S-B}$ and is almost thrice as large as the risk premium of the SMB (0.64%).

These results comfort our hypothesis about the specific industries that drive the size premium and selected them through the lasso is promising to explain the size premium. To sum up, it seems that the addition of the $L_{-}IND_{S-B}$ portfolio accounts for the size effect in the cross-section and provides additional explanatory power in capturing the corresponding risk premium.

Conclusion

Motivated by the renewed interest in the size effect in the literature, we proposed an explanation based of the particular impact of specific industries selected by use machine learning techniques. Our paper examines the hypothesis that the persistence or not of the size effect during the period of investigation might be due to specific industries that the Lasso helps select. Our results underline that the size premium seems to be driven by some specific industries which coincide for example with the electronic and Internet Bubble in 1980's and 2000's. As a direct implication, when these selected industries are used to form our $L_{-IND_{Size}}$ portfolios, the size premium is better explained compared with the SMB factor. Our results matter to explain the determinant of the size premium and might give new insights on the ongoing literature on the size effect which remains an actively debated topic in empirical finance.

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Π_0	MKT	SMB	HML	MOM	QMJ	$L_{-}IND_{S-B}$	R^2
1.54	-1.35						0.47
(3.1)	(-3.01)						
1.51	-1.31	-0.27					0.53
(1.6)	(-1.43)	(-2.8)					
0.83	-0.51		-2.98				0.82
(2.13)	(-1.32)		(-3.1)				
1.27	-1.03			3.51			0.59
(2.26)	(-1.91)			(1.06)			
1.2	-1.00				0.36		0.55
(1.52)	(-1.3)				(2.93)		
1.31	-1.11					-0.47	0.54
(1.39)	(-1.20)					(-1.87)	
1.69	-1.34	0.52	-3.97				0.88
(3.94)	(-3.2)	(3.35)	(-5.01)				
1.63	-1.28	0.48	-3.75			-0.64	0.87
(3.64)	(-2.94)	(2.9)	(-4.35)			(-0.78)	
1.76	-1.41	0.49	-3.8		1.01		0.85
(2.84)	(-2.25)	(2.14)	(-2.9)		(2.35)		
1.70	-1.36	0.45	-3.58		1.07	-0.71	0.98
(2.59)	(-2.04)	(1.84)	(-2.53)		(2.33)	(-0.71)	
1.97	-1.62	0.64	-4.8	-2.65			0.97
(8.99)	(-7.55)	(7.96)	(-11.16)	(-2.69)			
1.94	-1.59	0.65	-4.87	-2.67	0.66		0.96
(6.2)	(-5.02)	(5.36)	(-6.88)	(-2.41)	(2.84)		
1.91	-1.56	0.6	-4.58	-2.45		0.39	0.84
(11.47)	(-9.59)	(9.62)	(-13.57)	(-3.27)		(1.15)	
1.88	-1.53	0.62	-4.66	-2.47	0.72	0.42	0.98
(7.73)	(-6.22)	(6.4)	(-8.32)	(-2.85)	(3.93)	(0.99)	

Table 2.6: Full Period 10 Size Cross-Sectional

The table reports results from Fama and MacBeth regressions for the 10 Size portfolios over the full sample period, from July 1957 to October 2018. The regressions considered variables are the market, SMB, HML, MOM, QMJ factors and $L_{-}IND_{S-B}$ portfolios. All the figures are expressed in percentage and the t-stat of the estimates are given below within parenthesis. R^2 is the adjusted R-squared of the cross-sectional regression.

Π_0	MKT	SMB	HML	MOM	QMJ	$L_{-}IND_{S-B}$	R^2
1.74	-1.54				0.69	0.38	0.59
(1.64)	(-1.48)				(1.57)	(0.4)	
1.38	-1.17	-0.26				-2.15	0.61
(1.48)	(-1.27)	(-2.76)				(-1.35)	
1.60	-1.25		-3.98			0.52	0.84
(3.27)	(-2.63)		(-4.30)			(2.04)	
1.20	-0.97			3.45		-0.97	0.59
(1.25)	(-1.02)			(0.95)		(-1.58)	
1.74	-1.54				0.69	0.38	0.59
(1.64)	(-1.48)				(1.57)	(0.40)	
1.86	-1.51		-4.79	-2.68		1.61	0.95
(5.04)	(-4.20)		(-6.45)	(-1.62)		(3.43)	
1.37	-1.01		-4.46		0.69	0.13	0.91
(2.47)	(-1.83)		(-4.17)		(3.02)	(0.26)	
1.59	-1.22		-5.48	-2.71	0.36	1.2	0.96
(5.17)	(-4)		(-8.47)	-2.16	(2.37)	(2.99)	
2.42	-2.18	-0.13			1.72	-2.02	0.82
(3.64)	(-2.94)	(-2.90)			(3.28)	(-0.78)	
1.28	-1.04	-0.16		3.56		-2.15	0.65
(1.31)	(-1.07)	(-0.90)		(0.96)		(-1.40)	
2.59	-2.35	-0.12			1.68		0.79
(3.31)	(-3.09)	(-1.37)			(3.30)		

Table 2.7: Full Period 10 Size Cross-Sectional (Bis)

The table reports results from Fama and MacBeth regressions for the 10 Size portfolios over the full sample period, from July 1957 to October 2018. The regressions considered explanatory variables are the market, SMB, HML, MOM, QMJ factors and $L_{-}IND_{S-B}$ portfolios. All the figures are expressed in percentage and the t-stat of the estimates are given below within parenthesis. R^2 is the adjusted R-squared of the cross-sectional regression.

	Π_0	MKT	SMB	HML	MOM	QMJ	L_IND _{S-B}	R^2
	0.52	-0.43				•	5 5	0.47
	(3.09)	(-2.97)						
	0.73	-0.64					-0.08	0.54
	(1.51)	(-1.35)					(-1.84)	
	0.86	-0.74	0.02	-0.6				0.59
	(2.25)	(-2.00)	(0.23)	(-1.61)				
	0.55	-0.43	0.02	-0.38			-0.5	0.74
မီ	(1.64)	(-1.30)	(0.21)	(-1.19)			(-2.48)	
I A	-0.16	0.29	-0.11	0.05		0.41		0.96
ler	(-0.86)	(1.55)	(-3.03)	(0.36)		(11.45)		
old	-0.13	0.26	-0.1	0.04		0.39	-0.21	0.95
G	(-0.66)	(1.31)	(-2.47)	(0.25)		(8.12)	(-2.04)	
	0.84	-0.73	0.07	-0.77	-0.42			0.55
	(2.09)	(-1.86)	(0.56)	(-1.67)	(-0.43)			
	0.56	-0.44	0.03	-0.42	-0.19		-0.5	0.56
	(1.48)	(-1.18)	(0.25)	(-0.94)	(-0.22)		(-2.22)	
	-0.15	0.28	-0.1	0.01	-0.08	0.41		0.95
	(-0.72)	(1.34)	(-2.05)	(0.07)	(-0.23)	(10.41)		
	-0.13	0.25	-0.09	0.02	-0.06	0.39	-0.21	0.94
	(-0.55)	(1.11)	(-1.77)	(0.09)	(-0.16)	(6.95)	(-1.78)	
	-0.09	0.08						0.43
	(-2.91)	(2.81)					0.02	
	-0.08	0.07					-0.02	0.52
	(-2.02)	(1.81)	0.000	0.00			(-0.16)	0.90
n	-0.05	0.04	-0.003	0.08				0.39
tio	(-0.75)	(0.50)	(-0.16)	(1.12)			0.002	0.07
rec	-0.05	0.03	-0.003	0.08			-0.003	0.27
un	(-0.67)	(0.44)	(-0.13)	(1.02)		0.16	(-0.02)	0.76
Ses	-0.202	0.18	-0.042	(0.197)		-0.10		0.76
<u>14</u>	(-2.02)	(1.84)	(-1.01)	(2.24)		(-1.95)	0.00	0.46
	-0.2	(1.69)	-0.04	(2,01)		-0.10	(0.09)	0.40
	(-1.79)	(1.02)	(-1.43)	(2.01)	0.24	(-1.74)	(0.01)	0.75
	-0.04	(0.24)	0.01	(1.07)	(0.34)			0.70
	(-0.78)	(0.34)	(0.02)	(1.07)	(2.10)		0.08	0 56
	-0.03 (-0.60)	(0.13)	(0.72)	(1.09)	(2.66)		-0.08	0.00
	-0.04 (-0.78) -0.03 (-0.60)	$\begin{array}{c} 0.02 \\ (0.34) \\ 0.01 \\ (0.13) \end{array}$	$\begin{array}{c} 0.01 \\ (0.52) \\ 0.01 \\ (0.72) \end{array}$	$\begin{array}{c} 0.09 \\ (1.87) \\ 0.09 \\ (1.91) \end{array}$	$\begin{array}{c} 0.34 \\ (2.78) \\ 0.33 \\ (2.66) \end{array}$		-0.08 (-0.89)	0.75 0.56

Table 2.8: Golden Age and Resurrection 10 Size Cross-Sectional

The table reports results from Fama and MacBeth regressions for the 10 Size portfolios over the Golden Age period from Jan 1957 to Dec. 1979 and the Resurrection period from Jan 2000 to Oct. 2012. The regressions considered explanatory variables are the market, SMB, HML, MOM, QMJ factors and $L_{-IND_{S-B}}$ portfolios. All the figures are expressed in percentage and the t-stat of the estimates are given below within parenthesis. R^2 is the adjusted R-squared of the cross-sectional regression.

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Appendix 2.A The Lasso

Traditional statistical estimation procedures such as the Ordinary Least Squares (OLS) tend to perform poorly in high-dimensional problems. In particular, although OLS estimators typically have low bias, they tend to have high prediction variance, and may be difficult to interpret (Brown, 1993). In such situations, it is often beneficial to rely on shrinkage, *i.e.* to shrink the estimates towards zero, which amounts to bias the estimator so as to decrease its variance, with the net result of reducing its mean squared error. There are many shrinkage methods suggested in the literature. Tibshirani (1996) introduced the *least absolute shrinkage and selection operator* (Lasso), which has been a breakthrough in the field of sparse model estimation. The Lasso performs variable selection and coefficient shrinkage simultaneously.

Let us define the linear model :

$$y_t = \alpha + \sum_{i=1}^{I} \beta_i \cdot x_{i,t} + \varepsilon_t , \qquad (2.A.1)$$

where y is the response variable, x is the *I*-dimensional set of predictors, (α, β) is the set of parameters and $\varepsilon \sim IID(0, \sigma^2)$.

The Lasso shrinks some coefficients while setting others exactly to zero, and thus theoretical properties suggest that the Lasso enjoys the good features of both subset selection and ridge regression (Tibshirani, 1996). The Lasso estimator solves

$$\arg\min_{\alpha,\beta} \left(\frac{1}{2} \sum_{t=1}^{T} \left(y_t - \alpha - \sum_{i=1}^{I} \beta_i \cdot x_{i,t} \right)^2 + \lambda \sum_{i=1}^{I} |\beta_i| \right), \qquad (2.A.2)$$

where λ is a regularization parameter. When $\lambda = 0$, equation (2.A.2) reduces to the familiar OLS objective function. The presence of λ in equation (2.A.2) shrinks the slope estimates β_i 's. The Lasso penalty term allows for shrinkage to zero (for a sufficiently large λ), so that it performs variable selection. The penalty parameter λ required for the implementation of the Lasso estimator are found by the Akaike Information Criterion (AIC) as done by Han et al. (2018) but often rely upon crossvalidation (Ghosh, 2012; Zou et al., 2007; Chatterjee and Jafarov, 2015; Hastie et al., 2015).

Efron et al. (2004) developed an efficient algorithm known as *Least Angle Regression* (LARS) algorithm for finding the solution path of the lasso method (see also Hastie et al., 2007). We summarize it in Algorithm 2.1.

Appendix 2.B Additional Lasso results

In this appendix we regroup some auxiliary results. The Figures 2.B.1, 2.B.2, 2.B.3, 2.B.4, 2.B.5 and 2.B.6 summarize the Lasso procedure selection of industries that characterize the Small and Big size portfolios during the *Golden Age* (from July 1957 to December 1979), the *Embarrassment* (from January 1980 to December 1999) and

Algorithme 2.1 LARS algorithm

Step 1 : Standardize the predictors $\{x_i, i = 1, ..., I\}$ to have zero mean and unit variance. Start with the residual $r = y - \bar{y}, \beta_{i_1}, ..., \beta_I = 0$.

Step 2: Find the predictor x_i most correlated with r.

Step 3: Move β_i from 0 towards its least squares coefficient (x_i, r) , until some other competitor x_k has as much correlation with the current residual as does x_i .

Step 4: Move (β_i, β_k) in the direction defined by their joint least squares coefficient of the current residual on (x_i, x_k) until some other competitor x_l has as much correlation with the current residual.

Step 5 : If a non-zero coefficient hits zero, recompute the current joint least squares direction.

Step 6 : Continue in this way until all I predictors have been entered in the model and we arrive at the full least squares solution.

the *Resurrection* (from January 2000 to Dec 2012).

Appendix 2.C NBER recession and expansion results

In this section we present some preliminary results based on ex post US business cycle expansion and contraction data provided by the National Bureau of Economic Research (NBER). The Table 2.9 reports the average returns on small and big stock portfolios, as well as the resulting raw spread given by the equation (3.1). In Table 2.10 we provide the same statistics for the risk-adjusted spreads (see equation 3.3). All these statistics are computed the basis of the expansion and recession periods available on the NBER website.⁶

The results of Table 2.9 show that the size premium depends on the business cycle. For the FF small and big size portfolios, it is positive and significant during expansion periods (0.35% with a *t*-stat of 1.96) while it becomes negative, but not significant, during recession periods (-0.06% with a *t*-stat of -0.14). In comparison, for the Lasso portfolios, the trend is the opposite with 0.06% (resp. -0.08%) during recessions (resp. expansions) but these figures are never significant. Hence, controlled for the business cycle, the FF portfolios capture a size premium during expansion periods only while their Lasso proxies based on selected industries fail to capture this premium.

We notice however that, both FF and Lasso portfolios, exhibit a similar behavior through the business cycle. Indeed, irrespective of the way they are obtained (following FF definition or the Lasso), both small and big size portfolios exhibit insignificant average returns during recessions. On the contrary, both exhibit significantly positive returns during expansion periods.

The risk-adjusted spreads are reported on Table 2.10. First we notice that during recession periods for all considered models the $L_{-IND_{Small}}$ exhibit a positive and

⁶https://www.nber.org/cycles.html





The top left panel depicts a heatmap that represents the industry portfolios selected by the Lasso to explain the returns of the small size portfolio for the Resurrection from January 2000 to Dec 2012 (the darker the color the larger the value of the corresponding β_i). The evolution of the number of selected industries over time is plotted on the bottom left panel. The top right panel represents the fraction of time a given industry is selected over the total sample period.

Figure 2.B.2: Lasso of Big Size on 30 Industry Portfolios in Resurrection period.



Resurrection_Big

The top left panel depicts a heatmap that represents the industry portfolios selected by the Lasso to explain the returns of the small size portfolio for the Resurrection from January 2000 to Dec 2012 (the darker the color the larger the value of the corresponding β_i). The evolution of the number of selected industries over time is plotted on the bottom left panel. The top right panel represents the fraction of time a given industry is selected over the total sample period.

Figure 2.B.3: Lasso of Small Size on 30 Industry Portfolios in Embarrassment period.



Embarrassment_Small

The top left panel depicts a heatmap that represents the industry portfolios selected by the Lasso to explain the returns of the small size portfolio for the Embarrassment from January 1980 to December 1999 (the darker the color the larger the value of the corresponding β_i). The evolution of the number of selected industries over time is plotted on the bottom left panel. The top right panel represents the fraction of time a given industry is selected over the total sample period.

Figure 2.B.4: Lasso of Big Size on 30 Industry Portfolios in Embarrassment period..



Embarrassment_Big

The top left panel depicts a heatmap that represents the industry portfolios selected by the Lasso to explain the returns of the small size portfolio for the Embarrassment from January 1980 to December 1999 (the darker the color the larger the value of the corresponding β_i). The evolution of the number of selected industries over time is plotted on the bottom left panel. The top right panel represents the fraction of time a given industry is selected over the total sample period.



Figure 2.B.5: Lasso of Small Size on 30 Industry Portfolios in GA period.

The top left panel depicts a heatmap that represents the industry portfolios selected by the Lasso to explain the returns of the small size portfolio for the Golden Age from July 1957 to December 1979 (the darker the color the larger the value of the corresponding β_i). The evolution of the number of selected industries over time is plotted on the bottom left panel. The top right panel represents the fraction of time a given industry is selected over the total sample period.
Figure 2.B.6: Lasso of Big Size on 30 Industry Portfolios in GA period.



Golden Age_Big

The top left panel depicts a heatmap that represents the industry portfolios selected by the Lasso to explain the returns of the small size portfolio for Golden Age from July 1957 to December 1979 (the darker the color the larger the value of the corresponding β_i). The evolution of the number of selected industries over time is plotted on the bottom left panel. The top right panel represents the fraction of time a given industry is selected over the total sample period.

	Fa	ma-Frenc	h			Lasso	
ц	R_{Small}	R_S_S	R_{Big}	-	R_{Small}	R_S_S	R_{Big}
essio	-0.04	-0.07	0.03		0.25	0.06	0.19
Rec	(-0.05)	(-0.14)	(0.05)		(0.28)	(0.14)	(0.32)
				_			
Expansion	1.34 (5.89)	0.35 (1.96)	0.99 (6.55)		0.96 (4.33)	-0.08 (-0.98)	1.08 (7.07)

Table 2.9: Lasso selected Industries Raw Size Premium (2)

The table reports the mean returns (R_{Size}) of the small and big size portfolios and their raw spreads (Eq. 3.1) over the sample period accounting for US business cycle expansions and contractions data provided by the National Bureau of Economic Research (NBER). All the figures are expressed in percentage and the t-stat of the estimates are given below within parenthesis.

significant abnormal return (α_{Small}). In comparison the FF small size never exhibit a significant return. Hence with our approach the small cap better behave during recession. Consistent to the literature the Big cap for our Lasso as much as the FF portfolios deliver a significant positive return. However the Lasso portfolios exhibit higher abnormal returns.

During the expansion the results are more stable between the small and big cap. In fact except the insignificant case of the FF big size in the model (I) all the abnormal returns are significant with higher values for the Big size for both the Lasso as much as FF portfolios.

Overall as summarized in Table 2.10 the risk-adjusted size premium is only significant for the FF portfolios for the model (II) during the recession and for the model (III) and (IV) during the expansion.

Appendix 2.D Additional results

We consider in this part the results of the SMB regression on the market portfolio, HML, MOM, QMJ factors and $L_{-}IND_{Size}$ portfolio. To investigate the implication of the Lasso approach, we relate the SMB factor to these industry portfolios and also investigate the relation with other known pricing factors such the HML of Fama and French, the MOM of Carhart and the QMJ of Asness et al. in addition of the

			Lasso		Fa	ama-Frenc	h
		α_{Small}	$R_{-}A_{-}S$	α_{Big}	 α_{Small}	$R_{-}A_{-}S$	α_{Big}
	т	0.76	0.23	0.53	0.29	-0.08	0.36
	1	(2.38)	(0.73)	(5.41)	(0.71)	(-0.15)	(3.62)
ssion	ΤT	0.62	0.08	0.54	-0.10	-0.54	0.44
	11	(2.09)	(0.28)	(5.40)	(-0.51)	(-2.64)	(6.83)
ece	TTT	1.06	0.59	0.47	0.64	0.39	0.25
\mathbf{R}	111	(3.23)	(1.87)	(4.57)	(1.53)	(0.81)	(2.47)
	W	0.85	0.38	0.47	0.03	-0.34	0.37
	1 V	(2.75)	(1.28)	(4.49)	(0.16)	(-1.59)	(5.62)
	т	0.34	-0.11	0.43	 0.51	0.15	0.36
-	1	(3.09)	(-0.80)	(11.36)	(3.19)	(0.81)	(0.44)
ior	П	0.31	-0.13	0.44	0.41	0.03	0.38
uns	11	(3.01)	(-1.18)	(12.09)	(5.72)	(0.32)	(4.33)
tpa	III	0.51	0.16	0.35	1.04	0.80	0.24
Ê	111	(4.72)	(1.45)	(9.51)	(7.40)	(4.89)	(6.73)
	W	0.41	0.04	0.37	0.53	0.17	0.36
	ΤV	(3.83)	(0.35)	(10.14)	(7.31)	(2.44)	(3.53)

Table 2.10: Risk-adjusted size premium

The table reports the abnormal returns (α_{Size}) of the small and big size portfolios and their risk-adjusted spreads (R_A_S) over the business cycle (expansion and contraction periods follows the definition of the National Bureau of Economic Research). All the figures are expressed in percentage and the t-stat of the estimates are given below within parenthesis. Model (I) accounts for the Market, HML and MOM; Model (II) accounts for the Market, SMB, HML and MOM; Model (III) replaces SMB by QMJ; Model (IV) accounts for all the factors. Market.⁷

Hence we test :

$$SMB_t = \alpha_t + \beta_t \cdot Factor_t + \varepsilon_t. \tag{2.D.1}$$

 $Factor_t$ is a vector containing the return on the Market, HML, MOM, QMJ or $L_{-}IND_{Size}$ at date t. For β we refer to the vector containing the loadings of factors in $Factor_t$ accordingly to the tested model.

The idea behind the relation expressed by Eq. 2.D.1 is to study if their is any abnormal returns by controlling for the portfolio $L_{-IND_{Size}}$. In other word we investigate the impact of this portfolio on the size premium measured by SMB.

The Table 2.11 - 2.12 and 2.13 summarize the results for $L_{-IND_{S-B}}$, $L_{-IND_{Small}}$ and $L_{-IND_{Big}}$ regressed on the market portfolio, SMB, HML, MOM, QMJ factors portfolios. All over these table we can first notice that the Asness et al. (2018) results on size premium are recovered. In fact the addition of QMJ factor explains much of the size effect variation, transform it from a small and insignificant effect to a large and statistically strong one and also makes the size effect varies significantly through time.

When we consider our tested portfolios over the July 1957 to November 2018 time period, which is the Full sample period, as much as for the other periods the model that includes Market, HML, MOM, to capture value and momentum exposure, seems not to exhibit reliable size premium (through the α). For example for the full sample period the intercept from the regression is of 0.15 with a t-statistic of 1.36, which is insignificantly different from zero.

The inclusion of our $L_{-}IND_{S-B}$ portfolio exhibits a weak size effect with noticeable variation over time in comparison with the QMJ impact. In fact QMJ drives significant SMB's alpha and explains a substantial fraction of the variation in SMB's returns for the considered periods (Asness et al., 2018). Moreover even this portfolio inclusion has impact on size premium (from 0.05% to 0.27% with the *Embarrassment* period), this impact remains insignificant. However SMB loads very significantly and positively on $L_{-}IND_{S-B}$ for all tested periods except the Resurrection period. For instance for the *Golden Age* this $L_{-}IND_{S-B}$ portfolio always exhibits the most significant load (t-stat of 19.72) and allows to temper the QMJ impact which decreases. In fact as we can notice the inclusion of $L_{-}IND_{S-B}$ drives the SMB alpha from 0.60% to 0.26% and the QMJ significance decreases from 9.11 to 3.84. The R^2 even rises from 38 to 69% with the inclusion of this one additional $L_{-}IND_{S-B}$ portfolios.

Accordingly to the considered period the $L_{-}IND_{Small}$ and $L_{-}IND_{Big}$ portfolios also deliver similar features than the QMJ factor. Hence the selection by Lasso procedure of specific industries that drives the Small and Big cap explains a fraction of the variation in SMB's returns and allows to control the return premium to size induced by the QMJ.

⁷QMJ is long the average of the Small Quality and Big Quality portfolios and short the average of the Small Junk and Big Junk portfolios (see Asness et al. 2019 for further details).

	α	β_{MKT}	β_{HML}	β_{MOM}	β_{QMJ}	$L_{-}IND_{S-B}$	R^2		α	β_{MKT}	β_{HML}	β_{MOM}	β_{QMJ}	$L_{-}IND_{S-B}$	R^2
	0.15	0.18	-0.13	-0.01			0.09	-	0.32	0.29	0.05	-0.10			0.19
-	(1.42)	(6.96)	(-3.24)	(-0.06)				n)	(1.89)	(7.34)	(0.73)	(-1.98)			
rio	0.5	-0.01	-0.22	0.07	-0.71		0.27	\mathbf{A}_{90}	0.6	0.11	-0.22	-0.05	-0.91		0.38
$\mathbf{P}_{\mathbf{e}_{\mathbf{I}}}$	(4.92)	(-0.27)	(-5.95)	(2.68)	(-13.52)			'n	(3.98)	(2.75)	(-3.2)	(-1.16)	(-9.11)		
cal	0.17	0.08	-0.07	0.08		0.4	0.23	lde	0.15	0.08	-0.11	-0.13		0.91	0.67
Tot	(1.66)	(3.11)	(-1.85)	(3.28)		(11.57)		Ğ	(1.38)	(2.96)	(-2.48)	(-4.05)		(19.72)	
	0.43	-0.04	-0.16	0.11	-0.56	0.27	0.33		0.26	0.04	-0.19	-0.11	-0.31	0.82	0.69
	(4.45)	(-1.42)	(-4.38)	(4.57)	(-10.4)	(7.96)		_	(2.39)	(1.42)	(-3.87)	(-3.49)	(-3.84)	(16.12)	
	0.49	0.26	-0.29	0.14			0.2		0.05	0.05	-0.26	-0.09			0.08
u	(1.87)	(4.39)	(-3.85)	(3.02)				ent	(0.29)	(1.07)	(-3.6)	(-1.81)			
tio	0.88	-0.14	-0.22	0.19	-0.82		0.43	ŝme	0.66	-0.15	-0.47	-0.05	-0.86		0.28
rec	(3.85)	(-1.95)	(-3.38)	(4.83)	(-7.77)			ass	(3.75)	(-3.26)	(-6.74)	(-1.04)	(-8.19)		
sur	0.50	0.25	-0.28	0.15		0.03	0.2	arr	0.27	-0.05	-0.32	-0.04		0.70	0.34
Re	(1.90)	(3.57)	(-3.19)	(2.85)		(0.39)		qm	(1.21)	(-1.05)	(-3.27)	(-0.18)		(9.59)	
	0.86	-0.12	-0.25	0.18	-0.83	-0.05	0.43	님	0.74	-0.20	-0.48	-0.02	-7.28	0.53	0.47
	(3.76)	(-1.64)	(-3.36)	(3.96)	(-7.79)	(-0.80)			(4.46)	(-4.86)	(-6.33)	(0.47)	(-7.8)	(9.23)	

Table 2.11: SMB explained by Lasso selected Industries (1)

The table reports results from regressions for the SMB factor over the full sample period, from July 1957 to October 2018, from July 1957 to December 1979, the Embarrassment, from January 1980 to December 1999 and the Resurrection, from January 2000 to December 2012, periods (Asness et al., 2018). The regressions considered explanatory variables are the market, SMB, HML, MOM, QMJ factors and $L_{-IND_{SB}}$ portfolios. All the figures are expressed in percentage and the t-stat of the estimates are given below within parenthesis. R^2 is the adjusted R-squared of the cross-sectional regression.

	α	β_{MKT}	β_{HML}	β_{MOM}	β_{QMJ}	$L_{-}IND_{Small}$	R^2		α	β_{MKT}	β_{HML}	β_{MOM}	β_{QMJ}	$L_{-}IND_{Small}$	R^2
	0.15	0.18	-0.13	-0.01			0.09	-	0.32	0.29	0.05	-0.10			0.19
	(1.42)	(6.96)	(-3.24)	(-0.06)				0	(1.89)	(7.34)	(0.73)	(-1.98)			
rio.	0.50	-0.01	-0.22	0.07	-0.71		0.27	100	0.6	0.11	-0.22	-0.05	-0.91		0.38
$\mathbf{P}_{\mathbf{e}_{\mathbf{I}}}$	(4.92)	(-0.27)	(-5.95)	(2.68)	(-13.52)			'n	(3.98)	(2.75)	(-3.2)	(-1.16)	(-9.11)		
al	0.01	-0.24	-0.1	0.09		0.4	0.19	lde	-0.30	-0.93	-0.11	-0.09		1.02	0.69
Tot	(0.08)	(-4.77)	(-2.52)	(3.32)		(9.38)		Ğ	(-2.73)	(-14.4)	(-2.46)	(-2.70)		(20.42)	
	0.35	-0.27	-0.18	0.12	-0.61	0.24	0.32		-0.12	-0.87	-0.2	-0.07	-0.36	0.92	0.71
	(3.49)	(-5.85)	(-5.09)	(4.80)	(-11.67)	(6.82)		_	(-1.10)	(-13.87)	(-4.29)	(-2.20)	(-4.83)	(17.3)	
	0.49	0.26	-0.29	0.14			0.2		0.05	0.05	-0.26	-0.09			0.08
ц	(1.87)	(4.39)	(-3.85)	(3.02)				ent	(0.29)	(1.07)	(-3.6)	(-1.81)			
tio	0.88	-0.14	-0.22	0.19	-0.82		0.43	ŝme	0.66	-0.15	-0.47	-0.05	-0.86		0.28
rec	(3.85)	(-1.95)	(-3.38)	(4.83)	(-7.77)			ass	(3.75)	(-3.26)	(-6.74)	(-1.04)	(-8.19)		
sur	0.49	0.26	-0.3	0.14		0	0.20	arr	-0.15	-0.49	-0.23	0.01		0.49	0.25
Re	(1.86)	(2.20)	(-3.60)	(2.56)		(-0.03)		qm	(-0.92)	(-5.99)	(-3.49)	(0.02)		(7.45)	
	0.88	-0.10	-0.23	0.18	-0.82	-0.03	0.42	É	0.44	-0.64	-0.42	-0.04	-0.81	0.46	0.43
	(3.84)	(-0.91)	(-3.32)	(3.92)	(-7.76)	(-0.47)			(2.76)	(-8.73)	(-6.89)	(0.91)	(-8.72)	(8.02)	

Table 2.12: SMB explained by Lasso selected Industries (2)

The table reports results from regressions for the SMB factor over the full sample period, from July 1957 to October 2018, from July 1957 to December 1979, the Embarrassment, from January 1980 to December 1999 and the Resurrection, from January 2000 to December 2012, periods (Asness et al., 2018). The regressions considered explanatory variables are the market, SMB, HML, MOM, QMJ factors and $L_{-IND_{Small}}$ portfolios. All the figures are expressed in percentage and the t-stat of the estimates are given below within parenthesis. R^2 is the adjusted R-squared of the cross-sectional regression.

	α	β_{MKT}	β_{HML}	β_{MOM}	β_{QMJ}	$L_{-}IND_{Big}$	R^2		α	β_{MKT}	β_{HML}	β_{MOM}	β_{QMJ}	$L_{-}IND_{Big}$	R^2
	0.15	0.18	-0.13	0			0.09	-	0.32	0.29	0.05	-0.10			0.19
р	(1.42)	(6.96)	(-3.24)	(-0.06)					(1.89)	(7.34)	(0.73)	(-1.98)			
rio	0.5	-0.01	-0.22	0.07	-0.71		0.27	186	0.6	0.11	-0.22	-0.05	-0.91		0.38
Pe	(4.92)	(-0.27)	(-5.95)	(2.68)	(-13.52)			u 7	(3.98)	(2.75)	(-3.20)	(-1.16)	(-9.11)		
al	0.51	0.93	-0.09	-0.04		-0.79	0.15	lde	0.80	1.37	0.03	-0.16		-1.13	0.24
lot	(4.36)	(8.50)	(-2.18)	(-1.46)		(-7.06)		G	(4.00)	(5.35)	(0.37)	(-3.05)		(-4.27)	
	0.64	0.37	-0.19	0.04	-0.64	-0.38	0.29		0.77	0.55	-0.21	-0.08	-0.85	-0.45	0.39
	(5.92)	(3.39)	(-5.07)	(1.65)	(-11.79)	(-3.55)			(4.34)	(2.20)	(-3.09)	(-1.65)	(-8.02)	(-1.79)	
								-							
	0.49	0.26	-0.29	0.14			0.2	-	0.05	0.05	-0.26	-0.09			0.08
n	(1.87)	(4.39)	(-3.85)	(3.02)				ent	(0.29)	(1.07)	(-3.60)	(-1.81)			
tio	0.88	-0.14	-0.22	0.19	-0.82		0.43	me	0.66	-0.15	-0.47	-0.05	-0.86		0.28
rec	(3.85)	(-1.95)	(-3.38)	(4.83)	(-7.77)			ase	(3.75)	(-3.26)	(-6.74)	(-1.04)	(-8.19)		
nr	0.58	0.48	-0.26	0.13		-0.23	0.2	arr	0.77	1.07	-0.22	-0.13		-1.08	0.2
Ses	(2.12)	(2.37)	(-3.19)	(2.64)		(-1.12)		qm	(3.70)	(6.09)	(-3.19)	(-2.76)		(-5.99)	
щ	0.83	-0.29	-0.24	0.2	-0.85	0.15	0.43	E	1.01	0.54	-0.4	-0.08	-0.71	-0.69	0.32
	(3.52)	(-1.49)	(-3.45)	(4.85)	(-7.77)	(0.83)			(5.24)	(2.97)	(-5.86)	(-1.78)	(-6.62)	(-3.89)	

Table 2.13: SMB explained by Lasso selected Industries (3)

The table reports results from regressions for the SMB factor over the full sample period, from July 1957 to October 2018, during the Golden Age, from July 1957 to December 1979, the Embarrassment, from January 1980 to December 1999 and the Resurrection, from January 2000 to December 2012, periods (Asness et al., 2018). The regressions considered explanatory variables are the market, SMB, HML, MOM, QMJ factors and $L_{-IND_{Big}}$ portfolios. All the figures are expressed in percentage and the t-stat of the estimates are given below within parenthesis. R^2 is the adjusted R-squared of the cross-sectional regression.

Chapter 3

On the non-linear relation between the market and the common risk factors

Résumé

Dans cet essai nous conduisons une étude sur la relation entre le marché et les principaux autres facteurs de risque en considérant une approche non-linéaire de la relation entre le marché avec ces facteurs dans le but d'en réduire le nombre. Ainsi, nous mettons en évidence l'existence d'une transformation non-linéaire du facteur de marché qui conduit à distinguer, sans *a priori* et de manière statistiquement optimale, les rendements de marché positifs des rendements négatifs. En d'autres termes les facteurs de risque considérés peuvent être en partie pris en compte de façon non-linéaire par le facteur de marché. Nous explorons également dans quelle mesure le lien entre le marché et certains facteurs de risque peut s'avérer profitable en termes de stratégies d'investissements. Nous montrons qu'une exposition différenciée aux bêtas des rendements de marché positifs et (surtout) négatifs procure une rentabilité supérieure (et anormale) à celle du marché. En outre la performance d'un investissement basé sur ces stratégies s'avère particulièrement significative.

On the non-linear relation between the market and the common risk factors

Abstract

In this essay we conduct a study of the relationship between the market and other standard risk factors considering, in particular, a non-linear relation between the market and these factors in order to reduce their number. Thus, we highlight the existence of a non-linear transform of the market factor that leads to a distinction, without *a priori* and in a statistically optimal way, between upside and downside market returns. In other words, the considered risk factors may be partially taken into account in a non-linear way by the market factor. We also explore the extent to which the relation between the market and certain risk factors can be profitable in terms of investment strategies. We show that a differentiate exposure to upside and (especially) downside market betas leads to high (and abnormal) returns. Additionally, investments based on these strategies prove to be particularly profitable.

Introduction

The capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965) and Mossin (1966) supports a linear relation between the risk premium of an asset or portfolio and the market portfolio. However many other risk factors appear to be priced even though the CAPM states that the market risk should be the only relevant factor for investors (Cochrane, 2011; Harvey et al., 2016). In fact the phenomenological three factor model of Fama and French (1992, 1993) or the four factor of Carhart (1997) or the more recent five factor model of Fama and French (2015) account for the addition of other factors to the market factor.¹ These factors are considered as proxies for (unidentified) state variables that produce undiversifiable risks in returns that are not captured by the market portfolio. The study of the relation between the market and these other risk factors is a central issue in financial

¹The most standard risk factors are the Size (SMB), Value (HML), Momentum (WML), Profitability (RMW) and Investment (CMA) factors. SMB (Small minus Big) is the return on a diversified portfolio of small stocks minus the return on a diversified portfolio of big stocks; HML (High minus Low) is the difference between the returns on diversified portfolios of high and low Book-to-Market stocks, RWM (Robust minus Weak) is the difference between the returns on diversified portfolios of stocks with robust and weak profitability, CMA (Conservative minus Aggressive) is the difference between the returns on diversified portfolios of low and high investment stocks which are called respectively conservative and aggressive and WML (Winners minus Losers) is the difference between the returns on diversified portfolios of past winners and losers.

economics which has naturally received much attention and is especially relevant in a context of potential redundancies between these factors, as acknowledged by Fama and French (2015).² Furthermore Fama and French (2018a) argue that given the plethora of factors that might be included in a model, choosing among competing models is an open challenge and there is no consensus on the number or the type of factors in the literature (see also Cochrane, 2011; Harvey et al., 2016). Kan and Robotti (2012) also report that the unnecessary factors have an detrimental impact on the acceptance or rejection of existing asset pricing models. They argue that the standard errors of the risk-premium estimates associated with the genuine factors included in a model are affected by the presence of irrelevant factors.

Hence in this Chapter we contribute to the aforementioned debate by examining and shedding new light on the relationship between the market and the other risk factors. Our work builds on the literature that tries to develop an understanding of the relationship between the market and the pricing factors. In fact many procedures have been proposed to identify and extract the common uncorrelated components between the risk factors. Among others, the Principal Component Analysis (PCA) or the Gram-Schmidt orthogonalization method have been adopted to extract the linear dependence or eliminate the correlations between the risk factors (see Baker and Wurgler, 2006; Klein and Chow, 2013).³

Our work relates to but distinguishes from these previous articles dealing with the relationships between risk factors. In fact orthogonalization does not allow to eliminate the potential irrelevant factors when the resemblance between the original and orthogonalized factors is maintained. In addition it does not permit to account for the widely documented asymmetry (non-linearities) in asset return. Hence to overcome this issue we apply the *Alternating Conditional Expectations* (ACE) algorithm developed by Breiman and Friedman (1985) to address the problem of the (non-linear) relation between the market factor and the other seemingly priced risk factors. In fact the ACE estimates the optimal transforms for both risk factors and the market in a regression framework.⁴ To the best of our knowledge, this ACE approach has never been applied to analyze the optimal transform between risk factors in the context of asset pricing.

The ACE regression results show that the conditional expectations of the consid-

 $^{^{2}}$ Fama and French (2015) write:

[&]quot;[T]he factors are just diversified portfolios that provide different combinations of exposures to the unknown state variables[...] The role of [a] valuation model is to suggest factors that allow us to capture the expected return effects of state variables without naming them[...] Thus in the five-factor model HML seems to be **redundant** for explaining average returns."

³Fama and French (1993) already advocated the necessity to orthogonalize the market factor since its return is a mixture of the many other risk factors or to apply orthogonal transforms to the other risk factors to specify the role of each of them. Klein and Chow (2013) apply different orthogonal transforms to these factors – the Principal Component Analysis (PCA), the Gram-Schmidt (GS) orthogonalization process and the Schweinleir-Wigner/Lowdin symmetric procedure (SWL) among others – and specify the role of each of them.

⁴Optimal ACE transform always exists as argued by Breiman and Friedman (1985) and provides a powerful tool for exploratory data analysis.

ered risk factors can be expressed as piecewise-linear functions of the market returns. The location of the kink of these piecewise-linear functions is always found to be close to zero, meaning that the non-linear behavior of the considered risk factors mainly summarizes to a difference of sensitivity to the direction of the market movements. These results are consistent with Ang et al. (2006), among others, who point out that a fraction of the momentum effect (WML) could be specifically related to the drops in the market, thus illustrating the difference of sensitivity to the positive and negative market fluctuations. Harvey and Siddique (2000) and Chung et al. (2006) suggest that the size factor, and the book-to-market to a lesser extent, proxy for higher-order systematic co-moments of returns.⁵ The ACE results confirm that a part of these factors is indeed explained by, at least, the upside or downside market returns.

The other contribution of this Chapter is our analysis of the pricing implications and more importantly the investment opportunities stemming from the asymmetric relation between the risk factors and the market. In fact considering that asset or portfolio returns are non linearly explained by the market only, we aim to observe whether taking into account the asymmetric reactions to positive and negative returns allows to achieve better performance compared to an exposure to the market as a whole.⁶ Hence following Frazzini and Pedersen (2014), who propose a Betting-against-Beta (BAB) strategy, we extend their approach to account for the differentiated sensitivities to the market upside and downside returns. Frazzini and Pedersen's strategy exploits the deviation reported by Black et al. (1972), that is the security market line is flatter than predicted by the standard CAPM for U.S. equities. They form a portfolio that is long on low-beta stocks and short on highbeta stocks, thus maintaining an overall beta-neutral position. Hence we apply their approach based on the upside and downside market betas.

The empirical analysis is conducted on a set of portfolios obtained from Kenneth French's web site. We consider the daily returns on the market portfolio, the 5 Fama-French portfolios SMB, HML, WML, RMW and CMA, the 30 (and 48) Industry portfolios, the 10 portfolios sorted by decile of size, the 10 portfolios sorted by decile of book-to-market ratio and the 10 portfolios sorted by decile of operating profitability and the 10 portfolios sorted by decile of operating profitability and the 10 portfolios sorted by decile of investment.⁷ The sample covers the period from July 1963 to August 2018 and includes all NYSE, AMEX and NASDAQ listed stocks.

Regarding the beta sorted strategy, based on the 30 Industry portfolios as well as on the 10 Value and Operating Profitability portfolios, the lowest beta sorted portfolios not only deliver a greater return than the highest beta portfolios, consistent with Frazzini and Pedersen (2014), but also a higher average return than the Market. The Long-Short portfolio delivers a significant positive return reaching

⁵Chung et al. (2006) show that, adding a set of systematic co-moments (but not standard moments) reduces the explanatory power of the Fama-French factors to insignificance in almost every tested cases.

⁶This approach follows Ang et al. (2006) and Lettau et al. (2014) who consider the downside market moves to account for the asymmetric reactions to positive and negative returns.

⁷These 2 portfolios are only available in monthly data.

more than 4% for the 30 Industry portfolios.

However when it comes to consider the 10 Size, Momentum, Investment and Operating profitability portfolios, the results show that betting on the market upside and downside betas is much profitable than betting against the (whole) market beta. We can notice that for the 10 Momentum portfolios all the portfolio going long on the highest loading and short on the lowest one are positive and significant with a potential annualized return of 3%. For the 10 Size portfolios, the long-sort portfolio based on the downside (upside) sorted beta portfolio exhibits an annualized significant return of 3.44% (3.17%). These results for the 10 Size portfolios can be related to the fact that the BAB might account for a part of size premium as already noticed by Asness et al. (2018).

We also report the performance (cumulative return) of these portfolios over time. Those based on the downside market betas have consistently delivered positive returns. In sum, our results underline a real investment opportunity when accounting for the difference of sensitivities to the market upside and downside moves.

The rest of this paper is organized as follows. In Section 1 we expose the framework for studying the relation between the market and other risk factors. Section 2.3 presents the implications of the relation obtained between the market and risk factors in terms of investment opportunities. Appendices 3.A and 3.B detail the ACE procedure and the methodology for multiple hypothesis testing while Appendices 3.C and 3.D provide additional results.

1 Framework

1.1 Risk factors and non-parametric approach

Risk factors such as the Size (SMB), Value (HML), Momentum (WML), Profitability (RMW) and Investment (CMA) factors are argued to be needed in addition of the market factor, for explaining portfolio returns (Carhart, 1997; Fama and French, 2015, 1993). In spite of the variety of methods that have been adopted to extract the linear dependence between these factors (see Baker and Wurgler, 2006; Klein and Chow, 2013), no consensus is established regarding the relation, and the possible redundancies, between them. However Harvey and Siddique (2000) consider Fama-French factors as proxies for co-skewness while Hung (2007) shows that both momentum and size effects are attributable to higher order systematic co-moments. These results suggest a non-linear relation between the market factor and the other risk factors. Hence we look for the optimal transforms between each risk factor and the market to account for these asymmetry (non-linearities) in asset returns.

To this aim we test whether the risk factors carry specific features or simply proxy for non-linear reactions of asset prices to market movements by relying on an exploratory approach that yields maximum dependence between the market and the other risk factors: the ACE algorithm developed by Breiman and Friedman (1985).⁸ An Optimal ACE transform always exists, as argued by Breiman and

 $^{^8 \}mathrm{See}$ Appendix 3.A for further details on the ACE procedure.



Figure 1.1: ACE Market transformation

This figure represents the ACE optimal transforms of the market returns for the factors SMB, HML, WML, RMW and CMA over the time period from July 1963 to August 2018.

Friedman (1985), and provides a powerful tool to non-parametrically estimate this best mapping without *a priori*. We apply the ACE method in way to keep the market factor unchanged since it plays the role of a reference for market participants.

The ACE regression results show that the conditional expectations of the considered risk factors, given the market return, admit a particularly simple form. It can be stylized by simple linear functions of the upside and downside market returns. As summarized on Figure 1.1, the optimal transform between the market factor and the Size (SMB), Value (HML), Momentum (WML), Profitability (RMW) factors is asymmetric and denotes a difference of sensitivity to the positive and negative market returns. In blue, for instance, the relation between the factor WML and the market graphically exhibits a behavior consistent with Ang et al. (2006) who underlined that a fraction of the momentum effect could be related to the market downside risk. These results are also consistent with Harlow and Rao (1989) and Pettengill et al. (1995), among others, who advocated the asymmetry between market gains and losses. Finally, we can notice that the relation between the market and the Investment factor (CMA) remains essentially linear. It could explain the trouble raised by the potential redundancy of this factor.

1.2 Methodology

We now turn to the derivation and investigation of the analytical expression of the relation between the market and the risk factors. The ACE graphical results (Figure 1.1) shed new light on the non linear relation that can be optimally defined between the market and other risk factors in order to account for the asymmetric effect of the market risk. Given the simple shape the relations between the factors and the market returns, we consider the following model:

$$R_{Factor,t} = \begin{cases} \alpha + \beta_{+} \cdot (R_{M,t} - R_{f,t})^{+} + \beta_{-} \cdot (R_{M,t} - R_{f,t})^{-} + \varepsilon_{t}, \\ \alpha + \beta_{+} \cdot (R_{M,t} - R_{f,t}) + \delta_{-} \cdot (R_{M,t} - R_{f,t})^{-} + \varepsilon_{t}, \end{cases}$$
(1.1)

with β_{\pm} the upside and downside betas,

$$\delta_{-} = \beta_{-} - \beta_{+}, \tag{1.2}$$

and the notation x^+ (resp. x^-) stands for max (x, 0) (resp. min (x, 0)). R_M , R_{Factor} , and R_f stand for the returns on the market, the risk factors and the risk-free rate at date t. Equation (1.1) expresses the fact that each risk factor can be explained by the positive and negative returns of the market separately. Each factor is then assumed to exhibit different sensitivities (β_+ and β_-) to the upside and downside market returns. Similarly, we can consider that the factor can be explained by the market as a whole and, specifically, by the negative market returns (with sensitivity given by δ_-). This latter consideration can be related to Ang et al. (2006), among others, who argue that risk factors account for the negative market movements. Hence, the model (1.1) is not new, but, in contrast with the literature, our choice is not the result of a postulate but the consequence of the empirical observations resulting from the ACE procedure summarized on Figure 1.1.

As a benchmark, we also consider the model in which each risk factor is linearly related to the market portfolio:

$$R_{Factor_t} = \alpha + \beta \cdot (R_{M,t} - R_{f,t}) + \varepsilon_t \,. \tag{1.3}$$

This relation expresses that according to the sensitivity of the risk factor to the market, the considered risk factor can be explained by the market. The idea is to compare the linear and non-linear relations between the market and other risk factors.

We estimate Equations (1.1-1.3) using Ordinary Least Square (OLS) regression. Many papers underline the time variations of the asset betas (see Lee and Chen, 1982, among others). Then a popular way to address the problem is to let the beta varies over time as Jagannathan and Wang (1996) for instance. As a consequence, the rolling window approach is commonly used.⁹ All along of this Chapter, we consider a rolling period of 750 days with a lag of 125 days between two successive periods for the regressions. The main drawback of this approach is the necessity to

⁹In this approach, it is assumed that the near future is similar enough to the past otherwise the time-varying betas change too quickly in order to adapt to the changing market conditions.

perform multiple hypotheses testing (one for each rolling window) which requires an adapted methodology. Indeed, performing a (unique) hypothesis test at the 5% significance threshold, for example, means accepting to wrongly reject the null hypothesis once out of twenty (*i.e.* 5% of the time). It is easy to understand that repeating the test several times (multiple hypotheses) will lead to a rejection rate much higher than expected. It is therefore appropriate to adjust the significance threshold of the test to take into account the presence of multiple hypotheses. Bonferroni (1936) was the first to propose such a correction, but it is known to be too conservative. Thus we use the procedure of Benjamini and Yekutieli (2001) which allows us to determine a more accurate correction in case of multiple mutually dependent hypotheses. The Benjamini and Yekutieli (2001) adjustment method, based on the procedure that controls the False discovery rate (FDR), seems to be well adapted for the test of the overall significance of the regression parameters.¹⁰

1.3 Preliminary analysis

We first present, in Table (3.1), summary statistics and correlations relating to the market and the risk factors. Panel (I) of Table (3.1), reports the annualized mean, standard deviation and the two first higher order moments for the market, Fama and French and Carhart factors. Highest (lowest) annualized average returns are evidenced by WML (SMB), while the market (CMA) has the highest (lowest) annualized standard deviation of returns. The market, SMB, CMA and WML have negative skewness and all the factors exhibit excess kurtosis. The SMB and WML factors exhibit a greater skewness in absolute value while the factor SMB exhibits the highest the excess-kurtosis with a value of 21.41. The mean, standard deviation as well as the skewness and the excess-kurtosis for the Market and other risk factors are all significant.

In Panel (II) of Table (3.1), we report the correlations between the market, SMB, HML, WML, RMW and CMA factors. HML and CMA have a correlation of 55% suggesting that they capture common information which is consistent with the Fama and French (2015) note of caution regarding the redundancy between these factors. The profitability factor RMW has a negative correlation with the market, SMB and HML while this latter is negatively correlated with the market, WML and RMW. Overall the market has a significant negative relationship with all other risk factor except the CMA with which a higher positive correlation is exhibited (37%).

The presence of these significant correlation between the market and the risk factors partially motivates us to test the best transform of the market returns accounting for the changes in the risk factors. That is, we want to assess how much the risk factors depend on the upside/downside market returns and on the whole market returns. In order to derive a better understanding of the exposure of Fama and French and Carhart factors to the market, we examine the linear and non-linear models defined by equations ((1.3)-(1.1)) using moving windows of 750 days with a lag 125 days between two successive sub-periods.

¹⁰see Appendix 3.B for further details on the adjustment method of Benjamini and Yekutieli (2001).

		Panel (I)		
Factor	Mean	StDev.	Skew.	X-Kurt.	
$R_M - R_f$	6.46	15.52	-0.52	15.69	
SMB J	2.19	8.20	-0.96	21.41	
HML	4.05	7.91	0.36	10.48	
WML	7.72	11.08	-0.96	14.53	
RMW	3.15	5.76	0.32	10.07	
CMA	3.34	5.75	-0.38	11.21	
		Panel	(II)		
	$R_M - R_f$	SMB	HML	WML	RMW
SMB	-0.12				
HML	-0.21	0.07			
WML	-0.12	0.02	-0.26		
RMW	-0.21	-0.29	-0.06	0.13	
CMA	0.37	0.03	0.55	0.03	0.06

 Table 3.1: Pricing Factors Relations

We report the annualized mean and standard deviation (in percentage) of the Market and the considered risk factors over the time period from July 1963 to August 2018. The Skewness and the excess-Kurtosis are not annualized. We also report the correlation between the market portfolio, the SMB, HML, WML, RMW and CMA factors over the same period. All the reported correlations are statistically significant at the 5% level. We report, on Table 3.2, the average value of regression parameters (β , β_+ , $\beta_$ and δ_-). For each of these parameters we also report the significance at the level 5% (values in parenthesis on Table 3.2) which is obtained with the Benjamini and Yekutieli (2001) adjustment method. This significance represents the number of times a parameter is considered significant throughout the 105 sub-periods that span our whole sample. We also provide the significance when at least one of the market upside or downside beta is significant (the second figure within parenthesis in the column reporting β_+ in Table 3.2).

First we notice that the correlation between the market and others factors (represented by the β in equation (1.3)) is always negative except for the WML factor. These results are not fully consistent with the correlations reported in Table 3.1over the whole sample period. It illustrates the importance of the sliding window approach to account for the time variations of the parameters. Moreover, as expected the market plays an important role in explaining the risk factors as reflected by its significance (the highest one, observed for the SMB, reaches 96%). When we account, in addition to the market, for its downside moves, all the factors, except the RMW, appear positively correlated to these latter (see column δ_{-}). The impact of the market downside risk is more important for the SMB (significant 66% of the time) but is also related, to a lesser extent, to the WML (significant 27% of the time). Furthermore an inverted correlation between to the market upside (β_+) and downside (β_{-}) returns and these two risk factors can also be mentioned which is consistent with our ACE graphical results (Figure 1.1). Thus a possible explanation for this pattern could be that these risk factors introduce downside risk without comparable upside potential, creating an asymmetric risk profile. Our results also highlight the importance of the asymmetric reactions of the factors to the market moves by the fact that at least one of the upside or downside market beta is always significant (Table (3.2)). For example the SMB is always (100%) sensitive to either the positive or negative market returns while it is less when the whole market is considered (96%). HML is the only noticeable expectation : accounting for upside or downside market returns does not improve on the linear model; in both cases, the parameters are significant over 91.4% of sub-samples.

In sum, our results confirm that the risk factors capture in a non-linear way a part of the market risk, which is consistent with Harvey and Siddique (2000) and Klein and Chow 2013. Our results also underlie that the downside market returns are mostly related with the factor SMB and, to a lesser extent, with WML (consistent with Ang et al. 2006; Hung 2007).

In the next section we investigate the implications of our results by first considering a model based on the positive and negative market returns.

2 Empirical implications

2.1 Model

Most of the asset pricing models can be expressed as :

Factor	β	β_+	β_{-}	δ_{-}
SMB	-0.07	-0.19	0.05	0.24
	(96.2)	(85.7)	(77.1)	(66.7)
			(100)	
HML	-0.17	-0.18	-0.17	0.01
	(91.4)	(88.6)	(82.9)	(17.1)
			(91.4)	
WML	0.04	-0.04	0.11	0.15
	(84.8)	(81.0)	(72.4)	(27.6)
			(94.3)	
RMW	-0.03	-0.00	-0.056	-0.05
	(72.4)	(73.3)	(67.6)	(26.7)
			(86.7)	
CMA	-0.16	-0.17	-0.14	0.02
	(94.3)	(81.9)	(87.6)	(09.5)
			(95.2)	

 Table 3.2:
 Pricing Factors Relations

This Table reports the results of the OLS regression whose dependent variables are the daily returns of the risk factors SMB, HML, WML, RMW and CMA for the period from July 1963 to August 2018. The regression parameters (β , β_+ , β_- , δ_- in equations (1.3) and (1.1)) are averaged over the 105 sub-periods resulting from our rolling window approach (750 days of regression with a lag of 125 days). The significance of each parameter (in parenthesis) is obtained with Benjamini and Yekutieli (2001) adjustment method (at the 5% significance level). The second parenthesis in the column with β_+ corresponds to test of significance of at least one of the market upside or downside beta.

$$R_t - R_{f,t} = \alpha_1 + \beta \cdot (R_{M,t} - R_{f,t}) + \zeta \cdot Factor_t + \varepsilon_t, \qquad (2.1)$$

where R_t and R_f stand for the returns of the asset (or portfolio) under consideration and the risk-free rate. Factor_t is a vector that contains the returns, at date t, on the factors SMB and HML, for the Fama and French three-factor model (FF3), the factor WML, in addition, for the Carhart four-factor model (CAR4) and the factors RMW and CMA for the Fama and French five-factor model (FF5). The parameter ζ refers to the vector of corresponding factor loadings. Equation 2.1 nests the CAPM, the Fama and French 3 (5)-factor (FF3)-(FF5) and the Carhart 4-factor (CARH4) models.

Accounting for the results of the previous section, we look at the consequences of the replacement of the original factors in equation 2.1 by their counterpart in terms of upside and downside betas given by Equation (1.1). Hence we consider the model defined by:

$$R_{t} - R_{f,t} = \begin{cases} \alpha + \beta_{+} \cdot (R_{M,t} - R_{f,t})^{+} + \beta_{-} \cdot (R_{M,t} - R_{f,t})^{-} + \varepsilon_{t}, \\ \alpha + \beta_{+} \cdot (R_{M,t} - R_{f,t}) + \delta_{-} \cdot (R_{M,t} - R_{f,t})^{-} + \varepsilon_{t}. \end{cases}$$
(2.2)

Our approach can be viewed as close to the one defined by Lettau et al. (2014). The model (2.2) has, among other benefits, to account for the downside risk which is in line with Ang et al. (2006) considerations about the stylized fact that the exposure to high downside risk requires a compensation. It can also be related to Pettengill et al. (1995) and Isakov (1999) who estimate a positive (negative) market premium by considering in an *ad hoc* way, the sub-samples of positive (negative) excess market returns. However our approach, resulting from a statically robust procedure, accounts for both sub-samples at same time.

In the Appendix (3.D) we present and detail the cross-sectional results entailed with this model by considering the Fama and MacBeth (1973) procedure. In the next section 2.2, following Frazzini and Pedersen (2014) and Post et al. (2009) among others, who defined strategies based on regression parameters sorted portfolios, we compare the investment performance of strategies based on (whole) beta as well as upside and downside market betas sorting.

2.2 Investment opportunities

In the previous section we define a model (2.2) to account for the observed non linearities risk factors. Hence in this section we analyze the average returns of portfolios obtained by ranking the market betas, as in Frazzini and Pedersen (2014), and by extending this approach to the upside and downside betas. For the rest of this Chapter, when talking about sorting by beta, we mean the regression parameters β , β_+ , β_- and δ_- in the models defined in Subsection 2.1). Hence we sort the betas because we aim to observe whether the differentiate exposure to the upside and downside market betas allows to achieve better performances than the whole market. Moreover the main advantage of sorting the betas is that it maximizes the spread in the betas and the associated risk premiums, and it gives a first indication of the potential gross profitability of practical trading strategies. We employ a single-sorting routine for each of the four betas. Following Frazzini and Pedersen (2014) we sort our sample returns according to their previous market risk exposure (beta) using Model (2.2). To this aim we run a regression over the previous 750 days. Considering the month of July of year t as the beginning of our testing period the regression window will start in July of year t-5 and ends in June of year t (750) days). Then, our returns are sorted into beta-quintile portfolios, according to their previously experienced beta (from the lowest to the highest). For each portfolio, equally-weighted returns are calculated over the following 125 days. We repeat this procedure on the following rolling windows (of 750 days) with 125 days lag and so on. These portfolios are then rebalanced every 125 days. It ensures a reasonable turn-over. As in Frazzini and Pedersen (2014), we leverage our sorted portfolios to a market beta of 1 to keep the same level of systematic risk for all compared portfolios. In addition we also consider the market neutral (i.e. with a market beta of zero) portfolio that is long (short) on the lowest (highest) beta noted R_{Low} (R_{High}).¹¹ We provide the cumulative return for the lowest (highest) beta sorted portfolios as well as the market portfolio. The cumulative return of the Long-Short beta sorted portfolios are also reported.¹²

2.3 Results

In Table 3.3 we report the annualized mean and standard deviation of beta sorted strategies based on the 30 Industry, 10 Size, Value and Momentum portfolios (the results for 10 Operating Profitability and Investment portfolios are presented in Table 3.4 in Appendix 3.D). For all the tested portfolios, except the 10 Momentum portfolios, our results are consistent with the literature and particularly with Frazzini and Pedersen (2014). That is, there is significant evidence of a higher performance of the portfolios invested in the lowest betas (first quintile), compared to that obtained by portfolios invested in the highest betas (last quintile). This evidence holds, in particular, for the 10 Value portfolios (annualized mean return for Low-High of

$$R - R_f = \frac{1}{\theta} \left(R_{\beta^*} - R_f \right), \qquad (2.3)$$

where θ is the market beta obtained by the regressing each beta sorted portfolio (noted R_{β^*}) on the market.

Then they define the portfolio :

$$R_{Low} - R_{High} = \frac{1}{\theta_{Low}} \left(R_{\beta^*} - R_f \right)_{Low} - \frac{1}{\theta_{High}} \left(R_{\beta^*} - R_f \right)_{High},$$
(2.4)

where $R_{Low}(R_{High})$ denotes the portfolio formed with the lowest (highest) betas. ¹²The cumulative return is obtained with :

$$1 + R_T^{WC} = \prod_{t=1}^{T-1} (1 + R_t).$$
(2.5)

It allows to estimate the return over time of an investment of \$1 in this portfolio.

¹¹To form their Bet-against-Beta factor, Frazzini and Pedersen (2014) first leverage their beta sorted portfolios with

1.73%) and the 30 Industry portfolios (annualized mean return for Low-High of 4.29%) in Table 3.3 and the 10 Operating Profitability portfolios (annualized mean return for Low-High of 2.34%) in Table 3.4 in Appendix 3.D. The performance for the portfolios invested in the lowest β , β_+ , β_- and δ_- are also higher than the market portfolio for all tested portfolios except the 10 Momentum and Operating Profitability portfolios.

We point out in the previous section the correlation between the market downside returns and the SMB and WML factors. For the 10 Momentum portfolios, we observe that the β_{-} and δ_{-} based portfolios exhibit a significant mean spread (High-Low) of respectively 3% and 3.04% while for the 10 Size portfolios the δ_{-} sorted portfolio captures a premium of 3.44 per year. The results for the 10 Size portfolios seem to be consistent with what we pointed out above about the potential relation between the size effect and the downside market returns. Moreover for these portfolios the strategy based on β_{+} exhibits a clear increasing trend across the quintiles with a significant Low-High mean return of 3.17%. It can be also noticed that, for both the 10 Operating Profitability and Investment portfolios in Table 3.4 in Appendix 3.D, the (Low-High) β_{-} based strategy delivers a significant mean of respectively 3.29% and 2.98%. Thus the β_{-} as δ_{-} capture priced risk in relation with the market downside returns and these results provide additional support to the literature on downside risks (Ang et al., 2006; Post et al., 2009).

Figures 2.1, 2.2, 2.4 and 2.3 (see also Figures 3.C.1 and 3.C.2 in Appendix 3.C) show the cumulative returns of our beta-quintile sorted portfolios over time. Specifically, we have in the left (right) bottom panel the cumulative return of Low (High) beta sorted portfolios with the market cumulative return. In the top panel we represent on the left (resp. right) the Low-High δ_- (resp. β_-) -sorted portfolios and in the middle the portfolios sorted by β_+ . In addition in each of these three top panels, we present the Low-High β sorted portfolio. First we note that all the highest and lowest beta sorted portfolios, except the 10 Operating Operating (Figure 3.C.2 in Appendix 3.C), the highest δ_- -sorted portfolios (in red) deliver the highest performance over time.

Considering the Low-High β sorted portfolios an abnormal return over time is evidenced for the 30 Industry portfolios (Figure 2.1) which is consistent with Frazzini and Pedersen (2014) and justifies the real interest of investing or managing a strategy based on beta-sorted portfolios to outperform the market. However when it comes to consider the 10 Size and Momentum portfolios betting on the market downside beta seems to be more profitable than betting against the market beta.

For the 10 Momentum portfolios (Figure 2.4) and the 10 Size (Figure 2.2) the δ_{-} -based strategy exhibits a higher return than the β -based strategy over time. This is also true for the 10 Investment portfolios (Figure 3.C.1 in Appendix 3.C) while for the 10 Operating Profitability, the β_{-} -based portfolio outperforms the β_{-} based portfolio. An explanation could be a potential correlation between BAB, size premium and downside market returns. In fact, as argued by Asness et al. (2018), the BAB factor is related to the size premium which is also related to the market downside returns as above underlined. For the 10 value portfolios the β_{-} based

	β_+		β_		δ_{-}		β	
	Mean (%)	Sd (%)	Mean $(\%)$	Sd (%)	Mean $(\%)$	Sd (%)	Mean $(\%)$	Sd (%)
				Panel .	A : Size			
Low	11.78	16.5	10	15.85	8.35	14.7	10.5	16.29
2	9.94	15.67	9.57	15.34	9.27	15.09	10.23	15.65
3	10.01	15.35	10.46	15.35	10.02	15.45	10.19	15.28
4	9.82	15.32	9.96	15.39	10.56	15.99	10.25	15.4
High	8.6	15.18	9.69	15.64	11.79	16.89	8.76	15.28
Market	8.09	14.36						
Low-High	3.17 ***		0.31		-3.44***		1.74	
				Panel I	3 :Value			
Low	10.16	15.28	9.52	15.2	9.44	14.96	10.1	15.19
2	10.26	15.17	9.94	15.14	8.38	14.92	9.51	15.21
3	9.27	15.09	9.61	15.12	8.91	15.01	9.84	15.14
4	9.49	15.13	9.61	15.08	10.56	15.31	9.87	15.06
High	8.55	15.12	8.89	15.22	10.16	15.67	8.37	15.18
Low-High	1.6		0.63		-0.71		1.73^{*}	1.99
				Panel C :	Momentum			
Low	7.26	15.5	6.37	15.37	6.24	16.02	7.06	15.39
2	7.47	15.29	7.36	15.23	5.93	15.4	7.8	15.23
3	7.84	15.14	7.15	15.04	7.81	15.17	7.33	15.09
4	8.09	15.16	8.57	15.32	10.03	15.36	7.64	15.29
High	8.51	15.89	9.37	15.65	9.29	15.73	9.21	15.71
Low-High	-1.25		-3**		-3.04*		-2.15	
				Panel D	: Industry			
Low	9.82	20.66	9.59	20.63	7.25	17.56	10.49	20.84
2	8.62	17.87	9.02	18.04	8.36	17.19	8.62	17.96
3	7.57	17.53	7.97	17.3	8.43	17.39	8.06	17.45
4	7.69	17.12	7.12	17.12	6.79	17.6	6.91	17.17
High	6.22	16.9	6.27	17.05	7.69	18.58	6.19	16.94
Low-High	3.6		3.32		-0.45		4.29 *	

to the 10 daily Size, Value, Momentum and 30 Industry Portfolios. The mean and the standard deviation are all expressed on an annual basis. Significance Level: *** at 1%, ** at 5%, * at 10%. these portfolios are obtained by sorting the regression parameters in model (2.2) applied This table reports the performance of different beta-quintile sorted portfolios during the time period from November 1939 to August 2018. Following Frazzini and Pedersen (2014)

Table 3.3: Sorted Portfolios

strategy is outperformed by the β_+ -based strategy but the results are mitigated when the β_- and δ_- -based strategies are considered (Figure 2.3).

Conclusion

This study first investigates the relation between the market and the other non market risk factors. The non-parametric ACE regression of Breiman and Friedman (1985) gives us new insights about the relations between the risk factors and the market which admits a remarkably simple non-linear form. In light of these insights we propose a pricing model which only accounts for the market upside an downside returns to study the potential investment opportunities. As in Frazzini and Pedersen (2014), we show, by forming portfolios based on regression parameters, that the exposures to the upside and downside market betas allows an investor to earn substantially larger returns than the market portfolio with the same level of systematic risk.

In summary, our results suggest that a proper definition of the market relation with other risk factors and particularly the downside risk is a important force behind stock prices and provide additional support to this trend of literature.

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Figure 2.1: 30 Industry Cumulative Return

This figure plots the cumulative return of the beta sorted portfolios formed on the 30 Industry portfolios for the period from November 1939 to August 2018. The beta $(\beta, \beta_+, \beta_-, \delta_-)$ sorted portfolios are formed with the procedure described in Section 2.2 following Frazzini and Pedersen (2014). We show in the left (right) bottom panel the cumulative return of Low (High) beta-sorted portfolios with the market as a benchmark. In the top panel we represent in the left (right) Low-High δ_- (β_-) -sorted portfolios with the (β_+) one in the center. In each of the three top panels, we also present the Low-High β -sorted portfolio (as a Bet-against-Beta strategy). We have for β the Black line, β_+ the Green line, β_- the Blue line, δ_- the Red line.



Figure 2.2: 10 Size Cumulative Return

This figure plots the cumulative return of the beta sorted portfolios formed with the 10 Size portfolios for the period from November 1939 to August 2018. The beta (β , β_+ , β_- , δ_-) sorted portfolios are formed with the procedure described in Section 2.2 following Frazzini and Pedersen (2014). We show in the left (right) bottom panel the cumulative return of Low (High) beta-sorted portfolios with the market as a benchmark. In the top panel we represent in the left (right) Low-High δ_- (β_-) -sorted portfolios with the (β_+) one in the center. In each of the three top panels, we also present the Low-High β -sorted portfolio (as a Bet-against-Beta strategy). We have for β the Black line, β_+ the Green line, β_- the Blue line, δ_- the Red line.



Figure 2.3: 10 Value Cumulative Return

This figure plots the cumulative return of the beta sorted portfolios formed with the 10 Value portfolios for the period from November 1939 to August 2018. The beta (β , β_+ , β_- , δ_-) sorted portfolios are formed with the procedure described in Section 2.2 following Frazzini and Pedersen (2014). We show in the left (right) bottom panel the cumulative return of Low (High) beta-sorted portfolios with the market as a benchmark. In the top panel we represent in the left (right) Low-High δ_- (β_-) -sorted portfolios with the (β_+) one in the center. In each of the three top panels, we also present the Low-High β -sorted portfolio (as a Bet-against-Beta strategy). We have for β the Black line, β_+ the Green line, β_- the Blue line, δ_- the Red line.



Figure 2.4: 10 Momentum Cumulative Return

This figure plots the cumulative return of the beta sorted portfolios formed with the 10 Momentum portfolios for the period from November 1939 to August 2018. The beta $(\beta, \beta_+, \beta_-, \delta_-)$ sorted portfolios are formed with the procedure described in Section 2.2 following Frazzini and Pedersen (2014). We show in the left (right) bottom panel the cumulative return of Low (High) beta-sorted portfolios with the market as a benchmark. In the top panel we represent in the left (right) Low-High δ_- (β_-) -sorted portfolios with the (β_+) one in the center. In each of the three top panels, we also present the Low-High β -sorted portfolio (as a Bet-against-Beta strategy). We have for β the Black line, β_+ the Green line, β_- the Blue line, δ_- the Red line.

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Appendix 3.A ACE regression

Non-parametric regression techniques based on statistical and optimization theory have been developed to offer much more flexible data analysis tools for exploring the underlying relationships between dependent and independent variables. In this Appendix, we describe a very general and computationally efficient non-parametric regression algorithm called *Alternating Conditional Expectation* (ACE). The algorithm provides a method to estimate transformations in multiple regression without prior assumptions of a functional relationship. The ACE transformations are shown to be optimal and yield maximum correlation between the variables in the transformed space. Unlike conventional multiple regression, the proposed approach allows for data correction and equilibration for the dependent as well as independent variables. The power of the method lies in its ease of use, particularly for multivariate regression, and its ability to identify and correct for outliers without subjective assumptions. The ACE algorithm, originally proposed by Breiman and Friedman (1985), provides a method for estimating optimal transformations for multiple regression that result in a maximum correlation between a dependent (response) random variable and p independent (predictor) random variables.

Breiman and Friedman (1985) define the ACE regression as:

$$\Theta(Y) = \sum_{i=1}^{p} \Phi_i(X_i) + \varepsilon, \qquad (3.A.1)$$

where Θ is a function of the response variable, Y, Φ_i are functions of the predictors X_i , $i = 1, \ldots, p$ and ε is a random error component independent of the X_i .

A brief description of the theory of ACE and its implementation as applied to continuous random variables are given in the following sections (see Breiman and Friedman, 1985, for further details).

Optimal transforms

Optimal transforms minimize the variance of a linear relationship between the transformed dependent and independent variables. For a given data set consisting of a response variable Y and predictor variables X_1, \dots, X_p , the ACE algorithm starts out by defining arbitrary measurable functions $\Theta(Y), \Phi_1(X_1), \dots, \Phi_p(X_p)$ with zero-mean and finite variance:

$$\mathbf{E}\left[\Theta\left(Y\right)\right] = 0, \qquad (3.A.2)$$

$$\mathbf{E}\left[\Phi_{i}\left(X_{i}\right)\right] = 0, \qquad (3.A.3)$$

$$\mathbf{E}\left[\Theta\left(Y\right)^{2}\right] < \infty, \qquad (3.A.4)$$

$$\operatorname{E}\left[\Phi_{i}\left(X_{i}\right)^{2}\right] < \infty, \qquad (3.A.5)$$

for i = 1, ..., p.

The error variance e^2 that is not explained by regression of the transformed

dependent variable, $\Theta(Y)$, on the sum of the transformed independent variables, $\sum_{i=1}^{p} \Phi_i(X_i)$, reads

$$e^{2}(\Theta, \Phi_{1}, \dots, \Phi_{p}) = \mathbb{E}\left[\left(\Theta\left(Y\right) - \sum_{i=1}^{p} \Phi_{i}\left(X_{i}\right)\right)^{2}\right].$$
 (3.A.6)

The functions Θ^* , Φ_1^* , ..., Φ_p^* are said to be optimal for the regression if they satisfy:

$$e^{*2} = e^2 \left(\Theta^*, \Phi_1^*, \dots, \Phi_p^*\right) = \min_{\{\Theta, \Phi_1, \dots, \Phi_p\}} e^2 \left(\Theta, \Phi_1, \dots, \Phi_p\right) , \qquad (3.A.7)$$

under the constraint $E[\Theta(Y)^2] = 1$, which is just a convenient normalization.

We also consider the correlation coefficient between the transformed dependent variable and the sum of the transformed independent variables:

$$\rho\left(\Theta, \Phi_{S}\right) = \mathbb{E}\left[\Theta\left(Y\right) \cdot \Phi_{S}\left(X\right)\right], \qquad (3.A.8)$$

where $\Phi_S(X) = \sum_{i=1}^p \Phi_i(X_i)$. The function $\Theta^{**}, \Phi_1^{**}, \ldots, \Phi_p^{**}$ are said to be optimal for the correlation if

$$\rho^* = \rho\left(\Theta^{**}, \Phi_S^{**}\right) = \max_{\{\Theta, \Phi_1, \dots, \Phi_p\}} \rho\left(\Theta, \Phi_S\right) , \qquad (3.A.9)$$

under the constraints $E[\Theta(Y)^2] = 1$ and $E[\Phi_s^2] = 1$, which are convenient normalizations again.

It can be shown that $\Theta^{**}, \Phi_1^{**}, \ldots, \Phi_p^{**}$ are optimal for the correlation *if and only* if $\Theta^* = \Theta^{**}, \Phi_1^* = \rho^* \cdot \Phi_1^{**}, \ldots, \Phi_p^* = \rho^* \cdot \Phi_1^{**}$ are optimal for the regression. The minimum regression error and maximum correlation coefficient are then related by $e^{*2} = 1 - \rho^{*2}$. The proof of this statement and the existence of optimal transforms can be found in Breiman and Friedman (1985).

For clarity of the exposition, and without loss of generality, we now restrict the exposition to the bivariate case (p = 1). The objective is to minimize

$$e^{2}(\Theta, \Phi) = \mathbb{E}\left[\left(\Theta\left(Y\right) - \Phi\left(X\right)\right)^{2}\right]$$
(3.A.10)

under the constraint $E[\Theta(Y)^2] = 1$. The minimum with respect to Φ , for a given $\Theta(Y)$, is reached when

$$\Phi(X) = \mathbb{E}\left[\Theta(Y)|X\right], \qquad (3.A.11)$$

by definition of the conditional expectation. Similarly, the solution to the minimization problem with respect to Θ , for a given $\Phi(X)$, is

$$\Theta\left(Y\right) = \frac{\mathbb{E}\left[\Phi\left(X\right)|Y\right]}{\left\|\mathbb{E}\left[\Phi\left(X\right)|Y\right]\right\|_{2}},\tag{3.A.12}$$

where $\|\cdot\|_2 = \sqrt{E[\cdot^2]}$ is the Euclidean norm.

Equations (3.A.11) and (3.A.12) form the basis of the ACE algorithm. The procedure involves an iterative calculation of conditional expectations as follows:

Algorithme 3.1 ACE algorithm

Require: $\gamma > 0$ a given tolerance level. 1: Set $\Theta^0(Y) = \frac{Y}{||Y||_2}$. 2: Evaluate $\Phi^0(X) = E[\Theta^0(Y)|X]$. 3: Set $\Delta e^2 = 2\gamma$ and k = 0. 4: while $\Delta e^2 > \gamma$ do 5: k = k + 16: Evaluate $\Phi^k(X) = E[\Theta^{k-1}(Y)|X]$ 7: Evaluate $\Theta^k(Y) = \frac{E[\Phi^k(X)|Y]}{||\Phi^k(X)|Y||_2}$ 8: Evaluate $\Delta e^2 = |e^2(\Theta^k, \Phi^k) - e^2(\Theta^{k-1}, \Phi^{k-1})|$ 9: end while 10: The final (Θ^k, Φ^k) is the solution to the optimal transform (Θ^*, Φ^*) .

Smoother

Concept of data smoother

When the ACE algorithm is implemented on a finite data set, a data *smoother* is need to estimate the conditional expectations. Friedman and Stuetzle (1981) define a smoother as a procedure that operates on a bivariate data set $\{x_i, y_i, 1 \le i \le n\}$ and produces a decomposition

$$y_i = s\left(x_i\right) + \varepsilon_i \,, \tag{3.A.13}$$

where $s(\cdot)$ is a smooth function (or simply smoother) and ε_i are residuals.

If we assume that the data y_i is generated from a smooth function plus a random noise

$$y_i = f(x_i) + \varepsilon_i, \qquad (3.A.14)$$

then smoothers can be viewed as curve estimators and s is considered as an estimate for f. Examples of commonly used data smoothers are local averages, kernels... A smoother with desirable properties called a *super smoother* is used in Breiman and Friedman (1985) ACE transformations. The following is a brief description of the super smoother developed by Friedman and Stuetzle (1981). The goal of super smoother is to find a procedure that can approximate the function f as closely as desired given a dense enough data set and without any condition on f apart from being smooth.

Super smoother for ACE

For a finite data set, the ACE algorithm replaces the conditional expectation in Algorithm 3.1 by the data smoother s, such as the super smoother (see Friedman and Stuetzle, 1981, for further details). For bivariate data, the algorithm can be formulated as follows

Algorithme 3.2 Smoother algorithm

Require: $\gamma > 0$ a given tolerance level. 1: Set $\Theta^{0}(Y) = \frac{Y}{||Y||_{2}}$. 2: Evaluate $\Phi^{0}(X) = s_{X}(\Theta^{0}(Y))$. 3: Set $\Delta e^{2} = 2\gamma$ and k = 0. 4: while $\Delta e^{2} > \gamma$ do 5: k = k + 16: Evaluate $\Phi^{k}(X) = s_{X}(\Theta^{k-1}(Y))$ 7: Evaluate $\Theta^{k}(Y) = s_{Y}(\Phi^{k}(X))$ 8: Evaluate $\Delta e^{2} = |e^{2}(\Theta^{k}, \Phi^{k}) - e^{2}(\Theta^{k-1}, \Phi^{k-1})|$ 9: end while 10: The final (Θ^{k}, Φ^{k}) is the solution to the optimal transform (Θ^{*}, Φ^{*}) .

Appendix 3.B Adjustment methods

The Benjamini and Yekutieli (2001) adjustment method is based on the procedure that controls the *false discovery rate* (FDR) in multiple hypotheses testing. The FDR is controlled under positive dependence assumptions and based on Benjamini and Hochberg (1995) procedure which is valid for independent tests.

Considering a family of m null hypotheses, $H_1, ..., H_m$, and their corresponding individual p-values, $p_1, ..., p_m$, let

- *R* be the number of rejected null hypotheses (also called "*discoveries*", either true or false),
- V be the number of false positives (Type I error),
- and S be the number of true positives (also called "true discoveries").

We have R = V + S. Hence R is an observable random variable while S and V are unobservable random variables. The false discovery rate (FDR) is :

$$FDR = \begin{cases} E\left[\frac{V}{R}\right], & R > 0\\ 0 & R = 0. \end{cases}$$

We want the FDR to remain below a threshold α . Benjamini and Hochberg (1995) shows that algorithm (3.3) ensures FDR $\leq \frac{m_0}{m}\alpha$, where *m* is the number of tested hypothesis and m_0 is the number of true null hypotheses, end hence FDR $\leq \alpha$.

Appendix 3.C Additional results

In this section we report some additional annualized mean and standard deviation results for beta-quintile portfolios sorted and formed within 10 Investment and Operating Profitability portfolios (from July 1967 to August 2018). The spread between the lowest (first quintile) and highest (last quintile) portfolio returns mean is also Algorithme 3.3 Adjustment method for multiple hypothesis testing

- 1: Let $H_{(1)}, \cdot, H_{(m)}$, be m null hypotheses.
- 2: Define a threshold α .
- 3: Sort in ascending order the *p*-values $p_{(1)}, \ldots, p_{(m)}$ of the *m* null hypothesis.
- 4: Find the largest integer k such that $p_{(k)} \leq \frac{k}{m} \cdot \alpha$.
- 5: Reject the null hypothesis (*i.e.*, declare discoveries) for all $H_{(i)}$ with i = 1, ..., k.
- 6: If the *m* tests are dependent step (4) becomes: Find the largest *k* such that $p_{(k)} \leq \frac{k}{m \cdot \sum_{i=1}^{m} \frac{1}{i}} \cdot \alpha.$

given (Table 3.4). The cumulative return of these portfolios are also given (Figures 3.C.1-3.C.2)

Appendix 3.D Cross-sectional implications

In this section we consider the cross sectional implication of our tested model. The test-procedure is based on the well-known Fama and MacBeth (1973) methodology. The first stage regressions produce point estimates for the parameters β , β_+ , β_- , ζ and ζ^* using the 48 Industry portfolios. We perform rolling regressions over time spans of 750 days to estimate those parameters (see equation (2.1) and (2.2)). Between two successive rolling windows, we use apply a lag of 125 days. The second-stage regression is a cross-sectional regression of the average excess return of the assets, $\overline{R_i - R_f}$, against β , β_+ , β_- , ζ and ζ^* .

Focusing on equation (2.2), we compare the different risk premia when we specifically account, or not, for the downside market returns. Omitting the equation for the first-step regression, we consider the following second-step regression equations:

$$\overline{R_i - R_f} = \Pi_0 + \hat{\beta}_{+,i} \cdot \Pi_{\beta_+} + \hat{\beta}_{-,i} \cdot \Pi_{\hat{\beta}_{-}} + \varepsilon_i , \qquad (3.D.1)$$

and

$$\overline{R_i - R_f} = \Pi_1 + \hat{\beta}^*_{+,i} \cdot \Pi_{\beta^*_+} + \hat{\beta}^*_{-,i} \cdot \Pi_{\hat{\beta}^*_-} + \hat{\zeta}^*_i \cdot \Pi^*_{Factors} + \varepsilon_i , \qquad (3.D.2)$$

where $\hat{\zeta}$ is the vector of factor loadings obtained in the first-step regression when regressed against the factors SMB, HML, WML, RMW and CMA and Π^*_{Factor} is the vector of the corresponding risk-premia Π_{SMB} , Π_{HML} , Π_{WML} , Π_{RMW} and Π_{CMA} .¹³ We consider the CAPM, the Fama and French three-factor model (FF3), the fourfactor model of Carhart (CAR4), the Fama and French five-factor model (FF5)

$$R_{t} - R_{f,t} = \begin{cases} \alpha_{2} + \beta_{+}^{*} \cdot (R_{M,t} - R_{f,t})^{+} + \beta_{-}^{*} \cdot (R_{M,t} - R_{f,t})^{-} + \zeta^{*} \cdot Factor_{t} + \varepsilon_{t}, \\ \alpha_{2} + \beta^{*} \cdot (R_{M,t} - R_{f,t}) + \delta_{-}^{*} \cdot (R_{M,t} - R_{f,t})^{-} + \zeta^{*} \cdot Factor_{t} + \varepsilon_{t}, \end{cases}$$
(3.D.3)

 $^{^{13}\}mathrm{We}$ also have

	β_+		β_		δ_{-}		β	
	Mean $(\%)$	Sd (%)	Mean $(\%)$	Sd (%)	Mean $(\%)$	Sd (%)	Mean $(\%)$	Sd (%)
				Panel A :]	Investment			
Low	7.79	16.43	8.39	16.55	8.44	16.32	7.77	16.53
2	7.61	16.21	7.9	16.18	6.6	16.19	7.5	16.17
3	7.05	16.13	7.25	16.21	7.05	16.14	7.25	16.13
4	7.83	16.16	7.09	16.12	6.1	16.09	8.03	16.16
High	5.69	16.23	5.41	16.2	7.31	16.26	5.45	16.18
Market	6.47	15.15						
Low-High	2.11		2.98 **		1.13		2.32	
			Panel	B : Opera	ting Profitabi	lity		
Low	6.27	16.12	7.51	16.04	5.94	16.2	6.74	16.03
2	5.93	16.11	6.53	16.06	6.12	16.16	6.38	16.04
3	7.21	16.05	6.24	16.03	7.81	16.02	6.43	15.98
4	6.25	16.09	6.61	16.03	5.94	16.1	7.14	16.06
High	5.13	16.35	4.22	16.38	4.95	16.32	4.34	16.37
Low-High	1.14		3.29 ***		0.94		2.34 *	

 Table 3.4:
 Sorted Portfolios

This table reports the performance of different beta-quintile sorted portfolios during the November 1939- August 2018 period. Following Frazzini and Pedersen (2014) these portfolios are obtained by sorting the regression parameters related to the equations 2.2 applied to 10 monthly Operating Profitability and Investment Portfolios. The mean and the standard deviation are all annualized form a monthly basis (that is, multiplied by 12). Significance Level: *** at 1%, ** at 5%, * at 10%.


Figure 3.C.1: 10 Investment Cumulative Return

This figure plots the cumulative return of the beta sorted portfolios formed with the 10 Investment portfolios for the period covering July 1963 - August 2018. The beta (β , β_+ , β_- , δ_-) sorted portfolios are formed with the procedure described in Section 2.2 and following Frazzini and Pedersen (2014). Hence we have in the left (right) bottom panel the cumulative return of Low (High) beta sorted portfolios with also the market. In the top panel we represent in the left (right) Low-High (or High low) $\delta_-(\beta_-)$ sorted portfolios with the (β_+) one in the center. In addition in each of these 3 top panel graphs we also present the Low-High β sorted portfolio (as a bet against beta factor). We have for β the Black line, β_+ the Green line, β_- the Blue line, δ_- the Red line and the Market in Blue azure line.



Figure 3.C.2: 10 Operating Profitability Cumulative Return

This figure plots the cumulative return of the beta sorted portfolios formed with the 10 Operating Profitability portfolios for the period covering July 1963- August 2018. The beta $(\beta, \beta_+, \beta_-, \delta_-)$ sorted portfolios are formed with the procedure described in Section 2.2 and following Frazzini and Pedersen (2014). Hence we have in the left (right) bottom panel the cumulative return of Low (High) beta sorted portfolios with also the market. In the top panel we represent in the left (right) Low-High (or High low) $\delta_-(\beta_-)$ sorted portfolios with the (β_+) one in the center. In addition in each of these 3 top panel graphs we also present the Low-High β sorted portfolio (as a bet against beta factor). We have for β the Black line, β_+ the Green line, β_- the Blue line, δ_- the Red line and the Market in Blue azure line.

and, for each of them, the corresponding model with the market downside returns denoted CAPM-, FF3-, CAR4- and FF5-.

Tables 3.5, 3.6 and 3.7 summarize our results and present the intercepts $(|\Pi_0|)$ and risk premia (Π) obtained with the different alternative models. We first notice that, for all the tested models, the intercepts – and hence the unexplained expected returns – are more significant when we do not take specifically into account the downside market returns. In fact, when we consider the market factor only (CAPM), the significance level of the intercept is strongly reduced, going from 34% to 0.9%, when the market downside return is taken into account. This can be seen as that the relation between risk and return is partly restored when Market negative part is considered in addition. The results also point to some evidence of a positive and negative beta premium on the market with more or less significant level depending on the tested models and portfolios. The 10 Size portfolios exhibit the most significant risk premia with respect to studied models. Consistent with the literature, the Market premium is positive and significant for the Size portfolios. However, when the negative market returns are accounted for, the level of significance of the market decline (going from 33% to 2.8%).¹⁴ Among all other risk factors, the size factor (SMB), is the rare one to exhibit a significant premium. However, as noticed above, the negative market returns affects the SMB factor more than the positive returns. In fact, Tables 3.5 and 3.6 show that, for all the tested models, the significance level of SMB declines when the market negative returns are explicitly taken into account. For FF3, the significance level goes from 40% to 32%, while it goes from 39% to 17% for CAR4 and from 20% to 0.9% for FF5 models. These results show that the SMB factor seems to be strongly correlated to the negative market movements. The coefficients of the risk factors for both CAR4 and CAR4- models are consistent with what is documented in the empirical literature: significantly positive premia for value and momentum and a significantly negative premium for size. However, the premium for the negative market returns becomes less significant in almost all cases. We find some striking the results when the FF3 and FF5 models and its negative market considered related models. In contradiction with the literature, HML exhibits a negative risk premium which means that the premium of this factor shows

where ζ^* refers to the vector containing β_{SMB} , β_{HML} , β_{WML} with *Factor* defined above.

$$\overline{R_i - R_f} = \Pi_1 + \hat{\beta}_+ \cdot \Pi_{\beta_+} + \hat{\beta}_- \cdot \Pi_{\hat{\beta}_{-}} + \hat{\zeta^*} \cdot \Pi^*_{Factor} + \varepsilon_i.$$

One can easily verify that $\Pi_{\beta_{-}} = \Pi_{\delta_{-}}$.

Proof. In fact we replace in equation (3.D.1) $\hat{\beta}_{-}$ by $\hat{\beta}_{+} + \hat{\delta}_{-}$. Then :

$$\overline{R_i - R_f} = \Pi_1 + \hat{\beta}_+ \cdot \Pi_{\beta_+} + \left(\hat{\beta}_+ + \hat{\delta}_-\right) \cdot \Pi_{\beta_-} + \varepsilon_i , \qquad (3.D.4)$$

and then

$$\overline{R_i - R_f} = \Pi_1 + \hat{\beta}_+ \cdot \underbrace{\left(\Pi_{\beta_+} + \Pi_{\beta_-}\right)}_{\Pi_{\beta_+}} + \hat{\delta}_- \cdot \underbrace{\Pi_{\beta_-}}_{\Pi_{\delta_-}} + \varepsilon_i .$$
(3.D.5)

¹⁴The negative market returns exhibit a significant positive premium of 2.6%.

considerable variation across time. In parallel RMW and CMA exhibit positive premia which is some opposite with an possible redundancy between HML and these factors. It is beyond the scope of this appendix section to determine the origins of these variations and differences: sampling error, temporary market inefficiencies, variations in risk attitude, or other explanations.

What is relevant for this part is the specific impact of the negative market returns for the considered models. In addition, the multivariate regression only increases the economic and statistical significance of market negative linear piece-wise premiums. These results further support our conclusion that downside risk is relevant for stock investors and that systematic downside risk should be accounted for which is consistent with Ang et al. (2006) and Lettau et al. (2014).

		- (24)		~					
Models	$\Pi(\%)$			Sigr	Significance (%)				
	Size	Value	Mom	Size	Value	Mom			
				$ \Pi_0 $					
CAPM	0.02	0.04	0.01	34.29	11.43	13.33			
CAPM-	0.01	0.02	0.01	0.95	0.00	10.48			
FF3	0.01	0.03	0.04	0.00	0.00	0.00			
FF3-	0.00	0.03	0.04	0.00	0.00	0.00			
			1	MKT					
CAPM	1.08	-0.74	1.44	33.33	0.00	22.86			
CAPM-	1.76	0.35	1.79	2.86	0.00	20.95			
				M-	<i>A</i> -				
	2.60	4.44	-1.44	7.62	0.00	0.00			
			1	MKT	KT				
	1.90	0.49	-1.36	0.00	0.00	0.00			
	3.28	1.43	-0.52	0.00	0.00	0.00			
				M-					
	4.01	-1.24	-5.64	0.00	0.00	0.00			
			ç	SMB					
	0.61	1.28	-5.57	39.05	0.00	0.00			
	0.51	1.80	-4.86	17.14	0.00	0.00			
		HML							
CAR4	-0.07	1.06	-2.30	0.95	0.00	0.00			
CAR4-	1.27	0.88	-3.81	0.00	0.00	0.00			
			I	VML	ML				
	3.72	6.74	2.07	0.00	0.00	3.81			
	5.42	6.46	2.20	0.00	0.00	1.91			

Table 3.5: 10 Size, Value and Momentum portfolios Cross section

This table reports the absolute average value of the intercept, $|\Pi_0|$, and the Average value of the premia, Π results from Fama and MacBeth regressions (Equations (3.D.1) and (3.D.2)). The Fama and MacBeth procedure is applied, with rolling window approach (detailed in the Subsection 1.2) to the CAPM, the Fama and French (1993) three-factor model FF3, the four-factor model CAR4 of Carhart (1997), the five-factor Fama and French (2015) model FF5. We also consider models with the market negative part denoted CAPM-, FF3-, CAR4- and FF5-. The dependent variables are the daily returns of the 10 Size, Value and Momentum portfolios (from July 1963 to August 2018). The significance of each parameter (in parenthesis) is obtained with Benjamini and Yekutieli (2001) adjustment method (at the level 5%).

Models		$\Pi(\%)$		Sign	ificance	(%)			
	Size	Value	Mom	Size	Value	Mom			
			I	I_0					
CAR4	0.01	0.02	0.04	0.000	0.00	0.00			
CAR4-	0.01	0.01	0.03	0.000	0.00	0.00			
FF5	0.00	0.01	0.04	0.000	0.00	0.00			
FF5-	0.00	0.00	0.05	0.00	0.00	0.00			
	MKT								
	2.09	-0.76	-1.41	0.00	0.00	0.00			
	2.770	0.00	-0.94	0.95	0.00	0.00			
			$M_{\rm c}$	1-					
	1.34	-0.612	-2.57	0.00	0.00	0.00			
			\mathbf{SI}	MB					
FF3	0.55	1.57	-2.75	40.00	0.00	0.00			
FF3-	0.50	1.97	-2.33	32.38	0.00	0.00			
	HML								
	0.71	0.83	-0.53	0.00	0.00	0.00			
	1.11	0.69	-2.09	0.00	0.00	0.00			
		МКТ							
	2.84	1.36	-1.44	0.00	0.00	0.00			
	2.63	2.79	-2.45	0.95	0.00	0.00			
			$N_{\rm c}$	1-					
	2.68	1.64	-2.07	0.00	0.00	0.00			
			SI	MB					
	0.73	0.48	-3.93	20.95	0.00	0.00			
	0.671	0.42	-3.34	0.95	0.00	0.00			
			H	ML					
FF5	-0.63	1.24	-2.00	0.00	0.00	0.00			
FF5-	-0.96	0.92	-3.49	0.95	0.00	0.00			
		RMW							
	-0.27	0.39	1.27	0.00	0.00	0.00			
	-0.53	-0.58	1.78	0.00	0.00	0.00			
			CI	MA					
	-1.30	0.05	1.61	0.00	0.00	0.00			
	-1.77	0.04	0.87	0.00	0.00	0.00			

Table 3.6: 10 Size, Value and Momentum portfolios Cross section (Next)

This table reports the absolute average value of the intercept, $|\Pi_0|$, and the Average value of the premia, Π results from Fama and MacBeth regressions (Equations (3.D.1) and (3.D.2)). The Fama and MacBeth procedure is applied, with rolling window approach (detailed in the Subsection 1.2) to the CAPM, the Fama and French (1993) three-factor model FF3, the four-factor model CAR4 of Carhart (1997), the five-factor Fama and French (2015) model FF5. We also consider models with the market negative part denoted CAPM-, FF3-, CAR4- and FF5-. The dependent variables are the daily returns of the 10 Size, Value and Momentum portfolios (from July 1963 to August 2018). The significance of each parameter (in parenthesis) is obtained with Benjamini and Yekutieli (2001) adjustment method (at the level 5%).

Models	$\Pi(\%)$	Signif (%)	$\Pi_{All \ Sample}(t-\text{stat})$	Models	$\Pi(\%)$	$\operatorname{Signif}(\%)$	$\Pi_{All \ Sample}(t-\text{stat})$
		$ \Pi_0 $	Π_0			$ \Pi_0 $	Π_0
CAPM	0.04	37.6	-0.57 (-0.52)	FF3	0.04	21.5	-0.63 (-0.56)
CAPM-	0.04	36.6	$0.43\ (0.34)$	FF3-	0.04	21.5	0.62(0.41)
		MKT				MKT	
	1.20	51.6	$0.40 \ (0.36)$		1.83	41.9	$0.62 \ (0.52)$
	1.28	47.3	-0.41 (-0.34)		1.60	31.2	-0.59(-0.38)
		M-				M-	
	1.41	29.0	-6.16 (-1.47)		0.92	12.9	-6.39(-1.16)
						SMB	
					-1.03	49.5	-0.43 (-0.52)
					-1.13	41.9	-0.31 (-0.38)
						HML	
					-0.61	47.3	-0.82(-1.15)
					-0.56	46.2	-0.78 (-1.10)

Table 3.7: 48 Industry Cross section

This table reports the absolute average value of the intercept, $|\Pi_0|$, and the Average value of the premia, Π results from Fama and MacBeth regressions (Equations (3.D.1) and (3.D.2)). The Fama and MacBeth procedure is applied, with rolling window approach (detailed in the Subsection 1.2) to the CAPM, the Fama and French (1993) three-factor model FF3, the four-factor model CAR4 of Carhart (1997), the five-factor Fama and French (2015) model FF5. We also consider models with the market negative part denoted CAPM-, FF3-, CAR4- and FF5-. The dependent variables are the daily returns of the 48 Industry portfolios (from July 1963 to August 2018). The significance of each parameter (in parenthesis) is obtained with Benjamini and Yekutieli (2001) adjustment method (at the level 5%). We also report the absolute average value of the intercept, $|\Pi_0|$, and the Average value of the premia, Π results from Fama and MacBeth regressions applied to the Full Sample. T-stat is the statistic when the full sample is used for regression without rolling period.

Models	$\Pi(\%)$	Signif (%)	$\Pi_{All \ Sample}(t-\text{stat})$	Models	Π(%)	Signif(%)	$\Pi_{All \ Sample}(t-\text{stat})$
		$ \Pi_0 $	Π_0			$ \Pi_0 $	Π_0
CAR4	0.03	16.1	-0.81 (-0.49)	FF5	0.04	14.0	-0.88 (-0.50)
CAR4-	0.03	10.8	$0.00\ (0.00)$	FF5-	0.04	10.8	-0.32 (-0.18)
		MKT				MKT	
	2.12	36.6	$0.82 \ (0.46)$		1.80	30.1	$0.87 \ (0.51)$
	2.04	29.0	$0.14 \ (0.08)$		1.56	25.8	$0.22 \ (0.13)$
		M-				M-	
	-0.09	0.0	-7.62(-1.32)		0.13	2.2	-9.46(-1.49)
		SMB				SMB	
	-0.79	46.2	-0.40 (-0.48)		-0.99	41.9	-0.39(-0.46)
	-0.87	45.2	-0.15 (-0.18)		-1.10	41.9	-0.13(-0.16)
		HML			HML		
	0.10	37.6	-0.82 (-1.13)		-0.85	37.6	-0.80 (-0.96)
	0.13	36.6	-0.75 (-1.05)		-0.82	33.3	-1.11 (-1.32)
		WML				RMW	
	10.26	90.3	$0.83\ (0.21)$		0.42	22.6	-0.23(-0.21)
	10.16	88.1	2.42(0.60)		0.46	12.9	0.38(0.34)
					CMA		
					0.56	35.5	0.11(0.09)
					0.48	30.1	$0.55\ (0.45)$

Table 3.7: 48 Industry Cross section (Continued)

This table reports the absolute average value of the intercept, $|\Pi_0|$, and the Average value of the premia, Π results from Fama and MacBeth regressions (Equations (3.D.1) and (3.D.2)). The Fama and MacBeth procedure is applied, with rolling window approach (detailed in the Subsection 1.2) to the CAPM, the Fama and French (1993) three-factor model FF3, the four-factor model CAR4 of Carhart (1997), the five-factor Fama and French (2015) model FF5. We also consider models with the market negative part denoted CAPM-, FF3-, CAR4- and FF5-. The dependent variables are the daily returns of the 48 Industry portfolios (from July 1963 to August 2018). The significance of each parameter (in parenthesis) is obtained with Benjamini and Yekutieli (2001) adjustment method (at the level 5%).

Chapter 4

A brief survey on performance measures

Introduction

Portfolio management consists in building portfolios and then making them evolve in order to achieve the performance objectives specified by the investor, while adhering to the constraints of the strategy (target return, risk level, investment universe...).

In Chapter 1 we provided a description of portfolio returns assessment. In fact, ex post calculation of the return on a portfolio is only the first element needed to determine its performance. When we presented the methods for calculating the return on a portfolio, we noted that the value of the return on its own was not a sufficient criterion for appreciating the performance and that it was necessary to associate a measure of the risk to be incurred. Risk is an essential part of the investment process. It can differ considerably from one portfolio to another. In addition, it is likely to evolve over time.

Modern portfolio theory and the CAPM have established the quantitative relation between the risk and return of an investment. More specifically, these theories highlighted the notion of *risk reward*. Therefore, we are now endowed with the necessary elements for calculating indicators while taking both risk and return into account. The first indicators that have been developed come from portfolio theory and the CAPM. They are therefore more specifically related to equity portfolios. They allow us to evaluate a *risk-adjusted* performance. It is thus possible to compare the performance of funds with different levels of risk, while the return alone only enabled comparisons between funds with the same level of risk. This approach is the reference for measuring the performance of traditional investments.

Performance measurement is used not only to assess the portfolio managers' past performance, but also to forecast their future outcomes. For this reason, an emphasis has been placed on developing methods to provide investors with information that meets their needs, and it explains the growing amount of academic and professional research devoted to performance measurement. The topic of performance analysis is still in expansion, meeting the demands of both investors and portfolio managers.

This Chapter provides a brief review of the available performance measures in

relation with the models presented in Chapter 1. It is a background information that will help understand the performance measures that will be discussed in the next Chapter of the thesis. This analysis focuses only on the performance measures most frequently mentioned in the current financial literature. In fact, a huge number of different alternative risk-adjusted performance measures related to asset pricing models have been identified in the literature; Cogneau and Hübner (2009a,b) even identified more than 101 ways to measure performance.¹

These performance measures are mainly considered over the short term in the literature. However long-term horizons are more interesting according to Cochrane (2011), partly because they more strongly relate predictability to volatility, "bubbles" and the nature of price movements. Fama and French (2018b) and Bessembinder (2018) have recently shed new light on the properties of long-term return distributions and renewed interest in this subject. In addition divergent conclusions and interpretations emerged in the performance assessment and asset allocation literature when horizon is accounted for. In fact the asset pricing literature provides diverging conclusions regarding the benefits of time on asset return performance with Zakamouline and Koekebakker (2009) who argue that the Sharpe ratio increases on the long run while van Binsbergen and Koijen (2017) and Madan and Schoutens (2018) claim the opposite.

The performance measures discussed in this survey can be classified according to the risk-measures underpinning the pricing models. Hence we will consider the performance measures based on the volatility, the lower partial moments and the higher-order moments.

For the remainder of this Chapter, the organization is as follows. We first examine performance measures based on the CAPM (medium-variance framework) with the well-known Sharpe Ratio. We then discuss performance measures based on the lower partial moments. We proceed with modified Sharpe ratios derived from models that take into account higher-order moments and complete this overview with other performance measures related to factor models.²

1 Models application to performance measurement

In the past years risk-adjusted performance measures have gained increasing importance. One of the main reason for this development is the emergence of investment funds as a major investment class. A s a consequence, investors need an effective tool to evaluate the respective performance of the various funds compared to the risk taken by the fund managers to choose the right alternative for capital allocation (Weisman, 2002).

The literature is strongly concerned about the importance of choosing the right

¹Most of them attempt to remedy the shortcomings of the Sharpe ratio which heavily relies on normally distributed returns.

²Furthermore, one can distinguish between performance measures which assume normally distributed returns, measures which explicitly account for higher-order moments of a specific distribution and measures which implicitly account for higher-order moments without assumptions on the return distributions.

risk-adjusted performance measure. But it also seems that the impact of the investment horizon is very significant on the performance assessment and the asset allocation (Bessembinder, 2018). This section provides a brief overview of the current state of risk-adjusted performance measures by describing the most frequently used risk-adjusted performance measures in the asset-pricing literature reviewed in Chapter 1.

1.1 CAPM based performance measures

In spite of the criticisms it has to face, the CAPM is widely appreciated as an asset valuation model. The model highlights the relation between the risk and return of an asset and shows the importance of taking the risk into account. It provides an operational theory that allows the return on an asset to be evaluated relative to its risk. Hence, based on this model, a set of risk-adjusted performance measures can be defined and assessed.

Treynor Measure

The *Treynor ratio* (Treynor, 1962) is drawn directly from the CAPM. It measures the relationship between the return on the portfolio, above the risk free rate, and its systematic risk. This is defined by:

$$TR = \frac{\mathrm{E}\left[R\right] - R_f}{\beta},\qquad(1.1)$$

where E[R] denotes the expected return of the portfolio, R_f the return on the risk-free asset and β the systematic risk of the portfolio.

The Treynor ratio is particularly well-suited for appreciating the performance of a well-diversified portfolio, since it only takes the systematic risk of the portfolio into account. It is also for this reason that the Treynor ratio is the most appropriate indicator for evaluating the performance of a portfolio that only constitutes a part of the investor's assets: If the investor has diversified his investments, the systematic risk of his portfolio is all that matters. Calculating this indicator requires a market index to be chosen to estimate the beta of the portfolio.

Jensen's alpha

The *Jensen's alpha* measures the difference between the return on the portfolio in excess of the risk-free rate and the return explained by the market model. It is based on the CAPM and is calculated by carrying out the following regression:

$$R_t - R_f = \alpha + \beta \cdot (R_{M,t} - R_f) + \varepsilon_t \,. \tag{1.2}$$

The term $\beta \cdot (R_M - R_f)$ captures the return on the portfolio explained by the model. The intercept α measures the share of additional return that is due to the manager's choices. The statistical significance of α can be evaluated by calculating the *t*-statistic of the regression. The value of alpha is actually proportional to the level of risk taken, as measured by the beta. In fact varying the beta according to anticipated movements in the market also varies positively or negatively the Jensen's alpha (Le Sourd, 2007). As the Treynor ratio, the Jensen's alpha (only) takes into account the systematic risk. In this respect, both are subject to the same criticism regarding their dependence on the choice of market index.

Sharpe ratio

The most frequently used performance measure, which relies on the volatility as a risk measure is the *Sharpe ratio* (Sharpe, 1966). Due to its simplicity and thus ease of application, it has found widespread acceptance in the literature and in practice (Weisman, 2002). The Sharpe ratio measures risk as a function of volatility, reflecting the paradigm of Modern Portfolio Theory that prevailed when the Sharpe ratio was introduced. It is expressed as:

$$SR = \frac{\mathrm{E}\left[R\right] - R_f}{\sigma}, \qquad (1.3)$$

where σ denotes the volatility of the portfolio returns.

This ratio measures the return of a portfolio in excess of the risk-free rate, also called the risk premium, compared to the total risk of the portfolio, measured by its standard deviation. It has been the subject of generalizations since its initial definition. It thus offers interesting possibilities for evaluating portfolio performance, while remaining simple to calculate. One of the most common extension to this measure amounts to replace the risk-free rate with a the rate of return on a benchmark portfolio.

Although it is frequently used in theory and practice, the Sharpe ratio has a major drawback since it has been designed to be used in a mean-variance context and therefore requires that returns are normally distributed. The repeated financial crises have clearly shown that this assumption does not hold and that extreme events (most importantly large losses) are more likely to occur than assumed by the normal distribution. Volatility is used to measure risk when normal distributions of returns are assumed and does treat variability in gains and losses in the same way. Thus the Sharpe ratio penalizes both downside and upside returns. A rational investor, however, distinguishes between gains and losses and would rather consider high gain potential as attractive (Zakamouline, 2011).

Yet, measuring risk-adjusted return using the Sharpe ratio could result in an asset with high upward volatility being rated at a lower level than an asset with low downward volatility. Hence, to overcome the deficiencies of traditional performance measures when returns deviate from the normal distribution and particularly to account for the distinction between gains and losses, Lower Partial Moments (LPM) performance measures have been introduced. The Lower Partial Moments enable the risk to be captured by considering only deviations of returns that are below an ex-ante defined threshold (τ or R_f here). After Markowitz (1959) with the semi-variance, Hogan and Warren (1974) and Bawa and Lindenberg (1977) were the first to introduce this kind of measures before Harlow and Rao (1989), among

others, extended and generalized the Lower Partial Moment (LPM) framework and introduced an Asymmetric response model (ARM).

1.2 Downside risks based performance measures

The notion of semi-variance takes into account risk asymmetry by determining whether deviations from the mean occur above or below the mean. The calculation principle is the same as for the variance, except that only below-average returns are taken into account. It therefore provides an asymmetric measure of risk, which corresponds to the needs of investors, who are only interested in the risk of loss of their portfolio. It reads as follows:

$$SV = \mathbf{E}\left[\left(R - \mathbf{E}\left[R\right]\right)^{2} \middle| R \le \mathbf{E}\left[R\right]\right].$$
(1.4)

The lower partial moment generalizes the notion of semi-variance :

$$LMP_m(\tau) = \mathbb{E}\left[\min\left(R - \tau, 0\right)^m\right], \qquad (1.5)$$

where τ is a user-defined threshold (target return). We use the LPM notation defined by Hogan and Warren (1974) and Bawa and Lindenberg (1977). It measures the risk of falling below a target return chosen by the investor. The mean return is replaced in this formula by the value of the target return below which the investor does not wish to drop. This notion can then be used to calculate the risk-adjusted return indicators that are more specifically appropriate for asymmetrical return distributions.

Among the best-known LPM based performance measures are the Omega, Sortino and Kappa ratios (presented below). In fact, the use of the lower partial moments of order 1, 2 and 3 respectively leads to the definition of these performance measures. Originally the Omega and Sortino ratios did not explicitly relied on the LPM as a risk-measure. The categorization of the Omega, Sortino and Kappa ratios according to the order of the LPM traces back to Kaplan and Knowles (2004), who tried to find a more comprehensive description of performance measures.

Omega Ratio

The Omega ratio has been introduced by Shadwick and Keating (2002). It equals one plus the ratio of the excess return over a threshold τ to the LPM of order 1:

$$\Omega\left(\tau\right) = 1 + \frac{\mathrm{E}\left[R\right] - \tau}{LMP_{1}\left(\tau\right)}.$$
(1.6)

This definition is appealing in that it resembles the Sharpe ratio in its structure. That is why Kazemi et al. (2004) refer to the last term of the right-hand side as the *Sharpe-Omega ratio*. It usually appears in the context of portfolios of hedge funds and is consistent with traditional performance measures (Le Sourd, 2007).

Sortino ratio

Similar to Omega, downside deviations can be interpreted as the square root of the LPM of order 2 which finally leads to the version of the *Sortino ratio* (Sortino and Price, 1994) in which an LPM is used as a risk measure (Kaplan and Knowles, 2004). It is defined on the same principle as the Sharpe ratio. However, the risk-free rate is replaced by target return τ :

$$SOR(\tau) = \frac{\mathrm{E}[R] - \tau}{\sqrt{LMP_2(\tau)}}.$$
(1.7)

This measure allows a distinction between "good" and "bad" volatility: it does not penalize portfolios with returns that are larger than their target, as opposed to the Sharpe ratio.

Kappa ratio

Motivated to find a more comprehensive risk-adjusted performance measure, Kaplan and Knowles (2004) developed the *Kappa ratio*. The general form of Kappa is:

$$K_{n}(\tau) = \frac{\mathrm{E}\left[R\right] - \tau}{\sqrt[n]{LMP_{n}(\tau)}}.$$
(1.8)

Kaplan and Knowles (2004) showed that the Omega and Sortino ratios are only special cases of Kappa, with the order n determining whether the Sortino ratio, the Omega ratio, or another risk-adjusted return measure is generated. Choosing the parameter so that n = 1 – respectively n = 2 – yields Omega (= K_1) – respectively the Sortino ratio (= K_2). In general, any number is admissible for the parameter $n.^3$

Other downside performance ratios

Other downside measures have also been proposed to provide a more accurate picture of extreme risks and to overcome the drawbacks of the Sharpe ratio. For example the Value-at-Risk (VaR) has become an essential tool for communicating risk to managers, directors and shareholders as it captures downside risk in a single figure which is easy to interpret.⁴ Dowd (2000) proposed to express the excess return of an asset relative to its VaR.

However the VaR presents also some shortcomings. In fact this risk measure does not consider losses outside of the $(1 - \alpha)$ -confidence interval. Then the Conditional

$$VaR_{1-\alpha}(X) = \inf \left\{ t \in \mathbb{R}_+ : \Pr\left(X \le t\right) \ge 1 - \alpha \right\} .$$

³As long as $E[|R|^n] < \infty$.

⁴The popular concept of VaR describes the expected maximum loss over a target horizon within a given confidence level α :

Value-at-Risk (CVaR) has been developed.⁵ Agarwal and Naik (2003), considering hedge funds, used this risk measure to demonstrate the extent to which the mean-variance framework underestimate the impact tail risks in performance assessment.

Therefore all these performance measures, while taking into account the specificity of (downside) risks, fail to take into account the impact of higher-orders moments. Thus in the next section the higher-order moments based performance measures are presented.

1.3 Higher order moments context

To overcome the drawbacks of the assumption of normally distributed returns, higher-order moments have to be taken into account to develop performance measures that go beyond the mean-variance framework. In fact empirical evidence suggests that asset returns are positively skewed and have heavier tails than implied by a normal distribution (Fama, 1965). These results have many implications for both asset allocation and performance measurement. Early works suggested the use of the third moment, the skewness of expected returns, in addition to the original first two moments in the asset allocation models (Arditti and Levy, 1975). Jurczenko and Maillet (2006) summarize the discussion on the need for multi-moment portfolio theories and aim to put together previously scattered literature on the topic. We also reviewed this literature trend in the Chapter 1. As direct implications in terms of performance measurement, the account of higher-order moments has been suggested to overcome some shortcomings of the Sharpe ratio. A myriad of other risk-adjusted performance measures, most of which are not based on the normality assumption, have been developed in the literature to take into account the higherorder moments of asset returns (Cogneau and Hübner, 2009a,b). This was achieved by integrating information on higher-order moments like the skewness and kurtosis as well as by developing measures which do not make any distributional assumption and therefore are generally applicable – regardless of the return distribution.

Pezier and White (2006) were motivated by the limitations of the Sharpe ratio, especially those resulting from the assumption of normally distributed returns, and therefore suggested an Adjusted Sharpe ratio (ASR) to overcome this deficiency. The measure is derived from a Taylor series expansion of the expected utility with an exponential (CARA) utility function. Keeping the first four terms of the expansion leads to the formula of the ASR stated below, where SR stands for the original Sharpe ratio, Skew for skewness and Kurt for kurtosis (Pezier and White, 2006; Zakamouline and Koekebakker, 2009):

$$ASR = SR \cdot \left(1 + \frac{\text{Skew}}{6} \cdot SR - \frac{(\text{Kurt} - 3)}{24} \cdot SR^2\right).$$
(1.9)

$$CVaR_{1-\alpha}(X) = \frac{1}{\alpha} \int_0^\alpha VaR_{1-\gamma}(X) \, d\gamma$$

⁵The conditional Value-at-Risk, CVaR, describes the expected loss under the condition that VaR is exceeded:

The ASR belongs to the group of measures in which skewness and kurtosis are explicitly included. The higher-order moments have also been accounted for in the literature when the VaR is the considered risk measure. In fact the Modified Value-at-Risk (MVaR) which adjusts VaR for skewness and kurtosis is used by Gregoriou and Gueyie (2003) to measure risk-adjusted performance, the Modified Sharpe Ratio (MSR).⁶ They argue that that this modified Sharpe is lower and more accurate when examining non-normal returns in comparison to the Sharpe Ratio by ranking (30) funds of hedge funds. Due to the lack of theoretical foundation, these measures shall should be considered with caution (Cogneau and Hübner, 2009a,b).

1.4 Multi-factor application to performance measurement

Without questioning the contribution of the CAPM, the current consensus tends towards the idea that a single factor is not sufficient for explaining asset returns. Besides the market factor, other factors have been identified (the size and value of Fama and French for example). Some of these factors are very well known and have resulted in a family of models collectively referred to as multi factorial models (see Chapter 1). These models constitute an alternative theory to the CAPM, but do not replace it. They also allow asset returns to be explained by factors other than the market index, and thus provide more specific information on risk analysis and the evaluation of managers' performance. They are generally stated as:

$$R_{i,t} = \alpha_i + \sum_k \beta_{i,k} \cdot F_{k,t} + \varepsilon_{i,t} , \qquad (1.10)$$

with $\beta_{i,k}$ is the sensitivity of asset *i* to the factor F_k and ε_i the specific risk.

The multi-factor models have a direct application in investment fund performance measurement. The models provide more information for performance analysis than the Sharpe, Treynor and Jensen ratios (Le Sourd, 2007). Once the model has been selected, we can attribute the contribution of each factor to the overall portfolio performance. This is easily done when the factors are known, which is the case for models that use macroeconomic factors or fundamental factors. Practically speaking, the implementation of factor models is carried out in two stages with, for example, the Fama and MacBeth procedure. When the list of factors is established and the risk premium calculated, the fund performance is given by

$$\alpha = \mathbf{E} \left[R_i - R_f \right] - \sum_k \widehat{\beta_{i,k}} \cdot \lambda_k \,, \tag{1.11}$$

⁶The MSR yields the same results as the Excess Return on VaR if returns are normally distributed.The modified quantile is used to calculate the Modified Value-at Risk:

$$MVaR = -(r + z_{CF} \cdot \sigma)$$
,

with z_{CF} , the quantile obtained by Cornish-Fisher expansion, whose expression is given by

$$z_{CF} = z_{\alpha} + \left(z_{\alpha}^{2} - 1\right) \cdot \frac{Skew}{6} + \left(z_{\alpha}^{3} - 3z_{\alpha}\right) \cdot \frac{Kurt}{24} - \left(2z_{\alpha}^{3} - 5z_{\alpha}\right) \cdot \frac{Skew^{2}}{36},$$

where z_{α} denotes the α -quantile of the of the standard normal distribution and *Skew* (resp. *Kurt*) the skewness (resp. the excess kurtosis) of the distribution of asset returns (Favre and Galeano, 2002).

where $\widehat{\beta_{i,k}}$, λ_k are estimates of the sensitivity and the risk premium associated to the factor F_k .

2 Summary and comments

In this Chapter we considered a brief overview of performance measures in link with the asset pricing models reviewed in Chapter 1. Depending on the context and on the manager's strategies some methods seem to be more appropriate than others. For example we observed that risk-adjusted performance assessment based on the Sharpe ratio is not sufficient and that alternative performance measures deliver important additional information for investment decisions. Furthermore the literature on performance analysis does not clearly establish which performance measures under which circumstances and for what reasons lead to different results.

In the recent literature many papers, among them Pastor and Stambaugh (2012b), have shown that the volatility is increasing with the horizon. Fama and French (2018b) and Bessembinder (2018) recently showed that the higher-order moments, the skewness and kurtosis, are strongly impacted by investment horizon. Given the investment horizon has a large impact on the variance and the higher-order moments, it is expected that the horizon also influences the performance assessment and asset allocation. This expectation can be related to Bessembinder's note of caution about the necessity "to reassess standard methods of evaluating investment management performance" when dealing with long term horizons.

Pastor and Stambaugh (2012b) concluded that the variance exhibits an upward trend when the return horizon increases showing the impact of the horizon on the performance measure. Furthermore Cochrane (2011) considered long horizons as more interesting because they link return predictability to volatility, "bubbles," and the nature of price movements. It is thus interesting to investigate performance measures beyond the mean-variance framework when the investment horizon varies.

In fact, since the 90's and even before, the impact of horizon on the performance analysis has been source of diverging conclusion in the literature. For example, with the Sharpe ratio, the most widely used performance measure, many shortcomings have been highlighted. Divergences exist when the term structure of this performance measure is considered. Two opposite conclusions were drawn on the long-term behavior of the Sharpe ratio. On the one hand Zakamouline and Koekebakker (2009) argued that the Sharpe ratio increases on the long run while, on the other hand, van Binsbergen and Koijen (2017) and Madan and Schoutens (2018) claim the contrary.

Higher-order moments have been proposed to remedy the deficiencies of the Sharpe ratio. Bessembinder (2018) and Fama and French (2018b) recently showed that the long-term behavior of the return distributions and their (higher-order) moments are of great importance for a better assessment of the return characteristics. They showed that skewness, even when negative for short-term returns, becomes positive in the long run. A similar observation has been previously reported for symmetric distributions of short-term returns by Arditti and Levy (1975). These results have many implications. Then it is interesting to investigate if any explanation (reconciliation) can be given between these seemingly opposite results by considering the combined impact of horizon on both higher-order moments and the Sharpe ratio through the generalization provided by the ASR of Pezier and White (2006).

In the next Chapter we consider the compounding rule to investigate the properties of long term return distributions in order to analyze the implication on performance assessment. Additionally, as with the performance measures, the literature has also exhibited divergent point of views on optimal long term asset allocation. Many authors, as Thorley (1995) or Hansson and Persson (2000), are among the proponents of the time diversification and argue for an increasing market participation when the investment horizon increases. The opponents, as Gunthorpe and Levy (1994) or Bodie (1995), advocate a decreasing holdings in risky assets with the horizon. The later mainly rely on Merton and Samuelson (1974) argument against the fallacious use of the central limit theorem in the many-period problem solved by an expected utility maximizer while the former reject this argument on the basis of the common sense. Hence we also investigate the optimal asset allocation along horizon in the following Chapter.

Chapter 5

Higher-moments in the long run and implication for asset allocation

Résumé

Dans cet essai nous revenons sur la question de la diversification temporelle en analysant l'impact de l'horizon sur les propriétés des distribution de rendements composés – par opposition aux rendements simples. Dans un premier temps, nous généralisons l'expression analytique des moments d'ordre supérieurs de la distribution des rendements obtenue par Arditti and Levy (1975). Nous constatons qu'à mesure que nous allongeons l'horizon de placement, l'asymétrie de la distribution des rendements devient de plus en plus positive, ce qui implique, pour un investisseur de long terme, la nécessité d'adapter sa stratégie d'investissement. Nous montrons par ailleurs que l'effet de composition est le principal raison expliquant la forme des distributions de rendement d'actifs à long terme. A la lumière de ces considérations, nous examinons ensuite les questions d'allocation d'actifs et de mesure de performance à long terme pour proposer une explication pouvant réconcilier les points de vue apparemment opposés sur la question exprimés dans la littérature. Nos résultats apportent un nouvel éclairage à ces divergences d'opinions quant aux stratégies d'investissement à suivre sur le long terme.

Higher-moments in the long run and implication for asset allocation

Abstract

We revisit the issue of time diversification and analyze the impact of horizon on the properties of the distribution of compound returns – as opposed to simple returns. First, we generalize the analytical expression of the higher order moments of the return distributions obtained by Arditti and Levy (1975). We notice that the skewness of the distribution of returns becomes increasingly positive as we extend the investment horizon. For a long-term investor, it implies the need to adapt her investment strategy. We also show that the composition effect is the main reason for the shape of long-term return distributions. In light of these considerations, we then examine the questions of long-term asset allocation and performance measurement to propose an explanation that can reconcile the apparently dissenting views expressed in the literature on the issue. Our results shed new light on these divergences of opinion regarding long-term investment strategies.

Introduction

Fama and French (2018b) and Bessembinder (2018) have recently shed new light on the properties of long-term stock return distributions and renewed interest in this subject. In this Chapter, we show that the generally observed positive skewness of long-term stock returns is the result of the compounding effect and holds not only when the short-term returns are symmetric, as proved by Arditti and Levy (1975), but also for negatively skewed short-term asset return distributions. We make more precise the statement by Bessembinder (2018) according to which: "The positive skewness in long horizon returns is attributable both to skewness in the distribution of [short term] individual stock returns and to the fact that the compounding of random returns induces skewness." Indeed, we demonstrate that the main driving force of the positive skewness of long-term returns is the compounding rule itself; short-term asymmetry is only a second order effect whose impact on the sign of longterm asymmetry becomes non-negligible for unrealistically negative short-horizon skewness only. We then show that these results have substantial consequences on performance assessment and investment decisions.

A vast literature deals with the impact of the higher order moments on optimal allocation. It is well-known that investors value positive skewness and are averse to kurtosis. Jondeau and Rockinger (2006), Mitton and Vorkink (2007), Jurczenko et al. (2015), and Dahlquist et al. (2017) among many others, address the effect of higher order moments on asset allocation. However, the existing literature mainly focuses on higher order moments in the short run; probably because much less is known on the skewness and kurtosis of the return distributions on the long run. Apart from the study of the long-term behavior of the volatility by Pastor and Stambaugh (2012b) and the recent articles by Bessembinder (2018) and Fama and French (2018b), the long-term behavior of the stock return distributions and their (higher order) moments have remained largely unexplored. Another recent exception is Madan and Schoutens (2018) who consider pure jump processes with self-decomposable laws to investigate the long-term return characteristics.¹ Our paper is in line with this trend of literature and can be related to recent efforts to understand the properties of long-term returns and their impact on asset allocation and performance measurement.

We first derive the analytical expression of the higher moments of the return distributions as a function of the investment horizon. We extend the results of Arditti and Levy (1975) who focused on short-term returns with symmetric distributions and restricted their attention to the skewness. Here, we consider not only the skewness but also the kurtosis without distributional assumptions on the short-term returns. Our analytical expressions offer simple interpretations and allow us to derive a classification of the risky securities in terms of a limited set of admissible behaviors of the skewness and kurtosis when the investment horizon varies. In contrast with the long-term skewness and kurtosis of the continuously compounded returns (or log-returns) which converge to zero as the investment horizon increases, the skewness and kurtosis of the simple returns grow unbounded when the investment horizon increases.

Then, we focus on the assessment of the long-term performance of risky portfolios. It is well-known that investors need an effective tool to evaluate the respective performance of the various funds comprised in their investment universe and that the Sharpe ratio has found widespread acceptance among the professionals as a riskadjusted performance measure due to both its simplicity and ease of use. However, the literature has addressed several limits of the Sharpe ratio when the return distributions depart form the normal law and advocates taking into consideration the impact of higher moments to better capture the risk-adjusted performance (Hodges, 1998; Pezier, 2011; Zakamouline and Koekebakker, 2009). Given the investment horizon has a large impact on the higher moments, it is expected to influence the performance assessment. This expectation is consistent with Bessembinder's note of caution about the necessity "to reassess standard methods of evaluating investment management performance".

We adopt the generalized Sharpe ratios proposed by Pezier (2011) to account for the skewness and kurtosis in the performance measurement. Interestingly, the generalized Sharpe ratios appear to be less sensitive to the higher moments compared with their counterparts evaluated on the basis of the square root rule which is shown to yield inconsistent results on the long run. Our results also confirm that the Sharpe ratio reaches a maximum at finite, but large, horizon. This feature can explain the dissenting views in the literature regarding the increasing or decreasing behavior of the Sharpe ratio when the investment horizon becomes large (Zakamouline and

¹We have to mention that real estate is another noticeable exception due to the inherent long-term nature of this type of asset.

Koekebakker, 2009; van Binsbergen and Koijen, 2017).

Finally, we relate the behavior of the long-term higher order moments to the problem of the optimal asset allocation. The determination of the optimal holding period is a standard topic in finance (Atkins and Dyl, 1997; In et al., 2011) and related fields.² However, the literature provides diverging conclusions regarding the benefits of time diversification both from a theoretical and a practical point of view. The proponents of the time diversification argue for an increasing market participation when the investment horizon increases (Thorley, 1995; Hansson and Persson, 2000) while the opponents advocate decreasing holdings in risky assets (Gunthorpe and Levy, 1994; Bodie, 1995). The later mainly rely on Merton and Samuelson (1974) argument against the fallacious use of the central limit theorem in the many-period problem solved by an expected utility maximizer while the former reject this argument on the basis of the common sense and because investment horizon indifference is only a special case within expected utility theory.

This Chapter analyzes the optimal allocation solution on the long run. We show that the impact of the higher moments on the optimal allocation is much larger than observed in the case of the Sharpe ratio. In that, accounting for the higher moments qualitatively changes the behavior of the optimal allocation as a function of the investment horizon. It switches from a monotonically decreasing behavior in a mean-variance setting to a non-monotonic behavior when higher moments are accounted for suggesting that time diversification actually occurs at intermediate time scales but vanishes on the long run. These results are consistent with Madan and Schoutens (2018).

The empirical analysis is conducted on a set of 109 portfolios obtained from Kenneth French's web site. We consider the market portfolio, the 5 Fama-French portfolios SMB, HML, WML, RMW and CMA, the 48 Industry portfolios, the 10 portfolios sorted by decile of size, the 10 portfolios sorted by decile of book-to-market ratio, the 10 portfolios sorted by decile of momentum and the 25 Investment-Profitability sorted portfolios. We rely on daily, monthly and yearly data, depending on their availability. For illustration purpose, we focus on the Market portfolio and the Agriculture industry portfolio for reasons that will become clear in the following.

The remainder of the Chapter is organized as follows. We begin with a discussion of our motivation for long horizon higher moments in Section 1. Section 2 derives the relation between of the higher moments at two different time scales and analyzes its consequences. In Section 3 we study the impact of higher moments on the longterm risk-adjusted performance and asset allocation. Finally, the robustness of our results is studied in Section 4 before we conclude.

²Several attempts have been led to develop models that determine analytically the optimal holding period for real estate portfolios. Among others, Baroni et al. (2007) model the real estate asset through a standard diffusion process, and provide a closed form solution for the *ex ante* optimal holding period (*e.g.* for close-ended funds) while Amédée-Manesme et al. (2016) examine the impact of the market volatility on the optimal holding period of a real estate asset under risk aversion.

1 Motivation

Fama and French (2018b) and Bessembinder (2018) recently used bootstrap simulations to analyze the properties of long-term returns. Starting from a sample of monthly returns, they simulated return distributions for horizons up to 30 years. We use the same bootstrap procedure to generate long-term returns but we start from daily sample returns.³ We do not describe the procedure here to save space but refer the reader to Fama and French (2018b). The Figure 1.1 reports the results of the bootstrap procedure (blue dots) for the first four sample moments across time scales for the equally-weighted market portfolio returns between July 1963 and December 2016.^{4,5} We crosscheck our implementation of the bootstrap procedure with the monthly sample returns (green dots) and make sure that we get the same results as Fama and French (2018b). We also check the consistency of the results when we start from weekly (red dots) and yearly (black dots) returns instead of daily returns. Overall, the consistency is good, and even remarkably good for the mean and standard deviation, meaning that the presence of serial dependence does not impinge the procedure.

We notice that the bootstrapped values of both the mean, the standard deviation and the skewness exhibit an upward trend when the return horizon increases. This evolution is consistent with Pastor and Stambaugh (2012b). The skewness starts with negative values for short-term returns and becomes positive in the long run as observed by Bessembinder (2018) and Fama and French (2018b). In fact the skewness of the market portfolio starts with a negative value of -0.5, becomes positive after one to six months and reaches a level about $3 \sim 4$ at the 30 years time scale. On the contrary, the kurtosis exhibits a U-shape. It first decreases from a rather high level (about 20 at the daily time scale) to reach a minimum value, slightly above 3, between one to six months also and increases again to values beyond 20 at the horizon of 30 years.

We can notice that, between one and six months, the departure of the distribution of compounded returns from the Gaussian distribution remains limited with a skewness close to zero and a kurtosis not far from three. However, as reported in Table 5.1, a Jarque-Bera test rejects the normality assumption. Hence, even if the distribution of compounded returns seems close to the Gaussian distribution at the time scale of a few months, it cannot be confused with it statistically speaking.

The behavior illustrated here with the market portfolio is quite standard and representative of the results we obtain with industry portfolios or portfolios sorted by size, book-to-market and other usual firm characteristics. We come back to this result that appears as a stylized fact in the next section. In order to understand the reasons for the typical shapes of the moments we have just illustrated, let us now derive the theoretical expressions of these moments at a given arbitrary time scale.

³We report the results of the bootstrap procedure when we start from different initial time scales in Appendix. It allows us check the robustness of the bootstrap procedure *vis-a-vis* the presence of serial dependence in the data and the consistency of our results with the previous studies.

⁴The data are downloaded form Kenneth French's website.

⁵See also the Table in Appendix for complementary results.



Figure 1.1: Market portfolio four first moments

The figure depicts the mean return (upper left panel), the standard deviation (upper right panel), the skewness (lower left panel) and the kurtosis (lower right panel) of the market portfolio between July 1963 and December 2016 for eleven horizons ranging between 1 day and 30 years obtained by the Fama-French bootstrap procedure starting from samples of daily (blue), weekly (red), monthly (green) and yearly (black) data.

Horizon	Skewness	Kurtosis	JB-stat	<i>p</i> -value
1 Month	0.0364	3.6026	153.498	0.000
3 Months	0.1230	3.2705	55.733	0.000
6 Months	0.2772	3.2220	148.642	0.000
1 Year	0.4313	3.5129	419.691	0.000

Table 5.1: Normality test for the bootstrapped returns of the equally-weighted market portfolio between July 1963 and December 2016.

2 Moments across time scale

In this section, we derive the analytical expressions of the first four moments of the return distribution for any horizon T. We set a reference time scale, say one day for example, and express the duration in units of the reference time scale. At the reference time scale, we denote by r_t the (simple) return at date t. The returns are assumed to be independent and identically distributed. Of course, this assumption is not realistic but it is necessary for the tractability of the calculations. In addition, the previous illustration has shown that serial dependence do not have a significant impact on large scale moments. Indeed, Figure 1.1 has illustrated that starting form a reference time scale equal to one day or to one year does not significantly affect the estimated moments at larger time scales. A systematic study on dozens of portfolios have confirmed this result.

We denote by μ the expectation of the simple return r_t at the reference time scale, by σ^2 its variance, s^3 and k^4 its third and fourth (non-standardized) central moments.⁶ We denote by R_T the (simple) return at time scale T, that is,

$$1 + R_T \coloneqq (1 + r_1) \cdot (1 + r_2) \cdots (1 + r_T) . \tag{2.1}$$

All the proofs of the results exposed in this section are gathered in Appendices 5.A and 5.B.

2.1 Mean and variance

We start with the two first moments in order to remind the well-known expressions of the expected return and the variance at the horizon T (Goodman, 1962). The mean return of R_T is given by

$$\mathbf{E}[R_T] \coloneqq \mathbf{E}\left[\prod_{t=1}^T \left(1+r_t\right)\right] - 1, \qquad (2.2)$$

$$= (1+\mu)^T - 1.$$
 (2.3)

⁶The skewness and kurtosis are then respectively given by $\tilde{s}^3 := \frac{s^3}{\sigma^3}$ and $\tilde{k}^4 := \frac{k^4}{\sigma^4}$.

This relation holds for all integer horizon T but also generalizes to non-integer values as well as values of T less than one, that is less than the reference time scale. This remark is important because it shows that we can analyze the behavior of both the long-term and the short-term average return. Due to the accessibility of intraday data, an analytical expression for the (very) short-term returns seems not to be very useful, however if we want to analyze the short-term behavior of the returns before the availability of intraday data it is relevant. Besides, from a theoretical point of view it allows us to extrapolate the behavior of the mean return both at small and large time scales.

For the short-term (that is, as $T \to 0$), the behavior of the mean return becomes

$$E[R_T] = T \cdot \ln(1+\mu) + o(T).$$
 (2.4)

Unsurprisingly, we get a linear relation between the expected return and the horizon T as would be the case for continuously compounded returns.

The variance, at horizon T, reads

$$\operatorname{Var}\left(R_{T}\right) \coloneqq \operatorname{Var}\left[\prod_{t=1}^{T}\left(1+r_{t}\right)\right],\qquad(2.5)$$

$$= \sigma^{2T} \cdot \left[\left(1 + \zeta^2 \right)^T - \zeta^{2T} \right] \,, \tag{2.6}$$

with

$$\zeta \coloneqq \frac{1+\mu}{\sigma} \,. \tag{2.7}$$

This expression has been first considered by Tobin (1965) to account for the compounding effect in the assessment of the return variance. It shows that the variance, and hence the standard deviation, is a monotonically increasing function of the horizon as empirically observed by Bessembinder (2018) and Fama and French (2018b).

Again, for the short-term returns, the variance is a linear function of the horizon:

$$\operatorname{Var}\left(R_{T}\right) = T \cdot \ln\left(1 + \frac{1}{\zeta^{2}}\right) + o(T), \qquad (2.8)$$

as would be the case for the variance of the continuously compounded returns, while it grows exponentially fast on the long run:

$$\operatorname{Var}\left(R_{T}\right) \sim_{T \to \infty} \left[\sigma^{2}\left(1+\zeta^{2}\right)\right]^{T} . \tag{2.9}$$

2.2 Skewness

We now turn to the investigation of the properties of the skewness and derive our first set of results. Arditti and Levy (1975) were the first to obtain an analytical expression for the skewness of the long-term compounded returns under the assumption of a *symmetric* distribution of short-term returns. Here, we relax this assumption and obtain the following general expression for the (standardized) skewness at horizon T when the returns are independent and identically distributed:

$$\operatorname{Skew}(R_T) \coloneqq \frac{\operatorname{E}\left[\left(R_T - \operatorname{E}\left[R_T\right]\right)^3\right]}{\operatorname{Var}\left(R_T\right)^{3/2}},\tag{2.10}$$

$$=\frac{\left(\frac{\tilde{s}^{3}-\tilde{s}_{\min}^{3}}{(1+\zeta^{2})^{3/2}}\right)^{T}}{\left[1-\left(\frac{\zeta^{2}}{1+\zeta^{2}}\right)^{T}\right]^{3/2}}-\frac{\left(\frac{\zeta^{2}}{1+\zeta^{2}}\right)^{T/2}\cdot\left[3-2\cdot\left(\frac{\zeta^{2}}{1+\zeta^{2}}\right)^{T}\right]}{\left[1-\left(\frac{\zeta^{2}}{1+\zeta^{2}}\right)^{T}\right]^{3/2}},$$
(2.11)

where $\tilde{s}^3 \coloneqq \frac{s^3}{\sigma^3}$ denotes the skewness (or standardized third central moment) at the reference time scale and ζ is given by (2.7) while $\tilde{s}^3_{\min} \coloneqq -(\zeta^3 + 3\zeta)$ is the minimum value of the standardized skewness at this time scale.⁷

The rightmost term in equation (2.11) goes to zero as T goes to infinity while the leftmost term in the right-hand side behaves like $\left(\frac{\tilde{s}^3 - \tilde{s}_{\min}^3}{(1+\zeta^2)^{3/2}}\right)^T$ for large horizons since its denominator goes to one. It is always positive, since $\tilde{s}^3 > \tilde{s}_{\min}^3$, and is either larger than one, so that the standardized skewness grows to infinity or is smaller than one, so that the standardized skewness goes to zero. Thus, when T goes to infinity the skewness goes to plus infinity as soon as $\tilde{s}^3 > (1+\zeta^2)^{3/2} - (\zeta^3 + 3\zeta)$ and zero otherwise.⁸

Regarding the short-term horizons, equation (2.11) simplifies:

Skew
$$(R_T) = \frac{1}{\sqrt{T}} \cdot \left[\frac{\left[\ln \frac{\dot{s}^3 - \dot{s}^3_{\min}}{\zeta^3} \right] - 3 \left[\ln \left(1 + \frac{1}{\zeta^2} \right) \right]}{\left[\ln \left(1 + \frac{1}{\zeta^2} \right) \right]^{\frac{3}{2}}} \right] + O(T) .$$
 (2.12)

It shows that the skewness diverges to plus or minus infinity when T goes to zero. We notice that the numerator of the above equation is negative whenever

$$\left(\tilde{s}^3 - \tilde{s}^3_{\min}\right)^{1/3} < \zeta + \frac{1}{\zeta}$$
 (2.13)

If this condition holds, the skewness goes to minus infinity when T goes zeros and to plus infinity otherwise.

We can summarize the behavior of the skewness in the following proposition:

Proposition 1. The skewness of the distribution of compounded returns

1. is a U-shaped function of the horizon when

$$\tilde{s}^3 > \frac{3}{\zeta} + \frac{1}{\zeta^3},$$
(2.14)

⁷Since the simple return r_t admits -1 as a lower bound (we assume limited liability securities), $E\left[\left(1+r_t\right)^3\right]$ is necessarily positive which yields $\tilde{s}^3 > -\left(\zeta^3+3\zeta\right)$ irrespective of (μ,σ) . ⁸We can notice that $\left(1+\zeta^2\right)^{3/2}-\left(\zeta^3+3\zeta\right)=-\frac{3}{2}\zeta+O\left(\zeta^{-1}\right)$ meaning that the condition simplifies to $\tilde{s}^3 > -\frac{3}{2}\zeta$ when ζ is large enough.

2. grows from minus infinity to plus infinity when the horizon varies from zero to infinity whenever

$$\left(1+\zeta^{2}\right)^{3/2} - \left(\zeta^{3}+3\zeta\right) < \tilde{s}^{3} < \frac{3}{\zeta} + \frac{1}{\zeta^{3}}, \qquad (2.15)$$

3. grows from minus infinity to zero when the horizon varies from zero to infinity whenever

$$\tilde{s}^3 < \left(1+\zeta^2\right)^{3/2} - \left(\zeta^3+3\zeta\right)$$
 (2.16)

Our proposition nests and extends Arditti and Levy's result regarding the positive value of the long-term skewness for symmetric distributions of compounded returns. In fact, we can see that the condition 2 in the proposition 1 is satisfied for all reasonable values of the parameter ζ in the case of symmetric returns ($\tilde{s}^3 = 0$).⁹ However, the behavior of the skewness is more complex than Arditti and Levy's results suggests. It is not necessarily monotonic (point 1), meaning that the short-term skewness can also be large. Moreover, if the short-term skewness goes beyond the lower threshold defined by point 3, the long-term skewness converges to zero (as would be the case for continuously compounded returns whose long-term distribution is expected to converge toward the Gaussian distribution by virtue of the central limit theorem. See also Fama and French, 2018b, for an empirical verification).

The Figure 2.1 illustrates the behavior of the skewness for the Market portfolio and the Agriculture industry portfolio as a function of the horizon. The red dots represent the skewness of the portfolios obtained with the bootstrap procedure of Fama and French (2018b) while the blue curve is the theoretical skewness given by equation (2.11). The parameters μ and σ are estimated from the data at the daily time scale without any adjustment.¹⁰ We can notice the remarkable agreement between the theoretical values and the bootstrapped values. The figure exhibits the two distinctive behaviors mentioned at points 1 and 2 of proposition 1: The skewness of the Market portfolio is an increasing function of the horizon (point 2) while the skewness of the Agriculture industry portfolio is U-shaped (point 1). In both cases, the skewness reaches its minimum absolute value between one week and one month.

Among all the studied portfolios, the case of a skewness that converges to zero on the long run (point 3) has never been observed. The Figure 2.2 justifies this assertion. It depicts the estimated skewness \tilde{s}^3 of 109 portfolios (five of which are long-short portfolios) estimated at the daily time scale versus ζ (as defined by Eq. 2.7). The three regions defined by Proposition 1 are delineated by the red and blue curves : above the red curve stands the region defined at point 1 where the skewness is U-shaped, between the red and the blue curves stands the region defined at point 2 where the skewness grows from minus infinity to plus infinity and below the blue curve stands the region defined at point 3 where the skewness grows from

⁹With μ of the order of a few percents (in absolute value) and σ below a few tens of percent, $\zeta = \frac{1+\mu}{\sigma}$ is typically larger than one so that $(1+\zeta^2)^{3/2} - (\zeta^3 + 3\zeta) < 0$ and $\frac{3}{\zeta} + \frac{1}{\zeta^3} > 0$. ¹⁰I.e. the parameters μ and σ are *estimated* from the two first sample moments of the returns

¹⁰I.e. the parameters μ and σ are *estimated* from the two first sample moments of the returns at the daily time scale; they are not *calibrated* for the theoretical skewness to best fit the bootstrapped skewness.



Figure 2.1: Skewness of the market and agriculture portfolio

The figure depicts the skewness of the Market portfolio (left panel) and the Agriculture industry portfolio (right panel) as a function of the return horizons. Red dots represent the bootstrapped values following Fama and French (2018b) procedure while the blue curve is obtained from equation (2.11) without adjusted parameters.



Figure 2.2: Skewness and kurtosis regions

The figure depicts the (cubic root) of the empirical skewness of 109 portfolios versus ζ . The set of portfolios comprises the market portfolio (MARK), the 5 Fama-French portfolios SMB, HML, WML, RMW and CMA, the 48 Industry portfolios (INDU_48), the 10 portfolios sorted by decile of size (SIZE_10), the 10 portfolios sorted by decile of Book-to-Market ratio (BM_10), the 10 portfolios sorted by decile of Momentum (MOM_10) and the 25 Investment-Profitability sorted portfolios (OP-INV_10). The three regions defined in Proposition 1 are delineated by the red and blue curves. The green curve splits the two regions defined in Corollary 1.

minus infinity to zero.

As mentioned above, none of the portfolio skewness \tilde{s}^3 falls below the blue curve meaning that their long-term skewness always grow to infinity at large horizons. Additionally, we notice that a majority of the studied portfolios satisfy the condition 2. Among the industry portfolios, only a small half satisfies the condition 1 and are expected to exhibit a U-shaped skewness as a function of the horizon. This observation allows us to make more accurate Bessembinder's statement according to which the positive skewness of long-term returns results from both the skewness of the short-term returns and the compounding effect. As shown by Proposition 1, this assertion is theoretically true. However, the empirical results summarized by Figure 2.2 show that the compounding effect is the dominant driving force. Indeed, the estimated short-term skewness is never found negative enough to prevent the long-term skewness from being non-positive.

The practical implication of the result is important. Figure 2.2 shows that a



Figure 2.3: Skewness and kurtosis for long and short positions

The figure depicts the Skewness and kurtosis of the Market portfolio in Blue (long position) and Green (short position) as a function of the horizon. Red dots represent the bootstrapped values following Fama and French (2018b) procedure while the blue curve is obtained from equation 2.11-2.18 without adjusted parameters.

large positive skewness for long-term returns can be regarded as a stylized fact. Since most investors exhibit prudence (Kimball, 1990), they exhibit preference for positive skewness. As a consequence, the demand for risky securities or portfolios should be expected to increase as investment horizon increases since, everything else taken equal, the skewness increases on the long-run.

In order to check the robustness of our results on the sign of the short-term skewness, it is interesting to consider the change in the shape of the distribution of returns when we hold a short position in the asset. This consideration confirm our argument on the driving force of the compounding rule on explaining the long term positive skewness. In the left panel of Figure 2.3, we depict the skewness of a long exposure to the Market portfolio (in Blue) and to a short exposure (in Green) as a function of the horizon. As expected, the sign of the short-term skewness if the opposite but, on the long run, both positions converges to the same skewness.

2.3 Kurtosis

Let us now extend Arditti and Levy's results to the case of the kurtosis. At horizon T. the kurtosis reads

$$\operatorname{Kurt}(R_T) \coloneqq \frac{\operatorname{E}\left[\left(R_T - \operatorname{E}\left[R_T\right]\right)^4\right]}{\operatorname{Var}(R_T)^2}, \qquad (2.17)$$
$$= \frac{\left(\frac{\tilde{k}^4 - \tilde{k}_{\min}^4}{\left[1 + \zeta^2\right]^2}\right)^T - 4 \cdot \left(\frac{\zeta \cdot \left(\tilde{s}^3 - \tilde{s}_{\min}^3\right)}{\left[1 + \zeta^2\right]^2}\right)^T + 3 \cdot \left(\frac{\zeta^2}{1 + \zeta^2}\right)^T \cdot \left(2 - \left(\frac{\zeta^2}{1 + \zeta^2}\right)^T\right)}{\left(1 - \left(\frac{\zeta^2}{1 + \zeta^2}\right)^T\right)^2}, \quad (2.18)$$

with $\tilde{k}^4 \coloneqq \frac{k^4}{\sigma^4}$ and $\tilde{k}^4_{\min} \coloneqq -[4\tilde{s}^3\zeta + 6\zeta^2 + \zeta^4]$. The rightmost term in the numerator of equation (2.18) goes to zero as T goes to infinity while the denominator goes to one. As a consequence, the kurtosis of R_T is controlled by the term $\frac{\tilde{k}^4 - \tilde{k}_{\min}^4}{[1+\zeta^2]^2}$. Indeed, the term $\frac{\zeta \cdot (\tilde{s}^3 - \tilde{s}_{\min}^3)}{[1+\zeta^2]^2}$ can never be dominant on the long run: since it is necessarily positive – remind that $\tilde{s}^3 > \tilde{s}_{\min}^3$ - its contribution to the kurtosis is negative; if it was the leading contribution, the kurtosis would eventually be negative which is impossible since $E\left|\left(R_T - E\left[R_T\right]\right)^4\right|$ must be positive (for any non-degenerate random variables). As a consequence, the kurtosis goes either to zero or to infinity according as $\frac{\tilde{k}^4 - \tilde{k}_{\min}^4}{[1+\zeta^2]^2}$ is smaller than one or larger than one, that is

$$\operatorname{Kurt}(R_T) \to +\infty \qquad \Longleftrightarrow \qquad \tilde{k}^4 > \tilde{k}_{\min}^4 + \left(1 + \zeta^2\right)^2, \qquad (2.19)$$

$$\sim \qquad \tilde{k}^4 > 1 - 4\zeta \left(\tilde{s}^3 + \zeta \right) \,. \tag{2.20}$$

At short time scale, the kurtosis becomes

$$\operatorname{Kurt}\left(R_{T}\right) = \frac{1}{T} \cdot \left[\frac{\left[\ln\left(\frac{\tilde{k}^{4} - \tilde{k}_{\min}^{4}}{\zeta^{4}}\right) - 4\ln\left(\frac{\tilde{s}^{3} - \tilde{s}_{\min}^{3}}{\zeta^{3}}\right) + 6\ln\left(1 + \frac{1}{\zeta^{2}}\right)\right]}{\left[\ln\left(1 + \frac{1}{\zeta^{2}}\right)\right]^{2}}\right] + O\left(T\right) . \quad (2.21)$$

Hence, in the limit of short-term horizons, the kurtosis necessarily diverges to plus infinity since the numerator of the leading term cannot be negative. Accounting for the behavior of the kurtosis for both short-term and long-term horizons, we can state that :

Proposition 2. The kurtosis of the distribution of compounded returns

- 1. is a U-shaped function of the horizon when $\tilde{k}^4 > 1 4\zeta (\tilde{s}^3 + \zeta)$,
- 2. decreases from plus infinity to zero when the horizon varies from zero to infinity otherwise.

Similar to the skewness, the kurtosis exhibits several distinct behaviors. It is either non-monotonic and goes to infinity both at short and large horizons or decreases from plus infinity at short time scale to zero at large time scale. In the first case, the long-term behavior of the kurtosis of the simple returns is very different from the behavior of the long-term kurtosis of the continuously compounded returns which is known to converge to zero as a consequence of the central limit theorem.

The condition that distinguishes the two regimes stated in Proposition 2 is a bit more complicated than the ones encountered for the skewness. Indeed, the parameter ζ is not enough to delineate the different regions. However, since the kurtosis is always positive we can derive a simpler sufficient condition for the kurtosis to be U-shaped:

Corollary 1. The kurtosis of the distribution of compounded returns is a U-shaped function of the horizon whenever its skewness

- 1. satisfies $\tilde{s}^3 > \frac{1}{4\zeta} \zeta$,
- 2. or is a U-shaped function.

Proof. According to Proposition 2, the kurtosis is U-shaped as soon as $\tilde{k}^4 > 1 - 4\zeta (\tilde{s}^3 + \zeta)$. Since \tilde{k}^4 is positive, it is enough for the kurtosis to be U-shaped that $1 - 4\zeta (\tilde{s}^3 + \zeta) < 0$ or, equivalently, $\tilde{s}^3 > \frac{1}{4\zeta} - \zeta$ which proves the first statement. Now, when the skewness is U-shaped, Proposition 1 states that $\tilde{s}^3 > \frac{3}{\zeta} + \frac{1}{\zeta^3}$. In such a case, $1 - 4\zeta (\tilde{s}^3 + \zeta) < -11 - 4\zeta^2 - \frac{4}{\zeta^2} < 0$, which concludes the proof as a consequence of the first point of the corollary.

The Figure 2.4 illustrates the behavior of the kurtosis for the Market portfolio and the Agriculture industry portfolio as a function of the horizon. The red dots represent the kurtosis of the portfolios obtained with the bootstrap procedure of Fama and French (2018b) while the blue curve depicts the theoretical kurtosis given by equation (2.18). We still get a remarkable agreement between the theoretical values and the bootstrapped values without any parameter adjustment. The two panels exhibit a common U-shaped behavior with a minimum reached at horizons ranging between three months to one year. In fact, relying on condition 1 in Corollary 1, Figure 2.2 shows that all the 109 portfolios considered in the previous subsection satisfy the sufficient condition for a U-shaped kurtosis, that is their short-term skewness \tilde{s}^3 is larger than $\frac{1}{4\zeta} - \zeta$ (which is characterized by the region above the green curve).

Once again, this result provides valuable insights. Since most investors exhibit temperance (Kimball, 1992), they prefer securities with low kurtosis. This means that, everything else taken equal, these investors prefer intermediate investment horizons and will mitigate the impact of the preference for skewness and its consequences on long-term investment preference. We will investigate this trade-off in the next section. Before that, let us conclude this section with a deeper analysis of the short term behavior of the higher moments.

2.4 Short term extrapolation

The relations (2.11) and (2.18) provide the expression of the higher order moments for investment horizons larger than the reference time scale as well as shorter hori-



Figure 2.4: Kurtosis of the market and agriculture portfolios

The figure depicts the kurtosis of the Market portfolio (left panel) and the Agriculture industry portfolio (right panel) as a function of the return horizons. Red dots represent the bootstrapped values following Fama and French (2018b) procedure while the blue curve is obtained from equation (2.18) without adjusted parameters.
zons. This later property is possibly meaningful from a practical point of view in so far as the bootstrap procedure of Fama and French (2018b) only applies to horizons longer than the reference time scale and cannot be generalized to generate samples of returns at a time scale smaller than the reference time scale which constitutes a lower limit. Hence the inference of short-term moments can only be performed on the basis of the theoretical relations (2.11) and (2.18).

The accuracy of the equations (2.11) and (2.18) for long-term horizons has been validated in the previous sub-sections while the robustness of the Fama and French procedure to a change in the reference time scale has been illustrated on Figure 1.1 and will be further investigated in section 4. It thus makes sense to check the quality of the extrapolation provided by these formulae for short-term horizons. To this aim, we start from actual samples of monthly returns and estimate the parameters μ, σ, \tilde{s}^3 and \tilde{k}^4 at this monthly time scale. The blue curves on Figure 2.5 depict the theoretical values of the skewness and kurtosis for the Market portfolio and the Agriculture industry portfolio together with their bootstrapped estimates for horizons ranging from three months to thirty years (red circles). As previously, we get a very good agreement between theoretical and bootstrapped values. Then, we consider the weekly (resp. daily) actual returns and estimate their sample skewness and kurtosis. Their corresponding values are represented on Figure 2.5 by black (resp. pink) circles while the green part of the curves depict the extrapolation to short time scales of the skewness and kurtosis based on the theoretical expressions with parameter values estimated at the monthly time scale.

The extrapolation to small time scales only provides mixed results. While the discrepancy between the skewness and kurtosis of weekly returns and their extrapolated counterparts remains limited, the difference becomes large at the daily time scale. Overall, the extrapolated skewness seems to underestimate the sample skewness while the extrapolated kurtosis overestimate the sample kurtosis. In an attempt to improve the results, we calibrate the parameters $(\mu, \sigma, \tilde{s}, \tilde{k})$ which best fit the sample moments. To this aim, we solve the following problem

$$\min_{\left\{\mu,\sigma,\tilde{s},\tilde{k}\right\}} \qquad \sum_{n=1}^{4} \sum_{T} \left(\frac{\operatorname{Th} M_n\left(T;\mu,\sigma,\tilde{s},\tilde{k}\right)}{\operatorname{Sample} M_n\left(T\right)} - 1 \right)^2, \qquad (2.22)$$

where $\operatorname{Th} M_n(T; \mu, \sigma, \tilde{s}, \tilde{k})$ denotes the theoretical moments of order *n* at investment horizon *T* whose expression are given by equations (2.3), (2.6), (2.11) and (2.18) while $\operatorname{Sample} M_n(T)$ denotes their sample counterpart obtained by bootstrap.

The table 5.2 compares the values of the calibrated parameters to the sample ones for our two benchmark portfolios. The agreement is very good. However, the dashed black curves on Figure 2.5, which depict the theoretical skewness and kurtosis based on the calibrated parameters, do not allow to improve on the short time scale extrapolation of the higher moments.

Figure 2.5: Skewness and Kurtosis of the market and agriculture portfolios for short horizons



This figure represents the skewness (upper part) and the kurtosis (lower part) of the Market portfolio (left part) and the Agriculture industry portfolio (right part) as a function of the investment horizon. Monthly returns are used to estimate the bootstrapped moments (red dots) and the theoretical moments given by Eq. (2.11) and (2.18) beyond one month (blue curve) and below one month (green curve). The pink (resp. black) dot depicts the sample daily (resp. weekly) moments. The dashed curve depicts the theoretical skewness and kurtosis when the parameters are solution to the calibration equation (2.22).

	Market			Agriculture		
	Estimation	Calibration	-	Estimation	Calibration	
Mean	0.009	0.009	-	0.010	0.010	
St. dev.	0.044	0.041		0.065	0.055	
Skewness	-0.503	-0.479		0.013	0.013	
Kurtosis	4.953	4.959		4.770	4.805	

Table 5.2: Four first moments calibration

Estimated versus Calibrated moments of the monthly returns of the Market portfolio and Agriculture industry portfolio from July 1963 to December 2016. Calibrated moments are obtained as the solution to 2.22.

3 Implication for optimal allocation

We now address the consequences of the evolution of higher moments with the investment horizon for asset allocation and performance assessment. The departure of the return distributions from normality at both short and long time scales leads to significant changes in the performance assessment. In a Gaussian framework we expect the Sharpe ratio to grow with the investment horizon according to the square root rule. As we will see, due to the compounding rule and the increasing departure of the return distribution from normality, the Sharpe ratio behaves like a non-monotonic function of the investment horizon, *i.e.* it admits a maximum at a finite horizon, and grows at a slower pace than predicted by the square root rule which therefore appears as an overly optimistic way to extrapolate short-term Sharpe ratios to the long-term. Consequently, for a buy-and-hold strategy, the optimal mix between a risky portfolio and the risk-free asset does depend on the investment horizon but does not unambiguously tilts toward the risky portfolio when the investment horizon increases as suggested by common wisdom which relies on the misleading idea that risky securities appear less risky on the long run (the time diversification paradigm).

3.1 Term structure of the Sharpe ratio

There are many ways to assess the performance of asset returns. The most widespread is certainly the Sharpe ratio (SR). It is thus interesting to investigate its behavior when the investment horizon varies. In the following, we assume that the risk free rate of return is either equal to zero or that r_t denotes the excess return over the risk free rate. As a consequence, the Sharpe ratio reads

$$SR_T := \frac{\mathrm{E}\left[R_T\right]}{\sqrt{\mathrm{Var}\left(R_T\right)}}\,,\tag{3.1}$$

$$\stackrel{(2.3-2.6)}{=} \frac{1 - \frac{1}{(1+\mu)^T}}{\left[\left(1 + \frac{1}{\zeta^2} \right)^T - 1 \right]^{\frac{1}{2}}}.$$
(3.2)

This expression, known as Tobin's compounding rule, differs from the traditional formula based on the square root rule which is still the benchmark for many professional applications. Among many others, the famous investment research firm Morningstar reports that, starting March 2005, it uses the square root rule to annualize the Sharpe ratio estimated from monthly data instead of Tobin's compounding rule (Morningstar, 2005). As we will see in the sequel, the relevance of this change of methodology is highly questionable.

The major difference between the square root rule and the relation (3.2) lies in the existence of a maximum with the later expression while the square root rule provides a monotonically increasing function of the horizon. Indeed, we establish in Appendix 5.C that

Proposition 3. Provided that $\mu > 0$, the Sharpe ratio (3.2) admits a (unique) maximum as a function of the horizon.

The optimal horizon T^* that maximizes the Sharpe ratio (3.2) cannot be expressed in closed-form but a good approximation is given by (see Appendix 5.C)

$$T^* \simeq \frac{1}{\mu} \cdot \log\left(1 + \frac{2\mu}{\sigma^2}\right)$$
 (3.3)

The intuition for the existence of a maximum Sharpe ratio is the following. At large time scale, under the assumption $\mu > 0$, expression (3.2) behaves like

$$SR_T \sim \left(\frac{\zeta^2}{1+\zeta^2}\right)^{\frac{T}{2}},$$
(3.4)

that is, it asymptotically decays to zero. On the contrary, at small time scale,

$$SR_T = \sqrt{T} \cdot \frac{\ln\left(1+\mu\right)}{\sqrt{\ln\left(1+\frac{1}{\zeta^2}\right)}} + o(T), \qquad (3.5)$$

$$\propto \sqrt{T}$$
, (3.6)

meaning that the square root rule applies and implies that the Sharpe ratio is a function that starts from zero and grows with the horizon (still assuming μ positive). Hence, the Sharpe ratio is an increasing function of the horizon at small time scale and a decreasing function of the horizon at large time scale so that it must exhibit a maximum.

This result is important for two reasons at least. First, when sorting funds or investment opportunities on the basis of the Sharpe ratio, the use of the approximate square root rule delivers the same ranking irrespective of the horizon. In such a case, the knowledge of the ranking based on the monthly Sharpe ratio, say, provides the ranking at any other time scale. Such a property can be considered as very convenient for practical purpose but it is completely misleading. Indeed, when relying on the proper expression (3.2) of the Sharpe ratio, it appears that there is no reason for the ranking to be the same at different horizons since the Sharpe ratio is not a monotonic function of the horizon.

The second interesting consequence of Proposition 3 is the existence of an optimal horizon at which an investment performance is maximum while the square root rule suggests an unbounded growth. We illustrate on Figure 3.1 the evolution of the Sharpe ratio for the market portfolio across time scales (blue curve) together with the square root approximation (3.5) starting from one day (red curve) and one month (green curve). The purple vertical line depicts the approximate location of the maximum Sharpe ratio given by equation (3.3). The figure clearly shows the overestimation provided by the square root rule beyond an investment horizon of few months. The Sharpe ratio reaches a maximum value equals to 0.90, much smaller the value extrapolated from the square root rule at that horizon.

The previous analysis does not account for the departure of the return distribution from normality at large and small time scales. However, it is well-established that the Sharpe ratio is a relevant performance measure when the underlying return distribution is Gaussian (or close to) but fails to account for the impact of higher moments. As illustrated in the previous section, the departure of the return distribution from normality is quite significant for both short and large horizons. It is thus necessary to consider generalized Sharpe ratios to account for the skewness and kurtosis of the returns at short and large investment horizons. Following Pezier (2011), we consider the generalized Sharpe ratios

$$ASR_{1,T} := SR_T \cdot \left(1 + \frac{\text{Skew}_T}{6} \cdot SR_T\right), \qquad (3.7)$$

$$ASR_{2,T} := SR_T \cdot \left(1 + \frac{\text{Skew}_T}{6} \cdot SR_T - \frac{(\text{Kurt}_T - 3)}{24} \cdot SR_T^2\right), \quad (3.8)$$

where SR_T , Skew_T and Kurt_T denote the Sharpe ratio (3.2), the Skewness (2.11) and the Kurtosis (2.18) at the investment horizon T. With a positive (resp. negative) skewness, the generalized Sharpe ratio ASR_1 is larger (resp. smaller) than the traditional Sharpe ratio SR. For leptokurtic distributions, *i.e.* Kurt_T > 3, the generalized Sharpe ratio ASR_2 is less than ASR_1 . These changes in the Sharpe ratio are obviously consistent with the investors' preference for skewness and aversion for kurtosis. At short and long time scales, the Sharpe ratio SR is small (see equations. 3.4-3.5); we expect a negligible impact of the higher moment. At intermediate time scales, the corrections are expected to dominate so that the impact of the higher moments should be significant in this range.

The Figure 3.2 presents the evolution of the Sharpe ratio and its generaliza-

Figure 3.1: Market portfolio Sharpe ratio



Market Portfolio

This graph presents the evolution of the Sharpe ratio for the market portfolio across time scales (blue curve) together with the square root approximation 3.5 starting from one day (red curve) and one month (green curve). The purple vertical line depicts the approximate location of the maximum Sharpe ratio given by Proposition 3. Notice the logarithmic scale in abscissa which explains the convex shape of square root approximation provided by the red and green curves.

tions (3.7-3.8) as a function of the horizon for the Market portfolio (left panels) and the Agriculture industry portfolio (right panels). The Sharpe ratio is evaluated on the basis of Tobin's rule on the upper panels and on the basis of the square root rule on the lower panels. As expected and reported in the literature, the Sharpe ratio is impacted by the higher moments. More important, the impact of the higher moments is all the more significant the larger the Sharpe ratio. This assertion can be readily observed on the Figure 3.2 by comparison of the upper and lower panels. It is consistent with the analysis of the expressions (3.7-3.8) which show that the sensitivities of the generalized Sharpe ratios ASR_1 and ASR_2 to the skewness and kurtosis are proportional to the Sharpe ratio SR_T itself and the squared Sharpe ratio respectively. When the Sharpe ratio is evaluated by use of equation (3.2), the impact of the higher moments is almost negligible for horizons up to one year or so and are much more moderate than the impact observed when the Sharpe ratio is evaluated on the basis of the square root rule for horizons larger than one year. In addition, the location – in terms of horizon – of the maximum of the Sharpe ratio is much less sensitive to the higher moments when evaluated by use of Tobin's rule compared to the square root rule.

To sum up, the departure of the compounded return distribution from normality has an important impact on the Sharpe ratio in so far as it leads to the existence of a maximum at finite horizon and to a decaying Sharpe ratio on the long run as opposed to a monotonically increasing behavior when evaluated with the (approximate) square root rule. Compared with the approximation provided by the square root rule, the impact of the higher moments on the Sharpe ratio is much more limited. These findings are consistent with Hodges et al. (1997) and reconcile the seemingly opposite results reported by Zakamouline and Koekebakker (2009) who argue that the Sharpe ratio increases on the long run while van Binsbergen and Koijen (2017) and Madan and Schoutens (2018) claim the opposite.

3.2 Term structure of allocation weights

We now analyze how the horizon affects the optimal allocation between a risky portfolio and the risk-free asset. In the mean-variance framework, with normally distributed returns, the optimal allocation in the risky portfolio is independent of the horizon T and is given by $\frac{SR_T}{\gamma\sqrt{Var(R_T)}}$. However this result hurts the common sense according to which investment horizon impacts asset allocation (Thorley, 1995; Ferguson and Simaan, 1996, among many others). By virtue of the principle of time diversification, Hansson and Persson (2000) claims that investors with longer horizon should hold more stocks. In contrast Gunthorpe and Levy (1994) or Bodie (1995), among others, support the idea that long term investors should invest more in the risk-free asset. Accounting for the long term impact of the higher moments, we shed new light on this debate.

We consider an economic agent who follows a buy-and-hold strategy. This assumption may seem very restrictive but it is necessary to obtain simple closed-form expressions and, more important, is in line with the observed investment behavior of most individuals as exemplified by the low level of investment fund switching by



Figure 3.2: .Market and agriculture portfolios higher moments Sharpe ratios

The upper part of the graph presents the evolution of the Sharpe ratio for the Market (left panel) and Agriculture (right panel) portfolios when higher moments are accounted for. The lower part of the graph depicts the Sharpe ratio evaluated on the basis of the square root rule and the related cases with higher moments for the same two portfolios. Notice the logarithmic scale in abscissa.

pension plan participants (Collard, 2009, and references therein). Hence, we assume the agent invests his initial wealth W_0 up to time T. At time zero, he invests a fraction w_T of his wealth in a risky security (or portfolio) and the remainder in the risk free asset. The investor's wealth at time T is given by

$$W_T = W_0 \cdot [1 + r_f + w_T (R_T - r_f)] , \qquad (3.9)$$

where R_T and r_f denote the return on the risky asset and the risk free rate of return, respectively, at horizon T. The investor maximizes the expected utility of his terminal wealth, so that the optimal allocation w_T^* satisfies

$$\mathbf{E}\left[U\left(W_{T}^{*}\right)\right] = \max_{w_{T}} \mathbf{E}\left[U\left(W_{T}\right)\right].$$
(3.10)

A quadratic approximation of the utility function $U(x) \simeq x - \frac{\gamma}{2} \cdot x^2$, where γ denotes the relative risk aversion, yields the well-known result:

$$w_T^* = \frac{SR_T}{\gamma \cdot \sqrt{Var(R_T)}}, \qquad (3.11)$$

or, accounting for (2.6, 3.2),

$$w_T^* = \frac{1}{\gamma} \cdot \frac{1 - \frac{1}{(1+\mu)^T}}{\left(1+\mu\right)^T \cdot \left[\left(1 + \frac{1}{\zeta^2}\right)^T - 1\right]},$$
(3.12)

$$= \frac{1}{\gamma} \cdot \frac{(1+\mu)^T - 1}{\sigma^{2T} \cdot \left[(1+\zeta^2)^T - \zeta^{2T} \right]}.$$
 (3.13)

For short investment horizons

$$w_T^* = \frac{1}{\gamma} \cdot \frac{\ln\left(1+\mu\right)}{\ln\left(1+\frac{1}{\zeta^2}\right)} + O\left(T\right) \simeq \frac{1}{\gamma} \cdot \frac{\mu}{\sigma^2}, \qquad (3.14)$$

while w_T^* goes to zero at large time scale. Equation (3.12) shows that the optimal weight w_T^* is a monotonically decreasing function of the horizon T, meaning that long term investors should invest less in risky securities.

This result is consistent with the convergence of the Sharpe ratio to zero at large time scale and is in accordance with Gunthorpe and Levy (1994) and Bodie (1995). At the same time, and even if it can seem counter-intuitive, this result is at odd with the recommendation that investors with long investment horizons should tilt their portfolio toward stocks (Hansson and Persson, 2000). The counter-intuitive nature of the result comes from the idea that risky securities are less risky on the long run and, consequently, that their performance – expressed in terms of Sharpe ratio, for instance – is larger at longer horizon. As demonstrated in the previous sections, these statements are not correct.

For illustration purpose, the blue curves on Figure 3.3 depict the evolution of the



Figure 3.3: Market and agriculture portfolios allocation

The figure depicts the optimal weight as a function of the investment horizon for a buy-andhold strategy when the risky asset is the Market portfolio (left panel) and the Agriculture industry portfolio (right panel). The blue, black and red curves represent the optimal allocation weights, respectively, in the mean-variance framework (see Eq. 3.12), when skewness is accounted for (see Eq. 3.15) and when both skewness and kurtosis are taken into account (see Eq. 3.16). The investor's risk aversion coefficient is set to $\gamma = 2$, his relative preference for skewness $b_3 = 1.5$ and his relative aversion for and kurtosis $b_4 = 3$ as in Zakamouline and Koekebakker (2009).

optimal weight w_T^* when the risky asset is the market portfolio (left panel) and the Agriculture industry portfolio (right panel) and the risk aversion $\gamma = 2$.

The conclusion drawn up to now is not really satisfying. Indeed, the second order approximation of the problem (3.10) only makes sense when the realizations of the returns weakly depart from their mean value. As illustrated in the previous section, this assumption never holds for long term horizons since the return distributions exhibit large skewness and kurtosis. Hence, in an attempt to reconcile the two dissenting views regarding the variation of the optimal investment in risky securities with the horizon, let us now account for the higher moments. They could be responsible for a change in the behavior of w_T^* in so far as investors usually exhibit preference for skewness. In this respect, risky securities are more desirable in the long run since their skewness always increases with the holding horizon (see Proposition 1). Of course, the kurtosis also increases with the investment horizon (see Proposition 2), so that both effects can mitigate and maybe cancel out.

Following Zakamouline and Koekebakker (2009), approximate closed-form expres-

sions for the optimal weight that should be invested in the risky asset are given by:¹¹

$$w_{1,T}^* = w_T^* \cdot \left(1 + b_3 \frac{\text{Skew}_T}{2} \cdot SR_T \right) ,$$
 (3.15)

$$w_{2,T}^* = w_T^* \cdot \left(1 + b_3 \frac{\text{Skew}_T}{2} \cdot SR_T - b_4 \frac{(\text{Kurt}_T - 3)}{6} \cdot SR_T^2 \right), \quad (3.16)$$

where $w_{1,T}^*$ (resp. $w_{2,T}^*$) is the solution to the investor problem (3.10) based on the third (resp. fourth) order expansion of the utility function. The coefficients b_3 and b_4 denote the investor's relative preference for skewness and his relative aversion for kurtosis. These parameters are assumed to be positive. Zakamouline and Koekebakker (2009) show that for an investor with Hyperbolic Absolute Risk Aversion (HARA) utility, $b_3 = \left(1 + \frac{1}{\gamma}\right)$ and $b_4 = \left(1 + \frac{1}{\gamma}\right)\left(1 + \frac{2}{\gamma}\right)$. So, if we choose $\gamma = 2$, then we have $b_3 = 1.5$ and $b_4 = 3$.

The Figure 3.3 depicts the solution (3.15) in black and (3.16) in red. Even if the introduction of the higher moments does not qualitatively change the long term behavior of the demand for the risky portfolio, it is not monotonically decreasing anymore. Up to time scales equal to one to three years, investors should increase their holding in the risky portfolio when the investment horizon increases. The increase illustrated on Figure 3.3 remains moderate but can be much more significant for other portfolios (see Figure 5.E.6 in Appendix). Hence, time diversification does occur at moderate time scale but disappears on the long term. In this respect, our results make a bit more accurate Ferguson and Simaan's statement according to which any simple characterization of the relation between the portfolio composition and the investment horizon is "treacherous".

4 Robustness check

In order to check the robustness of the results presented in the previous section, we consider weekly and monthly data to test the consistency and the sensitivity of our results to different reference time scales. The Figure 1.1 already showed that the two first moments of the market portfolio are not sensitive to the reference time scale while the skewness and kurtosis remains very consistent. The larger sensitivity of the skewness and kurtosis to the reference time scale can probably be ascribed to their non-linear nature (both are ratios of moments and not just moments, which leads to inherent biases).

In this brief section, we use daily, weekly and monthly data to assess the impact of the reference time scale on the Sharpe ratio and the optimal allocation. The Figure 4.1 depicts the evolution of the Sharpe ratio (3.2) and the generalized Sharpe ratios (3.7-3.8) with respect to the investment horizon for the Market portfolio (left panel) and the Agriculture industry portfolio (right panel) when the reference time

¹¹We can easily show that, considering a utility function U(r), the effect of skewness can be accounted for as in Rubinstein (1973). More generally any HARA utility can be used to account for skewness and kurtosis as in Zakamouline and Koekebakker (2009).



Figure 4.1: Market and agriculture portfolios robust Sharpe ratio

This Figure presents the evolution of the Sharpe ratios of the Market portfolio (left panel) and the Agriculture industry portfolio (right panel) across time scales. The blue curves depict the Sharpe ratio (3.2) while the black (resp. red) curves depicts the generalized Sharpe ratio (3.7) (resp. 3.8).

scale varies. A very good agreement is found between the different reference time scales.

The figure 4.2 depicts the results of a similar exercise for the optimal allocation. Again the results are consistent and only weakly sensitive to the reference time scale. Additional results are presented in appendix 5.E and lead to the same conclusions.

Conclusion

The purpose of the Chapter was the investigation of the impact of the horizon on the higher moments of the return distributions and its consequences on asset allocation and performance assessment. First, we showed that, consistent with recent empirical studies by Bessembinder (2018) and Fama and French (2018b), the skewness becomes positive in the long run even if it is (strongly) negative at short time scales. In this respect, we complement the literature by justifying the fact that the compounding effect is the main driving force that explains what can be considered as a stylized fact.

Second, the analysis of the impact of the higher moments on the long term performance of risky assets proved that their impact is much less significant than usually assumed when the square root rule is applied. In particular, the existence and



Figure 4.2: Market and agriculture portfolios robust allocation

This Figure presents the evolution of the optimal allocation when the risky portfolio is the Market portfolio (left panel) and the Agriculture industry portfolio (right panel). The blue curves depict the optimal allocation in the mean-variance framework (3.12) while the black (resp. red) curves depicts optimal allocation when skewness (resp. kurtosis) is accounted for.

the location of an optimal horizon that maximizes the Sharpe ratio is only weakly sensitive to the higher moments.

Third, accounting for the higher moments qualitatively changes the behavior of the optimal mix between a risky portfolio and the risk-free asset as a function of the horizon. In the mean-variance framework, the optimal investment in the risky portfolio monotonically decreases when the horizon increases. On the contrary, accounting for the higher moments leads the investors to increase their holdings in the risky portfolio up the horizons of one to three years and to monotonically decrease their holdings beyond. Hence, accounting for the higher moments favor time diversification at short to moderate investment horizons.

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Appendix 5.A Moments of the distribution of compounded returns

Denoting r_t by the simple return the day (or month) t, the simple return for T days (or months) satisfies

$$R_T = \prod_{t=1}^T (1+r_t) - 1.$$
 (5.A.1)

We assume that the short term returns are iid random variables with finite fourth moment. We denote by $\mu := \mathbb{E}[r_t]$ the expected short term return, σ^2 the variance, s^3 and k^4 the third and fourth central moments. Notice that

$$E[1+r_t] = 1+\mu,$$
 (5.A.2)

$$\operatorname{Var}\left[1+r_{t}\right] = \operatorname{Var}\left[r_{t}\right] = \sigma^{2}, \qquad (5.A.3)$$

$$E\left[((1+r_t) - (1+\mu))^3\right] = E\left[(r_t - \mu)^3\right] = s^3, \qquad (5.A.4)$$

$$E\left[((1+r_t) - (1+\mu))^4\right] = E\left[(r_t - \mu)^4\right] = k^4, \qquad (5.A.5)$$

The expected return at horizon T simply reads

$$E[1 + R_T] = E\left[\prod_{t=1}^T (1 + r_t)\right],$$
 (5.A.6)

$$= (1+\mu)^T$$
, (5.A.7)

while the second moment satisfies

$$E\left[(1+R_T)^2\right] = E\left[\prod_{t=1}^T (1+r_t)^2\right],$$
 (5.A.8)

$$=\prod_{t=1}^{T} E\left[(1+r_t)^2 \right], \qquad (5.A.9)$$

$$= \left[(1+\mu)^2 + \sigma^2 \right]^T .$$
 (5.A.10)

Hence

$$\operatorname{Var}\left(R_{T}\right) = \operatorname{Var}\left(1 + R_{T}\right), \qquad (5.A.11)$$

$$= \mathbf{E}\left[(1+R_T)^2 \right] - \mathbf{E}\left[(1+R_T) \right]^2, \qquad (5.A.12)$$

$$= \left[(1+\mu)^2 + \sigma^2 \right]^T - (1+\mu)^{2T} , \qquad (5.A.13)$$

$$= \sigma^{2T} \cdot \left[\left(1 + \frac{(1+\mu)^2}{\sigma^2} \right)^T - \left(\frac{(1+\mu)^2}{\sigma^2} \right)^T \right], \qquad (5.A.14)$$

$$= \sigma^{2T} \cdot \left[\left(1 + \zeta^2 \right)^T - \zeta^{2T} \right] \,. \tag{5.A.15}$$

Given $|\mu| \ll 1$ and $\sigma \ll 1$, we get $\operatorname{Var}(R_T) = \sigma^2 \cdot T + o(\mu)$.

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The third order moment moment reads

$$E\left[(1+R_T)^3\right] = E\left[\prod_{t=1}^T (1+r_t)^3\right],$$
 (5.A.16)

$$= \prod_{t=1}^{I} \mathbb{E}\left[(1+r_t)^3 \right] , \qquad (5.A.17)$$

$$= \left[s^3 - s_{\min}^3\right]^T, \qquad (5.A.18)$$

with $s_{\min}^3 := -3\sigma^2 (1+\mu) - (1+\mu)^3$ and $s^3 \ge s_{\min}^3$ since $E\left[(1+\tilde{r})^3\right] = s^3 - s_{\min}^3$ is positive.

Given that the third order central moment $\mu_3(R_T)$ of R_T satisfies

$$\mu_3(R_T) = \mu_3(1 + R_T) , \qquad (5.A.19)$$

$$= \mathbb{E}\left[(1+R_T)^3 \right] - 3 \cdot \mathbb{E}\left[1+R_T \right] \cdot \text{Var} \left(1+R_T \right) - \mathbb{E}\left[1+R_T \right]^3, \quad (5.A.20)$$

$$= \mathbf{E}\left[\left(1+R_{T}\right)^{3}\right] - 3 \cdot \mathbf{E}\left[1+R_{T}\right] \cdot \operatorname{Var}\left(R_{T}\right) - \mathbf{E}\left[1+R_{T}\right]^{3}, \qquad (5.A.21)$$
$$= \left(s^{3}-s^{3}\right)^{T} - 3 \cdot \left(1+u\right)^{T}\left[\left(\sigma^{2}+\left(1+u\right)^{2}\right)^{T}-\left(1+u\right)^{2T}\right]$$

$$= (3^{-3} - 3^{-1})^{-3} + (1 + \mu)^{-1} \left[(0^{-1} + (1 + \mu)^{-1})^{-1} - (1 + \mu)^{-1} \right]$$

$$= -(1 + \mu)^{3T},$$

$$(5.A.22)$$

$$= \left(s^{3} - s_{\min}^{3}\right)^{T} - \left(1 + \mu\right)^{3T} \left[3 \cdot \left(\frac{\sigma^{2}}{\left(1 + \mu\right)^{2}} + 1\right)^{T} - 2\right], \qquad (5.A.23)$$

the skewness reads

$$\frac{\mu_{3}(R_{T})}{\operatorname{Var}(R_{T})^{3/2}} = \frac{\left[\frac{s^{3}-s_{\min}^{3}}{\left((1+\mu)^{2}+\sigma^{2}\right)^{3/2}}\right]^{T}}{\left[1-\frac{1}{\left(1+\frac{\sigma^{2}}{\left(1+\mu\right)^{2}}\right)^{T}}\right]^{3/2}} - \frac{3\left(1+\frac{\sigma^{2}}{\left(1+\mu\right)^{2}}\right)^{T}-2}{\left[\left(1+\frac{\sigma^{2}}{\left(1+\mu\right)^{2}}\right)^{T}-1\right]^{3/2}}, \quad (5.A.24)$$

$$= \frac{\left(\frac{\tilde{s}^{3}-\tilde{s}_{\min}^{3}}{\left(1+\zeta^{2}\right)^{3/2}}\right)^{T}}{\left[1-\left(\frac{\zeta^{2}}{1+\zeta^{2}}\right)^{T}\right]^{3/2}} - \frac{\left(\frac{\zeta^{2}}{1+\zeta^{2}}\right)^{T/2}\cdot\left[3-2\cdot\left(\frac{\zeta^{2}}{1+\zeta^{2}}\right)^{T}\right]}{\left[1-\left(\frac{\zeta^{2}}{1+\zeta^{2}}\right)^{T}\right]^{3/2}}, \quad (5.A.25)$$

where $\tilde{s}^3 := s^3 \cdot \sigma^{-3}$ denotes the (standardized) short term skewness, $\tilde{s}^3_{\min} := s^3_{\min} \cdot \sigma^{-3} = -(3\zeta + \zeta^3)$ and $\zeta := \frac{1+\mu}{\sigma}$.

The last relation shows that the skewness is positive at horizon ${\cal T}$ whenever the short term skewness satisfies

$$\tilde{s}^3 > \tilde{s}^3_{\min} + \zeta \cdot \left(1 + \zeta^2\right) \cdot \left[3 - 2 \cdot \left(\frac{\zeta^2}{1 + \zeta^2}\right)^T\right]^{1/T}$$
. (5.A.26)

The rightmost term in equation (5.A.24) goes to zero as $3 \cdot \left(\frac{\zeta^2}{1+\zeta^2}\right)^{T/2}$ as $T \to \infty$

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while the denominator in the right-hand side goes to one. As a consequence, the standardized skewness of R_T is controlled by the $\frac{\tilde{s}^3 - \tilde{s}_{\min}^3}{(1+\zeta^2)^{3/2}}$. It is always positive and is either larger than one and the standardized skewness grows to infinity or is smaller than one and standardized skewness goes to zero. Clearly, the skewness goes to infinity as soon as $\tilde{s} > \tilde{s}_{\min} + (1+\zeta^2)^{3/2}$ and zero otherwise.

The fourth order moment moment reads

$$E\left[(1+R_T)^4\right] = E\left[\prod_{t=1}^T (1+r_t)^4\right],$$
 (5.A.27)

$$= \prod_{t=1}^{I} \mathbf{E} \left[(1+r_t)^4 \right] , \qquad (5.A.28)$$

$$= \left[k^4 - k_{\min}^4\right]^T, \qquad (5.A.29)$$

with $k_{\min}^4 := -4s^3 (1+\mu) - 6\sigma^2 (1+\mu)^2 - (1+\mu)^4$.

The fourth order central moment $\mu_4(R_T)$ is given by

$$\mu_{4} (R_{T}) = \mu_{4} (1 + R_{T}) ,$$

$$= E \left[(1 + R_{T})^{4} \right] - 4 \cdot E \left[1 + R_{T} \right] \cdot \mu_{3} (R_{T})
- 6 \cdot E \left[1 + R_{T} \right]^{2} \cdot Var (R_{T}) - E \left[1 + R_{T} \right]^{4} ,$$

$$= \left(k^{4} - k_{\min}^{4} \right)^{T} - 6 \cdot (1 + \mu)^{2T} \cdot \left(\left[(1 + \mu)^{2} + \sigma^{2} \right]^{T} - (1 + \mu)^{2T} \right)
- 4 \cdot (1 + \mu)^{T} \cdot \left(\left(s^{3} - s_{\min}^{3} \right)^{T} - (1 + \mu) \left[3 \cdot \left(1 + \frac{\sigma^{2}}{(1 + \mu)^{2}} \right)^{T} - 2 \right] \right) \right)
- (1 + \mu)^{4T} ,$$

$$= \left(k^{4} - k_{\min}^{4} \right)^{T} - 4 \cdot (1 + \mu)^{T} \cdot \left(s^{3} - s_{\min}^{3} \right)^{T}
+ 3 \cdot (1 + \mu)^{4T} \cdot \left(2 \cdot \left(1 + \frac{\sigma^{2}}{(1 + \mu)^{2}} \right)^{T} - 1 \right) ,$$
(5.A.30)

(5.A.31)

(5.A.32)

(5.A.33)

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hence the kurtosis of R_T satisfies

$$\frac{\mu_4(R_T)}{\operatorname{Var}(R_T)^2} = \frac{(k^4 - k_{\min}^4)^T}{\left(\left[(1+\mu)^2 + \sigma^2\right]^T - (1+\mu)^{2T}\right)^2} - 4 \cdot \frac{(1+\mu)^T \cdot (s^3 - s_{\min}^3)^T}{\left(\left[(1+\mu)^2 + \sigma^2\right]^T - (1+\mu)^{2T}\right)^2} + 3 \cdot \frac{(1+\mu)^{4T} \cdot \left(2 \cdot \left(1 + \frac{\sigma^2}{(1+\mu)^2}\right)^T - 1\right)}{\left(\left[(1+\mu)^2 + \sigma^2\right]^T - (1+\mu)^{2T}\right)^2}, \quad (5.A.34)$$

$$= \frac{\left(\frac{k^4 - k_{\min}^4}{\left[(1+\mu)^2 + \sigma^2\right]^2}\right)^T}{\left(1 - \left(\frac{(1+\mu)^2}{(1+\mu)^2 + \sigma^2}\right)^T\right)^2} - 4 \cdot \frac{\left(\frac{(1+\mu) \cdot (s^3 - s_{\min}^3)}{\left[(1+\mu)^2 + \sigma^2\right]^2}\right)^T}{\left(1 - \left(\frac{(1+\mu)^2}{(1+\mu)^2 + \sigma^2}\right)^T\right)^2} + 3 \cdot \frac{\left(\frac{(1+\mu)^2}{(1+\mu)^2 + \sigma^2}\right)^T \cdot \left(2 - \left(\frac{(1+\mu)^2}{(1+\mu)^2 + \sigma^2}\right)^T\right)}{\left(1 - \left(\frac{(1+\mu)^2}{(1+\mu)^2 + \sigma^2}\right)^T\right)^2}, \quad (5.A.35)$$

$$= \frac{\left(\frac{k^4 - k_{\min}^4}{\left[1+\epsilon^2\right]^2}\right)^T}{\left(1 - \left(\frac{\epsilon^2}{(1+\epsilon^2)}\right)^T\right)^2} - 4 \cdot \frac{\left(\frac{\epsilon(s^3 - s_{\min}^3)}{(1+\epsilon^2)^2}\right)^T}{\left(1 - \left(\frac{\epsilon^2}{(1+\epsilon^2)}\right)^T\right)^2} + 3 \cdot \frac{\left(\frac{\epsilon^2}{(1+\epsilon^2)}\right)^T \cdot \left(2 - \left(\frac{\epsilon^2}{(1+\epsilon^2)}\right)^T\right)}{\left(1 - \left(\frac{\epsilon^2}{(1+\epsilon^2)}\right)^T\right)^2}, \quad (5.A.36)$$

with $\tilde{k}^4 := k^4 \sigma^{-4}$ and $\tilde{k}^4_{\min} := k^4_{\min} \sigma^{-4}$.

The rightmost term in the equation above goes to zero as $\left(\frac{\zeta^2}{1+\zeta^2}\right)^T$ as $T \to \infty$ while the denominator in the right-hand side goes to one. As a consequence, the kurtosis of R_T is controlled by the term $\frac{\tilde{k}^4 - \tilde{k}_{\min}^4}{[1+\zeta^2]^2}$. Indeed, the term $\frac{\zeta \cdot (\tilde{s}^3 - \tilde{s}_{\min}^3)}{[1+\zeta^2]^2}$ can never be dominant: since it is necessarily positive, its contribution to the kurtosis is negative ; if it was the leading contribution, the kurtosis would eventually be negative which is impossible since $\mu_4(R_T)$ must be non-negative. As a consequence, the kurtosis goes either to zero or infinity according as $\frac{\tilde{k}^4 - \tilde{k}_{\min}^4}{[1+\zeta^2]^2}$ is smaller or larger than one, that is,

$$\frac{\mu_4(R_T)}{\operatorname{Var}(R_T)^2} \to +\infty \iff \tilde{k}^4 > \tilde{k}_{\min}^4 + \left(1 + \zeta^2\right)^2.$$

Appendix 5.B Short-term behavior of the moments

We summarize the very short-term (that is, as $T\to 0)$ behavior of the moments of the compounded returns. We have

$$E[R_T] = (1+\mu)^T - 1,$$
 (5.B.1)

$$= \exp \left[T \cdot \ln \left(1 + \mu \right) \right] - 1, \qquad (5.B.2)$$

$$= T \cdot \ln (1 + \mu) + o(T), \qquad (5.B.3)$$

and

$$\operatorname{Var}(R_T) = \left[(1+\mu)^2 + \sigma^2 \right]^T - (1+\mu)^{2T} , \qquad (5.B.4)$$

$$= \exp\left[T \cdot \ln\left[(1+\mu)^2 + \sigma^2\right]\right] - \exp\left[T \cdot \ln\left(1+\mu\right)^2\right], \qquad (5.B.5)$$

$$= T \cdot \ln\left[(1+\mu)^{2} + \sigma^{2}\right] - T \cdot \ln(1+\mu)^{2} + o(T), \qquad (5.B.6)$$

$$= T \cdot \ln\left(1 + \left(\frac{\sigma}{1+\mu}\right)^2\right) + o(T), \qquad (5.B.7)$$

The third central moment satisfies

$$\mu_{3}(R_{T}) = \left(s^{3} - s_{\min}^{3}\right)^{T} - 3 \cdot (1+\mu)^{T} \left[\left(\sigma^{2} + (1+\mu)^{2}\right)^{T} - (1+\mu)^{2T}\right] - (1+\mu)^{3T}, \qquad (5.B.8) = \exp\left[T \cdot \ln\left(s^{3} - s_{\min}^{3}\right)\right] - 3 \exp\left[T \cdot \ln\left(1+\mu\right)\left(\sigma^{2} + (1+\mu)^{2}\right)\right] + 2 \exp\left[3T \cdot \ln\left(1+\mu\right)\right], \qquad (5.B.9)$$

$$= T \cdot \ln \left(s^3 - s_{\min}^3\right) - 3T \cdot \ln \left(1 + \mu\right) \left(\sigma^2 + (1 + \mu)^2\right) + 6T \cdot \ln \left(1 + \mu\right) + o(T) , \qquad (5.B.10)$$

$$= T \cdot \left[\ln \frac{s^3 - s_{\min}^3}{(1+\mu)^3} - 3\ln \left(1 + \left(\frac{\sigma}{1+\mu} \right)^2 \right) \right] + o(T) .$$
 (5.B.11)

We notice that the third central moment is negative as T goes to zero whenever

$$\frac{(s^3 - s_{\min}^3)^{1/3}}{1 + \mu} < 1 + \left(\frac{\sigma}{1 + \mu}\right)^2 \iff \left(\tilde{s}^3 - \tilde{s}_{\min}^3\right)^{1/3} < \zeta + \frac{1}{\zeta}.$$
 (5.B.12)

When this condition holds, the skewness goes the minus infinity when T goes zeros and to plus infinity otherwise.

The fourth central moment reads

$$\mu_{4} (R_{T}) = \left(k^{4} - k_{\min}^{4}\right)^{T} - 4 \cdot (1+\mu)^{T} \cdot \left(s^{3} - s_{\min}^{3}\right)^{T} \\ + 3 \cdot (1+\mu)^{4T} \cdot \left(2 \cdot \left(1 + \frac{\sigma^{2}}{(1+\mu)^{2}}\right)^{T} - 1\right), \quad (5.B.13)$$

$$= \exp\left[T \cdot \ln\left(k^{4} - k_{\min}^{4}\right)\right] - 4 \exp\left[T \cdot \ln\left(1+\mu\right)\left(s^{3} - s_{\min}^{3}\right)\right] \\ + 6 \exp\left[T \cdot \ln\left(1+\mu\right)^{2}\left(\sigma^{2} + (1+\mu)^{2}\right)\right] \\ - 3 \exp\left[T \cdot \ln\left(1+\mu\right)^{4}\right], \quad (5.B.14)$$

$$= T \cdot \left[\ln\left(k^{4} - k_{\min}^{4}\right) - 4\ln\left(s^{3} - s_{\min}^{3}\right) + 6\ln\left(\sigma^{2} + (1+\mu)^{2}\right) \\ - 4\ln\left(1+\mu\right)\right] + o(T), \quad (5.B.15)$$

$$= T \cdot \left[\ln\frac{k^{4} - k_{\min}^{4}}{(1+\mu)^{4}} - 4\ln\frac{s^{3} - s_{\min}^{3}}{(1+\mu)^{3}} \\ + 6\ln\left(1 + \left(\frac{\sigma}{1+\mu}\right)^{2}\right)\right] + o(T). \quad (5.B.16)$$

In the limit of short-term horizon, the kurtosis necessarily diverges to plus infinity since $\mu_4(R_T)$ cannot be negative.

Appendix 5.C Proof of Proposition 3

Assuming that the risk-free rate equals zero or that μ denotes the expected return over the risk-free rate, the Sharpe ratio at time scale T reads

$$SR_T = \frac{1 - \frac{1}{(1+\mu)^T}}{\left[\left(1 + \frac{1}{\zeta^2}\right)^T - 1\right]^{\frac{1}{2}}}.$$
 (5.C.1)

By differentiation with respect to T, we get

$$\frac{\partial SR_T}{\partial T} = -\frac{\left(1 - \frac{1}{(1+\mu)^T}\right) \left(1 + \frac{1}{\zeta^2}\right)^T \log\left(1 + \frac{1}{\zeta^2}\right)}{2\left[\left(1 + \frac{1}{\zeta^2}\right)^T - 1\right]^{3/2}} - \frac{\frac{1}{(1+\mu)^T} \log\frac{1}{(1+\mu)}}{\sqrt{\left(1 + \frac{1}{\zeta^2}\right)^T - 1}}, \quad (5.C.2)$$
$$= -\frac{\left(1 - \frac{1}{(1+\mu)^T}\right) \left(1 + \frac{1}{\zeta^2}\right)^T \log\left(1 + \frac{1}{\zeta^2}\right) + 2\left[\left(1 + \frac{1}{\zeta^2}\right)^T - 1\right] \cdot \frac{1}{(1+\mu)^T} \log\frac{1}{(1+\mu)}}{2\left[\left(1 + \frac{1}{\zeta^2}\right)^T - 1\right]^{3/2}}, \quad (5.C.3)$$

so that, assuming $\mu > 0$,

$$\frac{\partial SR_T}{\partial T} \ge 0 \iff 2\frac{\frac{1}{(1+\mu)^T}}{\frac{1}{(1+\mu)^T} - 1} \cdot \log \frac{1}{(1+\mu)^T} \ge \frac{\left(1 + \frac{1}{\zeta^2}\right)^T}{\left(1 + \frac{1}{\zeta^2}\right)^T - 1} \cdot \log \left(1 + \frac{1}{\zeta^2}\right)^T.$$
(5.C.4)

Setting $x \coloneqq \frac{1}{(1+\mu)}$ and $y \coloneqq \left(1 + \frac{1}{\zeta^2}\right)$, we are looking for T such that

$$2f\left(x^{T}\right) = f\left(y^{T}\right), \qquad (5.C.5)$$

where the function is defined as :

$$f(\bullet) := \frac{\bullet}{\bullet - 1} \cdot \log \bullet \tag{5.C.6}$$

over the range $[1, +\infty)$.¹²

Introducing the auxiliary variable $u \coloneqq e^{-T}$, equation (5.C.5) which reads

$$2\frac{\log x}{1-x^{-T}} = \frac{\log y}{1-y^{-T}},$$
(5.C.7)

becomes

$$\log x^2 \cdot u^{\log y} - \log y \cdot u^{\log x} = \log \frac{x^2}{y}.$$
(5.C.8)

We can notice that u = 1 is an obvious solution but it is not admissible since it would lead to T = 0 and equation (5.C.7) is not defined at that point.

Let us now introduce the variable $v \coloneqq \left(\frac{\log x^2}{\log \frac{x^2}{y}}\right)^{\frac{1}{\log y}} \cdot u$. By substitution into the above equation, we get

$$v^{\log y} - \frac{\log y}{\log \frac{x^2}{y}} \cdot \left(\frac{\log x^2}{\log \frac{x^2}{y}}\right)^{-\frac{\log x}{\log y}} \cdot v^{\log x} = 1.$$
(5.C.9)

It is well-known that the equation

$$z^{\alpha} + t \cdot z^{-\beta} = 1,$$
 (5.C.10)

with $(\alpha, \beta, t) \in \mathbb{R}^3_+$ admits two real solutions $z_{\pm}(t)$ when $t < t^* \coloneqq \frac{\alpha}{\beta} \cdot \left(\frac{\beta}{\alpha+\beta}\right)^{1+\frac{\beta}{\alpha}}$ and zero otherwise.¹³ The upper solution, $z_+(t)$, satisfies $z_+(0) = 1$ and decays to $z^* \coloneqq z(t^*) = \left(1 + \frac{\alpha}{\beta}\right)^{-\frac{1}{\alpha}}$ while the lower solution, $z_-(t)$, goes to zero as t goes to

¹³In the limit case
$$t = \frac{\alpha}{\beta} \cdot \left(\frac{\beta}{\alpha+\beta}\right)^{1+\frac{\beta}{\alpha}}$$
, equation (5.C.10) admits one single solution $z^* = \left(1+\frac{\alpha}{\beta}\right)^{-\frac{1}{\alpha}}$

¹²We assume $\mu > 0$, which is the interesting case, so that x is less than one while y is larger than one.

zero and grows to z^* as t goes to t^* . More precisely, for small t:

$$z_{-}(t) = t^{1/\beta} \left(1 + \frac{t^{\frac{\alpha}{\beta}}}{\beta - \alpha \cdot t^{\frac{\alpha}{\beta}}} + o\left(t^{\frac{\alpha}{\beta}}\right) \right) .$$
 (5.C.11)

The proof of this statement is straightforward and follows the line of Hochstadt (1986, pp. 81-86).

Hence, with $t = -\frac{\log y}{\log \frac{x^2}{y}} \cdot \left(\frac{\log x^2}{\log \frac{x^2}{y}}\right)^{-\frac{\log x}{\log y}} > 0, \alpha = \log y > 0$ and $\beta = -\log x > 0$, we can conclude that the Sharpe ratio admits a maximum if, and only if,

$$t = -\frac{\log y}{\log \frac{x^2}{y}} \cdot \left(\frac{\log x^2}{\log \frac{x^2}{y}}\right)^{-\frac{\log x}{\log y}} < t^* = -\frac{\log y}{\log x} \cdot \left(\frac{-\log x}{\log y - \log x}\right)^{1 - \frac{\log x}{\log y}}, \quad (5.C.12)$$

which is equivalent to

$$2 < \left(1 + \frac{1}{1 - \frac{\log y}{\log x}}\right)^{1 - \frac{\log y}{\log x}}.$$
 (5.C.13)

This condition holds if and only if μ is positive, so that $-\frac{\log y}{\log x} > 0$, since the function $g(\bullet) = \left(1 + \frac{1}{1+\bullet}\right)^{1+\bullet}$ is increasing and equals two when its argument equals zero.

Among the two possible solutions of equation (5.C.9), only the one on the lower branch z_{-} is admissible. Indeed, by definition, $u = e^{-T}$ is less than one so that vmust be less than $v_{\max} \coloneqq \left(\frac{\log x^2}{\log \frac{x^2}{y}}\right)^{\frac{1}{\log y}}$. Given that $v_{\max} > z^*$,¹⁴ the whole lower branch z_{-} is admissible while the obvious solution $z = v_{\max}$, which belong to the upper branch z_{+} , is not admissible (see supra).

As a consequence, given the condition $\mu > 0$ holds and accounting for expression (5.C.11), the solution v to the equation (5.C.9) is approximately equal to

$$v^* \simeq \left(-\frac{\log y}{\log \frac{x^2}{y}}\right)^{-\frac{1}{\log x}} \cdot \left(\frac{\log x^2}{\log \frac{x^2}{y}}\right)^{\frac{1}{\log y}}, \qquad (5.C.14)$$

so that

$$u^* \simeq \left(-\frac{\log y}{\log \frac{x^2}{y}}\right)^{-\frac{1}{\log x}},\qquad(5.C.15)$$

 14 We have

$$v_{\max} = \left(\frac{\log x^2}{\log \frac{x^2}{y}}\right)^{\frac{1}{\log y}} = \left(1 - \frac{1}{2} \cdot \frac{\log y}{\log x}\right)^{-\frac{1}{\log y}} > \left(1 - \frac{\log y}{\log x}\right)^{-\frac{1}{\log y}} = z^*$$

and

$$T^* \simeq \frac{1}{\log x} \cdot \log\left(-\frac{\log y}{\log \frac{x^2}{y}}\right),$$
 (5.C.16)

$$\simeq \frac{1}{\log\left(1+\mu\right)} \cdot \log\left(1+2 \cdot \frac{\log\left(1+\mu\right)}{\log\left(1+\frac{1}{\zeta^2}\right)}\right), \qquad (5.C.17)$$

$$\simeq \frac{1}{\mu} \cdot \log\left(1 + \frac{2\mu}{\sigma^2}\right) \,. \tag{5.C.18}$$

Appendix 5.D Additional summary statistics

This appendix synthesizes the results of the bootstrap procedure introduced by Fama and French (2018b) when applied to samples of returns on the market portfolio. We start from four samples of returns with different reference time scales : daily returns, weekly returns, monthly returns and yearly returns. For each sample, corresponding to a particular reference time scale, we generate samples of bootstrapped returns with time scales ranging from the reference time scale up to 30 years. The first four moments the bootstrapped returns are then estimated and reported in table 5.3. Despite the presence of serial dependence in the return volatility, the results are consistent irrespective of the reference time scale meaning that these serial dependence do not impair the procedure.

Appendix 5.E Robustness check

We present hereafter a collection of graphs obtained from two randomly chosen industry portfolios labeled Indu-1 and Indu-2 for two different initial time scales (daily and weekly). The shape of the graphs is perfectly consistent with the graphs presented in the main body of the text.

- 5.E.1 Daily reference time scale
- 5.E.2 Weekly reference time scale

	Daily data					Weekly data				
Horizon	Mean	St.dev. (%)	Skew.	Kurt.	Mea	n $\frac{\text{Sd.dev}}{(\%)}$	Skew.	Kurt.		
1 Day	1.0004	0.009	-0.515	18.599	_	_	_	—		
1 Week	1.002	0.020	-0.209	6.722	1.002	2 0.022	-0.365	7.562		
1 Month	1.009	0.046	0.011	3.569	1.00	8 0.042	-0.044	4.217		
3 Months	1.026	0.080	0.111	3.249	1.02	4 0.077	0.087	3.379		
6 Months	1.056	0.117	0.267	3.217	1.052	2 0.115	0.261	3.416		
1 Year	1.116	0.175	0.396	3.429	1.11^{-1}	4 0.180	0.409	3.545		
3 Years	1.385	0.383	0.825	4.297	1.38	1 0.387	0.875	4.556		
5 Years	1.713	0.628	1.147	5.491	1.70	5 0.623	1.077	4.970		
10 Years	2.921	1.539	1.643	7.600	2.92	7 1.560	1.592	7.270		
20 Years	8.660	6.811	3.070	25.895	8.42	8 6.613	2.504	13.883		
30 Years	25.318	27.231	4.184	37.253	24.67	7 25.120	3.273	20.797		
	Monthly data					Yearly data				
Horizon	Mean	St.dev. (%)	Skew.	Kurt.	Mea	n $\frac{\text{Sd.dev}}{(\%)}$	Skew.	Kurt.		
1 Month	1.009	0.044	-0.503	4.944	_	_	_	_		
3 Months	1.028	0.078	-0.13	3.508	_	_	_	_		
6 Months	1.056	0.113	0.048	3.158	_	_	—	_		
1 Year	1.114	0.170	0.331	3.461	1.11	7 0.173	0.653	4.429		
3 Years	1.381	0.363	0.655	3.809	1.39	5 0.372	0.832	4.332		
5 Years	1.709	0.599	1.09	5.681	1.73	9 0.608	1.029	5.036		
10 Years	2.949	1.535	1.649	8.021	3.02	7 1.552	1.412	6.690		
20 Years	8.584	6.582	2.461	13.300	9.23	0 7.093	2.025	10.090		
30 Years	25.464	24.924	3.256	21.463	26.74	9 26.373	3.142	16.806		

Table 5.3: Market portfolio summary statistics of the bootstrapped (gross) returns

Summary statistics of the bootstrapped (gross) returns for the Market portfolio between July 1963 and December 2016 for eleven horizons ranging between 1 day and 30 years.



Figure 5.E.1: Skewness and kurtosis of two daily industry portfolios over time

Skewness and kurtosis of two industry portfolios INDU-1, INDU-2. Equations. 2.11 and 2.18 are used to estimate the skewness and the kurtosis for each considered horizon. No adjusted parameters.



Figure 5.E.2: Sharpe ratio of two daily industry portfolios over time.

Sharpe ratio of two Industry portfolios INDU-1, INDU-2 obtained with the Eqs. (3.7) and generalized Sharpe ratios (3.7-3.8).



Figure 5.E.3: Allocation weights of two daily industry portfolios over time

Optimal allocations of two industry portfolios INDU-1, INDU-2 obtained with the Eqs. 3.15 and 3.16 .



Figure 5.E.4: Skewness and kurtosis of two weekly industry portfolios over time

Skewness and kurtosis of the industry portfolio INDU-1, INDU-2. Equations. 2.11 and 2.18 are used to estimate the skewness and the kurtosis for each considered horizon. No adjusted parameters.



Figure 5.E.5: Sharpe ratio of two weekly industry portfolios over time

Sharpe Ratio of two Industry portfolios INDU-1, INDU-2 obtained with the Eqs. (3.7) and generalized Sharpe ratios (3.7-3.8).



Figure 5.E.6: Allocation weights of two weekly industry portfolios over time

Optimal allocations obtained with the Eqs.3.12 3.15 and 3.16.

Conclusion and outlook

Many models have been proposed to estimate the return of an asset or a portfolio based, on the one hand, on the principle of what the price should be (positivism) – equilibrium-based pricing models such as the CAPM – and, on the other hand, on what the price actually is (normative approach) – arbitrage and/or multi-factorial models. Our study contributes to this literature by examining the determinants of asset returns. In fact, in this thesis, we investigate the determinants of asset returns such as the risk factors, the asymmetry of the distribution of returns and the investment horizon.

To this aim, we first critically review the positive-normative theories presented in the asset pricing literature that could be adopted to address these research questions. The literature on asset pricing revealed that the multi-factor models are the most important competitors to the equilibrium models, which include the CAPM. Indeed, the CAPM has been mostly criticized for not taking into account a number of anomalies, including the size effect. This anomaly reflects that small size firms (lower market capitalization) tend to have a higher return compared to the big ones.

Thus, in the first essay, we reconsidered the study of this anomaly in order to better take it into account. To this aim, we question the composition of the Fama-French size-sorted portfolios using the LASSO (Least Absolute Shrinkage and Selection Op*erator*) approach developed by Tibshirani (1996). On the basis of a penalized linear regression, the Lasso allows the selection of relevant explanatory variables by imposing constraints on the regression coefficients. We examined the hypothesis that the persistence (or not) of the size effect could be explained by specific industries. Our results indicate that not all industries and, in fact, only a few matter to explain size sorted portfolio returns over time and particularly the small and big cap portfolios. Around the years 2000s, for example, internet related firms are identified and selected to explain the return on the small size portfolio. Overall our results are important to explain the determinant of the size premium and would contribute to the current literature on the size effect. In addition, it has been shown that considering the portfolio formed with these specific industries has an impact on the factors proposed in the literature to take into account the size effect such as the SMB factor of Fama and French (1993) and the QMJ factor of Asness et al. (2019).

However, a number of research avenues can be identified to improve our approach. The first is methodological. Indeed, the Lasso has limitations when there are too strong correlations between the explanatory variables to be selected. The method may then encounter convergence issues. Zou and Hastie's (2005) *Elastic Net*, which combines Lasso and regularized regression (Tikhonov, 1943), could overcome this problem and strengthen the results we obtained, as in the work of Freyberger et al. (2018). Another possibility could be to generalize our study to the analysis of other anomalies. In the same way that common sense suggests, *a priori*, the existence of a connection between size and industries, such a relation seems to be conceivable between growth potential (*i.e.* value) or operating margins... and industries.

In the second essay of this thesis we considered other anomalies which, as the size effect, have been the subject of a vast literature. These anomalies have given rise to the proposal of risk factors to take into account certain regularities that are not covered by the CAPM. However, given the plethora of factors that could be included in a model, the choice of the relevant factors remains an open question (Fama and French, 2018a). In addition, the relationship or correlation between risk factors could affect the validity of the models (Kan and Robotti, 2012). Hence we examined the relationship between the market and the other common risk factors and provided new insights on their relationship. To this aim, we considered the *Alternating Conditional Expectation* (ACE), a non-parametric regression approach introduced by Breiman and Friedman (1985). This approach allowed us to evidence an optimal relation between these factors. Thus in this essay, we showed that a non-linear transform of the market factor allowed other risk factors to be taken into account. Potential investment opportunities, as in Frazzini and Pedersen (2014), can be established allowing an investor to achieve significantly abnormal returns.

However, even if our results suggest a better definition of the relationship between the market and other common risk factors, it remains room for improvement. In fact, in our essay, a limited number of risk factors were considered and we only sought to establish the existence of non-linear relationships between the market and these other risk factors. This approach may seem excessively simplistic insofar as it implies that the market is the sole variable that explains the expected returns. It would therefore be appropriate to extend our study to include possible non-linear relation between some of the other risk factors. In fact in our essay we consider a limited number of risk factors thus we could continue by testing with a larger number to better define the relationship between the market and risk factors.

Finally, in the third essay of this thesis, we confirm the impact of the horizon on the asset returns. First, based on the compounding rule, we studied the impact of the horizon on the higher order moments of long-term return distributions. We have shown that, according to the recent empirical studies of Bessembinder (2018) and Fama and French (2018b), asymmetry becomes positive in the long term and complements the literature by justifying the fact that the effect of compounding is the main driver of the positive asymmetry in the long term, which can be considered as a stylized fact. In a second time, we show that taking into account the higher order moments qualitatively changes the behavior of the optimal allocation between a risky portfolio and the risk-free asset when the horizon varies as well as long-term performance of risky assets. We have shown that taking into account higher order moments reconciles divergent views on optimal asset allocation over time (Bodie, 1995; Thorley, 1995) and on performance measurement (Zakamouline and Koekebakker, 2009; van Binsbergen and Koijen, 2017). However, the dynamic approach, rather than *Buy and Hold* strategy considered in this essay, is also used in practice. Thus, taking this approach into account as well as the use of other class of assets could be interesting to consider for future research.
Conclusion et perspectives

Il existe de nombreux modèles pour estimer le rendement d'un actif ou d'un portefeuille. Ces modèles cherchent à déterminer ce que devrait être le prix (approche positiviste), avec les modèles d'évaluation d'équilibre tel que le MEDAF, ou bien à représenter ce que le prix est en réalité (approche normative), avec les modèles d'arbitrage ou/et multifactoriels. Notre étude apporte une contribution à cette littérature sur l'évaluation des actifs en examinant les déterminants de leurs rendements. En effet au fil de cette thèse, nous avons cherché à définir le comportement des rentabilités des actifs financiers lorsque l'on considère les facteurs de risque, l'asymétrie des rendements et l'horizon d'investissement comme déterminants.

Pour ce faire, nous avons commencé par passer en revue de façon critique les théories présentées dans la littérature sur l'évaluation d'actifs. Il s'avère que, bien que non fondés théoriquement, les modèles multifactoriels permettent de rendre compte des rendements attendus de manière bien plus satisfaisante que la plupart des modèles d'équilibre. En effet le MEDAF a fait l'objet de critiques importantes quant à son incapacité à expliquer un certain nombre d'anomalies parmi lesquelles l'effet de taille. Cette anomalie se traduit par une rentabilité supérieure des entreprises de petite taille (selon le critère de la capitalisation boursière) comparée à celles de grande taille.

Ainsi nous nous sommes intéressés, à travers le premier essai, à cette problématique toujours d'actualité afin de mieux en cerner l'origine. Pour ce faire, nous avons envisagé une explication sectorielle de l'effet de taille et avons eu recours à la méthode dite du LASSO (Least Absolute Shrinkage and Selection Operator) développée par Tibshirani (1996) qui permet, sur la base d'une régression linéaire pénalisée, la sélection des variables explicatives pertinentes en exerçant des contraintes sur les coefficients de régression. Nous avons examiné l'hypothèse selon laquelle la persistance ou non de l'effet de taille pourrait être expliquée par des spécificités sectorielles. Les résultats obtenus mettent en évidence des relations significatives entre les critères de classifications en termes de taille et de secteurs d'activité et donnent aussi une idée de l'évolution de cet effet en fonction des secteurs industriels. Nous avons notamment montré que certains secteurs sont plus influents que d'autres et, en particuliers pendant la bulle internet, les secteurs liés aux nouvelles technologies de la communication et de l'information sont ceux qui étaient les plus pertinents pour expliquer l'effet de taille. Nos résultats sont importants pour expliquer le déterminant de la prime de taille et contribuent, à cet égard, à la littérature actuelle sur ce sujet. Par ailleurs, nous avons montré que la prise en compte de secteurs industriels spécifiques avait un impact sur les facteurs proposés dans la littérature, pour prendre en compte cet effet, tel que les facteurs SMB de Fama and French et QMJ de Asness et al..

Toutefois quelques pistes de recherches permettant d'améliorer notre approche peuvent être envisagées. La première est méthodologique. En effet, le LASSO présente des limites lorsqu'il existe de trop fortes corrélations entre les variables explicatives à sélectionner. La méthode peut alors rencontrer des problèmes de convergence. L'*Elastic Net* de Zou and Hastie (2005), qui combine LASSO et régression régularisée (Tikhonov, 1943), pourrait y palier et donner plus de robustesse aux résultats obtenus à l'instar des travaux de Freyberger et al. (2018). Une autre piste pourrait être la généralisation de notre étude à l'analyse d'autres anomalies. De la même façon que le bon sens suggère, *a priori*, l'existence d'un lien entre taille et secteur d'activité, un tel lien parait pouvoir être envisagé entre potentiel de croissance (en donc *value*) ou encore marge opérationnelle... et secteur d'activité.

Dans le deuxième essai de cette thèse nous avons considéré d'autres anomalies qui, comme l'effet de taille, ont fait l'objet d'une vaste littérature. Ces anomalies ont donné naissance à la proposition de facteurs de risque pour tenir compte de certaines régularités qui échappent au MEDAF. Toutefois, étant donné le nombre de facteurs qui pourraient être inclus dans un modèle, le choix des facteurs pertinents reste une question ouverte (Fama and French, 2018a). De plus, le lien ou la corrélation entre les facteurs de risque pourrait avoir une incidence sur la validité des modèles (Kan and Robotti, 2012). Ainsi nous avons examiné la dépendance entre le marché et certains autres facteurs de risque et avons donné un nouvel aperçu des relations entre les facteurs de risque et celui de marché. Pour ce faire nous avons eu recours à la régression non paramétrique, à savoir l'Alternating Conditional Expectation (ACE) de Breiman and Friedman (1985). Cette approche permet d'identifier une relation optimale entre variables. Ainsi, nous avons montré qu'une transformation non-linéaire du facteur de marché permettait d'expliquer, et donc de prendre en compte, une part des autres facteurs de risque. En effet, nous avons montré que les facteurs de risque considérés notamment dans les modèles de Fama and French (1993, 2015) ou Carhart (1997), peuvent être en partie pris en compte de façon non-linéaire par le facteur de marché. L'ACE nous à permis de montrer qu'une transformation linéaire par morceau du facteur de marché, qui distingue les rendements positifs des rendements négatifs, s'avère optimale pour prendre en compte ces autres facteurs de risque. De plus, nos résultats montrent que lorsque l'on distingue sensibilités aux rendements positifs et négatifs du marché, des opportunités d'investissement peuvent être exploitées, à l'image de la stratégie bet-aqainst-beta de Frazzini and Pedersen (2014), pour permettre à un investisseur d'obtenir des rendements anormaux économiquement significatifs.

Cependant, même si nos résultats suggèrent une meilleure définition de la relation entre le marché et les autres facteurs de risques usuels, nous pouvons évoquer certaines pistes d'amélioration. En effet, dans notre essai, un nombre limité de facteurs de risque a été considéré et nous avons seulement cherché à établir l'existence de relations non-linéaires entre le marché et ces autres facteurs de risque. Cette approche peut paraître excessivement réductrice dans la mesure où elle sous-entend que le marché constitue l'unique variable d'état explicative des rendements attendus. Il serait donc pertinent d'étendre notre étude pour intégrer d'éventuelles relations non-linéaires entre certains des autres facteurs de risques.

Enfin, dans le troisième essai de cette thèse, nous confirmons l'impact de l'horizon sur le rendement des actifs. Dans un premier temps, en nous basant sur la règle de composition des rendements, nous avons étudié l'impact de l'horizon sur les moments d'ordre supérieurs des distributions des rentabilités sur le long terme. Nous avons montré que, conformément aux récentes études empiriques de Bessembinder (2018) et Fama and French (2018b), l'asymétrie des rendements devient systématiquement positive à long terme. Ce résultat complète la littérature en justifiant le fait que l'effet de la composition est le principal moteur expliquant le caractère positif de l'asymétrie qui peut, dès lors, être considéré comme un fait stylisé. Dans un deuxième temps, nous montrons que la prise en compte des moments d'ordre supérieur modifie qualitativement le comportement à long terme de l'allocation optimale entre un portefeuille risqué et l'actif sans risque. Il en est même pour la mesure de performance à long terme des actifs risqués. Nous avons montré que la prise en compte des moments d'ordre supérieur permettait de réconcilier les points de vue divergents sur allocation optimale d'actifs en fonction de l'horizon (Bodie, 1995; Thorley, 1995) comme sur la mesure de performance (Zakamouline and Koekebakker, 2009; van Binsbergen and Koijen, 2017). Toutefois, l'approche dynamique, plutôt que Buy and Hold qui a été considéré dans cet essai, est aussi utilisée dans la pratique. Ainsi la prise en compte de cette approche comme le recours à d'autres types d'actifs pourrait constituer des pistes de recherches futures.

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Asset Return Determinants: Risk Factors, Asymmetry and Horizon consideration

Resumé Les déterminants du rendement des actifs demeurent un sujet de recherche actif dans la littérature financière. Cette thèse s'intéresse au rôle de certains facteurs de risque, de l'asymétrie de la distribution des rendements et de l'horizon d'investissement comme déterminants des rendements d'actifs. Nous démontrons d'abord que l'effet de taille peut être considéré comme étant partiellement le fait de certains secteurs industriels jugés statistiquement pertinents pour expliquer spécifiquement la performance des portefeuilles constitués d'entreprises de petites (grandes) tailles puis nous en étudions les implications empiriques sur les modèles d'évaluation des actifs. Nous considérons, dans un deuxième temps, la relation entre le marché et les principaux facteurs de risque proposés dans la littérature – dont le facteur SMB qui prend explicitement en compte l'effet de taille – et soulignons que les facteurs considérés peuvent être partiellement expliqués par le facteur de marché de manière non-linéaire. En outre, nous montrons que l'exploitation de la relation non-linéaire entre le marché et ces facteurs de risque peut être profitable en termes de stratégies d'investissement. La dernière partie de cette thèse s'intéresse à la question de la diversification temporelle et analyse l'impact de l'horizon sur les propriétés de la distribution des rendements composés pour montrer que l'effet de composition est la raison principale de la forme des distributions de rendement à long terme. Nous apportons alors un nouvel éclairage permettant d'expliquer les divergences d'opinions exprimées dans la littérature quant aux stratégies de placement à suivre sur le long terme.

Mots clés : rendements d'actifs, effet de taille, marché, facteurs de risque, asymétrie, horizon, mesures de performance, allocation optimale.

Abstract The determinants of asset returns remain an active research topic in the financial literature. This thesis focuses on the role of certain risk factors, of the asymmetry of the distribution of returns and of the investment horizon as determinants of asset returns. We first demonstrate that the size effect can be considered partially due to specific industries that are considered statistically relevant to explain the performance of the portfolios of small (big) firms and we study the empirical implications of this finding in terms of asset pricing. We then consider the relationship between the market and the main risk factors proposed in the literature – including the factor SMB that explicitly accounts for the size effect – and point out that the considered factors can be partially explained by a non-linear relation with the market factor. In addition, we show that exploiting the non-linear relationship between the market and these risk factors can be profitable in terms of investment strategies. The last part of this thesis focuses on the issue of time diversification and analyses the impact of the horizon on the properties of the compounded return distributions to show that the compounding effect is the main reason for the shape of the long-term return distributions. We then shed new light on the divergences of opinion expressed in the literature regarding long-term investment strategies.

Keywords : asset return, size effect, risk factors, asymmetry, horizon, performance measurement, optimal allocation.