Planification stratégique des aéroports en environnements incertains : approche utilisant la programmation dynamique et logique dual floue

Elena Mihaela Capitanul

THÈSE

En vue de l'obtention du:

DOCTORAT DE L'UNIVERSITÉ DE TOULOUSE

Délivré par: *l'Université Toulouse 3 Paul Sabatier (UT3 Paul Sabatier)*

Présentée et soutenue par:

ELENA MIHAELA **CĂPITANUL**

Le 30 Septembre 2016

Titre:

AIRPORT STRATEGIC PLANNING UNDER UNCERTAINTY: FUZZY DUAL DYNAMIC PROGRAMMING APPROACH

École doctorale:

MITT – Mathématiques, Informatique, Télécommunications de Toulouse

Unité de recherche:

Laboratoire de Mathématiques Appliquées, Informatique et Automatique pour l'Aérien

ÉCOLE NATIONALE DE L'AVIATION CIVILE

Directeur du Thèse:

Felix **MORA-CAMINO**

Jury:

Francisco A. de MORAES A. DORIA Federal University of Rio de Janeiro, Rapporteur

Rosa Maria ARNALDO VALDÉS, Polytechnic University of Madrid, Rapporteur

Sonia CAFIERI, Ecole Nationale de l'Aviation Civile, Examinateur

Alba Maria AGUSTIN, Public University of Navarre, Examinateur

Philippe MARTHON, National Polytechnic Institute of Toulouse, Examinateur

[Page intentionally left blank]

ACKNOWLEDGEMENTS

Professor Felix Mora-Camino, firstly I would like to thank you! The opportunity you gave me three years ago turned out to be the biggest professional challenge of my life so far. Thank you for your continuous guidance, support and shared knowledge! Moreover, most importantly, thank you for understanding my struggles and pushing me to strive, trusting more than me at times than I can achieve this. Your thoughtful direction, insightful vision and being a constant inspiration lead me to learn so much over the last three years and I am most grateful.

My appreciation to every one of my co-authors over the last three years, from whom I learned new techniques and research approaches: Professor Carlos Nunes Cosenza and Fabio Krykhtine, both from COPPE, Federal University of Rio de Janeiro, Professor Walid El Moudani from Lebanese University of Tripoli and Professor Hamdan Al-Fazari from Sohar University, Oman.

I want to thank everybody I met as a member of the MAIAA Laboratory. Friends and colleagues, each one of you taught me something I will carry with me forever. My time at ENAC would have been significantly less rewarding without the interactions, motivating and stimulating discussions I enjoyed with each one of you. Thank you for creating such a diverse and multicultural environment in which I felt so welcomed. Sharing the student life with you all, was a privilege and one of the most culturally enriching periods of my life.

Words are not enough to express my love for my family. Your unconditional support, especially in the most challenging moments, helped me push through and continue this endeavour. You helped me stayed motivated when I was not, showed me the positives when I saw only negatives, lifted me up when I was down. So deeply grateful for everything. In addition, I want to send all my love and gratitude to all my soul friends who encouraged me throughout the entirety of this process. Thank you all from the bottom of my heart!

To my husband,

To my parents and little brother,

To my very special friend, Iulian,

With all my love and gratitude…

ABSTRACT

Airports are critical connectors in the air transportation operational system. In order to meet their operational, economic and social obligations in a very volatile environment, airports need to embrace change rather than resist it. Like any other industry, airports face a wide array of risks, some specific to air transportation, other having only an indirect influence but powerful enough to disrupt airport activities.

Long-term airport planning has become a complex issue due to the constant growth in air traffic demand. A new dimension of complexity emerged when uncertainty began having a more and more disruptive and significantly costly impact on developing airport infrastructure. Historically, the ability of traditional risk and uncertainty mitigation tools proved inefficient. Countless unforeseen events like terrorist attacks, economic recession, natural disasters, had a dramatic impact on traffic levels, some with a global reach. To these highly improbable type of events can be added technological advancements, new airlines and airports business models, policy and regulation changes, increasing concern for environmental impact.

In this context, the thesis puts forward an innovative approach for addressing risk assessment and mitigation under uncertainty in long-term airport infrastructure development projects. The thesis expands on the newly developed formalism of fuzzy dual numbers as a key tool to address uncertainty. After a comprehensive review of the airport industry in the context of uncertain environments, fuzzy dual numbers and fuzzy dual calculus are introduced. Since the airport infrastructure development project is another case of multi-stage decision making problem, dynamic programming is considered in order to optimize the sequential decision making process. The originality of the approach resides in the fact that the entire process will be *fuzzified* and fuzzy dual dynamic programming components will be introduced. To validate our method, a study case will be developed.

Key words: airports, optimization, fuzzy logic, dynamic programming, financial risk

RÉSUMÉ

Les aéroports sont des connecteurs critiques dans le système opérationnel de transport aérien. Afin de répondre à leurs obligations opérationnelles, économiques et sociales dans un environnement très volatil, ont besoin d'aéroports à embrasser le changement plutôt que d'y résister. Comme toute autre industrie, font face à des aéroports un large éventail de risques, dont certains spécifiques au transport aérien, les autres ayant seulement une influence indirecte mais assez puissant pour perturber les activités aéroportuaires.

La planification longue terme de l'aéroport est devenue une question complexe en raison de la croissance constante de la demande de trafic aérien. Une nouvelle dimension de complexité est apparue lorsque l'incertitude a commencé à avoir un impact plus en plus perturbatrice, et significativement coûteuse sur le développement des infrastructures aéroportuaires.

Historiquement, la capacité des outils traditionnels pour atténuer le risque et l'incertitude ont avérée inefficace. D'innombrables événements imprévus comme les attaques terroristes, la récession économique, les catastrophes naturelles, ont eu un impact dramatique sur les niveaux de trafic, certains avec une portée mondiale. Pour ce type hautement improbable d'événements peut être ajouté les progrès technologiques, de nouveaux modèles d'affaires des compagnies aériennes et aéroports, les changements de politique et de réglementation, préoccupation croissante pour l'impact environnemental.

Dans ce contexte, la thèse met en avant une approche novatrice pour aborder l'évaluation des risques et de l'atténuation dans l'incertitude dans les projets de développement des infrastructures aéroportuaires à long-terme. La thèse se développe sur le formalisme récemment développé de nombres flous comme un outil clé pour aborder l'incertitude. Après un examen approfondi de l'industrie aéroportuaire dans le contexte des environnements incertains, nombres double flous et double floue arithmétiques sont introduits. Comme le projet de développement des infrastructures aéroportuaires est un autre cas de problème de prise de décision en plusieurs étapes, la programmation dynamique est prise en compte afin d'optimiser le processus

séquentiel de prise de décision. L'originalité de l'approche réside dans le fait que l'ensemble du processus sera floue et la composante double floue de la programmation dynamique sera introduite. Pour valider notre méthode, une étude de cas sera développée.

Mots-clés: aéroports, optimisation, logique floue, programmation dynamique, risque financier

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

A-CDM	Airport – Collaborative Decision Making
ACI	Airports Council International
ANSP	Air Navigation Service Provider
ERM	Enterprise Risk Management
FAA	Federal Aviation Administration
FLS	Fuzzy Logic System
GDP	Gross Domestic Product
ICAO	International Civil Aviation Organization
IATA	International Air Transportation Association
IT&C	Information Technology & Communication
NextGen	FAA Next Generation Air Transportation System
NFC	Near Field Communication
RPK	Revenue Passenger Kilometre
SESAR	Single European Sky ATM Research
NPV	Net Present Value

X	Universe of discourse
x	Generic elements of X
$\tilde{A}$	Fuzzy set
$\mu_{\tilde{A}}(x)$	Membership function

$(x, \mu_{\tilde{A}}(x))$	Singleton
$S(\tilde{A})$	Support set of the fuzzy set $\tilde{A}$
$hgt(\tilde{A})$	Height of $\tilde{A}$
A_α	α-cut set
$A_\alpha^{'}$	Strong α-cut set
$\left\|\tilde{A}\right\|$	Scalar cardinality of a fuzzy set (the power of $\tilde{A}$)
$\left\|\left\|\tilde{A}\right\|\right\|$	Relative cardinality
$\tilde{\Delta}$	Set of fuzzy dual numbers of the form $a + \varepsilon.b$
a	Primal part
b	Dual part
ε	Unity pure dual number
ρ	Shape parameter
$\succ$	Strong partial order
$\succeq$	Mean partial order
$\succsim$	Weak partial order
$\cong$	Fuzzy equality
c	Degree of certainty
α	Area of intersection of two fuzzy dual numbers
$\tilde{0}$	Fuzzy dual neutral element
$\tilde{1}$	Neural element of fuzzy dual multiplication

$\tilde{+}$	Fuzzy dual addition
$\tilde{\bullet}$	Fuzzy dual product
E	Euclidean space
$\tilde{E}$	Set of fuzzy dual vectors
E^+	The positive half space of E in its canonical basis
*	Inner product in $\tilde{E}$
.	Inner product in E
$\tilde{M}_n$	Set of fuzzy dual square matrices of order $n \times n$
$P(e_i)$	Fuzzy dual probability distribution of the event e_i
$P(\bar{e}_i)$	Fuzzy dual probability distribution associated with the complementary of event e_i
*	Probabilistic product
D_0	Fuzzy dual linear programming problem with fuzzy dual constraints and real decision variables
c_i	Cost coefficients
a_{ki}	Technical parameters
b_k	Constraint levels
d_i	Decision variables
$\underline{x}^*$	Solution of problem D_0
D_1	Fuzzy dual linear programming problem with fuzzy dual variables
X_n	Set of feasible decisions
x_n	A decision from the set of feasible decisions X_n

s_n	State of the process
n	A stage from a total of N stages
G	Fuzzy goal characterized by the fuzzy membership function $\mu_G(x)$
C	Fuzzy constraint characterized by the fuzzy membership function $\mu_C(x)$
D	Fuzzy decision characterized by the fuzzy membership function $\mu_D(x)$
d	Function representing the fuzzy operator *and*
$G^{'}$	Induced fuzzy goal
Γ	Transition function from state s_n to state s_{n+1}
K	Time horizon
I	Set of traffic activities
D_k^i	Predicted potential demand
T_k^i	Traffic
S_k^i	Mean capacity of aircraft type i
α_k^i	Mean load factor
r_k^i	Rate of return
C_k^{Pi}	Potential passenger processing capacity
C_k^{Ti}	Potential aircraft movements processing capacity
$\overline{D}_k^i$	Estimated level of demand
L_i	Candidate upgrades

θ_l^i	The period at which upgrade *l* for traffic type *i* is scheduled
Δ_k	Set of projects retained until period *k*
$c_l^{ik}(\Delta_k)$	Cost of upgrade
R_k^i	Revenues generated by traffic type *i* at period *k*
ρ	Rate of actualization
π	Expected net present value of the project
π^L	Primal part of the NPV fuzzy dual representation
π^D	Dual part of the NPV fuzzy dual representation
$\Delta\pi_{\max}$	Maximum allowed level of uncertainty
R_V	Residual value of airport equipment

CHAPTER I

GENERAL INTRODUCTION

1.1 Global overview

The air transportation industry is a large-scale, complex and highly technical system with airports as critical components. Today's airports evolved from basic elements of infrastructure of the global transportation system into dynamic businesses that operate in highly volatile and uncertain environments. As integral parts of the air transport value chain, their economic performance is a paramount indicator for efficient evidence-based decision making and comprehensive understanding of their long-term development.

As major economic drivers and catalysts for economic growth, airports are directly affected by economic trends. Therefore, while major economies remain in fragile state, continuing a period of unstable recovery for the global economy, uncertainty remains a common challenge for world airports. While the majority of advanced economies remained on their track towards recovery from persisting downside risks, emerging markets experienced a slowdown creating significant uncertainty regarding future direct investment, especially in infrastructure development projects like airports.

In spite of facing a highly uncertain environment and exposure to an imbalanced global economic revival since the beginning of the 21st century, passenger traffic remained resilient and on a steady ascending trend, consistently outstripping the growth in global economic output as shown in Fig. 1.1.

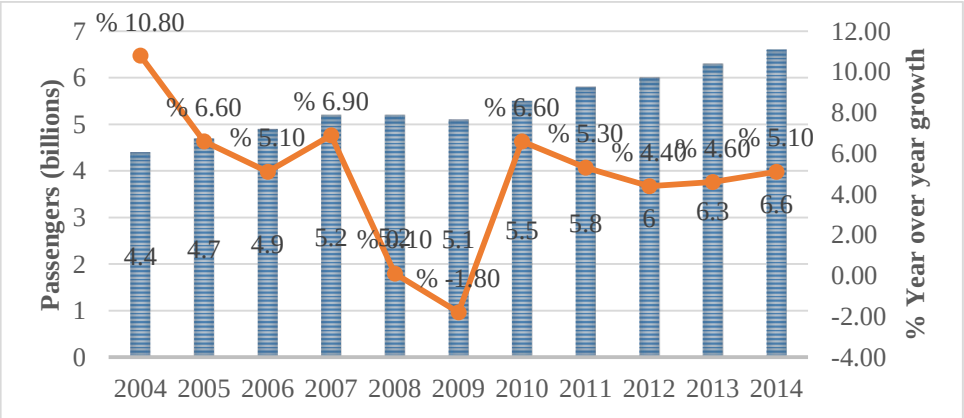

Fig. 1.1 Global passengers figures 2004-2014

The demand for air transport services has risen much faster than demand for most other goods and services in the world economy. Since 1970 air travel demand, measured by Revenue Passenger Kilometres flown (RPKs) has increased ten times compared to a three-four expansion of the world economy. Along the same period, international passenger and cargo demand, both reflecting and facilitating the globalization of business supply chains and economies generally, was multiplied forty times [IATA, 2013].

Currently, there are two forces at play in the global economy, pushing the pendulum in opposite directions. While global economies experience a slow but steady resurgence, the emerging markets know an opposite trend, their slowdown resulting in modest growth levels as depicted in Fig. 1.2.

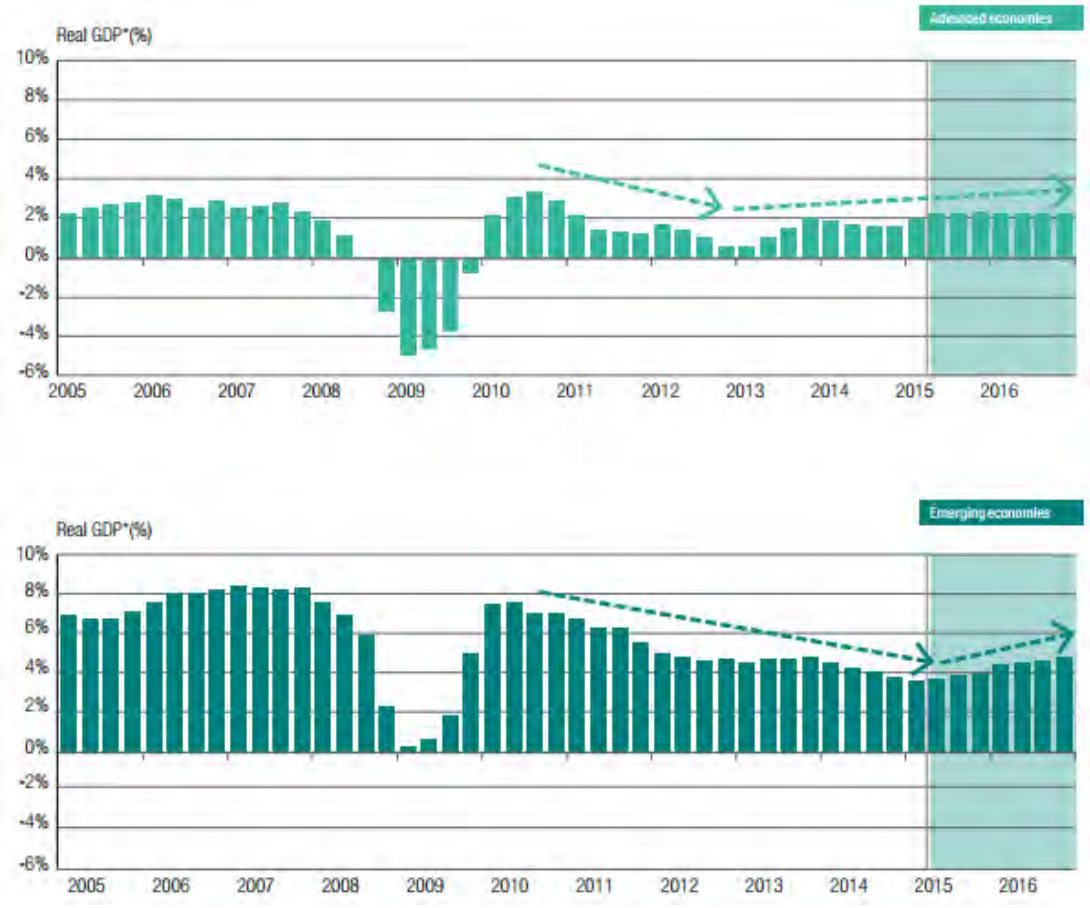

Fig. 1.2. Compared variation of GDP in advanced and emerging economies [Source: AIRBUS, 2011]

Even so, future growth in air transportation will be generated by emerging economies. The main reason is their demography. These countries account for over 85% of the world's population although half of the global GDP comes from developed economies. Over the period 2000 to 2013, the compound annual growth rate for passenger traffic was 9.1% for emerging markets compared to 1.5% for developed economies [ACI Annual Report, 2014]. The economic rising of emerging markets will not only help developing air transportation but will eventually create a paradigm shift in the next decades, as the more mature markets like Europe and North America cede rank to new airport hubs in regions like Middle East, Asia Pacific and Latin America. Rising incomes, liberalization and competition in the emerging markets corroborated with their sizeable population, will reshape air transportation in the decades to come.

One major consequence of these industry shifts is the increasing pressure for long-term airport development, as more and more potential passengers gain access to air travel.

Airports responded dynamically to all these industry shifts. Other factors of change are the uncertainty triggered events like terrorist threats, natural disasters, wars, political unrest, health pandemics and the tremendous financial challenges posed by the recent global economic downturn. The significant impact on airport operations of this complex mix of elements has consequences very difficult to quantify.

The challenge airports face is not only dealing with all these issues, but doing so while keeping sight of long-term priorities such as safety, security and sustainability and, more recently, business performance. Budgetary constraints are directing long-term airport development towards private sector funding with an increased focus on non-aeronautical revenues and will, eventually, determine governments to provide and/or improve the regulatory framework that will attract private capital.

1.2 Airports – Facts and Figures

Airports globally opted for implementing business models with diversified passenger-based revenue schemes. They shifted from being simple infrastructure providers to far-reaching profit driven enterprises. Compared to 2013 figures, industry revenues as a whole increased by 8.2%, surpassing US $142 billion in 2014. Considering

aeronautical sources of income, over 55% of every dollar was generated by passenger related charges. Also, non-aeronautical revenues, generated by retail concessions and parking just to name a few, count for 45% of the total revenue stream, a 7.2% growth in 2014 [ACI MR, 2016].

Table 1.1 Airport industry – Facts & Figures [Source: ACI Economic Report, 2016]

Airport industry key industry facts for the 2014 financial year	
Total passengers	6 633 494 648
Percent chance compared to 2013	5.1%
Total cargo	100 464 251
Percent change compared to 2013	4.5%
Global industry revenue growth year over year	8.2%
Global industry revenue	US $142.5 billion
Revenue per passenger growth year over year	3.2%
Distribution of global revenues:	Aeronautical – 55.5% Non-aeronautical – 40.4% Non-operating – 4.1%
Global airport revenue per passenger	US $21.22
Global aeronautical revenue per passenger	US $8.58
Total cost per passenger	US $16.82
Aircraft related charges	33.6%
Passenger related charges	55.8%
Other non-aeronautical related charges	10.6%
Distribution of non-aeronautical revenue:	Retail concession – 28% Car parking – 22% Real estate rent – 15%

Labour cost share of operating expenses	36%
Global debt-to-EBITDA ratio	5.03%
Industry net profit margin	16%
Global return on invested capital-ROIC	6.3%

From a different perspective, focusing solely on the global overview is not going to portray a very accurate reality of the industry. Even if the airport industry seems to be profitable on the aggregate level, with returns on invested capital surpassing 6%, the majority of airports are not in a healthy financial state. In figures, 67% of global airports operate at a net loss, 80% of these airports servicing less than one million passengers per year [ACI Annual Report, 2014]. Therefore, the profitability of the industry is practically generated by 20% of the airports that carry the most of the passenger traffic. To put things into perspective, in 2011, 42 airport cities were concentrating 90% of long-haul traffic [AIRBUS, 2011]. While high traffic volumes are concentrated in only a handful of airports, the rest are left to cope with the effects of economies of scale. On the other side, hub-like airports are facing more and more acute capacity crunches with little to no options to expand while traffic demand is on an ascending trend.

1.3 Airport industry – Future trends

Since the beginning of the 21st century, the following trends are dominating the air transportation industry:

- *Strong, sustainable long-term growth*. For the past twenty years, airports faced a constant 4-5% yearly growth globally. Air travel consistently became more and more affordable and flight safety improved dramatically. The obvious consequence was the increase in demand for expansion and development of airport infrastructure.
- *Globalization*. Traffic demand will continue to grow since the global market is far from saturation. Expected increases in population and living standards, the tendency of leaning towards flying in detriment of other transportation

modes, long-distance travel for business or leisure increase the general propensity to fly.

- *Organizational change*. Political (the open skies agreements) and economic deregulation changed the way the industry operated more than two decades ago. Liberalization of air travel permitted the emergence of low cost carriers, gave access to new markets, increased competition and open the door for private investment. The industry became more innovative and productive and airports had to adopt new business models. As deregulation continues to spread worldwide, various opportunities for growth are expected but in the same time, the dynamism of the market makes for an uncertain future. Airports consequently need to integrate flexibility in their planning projects in order to mitigate ongoing changes.
- *Privatization.* The governmental involvement in the air transport industry has diminished constantly. Airlines and airports transitioned from fully owned and regulated by government bodies to market regulated partially or fully privatized business. Worldwide, both airlines and airports converge to some form of public-private partnership. All these managerial shifts have a high impact on the way airports develop and operate. The once protected and beneficiary of public subsidies, airports now focus on economic performance which ultimately will influence the airport planning decision making process.
- *Technical improvements*. Technical advances in aircraft and air traffic management but also in complementary industries like IT&C push the airports' adaptability to the rapid changes the air transportation industry faces. From revolutionary new type of aircrafts like Airbus 380, Airbus 350, Boeing 787, electronic passengers processing and e-tail, to electronic border control, all lead to major revisions in regards to airport operations.
- *Uncertainty.* The high rate of change in our current world is something that has to be acknowledged. Living in a fast-paced world has become routine. Even the most complex forecasting methods cannot entirely accurate estimate short and medium term future trends. When it comes to elaborating forecast for long-term project the error increases exponentially. A steady growth trend can be reversed by an economic, politic or social disruptive event. Since

growth is speculative, committing to a long-term airport infrastructure project entails a significant amount of risk.

The multitude and variation of these trends create a very complex environment when it comes to airport long-term planning. With a continuously changing context, objectives and performance criteria, airports need to be responsive to a range of commercial and managerial factors in order to be cost effective, adaptable, flexible, profitable and efficient.

Another aspect became more and more prominent in the current years and that is shaping the future of the industry: *passenger – air travel interaction*. The web and mobile phone became top two sales channels for flights. Airports and airlines are taking this experience one-step forward by providing personalized experiences through their own mobile apps. Mobile check-in is offered by more than 90% of the airlines. In 2013, only 50% offered that feature [SITA, 2013]. 2D boarding passes and contactless technology like NFC (Near Field Communication) are used at different stages of the journey: boarding gates, security, retail, access to different passenger facilities like premium lounges. Customer service is becoming more mobile and more visible on social media platforms. Currently, airports are embracing and investing more and more in business intelligence solutions with the objective of improving customer service and satisfaction using personalized services.

The outlook for the future of the aviation industry is nothing but positive. From the latest biometric technologies to new security processes, the passenger journey will continue to improve over the coming decades. Moreover, passenger and freight numbers are set to continue this positive trend, according to [ACI GTFR, 2013]. Despite the short-term outlook for traffic growth looking sluggish, the report says that by the end of 2014 passenger numbers are expected to accelerate along the global economic growth, reaching 12.2 billion passengers by 2031 as shown in Fig. 1.2.

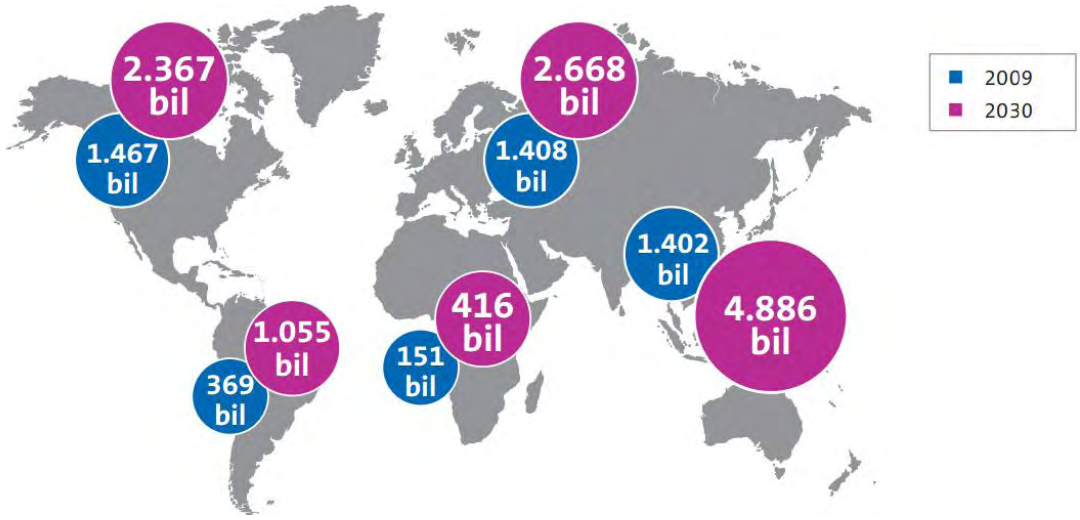

Fig. 1.3 Forecasted passenger traffic levels per region [Source: ACI GTFR, 2013]

Freight volumes are also looking promising for the next 20 years, with the ACI report predicting growth of 4.5% per annum on average during 2012-2031. Asia-Pacific will retain the title of largest freight market in the world with average growth of 5.8% per annum, while Latin America/Caribbean is also expected to grow steadily over the next two decades. European and North American freight markets will grow more slowly – 3.0% and 3.1% per annum respectively.

Aircraft movements will also increase by 2.9% per annum to 137 million by 2031. Asian airports will handle almost three times as many aircraft in 2031 compared with 2011, and aircraft size will be the highest in the Asia-Pacific region.

Airports create a large and innovative industry that is in the process of permanently redefining its organizational, technological and economic aspects. The airport of the future will be an intelligent, adaptive and responsive to its environment, most probably looking very different than the way it looks today.

1.4 Motivation

Our lives are changing at an unprecedented pace. Transformational shifts in our economic, environmental, geopolitical, societal and technological systems offer unparalleled opportunities, but the interconnections among them also imply enhanced systemic risks. Stakeholders from across business, government and civil society face an evolving imperative in understanding and managing emerging global risks that by

definition, respect no national boundaries. Conceptual models are required to define, characterize and measure the potential negative impacts of interconnected global risks.

To manage global risks effectively and build resilience to their impacts, better efforts are required to understand, quantify and foresee the evolution of interdependencies between risks, supplementing traditional risk-management tools with new concepts designed for uncertain environments. If global risks are not addressed effectively, their social, economic and political fallouts could be far-reaching. As seen in Fig. 1.4, the robustness of air transportation in face of disruptive events looks solid, with quick rebounds but with high financial costs. This is mostly due to the value passenger place on the benefits of air travel. As can be seen, in the last 10 years the market growth of air transportation surpassed 60%.

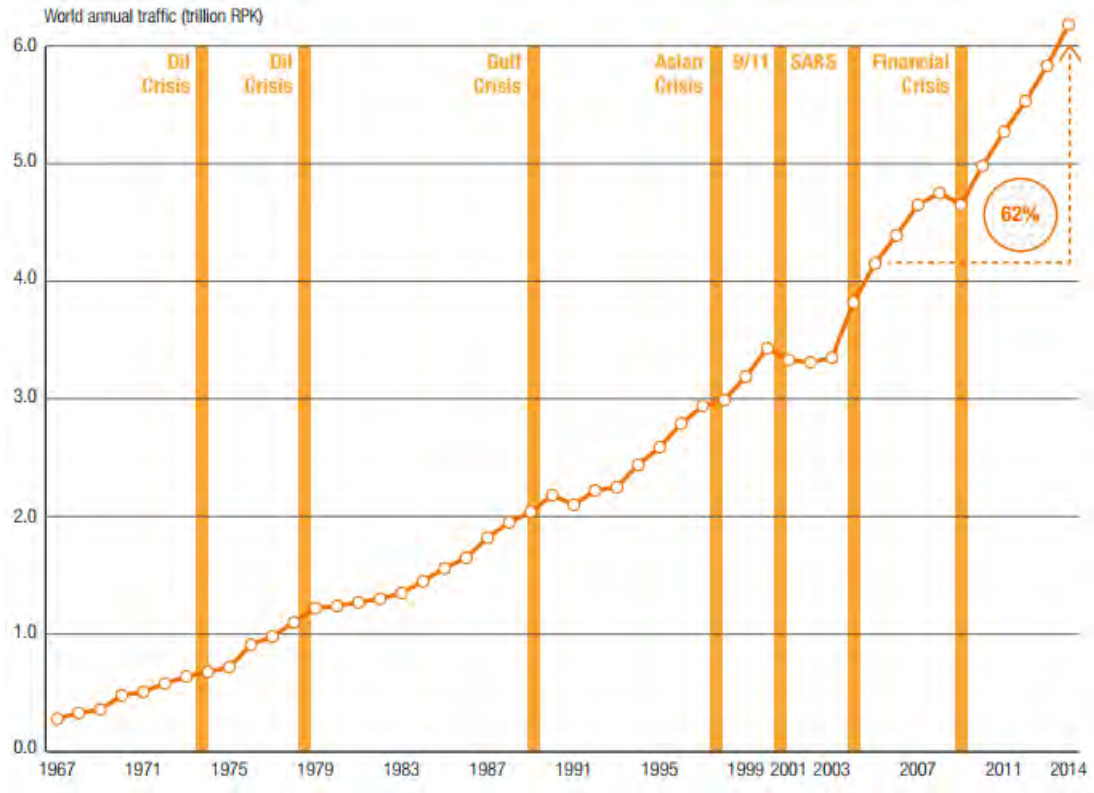

Fig. 1.4. World annual traffic evolution corroborated with global disruptive events [Source: AIRBUS, 2011]

The constant growth of air transportation translates into major airport infrastructure projects. It has become routine for airport planners to deal with increments in demand between 50 and 100 percent. Taking into consideration that the planning horizon for a large-scale airport infrastructure project can span up to 20 years and more due to its

complexity, risk and uncertainty mitigation can make the difference between a success story and a financial disaster.

Looking beyond statistics and forecasts, the scarcity of airport assets will create a domino effect that will reverberate well beyond the air transportation industry. Delays, cancelations, reduced connectivity on one side, local and national social, political and environmental restrictions can render airports incapable of coping with market demand and its potential abrupt fluctuations.

Airport developers will have to consider all these factors and integrate them in their long-term infrastructure development plan.

1.5 Definition of the problem

Airports are critical connectors in the air transportation system. In order to meet their operational, economic and social obligations in a very volatile environment, airports need to embrace change rather than resist it. Like any other industry, airports face unexpected challenges, some specific to air transportation, other having only an indirect influence but powerful enough to disrupt airport activities.

Long-term airport planning has become a complex issue due to the constant growth in air traffic demand. A new dimension of complexity emerged when uncertainty began having a disruptive and significantly costly impact on developing airport infrastructure.

Planning, operation and management of airports depends heavily on demand forecasting and evolution of the most impactful airport stakeholders: passengers, airlines, regulators, and the business community over a long-term horizon.

Historically, the ability of traditional risk and uncertainty mitigation tools proved inefficient. Countless unforeseen events like terrorist attacks, economic recession, natural disasters, had a dramatic impact on traffic levels, some with a global reach. To these highly improbable type of events can be added technological advancements, new airlines and airports business models, policy and regulation changes, increasing concern for environmental impact.

While the majority of the airports still rely on traditional forecasting techniques to guide their decision making in their planning process, it became more and more apparent that treating uncertainty as a minor perturbation to the general trend line is far from accurate. Realistically, the cases where airport traffic levels match the long-term forecasted demand or the timing at which the traffic reaches the critical level requiring new capacity are the exception, not the rule.

Airport long-term infrastructure planning can be reduced to a decision making in uncertain environment problem. The efficiency and feasibility of the sequential decision making process is affected by the decisions the stakeholders make at each stage of the project. This type of problem as we are formulating it in the context of this thesis, assumes uncertainty is an ubiquitous aspect of the decision making process. The ultimate goal of the decision-maker is to successfully close the project, following a sequence of feasible states at each particular stage. In the case of long-term projects, this more probably will translate not in reaching the final objective at any cost but finding the best trade-off between infrastructure development and uncertainty mitigation at every stage of the project.

1.6 Research objectives

Given the motivation presented in the previous section, the objective of the thesis is to put forward a new approach in assessing and mitigating risk in uncertain environments in the context of long-term airport infrastructure planning.

Our objectives are:

- ✈ Investigate the impact uncertainty has on long-term airport infrastructure development projects,
- ✈ Introduce a new perspective when it comes to understanding risk and uncertainty impact on long-term airport infrastructure projects,
- ✈ Expand the formalism of fuzzy dual numbers introduced by Mora and Cosenza,
- ✈ Introduce fuzzy dual dynamic programming as an innovative tool to address risk and uncertainty on long-term airport infrastructure projects,

- ✈ Provide the decision-maker with a tool capable of pointing him the best option at each stage of the project and, ultimately, successfully achieving his long-term goal.

1.7 Outline of the thesis

The thesis is structured in eight chapters.

Chapter I begins with a brief overview of the global air transportation industry, moving on to a snap view of the airport industry facts and continuing with airport future trends. The second half of the chapter details our motivation for pursuing this issue, our approach and research objectives, and concludes with the outline of the thesis.

Chapter II presents background information on the long-term airport planning issue from the business perspective. We give a brief but concise introduction on aspects like airport business models, airport investor profile and a global review of ongoing airport development projects. In the second half, we present the long-term airport planning process and its evolution for the past decades.

Chapter III introduces traditional techniques employed for addressing and mitigating risk in long-term airport infrastructure development plans. We place our problem in the context of global risk and detail on the limitations of traditional risk mitigation tools.

Chapter IV expands on the concept of uncertainty: definition, topology, sources of uncertainty, planning, management and decision making under uncertainty. To make the transition towards our proposed mathematical approach, the chapter ends with a brief mention of the paradigm shift triggered by the seminal work of Lotfi Zadeh on how we understand and address uncertainty.

Chapter V starts with a theoretical round up of fuzzy logic constructs commencing with fuzzy logic, fuzzy logic systems and fuzzy numbers as theoretical concepts and continues with fuzzy set theory. This creates the appropriate context to introduce the innovative formalism of fuzzy dual numbers and to expand on it introducing concepts

like fuzzy dual numbers comparison, fuzzy dual calculus, fuzzy dual vectors, fuzzy dual matrices and fuzzy dual probabilities.

Chapter VI takes the innovative fuzzy dual logic concept introduced in the previous chapter and merges it with a classic technique: dynamic programming. First, we introduce linear programming with fuzzy dual parameters and fuzzy dual variables. Then, after a brief mention of dynamic programming and its fuzzy aspect, we continue with the fuzzy dynamic programming formalizations of the Bellman and Zadeh's approach and continue with Kacprzyk take on fuzzy multistage decision making. We conclude with introducing the fuzzy dual dynamic programming formalism as a core concept of the thesis.

In *Chapter VII,* we construct a theoretical model to show the capability of our proposed formalisms. We define a planning context and the adopted assumptions, continuing with the deterministic problem formulation.

Finally, *Chapter VIII* presents the conclusions, the contributions of this research and potential future development possibilities.

CHAPTER II

THE AIRPORT BUSINESS AND THE LONG-TERM AIRPORT PLANNING PROCESS

2.1 Introduction

Airports evolved tremendously in the last decades from basic infrastructure providers to complex businesses, in a continuous competition for traffic and business opportunities. This approach has a major impact on the way airport infrastructure planning evolves in the context of global market deregulation and liberalization.

Air transportation is growing at a fast stable rate. Historically, airport planning was following a rigid set of rules, which no longer satisfy the reality airports need to face in the twenty first century. Standard master planning is no longer applicable in a highly volatile, competitive and uncertain environment airports operate nowadays.

The challenge to fund expansive long-term airport development projects has become more and more strenuous considering the fact that public financing has become limited due to increased budgetary constraints while access to capital markets is rather difficult due to restrictive financial trends.

If airlines have a very dynamic response to capacity expansion by acquiring new aircraft and slots to operate, airports react much slower to an increase of demand. Economic, political, social and environmental factors – all weigh in when it comes to long-term airport planning decision making. For a better understanding of this process, this chapter gives a brief overview of the specifics that create the environment in which airports plan their environment and their impact on this complex endeavour.

2.2 Airport business models

Since the liberalisation of the aviation sector and market deregulation, airports underwent a full transformation, embracing complex business models and aiming for profitability.

The global process of ‘commercialization’ of airports has far-reaching consequences on the long-term airport planning process. While airports compete for route development, traffic growth and various business opportunities they have to balance these endeavours against increased efficiency, top service quality and optimal investment solutions.

The dynamism of the airport market is oblivious to the ownership of the airport itself, both publicly and privately owned airports operating in the same uncertain environment. This is the main reason 78% of European airports are *corporatized* - structured as independent commercial entities and the distinction between public and private ownership is losing its significance with some of the most active airport investors being airport operators themselves, with some percent of public participation, like Aéroports de Paris and TAV [ACI EU ROEA, 2016]. Globally though, the debate between the advantages and disadvantages of private airport ownership has not reach a definitive conclusion.

Table 2.1 below summarizes the most common ownership models found in the industry.

Table 2.1 Airport ownership models [Source: ACI EU ROEA, 2016]

Airport operator	Entity responsible for daily operations of airport services and facilities. Can be considered part of the public administration if it is functionally dependent on the regional/national administration like the Ministry of Transport, Local/Regional Councils, etc. This implies that the airport's executive management has limited independence.
Corporatized airport operator	A public operator is considered *corporatized* if its acting like an independent economic enterprise, structured and complying with commercial laws, whose shares are completely owned by public authorities of the country in which the airport is located.
Full public ownership	The airport operator is fully owned by a public authority or a mixture of public authorities at a local, regional, national or trans national level.
PPP – public-private partnership	PPP implies that the airport operator is owned by an independently acting enterprise, structured and complying with the commercial laws, whose shares are owned by a combination of private investor(s) and the public authority where the airport is located. The private partner is usually expected to provide funding for the necessary infrastructure, easing budgetary constraints and also bringing specialized expertise and know-how while the public partner offers a risk controlled environment.

Full private ownership	The airport operator is fully owned by private individuals or enterprises. Any ownership by entities that are themselves owned completely or partially by public authorities will be considered *private* if these entities originate from a different country or region than where the airport in question is located.
Concession	An airport concession is considered the legal framework within which the operator is entitled to operate the airport, granted by public authorities in the instances where the airport operator does not own the land.
Lease	Existing facilities or/and land are leased to a private entity which will directly provide services to customers or off takers. Includes aeronautical and non-aeronautical leases, land leases, fixed-based operator leases, hangar rental leases, airline leases, subleases, etc.
Divestiture	The assets are sold to a private entity who provides services directly to the customers.
Developer finance and operations	BOT – Build Operate Transfer BOO – Build Own Operate BOOT – Build Own Operate Transfer DCMF – Design Construct Manage Finance This variety of developer financing and operations implies the existence of a private investor who finances and refurbishes / builds a facility in order to provide services to large public off takers or directly to customers. It can range from passenger terminals to cargo facilities, car parks and fuel systems or any other major facilities.

In the case of European airports, the trend is more than obvious, as seen in Fig. 2.1.

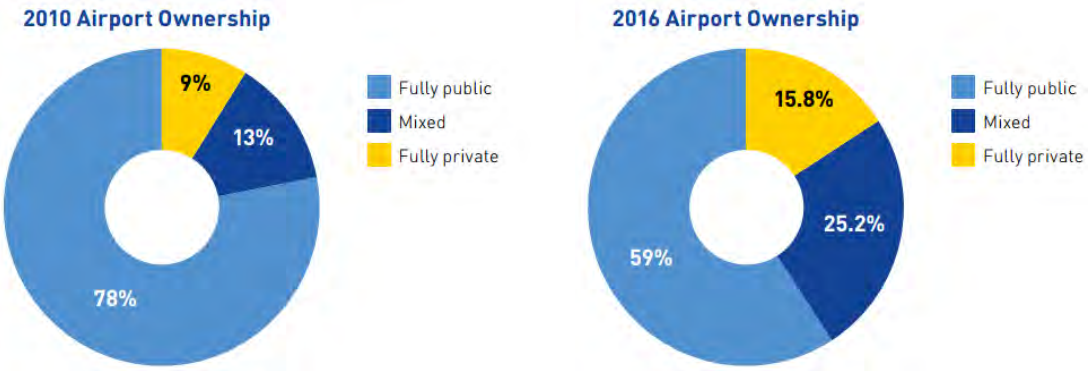

Fig. 2.1 European airports ownership over a six-year span [Source: ACI EU ROEA, 2016]

In only a six-year span, even though the majority of European airports are still publicly owned, this proportion dropped significantly. In this short period, airports opted for various forms of PPPs or even full private ownership. Private participation prevails at larger airports, though. Overall, more than 40% or European airports have some form of private involvement but they also handle approximately 75% of annual traffic. This implies that larger airports are more attractive to private investors due to their profitability.

Airport privatization has the potential to bring a specific set of benefits for the stakeholders involved:

- access to private capital for infrastructure development,
- operational efficiency: private business-oriented management is far more keen to cut cost and boost revenues than public ownership,
- enables long-term focus to meet customer-oriented managerial tools: the use of new techniques, “know how” and a customer oriented set of skills improved decision making process,
- extract an upfront or ongoing payment for the airport asset (monetize the asset),
- stimulate air service and airline competition,
- introduce more innovation and creativity, including entrepreneurial ideas in the development of non-airline revenue, secure long-term efficiencies in

operation and maintenance and enhance customer service, shift the risk of debt, capital development, and/or operations to the private sector,

- ✈ accelerate project delivery and reduce construction costs,
- ✈ reduce reliance on general tax levies, and de-politicize airport decision making.

The trend towards partial or full privatization has now spread globally even though the majority of airports are owned and operated by local or national authorities. After a period of financial downturn that triggered a decline in the number of airport deals as well as in deal value, airport investment is currently above pre-crisis levels due to economic recovery and transactions in emerging markets.

2.2.1 Airport investor profile

The airport investor profile looks very different in the current economic environment. Traditionally, airport development investors were infrastructure funds and major developers. Now the investor profile is far more diverse with pension funds, logistic groups, private equity houses, consortia including financial institutions and operational experts, are all part of the process. A suitable example is the infrastructure company Global Investment Partners. Founded by Credit Suisse, General Electric Company and an independent senior management team, it acquired Gatwick airport in 2009 and Edinburg airport in 2012 [Chow and Smith, 2012].

The major trigger for this evolution of the airport investor profile was actually the financial crisis that crippled airport developers, traditionally the construction companies. Spain was a victim of this kind of scenario, when the economy collapsed leaving numerous airport investment projects in major financial difficulty.

Another distinctive feature of today's airport investor is the criteria they apply when selecting potential investment projects. Long-term airport development projects are appealing to pension funds because they are in the position of ensuring longer-term returns. Therefore, they will go for airports that serve more than five million passengers per year, with more than one terminal. Contrary of what private equity firms are looking for – small airports, one terminal, less than 5 million passengers with strong potential for rapid growth and a relative short return of investment.

Today's investors are also analysing carefully the revenue mix of an airport before making an investment decision. While most of the revenues are generated via aeronautical streams, non-aeronautical sources like retail, parking and real estate become more and more significant sources of growth.

Today's economic environment has created the perfect set of conditions for encouraging airport investment – governments are feeling compelled in reducing their debt, regulators' objective is to have healthy airport competition and quality service levels, traffic demand for both passengers and cargo is on a steady positive trend, infrastructure assets is of interest for all types of investors. This unique mix is redefining the fundamentals of the market by creating new opportunities in both mature and emerging airport markets.

However, like any other investment, airport investment has its own risks. A complex mix of factors ranging from location, catchment area, airline mix, to business model, determines airport performance. Ignoring these aspects will lead to an overvaluation of the airport's performance capability. Overvaluation risk is not the only risk to avoid. Investors focused on emerging markets are currently facing an uncertain environment caused by an economic slowdown and unclear travel trends. In addition, even though non-aeronautical revenues are increasing, airlines are still generating the majority share and now, much more than before, airlines have a decisive role in an airport's profitability.

In conclusion, each airport is a unique complex system operating in an uncertain environment. Its ownership is not a guarantee of certain success, each approach coming with a set of benefits and pitfalls that require skilful balancing in order to achieve operational and financial success. In consequence, airports should adopt a business model based on their objectives and strategic development plans, balancing carefully the advantages and disadvantages of each option while assessing thoroughly the risk to which they are exposing themselves.

Table 2.2 exemplifies, with selected airports, different business models and various investor consortia in European airports. Each of the airports has a different particularity, showing the diversity of approaches for airport investment.

Table 2.2 Ownership of selected European airports [Source: ACI ROEA, 2016]

Airport	Operator & Ownership	Shares %	Shareholders
Vienna International Airport	Flughafen Wien AG	29.9%	Airports Group Europe
	Mostly private	20.1 %	Free float
		20%	Wien Holding Gmbh
		20%	Province of Lower Austria
		10%	Employee financial participation
Brussels Airport	Brussels Airport Company NV	39%	Ontario Teacher's Pension Plan
	Mostly private	36%	Macquire European Investment Funds
		25%	Belgian State
Zagreb International Airport	MZLZ d.d.	20.77%	Aéroports de Paris Management S.A.
	Fully private	20.77%	Bouygues Bâtiment International
		20.77%	Marguerite Fund
		17.58%	IFC
		15%	TAV Airport Holdings Co.
		5.11%	Viadukt
Vaclav Havel Airport Prague	Letiště Praha a.s.	100%	Ministry of Finance
	Fully public		
	Corporatized		
Paris Orly Airport	Aéroports de Paris	50.63%	State of France
	Mostly public	21.49%	Institutional investors
		8%	Schiphol
		8%	VINCI Airports

		4.81%	PREDICA
		3.04%	Others
		2.33%	Retail investors
		1.69%	Employees
Toulouse Blagnac Airport	Aéroport Toulouse-Blagnac Mostly public	49.9%	CASIL Europe
		25%	Chamber of Commerce and Industry of Toulouse
		10.1%	State of France
		5%	Regional Council of Midi-Pyrénéés
		5%	Departmental Council of Haute-Garonne
		5%	Greater Toulouse Urban Area Community
Dűseldorf Airport	Flughafen Dűseldorf GmbH Equal public & private	50%	Land Capital Dűseldorf
		20%	AviAlliance GmbH
		20%	ARI Aer Rianta International
		10%	Airport Partners Holding Verwaltungs Gmbh
Frankfurt Airport	Fraport AG Mostly public	31.35%	Land Hessen
		20.02%	City of Frankfurt
		8.45%	Deutsche Lufthansa AG
		2.99%	RARE Infrastructure Ltd.
		37.19%	Free float
Budapest Airport	Budapest Airport Zrt.	52.66%	AviAlliance GmbH

	Fully pivate	22.17%	Malton Investment Pte Ltd.
		20.17%	Caisse de dépôt et placement de Quebec
		5%	KfW IPEX Bank GmbH
Heathrow Airport	Heathrow Airport Limited Fully private	25%	Ferrovial S.A.
		20%	Qatar Holdings
		13%	Caisse de dépôt et placement de Quebec
		11%	Government of Singapore
		11%	Alinda Capital Partners
		10%	China Investment Corp.
		10%	Universities Superannuation Scheme (USS)
Schiphol Airport Amsterdam	Schiphol Group Mostly public	70%	Ministry of Finance
		20%	City of Amsterdam
		8%	Aéroports de Paris
		2%	City of Rotterdam

2.2.2 2015 Global airport development projects

At the beginning of 2015 there were over 2300 airport construction projects worldwide worth USD 534 billion, according to the Centre for Aviation report [CAPA, 2015]. Some are new projects, some are part of previous master plans, with budgets raging between USD 1 million and USD 20 billion and projects expanding on very long-term like Stockholm - 2043, Rome - 2044 or Mexico - 2069.

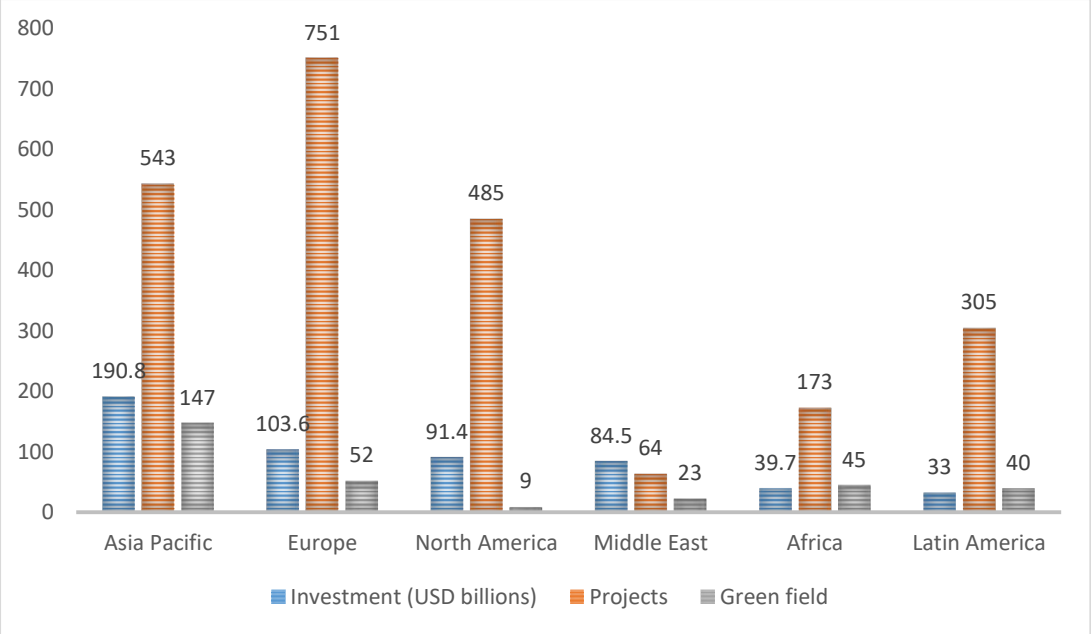

Fig. 2.2 Global airport development projects and investment by region (Jan. 2015) [Source: Centre for Aviation, Airport Construction & Cap Ex Database (2015)]

Fig. 2.2 reveals substantial discrepancies between the number of projects undergoing in every region and their investment value. The main reason for this apparent inconsistency is the difference in calibre of the projects accounted for, ranging from runways extensions or terminal upgrades to an entire airport city. The major airport long-term development projects with the highest investment amounts are concentrated in the Asia Pacific region and Middle East, while Europe leads on the number of projects currently underway. In addition, there are not many green-field airport development projects. As expected, emergent markets like China and India are on the front line, looking to bring air connectivity to a large percentage of their population. While the spike in Africa is justified by new, less than 1 million passengers per year airports, Europe is not necessarily addressing its capacity crunch since the green field projects are located in Central and Eastern part of the continent.

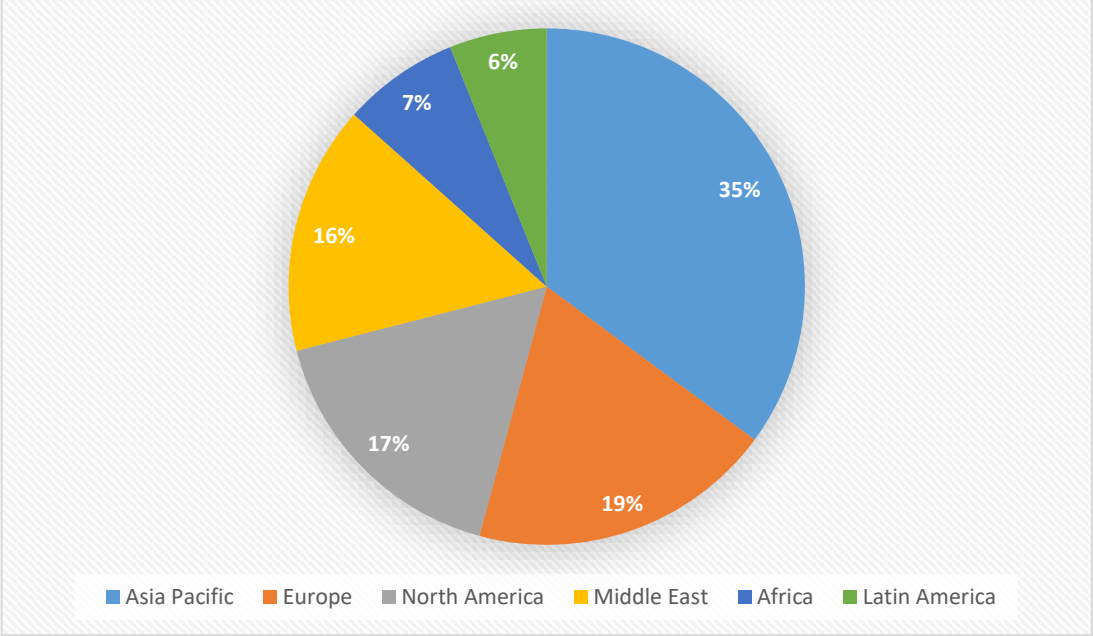

Fig. 2.3 Total investment apportioned globally [Source: Centre for Aviation, Airport Construction & Cap Ex Database (2015)]

As seen in Fig. 2.3, **Asia Pacific** is leading the pack on the overall value of investment in long-term airport infrastructure development due to emerging economic markets like China and India.

China leads the field with two of the world's largest green field airport construction projects: Beijing Daxing and Chengdu, with investment values summing up to USD 13.1 billion and USD 12.1 billion, respectively. The very dynamic Chinese air transportation market is in full expansion mode, with numerous regional airport projects initiated with the purpose of boosting regional economy. Overall, the total investment closes in to USD 60 million. However, lack of profitability of Chinese airports remains a critical issue with only 25% of Chinese airports turning a profit.

India has its focus on secondary and "low cost" airports; the main objective is providing air transportation to 70% of the country's population who lacks air connectivity. Total investment amount is approximately USD 8.5 billion, including Mumbai and New Delhi major expansion projects valued at USD 3 billion and USD 1.8 billion, respectively. Navi Mumbai is also adding USD 2.4 billion to the total, a public-private partnership green field project, delayed several times due to land

acquisition issues and environmental problems, with construction work projected to begin in 2016.

Australia is also one of the big South Pacific players, with airport infrastructure development projects worth approximately USD 19 billion. While Melbourne airport is in the middle of an USD 8 billion expansion, Sydney follows with a second airport project worth USD 1.6 billion.

Other major Asian airport infrastructure markets are Singapore – who continues the expansion of Changi airport forth terminal and other construction work, USD 2.2 billion; Indonesia is facing a capacity crunch who needs infrastructure development worth USD 15.3 billion, relying on public-private partnerships to cope with the increasing demand; Vietnam, Thailand, Pakistan also have airport infrastructure development programmes undergoing.

Japan is fully embracing mass airport privatization with few major projects on the radar, South Korea's Incheon airport is in its third construction phase with an overall cost of USD 3.3 billion. Taiwan's Taoyuan Aerotropolis project is worth USD 2.3 billion and is scheduled to receive in 2021, 77 million passengers per year.

Middle East has been for many years now a hot point on airport infrastructure map due to the Gulf area with United Arab Emirates' Dubai and Abu Dhabi and Qatar's Doha airports, who are undergoing major expansion projects.

Dubai International, the airport with the highest number of international passengers processed per year just opened concourse D, worth USD 10.9 billion, adding an extra capacity of 18 million passengers per year. Al Maktoum International Airport, 15 kilometres away from Dubai International, Dubai's Airport City, has the objective of handling 120 million passengers per year, with the option to expand up to 240 million. The initial phase of the 56 square kilometres' mammoth project is worth approximately USD 33 billion.

Both Abu Dhabi and Doha are focused on capacity expansion. Abu Dhabi will be able to receive 40 million passengers per year in 2017 after an overall USD 6.8 billion investment, while Doha Hamad International Airport allocated USD 3 billion for the

second phase of the development of the airport city which ultimately will have a capacity to 65 million passengers per year.

All other Middle East countries are investing in their major international gateways: Kuwait International Airport is undergoing a USD 4.8 billion development project, Muscat a USD 4.7 billion one, Tehran USD 2.8 billion. Saudi Arabia's objective to become a major domestic and international gateway by 2020 is backed by projects worth USD 8 billion, with expansion works at Riyadh King Khaled International Airport and the construction of the new King Abdulaziz International Airport.

Europe, as seen in Fig. 2.3, leads in number of undergoing development projects. Most of Europe's airports are involved in governmental airport infrastructure development programmes. Some are considerably extensive, costly and expand over a long period of time, but the majority are moderate in value.

Heathrow, Europe's busiest airport in terms of passengers, is closing in its' five year, USD 13 billion expansion scheme, due to finalize in 2019. Gatwick, the major 'low-cost' airport, after a USD 3 billion investment, is waiting for Airports Commission approval for a second runway which will trigger a USD 10.6 billion investment.

Berlin's Brandenburg Airport turned out to be a financial disaster, more than doubling its initial cost to around USD 6.4 billion. Initially scheduled to open in 2011, finally is projected to receive its first passengers in the second half of 2017.

In order to cope with future potential demand of 90 million passengers by 2020, in case of Frankfurt International Airport, and 50 million passengers by 2017, respectively, by Munich International Airport, extensive master plans are underway. Since Frankfurt is expanding its airport city infrastructure including the Cargo City and a fourth runway, the total costs will rise up to USD 10 billion.

Schiphol Airport, another major European hub, will invest USD 1.3 billion in upgrading projects as part of the partnership with KLM.

Scandinavian countries are investing considerably in their gateways. Stockholm Arlanda Airport has an extensive master plan, spanning until 2043, with a total cost of USD 2 billion. Copenhagen Airport is targeting passengers from Russia and China

in order to bring its passenger count to 40 million and for that is planning to invest USD 3.6 billion.

Rome Fiumicino Airport is looking far into the future with a multi stage long-term airport infrastructure project worth USD 12 billion, reaching its completion in 2044. First stage is a capacity increase to over 50 million passengers until 2021.

Vienna Airport, is currently upgrading facilities investing USD 1.3 billion but their long-term investment strategy includes a USD 1 billion third runway.

Istanbul Grand wants to become an intercontinental major hub and it is on target to open at the end of 2017, carrying a tag price of USD 26 billion. With an initial capacity of 90 million passengers annually, it has expansion capabilities up to 150 million once the 80 million mark is reached, triggering the first of two development phases to be set in motion.

Russia allocated approximately USD 10 billion for airport modernization since it is going to be the host of 2018 Soccer World Cup. Both Moscow's airports are undergoing expansions projects that look beyond 2018 worth over USD 5 billion.

North America region can always count on Atlanta airport for representation, the world's busiest airport for more than 15 years. More than 101 million passengers were handled there in 2015. As its USD 9 billion long-term expansion programme concluded, a 20-year Master Plan for an Airport City was announced. Other multibillion projects currently undergoing are Philadelphia Airport, Los Angeles Airport, Washington Dulles, Tampa and Orlando. While Canada's Calgary USD 1.7 billion development project concludes, Vancouver has a USD 3 billion strategic plan looking to develop its airside.

Latin America, as the Asia Pacific region, is a booming air transportation market. Brazil is still going on the momentum created by the Soccer World Cup while preparing for the 2016 Olympic Games. The main objective is to bring air connectivity to 96% of the population while rebuilding 270 regional airports. Rio de Janeiro is singled out as the major airport infrastructure project in Brazil at the moment with a total cost of USD 2.2 billion which will ensure an increase in capacity from 17 million

passengers to 40 million by mid-2016. Sao Paolo is going to be serviced by a privately operated airport by 2025 at an estimated cost of USD 3.8 billion.

Not only the biggest airport infrastructure development project in Mexico, but in the entire world, Mexico City International Airport will come with a price tag of USD 9.1 billion and expand over a period of more than fifty years. The new six runways will ensure a capacity processing capability of 120 million passengers per year.

Africa sums up airport development projects of USD 40 billion overall. The largest project on the continent and in the same time one of the most extensive in world is Cairo Airport Company airport city scheme with an overall cost of USD 11.8 billion. Angola is another country who invested USD 2.16 billion in 30 airport projects, including 16 new ones over de 2013-2016 period.

2.3 Airport costs and revenues breakdown

2.3.1 Airport Costs

Airport cost breaks down into two major categories of high fixed costs related to the operation and maintenance of airport infrastructure:

1. ***Operating costs***
2. ***Capital costs***

Currently, more and more airports are faced with the risk of reaching their capacity limitations as traffic demand continues its positive steady trend globally. Necessary expansion of fixed assets, like terminals or runways, automatically trigger an increase in labour expenses, maintenance costs and depreciation related to their operation. As Fig. 2.4 details, personnel expenses remain the predominant cost related to operations and approximately a fifth of the total, while depreciation is actually the most costly expense on the aggregate.

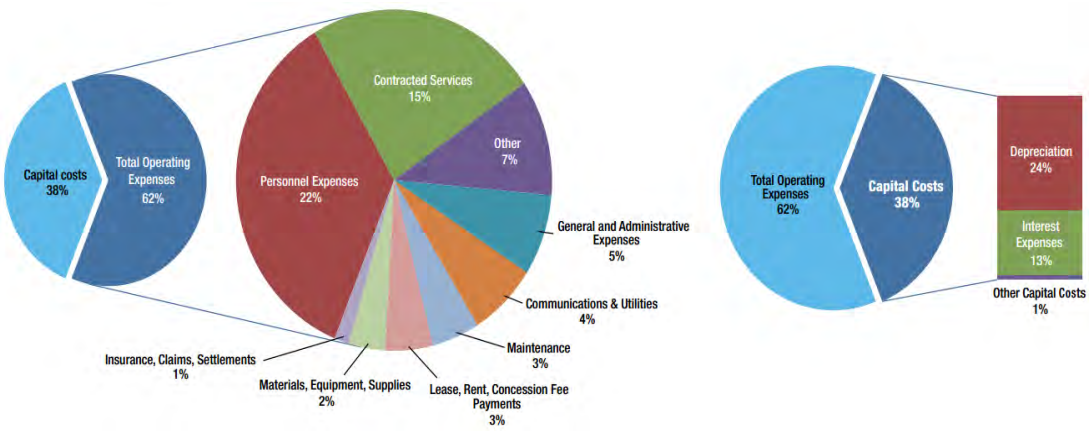

Fig. 2.4 Distribution of operating expenses and distribution of capital costs in 2012 [Source: ACI ER, 2013]

2.3.2 Airport revenues

Traditionally, airports were seen as a facilitator for airline operations with no interest in diversifying their revenue stream. Consequently, aeronautical revenues were considered the most important source of income. Currently, aeronautical revenues are still the prevalent source of income for airports, but this is on a rapid descending trend. Now, at the centre of the airport business is the passenger, this generating a complex multi-service non-aeronautical sector.

Airport revenues breakdown into three major categories:

1. ***Aeronautical revenues***
2. ***Non-aeronautical revenues***
3. ***Non-operating revenues***

Aeronautical revenues are generated by the specific charges and fees levied on users of airport facilities and services. Fig. 2.5 provides a detailed breakdown of global aeronautical revenues.

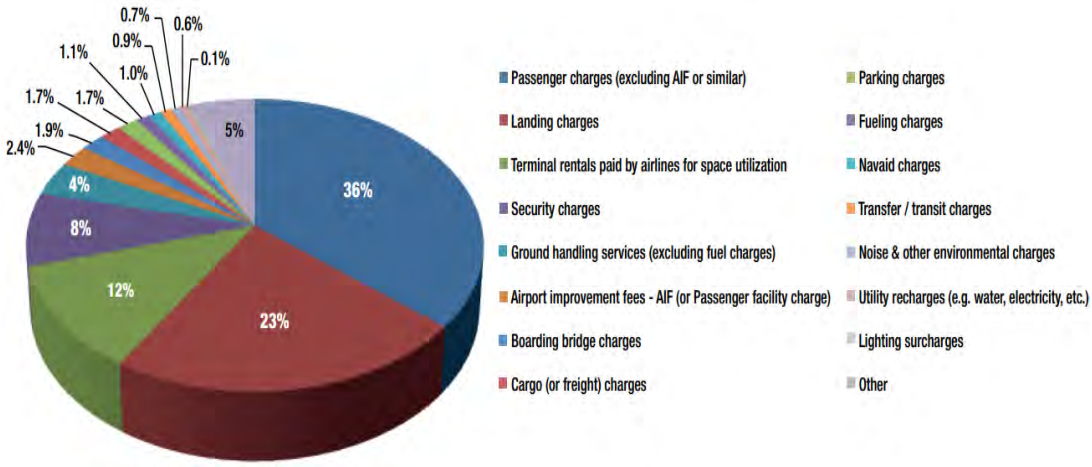

Fig. 2.5 Global aeronautical revenues for the financial year 2012 [Source: ACI ER, 2013]

Non-aeronautical revenues are becoming increasingly important on the overall financial health of an airport. Currently, this sector is becoming more and more diverse and its profitability is a paramount indicator of airport performance, especially considering that non-aeronautical sources of revenue tend to generate higher net profit margins than aeronautical revenues and be an attractive aspect for potential investors [ACI ER, 2013]. Fig. 2.6 provides a detailed breakdown of global non-aeronautical sources of revenue.

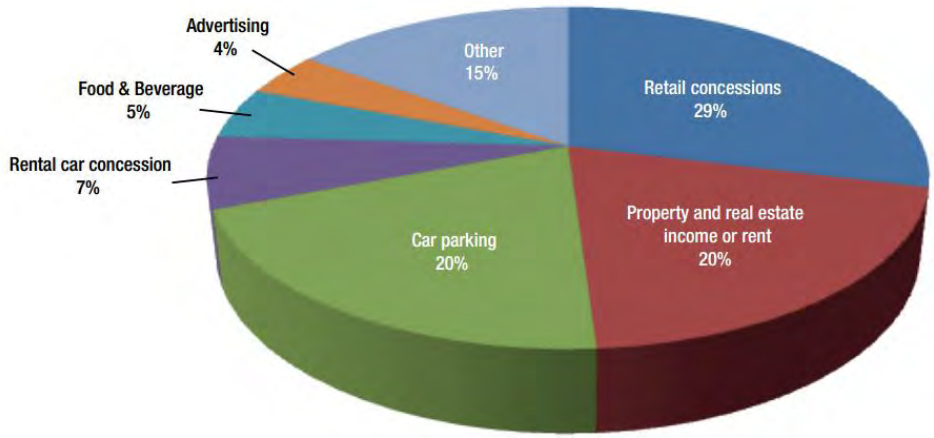

Fig. 2.6 Global non-aeronautical revenues for the financial year 2012 [Source: ACI ER, 2013]

As can be seen, retail concessions are the leading source of non-aeronautical income for airports but only on aggregate level. Table 2.3 shows how non-aeronautical revenues are distributed globally and significant regional variation can be noticed.

Table 2.3 Regional distribution of non-aeronautical sources of revenue [ACI ER, 2013]

Region	Retail concessions	Food & Beverage	Car parking	Car rental concessions	Real estate	Advertising	Other
Europe	34.2%	3.9%	14.4%	3.3%	22.5%	2.6%	19%
N. Am.	61.2%	3.8%	6.4%	0.8%	9.8%	1.5%	16.5%
Asia Pac.	44.5%	3.9%	10.6%	1.8%	23.1%	4.9%	11.2%
L. Am.	28.9%	6.7%	7.9%	3.1%	19.2%	4.9%	29.4%
Africa	42.9%	2.2%	14.6%	4.5%	20.9%	7.4%	7.5%
Middle East	7.7%	6.7%	39.1%	16.8%	15.1%	5.8%	8.8%
WORLD	**28.9%**	**4.8%**	**20.2%**	**6.8%**	**20.1%**	**4.1%**	**15.1%**

2.4. The long-term airport planning process

Airports constitute a paramount piece of the global infrastructure puzzle. They are significant economic drivers with multiplier effects on national and regional economies. In the same time, airports are at the core of a dynamic and complex system, facing constant change and a very competitive environment. Embracing the business culture had become the new normal for airports.

As the world economy is going through successive economic downturns, the air transport industry is expected to continue to grow steadily on the long run. Following this trend, airports are expected to expand accordingly.

Airport planning is, in general, a long-term planning issue which has at its core the following objectives:

- optimized infrastructure development costs and functionality,
- optimized economic and operational performance,
- high degree of flexibility in order to integrate all the shifts in demand and potential disturbances according to the airport future needs and level of growth.

The new business culture concepts that airports need to embrace includes strong air service competitor advantages, capability of taking long-term risks, adopting the stakeholder collaborative decision making culture, diversifying the revenues sources and, most of all, placing the passenger at the core of the business.

The construction of a new airport or the extension of an existing one requires significant investments and many times public-private partnerships are considered the best option in order to make feasible such projects. One characteristic of these projects is uncertainty with respect to financial and environmental impact on the medium to long-term planning. Another one is the multistage nature of these types of projects.

Airports were traditionally seen as the responsibility of governments to manage and operate, typically in line with strategic economic policies [IATA, 2013]. In the more recent economic environment, a paradigm shift occurred were private stakeholders emerged as investors evolving from decision makers in airport planning and development to full owners and operators. Privatization of airports emerged as the tool "to go to" for governments looking for strategies to make the local aviation market more dynamic and to achieve their long-term planning goals when the costs of funding new infrastructure or maintaining the existing one exceeds their resources. The privatization of airports makes for a governance space where different governance modes intersect and overlap as noted by [Donnet and Keast, 2011].

The long-term airport planning process is a complex endeavour due to the intricacies of the airport system, stakeholders involved and the significant degree of uncertainty. In a highly volatile economic context, the planning process needs to be constantly adjusted to the realities of the market the airport will serve. Quantities such as "demand" and "capacity" need to be re-thought in a dynamic context to compute the operational parameters of the future airport. The fact that long-term airport planning is a multi-billion business investment requiring a systemic and flexible approach must be acknowledged from inception.

Long-term airport planning has to integrate tools capable of ensuring efficient operability and strong financial performance while providing a framework where future airport strategies, objectives, and the steps to achieve them are clearly defined.

Following the saying that the “the forecast is always wrong”, innovative long-term airport planning approaches shift the paradigm from a single scenario master plan to an extended range of possible futures and scenarios of operation while analysing the feasibility of each alternate development option. In this case, the decision maker is better informed about the profile of risks and benefits he should expect. The ultimate goal when undertaking long-term airport infrastructure development projects is positioning the airport to maximize its performance by seizing opportunity while avoid unnecessary developments.

Making assumptions about what the future holds in a continuously evolving industry is very challenging. Events like airline mergers, restructuring or bankruptcies, economic crises, new policies or regulatory requirements, the low cost carriers' consolidation, constant technical advancements, all these force airports to rethink the way they position themselves on the market and as a global infrastructure provider. Flexibility and adaptability are priorities for the new business model airports need to embrace.

Another very important factor to consider in long-term airport planning is the local setting. This gives a certain uniqueness to the airport. Characteristics such as location, size, and type of operations, governing structure, organizational values and culture, all shape the entire planning process.

Long-term Airport Planning requires (1) collaborative participation of all the parties affected directly and indirectly by the outcomes of the project and (2) a large spectrum of data from various sources, which will allow detailed multiple scenario analysis. Each change that will inadvertently appear in the environment in which the planning process takes place may cause disturbances to a certain extent. Therefore, the planning process needs to constantly updated, in order to integrate all the uncertainties that arise.

Planning solicits the input of a diverse group of stakeholders. Each stakeholder plays a very specific role in the process and it has its own interests to protect which in many cases are conflicting with the interests of some of the other stakeholders. Accommodating the needs and priorities of all the stakeholders during an infrastructure planning process adds a new dimension to the complexity of the project.

The *Long-term Airport Planning Process* main characteristic is to take ideas and concepts into actionable steps towards materializing a complex endeavour. The *Long-Term Airport Planning Process* is structured on four major pillars:

I. *Preplanning*. The preplanning phase defines the objectives of the planning process. During this phase are identified the reasons for initiating such a plan and the airports' readiness to undertake such a challenge. In addition, the role of all the stakeholders is established.

II. *Analysis*. The analysis phase provides a scan of the environment the airport operates. This creates the opportunity for identifying gaps between the current airport performance and the objectives set. In addition, it gives the chance for reassessing different goals if the initial ones were not in accordance with the airports' capabilities and creates awareness on a range of future possible scenarios that might create operational disruptions.

III. *Implementation*. The implementation phase constitutes the most dynamic part of the entire process. A global day-to-day action plan is put in motion following a timeline with specific milestones in order to bring the entire vision to reality. Short and long-term objectives are set, prioritised, assigned and implemented. Key performance indicators are defined for evaluating performance levels and for offering a quantifiable view on the evolution of the project.

IV. *Monitoring*. The monitoring phase is an ongoing activity throughout the entire duration of the project. The feasibility of the entire undertaking is assessed using the key performance indicators defined during the initial phases and adjustments are made, if necessary.

2.4.1 Planning concepts

The concept of airport planning is standardized. ICAO, FAA, EASA, all provide the most commonly used standards and recommended practices. Even though the manuals are fundamentally the same, specific details differentiate them. Traditionally, these are the tools commonly used in airport long-term planning projects.

The complexity of the industry and the shortcomings of traditional airport master planning, corroborated with the high degree of uncertainty that impacts both short-term and long-term undertakings when it comes to airport infrastructure projects, determined researchers to develop alternative approaches like *Dynamic Strategic Planning* [de Neufville and Odoni, 2003], *Flexible Strategic Planning* [Burghouwt, 2007], *Adaptive Airport Strategic Planning* [Kwakkel et al., 2010] and *Real Options.* The common ground of all these new alternatives is introducing flexibility and adaptability as key elements of the planning process. While flexibility can be defined in various ways, the common premise is that flexibility allows a system to undergo change with greater ease or at lower costs than if no flexible options are considered [McConnell, 2007]. Even so, these options to traditional master planning remain conceptual and empirically based, with no standardized operating procedures supporting them.

All the above-mentioned airport planning concepts are detailed below.

2.4.1.1 Plans

In the majority of contexts, planning is a "top-down" type of activity where usually government officials prepared extensive set of documents, which are forwarded for implementation to the interested parties.

In some cases, planning is approached "bottom-up", like in the case of United States where local authorities prepare their own plans and forward them to the National Plan of Integrated Airport Systems with no guarantees that they will receive funding.

2.4.1.2 Master Plans

The International Civil Aviation Organization (ICAO) in DOC 9814-AN/902 Airport Planning Manual very specifically details the concept of master plan [ICAO, 1987]:

"A generally accepted definition states that an airport master plan *presents the planner conception of the ultimate development of a specific airport. It effectively presents the research and logic from which the plan was evolved and artfully displays the plan in a graphic and written report. Master plans are applied to the modernization and expansion of existing airports and to the construction of new airports, regardless of their size or functional role."*

The typical master plan has a linear view of the process and the way of how it will unfold. The major flaw of this approach is considering the initial forecasting still valid through the development of the project. In a fast-paced evolving industry, master planes based on these principles become obsolete very fast. Long-term airport planning can span up to a 30 years long period. Inflexibility and failure to integrate potential risks and uncertainty can cause a master plan to fail even in the preplanning phase.

2.4.1.3 Strategic Planning

Strategic planning, in general terms, is defined as the process undertaken by an organization to define its future and formulate a road map to guide the organization from its current state to its vision for the future [ACRP Report 20, 2009].

The fundamental concept on which strategic planning is based refers to the impact present actions have on the future of the organization. Defining this process includes specific key elements that help the organization identify existing and potential challenges and develop a vision for the future. Those key elements include:

- concise and quantifiable definitions of the organization's mission, values and vision,
- a comprehensive SWOT (strengths-weaknesses-opportunities-threats) analysis,
- definition of strategic issues that will be addressed during the implementation of the strategic plan,
- definition of short-term and long-term action plans that will materialize the organization's vision,
- identification of key performance indicators to monitor and evaluate the progress made toward achieving the objectives.

Airport strategic planning looks beyond the simplistic infrastructure provider role airports were traditionally labelled with. It is a continuous and dynamic process, incorporating elements of the "bottom-up" approach, which seeks consensus among stakeholders.

Today, seeking strong financial performance is a core objective for business driven airports and airport-systems world-wide.

2.4.1.4 Dynamic Strategic Planning

In [de Neufville and Odoni, 2003] dynamic strategic planning is defined as "a marriage of the best elements of both master and strategic planning". The authors emphasize the compatibility between dynamic strategic planning and traditional approaches like master and strategic planning. The authors build in the traditional approach by considering a range of forecasts, rather than just one as in case of master and strategic planning. This allows for relative seamless adjustments in case of any type of change. De Neufville and Odoni outline the following key elements for developing a dynamic strategic plan:

- ✈ overview of existing conditions,
- ✈ development of a forecast range of future traffic, including possible scenarios for every traffic type (international, domestic, transfer, cargo),
- ✈ evaluation of facility requirements suitable for current and potential different levels and types of traffic,
- ✈ do a comparative analysis based on different alternative scenarios,
- ✈ select the most suitable initial development, capable of integrating flexible responses to possible future conditions [de Neufville and Odoni, 2003].

"Dynamic Strategic Planning is the approach recommended for airport development. It recognises that the airline/airport industry is highly uncertain; [...] leads to a flexible development strategy that positions airports to minimize risks, take advantage of opportunities as they arise, and thus maximize expected value" [de Neufville and Odoni, 2013].

2.4.1.5 Flexible Strategic Planning

Burghouwt's vision relies heavily on the principles iterated by de Neufville and Odoni. However, at the center of this approach is proactive re-adaptive demand driven planning in order to integrate uncertainties related not only to traffic fluctuations, but

also to a much broader range of disruptive factors like airport competition or regulatory changes. His concept detailed in [Burghouwt, 2007] embraces risk and considers it an opportunity rather than a disruptive factor, with scenario planning, decision analysis and real options as preferred analytical tools.

2.4.1.6 Adaptive Airport Strategic Planning

Adaptive Airport Strategic Planning - AASP [Kwakkel et al., 2008] [Kwakkel et al., 2010] employs ideas from both dynamic and flexible strategic planning and merges them with the concept of adaptive policymaking.

Adaptive policymaking is a generic approach for organizations trying to integrate and mitigate the uncertainties impact by creating a common operational ground in order to facilitate adaptability in the face of future unplanned conditions and developments [Walker, 2000], [Walker et al, 2001].

As stated in [Kwakkel et al., 2010], the central idea of AASP is to have a plan that is flexible and over time adapt to the changing conditions under which an airport must operate, offering a framework and stepwise approach for making such adaptive and flexible plans. The authors developed a model for Amsterdam's Schiphol Airport using exploratory modeling (EM), an operational research technique used to improve the flexibility of the airport planning process.

2.4.1.7 Real Options

Real options is a technique borrowed from the financial world, based and developed from the concept of financial options. A concise definition refers to real options as the possibility but not the obligation to take a certain course of action. As noted in [de Neufville and Odoni, 2003] an important feature of real options is the fact that their value increases with risk, which is exactly the opposite of the majority other types of assets, which decrease in value the riskier they become.

Even if *real options* is not prevalent as a concept in airport planning, like the above mentioned non-traditional approaches, airports worldwide applied variations specific to this technique.

The most common examples of *real options* are the shared use of facilities and equipment by multiple users, which reduces space requirements and allows greater flexibility in airport design, incremental development options and multi-functionality.

2.5 Conclusions

This chapter gives an overview of the current airport industry and the global context in which is performing, with a focus on the transition airports made from basic infrastructure providers to performance driven businesses. The data presented had the purpose to create context and depict a current relatable image of the airport industry with a focus on long-term development projects and their impact. The chapter concluded with a brief run-down of the evolution of airport master planning concepts. Now that a perspective on the global airport market was given, the next chapter will follow the same line addressing risk in the context of airport long-term infrastructure development projects.

CHAPTER III

TRADITIONAL RISK ASSESSMENT IN LONG-TERM AIRPORT PLANNING PROJECTS

3.1 Introduction

Development of air transportation infrastructure projects is a risk sensitive industry due to the significant impact of project failure on the financial health of the developer, owner, local communities and the environment.

All long-term airport development projects have one high impact element in common – continuous subjection to risk while dwelling in a highly uncertain environment. There are numerous examples of airport development projects who substantially exceeded their cost, failed to meet their completion deadlines repeatedly or were abandoned before completion as detailed in the previous chapter. The consequences of such major failures, besides the obvious costs and time overspending, most of the time lead to costly litigations, contractual penalties or/and cancellations.

Every long-term development project is unique no matter how many other projects alike were prior completed. That is because its exposure to the unknown, with a different set of risks and various degrees of uncertainty, which eventually influence in a very different mater the commercial, administrative or physical aspects of the project.

An essential element of long-term airport development is risk assessment. Ranging from minor inconveniences to major project disasters, risks need to be identified and their probability and severity assessed and mitigated in order to reduce their possible impact or even avoid them all together. A certainty when it comes to risk and uncertain environments is that major disruptive events cannot be predicted with one hundred percent accuracy.

Risk affecting the course of a long-term airport infrastructure project can occur at any stage. Specific risks can be associated with certain project tasks or stages, while other originate from external causes, either with the possibility of manifesting at any time. Typically, the later in the project development a risk event occurs, the costly that ends up to be in terms of time and money compared with a similar event occurring closer to the start of the project. That is the simple consequence of the fact that the further along the project is, the higher the sunk costs and value of work invested, therefore a higher value at risk of damage or/and loss.

Long-term airport infrastructure development projects have, in greater extent, elements of novelty who are obviously complex and large so they strongly need a risk strategy in order to identify most of the potential risk they are exposed to and elaborate approachable ways to mitigate them.

3.2 The global risk landscape

Airports are paramount connecting points of the global infrastructure network. Building resilience against global risks requires having a common understanding among stakeholders of what those risks are and how they are affecting operations, management and overall airport performance. The commercialization of airports, more and more prevalent in the last years, propelled the internationalization factor of airport business and consequently, increased airports exposure to global risks.

A particularity of the airport business is their increased vulnerability to global risks regardless of geographical proximity of the potential risk. Airports are parts of the aviation value chain, therefore economic sustainability has to be achieved by every sector – airlines, airports, air navigation service providers, suppliers, manufacturers, etc. In addition, the robustness of one sector is heavily dependent on the robustness of the others. These aspects add another dimension to the complexity of the problem and creates another layer of uncertainty due to interdependencies.

Fig. 3.1 is the upper right snapshot of the global risks graph, as perceived to evolve and interact as of 2016 [WEF GRR, 2016]. The scale ranges from 1- risk unlikely to happen or with no impact, to 7- risk highly likely to occur and with a devastating impact. 2016 is a year that puts on the map risks that were considered only probable ten years back. Global warming is a scientific fact and both airlines and airports have made it their mission to reduce their carbon footprints. Geopolitical volatility generating the largest forced migration in recent history causes uncertainty regarding the international security landscape, one of the pillars of air transportation already shaken by acts of terrorism. Not only these threats are highly disruptive on their own but they also can give rise to cascading risks, increasing even more the degree of uncertainty of the environment airports operate in. The cascading risks strongly emerging are the ones related to climate changes like the water crisis and food security

issues. In addition, global disease outbreaks are constantly challenging the preparedness and response measures of affected airports. There are also, regional nuances. While European countries are more concerned with economic and geopolitical risks (fiscal crisis, unemployment, inflation), Unites States is more concerned with the cyber-crimes and their impact.

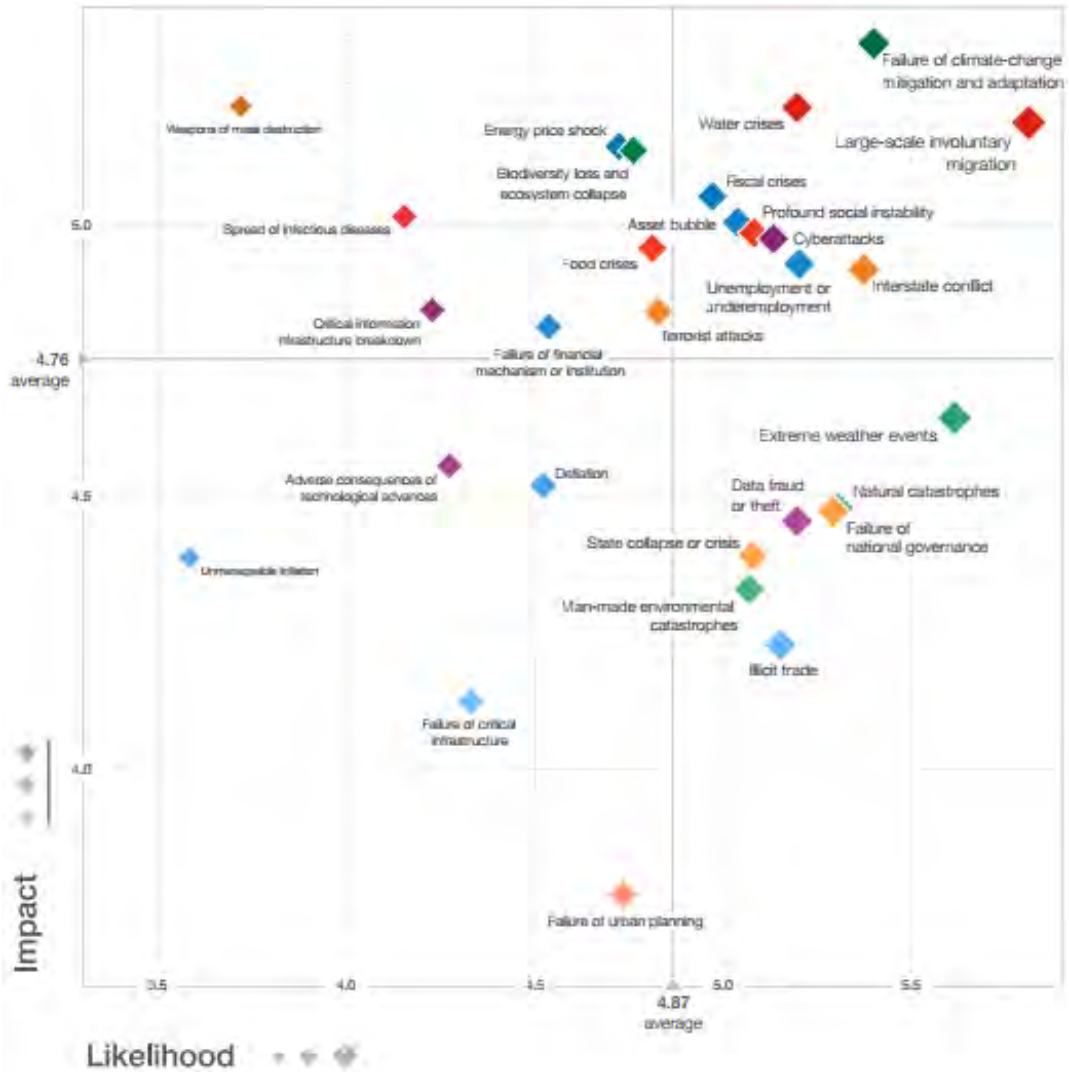

Fig. 3.1 The global risks landscape in 2016 [Source: WEF GRR, 2016]

3.3 Identifying risk

Tackling risk is a very complex process and there is no standardized one-fits-all approach. A superficial approach to risk assessment and mitigation will most likely lead to organizational downfall. Regardless of the approach taken to performing any type of task, key performance indicators will always be impacted by risk.

Classification of risks is also adaptive and specific to the associated project.

Risk analysis can be either qualitative or quantitative. Qualitative risk analysis will approach risks in a very descriptive way (Fig. 3.2), unlike quantitative risk analysis, which takes things one-step further by providing quantifiable outcomes to a risk event.

Potential impact			
	Severe impact risk Low chance of occurrence	Severe impact risk Medium chance of occurrence	Severe impact risk High chance of occurrence
	Medium impact risk Low chance of occurrence	Medium impact risk Medium chance of occurrence	Medium impact risk High chance of occurrence
	Low impact risk Low chance of occurrence	Low impact risk Medium chance of occurrence	Low impact risk High chance of occurrence
	Chance of occurrence		

Fig. 3.2 Qualitative risk classification matrix

In general terms, we can categorize them as *exo-industry* and *endo-industry risks*.

The main exo-industry risks are:

- Volatility of the economic environment with major market shifts: The traditionally strong and robust North American and European markets have become stagnant while emergent Asian and Latin American markets are soaring. Air traffic evolution follows economic trends.
- Political policy and regulation regarding environment, taxation, security regulations, and bilateral and open skies agreements, all have the potential to be either a major constraint for future airport development or a facilitator.
- "Black swans" are events or occurrences that deviate beyond what is normally expected of a situation and that would be extremely difficult to predict. [Taleb, 2007] popularized this term. The following events are considered as such: the terrorist attacks of September 2011, the SARS outbreak (2003), the Indian Ocean tsunami (2004), Hurricane Katrina (2005), the global financial crisis (2008), the volcanic eruption of Eyjafjallajökull (2010), Arabic spring, the Japanese tsunami that caused the Fukushima nuclear disaster (2011).
- Social and cultural aspects have a powerful impact on local communities. Public awareness on aviation environmental impact, the prevalence of Internet

video conferencing over business travel, the living standard, all these factors affect decisively the propensity to fly. In addition, public perception in the era of information and social media can burry an airline and cause major financial losses to the base airport like and all this with no apparent or proven fault of neither the airline nor the airport. Unfortunately, this is the case of Malaysian Airlines, following the tragedies of flights MH370 and MH17, who also affected the performance of Kuala Lumpur International Airport, the Malaysian Airlines hub, and other Malaysian airports as well.

The main endo-industry risks are:

- The airport performance is strongly dependent on airline operations. Airports are impacted by the operational, financial and overall business models of airlines (legacy, low-cost, start-up). To all these aspects the trending airlines alliance model can rapidly turn from an opportunity or strength, to a weakness or a threat, depending on the context the airport finds itself in. Powerful alliances offer to the airport the opportunity to reach a larger and more diverse market but also internal instability within an alliance can significantly complicate airport future development plans. In conclusion, airports should take all the necessary steps to minimize the disruptions to which the airline industry is exposed.

- The emergence of private investors in the airport market, ranging from partial privatization to full ownership and operation, brings a new degree of uncertainty and risk to the system due to investor profile diversity and to increased scepticism in considering airports a very secure and profitable undertaking, unlike the pre-financial crisis era.

- Airport competition is emerging as a serious pressure point in the industry with more visibility between primary and secondary airports and is even more pronounced for cargo airports;

- Technological advancements determine airports to adjust their infrastructure in order to keep up with the new aircrafts which gain popularity in a far more accelerated pace than the specific airport infrastructure (Airbus A380, Airbus

A350, Boeing 787). Also major operational improvements like A-CDM (Airport – Collaborative Decision Making), SESAR (Single European Sky – ATM Research) or NextGen are pushing airports forward in terms of infrastructure and operational advancements.

- Forecasting errors, statistical and modeling errors, misinterpretation of data, errors in the data, are adding to the overall error margin for mid and long-term forecasting.

The increased frequency of highly disruptive events with catastrophic consequences are causing not only an overall increase of uncertainty but also, an increase of the level of exposure. The biggest trigger behind the rise in extreme risk events is the rise of the human-built environment, which currently expanded massively compared to twenty years ago [PWC RR, 2012].

In this context, airport development projects are exposed to a very complex and dynamic environment, characterized by a significant degree of uncertainty and risk. To finish a project successfully, the complexity of the environment must be approached systematically, having comprehensive vertical and horizontal awareness and thoroughly understanding the adaptability of all moving parts involved.

3.4 Limitations of airport forecasting

Airport planning and management relies to great extent on projections of future demand: passengers (domestic or/and international), cargo, aircraft movements, etc. Future demand has to balance perspectives of all the stakeholders involved – airlines, passengers, on site businesses, regulators, authorities, etc. – over a long period. Airport facilities have long life spans of at least 20 years. Investment decisions like the development of a new runway or a new terminal are determining the airport service level and operations costs for an extensive period.

Forecasting future airport levels of demand is paramount for effective decision making in airport planning. Accurate forecasts are drivers for drafting and implementing feasible investment policies, adding value for the airport and its users. Conversely, forecasting inaccuracies can induce poor investment decisions with consequences ranging from higher operational and financial costs to total financial disasters.

The ability of traditional forecasting tools to predict accurately traffic demands for extensive periods has proved repeatedly to be unreliable. Beyond the typical factors exposed to uncertainty like socio-economic and environmental aspects, dynamics of the air transportation industry or exogenous variables with high impact like "black swan" type of events, there is one aspect that is more difficult to predict than atomic particle dynamics – that is *people's behaviour* [Trani, 2015].

According to [Maldonado, 1990], forecasts are always wrong, with large errors that became even larger in case of long-term predictions and a notable aspect is the lack of relevance of the airport size on the forecasting error. For a five-year period, the average difference between the forecast and actual demand was 22% [Trani, 2015], while for a ten-year period the average difference between forecasts and actual demand was 40%, for fifteen year periods the average difference surpassing even 75% [Nishimura, 1999].

In addition, the longer the time span for a forecast, the less accurate the prediction regarding technological advancements and infrastructure requirements necessary to sustain them.

3.5 Traditional tools for integrating risk in airport planning

Airport developers are dealing with massive amounts of data in order to keep track of numerous moving parts. Accurate, on the spot decision making in case of large-scale disruption caused by external factors can save a development project from major delays, cost overruns or legal disputes. Integrating flexibility at every stage of the project will ensure responsiveness to unforeseen events.

Airport development projects' risk of failure is higher, compared to other infrastructure projects due to their complexity, novelty and susceptibility to change [Pichott and Scott, 2014]. They involve a large number of stakeholders and a wide variety of costs, spanning over extensive periods of time and are typically very large in scope. Therefore, change is highly likely.

The major challenge in case of airport development projects is choosing an approach that will allow risk transfer to specialist third parties (designers, contractors, operators) in order to increase resilience in a volatile environment. While the search for the

perfect support tool continues, industry professionals consider experience irreplaceable. Past mistakes always paved the way for better decision making in case of future projects.

3.5.1 Traditional air traffic demand forecasting

In the air transportation industry, risk assessment and uncertainty are integrated in the methods used for developing demand forecasting, with a more of an ancillary role rather than a primary focus. The two concepts of risk and uncertainty are often interchangeable and not thoroughly defined.

The state of practice when it comes to incorporating risk into aviation demand forecasting is including techniques like:

- High and low forecasts,
- "What if" analysis,
- Sensitivity analysis,
- Data driven procedures,
- Judgement driven procedures [Kincaid et al., 2012].

While [Spitz and Golaszewski, 2007] grouped more complex approaches, as follows:

- Time series methods,
- Econometric modeling with explanatory variables,
- Market share analysis,
- Simulations.

3.5.2 Traditional risk mitigation tools

Once identified, risks are ranked according to their probability of occurrence and impact severity. This step will bring into focus the events that are more likely to happen and have the greatest disruptive impact on project operations. Cleary stating the possible causes and effects of every risk will broaden the understanding and ensures a better mitigation approach in case of occurrence.

During the life span of a development project, exposure to risk and the uncertain environment will prompt the necessity of not only identifying and classifying risks but also evaluate their combined effects.

Airports have grown more proficient at managing internal risks by implementing *Enterprise Risk Management* (ERM) programs, helping them identify and manage a range of risks. However, this approach proved itself les effective in face of exo-industry risks mentioned in the section above. Another key area where airports have a slow response to is emerging risks. These appear on the airport radar but they are difficult to assess and manage due to the fact they tend to unfold in unexpected ways. More sophisticated ERM systems can aggregate data from across the industry in order to identify the most significant risks an airport is exposed to but external risks are increasingly difficult to quantify and exclusive reliance on readily available data should be avoided.

Risk mitigating tools are classified into three basic categories:

1. risk identification and tracking: continuous horizon scanning and early-warning capabilities will potentially allow early identification and tracking of emerging risks;

2. risk forecasting and analysis: uses traditionally scenario planning due to the fact the technique shows the impact of alternative assumptions rather than providing one precise forecast, simulation models are also used to assess performance under different economic, political, environmental scenarios; one of the most popular tools for analyzing unpredictable risk is *reverse stress testing* which goes backward from an assumed consequence but that will be completely futile in the case of a tsunami, earthquake or volcanic eruption;

3. risk mitigation: the more disruptive the risk and with more dire consequences, the lower the probability to foresee it so the efforts are focused not to detect these kind of events but rather towards mitigation tools and ways to increase resilience and robustness. Simulations, case studies, aligning risk and strategy are approaches that can help assessing risks more accurately.

3.6 Conclusions

Fully accepting the fact that unpredicted risk events are a daily reality and understanding that building stability has to make way to building robustness and resilience, will ultimately determine airports to go beyond the disaster recovery mindset to ensure they can effectively manage disruptive events and ensure operational continuity while also creating stability throughout the entire industry value chain. The only highly probable thing is that this era of volatility is here to stay so integrating risk and uncertainty it is imperative in order to insure positive performance. This chapter focused on risk as it is perceived, understood and addressed by the industry. The following chapter will deal with uncertainty as an aspect independent of risk and presented from the mathematical perspective.

CHAPTER IV

DECISION MAKING UNDER UNCERTAINTY

4.1 Introduction

Uncertainty is one the most abstract concepts to grasp, quantify, manage and mitigate. Uncertainty intrudes in the plans for the future, interpretations of the past and decisions in the present [Wierman, 2010].

As [Shackle, 1961] so eloquently explains "in a predestinate world, decision would be *illusory*, in a world of a perfect fore-knowledge, *empty*, in a world without natural order, *powerless*; our intuitive attitude to life implies non-illusory, non-empty, non-powerless decision…Since decision in this sense excludes both perfect foresight and anarchy in nature, it must be defined as choice in face of bounded uncertainty."

Decision making is arguably the most important human reasoning capability and uncertainty governs our daily lives, therefore decision making under uncertainty is a process that governs our entire existence.

Scientists did not traditionally share this view until the end of 19th century, on the contrary, science without uncertainty was an ideal for which science should strive [Klir, 2006]. This attitude began to change at the beginning of 20th century when the complexity of the studied processes pushed researchers to find new approaches in dealing with complex systems.

Decision making under uncertainty became a transversal field with an extremely vast area of application and, in the same time, relying heavily on mathematics, statistics, economics and business management, operations research, computer science, engineering. This combination creates and extremely large spectrum of possibilities and directions of research.

4.2 Defining Uncertainty

Understanding uncertainty has posed a challenge for scientists in all fields of research. Uncertainty is ubiquitous and this may well be the cause for the lack of a generalized definition for the term.

The typical approach in assessing uncertainty is by balancing it against the sought precision. Expecting high levels of precision comes with high costs. The higher the degree of complexity of a problem or a system, the more imprecise the information

available to describe it, consequently, the higher the degree of uncertainty. Therefore, there is a direct connexion between uncertainty and precision or as Lotfi Zadeh said: "*we must exploit our tolerance for imprecision*" [Zadeh, 1973].

4.2.1 Uncertainty typology

Probability theory is a widespread tool for tackling uncertainty. Its popularity stems from its long history starting two and a half centuries ago with the seminal work of Thomas Bayes and since then, developed continuously. However, while uncertainty understanding and formulation grew more and more complex, new theories emerged, shifting from the random approach to more diverse typologies.

Jan Lukasiewicz developed a discrete, multi-valued type of logic, giving the first rigorous formulation of many-valued logic, at the beginning of the twentieth century (approx. 1930) [SEP, 2014].

Lotfi Zadeh introduced the concept of fuzzy logic in his seminal work *Fuzzy set theory* [Zadeh, 1965]

Arthur Dempster developed a *theory of evidence* that included for the first time an assessment of ignorance [Dempster, 1968].

Dempster's work was extended by Glen Shafer [Shafer, 1976], developing a comprehensive theory of evidence dealing with multi-source information.

[Morgan and Henrion, 1990] is addressing aspects best dealt probabilistically like random error and statistical variation, as well as aspects that are best modelled with fuzzy set theory.

According to Smithson typology [Smithson, 1990], which comes from behavioural sciences, ignorance is the root of all uncertainty, making it close to impossible to plan for future disasters.

With the emergence of computer science in the second half of the 20th century, the development of tools for uncertainty assessment expanded also. [Klir and Wierman, 1998] focused on the development of uncertainty measures in mathematical systems. They classified uncertainty in two major categories:

- *Fuzziness*, which deals with information that is indistinct,
- *Ambiguity*, which deals with multiplicity.

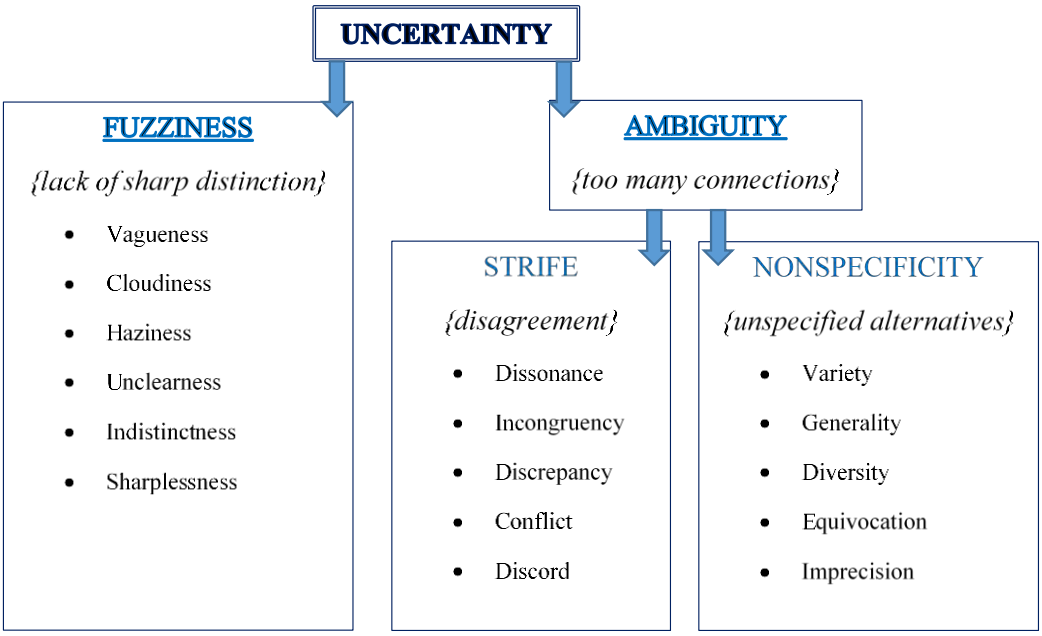

Fig. 4.1 Uncertainty typology [Wierman, 2010]

4.2.2 Specific definitions

Information "refers to a collection of symbols or signs produced either through the observation of natural or artificial phenomena or by cognitive human activity with a view to help an agent understand the world or current situation, making decisions or communicating with other human or artificial agents." [Dubois et al., 2009]

Certainty is operationally defined as determinism, "a state, such that evidence to the contrary is below a threshold of disputation" [Booker and Ross, 2011].

Precision is the capability of reproducing accurately the behaviour of a system in real world conditions. This capability coincides with a high degree of certainty.

Uncertainty does not have one unified definition. Typically, the term comprises all that is not known with certitude. [Zimmermann, 2000] proposes the following definition for uncertainty:

"*Uncertainty implies that in a certain situation a person does not dispose about information which quantitatively and qualitatively is appropriate to describe, prescribe or predict deterministically and numerically a system, its behaviour or other characteristica.*"

Uncertainty Quantification is an evaluation process based on different methods or models that could yield either a numerical statement or a linguistic one, which ultimately will be interpreted in an appropriate context by the decision maker. Probability theory is the most popular tool in addressing this matter.

Total Uncertainty represents the aggregation of all relevant uncertainties relative to a problem placed in a specific context.

4.2.3 Types of information

The type of information available dictates which kind of approach towards tackling uncertainty yields the best outcomes.

1. *Numerical information* – Typically, is the most abundant type of information this being the main reason why this type of information needs to be scaled in order for the appropriate mathematical tools to be applied.

2. *Interval information* – Interval arithmetic is used with interval-valued information to be obtained as outcome. None the less, this information is considered to be exact in the sense that the boundaries of the intervals are crisp.

3. *Linguistic information* – Linguistic information in the sense of [Bellman and Zadeh, 1970] refers to the information provided by natural language.

4. *Symbolic information* – Is provided by an aggregation of numbers, letters or pictures carrying a specific meaning.

5. *Inference* – Inference uncertainty emerges from the concept of making inferences, defined as the difference between the observable quantity (what is measured) and what is desired (unobservable quantity). Therefore, inference uncertainty is defined as the uncertainty induced by the act or process of deriving a conclusion about an entity that is unmeasured or unavailable based on what one has been or what can be observed and measured or made available [Booker and Ross, 2011].

It has become evident that reduced doubt and uncertainty are equivalent to increased understanding and certainty – opposite notions and inversely related, as one increases the other one decreases. In dealing with real world situations, the decision maker needs to know not only the expected result of a simulation but also the degree of certainty with which his results will replicate real world behaviour, under similar conditions.

Being able to express a level of certainty in a prediction is analogous to being able to express the level of uncertainty in that prediction. However, certainty and its inverse, uncertainty, are relative concepts that need to be formulated with respect to a specific context and standard. [Ross, 2003] proposed such a standard by stating that all uncertainty should scale between two extremes or boundary conditions on uncertainty, i.e. between the case of no uncertainty and maximum uncertainty.

4.2.4 Sources of uncertainty

Uncertainty is caused by a disrupted information flow regarding the observed system. In a predefined situational occurrence, uncertainty is the inverse of information in regards to the parameters of the system in different states as well as the inverse of predefined expectations of the observer regarding those states. Information about a particular problem may be incomplete, imprecise, fragmentary, unreliable, vague, contradictory or deficient in some other way [Klir and Yuan, 1995].

Sources of uncertainty must be identified and understood so they can be accurately conveyed to decision makers. Uncertainty may be caused by a multitude of factors form poorly understood initial conditions to random, uncontrollable or unknown effects. However, there are additional sources of uncertainty from incomplete information, lack of knowledge, vagueness and ambiguity. Sources of these kind of uncertainty include physical models, mathematical models, statistical models, computational models, currently known theory, decisions, interpretations, extrapolations, interpolations, predictions, indirect observable quantities, inferences, contradicting data or models, indirect observable quantities, inferences being made [Booker and Ross, 2011].

Since there is no universal taxonomy nor a standard definition of uncertainty sources, ultimately is up to the decision maker to construct a framework that suits his expectations in approaching the problem.

The following classification identifies the most common sources of uncertainty.

1. *Objective information* versus *Subjective information* – The so-called objective information stems from direct measurements, while the subjective information relies on perception of events or is obtained without resorting to direct observations.

2. *Information quantity* versus *information quality* – The most common cause of uncertainty is lack of information and no information is the most unfortunate situation a decision maker can be. Quantitative information is typically presented numerically while the qualitative information is symbolic, expressed in natural language. However, subjective information can be numerical and objective information can be qualitative. Approximation enters this category. In this case, the decision maker is the one who does not gather all the relevant information available, either because he cannot or will not, even though he may have this option. On the other hand, an abundance of information leads to complexity. This type of uncertainty is caused by the limited human simultaneous perception and processing capabilities of extensive amount of data [Newell and Simon, 1972]. Ambiguity also enters this category. Ambiguity describes a situation in which, mathematically speaking, we have a one-to-many mapping. Information quality – Knowing the probability, with which different system states can occur, gives the decision maker a wider range of choices.

3. *Singular information* versus *Generic information* – Singular information refers to a rendition of one particular state of the system at a very specific time. Generic information refers to a broader spectrum of situations ranging from axiomatic knowledge to a representative sample of observations.

4. *Contradicting information* – uncertainty arises also when the decision maker is faced with sets of information that point to conflicting system behaviour.

More information will more likely increase the conflict. The cause of such situations can be incorrect or irrelevant information but not identifiable as such, neither by the system or the observer.

5. *Human belief systems* – The decision maker is assumed to possess some information about the reality of the environment in which he is exerting the decision making. Belief stems from experience and is filtered by reason in order to be assimilated. However, a certain belief makes sense in a specific dynamic context, changing constantly as new experiences continue to shape beliefs and the ways of reasoning. Therefore, there is the possibility that objective data be interpreted in a subjective way, altering the outputs of the system. According to [Dubois et al., 2004] the epistemic state of the decision-maker consists of the following three components: generic knowledge, singular observations and beliefs.

4.2.5 Uncertainty management

Since there is no standardized approach in addressing and aggregating all types of uncertainty, the most efficient approach to manage uncertainty is by finding feasible solutions to mitigate uncertainty impacts and manage them overall.

In order to accurately identifying the measures necessary to mitigate the impact uncertainty has, several steps need to be followed:

1. Become aware of the uncertainty sources and types the decision maker is dealing with. This is a problematic step considering the variety of uncertainty types, many of them considered very difficult to assess.

2. Determine what information, knowledge and theory are applicable to the type of uncertainty identified. A common uncertainty is the lack of information, stated simply as that which we do not know. This type of uncertainty is not the kind probability theory is designed to quantify. Here is where the seminal work of Lotfi Zadeh [Zadeh, 1965] created a paradigm shift in understanding and addressing uncertainty.

3. Choosing the appropriate approach and metrics, including methods for combining uncertainties and stating clearly how total uncertainty will be understood and interpreted by the decision-maker.

4.3 Decision making under uncertainty

Decision making is a process governed by the quality and the quantity of the information available at the onset of the problem. In the situation when the decision maker has full knowledge regarding the states of the process, the options and the outcomes, decision making becomes a completely deterministic process with the objective to evaluate and optimize the decision criteria by either maximizing the utility function or minimizing the cost function. These types of problems are classified as *decision making under certainty.*

In the real world we cannot have a deterministic approach. Reality is dynamic, complex and uncertain, this affecting not only the way decisions are made and implemented, but also influencing the evolution of the entire system governed by those decisions.

[Danzig, 1955] and [Charnes and Cooper, 1959] set the foundation for what today is considered to be the field of decision making under uncertainty, by tackling uncertainty regarding demand using linear programming and, stochastic programming and optimization, respectively. Even though these authors had a different approach, they share the same assumption that the probability distributions of the random variables are known with certainty. Aspect that remained unchanged in the following decades, literature review showing that decision making under uncertainty relied on the precise knowledge of the underlying probabilities [Bertsimas and Thiele, 2006].

Considering these assumptions, computational complexity was an issue due to the large-scale nature of stochastic programming problems. Solutions were proposed in [Shaphiro et al., 2009], [Birge and Louveaux, 2011], [Kall and Mayer, 2011]. Currently, stochastic programming is a well-established modeling tool for addressing problems that involve an accurate description of random quantities. However, this approach does not depict a very realistic image of the environment a decision-maker is exposed to in the situation of real-life applications. In highly volatile environments,

stochastic programming has its limitations. Therefore, the need for an alternative, non-probabilistic approach became more and more pressing.

Dealing with uncertain information impacts negatively the solution in two ways: the solution yielded is not feasible at the moment the decision maker decides to implement it and, if feasible, may not be optimal (implementing it would be either too costly or with a too small rate of return). This issue of potential lack of solution feasibility was first addressed by [Soyster, 1973], who used the ultra-conservative approach of associating every uncertain variable in convex programming problems with its worst-case value in given set. This indeed, minimized the effect of dealing with uncertain parameters but deemed the solution impractical to implement. Further research addressing the issues on ultra-conservationism by limiting parameters to worst case scenario values within a set, has developed into the field called today *robust optimization* [Bertsimas and Sim, 2003], [Bertsimas and Sim, 2004], [Bertsimas et al., 2004], [Bertsimas and Brown, 2005].

Research on this field expanded significantly in many different directions, establishing theories and methods for uncertainty modeling. Most of these approaches are focused either on specific *types of uncertainty* defined by their causes or they imply certain causes, requiring also specific types of information quality depending on the type of information processing is employed [Zimmermann, 2000].

Currently, the body of knowledge comprising theories, models, and paradigms dealing with uncertainty is extensive. Among the most prominent ones are:

- Probability Theories like the Bayesian Probability [Jaynes, 1986], [Bernardo and Smith, 2000] and the ones postulated by [Kolmogoroff, 1956], [de Finetti, 1974], [Koopman, 1940];
- Possibility Theory [Dubois and Prade, 1988],
- Set theory [Cantor, 1874],
- Fuzzy set theory [Bellman and Zadeh, 1970],
- Evidence Theory [Shafer, 1976],
- Intuitionistic set theory [Atanassov, 1986],

- Rough set theory [Pawlack, 1985],
- Interval analysis [Moore, 1966].

A common limitation to these approaches is their expansiveness and complexity as well as the tendency to not cumulatively address long-term and uncertainty aspects of multi stage decision making problems.

4.4 Planning under uncertainty

According to [Horner, 1999], Dantzig was considering planning under uncertainty one of the most important open problems in optimization. It continues to remain so, not only due to its complexity and to the large array of applications, but mostly because of the heightened degree of uncertainty of the environmental setting the addressed problems are unfolding in.

The classical approach to this problem is system optimization that focuses primarily on maximizing system performance by optimizing resource allocation using linear, non-linear or integer programming.

A more novel approach involves decision analysis that takes explicitly into account risk and uncertainty. This allows flexibility to be integrated in the process, allowing potential opportunities for growth to be taken advantage of or avoiding the consequences of a disruptive event. Constructing a comprehensive decision analysis model is a strenuous endeavour due to its complexity. All the possible alternatives need to be identified and assessed, their degree of flexibility needs to be evaluated by employing various analytical tools and finally, selecting the most effective scenario.

The probabilistic approach to solve sequential decision problems is by using the *Markov Decision Process* formalism, who provides a mathematical framework for modeling decision making using linear or dynamic programming. In many real world applications constructing an accurate Markovian model is challenging due to the difficulty in estimating the occurrence probability of uncertain events, which will make decision making a very challenging task.

4.4.1 Airport planning under uncertainty

Airport planning under uncertainty is far from being a saturated topic. On the contrary. The topic of long-term airport planning under uncertainty is more often addressed from the industry's point of view, which assumes a more practical approach and less a mathematical perspective.

[Maldonado, 1990] used dynamic strategic planning, a concept introduced in [de Neuffville, 1990], to address the performance of selected airports development projects in the USA and proposed this alternative approach that takes into consideration uncertainty via staged development.

[Kwackel et al., 2010] introduced the concept of adaptive airport strategic planning as a generic approach for the treatment of uncertainty via hedging and mitigation actions and applied this concept to Schiphol Airport long-term development vision.

4.5 A paradigm shift and the work of Lotfi Zadeh

As previously detailed, much of what was conceptualized regarding uncertainty was guided by the principles and axioms of probability theory. In the beginning of twentieth century non-binary logic was introduced, logic which is not constrained to the confinements of probability theory.

The fast pace development of computer science and information theory reignited the interest for human reasoning and representation. However, a new dimension was introduced by acknowledging the implications of imprecision, contradictions and uncertainty overall, moving away from the probabilistic approach and focusing on qualitative logical formalisms and the representation of gradual nature of linguistic information – feature achieved especially by the fuzzy set theory.

The basis of the logical approach is the idea of incomplete knowledge in contradiction with the use of a unique probability distribution specific to Bayesian theory.

The seminal work of Lotfi Zadeh [Zadeh, 1965] created a paradigm shift in understanding and addressing uncertainty through his fundamental insight on the fact that it is not possible nor rational to model uncertainty in every situation probabilistically.

Lotfi Zadeh's response to this challenge has been the development of the theory of *Fuzzy Logic* and *Fuzzy Sets*. His objective was to develop a theoretical framework for the understanding the behaviour of systems requiring a less rigid description.

4.6 Conclusions

Ignoring uncertainty when addressing any scientific problem is not an option. Solving it without taking into consideration uncertainty will only yield incorrect solutions. An integrated approach to managing uncertainty would use all available information. In addition, would detail why and how uncertainty is determined and provide methods to mitigate its impact. After presenting how human reasoning understands and relates to uncertainty, the next chapter will introduce an innovative mathematical approach to this issue based on the seminal work of Lotfi Zadeh - *fuzzy logic and fuzzy sets*.

CHAPTER V

FUZZY DUAL NUMBERS AND UNCERTAINTY

5.1 Introduction

Starting with the seminal work of Lotfi Zadeh in 1965 [Zadeh, 1965], fuzzy logic and fuzzy set theory evolved in many directions, with applications in very diverse fields of research where the conventional techniques are obsolete in dealing with too complex or too vaguely defined issues. By providing the basis for a systematic approach to approximate reasoning [Zadeh et. al, 1975], fuzzy set theory ended up having a significant impact on research domains like economics, medicine, decision analysis and artificial intelligence.

Three stages can be identified in mathematics of fuzziness since its emergence: the straightforward fuzzification during the seventies (fuzzy topology, fuzzy algebraic structures such as fuzzy groups, and fuzzy vector spaces, fuzzy arithmetic, fuzzy measure theory, fuzzy relational calculus), the explosion of possible choices in the generalization process during the eighties (the concept of *fuzzy number*, discovery of *triangular norms and co-norms*, deep study of alternative operations on logical level i.e. *negation, disjunction, conjunction, implication* and also on fuzzy set theory level i.e. *complementation, union, intersection, inclusion*) and currently, the standardization, axiomatization, L-Fuzzification and fuzzy representation and manipulation of imprecision and uncertainty [Kere, 1995].

The term of *soft computing* or *computational intelligence* entered the literature in 1992, including fuzzy set theory, neural networks and evolutionary computing (genetic algorithms) [Zimmerman, 2010]. The soft computing umbrella gathers underneath tools for addressing imprecision and uncertainty.

Traditionally, the techniques used for modeling, reasoning and computing are crisp, deterministic and very precise. Bivalent logic states that a statement can be either true or false. In traditional set theory, an element is either belonging to a set or not. In classical optimization theory a solution is either feasible or not. Following this type of reasoning, we implicitly assume that the parameters of the system mirror precisely the real world version we are modeling. Therefore, the model does not contain any uncertainties. Placing the system in a certain environment implies that the parameters of the model are precisely defined, with no vagueness regarding their values or occurrence.

Modeling reality is not by far as easy because of two simple reasons: reality is neither crisp nor certain. Even more, an exhaustive description of a real system would require a significant amount of data so complex that processing time and cost will be prohibitive.

"As the complexity of a system increases, our ability of making precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics" [Zadeh, 1973].

Yet, when it comes to modeling reality and its uncertainties, probability and statistics are the tools generally used [Zadeh, 2005], but both of these tools fail when it comes to accuracy and robustness, especially in the case of long-term planning horizons.

5.2 Fuzzy logic

Human language uses a generous amount of vagueness and imprecision, which can also be referred to as fuzziness. The challenge is to represent and manipulate inferences using this kind of information.

As introduced by Zadeh in 1965 [Zadeh, 1965], [Zadeh, 1968], [Zadeh, 1973], *Fuzzy logic* is a multivalued type of logic in which the truth values of variables may take any value between 0 and 1, therefore considered to be *fuzzy*, in contrast with *Boolean logic*, where the truth values, often called *crisp*, may only be 0 or 1.

Fuzzy logic adds to bivalent logic an important capability – the capability to reason precisely with imperfect information, which in one or more respects is imprecise, uncertain, incomplete, unreliable, vague or partially true [Zadeh, 2009]. This process is practically daily human reasoning and that is why fuzzy logic is part of Artificial Intelligence.

While, traditional variables take numerical values, fuzzy logic deals with *linguistic variables.* Linguistic variables are the input or output variables of the system whose values are words or sentences from natural language and it is usually decomposed into a set of linguistic terms, as exemplified in Fig. 5.1.

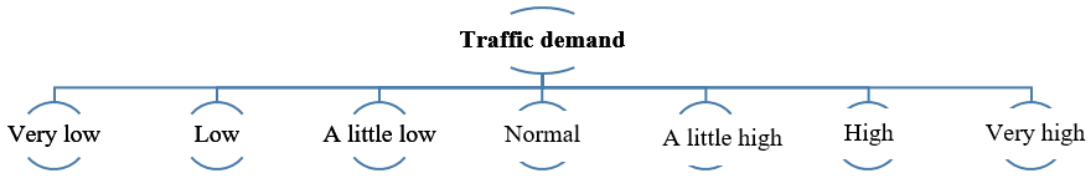

Fig. 5.1 Representation of the linguistic variable “traffic demand”

5.3 Fuzzy Logic Systems

A Fuzzy Logic System – FLS, whose architecture is presented in Fig. 5.2, is designed to process deterministic data through fuzzy logic and to put into practice the knowledge gathered in a base of fuzzy rules. This implies the necessity to fuzzify input data and when the processing is completed, to provide a deterministic result through defuzzification.

Membership functions are used in the fuzzification and defuzzification steps of a *Fuzzy Logic System*. Their role is to map the crisp variables to fuzzy linguistic terms in the fuzzification phase and do the exact opposite in the defuzzification phase.

The fuzzification module transforms the inputs, which have crisp values, into fuzzy sets. In a FLS, the base of rules is developed to manipulate the input variable. In general, FLS incorporate more than one rule that describe knowledge. The most common is the IF-THEN rule. IF a set of conditions is satisfied, THEN a set of consequences can be inferred.

Fuzzy reasoning is divided in two parts: evaluating the antecedent - IF part of the rule and applying the result to the consequent - THEN part of the rule.

The evaluation of the fuzzy rules and aggregation of the results of the individual rules is performed using fuzzy set operations – this process is called inference. The result of the inference step is a fuzzy value, obtained by the aggregation of all output fuzzy sets into a single output fuzzy set, obtained by applying all the rules in the rule base. In order to obtain a crisp value, defuzzification is performed in accordance to the membership function of the output variable.

The *Fuzzy Logic System* can be seen as a non-linear mapping of an input data set to a scalar output data [Mendel, 1995].

The algorithm behind FLS performs the following steps:

1. Definition of the linguistic variables;
2. Definition of the membership functions;
3. Development of the base of rules;
4. Fuzzification (convert crisp data into fuzzy values using membership functions);
5. Inference (rule base evaluation and result aggregation);
6. Defuzzification (convert output data into non-fuzzy values).

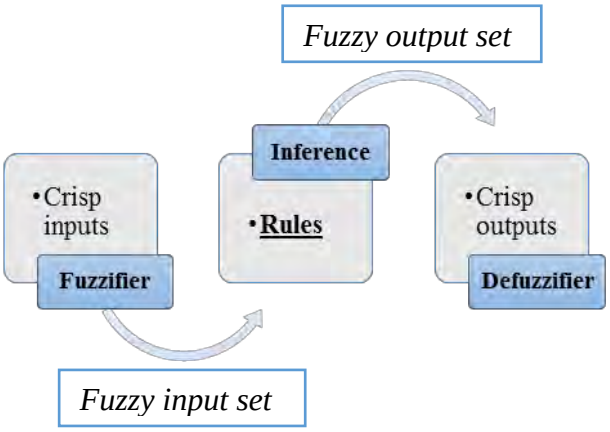

Fig. 5.2 The fuzzy logic system architecture

5.4 Fuzzy set theory

Fuzzy set theory is one of the tools developed with the purpose of modeling reality more accurately than the traditional options. The theory of fuzzy sets relates to classes of objects with un-sharp boundaries in which membership is a matter of degree. In fuzzy logic, the truth of any statement becomes a matter of degree.

"The notion of a fuzzy set provides a convenient point of departure for the construction of a conceptual framework which parallels in many aspects the framework used in the case of ordinary sets, but is more general than the latter and, potentially, may prove to have a much wider scope of applicability, particularly in the fields of pattern classification and information processing. Essentially, such a framework provides a

natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables" [Zadeh, 1965].

Zadeh refers to *imprecision*, in his seminal work quoted above, in the sense of *vagueness* rather than in the sense of a lack of knowledge about the value of a specific parameter.

Fuzzy set theory provides a precise and rigorous mathematical framework for the study of vague conceptual phenomena and decision making based on their evaluation.

For the past forty years, fuzzy set theory grew in popularity considerably due to the success of fuzzy control applications. According to [Zimmermann, 2010] fuzzy set theory developed roughly around two directions:

1. A formal theoretic approach that became more and more complex and enlarged by the inclusion of original ideas and concepts, and merging with classical mathematical areas like algebra [Dubois and Prade, 1979], graph theory [Kim and Roush, 1982], mathematical programming, either by generalizing them or *fuzzifying* them.

2. An application oriented 'fuzzy technology', a tool for modeling, problem solving and data mining that has been proven superior to existing methods in many cases and a feasible addition to classical approaches in other cases.

5.4.1 Definition and representation

If *X* is a collection of objects, called the *universe of discourse*, whose generic elements are denoted by *x*, then a *fuzzy set* $\tilde{A}$ in *X* is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x) | x \in X)\} \tag{5.1}$$

,where $\mu_{\tilde{A}}(x)$ is the *membership function* and each pair $(x, \mu_{\tilde{A}}(x))$ represents a *singleton*.

The *grade of membership*, i.e. the value $\mu_{\tilde{A}}(x)$ at *x*, represents the degree to which *x* belongs to $\tilde{A}$. Therefore, the closer the value of $\mu_{\tilde{A}}(x)$ is to 1, the more *x* belongs to $\tilde{A}$.

A crisp subset of *X* can be viewed as a fuzzy set in *X*, with its membership function given by:

$$\mu_{\tilde{A}} = \begin{cases} 0, x \notin \tilde{A}, \\ 1, x \in \tilde{A}. \end{cases} \tag{5.2}$$

Depending if *X*, the universe of discourse, is discrete or continuous, we can have alternative notations for the fuzzy sets to indicate the *union* of the fuzzy set: *summations* or *integrals*. The adopted notation of a fuzzy set with a discrete universe of discourse is:

$$\tilde{A} = \sum_{x_i \in X} \mu_{\tilde{A}}(x) / x_i \tag{5.3}$$

, which is the union of all singletons.

For a continuous universe of discourse, the adopted notation of a fuzzy set is:

$$\tilde{A} = \int_X \mu_{\tilde{A}}(x) / x \tag{5.4}$$

, where the integral sign indicates the union of all $\mu_{\tilde{A}}(x) / x$ singletons.

Fuzzy sets with crisply defined membership functions are called *ordinary fuzzy sets.*

5.4.2 Graphical representation of membership functions

Graphical representations of fuzzy sets are suitable in the case when *X* is one or two-dimensional Euclidean space, as seen in examples below.

5.4.2.1 Triangular membership functions

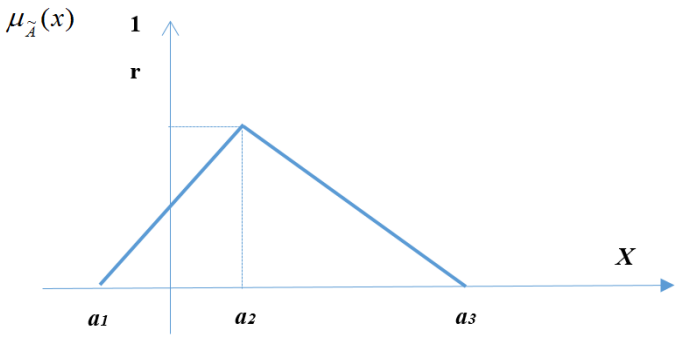

Fig. 5.3 Graphical representation of a triangular membership function

The analytical representation is given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} r, & \text{if } a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} r, & \text{if } a_2 \le x \le a_3 \\ 0 & \text{otherwise.} \end{cases} \tag{5.5}$$

5.4.2.2 Trapezoidal membership functions

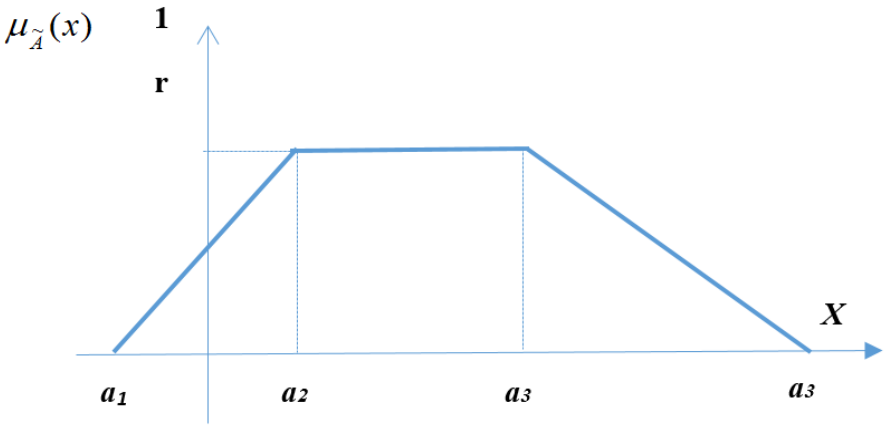

Fig. 5.4 Graphical representation of a trapezoidal membership function

The analytical representation is given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} r, & \text{if } a_1 \le x \le a_2 \\ r, & \text{if } a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} r, & \text{if } a_3 \le x \le a_4 \\ 0 & \text{otherwise.} \end{cases} \tag{5.6}$$

5.4.2.3 S-shaped membership functions

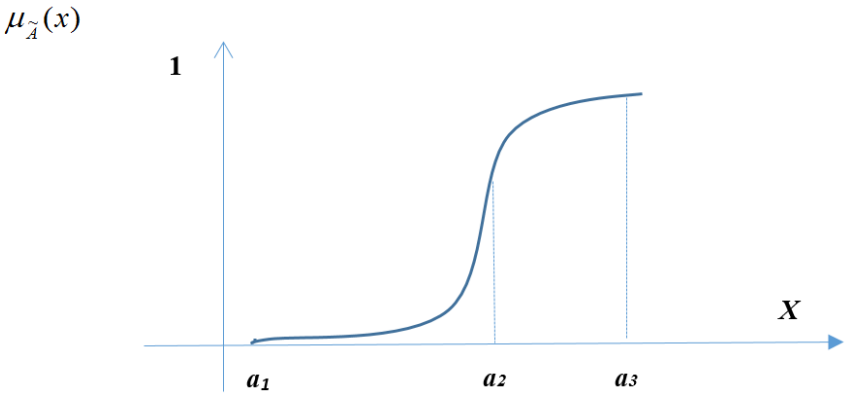

Fig. 5.5 Graphical representation of an S-shaped membership function

The analytical representation is given by:

$$\mu_{\ddot{A}}(x) = \begin{cases} 0, & \text{if } x \le a_1 \\ 2\left(\dfrac{x - a_1}{a_3 - a_1}\right)^2 & \text{if } a_1 < x < a_2 \\ 1 - 2\left(\dfrac{x - a_1}{a_3 - a_1}\right)^2 & \text{if } a_2 \le x \le a_3 \\ 1, & \text{if } a_3 \le x. \end{cases} \tag{5.7}$$

5.4.2.4 Bell-shaped membership functions

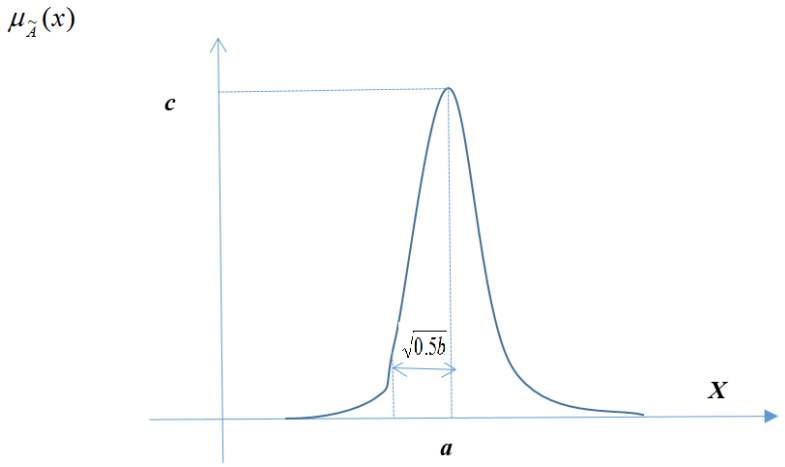

Fig. 5.6 Graphical representation of a Bell-shaped membership function

The analytical representation is given by:

$$\mu_{\tilde{A}}(x) = c \cdot \exp\left(-\frac{(x-a)^2}{b}\right) \tag{5.8}$$

5.4.2.5 The case of an infinite universe of discourse

In the case of an infinite universe of discourse, the above-mentioned representations of the membership functions are ineffective. The solution is to opt for an analytical representation instead.

There are several methods for constructing membership functions and they are classified as follows, according to [Aliev, 2013]:

1. Membership functions based on heuristics;
2. Membership functions based on reliability concepts with respect to the particular problem;
3. Membership functions based on more theoretical considerations;
4. Membership functions as a model for human concepts;
5. Membership functions based on intensive data processing, neural networks in general.

5.4.3 Fuzzy set operations

In [Zadeh, 1965] the following operations for fuzzy sets were defined:

Intersection (AND) $\mu_{\tilde{A}\cap\tilde{B}}(X) = Min(\mu_{\tilde{A}}(X), \mu_{\tilde{B}}(X)) \forall x \in X$ (5.9)

Union (exclusive OR) $\mu_{\tilde{A}\cup\tilde{B}}(X) = Max(\mu_{\tilde{A}}(X), \mu_{\tilde{B}}(X)) \forall x \in X$ (5.10)

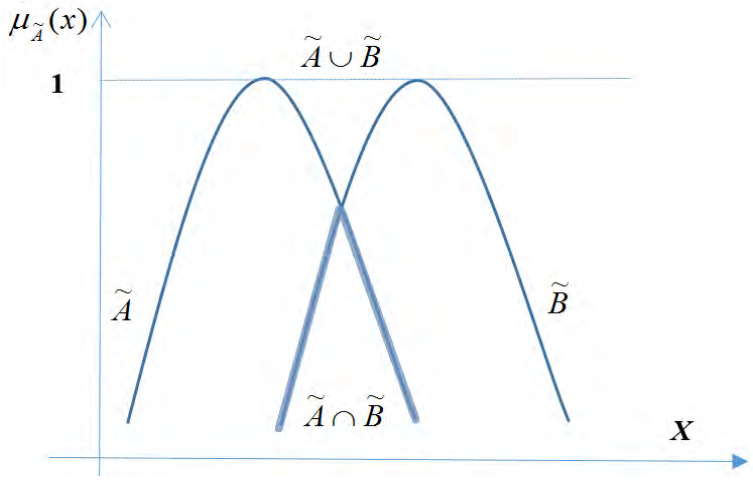

Fig. 5.7 Intersection and union of two fuzzy sets

Complement (NOT) $\mu_{\tilde{A}}(X) = 1 - \mu_{\tilde{A}}(X) \forall x \in X$ (5.11)

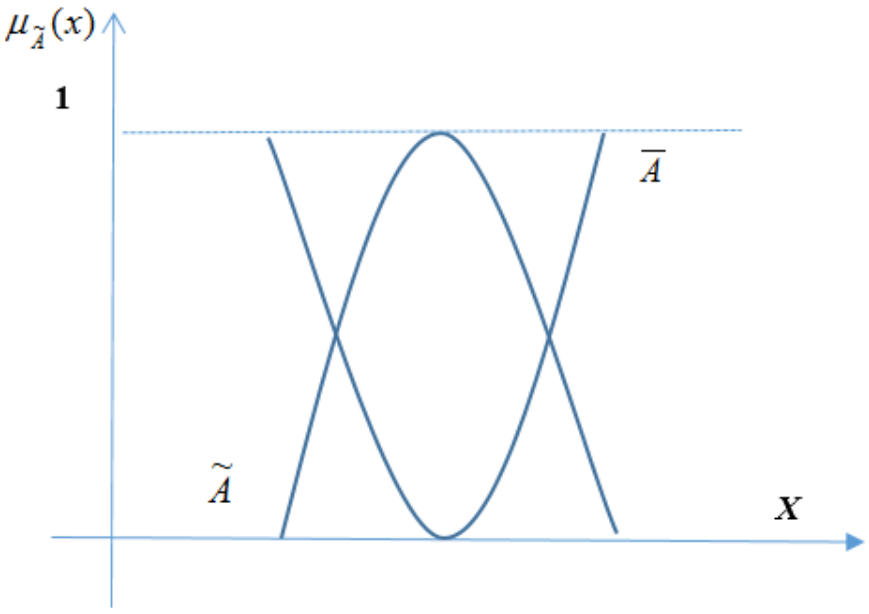

Fig. 5.8 The complement of a fuzzy set

The following properties of crisp sets hold also for fuzzy sets:

Commutativity: $\tilde{A} \cup \tilde{B} = \tilde{B} \cup \tilde{A}$ (5.12)

Associativity: $(\tilde{A} \cup \tilde{B}) \cup \tilde{C} = \tilde{A} \cup (\tilde{B} \cup \tilde{C})$; $(\tilde{A} \cap \tilde{B}) \cap \tilde{C} = \tilde{A} \cap (\tilde{B} \cap \tilde{C})$ (5.13)

De Morgan's laws: $\overline{A \cup B} = \overline{A} \cap \overline{B}$; $\overline{A \cap B} = \overline{A} \cup \overline{B}$; (5.14)

Distributive laws: $\tilde{A} \cup (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cup \tilde{B}) \cap (\tilde{A} \cup \tilde{C})$; (5.15)

$$\tilde{A} \cap (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cup (\tilde{A} \cap \tilde{C}) \quad (5.16)$$

The following two properties of crisp sets do not stand for fuzzy sets:

Law of contradiction: $\tilde{A} \cap \overline{A} = \emptyset$, (5.17)

Law of excluded middle: $\tilde{A} \cup \overline{A} = X$ (5.18)

5.4.4 Fuzzy set properties

5.4.4.1 The support of a fuzzy set

The *support set* of a fuzzy set $\tilde{A}, S(\tilde{A})$ is the crisp set of all elements $x \in X$ such that their membership function is different from zero, i.e. $\mu_{\tilde{A}}(x) > 0$:

$$S(\tilde{A}) = \{x \in X : \mu_{\tilde{A}}(x) > 0\} \quad (5.19)$$

5.4.4.2 The crossover point of a fuzzy set

The elements of *x* such as $\mu_{\tilde{A}}(x) = 1/2$ are the crossover points of $\tilde{A}$.

A fuzzy set that has only one point in *X* with $\mu_{\tilde{A}}(x) = 1$ as its support is called a *singleton*.

5.4.4.3 The height of a fuzzy set

The height of $\tilde{A}$ is defined as the least upper bound of $\mu_{\tilde{A}}(x)$, i.e.

$$hgt(\tilde{A}) = \sup_{x \in X} \mu_{\tilde{A}}(X) \tag{5.20}$$

$\tilde{A}$ is set to be n*ormalized* (or called a *normal* fuzzy set) if and only if $\exists x \in X, \mu_{\tilde{A}}(x) = 1$, implying $hgt(\tilde{A}) = 1$. Otherwise, $\tilde{A}$ is called *subnormal fuzzy set*.

5.4.4.4 The empty fuzzy set

A fuzzy set is empty, Ø, if its membership function is identically zero, i.e.:

$$\mu_{\tilde{A}}(x) = 0 \text{ for all } x \in X \tag{5.21}$$

5.4.4.5 The α-Level fuzzy sets

The *α-cut* method is an important way of representing fuzzy sets. This type of representation allows the use of crisp sets properties and operations in fuzzy set theory.

The crisp set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α - *cut* set:

$$A_\alpha = \{x \in X \,|\, \mu_{\tilde{A}}(x) \geq \alpha\} \tag{5.22}$$

A *strong* α *-cut* is defined as follows: $A'_\alpha = \{x \in X \,|\, \mu_{\tilde{A}}(x) > \alpha\}$ (5.23)

All α-cuts of any fuzzy set form families of crisp sets, which can be used to represent a given fuzzy set $\tilde{A}$ in X [Bector and Chandra, 2005].

5.4.4.6 Convexity of fuzzy sets

The convexity of fuzzy sets plays a paramount role in the definition of *fuzzy numbers* and the derived *fuzzy arithmetic* [Bector and Chandra, 2005].

A fuzzy set $\tilde{A}$ is convex if and only if:

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2 > \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \tag{5.24}$$

, for all $x_1, x_2 \in R, \lambda \in [0,1]$.

Alternatively, a fuzzy set $\tilde{A}$ on *R* is convex if and only if all its α-level sets are convex in the classical sense.

The convexity of a fuzzy set does not imply that its membership function $\mu_{\tilde{A}}$ is a convex function in the crisp sense.

5.4.4.7 The cardinality of fuzzy sets

When *X* is a finite set, the scalar cardinality $\left|\tilde{A}\right|$ of a fuzzy set $\tilde{A}$ on *X* is defined as:

$$\left|\tilde{A}\right| = \sum_{x \in \tilde{A}} \mu_{\tilde{A}}(x) \tag{5.25}$$

$\left|\tilde{A}\right|$ is also referred to as the power of $\tilde{A}$.

The relative cardinality is defined as:

$$\left\|\tilde{A}\right\| = \left|\tilde{A}\right| / \left|X\right| \tag{5.26}$$

when *X* is defined as:

$$\left|\tilde{A}\right| = \int_{x} \mu_{\tilde{A}}(x) dx \tag{5.27}$$

5.4.4.8 Equality of fuzzy sets

Two fuzzy sets $\tilde{A}$ and $\tilde{B}$ are set to be equal if and only if:

$$\forall x \in X,\ \mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x) \ \ \tilde{A} = \tilde{B} \tag{5.28}$$

5.4.4.9 Inclusion of fuzzy sets

Given the fuzzy sets $\tilde{A}, \tilde{B} \in \tilde{C}(X)$, $\tilde{A}$ is said to be included in $\tilde{B}(\tilde{A} \subseteq \tilde{B})$ or $\tilde{A}$ is a subset of $\tilde{B}$ if $\forall x \in X, \mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$.

5.4.6.10 Triangular-Norm and Triangular Co-Norm

t-norm is a function *t*: [0,1] x [0,1] → [0,1] that satisfies the following axioms:

The boundary condition: $t(\mu_{\tilde{A}}(x),1) = \mu_{\tilde{A}}(x)$ (5.29)

Monocity $if \mu_{\tilde{A}}(x) \le \mu_{\tilde{C}}(x) and \mu_{\tilde{B}}(x) \le \mu_{\tilde{D}}(x)\ then\ t(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) \le t(\mu_{\tilde{C}}(x), \mu_{\tilde{D}}(x))$

Commutativity: $t(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) = t(\mu_{\tilde{B}}(x), \mu_{\tilde{A}}(x))$ (5.30)

Associativity: $t(\mu_{\tilde{A}}(x), t(\mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x))) = t(t(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x)))$ (5.31)

The function *t* takes as its arguments the pair consisting of the element membership grades in the sets $\tilde{A}$ and $\tilde{B}$, and yields membership grades of the elements in the

$$\tilde{A} \cap \tilde{B}: (\tilde{A} \cap \tilde{B})(x) = t[\tilde{A}(x), \tilde{B}(x)] \qquad \forall x \in X \tag{5.32}$$

Frequently used t-norm based fuzzy intersection operations are:

Standard intersection $t_0(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) = \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$ (5.33)

Algebraic product $t_1(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) = \mu_{\tilde{A}}(x) \cdot \mu_{\tilde{B}}(x)$ (5.34)

Bounded difference $t_2(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) = \mu_{\tilde{A} \cap \tilde{B}}(x) = \max(0, \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) - 1)$ (5.35)

Drastic intersection $t_3(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) = \begin{cases} \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\} & \\ & if \mu_{\tilde{A}}(x) = 1 \\ & or \mu_{\tilde{B}}(x) = 1 \\ 0 & \text{otherwise.} \end{cases}$ (5.36)

For four fuzzy intersections, the following is true:

$$t_3(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) \le t_2(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) \le t_1(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) \le t_0(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)) \tag{5.37}$$

t-conorm is a function *s*: [0,1] x [0,1] → [0,1] which is commutative, associative and monotonic in every variable, with the following boundary condition:

$$s(\mu_{\tilde{A}}(x), 0) = \mu_{\tilde{A}}(x) \tag{5.38}$$

5.5. Fuzzy numbers

In areas like optimization and decision making, dealing with crisp numbers and crisp intervals is not a feasible option, the alternative being “approximate” numbers or intervals which are close to a given real number or interval.

Before giving the definition of a fuzzy number, the following aspects should be considered. The objective is to define *numbers that are close to a given real number r*. The real number *r* is obviously close to *r* itself, therefore the fuzzy set defined as such must be a normal fuzzy set, i.e. will have the following property: $\mu_{\tilde{A}}(r) = 1.$

In addition, the intervals should be considered at varying levels $\alpha \in (0,1]$ to have the proper gradation, that is the α-cuts of $\tilde{A}$ must be closed intervals of the type $[a_\alpha^L, a_\alpha^R]$. In order to perform interval arithmetic, the intervals $[a_\alpha^L, a_\alpha^R]$ for $\alpha \in (0,1]$ must be of finite length and that implies that the support of $\tilde{A}$ is bounded.

A fuzzy set $\tilde{A}$ in **R** is called a *fuzzy number* if it satisfies the following conditions:

1. $\tilde{A}$ is normal fuzzy set,
2. $\tilde{A}$ is convex,
3. $\tilde{A}_\alpha$ is a closed interval for every $\alpha \in (0,1]$,
4. the support of $\tilde{A}$ is bounded.

Let $\tilde{A}$ be a fuzzy set in **R.** $\tilde{A}$ is a fuzzy number if and only if there exist a closed interval (that may be a singleton) $[a,b] \neq \emptyset$ such that:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1, & x \in [a,b] \\ l(x), & x \in (b,\infty) \\ r(x), & x \in (-\infty,a), \end{cases} \tag{5.39}$$

where $l:(-\infty,a) \to [0,1]$ is *increasing, continuous from the right* and *l(x)=0* for $x \in (-\infty, w_1), w_1 < a$

and $r:(b,\infty) \to [0,1]$ is *decreasing, continuous from the left* and *r(x)=0* for $x \in (w_2,\infty), w_2 > b$.

Here, the term *increasing* is used in the sense that $x \geq y \Rightarrow l(x) \geq l(y)$, i.e. *l* is *non-decreasing.*

In the majority of real life applications, the functions *l(x)* and *r(x)* are continuous, making the membership function continuous as well.

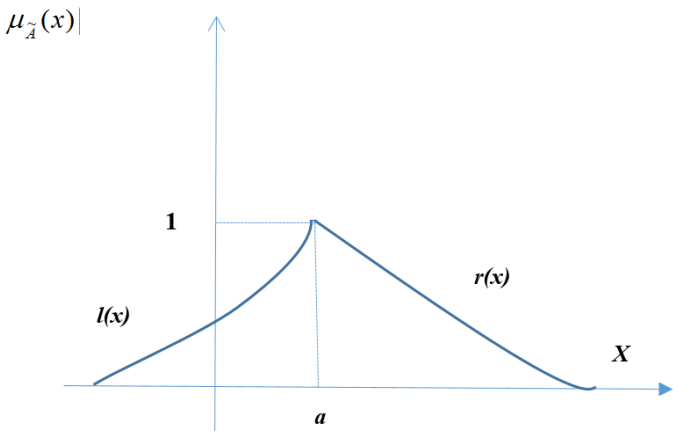

Fig. 5.9 Representation of a fuzzy number *a* with continuous *l* and *r*

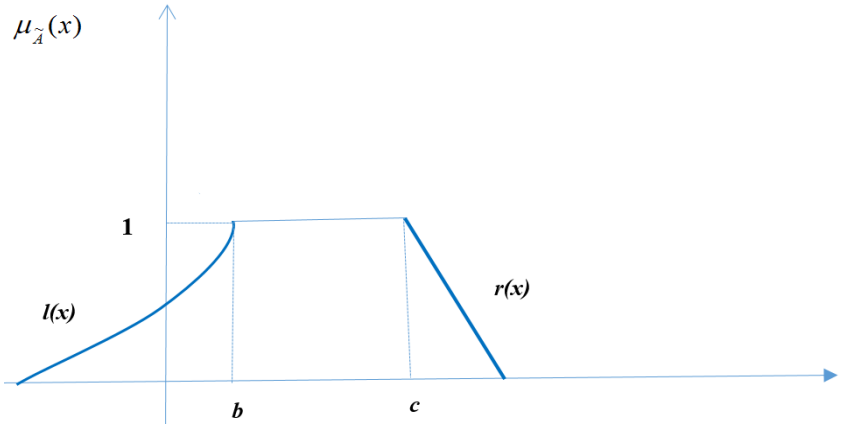

Fig. 5.10 Fuzzy interval [b, c] with continuous *l* and *r*

5.5.1 Triangular fuzzy numbers

A *triangular fuzzy number* $\tilde{A}$, as shown in Fig. 5.11, denoted by the triplet $\tilde{A} = (a_l, a, a_u)$, has the following membership function:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_l, x > a_u, \\ \dfrac{x - a_l}{a - a_l}, & a_l \le x \le a, \\ \dfrac{a_u - x}{a_u - a}, & a < x \le a_u. \end{cases} \tag{5.40}$$

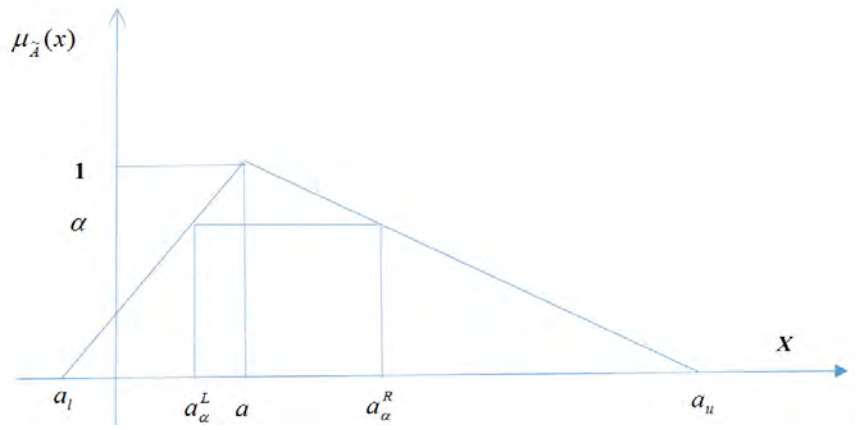

Fig. 5.11 Representation of a triangular fuzzy number

The α-cut of a triangular fuzzy number is the closed interval:

$$\tilde{A}_\alpha = [a_\alpha^L, a_\alpha^R] = [\alpha(a - a_1) + a_l, -\alpha(a_u - a) + a_u],\ \alpha \in (0,1] \tag{5.41}$$

5.5.2 Trapezoidal fuzzy numbers

A *trapezoidal fuzzy number* $\tilde{A}$, as shown in Fig. 5.12, denoted by the quadruplet $\tilde{A} = (a_l, a', a'', a_u)$, has the following membership function:

$$\mu_{\tilde{A}} = \begin{cases} 0, & x < a_l, x > a_u, \\ \dfrac{x - a_l}{a' - a_l}, & a_l \le x < a', \\ 1, & a' \le x \le a'' \\ \dfrac{a_u - x}{a_u - a''}, & a'' < x \le a_u. \end{cases} \tag{5.42}$$

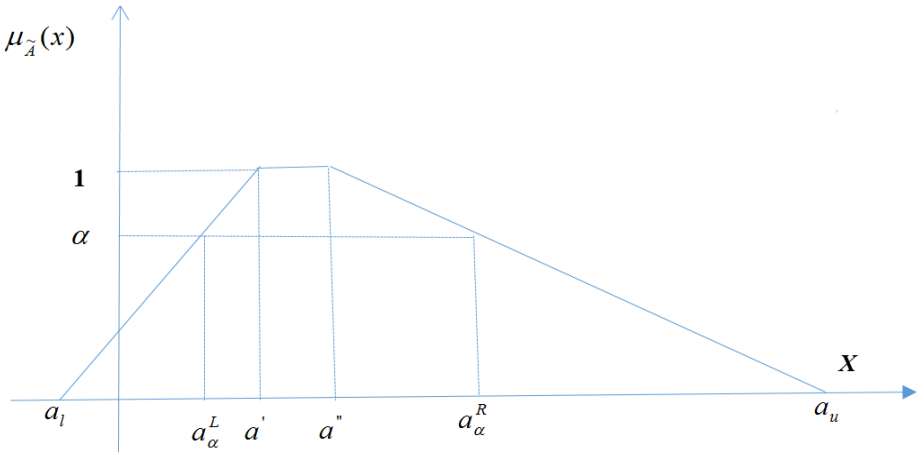

Fig. 5.12 Representation of a trapezoidal fuzzy number

The α-cut of a triangular fuzzy number is the closed interval:

$$\tilde{A}_\alpha = [a_\alpha^L, a_\alpha^R] = [\alpha(a' - a_1) + a_l, -\alpha(a_u - a'') + a_u],\ \alpha \in (0,1] \tag{5.43}$$

5.5.3 L-R fuzzy numbers

A *L-R fuzzy number* as shown in Fig. 5.13, denoted by $\tilde{A} = (a, b, \alpha, \beta)$, has the following membership function:

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\dfrac{x-a}{\alpha}\right), & (a-\alpha) \le x < a, \alpha > 0, \\ R\left(\dfrac{x-b}{\beta}\right), & b < x \le (b+\beta), \beta > 0, \\ 1, & a \le x \le b \\ 0 & otherwise \end{cases} \tag{5.44}$$

, where *L, the left reference function*, and *R, the right reference function*, are piecewise continuous functions, *L* is increasing, *R* is decreasing and $L(0) = R(0) = 1$. Also, *a* and *b* are the *starting and the end points of the interval*, α is the *left spread*, while β is the *right spread*.

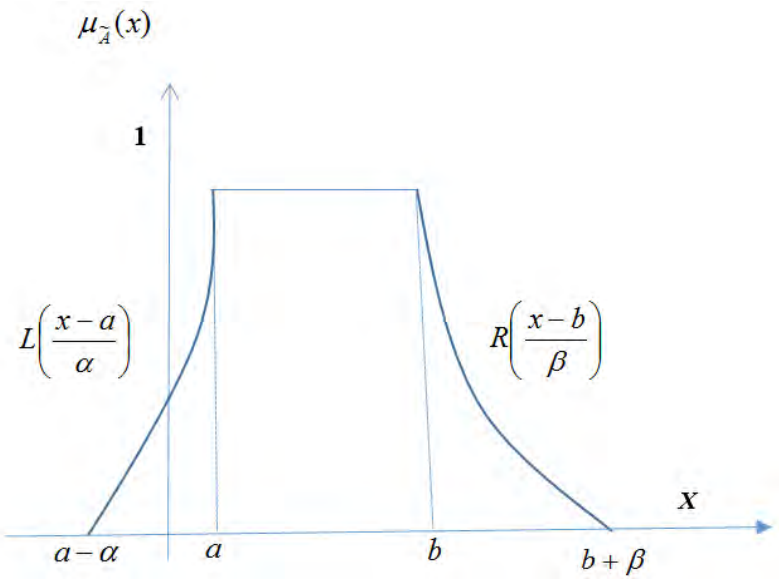

Fig. 5.13 Representation of an L-R fuzzy number

5.6 Fuzzy dual numbers

Before introducing the concept of *fuzzy dual numbers*, [Clifford, 1873]'s seminal work on *dual numbers* has to be mentioned. Part of the *Theory of engines*, dual numbers is a concept based on the use of a nilpotent operator noted $\boldsymbol{\varepsilon}$ to define dual quaternions in order to represent the movement of screwing of a mechanical system. The applicability of dual numbers ranges from screw systems and plane joints modeling to iterative methods for displacement analysis of spatial mechanisms, inertial force analysis of spatial mechanisms [Kandasamy and Smarandache, 2012] or kinematic and dynamic modeling of robotic manipulators [Herrera et al., 2012].

More recently, [Cosenza and Mora-Camino, 2011], [Cosenza and Mora-Camino, 2012] and [Cosenza et. al., 2012], proposed a new approach on dual numbers, *fuzzy dual numbers*, concept developed with the objective to address efficiently uncertainty in general decision making problems.

Expanding on this concept, fuzzy dual comparison and fuzzy dual calculus are introduced, to treat parameter uncertainty and solution diversion in mathematical optimization problems through a better trade-off between complexity and feasibility of the proposed solution. In addition, the concepts of fuzzy dual vectors and fuzzy dual matrices are formalized. Fuzzy dual probabilities are also introduced, with the

purpose to take into account uncertainty present in a priory probability distributions used for prediction purposes, leading to the concept of fuzzy dual entropy.

5.6.1 Definitions and representation

A set of *fuzzy dual numbers* is defined as the set $\tilde{\Delta}$ of numbers of the form $a+\varepsilon.b$, where a is the *primal part* and b is the *dual part* of the fuzzy dual number $\forall a \in R, \forall b \in R^{+}$.

ε represents the *unity pure dual number*.

A fuzzy dual number loses both its dual and fuzzy attributes if b equals zero. The lower and upper bounds of $a+\varepsilon.b$ are given by $B^{low}(a+\varepsilon.b)=a-b$ and $B^{high}(a+\varepsilon.b)=a+b$.

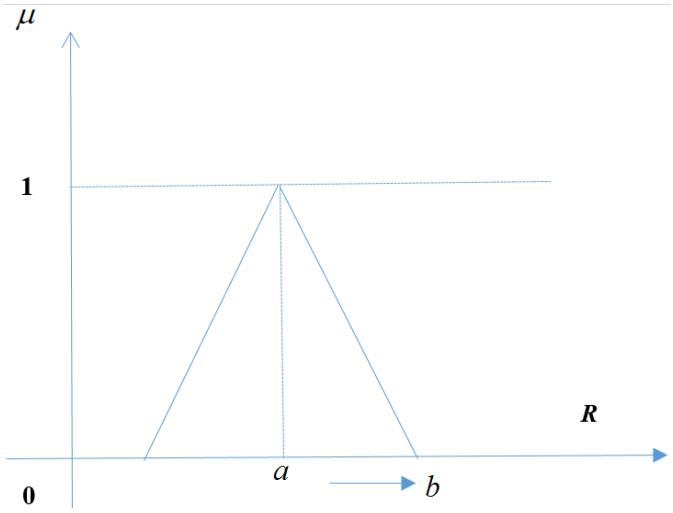

Fig. 5.14 Graphical representation of a triangular fuzzy dual number

The *pseudo norm* of a fuzzy dual number is given by $\|a+\varepsilon.b\| = |a| + \rho.b \in R^{+}$, where $\rho > 0$ is the shape parameter.

The shape parameter is given by $\rho = (1/b)\int_{-b}^{+b} \mu(u)du$, where μ is the membership function.

Fig. 5.15 depicts several graphical representations of fuzzy dual numbers with different shape parameters.

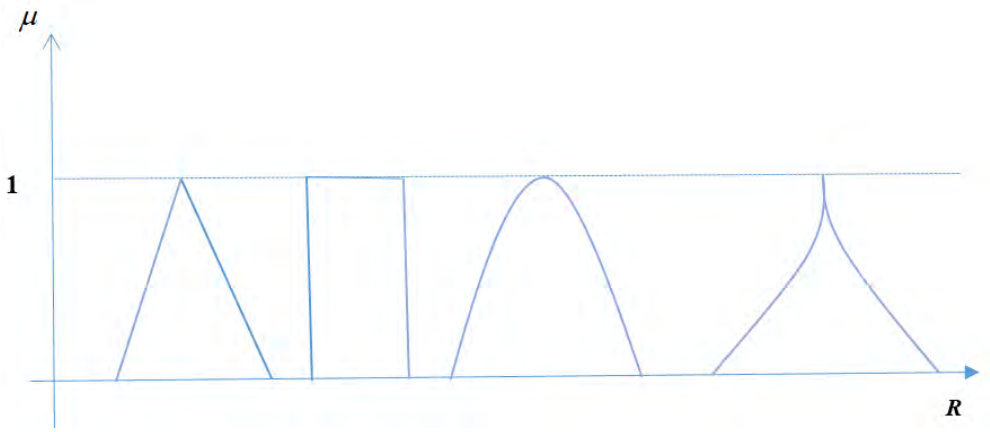

Fig. 5.15 Examples of fuzzy dual numbers with different shape parameters

The following properties of the pseudo norm are maintained, no matter the values the shape parameters take:

$$\forall a+\varepsilon.b \in \tilde{\Delta} : \|a+\varepsilon.b\| \geq 0 \tag{5.45}$$

$$\forall a \in R, \forall b \in R^{+} \ \|a+\varepsilon.b\| = 0 \Rightarrow a = b = 0 \tag{5.46}$$

$$\|(a+\varepsilon.b)+(\alpha+\varepsilon.\beta)\| \leq \|a+\varepsilon.b\| + \|\alpha+\varepsilon.\beta\| \ \ \forall a,\alpha \in R, \forall b,\beta \in R^{+} \tag{5.47}$$

$$\|\lambda.(a+\varepsilon.b)\| = \lambda.\|a+\varepsilon.b\| \ \ \forall a \in R, \forall b,\lambda \in R^{+} \tag{5.48}$$

5.6.2 Orders between fuzzy dual numbers

When comparing two fuzzy dual numbers, only four different situations appear. They are represented in figure 5.16

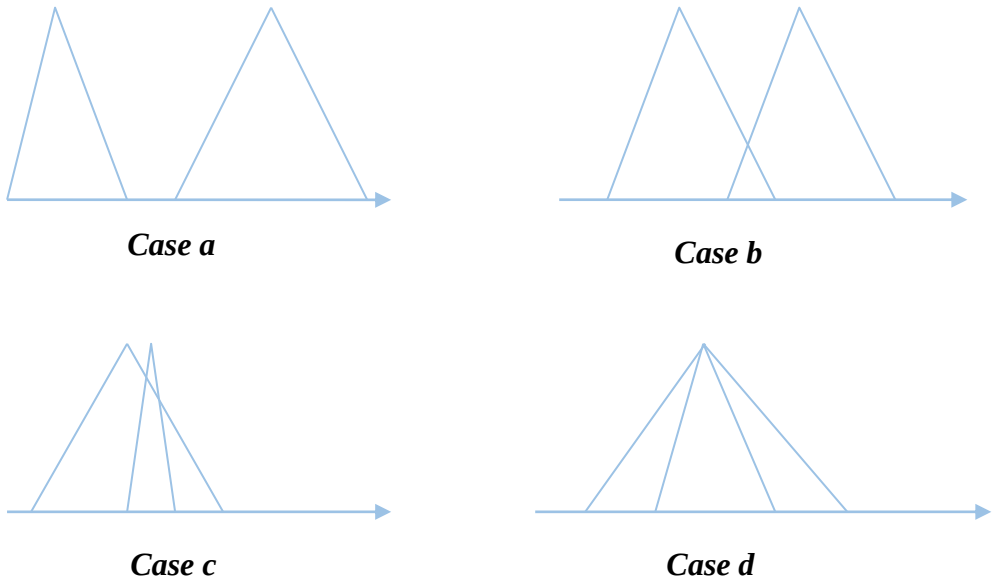

Fig. 5.16 Relative situations of two fuzzy dual numbers

Case a, corresponds to *strong* partial order, written $\breve{\succ}$, which is be defined over $\tilde{\Delta}$ by:

$$\forall a_1+\varepsilon b_1, a_2+\varepsilon b_2 \in \tilde{\Delta} : a_1+\varepsilon b_1 \breve{\succ} a_2+\varepsilon b_2 \Leftrightarrow a_1-\rho b_1 \succ a_2+\rho b_2 \quad (5.49)$$

The *mean* partial order of *case b*, written $\hat{\succeq}$, is defined over $\tilde{\Delta}$ by:

$$\forall a_1+\varepsilon b_1, a_2+\varepsilon b_2 \in \tilde{\Delta} : a_1+\varepsilon b_1 \hat{\succ} a_1+\varepsilon b_2 \Leftrightarrow a_1+\rho b_1 \succ a_2+\rho b_2 \succ a_1-\rho b_1 \quad (5.50)$$

The *weak* partial order of *case c*, written $\tilde{\succ}$, is such as:

$$a_1 > a_2,\ a_1-\rho b_1 \succ a_2-\rho b_2,\ a_1+\rho b_1 \prec a_2-\rho b_2 \quad (5.51)$$

The *fuzzy equality* between two fuzzy dual numbers, corresponding to case *d*, is symbolized by $\cong$ and is characterized by:

$$a_1 = a_2 \quad \text{and} \quad b_1 = b_2 \quad (5.52)$$

Then, it appears that it is *always possible* to rank two fuzzy dual numbers and to assign a qualitative evaluation to this comparison (strong, mean or weak). When either (5.49), (5.50) or (5.51) is satisfied, it will be said that *fuzzy dual number* $a_1 + \varepsilon b_1$ *is greater than fuzzy dual number* $a_2 + \varepsilon b_2$ and we will write:

$$a_1 + \varepsilon b_1 \succ a_2 + \varepsilon b_2 \tag{5.53}$$

A *degree of certainty c* can be attached to this assertion. A candidate expression for this degree is given by:

$$c = 1 - \frac{1}{2}\min\left\{\frac{\alpha}{b_1}, \frac{\alpha}{b_2}\right\} \text{ if } a_1 \geq a_2 \quad \text{and} \quad c = \frac{1}{2}\min\left\{\frac{\alpha}{b_1}, \frac{\alpha}{b_2}\right\} \text{ if } a_1 < a_2 \tag{5.54}$$

where α is the area of the intersection between fuzzy dual numbers $a_1 + \varepsilon b_1$ and $a_2 + \varepsilon b_2$.

In Fig. 5.16, in case a: c=1, in case b: c=0.9, in case c: c=0.7 and in case d: c=0.5.

5.7 Fuzzy dual calculus

The *fuzzy dual neutral element* is given by: $\tilde{0} = (0 + \varepsilon 0)$.

The *neutral element of fuzzy dual multiplication* is given by $\tilde{1} = (1 + \varepsilon 0)$ and only non-zero crisp numbers have an inverse.

The fuzzy dual addition of two fuzzy dual numbers, denoted by $\tilde{+}$, identical to the dual numbers addition, is given by:

$$(a_1 + \varepsilon b_1) \tilde{+} (a_2 + \varepsilon b_2) = (a_1 + a_2) + \varepsilon(b_1 + b_2) \tag{5.55}$$

The fuzzy dual product of two fuzzy dual umbers, denoted by $\tilde{\bullet}$, is given by:

$$(a_1 + \varepsilon b_1) \tilde{\bullet} (a_2 + \varepsilon b_2) = (a_1 \cdot a_2 + \varepsilon(|a_1| \cdot b_2 + |a_2| b_1)) \tag{5.56}$$

The fuzzy dual product is constructed in a way that the fuzzy interpretation of the dual part is preserved but is different from dual calculus.

Both fuzzy dual addition and fuzzy dual multiplication are commutative and associative, while fuzzy dual multiplication is also distributive with respect to the fuzzy dual addition.

The *nilpotent* property of fuzzy dual calculus for operator ε is maintained:

$$\varepsilon \tilde{\bullet} \varepsilon = \varepsilon^2 = \tilde{0} \tag{5.57}$$

5.8 Fuzzy dual vectors

Let *E* be a Euclidean space of dimension *p* over *R*, we construct a set $\tilde{E}$ composed of pairs of vectors, which are called dual fuzzy vectors taken from the Cartesian product $E \times E^+$, where E^+ is the positive half-space of *E* in its canonical basis. The following operations are defined over $\tilde{E}$:

Addition: $(a,b)+(c,d)=(a+c,b+d) \qquad \forall a,b \in E \quad \forall c,d \in E^+$ (5.58)

Multiplication by a fuzzy dual scalar $\lambda + \varepsilon\ \mu$:

$$(\lambda + \varepsilon\ \mu)\ (a,b) = (\lambda a, |\lambda|\ b + \mu |a|) \quad \forall \lambda + \varepsilon\ \mu \in \tilde{\Delta}, \forall (a,b) \in \tilde{E} \tag{5.59}$$

Then $(a,b) = a + \varepsilon b \qquad \forall (a,b) \in \tilde{E}$ (5.60)

, where the real dual part of the fuzzy dual vector $a + \varepsilon b$ are given by:

$r(a + \varepsilon\ b)) = a$ and $d(a + \varepsilon\ b)) = b$, respectively.

The pseudo-dual scalar product is defined as:

$$u * v = R(u).R(v) + \varepsilon\ (|R(u)|.D(v) + D(u).|R(v)|) \quad \forall\ u, v \in \tilde{E} \tag{5.61}$$

, where "*" represents the inner product in $\tilde{E}$ and "." represents the inner product in *E*.

Two fuzzy dual vectors *u* and *v* are said to be orthogonal if $u * v = \tilde{0}$ where $\tilde{0}$ is the neutral element.

For a dual vector u in $\tilde{E}$ with $R(u) \neq 0$, the Euclidean norm $\| \ \|$ associated to E is given by: $\|u\|_D = \|R(u)\| + \rho \, |R(u)| \cdot D(u) / \|R(u)\|$ (5.62)

If $u = \tilde{0}$, $\|u\|_D = \tilde{0}$ implies the existence of the pseudo-fuzzy dual norm while the orthonormal basis can be considered in $\tilde{E}$.

5.9 Fuzzy dual matrices

The set $\tilde{M}_n$ of fuzzy dual square matrices of order $n \times n$ is constructed on the same logic as fuzzy dual numbers and fuzzy dual vectors. Hence, a fuzzy dual matrix will be defined as:

$$A = [a_{ij}] = [r(a_{ij}) + \varepsilon \, d(a_{ij})] = r(A) + \varepsilon \, d(A) \tag{5.63}$$

, where $r(A)$ is a $R^{n \times n}$ matrix and $d(A)$ is a positive $R^{n \times n}$ matrix.

The basic operations over dual square matrices will be defined as follows:

$$A + B = R(A) + R(B) + \varepsilon \, (D(A) + D(B)) \qquad \forall A, B \in \tilde{M} \tag{5.64}$$

$$A \bullet B = R(A) \, R(B) + \varepsilon \, (|R(A)| \cdot D(B) + D(A) |R(B)|) \quad \forall A, B \in \tilde{M} \tag{5.65}$$

$$\lambda \, A = R(\lambda) \, R(A) + \varepsilon \, (|R(\lambda)| \, D(A) + D(\lambda) \, |R(A)|) \; \forall \lambda \in \tilde{\Delta}, \; \forall A \in \tilde{M}_3 \tag{5.66}$$

The product of a fuzzy dual square matrix by a fuzzy dual vector *u* is considered in this context to be a fuzzy dual vector given by:

$$A \times u = R(A) R(u) + \varepsilon \, (|R(A)| \, D(u) + D(A) \, |R(u)|) \tag{5.67}$$

5.10 Fuzzy dual probabilities

Considering a complete set of *n* discrete events, we introduce here the notion of fuzzy dual probability distribution by considering that to each event e_i, i = 1 to n, is assigned a dual number $P(e_i) = p_i + \varepsilon P_i$. These dual numbers are supposed to satisfy the conditions:

$p_i \in [0,1]$ and $0 \le P_i \le \min\{p_i, 1-p_i\}$ with $\sum_{i=1}^{n} p_i = 1$ (5.68)

Then $\{p_i + \varepsilon P_i, i = 1, \cdots, n\}$ is a fuzzy dual probability distribution while $p_i + \varepsilon P_i$ is a fuzzy dual probability value.

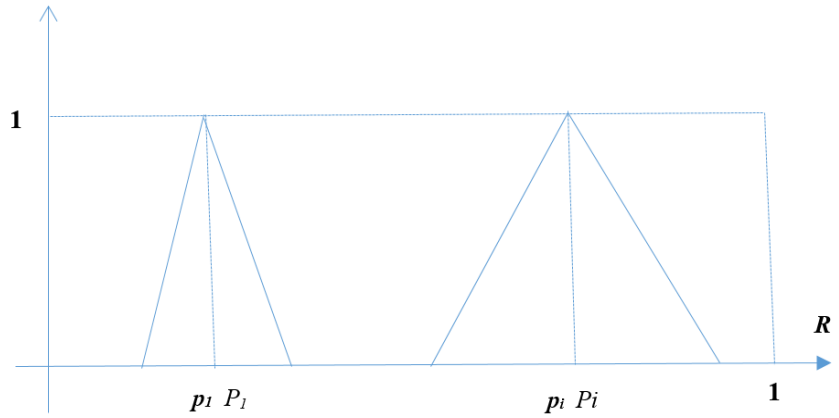

Fig. 5.17 Fuzzy dual probability values

Let $\pi_i \in [-1, +1], i = 1, \cdots, n$, be such as $\sum_{i=1}^{n} \pi_i P_i = 0$, then, $\{p_i + \pi_i P_i, i = 1, \cdots, n\}$ is called a perfect realization of the fuzzy dual probability distribution since:

$0 \le p_i + \pi_i P_i \le 1$ and $\sum_{i=1}^{n} (p_i + \pi_i P_i) = 1$ (5.69)

The set R_e of all perfect realizations associated to the fuzzy dual probability distribution $\{p_i + \varepsilon P_i, i = 1, \cdots, n\}$ is a polyhedron in the R^n space of the π_i's and hence is a convex set.

The fuzzy dual probability associated with the complementary of event e_i is then:

$P(\bar{e}_i) = 1 - P(e_i) = (1 - p_i) + \varepsilon (1 - P_i)$ (5.70)

The fuzzy dual probability associated with the occurrence of independent events e_i and e_j is then:

$P(e_i \wedge e_j) = P(e_i) * P(e_j) = p_i \cdot p_j + \varepsilon P_i \cdot P_j$ (5.71)

, where the *probabilistic product* '$*$' is defined there.

The fuzzy dual probability associated with the occurrence of independent events e_i or e_j is then:

$$P(e_i \vee e_j) = P(e_i) + P(e_j) = p_i + p_j + \varepsilon(P_j + P_i) \quad (5.72)$$

5.11 Conclusions

This chapter gave a brief review of the fundamentals of fuzzy logic, fuzzy numbers and fuzzy sets in order to create the basis for properly introducing a new approach in the field: fuzzy dual numbers. This innovative concept creates a new spectrum of possibilities to address complex issues dealing with uncertainty in decision making problems. Building on the new fuzzy dual numbers formalism and expanding on it to create an innovative framework and direction of research rests at the core of the thesis. The next chapter pairs fuzzy dual logic with a classic mathematical formalism – dynamic programming with the objective to address multi-stage decision making problems.

CHAPTER VI

FUZZY DUAL MATHEMATICAL PROGRAMMING

6.1 Introduction

Expanding on the basis of the fuzzy dual logic formalism and fuzzy dual numbers introduced in Chapter V, the next step is to create the framework for addressing the airport long-term planning problem merging the fuzzy dual logic formalism with the classic dynamic programming technique.

Mirroring real world problems through a mathematical perspective cannot be done applying a deterministic optimization approach simply because of the degree of uncertainty regarding the parameters of such problems. Long-term airport planning fits in this category, firstly due to the intrinsic complexity of the system, secondly due the extensive time span of the project.

A solution to tackle these problems can be robust optimization as long as the parameters are confined within given bounds [Ben-Tal et al., 2009].
In the situation when probability distributions of the parameter values are available, stochastic optimization techniques provide feasible solutions [Ruszczynski and Shapiro, 2003].

[Tanaka et al., 1974] and [Zimmerman, 1974] pioneered a new approach in mathematical programing, allowing flexibility in constraints and fuzziness in the objective function in linear and nonlinear programming, fully embracing the understanding that in the case of real world problems involving large scale systems, the major source of imprecision should be more properly labeled as *fuzziness* rather than *randomness*.

Typically, these three main approaches lead to unmanageable computations. In addition, in many situations the proposed optimal solution has limited practicality because of different implementation constraints that have not been considered explicitly in the formulation of the problem. In these situations, post optimization sensibility analysis is performed which adds to the computational complexity of the problem.

This chapter puts forward a new formalism to treat parameter uncertainty and solution diversion in mathematical optimization problems: fuzzy dual mathematical programming. Both the linear and dynamic perspective are approached and also their limitations are addressed.

6.2 Fuzzy Dual Linear Programming

6.2.1 Linear Programming

Linear programming covers a broad class of optimization problems in which both the constraints and the optimization criteria are linear functions. A significant number of problems can be formulated using this formalism:

$$\max_{\underline{x} \in R^{n+}} \sum_{i=1}^{n} c_i \, x_i \tag{6.1}$$

under constraints:

$$\sum_{i=1}^{n} a_{ki} \, x_i = b_k \quad k \in \{1, \cdots, m\} \tag{6.2}$$

, where a_{ki}, b_k and c_i are real numbers and where either $x_i \in R^+ \quad i \in \{1, \cdots, n\}$ or $x_i \in Z \quad i \in \{1, \cdots, n\}$, in which case x_i are real variables.

Many real life decision making problems, of both technical and economic nature, can be addressed using this general formalism: production problems, transportation, communication, composition, design.

[Kantorovich, 1939] and [Koopmans, 1960] introduced linear programming to tackle production planning problems of increased complexity. They were awarded jointly the 1975 Nobel Prize in Economic Sciences for their contribution to the field of resource allocation, specifically the theory of optimal use of resources. The theory of Kantorovich spaces led immediately to the discovery of linear programming [Kutateladze, 2012].

Dantzig independently developed, between 1946 and 1947, the general linear programming formulation as a tool to address planning problems. He also created, in 1947, the *simplex method* who tackled efficiently for the first time the linear

programming problem. The method has been further on used to solve various problems with increased complexity. The *simplex method* generally has an acceptable computation time even for large problems. However, in some cases, while the size of the problem is far from excessive, this method can introduce unacceptable calculation times.

6.2.2 Computational complexity

Computer scientists have developed the complexity theory for optimization problems in the 1970s. [Cook, 1971] formalized the notions of polynomial-time reduction known as Cook-reduction and NP-Completeness and proved the existence of an NP-complete problem by proving that the Boolean satisfiability problem is NP-complete. In the landmark paper [Karp, 1972], twenty-one problems are proved to be NP-complete. [Garey and Johnson, 1979] was the first text book to exclusively address NP-completeness and computational intractability, being also the most cited reference in computer science literature. It is both theoretically and practically relevant to know the complexity class of the general linear programming problems.

[Khachiyan, 1979] showed for the first time that the linear programming problem is solvable in polynomial time. He proposed an algorithm that theoretically guaranteed the calculation time is bounded by a polynomial expression of the size of the considered instance of the problem. However, the numerical experiments that were conducted were very disappointing and it was [Karmarkar, 1984] that introduced his new interior point method which proved that these problems are of class P-complexity.

When considering the cases in which $x_i \in Z \quad i \in \{1,\cdots,n\}$, the resulting integer linear programming problems are, in general, of combinatorial nature and belong to the NP-Complex class. Exactly tailored methods such as back-stepping, branch and bound and dynamic programming, have been developed to solve more efficiently these problems by avoiding repetitive tests. However, in the case of large instances of these integer linear programming problems, heuristics must be used to get a rather good solution in an acceptable computing time.

6.2.3 Linear Programming with fuzzy dual parameters

Fuzzy dual formulations of uncertain mathematical programming problems are considered to address, in this section, the case of linear programming, but the formalism can be applied to other classes of objective functions and restrictions.

Further, we define problem D_0 as a fuzzy dual linear programming problem with fuzzy dual constraints and real decision variables:

$$\max_{\underline{x}\in R^{n+}} \left\| \sum_{i=1}^{n} (c_i + \varepsilon d_i) x_i \right\| \tag{6.3}$$

under strong constraints:

$$\sum_{i=1}^{n} (a_{ki} + \varepsilon \alpha_{ki}) x_i \tilde{\geq} b_k + \varepsilon \beta_k \quad k \in \{1, \cdots, m\} \tag{6.4}$$

$$\text{and } x_i \in R^+ \quad i \in \{1, \cdots, n\} \tag{6.5}$$

In this case, uncertainty is attached to cost coefficients c_i, to technical parameters a_{ki} and to constraint levels b_k.

The above problem corresponds to the minimization of the worst estimate of total cost with satisfaction of strong level constraints. Here variables x_i belong to $\Re^+$ but they could be either fully real or integer. In the case in which the d_i are zero, the fuzziness is restricted to the feasible set.

Problem D_0 is equivalent to the following problem in $\Re^{+n}$:

$$\max_{\underline{x}\in R^{n+}} \left| \sum_{i=1}^{n} c_i x_i \right| + \rho \sum_{i=1}^{n} d_i x_i \tag{6.6}$$

under the constraints:

$$\sum_{i=1}^{n} (a_{ki} - \rho \alpha_{ki}) x_i \geq b_k + \rho \beta_k \quad k \in \{1, \cdots, m\} \tag{6.7}$$

$$\text{and } x_i \geq 0 \quad i \in \{1, \cdots, n\} \tag{6.8}$$

Then it can be seen that the proposed formulation leads to minimize a combination of the values of the nominal criterion and of its degree of uncertainty. In the case in which the cost coefficients are positive, this problem reduces to a classical linear programming problem over $\Re^{+n}$. In the general case, since the quantity $\sum_{i=1}^{n} c_i x_i$ will have at solution a particular sign, the solution $\underline{x}^*$ of problem D_0 will be such as:

$$\arg\max\left\{ \max_{\underline{x}\in R^{n+}} \left(\sum_{i=1}^{n} c_i \bar{\bar{x}}_i + \rho \sum_{i=1}^{n} d_i \bar{\bar{x}}_i\right), \max\left(\rho \sum_{i=1}^{n} d_i \bar{\bar{x}}_i - \sum_{i=1}^{n} c_i \bar{\bar{x}}_i\right) \right\} \tag{6.9}$$

where $\bar{\bar{x}}$ is solution of problem:

$$\max_{\underline{x}\in R^{n+}} \left(\sum_{i=1}^{n} c_i x_i + \rho \sum_{i=1}^{n} d_i x_i\right) \tag{6.10}$$

under the constraints:

$$\sum_{i=1}^{n} (a_{ki} - \rho\, \alpha_{ki})\, x_i \geq b_k + \rho\, \beta_k \quad k \in \{1, \cdots, m\} \tag{6.11}$$

$$\sum_{i=1}^{n} c_i x_i \geq 0 \quad \text{and} \quad x_i \geq 0 \quad i \in \{1, \cdots, n\} \tag{6.12}$$

and where $\bar{\bar{x}}$ is solution of problem:

$$\min_{\underline{x}\in R^{n+}} \left(\rho \sum_{i=1}^{n} d_i x_i - \sum_{i=1}^{n} c_i x_i\right) \tag{6.13}$$

under the constraints:

$$\sum_{i=1}^{n} (a_{ki} - \rho\, \alpha_{ki})\, x_i \geq b_k + \rho\, \beta_k \quad k \in \{1, \cdots, m\} \tag{6.14}$$

$$\sum_{i=1}^{n} c_i x_i \leq 0 \quad \text{and} \quad x_i \geq 0 \quad i \in \{1, \cdots, n\} \tag{6.15}$$

The fuzzy dual optimal performance of this program will be given by:

$$\sum_{i=1}^{n} (c_i + \varepsilon\, d_i)\, x_i^* = \sum_{i=1}^{n} c_i x_i^* + \varepsilon \sum_{i=1}^{n} d_i x_i^* \tag{6.16}$$

Considering other linear constraints involving the other partial order relations over $\tilde{\Lambda}$ (weak inequality and fuzzy equality) the solution of the fuzzy dual programming problem will lead to the consideration of at most two classical linear programming problems. The integer version of problem D_0 will lead also to classical integer linear programming problems.

6.2.4 Linear Programming with fuzzy dual variables

Now we consider fuzzy dual programming problems with fuzzy dual variables. In this case, we formulate problem D_1 :

$$\max_{\underline{x}\in R^n, \underline{y}\in R^{n+}} \left\| \sum_{i=1}^{n} (c_i + \varepsilon d_i)(x_i + \varepsilon y_i) \right\| \tag{6.17}$$

under the strong constraints:

$$\sum_{i=1}^{n} (a_{ki} + \varepsilon \alpha_{ki})(x_i + \varepsilon y_i) \breve{\geq} b_k + \varepsilon\beta_k \quad k \in \{1, \cdots, m\} \tag{6.18}$$

and $\quad x_i \in R,\ y_i \geq 0 \quad i \in \{1, \cdots, n\}$ (6.19)

The above problem corresponds to the minimization of the worst estimate of total cost with satisfaction of strong level constraints when there is some uncertainty not only on the values of the parameters but also on the capability to implement exactly the best solution.

Problem D_1 can be rewritten as:

$$\max_{\underline{x}\in R^n, \underline{y}\in R^{n+}} \left\| \sum_{i=1}^{n} (c_i\ x_i + \varepsilon(|x_i| d_i + |c_i| y_i) \right\| \tag{6.20}$$

under constraints 6.18 and 6.19, and:

$$\sum_{i=1}^{n} (a_{ki}\ x_i + \varepsilon(\alpha_{ki}|x_i| + |a_{ki}| y_i)) \breve{\geq} b_k + \varepsilon\beta_k \quad k \in \{1, \cdots, m\} \tag{6.21}$$

which is equivalent in $R^n \times R^{n+}$ to the following mathematical programming problem:

$$\min_{\underline{x} \in R, \underline{y} \in R^{n+}} C(\underline{x}, \underline{y}) = \left| \sum_{i=1}^{n} c_i x_i \right| + \rho \sum_{i=1}^{n} (d_i |x_i| + |c_i| y_i) \tag{6.22}$$

under constraints 6.18 and 6.19, and:

$$\sum_{i=1}^{n} (a_{ki}\, x_i - \rho(\alpha_{ki} |x_i| + |a_{ki}| y_i)) \geq b_k + \rho \beta_k \quad k \in \{1, \cdots, m\} \tag{6.23}$$

Let

$$A(\underline{x}, \underline{y}) = \left\{ \underline{x} \in R^n, \underline{y} \in R^{+n} : \sum_{i=1}^{n} (a_{ki}\, x_i - \rho(\alpha_{ki} |x_i| + |a_{ki}| y_i)) \geq b_k + \rho \beta_k \quad k \in \{1, \cdots, m\} \right\} \tag{6.24}$$

since

$$\forall x \in R^n, \forall y \in R^{n+} \qquad A(\underline{x}, \underline{y}) \subset A(\underline{x}, \underline{0}) \text{ and } C(\underline{x}, \underline{y}) \geq C(\underline{x}, \underline{0}) \tag{6.25}$$

The case of no diversion of the nominal solution is expected to be always preferable. In the case in which the diversion from the nominal solution is fixed to $\bar{y}_i, i \in \{1, \cdots, n\}$, problem *D1* has the same solution than problem $D_1{}'$:

$$\max_{\underline{x} \in R^n} \left| \sum_{i=1}^{n} c_i x_i \right| + \rho \sum_{i=1}^{n} d_i |x_i| \tag{6.26}$$

under constraints 6.18 and 6.19, and

$$\sum_{i=1}^{n} (a_{ki}\, x_i - \rho\, \alpha_{ki} |x_i|) \geq b_k + \rho (\beta_k + \sum_{i=1}^{n} |a_{ki}| \bar{y}_i) \quad k \in \{1, \cdots, m\} \tag{6.27}$$

The fuzzy dual optimal performance of problem (6.22) will be given by:

$$\sum_{i=1}^{n} c_i\, x_i^* + \varepsilon \sum_{i=1}^{n} (|x_i^*| d_i + |c_i| y_i) \tag{6.28}$$

, where $\underline{x}^*$ is the solution of problem $D_0^{'}$.

In the case in which *p* of the *n* decision variables are of undetermined sign, the solution of this problem is obtained by solving 2^{p+1} classical linear programming problems. Here other linear constraints involving the other partial order relations over $\tilde{\Delta}$ (weak inequality and fuzzy equality) could be introduced in the formulation of problem D_1 while the consideration of the integer version of problem D_1 will lead to solve also, families of classical integer linear programming problems.

The performance of the solution of problem D_1 will be potentially diminished by the reduction of the feasible set defined by 6.18, 6.19 and 6.27.

6.3 Dynamic Programming

Since its publication in the late 1950's by R. Bellman [Bellman, 1957], Dynamic Programming has become very quickly a widely applied mathematical formalism in decision making processes.

Dynamic Programming is a mathematical technique for making a sequence of interrelated decisions, providing a systematic procedure for determining the optimal combination of resources [Hillier and Lieberman, 2010].

The objective of dynamic programming is to optimize *sequential decision making* processes, common feature of the operational aspect in a multitude of fields and industries, ranging from economics to engineering. From the mathematical perspective, it can be applied to linear or nonlinear problems involving either real or integer variables. The only applicability condition consists in the separability of objective and constraints functions with respect to the decision variables.

Dynamic Programming can tackle processes either deterministic or stochastic in nature, with a continuous or a discrete stage evolution, with both finite and infinite problem duration.

Currently, the field of application of dynamic programming has become even more diverse, targeting optimization problems that can be reformulated as *multi-stage decision processes.* Some of the main areas of decision making such as Artificial Intelligence, Automatic control and Operations Research, make use of the paradigm of Dynamic Programming.

A *multi-stage decision process* can be briefly introduced as follows: consider a physical system *S*, described by a vector of *states*, *p*, at any time, *t*. In a theoretical scenario, the components of vector *p*, are definite quantities, like capacity or demand, for example, but in a realistic scenario, the components of *p* may have a certain amount of uncertainty. Time passing implies changes to the system, which can be either deterministic or stochastic in nature. To all these dynamic factors, we intervene with a choice of which kind of transformation will be applied to the system. Therefore, we make a decision, a decision being equivalent to a transformation. Complex systems require a sequence of decisions, therefore the term *multi-stage decision process*. Each decision implies that a certain transformation will occur impacting specific parameters, grouping all these decisions or sequence of choices generates what is a called a *policy.*

An *optimal policy* has the property that whatever the initial state and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision [Bellman 1957]. Or adapted for the current state and decision by [Bradley et al., 1977], any *optimal policy* has the property that, whatever the current state and decision, the remaining decisions must constitute an optimal policy with regard to the state resulting from the current decision. This is illustrated in the case of the search for a minimum length path in a directed graph:

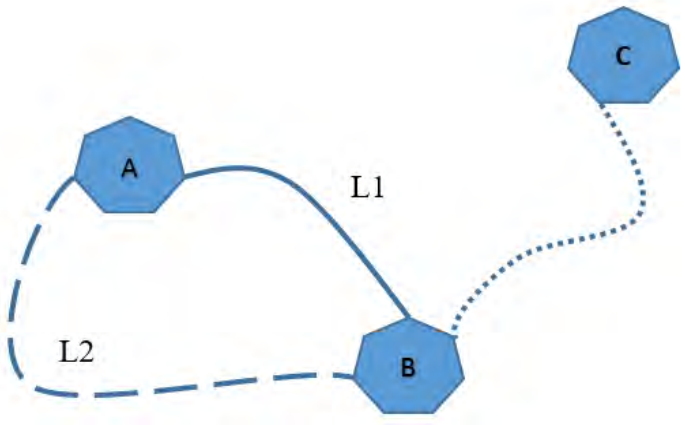

Fig. 6.1 Search for the minimum length path between A and C

Supposing that L1< L2, all paths between A and C going through B along L2 will be worse than all paths between A and C going through B along L1. Then all these paths can be deleted from the search for the minimum length path between A and C. Then

this property allows facing the common explosion of the number of candidate solutions in a combinatorial optimization problem.

Dynamic Programming has the distinctive characteristic of dividing the optimization problem into multiple *stages* with the objective of solving them sequentially, one stage at a time. The corresponding solution of each *stage* helps define the parameters of the next stage's problem. Typically, the stages represent specific time milestones in a problems' planning horizon.

Each stage of the problem to be solved has specific *states*. The states of the process should reveal the information necessary to assess the impact the current decision has upon future actions and ensure seamless decision making regardless of how the process reached the current state. Therefore, defining the states of the system is a critical aspect in designing the dynamic programming model. In addition, another critical aspect that must be noted, is the number of state variables, which should be small due to significant computational effort that considerably limits the applicability of dynamic programming in practice.

The core of the dynamic programming approach is the optimization procedure itself, which reaches a solution of the overall *N*-stage problem by repeatedly solving one-stage problems until the overall optimum is found. This approach is based either on *backward induction*, where the first stage to be analysed is the final stage of the problem and by backtracking one stage at a time until all stages are included, or *forward induction*, where the initial stage needs to be solved and then moving forward one stage at a time until all stages are included. No matter the optimization procedure applied, the principle of optimality stated above is governing the entire process.

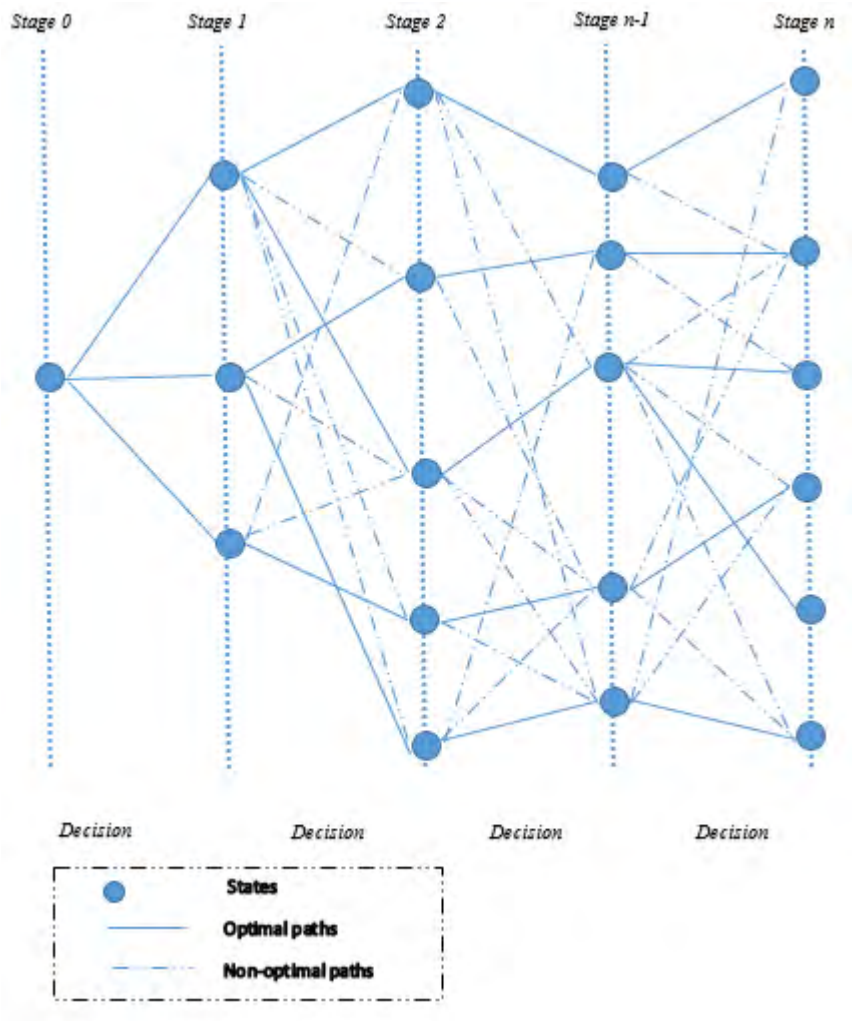

Fig. 6.2 Dynamic programming decision process

6.3.1 Formalization of the dynamic programming process

Considering a multi-stage decision process where the global return for a particular stage is given by:

$$\sum_{n=1}^{N-1} f_n(s_n, x_n) \tag{6.29}$$

, x_n is a decision from the set of feasible decisions X_n and s_n is the state of the process with *n* stages to go for a total of *N* stages.

In the case of a deterministic process, the next state is completely determined by the current state and the decision taken at that time. Therefore, the *transition function* or

functional equation can be defined such that, given s_n, the state of the process with *n* stages to go, the following state of the process with *N-(n+1)* stages is:

$$s_{n+1} = t_n(x_n, s_n) \tag{6.30}$$

The expected total return from the present stage until the end of the planning horizon is given by the *value function*.

Given the current state s_n, the objective is to maximize the total return moving forward with the remaining stages. The decision x_n, chosen from the set of feasible decisions X_n, yields a return at this stage, $f_n(s_n, x_n)$, resulting in a new state s_{n+1} with *N-(n+1)* stages to go, as can be deduced from Fig. 6.3. Stage returns are independent of one another.

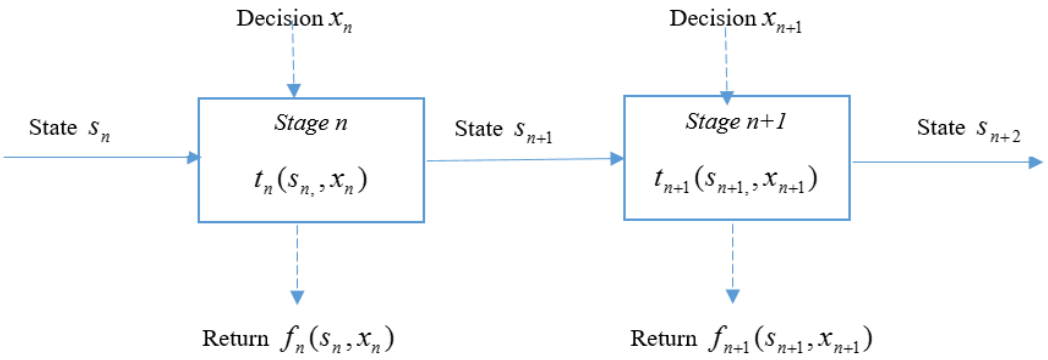

Fig. 6.3 Forward induction multistage decision process

6.4 Fuzzy dynamic programming

Fuzzy set theory developed by [Zadeh, 1965] established itself as the mathematical tool to address uncertainties and imprecision in tackling real world problems and dynamic programming was one of the earliest fundamental methodologies to which fuzzy sets was applied [Bellman and Zadeh, 1970], leading to what is presently called *fuzzy dynamic programming*.

Fuzzy dynamic programming has been applied successfully to multi stage decision making problems in a multitude of areas, with real world applications like civil and

environmental engineering (integrated regional development, water resources operation and design, pollution control modeling), transportation (traffic planning and routing), energetic systems, health care, control systems, aerospace systems, etc.

A significant body of work emerged since dynamic programming started being applied in conjunction with fuzzy representation. Currently we can find in literature a large spectrum of dynamic programming models in which various elements have been fuzzified, notably the goals and constraints, but also the states and policy, state transitions, planning horizon, etc. [Kacprzyk and Esogbue, 1996].

Addressing general problems, the pioneer work of [Esogbue and Ramesh, 1970] needs mentioning, who tackled general resource allocation with fuzzy goals and constraints and described the first computational algorithm for fuzzy dynamic programs.

On more specific problems, [Esogbue, 1983] work on general control problems with limited resource allocation with the objective to attain specific goals distributed over time, paved the way for a much wider class of applications. Also, in a series of papers, [Kacprzyk and Straszack, 1981, 1982a, 1982b, 1984] proposed a fuzzy dynamic programming model for determining socio-economic regional development strategies taking into account limited resources, efficiency and sustainability.

Several recent developments in the field are mentioned below.

[Faye et al., 2002] applied dynamic programming to address long-term management issues of water resource systems where the weighting parameters of the optimization criterion where computed using fuzzy representation of the different goals.

[Abo-Sinna, 2004] reviewed the major concepts used in multi-objective dynamic programming and fuzzy multi-objective dynamic programming, examining the progress made in theory and methodology.

[Schweickardt and Miranda, 2007] put forward a new fuzzy dynamic programming model to calculate solution of problems with uncertainties in data represented by fuzzy sets, with the objective to help a regulatory authority in fixing levels of efficiency, targets and penalties to a regulated market by computing the distribution system expansion costs.

[Parida, 2013] developed a fuzzy dynamic system approach to solve multi-stage decision making problems, analysing the deterministic, stochastic, fuzzy planning horizon and fuzzy criterion sets cases. He emphasized the importance of developing efficient fuzzy dynamic programming algorithms by analysing the computational complexity of [Esogbue, 1999], [Kacprzyk, 1977] and [Stein, 1980] approaches, concluding by showing the superiority of Stein's model from both space and time considerations.

6.4.1 Formalization of the fuzzy dynamic programming model – Bellman and Zadeh's approach

In this section the basic elements of [Bellman and Zadeh, 1970] general approach for fuzzy dynamic programming are briefly presented.

Let X be a space of options, then, given a fuzzy goal G in X characterized by the fuzzy membership function $\mu_G(x)$ and a fuzzy constraint C in X characterized by the fuzzy membership function $\mu_C(x)$, a fuzzy decision D in X which satisfies C while achieving G will have a fuzzy membership function $\mu_D(x)$ defined by:

$$\mu_D(x) = d(\mu_C(x), \mu_G(x)) \in [0,1], \forall x \in X \tag{6.31}$$

, which provides for each $x \in X$ a measure of performance from 1, for an excellent feasible decision to 0, for a very bad or unfeasible decisions, with intermediate values.

In [Parida, 2013] function d is the fuzzy operator *and*, which can be taken such as:

$$\mu_C(x) \wedge \mu_G(x) = \min\left(\mu_C(x), \mu_G(x)\right) \quad \forall x \in X \tag{6.32}$$

In that case, the optimal decision with respect to $x \in X$ will be such that:

$$\mu_D(x^*) = \sup_{x \in X}(\mu_C(x) \wedge \mu_G(x)) \tag{6.33}$$

Another common realization of the fuzzy *and* is:

$$\mu_C(x) \wedge \mu_G(x) = \mu_C(x) \cdot \mu_G(x) \quad \forall x \in X \tag{6.34}$$

It appears that in both cases, the constraint and the goal are treated in the same level since:

$$\mu_C(x) \wedge \mu_G(x) = \mu_G(x) \wedge \mu_C(x) \text{ and } \mu_C(x) \cdot \mu_G(x) = \mu_G(x) \cdot \mu_C(x) \quad \forall x \in X \tag{6.35}$$

However, for many applications, feasibility is a condition to be considered prior to any assessment of the degree of achievement of the goal.

What is to be expected is that $d(\mu_C, \mu_G)$ to fulfil the following conditions:

- $d(0, \mu_G) = 0$ and $d(\mu_C, 0) = 0$;
- *d* is increasing with respect to both arguments;
- *d* is not a symmetric function with respect to its arguments.

Examples of candidate *d* functions are:

- $d(\mu_C, \mu_G) = {\mu_C}^{\alpha} \cdot \mu_G$ with $\alpha > 1$ (6.36)
- $d(\mu_C, \mu_G) = \dfrac{e^{\mu_C} - 1}{e - 1} \cdot \mu_G$ (6.37)
- $$d(\mu_C, \mu_G) = \begin{cases} 0 \ \ if \ \mu_C < s_{\min} \\ \mu_C \cdot \mu_G \quad if \ s_{\min} \le \mu_C \le \cdot s_{\max} \\ \mu_G \ if \ \mu_C > s_{\max} \end{cases} \tag{6.38}$$

  with $0 \le s_{\min} < s_{\max} \le 1.$

Here it will be considered that an analogous reasoning stands for the case of multiple fuzzy constraints and fuzzy goals, even if is defined in different spaces.

Suppose that the fuzzy constraint *C* is defined on a fuzzy set in $X=\{x\}$, the fuzzy goal *G* is defined on a fuzzy set $Y=\{y\}$, and a function $f : X \to Y, y = f(x)$ is known. Typically, *X* and *Y* are decisions sets and their outcomes, respectively. Now the induced fuzzy goal *G'* in *X* generated by *G* in *Y* is given by:

$$\mu_{G'}(x) = \mu_G(f(x)), \text{ for each } x \in X \tag{6.39}$$

, with both *G'* and *C* being defined as fuzzy sets in the same space *X*.

The *min-type fuzzy decision* is

$$\mu_D(x) = \mu_C(x) \hat{\wedge} \mu_{G'}(x) = \mu_C(x) \hat{\wedge} \mu_G(f(x)), \text{ for each } x \in X \qquad (6.40)$$

Then, for n fuzzy constraints defined in X, C_1, …, C_n, m fuzzy goals defined in Y, G_1 , …, G_m , and a function $y = f(x)$, then the *min-type fuzzy decision* is given by:

$$\mu_D(x) = (\mu_{C_1}(x) \wedge \cdots \wedge \mu_{C_m}(x)) \hat{\wedge} (\mu_{G_1}(f(x)) \wedge \ldots \wedge \mu_{G_n}(f(x))) \quad \forall x \in X \qquad (6.41)$$

6.4.2 Fuzzy Multistage Decision Making – Kacprzyk approach

A general framework for multi-stage decision making under fuzziness as described by [Bellman and Zadeh, 1970] and [Kacprzyk, 1983] can be introduced at this stage.

First, the state transition equation describing the dynamics of the deterministic dynamic system is given by:

$$s_{t+1}=f(s_t, x_t),\ t=0,1,\ldots \qquad (6.42)$$

, where $s_t, s_{t+1} \in S = \{s_1,..., s_n\}$ are the states at stage (time) t and $t+1$, respectively and $x \in X = \{x_1,...x_m\}$ is the decision at stage t, with X and S are assumed to be finite.

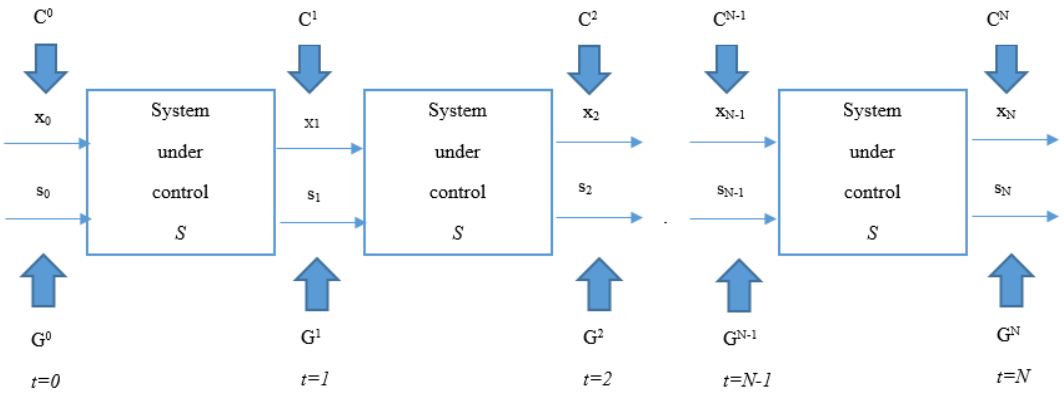

Fig. 6.4 General framework for multistage decision making under fuzziness [Kacprzyk, 1997]

Fig. 6.4 illustrates the concept of multi stage decision making under fuzziness. As seen, the starting point is the state s_0 at stage $t=0$, where a decision is made, x_0, generating the next state at $t=1$, finally, reaching the stage $t=N-1$ in the state s_{N-1}, decision x_{N-1} is made to reach the last state, s_N.

The state transitions are given by (6.42) while the consecutive decisions u_t are subjected to fuzzy constraints C^t and fuzzy goals G^{t+1} are imposed on the states x_{t+1}, with $t=0,1,...,N-1$.

The performance of the multi stage decision making process is evaluated by the fuzzy decision given by:

$$\begin{aligned}\mu_D(x_0,...,x_{N-1}|s_0) &= (\mu_C^0(x_0)\hat{\wedge}\mu_{G^1}(s_1))\wedge\cdots\wedge(\mu_{C^{N-1}}(x_{N-1})\hat{\wedge}\mu_{G^N}(s_N)) \\ &= \underset{t=0}{\overset{N-1}{\Lambda}}[\mu_{C^t}(x_t)\hat{\wedge}\mu_{G^{t+1}}(s_{t+1})]\end{aligned} \tag{6.43}$$

, where N is a specified planning horizon.

The problem is to find the optimal sequence of decisions $x_0^*,...,x_{N-1}^*$ such that

$$\begin{aligned}\mu_D(x_0^*,...,x_{N-1}^*|s_0) &= \max_{u_0,...u_{N-1}}(\mu_C^0(x_0)\hat{\wedge}\mu_{G^1}(s_1))\wedge\cdots\wedge(\mu_{C^{N-1}}(x_{N-1})\hat{\wedge}\mu_{G^N}(s_N)) \\ &= \max_{u_0,...u_{N-1}}\underset{t=0}{\overset{N-1}{\Lambda}}[\mu_{C^t}(x_t)\hat{\wedge}\mu_{G^{t+1}}(s_{t+1})]\end{aligned} \tag{6.44}$$

This general problem formulation can be extended such that the planning horizon, alongside the fixed and specified value can also be fuzzy, implicitly given by entering a termination set of sets or even infinite. In addition, the type of dynamic system can be deterministic, stochastic, fuzzy or fuzzy-stochastic and finally, the type of objective function can refer to cost minimization, profit maximization or a fuzzy criterion set based satisfactory degree of maximization [Kacprzyk, 1997, 1983b].

The optimal solution can be constructed recursively by considering the equations:

$$\begin{cases}\mu_{G^{N-i}}(x_{N-i}) = \max_{u_{N-i}}[\mu_{C^{N-i}}(x_{N-i})\wedge\mu_{G^{N-i+1}}(s_{N-i+1})] \\ s_{N-i+1} = f(s_{N-i},x_{N-i});i=1,...,N\end{cases} \tag{6.45}$$

, where $\mu_{G^{N-i}}$ is a fuzzy goal generated at $t=N-i$ by a fuzzy goal at $t=N-i+1$.

An optimal sequence of decisions sought, $u_0^*,...,u_{N-1}^*$, is given by the successive maximization of u_{N-i} values in (6.45).

6.4.3 Limitations of the fuzzy dynamic programming approach

Looking beyond the extended benefits and applicability of fuzzy dynamic programming, there are however several limitations to consider.

First, the effective analytic solution of a large number of relative simple equations is a difficult endeavour, even if we are considering a computational solution. Therefore, in the case of a large number of variables, reaching an optimum solution is not an easy task. This may be summed up as the dreaded *curse of dimensionality,* which is an inherent characteristic of dynamic programming. Improved computational procedures need to be developed in order to alleviate the dimensionality problem, which is even more cumbersome in the case of multi-stage and multi-objective dynamic programming.

Second, whatever the difficulties arisen in the deterministic case assumed above, they are compounded in the stochastic case, where the outcome of a decision is a random variable.

By adding the fuzzy dimension, the scope is to replicate as close as possible the real world. Including too many features of reality into a mathematical model will add extreme complexity, with numerous unknown parameters and functions. On the other hand, constructing a too simple model will fail to provide accuracy and robustness.

Computational complexity of fuzzy dynamic programs is an important issue in the field of dynamic programming but not often addressed. [Esogbue, 1999] made a computational complexity analysis using the algorithms developed by [Kacprzyk, 1977] and [Stein, 1980] addressing the case of a fuzzy planning horizon.

In the case of [Kacprzyk, 1977] algorithm, for the storage complexity *S,* each variable takes a unit of storage space. This allows determining the demand for the storage tables during computations, ignoring all input tables and intermediate variables created during processing. Computational complexity *T* is a function of the basic operations: comparison, mod operation, assignment, arithmetic operations, in order to generate the performance profile of the algorithm. Is assumed that the total number of all the above-mentioned operations is roughly proportional to the number of comparisons. Therefore, the number of comparisons approximates the computational

complexity of an algorithm. [Esogbue, 1999] proved that the dynamic programming approach presented by [Kacpryzk, 1977] requires *N(N+1)/2* iterations while the one proposed by [Stein, 1980] requires only *N* and so computationally more efficient.

For exemplification, the following model is considered: *n* equations, u_{N-1} decisions each assuming *m* values, and time *t=N-1,…,K*. The total number of operations involved is

$$n(2mt - 2mK - 2m - t + K) + n(N - K + 1) + (K - 1)n(2m - 1) \quad (6.46)$$

In case of m=K=t, the order is $O(2K^3)$.

The formulation proposed by [Stein, 1980] is the same except for the structure of recurrence equations that require *N* only iterations of the optimizing process as opposed to *N*(*N+1)/2* required in Kacpryzk's algorithm. In Stein's case, the time and space complexities are of order *O*(*n*) and *O*(*mn*), respectively, with the time complexity given by *O*((*2m-1*)(*N-K*)), having the order is O(K^2). The total memory demand is *n*(*N-K+1*)(*N-K+2)/2+2n+n*(*K-1*), having the order *O*(*K*).

As seen, the later model proposed by [Stein, 1980] is computationally superior from both time and space considerations, taking up O(*K*) memory spaces in O(K^2) operations, as opposed to O(K^2) memory spaces in O($2K^3$) operations in [Kacprzyk, 1977] model.

The conclusion is that the difference between the earliest and the latest possible termination times affects considerably the computational burden of the dynamic programming process.

6.5 Fuzzy Dual Dynamic Programming

In this case, we consider the following fuzzy dual formulation of an optimization problem:

$$\max \sum_{n=1}^{N-1} (c_n(s_n, x_n) + \varepsilon \cdot d_n(s_n, x_n)) \quad (6.47)$$

, with $s_{n+1} = \Gamma(s_n, x_n) \in S$ and $x_n \in X_{S_n}, s_1$ given. (6.48)

Here Γ represents the transition of the process from state s_n when decision x_n is taken to the resulting state s_{n+1}. X_{S_n} is the set of feasible decisions according to current state s_n of the process.

In addition, a transition graph $G = [S, X]$ is built from the initial state s_1 by considering all feasible decisions from each state of each stage to the next stage:

$$S = \bigcup_{n=0}^{N-1} \Gamma^n(s_1) \text{ and } X = \bigcup_{n=0}^{N-1} \left(\Gamma^n(s_1), X_{S_n}\right) \tag{6.49}$$

The optimality principle of dynamic programming can be put into action here to generate from stage to stage an optimal solution tree since fuzzy dual performances can always be compared according to (5.49), (5.50), (5.51) and (5.52) respectively.

Therefore, the fuzzy dual comparison proposed in Chapter V is used. When the performance of a path to a state is considered superior to any other path to this state with a degree of certainty c higher than 0.6, this path with the corresponding decision to reach it from the previous stage is retained. While, when $0.4 \leq c \leq 0.6$, the two fuzzy dual performances are considered very close and any of them can be taken as superior.

Then, supposing that Γ_{nj} is the set of states of stage *n*-1 from which it is possible to reach state *j* of stage *n*, the retained decision from stage *n*-1 to state *j* of stage *n* will be associated to a state of stage *n*-1 such as:

$$k_n^* = \arg\max_{k \in \Gamma_{n-1\,j}} \left\{ G_{n-1}^k + g_n(k,(k,j)) \right\} \tag{6.50}$$

, where

$$G_{n-1}^k = \sum_{m=1}^{n-1} g_m(k_{m-1}^*, (k_{m-1}^*, k_m^*)) \tag{6.51}$$

and where a resulting degree of certainty is given by:

$$c_n^j = \min_{k \in \Gamma_{n-1,\,j}, k \neq k_n^*} c_{n,k,j} \tag{6.52}$$

, where $c_{n,k,j}$ is attached to the degree of certainty of the fuzzy dual comparison of $G_{n-1}^k + g_n(k,(k,j))$ with $G_{n-1}^{k_n^*} + g_n(k_n^*,(k_n^*,j))$.

Then to each state *j* of each stage *n* is attached:

- a fuzzy dual performance given by $G_{n-1}^{k_n^*} + g_n(k_n^*,(k_n^*, j))$, representing the deterministic aspects (the real part of the performance index) as well as the degree of uncertainty (the dual part of the performance index),
- and a degree of certainty c_n^j of having chosen the best solution to reach state *j* at stage *n*.

The optimal sequence of decisions will follow from one stage to the next, the path from the initial state at the initial stage to a best performance state at the final stage as seen in Fig. 6.5.

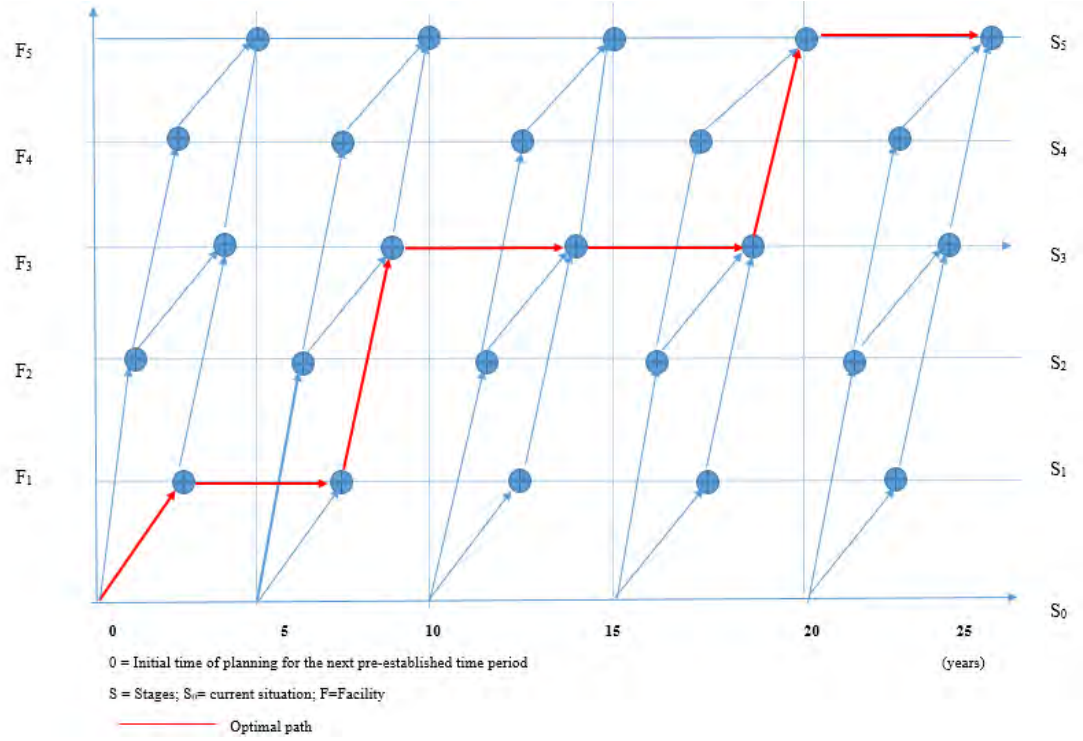

Fig. 6.5 Optimal sequence of decisions

6.6 Conclusions

This chapter formalized innovative concepts like linear programming with fuzzy dual parameters and fuzzy dual variables and placed them in the broader context of fuzzy mathematical programming.

Fuzzy dual dynamic programming was introduced as a new technique for addressing multi stage optimization problems, capable of offering the best trade-off between accuracy in uncertainty representation and computational complexity. The dynamic programming aspect employed during the solution process ensures optimal decision making stage by stage, while the fuzzy dual aspect addresses the complexities of uncertainty in both variables and parameters values.

In addition, this approach can constitute the basis for addressing large class of stochastic optimization problems and give a new direction of research in the field of fuzzy mathematical programming.

CHAPTER VII

SOLUTION APPROACH AND CASE STUDY

7.1 Introduction

If Chapter V introduced the concept of fuzzy dual numbers and fuzzy dual logic, and in Chapter VI a model for risk assessment with the use of dynamic programming was constructed, Chapter VII illustrates the ability of this tool to assess financial risk associated with airport long-term planning.

The starting point of any airport planning project and its financing are its current state and the potential demand evolution forecast. The forecast generally covers the time horizon of the project and over. It includes potential demands for the annual volumes of international and domestic scheduled and non-scheduled passengers, freight and aircraft movements. In addition, daily and monthly traffic distributions are required in order to identify traffic trends and peaking patterns along with the fleet mix. Of paramount importance is the integration of uncertainty in demand forecasting since the decisions taken at a specific step of the development plan can have a long-term impact over the general outcome of the project.

The considered case consists in constructing a Master Plan who will incorporate the main elements encountered in airport projects, focusing on infrastructure needs. It sets the problem of the timing of the construction of facilities in order to meet future traffic demand, covering a 25-year time span. The Master Plan is built on a flexible framework by no committing in advance to any particular project, but following a comprehensive decision making process that will avoid situations in which short-term initiatives could preclude long-term opportunities.

Major associated risks that need to be assessed and mitigated during the implementation of the master plan include:

- deficit in airport capacity leading to unsustainable levels of traffic and airport economic performance over long-term,
- generation of unacceptable environmental impacts,
- failing to achieve transport integration with the surrounding multimodal ground transportation system,
- lack of quantifiable economic benefits for the region the airport serves.

In order to sustain all the forecasted traffic, targeted investment should focus on:

- construction of the second runway and, eventually, construction of a third one,
- increasing airfield capacity,
- increasing passenger terminal capacity and construction of a second one,
- construction of dedicated cargo terminal,
- add necessary airside facilities for ground handling operations support,
- add necessary landside facilities for airport related activities support,
- improve surface access to the airport by all modes of transportation.

The major constraint the airport development project is facing is the fact that the airport operational area is restricted by the land the airport owns. For the initial stages of the development project, additional land has already been acquired to facilitate infrastructure expansion. Further land will be acquired to allow or safeguard the potential airport expansion as long as it remains a commercially viable option. A factor to be noted is the location of the airport in an urban area, which imposes aerodrome and navigational constraints beyond the boundary of the airport operational area. Also, the operational area is currently constrained by the adjoined land use, including rail network and highway. Completing the 25-year Master Plan based on the potential traffic will definitely require acquisition of land to the south and safeguarding also land to the east as a way of not risking future airport and airport-related development projects.

As seen, the traffic mix is generating specific costs and revenues, with primary focus on passengers and freight flows as well as aircraft traffic that is related with the level of these flows.

7.2 Generic problem formulation

The generic problem formulation is built in two steps: first a deterministic formulation is developed and then uncertainty levels are introduced according to fuzzy dual formalism.

7.2.1 Deterministic problem formulation

Let the level of predicted potential demand for traffic type *i* along the planning horizon K be given by $D_k^i, i \in I, k \in \{1,2,\cdots,K\}$, where I is the set of traffic activities. The necessary aircraft traffic T_k^i to cope with a predicted passenger demand level D_k^i, can be approximated by:

$$T_k^i = D_k^i / (S_k^i \alpha_k^i) \tag{7.1}$$

where S_k^i is the mean capacity of aircraft type *i* at time *k* corrected by the expected mean load factor α_k^i. The rate of return r_k^i, associated with the traffic of type *i* at time *k*, depends on the investments made until that period. Let the potential airport passenger processing capacity be C_k^{Pi} and the potential aircraft movements processing capacity be C_k^{Ti}, then the estimated level of demand of type *i* at period *k*, $\overline{D}_k^i$, is such as:

$$\overline{D}_k^i = \min\{D_k^i, C_k^{Pi}, S_k^i C_k^{T_i}\} \tag{7.2}$$

Let L_i be the number of candidate upgrades that can be performed for traffic type *i* at the considered airport.

Let θ_l^i be the period (an integer) at which upgrade *l* for traffic type *i* is scheduled. When a project is retained, the corresponding value of θ_l^i is within the set $\{1,2,...,K\}$ and when it is not retained $\theta_l^i = K+1$, $l \in \{1,2,...,L_i\}$.

Different types of constraints may be found between interrelated projects:

- Sequential constraints: technical considerations generally impose sequential constraints, so it is supposed that for given a type of traffic *i* and a pair of projects (*l* , *l'*), there may be constraints such as:

$$\exists l, l' \in \{1, \cdots, L_i - 1\}, i \in I : \theta_l^i \le \theta_{l'}^i \quad (7.3.a)$$

- Exclusion constraints: if project *l* for traffic type *i* is retained, a set of concurrent or contradictory projects will be dismissed:

$$\theta_l^i \in \{1,2,..., K\} \Rightarrow \theta_{l'}^i = K+1,\ l' \in \Lambda_l^i \subset \{1, \cdots, L_i\} \quad (7.3.b)$$

- Inclusion constraints: if project *l* for traffic type *i* is retained, a set of complementary projects related with other traffic should be performed altogether:

$$\theta_l^i \in \{1,2,..., K\} \Rightarrow \theta_{l'}^j = \theta_l^i\ ,\ l' \in M_l^i \subset \{1, \cdots, L_j\} \quad (7.3.c)$$

Since the different types of traffic may use common resources in the airport, global capacity constraints must be satisfied.

Let Δ_k be the set of projects which have been retained until period *k*, then the corresponding capacities with respect to passengers and flights are $C_k^{Pi}(\Delta_k)$ and $C_k^{T_i}(\Delta_k)$.

Let $c_l^{ik}(\Delta_k)$ be the cost of upgrade *l* with respect to traffic type *i* when performed at period *k*.

Revenues R_k^i from traffic type *i* at period *k* are given by:

$$R_k^i = r_k^i \cdot \overline{D}_k^i(\Delta_k) \quad (7.4)$$

, where r_k^i is the corresponding service rates.

The adopted strategy develops at first a deterministic approach, which leads to the formulation of an optimization problem. Then, the parameters and variables subject

to significant uncertainty are pointed out and a fuzzy dual based model of their uncertainty is established. Finally, a fuzzy dual formulation of the airport planning problem is proposed.

The deterministic formulation of the optimal programming problem associated to airport planning can be such as:

$$\max_{\theta_l^i} \quad \pi([\theta_l^i], l \in \{1, \cdots, L_i\}, i \in I) \tag{7.5}$$

, under constraints (7.3.a), (7.3.b) and (7.3.c).

Here the expected net present value of whole project is given by:

$$\pi([\theta_l^i], l \in \{1, \cdots, L_i\}, i \in I = \sum_{i \in I} \left(\sum_{k=1}^{K} \left(\frac{r_k^i}{(1+\rho)^k} \overline{D}_k^i(\Delta_k) \right) - \sum_{\substack{l=1 \\ \theta_l^i \le K}}^{L_i} \left(\frac{c_l^{ik}(\Delta_k)}{(1+\rho)^{\theta_\rho^i}} \right) \right) + \frac{R_V(\Delta_K)}{(1+\rho)^K} \tag{7.6}$$

, where ρ is the rate of actualization and $\frac{R_V(\Delta_K)}{(1+\rho)^K}$ is the residual value of airport equipment.

Observe that, according to expression (7.2), the estimation of demand levels at period *k* will depend of previous planning decisions.

7.2.2 Fuzzy dual representation of uncertainty for airport planning

Let the fuzzy dual representations of the effective levels of demand, the rates of net return and the upgrade costs be given by:

$$r_k = r_k^L + \varepsilon\, r_k^D \tag{7.7}$$

$$\overline{D}_k^i(\Delta_k) = \overline{D}_k^{iL}(\Delta_k) + \varepsilon\, \overline{D}_k^{iD}(\Delta_k) \tag{7.8}$$

$$c_l^{ik}(\Delta_k) = c_l^{ikL}(\Delta_k) + \varepsilon\, c_l^{ikD}(\Delta_k) \tag{7.9}$$

where the likely components are indexed by *L* and the dual components are indexed by *D*.

In many situations, the likely components can be associated with mean estimated values while the dual components can be associated with their corresponding standard deviations.

The expression of the fuzzy dual net present value is given by:

$$\pi([\theta_l^i], l \in \{1,\cdots,L_i\}, i \in I) = \pi^L([\theta_l^i], l \in \{1,\cdots,L_i\}, i \in I) + \varepsilon\, \pi^D([\theta_l^i], l \in \{1,\cdots,L_i\}, i \in I) \quad (7.10)$$

where:

$$\pi^L([\theta_l^i], l \in \{1,\cdots,L_i\}, i \in I) = \sum_{i \in I} \left(\sum_{k=1}^{K} \left(\frac{r_k^L}{(1+\rho)^k} \overline{D}_k^{iL}(\Delta_k) \right) - \sum_{\substack{l=1 \\ \theta_l^i \le K}}^{L_i} \left(\frac{c_l^{ikL}(\Delta_k)}{(1+\rho)^{\theta_\rho^i}} \right) \right) + \frac{R_V^L(\Delta_K)}{(1+\rho)^K} \quad (7.11)$$

and

$$\pi^D([\theta_l^i], l \in \{1,\cdots,L_i\}, i \in I) = \sum_{i \in I} \left(\sum_{k=1}^{K} \left(\frac{r_k^L \cdot \overline{D}_k^{iD}(\Delta_k) + r_k^D \cdot \overline{D}_k^{iL}(\Delta_k)}{(1+\rho)^k} \right) - \sum_{\substack{l=1 \\ \theta_l^i \le K}}^{L_i} \left(\frac{c_l^{ikD}(\Delta_k)}{(1+\rho)^{\theta_\rho^i}} \right) \right) + \frac{R_V^D(\Delta_K)}{(1+\rho)^K} \quad (7.12)$$

, where $\frac{R_V^L(\Delta_K)}{(1+\rho)^K} + \varepsilon \frac{R_V^D(\Delta_K)}{(1+\rho)^K}$ is the current fuzzy dual residual value of airport equipment.

7.3 Airport Planning with Fuzzy Dual Framework

In the case in which only sequencing decisions are taken into account for the set of possible projects, the problem reduces to a time scheduling problem.

Then, once a development scenario has been chosen by setting the decision variables $[\theta_l^i], l \in \{1,\cdots,L_i\}, i \in I$, the likely net present value as well as its attached uncertainty can be computed according to a step by step process as detailed in Fig. 7.1, where current capacity and current and future demand for each type of airport traffic are estimated. Then sensitivity analysis can be performed with respect to the timing of different projects.

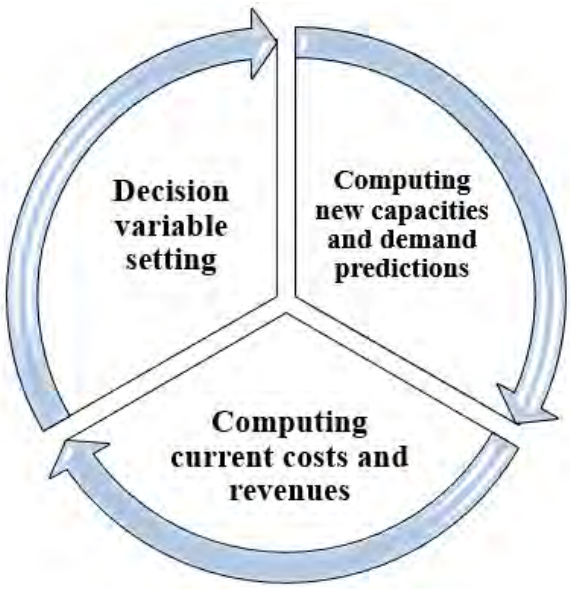

Fig. 7.1 The airport planning loop

Now, the programming problem associated to airport planning which takes into account the level of uncertainty can be performed as a multi criteria problem by considering on one side the maximization of the likely net present value and on the other side the minimization of uncertainty on this value. However, introducing a maximum uncertainty level, it can be formulated as:

$$\max_{\theta_l^i} \quad \pi^L([\theta_l^i], l \in \{1, \cdots, L_i\}, i \in I) \tag{7.13}$$

under constraints (7.3) and a global uncertainty level constraint such as :

$$\pi^D([\theta_l^i], l \in \{1, \cdots, L_i\}, i \in I) \leq \Delta\pi_{\max} \tag{7.14}$$

where $\Delta\pi_{\max}$ represent the maximum allowed level of uncertainty.

While solving one of the above problems, the global airport investment plan is considered safe in absolute terms when:

$$\pi^{L^*}([\theta_l^i], l \in \{1, \cdots, L_i\}, i \in I) > \pi^{D^*}([\theta_l^i], l \in \{1, \cdots, L_i\}, i \in I) \tag{7.15}$$

A risk degree between 0 and 100% is attached to any solution, either optimal or approximate, for obtaining a present net value equal to π^{L^*}:

$$risk = \begin{cases} 0 \quad if \quad \pi^{L^*} - \pi^{D^*} > 0 \\ 100 \cdot \left(\dfrac{\pi^{D^*} - \pi^{L^*}}{2\pi^{L^*}} \right) \quad if \quad \pi^{L^*} - \pi^{D^*} < 0 \le \pi^{L^*} \le \pi^{L^*} + \pi^{D^*} \\ 100 \cdot \left(\dfrac{\pi^{L^*} + \pi^{D^*}}{2\pi^{D^*}} \right) \quad if \quad \pi^{L^*} < 0 \le \pi^{L^*} + \pi^{D^*} \\ 100 \quad if \quad \pi^{L^*} + \pi^{D^*} < 0 \end{cases} \tag{7.16}$$

In addition, it can be interesting to consider the risk level at different stages of the planning process.

7.4 The airport planning scenario

In this section, the overall assumptions allowing to characterize the airport planning case study are established.

The region the airport is serving is expected to become increasingly important at regional and national level with a catchment area of 8 million people living within one-hour travel time of the airport, and 40 million living within two-hours travel time. Currently, less than 40% of the region's demand for air travel is served by the local airport. A significant air travel demand is therefore underserved in the region, contributing to an overgrowing number of unnecessary surface trips and congestion. An overall unsustainable situation is expected within a decade. In this context, guaranteed access to markets are more and more relevant for economic development both from a business and commercial perspective but also for boosting tourism and creating a more efficient transportation system.

The airport is strategically located, which generates the potential of becoming the principal international gateway for the region it is serving. The need for access to sustainable air travel is expected to continue its positive trend, the airport becoming a basic driver for economic growth in the region. The airport is already providing access to air travel in an integrated way, acting as a regional transport hub with interchange facilities across all modes.

The airport has a mixed ownership with the majority share belonging to private investors.

A Master Plan covering a 25 year time-span details future airside and landside infrastructure requirements and flexible and sustainable expansion strategies necessary to implement in order to accommodate the forecasted traffic growth while mitigating potential risks that may jeopardize irreversibly the chances of success of the entire development project. The main objective of the airport is to claw back traffic, which currently travels to other regions for access to air travel with the benefit of decongesting the over capacitated airports and creating the premises for a sustainable regional economic development and increased environment awareness and mitigation.

Current passenger throughput is 9 million, expected to reach the 35 million passengers level in 25 years, as presented in Table 7.1. This will translate in a 20% increase in the airports capability to satisfy air traffic demand for the region, up to 60%. This will suggest the addition of a new runway at the 10-year mark and the possibility of adding a new terminal building to the current airport configuration. The traffic forecast provides estimates every five years. This forecast is one of the key indicators that will deem which phase of the master plan is the best trade-off between commercial viability and associated risks.

The airport has experienced strong growth of passenger traffic, over the last two decades averaging at 8% per year, with the national market share increasing from 3% to 4%.

Currently, the air traffic breakdown by market sector at this airport is:

- ✈ Low cost - 45%
- ✈ Short haul – 35%
- ✈ Long haul – 10%
- ✈ Charter – 10%.

Long-haul is expected to be the most potent sector of growth. This sector is currently limited by the lack of proper airside infrastructure, the existing length of the runway is precluding operation of commercial flights both east and west and severely limits access to emerging markets. Short haul traffic historically has been the fastest growing

market sector for the airport and going forward the assumption that the sector will continue its steady growth is considered. A similar trend can be identified for the low cost sector who is looking to further expand its network. The only sector who is predicted to contract will be the charter flights due to continuous consolidation and expansion of low-cost carriers.

Overall, the focus and opportunities for growth are identified solely in the international sector, while domestic traffic is forecasted to have the slowest growth, reaching complete maturity.

The forecasted growth of long-haul flights will also trigger an increase of future freight activity. This is also supported by the progressive addition of new routes, giving the airport access to new markets and positioning it as a regional cargo hub.

Current air transport movements (ATM) are 100 000 per year. This translates in 90 passengers per ATM in average with a predicted average in 25 years of 160.

Table 7.1 Forecast of nominal passenger, ATM and freight activity levels

	Pax	*ATM*	*Freight*
Current	9 million	100,000	15,000 t
5 year mark	12 million	130,000	30,000 t
10 year mark	15 million	160,000	55,000 t
15 year mark	20 million	180,000	80,000 t
20 year mark	25 million	200,000	100,000 t
25 year mark	35 million	220,000	125,000 t

Table 7.2 Forecast of uncertainty for passenger, ATM and freight activity levels

	$\delta Pax/Pax$	$\delta ATM/ATM$	$\delta Freight/Freight$
Current	0%	0%	0%
5 year mark	10%	9%	6%
10 year mark	15%	12%	10%
15 year mark	20%	18%	15%
20 year mark	25%	20%	16%
25 year mark	30%	28%	20%

A fuzzy dual demand level will be associated with the uncertainty levels given in Table 7.2. For instance in the case of passenger demand we have:

$$\overline{D}_k^{Pax} = \overline{D}_k^{Pax} + \varepsilon \left(\delta Pax / Pax\right) \cdot \overline{D}_k^{Pax} \tag{7.17}$$

Here ρ is taken equal to 0.03.

7.5 Numerical Application

The considered airport plan development includes two new runways, two new terminal buildings (one passengers, one cargo) over a period of 25 years divided in five stages of five years duration and corresponding to five different operational configurations for the airport. Ancillary facilities such as control buildings, fire and rescue facilities, multi-store car parks, taxiways, hangars, rail access are aggregated to the corresponding terminals and runways development phases. Due to technical and capacity considerations, runway number three will be constructed only after runway number two, second terminal and cargo terminal completion. Cargo terminal phase will begin only after runway number two has been completed. Also, the second passenger terminal will be constructed only after runway number two is ready.

Figure 7.2 displays the resulting dynamic programming decision graph.

Here 31 different paths lead to the states of the final stage while 20 different states at equal or different stages must be evaluated following relations (7.10), (7.11) and (7.12). To each state is associated the corresponding passengers and cargo capacity.

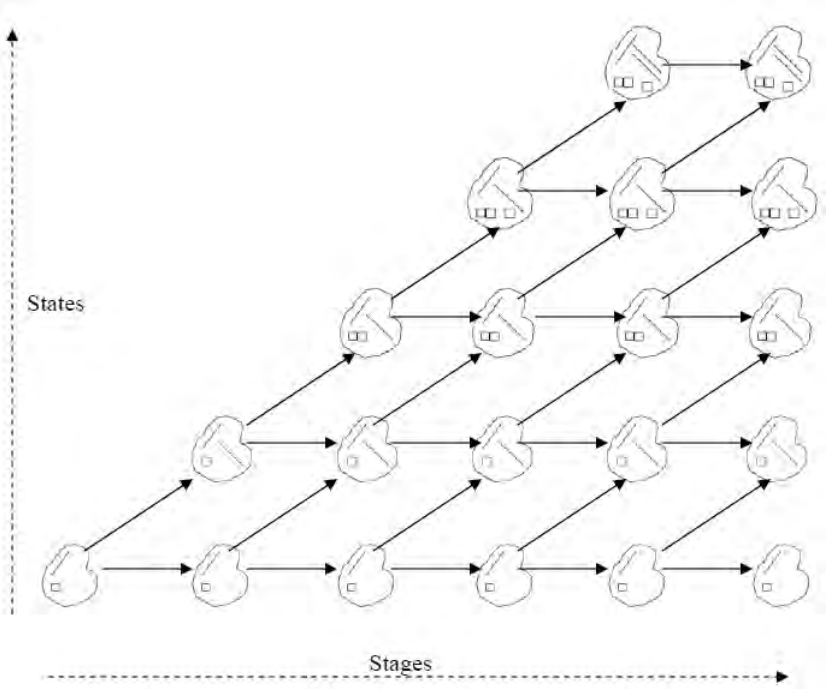

Fig. 7.2 Dynamic Programming Decision Graph

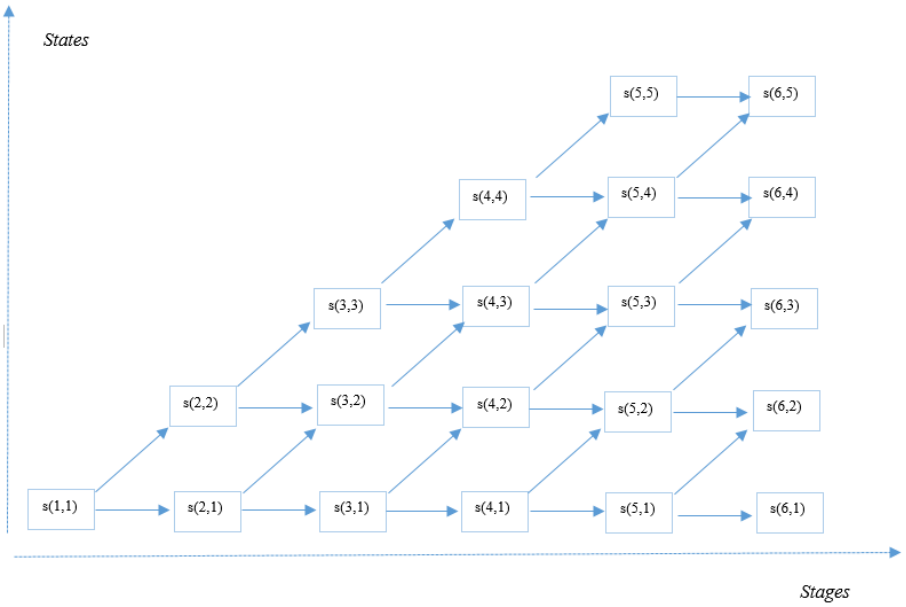

Fig. 7.3 Coding of potential options (stage, state) for the airport infrastructure development plan

The expected passengers and cargo capacities associated to each of these states are the following:

states (i, 1) - Passenger capacity: 10 million;

- Cargo capacity: 30, 000 t.

states (i+1, 2) - Passenger capacity: 15 million;

- Cargo capacity: 45, 000 t.

states (i+2, 3) - Passenger capacity: 25 million;

- Cargo capacity: 65, 000 t.

states (i+3, 4) – Passenger capacity: 25 million;

- Cargo capacity: 125, 000 t.

states (i+4, 5) – Passenger capacity: 35 million;

- Cargo capacity: 135, 000 t.

The application of the proposed fuzzy dual dynamic programming approach leads to the following optimal decision tree represented in Fig. 7.4 where each potential state corresponding to every stage has associated a fuzzy dual performance, a degree of certainty and a fuzzy dual net present value.

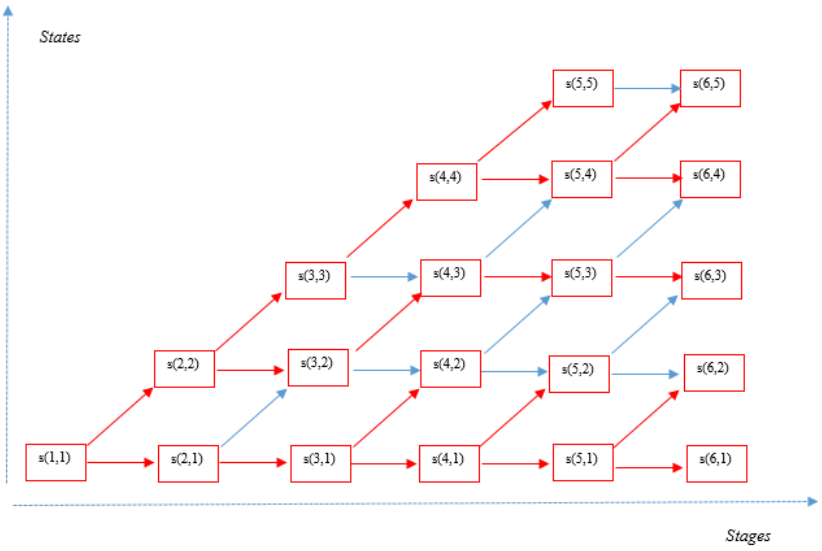

Fig. 7.4 Fuzzy dual dynamic programming solutions tree

The breakdown for every stage and states in the optimal decision tree is detailed bellow.

Stage 1: represents current airport situation, with the following associated parameters:

state (1,1) – current airport parameters

- ✈ Fuzzy dual performance: 0 + ε 0
- ✈ Degree of certainty =1.
- ✈ Fuzzy dual NPV: 1000 + ε 0 .

Stage 2: five-year milestone

state (2,1) – no facilities added

- ✈ Fuzzy dual performance: 150 + ε 20
- ✈ Degree of certainty = 1
- ✈ Fuzzy dual NPV: 970 + ε 150

state (2,2) - addition of the second runway

- ✈ Fuzzy dual performance: -250 + ε 30
- ✈ Degree of certainty=1
- ✈ Fuzzy dual NPV: 1280 + ε 140

Stage 3: ten-year milestone

state (3,1) – no facilities added

- ✈ Fuzzy dual performance: 135 + ε 32
- ✈ Degree of certainty=1.
- ✈ Fuzzy dual NPV: 950 + ε 310

state (3,2) – addition of the second runway

- ✈ Fuzzy dual performance: 125 + ε 34

- ✈ Degree of certainty= 0.90
- ✈ Fuzzy dual NPV: 1210 + ε 275

state (3,3) – addition of the second passenger terminal

- ✈ Fuzzy dual performance: -230 + ε 35
- ✈ Degree of certainty : 1
- ✈ Fuzzy dual NPV: 1450 + ε 190

Stage 4 – fifteen-year milestone

state (4,1) – no facilities added

- ✈ Fuzzy dual performance: 128 + ε 56
- ✈ Degree of certainty=1.
- ✈ Fuzzy dual NPV: 925 + ε 525

state (4,2) – addition of the second runway

- ✈ Fuzzy dual performance: -235 + ε 48
- ✈ Degree of certainty=0.84
- ✈ Fuzzy dual NPV: 1210 + ε 490

state (4,3) – addition of the second passenger terminal

- ✈ Fuzzy dual performance: -25 +ε 41
- ✈ Degree of certainty: 0.83
- ✈ Fuzzy dual NPV:1400 + ε 320

state (4,4) - addition of the cargo terminal

- ✈ Fuzzy dual performance: -220 + ε 35
- ✈ Degree of certainty : 1
- ✈ Fuzzy dual NPV: 1750 + ε 260

Stage 5 – twenty-year milestone

state (5,1) – no facilities added

- ✈ Fuzzy dual performance: 123 + ε 97
- ✈ Degree of certainty =1.
- ✈ Fuzzy dual NPV: 905 + ε 840

state *(5,2)* – addition of the second runway

- ✈ Fuzzy dual performance: -227+ ε 84
- ✈ Degree of certainty = 0.75
- ✈ Fuzzy dual NPV: 1195 + ε 766

state (5,3) – addition of the second passenger terminal

- ✈ Fuzzy dual performance: 115 + ε 73
- ✈ Degree of certainty: 0.75
- ✈ Fuzzy dual NPV: 1380 + ε 470

state (5,4) - addition of the cargo terminal

- ✈ Fuzzy dual performance: 110 + ε 42
- ✈ Degree of certainty: 0.77
- ✈ Fuzzy dual NPV: 1675 + ε 365

state (5,5) – addition of the third runway

- ✈ Fuzzy dual performance: -210 + ε 55
- ✈ Degree of certainty: 1
- ✈ Fuzzy dual NPV: 1800 + ε 466

Stage 6: - twenty-five-year milestone

state (6,1) – no facilities added

- ✈ Fuzzy dual performance: 120 + ε 129
- ✈ Degree of certainty between=1.
- ✈ Fuzzy dual NPV: 894+ ε 962

state (6,2) – addition of the second runway

- ✈ Fuzzy dual performance: 115 + ε 105
- ✈ Degree of certainty = 0.66
- ✈ Fuzzy dual NPV: 1185 + ε 971

state (6,3) – addition of the second passenger terminal

- ✈ Fuzzy dual performance: 110 + ε 92
- ✈ Degree of certainty: 0.59
- ✈ Fuzzy dual NPV: 1370 + ε 750

state (6,4) - addition of the cargo terminal

- ✈ Fuzzy dual performance: 108 + ε 65
- ✈ Degree of certainty: 0.68
- ✈ Fuzzy dual NPV: 1650 + ε 582

state (6,5) – addition of the third runway

- ✈ Fuzzy dual performance: -200 + ε 75
- ✈ Degree of certainty : 0.67
- ✈ Fuzzy dual NPV: 1810 + ε 684

Then it appears that to get at the horizon of 25 years (degree of certainty 0.67) with the project entirely complete (i.e. airport with three runways, two passenger terminals and a cargo terminal) the best solution is to start immediately the construction process by adding each five years a new element (second runway, second passenger terminal, cargo terminal in this particular order), then wait for five years before constructing the third runway. As defined by relation (7.16), there is no financial risk in this case.

In the case in which it is considered that the third runway will not be taken into consideration (traffic deficit, environmental considerations, lack of quantifiable economic benefits, difficulties in funding, etc.), then the best solution appears to be (degree of certainty 0.59) starting as soon as possible the second runway (+5), the second passenger terminal (+10) and the cargo terminal (+15). Here also, there is no financial risk attached according to relation (7.16).

However, the do nothing solution (state (6,1)) has a financial risk attached according to relation (7.16). In this particular case, airport congestion will generate increasing operating costs.

7.6 Conclusions

This chapter illustrates the applicability of the proposed approach in addressing uncertainty in airport infrastructure development projects using fuzzy dual representation of uncertainty.

Finding the balance between maximization of expected net present value and minimization of its uncertainty level has traditionally been a complex task and, in many cases not very successful in the context of long-term projects. Therefore, this problem was imbedded in a multi criteria assessment context. The dynamic programming dimension allows the unfolding of the multi-stage decision making process while the generated uncertainty is assessed with a limited computational effort by the use of fuzzy dual performance indicators.

The proposed approach can be adapted to major development projects in other fields of activity (manufacturing, energy, other transportation modes) and can integrate different types of risks such as environmental and social.

The proposed tool is perfectible but none the less can be considered a starting point for further research in this field.

CHAPTER VIII

CONCLUSIONS & PERSPECTIVES

Operating and planning in an uncertain environment has become a more and more difficult exercise for the aviation industry, as volatility has grown as a fundamental characteristic of the social, economic and political environment.

For an airport long-term development project to be successful, the decision makers should have:

- a deep understanding of the industry and the market dynamics, the airport position and interaction on the aviation value chain,
- a comprehensive understanding of business planning, programming implementation,
- and, as it became more and more stringent, a deep understanding of related risk and operating in uncertain environments.

The impact of under-performing development plans is multi-fold. The most easily quantifiable is the financial impact. Lack of accurate forecasting in long-term development projects and disregard for major disruptive events, can make or break a project.

On another hand, trying to translate real world complexities in mathematical terms is a highly challenging task, leading to unmanageable computations.

The novelty of the idea this thesis puts forward, is to take a concept which became increasingly popular in other areas like automatization, robotics, environmental sciences, medicine – fuzzy logic - and propose an innovative approach in a field were efficient results are lacking – long-term airport infrastructure planning.

Starting from these premises, we expanded on the new formalism of fuzzy dual numbers and introduced fuzzy dual calculus as a possible solution to treat parameter uncertainty and solution diversion in mathematical optimization problems, with the objective of offering better trade-off between complexity and effectiveness. Once the uncertainty aspect of the problem is addressed, the use of dynamic programming has been considered since long-term airport development projects are sequential problems and dynamic programming is an effective technique to obtain the solution of optimal sequential decision making processes. Fuzzy dynamic programming has been

considered to address real world multi-stage decision making problems, but the computational complexity associated with its implementation has limited its use.

The originality of the approach proposed in this thesis is the introduction of the fuzzy dual representation in the decision process of dynamic programming with an immediate effect not only on the computational burden but also on the volume of input data to start the process.

The integration of different mathematical techniques led us to design a rather original method to cope with sequential decision problems. This method offers the decision-maker a mapping of the decision space, helping him navigate from one stage to another by assessing the uncertainty associated to the different states to ultimately choose a sequence of decisions. This approach should allow the decision maker to face efficiently complexity and uncertainty and balance different solutions.

REFERENCES

[Abo-Sinna, 2004] Abo-Sinna, M., A., *Multi-objective (fuzzy) dynamic programming problems: a survey and some applications,* Applied Mathematics and Computation 157 (2004) 861-888.

[ACI ER, 2013] Airports Council International, *Economics Report – Preview edition,* 2013.

[ACI GTFR, 2013] Airports Council International, *Global Traffic Forecast Report 2012-2031,* 2013.

[ACI Annual Report, 2014] Airports Council International *Annual Report* 2014, p.18.

[ACI EU ROEA, 2016] Airports Council International Europe, *Report on the Ownership of European Airports,* 2016.

[ACI MR, 2016] Airports Council International, Media Release – March 7th 2016, http://www.aci.aero/News/Releases/Most-Recent/2016/03/07/ACI-releases-the-20th-edition-of-the-Airport-Economics-Report-and-Key-Performance-Indicators.

[ACRP Report 20, 2009] Airport Cooperative Research Program, Report 20, *Strategic Planning in the Airport Industry,* Transportation Research Board, 2009, page 6.

[ACRP WOD, 2015] Airport Cooperative Research Program, Web Only Document 22, *Passenger value of time, benefit-cost analysis and airport capital investment decisions,* Vol.3, Transportation Research Board, 2015.

[AIRBUS, 2011] AIRBUS, *Global Market Forecast 2012-2031,* 2011.

[Aliev, 2013] Aliev A., R., *Fundamentals of the Fuzzy Logic-based generalized theory of decisions,* Studies in Fuzziness and Soft Computing, Springer, 2013.

[Atanassov, 1986] Atanassov, K., T., *Intuitonistic fuzzy sets,* Fuzzy sets and systems 20, 87-96.

[Bector and Chandra, 2005] Bector, C., R., Chandra, S., *Fuzzy Mathematical Programming and Fuzzy Matrix Games,* Studies in Fuzziness and Soft Computing, vol. 169, 2005, p. 24.

[Bellman, 1957] Bellman, R., *Dynamic Programming,* First Princeton Landmarks in Mathematics ed., with a new introduction, Princeton University Press, Princeton, 2010.

[Bellman and Zadeh, 1970] Bellman, R., Zadeh L., A., *Decision making in a fuzzy environment,* 1970, Management Science 17B, 141-164.

[Ben-Tal et. al., 2009] Ben-Tal, A., El Ghaoui, L., Nemirovski, A., *Robust optimization,* Princeton University Press, Princeton Series in Applied Mathematics, 2009.

[Bernardo and Smith, 2000] Bernardo, J., M., Smith, A., F., M., *Bayesian Theory,* Wiley Series in Probability and Statistics, Wiley & Sons, NY, 2000.

[Bertsimas and Sim, 2003] Bertsimas, D., Sim, M., *Robust discrete optimization and network flows,* Mathematical programming, 98:48-71, 2003.

[Bertsimas and Sim, 2003] Bertsimas, D., Sim, M., *The price of robustness,* Operations Research, 52(1):35-53, 2004.

[Bertsimas and et al., 2004] Bertsimas, D., Pachamanova, D., Sim, M., *Robust linear optimization under general norms,* Operations Research Letters, 32(6):510-516, 2004.

[Bertsimas and Brown, 2005] Bertsimas, D., Brown., D., *Robust linear optimization and coherent risk measures,* Technical report, Massachusetts Institute of Technology, 2006.

[Bertsimas and Thiele, 2006] Bertsimas, D., Thiele, A., *Robust and Data-Driven Optimization: Modern Decision Making under uncertainty,* 2006.

[Birge and Louveaux, 2011] Birge, J., Louveaux F., *Introduction to stochastic programming,* Springer Science+Business Media, LLC, 2nd ed., 2011.

[Booker and Ross, 2011] Booker, J., M., Ross, T.J., *An evolution of uncertainty assessment and quantification,* 2011, Scientia Iranica D 18 (3), 669-676.

[Bradley et al., 1977] Bradley, S., P., Hax, A., C., Magnanti, T., L., *Applied Mathematical Programming,* Addison-Wesley, 1977.

[Brodsky and Shoham, 2000] Brodsky, V., Shoham, M., *Dual numbers representation of rigid body dynamics*, Mechanism and Machine Theory, 34(5) 693-718, 2000.

[Burghouwt, 2007] Burghouwt, G., *Airline network development and its implications for airport planning,* Ashgate Publishing Company, Burlington, Vermont, 2007.

[Cantor, 1874] Cantor, G., *On a property of the Collection of all Real Algebraic Numbers,* J. Reine Angew. Math. 77:258-262, 1874.

[CAPA, 2015] Centre for Aviation, *The world's biggest airport construction projects 2015, Part 1 and 2,* Jan. 2015, www.centreforaviation.com.

[Chow and Smith, 2012] Chow, B., Smith, C., *Airport transactions: taking off around the world,* The new normal for airport investment, PWC, 2012.

[Clifford, 1873] Clifford, W., K., Proc. London Mathematic Society, 4, 381, 1873.

[Cook, 1971] Cook, S.,A., *The complexity of theorem-proving procedures,* University of Toronto, 1971.

[Copeland and Weston, 1988] Copeland, T., E., Weston J., F., *Financial theory and corporate policy,* Addison-Wesley Publishing Company, USA, 1988.

[Cosenza and Mora-Camino, 2011] Cosenza, C., A., N., Mora-Camino, F., *Nombres et ensembles duaux flous et applications.* Technical report, LMF Laboratory, COPPE, Universidad Federal de Rio de Janeiro, 2011.

[Cosenza and Mora-Camino, 2012] Cosenza, C., A., N., Mora-Camino, F., *Fuzzy dual entropy and trip distribution in transportation systems,* Proceedings of the 17th International Conference of Hong Kong Society for Transportation Studies, Hong Kong, 15-17 Dec. 2012.

[Cosenza et al., 2012] Cosenza, C., A., N., Lenguerke, O., Mora-Camino, F., *Fuzzy sets and dual numbers: an integrated approach*, Proceedings of 9th International Conference on Fuzzy Sets and Knowledge Discovery, Chongqing, pp.81-86, 2012.

[Charnes and Cooper, 1959] Charnes A., Cooper, W., *Chance constrained programming,* Management Science, 6(1):73-79, 1959.

[Dantzig, 1955] Dantzig, G., Linear programming under uncertainty, Management science, 1(3-4):197-206, 1955.

[de Finetti, 1974] de Finetti, B., *Theory of probability: a critical introductory treatment,* Wiley & Sons, NY, 1974.

[de Neufville, 1990] de Neufville, R, *Succesfull sitting of airports: the Sydney example*, Journal of Transportation Engineering ASCE, Vol. 116, No.1, pp 37-48.

[de Neufville and Odoni, 2013] de Neufville, R. and Odoni, A., *Airport Systems:Planning, Design and Management,* McGraw-Hill, New York, 2003.

[de Neufville and Odoni, 2013] de Neufville, R. and Odoni, A., *Airports Systems: Planning, Design and Management*, 2013, McGraw Hill Education LLC, 2d. ed., page 79.

[Dempster, 1968] Dempster, A., P., *A generalization of Bayesian inference*, 1968, J. Roy, Statist., Soc., Ser., B., 30, 205-247.

[Dimentberg, 1965] Dimentberg, F., M., *The screw calculus and its applications in mechanics,* Izdat. Nauka Moscow, 1965, English translation AD680993, Clearinghouse for Federal and Scientific Technical Information.

[Donnet and Keast, 2011] Donnet, T., Keast, R., *fitting airport privatization to purpose: aligning governance, time and management focus,* Issue 11(2), 2011, pp. 98-114, www.ejtir.tbm.tudelft.nl.

[Dubois and Prade, 1979] Dubois D., Prade H., *Fuzzy real algebra: some results*, Fuzzy Set Systems 1979, 2:327-348.

[Dubois and Prade, 1988] Dubois, D., Prade, H., *Possibility theory,* NY, 1988.

[Dubois and Prade, 1979] Dubois D., Prade H., *Towards fuzzy differential calculus: part 1 integration of fuzzy mappings. Part 2: integration of fuzzy intervals. Part 3: Differentiation,* Fuzzy Set Systems, 1982, 8:1-17; 105-116, 225-233.

[Dubois et al., 2004] Dubois D., Fargier H., Prade H., *Ordinal and probabilistic representations of acceptance,* 2004, Journal of Artificial Intelligence Research, 22:23-56.

[Dubois et al., 2009] Dubois D., Bouyssou D., Pirlot M., Prade H., eds, *Decision Making Process*, 2009, ISTE, London, UK & Wiley, Hoboken, N.J. USA.

[Esogbue, 1983] Esogbue, A.,O., *Dynamic programming, fuzzy sets and the modeling of R&D management control systems,* IEEE Trans. on Systems, Man, and Cybernetics, SMC-13, 18-30.

[Esogbue, 1986] Esogbue, A.,O., *Optimal clustering of fuzzy data via fuzzy dynamic programming*, Fuzzy sets and Systems, 18, 283-298.

[Esogbue, 1999] Esogbue A., O., *The computational complexity of some fuzzy dynamic programs,* The International Journal of Computers and Mathematics with Applications 37:47-51, 1999.

[Esogbue and Bellman, 1981] Esogbue, A.,O., Bellman, R., *A fuzzy dynamic programming algorithm for clustering non-quantitative data arising in water pollution control planning*, Proceedings of 3rd International Conference on Mathematical Modeling, LA, California, USA, 1981.

[Esogbue and Bellman, 1984] Esogbue, A.,O., Bellman, R., *Fuzzy dynamic programming and its extensions*, TIMS/Studies in the Management Sciences, 20, 147-167.

[Esogbue and Ramesh, 1970] Esogbue, A.,O., Ramesh, V., *Dynamic programming and fuzzy allocation processes*, Technical Memo No. 202, Department of Operations Research, Case Western University, Cleveland, USA, 1970.

[Harvey, 1995] Harvey, C., R., *Finance – lecture notes,* Duke University, http://www.duke.edu/~charvey/Classes/ba350/project/project.htm, 1995.

[Hillier and Lieberman, 2010] Hillier, S., F., Lieberman, J., G., *Introduction to operations research,* 9th ed., McGraw Hill International Edition, Singapore, 2010, p. 424.

[Faye et al., 2002] Faye, R., M., Sawadogo, S., Mora-Camino, F., *Logique floue appliquée à la gestion à long-terme des resources en eau,* Revue des sciences de l'eau, vol. 15, no.3, 2002, p.579-596.

[Garey and Johnson, 1979] Garey, M., R., Johnson, D., S., *Computers and Intractability: a guide to the Theory of NP-Completeness,* A Series of Books in the Mathematical Sciences, San Francisco, Calif., W.H. Freeman and Co..

[Herrera et al., 2012] Herrera, R., T., Alcantara, S., M., Meda-Campana, J., A., Velasquez, A., S., *Kinematic and dynamic modeling of serial robotic manipulators using dual number algebra,* Instituto Politecnico Nacional de Mexico, Serial and Parallel Robot Manipulators – Kinematics, Dynamics, Control and Optimization, 2012.

[IATA, 2013] International Air Transport Association – IATA, Pearce, B., IATA Economics Briefing No.10, *Profitability and the air transport value chain,* June 2013, v. 1.1, www.iata.org.

[ICAO, 1987] International Civil Aviation Organization, DOC 9814-AN/902 *Airport Planning Manual, Part 1: Master Planning,* 1987, 2d. ed., ICAO, Montreal, Canada, page 1-2.

[Jaynes, 1986] Jaynes, E., T., *Bayesian Methods: General Background,* Maximum Entropy and Bayesian Methods in applied Statistics, ed. Justice, J. H., Proceedings of the 4th Maximum Entropy Workshop, 1984, Cambridge University Press, 1996.

[Kacprzyk, 1977] Kacprzyk, J., *Decision-makin of a non-fuzzy system in a fuzzy environment with fuzzy termination time,* Systems Science, 3, 325-341, 1977.

[Kacprzyk, 1983b] Kacprzyk, J., *Multistage decision making under fuzziness,* Verlag TÜV, Rheinland, Cologne, 1983.

[Kacprzyk, 1997] Kacprzyk, J., *Multi-stage fuzzy control: A model based approach to control and decision making,* Wiley, Chichester, 1997.

[Kacprzyk and Esogbue, 1996] Kacprzyk, J., Esogbue, A., O., *Fuzzy dynamic programming: Main developments and applications,* Fuzzy sets and systems 81 (1996) 31-45.

[Kacprzyk and Straszak, 1981] Kacprzyk, J., Straszak, A., *Application of fuzzy decision making models for determining optimal policies in 'stable', integrated regional development,* In P.P. Wang and S.K. Chang (Eds.):Fuzzy Sets Theory and

Applications to Policy Analysis and Information Systems, Plenum Press, New York, 321-328, 1981.

[Kacprzyk and Straszak, 1982a] Kacprzyk, J., Straszak, A., *A fuzzy approach to the stability of integrated regional development,* In G.E. Lasker (Ed.): Applied Systems and Cybernetics, Vol.6, Pergamon Press, New York, 2997-3004, 1982.

[Kacprzyk and Straszak, 1982b] Kacprzyk, J., Straszak, A., *Determination of 'stable' regional development trajectories via a fuzzy decision making model,* In R.R. Yager (Ed.): Recent developments in Fuzzy Sets and Possibility Theory, Pergamon Press, New York, 531-541, 1982.

[Kacprzyk and Straszak, 1982c] Kacprzyk, J., Straszak, A., *Determination of stable trajectories for integrated regional development using fuzzy decision models,* IEEE Transactions on Systems, Man and Cybernetics, SMC-14, 310-313, 1984.

[Kall and Mayer, 2011] Kall, P., Mayer, J., *Stocahstic Linear Programming: models, theory and computation,* International Series in Operations Research and Management Science, 2nd ed., 2011.

[Kandasamy and Smarandescu, 2012] Kandasamy, W., B., V., Smarandescu, F., *Dual Numbers,* ZIP Publishing, Ohio, 2012.

[Kantorovich, 1939] Kantorovich, L.V., *Mathematical Methods of Organizing and Planning Production*, Management Science 6, no. 4, 366-422, published in 1960.

[Karmarkar, 1984] Karmakar, N., *A new polynomial-time algorithm for linear programming,* AT&T Laboratories, Murray Hill, New Jersey, 1984.

[Karp, 1972] Karp, R.., M., *Reducibility among combinatorial problems,* in Miller and Thatcher (editors), Complexity of Computer Computations, NY:Plenum, pp.85-103.

[Kere, 1995] Kere, E., *On the evolution of the mathematics of fuzziness,* Fuzzy set theory and advanced mathematical applications, Springer Science+Business Media, New York, USA, 1995.

[Khachiyan, 1979] Khachiyan, L., G., *Polynomial algorithms in linear programming,* USSR Comput. Maths. Math. Phys., Vol. 20, No. 1, 53-72, 1979.

[Kim and Roush, 1982] Kim, K., H., Roush, F., W., *Fuzzy flows in networks*, Fuzzy Set Systems, 1982, 8:33-38.

[Kincaid et al., 2012] Kincaid, I., Tretheway, M., Gros., S., Lewis, D., *ACRP Report 76: Addressing uncertainty about future airport activity levels in Airport Decision Making*, Transportation Research Board, Washington D.C., 2012.

[Klir, 2006] Klir, G., J., *Uncertainty and information – Foundations of generalized information theory*, Wiley Interscience, New Jersey, 2006.

[Klir and Wierman, 1998] Klir, G., J., Wierman, M., J., *Uncertainty-based information*, New York: Physica-Verlag, 1998.

[Klir and Yuan, 1995] Klir G., J., Yuan, B., *Fuzzy sets and fuzzy logic: Theory and applications*, 1995, Prentice Hall, Upper Saddle River, NJ.

[Kolmogorov, 1956] Kolmogorov, A., N., *Foundations of the theory of probability*, Chelsea Publishing Company, NY, 1956.

[Koopman, 1940] Koopman, B., O., *The bases of probability*, Bulletin of the American Mathematical Society, 46, 763-774, 1940.

[Koopmans, 1960] Koopmans, T., C., *A note about Kantorovich's paper: Mathematical Methods of Organizing and Planning Production*, Management Science 6, no. 4, 363-365, published in 1960.

[Kutateladze, 2012] Kutateladze, S., S., *Mathematics and Economics of Leonid Kantorovich*, Siberian Mathematical Journal, Vol. 53, No.1, pp.2012, 2012.

[Kwakkel et al., 2008] Kwakkel, J., Walker, E., Wijnen, R.A.A, The treatment of uncertainty in airport strategic planning: the case of Schiphol airport's long-term vision, 12th Air Transport Research Society World Conference, Athens, Greece, 2008.

[Kwakkel et al., 2010] Kwakkel, J., Walker, W., E., Marchau, V., Assessing the efficiency of adaptive airport strategic planning: results from computational experiements, submitted to Transport Policy, 2010.

[Maldonado, 1990] Maldonado J., *Strategic planning: An approach to improving airport planning under uncertainty,* Master Thesis, Massachusetts Institute of Technology, Cambridge, MA (1990).

[McConnell, 2007] McConnell, J., *A life-cycle flexibility framework for designing, evaluating and managing complex real options,* PhD dissertation, MIT Engineering Systems Division, Cambridge, 2007.

[Mendel, 1995] Mendel, J., *Fuzzy logic systems for engineering: a tutorial,* Proceedings of the IEEE, 83(3):345-377, 1995.

[Moore, 1966] Moore, R., E., *Interval Analysis*, Englewood Cliff, New Jersey: Prentice-Hall, 1966.

[Morgan and Henrion, 1990] Morgan, M., G., Henrion, M., *Uncertainty*, Cambridge University Press, 1990, New York.

[Newell and Simon, 1972] Newell, A., Simon, H., A., *Human Problem Solving*, 1972, Prentice-Hall, Englewood Cliffs, NJ.

[Nishimura, 1999] Nishimura T., *Dynamic strategic planning for transportation infrastructure investment,* Master Thesis, Massachusetts Institute of Technology, Cambridge, MA (1999).

[Parida, 2013] Parida, P., K., *Fuzzy dynamic system approach to multistage decision making problems,* Ultra Scientist Vol. 25(2)A, 350-360, 2013.

[Pawlak, 1985] Pawlak, Z, *Rough sets*, Fuzzy sets and systems, 17, 99-102.

[Pichott and Scott, 2014], Pichott, P.-E., Scott, R., *Keeping airport projects on course in a turbulent world,* Price Waterhouse Coopers, 2014.

[PWC RR, 2012] Price Waterhouse Coopers, *Risk in review: coping with the unknown risk-risk management strategies for an uncertain world,* 2012

[Ross, 2003] Ross, T., J., *An enhanced reliability index for assessing margin of safety in structures,* 2003, Conference on Relations and Optimization in Structural Systems, Banff, Canada, A.A. Balkema Publishers, Netherlands, 291-298.

[Ruszczynski and Shapiro, 2003] Ruszczynski, A., Shapiro, A., *Stochastic Programming Models*, Handbooks in Operations Research and Management Science, Vol. 10, Chapter 1, 2003.

[Schweickardt and Miranda, 2007] Schweickardt, G., A., Miranda, V., *A fuzzy dynamic programming approach for evaluation of expansion distribution cost in uncertainty environments,* Latin American Applied Research, 2007, 32:227-234.

[SEP, 2014] Stanford Encyclopaedia of Philosophy, Jan Lucasiewicz, First published Thu May 15, 2014; substantive revision Fri Jun 6, 2014, web address: http://plato.stanford.edu/entries/lukasiewicz/, accessed 10th Nov. 2015.

[Shackle, 1961] Shackle, G., L., S., *Decision, order and time in human affairs,* Cambridge University Press, NY. 1961.

[Shafer, 1976] Shafer G., A., *A mathematical theory of evidence,* 1976, Princeton University Press, Princeton, NJ.

[Shapiro et al., 2009] Shapiro, A., Dentcheva, D., Ruszczynski, A., *Lectures on stochastic programming: modeling and theory,* Society for Industrial and Applied Mathematics and the Mathematical Programming Society, 2009.

[SITA, 2013] Société Internationale de Télécommunication Aéronautiques, *Flying into the future,* Air Transport Industry Insights, 2013.

[Smithson, 1990], Smithson, M., *Ignorance and disasters,* International Journal of Mass Emergencies and Disasters 8(3), 1990, 207-235.

[Soyster, 1973] Soyster, A., *Convex programming with set-inclusive constraints and applications to inexact linear programming,*

[Spitz and Golaszewski, 2007] Spitz, W., Golaszewski, R., *ACRP Synthesis 2: Airport aviation activity forecasting,* Transportation Research Board of the National Academies, Washington, D.C., 2007.

[Stein, 1980] Stein W., E., *Optimal stopping in a fuzzy environment,* Fuzzy Sets and Systems, 3, 252-259, 1968.

[Study, 1901] Study, E., *Geommetry der Dynamen,* Leipzig, 1901.

[Taleb, 2007] Taleb, N., N., *The Black Swan: the impact of highly improbable,* 1st ed., Random House, NY, USA, 2007.

[Tanaka et al., 1974] Tanaka, H., Ichihashi, H., Asai, K., *On fuzzy mathematical programming,* J. Cybernet. 3, 37-46, 1974.

[Trani, 2015] Trani, A., *Demand forecast uncertainty*, Airport Planning and Design, CEE 4674, Air Transportation Systems Laboratory, Virginia Tech, 2015.

[Walker, 2000] Walker, W., E., *Policy analysis: a systematic approach to supporting policy making in the public sector,* Journal of Multicriteria Decision Analysis, Vol 9 (2000), pp.11-27.

[Walker et al., 2001] Walker, W., E., Rahman, S., A., Cave J., *Adaptive policies, policy analysis and policy-making,* European Journal of Operational Research, Vol. 128 (2001), pp. 282-289.

[WEF GRR, 2016] World Economic Forum, *The global risks report 2016,* 11th ed., http://wef.ch/risks2016

[Wierman, 2010] Wierman, J., M., *An introduction to mathematics of uncertainty,* Creighton University, 2010.

[Zadeh, 1965] Zadeh, L., A., *Fuzzy sets,* Information and Control, 1965, 8:338-353.

[Zadeh, 1968] Zadeh, L., A., *Fuzzy algorithms,* Information and Control, 1968, 12:94-102.

[Zadeh, 1973] Zadeh, L., A., *Outline of a new approach to the analysis of complex systems and decision processes,* IEEE Transportation Systems, Man., Cybernetics, SMC-3, 1973, pages 28-44.

[Zadeh et. al, 1975] Zadeh, A., L., Fu, K., Tanaka, K., Shimura, M., *Fuzzy sets and their applications to cognitive and decision processes,* Academic Press, Inc., USA, 1975, page ix.

[Zadeh, 2005] Zadeh, L., A., *From imprecise to granular probabilities,* Fuzzy set systems, 2005, 154:370-374.

[Zadeh, 2009] Zadeh, A., L., *Toward extended fuzzy logic – A first step*, Fuzzy Sets and Systems, 160 (2009), 3175-3181.

[Zimmermann, 1974] Zimmermann, H., J., *Optimization in fuzzy environment*, Presented at XXI Intl. TIMS and 46th ORSA Conference, San Juan, Puerto Rico, 1974.

[Zimmermann, 2000] Zimmermann, H., J., *An application-oriented view of modeling uncertainty*, European Journal of Operational Research, 2000, 122: 190-198, p. 192.

[Zimmermann, 2010] Zimmermann, H., J., *Fuzzy set theory*, John Wiley & Sons, 2010, vol. 2, May/June, p. 317.

APPENDIX A

DUAL NUMBERS

Dual numbers were introduced in the 19th century by [Cliford, 1873], with a subsequent generalization of their application to rigid body kinematics by Kotelnikov and Study in their Principle of transference [Study, 1901], [Dimentberg, 1965].

The *principle of transference* states that when dual numbers replace real ones, all relations of vector algebra for intersecting lines are valid for skew lines. This implies that all rules of vector algebra for kinematics for a rigid body with a fixed point (spherical kinematics) also hold for motor algebra of a free rigid body (spatial kinematics). *As a result, a general rigid body motion can be described by only three dual equations rather than six real ones* [Brodsky and Shoham, 2000].

In linear algebra, the dual numbers extend the real numbers by adjoining one new element ε with the property $\varepsilon^2=0$ (ε is nilpotent). The collection of dual numbers forms a particular two-dimensional commutative unital associative algebra over the real numbers.

Every dual number has the form $z=a+\varepsilon b$, where a and b are uniquely determined real numbers. Dual numbers can also be thought of as the exterior algebra of a one-dimensional vector space.

Dual numbers form the coefficients of quaternions.

Dual numbers algebra

Two dual numbers are equal if and only if their real and dual parts are equal, respectively.

$$(a+\varepsilon b)+(x+\varepsilon y)=(a+x)+\varepsilon(b+y) \quad . \text{ (b1)}$$

Multiplication of two dual numbers results in:

$$(a+\varepsilon b)(x+\varepsilon y)=ax+\varepsilon(bx+ay) \quad \text{(b2)}$$

Division of dual numbers, $\frac{a}{b}$, is defined as the inverse operation of multiplication. Due to the fact $\varepsilon^2=0$, division is possible and unambiguous only if $b \neq 0$.

$$\frac{z}{w}=\frac{a}{b}+\varepsilon\left(\frac{a'}{b}-\frac{ab'}{b^2}\right), \qquad b \neq 0. \quad \text{(b3)}$$

Dual function of dual numbers

The *dual function* of dual number presents a mapping of a dual numbers space on itself, namely: $f(z) = \hat{f}(a,b) + \varepsilon\tilde{f}(a,b)$ (b4)

, where $z=a+\varepsilon b$ is a dual variable, $\hat{f}$ and $\tilde{f}$ are two, generally different, functions of two variables.

[Dimentberg, 1965] gave a comprehensive analysis of the properties of dual functions. The general expression for dual analytic function as given by Dimentberg is:

$$f(a+\varepsilon b) = \hat{f} + \varepsilon\tilde{f} = f(a) + \varepsilon(bf'(a) + \dddot{f}(a)) \quad \text{(b5)}$$

, where $\dddot{f}$ is an arbitrary function of a real part of a dual variable.

The analytic condition of a dual function is:

$$\frac{\partial\tilde{f}}{\partial b} = \frac{\partial\hat{f}}{\partial a} \quad \text{(b6)}$$

The derivate of such a dual function with respect to a dual variable is:

$$\frac{df(z)}{dz} = \frac{\partial\hat{f}}{\partial a} + \varepsilon\frac{\partial\tilde{f}}{\partial b} = f'(a) + \varepsilon(bf''(a) + \dddot{f}'(a)) \quad \text{(b7)}$$

The above definition allows the formulation of dual forms of different functions, for example:

$$\sin(a+\varepsilon b) = \sin(a) + \varepsilon b\cos(a),$$
$$\cos(a+\varepsilon b) = \cos(a) - \varepsilon b\sin(a),$$
$$e^{a+\varepsilon b} = e^a \cdot e^{\varepsilon b} = e^a(1+\varepsilon b) = e^a + \varepsilon b e^a,$$
$$\sqrt{z} = \sqrt{a+\varepsilon b} = \sqrt{a} + \frac{\varepsilon b}{2\sqrt{x}}, x > 0.$$

The above formulas are widely applied in kinematics.

APPENDIX B

FINANCIAL DECISION MAKING

This annex displays classical concepts and techniques for investment projects evaluation from the financial point of view.

Investment projects, depending on how they influence the investment decision making process, are classified in three categories according to [ACRP WOD 22, 2015]:

- ***Independent projects***
- ***Mutually exclusive projects***
- ***Contingent projects***

Independent projects are not influencing in any way the decision to pursue or no other projects. These types of projects can be evaluated independently and the decision is made depending on the added value they bring to the company.

Mutually exclusive projects imply that the acceptance of one prevents the pursue of the alternative option. Therefore, this mutually exclusive projects involve a 'either-or' type of decision. The projects can be evaluated separately and the decision should point to the one that yields the highest net present value. The major risk in dealing with mutually exclusive projects is not identifying the presumptive projects as such, which will lead to a significant loss of resources and ultimately can irremediably affect the financial health of a company.

Contingent projects acceptance or rejection is dependent on the decision to accept or reject one or more other projects. Contingent projects are either complementary or substitutes. Complementary projects enhance each other's cash flows. In the case of a substitute project, its success or even failure depends on the decision to reject the other project. When evaluating contingent projects, the cash flow interactions between all projects should be analysed.

Traditional investment decision tools used in airport capital finance budgeting are:

- ***Payback period***
- ***Net Present Value (NPV)***
- ***Internal Rate of Return (IRR)***

- ***Benefit / Cost Ratio*** (also known as *present value index* and *profitability index*)

The above mentioned techniques assist the decision maker in selecting one project over another, prioritize investment projects and choose among mutually exclusive alternatives.

According to [Copeland and Weston, 1988] the best technique employed should maximize value to investors by satisfying the following criteria:

- all investment related cash flows should be taken into consideration with the exception of interest payments on borrowings; interest represents cost of capital and is accounted for by discounting.
- the cash flows should be discounted at the opportunity cost of capital; the value of cash today is greater than the value of same amount tomorrow due to possibility of investment that could generate future returns; the process of discounting takes into consideration the time value of money.
- the technique used should give the decision maker a clear option from a set of mutually exclusive projects; projects become mutually exclusive when choosing one option precludes implementing the others, such as alternative means of achieving the same objective.
- The technique should also permit the decision maker to consider one project independently from all others, known as *the value-additivy principle*; the projects are *independent* when the decision to pursue one does not affect the decision to pursue another; this gives the decision maker the possibility to pursue one or all of the projects, as opposed to *contingent* projects that need to be carried out together or not at all and therefore should be analysed as a single project.

Payback period is one of the most frequently used instruments for decision making in capital investment. The payback period is the number of years it takes to recover the initial cash outlay on a project without taking interest (or discounting) into account. The decision to pursue a project is made on the assumption that the payback period is

less or equal to an acceptable time limit. The main advantage of this technique is its simple calculation process. On the other hand, deciding an acceptable deadline for payback is rather arbitrary and when deciding between alternatives, can lead to wrong choices.

Net Present Value (NPV) technique discounts cash flows to take into account the time value of money.

$$NPV = \sum_{t=1}^{N} \frac{Net\_cash\_flow_t}{(1+r)^t} - Initial\_outlay = \sum_{t=1}^{N} \frac{Cash\_proceeds_t}{(1+r)^t} - \sum_{t=1}^{N} \frac{Cash\_outlays_t}{(1+r)^t}$$

, where *N* is the number of years in the project's evaluation period and *r* is the discount rate.

To calculate it an appropriate discount rate needs to be chosen, the present value of the cash proceeds expected from the investment needs to be calculated along with the present value of the cash outlays required by the investment. Obviously, the NPV's value has to be positive for the project to be valid and in case of mutually exclusive options, the one with the highest NPV should be retained. To be mentioned that the NPV method among the four discussed, always leads to an investment option that will maximize value.

Internal Rate of Return (IRR) is the discount rate that makes the present value of annual net cash flows equal to the initial outlay, or using a different approach, the IRR is the discount rate that makes the project NPV equal to zero. Obviously, the project with the highest IRR should be retained. This method is superior to the discounted payback period technique because it considers all cash flows. On the other hand, unlike NPV method, the IRR does not show the currency value of the net financial payoff resulting from the investment. According to [Harvey, 1995] the limitations of IRR technique can lead to undesirable outcomes in the cases when the investment has a non-uniform term structure, when considering mutually exclusive projects with significant scale differences or significant differences in the timing of cash flows.

Benefit / Cost Ratio (BCR) is a performance indicator used in cost-benefit analysis that gives the return in present value terms per unit invested.

BCR = present value of cash inflows / present value of cash outflows

Obviously, for a project to be retained the BCR has to be greater than one and in the case of mutually exclusive projects, the one with the greater BCR should be retained. The limitations of this technique consist in pointing to the choice with highest ratio that ultimately may not yield the largest return in absolute value and also, there is the possibility of altering the final result according to the analyst allocation of cash flows in the case of multiple alternative projects (by including or excluding certain costs or benefits that are constant across all projects).

From the four techniques described above, the NPV method is the only investment decision instrument that always leads to an investment option that will maximize value.

Overall, these techniques are cash flow based, hence, they are suitable for projects that will generate directly revenue or reduce financial costs, like operating and maintenance costs.

LIST OF PUBLICATIONS

[1] Elena M. Capitanul, Carlos Albert Nunes Cosenza, Walid El Moudani, Félix Mora-Camino, *Airport Investment Risk Assessment under Uncertainty,* **International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering, World Academy of Science, Engineering and Technology (WASET), 2014, 8 (9), pp 1202-1206.**

[2] Elena M. Capitanul, Carlos Albert Nunes Cosenza, Walid El Moudani, Félix Mora-Camino, *A fuzzy dual approach for airport development risk assessment,* **19th International Conference of Hong Kong Society for Transportation Studies 13-15 Dec. 2014, Hong Kong, China.**

[3] E. M. Capitanul, H. Al-Fazari, C. A. Nunes Cosenza, W. El Moudani, F. Mora-Camino, *Fuzzy risk assessment for airport strategic planning,* **18th International Conference on Aerospace Engineering and Management, Dubai, UAE, 25-26 Feb. 2016.**

[4] E. M. Capitanul, F. Mora Camino, F. Krykhtine, C.A. Nunes Cosenza, *Fuzzy dual dynamic programming,* accepted to **12th International Conference on Natural Computation, Fuzzy Systems and Knowledge Discovery (ICNC-FSKD 2016) 13-15 August 2016 in Changsha, China.**

[5] E. M. Capitanul, F. Krykhtine, H. Al-Fazari C.A. Nunes Cosenza, F. Mora Camino, *Airport planning using fuzzy dual dynamic programming,* accepted to **XV SITRAER – Simposio de Transporte Aereo, 24-26 October 2016, Sao-Paolo, Brazil.**

[Page intentionally left blank]

Doctoral Researcher - Elena Mihaela CĂPITANUL

Doctoral Supervisor - Felix MORA-CAMINO

ABSTRACT

Airports are critical connectors in the air transportation operational system. In order to meet their operational, economic and social obligations in a very volatile environment, airports need to embrace change rather than resist it. Like any other industry, airports face a wide array of risks, some specific to air transportation, other having only an indirect influence but powerful enough to disrupt airport activities.

Long-term airport planning has become a complex issue due to the constant growth in air traffic demand. A new dimension of complexity emerged when uncertainty began having a more and more disruptive and significantly costly impact on developing airport infrastructure. Historically, the ability of traditional risk and uncertainty mitigation tools proved inefficient. Countless unforeseen events like terrorist attacks, economic recession, natural disasters, had a dramatic impact on traffic levels, some with a global reach. To these highly improbable type of events can be added technological advancements, new airlines and airports business models, policy and regulation changes, increasing concern for environmental impact.

In this context, the thesis puts forward an innovative approach for addressing risk assessment and mitigation under uncertainty in long-term airport infrastructure development projects. The thesis expands on the newly developed formalism of fuzzy dual numbers as a key tool to address uncertainty. After a comprehensive review of the airport industry in the context of uncertain environments, fuzzy dual numbers and fuzzy dual calculus are introduced. Since the airport infrastructure development project is another case of multi-stage decision making problem, dynamic programming is considered in order to optimize the sequential decision making process. The originality of the approach resides in the fact that the entire process will be *fuzzified* and fuzzy dual dynamic programming components will be introduced. To validate our method, a study case will be developed.

Key words: airports, optimization, fuzzy logic, dynamic programming, financial risk