



Designing new auction mechanisms for spectrum sharing in 5G networks

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**Jeux d'acteurs et mécanismes d'enchères dynamiques pour
la gestion du spectre dans les réseaux 5G**

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Résumé

Compte tenu des modèles de prévision de trafic pour les services data mobiles, une saturation des réseaux mobiles est prévue à horizon 2020. Le déploiement des systèmes de prochaine génération 5G a pour but d'adresser ce problème tout en permettant aux utilisateurs d'accéder à de nouveaux services très haut débit avec une qualité de service et d'expérience accrue. L'implémentation des systèmes 5G se fonde notamment sur une gestion optimisée des ressources des réseaux d'accès radio et donc des bandes de fréquences. En effet, actuellement les licences pour utiliser certaines bandes de fréquences sont achetées pour des dizaines d'années par les opérateurs via des mécanismes d'enchères, et l'utilisation de ces bandes leur est exclusivement réservée. Les années à venir vont voir la libéralisation de l'utilisation du spectre, dictée par la technologie, la régulation mais aussi la nécessité d'augmenter les ressources disponibles pour pouvoir supporter plus de trafic.

Plusieurs techniques sont en cours de développement, à commencer par le "Dynamic Spectrum Access" (DSA), DSA fait référence à la situation dans laquelle un utilisateur principal, qui a le droit exclusif d'utiliser la bande, partage sa bande de fréquence avec des utilisateurs secondaires. Les utilisateurs secondaires doivent permettre à l'utilisateur principal d'utiliser son spectre sans le perturber. Pour cela, ces systèmes utilisent généralement la radio cognitive: les utilisateurs secondaires - les opérateurs de réseaux mobiles dans notre contexte - peuvent détecter intelligemment les canaux de communication qui sont utilisés et ceux qui ne le sont pas, et passer aux canaux inutilisés. Cependant, pour les opérateurs mobiles, cette approche est risquée car ni l'accès au spectre ni la qualité de service (protection contre les interférences) ne sont garantis. Ainsi, le "Licensed Shared Access" (LSA) a été proposé comme une autre solution alternative pour le partage du spectre.

Le LSA a été proposé par le Radio Spectrum Policy Group (RSPG) en novembre 2013. Le concept implique trois parties: le propriétaire d'une bande de fréquence, les utilisateurs secondaires (des opérateurs) qui sont appelés titulaires d'une licence LSA et le régulateur. Contrairement au DSA, à travers le LSA, un utilisateur secondaire doit obtenir une licence auprès du régulateur avant d'accéder au spectre du titulaire. La licence comprend les conditions de partage, notamment en terme de durée, de fréquence et de zone géographique. Le concept LSA garantit au titulaire et au licencié LSA un certain niveau de QoS spécifié dans la licence LSA. Le déploiement d'un système LSA nécessite l'introduction de deux nouveaux blocs: le référentiel LSA et le contrôleur LSA.

Le référentiel LSA est une base de données qui contient des informations sur la bande LSA ainsi que les conditions de partage. Il est contrôlé par le régulateur et le titulaire initial, et est tenu de fournir les informations sur la disponibilité de la bande LSA. Le contrôleur LSA réside dans le domaine de l'opérateur et contrôle l'accès à la bande LSA en suivant les instructions reçues du référentiel LSA. Chaque opérateur doit avoir son propre contrôleur LSA.

Le concept LSA comporte deux différences majeures par rapport à l'attribution du spectre 3G ou 4G aux opérateurs. Tout d'abord, l'allocation doit fonctionner à une échelle de temps plus rapide, car la disponibilité du spectre LSA sera modifiée par le titulaire via le référentiel LSA, éventuellement plusieurs fois par heure, et le régulateur doit allouer le spectre LSA dès que son titulaire le libère afin d'améliorer l'utilisation du spectre. Deuxièmement, la réutilisation spatiale (les opérateurs qui n'interfèrent pas peuvent utiliser les mêmes bandes simultanément), doit être prise en compte.

L'attribution de la bande LSA se compose de quatre étapes: la première étape est la phase de préparation au cours de laquelle le titulaire identifie les parties de sa bande de fréquence qu'il pourrait partager avec d'autres en utilisant l'approche LSA. Dans la deuxième étape, le régulateur alloue le spectre LSA aux opérateurs via une licence. La phase de déploiement est la troisième étape. Dans cette étape, le titulaire efface la bande LSA de son propre usage et signale cette bande au référentiel LSA. Les opérateurs obtiennent la connaissance de la disponibilité du spectre LSA via le référentiel LSA et commencent à utiliser la bande LSA. Enfin, dans la phase de libération, les opérateurs libèrent la bande LSA lorsque la licence LSA expire.

La première bande candidate pour le contexte LSA est la bande 2.3-2.4 GHz. En France, cette bande est aujourd'hui utilisée par le ministère de la Défense pour certaines applications, notamment de télémesures aéronautiques. Le concept LSA semble être une solution gagnant-gagnant: pour les opérateurs, ils peuvent étendre leurs capacités et pour le titulaire, il peut gagner plus d'argent en libérant sa bande temporairement. Cependant, un aspect important qui devrait être pris en compte est la disponibilité de la bande LSA. En effet, les opérateurs peuvent ne pas être intéressés à utiliser une très petite quantité de spectre ou à utiliser le spectre pendant une trop courte période de temps. La mise en œuvre dépend de l'activité du titulaire de la bande, à titre d'exemple en France, on peut améliorer l'utilisation de la bande 2,3-2,4 GHz puisque 80% de cette bande pourraient être disponibles dans certaines zones géographiques. Plusieurs essais du concept LSA

ont eu lieu en Europe ¹ et ont montré son applicabilité. LSA en est actuellement aux étapes finales de normalisation et de validation sur le terrain, mais les détails sur la façon d'allouer et de tarifier le spectre entre plusieurs opérateurs potentiels restent ouverts.

En général, l'attribution des licences se fait selon deux approches: les démarches administratives (par exemple, comparaison des candidatures ou concours de beauté dans lequel un comité établit un certain nombre de critères et la licence est attribuée à l'opérateur avec la meilleure combinaison de ces critères) et les approches basées sur le marché (par exemple, *enchères*). Les approches administratives sont utilisées lorsque la demande est inférieure à l'offre ou lorsque le régulateur et les opérateurs peuvent trouver un accord pour diviser le spectre à un certain prix. Cependant, lorsqu'on ne peut pas satisfaire tous les opérateurs ou en cas de manque de ressources, les enchères sont le moyen le plus équitable d'attribution de spectre: puisque le régulateur ignore la valeur que la bande a pour les opérateurs, une approche naturelle est de leur faire déclarer cette valeur, via un mécanisme d'enchère et ensuite il peut allouer les ressources de la manière la plus efficace, afin de maximiser la valeur résultante pour le marché.

La bande 2,3-2,4 GHz est considérée comme une ressource très précieuse grâce à sa capacité de propagation et à offrir un débit élevé. De plus, selon une étude économique [1], les économies réalisées sur les coûts d'infrastructure s'élèvent à environ 12 milliards € lorsque la bande LSA est disponible pour les opérateurs. Ainsi, les enchères sont plus adéquates pour allouer du spectre aux opérateurs dans ce contexte.

Le régulateur définit les règles de l'enchère (attribution des fréquences LSA et paiements) et gère l'enchère. Une question fondamentale est: comment définir le mécanisme d'enchère? La définition d'un mécanisme d'enchère dépend des objectifs du régulateur. Par exemple, si le régulateur veut simplifier l'enchère aux opérateurs alors il doit concevoir un mécanisme d'enchère **sincère**. Un mécanisme d'enchère est sincère ("truthful") lorsque chaque opérateur maximise son gain en révélant ses préférences au régulateur. C'est une propriété importante car les opérateurs n'ont pas à construire des stratégies complexes pour les enchères. En particulier, cette propriété induit une certaine équité dans la participation: les participants plus riches ne peuvent pas prendre l'avantage sur leurs concurrents en mettant en œuvre des mesures coûteuses pour optimiser leur stratégie d'enchères. D'autres considérations peuvent être prises en compte, afin d'évaluer un mécanisme d'enchère, tel que le **revenu** du régulateur, l'**équité** et l'**efficacité** de l'allocation.

Il existe deux grandes catégories d'enchères: les enchères fermées et les enchères ou-

¹<https://www.cept.org/ecc/topics/lisa-implementation>

vertes. Une enchère fermée est composée d'un seul tour, le processus est asynchrone et les participants ont une seule chance pour interagir avec le commissaire-priseur. Les enchères ouvertes nécessitent la présence (physique ou électronique) de tous les participants en même temps. À chaque tour, les prix peuvent augmenter (enchère ascendante) ou diminuer (enchère descendante). Les enchères ouvertes ont été utilisées avec grand succès pour l'allocation du spectre et elles sont préférées aux enchères fermées. En effet, ces enchères présentent plusieurs avantages: elles préservent la confidentialité du ou des enchérisseurs gagnants car le ou les gagnants n'ont pas besoin de révéler leurs valeurs. De plus, elles sont plus transparentes: chaque enchérisseur voit l'évolution de l'enchère.

Les enchères traditionnelles sont largement discutées mais la plupart d'entre elles ne peuvent pas être directement appliquées à notre scénario car elles ne prennent pas en compte l'aspect dynamique de LSA (l'exécution de ces enchères peut prendre un temps énorme par rapport à la disponibilité de la bande LSA) ou à la réutilisation du spectre: deux stations de base qui n'interfèrent pas peuvent utiliser la bande en même temps. Une spécificité des licences LSA réside dans les interactions entre les opérateurs, en raison du chevauchement éventuel des zones de couverture, d'où la possibilité limitée d'allouer le même spectre à plusieurs opérateurs. L'interférence entre les stations de base est gérée par des groupes. Un groupe est un ensemble de stations de base de différents opérateurs de sorte que deux stations de base dans le même groupe n'interfèrent pas, par conséquent le spectre attribué à un groupe peut être utilisé par tous les membres du groupe. La création des groupes est effectuée par le régulateur avant le déroulement de l'enchère. Une fois les groupes constitués, chaque station de base soumet son bid au régulateur et celui-ci effectue les allocations et les paiements à l'aide d'un mécanisme d'enchère. La réutilisation du spectre complique l'analyse des enchères: en général, les enchères ont été interprétées comme un jeu compétitif car les enchérisseurs sont en concurrence pour obtenir des articles (chaque article est attribué à un et un seul enchérisseur). Cependant, dans notre scénario, il n'y a pas nécessairement de concurrence ou de conflit (au sens de l'allocation) entre les opérateurs qui n'interfèrent pas entre eux.

Récemment, de nombreux travaux ont été proposés qui prennent en compte la réutilisation du spectre et peuvent être appliqués au contexte LSA. Cependant, l'enquête sur les revenus et l'équité de l'allocation n'a pas suffisamment retenu l'attention. Une autre limitation de ces travaux est que lorsqu'ils envisagent la réutilisation du spectre, ils séparent les stations de base en groupes de telle sorte que chaque station de base appartient à un et à un seul groupe. Cependant, permettre les stations de base à appartenir aux plusieurs

groupes peut conduire à améliorer les performances de l'enchère. De plus, ces mécanismes de la littérature sont tout ou rien, c'est-à-dire que toute la quantité disponible de spectre LSA est considérée comme un bloc et allouée à un et un seul groupe. Une question peut se poser: peut-on améliorer le résultat de l'enchère en divisant le spectre disponible en plusieurs blocs? Si oui, comment convertir les mécanismes d'enchères tout ou rien en mécanismes d'enchères multi-blocs? Aussi, ce sont des enchères fermées. En général, les opérateurs préfèrent les enchères ouvertes où les informations sont révélées au cours d'une phase de convergence. Dans cette thèse on investigate la possibilité d'extension de ces hypothèses et ces choix. Le manuscrit est composé de sept chapitres.

Chapitre 1 introduit les motivations de la thèse ainsi que le vocabulaire associé à LSA et aux enchères.

Chapitre 2 donne la définition d'un mécanisme d'enchère pour LSA et formalise les propriétés souhaitées pour un tel mécanisme. Les notions essentielles de sincérité, d'efficacité, de rationalité individuelle et d'équité sont expliquées dans ce chapitre. Les étapes des enchères étudiées dans le manuscrit, incluant la notion d'enchère du groupe, sont listées. Le modèle d'interférence choisi pour la thèse est également détaillé dans ce chapitre.

Chapitre 3 décrit une méthode d'estimation de la valeur du spectre, une donnée nécessaire aux opérateurs pour participer aux enchères. La méthode proposée établit les liens qui unissent la qualité de service dans le réseau, la satisfaction des utilisateurs et le revenu pour l'opérateur. On choisit des modèles bien établis dans la littérature pour montrer comment le spectre LSA peut être valorisé.

Dans le chapitre 4, on illustre les notions de sincérité, de revenu, d'efficacité et d'équité. On étudie les mécanismes d'enchères de la littérature qui sont des candidats potentiels pour allouer le spectre dans contexte LSA. Ces mécanismes attribuent la bande au groupe ayant le group-bid (le bid global d'un groupe) le plus élevé. Pour chaque groupe, TAMES [2] et TRUST [3] calculent le bid du groupe en fonction du bid minimal de sorte que la décision prise pour un groupe (qu'il s'agisse du groupe gagnant ou non) est basée uniquement sur ce bid minimal. Afin d'impliquer davantage des stations de base dans le calcul du group-bid, le LSAA [4] a été proposé comme solution alternative. On montre que ce mécanisme n'est pas sincère et deux variantes sincères de LSAA sont proposées: TLSAA et TLSAA2. TLSAA surpasse LSAA en terme d'efficacité et TLSAA2 a les mêmes revenus que LSAA en supposant une enchère sincère. Le principal problème des mécanismes mentionnés ci-dessus est qu'on n'a pas de garantie en termes d'efficacité qui peut être indésirable du point de vue du régulateur. Par conséquent, on montre

comment obtenir une efficacité optimale en implémentant le mécanisme Vickrey-Clarke-Groves (VCG) [5–7]). Comme le régulateur peut également être sensible à son revenu, on montre comment augmenter le revenu en ajoutant un prix de réserve R par station de base, c’est-à-dire que chaque station de base doit proposer au moins ce montant pour participer à l’enchère.

Dans un deuxième temps, on propose PAM, un mécanisme d’enchère qui partage la bande LSA proportionnellement entre les groupes. PAM surpasse les autres mécanismes en terme d’équité. Aussi, lorsque $R = 0$, contrairement aux autres mécanismes (sauf VCG), l’efficacité de PAM ne peut pas être arbitraire loin de l’optimale. Aussi, on montre que l’allocation PAM correspond au “Nash bargaining solution”: supposons qu’il n’y ait pas de processus d’enchère et que les groupes doivent négocier afin de partager la bande LSA sinon (si aucun accord n’est atteint), chaque groupe obtient zéro. Les groupes doivent s’entendre sur certains axiomes “rationnels” comme par exemple l’axiome de symétrie: si deux groupes ont le même group-bid, ils doivent obtenir la même quantité. En fait, Nash bargaining solution est adéquate dans ce cas car elle représente la solution unique qui satisfait quatre axiomes: symétrie, indépendance des alternatives non pertinentes, optimalité de Pareto (personne ne peut avoir de meilleurs résultats sans nuire à l’autre) et l’invariance à la transformation affine. Cela représente un avantage de l’allocation de PAM car cette allocation semble en quelque sorte naturelle et non imposée aux groupes.

PAM présente plusieurs avantages sous l’hypothèse que chaque groupe peut accepter n’importe quelle fraction de la bande même si elle est trop petite ce qui n’est pas forcément le cas en pratique. Afin de rendre le mécanisme plus approprié pour être appliqué dans la vie réelle où de très petites portions de spectre pourraient ne pas être utiles aux opérateurs, on propose PAM $^\sigma$. Le régulateur fixe une fraction minimale σ : chaque station de base doit obtenir au moins cette fraction ou elle ne reçoit rien. On analyse le trade-off entre l’efficacité et l’équité après l’introduction de σ : PAM $^\sigma$ offre une efficacité plus élevée et une équité inférieure par rapport à PAM, cela est naturel car certains groupes seront exclus (ce qui conduit à diminuer l’équité) et la fraction d’origine allouée à ces groupes (par PAM) sera utilisée par d’autres groupes dont les valuations sont plus élevées (ce qui augmente l’efficacité). En particulier, quand $\sigma > 0.5$ on montre que PAM $^\sigma$ est égale à VCG.

Dans le chapitre 5, afin de donner au régulateur plus de flexibilité dans la construction du groupe, on relaxe l’hypothèse “chaque station de base doit appartenir à un et un seul groupe”. On montre comment adapter les règles de paiement des mécanismes

précédents, lorsque cela est possible, pour maintenir une enchère sincère sans modifier la règle d'allocation. Cette relaxation a un impact sur le résultat de l'enchère, par exemple, elle peut augmenter l'efficacité. Dans un deuxième temps, nous avons défini un opérateur qui coordonne plusieurs stations de base comme un joueur, cela peut être plus proche de la réalité. Cependant, la plupart des résultats étaient négatifs, nous avons montré qu'avec cette hypothèse, seul VCG peut être appliqué. Pour les autres mécanismes ils ne sont plus sincères.

Dans le Chapitre 6, on suppose que la quantité du spectre LSA peut être divisée en K blocs de même taille. De plus, on suppose que le spectre dans la bande 2,3 - 2,4 GHz est homogène, c'est-à-dire que les opérateurs ne sont sensibles qu'à la quantité de spectre obtenue et non à une bande de spectre spécifique; et donc ces blocs sont identiques. De plus, on suppose que chaque station de base a une fonction de valuation concave. Dans ce scénario, chaque station de base soumet au régulateur un vecteur de bids composé par K éléments. Le n^{th} élément représente le bid pour un $n^{\text{ième}}$ bloc supplémentaire étant donné qu'elle a déjà $n - 1$ blocs.

On montre comment convertir un mécanisme d'enchère tout ou rien en un mécanisme multi-blocs. De plus, on étudie le fractionnement du spectre, c'est-à-dire que pour une quantité donnée de spectre, on montre comment définir K afin de maximiser l'utilité du régulateur. De plus, lorsqu'une station de base peut appartenir à plusieurs groupes, on prouve que les problèmes initiaux (construction des groupes et allocation) sont NP-difficiles. Par conséquent, on propose une heuristique pour le regroupement. Cette heuristique est facile et rapide à construire mais peut avoir un impact négatif sur l'efficacité. La perte en terme d'efficacité est quantifiée. La simulation suggère que l'heuristique fonctionne bien et que son efficacité est au moins 60 % de celle de l'optimale.

Dans Le Chapitre 7, on suppose à nouveau que le spectre LSA est composé par K blocs identiques, mais contrairement aux travaux précédents mentionnés qui sont des enchères fermées, on se concentre sur les "enchères ascendantes" où l'information est révélée par les stations de base au cours d'une phase de convergence. On commence d'abord par étudier un mécanisme, qui est équivalent à VCG (en terme d'allocation et paiement), proposé par Mishra et Parkes [8], on montre comment l'adapter au contexte LSA. Comme on peut avoir une certaine complexité de calcul, on utilise une autre approche "l'approche de clinching" proposée par Ausubel [9]. À chaque tour, le régulateur déclare un prix et les enchérisseurs répondent en demandant une quantité (à chaque tour la quantité demandée ne peut pas dépasser la quantité demandée au tour précédent) à ce prix, le prix augmente

jusqu'à ce que la somme des demandes ne soit pas supérieure à K . Les paiements sont calculés au cours de l'enchère. On montre, en utilisant l'approche de clinching, comment convertir des mécanismes d'enchères fermés en des mécanismes d'enchères ascendantes. Deux méthodes sont proposées pour cette conversion. La première méthode consiste à introduire un représentant par groupe afin que chaque représentant agisse au nom des membres de son groupe. Cette approche préserve la confidentialité des valuations des gagnants. Cependant, les stations de base ne voient pas l'évolution de l'enchère qui peut être indésirable. Dans la deuxième méthode, on montre comment implémenter la version ascendante lors de la suppression des représentants afin que l'enchère se déroule directement entre les stations de base et le régulateur et donc la transparence est garantie.

Le travail de cette thèse a de nombreuses ouvertures pour de futures extensions. Tout d'abord, on a considéré une interférence binaire, c'est-à-dire que pour chacune des deux stations de base, ou bien elles interfèrent ou bien elles n'interfèrent pas. Il serait intéressant de considérer des modèles où l'interférence est traitée en fonction de la puissance d'émission. Il serait également intéressant de traiter l'enchère LSA comme un jeu répété car le régulateur peut exécuter l'enchère plusieurs fois par jour dans certaines configurations. Un autre critère important est de savoir comment partager les revenus entre le titulaire de la bande et le régulateur. Il serait également important de faire fonctionner les mécanismes d'enchères en contexte réel afin de tester les performances. Les auteurs de [10] ont proposé une plate-forme du système d'enchères en ligne pour le partage de spectre LSA (ALS3). Il serait intéressant de mettre en œuvre les propositions sur cette plate-forme.

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Part I

Introduction and mathematical background

Chapter 1

Introduction

Accommodating exploding data traffic is among the greatest challenges for fifth generation (5G) [11]. According to some estimations, data rates will be multiplied by 10 compared to 4G [12], latency must go down to one millisecond or less [13, 14]. 5G use cases can be mapped to three different classes: the first class is Machine-Type Communications (MTC) [15] which will create an environment of smart cities based on a new concept called internet of everything [16]. Ultra-Reliable Low-Latency Communications (URLLC) is the second class. This category includes new services, such as connected autonomous vehicles, in which the level of latency is vital [17]. The third class is enhanced Mobile BroadBand (eMBB).

The third category of use cases, eMBB, can be interpreted as an extension or an evolution of 4G networks. Compared to 4G networks, 5G networks have to offer higher user mobility, enhanced connectivity and higher capacity [18]. According to [19], cellular network capacity may need to deliver as much as 1000 times the capacity of 4G. Accommodating that traffic needs much larger bandwidths than the actual ones. Therefore, millimeter wave bands (frequency spectrum from 30 GHz to 300 GHz¹) will be deployed. In the millimeter-wave range, data rates can reach 10 Gbits/s and more. However millimeter waves cannot be used for all types of wireless applications because it has a limited range. Contrary to millimeter waves, spectrum below 6 GHz provides a wide-area coverage. Currently some holders (through a licence) of those frequencies (e.g., military, satellites, some commercial users), which are called incumbents, do not always use all their frequencies: usage varies with time and geographical location [20, 21]. Hence there

¹<https://www.electronicdesign.com/technologies/communications/article/21796260/millimeter-waves-will-expand-the-wireless-future>

is some room for improvement which has given rise to the proposal of the concept of Dynamic Spectrum Access (DSA) [22,23].

DSA refers to the situation in where a primary user, who has an exclusive right to use the band, shares his bandwidth with a secondary user [24]. Secondary users must allow the primary user to use his spectrum without disrupting it. For this, these systems typically use cognitive radio [25,26]: secondary users—Mobile Networks Operators (MNOs) in our context—can intelligently detect those communication channels that are in use and those that are not, and move to unused channels [27]. However, for MNOs this approach is risky because neither the access to spectrum nor the quality of service (protection from interference) are guaranteed [28]. Thus, the Licensed Shared Access (LSA) was proposed as another alternative solution for spectrum sharing.

1.1 Licensed Shared Access for dynamic spectrum access

In November 2011, in order to support the deployments of 5G systems, the Radio Spectrum Policy Group (RSPG) has proposed a new sharing concept called Licensed Shared Access (LSA) [29–31] (Fig. 1 summarizes the relation between LSA concept and 5G). The concept involves three stakeholders: the owner of the 2.3 – 2.4 GHz bandwidth (the incumbent), the secondary user which is called LSA licensee, and the regulator [22]. The frequency band is used by different incumbents in Europe [32] (e.g. in France it is used by the military). Contrary to DSA, under the LSA approach, the secondary user needs to obtain a license from the regulator before accessing the spectrum of the incumbent. The license includes the conditions of sharing, in particular in terms of time, frequency and geographic region. The LSA concept guarantees to the incumbent and to the LSA licensee a certain level of QoS [33] specified in the LSA license. The LSA licensee is typically an MNO. Deploying an LSA system requires the introduction of two new architectural building blocks [34]: the LSA repository and the LSA controller. The LSA repository is a database which contains information about LSA spectrum bands together with their conditions of sharing. It is controlled by the regulator and the incumbent, and is required to deliver the information on spectrum availability based on the incumbent spectrum use and associated conditions for sharing. The LSA controller resides in the network operator’s domain and controls the access to the incumbent’s spectrum by following the instructions received from the LSA repository. Each MNO has to have his own LSA

controller.

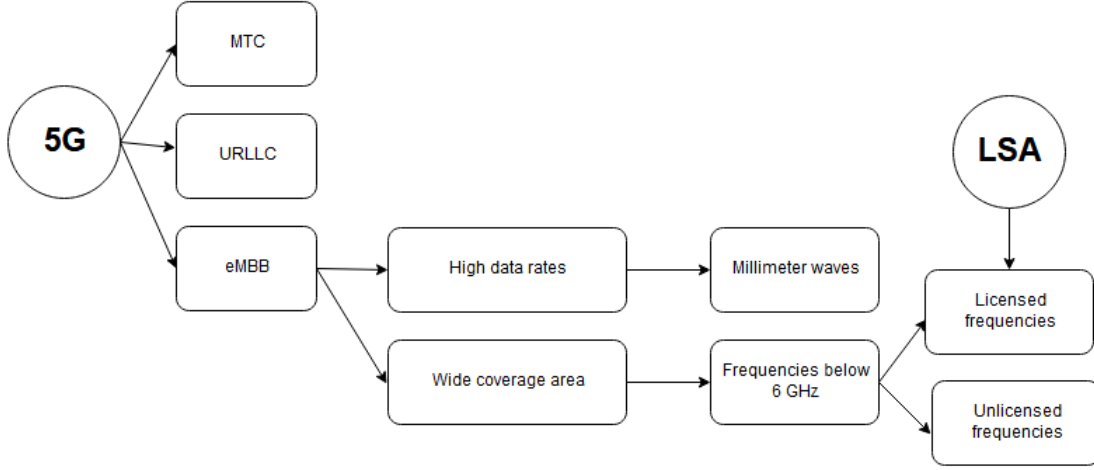


Figure 1 – LSA and 5G relation

The LSA concept involves two major differences with regard to the allocation of 3G or 4G spectrum to operators. First, the allocation needs to work at a faster time scale, since the availability of LSA spectrum will be changed by the incumbent via the LSA repository, possibly several times per hour, and the regulator has to allocate the LSA spectrum for potential LSA licensees as soon as the incumbent releases his spectrum in order to improve the use of the spectrum. Second, spatial re-usability (MNOs who do not interfere can use the same spectrum bands simultaneously), should be leveraged, more will be said about it in the next chapter.

1.1.1 LSA workflow

The LSA workflow is composed of four steps: the first step is the preparation phase in which the incumbent identifies parts of the spectrum bands that it could share with others using the LSA approach, then it reports initial sharing conditions to the regulator. In the second step the regulator allocates the LSA spectrum to MNOs via a license. The deployment phase is the third step, in this step the incumbent clears the LSA bands from its own use and reports these bands to the LSA repository. The MNO obtains knowledge of LSA spectrum availability from the LSA controller via the LSA repository. The MNO plans the Radio Access Network (RAN) for the LSA band according to the license rules. Finally, in the release phase, the MNO clears the LSA band when the LSA license expires.

The MNO informs the incumbent when the band is released to allow the incumbent to use it for its own operations.

An important problem that should be addressed is the availability of the LSA band i.e., the activity of the incumbent. Indeed, MNOs may be not interested in using a very small amount of spectrum or to use spectrum for a short period of time. The investigation of the availability of LSA bandwidth for some European country is given in [1]. The implementation depends on the activity of the incumbent as an example in France, we can improve the use of the 2.3 – 2.4 GHz since 80% of that band could be available in some geographical areas which are important for MNO (since those areas cover 80 % of population). Several trials of the LSA concept have taken place in Europe ¹ and have shown its applicability. LSA is now under the final stages of standardization and field validation [35] as regards the technical aspects, but the specifics of how to allocate and price spectrum among several potential secondary users remain open.

A key objective for LSA is to allocate the spectrum in the most efficient way, so as to maximize the resulting value to the market. In general the attribution of licences is done via two approaches: administrative approaches (e.g., comparison of candidacies or beauty contest in which a committee sets a number of criteria and the license is attributed to the MNO with the best mix of those criteria [36]) and market-based approaches (e.g., *auctions*) [37]. Administrative approaches are used when demands are lower than supply or when the regulator and MNOs can find an agreement to split spectrum at some price [38]. However, when we cannot satisfy all MNOs or there is a lack of resources, auctions are the fairest means for spectrum assignment: since the regulator ignores the valuation that the bandwidth has for operators, a natural approach is to have them declare that valuation, through an auction mechanism and then he can allocate resources in the most efficient way, so as to maximize the resulting value to the market.

The 2.3 – 2.4 GHz is considered as a very valuable (and therefore scarce) resource thanks to its ability to travel far and offer high capacity. In addition, according to an economic study [1], the benefits from savings in infrastructure costs are around 12 billion € when allowing that bandwidth available for MNOs. Ericsson, NSN and Qualcomm encourage European regulators to consider that study. Thus, auctions are more adequate to allocate spectrum for MNOs in this context.

¹<https://www.cept.org/ecc/topics/lsa-implementation>

1.1.2 Properties and indicators for LSA frequency allocation auctions

The participants in the auction are the *auctioneer* (the regulator in the LSA context) and the *bidders* (MNOs) who compete to obtain the item(s) (LSA spectrum). The auctioneer defines the rules of the auction (allocation of item(s) and payments) and runs the auction. Also, before participating in the auction, each MNO has to compute his valuation for the LSA spectrum (how much is it worth). In the literature [2–4], it is common to suppose that each bidder knows his valuation. In reality, MNOs may face significant challenges in valuing LSA spectrum. Indeed, valuing spectrum is a very complex task because many factors can affect that valuation [39].

In order to evaluate the performance of an auction, several metrics can be used, we focus on the most used metrics and properties in the literature [40, 41] which are:

- The *revenue* of the auction which is the sum of payments of bidders.
- *Individual rationality*, which means that a bidder has a bidding strategy that ensures him to get a non-negative payoff so he is always better off participating in the auction than staying out [42].
- The *fairness of the allocation*. (we detail that property in the next chapter).
- The *efficiency* of the auction which is defined as the sum of the valuations served [43].
- *Truthfulness*: an auction mechanism is truthful or incentive compatible if and only if for each bidder, declaring truthfully one's preferences maximizes one's utility given any fixed bids of the other bidders.

Truthfulness is very important because it reduces the complexity of bidding process, since the strategies to use are very simple (just declare one's preferences). In particular, that property induces some fairness in participation, in the sense that wealthier bidders cannot get an edge over competitors by implementing costly measures to optimize their bidding strategy. Also, this property is desirable from the auctioneer point of view because it is simpler to base the allocation optimization (with respect to any indicator) on real utilities rather than unfaithful ones.

As for any multi-constraint problem, it is not possible to jointly maximize satisfaction of all properties. It was proven in [44] that the Vickrey-Clarke-Groves (VCG) [5–7] mechanism is the unique truthful auction mechanism which is individually rational and maximizes efficiency. However, as for most proposed auction mechanisms for the LSA context [3, 4, 45], there are no guarantees in terms of revenue for VCG.

Auctions can be interpreted from different angles. From mechanism design perspectives, auctions are allocation mechanisms. From game theory perspectives: each auction mechanism is defined by a set of rules, since the choice of each bidder will affect the outcome of other bidders, those rules can be mapped to a mathematical formulation which we call game. A game is defined by the set of players, strategies and payoffs [46]. Players are bidders, a strategy for a player determines the action that he has to take (proposing a bid) and payoff of a bidder which is his utility. Thus finding the optimal strategies for players in the game, if they exist, leads to finding the optimal strategies of bidding in the auction.

There are two major categories of auctions [47]: *one-shot* auctions and *multi-round* auctions. A one-shot auction is composed of a single round. In general, they are also called *sealed-bid auction* [48], each bidder submits his bids without knowing the bids of other bidders. Each bidder has one chance to interact with the auctioneer. The traditional reasons for the sealed auctions are: simplicity (the seller has to announce just the form of bids), they are easier and faster to implement and finally there is no need to bring bidders together (this reason is irrelevant nowadays thanks to technology).

When there are many objects to allocate, we can use combinatorial auctions [49] so each bidder submits a bid for each bundle of objects. If the objective of the regulator is to maximize efficiency, then the allocation is done by solving the winner determination problem [50, 51] in order to select the value-maximizing allocation of disjoint bids.

Contrary to one-shot auctions, multi-round auctions are more transparent because bidders see the evolution of the auction. At each round, prices may increase or decrease: there are two standard types of multi-round auction for a single item: the ascending auction [52] and the descending auction [8]. In the ascending auction, the price increases until one bidder remains so that bidder is the winner and he pays the final price. The descending auction works in the opposite way i.e., at the first round the price is very high, the auctioneer reduces the price at each round. The first bidder who accepts the current price wins the auction and pays that amount.

For the general case when the auctioneer has many items, there are different types of

auctions [52] [53] [54] that could be applied for different scenarios. Indeed, an auction designer has to take into account: the objectives of the auctioneer, the nature of items (e.g., identical or not) and the utilities of bidders. In addition, in multi-round auctions, in order to avoid sniping –which happens when bidders start bidding at the end of the auctions– the auctioneer introduces some activity rules [55] to force bidders to be active at each round. Sniping is undesirable because it limits the information discovery for some bidders.

1.1.3 Why designing new auction mechanisms for LSA?

Traditional auctions are widely discussed (see e.g. [56, 57]), however most of them cannot be directly applied to our scenario because they do not take into account either the dynamic aspect of LSA (running those auctions may take a huge time compared to the availability of the LSA band) or wireless spectrum re-usability specific consideration. Spectrum re-usability complicates the auction analysis: in general, auctions were interpreted as a competitive game because bidders are in competition to obtain items (each item is attributed to one and only one bidder). However in our scenario, there is not necessarily a competition or conflict (in the allocation sense) between MNOs who do not interfere with each other. As example, as shown in Fig. 2, base station (BS) 1 and BS 3 can use the same bandwidth simultaneously since there is no interference between them.

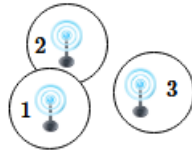


Figure 2 – Some base stations with their coverage areas

Recently, many works have been proposed which take into account spectrum re-usability [2, 3, 58] and can be applied for the LSA context. However, the investigation on the revenue and the fairness of the allocation have not drawn enough attention.

Another limitation of those works is that when they consider spectrum re-usability, they separate BSs into non-overlapping groups (partition). As an example, for Fig. 2, a possible partition is to put BS 1 and BS 3 in a group and BS 2 in another group. In other words, all previous works do not allow overlapping among group (otherwise they are not anymore truthful). However, allowing overlapping among groups (example: put BS 1 and

BS 3 in a group and BS 2 and BS 3 in another group) may leads to increase the outcome of the auction. Therefore, the impact of overlapping needs to be investigated. Next we summarize the document structure and the main contributions.

1.2 Document structure and contributions

The document is organized as follows: after providing some mathematical background in Chapter 2, in Chapter 3, we address the LSA spectrum valuation. A simple model is proposed and then used as a basis for possible application when real data is available. Part II is composed of Chapter 4 and Chapter 5, we suppose that the regulator has a small quantity of LSA spectrum to allocate. In Chapter 4, we investigate the auction mechanisms which were proposed as candidates for LSA context. We propose two truthful variants of LSAA, the first auction mechanism which was proposed as candidate for the LSA context, we show also how to increase the revenue of all-or-nothing mechanisms while maintaining truthfulness by introducing a minimum amount paid per bidder, a contribution presented at IFIP Performance'18 [59]. At a second time, we show how to increase fairness of the allocation by sharing the available LSA spectrum among all base stations, this work was presented at PIMRC'18 conference [60]. In Chapter 5, we investigate the impact of groups overlapping. Overlapping complicates the auction analysis; we show how to adapt the payment rule when it is possible so that mechanisms are still truthful.

Part III is composed by two chapters: Chapter 6 and Chapter 7. In Chapter 6, we suppose that the available quantity of spectrum can be split into K identical blocks. We show how to adapt the one-shot version of VCG and study the impact of allowing a BS to be in several groups on the complexity of the allocation, a work published in NETGCOOP'18 [61]. Then, we show how to convert all-or-nothing auction mechanisms into a multi-blocks auction mechanisms. This work is currently under review by the journal Netnomics. In Chapter 7, we show how to implement the equivalent ascending version of one-shot multi-blocks auction mechanisms. Finally in part IV, we conclude the document and derive some perspectives.

Chapter 2

Mathematical background

In this chapter, we start by defining what is an auction mechanism for the LSA context and we formalize the required properties of LSA auctions in Section 2.1. Then, we present in Section 2.2 the model used in the literature in order to manage interference among BSs.

2.1 Auction mechanisms and properties

In this section we provide the definition of an auction mechanism and the most desirable properties. In this section as in the rest of the document, we shall use bidder, player and BS interchangeably (unless where specifically mentioned).

2.1.1 Auction mechanism

An auction mechanism is an allocation mechanism used in order to assign some item(s) (LSA spectrum in our context) for bidders. An auction mechanism (MEC) takes some bids submitted by N bidders under a predetermined format and returns two components [62]:

- an allocation of the goods among the bidders $\alpha^{\text{MEC}} = (\alpha_1^{\text{MEC}}, \dots, \alpha_N^{\text{MEC}})$.
- a payment vector $p^{\text{MEC}} = (p_1^{\text{MEC}}, \dots, p_N^{\text{MEC}})$, where p_i^{MEC} is the price that player i is charged.

Note that in the ascending auction, the auctioneer may collect bids at each round.

In the following section, we describe the most desirable properties from the point of view of the regulator.

2.1.2 Properties

In this section, we list the properties that a mechanism may satisfy, which are the most used in the literature [40, 41]. As for any multi-constraint problem, it is not possible to jointly satisfy all properties, hence the auction designer has to set a trade-off between them. For a bidder i , we denote by v_i his valuation for the item(s), by u_i his utility and by b_i his bid.

Truthfulness or Incentive Compatibility

A mechanism is truthful or incentive compatible if and only if for each player i , declaring truthfully one's preferences maximizes one's utility given any fixed bids of the other players. i.e.,

$$u_i(v_i, b_{-i}, v_i) \geq u_i(b_i, b_{-i}, v_i) \quad \forall b_{-i}.b_i, \quad (2.1)$$

where $b_{-i} = (b_1, \dots, b_{i-1}, b_{i+1}, \dots, b_N)$. Luckily, when bidders' allocation is one-dimensional, this property can be guaranteed in a quite general setting. Let us first introduce the following definition.

Definition 1. (*Implementable allocation rule*) [63] *An allocation rule is implementable if there is a payment rule such that each bidder can maximize his utility by reporting his valuation independently of bids of other bidders. .*

Myerson indeed showed in a lemma [64] that an allocation rule $\alpha_i(b_1, \dots, b_N)$ is implementable if and only if it is monotone. An allocation rule is monotone if for each player i and bids b_{-i} , $\alpha_i(b_i, b_{-i})$ is non-decreasing in b_i . In addition, if we add the constraint that a zero bid implies a zero payment, the payment rule is unique. Roughgarden details that payment rule in the general case [65]: given a piecewise constant monotone allocation curve as shown in Fig. 3a, each player i should pay a price as a function of the corresponding *breaking points* (points at which i 's allocation changes) in the range $[0, b_i]$. Specifically, if there are X breaking points (z_j) then the payment is given by:

$$p_i(b_i, b_{-i}) = \sum_{j=1}^X z_j \cdot (\text{jump in } \alpha(\cdot, b_{-i}) \text{ at } z_j). \quad (2.2)$$

This price corresponds to the greyed surface in Fig. 3a ($X = 3$). In particular, if there is one and only one indivisible item, i.e., the allocation is either 1 or 0 as shown in Fig. 3b

then there is one and only one breaking point for each player, that is his minimum bid to win the auction. As an example, in the second-price auction the breaking point for each player is the maximum bid of the other players. In addition, without losing truthfulness

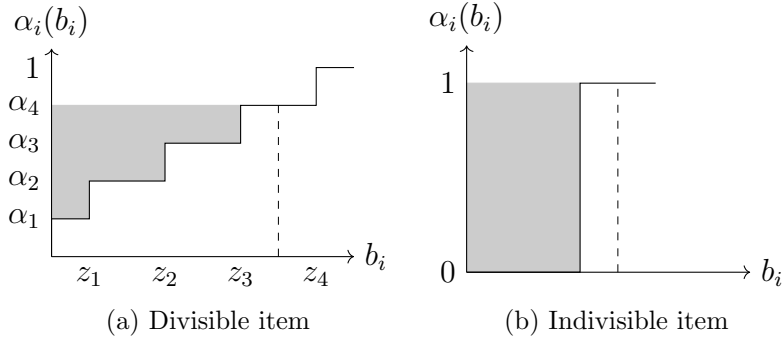


Figure 3 – Some piece-wise constant monotone allocation curves (solid curves). The grey area represents the price paid when the bid b_i has value .

and in order to protect himself from low revenues, the auctioneer may introduce a “reserve price per bidder”, imposing in the allocation rule that bids strictly below that price be allocated no resource [59]. By applying Myerson’s result above (since the allocation is still monotone), this involves that the per-unit revenue from each player is at least that reserve price: any bid strictly below it leading to a null allocation, the breaking point(s) for each winning player must at least equal that reserve price.

Individual rationality

This property means that a player has a bidding strategy (a function that transforms valuation to bid) that ensures him to get a non-negative utility, hence he is always better off participating in the auction than staying out of the mechanism.

Fairness of the allocation

According to Oxford English dictionary Fairness is “the quality of treating people equally or in a way that is reasonable”. This definition seems ambiguous: suppose we have two items and two bidders. Each bidder wants both items. For each item, the first is willing to pay 10 and the second is willing to pay 1. Treating bidders equally leads to give one item to each one. Treating people reasonably may lead to give both items to the first one. We believe that people perceive equality in the same way but for the reasonability

axiom, each one may have his own perception. Therefore, in this document we refer to the fairness as “the quality of treating people equally”. There exist several measures of fairness such as *max-min fairness*, *proportional fairness* and *Jain’s index* [66]. A detailed description of those measures is given in [67]. We will use Jain’s index which is given by:

$$J(\alpha^{\text{MEC}}) = \frac{(\sum_{i=1}^N \alpha_i^{\text{MEC}})^2}{N \sum_{i=1}^N (\alpha_i^{\text{MEC}})^2}.$$

This index is a continuous function of the allocations, with values in $[\frac{1}{N}, 1]$: it achieves its maximum 1 if all players obtain the same amount, and is minimum and equal to $\frac{1}{N}$ if one and only one player obtains some good. As another reference, a situation in which $a\%$ of users receive equal allocation and the remaining $(100 - a)\%$ receive zero [68] gives a Jain index of $a/100$. Motivated by those features we will use this index to measure the fairness of a mechanism’s allocation.

Efficiency

Efficiency E^{MEC} , of a given mechanism MEC, is defined as the sum of the valuations served [43]:

$$E^{\text{MEC}} = \sum_{i=1}^N \alpha_i^{\text{MEC}} v_i. \quad (2.3)$$

This means that the social value of the good being sold equals the maximum of the potential buyers’ individual valuations [69]. Since valuations are private (so they are unknown to the auctioneer), one may wonder how to maximize efficiency? i.e., how to set α ?. Indeed, if the auction is truthful then maximizing efficiency could be done based on bids instead of valuations.

2.2 Grouping discussion and model

In this section, we present the grouping model and preferences of bidders.

2.2.1 Grouping operators before the auction

Most LSA auction mechanisms which involve spectrum re-usability act as follows: they consider a scenario in which multiple BSs of different operators compete for LSA spectrum at a defined period of time in a particular geographical area; two BSs can use the same bandwidth simultaneously if they do not interfere with each other. This can be captured in a model by using an interference graph. Fig. 4 shows an example of an interference graph: BSs are represented by vertices, an edge between two vertices means that those BSs interfere. For example, in Fig. 4 BSs $\{3, 5\}$ can use the same fraction of bandwidth simultaneously. The competition between the BSs is transformed into a competition between groups in such a way that two BSs in the same group do not interfere, hence the spectrum allocated to a group is used by all the members of the group. An example of group constitution for the interference graph of Fig. 4 is: $g_1 = \{1, 2, 4, 6\}$ and $g_2 = \{3, 5\}$. Groups are formed and their composition sent to bidders before the auction takes place. Note that in some works [4, 70], grouping is done after receiving bids. In such a situation, bidders may try to manipulate their bids to change the groups formed so we may lose truthfulness.

Moreover, works schemes proposed in the literature [3, 58, 71] introduce the following two key assumptions. Without those assumptions, those works are not anymore truthful.

Assumption 1. *The grouping is made such that each base station belongs to one and only one group.*

Assumption 2. *Each base station is controlled by a different player, i.e., we assume non-coordination among bids submitted by base stations.*

A relaxation of assumption 1 means that each base station can be in several groups. A relaxation of assumption 2 means that each MNOs can coordinate several bids of BSs. We discuss those relaxation in chapter 5.

Grouping is very important because it has a direct impact on the outcome of the auction. As an example, if we want to maximize efficiency then a sufficient condition is to extract the maximum independent sets from the interference graph (groups are the maximum independent sets [72]): It consists in finding the largest subsets in a graph, such that there is no edge between two nodes of each subset and such that each subset is not included in another. In our context, it leads to the creation of the largest groups such that, each two base stations which do not interfere with each other belong to at least one same group. However there are two problems:

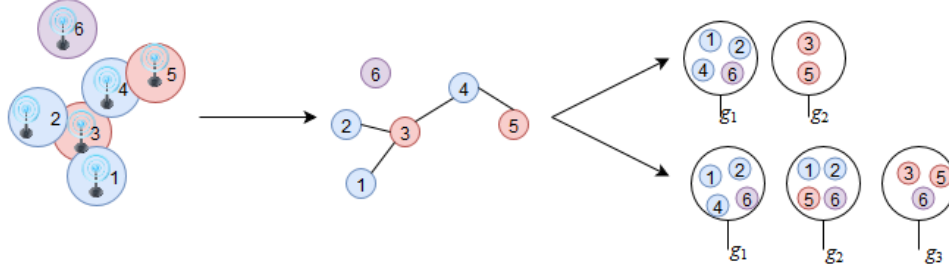


Figure 4 – Some BSs with their coverage areas (*left*), the corresponding interference graph (*center*), and two possible group configurations (*right*).

- First, extracting maximum independent sets is known to be an NP-hard problem [73, 74].
- It violates assumption 1 because BSs may belong to more than one group except for two special cases:
 - The interference graph is complete [75] i.e., there is an interference between each two BSs. In this situation we have M different groups, each group contains one and only one BS.
 - The interference graph is edgeless i.e., each two BSs do not interfere with each other. In this situation we have only one group composed of all BSs.

Therefore most of candidate mechanisms for LSA are not anymore truthful (except for the two special cases).

In the following we provide a simple algorithm for grouping.

2.2.2 Heuristic method for grouping

We sort in an ascending order all the BSs by the degree of interference. BSs with low degree are ranked first and grouped together as much as possible. This heuristic will lead to a group repartition in which each base station belongs to one and only one group, and that may have a negative impact on some properties such as efficiency. But, is easy and fast to build, and will create large groups to get close to the maximum independent sets approach.

Algorithm 1 Grouping algorithm implemented in this chapter

Ω : sorted BSs by increasing degree in the interference graph (ties broken randomly)

Let us denote by h the current number of groups and set $h = 1$

for $i = 1, i \leq N, i++$ **do**

for $j = 1, j \leq h, j++$ **do**

if vertex $\Omega(i)$ has no interference to any node in group g_j **then**

 Put the vertex (i) into g_j

 break

else

if $j == h$ **then**

$h++$

 create one more group g_h , put $\Omega(i)$ into g_h

end if

end if

end for

end for

2.2.3 Bidders preferences and group-bid

We suppose that spectrum in the 2.3 – 2.4 GHz is homogeneous i.e., MNOs are only sensitive to the amount of obtained spectrum and not to a specific spectrum band they can use. Also, we suppose that each bidder i has a concave valuation for spectrum as shown in figure 5 and quasi-linear utility (payoff) function which is common in the literature [76–78]: if it obtains a fraction $\alpha_i > 0$ of the available bandwidth and pays p_i , his utility u_i is his valuation for the obtained amount of spectrum minus the price paid for that amount.

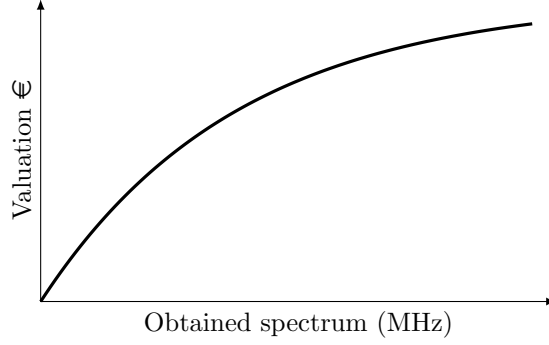


Figure 5 – An example of a concave valuation function of obtained spectrum for a bidder i .

The competition of bidders is through submissions of *bids*. Since the utility of each bidder depends on allocations and prices (computed based on bids), it is reasonable to assume that players will try to bid strategically to maximize their utility.

Before summarizing steps of the auction, let us first introduce the definition of the “group-bid”, which is a mechanism-specific quantity. Group-bid is very important because it will impact the outcome of the auction: after receiving bids of each group, the auctioneer constructs the group-bid (of that group) based on bids of its members. Then, he makes the allocation for groups based on those group-bids.

Definition 2. *The group-bid B_h^{MEC} of each mechanism MEC is a positive real obtained via a function f_h^{MEC} that takes bids from bidders of a group h (which contains n_h bidders) and returns a positive real.*

$$\begin{aligned} f_h^{\text{MEC}} : \quad \mathbb{R}^{n_h} &\rightarrow \mathbb{R} \\ (b_1, \dots, b_{n_h}) &\rightarrow B_h^{\text{MEC}} = f_h^{\text{MEC}}(b_1, \dots, b_{n_h}) \end{aligned} \quad (2.4)$$

We suppose that f_h^{MEC} is a continuous and non-decreasing with respect to each variable.

2.2.4 Steps of the auction

Steps of the auction are summarized in the following.

1. Group construction: from the interference graph, the regulator constructs groups.
2. Bid collection: bidders are asked to declare their bids.

3. Allocation: each base station is allocated some fraction α_i^{MEC} of the available spectrum.
4. Payment: each player i is charged a price p_i^{MEC} .

Note that for the case of ascending auction, the second and the third steps may be performed at each round. (The allocation at each round is temporary).

In the next chapter, we discuss the problem of assessing the value of spectrum for the MNOs.

Chapter 3

LSA spectrum valuation

3.1 Introduction

In this chapter, we address LSA spectrum valuation. There are several approaches for spectrum valuation [79] such as the engineering value [80], which refers to the potential saving in the infrastructure of the operator's network as a result of acquiring additional spectrum resources, and the economic value [81] which is the revenue surplus from the market when using that spectrum. Each approach may have its advantages and disadvantages; and also its use cases. But it has to be based on cogent reasoning, logic and scientific method. In this chapter, we propose to develop a model in order to compute the economic value of an LSA bandwidth.

3.2 LSA spectrum valuation model

In the following, we present our model for computing the LSA spectrum using the revenue surplus approach. We denote by W_n the normal bandwidth of base station i and by W_{tot} , the total bandwidth i.e., the package composed of the normal bandwidth and the LSA bandwidth. The valuation v_i is given by

$$v_i = V_i(W_{\text{tot}}) - V_i(W_n), \quad (3.1)$$

where $V_i(W_{\text{tot}})$ is the valuation of the total bandwidth and $V_i(W_n)$ is the valuation of the normal bandwidth. Now the question is: given a bandwidth W , how to compute $V(W)$?

We suppose that the valuation of a bandwidth during a period t is just the average

revenue from a user multiplied by the average number of users served during that period.

$$V(W) = N_t \tilde{\text{r\`ev}}, \quad (3.2)$$

where $\tilde{\text{r\`ev}}$ is the average revenue from a user and N_t is the average number of users served during t .

Authors in [82] have supposed that the revenue from a user depends on his satisfaction: the more he is satisfied, the more he pays. In addition, authors in [83] have noted that spectrum has more valuation in high-income region. Combining those assumptions we suggest the following representation of average revenue from a user:

$$\tilde{\text{r\`ev}} = c_u^z \tilde{S}, \quad (3.3)$$

where \tilde{S} is the average satisfaction and c_u^z is a constant in euro per unit of satisfaction in a given geographical zone.

User satisfaction depends on their perceived QoS [78, 84]. Indeed, many parameters (such as data rate, bit error rate) have an impact on user perceived QoS. However, it's common in the literature [78, 82, 85, 86] to express user satisfaction as a function of data rate. Results given in [87] suggest that user satisfaction keeps increasing with the data rate but more and more slowly. That approach is used in [85, 86]. In [78] authors have proposed the following formulation of user satisfaction:

$$S = 1 - e^{-\left(\frac{d}{d_{\text{com}}}\right)}, \quad (3.4)$$

where d is his data rate and d_{com} is a comfort data rate (can be interpreted as the mean data rate beyond which, user satisfaction exceeds 63% of maximum satisfaction [88]). Among the many factors that can affect the data rate of a user who belongs to BS i , we suppose that the main factors are:

- The total throughput D of BS i , D depends on the bandwidth W and other factors such as the digital modulation.
- The number of users connected to BS i .
- The maximum number N_m of users that can be served simultaneously by BS i .
- The scheduling i.e., how resources are divided when there are n users connected to BS i , we suppose that when there are n users, resources are allocated in such a way

that all users perceive the same data rate.

Now after defining the factors that impact user satisfaction, in the following we provide a model which computes the average user satisfaction from those parameters (the final formula is given in (3.6)). We use the Poisson process which is a simple and widely used stochastic process for modeling inter-arrival times [89]. In addition, we suppose that there are two types of users

- Type 1: Users with arrival rate λ_1 , each one is connected for a duration exponentially distributed with mean $\frac{1}{\mu}$ so the service rate is independent of the BS throughput (watching a video).
- Type 2: Users with arrival rate λ_2 , each one consumes a quantity of data exponentially distributed with mean m so the service rate depends on the available BS throughput (downloading an application, loading a web page).

At each time, when there are n users, resources are allocated in such a way that all users, independently of their types, perceive the same data rate. Therefore if there are i users of Type 1 and j users of Type 2, we can establish the following results:

- The service rate of users of Type 1 is $\mu_{i,j}^1 = i \times \mu$ (there are i users of Type 1 and the service time of each one is $\frac{1}{\mu}$).
- The service rate of users of Type 2, $\mu_{i,j}^2$ depends on their data rate. That data rate depends on the total throughput D and the total number of connected user (we have $i + j$ users). There are j users of Type 2, the service rate of each one is his throughput divided by m : $\mu_{i,j}^2 = \frac{\frac{D}{i+j}}{m} \times j$.

The number of connected users can be interpreted as a random process. Since, the evolution of that process depends only on the actual state then it can be considered as a Markov process [90]. As an example, if $N_m = 3$ the associated Markov chain of this process is given at Fig. 6 ($\mu_{\text{sat}} = \frac{D}{m}$).

We suppose that λ_1 , λ_2 , D , m and N_m are known so we can compute service rates. We denote by $\Pi_{i,j}$ the steady state probability when there are i users of Type 1 and j users of Type 2. The associated balance equations of the Markov chain can be established as follows:

- $(\lambda_1 + \lambda_2)\Pi_{0,0} = \mu_{1,0}^1\Pi_{1,0} + \mu_{0,1}^2\Pi_{0,1}$

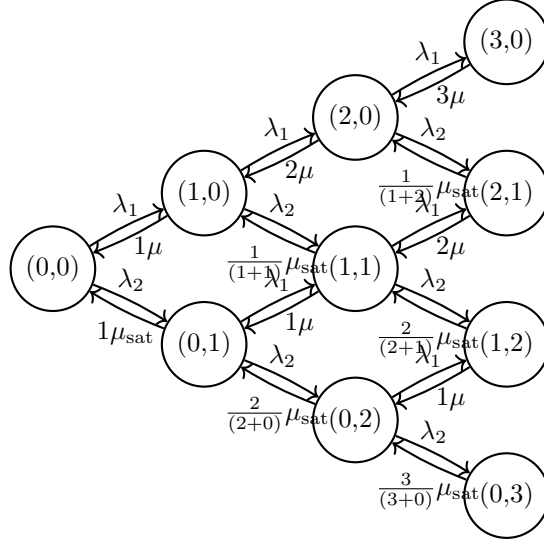


Figure 6 – Markov chain for $N_m = 3$

- $(\lambda_1 + \lambda_2 + \mu_{i,0}^1)\Pi_{i,0} = \mu_{i+1,0}^1\Pi_{i+1,0} + \mu_{i,1}^2\Pi_{i,1} + \lambda_1\Pi_{i-1,0}; i < N_m$
- $(\lambda_1 + \lambda_2 + \mu_{0,j}^2)\Pi_{0,j} = \mu_{0,j+1}^2\Pi_{0,j+1} + \mu_{1,j}^1\Pi_{1,j} + \lambda_2\Pi_{0,j-1}; j < N_m$
- $\lambda_1\Pi_{N_m-1,0} = \mu_{N_m,0}^1\Pi_{N_m,0}$
- $\lambda_2\Pi_{0,N_m-1} = \mu_{0,N_m}^2\Pi_{0,N_m}$
- $(\mu_{i,j}^1 + \mu_{i,j}^2)\Pi_{i,j} = \lambda_1\Pi_{i-1,j} + \lambda_2\Pi_{i,j-1}; i + j = N_m; i, j < N_m$
- $(\lambda_1 + \lambda_2 + \mu_{i,j}^1 + \mu_{i,j}^2)\Pi_{i,j} = \lambda_1\Pi_{i-1,j} + \lambda_2\Pi_{i,j-1} + \mu_{i+1,j}^1\Pi_{i+1,j} + \mu_{i,j+1}^1\Pi_{i,j+1}; 0 < i, j < N_m$

From those equations, we built a matrix A such that $\Pi A = \Pi$ then we look for the eigenvector of constant sign associated with 1 and we normalize it to find Π . Once Π is computed the average satisfaction can be written as:

$$\tilde{S} = \sum_{k=0}^{N_m} \left(\sum_{i=1}^k \Pi_{i,k-i} \right) (1 - \exp(-\frac{d_k}{d_{com}})),$$

with $d_k = \frac{D}{k}$ the data rate of each user when there are k users.

In order to consider the refused users, we propose to penalize BS i , as in [82], by

subtraction p_e from its revenue. Therefore, the average revenue $\tilde{\text{rev}}$ from a user is:

$$\tilde{\text{rev}} = c_u^z \tilde{S} - \Pi_{N_m} p_e \quad (3.5)$$

Finally, the valuation of W during t is:

$$V_i(W) = N_t(c_u^z \tilde{S} - \Pi_{N_m} p_e) \quad (3.6)$$

Fig. 7 summarizes steps that we have done in order to compute the valuation.

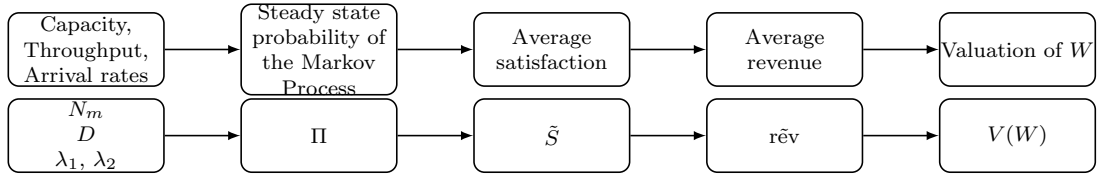


Figure 7 – Steps for computing the valuation for the bandwidth during a period t

Illustration

The objective of this illustration is to show how this model could be used. We fix $N_m = 100$, $c_u^z = 1$ euro/unit of satisfaction, $d_{\text{com}} = 5$ Mb/s, $t = 300$ s and $p_e = 0.2$ euro. We suppose that all users are of Type 2 with $m = 50$ Mb and arrival rate λ . We fix three possible values for λ . For each value, we compute the valuation as a function of the throughput as shown in Fig. 8. Suppose that the normal bandwidth generates a throughput 40 Mb/s and by obtaining the LSA license, the total throughput will be 60 Mb/s then: if $\lambda = 1$ s⁻¹, we can conclude that v_i is very small. This can be interpreted as follows: with the normal bandwidth we can well satisfy users connected to BS i so there is no need to additional bandwidth. On the other hand if $\lambda = 5$ s⁻¹, then by obtaining the license, BS i can double its revenue so v_i is important.

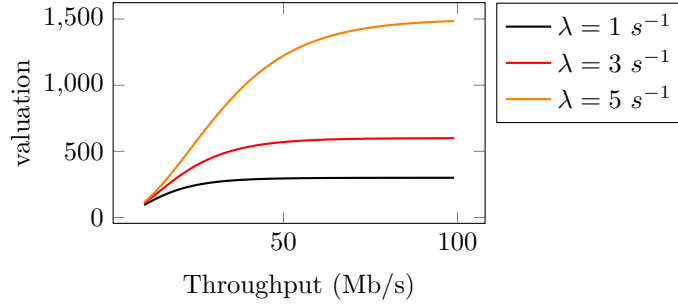
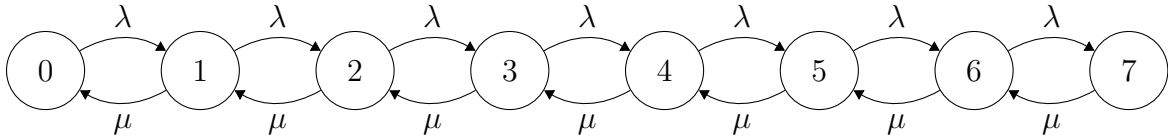


Figure 8 – Valuation as a function of the throughput for $N_m = 100$

3.3 Deriving the valuation of an LSA bandwidth in real world

We suppose that there is an LSA bandwidth which can be used for a duration t in a particular geographical area. Let's consider a BS i , for simplicity, we suppose that all users are of Type 2. The corresponding Markov chain is:



This chain is equivalent to the one of $M/M/1/N_m$ (here $\mu = \frac{D}{m}$), Thus the stationary distribution is:

- $\Pi_n = \frac{(\rho^n)}{n!} \times \Pi_0$ for $1 \leq n \leq N_m$ and $\rho = \frac{\lambda}{\mu}$
- $\Pi_0 = \left(\sum_{n=0}^{N_m} \frac{(\rho^n)}{n!} \right)^{-1}$

We want to compute $v_i = V_i(W_{\text{tot}}) - V_i(W_n)$. We start by computing the valuation of the normal bandwidth which is the one without the added LSA spectrum.

3.3.1 Deriving the valuation of the normal bandwidth

We denote by D_n the throughput of the normal bandwidth W_n . In practice, from the point of view of BS, it can compute the average arrival rate λ since it knows the average number of presented users N_p during t , therefore we can set λ as $\lambda = \frac{N_p}{t}$. However, in order to compute μ , it has to compute m ($\mu = \frac{D}{m}$) which is not necessarily simple. In practice during a period t , from real data, we have the average number of connected users \tilde{N}_c ; $\tilde{N}_c = \sum_{i=0}^{N_m} i\Pi_i$. By using equation (3.7) presented in [91] we can find ρ .

$$\sum_{i=0}^{N_m} i\Pi_i = \frac{\rho}{1-\rho} \frac{1 - (N+1)\rho^N + N\rho^{N+1}}{1 - \rho^{N+1}}. \quad (3.7)$$

Once $\rho = \frac{\lambda}{\mu}$ is computed, we compute the stationary distribution and we can therefore compute the revenue from the bandwidth W_n using (3.6).

3.3.2 Deriving the valuation of the total bandwidth

We denote by D_{tot} the throughput generated by the package composed of the normal bandwidth and the LSA bandwidth. After introducing the LSA bandwidth, the total throughput of a BS will increase. In addition the stationary distribution is not anymore the same (since service time of users depends on their throughput which will be increased). We denote μ' the new service rate. In order to compute μ' we proceed as follows:

1. We compute the normal service rate (since we know λ and ρ) $\mu = \rho \times \lambda$
2. From μ , we can find m , which is $\frac{D_n}{\mu}$
3. Finally, we compute $\mu' = \frac{D_{\text{tot}}}{m}$
4. We set $\rho' = \frac{\lambda}{\mu'}$ and compute the new stationary distribution Π'

Once the new stationary distribution is found, we can apply (3.6) to find $V(W_{\text{tot}})$ and then compute v_i .

3.4 Conclusion

Valuing spectrum is a complex task because a lot of factors can be introduced. In our study, we have developed a simple model: we have supposed that the valuation of an LSA bandwidth for a BS is its surplus i.e., the revenue with that bandwidth minus its revenue without that bandwidth. We have supposed that the revenue from a user depends on his satisfaction which depends on his data rate.

We have defined the number of connected users as a Markov process and show how to derive the steady state probability distribution so that from that probability distribution we can derive the average satisfaction and therefore the revenue from a user. In order to compute the steady state probability, we have supposed that there are two types of users and we have derived some theoretical results. Results suggest that the valuation of an LSA bandwidth varies and can be very high when users are not satisfied with the normal bandwidth. On the other hand, it can be very low when users are well satisfied with the normal bandwidth. We finally show how our model can be used to provide estimation of valuation when real data is available.

In next chapters, we suppose that each BS knows its own valuation. In the next part, we focus on one-shot auction for the case where the available quantity of LSA spectrum is small.

Part II

Geographical zones with little available spectrum

Chapter 4

From all-or-nothing auction mechanisms to proportional allocation

4.1 Introduction

In this chapter, we suppose that the regulator has a small quantity of LSA spectrum to allocate for M groups. Since each BS may not accept an infinitesimal amount of spectrum, then at a first time we suppose that all the available LSA spectrum will be considered as one block so that each BS can either obtain that block or nothing i.e., for each auction mechanism α_i^{MEC} is either 0 or 1. We denote by v_i the valuation of player i for all the available quantity of LSA spectrum, therefore the utility of a BS i is $v_i - p_i^{\text{MEC}}$ or 0. The organisation of this chapter is as follows: we start by reviewing auction mechanisms (for LSA) from the literature in Section 4.2. Then in Section 4.3, we show how to ensure truthfulness to the first auction mechanism which was proposed as a candidate for the LSA auction. The revenue generated by several auctions is analyzed in Section 4.5.

At a second time, we suppose that the available LSA spectrum can be split. Under this assumption, a new auction mechanism which shares all the available bandwidth among BSs is presented in Section 4.6. Simulations results are presented in Section 4.7. Finally we conclude the chapter in Section 4.8.

In this chapter we consider two key assumptions introduced in the literature: assumption 1 (each BS belongs to one and only one group) and assumption 2 (each base station is controlled by a different player, i.e., we assume non-coordination among bids submitted

by base stations).

4.2 State of the art

In this section we present the auction mechanisms which were proposed as candidates to allocate LSA spectrum. Each bidder with valuation v_i submits a bid b_i which represents the willingness to pay for all the available LSA spectrum. After receiving bids from bidders, the auctioneer computes the group-bid of each group and attributes the bandwidth to the group with the highest group-bid.

4.2.1 TAMES

TAMES [2] computes the group-bid B_h^{TAMES} of each g_h as

$$B_h^{\text{TAMES}} = (|g_h| - 1) \min_{i \in g_h} b_i,$$

where $|g_h|$ is the cardinal of group h . All players of the highest group-bid group are winners, except the one with the lowest bid of that group. Each winning player pays the same price, that is the lowest bid in their group. This mechanism is truthful and individually rational. However, under TAMES the bidder with the lowest bid who decide whether his group is a winning group or not is not allowed to use the bandwidth. This is contrary to TRUST in which all players of the winning group can use the bandwidth.

4.2.2 TRUST

TRUST [3] works quite similarly to TAMES. It computes the group-bid as:

$$B_h^{\text{TRUST}} = |g_h| \min_{i \in g_h} b_i.$$

All players of the group with the highest group-bid are winners. Winners pay equitably the second-highest group-bid (each winner pays a proportion $1/|g_h|$ of it). This mechanism is truthful and individual rational.

Both Tames and TRUST are truthful and individually rational. However, they compute the group-bid based on the player with the lowest bid. From the point of view of bidders, those mechanisms are unfair because the bidder with the lowest bid decide for

the members of his group i.e., if that bidder has a very low bid then other bidders of his group cannot do anything in order to change the outcome (from losing to winning). Contrary to TAMES and TRUST, in LSAA [4] each bidder of the group is involved in the definition of the group-bid i.e., each bidder can increase the group-bid of his group.

4.2.3 LSAA

In LSAA [4], bids in each group are sorted in a non-ascending order. The group-bid of a group g_h is computed as:

$$B_h^{\text{LSAA}} = \max_{i \in g_h} \text{rank}(b_i) b_i,$$

where $\text{rank}(b_i)$ is the rank of player i 's bid in the group. This group-bid can be interpreted as follows: it represents the maximum amount that a subset of g_h can share equitably.

The authors define an index j such that:

$$j = \max \left\{ \text{rank}(b_l), l \in \arg \max_{i \in g_h} (\text{rank}(b_i) b_i) \right\}. \quad (4.1)$$

If g_h is the winning group, then only players with rank below or equal to j are winners. Winners pay the second highest group-bid equally.

Authors of LSAA claim that LSAA is truthful. However, it is not the case as we will show in the following proposition.

Proposition 1. *LSAA is not truthful.*

Proof. suppose we have two groups, the first group is composed of three bidders with bids respectively $\{30, 19, 10\}$ and the second is composed of two bidders with bids $\{15, 10\}$. Group one is the winning group because it has the highest group-bid $\max\{30 \times 1, 19 \times 2, 10 \times 3\} = 38$. The group-bid of the second group is $\max\{15 \times 1, 10 \times 2\} = 20$. Player 3 in group one is a loser. Players 1 and 2 are winners and each one pays $\frac{20}{2} = 10$. If player 3 had proposed 15 instead of 10 he would have been a winning player and he would have paid $\frac{20}{3}$, yielding him the strictly positive utility $10 - \frac{20}{3}$. \square

Assuming truthful bidding, this mechanism outperforms the previous one in terms of efficiency and revenue. In the next section we show how to modify the payment rule of LSAA to render it truthful.

4.3 TLSAA and TLSAA2: Two truthful variants of LSAA

In this section we propose two variants that are truthful: TLSAA and TLSAA2. Both mechanisms compute the group-bid of each group in the same way as LSAA however:

1. TLSAA is more efficient than LSAA but has a lower revenue.
2. TLSAA2 has the same revenue as LSAA but it is less efficient than TLSAA.

4.3.1 TLSAA

We preserve LSAA's method of group-bid computation and allocation. All bidders of the winner group can use the bandwidth. We propose a new payment rule which ensures truthful bidding: since the allocation rule is monotone, we can implement the truthful payment rule given in (2.2). This gives

$$p_i^{\text{TLSAA}} = \min\{b_i \text{ s.t. } \alpha_i^{\text{LSAA}}(b_i) = 1\}. \quad (4.2)$$

Each player pays his break point (the point at which his allocation changes from zero to one).

Example 1. We illustrate that rule with an example: suppose we have two groups with bids respectively $\{20, 10, 9, 6\}$ and $\{20, 8, 7\}$. The first group wins the auction since it has the highest group-bid (with value 27). Let us compute the payment of the first player (the one with bid 20): by proposing a bid lower than 5.25 player 1 would be a losing player because the group-bid of his group would then be below the second group-bid 21, and by proposing a bid higher than 5.25 group 1 wins the auction. So Player 1 should pay 5.25. Note that for the second and the third player the same reasoning can be made and each one should pay 5.25, however the fourth player should pay 0 because his group is a winning group whether he is present or not (there is no break point for him).

In LSAA, the revenue is given by the second highest group-bid. A question which may arise regards the revenue of this modified version of LSAA. We show below that truthfulness comes at a cost, since revenue may decrease with respect to the initial version (that was assuming truthful bidding).

Proposition 2. *The revenue of TLSAA cannot be higher than the second-highest group-bid.*

Proof. We denote by g_w^{LSAA} the winning group and by $B_{\text{second}}^{\text{LSAA}}$ the second highest group-bid. Let us define j' such that:

$$j' = \max \{ \text{rank}(b_i), i \in g_w^{\text{LSAA}} \text{ and } \text{rank}(b_i)b_i \geq B_{\text{second}}^{\text{LSAA}} \}. \quad (4.3)$$

Consider a player i in the winning group:

- if $\text{rank}(b_i)$ is strictly above j' then that player pays 0, because his group always wins whatever his bid (there is no breaking point for him);
- if $\text{rank}(b_i)$ is below j' then we can distinguish two cases:
 1. if his group remains the winning group without i 's bid, that player pays 0.
 2. if his group is a losing group if i is not there (winning group only with his presence), his breaking point is exactly $\frac{B_{\text{second}}^{\text{LSAA}}}{j'}$.

Hence the maximum revenue is $\frac{B_{\text{second}}^{\text{LSAA}}}{j'} \cdot j' = B_{\text{second}}^{\text{LSAA}}$. □

One may then wonder whether we can find an allocation rule that ensures the same revenue as LSAA. To reach that goal, we propose TLSAA2, in which the group-bid is defined as in LSAA, but we modify the allocation rule and still apply the payment rule ensuring truthful bidding, given in (2.2).

4.3.2 TLSAA2

We propose TLSAA2, in which the group-bid is defined as in LSAA, but we modify the allocation rule. The allocation rule is defined as follows: a winning player should not only belong to the winning group but also bid at least as high as player j' (see (4.3)). In turn, the payment rule is defined as follows: each winning player pays

$$p_i^{\text{LSAA}} = \frac{B_{\text{second}}^{\text{LSAA}}}{j'}. \quad (4.4)$$

Through the following proposition we prove that TLSAA2 is a truthful mechanism with revenue achieved by the seller equal to $B_{\text{second}}^{\text{LSAA}}$.

Proposition 3. *TLSAA2 is truthful with revenue equal to $B_{\text{second}}^{\text{LSAA}}$.*

Proof. For the revenue, it is clear that it is equal to $\frac{B_{\text{second}}^{\text{LSAA}}}{j'} j' = B_{\text{second}}^{\text{LSAA}}$. This payment rule ensures a truthful bidding because the allocation rule is monotone (the allocation rule of TLSAA2 is just the allocation rule of TLSAA with constraint given by (4.3)), and the payment rule corresponds to Equation (2.2). \square

It was shown in [4] that LSAA outperforms TAMES and TRUST in terms of efficiency. Therefore we can conclude that TLSAA is the best one in terms of efficiency. However this is relative because the real question is: how much far are we from the optimal efficiency? Before providing an answer in proposition 4, let us first compute the optimal efficiency: we have one block to attribute to one group. Clearly, using equation (2.3), the optimal efficiency can be obtained by allocating that block to the group with the highest sum of bids i.e., we define the group-bid of each group as the sum of bids of its members and we allocate the bandwidth to the group with the highest group-bid.

Proposition 4. *For TLSAA, efficiency can be arbitrary far from the optimal one*

Proof. Suppose we have two groups. The first group is composed of n_1 bidders with valuation $\{n_1, \frac{n_1}{2}, \frac{n_1}{3}, \dots, 1\}$. Therefore the group-bid of this group is n_1 . The second group is composed of one bidder with group-bid $n_1 + 1$. Clearly TLSAA's efficiency is $n_1 + 1$ (since group two has the highest group-bid). The optimal efficiency is $\sum_{i=1}^{n_1} \frac{n_1}{i}$. The ratio between those two quantity is: $\frac{n_1+1}{n_1 \sum_{i=1}^{n_1} \frac{1}{i}} \leq \frac{n_1+1}{n_1 \ln(n_1+1)}$. Clearly, this ratio goes to zero when n_1 goes to infinity. \square

Obtaining the optimal efficiency while ensuring truthful telling could be done by implementing VCG. In the next section, we show how to implement VCG for the setting of this chapter.

4.4 VCG

We present the general principle of VCG ([5–7]), we show how to implement it for the LSA context in the next section. The principle of VCG is to allocate resources to maximize the “declared” efficiency (since computed based on submitted bids) and charge each bidder the loss of declared efficiency his presence causes to the others. We denote by E_a^{-i} the efficiency when bidder i is absent, and by E_p^{-i} the efficiency when bidder i is

present but without counting him. The payment rule is therefore:

$$p_i^{VCG} = E_a^{-i} - E_p^{-i}.$$

Contrary to TRUST and TAMES which may harm efficiency, VCG is known to be the unique mechanism which is truthful, individually rational and maximizes efficiency.

An implementation of VCG could be done by defining the group-bid of a group g_h as

$$B_h^{VCG} = \sum_{i \in g_h} b_i;$$

Another advantage of VCG compared to other mechanisms is that each bidder participates in the computation of the group-bid of his group. The winning group is then the group with the highest group-bid.

For the payment we perform as follows: if a player belongs to a losing group he pays 0 because whether he is present or not the winning group is the same. If a player belongs to the winning group g_{win} with group-bid B_{win}^{VCG} then we can distinguish two cases: if his presence does not change the outcome, i.e., $B_{\text{win}}^{VCG-i} \geq B_{\text{second}}^{VCG}$ (with B_{second}^{VCG} the second-highest group-bid and B_{win}^{VCG-i} the group-bid of the winning group when player i is absent) then he pays 0 otherwise he pays $B_{\text{second}}^{VCG} - B_{\text{win}}^{VCG-i}$. To summarize:

$$p_i^{VCG} = [B_{\text{second}}^{VCG} - B_{\text{win}}^{VCG-i}]^+. \quad (4.5)$$

Efficiency is important. But, revenue also is an important criterion. However, all the previous mechanisms could generate an extremely low revenue as we illustrate in the following example.

Example 2. Suppose we have two groups g_1 and g_2 . The first group is composed of three players with bids respectively 3, 2. The second group is composed of four players with bids respectively 30, 20, 15 and 0.75 .

1. When TAMES is applied, the group-bid of the first group is $2 \times 1 = 2$ and the group-bid of the second group is $0.75 \times 3 = 2.25$, thus group two (except bidder with bid 0.75) wins the auction and each winner pays 0.75.
2. When TRUST is applied, the group-bid of the first group is $2 \times 2 = 4$ and the group-bid of the second group is $0.75 \times 4 = 3$, thus group one wins the auction and each bidder pays 1.5.

3. When VCG is applied, the group-bid of the first group is 5 and the group-bid of the second group is $30 + 20 + 15 + 0.75 = 65.75$, thus group two wins the auction. Each bidder of the second group pays zero, therefore the revenue is 0.
4. For LSAA, the group-bid of the first group is 4 and 45 for the second group. For TLSAA, the revenue is 0 and for TLSAA2 the revenue is 4.

In the following section, we show how to increase the revenue of any mechanism with all-or-nothing allocation rule.

4.5 Improving revenue using minimum prices per buyer

In order to increase VCG's revenue, the authors in [92] have introduced a reserve price per bidder (minimum amount per bidder) R . In [59], we have extended that approach for other mechanisms and developed the analytical expressions of the average revenue when operators' valuations for the spectrum are independent and identically distributed from a uniform distribution. In the following proposition, we generalize [59] for any mechanism with a monotone and all-or-nothing allocation rule.

Proposition 5. *Consider a mechanism with a monotone all-or-nothing allocation rule (α_i^{MEC} is either 0 or 1 for each player i). We denote by $p_i^{\text{MEC}'}$ the corresponding truthful payment rule. For any non-negative value R , the mechanism MEC' defined as follows is truthful:*

- the allocation rule $\alpha^{\text{MEC}'}$ is simply the rule α^{MEC} , ignoring all bids strictly below R ;
- the payment rule consists in charging player i a price

$$p_i^{\text{MEC}'}(b_i) = \begin{cases} \max\{R, p_i^{\text{MEC}}(b_i)\}, & \text{if } \alpha_i^{\text{MEC}'} = 1 \\ 0, & \text{if } \alpha_i^{\text{MEC}'} = 0, \end{cases} \quad (4.6)$$

Additionally, this modification ensures that the per-unit price paid by players is at least R .

Proof. The allocation rule α' is still monotone, therefore there must exist a payment rule p' which renders the mechanism truthful [93].

Let us fix a player i with valuation v_i . If $v_i < R$, bidding truthfully ensures a utility equal to 0 otherwise he obtains either a negative utility or a utility equal to 0.

We distinguish two cases for a winning player with $v_i > R$:

- $p_i^{\text{MEC}}(v_i) \geq R$: this situation corresponds to the original mechanism facing only bidders with valuations above R , hence proposing a bid $b_i = v_i$ maximizes his utility.
- $0 \leq p_i^{\text{MEC}}(v_i) < R$: bidding truthfully generates a utility $v_i - R$, any other bid b_i leads to a lower utility since the bidder would either get no resource (hence utility 0), or still be a winner and pay at least R .

For a losing player, the outcome corresponds to the original mechanism MEC (now facing only bidders with valuations above R). Since MEC is truthful, and MEC' only has larger payments than MEC, bidding truthfully—and losing—remains a best strategy. \square

From Proposition 5, we can introduce to all previous mechanisms a reserve price per bidder and the payment rule for each mechanism is given by (4.6).

Providing all the available bandwidth to the group with the highest valuation may maximize efficiency but would result in the dissatisfaction of other groups. It is to allow a trade-off between efficiency and fairness that we propose the Proportional Allocation Mechanism (PAM).

4.6 The Proportional Allocation Mechanism (PAM)

Recall that we have supposed that valuation for spectrum is an increasing concave function. However, since that amount of spectrum is small then we can suppose that valuation for spectrum is linear as shown in Fig. 9. If a BS obtains a fraction $\alpha_i > 0$ of the available bandwidth and pays p_i , then its utility u_i is approximated by:

$$u_i(\alpha_i, p_i) = \alpha_i v_i - p_i.$$

Moreover, if we suppose that each BS will accept any amount of spectrum even a small one then in this situation we propose PAM: the Proportional Allocation Mechanism. The general principle of our proposition is to share spectrum among groups proportionally to their declared valuations. Sharing resource in proportion to group-bids ensures that

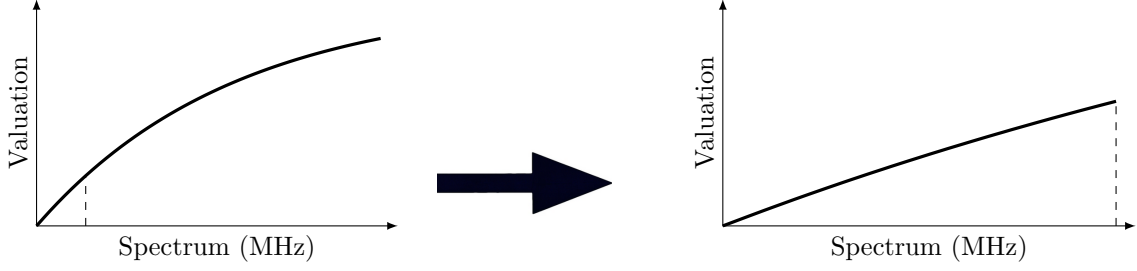


Figure 9 – Valuation for spectrum can be assumed to be linear when the amount of available spectrum is small

highest-bidding groups get more resource than the others (leaning toward efficiency), but each group will receive a non-null allocation (leaning toward fairness).

4.6.1 Allocation

Similarly to the previous mechanisms, we introduce a minimum bid amount per bidder. The group-bid B_h^{PAM} of each group is similar to the group-bid of VCG. $B_h^{\text{PAM}} = B_h^{\text{VCG}} = \sum_{i \in g_h} b_i$. Denoting by $B_{\text{Tot}}^{\text{PAM}} = \sum_{h=1}^M B_h^{\text{PAM}}$ the sum of all group-bids, we propose to allocate to each group g_h a proportion γ_h^{PAM} of the available spectrum, with $\gamma_h^{\text{PAM}} = \frac{B_h^{\text{PAM}}}{B_{\text{Tot}}^{\text{PAM}}}$.

The fraction α_i^{PAM} allocated to each player i can be expressed as follows:

$$\alpha_i^{\text{PAM}} = \frac{b_i + B_g^{\text{PAM}-i}}{b_i + B_{\text{Tot}}^{\text{PAM}-i}} = \frac{b_i + \sum_{j=1, j \neq i}^{n_h} b_j}{b_i + \sum_{j=1, j \neq i}^N b_j}. \quad (4.7)$$

Where $B_{\text{Tot}}^{\text{PAM}-i}$ (resp. $B_g^{\text{PAM}-i}$) represents the aggregated group-bid (resp, the group-bid for i 's group) had bidder i be absent. If a player i belongs to g_h then $\alpha_i^{\text{PAM}} = \gamma_h^{\text{PAM}}$

Remark: PAM and the Nash bargaining solution

Suppose that there is no auction process and groups must negotiate in order to share the bandwidth otherwise (if no agreement is reached) each group gets zero. The question is how groups will behave? Groups should agree with respect to some “rational” axioms like for example a symmetry axiom: if two groups have the same group-bid then they

should obtain the same amount. In fact, the *Nash bargaining solution* [94] is adequate for this case because it represents the unique solution that satisfy four axioms: symmetry, independence of irrelevant alternatives, Pareto optimality (no one can have better pay off without harming other one) and invariance to affine transformation. In our case the Nash Bargaining solution (where the bargaining power of each group is $\frac{B_h}{B_{\text{Tot}}}$) is just PAM's allocation: we have M groups who want to share the LSA available bandwidth. Each group h , with valuation $V_h = \sum_{i=1}^{n_h} v_i$ has to accept some amount x_h , if no agreement is reached, then each one gets 0. We denote by V_{tot} the sum of all valuations of groups and by γ_h the bargaining power of group h . $\gamma_h = \frac{V_h}{V_{\text{Tot}}}$.

The Nash bargaining solution solves the following optimization problem

$$\begin{aligned} & \underset{X}{\text{maximize}} && \prod_{h=1}^M (x_h V_h)^{\gamma_h} \\ & \text{subject to} && \sum_{h=1}^M x_h = 1 \\ & && x_h \geq 0 \end{aligned}$$

Which is equivalent to

$$\begin{aligned} & \underset{X}{\text{maximize}} && \sum_{h=1}^M \gamma_h \log x_h \\ & \text{subject to} && \sum_{h=1}^M x_h = 1 \\ & && x_h \geq 0 \end{aligned}$$

The Lagrangian is:

$$\mathcal{L}(X, \lambda) = \sum_{h=1}^M \gamma_h \log x_h + \lambda(1 - \sum_{h=1}^M x_h)$$

Setting the gradient $\nabla \mathcal{L}$ equal to the 0 vector we get:

$$\frac{\mathcal{L}}{\partial \lambda} = (1 - \sum_{h=1}^M x_h) = 0 \tag{4.8}$$

$$\frac{\mathcal{L}}{\partial x_h} = \frac{\gamma_h}{x_h} - \lambda = 0, \quad \forall h \tag{4.9}$$

(3.8) implies

$$\sum_{h=1}^M \gamma_h = \lambda \sum_{h=1}^M x_h \quad (4.10)$$

which means $\lambda = 1$, hence by (3.8) we get $x_h = \gamma_h \forall h$

Thus the Nash bargaining solution is given by setting $x_h = \gamma_h$ which correspond to the PAM allocation.

This represents an advantage of PAM's allocation because that allocation, somehow seems to be natural and not imposed for groups

4.6.2 Payments: building a truthful payment rule

We provide the payment rule that ensures truthful bidding for each player. Let us write the payment p_i^{PAM} for bidder i (who obtains a proportion α_i^{PAM} of the available spectrum) under the form:

$$p_i^{\text{PAM}} = \alpha_i^{\text{PAM}} R + S_i(b_i),$$

where $S_i(b_i)$ is an adjustment price which is introduced to ensure truthfulness and $S_i(b_i) \geq 0$ to ensure that no bidder pays a unit price below R as we see next (this also proves that players with $v_i < R$ would get a strictly negative utility if bidding above R). Note that α_i^{PAM} and S_i depend also on b_{-i} but to simplify notation we only write the dependency on b_i .

Consider given bids $b_{-i} = (b_1, \dots, b_{i-1}, b_{i+1}, \dots, b_N)$ of bidders different from i . We want truthful bidding to be a dominant strategy for bidder i . First, recall that the utility u_i of bidder i when bidding b_i is, as seen as a function of b_i ,

$$\begin{aligned} u_i(b_i) &= \alpha_i^{\text{PAM}}(b_i) v_i - (\alpha_i^{\text{PAM}}(b_i) R + S_i(b_i)) \\ &= \alpha_i^{\text{PAM}}(b_i) (v_i - R) - S_i(b_i). \end{aligned} \quad (4.11)$$

Like α_i^{PAM} and S_i , u_i depends also on b_{-i} but we write $u_i(b_i)$. The objective is to find the adjustment price $S_i(b_i)$ for each base station i so that its utility is maximized for $b_i = v_i$. We can formulate the problem as follows:

$$\begin{aligned} \text{Find } & S_i(b_i) \\ \text{s.t. } & \underset{b_i}{\operatorname{argmax}} (v_i - R) \alpha_i^{\text{PAM}}(b_i) - S_i(b_i) = v_i \end{aligned} \quad (4.12)$$

In particular, if S_i is differentiable in b_i , then u_i being maximized for $b_i = v_i$ means $u'(v_i) = 0$. Let us first rewrite (4.11) using (4.7), as

$$u_i(b_i) = \frac{b_i + B_g^{\text{PAM}^{-i}}}{b_i + B_{\text{Tot}}^{\text{PAM}^{-i}}}(v_i - R) - S_i(b_i).$$

Differentiating, we get

$$u'_i(b_i) = (v_i - R) \frac{B_{\text{Tot}}^{\text{PAM}^{-i}} - B_g^{\text{PAM}^{-i}}}{(b_i + B_{\text{Tot}}^{\text{PAM}^{-i}})^2} - S'_i(b_i). \quad (4.13)$$

Therefore, to satisfy $u'_i(v_i) = 0$ we must have for all v_i

$$S'_i(v_i) = (v_i - R) \frac{B_{\text{Tot}}^{\text{PAM}^{-i}} - B_g^{\text{PAM}^{-i}}}{(v_i + B_{\text{Tot}}^{\text{PAM}^{-i}})^2}. \quad (4.14)$$

Integrating, we obtain:

$$S_i(x) = B_{\text{Tot}}^{\text{PAM}^{-i}} - B_g^{\text{PAM}^{-i}} \left(\ln(x + B_{\text{Tot}}^{\text{PAM}^{-i}}) + \frac{B_{\text{Tot}}^{\text{PAM}^{-i}} + R}{x + B_{\text{Tot}}^{\text{PAM}^{-i}}} \right) + C_i, \quad (4.15)$$

where C_i is a constant. Note that v_i is indeed a maximum because $u'(v_i) = 0$ and $\frac{\partial u_i}{\partial b_i} = \frac{B_{\text{Tot}}^{\text{PAM}^{-i}} - B_g^{\text{PAM}^{-i}}}{(b_i + B_{\text{Tot}}^{\text{PAM}^{-i}})^2}(v_i - b_i)$, so for $b_i \leq v_i$, u_i is an increasing function and for $b_i \geq v_i$, u_i is decreasing.

We now focus on the value of the constant C_i so that when $v_i \geq R$, bidding truthfully yields a higher utility than bidding below R (which ensures utility 0). In particular, each player submitting a bid $b_i = R$ should pay exactly a unit price R , since we imposed the paid unit price to be above R and raising it more would lead to a strictly negative utility for players with $v_i = R$. Hence $S_i(R) = 0$, which leads to

$$C_i = -B_{\text{Tot}}^{\text{PAM}^{-i}} - B_g^{\text{PAM}^{-i}} \left(\ln(R + B_{\text{Tot}}^{\text{PAM}^{-i}}) + 1 \right).$$

So finally we obtain the price to be paid by base station i :

$$p_i^{\text{PAM}} = \frac{b_i + B_g^{\text{PAM}^{-i}}}{b_i + B_{\text{Tot}}^{\text{PAM}^{-i}}} R + B_{\text{Tot}}^{\text{PAM}^{-i}} - B_g^{\text{PAM}^{-i}} \left(\ln \frac{b_i + B_{\text{Tot}}^{\text{PAM}^{-i}}}{R + B_{\text{Tot}}^{\text{PAM}^{-i}}} + \frac{R + B_{\text{Tot}}^{\text{PAM}^{-i}}}{b_i + B_{\text{Tot}}^{\text{PAM}^{-i}}} - 1 \right). \quad (4.16)$$

Remark: PAM's payment and Myerson lemma

We show how to obtain PAM's payment using Myerson's lemma. The payment corresponds to the colored surface in Fig. 10 (it is equal to the area bounded by the allocation curve and $y = \alpha_i^{\text{PAM}}$). The colored surface composed of the grey surface and the black surface.

- The grey surface is equal to

$$\alpha_i^{\text{PAM}}(b_i) \times R = \frac{b_i + B_g^{\text{PAM}^{-i}}}{b_i + B_{\text{Tot}}^{\text{PAM}^{-i}}} R,$$

which corresponds to the first term of equation (4.16).

- The black surface S_{black} is equal to $(b_i - R)\alpha_i^{\text{PAM}}(b_i) - \int_R^{b_i} \alpha_i^{\text{PAM}}(x) dx$:

$$\begin{aligned} S_{\text{black}} &= (b_i - R)\alpha_i^{\text{PAM}}(b_i) - \left((b_i - R) + (B_{\text{Tot}}^{\text{PAM}^{-i}} - B_g^{\text{PAM}^{-i}}) \ln \frac{b_i + B_{\text{Tot}}^{\text{PAM}^{-i}}}{R + B_{\text{Tot}}^{\text{PAM}^{-i}}} \right) \\ &= (B_{\text{Tot}}^{\text{PAM}^{-i}} - B_g^{\text{PAM}^{-i}}) \left(\ln \frac{b_i + B_{\text{Tot}}^{\text{PAM}^{-i}}}{R + B_{\text{Tot}}^{\text{PAM}^{-i}}} + (b_i - R) \frac{(\alpha_i^{\text{PAM}}(b_i) - 1)}{B_{\text{Tot}}^{\text{PAM}^{-i}} - B_g^{\text{PAM}^{-i}}} \right) \\ &= (B_{\text{Tot}}^{\text{PAM}^{-i}} - B_g^{\text{PAM}^{-i}}) \left(\ln \frac{b_i + B_{\text{Tot}}^{\text{PAM}^{-i}}}{R + B_{\text{Tot}}^{\text{PAM}^{-i}}} + \frac{R - b_i}{b_i + B_{\text{Tot}}^{\text{PAM}^{-i}}} \right), \end{aligned}$$

which corresponds to the second term of (4.16).

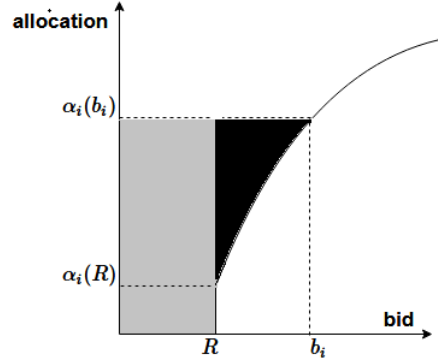


Figure 10 – The colored surface represents PAM’s payment

4.6.3 Properties

In this subsection we illustrate PAM’s properties.

Truthfulness

By construction (see Subsection 4.6.2), we have the following proposition.

Proposition 6. *PAM is truthful, i.e., for each bidder $i = 1, \dots, N$, bidding one’s true valuation $b_i = v_i$ is a dominant strategy.*

Individual rationality

Since players have a dominant strategy, this translates into $u_i(v_i) \geq 0 \forall v_i \geq 0$, since bidding $b_i = 0$ ensures a non-negative utility. Hence the following proposition.

Proposition 7. *PAM is individually rational.*

Efficiency

We show in the next proposition that when $R = 0$, the normalized efficiency for PAM allocations is lower-bounded, i.e., allocations cannot be arbitrarily far from the optimal ones.

Proposition 8. *If the reserve price per bidder $R = 0$, then the normalized efficiency of PAM is at least $\frac{2}{\sqrt{M+1}}$.*

Proof. Without loss of generality, we suppose that group 1 has the highest group-bid. Let $1, X_2, \dots, X_M$ be the normalized valuations, i.e., $X_i = \frac{B_i^{\text{PAM}}}{B_1^{\text{PAM}}}$. PAM's normalized efficiency can then be computed as:

$$E_N^{\text{PAM}} = \frac{E^{\text{PAM}}}{B_1^{\text{PAM}}} = \frac{\sum_{i=1}^N \alpha_i^{\text{PAM}} b_i}{B_1^{\text{PAM}}} = \frac{\sum_{i=1}^M \gamma_i^{\text{PAM}} B_i^{\text{PAM}}}{B_1^{\text{PAM}}} = \sum_{i=1}^M \gamma_i^{\text{PAM}} X_i.$$

Applying the PAM allocation rule, we have $\gamma_i = \frac{X_i}{\sum_{i=1}^M X_i}$, therefore

$$E_N^{\text{PAM}} = \frac{\sum_{i=1}^M X_i^2}{\sum_{i=1}^M X_i} = \frac{1 + \sum_{i=1}^{M-1} X_{i+1}^2}{1 + \sum_{i=1}^{M-1} X_{i+1}}.$$

Using the Cauchy-Schwarz inequality, we can deduce that:

$$E_N^{\text{PAM}} = \frac{1 + \sum_{i=1}^{M-1} X_{i+1}^2}{1 + \sum_{i=1}^{M-1} X_{i+1}} \geq \frac{1 + \frac{(\sum_{i=1}^{M-1} X_{i+1})^2}{M-1}}{1 + \sum_{i=1}^{M-1} X_{i+1}}.$$

Let $Y = \sum_{i=1}^{M-1} X_{i+1}$ and let $g(Y) = \frac{1 + \frac{Y^2}{M-1}}{1 + Y}$, we observe that $g(Y)$ has a minimum at $Y = Y_0 = \sqrt{M} - 1$ and $g(Y_0) = \frac{2}{\sqrt{M+1}}$. Thus, the efficiency of the mechanism is lower bounded by $\frac{2}{\sqrt{M+1}}$. Note additionally that this bound is tight: we attain it for $X_i = \frac{1}{\sqrt{M+1}}$ for $i \in 2, \dots, M$ as an example, if the group-bid of the first group is 1 and the group-bid, for $i \in 2, \dots, M$, is $\frac{1}{\sqrt{M+1}}$ then the normalized efficiency is $\frac{2}{\sqrt{M+1}}$. \square

4.6.4 PAM^σ : guaranteeing winners get at least a minimum amount of spectrum

In the following, we show how to make the mechanism more suitable to be applied in real life, where very small portions of spectrum might not be useful for operators. The regulator fixes a minimum amount σ : each player must get at least that amount or he gets nothing. We call this extension PAM^σ . For the allocation, we keep excluding groups with low group-bids (we start by the one with the lowest group-bid) until obtaining a

feasible allocation.

Example 3. Suppose there are three groups, the first group is composed of two players with bids 12 and 5 respectively. The second group is composed of one player with bid 14 and the third group is composed of one player with bid 15. Suppose that $\sigma = 0.4$. We start by computing the fraction allocated to group two which is lower than 0.4 ($\frac{14}{46} < 0.4$). Thus, group two will be excluded and the final allocation is $\frac{17}{32}$ for group one and $\frac{15}{32}$ for group three. Also, in order to exclude the third group, the second bidder of the first group (whose valuation is equal to 5) has to propose at least a bid b_2 such that $\frac{15}{15+12+b_2} < 0.4$ i.e., $b_2 = 10.5$.

In the following, we investigate PAM^σ 's efficiency.

Proposition 9. *As σ gets closer to 0.5, PAM^σ 's efficiency gets closer to VCG's efficiency. Moreover, when $\sigma \geq 0.5$, PAM^σ 's efficiency is equal to VCG's efficiency.*

Proof. We can distinguish three cases:

- $\sigma < 0.5$, we denote by M^r the number of remaining groups i.e., groups with allocation higher than σ . Please note that the maximum number of remaining groups is $\left\lfloor \frac{1}{\sigma} \right\rfloor$, and that as we increase sigma, we exclude groups with low group-bids. Therefore, groups with high group-bids will obtain more spectrum which leads to increase efficiency.
- $\sigma = 0.5$ in this situation M^r is either:
 - 1, when the two highest group-bids have different value. In this situation the group with the highest group-bid obtains all the available bandwidth and we obtain the same efficiency as VCG.
 - 2, when the two highest group-bids are equal. In this situation, each group obtains a fraction equal to 0.5 (each bidder pays $\frac{b_i}{2}$) and in this situation we obtain the same efficiency as VCG but we increase the fairness of the allocation.
- $\sigma > 0.5$, all the available bandwidth must be allocated to one and only one group. The remaining group is the group with the highest group-bid (if there are many, we choose one randomly). Therefore we obtain the same efficiency as VCG.

□

In the following, we derive the analytical expression of payments for $\sigma < 0.5$. We denote by $S_i(B_{\text{Tot}}^{-i}, R, B_g^{-i}, b_i) = (B_{\text{Tot}}^{-i} - B_g^{-i}) \times \left(\ln \left(\frac{b_i + B_{\text{Tot}}^{-i}}{R + B_{\text{Tot}}^{-i}} \right) + \frac{R + B_{\text{Tot}}^{-i}}{b_i + B_{\text{Tot}}^{-i}} - 1 \right)$.

Let us fix a player i , we denote by b_i^* the minimum bid that allows him to obtain σ , if b_i^* is lower than R then we set it to R (and he gets more than σ). We sort the group-bids of other groups in a non-decreasing order. Player i can exclude some groups by increasing his bid. We suppose that player i can exclude m_i groups, i.e., there are $(M-1) - m_i$ groups excluded without the participation of player i . Without loss of generality, we suppose that player i can exclude the first m_i groups (which are sorted in a non-decreasing order). We denote by $\{c_1, \dots, c_{m_i}\}$, the breaking points, i.e. if player i proposes a bid higher than c_j , then group j (and groups below) will be excluded. We denote by $(B_{\text{Tot}}^{-i})^0$ the sum of all group-bids except the bid of player i , when player i does not exclude any group (of the m_i groups that he can exclude). We denote by $(B_{\text{Tot}}^{-i})^j$ the sum of all bids after excluding group j (and groups below), $(B_{\text{Tot}}^{-i})^j = (B_{\text{Tot}}^{-i})^0 - \sum_{i=1}^j B_i$. Let us compute those breaking points: in order to exclude the first group the following conditions must hold:

1. $\frac{B_1}{(B_{\text{Tot}}^{-i})^0 + b_i} < \sigma$
2. $b_i \geq b_i^*$.

Thus, $c_1 = \max\{b_i^*, \frac{B_1}{\sigma} - (B_{\text{Tot}}^{-i})^0\}$. In general $c_j = \max\{b_i^*, \frac{B_j}{\sigma} - (B_{\text{Tot}}^{-i})^{j-1}\}$. We denote by l the number of groups excluded when $b_i = b_i^*$. The payment rule is given by:

$$p_i(b_i) = \begin{cases} P_i^{\text{PAM}}(b_i, (B_{\text{Tot}}^{-i})^l, b_i^*), & \text{if } b_i^* \leq b_i \leq c_{l+1} \\ p_i(c_k) + S_i^k(b_i) + J_i^k(b_i) & \text{if } c_k < b_i \leq c_{k+1}, \\ & k \in \llbracket l+1 ; m_i - 1 \rrbracket \\ p_i(c_{m_i}) + J_i^{m_i}, & \text{if } b_i > c_{m_i}, \end{cases} \quad (4.17)$$

where:

- $P_i^{\text{PAM}}(b_i, (B_{\text{Tot}}^{-i})^l, b_i^*)$ is given by Equation (4.16) when replacing R by b_i^* and B_{Tot}^{-i} by $(B_{\text{Tot}}^{-i})^l$,
- $S_i^k(b_i) = S_i\left((B_{\text{Tot}}^{-i})^k, c_k, B_g^{-i}, b_i\right)$,
- $J_i^k(b_i) = c_k \left(\frac{b_i + B_g^{-i}}{b_i + (B_{\text{Tot}}^{-i})^k} - \frac{c_k + B_g^{-i}}{c_k + (B_{\text{Tot}}^{-i})^{k-1}} \right)$, and
- $J_i^{m_i} = c_{m_i} \left(1 - \frac{c_{m_i} + B_g^{-i}}{c_{m_i} + (B_{\text{Tot}}^{-i})^{m_i-1}} \right)$.

As an example if $c_{l+1} < b_i \leq c_{l+2}$ then $p_i(b_i)$ is equal to the sum of the following terms:

- $p_i(c_{l+1}) =$

$$\frac{c_{l+1} + B_g^{-i}}{c_{l+1} + B_{\text{Tot}}^{-i}} b_i^* + \left(B_{\text{Tot}}^{-i} - B_g^{-i} \right) \left(\ln \left(\frac{c_{l+1} + B_{\text{Tot}}^{-i}}{b_i^* + B_{\text{Tot}}^{-i}} \right) + \frac{b_i^* + B_{\text{Tot}}^{-i}}{c_{l+1} + B_{\text{Tot}}^{-i}} - 1 \right),$$

represented by the white area (S_1) in Fig. 11.

- $J_i^{l+1}(b_i) =$

$$c_{l+1} \left(\frac{b_i + B_g^{-i}}{b_i + (B_{\text{Tot}}^{-i})^{l+1}} - \frac{c_{l+1} + B_g^{-i}}{c_{l+1} + (B_{\text{Tot}}^{-i})^l} \right),$$

represented by the black area in Fig. 11.

- $S_i^{l+1}(b_i) =$

$$\left((B_{\text{Tot}}^{-i})^{l+1} - B_g^{-i} \right) \times \left(\ln \left(\frac{b_i + (B_{\text{Tot}}^{-i})^{l+1}}{c_{l+1} + (B_{\text{Tot}}^{-i})^{l+1}} \right) + \frac{c_{l+1} + (B_{\text{Tot}}^{-i})^{l+1}}{b_i + (B_{\text{Tot}}^{-i})^{l+1}} - 1 \right),$$

represented by the grey area in Fig. 11.

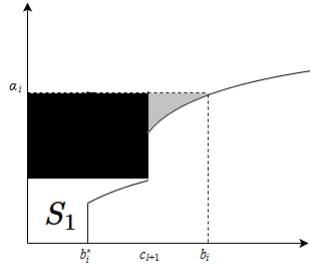


Figure 11 – Payment of player i when $c_{l+1} < b_i \leq c_{l+2}$. Payment is the sum of S_1 , the black surface and the grey surface.

4.7 Performance evaluation

This section compares the performance of the different aforementioned truthful mechanisms. Table 1 summarizes state-of-the-art mechanisms as well as mechanisms proposed in this chapter for truthful bandwidth allocation auctions in the LSA context.

Mech.	group-bid	Allocation	Payment
TAMES	$(g_h - 1) \min_{i \in g_h} b_i$	group with the highest group-bid except the one with the lowest bid	each winning player pays the lowest bid of his group
TRUST	$ g_h \min_{i \in g_h} b_i$	group with the highest group-bid	winners pay the second highest group-bid equally
VCG	$\sum_{i \in g_h} b_i$	group with the highest group-bid	see Eq. (4.5)
PAM	$\sum_{i \in g_h} b_i$	each group obtains a fraction in proportion to its group-bid	see Eq. (4.16)
PAM $^\sigma$	$\sum_{i \in g_h} b_i$	each group obtains either a fraction (at least equal to σ) in proportion to its group-bid or nothing	see Eq. (4.17)
TLSAA	$\max_{i \in g_h} \text{rank}(b_i) b_i$	group with the highest group-bid	see Eq. (4.2)
TLSAA2	$\max_{i \in g_h} \text{rank}(b_i) b_i$	players (of the group with the highest group-bid) and with rank below j' see (4.3)	see Eq. (4.4)

Table 1 – Summary of truthful auction mechanisms that can be applied to allocate LSA spectrum.

The performance evaluation is based on simulations. We are particularly interested in average efficiency and fairness metrics, as well as in the average revenue of the auctioneer (which we have derived analytically in [59] for some mechanisms).

Simulation settings

We recall that this chapter considers the scenario where each base station belongs to one and only one group. We have thus fixed two groups from the interference graph of Fig. 4: $g_1 = \{1, 2, 4, 6\}$ and $g_2 = \{3, 5\}$. The marginal valuations of base stations are drawn from the uniform distribution over the interval $[0, 100]$. For each mechanism and for each minimum amount R , we have computed the average (with respect to each metric) over 10.000 draws.

Results

Simulation results (Fig. 12) show that PAM outperforms the other schemes in terms of fairness. In terms of revenue, Fig. 13 shows that TLSAA2 could offer the highest revenue by playing on the reserve price. In terms of efficiency, VCG is efficient by construction.

In terms of fairness, we can observe that the curve is non-increasing, this can be intuitively explained as follows: as we increase R , we increase the number of excluded players (with valuation lower than R). On the other hand, the bandwidth will be allocated for the remaining players (or for a set of the remaining players) hence we increase the gap (of allocation) between players which means Jain's index decreases.

We also observe a non-monotonicity on the average revenue and efficiency which can be explained as follows: by increasing R the revenue from a player i keeps increasing until reaching $R = v_i$. Intuitively for efficiency, as we increase the minimum amount, bidders with low valuations will be excluded and then bidders with higher valuations will be allocated more resource, then we increase the efficiency; however if we set a too high reserve price then many players will be excluded, reducing efficiency.

In addition for PAM^σ , it offers higher efficiency and lower fairness compared to PAM, this is natural since some groups will be excluded (which leads to decrease fairness) and the original fraction allocated to those groups (by PAM) will be used by other groups with higher valuations (leads to increase efficiency).

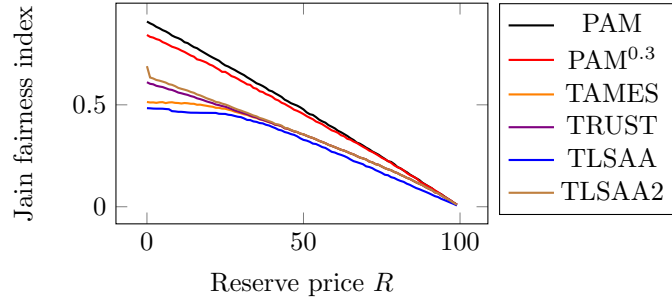


Figure 12 – Average fairness as a function of the reserve price R

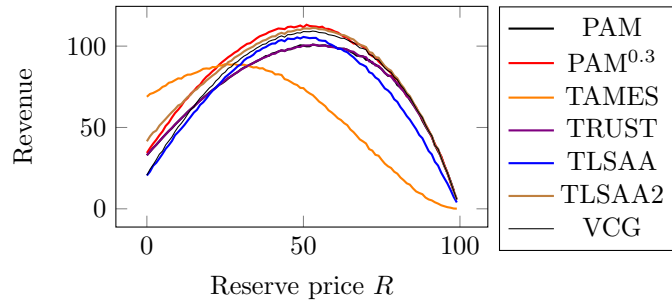


Figure 13 – Average Revenue as a function of the reserve price R

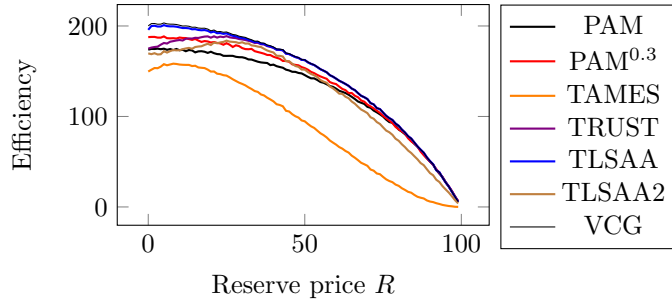


Figure 14 – Average efficiency as a function of the reserve price R

Simulations results show that, as aforementioned, there is no one single mechanism which outperforms the others at all metrics. Which mechanism to chose should be thus based on the sensibility of the auctioneer to each criteria.

4.8 Conclusion

We have studied the case where the auctioneer has a small quantity of LSA spectrum to auction for. In this case it is reasonable to assume all or nothing allocations. We

have reviewed auction mechanisms from the literature which are potential candidates to allocate spectrum for the LSA context. Those mechanisms attribute the bandwidth to the group with the highest group-bid. TAMES and TRUST compute the group-bid of each group based on the bidder with the lowest bid so that the decision made for a group (either it is the winning group or not) is based only on the lowest bid. In order to involve more bidders in the group-bid computation (so that for the decision) LSAA was proposed as an alternative solution. We have shown that this mechanism is not truthful and therefore we have proposed two truthful variants of LSAA: TLSAA and TLSAA2. TLSAA outperforms LSAA in terms of efficiency and TLSAA2 has the same revenue as LSAA assuming truthful bidding. The main problem of the previous mentioned mechanism is that we do not have guarantees in terms of efficiency which may be undesirable from the point of view of the auctioneer. Therefore we have shown how to obtain the optimal efficiency by implementing VCG.

The regulator may be sensitive also to his revenue, we have shown how to increase revenue by adding a reserve price R per bidder i.e., each bidder has to propose at least that amount in order to participate in the auction.

At a second time, since the quantity of the available spectrum is small, we have supposed that utility can be approximated to be linear. We have proposed PAM, a truthful auction mechanism which shares the bandwidth proportionally among groups. PAM outperforms the other mechanisms in terms of fairness. Also, when $R = 0$, contrary to other mechanism (except VCG), PAM's efficiency can not be arbitrary far from the optimal one. We have shown how to adapt PAM, by proposing PAM^σ when each bidder has to obtain at least a minimum amount (or nothing).

The studied mechanisms have different properties so the regulator can choose one with respect to his preferences. We further investigate on it and report several simulation results in the following chapter.

Chapter 5

Extensions and limitations

5.1 Introduction

In this chapter, we consider first relaxing assumption 1 so that BS grouping allows a BS to belong to several groups. This relaxation may improve efficiency of the allocation but complicates the mechanism analysis (ensuring truthfulness becomes harder). Indeed, the intuition behind the complexity is that in general payment of each BS which belongs to the winner group is a function of bids of other losing group(s). When relaxing that assumption a winner BS could be also in another losing group(s) therefore its bid may impact its final payment. Also, when allowing a BS to belong to more than one group, we increase efficiency: suppose that there is a BS which is not causing interference to any other BS, clearly this BS should belong to all groups. In the following, we investigate the truthfulness of the previous mechanisms when removing this restriction, by addressing the following question: given the allocation rule and the hypothesis that a bidder can belong to more than one group, is there a payment rule such that those mechanisms are still truthful? We shall add a star to the original mechanism to denote the new version. Note that the difference between MEC and MEC* (if it exists) resides only in the payment rule.

As a next step, we investigate the relaxation of assumption 2 i.e., we treat the case when several BSs are controlled by a common entity: Each BS belongs to an MNO, thus we suppose that each MNO can coordinates the bids of his BSs which will participate in the auction process.

5.2 Extension: relaxing the one-group-per-player assumption

In this section, we investigate the implementation of the previous studied mechanisms in this new scenario.

5.2.1 Candidate mechanisms are not all adaptable

In this subsection, we show that we can adapt all previous mechanisms except TAMES and TLSAA2. Before that, let us introduce the following proposition.

Proposition 10. *Given a truthful monotone allocation rule, if a player belongs to all groups then he pays 0.*

Proof. Direct application of Myerson's lemma (there is no breaking point for this player because he is always a winning player). \square

We now analyze the extension of each one of the studied mechanisms to the case where assumption 1 is relaxed.

- TAMES:

Under TAMES, all players of the group with the highest group-bid are winners except the player with the lowest bid. With the assumption that a player can belong to more than one group, the allocation rule is non-monotone. Indeed, consider the following example:

Example 4. A player with valuation equal to **15**, belonging to two groups with bids respectively $\{\mathbf{15}, 20, 25\}$ and $\{9, 10, \mathbf{15}, 20\}$. Bidding truthfully leads to a utility equal to 0 because he is a losing player (group one wins and player two and three pays 15 each one). However, any bid lower than 12 and higher than 9 leads to a higher utility because in that situation this player is a winning player (he wins and pays 9). Since the allocation rule is not monotone anymore, we cannot find a truthful payment rule [93].

- TRUST*:

Under TRUST* all players of the group with the highest group bid are winners.

Clearly, the allocation rule is monotone. Thus we can find a truthful payment rule. The break-point for player i is given by the minimum bid that allows i to win the auction. We denote by $B_{\max}^{\text{TRUST}^{-i}}$ the highest group-bid of groups to which i does not belong.

$$p_i^{\text{TRUST}} = \min\{b_i, \exists g_h \text{ s.t } i \in g_h \text{ and } B_h^{\text{TRUST}} \geq B_{\max}^{\text{TRUST}^{-i}}\} \quad (5.1)$$

In order to compute that quantity we can proceed as follows, for each group h which i belongs to, we compute the minimum bid, if it exists, which allows to have $B_h^{\text{TRUST}} \geq B_{\max}^{\text{TRUST}^{-i}}$. Then we take the minimum of all those minimum bids.

- VCG*:

We propose to apply VCG in this context. The winning group is the group with the highest group-bid. We denote by $B_{\max}^{\text{VCG}^{-i}}$ the highest group-bid of groups to which i does not belong. If the player belongs to the winning group $g_{\text{win}}^{\text{VCG}}$ with group-bid $B_{\text{win}}^{\text{VCG}}$ then we can distinguish two cases: if i 's presence does not change the outcome i.e., $B_{\text{win}}^{\text{VCG}^{-i}} \geq B_{\max}^{\text{VCG}^{-i}}$ then he pays 0 otherwise he pays $B_{\max}^{\text{VCG}^{-i}} - B_{\text{win}}^{\text{VCG}^{-i}}$. To summarize:

$$p_i^{\text{VCG}} = [B_{\max}^{\text{VCG}^{-i}} - B_{\text{win}}^{\text{VCG}^{-i}}]^+. \quad (5.2)$$

- PAM*:

We denote by n_i the number of groups which i belongs to. The initial version of PAM in [60] was actually designed under this assumption. The payment rule is given by:

$$p_i^{\text{PAM}} = \frac{n_i b_i + B_g^{\text{PAM}^{-i}}}{n_i b_i + B_{\text{Tot}}^{\text{PAM}^{-i}}} R + \frac{B_{\text{Tot}}^{\text{PAM}^{-i}} - B_g^{\text{PAM}^{-i}}}{n_i} \left(\ln \frac{n_i b_i + B_{\text{Tot}}^{\text{PAM}^{-i}}}{n_i R + B_{\text{Tot}}^{\text{PAM}^{-i}}} + \frac{n_i R + B_{\text{Tot}}^{\text{PAM}^{-i}}}{n_i b_i + B_{\text{Tot}}^{\text{PAM}^{-i}}} - 1 \right). \quad (5.3)$$

- TLSAA*:

Under TLSAA* all players of the group with the highest group-bid are winners. The allocation rule is monotone. Thus we can find a truthful payment rule (the minimum bid which allows him to win the auction) by analogy to TRUST.

$$p_i^{\text{TLSAA}} = \min\{b_i, \exists g_h \text{ s.t } i \in g_h \text{ and } B_h^{\text{TLSAA}} \geq B_{\max}^{\text{TLSAA}^{-i}}\} \quad (5.4)$$

- TLSAA2:

We cannot find a truthful payment rule since the allocation rule is non-monotone, which can be seen on the following example.

Example 5. Suppose we have two groups with bids respectively $\{15, \mathbf{5}, 3, 2.5\}$ and $\{7, \mathbf{5}, 4\}$, where $\mathbf{5}$ is the valuation of a unique bidder belonging to both groups. Clearly that bidder is a losing player (the first group wins the auction and only the first player is a winning player and he pays 12). However, if that bidder had proposed $b_i = 2.5$ instead of 5 then he would be a winning player because in this situation all players of the first group would be winners and each one would pay 2.

Table 2 summarizes the different truthful mechanisms for LSA with and without assumption 1.

Scenario	Candidate truthful mechanisms
Each BS belongs to one and only one group	TAMES, TRUST, TLSAA, TLSAA2, VCG, PAM
Each BS can belong to more than one group	TRUST*, TLSAA*, VCG*, PAM*

Table 2 – Candidate truthful mechanisms.

In the following we numerically evaluate the impact of assumption 1. We compare MEC and MEC*, without considering TAMES and TLSAA2 since as we have shown they cannot be extended preserving truthfulness.

5.2.2 Impact of allowing a BS to be in several groups

In this section we study the impact of assumption 1 on each mechanism using the following simulations: we fix two possible group configurations from the interference graph of Fig. 4: In the first configuration C_1 , we have two groups $g_1 = \{1, 2, 4, 6\}$ and $g_2 = \{3, 5\}$. For the second configuration C_2 we have three groups $g_1 = \{1, 2, 4, 6\}$, $g_2 = \{1, 2, 5\}$ and $g_3 = \{3, 5, 6\}$. The marginal valuations of BSs are drawn from the uniform distribution over the interval $[0, 100]$. For each mechanism and for each reserve price R , we compute the average (with respect to each metric) over 10000 draws. Results are shown on Figs. 15 to 18.

As we can see from these figures, for the all-or-nothing mechanisms, efficiency and fairness are higher when a player can belong to more than one group: the cardinal of the winning group will be higher in average with this assumption, yielding larger fairness and efficiency. Hence first conclusion is: if the regulator wants to adapt an all-or-nothing mechanism and his objectives are to maximize efficiency and fairness, he should choose MEC* instead of MEC, i.e., construct groups by allowing BSs to belong to several groups.

When the allocation is not all-or-nothing (PAM and PAM*), it seems to be natural that efficiency and fairness have opposite trends. Indeed, increasing efficiency means that, in average, players with the highest valuations obtain more allocation (and hence players with lower valuations obtain less), i.e., the allocation gap between players increases hence a smaller Jain index (fairness).

In terms of revenue, as we can see from on Figs. 15 to 18, for some reserve prices the revenue generated by MEC is higher than the revenue generated by MEC*, and we have the opposite for other reserve prices. Thus we are not able to conclude whether MEC* is better than MEC* (or the opposite) in terms of revenue.

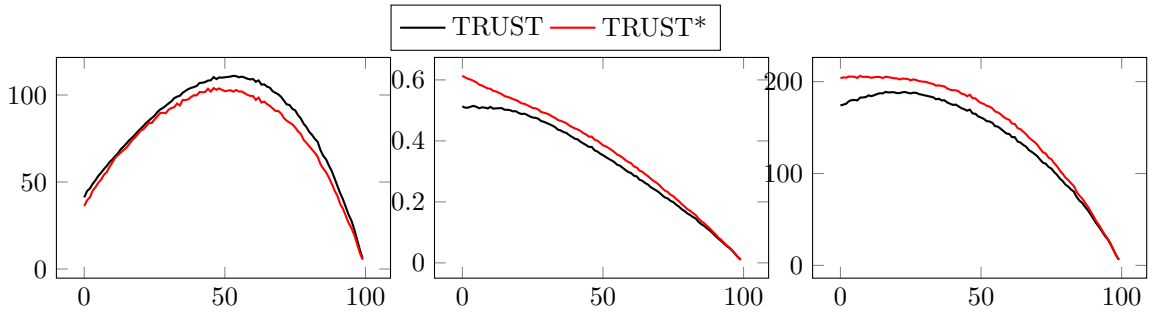


Figure 15 – Average revenue (*left*), fairness (*center*) and efficiency as a function of the reserve price.

5.2.3 Choosing the best mechanism: revenue/fairness/efficiency tradeoffs

We assume that the regulator is sensitive to the revenue from the auction, the allocation's fairness and efficiency. More specifically, we suppose that, given a mechanism, the normalized utility of the regulator $U_{\text{Reg}}^{\text{Mec}}$ is of the form:

$$U_{\text{Reg}}^{\text{Mec}} = \beta_1 \frac{\text{Rev}^{\text{Mec}}}{\text{Rev}_{\text{max}}} + \beta_2 J(\alpha) + \beta_3 \frac{E^{\text{Mec}}}{E_{\text{max}}}, \quad (5.5)$$

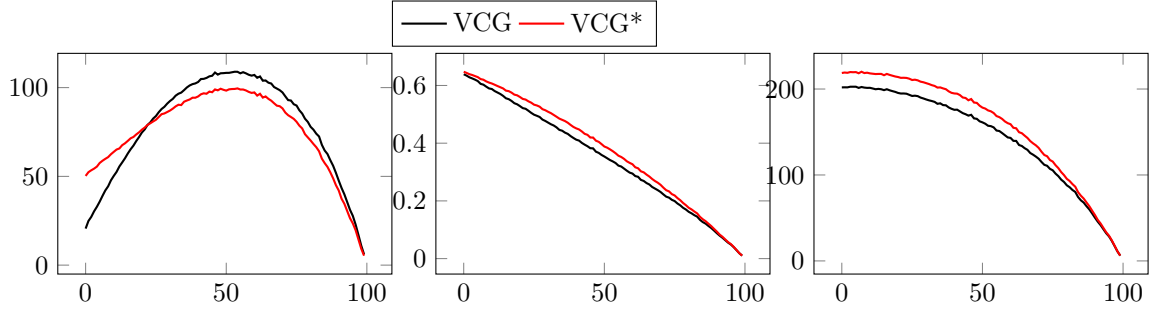


Figure 16 – Average revenue (*left*), fairness (*center*) and efficiency (*right*) as a function of the reserve price.

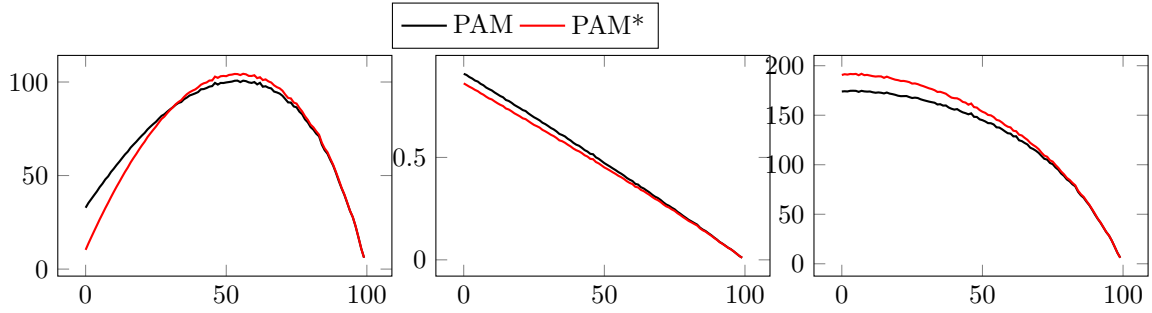


Figure 17 – Average revenue (*left*), fairness (*center*) and efficiency (*right*) as a function of the reserve price.

where β_1 is the weight that the regulator puts on revenue, β_2 is the weight that he puts on fairness and β_3 is the weight that he puts on efficiency. $\beta = (\beta_1, \beta_2, \beta_3)$, $\beta_1 + \beta_2 + \beta_3 = 1$, Rev^{\max} and Eff^{\max} are respectively the maximum revenue and maximum efficiency over the set of candidate mechanisms that we use to normalize the revenue criterion in (5.5). Table 3 shows, for a fixed value of β , how the regulator can maximize his utility by choosing a specific mechanism and by applying an appropriate configuration and reserve price (we have fixed two possible group configurations from the interference graph of Fig. 4: In the first configuration C_1 , we have two groups $g_1 = \{1, 2, 4, 6\}$ and $g_2 = \{3, 5\}$. For the second configuration C_2 we have three groups $g_1 = \{1, 2, 4, 6\}$, $g_2 = \{1, 2, 5\}$ and $g_3 = \{3, 5, 6\}$). Also after allowing a BS to be in several groups, we provide the gain of the operator i.e., how much his utility is increased compared to the scenario in which there is only the configuration C_1 . In most cases the extension increases the utility of the regulator except when his objective is to maximize the fairness of the allocation only: as mentioned before, PAM is the most fair mechanism and it works better when each BS

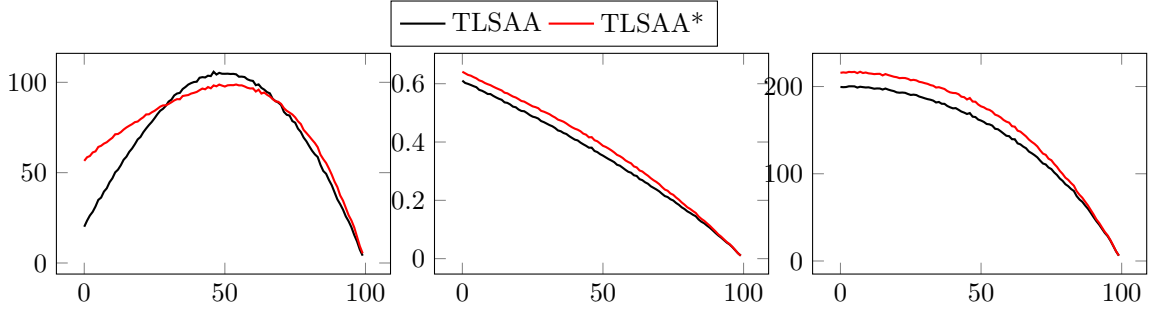


Figure 18 – Average revenue (*left*), fairness (*center*) and efficiency (*right*) as a function of the reserve price.

$(\beta_1, \beta_2, \beta_3)$	Optimal mechanism	Group configuration	R	U_{Reg}	Gain
$(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$	VCG*	C_2	27	0.765	8%
$(0.5, 0.5, 0)$	PAM*	C_2	46	0.7384	4%
$(0.5, 0, 0.5)$	TRUST*	C_2	41	0.92	9.5%
$(0, 0.5, 0.5)$	PAM*	C_2	0	0.8638	3%
$(1, 0, 0)$	PAM*	C_2	56	1	2%
$(0, 1, 0)$	PAM	C_1	0	0.94	\times
$(0, 0, 1)$	VCG*	C_2	0	1	23%
$(0.2, 0.2, 0.6)$	TLSAA*	C_2	16	0.845	14%

Table 3 – Optimal mechanisms, reserve prices and configuration for some specific values of β_1, β_2 and β_3 .

belongs to one and only one group.

5.3 Difficulty to ensure truthfulness when an MNO is defined as a player who coordinates several BSs

In the literature, it's common to suppose that a BS is player. In game theory, a player is defined as a strategic agent who takes actions. In reality the action (proposing a bid) is taken by the MNO who controls that BS. One may wonder why all those works do not assume that a player is an MNO who coordinates several BSs. We can explain that as follows: in order to obtain theoretical results and construct a truthful mechanism we use Myerson lemma which states that for any monotone allocation rule we can find the payment rule that ensures truthful bidding. However, this lemma does not work beyond single parameter environments [95], for the general case when players submit bids in more

than one dimension, there are no theoretical results, (except VCG), on how to implement a truthful payment rule. In addition, the notion of monotonicity is not anymore clear and needs to be defined.

But, for the special case where all BS of each MNO belong to one and only one group (the same group), we can consider each MNO as a big BS with a bid equal to the sum of bids of its BS and therefore we return to the original version (one dimension parameter).

In this section we consider relaxing assumption 2, we redefine an operator as a player who coordinates several BSs. We suppose that the utility U_I of an operator I which has N_I BSs is:

$$U_I = \sum_{i=1}^{N_I} \alpha_i v_i - p_I.$$

Note that we will not change the system model i.e., the regulator collects bids from BSs and then charges each BS its payment. The question which may arise is: under the new definition of a player, do the previous mechanisms preserve truthfulness?

In this new definition of players, truthfulness means that for each operator I who has N_I BSs, proposing a bid vector equal to the valuation vector, i.e., proposing $(b_1, \dots, b_{N_I}) = (v_1, \dots, v_{N_I})$ maximizes his utility.

Also, under this new definition of players, one may wonder the impact of assumption 1 on truthfulness. We show in the following proposition that relaxing assumption 1 is not anymore challenging i.e., if we can find a mechanism which is truthful under assumption 1 then it is still truthful when assumption 1 is relaxed.

Proposition 11. *Under the new definition of player, if we can design a truthful auction mechanism under assumption 1 then this mechanism can be adapted (we can find the truthful payment rule) when assumption 1 is relaxed.*

Proof. Let us fix an operator I with N_I BSs with a vector of valuations $V_I = (v_1, \dots, v_{N_I})$ under the assumption that a BS can belong to more than one group. The utility of the operator is given by:

$$U_I = \sum_{i=1}^{N_I} \alpha_i(B_I) v_i - P_I(B_I),$$

where $B_I = (b_1, \dots, b_{N_I})$ is the bid vector. We call this scenario scenario 1. Our objective is to find $P_I(B_I)$ which elicits truth telling. From scenario 1, we can find a similar scenario (scenario 2) in terms of group allocation by considering each BS i with valuation v_i and bid b_i and which belongs to n_i groups in scenario 1 as n_i different BSs in scenario 2 with the same valuation v_i and bid b_i . In total we have $N_I^* = \sum_i^{N_I} n_i$ BSs; and $\alpha_i^{\text{first}} = \sum_{j=1}^{n_i} \alpha_j^{\text{second}}$ where α^{first} is the allocation vector in the first scenario and α^{second} is the allocation vector in the second scenario. We denote by B_I^* and V_I^* the bid and valuation vectors of those N_I^* BSs.

Example 6. In the following example as shown in Fig. 19, the MNO has two BSs in scenario 1: BS 1 with bid b_1 belongs to g_1 , g_2 and g_3 and BS 2 with bid b_2 belongs to g_1 and g_2 . BS 1 which belongs to three groups in the first scenario is considered as three different BSs in the second scenario BS 1, BS 3 and BS 4; BS 2 in the first scenario is considered as two different BSs in the second scenario (BS 2 and BS 5). Since group-bids are the same in both scenario (by construction) therefore the fraction allocated to BS 1 in the first scenario is just the sum of fraction allocated to BS 1, BS 3 and BS 4 in the second scenario. Similarly, the fraction allocated for BS 2 in the first scenario is the sum of fractions allocated to BS 2 and BS 5 in the second scenario.

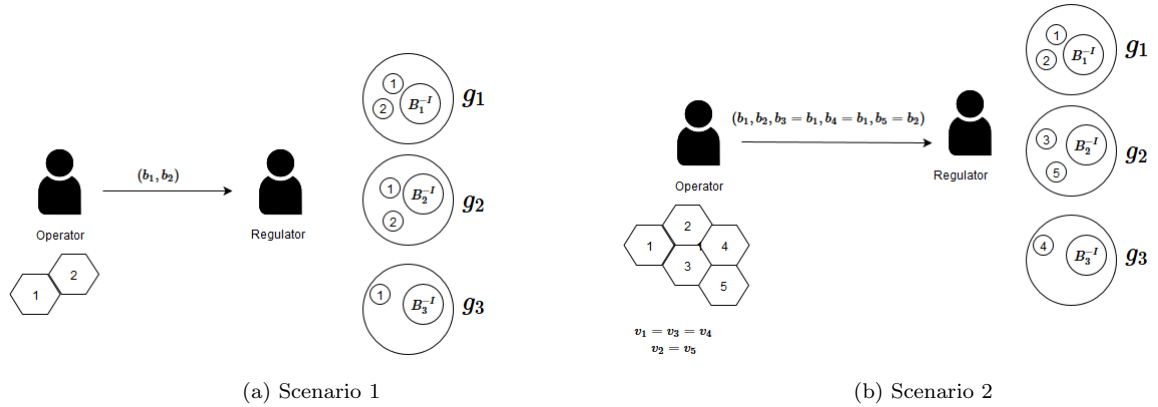


Figure 19 – Two equivalent scenarios in terms of group allocation

By assumption, for the second scenario, we can find the payment rule P_I^* which ensures truthful bidding ($B_I^* = V_I^*$). Then, by construction $P_I(B_I) = P_I^*(B_I^*)$ must ensure a truth

telling. Indeed:

$$\begin{aligned}
U_I &= \sum_{i=1}^{N_I} \alpha_i^{\text{first}}(B_I) v_i - P_I^*(B_I^*) \\
&= \sum_{i=1}^{N_I^*} \alpha_i^{\text{second}}(B_I^*) v_i^* - P_I^*(B_I^*)
\end{aligned}$$

meaning that U_i is maximized for $B_I^* = V_I^*$, by setting $B_I = V_I$ we obtain that maximum utility. \square

5.3.1 Most candidate mechanisms are not truthful

In the following proposition, we show that all the previous mechanisms, except VCG, could not be applied.

Proposition 12. *For all the previous mechanisms except VCG, there is no payment rule ensuring truthful bidding when an MNO is defined as a player who coordinates several bids of BSs.*

Proof. We show the non-existence for each of the other schemes.

- TAMES and TLSAA2: it is a consequence of Proposition 11, and the fact that these mechanisms need assumption 1 to be truthful. A negation of Proposition 11 is: if a mechanism is not truthful when assumption 1 is relaxed then it could not be truthful under assumption 1. Since TAMES and TLSAA2 are not truthful when assumption 1 is relaxed (and also in a special case when each operator has only one BS) then they could not be truthful in this new scenario.
- TRUST: consider two groups, and suppose that operator I has two BSs with valuations in bold and which are in two groups with valuations respectively $\{\mathbf{30}, 3\}$ and $\{\mathbf{5}, 4\}$. By bidding truthfully the second group wins the auction so the utility of the operator I is $5 - p_I$, however if that operator proposes a bid vector $\{\mathbf{30}, \mathbf{0}\}$ then we are back to the situation where a BS is a player (since each operator has one and only one BS): group one wins the auction and in this situation the utility of the operator i is $30 - 2 = 28$ which is strictly better than the previous one.
- PAM: consider an operator I with two BSs in two different groups. The first group is composed by two BSs of different operators and the second contains only one BS. We

denote by $v_{I,1}$ the valuation of the first BS and by $v_{I,2}$ the valuation of the second. Suppose that $v_{I,1} > v_{I,2}$ and the reserve price per bidder is zero. The utility of the operator is given by:

$$U_I = \alpha_1 v_{I,1} + (1 - \alpha_1) v_{I,2} - p_I. \quad (5.6)$$

But proposing any bid $b_I = (b_{I,1}, 0)$ ensures a maximum utility which is equal to $v_{I,1}$ because in this situation, a player is a BS and we have only one group thus the payment is zero.

- TLSAA: we consider two groups with bids respectively $\{\mathbf{16}, 20\}$ and $\{15, \mathbf{30}\}$. By bidding truthfully, operator I gets a utility lower than 16. However, if he reports only his bid for the second BS then he obtains a utility $30 - 10 = 20$.

□

5.3.2 Applying VCG

We apply the VCG mechanism in when an operator coordinates several bids of BSs. The bandwidth is allocated to the group with the highest group-bid. The payment of operator I , p_I^{VCG} is given by

$$p_I^{\text{VCG}} = E^{-I} - (E^I - \sum_{i=1}^{N_I} b_i \alpha_i),$$

where E^{-I} is efficiency when the operator I is absent, E^I is efficiency when he I is present.

Example: Consider two groups with bids respectively $\{\mathbf{5}, 30\}$ and $\{\mathbf{12}, \mathbf{8}, 25\}$. The payment of operator I (which has BSs with bold-written bids) is:

$$30 - (45 - (12 + 8)) = 5$$

5.4 Conclusion

In this chapter, in order to give the regulator more flexibility in the group construction, we have relaxed assumption 1 by allowing each BS to be in multiple group. We have shown how to adapt the payment rules of the previous mechanisms, when possible, to maintain truthful bidding without modifying the allocation rule. This relaxation impact the outcome of the auction as an example it can increase efficiency.

At a second time, we have defined a player as an operator who coordinates several BSs, this may be closer to reality. However, most of results were negative, we have shown that under this assumption, only VCG can be applied and elicit truthful bidding.

We have supposed that the available quantity of available spectrum is small. However, if the available LSA spectrum is important. Then, the auctioneer has to decide if that quantity will be allocated as one block or it will be split to several blocks with equal [96] or different [97] sizes. Splitting spectrum is important because it may increase the outcome of the auction with respect to any metric, as an example it can increase fairness of the allocation since spectrum will not be necessarily allocated to one and only one group, but it may complicate the auction analysis. In the next chapter we treat the case where the auctioneer has K blocks to allocate.

Part III

Geographical zones with more available resource

Chapter 6

One-shot auctions for multiple blocks

6.1 Introduction

In the previous two chapters, we have designed mechanisms to allocate LSA spectrum, that can be applied when there is only one block to allocate or valuation is linear. In this chapter, we will take into account the concavity of valuations. We suppose that spectrum is split into K identical blocks, identical meaning that BSs do not have preferences over blocks [70, 98]. Each BS i has a private vector-valuation \mathbf{v}_i composed of K elements: the first element $\mathbf{v}_{i,1}$ represents the value of one block and the n^{th} element $\mathbf{v}_{i,n}$ ($n > 1$) represents the value of the BS i for an n^{th} extra block given that it has already $n-1$ blocks. The value of a block, for a BS, decreases with the number of blocks already obtained. This corresponds to a discretization of concave valuation functions for spectrum [78], as illustrated in Fig. 20. Finally, we adopt a quasi-linear utility model, if a BS i obtains n_i^b blocks and pays \mathbf{p}_i , its utility is then:

$$u_i = \sum_{n=1}^{n_i^b} \mathbf{v}_{i,n} - \mathbf{p}_i.$$

In particular, an operator obtaining no block gets a utility equal to zero.

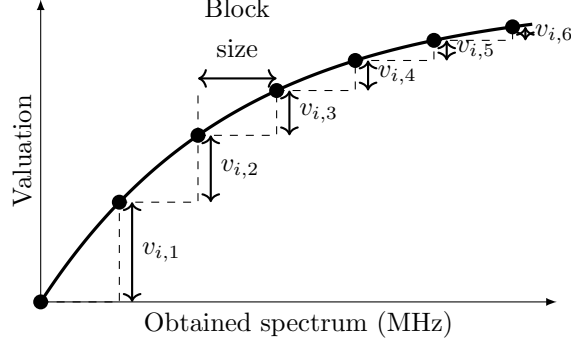


Figure 20 – An example of a concave valuation function of obtained spectrum

The chapter is composed as follows: in Section 6.2, we show how convert all-or-nothing auction mechanisms, under assumption 1 and assumption 2, to block allocation schemes. Then in Section 6.4, we derive attention on the complexity of allocation for multi-blocks scenario and we show that when assumption 1 is relaxed, the optimal allocation (with respect to efficiency) is an NP-hard problem. Section 6.5 concludes the chapter.

6.2 Converting all-or-nothing schemes to block allocation schemes

We suppose that each BS submits to the auctioneer a bid vector \mathbf{b}_i composed of K elements in non increasing order. $\mathbf{b}_{i,n}$ represents the bid of BS i for an n^{th} extra block given that it has already $n - 1$ blocks. We denote by $\mathbf{B}_h^{\text{MEC}}$ the group-bid vector of g_h . $\mathbf{B}_h^{\text{MEC}}$ is composed of K elements. The n^{th} $\mathbf{B}_{h,n}^{\text{MEC}}$ represents the bid of g_h for an n^{th} extra block given that it has already $n - 1$ blocks. We define $\mathbf{B}_h^{\text{MEC}}$, the group-bid vector of g_h which is composed of n_h bidders, as follows (without loss of generality, we suppose that g_h is composed of the first n_h bidders):

$$\mathbf{B}_{h,n}^{\text{MEC}} = f_h^{\text{MEC}}(\mathbf{b}_{1,n}, \dots, \mathbf{b}_{n_h,n}). \quad (6.1)$$

We suppose that f_h^{MEC} is a continuous and non-decreasing function with respect to each variable.

6.2.1 Allocation

Once group-bid vectors are computed from bids of players, the regulator obtains $M \times K$ bids in total (we have M group-bid vectors and each vector is composed of K elements). Blocks are allocated to the highest K bids among those $M \times K$ bids. We denote by C_h^{MEC} the vector of competing bids facing group h , i.e. C_h^{MEC} is composed of the highest K bids of other group. C_h^{MEC} is sorted in an ascending order. The number of blocks that a group wins is the number of competing bids he defeats:

$$\text{if } \mathbf{B}_{h,n}^{\text{MEC}} > C_{h,n}^{\text{MEC}} \text{ then } g_h \text{ obtains an } n^{\text{th}} \text{ block.} \quad (6.2)$$

Example 7. Suppose we have five blocks to allocate to three groups which are composed of one, two and three players respectively. Bids of players are assumed as follows:

- bids of player of the first group: $\{(25, 19, 10, 8, 2)\}$
- bids of players of the second group: $\{(10, 9, 4, 3, 2), (11, 8, 3, 2, 1)\}$
- bids of players of the third group:
 $\{(13, 10, 9, 8, 5), (11, 8, 6, 5, 2), (9, 8, 5, 3, 2)\}$ ‘

We propose to apply VCG i.e., $(\mathbf{B}_{h,n}^{\text{VCG}} = \sum_{i=1}^{n_h} \mathbf{b}_{i,n})$. Group-bids are defined as follows:

- $\mathbf{B}_1^{\text{VCG}} = \{25, 19, 10, 8, 2\}$
- $\mathbf{B}_2^{\text{VCG}} = \{21, 17, 7, 5, 3\}$
- $\mathbf{B}_3^{\text{VCG}} = \{33, 26, 20, 16, 9\}$

Now let us compute the allocations: the five highest group-bid are $\{33, 26, 25, 21, 20\}$. Three of those components (33, 26, 20) are in $\mathbf{B}_3^{\text{VCG}}$ therefore the third group obtains three blocks. One component (21) is in $\mathbf{B}_2^{\text{VCG}}$ then the second group obtains one block and one component (25) is in $\mathbf{B}_1^{\text{VCG}}$ then the first group obtains one block.

We can also performs allocations based on group-bid vectors and facing-bid vectors as follows. Let us first introduce facing-bids:

- $C_1^{\text{VCG}} = \{17, 20, 21, 26, 33\}$

- $C_2^{\text{VCG}} = \{19, 20, 25, 26, 33\}$
- $C_3^{\text{VCG}} = \{10, 17, 19, 21, 25\}$

The allocation can be performed as follows: B_1^{VCG} defeats only one elements of C_1^{VCG} ($25 > 17$), therefore the first group obtains one block. B_2^{VCG} defeats only one element of C_2^{VCG} ($21 > 19$), therefore the second group obtains one block. B_3^{VCG} defeats three elements of C_3^{VCG} ($33 > 10, 26 > 17, 20 > 19$), therefore the third group obtains three blocks.

6.2.2 Payments

We denote by p_i^{MEC} the payment vector of bidder i when the mechanism MEC is applied. $p_{i,n}^{\text{MEC}}$ represents the payment of bidder i for his n^{th} block. Here we propose a payment rule that will ensure a truthful bidding as we will show next. The payment of player i , who belongs to group h , is given by :

$$p_{i,n}^{\text{MEC}} = \min\{r : f_h^{\text{MEC}}(\mathbf{b}_{1,n}, \underbrace{\dots, r}_{i}, \dots, \mathbf{b}_{n_h,n}) \geq C_{h,n}^{\text{MEC}}\} \quad (6.3)$$

This payment can be interpreted as follows: for his n^{th} block, bidder i pays the minimum amount r that allows him to obtain that block i.e., the minimum amount such that $B_{h,n}^{\text{MEC}} = f_h^{\text{MEC}}(\mathbf{b}_{1,n}, \underbrace{\dots, r}_{i}, \dots, \mathbf{b}_{n_h,n}) \geq C_{h,n}^{\text{MEC}}$.

Example 8. The payment rule of VCG using (6.3) is:

$$\begin{aligned} p_{i,n}^{\text{VCG}} &= \min\{r : r + \sum_{j=1, j \neq i}^{n_h} b_{j,n} \geq C_{h,n}^{\text{VCG}}\} \\ &= [C_{h,n}^{\text{VCG}} - \sum_{j=1, j \neq i}^{n_h} b_{j,n}]^+ \end{aligned}$$

This payment can be interpreted as follows: $p_{i,n}^{\text{VCG}}$ equal zero when the presence of bidder i does not change the outcome i.e., there is no damage for other bidders, $p_{i,n}^{\text{VCG}}$ is strictly positive only if the presence of bidder i causes damage to other bidders i.e., when he is present that block is allocated to g_h instead to another group (which has $C_{h,n}^{\text{VCG}}$). In this situation the damage caused to other bidders is $C_{h,n}^{\text{VCG}}$, but his presence is beneficial to

the members of his group (the gain of his presence is $\sum_{j=1, j \neq i}^{n_h} \mathbf{b}_{j,n}$). The difference between those two quantities is $(C_{h,n}^{\text{VCG}} - \sum_{j=1, j \neq i}^{n_h} \mathbf{b}_{j,n})$.

Let us illustrate the payment of the first player of g_3 of example 7, we denote by $\mathbf{p}_i^{\text{VCG}}$ that payment:

$$\mathbf{p}_i^{\text{VCG}} = \mathbf{p}_{1,1}^{\text{VCG}} + \mathbf{p}_{1,2}^{\text{VCG}} + \mathbf{p}_{1,3}^{\text{VCG}} = [10 - 20]^+ + [17 - 16]^+ + [19 - 11]^+ = 0 + 1 + 8 = 9$$

The gray surface in Fig. 21 illustrates how much player one of the third group has to pay after obtaining three blocks. As this figure shows, for his first block he pays nothing because his group obtains this block whether he is present or not. However, for the second and the third block, he pays because his presence has changed the outcome.

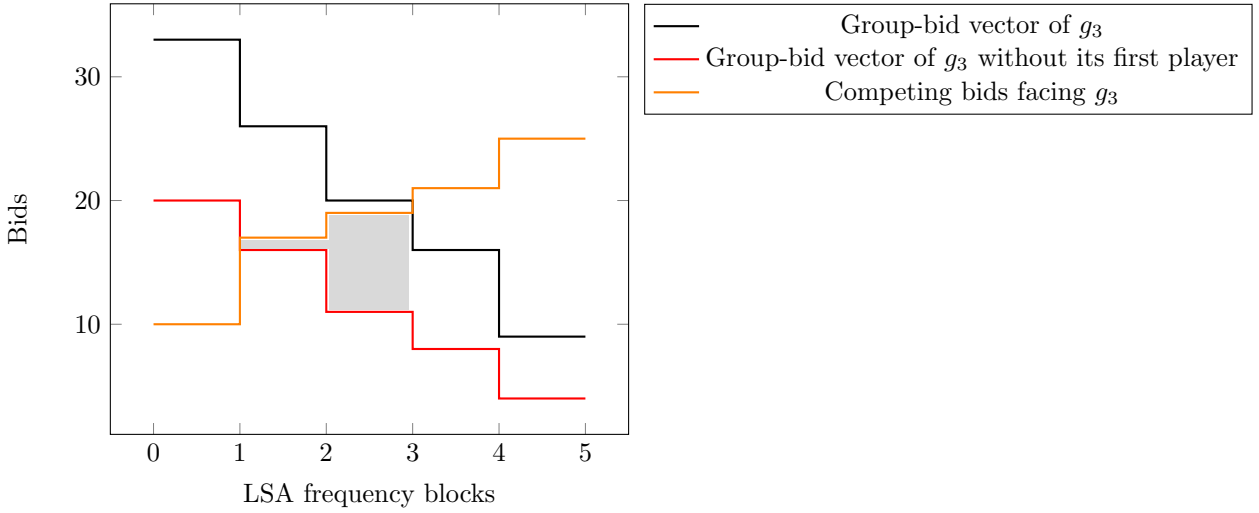


Figure 21 – Graphical illustration of the payments of the first player of the third group, noted g_3

We now prove that the proposed mechanism with allocation given in equation (6.2) and payment given in equation (6.3) is truthful. Let us first introduce the two following lemmas.

Lemma 1. *Payments for blocks can only increase i.e., if player i pays $\mathbf{p}_{i,n}^{\text{MEC}}$ for his n -th block then he pays $\mathbf{p}_{i,n+1}^{\text{MEC}} \geq \mathbf{p}_{i,n}^{\text{MEC}}$ for his $(n+1)$ -th block.*

Proof. This is a consequence from the payment rule.

$$\mathbf{p}_{i,n}^{\text{MEC}} = \min\{r : f_h^{\text{MEC}}(\mathbf{b}_{1,n}, \dots, \underbrace{\mathbf{b}_{i,n}}_r, \dots, \mathbf{b}_{n_h,n}) \geq C_{h,n}^{\text{MEC}}\} \quad (6.4)$$

$$\leq \min\{r : f_h^{\text{MEC}}(\mathbf{b}_{1,n}, \dots, \underbrace{\mathbf{b}_{i,n}}_r, \dots, \mathbf{b}_{n_h,n}) \geq C_{h,n+1}^{\text{MEC}}\} \quad (6.5)$$

$$\leq \min\{r : f_h^{\text{MEC}}(\mathbf{b}_{1,n+1}, \dots, \underbrace{\mathbf{b}_{i,n+1}}_r, \dots, \mathbf{b}_{n_h,n+1}) \geq C_{h,n+1}^{\text{MEC}}\} \quad (6.6)$$

$$= \mathbf{p}_{i,n+1}^{\text{MEC}} \quad (6.7)$$

□

For the first inequality we have used the fact that $C_{h,n+1}^{\text{MEC}} \geq C_{h,n}^{\text{MEC}}$. For the second inequality, we have used the fact that for any bidder j $\mathbf{b}_{j,n} \geq \mathbf{b}_{j,n+1}$ and the fact that f_h^{MEC} is an increasing function.

In the following, we call a component $\mathbf{b}_{i,n}$ of the bid vector of player i a *winning component* if $\mathbf{b}_{i,n} \geq \mathbf{p}_{i,n}^{\text{MEC}}$ and a *losing component* if $\mathbf{b}_{i,n} < \mathbf{p}_{i,n}^{\text{MEC}}$.

Lemma 2. *If a player gets n blocks, then the winning components of his bid vector are exactly his first n components.*

Proof. Assume that there is a situation in which $\mathbf{b}_{i,n}$ is a losing bid and $\mathbf{b}_{i,n+1}$ is a winning bid: $\mathbf{b}_{i,n}$ being a losing bid means that $\mathbf{b}_{i,n} < \mathbf{p}_{i,n}^{\text{MEC}}$ with $\mathbf{p}_{i,n}^{\text{MEC}}$ the price that he would have pay for the n th block, and $\mathbf{b}_{i,n+1}$ being a winning bid means that $\mathbf{b}_{i,n+1} \geq \mathbf{p}_{i,n+1}^{\text{MEC}}$, hence from Lemma 1, $\mathbf{b}_{i,n+1} \geq \mathbf{p}_{i,n}^{\text{MEC}}$ and then $\mathbf{b}_{i,n+1} > \mathbf{b}_{i,n}$, a contradiction. □

We are now ready to establish the main result of this section.

Proposition 13. *For the proposed mechanism with allocation given in equation (6.2) and payment given in equation (6.3), truthful bidding, is a dominant strategy i.e., for each bidder i proposing $(\mathbf{b}_{i,1}, \dots, \mathbf{b}_{i,K}) = (\mathbf{v}_{i,1}, \dots, \mathbf{v}_{i,K})$ maximizes his utility.*

Proof. Suppose that by bidding truthfully, player i (who belongs to group h) gets n blocks. From lemma 2 his winning bids are the first n bids. For his first n bids, player i cannot do better than proposing his true valuations: lowering the corresponding bids could make him lose blocks that are charged below his valuation for them, and increasing those bids would have no impact because he pays the same amount.

Player i does not obtain an $(n + 1)^{\text{th}}$ block, so we have

$$B_{h,n+1}^{\text{MEC}} = f_h^{\text{MEC}}(\mathbf{b}_{1,n+1}, \dots, \mathbf{v}_{i,n+1}, \dots, \mathbf{b}_{n_h,n+1}) < C_{h,n+1}^{\text{MEC}}. \quad (6.8)$$

If player i wants an $(n + 1)^{\text{th}}$ block then he has to propose a bid $\mathbf{b}_{i,n+1}$ such that $B_{h,n+1}^{\text{MEC}} \geq C_{h,n+1}^{\text{MEC}}$, however this leads to a lower utility: in this situation, player i pays an amount r higher than $\mathbf{v}_{i,n+1}$ because $f_h^{\text{MEC}}(\mathbf{b}_{1,n+1}, \dots, \mathbf{v}_{i,n+1}, \dots, \mathbf{b}_{n_h,n+1}) \leq C_{h,n+1}^{\text{MEC}}$, $f_h^{\text{MEC}}(\mathbf{b}_{1,n+1}, \dots, \underbrace{\mathbf{v}_{i,n+1}}_r, \dots, \mathbf{b}_{n_h,n+1}) \geq C_{h,n+1}^{\text{MEC}}$ and f_h^{MEC} is an increasing function. \square

Remarks

In the approach that we propose, if a group gets a block, then all bidders of that group can use that block. However in the original versions of TAMES and TLSAA2, there are some bidders which will be scarified. In other word, the payment rule that we propose is different from the one proposed in the original version (since allocation is not the same).

- For TAMES, bidder with the lowest bid is excluded. A question may arise is: can we generate the original payment of TAMES and keep excluding, for each winning block, player with the lowest bid for that block and charging the winners with that amount? That approach can not be generalized when there are K blocks because we may loose truthfulness as we illustrate in the following example.

Example 9. Suppose there are two blocks and two groups the first group is composed of two bidders: bids of the first bidder are $\{9, 5\}$ and $\{8, 6\}$ for the second bidder. The second group is composed of two bidders with bids respectively $\{3, 2\}$ and $\{1, 1\}$. Applying TAMES means that the first group wins both blocks. Applying the original formula of payment means that for the first block, bidder one obtains that block and pays 8, the second bidder obtains the second block and pays 5. However, if the first bidder has proposed $\{7, 7\}$, then he will obtain the second block (as before he obtains only one block) and pays 6 which increases his utility.

- Under TLSAA2 if a group obtains an n^{th} block, then not all players of that group will use that block. Contrary to TAMES in which we have shown that we can not extend its payment rule for multi-blocks scenario, in the following we show how to adapt TLSAA2. Let us fix j_n such that:

$$j_n = \max \{ \text{rank}(\mathbf{b}_{i,n}), i \in g_h \text{ and } \text{rank}(\mathbf{b}_{i,n}) \mathbf{b}_{i,n} \geq C_{h,n} \} \quad (6.9)$$

Our proposed payment rule is then that each player with rank below to j_n pays an amount to $C_{h,n}/j_n$, players with rank above j_n will not get that block.

Proposition 14. *The proposed payment rule (6.9) ensures a truthful bidding for TLSAA2 adapted for multi-blocks scenario.*

Proof. Suppose that by bidding truthfully, player i (who belongs to group h) gets n_i^b blocks. For his first n_i^b bids, player i cannot do better than proposing his true valuations: lowering the corresponding bids could make him lose blocks that are charged below his valuation for them, and increasing those bids would have no impact because the payments are independent of his bids.

Player i does not obtain an $(n_i^b + 1)^{\text{th}}$ block, so we have

$$\mathbf{v}_{i,n_i^b+1} \text{rank}(\mathbf{v}_{i,n_i^b+1}) \leq C_{h,n_i^b+1}^{\text{TLSAA2}} \quad (6.10)$$

where $\text{rank}(\mathbf{v}_{i,n_i^b+1})$ is just $\text{rank}(\mathbf{b}_{i,n_i^b+1})$ when $\mathbf{b}_{i,n_i^b+1} = \mathbf{v}_{i,n_i^b+1}$. If player i wants an $(n_i^b + 1)^{\text{th}}$ block then he has to propose a bid $\mathbf{b}_{i,n_i^b+1} \geq \mathbf{v}_{i,n_i^b+1}$ such that $\mathbf{B}_{h,n_i^b+1}^{\text{TLSAA2}} \geq \frac{C_{h,n_i^b+1}^{\text{TLSAA2}}}{j_{n_i^b+1}}$, however this leads to a lower utility: in this situation, player i pays $\frac{C_{h,n_i^b+1}^{\text{TLSAA2}}}{j_{n_i^b+1}}$ such that $j_{n_i^b+1} \leq \text{rank}(\mathbf{v}_{i,n_i^b+1})$ (see (6.9)). Thus player i pays at least $\frac{C_{h,n_i^b+1}^{\text{TLSAA2}}}{\text{rank}(\mathbf{v}_{i,n_i^b+1})}$ for that block, which is higher than \mathbf{v}_{i,n_i^b+1} (from (6.10)). \square

In the following, we refer to a mechanism MEC to be a pay-facing-bid mechanism if the payment of a group for its n^{th} block is exactly $C_{h,n}^{\text{MEC}}$ (winning bidders of that group pay that amount). As an example TLSAA2 is pay-facing-bid mechanisms. We establish some revenue guarantees for pay-facing-bid mechanisms.

6.2.3 Revenue guarantees for pay-facing-bid mechanisms

For pay-facing-bid mechanism, if a group obtains an n^{th} block i.e., $\mathbf{B}_{h,n}^{\text{MEC}} > C_{h,n}^{\text{MEC}}$ then it pays $C_{h,n}^{\text{MEC}}$. We have $\mathbf{B}_1^{\text{MEC}}, \dots, \mathbf{B}_M^{\text{MEC}}$ group-bid vectors. Each element is composed of K components. In total we have KM components. We sort all those components in a non increasing order to form a bid vector BS^{MEC} of size KM , $\text{BS}^{\text{MEC}} = \{\text{BS}_1^{\text{MEC}}, \dots, \text{BS}_{KM}^{\text{MEC}}\}$. Let us first introduce the following lemma.

Lemma 3. *If a group h wins n blocks then he pays the first n bids in $\{BS_{K+1}^{\text{MEC}}, \dots, BS_{2k}^{\text{MEC}}\}$ which are proposed by other groups.*

Proof. C_h^{MEC} is composed of the highest K bids of other groups which are a subset in $\{BS_1^{\text{MEC}}, \dots, BS_{2k}^{\text{MEC}}\}$. $\{C_{h,n+1}^{\text{MEC}}, \dots, C_{h,K}^{\text{MEC}}\}$ are in $\{BS_1^{\text{MEC}}, \dots, BS_K^{\text{MEC}}\}$ because they are winning bids (group h cannot defeat those bids). The first n components of C_h^{MEC} (which group h will pay) are in $\{BS_{K+1}^{\text{MEC}}, \dots, BS_{2k}^{\text{MEC}}\}$ because they are defeated bids and by definition since C_h^{MEC} is composed of the highest K bids of other groups, the first n components of C_h must be the first n bids in $\{BS_{K+1}^{\text{MEC}}, \dots, BS_{2k}^{\text{MEC}}\}$ which are proposed by other groups. \square

Proposition 15. *The revenue of each pay-facing-bid mechanism adapted from MEC to the multi-block setting is in $[\sum_{i=K+1}^{2K} BS_i^{\text{MEC}}, K \times BS_{K+1}^{\text{MEC}}]$.*

Proof. Clearly for each group, the facing vector cannot have a component lower than BS_{2K}^{MEC} hence the revenue from each block is higher than BS_{2K}^{MEC} , also the first K components $\{BS_1^{\text{MEC}}, \dots, BS_K^{\text{MEC}}\}$ are the winning bids thus the maximum revenue from each block is BS_{K+1}^{MEC} .

- Upper bound: The best case in terms of revenue is when all blocks are allocated to K different groups (here we suppose that M higher than K) and BS_{K+1}^{MEC} is a bid from another group who does not get any block, in this situation the revenue is $K \times BS_{K+1}^{\text{MEC}}$.

- Lower bound: Suppose that the revenue could be lower than $\sum_{i=K+1}^{2K} BS_i^{\text{MEC}}$, this means that there exists at least BS_j^{MEC} ($j \in \{K, \dots, 2K-1\}$) which will not be paid and at least a component BS_i^{MEC} lower than BS_j^{MEC} which will be paid at least twice by a group h and another group h' . This is a contradiction because:

1. if BS_j^{MEC} is not a bid of group h then in this situation and using lemma 3, since group h pays BS_i^{MEC} then it must pay BS_j^{MEC} .
2. if BS_j^{MEC} is a bid of group h then in this situation and using lemma 3, since group h' pays BS_i^{MEC} then it must pay BS_j^{MEC} .

Thus, BS_j^{MEC} must be paid by some group. Hence the revenue cannot be lower than

$$\sum_{i=K+1}^{2K} BS_i^{\text{MEC}}. \quad \square$$

6.2.4 Performance analysis

In the following section, we suppose that the auctioneer has K blocks to allocate and we compare the previous mechanisms in terms of average efficiency, average revenue and average fairness.

Simulation settings

For our simulation we go through the following steps:

1. Fix the number of blocks and the number of groups.
2. The number of players is chosen randomly from the discrete uniform distribution of integer values in the interval $[1 ; 30]$
3. For each player i we create the bid vector \mathbf{b}_i which is composed of K elements: the first bid is drawn from the uniform distribution over the interval $[0, 100]$ and the n -th element ($n > 1$) is drawn from the uniform distribution $[0, \mathbf{b}_{i,n-1}]$.

For each number of blocks and number of groups, the average revenue and social welfare are computed over 10 000 draws. A draw means that we generate the number of players for each group then we generate the vector bid of each player.

Simulation results

K	1	2	4	8	16	50	100
VCG	0.18	0.34	0.59	0.758	0.881	0.966	0.98
TRUST	0.12	0.21	0.39	0.61	0.78	0.92	0.96
TLsAA	0.17	0.35	0.58	0.78	0.896	0.965	0.97
TLsAA2	0.11	0.23	0.49	0.65	0.782	0.89	0.93

Table 4 – Average fairness as a function of the number of blocks for $M = 10$

In terms of fairness, Table 4 shows that as we keep increasing the number of blocks, fairness increases this seems to be natural since as we keep increasing the number of blocks there are more chances that each group will be served which leads to increasing Jain's index.

K	1	2	4	8	16	50	100
VCG	1	1	1	1	1	1	1
TRUST	0.66	0.658	0.69	0.74	0.83	0.96	0.99
TLSAA	0.97	0.98	0.99	0.99	0.99	0.999	0.99
TLSAA2	0.86	0.88	0.91	0.93	0.95	0.99	0.99

Table 5 – Average normalized efficiency as a function of the number of blocks for $M = 10$

K	1	2	4	8	16	50	100
VCG	0.26	0.18	0.122	0.10	0.102	0.12	0.14
TRUST	0.23	0.208	0.166	0.12	0.068	0.005	0.004
TLSAA	0.31	0.23	0.15	0.12	0.118	0.13	0.15
TLSAA2	1	1	1	1	1	1	1

Table 6 – Average normalized revenue as a function of the number of blocks for $M = 10$

K	1	2	4	8	16	50	100
TLSAA2	722	1278.81	1970	2433	1067.5	599	26.07

Table 7 – Average revenue of TLSAA2 as a function of the number of blocks for $M = 10$

In terms of efficiency, Table 5 shows that TRUST, TLSAA and TLSAA2 converge to the optimal efficiency as we keep increasing the number of blocks which can be justified as follows: we can see from Fig. 20 that if we keep moving on to the right side by adding blocks, the value of an extra block is very small. i.e., adding a block has a very low impact on social welfare. In terms of revenue, as we can see from Table 6 TLSAA2 offers the highest revenue. However, if we keep increasing K , the revenue converges to zero (Table 7), this can be explained from Fig. 15, as we increase the number of blocks BS_K^{MEC} converges to zero, which justifies the low revenue.

As we can see from Table 7, for $K = 100$, revenue is very low. If the objective of the regulator is to maximize his revenue then by allocating 8 blocks he obtains a higher revenue. Therefore a natural question that may arise is: how to divide spectrum?

6.3 How to set the number of blocks for a given quantity of spectrum?

Choosing the number of blocks is very important because it may impact the outcome of the auction especially in terms of revenue as we show in the following example: we suppose that the auctioneer has a quantity of spectrum that could be split into two blocks so he can allocate that quantity as a one block or two blocks. There are three groups, each group is composed of one bidder.

Example 10.

- For the first case, we suppose that valuation vectors of bidders for two blocks are: $\{30, 1\}$, $\{25, 3\}$ and $\{4, 4\}$. The valuation of bidders for the whole spectrum as a one block are 31, 28 and 6. In such a situation, if the auctioneer allocates spectrum as a one block, then the first bidder obtains that block and pays 28. On the other hand, if the auctioneer allocates the spectrum as two blocks then bidder one and bidder two obtain one block each one and each bidder pays 4. The revenue is 8.
- For the second case, we suppose that valuation vectors of bidders for two blocks are: $\{10, 5\}$, $\{10, 1\}$ and $\{9, 0\}$. The valuation of bidders for the whole spectrum as a one block are 15, 11 and 9. In such a situation, if the auctioneer allocates spectrum as a one block then the first bidder obtains that block and pays 11. On the other hand, if the auctioneer allocates the spectrum as two blocks then bidder one and bidder two obtain one block each one, and each bidder pays 9 so the revenue is 18. In this situation, allocating spectrum as a two blocks maximizes the revenue.

Therefore, splitting spectrum is very important. In the following simulation, we suppose that there are 100 blocks with the minimum allowed size. Those blocks can be seen as $K' = 50$ blocks of double size or $K' = 25$ blocks of quadruple size etc.

We evaluate the outcome of the auction by changing the number of blocks. We conclude that by setting $K' = 5$, and by applying TLSAA2, the auctioneer multiplies his revenue by 1000 in average (Table 8) for a loss of efficiency lower than 35% (Table 9) compared to the optimal efficiency (Applying VCG and set $K' = 100$) and a loss of fairness lower than 50 % (Table 10).

In the following, we focus on efficiency. We study the impact of relaxing assumption 1 on the complexity of VCG's allocation. Indeed, when assumption 1 is relaxed, then the allocation is not necessarily trivial since there is a correlation among group-bid vectors.

K'	100	10	5	4	2	1
VCG	3.82	1.5	422	458	488	368
TRUST	0.01	548	563	450	306	275
TLSAA	4.08	1.17	492	534	525	380
TLSAA2	26.8	40	3740	3523	2292	1304

Table 8 – Average revenue as a function of the number of blocks for $M = 10$

K'	100	10	5	4	2	1
VCG	15574	15487	11337	9962	5469	2945
TRUST	15535	15481	8233	6830	3606	1908
TLSAA	15574	15486	11271	9588	5379	2887
TLSAA2	15570	15486	10572	8862	4800	2558

Table 9 – Average efficiency as a function of the number of blocks for $M = 10$

K'	100	10	5	4	2	1
VCG	0.98	0.99	0.71	0.6	0.33	0.18
TRUST	0.96	0.99	0.5	0.4	0.21	0.12
TLSAA	0.97	0.99	0.0.71	0.61	0.33	0.17
TLSAA2	0.93	0.99	0.54	0.44	0.22	0.11

Table 10 – Average fairness as a function of the number of blocks for $M = 10$

6.4 Impact of allowing a BS to be in several groups on the complexity of VCG's allocation

In this section, we treat the case when assumption 1 is relaxed so each BS i may belong to several groups. We are interested in the implementation of VCG. We denote by X the allocation vector for groups, $X = \{x_1, \dots, x_M\}$ and $\sum_{i=1}^M x_i = K$. In the following proposition, we show that when assumption 1 is relaxed, VCG's allocation is an NP-hard problem.

Proposition 16. *For a given configuration of groups and with the hypothesis that a player can belong to more than one group, allocating resources in an efficient manner is an NP Hard problem.*

Proof. We show this by the reduction of the maximum coverage problem [99]. The max-

imum coverage problem is known to be NP Hard. It can be described as follows: given a collection of sets $S = S_1, S_2, \dots, S_M$ which may have common elements, select at most K of these sets such that the maximum number of elements are covered i.e., the union of the selected sets has maximal size. We consider an instance of the previous problem and reduce it to our problem. The reduction may be made as follows:

- Each set is a group.
- An element of a set is a player.
- The number of sets to select is the number of blocks to allocate.
- Each player wants exactly one block and his valuation equals 1.

Clearly, solving our problem leads to solving the previous problem. \square

Relaxing assumption 1 may lead to increase efficiency but it may induce some computational complexity. On the other hand, when we consider assumption 1, the allocation is easy: We have just to sort the KM components and attributing blocks to groups with the K highest components (since there is no correlation among those components) so the complexity is just the complexity of sorting KM components which is $\mathcal{O}(KM \log KM)$, but we may loose in terms of efficiency.

we evaluate the performance of the proposed method of grouping (Algorithm 1 in Chapter 2 and compare it to the optimal one (assumption 1 is relaxed) in terms of efficiency. We define the normalized efficiency E_{Nor} as follows:

$$E_{\text{Nor}} := \frac{E_{\text{prop}}}{E_{\text{opt}}}, \quad (6.11)$$

where E_{prop} is the efficiency generated with algorithm 1 and E_{opt} is the optimal efficiency which is obtained in two steps:

1. Extracting all maximal independent sets of the interference graph. This can be done using a software like Julia. Notice that this step depends on the density of the graph i.e., the number of edges divided by all possible edges which is equal to $\frac{N(N-1)}{2}$.
2. Computing an optimal allocation for that set of groups via a solver. To obtain the optimal allocation, we have to solve the following problem:

$$\begin{array}{ll} \underset{X}{\text{maximize}} & \sum_{i=1}^N \sum_{j=1}^{n_i^b} \mathbf{b}_{i,j} \end{array} \quad (6.12)$$

$$\begin{array}{ll} \text{subject to} & \sum_{h=1}^M x_h = K, \end{array} \quad (6.13)$$

$$n_i^b = \sum_{h=1}^M \gamma_{i,h} x_h, \forall i \in \llbracket 1; N \rrbracket \quad (6.14)$$

where $\gamma_{i,h} = 1$ if player i belongs to the group h . The first constraint ensures that the sum of allocated blocks for groups is K . The second constraint means that the number of blocks obtained by bidder i is the sum of blocks obtained by groups to which he belongs to. This problem is a combinatorial and non-linear problem because the objective function is not linear. Having a linear formulation is an important task to solve effectively the problem with classic solvers. Transforming the problem into a Integer Linear Problem (ILP) problem consists in developing an equivalent expression where objective function and all constraints are linear. In order to obtain a linear objective function, we have introduced constraints (6.17), (6.18) to linearize the objective function. We have introduced a binary variable $y_{i,j}$, $y_{i,j} = 1$ if $j \leq n_i^b$. Thus, the linear problem is represented as follows:

$$\begin{array}{ll} \underset{X}{\text{maximize}} & \sum_{i=1}^N w_i \end{array} \quad (6.15)$$

$$\begin{array}{ll} \text{subject to} & \sum_{h=1}^M x_h = K, \end{array} \quad (6.16)$$

$$w_i = \sum_{j=1}^K y_{i,j} \mathbf{b}_{i,j}, \forall i \in \llbracket 1; N \rrbracket, \quad (6.17)$$

$$y_{i,j} \leq n_i^b, \forall i \in \llbracket 1; N \rrbracket, \forall j \in \llbracket 1; K \rrbracket, \quad (6.18)$$

$$n_i^b = \sum_{h=1}^M \gamma_{i,h} x_h, \forall i \in \llbracket 1; N \rrbracket \quad (6.19)$$

Finally, the optimisation problem can be written as follows:

$$\begin{aligned} & \underset{\mathbf{X}}{\text{maximize}} && \sum_{i=1}^N \sum_{j=1}^K y_{i,j} \mathbf{b}_{i,j}, \end{aligned} \quad (6.20)$$

$$\begin{aligned} & \text{subject to} && \sum_{h=1}^M x_h = K, \end{aligned} \quad (6.21)$$

$$y_{i,j} \leq \sum_{h=1}^M \gamma_{i,h} x_h, \forall i \in \llbracket 1; N \rrbracket, \forall j \in \llbracket 1; K \rrbracket, \quad (6.22)$$

Simulation settings

As for the test settings, the computations have been made on a server of 16 processors Intel Xeon of CPU 5110 and clocked at 1.6 GHz each. The code has been written in Julia 0.5.0, and the solver used is Cplex 12.6 (default branch-and-cut algorithm [100]).

The steps of simulation could be summarized as follows:

- Fix the density of the graph d (the number of edges divided by the maximum possible number of edges), N and K .
- Generate an interference graph randomly with respect to N and d .
- Create groups in two manners: by the proposed method and by extracting all maximum independent sets: the generation of the interference graph and the extraction of all maximum independent sets are made by Julia.
- Generate bids: For each player i we create the bid vector which is composed of K elements: the first bid is drawn from the uniform distribution over the interval $[0, 100]$ and the n -th element ($n > 1$) is drawn from the uniform distribution $[0, \mathbf{b}_{i,n-1}]$.
- Allocate blocks with respect to the two grouping methods.

Simulations were made over 100 independent draws. A draw means that we generate a graph with respect to d and N , and for each player i we generate his bid-vector. We denote by t_1 the average resolution time for the optimal solution (groups with maximum independent sets) and t_2 the resolution time with the proposed method of grouping.

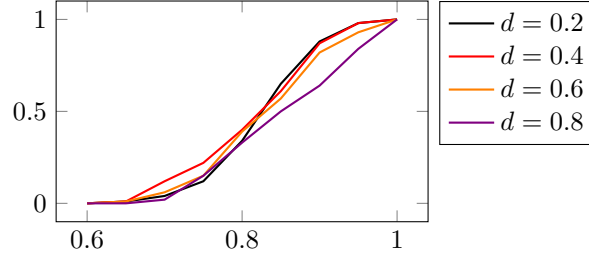


Figure 22 – Cumulative density function of E_{Nor} as a function of the density of the graph for $K = 2$ and $N = 50$

K	1	2	4	8
Average E_{Nor}	0.71	0.743	0.773	0.81
$t_1(s)$	0.5	3.2	62.3	450
$t_2(s)$	0.2	0.55	1.1	1.9

Table 11 – Average normalized efficiency and resolution time as a function of the number of blocks for $N = 50$ and $d = 0.2$

Simulation results

As we see from Fig. 22 and Table 11, with the proposed method of grouping, efficiency is at least 60 % of the optimal one. Table 11 shows the average resolution time of the optimal allocation and the average resolution time of the allocation with the proposed method of grouping. As we keep increasing the number of block, the gap between those time increase exponentially, therefore when the number of block is high, allowing a BS to be in one and only one group may be a necessarily assumption especially when duration of license is near to the optimal resolution time otherwise we may loose time looking for the optimal allocation.

6.5 Conclusion

In this chapter, we have studied the case when a regulator has several identical blocks to allocate in the context of LSA. We have shown how to adapt any all-or-nothing auction mechanism into a multi-blocks mechanism. Also, we have investigated spectrum splitting i.e., for a given a quantity of spectrum we have shown how to set K in order to

maximize the utility of the regulator. In addition, when assumption 1 is relaxed the initial problems (group construction and allocation) are NP-Hard. Therefore, we have proposed an heuristic for grouping. This heuristic is easy and fast to built but may have a negative impact on efficiency. We have quantity the loss in terms of efficiency. Simulation suggest that the heuristic performs well and efficiency is at least 60 % of the optimal one. In the next chapter, we focus on ascending auctions.

Chapter 7

Ascending auction for multiple blocks

7.1 Introduction

In this chapter, we again suppose that the auctioned LSA spectrum is composed of K identical blocks, but contrary to the mentioned previous work which are sealed-bid one-shot auctions, we focus on “ascending auctions” where information is revealed by bidders during some convergence phase. Ascending auctions have been used with great success to auction spectrum and they are sometimes preferred to sealed auctions [52]. Compared with one-shot auctions, ascending auctions have several advantages: they preserve the privacy of the winning bidder(s) because the winner(s) do(es) not reveal his/her valuation(s). Also, they give bidders the opportunity to adjust their valuations over the convergence phase. This benefit of price discovery is ignored in one-shot auctions, which assume that each bidder perfectly knows his valuation. Another advantage is the transparency because each bidder sees the evolution of the auction. In this document we first present an auction mechanism proposed by Mishra and Parkes, we show how to adapt it to the LSA context. Since we may have some computational complexity, we use another approach: Ausubel [9] has developed a mechanism which achieves the outcome of VCG. This mechanism is based on the “clinching approach”. We show how to adapt the clinching approach for the LSA context using two approaches: the first approach is by adding a representative per group so that the auction will be between the auctioneer and those representatives and the second approach is by removing those representatives i.e., BSs communicate directly with the auctioneer.

We assume (without much loss of generality, since one can select the monetary unit) that all valuations are integers. Also, we consider assumption 1 and assumption 2.

7.2 Adapting Mishra and Parkes's mechanism

In this section we start by presenting Mishra and Parkes's mechanism then we show how to adapt it for the LSA context.

7.2.1 Background: Mishra and Parkes' UCE mechanism

The mechanism proposed by Mishra and Parkes can be applied for general valuations (items may be different), this auction mechanism is equivalent to VCG (same allocations and payments). Consider N bidders and a set of different items $\mathbf{I} = \{1, \dots, K\}$, and denote by $\Omega = \{S \subseteq \mathbf{I}\}$ the set of all bundles of items. Mishra and Parkes [54] define an ascending auction –which we will call UCE– as a price path that starts from round 0 with vector price P^0 and ends at some round T with vector price P^T . The authors develop an ascending auction that is equivalent to the VCG mechanism by introducing the concept of Universal Competitive Equilibrium (UCE). The price vector P^t at each round t is of dimension $N2^K$ (each player i faces a price vector P_i^t of 2^K elements, one for each bundle), and is therefore non-anonymous (each player sees a different price for the same bundle). Before presenting the auction, we introduce some necessary notations and definitions.

Notations and definitions for UCE

We denote by $v_i(S) \geq 0$ the valuation of player i for the bundle of items S and by $p_i(S)$ the price paid by buyer i when obtaining that bundle.

Feasible allocation An allocation is a vector of bundles on buyers, the set of feasible allocations is denoted by \mathbb{X} .

Demand set The demand set of a player i , d_i^t at round t is defined as the set of bundles that maximize his profit at price vector P_i^t , i.e., mathematically,

$$d_i(P) := \arg \max_{S \in \Omega} (v_i(S) - P_i^t(S)).$$

Note that $d_i(P)$ can contain several elements if the player has several utility-maximizing bundles. Each player reports his demand set at each round; if a buyer demands the empty bundle, then this player is called inactive, and must receive a zero utility from any obtained bundle, i.e., $P_i(S) \geq V_i(S) \forall S \in \Omega$.

Supply set The supply set is the set of allocations that maximize the payoff of the seller, which is the sum of payments of all players at price P :

$$L(P) := \arg \max_{X \in \mathbb{X}} \sum P_i(X_i)$$

Note that those are not the real paid prices: the price vector aims at eliciting preference revelation, so each player will have a discount at the end of the auction. The final payment by each player is indeed defined in [54] as:

$$p_i = P_i(S) - \underbrace{\left(\Pi(P) - \Pi(P_{-i}) \right)}_{\text{discount}}, \quad (7.1)$$

where $\Pi(P)$ is the sum of prices (based on P) of players for the final allocation, and $\Pi(P_{-i})$ is the sum of payments of all players when i is absent (i.e., for a new revenue-maximizing allocation ignoring i).

Definition 3. *Competitive Equilibrium (CE): a price P and an allocation X are a competitive equilibrium (CE) if $X \in L(P)$ and $X_i \in d_i(P_i)$ for every buyer i , i.e., for this price the allocation both maximizes revenue and satisfies each buyer.*

Definition 4. *Universal Competitive Equilibrium (UCE): a price P is a UCE if it is a CE and the projection of P on every marginal economy (that is, the same situation but removing one player: in total there are N marginal economies) is a CE. This means that we can always satisfy all buyers while maximizing the revenue of the regulator after excluding any individual player.*

Achieving a universal competitive equilibrium price is very important because as is proved in [54], Vickrey payments can be computed from P if and only if P is a UCE price vector.

We illustrate these notions using an example with two items (A and B) and three

players having the following valuations:

$$\begin{pmatrix} v_1(A) & v_1(B) & v_1(A, B) \\ v_2(A) & v_2(B) & v_2(A, B) \\ v_3(A) & v_3(B) & v_3(A, B) \end{pmatrix} = \begin{pmatrix} 4 & 0 & 4 \\ 0 & 5 & 5 \\ 0 & 2 & 4 \end{pmatrix}.$$

We also suppose that $P_1(\emptyset) = P_2(\emptyset) = P_3(\emptyset) = 0$. Now consider the price vectors in $P = \begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 2 \\ 0 & 2 & 4 \end{pmatrix}$. The player demands are then $d_1(P) = \{A, \{A, B\}\}$, $d_2(P) = \{B, \{A, B\}\}$ and $d_3(P) = \{\emptyset, A, B, \{A, B\}\}$. In this example (P, X) –where X is the allocation that assigns item A to player one and item B to player two– is a CE equilibrium for the main economy because the seller can maximize his revenue and satisfy the players. However, after excluding player one, P is not anymore a competitive equilibrium in the resulting marginal economy because maximizing revenue implies allocating both items to player three, in which case player two is not satisfied.

On the other hand, the price $P = \begin{pmatrix} 2 & 0 & 4 \\ 0 & 4 & 4 \\ 0 & 2 & 4 \end{pmatrix}$ is a universal competitive equilibrium:

in the main economy composed by all players, the seller can maximize his revenue (6) while satisfying all buyers by allocating item A for player one and item B for player two. In addition:

- this still holds in the marginal economy where player three is removed;
- in the economy composed by player two and three, revenue is maximized by allocating both items (or only item B) to player two, both players having no better option;
- similarly in the economy composed by player one and three, the seller can allocate both items to player one.

Now applying (7.1), we get the paid prices $p_1 = 2 - (6 - 4) = 0$, $p_2 = 4 - (6 - 4) = 2$, and $p_3 = 0$.

The UCE auction steps

We now explain how the UCE auction can be implemented in practice.

The auction starts with all prices set to 0 in P^0 . At each round t , the seller asks players' demands for the price vector, and checks whether a Universal Competitive Equilibrium is reached. If it is not the case, a subset of active players (i.e., not having \emptyset in their demand set) is selected and all the prices of their demand sets are increased by one unit in the next price P^{t+1} . How to choose this subset opens some trade-offs, larger subsets speeding up the convergence while possibly increasing the communication overhead and the revealed valuations of players. An example of choosing that subset is given in Section 7.2.2.

We can summarize the auction steps as follows:

1. At every round t , each buyer reports his demand set for the price vector P^t . Players should respect two activity rules:
 - Round Monotonicity: for every buyer $d_i(P^t) \subset d_i(P^{t+1})$.
 - Bundle Monotonicity: if $S \subset T$ and $S \in d_i(P^t)$ then $T \in d_i(P^t)$.

Note that these rules are satisfiable because valuations are integers and prices only increase by one unit (or zero) between t and $t + 1$ for bundles in $d_i(P^t)$, and never increase if $\emptyset \in d_i(P^t)$.

2. The seller computes the supply. If the situation is not an UCE, the auctioneer chooses a set of players who will see a price increase at each demanded bundle.
3. The auction ends when a UCE price vector P is reached; then a CE allocation is chosen, i.e., revenue is maximized and every buyer gets a bundle from his demand set.
4. Each buyer is charged an amount p_i computed from the final price P , applying (7.1) for the chosen allocation.

7.2.2 Adapting Mishra and computational complexity

To implement the UCE auction mechanism, we need to take into account a set of feasible allocations different from the original design since the allocation will be made for groups instead of individual bidders while bids are individual.

LSA-UCE rules

We denote by \mathbb{X}^g the set of feasible allocations, that assign to each player i a bundle X_i^g such that $X_i^g = X_j^g$ if players i and j are in the same group and $X_i^g \cap X_j^g = \emptyset$ otherwise. Note that all the demonstrations of [54] are still valid when replacing \mathbb{X} with \mathbb{X}^g so the truthfulness and social welfare maximization properties can be easily proved. The auction steps are exactly the same as those presented in subsection 7.2.1, except that the allocation is made for groups i.e., players of the same group obtain the same bundle. We illustrate in the following how we can adapt Mishra and Parkes' model in the LSA context.

Note that the price vector for each player is composed of K components instead of 2^K (since items are identical), the first component representing the price for all K blocks and the K^{th} component representing the price for only one block.

An example is provided in Tab. 12. At each round, following a proposition in [54], we take a minimum set of buyers who cannot be jointly satisfied until a CE, i.e., until Round 7. Then we pick one of the active players (those having strictly positive utilities) until a UCE is reached, at Round 9: in the marginal economies where one player of Group 1 is removed, revenue is maximized by giving one block to each player, and if player four is removed each player can get two blocks. Finally, Group 1 obtains two blocks since this maximizes revenue, and the payments yield: $p_1 = 3 - (9 - 7) = 1$, $p_2 = 3 - (9 - 7) = 1$, $p_3 = 3 - (9 - 8) = 2$, and $p_4 = 0 - (9 - 9) = 0$.

Computational complexity problems with U-LSA

Even if the original problem of computing an optimal allocation is not NP-hard when we know the players' valuations, implementing UCE may involve having to solve NP-hard problems, which prevents its use in practice.

Proposition 17. *Even when blocks are identical, an implementation of UCE can lead to the regulator having to solve NP-hard problems.*

Proof. We show that the step of finding a revenue- maximizing allocation can correspond to solving a knapsack optimization problem with N items, item i ($i = 1, \dots, N$) having weight w_i and value \tilde{v}_i and maximum allowed weight W . Note that we assume $W < \sum_i w_i$ otherwise the problem is trivial.

We consider an instance of the knapsack problem and reduce it to our problem:

Round	$V_{i,2}, V_{i,1}$	Group 1			Group 2
		buyer 1 5, 3	buyer 2 6, 4	buyer 3 6, 3	buyer 4 7, 6
0	Price Utility	0, 0 (5), 3	0, 0 (6), 4	0, 0 (6), 3	0, 0 (7), 6
1	Price Utility	1, 0 (4), 3	0, 0 (6), 4	0, 0 (6), 3	1, 0 (6), (6)
2	Price Utility	1, 0 (4), 3	0, 0 (6), 4	1, 0 (5), 3	2, 1 (5), (5)
3	Price Utility	2, 0 (3), (3)	0, 0 (6), 4	1, 0 (5), 3	3, 2 (4), (4)
4	Price Surplus	2, 0 (3), (3)	1, 0 (5), 4	1, 0 (5), 3	4, 3 (3), (3)
5	Price Utility	2, 0 (3), (3)	1, 0 (5), 4	2, 0 (4), 3	5, 4 (2), (2)
6	Price Utility	2, 0 (3), (3)	2, 0 (4), (4)	2, 0 (4), 3	6, 5 (1), (1)
7	Price Utility	2, 0 (3), (3)	2, 0 (4), (4)	3, 0 (3), (3)	7, 6 (0), (0)
8	Price Utility	2, 0 (3), (3)	3, 1 (3), (3)	3, 0 (3), (3)	7, 6 (0), (0)
9	Price Utility	3, 1 (2), (2)	3, 1 (3), (3)	3, 0 (3), (3)	7, 6 (0), (0)

Table 12 – Example of an UCE auction in the LSA context for two resource blocks and two groups ($V_{i,2}$ is the valuation of bidder i for two blocks and $V_{i,1}$ is his valuation for one block). At each round, bidder utility in parentheses indicate the demand sets (as example, in the round 0 player one demands two blocks because he maximizes his utility if he obtains two blocks), and grayed cells indicate the bidders whose prices (of the demand set) will be raised by one unit (as example, in the round 0 player one demands two blocks and player four demands two blocks, we choose them as minimum set, thus prices of their demand sets will increase in the next round).

- The maximum allowed weight W is the number of blocks K to allocate.
- Each item of weight w_i and value \tilde{v}_i corresponds to a group which contains only one player with vector of valuations v_i such that $v_{i,n} = \tilde{v}_i \mathbb{1}_{n \leq w_i}$, this means that the valuation of player i for n_i blocks is $n_i \tilde{v}_i$ if $n_i \leq w_i$ and $w_i \tilde{v}_i$ if $n_i \geq w_i$.

Then, as long as its price is below \tilde{v}_i , player i will keep asking for w_i blocks or more, and the corresponding prices will be of the form “ p_i for w_i blocks or more, and 0 otherwise”. Since the algorithm does not specify whose (unsatisfied) bidder prices will be raised, it can happen¹ that the prices of each bidder i reach \tilde{v}_i , and finding an allocation maximizing revenue then corresponds to solving the knapsack problem. \square

Thus, we propose in the following mechanism a simpler allocation based on the concept of clinching.

7.3 Adapting the clinching approach to the LSA context

We start by presenting the clinching auction and then we show how to adapt it for the LSA context.

7.3.1 Background: clinching auction

The clinching auction is an ascendant auction for K homogeneous goods, where bidders have decreasing marginal valuations: the willingness-to-pay for an extra item decreases with the number of items already obtained. At each round t , the auctioneer declares a price p^t and bidders respond by asking for a quantity (at each round the demanded quantity cannot exceed the demanded quantity in the previous round) at that price, the price increasing (in general we can increment by $\epsilon > 0$ but here with integer valuations we take $p^{t+1} = p^t + 1$) until demand is no greater than supply K . Bidders’ payments are computed during the auction: an active bidder clinches (obtains) an item at price p if the demand of the other players at that price is less than the supply. The seller computes two

¹As an example suppose that $W = 6$ and consider an item of weight $w_i = 3$ and value $\tilde{v}_i = 5$, this object corresponds to a player with $v_i = (5, 5, 5, 0, 0, 0)$, or $V_i = (15, 15, 15, 15, 10, 5)$. The price starts with $(0, 0, 0, 0, 0, 0)$, player i keeps asking for 3, 4, 5 or 6 blocks, thus the price can reach $(5, 5, 5, 5, 0, 0)$.

quantities namely *cumulative clinch* and *current clinch*, defined as follows. The cumulative clinch Cl_i^t of player i at round t is defined as:

$$\text{Cl}_i^t = \max\{0, K - \sum_{j \neq i} d_j^t\}, \quad (7.2)$$

with d_j^t the demand of player j at round t . The increment of the obtained blocks is called the current clinch at round t of player i , and denoted by cl_i^t :

$$\text{cl}_i^t = \text{Cl}_i^t - \text{Cl}_i^{t-1}. \quad (7.3)$$

When the auction ends, each bidder i obtains a quantity equal to his cumulative clinch Cl_i , and his payment p_i is:

$$p_i = \sum_{t=0}^T p^t \text{cl}_i^t. \quad (7.4)$$

It was proven in [9] that the clinching auction achieves the outcome of VCG i.e., it ensures an efficient allocation, charges each player with his Vickrey payment and bidding truthfully is an ex post Nash equilibrium. Truthful bidding is an ex-post equilibrium when each player knows that bidding truthfully is a best strategy if all other players also bid truthfully and without knowing the other players' valuations [101]. Here bidding truthfully means that each player reports his demand with respect to his valuations: $d_i(p) = \max\{n \text{ such that } \mathbf{v}_{i,n} > p\}$, for a given declared price p .

An illustrative example is provided in Tab. 13, with three items, and three players with respective marginal valuations $\{6, 4, 0\}$, $\{5, 3, 2\}$, $\{2, 1, 0\}$. (Note that we will suppose that players are not willing to pay a price per block equal to the valuation of that block, as an example if player one gets one block for a price $p^t = 6$ then his utility is zero, hence we will suppose that for $p^t = 6$ player one will not demand any block i.e., his demand is zero.)

For $p = 2$, the sum of demands of player two and three is equal to 2, hence, $\text{cl}_1^2 = 1 - 0 = 1$,

player one clinches his first block at price 2. Similarly, player two clinches his block at the same price. At $p = 3$, $\text{cl}_1^3 = 2 - 1 = 1$, thus player one clinches his second block. Finally the auction concludes at price $p = 3$ ($d_1 + d_2 + d_3 = 3$), player one obtains two blocks and pays $2 + 3 = 5$ and player two obtains one block and pays 2.

Round	0	1	2	3
Price	0	1	2	3
Total demand	7	6	4	$3 = K$
d_1	2	2	2	2
Cl_1^t	0	0	1	2
cl_1^t	0	0	1	1
p_1	0	0	2	$2+3$
d_2	3	3	2	1
Cl_2^t	0	0	1	1
cl_2^t	0	0	1	0
p_2	0	0	2	2
d_3	2	1	0	0
Cl_3^t	0	0	0	0
cl_3^t	0	0	0	0
p_3	0	0	0	0

Table 13 – An example of clinching auction for $K = 3$ items.

Remarks

1. We illustrate in the following example why truthful telling is not a dominant strategy. We suppose we have two blocks and two players, where valuations of the first player are $\{3, 2\}$ and the second $\{2, 1\}$. Suppose that the second player uses the following strategy: if the first player demands two blocks at the first round then he will continue to demand 2 blocks until the end of the auction (even though he will obtain a negative utility), otherwise he demands one block. Clearly, given that strategy, player one has to demand only one block at the first round so at the second round the auction clears (ends) and he obtains one block leading him to utility $3 - 1 = 2$.
2. We denote by c_i the highest K valuations of other players facing player i . If a player obtains his n^{th} block at a price p then p is the minimum amount such that the sum of demands of all other players is $K - n$, clearly p corresponds to $c_{i,n}$.

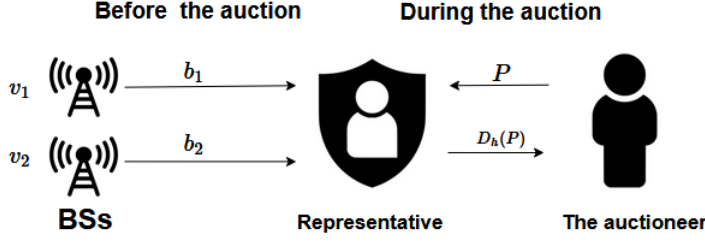


Figure 23 – The first approach (with a representative per group)

7.3.2 Adapting the clinching approach for the LSA context

We propose to implement the ascendant version of VCG, based on the clinching approach using two approaches: the first version is by introducing a representative per group, an intermediary between bidders and the auctioneer. In the second approach, we remove those representative so that the auction will be between the auctioneer and bidders.

With representatives

We propose to introduce a representative per group which will act on behalf of members of that group. It can be an interface between BSs and the auctioneer. There is no communication between BSs and the auctioneer (see Fig. 23). The auction will be between the M representatives and the auctioneer. Before the auction takes place, each BS i transmits to the representative of its group its bids vector \mathbf{b}_i which can be different from \mathbf{v}_i , then each representative h constructs the group-bid vector based on the received bids ($\mathbf{B}_{h,n}^{\text{VCG}} = f_h^{\text{VCG}}(\mathbf{b}_{1,n}, \dots, \mathbf{b}_{n_h,n}) = \sum_{i=1}^{n_h} \mathbf{b}_{i,n}$).

As in the initial clinching auction, the auctioneer broadcasts a per-block price P starting with $P = 0$ (to simplify notation we write P instead of P^t), each representative h responds with its demand $D_h(P)$, that is, a number of blocks the group is willing to buy at that price:

$$D_h(P) = \max\{n, \mathbf{B}_{h,n}^{\text{VCG}} > P\}. \quad (7.5)$$

The auctioneer keeps increasing P by one unit until the sum of demands of all representatives is equal to or below K . To perform clinching (i.e., decide on block allocations), we use the same model as before but adapt it to groups (representatives): the cumulative

clinch Cl_h^t of representative h is then defined as:

$$\text{Cl}_h(P) := \max\{0, K - \sum_{j \neq h} D_j(P)\}. \quad (7.6)$$

As in the original scheme, the current clinch at time t for representative h is the increment of Cl_h .

$$\text{cl}_h(P) = \text{Cl}_h(P) - \text{Cl}_h(P - 1). \quad (7.7)$$

After obtaining a block at price P , each representative charges each BS of its group as follow:

$$\mathbf{p}_{i,n}^h = \min\{r : f_h^{\text{VCG}}(\mathbf{b}_{1,n}, \dots, \underbrace{r}_i, \dots, \mathbf{b}_{n_h,n}) \geq P\} = [P - (\mathbf{B}_{h,n}^{\text{VCG}})^{-i}]^+ \quad (7.8)$$

Summarizing, the ascending auction we propose would follow the following steps:

1. Each BS reports to the corresponding representative its bids vector.
2. The representative constructs his vector of valuations.
3. At each round, each representative reports his demand $D_h(P)$ to the auctioneer.
4. The auctioneer computes the cumulative clinch Cl_h^t of each representative h at round t which is defined as:

$$\text{Cl}_h^t = \max\{0, K - \sum_{j \neq i} D_j^t\}, \quad (7.9)$$

with D_j^t the demand of other representative j at round t . the current clinch at round t of the representative h is denoted by cl_h^t :

$$\text{cl}_h^t = \text{Cl}_h^t - \text{Cl}_h^{t-1}. \quad (7.10)$$

5. If a representative clinches a block at a price P then it charges each BS i of its group a price given in (7.8) and sends that amount to the auctioneer.
6. If the demands of all representatives is higher than K , then the auctioneer increases P at the next round and we go back to step 3, otherwise the auction ends.

In the following, we investigate truthfulness of the proposed mechanism. Let us first introduce the following proposition.

Proposition 18. *From the point of view of a BS, proposing a bid \mathbf{b}_i to the representative is the same as proposing a bid to the auctioneer in the one shot version because both auction will generate the same outcome (in terms of allocations and payments).*

Proof. After receiving bids from BSs, each representative of g_h computes the group-bid vector of g_h , (before that step is made by the auctioneer). A representative obtains his n^{th} block at price P means that the demand of other group at that price is $K - n$ and at $P - 1$ is $K + 1 - n$, therefore P must be equal to $C_{h,n}$. Thus the auction with representatives leads to the same allocation as the one-shot version. Now, for his n^{th} block i.e., when $P = C_{h,n}$, player i will be charged, by the representative, with respect to (7.8). That amount is the same as before (see (6.3)). Therefore both auction lead to the same allocations and payments. \square

Proposition 19. *After introducing representatives, for each bidder i , reporting his true valuation vector to the corresponding representative is a dominant strategy.*

Proof. Since in the one shot version bidding truthfully is a dominant strategy, then here also bidding truthfully is a dominant strategy. \square

Example 11. Consider three groups and $K = 4$ blocks, with the following valuation vectors:

- in Group 1 (3 players): $\{9, 7, 6, 5\}, \{7, 7, 5, 2\}, \{6, 3, 3, 2\}$.
- in Group 2 (2 players): $\{7, 6, 4, 3\} \{9, 7, 5, 2\}$
- in Group 3 (1 player): $\{15, 10, 3, 1\}$

Group-bids are: $\mathbf{B}_1^{\text{VCG}} = \{22, 17, 14, 9\}$, $\mathbf{B}_2^{\text{VCG}} = \{16, 13, 9, 5\}$ and $\mathbf{B}_3^{\text{VCG}} = \{15, 10, 3, 1\}$.

We conclude that the auction stops at $P = 14$ (when the sum of demands equals 4):

Price	1	2	3	4	5	6	7	8	9	10	11	12	13	14
D_1	4	4	4	4	4	4	4	4	3	3	3	3	3	2
D_2	4	4	4	4	3	3	3	3	2	2	2	2	1	1
D_3	3	3	2	2	2	2	2	2	2	1	1	1	1	1

Table 14 – Demand and price evolution for VCG

1. The first group clinch his first block at $P = 10$. Each one pays zero.

2. At $P = 13$ the first group clinches its second block. The first bidder pays $13 - (7 + 3) = 3$, the second bidder pays 3 and the third bidder pays $[13 - 14]^+ = 0$.
3. At $P = 14$, the second and the third group clinch their first block. For the first bidder of the second group he pays $14 - 9 = 5$. The second bidder pays $14 - 7 = 7$. The bidder of the third group pays 14.

The implementation with representatives has the following advantages: truthful telling is a dominant strategy and the auctioneer could not have a precised idea about valuations of BSs, he may have only an idea about the total valuation of group h for an n^{th} block but he can not see the valuation of each BS. In practice, it may be difficult to introduce those representatives because we may have “the black box effect”: from the point of view of players, they cannot see the evolution of the auction (they are just asked to pay an amount for an obtained block). For the auctioneer, he cannot see how each BS is charged; in the original version of clinching (without groups), each player pays the clinching price. In our situation, the clinching price is the maximum amount that a representative can pay.

Proposition 20. *If a group (representative) clinches his n^{th} block at price P then the sum of payments of players of that group cannot be higher than P .*

Proof. We can distinguish two cases

1. There exists a player i such that $\mathbf{b}_{i,n} > P$, then in this situation, each player j (in the same group as i) except i pays zero because $\mathbf{B}_{h,n}^{\text{VCG}^{-j}} > P$, for player i he will pay $[P - \mathbf{B}_{h,n}^{\text{VCG}^{-i}}]^+ < P$ thus the revenue in this situation is lower than P .
2. $\forall i, \mathbf{b}_{i,n} < P$, we take any set S_h such that the sum of bids of its members is higher than P and lower than P when removing any player of the set i.e., $\sum_{i=1, i \in S_h} \mathbf{b}_{i,n} \geq P$ and $\forall j \in S_h \sum_{i=1, i \neq j, i \in S_h} \mathbf{b}_{i,n} \leq P$, we can obtain that set as follows: we sort bids of group h in a non increasing order. In the beginning S_h is composed by the player with the highest bid. We keep extending S_h by adding players until both conditions hold. In this situation, each player of group h who does not belong to S_h pays zero, and the payment of group h

is given by:

$$P_{h,n} = \sum_{i=1}^{|S_h|} (P - \sum_{j \neq i}^{n_h} \mathbf{b}_{j,n}) \quad (7.11)$$

$$= |S_h|P - \sum_{i=1}^{|S_h|} \sum_{j \neq i}^{n_h} \mathbf{b}_{j,n} \quad (7.12)$$

Since

$$\sum_{i=1}^{|S_h|} \sum_{j \neq i}^{n_h} \mathbf{b}_{j,n} = \sum_{i=1}^{|S_h|} (\sum_{j \neq i}^{n_h} \mathbf{b}_{j,n} + \mathbf{b}_{i,n} - \mathbf{b}_{i,n}) \quad (7.13)$$

$$= |S_h| \mathbf{B}_{h,n}^{\text{VCG}} - \sum_{i=1}^{|S_h|} \mathbf{b}_{i,n} , \quad (7.14)$$

we obtain

$$P_{h,n} = |S_h|P - |S_h| \mathbf{B}_{h,n}^{\text{VCG}} + \sum_{i=1}^{|S_h|} \mathbf{b}_{i,n} \quad (7.15)$$

$$\leq |S_h|P - |S_h| \sum_{i=1}^{|S_h|} \mathbf{b}_{i,n} + \sum_{i=1}^{|S_h|} \mathbf{b}_{i,n} \quad (7.16)$$

$$= (|S_h| - 1)(P - \sum_{i=1}^{|S_h|} \mathbf{b}_{i,n}) + P \quad (7.17)$$

$$\leq P. \quad (7.18)$$

□

So if the auctioneer provides a representative a block at price P , then his revenue varies from 0 to P .

In the following we show how to implement the ascending version when removing those representatives so that the auction will be held between the auctioneer and BSs directly.

Without representatives

In this scenario, the auction will be between the regulator and BSs. Similarly to what was presented before, the auctioneer fixes a price P and keeps increasing P until demand

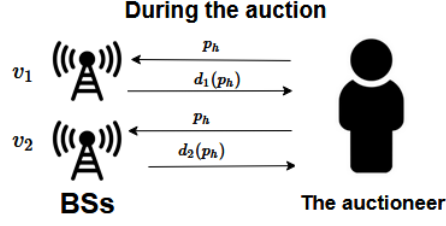


Figure 24 – The second approach

of groups is no higher than supply. The question here is how to compute the demand of groups? We propose to introduce a price p_h per group and perform as follows: for each price P , the auctioneer keeps increasing p_h and asks each player of group h his demand $d_i(p_h)$ (see Fig. 24), until he can compute the demand of group h $D_h(P)$ i.e., demands of groups will be computed from demands of players.

$$D_h(P) = \max\{n : \exists (r_1, \dots, r_{n_h}) \in \mathbb{R}^{|n_h|} \text{ s.t. } d_i(r_i) = n \text{ and } f_h^{\text{VCG}}(r_1, \dots, r_{n_h}) > P\} \quad (7.19)$$

Note that bidder i reports his demand truthfully means that:

$$d_i(p_h) = \max\{n, \mathbf{v}_{i,n} > p_h\}. \quad (7.20)$$

We show in the following proposition that this demand is the same as the one presented before (with representatives).

Proposition 21. *The demand of a group h given in (7.19) is equivalent to the demand given in ((7.5)).*

Proof. In order to prove the equivalence we show the following equivalence:

$$B_{h,n}^{\text{VCG}} > P \iff \exists (r_1, \dots, r_{n_h}) \in \mathbb{R}^{|n_h|} \text{ s.t. } d_i(r_i) = n \text{ and } f_h^{\text{VCG}}(r_1, \dots, r_{n_h}) > P$$

- $\Rightarrow B_{h,n}^{\text{VCG}} > P$: this means that $f_h^{\text{VCG}}(\mathbf{b}_{1,n}, \dots, \mathbf{b}_{n_h,n}) > P$, since f_h^{VCG} is a continuous function then we can find $(\epsilon_i, \dots, \epsilon_{n_h})$ such that $f_h^{\text{VCG}}(\mathbf{b}_{1,n} - \epsilon_1, \dots, \mathbf{b}_{n_h,n} - \epsilon_{n_h}) > P$. Thus by setting $r_i = \mathbf{b}_{i,n} - \epsilon_i$ we obtain $f_h^{\text{VCG}}(r_1, \dots, r_{n_h}) > P$ and $d_i(r_i) = n$.
- $\Leftarrow \exists (r_1, \dots, r_{n_h}) \in \mathbb{R}^{|n_h|} \text{ s.t. } d_i(r_i) = n \text{ and } f_h^{\text{VCG}}(r_1, \dots, r_{n_h}) > P$: means that $\mathbf{b}_{i,n} > r_i \forall i$ (because $d_i(\mathbf{b}_{i,n}) = n - 1$) since $f_h^{\text{VCG}}(r_1, \dots, r_{n_h}) > P$ then $f_h^{\text{VCG}}(\mathbf{b}_{i,1}, \dots, \mathbf{b}_{i,n_i}) > P$.

□

The auctioneer keeps increasing p_h until he can compute the demand of group h at price P as illustrated in Fig. 25.

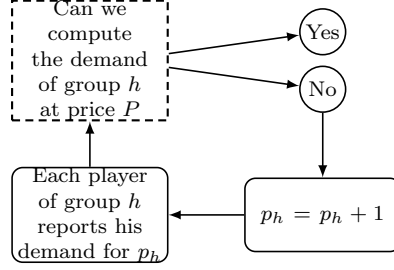


Figure 25 – Relation between P and p_h

Remark

Preserving valuations of players is one of the most desirable properties of multi-round auctions, however the auctioneer may have an idea about valuations of some players. As an example, in the English auction, if a player drops out at a price, then that price corresponds exactly to his valuation. Similarly here, when introducing prices per group, the auctioneer can have some idea about valuations of some players: if $d_i(p_h) = n$ and $d_i(p_h + 1) = n - 1$ then $\mathbf{b}_{i,n} = p_h + 1$.

Now the question is how to charge each player? We will use the same logic as before: Let us fix the first bidder of g_h . If g_h can clinch an n^{th} block without the participation of that bidder i.e., we achieve some vector $(0, r_2, \dots, r_{n_h})$ such that $f_h^{\text{VCG}}(0, r_2, \dots, r_{n_h}) > P$, then the first bidder pays zero. Otherwise group h cannot clinch that block therefore we can compute the maximum amount that it can pay for that block i.e., from demand of players we can compute $\mathbf{B}_{h,n}^{\text{MEC}-1}$ and therefore player 1 pays $P - \mathbf{B}_{h,n}^{\text{MEC}-1}$.

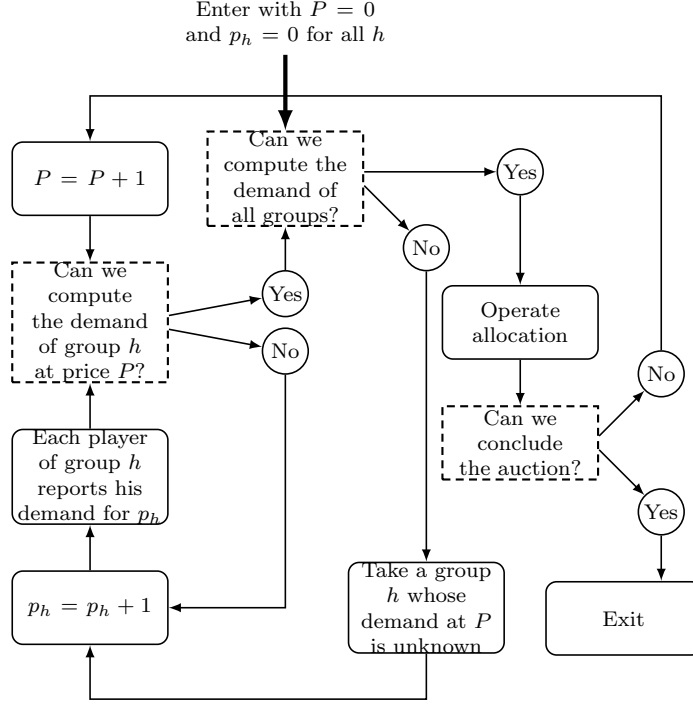


Figure 26 – Applying the clinching approach in the LSA context.

In the following proposition, we show that truthful telling is an ex post-Nash equilibrium.

Proposition 22. *In the ascending implementation of MEC without representative, truthful telling is an ex post-Nash equilibrium.*

Proof. Let us fix a player i , suppose that all other BSs report their demand truthfully during the auction, by reporting its true demand player i will obtain the same utility as in the auction with the representatives since both auctions have the same allocations (same demands) and payments (by construction). We denote by u_1 that utility. Now we have to show that any other strategy of demanding for player i will reduce its utility i.e., it obtains a utility $u_2 \leq u_1$. Suppose that that strategy generates a higher utility, this means that player i could obtain the same utility in the first implementation (with representatives) by proposing a bid vector with respect to his reported demands i.e., if $d_i(p_h) = n - 1$ and $d_i(p_h - 1) = n$ then he fixes $b_{i,n} = p_h$. This is a contradiction because in the first implementation, proposing the valuation is a dominant strategy. Thus $u_1 \geq u_2$. \square

Example 12. We take the same configuration as example 11 To compute $D_1(P)$ for $P = 1$, we start with $p_1 = 1$ and ask players their demand at p_1 . All players are willing to

buy 4 blocks at that price, hence we know that $D_1(P) = 4$ as long as $P \leq 3$, each player is willing to pay a unit price $r_1 = r_2 = r_3 = 1$.

For $P = 4$ we need to ask individual demands at $p_1 = 2$. Truthful answers give $d_1(2) = 4$ and $d_2(2) = d_3(2) = 3$, so we know that $D_1(P) = 4$ as long as $P \leq 5$.

For $P = 6$, we know from the responses for $p_1 = 2$ that $D_1(P) \geq 3$. We increase p_1 ($p_1 = 3$). Since player one is willing to buy 4 blocks at a unit price 3, then $D_1(P) = 4$, ($r_1 = 3, r_2 = r_3 = 1.5$).

Following that process, we derive the group demands. $D_1(P) = 4$ for $P < 9$, $D_1(P) = 3$ for $9 \leq P \leq 13$, $D_1(P) = 2$ for $14 \leq P < 17$, $D_1(P) = 1$ for $17 \leq P < 22$ and $D_1(P) = 0$ for $P \geq 22$.

At each P , we compute the demand of all groups as the first group, until total demand gets equal to or below K . We conclude that the auction stops at $P = 14$ (when the sum

Price	1	2	3	4	5	6	7	8	9	10	11	12	13	14
D_1	4	4	4	4	4	4	4	4	3	3	3	3	3	2
D_2	4	4	4	4	3	3	3	3	2	2	2	2	1	1
D_3	3	3	2	2	2	2	2	2	2	1	1	1	1	1

Table 15 – Demand and price evolution for VCG

of demands equals 4):

1. The first group clinches his first block at $P = 10$. Each one pays zero because when he is absent the other players could obtain that block.
2. At $P = 13$ the first group clinches its second block. The first bidder pays $13 - (7 + 3) = 3$, because from $p_1 = 8$ we can see that bidder two can pay a maximum amount 7 for a second block and the third bidder can pay a maximum amount 3. The second bidder pays 3 and the third bidder pays 0.
3. At $P = 14$, the second and the third group clinch their first block. For the first bidder of the second group he pays $14 - 9 = 5$ (p_2 was incremented till 9 which corresponds to the maximum amount that bidder can pay in order to obtain one block. The second bidder can not obtain that block when he is alone). The second bidder pays $14 - 7 = 7$. The bidder of the third group pays 14.

Remark We can generalize those approaches to any auction mechanism with group-bid function f_h^{MEC} . Indeed, we have just to replace f_h^{VCG} by f_h^{MEC} and all demonstrations

are still valid (truthfulness is an ex-post Nash equilibrium). In the following example we show how to implement the ascending version of TLSAA.

Example 13. We take the configuration of example 11. Recall that:

- $f_h^{\text{TLSAA}}(r_1, \dots, r_h) = \max\{r_i \text{rank}(r_i)\}$ which can be interpreted as the maximum amount that a subset of players of group h can pay equitably.

TLSAA implementation with representatives

Group-bids are: $B_1^{\text{TLSAA}} = \{18, 14, 10, 6\}$, $B_2^{\text{TLSAA}} = \{14, 12, 8, 4\}$ and $B_3^{\text{TLSAA}} = \{15, 10, 3, 1\}$.

Price	1	2	3	4	5	6	7	8	9	10	11	12
D_1	4	4	4	4	4	3	3	3	3	2	2	2
D_2	4	4	4	3	3	3	3	2	2	2	2	1
D_3	3	3	2	2	2	2	2	2	2	1	1	1

Table 16 – Demand and price evolution for TLSAA

We conclude that the auction stops at $P = 12$ (when the sum of demands equals 4): the first and the second group clinches their first block each one at $P = 10$. For the first group, each player pays 0. For the second group, each bidder pays 5.

At $P = 12$ the first group clinches its second block and the third group clinches its first block. Player one and two of the first group pay 6 each one for their second block and player three pays 0 for his second block. The player of the third group pays 12.

TLSAA implementation without representatives

The demand of g_h under TLSAA can be written as:

$$\begin{aligned}
D_h(P) &= \max\{n : \exists (r_1, \dots, r_{n_h}) \in \mathbb{R}^{n_h} \text{ s.t. } d_i(r_i) = n \text{ and } f_h^{\text{LSAA}}(r_1, \dots, r_{n_h}) > P\} \\
&= \max\{n : \exists (r_1, \dots, r_{n_h}) \in \mathbb{R}^{n_h} \text{ s.t. } d_i(r_i) = n \text{ and } \max\{r_i \text{rank}(r_i)\} > P\} \\
&= \max\{n : \exists \omega \subset g_h \text{ and } r \in \mathbb{R} \text{ s.t. } d_i(r) = n \text{ and } r|\omega| > P\}
\end{aligned}$$

$D_h(P)$ is the maximum number of blocks that a subset of players in group h are willing to buy if they equally share the unit price P .

To compute $D_1(P)$ for $P = 1$, we start with $p_1 = 1$ and ask players their demand at p_1 . All players are willing to buy 4 blocks at that price, hence we know that $D_1(P) = 4$ as long as $P \leq 3$, each player is willing to pay a unit price $r = P/3 \leq 1$.

For $P = 4$ we need to ask individual demands at $p_1 = 2$. Truthful answers give $d_1(2) = 4$ and $d_2(2) = d_3(2) = 3$, so we know that $D_1(P) = 4$ as long as $r = P/3 < 2$, i.e., when $P \leq 5$.

For $P = 6$, we know from the responses for $p_1 = 2$ that $D_1(P) \geq 3$, since all three players are interested to buy 3 blocks at a unit price $6/3 = 2$. But possibly $D_1(P) = 4$, if player one is willing to buy 4 blocks at a unit price 6. So we increase p_1 and ask players their demand until either 6 is reached or $d_1(p_1) < 4$. The latter occurs first, for $p_1 = 5$, which leads to the conclusion that $D_1(6) = 3$ (each player is willing to pay $r = 2$).

Following that process, we derive the group demands $D_1(P) = 3$ for $P = 7, 8, 9$, as an example for $P = 9$ we have $r = 4.5$ and ω is composed by the first and the second bidder since the third bidder is not willing to pay 3 in order to have 3 blocks.

Finally, we have $D_1(P) = 2$ for $10 \leq P \leq 13$, $D_1(P) = 1$ for $14 \leq P \leq 17$, and $D_1(P) = 0$ for $P \geq 18$.

At each P , we compute the demand of all groups as the first group, until total demand gets equal to or below K . We conclude that the auction stops at $P = 12$ (when the sum

Price	1	2	3	4	5	6	7	8	9	10	11	12
D_1	4	4	4	4	4	3	3	3	3	2	2	2
D_2	4	4	4	3	3	3	3	2	2	2	2	1
D_3	3	3	2	2	2	2	2	2	2	1	1	1

Table 17 – Demand and price evolution for TLSAA

of demands equals 4):

1. The first and the second group clinch their first block at $P = 10$. For the first group, each player pays 0 because when he is absent, the other two bidders can obtain that block for $r = 5$. For the second group, it obtains its first block and each bidder pays 5, p was incremented till 9 so we know that 5 is the minimum amount that each one has to propose in order to obtain that block.
2. At $P = 12$ the first group clinches its second block and the third group clinches its first block. Player one and two of the first group pay 6 each one for their second

block: when one of them is absent the group will not obtain that block ($p_1 = 9$). When they are present, $r = 6$ is the minimum amount that allow each one of them to win that block. Player three pays 0 for his second block). The player of the third group pays 12.

Note that, in order to evaluate the performances of an auction mechanism, there is no need to implement the ascending version, we can use the one-shot auction since both auction leads to the same outcome.

7.4 Conclusion

In this chapter, we have shown how to implement the ascending version of VCG using the UCE auction and highlighted some computational complexity issues with the resulting mechanism, even when auctioned items (spectrum blocks) are identical. To deal with computation complexity of UCE, we have proposed to adapt the clinching approach for the LSA context. That approach can be used to develop the equivalent ascending auction of any one-shot truthful mechanism that could be applied for LSA. we have proposed two implementations: For the first implementation, we have introduced a representative per group. At each round, each representative transmits to the regulator the demand of its group based on bids of its members. Each BS is charged a price computed by the representative of its group. There are two advantages of this implementation. First, truthful telling is a dominant strategy and second we preserve privacy of valuations of BSs. However, it can be difficult to introduce those representatives in practice. Thus, at a second time, we have proposed another ascending implementation without those representatives and in which communication is directly between the auctioneer and BSs. We have introduced a price per group and show how to compute the payment of each player. In the second approach truthful telling is an ex post Nash equilibrium. Transparency is the main advantage of the second implementation because each BS sees the evolution of the auction.

Part IV

Conclusions and perspectives

Chapter 8

Conclusion and perspectives

8.1 Conclusion

In this thesis we have studied efficient frequency allocation in the context of 5G LSA. Under LSA the owner of the 2.3 – 2.4 GHz bandwidth can share his bandwidth with MNOs. Sharing is done after obtaining a license from the regulator.

How to attribute licenses is not specified yet. In general, in the licensing process, the regulator can use administrative approaches when demand is lower than supply and auctions when demand exceeds the amount of spectrum to offer. The 2.3 – 2.4 GHz bandwidth is considered as a valuable resource thanks to its ability to travel far and offer high capacity. Therefore, for the licensing process in the LSA context, auctions are more adequate.

A specificity of LSA licenses lies in the interactions among buyers, due to possibly overlapping coverage areas, hence there are limited possibilities of allocating the same spectrum to several MNOs. Interference among coverage areas is managed through **groups**. A group is a set of base stations of different MNOs such that two base stations in the same group do not interfere, hence the spectrum allocated to a group can be used by all the members of the group. The group creation is performed by the auctioneer from the interference graph before the actual auction takes place.

Once groups are formed, base stations submit to the auctioneer their bids and the auctioneer performs allocations and payments using an auction mechanism. An auction mechanism needs to be well designed otherwise the benefits of the auction can be lost. A well designed auction mechanism should be **truthful** so it prevents strategic behaviour and therefore bidders do not have to construct complex strategies for bidding. Other

considerations can be taken into account, in order to evaluate an auction mechanism, such as **revenue** of auctioneer, **fairness** and **efficiency** of the allocation.

Potential candidate auction mechanisms for the LSA context are truthful **one-shot** auctions. A one-shot auction mechanism is composed of a single round so bidders have only one chance to submit bids for the auctioneer. In addition, those mechanisms of the literature attribute all the available spectrum as a **one block** for only one group. Also, they make **two main key assumptions**, without those assumptions they are not anymore truthful. The first assumption is that each base station has to be in one and only one group. The second assumption is that when modeling interaction between base stations, each base station is considered as player. Finally, they do not draw enough attention for the revenue.

In this thesis, we investigate the above mentioned considerations through these questions: can we design alternative mechanisms with better performances? Is it possible to relax the assumptions made while preserving truthfulness? If yes what are the impacts on the outcome of the auction? We have provided answers for those questions in Part II of this document. Other questions which may arise are: can we improve the outcome of the auction by splitting the available spectrum and how to convert all-or-nothing auction mechanisms to multi-block auction mechanism? Can we develop equivalent ascending versions of those mechanisms? In Part III of the document we have treated those questions.

In Part II of this document, we have treated the case when the quantity of spectrum is relatively small so it can be allocated as a one block. After studying the potential candidate mechanisms for LSA and proposing two truthful variants of LSAA, we have shown how to increase revenue while maintaining truthfulness by introducing a reserve price per bidder and how to maximize efficiency by showing how to implement VCG. Also, we have proposed PAM^σ , an auction mechanism which allocates the bandwidth proportionally among groups such that each group obtains at least a fraction equal to σ , or nothing. PAM^σ is flexible: a fairness oriented regulator could set $\sigma = 0$ and $R = 0$; in this case simulation results suggest that Jain's fairness index achieves 90% of its maximum value. On the other hand, an efficiency and revenue oriented regulator can trade-off them by playing on R and σ .

At a second time we have supposed that each base station can be in several groups so that we give the regulator more flexibility in the group construction phase. We have shown how to adapt the payment rules of the previous mechanisms, when possible, to maintain

truthful bidding without modifying the allocation rule. In most cases, simulations suggest that the relaxation has a positive impact on the outcome of the auction. The gain of the auctioneer which we define as a linear combination of efficiency, revenue and fairness could be increased more than 20%. After, in order to provide a more realistic assumption, we have considered no longer that players are given by BSs but that a player coordinates several BSs (thus MNOs are the players), under this new definition of players, most of our results were negative except for VCG. We have shown how to apply VCG in this new scenario.

In Part III of the document, we have supposed that the available quantity of LSA spectrum allows to split it into several blocks. We have shown that the choice of the number of blocks is very important. This choice has a direct impact on the outcome of the auction. As an example it can increase efficiency or fairness but it can also decrease revenue therefore the auctioneer has to set a trade-off between those metrics. Then, we have shown, when it is possible and under the assumption that each base station must belong to one and only one group, how to convert all-or-nothing auction schemes to multi-blocks schemes while preserving truthful bidding.

At a second time, we have considered the relaxation of allowing a base station to be in several groups. We have studied its impact on the complexity of the allocation and we have shown that the allocation is an NP-hard problem. Therefore to avoid heavy computation steps, we have supposed that each base station must belong to one and only one group. Avoiding heavy computation steps may be vital in the LSA context especially when the resolution time for the optimal allocation is close to the duration of licenses. We have proposed a grouping method under the assumption that each base station must belong to one group. This heuristic is easy and fast to build but it may have a negative impact on efficiency. We have quantified that impact through simulations. Simulations suggest that the average minimum efficiency is 67 % compared to the optimal one. Also as we keep increasing the number of blocks, we keep getting closer to the optimal efficiency.

In the second chapter of this part, we have considered ascending auctions. Ascending auctions are more transparent than one-shot auctions because bidders see the evolution of the auction. In addition, ascending auctions preserve privacy because bidders do not reveal necessarily their valuations. We started by studying Mishra and Parkes's mechanism, an ascending truthful auction mechanism which maximizes efficiency. Then we have shown how to adapt that mechanism for the LSA context. Since that implementation may lead to solve an NP-hard problem, we have studied another ascending-price auction mechanism,

proposed by Ausubel, that stops once demand is no higher than supply. That mechanism was proven to be equivalent to VCG. We have shown, by using the approach proposed by Ausubel which is called “the clinching approach”, how to convert one-shot multi-block auction mechanisms to ascending multi-block auction mechanisms. We have proposed two methods for that conversion. The first method is by introducing a representative part per group so that each representative will act on behalf of the members of its group. Since bidders do not see the evolution of the auction which may be undesirable, in the second method, we have shown how to implement the ascending version when removing representatives so that the auction is run directly between bidders and the auctioneer and therefore we ensure transparency.

8.2 Perspectives

We believe this thesis work has many interesting openings for future extensions. First, in our work, we have considered binary interference i.e., for each two base stations either they interfere with each other or not. It would be interesting to consider models where interference has more flexibility: the interference between BSs would depend on their respective coverage areas. The coverage area of BSs depends on their transmission power therefore a better approach is to model the interference with respect to the transmission powers and BSs positions. On the other hand, reducing the transmission power may reduce the coverage which may reduce the valuation, in such a situation each BS submits a valuation function to the auctioneer (valuation as a function of the transmitted power). This may improve the outcome of the auction but would need more elaborate auction analysis.

It would be also interesting to treat LSA auction as a repeated game [102], since the auctioneer may run the auction many times per day in some configurations. Repeated games have a larger space of strategies therefore truthfulness is not anymore dominant strategy: in repeated game bidders may decide to make a cooperation with other bidders. This occurs because bidders can maximize their utilities by making such a cooperation and also they want to avoid future punishment, such punishment in repeated games context is known as trigger punishment.

Another important criterion is how to share the revenue between the incumbent and the regulator. Indeed, if we want to divide that revenue in a fair manner then Shapely value, which is a solution concept in cooperative games used to share gains and costs

fairly between several agents working in a coalition [103], could be a good solution. But several other exist and many metrics can be considered to evaluate the method. This should be carefully studied as a cooperative game, and trade-offs between metrics should be analyzed.

Also, it would be interesting to run our auction mechanisms in real context in order to test its performances. Authors in [10] have proposed an online Auction system platform as a marketplace for LSA-based Short-term Spectrum Sharing (ALS3). According to the authors, ALS3 allows easy addition of new auction mechanism. It would be interesting to implement our proposed algorithms on that platform.

In this thesis, we consider that groups are already set and provide some elements for the constitution of the groups. We focus on auctions mechanisms and their performance. Yet, it seems evident that the choice of groups has an important effect on the behavior of auctions and their results. Indeed, group configuration depends on the objective of the auctioneer. Our results allow each base station to be in several groups, therefore our results still hold for any resulting group configuration.

Also, when there is a small quantity of LSA spectrum, we have supposed that the auctioneer fixes a minimum amount so that each bidder may obtain at least that amount, or nothing. A question may arise is: how to adapt PAM when each BS has its own minimum amount? And for the case when spectrum allows to be split to several blocks, we have supposed that blocks are identical. It would be interesting to investigate the case when blocks have different size.

Last but not least, once the 5G specific LSA application framework is decided by regulation authorities, we think that the algorithms and models developed in this thesis can be updated in order to have the best efficiency in the practical 5G framework.

Appendix A

Abbreviations

- BS: Base Station
- CE: Competitive Equilibrium
- DSA: Dynamic Spectrum Access
- eMBB: enhanced Mobile BroadBand
- LSA: Licensed Shared Access
- LSAA: LSA Auction
- MNO: Mobile Network Operator
- MTC: Machine-Type Communications
- PAM: Proportional Allocation Mechanism
- RAN: Radio Access Network
- RSPG: Radio Spectrum Policy Group
- TAMES: Truthful Auction Mechanism for hEterogeneous Spectrum allocation
- TLSAA: Truthful LSAA
- TRUST: TRuthful doUble Spectrum aucTions
- UCE: Universal Competitive Equilibrium

- URLLC: Ultra-Reliable Low-Latency Communications
- VCG: Vickrey-Clarke-Groves

Appendix B

Notations

- N : number of bidders
- M : number of groups
- v_i : valuation of bidder i for the available LSA spectrum when there is only one block.
- b_i : bid of bidder i for the available LSA spectrum when there is only one block.
- α_i^{MEC} : fraction allocated to bidder i when the mechanism MEC is applied
- p_i^{MEC} : payment of bidder i when the mechanism MEC is applied
- R : reserve price per bidder
- u_i : utility of bidder i
- g_h : group h which contains a subset of players.
- n_h : number of bidders in g_h
- B_h^{MEC} : group-bid of g_h when the mechanism MEC is applied and when there is only one block of LSA spectrum.
- Rev^{MEC} : revenue of the auctioneer when the mechanism MEC is applied.
- E^{MEC} : efficiency when the mechanism MEC is applied.
- K : number of blocks

- n_h^b : number of blocks that g_h obtains
- n_i^b : number of blocks that bidder i obtains.
- \mathbf{v}_i : vector-valuation of BS i when there are more than one block. $\mathbf{v}_{i,n}$: valuation of the BS i for an n^{th} extra block given that it has already $n - 1$ blocks.
- $\mathbf{B}_h^{\text{MEC}}$: group-bid vector of g_h when the mechanism MEC is applied and when there are more than one block.
- $\mathbf{B}_{h,n}^{\text{MEC}}$: group-bid of g_h , when the mechanism MEC is applied, for an n^{th} extra block given that it has already $n - 1$ blocks.

Publications during this thesis

- [1] A. Chouayakh, A. Bechler, I. Amigo, L. Nuaymi and P. Maillé, (2018). “PAM: A Fair and Truthful Mechanism for 5G Dynamic Spectrum Allocation”, *IEEE 29th Annual International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, (pp. 1-6)
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Titre : Jeux d'acteurs et mécanismes d'enchères dynamiques pour la gestion du spectre dans les réseaux 5G

Mot clés : Enchères, théorie des jeux, partage des fréquences, partage sous licence

Résumé : Dans le cadre de la 5G, des nouveaux modes de partage dynamique de spectre comme le “Licensed Shared Access” (LSA) vont être implémentés. A travers ces modes, les opérateurs peuvent utiliser temporairement une bande de fréquence attribuée initialement à un autre utilisateur. L'accès à la bande se fait après l'obtention d'une licence. L'attribution des licences se fait à travers des mécanismes d'enchères. Les mécanismes proposés dans la littérature pour le contexte LSA sont des mécanismes d'enchères à un seul tour qui attribuent tout le spectre disponible comme un seul bloc.

Dans cette thèse, nous montrons d'abord comment améliorer les performances de ces enchères (en termes de revenus, d'efficacité et d'équité de l'allocation) -tout en préservant la sincérité des enchères- en divisant le spectre et en convertissant les enchères mono-bloc en des enchères multi-blocs. Ensuite, nous montrons comment convertir des mécanismes à un seul tour en mécanismes ascendants équivalents (en termes d'allocations et de paiements) afin d'ajouter la transparence et la confidentialité à l'enchère.

Title: Designing new auction mechanisms for spectrum sharing in 5G networks

Keywords: auctions, game theory, spectrum sharing, Licensed Shared Access

Abstract: In 5G networks, new spectrum sharing concepts such as Licensed Shared Access (LSA) will be implemented in order to optimize spectrum usage: a Mobile Network Operator can access temporarily to other incumbent's spectrum after obtaining a license. The LSA concept guarantees to the incumbent and the LSA licensee a certain level of QoS according to the LSA license. The licensing process is made via an auction mechanism. The mechanisms proposed in the literature for the LSA context are one-shot auction mechanisms which allocate all the available spectrum as a one block.

In this thesis, first we show how to increase the performances of those auctions (in terms of revenue, efficiency and fairness of the allocation)–while preserving truthful bidding–by splitting spectrum and converting single block auctions to multi-block auctions. Then, we show how to convert one-shot mechanisms to equivalent ascending mechanisms (in terms of allocations and payments) so that we add transparency and privacy to the auction.