Sustainable metal extraction, steady-state good production and cooperative wealth allocation among nations and generations : a transdisciplinary approach

Fatma Zahra Rostom

UNIVERSITÉ PARIS I PANTHÉON SORBONNE
UFR D'ÉCONOMIE
Laboratoire de rattachement : Centre d'Économie de la Sorbonne

THÈSE
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Fatma Zahra ROSTOM

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Sous la direction de M. Gaël Giraud
Directeur de Recherche CNRS, Centre d'Économie de la Sorbonne, co-directeur de la Chaire Énergie et Prospérité
Et de M. Olivier Vidal (co-directeur)
Directeur de Recherche CNRS, ISTerre, Université Grenoble Alpes

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Professeur, University College London

M. Mouez Fodha
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Mme. Céline Guivarch
Directrice de Recherche, UMR CIRED, École des Ponts ParisTech
Ingénieur en Chef des Ponts Eaux et Forêts

M. Francesco Ricci, *Rapporteur*
Professeur, Université de Montpellier

M. Stéphane Zuber
Directeur de Recherche CNRS, Centre d'Économie de la Sorbonne, Université Paris I Panthéon Sorbonne

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Jadis, je pensais le monde académique terne et dédaigneux, et j'imaginais les Docteurs de l'Université en longue toge noire, se regardant (le nombril) du bout de leurs lunettes pincées, l'auriculaire soulevé. Il m'a fallu cinq années et des brouettes pour les observer et en conclure que finalement, un nombre non négligeable de ces spécimens du genre *Homo* se promènent également, comme d'autres, avec leur âme d'enfant rayonnant dans leur poitrine. Cette thèse a été une très belle promenade au bord du fleuve du temps, mais je ne serai jamais arrivée à l'estuaire sans le soutien et l'amour de mes congénères. Le moment est venu de vous remercier et, puisque cette thèse m'a fait réfléchir très fort – si fort que mes cheveux sont devenus fous – au lien entre les générations, j'ai voulu faire des remerciements générationnels – ils décideront peut-être enfin d'obéir à la pesanteur. En tunisien, remercier un semblable en lui disant عيشك revient littéralement à lui souhaiter une longue vie, puisque rien n'est plus précieux que le temps, seule ressource sans nul doute épuisable dans une vie. Par conséquent, si vous voulez une petite rallonge, il vous faudra apprendre le tunisien. Pour vous aidez – ma bonté est quant à elle sans limite – j'ai écrit vos noms en tunisien – ce qui veut dire que vous apprendrez un tunisien très personnel. Le reste est en français, parce que je souhaite aussi très longue vie à la France, ma terre d'adoption. Je souhaite par contre une très courte vie à ses frontières.

Alors, longue vie aux générations d'avant, celles qui ont peut-être connu un monde sans Google, sans Starbucks à chaque coin de rue, sans drônes, sans GIEC. Je souhaite d'abord très longue vie à mes deux guideurs de thèse, l'éclec(sias)tique غائال et le gé(nia)ologique أوليفيا. Très solennellement longue vie aux membres de mon jury qui ont offert beaucoup de leur temps : les rapporteurs رايموند et فرنشاسكو, معز qui a suivi l'évolution de mon travail tout au long de ma thèse, et enfin سالين et ستافان que j'admire beaucoup. Interminable vie sans arbre à صندري, et avec à كزافيا, et toute la bande d'énergéticiens, pour m'avoir toujours encouragée à poursuivre. Longue vie à لويك, جون كريستوف, أسامة, أمانوال et كرستين qui font comme tant d'autres intrépides administratifs un travail de fourmi géante pour permettre aux profs de se la couler douce, et aussi à آدريان et أنتونان pour leurs conseils au début de ma thèse. Longue vie à فرانصوى et ساسيل, sans qui je n'aurais pas rencontré Olivier et Gaël, respectivement, ainsi que tous mes profs pour leurs enseignements précieux, et mes maîtres de stage pour la confiance qu'ils m'ont accordée. Longue vie à mes chargés de cours, qui ont bien voulu que je plante des graines dans les esprits de mes étudiants. Pas besoin de souhaiter une longue vie à جوال, l'Immortel Dieu de la MSE.

Une éternelle vie à ma maman بلقيص, qui a toujours été là pour tout donner, y compris des menaces de coups de sabots pour que je fasse mes devoirs, je n'en serais pas là sans ça. Une aussi longue pour toutes mes autres mamans : la kalaa-grandoise شلبية, la turquoise طاطا, la marsoise راجى et la sétoise لولو. Et bien sûr une perpétuelle vie à mon unique papa شكري, pour m'avoir légué son goût pour l'aventure et son amour des grands éclats de rire. Longue vie à toute la عائلة, pour m'avoir soutenue et encouragée, et en particulier à هند et son amie pour la relecture de mes calculs infernaux. Spéciale dédicace à مارتين pour les cafés du matin, et surtout à ses oreilles qui n'ont pas trop saignées pendant nos cours de chant, et aussi à أمانوال et إيزور pour avoir fait vivre des moments princiers à de valeureux vétérans. Enfin, très longue vie à كندالى, que votre talent à sonder l'âme et l'esprit puisse encore guérir de nombreux étudiants.

A ma génération, longue vie ! Je commence avec le merveilleux mathémato-écono-pouète ستافان, et sa non moins merveilleuse كلار. Sans vos lumières, je ne serai jamais arrivée à bon port. Très longue vie également à l'autre couple de ma vie, شرين et حاطم, pour votre amour bien sûr, et parce que vous maîtrisez deux de mes grandes peurs, le feu et la machine. Longue vie au Tronc, mort à la CES, et vive la caravane ! Cette fois, pas besoin de business plan صابر. Ensuite, longue, très longue vie à دانيال et أمانوال (et كلامنس qui supporte ce dernier) pour leurs encouragements et leur aide précieuse avec le chapitre 2 qui m'en a fait baver un océan. Le beau Cyrano آدوار m'a aussi bien aidé avec mes équilibres pleins de panache, مالو avec son cours de compta, ainsi que le fanclub du cuivre, آدريان et le licornesque سيريل. Très longue également à tous mes relecteurs intrépides : d'abord mes colocs d'amour et de toujours, comment faire le poids devant tant de beauté et de cœur, هادريان et آكسال (et يائال qui le fait rire), les gas, je vous aime d'amour ; ensuite au meilleur ex-voisin, le très sexy-bearded أورليان, à ma meilleure amie de la Tunisie فريال, à ma meilleure amie de la France مريان, à ma meilleure amie de l'Éthiopie شامى et enfin au meilleur cousin مهدي. J'en profite pour faire une spéciale dédicace à tous mes zincous du monde ! Longue vie alcoolisée à tous les poteaux de la MSE, parce que vous êtes la joie de ce bâtiment étrange. D'abord ceux du/que j'associe au cinquième avec qui j'ai fait mes premiers pas dans l'aventure, entre autres à ma double italienne فدريكى et tous ses vélos, à فلورون et ses blind-tests (et son frère magicien des cervicales), à ma prodigieuse sirène تائيز et nos délires existentiels, à mon frère capillaire لورانزو et son accent ravageur, à كزافيا et ses rollers, à نيكوس et son île magique, à كاتيا et ses blagues russes, à آنطوان et son mac, à كلامي et ses chaussures d'escalade, et enfin à la blonde la plus intelligente de l'univers mais aussi la plus belle ماطيلد (tu abuses meuf). Ensuite ceux du deuxième, entre autres mon bébé نيلس, mon musicien fou بياريك, ma sociologue préférée سوفي, mon innefficient du marché طومى et l'espionne iranienne la plus drôle de l'univers مونى. Une longue vie syndicalisée aux Doctorant.e.s Mobilisé.e.s, qui m'ont valeureusement défendue contre l'infâmie administrative, en particulier غيوم qui est toujours debout. Longue vie à tous ceux que j'ai oublié de citer aussi, je fais tout à la dernière minute les gas ! Je pense d'un coup à يارى et notre fanclub de Joan Robinson, longue vie à toi et à l'amour tuniso-libanais. Très longue vie à notre gang des doctorants musiciens, philosophes mais avant tout gourmands : كونتان et sa fondue, agapi mou سوفيى et sa rhubarbe, جاف et son hamburger what else ?, سيمونى qui a tout mangé mes sardines, et enfin mon futur ex-mari d'amour آنطوان et sa Normandie toute entière. Spéciale dédicace aux mineurs de Paris pour nos rires cataphiles, et surtout au gang des géologues de Lyon, j'ai tellement appris avec vous, ma bande de vadrouilleurs géozoophytophiles.

Heureusement, il n'y a pas que la thèse dans une thèse, et celle-ci m'a fait découvrir une des choses les plus merveilleuses qui soit, la musique. Une vie de géant à toi et aux tiens, أنيس, pour m'avoir ouvert cette porte que je pensais verrouillée à jamais. Une vie toute aussi grande à la crew de notre *Boat* rêvé, que chacune de nos journées soient baignées de sons magiques et de voiles féeriques. Spéciale dédicace à mes chers amis du Gamelan, l'instrument qui vous donne la réponse à La grande question sur la vie, l'univers, et le reste. Et puis bien sûr, vive la danse ! La deuxième chose merveilleuse que j'ai découverte pendant cette thèse, c'est planter des petits pois, entre autres miracles de la nature. Et pour ça, ma reconnaissance éternelle aux super-héros paysans ألويز et كلامون, qui personnifient pour moi le courage, et qui ont fait d'une idée vague une réalité et un exemple à suivre. Encore une spéciale dédicace aux non moins courageux fermiers de Villetaneuse, je prie Tanit que cessent les manigances politiques et que vous puissiez continuer à semer l'espoir au fil des saisons blanches et sèches.

Ma thèse m'a aussi fait réfléchir très fort à la notion de foyer. Et si je considère que la Terre entière est le foyer de tous, question d'échelle, il n'empêche qu'il y a des endroits où le feu est plus chaud, où les personnes sont plus proches. Longue vie d'abord à mes précieux شيماء et فاروق, qu'elle vous fasse vivre plein d'aventures. Longue vie aussi à كوكوت et بوس, mes amitiés les plus vieilles, ainsi qu'à باديس et مهدي فيليب , mes amitiés les plus délurées. Comme ce qui compte, ce n'est pas la taille, et pas non plus la quantité, merveilleuse vie à toi عمبر, mon oiseau-chat des îles qui m'a emmenée, à bord d'un tapis de plumes volant, découvrir les mondes enchantés par-delà la Ville, puis m'a déposée dans la belle maison de St-Leu-La-Forêt. Là est mon actuel foyer, avec ses habitants : مايى la bleue, ويلو la grise, نومى le noir-cuivré, ناوكي qui-vous-fait-fondre et ديفو le presque-blanc. Je vous souhaite une chatte de vie ! Enfin, une resplendissante vie à آلكصندرو, qui illumine mes jours et mes nuits de ses rires et mélodies. Quelque soit l'île où on ira, je me sentirai chez moi avec toi.

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Aux générations qui arrivent, bienvenue et courage pour ce qui vient ! Vous serez, c’est mon espoir, plus informés et intelligents que nous. C’est en tout cas ce que j’ai pu observer car durant ma thèse, j’ai adopté plusieurs petits humains – chut, leurs parents vont être jaloux. Longue vie à mes apprentis sorciers stéphanois روبان et شرلين, coulannais ليون, زالي et جوليوس, crétois نفالي et saintlupien بول. Longue vie aussi à tous mes Petits Débrouillards, vous êtes merveilleux ! Longue vie à mon petit bout de cuivre aussi, laissez-en au moins assez pour la liberté sans hormone, les terriens. Pour finir, longue vie à mon vaillant petit عود qui va bien se promener désormais, pour ta tendresse dans mes moments de mélancolie et la sérénité que ton espèce apporte à nous autres, pris dans une course folle.

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Fatma Zahra Rostom
ED 465,
Centre d'Économie de la Sorbonne,
Université Paris I Panthéon-Sorbonne
Chaire Énergie et Prospérité

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Contents

This thesis contains four separate articles in English :

- *Global Trends in Metal Consumption and Supply: The Raw Material-Energy Nexus* published in *Elements* in 2017. This article is co-authored by Olivier Vidal [1], Cyril François, Gaël Giraud [2] and myself.
- *Prey-Predator Long-Term Modelling of Copper Reserves, Production, Recycling, Price and Cost of Production* published in *Environmental Science and Technology* in 2019, by the same co-authors.
- *Steady-State Extraction and Production Growth in a Goodwin-Class Model* co-authored by Cheryn Ali [3], Gaël Giraud and myself.
- *A Dynamic Cooperative Game for International and Intergenerational Allocation with Exhaustible Natural Resources* co-authored by Stéphane Gonzalez[4] and myself.

A **summary in French** is first presented, a **detailed introduction in English** is further available. **The first chapter** regroups the first two articles and focuses on the mining sector. **The next two chapters** correspond to the other two articles.

[1]CNRS - ISTerre, Université Grenoble Alpes

[2]CNRS - Centre d'Économie de la Sorbonne, Université Paris 1 Panthéon Sorbonne - Chaire Énergie et Prospérité

[3]Bio-Informatics and Systems Biology Master student at Université Paul Sabatier, Toulouse

[4]Université Lyon, UJM Saint-Etienne, CNRS, GATE L-SE UMR 5824

Résumé

Extraction Durable des Métaux, Production Stationnaire des Biens et Allocation Coopérative des Richesses : une Approche Transdisciplinaire de la Modélisation

Les matières premières minérales constituent la substance des biens que nous consommons et des infrastructures que nous utilisons. Ces richesses naturelles sont sources et conditions du développement matériel des sociétés, que plusieurs indicateurs nous permettent de mesurer. Le plus usité est le Produit Intérieur Brut ou PIB. Ce dernier a connu une progression extrêmement rapide ces cinquante dernières années, traduisant une amélioration significative du niveau de vie mondial moyen, qui s'est cependant accompagnée de dégradations alarmantes des milieux naturels. Si les pressions environnementales les plus importantes sont liées au changement climatique, il faut toutefois également citer l'effet des pollutions locales ainsi que les destructions d'habitats naturels qui font aujourd'hui craindre aux scientifiques une possible sixième extinction de masse liée à l'activité anthropique. La notion de capacité de charge de notre planète prend quant à elle non seulement en compte les dommages liés au développement d'une espèce, mais également les limites des ressources qu'elle puise. Le débat public autour de la question de la compatibilité entre la croissance en volume (PIB) et cette capacité de charge s'est intensifié, et la « croissance verte » est devenue le *Leitmotiv* de nombreuses études, ainsi que l'objectif de plusieurs organisations internationales[5].

Le développement matériel des sociétés modernes est directement lié à une consommation exponentielle des ressources non renouvelables (voir Fig. 1). La première des ressources qui vient à l'esprit est communément « l'or noir », c'est-à-dire le pétrole, l'accès aux matériaux, et plus spécifiquement aux métaux, pouvant paraître moins cruciale. Pourtant, la question de leur approvisionnement est aujourd'hui sujette, pour les décideurs[6] comme pour les industriels[7],

[5] Par exemple, l'objectif 8 de développement durable des Nations Unies.

[6] La Commission Européenne a publié un rapport conséquent sur les métaux critiques (Source : Ad-Hoc Working Group On Defining Critical Raw Materials, *Critical raw materials for the EU*, 2010), et les États-Unis ont publié un « American Minerals Security Act » en mai 2019 (Source : *https ://www.reuters.com/article/us-usa-minerals-electric/u-s-senate-moves-forward-on-plan-to-develop-electric-vehicle-supply-chain-idUSKCN1SK0L7*)

[7] Voir par exemple le rapport PwC (Source : PwC, *Minerals and metals scarcity in manufacturing : the ticking timebomb*, 2011) sur la perception de la rareté des ressources par les directeurs de grandes entreprises, ou

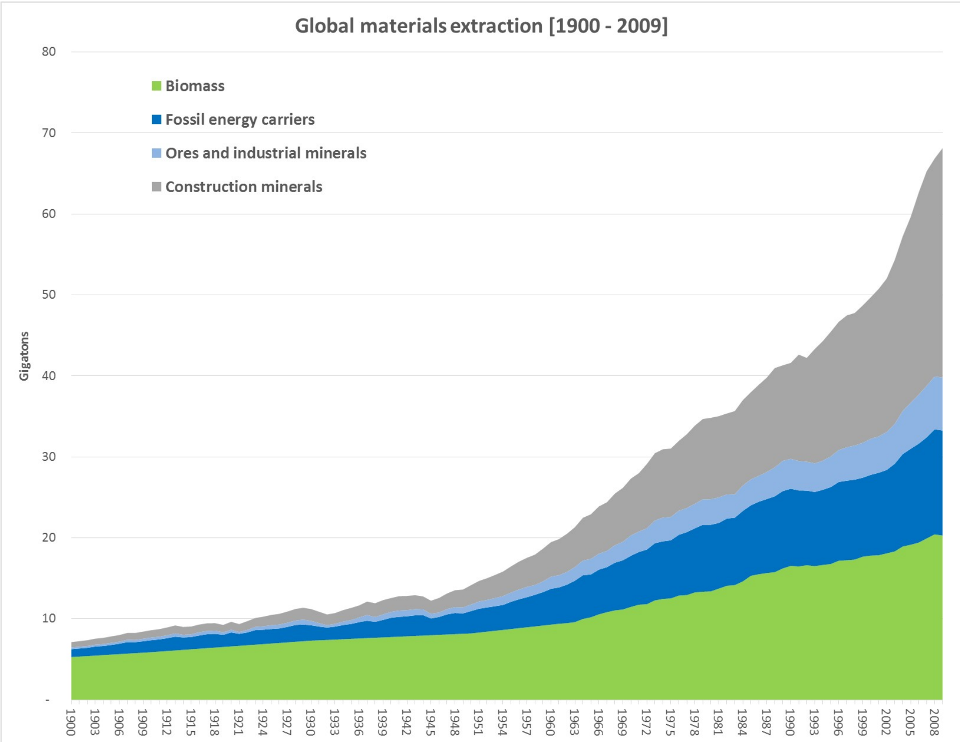

FIGURE 1 : Evolution de l'extraction des matières premières minérales depuis 1900 (Source : Krausmann et al., *Growth in global materials use, GDP and population during the 20th century*, Ecological economics, 68 (2009), pp. 2696-2705).

à des préoccupations grandissantes, quoique concernant des risques à court terme. La disponibilité au long terme de ces ressources non renouvelables est quant à elle débattue au sein de la communauté académique, qui reste divisée sur ce sujet. Contrairement aux questions de pressions environnementales citées précédemment, la possibilité d'une déplétion des ressources non renouvelables et ses conséquences sur le développement ne font pas l'objet d'un consensus clair, ni du côté des sciences de la Terre, ni de celui des sciences économiques.

Cette thèse examine d'abord les raisons des divisions sur ce sujet, entre et au sein des deux communautés. En combinant des connaissances géologiques sur la distribution de ces métaux dans la croûte terrestre, des données techniques sur l'industrie minière ainsi que la modélisation d'indicateurs économiques, elle analyse la plausibilité d'une menace de déplétion de certains métaux clés avant la fin du siècle. Elle explore ensuite ses possibles conséquences en termes de croissance à long terme et d'allocation internationale et intergénérationelle des richesses, fruits de l'extraction.

Nous adoptons pour cela le point de vue du *Vaisseau Terre*[8]. La planète y est considérée dans son ensemble, c'est-à-dire comme un système fermé où ses habitants extraient, transforment et échangent des ressources. Dès lors, les questions liées à la durabilité de ce système peuvent se formuler de la manière suivante :

- Les connaissances que nous avons sur les processus de formation de la Terre peuvent-elles nous éclairer sur l'existence d'éventuelles « limites planétaires » liées à l'épuisement des ressources non renouvelables ?
- Les sociétés actuelles peuvent-elles poursuivre leur développement matériel (mesuré par le PIB) au vu de ces limites planétaires ?
- Qu'impliquent ces limites planétaires en matière de gouvernance mondiale ?

Cette thèse propose une approche transdisciplinaire visant à nourrir le débat suscité par ces questions, ce qui nécessite une attention particulière à la définition du temps de l'étude. La notion de temps diverge en effet fortement entre les disciplines de la Géologie et de l'Économie. Pour cette dernière, la notion de long terme se réduit à l'échelle d'une vie humaine. Sur les marchés financiers, les obligations durent en moyenne dix ans, celles dites de long terme durent environ 30 ans, quelques rares exceptions allant jusqu'à 100 ans. L'échelle des « temps géologiques » est d'une tout autre mesure, la Terre ayant environ 4,55 milliards d'années. Il est important d'avoir ces ordres de grandeur à l'esprit. Les trois chapitres de cette thèse intègrent le concept de temps à l'aide d'outils de modélisation dynamique appliqués différemment selon les questions posées et les données disponibles. Le premier chapitre disposant de séries de données temporelles très longues, il est possible d'y quantifier l'unité de temps. En revanche, les deuxième et troisième chapitres ne considèrent le temps que théoriquement, à la différence près que le troisième chapitre ne se préoccupe pas de comment le long terme *sera*, mais de comment il *devrait être*. Examinons maintenant plus en détail les questions précises posées par chaque chapitre, la méthodologie développée pour y répondre, ainsi que leur contribution respective à la littérature.

la récente annonce de Tesla sur le lithium : *https ://www.reuters.com/article/usa-lithium-electric-tesla-exclusive-int/exclusive-tesla-expects-global-shortage-of-electric-vehicle-battery-minerals-sources-idUSKCN1S81QI.*

[8]Source : Kenneth E. Boulding, *The economics of the coming spaceship earth*, in Sixth Re- sources for the Future Forum on Environmental Quality in a Growing Economy, 1966.

Le premier chapitre est composé de deux articles se focalisant sur le secteur minier à l'échelle mondiale et traitant de l'offre et de la demande des matériaux d'origine fossile, ainsi que du fonctionnement de ce secteur.

Le premier article retrace l'évolution longue de l'approvisionnement en différents matériaux de base comme l'acier, l'aluminium, le cuivre, le zinc et de nombreuses Terres Rares. Un aspect crucial de leur disponibilité est l'interdépendance entre les secteurs miniers et énergétiques. En effet, la demande pour ces matériaux dépend du type d'infrastructure énergétique déployée. Si les énergies dites renouvelables sont souvent décrites comme étant des sources « propres » du point de vue des émissions de gaz à effet de serre, elles sont bien plus intensives en matériaux que celles issues de sources fossiles. Ce premier article traite de l'impact de différents scénarios mondiaux de transition énergétique sur la consommation de matériaux. Il montre que la consommation passée a suivi une tendance exponentielle depuis le début du siècle dernier, et augmenterait d'autant plus dans le futur si l'ensemble du système énergétique reposait sur des énergies renouvelables. L'autre pendant de l'interconnexion matériaux-énergie comprend les besoins énergétiques du secteur minier nécessaires à l'extraction des éléments des gisements et à l'obtention des concentrés. Nous montrons qu'il existe une relation empirique entre la dilution des éléments dans les gisements et l'énergie nécessaire à leur extraction. Cette relation se retrouve également avec les prix, possiblement par le canal des coûts.

Le deuxième article s'appuie sur l'exemple du cuivre pour conceptualiser le secteur minier d'un point de vue géologique et économique. Ce métal structurel possède en effet des caractéristiques physico-chimiques particulières, qui en font un des métaux les plus importants et les moins remplaçables des économies passées et présentes. Le but de cet article est de savoir si le taux d'extraction observé actuellement peut être soutenu jusqu'à la fin de ce siècle, et dans le cas contraire, quelles seraient les conditions requises pour un maintien stable de l'extraction. Pour ce faire, nous proposons un modèle assez simple pour décrire les stocks et les flux de cuivre disponibles et extraits, ainsi que ceux de variables économiques comme les profits du secteur minier. Nous modélisons par exemple l'évolution des réserves en cuivre. Contrairement à de nombreuses études prédisant un pic de cuivre et se fondant sur la théorie de Hubbert[9], nous ne considérons pas ici les estimations actuelles de ressources disponibles comme une limite absolue, dans la mesure où de nouveaux gisements sont découverts et où la diminution des concentrations permise par l'amélioration technologique conduit à une augmentation continuelle des stocks estimés. La rareté n'est plus alors une question d'épuisement à proprement parler, mais une course entre l'augmentation des réserves et l'extraction. Nous modélisons également la « richesse » du secteur minier, correspondant à ses profits cumulés. Il s'agit de la capacité productive de ce secteur, c'est-à-dire de l'infrastructure déployée pour transformer le cuivre d'un état de gisement à un concentré délivré sur le marché. Ce modèle, inspiré de la dynamique des populations de proies et de prédateurs, a été calibré par de nombreuses séries de données et utilisé afin de simuler des scénarios de production de cuivre. Nos résultats suggèrent que la consommation exponentielle de cuivre pourrait ne pas durer, même avec des taux de recyclage très élevés. Le modèle nous permet cependant de trouver des conditions nécessaires à l'établissement d'un plateau d'extraction plutôt qu'un pic, moyennant une stabilisation de la demande en cuivre. Si l'hypothèse d'une saturation de la consommation de cuivre pour des niveaux de développement élevés est correcte, alors une condition est que le PIB mondial se stabilise après 2100. Un effort important sera également à fournir en matière

[9] M. K. Hubbert, *Nuclear energy and the fossil fuel*, 1956.

de collecte et de recyclage : une deuxième condition est que le taux moyen atteigne 70% d'ici la fin du siècle, contre 40 % aujourd'hui.

Le second chapitre place le secteur minier dans l'économie globale. Ce travail de recherche a en effet pour but d'évaluer de manière théorique les conséquences à long terme de la dynamique de ce secteur sur la croissance de la production. Nous construisons pour cela un modèle de croissance stock-flux cohérent basé sur la dynamique proposée par Goodwin[10]. La rareté des ressources naturelles non renouvelables y est explicitée par le modèle proie-prédateur décrit précédemment : le secteur minier extrait des ressources pouvant être découvertes à une vitesse donnée, et les vend au secteur final qui produit des biens à partir de ces ressources, de la force de travail engagée et de sa capacité productive ou capital. Un des objectifs de ce chapitre est d'expliciter plus en détail la notion de « richesse » du secteur minier, qui a la possibilité de s'endetter pour financer son activité. Nous étudions ensuite analytiquement les équilibres de long terme de cette dynamique globale et identifions les conditions nécessaires à un équilibre désirable. Nous montrons que, dans cette configuration spécifique, un sentier désirable ne peut émerger que si la croissance à long terme est nulle : l'unique équilibre désirable à long terme est donc stationnaire. Nous montrons également que d'autres équilibres peuvent être atteints, dont un équilibre déflationniste où il n'y a plus de production ni d'emploi, et où la dette prend une valeur non finie. Dans ce deuxième chapitre, la production est donnée par une fonction de production CES qui permet d'évaluer la substituabilité entre les différents facteurs de production (capital, travail et ressources). Nous proposons d'estimer empiriquement les paramètres de cette fonction, grâce à un *proxy* de l'économie mondiale qui agrège 37 pays comptant pour 83% du PIB mondial moyen entre 1990 et 2017. Nous montrons que trois formes fonctionnelles sont envisageables et discutons des limites à l'utilisation d'une telle fonction pour modéliser la production.

Le troisième chapitre explore les implications induites par la rareté des ressources non renouvelables en matière de commerce et de coopération internationale. Nous cherchons ici à savoir si une stratégie de coopération à long terme pourrait être mise en place à l'échelle mondiale si les ressources et les capacités de production étaient mises en commun. Pour cela, nous nous plaçons dans le cadre d'un jeu coopératif où les pays peuvent former des coalitions afin d'optimiser leur consommation sur le long terme dans les limites de leur stock de ressources et de capital. Contrairement aux chapitres précédents, le stock de ressources est ici fini puisque nous nous intéressons à des générations infinies. Une coalition décide de son plan d'investissement et d'extraction, et en déduit les quantités de ressources qu'elle souhaite échanger à l'intérieur de ses frontières. Elle distribue ensuite les biens de consommation produits entre ses membres. Notons que ni la forme fonctionnelle de la fonction de production, ni la détermination des prix, n'ont besoin d'être explicitées. De plus, le contrat n'est pas exécutoire dans ce jeu coopératif : chaque membre peut décider de rompre une alliance si elle n'est pas à son avantage. Nous recherchons donc la coalition la plus efficace et la plus stable, qui pourra allouer les biens de consommation produits sur le court terme tout en garantissant une consommation suffisante aux générations suivantes. Pour cela, nous utilisons le concept de « cœur récursif », qui satisfait des propriétés de stabilité coalitionelle et de consistance temporelle. Nous montrons que cet ensemble contient au moins deux éléments. En d'autres termes, si les pays membres d'une coalition suivent une stratégie de coopération pour leurs choix d'investissement, d'extraction et d'échange de ressources, alors la meilleure manière d'optimiser leur propre consommation

[10]Richard M. Goodwin. *A growth cycle. In Socialism, Capitalism and Growth*, C.H. Feinstein (ed.). Cambridge University Press, Cambridge, 1967

tout en se souciant des générations futures est de former une coalition internationale. Le cœur récursif est composé des sentiers de consommation à allouer à chaque pays de cette coalition mondiale. Puisqu'il n'est pas réduit à un singleton, il sera possible de faire un choix entre toutes les allocations envisageables. Ce choix pourra se faire par exemple en suivant certains critères de justice. Le présent modèle peut donc être vu comme un outil permettant de porter un regard nouveau sur notre manière d'appréhender l'opposition Nord-Sud. Il pose un cadre pour l'exploration d'un partage juste des fruits de la croissance économique mondiale.

Cette thèse conclut que l'extraction du cuivre, l'un des métaux essentiels au développement matériel des sociétés, ne pourra pas se poursuivre au taux observé aujourd'hui. Après avoir exploré les conséquences de cet argument sur la croissance à long terme, elle infirme la possibilité d'un équilibre de long terme désirable avec une croissance positive, dans la configuration proposée. Elle propose enfin une nouvelle manière de réfléchir au commerce dans un contexte de ressources finies, et montre qu'une coalition mondiale où les richesses naturelles et productives des pays sont mises en commun peut être à l'avantage de tous.

Introduction

There is perhaps no better demonstration of the folly of human conceits than this distant image of our tiny world. To me, it underscores our responsibility to deal more kindly with one another, and to preserve and cherish the pale blue dot, the only home we've ever known.

— Carl Sagan, *Pale Blue Dot*

Mineral materials constitute the structural basis of the goods we produce and the infrastructures we use. This natural wealth is a source and condition to the material development of societies. The latter can be measured, or at least approximated, by various indicators. Population growth is the most obvious of them, and the most used nowadays is the Gross Domestic Product, or GDP. The last half-century has witnessed an extremely rapid progression of both indicators at the world scale, together with significant improvements in living standards. However, other features have accompanied this evolution, the most prominent being global warming. The average global temperature has increased by about 1°C above pre-industrial levels. A significant part of the warming has occurred since 1975, and it currenlty increases at a rate of roughly 0.20°C per decade (Intergovernmental Panel on Climate Change, 2018), which is mostly explained by the exponential consumption of fossil energy (10 EJ/year in 1880, 70 EJ/year in 1950 and 450 EJ/year in 2010 (Court, 2016)). Society is indeed facing, at the dawn of this third millennium, increasing environmental pressure. Over the last decades, a consensus among natural scientists has emerged, raising concerns about the possibly irreversible damages caused by anthropic activity. The environmental threats include, of course, the consequences of climate change, but also the various forms of pollution and the destruction of species' habitat, leading to reversible or permanent biodiversity losses. As a consequence, the public debate around the long-lasting compatibility between growth in volume and the planet's carrying capacity has intensified. "Green growth" or "inclusive and sustainable growth", as the United Nations puts it[11] , has become the *Leitmotiv* of many studies and the objective of many international institutions.

The concept of carrying capacity of the planet also encompasses the natural sources we draw, and material development is directly linked to the exponential consumption of non-renewable resources (as an illustration, see Fig. 2). Of course, modern society's dependency on

[11] Sustainable Development Goal Number 8

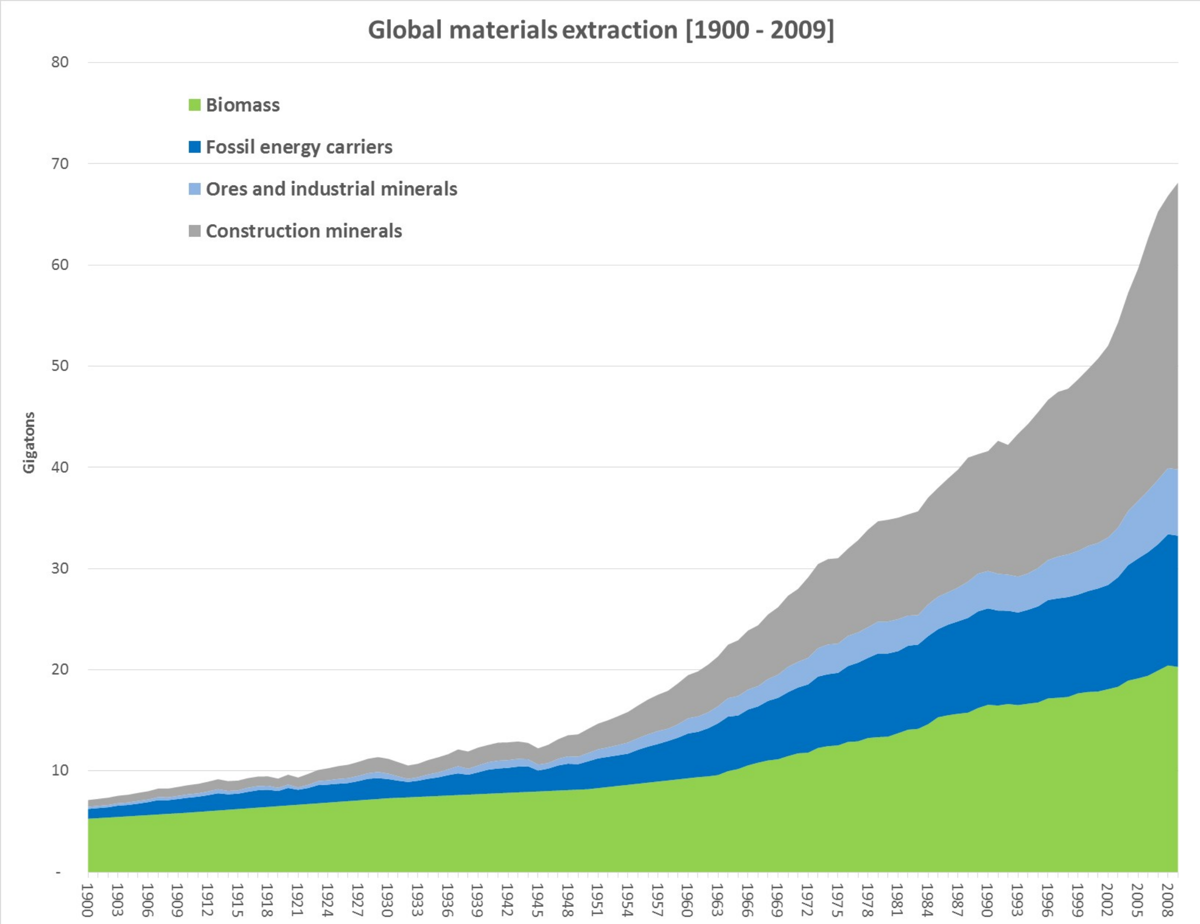

Figure 2: Long-term trends in global mineral extraction (from (Krausmann et al., 2009))

fossil fuels or 'black gold' first comes to mind, and fossil materials are regarded with less attention. Though, the question of their limitation has always been a matter of concern throughout modern history. A careful attention to the current public debate shows that it is still seriously considered, be it by decision makers for geostrategic purposes[12] or by the industry, the first to be concerned by a supply shortage in materials[13]. But these concerns only consider short-term risks. The long-run availability of these resources is debated within the academic community, who remains divided on that matter. Unlike the environmental threats cited above, no clear consensus on a plausible depletion threat and its possible consequences on development has emerged neither from Earth sciences, nor from Economics.

This dissertation analyzes the reasons of this divide in both communities. By combining the specific knowledge of Earth sciences on the distribution of metals in the Earth's crust, technical information on the evolution of mined ore grades and the modeling of economic variables, this work investigates the plausibility of a depletion threat of key metal resources within the current century. It further explores the consequences this would imply in terms of global long-term growth and international allocation of wealth.

[12]The European Commission (Ad-Hoc Working Group on Defining Critical Raw Materials, 2010) published an interesting report on criticality, and the U.S. introduced a new American Minerals Security Act in May, 2019.

[13]See for instance the PwC survey (PwC, 2011) on how resource scarcity is perceived by senior executives.

1 Are Metal Resources Exhaustible? On Geologic Scarcity and Economic Abundance

1.1 Geology of Mineral Resources

Natural resources can be classified according to the time needed for their renewal. Renewable resources are the resources that can be replenished on a human timescale. They include food, air, and some sources of energies, as well as some structural materials such as wood, fibers and so on. Non-renewable or fossil resources can be replenished only on a geological timescale and include fossil materials (metals, sand and other construction materials) and energy sources (coal, oil, gas and nuclear). The latter are left apart here because they are consumed in the production process and are partial substitutes to renewable energies.

An ore can be defined as "a naturally occuring solid material containing a useful commodity that can be extracted at a profit" (Arndt and Ganino, 2012). An ore deposit is therefore an accumulation of this commodity that is large and concentrated enough to render the extraction profitable. Note that this definition already encompasses geologic and economic notions. Recall that apart from its main components, the content of most elements in the earth's crust is very small: 0.007 % of copper for instance. The grade and the tonnage of a rock are the principal characteristics used to describe an ore. The grade is the content by weight of an element in an ore, while the tonnage is the total mass of rock. It is established that the relationship between the grade and tonnage of different rock types is lognormal (Ahrens, 1954). This distribution peaks at very low grades which correspond to common rocks, the tonnage of currently mined ores being at the tail of the distribution. Geologists define the mineralogical barrier as the jump in energy that separates the ore from the common rocks. This energy is necessary to extract the element's concentrate from the gangue made of an "enclosing mixture of valueless silicate minerals" (Gordon et al., 1987). For copper, this barrier is established at 0.1 %. It should be distinguished from the mine-specific cut-off grade identified by the mining industry that reflects the ore's profitability limit.

Observe that as far as mining is concerned, the geological and economic notions are entwined. Yet, the natural processes needed to obtain an ore are purely of geological nature: the process of concentration is key and induces world geographical disparities (Fig. 3). The modern genetic classification of ore deposits is based on the ore-forming process. The first class is magmatic: the ores form by accumulation of minerals that crystallize from magma. Copper ore in carbonatites belong to that class. A second important class includes the deposits associated with the circulation of hydrothermal fluids which mobilize the elements that further precipitate ; porphyry copper of Chile is one of them. The last major class of interest encompasses sedimentary deposits, including the copperbelt in sandstones in Central Africa.

Therefore, tectonic and geomorphological settings determine the location of deposits. This non-uniform distribution strongly influences metal prices and global trade, and "lies at the heart of many of the alliances and conflicts that govern relationships between countries around the world" (Arndt and Ganino, 2012).

1.2 Extraction and Society

Early stages of mankind are named after the materials from which tools and weapons were made of. For instance, the Bronze Age, which arose fully around 3000 BCE, was preceded by

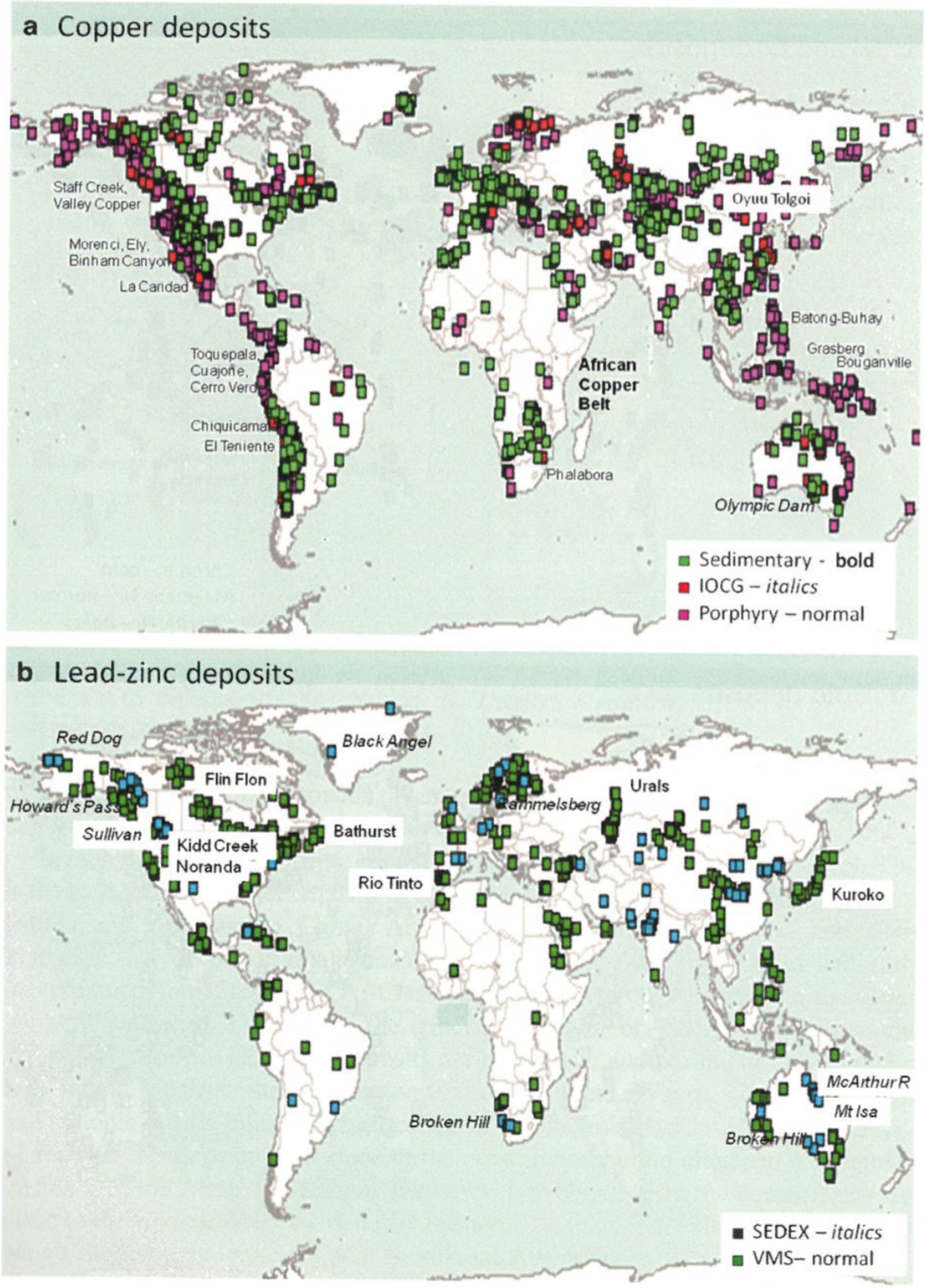

Figure 3: Copper and lead-zinc ore world distribution (from (Arndt and Ganino, 2012))

the Chalcolithic or Copper Age. The Near East was the “kernel of the Age of Metals", but was poorly endowed with these materials. The “valley urban societies" therefore had to exchange with the “barbarians" (in the technical archaeological sense), and the search for metals lead them to Oman, Pakistan or Cyprus among other regions. This often required sea shipments: the transport and exchange of commodities established “trading posts, and even colonies". The trade expanded from the Near East to Europe, mainly through Troy, by sea but also by land, with caravans carrying important loads (up to 12 tons of tin in one caravan). The “first city ‘civilization’ in Europe" was founded in Crete, the Minoan culture resulting “from this activity of exchange". Iron then progressively replaced bronze, as “difficulties in importing tin" appeared. Iron was locally available and already “known as a workable metal", but was much more labor-intensive, and therefore costly. Iron use increased significantly with the invention of its carburized form which lowers its melting point. It is therefore the mastery of the ore smelting process that resulted in large disparities between regions (Goody, 2012).

Modern extraction, trade and consumption are of much higher orders of magnitude and different structure. Mining techniques have completely changed in scale, even if they still rely on quite similar principles. The ore is hauled from open-pit or underground mines, then processed by milling, smelting and refining. Important improvements in technological efficiency have significantly lowered the grade that can be worked economically, which in turn increases the tonnage. New techniques have also been developed, such as *in-situ* leaching which consists in dissolving the ore directly in the mine. The concentrates are produced by a limited number of countries and are then available on the world market. Raw materials represented 22.5% of the world total merchandise trade in 2008 (WTO, 2010). The development of trade is necessary to meet an ever-growing demand. At the global scale, mineral resource consumption has increased exponentially over the last century, with growth rates spanning from 3% for copper, 5% for aluminum to 6% for cement and steel. Indeed, the first stages of industrialization are characterized by the construction of infrastructures in the sectors of heavy industry, housing, transport and communication, and in the sectors of production, transport and use of energy. This development phase consumes mainly ‘structural’ raw materials such as concrete, steel and iron, aluminium, copper, manganese, zinc, chromium, lead, titanium, and nickel. The industrialization of developing countries will therefore inevitably be associated with an increase in the consumption of raw materials, as this has been the case since the late 90’s with the rapid industrialization of China, which has doped the growing rate of global material extraction. In parallel to the building of their base infrastructure, most economies move towards high technologies. At the beginning of the 20th century, metal consumption was limited mainly to Fe, Cu, Pb, Zn, Ag and Au, which had the desired basic physical and chemical properties at this time. High technologies now require new raw materials and metals since they use many additional properties, including the electronic structure, the catalytic, quantum or semi-conductive properties specific to almost all elements of the periodic table.

1.3 Geological scarcity

The notion of scarcity is intrinsically linked to the natural processes that induce the concentration of elements in the Earth’s crust, but also to the technical conditions of extraction and its profitability. The question of the inventory and the quantitative assessment of what is available is subsequently raised. Indeed, several studies suggest that the future supply of raw materials will not match the demand since the amount of mineable fossil resources theoretically decreases with time, and production of several metals has either peaked already or will peak in a foreseeable future (Sverdrup et al., 2014). This classical modeling of fossil resources assumes that production follows a symmetric bell-shaped curve of normal distribution: it is the Hubbert peak theory (Hubbert, 1956), which relies on the assumption that the Ultimately Recoverable Resources (URR) are finite and quantifiable. It is true that the stock of fossil mineral resources on Earth is finite and not renewable at a human scale. However, the whole continental crust is composed of minerals that could eventually be extracted to produce metals. Improvements of technology allow new discoveries and the exploitation at economically viable costs of resources that were not exploitable with older technologies. It is the fundamental difference between *resources* and *reserves*, the latter being the amount of resources that can be extracted at viable cost, given the current state of technology. It follows that the reserves are larger now than they were 50 or 100 years ago. For this reason, the time before exhaustion of the known reserves at the current level of production has remained fairly constant since 70 years. These difficulties in the quantitative assessement are well illustrated by the case of copper: the URR values estimated in 2010 by calibrating logistic functions on the historical data of global production ranged from about 1 Gt (The oil drum: Europe, 2010) to 3.8 Gt (Frimmel and Müller, 2011). In 2010, the identified copper resources ranged from 1.1 Gt (Raw Materials Database) to 1.5 Gt (United State Geological Survey, USGS hereafter). Three years later, the USGS completed its geology-based assessment of global copper resources and proposed that about 3.5 Gt of undiscovered copper should be added to the 2.1 Gt identified resources (Johnson et al., 2014). USGS geologists subsequently published an article where they fiercely criticized the use of such data to predict future production (Meinert et al., 2016). More recently, Singer (Singer, 2017) has estimated that 4.35 Gt of copper were present in undiscovered mineral deposits, and Henckens et al. (Henckens et al., 2016) have reported an amount of extractable global resources equal to 7.5 Gt. Finally, based on geodynamic arguments, Kesler et al. (Kesler and Wilkinson, 2008) estimated that ultimate copper reserves in porphyry deposits could be as high as 1300 Gt, among which 89 Gt would be exploitable if mining in the future could reach depths of around 3.3 km. This non-exhaustive comparison of data published during the last decade clearly shows that URR estimated from geological constraints or from historical data of production span a huge range of values and increase with time. They only provide a crude estimate of the total amount of available copper and cannot be used to produce robust estimates of future production.

1.4 Economic scarcity

In the field of Economics, the notion of scarcity is often reflected in relative prices. The debate around this notion goes back to the controversy between Malthus and Ricardo, who had two different views on this question. Malthus postulated absolute limits to resources, while Ricardo looked at their decreasing quality (Hall and Hall, 1984). Hotelling's seminal paper (Hotelling, 1931), for instance, considered a Malthusian scarcity, since resources are supposed to have an ultimate limit. The reference work of Harold Barnett and Chandler Morse (Barnett and Morse, 1963), published in 1963, consisted in a measure of the Ricardian scarcity of minerals through their unit costs. The authors concluded that technological change pushes unit costs downwards, demonstrating the decreasing Ricardian scarcity of minerals.

The scarcity controversy continued though, and reflected an important philosophical break-up, between on one hand the so-called 'cornucopians', represented by Julian Simon notably, and the so-called 'doomsters' on the other hand, who counted among other advocates Paul

Ehrlich. At the end of the 70's, after a decade of fierce debates around the possibility for societies to reach the Earth's carrying capacity in a foreseeable future, a \$ 10,000 bet was published in the *Social Science Quaterly Journal*. Simon gambled on the decrease in price of a basket of 5 metals in ten years, as a proof of their diminishing scarcity. Ehrlich took the bet and lost ten years later. Had the bet been set on another ten-year period or had the basket contained more commodities, Ehrlich would have won. This anecdote illustrates why that question cannot be solved only by a price or cost analysis.

The mining sector corresponds to a small part of the economy, but metals constitute the bones and nerves of the infrastructure and are essential to development. The issue of their scarcity therefore raises an important concern on long term growth of production. The debate of the impact of a possible depletion on future development is explored in the next section.

2 Is Exhaustibility Compatible with a Sustained Growth of Production? On the Origins of a Divide in the Economics of Sustainability

2.1 Infinite or Limited Growth

Growth theory pioneer Robert Solow exposes in 1992 his idea of "heaven" as "an occasion when a piece of pretty economic theory turns out to suggest a program of empirical research and to have implications for the formulation of public policy" (Solow, (1992). His pretty theory became the bedrock of modern vision of economic growth. In 1956 (Solow, 1956) and during the following years, he published, along with Trevor Swan, his famous exogenous growth model. Production is described as a function of labor and capital. Agents save a given share of their revenues to invest. Investment accumulates into the stock of capital. Production can therefore grow in volume, at a rate given by technical change and population dynamics. A key point of this model is that technological advances compensate capital diminishing return effects. From the beginning, criticisms were addressed to the model for it disregarded the effect of land and exhaustible natural resources. Swan concluded that technological change was necessary to compensate diminishing returns of land (Swan, 1956). This theory provides the basic structure of neoclassical conception of growth. Growth theorists subsequently endogenized the exogenous variables of the model, giving birth to the endogenous growth theory (Romer, 1986) and the micro-founded growth theory (Cass, 1965; Koopmans et al., 1963), among the most famous research programs. Solow's model initiated a tremendous number of empirical research (Mankiw et al., 1992).

In 1972, Donella and Dennis Meadows and colleagues published their famous report (Meadows et al., 1972), whose conclusions were much more nuanced. Relying on non-linear dynamical systems, the MIT team exhibited scenarios leading to a worldwide collapse due to the exhaustion of non-renewable resources, as well as alternative narratives compatible with sustained growth. The world model used in this report was based on the emerging science of system dynamics, using computer simulations to better understand dynamic behaviors of the complex non-linear "Nature-Capital" system. The point was to determine "broad behavior modes" or the tendencies of global variables to change as time flows. The international success of this book led Robert Solow to organize a symposium on the economics of exhaustible resources in 1974 (Couix, 2019). There, Solow presented a new version of his 1956 seminal model (Solow, 1956), refining the workhorse production function with a flow of extracted resources. His conclusion, however, was that substitution between capital and resources would ensure a constant per-capita-consumption for future generations (Solow, 1974b). A similar conclusion was reached by Joseph Stiglitz in (Stiglitz, 1974b), where "resource-augmenting productivity" can overcome their exhaustibility. The debate, nevertheless, was not over as, in 1975 (Georgescu-Roegen, 1975) and 1979 (Georgescu-Roegen, 1979), these two models received a fierce criticism by Nicholas Georgescu-Roegen, who stressed that thermodynamical limits to efficiency might prevent progress in technology from compensating the exhaustion of non-renewable resources. This controversy led the discipline to split up between environmental and ecological economists. A confrontation was launched again in the 90's by Herman Daly (Daly, 1997) - prolonging Georgescu-Roegen's criticism - followed by responses from Solow (Solow, 1997) and Stiglitz (Stiglitz, 1997), but this renewed controversy did not lead to a consensus either, not even to a common theoretical ground. Environmental economists, for whom technological and natural capital are substitutes, recommend weak sustainability, whereas ecological economists defend strong sustainability, emphasizing that technology and nature are hardly substitutable. Each community has its own "heaven", leading to conflicting theories, empirical studies and policy implications.

2.2 The Specific Case of Metals

When it comes to applying non-renewable resource theories to metals, several points need to be taken into account.

First, it is important to distinguish structural or base metals such as steel, copper or aluminum from high-technology metals. Base metals are essential to the building of infrastructures and therefore to the basic functioning of economies. Their use depends on their specific physical and chemical properties. Copper, for instance, has a high electrical and thermal conductivity, resists well to corrosion and is the base of many alloys. In 2010, 42 % of copper is used in power grids and electronic appliances, 28 % in construction, 12 % in transport, 9 % in industrial machinery and the rest in other applications such as coins, medecines or fungicides (Arndt and Ganino, 2012). This feature constitutes an important limit to the substitution argument (see Fig. 4).

Of course, a lot can be done to increase metal efficiency. The UNEP's International Resource Panel has developed the concept of "decoupling" applied to material resource use (International Resource Panel, 2011). Not only technological innovation should permit a reduction of use, but also correct incentives towards policy makers, corporate leaders and consumers.

Moreover, in contrast to fossil fuels, primary metals are not lost when they are used and metal-bearing goods manufactured today are the reserves for tomorrow's recycling (see Fig. 5). Increasing the share of recycling is not sufficient to meet the demand in a period of growth because we can only recycle some of the consumer goods and equipment that were created several decades ago. In practice, the recycling potential is limited by economic factors, such as the difference between the price of metals and the cost of their recycling. Only metals present in sufficiently high concentrations in the end-of-life products are recycled, because the recycling cost of more diluted metals remains uncompetitive compared to primary production. In that sense, a future increase in price due to an eventual depletion of primary reserves would foster recycling.

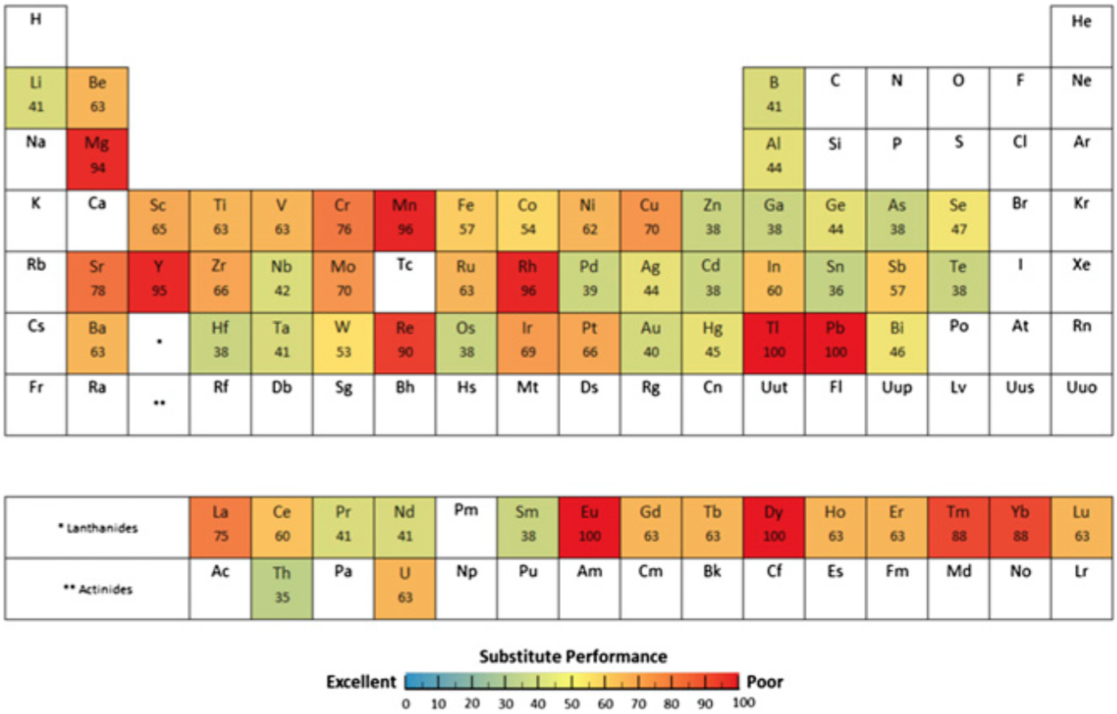

Figure 4: The peridiodic table of substitute performance (from (Graedel et al., 2013))

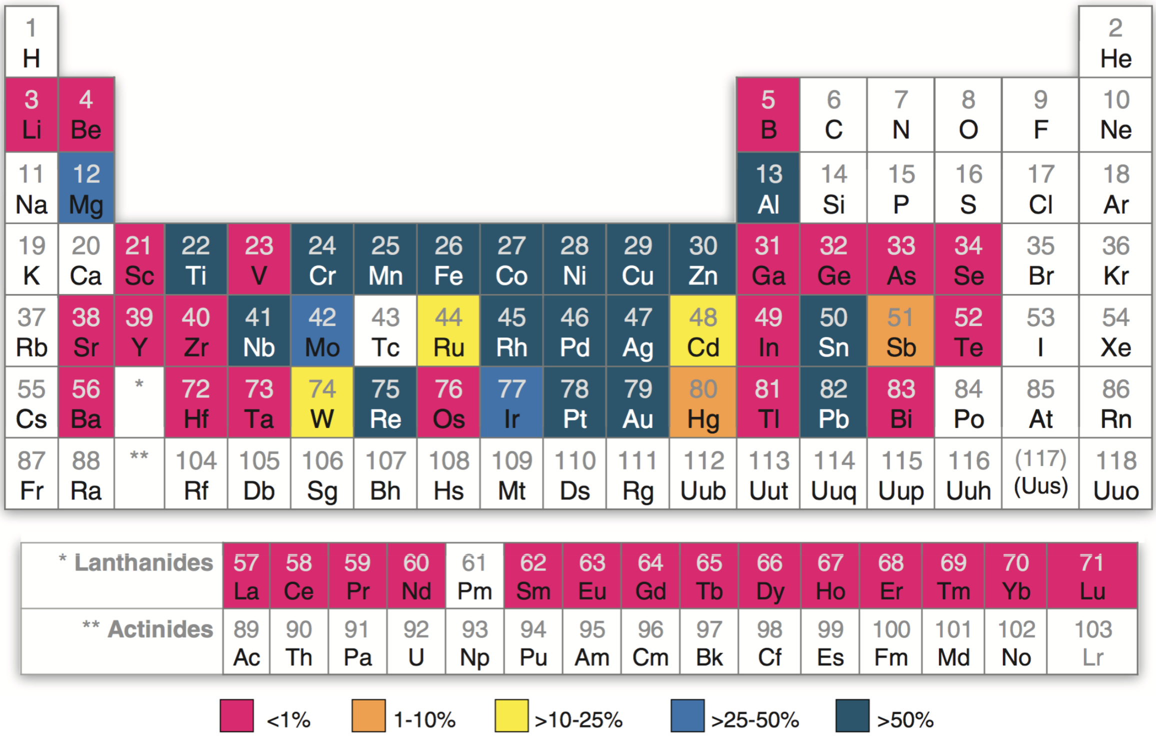

Figure 5: The peridiodic table of end-of-life recycling rates (from (Reck and Graedel, 2012))

Altogether, the overall stocks of all metals to be produced by 2050 and the metal flow into use in 2050 could reach 5 to 10 times the current levels (Graedel, 2011). This means that the cumulative amount of metals to be produced in the next 30 years would exceed the cumulative amount that has been produced until today. In 1987, two Nobel-awarded economists (Tjalling Koopmans and William Nordhaus) and two eminent geologists (Robert Gordon and Brian Skinner) published a book that has gone quite unnoticed (Gordon et al., 1987). The authors propose "a step toward resolving the debate between geologists ans economists" and an answer to "whether economic growth will exhaust our precious heritage of fuels and nonfuel minerals". They focus on copper through "materials modeling", considering copper's distribution and average grade, but also substitution and recycling possibilities. The results obtained for their base case suggest that the ores containing copper minerals could be mostly exhausted by 2070. They open their discussion on the assumption that the globe's economies would gradually rely on geochemically abundant metals and might enter a "New Iron Age". The authors do not discuss the differential impacts this "Age" would have between, on one hand, modern industrialized economies and, on the other, countries who still are at early stages of development.

3 Why do Exhaustion and Disparity require efficient and fair international strategies? On Intergenerational Inequity, Geographical Inequality and Trade

3.1 Generational Equity and Sustainability

According to Solow, the question raised by the possible depletion of non-renewable natural resources takes, at the "popular level", the following form: "how much of the world's - or a country's - endowment of nonrenewable resources is it fair for the current generation to use up, and how much should be left for generations to come who have no active voice in contemporary decisions?" (Solow, 1986). This formulation is "damaging" according to him, since it implies that the current generation "owes" a share of resources to the following generations, whereas it only owes a productive capacity. This shapes a particular definition of sustainability, that can be viewed as weak, since natural resources are viewed as pure instruments of wealth that can be combined to satisfy some needs. Therefore, and as long as these instruments of wealth are substitutes, the question of 'equity' converts into a question of pure 'efficiency' of resource allocation.

How to guarantee intergenerational justice in this view of sustainability? The choice of a 'fair' criterion is key, and intensively debated (Chichilnisky, 1996; Asheim, 2017). Solow originally focused on an egalitarian rule, based on the Rawlsian max-min principle, that "the standard of consumption should be achieved by the least well-off generation" (Solow, 1986). This criterion requires that consumption per-capita remains constant over time. He opposed the utilitarian norm introduced by the Ramsey model (Ramsey, 1928), for whom social performance is the sum over time of utilities of consumption. Optimal depletion issue is mostly tackled in the intertemporal allocation framework. A large number of studies are based on the DHSS growth model (Dasgupta and Heal, 1974; Solow, 1974a; Stiglitz, 1974a), but use either of these criteria.

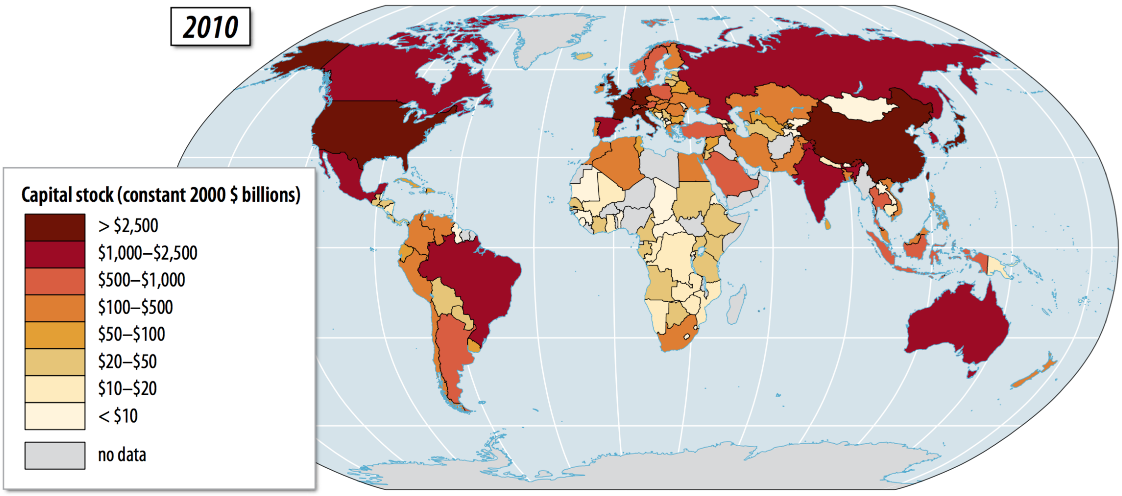

Figure 6: The distribution of capital stocks (from (World Bank, 2013))

The discounted-utility approach has become a convention for these sets of questions. Recall that in his original paper, Ramsey did not use a discount rate. He assumed instead that there was an upper-bound to the level of "enjoyment" attainable by agents he calls "bliss".

3.2 Geographical Distribution of Geological and Economic Wealth

Natural processes have resulted in an unequal distribution of resource wealth, as can be seen in Fig. 3. But countries are also unequally endowed in productive capacity. It can be approximated by the measure of capital stocks, calculated by the perpetual inventory method. Its distribution in 2010 is displayed in Fig. 6. Since 1980, it has gradually expanded, mostly to Eastern Asia. This is explained by the ever-growing share of global manufacturing investment (about 75 % between 2005 and 2007 (World Bank, 2013)) devoted to this region.

We observe that the geographical distribution of extraction does not correspond to the geographic distribution of manufacturing capacity, with some exceptions like China, Australia or the U.S.

3.3 Sustainability and Trade, competition or cooperation?

International trade structure is marked with complexity. Scholars analyze the value-added chain, which is "the process by which technology is combined with material and labor inputs, and then processed inputs are assembled, marketed, and distributed" (Kogut, 1985). The physical separation of different parts of a production process is called "fragmentation" (Arndt and Kierzkowski, 2001), "desintegration of production" (Feenstra, 1998) or "global production sharing" (Yeats, 1999). Most industrialized countries export high value-added manufactured goods, while developing countries mostly export primary products. Latin America earned 40 % of its export revenues from mineral and fuels products, 70 % for Africa in 2006 (International Resource Panel, 2011).

This considerable development of trade is directly linked to the institutional implementation of free trade agreements in free trade areas. The "rationale for free trade" is over two centuries old, and was formally defended by the “Ricardian analysis" of comparative advantages when a free trade policy is “adopted in an institutional setting" (Bhagwati, 2003). Later, Samuelson showed that free trade was Pareto-superior than autarky in static analysis (Samuelson, 1962). In the 90’s, the benefits from free trade seemed to reach a strong consensus among economists. Its effects on growth were the subject of rich theoretical and empirical literature (Ben-David and Loewy, 1998; Frankel and Romer, 1999), but its social and environmental costs were also fiercely criticized (Rodrik, 2017; Daly, 1993).

The debate on the “trade-environment divide" is mostly focused on pollution issues (Esty, 2001; Copeland and Taylor, 1994). The question of the interactions between trade and materially sustainable growth (Dupuy, 2014) was studied by Asheim (Asheim, 1986) who looked at the effects of opening economies on the Hartwick rule. Other models focus on two-region trade, the industrialized North versus the resource exporter South (Chichilnisky, 1993). The relationship between optimal depletion theory and trade was also studied, but mostly between two countries (Kagan et al., 2015; Tamasiga and Bondarev, 2014).

It is worth noticing that in a context of possible resource depletion, the question of spatial justice has not been as studied as temporal equity. This is obviously due to the way trade was historically designed and implemented, and international competition may seem to be a rule. But competition is only one side of the coin. Free trade agreements can also be seen from a cooperative point of view. In that sense, both natural and productive endowments are shared in a common pool. This flip of a coin would renew the perspective on the convergence of living standards between countries, and open a space for discussion on the equity of trade in times of environmental threats.

4 Contribution and Transdisciplinary Methodology

4.1 Research Questions and Methodologies: Time and Dynamics

The three chapters of this dissertation aim at shedding light on specific aspects of the broad issues raised by non-renewable resources depletion and its consequences on the long-run. For that purpose, it adopts Boulding’s “Spaceship Earth” point of view (Boulding, 1966). Earth is seen as a closed system whose inhabitants extract, transform and exchange natural resources and goods. The broad questions, previously explored in this introduction, are:

- Does our knowledge of the Earth’s formation processes provide us any valuable information on possible planetary boundaries induced by the exhaustibility of non-renewable natural resources?
- Can this information shed light on the ability of societies to continue their material development - measured through GDP - within these planetary boundaries?
- What would these planetary boundaries imply in terms of global governance?

This dissertation offers a transdisciplinary modeling approach to feed the debate revolving around these questions, which have in common the imperative to delimit space and time scales. The latter is crucial, especially when it comes to bridging two academic fields which have radically different sense of time. The notion of long-term in Economics is limited to a human timescale. In financial markets, bonds last 10 years on average. Long-term bonds last 30 years, even if some countries and big companies can issue 100-year bonds. Geologists do have a much wider definition of long-term, known as "geological times". Recall that the Earth is 4.55 billion years old. All three chapters use dynamic modeling, but this tool is applied very differently according to the question and the available data. Since it uses very long-span data, the first chapter is able to give a quantitative information on the time unit. The second and third chapters only consider the long-term theoretically, with the difference that the third chapter does not ask how the long-term could be, but how it should be. Let us now see in details what specific questions these three chapters intend to tackle, and what dynamic methodology was chosen.

The first chapter is composed of two articles dealing with the supply and demand of fossil materials, and the functioning of the mining sector. It adopts a world scale, but it is restricted to this specific sector. The first article is an overview of the hard facts on the past evolution of the global supply of different base materials such as steel, aluminum, copper, zinc and more technological metals like Rare Earth Elements. The purpose is not to provide a conclusive answer to the very complex issues raised by mineral resources consumption and supply, but rather to identify some of the key points that seem important to consider. One is the inter-dependency between the metal and energy sectors. Indeed, the demand for materials crucially depend on the structure of our energy-providing system. If renewable resources are seen as 'clean energy' from the point of view of greenhouse gases, they are much more material intensive than fossil sources. One issue tackled in this article is therefore the impact of different world energy transition scenarios on the future consumption of materials. The other side of the material-energy nexus is the energy needed to extract materials from their ore deposit and obtain concentrates. We ask whether there exists any empirical relationship between the dilution of elements in the ore and the cumulative energy needed to extract them, and if this has an impact on their price through a cost channel. The second article focuses on one metal, copper. We chose it as an experimental case to have a clear conceptualization of mining, both geologically and economically speaking. As we have seen, this structural metal presents a broad range of uses in past and present economies. It is used for its specific chemical and physical properties, which leaves little room for substitution. If long-term supply issues were to be encountered for this metal, it would impact the economy as a whole. The goal of this article is to check whether the current rates of copper extraction are sustainable until the end of this century, and if not, what would be the condition for a sustained copper extraction. Note that 'sustainable' here has its original sense of continuity with past trends, whereas 'sustained' has the sense of stability. To answer this complex issue we propose a mind-sized model, giving a dynamic description of stocks and flows of both geological and economic variables. The evolution of primary copper reserves and production, recycling, price, revenues and costs of production are modeled with a 'prey-predator' dynamics. The model was constrained with a wide variety of historical data - including primary production and reserve, mass flow data, price, industrial revenue, cost of production and profits - and was used to simulate a range of possible future paths of production.

The second chapter embeds the extracting sector, as modeled in the previous chapter, into the whole economy. Indeed, this work aims at theoretically evaluating the long-term consequences of mining dynamics on long-term growth. For that purpose, we construct a stock-flow consistent continuous-time macro-dynamics built on (Grasselli and Lima, 2012) and (Bovari et al., 2018), where the scarcity of natural resources is made explicit through the simple prey-predator dynamics previously introduced. A mining sector extracts some natural resource that can only be discovered at finite speed and sells it to the final good sector which produces the consumption good out of capital, labor and this natural resource. This model enables to assess the ability of factor substitution to circumvent a collapse *à la* Meadows and to sustain an ever growing path. As a proof of concept, we analytically examine the long-run steady states of our vector field and identify conditions under which a stationary balanced path can be reached. We further provide a proxy of the world economy by aggregating 37 countries, which account for 83% of the average world GDP between 1990 and 2017. We estimate the parameters of the production function, and discuss factor's substitution and weight.

The third chapter explores the implications of natural resource scarcity in terms of global cooperation and trade. We investigate whether an international long-term cooperation strategy, where natural resources and productive capacity are shared in a common pool, can be put in place. For that purpose, we use a cooperative game framework, where countries can form coalitions in order to optimize their discounted consumption stream in the long-run, within the limits of their stock of natural resources. The coalition decides its extraction and investment plan, and deduces the amounts of resources it wishes to trade. It then distributes consumption goods to its members, following some criteria. Of course, members can break an alliance according to their interest. In this work, we look for the most efficient and stable coalition, and how it can allocate the consumption stream it produced among its members in the short run, while taking into account the consumption left for future generations in the long run. For this purpose, we use the concept of *recursive core*, that satisfies both coalitional stability and time consistency.

4.2 Variables and Parameters: the Construction of a Model

The three models described in this dissertation present variables that may seem very similar at first sight. If they do refer to the same notion, their detailed conception depends on the tackled question, the methodology and the timescale.

Resources

In the first chapter, the definition of the variable R is key. Indeed, in the simplest model of fossil resources proposed by Hubbert (Hubbert, 1956) and its derivatives, resources are viewed as a finite and quantifiable stock (the URR) that can be depleted. It assumes that geology is the sole driver of production while in reality, the main driver is the capacity of the industry to make profit for a given level of demand. But the URR is not "all there is" (Meinert et al., 2016), new reserves are discovered, and the decrease in ore grade allowed by technological change induces a constant increase in the assessed stocks. That is why we chose to model *reserves* as opposed to *resources*. The stock of reserves can regenerate with time in response to the discovery of new copper deposits and the decrease of average grade and cut-off grade of exploited deposits. Consequently, scarcity is not a depletion question *per se*, but a race between reserve increase and extraction. The second chapter follows the same reasoning. In chapter 3 though, we chose a finite stock $\bar{\Omega}$, because the study focuses on long-term strategies when time goes to infinity and that 'all there is' might be reached.

Wealth

In the first chapter, the stock of wealth W is an aggregate of economic resources used to produce primary copper. It is the productive capacity of the copper sector. It encompasses the industrial infrastructures and all other forms of capital, but also some public infrastructures used by the different industrial sectors from mining to recoverable copper delivered on market. It corresponds to cumulative profits, since its variation is obtained from the difference between revenues and costs. One of the goals of the second chapter is to give more details on the financial dynamics behind this notion, synthesized now by the capital variable K_R. Indeed, the mining industry is able to finance its activity by leveraging and financial constraints are captured following accounting principles. The non-mining wealth or capital K follows the same rules. In the third chapter, the mining industry does not need to be modeled and K aggregates all sectors.

Production

Good production, in the first chapter, is exogenously fixed as a driver of copper demand. Production there refers to copper production, as the model is sector-specific. In the second chapter, it is given by a CES production function that encompasses non-mining capital K, labor L and material flow M, which gives the opportunity to evaluate the possible substitution between factors, and discuss the limits of using a production function (Pottier, 2014). In the third chapter, the functional form of the production function needs not to be fixed, it only needs to fulfill some assumptions on continuity and concavity.

Price

The dynamics of prices is of course key in matter of scarcity. In the first chapter, copper price reflects the trade-off between decreasing ore grades and technological advances. Its possible future evolution depends on the scenario chosen. In the second chapter, both prices of copper and goods follow a dynamics that depend upon their respective production costs. This is an important departure from the conventional vision of market-clearing prices. The interesting feature of the last chapter is that no notion of price is needed to obtain our results.

Technological change

As we have just mentioned, technological change is partly captured in the evolution of copper price in the first chapter. But it is also captured by the model parameter β associated to extraction that is the efficiency of wealth to produce copper. In chapter two, technological change is given by the 'Total Factor Productivity' in the production function. A limit of the third chapter is that no technological change is modeled. Countries have different technologies since their production function is specific, but the relationship between capital and resources is fixed.

4.3 Contribution to the on-going debates

This dissertation provides a discussion on three dimension of non-renewable resources scarcity, through original modeling methodologies and both physical and monetary database construction.

The first chapter shows that the consumption of fossil materials has followed an exponential evolution during the last century, and would grow even more if the entire energy system was based on renewable sources. It further provides a strong empirical relationship between dilution, extraction energy and commodity prices. The results of the second part of this chapter suggest that the exponential consumption of copper is not sustainable, even with high recycling rates. Business-as-usual scenarios assuming a constant growth of inelastic demand and an evolution of model parameters in line with the 1900-2015 trends lead to a bell-shape copper production that peaks by the mid-century. However, the model provides conditions under which extraction can be sustained and can lead to a plateau instead of a peak and/or postpone the eventual peak of production after 2200. In that purpose, the demand for copper should first be stabilized. If the assumption on a stabilization of consumption of copper at high GDP is correct, then world GDP should also be constant after 2100. The second important condition is on recycling rates: they should reach 70 % by the end of the century.

The second chapter shows that, when the previous dynamics is set into a Goodwin-class model, no long-run balanced growth path can emerge with a positive growth rate of GDP. In other words, as soon as due account is taken of the dependence of our economies on natural resources, the unique meaningful long-run steady state *must* be stationary. The second finding of this chapter is that this steady state is not an isolated zero of our vector field. Moreover, our dynamics admit other equilibria, among which a debt-deflationary equilibrium with zero production, employment and unbounded private debt. Our empirical estimation of the production function provides three possible functional forms with different substitution elasticities between capital, labor and copper.

The third chapter demonstrates that the recursive core of the cooperative game we propose contains at least two elements. This means that, if the countries of a coalition follow an long-term cooperation strategy in terms of extraction, investment and induced trade, then the best way to optimize their own consumption stream while caring about future generations is to form a global coalition. The recursive core is made of the streams of consumption to be allocated to each country. Since this core is not reduced to a singleton, it leaves room to a choice among these allocations, for instance by using fairness criteria. The presented model can therefore be viewed as a tool to refresh the common look at the North-South opposition and sets the conceptual framework for the exploration of a fair sharing of the fruits of global economic growth.

This dissertation gives strong arguments that the extraction of one essential metal at present rates is not sustainable within the current century. It consequently investigates the effects on long-term growth and disproves the possibility, within our particular setting, of a desirable positive growth path. It finally proposes a new way of considering trade if a resource is exhaustible and finds that a global coalition where natural and productive wealth are shared in common is to the advantage of all countries.

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CHAPTER 1

Sustainable Mineral Extraction: Hard Facts and Possible Futures

Vous confondez paperasse et capital, inspecteur. Une erreur fréquente. [...] Non, je vous parle d'une richesse arrachée à même la terre. Et comme pour toute offrande... pour tout pacte infernal... ce genre de transaction se paie avec du sang. Avez-vous entendu parler du Cerro Rico?

— Jonathan Hickman, Tomm Cocker, *The Black Monday Murders* (trad. Maxime Le Dain)

This first chapter investigates the functioning of the mining sector. It is composed of two parts, corresponding to two articles. The goal of this chapter is to tackle the issue of exhaustibility from a sectoral point of view, in light of the current knowledge on the distribution of minerals in the Earth's crust, but also past trends in extraction, ore grade and mining finance.

The first part is an overview of the hard facts on the past evolution of the global supply of different base materials such as steel, aluminum, copper, zinc and more high technology metals like Rare Earth Elements. It further explores nexus between minerals and energy. Indeed, the demand for materials crucially depend on the structure of energy-providing systems. If renewable resources are seen as 'clean energy' from the point of view of greenhouse gases, they are much more material-intensive than fossil sources, as we shall see. One issue tackled in this article is therefore the impact of different world energy transition scenarios on the future consumption of materials. We show that it would have major impacts in terms of mineral consumption. The other side of the material-energy nexus is the energy needed to extract materials from their ore deposit and obtain concentrates. We find a strong empirical relationship between the dilution of elements in the ore and the cumulative energy needed to extract them, and if this has an impact on their price through a cost channel.

The second part focuses on one metal, copper, since it presents a broad range of uses especially in infrastructures. The goal of this article is to check whether the current rates of copper extraction are sustainable until the end of this century, and if not, what would be the conditions for a sustained copper extraction. 'Sustainable' here has to be understood in the sense of continuity with past trends, whereas 'sustained' has the sense of stability. To answer this complex issue we propose a mind-sized model, giving a dynamic description of stocks and flows of both geological and economic variables. We justified in the general introduction the choice of our variables. The stock from which copper is extracted corresponds to its reserves R, which can increase because the ore tonnage increases with lowering ore grade. There is therefore no absolute and fixed limit. The extractive capacity corresponds to the wealth W of the mining sector, and equals its cumulative profits, since its variation is obtained from the difference between revenues and costs. A much detailed dynamics will be presented in the next chapter. The dynamics of prices is of course key in the matter of scarcity. The copper price is also modeled in an original way. We calculate a 'reference price' of copper incorporating both the effects of embodied energy increase with lowering ore grade and technological improvements. The key question of recycling is also taken into account and the exogenous total demand can be met either by primary or secondary production. The evolution of primary copper production is modeled with a 'prey-predator' dynamics. These famous coupled system was introduced first in the field of Ecology to understand the dynamics of two cohabiting populations linked by a predation relationship. It was widely studied in the field of Mathematics and exhibits interesting cyclical dynamics. It was used in different fields, including Economics as we shall see in the next chapter. Using this analogy for renewable resources is quite intuitive. The harvesting capacity can be seen as a predator to the resource: the 'birth' rate of resources is exogenous and their 'death' depends on the size of the harvesting capacity. The 'birth' rate of the harvesting capacity depends on the size of the stock to harvest, and its 'death' is independent. In this model, we extend this view to non-renewable resources by interpreting the 'birth' of reserves as the increase in ore tonnage when ore grade decreases. The model was actually constrained with a wide variety of historical data - including primary production and reserve, mass flow data, price, industrial revenue, cost of production and profits - and was used to simulate a range of possible future paths of production. The results suggest that the exponential consumption of copper is not sustainable, even with high recycling rates. Business-as-usual scenarios assuming a constant growth of inelastic demand and an evolution of model parameters in line with the 1900-2015 trends lead to a bell-shape copper production that peaks between by the mid-century. However, the model provided conditions for which the extraction can be sustained and that can lead to a plateau or postpone the eventual peak of production after 2200. The demand in copper should first be stabilized. If the assumption on a saturation of consumption of copper at high GDP is correct, then world GDP should also be constant after 2100. Then, recycling rates should reach 70 % by the end of the century, as opposed to 40 % today. The question of the energy price is also key: we assumed here a constant price. Ensuring a plateau or postponing the peak would enable smoother impacts on the productive sectors for whom copper is crucial, which are themselves essential to the whole economy.

Global Trends in Metal Consumption and Supply: The Raw Material–Energy Nexus

Olivier Vidal[1], Fatma Rostom[2], Cyril François[1] and Gael Giraud[2]

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The consumption of mineral resources and energy has increased exponentially over the last 100 years. Further growth is expected until at least the middle of the 21st century as the demand for minerals is stimulated by the industrialization of poor countries, increasing urbanization, penetration of rapidly evolving high technologies, and the transition to low-carbon energies. In order to meet this demand, more metals will have to be produced by 2050 than over the last 100 years, which raises questions about the sustainability and conditions of supply. The answers to these questions are not only a matter of available reserves. Major effort will be required to develop new approaches and dynamic models to address social, economic, environmental, geological, technological, legal and geopolitical impacts of the need for resources.

Keywords: mineral resources, raw materials, energy

INTRODUCTION

The global acceleration of industrialization during the 20th century can be illustrated by the exponential growth of all the indicators of human activity, prosperity, and environmental impacts (Steffen et al. 2014). The growth of populations, particularly urban populations, and the increase in economic activity, average income and standard of living are all associated with an equally exponential growth in the need for energy and raw materials (Fig. 1). Mineral resources (gravels and sand, cement, ores, industrial minerals) are at the forefront of this growth because they are used to build urban and transport infrastructures, to manufacture consumer goods, and to produce and use energy. Per-capita levels of resource consumption are at their highest levels in history, and humanity is now using mineral resources at the unprecedented level of 70 Gt/year (Wiedmann et al. 2015). This great acceleration naturally raises the question of resource sustainability. Maintaining exponential trends is simply impossible in a finite world. Nevertheless, further growth in raw material and energy consumption is expected because the poorest nations aspire to increase their living standards and welfare, and because new advanced technologies are expanding rapidly.

These different issues are discussed herein, including a brief review of society's resource needs and some modelling of future metal production. Our purpose is not to provide a conclusive answer to the very complex issues raised by mineral resource consumption and supply, but rather to identify some of the key points that seem important to consider.

1 Institut des Sciences de la Terre-ISTerre
University Grenoble Alpes, CNRS
Grenoble, France
E-mail: olivier.vidal@univ-grenoble-alpes.fr

2 Centre d'économie de la Sorbonne, Université Paris Panthéon-Sorbonne, France
Chair Energy & Prosperity, Agence Française de Développement
Paris, France

TRENDS IN MINERAL RESOURCE CONSUMPTION

The first stage in industrialization is characterized by the construction of infrastructures in the sectors of heavy industry, housing, transport and communication, and in the sectors of production, transport and use of energy. This development phase consumes mainly 'structural' raw materials produced in global amounts of more than one million tons/year, such as concrete, steel and iron, aluminium, copper, manganese, zinc, chromium, lead, titanium, and nickel. After a period of increased consumption, the yearly consumption of structural raw materials stabilizes when the gross domestic product (GDP) per capita reaches about US$15,000–$20,000 per capita for steel and concrete consumption, and US$20,000–$25,000 for aluminium and copper consumption (Bleischwitz and Nechifor 2016). Many countries with large populations (e.g. China, Indonesia, India, Pakistan and many African countries) currently have GDP/capita of less than US$15,000. Their industrialization will inevitably be associated with an increase in the consumption of raw materials. This has clearly been the case since the late 1990s with the rapid industrialization of China, which led to dramatic growth in the rate of global steel and concrete consumption (6%/year), Al (5%/year), Cu (3%/year) (Fig. 1B and 1C), and Cr (5%/year), Mn (6%/year), Ni (5%/year) and Zn (4%/year). The maximum stock of iron and steel in developed societies is estimated to be about 10 t/capita (Wiedmann et al. 2015). In order to increase the current stock of 2.7 t/capita for a population of 7 billion individuals to a stock of 10 t/capita for 9 billion individuals, it would be necessary to produce 71 Gt of iron, which represents 85% of the known crude ore reserves (USGS 2017). For this evolution to take place in 35 years, the average steel production must reach 3 Gt/year by 2050, twice the current production rate.

Subsequent to, or in parallel with, the building of its base infrastructure, a country's economy will move towards high technologies. At the beginning of the 20th century, metal consumption was limited mainly to Fe, Cu, Pb, Zn, and Ag, which had the desired basic physical and chemical properties needed for the technologies of the time. High technologies now require new raw materials and mineral resources because they utilise many additional properties, including electronic structure, catalytic properties, quantum and semi-conductor properties specific to almost all the elements of the periodic table. The annual growth in the production of rare metals is at record levels: around

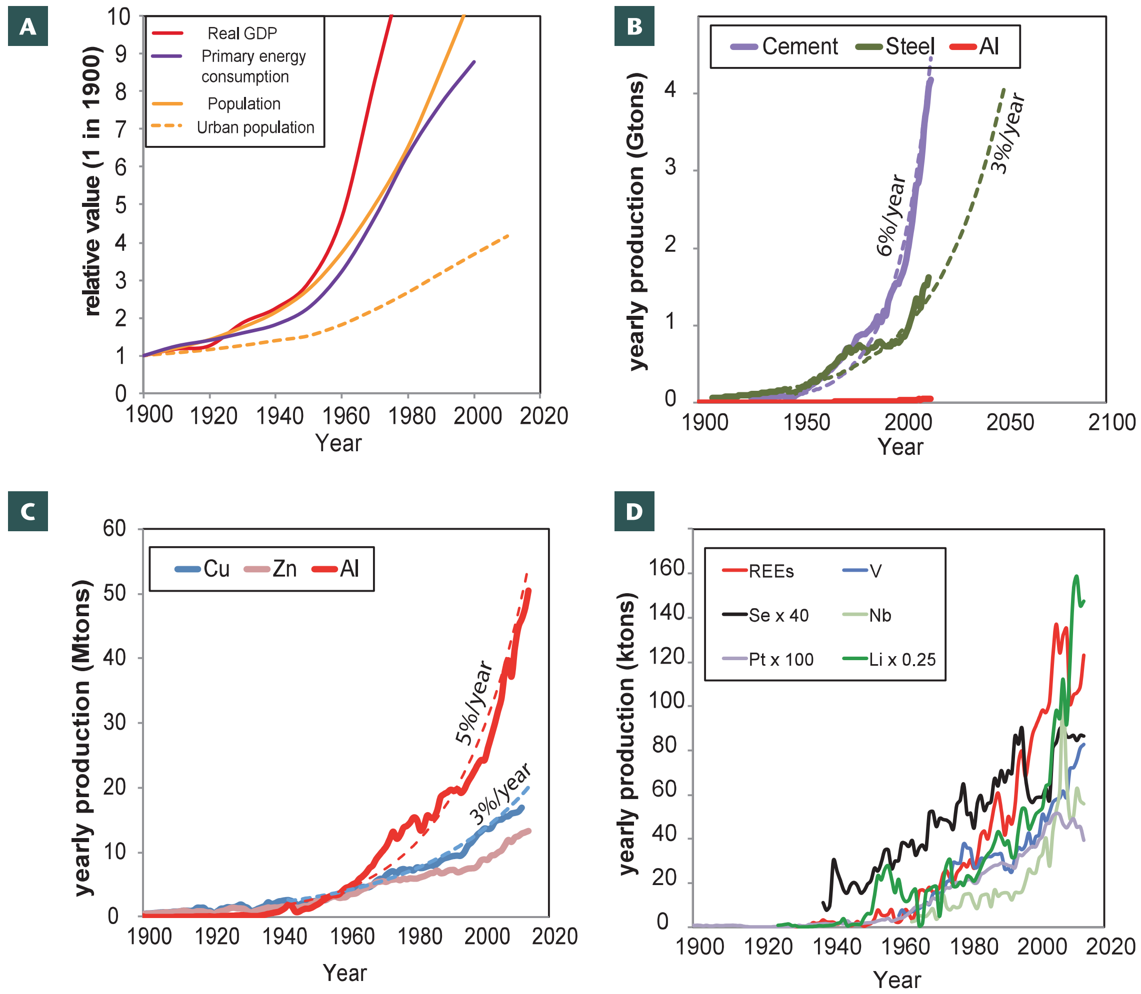

Figure 1 (**A**) Historical evolution of various indicators of prosperity and human activity. From Steffen et al. (2014). (**B**) Yearly production, between 1900 and 2015 of cement, steel and Al. (**C**) Yearly production, between 1900 and 2015 of Cu, Zn and Al. (**D**) Yearly production of rare-earth elements (REEs), V, Se, Nb, Pt and Li. The dashed lines in (B–C) show the evolution of production calculated for the indicated growth rates. Data in parts 1B, 1C and 1D from Kelly et al. (2017).

10%/year for Sb, Be, Co, Ga, Ge, Li, Mo and some of the rare-earth elements (REEs) (Fig. 1D). These huge growth rates and the inevitable problems of supply have become a focus of attention during the last decade. But evaluating the future of rare metals is extremely difficult because the demand for such metals depends on the rapid development of new technologies. For instance, Hitachi in 2012 manufactured high-efficiency permanent magnet synchronous motors that are free of REEs. Reluctance motors using REE-free electromagnets instead of permanent magnets are also an option for the future. Thus, should the supply of REEs become an issue, then technological innovations might develop that do not require essential REEs in their manufacture. This is an important difference between the 'high-tech' metals and the 'structural metals' such as steel, copper and aluminium: it would be very difficult to find adequate substitutes for the latter group. Another difference is that rare metals are often extracted as by-products from the production of the major metals that are the economic basis for most mines. If the demand of a co-produced metal rises, mining larger quantities is not possible because the major metal determines the volumes.

Until now, industrial development has been possible due to the access to cheap and abundant fossil energy. This situation is likely to change because the emissions of carbon dioxide and other combustion-related components have worrying environmental consequences. The Paris Agreements of COP 21 (the 21st meeting of the Conference of the Parties), which plans to achieve 'carbon neutrality' sometime between 2050 and 2100, will involve a massive reduction of carbon dioxide emissions and an in-depth revision of the existing global fossil energy–based system. New low-carbon energy infrastructures require more raw materials per megawatt of installed capacity than the existing fossil fuel–based facilities (Kleijn et al. 2011; Garcia-Olivares et al. 2012; Vidal et al. 2013; Hertwich et al. 2015) (Fig. 2). Therefore, large amounts of structural mineral resources (Fig. 3) and high-tech metals will be consumed, such as REEs in the super-magnets of wind turbines; Ga, In, Se and Te in photovoltaic thin films; Li in batteries of hybrid or electric vehicles; and platinum-group metals (PGMs) in fuel cells. Garcia-Olivares et al. (2012) have estimated that a complete shift to electricity produced by renewable resources as the sole source of used energy could require as much as 330 Mt of Cu (almost 20 times the current global yearly production), 8 Mt of Li (190 times), 66 Mt of Ni (30 times) and 31 kt of Pt (15 times). The supply of such huge amounts of metals, while preserving the supply for other needs, hardly seems achievable by 2050. Even the more conservative scenarios of future energy production that maintain a share of fossil fuels point out that very large quantities of metals will be needed (Fig. 3). The amounts of steel, aluminium and copper required to build the infrastructure needed for energy production according

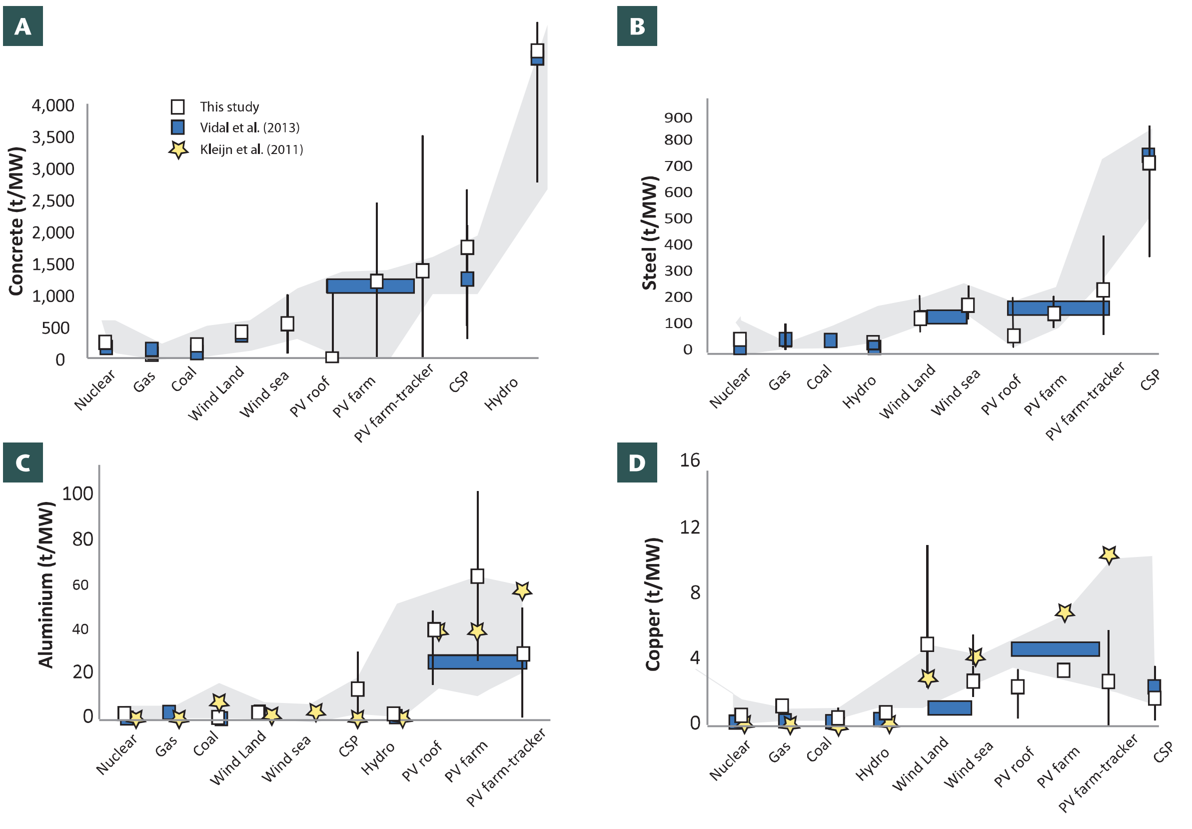

Figure 2 The amount of raw material per megawatt (t/MW) required for a range of energy production facilities: (**A**) concrete, (**B**) steel, (**C**) aluminium, and (**D**) copper. Abbreviations are as follows: PV = photovoltaic cell; CSP = concentrated solar power. The values labelled 'this study' were averaged from about one-hundred published sets of data, the error bars showing the spread of data. Although the spread of data is large, an average increase of the amount of raw material per megawatt is observed between fossil-fuel energy production technologies and technologies using renewable energy. Data derived from Kleijn et al. (2011), Vidal et al. (2013) and present study.

to the Bluemap scenario of the International Energy Agency (IEA 2010) represent 0.7, 3 and 2 times the global 2010 supply of these metals, respectively. The amounts of steel, aluminium and copper required for the Ecofys–WWF[3] scenario (see Deng et al. 2011) correspond to 2.5, 6 and 4.4 times the 2010 supply, respectively, in addition to 10 Mt of lithium. The ever-increasing need for rare metals is also worrying (Örlund 2011): forecasts for 2030 indicate that the yearly global demand in Ga, In, Se, Te, Dy, Nd, Pr and Tb for photovoltaic cells and wind turbines will require production levels of these metals to be increased by between 10% to 230% of the 2010 world supply.

Altogether, the overall stocks of all metals needed to be produced by 2050 and the use of these metals in all manner of products in 2050 could be 5 to 10 times the current stocks (Graedel and Cao 2010; Graedel 2011). This means that the cumulative amount of metals to be produced over the next 35 years would exceed the cumulative amount that has been produced to date. Is this possible?

WILL THE SUPPLY MATCH THE DEMAND?

Several studies suggest that the future supply of raw materials will not match the demand because the amount of mineable fossil resources *apparently* decreases with time (see counterargument below) and that the production of several metals has either already peaked or will peak in the foreseeable future (Meadows et al. 1972; Laherrère 2010; Kerr 2014; Northey et al. 2014; Sverdrup and Ragnasdottir 2014). Classical modelling of fossil resources assumes that production follows a symmetric bell-shaped curve of normal distribution (Hubbert 1956; Laherrère 2010; Frimmel and Müller 2011). The background to this assumption is that production continuously increases at an exponential rate when easily accessible resources can be extracted, and it later collapses when the stock of mineable resources (what is termed the 'ultimately recoverable resources') becomes depleted. According to these assumptions, the decline of the growth rate of base metals production between 1970 and 2000 (Fig. 1) could have been erroneously interpreted as a sign of reserve depletion – 'reserves' being the part of a resource that can be economically extracted using the technology of the time. In reality, the stabilisation of production after the strong growth that followed World War II resulted from a lower growth-rate in demand. China's sharp increase in metal consumption starting during the late 1990s was matched by an equally strong increase in production, with no supply constraints due to depletion of reserves. It seems, therefore, nonsensical to assume that the demand and production of a mineral resource will continuously increase at an exponential rate until the reserves are depleted. The Hubbert peak theory (Hubbert 1956) assumes that the ultimately recoverable resources are finite and quantifiable. While it is true that the stock of mineral resources in Earth is finite and is not being renewed on a human timescale, it is also true that the whole continental crust is composed of minerals that could

3 Ecofys is a Netherlands-based renewable energy consultancy organisation; WWF is the World Wide Fund for Nature (formerly the World Wildlife Fund), which is an international non-governmental organisation dedicated to conservation.

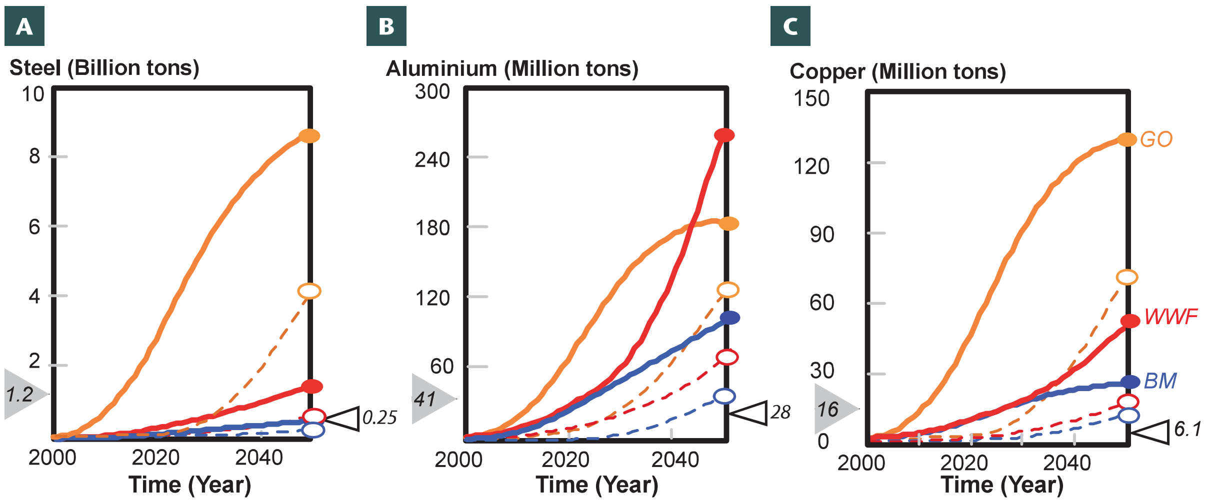

FIGURE 3 Cumulative amounts of three key metals that will be primary mined (continuous lines) and recycled (dashed lines), as projected between the years 2000 and 2050 and that are, or will be, used to build the infrastructure of energy generation: (**A**) steel, (**B**) aluminium, and (**C**) copper. Abbreviations refer to reference sources used: GO = Garcia-Olivares et al. (2012); BM = bluemap of IEA (2010); WWF = World Wildlife Fund and Ecofys of Deng et al. (2011). The grey-filled and open triangles show the 2010 global supply of primary and recycled metals, respectively.

eventually be extracted to produce metals. Improvements in technology will allow new discoveries to be made and make economically viable the exploitation of resources that were previously impossible to extract. It follows that the ultimately recoverable resources and the reserves are, in fact, larger now than they were 50 or 100 years ago. The increase of ultimately recoverable resources with time is well illustrated by the case of copper: the recovery values estimated in 2010 were made by calibrating logistic functions on the historic data of global production, and the derived recovery values that were calculated ranged from about 1 Gt (Laherrère 2010) to 3.8 Gt (Frimmel and Müller 2011). At the same time, the US Geological Survey estimated copper resources at 1.5 Gt. Three years later in 2014, the US Geological Survey completed its geology-based assessment of global copper resources and suggested that an amount of ~ 3.5 Gt of undiscovered copper should be added to the 2.1 Gt of already identified resources (Johnson et al. 2014). Thus, the demand for a metal and the reserves of that metal both fluctuate over time.

At least two other points must be considered when dealing with the future evolution of metal production, and these are dealt with below.

Energy Required to Produce a Mineral Resource and Associated Environmental Impacts

Mineral availability is related to accessibility, which is tightly connected to the energy required to extract/produce that mineral. Currently, about 10% of the global energy consumption and about 25% of the energy consumed by industry worldwide is used for the production of steel, cement and aluminium (EIA 2013). The production of mineral resources is, therefore, very energy intensive. Because the production costs depend on the cumulative energy of production (E_{cum}) and because the reserves depend on the production costs, any future reserves are connected to the evolution of E_{cum}. To a first order, both E_{cum} and the price of metals vary as a power-law of the dilution of the metal (inverse of ore grade) (Phillips and Edwards 1976; Johnson et al. 2007; Gutowski et al. 2013) (Fig. 4). It follows that, at constant technology, E_{cum} is expected to increase exponentially with decreasing ore grade (Mudd 2010; Norgate and Jahanshahi 2010). This need for energy is often seen as a future limit to extraction. However, the increase in E_{cum} can be compensated by improving the energy efficiency. For example, the E_{cum} of steel has been reduced from about 50 MJ/kg in 1900 to 25 MJ/kg in 2010 and the E_{cum} of primary aluminium has decreased from 50 kWh/kg in 1950 to 15 kWh/kg in 2010 (Gutowski et al. 2013). Further improvements in energy efficiency will likely be possible, even though the *rate* of improvement will necessarily decline as the thermodynamic limit of primary production is approached (about 10MJ/kg for steel, and 10 kWh/kg for Al). If we were to use less efficient, but nonetheless less polluting, renewable and free energy sources at a low price in the future, then the question of mineral scarcity must be viewed differently. When the infrastructure of renewable energy matures, the price of energy will drop and the contribution of E_{cum} in the cost of raw materials production will drop as well. We could afford to exploit resources not currently open to development. It is not certain whether this constitutes real progress, however, because cheaper energy and resources would entail an increase in demand and consumption, and a concomitant increase in environmental impacts other than those deriving from energy use. The 5 November 2015 Samarco company tailings dam collapse in Brazil is an illustration of the risks of large-scale mining. The decision in April 2017 by El Salvador to ban all metal mining on its territory is another example of popular mobilisation against environmentally destructive projects that had been responsible for heavily polluting the country's surface waters. The future production of mineral resources will depend on social acceptance, which requires improvement of the levels of environmental protection and governance (Ali et al. 2017), the respect of safeguards and the systematic use of cutting-edge, although expensive, technologies that reduce the damage from mining and mineral processing activities.

The Potential of Recycling

The stabilization, or even the decline, of primary production does not mean that the world will suddenly run out of metals. In contrast to fossil fuels, primary metals are not lost when they are used, and metal-bearing goods that are manufactured today can be viewed as the reserves for tomorrow's recycling. An increase in recycling, however,

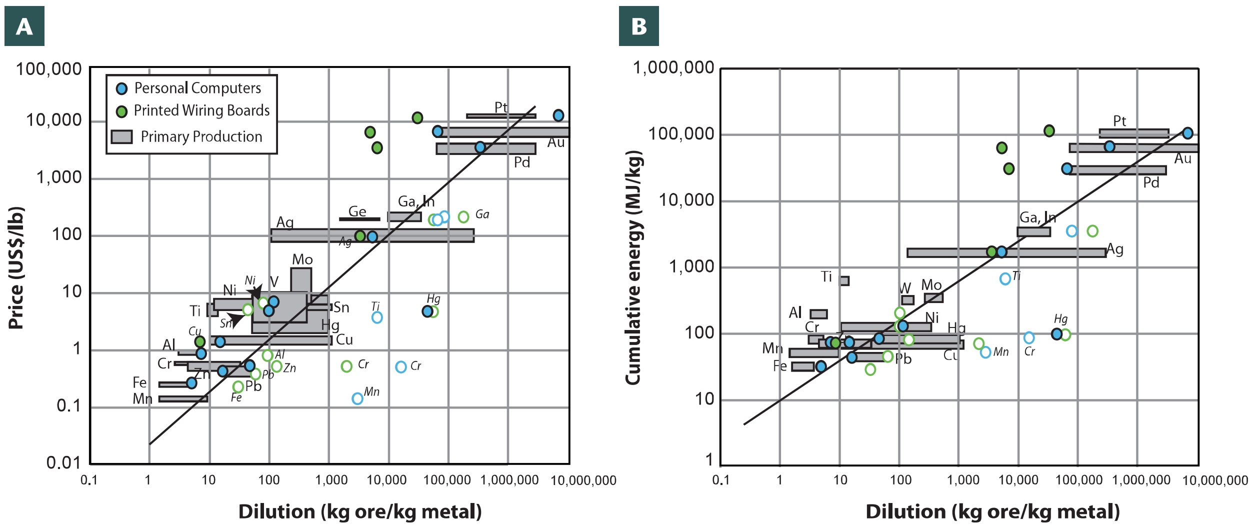

Figure 4 (**A**) Graph of price (in US$ per pound of metal) in terms of the dilution of that metal in its ore deposit (grey boxes) and that metal in personal computers (filled blue circles) and in printed circuit boards (filled green circles). Black line defines the average price vs dilution relationship obtained from primary metal production. Filled circles above this line are metals predominantly recycled; those metals below the black line (signified by empty circles) are too diluted compared to their concentration in ore deposits to be recycled at affordable price and energy cost. A noticeable exception of this trend is Hg, which is recycled for regulation reasons. (**B**) Graph of cumulative energy of metal production (E_{cum},) versus metal dilution in original ore body. Symbols as for 4A. Data from Phillips and Edwards (1976), Gutowski et al. (2013), and Kelly et al. (2017).

will not be sufficient to meet demand during future periods of economic growth because we can only recycle some of the consumer goods and equipment that were created within the last few decades. But when most countries in the world reach a GDP/capita that corresponds to the saturation level, then recycling of many metals may, in theory, become the most important source of raw materials. In practice, recycling potential is limited by economic factors, such as the difference between the price of metals and the cost of their recycling. Only the metals present in sufficiently high concentrations in their end-of-life products can be recycled: the recycling cost and E_{cum} of the most diluted metals will remain uncompetitive compared to the costs of their primary production (Fig. 4). This is why most rare metals used in high technologies are not recycled today. There are also examples for base metals whose recycling is limited by prohibitive costs. Falconer (2009) estimated that 60% to 80% of the copper in the undersea cables that link offshore wind turbines to Great Britain will not be recycled at the present cost of copper. However, a future increase of price due to an eventual depletion of primary reserves will foster recycling, which is the good news about mineral resources scarcity.

LONG-TERM FUTURE TRENDS

Modelling of mineral resource should not rely solely on factors such as a knowledge of present geological availability, historical data of production, and amount of ultimately recoverable resources. Modelling also requires assumptions regarding the growth rate of demand and the increase in levels of technology. Dynamic models are urgently needed, ones that will integrate the full complexity of the value chain, from primary production to recycling, and that will link together the energy requirements with the economic, geological, environmental, engineering, social and geopolitical dimensions. Such models are inherently complicated (Sverdrup and Ragnasdottir 2014), but it would be delusory to attempt to deal with the very complex issues with simple empirical and deterministic models such as that used by Hubbert for conventional oil in the 1950s. Olivier Vidal and Nicholas Arndt proposed a 'prey–predator' dynamic model to tackle some of these complexities (see S1 supplementary information in Ali et al. 2017). They showed that a business-as-usual scenario of copper demand and costs of production at constant price lead to a peak of primary production in 2040, followed by a collapse of primary supply. This evolution is consistent with estimates made by Northey et al. (2014), who based their model on geological availability and future mines capacity. However, many other scenarios of demand can be envisaged. If the demand stabilizes at about 30 Mt/year from 2030 onwards (16 Mt/year in 2015), no production peak is observed with the Vidal and Arndt model, and the primary supply stabilizes until 2100 at constant copper price. Recycling is predicted to compensate for the decline of primary production even in cases where primary production collapses: the cumulative flow of primary and recycled copper, for example, remains higher than twice the present level. The Vidal and Arndt model, when applied to other metals, suggests that the supply of most base and precious metals (except gold) should meet demand until the end of the 21st century. The situation for rare metals is not as clear because the historical data on reserves and production are either lacking, imprecise, or they span too short a period of time to be used as reliable constrains for the models.

PERSPECTIVES

Sustaining the supply of raw materials is not only a matter of primary resources and reserves. The Earth remains an immense reservoir of elements, even though their accessibility is limited by our ability to identify and access the resources when they are deep and hidden, by our ability to anticipate geopolitical risks, and by our ability to control the environmental and social impacts associated with element extraction. Energy issues must be taken into account in addition to the economic and social conditions of primary production, recycling, and the eventual depletion of reserves. The evolution of demand must also be analysed for the two kinds of mineral resources identified above. The supply of *structural raw materials* seems to be secured for the next 50 years because the reserves are huge. By contrast, the supply of most *high-tech metals* is often considered to be critical. However, it is the production of the structural raw materials that bear most of the environmental impacts and energy needs, and this is important because the long-term demand for structural raw materials is inevitable as poor countries become ever-more industrialized. The supply of rare metals is a shorter-term issue that results from four factors: 1) the decoupling of material and technological innovations from the reality of primary supply; 2) the difficulty of recycling complex and diluted products; 3) poor historical knowledge regarding rare-metal reserves, because these metals have not been in use for very long; 4) the fact that many high-tech metals are by-products of major' metal production.

Despite the enormous stakes in, and complexities of, mineral commodities supply, simplistic empirical models are still being used to assess future supplies of mineral resources. Moreover, modelling to date has tended to focus on the catastrophic dimension of exponential growths. More accurate modelling must analyze the conditions of a more sustainable supply and also integrate the full complexity of the raw material value chain. This is equally true for the transition to low-carbon energies, whose impacts must be modelled not only to find the best techno-economic optimum but also to optimize the environmental impacts and the raw materials supply methods. This can be done by defining the technologies that have minimal overall impact (on multiple factors in their production) and attempting to quantify the pace of when these technologies might come onstream.

Thus, the multi-faceted nature of producing and using raw materials needs to be integrated into dynamic models that will combine together knowledge from different scientific disciplines – Earth and environmental sciences, material sciences, engineering, economics, social sciences, and others – to get a better idea of what the future for metal production of all types really holds.

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Prey−Predator Long-Term Modeling of Copper Reserves, Production, Recycling, Price, and Cost of Production

Olivier Vidal,[*,†,‡] Fatma Zahra Rostom,[§,∥] Cyril François,[†] and Gaël Giraud[‡,∥,⊥]

[†]ISTerre, Université Grenoble Alpes, 38400 Gières, France

[‡]CNRS, 75016 Paris, France

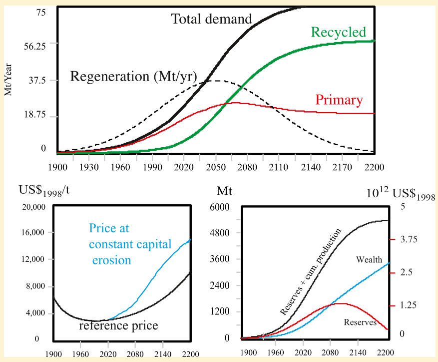
[§]Université Paris 1—Panthéon Sorbonne, 75013 Paris, France

[∥]Chaire Energie et Prospérité, 75002 Paris, France

[⊥]Agence Française du Développement, 75012 Paris, France

Supporting Information

ABSTRACT: The dynamics of copper production is modeled with a prey−predator approach linking the evolution of reserves to that of industrial wealth. Our model differs from earlier approaches in that it does not require a priori knowledge of the initial stock of resources. The model variables and a long-term reference price are estimated from historical data, taking into account the combined effects on price and reserve of technological improvements and changes in ore grade. The business-as-usual scenarios invariably lead to a peak of primary production by the middle of the century. The peak of production is not the result of the complete exhaustion of exploitable copper but of the combination of (1) the deviation of growth of reserves from the exponential historical trend and (2) the incapacity of technological improvements to offset the increase in production costs. In the leveled-off-demand scenario for which future demand is simulated based on assumed evolutions of world population and gross domestic product per capita, no collapse of primary production is observed within the century for optimistic regeneration of reserves and a collection-recycling rate reaching 70% by 2100, at constant energy prices.



INTRODUCTION

The strong increase of demand for mineral resources and metals observed since 100 years will be maintained in the future decades to satisfy the needs from increasing global population, economic growth, and urbanization.[1−4] Many studies raise concerns that the future supply will not keep up with the demand because the exhaustion of fossil resources will soon become a limiting factor to production. These studies predict that production of many metals has already peaked or will peak in a near future.[5−13] Until now, technological progress has allowed the exploitation of new resources that were not exploitable with older technologies. Irrespective of pressures on the mining industry, the metal reserves, the part of global mineral resources that can be extracted at economically viable conditions using the current technologies, have grown at a rate comparable to that of consumption.[14−19] On the medium run, historical trends seem to invalidate the production peak theory, and so far, the only mineral commodity that has experienced a decrease in production is mercury, the demand for which has plummeted because of its toxicity. However, reserves and production cannot keep growing forever in a finite world, and on the long term, exhaustion of easily accessible high-quality mineral deposits leading to poorer-quality resources being available is a true matter of concern.[3,20−22] In addition to the question of availability, the increasing energy demand and environmental impacts of the extraction from low-grade ore deposits are worrying.[23,24] These evolutions raise the question of how long the improvement of technology and market regulation forces will be sufficient to renew the future metal reserves at the same rate as in the past.

Modeling the future of metal production at a global scale must incorporate the interdependencies among production, average ore grade (OG) and reserves, price and production costs, population, and average economic development. This requires dynamic models that describe the evolution of material stocks (resources, reserves, metals in the society) and flows (yearly primary production and recycling, flow of resources from reserves, and flows toward the stocks of end-of-life products), as well as their links with economic variables. Powerful and very complete dynamic models incorporating all of these dimensions have been already developed (e.g., the World model[11]). Unfortunately, their high level of complexity makes them difficult to understand for nonspecialists. The myriad of feedback loops complicates the identification of the

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most important variables controlling the evolution of production, and the procedure used to constrain these variables is not always straightforward.

In the present study, we propose a much simpler approach than that adopted in the World and similar system dynamics models. The evolutions of reserves, production, industrial wealth, cost of production, and price are modeled with a prey–predator approach involving only two differential equations and four variables that can be constrained by historical data. We show that two variables, the metal price and the average ore grade of ore deposits, have a major impact on the outcome. A range of possible scenarios is proposed for different assumptions regarding the rates of recycling and regeneration of reserves, for different imposed future demands. Our study focuses on copper, a strategic metal with a myriad of applications in the energy and ICT sectors. Copper is a vital commodity for the transition toward low-carbon energies;[25−27] it is mined as the sole or major metal in many deposits; and there are rich historical records of production, reserves, and price. Moreover, several forecasts of copper peak production occurring in the near future have been recently published.[9,11,13,20,22,28]

BRIEF OVERVIEW OF PRIMARY PRODUCTION MODELS ASSUMING A STATIC STOCK OF ULTIMATE RECOVERABLE RESOURCE (URR)

Hubbert[5,29] proposed a simple model of fossil resource production and popularized the notion of "peak oil". He estimated the ultimate recoverable resource (URR) of oil in the lower 48 US states using the historical annual oil production data, modeled as a logistic equation. With this formalism, production follows a bell-shaped curve, and if URR is known, the date and magnitude of the production peak can be determined.

An important weakness of Hubbert's approach or its derivatives[30−32] lies in its empirical nature and the lack of connection among demand, production, price, and reserves. It assumes that geology is the sole driver of both reserves and production, while in reality, the main driver for production is the capacity of the industry to make a profit at a given level of demand. For structural raw materials, the demand increases with gross domestic product (GDP) per capita during the early stages of economic development. Bleischwitz et al.[33,34] argue that this yearly consumption levels off when GDP per capita reaches about 15 000 to 25 000 US-$. This would explain why the global increase of consumption slowed down between 1970 and 2000, when presently developed countries had achieved to build their base infrastructure, compared to the period 1950–1970. The global slowdown of metal demand and supply between 1970 and 2000 was in no way indicative of reserve depletion, as this could have been erroneously interpreted with Hubbert's approach. This downturn in demand triggered a drop in price, while reserve depletion would have triggered an increase.

The use of a static initial stock of exploitable resources (URR) is another important weakness. In the case of copper, the values of URR estimated since 2010 range from about 1 Gt[8] to 3.8 Gt.[32] In 2010, the identified copper resources ranged from 1.1 Gt (Raw Materials Database) to 1.5 Gt (USGS). Three years later, the USGS completed its geology-based assessment of global copper resources and proposed that about 3.5 Gt of undiscovered copper should be added to the 2.1 Gt identified resources.[35] More recently, Singer[36] has estimated that 4.35 Gt of copper was present in undiscovered mineral deposits, and Henckens et al.[18] have reported an amount of extractable global resources equal to 7.5 Gt. Finally, based on geodynamic arguments, Kesler et al.[37] estimated that ultimate copper reserves in porphyry deposits could be as high as 1300 Gt, among which 89 Gt would be exploitable if mining in the future could reach depths of around 3.3 km. This nonexhaustive comparison of data published during the last decade clearly shows that URR estimated from geological constraints or from historical data of production span a huge range of values and increase with time. They only provide a crude estimate of the total amount of available copper and cannot be used to produce robust estimates of future production.

Bell-shaped production curves are also obtained with nonempirical prey–predator-like models, which were developed by Lotka and Volterra[38,39] to describe the dynamics of competition in simple biological systems, such as between two species like wolves (W) and rabbits (R)

$$\frac{dR}{dt} = \alpha R - \beta WR \tag{1}$$

$$\frac{dW}{dt} = \delta RW - \gamma W \tag{2}$$

where α and β are the rabbits' birth and death per wolf rates, respectively, and δ and γ are the wolves' birth per rabbit and death rates, respectively. At constant values of α, β, δ, and γ, the equations have periodic solutions, the periodic variation of the predator population $W(t)$ lagging behind the prey population $R(t)$. Bardi and Lavacchi[40] examined various situations where the production of a natural resource (the prey) depends on the capital stock (the predator) employed in its production. In all cases, the model generates a Hubbert-like curve. However, in contrast to Hubbert's empirical approach, the reasons for growth and decline are explicit. The system dynamics is controlled by two internal feedbacks: a positive feedback that results from the reinvestment of profits generated by resource production and a negative feedback that results from the gradual depletion of resources. Another common feature of Bardi and Lavacchi[40] and Hubbert-like approaches is that the stock of fossil resources is considered to be finite and must be known (number of prey at t_0 = URR) because the rate of reserve regeneration was assumed to be zero for fossil resources. In the case of copper, reserves have been increasing from about 25 Mt in 1900 to about 700 Mt today at a rate sufficient to compensate for depletion due to extraction. The growth rate of fossil reserves is therefore an important variable to consider, and α should not be assumed to be zero as proposed in previous works. As we show below, the assumption of constant β, δ, and γ is also not consistent with the historical evolution of production, reserves, production cost, and price of copper, which change with the average concentration of exploited deposits and the improvement of technology.

MATERIALS AND METHODS

Stock-Flow Model with a Prey–Predator Dynamics Adapted to Fossil Resource Extraction. Our study focuses on primary production, but the contribution of recycling is included to compare the modeled future demand with total production (primary and secondary). The copper life-cycle is modeled using the simplified stock-flow model shown in f1

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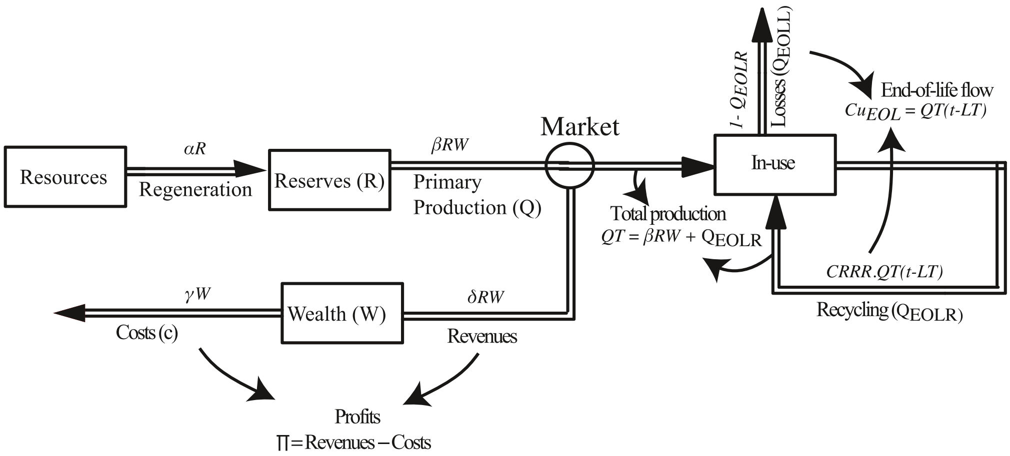

Figure 1. Modeled copper life-cycle. The boxes and pipes represent stocks and flows, respectively. Wealth W varies by profit accumulation, the flow Π being equal to revenues δRW minus costs $c = \gamma W$. Reserves R increase by regeneration αR and are depleted by primary production $Q = \beta RW$. This freshly extracted copper, as well as the recycled copper Q_{EOL}, is embodied in goods: the flow QT of total copper accumulates into the in-use stock. At the end of its lifetime LT, the embodied copper (Cu_{EOL}) is either recycled (Q_{EOLR}) or lost (Q_{EOLL}).

Figure 1, in which the end-of-life flow of copper is proportional to primary production with a lag of 20 years.

The production of primary copper is modeled with eqs 1 and 2, where the stock of predators W is now the wealth of the mining industry and the stock of preys R represents the copper reserves. A list of all variables and parameters of the present model, as well as their units, is available in Table 1. They are compared to the original prey–predator model. The stock of wealth W is an aggregation of economic resources used to produce primary copper. It encompasses the industrial infrastructures and all other forms of capital and also some public infrastructures used by the different industrial sectors from mining to recoverable copper delivered on market. The stock of reserves is allowed to regenerate with time in response to the discovery of new copper deposits and the decrease of average grade and cut-off grade of exploited deposits.[41,42] Both these effects are captured in the first term αR of eq 1, where α is the yearly rate of regeneration. It is clear that the renewal of metal reserves is not regeneration in the sense applied to renewable resources, mineral deposits cannot be renewed in the way rabbits are born or forests are replanted, but we will show that they can be modeled as such. The second term of eq 1 is the annual production, where β is equivalent to the predator predation rate and represents the efficiency of wealth to extract copper at given levels of reserves and wealth.

The evolution of industrial wealth with time is given by eq 2, where the first term represents the annual revenues of the mining industry and the second term represents the aggregated costs of production, calculated as a fraction of W. The annual revenues are proportional to δ, which describes how efficiently the extracted copper is transformed into wealth. This efficiency to transform copper into money is naturally demand- and price-dependent. The revenues are also given by the copper production Q multiplied by the price p, so that

$$\text{revenues} = \delta RW = pQ = p\beta RW \tag{3}$$

eq 3 can be rearranged to express δ as a function of price and β

$$\delta = p\beta \tag{4}$$

The costs of production are an aggregation of all costs from mining to recoverable copper delivered on market, depreciation and amortization, corporate overheads, royalties, and other financial interests. In the following, the share of costs f is set as the ratio of the unitary cost per tonne of copper $c_{\text{per-tonne}}$ to price

$$f = \frac{c_{\text{per-tonne}}}{p} = 1 - m \tag{5}$$

where m is the net margin. The yearly total costs c and the per-tonne costs read

$$c = \gamma W = f\,\text{revenues} = fpQ = f\delta RW \tag{6}$$

$$c_{\text{per-tonne}} = \frac{c}{Q} = \frac{\gamma}{\beta R} = fp \tag{7}$$

Finally, the yearly profits of copper sales Π and the per-tonne profits $\Pi_{\text{per-tonne}}$ read

$$\Pi = \text{revenues} - \text{costs} = (1 - f)pQ \tag{8}$$

$$\Pi_{\text{per-tonne}} = (1 - f)p \tag{9}$$

Since prices of mineral resources vary with time, δ and/or β are also time-dependent. Similarly, α must be allowed to change with time because in the absence of predators ($W = 0$), reserves would grow forever for $\alpha \neq 0$, which does not make sense.

The stock of copper embodied in goods (in-use-copper) can be estimated by integrating the difference between the inflow of produced copper QT (primary and secondary production) minus the outflow of copper in end-of-life products Cu_{EOL} (Figure 1)

$$(\text{in-use-copper}) = \int_T (QT(t) - Cu_{EOL}(t))\mathrm{d}t \tag{10}$$

The outflow corresponds to the amount of copper incorporated in goods at the time they were produced, so that

$$Cu_{EOL}(t) = QT(t - \text{LT}) \tag{11}$$

where LT stands for the average lifetime of goods. The yearly amount of copper recycled from old scrap today Q_{EOLR} is therefore equal to the amount produced LT years ago multiplied by a recycling rate CRRR synthesizing collecting, processing, and recycling rates

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Table 1. List of Symbols and Abbreviations

symbols	original prey−predator model	present model	units
t	time	time	year
α	preys birth rate	rate of reserves regeneration	year^{-1}
β	predation rate	efficiency of wealth to produce copper	($\$_{1998}$ year)$^{-1}$
δ	predators birth rate per prey	efficiency of copper exploitation to create wealth	(tonne year)$^{-1}$
γ	predators death rate	rate of wealth erosion	year^{-1}
R	number of preys	reserves	tonnes (metric tons) Mt = 10^6 tonnes Gt = 10^9 tonnes
W	number of predators	wealth	$\$_{1998}$
$Q = \beta WR$	yearly number of killed preys	yearly primary production	tonnes/year
Cu_{EOL}		copper embodied in end-of-life products	tonnes/year
Q_{EOLR}		yearly secondary production	tonnes/year
Q_{EOLL}		yearly lost copper	tonnes/year
QT		yearly total production	tonnes/year
D		yearly total demand	tonnes/year
YACC		yearly average consumption of copper per capita	kg/capita/year
WP		world population	beings
GDP		gross domestic product	$\$_{1998}$/year
δRW	yearly births of predators	yearly revenues	$\$_{1998}$/year
γW	yearly deaths of predators	yearly wealth erosion	$\$_{1998}$/year
αR	yearly births of preys	yearly regeneration of reserves	tonnes/year
Cu_{OG}		yearly regeneration of reserves as a function of OG	tonnes/year
$p = \delta/\beta$		unit price	$\$_{1998}$/tonne
m		net margin	
$f = 1 - m$		share of the costs of production in the revenues	
$c = fpQ$		yearly costs of production	$\$_{1998}$/year
$\Pi = mpQ$		yearly profits	$\$_{1998}$/year
$c_{per\text{-}tonne}$		per-tonne unit cost	$\$_{1998}$/tonne (or simply $\$_{1998}$/t)
$\Pi_{per\text{-}tonne}$		per-tonne profits	$\$_{1998}$/tonne
OG		ore grade	%
OT		ore tonnage	Mt
α_{OG}, β_{OG}, δ_{OG}, γ_{OG}		parameters derived from the evolution of OG	
α_{FD}, β_{FD}, δ_{FD}, γ_{FD}		parameters derived from the scenario of future demand	
p_{CT}		reference price at constant technology	$\$_{1998}$/tonne
p_{TI}		reference price with improving technology	$\$_{1998}$/tonne
ETIP		effect of technological improvement on price	
CRRR		collection rate-recycling rate	%
EOL-RIR		end-of-life recycling input rate	%
EOL-RR		end-of-life recycling rate	%
LT		lifetime	year

$$Q_{EOLR}(t) = CRRR^{*}Cu_{EOL}(t) \quad (12)$$

and the yearly flow of lost copper is

$$Q_{EOLL}(t) = (1 - CRRR)^{*}Cu_{EOL}(t) \quad (13)$$

CRRR involves the proportion of copper produced at time t that will be recycled LT years later; it corresponds, modulo a lag of LT years, to the end-of-life recycling rate EOL-RR described in the literature.[43] For a stock of copper in goods equal to 20 Mt in 1900, the in-use, recycled, and lost stocks are fairly well reproduced with a constant CRRR of 40% from 1900 to 2015 and an average LT equal to 20−25 years (Figure 2). The end-of-life recycling input rate (EOL-RIR) corresponding to the proportion of metal produced from old scrap (a metallurgical indicator) at time t is given by

$$\text{EOL-RIR} = \frac{Q_{EOLR}(t)}{Q(t)} \quad (14)$$

EOL-RIR is estimated to span between 18 and 20% from 1920 to 2015 (Figure 2), in agreement with the values reported in the literature[43,44] at the global scale. However, copper EOL-RIR is higher in rich countries than the world average. Soulier et al.[45] estimated for instance that between 2005 and 2014, 50% of the copper refined and remelted in the EU was from secondary sources.

Estimation of the Future Global Copper Demand. The future demand for copper can be estimated from historical data of copper consumption versus GDP combined with assumed evolutions of population and GDP. The growth rates of population and GDP per capita from 1900 to 2015 are given by numerous long-time series.[46−48] The United Nations foresee a growth of the world population from 7.3 billion individuals in 2015 to 11 billion in 2100 (medium scenario), and the GDP per capita is assumed to follow a similar trend from 7000 US$_{1998}$ in 2015 to 12 500 US$_{1998}$ in 2100 (Figure 3a). Both population and GDP per capita were assumed to be steady after 2100. The annual copper consumption increases

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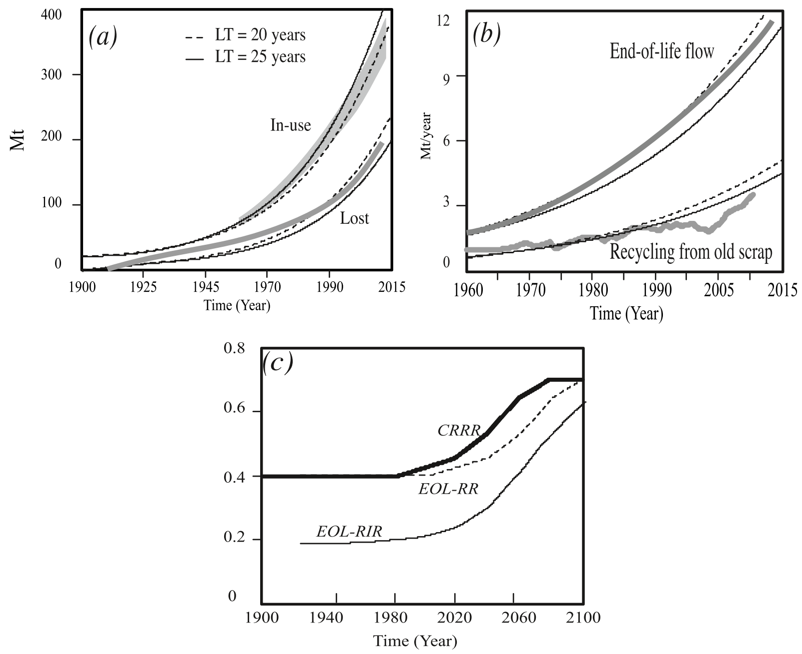

Figure 2. (a) Stocks of in-use and lost copper, (b) yearly end-of-life flows and recycled copper from old scraps, and (c) evolution of the different recycling rates CRRR, EOL-RR, and EOL-RIR for the evolution of GDP per capita shown in Figure 3. The thin lines in (a) and (b) show the values calculated for an average liftetime, LT, of 20 years (dashed) or 25 years (continuous) and CRRR of 40%. The gray areas and lines show the values reported in the literature.[43,44]

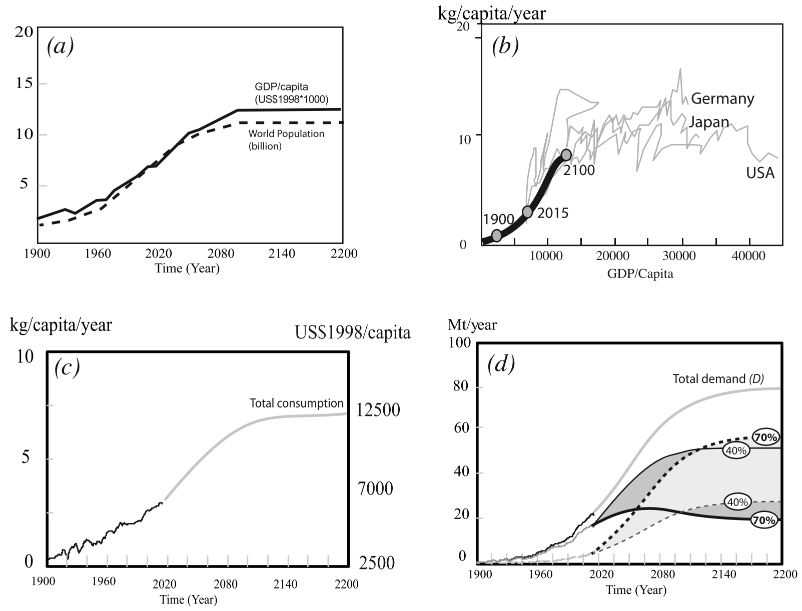

Figure 3. (a) Evolution of population and GDP per capita, (b) yearly copper consumption per capita versus GDP per capita, (c) evolution of yearly copper consumption per capita, and (d) historical and simulated total demand *D*, primary production (continuous lines and deep gray area), and recycled production (dashed lines and light gray area). The future primary and secondary productions are labeled to separate scenarios for CRRR = 40 % or increasing to 70% in 2100.

with growing GDP per capita and levels off at about 10 kg/capita/year for a GDP per capita above 15 000 US $\$_{1998}$[33,34,49−51] (Figure 3b). By combining the evolution of the world population (WP) and GDP per capita with the copper intensity per capita, the yearly average consumption of copper per inhabitant (YACC) is calculated to increase from 3 kg/capita/year in 2015 for an average GDP per capita of 7 000 US$\$_{1998}$ to 7 kg/capita/year in 2100 for 12 500 US$\$_{1998}$ (Figure 3c). The total global demand for copper *D* (in Mt/year) is modeled with the following logistic function (Figure 3d)

$$D(t) = \frac{\mathrm{YACC}(2100) \times \mathrm{WP}}{\frac{1 + \mathrm{YACC}(2100) \times \mathrm{WP}}{QT(1900) - 1} e^{-\tau(t-1900)}} \quad (15)$$

where τ is the average rate of production growth and $QT(1900)$ is the primary and recycled copper production in 1900. The rate τ = 3.75% and $QT(1900)$ = 0.45 Mt were

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adjusted to fit the historical data of global production. Total global copper demand is found to be 45 Mt/year in 2050, in fair agreement with values estimated by Elshkaki et al. for the MF and PF GEO-4 scenarios.[3] It further increases to 75 Mt/year in 2100 and stabilizes at 80 Mt/year in 2200, in agreement with the values of the SSP4 scenario estimated by Schipper et al.[52] The amount of copper recycled from old scrap Q_{EOLR} is obtained by eq 12, and the required primary production is given by the difference between the total demand D and Q_{EOLR}. The demands for primary and secondary copper were estimated for the two evolutions of CRRR illustrated in Figure 3d, either at steady CRRR of 40% or assuming an increase to 70% in 2100. In the first case, the demand for primary copper reaches 50 Mt/year in 2100, while in the second case, it peaks at 26.5 Mt/year in 2060 and decreases to about 24 Mt/year after 2100, in agreement with the scenario SSP4.[52]

Calibration of the Model for Primary Production. *Estimation of the Quantity of Exploitable Copper and Reserve Regeneration.* The value R_{1900} and the yearly evolution of α were estimated from the 1900 to 2015 historical data of reserves reported by numerous studies[14,44,53] and the compilation of Schodde.[54] Copper reserves have grown exponentially between 1900 and 2015, at an average rate of 2.85 %/year. From eq 1, the value of α from 1900 to 2015 can therefore be approximated by

$$\alpha = \ln(1.0285) + \frac{Q}{R} \tag{16}$$

with the values of production reported by the ICSG and the USGS.[44,55]

However, a constant rate of growth of reserves cannot be assumed to model the future availability of primary copper. Indeed, the average ore grade (OG) of exploited copper deposits is observed to decrease continuously since 1900,[24,56] and the exponential increase in reserves is only valid for a specific range of copper ore grades. The observed variation of OG (in %) in time can be fitted by the following exponential function (Figure 5a)

$$\mathrm{OG} = 8 \times 10^{10} \mathrm{e}^{-0.0125t} \tag{17}$$

Below OG = 0.5%, the uni- or bimodal nature of copper distribution in natural rocks is still debated.[19,41] The bimodal hypothesis involves two distributions, one centered at the average grade of copper in the crust (OG ≈ 30 ppm[57]) and another centered at OG ≈ 0.3−0.5 % for ore deposits.[41] The OT-versus-OG relationship in ore deposits is log-Gaussian[41,58] with OT being the ore tonnage given by

$$\mathrm{OT} = \frac{A}{\mathrm{OG}\sigma\sqrt{2\pi}} \exp\left(\frac{-\log(\mathrm{OG}) - \mu^2}{2\sigma^2}\right) \tag{18}$$

where μ is the central tendency, σ is the dispersion, and A is the scaling factor that determines the function amplitude. The additional amount of copper Cu_{OG} that can be extracted from a given OT at a given OG reads

$$\mathrm{Cu_{OG}} = \mathrm{OT} \times \frac{\mathrm{OG}}{100} \tag{19}$$

For the imposed variation of OG with time given by eq 17, Cu_{OG} represents the yearly amount of additional available copper, and a plot of the integral of Cu_{OG} with time shows the evolution of reserves summed with cumulative production. The future rate of reserve regeneration α_{OG} reads

$$\alpha_{OG} = \frac{\mathrm{Cu_{OG}}}{R} \tag{20}$$

eq 20 must be used instead of eq 16 at low concentrations because the OT-versus-OG relationship in eq 18 implies that α_{OG} is no longer constant. The historical data of ore tonnage, copper reserves, and production from 1900 to 2015 were best fitted with the parameters A_1 = 6500 Mt, μ_1 = −0.55, and σ_1 = 0.7 in eq 18. A second set of parameters was obtained from the highest possible evolution of reserves still in reasonable agreement with historical data (A_2 = 9350 Mt, μ_2 = −0.72, and σ_2 = 0.75). Both sets of parameters lead to OT-versus-OG evolutions compatible with the range of values estimated by Gerst.[41] They also reproduce the historical evolution of the integral of Cu_{OG} calculated as the sum of the reserves plus the cumulative production. The evolution of ore tonnage with time follows a bell-shaped curve. Cumulated Cu_{OG} in traditional deposits (volcanic massive sulfide and sediment-hosted ores, sulfide and oxide porphyry) is asymptotic to 5 Gt for the best fit (curves 1 in Figure 4) or 7.5 Gt for the highest ore tonnage hypothesis (curves 2 in Figure 4). These amounts of copper are in the range of the 5 Gt of identified and undiscovered resources estimated by Johnson et al.[35] and the 6.3−7.5 Gt of mineable copper estimated more recently.[18,19,36]

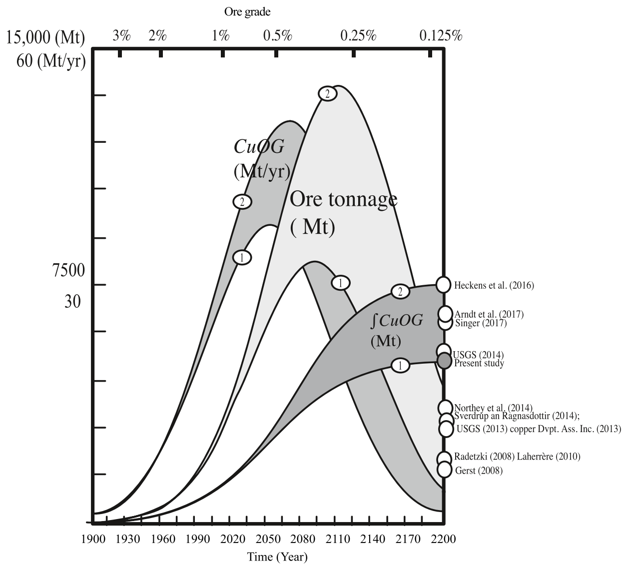

Figure 4. Evolution of the ore tonnage, the additional amount of copper that can be extracted Cu_{OG} and the integral of Cu_{OG} (historical reserve and cumulative production), as a function of time (lower scale) and ore grade (upper scale). The gray areas show the range of possible values between curves (1) obtained from the best fit of historical data and curves (2) obtained from the highest possible evolution of reserves still in reasonable agreement with the historical data. The white circles show different estimates of URR from continental crust above 1 km depth. The gray circle is the amount of reserves in 2200 estimated for the best-fit case.

Estimation of the Evolution of Wealth Creation in the Mining Industry. The stock of wealth was estimated from the cumulative yearly profits

$$W = W_{1900} + \int_{1900}^{t} \Pi(t)\mathrm{d}t \tag{21}$$

A discussion of the available literature[59−61] and estimation procedure of the profits and costs of the copper sector from

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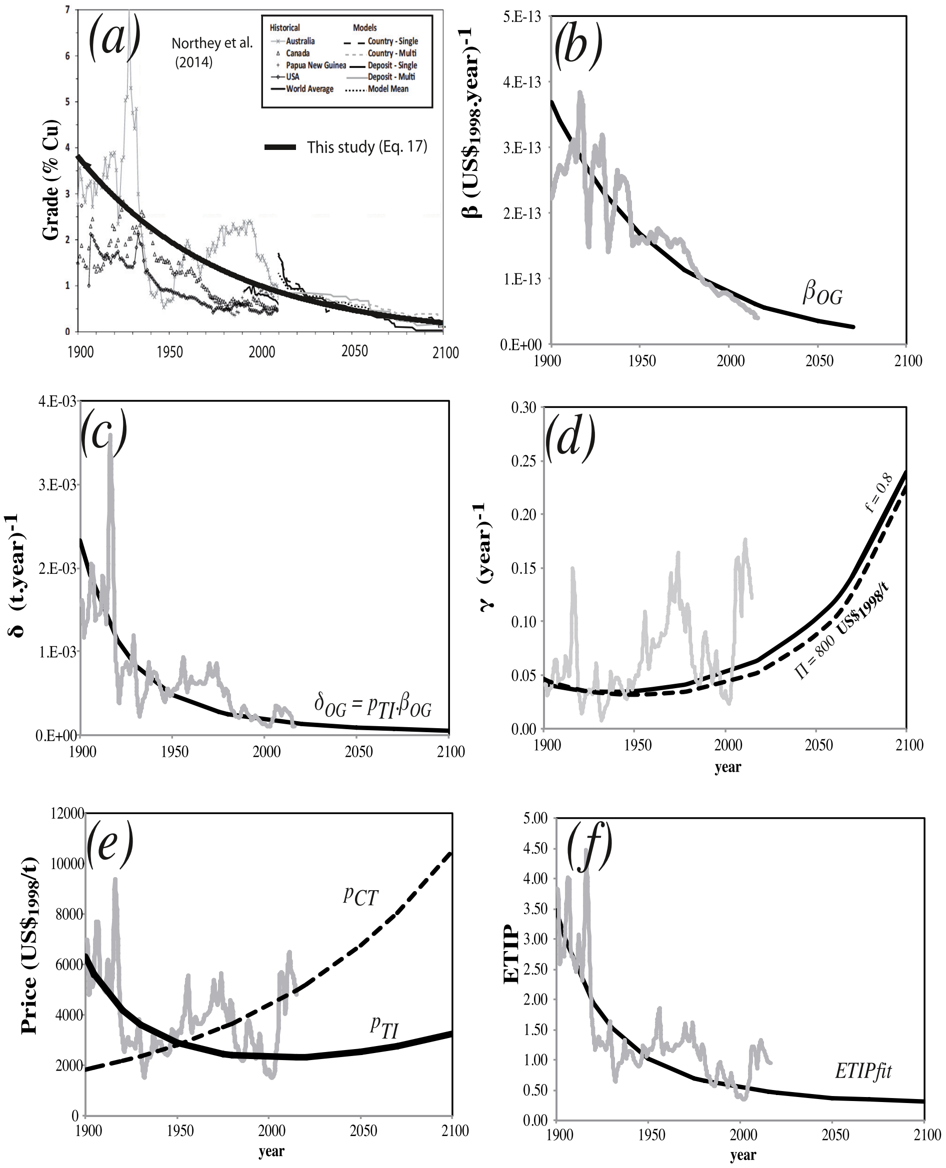

Figure 5. (a) Evolution of the ore grade OG (% of copper), (b)−(d) the model parameters β_{OG}, δ_{OG}, and γ_{OG} (for constant per-tonne profit Π or constant share of costs f), (e) price at constant technology p_{CT} and reference price p_{TI}, and (f) the technological effect ETIP. The gray lines show the historical data of prices in (e) or the model variables constrained by historical evolution. The black lines show fitted variables.

1900 to 2015, as well as the table of the database used in this study, are provided in the Supporting Information. The costs of copper production increased from 1500 US-$\$_{1998}$/tonne in 1930–4000 US-$\$_{1998}$/tonne in 1970. It then decreased to about 1500 US-$\$_{1998}$/tonne in 2000 and increased again to 5000 US-$\$_{1998}$/tonne in 2010. This evolution is fairly reproduced with constant average $\Pi_{\text{per-tonne}}$ = 800 US-$\$_{1998}$/tonne and f = 0.8. In the following, W was calculated for these two situations, i.e., assuming either a constant $\Pi_{\text{per-tonne}}$ of 800 US-$\$_{1998}$/tonne, in which case f varies with price while profits vary with production (eqs 8 and 9)

$$\Pi = 800Q \tag{22}$$

$$f = 1 - \frac{800}{p} \tag{23}$$

or assuming f constant, in which case $\Pi_{\text{per-tonne}}$ and $c_{\text{per-tonne}}$ are proportional to price.

The remaining variables of the model were estimated using the prices listed in US-$\$_{1998}$ by the USGS for the period 1900–2015. The values of $\beta(t)$, $\delta(t)$, and $\gamma(t)$ are slightly different for the two assumptions but show the same variations in time. Strong oscillations of all variables between 1900 and 1950 (gray lines in Figure 5) are required to reproduce the equally huge variations of copper production within a few months, which cannot be due to abrupt changes in reserves or wealth (Figure 6). Since 1900, the price of copper has shown strong short-time variations driven by global socioeconomic changes, oil crises, and wars. However, it remained on the long run fairly stable at around 3500 US-$\$_{1998}$/tonne, so that δ decreases with time proportionally to β (Figure 5). The rate of wealth erosion γ shows the same short-term variations as δ; it peaked during World War I, the seventies (oil crises), and in 2010, when the production costs were pulled up by investments in new operations.

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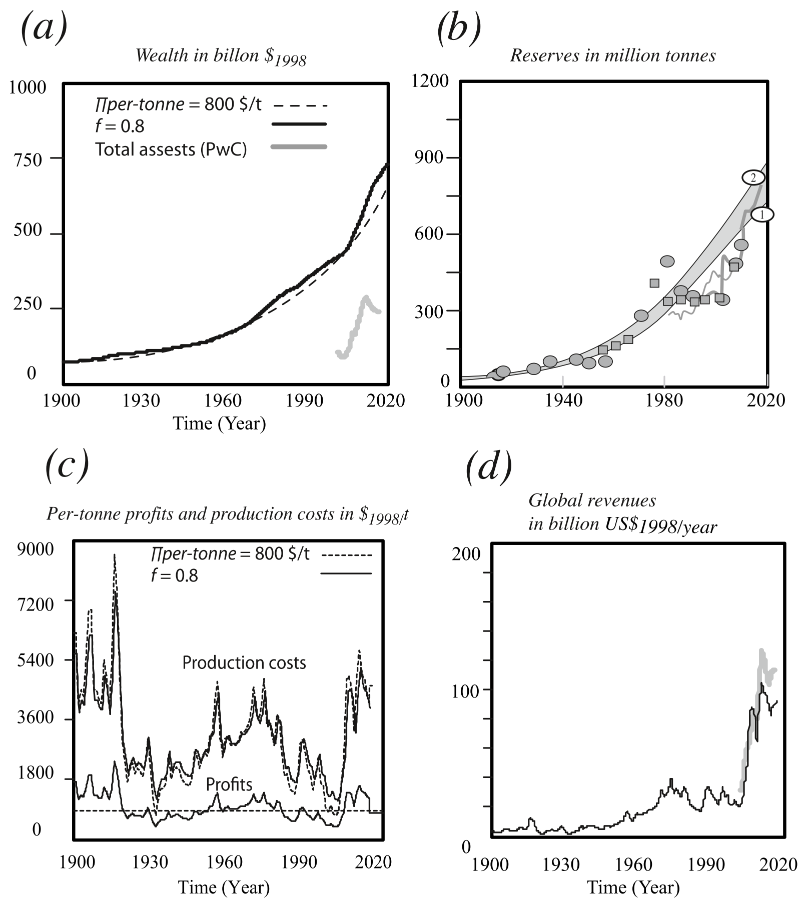

Figure 6. Evolution of (a) wealth W, (b) reserves R calculated for the two ore-tonnage-versus-ore-grade relationships, (c) per-tonne profit $\Pi_{\text{per-tonne}}$ and costs $c_{\text{per-tonne}}$, and (d) global revenues calculated with the values of α, β, δ, and γ estimated for the two assumptions of constant $\Pi_{\text{per-tonne}}$ = 800 US-$\$_{1998}$/tonne or share of costs f = 0.8. The thick gray lines in (a) and (d) show the historical total assets and copper revenues, respectively. The gray symbols in (b) show the observed historical reserves. The global revenues in (d) calculated for constant $\Pi_{\text{per-tonne}}$ = 800 US-$\$_{1998}$/tonne or f = 0.8 are indistinguishable.

The calculated wealth is similar for the two assumptions of constant per-tonne profit and constant f (Figure 6a). In both cases, the calculated wealth in 2010 is two times higher than the total assets of the copper mining industry estimated from the PwC data. Wealth considered in the present study encompasses not only the private infrastructure but also the part of public infrastructure used by the industry. A higher value of calculated wealth compared to the total assets is therefore not surprising. However, this difference suggests that the per-tonne average profit of copper sales might be lower than 800 US-$\$_{1998}$/tonne. Similar values of W and total assets can be obtained for an average per-tonne profit of 500 US-$\$_{1998}$/tonne or f > 0.8. The global revenues (=δRW) show a strong increase at the beginning of the years 2000, in good agreement with the revenues estimated from the PwC reports (thick gray line in Figure 6d).

■ RESULTS AND DISCUSSION

Ore Grade and Technological Improvement as Drivers of the Model Variables. At given reserve and wealth stocks, the yearly production is proportional to β, which is the efficiency of wealth to produce copper, equivalent to the predation rate of predators on preys in biological systems. The effort that wolves must produce to catch the same number of rabbits dispersed in a large area is higher than if the rabbits were concentrated in a small area. It follows that the predation rate is expected to decrease with dilution, corresponding to the decrease of the average ore grade (OG) of exploited deposits observed for hundreds of years.[24,56] Like OG, β is also expected to decrease exponentially with time (Figure 5b). An exponential fit of β from historical data leads to

$$\beta(t) = 2.97\mathrm{e}^{-0.01564t} \tag{24}$$

so that the evolution of β with ore grade reads

$$\beta(\mathrm{OG}) = \beta_{\mathrm{OG}} = 6.77 \times 10^{-14}\mathrm{OG}^{1.25} \tag{25}$$

Decreasing the average ore grade of exploited deposits at constant technology also changes the embodied energy in production and the metal price, which both increase as a power law of dilution.[23,56,62−68] If the same extraction technology had been used since 1900, the embodied energy and the price of copper would have increased exponentially. During the last century, the prices of base metals have not followed this expected exponential increase, which implies that the additional energy required to mine metals from lower-grade deposits has been compensated by the improvements in energy efficiency of production. The price at constant technology p_{CT} (in US-$\$_{1998}$/tonne) can be calculated as a function of ore grade from the following equation, which was derived from the

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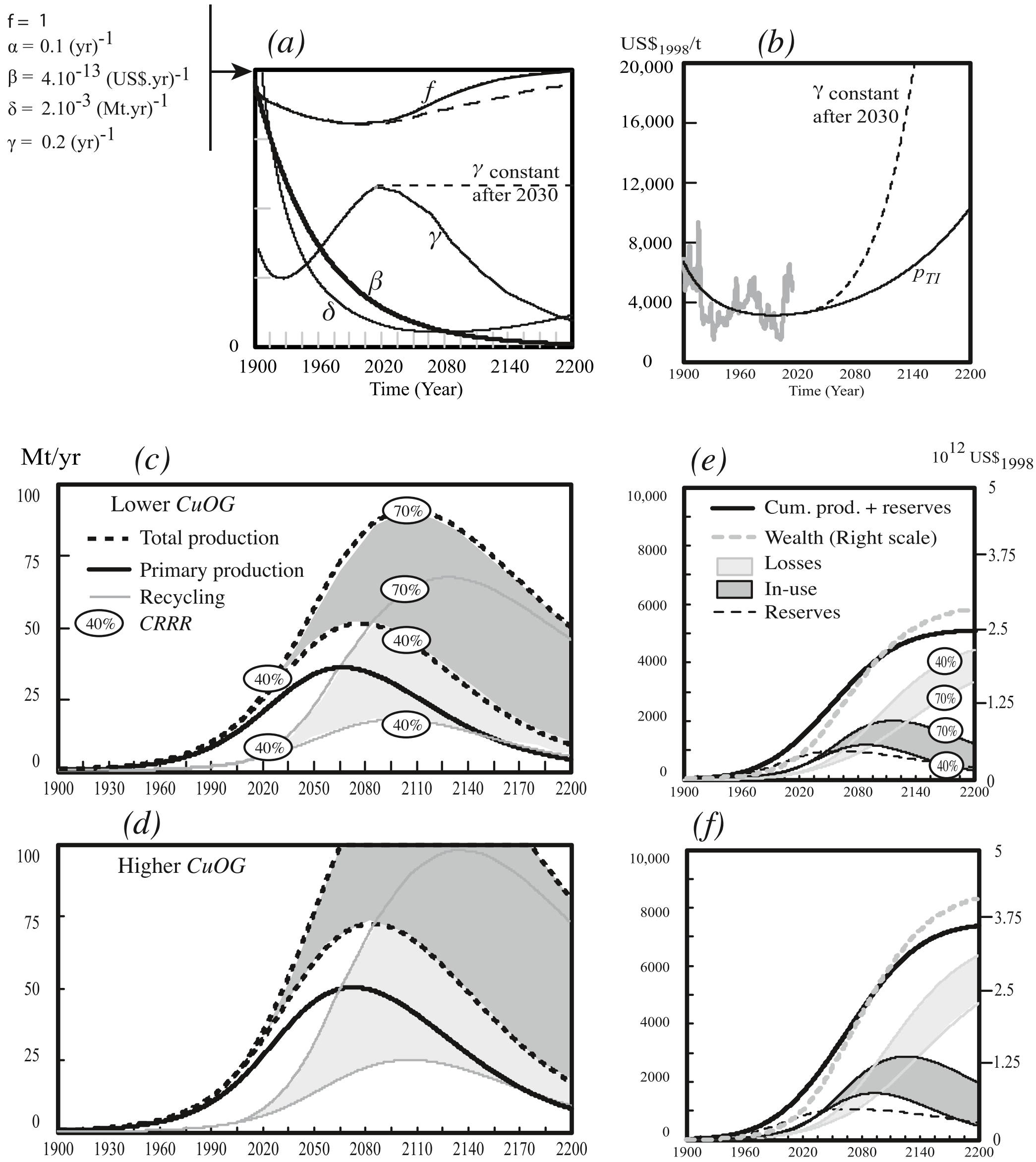

Figure 7. Evolution of (a) the model variables β, δ, γ, and f, (b) reference price p_{TI}, (c) and (d) production, and (e) and (f) wealth, reserve, and in-use and lost stocks, for the business-as-usual scenarios. In (a) and (b), the evolutions of γ and f are shown by continuous and dashed lines, for imposed reference price p_{TI} or constant γ after 2030, respectively. (c) and (e) were computed using the low regeneration path for Cu_{OG}; (d) and (f) were computed using the high regeneration path for Cu_{OG}. The gray areas in (c) to (f) show the differences in total production, recycling, in-use, and lost copper when calculated for recycling rate CRRR = 40% or 70% in 2100, respectively.

original price-versus-dilution relationship proposed by Johnson[65]

$$p_{CT}(OG) = 4700 \times OG^{-0.7} \tag{26}$$

or as a function of time (Figure 5e)

$$p_{CT}(t) = 10^{-53} t^{17.2} \tag{27}$$

The effect of technological improvements on price ETIP (Figure 5f) calculated as the ratio $\frac{p}{p_{CT}}$ varies exponentially with OG and time. The reference price of copper p_{TI} (Figure 5e) incorporating both the effects of embodied energy increase with lowering ore grade at constant technology and technological improvements can be calculated using p_{CT} and the exponential fit of ETIP ($ETIP_{fit} = 0.25\ e^{-0.678OG}$) as

$$p_{TI} = p_{CT} \times ETIP_{fit} \tag{28}$$

The results of the calculation show that p_{TI} follows a classical U-shaped curve with a first period of decrease between 1900 and 2010, when the improvements in technology overwhelm the negative effect of ore-grade drop (Figure 5e). During this period, p_{TI} decreases from 6300 US-$\$_{1998}$/tonne in 1900 to 2300 US-$\$_{1998}$/tonne in 2010, at a constant rate of −1%/year. This decay is of the same order of magnitude as the decay in embodied energy observed for steel and aluminum production from 1900 to 2010[66,69] and for refined copper produced from porphyry between 1963 (94.5 MJ/kg[70]) and 2013 (57 MJ/kg[71]). The situation is different after 2010, when the negative effect of dilution overwhelms the positive effect of technological improvements. The combined effects of technological improvements and OG reduction result in a decrease in ETIP

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(Figure 5f), and after 2020, p_{TI} does not decrease anymore but increases at a rate of 0.6 to 0.8 %/year.

Naturally, p_{TI} is a reference price that does not consider the demand/supply variations or any other event such as oil crisis, wars, economic competition, production monopoly, import tariffs and quotas, export controls, cartels, nationalization, and so forth. It also assumes that energy is available at a constant price of about 25 US-$\$_{1998}$/Brent-oil barrel, as it was the case in 1910, 1925, 1950, 1995, and 2005, the dates at which $p_{TI} = p$.

Exploring the Future Global Copper Production. Having constrained the evolution of the model variables (Figures 4 and 5), it is now possible to explore the future of copper production depending on the constraints on demand. In each set of scenarios, four cases are studied, which correspond to the four combinations of higher and lower regeneration of reserves with higher and lower CRRR.

The Business-as-Usual Scenarios (No Constraint on the Demand Side). In this set of scenarios, the primary production is calculated with β_{OG}, δ_{OG}, γ_{OG} (Figure 7a), and p_{TI} (Figure 7b) derived from the above historical analysis, for the low and high rates of reserve regeneration (Figure 7c,d). A per-tonne profit of 600 US-$\$_{1998}$/tonne was assumed to reduce the difference between the total assets reported in PwC reports and the modeled wealth. The modeled reserves (Figure 7e,f) follow the historical data and increase until the date of the inflection point of the ore-tonnage-versus-time curve shown in Figure 4. After this date, the growth of reserves with time is not exponential anymore, and reserves are consumed faster than they regenerate if production keeps growing at a constant rate. The peak of reserves is followed 10 years later by the peak of primary production at 37−45 Mt/year, in fair agreement with the date and magnitude of production peaks estimated by various authors.[9−11,13,32] The production then declines to 4.3 Mt/year in 2200 (Figure 7c), while 445 Mt of reserves is still available. The reserves in 2200 are thus equal to the reserves in 1992, when the production was close to 9 Mt/year. This observed decline of the production/reserve ratio (=β_{OG}*W) indicates that the 7-fold increase in wealth from 1992 to 2200 does not balance the effect of lowering ore grade on β_{OG}. The necessary investment to cope with the decrease of ore grade cannot be achieved for the expected evolution of p_{TI} and future costs of production.

Similar results are obtained for both the high and low evolutions of Cu_{OG}: the exponential growth of total copper production cannot be maintained for very long. For the low Cu_{OG} evolution, the 80 Mt/year of estimated total demand in 2100 is not met by production if the recycling rate of copper remains at the present value (CRRR = 40%). To satisfy the demand, 50 Mt/year of primary copper is needed from 2100 onward, which is not compatible with the expected peak of production at 37 Mt/year in 2070. About 50 Mt/year of primary copper can be produced for the high Cu_{OG} evolution, so that the total production in 2100 is close to the needed 80 Mt/year. However, the rapid decline of primary production after this date would not compensate the losses of recycling, which are significant for CRRR = 40%. This is illustrated in Figure 7e and f, which shows that the cumulative amount of lost copper becomes higher than the stock of copper in-use after 2060-2070. The only way to reduce the amount of lost copper and the demand for primary copper is to increase the share of recycling. Increasing CRRR from 40 to 70% between 2015 and 2100 postpones the peak of total production by 40 to 50 years. However, production decreases rapidly after the peak and tends to zero in the first half of the XIInd century.

At constant per-tonne profit, the rate of wealth erosion γ_{OG} is calculated to decrease after 2030 (Figure 7a), which implies that the industry is able to decrease the proportion of its costs relative to the size of its wealth ($\gamma = c/W$ in eq 6). The effect on price of a γ_{OG} assumed constant after 2030 is illustrated by the dashed line in Figure 7b. This case would reproduce a situation where energy price increase was compensated by labor cost cuts. A third situation can be modeled by forcing the price to follow p_{TI} and γ_{OG} to remain constant after 2030. In that case, the calculated per-tonne profit becomes rapidly negative because the costs become higher than the revenues. The industrial wealth is consumed, which is equivalent to bankruptcy, and the peak of production occurs earlier and is lower than in the previous cases. Naturally, this last situation is very unlikely at the global scale, but it applies at the local scale, when the market price of copper is too low for mines to cover their local costs of production.

These results suggest that irrespective of the increasing environmental consequences associated with copper production from more diluted sources, the business-as-usual primary production cannot be maintained long on historical trends. This conclusion is in line with numerous previous works, including those using Hubbert's approach.[9,11−13] The peak and later collapse of production are due to the departure of the ore-tonnage-versus-time curve from an exponential growth. The declining quality of reserves is the second reason. For ETIP shown in Figure 5f, the increasing costs of production after 2020 are no longer compensated by technological improvements. If the mining industry is not able to reduce the rate of wealth erosion γ_{OG}, a collapse of production will result from the impossibility to maintain the conditions of an economically viable extraction without a huge increase of price.

Leveled-off Demand Scenarios. In contrast with the previous scenarios where the production was estimated for a known evolution of β_{OG}, the efficiency of wealth to produce copper at fixed demand β_{FD} is now adjusted so that total production does not exceed the leveled-off demand. To reduce production for the same levels of reserves and wealth, β_{FD} must be lower than β_{OG} (Figure 8a). As the regeneration of reserves is still constrained by eq 19, lower production results in a higher available copper stock than in the previous scenarios (Figure 9). This situation lasts until the regeneration rate begins to decline, when the ore grade of exploited deposits falls below 0.3%. At this stage, the stock of reserves also begins to

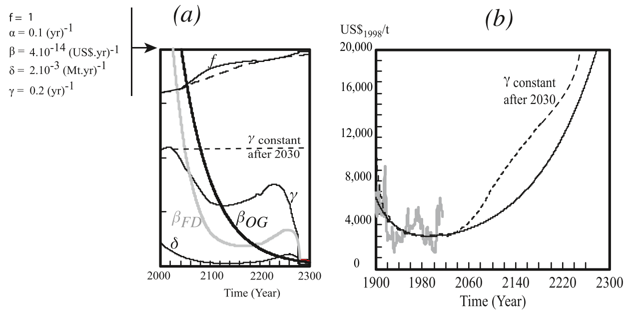

Figure 8. Evolution of (a) the model variables β_{OG}, β_{FD}, δ, γ, and f and (b) price where the gray, black, and dashed lines show historical data, conditions used to follow the reference price p_{TI}, and the case of constant γ_{OG} after 2030, respectively.

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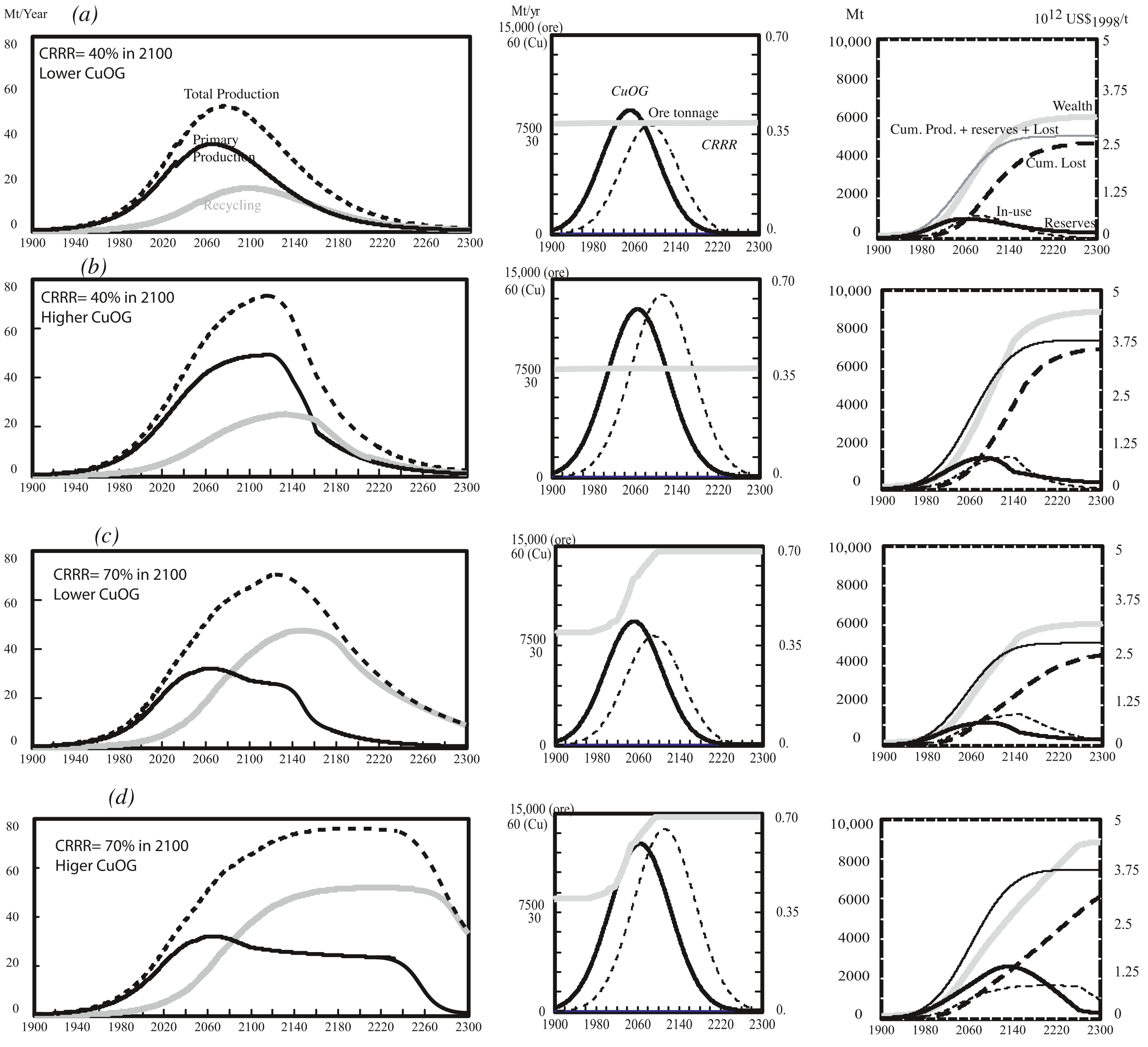

Figure 9. Evolution of production, regeneration, and copper and wealth stocks for the four scenarios of leveled-off demand (different recycling rates CRRR and regeneration Cu_{OG}).

decline, as the consumption of reserves (primary production) no longer balances its regeneration. To compensate for the decrease in reserves while maintaining the level of production, β_{FD} becomes equal to and finally slightly higher than β_{OG} (Figure 8a). This evolution of β_{FD} is possible because fewer reserves were consumed between 2020 and 2100 than in the business-as-usual scenarios, so the average OG of the remaining reserves is slightly higher. As a result, β_{FD} does not have to decrease over time at the same rate as β_{OG}. However, β_{FD} cannot remain larger than β_{OG} for very long, it eventually decreases rapidly and becomes equal to β_{OG} when the average OG and reserve stock are equal to those calculated in the business-as-usual scenarios (Figure 8a). The rapid decline of β_{FD} is illustrated by the equally rapid decline of primary production in 2120−2180 (Figure 9b,c) or 2240−2280 (Figure 9d). After this phase of production decline, production and reserve evolutions are controlled by the regeneration-versus-OG curve, as in the business-as-usual scenarios.

The results of the modeling with the four possible combinations of CRRR and reserve regeneration show quite contrasted trends. At low reserve regeneration and constant CRRR of 40% (Figure 9a), the evolution of production is identical to that observed in Figure 7c because the production modeled with β_{OG} did not exceed the leveled-off demand. At low reserve regeneration and high CRRR (70% in 2100, Figure 9c), much less primary copper is needed, but primary production still collapses from 2140 onward. The only way to maintain total production at the level of the expected demand until 2260 is to combine a high level of recycling with a high regeneration of reserves (Figure 9d). In this case, the classical pattern of a sudden peak in primary production followed by a collapse before the end of the century is avoided. This does not mean that sustainable copper production is assured in the very long run, and even in this optimistic

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scenario, the rapid decline in traditional reserves (volcanic massive sulfide and sediment-hosted ores, sulfide and oxide porphyry) after 2200 leads to a collapse in production after 2260.

In the case of high recycling and regeneration rates, γ_{FD} is found to decrease after 2030 to follow the reference price p_{TI} at constant per-tonne profit of 600 US-$\$_{1998}$/tonne. However, this drop in γ_{FD} is less pronounced than in the business-as-usual scenarios and the increase in price to keep γ_{FD} constant after 2020 is much lower. The price reaches 8000 US-$\$_{1998}$/tonne in 2100, half the price estimated in the business-as-usual scenarios, a value probably acceptable without a strong impact on demand if copper remains hardly substitutable by cheaper metals for the same functionality.[72]

The comparison of Figure 9b,c shows that increasing CRRR from 40 to 70 % has almost the same effect on total production as a 50% increase in primary reserves. However, the environmental impacts are very different in both cases, as recycling is much less energy- and water-intensive than primary production; these criteria will be of the utmost importance in a context of climate change mitigation and adaptation. In addition, the cumulative amounts of metal lost would be significantly reduced, from 4000 Mt in 2100 or 7000 Mt in 2300 (Figure 9b) to 2500 or 4500 Mt (Figure 9c), respectively. These considerations are an urgent call for the implementation of an efficient metal collecting, processing, and recycling infrastructure.

Interests and Limitations of the Prey−Predator Dynamics. The prey−predator dynamics used in the present study is able to reproduce the 1900−2015 evolutions of copper production, reserves, price, costs of production, revenues, and profits of the copper industry, as well as the cost- and price-reducing effects of improved technologies and the cost- and price-increasing effects of decreasing ore grade. The model provides a simple way to link materials to monetary flows and stocks, which is critical to estimate the future of natural resources. All model parameters change with time, in response to the exponential decay of the average grade of exploited ore deposits. The ratio δ/β (prey death rate/predator birth rate) is constant in biological systems, while it corresponds to the price in our model. The price is therefore an adjustment variable that stabilizes or increases wealth creation (predator birth), while reserves (prey population) and production both decrease. This dynamics contributes to decouple copper production from the geological reality and the depletion of high-quality reserves.

These differences between the original prey−predator and the present reserve−wealth formalisms introduce complexity and uncertainties, which are certainly large but difficult to evaluate on the time horizon considered in the present study. In particular, the demand was assumed to be inelastic, which is not realistic and constitutes an obvious limitation of our modeling. Moreover, all of the discussed scenarios assume a constant long-term energy price, in the range of 25 US-$\$_{1998}$/Brent-oil barrel. Should the price of energy increase significantly in the future, the production costs and price of copper would increase more rapidly, and this would naturally affect the results.

Another important source of uncertainty concerns the rate of reserve regeneration. In the long run, Arndt et al.[19] recently argued that the distribution of copper in the crust is not bimodal but unimodal, in which case the OT-versus-OG relationship used in the present study would underestimate the growth of reserves at OG < 0.5%. Copper from the oceanic crust as well as deep continental deposits might further expand the future reserves. However, exploiting such resources requires significant investments in new technologies. This challenges the traditional belief that the cost-cutting effects of technology improvements observed in the past will continue in the future. Further improvements of technology are obviously possible, but to ensure steady growth in primary production over the long term, the annual rate of technological improvement will have to be higher than it has been over the past 50 years. The transformation of resources into reserves also depends on many parameters not considered in the above equations, including the geopolitical situation of producing countries, the environmental impacts of extraction, and the need for additional resources such as water. The latter is essential in remote producing regions that may be affected by significant changes in precipitation due to global warming.

A major source of uncertainty concerns future demand, for which we have assumed to follow past trends of per-capita consumption. Yet, new uses of copper and the shift to a numerical world where the share of renewable energy is increasing could deviate the trend. Similarly, there are uncertainties about future population and fertility rates, and the evolution of GDP is a matter of social choice.

Finally, an important question concerns the expected price of primary copper in a context of high recycling. Currently, the price of recycled copper follows that of primary copper. However, this situation could change if recycled copper becomes the most abundant source. Copper recycling is significantly less demanding in energy, and the eventual competition between recycling and primary production leading to a stabilization or even a decrease in copper price after 2050 could be detrimental for primary production.

■ ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge on the ACS Publications website at DOI: 10.1021/acs.est.9b03883.

Monetary data of prices, revenues, profits, and costs of production; monetary database (Table S1) (PDF)

■ AUTHOR INFORMATION

Corresponding Author

*E-mail: olivier.vidal@univ-grenoble-alpes.fr.

ORCID

Olivier Vidal: 0000-0001-8520-0359

Notes

The authors declare no competing financial interest.

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CHAPTER 2

Steady-State Extraction and Production Growth in a Goodwin-Class Model

INSUFFICIENT DATA FOR MEANINGFUL ANSWER.
— Isaac Asimov, *The Last Question*

The present chapter embeds the extracting sector, as modeled in the previous chapter, in the whole global economy. It aims at evaluating the consequences on long-term growth of that mining dynamics in a particular class of growth models.

Indeed, as neoclassical approaches based on the general equilibrium paradigm are subject to serious criticisms in light of the financial crisis (Stiglitz, 2011; Lavoie, 2014; Romer, 2016; Brancaccio and Saraceno, 2017), and as environmental constraints are gaining momentum in the public debate, post-keynesian economists attempted to find a common ground with ecological economists (Kronenberg, 2010). Some authors (Berg et al., 2015) proposed an input-output production combined with a stock-flow consistent model to explore the effects of energy price shocks in a monetary economy. Others (Dafermos et al., 2017) chose to combine the stock-flow consistency approach with Georgescu-Roegen's flow-fund model, building a tool that takes in consideration accounting and thermodynamic principles. This model is calibrated and used to analyze green finance policies. This literature is promising, yet still emerging and with no specific attention given to the root controversy on exhaustible resources.

Another emerging branch of the literature revives Goodwin's seminal model (Goodwin, 1967), which adapts the prey-predator dynamics to describe the interplay between the wage share and employment. This intuition was brought up to date by van der Ploeg's research program who extended these income distribution issues to a neoclassical production function (van der Ploeg, 1983, 1985, 1987). There, he describes Goodwin's model as a "beautiful theory" which "discusses the symbiotic contradictions of capitalism in one complete model of economic growth and perpetual cycles". It was later enriched (Keen, 1995) by adding with the dynamics of private debt. The model then displays the possibility for the economy to be trapped in the basin of a attraction of a deflationary long-run steady state. Various extensions were proposed more recently (Grasselli and Lima, 2012; Grasselli and Nguyen-Huu, 2016), including one version with a climate module that aims at assessing the effect of damages on long-term growth (Bovari et al., 2018). An interesting extension of this model describes the effect of factor substitution on the different basins of attraction (Bastidas et al., 2018). However, this literature neglected to investigate the effects of mineral resources scarcity on the structure of long-term equilibria, which is what we propose in this chapter.

For that purpose, we construct a stock-flow consistent continuous-time macro-dynamics where the scarcity of natural resources is made explicit through the simple prey-predator dynamics previously introduced. The main departures, here, is that mineral "wealth" is much more detailed and the mining industry is able to finance its activity by leveraging. Moreover, the complex dynamics of the birth/death parameters is neglected for simplicity. The mining sector extracts some natural resource that can only be discovered at finite speed and sells it to the final good sector which produces the consumption good out of capital, labor and this natural resource. At variance with the equilibrium-based approach, we consider a full-blown non-linear macro-dynamics, where the conditions for long-run equilibria to be reached can be made explicit. Apart from this departure with the tradition, we keep with most neoclassical features: production factors are substitutable and a supply-driven axiom is adopted, even though we acknowledge that the essential demand determination of output is missing and leave it for further analysis. The aim is to assess the ability of factor substitution to circumvent a collapse *à la* Meadows and to sustain an ever growing path. As a proof of concept, we identify conditions under which a stationary balanced path can be reached. We show that, conversely, whenever one of these conditions fails, the world economy may be attracted by a catastrophic long-run steady state due to the interplay between the scarcity of natural resources and the dynamics of debt. We further provide a calibration of the world production technology. For that purpose, we construct a proxy of the world economy by aggregating 37 countries, which account for 83% of the average world GDP between 1990 and 2017. Then, we estimate the parameters of the CES production function. There is an ongoing debate about the empirical measure of the elasticity of substitution between capital, K, and labor, L. Here, this issue is reexamined within a (K, L, M) framework, where production of the final consumption good also relies in a non-trivial way on the flow, M, of 'material'. We discuss factor's substitution and weight.

The chapter is organized as follows. The next section sets the scene by describing the fundamentals of our model. Section 3 is devoted to displaying its stock-flow consistency. The following section contains the analysis of the long-run dynamics of our model: our main two conclusions are presented there. Section 5 provides details about how we constructed our proxy of the world economy using a long-span dataset of world demographic, macroeconomic and geologic variables. A specific dataset for the mining industry is also constructed in order to analyze past evolutions of this crucial sector. The last section presents our estimation of the production function based on this data set.

1 The model

Consider a closed economy where a unique consumption good is produced out of materials, capital and labour. A material is extracted by a specific mining sector, and then sold to the rest of the economy. The material stands for any exhaustible natural resource entering into the production process of the consumption good, such as oil, sand or any mineral. When going to the data in section 5, we shall consider a more specific setting where the material will be interpreted as copper. Apart from the extractive sector, there are three other sectors in our

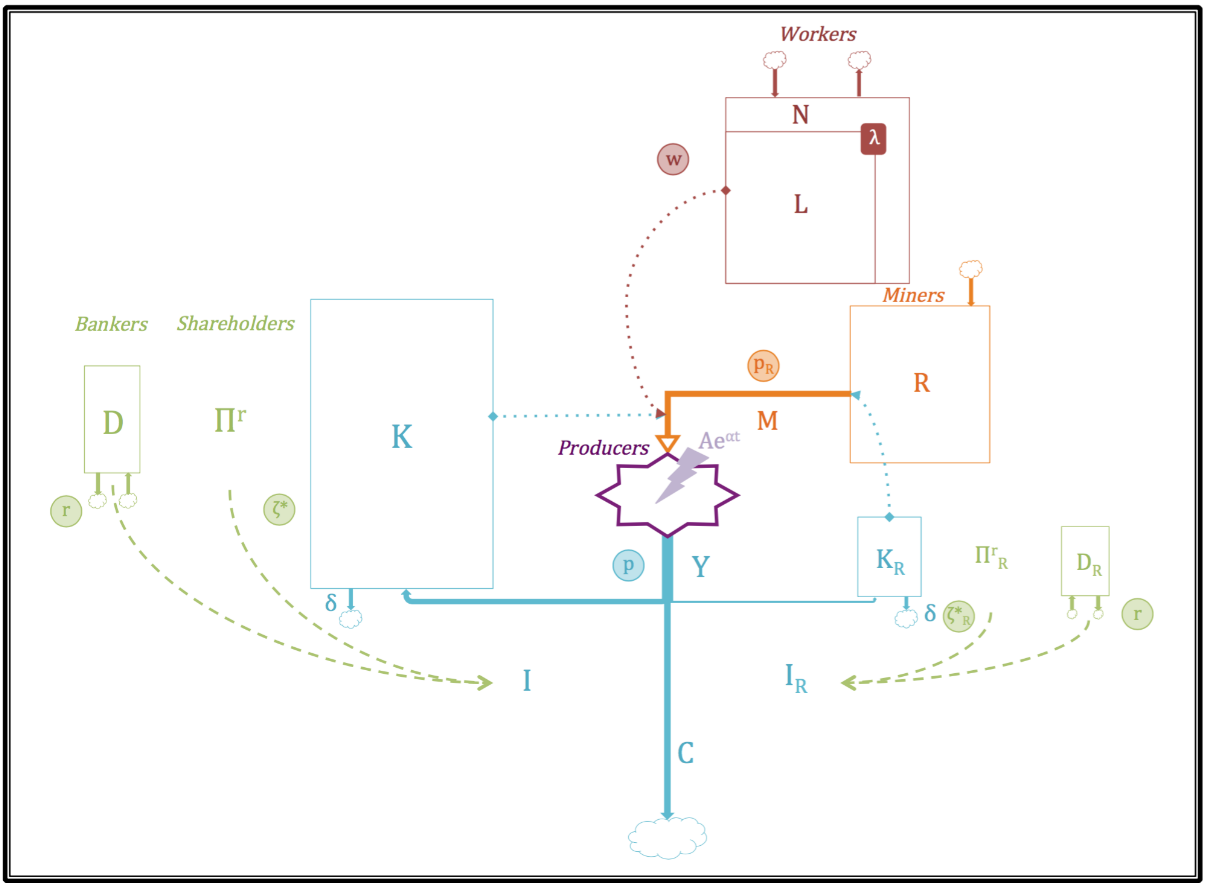

Figure 2.1: Sketch of the model engine. The boxes and pipes represent stocks and flows, respectively. Clouds are exogenous creations or losses, depending on the direction of the flow. Dots connectors represent actions (mining, worker or machine working). Dashed connectors are sources of funds. Circles surrounds factor compensations.

economy: households, final good producing firms, and banks. The government is omitted in order to keep the model as simple as possible.

Tab. 2.1 presents all variables and parameters used in the present model, along with their corresponding units. These units are important for the graphical representation of the stocks, flows and values in Fig. 2.1. Blue flows and stocks are consumption goods, orange ones are materials, red ones are workers and green ones monetary stocks and flows.

Let us begin with the mining sector.

1.1 Mining

The dynamics of the extractive sector is similar to the one studied in Chapter 1. It is based on a capital-intensive industry having at the starting time a capital stock, $K_R(0) > 0$, and a reserve, $R(0) > 0$. We call $M(t)$ the quantity of materials extracted at a given point of time $t \geq 0$, which is determined by an extraction function depending upon the current reserves, $R(t)$, the capital of mining firms, $K_R(t)$, and the (constant) extractive productivity, $b > 0$. For simplicity, the time index will be suppressed when there is no risk of confusion. Instantaneous extraction is given by:

Table 2.1: List of symbols

Notation	Variable/Parameter	Units
Goods production		
Y	output	consumption units/year
g	output growth rate	year^{-1}
M	extracted raw materials	tonnes/year
g_R	extraction growth rate	year^{-1}
L	total employed labor	beings
L^*	augmented labor	beings
K	installed capacity of production or capital	consumption units
K^*	material-capital aggregate	aggregated units
α	rate of unexplained change	year^{-1}
A	scaling parameter	conversion units
θ, ψ	distribution parameters	conversion units
ρ^*, ρ	substitution parameters	conversion units
ℓ	labor productivity	consumption units/(beings $\times$ year)
ν	material-capital-to-output ratio	aggregated units
C	consumption	consumption units/year
Mineral extraction		
R	reserves	tonnes
K_R	installed capacity of extraction or capital	consumption units
a	exploration efficiency	year^{-1}
b	extraction efficiency	(consumption units $\times$ year)$^{-1}$
Labor stabilization		
N	pool of people of working age	beings
N_{max}	long-term number of people of working age	beings
γ	population logistic growth rate	year^{-1}
$\beta(N)$	population growth rate	year^{-1}
Capital accumulation		
I	investment in the non-mining sector	consumption units/year
I_R	investment in the mining sector	consumption units/year
$\kappa(\pi)$	investment ratio	-
$\kappa_R(\pi_R)$	mining investment ratio	-
δ	depreciation rate	year^{-1}
Factors valuation		
p	consumption price	PPP/consumption unit
i	consumption price growth rate	year^{-1}
p_R	raw material price	PPP/tonne
i_R	raw material price growth rate	year^{-1}
φ	share of extraction costs in revenues	-
φ_0	extraction cost parameter	-
u, u_R	markups	-
η, η_R	relaxation coefficients	year^{-1}
w	wage	PPP/(year $\times$ being)
$\phi(\lambda)$	negotiated wage growth rate	year^{-1}
r	interest rate	year^{-1}
$\zeta(\pi), \zeta^*$	dividends payout ratio	-
$\zeta_R(\pi_R), \zeta_R^*$	mining dividends payout ratio	-
Profits calculation		
Π	profits before interests	PPP/year
Π^n	net profits	PPP/year
Π^r	remaining profits	PPP/year
Π_R	mining profits before interests	PPP/year
Π_R^n	mining net profits	PPP/year
Π_R^r	mining remaining profits	PPP/year
Δ	dividends	PPP/year
Δ_R	mining dividends	PPP/year
D	debt stock	PPP
D_R	mining debt stock	PPP
Transactions		
O_i	deposits of sector i	PPP
L_i	loans of sector i	PPP
A_i	equity of sector i	PPP
NW_i	net worth of sector i	PPP
S_i	savings of sector i	PPP/year
Ratios		
ω	wage share	-
λ	employment rate	-
d	debt-to-output ratio	year
d_R	mining debt-to-output ratio	year
m	raw material share	-
μ_R	extraction-to-reserve ratio	year^{-1}
μ_K	extraction-to-capital ratio	tonnes/(year $\times$ consumption unit)
μ_{K_R}	extraction-to-mining-capital ratio	tonnes/(year $\times$ consumption unit)
π	profit share	-
π_R	mining profit share	-
r_p	price ratio	consumption units/tonne

$$M = bRK_R. \tag{2.1}$$

The stock of fossil reserves, $R(t)$, increases with reserve discoveries and lowering ore grade through parameter $a > 0$ (which reflects the reserve quality), and decreases with extraction:

$$\dot{R} = aR - M. \tag{2.2}$$

For simplicity, we suppose that extractive firms are myopic, whereas the stock of extracting capital is used at full capacity, whatever being the market price of the material resource.[1] By selling the material $M(t)$ at price $p_R(t)$, extractive firms earn a profit before interest, $\Pi_R(t)$:

$$\Pi_R := (1 - \varphi)p_R M - p\delta K_R, \tag{2.3}$$

where $p(t) \geq 0$ is the price of mining capital, $\varphi(t) > 0$ is the share of the extracting cost of a unit of matter, and $\delta > 0$ is the (constant) depreciation rate. The net profit obtains after debt servicing with a fixed short-run, nominal interest rate $r > 0$:

$$\Pi_R^n := \Pi_R - rD_R, \tag{2.4}$$

where $D_R(t)$ denotes the current private debt of the mining sector. This net profit is used to compute the profit-to-revenue ratio,

$$\begin{aligned} \pi_R &:= \frac{\Pi_R^n + \delta p K_R}{p_R M} \\ &= 1 - \varphi - r d_R, \end{aligned} \tag{2.5}$$

where $d_R := \frac{D_R}{p_R M}$ is the debt-to-revenue ratio. The profit-to-revenue ratio will be the key determinant of aggregate investment, I_R, in the mining sector

$$I_R := \kappa_R(\pi_R) r_p M, \tag{2.6}$$

for some function $\kappa_R(\cdot)$ taking values in $(0, 1)$, and where $r_p := \frac{p_R}{p}$ is the price of natural resources normalized by the consumption price. As for the mining capacity, K_R, it varies according to the standard accumulation equation:

$$\dot{K}_R = I_R - \delta K_R. \tag{2.7}$$

Part of the net profit, Π_R^n, is distributed to the shareholders of the mining sector in proportion to the performance ratio, π_R, and the gross revenues, $p_R M$. Dividends from the extractive field are therefore given by

$$\Delta_R := \zeta_R(\pi_R) p_R M,$$

with $\zeta_R(\cdot)$ taking value in $[0, 1]$. The remaining profits

[1] Adaptive and consistent expectations are introduced in (Dossetto and Giraud, 2019) in a model without exhaustible resources. There, it is shown that dropping myopia does not qualitatively modify the phase space of the dynamics. A similar conclusion would presumably hold here but exploring this issue is left for further research. The usage rate of producing capital will be made endogenous in a subsequent section at the world level. For simplicity, we kept constant the utilization rate of mining capital in the extractive sector.

$$\begin{aligned}\Pi_R^r &:= \Pi_R^n - \Delta_R \\ &= \Big(1 - \varphi - \zeta_R(\pi_R)\Big) p_R M - r D_R - \delta p K_R,\end{aligned}$$

will provide self-financing to the mining sector. The possible gap between investment needs, pI_R, and self-financing is filled with additional debt:

$$\dot{D}_R = pI_R - (\Pi_R^r + \delta p K_R). \tag{2.8}$$

Depreciation is a cost in the net result but a source of liquidity in cash flows —hence its presence in the rhs of (2.8). Whenever $\dot{D}_R < 0$, this means that the remaining cash after investment has been financed is used by extractive firms to repay back the debt of the mining sector.

Let us now turn to the non-extractive world economy.

1.2 Production of the final good

At time t, the output on the final good market is $Y(t)$. It has been produced courtesy of some flow, $M(t)$, of materials and labor, $L(t)$, together with a capital stock, $K(t)$. The aggregate profit before interests, Π, earned by the final good sector, results from the firms' income minus the wage bill, the intermediate consumption bill and depreciation costs:

$$\Pi = pY - wL - (1 - \varphi) p_R M - \delta p K, \tag{2.9}$$

where $p(t)$ is the final good price[2] while $w(t)$ stands for the unitary money wage, and δ is the depreciation rate.[3] Notice that the extraction cost, $\varphi(t) p_R(t) M(t)$, is paid by the mining sector to the rest of the world, hence enters positively in the profit of the non-extractive production sector.

In a way similar to the mining sector, the net profit is calculated after interests have been paid:[4]

$$\Pi^n = \Pi - rD, \tag{2.10}$$

where D stands for the aggregate private debt of the final good production sector. The profit-to-output ratio

$$\begin{aligned}\pi &:= \frac{\Pi^n + \delta p K}{pY} \\ &= 1 - \omega - (1 - \varphi) m - rd \end{aligned} \tag{2.11}$$

can now be defined, where $\omega := wL/pY$[5], $m := M/pY$ and $d := D/pY$ are the wage-, intermediate-consumption- and debt-to-output ratios respectively. The current level of aggregate investment, $I(t)$, is set according to some continuous function, $\kappa(\cdot)$ of π:

[2] We therefore extend the standard, neoclassical simplification by identifying the price of (mining and non-extractive) capital with that of the final good.

[3] For simplicity, we assume the depreciation rates to be equal in the mining and final good sectors. Dropping this restriction would just complicate notations without altering any results to follow.

[4] The same remark as for δ holds for r, see the previous footnote.

[5] The wage share, ω, should not be confused with the unitary nominal wage, w.

$$I = \kappa(\pi)Y.$$

As usual, capital evolves according to $\dot{K} = I - \delta K$. Part of the net profit is distributed to shareholders of the final good production sector according to some continuous and increasing function, ζ, of π taking values in $[0, 1]$: $\Delta := \zeta(\pi)pY$. Remaining profits therefore write:

$$\begin{aligned}\Pi^r &:= \Pi^n - \Delta \\ &= (1 - \zeta(\pi))pY - wL - (1 - \varphi)p_R M - rD - \delta pK.\end{aligned}$$

Like in the mining sector, changes in the (private) debt of the final good production field reflect the adjustment between investment needs, pI, and self-financing after having taken due account of the remaining profits and depreciation:

$$\dot{D} = pI - (\Pi^r + \delta pK). \tag{2.12}$$

It readily follows from (2.12), that $\dot{D} = 0$ if, and only if, $p\dot{K} = \Pi^r$.

1.3 Labour

For the empirical assessment of our model, we shall assume that the workforce, N, follows the UN demographic median scenario of the world population at working age[6], which states that working population should reach a plateau around 11 billion people before the end of the century. Its dynamics will therefore be taken as following some exogenous "S-shaped" function:

$$\hat{N} = \gamma(1 - \frac{N}{N_{max}}) = \beta(N), \tag{2.13}$$

where, as usual, $\hat{x} := \dot{x}/x$.

The employment rate is defined as $\lambda := \frac{L}{N}$. The bargaining power of workers for wages is assumed to depend upon the employment rate λ, according to some continuous short-run Phillips curve, ϕ:

$$\hat{w} = \phi(\lambda). \tag{2.14}$$

The final good sector maximizes its instantaneous profit taking as given the set of prices, $p(t)$, for the final good, wages $w(t)$, the price $p_R(t)$ for the material resource and the short-run nominal interest, r, on its debt, as well as the stock of capital, $K(t)$, and the available flow of matter, $M(t)$. The final good sector chooses the quantity, L, of hired workers so as to maximize its profit, i.e., solves $\max_L \Pi^n$. The first order condition yields

$$\frac{\partial Y}{\partial L}(t) = \frac{w(t)}{p(t)}, \tag{2.15}$$

according to which firms set the optimal amount, $L^*(t)$, of hired labour. This is consistent with (van der Ploeg, 1985) reinterpretation of Goodwin's seminal dynamics. This profit-maximization assumption on the final good sector is our main departure with the literature

[6] That is, from 16 to 64 years old.

devoted to system dynamics and resource exhaustion, as well as with most of post-keynesian literature.[7]

1.4 Consumption

Since Say's law is assumed[8], the level, C, of aggregate consumption will adjust so as to clear the final good market:

$$Y = (I + I_R) + C. \tag{2.16}$$

The final good is sold on the consumption market, where it is given a price p. The price dynamics is set so that, if the latter was constant, p would converge toward some long-term value given by a markup $u \geq 1$ and a unit cost, $\omega + \Big(1 - \varphi)m$, capturing labor and intermediate raw materials costs, with a relaxation time $1/\eta > 0$. In other words, the inflation rate, i, is given by

$$i := \hat{p} = \eta\Big(u\Big[\omega + \Big(1 - \varphi)m\Big] - 1\Big)\mathbf{1}_{\{p>0\}}. \tag{2.17}$$

Since, however, the unit cost of production, $\Big[\omega+\Big(1-\varphi)m\Big]$, will endogenously vary across time, (2.17) provides a potentially rich price dynamics. Whenever the markup satisfies $u > 1$, this means that imperfect competition prevails on the consumption market. Since η is finite, prices do not adjust instantaneously to a change in the unitary cost, reflecting some stickiness which is reminiscent of (Calvo, 1983). This modelling is consistent with the empirical body of work on sticky prices provided by (Blinder, 1981), as well as with the view Classical economists of the early nineteenth century shared on market prices. For simplicity, the zero boundary works as an absorbing state: once it has been reached by p, inflation vanishes so that the price stays at zero for ever. While Solow's seminal model was a real economy, (2.17) is the main point of departure of our model with a purely neo-classical approach, where prices would be assumed to play the adjustment role for market clearing.

The extracted quantity of matter, M, is sold on the market for commodities, where it is given a value which will follow a similar dynamics. The price, p_R, of the resource R therefore fluctuates according to

$$i_R := \hat{p_R} := \eta_R(u_R\varphi - 1)\mathbf{1}_{\{p_R>0\}}, \tag{2.18}$$

where $\eta_R > 0$ is the relaxation parameter, $u_R \geq 1$ is the markup and $\varphi(t)$ is, as already said, the unit cost of extraction which aggregates all operating costs.

2 Stock-Flow consistency

Tab. 2.2 spells out the stock-flow consistency of the model. The economy is made of four sectors with households (subscript H), non-extractive firms dedicated to consumption and capital

[7]In (Meadows et al., 1972), for instance, prices and profits are absent from the modelling picture, whereas the behaviour of the production sector is assumed to follow some rule-of-thumb. On the other hand, in most of the literature devoted to Goodwin's model (including (Bovari et al., 2018)), production is assumed to be Leontief, so that no maximization program is needed to solve the trade-off between capital and labor.

[8]This restriction is dropped in (Grasselli and Nguyen-Huu, 2016). This turns out to deeply modify the phase space of the resulting dynamical system. This extension to our setting is left for further research.

goods production (subscript F if needed), mining firms (subscript R) and banks (subscript B).[9] The net borrowing of each sector is the difference between its loans, L, and its deposits, O.

We assume that both types of firms as well as banks are privately owned by households. As a consequence, the net worth of households is comprised of all equities $A_F+A_R+A_B$ as well as their own saving stock O_H-L_H. All other net worths are therefore equal to zero, since equities are balancing variables: $A_F = pK + O_F - L_F$, $A_R = pK_R + p_R R + O_R - L_R$ (mining firms own the extractive capital and mines as assets) and $A_B = (L_H-O_H)+(L_F-O_F)+(L_R-O_R)$ (loans are the assets of the banking sector, deposits are its liabilities). Hence, the total net worth or wealth of the economy, NW, is owned by households:

$$NW = NW_H = p(K + K_R) + p_R R.$$

Notice that the households' saving flow is given by:

$$S_H := -\dot{D}_H = \dot{O}_H - \dot{L}_H = (wL + \Delta + \Delta_R + \Delta_B - rD_H) - pC.$$

The savings of non-mining and mining firms are equal to their remaining profits, Π^r and Π^r_R, respectively. Whenever investment needs turn out to exceed their remaining profits, they contract loans from the banking sector:

$$p\dot{K} = \dot{D} + \Pi^r$$

$$p\dot{K}_R = \dot{D}_R + \Pi^r_R$$

Finally, the banking sector distributes all its profits (i.e., interests on net borrowings[10]) to its shareholders through dividends:

$$\Delta_B := r(D_H + D + D_R).$$

Therefore,

$$S_H = \Big(wL + \Delta + \Delta_R + r(D + D_R)\Big) - pC.$$

Note that $S_H + S_F + S_R + S_B = S_H + \Pi^r + \Pi^r_R = p(\dot{K} + \dot{K}_R)$, so that savings always equal net investments in the economy.

3 Exploring possible futures

Combining the neoclassical production function with the dynamics of its inputs leads to a complex non-linear dynamical system. In this section, we discuss the long-term qualitative behavior of this system by exploring possible steady states. The conditions for local stability of these complex equilibria are left for further analysis.

[9]Observe that, for simplicity, both mining and non-mining capitals are built by the non-extractive production sector.

[10]In words, net interest payments (the difference between interest received and interest paid) are equal to banks' income, because banks also have to pay interest to customers, who hold money in their bank accounts. Fees and commissions as further sources of income for banks are neglected.

Table 2.2: Stock-Flow Table

	Households (H)	Firms (F)		Mining Firms (R)		Banks (B)	Sum
Balance sheet							
Deposits	$+O_H$	$+O_F$		$+O_R$		$-(O_H + O_F + O_R)$	0
Loans	$-L_H$	$-L_F$		$-L_R$		$+(L_H + L_F + L_R)$	0
Fixed Capital		$+pK$		$+pK_R$			$+p(K + K_R)$
Natural Reserves				$+p_R R$			$+p_R R$
Equity	$+A_F + A_R + A_B$	$-A_F$		$-A_R$		$-A_B$	0
Sum (Net Worth)	NW_H	$NW_F = 0$		$NW_R = 0$		$NW_B = 0$	$NW = +p(K + K_R) + p_R R$
Transaction		**Current**	**Capital**	**Current**	**Capital**	**Current**	
Consumption	$-pC$	$+pC$					0
Investment		$+p(I + I_R)$	$-pI$		$-pI_R$		0
Intermediary Consumption		$-(1-\varphi)p_R M$		$+(1-\varphi)p_R M$			0
Accounting memo [GDP]		$[+p(C + I + I_R) - (1-\varphi)p_R M]$		$[+(1-\varphi)p_R M]$			$[pY]$
Wages	$+wL$	$-wL$					0
Capital Depreciation		$-p\delta K$	$+p\delta K$	$-p\delta K_R$	$+p\delta K_R$		0
Interest on Deposits	$+rO_H$	$+rO_F$		$+rO_R$		$-r(O_H + O_F + O_R)$	0
Interest on Loans	$-rL_H$	$-rL_F$		$-rL_R$		$+r(L_H + L_F + L_R)$	0
Dividends	$+\Delta + \Delta_R + \Delta_B$	$-\Delta$		$-\Delta_R$		$-\Delta_B$	0
Sum (Balance)	S_H	$S_F = \Pi^r$	$-p(I - \delta K)$	$S_R = \Pi^r_R$	$-p(I_R - \delta K_R)$	$S_B = 0$	0
Flow of funds							
Deposits	$+\dot{O}_H$	$+\dot{O}_F$		$+\dot{O}_R$		$-(\dot{O}_H + \dot{O}_F + \dot{O}_R)$	0
Loans	$-\dot{L}_H$	$-\dot{L}_F$		$-\dot{L}_R$		$+(\dot{L}_H + \dot{L}_F + \dot{L}_R)$	0
Fixed Capital		$+p\dot{K}$		$+p\dot{K}_R$			$+p(\dot{K} + \dot{K}_R)$
Sum (Savings)	S_H	S_F		S_R		S_B	$+p(\dot{K} + \dot{K}_R)$
Revaluation Matrix							
Fixed Capital		$+\dot{p}K$		$+\dot{p}K_R$			$+\dot{p}(K + K_R)$
Natural Reserves				$+p_R\dot{R} + \dot{p}_R R$			$+p_R\dot{R} + \dot{p}_R R$
Equity	$+\dot{A}_F + \dot{A}_R + \dot{A}_B$	$-\dot{A}_F = -(S_F + \dot{p}K)$		$-\dot{A}_R =$ $-(S_R + \dot{p}K_R + p_R\dot{R} + \dot{p}_R R)$		$-\dot{A}_B = 0$	0
Change in Net Worth	$+\dot{A}_F + \dot{A}_R + \dot{A}_B$ $+S_H$	0		0		0	$+p(\dot{K} + \dot{K}_R) + \dot{p}(K + K_R)$ $+p_R\dot{R} + \dot{p}_R R$

3.1 Reduced form of the dynamics

In order to make the analysis tractable, let us assume that the production function is of the form

$$Y = A\Big[\psi K^{*-\rho} + (1-\psi)L^{*-\rho}\Big]^{-1/\rho},$$

where $K^* := K^\theta M^{1-\theta}$, called "material-capital" hereafter, $L^* := e^{\alpha t}L$, while $\psi, \theta \in (0,1)$ and $\rho \in (-1, +\infty)$, as usual. In words, we consider a Cobb-Douglas functional form between K and M, nested in a constant-returns-to-scale CES production function together with labor-augmenting technological progress, improving at rate $\alpha \geq 0$.

Observe that the behavior of firms producing the final good is fully accommodating in the sense that, given the flow, M, of materials extracted by the mining sector, it will be automatically sold as an intermediate consumption for the final good sector, precluding more general specifications of the relationships between both sectors. This obvious shortcoming is consistent with the one that is common to almost all the literature dedicated to Goodwin's seminal model, where Say's law is in force: households always accommodate their consumption to production. Here, assuming Say's law on the intermediate consumption market can be partially justified by the fact that we want to find the simplest model of long-term growth with material constraints.

Motion equations

We first compute the motion of key variables of our economy. They will be useful when computing the reduced system of our dynamics.

Labor productivity

By derivating the production function with respect to L, we obtain:

$$\frac{\partial Y}{\partial L^*} = \frac{1-\psi}{A^\rho}\left(\frac{Y}{L^*}\right)^{1+\rho}.$$

Let $\ell := Y/L$ denote instantaneous labor productivity. The first order condition (2.15) leads to[11]

$$\ell = Ae^{\alpha t}\left(\frac{\omega}{1-\psi}\right)^{1/\rho}.$$

The growth rate of labor productivity is therefore equal to:

$$\hat{\ell} = \frac{1}{\rho}\hat{\omega} + \alpha.$$

Output

By derivating the production function with respect to K^*, we get

$$\frac{\partial Y}{\partial K^*} = \left(\frac{Y}{K^*}\right)^{1+\rho}\frac{\psi}{A^\rho}.$$

[11] Remember that $\omega := \frac{wL}{pY}$ refers to the wage share in the world output. Due to the partial substitutability between capital, labor and matter, it is endogenously determined by the whole economic dynamics.

Let ν designate the matter-capital-to-output ratio $\nu := K^*/Y$. The CES production function being homogeneous of degree one, the Euler equation gives:

$$\frac{\partial Y}{\partial K^*} = \frac{1}{\nu}(1-\omega).$$

Combining both expressions leads to:

$$\nu = \frac{1}{A}\left(\frac{1-\omega}{\psi}\right)^{-1/\rho}. \tag{2.19}$$

To emphasize that, at variance with several Goodwin-class models dealing with a Leontieff production function, the matter-capital-to-output ratio is a function of ω, we shall denote it $\nu(\omega)$. Its growth rate is:

$$\hat{\nu}(\omega) = \frac{1}{\rho}\frac{\dot{\omega}}{1-\omega}, \tag{2.20}$$

while the growth rate of matter-capital is given by:

$$\hat{K^*} = \theta\hat{K} + (1-\theta)\hat{M}.$$

Thanks to the capital accumulation dynamics, one gets:

$$\begin{aligned}\hat{K} &= \frac{\kappa(\pi)}{\nu(\omega)}\mu_K^{1-\theta} - \delta \\ &= \kappa(\pi)A\left(\frac{1-\omega}{\psi}\right)^{1/\rho}\mu_K^{1-\theta} - \delta,\end{aligned}$$

where $\mu_K := M/K$. Using (2.2), (2.6), and (2.7), we obtain the growth rate of the flow, M, of matter defined by (2.1):

$$\begin{aligned}g_R := \hat{M} &= \hat{R} + \hat{K_R} \\ &= a - \mu_R + \kappa_R(\pi_R)r_p\mu_{K_R} - \delta \\ &= a - \mu_R + \hat{\mu_R},\end{aligned} \tag{2.21}$$

where $\mu_{K_R} := \frac{M}{K_R} = bR$ and $\mu_R := \frac{M}{R} = bK_R$.

The output growth rate can now be written, courtesy of (2.20)

$$\begin{aligned}g := \hat{Y} &= \hat{K^*} - \hat{\nu}(\omega) \\ &= \theta\Big[\frac{\kappa(\pi)}{\nu(\omega)}\mu_K^{1-\theta} - \delta\Big] + (1-\theta)\Big[a - \mu_R + \hat{\mu_R}\Big] - \frac{1}{\rho}\frac{\dot{\omega}}{1-\omega} \\ &= \theta\frac{\kappa(\pi)}{\nu(\omega)}\mu_K^{1-\theta} + (1-\theta)\Big[a - \mu_R + \kappa_R(\pi_R)r_p\mu_{K_R}\Big] - \delta - \frac{1}{\rho}\frac{\dot{\omega}}{1-\omega}.\end{aligned} \tag{2.22}$$

Note that g is a function of $\pi, \pi_R, \omega, \mu_K, \mu_{K_R}, r_p$ and μ_R.

Wage ratio

By definition of the wage share ω, $\hat{\omega} = \hat{w} - \hat{\ell} - i$. Therefore,

$$\begin{aligned}\hat{\omega} &= \frac{\rho}{1+\rho}\left(\phi(\lambda) - \alpha - i\right)\\ &= \frac{\rho}{1+\rho}\Big(\phi(\lambda) - \alpha - \eta\Big[u\Big(\omega + \Big(1-\varphi\Big)m\Big) - 1\Big]\Big)\end{aligned}$$

Employment

By definition of λ, $\hat{\lambda} = \hat{Y} - \hat{\ell} - \hat{N}$. This leads to:

$$\begin{aligned}\hat{\lambda} &= g - \frac{1}{\rho}\hat{\omega} - \alpha - \beta\\ &= \theta\frac{\kappa(\pi)}{\nu(\omega)}\mu_K^{1-\theta} + (1-\theta)\Big[a - \mu_R + \kappa_R(\pi_R)r_p\mu_{K_R}\Big]\\ &\qquad\qquad - \frac{1}{\rho}\frac{\dot{\omega}}{\omega(1-\omega)} - \delta - \alpha - \beta. \end{aligned} \tag{2.23}$$

Debt ratio

Let $d := \frac{D}{pY}$ be the private debt-to-output ratio. The debt dynamics is given by:

$$\hat{D} = \frac{1}{d}\Big[\kappa(\pi) - \Big(1 - \omega - \Big(1-\varphi\Big)m - \zeta(\pi)\Big)\Big] + r, \tag{2.24}$$

where $m := \frac{p_R M}{pY}$. As a consequence,

$$\begin{aligned}\hat{d} &= \frac{1}{d}\Big[\kappa(\pi) + \zeta(\pi) - \pi\Big] - (g+i)\\ &= \frac{1}{d}\Big[\kappa(\pi) - \Big(1 - \omega - \Big(1-\varphi\Big)m - \zeta(\pi)\Big)\Big] - \theta\frac{\kappa(\pi)}{\nu(\omega)}\mu_K^{1-\theta} - (1-\theta)\Big[a - \mu_R + \kappa_R(\pi_R)r_p\mu_{K_R}\Big]\\ &\qquad + \frac{1}{\rho}\frac{\dot{\omega}}{1-\omega} + \delta + r - \eta\Big[u(\omega + \Big(1-\varphi\Big)m) - 1\Big] \end{aligned} \tag{2.25}$$

Remember that the debt of the extractive sector is not included in D. It will therefore follow a specific dynamics, to be described below.

Resource share

By definition of m, we have

$$\begin{aligned}\hat{m} &= g_R - g + i_R - i\\ &= \theta\Big[\kappa_R(\pi_R)r_p\mu_{K_R} + a - \mu_R - \frac{\kappa(\pi)}{\nu(\omega)}\mu_K^{1-\theta}\Big] + \frac{1}{\rho}\frac{\dot{\omega}}{1-\omega}\\ &\qquad + \eta_R\Big[u_R\varphi - 1\Big] - \eta\Big[u\Big(\omega + \Big(1-\varphi\Big)m\Big) - 1\Big]. \end{aligned} \tag{2.26}$$

Mining debt ratio

Let $d_R := \frac{D_R}{p_R M}$ be the debt ratio of the extractive sector. The debt dynamics is given by

$$\hat{D_R} = \frac{1}{d_R}\Big[\kappa_R(\pi_R) - \Big(1 - \varphi - \zeta_R(\pi_R)\Big)\Big] + r,$$

from which the dynamics of the debt ratio follows:

$$\begin{aligned}\hat{d_R} &= \frac{1}{d_R}\Big[\kappa_R(\pi_R) + \zeta_R(\pi_R) - \pi_R\Big] - (g_R + i_R) \\ &= \frac{1}{d_R}\Big[\kappa_R(\pi_R) - \Big(1 - \varphi - \zeta_R(\pi_R)\Big)\Big] - \kappa_R(\pi_R) r_p \mu_{K_R} + \mu_R - a + \delta + r - \eta_R\Big[u_R\varphi - 1\Big].\end{aligned} \tag{2.27}$$

The dynamical system

The dynamics boils down to a nine-dimensional system:

$$\begin{array}{rcl} \dot{\omega} &=& \omega\frac{\rho}{1+\rho}\Big(\phi(\lambda) - \alpha - i\Big) \\ \dot{\lambda} &=& \lambda\Big[g - \frac{1}{\rho}\hat{\omega} - \alpha - \beta\Big] \\ \dot{d} &=& \Big[\kappa(\pi) + \zeta(\pi) - \pi\Big] - d(g+i) \\ \dot{m} &=& m\Big[g_R - g + i_R - i\Big] \\ \dot{\mu_K} &=& \mu_K g_R - \mu_K\Big[\frac{\kappa(\pi)}{\nu(\omega)}\mu_K^{1-\theta} - \delta\Big] \\ \dot{\mu_R} &=& \mu_R\Big[\kappa_R(\pi_R) r_p \mu_{K_R} - \delta\Big] \\ \dot{d_R} &=& \Big[\kappa_R(\pi_R) + \zeta_R(\pi_R) - \pi_R\Big] - d_R(g_R + i_R) \\ \dot{\mu_{K_R}} &=& \mu_{K_R}\Big(a - \mu_R\Big) \\ \dot{r_p} &=& r_p\Big(i_R - i\Big), \end{array} \tag{2.28}$$

with the following auxiliary variables: the profit shares, π and π_R, defined by (2.11) and (2.5) respectively, the inflation rates, i and i_R (cf. (2.17) and (2.18)), and the extraction and production dynamics:

$$g_R = a - \mu_R + \hat{\mu_R}, \tag{2.29}$$

$$g = \theta\Big[\frac{\kappa(\pi)}{\nu(\omega)}\mu_K^{1-\theta} - \delta\Big] + (1-\theta)g_R - \frac{1}{\rho}\frac{\dot{\omega}}{1-\omega}. \tag{2.30}$$

Observe that the price ratio r_p is fundamental to understand the interplay between the mining and the non-mining sector, and raises an interesting discussion on the limitations of transdisciplinary work. This variable was added to have a model consistent in units, adopting a physicist's viewpoint. Indeed, in equation (2.6), the left-hand side is expressed in units of final good (or, equivalently, capital, as both are perfect substitutes), while M is expressed in tonnes of matter. From an economist's viewpoint, money is fungible, whatever its use. This is the reason why he/she usually pays little attention to the units involved in equations. If r_p is not used in equation (2.6), then the dynamics can be disentangled into two coupled dynamical subsystems:

$$\begin{aligned}
\dot{\omega} &= \omega\frac{\rho}{1+\rho}\Big(\phi(\lambda)-\alpha-i\Big)\\
\dot{\lambda} &= \lambda\Big[g-\frac{1}{\rho}\hat{\omega}-\alpha-\beta\Big]\\
\dot{d} &= \Big[\kappa(\pi)+\zeta(\pi)-\pi\Big]-d(g+i)\\
\dot{m} &= m\Big[g_R-g+i_R-i\Big]\\
\dot{\mu_K} &= \mu_K g_R-\mu_K\Big[\frac{\kappa(\pi)}{\nu(\omega)}\mu_K^{1-\theta}-\delta\Big]
\end{aligned}\tag{2.31}$$

and

$$\begin{aligned}
\dot{\mu_R} &= \mu_R\Big[\kappa_R(\pi_R)\mu_{K_R}-\delta\Big]\\
\dot{\mu_{K_R}} &= \mu_{K_R}\Big(a-\mu_R\Big)\\
\dot{d_R} &= \Big[\kappa_R(\pi_R)+\zeta_R(\pi_R)-\pi_R\Big]-d_R(g_R+i_R)
\end{aligned}\tag{2.32}$$

Indeed, the variables in (2.31) do not affect the variables of (2.32), so that the reduced system (2.32) can be solved separately. Moreover, the trajectories of (μ_R,μ_{K_R},d_R) arising as solutions of (2.32) can be treated as time-dependent coefficients for the differential equations of (2.31). Our contention, here, is to show the kind of complication that arises when doing a (small) step on the bridge of the disciplines. Going further in the direction of considering the difference between tonnes of matter and units of final good would ultimately lead us to revisit part of the Cambridge controversy, presumably forcing us to go beyond the modeling short-cut of a production function.

3.2 Long-term equilibrium analysis

Our aim in this section is to characterize long-run equilibria of (2.31) and (2.32). For that purpose, and to keep things tractable, we assume that the time-dependent function $\varphi(t)$ is such that $\lim_{t\to+\infty}\varphi(t)\equiv\overline{\varphi}$, where $\overline{\varphi}$ is a constant function, with value

$$\overline{\varphi}>\frac{1}{u_R},$$

which ensures that $i_R(t)>0$ as time grows to infinity. This amounts to imposing a lower-bound on extracting costs.

Furthermore, to guarantee that the employment rate satisfies $0 \leq \lambda(t) \leq 1$ at all times, we make the following standard assumption on the short-run Phillips function ϕ

$$\phi'(\lambda) > 0 \text{ on } (0,1), \quad \phi(0) \leq \alpha, \quad \text{and } \lim_{\lambda \to 1^-} \phi(\lambda) = +\infty.$$

Finally, to ensure the existence of a balanced path, we shall also need two minor restrictions. First,

$$u > \frac{\bar{i}_R}{\eta} + 1, \tag{2.33}$$

where $\bar{i}_R$ is given by (2.18) at the asymptotic value, $\overline{\varphi}$. This condition is satisfied for a wide range of economically plausible parameters[12]. Second,

$$\kappa(1) + \zeta(1) > 1, \tag{2.34}$$

which amounts to requiring that investment and dividends in the final good sector be sufficiently high whenever its profit share, π, peaks at 100%. Otherwise stated, as follows from (2.12), final good firms will feel sufficiently confident to borrow money for π close enough to 1.

Later on, we shall refer to these restrictions as standing assumptions.

Balanced paths

First, observe that whenever there is non-zero employment in the long-run, the equilibrium condition on λ in (2.28) implies $\overline{g} = \alpha$[13]. In other words, the non-extractive part of the economy must grow at the pace of labor productivity, as in every Solow-type model.

Second, as soon as $\mu_{K_R} > 0$ in the long-run, the equilibrium condition on this matter-to-mining-capital ratio in (2.28) implies $\overline{\mu}_R = a$: the matter-to-reserve ratio must equal the pace at which new discoveries are made to increase available reserves. Equation (2.29) then implies that $\overline{g}_R = 0$. At an interior equilibrium, the mining sector must remain stationary: the flow of extraction, M, is exactly equal to aR, the quantity of reserves newly discovered (see (2.2)), so that R remains constant as well.

Third, the condition for a non-slavery steady state ($\omega \neq 0$) is

$$\overline{\lambda} = \phi^{-1}(\alpha + \bar{i}). \tag{2.35}$$

Our standing assumptions on ϕ guarantee that a solution, $\overline{\lambda}$, to (2.35) exists, provided $\bar{i} \geq 0$[14].

Fourth, the long-run value of the resource inflation is $\bar{i}_R$, given by (2.18) at the asymptotic value, $\overline{\varphi}$.

Suppose $\alpha > 0$. At an interior steady state, since M remains constant while Y keeps growing in the long run, then, as long as p remains bounded away from 0, m converges exponentially to zero at speed $\alpha + \bar{i} - \bar{i}_R$. An obvious candidate for a balanced long-run path is therefore $\overline{m} = 0$ and $(\overline{\omega}, \overline{\lambda}, \overline{d}, \overline{\mu}_K)$ such that

[12] For instance, for $u \sim 1.3$ and $\bar{i}_R \sim 2\%$, (2.33) imposes $\eta > 6 \times 10^{-2}$.

[13] Since $\beta(N(t)) \to 0^+$ as $t \to \infty$, see (2.13).

[14] As an illustration, suppose $\alpha \sim 1\%$, $\overline{\omega} \sim 0.75$, $u \sim 1.35$ (as is often observed), then $u\overline{\omega} - 1 > 0$.

$$\overline{\omega} = 1 - \overline{\pi} - r\overline{d},$$

$$\bar{i} = \eta\Big(u\overline{\omega} - 1\Big),$$

$$\overline{d} = \frac{\kappa(\overline{\pi}) + \zeta(\overline{\pi}) - \overline{\pi}}{\alpha + \bar{i}}, \tag{2.36}$$

The long-run value of μ_K needs to be clarified. At an interior equilibrium,

$$\hat{Y} = \theta\hat{K} + (1-\theta)\hat{M} - \hat{\nu}(\omega) = \theta\hat{K},$$

so that

$$\hat{K} = \frac{\alpha}{\theta}.$$

If $\alpha > 0$, this means that K is exponentially growing at infinity. Therefore, $\mu_K \to 0^+$ as time goes. Observe that, at such a steady state, (2.22) would lead to $\overline{g} = -\theta\delta < 0$, in contradiction with $\alpha > 0$.

Suppose now $\alpha = 0$. For such a stationary path, $\overline{\lambda}$ is still given by (2.35). The equilibrium condition on λ in (2.31) becomes trivial ($g = 0$), so that the remaining 4-tuple $(\overline{\omega}, \overline{d}, \overline{m}, \overline{\mu}_K)$ is determined by the 3 equilibrium conditions on d, m and μ_K only. This indeterminacy leaves us with one degree of freedom, so that the manifold of stationary steady states can be indexed by $\overline{\omega} \in (0,1)$. Next, fix some provisional m, and solve the equilibrium condition on d for $\overline{\pi}$:

$$\begin{aligned}\kappa(\pi) + \zeta(\pi) - \pi &= \bar{i}\overline{d} \\ &= \frac{\bar{i}}{r}\Big(1 - \overline{\omega} - (1-\overline{\varphi})m - \pi\Big).\end{aligned}$$

The continuous mapping $f(\pi) := \kappa(\pi) + \zeta(\pi) - \pi - \frac{\bar{i}}{r}\Big(1 - \omega - (1-\overline{\varphi})m - \pi\Big)$ verifies $f(1) > 0$ because of (2.34), and $f(0) < 0$, provided $1 - \overline{\omega} - (1-\overline{\varphi})m > 0$. Therefore, under the latter condition, there exists some $\overline{\pi} \in (0,1)$ such that $f(\overline{\pi}) = 0$. The long-run debt ratio readily follows

$$\overline{d} = \frac{1}{\bar{i}}\Big(\kappa(\overline{\pi}) + \zeta(\overline{\pi}) - \overline{\pi}\Big).$$

The matter-to-output ratio, $\overline{m}$, obtains from $\bar{i}_R = \bar{i}$ (equilibrium condition on m),

$$\overline{m} = \frac{1}{u(1-\overline{\varphi})}\left(\frac{\bar{i}_R}{\eta} + 1 - u\overline{\omega}\right),$$

while $\overline{\mu}_K$ can be found from the equilibrium condition on μ_K:

$$\overline{\mu}_K = \left[\frac{\delta\nu(\overline{\omega})}{\kappa(\overline{\pi})}\right]^{\frac{1}{1-\theta}}.$$

Finally, the above stated condition, $1 - \overline{\omega} - (1 - \overline{\varphi})m > 0$, holds for $m = \overline{m}$ thanks to (2.33). Therefore, there is a one-dimensional manifold of stationary steady states, indexed by $\overline{\omega} \in (0, 1)$, compatible in the long run with the constraints imposed by the relative scarcity of matter.

The argument exposed in this section also hold for (2.28): no balanced path with positive growth is compatible with our equilibrium equations. Indeed, the mining sector would necessarily be stationary, as before, so that $\mu_K \to 0^+$, which would contradict the ever growing assumption of Y and K. Therefore, a balanced path can only occur for $\alpha = 0$.

Debt-deflationary equilibria

In order to ensure the existence of alternative equilibria to the curve of stationary balanced paths examined in the previous section, we assume an innocuous restriction on $\kappa(\cdot)$, namely that $\lim_{\pi \to -\infty} \kappa(\pi) =: \kappa(-\infty)$ exists in $\mathbb{R}$.[15]

Making the usual change of variable in each sector, $d \to q := 1/d$ and $d_R \to q_R := 1/d_R$ leads to a modified pair of dynamical systems for which $(\overline{\omega}_2, \overline{\lambda}_2, \overline{q}_2, \overline{m}_2, \overline{\mu}_{K2}) = (0, 0, 0, 0, 0)$ and $(\overline{\mu}_{R2}, \overline{\mu}_{K_R 2}, \overline{q}_{R2} = (0, 0, 0)$ turn out to be solutions, as can be readily checked.

Since the previous section has proven that a (stationary) balanced growth path is compatible with the material constraints faced by the mining sector, hence also by the world economy, this crash equilibrium cannot be interpreted as being solely due to the scarcity of matter. It is rather, as in the previous papers inspired from (Keen, 1995), a combination of this scarcity and an overhang of private debt.

4 Past evolution of world demographic, macroeconomic and geologic variables: dataset construction

4.1 World demographic and macroeconomic dataset

A large dataset synthesizing demographic, macroeconomic and geologic data was constructed from different sources. The data span mostly from 1980 to 2017 (precise time spans are indicated in Tab. 2.3 and 2.4) and comprise 37 countries, for whom long-span reliable data are available[16]. Tab. 2.3 presents the variables composing the dataset, their units and their sources. Economic variables are all expressed in 2011 real terms, using the 2011 US$-PPP benchmark, as compiled by the Penn World Table 9.1 research group. A long-span country-specific database for paid dividends is missing to have accurate estimates of this variable. Hence, the U.S. payout ratio, calculated from the FED data and substantiated by the literature, was chosen as a benchmark. The real long-term interest rate is a cross-country median.

These 37 countries account for about 83 and 78 % of the 1990-2017 average world GDP and copper production, respectively, but only 66% of the world population. This issue is even more significant in the long run since their population is expected to decline from 2050 onward while

[15]Assuming that investment becomes negative as $\pi \to -\infty$ amounts to saying that the non-extractive sector will divest in the hope of recovering some cash whenever its losses become too high.

[16]Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Czech Republic, Denmark, Finland, France, Germany, Greece, Hong Kong, Hungary, India, Indonesia, Ireland, Italy, Japan, the Republic of Korea, Luxembourg, Mexico, Netherlands, Norway, Poland, Portugal, the Russian Federation, Singapore, South Africa, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States.

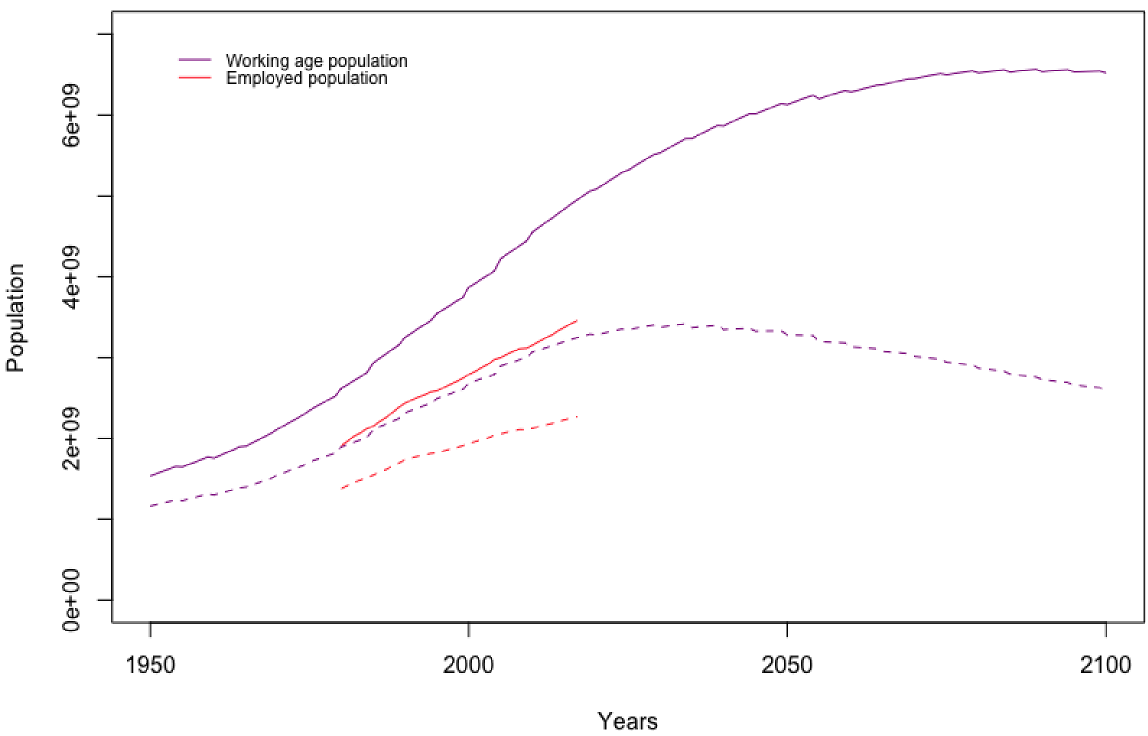

Figure 2.2: Evolution of the working age (purple) and employed population (red), for the 37 countries (dotted lines) and the world (plain lines).

total population should stabilize (Fig. 2.2) by the end of the century (medium fertility variant UN scenario). Variables were therefore scaled in order to fill the gap of missing data while keeping orders of magnitude in a correct range. Tab. 2.4 presents the variables composing the global scaled dataset, their units and their sources. The aggregate employment rate of the 37 countries, calculated from PWT and UN databases, was used to derive the world labor force[17]. All monetary flows were scaled by the 1990-2017 average ratio of the PWT world GDP to the PWT 37 countries GDP. This ratio was roughly stable at about 1.21. The world capital was deduced from the fundamental accumulation equation: K_{1980} is scaled by the 1.21 ratio, then capital accumulates through scaled investment and the world average depreciation rate. Private debt was calibrated so that the private-debt-to-output ratio be equal to that of the 37 countries. Geologic data were extracted from the USGS and BGS databases (Fig. 2.3).

[17] A more relevant measure of unemployment would be the number of actual hours of work. It should take into account the informal sector — a challenge for every statistics related to the labour market in the globalized South. This refinement of the measure used here is left for further research.

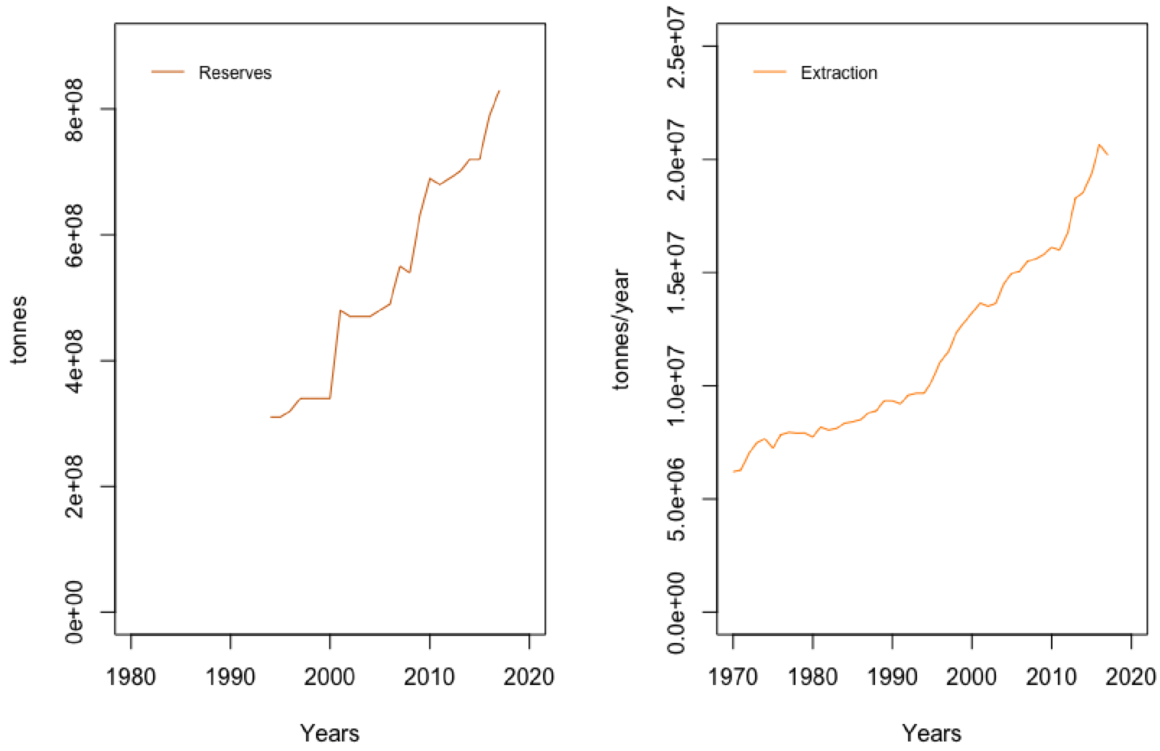

Figure 2.3: Evolution of the world copper reserves and extraction

Table 2.3: Description of the 37 countries database

Variables	Average 1990-2015	Units	Time span	Sources
Demographic variables				
Total population	4.19×10^9	beings	1950-2100	United Nations (UN)
Population at working age	2.76×10^9	beings	1950-2100	UN
Employed population	1.99×10^9	beings	1980-2017	Penn World Table 9.1 (PWT)
Macroeconomic variables				
Capital	2.00×10^{14}	PPP_{2011}	1980-2017	PWT
Private debt	6.74×10^{13}	PPP_{2011}	1990-2017	International Monetary Fund (IMF)
Output-side GDP	5.50×10^{13}	PPP_{2011}/year	1980-2017	PWT
Taxes on products	3.46×10^{12}	PPP_{2011}/year	1990-2017	World Bank Indicators (WBI)
GDP at factor cost	5.15×10^{13}	PPP_{2011}/year	1990-2017	Calculated using GDP and Taxes
Investment	2.02×10^{13}	PPP_{2011}/year	1980-2016	Calculated using Capital and Depreciation
Wages	3.15×10^{13}	PPP_{2011}/year	1980-2017	PWT
Depreciation rate	3.85	%	1980-2017	PWT
Real long-term interest rate (cross-country)	2.37	%	1976-2016	BIS Working Papers
Payout ratio (USA)	61.86	%	1980-2018	Federal Reserve Economic Data
Geological variables				
Copper primary production	1.07×10^7	tons	1970-2017	British Geological Survey (BGS)

Table 2.4: Description of the scaled database

Variable	Average 1990-2015	Units	Time span	Sources
Demographic variables				
Total population	6.35×10^9	beings	1950-2100	UN
Population at working age	4.03×10^9	beings	1950-2100	UN
Employment rate	72.03	%	1980-2017	Calculated for 37 countries
Employed population	2.89×10^9	beings	1980-2017	Scaled
Economic variables				
Capital	2.44×10^{14}	PPP_{2011}	1980-2016	Calculated using scaled Investment
Private debt	8.16×10^{13}	PPP_{2011}	1990-2017	Scaled
Output-side GDP	6.69×10^{13}	PPP_{2011}/year	1990-2017	PWT
	6.65×10^{13}	PPP_{2011}/year	1980-2017	Scaled
Taxes on products	4.19×10^{12}	PPP_{2011}/year	1990-2017	Scaled
GDP at factor cost	6.23×10^{13}	PPP_{2011}/year	1990-2017	Scaled
Investment	2.44×10^{13}	PPP_{2011}/year	1980-2016	Scaled
Wages	3.81×10^{13}	PPP_{2011}/year	1980-2017	Scaled
Depreciation rate	3.85	%	1980-2017	PWT
Real long-term interest rate (cross-country)	2.37	%	1976-2016	BIS Working Papers
Payout ratio (USA)	61.86	%	1980-2018	Federal Reserve Economic Data
Geologic variables				
Copper reserves	4.96×10^8	tons	1950-2017	U.S. Geological Survey (USGS)
Copper primary production	1.37×10^7	tons	1970-2017	British Geological Survey (BGS)

4.2 World copper dataset

A financial database for the copper sector was constructed courtesy of the long-span data of production, price and costs, as well as the aggregate depreciation rate and the U.S. payout ratio (Tab. 2.5). All data resulting from this process were substantiated with the partial data of the PwC financial reports on the mining sector. The stock-flow consistent model described in section 1 was used to link the variables. Note that data are in real terms, hence all corresponding model variables must be divided by the consumption price p. In nominal terms, the revenues are given by $p_R M$, derived from the data of copper production and unit price. Production costs are given by the unit cost C1, representing all costs incurred at mine level, from mining through to recoverable copper delivered to market, minus net by-product credits. Depreciation costs are recovered from the unit operating cost C2, which is the sum of C1 and depreciation and amortization. Financial and other costs are deduced from the unit total cost C3, which is the operating cost C2 plus corporate overheads, royalties, other indirect expenses and financial interests (Fig. 2.4). Net profits are therefore deduced from revenues and total costs. Using the U.S. payout ratio, the level of distributed dividends can be estimated. Remaining profits are the difference between net profits and dividends (Fig. 2.5). The capital of the copper sector is estimated from the depreciation costs and the aggregate depreciation rate. Note that the average capital-to-revenue ratio is 2.88, which is quite consistent with the usual capital-to-output ratio discussed in the literature. Moreover, the share of copper capital in the total world capital is 0.07% on average, which is quite consistent with the 0.14% of copper production value in the global GDP of our 37 countries in 2011. Investment is derived from the fundamental accumulation equation. Changes in debt are deduced from the difference between investment and the sum of remaining profits and provisions. The level of debt is obtained from the debt dynamics and by estimating D_{1980} with the financial cost in 1980 divided by r=3%. The average copper-to-world debt of 0.22 % is an acceptable order of magnitude.

4.3 World non-copper dataset

A financial database for the non-copper sector was constructed by using the scaled total productive data and the financial variables from the copper sector (Tab. 2.6). The time evolution of investments and profits is displayed on Fig. 2.6. Again, we relied on our stock-flow consistent model to link the variables to one another. Note that like before, the data are in real terms, hence all corresponding model variables must be divided by the consumption price p. The stock-flow consistency is validated: the two ways to calculate the change in debt are of the same order of magnitude.

Table 2.5: Description of the financial copper database

Variable	Model	Average 1990-2015	Units	Time span	Sources/Calculus
Source variables					
Copper primary production	M	1.37×10^{7}	tons	1970-2017	British Geological Survey (BGS)
Copper price	$\frac{p_R}{p}$	4666	PPP_{2011}/ton	1980-2017	Average LME price (USGS)
Copper costs:				1980-2016	Aguirregabiria and Luengo (2016)
Production costs C1	$\varphi \frac{p_R}{p}$	2022	PPP_{2011}/ton		Chilean Copper Commission
Operating costs C2	$\varphi \frac{p_R}{p} + \delta \frac{K_R}{M}$	2468	PPP_{2011}/ton		
Total costs C3	$\varphi \frac{p_R}{p} + \delta \frac{K_R}{M} + r \frac{D_R}{pM}$	2942	PPP_{2011}/ton		
Payout ratio (USA)	ζ_R^*	61.86	%	1980-2018	Federal Reserve Economic Data
Depreciation rate	δ	3.85	%	1980-2017	PWT
Constructed variables					
Revenues	$\frac{p_R}{p} M$	6.82×10^{10}	PPP_{2011}/year	1980-2017	$\frac{p_R}{p} \times M$
Production costs	$\varphi \frac{p_R}{p} M$	2.85×10^{10}	PPP_{2011}/year	1980-2016	$C1 \times M$
Depreciation costs	δK_R	6.39×10^{9}	PPP_{2011}/year	1980-2016	$(C2 - C1) \times M$
Financial costs	$r \frac{D_R}{p}$	6.98×10^{9}	PPP_{2011}/year	1980-2016	$(C3 - C2) \times M$
Net profits	$\frac{\Pi_R^n}{p}$	2.63×10^{10}	PPP_{2011}/year	1980-2016	$(\frac{p_R}{p} - C3) \times M$
Paid dividends	$\frac{\Delta_R}{p}$	1.48×10^{10}	PPP_{2011}/year	1980-2016	$\zeta_R^* \times \frac{\Pi_R^n}{p}$
Remaining profits	$\frac{\Pi_R^r}{p}$	1.14×10^{10}	PPP_{2011}/year	1980-2016	$\frac{\Pi_R^n - \Delta_R}{p}$
Capital	K_R	1.61×10^{11}	PPP_{2011}	1980-2016	$\frac{(C2 - C1) \times M}{\delta}$
Investment	I_R	1.33×10^{10}	PPP_{2011}/year	1980-2015	Calculated using K_R and δ
Debt variation	$\frac{\dot{D}_R}{p}$	-4.51×10^{9}	PPP_{2011}/year	1980-2015	$I_R - \frac{\Pi_R^r + \delta p K_R}{p}$
Debt	$\frac{D_R}{p}$	1.53×10^{11}	PPP_{2011}	1980-2016	Calculated using $\frac{\dot{D}_R}{p}$

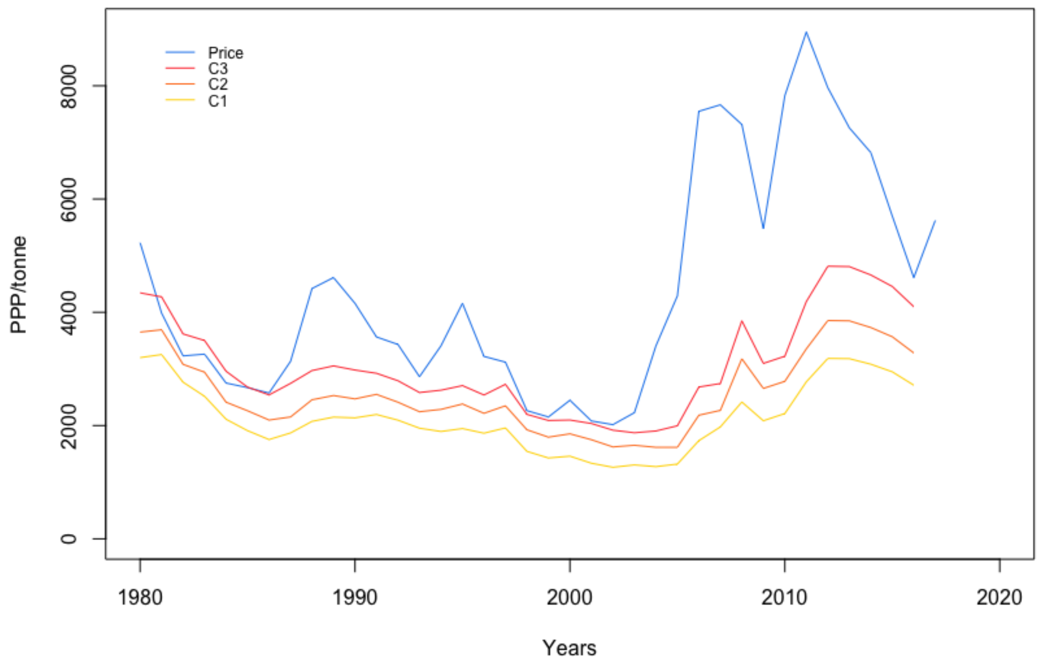

Figure 2.4: Evolution of copper unit prices and costs

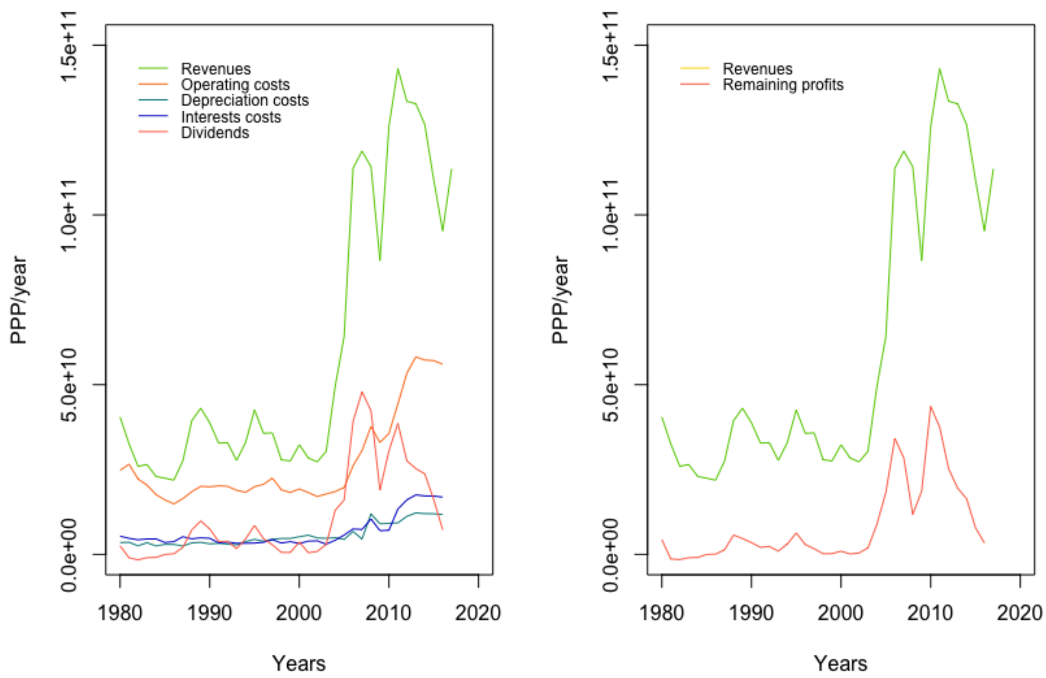

Figure 2.5: Evolution of the revenues and spendings of the copper sector

Table 2.6: Description of the financial non-copper database

Variable	Model	Average 1990-2015	Units	Time span	Sources/Calculus
Source variables					
Capital	K	2.44×10^{14}	PPP_{2011}	1980-2016	Total capital-K_R
Debt	$\frac{D}{p}$	8.14×10^{13}	PPP_{2011}	1990-2016	Total Debt-D_R
GDP at factor cost	Y	6.23×10^{13}	PPP_{2011}/year	1990-2017	Scaled
Wages	$\frac{w}{p}L$	3.81×10^{13}	PPP_{2011}/year	1980-2017	Scaled
Copper Revenues	$\frac{p_R}{p}M$	6.82×10^{10}	PPP_{2011}/year	1980-2017	Copper database
Copper Production costs	$\varphi\frac{p_R}{p}M$	2.85×10^{10}	PPP_{2011}/year	1980-2016	Copper database
Investment	I	2.44×10^{13}	$PPP_{2011}/year$	1980-2015	Total investment-I_R
Consumption	C	3.79×10^{13}	$PPP_{2011}/year$	1990-2015	Adjustment: $Y - I - I_R$
Depreciation rate	δ	3.85	%	1980-2017	PWT
Profits before interests	$\frac{\Pi}{p}$	1.45×10^{13}	PPP_{2011}/year	1990-2016	$Y - \frac{w}{p}L - (1-\varphi)\frac{p_R}{p}M - \delta K$
Real long-term interest rate	r	2.37	%	1976-2016	BIS Working Papers
Net profits	$\frac{\Pi^n}{p}$	1.30×10^{13}	PPP_{2011}/year	1990-2016	$\frac{\Pi}{p} - r\frac{D}{p}$
Payout ratio	ζ^*	61.86	%	1980-2018	Federal Reserve Economic Data
Paid dividends	$\frac{\Delta}{p}$	7.89×10^{12}	PPP_{2011}/year	1990-2016	$\zeta^* \times \frac{\Pi^n}{p}$
Remaining profits	$\frac{\Pi^r}{p}$	5.08×10^{12}	PPP_{2011}/year	1990-2016	$\frac{\Pi^n - \Delta}{p}$
Debt variation	$\frac{\dot{D}}{p}$	3.99×10^{12}	PPP_{2011}/year	1990-2015	Variation of $\frac{D}{p}$
		9.64×10^{12}	PPP_{2011}/year	1990-2015	$I - \frac{\Pi^r + \delta pK}{p}$

5 Estimation of the K-L-M production function

5.1 The CES production function with three production factors

The production technology in the final good sector through a CES production function with three factors: mineral resources, M (the quantity of extracted resources, reduced to copper in the present model, in tons), labor, L (the number of persons employed in this first version, or the number of working hours) and the capital stock, K. The latter can be interpreted as a stock of machines or the value of firms' assets. This functional form was chosen in order to show all substitution possibilities between inputs, following the Solovian line of thought.

Klump et al. (Klump et al., 2007) provide precious information on the sucCESs (*sic*) of this function in neoclassical growth theory. Humphrey and Moroney (Humphrey and Moroney, 1975) estimate the function using the cost share of inputs and discuss capital, labor and natural resources substitution. However, the cost share theorem is subject to strong criticisms (Frondel and Schmidt, 2002; Felipe and McCombie, 2005; Kümmel et al., 2008; Giraud and Kahraman, 2014; Pottier, 2014), the estimation of parameters using this method could be "statistical artefacts" and cannot therefore be interpreted as technical constraints to production (Felipe and McCombie, 2001). In the following, we propose an estimation of these parameters by directly using data of input physical quantity.

The general form of an n-input CES production function is:

$$Y = Ae^{\alpha t}\Big(\sum_{i=1}^{n}\theta_i X_i^{-\rho}\Big)^{-1/\rho}$$

where X_i correspond to the inputs of the production function. Technological change is captured by $Ae^{\alpha t}$, the time-dependent global factor productivity. $\theta_i \in [0, 1]$ correspond to the distribution parameters which govern how much the input factor i contributes to output, and $\rho > -1$ determines the elasticity of substitution.

5.2 Estimation procedure

The task we perform in this section is to estimate the parameters of the CES production function without relying on any assumption on the cost share of each input. Being not linear in its parameters, the CES cannot be linearised analytically, which is why many papers "guesstimate" its parameters or, at most, approximate them using the "Kmenta approximation" technique. We choose to use a direct estimation approach proposed by the R package **micEconCES** (Henningsen and Henningsen, 2011).**micEconCES** proposes a non-linear least-square estimation using different optimization algorithms. The Levenberg-Marquardt algorithm combines two non-linear optimization algorithms in order to increase the probability of converence of the approximation process.This method outperforms the others in estimating the distribution parameters, but it does as poorly in estimating the elasticity of substitution. By contrast, the so-called PORT-routine is a gradient-based optimization algorithm that allows for constraints on the parameter space. This package also allows for the estimation of Hicks-neutral technological change. Unfortunately, the current literature on factor-augmenting technological change estimation did not reach a state-of-the-art approach, which prevents us from having a factor-specific analysis of productivity. This task is left for further developments.

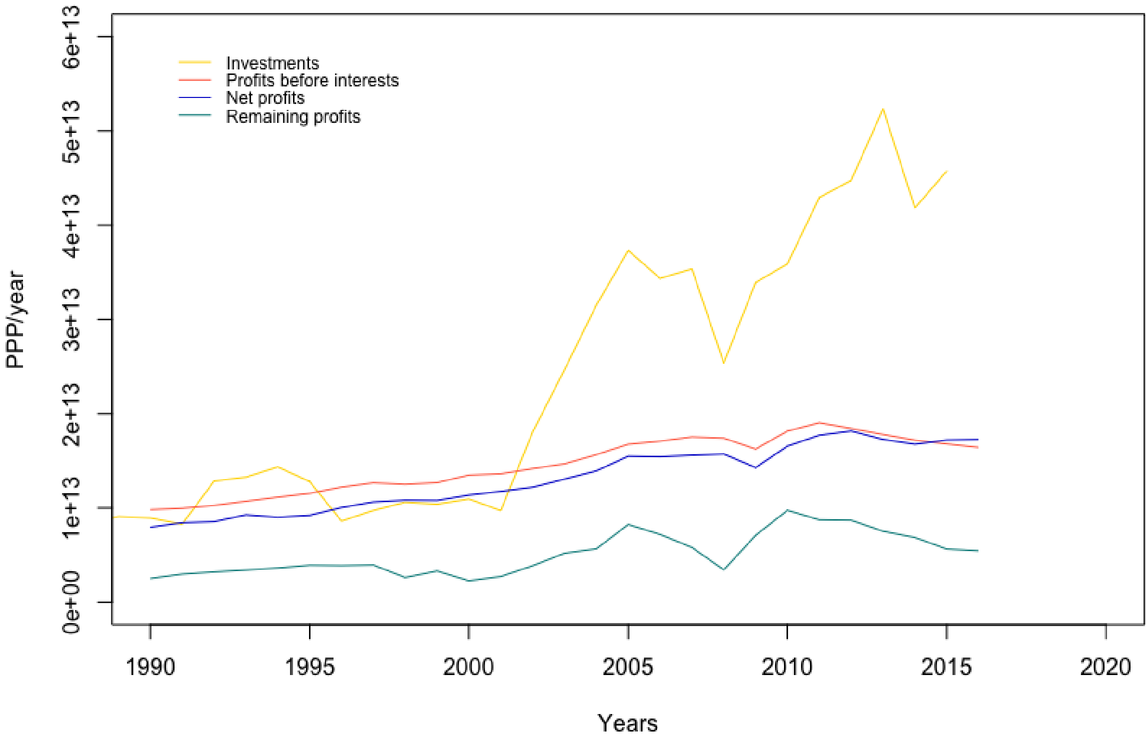

Figure 2.6: Evolution of the investments and profits of the non-mining sector

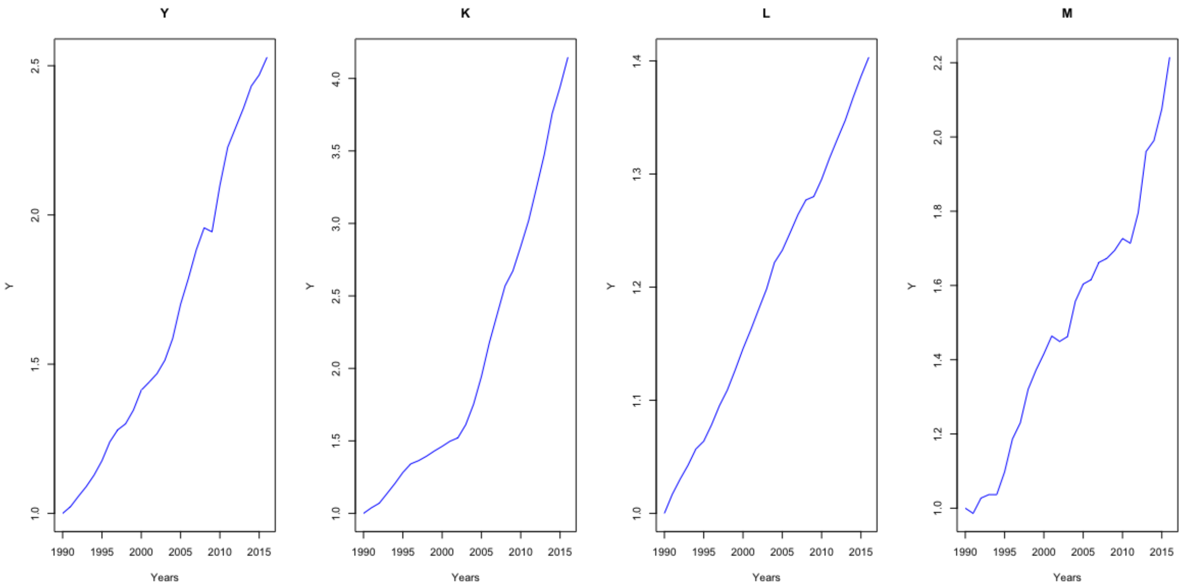

Figure 2.7: Plot of the data used for the estimation; Y is the dimensionied world output at factor cost, K is the scaled world non-copper capital, L is the number of employed people and M is the copper production.

Data were selected from the world dataset described in the previous section (Fig. 2.7). The non-mining capital stock, K, was matched with non-copper capital at the world level, whereas labor L is represented by the number of persons employed. Different options were tested for the less common input M. In the theoretical model of section 1 and the analysis of its dynamics, we chose to match M with the flow of extracted mineral resource (copper in our case). But M could also stand for the cumulative in-use amount of copper in goods, or the production weighted by the relative price of copper. Both output and inputs data were normalized by their starting-year value in order to avoid unit biases. Convergence was achieved in all runs.

As showed by Sato (Sato, 1967), multiple-input CES production function imposes strict conditions for the values of elasticities of substitution. We therefore chose a two-level nested CES, and different nesting structures were tested. The output of the final good sector is modeled by the following nested constant-returns-to-scale CES function with Hicks-neutral technological change:

$$Y = Ae^{\alpha t}\Big[\psi\Big(\theta X_1^{-\rho^*} + (1-\theta)X_2^{-\rho^*}\Big)^{\rho/\rho^*} + (1-\psi)X_3^{-\rho}\Big]^{-1/\rho} \tag{2.37}$$

where X_1, X_2 and X_3 stand for K, L or M. ψ and θ in $[0,1]$ are distribution parameters, and ρ and ρ^* in $[-1,+\infty)$ determine the (constant) elasticities of substitution: $\sigma^* = \frac{1}{1+\rho^*}$ is the elasticity of substitution between X_1 and X_2, and $\sigma = \frac{1}{1+\rho}$ the elasticity of substitution between the aggregate (X_1, X_2) and X_3.

5.3 Results

Our findings using the PORT routine are presented in Fig.2.8 and Tab.2.7. All nesting structures present the same residual error, but the estimation is more significant for the $(KL) + M$

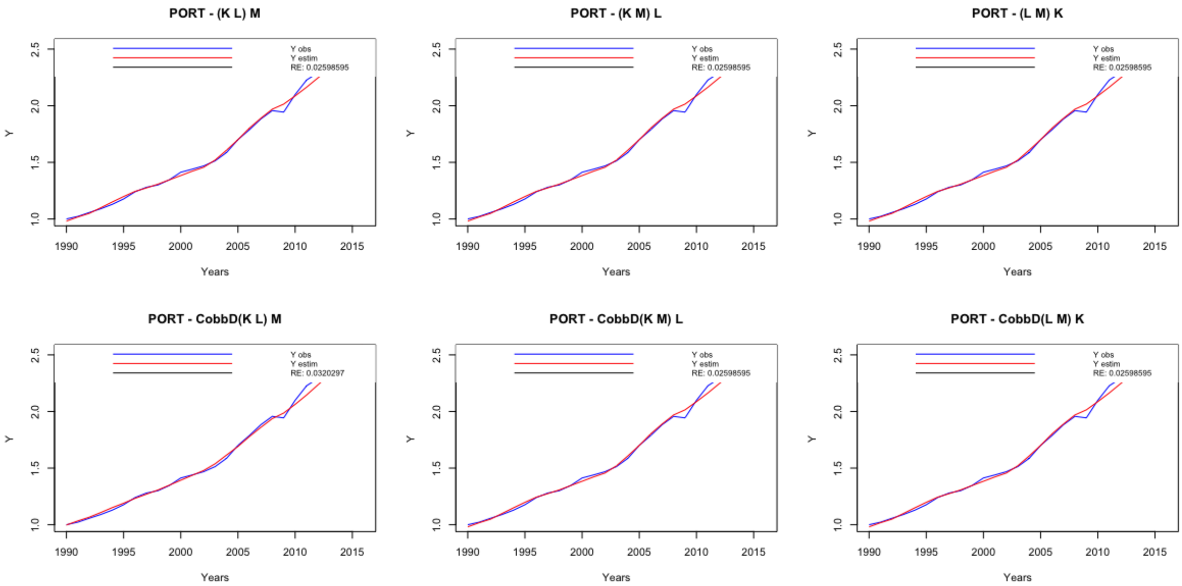

Figure 2.8: Y observed and estimated for the three-input CES with PORT routine

structure. They all tend to exclude M from the estimation[18].

In an attempt to prevent the optimization algorithm from systematically excluding M, the estimation was also conducted with the Levenberg-Marquardt routine. The results are presented in Tab.2.8. The structures $(KL) + M$ and $(KM) + L$ should be excluded from the analysis since the substitution parameters are out of their economic boundaries ($\rho < -1$ and $\rho^* < -1$, respectively). However, the $(KM) + L$ structure exhibits the smallest residual standard error and was therefore chosen to test different substitution possibilities between K and M, through the parameter ρ^*: $\rho^* = -1$ for the linear case, $\rho^* = 0$ for the Cobb-Douglas case and $\rho^* = 100$ (the upper-bound of the algorithm) to approximate the complementary case. Results are shown in Tab. 2.9. The Cobb-Douglas and complementary configurations should be excluded from the analysis since, again, the distribution parameters are out of their economic boundaries ($\theta > 1$ and $\psi < 0$, respectively).

These results therefore lead us to two plausible structures for the three-input CES[19]: the free $(L, M) + K$ nesting (Tab.2.8) or the $(KM) + L$ nesting with $\rho^* = -1$ (Tab. 2.9). The first choice corresponds to the following function [20]:

$$Y = A_n e^{\alpha t}\Big[\psi\Big(\theta L^{-\rho^*} + (1-\theta)M^{-\rho^*}\Big)^{\rho/\rho^*} + (1-\psi)K^{-\rho}\Big]^{-1/\rho} \tag{2.38}$$

The estimation of this structure shows quite high coefficients (ρ^* = 14.244 and ρ = 1.1489), which implies a very low substitutability between L and M, and low substitutability between the aggregate (L, M) and K. It gives a higher weight to L (θ = 0.5729) compared to M and to capital compared to the aggregate ($1 - \psi$ = 0.5366). The total factor productivity is very low compared to standard estimates (α = 0.66%).

The second option corresponds to the following production function, which is close to the

[18]That is, the distribution parameter associated to M is estimated to be equal to zero.
[19]All possible structures have not been explored however.
[20]A_n is a scaling coefficient since data were normalized.

Table 2.7: CES estimation: three inputs - PORT

Run	Residual Std Error	Parameters	Estimate	Std Error	P value
PORT	0.02599	A	0.9812	0.01194	***
$X_1 = K$		α	0.008585	0.004214	*
$X_2 = L$		θ	0.5390	0.1376	***
$X_3 = M$		ψ	1.0000	0.1222	***
		ρ^*	1.3172	0.3853	***
		ρ	0.2312	$5.824\ 10^{12}$	
PORT	0.02599	A	0.9812	0.01175	***
$X_1 = K$		α	0.008585	0.004280	*
$X_2 = M$		θ	1.0000	0.2624	***
$X_3 = L$		ψ	0.5390	0.1944	**
		ρ^*	0.2299	$1.428\ 10^{13}$	
		ρ	1.3172	0.4789	**
PORT	0.02599	A	0.9812	0.01126	***
$X_1 = L$		α	0.008585	0.004013	*
$X_2 = M$		θ	1.0000	0.1727	***
$X_3 = K$		ψ	0.4610	0.1097	***
		ρ^*	0.3448	$1.582\ 10^{13}$	
		ρ	1.3172	0.4403	**

Table 2.8: CES estimation: three inputs - Levenberg-Marquardt (LM)

Run	Residual Std Error	Parameters	Estimate	Std Error	P value
LM	0.02448	A	0.9941	0.01543	***
$X_1 = K$		α	0.001153	0.002523	
$X_2 = L$		θ	0.7921	0.1063	***
$X_3 = M$		ψ	0.5025	0.2211	*
		ρ^*	1.9783	0.5820	***
		ρ	-20.464	21.41	
LM	0.02301	A	1.0032	0.01325	***
$X_1 = K$		α	0.003523	0.004259	
$X_2 = M$		θ	0.3315	0.2650	
$X_3 = L$		ψ	0.7894	0.1831	***
		ρ^*	-8.740	10.434	
		ρ	2.5139	1.097	*
LM	0.02570	A	0.9838	0.01201	***
$X_1 = L$		α	0.006631	0.004265	
$X_2 = M$		θ	0.5729	0.6235	
$X_3 = K$		ψ	0.4634	0.08105	***
		ρ^*	14.244	42.202	
		ρ	1.1489	0.3979	**

Table 2.9: CES estimation: three inputs - Levenberg-Marquardt (LM) - Linear, Cobb-Douglas, Complementary

Run	Residual Std Error	Parameters	Estimate	Std Error	P value
LM	0.02582	A	0.9832	0.01299	***
$X_1 = K$		α	0.006456	0.004265	
$X_2 = M$		θ	0.8482	0.2724	**
$X_3 = L$		ψ	0.6484	0.1877	***
ρ^*=-1		ρ	1.6465	0.6591	*
LM	0.02596	A	0.9808	0.01249	***
$X_1 = K$		α	0.009477	0.004317	*
$X_2 = M$		θ	1.7244	0.4155	**
$X_3 = L$		ψ	0.4917	0.2021	*
ρ^*=0		ρ	1.2069	0.6012	*
LM	0.03337	A	0.9810	0.02090	***
$X_1 = K$		α	0.03363	0.002333	***
$X_2 = M$		θ	0.9987	0.8060	***
$X_3 = L$		ψ	-0.5161	0.1627	**
ρ^*=100		ρ	0.1082	0.4845	

one we adopted for our theoretical analysis in section 3:

$$Y = A_n e^{\alpha t}\Big[\psi\Big(\theta K + (1-\theta)M\Big)^{-\rho} + (1-\psi)L^{-\rho}\Big]^{-1/\rho} \tag{2.39}$$

The relationship between K and M in this structure is linear: they are perfect substitutes — which makes perfect sense economically since, actually, matter enters the production of capital. The estimation exhibits a coefficient $\rho > 1$, suggesting relatively low substitutability between the aggregate (K, M) and L. It gives a very important weight to K compared to M (θ = 0.8482) and an important weight to the aggregate KM compared to the L (ψ = 0.6484). The total factor productivity is, as in the previous structure, very low (α = 0.65%).

These results should however be taken with due care. First, the capital series is built according to the standard perpetual inventory method. The monetary aggregation implicit in this way of proceeding does not answer the crucial questions raised by the famous Cambridge Capital Controversy on the ambivalence of capital as a production capacity(Robinson, 1953; Cohen and Harcourt, 2003). In particular, a simple unit homogeneity analysis results in a crucial inconsistency in the combination of inputs of totally different natures such as infrastructure, real estate, production machines, financial assets, land, etc. This unit inconsistency highlights the lack of a proper modelling of the transformation process occurring from inputs to output. In addition, the relevance of the substitution interpretation is disputable. One can imagine substituting labor services with machinery, but not materials with labor, as suggested by our results. Last but most important, one crucial input is missing from our interpretation of M as copper, namely energy, without which no transformation is possible (Stern, 2010). Setting a proper analytical framework where these issues could be dealt with obviously goes beyond the scope of the present paper.

Discussion and Conclusion

We constructed a stock-flow consistent macro model where the scarcity of natural resources is made explicit through the simple prey-predator dynamics introduced in the previous chapter. As a proof of concept, we analytically examined the long-run stead states of our vector field and identified conditions under which a stationary balanced path can be reached.

First, we showed that our model enables to reach an impossibility result: no long-run balanced growth path can emerge with a positive growth rate of GDP. The strength of our finding is that it does not depend on some *ad hoc* non-substitutability restriction: capital, labour and natural resources can be made arbitrarily substitutable without restoring existence of an ever growing steady state. In other words, as soon as due account is taken of the dependence of our economies on natural resources, the unique desirable and meaningful long-run steady state is stationary. We use the term stationary state to describe zero growth in GDP. A steady state is a zero of the vector field, and refers to a long-run equilibrium of our macro-dynamics. Thus, consistently with most of the literature, a "steady state growth" refers to a long-run equilibrium with positive GDP growth (Solow, 1974). This terminology departs from (Daly, 1991) where the word "steady state" is used in place of what we here call a stationary steady state.

The second finding of this chapter is that this steady state is not an isolated zero of our vector field. Rather, indeterminacy is the rule: We show that there exists a continuum of stationary states with a finite level of debt and strictly positive employment rate, which may be indexed by the wage share in GDP.

Moreover, our dynamics admits other equilibria, distinct from this stationary attractor, among which a debt-deflationary equilibrium with zero production, employment and unbounded private debt. This feature opens the door to a renewed understanding of public policy based on two dimensions: first, how to drive the economy into the basin of attraction of the attractor of desirable equilibria? Second, once this basin has been reached, which point should we choose among the continuum of stationary states? The latter question refers to the old debate on the distribution of wealth between labor and capital: our indeterminacy result thus suggests that stationarity also opens the door for a discussion in terms of social equity. We shall not develop this public policy aspect in this paper but it should be kept in mind.

We further constructed a proxy of the world economy by aggregating 37 countries, and estimated the parameters of the three-input CES production function. This empirical estimation provides two possible functional forms with different substitution elasticities between capital, labor and copper: either an $(LM) + K$ structure with a low substitutability between L and M, and (LM) and K, or an $(KM) + L$ structure with perfect substitutability between K and M, and relatively low between (KL) and L. We note however that the best fit excludes M from the estimation.

Our main conclusion echoes the strong plea made in favour of a stationary state by Herman E. Daly (Daly, 1991) and others. It therefore invites to address carefully the 'post-growth challenge' it poses.

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Appendix: Estimation of the CES production function

Two-input CES

We tested a two-input CES with constant returns to scale, in the form:

$$Y = Ae^{\alpha t}\Big[\theta X_1^{-\rho} + (1-\theta)X_2^{-\rho}\Big]^{-1/\rho} \tag{2.40}$$

where X_1 and X_2 correspond to either K, L or M. θ is the distribution parameter and ρ determines the (constant) elasticity of substitution. Results are presented in Tab.2.10 and Fig.2.9. The best fit with the best significance is obtained for K and L. The results of the estimation with the series of cumulative copper production and copper revenues are presented in Tab.2.11 and 2.12. The best fit with the best significance obtains when M stands for cumulative production.

Three-input CES

The results using the PORT routine are presented in Tab.2.13 for free or imposed $\rho^* = 0$ (nested Cobb-Douglas). The results of the estimation with the series of cumulative copper production and copper revenues are presented in Tab.2.14.

Table 2.10: CES estimation: two inputs

Run	Residual Std Error	Parameters	Estimate	Std Error	P value
PORT	0.02599	A	0.9812	0.01114	***
$X_1 = K$, $X_2 = L$		α	0.008585	0.002404	***
		θ	0.5390	0.08446	***
		ρ	1.3172	0.3294	***
PORT	0.04950	A	1.0147	0.02069	***
$X_1 = K$, $X_2 = M$		α	-0.005961	0.002098	**
		θ	0.7532	0.1100	***
		ρ	3.3712	1.5878	*
PORT	0.04276	A	0.9603	0.01218	***
$X_1 = L$, $X_2 = M$		α	0.02523	0.002321	***
		θ	1.0000	0.1477	***
		ρ	0.2818	$1.021\ 10^{13}$	

Table 2.11: CES estimation: two inputs (KM) and different series of M

Run	Residual Std Error	Parameters	Estimate	Std Error	P value
PORT	0.04950	A	1.0147	0.02069	***
$X_1 = K$, $X_2 = M$		α	-0.005961	0.002098	**
Production		θ	0.7532	0.1100	***
		ρ	3.3712	1.5878	*
PORT	0.03419	A	0.94643	0.02741	***
$X_1 = K$, $X_2 = M$		α	-0.02933	0.0008233	***
Cumulative production		θ	0.9555	0.02089	***
		ρ	-1.0000	0.2843	***
PORT	0.09174	A	1.1060	0.03208	***
$X_1 = K$, $X_2 = M$		α	-0.02102	0.001431	***
Revenues		θ	1.0000	0.03531	***
		ρ	-0.01339	$2.673\ 10^{10}$	

Table 2.12: CES estimation: two inputs (LM) and different series of M

Run	Residual Std Error	Parameters	Estimate	Std Error	P value
PORT	0.04276	A	0.9603	0.01218	***
$X_1 = L$, $X_2 = M$		α	0.02523	0.002321	***
Production		θ	1.0000	0.1477	***
		ρ	0.2818	$1.021\ 10^{13}$	
PORT	0.04263	A	0.9602	0.03314	***
$X_1 = L$, $X_2 = M$		α	0.02522	0.001281	***
Cumulative production		θ	1.0000	0.03034	***
		ρ	0.38991699	$1.903\ 10^{13}$	
PORT	0.02383	A	0.9863	0.009168	***
$X_1 = L$, $X_2 = M$		α	0.02180	0.0006321	***
Revenues		θ	0.9572	0.01353	***
		ρ	-1.0000	0.6082	

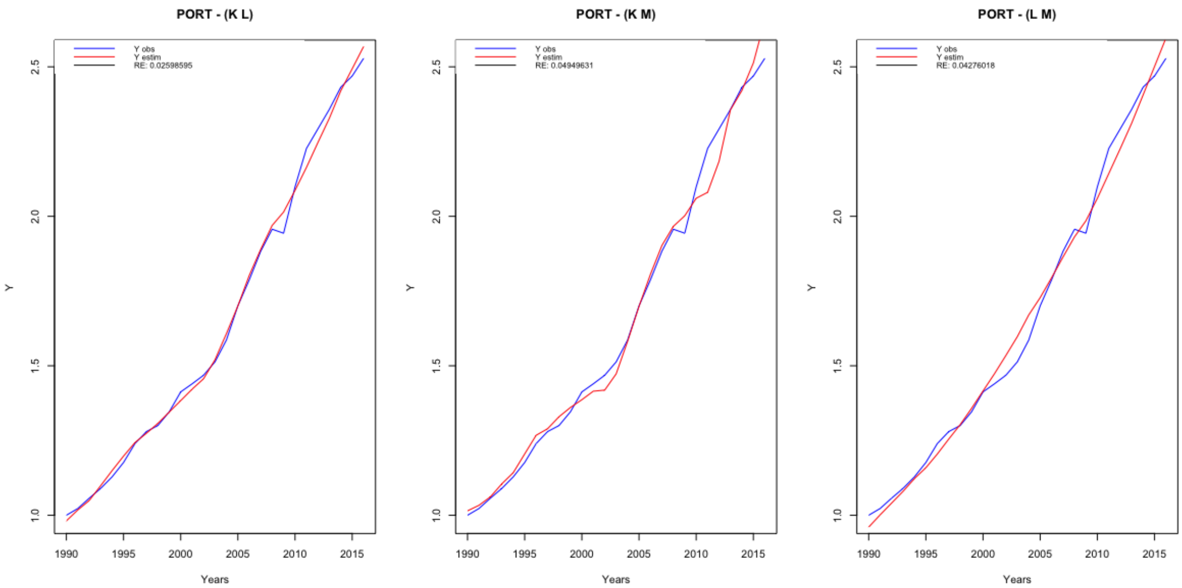

Figure 2.9: Y observed and estimated for the two-input CES

Table 2.13: CES estimation: three inputs - PORT - nested Cobb-Douglas

Run	Residual Std Error	Parameters	Estimate	Std Error	P value
PORT	0.03203	A	0.9976	0.01364	***
$X_1 = K$		α	0.01385	0.005044	**
$X_2 = L$		θ	0.2428	0.1480	
$X_3 = M$		ψ	1.0000	0.1288	***
$\rho^* = 0$		ρ	0.2245	$4.901\ 10^{12}$	
PORT	0.02599	A	0.9812	NA	
$X_1 = K$		α	0.008585	NA	
$X_2 = M$		θ	1.0000	NA	
$X_3 = L$		ψ	0.5390	NA	
$\rho^* = 0$		ρ	1.3172	NA	
PORT	0.02599	A	0.9812	NA	
$X_1 = L$		α	0.008585	NA	
$X_2 = M$		θ	1.0000	NA	
$X_3 = K$		ψ	0.4610	NA	
$\rho^* = 0$		ρ	1.3172	NA	

Table 2.14: CES estimation: three inputs - different M series

Run	Residual Std Error	Parameters	Estimate	Std Error	P value
PORT	0.02599	A	0.9812	0.01175	***
$X_1 = K$		α	0.008585	0.004280	*
$X_2 = M$		θ	1.0000	0.2624	***
$X_3 = L$		ψ	0.5390	0.1944	**
Production		ρ^*	0.2299	$1.428\ 10^{13}$	
		ρ	1.3172	0.4789	**
PORT	0.02620	A	0.9811	0.02131	***
$X_1 = K$		α	0.00874	0.004335	*
$X_2 = M$		θ	1.0000	0.03950	***
$X_3 = L$		ψ	0.5356	0.09219	***
Cumulative production		ρ^*	-0.1398	$3.040\ 10^{12}$	
		ρ	1.325	0.5598	*
PORT	0.01898	A	1.0068	0.008236	***
$X_1 = K$		α	0.01790	0.001383	***
$X_2 = M$		θ	0.5718	0.08925	***
$X_3 = L$		ψ	0.09275	0.03402	**
Revenues		ρ^*	-0.3331	1.6606	
		ρ	-1.0000	0.5247	

CHAPTER 3

A Dynamic Cooperative Game for International and Intergenerational Allocation with Exhaustible Natural Resources

Mais le vrai voyage est intérieur. La musique se nourrit à cette source intime, au creux du cœur, sous l'étoile, car il n'y a pas d'ailleurs meilleur, ni d'âge d'or dans le passé. Chaque jour, sous un ciel de cuivre, on reprend la route, creusant un peu plus pour trouver le pain, le sel et l'or du chant profond.

— Thierry "Titi" Robin, *Alezane*

The present chapter explores the implications of natural resource scarcity on global cooperation and trade.

The interaction between trade and materially sustainable growth (Dupuy, 2014) was studied by Asheim (Asheim, 1986) who looked at the effects of opening economies on the Hartwick rule. Other models focus on two-region trade, the industrialized North versus the resource exporter South (Chichilnisky, 1993). A rich literature studies the so-called "resource curse" (Gaitan and Roe, 2012) or the formation of cartels (Kemp and Van Long, 1984) through the traditional Heckscher and Ohlin model (Heckscher and Ohlin, 1991). A strategic game theory approach was also used to explore finite resources issues (Van Long, 2011), and the relationship with trade was also studied with two countries (Kagan et al., 2015; Tamasiga and Bondarev, 2014). However, and despite the call of some game theorists to develop cooperative game theory (Samuelson, 2016), the literature on material sustainability neglected to treat the issue through cooperative lenses. Trade can indeed be analyzed through three concepts: "pure competition", "coalitional power" and "fair division". Shapley and Shubik (Shapley and Shubik, 1969) provide a static market game for the three varieties of solution and show that, under certain assumptions, the outcomes converge. In a distinct stream of literature of cooperative games, the question of renewable and common-pool resources was studied (Funaki and Yamato, 1999), mostly for fishery or river sharing (Ambec and Sprumont, 2002; Béal et al., 2013) problems. However, these models do not take into account the dynamic aspects of the game.

The goal of this chapter is to provide a dynamic cooperative framework to trade when traded natural resources are limited, and therefore raise the issue of intergenerational equity. The literature on dynamic cooperative game is quite limited. Recent papers tackle the issue of dynamic core concepts (Predtetchinski et al., 2004; Kranich et al., 2005; Lehrer and Scarsini, 2013), but the first studies date back to the 70's with the introduction of core concepts for production economies (Boehm, 1974; Becker, 1982). In particular, the concept of recursive core (Becker and Chakrabarti, 1995) is defined as a set of allocations for which no coalition can improve upon its consumption stream at any time given its accumulation of assets up to that period. Some dynamic cooperative approaches were used to tackle environment issues, especially fisheries (Munro, 1979) and pollution (Jørgensen and Zaccour, 2001; Hoofe, 2019). A model most closely related to our motivation is presented by d'Albis and Ambec (d'Albis and Ambec, 2010), and studies fair allocations of overlapping generations depleting a natural resource over an infinite future. But the authors do not include any international aspects in the model.

Though, one key feature of natural resources is their geologic unequal distribution among countries. The idea of our study is to take advantage of the disparities between a finite number of countries to have a fresh view on the nexus between international trade and intergenerational equity. We model the geographical heterogeneity of natural endowments that results from geological processes, as well as the geographical heterogeneity of capital and technological endowments that results from historical processes. Trade is viewed as a market cooperative TU game à la Shapley-Shubik (Shapley and Shubik, 1969) where non-renewable natural resources inputs are exchanged in such a way as to maximize the total value to be shared among the coalition.

The question we intend to tackle is whether an international long-term cooperation strategy, where natural resources and productive capacity are shared in a common pool, can be put in place. For that purpose, we settle a cooperative game where countries can form coalitions in order to optimize their discounted consumption stream in the long-run, within the limits of their stock of natural resources. The game is cast into a traditional Ramsey-type model of intergenerational equity, where the Bellman's dynamic programming framework is used. An international long-term policy is translated into:

- An *action strategy* taken by a coalition S. It consists in a quota of natural resources extraction, a commercial quota and an investment plan decided for each country at each time t. An action strategy is *optimal* for S if the commercial quota optimizes the current collective production of S, and if the extraction quota and the investment plan ensure the maximization of long-term consumption.

- A *distribution policy* of collectively produced consumption goods. Such a policy is said to be *efficient* and *coalitionally rational* if, from some date t onward, a coalition cannot block an international agreement by undertaking a policy increasing its own long-term consumption.

In other words, the coalition decides its extraction and investment plan, and deduces the amounts of resources it wishes to trade. It then distributes consumption goods to its members. Of course, members could break an alliance according to their interest at any point in time. In this work, we look for the most efficient and stable coalition. For this purpose, we use the concept of recursive core.

Traditionally, the core is the set of allocations which cannot be improved upon by any coalition of agents. The core allocations defined in this way, however, do not deal with the dynamic issues relevant to the intergenerational sharing of benefits from resource extraction and economic growth. In such a dynamic framework, the twin issues of coalitional stability and time consistency have to be handled at once. First, it is a question of designing a policy that no coalition of the present generation has an incentive to refuse. Second it is a question of ensuring that at no time the policy initiated then departs from the one planned originally. We therefore look for the streams of consumption allocated to agents which are coalitionaly stable on one hand, and furthermore satisfy the time consistency property. That is, the core allocations must be such that at no moment will coalitions of agents ever want to block or break a contract in the core. The core concept that satisfies this property is called the recursive core.

The first section of this chapter states the mathematical bricks used to build the model. Then, two versions are presented. The Bronze Age model is a simplified version where extraction and trade are the only considered actions. This model is presented in the second section, as a pedagogical tool for the reader to have a easier understanding of the reasoning. In section 3, the Capital Age model, which is the heart of the chapter, is presented. In this version, the dynamics of capital accumulation is set through an investment strategy. All proofs are based on the same reasoning as section 2, and are available in the appendix. The last section is a discussion on the model, its possible refinements and the research avenue it opens.

1 Preliminaries

1.1 Static Cooperative Games

Let N denote a fixed finite nonempty set with n members, who will be called agents or players. *Coalitions of players* are nonempty subsets of N.

A *transferable utility* (TU) *game* on N is a pair (N, v) where v is a mapping $v : 2^N \to \mathbb{R}$ satisfying $v(\emptyset) = 0$. We denote by $\mathcal{G}(N)$ the set of all games over N. For any coalition S, $v(S)$ represents the *worth* of S, i.e., what coalition S could earn regardless of other players.

A payoff vector is a vector $x \in \mathbb{R}^N$ that assigns to agent i the payoff x_i. A payoff vector is *efficient* with respect to (N, v) if $\sum_{i\in N} x_i = v(N)$; it is *coalitionally rational* if $\sum_{i\in S} x_i \geq v(S)$ for every possible coalition S.

The *core* of (N, v), denoted by $\mathsf{C}(N, v)$, is the set, possibly empty, of efficient and coalitionally rational payoff vectors:

$$\mathsf{C}(N, v) = \left\{ x \in \mathbb{R}^N : \forall S \subseteq N,\ \sum_{i\in S} x_i \geq v(S) \text{ and } \sum_{i\in N} x_i = v(N) \right\}.$$

The interpretation of the core is that no group of agents has an incentive to split from the grand coalition N and form a smaller coalition S since they collectively receive at least as much as what they can obtain for themselves as a coalition.

The so-called Bondareva-Shapley theorem (Bondareva, 1963; Shapley, 1967) provides a sufficient and necessary condition under which the core of a TU-game is nonempty. First, we introduce the concept of balanced maps. A *balanced map* $\lambda : 2^N \longrightarrow [0, 1]$ is such that:

$$\lambda(\emptyset) = 0, \text{ and } \forall i \in N, \quad \sum_{S\ni i} \lambda(S) = 1.$$

Denote by $\mathbb{B}(N)$ the set of balanced maps over N.

Proposition 1. (Bondareva-Shapley Theorem)
For each TU-game (N, v), $\mathsf{C}(N, v) \neq \emptyset$ if and only if for each balanced map $\lambda \in \mathbb{B}(N)$, the following inequality holds:

$$\sum_{S \subseteq N} \lambda(S) v(S) \leq v(N). \tag{3.1}$$

The following proposition is a direct consequence of Corollary 1 in (Gonzalez and Grabisch, 2015).

Proposition 2. If, for each balanced map $\lambda \in \mathbb{B}(N)$ such that $\lambda(N) \neq 1$ the following strict inequality holds:

$$\sum_{S \subseteq N} \lambda(S) v(S) < v(N),$$

then $\mathsf{C}(N, v)$ is not reduced to a singleton.

1.2 Dynamic Set-Up

Definition 1. (Discounted dynamic programming problem: the Bellman framework) A stationary discounted dynamic programming problem is specified by a tuple $\{\mathcal{S}, \mathcal{A}, \Phi, f, r, \delta\}$, where for some n, $\mathcal{S} \subseteq \mathbb{R}^n$ is the state space with generic element s ; for some k, $\mathcal{A} \subseteq \mathbb{R}^k$ is the action space with generic element a ; $\Phi : \mathcal{S} \mapsto 2^{\mathcal{A}}$ is the feasible action correspondence ; $f : \mathcal{S} \times \mathcal{A} \mapsto \mathcal{S}$ is the transition function ; $r : \mathcal{S} \times \mathcal{A} \mapsto \mathbb{R}$ is the reward function ; $\delta \in [0, 1)$ is the discount factor. The value function V of this dynamic programming problem takes the following form:

$$V(s_0) = \max_{(a_t)_{t \in \mathbb{N}}} \sum_{t \in \mathbb{N}} \delta^t r(s_t, a_t)$$

subject to

$$a_t \in \Phi(s_t) \ t \in \mathbb{N}$$
$$s_{t+1} = f(s_t, a_t)$$

The following proposition is a well-established theorem of existence (see for instance Theorem 12.19 in Sundaram (1996, p.298)).

Proposition 3. (Existence of an optimal path) Suppose that $\{\mathcal{S}, \mathcal{A}, \Phi, f, r, \delta\}$ satisfies the following conditions:

(i) $r : \mathcal{S} \times \mathcal{A} \mapsto \mathbb{R}$ is continuous and bounded on $\mathcal{S} \times \mathcal{A}$,

(ii) $f : \mathcal{S} \times \mathcal{A} \mapsto \mathcal{S}$ is continuous on $\mathcal{S} \times \mathcal{A}$,

(iii) $\Phi : \mathcal{S} \mapsto 2^{\mathcal{A}}$ is a continuous compact-valued correspondence.

Then, there exists an optimal path $(a_t^*)_{t \in \mathbb{N}}$ such that $V(s_0) = \sum_{t \in \mathbb{N}} \delta^t r(s_t, a_t^*)$.

Definition 2. (The recursive core) Let (N, v_t) be a TU game. Suppose that for each $t \in \mathbb{N}$, $v_t(N) = r(s_t^*(N), a_t^*(N))$ where $(a_t^*(N))_{t \in \mathbb{N}}$ is an optimal solution taken by N, and $(s_t^*(N))_{t \in \mathbb{N}}$

the associated state. A distribution policy, π, is a mapping which associates with each initial state $\mathfrak{I}$ a payoff stream $(\pi_{i,t}(\mathfrak{I}))_{i\in N,t\in\mathbb{N}}$ such that $\forall t \in \mathbb{N}$:

$$\sum_{i\in N} \pi_{i,t}(\mathfrak{I}) = v_t(N)$$

A policy π is dominated by S at date T if there exists an initial state $\mathfrak{I}$ such that:

$$\sum_{t=0}^{+\infty} \delta^t \sum_{i\in S} \pi_{i,t+T}(\mathfrak{I}) < V(s^*_{S,T}(N))(S),$$

where for each $t \in \mathbb{N}$, $s^*_{S,t}(N)$ is the projection $(s^*_{i,t}(N))_{i\in S}$ and, for each state s, $V(s)(S)$ is the value function of the dynamic programming problem for coalition S.

The *recursive core* $\mathcal{C}(\mathfrak{I})$ is the set of undominated policies when the initial state is $\mathfrak{I}$.

1.3 Maximum Under Convexity

We state the following version of Berge's Maximum Theorem under Convexity (Sundaram, 1996, p.237).

Proposition 4. (Maximum Theorem under Convexity) For some k and l, if f is a continuous and strictly concave function on $S \times \Theta \subsetneq \mathbb{R}^k \times \mathbb{R}^l$ and D is a compact-valued continuous correspondence on Θ with a convex graph then $f^*(\theta) = \max\{f(x,\theta), x \in D(\theta)\}$ is strictly concave and $D^*(\theta) = \underset{x\in D(\theta)}{\operatorname{argmax}} f(x,\theta)$ is a continuous function[1].

2 The Bronze Age Model: Intergenerational Market Game of Wealth Creation from Traded Mineral Resources

2.1 Game Set-Up

Let N be a fixed and finite non-empty set of countries, who are the game players. Countries have the possibility to form a coalition which can be viewed as a free-trade area. Each country $i \in N$ has a natural resource[2] that can be exploited by successive generations indexed by their birth date $t \in \mathbb{N}$. We denote by $\Omega_{i,t}$ the stock of resources available to the country $i \in N$ at generation $t \in \mathbb{N}$. The cost of extraction $c_i(e_{i,t}, \Omega_{i,t})$ of $i \in N$, expressed in consumption units, is represented through a continuous convex function depending both on the quantity $\Omega_{i,t}$ available to the country i and the quantity $e_{i,t} \in [0, \Omega_{i,t}]$ extracted at date t. The dynamics of i's resource stock is given by the following law of motion:

$$\Omega_{i,t+1} = \Omega_{i,t} - e_{i,t}$$

Each generation t and each country i uses the resource as an input to produce consumption units through a production function f_i. We assume that f_i is continuous, strictly increasing

[1] In particular, $D^*(\theta)$ is single-valued everywhere.

[2] Note that this model could be extended to a finite number of resources

and strictly concave with respect to the quantity of resource used. Furthermore, we assume that resources are freely tradable with consumption units within all nonempty coalition $S \subseteq N$ with no transaction costs. After the commercial exchange, each country i has a quantity $z_{i,t}$ of resource input.

Let $\Omega(S) = (\Omega_i)_{i \in S}$ (or simply Ω if there is no danger of confusion) be the vector of natural resource stocks. The state space $\mathcal{S}(S)$ is defined by:

$$\mathcal{S}(S) := \{\Omega(S) \in [0, \bar{\Omega}]^S | \sum_{i \in S} \Omega_i < \bar{\Omega}\},$$

where $\bar{\Omega}$ is the total amount of resources on Earth. Let $e(S) = (e_i)_{i \in S}$ (or simply e if there is no danger of confusion) be the vector of extraction. The action space $\mathcal{A}(S)$ is defined by:

$$\mathcal{A}(S) := \{e(S) \,|\, e(S) \in [0, \bar{\Omega}]^S\},$$

We define the set of couples (Ω, e) describing the extractive sector as:

$$\mathcal{O}(S) := \mathcal{S}(S) \times \mathcal{A}(S).$$

The set of feasible vectors of extraction $e(S)$ for a coalition S with a stock described by $\Omega(S)$ is

$$E^{\Omega}(S) := \{e(S) \in (\mathbb{R}_+)^S,\ e_i \leq \Omega_i\}.$$

The set of available resource input after the exchange $z(S) = (z_i)_{i \in S}$ (or simply z if there is no danger of confusion) is

$$Z^e(S) := \{z(S) \in (\mathbb{R}_+)^S,\ \sum_{i \in S} z_i = \sum_{i \in S} e_i\}.$$

It follows that the output - expressed in consumption units - of the countries belonging to S when the stock is $\Omega(S)$ and the extraction is $e \in E^{\Omega}(S)$ shall not be more than:

$$v^{\Omega,e}(S) = \max\{\sum_{i \in S}(f_i(z_i) - c_i(e_i, \Omega_i)),\ (z_i)_{i \in S} \in Z^e(S)\}.$$

Let $\delta \in (0, 1)$ represent the discount factor. The intergenerational worth $V^{\Omega_0}(S)$ of S is defined as the value of the following dynamic programming problem:

$$\begin{aligned} &\underset{(e_t)_{t \in \mathbb{N}}}{\text{maximize}} && \sum_{t \in \mathbb{N}} \delta^t v^{\Omega_t, e_t}(S) \\ &\text{subject to} && \\ & && \Omega_{i,t+1} = \Omega_{i,t} - e_{i,t},\ i \in S\ \ t \in \mathbb{N} \\ & && e_{i,t} \leq \Omega_{i,t},\ i \in S,\ t \in \mathbb{N}. \end{aligned} \tag{3.2}$$

2.2 Optimal Action Strategy

We first show that, given a flow of resource extracted by each country, there exists a unique optimal way to trade that can be interpreted as an optimal commercial quota.

Proposition 5. The function $(\Omega, e) \mapsto v^{(\Omega,e)}(S)$ is continuous and strictly concave over $\mathcal{O}(S)$. Furthermore, for each $(\Omega, e) \in \mathcal{O}(S)$, there exists a unique $z^* \in Z^e(S)$ – continuously varying with (Ω, e) – such that

$$v^{\Omega,e}(S) = \sum_{i \in S}(f_i(z_i^*) - c_i(e_i, \Omega_i)).$$

Proof. Let $g : \mathcal{O}(S) \times \mathbb{R}_+^S \to \mathbb{R}_+$ be the function defined as

$$g((\Omega, e), z) = \sum_{i \in S}(f_i(z_i) - c_i(e_i, \Omega_i)),$$

and $C : \mathcal{O}(S) \rightrightarrows \mathbb{R}_+$ be the correspondence defined as

$$C(\Omega, e) = Z^e(S).$$

By continuity and strict concavity of f_i and $-c_i$ for each $i \in S$, function g is continuous and strictly concave on $\mathcal{O}(S) \times \mathbb{R}_+^S$. Clearly, C is a compact-valued continuous correspondence on $\mathcal{O}(S)$ and has a convex graph. We have the needed conditions to apply Proposition 4. We conclude that $v^{\Omega,e}(S) = \max\{g((\Omega, e), z),\ z \in C(\Omega, e)\}$ is a continuous and strictly concave function on $\mathcal{O}(S)$ and $C^*(\Omega, e) = \operatorname{argmax}\{\, g((\Omega, e), z),\ z \in C(\Omega, e)\}$ is a continuous single-valued function.

□

We now show that, given an initial resource stock, there exists a unique optimal way to extract that can be interpreted as an optimal extraction quota.

Proposition 6. For each nonempty coalition $S \subseteq N$, and each $(\Omega_{i,0})_{i \in S} \in [0, \bar{\Omega}]^S$, there exists a unique optimal path of extraction to the dynamic programming problem (3.2).

Proof. **Existence.** The existence of an optimal path of extraction is a direct application of Proposition 3. Indeed,

(i) By Proposition 5, $(\Omega, e) \mapsto v^{\Omega,e}$ is continuous on $\mathcal{O}(S)$. It follows, from the compacity of $\mathcal{O}(S)$, that $(\Omega, e) \mapsto v^{\Omega,e}$ is also bounded.

(ii) It is clear that $(\Omega, e) \mapsto \Omega - e$ is continuous from $\mathcal{O}(S)$ to $\mathbb{R}_+^S$.

(iii) It is straightforward to see that $\Omega \mapsto E^\Omega(S)$ is a compact-valued, continuous correspondence.

Uniqueness. Assume by way of contradiction that there exist two different optimal paths of extraction $(e_t^0)_{t \in \mathbb{N}}$ and $(e_t^1)_{t \in \mathbb{N}}$ when the initial stock vector is Ω_0. We denote by $(\Omega_t^0)_{t \in \mathbb{N}}$ and $(\Omega_t^1)_{t \in \mathbb{N}}$ the respective sequence of stock vectors defined for each $t \in \mathbb{N}$ by $\Omega_{t+1}^0 = \Omega_t^0 - e_t^0$ and $\Omega_{t+1}^1 = \Omega_t^1 - e_t^1$.

For each $\alpha \in [0, 1]$, we define the path of extraction $(e_t^\alpha)_{t \in \mathbb{N}}$ as follows:

$$\forall t \in \mathbb{N},\ e_t^\alpha = \alpha e_t^1 + (1 - \alpha) e_t^0.$$

Let $(\Omega_t^\alpha)_{t \in \mathbb{N}}$ be the sequence of stock vectors defined by $\Omega_0^\alpha = \Omega_0$ and for each $t \in \mathbb{N}$ by $\Omega_{t+1}^\alpha = \Omega_t^\alpha - e_t^\alpha$. A simple induction leads to $\Omega_t^\alpha = \alpha \Omega_t^1 + (1 - \alpha)\Omega_t^0$.

It is clear that

$$e_0^\alpha \leq \Omega_0,$$

and for each $t \in \mathbb{N}$,

$$e_{t+1}^\alpha = \alpha e_{t+1}^1 + (1-\alpha)e_{t+1}^0 \leq \alpha(\Omega_t^1 - e_t^1) + (1-\alpha)(\Omega_t^0 - e_t^0) = \Omega_t^\alpha - e_t^\alpha.$$

Hence, $(e_t^\alpha)_{t\in\mathbb{N}}$ is a feasible path of extraction. It follows that

$$\begin{aligned}
V^{\Omega_0}(S) &\geq \sum_{t\in\mathbb{N}} \delta^t v^{\Omega_t^\alpha, e_t^\alpha}(S) \\
&> \sum_{t\in\mathbb{N}} \delta^t (\alpha v^{\Omega_t^1, e_t^1}(S) + (1-\alpha) v^{\Omega_t^0, e_t^0}(S)) \\
&= \alpha V^{\Omega_0}(S) + (1-\alpha)V^{\Omega_0}(S) \\
&= V^{\Omega_0}(S).
\end{aligned}$$

The strict inequality, deduced from Proposition 5, leads to a contradiction.

□

2.3 Efficient and Coalitionally Rational Distribution Policy

By Proposition 6, for each initial stock vector $\Omega \in [0, \bar{\Omega}]^N$, there exists a unique optimal path of extraction $(e_t^*(N))_{t\in\mathbb{N}}$ to the dynamic programming problem (3.2) for the grand coalition N. Denote by $(\Omega_t^*(N))_{t\in\mathbb{N}}$ the sequence of stock vectors defined by $\Omega_0^*(N) = \Omega$ and for each $t \in \mathbb{N}$, $\Omega_{t+1}^*(N) = \Omega_t^*(N) - e_t^*(N)$. We denote by $\Omega_{S,t}^*(N)$ the projection $(\Omega_{i,t}^*(N))_{i\in S}$ of $\Omega_t^*(N)$. As traded quantities are also unique, we can conclude that there exists a unique path for collective production in the coalition N. Note that the consumption path of each country, for its part, is not unique and depends upon the sharing rule decided by the coalition.

An international distribution policy, π, is a mapping which associates with each initial state $\Omega \in [0; \bar{\Omega}]^N$ a path of consumption vector $(\pi_{i,t}(\Omega))_{i\in N, t\in\mathbb{N}}$ such that $\forall t \in \mathbb{N}$:

$$\sum_{i\in N} \pi_{i,t}(\Omega) = v^{\Omega_t^*(N), e_t^*(N)}(N).$$

A policy π is dominated by S at date T if there exists an initial state Ω such that:

$$\sum_{t=0}^{+\infty} \delta^t \sum_{i\in S} \pi_{i,t+T}(\Omega) < V^{\Omega_{S,T}^*(N)}(S).$$

The *recursive core* $\mathcal{C}(\Omega)$ is the set of undominated policies when the initial state is Ω.

Theorem 1. For each $\Omega \in [0, \bar{\Omega}]^N$, $\mathcal{C}(\Omega)$ contains at least two elements.

Proof. We proceed in two steps.
Step 1: For each $\Omega_0(N) \in [0, \bar{\Omega}]^N$, $\mathsf{C}(N, V^{\Omega_0(N)}) \neq \emptyset$ and $\mathsf{C}(N, V^{\Omega_0(N)})$ is not reduced to a singleton.
By Proposition 6, for each nonempty coalition $S \subseteq N$, and each initial stock vector $\Omega_0(S) \in [0, \bar{\Omega}]^S$, there exists a unique optimal path of extraction $(e_t^*(S))_{t\in\mathbb{N}}$ to the dynamic programming problem (3.2). For each nonempty coalition $S \subseteq N$, denote by $(\Omega_t^*(S))_{t\in\mathbb{N}}$ the sequence of stock vectors defined by $\Omega_0^*(S) = \Omega_0(S)$ and for each $t \in \mathbb{N}$, $\Omega_{t+1}^*(S) = \Omega_t^*(S) - e_t^*(S)$.

Let $\lambda \in \mathbb{B}(N)$ be a balanced system of N. If $\lambda(N) = 1$, it is clear that

$$V^{\Omega_0(N)}(N) = \sum_{S \subseteq N} \lambda(S) V^{\Omega_0(S)}(S).$$

Assume $\lambda(N) \neq 1$. We define $(\tilde{\Omega}_t(N))_{t\in\mathbb{N}} \in (\mathbb{R}^N_+)^{\mathbb{N}}$ and $(\tilde{e}_t(N))_{t\in\mathbb{N}} \in (\mathbb{R}^N_+)^{\mathbb{N}}$ as follows:

$$\forall i \in N, \forall t \in \mathbb{N}, \quad \tilde{\Omega}_{i,t}(N) = \sum_{S \ni i} \lambda(S) \Omega^*_{i,t}(S),$$

$$\forall i \in N, \forall t \in \mathbb{N}, \quad \tilde{e}_{i,t}(N) = \sum_{S \ni i} \lambda(S) e^*_{i,t}(S),$$

Observe that for each $i \in N$ and each $t \in \mathbb{N}$, we have $\tilde{\Omega}_{i,t+1}(N) = \tilde{\Omega}_{i,t}(N) - \tilde{e}_{i,t}(N)$ and $\tilde{\Omega}_{i,t}(N) \in [0, \bar{\Omega}]$ because $\sum_{S \ni i} \lambda(S) = 1$. It follows that $\tilde{e}_t(N) \in E^{\tilde{\Omega}_t(N)}$ for each $t \in \mathbb{N}$. Since $(\Omega^*_t(S), e^*_t(S)) \in \mathcal{O}(S)$, by Proposition 5, for each $t \in \mathbb{N}$ and each nonempty coalition $S \subseteq N$ there exists a unique $z^*_t(S) \in Z^{e^*_t(S)}(S)$ such that

$$v^{\Omega^*_t(S), e^*_t(S)}(S) = \sum_{i \in S} (f_i(z^*_{i,t}) - c_i(e^*_{i,t}, \Omega^*_{i,t})).$$

Define $(\tilde{z}_t(N))_{t\in\mathbb{N}} \in (\mathbb{R}^N_+)^{\mathbb{N}}$ as follows:

$$\forall i \in N, \forall t \in \mathbb{N}, \quad \tilde{z}_{i,t}(N) = \sum_{S \ni i} \lambda(S) z^*_{i,t}(S)$$

The following equalities hold:

$$\begin{aligned} \sum_{i \in N} \tilde{e}_{i,t}(N) &= \sum_{i \in N} \sum_{S \ni i} \lambda(S) e^*_{i,t}(S) \\ &= \sum_{S \subseteq N} \lambda(S) \sum_{i \in S} e^*_{i,t}(S). \end{aligned}$$

Since $z^*_t(S) \in Z^{e^*_t(S)}(S)$, it follows that

$$\begin{aligned} \sum_{S \subseteq N} \lambda(S) \sum_{i \in S} e^*_{i,t}(S) &= \sum_{S \subseteq N} \lambda(S) \sum_{i \in S} z^*_{i,t}(S) \\ &= \sum_{i \in N} \sum_{S \ni i} \lambda(S) z^*_{i,t}(S) \\ &= \sum_{i \in N} \tilde{z}_{i,t}(N). \end{aligned}$$

from which we deduce that $\tilde{z}_t(N) \in Z^{\tilde{e}_t}(N)$ for each $t \in \mathbb{N}$.

By definition of $v^{\tilde{\Omega}_t(N), \tilde{e}_t(N)}(N)$, for each $t \in \mathbb{N}$, we have

$$v^{\tilde{\Omega}_t(N), \tilde{e}_t(N)}(N) \geq \sum_{i \in N} \Big(f_i(\tilde{z}_{i,t}(N)) - c_i(\tilde{e}_{i,t}(N), \tilde{\Omega}_{i,t}(N)) \Big).$$

The hypothesis of strict concavity ensures that

$$\sum_{i \in N} \Big(f_i(\tilde{z}_{i,t}(N)) - c_i(\tilde{e}_{i,t}(N), \tilde{\Omega}_{i,t}(N)) \Big) > \sum_{i \in N} \sum_{S \ni i} \lambda(S) \Big(f_i(z^*_{i,t}(S)) - c_i(e^*_{i,t}(S), \Omega^*_{i,t}(S)) \Big)$$

because for each $i \in N$ and $t \in \mathbb{N}$, $\tilde{z}_{i,t}(N)$ (resp. $(\tilde{e}_{i,t}(N), \tilde{\Omega}_{i,t}(N))$) is a convex combination of $(z^*_{i,t}(S))_{S \ni i}$ (resp. $(e^*_{i,t}(S), \Omega^*_{i,t}(S))_{S \ni i}$). Combining the two last inequalities, we have

$$\begin{aligned} v^{\tilde{\Omega}_t(N),\tilde{e}_t(N)}(N) &> \sum_{i \in N} \sum_{S \ni i} \lambda(S) \Big(f_i(z^*_{i,t}(S)) - c_i(e^*_{i,t}(S), \Omega^*_{i,t}(S)) \Big) \\ &= \sum_{S \subseteq N} \lambda(S) \sum_{i \in S} \Big(f_i(z^*_{i,t}(S)) - c_i(e^*_{i,t}(S), \Omega^*_{i,t}(S)) \Big) \\ &= \sum_{S \subseteq N} \lambda(S) v^{\Omega^*_t(S), e^*_t(S)}(S) \end{aligned}$$

Since $(\tilde{e}_t(N))_{t \in \mathbb{N}}$ is a feasible path and $\tilde{\Omega}_0(N) = \Omega_0(N)$, the definition of $V^{\Omega_0(N)}(N)$ ensures that

$$\begin{aligned} V^{\Omega_0(N)}(N) &\geq \sum_{t \in \mathbb{N}} \delta^t v^{\tilde{\Omega}_t(N),\tilde{e}_t(N)}(N) \\ &> \sum_{t \in \mathbb{N}} \delta^t \sum_{S \subseteq N} \lambda(S) v^{\Omega^*_t(S), e^*_t(S)}(S) \\ &= \sum_{S \subseteq N} \lambda(S) \sum_{t \in \mathbb{N}} \delta^t v^{\Omega^*_t(S), e^*_t(S)}(S) \\ &= \sum_{S \subseteq N} \lambda(S) V^{\Omega_0(S)}(S). \end{aligned}$$

By Proposition 1 stating the Bondareva-Shapley theorem, $\mathsf{C}(N, V^{\Omega_0(N)}) \neq \emptyset$ and by Proposition 2, $\mathsf{C}(N, V^{\Omega_0(N)})$ is not reduced to a singleton.

Step 2: Existence and non uniqueness of the solution.

Step 1 states that for each state $\Omega \in [0; \bar{\Omega}]^N$, the core $\mathsf{C}(N, V^{\Omega})$ is nonempty. Hence, for each initial state $\Omega \in [0; \bar{\Omega}]^N$ we can build a sequence $((X_{i,t}(\Omega))_{i \in N})_{t \in \mathbb{N}}$ - or simply $((X_{i,t})_{i \in N})_{t \in \mathbb{N}}$ - such that for each $t \in \mathbb{N}$:

(i) $\sum_{i \in N} X_{i,t} = V^{\Omega^*_t(N)}(N)$,

(ii) $\sum_{i \in S} X_{i,t} \geq V^{\Omega^*_{S,t}(N)}(S)$,

where $(\Omega^*_t(N))_{t \in \mathbb{N}}$ is the sequence of stock vectors associated with the optimal extraction path of N when the initial state is Ω.

Let the instantaneous allocation path for each country be $(x_{i,t}(\Omega))_{i \in N, t \in \mathbb{N}}$ - or simply $(x_{i,t})_{i \in N, t \in \mathbb{N}}$ - defined for each initial state $\Omega \in [0; \bar{\Omega}]^N$ such that for each $t \in \mathbb{N}$, $x_{i,t} = X_{i,t} - \delta X_{i,t+1}$.

Let us first show that $(x_{i,t})_{i \in N, t \in \mathbb{N}}$ is an international distribution policy. For each $t \in \mathbb{N}$,

$$\begin{aligned} \sum_{i \in N} x_{i,t} &= \sum_{i \in N} X_{i,t} - \delta \sum_{i \in N} X_{i,t+1} \\ &= V^{\Omega^*_t(N)}(N) - \delta V^{\Omega^*_{t+1}(N)}(N) \end{aligned}$$

and the Bellman equation ensures that:

$$V^{\Omega^*_t(N)}(N) = v^{\Omega^*_t(N), e^*_t(N)}(N) + \delta V^{\Omega^*_{t+1}(N)}(N)$$

Therefore, we have for each $t \in \mathbb{N}$,

$$\sum_{i \in N} x_{i,t} = v^{\Omega_t^*(N), e_t^*(N)}(N).$$

Let us now prove that $(x_{i,t})_{i \in N, t \in \mathbb{N}}$ is an undominated policy. For each $T \in \mathbb{N}$, we have

$$\begin{aligned}
\sum_{i \in S} \sum_{t=0}^{\infty} \delta^t x_{i,t+T} &= \sum_{i \in S} \sum_{t=0}^{\infty} \left(\delta^t X_{i,t+T} - \delta^{t+1} X_{i,t+T+1} \right) \\
&= \sum_{i \in S} X_{i,T} \\
&\geq V^{\Omega_{S,T}^*(N)}(S)
\end{aligned}$$

We can therefore conclude that the set $\mathcal{C}(\Omega)$ of undominated policies when the initial state is Ω is nonempty. Since we established in Step 1 that for each state $\Omega \in [0; \bar{\Omega}]^N$, $\mathsf{C}(N, V^{\Omega})$ is not reduced to a singleton, we deduce by construction of $(x_{i,t})_{i \in N, t \in \mathbb{N}}$ that $\mathcal{C}(\Omega)$ is not reduced to a singleton.

□

3 The Capital Age Model: Optimal growth with Traded Mineral Resources and Capital Accumulation

3.1 Game Set-Up

In the previous section, growth was constrained by natural resource availability and costs of extraction, as well as the country-specific form of the production function. In the present section, we add a new growth engine: capital accumulation. Countries have three decisions to make at each period: resource extraction, resource trade and investment level through savings that will increase the production capacity of the next period. The remaining created flow of wealth is consumed, and generations seek to maximize their present consumption as well as the discounted consumption of their successors. The present model is therefore a cooperative games adaptation of Ramsey's benchmark model with traded mineral resources.

At period t and for coalition S, consider an amount $\breve{y}_t(S)$ of a single final good, which can be viewed as Ricardo's metaphorical corn in neoclassical theory. The coalition chooses the amount $\eta_{i,t}$ each country i will invest for tomorrow (the seeds to be sowed next year) to increase the country's stock of capital $K_{i,t}$. This leaves $\breve{y}_t(S) - \sum_{i \in S} \eta_{i,t}$ to the present consumption of the coalition. The dynamics of i's capital stock is given by the following law of motion, τ being the depreciation rate:

$$K_{i,t+1} = K_{i,t} + \eta_{i,t} - \tau K_{i,t}$$

The coalition also chooses the quantity $e_{i,t}$ of resources each country will extract from its available stock $\Omega_{i,t}$. The dynamics of i's resource stock is given by the following law of motion:

$$\Omega_{i,t+1} = \Omega_{i,t} - e_{i,t}$$

Extracted resources are exchanged in a free-trade area (no transaction costs), and country i ends up with a quantity $z_{i,t}$ of input for its domestic production. The country's productive sector then uses its production capacity $K_{i,t}$ and $z_{i,t}$ to produce $f_i(z_i, K_i)$ through a raw production function f_i (to be seen as the gross production of Leontief's input-output theory). We assume, for each $i \in S$, $f_i : \mathbb{R}_+ \times \mathbb{R}_+ \mapsto \mathbb{R}_+$ satisfies:

(i) f_i is continuous, non-decreasing, strictly concave,

(ii) $f_i(z_i, 0) = 0$ for each $z_i \in \mathbb{R}_+$,

(iii) f_i is differentiable with respect to K_i,

(iv) $\lim\limits_{K_i \to 0} \dfrac{\partial f_i(z_i, K_i)}{\partial K_i} > \tau$ for each $z_i \in \mathbb{R}_+$,

(v) $\lim\limits_{K_i \to \infty} \dfrac{\partial f_i(z_i, K_i)}{\partial K_i} < \tau$ for each $z_i \in \mathbb{R}_+$.

Note that these conditions are weaker than the Inada conditions. The cost of extraction $c_i(e_{i,t}, \Omega_{i,t})$ of $i \in N$, expressed in consumption units, is represented through a continuous convex function depending both on the quantity $\Omega_{i,t}$ available to the country i and the quantity $e_{i,t} \in [0, \Omega_{i,t}]$ extracted at date t. This cost can be viewed as intermediary consumption lost in the production process (international stock-flow coherence to be checked). We consider that each country i has an initial endowment $\breve{y}_{i,0}$ and that

$$\breve{y}_0(S) = \sum_{i \in S} \breve{y}_{i,0}.$$

The production process takes one period to be achieved, since time is necessary for natural resources transportation and capital usage.[3]. The choice of the trading strategy is made such that the total net production $\breve{y}_{t+1}(S)$ of each coalition S is maximized through the following static optimization problem:

$$\begin{aligned} &\underset{(z_{i,t})_{i \in S}}{\text{maximize}} \quad \sum_{i \in S} f_i(z_{i,t}, K_{i,t}) - c_i(e_{i,t}, \Omega_{i,t}) \\ &\text{subject to} \\ &\qquad\qquad \sum_{i \in S} z_{i,t} = \sum_{i \in S} e_{i,t}. \end{aligned}$$

Fig. 3.1 presents a scheme of the model at t and $t + 1$.

3.2 Significant Sets

We define $\bar{\Omega} \in \mathbb{R}_+$ as the total and finite amount of resources on Earth. Following neoclassical arguments, the assumptions on the function f_i as well as the dynamics of K_i ensure that there exists $\bar{K} \in \mathbb{R}_+$ such that for each $i \in S$ and for each $t \in \mathbb{N}$, $K_{i,t} \leq \bar{K}$. As a consequence, there exists $\bar{\breve{y}} \in \mathbb{R}_+$ such that for each $t \in \mathbb{N}$, $\breve{y}_t(S) \leq \bar{\breve{y}}$.

[3]Therefore, unlike the previous section, t must now be viewed as a period and not a generation.

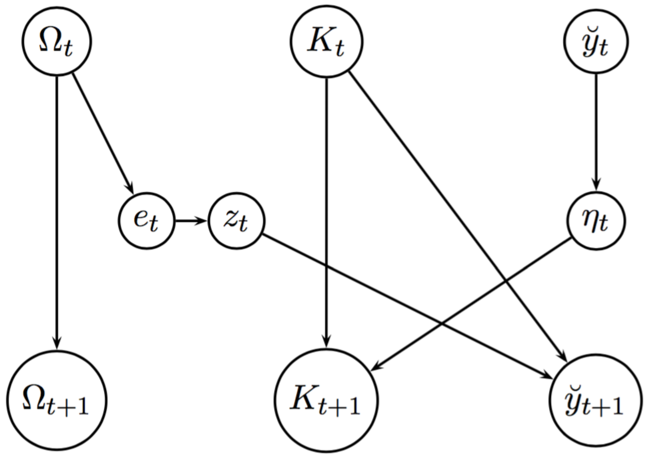

Figure 3.1: Scheme of the state and action dynamics

Let $\Omega(S) = (\Omega_i)_{i\in S}$ (or simply Ω if there is no danger of confusion) be the vector of natural resources stocks and $K(S) = (K_i)_{i\in S}$ (or simply K if there is no danger of confusion) be the vector of capital stocks. The state space $\mathcal{S}(S)$ is defined by:

$$\mathcal{S}(S) := \{(\Omega(S), K(S), \breve{y}(S)) \in \mathbb{R}_+^S \times \mathbb{R}_+^S \times \mathbb{R}_+,\ \breve{y}(S) \leq \bar{\breve{y}} \text{ and } \forall i \in S,\ \Omega_i \leq \bar{\Omega}, K_i \leq \bar{K}\}.$$

Let $e(S) = (e_i)_{i\in S}$ (or simply e if there is no danger of confusion) be the vector of extraction and $\eta(S) = (\eta_i)_{i\in S}$ (or simply η if there is no danger of confusion) be the vector of investment. The action space $\mathcal{A}(S)$ is defined by:

$$\mathcal{A}(S) := \{(e(S), \eta(S)) \in \mathbb{R}_+^S \times \mathbb{R}_+^S,\ \forall i \in S,\ e_i \leq \bar{\Omega}, \eta_i \leq \bar{\breve{y}}\}.$$

As in the previous section, the set of feasible vectors of extraction $e(S)$ for a coalition S with a stock described by $\Omega(S)$ is

$$E^{\Omega}(S) := \{e(S) \in (\mathbb{R}_+)^S,\ e_i \leq \Omega_i\}.$$

We now define the set of feasible vectors of investment $\eta(S)$ for a coalition S by

$$\mathcal{I}^{\breve{y}(S)}(S) := \{\eta(S) \in (\mathbb{R}_+)^S, \sum_{i\in S} \eta_i \leq \breve{y}(S)\}.$$

We can therefore define the set of couples $(e(S), \eta(S))$ of feasible actions by:

$$\Phi^{\Omega,\breve{y}(S)}(S) := \{(e(S), \eta(S)) \,|\, e(S) \in E^{\Omega}(S), \eta(S) \in \mathcal{I}^{\breve{y}(S)}(S)\},$$

Now, the set of available resource input after the exchange $z(S) = (z_i)_{i\in S}$ (or simply z if there is no danger of confusion) is:

$$Z^e(S) := \{z(S) \in (\mathbb{R}_+)^S, \sum_{i\in S} z_i = \sum_{i\in S} e_i\}.$$

3.3 Coalitional and Intergenerational Worth

At each period, countries belonging to a coalition S when the production is $\breve{y}(S)$ and the investment is $\eta \in \mathcal{I}^{\breve{y}(S)}(S)$ will earn a worth that shall not be more than their consumption:

$$\forall \breve{y}(S) \in \mathbb{R}_+, \forall \eta \in \mathcal{I}^{\breve{y}(S)}(S) \;\; v^{\breve{y}(S),\eta}(S) = \breve{y}(S) - \sum_{i\in S} \eta_i.$$

Let us now zoom up in the time scale, and describe the behavior of countries from an intertemporal viewpoint. Let $\delta \in]0,1[$ be the discount factor. The intertemporal worth $V^{\Omega_0,K_0,\breve{y}_0(S)}(S)$ of S is defined as the value of the following dynamic programming problem:

$$\begin{aligned}
&\underset{(e_t,\eta_t)_{t\in\mathbb{N}}}{\text{maximize}} && \sum_{t\in\mathbb{N}} \delta^t v^{\breve{y}_t(S),\eta_t}(S) \\
&\text{subject to} && \\
& && \Omega_{t+1} = \Omega_t - e_t, \;\; t \in \mathbb{N} \\
& && K_{t+1} = (1-\tau)K_t + \eta_t, \;\; t \in \mathbb{N} \\
& && \breve{y}_{t+1}(S) = \max\left\{\sum_{i\in S} f_i(z_{i,t}, K_{i,t}) - c_i(e_{i,t}, \Omega_{i,t}),\; (z_{i,t})_{i\in S} \in Z^e(S)\right\}, \;\; t \in \mathbb{N} \\
& && (e_t, \eta_t) \in \Phi^{\Omega_t, \breve{y}_t(S)}(S), \; t \in \mathbb{N}
\end{aligned} \tag{3.3}$$

3.4 Optimal Action Strategy

We first show that, given a flow of resource extracted by each country, there exists a unique optimal way to trade that can be interpreted as an optimal commercial quota.

Proposition 7. The function $(\Omega, e, K) \mapsto \max\left\{\sum_{i\in S} f_i(z_i, K_i) - c_i(e_i, \Omega_i),\; (z_i)_{i\in S} \in Z^e(S)\right\}$ is continuous and strictly concave over $[0,\bar{\Omega}]^S \times [0,\bar{\Omega}]^S \times [0,\bar{K}]^S$. Furthermore, for each $(\Omega, e, K) \in [0,\bar{\Omega}]^S \times [0,\bar{\Omega}]^S \times [0,\bar{K}]^S$, there exists a unique $z^* \in Z^e(S)$ – continuously varying with (Ω, e, K) – such that

$$\max\left\{\sum_{i\in S} f_i(z_i, K_i) - c_i(e_i, \Omega_i),\; (z_i)_{i\in S} \in Z^e(S)\right\} = \sum_{i\in S} f_i(z_i^*, K_i) - c_i(e_i, \Omega_i).$$

Proof. Let $g : [0,\bar{\Omega}]^S \times [0,\bar{\Omega}]^S \times [0,\bar{K}]^S \times \mathbb{R}_+^S \to \mathbb{R}_+$ be the function defined as

$$g((\Omega, e, K), z) = \sum_{i\in S} \Big(f_i(z_i, K_i) - c_i(e_i, \Omega_i)\Big),$$

and $C : [0,\bar{\Omega}]^S \times [0,\bar{\Omega}]^S \times [0,\bar{K}]^S \rightrightarrows \mathbb{R}_+^S$ be the correspondence defined as

$$C(\Omega, e, K) = Z^e(S).$$

By continuity and strict concavity of f_i and $-c_i$ for each $i \in S$, function g is continuous and strictly concave on $[0,\bar{\Omega}]^S \times [0,\bar{\Omega}]^S \times [0,\bar{K}]^S \times \mathbb{R}_+^S$. Clearly, C is a compact-valued continuous correspondence on $[0,\bar{\Omega}]^S \times [0,\bar{\Omega}]^S \times [0,\bar{K}]^S$ and has a convex graph. We have the needed conditions to apply Proposition 4. We conclude that $\max\{g\Big((\Omega,e,K),z\Big),\ z \in C(\Omega,e,K)\}$ is a continuous and strictly concave function on $[0,\bar{\Omega}]^S \times [0,\bar{\Omega}]^S \times [0,\bar{K}]^S$ and $C^*(\Omega,e,K) = \operatorname{argmax}\{\, g\Big((\Omega,e,K),z\Big),\ z \in C(\Omega,e,K)\}$ is a continuous single-valued function. □

We now show that, given the initial endowments of each country, there exists a unique optimal way to extract and to invest that can be interpreted as an optimal extraction quota and investment plan.

Proposition 8. For each nonempty coalition $S \subseteq N$, and each $(\Omega_{i,0})_{i\in S} \in [0,\bar{\Omega}]^S$, $(K_{i,0})_{i\in S} \in [0,\bar{K}]^S$ and $\breve{y}_0(S) \in [0,\bar{\breve{y}}]$, there exists a unique optimal path of extraction and investment to the dynamic programming problem (3.3).

3.5 Efficient and Coalitionally Rational Distribution Policy

By Proposition 8, for each initial state $(\Omega,K,\breve{y}) \in [0;\bar{\Omega}]^N \times [0,\bar{K}]^N \times [0,\bar{\breve{y}}]$ of the grand coalition N, there exists a unique optimal extraction and investment solution $\Big(e^*_t(N),\eta^*_t(N)\Big)_{t\in\mathbb{N}}$ to the dynamic programming problem (3.3). Let $\Big(\Omega^*_t(N),K^*_t(N),\breve{y}^*_t(N)\Big)_{t\in\mathbb{N}}$ be the state evolution corresponding to the optimal path when the initial state is $(\Omega,K,\breve{y})$. Denote by $\Big(\Omega^*_{S,t}(N),K^*_{S,t}(N),\breve{y}^*_{S,t}(N)\Big)$ the projection $\Big(\Omega^*_{i,t}(N),K^*_{i,t}(N),\breve{y}^*_{i,t}(N)\Big)_{i\in S}$ of $\Big(\Omega^*_t(N),K^*_t(N),\breve{y}^*_t(N)\Big)$. Note that the consumption path, for its part, is not unique. An international distribution policy, π, is a mapping which associates with each initial state $(\Omega,K,\breve{y})$ a path of consumption vector $(\pi_{i,t}(\Omega,K,\breve{y}))_{i\in N,t\in\mathbb{N}}$ such that $\forall t \in \mathbb{N}$:

$$\sum_{i\in N} \pi_{i,t}(\Omega,K,\breve{y}) = \breve{y}^*_t(N) - \sum_{i\in N} \eta^*_{i,t}(N)$$

A policy π is dominated by S at date T if there exists an initial state $(\Omega,K,\breve{y})$ such that:

$$\sum_{t=0}^{+\infty} \delta^t \sum_{i\in S} \pi_{i,t+T}(\Omega,K,\breve{y}) < V^{\Omega^*_{S,T}(N),K^*_{S,T}(N),\breve{y}^*_{S,T}(N)}(S).$$

The *recursive core* $\mathcal{C}(\Omega,K,\breve{y})$ is the set of undominated policies when the initial state is $(\Omega,K,\breve{y})$.

Theorem 2. For each $(\Omega,K,\breve{y}) \in [0;\bar{\Omega}]^N \times [0,\bar{K}]^N \times [0,\bar{\breve{y}}]$, $\mathcal{C}(\Omega,K,\breve{y})$ contains at least two elements.

We showed that there exists a unique optimal path of extraction and investment for each country, which could be interpreted as quotas imposed by a social planner. This leads to a unique path of resource and capital stocks. As traded quantities are also unique, we can conclude that there exists a unique path for collective production in a coalition S. The consumption path of each country, for its part, is not unique and depends upon the sharing rule decided by the coalition.

This view corresponds to a benevolent planner interpretation, and one could object that imposing commercial quotas is not consistent with a common definition of free trade. This objection is fully anchored in the socialist calculation debate (O'Neill, 2004; Lavoie, 1985;

Stiglitz, 1996; Bardhan and Roemer, 1992; Nove, 2003; Hahnel, 2013) and is not the subject of this study. Note however that, by definition of optimality, agents should not deviate from these quotas even in a free market interpretation. Another alternative interpretation would be the construction of a negotiation procedure between countries. In all cases, no countries would have the incentive to refuse a core allocation. Note that the question of a strategic behavior is neglected by this study, and an implementation of this cooperative game leading to a strong-Nash solution could be the subject of a subsequent paper.

4 Discussion and Conclusion

The present model is not exhaustive and allows for a multitude of refinements. First, technological progress is not taken into account, and should be through a time-dependent production function or an endogenous knowledge capital accumulation. An interesting question is to characterize technological catch-up effects between countries and how it affects the balance of power. In order to better take into account this balance of power, and in view of the geostrategic nature of natural resources, a major refinement to this model would be to add the risk for war (Martin et al., 2008). Studying monetary and/or ecological debt in this context is also a lead to explore, as well as the notion of trust in a monetary economy (Gale, 1978). Another possible refinement is to dig more into environmental issues. Modeling recycling, or the effects of pollution can be easily implemented. The model also offers an interesting tool to study the effects of a bliss point on intergenerational allocations and the depletion of exhaustible resources. Cooperation costs can also be introduced in this model. In the original market game, they entail core vacuity. However, in an infinite horizon model, costs can be allocated throughout generations, ensuring the existence of core distribution policies.

A more profound analysis of the short-run efficient and coalitionaly rational policies is also required. In fact, the instantaneous cores can be empty even if the overall core is not (Lehrer and Scarsini, 2013). A characterization of the distribution policies is therefore needed. This could be implemented by numerically simulating specific examples, and would help to highlight the effects of the different variables and parameters on the solution.

This third chapter demonstrates that the recursive core of the cooperative game we propose contains at least two elements. This means that, if the countries of a coalition follow a long-term cooperation strategy in terms of extraction, investment and induced trade, then the best way to optimize their own consumption stream while caring about future generations is to form a global coalition. The recursive core is constituted of the streams of consumption to be allocated to each country. Since this core is not reduced to a singleton, it leaves room for a choice among these allocations, for instance by using some fairness criteria. The important missing step is now the proposition of an axiomatic for core allocations, exploring possible ethic consumption paths. An interesting lead is to study the nucleolus solution concept (Serrano, 1995), which maximizes recursively the 'welfare' of the worst treated coalitions and can be understood as an application of the Rawlsian social welfare function. Another lead would be to study the Shapley value. The presented model can therefore be viewed as a tool to refresh the common look at the North-South opposition and sets the conceptual framework for the exploration of a fair sharing of the fruits of global economic growth.

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Appendix

Proposition 8

Proof. **Existence.** The existence of an optimal path of extraction and investment is a direct application of Proposition 3. Indeed,

(i) The reward function $(\breve{y}(S), \eta) \mapsto v^{\breve{y}(S),\eta}(S)$ is continuous and bounded on $[0, \bar{\breve{y}}] \times [0, \bar{\breve{y}}]^S$.

(ii) It is clear that $(\Omega, e) \mapsto \Omega - e$ is continuous from $[0, \bar{\Omega}]^S \times [0, \bar{\Omega}]^S$ to $[0, \bar{\Omega}]^S$. In addition, $(K, \eta) \mapsto (1 - \tau)K + \eta$ is continuous from $[0, \bar{K}]^S \times [0, \bar{\breve{y}}]^S$ to $[0, \bar{K}]^S$. By Proposition 7, $(\Omega, e, K) \mapsto \max\left\{\sum_{i\in S} f_i(z_i, K_i) - c_i(e_i, \Omega_i),\ (z_i)_{i\in S} \in Z^e(S)\right\}$ is continuous on $[0, \bar{\Omega}]^S \times [0, \bar{\Omega}]^S \times [0, \bar{K}]^S$. Hence, the transition function is continuous.

(iii) The feasible action correspondence $(\Omega, \breve{y}(S)) \mapsto \Phi^{\Omega,\breve{y}(S)}(S)$ is compact-valued and continuous.

Uniqueness. Assume by way of contradiction that there exist two different optimal paths $(e^0_t, \eta^0_t)_{t\in\mathbb{N}}$ and $(e^1_t, \eta^1_t)_{t\in\mathbb{N}}$ when the initial vector is $(\Omega_0, K_0, \breve{y}_0(S))$ for a coalition S.

We denote by $(\Omega^0_t)_{t\in\mathbb{N}}$ and $(\Omega^1_t)_{t\in\mathbb{N}}$ the respective sequence of resource stock vectors defined by $\Omega^0_0 = \Omega_0$, $\Omega^1_0 = \Omega_0$ and for each $t > 0$, $\Omega^0_{t+1} = \Omega^0_t - e^0_t$ and $\Omega^1_{t+1} = \Omega^1_t - e^1_t$. We denote by $(K^0_t)_{t\in\mathbb{N}}$ and $(K^1_t)_{t\in\mathbb{N}}$ the respective sequence of capital stock vectors defined by $K^0_0 = K_0$, $K^1_0 = K_0$ and for each $t > 0$, $K^0_{t+1} = (1 - \tau)K^0_t + \eta^0_t$ and $K^1_{t+1} = (1 - \tau)K^1_t + \eta^1_t$. For each $\alpha \in [0, 1]$, we define the path of extraction $(e^\alpha_t)_{t\in\mathbb{N}}$ as follows:

$$\forall t \in \mathbb{N},\ e^\alpha_t = \alpha e^1_t + (1 - \alpha)e^0_t;$$

the path of investment $(\eta^\alpha_t)_{t\in\mathbb{N}}$ is defined as:

$$\forall t \in \mathbb{N},\ \eta^\alpha_t = \alpha \eta^1_t + (1 - \alpha)\eta^0_t.$$

Let $(\Omega^\alpha_t)_{t\in\mathbb{N}}$ be the sequence of resource stock vectors defined by $\Omega^\alpha_0 = \Omega_0$ and for each $t \in \mathbb{N}$ by $\Omega^\alpha_{t+1} = \Omega^\alpha_t - e^\alpha_t$. A simple induction leads to $\Omega^\alpha_t = \alpha\Omega^1_t + (1 - \alpha)\Omega^0_t$ for each $t \in \mathbb{N}$. Let $(K^\alpha_t)_{t\in\mathbb{N}}$ be the sequence of capital stock vectors defined by $K^\alpha_0 = K_0$ and for each $t \in \mathbb{N}$ by $K^\alpha_{t+1} = (1 - \tau)K^\alpha_t + \eta^\alpha_t$. A simple induction leads to $K^\alpha_t = \alpha K^1_t + (1 - \alpha)K^0_t$ for each $t \in \mathbb{N}$.

We set $\breve{y}^\alpha_0(S) = \breve{y}_0(S)$ and for each $t > 0$,

$$\breve{y}^\alpha_t(S) = \max\left\{\sum_{i\in S} f_i(z_{i,t-1}, K^\alpha_{i,t-1}) - c_i(e^\alpha_{i,t-1}, \Omega^\alpha_{i,t-1}),\ (z_{i,t-1})_{i\in S} \in Z^{e^\alpha_{t-1}}(S)\right\}.$$

We define for each $t \in \mathbb{N}$ the following vectors[4]:

$$(z^{*0}_{i,t})_{i\in S} = \operatorname*{argmax}_{(z_{i,t})_{i\in S}\in Z^{e^0_t}(S)} \sum_{i\in S} f_i(z_{i,t}, K^0_{i,t}) - c_i(e^0_{i,t}, \Omega^0_{i,t}),$$

[4] It follows from Proposition 7 that both vectors are uniquely defined.

and

$$(z_{i,t}^{*1})_{i\in S} = \underset{(z_{i,t})_{i\in S}\in Z^{e_t^1}(S)}{\text{argmax}} \sum_{i\in S} f_i(z_{i,t}, K_{i,t}^1) - c_i(e_{i,t}^1, \Omega_{i,t}^1).$$

We also define $(z_t^\alpha)_{t\in\mathbb{N}}$ as follows:

$$\forall t \in \mathbb{N},\ z_t^\alpha = \alpha z_t^{*1} + (1-\alpha) z_t^{*0}.$$

It is straightforward to see that

$$\sum_{i\in S} \eta_{i,0}^\alpha \leq \breve{y}_0 = \breve{y}_0^\alpha, \tag{3.4}$$

and for each $t \in \mathbb{N}$,

$$\begin{aligned}\sum_{i\in S} \eta_{i,t+1}^\alpha &= \alpha \sum_{i\in S} \eta_{i,t+1}^1 + (1-\alpha)\sum_{i\in S}\eta_{i,t+1}^0 \\ &\leq \alpha \sum_{i\in S}\Big(f_i(z_{i,t}^{*1}, K_{i,t}^1) - c_i(e_{i,t}^1, \Omega_{i,t}^1)\Big) + (1-\alpha)\sum_{i\in S}\Big(f_i(z_{i,t}^{*0}, K_{i,t}^0) - c_i(e_{i,t}^0, \Omega_{i,t}^0)\Big).\end{aligned}$$

By concavity of f_i and $-c_i$, we therefore have

$$\sum_{i\in S}\eta_{i,t+1}^\alpha \leq \sum_{i\in S}\Big(f_i(z_{i,t}^\alpha, K_{i,t}^\alpha) - c_i(e_{i,t}^\alpha, \Omega_{i,t}^\alpha)\Big). \tag{3.5}$$

Since for each $t\in\mathbb{N}$, $(z_{i,t}^\alpha)_{i\in S} \in Z^{e_t^\alpha}(S)$, it follows that

$$\begin{aligned}\sum_{i\in S}\Big(f_i(z_{i,t}^\alpha, K_{i,t}^\alpha) - c_i(e_{i,t}^\alpha, \Omega_{i,t}^\alpha)\Big) &\leq \max\Big\{\sum_{i\in S} f_i(z_{i,t}, K_{i,t}^\alpha) - c_i(e_{i,t}^\alpha, \Omega_{i,t}^\alpha),\ (z_{i,t})_{i\in S}\in Z^{e_t^\alpha}(S)\Big\} \\ &= \breve{y}_{t+1}^\alpha. \end{aligned}\tag{3.6}$$

By combining Eqs 3.4, 3.5 and 3.6, we deduce that for each $t\in\mathbb{N}$,

$$\sum_{i\in S}\eta_{i,t}^\alpha \leq \breve{y}_t^\alpha. \tag{3.7}$$

On the other hand, it is clear that

$$e_0^\alpha \leq \Omega_0,$$

and for each $t\in\mathbb{N}$,

$$e_{t+1}^\alpha = \alpha e_{t+1}^1 + (1-\alpha)e_{t+1}^0 \leq \alpha(\Omega_t^1 - e_t^1) + (1-\alpha)(\Omega_t^0 - e_t^0) = \Omega_t^\alpha - e_t^\alpha. \tag{3.8}$$

Therefore, from Eqs 3.7 and 3.8, we can conclude that $(e_t^\alpha, \eta_t^\alpha)_{t\in\mathbb{N}}$ is a feasible path. It follows that

$$\begin{aligned}V^{\Omega_0, K_0, \breve{y}_0(S)}(S) &\geq \sum_{t\in\mathbb{N}}\delta^t v^{\breve{y}_t^\alpha(S), \eta_t^\alpha}(S) \\ &= \sum_{t\in\mathbb{N}}\delta^t\Big(\breve{y}_t^\alpha(S) - \sum_{i\in S}\eta_{i,t}^\alpha\Big) \\ &\geq \Big(\breve{y}_0(S) - \sum_{i\in S}\eta_{i,0}^\alpha\Big) + \sum_{t>0}\delta^t\Bigg(\sum_{i\in S}\Big(f_i(z_{i,t-1}^\alpha, K_{i,t-1}^\alpha) - c_i(e_{i,t-1}^\alpha, \Omega_{i,t-1}^\alpha)\Big) - \eta_{i,t}^\alpha\Bigg).\end{aligned}\tag{3.9}$$

By strict concavity of f_i and $-c_i$, we have for each $t > 0$:

$$\sum_{i\in S}\Big(f_i(z^{\alpha}_{i,t-1},K^{\alpha}_{i,t-1})-c_i(e^{\alpha}_{i,t-1},\Omega^{\alpha}_{i,t-1})\Big) > \alpha\sum_{i\in S}\Big(f_i(z^{*1}_{i,t-1},K^{1}_{i,t-1})-c_i(e^{1}_{i,t-1},\Omega^{1}_{i,t-1})\Big)$$
$$+(1-\alpha)\sum_{i\in S}\Big(f_i(z^{*0}_{i,t-1},K^{0}_{i,t-1})-c_i(e^{0}_{i,t-1},\Omega^{0}_{i,t-1})\Big). \tag{3.10}$$

Combining Eqs 3.9 and 3.10 leads to:

$$\begin{aligned}
V^{\Omega_0,K_0,\breve{y}_0(S)}(S) &> \alpha\Bigg(\Big(\breve{y}_0(S)-\sum_{i\in S}\eta^1_{i,0}\Big)+\sum_{t>0}\delta^t\sum_{i\in S}\Big(f_i(z^{*1}_{i,t-1},K^{1}_{i,t-1})-c_i(e^{1}_{i,t-1},\Omega^{1}_{i,t-1})\Big)-\eta^1_{i,t}\Big)\Bigg)\\
&+(1-\alpha)\Bigg(\Big(\breve{y}_0(S)-\sum_{i\in S}\eta^0_{i,0}\Big)+\sum_{t>0}\delta^t\sum_{i\in S}\Big(f_i(z^{*0}_{i,t-1},K^{0}_{i,t-1})-c_i(e^{0}_{i,t-1},\Omega^{0}_{i,t-1})\Big)-\eta^0_{i,t}\Big)\Bigg)\\
&=\alpha V^{\Omega_0,K_0,\breve{y}_0(S)}(S)+(1-\alpha)V^{\Omega_0,K_0,\breve{y}_0(S)}(S)\\
&=V^{\Omega_0,K_0,\breve{y}_0(S)}(S).
\end{aligned}$$

The strict inequality leads to a contradiction. □

Theorem 2

Proof. We proceed in two steps.
Step 1: For each $\Omega_0(N)\in[0,\bar{\Omega}]^N$, $K_0(N)\in[0,\bar{K}]^N$ and $\breve{y}_0(N)\in[0,\bar{\breve{y}}]$, $\mathsf{C}(N,V^{\Omega_0(N),K_0(N),\breve{y}_0(N)})\neq\emptyset$ and is not reduced to a singleton.

By Proposition 8, for each nonempty coalition $S\subseteq N$, and each initial stock vector $\Big(\Omega_0(S),K_0(S),\breve{y}_0(S)\Big)\in[0,\bar{\Omega}]^S\times[0,\bar{K}]^S\times[0,\bar{\breve{y}}]$, there exists a unique optimal path $\Big(e^*_t(S),\eta^*_t(S)\Big)_{t\in\mathbb{N}}$ to the dynamic programming problem (3.3). For each nonempty coalition $S\subseteq N$, denote by $(\Omega^*_t(S))_{t\in\mathbb{N}}$ the sequence of resource stock vectors defined by $\Omega^*_0(S)=\Omega_0(S)$ and for each $t\in\mathbb{N}$, $\Omega^*_{t+1}(S)=\Omega^*_t(S)-e^*_t(S)$. Denote as well by $(K^*_t(S))_{t\in\mathbb{N}}$ the sequence of capital stock vectors defined by $K^*_0(S)=K_0(S)$ and for each $t\in\mathbb{N}$, $K^*_{t+1}(S)=(1-\tau)K^*_t(S)+\eta^*_t(S)$. Finally, denote $(\breve{y}^*_t(S))_{t\in\mathbb{N}}$ the sequence of production flow vectors defined by $\breve{y}^*_0(S)=\breve{y}_0(S)$ and for each $t\in\mathbb{N}$, $\breve{y}^*_{t+1}(S)=\max\Big\{\sum_{i\in S}f_i(z_{i,t}(S),K^*_{i,t}(S))-c_i(e^*_{i,t}(S),\Omega^*_{i,t}(S)),\ (z_{i,t}(S))_{i\in S}\in Z^{e^*_t(S)}(S)\Big\}$.

Let $\lambda\in\mathbb{B}(N)$ be a balanced system of N. If $\lambda(N)=1$, it is clear that

$$V^{\Omega_0(N),K_0(N),\breve{y}_0(N)}(N)=\sum_{S\subseteq N}\lambda(S)V^{\Omega_0(S),K_0(S),\breve{y}_0(S)}(S).$$

Assume $\lambda(N)\neq 1$. We define $(e^{\lambda}_t(N))_{t\in\mathbb{N}}\in(\mathbb{R}^N_+)^{\mathbb{N}}$ and $(\Omega^{\lambda}_t(N))_{t\in\mathbb{N}}\in(\mathbb{R}^N_+)^{\mathbb{N}}$ as follows:

$$\forall i\in N,\ \forall t\in\mathbb{N},\quad e^{\lambda}_{i,t}(N)=\sum_{S\ni i}\lambda(S)e^*_{i,t}(S),$$

$$\forall i \in N, \forall t \in \mathbb{N}, \quad \Omega^{\lambda}_{i,t}(N) = \sum_{S \ni i} \lambda(S) \Omega^{*}_{i,t}(S).$$

Observe that for each $i \in N$ and each $t \in \mathbb{N}$, we have $\Omega^{\lambda}_{i,t}(N) \in [0, \bar{\Omega}]$ since $\sum_{S \ni i} \lambda(S) = 1$ and $\Omega^{\lambda}_{i,t+1}(N) = \Omega^{\lambda}_{i,t}(N) - e^{\lambda}_{i,t}(N)$. We also can assert that for each $i \in N$ and each $t \in \mathbb{N}$,

$$e^{\lambda}_{i,t}(N) \leq \Omega^{\lambda}_{i,t}(N). \tag{3.11}$$

We define as well $(\eta^{\lambda}_t(N))_{t \in \mathbb{N}} \in (\mathbb{R}^N_+)^{\mathbb{N}}$ and $(K^{\lambda}_t(N))_{t \in \mathbb{N}} \in (\mathbb{R}^N_+)^{\mathbb{N}}$ as follows:

$$\forall i \in N, \forall t \in \mathbb{N}, \quad \eta^{\lambda}_{i,t}(N) = \sum_{S \ni i} \lambda(S) \eta^{*}_{i,t}(S),$$

$$\forall i \in N, \forall t \in \mathbb{N}, \quad K^{\lambda}_{i,t}(N) = \sum_{S \ni i} \lambda(S) K^{*}_{i,t}(S).$$

Observe that for each $i \in N$ and each $t \in \mathbb{N}$, we have $K^{\lambda}_{i,t}(N) \in [0, \bar{K}]$. We set $\breve{y}^{\lambda}_0(N) = \breve{y}_0(N)$ and for each $t > 0$,

$$\breve{y}^{\lambda}_t(N) = \max\left\{ \sum_{i \in N} f_i(z_{i,t-1}(N), K^{\lambda}_{i,t-1}(N)) - c_i(e^{\lambda}_{i,t-1}(N), \Omega^{\lambda}_{i,t-1}(N)),\ (z_{i,t-1}(N))_{i \in N} \in Z^{e^{\lambda}_{t-1}(N)}(N) \right\}.$$

Observe that for each $i \in N$ and each $t \in \mathbb{N}$, we have $\breve{y}^{\lambda}_{i,t}(N) \in [0, \bar{\breve{y}}]$. It is straightforward to see that

$$\sum_{i \in N} \eta^{\lambda}_{i,0}(N) \leq \breve{y}_0(N) = \breve{y}^{\lambda}_0(N), \tag{3.12}$$

and for each $t \in \mathbb{N}$,

$$\begin{aligned}
\sum_{i \in N} \eta^{\lambda}_{i,t+1}(N) &= \sum_{i \in N} \sum_{S \ni i} \lambda(S) \eta^{*}_{i,t+1}(S) \\
&= \sum_{S \subseteq N} \lambda(S) \sum_{i \in S} \eta^{*}_{i,t+1}(S) \\
&\leq \sum_{S \subseteq N} \lambda(S) \breve{y}^{*}_{i,t+1}(S) \\
&= \sum_{S \subseteq N} \lambda(S) \sum_{i \in S} \Big(f_i(z^{*}_{i,t}(S), K^{*}_{i,t}(S)) - c_i(e^{*}_{i,t}(S), \Omega^{*}_{i,t}(S) \Big) \\
&= \sum_{i \in N} \sum_{S \ni i} \lambda(S) \Big(f_i(z^{*}_{i,t}(S), K^{*}_{i,t}(S)) - c_i(e^{*}_{i,t}(S), \Omega^{*}_{i,t}(S) \Big),
\end{aligned} \tag{3.13}$$

where we define, for each $t \in \mathbb{N}$,

$$z^{*}_t(S) = \underset{(z_{i,t}(S))_{i \in S} \in Z^{e^{*}_t}(S)}{\operatorname{argmax}} \sum_{i \in S} f_i(z_{i,t}, K^{*}_{i,t}) - c_i(e^{*}_{i,t}, \Omega^{*}_{i,t}).$$

By concavity of f_i and $-c_i$, we therefore have

$$\sum_{i \in N} \eta^{\lambda}_{i,t+1}(N) \leq \sum_{i \in N} \Big(f_i(z^{\lambda}_{i,t}(N), K^{\lambda}_{i,t}(N)) - c_i(e^{\lambda}_{i,t}(N), \Omega^{\lambda}_{i,t}(N)) \Big) \tag{3.14}$$

where $(z^{\lambda}_t(N))_{t \in \mathbb{N}} \in (\mathbb{R}^N_+)^{\mathbb{N}}$ is defined as

$$\forall i \in N, \forall t \in \mathbb{N}, \quad z^{\lambda}_{i,t}(N) = \sum_{S \ni i} \lambda(S) z^{*}_{i,t}(S).$$

The following equalities hold:

$$\sum_{i\in N} e^{\lambda}_{i,t}(N) = \sum_{i\in N}\sum_{S\ni i}\lambda(S)e^{*}_{i,t}(S)$$
$$= \sum_{S\subseteq N}\lambda(S)\sum_{i\in S}e^{*}_{i,t}(S).$$

Since $z^*_t(S) \in Z^{e^*_t}(S)$, it follows that

$$\begin{aligned}\sum_{S\subseteq N}\lambda(S)\sum_{i\in S}e^{*}_{i,t}(S) &= \sum_{S\subseteq N}\lambda(S)\sum_{i\in S}z^{*}_{i,t}(S)\\ &= \sum_{i\in N}\sum_{S\ni i}\lambda(S)z^{*}_{i,t}(S)\\ &= \sum_{i\in N}z^{\lambda}_{i,t}(N),\end{aligned}$$

from which we deduce that $z^{\lambda}_t(N) \in Z^{e^{\lambda}_t(N)}(N)$ for each $t \in \mathbb{N}$. It follows that

$$\begin{aligned}\sum_{i\in N}\Big(f_i(z^{\lambda}_{i,t}(N), K^{\lambda}_{i,t}(N)) - c_i(e^{\lambda}_{i,t}(N), \Omega^{\lambda}_{i,t}(N))\Big) \leq \max\Big\{&\sum_{i\in N} f_i(z_{i,t}(N), K^{\lambda}_{i,t}(N))\\ &- c_i(e^{\lambda}_{i,t}(N), \Omega^{\lambda}_{i,t}(N)),\ (z_{i,t}(N))_{i\in N}\in Z^{e^{\lambda}_t(N)}(N)\Big\}\\ &= \breve{y}^{\lambda}_{t+1}(N). \qquad (3.15)\end{aligned}$$

By combining Eqs 3.12, 3.14 and 3.15, we deduce that for each $t \in \mathbb{N}$,

$$\sum_{i\in N}\eta^{\lambda}_{i,t}(N) \leq \breve{y}^{\lambda}_t(N). \qquad (3.16)$$

We can conclude from Eqs 3.11 and 3.16 that $(e^{\lambda}_t(N), \eta^{\lambda}_t(N))_{t\in\mathbb{N}}$ is a feasible path.

By definition of $\breve{y}^{\lambda}_t(N)$ for each $t \in \mathbb{N}$, and since $z^{\lambda}_t(N) \in Z^{e^{\lambda}_t(N)}(N)$ for each $t \in \mathbb{N}$, we have

$$\breve{y}^{\lambda}_{t+1}(N) \geq \sum_{i\in N}\Big(f_i(z^{\lambda}_{i,t}(N), K^{\lambda}_{i,t}(N)) - c_i(e^{\lambda}_{i,t}(N), \Omega^{\lambda}_{i,t}(N))\Big). \qquad (3.17)$$

The hypothesis of strict concavity ensures that

$$\sum_{i\in N}\Big(f_i(z^{\lambda}_{i,t}(N), K^{\lambda}_{i,t}(N)) - c_i(e^{\lambda}_{i,t}(N), \Omega^{\lambda}_{i,t}(N))\Big) > \sum_{i\in N}\sum_{S\ni i}\lambda(S)\Big(f_i(z^{*}_{i,t}(S), K^{*}_{i,t}(S)) - c_i(e^{*}_{i,t}(S), \Omega^{*}_{i,t}(S))\Big) \qquad (3.18)$$

because for each $i \in N$ and $t \in \mathbb{N}$, $z^{\lambda}_{i,t}(N)$ (resp. $K^{\lambda}_{i,t}(N), e^{\lambda}_{i,t}(N), \Omega^{\lambda}_{i,t}(N)$) is a convex combination of $(z^*_{i,t}(S))_{S\ni i}$ (resp. $(K^*_{i,t}(S))_{S\ni i}, (e^*_{i,t}(S))_{S\ni i}, (\Omega^*_{i,t}(S))_{S\ni i}$).

Combining Eqs. 3.17 and 3.18, we have:

$$\begin{aligned}\breve{y}^{\lambda}_{t+1}(N) &> \sum_{i\in N}\sum_{S\ni i}\lambda(S)\Big(f_i(z^{*}_{i,t}(S), K^{*}_{i,t}(S)) - c_i(e^{*}_{i,t}(S), \Omega^{*}_{i,t}(S))\Big)\\ &= \sum_{S\subseteq N}\lambda(S)\sum_{i\in S}\Big(f_i(z^{*}_{i,t}(S), K^{*}_{i,t}(S)) - c_i(e^{*}_{i,t}(S), \Omega^{*}_{i,t}(S))\Big)\\ &= \sum_{S\subseteq N}\lambda(S)\breve{y}^{*}_{t+1}(S). \qquad (3.19)\end{aligned}$$

Since $(e_t^\lambda(N), \eta_t^\lambda(N))_{t\in\mathbb{N}}$ is a feasible path, and that $\Omega_0^\lambda(N) = \Omega_0(N)$, $K_0^\lambda(N) = K_0(N)$ and $\breve{y}_0^\lambda(N) = \breve{y}_0(N)$, the definition of $V^{\Omega_0(N),K_0(N),\breve{y}_0(N)}(N)$ ensures that

$$V^{\Omega_0(N),K_0(N),\breve{y}_0(N)}(N) \geq \sum_{t\in\mathbb{N}} \delta^t \Big(\breve{y}_t^\lambda(N) - \sum_{i\in N} \eta_{i,t}^\lambda(N) \Big).$$

Moreover, observe that

$$\breve{y}_0(N) = \sum_{S\subseteq N} \lambda(S)\breve{y}_0(S). \tag{3.20}$$

Indeed,

$$\begin{aligned}
\sum_{S\subseteq N} \lambda(S)\breve{y}_0(S) &= \sum_{S\subseteq N} \lambda(S) \sum_{i\in S} \breve{y}_{i,0} \\
&= \sum_{i\in N} \sum_{S\ni i} \lambda(S)\breve{y}_{i,0} \\
&= \sum_{i\in N} \breve{y}_{i,0} \sum_{S\ni i} \lambda(S) \\
&= \sum_{i\in N} \breve{y}_{i,0}.
\end{aligned}$$

Using Eqs. 3.20, 3.13 and 3.19, we obtain

$$\begin{aligned}
V^{\Omega_0(N),K_0(N),\breve{y}_0(N)}(N) &= \left(\breve{y}_0(N) - \sum_{i\in N} \eta_0^\lambda(N) \right) + \sum_{t>0} \delta^t \Big(\breve{y}_t^\lambda(N) - \sum_{i\in N} \eta_{i,t}^\lambda(N) \Big) \\
&> \left(\sum_{S\subseteq N} \lambda(S)\breve{y}_0(S) - \sum_{S\subseteq N} \lambda(S) \sum_{i\in S} \eta_{i,0}^*(S) \right) \\
&\quad + \sum_{t>0} \delta^t \Big(\sum_{S\subseteq N} \lambda(S)\breve{y}_t^*(S) - \sum_{S\subseteq N} \lambda(S) \sum_{i\in S} \eta_{i,t}^*(S) \Big) \\
&= \sum_{S\subseteq N} \lambda(S) \sum_{t\in\mathbb{N}} \delta^t \Big(\breve{y}_t^*(S) - \sum_{i\in S} \eta_{i,t}^*(S) \Big) \\
&= \sum_{S\subseteq N} \lambda(S) V^{\Omega_0(S),K_0(S),\breve{y}_0(S)}(S).
\end{aligned}$$

The strict inequality holds by strict concavity and since $\lambda(N) \neq 1$.

By Proposition 1 stating the Bondareva-Shapley theorem, $\mathsf{C}(N, V^{\Omega_0(N),K_0(N),\breve{y}_0(N)}) \neq \emptyset$ and by Proposition 2, $\mathsf{C}(N, V^{\Omega_0(N),K_0(N),\breve{y}_0(N)})$ is not reduced to a singleton.

Step 2: Existence and non uniqueness of the solution.

Step 1 states that for each state $(\Omega, K, \breve{y})$, the core $\mathsf{C}(N, V^{\Omega,K,\breve{y}})$ is nonempty. Hence, for each initial state $(\Omega, K, \breve{y})$ we can build a sequence $((X_{i,t}(\Omega, K, \breve{y}))_{i\in N})_{t\in\mathbb{N}}$ - or simply $((X_{i,t})_{i\in N})_{t\in\mathbb{N}}$ - such that for each $t \in \mathbb{N}$:

(i) $\sum_{i\in N} X_{i,t} = V^{\Omega_t^*(N),K_t^*(N),\breve{y}_t^*(N)}(N)$,

(ii) $\sum_{i\in S} X_{i,t} \geq V^{\Omega_{S,t}^*(N),K_{S,t}^*(N),\breve{y}_{S,t}^*(N)}(S)$,

where $(\Omega_t^*(N), K_t^*(N), \breve{y}_t^*(N))_{t\in\mathbb{N}}$ is the sequence of state vectors associated with the optimal extraction and investment path of N when the initial state is $(\Omega, K, \breve{y})$.

Let the instantaneous allocation path for each country be $(x_{i,t}(\Omega, K, \breve{y}))_{i\in N,t\in\mathbb{N}}$ - or simply $(x_{i,t})_{i\in N,t\in\mathbb{N}}$ - defined for each initial state $(\Omega, K, \breve{y})$ such that for each $t \in \mathbb{N}$, $x_{i,t} = X_{i,t} - \delta X_{i,t+1}$.

Let us first show that $(x_{i,t})_{i\in N,t\in\mathbb{N}}$ is an international distribution policy. For each $t \in \mathbb{N}$,

$$\begin{aligned}\sum_{i\in N} x_{i,t} &= \sum_{i\in N} X_{i,t} - \delta \sum_{i\in N} X_{i,t+1} \\ &= V^{\Omega_t^*(N),K_t^*(N),\breve{y}_t^*(N)}(N) - \delta V^{\Omega_{t+1}^*(N),K_{t+1}^*(N),\breve{y}_{t+1}^*(N)}(N)\end{aligned}$$

and the Bellman equation ensures that:

$$V^{\Omega_t^*(N),K_t^*(N),\breve{y}_t^*(N)}(N) = v^{\breve{y}_t^*(N),\eta_t^*(N)}(N) + \delta V^{\Omega_{t+1}^*(N),K_{t+1}^*(N),\breve{y}_{t+1}^*(N)}(N).$$

Therefore, we have for each $t \in \mathbb{N}$,

$$\sum_{i\in N} x_{i,t} = v^{\breve{y}_t^*(N),\eta_t^*(N)}(N).$$

Let us now prove that $(x_{i,t})_{i\in N,t\in\mathbb{N}}$ is an undominated policy. For each $T \in \mathbb{N}$, we have

$$\begin{aligned}\sum_{i\in S}\sum_{t=0}^{\infty} \delta^t x_{i,t+T} &= \sum_{i\in S}\sum_{t=0}^{\infty}\left(\delta^t X_{i,t+T} - \delta^{t+1} X_{i,t+T+1}\right) \\ &= \sum_{i\in S} X_{i,T} \\ &\geq V^{\Omega_{S,T}^*(N),K_{S,T}^*(N),\breve{y}_{S,T}^*(N)}(S).\end{aligned}$$

We can therefore conclude that the set $\mathcal{C}(\Omega, K, \breve{y})$ of undominated policies when the initial state is $(\Omega, K, \breve{y})$ is nonempty. Since we established in Step 1 that for each state $(\Omega, K, \breve{y})$, $\mathsf{C}(N, V^{\Omega,K,\breve{y}})$ is not reduced to a singleton, we deduce by construction of $(x_{i,t})_{i\in N,t\in\mathbb{N}}$ that $\mathcal{C}(\Omega, K, \breve{y})$ is not reduced to a singleton.

$\square$

Conclusion

Charos mourait à chaque instant pour renaître aussitôt, comme la vie. Il y a des milliers d'années que filles et garçons dansent sous les arbres au feuillage neuf – peupliers, sapins, chênes, platanes, palmiers élancés –, et ils danseront des milliers d'années encore, dévorés de désir. Les visages se perdent sous terre, changent tous les vingt ans, laissent la place à d'autres. Mais l'essence restera toujours, unique, identique à elle-même, dansant dans ses vingt ans, amoureuse, immortelle.

— Nikos Kazantzaki, *Alexis Zorba* (trad. René Bouchet)

This dissertation offers a perspective on the long-run availability of the materials that constitute the bones and nerves of every infrastructure. It provides different methodologies to analyze non-renewable natural resource scarcity and its consequences. This work is rooted in very old debates, which have been revived throughout the history of economic thought, making Economics such a living science. I looked at it with the eyes of a geologist first, who is someone with a completely different sense of time an space compared to an economist. However, both are amazed by complexity, whether this wonder comes from the observation of natural processes or from the scrutiny of human development.

The first part of this work spotlights Earth sciences, but also shows that the scarcity issue cannot be disentangled from its economic side. It adopts a world scale, but restricted to the specific sector of mining. We show that the consumption of fossil materials has followed an exponential evolution during the last century, and would grow even more if the entire energy system was based on renewable sources. This work further provides a strong empirical relationship between dilution, extraction energy and commodity prices. The modeling of the copper sector suggests that its exponential consumption is not sustainable, even with high recycling rates and optimistic assessment of copper Earth's crust distribution. Business-as-usual scenarios assuming a constant growth of inelastic demand and an evolution of model parameters in line with the 1900-2015 trends lead to a bell-shape copper production that peaks between by the mid-century. However, the model provided conditions for which the extraction can be sustained and that can lead to a plateau instead of a peak and/or postpone the eventual peak of production after 2200. The demand in copper should first be stabilized. If the assumption on a stabilization of consumption of copper at high GDP is correct, then world GDP should also be constant after 2100. The second condition is that recycling rates should reach 70% by the end of the century, as opposed to 40% today.

These results give strong arguments that the extraction of one essential metal at present rates is not sustainable within the current century. However, it is worth reminding that, like any other model, its relies in a certain number of assumptions, which only give an approximation of reality. For instance, one key assumption of our model is that energy prices are constant, and a potential change would certainly affect the results. Moreover, the wealth of the productive sector is only its cumulative profits, and do not encompasses other sources of financing. Moreover, from the empirical point of view, the financial data taken as reference points do not correspond to this variable. These two points are explored in the second chapter. The question of the 'reference price' is also crucial, and even though it is an empirical finding, the economic theory behind should be deepened. Finally, an extensive sensitivity analysis should be conducted on this model to show its robustness. Even though most variables are controlled by historical data, non-linear dynamical system tend to be very sensitive. However, note that an initial analysis can already be drawn from the available scenarios we propose, and suggests that we can be quite confident of our results.

The second part embeds the mining model in a world economy to investigate the effects of its dynamics on long-term growth. We show that, when the mining sector is set into a Goodwin-class model, no long-run balanced growth of production can emerge with a positive growth rate of GDP. Moreover, the dynamics admits other equilibria, among which a debt-deflationary equilibrium with zero production, employment and unbounded private debt. In other words, the unique desirable long-run steady state is stationary. We further provide an empirical estimation of the production function, giving three possible functional forms with different substitution elasticities between capital, labor and copper.

As for the previous model, due caution needs to be taken on these results. First, the Goodwin-class model, as many other macroeconomic models, relies on a assumptions that are difficult to check. There is for instance a large debate in Economics on the existence of a Phillips curve. Moreover, this model misses an important feature of mineral resource scarcity for the question of recycling was here neglected. Last but not least, the production function is key in this model and still presents important weaknesses that prevents from giving a definitive answer on the matter of long-term growth. Not only the Cambridge controversy issues has not been resolved, but they appear even more clearly when materials are taken into account. The substitution of capital and material for instance may seem absurd, since capital is made out of materials. The production function, as it is, do not incorporate all the complexity of the transformation process. The flow-fund model is more adapted to that purpose. The long-span dataset we constructed is also to be taken with due precaution since it is a simplistic aggregate of a limited number of countries. A promising avenue for research is therefore to develop the input-output theory, and to combine it with material flow analysis in a dynamic framework. Such a consideration of physical and monetary data would interestingly bridge many weaknesses of the neoclassical production function. Unfortunately, a global long-span dataset is missing, and we call for an effort to build it.

The last part explores the implications of natural resource scarcity through cooperative lenses. We show that an international long-term cooperation strategy, where wealth derived from natural resources and productive capacity is shared in a common pool, can be put in place. We build for that purpose a dynamic cooperative game, and look for its recursive core. We show that it contains at least two elements, which means that, if the countries of a coalition follow an long-term cooperation strategy in terms of extraction, investment and induced trade, then the best way to optimize their own consumption stream while caring about future generations is to form a global coalition. Since this core is not reduced to a singleton, it leaves room to a choice among these allocations, for instance by using fairness criteria. The presented model can therefore be viewed as a tool to refresh the common look at the North-South opposition. This change of perspective opens a space for discussion on the equity of trade in times of environmental threats.

Indeed, mineral scarcity and environmental damages are entwined. First because the extraction process has strong local environmental (and social) impacts. Water use and pollution are of the most worrying issue. But the extraction is also energy-intensive. It consequently strongly contributes to the extensive use of fossil fuels, and therefore to global warming. At the same time, the essential transition to low carbon energy structure will increase the demand for these materials.

This dissertation calls for an urgent large-scale development of an efficient collecting and recycling infrastructure. Another crucial leverage is the increase in the lifetime of goods, that will reduce depreciation effects. It calls as well for explicit policies to reduce material intensity. However, these policies should take into consideration the geographical inequalities in terms of material development. A significant part of the developing world is still at earlier stages and do not have a full access to proper essential infrastructures like water or electricity. In 1930, in times of extreme economic pessimism, John Maynard Keynes had colorful visions of the future[5]. According to him, if only "our willingness to entrust to science the direction of those matters which are properly the concern of science" and "our determination to avoid wars and civil dissensions" was achieved, we would attain an "economic bliss". Maybe time has come for societies to decide what this bliss is made of.

[5] John Maynard Keynes, *Economic Possibilities for our Grandchildren*,1930

Summary

Keywords: *Non-renewable natural resources, Metals, Mining Sector, Growth Theory, Cooperative Games, Sustainability*.

This dissertation offers a transdisciplinary modeling approach to feed the debates raised by the long-run availability of mineral materials. It investigates the plausibility of a depletion threat posed to key metal resources within the current century. The first chapter studies the supply and demand of fossil materials at a global scale and further focuses on the functioning of the copper mining sector. Our work provides conditions under which the extraction of copper can be sustained and lead to a plateau instead of a peak, among which the stabilization of the demand for copper and the significant increase in collecting and recycling rates. The second chapter embeds the extracting sector into the whole global economy and evaluates theoretically the consequences of the mining dynamics on the long-run growth of output. We demonstrate that the unique desirable long-run steady state is stationary. The third chapter explores the implications of natural resource scarcity in terms of global trade and international cooperation. We show that if the countries of a coalition follow a long-term cooperation strategy in terms of extraction, investment, and resource trade, then the best way to optimize their own consumption while caring about future generations is to form a global coalition. This dissertation proposes a new way of considering global trade in the setting of an exhaustible resource and demonstrates that a global coalition where natural and productive wealth is commonly shared is to the advantage of all countries.

Mots-Clés : *Ressources Naturelles Non Renouvelables, Métaux, Secteur Minier, Théorie de la croissance, Jeux Coopératifs, Soutenabilité*.

Cette thèse propose une approche transdisciplinaire visant à nourrir les débats suscités par la disponibilité à long terme des matières premières minérales. Elle analyse la plausibilité d'une menace de déplétion de certains métaux clés avant la fin du siècle. Le premier chapitre se focalise sur le fonctionnement du secteur minier et traite de l'offre et de la demande des matériaux fossiles à l'échelle mondiale, en s'appuyant sur l'exemple du cuivre. Nos résultats nous permettent de poser des conditions nécessaires à l'établissement d'un plateau d'extraction plutôt qu'un pic, moyennant une stabilisation de la demande totale en cuivre. Le second chapitre place le secteur minier dans l'économie globale et évalue de manière théorique les conséquences à long terme de la dynamique de ce secteur sur la croissance de la production. Nous montrons que, dans la configuration proposée, l'unique équilibre désirable à long terme est stationnaire. Le troisième chapitre explore les implications induites par la rareté des ressources non renouvelables en matière de coopération et de commerce international. Nous montrons que si les pays membres d'une coalition suivent une stratégie de coopération pour leurs choix d'investissement, d'extraction et d'échange de ressources, alors la meilleure manière d'optimiser leur propre consommation tout en se souciant des générations futures serait de former la coalition la plus large. Cette thèse propose une nouvelle manière de réfléchir au commerce international dans un contexte de ressources finies, et montre qu'une coalition mondiale où les richesses naturelles et productives des pays sont mises en commun peut être à l'avantage de tous.