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Teachers' Knowledge for Integrating Dynamic Geometry Software into Mathematics Lessons : contrasting Chinese and French Cases

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Teachers' Knowledge for Integrating Dynamic Geometry Software into Mathematics Lessons: Contrasting Chinese and French Cases

Connaissances des Enseignants pour L'intégration de Logiciels de Géométrie

Dynamique dans Les Cours de Mathématiques: Contraster Les Cas Chinois et Français

数学课堂中整合动态几何软件所需的教师知识：中法案例对比

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**Teachers' Knowledge for Integrating Dynamic Geometry Software
into Mathematics Lessons: Contrasting Chinese and French Cases**

Fangchun Zhu

Abstract

Due to the development of the use of new technologies such as computer science in the teaching of mathematics, it is important to study one of the critical aspects which conditions the practice of teachers in the classroom: teacher knowledge. There is a great deal of research into the description and definition of what teachers' knowledge is or what kind of knowledge they need. Several of them point to the important role of technology in describing teacher knowledge. Teacher knowledge can be reflected in their classroom activities. In this research, I have selected one of the most important technologies in the teaching of mathematics: dynamic geometry software, in order to study teacher knowledge involved in their class practices.

- Taking instrumental orchestration (Trouche, 2004) as a theoretical framework, this research is based on the observation of mathematics lessons from 5 Chinese teachers and 1 French teacher in order to analyze the knowledge they demonstrate in their practice with the technology. Instrumental orchestration takes into account all aspects of teaching practice, from lesson preparation to class conduct.

Many works have dealt with dynamic geometry and have characterized how teachers use it. For example, dynamic geometry software plays two distinct roles in mathematical tasks and problem solving: that of amplifier or that of generator (Laborde, 2001). This type of software also affects teacher-student interaction (Hollebrands & Lee, 2016). Finally, by using software, the teacher shows whether he focuses rather on mathematics or rather on mathematics related to software (Hollebrands & Lee, 2016).

- Focus on technology
- Focus on technology to notice mathematics
- Focus on mathematics with use of technology
- Focus on mathematics

The assumption that we made is that

- Teacher knowledge is revealed in their instrumental orchestrations with dynamic geometry and that we can characterize and describe them by

contrasting the cases of Chinese and French teachers.

- More precisely:
 - What can we learn from the analysis of the roles of dynamic geometry in the mathematical tasks selected by the teachers?
 - What can we learn from the organization of tasks by contrasting the teaching practice of Chinese teachers and French teachers?
 - What can we learn from the teacher-student interaction by contrasting the French and Chinese cases?

In total, 11 lessons from the 6 teachers in the study were observed and recorded. They were also interviewed before and after the observed math class, in order to understand in depth their opinions on dynamic geometry.

The result is that most teachers see software as an amplifier for presenting content during the teaching process, although software can change learners' strategies for solving tasks. During class and during interaction with students, most teachers focus on mathematical content regardless of technology, although they use different instrumental orchestrations during lessons. The interviews reveal that for teachers, there are many factors that can affect the use of software by teachers, such as: the physical configuration of the class, the capacity of the students, the educational objective, the exam. These considerations require further study. For example, if the teacher conducts a lesson in a computer lab, allowing each student to use dynamic geometry individually, then the teacher can use the software more to create new learning situations for students, while he can control it more in general class.

Résumé

En raison du développement de l'utilisation de nouvelles technologies telles que l'informatique dans l'enseignement des mathématiques, il est important d'étudier l'un des aspects critiques qui conditionne la pratique des enseignants en classe : les connaissances des enseignants. Il existe de nombreuses recherches portant sur la description et la définition de ce que sont les connaissances des enseignants ou sur le type de connaissances dont les enseignants ont besoin. Plusieurs d'entre elles indiquent le rôle important des technologies pour décrire les connaissances des enseignants. Les connaissances des enseignants peuvent se refléter dans leurs activités en classe. Pour cette recherche nous avons retenu l'une des technologies les plus importantes de l'enseignement des mathématiques : la géométrie dynamique, afin d'étudier les connaissances des enseignants mobilisées dans leur pratiques de classe.

- Prenant l'orchestration instrumentale (Trouche, 2004) comme cadre théorique, cette recherche s'appuie sur l'observation des cours de mathématiques de 5 professeurs chinois et 1 professeur français afin d'analyser les connaissances qu'ils démontrent dans leur pratique avec la technologie. L'orchestration instrumentale permet de prendre en compte tous les aspects de la pratique enseignante, de la préparation des leçons à la conduite de classe.

De nombreux travaux ont traité de la géométrie dynamique et ont caractérisé l'usage qu'en font les enseignants. Par exemple, le logiciel de géométrie dynamique joue deux rôles distinct dans les tâches mathématiques et la résolution de problème : celui d'amplificateur ou celui de générateur (Laborde, 2001). Ce type de logiciel affecte également l'interaction enseignant-élève (Hollebrands & Lee, 2016). Enfin, en utilisant un logiciel, l'enseignant montre s'il se concentre plutôt sur les mathématiques ou plutôt sur les mathématiques liées aux logiciels (Hollebrands & Lee, 2016).

- Focus sur la technologie
- Focus sur la technologie pour remarquer les mathématiques
- Focus sur les mathématiques avec l'utilisation de la technologie
- Focus sur les mathématiques

L'hypothèse que nous avons faite est que

- les connaissances des enseignants sont révélées dans leurs orchestrations instrumentales avec la géométrie dynamique et que nous pouvons les caractériser et les décrire en contrastant les cas des enseignants chinois et français.
- Plus précisément :
 - Que pouvons-nous apprendre de l'analyse des rôles de la géométrie dynamique dans les tâches mathématiques sélectionnées par les enseignants ?
 - Que pouvons-nous apprendre de l'organisation des tâches en contrastant la pratique pédagogique des enseignants chinois et des enseignants français ?
 - Que pouvons-nous apprendre de l'interaction enseignant-élève en contrastant les cas français et chinois ?

Au total, 11 leçons des 6 enseignants de l'étude ont été observées et enregistrées. Ils ont également été interviewés avant et après le cours de mathématiques observé, afin de comprendre en profondeur leurs opinions sur la géométrie dynamique.

Le résultat obtenu est que la plupart des enseignants considère le logiciel comme un amplificateur de présentation des contenus au cours du processus d'enseignement, bien que le logiciel puisse changer la stratégie des apprenants pour résoudre les tâches. Pendant la classe et au cours de l'interaction avec les élèves, la plupart des enseignants se concentre sur des contenus mathématiques indépendamment de la technologie, bien qu'ils mobilisent différentes orchestrations instrumentales lors des leçons. Les interviews révèlent que pour les enseignants, il existe de nombreux facteurs qui peuvent affecter l'utilisation des logiciels par les enseignants, comme : la configuration physique de la classe, la capacité des élèves, l'objectif pédagogique, l'examen. Ces considérations nécessitent des études supplémentaires. Par exemple, si l'enseignant conduit une leçon en salle informatique, permettant à chaque élève d'utiliser individuellement la géométrie dynamique, alors l'enseignant pourra utiliser davantage le logiciel pour créer de nouvelles situations d'apprentissage pour les élèves, alors qu'il peut le contrôler davantage en classe générale.

摘要

由于在数学教学中使用了诸如计算机科学之类的新技术，因此，研究制约课堂教学实践的关键因素之一：教师知识，就变的非常重要。有很多研究都在关注教师知识是什么或教师需要什么样的知识进行教学。其中，某些研究指出我们需要关注技术在教师知识方面起到的重要作用。教师知识可以反映在他们的课堂活动中。因此，本研究选择了数学教学中最重要的技术之一：动态几何软件，以研究参与课堂实践的教师知识。

- 本研究以工具编配 (Trouche, 2004 年) 作为理论框架，基于对 5 位中国老师和 1 位法国老师的数学课的观察，分析他们在使用该技术的教学实践中所呈现的知识。工具编配充分涉及了教学实践的方方面面，从教师备课到实际的课堂行为等。

有许多研究涉及动态几何软件，并且分析了教师如何使用它。例如，动态几何软件在数学任务和问题解决中扮演着两个不同的角色：放大器或发生器 (Laborde, 2001)。这类软件还同时影响着师生互动 (Hollebrands & Lee, 2016)。最后，通过使用软件，教师可以显示他是专注于数学还是专注于与软件相关的数学 (Hollebrands & Lee, 2016)。

- 关注技术
- 关注技术，同时涉及数学
- 关注数学，同时需要使用技术进行回答
- 关注数学

本文所做的假设是：

- 教师知识通过使用动态几何软件进行工具编配的过程中得以展现，我们可以通过对比中法教师的教学实践来表征和描述他们的教师知识。
- 具体地说：
 - 通过分析动态几何在教师选择的数学任务中的作用，我们可以了解什么？
 - 通过对比中国老师和法国老师的教学实践，我们可以从任务安排中了解什么？
 - 通过比较法国和中国的案例，我们可以从师生互动中了解什么？

本研究总共观察并记录了 6 位老师的 11 堂数学课。在观察的数学课之前和之后，还对所有教师进行了访谈，以深入了解他们对动态几何的看法。

结果显示，尽管动态几何软件可以改变学习者解决任务的策略，但大多数教师将软件视为在教学过程中呈现内容的辅助工具。在课堂上以及与学生的互动过程中，尽管他们在上课时使用不同的工具编配形式，但大多数教师都将重点放在数学内容上，而不论其技术如何。另外，通过访谈，对于教师而言，有许多因素会影响教师对软件的使用，例如：班级的物理环境，学生的能力，教育目标，考试。当然这些因素还需要进一步研究进行论证。例如，如果老师在计算机房里上课，他更可能允许每个学生分别使用动态几何软件，那么老师可以更多地使用该软件为学生创建新的学习环境，而他可以在普通课堂上对其进行更多控制。

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1. Introduction

1.1 Setting the problem

As our society evolves, Information and Communication Technology (ICT) has become very important in our lives. In almost every field, we can see the changes that technology has brought to us. For example, computers help us to handle complex data easily and the Internet allows us to communicate with our friends easily.

In education, of course, technology changes teaching methods during the years. More and more teachers use computer, software or other technologies to support their teaching practices and in many schools, especially in colleges, IT staffs develop some websites or internet platforms for teachers, with these platforms, students can download or upload resources which are useful for their learning. Technology also allows the learner to choose when and where to study. And with technology, students are not only learners but also transmitters of information during lessons. This requires our teachers to change their teaching methods to meet the needs of different students. In the beginning, a teacher may only use slides to present important information during a lesson. However, they now use many different software or devices such as Dynamic Geometry Software (DGS) or even iPhones or iPads to download or store information during lessons so that they can view these contents at any time. Some technologies such as WeChat, QQ (a Chinese software similar to WhatsApp) and other instant messaging tools also affect the teachers' work. For example, they can use the software to connect with colleagues to discuss how to organize their lessons and share problems they have in the classroom. Easy access to the internet gives our teachers more opportunities to find useful resources and integrate them into their lessons, and they also upload some of their lesson videos so that other teachers can download them and offer their own opinions. In Shanghai, for example, all the math teachers have created different working groups through these technologies. They can use these technologies to share their own resources or

experiences and get some advice from expert teachers.

As a result of the widespread use of technology in teaching and learning practices, mathematics educators are placing greater emphasis on the effects that technology brings to the mathematics learning process. For example, ICT can be seen as an aid to students' learning of mathematics; as a tool for teachers to prepare lessons, design tasks or search for useful pedagogical information; and as a medium for teachers to communicate and collaborate with each other (Ponte et al., 2002). Since the 1980s, the impact of ICT on learning and teaching mathematics has taken a major place in the corporate, pedagogical and research literature for at least three reasons (Trouche, 2003): the potential of new tools; the evolution of student equipment; institutional injunctions.

However, the question of how ICT affects the teaching and learning of mathematics is far from being explained. There is some evidence that ICT can bring many advantages to the mathematical activities of teachers and students, and researchers have made many efforts to help teachers use technology in the classroom, but teachers still lack the skills and knowledge necessary to successfully integrate it into the lessons (Koehler et al., 2007). It also shows that, despite the recognition of the potential of technology in teaching and learning, its integration with mathematics education lags behind the high expectations many researchers and educators had decades ago (Drijvers et al., 2010) and many teachers have little or no desire to use these technologies. It is crucial that if a teacher feels that integrating technology into his/her classroom does not help his/her teaching process, he/she will no longer use it unless he/she faces some other requirements, such as mandates from the administration.

Another important phenomenon we need to be aware of is that different teachers always choose to use technology in different ways based on their experience. Some teachers simply use these resources to assign homework while others may allow students to explore new knowledge by using technology, and during their careers, teachers may change the way they use new technology. So, we need to reflect on the following questions: Why do teachers use technology in different ways? What are

the challenges for teachers to incorporate technology in mathematics teaching? According to the researchers, there are too many variables for teachers to consider when using technology in the classroom, one of which is the need for teachers to translate between old and new teaching practices. (Assude & Gelis, 2002; Lagrange et al., 2003). This means that the use of technology in teaching and learning often involves changes in the environment in which the course works: changes in classroom location and physical layout, changes in classroom organization and procedures (Jenson & Rose, 2006) and the resources they need to use in the classroom, such as mathematical tasks (Hegedus et al., 2017). Specially in the case of dynamic geometry systems, we can see the shift in considering mathematical activity and teacher profession caused by the introduction of ICT into mathematics classroom (Trgalova et al., 2009).

Thus, the main issue for teachers integrating technology into their curriculum is changing the teaching methods they are familiar with. In order to help teachers use technology effectively, it is helpful to understand the realities of how teachers use technology in their daily teaching. In this study, I would like to provide a clear description of teachers' use of technology and analyze the knowledge relating to information technology that lies behind their pedagogical processes.

1.2 Overview of this study

There are seven chapters in this thesis. The second chapter presents the two main concepts of this study: teacher knowledge and a particular technology: dynamic geometry software. The chapter begins by describing the existing research on teacher knowledge and why dynamic geometry software was chosen as the key element of this thesis. The chapter then describes the following question: what teacher knowledge is needed to integrate DGS into a mathematics lesson.

Chapter 3 talks about the main theoretical framework of this study. Based on the literature review in Chapter 2, I use an instrumental orchestration to analyze teachers' practices in using DGS. It also discusses the different roles that DGS plays in

mathematical tasks and the different emphases it shows in teacher-student interactions.

Chapter 4 explores the research design based on the theoretical framework talked about in Chapter 3 and presents the key information about the subjects in this study.

Then in Chapters 5 and 6, the main findings of this study are presented. They show the different ways in which teachers use DGS for classroom teaching and the teacher knowledge demonstrated in their practice of using the software in their courses.

The final chapter, Chapter 7, provides a brief discussion of the results of this study. And some suggestions are made for future research on teachers' practice using DGS.

2 Literature review: Teacher knowledge and Dynamic Geometry Software

Teacher knowledge has always been one of the important topics in educational research. In the 21st century, the number of studies on teacher knowledge has increased rapidly. Researchers in almost all disciplines are concerned with studying teacher knowledge in order to improve teaching and learning in schools. Based on the themes of these studies, the following three issues (Fan, 2014) are of critical importance to us: 1. What knowledge do teachers need? 2. What knowledge do teachers have? 3. How do teachers develop their knowledge?

This study is primarily concerned with the knowledge that teachers exhibit in their classroom practice. I will discuss these three main issues in this chapter in order to provide a clear understanding of the existing research on teacher knowledge. It will also explain the position this research fits within the previous research work and how this research will contribute to it. In section 2.1, I will begin by discussing what knowledge is from a historical perspective, before I define what teacher knowledge is. It answers the question of what knowledge teachers need in the teaching process. Section 2.2 then discusses DGS, showing the main research on this mathematical tool and discussing the effects it brings in mathematics teaching. Based on these two ideas, in section 2.3 I relate teacher knowledge to DGS and suggest what knowledge teachers need when integrating DGS into their mathematics lessons. Section 2.4 will discuss the main question of this study.

2.1 What knowledge teacher needs?

2.1.1 What is knowledge

To answer this question, we must first look to history and consider what is the definition of knowledge. With the development of human being, knowledge has always received great attention from philosophers like Plato, Aristotle, Descartes,

Kant, Russell, and Dewey. They all made important contributions to the theory of knowledge (Chisholm, 1989). Researchers today still attempt to provide a clear definition of knowledge. Although we all use *knowledge* in our lives, for example parents always ask their children what they learned in class, no one can give a clear definition of knowledge. Many experts such as Russell believe that *knowledge* is a *highly ambiguous* term that is *incapable of precision*. Dewey and Bentley also called the term a *loose name* or a *vague term*. Wilson and Pritchard, on the other hand, argue that knowledge is a primitive and undefinable concept (Fan, 2014).

Because the term is difficult to define, some researchers have tried to explain the term from different perspectives so that we can understand what knowledge is. For example, in epistemology, if a person said: “1. he/she must believe that is the case; 2. indeed, that is the case; and 3. he/she can justify that is the case. (Fan, 2014, p.11)” We can say that the person has knowledge about the cases. Here we can give a mathematical example, if we know that “the sum of the interior angles of a triangle is 180 degrees”, that means first we believe that “the sum of the interior angles of a triangle is 180 degrees”, second the proposition “the sum of the interior angles of a triangle is 180 degrees” is true, and third we can prove that “the sum of the interior angles of a triangle is 180 degrees”, for example we can measure the degrees of each interior angle of the triangle, then add them up. Later epistemological researchers, on the other hand, have broadened the definition of knowledge. For them, knowledge is not only *justified true beliefs*, but also *objectively grounded* beliefs, or beliefs that are *adequately supported*. This means that if a claim is justified by evidence, whether direct or indirect, it can be knowledge to the claimer (Chisholm, 1989).

But as we can see, this definition of knowledge is not sufficient to relate only to propositional knowledge. In daily life, there is a wide range of definitions of knowledge. We always say that we know a person, a place or a news. For example, if someone says he knows Shanghai, it means that he knows the city well, perhaps he has lived there for many years. Or he knows about World War II, meaning, for example, that he knows that World War II ended in 1945, because he has read some books about it. So these are all different ideas and different types of knowledge.

Knowing these is an important step in answering the question of what is knowledge. And it is also helpful and often necessary. Because there are so many meanings to knowledge, this leads to the fact that we may focus on some knowledge and ignore others. So, one way to define knowledge is to categorize the term.

For example, Russell (cited in Fan, 2014) divides knowledge into two different kinds of knowledge based on epistemology: *knowledge of truths* (roughly equivalent to propositional knowledge) and *knowledge of things*. Then *knowledge of truth* can be divided into two sub sets: *immediate knowledge* (direct knowledge), which can be presented in intuitive statements such as direct judgment of perception and mathematical axioms; and *mediate knowledge* (indirect knowledge), which can be stated in demonstrable necessary propositions and inferred empirical statements such as mathematical theorems. For *knowledge of things*, he distinguished two kinds: *knowledge by acquaintance* and *knowledge by description*. Another important classification of knowledge is made by Ryle (cited in Fan, 2014): *knowing how* and *knowing that*. The former is similar to the *knowledge of truth*, but knowing how is the more important idea of knowledge. Based on the above definition of knowledge, for traditional theorists, especially the ancient Greeks, they placed too much emphasis on focusing on the *knowledge of truth* while ignoring and underestimating the ability to perform a task, which *knowing how* is about.

In addition to these classifications, other researchers make their own classifications based on their own needs. These classifications may not be accurate, but they are easier for us to understand. For example, someone distinguished *theoretical knowledge* from *practical (or empirical) knowledge*, someone talked about *craft knowledge* and *research knowledge*, someone presented *conceptual knowledge* and *methodological (or procedural, or process) knowledge*, someone used *professional knowledge* from *general knowledge*, and others have talked about *personal or local knowledge* and *public knowledge* (Cochran-Smith & Lytle, 1993; Eraut, 1994; Grimmitt & Mackinnon, 1992; Leinhardt, 1990; Machlup, 1980). Another classification is based on the different situations, such as the *professional knowledge* of the expert, the specialist, and as *scientific and technical knowledge*

(Vergnaud, 2011)

All of the above studies have attempted to classify knowledge in different ways without defining the term. Therefore, some researchers have tried to find a suitable way to define what knowledge is. They argue that there are three important elements: the knower, which refers to the subject of knowledge (who knows); the known, which refers to the object of knowledge (what is known); and the knowing, which represents the interaction between subject and object (how to know) (Fan, 2014). Similarly, Mercier and Le Maréchal (2006) also consider knowledge as an interaction between people (knower) and objects (known). They also mention that, in a broad sense, the object of knowledge is not limited to material objects. If we analyze the categories above, we can find most of these categories are based on the object of knowledge, such as *knowing that* and *knowing how*, *theoretical knowledge* and *practical knowledge*. Some are based on the knowing, such as *craft knowledge* and *research knowledge*, *direct knowledge* and *indirect knowledge*. Still some are based on the knower, such as *professional knowledge* and *general knowledge*, *personal knowledge* and *public knowledge* (Fan, 2014).

Based on the three main elements, some categories of the knowledge mentioned above like individual knowledge can be defined as relations (knowing) between people (knower) and objects (known) and the shared knowledge, which comes from a description of collective, official or institutional relations, to objects. From this point of view knowledge is seen as a means of action and means of control of action (Sabra, 2011). Other researchers think knowledge can be seen as a property of a system made by the knower and the known, or the knowing (Fan, 2014). This study views knowledge in similar ways. Since knowledge here, is a property that is expressed in the interaction process between the knower and the known, it is necessary to analyze this interaction process.

2.1.2 Teacher knowledge

After a discussion of knowledge, we can identify a critical term in this study:

teacher knowledge. Researcher argues that teacher knowledge can assist teachers in managing lesson contents, teaching and learning tasks in the classroom and make these content accessible to students (Subramaniam, 2014). But what kind of teacher knowledge teacher needs in his/her work? As Znaniecki said that “every individual who performs any social role is supposed . . . to possess the knowledge indispensable for its normal performance” (cited in Fan, 2014). Obviously, the teacher needs some special knowledge to ensure that his/her teaching is more effective, but the question is: what knowledge does the teacher need and how much knowledge is enough for them?

According to the existing literatures on knowledge, two ways exist to describe knowledge, the first is to classify it in different ways, which helps us to know from which perspective we are discussing knowledge; the second is to define it, which means that we can check what is knowledge and what is not. For the term teacher knowledge, we can still describe it in these two ways.

The first approach is used by many researchers who want to study the different kinds of knowledge that teachers need in their teaching practice. They argue that in studying teacher knowledge, teachers first need to know what they are trying to teach in the classroom. This knowledge can be described as a type of knowledge called *subject matter knowledge*. However, it is also believed that *subject matter knowledge* alone is not sufficient for effective teaching. Teachers also need to know how to explain the contents to students in an appropriate way so that they can understand it more easily. For example, in mathematics, it is not enough for a mathematics teacher to have knowledge of mathematics, educational theory and mathematics education. But, there is no single answer to the question of what kind of mathematical knowledge is sufficient or essential for teachers. (Fan, 2014).

From 1980s, teacher knowledge became a key issue in teacher education. A growing body of educational research has attempted to categorize teacher knowledge, such as “knowledge base for teaching”, “professional knowledge” and “practical knowledge”. The most important category of teacher knowledge comes from Schulman, who distinguished seven types of knowledge needed for teaching.

(Shulman, 1987): (1) *content knowledge*, which mainly refers to subject matter knowledge, such as mathematics teacher needs know well about mathematics; (2) *general pedagogical knowledge*, which means teacher needs to know well about the principles and strategies of classroom organization and teaching process which is not related to subject he/she teach; (3) *curriculum knowledge*, means teacher need to master the materials or programs which support his/her teaching; (4) *pedagogical content knowledge*, teacher also needs to know how to use their knowledge about the strategies to manage classroom teaching based on his/her subject background, including how particular topics, problems, or issues are organized, represented. For example, mathematics teacher should know which kinds of strategies can help students learn mathematics contents like function effectively; (5) *knowledge of learners* such as their characteristics, interests, abilities and so on; (6) *knowledge of educational contexts*, ranging from the environment of classroom, the governance and financing of school districts, to the character of communities and cultures; and (7) *knowledge of educational ends, purposes, and values*.

This framework has been modified and used by many other researchers based on their rich bodies of researches. For example, Howey & Grossman (1989) integrated the seven types mentioned above into four types of teacher knowledge: *general pedagogical knowledge*, *subject matter knowledge*, *pedagogical content knowledge*, and *knowledge of context*. Cochran, DeRuiter, and King (1993) proposed a comprehensive model of *pedagogical content knowing* that draws on and refines Shulman's concept of pedagogical content knowledge. It emphasizes the importance of the teacher knowledge of how students learn and the context in which they teach. Other researchers have attempted to add other elements to the original framework to make it more appropriate for today's teaching and learning situations, such as putting subjects such as mathematics into the framework in an attempt to make the framework explain the specific knowledge needed by mathematics teachers like MPCK, or adding technology into the framework based on developments in technology such as TPACK (Koehler et al., 2007). These studies give us more perspectives to understand the knowledge that teachers have in their daily teaching.

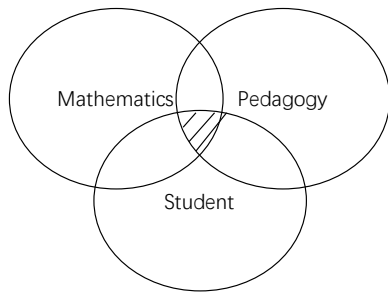


Figure 2.1: Elements in teaching process

According to the existing literatures, this figure shows three key elements in the mathematics classroom and reflects the knowledge that mathematics teachers need to teach. The shaded part of the figure means that the teacher’s work is always based on the interaction between these three elements. It also means that the teacher needs to have at least mathematical knowledge, pedagogical knowledge and knowledge about student. So that teacher knowledge is not a separate concept, it consists of different elements and the interaction between these elements constitutes the overall concept of teacher knowledge.

Based on mathematical teaching and learning, Bromme posed one more comprehensive categorization. The following five fields (cited in Fan, 2014) of teachers’ knowledge are needed for teaching mathematics: (1) knowledge about mathematics as a discipline, which contains mathematical propositions, rules, mathematical modes of thinking, and methods which is related to content knowledge by Shulman; (2) knowledge about school mathematics, which has a “life of its own” with its own logic, not just simplifications of mathematics as it is taught in universities; (3) the philosophy of school mathematics, which are ideas about the epistemological foundations of mathematics and mathematics learning and about the relationship between mathematics and other fields of human life and knowledge; (4) general pedagogical (and psychological) knowledge, which mainly includes general classroom organization and instructional communication, and has a relatively independent validity separate from the school subjects. This is main related to pedagogical knowledge by Shulman; and (5) subject-matter-specific pedagogical knowledge, similar to what Shulman termed “pedagogical content knowledge”. From these main researchers, two knowledge is common used: *subject matter*

knowledge, and *pedagogical knowledge*, although sometimes they are termed differently. These classifications are based only on the self-beliefs and experiences of the researchers and there is also a huge variation between these studies. This is why there are so many different categories of teacher knowledge. Therefore, we need a unified definition of teacher knowledge, not just make some categories.

Chinese researchers have also paid attention to classifying teachers according to their knowledge needs in the teaching process. However, unlike western countries, the 21st century, teacher knowledge has begun to become one of the topics of educational research in China (Jiang & Wang, 2016)¹. In these studies, some discuss what teacher knowledge is and its components, some talk about subject and practical knowledge (Li & Chen, 2016; Shao & Zhang, 2008; Zou & Chen, 2005)²; and some review the researches related to teacher knowledge in China and Western countries (Kang, 2012; Li & Shen, 2011)³. Then, the interest of Chinese researchers gradually shifted from what knowledge teachers need to how teachers use knowledge in actual practice and what is teacher knowledge (Peng & Zhu, 2009)⁴. They also agree that teacher knowledge is contextualized and the teaching process is very complex, so teachers cannot use knowledge without the teaching environment (Zhang, 2018)⁵. For example, Han and Ma analyzed mathematics teachers' knowledge based on the types mentioned above, such as PCK and MCK. They found that teachers in high level schools have more knowledge than others (Han & Ma, 2016)⁶.

¹ All of the Chinese articles (hereafter) would be translated into English in the reference list and put the Chinese version in the footnote 姜霞, 王雪梅 (2016). 我国外语教师知识研究:回顾与展望——基于外语类和教育类CSSCI 期刊论文的分析. *外语界*,(06), 33-41.

² 李四清, & 陈坚林. (2016). 高校外语教师知识结构与教学自主的关系探究. *外语与外语教学*(5), 88-96. 邵光华, & 张明慧. (2008). 个人知识观下的教师专业知识发展研究. *宁波大学学报(教育科学版)*,30(01), 74-79. 邹斌, & 陈向明. (2005). 教师知识概念的溯源. *课程.教材.教法*,24(6), 85-89.

³ 康晓伟. (2012). 当代西方教师知识研究述评. *外国教育研究*(8), 84-91. 李长吉, & 沈晓燕. (2011). 教师知识研究的进展和趋势. *当代教师教育*(03), 5-10.

⁴ 彭伟强, 朱晓燕(2009). 国外不同研究路向的外语教师知识研究. *中国外语教育*,(2), 28-36.

⁵ 张光陆(2018).学生核心素养视角下的教师知识:特征与发展. *课程.教材.教法*,38(3), 64-69+82.

⁶ 韩继伟&马云鹏. (2016). 中学数学教师的教师知识状况的调查研究. *全球教育展望*(4), 106-117.

One problem that can be identified with these classifications is that the integration of digital technologies in mathematics teaching and learning has not been addressed in almost every study of mathematics teacher knowledge (Niess et al., 2009). In the 21st century, technology has become more important in math education. Teachers also need to learn how to use technology to teach with their students. Therefore, there seems to be a need to include technology and locate it appropriately in the structure of teacher knowledge (Fan, 2014). In the following section I would describe the knowledge teachers needed about technology.

In addition to classifying teacher knowledge, a number of other researchers have given us a possible way to define teacher knowledge. An important point in their work is that teacher knowledge is situated in a particular *situation or context*. That is, they believed that teacher knowledge is situated in a complex situation including classrooms, teaching activities and events (Yamagata-Lynch, 2003). As we have discussed, knowledge may be correct in one context, but not in others. So for teachers, their knowledge does not exist independently of the situations in which they use it to teach. Like what Yamagata-Lynch said “Facing with these complex situations, it is not enough for teachers only apply a set of predefined principles, they need to plan and use different teaching process and strategies in order to be suitable in these situations” (Yamagata-Lynch, 2003, p. 34). This tells us that it is not enough to train teachers in situations that are not relevant to their classroom teaching. Teachers will not find useful information from these training programs. (Cochran et al., 1993; Yamagata-Lynch, 2003).

The fact that teacher knowledge is profoundly linked to his or her pedagogical practices and reflections based on his or her experiences which means that for pre-service teachers, they need the imagination and skills to work in a variety of settings and they also need contextual experiences that are as close as possible to real teaching practice. (Cochran et al., 1993; Ponte et al., 2002). It can be seen as a reciprocal process of interaction between the teacher pedagogical knowledge, teaching and learning activities, and the mathematical contents and thus influences students’ learning activities (Subramaniam, 2014). Indeed, the key issue is that the meaning of a piece of

knowledge cannot be reduced from behaviors, whereas meaning cannot be characterized, diagnosed or taught without linking it to behaviors (Balacheff, 2013). It comes mainly from teacher experience and situations encountered (Vergnaud, 2011).

Balacheff suggests one way to determine knowledge when we consider situation. While his work focuses more on knowledge about learners when they are confronted with different learning situations, it can still help us understand knowledge from another perspective: teacher. He posed a model to definite knowledge which named: $cK\phi$ (in which c replaces conception, K replaces knowing and ϕ replaces concept). In this model, conception can be definite through the following way which is written in French: *“La conception est l’état d’équilibre d’un système, et plus précisément l’état d’équilibre d’une boucle action/rétroaction du système [sujet<>milieu] sous des contraintes proscriptives de viabilité.”*⁷(Balacheff & Margolinas, 2005 p.6) and if we translate it into English and try to make it easier to understand, the conception can be seen as a dynamic equilibrium of an action/feedback (interaction) between the subject (which means knower: teacher, learner or any person) and the *milieu* (in English maybe we can still use the words *milieu*) within a set of constraints. This means that knowing can be viewed as a set of conceptions that can be activated by different situations that the observer perceives as conceptually the same (Balacheff 2013). The following figure shows the main idea of conception.

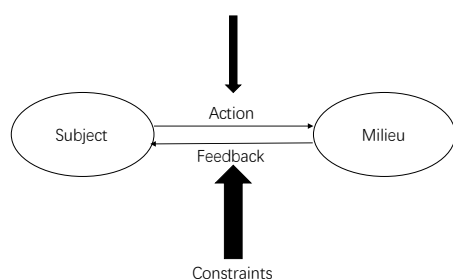


Figure 2.2: Knowledge by Balacheff (Balacheff & Margolinas, 2005)

In this definition, we find a critical element: *milieu* (in French) which comes from

⁷ English translation: “Conception is the equilibrium state of a system, and more precisely the equilibrium state of an action / feedback loop of the system [subject <> milieu] under proscriptive constraints of viability”

the work of Brousseau. For Brousseau, the milieu is the learner's antagonist system in the learning process:

*“Le système antagoniste du joueur dans une situation est pour le joueur comme pour l'observateur, une modélisation de la partie de l'univers à laquelle se réfère la connaissance en jeu et les interactions qu'elle détermine. C'est ce système antagoniste que nous avons proposé d'appeler milieu”*⁸ (Brousseau, 1990 p320)

The learner acts on the milieu and he is also influenced by feedback from it. Figure 2.2 depicts this interaction between the subject and the milieu. First, the subject acts on the milieu and also the milieu gives some feedback which influences the subject's subsequent actions. It also illustrates that this interaction is influenced by the constraints in the milieu. The whole process shows the knowing process of the subject, like what Balacheff said:

From this definition of conception, I can derive a definition of knowing as the characterization of a dynamic set of conceptions. This definition has the advantage of being in line with our usual use of the word “knowing” while providing grounds to understand the possible contradictions evidenced by learners' behaviours and their variable mathematical development. A conception is a situated knowing; in other words, it is the instantiation of a knowing in a specific situation detailed by the properties of the milieu and the constraints on the relations (action/feedback) between this milieu and the subject. (Balacheff 2010, p. 18)

Based on figure 2.2, Brousseau posed a new possible way to define what is knowledge. Then we can discuss what is teacher knowledge according to this definition. First, we need to know what is *milieu*. It is suitable for many kinds of context. Teacher knowledge exists in a certain context made by teacher, student and milieu. For Brousseau milieu in this context can be seen “external” to teaching which is stripped of didactical intentions and presuppositions (Brousseau, 1997). Another researcher Bloch

⁸ English translation: “The player's antagonistic system in a situation is both for the player and for the observer, a modeling of the part of the universe which refers the knowledge and the interactions it determines. It is this antagonistic system that we have proposed to call “milieu”.”

in his French paper thought that milieu for teaching contains: (1) situation that which teacher has implemented for the “Student”; (2) the students. And we consider that teacher knowledge exists in that milieu (Bloch, 1999). Since knowledge makes its meaning in the interaction between the subject and the milieu, we can therefore know that the milieu does not exist independently of the subject. And the elements contained in the milieu can be different for different subjects. For example, the number 3 for one subject means only a number in his milieu, but for another subject it may mean a representation of an integer. This feature of the milieu is one of the reasons why we are able to describe knowledge from it.

Other researchers have attempted to define teacher knowledge with similar elements. As we discussed above, knowledge can be seen as a process of interaction between the knower and the known, where the interaction is called knowing. From this we can find it is similar to what Balacheff said in his study. Here the knower is the subject, the known is the milieu, and the knower is the interaction. On the basis of these works, knowledge can be seen as a dynamic process and defined as the subject’s knowledge of the object as an interaction between the knower and the known, where the process of interaction is called knowing. According to this definition, for teacher’s pedagogical knowledge: the teacher here is the knower, the pedagogy of his teaching is the known, and interaction is the process by which the teacher generates his own knowledge or learns or receives the pedagogical knowledge of others (Fan, 2014).

In this research, I focus on two descriptions about knowledge made by Balacheff and Fan. By contrasting the two descriptions, I found both of them think knowledge is a property of a system. For Balacheff, knowledge is reflected in the system which is constructed by subject and milieu (figure 2.2) which is affected by constraints. For Fan they system contains: the knower means the subject of knowledge; the known means the object of knowledge; and the knowing means the interaction between subject and object. Both of these two systems pay attention to the interaction between subject and object of knowledge. So in my research, teacher knowledge is a property of a system made by teacher and didactical situation (figure 2.3).

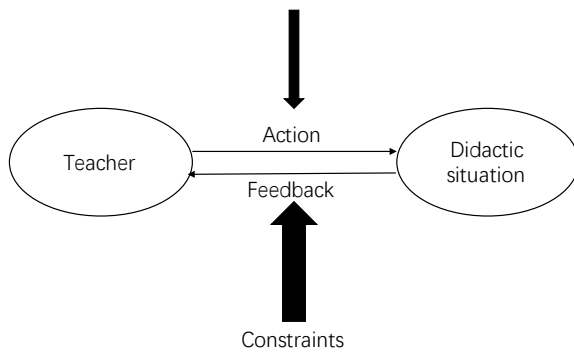


Figure 2.3: Definition of teacher knowledge

Here didactical situation means: “In didactical Situations, the teacher maintains direct responsibility for all stages of the lesson. She tells the students her intentions, what they will have to do, and what the result should be. She intervenes freely to keep the class travelling on the desired route.” (Brousseau et al., 2014 p.147).

From these definitions or categories of teacher knowledge, it is clear that in order to have a clear understanding of what knowledge teachers need in their daily practice with students, we need to focus on the situation in which teachers work and try to describe how they act in these situations. This is one of the reasons why this study concerns teachers’ practice in real classroom. Three elements are critical in the daily practice of teachers: the teacher, the students and the content. Thus, for example, in German mathematics education research, teacher knowledge is conceptualized as an interaction between the teacher, students and mathematics contents (Subramaniam, 2014). During the interaction, teachers are free to choose appropriate actions and strategies to teach, while the pedagogical situation provides them with an uncertain space in which teacher knowledge can be constructed (Mackrell et al., 2013).

Before presenting the existing research related to the knowledge teachers need to use DGS in their lessons, I will begin the next section by describing what researchers have noticed about DGS and why I choose to focus on this kind of software in this research and then I will describe the knowledge used for using DGS.

2.2 DGS in mathematics education

In the 21st century, information technology has become very important in all areas

of our life. In the field of education, especially in mathematics education, a lot of software are designed to support teaching mathematics in schools. These kinds software are designed to make mathematics as *explicit* as possible and it shows not only the *results* of mathematical activities, but also the *process* of mathematics. (Tall, 1986). For example, a number of information technologies such as calculators, microcomputer software and other instruments are used in mathematics lessons. They have become very important in the U.S. and other Western countries since the 1970s (Research Group of the Core Contents and the Teaching in Primary and Secondary Schools, 2012)⁹. Early in 1985, American Education Development Center developed a software for teaching mathematics called *geometric exploration*. Since then teachers began to use this kind of software in their lessons. And then in 1989, the American company established one of the DGS, Geometer's Sketchpad, which then became a very widely used software in mathematics education (Mou, 2012)¹⁰. These technologies allow students to learn geometry more effectively than traditional methods. Also in China, Geometer's Sketchpad is also used to teach mathematics. For example, secondary school teachers always choose to use this software when teaching geometry. Some teachers believe that Geometer's Sketchpad has several key features such as *free to use*, *simplicity*, *exploratory*, *interactive* (Hu, 2005)¹¹, *accuracy* (Mou, 2012)¹².

In this study, I focus only on one widely used technology in mathematics teaching: DGS. Because of the idea of direct manipulation, DGS has been developed since the 1980s and allows learners to verify relevant geometric properties by dragging geometric objects, while visualizing abstract geometric contents (Laborde & Laborde, 2008; N. Sinclair et al., 2016).

⁹ “中小学数学课程核心内容及其教学的研究” 课题组 (2012). 数学·信息技术·数学教学. 课程.教材.教法 (12), 62-66.

¹⁰ 牟丽华. (2012). 几何画板优化初中数学教学的案例研究.重庆师范大学, 重庆.

¹¹ 胡卫华. (2005). 基于《几何画板》的数学探究式教学模式的实证研究. 广西师范大学, 桂林.

¹² 牟丽华. (2012). 几何画板优化初中数学教学的案例研究.重庆师范大学, 重庆.

2.2.1 Why choosing Dynamic Geometry Software

In order to find the effect that DGS brings to the process of teaching and learning mathematics, some researchers argue that we first need to better understand what geometry is (Straesser, 2001). First: geometry, as a sub-discipline of mathematics, has logical relationships and intuitive understanding, which is also known as *deductive* geometry (Hilbert & Cohn-Vossen, 1996). Second: Geometry is seen as a tool for exploring real life, which implies *practical* geometry (Grunbaum, 1981). And then there are several questions (Straesser, 2001): Do we have a special geometry incorporated in DGS? Or does a computer-based DGS basically incorporate the “same” paper-and-pencil based geometry?

One of the answers is that it is difficult to help students construct knowledge in traditional learning situations (paper-pencil). However, with DGS or advanced graphing calculator with a computer algebra system (CAS), teachers can use them to teach mathematics modelling and provide examples. With them, students can also imitate teacher’s actions and use the technology to verify, demonstrate, drill, and practice (Niess et al., 2009).

The following areas shows the interests of different researches about DGS in mathematics education (Sinclair et al., 2016):

- (1) introducing or designing new technology;
- (2) the role of different technologies; and
- (3) empirical studies on the use of technology in teaching and learning mathematics (Healy, 2000; Mariotti, 2000; Ruthven, 2005).

Dynamic geometry software creates a new situation that allows the user to construct and manipulate Euclidean geometries (Baulac et al., 1988). It contains primitives for drawing basic geometric objects, such as points, lines, and segments. With these tools, the user can use DGS to construct new objects, such as parallel lines or bisectors of angles (Olivero, 2002). For mathematics teaching, one of the main features of DGS is *dragging*, a direct manipulation of the software. It implies a simultaneity between the students’ actions and feedbacks from DGS (Sinclair et al.,

2016). This simultaneity can address the gap between experimental and theoretical mathematics, or deal with how to transfer from conjecture to formalization (Leung, 2003). When students drag a geometric object with DGS, some of the geometric properties of the graph defined in its construction are preserved. So that dragging is often seen as a problem-solving tool that includes constructing, finding commonalities, conjecturing, and proving or disproving (Healy & Hoyles, 2001; Hölzl, 1996). There are three main modes of dragging identified in the literature (Healy, 2000; Laborde, 2001):

1) *dragging for verifying*, is dragging to check the correctness of the supposed (known) geometrical properties in the figure. The uses of DGS in classroom are often limited to this modality, students are expected to drag figures to confirm empirically the properties which are more or less given (Hölzl, 2001).

2) *dragging for conjecturing*, is dragging to look for new geometry properties through the perception of what remains invariant when dragging.

3) *dragging for validating/invalidating*, is dragging to check whether the construction preserves its geometrical properties when dragging.

These different kinds of dragging modes can affect or cause different teaching practices, from which we can have an understanding about teacher knowledge revealed in teacher's interaction with DGS. This is the most important reason why in this research, I choose to focus DGS among all of the technologies in mathematics education.

Many different issues can be identified in researches on DGS. For example, some studies have focused on what users can do with DGS, but cannot do with paper and pencil. Other studies have focused on how to use dragging, or the interplay between experience and theoretical properties or DGS to guess and prove (Olivero, 2002). Following sections will discuss the effects of DGS in teaching and learning mathematics.

2.2.2 DGS in teaching and learning mathematics

Based on previous research work, mathematics teachers need to change their traditional teaching methods in the new environment with technology. For example, students can interact directly with DGS. And these interaction processes with the software are very complex. Therefore, teachers may find that students' interaction processes with the software can lead them to produce results that are not what they expect (Olivero, 2002). DGS not only affects the learning process of the students but also affects the teaching methods at the same time. Therefore, teachers need help to use the software for their teaching practices. Some researchers have focused on how to help teachers adapt to the new situation created by DGS, such as building dynamic geometry repositories to help teachers arrange the teaching and learning process (Trgalová & Jahn, 2013). Or they have tried to find effective ways to embed DGS, such as Geometer's Sketchpad, into math lessons.

In China, it has been argued that the use of DGS will change the teacher's opinions on teaching (Hu, 2005)¹³. This means that teacher needs to change their roles in the classroom, from a leader to a learning supporter. They need to recognize that students are the center during teaching and they need to give them more time and space to explore and learn on their own. Teachers also need to find their own ways of using software and try to find creative strategies of teaching with DGS. In a quantitative study (Hu, 2005)¹⁴, more than 75% of teachers felt that their opinions on teaching had changed, and about half of them felt that using mathematics software could improve their teaching methods. For example, teachers used to tell students directly about specific mathematical concepts, such as functions (Guo, 2012)¹⁵. This software, however, allows teachers to let students experience and explore important

¹³ 胡卫华. (2005). 基于《几何画板》的数学探究式教学模式的实证研究. 广西师范大学, 桂林.

¹⁴ 胡卫华. (2005). 基于《几何画板》的数学探究式教学模式的实证研究. 广西师范大学, 桂林.

¹⁵ 郭建峰. (2012). 让计算机辅助教学(CAI)为课堂教学插翅添翼——浅谈《几何画板》的特点及在高中数学教学中的应用. *数学教学通讯*, (27), 16 - 18.

mathematical knowledge on their own(Fan, 2003)¹⁶. In traditional classroom, sometimes students may have difficulty in understanding abstract mathematical concepts. But Geometer’s Sketchpad is a powerful supplement to the mathematics lessons and it can greatly improve mathematics teaching and learning (Zhao et al., 2012)¹⁷. Therefore, in China, both teachers and students are willing to use the software to learn and teach mathematics.

2.2.3 Affects brought by DGS

Some studies have also mentioned that the development of digital technology has an impact on teaching and learning mathematics, however, even though technology has been developed for many years, its use in the mathematics classroom is not so spread (Gueudet & Trouche, 2011). In the classroom, not much changes were made to students’ learning contexts or teachers’ teaching practices. There is also a lack of adequate strategies to effectively integrate them into teaching and learning mathematics. (Niess et al., 2009). Others like Kortenkamp et al. (2009) point out: “still, the adoption of DGS at school is often difficult. Despite the fact that a lot of DGS class material exists, Interactive Geometry is still not used in classrooms regularly. Many teachers do not seem to know about the new possibilities, or they do not have access to the software and/or resources” (p. 1,150). Laborde et al (2011) also noted a gap between teachers’ daily practice with dynamic geometry software and what is envisioned in educational research work. In today’s mathematics classrooms, teachers always present some of the results they have found with their software. Only a few of them choose to let their students operate Geometer’s Sketchpad or other software to explore mathematics contents (Hu, 2005)¹⁸. Many students also indicated that they did not interact with each other during the lessons.

¹⁶ 范文贵. (2003). 利用几何画板开展探究性数学学习的案例分析. *中国电化教育*, (4), 34–36.

<https://doi.org/10.3969/j.issn.1006-9860.2003.04.010>

¹⁷ 赵生初, 杜薇薇, & 卢秀敏. (2012). 《几何画板》在初中数学教学中的实践与探索. *中国电化教育*, 20(3), 104–107. <https://doi.org/10.3969/j.issn.1006-9860.2012.03.020>

¹⁸ 胡卫华. (2005). 基于《几何画板》的数学探究式教学模式的实证研究. 广西师范大学, 桂林.

In these mathematics classes, students only see the computer and watch teachers construct diagrams through the software until the end of the class. They do not have the opportunity to share their findings about particular problems or even not know whether what they are doing is correctly or not (Liu, 2009) ¹⁹.

In the case of DGS, there are several reasons that could explain this phenomenon. One possible answer to the gap between research and pedagogical practice is that geometry itself changes when DGS such as Cabri-géomètre is used. And the answer obviously depends on the understanding of geometry (Straesser, 2001). Lagrange mentioned another reason that may be more important is the shift in consideration of mathematical activities and the teaching profession caused by the introduction of technology into the mathematics classroom (cited in Trgalova et al., 2009). Some researchers have found that teachers were concerned that the use of DGS might complicate mathematics tasks, especially in the area of assessment (Artigue, 1997). This requires teachers to think about how to use new types of tasks to integrate DGS effectively. On the other hand, it also asks teachers to think about the relationship between tasks with paper-pencil and tasks with DGS, so that they can be inserted into the daily teaching process (Assude & Gelis, 2002). These changes that teachers need to make when using DGS are simultaneously affected by factors such as changes in the curriculum program, teachers' own experiences, and teaching resources (Sabra, 2011). So that teachers need more specific skills or knowledge to use these technologies to organize their classrooms (Ertmer et al., 2003). They need to think about how and why they use technology in their mathematics lessons in order to develop their own ideas of using technology. Overall, the use of DGS requires teachers to rethink their teaching methods in order to deal with unfamiliar or even new teaching situations (Guerrero, 2010; Stoilescu, 2014) and play a more critical role in designing activities or tasks for their students. (Hoyles & Lagrange, 2010).

Other reasons such as economic background also affect the integration of DGS in

¹⁹ 刘华. (2009). 信息技术与数学课程整合——几何画板篇——几何画板能为高中数学教学做什么. *中学数学月刊*, (9), 36–38. <https://doi.org/10.3969/j.issn.1004-1176.2009.09.017>

mathematics lessons. For example, in some parts of China, especially in rural areas, where teachers and students only have a few computers, so many rural teachers do not have the opportunity to learn new technologies. This may be a critical constraint for some of the teachers to use technologies like software in teaching mathematics. In contrast, Ministry of Education of China issued new mathematics curriculum standards (Ministry of Education of the People's Republic of China, 2011)²⁰ to encourage teachers to use mathematics software to support their lessons and give students more opportunities to explore knowledge on their own.

From the literatures, we can see that many researchers believe that DGS can improve teachers' teaching methods. However, no one has attempted to study the extent to which the software can improve teaching. In some of studies, they have tried to design an evaluation system to assess the benefits of these software to teachers (Trgalova et al., 2011). In China, only a few studies have investigated the actual use of mathematics software by teachers. According to the researchers, an important lesson is that research on the use of digital technologies in mathematics education should not be limited to the study of the tools alone, but should look at the tasks and their embedding in teaching and learning as a whole in order to understand what works, and why it works (Drijvers, 2015).

Therefore, the question is no longer whether teachers should incorporate DGS into their teaching process, but how they can use DGS to create new opportunities for students' learning process (Angeli & Valanides, 2009). So in this study, I try to identify the changes in teaching activities with DGS. The main issue is how teachers integrate DGS into their daily classroom and how they use their knowledge when using this kind of software in their mathematics lessons.

2.3 Knowledge needs by teachers with DGS

In mathematics education, the potential of technology is being discussed due to

²⁰ 中华人民共和国教育部. (2011). *数学课程标准*. 北京师范大学出版社.

its advantages in teaching and learning. Teachers need special knowledge in order to successfully integrate technological tools. This study focuses specifically on teacher knowledge of one of these technologies: DGS. The topic of teacher knowledge is one of the important issues in the field of educational research. Teacher education and in-service teacher professional development are key to high quality education. Because technology has become an important component or tool for learning and teaching, teachers should have a general idea of what their subject is like by using technology and what it means in teaching and learning (Niess, 2005). Researchers tried to find factors that can influence the quality of mathematics teaching and learning, such as mathematical knowledge, pedagogical skills, curriculum knowledge and teacher beliefs (Drijvers et al., 2013). In this study, one of the main issues is to discuss teacher knowledge in a digital situation. If a teacher integrates technology without sufficient necessary knowledge about the curriculum and technology, he/she may not be able to use these tools to organize classroom instruction effectively. (Kastberg & Leatham, 2005). In this section, I would present the existing researches related to this issue. One thing that has been realized is that mathematics teachers need to know how to use information and communication technology (ICT) tools, including subject-specific educational software (Ponte et al., 2002). And we also need to notice that in the teacher education literatures, researchers agree that mathematics knowledge is necessary for effective teaching (National Mathematics Advisory Panel, 2008). However, only mathematics knowledge is not sufficient at all, and there is less agreement in this literature about what mathematics knowledge is, and especially how much knowledge is necessary for teaching (Copur-Gencturk & Lubienski, 2013). In order to help teachers use technology to teach effectively, we need to analyze what knowledge is necessary in order to use these technologies effectively.

Some researchers have addressed different ideas towards the knowledge of using technology by teachers under different type of knowledge, like information and communication (ICT)-related PCK (Angeli & Valanides, 2005); Technological Content Knowledge (Koehler et al., 2007); and electronic PCK or e-PCK (Irving, 2006). Researchers believe that technological knowledge cannot be treated as a

knowledge unrelated to or separate from pedagogical task and situations. It is not only about what technology can do, but also, and perhaps more importantly, what it can do for the classroom teaching (Koehler et al., 2007). Therefore, teachers need to have a deep understanding of the software in order to effectively integrate it into their classroom. For example, 1, they need to know how to operate the software; 2, they need to know exactly what they need to teach; 3, they need to know what is appropriate for teaching with technology and what is not; 4, they need to know how their teaching strategies need to change in order to use the technology; 5, they need to know their students and know whether these students are prepared to learn with technology. All of these, of course, reflect the teachers' perceptions of how to use technology in the classroom for teaching and learning and influence their technology practices. For example, Angeli & Valanides (2009) proposed one framework of the knowledge teachers need to have (ICT-TPCK) when using ICT in their classroom.

Within mathematics, some other researchers analyzed teacher knowledge from the following four points of views (Niess et al., 2009): knowledge about technology, for example, whether teachers can operate computer software (e.g., MS Word, PowerPoint, Internet browsers, email) and whether they can install or remove peripheral devices (e.g., USB drives, microphones) & software. We can call this knowledge as technology knowledge (TK); knowledge about mathematics contents, for example, whether teachers have sufficient mathematics knowledge about teaching subject. Whether they are able to gain deeper understanding about the mathematics contents they need to teach. Whether they are confident to teach the content knowledge. We can call it mathematics content knowledge (MCK); knowledge about the pedagogical strategies with technology, for example, they need to think about how to motivate students through technology or how to use different instruction strategies when technology is used. We can call this pedagogical technology knowledge (PTK) and also we need to notice that teacher also needs to know about the existence of software for particular tasks (ability to design or choose suitable tasks) and knowledge about how to promote students learning with technology, like how students construct and use their understandings of mathematics

with technologies (cited in Cochran, DeRuiter, & King, 1993). Based on enough understanding of students' complex construction of knowledge, teachers can reflect their teaching process to make effective using of technologies (Resnick, 1989, Reynolds, 1992).

In the classroom, teachers will design learning activities to guide students' learning process. So that they need to know how to design appropriate activities to help students construct mathematical knowledge by using appropriate ICT tools. For example, using ICT tools to design self-directed learning activities for students or using them to design inquiry activities to guide students' understanding of mathematical contents. They also need to know how to design lessons that appropriately integrate content, technology and pedagogy to achieve student-centered learning and so on. These require teachers to be knowledgeable enough to integrate technology into mathematics lessons.

One of the models of teachers' professional knowledge was developed by Shulman. His model distinguished between two kinds of knowledge in teaching: *content knowledge* (like mathematics knowledge in mathematics teaching) and *pedagogical knowledge* (Shulman, 1987). Pedagogical content knowledge (PCK) combines knowledge of content and pedagogy. It is a new type of knowledge that cannot be reduced to either pedagogical or content knowledge. It includes domain-specific pedagogical insights and refers to the need for teachers to know how to organize teaching and learning processes according to specific subject content.

However, only subject content knowledge, pedagogical knowledge, and pedagogical content knowledge are not sufficient to describe the knowledge teachers need to integrate technology. It is important to focus on one specific kind of knowledge that can help teachers integrate new technologies into their mathematics lessons. Koehler, Mishra and Yahya (2007) therefore proposed a new model that combines knowledge of technology, pedagogy and content, abbreviated as TPACK. TPACK is a coherent way of describing the body of knowledge and skills that are needed to teach by using technology. TPACK includes teachers' understanding of the mathematics teaching and understanding of using technology (Koehler et al., 2007).

For example, it describes the teacher's beliefs about mathematics, how he/she believes mathematical problems can best be solved through the use of technology, and the importance of learning mathematics through the use of technology for students (Guerrero, 2010).

There are many different definitions of TPACK concepts. One of the definition made by Koehler and Mishra (2007) have been widely used. Based on Shulman's PCK framework, pedagogical knowledge (PK) is knowledge about using what kind of strategies or process to teach, content knowledge (CK) is knowledge about the specific subject, which students need to know. and PCK is pedagogical-content knowledge. Then another critical knowledge, technological knowledge (TK), appears. It refers to knowledge of operating technology or solving the technical problems such as operating computer software like spreadsheets email or DGS. Technological pedagogical knowledge (TPK) encompasses the knowledge of using technology for instruction, for example, teachers need to know how to modify their instructional processes with the help of DGS. Technological content knowledge (TCK) is knowledge about how technology and content are interrelated. For example, how to use DGS to solve geometric problems such as drawing symmetrical objects. Technological pedagogical content knowledge (TPACK), finally, finally, includes an understanding of using technology to represent concepts or using technology in constructive ways to teach contents such as geometry; an understanding of what makes geometry difficult or easy to learn and how DGS can help correct some of the problems students face; an understanding of students' prior knowledge and epistemological theories; and an understanding of how to use DGS to build on the develop new or strengthen old knowledge based on existing knowledge of geometry.

In China, researchers have also identified the need for a consistent definition of teachers' technology-related knowledge and have observed little use of TPACK for specific disciplines such as mathematics (Wang, 2015)²¹.

²¹ 王白萍. (2015). 整合技术学科教学法知识(TPACK)研究现状与发展趋势. *现代基础教育研究*, 20, 61-70.

Duan, Yan and Zhang (2015)²² conducted an empirical study on the composition of TPACK in Chinese practice through a questionnaire survey with junior high school mathematics teachers. She found that TPACK for junior high school mathematics teachers consists of five elements: understanding the role of technology in mathematics teaching; awareness of student understanding; awareness of curriculum and resources; awareness of teaching assessment; and awareness of teaching strategies.

Ruan and Yang (2014)²³ posed new framework of knowledge named: TSACK (Technological, Strategic and Content Knowledge) and TMACK (Technological Methodological and Content Knowledge) based on TPACK.

In order to help teachers use technology, we need to design programs that let teachers rethink their teaching process by using different technological tools (Wang, 2015)²⁴. And we also need to focus on the behavioral characteristics of teachers' use of technology, which can present teachers' knowledge (Li et al., 2015)²⁵. However, only teaching isolated technology skills (the T in TPACK) in pre-service teacher programs does little to help develop sufficient knowledge about using technology for more effective instruction. Its relationship to subject content and how to help students learn to use technology appropriately while meeting specific course content standards (Harris et al., 2009) is also crucial. Teacher knowledge about technology cannot be seen as a kind of knowledge which is isolated with situation. Good teaching requires an understanding of how technology relates to pedagogy and content. (Hughes, 2005; Koehler et al., 2007; Margerum-Leys & Marx, 2000).

Returning to the two terms used in this study: teacher knowledge and DGS, according to the literature review, teacher knowledge in dynamic geometry situation is a complex area of research in mathematics education. Many different theoretical

²² 段元美, 闫志明, & 张克俊. (2015). 初中数学教师 TPACK 构成研究. *电化教育研究*(04), 116-122.

²³ 阮全友, & 杨玉芹. (2014). 整合技术的学科和教学知识框架的发展:从 TPACK、TSACK 到 TMACK. *中国远程教育*(11), 20-26.

²⁴ 王白萍. (2015). 整合技术学科教学法知识(TPACK)研究现状与发展趋势. *现代基础教育研究*, 20, 61-70.

²⁵ 李建珍, 冯利珍, & 袁玉飞. (2015). 国内 TPACK 研究的分析与思考. *电化教育研究*, 36(11), 104-110.

frameworks have attempted to explain and describe the different elements that make up this kind of teacher knowledge. An important part of teachers' practice with technology is particularly relevant to their knowledge: the *institutional situation* of their practice, their practice in and out of the classroom, their resources, and their professional development (Hoyles & Lagrange, 2010). In this study, I hypothesize that teachers' use of instructional strategies, course materials, and content is mediated by their pedagogical knowledge (Beswick & Goos, 2018). Before we can analyze teachers' practices, we need to know what teachers do in their instructional process. According to Thames and Ball (2010), mathematics teachers engage in the following tasks: asking mathematical questions; giving and evaluating explanations; choosing or designing tasks (Sabra, 2011); using and selecting representations; recording mathematical assignments on the board; selecting and arranging examples; analyzing students' errors; evaluating students' unconventional ideas; mediating discussions; attending to and using mathematical language. Also select appropriate resources and their combination or modification during teaching process (Gueudet & Trouche, 2009a).

In this study, rather than trying to categorize what knowledge teachers need to use technology, especially DGS, in the classroom, like what other frameworks such as TPACK do. I consider teacher knowledge as the whole process by which teachers interact with DGS in mathematics lessons. I still describe teacher knowledge relating to DGS as a property of the system constituted by teacher and the didactic situation, and attempt to integrate DGS into the new system, as shown below.

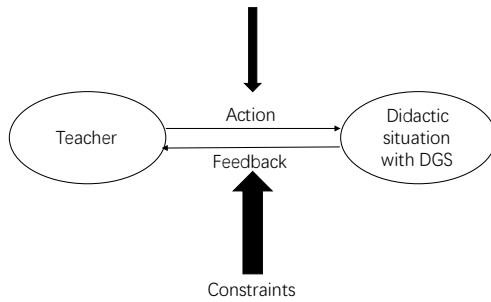


Figure 2.4: Teacher knowledge in dynamic geometry environment

2.4 Aims and research questions

In this section, I will address the main aims and research questions of this study. As in the previous section, two key terms: teacher knowledge and DGS form the main focus of this study. Based on the hypothesis, teacher knowledge relating to DGS is a property of the interaction between teacher and didactical situation with DGS. And I focus on observable elements of teacher knowledge during teaching practice with DGS which can make tacit knowledge explicit. So that this study talks about the interaction (the knowing) between the teacher (the knower) and the didactical situation (the known) created by using DGS. As discussed above, due to the complexity of teacher practice, I focused on three dimensions of teaching behaviors in this study: selecting mathematical tasks with DGS, organizing lessons with DGS, and teacher-student interaction. After studying the activities of teachers by using DGS in the classroom, I wanted to address the following questions

- What teacher knowledge is related to teaching practice with DGS in cases from China and France?
 - Which roles DGS plays in mathematics tasks in cases from China and France?
 - Which types of instrumental orchestration exist in teaching with DGS in cases from China and France?
 - What teachers focus on during the teacher-student interaction in cases from China and France?

3 Theoretical framework: Teacher knowledge within instrumental orchestration

In this chapter, I will present the main theoretical framework of this study. The use of instrumental orchestration allows us to analyze teachers' practice with DGS. The chapter begins with an introduction of instrumental orchestration, its origins and development. It then introduces the different types of instrumental orchestration presented in the teachers' lessons. After that, I will show the relationship between instrumental orchestration and teacher knowledge. Instrumental orchestration consists of three main elements: didactic configuration, exploitation mode and didactical performance. Based on them, this section will also present the relationship between the question in this study and the three elements. Didactic configuration and exploitation mode show how mathematics teachers organize their classrooms and how they plan to present tasks to their students. For example, what kind of tools they use or what kind of tasks they design for their students. Didactic configuration and exploitation mode also show the different roles that DGS play in mathematics coursework. These modes are discussed in Section 3.4. In order to analyze teachers' practices in didactical performance, Section 3.5 shows the different foci of interaction between teacher and students. This section discusses the questions raised by mathematics teachers and the feedback given to students.

3.1 Instrumental orchestration

In this study, I describe teacher knowledge which relating to using DGS as a property of the system constructed by teacher and didactic situation with DGS, and I argue that teacher's interaction with the didactic situation reflects the knowledge he or she needs to teach mathematics with DGS. In order to analyze this interaction, I chose to use the instrumental orchestration proposed by Trouche. First, because this framework was created for the study of technology-rich instructional environments, which is also the

case in this study. Second, it encompasses the entire practice of the teacher from the beginning of lesson planning to real practice in the classroom.

3.1.1 Introduction

Instrumental orchestration, which comes from instrumental approach, can be used to analyze teachers' pedagogical practices. Before describing it and showing how it will be used in this study, I need to distinguish between two important concepts before introducing this approach: artifact and instrument. Many researchers have focused on these concepts since the end of the last century. Verillon and Rabardel (1995) compared them in their studies. For them artifact means a given object, for example, computer, calculator, blackboard, chalk and so on even language can also be seen as an artefact. And when someone is told how to use these artefacts, he becomes to construct a *scheme* of these artefacts. *Scheme* is defined by Vergnaud (1998) as invariant organization of user's activities with an artefact in a certain class of situations in order to solve a certain task. This *scheme* is affected by many factors such as experience, belief, interaction with others, objective and so on (Tabach, 2011). According to Verillon and Rabardel (1995), artefact and scheme construct the concept of instrument. An instrument is a psychological structure. It does not exist in itself and when the subject is able to claim an artefact for himself and combine it with his own activities, it becomes an instrument. An instrument is related to a task and an activity. It is something that can be used to solve a task. Recent researches try to take teacher as a learner and find that in order to appropriate a tool, how learners develop their own knowledge of how to use it, which turns the tool into an instrument that mediates an activity between learners and a task (Alqahtani & Powell, 2017, 2015). Here is a simple example, a computer, if it is not used, it is only one of the artefacts. But if one teacher uses it to design his teaching process, the computer becomes an instrument. So how to use an instrument is different among people, although they may use the same instrument. This shows that instrument is not just a tool or an artifact, it is a mixed entity and made up of an artifact component and a scheme component (Alqahtani & Powell, 2017).

The construction of this process, named instrumental genesis, is a complex process, many factors like time, and linked to the artifact characteristics (its potentialities and its constraints) and to the subject's activity, his/her knowledge and former method of working (Trouche, 2004) can affect this process. At the same time, an instrument which at the same time facilitates action if the user holds the required competence, and on the other hand limits this action because of its own constraints (Rabardel, 1995b). This shows instrumental genesis works in two directions. Firstly, it is directed towards the artefact, loading it progressively with potentialities, and eventually transforming it for specific uses; this is called the instrumentalization of the artefact. Secondly, instrumental genesis is directed towards the subject, leading to the development or appropriation of schemes of instrumented action which progressively take shape as techniques that permit an effective response to given tasks. The latter direction is properly called instrumentation like the following figure shows (Artigue, 2002; Trouche, 2004).

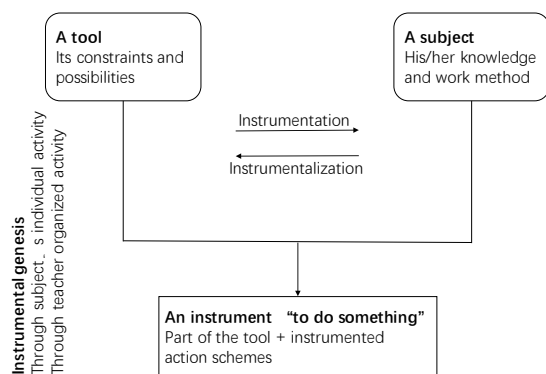


Figure 3.1: Instrumental genesis by Trouche

It describes the complexity of using technologies in solving tasks in different situations (Artigue, 2002). This approach can investigate how learners use technologies under the influence of their mathematical knowledge and knowledge relating to the technology. It also can be used to analyze the teaching takes into account the construction of the instrument and its relationships with the learning of mathematics (Lagrange et al., 2001). Teacher has to take care of these processes among their students.

3.1.2 Instrumental orchestration

From the literature, researchers always focus on the instrumental genesis of students and how it affects their learning activities. And students' instrumental genesis is guided by teacher and it is affected by teaching plan. Luc Trouche (2004) describes the necessity of guidance for the instrumental success of students. Nowadays, mathematics teachers are encouraged to use technology in the classroom. In order to help them integrate technology into the mathematics teaching and learning process, we need to gain a better understanding of their use of technology to manage mathematics lessons. To account for the social aspect, Trouche (2004) introduces instrumental orchestration to describe this process. For Trouche, instrumental orchestration tries to answer questions about what technological artifacts mathematics educators should introduce to learners and what guidance they should provide so learners can appropriate and use artifacts as instruments to mediate their activity. Drijvers and his colleagues (2010) describe instrumental orchestration as “the intentional and systematic organization of the various artifacts available in a computerized learning environment by the teacher for a given mathematical situation, in order to guide students' instrumental geneses” (Drijvers et al., 2010, p.214). They distinguish instrumental orchestration into three different elements: a didactic configuration, an exploitation mode and a didactical performance (Drijvers et al., 2010).

A didactic configuration refers to an arrangement of artefacts in the environment, in other words, a configuration of the teaching setting and the artefacts involved in it. Like in the musical metaphor of orchestration, the didactic configuration can be seen as choosing what musical instruments need to be included, and how to arrange them in space so that the different sounds result in polyphonic music. In the mathematics classroom, it might come down to a sound and converging mathematical discourse.

An exploitation mode means the way teachers decide to exploit a didactic configuration for the benefit of his or her didactical intentions. This includes

decisions on the way a task is introduced and worked through, on the possible roles to be played by the artefacts, and on the schemes and techniques to be developed and established by the students.

A didactical performance involves the ad hoc decisions taken while teaching on how to actually perform in the chosen didactic configuration and exploitation mode: what question to pose now, how to do justice to (or to set aside) any particular student input, how to deal with an unexpected aspect of the mathematical task or the technological tool, or other emerging goals.

The following table shows the relations between research questions and instrument orchestration:

Table 3.1: Elements of instrumental orchestration

Instrumental orchestration		Focus in this research
Didactic configuration	An arrangement of artefacts in the environment, or in other words, a configuration of the teaching setting and the artefacts involved in it.	The process of choosing and using different kinds mathematics tasks or software and how to configure them in the classroom.
Exploitation mode	The way teacher decides to exploit a didactic configuration for the benefit of his or her didactical intentions.	How teacher organizes teaching process with the tasks and software chose in the classroom.
Didactical performance	The ad hoc decisions taken while teaching on how to actually perform in the chosen didactic configuration and exploitation mode.	The interaction between teacher and students: questions teacher poses, feedbacks teacher gives and purpose of each question.

3.1.3 Why choosing instrumental orchestration?

As stated in Chapter 2, in this thesis, teacher knowledge is manifested in the interaction between teacher and didactic situation. This study only focuses on some of the elements contained in the didactic situation: mathematics task and DGS. Then teacher knowledge is shown in the interaction between the teacher and didactic

situation created by the mathematical task and the DGS. According to the definition of instrumental approach, artefacts can be physical or symbolic objects, such as algorithms, while instrument is the method that contain the subject's thinking in order to solve problems (Hollebrands & Okumuş, 2017). So in this study, we can see DGS and mathematics task as two important artefacts, and then the instrument contains the ways in which teachers use both of them to solve real problems in the teaching and learning process. This is the first reason why we use the instrumental orchestration in this study to describe the ways in which teachers use the mathematical task and the DGS.

Second, as shown in Figure 3.1, the way a subject uses a tool is influenced by his/her knowledge. That is, instrumental orchestration can be used to describe the actions that a subject takes with a tool in order to solve a task, and in these actions we can analyze his/her knowledge. In the next section, I will describe in depth the connection between teacher knowledge and instrumental orchestration and show some existing research that relates to these two terms in mathematics education.

3.2 Types of instrumental orchestration

As technology evolves, teachers spend different amounts of time on different classroom organizations. For example, they may spend less time on teaching ideas, less time on explaining mathematical contents, and more time on explaining technical problems, and they may also make more assertions and instructions/initiating remarks in technology lessons (Monaghan, 2004). Drijvers has designed various studies to analyze how teachers use technology to organize teaching activities. He and his colleagues argue that, based on empirical data, the following types of different classroom orchestration can be used to describe the use of technology in mathematics teaching: Technical-demo, Explain-the-screen, Link-screen-board, Discuss-the-screen, Spot-and-show, Sherpa-at-work (Drijvers et al., 2010), Guide-and-explain and Board-instruction (Drijvers et al., 2013). In all cases open mathematical tools such as DGS were used in a computerized environment. For

almost all orchestration types, the didactic configuration involves a whole-class setting in which the students sit facing one central screen. The following part shows the main features of these different types of instrumental orchestration mentioned above. Then, in this study, I will use this criterion to analyze Chinese and French mathematics lessons to see if the teachers integrate dynamic geometry software in these ways. Each type is described by the first two elements: didactic configuration and exploitation mode. The ad hoc nature of the third elements: didactical performance made it unsuitable for introduction here in a global way (Drijvers et al., 2010). I will then show some examples of each type displayed in the classrooms to illustrate how didactical performance can occur in a mathematics lesson.

Technical-demo

- Description: It concerns the demonstration of tool.
- Didactic configuration: Digital mathematics environment, including computer, internet, screen, software is necessary and also one projector to show tasks on the screen is also critical. The classroom arrangement needs allow all the students can follow the teachers' demonstration
- Exploitation modes: One of the possible way to exploit is that teachers can introduce one technology in a situation or with a task, also students' work can also be the departure of the teaching process
- Characteristics: The teaching process controlled by teacher; Students watch and apply what teacher said; The main issue is about the problems of technology; Topic of this process comes from students or difficulties identified by teacher.

Explain-the-screen

- Didactic configuration: Teachers explain what happened on the screen to the whole class by operating the software.
- Didactic configuration: It is similar to the Technical- demo ones.
- Exploitation modes: Teachers can take students' work or their own solution of the task as the topic of the teaching process.
- Characteristics: The teaching process controlled by teacher or student; Other

students just listen to the teacher; The issue is not just about technology in relation with the problems of mathematics; The topics come from teacher or students' work

Guide-and-explain

- Description: Students begin to explain the mathematics contents shown on the screen guided by the questions posed by teachers.
- Didactic configuration: Similar to explain-the-screen
- Exploitation modes: Teachers can take students' work or their own solution of the task as the topic of the teaching process and pose questions while teaching.
- Characteristics: The teaching process controlled by teacher; Students answer questions; The issue is not just about technology in relation with the problems of mathematics; The topics come from teacher or students' work

Link-screen-board

- Description: This type begins to concern the relationship between two main resources in classroom: screen and board. Teachers begin to think how to represent what happens in the technological environment only with paper, book and blackboard.
- Didactic configuration: The configuration is also similar to the above two types, digital mathematics environment with a projector is critical. In this type, some traditional resource like blackboard is needed also. And we need to let both blackboard and screen are visible to the students.
- Exploitation modes: Similar to "explain-the-screen", students' work or teachers' own solution to a task can be the departure of the teaching process
- Characteristics: The teaching process controlled by teachers; Students just listen to the teacher; use more than one resources in class such as technology, board, chalk and so on, issues related to the two types of representations; The topic comes from students' work or a task

Discuss-the-screen

- Description: Students participate into the discussion. Teachers may just help

or talk with the students if they need help.

- Didactic configuration: Digital mathematics environment and projector which can show tasks or students' work are necessary. And the classroom arrangement need to make students discuss easily
- Exploitation modes: Students' work, a task, or a problem or approach set by the teacher can be the topic of discussion
- Characteristics: Students began to participate in the process; teachers are not the center of the class; Teacher or other students give feedbacks; The topic is about mathematics; The issue comes from students' work or a task

Spot-and-show

- Description: one or more students explain their work to the whole class and others or teacher give some feedback or ask some questions.
- Didactic configuration: Including digital mathematics environment
- Exploitation modes: Teachers choose some students whose work are shown on the screen to explain their solutions, other students or themselves give some feedbacks.
- Characteristics: Students themselves make the explanations not the teacher; Teacher and other students give feedback; The topic is about mathematics; Students' work was shown on the screen as a topic of the following class

Sherpa-at-work

- Description: Teachers can allow students to share his/her construction steps to the whole class. This student is called a Sherpa student (Trouche, 2004). He/she uses the technology to present his or her work, or do what teachers or others tell him or her to do with software.
- Didactic configuration: The facilities are similar to "Discuss-the-screen". The classroom need to allow the so-called Sherpa-student control the software and other students can follow his operation.
- Exploitation modes: Teachers can let Sherpa-student explain his own work or ask him to carry out the activities of other students.
- Characteristics: One student (Sherpa student) control the software; Teacher

or other students give instructions; Topic is about mathematics or technology;
Present the work or do what teacher asked to do.

Board-instruction

- Description: It is a traditional teaching orchestration in classroom. In this type, technology is not critical. Teachers use traditional resources like board to explain what they need their students learn during this lesson.
- Didactic configuration: Didactic configuration is more similar to the types above, but technology is not essential in this type.
- Exploitation modes: Teachers may stand in front of the board and teach.
- Characteristics: Teacher controls the teaching process; Students listen to the teacher and answer questions; Teaching content is about mathematics; The content may come from students' work or teacher's preparation

Not-use-tech (Tabach, 2011)

- Description: No technology used in the teaching process.
- Didactic configuration: Whole-class setting, one central screen.
- Exploitation modes: The technology is available but the teacher chooses not to use it.
- Characteristics: No

In my research, this instrumental orchestration framework here is not a closed framework, and as teachers progress in their careers, we need to amplify this framework and add more perspectives to demonstrate their knowledge in different teaching situations.

3.3 Link between knowledge and instrumental orchestration

As talked above, when we use a tool in an activity to solve the problem we face, we develop scheme by ourselves. "The scheme of utilization integrates practice (how using selected resources for teaching a given subject) and knowledge (on mathematics, on mathematics teaching, on students, on technology)." (Gueudet & Trouche, 2011 p. 402) Which means in order to appropriate a tool, the users (here in this research mathematics

teachers) develop their own knowledge of how to use it. This process turns the tools we use into an instrument that mediates an activity between users and a task (Alqahtani & Powell, 2017). In order to understand a scheme's function and dynamics, the following components are essential for us: the objects and the anticipations, the rules of action, gathering of information, control-taking and the operative invariants. In which "the operative invariants are the implicit knowledge contained in the schemes: theorems-in-action, that is, propositions believed to be true." (Trouche, 2004 p. 286)

Some researchers have attempted to find the relationship between teachers' practices and their knowledge. For example, Assude (2007) tried to find different knowledge from teachers' classroom orchestration by using DGS, which she named instrumental knowledge (IK) and mathematical knowledge (MK), and she also focused on the relationship between these two kinds of knowledge.

In this study, I used instrumental orchestration to analyze teachers' practice with DGS. Therefore, it is necessary to consider the links between instrumental orchestration and teacher knowledge. When using different instruments in mathematics lessons, teachers develop their knowledge by constructing the ways of using them. In the use of instruments there are two parts: the observable part and the unobservable part of the patterns of use. The observable part of the patterns of use are the regularities in the activities of teachers in various contexts, reporting the same class of situation (Gueudet & Trouche, 2009b). Here I pay attention to the invisible part which is essentially the knowledge of the teacher (Sabra, 2011).

Other researchers have shown that there are two kinds of utilization schemes when interacting with different artefacts. They each constitute different kinds of knowledge. The first type of utilization scheme is the usage scheme, which is directly related to the artefact. It constitutes basic knowledge of how to operate or use the artefact. For example, knowledge of how to operate a computer includes basic knowledge of how to start software. The second type of utilization scheme is an instrumental-mediated action scheme, which has more to do with the transformations that can be made to the objects. This type of scheme is concerned with activities that can be individual or collective, and it will guide the user, using the artefact, to reach the desired goal. It's like

knowledge about the situation. In the case of driving a car, for example, an instrumental-mediated action scheme will be more concerned with other variables on the road, which the driver needs to be aware of and react to in order to be able to reach the final destination (Alqahtani & Powell, 2017; Lonchamp, 2012)

For DGS, the usage scheme includes knowledge relating to the use of the software and its functions. The instrumental-mediated action scheme included knowledge of geometry and dependencies (Alqahtani & Powell, 2017, 2015). When teachers use DGS as an instrument in mathematics class, they are able to use it to manage the class and show students geometry concepts or solve geometry problems. This appropriation process will allow teachers to know how to use the DGS in mathematics instruction. Within DGS, Straesser (2001) sees that geometry is “lived in differently, broader scope, has a new, more flexible structure, [and] offers easy access to certain heuristic strategies.” (p. 331).

Knowledge about pedagogical strategies would be shown in the types of orchestration of classroom. For example, a teacher who prefer to use student-center strategies may be exploited into Discuss- the-screen, Spot-and-show or Sherpa-at-work types. And a teacher used more teacher-centered strategies may be exploited into Technical-demo or Explain-the-screen type (Drijvers et al., 2013). Teachers may need knowledge to solve the problems during the lessons like:

- Address the common misconceptions students have.
- How to select effective teaching approaches to guide student thinking and learning.
- Help students to understand the content knowledge through various ways.
- Address the common learning difficulties my students have.
-

In addition, we need to consider the relationship between different types of instrumental orchestration and different types of teacher knowledge. Drijvers et al (2013) attempt to study teacher knowledge, which is evident in their process of different instrumental orchestration. Based on TPACK model, they examined the relationship between teacher knowledge and different types of instrumental

orchestration. For example, the teacher's technical knowledge (TK) plays a crucial role in technical-demo. It is evident that teachers need more technological knowledge to address students' difficulties when using mathematics software like DGS to learn mathematics. And also, teacher needs more knowledge about how to teach technology (TPK). In the guided-and-explain type, teachers need more knowledge about how to organize instructional processes according to different mathematical contents (PCK) and how to use technology to teach these contents (TPACK). After observing several teachers for over one year, they found that, based on the TPACK model, PCK was mentioned more often by teachers in order to integrate technology into the math classroom, followed by TPACK, TK, and TPK. As time follows, TK became less important. This may be because the students are not familiar with the new technologies at first. So they encounter more difficulties in learning mathematics by using these technological tools. As a result, teachers need to spend more time addressing the technological problems. This means that teachers need more TK when they start using technology for math instruction. After a few lessons, students have mastered technologies and teachers can focus more on thinking about how to use these tools to make students learn mathematics more effectively. In relation to instrumental orchestration, TK is more needed in "technical-demo" or "tech-support" orchestration. And "guide-and-explain" orchestration needs more PCK or TPACK. And in their study, even when teachers integrate technology into their mathematics lessons, they still use more pedagogical content knowledge (PCK). That means the most important thing for teachers is how to teach more effectively, and technology remains a support for instruction rather than a tool for student learning.

In my study, teacher knowledge encompasses the knower (teacher), the known (pedagogy for teaching with DGS), and the knowing (interaction between teacher and pedagogy). Since teacher knowledge is manifested in their pedagogical practices, in this study, the main focus is on the process of interaction between teacher and pedagogy. I will use instrumental orchestration to analyze the actual practice of teachers when using DGS in mathematics teaching process. Based on this theory, we can divide teacher's practice into three part: didactic configuration which shows how

teacher use different tasks with DGS, exploitation mode which shows how teacher organizes the tasks and how they use DGS when teaching and didactical performance which shows the ad hoc decision with DGS in the existing didactic configuration and these decisions means the questions and feedbacks teacher makes during the practice. These three parts reflect teacher knowledge relating to the role of DGS (didactic configuration), teacher knowledge relating to class orchestration with DGS (exploitation mode) and teacher knowledge relating to interaction with students (didactical performance). The following sections present how to analyze these practices based on different points of view. Section 3.4 describes the role of DGS (didactic configuration). Section 3.5 describes the different types of instrumental orchestration (exploitation mode). And section 3.6 describes the interaction with students (didactical performance).

3.4 Role of DGS in mathematics task

Mathematical problems and tasks are key elements of the didactical situation. In all fields of education, tasks always play a crucial role for teachers and students. They can affect students' learning of mathematics (Stein et al., 1996). They can also help teachers understand students' learning outcomes and reflect on their own teaching process to enable students to succeed in school. Tasks also influence or determine the degree of learning opportunities available to students. The design and choice of different tasks by teachers often demonstrate their different concerns about teaching and learning (Haggarty & Pepin, 2002; Törnroos, 2005). There is evidence in the literature that teachers' mathematical knowledge and ability to design mathematical practices are developed when they interact with teaching resources (e.g. mathematical tasks), adapting and appropriating them (Pepin, 2015).

New technologies, like computers or software, emerge through the use of new tools and interact with the usual ones. In new technological environment teachers need to reconsider their organizations, for example, sets of tasks and techniques,

managed in traditional classroom like paper-pencil environment. Mathematics task is one of the key elements of teaching which affects students' learning of mathematics (Stein et al., 1996). Teachers design and select different tasks, which often reveal their different pedagogical concerns (Haggarty & Pepin, 2002; Törnroos, 2005). Thus, an analysis of the tasks can help to understand their considerations in the new technological situation. In addition, with the use of DGS, it leads to more changes in the elements of the task and add more dynamic transformations of the tasks (Leikin & Grossman, 2013). However, the use of tools in instructional tasks is still not widespread (Trouche, 2004).

Because we focus on the interaction between teacher and DGS, it is helpful to think about the different roles DGS plays in mathematics teaching and learning. Pea (1987) proposed the distinction between two roles of technologies in teaching and learning: amplifier and reorganizer. Generally, technologies are mainly seen as *cultural amplifiers* of the intellect, like the tools can improve cognitive capacities and they can help to solve the tasks more quickly and more efficiently. Only letting technologies be an amplifier is not enough for teaching and learning mathematics. They can also change the organization of human relationship at work. This means human needs to change his/her actions required when solving the tasks with the environment created by DGS (Özen & Köse, 2013). For example, it can help students manipulate geometry objects by measuring and dragging tools to establish relationships, observe features of the objects, make conjectures, investigate theories and their relationships, and justify them. (Healy & Hoyles, 2001; Marrades & Gutiérrez, 2000). And DGS also provides teachers with environments that help their students to experiment more easily than with other technological environments or traditional paper-pencil environment. Students can observe whether the mathematical relations change or not, construct and verify their own hypothesis (Marrades & Gutiérrez, 2000; Straesser, 2001). As mentioned by Christou et al: "...DGS as a mediation tool, encourage students to use in problem solving and posing the processes of modeling, conjecturing, experimenting and generalizing..." (Christou et al., 2005 p.141).

In order to make relevant use of DGS in mathematics teaching and learning, researchers think that it is necessary for teachers to design appropriate tasks, such as designing non-routine mathematical tasks, so that students have more opportunities to explore mathematics and make connections between their work with DGS and the mathematics they are learning (Hollebrands & Okumuş, 2017; Laborde, 2001). Teachers also have an important role in determining whether the tasks which students encounter require higher or lower level of cognitive needs. (Cayton et al., 2017).

When a continuous integration of DGS in mathematics teaching and learning occurs, the use of DGS evolved over time from being a visual amplifier to a fundamental component that enhances conceptual understanding (Koyuncu et al., 2015). Laborde proposed four different roles DGS played in mathematics tasks.

1. DGS is used mainly as *facilitating material aspects* of the task while not changing it conceptually. Like constructing a triangle in software, the midpoints of its sides, and then its medians to display the diagram on a board or print it on a paper.
2. DGS is supposed to *facilitate the mathematical task* that is considered as unchanged: this is the case when DGS is used as a visual amplifier in the task of identifying properties. It is assumed that it is easier to observe that three lines always intersect in one point during the deformation of the diagram by the drag mode than in a static paper-and-pencil diagram.

These two first roles can be seen as an amplifier in mathematics tasks (Soury-Lavergne, 2017).

DGS has also offered new tasks, whose resolution engages geometric knowledge in a way that would not be possible without it (Laborde, 2001).

3. DGS can also be used to *modify the solving strategies* of the task due to the availability of some of its tools and the possibility to hide others. It change the task, which might be more or less difficult: like creating a parallelogram without using parallel line tools in DGS.
4. The task itself takes its meaning or its 'raison d'être' from Cabri. Like "Boite-noire-sourire". Laborde (2001) illustrated: "In the black box situations, the

students are given a diagram on the screen of the computer and they are asked questions about it. This kind of situation was used in our scenarios for introducing new transformations. A point P and its image P' through the unknown transformation were given to the students. They could move P and observe the subsequent effect on P' . Students were asked to find the properties of the unknown transformation by means of this black box. In such a task, students must ask themselves questions about the transformation.” (p. 302)

These two can show that DGS is the generator of new tasks in mathematics (Soury-Lavergne, 2017).

3.5 Interaction between teacher and students

The interaction between teacher and students during the lessons constitutes another key aspect of the teacher’s practice. It is one of the elements shown in teaching performance and one of the three elements in instrumental orchestration. It also provides a means of expressing some elements of teacher knowledge, such as knowledge of students or knowledge of how to present mathematical contents. Wertsch (1998) describes teacher-student interaction in a formal teaching situation as the following process: the teacher asks a series of questions, then students answered them, followed by an evaluation or a feedback from the teacher. The use of technology may affect the interaction between teacher and students. It may cause that students pose more specific, technical questions. And teachers may not prepare these situations, and the lessons may deviate from what they prepared before. And technology may also provide opportunities to build and support students’ mathematical thinking (Cayton et al., 2017). Some researchers have noted that in non-technology classes, when students work in pairs, the teacher may only talk to one of the pairs. But in a technology class, the teacher may talk to all the students around a single computer (Monaghan, 2004).

When we analyze the process of teaching and learning mathematics in the classroom, the three elements: teacher, student, and mathematics, form the basic

relationship in mathematics classroom. We can see the interaction and relationship among these three elements (Hollebrands & Lee, 2016). Now the emerging of technology in didactic situation requires to consider the role technology played in the existing mathematics teaching process. Tall (1986) gave a new way to consider the interactions occurred among teacher, student, mathematics and technology. He extended the didactic triangle into tetrahedron (Figure 3.2). Each vertex represents teacher, student, mathematics and technology. The diagram, helps distinguish between the different foci of the teaching process with technology. For example, a teacher may focus on technology and provide few opportunities for students to learn mathematics. On the other hand, they may focus primarily on mathematics and not provide any instruction in technology (Assude, 2007; Erfjord, 2011).

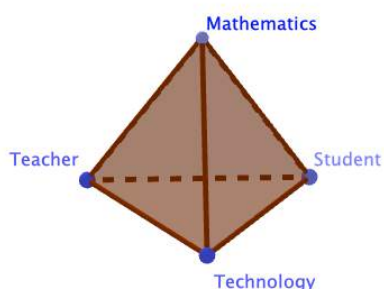


Figure 3.2: Teaching tetrahedron (Hollebrands & Lee, 2016)

Opening the tetrahedron, we can find different focuses of the interaction between teacher and students. There are four basic focuses of the interactions: focus on technology, focus on technology to notice mathematics, focus on mathematics with the use of technology and focus on mathematics (Table 3.2) (Hollebrands & Lee, 2016).

Table 3.2: Initial focus of questions and feedbacks

Focus	Description	Coding diagram
Focus on technology	Teacher's question or feedback is only about the technology	

Focus on technology to notice mathematics	Teacher's question or feedback is related to a technological action which is related to a mathematical idea	
Focus on mathematics with use of technology	Teacher's question or feedback is about mathematics and he/she has an assumption that students need to use technology to make answers	
Focus on mathematics	Teacher's question or feedback is only about mathematics. And students can answer them without the help of technology	

Obviously, focus on technology means that teacher only raises technical issues or statements that are not related to mathematics. For example, the teacher presents the advantages or limitations of technology. Focus on mathematics, obviously, means that teacher poses questions or statements concerning only pure mathematics knowledge, such as making mathematical conclusions. Focus on technology to notice mathematics means teacher begins to consider how to use technologies to construct mathematical objects like using DGS to create segment or circle. Focus on mathematics with the use of technology is another kind of interaction which considers both technology and mathematics. But these kinds of questions or statements focus on how to solve mathematics problems with technology. In other words, these questions exist only in a technological environment, which means that without the help of technology, students will have difficulties in finding solutions.

Dynamic geometry software, due to the changes in the teaching situation, can show different trends in the oral interactions between teacher and students. In order

to analyze the whole teaching process and teachers' decisions, we need to compare the interactions in different situation. As mentioned above, in this study, teachers, students, mathematics and DGS constructed the instructional situation.

From figure 3.2, we can find four different routes to demonstrate how teacher interacts with students in mathematics lessons with technologies. First, these are mathematics lessons and the most important aim is to let students master mathematical contents. So the following route: teacher-mathematics-student is obvious. Second, because in this study, teachers use DGS as an important resource in their lessons. Teachers also need to make students aware of this type of software and what they can do during the lessons. So the second route could be: teacher-technology-students. Third, as a result of involving technology, teachers begin to think about how to integrate technology into the lesson in order to help students learn mathematics easily. However, depending on their teaching goals, teachers can ask different questions for their students in the classroom: when they want their students to know how to use technology, they can use some mathematical contents as examples, and when they want their students to know how to solve mathematical tasks with the help of technology, they can manipulate software to draw their students' attention to the characteristics of mathematics. So here we have the third route, teacher-technology-mathematics-students, and the fourth route: teacher-mathematics-technology-students. We can use table 3.2 to summarize these different focuses of questions in the lessons.

These different focuses would cause different kinds of feedback or questions given by teachers: feedback linked to the modeled situation embodying mathematics; feedback depending on the solution of the student, in particular feedback showing to the student the effect of his/her answer; feedback in terms of correct/incorrect (Laborde & Laborde, 2011).

National Council of Teachers in Mathematics (NCTM) emphasized that it is important for teachers to choose worthwhile mathematics tasks and pose quality questions to make students learn mathematics with the help of technology (Hollebrands & Lee, 2016). With geometry software, the questions can be different

from paper-pencil contexts. They classify the questions into the following types (Hollebrands & Lee, 2016):

- Focus on technology, “Try to drag click circle button and draw a circle on your screen”;
- Focus on technology to notice mathematics, “if you select the center of the circle you had drawn and move it, what did you see”;
- Focus on mathematics with the use of technology, “can you make a circle that circumscribes the triangle given on your screen”;
- Focus on mathematics, “try to conclude the ways to prove the congruent triangles”.

Each of these questions has different purpose like: managing, such questions as “Open the software and see the requirement of the tasks”; checking, means if students understand the mathematics knowledge (What shape is ABCD? Explain); focusing attention, means making students pay attention to the software and screen (Drag each of the red points A, B, and C and observe the diagram); giving clues, such as: Use your conclusions and the appropriate congruency theorem to prove that AMD is congruent to CNB (Sinclair, 2003).

In addition to the questions, in this research, I also focused on the teacher’s feedback to the students who answered the questions. Such as feedback just like “yes” or “no”, or further questions, or correct answers given by the teachers themselves, or explanations of the answers, etc.

Now, I want to reconsider the research questions and describe the relation between these questions and instrumental orchestration. The main research question: *What can we learn about the teachers’ knowledge shown in their instrumental orchestration by contrasting the different usage of DGS between Chinese and French teachers?* can be divided into three sub ones which relates to the three elements of instrumental orchestration. The first one wants to analyze the role of DGS in mathematics tasks which can be shown both in didactic configuration and exploitation mode. The second analyze the organization of classroom which related to exploitation mode and the third one is about the interaction between teacher and

students which related to didactical performance.

4 Research process

This chapter describes the whole research process and how the methodology was designed to determine what type of data was needed, how it was collected and how it was analyzed. This study explored the ways in which teachers organized their classroom activities. Researchers designed different methods and collecting various of data to analyze teacher's use of technologies (Lagrange et al., 2001). The data includes teachers' lesson plans, classroom interactions, use of written support materials, lesson videos and interview audio with teachers.

From the literature review, many researches put attention to different types of teacher knowledge which comes from Shulman (e.g. Angeli & Valanides, 2005; Cochran, DeRuiter, & King, 1993; Guerrero, 2010; Niess, 2005) Most of these researches used questionnaire or interview (e.g. Koehler, Mishra, & Yahya, 2007; Nilsson & Vikström, 2015; Norton, 2012) to analyze what kinds of the knowledge teacher needs or has during his/her daily practice or how to help teachers develop their knowledge. In China, some researchers also use questionnaire based on foreign researches to analyze Chinese teacher knowledge with technologies (e.g. Zhan, 2011; Zhang, 2011)²⁶

Few studies which have focused on the observable elements of how teachers use their knowledge in the classroom. Since this study focuses on teachers' interaction with DGS, as described above, the questionnaire alone could not provide insight into teachers' interaction processes. Therefore, based on the research questions and the three components of teachers' use of DGS practices I have selected (choosing task, classroom orchestration, and teacher-student interaction), for each question I designed different methods to collect the required data.

²⁶ 詹艺(2011). 培养师范生“整合技术的学科教学知识”(TPACK)的研究.华东师范大学, 上海. 张育桂(2011). 小学数学教师整合技术的学科教学知识(TPACK)研究.信阳师范学院, 河南: 信阳.

4.1 Data collection

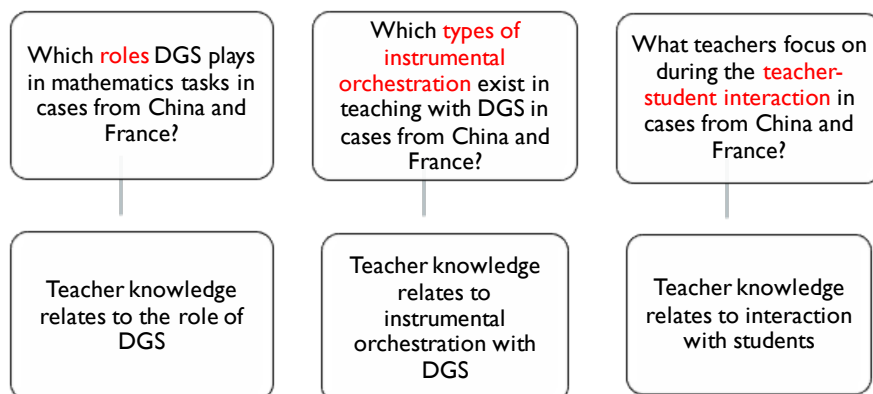


Figure 4.1: Relations between research questions and teacher knowledge

Based on the relations between research questions and teacher knowledge (Figure 4.1), I designed the following research methods to collect data needed to analyze teacher knowledge revealed in their teaching behaviors with DGS.

For first research question, choosing tasks is a key part of teacher' lesson preparation and in order to analyze what kinds of tasks teacher used with DGS, I need to collect data like task list and other supplementary documents like diagrams with DGS, so I first made a preparation meeting with all the teachers to collect lesson plans; mathematics tasks list and diagrams designed by teachers with DGS. All of the documents were designed by the teachers themselves with DGS. These tasks would be analyzed to see what role DGS plays in these tasks and the sequence in which they are presented. Classroom observations also help me to analyze how teachers use these tasks during the teaching and how they deal with teaching incidents for which they were not prepared beforehand. Analysis of these data will describe teacher knowledge which relates to different types of mathematical tasks with DGS.

For classroom orchestration (second question), in this research, I suppose that teacher knowledge is a property of teacher◊milieu system. Classroom orchestration is one of the elements of teacher◊milieu system. And their classroom behavior can be seen as an observable part of classroom orchestration. So I suppose observing teacher's behavior in the classroom with DGS can help us to characterize their

orchestration with DGS and from which we can analyze teacher knowledge. And because of the complex of teaching practice in classroom, during the observation there may be some ambiguities. To deal with these ambiguities I designed semi-structured interviews to discuss with teachers in order to remove them. By analyzing these data, we can describe teacher knowledge relating to classroom orchestration with DGS.

For the third research question about teacher-student interaction, which, in my research, is another observable part of teacher $\langle \rangle$ milieu system and from which we can see some characteristics of teacher knowledge. I use class observation to record all the questions, feedbacks and answers during the communication in the lessons with DGS. All these data can help me know what teachers focus during their lessons. I also used semi-structured interviews after the lesson to find out why teachers choose to ask students these questions and to give this feedback during the lessons. This process can help us describe teacher knowledge relating to teacher-student interaction.

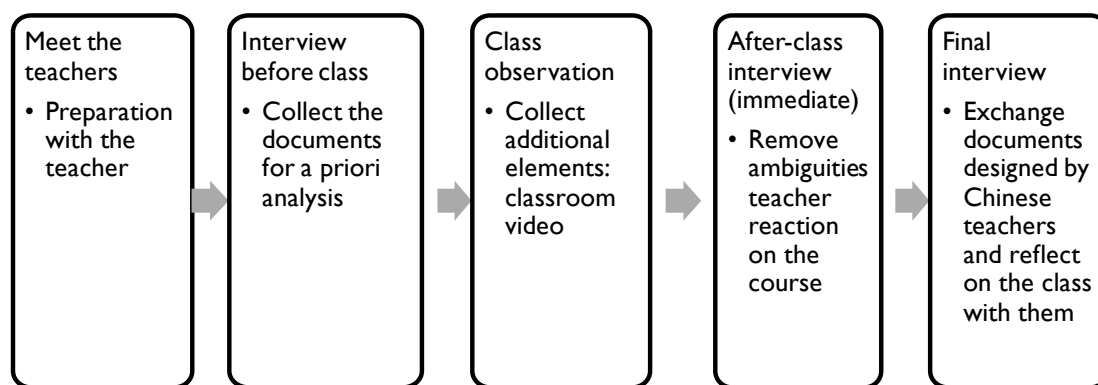


Figure 4.2: Research process

Figure 4.2 presents the whole process for collecting all the data in this research. I first met all the teachers to make preparation with them, for example, ask them when I can make class observation. Then I made an interview before the lessons to collect documents like lesson plans to make a priori analysis. After I made class observation to collect additional elements like classroom videos. Then after each class, I made an immediate interview to remove ambiguities teacher reacts on the lessons. Then I made a final interview with all the teacher to exchange documents designed by

Chinese and French teachers and make reflection of the lessons with them.

In the following sections, I will describe all of the above research process in depth. I begin with describing the interview process with the teachers. All teachers in this study participated in a total of two interviews, one immediately after observing their lessons and the second after collecting all the data from all teachers. Section 4.3 describes the classroom observations. It is about how to record the whole teaching process and the main focus of each video record. In Section 4.4, I present some main information about all the teachers who participated in this study, including five secondary school mathematics teachers in Shanghai, China (Mrs. J, Mr. ZH, Mrs. Y, Mr. W & Mr. X) and 1 French secondary school mathematics teacher (Mrs. Louisa). Then, in Section 4.5, I describe the whole process and methodology of data analysis in order to answer the questions of this study.

4.2 Semi-structured Interview

Merriam (2009) designed a case study to explore mathematics teachers' integration of DGS. According to Merriam "a qualitative case study is an intensive, holistic description and analysis of a bounded phenomenon such as a program, an institution, a person, a process, or a social unit" (p. x). This section describes the first research method I used in this study: semi-structured interviews. I first made a preparation with all the teachers before the classes (section 4.2.1) and then made two semi-structured interview (section 4.2.1 and 4.2.3) in order to gain insight into their views on DGS and made audio recordings.

Both the Chinese and French teachers would answer the same questions and talk about their views on the utility of DGS. And during the interviews, I would ask them to describe how they want use the software to support their lessons and to show their process of teaching practices inside and outside the school. All interviews would be recorded for further analysis to reflect mathematics teachers' perceptions of DGS and to identify differences between Chinese and French teachers. From these data, I can see the process of lesson preparation and the support teachers receive from other

colleagues or experts. At the same time, I would spend some time working with these teachers to find out more details about their use of mathematics software. I may also find out if there are differences in the development of their teaching. During the research process these teachers would also talk about some of their teaching experiences to discover how they develop their knowledge during the career. The following sections described these interviews in detail.

4.2.1 Preparation meeting and interview with teachers before lesson

As mentioned above, I have contacted each teacher and made interviews with him or her before the classes. The interviews have been structured around the following parts: the first part would take place before their lessons. In this part, I would discuss their lesson planning with these teachers, including the teaching process of the lessons they have designed, the mathematics tasks they have prepared for their students, and the teaching resources they would use in their lessons, particularly the diagram of DGS. This meeting would not last too long. During the research, I did not interrupt their daily work during the research process. So their teaching process was not different from the normal teaching process. The questions in this part of interview were:

- Which session could I attend and observe?
- In which chapter for which level (grade 6 to grade 9) do you use the DGS?
- How do you organize the sequence of the chapters for the student?
- How many sessions and for which sessions do you use the DGS?

And during the interview, I also discuss with the teachers to determine how the classroom observation can be organized, such as setting a date, authorizing audio and video recordings of a particular group of students, asking which students are most suitable for recording, and videoing the professor.

4.2.2 Interviews after lesson

The interview I made with teachers after their lessons in this research contains

two parts. All these teachers had to answer a few short questions after each lesson observed to remove ambiguities. These questions allowed teachers to reflect on teaching progress, lesson objectives (whether or not achieved), and the role of the DGS in the proposed assignment, curriculum, and sequence of tasks.

During my PhD, I spent four years in France and I had the opportunity to attend some math lessons before designing my methodology. I've observed many differences between Chinese and French mathematics lessons. For example, in China, students are quiet in the classroom. They may not have much time to present their ideas. But in France, I found a completely different phenomenon. Students can answer questions at any time without disturbing the teaching process. And French teachers use a diversity of resources in the classroom, especially DGS, although not every student operates software to learn mathematics. In addition, Chinese and French teachers have completely different cultures regarding mathematics education, which can affect their practice in the classroom. Based on these differences, it would be interesting to contrast these two countries and introduce the French example to Chinese teachers in order to make them think about teaching mathematics in a different way.

So in the second part, I would prepare for a final discussion with these teachers to analyze their knowledge relating to DGS. The main methodological element of this final discussion, is to provide teachers with new teaching situations considered to be very different from their own experiences. I will submit them DGS tasks from the other country. By confronting a teacher to a new teaching process different from his previous experiences, I've expected that I could obtain additional elements about their teaching action. Following Schön arguments, they can take account of the unintended changes by forming new appreciations and understandings and by making new moves (Schön, 1984). This also can help the inquirer reflect on the similarities and differences he has perceived or enacted (Schön, 1984). So I think having Chinese teachers analyze the French lessons from a Chinese point of view will help us to understand more about the differences between Chinese and French teaching cultures and what these two countries focus on in mathematics education.

In this final interview, the teachers were confronted with difficult problems and issues that did not fit into their known categories, but they had their own knowledge that might explain these phenomena (Schön, 1984). In such cases, the practitioner experiences a surprise that causes him-her to rethink him-her knowing-in-action in ways that go beyond existing rules, facts, theories, and operations. S-he responds to these surprises or anomalies by adapting some of his-her strategies of action, theories of phenomena, or ways of framing problems and she also invents field experiments to test her new understandings (Schön, 1987).

During the final interview, I would give them documents such as lesson plans or task lists designed by other teachers and discuss with them how they would organize or modify these tasks if they decided to use these documents in their own classrooms. For example, I would give the Chinese tasks to the French teacher and the French tasks to the Chinese teachers. Teachers can talk about their opinions based on their own knowledge and teaching culture. This process is more about allowing teachers to reflect their own teaching practices from different perspectives. Afterwards, I would show some video clips from classroom observations and discuss them with the teachers so that they can reflect on or evaluate the organization of different lessons. These teachers would see video clips produced by others. The most important purpose of this part is to allow the Chinese teachers to see and give their opinions on the French teaching clips and for the French teachers to evaluate the Chinese teaching clips. The purpose of this interview is to gather teachers' views on the role of DGS and to find out what other possibilities teachers envisage.

We have discussed questions about the resource materials, their format, how they are organized and used (Sabra, 2011). The teachers were also asked to talk about their perceptions of the constraints and affordances of existing technology, their preparation, their descriptions of lessons, their descriptions of what actually happened, their descriptions of what they did afterwards (Bueno-Ravel & Gueudet, 2007).

This final interview contained two parts. First we have discussed their opinions towards mathematics software and their lesson organization with these technologies.

I want to discuss with the teachers to reflect the process of the class based on the questions like: Do these teachers finish the objectives that are determined before the classes?

4.3 Class observation

In this section, I would describe the classroom observation process of this study. Some researchers have proposed the concept of observational pedagogical knowledge, which comes from teachers' observations and reflections based on students' mathematical activities during their teaching process (Miyakawa & Winsløw, 2013). When observing teachers' practices in integrating ICT into the lessons, the following points can be included in the observation list from which we can analyze what they know about DGS: the type of task (with DGS), the relationship between paper-pencil tasks and DGS tasks, and the relationship between different tools like DGS and paper-pencil (Bueno-Ravel & Gueudet, 2007). And also the teacher's behavior is videotaped, both in classroom teaching and in individual interaction with students (Kynigos, 2015); the organization and configuration of the class; the resources mobilized by teachers and students (especially in this research: DGS); deviations from the forecast; comments and possible remarks made by both teacher and students (Sabra, 2011). The purpose of this classroom observation is to:

- observe the implementation of the document constructed by the teacher;
- identify student and teacher reactions that would ultimately have an impact on the new uses of DGS;
- and infer teacher knowledge relating to his or her teaching activities (Sabra, 2011).

In this research, I would observe French and Chinese mathematics lessons. When analyzing this data, just from the pictures and the data from the interviews may lead to overlook some details. By observing these lessons and videotaping them, it would be possible to find out how the teachers change their teaching plans. Also, the classroom videos allowed for analysis of how these teachers used the mathematics

software and identifying the differences between Chinese and French teachers. In China, generally mathematics lessons are held in regular classrooms, so I prepared one camera to record the entire classroom instruction. Due to the features of the Chinese classroom, I could not prepare another camera to follow the teacher, which would interrupt the teaching activities in the classroom. Also, Chinese teachers seldom move around during their teaching and their activity area is usually in the front of the classroom. They come close to the students to check their work when they do some exercises in classes. So one camera would have fulfilled the requirements of this study. And because of the research objectives, the video recording focuses on the teacher rather than the students. Also to meet the ethical issues in educational research, only the teacher's face, not the student's face, is shown in the videos. To protect the privacy of all students, the entire study ensured that students only had their backs to the camera.

The situation is different in France. Some lessons with DGS may take place in the computer lab and teacher student close interactions are frequent and constitute a main part of the teaching activity. In the two French lessons in computer laboratory, the teacher was always walking around the classroom to discuss with the students and using software to check their work. In order to analyze teacher's activities with DGS in detail, I needed an additional camera to follow the teacher and record the actions of the teacher and students. One camera recorded the whole classroom and the other to follow the teacher and record details of the activities of the teacher and the students, while involving what was happening on the students' screens. In the lessons which are also held in a regular classroom, just like Chinese mathematics lessons. There is only one camera to record the main information of the class. And all the students have their backs to the camera. Based on this kind of teaching situation, I created one video for each of Chinese and French mathematics lessons in the regular classroom and two videos for the two French lessons which is held in the computer lab.

All the details of the lessons in this research is like the following table:

Table 4.1: Basic information of the lessons

Content of lesson	Teacher	Nationality	Grade	Class ²⁷	School ²⁸
Review test	Mrs. J	China	Grade 9	Class 1	School 1
Review test	Mrs. J	China	Grade 9	Class 2	School 1
Moving point	Mr. ZH	China	Grade 8	Class 3	School 1
Relation among the sides of triangle	Mr. ZH	China	Grade 7	Class 4	School 2
Congruent triangle	Mrs. Y	China	Grade 7	Class 5	School 3
Isosceles triangle	Mr. W	China	Grade 7	Class 6	School 4
Review test	Mr. X	China	Grade 8	Class 7	School 5
Circle	Madame Louisa	France	Grade 6	Class 8	School 6
Construct triangle	Madame Louisa	France	Grade 6	Class 8	School 6

During class observation, I was one of the participants in the classroom, even though I did not interact with the students or the teacher. I get to observe all the students, their work, notes, etc. And also take notes on things of interest, such as some incidents that the teacher did not prepare for, so that I can discuss them after the lesson to determine if they affected their teaching process and how they dealt with them during the lesson. Since only one camera was used for this lesson, I also needed to focus on teacher' and student' activities outside of the camera. These notes are used to support the following analysis based on the lesson video.

The following figure presents the whole research process I designed for collecting data:

²⁷ The same class number means same students who are in the lessons

²⁸ The same school number means students come from the same school (all the detail information about the schools are in section 4.3)

The following table presents the timeline in this study:

Table 4.2: Timeline of the work

Date	Place	Work	Teacher	Data
Before 04/05/2018	Shanghai	Preparation meeting	All Chinese teachers	Arrangement of the class observation
25/04/2018	Shanghai	Class observation	Mrs. J	Lesson video, tasks, software
25/04/2018	Shanghai	Class observation	Mr. ZH	Lesson video, tasks, software (Sketchpad)
25/04/2018	Shanghai	Interview immediately after lesson	Mr. ZH	Interview record
14/05/2018	Shanghai	Class observation	Mr. W	Lesson video, tasks, software (Sketchpad)
14/05/2018	Shanghai	Interview immediately after lesson	Mr. W	Interview record
15/05/2018	Shanghai	Class observation	Mrs. J	Lesson video, tasks, software
15/05/2018	Shanghai	Interview immediately after lesson	Mrs. J	Interview record
23/05/2018	Shanghai	Class observation	Mrs. Y	Lesson video, tasks, software (GeoGebra)
25/05/2018	Shanghai	Class observation	Mrs. Y	Lesson video, tasks, software (GeoGebra)
25/05/2018	Shanghai	Interview immediately after lesson	Mrs. Y	Interview record
15/11/2018	Lyon	Preparation meeting	Louisa	Arrangement of the class observation
26/11/2018 ²⁹	Shanghai	Class observation	Mr. X	Lesson video, tasks, software (GeoGebra)
26/11/2018	Shanghai	Interview immediately after lesson	Mr. X	Interview record
11/01/2019	Grenoble	Class observation	Louisa	Tasks, lesson video, software (GeoGebra)
11/01/2019	Grenoble	Interview	Louisa	Interview record

²⁹ The lesson video is recorded by my partner.

		immediately after lesson		
01/02/2019	Grenoble	Class observation	Louisa	Tasks, lesson video, software (GeoGebra)
01/02/2019	Grenoble	Final interview	Louisa	Interview record
16/03/2019	Shanghai	Final interview	Mr. W	Interview record
29/03/2019	Shanghai	Final interview	Mrs. J	Interview record
01/04/2019	Shanghai	Final interview	Mr. X	Interview record
16/05/2019	Shanghai	Final interview	Mrs. Y	Interview record
21/05/2019	Shanghai	Final interview	Mr. ZH	Interview record

4.4 Teacher information

Merriam (2009) said: “One technique for assessing the boundedness of the topic is to ask how finite the data collection would be ... If there is no end, actually or theoretically, to the number of people who could be interviewed or to observations that could be conducted, then the phenomenon is not bounded enough to qualify as a case” (p.41). In order to gain insight into teacher knowledge relating to integrating DGS in mathematics lessons, several teachers from China and France were selected for this study. According to the research recommendations, teachers with extensive teaching experience are crucial to our understanding of how teachers integrate DGS according to their teaching objectives. The young teachers who are just starting their careers are also important in order to have an overall impression of their teaching practice. By analyzing the practice of young teachers and comparing it with that of experienced teachers, we can get a sense of how teachers use DGS to develop knowledge in their teaching.

Due to the different teaching backgrounds, it was difficult to find an equal number of teachers in both countries. Before the research, I contacted many teachers, but only one French teacher allowed me to make lesson video. Since this teacher was an experienced teacher who was familiar with DGS, in order to balance two countries, I also needed to find some experienced teachers who were also good at using DGS in China, so I chose two experienced Chinese teachers. And in order to describe

teacher knowledge in as much detail as possible, I also chose some young teachers. The following section describes the basic information about the teachers included in this study. The following is basic information about these teachers. Six mathematics teachers, one from France and five from China, participated in this study.

Mrs. Luisa teaches mathematics in a public secondary school in France. She received her PhD from the University in the field of mathematics teaching and technology a year before joining this study. She is very familiar with DGS and has more than 20 years of teaching experience in French secondary schools and has served as a teacher educator for research programs. All of her students are intermediate level in France.

Five Chinese mathematics teachers participated in this study. Two of them are experienced teachers who are familiar with DGS, the other two has only 3 years of teaching experience, one of them has used DGS at the beginning of his career and the other has no experience of teaching with DGS before this study. The last teacher has been working in secondary schools for about ten years and has also used DGS extensively in his classroom.

The first Chinese teacher, Mr. ZH, belongs to a private secondary school in Shanghai and has about 20 years of teaching experience. His school encourages teachers to use new technologies such as iPad or smartphone in the classroom. The school ministry has purchased a number of software to support teachers' innovative teaching methods, such as the Geometer's Sketchpad, which is used in almost all schools in Shanghai. Before lessons, Mr. ZH often allows students to learn new contents by themselves and complete some exercises with the help of videos which Mrs. ZH has prepared before. Then, during class, he can start with the difficulties students have when completing the exercises. From Mr. ZH's practice, we can see the possibility of incorporating dynamic geometry into mathematics lessons.

Mrs. J also comes from the same private secondary school. She is one of the school leaders, so she always spends some time handling administrative duties. She has over 20 years of experience of teaching mathematics. At the beginning of her career, she worked in a public school, and about ten years later she came to work at

her current school. Again, she is very familiar with DGS and always uses technology to design her own lesson activities and then tries to share her designs with other young teachers to help them organize their lessons with different technologies. During the year she participated in this study, she taught two classes. She also had an elective course where students could learn how to construct different objects with DGS such as Geometer's Sketchpad.

Both of the above-mentioned teachers are experienced mathematics teachers in China, and in order to get more information about the teacher knowledge relating to DGS integration and to understand the differences between experienced and new teachers, two young teachers also participated in this study.

The first, Mrs. Y, who began her teaching career three years ago after earning her master degree in mathematics education, comes from a public secondary school in Shanghai that does not have as many technological resources as the schools of Mr. ZH and Mrs. J. She does not know much about how to use DGS before participating in this study. And in her class, she did not see the need to use too much information technology. So during the study, she began working with me to learn about different software and trying to incorporate it into her classroom instruction. In her school, she had a supervisor. In the Chinese schools, all new teachers graduating from college have a supervisor at their school to help them start their career and prepare them for teaching. Mrs. Y's supervisor is experienced in teaching mathematics and is good at using technology like DGS in her school. So in this research, Mrs. Y got a lot of useful advices from her supervisor about using technology in mathematics teaching.

Mr. W was also a young teacher in a public secondary school, which is similar to Mrs. Y's, without many technological resources available to him during lessons. He also got his undergraduate degree three years before this study and he then found job at his school. While in college, he learned about software used in teaching mathematics, such as Geometer's Sketchpad. So when he started working as a mathematics teacher, Mr. W had some experience with technology. Before the research, Mr. W began to think about how to use software and has been using it in his classes. Due to the lack of teachers at his school, Mr. W also teaches geography.

In China, it is not common for one teacher to teach multiple subjects. So Mr. W. told me that sometimes he finds it difficult to prepare for two different subjects at the same time.

Mr. X, the last teacher in China to participate in this study, has about 10 years of teaching experience and is familiar with the use of DGS in mathematics teaching. He comes from another private secondary school in Shanghai. However, his school does not have much digital resources to help teachers integrate technology into the classroom. For example, he cannot use a smartphone or iPad like the first two experienced teachers.

In this study, these six teachers can be categorized into three groups based on their teaching experience, Mrs. J and Mr. ZH from China and Mrs. Luisa from France can be placed in the same group and considered as experienced teachers. They all have more than 20 years of teaching experience. At the beginning of their teaching careers, there were not many resources available for the teaching process. At that time, the teachers only used traditional resources such as blackboard and chalk. They needed to spend a lot of time copying tasks and contents on the blackboard. With the advancement of technology, digital resources became more and more important in their teaching careers. So these teachers began to look at how they could use digital resources to help them in their teaching. These teachers are familiar with both traditional and digital teaching environments. Mr. X can be placed in the second group, because he has about 10 years of teaching experience and also specializes in DGS. The last two teachers, Mrs. Y and Mr. W, form the third group according to their career length.

4.5 Data analysis

In this section, I will describe how I deal with the collection of all the data in this study based on the methodology described above. In this study, the data including:

- task lists;
- interview audios and transcripts with teachers;

- courseware designed with DGS;
- lesson videos (8 lessons from China and 3 from France);
- lesson notes made by myself during the observation and video transcripts which record all the interactions between teacher and students.

All lesson videos will be transcribed first, recording all teacher and student words according to the timeline. The recording of each lesson includes the teachers' interaction with the students, the actions of the teachers with DGS, and the diagrams displayed to the students on the screen. For the two French lessons held in the computer lab, the recordings also included the student's computer screen and their answers from the task list.

4.5.1 Method of analysis of lesson video

The videos are then re-analyzed according to the theory of instrumental orchestration and the types of instrumental orchestration described in section 3.2. The following methodological issues are crucial in the analysis process: When can it be claimed that a particular orchestration type is a variant of another orchestration type (Tabach, 2013)? For me, if the didactic configuration and the didactical exploitation of teaching processes are the same, these two processes can be seen as the same orchestration type. Otherwise we can identify two different orchestration types (Tabach, 2013). For the initial analysis, I will count the number of each types of instrumental orchestration used in these lessons, as well as how long each type lasts in the lesson, so that I can get a basic impression of how each teacher uses DGS in their lessons.

These videos of general classroom lessons are cut into short instructional episodes based on the different mathematics tasks used in this lesson. Each instructional episode includes one mathematics task. Then, if an episode contains a different type of instrumental orchestration, they would be cut into more sub-episodes. For those lessons in the computer lab, the videos are cut into episodes based on the interactions between the teacher and different students, because in these lessons the students have

their computers to solve the tasks and the teacher (Louisa) walks around to check if the students have finished the tasks. So the interaction always happens between her and different students. This made it easier for me to cut the videos. For example, if the teacher discusses with two different students, the video will be cut into two different episodes.

4.5.2 Example of analysis of classroom interaction

According to the interaction framework mentioned in section 3.5, interactions between teachers and students would be analyzed in the following ways. For example, if the questions or feedbacks focus on technology, then they would be located on the “student-teacher-technology”. If the questions or feedbacks focus the student’s attention on mathematics, then we will describe this interaction as belonging to the “student-mathematics-teacher”. Teachers’ questions and feedback could also focus on both mathematics and technology. For example, they could focus on how to use technology to solve mathematical tasks, which would be located on the “teacher-mathematics-technology-student”. Hollebrands and Lee (2016) called it focus on mathematics with the use of technology. For example, teachers may exploit mathematical knowledge with the use of technology such as focusing on variance/invariance and making a connection between mathematics and technological representations. Or they could also focus on using mathematical contents to present the function of technology, which can be located on “teacher-technology-mathematics-student”. This can be called focusing on technology to notice mathematics. Focus on technology involves teacher actions such as giving information about a teacher-created sketch beforehand, and creating and/or measuring geometric objects. Focus on mathematics involves little use of technology such as constructing a mathematical argument and recalling a definition/property (Hollebrands & Lee, 2016).

First, I would describe how the Chinese teachers used DGS to ask questions to their students and see what feedbacks they gave to their students. The following is

an example of Mr. ZH's first lesson to illustrate how I analyzed the interaction between students based on the framework in Section 3.

T: You finish? OK, let the diagram stop. OK, the first one, what is the maximum of side C? I did not ask the ones on the right side, try to answer it.

St 1: The maximum is 10

T: OK, maximum is 10. I have said in the video, so the maximum is 10. Now what is the relation?

St 1: They are all on the same line.

T: Good, he said on the same line. Let us see, this is position. OK, now C is 10 and they are all on the same line. But what is the mathematical relation between the three sides?

T: $B+C=A$, good, sit down. $B+C=A$, we can also say $A-B=C$. OK, now, I want you to think, if I want it to become a triangle, what is the relation? Let me make a triangle. What is the conclusion? OK, can you answer?

St 2: We can find $A+B$ is bigger than C

T: Now you also find $A+B$ is bigger than C ? OK, sit down.

The following table shows the focus of the questions and feedbacks of this teacher during the episode.

Table 4.3: Analysis of question and feedback

Questions and feedbacks	Focus
<i>You finish?</i>	Not focus on mathematics or technology
<i>OK, the first one, what is the maximum of side C?</i>	Only focus on mathematics
<i>St 1: The maximum is 10</i>	Teacher's feedback is just repeating what student answered. The answer is only focus on mathematics
<i>T: OK, maximum is 10. I have said in the video, so the maximum is 10</i>	Only focus on mathematics
<i>T: Now what is the relation?</i>	Teacher's feedback is just repeating what student answered. The answer is only focus on mathematics
<i>St 1: They are all on the same line.</i>	Teacher's feedback is just repeating what student answered. The answer is only focus on mathematics
<i>T: Good, he said on the same line.</i>	Teacher moves the point on the screen at that time. The feedback focuses on technology to notice mathematics
<i>T: Let us see, this is position. OK, now C is 10 and they are all on the same line.</i>	

<p><i>T: But what is the mathematical relation between the three sides?</i></p> <p><i>St 1: $B+C=A$</i></p> <p><i>T: $B+C=A$, good, sit down. $B+C=A$, we can also say $A-B=C$.</i></p> <p><i>T: OK, now, I want you to think, if I want it to become a triangle, what is the relation?</i></p> <p><i>T: Let me make a triangle. What is the conclusion?</i></p> <p><i>St 2: We can find $A+B$ is bigger than C</i></p> <p><i>T: Now you also find $A+B$ is bigger than C?</i></p> <p><i>T: OK, sit down</i></p>	<p>Only focus on mathematics</p> <p>Teacher's feedback is just repeating what student answered. The answer is only focus on mathematics</p> <p>Teacher moves the point on the screen at that time. The feedback focuses on technology to notice mathematics</p> <p>Teacher moves the point on the screen at that time. The feedback focuses on technology to notice mathematics</p> <p>Teacher's feedback is just repeating what student answered. The answer is only focus on mathematics</p> <p>Not focus on mathematics or technology</p>
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4.5.3 Example of analysis of the interviews

In this study, all the interview transcripts were also further analyzed. Based on the theoretical framework discussed in Chapters 2 and 3, the interview transcripts would first be cut into short sections based on the questions designed in the interview list. And each of the teacher's responses would first be cut into sentences, and then each sentence would be abstracted into a few key words mentioned by the teacher, thus presenting the meaning of each sentence.

An example of analysis of transcript teachers' interview (from the interview with Mrs. J):

Which one is suitable? I like to use, if in preparation or in lecture, maybe PPT, if I have lecture for other teachers, because at that time I just need to present. But in general lessons, I think I use express everything more times. Because I can operate it during the lessons or I can let students operate it with iPad. I can have more time face to my students with this software. I think it is better for teachers reduce the time for back to students. We need to observe students'

facial expression, then you can know whether they understand or not. So I use this one more times.

Step 1: I divide this transcription into the following sentences:

1 Which one is suitable?

2 I like to use, if in preparation or in lecture, maybe PPT, if I have lecture for other teachers, because at that time I just need to present.

3 But in general lessons, I think I use expresseverything more times.

4 Because I can operate it during the lessons or I can let students operate it with iPad.

5 I can have more time face to my students with this software.

6 I think it is better for teachers reduce the time for back to students.

7 We need to observe students' facial expression, then you can know whether they understand or not.

8 So I use this one more times.

Step 2: for each sentence, I find the first one is repeating what I asked during the interview, so it can be delete when I analyze the data. Second one, the teacher mentions one software: PPT, but it is used for teacher education program like the teacher said: *if I have lecture for other teachers*, so it can also be deleted. Then the third sentence, this teacher mentioned another software: expresseverything which is held on iPad for presenting the contents during the lessons. This sentence is related to the software. The forth one, I find that the teacher describes how she use the software during the lessons: *I can operate it or I can let students operate*. It shows that this sentence is about how to organize the class with software. The next one, it contains one key word: student. The teacher describes why she uses this software in the lessons. So it can be related to the factors which can affect teacher's practice with technology. Next, this teacher emphasizes the reason again. Then she mentioned that during the lesson teachers need to notice students' facial expression, which related to the interaction with students. And the last one is for repeating that she uses this software for her lessons.

Step 3: I find sentence 3 to 7 related to my research questions. Sentence 3 and 4

are related to question 1.2: Which types of organization Chinese and French teachers show in teaching practice with DGS? Sentence 5, 6 and sentence 7 are related to question 1.3: What Chinese and French teachers focus on during the teacher-student interaction with DGS?

4.5.4 A pilot study

Before starting this study, I conducted a pilot study by using the research methods mentioned above in order to see if the research design would work and to see what kind of knowledge I could analyze in the process. For the pilot study, I chose a French mathematics teacher from a secondary school and observed one of his lessons. After the lesson, we discussed the questions about his lesson plan, classroom organization, and interaction with students together based on the research questions.

Based on classroom observations and interviews during the pilot study, I found it is difficult to make a clear category of teacher knowledge in teacher's daily practice. For example, each teacher presented their knowledge as a whole when they taught their students, and their knowledge could not exist apart from their disciplinary context. So we cannot talk about mathematics teacher knowledge apart from their mathematical thinking, which means that we cannot find purely pedagogical content knowledge in their curricular practices. Even if we ask teachers to reflect their use of knowledge in their lessons, they will all talk about it in the context of mathematics. Here, therefore, I do not attempt to distinguish between different kinds of teacher knowledge, as other theoretical frameworks do, but to look at teacher knowledge as a whole and try to analyze it from different perspectives. And also I need to think about the knowledge presented in the technological situation.

5 Stories of Chinese and French mathematics teachers with DGS

This chapter presents the results of the data analysis of this study based on the teaching process of different teachers. According to the research questions, this chapter contains three sections. First, in section 5.2, I talked about the role of DGS in mathematics tasks. It shows mainly the tasks used in all the observation lessons. And based on the category of Laborde (Laborde, 2001; Soury-Lavergne, 2017), I analyze each task to find the role of DGS. Section 5.3 then refers to practices regarding to how to use of DGS in the lessons. It shows their instrumental orchestration and how they interact with DGS in the teaching of mathematics. Section 5.4 focuses on the interaction between teacher and students in a DGS situation. This section discusses the different objectives of the questions and feedbacks from teachers. We can see whether teachers' questions are more mathematics-related or more technology-related, and whether their questions link mathematics with DGS in the classroom. From this we can find out what role DGS plays in the mathematics lessons of different teachers in China and France.

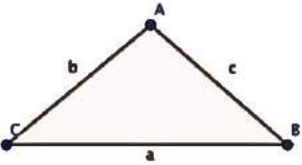
5.1 Roles of DGS in mathematics tasks

Of course, tasks also present teacher knowledge of mathematics contents. Which tasks are the best examples of representing mathematical theory, and in what sequence the tasks are presented to students, can make it easier for students to understand the mathematical content.

5.1.1 Task in Mr. ZH's lesson

The following section shows the tasks ZH used during the two lessons.

Table 5.1: Task used in first lesson

	Task ³⁰
	<p>ZH designed one diagram of triangle whose length of side AC and BC are determined ($AC = 4$, $BC = 6$). Point A can be moved while points B and C are stable. From the diagram, the trace of point A is like a circle. Main question: what is the relation among the three sides of triangle</p> <p>Question 1: Looking for the maximum of the length of AB and what is the relation among the three sides at that time?</p> <p>Question 2: Looking for the minimum of the length of AB and what is the relation among the three sides at that time?</p> <p>Question 3: What conclusion you can find based on the first two questions.</p>

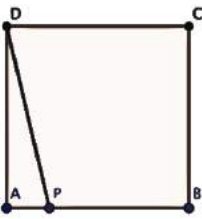
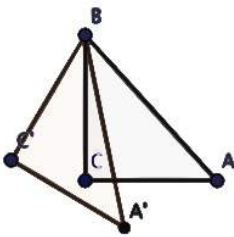
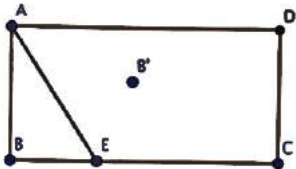
This table presents the task that Mr. ZH used in his first video lesson, and after students completed the first three questions, ZH added a fourth one. He changed the conditions of this task. In the first three questions, the lengths of BC and AC did not change as point A moved, which means that points B and C are fixed. But in question 4, all the points can move. Students need to discover whether their previous conclusions are also correct in the new situation. Therefore, it is not the diagram that changes, but the way the teacher moves. Mr. ZH chooses to move one point or the other in order to create new situation for students and make them reflect their previous answers. He lets students explore the diagram in a different way.

In the second lesson, Mr. ZH designed the tasks not to address new mathematical contents, which means that the knowledge needed to solve the tasks in this lesson has been learned by the students before. All of the tasks contain a key element: moving point, which is one of the most important types of tasks that Chinese students need to solve in order to pass the examination. Mr. ZH spends one lesson (about 45 minutes) before to introduce the solving strategies. This lesson is more like a practice

³⁰ All the tasks in this section are translated from Chinese and French. The original version of the tasks is put in the appendix

session where the teacher rather than the students asks all the questions. During the lesson, students need to solve these tasks on their own or in a group, which was up to the teacher. He designed three tasks that all contained one or more different types of moving points.

Table 5.2: Tasks used in second lesson

Task	
	<p>The first task contained one moving point P. At first, point P is coinciding with point A then it moves from A to B to C to D. Students need to find out the mathematical relation between the area of triangle APD and the length of moving trace.</p>
	<p>The second task contained two moving points, point A and C, but these points do not move according to line but like a circle. There is a right triangle ABC, $\angle C=90^\circ$, $BC=4$, $AC=3$. ZH makes triangle ABC turns around point B to bring point A on the line BC (point A'), students need to find the length of segment AA'</p>
	<p>The third task, ZH designed one rectangle ABCD. There is a point E which moves along segment BC, from point B to C. Point B' is a symmetry point of point B according to segment AE like the figure shows. Students need to find what the trace of point B' moves when point E is moving along BC</p>

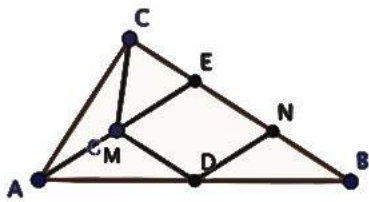
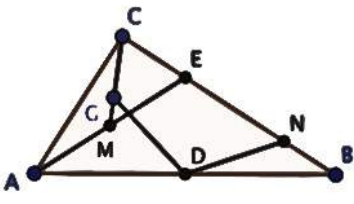
The DGS acts as a generator in these tasks. It changes the task solving strategy of the students to find answers. Without DGS, our students would only solve these tasks with paper and pencil. They need to construct different diagrams based on the task situation and imagine how these points move based on their previous mathematical knowledge. Then with these constructed diagrams, students need to find an appropriate way to solve the task. But with DGS, they can change the strategy. Some geometric properties of the task can be displayed directly on the screen by dragging special points. For tasks like these, students can drag the moving points and see how the triangles move, and easily find out what does not change during the operation to find the results of the moving points (Güven, 2008). And the software can give them

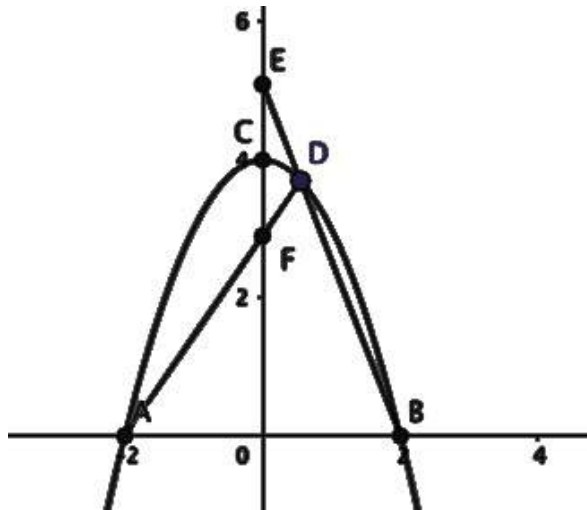
direct feedback so they can check if their conjectures are correct.

5.1.2 Task in Mrs. J's lesson

The following section describes the tasks in Mrs. J' lessons. Mrs. J. is an experienced teacher in China. These two lessons are about revision. After the students had completed their homework at home, Mrs. J reviewed the tasks in class and discussed them with her students to find the correct answers. Due to the complexity of the tasks, this teacher has only one or two tasks for the students.

Table 5.3: Task in Mrs. J's first lesson

	Task
 <p data-bbox="252 1144 523 1178">Figure for question 1</p>	<p data-bbox="901 813 1308 1137">Task 1: In the picture, there is a right triangle ABC, $\angle ACB=90^\circ$, point D is the midpoint of AB, point E is on segment BC, $AE=BE$, point M is the midpoint of AE, point G is on segment CM, $\angle GDN=\angle AEB$</p> <p data-bbox="901 1144 1308 1346">Question 1: like the picture 1, if point G coincides with point M, prove that quadrilateral DMEN is a rhombus.</p> <p data-bbox="901 1352 1308 1597">Question 2: like the picture 2, when point G does not coincide with point M and point C, prove when point G is moving, $GD=DN$ is always true.</p>
 <p data-bbox="252 1480 523 1514">Figure for question 2</p>	



Task 2: In the following picture, quadratic function $y=ax^2+4$ ($a \neq 0$) intersects axis x at point A and $B(2,0)$, and intersects axis y at point C , point D is on this quadratic function and in the first quadrant.

Question 1: if the area of triangle ABD is 4,

Sub question 1: find the coordinate of point D .

Sub question 2: link point O and point D , point M is on this quadratic function, $\angle MDO = \angle BOD$, find the coordinates of point M .

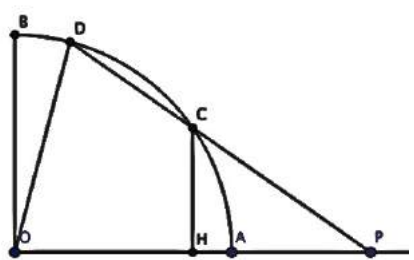
Question 2: line BD and line AD intersect axis y at point E and point F , does the value of $OF+OE$ change? Explain the reason

These two tasks also contain moving processes like other tasks. They always appear at the end of the examination because of their difficulty level. As I mentioned above, when students try to solve these tasks they cannot use any technology, including calculators, even though DGS can show them the moving process directly in the task. So students need to construct several figures based on the situation to understand how these geometry objects change. The tasks have some similar properties, for example, they contain several questions within a situation. Like the first task, it contains two questions, the second task contains two questions and two sub-questions. These questions are not independent of each other. In the words of Mrs. J, the first questions always give some hints to the students on how to solve the following ones. For example, in the first task, questions 1 is a special case according to the situation, and by solving the question, students can translate their strategies to question 2. And during the lesson, Mrs. J also emphasized that the methods in question 1 can be used in question 2.

DGS acts as a generator in these tasks. This is not to say that these tasks cannot be solved without software, as our students can solve these tasks with paper and pencil,

which shows that there are some ways for students to solve these tasks without any technology. I think DGS acts as a generator because it can change students' strategies that they use and also change their attentions. With DGS, some of the geometric properties of this task situation can be displayed directly and it is no longer a problem for the students to find them. Without software, students need to make some diagrams to imagine the process of moving and the changes in geometric objects. This process is like what we call *photo-dragging* (Olivero, 2002). Here, students use paper and pencil to construct pictures to complete the dragging process. This requires more skills on the part of the students to construct as accurate a draft as possible. If they fail to do this, they will find it more difficult to solve the problem. Because of the function of DGS, students can see the whole dragging process. It is like *film-dragging* (Olivero, 2002). From this point of view, we can see the changes that DGS brings to the students when solving the tasks. For example, in task 1, it is easy for students to find that when point G is moving, GD and DN are always in the same length.

Table 5.4: Task in J's second lesson

	Task
	<p>Task: As the following figure shown, in a sector AOB, $\angle AOB=90^\circ$, point P is on the line OA, point H is the midpoint of OP and $HC \perp OP$, points P, C, D are on the same line.</p> <p>Question 1: if $\text{arc } AC = \text{arc } CD$, what is the length of CD</p> <p>Question 2: if point C is on arc AD and $PA=x$, $CD=y$, find the function between x and y, and then find the range of x</p> <p>Question 3: if the midpoint of CD is E, the intersect point of ray HE and OD is F, if $DF = \frac{1}{4}$, find the cotangent of $\angle P$</p>

This type of task is always used as the last one in the examination, which means it is the most difficult task and students need to solve it with paper and pencil in the examination, although we can see that there are many moving points in the task, such as point P, point H, etc. Students need to create several diagrams of how the points

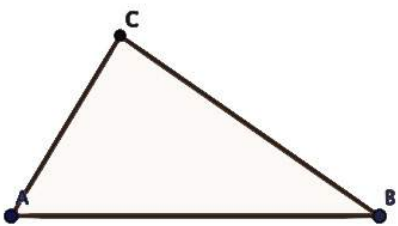
move based on the description of the task and draw some figures based on their imagination, just like what they do in the previous tasks. Just for the task itself, the DGS acts as a generator. With DGS, students need to change their strategies in new situations. They can manipulate the software to see how the points move and what kind of results they can get based on the software. Without DGS, they need do what some scholars call *photo-dragging*, like previous tasks, although they don't actually drag the points.

Although in her lessons, Mrs. J used only one task, it contained three different questions in the same task situation. These questions are presented from simple to complex. In these questions, we can also see the *photo dragging*. For example, there is no moving part in the first question. The designer fixes the points so that the student solves the question based on a specific moment during the movement, like here he says if $\text{arc } AC = \text{arc } CD$. This allows the student to not think about which point to move and how to move, but what the graph looks like at this moment. And the same thing happens in the third question.

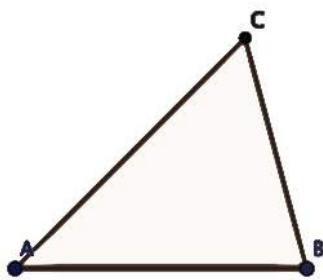
5.1.3 Task in Mrs. Y's lesson

After two experiences teachers, the following part shows the tasks of the first young teacher Mrs. Y's tasks.

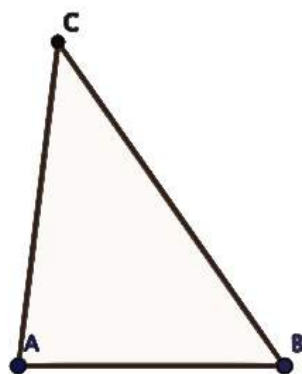
Table 5.5: Tasks in Mrs. Y's lesson

	Task
	Task 1 ³¹ : Using DGS to construct a new triangle, which is congruent with the given triangle ABC, in which $AB=7$, $AC=4$, $BC=6$.

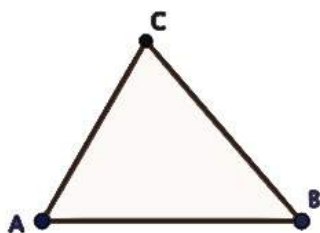
³¹ In this triangle, point A and B could be dragged while point C is stable. When point A is dragged the whole triangle moves without changing its type and size. When point B is dragged, point A is not moving. The trajectory of point B is a circle whose center is point A and radius is the length of AB.



Task 2: Using DGS to construct a new triangle, which is congruent with the given triangle ABC, in which $AB=6$, $\angle A=45^\circ$, $\angle B=75^\circ$.



Task 3: Using DGS to construct a new triangle, which is congruent with the given triangle ABC, in which $AB=5$, $\angle C=42^\circ$, $\angle A=83^\circ$.



Task 4: Using DGS to construct a new triangle, which is congruent with the given triangle ABC, in which $AB=5$, $AC=4$, $\angle A=60^\circ$.

From the task, Mrs. Y constructed some sample triangles before the lesson and gave some information, such as the length of each side or the degree of each angle. Students need to use DGS to construct another triangle that is congruent with the sample ones. In these tasks, DGS is the generator of the task. Although students can construct the same triangle with paper and pencil, they cannot drag any points to check if their construction is congruent with the sample triangle without DGS. At the beginning, Mrs. Y gave students the data for these triangles and asked them to construct new triangles. This meant that students could only use the strategies

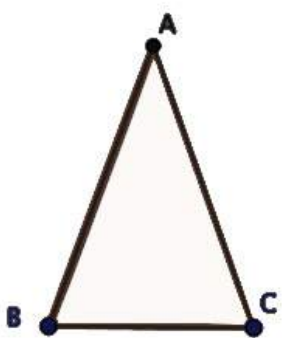
prescribed to solve this task.

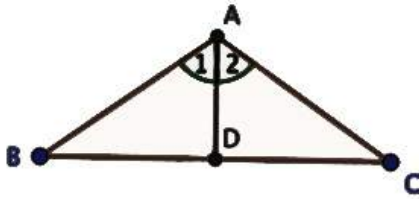
In the beginning, Mrs. Y operated the computer by herself to solve the first task because she wanted to show students how to solve it with DGS. She did this by asking “How to draw it?” ; “ How to draw this point on side AC, that is side DF?” and other questions that allow students to review drawing strategies. However, most of the time, Mrs. Y just tells students where to find the right tool or what to do with the software. So, although DGS is the generator in the task, Mrs. Y uses it more as an amplifier in her teaching. In the next task, she changes the role of DGS. She let the students directly manipulate the software themselves. Although only two students could operate the software in class due to the limitations of the computer in Mrs. Y’s class, her teaching process can be seen as a creation of Chinese teaching strategies.

From the data, students have more opportunities to interact with DGS during Mrs. Y’s lessons. The software is used to create a new learning situation for students to explore mathematics. This is also a new strategy for Chinese teachers to use DGS in their classrooms.

5.1.4 Task in Mr. W’s lesson

Table 5.6: Task in Mr. W’s lesson

	Task
	Task1: In the following picture, we know $AB=AC$ and $\angle B=70^\circ$ Question1: How much is $\angle C$ Question2: How much is $\angle A$



Task2: In the following picture, we know we know $AB=AC$ and $\angle BAC=110^\circ$, segment AD is the midline of triangle ABC

Question1: How much is $\angle 1$ and $\angle 2$

Question2: Is AD perpendicular with BC and why?

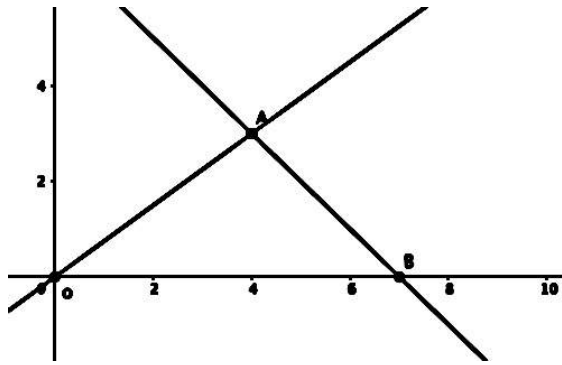
Mr. W. designed the above tasks for his students in his class, but these tasks have different characteristics if compared with tasks of other teachers, Mr. W. is the only teacher who designed the tasks with DGS without any moving process. In China, these tasks are very common for both students and teachers. We can find similar tasks in almost every mathematics exercise book. These tasks do not require students to use any technology to solve them, although some students may need a calculator to help them with basic calculations. In these tasks, technology is not necessary. Based on the role of DGS in mathematics tasks, we can see that in these tasks DGS simply acts as an amplifier for paper and pencil. It does not change anything for the students. It does not change the students' problem solving process or strategy. With or without DGS, students are using the same method to find the answer.

5.1.5 Task in Mr. X's lesson

Here is Mr. X's task for his lesson, and just like Mrs. J, his task for this lesson is to review students' homework. And because this students' task are very complex, Mr. X has only one task prepared for this lesson.

Table 5.7: Task in Mr. X's lesson

Task



The original figure

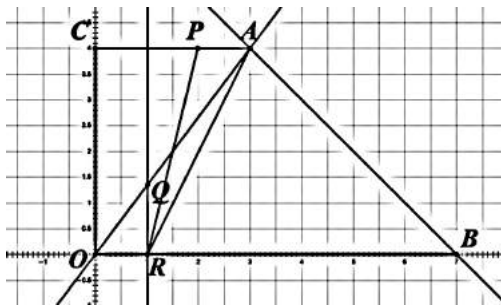


Figure for sub question 1

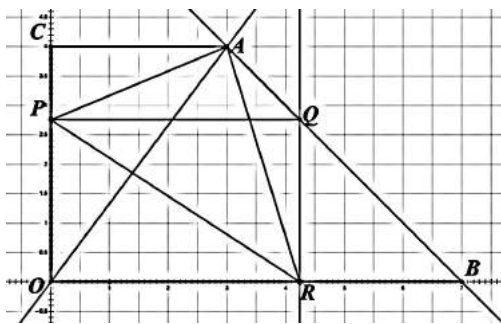


Figure for sub question 2

Task: In the following figure, function $y=-x+7$ intersects function $y=\frac{4}{3}x$ at point A and axis x at point B.

Question 1: what are the coordinates of point A and B

Question 2: if $AC \perp$ axis y at point C, and line l parallels with axis y. The speed of point P which moves from point O is 1 unit per second. It moves from point O to C to A. At the same time line l moves from point B and its speed is the same as point P. Line l intersect axis x at point R, intersect segment BA or segment AO at point Q. Point P stops when arriving at point A and the same time line l also stops. If the time of moving is t, then

Sub question 1: when the area of triangle APR is 8, find the value of t

Sub question 2: whether triangle APQ can be an isosceles triangle or not? If yes, find the value of t, if not, please find the reason.

This task comes from the Chinese senior middle school entrance examination, which is one of the most important examinations for secondary school students. It is the last task, which means it is one of the most difficult tasks for students. As we said, Chinese students have never used DGS to solve this kind of tasks. Of course some teachers may use DGS in their lessons, but they also take care to think about how to make their students not rely on this software when solving the tasks. Depending on the task, DGS can be seen as a generator for new tasks. With or without DGS, students need to use different solving strategies to find the answer. As mentioned above, students often cannot use the software when solving tasks, so they need to

draw several diagrams to simulate the moving process, depending on the problem situation. This strategy is like what we call *photo dragging*, where students solve tasks based on particular moments during the movement. With DGS, on the other hand, students do not need to construct several diagrams. They can manipulate the software to drag the points and see how they move based on what they do. This will help them understand how the whole geometric object changes. Therefore, students have two different situations: solve the task with the help of a diagram given by the software or construct a diagram by using paper and pencil to find a solution. According to the software, the student can see the moving process, which we call *film dragging*. We can say that because of DGS, students can solve the task in a new way and no longer have to work with paper and pencil as before.

5.1.6 Task in Madame Louisa's lesson

After describing Chinese tasks in this research, the following part shows the French tasks used in Louisa's three lessons.

Topic Dynamic Geometry

Instructions: GeoGebra software is used. <http://www.geogebra.org>

For each of the following building programs:

1. Build a freehand figure with pencil in the center column;
2. Build an instrument and pencil figure in the right column;
3. Build a computer figure by using the software, then move the points. After each construction, call the teacher and record your work.

Circles

Task 1

Program	Freehand figure with pencil	Instrument and pencil figure
1. Place two points A and B. Draw the segment [AB]. 2. Draw the center A circle and go through B. 3. Draw the circle from center B through A.		



Task 2

Program	Freehand figure with pencil	Instrument and pencil figure
1. Draw a circle of center A and radius 3cm. 2. Place two points B and C on this circle. 3. Draw the segment [BC].		



Task 3

Program	Freehand figure with pencil	Instrument and pencil figure
1. Place two points A and B. Draw the segment [AB]. 2. Draw the circle of diameter [AB].		

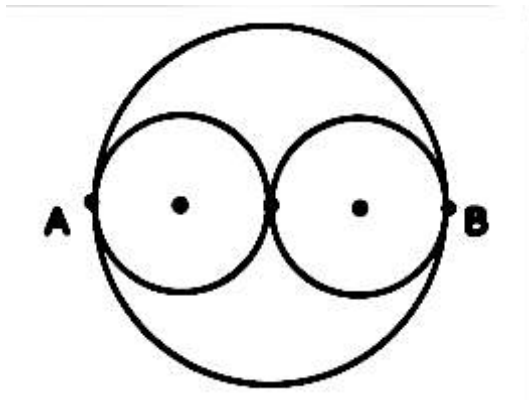


Task 4

Review of the menus used: Write the function of each icon:



Task 5





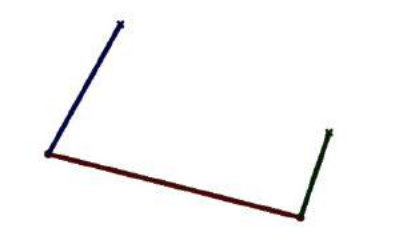
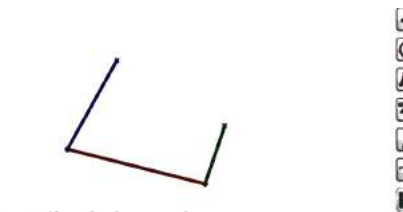
1. Write a construction program of the figure above starting with:
Build two points A and B ...
2. Build the figure with the instruments and also with paper-pencil.
3. Build a computer figure using the software.

The above list shows the tasks used in Mrs. Louisa's first lesson. It shows the second type of roles of DGS: the generator. In these tasks, the teacher always gives the students all the steps to construct geometric objects with DGS. The purpose of this part is to let the students be familiar with the different tools in GeoGebra and to construct circles in different ways. And after completing the first four tasks, students need to construct the same diagram in Task 5, which is the most complex task in this lesson by using the above-mentioned tools. Students need to use the strategies they learned before the lesson to construct the same circles that meets the teacher's requirements. Of course there are no guidelines for students to imitate in task 5. They first needed to think about the properties of the object and then use the strategies they had learned before to construct the same diagram. In this lesson, Louisa does not just have her students use DGS to complete the tasks. She also has them solve the tasks by using other physical tools such as paper-pencil and compass. This arrangement helps students realize the difference between the different tools they use to solve the tasks (like compass and DGS). When solving the tasks, students need to think about the situation they are facing and modify their strategies.

The following part describes Louisa's tasks for the second and the third lessons. These tasks are from the courseware that Louisa designed with DGS before the classes and students will need to solve these tasks by using the same software in the

second lesson and then discuss them with Louisa in the third lesson. The second lesson took place in the computer lab, where students could manipulate the software, and the third lesson took place in the regular classroom.

Table 5.8: Task in Louisa's second lesson

	Example tasks
	<p>Example 1: Can these three segments be the sides of one triangle? (These segments can be moved and rotated by the students)</p>
	<p>Example 2: In this task, these three segments cannot be rotated anymore. How can you know if these segments could be the sides of one triangle? And explain your conclusion.</p>
	<p>Example 3: Can these three segments be the sides of one triangle? (The blue and green³² segments can be rotated but cannot be moved) Yes, or No</p>
	<p>Example 4: In this task, the blue and green segments cannot be rotated. With the help of possible tools besides, can you find if these three segments are the sides of one triangle. And explain your finding.</p>

This table shows four sample tasks from Louisa's second and third lessons with DGS. She designed a total of seven tasks for students. Some of these tasks were similar, but Louisa changed the tools available to the students. As shown below (figure 5.1), they are the same tasks as the ones shown in Example 4. Students also need to find out if the three lines (blue, red, and green) are sides of a triangle. On the right, however, the tools available to students to solve this task are different. In the

³² the blue and green segments are at the extremities of the broken line

diagram (and in Example 4), we can see that Louisa has changed the tools available to her students, so they need to rethink their strategies to find the right answer. They need to use these different tools to imitate the rotation of the segments, just as what they did with the software in Example 3.

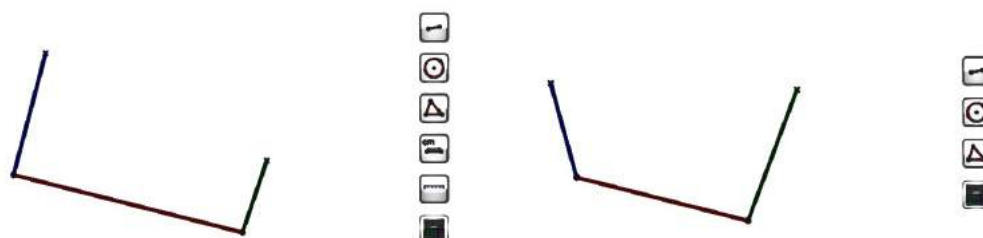


Figure 5.1: Diagrams of example 3

From these tasks, we can see that Louisa uses DGS to create learning situations for students to explore triangles. The purpose of these tasks is for students to find the relationship between the three sides of a triangle. After finishing all of the tasks, Louisa wanted students to know what kind of segments would make a triangle. To accomplish this, Louisa first had students move the line segments to construct the triangle as they wished, and then she added several constraints to allow the segments to move and rotate under special conditions. In the last three tasks, all of the segments could no longer be moved or rotated, and students needed to use special tools to imitate the moves and rotations in the previous tasks. To encourage students to think deeply, Louisa also changed the tools available as shown in the diagram. By doing so, students should reflect on their strategies to meet the new requirements. DGS, in these tasks, acted as a generator. This is because students first need to solve the tasks with pencil and paper in order to realize the difference between DGS and physical resources. Then, they need to modify their strategies because only with paper and pencil they cannot drag the points to change the objects they have constructed. This means that without the software, these tasks would not exist. Some tasks can only be solved with tangible tools such as paper and pencil and compass, while others require the help of software.

The next section presents how these teachers organize their lessons with these tasks and the software.

5.2 Classroom orchestration with DGS

In this section, I also present the analysis according to the sequences of teachers one by one. First let us see Mr. ZH's classes with DGS.

5.2.1 Main types of instrumental orchestration in Mr. ZH's lessons

Based on the framework of instrumental orchestration (Drijvers et al., 2010; Trouche, 2003), the following analysis of ZH's lesson was divided into three parts: didactic configuration, exploitation mode, didactical performance. This section introduced the process of teaching mathematics with dynamic geometry in China.

The following table shows the types of instrumental orchestration used in ZH's two mathematics lessons.

Table 5.9: Types of instrumental orchestration in two mathematics lessons

Type of instrumental orchestration	First lesson	Duration ³³	Second lesson	Duration
Technical-demo				
Explain-the-screen	5	7 min	7	8 min
Guide-and-explain	3	11 min	3	5 min
Link-screen-board	1	30 sec		
Discuss-the-screen	2	2 min		
Spot-and-show			2	5 min
Sherpa-at-work				
Board-instruction	1	5 min	1	2 min

As we can see from the table, Mr. ZH spent half of the classroom time teaching

³³ Duration in each table (table 5.9 to table 5.14) means: total time of using each type of instrumental orchestration in each lesson

with dynamic geometry (about 20 minutes), and the rest of the time Mr. ZH also used techniques such as slides or videos, but this study only focuses on DGS. So here we do not analyze the teaching process without dynamic geometry. During these processes, ZH used to organize his lessons in *explain-the-screen* or *guide-and-explain* type. He used these two types 8 times in the first lesson, lasting 18 minutes, and 10 times in the second lesson, lasting 13 minutes. This means that his lessons were more like teacher-centered lessons (Drijvers et al., 2010). He used most of the class time to control the teaching process. Although ZH choose to controll the teaching, it did not mean that his students could not participate in the teaching process in other ways. For example, he often asks several questions to guide students to make explanations. This means that ZH began to notice that students needed more time to present their ideas. In the second lesson, ZH used a type of instrumental orchestration more like student-centered: *spot-and-show*. He put some of the students' answers on the screen and asked them to tell others about their solutions to the tasks. However, this type of instrumental orchestration is not common in ZH's two lessons (only 5 minutes in total in the second lesson).

Didactic configuration

In the video, we can see that the didactic configuration includes a digital mathematics environment (computer, screen, internet, DGS, etc.) as well as traditional resources such as blackboard, chalk, textbooks, etc. Of course, all students can see the screen and clearly follow the teacher's explanations. This didactic configuration is similar to other mathematics classes in China. In these lessons, the didactic configuration does not change during the teaching. The teacher prepares all the resources or equipment needed before the lessons, and never adds or removes new resources during the instruction. The second lesson has an important feature. Mr. ZH can use smartphone in his teaching process. Through it, ZH can take pictures of the students' work and put them on the screen as the next topic to be discussed. This is one of the reasons why he organized a *spot-and-show* like the episodes in the second lesson, which did not appear in the first lesson.

Since the didactic configuration of a lesson ever changes, the following analysis

of the instrumental orchestration in these two Chinese lessons is based on the last two elements: exploitation mode and didactical performance. First of all, I will show some example episodes from Mr. ZH's two lessons and analyze how he uses different types of instrumental orchestration.

First is *discuss-the-screen*.

In ZH's first lesson, the figure below shows how students discuss what is happening on the screen. In this lesson, all the students were divided into groups (two or three students) as they wished, which made it easier for them to discuss. As a exploitation mode, ZH shows the tasks and diagrams that students need to discuss and allows the diagrams to move automatically. In this lesson, ZH did not control the process of discussion. He just decided the topics to be discussed and the total time for the discussion. The topic of this discussion is a mathematics task Mr. ZH prepared before class which is about the relationship between the three sides of a triangle. During the discussion, ZH walked around and listened to the students' discussion. there was no oral interaction between ZH and the students. DGS played as a projector and the students did not operate the software anymore. These features fit the main characteristics of the *discuss-the-screen*, so this teaching process can be classified as this type of instrumental orchestration. This type occurred once during ZH's first lesson.



Figure 5.2: Screen in the first lesson

Explain-the-screen and *guide-and-explain*

In these two lessons, ZH preferred to use these two types of instrumental orchestration. For example, before the discussion in the first lesson, the ZH presents the task with the help of DGS, during which there is no oral interaction between the

ZH and the students. This indicates that the entire teaching process is controlled by ZH himself. He explains the task by manipulating the software and displaying the diagrams on the screen. This process can be seen as *Explain-the-screen*. A similar process occurs in the second lesson. ZH used computer to show some of the students' worksheets and tried to explain the students' ideas by himself. There was no interaction between the teacher and the students during this explanation process.

As we can see from the video, ZH chose to control the explanation process only when he needed to present or explain the requirements of the tasks. After that, ZH changed the way he organized the explanation. He asked several questions to let the students make explanation. He chose some of the students to answer these questions in order to let others know what was happening on the screen. The following episode can show what happened during this teaching process.

In the first lesson, ZH put the task on the screen and at first the diagram did not move, it was only when he noticed some misunderstandings of the students that ZH started to manipulate the software, dragging a point in order to show why they were wrong. For example, a student tries to answer the second sub-question of this task (the minimum value on the length of AB). He first gave an incorrect answer: 2.2. ZH dragged the point A to let this student notice his answer is not correct. In this episode, the interaction takes place between two people: the student selected to answer the question and ZH. The other students just listen to them and do not give any feedback. The topic of this episode is about the same mathematics task. Mr. ZH asks a series of questions and asks the students to interpret the diagram by answering these questions. So we can think this teaching process as *guide-and-explain*. It involves the whole class explaining what is happening on the screen and the guided explanation by the questions ZH has prepared. The explanation is not only about technology but also about mathematical contents. After all students have completed their explanations, the ZH begins to make conclusions or to explain any incorrect answers.

The above type of teaching process also happened in his second lesson, for example, ZH put a diagram designed based on the task situation with the help of

DGS on the screen, as shown in the figure. The following instructional process is based on this diagram. As an didactical performance, as shown in the figure, one student is asked to explain what is happening on the screen. During this time, we can see that point P begins to move from point A to point B and then, from point B to point C.

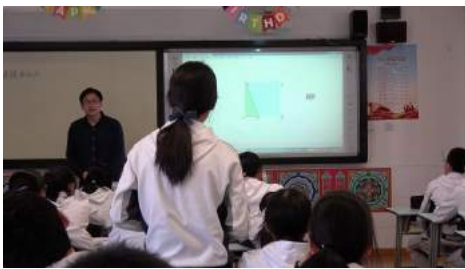


Figure 5.3: Student answered questions

What this student needs to do is explaining why point P move as they appear on the screen and answer the questions. In the meantime, ZH himself controls the software to make point P move in the way designed before the lesson. Just like in the first lesson, the interaction takes place between this student and ZH. The others are just watching what is happening on the screen and listening to the student's explanation. We can see that it is not the teacher, but the student who is trying to explain what is happening on the screen to the whole class during that time, and this explanation is guided by a series of questions. This also fits with the main properties of *guide-and-explain*. We should note that although the process is more teacher-centered, students are not always in a passive position. They are given the opportunity to present their ideas under the guidance of the teacher.

Spot-and-show

This type only occurs during the second lesson. Because ZH can use his smartphone to take pictures of the students' work. But in the first lesson, he could not do so. After ZH explains the students' ideas, he asks these students to tell others how they solved the tasks. As we discussed above, the didactic configuration of a mathematics lesson is always the same. As exploitation mode, ZH uses students' answers as a topic for the following instruction. For example, in the figure below, a student's work is displayed on the screen as she attempts to explain why she solved

the task in the way she did.



Figure 5.4: Student explained her work shown on the screen in ZH's Class

In this episode, ZH gave the students some feedback or asked some questions for them to think about in depth. The questions come from the work that the students have done based on the task situation. So by definition, this kind of teaching process can be explicitly of *spot-and-show*. However, in this lesson, what happened was also different from the category of Drijvers and his colleagues. For example, in their work, *spot-and-show* refers to a teacher selecting one or more students' work during the preparation for a lesson and then using it as a topic during the lesson. In Mr. ZH lesson, however, instead of selecting student work prior to the lesson, he chose it when students solved the task and at the same time he put them on the screen and made them as a topic of following discussion.

Board-instruction

In these two lessons, ZH uses the board and pencil as the most critical resources when he wants his students to know some important conclusions based on the task situation. In the following section, we can see how ZH used only the board and pencil to introduce some concepts in the first lesson.

In this process, ZH wants to explain a problem-solving strategy that is a little difficult for students to understand. So he needs to write down all the steps to make the explanation clear. Like the figure below, this strategy is a type of algebraic approach. It has nothing to do with geometry. So DGS becomes less critical in the process.

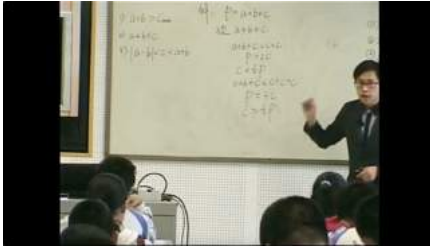


Figure 5.5: ZH wrote solving strategies on the board

As we can see, ZH did not use digital resources in this process when he explained the problem solving strategies. He just wrote all the information on the board for the students. This is just like the regular Chinese lessons where no digital resources are used and the teaching process is controlled by the teacher.

5.2.2 Main types of instrumental orchestration in Mrs. J's lessons

In this section, I present two different lessons from Mrs. J, who is also an experienced mathematics teacher at the same school as Mr. ZH. The table below shows a general overview of the different types of instrumental orchestration exist in Mrs. J's two lessons.

Table 5.10: Types of instrumental orchestration in Mrs. J's lessons

Type of instrumental orchestration	First lesson	Duration	Second lesson	Duration
Technical-demo				
Explain-the-screen	2	4 min	6	17 min
Guide-and-explain	7	26 min	9	13 min
Link-screen-board	4	6 min	1	1 min
Discuss-the-screen			1	1 min
Spot-and-show				
Sherpa-at-work			3	6 min
Board-instruction	1	3 min		

In the table, in contrasting to Mr. ZH, we can see that Mrs. J's first lesson is more

teacher-centered. Both of the two teachers are experienced and know much about how to use technology in the mathematics teaching process. However, in general, their orchestration of DGS is different. This phenomenon indicates their different perceptions of the role of DGS. In both of Ms. J's classes, students did not have much time to discuss what was happening on the screen, even though she also divided the students into different groups. Of course, Mrs. J would engage students in discussion, but the topics were about their homework not the diagram on the screen. There is no interaction between the software and the students during the discussion. Mrs. J uses mainly *guide-and-explain* in her lessons (26 minutes in the first lesson and 13 minutes in the second lesson). The second was *explain-the-screen*. In the second lesson she used this type 17 minutes. Another key feature of J's lesson, particularly in the second lesson, is that J tried to let students manipulate the software, and she chose three different students to manipulate the software and demonstrate their problem solving strategies (J used *Sherpa-at-work* three times in the second lesson, lasting a total of 6 minutes). However, if we analyze the course of these episodes, we also find some differences. In J's lesson, these three students did not play the role of *Sherpa student*, but rather, Mrs. J played this role. That is, in her class, the student tells the teacher (in this case, Mrs. J) how to operate the software based on the task situation, and then the teacher does what the student says to the software. We can say the teacher plays a new role in this process. I will show more of details of these brief episodes later in this section. Like Mr. ZH, Mrs. J also uses several questions to guide the process and give students the opportunity to talk about their ideas and participate in the lesson activities.

Explain-the-screen and guide-and-explain

There are also many common resources in J's classroom such as computer with DGS, chalk, whiteboard, textbook, projector and iPad which is only available at her school. In both classes, J had her students sit in different groups (4-5 students in a small group, as shown below). She told me that this arrangement could help students discuss their ideas more easily. In both lessons, J also use more times *explain-the-screen* and *guide-and-explain* to organize her teaching process with DGS. And she

had more students explain the screens during the class.



Figure 5.6: Classroom configuration and diagram on the screen

Now let us show the details of how these two lessons were taught. Both lessons are about revision, meaning that students have already learned all of the mathematics contents before this lesson, and the objective of this lesson is to show students how to use these contents to solve some complex tasks. Before the lessons, the students had been given the tasks and attempted to solve them as homework. So in the beginning, Mrs. J did not need to introduce or explain these tasks to the students like what Mr. ZH did. This helped her save more time to provide in-depth explanations of how to use students' prior knowledge to solve the tasks. Before introducing the problem-solving strategies, she had students review their homework, identify any difficulties, and then discuss them with the group.

In this episode, Mrs. J guides the student's learning activities by asking a series of questions, like *what type is the quadrilateral, how can you know it, why you justify DN is parallel to ME* (diagram) and so on. This student attempts to answer the questions one by one in order to explain step by step to other students how he solved the task. This episode can be called guide-and-explain. The entire teaching process is guided by the teacher with a sequence of questions. And the student explains what happens on the screen by answering these questions.

In this episode, this student did not solve the task correctly because of some mistakes and instead of trying to help him with his difficulties, Mrs. J asked another student to explain his strategy. Mrs. J told me after the lesson because the student had already completed the work at home and she had also asked the student to discuss their solution before the lesson to find out if they had a problem. So she believes that if the student did not answer her questions successfully, that is because this student

did not meet her expectations and did not complete the homework. Just like what she said after this student: *look, I let you go through your work before*. For Mrs. J, this student did not answer her question because he did not review his homework clearly enough and discuss it with the other students. So in this lesson, she did not want to spend too much time dealing with the mistakes the student made, but instead she asked another student to complete the answer.

After this incident, Mrs. J had another student make a conclusion. During this time, she changed her teaching process. Before making the conclusion, she guided the instructional process with a sequence of questions for the student to answer. We turned the process into *guide-and-explain*. When the students began to make their own conclusions, Mrs. J stopped asking questions and instead allowed the students to control the explanation. During the interaction, Mrs. J barely asked questions and she just simply said “then,” “after,” and “so,” to allow students to continue or repeat what they had said. Here, we put this episode into the *explain-the-screen*, but it is important to note that in Mrs. J’s class, it is the student, not the teacher, who controls the explanation process. At the same time these two instrumental orchestration: *guide-and-explain* and *explain-the-screen*, alternated in Mrs. J’s two lessons.

Link-screen-board

In Mrs. J’s lessons, she takes care to connect traditional resources (e.g., blackboard, chalk, textbook) with digital resources (e.g., DGS). In this section, we will see how Mrs. J connects these different resources in her classes. As shown, Mrs. J drew the same function diagram and explained to the class what the students needed to understand. This episode occurred when a student tried to come up with his own idea that was different from what the teacher had said. In order to show him why his answer was incorrect, Mrs. J drew a new diagram on the board. She thought the diagram on the screen was not clear enough to help the student understand why he was wrong. So she drew a larger one and explained the process based on this new diagram.



Figure 5.7: Teacher explain with DGS and blackboard

During the teaching process, the interaction occurred only between Mrs. J and this student. The other students just sit there and listen to their talking. Here, this interaction was not only controlled by Mrs. J. This student himself were given enough time to express his ideas, even though they may not fit the task. During the discussion, Mrs. J listened patiently and after he finished, she began to explain why his answer was inaccurate with another example, like the figure bellow. This was because Mrs. J found that the student had difficulty to understand this complex problem. She then presented an example that was similar to the problem and familiar to the student, which she thought would help him understand the problem.



Figure 5.8: Link-screen-board

The reason we put this episode in the *link-screen-board* is because Mrs. J uses both traditional and digital resources and tries to link them together. When she found that the screen was not clear enough for her students to see and follow her teaching, she discovered that she could draw a new figure on the board. And the board itself was large enough for her to write down everything she wanted to write. With the help of the board, Mrs. J had the moving figure stop and then discussed with the student what was happening at that time.

Sherpa-at-work

This type appears only in Mrs. J's second lesson. It is an important feature in J's lessons. In the first lesson, J only had her students answer questions or discuss with each other, and she used these strategies to engage her students in her teaching

process. In the second lesson, Mrs. J changed the way she orchestrated her lesson in an attempt to get her students to participate more actively in her lesson. Thus, she began this lesson by having a student show others how to construct a diagram based on the task situation with the help of DGS, as shown in the figure below.

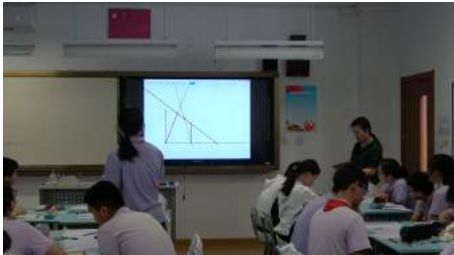


Figure 5.9: Sherpa student explains the construction

Here we can see it is Mrs. J who operates the software with iPad in hand. The student describes how to construct this diagram based on the feedback on the screen. The other students just sat and watched what was happening. At that time, Mrs. J did not ask any questions, she just said “ok”, “then”, “after” for the girl to continue or repeat what the student said. In this episode, the teacher’s main role is like that of a robot, responding with software based on the student’s instructions. Depending on the type of instrumental orchestration mentioned above, this episode could be placed in *Sherpa-at-work*. But in this lesson, it is not the student but the teacher (Mrs. J) who plays this *Sherpa student* role.

In Mrs. J’s second lesson, this instrumental orchestration occurred three times. On two of these episodes, Mrs. J acted as *Sherpa student* and responded to the students’ instructions. And in the third time, she gave her iPad to a group of students (shown in the figure below) and asked them to describe their actions based on the task situation. Here, as can be seen in the figure, two of the students are manipulating the iPad and trying to make the diagram on the screen move in order to successfully solve the task.



Figure 5.10: Sherpa student drag the points with iPad

In this process, Mrs. J also asks a series of questions for these two students to explain why the diagram can move as they show, and what kind of results can be found when dragging the diagram. So this process fits perfectly with the description of *Sherpa-at-work* in the work of Drijvers and colleagues (Drijvers et al., 2010).

5.2.3 Main types of instrumental orchestration in Mrs. Y's lessons

Here, we take a general look at a young mathematics teacher in Shanghai. We can use her orchestration of DGS as an example to understand how young Chinese teachers integrate information technology into their mathematics lessons. We can also contrast her with other elder teachers in China and France to see the differences between mathematics teachers and to gain insight into their knowledge relating to mathematics teaching with DGS.

Table 5.11: Types of instrumental orchestration in Mrs. Y's lessons

Type of instrumental orchestration	First lesson (draw triangle)	Duration	Second lesson (congruent triangle)	Duration
Technical-demo				
Explain-the-screen	6	15 min	6	16 min
Guide-and-explain				
Link-screen-board	1	6 min		
Discuss-the-screen				
Spot-and-show				

Sherpa-at-work			3	12 min
Board-instruction	5	23 min	4	10 min

As can be seen from the table, Mrs. Y's class did not show many types of instrumental orchestration. As with other Chinese teachers, she used *explain-the-screen* for several times. The lack of *guide-and-explain* meant that in the classroom, Mrs. Y controlled the entire teaching process. And her students could not interact directly with the DGS during her lessons, especially during the first lesson (drawing triangles). Perhaps because it was the first time for Mrs. Y to use DGS in her lessons and she was unfamiliar with organizing instruction by using the new software, I found that the frequency of *board-instruction* in her two lessons was significantly higher than that of other Chinese mathematics teachers. Mrs. Y used this type of instruction for 23 minutes in the first lesson and 10 minutes in the second lesson. Another important feature was that in these two lessons, Mrs. Y did not arrange any DGS-based discussions with her students. However, in the second lesson, Mrs. Y made some changes. Although *explain-the-screen* and *board-instruction* were used most often and no *discuss-the-screen* were present, Y used *Sherpa-at-work* (3 times in the second lesson), which was used infrequently by other Chinese teachers in mathematics lessons. This shows that Mrs. Y's courage to use DGS in her math lesson was strong. And as can be seen from the table, the strongest change was that Y added a new instrumental orchestration (*Sherpa-at-work*) to integrate DGS in her second lesson. For her, there was no ready-made model to learn to help her teach with dynamic geometry because not many teachers in her school use DGS in their lessons. Therefore, Y did not have the opportunity to work with the experienced teachers (except her supervisor) and discuss with them when she had some troubles.

Didactic configuration

In this lesson, the classroom is equipped with a computer already installed with DGS (GeoGebra for this lesson) and a blackboard for the teacher to write important ideas on. The students sit like any other regular classes in China, and everyone has a clear view of the board and screen. As we discussed in the other two teachers' classes

we mentioned earlier, Mrs. Y's classroom configuration is similar to other classrooms. A computer was a necessity for her. If she did not allow her students to operate the software, then only she could use it during class. There is also a screen so that students can see what the teacher is doing with the computer. Other traditional resources are also included in the classroom. As with the other classrooms, there are about 30 students who sit in pairs during these classes. The physical configuration of the classroom does not change during the course. Unlike two experienced teachers who work in a private school, Mrs. Y cannot use smartphone or iPad to help with her teaching practice. This is because in most middle schools, neither teachers nor students are allowed to bring smartphones into the classroom. Of course, these technologies are rarely used in the classroom.

I would now describe the main types of orchestration exhibited by Mrs. Y's two lessons, which, as mentioned above, were most frequently used in the classroom as *explain-the-screen*, indicating that the lessons were centered on the teacher rather than the students. And because she was not very familiar with DGS, Mrs. Y also only used the board to teach during the lesson, although she integrated DGS into two mathematics lessons in this study. At that time, DGS became less important in the teaching process and was never used. One feature to note in her lessons is that Mrs. Y tried *Sherpa-at-work* in the second lesson, where three students were chosen to manipulate the software in front of the class. This is not common in Chinese mathematics classrooms (only Mrs. J also uses this type). Here, I would like to put up some teaching examples to analyze how Mrs. Y organizes her classroom with DGS and the differences between her practice and the type proposed by Drijvers and his colleagues.

Explain-the-screen

The first lesson was the first time Mrs. Y used DGS in her classes. Before this lesson, she did not know much about DGS and how to use it into her lessons. The teaching resources she often used were traditional such as chalk, textbook, and blackboard. The only software she uses is slides or word, which helps her present contents to her students. So from the general information that she controls everything

during the lesson, the lesson is more teacher-centered. Here, this section presents several episodes from the first lesson to demonstrate the details of her orchestration with DGS. As shown in the figure below, in the first lesson, Mrs. Y orchestrated the lesson in a way of *explain-the-screen*.



Figure 5.11: Explain-the-screen

Here the students are sitting and looking at the screen. There are several triangles on the screen, constructed by Mrs. Y when she prepared this lesson before. Mrs. Y manipulates the software to drag some vertices around so that students can see how the triangles change and students can know that they cannot determine the size and type of a triangle by only one side and one angle.



Figure 5.12: Mrs. Y control the software

In this short episode, Mrs. Y was always in control of the software, operating it according to her understanding and arrangement of the lesson. Her students did not interact with the software in any way and never discussed or determined the movement of the triangles by dragging the points. During this time, the student only answered simple questions such as what can I move? What is changing? Most of the time DGS was held by Mrs. Y. When she dragged one of the vertices of the triangle, she showed the student and explained the dragging of the point. This process was different from the two elder teachers mentioned above. There was little interaction between Mrs. Y and her students. She did not allow students to use the software to

answer or present their ideas.

The same type of instrumental orchestration was used by Mrs. Y in the second lesson. This episode was about constructing triangles with DGS. It lasted about 3 minutes. During this episode, Mrs. Y kept control of the software and her students sat there and saw what was happening on the screen, as shown in the picture below:



Figure 5.13: Mrs. Y construct triangle example

In this figure, Mrs. Y began to construct another triangle based on the three sides given. However, she did not tell the students the steps of constructing and she asked some questions before the problem solving process, like “let us review it” and asked the students to review the strategies they had already learned before. Then she asked “How do you draw it?” By asking questions like this, she wanted the students to tell her how they did it rather than sit back and wait for teacher’s answer. Depending on the type of instrumental orchestration, Y used *explain-the-screen* to organize the teaching with this task, and she used dynamics software in this lesson.

In this short episode, Mrs. Y asked several questions to have students review how to construct triangles with DGS. She did not have students stand up and explain to others what was on the screen. In contrasting to the two experienced teachers mentioned above, in Mrs. Y’s lessons, when Mrs. Y used *explain-the-screen*, the students were not always involved in the process. She never asks students to control the explanation process, but asks a few simple questions to the whole class and displays the answers on the screen to see if they are listening carefully. In this episode, Mrs. Y controlled the teaching process from the beginning and The level of student engagement was lower than in the other Chinese lessons mentioned above. This episode demonstrates her teacher-centered teaching process in two of her lessons.

The figure below showed another episode of *explain-the-screen* in Mrs. Y's lesson.



Figure 5.14: Explain-the-screen in the second lesson

In this episode, there was no oral interaction between Mrs. Y and the students. Mrs. Y controlled all the process and did not let the students to answer the questions. Although she asked several questions such as How do you construct a triangle? Is it the same? and so on, but she just gives the answers herself immediately, no students are involved in the teaching process. It was her own who made explanation to all the students. There was no oral interaction between her and the students in this teaching process. And the software was controlled by Mrs. Y. The students just sat there, watching and taking notes, just like what other students did in any other mathematics class.

Board-instruction

In both lessons, although she began to use DGS in her teaching, Mrs. Y was still preferred to use traditional resources. From the general information table for the instrumental orchestration of her lessons (Table 5.11). There were several instances where Mrs. Y was teaching only with the board, although she could use DGS. For example, in the following episode of the first lesson:



Figure 5.15: Board-instruction

At that time, Mrs. Y moved the board in order to cover the screen. This means

that Mrs. Y would not want to teach with DGS at that time and she did not want the software to interrupt the students' attention and wanted them to pay attention to what she wrote on the board. The episode was for students to construct a triangle by using paper and pencil based on the data written on the board. After describing the construction requirements, Y had one student construct the triangle on the board and the other students completed this task on their notebooks.



Figure 5.16: Student work on blackboard

As you can see in the figure, Mrs. Y is standing at the side of the classroom (not visible in the figure), giving this student more freedom to solve the problem on his own. The other students, as shown in the figure, were just constructing triangles on their notebooks and were not watching this student's solution. All the students were just completing their own work. In this case, there was no discussion or interaction between Mrs. Y and her students. She did not walk around and check the students' work to see if there were any problems like other teachers did when students were doing the exercises. Instead, she stood by and just watched what the student was doing on the board and she was focused on the construction on the board.

Here we should notice that in the type of instrumental orchestration proposed by Drijvers and his colleagues, *board-instruction* means that the teacher teaches without using technology in a technical situation. In this type of teaching, the teacher should be at the center. But in this lesson, it is not the teacher, at least not all the time, who is the center of the lesson. At this time, Mrs. Y is clearly at the edge of this instructional process when this student constructs the triangle on the board as shown above. Students do the exercise on their own. This time is for them to schedule and complete the task. The teacher does not interrupt the student's problem solving

process. She just waits and see if the students have finished and then begins the next teaching process.

In the next lesson, Mrs. Y also used *board-instruction* when she integrated DGS into her teaching process. However, in this lesson, instead of having the students solve the task on the board, she wrote down the answers herself as in the figure below.



Figure 5.17: Board-instruction in the second lesson

In this figure, we can see that the students cannot see the screen and the software as they did in the previous lesson, and Mrs. Y moved the blackboard to cover the screen. In this process, Mrs. Y did not use the software. At that moment, DGS was not so important anymore. Even through the figure, we can see that the software is still working. And only when Mrs. Y needed to look up some information on the computer, she would look at the computer and write it down on the board, as shown in the figure below.



Figure 5.18: Mrs. Y check information with software

And there were no other interactions between her and the computer, nor were there simultaneous interactions between the students and the computer. The same orchestration occurred a total of nine times in Mrs. Y's two math classes. This indicates that she was not yet familiar with incorporating DGS into her mathematics class.

Explain-the-screen and *board-instruction* were most common in Mrs. Y's lessons which suggested her lessons are more teacher-center according to Drijvers (2010, 2013) and his colleagues. From these episodes, DGS did not affect the students' learning activities. It simply acted as a projector and functioned to present contents or tasks to the students. In Mrs. Y's lessons, there was no direct interaction between the software and the students. Unlike the other two elder teachers mentioned above, Mrs. Y did not make the students the center of the course. She never let the student explain the screen to others. She wants to control everything so that the students can understand what she is saying.

Of course we need to notice that in the first lesson, Mrs. Y chose a few students to construct some triangles on the board, which shows that Mrs. Y also believes that she needs to give her students more time to participate in the classroom. She does not just think that students are negative in mathematics classes. She needs to design several activities to make them more active in mathematics learning. And after thinking about it, with the help of DGS, she found some solutions and designed some activities for that purpose.

So in the second lesson, she made the students the center of the lesson in some episodes. She chose a few students to go to the platform and manipulate the software to construct some triangles as she requested. Although this is not common in her lessons, it still shows that Mrs. Y notices the needs to give her students more time to interact directly with the software, which may have a positive effect on students as they learn geometry in class. This change is described below.

Sherpa-at-work

Sherpa students only exist in the second lesson. Mrs. Y asked three students to manipulate the software to construct the same triangle shown on the screen in the lesson. The figure below shows the process of students constructing the triangles in front of the classroom.



Figure 5.19: One Sherpa student work with DGS

Here we can see that while the student was constructing triangle with the computer, Mrs. Y just stood by and did not talk to him, and in the process Mrs. Y did not interrupt the student to tell him what to do. When the first *Sherpa student* had some difficulty with his construction, Mrs. Y asked him to reflect the previous lesson in which Mrs. Y manipulated the software to construct triangles for the student. She posed some questions for the students to try on their own.

St: How to draw an angle? 34

T: Draw an angle, here, it is angle, you can look at it. The segment is below, in the tool "line". You can click this arrow, yes, arrow, you can click it, and write on it ok, then the angles, you look at the degree. Its degree is given, look, which one? Attention, here, the angle, the vertex point, how many letters? Is the middle letter? So you need to draw angle D, what segment do you click first? You can try it, try.

This short episode shows how Mrs. Y helps a student construct a triangle. She asked several questions to help the student choose which tools in the software would help him make the correct construction. These questions were more about the construction process with DGS. And Mrs. Y shows him how to click the mouse and where to click, for example, *the segment is below, in the tool "line". You can click this arrow, yes, arrow, you can click it.* And the other students were sitting there watching what was happening on the screen. Mrs. Y told them before the teaching process not to talk to the *Sherpa student* who was constructing the triangle. So we

³⁴ All the transcripts (here and after) are translated into English from Chinese and French.

can see that no student gave him some advices.



Figure 5.20: Mrs. Y discussed with Sherpa student

As you can see from the figure, this student had some troubles using DGS to solve the task. Mrs. Y is trying to show him how to manipulate the software to find the right angle for a triangle. In order to introduce the operation, Mrs. Y clicked on the tool with DGS. Several questions were asked so that he would know the procedure for operating the software to construct the required triangles. This was the first time she had a student operate DGS in her classroom, and this student was unfamiliar with the software because he had not learned how to use it in his previous mathematics class. In that lesson, Mrs. Y told her students how to use software to construct some geometric objects, but never had them manipulate them during the lesson. In this lesson, Mrs. Y wanted to see if the students understood what she had done in the previous lesson, so she chose some of them to manipulate the software to construct triangles. However, she did not anticipate that the students would have a lot of difficulties in constructing geometric objects in different contexts. Although they knew how to draw triangles with paper and pencil, they had no idea about how to construct triangles with DGS. So this student could not solve it at the beginning. Mrs. Y chose to reconstruct the triangles, show the method again, and ask him to repeat what he did before. Mrs. Y also told other students not to give any advices. Here, the students had different opinions on how to solve the problem. As shown in the figure below, Mrs. Y again manipulates the software to show the students the correct construction.



Figure 5.21: Mrs. Y operated the software

After, Mrs. Y asked another *Sherpa student* to construct another triangle with DGS. She did not talk with this student and also told others not to give this *Sherpa student* some advices during the lesson. As shown in the figure below, Mrs. Y was standing off screen and never interacted with this student. She left him to think about all the questions on his own, rather than operating the software on his behalf.



Figure 5.22: Another Sherpa student

It is important to note here some of the key differences between the practices demonstrated in Mrs. Y's two mathematics lessons and the types of instrument orchestration detailed by Drijvers and his colleagues. In Mrs. Y's case, the *Sherpa student* was operating the software in front of the class. No one gave him any advice or told him how to construct the triangles during this process. In the second example episode in particular, the teacher was standing far away from the student, as shown in the figure. And before asking him to operate the software, she tells the other students not to tell him how to construct triangles with the software. So we can see that no interaction took place between Mrs. Y and this student, nor between this student and anyone else. Even if this student made some mistakes that led to his failure in the construction, Mrs. Y just left him to think by himself and we can only say that this student is an atypical *Sherpa student* because according to Drijvers and his colleague, a *Sherpa student* is someone who follows the instructions of others to

work with the software. What they do with the technology is based on what others including students and teachers tell them. So this brings us to the question of whether we can treat the teaching process as *Sherpa-at-work* based on the type of instrumental orchestration proposed by Drijvers, or do we need to name a new type of instrumental orchestration with DGS?

5.2.4 Main types of instrumental orchestration in Mr. W's lessons

In this study, I followed just one mathematics lesson of Mr. W. Mr. W. is one of the youngest teachers in this study. His use of DGS to orchestrate his mathematics lesson can represent the state of teachers who are just beginning their teaching careers and who use different technologies in their daily practice.

Table 5.12: Types of instrumental orchestration in Mr. W's lesson

Type of instrumental orchestration	First lesson	Duration
Technical-demo		
Explain-the-screen	3	5 min
Guide-and-explain	2	7 min
Link-screen-board	1	3 min
Discuss-the-screen		
Spot-and-show		
Sherpa-at-work		
Board-instruction	4	20 min

In this table, we can see that lessons using DGS are organized differently compared to the first two teachers. In Mr. W's class, although he told me that he knew how to use the software and would prepare some lessons beforehand, he only showed very little time to use the DGS to help him in his lessons. As we can see, he does not use many types of orchestration in his lessons and prefers to use explain-the-screen to orchestrate his teaching process. Also like other teachers, he chooses to control all of his teaching time himself. He let some of his students answer questions displayed on the screen. But this type of orchestration is not common in his class. We found this type of situation only twice in this class. Another important feature is

that in this lesson, Mr. W primarily used board instruction (20 minutes in total), although this type is also found in other teachers' lessons, but not as much as in Mr. W's, suggesting that Mr. W, although using DGS, is still familiar with traditional teaching resources. Next part, I would analyze his lesson in depth to see how Mr. W uses DGS to make instructional decisions.

Didactic configuration

In China, most secondary schools are always in the same physical configuration: computer, screen, projector, and other equipment and traditional resources are almost identical in all of the schools. In Mr. W's classroom, of course, the physical configuration is the same. But there are only 25 students in Mr. W's classroom. In Shanghai, the number of his students is not too many. So they do not sit in pairs like other schools, they just sit alone. This makes it difficult for students to discuss in class. This may be one of the reasons why Mr. W organized her lessons in this study in a teacher-centered way. Like Mrs. Y, he could not use technology such as smartphone or iPad in his lessons.

From the general information about the teaching process of Mr. W's mathematics class, we can see that he mainly used *board-instruction* and *explain-the-screen*. He does not use DGS very often. And even when he uses the software in his teaching, he chooses to keep it at the edge of his practice rather than at the center, although he learned how to use DGS in university before becoming a mathematics teacher. He does not let the students operate the software as Mrs. Y does, and the whole teaching process is controlled by himself, although sometimes the students ask questions based on the diagram shown on the screen. Then I would put up some teaching episodes to demonstrate the organization of his class with DGS.

Explain-the-screen

The main instrumental orchestration used in this lesson, like the table shown above, is *explain-the-screen*. This is not the first time for Mr. W. to use DGS such as Geometer's Sketchpad in classes. He told me during this study that he learned a lot about DGS in university, when he wanted to become a mathematics teacher in the future. After graduation, he finally started his teaching career and started thinking

about how he could incorporate this software into his mathematics classes so that they would fit into the Chinese curriculum. This lesson provides us with an example of how Mr. W. chose to organize his lessons with DGS.

In this section, I begin by discussing a common type of instrumental orchestration shown in his lesson: *explain-the-screen*. The following figure shows an example of this type of instrumental orchestration in this lesson.



Figure 5.23: Explain-the-screen

This picture shows the second task that Mr. W. used in this lesson. Here, we can see that the computer is equipped at the front of the classroom. Only the teacher can operate it during the teaching process. All the students are sitting in their seats just like in any other mathematics classes in China. In this figure, Mr. W is introducing the task to the students and explaining to them what they need to do in order to solve the task. In the meantime, DGS plays like a projector. Mr. W uses it to introduce the task and geometric objects. He prepares them before class and shows them to his students when he wants. Although in this picture we can see that there are some buttons that make the triangles move if we click on them, when Mr. W explained the task, he seldom clicked on one of the buttons and make the triangle move according to the design he had prepared before the lesson.

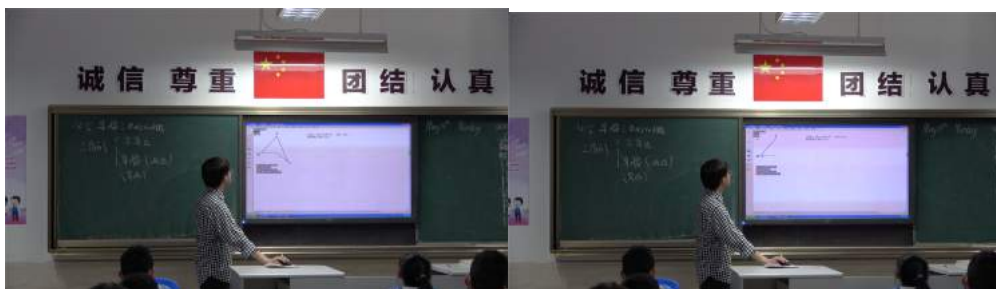


Figure 5.24: Mr. W operated the software

As shown in these two figures, we can see that points B and A are stable and point

C is moving so that the shape of the triangle changes. Mr. W. wanted to show students that in an isosceles triangle, the two angles (angle B and angle C in the figure) are equal. This process lasted for about a minute, during that time the students did not have time to talk about the task and Mr. W did not ask any questions for the students to think about. The entire instructional process was controlled by Mr. W. There was no interaction between Mr. W and the students other than verbal communication (just the teacher talking and the students listening).

T: In one triangle, AD is one edge of AC, it cannot reach here, so there are three isosceles triangles, three waists, bottom edges, top angles. Ok, let us discuss the measure of angle 1 after learning the lesson. Then, in triangle ABC, AB equal to AC, you have already known the conclusion, angle B equal to angle C. ok, think about it, if I fold this triangle, you can see angle B is coincide with angle C. Which kind of movement we call it?

All St: Fold

T: Through folding, we can make angle B and angle C coincide, then, if I need to add one line, which line I can add?

All St: Perpendicular bisector

Here is Mr. W's interaction with the students as he explains this task in this episode. From here we can see that he asks two questions: *you can see angle B is coincide with angle C. Which kind of movement we call it?* and *if I need to add one line, which line I can add?* Instead of asking one student to answer, he asked all students to give their answers at the same time. Mr. W also did not ask other deeper questions to let students to think more about the task, but chose to continue his teaching process as arranged before the lesson. In this episode, Mr. W is the center. He made the decision to allow the lesson to continue based on his own design. And he did not interrupt or disrupt his teaching program because of some unexpected occurrences. Based on the type of instrumental orchestration that mentioned above, we put this episode into *explain-the-screen*. Because Mr. W controls the entire teaching process, he rarely interacts with his students (and hardly ever answers questions). His students just sat and listened during the teaching of this episode. This

process is an example of a teacher-centered episode in his lesson.

The following figure shows another example of *explain-the-screen* in his lesson.



Figure 5.25: Screen shows the answers

This is another task he presents in this lesson. In this episode, Mr. W uses the computer to manipulate the software, but he does not change the triangles from the previous episode mentioned above. He just added some text showing how to solve the problem with the software, as shown below.

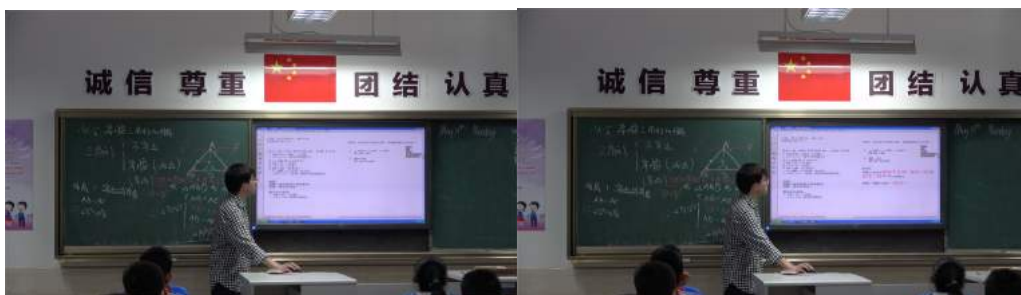


Figure 5.26: Screen shows the contents

And as we can see from the figures, sometimes Mr. W does not even show the triangle to the students. When we discussed it after class, he told me that the students were already impressed with the triangle after completing the first task. Since the type of triangle is always the same in the task, it does not matter if he does not show them the triangle. And he also just added some words on the screen. In this episode, the triangle is not the key point. After this episode, Mr. W uses the software to create the slides just like any other teacher. He just puts everything in the software and shows it to his students in class. This episode does not involve any dynamic process of the software. So the software was playing as a projector at the time. Also Mr. W controlled the entire lecture process while his students were in a negative position. He seldom asked questions to his students to engage them in his teaching process. There was not much interaction between him and his students. Like in the first

episode, Mr. W did not ask a particular student to answer a question during that time. If the students knew the answer, they could always answer the question. Mr. W also did not ask other deeper questions to get the students to think more about the task. He chose to continue his teaching process as he had arranged before the lesson.

Guide-and-explain

This section demonstrates another type of instrumental orchestration in Mr. W's lesson. From the general information mentioned above, we find that Mr. W prefers to organize his lessons in a teacher-centered way. *Explain-the-screen* and *guide-and-explain* are both typical types of teacher-centered instrumental orchestration with dynamic geometry. The following episode will show in more detail how Mr. W guides his students in explaining what is happening on the screen.



Figure 5.27: Mr. W let one student answer the questions

This figure shows an episode at the beginning of the course. From this we can see the screen presenting the task to the students. Each student can clearly see this screen. After presenting the main requirements of the task and telling the students what to do next, Mr. W began to select some of the students to answer his questions. In this episode, the DGS is still a projector to display the task and diagrams on the screen. This instructional process does not include any dynamic parts. Mr. W does not want the triangles to move when he uses the software here.

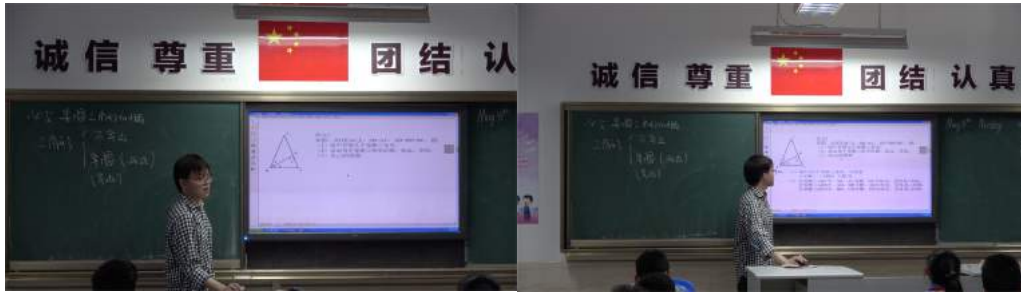


Figure 5.28: Screen shows the conclusions

Instead, he used it to display text on the screen that presented some of the questions he wanted students to answer or some of the things they needed to learn in the lesson, as shown in the figure.



Figure 5.29: Mr. W guided student's answer

This is another episode of the lesson. It also shows Mr. W's guided process with some questions. As in the figure above, on the screen we can see the task and the diagram that will be used by the students to solve the task. Also, in this process, Mr. W does not make the triangle change or move. The dynamic geometry simply presents a static object to the students. This is like a projector in any other mathematics classes where the teacher likes to use slides to show the contents or tasks. Here, we can see that Mr. W has not changed his approach compared to other teachers.

Mr. W also asked several questions for students to answer based on the tasks situation presented on the screen. During the interaction, Mr. W controlled the entire teaching process according to the plan he had designed before the lesson to keep the lesson continue. He really did not want any incidents during his lesson which would get him in trouble. Everything was prepared before the lesson and Mr. W made few changes during the teaching process. Probably due to his lack of teaching experience, he needs to deal with too many things in class. So organizing the lessons in a step-

by-step manner could have made his job easier and made him feel relaxed.

Board-instruction

In this lesson, although DGS was used to orchestrate the teaching process, as described in several examples above, it acted more as a projector, and Mr. W used it primarily to present the tasks or contents, rarely allowing the software to show the dynamic parts to the students. In the next part of the lesson, we can see that Mr. W returned to using traditional resources such as blackboard, although there were no problems with DGS.



Figure 5.30: Mr. W wrote the answer on blackboard

In this figure, it comes from the process of teaching after the first task in the lesson (mentioned in *explain-the-screen*). After using DGS to explain the requirements of this task. Mr. W began to show students how to use their knowledge to solve it. Although DGS was still displaying diagrams on the screen at this point, it was starting to become the edge of this teaching process. Mr. W. wrote down all the problem solving procedures on the board and did not interact with the DGS in any way. His students also only looked at the board and not the screen. Only when Mr. W needed to view some information about the task did he look at the graphs instead of manipulating the software.



Figure 5.31: Mr. W explained the contents on the blackboard

There were no changes on the screen during the instructional process shown in

this figure. This means that during this time, Mr. W was not operating the software. Instead, his teaching process was based around what on the board. For example, here he was pointing to the words written on the board to explain to the students the meaning of this task. In this episode, Mr. W did not change the way he interacted with the students. According to his lesson plan, he posed several questions, which he had designed before class and answered by one or more students. And Mr. W wrote down these answers on the board so that others would know what he had said. The questions were about strategies for solving the task displayed on the screen. Because the task requires students to give a formal mathematical proof of a geometric property, Mr. W needs to show students how to write down the proof on paper, which is one of the requirements in examinations. This is a major reason why Mr. W did not use DGS in this process. He believes that blackboard is better for students to see the format of mathematical proofs. He believes that blackboard is better to show the format and he can use it to show the proofs step by step to make them clearer.

Link-screen-board

Mr. W only used this type once in his lesson. The following section shows the details of this episode.



Figure 5.32: Mr. W operated the software

After writing the proof on the board, Mr. W used the software to show why the proof was correct in this task. And draw his students' attention to other strategies for solving this task.

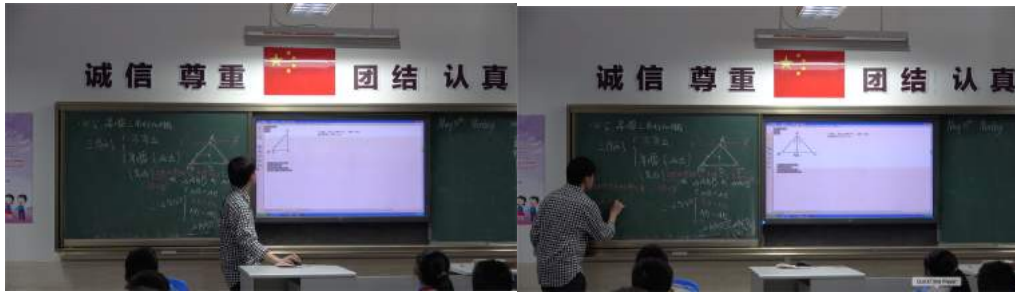


Figure 5.33: Link-screen-board

In the figures we can see that Mr. W wrote down on the board according to the method on the screen. He translates the geometry on the screen into mathematical language on the board. During this process, Mr. W often switches between the board and DGS. He first showed the students a few simple tasks on the screen. Then, he begins his main lesson. In this lesson, he needs students to know the properties of isosceles triangles and how to prove these properties in mathematical language. When he begins telling his students how to write down the proof process in mathematics, he chooses to teach it by using only the blackboard. For him, the blackboard was better suited to his teaching objectives at the time. Students could follow him step by step and know exactly what they needed to write first. DGS was not critical in this process. No one in the classroom was paying attention to the screen or what was happening. And through DGS, he just shows some of the answers to the task.

Mr. W mainly used DGS to show students what he wanted to show them. Throughout the lesson, he only had the objects on the screen move once, which is shown in the first episode of this section. At other moments, the geometric objects are static. He uses the software as a projector during the lesson. This is an important difference compared to other Chinese teachers.

5.2.5 Main types of instrumental orchestration in Mr. X's lessons

In this section, we will discuss the last Chinese teacher's practice of using DGS in her lessons. The following table shows the overall situation of the integration with DGS in X's lesson.

Table 5.13: Types of instrumental orchestration in Mr. X's lesson

Type of instrumental orchestration	First lesson	Duration
Technical-demo		
Explain-the-screen	4	13 min
Guide-and-explain		
Link-screen-board	2	13 min
Discuss-the-screen	1	1 min
Spot-and-show		
Sherpa-at-work		
Board-instruction	2	12 min

Like Mrs. J and Mr. ZH, Mr. X also comes from a private secondary school in Shanghai. However, he showed different organizational skills compared to the other two private school teachers. First of all, he did not use too many types of instrumental orchestration in his lessons. As we can see from the table, he mainly used *explain-the-screen* to orchestration his lessons with dynamic geometry in this study (4 times, 13 minutes in total). Unlike the other experienced Chinese teachers, he did not ask his students to explain the screen during the lessons. Therefore, in his lesson, the teaching process was under his control. Compared to other Chinese mathematics lessons, his two lessons were more closed and teacher-centered. Mr. X also paid attention to linking traditional resources such as paper-pencil and DGS. He used *link-screen-board* twice in his lesson, which lasted 13 minutes. Overall, he also had students discuss the task once during class by using DGS. And *board-instruction* also appeared 2 times in his class.

Didactic configuration

As mentioned above, the didactic configuration is always the same between schools, and some secondary schools may encourage teachers to use many technological resources, such as the school where Mrs. J and Mr. ZH are, so we can see them using iPad or smartphone in the classroom. In most secondary schools, the resources are similar. Computers are one of the most important technologies in every school. Students can see what is happening on the computer through the screen at the front of the classroom. Teachers can use DGS if this software is installed. Mr. X's

school is the same. He has about 40 students in his class and they sit in pairs like other schools in Shanghai, which makes it easier for them to discuss during the classes. Traditional resources are also included in his classroom. Smartphone and iPad cannot be used in this school.

As shown in the general information about instrumental orchestration in Mr. X's lessons, *Explain-the-screen* is also his primary type. He also used *discuss-the-screen* with students during his lessons. Not many types are displayed during his teaching process, and Mr. X does not spend much time on using DGS.

Explain-the-screen

The most common type is still *explain-the-screen*, which indicates that the lesson is more teacher-centered. This type was also the most preferred type in other Chinese mathematics lessons. Following I will discuss how Mr. X uses DGS to explain to his students and show how he differs from other Chinese teachers.

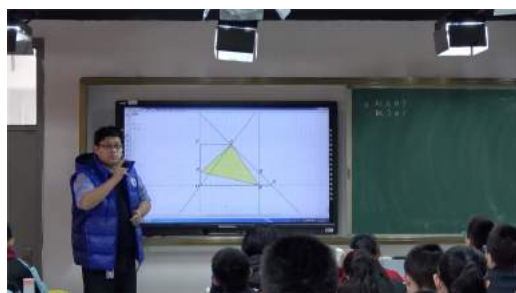


Figure 5.34: Mr. X explained the screen

This figure shows a general picture of Mr. X using DGS in this episode. As with the other lessons (Mr. W and Mr. ZH), the software is controlled by the teacher and students do not have the opportunity to operate the computer or determine the movement of objects. Their work is to listen to what the teacher says. In this episode, the object displayed on the screen does not change its shape. Mr. X used the software as projector to present the information. Because in this process, Mr. X is only trying to explain the first question of the task, which does not include any dynamic process. And the diagram is just used to allow students to check if what they have constructed correct figure on paper. So Mr. X does not think it is necessary to make the object move anymore.

This episode lasts about 7 minutes. Afterwards, Mr. X asked the students to solve

the following questions on their own based on this diagram on the screen, and he walked around to check students' solutions or answer their questions. Since neither Mr. X nor the students were using DGS at the time, I did not put it into any type of instrumental orchestration or describe it in more details. After finishing the questions, Mr. X began the following teaching process with DGS.



Figure 5.35: Explain-the-screen

Here, the didactic configuration does not change at all. From this figure, all the students are sitting in their seats, looking at the screen and the board. Mr. X is standing in the front of the classroom, using DGS to deliver his lecture. In this short episode, DGS still acts as a projector to display geometric objects through the screen. The diagram on the screen is static. And during the whole instruction, there is almost no interaction between Mr. X and his students. Mr. X just asks the students a few simple questions, just as what Mr. W did, which have nothing to do with DGS. The entire teaching process was controlled by the teacher. DGS was not a crucial part of this episode and it only showed pictures to the students and there was no interaction between them.

In this lesson, Mr. X organizes his instruction according to the requirements of the task that the students need to solve. After explaining the simple question in this task, Mr. X realized that he needed to make the object move in order to tell the students what to do next, so although he still controlled the didactic process, he had DGS show the students its dynamic process according to the task.

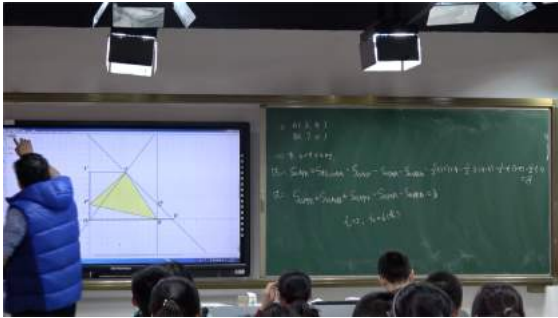


Figure 5.36: Mr. W operated the software

He used the software to click the button and point P moved from point O to point C to point A, while point R moved from point B to point O and point Q moved from point B to point A, according to his lesson plan.

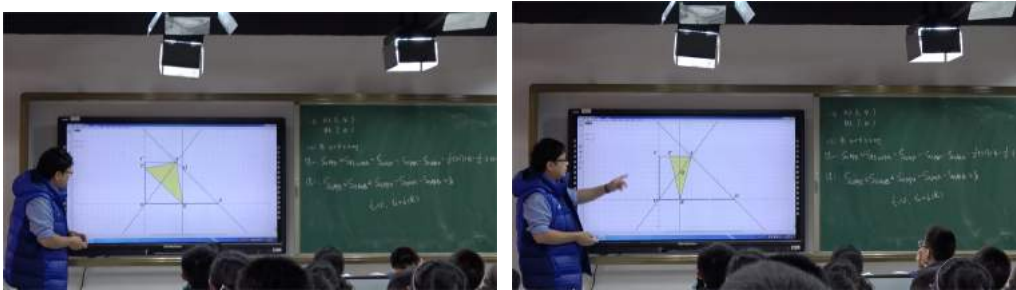
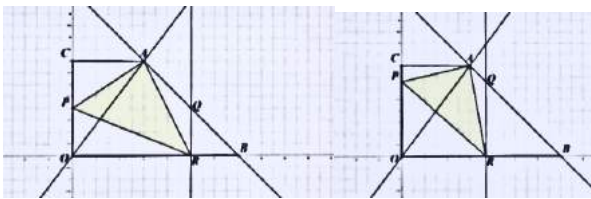


Figure 5.37: Mr. W explained the diagram

T: Good, no matter which method, we can find t_1 is 2, t_2 is 6. Because t is between 0 and 4. So delete 6, so t is 2. Ok, this is which situation? Which situation? $0 < t \leq 4$. So we have another situation right? Ok, I construct a simple diagram. Look at the moving process, I make it slowly, slowly. It is not cartoon. Ok, we can find, after, the object becomes slim. [triangle on the screen moves



] Ok, it turns back to the beginning. I have seen, many students construct the objects, in fact, when you construct the second figure, do you notice, this figure is a little easier?

St 1: The area is not changed

T: The second area not change?

St 1: It changes

T: Ok, let us see it again. Look at the yellow part, the area is intuitive. Now it

is large, then it becomes small and small

All St: It changes

T: It will change, but what you have said is partially correct

St 2: The height does not change

T: Which does not change?

All St: The height

From these figures, the objects on the screen are moving automatically. Mr. X designs everything before the lesson and shows it to the students during the class. All the students look at the screen to see what is happening, they cannot decide how the objects move and interact directly with the software. When the object moves, Mr. X explains to the students what is happening. Just like in the above interaction, we can see that the whole teaching process was controlled by Mr. X. Like other teachers, Mr. X did not change the way he interacted with the students. He asked a few questions, but they were not related to the DGS (I would analyze it in detail in section 5.3).

Board-instruction

In this research, I found that in Chinese lessons, teachers always use *board-instruction* when using DGS in their lessons. Mr. X is also. Like the following scenario



Figure 5.38: Mr. X explained the contents on the board

In this short episode, Mr. X wrote the answer to the task on the board and shared it with the students. As we can see from the figure, all the students were looking at the right side of the board and no one was paying attention to what was happening on the screen. Although DGS was still working during that time, it became less important. Neither Mr. X nor the students were interacting with the software. Mr. X

just uses it to present the diagrams of the task. He was focused on the board, writing the steps of solving the task on the board for the students to see. At that time, he was just standing on the right side of the classroom and did not operate the software anymore. His objective was to show the students how to solve the task and to learn how to write it down on the paper. During the teaching process, Mr. X controlled the teaching process. In other words, he was the center of the teaching process. His students only answered simple questions as necessary. The following episode is another example of this teacher-centered process during Mr. X's lesson. It takes place at the end of the lesson. Mr. X begins to explain the most difficult question of the task. At this point, he chose to write it down on the board and did not interact with the DGS again.

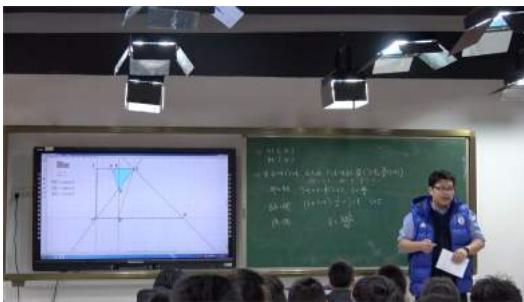


Figure 5.39: Mr. X wrote down the answers

At the beginning, DGS was still working like the figure. all the students just looked at the right part of the screen and focused on what Mr. X wrote on the board. No one paid attention to the screen and the diagram as they did in the first episode in this section. Then Mr. X made the conclusion of the task. He chose to move the position on the board like in the figure below.



Figure 5.40: Mr. X moved the blackboard

At the time, the students could not see anything on the screen. Although we could not find out DGS through the figure, it still worked. Mr. X chose to set it aside and

draw the students' attention to the conclusions he had written on the board. These episodes show an important element of Mr. X's lesson. For him, DGS is only a supporter of classroom mathematics teaching. He only turns on the software when he thinks it is necessary to show his students the diagrams.

T: Some things need to do by yourselves. Do not always try to find easy methods. Maybe it has, maybe not.

(students' working)

T: You can discuss the equations, the formulas.

(students' working)

T: When you discuss, check it step by step. Is your equation right? Where is the mistakes?

(Discuss)

(students' working)

T: After discussion, you can find where I make mistakes, where I have problems, the result is $\frac{226}{43}$, who get it? what I want to say is, this is complex, because we use distance formula between two points. When we learn similar triangle after, we can use some easy methods. After learning similar triangle. And what can we do after, we will talk it later. But some of you maybe not find out the results, maybe you make some mistakes when calculating. Wang, when he solved the problems, he found some mistakes, but some of you may not find. So I give you suggestions, when you come back home, today, you need to calculate the result again. The calculation is complex, you can use calculator, but you need to make sure that the equation is correct, then calculator can help you. Otherwise, you only can find some strange numbers. Of course you can search the answer from internet, but you can only find some similar triangles. First, you need to make sure your equations are right, then solve it. If the equation is not correct, you cannot find the right answer. So, today, we have talked more about Isosceles triangle, the distance formula between two points, do we have other methods? We can use geometry methods right?

Digital combination. Ok, geometry, you can make justification to make the calculation easier. We use it to know the two segments are equal. Then the coordinates in the coordinate system he represents represent what the connotation is, equal, and equal. Become a one-time equation. So today, we solve this kind of problem first time. Geometry sketchpad can help you. Today's homework, you need to solve all the problems, and another one is the learning plan for tomorrow, application of linear function. Ok, class is over.



Figure 5.41: Interaction during students' work

This interaction occurred at the end of the lesson. During that time Mr. X gave up using DGS. He wanted to make conclusion of his lesson. He did not ask too many questions during the interaction. First, he gave the students some time to complete the last question of this examination, which was the most complex question in the lesson. During this time, he checked the students' work and let them discuss if they had any difficulties. As you can see from the figure, DGS does not play any role while the students solve the task on their own. They could talk to each other based on their problem solving, but no one could operate the software to see what was happening. For Mr. X at the time, he needed to show his students how to solve these types of tasks with paper and pencil, as they would do in the examination. This was one of the most important objectives of the Chinese mathematics classes. In Mr. X's class, this objective accompanied the entire teaching process. And it is also a teaching belief in his career that students need to be succeed in examination. Therefore, he needs to create a similar learning situation for his students so that they are suitable for examinations.

After students finishing their work, Mr. X began to make his own conclusions. He

took control of the entire presentation to keep the students' attention on him. He moved the board so as not to interrupt the students' attention. He talked about the difficulties he found when students solved the task by themselves. He did not ask any questions to interact with the students. He just wanted to use the last few minutes to remind students of what they had already learned during this lesson. We saw that no students talked to Mr. X. They just took notes and listened to the teacher.

Link-screen-board

The blackboard is always one of the most important resources for mathematics teachers in the classroom. Our teachers often write what they think is important on the board to make their lessons clearer. Although DGS is used in many mathematics lessons, it does not mean that the blackboard is less important to teachers. In China, traditional resources such as the blackboard still play an important role in mathematics lessons. In this study, I find that although teachers choose to organize DGS lessons in different ways, they still need the blackboard for organize teaching process. Mr. X also tries to make a connection between DGS and blackboard. As in the following episode, he wrote down information based on the diagram on the screen.

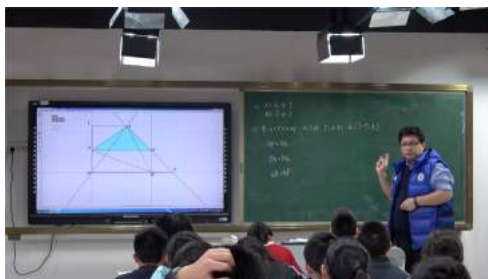


Figure 5.42: Mr. X explained the diagram with board

The didactic configuration here is like *board-instruction* we mentioned in the previous section. At first glance, the two episodes are very similar. But I noticed some differences in the process. In the beginning, Mr. X wrote down some answers on the right side of the board, like what he did in other episodes. And then he communicated with the students, all of whom pay attention to the information on the board and check it against their answers. after, unlike the above episodes, Mr. X moves to the screen and works the software to show the students how to get the answers on the board.

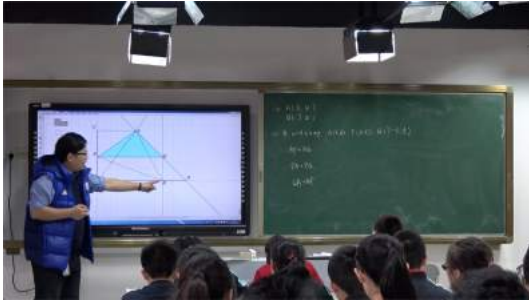


Figure 5.43: Mr. X operated the software

As shown in the figure, Mr. X pointed to the screen and explained what was displayed on the screen. The students' attention shifted to the left part. He then manipulated the software to show how the geometric objects moved and talked to the students. This process occurs several times in this episode. This is the most important difference from the episodes above. This is why this episode can be seen as *link-board-screen*, while the above episode is *board-instruction*. Because in the episode above, Mr. X and his student did not interact with the software in any way. He did not even change his position. He just stood on the right part of the classroom. As with the other episodes described in this section, Mr. X remained in control of the teaching process. He just asked a few simple questions that some of the students had to answer. The students' answers are very short and simple. They often had only one word or one number. Most of the time, the right to speak was held by Mr. X. He talked to the students and let them know the strategies for solving the task. This interaction occurs throughout the lesson, and Mr. X rarely changes the way he interacts with his students, although he integrates DGS into the lesson in different ways.

Thus, from the videos, DGS is not essential in Mr. X's class. His lessons are designed according to the syllabus and examinations. So DGS is only an aid for him. Students need to understand the required knowledge before using it to solve the task.

5.2.6 Main types of instrumental orchestration in Madame Louisa's lessons

In this class, Louisa divided the students into two groups. Each group of students works in the computer lab for about an hour while the other group is in a regular

classroom with another mathematics teacher from the school. In this study I would show one group of these students, about 14 students, working in the computer lab and analyze how Louisa organizes the lessons. Here, I would only focus on the instructional episodes in which the teacher and the students interact together with DGS. Therefore, if the students were just using the software on their own, the teaching process would not be presented in my study.

Table 5.14: Types of instrumental orchestration in Louisa's lessons

Type of instrumental orchestration	First lesson	Duration	Second lesson	Duration	Third lesson	Duration
Technical-demo			2	52 secs		
Explanation-the-screen	1 2	10 min	10	20 min	6	7 min
Guide-and-explain	5	1 min	11	16 min	1	50 secs
Link-screen-board	8	6 min	1	38 secs		
Discuss-the-screen			1	1 min		
Spot-and-show						
Sherpa-at-work	2 5	10 min			6	7 min
Board-instruction	1	1 min	3	2 min		

As we can see from the table, the first lesson took place in the computer lab, where Louisa had the students operate DGS for most of the lesson time, and she preferred to use *Sherpa-at-work* with the students before make explanation. An important feature that we should also notice is that although *Sherpa-at-work* was used twice as

often as *explain-the-screen*, the total time spent with both types was similar. From this we know that Louisa spent more time on explaining the screen to her students. And in the next section, I would describe the two types of instrumental orchestration in more details. In this lesson, Louisa also linked paper and pencil with DGS, as one of the objectives of the lesson is to show students how to shift their problem solving strategies between different problem solving situations. At some point, she tried to show students the connections between the different problem-solving situations they face to help them solve the same tasks with paper and pencil and with DGS.

In the second lesson, we could see a sudden change in the instrumental orchestration in her class. I recorded the same group of students for the first two lessons. And the two lessons were conducted in the same computer lab. This means that there was no differences in the didactic configuration of these two lessons. So it was interesting to know why Louisa made such a big change in the second lesson, with the same physical environment and with the same students. In the second lesson, she never used *Sherpa-at-work*, meaning that she did not tell her students too much about how to work with the software, as she did in the first lesson, but rather than had them follow her instructions. Instead, she spent most of her time on explaining screens to the students or guiding them to find problem solving strategies by using DGS. From the table, *explain-the-screen* occurred 10 times and *guide-and-explain* occurred 11 times. These two types took up the majority of the instructional time. The total time spent on these types is about 36 minutes, of which over 19 minutes is spent on *explain-the-screen* and the rest of the time is spent on guiding students to find appropriate solutions or checking their errors. In the next section, I would show some teaching episodes both in these two lessons, and compare the differences between them.

In the third lesson, when Louisa moved her class to a regular classroom, the instructional environment changes and so we can see her adjusting her lesson organization to fit the new didactic configuration. As we can see from the table, Louisa's total time spent on using the software was much reduced. She only spent about 15 minutes on using the software. And she only used three types of

instrumental orchestration in the lesson, with *explain-the-screen* and *Sherpa-at-work* still being the predominant types. Although the total frequency of each type reduced compared with the first two lessons (only six times for each type), the total time was about the same as the first lesson. We can see that the second lesson is about exploring new knowledge. So she used more teaching time to guide or explain during the second lesson, both in terms of frequency and total time. The third lesson comes after the second one. The students had already learned the main knowledge through solving the tasks with DGS earlier. So Louisa just needs to have them repeat what they did with DGS and give the students a few conclusions. So in this lesson she spent more time on getting students to do the exercises on their own, which would not be analyzed in this study because there was no DGS involved during this time.

The first two lessons took place in the computer lab. Each student had a computer to use. The teacher did not compose her lessons as a whole. She used many individual types of instrumental orchestration. This is a key difference between Louisa's first lesson and the Chinese lessons described above. Because of this key difference, we can identify different classroom orchestrations in French mathematics classes. Because Louisa divided her students into two groups, each student had access to her own computer to work on her own during the classes. At the beginning of the lesson, Louisa, began by explaining to all the students what was expected of them for the lesson and the activities they needed to complete. In this computer lab, the students have their own computers to use, so unlike a regular class, there is no need for a projector. Louisa did not choose to use this device to show something to all the students.

Didactic configuration

The French lessons in this study were held in two different classrooms compared to the Chinese lessons. The first two lessons took place in a computer lab and the last lesson took place in a regular classroom, similar to the Chinese classroom. In the computer lab, the French students were able to operate their own computers. As shown in the picture, each student had his or her own computer during the class. This is an important difference between the Chinese classroom and the French classroom.

As students learn in the computer lab, Louisa needed to design a new teaching process to make it more productive for them. Therefore, this new didactic configuration would affect her lesson process below. In the next section, I would describe the organization of her classes with DGS in more details.



Figure 5.44: Class configuration

The third lesson took place in a regular classroom, just like the one in China. The configuration of the classroom is similar. There is a computer in the classroom and a screen next to the blackboard so that all the students can see it clearly. The teacher spent most of the time standing at the front of the room and talking to the students. All students were seated in pairs. The whole configuration is shown in the figure below.



Figure 5.45: Class configuration in the third lesson

In this figure, we cannot see the computer clearly because it is put in the corner. This design makes the teacher not block the students' view which let them cannot see the screen, when the teacher operates the computer. But it has a weakness, for example, if the teacher needs to operate the computer during class, he cannot see the students' reactions, especially their body language. In China, the computer is always placed at the front of the classroom. There is a table, which we call a platform, like the Chinese classroom in the figure above. When the teacher operates the computer, he is still facing his students, which helps him to notice their reactions more easily.

In China, it is very important that the teacher spends most of his teaching time facing his students. Before every young teacher beginning his or her teaching career, he or she is told that it is better for the teacher to face the students than to turn his or her back to them.

In what follows, I would select a few short teaching episodes to see how Louisa arranges her lessons in these two different rooms. And I would contrast them with the Chinese ones to find out how the Chinese teachers differ from the French one.

Sherpa-at-work

Due to the different teaching environments, the French teacher showed a huge difference in how she organized her lessons with DGS. Her students, as we discussed above, could interact directly with the software during the lessons to construct several geometric objects by using the computer. This learning activity is different from the tasks that required students solve some mathematical problems, such as the China problem. Of course, students need to think about the suitability of their learning strategies for this new learning situation.

These French students had already learned how to use this DGS, so they had no problem with the basic operations of the software, such as how to construct segments or triangles. And because of the small number of students, Louisa can spend more time talking to all of her students and helping those who are struggling with the tasks. This is difficult for Chinese teachers to do. Here I have selected some typical episodes to see how Louisa uses DGS to teach in the computer lab.

In the first lesson, the main focus was to show students how to construct a circle by using different tools such as compass or DGS. All the mathematics knowledge required for this lesson has been learnt by the students and they were familiar with the software so they did not have any difficulty with the basic operations of the software. In this lesson, Louisa started by having the students solve the task on their own. She gave all the students a list of tasks and asked them to complete each of them according to the requirements written on the list, which was described in the chapter 4. The student who completed the tasks must raise his or her hand and tell Louisa. She would then come to the students and check their work. Louisa also

worked face to face with students who had difficulties. When meeting students face to face, Louisa would have them do some operations based on what she asked them to do and checked whether their work was correct. This instruction allows students to do what they are asked to do, just like *Sherpa student*. As in the figures shown below, Louisa had this student drag one of the points to check that the construction is correct.



Figure 5.46: Student operated the software

We can then find that this student dragged the point and the software shows the results of what this operation did. Based on these results, Louisa and the student could determine if she is having some trouble constructing the circle.

This episode only lasted a short time because this student made no mistakes in constructing the circle in this task. So Louisa did not need to spend more time teaching her more contents. In this lesson, this kind of brief episode happens a lot when Louisa had students check their work. If the work was correct, Louisa would have them move on to the following tasks without taking too much time.

The following episode shows another example of *Sherpa students* working with Louisa. It went on for much longer. We can see more about this type in this episode.



Figure 5.47: Another student as a Sherpa student

It started with a student solving a task he encountered in the lesson. First, Louisa had him repeat the requirements of the task and know what he needed to construct at

the end of the operation with the software. Then, Louisa had him perform some operations. She let the student click and select the appropriate tool from the software to construct the circle. She told the students: *click here and center, but no it's not there what you have done here* and the student make respond “ok”. Louisa then used the word “trace” to show him how to make the circle in a right way.



Figure 5.48: Louisa told student how to choose the tools

Then Louisa continued to tell the student the program he needed to do with the software. For example, she said: *Now we want a circle going on another circle, circle, here, it is not this one. That for center opens the circle and goes, you do not want it to go where? Go and put your arrow there or, no, no, but go to there, so then what you want ?* And after she let student drag one of the point: *move the points you see that it stays well.* Here, in this episode, all of this student’s action towards DGS are based on teacher’s guideline. He did what Louisa told him to do and finally finished the task.

If Louisa found that her students were struggling or making mistakes in the lesson, she preferred to use other types of instruction to show students how to correct those mistakes, such as *explain-the-screen*.

Explain-the-screen

In this section, I would describe how Louisa uses DGS to tell students the right way to solve the tasks with DGS when they made some mistakes in this lesson. Here, I also show some examples for description.

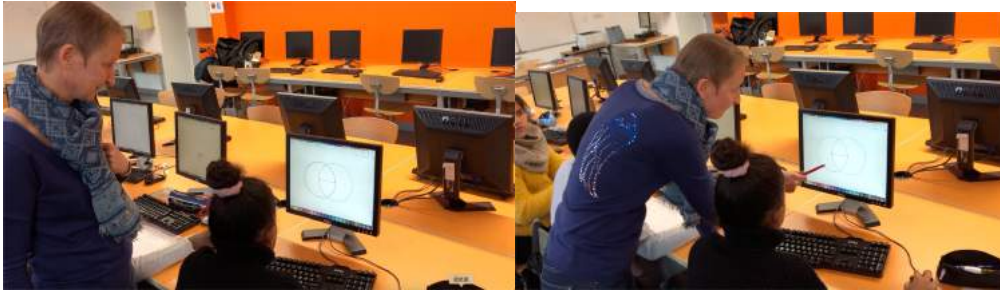


Figure 5.49: Louisa explained the screen with student

This figure shows an episode in which a student made an error while constructing a circle. In the figure, we can see that the student dragged the point to check her work. And Louisa found that the work was out of place. She began to tell her why she made these mistakes and how to correct them. At first, she asked this student: *Do you know why it does not work? Because you did not exactly tell it each time you want the circle that goes through this point. So we will erase it and I'll show you something (to make) you know if you can click on it.*

And then she pointed to the screen and explained the object to this student like the figure shown.

At the same time, she told the student how to make the correct construction with DGS. *If you want a circle of center A, and then if I want to tell it that it goes through B, I have to go on B and it is not to create another point where I say to myself, well, it's a little near, you see it's not good, for example, so hop, erase all these circles. So we're going to construct a circle of center A which goes through B, so I tell it B, so it'll stay, you see?* In this episode, there is no other interactions between Louisa and this student. The teaching process was controlled by Louisa and the student just looked at the screen and listened to what Louisa had said. During this time no one was operating the software, Louisa's teaching process was based on the diagram displayed through the screen and she did not make this diagram change by operating the software. After she told the students the correct construction steps. She began to control the software itself to display the construction shown below.



Figure 5.50: Louisa operated the software

As before, the student just sat there watching what Louisa did with the software and tried to remember all the steps in order to correctly solve the subsequent tasks. The main difference between this episode and the other two above is that Louisa is at the center of the process. Everything is under her control, such as operating the software and doing the explanations. The students become the edges of this teaching process. They do not interact directly with the computer. Their job is just to listen and watch. So here we can see that the teacher is the center of this teaching and learning process. I would now show other short examples of *explain-the-screen* from Louisa's course.

The figure below shows another episode of this lesson where Louisa noticed that this student made some errors in constructing the midpoint of a segment. Here, she replaced this student to manipulate the software, through which she could show the construction process, and she also repeated the steps the student needed to remember to solve the problem. As in the episodes above, during this time Louisa controlled the teaching process and the student just watched what she did with the computer. There was no other interactions that took place between the student and Louisa. Louisa simply said that there were no questions and did not let the student to present her ideas. *So ... not here. Left, in this menu. No. Approach the segment. Here, it put the midpoint, you see it is black which means that you cannot move it. Ah but, we do not have the right circle.* After finished, this student started repeating what Louisa had done before. And the teacher watched what he did with the software.



Figure 5.51: Louisa operated the software

At this point, Louisa also talked about how the software made feedbacks through the screen based on what the students had done. *We will remove it and after you do it again. So now you are looking for the middle of this segment here. Middle. Click on segment. No, no, you touched it twice, so it did (like this). There ... it just needs to click once on segment. Ok? And after, now you will be able to tell it to draw the circle of center and passing by ... and there yes it will stay.* As the episode shows, this student said nothing throughout the lesson, which shows a situation similar to the above episode.

In the second lesson, Louisa showed more times *explain-the-screen* during the lesson because, unlike the first lesson, the second lesson was new knowledge for these students. The tasks that the students faced were new to them. So Louisa chose to explain the whole teaching process of her teaching at the beginning of the lesson, which was not shown in the first lesson. As in the following episode, Louisa used a projector to print the task on the main screen so that it was clearly visible by all the students.

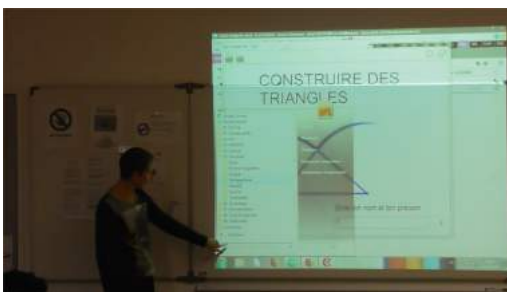


Figure 5.52: Louisa explained the software

This figure shows the beginning of the lesson. Louisa pointed to the screen and told the students how to open the tasks and then explained each task to them. Due to the arrangement of this classroom, when Louisa wants to operate the computer, she

needs to go to the right side of the classroom and cannot see the screen at the same time as shown in this picture.

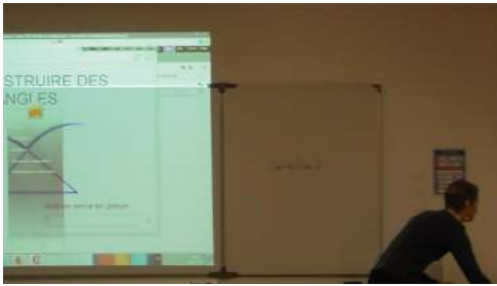


Figure 5.53: Louisa operated the software

It seemed inconvenient for the teacher to present the contents through the computer, because of the constraints of this environment, Louisa needed to walk between the computer and the screen to present the contents. During this time, students were looking at the screen and had no interaction with their own computers until Louisa told them to open the tasks. The instruction process was controlled by Louisa. She did not ask the students questions to make them think about what they needed to learn. This teaching process is similar to the Chinese examples. In China, as we mentioned above, when the teacher explains the screen or content, they control the whole process themselves, making the teaching more teacher-centered. This helps them to make the lesson continue more smoothly and without any incidents during the teaching process. Here, Louisa has the same objective in mind. Because she needed to make her students understand more about the tasks, she chose to control the teaching and not allow any accidents to happen at that point. That is why she adopted this type of instrumental orchestration.

As I said above, in the second lesson, Louisa used the *explain-the-screen* more often, although she spent more time teaching the students face-to-face. The following episode is another example of *explain-the-screen* from this lesson. And the same teaching process is common in this lesson.

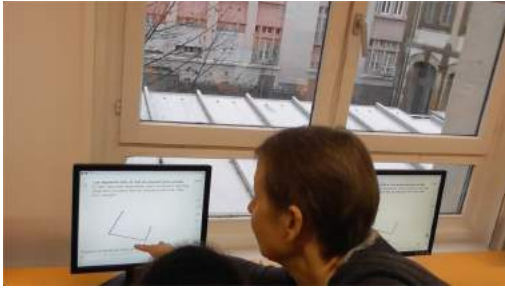


Figure 5.54: Louisa pointed to the screen to student

Here, Louisa pointed to the screen to explain the task to this student, during which time the student did not do anything with the computer. She just looked at the screen and listened to Louisa's explanation. Louisa did not control the whole explanation all the time, she would also ask the student to tell her about what happened on the screen. By doing this, the student would have more time to present her ideas. For example, at the beginning, Louisa asked the student why she got the answer and let her make her own explanation. In this process, Louisa did not interrupt the student as Chinese teachers do in their classes. In this episode, Louisa and the student always change their roles frequently. At first, the students do their own explanations, and then Louisa starts the explanation process, the students come back as interpreters. This also has similarities with the Chinese teaching episodes. The teacher and students share the control of the explanation and change roles frequently during the teaching process.

Guide-and-explain

At some point, Louisa also let students talk about the screen by answering a few questions. This happened more frequently in the second lesson. Because in this lesson the tasks required students to think about why they solved the problem this way. Louisa spent some instructional time asking students to explain their problem-solving strategies and answers.

Different from *explain-the-screen*, Louisa prepared several questions and used them to let students explain what was on the screen and to think more deeply about it in relation to the contents. Generally, Louisa asked questions consecutively during this instructional process. These questions are related to the content on the screen. As shown in the following scenario.



Figure 5.55: Louisa help the student answer the questions

In this episode, Louisa began her teaching by having the student explain his approach to solving the task, and then she asked the student a question to let him think about what tools he needed and how he could use them to complete the task. *Like with GeoGebra, follow it, well, what did you do here, what tools did you (choose)? When you turn the segment finally, (it likes to) construct what?* With these questions in mind, Louisa had the student repeat his solution step by step. And when she found some errors in his solution, Louisa had him review the previous task to find the right idea. *Well, then, good, on this page, it has done, so, how are you going to use the good tool to construct (the circle like) page 4? That's why, construct the triangle (with) triangle tool, it's the triangle, so you've done that, see page 4, and look how do you do to move the (segment and how to) find the center, then try it.* From the transcript, there is an important difference between *explain-the-screen* and *guide-and-explain*. In this episode, Louisa asked questions when the student made his explanation. But in *explain-the-screen*, Louisa just asked a question at the beginning, like “why?”, and then she gives the students enough time to present their ideas without being interrupted like the episode above. That is why the episode here is put into a *guide-and-explain* like the following figure.

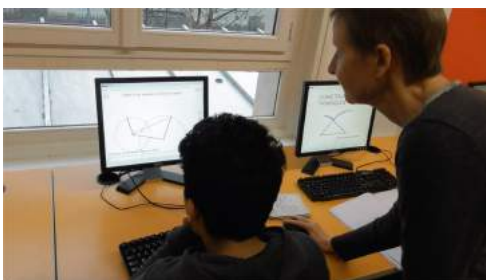


Figure 5.56: Louisa operated the software

Link-screen-board

In the first lesson, Louisa's objective is to let students know how to construct circles by using different tools. Students needed to think about how to transfer their construction procedures between different contexts. At the beginning, Louisa gave them a list of tasks that they needed to do first with paper and pencil and then with DGS to construct circles. Because of this arrangement, Louisa connected the different contexts during her instruction. For example, in the next episode, Louisa wanted students to know how to construct a circle with a given diameter. She started her teaching by asking students to review his solution with compass. She asked: *How did you do with the instruments to draw the segment AB and after how you know where you are going to put your compass.* This question let the student know that he could translate his process of solving the problem with paper and pencil to the DGS environment. As we can see from the figure, at first Louisa discussed the paper solution with this student and had him repeat the process with compass on how to determine the center of a circle of a given diameter. Afterwards, the student stated how he found the center of the circle on the paper. *St: Er... I [inaudible] the length after I divide by two.* Louisa let him use mathematical language to defining the point, *here you are divided by length. So what are you looking for, what is (the name of) this point? In a segment it's not the center but the? ... St: midpoint.*



Figure 5.57: Louisa asked questions based on the list

Then Louisa told the student that he could do the same thing with DGS. *Yes here it is necessary that you find the middle of the segment.* Louisa used this process to let students make connections between DGS and paper-pencil. She told the student to shift his strategy when solving tasks with different tools, which is the main objective of the lesson.

This teaching process happens more than once, and here is another example.



Figure 5.58: Louisa asked questions based on the list

This student was also having trouble constructing circles with DGS. Louisa asked her to think about why she constructed circles in this way. *But why you do not do it, Cadem, You wrote to me: “build at and B then draw a circle AB diameter” why do not you do that like what on the screen?* She pointed to the task list and then the screen showed some connection between the two different contexts.

From these two short episodes, we can see that Louisa is aware of the connections between different tools or resources during the lesson due to the objectives of her lessons. These tools serve for the instructional objectives of the mathematics lesson. This is one of the most important factors that influenced Louisa to organize her teaching with DGS in these different types.

This is one example of *link-board-screen* from the second lesson. As mentioned above, in this lesson Louisa did not give students a list of tasks on paper as she did at the beginning of the first lesson. All students started by interacting directly with DGS. They did not solve problems first with paper and pencil and then with DGS. At the end of the lesson, when most students were finishing tasks with DGS, Louisa gave them a new task list and then the students solved the problem with paper and pencil. So in this lesson, we did not see much time where Louisa connected traditional resources with technological resources. There was only one time where she tried to show students that they could use computer-based strategies to solve a similar task with paper-pencil. At that point, she began to explain the tasks based on the work of DGS.



Figure 5.59: Louisa let students think about paper-pencil tasks

As shown in the figure, first this student had trouble solving the task with the tangible tools. She needed to imagine how to achieve the same rotation with paper-pencil as she did with DGS. Louisa told her what she had done with the computer: *so show me, what (you) do, with what, can you show (me) with the computer, what you draw?* From this, Louisa connected the tangible tools with the computer environment to give students a better understanding of how to transform strategies in different situations. After a discussion between Louisa and other students, she found a solution to this problem.

St1: Because on the screen we have the three small segments

Louisa: As with the computer, what do you do?

St2: Madam, it's too small as it is

Louisa: And good, go ahead

St2: Oh yes, yes

Louisa: Is it good? You can tell it.

Board-instruction

In the second lesson, according to the lesson plan, after the students finished their work with DGS, Louisa had them do other tasks with tangible tools such as compass. These tasks are similar to those on the computer. Students also had to determine if some segments could construct triangles. They needed to change their strategies as the problem solving situation changed. Therefore, Louisa had also made some changes to organize her lesson. In this section, I would show examples of these teaching episodes to analyze these changes. Below is an example of one of the *board-instruction* examples.



Figure 5.60: Louisa used tangible tools

This student had already completed all the tasks using the software. Louisa then gave her a new list of tasks and told her what to do next. As we can see in the figure, Louisa told the student to choose tangible tools, such as compass or ruler, to complete these tasks. They do not need to operate software to check their work. *So, on paper it cannot be moved, but so, you (can) use the geometry tools and the compass (to emulate the moving process)*. Louisa told students exactly how to rotate with these tools, just as she did with DGS. During that time, her teaching had nothing to do with computers.

St1: Ah ?????? on the ebook

Louisa: Try to move them because actually we do not move them (only with paper-pencil), you will draw on your notebook,

St2: Dimension, three dimensions

St1: Like that

Louisa: That's it.

From the information here, it is clear that Louisa thought at that time she could teach without DGS and go back to a traditional setting due to the changes in tasks. Although the course still took place in the computer lab, DGS is not critical to this student in solving the problem.

In the following episode, the teaching process is the same. Louisa pointed to the paper and asked the student to think about how to solve the problem on this paper without DGS.



Figure 5.61: Louisa asked questions based on the list

In this figure, Louisa pointed to the paper and asked her student: *Noi, it's like what you did in the computerized notebook*. She wanted him to think about similar tasks with software, how he solved them, and then he could do similar things with paper and pencil. *Well, you do not do the same thing*. Here, she did not directly tell students the problem-solving strategy, but allowed them to reflect on their own approach.

Based on the analysis of the classroom orchestration of these teachers, some more questions need to notice: Do we need to put the episode into new types of instrumental orchestration if only one of the characteristics (didactic configuration, exploitation mode and didactical performance) has changed? And in order to identify the orchestration as a variant, do we need to let didactic configuration be fixed and allow only for variation in the didactical performance or exploitation mode? Or is it didactical exploitation the core of the orchestration type, while changing in didactic configuration is less important? (Tabach, 2013)

5.3 Interaction between teacher and students with DGS

Interaction between teacher and students in the classroom is one of the key aspects of the teaching and learning process. Classroom interactions include behavioral, emotional and cognitive aspects. Behavior can be divided into verbal and nonverbal behavior. Teacher-student interaction is a form of verbal behavior in the classroom. Generally, the teacher always participates in the class by asking several questions to communicate with the students or by letting them discuss in groups. Except verbal interaction, there are other kinds of interactions include gestures, eye expressions, etc. These all constitute the teaching process. How teachers interact with students in

different instructional contexts reflects their knowledge revealed in teaching practice. And with the integration of technology, especially DGS as mentioned above, it has changed the familiar teaching process, which has forced teachers to think about how they can adapt their teaching strategies to new contexts. Of course, these changes have also led to some new ways of interacting with students, both linguistically and other types. In this section, I would analyze the classroom interactions of Chinese and French teachers with DGS to find the differences between the two countries. From the analysis, I would show a general image of the teachers' perceptions of how they interact with students and with technology.

In this section, I have chosen several episodes from different teachers to analyze the interactions between these teachers and their students. From these episodes, we can see how teachers interact with students in dynamic geometry situations. Verbal interaction is one of the most important aspects of the teaching process. In all teaching and learning processes, knowledge is transferred from the teacher to the students through verbal interaction. Depending on the teaching context, the interaction occurs in different ways. For example, in China, mathematics lessons often take place in regular classrooms. Information transfers between one teacher and several students. By asking and answering questions, the teacher can get a clear picture of whether the students have mastered mathematics contents and whether they have difficulty in solving the tasks during the lessons. Teacher can also reorganize the teaching process based on the feedback received from the students. Therefore, verbal interaction is very important for mathematics teachers. This section describes the interactions between teachers and students when using different types of instrumental orchestration. As in the section above, I present the Chinese case first and then the French case. Each teaching episode is based on the type of instrumental orchestration used in these lessons. For example, in Mr. ZH's lesson, I start with an interaction in *discuss-the-screen* and then an interaction in *explain-the-screen*. I put this sequence of interactions in order to find if there were some differences in the focus of each question and feedback and its purpose between the teachers' different teaching styles. I would try to analyze the focus of each question to find issues that

teachers pay attention to when teaching with DGS.

5.3.1 Interaction in Mr. ZH's lesson

First I would show the interaction between Mr. ZH and his students when he let them discuss the questions displayed on the screen:

Questions:

1. What is the maximum of side C? And at that time, what is the relation among side A, B, C? What shape is it now?
2. What is the minimum of side C? At that time, what is the relation among side A, B, C? What shape is it now?
3. Based on the conclusions of question one and two, what you can find?

At the beginning, ZH let students understand the questions which they need to discuss with their classmates. As shown above, these questions are primarily about mathematical contents and have little to do with DGS. Students need to find a mathematical relationship between the elements of a geometric object and they do not need to interact with the software to find the answer. In line with the focus mentioned in Section 3.5, these questions indicate that Mr. ZH is more concerned with the mathematics itself and DGS is simply an aid tool for students to find the answer without further manipulation of the software. The following part is a transcript of ZH's conversation with the students during the teaching process:

T: You can discuss with each other and then I let one of you answer this question. You can think while seeing the diagram.

(Discussion)

T: Is there anyone have the answer? If you have, you can raise your hand.

You finish?

ZH did not spend much time talking to the students during the discussion, but told the students at the beginning of the discussion what they needed to do next and what he wanted from the discussion. Just as the transcript shows, ZH just told the student to start the discussion: *You can discuss with each other* and tell them what he wanted

them to do after the discussion: *I let one of you answer this question.* When the students began to work with their classmates, ZH stopped talking and walked around the classroom to see what they were doing during that time. No verbal interaction took place between ZH and the students. He only acted as a listener and did not participate in the students' work. When he noticed that the students had completed their work, he began his next teaching process with the following sentence: *Is there anyone have the answer? If you have, you can raise your hand. You finish?* It is evident from the interaction that ZH was focused on mathematics when he had the students discuss the mathematical questions. There was no DGS in these questions. Students only see the numbers on the screen and no longer manipulate the software.

This process is a typical teaching episode in almost all Chinese classes. When the teacher has students work together to solve a task or discuss an issue they have raised, the teacher always becomes a listener rather than a frequent participant in the conversation. Although they may use DGS for instruction, the software does not play an important role in the discussion process.

Next part shows the interaction happened in the type: *explain-the-screen* and *guide-and-screen*.

T: OK, the first one, what is the maximum of side C? Let me ask one student..... you, try to answer it.

St135: The maximum is 10

T: OK, maximum is 10. I have said in the video, so the maximum is 10. Now what is the relation?

St1: They are all on the same line.

T: Good, he said on the same line. Let us see, this is position. OK, now C is 10

[the teacher moves the mouse, and moves point A clockwise] and they are all

³⁵ Here the number of student (in all the transcripts) means different student who talked during the interaction. Same number in one transcript means same student. But same number in different transcripts does not always mean same student.

on the same line. But what is the mathematical relation among the three sides?

St1: $a+b=c$

T: Good, $A+B=C$, now it is not a triangle?

St1: No

T: Good, not a triangle. If I need it become a triangle, I need it be a triangle, you can make what conclusion? You can see now it is a triangle, what conclusion you make? [the teacher moves point A, to make a triangle ABC]

St1: $A+B$ is smaller than C .

T: $A+B$ is smaller than C ?

St1: Ah, no, it is bigger.

T: Yes, it is bigger. Good, now we let it become a triangle, and we find $A+B$ is bigger than C . OK, sit down.

This interaction took place right after the discussion mentioned above, where ZH repeated to the students the question to be answered: *what is the maximum of side C?* He then wanted to choose a student (St1) to answer the question. It is clear from the conversation that ZH's focus is still on the mathematics itself. His main questions are the same as those in the discussion. There is no DGS in these questions: *Now what is the relation? What is the mathematical relation among the three sides?* In contrast to the discussion process, ZH manipulated the software when he asked questions. For example, he used the software to explain whether a student's answer is correct: *Good, he said on the same line. Let us see, this is position. OK, now C is 10. [The teacher moves the mouse, and moves point A clockwise]* And also when he let this student make conclusion he operated the software again: *Good, not a triangle. If I need it become a triangle, I need it be a triangle, you can make what conclusion? You can see now it is a triangle, what conclusion you make? [the teacher moves point A, to make a triangle ABC]* But I also find that while Mr. ZH manipulated the software, his questions and feedbacks were still only focused on the mathematics, and there is still no DGS in those questions.

Next transcript comes from the second lesson of Mr. ZH. He also uses *explain-the-screen* and *guide-and-explain* in this teaching episode.

T: Ok, let me choose this student to explain. It is your answer, can you tell us, why you draw these pictures?

St2: Because, I think, it is moving along AB, it moves anti clockwise. So I can divide it into some parts

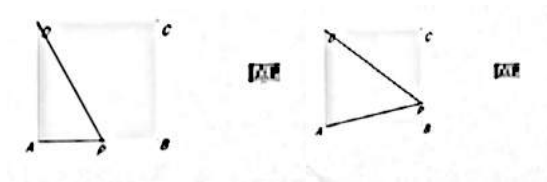
T: This point is moving right? How you can draw it like this?

St2: See this point as a fixed point

T: Why you see it as a fixed point?

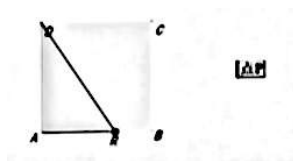
St2: It is on

T: Look, [there is a square on the screen] this point is moving, it is moving. [the teacher click the bottom on the screen then the point P began to move, like the figure]



St2: But I divided it into several parts, in these situations, it is when point P is moving on segment AB

T: Point P is moving on the segment AB, for example [teacher dragged the point P] point P is moving, she explained what she drawn, what she has drawn? It means one special time right?



T: At this special time the point P is fixed right? So we can study, in this special time, first, we need to find the special time and see the moving points as a fixed point. Then we can solve it, it is fixed, right? For example, on the AB, no matter how the point P moved the points can construct right triangle APD right? But the length of the side is changing, right? Ok.

During the interaction, the purpose of the interaction focused on solving mathematical tasks with the help of DGS. At the beginning, ZH began with the question: *It is your answer, can you tell us, why you draw these pictures?* And then

had this student explain how she solved this task. This question here is only about mathematics content. There is no DGS shown in the question. We could not find any words related to the software, such as *drag*, *moving*, etc. After that, ZH chose to operate the software, which led to a change in his focus. Like this question: *This point is moving right? How you can draw it like this? Or Look, [there is a square on the screen] this point is moving, it is moving. [the teacher click the bottom on the screen then the point P began to move, like the figure]* In these questions or statements, I found that ZH used some words like *move* and *look* to focus the students' attention on the diagrams on the screen. Also, he dragged the point P to move it along the square. Here, his focus is on letting students know how to solve the task with the help of DGS, even though they do not know how to operate the software directly. As ZH moves the point, they can see, think, and observe the situation directly. At the end of the interaction, when ZH made an explanation based on the students' own work. He continued to move the points and asked students to notice what was changing and what was not changing in the diagram. For example, *on the AB, no matter how the point P moved the points can construct right triangle APD right? But the length of the side is changing, right? Ok.*

Through interaction, we can see small changes in ZH's lessons. DGS becomes more and more important in the interaction with the students. In the first lesson, although he used the software, his questions were only about mathematics. But in the second lesson, he was manipulating the software more and his questions or statements were more related to DGS. And he hoped it would help his students pay attention to the answers to the questions.

In the next episode, ZH uses *spot-and-show* to organize his lesson. The transcript describes how one student explained her solution based on the questions ZH poses:

T: Ok, Ma, can you tell us, why you draw so many right triangles?

St4: We can see, in the problem, there is one sentence that the point moves on BC, then, it means the area where point E can move

.....

St4: I find the area, then, I find it means, point B need to arrive point B', it is

the basis of fold it is determined by AF. So.....

T: Point B' is determined by AF, it determined the trace right?

St4: Yes

T: Then?

St4: So, I think I can find something not change,

T: She said, what she said is she found something not change. What is the principle? What is the things not change?

St4: It means, we know the right angle in this triangle and AB

T: Ok, she said in this right triangle ABE, here is E, in triangle ABE, she found something not change, for example, AB and the right angle B, why you find these objects?

St4: Because, I, two points, think it is a little difficult. If we face something not change, then we may find the regular of the movement of point B'. We have already known the position of point F

T: very good

It is important to note in this teaching process that Mr. ZH used DGS to present the students' answers on the screen. And students just solved the tasks with paper and pencil, which could never be moved or changed. Therefore, ZH did not manipulate the software again in the rest of the lesson. ZH's questions and statements here returned to focus on the mathematical contents, for example, he began with *can you tell us, why you draw so many right triangles?* to let student talked about her solving strategies. There is no DGS in the questions, and we cannot find any words related to software, such as *move* or *drag*. This student and ZH, they talked more about the task itself around mathematics. *St4: We can see, in the problem, there is one sentence that the point moves on BC, then, it means the area where point E can move.....St4: I find the area, then, I find it means, point B need to arrive point B', it is the basis of fold it is determined by AF. So.....* Here, this student talked about a solution based on her answer with paper-pencil, and she did not use any software to solve the task, nor did any other students. Thus, although she used some words like *move* or *change* here, the statements still had nothing to do with the software and she

did not interact directly with DGS. The same phenomenon manifested itself in their next conversation. Mr. ZH's question was still about mathematics: *She said, what she said is she found something not change. What is the principle? What is the things not change? she found something not change, for example, AB and the right angle B, why you find these objects?*

5.3.2 Interaction in Mrs. J's lesson

This section describes the classroom interaction of another experienced teacher (Mrs. J) in China, who is from the same secondary school as Mr. ZH. As mentioned above, Mrs. J is also very familiar with many information technologies in mathematics education. During the interview for this study, she told me that she often used some software in her lessons, such as *Express Everything*, which is one of the software that helps teachers to draw geometric objects with iPad or Sketchpad. And she is very interested in exploring different software. After lessons, she often explores the software on her own and tries to construct different diagrams. Afterwards she would drag the points to see how the diagrams changes. Sometimes she would also search on the website for different tasks or read books to learn how to use different software in her courses.

I also begin my description based on the type of instrumental orchestration that Mrs. J used in her two lessons. The following recording shows the interaction between Mrs. J and one of the students who attempted to introduce his strategy guided by the questions. It is from *explain-the-screen* and *guide-and-explain*.

T: You need to justify what type is the quadrilateral

St1: It is diamond

T: Diamond. How can you know it?

St1: First point G and M are coincided.

T: Yes

St1: Then, DM, point D is the midpoint of AB, point M is the midpoint of AE, so DM the median line of this triangle.

T: Do you agree? DM is the median line of triangle ABE. You know it is median line, then?

St1: Then DM is parallel to BC

T: DM is parallel to BE

St1: So angle MDN is add angle END is 180

T: It is this angle, angle 1 and angle 2, angle 1 plus angle 2 is 180

St1: Then, it said, angle GEN is equal to angle MDN and AEB

T: Angle 1 is equal to this angle, angle 3. It means this angle is equal to angle 3, it change

St1: So angle 2 plus angle 3 is 180, then DN is parallel to ME

T: Why you justify DN is parallel to ME?

St1: So NDME is parallelogram

This interaction was around the first task: how to prove that the quadrilateral DMEN is a rhombus (as shown in Section 5.2.2). Mrs. J first asked one student to explain his proof of the task. At that time, Mrs. J did not ask the students too many questions. Most of the time, she just repeated the answer or used *then, ok* to let the student continue his explanation. Or she would pay attention to other students to see if they understood this student's answer, just as she asked other students: *Do you agree?* Neither mathematics nor technology was involved in this kind of question. The main questions that Mrs. J. asked at the time were more mathematically oriented, and we could not find DGS in them, like the question: *Diamond. How can you know it?* or *Why you justify DN is parallel to ME?* These questions are presented to let students make formal proofs of geometric contents, and they do not need to manipulate software to help them find the correct answer.

The next interaction followed when this student (St1) fail to make formal proof to the question. Mrs. J asked another one to continue explanation.

St2: In the conditions, AE is equal to BE

T: AE is equal to BE, what it means?

St2: So we know MD is the median line. Then, MD is 1/2 BE

T: Ok, the median line can help us know not only this one. But also DM is

equal to $1/2$ BE.

St2: Because point M and N, M is the midpoint of AE, then, so, it is also useful, it means AM is equal to ME and $1/2$ AE

T: Equal to $1/2$ AE

St2: Then, AE is equal to BE, so MD is equal to ME

T: MD is equal to

St2: ME

T: So now, when AE is equal to BE, this condition is also useful, AE is equal to BE, another one is ME is equal to, we need another condition, it is ME is equal to $1/2$ AE. Ok, from these two conditions, we can find ME is equal to DM, then it can be a diamond

Here we can find that this interaction is similar to the one above. Mrs. J still did not ask the student too many questions and she chose to give him enough time to present his ideas. She just repeated this student's answers or statements so that other students could clearly understand the explanation. During the interaction, we cannot find any words related to the software or operations like Mr. ZH did (he used some words like *drag* and *move* to show what he was doing with the software at that time). So this interaction shows that the teacher's focus was on the mathematical contents and not on the DGS.

The following transcript is from another instrumental orchestration from Mrs. J's class, where she had a student present the construction program for a diagram and manipulate the software based on the student's guidelines.

T: Now let us make a review. Who can tell us how we draw this figure? Huang.

St: First from point O, we put another point P randomly

T: Put point P randomly

St: Then

T: Then?

St: Then find the midpoint of OP

T: Then find the midpoint of OP, it is?

St: H

T: H

St: Construct the perpendicular line of OA which passes point H

T: Perpendicular line

St: It intersects with AB at point C

T: The intersects, point C, and then...

St: Then construct ray PC

T: Construct ray PC, is that right?

St: Intersect with AB at D

T: Intersect with AB at D, and then...

St: Then, link OD

T: Link OD, it is...?

St: The radium

T: Radium, and then, we can make the line bold to make them more clearly.

This recording shows the interaction in which Mrs. J had a student tell her how to construct a diagram based on the task situation (Section 5.2.2: the task in the second lesson), while she herself operated the software based on the student's answers. Like other interactions, Mrs. J did not ask too many questions when students presented their answers. Instead, she chose to repeat the answers or ask other students if they understood or agreed with the idea. The transcript shows that both the teacher and the student were more focused on the mathematical contents during the presentation. And they did not mention the main function of DGS: *dragging mode* which is the most important feature for users to explore geometry. While we can see some verbs, such as *put* and *find*, these words do not point directly to the software. If the teacher is not using any DGS in the classroom, we can still find these words in the interaction. In the case of paper and pencil, the construction program is also present. This is because for the students, they are not using the software to construct the diagrams as they solve the problems. During the classes, Mrs. J used technology such as DGS and iPads. However, her interaction with the students is more mathematical and less related to DGS, unlike Mr. ZH (shown in section 5.4.1).

5.3.3 Interaction in Mrs. Y' lesson

After seeing two experienced teachers' interaction with students in a DGS situation, we need to focus on another group of teachers in China who do not have as much experience with DGS in their classes. The other two secondary school teachers in this study have only three years of experience as mathematics teachers. One of the teachers has not studied any technology, especially DGS, before the study. Therefore, I am interested in analyzing their interaction with students by using DGS in the lessons. This, in turn, helps us to understand in detail the teachers' practices with DGS. The following interaction between Mrs. Y and her students demonstrates how students were engaged in the process of operating the software and making explanation.

T: Here, this angle A is 30° , AB is also given, it is marked in black. So which one can I move? Which one? Which one?



St 1: Point B

T: Point B, angle C can also change right. Point A can also, [teacher drag

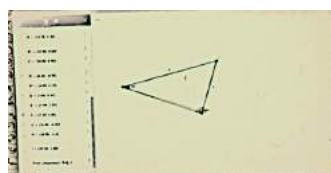


point B and C to show the change of the triangle]

it means only this angle 35° and this one do not change, angle B changes also.

Only these two unchanged. So if we determine two elements, the type of the triangle cannot be determined. Ok, then, look, if two angles are given. Which one?

St 2: That



T: Look, this one. This one is enlarged, look.

This

one is unchanged. Why we cannot change it?

St 3: Because this angle is locked, but we can enlarge it.

T: Because 35° , this angle E and D, these two angles are determined, and the sum of three inner angles of a triangle is 180, so the other angle is also given.

So it is unchanged. So all the three are unchanged. So what can be changed?

St 4: Length

T: Length of sides, is that?

St 5: Yes, similar

T: Because, yes, we will learn similar triangle in grade 9. Two angles are unchanged, and the sides can change. We want to determine the type of triangles. Here its form is not changed but the size is changing.

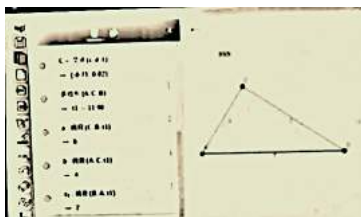
During the interaction, we can see that Mrs. Y was explaining to the students what was on the screen as well as operating the software. From the interaction, Mrs. Y asked students to think about which points could be moved with the software: *so which one can I move? Or why we cannot change it?* She wanted students to think about the tasks based on the DGS situation when she interacted with them, as shown in the diagram in the transcript. In addition to focusing on mathematical contents, these questions also expect students to use technology to find the geometric properties of triangles. Through the questions, Mrs. Y wants students to notice how to determine the size and shape of a triangle and to use DGS to check if their answers are correct. And from the students' statements, we can see that their answers also include some elements of DGS, as one of the students (St 3) said: *St 3: Because this angle is locked, but we can enlarge it.* He used words like *enlarge* which described the main function of DGS. What he meant was that he could drag one of the points to magnify the size of the triangle so that he could not determine the size or shape of the triangle.

If compared to experienced teachers, we found some differences in their interactions with students, although the type of instrumental orchestration was the same. For example, when *explain-the-screen*, Mrs. J focused on the mathematical contents, which was the same as for Mr. ZH. This shows that mathematical

knowledge is most important for these two experienced teachers. Of course, Mrs. Y also wanted her students to learn specific mathematical knowledge after the lesson, but this objective was obtained through the use of DGS in the teaching process. Her explanation is about using DGS to find the geometric features of triangles, and she uses more of verbs which is related to the software when interacting with her students.

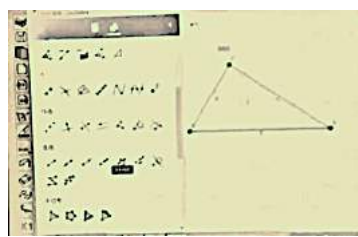
The following interaction is another example of *explain-the-screen*. During the interaction, the main question is about how to construct a triangle with three sides by using DGS. Mrs. Y asked several questions to involve the students in the construction process, and she explained the method while working with the software.

T: Let's draw another triangle whose sides are also 4, 7, 6. Then we can overlap the two triangles and see whether the two triangles are coincided, if they are coincided, then the two triangles are the same, congruent, ok, let us review it, (in) triangles, if the three sides are equal. How to draw it?

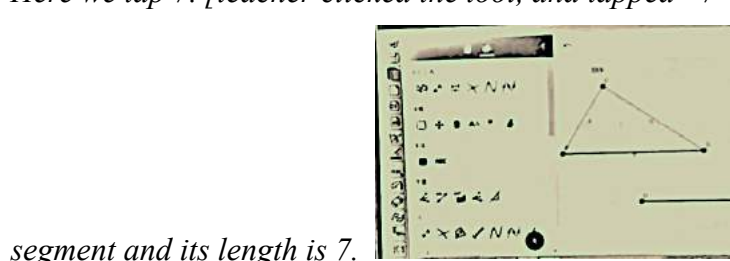


St 1: First draw one base side

T: First draw one base side. [teacher operated the computer, choose the tool



“segment”] We can find the tool “segments with given length” in the tool “lines”. Then here, we draw a segment, 7, right? Here we tap 7. [teacher clicked the tool, and tapped “7”] Now you can see a



segment and its length is 7.

St 2: Then we draw

T: Ok, then, how to draw this point on side AC, that is side DF?

St 3: Use compass

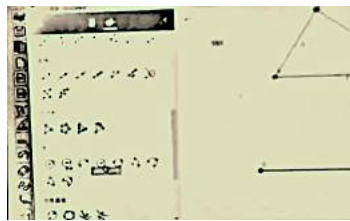
T: Use compass, it means its radius is?

All St: 4

T: 4, which one is the center?

St 4: D

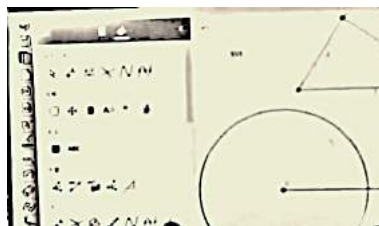
T: D, ok, now we can find circle. [teacher operated the computer, find the tool



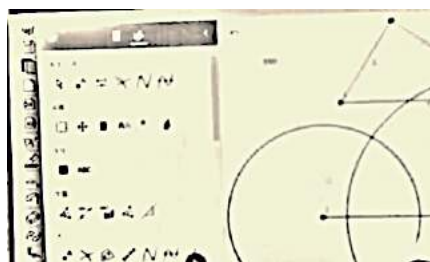
“circle”] Then you can click it, it showed “one center and one point”. We need to find the circle and radius, it means the radius is determined, click it. [teacher clicked the tool “circle with center and radius”, then looked at the students] Ok, now point D is the center, click it. What is the radius?

All St: 4

T: Then here tap “4”, then you can see a circle. [teacher tapped “4”, the screen showed one circle whose center is D and radius is 4



] Ok, then is this side, its length is 6. Ok, its center is point E, what is the radius? 6. [teacher clicked “circle with center



and radius”, tapped 6] Ok, now you can find the intersect point of the two circle?

St 5: Two

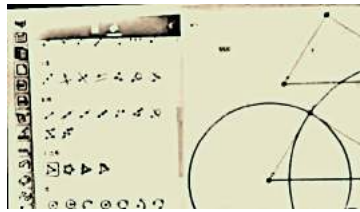
T: Yes, two, it's the same with the two points up or down

St 6: Yes, it is same

T: Ok, same, here I just draw one, this point, which is the intersect point?

St 7: F

T: F, it is F here, then, we can draw this triangle. [teacher clicked the intersect point] Link these three points, DEF. [teacher operated the mouse, linked point



D, E, F] This triangle, ok, these lines are supportive, we can delete them.

If we focus on the questions, we can see that Mrs. Y asked a lot of questions that are related to mathematical contents. For example, she asked students: *it means its radius is? Which one is the center? Which is the intersect point?* The DGS did not exist in these questions at that time, nor did the students need DGS to find the correct answer. Although DGS was working during that time when Mrs. Y introduced her students how to construct triangles, the questions only focused on the mathematical contents. We should also note that Mrs. Y did not ask too many technical questions, but there were some remarks about how to operate the software. For example, when she asked her students to note which tools could be used to construct triangles with DGS, she said: *We can find the tool "segments with given length" in the tool "lines". Ok, now we can find circle. [teacher operated the computer, find the tool "circle"] Then you can click it, it showed "one center and one point". We need to find the circle and radius, it means the radius is determined, click it. [teacher clicked the tool "circle with center and radius", then looked at the students] This triangle, ok, these lines are supportive, we can delete them.* From the interactions, Mrs. Y focused more on mathematical contents and DGS was one of the supporting tools in teaching process. When she needed to operate the software, her interactions with the students at the time were more related to DGS, like what was shown in the transcripts. For her, DGS is a tool which can help students know how to construct geometric objects

with paper and pencil. She told students in class: *If you can draw this one (DGS), then you can also draw it with paper pencil.* For us, we do not check if Mrs. Y is right, but we can note that DGS can help her students get what Mrs. Y wants them to know in the curriculum.

Then in Mrs. Y's second lesson, I pay attention to a different type of instrumental orchestration. The main task is still about how to construct triangles with DGS with given data. In this type, a student plays the role of *Sherpa student* to manipulate the software and solve the task by himself. The following interaction took place when this student had some difficulties in constructing angles with DGS and Mrs. Y explained it to him by using DGS.

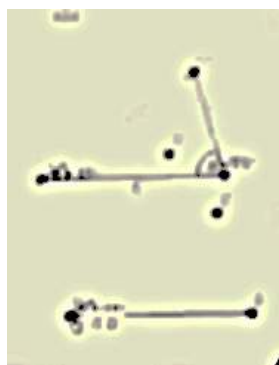
T: I can do it first. Look what to do next. When we draw an angle, like angle D, here point D. This letter should be in the middle, so, what we need to click first?

St 1: E

T: Because DE is one side of this angle, we already have one side, so we need to find another one. Ok, what is this angle? Clockwise or anticlockwise?

St 1: Anticlockwise

T: Ok, then, it means [teacher clicked "anticlockwise", screen showed one

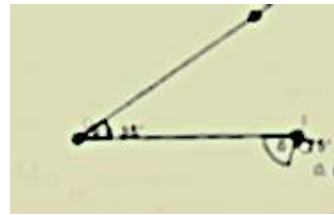


new angle and point E'] D, here we have point E', it is on another side of this angle. So, we first represent this side, then we find [teacher choose the tool "line"] the line, you click this point. You first determine another side, on it. It means point F should be on this line. Ok, try to draw another one

[Student working time: student choose the tool "angle with degree given", tapped the degree, teacher saw what the student do]

T: If you can draw this one, then you can also draw it with paper pencil

[Student working time: student showed the angle with hands, showed the



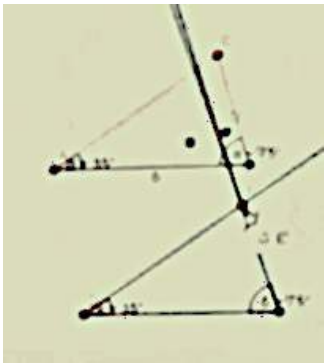
angle with hands, choose anticlockwise, laugh]

T: It should like this

[Student working time: student choose the tool “angle with degree given”, clicked point D and E, tapped the degree]

T: Look, you have this side and you need to find another

[Student working time: student choose clockwise, teacher showed clockwise with hands, choose suitable tools, clicked the tool “ray”, clicked point E and E’, deleted the ray, choose suitable tools, turned the angle EDE’ with mouse, clicked the tool “ray”, clicked the two points on the screen, deleted the ray



St 2: Clockwise, clockwise

St 3: Point E’

T: Point E’ is correct? [student clicked point E’ again, someone laughed



, teacher let the student go back, operated the computer]

It is not point E’, look, why not point E’. Here line DE’ is for assistance,

because point F is on this line.

St 4: This line

T: Then, this angle, look, if you enlarge it, you can find some mistakes. If you link point E. Look, here, you can find a little space, it means that this line is not the side of the angle, which is? You can link this line. It is the angle you want.

Because in this interaction, a student needed to manipulate the software to construct triangles with given data, neither Mrs. Y nor her students had much experience with DGS before participating in this study. Therefore, it is common for students to have many difficulties in solving tasks with DGS during the class. Here, Mrs. Y would like to tell this student the correct way to solve the problem with DGS. Analyzing the questions and statements, I find that Mrs. Y was focusing on letting the students know how to solve mathematics tasks with DGS. Her questions and statements were focused on the use of technology. For example, she asked student: *so, what we need to click first? Clockwise or anticlockwise?* (which is shown on the screen)? These questions do not exist if the students construct geometric objects only with paper and pencil. And at the end, Mrs. Y focuses on the main function of DGS, *dragging mode*, which changes the size of the triangles students constructed and shows the errors in those diagrams. *Then, this angle, look, if you enlarge it, you can find some mistakes. If you link point E. Look, here, you can find a little space, it means that this line is not the side of the angle, which is? You can link this line. It is the angle you want.*

Unlike experienced teachers, the different instrumental orchestration affected how Mrs. Y interacted with her students. When she was explaining the contents to her students, her focus was more on the mathematics itself, but when she had her students manipulate the software, DGS became a focus for her. If we compare her teaching to the two experienced teachers above, we can see that both two experienced teachers do not change their interactions with students no matter what type of instrumental orchestration they used in their lessons. However, the type has a strong influence on Mrs. Y.

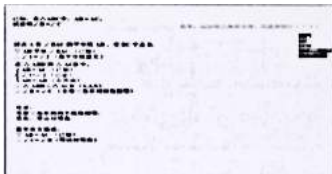
5.3.4 Interaction in Mr. W's lesson

In this section, another example of a young teacher's lesson shows how he interacts with students in a DGS situation. We would begin with the interaction in *explain-the-screen*. This is the first task described in section 5.2.4. Mr. W wanted to show students the properties of isosceles triangles.

T: Ok, equal edge to equal angle, I write it on the board. Look, how we write it, we can write it like this, because AB equal to AC , so angle C equal to angle B . Because we know equal edge to equal angle, what is the opposite angle to edge AB ? It is angle C ? so, the opposite angle of AC is?

All St: Angle B

T: Next, we know congruent triangle, then we can get any other conclusion? We have talked about one of them. Here it is the mid line. Ok, except BD equal to CD , can we get anything else? We know the edges are equal, what else?



All St: Angles are equal

T: Angles are equal, so, now except two bottom angles are equal, any other angles are also equal?

St 1: ABD equal to ACD

T: It is the second method, we added angle bisector here, ok, do we have any other equal angles?

All St: ADB equal to ADC

T: ADB and?

All St: ADC

T: These two angles are equal, so what is their position?

One St: adjacent supplementary angle

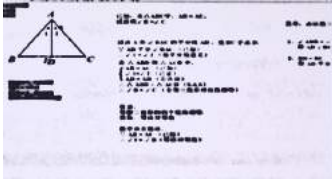
T: Supplementary angle, so the sum is?

All St: 180

T: 180, so they are all...?

All St: 90

T: So here because they are congruent, we also know the angle is 90° , it means what is the relation between AD and BC? Perpendicular. Ok, we have said, the second, angle bisector. We have talked it, not, AD, look, this line AD is...? AD, we have known it is perpendicular to BC, so AD is ... of edge BC?



St 2: Height

T: Height, because we know BD equal to DC, so AD divides BC into same parts, so AD is what? En? AD divides the edge.

St 3: Mid line

T: Mid line, it is the mid line of BC. Ok, according to the line we add here, AD is what of angle BAC? What line?

St 4: Angle bisector

T: So AD has how many different roles here?

All St: Three

T: Three roles, it is the...of top angle?

All St: Angle bisector

T: Angle bisector, it is also the...of bottom edge?

T: Mid line

T: Mid line, and it is also?

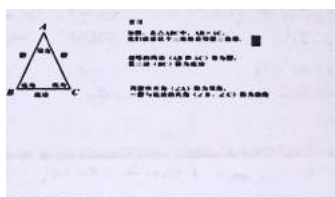
All St: Height

With this transcript, Mr. W, like other teachers, focused primarily on mathematical contents. We do not see any DGS in the questions posed to the students. Mr. W began with *I write it on the board. Look, how we write it, we can write it like this, because AB equal to AC, so angle C equal to angle B*. His emphasis on the language of mathematics to his students suggests that his main focus during that time was on

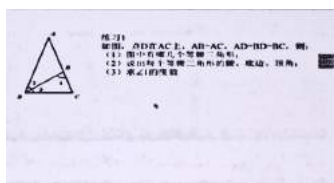
mathematical knowledge, not DGS. After, his questions, like: *Ok, except BD equal to CD, can we get anything else? We know the edges are equal, what else? Any other angles are also equal? So they are all...? What is the relation between AD and BC?* are all about the properties of isosceles triangle. We cannot find any words related to DGS, such as *move* and *drag* used by Mr. ZH in class and *click* used by Mrs. Y. From the figures in the transcript, it is clear that DGS was acting as a projector to present the words and answers to the students, and there was no movement during the teaching, nor did Mr. W operate it. All of this shows that Mr. W saw DGS as a tool to show students the mathematical contents, not as a tool to help them learn mathematics. Also unlike the other teachers, even though he was explaining the contents through the screen, he did not operate DGS during that time. Other teachers, on the other hand, interacted directly with DGS when they were explaining mathematics to their students.

In the following interaction, a similar phenomenon occurred in Mr. W's lesson, but at this time he posed a few questions and asked three students to explain what was on the screen via DGS.

T: Look at the definition of top angles, it is the clip angle of two waists. The clip angle one waist and bottom edge is bottom angle, any question? [teacher



clicked the bottom] Now look at this figure, $AB = AC$, $AD = BD = BC$, how many isosceles triangles? Tell me one. [New



questions]

St 1: ABC

T: ABC is an isosceles triangle, so its waist is?

St 1: AB, AC

T: AB, AC, bottom edge?

St 1: BC

T: BC, top angle?

St 1: Angle A

T: Angle A, ok, sit down. Gu anything else? Except ABC. Ah it is too small?

Come here, first you know isosceles triangles have two equal edges. So now here BC is equal to what?

St 2: BC is equal to BD

T: DB, so which one is isosceles triangle?

St 2: BCD

T: BCD, it is an isosceles triangle, which is waist? Li, help him

St 3: BC, BD

T: BC, BD, bottom edge?

St 3: CD

T: Top angle?

St 3: Angle DBC

T: Angle DBC, sit down

If we pay attention to the questions, we would see the same focus as the interaction above. All the questions are only about mathematics, like: *how many isosceles triangles? So now here BC is equal to what? Which one is isosceles triangle?* It is clear that DGS did not play a critical role in the teaching process. Like we described in section 5.3.4, Mr. W used the DGS primarily to show the tasks and answers to students. It mainly displayed static figures. During the interaction, what Mr. W did is click on a button, as in the transcript shown above, to present the students the conclusion and answer to the questions. Through Mr. W's lessons with DGS, we cannot see the moving process. This is one main difference compared to other Chinese teachers.

DGS becomes less important to Mr. W and his students when he used only the blackboard to teach. All the students paid attention to what Mr. W wrote on the board:

T: So which way can we use (to solve the problem)?

All St: Congruent

T: Only congruent, ok, we can put angle B into which triangle?

All St: ABD

T: Triangle ABD, angle C?

All St: ACD

T: Triangle ACD, then, if I want to check these two triangles are congruent, how many conditions do we need?

All St: Three

T: Now we have how many?

All St: Two

T: Who are they?

All St: AD equal to AD

T: AB equal to...?

All St: AC

T: AB equal to...?

All St: AC

T: This is one

St 1: AD equal to AD

T: AD equal to AD, then?

All St: Angle ADC equal to angle ADB

T: ADB equal to ADC, now I do not write this down, not write down the process, look, now, AB is here, AD is in both triangles and we have one more angle, what is the method? [teacher wrote the contents on the board



St 2: Side, side, angle

T: Is the method ok?

All St: No

T: What can we do?

St 3: BD equal to DC

T: BD equal to DC , it means how can we do that?

St 4: Mid line

T: We add...what on BC ?

St 5: Vertical line

T: Vertical line?

St 6: Mid line

T: Here we only need the equal edge, right. So here we just add the mid line.



Means we add...of BC and pass A ?

All St: Mid line

T: Mid line, what can we call it?

St 7: AD

T: AD , then it intersects BC on D , ok, now in these two triangles, we have



which condition? Here, one more

All St: BD equal to DC

T: BD equal to CD . So now the condition should be written above, because we add mid line, what we need to add? So...

All St: So, BD equal to CD

T: BD equal to CD , then, justify congruent triangles, so triangle ABC is congruent with triangle ACD , why?

All St: SSS

T: Three edges, so

All St: Angle B equal to angle C

The type of instrumental orchestration shown in his lessons did not change his interaction with the students. The main questions were still focused more on the

mathematical contents without any DGS. It was more evident in *board-instruction*. Mr. W abandoned technology and went back to using traditional resources, making his lessons more like traditional lessons without the use of computer science. What Mr. W did in this process was to write the proof process on the board. This reflects the intent of his lesson. He did not change the way he interacted with the students, but of course he did not need to do so because he was not using DGS.

Only when he needed to make connections between different resources: DGS and blackboard in this class, he would talk about something software-related, as the following transcript shows.

T: Yes, mid line is ok, any different ways?

St 1: Angle bisector

T: We have seen when we fold the triangle, look, when we fold it, angle BAC, if we link this edge AC, look, here, angle BAC, this point E, what is the relation between BAE and angle BAC?

St 2: Ah, we can add angle bisector

T: What is the relation between BAC and BAE?

St 3: BAE is the half of angle BAC

T: It means double, right? The big one is double of the small, so, we can add?

All St: Angle bisector

T: Angle bisector, so how can we use the second method? How to write it?

From point A we can add what?

One St: Bisector

T: Bisector, ok, we can remove this, ah. Ok, we can add one angle bisector AD. Ok, it means we add the bisector AE of angle BAC which passes point A, and it intersect with BC on point E, next? AB equal to AC, AD equal to AD, they are the same, what we need more?

All St: Angle 1 equal to angle 2

T: Because angle 1 equal to angle 2, so they are congruent, how can we check it?

All St: SAS

T: So, we can also know angle B equal to angle C. Look, here the first sentence, we add angle bisector which passes point A, then we have these two angles are equal, then we know the triangles are congruent. So here we get the first property of isosceles triangle, its two bottom angles are equal. Ok, we can call it?

All St: Equal edge to equal angle

During this interaction, he used some verbs related to manipulating the software, such as *fold* and *remove*, although there were not many of them. During this time, Mr. W manipulated the software to show the students the diagrams, just like the figure in the transcript. The main interaction was about mathematical contents. The questions include: *What is the relation between BAE and angle BAC? What is the relation between BAC and BAE?* This reflects his objective in this interaction to show students how to make formal proofs on the examination and write them down on paper. Based on this objective, DGS was certainly not a critical factor. And it reflects Mr. W's view of the role DGS plays in the teaching of mathematics. He talked during the interview, *I think maybe students will rely on the using of DGS during their learning and never think by themselves when solving the tasks.* His opinions on technology are indicative of the main thinking of Chinese mathematics teachers, who, due to the limitations of the examination (students are not allowed to use any technology, including calculators), have chosen to use DGS more carefully in their lessons to avoid making students rely on it when solving problems, although DGS has played an important role in some tasks.

In this study, Mr. W did not show more changes in his interactions with the students during the lessons. In these episodes, although the way he used DGS was different, his interaction with the students was almost identical. Several questions are posed and one or more students are asked to answer them in order to achieve his objective for the lesson. From these interactions, the questions Mr. W asked were more about mathematics and had nothing to do with the DGS.

5.3.5 Interaction in Mr. X's lesson

This section describes the last Chinese teacher's interaction with DGS. Like the first two teachers in this study, Mr. X is familiar with technology, although he does not have as much teaching experience. For him, technology can be an important supporter of mathematics learning for both teachers and students. We begin with the interactions in *explain-the-screen* of this lesson.

This interaction happens after students' solving the task by themselves. It is about how to find the area of the triangle APR in the task shown in section 5.2.5.

T: Attention, we, first, discuss the methods. Anyone use "large minus small"? Most of you use it. I have seen one use bottom times height. It maybe a little difficult. You did not know the height. We have seen "large minus small", but it has different situation. First OCAB is one trapezoid minus three triangles, others said OCAR, it is a trapezoid moving minus three triangles. Which one you choose?

St 1: Moving

T: Moving or not moving

St 2: Moving

T: Ok, both are right. On my opinion, I will choose the one not moving. Why? If you make mistakes, the bigger one can help you to get some scores, it is fixed. The small ones maybe a little difficult. After finding out the first one, another one, tell me

St 3: I prolong CA, and then

T: Prolong CA, oh, I know, we have said, we can make it a ...

All St: Rectangle

T: Rectangle, square or rectangle?

All St: Rectangle

T: Make it a rectangle, it also minus three objects. This rectangle and that trapezoid are both fixed. Their area is fixed. So this method, "large minus small", contains these situations, trapezoid minus three objects, trapezoid

minus two objects, rectangle minus three objects. So here, we use most is trapezoid minus three objects.

From this transcript, Mr. X wanted to explain the main methods that could help students know how to solve a set of tasks with similar conditions. His questions were more about general methods that were not just for solving specific tasks like the tasks in this lesson, his objective was to help students summarize the process of solving the problem. So in the beginning, Mr. X did not specifically ask questions related to this task: *Attention, we, first, discuss the methods. Anyone use “large minus small”? Most of you use it.* Here he mentioned one of the methods which was common used in solving the tasks related to the area of complex geometry objects “large minus small” in China. Certainly from the statements, we do not see any technology or DGS. After, he mentioned this specific task in class, he said: *First OCAB is one trapezoid minus three triangles, others said OCAR, it is a trapezoid moving minus three triangles. Which one you choose?* Here, although he used the word *change* which might lead us to think it was about DGS, Mr. X was not manipulating the software to make the diagrams and screens change when he was talking to the students. So I still think the focus of the question is on the mathematical contents. And judging from the following statements, DGS is no longer there. Their conversation was about mathematical methods rather than focusing on using DGS to solve problems.

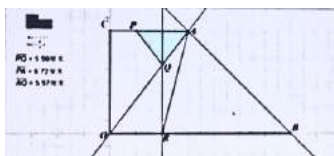
A similar phenomenon occurred in another interaction. At that time, Mr. X explained the answer to sub-question 1 of this task. He mainly used the board in his teaching and when he wanted his students to understand the changes in the diagram, he manipulated the software to move the points to the position he wanted.

T: So what is AQ? First, some of you find the coordinate of point A is 3 and 4. Someone said 3, 4, 5, they are the numbers of hyphens which we are very familiar with. It means OA is one number of hyphens right? Now, we want to find AQ, we only need to find OQ then it can be found. So what do we know from the coordinate of point Q? We can know the length of OR and RQ. This

is $7-t$, this is $\frac{4}{3}(7-t)$. So this one? $\frac{5}{3}(7-t)$, is it easy? Ok this one is? $5 - \frac{5}{3}(7-t)$. So here we can do not so much complex calculation. We can use only one one-dimensional equation to solve it. $7-t = 5 - \frac{5}{3}(7-t)$, so t is...? $\frac{41}{8}$. Who find this one? Raise your hands, no matter how you find it. Raise your hands if you find it. ok, good. You need to be more confident. It means you have the abilities. Ok, next, PA equal to PQ , how to do it?

St 1: Calculation

T: Calculation, ok, no other way. Ok, we talk it later, look at the last one, we also can use geometry methods, I can give you some suggestions, geometry methods. Think about it, can we not use distance formula between two points. What is this figure like, when QA equal to QP . In order to help you, I make the figure like that. [teacher drag the point P to move the figure



] When QA equal to QP , do you have some special conclusion? Use what we have learned before. This line L , what situation can you find? QA equal to QP , this line, what conclusion? Who find it? Do you find it?

St 2: I think...

T: Say it

St 2: If it is not right...

T: No problem, we learn it together, say it

St 2: It is the abscissa of the midpoint of AP , point Q

T: What he said is that it is the abscissa of the midpoint of AP , if we translate it into geometry sentences, what is it? This line is ... of AP ?

St 3: Vertical bisector

T: Good, vertical bisector. So QA is equal to QP .

Most of the questions here are still focused on mathematics, and Mr. X began by asking the question of how to present the length of AQ in mathematical language: So

what is AQ? After that, Mr. X presented the answer on the board. During this time, he talked about the mathematical solution to the task. I did not find DGS in the interaction. Then he noticed that the students had some difficulties in understanding the solution and the method. He came to the platform and operated the software: *What is this figure like, when QA equal to QP. In order to help you, I make the figure like that. When QA equal to QP, do you have some special conclusion?* This was the only time he mentioned DGS during the interaction. then he went back to the mathematical contents and asked students to make conclusions based on the diagram: *QA equal to QP, this line, what conclusion? Who find it? Do you find it?* These questions are for students to make mathematical conclusions, but without using any technology, including DGS. During this time, DGS simply presents the figures for the task and students focus on what Mr. X wrote on the board.

The following transcript is from *link-screen-board*, in which Mr. X both use DGS and board for his explanation of the answer.

T: Ok, we talk it next lesson. First, some of you talked with me during the break. In the situation like this figure, isosceles triangle, which method can you use? First you need make classification. Ok, how to do it? how many situations? Three. $AP = AQ$, $PA = PQ$, $QA = QP$, three situations. Ok, if you want to use the distance formula between two points, we need to write the coordinate of the three points. Coordinate of point A is 3,4; P is what? 0,t; Q is? 7-t,0. Ok?

St 1: t

T: Ok, who can tell me, how can you find the coordinate? How can you find them? Here, there is a process, the coordinate of R is (7-t,0), you use it when calculating the area. Then you find this line, $y=x+t$. and you can find this angle is 45° . So here, it is t, this is also t, so you can get 7-t and t. Ok, next, it is a calculating process. Is it difficulty? Do you remember, when I talked about classification discussion, how can we make it a little easier? First, you will have roots when using distance formula between two points. But here do we need roots? No, we can solve it with square. So do you try to calculate it

during the break? $AP = AQ$, do you solve it? what is the result?

St 2: 1

T: Ok, $t_1 = 1$, $t_2 = 7$. We remove t_2 , why? It is not in this range. Ok, what about $PA = PQ$? t_1 is 4, then?

St 3: No, teacher, it should be smaller than 4

T: Smaller

St 3: If it is 4, point Q is coincide with point A

T: Oh

St 4: It does not exist

T: Here, you find there are some different. When we make classification above, why 4 is contained here? Because we want to calculate the area of APR . But now we want to calculate the area of APQ . So here, it does not contain equal. Maybe you will ask, if I forget, what can I do? And I did not check it in the exam. What can I do? We can put 4 into the formula, $t=4$. Now point P arrived at point C , point Q arrived at A , there is no triangle. So here we have two methods to remove this. First, if you, like Yang, find the situation is $t > 0$ and $t < 4$, we remove it because it is not in the range. If you do not find it, and still think $t \leq 4$, you can put 4 into the formula and find it is not a triangle, so remove. So this answer should be removed.

Here, although Mr. X was teaching by associating the board with DGS and he talks about the diagrams shown on the screen, he did not interact directly with DGS. He only talked about the diagrams at a particular moment when the point moved to a particular position on the screen. This is also reflected in the questions asked during the interaction. For example, his questions were more about mathematical contents, such as: Coordinate of point A is 3,4; P is what? $0,t$; Q is? $7-t,0$. Ok? So do you try to calculate it during the break? $AP = AQ$, do you solve it? what is the result? The students solved the task during the break between lessons, but of course they did not use any technology other than calculator. And during the lessons, Mr. X did not ask students to use or interact with DGS when solving or answering the questions. He asked questions in mathematical language that had nothing to do with DGS so that

students would not rely on technology to solve mathematics tasks and affect their scores in the examination. This is a common perception among Chinese teachers who use DGS in their lessons.

5.3.6 Interaction in Louisa's lesson

This section describes the French teacher's use of DGS to interact with students in her lessons. In this study, it shows some different foci compared to the Chinese teachers. I also present this teacher based on the type of instrumental orchestration she used in her mathematics lessons. Since two of her lessons took place in the computer lab, the following interactions took place mainly between one student and this teacher.

First of all, I would introduce the interaction of the main type of instrumental orchestration in French lessons: *Sherpa-at-work*. This is the most important difference compared to Chinese lessons. We do not see Chinese teachers allowing their students to spend too much time working with software. One reason is that there is only one computer in the Chinese classroom, and mathematics lessons rarely take place in the computer lab. Another reason is that, as I described in the Chinese example, DGS is primarily an aid tool to mathematics instruction. Chinese teachers avoid having their students rely on DGS for mathematics because of the examination.

St 1: I do not understand

*Louisa: Well, remove this circle, so now, what do you want to do? A circle, ok, go on circle. Are you sure what you want? You want a circle, so you know its center and it passes through one point. Or you know its center and radius?
[teacher pointed to the screen to let student know which tool to use*



St 1: The center, and it is a circle which goes through a point.

Louisa: Ok, click here and the center, good. But it is not here. What do you do

here? Redo all of these correctly and I will come here again

St 1: Ok

Louisa: Draw, no, it is not you have a segment AB at beginning then you construct a circle

St 1: Here

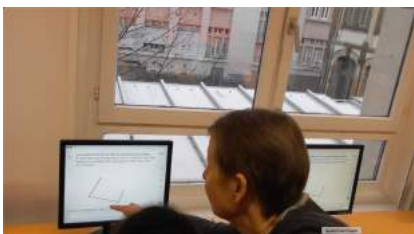
This interaction begins with a student's difficulty when constructing a circle with the center and the given point. As we can see, at first the teacher posed some questions to get the student to think about the purpose of this task., like "*what do you want to do*", "*Are you sure that's what you want?*". After the student answered the question, she let him click the mouse and pointed to the screen to show him how to manipulate the software. And her questions were related to dynamic geometry, where she wanted the student to think about strategies for solving tasks with technology. To make the construction clear to the student, Louisa always pointed to the screen to show which tools the student could use to complete the construction or which point needed to be dragged, just like the figures shown in the transcript, and to guide the student's activities and instructional process, like "*now you move the arrow (mouse) here and then move the points*" and "*Go put your arrow (mouse) there*". At this point her questions were more focused on technology and of course mathematics was the focus of the class. For example, the teacher always had the students explore the mathematical contents as well as the technology activities, "*move the points, you see that it is well stayed. Now you understand*". During the interaction, Louisa used the words like *click* or *draw* to tell students what they needed to do. These words were directed to DGS, indicating that Louisa paid her attention to using DGS to solve mathematics tasks and that her questions or statements at that time were not only about the mathematical contents, but were also about DGS. Student activities with technology were helping them understand and learn the mathematical contents.

Louisa also paid more attention to the connections between different situations during the lesson. For example, in this lesson, the tasks not only required students to think about how to solve problems with techniques such as dynamic geometry, but also required them to solve problems with paper and pencil. By transforming

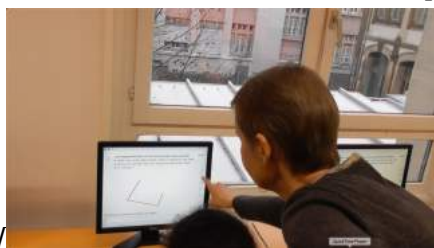
different solving strategies into different situations, students can notice and learn how to solve mathematical problems in a given situation, which is important for teachers to think about with their students.

The following interaction took place in another type of instrumental orchestration: *explain-the-screen*. Here, instead of having the student manipulate DGS and explain what happened on the screen, Louisa, herself, operated the software to this student.

Louisa: Well, it's like a compass, here it draws a circle. So, indeed, they will not join. So mark it. Next page. There, you are told that you cannot do as before, you cannot rotate them. [teacher pointed to the screen



]But you have to use the tools.



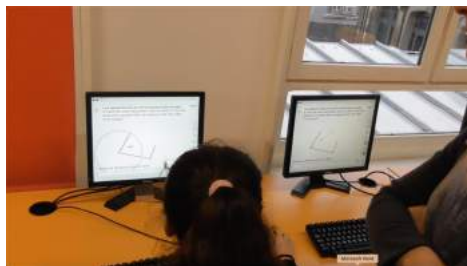
[So what are you going to be able to do with what you observed on the previous page?

St 1: They cannot rotate.

Louisa: That's it, so go ahead. This is where the center of the circle. Look on the previous page. Return before. Where is the center of the circle? That's good.

St 1: Ah okay

Louisa: And you Lilou, the center of the circle, where is it?



St 2: the[

Louisa: That's it. You have only drawn one (circle). That's enough? So go

ahead, click, here, click here then, Are you sure it's the right size (of) your circle? So, Lilou, could we make a triangle or not?

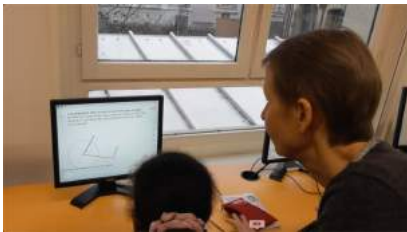
St 2: Em, no, Ah, here [student pointed to the screen



Louisa: Then draw it with the tool "triangle", as in GeoGebra. [teacher operated the software and pointed to the screen



] You put the three vertices and draw it. If clicks clicks the button, easy, as in the GeoGebra, clicks, and you see, resume Malis, [teacher operated the software



] That's it. Now, explain your steps. You say "I drew that, and then I draw that." Malis, that's good, good, on page 5.

St 3: Inaudible

Louisa: Yes, [inaudible] like that, hey, it exists or not, shows [inaudible] me, with tools, draw it to me with the little triangle, (you can) move it for drawing circle, And the third vertex is where? So, well, explain me your steps, "you [inaudible] to remake small, to move your circle, then, to try to make there and to draw the triangle, go ahead." After explains steps, that's it, it's good, now explains steps "I draw that, I draw that"

St 3: [inaudible]

Louisa: If you want, because you [inaudible]

St 1: Madam,

Louisa: Sorry?

St 1: The [inaudible]

Louisa: [inaudible] because in fact, you see a [inaudible]

At first, Louisa clicked the tools in the software and draw a circle to her student: *Well, it's like a compass, here it draws a circle.* And after, she told students how to interact with DGS to construct the same diagrams on the screen: *There, you are told that you cannot do as before, you cannot rotate them. But you have to use the tools. So what are you going to be able to do with what you observed on the previous page?* These statements indicate that Louisa focused her attention primarily on constructing circles with DGS and wanted to teach students the procedures for doing so. At that time, DGS was a tool for students to understand mathematics. As in the interaction above, Louisa also used verbs directly related to DGS, such as *click*, *draw*, or *drag*, in contrast to the Chinese examples, where teachers rarely used verbs that required them to manipulate the software. Of course, Louisa also asked questions that were specific to the mathematical content, such as: *Where is the center of the circle?* But this did not happen much times. When Louisa had students conclude his construction, she also focused on learning mathematics with DGS: *Now, explain your steps. You say "I drew that, and then I draw that."* The results student interpretations should be based on the operations with DGS. In China, on the other hand, teachers expect students to make conclusions primarily about mathematical contents. There is no relationship between the conclusions and DGS. This indicates the different concerns of Chinese and French teachers and the different roles that DGS plays in the lessons of the two countries.

In Louisa's lesson, these kinds of interactions happened many times, like the following ones: *If you want a circle of center a and then if I want to tell him that he goes through B I have to go on B is not to create another point where I say to myself ben it is a little near you see there c is not good for example so hop clear all these circles So we're going to make the center circle a and that goes through B so I tell*

him B so he'll stay you see? She tries to explain the construction program to the students as if it were in the first interaction. She said to the student: *you need to tell the software* This clearly shows that her focus is not just on mathematics, but that DGS is key to learning mathematics. Regardless in which instrumental orchestration she used in her lessons, the focus of the questions remained the same.

And in the second lesson, as we described in section 5.3.6, Louisa made some changes in the orchestration of the class. She used more time *guide-and-explain* and never let students to play the role of *Sherpa student* in this lesson. This is because the first lesson is a review lesson where all the contents have been learned by the students, but in the second lesson the knowledge is new to them.

The following interaction happened in *guide-and-explain*

Louisa: Well, then, good, by the page here it's done, so, how do you think to construct here by using the good tools of drawing like you see on page 4.

St 1: (I think) I want to use the compass

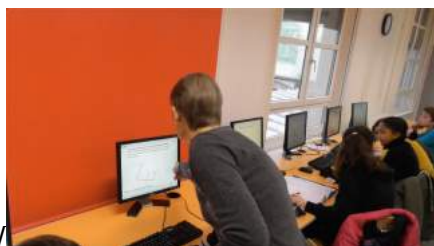
Louisa: There is no compass on this page,, They cannot be rotated anymore. [teacher pointed to the screen



] Because it is marked. So, that's it, yes,, the center of circle, you put it where? I [inaudible] the exercises, Noi, do you start there? And after you just move it where? [inaudible] draw me the circle.

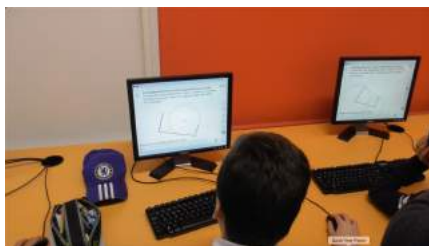
St 2: Madam, I do more pivot.

Louisa: That's why. Draw the triangle, the triangle tool, where is the triangle?



[So you've done that, we see what it gives,

see page 4, and look how do you do to move, do you find where is the center,



then try [inaudible]

Here, I still notice that the main focus is on how to use the DGS to explore mathematical contents for students. For example, she used words such as: *draw*, *turn*, *use tools*, and *put* which are directly related to the operation towards DGS. She repeatedly told the student that he needed to find the right tools in the software to help him solve the problem: *Well, then, good, by the page here it's done, so, how do you think to construct here by using the good tools of drawing like you see on page 4. Draw the triangle, the triangle tool, where is the triangle?* These exemplify Louisa's main focus in this lesson. Using DGS to help her create new learning situations for students to explore mathematics by themselves. In both lessons, this is the main role of the DGS.

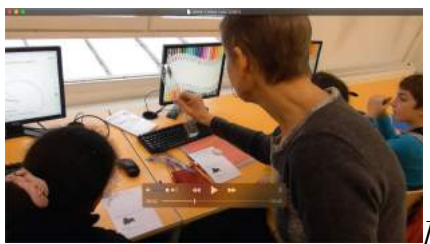
Although Louisa used paper and pencil during this time to communicate with students based on their problem solving, her questions or statements were also related to DGS. For example, the following interaction:

Louisa: So on paper you cannot move (these) three segments [teacher pointed



to the paper list] but so, you (can)

use the geometry tools and the compass. [teacher hold the tools



St 1: Ah [inaudible] on the notebook

Louisa: To try to (imitate) the movement of them (segments), because actually we cannot move them, you need to draw on your notebook, you go

St 2: Dimension, three dimensions

St 3: Like that

Louisa: That's it.

Here, Louisa wants her students to think about how to use physical tools like compass to move the segments, just like what they do with DGS. She told her students that they could not move segments on paper. But they could think about how to do the same thing by using compass. This shows that DGS is always at the center in Louisa's lessons, and no matter how she organizes her lessons, she focuses on letting her students to use DGS to explore mathematics and help them pay attention to what they need to learn during the lesson. This helps her students to be more active in their mathematics in the classroom.

6 Knowledge within instrumental orchestration of Chinese and French mathematics teachers

In this section, based on the analysis presented in Section 5, I would describe the knowledge that the teachers in these cases needed to use DGS in their mathematics teaching. Based on classroom observations and interviews with these teachers, I would present their knowledge in the following three sections: section 6.2 focuses on the teachers' choices of different mathematics tasks and the role DGS played in these tasks; section 6.3 describes how the teachers used their knowledge revealed in organizing their classroom practice with DGS and what role DGS played in their teaching process; and section 6.4 introduces teacher knowledge revealed in the interacting with students in the DGS situation. In each section, I also contrast mathematics teachers in China and France to show the differences between teachers in these two different cultural settings. In each section, I would present the teachers' views on the main topics of using DGS first and then abstract the focus of using DGS in mathematics lessons. At the end of each section, I would provide a short conclusion about teacher knowledge appropriate for integrating DGS into the mathematics teaching process.

6.1 Differences between French teacher and Chinese teacher in roles of Dynamic Geometry Software

In this section, I first show the perceptions of Chinese and French teachers about the tasks with DGS and then, based on these perceptions, we can see the role of DGS in different mathematics teaching contexts in sections 6.1.2, 6.1.3 and 6.1.4.

6.1.1 Teacher's opinion towards different tasks

According to the research design, the teachers were asked to look at the tasks from other countries and I would discuss these tasks with them based on their own teaching

culture. Based on the discussion of the different tasks, I could find out how the teachers thought about which mathematics tasks were more appropriate to use DGS. First, I would talk about the Chinese teachers' views on the French tasks used in this study. Then I show the French teacher's views on the Chinese tasks

When talking about the French tasks mentioned above about constructing circles with different tools in DGS, Mrs. J, an experienced teacher in China, first considered these tasks from the students' perspective. She did not think about whether the tasks were appropriate for DGS, but rather about how the students solved them during the lesson.

Maybe drawing the chord is more difficult for students. Because drawing a circle with a given diameter is easy. I just think about when I have lessons with Sketchpad in this situation, students will have what kinds of problems when constructing these tasks.

This suggests that, for Mrs. J, these tasks, which allow students to construct circles with DGS, can be used in her lessons if she has time. Mrs. Y presented a similar opinion in the interview:

I think these tasks may be difficult compared with our tasks. Because there are three segments separately, students need to move them together. They need to put the two endpoints like the blue one and the yellow one together. Some students may fix one of the segments and then try to move another two segments. They need to move them all the time.

And for Chinese teachers, the mathematics task is to help students understand particular mathematical contents, which the teacher uses to know if the students understand what is being taught in class. So the first question that Mrs. J asked in the interview was:

What is the objective of the task? What students need to learn?

This issue is very important for Chinese teachers. For example, when they teach circle in their own lessons, the tasks they choose are always about how to determine the center or radius of a circle. Obviously, this kind of tasks is completely different from the French tasks, so Mrs. J had some doubts as to why the French teacher chose

to use this kind of tasks in their own lessons. Another teacher, Mrs. Y, raised the same question in the interview.

What is her objective? What is the goal of French lessons? What kind of mathematics knowledge they need to know? The diameter or the radius?

This shows that she also pays attention to mathematical contents and for her the task is in the service of mathematical concepts or theories. The student needs to learn something from it and this kind of knowledge is always presented through sentences such as what is a circle and what is radius.

For Chinese teachers, mathematics tasks can play different roles in different lessons depending on the lesson objectives. Mr. ZH does not always design many tasks for his students in his classes. In his opinion, teachers need to determine what the objectives of their lessons are. He said that there are three main objectives for mathematics teachers: to introduce a new lesson; to do exercises in the lesson; and to explore mathematics in the lesson. For different objectives, teachers need to design different tasks. If teachers wanted students do some exercises, he said, they would design enough tasks for students to know how to apply their new knowledge to solve tasks. But in the other two types of lessons, he would not design too many tasks.

For me, I do not always design too many tasks in the lessons. For example, if this lesson is for introducing new knowledge, I may only choose one or two example tasks. Because I may pay more time to introduce the basic knowledge and theories. They are the main part of the lessons. And tasks, I may show one or two. And for me, tasks on the textbooks are important because they are designed by many experts. So it is enough for choosing one or two of them in the lessons. And we also have exercise lessons, in these lessons, we design many tasks for students to understand and train. So we need to know what roles tasks need to have in learning mathematics. And in these lessons, the difficult level of tasks is also different. For example, in new lessons, the tasks are not too difficult, they are just for students to understand new knowledge. And in exercise lessons, the tasks may be more complex, because they need to help students pass the examinations.

When talking about the role of DGS in mathematics tasks, Chinese teachers highlighted the support they received from DGS in the classroom. For example, Mr. ZH mentioned that DGS was one of the resources he used in his lessons. However, DGS would not be used if the lesson was just for the students' exercises. Only when he needed to explain complex tasks, he would choose to use the software to help him explain the tasks.

Generally, we may use DGS when explaining complex tasks, at least, for me, I will use software only in this situation. So in the exercise lessons, I will not use software for many times.

So for Mr. ZH, DGS is used more for explaining complex tasks.

Another teacher, Mr. X said:

In our syllabus, we do not have this objective. We do not need our students learn to use this software (DGS), although there are some contents related to information technologies. These contents are not mandatory. The technologies are more for supporting.

If he designed the lesson with these tasks, he would let students first construct any figures as what they wanted. Which means he did not let his students construct what he designed, but let them explore the software first to know how to construct figures with it. Some teachers may also choose to begin their lessons with a blank sketch which allow students to construct any geometry objects as they like (Erfjord, 2011; Ruthven et al., 2008). For example, Mr. X told me:

Recently, I let them learn Geometry Sketchpad, I do not design any figures for students. Because Geometry Sketchpad contains many tools like button, segments and so on. I let them explore by themselves and then discuss the functions of each tools in the software.

Researchers named this kind of sketch as student-created which allow students use their knowledge to create geometric objects (Dove & Hollenbrands, 2014; Erfjord, 2011).

Another important advantage for teachers to use DGS like Geometer's Sketchpad is that this technology can help teachers show students the process of movement

when learning geometry. Many years ago, when technology was not as accessible as it is today, it was always difficult for teachers to show students how geometric objects moves. At that time, it was difficult for students to make a visual image of the motion of geometry. Teachers needed to draw as many figures as possible and break the movement process into different moments to help students understand what they needed to learn in the course. Now with the help of DGS, everything is much easier. In the interview, Mr. ZH told me his thoughts on how and when to use Geometer's Sketchpad in Chinese secondary schools. He said:

Geometry Sketchpad, its main function is presenting moving process. We do not need to present static parts of geometry with this software. In grade 9, I think many contents need the help of Sketchpad. Like moving points in function, such as parabola and circle.

This gives us a clear picture of how Geometer's Sketchpad is used in ZH lessons. For him, the software is only integrated into mathematics lessons when he needs to present problems containing moving points. So based on what is taught in Chinese secondary schools, he believes that teachers use the software much more often in the ninth grade, the last year of secondary school. This is because in the last year, students finish learning new contents and start preparing for final examination in order to apply for senior middle school. Therefore, students need to know how to solve more complex tasks which they have not faced with before. And usually these tasks are about moving points. Students need to imagine how the points move based on the descriptions of the tasks and find appropriate answers to the questions. In order to make it easy for students to solve these kinds of tasks, DGS has become an important tool in mathematics classes. With the help of the software, students can solve complex tasks that may have been difficult before. With regard to the understanding of the role of DGS, Chinese teachers had similar views on the selection of appropriate contents to use DGS in their lessons. One such topic is moving point mentioned by Mr. ZH. These tasks have always been the most difficult in Chinese secondary schools, and both teachers and students find it complicated to solve them. Mr. X also told me that he usually uses DGS such as Geometer's

Sketchpad when introducing the last task of the examination. These tasks are always the most difficult tasks and they always contain the problems which need students move the points and make categories of different situations. At that time, we use Geometry Sketchpad

But ZH also mentioned a concern he had about the use of DGS in students' problem solving process. He said

In fact, in China, students need to pass the exams. In these exams, we cannot show the moving process with software. So students need to imagine the figures. They need to think how the point move. It moves from where to where, how it changes the figure. I am worried that some students can solve the problems with DGS, but without the software they cannot solve them anymore.

ZH's concern is also one of the reasons why Chinese teachers believe that DGS can only be a teachers' supporter during the lessons and should be controlled by teachers not students. Mr. W also shows the same concern:

But if teachers do not guide students to think about the changing process and only show the result of the moving process, I think students may rely on the software at the end and they may not think about it by themselves when solving other tasks.

He believes that teachers should not only show the process of moving with the help of the software during lessons, but also make students think about why the points move as the software shows. Only by this way students would not always rely on the software when solving tasks and get high scores in the final examinations. He added:

Yes, because we just use the software to help us show the moving process for students. When in the examinations, we cannot use the software to help us. So at the end, teachers need to help students imagine the moving process by themselves with the help of Geometry Sketchpad. I think it may have advantages for students in learning mathematics.

For Chinese secondary school students, this is one of the key skills if they want to get good scores in their final examinations.

Other Chinese teachers had similar views about using DGS in their classes. In

order to be successful in examinations, students need not to rely on the software. One teacher, Mr. X, believes that DGS also has its limitations. It can do a lot of complex things to make the task easier for students, but it cannot present accurate answers. He said that

Students need to calculate the results by themselves. For example, I can only see the figures with the software, but I cannot find the (mathematics) results with it.

For Chinese tasks, students always need to find an exact number to answer a question. For example, they need to find the length of a segment or the area of a geometric object. For Mr. X, DGS cannot show the exact answer for the students, of course we can use DGS to know for example the area of a triangle or the length of a segment, but this number is not accurate. It is always just an estimate, which is not what we want when solving a task. So students also need to be aware of the limitations of the software and get rid of their reliance on it when learning mathematics.

In China, teachers seldom used the tasks to let students construct geometry objects with DGS. Mrs. Y told me that

For the tasks about constructing circles, I think we do not have this kind of lessons.

It was difficult for the Chinese teacher to think of the objectives of these tasks. and Mrs. Y also thought they were a bit complicated for her students. They had not used DGS before. so she made some modifications to it, I think

I will also use physical resources together with the software. I may prepare some sticks for the students when I show this task to them. And let them use both of the resources (DGS and physical ones) to solve the problems. Or maybe I will tell them the length of the segments and do not let the students move them, then I will let them think whether they can construct one triangle or not. It may be easy for students.

In her classes she not only uses DGS to allow students to explore mathematical knowledge, but also uses some physical materials to help them understand geometric

theory. She believes this is a more appropriate way for students to learn mathematics. And the tasks she designed are not as complicated as the French tasks. Because Mrs. Y did not use or think of using these types of tasks in her classes, she did not have the opportunity to encounter the errors in the video of the French lessons.

But I think this student does not have the time to operate the software. He just looks at how teacher operates it in the classroom. So he may not make deep understanding of the software. he may just redo what teacher does before. If he can understand what is “S”, what is “SAS”, what is “AAS”, I think he may not have this kind of mistake. She talked to me that I will let students think about the mistake by themselves first. And I think I will also use the same strategy like the French teacher. I will let students tell me how they draw it with paper and pencil. Maybe I will forget this situation. I may ask students directly how they draw it with software. So I may use the same strategy like the video. First ask them how they draw it on paper and then let them tell me is the two situations (DGS and paper-pencil) are the same?

The French teacher’s approach gave Mrs. Y some advices on how to think about solving the task. Shen mentioned a trend among Chinese mathematics teachers:

But I think maybe Chinese teachers will not let the students think by themselves, they may tell them the answers directly and tell them how to construct the circles. This may let student not understand the construction and students may only repeat the construction without understanding it.

From the interviews, we could see that the time constraints and examinations were the main reasons why Chinese teachers did not use the French tasks. Elder teachers may try to modify these tasks to meet our requirements, as what Mrs. J and Mr. ZH did. However, younger teachers, who had limited teaching experience, chose to use the tasks they were familiar with rather than the French tasks during their lessons. The pressure from examination was one of the main reasons. Only the tasks which are similar with those in the examination would appear in their mathematics lessons. Young teachers choose to take more control over their lessons and avoid making changes.

From this, we know that these Chinese teachers are interested in French tasks. They tried to use and design their lessons based on these tasks. But I found that their focus was different from that of the French teachers. The Chinese teachers considered the mathematical contents rather than showing students how to transform strategies in different situations.

Here is what the French teacher had said about the tasks in her classes as well as the tasks in the Chinese classes. Like the Chinese teacher, Louisa first designed the tasks based on the mathematics students had learned in the previous lessons.

So, it is because at this moment we study the circle in class with the vocabulary, radius, diameter, chord. In class they have drawn circles with the exercises in their book, with given rays, given centers passing through a point, given diameters, and at the same time they also do it with dynamic geometry to discover the circle tool.

One thing that is different in the French course is that DGS is also one of the objectives of the lessons, which does not appear in the Chinese courses. Louisa told me in the interview that the task is for the students to discover the circle tool of dynamic geometry. This shows an important difference between lessons with DGS in China and France. In France, especially in these lessons, DGS is to create learning situations for students, so the software itself becomes one of the objectives of the task and the lesson, whereas in China, DGS is a supporter of teacher practice.

So in order to let students learn mathematics with DGS, Louisa designed the task to have students use tools and software to construct some objects.

Afterwards, they try to use what they have learned about the circle tool, to make that figure. There, they can control whether it resists movement or not.

Another important focus that Louisa mentioned in the interview is her attention to comparing two different solving situations: paper-pencil and DGS.

When they (students) draw it with paper pencil, when they place a midpoint, for them, it is the half of the segment which means that it is not the middle point. They place themselves at the half of the segment, which is easier for them to apprehend. There (with DGS), they place it a little at random and

after they realize that it does not work.

She imagined how students could use these two different tools in class to finish these tasks:

They draw the segment, they take half in length to point their compass and they draw the circle of diameter. Finally, in GeoGebra, it's more difficult because you have to use the midpoint.

This was also not shown when the Chinese teachers designed their tasks with DGS. Only when asked to reflect on the connection between paper and pencil and DGS, the Chinese teachers began to talk about their perceptions of these two different problem solving tools for the students in this study.

Of course, Louisa would also design tasks based on students' level. Like the Chinese teacher, she would also consider the difficulties that students may encounter in solving these tasks. She said that

I think that the problems for them is actually to put the midpoint of the segment by using "middle" (tools with GeoGebra).

When dealing with students' difficulties in the lessons, Louisa believes that the main problem for students is a lack of deep understanding of mathematical contents. She said:

It's really important to understand the notion of environment. And after, if they do not put the points in the middle and they do everything randomly, perceptively, the figure does not resist movement. But here, I think that on GeoGebra they are forced to understand more about the concept of diameter, the middle of the segment and all that.

Only when they have enough mathematical knowledge, they can use DGS to successfully solve the tasks and construct the correct diagrams by themselves.

For Louisa, DGS is also a medium for students to learn about how to use the same physical tool in different ways. In the second lesson, she thought the software could help students know how to rotate segments with compass. She said that:

The role of the software is that (to helps students) identify the fact that the segments had to turn After, the idea is to show them that when (only

having) paper and pencil, as students can no longer catch the segment to rotate it, they will take the compass to replace this displacement by rotation. In fact, the goal of the software is to bring the displacement by rotation.

After seeing the tasks in China, Louisa made some modifications based on the requirements of her French mathematics lessons and her teaching experience. And tried to use these tasks with DGS, just as she did in the lesson in this research. She told to me that:

I think that in France, we would ask them the area as a function of time, to build a graph. In GeoGebra, you can display the area. So to display the area, to have several values and to try to draw the graph: the curve for students to see that increases. Perhaps we would ask the position of point B for which the area is maximal. We would look for a maximum of a function.

Compared to the Chinese questions which were more complex for students, Louisa made the tasks more detailed, such as finding the maximum value of an area or finding some value to construct the whole figure. Like the Chinese teachers, the difficulty of the task is one of the main points Louisa presented during the lesson.

It's (the first two tasks) the same level of difficulty. I think if we put this one before this one, it might be the same. This one (third task), it is more difficult.

The Chinese teachers mentioned not only their views on DGS, but also on other techniques they might use in teaching practice. Mr. X believes that students are now familiar with computer science and are good at searching for information through websites.

So before, I told my students that there are many useful resources on the internet. Except my lessons, they can find many other lessons and find the answers of many tasks. And they can learn many things from the websites.

Modern internet technology can develop many different skills in students, such as searching for information; exploring and so on. These skills are also critical in students' daily lives. Miss. Y said

Technology can let students more active in the lessons and make them

understand the mathematics more clearly. It also helps students learn more skills in some aspects. They can learn how to design projects. Maybe someone thinks it may take too much time, but students can develop their skills through technologies in a long time. They can get many advantages by using them in the lessons.

Mrs. J also told me that she finds that foreign children always have a higher capacity in exploring than Chinese children. She said one of the reasons may be that they used technologies in their lessons. She said:

It shows students' exploration skills, although teacher may also tell them how to operate the software directly. But the moving process can make deep imagine for students. They can learn a lot from this. You can see in our country that students spend too much time to do exercises. Maybe they spend these time to let students explore mathematics.

6.1.2 Different role of dragging mode in China and France

Based on the different tasks chosen by the teachers in this study and their use of these tasks in the classroom, we found that *dragging mode*, as the most important function of DGS, plays a different role in Chinese and French tasks. Holzl (1996) suggested that the dragging mode has two roles in mathematics teaching: it can help us check the properties of mathematics and it can help us explore mathematics.

Generally, in China, students would not use technologies in mathematics lessons, and our teachers try to give them as much ability to understand geometric thinking as possible. When facing with tasks with moving points, they need to imagine the movement of a geometric object based on the situation. Our teachers are also used to explaining these tasks without using technologies. They choose to show students the result of the movement rather than the moving process. This strategy is more like the *photo dragging* introduced by Olivero (2002). His work distinguished two types of dragging: *photo dragging* “Modalities which suggest a discrete sequence of images over time: the subject looks at the initial and final state of the figure, without paying

attention to the intermediate instances. The aim is to get a particular figure.” (Olivero, 2002, p.141) and *film dragging* “Modalities which suggest a film: the subject looks at the variation of the figure while moving and the relationships among the elements of the figure. The aim of dragging is the variation of the figure itself.” (ibid., p.141) So based on these definitions, we can see that without DGS, teachers would not be able to show the whole moving process to their students like *film dragging*. This means that with technology, the teacher will change his teaching strategy when dealing with the same tasks. In the study here, the mentioned task shows the same thing. With the help of DGS, teachers and students have changed their approaches to solving tasks.

Dragging mode was used in French lessons to check whether students constructed the objects correctly. I found that in France, students worked individually with DGS and rarely discussed with each other during the lecture. The teacher walked around the classroom to check students’ work. Interaction between teacher and students occurred when students completed their constructions or met some difficulties. Because the lesson let students work with DGS, the teacher did not spend much time organizing the whole class teaching process. Instead, the instructional process always took place between one student and the teacher. In this study, each interaction was marked as an episode. So the following analysis is based on these episodes. The following transcript shows how the teacher used *dragging mode* to examine students’ dynamic geometric work.

Prof.: Maylis, Ok, drag point A and B. Perfect, you see that it stays correct because you told the software the center of the circle and the point it passes through.

From the episode I find that the teacher first asked the students to drag one or two points of their structure, for example dragging points A and B. Then she shows them what happens when they dragged the particular point. She let them see what happened on the screen. During these processes, the teacher wants the student to use the special tool of dragging to check if his/her construction is correct. This action allows the student to get a visual impression of the geometry properties. During the

lessons, dragging mode is used to demonstrate that the students' work is correct. Louisa told me that

Once students finish and they try to check..... All alone, they can check that their construction is not correct finally.

Of course, students did not always drag the points when the teacher asked them to do. When they constructed their figures, they also checked their construction process step by step by dragging the pattern themselves. As in the following episode, one of the students, while dragging the point, noticed that the construction did not change in the way he had expected. He was confused and wondered why the software was giving him such feedback and how he could modify his strategy to construct the diagram as required by the task.

In French classes, we noticed that students were working with dynamic geometry individually, so we used the *dragging mode* to check if their constructions were correct. During the lessons, Louisa first let students drag one or two points in the construction: *Drag point A and point B*. Then students can observe what happens when they drag a particular point. This indicates that Louisa wants her students to use the dragging mode to check whether their constructions are appropriate for the task. Through this action, students can form a visual image of the geometric properties. Louisa told me: *Once students are finished, they try to check.....All alone, they can check that their construction is not correct finally.*

6.1.3 DGS creates new kinds of mathematics tasks

As stated in section 5.2, both the Chinese and French tasks suggest that DGS created new types of tasks for students. However, it created these tasks in a different way. The Chinese tasks show a strategy for integrating DGS into mathematics lessons. Based on the categories of the roles, these tasks were difficult for students to solve without technical assistance. They can be seen as the second category of tasks based on the different roles of dynamic geometry (Laborde, 2001) in the mathematics tasks because it required students to think a new strategy to solve these

tasks, which was not the way students solved with paper and pencil.

In the French tasks, DGS is also the generator of new types of mathematics tasks. These tasks show the different expectations of this French mathematics teacher. Louisa integrated DGS in her lessons, but does not want the geometric objects to move like Chinese tasks. These tasks are for students to construct different diagrams with the help of DGS. From the tasks shown above, we can see that the teacher shows the construction programs step by step, like the first list of tasks in section 5.2.6. This shows that the teacher thinks that problem solving is not a simple transplantation of the paper-pencil procedure. The involvement of dynamic geometry changes the problem solving strategies. Therefore, these tasks show a new kind of mathematical tasks with dynamic geometry. Like the work of Laborde (2001), in these tasks, DGS caused the change of problem solving strategy when some of its tools were used and under this situation, the task may become more difficult for the students. However, the challenge is only to produce a triangle figure, without the choice of construction tools being problematized for the student. For the same type of construction, other task sequences with DGS make it possible to give another role to the technology, as in the example of triangle constructions by Voltolini (2014). The use of DGS remains the same in both tasks, they differ only in the details and the second task is considered to be the application of the constructing procedure described in the first task. These tasks are technology oriented, but unlike the Chinese examples, the students have to directly manipulate DGS. In this study, the second French task is also an example of a task that requires students to connect different tools like compass and DGS.

In France, we can see a completely different situation with DGS. In the example lessons, the French teacher used DGS as a tool for students to learn mathematics. The lessons took place in a computer lab. Each student can interact directly with the computer. The teacher combined different problem-solving situations. Students first solved the tasks with paper and pencil and then tried to find an appropriate solution with the help of the computer software. The objective of these tasks is to show students the differences and connections between different tools. Students need to think about how to transfer their strategies with paper and pencil to a computer

situation. The teacher sees this as a challenge for the students and she needs to help them understand more. This is also an important difference in the process of teaching mathematics in Chinese and French lessons. When I interviewed the French teacher after the lessons, she told me that because of using DGS, teachers can design many new tasks, it is more like problem solving for the students. They need to think about how to use DGS to create the suitable situation for students to explore or apply their mathematical knowledge to solve real-world problems. Thus, in the Chinese tasks, DGS was designed to change the student's problem-solving strategies, whereas in the French tasks, the DGS was designed to create a situation for the students to explore mathematics. These two different emphases show two different ways of using the DGS, with Chinese teachers using dynamic geometry to train students and French teachers using dynamic geometry to explore concepts.

6.1.4 Different focus of mathematics tasks with DGS

From the tasks chosen by the mathematics teachers in these two different countries and their pedagogical performance in using these tasks in the classroom, we can see that the teachers showed two different concerns. In China, not only in the lessons analyzed in this study, but also when I asked teachers to design or select appropriate mathematical tasks with the help of DGS, teachers always designed the tasks first and then thought about how they could use DGS to help students understand the solutions of these tasks. From the lessons, at first the Chinese students could only use paper and pencil to try to solve these tasks, then after the students completed their work, the teacher started to use DGS to explain the solutions to the students and asked them to answer some questions based on the diagrams on the screen. This phenomenon can help us to make a deep understanding of the application of DGS in Chinese mathematics lessons. It suggests that DGS plays as a supporting tool for mathematics teachers in these lessons. In this study, the teachers told me that an important objective for mathematics teachers in China is to let students get good scores in final examinations where DGS is not used. One of the teachers told his

students directly that they could not use dynamic geometry in the examinations, so they needed to imagine how geometric objects would move depending on the situation. This may explain why Chinese teachers use DGS as an aid in teaching mathematics. They need to avoid students' over-reliance on DGS, so they need to reduce the amount of time they spend on using the software.

Another important feature of these tasks is that the Chinese tasks can also be solved with paper and pencil, meaning that the students can solve the tasks without DGS. When I presented these tasks to the French teacher, she was interested in knowing how the Chinese students solved the problems without the help of DGS. For her, these tasks were complex for the students because they contained so many variables, some of which were changing. She told me that these tasks can be a bit difficult for French students, where they are rarely faced with so many moving points and variables.

In the French tasks, students are asked to construct different geometric diagrams with DGS. In the case of paper and pencil, these constructions are not available. Of course, students can construct geometric objects with paper and pencil, but these objects cannot be dragged. They are static. This means that the student's constructions may just look like the same as the teacher's works. They may not have the same properties. When students need to construct the same diagram by using DGS and use dragging mode to check if their construction is correct, there can be a lot of problems that do not exist in traditional situations.

This is different compared to the Chinese tasks. Because DGS provides new characteristics, both teachers and students need to think about how they can use the new tools to create dynamic objects that will help students learn mathematics. As Louisa says in this study, students are told to use compass to construct a figure, but they are not told why they should use compass. So the main point of her lesson is to show the students how to rotate the segments with different tools. The students need to do the same thing, like what they did with compass, with DGS to simulate the rotation of a segment. It is explained here that DGS are used to create learning situation and students can use them to construct geometric objects. They need to

interact directly with the software and think about the connections and differences between traditional situation and DGS situation.

6.1.5 Teacher knowledge revealed in choosing different types of tasks

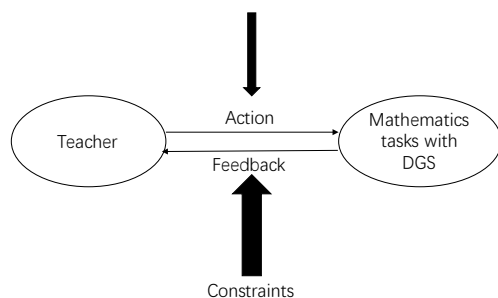


Figure 6.1: Teacher knowledge revealed in mathematics task with DGS

According to the definition of teacher knowledge in this study, teacher knowledge related to DGS are seen as the properties in the interaction between teachers and DGS in a given teaching situation. This interaction is expressed in three main parts: task selection with DGS, classroom organization with DGS, and teacher-student interaction with DGS. In this section, I describe the knowledge revealed in mathematics tasks based on the above analysis. In the study, teachers can choose different ways to use DGS for geometry instruction (Gueudet & Trouche, 2011): First, they may introduce a new mathematics notion and build a figure with DGS, and then drag the points to students in order to let them make some conjectures and at end let students make proof with paper-pencil. This way can be seen as a more classic one. Second, they may construct a figure and let students discuss how to build the same figure with DGS. This can be called a black-box type. These ways for using DGS correspond with the different types of tasks introduced by Laborde. In the first way DGS is an amplifier and the second is a generator.

As we said, DGS plays an important role in all the tasks in this study and it helps the teachers to design new types of tasks which are difficult to solve in traditional situation. Both in China and in France, DGS is a generator of mathematics tasks in different ways. In China, the generator is more prominent in changing students' problem-solving strategies in paper-pencil situation, with the use of DGS, it allows students to create

new ways of solving tasks. They can spend time doing things they could not do with paper and pencil. According to the lesson videos, students have to construct many figures to show some particular moments during the movement without DGS, which is similar to *photo dragging*, whereas with software, the moving process can be shown to the students, which is similar to *film dragging*. Students will feel much more easy when they need to find geometric properties and answers. And when the teachers find some troubles or mistakes, they drag the object to show the reason why the answer is incorrect. And they will also drag objects to help students answer the questions. Only in Mrs. Y's class, the dragging mode was to check students' work after they had finished the tasks at that time, because she designed some building tasks for the students. At other times, Mrs. Y chose to drag points to show students some properties, just like other Chinese teachers' lessons.

In France, the generator of a new type of tasks in different ways. These French tasks do not exist in paper and pencil situation. In mathematics classes, students were asked to construct diagrams with DGS. When using paper and pencil, they could only construct a static one, not drag anything around. In Louisa's lessons, the dragging mode is used to check if the students' constructions are correctly. This software is designed in a manner that makes the mathematics as *explicit* as possible. It must show the *processes* of the mathematics as well as giving the final results of any calculation (Tall, 1986).

These two ways of generating tasks also illustrate the two different roles of the dragging mode when teachers use DGS to teach mathematics. In China, the teacher uses the dragging mode to present geometric properties directly to the students, whereas in France it is to check if the students' work is correct. Of course, in the second French lesson, the dragging mode also showed the same role as the Chinese lessons.

Although DGS is used to create new types of tasks in both countries, in the actual classroom teachers integrate them in different ways. Based on the lesson observations, I find that in China, teachers prefer to organize their lessons in a teacher-centered way. During the lessons, teachers control the software in the way they prepare the lessons. For all Chinese teachers, they prepare everything before the lesson, including teaching

resources, tasks, questions, activities, and even students' answers. This is because they do not want too many unexpected events to happen in the classroom. That is why they like to integrate DGS as a projector and present only the contents or tasks to the students, so that the lesson continues smoothly according to their teaching plan. Mr. W, a young teacher in this study, is a prime example of how a teacher can take control of everything and be the center of the lesson. He is the only teacher in the study who never used any student-centered type of instrumental orchestration in his lessons and rarely used the dragging mode of DGS for his students. While the other teachers spent most of their time controlling the software, the students still had some opportunities to engage with the software. For example, Mr. ZH had students explain their work through the screen, and Mrs. J and Mrs. Y had students directly control the computer during class.

In China, most of the tasks (Section 5.2) are complex for students. They always appear as the last one or two tasks on the examination, which require students to be skillful enough to find the right answer, and students never use software when solving these tasks. This is why these Chinese teachers chose to integrate DGS in a teacher-centered way (I would describe this reason in more detail in Chapter 7). They do not want their students to rely on manipulating the software to help them find answers so that their examination performance can be unaffected. I notice that Mrs. J and Mrs. Y had students manipulate the software because the task included different elements compared to other Chinese tasks. The students needed to construct their own objects to help them solve the problem based on the task situation. This caused these two teachers to have students construct figures directly with DGS.

In short, DGS creates new tasks in Chinese mathematics classes by changing the way students solve tasks. However, during lessons, it is not a tool for students to solve tasks, but a tool to assist teachers in teaching.

In France, the tasks show DGS as a generator. Without the software, teachers would not be able to use these tasks in the classroom and students would not be able to solve them. Therefore, Louisa chose to use these tasks in the computer lab so that all students would have enough time to work with the software to explore and solve the tasks. In contrast to the Chinese teacher, Louisa's lessons are student-centered. She acts as a

supporter to the students in the classroom, making students more active. She uses DGS to create new learning situation so that students can learn mathematics on their own. Even in the regular classroom (Louisa's third lesson in the study), she had some students come to the platform and operate the software to explain their answers. This suggests that it was not the physical configuration of the classroom that allowed Louisa to integrate the software in a different way. Even though there was only one computer in the classroom, she still allowed students to manipulate the software. This approach was also evident in Mrs. Y's second lesson. However, Louisa used this approach more often. Having only one computer does not mean that teachers can only use the software in a teacher-centered way. It was their knowledge related to the software that influenced the integration of DGS in the lessons. Like these Chinese teachers, they see DGS as an aid, whereas Louisa sees it as one of the tools that students use to explore mathematics.

6.2 Differences between French teacher and Chinese teacher in classroom organization

This section discusses the organization of classrooms with DGS both in Chinese and French schools based on lesson videos and interviews. First, I present the teachers' reflections on their own lessons and also on other teachers' lessons and then talk about some of the main features of the classroom instruction with DGS from these lessons and the knowledge reflected in these processes.

6.2.1 Teacher's opinion towards classroom orchestration with DGS

The Chinese teachers in this study showed the same views about the use of DGS in mathematics teaching and learning, and both young and experienced teachers believed that DGS in mathematics teaching and learning should be a supporter to help teachers to organize lessons and helping students to learn mathematics more directly. Compared to the teaching practice 10 years ago, when technology was not common in schools, mathematics teachers needed to create different tools to present

mathematical contents to their students. The most common resources they used were chalk and blackboard. During that time, it was difficult for teachers teaching geometry. They did not have as many useful tools to present different objects to their students to make teaching more visual.

More than one teacher told me that they considered DGS such as Geometer's Sketchpad and GeoGebra to be one of the supporters of mathematics teachers. They find these tools useful and make their teaching easier and more efficient. For example, Mrs. J told me in the interview that:

Media I think it is just a supporter for teachers, but it will play a role more and more important in the future. ... But it still needs teachers' guidance and control in the lessons. So it cannot replace classroom teaching.

In her opinion, although technology is becoming increasingly important in the mathematics teaching process. However, the center of the classroom should still be the teacher (Guerrero, 2010) and she needs to think about how to control the use of these software in order to make her lessons more effective. Technology cannot replace the teacher in the lessons.

When talking about the advantages that DGS brings to the organization of mathematics lessons, Chinese teachers believe that there are several reasons why teachers choose to use DGS for their lessons. The first one is efficiency. DGS can help teachers save more time during the classes, at least teachers do not have to write all the contents or tasks on the blackboard as what they did many years ago. This advantage is also talked about by other teachers in China, as Mr. X said:

In fact, the most important affects for DGS is saving the time. Because if I draw figures with chalk and ruler, it will take most of the class time.

He believes that for teachers, drawing and writing costs too much time in traditional lessons, but with software, they can save more time and students have more opportunities to discuss and communicate.

Second is excitement, with technology teachers can upload students' work at any time. Traditionally, teachers would also have students solve some tasks on the board, but students could only solve one task when they were chosen to write on the board.

Mrs. J says this is a loss for these students.

I think, only with blackboard, students may not have been encouraged by teachers. For example, one student can solve one problem on the board, he will be happy. But he also loses the time to solve other tasks. Like if I design three tasks for students, he may finish one of them on the board, but he loses the time to solve other ones.

And she thinks that for students, when they give the teacher some advices or ask some questions, that is when they need more feedbacks, and if they find that no one answers their questions, they might be a little bit lost. Now with technology, it is easier and more immediate for the teacher or other students to give feedbacks. It is also an encouragement to the students.

The third is storage. With only the blackboard, if the teacher wants to add something, they need to remove what they wrote down before. If a student does not notice these things, they would not have the opportunity to see and take notes again. But now with the help of technology, if a student did not take notes during class, they can ask the teacher to review them because the software can store everything the teacher wants to show the student at any time.

In addition to these three advantages mentioned above, Mr. ZH told me about another important advantage of the DGS: intuitive. He said:

First I think is intuitive. Because only with oral language, students may not understand clearly. For example, I let students fold it twice, maybe eight students will have eight different results. But with software, they can see what result I need from the screen. So they can understand clearly about the results. So the first is intuitive.

DGS can present things which only using words cannot do. Mr. ZH believes that words may be ambiguous to students and they may interpret the same things mentioned by the teacher differently, but figures made with software will not cause these ambiguities. Students can look at what the teacher wants them to do and then react the same way based on the figures. Other Chinese teachers agree with this in their interviews, Mrs. Y said:

First, I think is saving time. Second I think it can help students make clear understanding. For example, maybe what I said will make them misunderstanding the knowledge. But letting them see the moving figure, for most of them, I think they can understand easily.

Just with words may make students feel vague and abstract when it comes to understanding geometric theory in mathematics class. Some students may also have poor skills of space imagination, and with the help of the software they can look directly at the diagrams to make learning mathematics a little easier.

Mr. X gave an example of why he thought DGS could make geometry more intuitive. In one class he said:

For example, the task today, some of the students said there was no solution. But when they see the diagram, they found that in fact it had one solution. So we can use software to make the answer more intuitive for students.

He believes that DGS can also present all the different situations depending on the task situation. It helps students know what they are doing is correct or not and gives them some links to successfully solve the task. Also in his class, he has the software show different situations to students.

In fact, DGS is for linking first and second questions in the task. For example, it helps use know how to classify different situations. And in the task about the area, DGS can help use know how to calculate the area of this object. In the lesson, students said they used "large minus small", it contained many types. It includes a new solution which mentioned by some students. They did two subtractions after spelling the objects up. Then, second, among the areas, the second one, it is moving on one side, that means the side is moving but the height is constant, and this is also obvious through the Sketchpad, which is seen constant. And when the triangle is isosceles triangle, at that time, in fact, it is more that he can find out that there are three situations in isosceles triangle. In which case there is no solution, in which there is a solution.

With the help of DGS, students can notice possible answers and find what they want to do based on the questions. Mr. W also told me that:

If we use Geometry Sketchpad, we can let student see the moving process more directly. Or because if we just draw the figure with paper and pencil, students may not see how it moves. But with software it can show the process to the students directly.

He agreed with the elder teachers that it was difficult for teachers to show the moving process to their students only with physical resources.

DGS is not only beneficial for students' learning mathematics, but also for teachers in their mathematics lessons. For the integration of DGS into the mathematics teaching and learning process, it has changed the teachers' teaching process. Whereas in previous years, teachers had only chalk and blackboard as their main teaching resources, they can now find more and more new technologies such as DGS to help them teach and make their classes more attractive. These changes require our teachers to change their teaching methods to adapt to the new situation. Teachers can use more technologies to present what they want their students to learn, and with the help of technologies they can make their lessons much more interesting.

Like Mrs. Y told me she thought:

DGS could improve teachers' skills, they do not only need to write all the things on the board and think about how to attract students. Some students may feel boring only seeing the board and just sit in the classroom. Of course, some of the teachers may have enough skills to attract students' interests. But students may be more interested in the software and make deep imagines.

In her opinion, mathematics teachers cannot just write everything on the board and only know how to solve tasks like what they did many years ago. Now, teachers need to focus on using technology to help students during lessons. Students may find it boring to see many words on the board, and just listening to the teacher can also make them think that the teacher is not a good teacher.

As in Section 6.1, I also showed the French lesson videos to Chinese teachers and discussed them in this study. Because in China, mathematics teachers never or rarely use tasks, like the French teacher did, to have their students construct objects with DGS. It is difficult for them to think about how to use these tasks in their lessons or

how to deal with the problems students encountered in solving these problems. During the interviews, I asked the teachers to design a mathematics lesson based on the French tasks and how they would modify the organization of the lesson to meet the requirements of mathematics lessons in China. Some teachers said that they would not organize the lesson like Louisa did in the video. For example, Mr. ZH told me:

I may not use the tasks like the French teacher. Why? Because, there are three tasks. All of them need students construct figures by hands, by compass and software. For me, I think only the first task lets students construct according to this sequence is enough, first by hands, then compass and then software. This can help them know the process and geometry properties, like how to construct one circle.

He changed the situation in which the tasks were solved, and although in the first task (Task 1 in Section 5.2.6) he would also have students construct the diagrams first with paper and pencil, after this he said that he did not have students construct with paper and pencil again in the following tasks:

After that, I think students need not to construct by hands and compass. They can directly construct the figure by software. Of course I will organize the process according to their skills.

If students can understand the main methods needed to solve the tasks, they can construct diagrams directly with DGS.

One of the X teachers said also that he would not use these French tasks because in China we do not require students to solve these kinds of tasks in class. And if he did use these tasks in class, he would not use them in sixth grade like the French teacher did:

I find they have this lesson in Grade 6, but in Shanghai, this content appears in Grade 8. Because at that time, students have learned what is trajectory, after that they can understand many moving process of points in the tasks. So at that time, you can tell them why we construct midpoint in this figure more easily. Because students need to know what is perpendicular bisector and

some basic knowledge of circle. After students can use software to construct circle rather than using software to explore or prove this knowledge.

Another teacher, Mrs. J, told me that

Like this task, I think it is difficult for us to use in our lessons. Because we cannot let student spend lots of time on exploring mathematics. Second, if I want to use it, I think in elective lessons may be suitable. I think I can try to do it in these lessons.

From her talk, we could see that it is difficult for Chinese teachers to use French tasks in “regular lessons”. One of the reasons, as Mrs. J said, is that we do not have as much time for students to explore mathematics with DGS in the computer lab. In China, teachers need to think about how to give students enough knowledge in the limited time they have. In each lesson, the teacher needs to talk about a lot of things, so she needs to save time and not allow the students to work with the software. In her school, students are lucky because they have the option to take elective lessons to gain more knowledge that is not in their textbooks. So in these kinds of lessons, Mrs. J tries to design the same type of tasks for the students to operate the software.

For example, I let students construct some simple objects like square. When they solve the tasks, some of them also find the same mistakes like the French student. Their constructions just look like correct but in fact it is not correct. And I also let them construct rectangles. I tell them the ratio of each sides of rectangle like one side is 6 and another one is 4. Some students may construct different rectangle with software.

We also talked about the mistakes that French students made, which Mr. ZH believes are due to their lack of technological knowledge rather than mathematical knowledge.

I think it is because he does not understand the properties of circle. He may not understand what is circle and does not know how to operate the software to construct figures. He may not master enough skills about the software. He said software knowledge is more important for students when they tried to solve the tasks with technology. Like this circle, it is obvious that it just looks

like correct. This point is not fixed any more. The student just clicks it because he thinks it is fixed. For example, if you have a segment and construct a circle with it. The students may have difficult in understanding how to put the point on the circle. I think there are two kinds of difficulties for this student.

The French student, who may have been able to construct the diagram with paper and pencil, did not understand the main method when he solved it with DGS. This caused him to have some troubles in solving the task. To solve this problem, ZH stated:

If I have this lessons, I will construct one circle first according to the program. Generally, if students are familiar with the software, they will not have the problems. And I think if my students make the same mistakes, I cannot make too much time explain to him. Because it is not the main part of my lessons.

Like Mrs. J, Mr. ZH also pays more attention to thinking about what students need to learn after the lessons.

Because each lesson has its own objective.

So such errors are not central to the mathematics lessons because they are caused by technology, not mathematics. ZH said:

Maybe I will explain to this student after lesson, and during the lessons I will tell him how to construct the circle, it is the main part I think.

Mrs. J told me that in her elective lessons, she would use similar tasks for students to manipulate software to construct geometric objects. As a result, she found that Chinese students were making several similar mistakes as their French counterparts. For her, these problems were caused by a lack of technical knowledge like the one Mr. ZH described. Her students had not learned much technical knowledge before, so it was a bit difficult for them to operate the software.

My students also make the same mistakes like this French student. Their constructions may look like correct. But the circle is not pass through the point. The students just think it passes. Then, I may drag the circle and let them see whether it is correct or not. The function of the software is to help us construct the correct figure in all the situations when we drag it. If the

figure is not correct, it may be because you make mistakes when you construct the figure.

To solve these problems, Mrs. J would take the same action as the French teacher. She said, like the French teacher:

I will let students review their construction to find out the errors.

But there were some differences between them, the French teacher, Mrs. J though she was just telling her students how to construct figures. But for Mrs. J, she would not take the same action:

Maybe I will pay attention to find what is the error in your operation, and not tell students what they need to do with the software.

She needs to let her students know why they are facing these problems and then let them solve them by themselves. Mrs. J wants to use these French tasks in her courses. She has found many benefits in using these tasks.

But I think, I will try to have the same lessons like the French teacher, I think it can have positive effects for students to understand mathematics.

With Louisa, we also talked about the use of DGS for classroom orchestration in Chinese classes. She began by describing her design for her two lessons. In the same situation, perhaps some French teachers would use the same process to use DGS for their lessons. Because the students had already learned something about working with software, Louisa did not tell them the procedure step by step:

Most teachers say "draw a triangle to the ruler and the compass, you draw a segment, you open the compass and trace". And they are just learning a procedure.

Louisa, in turn, presented tasks based on her experience of students' difficulties in learning mathematics with DGS at secondary school. These tasks were presented based on the level of complexity that Louisa thought was appropriate for her students.

So in fact it's gradual (based on difficulties). It's easy, it's more difficult and it's even more difficult. we're going more and more difficulty.

Louisa also looks for the practices of other teachers who use DGS that can give her useful information on how to use the software. Of course, Louisa also often design

different teaching processes to her students based on her own teaching objectives:

Originally, it was at least 10 years ago, I had seen some people doing GeoGebra software in Grade six, they (students) were making small freehand plots to geometry instruments and software to discover each tool little by little. Afterwards, I took it back, I kept some of their processes After, I added (some other things).

This provided her with new ideas for integrating DGS into mathematics lessons and using software to help students explore new mathematical knowledge. It also provided us with a pilot of how Louisa used DGS to design her lessons. She finds tasks online and then removes or adds new constraints because she thinks some of them might take too much time, and then she creates a sequence based on the difficulty of the tasks and organizes the lesson with DGS. And since she has started the course, she believes that she should not change the process:

I have been doing for a long time so I have some experience. I know that normally they (students) should succeed. Often, I do not change.

The use of DGS also needs to be consistent with the requirements of mathematics education. Like Chinese teachers, French teachers need to consider whether the content is appropriate for the DGS, which Louisa says may not be the primary resource for teachers today:

Now you have to do more Scratch, spreadsheet, you have to share. So, we do less dynamic geometry..... dynamic geometry loses a bit.

When looking at what Chinese teachers do with DGS, she thinks these tasks are appropriate for DGS. She said she would use the computer lab for such classes, which can help students solve problems with software. However, in China, the computer lab is rarely used for mathematics classes. Louisa also agreed with ZH's teaching process:

So, in France, there are also teachers who do like this. I know but not with their mobile phone, but with small "Flexcam". Afterwards, they put on the computer and it shows the paper pencil work of the student.

It is a way to get students to be more active in mathematics class without having to operate the software, and we can also have the software play an important role in the

lesson. And she also designed a process for teaching which is similar to the process used in Mrs. Y's class.

Sometimes, if you do it in class like that, a student can come to the teacher's computer and move the point to give explanations if it's a task to be solved.....

If this is a problem to solve, the students will actually manipulate on the teacher's computer. Of course, teacher explains the screen is common for French teachers. If we use dynamic geometry to illustrate a subject it is just the teacher who moves at the same time as he explains, it is to show students.

It demonstrates two main ways of using DGS in the classroom. We can see that when introducing new contents, such as a new theory or a new property, Louisa prefers to teach like a Chinese teacher, while she tries to have some of her students operate the software in class when she lets them solve the tasks. This shows that DGS is one of the tools for her to help students solve or explore mathematics problems. However, in China, it is used more for presenting information to students. Because teachers in China rarely let students use the software, and even if some teachers, like Mrs. J, give students some time to operate the software, it happens when students have already completed the task and the lesson is for discussing the solutions.

6.2.2 Classroom organization model of teachers with DGS

For the two Chinese teachers, Mr. X and Mr. W, it is difficult for me to model the teaching process because I did not record enough lesson for them. So the following discussion is based on the other three Chinese teachers and the French teacher.

From the two lessons, although Mr. ZH is dealing with different students and different mathematical contents, he has made a similar arrangement in these two lessons. In terms of didactic configuration, the two lessons are similar: a computer, a screen that can display contents, and a regular classroom. In both lessons, *explain-the-screen* was common in ZH's classroom. *Spot-and-show* is another important type of instrumental orchestration with DGS for ZH, although we do not find it in the first

lesson, which is more due to the physical limitations of the first lesson. In this lesson, he does not make the lesson in his own school. This means that all the students and equipment are new to him. He needed to change his teaching activities to meet the technological limitations of this lesson. For example, he could not upload his students' work to the internet during the first lesson with his smartphone, which is an important tool for his own school.

The following model could be used to present the process of ZH's didactical practice: *whole-class-discussion* ----- *spot-and-show (if possible)* ----- *explain-the-screen by one or more students* ----- *explain-the-screen by the teacher*. This means that from these two lessons, first, to solve the task, he asks the students to discuss it with each other so that almost every student has a chance to say something about the task. Then, if he had a smartphone, he would select some students and put their work on the screen as the topic of the following explanation. One or two students would then be asked to explain what was happening on the screen based on a series of questions ZH posed during the explanation. After that, regardless of whether the students' answer was correct or not, ZH would explain it again by himself. And only in this arrangement dynamic geometry was used to help students understand the properties more clearly.

Based on the description of the teaching process in J's lessons, we can also find some characteristics, for example, in these two lessons, Mrs. J prefers to spend more time for students to give their own opinions on mathematics tasks. So instead of controlling the process in her lessons, she chose to play as a supporter. She tries to let students think in depth with a sequence of questions. Just like in her first lesson, she asked questions such as *what type is the quadrilateral, how can you know it, why you justify DN is parallel to ME* , by answering these questions, students can gain a deep understanding of mathematical problem-solving strategies.

We could use the following process to describe the general organization of J's mathematics lessons: *Sherpa-at-work* ----- *guide-and-explain* ----- *explain-the-screen*. Unlike Mr. ZH's lessons, Mrs. J did not give the students much time to discuss the tasks before teaching because in Mrs. J's classes, all the tasks were solved

by the students before the lesson. So the students had already figured out a strategy for solving the problems, all Mrs. J had the student do was go over the homework and try to answer her questions. After the students answer the questions, Mr. ZH makes his own conclusions, but in Mrs. J's classes she gives the opportunity to one or more students to make conclusions as she interacts with the student. During the students' presentation, Mrs. J did not ask too many questions to interrupt their thought process. She chose to allow all students to express what they think. Just as she did in the first lesson, she simply used words like then, and, and next, or repeated the students' statements to keep the explanation continue.

In her second lesson, based on her general teaching process (using *explain-the-screen* and *guide-and-explain*), she tried to make some changes. She found that she could have students manipulate the software and interact directly with DGS. She even had two students show her how to construct a diagram based on the task situation, while she acted out the steps according to what the students said. In China, it is not common for teachers in mathematics classes to have students operate software. Usually, in schools, there is only one computer in the classroom. And it was always controlled by the teacher, and in most cases, the students had to sit in the classroom and watch how the teacher operate the software. They could not decide for themselves how to operate the software. In Mrs. J's second lesson, she promoted it and changed her usual teaching strategy to make the students the operators of DGS. Although only one or two students in the classroom had the opportunity to operate the software directly, the other students had to think about how to move the points with DGS. They were also involved in the operation in another way.

In this study, we observed two of Mrs. Y's mathematics lessons in order to describe her approach to using DGS in her mathematics lessons. From these two lessons we can see that Mrs. Y organizes her lessons in different ways. Although her second lesson is taken one day later, it shows a huge difference between the two lessons. In the first lesson, she uses more of *explain-the-screen* to organize lesson with DGS. And there were not too many other types of instrumental orchestration in her practice. I also found that DGS was always working in this lesson, but over a

period of time Mrs. Y did not interact with this software in any way. She chose to teach her students with traditional resources such as blackboard and chalk. This is because for her, DGS is new. Before the research she did not know much about using software to teach mathematics. This class was her first time programming a lesson by using DGS. Before teaching, she tried to learn how to use the software as quickly as possible. In order to better integrate DGS, she contacted teachers and other colleagues in her school for some suggestions. However, it was difficult for her to use it because there were no available models to learn from. Unlike the two elder teachers in this study, DGS was not used regularly in mathematics lessons at Mrs. Y's school. All teaching processes needed to be designed by herself.

In the second lesson, although the main type used was still *explain-the-screen*, Mrs. Y added a new type in her lesson. She had two students work with the software during the teaching to construct some diagrams on their own. I put these teaching processes into *Sherpa-at-work*, which is not common in Chinese mathematics classes. The *Sherpa-at-work* orchestration as Drijvers et al. (2010) characterize may guide students to develop their instrumental abilities if the teacher asks the *Sherpa* student to “carry out specific actions in the technological environment” (p.220). This requires the teacher to have enough skills to organize a suitable teaching process. In this study, only two teachers attempted to use this type, one was Mrs. Y and the other was an elder teacher Mrs. J. So for Mrs. Y, getting students to operate the software in the classroom is a big challenge. Many mathematics teachers in China are concerned that if they use DGS too often, or let their students operate the computer, they have no control over what happens and the students may not learn anything at the end of the lesson. They need to make sure that their lesson objectives are met after the lessons. Otherwise, although the use of software can make mathematics lessons easier and more attractive, it is not just a school principle but parents may not agree with. This is one of the main reasons why Chinese teachers do not use software in classes very often. Therefore, I was surprised that Mrs. Y allowed students to operate the software during class.

Unlike the two experienced teachers, we could not find a model for presenting

Y's lesson practices with DGS. In each lesson, the *explain-the-screen* was the main type. The fact that Mrs. Y herself controlled the whole explanation process made her lessons more teacher-centered, especially her first lesson. Students did not have the opportunity to participate in her teaching process with DGS. They just sat there and listened to Mrs. Y's explanations. For some time, DGS was not critical to the teaching process. Mrs. Y went back to using traditional resources such as the blackboard and chalk. Although she tried to make some changes in the second lesson by having the students construct some diagrams themselves by using DGS. However, I still found that *board-instruction* appeared five times in this lesson. From these two lessons, the DGS is a new resource in Mrs. Y's teaching career. She is still trying to find a more appropriate way to use this new resource in her lessons to make her teaching more effective.

In this study, as mentioned earlier, the Chinese teachers were divided into three groups based on their teaching experience: two elder teachers with over 20 years of teaching experience as well as lots of experience with technology; two young teachers, one of whom had extensive experience with technology while the other did not; and final one with 10 years of teaching experience and familiarity with technology. A simple comparison of these Chinese teachers shows me that, in general, Chinese teachers are used to organizing technology lessons with *explain-the-screen*. This type is found in all Chinese lessons. This is because in China, teachers always act as the controller of the classroom. They decide what to do in the classroom and like to control the process of students' work in all subjects. If we analyze these lessons according to the different groups, we will find that elder teachers show more types of instrumental orchestration with DGS. Younger teachers, on the other hand, tended to use a smaller number of instrumental orchestration types if they found them to be appropriate. Although both the elder and young teachers used *explain-the-screen* during their lessons, the elder teachers often had their students answer questions by using DGS or they operated software to help them answer questions if they were having difficulties, whereas these processes did not occur with the young teachers. So from the above table, *guide-and-explain* appears only in the classes of

the elder teachers. Although one of the young teachers, Mr. W, also used this to arrange his lessons, it was not common and always lasted only about one or two minutes. Chinese teachers, especially the young teachers, rarely allow students to discuss what is happening on the screen during classes based on the DGS. Elder teachers may have students discuss in class, but this is not the predominant type of instrumental orchestration in China, and this type is only present in the *open lesson* in this study. In general, Chinese *open lesson* needs to show some differences compared to regular lessons and teachers need to make students more active in these classes. So we can find that students have more time to discuss.

Another important thing to be noticed is that Chinese teachers did not change much across the different lessons in which they used DGS. That is, there was not strong changes in these types of instrumental orchestration. For example, all of the Chinese lessons made greater use of *explain-the-screen* although the content of these lessons varied. Even if one teacher faced with different mathematical contents, his arrangement of the DGS is similar. Of course, in some cases, teachers will make changes in the classroom. For example, a teacher might change the way he uses DGS when he feels he needs to deal with a student's problem by showing some of the student's work on a screen and let the students explain their problem solving strategies. At that point, the teacher will change the way they use DGS. But if you look at the whole lesson, the main process is the same. The teacher controls the lesson and operates the software to help teach it.

Louisa used a different approach to organize the teaching activities with the help of DGS due to the change in resources and lesson objectives. It was clear that the way the lessons were organized with DGS was completely different in these two lessons. In the first lesson, the objective of the course was to show the students how to transform strategies in different situations. Louisa was careful to make connections between the different situations to help students know the differences. In the second lesson, however, Louisa needs to have her students learn something new. With the help of software, she designed a learning situation that allowed students to explore mathematics on their own. Louisa changed a lot of the programming to help students

be successful after the lesson. When students encounter some problems in solving the task, she guides them to the solution or just tells them what to do. She does not let students operate the software for too long when they have problems. When she explains the mistakes to her students, Louisa becomes the center and the students are not as active as they were in the first lesson. As we discussed in previous chapter, this change was caused by the objectives of the lesson. The first lesson was for students to experience different situations to solve tasks, so they needed more time to work with the software. Whereas the second lesson is for students to learn new contents, so they need more guidance on what they need to do, rather than just exploring randomly.

6.2.3 Main features of the instrumental orchestration in mathematics classes

In this section, I talk about some of the main characteristics of Chinese and French teachers' practice of using DGS in schools. According to the data from this study, Chinese teachers firstly show more control in their classrooms compared to French teachers. Both their organization and their perceptions of DGS showed that the Chinese teachers had a more teacher-centered lesson.

If we look only at the types of instrumental orchestration shown in the Chinese lessons, we can easily conclude that these lessons are more teacher-centered. Based on the above analysis, *explain-the-screen* and *guide-and-explain* are used more by Chinese teachers. According to Drijvers, *explain-the-screen*, *link-screen-board* and *guide-and-explain* imply more control by the teacher in the teaching process. Due to the equipment in these lessons, only the teacher can operate DGS during the teaching process. Students have less time to interact with the technology. This instructional process is more teacher-centered (Drijvers et al., 2010). The student's work is to sit in the classroom, listen to the teacher and write down the main points of the lesson. Chinese teachers, on the other hand, often try to organize their teaching in the traditional way. What they do is add technology into the traditional way, but they do not change the teaching process much more.

Chinese teachers are always the center of the classroom. They design all the teaching process first and teach according to these plans without expecting any incidents during the lesson. Even DGS is becoming more and more important in mathematics teaching. Teachers still see it as a supporter to their teaching. Rather than using them to create new learning situations for their students but add them into the lessons which without any technology.

For example, in Mr. W's class, I talked about how DGS was used as a slide to show the contents. It did not show the main functions which could help students learn mathematics, such as displaying diagrams and changing objects based on the user's actions with the computer. Mr. W did not make the graphs on the screen move. Almost every diagram was stable, just as the teacher had shown it through the board. With this design, Mr. W could control the lesson so that it continued according to what he wanted. Both the experienced and young teachers in this study organized their teaching with *explain-the-screen*, *board-instruction* and *guide-and-explain*.

The DGS was always under the control of the teacher, except for Mrs. Y, who had some of the students operate the software during the lessons. For example, the software was still controlled by Mrs. J, although she also used *Sherpa-at-work* during the lesson, in which she acted as a *Sherpa student* (see section 5.3.3). In particular, Mr. W used DGS, but did not do much more than display the tasks and answers. The students also did not have time to interact with the software during the lesson. They were faced with the same learning situation like that in traditional classroom and this left them without much change on their learning activities in response to the new situation with DGS.

Except for Mr. X, other teachers sometimes asked their students to explain the screen based on the questions they asked. In Mr. X's class, his students did not answer many questions during the lesson compared to other Chinese teachers such as Mrs. Y and Mrs. J. This shows that the Chinese teachers' view of mathematics teaching is that the teacher should be the center of the class. They need to design and prepare all the things of the lesson according to their own plans before and keep the lesson going smoothly. Allowing students to operate the software may cause some accidents in

the teaching process, which is not what Chinese teachers want. This is because for Chinese mathematics teachers, they all face the problem of having a limited amount of class time and students needing to learn a lot of contents.

Although Mrs. Y and Mrs. J tried to have students manipulate the software during their lessons, it was still infrequent. In general, their lessons are still controlled by the teacher. Our teachers also knew that the students needed to be the center of the mathematics classes, but being stuck with the examination, all they could do was to have the students answer more questions and do more exercises during the classes. From the study, it is clear that Chinese teachers use *guide-and-explain* as a second major type of instrumental orchestration to organize their lessons. And although DGS is part of the mathematics lessons, there were some times in this study when teachers were still only using traditional resources for teaching. All of the Chinese teachers in the lessons using DGS used *board-instruction*, which meant that they did not operate DGS again during this time, although the software still showed some objects and figures for their students. But they were all focused on the board and the teacher. No one was interacting with the software.

As with the other lessons in this study, although we can place these teaching episodes in the category of Drijvers' work with DGS, there are still some differences in the Chinese mathematics lessons. Thus, the second main feature of the Chinese lessons was that there were some new characteristics that were not presented in Drijvers' work. For example, in ZH's second lesson, he made some orchestration by using a type that is not described in Drijvers' work.

The didactic configuration of this lesson is similar to the above example: a computer with DGS connected to the Internet, a screen displaying students' works or mathematics tasks, and students solving problems with paper and pencil. During the teaching, ZH places the mathematics task on the screen via DGS, but does not move the diagram in the following episode. As a didactical performance, ZH starts the second episode by having the students solve the problem with paper and pencil, although ZH is also operating the computer, but all he does is display a static diagram through the DGS. In these episodes, the students solved the task individually or in

pairs as they wished (in fact almost all of them solved the task individually and then when several students completed the task they started helping and discussing it with their tablemates). In these episodes, no interaction occurred between ZH and the students. This orchestration differed from *discuss-the-screen* because there was no interaction between students or between students and ZH. During this time, no one controls the software. The software acted like a projector, displaying diagrams or tasks on the screen. The students solve the task with paper and pencil. They can see the diagrams and imagine what would happen if they manipulated the software by dragging the points according to the task. So these activities are different from the types of instrumental orchestration mentioned in the Drijvers' paper.

The same process occurred for all the tasks in the second lesson, and ZH found that many students had difficulty solving the problem. To help students understand the problem-solving strategy, he chose one or more students who had successfully completed the task as an example to help others work. After ZH put the student's work on the screen, no discussion occurred; other students simply worked alone, and this student's work served only as a guide or a sample to help others. In addition, the dynamic geometry in this activity just serves as a projector to present the student's answers on the screen. No dragging occurred. In the second lesson, though, ZH also had students discuss the solution of the tasks before explaining how to solve the problems in this lesson. All discussion is based on paper and pencil. ZH just puts the diagram or task on the screen via DGS. The diagram itself is no longer moving. So the discussion or solution process is not like *discuss-the-screen* type.

The other examples are from Mrs. Y's class which shows few types in her practice. I only found *explain-the-screen*, *board-instruction*, *link-screen-board*, and *Sherpa-at-work* in her lessons. Comparing Mrs. Y's lessons with Drijvers' study, there was not much difference between *explain-the-screen* and *board-instruction*. In both types, the whole teaching practice is controlled by the teacher. However, I found some differences in *Sherpa-at-work*. In Drijvers' study, the *Sherpa student* is a so-called "robot" when it comes to operating the software. His operations are based on commands from the teacher or other students. He just does what he is told to do. But

in Mrs. Y's class, the *Sherpa student* was doing things on his own. In the description above, Mrs. Y told others not to talk to him when she had the student go to the front of the classroom to construct a triangle by using software, so he could solve the problem himself. This means that in the teaching episode, the *Sherpa student* in this lesson does not just play the role of a "robot" and act according to a program that has already been designed. The *Sherpa student* has his own ideas about what he can decide to do and think about how to solve the problem. Only when he tells his teacher that he finds problems he cannot solve, Mrs. Y goes to him and helps him deal with these difficulties. But when Mrs. Y helps the student, she also does not tell him what to do, but guides him to find his own solutions. This is the main difference between Mrs. Y's practice and Drijvers' work.

A third feature of Chinese lessons is that teachers are beginning to notice that students need more time to engage with the lessons. To provide a deeper understanding of the different strategies Chinese mathematics teachers use to integrate technology, I have chosen an example of a lesson from an experienced teacher, Mr. ZH, which we call the *China Open Lesson*.

In China, *Open Lesson* is vital for every teacher, whether experienced or young teachers. Almost every year, teachers are required to attend *Open Lesson* in schools. These *Open Lessons* give teachers the opportunity to promote their teaching practice because in these lectures other expert teachers can focus on their teaching practice and make suggestions at the end. Another important purpose of *Open Lesson* in China is that the Chinese Education Commission wants to provide some models of "good lessons" for teachers. In other words, the Education Commission chooses some experienced teachers to give *Open Lesson* and uses these classes as a good teaching practice for other teachers to learn from. So these *Open Lessons* always reflect what the trend is in Chinese mathematics education. For example, Mr. ZH's first lesson was an *Open Lesson*. There are many expert teachers involved in the teaching process.

We can see from this lesson that Mr. ZH began to allow more time for students to present their ideas. He wanted to change the traditional strategy in Chinese

mathematics classes that the teacher is the center of the teaching process. He began to notice that the students could explain the mathematical contents by themselves. Although he still used many teacher-centered instrumentation orchestration, such as *explain-the-screen*, he also tried to guide students in explaining mathematics themselves. So we can see that Mr. ZH gave students more time to participate in the mathematical learning process which is one of the important tendencies in Chinese mathematics education. And in the second lesson, ZH showed this tendency in his regular lesson as he started to not only let the students learn mathematics under the guidance of his questions, but also asked them to talk about their problem solving strategies based on their homework. So we can notice the use of *spot-and-show* in his class. ZH also asked many questions when explaining the tasks, and tried to select a few students to explain their problem solving methods. However, these processes did not continue for much long in these two lessons.

Another experienced teacher, Mrs. J, also noticed that she needed to give her students more space to think about or engage in teaching activities. In her first lesson, she made more use of *explain-the-screen* and *guide-and-explain* to organize the lesson. On the surface, it would be easy to define this type of instrumental orchestration as teacher-centered, according to the definition that Drijvers has given. But it is not enough to classify Mrs. J's class as teacher-centered in this study. Based on the deep analysis of the details presented in each instrumental orchestration, I find that Mrs. J does not always control the explanation. She gave more time to her students and allowed them to become the controller of the teaching process. In Drijvers' work, when the teacher uses *explain-the-screen*, it means that the teacher acts as an explainer while the student is in a passive position during the instructional time. So it is easy to think of this process as a teacher-centered lesson. But here, in Mrs. J's class, we need to notice the differences. When she arranges her lessons through *explain-the-screen*, it is not only she but also her students who are at the center of the process. Her role at this point is a guide or supporter to help the students make the right solution. At first, she asks questions to make students think deeply, then she gives them more space to think and never interrupts them as they make their

conclusions to the class. From this we found that the students in both lessons did not always play a passive role in the lesson.

And in the second lesson, Mrs. J makes a step further. She started letting her students work with the software on their own. Her students became more active in the second lesson. Although there was only one computer and one iPad in this class, Mrs. J tried to create a more student-centered situation so that her students would have enough time to experience learning mathematics. As mentioned above, in her school, a private secondary school in Shanghai, the mathematics teachers like Mr. ZH has access to more advanced digital resources such as iPads and smartphone in classes. With the iPad, Mrs. J can easily have students operate the software. Students do not need to stand in front of their classmates and demonstrate how to operate the computer, they simply sit in their seats and click on the iPad. These digital resources can help teachers create a more student-centered situation and make students more actively in mathematics lessons. Mrs. J's second lesson can help other teachers think about how to organize their classrooms and give students the opportunity to interact with DGS.

In other lessons, teachers are beginning to experiment with giving students more time to participate in the teaching process, like Mrs. J. *Explain-the-screen* is the main type of classroom arrangement in all Chinese mathematics classes with DGS. They often integrate the software into the traditional lesson organization. In a traditional lesson, the teacher usually asks the students questions. These questions help the teacher understand if the students have learned what they are talking in the classroom. Therefore, in this study, these teachers also ask several questions to their students when they operate the software during classes. These questions are about what happens on the screen. These questions allow them to let students think about mathematical problems, even though they cannot operate the software themselves. In terms of the teaching episodes, the two elder teachers used *guide-and-explain* more than once in their lessons. And this type was always used in conjunction with the *explain-the-screen*. First, these two teachers had some of their students try to explain the screen by posing several questions, and then they would make their own

conclusions.

As I mentioned above, Mrs. Y made changes between lessons. In the first lesson, she is the center of her teaching process. This lesson is what we call a teacher-centered lesson. This can be seen in the type of instrumental orchestration she uses. And in this lesson we did not find that her students had time to interpret the screen guidance through the questions that Mrs. Y asked. Of course Mrs. Y asked many questions during the lesson, but she did not select some students to stand up and answer questions like other teachers did in the classroom. Her students always answered questions together or randomly, which made her classroom a little noisy and uncontrolled. This difference suggests that Mrs. Y needs to improve her teaching experience in the future. She needs to think about how to give her students more opportunities to express their ideas and make her lessons more orderly.

In the second lesson, she began to make her students more actively in her lessons in a different way. To do this, she made a big change in her lesson. First, she manipulated the software to construct some objects for her students. By doing this, she wanted her students to know the procedure for using software to construct triangles. And then she explained how to use the software in order to make sure that all her students understood the steps she was talking about. During the explanation, she asked some questions, but most of them were answered by herself. This episode can be described as an *explain-the-screen*. There is not much different compared to her first lesson. But after that, she let some students come to the front of the classroom and try to construct a given object on their own by using software, just like she did. At that time, the other students were just watching what this student was doing on the screen. She told the other students not to talk to him or give him any advices. In this study, I put this instructional process into *Sherpa-at-work*, although it differs somewhat from Drijvers' study. In Mrs. Y's lesson, there was a *Sherpa student* who operated the software on his own, but in Drijvers' work, this *Sherpa student* operated the software based on others' guidance. Neither the teacher nor the other students gave him any advice or told him what to do in Mrs. Y's lesson. This teaching process is not common in China.

As we have said before, mathematics classes in China are held in regular classrooms, and these classrooms have only one computer. As a result, the software is controlled by the teacher and not by the students, which results in Chinese students having very few opportunities to interact directly with DGS. Thus, for Mrs. Y, she was a pioneer in using DGS. Although one of the elder teachers in this study, Mrs. J, also used *Sherpa-at-work* in her lessons, there were many differences between the two lessons. First, Mrs. J had iPad for her lessons, which made it easier for her to let her students work with the software. All she has to do is giving the iPad to the students and they can then work on it anywhere. However, Mrs. Y is not allowed to use iPad and has to have students come to the front of the classroom to work with the software. This made it more difficult for Mrs. Y to use this type in her lessons. Second, sometimes Mrs. J herself acted as a *Sherpa student* in her lesson, she had a student show her how to construct the required diagrams for the task and she operated the software based on the answers given by the student. This is more like Drijvers' *Sherpa-at-work*, but it is the teacher rather than the student who plays the role of *Sherpa student*. In Mrs. Y's class, the students operate the software based on their own. Others, including Mrs. Y, do not tell them how to operate the software. This was the main feature in Mrs. Y's lesson. From these two lessons, we can see that Mrs. Y is also trying to make her lessons more open to her students. And she tried to build a learning situation with DGS to make her students more active and involved in the mathematics learning process.

In some lessons, although Chinese teachers chose to use DGS to organize their lessons, it was forgotten by them for some time. As we can see, throughout the lessons, DGS did not present any problems that affected the teachers' use it for teaching. However, teachers like Mr. W, at some time, did not operate the software anymore. We can see that this phenomenon appeared in all the Chinese mathematics lessons in this study. Especially in the lessons of young teachers, according to the descriptions, elder teachers use DGS more times than younger teachers. DGS played a more important role in their classrooms.

In this study, I also found that elder teachers are more familiar with different

teaching resources such as blackboard or DGS. They notice the connections between these resources and try to integrate them together to arrange lessons. They make different resources work according to the mathematical contents in the classroom and make these resources play an important role in their teaching. So, we found that they use *link-screen-board* in some of their lessons. This type of lesson is not common in the lessons of young teachers.

6.2.4 DGS is for presenting contents in China cases while checking and exploring in France cases

Based on the analysis of classroom observations in this study. In China and France, DGS plays a different role in the teaching and learning of mathematics. One of the important functions of DGS is that users can drag objects on the screen to explore or examine mathematical theories. In these two countries, this mode shows its different roles in this study. In China, teachers used DGS more often to present contents during instruction and young teachers were more likely to show this kind of use in their mathematics lessons.

As Mr. X told me in the post-lesson interview, the DGS acted as a supporter of the mathematics teachers for the Chinese teachers. This was also shown in their teaching in this study. For example, in Ms. W's class, the DGS did not show the students too many moving processes during the lesson. As we discussed above, in this lesson, Mr. W used the DGS just like slides which is common in Chinese mathematics classes. The teacher prepares everything on the slides before the lesson, including the answers to the tasks or problems. During the lesson, they show these slides one by one. By doing this, teachers can save a lot of time, for example they do not have to write down the tasks on the board and they can use that time to let students do more exercises. Mr. W did the same thing, he wrote down all the tasks and answers with DGS. During the lesson, he showed each of these tasks and answers. He asked students to answer the questions and then displayed the answers to check if they were correct. Although we know that DGS can show the moving process, which is one of

the most important features of the software, Mr. W did not show this feature too many times. He only showed the students the moving process once in his lesson, which was not a critical part of the instructional process. The rest of the time, DGS just acted as a projector for the mathematics instruction.

Also in Mr. X's class, he prepares everything including tasks, software and diagrams before the class. During the class, he operates the software according to his design. Everything is under his program. When Mr. X felt that he needed to get students to visualize the task clearly, he chose to open the software and manipulate it to change the objects according to the task. And these changes, of course, were planned by Mr. X before the lesson. Neither during the lesson nor during the exercises did the students interact directly with the software during this lesson. All of them just watched what Mr. X move with the DGS. For most of the lessons, DGS simply presented the diagrams to the students. When students attempted to solve the problem, the diagrams did not change again, which means that the software did not affect the students' work in the classroom. Their learning activities were the same as they would have been without the use of DGS. This means that in this lesson, DGS played the role of a facilitator, and Mr. X did not use it to create or change the learning situation for the students. In these lessons, the DGS did not change anything for the students' learning of mathematics. He used the DGS to present charts based on the tasks described above. He arranged different types of instructional processes such as *explain-the-screen*, *link-screen-board*, and *board-instruction*. As we said, the classroom is more teacher-centered. His students do not have time to interact with DGS. They can only look at the screen to see what Mr. X wants to show. So we can say that DGS acted as a supporter of Mr. X in these lessons and rarely changed the learning activities of the students.

In China, some teachers such as Mrs. Y and Mrs. J tried to change the way it was used in mathematics instruction by having students manipulate the software to solve tasks, which changed the problem-solving situation for students. This led them to change their strategy and they needed to transform between two different situations, paper-pencil and DGS. For example, Mrs. Y has two students construct triangles with DGS. The students were faced with two different situations when using the software.

Mrs. J not only had the students operate the software, but also had some of the students show the construction program to other students, and she operated the software according to the students' programs to check if the students' answers were correct. This makes students think not only about how to solve problems with software, but also how to present them in a clear way that others can understand. This is a difficult task for students, and in other lessons they do not meet this kind of task. So, both Mrs. Y and Mrs. J tried to create a new type of learning situation with DGS by changing some of the lesson plans to allow students to move between different contexts. However, this is not common in Chinese mathematics classes. In this study, the other students were only facing one situation in the classroom and they did not need to think more about how to change their learning activities in different contexts. So for the Chinese teachers, Mrs. Y and Mrs. J created a new example for integrating DGS into a mathematics lessons and making the students the center of the lesson.

In France, from the video of the lessons, Louisa demonstrates the different uses of DGS in mathematics instruction. First, Louisa had students solve the tasks by themselves with paper and pencil and then with DGS. In the first lesson, all tasks are completed without teacher guidance. Louisa would only come to see what was going on if the student completed the work or had troubles. During this instruction, Louisa always let students drag the points on the screen to check if their work was correct, regardless of the different types of instrumental orchestration. In this lesson, the dragging mode is used to check students' work. In the second lesson, however, its function changes. As in the above episode, in second lesson, students use the software to drag points not only to check their work, but also to explore the task and knowledge. In the task, students needed to think about whether some of the segments given could be in one triangle. From the video, I found that they dragged the segments and tried to construct them into a triangle, as shown in the figure. This student is moving the segments so that they can construct a triangle. From the process, the dragging mode played a different role compared to what in the first lesson.



Figure 6.2: Student moved the segment with DGS

6.2.5 Teacher knowledge revealed in class orchestration with DGS

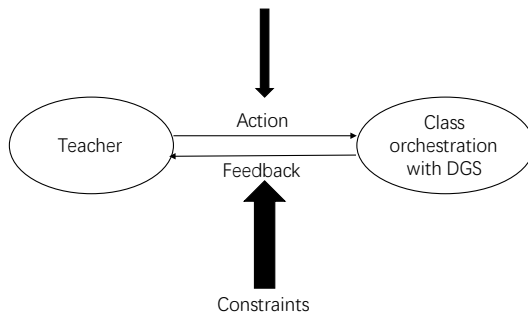


Figure 6.3: Teacher knowledge revealed in class orchestration with DGS

In China, according to the analysis of this study. We can see that the configuration of the teaching environment is similar and never changes during the lessons. All the classrooms have only one computer and it is mainly for used by teachers. Students sat in pairs and only in Mrs. J's class they sat together in small groups to make discussion more easy to happen. The resources used by the teachers were also similar, except that in Mr. ZH and Mrs. J's private school, they had access to iPad and smartphone during the lessons. With similar didactic configurations, most of the teachers in this study, their practice of DGS presented their own processes. Like Mr. ZH and Mrs. J, we can use the process in Section 6.3.2 to describe their practices of using DGS. Even confronted with different mathematical contents, they used similar instrumental orchestration in their classrooms. Other Chinese teachers, such as Mrs. W and Mrs. X, still showed the same characteristics as the other teachers, although we could not use a specific process to show their teaching practices by using DGS. They repeatedly used the same types, for example, *explain-the-screen*. Only Mrs. Y produced two completely different

lessons in this study. In the first lesson, like the other teachers, she organized the teaching process in a teacher-centered way in order to make preparations for lessons, prepare administrative documents and change the environment in their courses (Demir, 2011; Palak & Walls, 2009). And in the second lesson, she had the students come to manipulate the software and construct some triangles in front of the class. Mrs. J also showed *Sherpa-at-work* in her class, but one major difference was that in her class, she was the only one controlling the software, and the students were just telling her what to do. Whereas in Mrs. Y's class, the student controls the software and the others never speak to them during the construction process.

In the Chinese lessons, there is a phenomenon where students' work is almost exclusively about remembering facts, theories, or applying formulas, algorithms, and procedures that the teacher has taught them, without focusing on why or when it makes sense to do so. Students who follow the teacher's activities may not understand the purpose of the activities, and this teacher-led behavior may not help students apply their own methods to overcome the difficulty (Assude, 2007; Erfjord, 2011). In these lessons, "Doing mathematics means following rules laid down by the teacher; knowing mathematics means remembering and applying the correct rule when the teacher asks a question, and mathematical truth is determined when the answer is ratified by the teacher." (Lampert, 1990, p.31)

In France, first of all, the didactic configuration is different. The first two classes are held in a computer lab, where students can have their own computers to work on. This resulted in a different orchestration for Louisa's lessons. She used more *Sherpa-at-work* in the first lesson. And there is another main reason for this orchestration. The students were already familiar with DGS to construct diagrams and they had already learned the mathematics needed in the first lesson, so Louisa could have them interact with the software. In the second lesson, where the content is new to the students, Louisa uses more of a teacher-centered approach, such as *explain-the-screen* and no longer using *Sherpa-at-work*.

The Chinese and French teachers showed different ways of organizing the teaching process, as shown by the example of the lessons. All the Chinese lessons and the third

French lesson took place in a regular classroom, with little difference in the physical environment, while the teachers used different instrumental orchestration. This phenomenon can be explained by the fact that it is not (or not only) the physical environment, but teacher knowledge determines the teachers' interaction with DGS. Their different instrumental orchestrations are based on their perceptions of DGS. For example, the Chinese teacher tended to believe that DGS was an aid and that the use of software should be controlled by the teacher, so student-centered instrumental orchestration types such as *Sherpa-at-work* or *spot-and-show* were rarely used in their lessons, and they were more likely to use teacher-centered types. However, this French teacher believes that students should learn mathematics through exploration, so *Sherpa-at-work* or *spot-and-show* are used most frequently in her classes, even in the regular classroom.

Based on Drijvers' work in 2013 (Drijvers, 2013), teachers show their knowledge relating to technology in types like in *technical-demo*, and their knowledge relating to pedagogy in types like *guide-and-explain*. Back to the above lessons, Chinese teachers do not show the knowledge relating to technology which related to solve the technical problems when using DGS, in contrast, they show more knowledge relating to pedagogy during the lessons based on the types they choose with DGS. Chinese teachers show knowledge relating to general didactical process and make DGS as a supporter (Hughes, 2005). We also have the hypothesis that in these Chinese cases, they do not need to talk much about technology in preparing their mathematics lessons and show little this kind of knowledge, which is part of using technology to create meaningful learning situations for students. In contrast, the French cases show more relevant knowledge.

In the Chinese lessons, we can also see some changes in teachers' practice in the use of DGS. Some teachers started to think about how to make their students more active in the classroom and tried to give them more time to participate into the teaching process. They feel that teachers could allow students to express their ideas. As a result, a student-centered type of instrumental orchestration began to appear in Chinese classrooms, reflecting the trend in Chinese mathematics education.

Based on the analysis in this study, knowledge is shown in teachers' interaction with the teaching situation (including technology). From the above example, the teachers demonstrate different ways in which they organize their teaching process. Their activities in the instructional situation may be differently. Some teachers, such as Mrs. J, have students sit together in small groups to make discussion happened easily, while most teachers still have students sit in pairs. With similar didactic configuration, most of the teachers in this study, their practice with technological resources showed that their teaching process was stable. Even when confronted with different mathematical contents, they used similar strategies to integrate technological resources into their mathematics teaching, suggesting that it is not (or not only) the physical environment, but teacher knowledge that influences the teacher's interaction with teaching resources. They prefer to use more teacher-center ways in order to make preparations for lessons, prepare administrative documents and change the environment in their courses. These teachers see technological resources as supporter and should be controlled by teachers (Palak & Walls, 2009). It was obviously that this use of technologies only changed physical classroom environment without leading any changes in learning-teaching routines. These approach reflects that whether students are given access to technology like graphing calculators is connected with teachers' opinions towards the role technology play in learning process (Doerr & Zangor, 2000; Leatham, 2002, 2007). These Chinese teachers believe that technology need to be used as a supporter for teaching and used after students mastering mathematical concepts and can solve problems by paper-pencil (Kastberg & Leatham, 2005).

6.3 Differences between French teacher and Chinese teacher in interaction between teacher and students

In the following section, after analyzing the questions and feedbacks given by teachers in the classroom. We need to focus on other kinds of interactions between teachers and students during the teaching process. As mentioned in the interviews with

Chinese and French teachers in this study. They all talked about how students could present their ideas with others during the discussion process. The teachers, in turn, often move around the classroom to listen and participate in what the students have discussed. Interaction during these times should be different from the question and answer process. Therefore, it is helpful to focus on these times of interaction to gain an overall understanding of the knowledge that teachers demonstrate in their conversations with students.

6.3.1 Teachers' opinion towards the interaction with students

After analyzing the different types of mathematics tasks used in China and France, in addition to identifying the different ways of using DGS, I also found different questions posed in the lessons. In China we can see that the questions that teachers ask students to answer are more about finding a mathematical relationship between some geometric variables. The students are concerned with finding an appropriate way to solve the problem. Chinese teachers always tell their students to try to draw some figures to show some particular moments when some points move. Then they try to answer all the questions using their geometrical knowledge. They do not need to use technology in the problem solving process.

Like the sections above, I begin with the teachers' perceptions of the interaction with students in dynamic geometry situation. When we talked about interactions with students, teachers mentioned not only verbal interactions, but also other types of interactions such as gestures. In China, generally, the teacher stands at the front of the classroom during most of the class time. And in every class, both in France and China, almost every teacher would let students do some exercises on their own. At that time, the teacher always walks around the classroom to see how the students are doing on their own. Then some different types of interactions take place between the teacher and always one particular student. Here, we take a look at how these sample teachers handle these exercise times and how they interact with their students.

For Chinese teachers, they need to spend more time face-to-face with their

students in the classroom during instruction. This allows the teacher to notice students' facial expressions and gestures to determine if they understand the contents. It also shows up during their classroom discussions. For example, Mrs. J said to me:

First I will walk around the classroom to see students' body language, because if they have some special body language, it may show that they have some difficulties.

When she finds that some students have difficulties to solve some tasks or problems, Mrs. J will walk to them, listen to them, and answer their questions in order to help them successfully solve the tasks. This is the first focus that Mrs. J pays attention to when she has group discussions with students in the classroom. Second, she will also look at the answers that the students have written on their paper and see the students' problem solving process. Mr. ZH, another Chinese teacher, also considers the interaction with students is important in the classroom. He told me:

Teacher needs to look around the students. It is one kind of interaction. You cannot stand on the platform all of the time.

He also designed some activities that allowed students to work in groups, just like in Mrs. J's classes.

Because like our school, we always let students learn mathematics in groups.

There are some similarities in his interactions with students compared to Mrs. J. He also collects students' problems during the task-solving process and thinks how to deal with them in his lessons.

Students' errors or mistakes are always the most important thing for teachers to focus on. Almost every Chinese teacher chooses to collect students' mistakes when they ask them to do some exercises in the lessons. Our teachers see these mistakes as valuable topics for students to discuss, and they want students to learn something that will help them through these mistakes. They want students to discuss their partner's mistakes and learn the strategies that help them avoid those mistakes in the future. At the end of the discussion, teachers like Mrs. J, collect these answers but does not tell the students whether they are correct or not. Once she has collected all the answers she needs to teach, she will analyze them based on her own teaching

experience. If she finds there exist some errors in these answers, she will take different steps to deal with them depending on the student's mistakes. For example:

If during the exercises, some students make mistakes but others not, I will let them discuss by themselves. Because they need to check the answer with each other in the groups. Or if she found the problems were common in her classes, if I find many of the students have same problems in solving the tasks, I will collect the problems, for example, I will take photos and then after they finish all the exercises, I will let them see whether the answers are correct and pose some questions. Because she believes if one third of the students have same problems, teachers need to pay attention to it during the lesson.

We can clearly see how Mrs. J interacts with students during the lessons. She never interrupted students' talking. Her role in the discussion was a listener, which allowed her to listen to students' ideas. And she always did not evaluate these answers which made students feel relaxed when presenting their ideas. Her students do not need to worry about whether their answers are correct or not. Mrs. J encourages students to talk about their understanding during the lessons.

Like Mrs. J, if Mr. ZH finds that students' difficulties are only manifesting in some of the students, he will ask the rest of the group to help them understand why their answer is incorrect. He also told me that:

Generally, according to the lesson objectives, if the tasks are easy, I will spend more time on the students who are not good at mathematics and see whether they have some problems, like basic theories or just make simple mistakes or other difficulties. ... Or if the tasks are difficult, some students may not take part in the learning process. At that time, I will see who solve the answer correctly and find if they have different methods or whether they have some more simple methods.

Here we find a difference between him and Mrs. J. Here he mentions that he chooses different teaching priorities depending on the level of different students and depending on the contents. He believes that the teacher needs to engage all the students during the lesson. He needs to let those students who are not good at mathematics to achieve basic

objectives after his lessons, for example, they can solve some simple tasks. So if the lesson is not too difficult for his students. He will pay more attention to those students who are below average. And if he needs to introduce more difficult contents, he will focus on the good students and explain their work to help other students understand more after the lessons and encourage them to participate in his teaching activities.

Like the two experienced teachers above, Mr. X said that he would focus on if a student makes a few mistakes in practice.

Generally, I will look at whether their answers are correct.

But unlike the two teachers above, if Mr. X found that only some of his students made the same mistakes, he would not let them discuss it with their classmates in groups, but he would asked those students why they had those problems.

If only few of the students have this kind of difficulty, I will ask them separately.

For example, why they choose this answer of this multiple choice question.

We can see that the other two elder teachers, who would have their students discuss the problems in groups when they noticed some simple errors in solving the tasks. They never chose to tell the students how to modify the solution directly, but asked other students to help them. Mr. X, on the other hand, chooses another way to help these students. He tells them directly what is the right answer. In addition to the students' problems, in China, because many tasks have more than one solution or more than one method to solve them. So Mr. X also tries to find different ways to solve the tasks presented by the students.

And I will also see whether students have different answers. For example, the answer may be the same but they may have different strategies. I need to pay attention to these students, like multiple solutions or some special ideas.

For him, although students may use different methods to find the correct answers, this does not mean that they have no problems in solving the tasks. This is because some students may not use suitable methods to solve the tasks which may lead them to the wrong answer next time. Mr. X believes that teachers need to help students use the appropriate methods that are common in examinations to ensure that they can get good scores.

You need to explain to the students when they make mistakes. If it is not the content in middle school entrance examination, it may mean students have correct answers but the strategies may not suitable. At that time teacher needs to correct them.

However, unlike the two teachers above, Mr. X did not say that he was concerned about the body language of his students. During the discussion, he felt that the most important thing was to identify students' mistakes and to decide which kinds of mistakes he needs explained to all the students and which mistakes only needed to be talked with the particular student himself.

I, generally, see students' solving process to find it they make mistakes. If more than half of the students make the same mistakes, I will explain to them after the exercises. If only few of them have this kind of difficulties, I will just walk to them and discuss to them directly.

Mrs. Y gave me a short introduction about how she moves around the class during discussions or exercises. She said:

First, I will pay attention to good students, and then to see those who are not good at mathematics. I want to see whether they can finish the tasks. If most of the students cannot solve the questions, I will explain to them after.

She did not give too many details in the interview about how to deal with this teaching process. But we can still find some important things. First of all, she said that in her classes, she did not focus on the students who were good at mathematics, but those who were poor at mathematics. This shows that for her, she needs to make sure that all of her students are successful in learning mathematics. This reflects one of her goals in her teaching career, which is the scores that students get in the final examinations. She believes that students' scores are one of the indicators which can evaluate teachers' work. Therefore, she needs to pay more attention to those students who are not doing well in mathematics to ensure that they can get high scores in the examination. Second, also like the elder teachers in this study, she believes that if the students make mistakes which are common in the class, she needs to explain them to the whole class after discussion.

From the brief description of another young teacher in this study, we can see that these two young teachers do not notice as many things as their elder colleagues, such as gestures and facial expressions. They only focused on the solutions students got when they solved the task during exercises time. Mr. W told me:

At first, I see the students randomly. And if I find some misstates during the time, or many students have difficulties, it may let me put more attention. But I do not just focus on good students or poor students.

So he does not only focus on students who are not good at mathematics. He said he sees students' answers randomly, which is different from what other teachers do in the classroom. But he also mentioned that the students' difficulties in solving is one of the most important things for the teacher. From his description, I find out if he noticed several mistakes of the students, he also chose to explain to the whole class.

Here, we can see that the young teachers always communicate verbally with their students and do not pay more attention to their students' gestures or facial expressions as the elder teachers do. This is because for these young teachers, they do not have much experience in teaching. There are too many things that they need to focus on during lessons such as questions, objectives and contents. They do not know how to deal with these elements and they do not have the time to pay attention to the students' performance. And they believe that students' answers reflect their academic performance. For example, if a student presents a correct answer, the teacher can easily think that this student has understood what is presented in the lesson. One of the weaknesses, however, is that teachers typically select only a few of students to answer questions during the lessons, and most students do not have enough opportunity to present their answers. So elder teachers tend to focus not only on oral interaction but also on the body language of their students. This is an area that young teachers need to reflect on in their future career.

Louisa did not spend much time in the interview talking about her interactions with students. One important thing she mentioned is that students will react differently in the computer lab.

They (students) do not spend their time calling us (in computer lab) while in class (general room) they would spend their time calling us.

Because, in the computer lab, they can manipulate the software and, by dragging the points, they can check their own answers. If they fail to solve a question, they will automatically redo it. This is also reflected in the lesson videos, where Louisa only talked to students when they have finished answering a question, and Louisa did not interrupt them when they were solving the task, unless they had some troubles. The students just think by themselves. This was the same in China, where the Chinese students never interacted with their teachers when they were doing the exercises unless they noticed that they were having difficulties. This means that, in fact, in this study, it was not DGS but the students' answers that determined when and how the teacher interacted with them.

Louisa prefers to let students help each other during the lessons.

(If) They (students) who go faster, I ask them to help others. Afterwards, often when they go to help others, they tell them "you do this, you do that". So I do not know if it's help but here it is.

And as we can see, this process is important to Louisa, who chooses to use this strategy in her lessons. Whereas in a regular classroom, when she asks one student to manipulate the software, like Mrs. Y in China, she does not let other students discuss it with him.

We, when the student comes on the board, does his work and the others do not have to intervene when he does, they do not have the right to say "it's wrong, do like that, like that". He does by himself. Only when he finishes all the things, others can make their own ideas towards the task. Once he has finished doing his work, the others can intervene, they raise their hands and at that moment, they say "I do not agree, I think it's wrong I would have done like this or I did differently". At that moment, they come to propose their solution.

6.3.2 Teacher's interaction with students are similar

From the lessons, Chinese teachers rarely change the pattern of interaction with their students, regardless of how they organize their lessons with DGS. Like Mr. X, although he organizes his lessons with DGS in different ways, the pattern he interacts with his students does not change. From the above transcripts, we found that Mr. X, compared to other teachers like Mrs. J and Mr. ZH, always controlled the interaction and did not give the students much time to express their ideas. When interacting with the students, he would often ask a few short questions.

During the interaction, Mr. X talked a lot, and then asked the students some short questions, and after repeating the students' response, he controlled the process again. This interaction occurred many times throughout the lesson. Although Mr. X used different ways to integrate DGS, I did not see any change in these interactions. When he explained the screen, he asked a few short questions, but when he made his conclusion at the end, we did not that he asked any questions of the students. This shows that Mr. X was always the center of the mathematics lesson. One of the reasons maybe that the type of integration of DGS that Mr. X uses is a more teacher-centered, whereas he does not use *guide-and-explain* like other teachers.

Other teachers such as Mrs. J, when she used *guide-and-explain* to structure her lessons. She posed a series of questions to get students to interact with DGS. Like in the following transcript:

T: you need to justify what type is the quadrilateral

St1: it is diamond

T: diamond. How can you know it?

St1: first point G and M are coincided.

T: yes

St1: then, DM, point D is the midpoint of AB, point M is the midpoint of AE, so DM the median line of this triangle.

T: do you agree? DM is the median line of triangle ABE. You know it is median line, then?

St1: then DM is parallel to BC

In contrast to the interaction in Mr. X's lesson, we can see that her students are at the center of the interaction. Mrs. J simply repeated their answers or asked short questions so that the students could tell others how they solved the task. Of course we cannot say that in Mrs. J's lessons that the students control the teaching process. But in a few short teaching episodes, students began to have more time to communicate and present their ideas.

One of the characteristics of Chinese teachers' interaction with students is that the interaction always takes place in the following sequence: question-answer-feedback-question-answer-feedback. However, different teachers play different roles in the interaction process. Mrs. J, I found that students participated in the teaching practice by answering questions, and Mr. X seldom asked students to answer questions. And in some cases, all students answered teachers' question together, as in the following transcript:

T: So which way can we use (to solve the problem)?

All St: Congruent

T: Only congruent, ok, we can put angle B into which triangle?

All St: ABD

T: Triangle ABD, angle C?

All St: ACD

T: Triangle ACD, then, if I want to check these two triangles are congruent, how many conditions do we need?

All St: Three

T: Now we have how many?

All St: Two

T: Who are they?

All St: AD equal to AD

T: AB equal to...?

All St: AC

T: AB equal to...?

All St: AC

From the video, it is impossible to distinguish who answered which question as what I can do in other teachers' classes. And we can use another teacher's interaction as an example to see the differences between the two teachers. For example, the following transcript from Mrs. J's lesson:

T: you need to justify what type is the quadrilateral

St1: it is diamond

T: diamond. How can you know it?

St1: first point G and M are coincided.

T: yes

St1: then, DM, point D is the midpoint of AB, point M is the midpoint of AE, so DM the median line of this triangle.

T: do you agree? DM is the median line of triangle ABE. You know it is median line, then?

St1: then DM is parallel to BC

T: DM is parallel to BE

St1: so angle MDN is add angle END is 180

Here we see that Mrs. J used words like yes or then to allow the student to continue his explanation, or she chose to repeat his answer to other students during the explanation. However, Mr. W asked a series of questions when interacting with his students. This meant that his students were faced with new questions after their answers and did not think deeply about them. It is also evident that Mrs. J tried to give the students a voice and more time to express their ideas, while Mr. W controlled the interaction and wanted to continue according to his lesson plan.

6.3.3 Different focus during the interaction with students

From the analysis of the type or focus of questions posed by teachers with DGS, Chinese teachers always focus more on how to abstract mathematical knowledge from examples in their lessons (Cai & Wang, 2010). And the questions they pose are

more focused on mathematical contents and have little to do with DGS. In the lesson videos, when the students were answering some of the questions, I noticed that nothing changed on the screen, which means that these questions can be answered by the students without interacting with DGS, or what they needed to do was to see what was displayed on the screen. And during the interaction, the diagram is not moving. It only moves when students discuss what is happening on the screen. Like ZH's first lesson, only one or two questions are related to DGS. For example, ZH said:

Is the conclusion right only in this situation? If I change A and B the conclusion is right or not?

From this question we can see that ZH expects students to answer questions when he uses software to move and change the diagram. Another focus of the questions in ZH's classroom is to control the class or we can say that ZH uses these questions to choose the students who need to answer the questions. And ZH always used more positive words such as *good*, *ok* or *yes* when giving feedback to students. Another important key feature of the interaction is that he always repeats his students' answers. The same kind of focus was shown in Mrs. J's lesson. As mentioned above, the questions were also more focused on the mathematical contents. However, Mr. ZH used more words, such as *drag*, in his lessons that relates to the actions with DGS. For example, in the second lesson, he manipulated the software more often and his questions or statements were more related to DGS.

One of the most important moments that teachers need to interact with their students is during exercises time. In every mathematics lesson, teachers spend some minutes letting students do some exercises based on what they are teaching. For example, in Mrs. J's class, she divided her students in groups to make it easy for them to discuss. And before solving each task, Mrs. J would have her students discuss it and then walk around to look and listen to what students were discussing. During the interview, Mrs. J told me that she did not spend much time participating into students' discussion when she walked around the classroom. During that time, her role is not a teacher but a listener. She looked at the students' reactions and checked whether

they understood the contents. For her, the students' reactions are often evident in their facial expressions and body language, and by paying attention to these, Mrs. J can know if they understand what she has said.

In her opinion, she needs to reduce the amount of time she spends facing the blackboard and avoid letting her students only see her back. It is important for the mathematics teachers to face her students in the classroom. Therefore, she can watch all of her students and reorganize the lesson if she finds that many of them cannot keep up with her teaching. She also told me that because of DGS and other computer technologies, she can save a lot of time presenting tasks to her students. She compared her teaching process to not using DGS and told me, at least, with technology, she did not have to write everything down on the board like what she did when she began her teaching career. During that time, the mathematics teachers only had chalk and a notebook for their teaching, and she needed to write a lot of things down on the board during each lesson to give her students enough time to understand the important contents and solve the tasks. But now she only needs to prepare them before the lesson and display them during the teaching, and her students do not need to spend much time taking notes, even if they do not understand everything, they can review them after the lesson because of the storage function of computer.

Not only elder teachers, but also younger teachers in public schools focus more on mathematical contents than on technology. Like Mr. W, we can call him a special teacher because about half of the students in his school choose to become an athlete rather than take final examination when they finish secondary school learning. These students may return to their hometowns for further study. This means that all them will not take the final examination and thus will not be able to attend senior high school in Shanghai. So for this school, the teachers focus more on another half of the students who will take their final examination in Shanghai, which determines the final level of the school and whether it will be able to attract high level students in the future. So in China, we can call these students the "lifeline" of the school and the teachers. This different background of this school is a special sample in this study, from which we can see some interesting findings.

In his class, due to the small class size (about 25 students), Mr. W did not have students sit in groups. Throughout the class, all students worked individually. In his lesson, I did not find that students spent time discussing. The interaction took place only between Mr. W and one of the students. Although, like any other teacher, Mr. W would have students do some exercises during his lessons. In his classroom, however, he could see almost all of his students' work and check if they had difficulties in solving the tasks. He told me during the interview that he first chose to see if the mistakes made by students were common in his class. If only one or two students had such errors, W did not discuss the mistakes with all of his students, instead he chose to talk to the students face-to-face. And if he found that most of the students had the same mistakes, it was a sign that such troubles needed attention. So he would stop the students' work and explain it to the whole class.

From the interview, unlike the elder teachers above, Mr. W does not pay much attention to his students' body language. Perhaps because there are fewer students in his class than in other teachers' class, he can interact with as many students as possible. So he does not need to analyze the body language of his students to see if they understand his lessons well.

Mr. X also took a few minutes to have the student solve the tasks he had designed before the lesson. In the example lesson, Mr. X prepared a complex task for his students. In China, these tasks are always the last task in the examination, which means that they are the most difficult tasks for students. So Chinese mathematics teachers tend to spend most of their teaching time telling students how to solve these kinds of tasks, especially in the last two years of secondary school, when these students will soon attend an important examination. In the lesson, Mr. X divided this task into several small questions in order for the students to follow his explanation. First he asked students to solve the question one by one, for Chinese teachers it is not enough to just tell students how to solve the tasks directly, they always let students try to solve the task by themselves before explaining. For teachers, independent thinking is one of the important goals of teaching. So during the lesson, students have about 5 minutes to think about the task on their own. During this time,

Mr. X, like other teachers, walks around the classroom to see if his students are doing well or if they are making some mistakes. In the interview, he told me that when he noticed that some of his students were struggling or making some mistakes, he would first distinguish between several errors. If the mistakes were only in one or two students' work, he would come to them and explain face to face without interrupting anyone. If he found that more than half of the students in the class made same kinds of error, he would stop students' work and explained the mistakes to the whole class. In the video, we can see that Mr. X chose to explain to the students face-to-face many times, but of course this was because he found that he did not need to spend much time explaining to the class just for the mistakes which were only in one or two students' work. He also told me that this was because he thought this interaction was useful for some students. With this interaction, Mr. X does not need to let all the students know about others' mistakes, which can make some students who may not want others knowing about their mistakes feel relaxed.

In the interview, Mr. X, unlike Mrs. J, did not mention that he needed to spend more time face to his students. One of the reasons maybe in his school there was not so many digital resources such as iPad or smartphone like Mrs. J used in her lessons, so the teacher still needed to write down many things on the board, although they also felt that this was inconvenient for their teaching. We can see that changing the resources or teaching environment can affect the practice of teachers. Comparing these Chinese teachers, we can see that these five teachers use different digital resources, both public and private schools. In general, private schools can provide more resources than public schools. These resources also changed the daily practices of these teachers and students.

In the French case, Louisa was not only concerned with mathematics but also with how to use DGS to explore or solve mathematics tasks. Louisa always used the words like *click* or *draw* during the interaction, which was directly related to the manipulation of DGS, meaning that she focused on letting students use DGS to solve mathematics tasks, and her questions were not only about mathematical contents as in the Chinese teacher, but also related to DGS. Of course, Louisa also asks some

questions that focus only on the mathematical contents, such as where is the center of the circle? But unlike the Chinese teachers, Louisa wants her students to not only have mathematical conclusions, but also to learn to use DGS, for example, by having them explain the steps of constructing a geometric object with DGS. This shows the different concerns of the Chinese and French teachers and the different roles that DGS plays in the lessons from the two countries.

Louisa also notes the connection between physics tools and DGS. In her lesson videos, she reviews students' actions on DGS and gets them to think about how they use physical tools such as compass to move segments. No matter how she organized her lessons with DGS, the software is always at the heart of her interactions with students, and she focuses on letting students explore mathematics by using DGS.

6.3.4 Teacher knowledge revealed in interaction with students

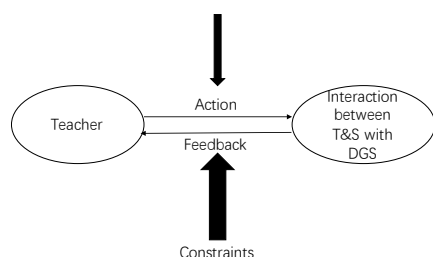


Figure 6.4: Teacher knowledge revealed in classroom interaction with DGS

Chinese and French teachers showed different ways of interacting with students based on DGS. And we can see these ways based on perspective of interaction system constructed by teachers and didactic situation with DGS (Figure 6.4). Chinese teachers consider mathematical contents to be the focus of their teaching. Judging from the questions they asked in class, they rarely connected the questions with the DGS. This means that when interacting with students, teachers do not see DGS as a medium to connect teachers and students. The main questions they posed in their interactions are about math. Louisa, on the other hand, focused more on how to make DGS a learning tool for her students. Her questions make a connection to DGS by using words that are directly related to the software, such as *drag*, *rotate*, etc., so that students can manipulate the software during class. She also posed some questions

about mathematics, but not too many.

Another way of interacting with students in China is that some teachers choose not to interrupt students when they are answering a question. Like Mrs. J, she just uses some words such as *then*, *ok*, *next* and lets the students continue talking without evaluating their conclusions. By interacting in this way, the teacher allows herself to be a listener during the lesson and the student to be the speaker. This also shifted the lesson from being more teacher-centered to student-centered.

Although the Chinese teachers used different types of instrumental orchestration in their classes, the pattern of teacher-student interaction with DGS was stable in the lesson. They do not change the focus of their interactions with students especially when they use *explain-the-screen* and *guide-and-explain*. They do not change the focus of their interactions with students, whether they use DGS or not. They all focus on the mathematics and put the DGS away. Only when some of them use more student-centered types, such as *spot-and-show* or *Sherpa-at-work*, they change the way they interact with students slightly. At that time, they would use the words related to software manipulation, such as *move* and *construct*.

In the case of France, Louisa also do not change the pattern of interaction with the students. Her focus is on how to use DGS to solve mathematical problems. She always asks the same questions about the same task to different students, and the order also does not change. She always has the student drag the points first and then asks him what the element on the screen is called in mathematical language. She changes her interaction only if the student makes some mistakes because different students may have different difficulties. She needs to check what the students' problem is.

Not only the oral interactions, but also other types of interactions mentioned by the teachers. Like the Chinese teachers, the gestures or emotions of the students are also their focuses. From the gestures, they can know whether the students understand the knowledge or not. This was not mentioned by Louisa. The Chinese teachers also let the students discuss or solve the task (in groups or not) by themselves. During this time, the teachers always not communicate with the students, but just watched what

they did and checked if they had any problems. They always collected the problems so that they could explain to the students at the end of the lesson. However, from the lesson videos, Louisa also had students do some exercises during the lessons. At that time Louisa not only check the students' work but also discussed with them if they were having difficulties. These are two different ways of interacting with students during the lesson.

As we discussed in the previous chapter, the Chinese teachers demonstrate their mathematical knowledge but do not demonstrate knowledge related to using technology to learn mathematics. The French teacher demonstrate her awareness of creating a new learning situation for her students and using technology as a key part of that situation. They also showed the same phenomenon when interacting with students. The Chinese teacher focused on mathematics and the French teacher focused on mathematics related to DGS. From their teaching process, all teachers tended to allow students present their ideas, but in different ways. The Chinese teachers chose to have their students answer questions or explain how to solve problems, while the French teachers had their students manipulate DGS. This also shows that DGS played different roles in their practice. The Chinese case does not talk more about technology-related knowledge in mathematics lessons, while the French case talks more about it.

7 Discussion and implication

After analyzing the cases from China and France, we see how teacher knowledge is demonstrated when they use DGS for mathematics teaching. The final section presents some of the issues that emerged from these cases regarding to the role of DGS in mathematics lessons, as well as some implications for future research to help us gain insight into teacher knowledge revealed in using DGS. It suggests that DGS can create many new tasks for teachers in which DGS allows students to explore mathematics with the software. But if we look at teachers' practices in the classroom, DGS just acts as a projector for teachers, especially if they think the task is complex for their students. And when the task itself is not complex, teachers are willing to make their lessons more student-centered. This is one of the important gaps between teachers' real practice and the theoretical role of DGS in mathematics. This chapter also describes some of the factors that influences teachers' use of DGS in their lessons based on the idea of constraints (Balacheff & Margolinas, 2005). These factors were mentioned in the interviews with teachers.

7.1 Dynamic geometry creates new kinds of mathematics tasks

By observing and analyzing the teachers who participate into this research, although I have limited data (5 Chinese teachers and 1 French teacher) to make general conclusion about teacher knowledge, I still find some characteristics by contrasting these teachers.

7.1.1 Different role of dragging mode in China and France

Researchers state that teachers' adaptation to teaching practice is an assimilation process and this adaptation comes from the changes in their way of thinking (Niess, 2005). So, technology makes us rethink how to teach mathematics. The National Council of Teachers of Mathematics (NCTM) (National Council of Teachers of Mathematics (NCTM), 2000) states: technologies like calculators and computers are

essential for teaching, learning and doing mathematics in school. For example, students can use different technological tools to get a visual image of mathematics, or to organize and analyze data, or to calculate efficiently and accurately. In turn, students can become more focused on making decisions, thinking and solving problems.

Based on the different tasks chosen by the teachers in this study and their use of these tasks in the classroom, it was found that *dragging mode*, the most important function of the DGS, plays a different role in the Chinese and French tasks. Holzl (1996) suggests that the dragging mode has two roles in mathematics teaching: first, it can help us test mathematics properties, and second, it can help us explore mathematics.

Generally, in China, where students do not use any technology in mathematics classes, Chinese teachers try to develop the ability to better understanding of geometric thinking. They need to imagine the process of moving a geometric object when faced with tasks that have moving points, such the tasks in this study. Chinese teachers are also used to explain these tasks without using any technology. They choose to demonstrate the result of the movement rather than the moving process. This strategy is more like the *photo dragging* (discrete dragging) introduced by Olivero (2002). We can infer that without technology, teachers cannot demonstrate the complete moving process like playing a film to their students. This implies that using technology can change our solving methods.

In French lessons, dragging mode not only allows students to check their constructions, but also allows them to explore new properties in mathematics lessons. In the second, for example, students explore the relationship among the three sides of a triangle with the aid of DGS. They can drag different segments to check if they could construct a triangle based on the task (as shown in section 5.2.6) and then they need to make conclusions about the relationship between the three sides of the triangle. As in 5.2.6, there were some questions which let students tell the teacher the reasons why they got their answers. It suggests that the purpose of the lesson is to find new mathematical knowledge for students. Comparing the two lessons in France, *dragging mode* works differently.

7.1.2 Gap between roles of DGS in task and the way teacher use it

The above tasks demonstrate that DGS can create different tasks by creating new learning situation or changing students' problem solving strategies. The Chinese task can be seen as the third type of task based on the different roles of DGS (Laborde, 2001) in mathematics tasks because it requires students to use a new strategy instead of which with paper and pencil. These tasks demonstrate a strategy for using dynamic geometry into mathematics lessons.

Among the French tasks, dynamic geometry is also the generator of new types of mathematics tasks. These tasks show the different expectations of the French mathematics teacher. This French teacher incorporated dynamic geometry in her lessons also, however, she did not want geometric objects to move like the Chinese ones. These tasks are designed to allow students to construct different diagrams with the help of dynamic geometry. From the tasks shown above, we can observe that the teacher systematically demonstrates the whole construction program. This proves that the teacher anticipates the solution is not a simple transformation of the paper and pencil procedure. Thus, these tasks show a new kind of mathematical task with dynamic geometry. Similar to the work of Laborde (2001), in these tasks dynamic geometry changed the problem solving strategy due to the use of some of its tools which may cause the task become more difficult. However, the challenge was simply to produce a triangle figure without problematizing the student's choice of construction tools. For the same type of construction, other sequences of tasks with dynamic geometry make it possible for the technique to play another role, as in the case of Voltolini (2014). The use of technology remains the same in both tasks, differing only in the details of the construction instructions, and in the first French lesson, the second task is considered to be the same application of the procedure described in the first task. These tasks were technology-oriented. Unlike the Chinese example, students had to directly manipulate the dynamic geometric environment. In addition, the second example of the French task in this study required the student to choose a different tool with the dynamic geometry software.

Using dynamic geometry software does not mean that teachers will change the way they interact with students more. One reason for this may be due to the tasks they design for their lessons. Although teachers already use DGS for mathematics teaching in both China and France, there is still a gap between the use in the research work and in everyday practice, and researchers suggest the provision of ready-made resources to help teachers take more advantage of dynamic mathematics environments (Laborde & Laborde, 2011; Mackrell et al., 2013).

By analyzing Chinese and French tasks, as we have analyzed in this study, dynamic geometry plays two important roles: the role of amplifier and the role of generator. Based on these two main roles, in the example of the Chinese mathematics tasks chosen in this study, DGS certainly acts as a generator of a new type of mathematical task by changing the strategies of these tasks. For example, these Chinese tasks contain one or more moving points, and in general, Chinese mathematics teachers always ask their students to imagine the process and result of moving process and draw some figures to get the answer. But with DGS, Chinese teachers can show the whole process of moving directly to their students. This makes it easier for them to see the relationships between different geometric objects.

While we can see the changes that DGS has brought to mathematics task, Chinese teachers' lessons are more teacher-centered in this research. Students did not use software to solve these tasks in the lessons. Teachers did not need to change their interaction with DGS. They remained focused on mathematics and used the software as their support. Only when they design special tasks, such as the construction of geometric objects, teachers need to consider new pattern of interactions with their students. Because they need the support of the software to explain the solutions, during class, it is the teacher who operates the software and decides how and when to use the software. The students are more passive during the lessons. The videos in this study give us a possible way for teachers to use DGS to explain how to solve geometry tasks. In these videos, the teacher primarily uses DGS to show the results of the problem solving process, although DGS could do more for us to solve the task. In fact, in the Chinese example, the task in which DGS plays the role of generator of a new solution

strategy is accompanied by a reinforced control by the teacher, whereas in the French cases the task can be problematic, very technical, and is accompanied by a greater autonomy left to the students. The intervention in the French class is piloted with a step-by-step procedure on paper. The French teacher's lessons are more student-centered. These tasks are used in the computer lab, which is completely different from the Chinese lessons. This physical environment allows students to operate the computer. With these examples, we can envisage an analysis based on the data that we will present to the teachers involved in the study. Thus, for some researchers, the increased complexity of classroom orchestration events can lead to teacher imposition of procedures (Stein et al., 1996).

In short, DGS creates new tasks in Chinese mathematics classes by changing the way students solve tasks. However, it is not a tool that students can use to solve tasks during the lessons, but rather a tool that aids teachers in practicing. In France, DGS is also a generator for the tasks. Without the software, teachers would not be able to use these tasks in the classroom and students would not be able to solve them. Therefore, the French teacher chose to use these tasks in the computer lab so that all students would have enough time to manipulate the software to explore and solve the tasks.

7.2 Instrumental orchestration

Based on the data collected in this study, although we cannot make a general conclusion, I still abstracted some characteristics of Chinese and French teachers' lesson orchestration with DGS.

7.2.1 Different strategies in Chinese and French cases

Teachers always choose different strategies to use DGS to teach geometry (Gueudet & Trouche, 2011): First, in a more classic way, teachers introduce a new mathematics notion and build a figure with DGS and subsequently drag the points so that the students can make some conjectures and in the end let the students create proofs with paper-pencil. Second, they may construct a figure and let students discuss how to build the

same figure with DGS, which can be called a black-box type. These methods for using DGS correspond with the different types of tasks introduced by Laborde (2001). In the first method, DGS is an amplifier and in the second, it is a generator.

In China, when we asked teachers to design or select appropriate mathematics tasks, they always designed the tasks first and then considered how to use dynamic geometry to help students or how to explain to the students the solutions with dynamic geometry. From the cases I find that initially the Chinese students only tried to solve these tasks with paper and pencil, and when they completed the work, the teacher started to explain the solution to the students with dynamic geometry and based on the figures on the screen. And during the lessons, DGS sometimes only presented static figures on the screen. When students solve the task, neither the teacher nor the students use it. This phenomenon can help us to understand how teachers use DGS in Chinese mathematics lessons, which proves that DGS is a supporting tool for mathematics teachers. Many Chinese mathematics teachers mentioned that the most important objective for mathematics teachers in China is to ensure that students get good scores on final examinations where they are unable to use DGS. Thus, in a mathematics lesson, the teacher directly tells the students that they cannot use dynamic geometry in the test and that they need to imagine how dynamic geometry moves according to the task situation. This could explain why Chinese teachers use dynamic geometry as an aid in mathematics teaching. Teachers want to avoid over-reliance of students on dynamic geometry and therefore they choose to reduce the amount of time spent on using this software.

In the French case, I see a completely different situation with DGS. In the example lessons, the French teacher uses dynamic geometry as a tool for students to explore and learn mathematics. The lessons take place in a computer lab, where each student can interact directly with the software. The tasks are designed to show the differences and connections between the different solving situations. Students must analyze strategies on how to replicate the use of paper and pencil in a computer environment. The teacher believes that this can be a challenge for the students and she needs to help them with more information about this. This is a key difference in the process of teaching Chinese

and French mathematics.

7.2.2 Different perspective of teaching and learning in Chinese and French cases

In the Chinese cases, I find that only some of the teachers (Mrs. J and Mrs. Y) allowed their students to interact directly with technological resources such as computer software during the teaching process. These students could interact with the computer and control their own learning process, or they could explore the software or use it to explore their learning contents according to the teacher's lesson preparation (Alper Ardiç & İşleyen, 2017). Based on these processes, technology has transformed the way teachers teach mathematics, allowing students to focus on deeper conceptual understanding by making decisions, such as which tool to use to construct the same triangle in Mrs. Y's class. Such learning situation with technology make exploring core mathematical concepts more tangible and interactive for students. This type of environment allows students to construct geometric objects, figures, and diagrams which technology can bridge the gap between concrete and abstract mathematics. (Guerrero, 2010). And similarly, in Mrs. J's class, students use handheld devices such as iPads to explore variations in geometry and gain a deeper understanding of mathematical concepts. However, this use is rarely seen in Chinese lessons.

Some of the teachers in this study also think about how to make students more active in their lessons and try to give them as much time as possible to participate into the teaching process. They believe that teachers need to allow students to express their ideas. However, unlike the situation in France, these Chinese teachers allowed their students to participate in the teaching activities by answering questions. They also acknowledge the need for flexibility in the classroom, not just read lesson plans, but their lessons are often limited by the required content coverage, a rapid teaching pace, and large class size (Cai & Wang, 2010). This makes them rethink whether it was possible for students to interact directly with technology in their lessons. On the other hand, these Chinese cases tend to view effective teaching as a form of teacher

instruction with a coherent structure. In these cases, technology was used more as a presentation tool to support the classroom process, so it was not possible to move away from a direct teacher-centered approach (Alper Ardiç & İşleyen, 2017). However, researchers think that using technologies in teaching environments can help to create more efficient learning situations for students. In this way, teachers need to change their classroom routines to help students deepen their understanding through their own efforts (Alper Ardiç & İşleyen, 2017).

These Chinese lessons also reveal another phenomenon of teachers' perceptions of the role of memory and practice in learning mathematics (Cai & Wang, 2010). This is influenced by the Chinese cultural system, which is known as Confucius. In China, we see that knowledge should be generated by learning from an authority, such as a teacher, rather than by the learners themselves. Therefore, in teaching, the Chinese tradition emphasizes the authority of the teacher and the diligence of the student during the lesson (Tweed, 2000). This could explain why these teachers organize their lessons in such a way because students can remember the ways in which teachers use mathematical facts, theories, formulas, algorithms, or procedures and techniques to solve tasks without paying attention to why or when it makes sense to do so.

In the case of this study, compared to Chinese students, students in French classes spend most of their class time interacting with computer software rather than with the teacher. In these French lessons, this teacher expects students to actively explore visual images, discuss, analyze, and communicate their findings. In order to ensure this focused and productive activity, teachers need to develop valuable tasks that take into account the realities of this particular learning environment (Sinclair, 2003). However, in China, the teachers' practices in this study did not show this desire to use DGS to create new situations for students to explore and solve problems. In the French example, the DGS was used more for students to explore mathematical contents. Teachers used it as a tool for students to learn mathematics. In the Chinese example, teachers do not design their classrooms based on DGS. Rather, they first develop their own teaching plan and then think about how to use DGS.

7.2.3 New characteristics in Chinese classroom orchestration with DGS

For mathematics learning, interactivity in a mathematical digital environment means a reaction of the environment to the user's actions related to mathematics. The user (we assume here: students) must perceive these reactions and get information on his/her actions (Laborde, 2014). However, when teachers in this study used DGS, the software was more about presenting the contents during lessons. The dragging mode is also controlled by the teacher to present or explain what is happening on the screen. Which makes the meaning of interactivity of the DGS became loose and it is simply used for pressing keys or buttons or touching objects on the screen (Laborde, 2014). This shows that DGS plays a supporting role to the Chinese mathematics teachers. This was also shown in their teaching process in this study. Although we know that DGS can show the moving process, which is one of the most important features of the software, these teachers do not use this feature much during their interaction process. For the most part, DGS simply acted as a projector in mathematics lessons. When teachers think that students may not be able to visualize the task clearly, they begin to manipulate the software and show them the moving process so that they can easily understand it.

Throughout the lessons, students do not interact directly with the software. All the students just watch the teachers' actions with DGS. This means that the software did not affect their problem solving process in the classroom. Their performances are the same as what they did without DGS. This means that DGS played the role of a supporter for teachers and these teachers do not use it to create a new learning situation for their students. DGS do not make any changes to the students' mathematics learning in the lessons.

In this study, two Chinese teachers (Mrs. J and Mrs. Y) attempted to change the way they used the software in their mathematics lessons. One of the teachers had some of her students manipulate the software to solve the task, which changed the problem-solving situation of the students. This strategy is more like what Drijvers and his colleagues described in their study: *Sherpa-at-work*, it allows students to change their strategies. They need to transform between two different environments: paper-pencil

and DGS. The reason DGS is used this way is because of the tasks she designed for her students required them to construct geometric objects with the software, which is one of the most important differences from other tasks. The students found that their strategies with paper and pencil were not appropriate for the new situation and they needed to think of new strategies to solve the tasks. The teacher not only asked students to think about how to solve problems with software, but also how to explain their solution in a clear way for others to understand. This is a difficult task for the students, and they do not face it anymore in other classes. So this teacher tries to create a new type of learning situation with DGS by changing some of the lesson plans to allow students to move between different contexts. But this is not common in Chinese mathematics class.

One teacher in another changed her teaching strategy in a way that was not observed in other studies. In her class, instead of having her students work directly with the software, she had several of them act as an instructor, instructing others to construct geometric objects with the aid of the software, while the teacher herself acted as a student, manipulating the software according to these students' instructions. This was interesting because I do not find this type of instructional activity in any other lessons in this study. In future research, we need to find more data to see whether this teaching process is a special example for this teacher or if it is also used by other Chinese teachers. And this activity also provides some suggestions for teachers to incorporate DGS into their mathematics teaching if there is only one computer in the classroom.

7.3 Interaction between teacher and students

By contrasting these Chinese and French cases, I found some differences in the interaction between teacher and students. Also I found Chinese and French teachers have some similarities in the patterns of teacher-student interaction with DGS.

7.3.1 Different focus of questions in Chinese and French cases

As analyzed above, in the Chinese cases, the teachers asked more mathematical questions that were rarely linked to the DGS. When students answered the questions in classes, there was no direct interaction between the students and DGS, which meant that these questions could be answered by the students without DGS, or what they needed to do was looking at what was displayed on the screen. Also these teachers do not want DGS included in the student's problem solving process. During the interaction, DGS plays more like a projector or an amplifier (Laborde, 2001; Soury-Lavergne, 2017) to present diagrams without moving anymore. This might be related to the knowledge of teachers and they may lack the ability to apply their knowledge to the students' thinking that was emerging from a technological-based task (Hollebrands & Lee, 2016). Another important factor may be that the situation in which our teachers learn mathematics is completely different from what students encounter in school today, so it is difficult for teachers to consider the student's learning process (Hollebrands & Lee, 2016). I have also noticed that teachers may be concerned that students are unfamiliar with technology, or that they may engage in extracurricular activities that the teacher does not want to see (Zbiek & Hollebrands, 2008). These facts also affect teachers' practice of using DGS in mathematics. Also, many of the tasks used by teachers are created for students to use the technology in a step-by-step instruction and rarely allow students to make mathematical reasoning, which could explain why teachers only ask more questions about mathematics (Cayton et al., 2017).

In the French case, the questions are more about using DGS to explore mathematics. Due to the different teaching environment, the French case takes place in a computer lab, allowing students to interact directly with DGS in the classroom. So this French teacher needed to think about how to help students learn mathematics with the software. This pedagogical situation led the teacher to ask different questions in this study compared to the Chinese cases. During the course, this teacher used many words, such as *move*, *change*, and *click*, which meant that she wanted her students to manipulate the software in order to observe what was happening on the screen and to notice the relevant

mathematical contents. These words are rarely used in Chinese lessons. This is because, as I mentioned above, in these cases, Chinese teachers want their students to be able to solve these tasks without the help of DGS, even though this software might make the task easier and give students a deeper understanding of mathematics. These Chinese teachers propose different ways of interacting with their students based on DGS. The Chinese teachers consider the mathematical contents to be the main focus of their lessons. In their interactions with students, it is evident that these teachers do not see DGS as a medium to connect them with their students. For the Chinese teachers, the questions they asked about in their teaching indicate that they want their students to focus on the results of the software rather than thinking about how to use DGS to interpret those results or to consider the concept of the mathematics tasks (Heid et al., 1998).

7.3.2 Stable teacher-student communication pattern

Researchers believe that technology based teaching environments have the potential to create learning opportunities for students where the mathematics is embedded within the technological tool (Cayton et al., 2017). However, in this study, from the teachers' interactions with students when using DGS in their lessons, they rarely connected the issues with DGS. The main topic they talked about in their interactions is about mathematics. From this study, we can see the main pattern that occurs during the interaction between these teachers and students (Steinbring, 1989): the teachers guided the interactions in an Initiation-Response-Evaluation pattern (Drijvers et al., 2010). During this kind of teaching, teachers often pose questions based on what they want to hear rather than on what students know or think about the lesson (Cayton et al., 2017).

Then with the data, the pattern of teacher-student interaction in a Chinese lesson can be described in the following order: the interaction begins with a mathematical question based on a teaching activity, and then one or more students give an answer. If the answer is correct, the teachers ask another question, but if the student makes a mistake, the teachers ask more students to answer until someone gives the correct answer, or they

answer on their own and then pose a new question. Regardless of the type of instrumental orchestration, and whether or not DGS is used, the pattern will never change and the teacher's focus in lessons will be the same. In the French case, I have found a similar pattern of lessons. Both the French and Chinese teachers rarely begin their communication with questions posed by the students. Although the French students would operate the software and would find something interesting, the French teacher always continued the lesson as she had prepared it without interrupting the students.

7.4 Some factors affect teacher's practice with DGS

In this study, I addressed one of the factors that influenced teachers' use of DGS in mathematics lessons: teacher knowledge. According to Balacheff, there are constraints in the subject-milieu system. Some researchers have suggested that there are a number of factors that play an important role in the use of technology. Like Hennessy et al. (2005, p. 162) note "include lack of confidence, experience, motivation, and training; access to resources and timetabled use of dedicated ICT class-rooms; unreliability of equipment; classroom practices which clash with the culture of student exploration, collaboration, debate, and interactivity within which much technology-based activity is said to be situated." For example, DGS may reduce the motivation for students to construct proofs because they can see directly from the software whether a mathematical hypothesis is correct, and they believe that teachers need to address the needs of the students, the content, and instruction and then decide which technology best accomplishes all these goals (Guerrero, 2010). Some researchers also think DGS also can be a way to promote students' understanding of the need for making proof in mathematics (Olivero, 2002). Time constraints and management of classroom operations lead the teacher to express computer requirements in turn (Bellemain, 1992).

In this study, based on the data collected, the following factors may influence teachers' DGS practices in mathematics lessons. And all of these factors can be seen as part of the constraints mentioned by Balacheff. Perhaps in the future, we can focus on

how these parts of the constraints affect teacher knowledge.

7.4.1 The first factor: student

In the practice of mathematics teaching, teacher, student, and mathematics form the basic classroom instruction. The student, as one of the main components of every mathematics lesson, needs to pay close attention to (Cai & Wang, 2010). The main aim of mathematics teaching is to make children master basic mathematical contents and to use them to solve the problems they face in life. The quality of how teachers use ICT depends on how well they adapt to the learning needs of their students (Yeh et al., 2015). So, of course, teachers need to pay attention to students' reactions and modify their lessons based on these feedbacks in order to achieve the main objective. In China, students are also a key factor which can influence mathematics lessons and teaching practice. In order to become qualified teachers, when they study at university, these student teachers are told to be student-centered. In China, an experienced teacher must be familiar with his or her students, including their personalities, their backgrounds, their strengths and weaknesses.

When Chinese teachers prepare their lessons, they always reflect what their students have learned before and what is more suitable for them now. For example, I did an interview with one Chinese mathematics teachers and ask her for what reasons she decide to choose the tasks (which are shown in the teaching plan she prepared before the class) and make the sequence of them like she did in the classroom. She told me each time she prepared her mathematics lessons, she would search for enough tasks and materials to use in the class. And then she will analyze which of these tasks is suitable for her students (her knowledge relating to students). She said when she taught the students whose ability is below average, she would choose easier tasks and first showed easy tasks on the screen, and explain to students. And let students do some exercises which are similar with the examples. And all the tasks she used are not too difficult.

In China, almost every mathematics teacher has to teach more than one class of

students. This makes it necessary for them to present the same contents to two different classes twice a day. If we observe these lessons, we can see that the teachers may change their approaches when they are dealing with different students. For those students whom the teacher considered to be good at math, the teacher would present more complex tasks, while for other students, the teacher might present only basic problems. Teachers always first consider their students' abilities then design different tasks to meet students' abilities and compile them into a sequence based on their level of difficulty. Evaluating students before each lesson is one of the main abilities of Chinese mathematics teachers. All the contents prepared for each lesson should be suitable for the students in order to get satisfactory results after the lesson. And they need to ensure that all students have time to participate in the activities, regardless of their level of mathematics. Therefore, this requires them to be familiar with all students, otherwise they cannot design appropriate activities for them in his lessons.

In China, not only elder teachers but also younger teachers believe that students are very important in the classroom. During their university studies, in preparation for their future teaching jobs, more than one expert teachers told them to focus on different students in the classroom. As these young teachers began their school life, they began to prepare lessons based on what they had learned during their college years. Although they may not be able to make a clear impression on all of their students or know what their strengths and weaknesses are, they still try to think from their students' perspective and try to understand their needs in order for them to succeed in mathematics.

These teachers also believe that DGS is not suitable for all the students because every student has his own personality and ability to learn mathematics. So for teachers, DGS can support students in geometry. But for those students who are not good at space imagine, DGS may not be as strong as we think it is. So from this we can see teachers always try to decide whether they use DGS or not from the students' point of view. This is important and shows a philosophy of Chinese mathematics teachers. As we have talked about, Chinese teachers always taught two classes of

students during the year. They needed to make some changes when meeting students from these two different classes, even though the contents of the lessons were similar.

When discussing French problems with Chinese teachers, I found they also analyze these tasks through the point of students. When they read the first French task, it was about constructing different circles based on different conditions. Teachers would first imagine how their students would react if they did the same thing in China. They tried to evaluate these tasks and wonders if students were unable to solve them. Because in their experience, these tasks are less complex than the ones they choose in lessons. Most of them asked me if the students were having difficulty. As a mathematics teacher, they need to think about what her students need and what they can learn after the lessons.

When Chinese teachers noticed that the French teacher only told the student how to work the software, how to choose the tools, how to click the points, but not why they need to do these things. If a student made the same mistake, our teachers would explain it to them in a completely different way. They would first to show them why they are not doing it correctly, and then introduce the knowledge needed before having the student build with the software. Because Chinese teachers believe that it is more important for students to know why rather than to know how to operate the software.

We can make a short summary here that students have played a crucial role in all the Chinese lessons. As some of the teachers mentioned in the interview, they prepare the lessons based on the students. Their first concern is whether students can understand the contents or the tasks and whether they are appropriate for these teaching practices. They do not take mathematics classes like a French teacher unless their students are getting what they need.

Others may not imagine how students would solve the tasks or whether they would have some troubles. Instead, they tried to think about what students would get out of solving these tasks and how they would understand what they needed to solve them. They focus more on whether the students can understand the solving process. When I asked them to design a lesson with these French tasks, they mentioned that

the students' familiarity with the software was another key issue when they considered if using technology in this lesson.

7.4.2 The second factor: links between DGS and paper-pencil

The teachers also believe that the distance between the paper and pencil environment and the dynamic geometry environment also influenced their practice of using DGS in their mathematics classes. By using DGS, the teachers found many advantages for both students and teachers in their lessons. As the Chinese teachers said, DGS saves teaching time, giving teachers more time to interact with students and to understand their performance more clearly. It also provides students with a visual understanding of geometry, helping them learn geometry more clearly and easily. In addition to these advantages that DGS brings to the teaching of mathematics, it also brings some changes to the teaching situation. For example, in China, many years ago, when DGS was not yet common used in mathematics lessons, students had to use paper and pencil to solve geometry tasks, which made it difficult for them to understand the process of movement of geometric objects in the task. They needed to draw many different figures that represented different moments in the movement process in order to understand the task situation. Now, with the help of DGS, everything becomes simple. The software can directly present the entire movement and students can also drag the points to see what is happen. This makes the process of problem solving much easier for the students to understand. But as some teachers mentioned, because DGS changes the problem solving context, students also need to think about whether their problem solving strategy with paper and pencil is still appropriate for DGS and pay attention to the difference between the two situations. It is also a key piece of knowledge for teachers.

As we can see from the above mentioned tasks in China and France with DGS, the French task includes these two different situations, allowing students to think about the differences between the two situations and noting that they need to make some modifications to their strategies in solving the task. Of course, there are some

connections between these two situations, so students can also transform some strategy between them.

In this study, I asked Chinese teachers to talk about how they understood the connections and differences between the two situations. In general, they did not show much understanding in their teaching. Generally, we cannot see the two situations together from the task. And students always need to use paper and pencil to solve tasks. This is because of the culture of examination in China. We know that examinations are very important for students and teachers in China. All the contents and processes designed by the teachers are for helping students get higher scores in the final examinations. We need to note that during the examination, students are not allowed to use any technological tools. It requires students to be familiar with strategies about how to solve geometry tasks with paper and pencil, so teachers should never use DGS as a problem-solving tool for students during lessons. As some teachers say, students may rely on the software to solve the task after using it. This is what Chinese teachers need to prevent in their teaching process. So the Chinese task is different from the French one in that it does not contain both solving situations and does not make students think about the connections and differences between them.

The relationship between paper-pencil and DGS also depends on the tasks designed by the teacher during the lesson. After looking at the tasks in the French class, Chinese teachers told me that regardless of the situation in which students solve the tasks, the most important thing is that students need to learn the basic concepts of solving different tasks. Like the French task, which was all about constructing circles, they needed to tell the students what a circle is in mathematics before having them construct a circle with paper and pencil or DGS. When students really understand these concepts, they can then know how to construct a circle themselves. As an example, one teacher said that once students learn about circles, they can know when they want to construct a circle, they first need to determine the center and find the radius. In both cases, the basic strategy is similar, with the difference being that students need to use different tools to find the center and radius of the circle. This is

because Chinese teachers believe that students need to learn mathematical concepts before they can solve mathematical tasks. This shows that Chinese teachers hold an opinion that technology should be used after learning related mathematics knowledge and skills (Turner & Chauvot, 1995). So situations are not critical for them. For students, they can try to modify their constructions with paper and pencil. But these constructions are not accurate enough. But with DGS, these constructions can be more accurate than paper and pencil. Only after understanding these concepts students can know how to construct different figures and do it successfully.

However, in China, we do not have our students construct them in the same way. Our student needs to know how to determine the midpoint by constructing a vertical bisector. That is why this student could not solve the task with DGS. His strategy with paper and pencil does not translate to DGS, but in China the student's strategy is different. Of course for him, constructing with paper and pencil can help the student solve the task with DGS. But they need to know the appropriate strategy and choose one of the best ways to solve the task.

7.4.3 The third factor: examination

There are other factors that can affect teachers' use of DGS in China, such as examinations. Teachers also mentioned that teachers encounter too much pressure in their teaching career. In China, almost everyone is concerned about examinations. Parents are always asking if their children can score higher and want teachers to do their best to help their children get good scores. So although teachers believe that the ability to explore mathematics is critical for students, they rarely give students the time to explore content in the lessons. It is why they did not want to use these French tasks in their lessons.

Young teachers like Mr. W also would not use these tasks because first of all he does not think he has extra time for his students to explore mathematics with DGS and the exploration process always takes too much time for the students. He has many objectives in each lesson. And students are also not required to construct or

explore geometric objects with DGS in the examinations. And this software cannot be used in the examination. He thinks that if students are interested in the software, they might be allowed to do some exploration after class not during the class.

In China, examinations are very important in all stages of a student's learning. Usually the way that teachers and parents know whether a student is successful in learning mathematics comes from the scores that they get in the examinations, so how to get high marks is one of the goals of the students and the teachers always try to help them succeed in these examinations. Of course, in almost all the examinations in China, computers cannot be used. So, due to this phenomenon, the use of software in mathematics lessons is limited.

During my research, many teachers told me that examinations were the most important purpose of their classes. All their class plans are based on the requirements of the final examination. So teachers do not always want their students to use DGS in class. The teachers do not want the students to be too dependent on the software and not be able to visualize the geometric properties themselves.

7.5 Discussion and future study

This study is concerned with describing teacher knowledge when using DGS for mathematics instruction. According to the definition of teacher knowledge, teachers in different countries show different ways of interacting with this technology in the classroom. Chinese teachers viewed DGS as an aid to classroom teaching and so used DGS to present mathematical contents in practice in a more teacher-centered way, while French teachers viewed DGS as a tool for student learning and so used it to create learning situations for students. However, Chinese teachers have noticed and made some changes to allow more time for students to present their ideas and even use the software directly.

Another feature of teacher knowledge, in the case of using DGS, the Chinese teachers focused more on mathematical contents without using DGS, while the French teachers linked mathematics and technology. This was most evident in their interactions

with students during their lessons. Almost all of the Chinese questions were about mathematical contents, whereas the French questions focused on exploring geometric properties by using DGS. This feature also reflects different views on the role of DGS in mathematics teaching.

As teachers mentioned in their interviews (mostly from Chinese teachers), they need to consider many factors when preparing to use DGS in the lessons, such as students' abilities and examinations. In this study, I do not conduct a deep analysis of how these factors affect teachers' classroom practices and how they use DGS to address these factors. It may need further researches to find the relations between these factors and teacher's practice with DGS.

Due to limited data, I do not conduct a comparative study of the knowledge of Chinese or French teachers, nor do I make general conclusions. This study is more of a contrasting study in order to reveal some characteristics of the selected teachers in France and China. Moreover, due to the complexity of teacher knowledge in teacher's practice, I would like to choose to model teacher knowledge by examining the three dimensions of teacher classroom practice mentioned above: choosing mathematics tasks with DGS; types of instrumental orchestration and teacher-student interaction. So in future research, we can extend the study to other aspects of teacher's practice using the same approach in order to analyze teacher knowledge in more detail. It is worthwhile to consider the integration of instrumental orchestration and other theoretical approach like Saxe's (1991) four parameters model or notion of teacher resources and documents (Gueudet & Trouche, 2009).

In this study, I try to use instrumental orchestration to analyze teacher practice in different culture: China and France. Based on the results, it is seemed that this framework is suitable and it appears to do justice to the subtleties in the preparation and performance of teaching techniques (Drijvers et al, 2010). And I also found several new features when the selected teachers incorporated DGS into their teaching activities. As in the Chinese case, sometimes when teachers asked students to discuss tasks, neither teachers nor students used DGS anymore, and it was interesting to see why the Chinese teachers used DGS in this way, even though the tasks that the students needed to discuss

included a dynamic element. Another characteristic is that in the Chinese case, I found that sometimes it was the teacher who operated the DGS according to the students' instructions. In the future, more data are needed to find out if this phenomenon only existed in this teacher's class or if many Chinese teachers had the same characteristics.

This study suggests that there is a gap between the role of dynamic geometry in mathematics tasks and the ways in which teachers integrate it into the lessons. As mentioned above, dynamic geometry plays two important roles in mathematics tasks: amplifier and generator. Although dynamic geometry played the role of a generator of new types of mathematics tasks by changing the solving strategy of these tasks in the Chinese tasks selected for this study, the Chinese teachers still organized the lessons in teacher-centered ways. It is the teacher who operates the software and decides if and when to use it, and the students are more passive during the lessons. This provides a possible means of explaining how to solve geometric tasks using dynamic geometry: like a projector. In fact, in the Chinese example, the task of dynamic geometry as a generator of new problem-solving strategies was accompanied by intensive control by the teacher.

In the French case, dynamic geometry plays the same role in mathematics tasks as it does in China. Moreover, these tasks can be problematic and highly technical, accompanied by leaving greater autonomy to the students. As a result, French teachers organize their lessons in a more student-centered way. All these tasks are used in a computer lab, which is completely different from the Chinese courses. This teaching environment allows students to operate the computer.

In future study, it is needed to concern the elaboration of a model of analysis that more directly relates the role of DGS in the tasks and the complexity that this represents from the point of view of the teachers, with the different orchestrations and the control teacher or less strong teacher. The pertinent question, however, is can teachers make advanced use of dynamic geometry at the cost of less student autonomy?, which needs to be solved in future studies.

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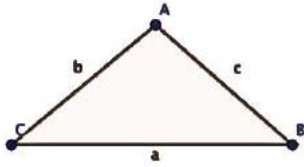
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Appendix

Task in Mr. ZH's lesson

Task in first lesson

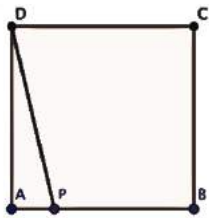
Task



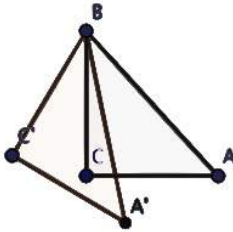
如图，在 $\triangle ABC$ 中， $AC = 4$ ， $BC = 6$ ，问在点 A 运动的过程中 AB，AC，BC 之间的关系。

Task in second lesson

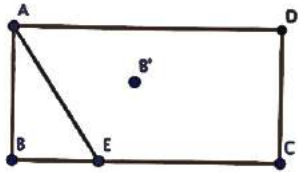
Task



边长为 4m 的正方形 ABCD 中，点 P 以 1m/s 的速度从 A 点出发，沿着 A-B-C-D-A 的方向运动，若点 A，D，P 围成的三角形面积为 S，求 $\triangle ADP$ 的面积 S 与 P 点运动的时间 t 之间的函数关系



在 $Rt\triangle ABC$ 中， $\angle C=90^\circ$ ， $BC=4$ ， $AC=3$ ，将 $\triangle ABC$ 绕着点 B 旋转后点 A 落在直线 BC 上的点 A' ，点 C 落在 C' 处，那么 AA' 的值为 _____



如图，在矩形 ABCD 中， $AB=4$ ， $BC=4\sqrt{3}$ ，点 E 是 BC 边上一动点，联结 AE，把 $\angle B$ 沿 AE 折叠，使点 B 落在点 B' 处，则点 B' 的轨迹长度是多少？

Task in Mrs. J's lesson

First task

Task

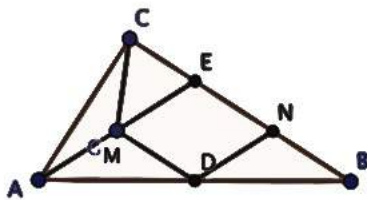


图 1

如图 1，在 $Rt\triangle ABC$ 中， $\angle ACB=90^\circ$ ，点 D 是边 AB 的中点，点 E 在边 BC 上， $AE=BE$ ，点 M 是 AE 的中点，联结 CM，点 G 在线段 CM 上，作 $\angle GDN=\angle AEB$ 交边 BC 于 N

- (1) 如图 2，当点 G 和点 M 重合时，求证：四边形 DMEN 是菱形
- (2) 如图 1，当点 G 和点 M、C 不重合时，求证： $DG=DN$

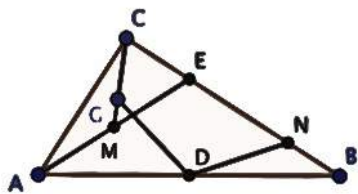
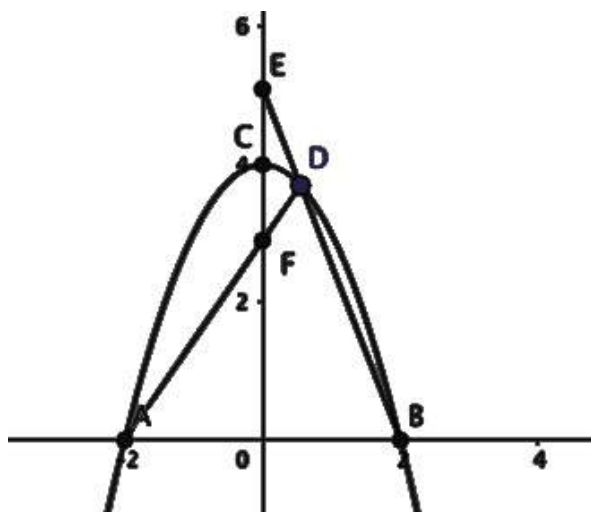


图 2

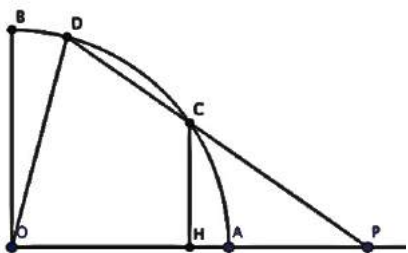


如图，已知抛物线 $y = ax^2 + 4$ ($a \neq 0$)与 x 轴交于点 A 和点 $B(2,0)$ ，与 y 轴交于点 C ，点 D 是抛物线在第一象限的点

- (1) 当 $\triangle ABD$ 的面积为4时，1)求点 D 的坐标；
2)联结 OD ，点 M 是该抛物线上的点，且 $\angle MDO = \angle BOD$ ，求点 M 的坐标
- (2) 直线 BD 、 AD 分别交 y 轴于点 E 、 F ，那么 $OE + OF$ 的值是否变化，请说明理由

Second task

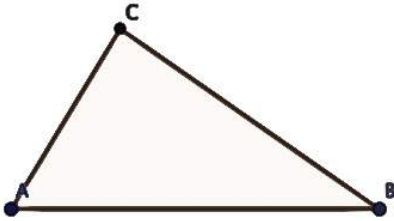
Task



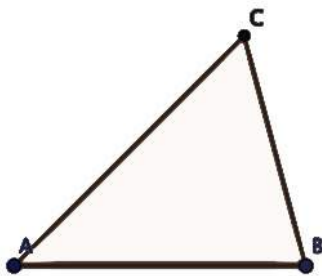
在半径为2的扇形 AOB 中， $\angle AOB = 90^\circ$ ， P 是 OA 延长线上的一点，过线段 OP 的中点 H 作 OP 的垂线交弧 AB 于点 C ，射线 PC 交弧 AB 于点 D ，联结 OD ，

- (1) 如图，当弧 $AC =$ 弧 CD 时，求弦 CD 的长；
- (2) 如图，当点 C 在弧 AD 上时，设 $PA = x$ ， $CD = y$ ，求 y 与 x 的函数关系式，并写出 x 的取值范围；
- (3) 设 CD 的中点为 E ，射线 HE 与射线 OD 交于点 F ，当 $DF = \frac{1}{4}$ 时，请直接写出 $\angle P$ 的余切值

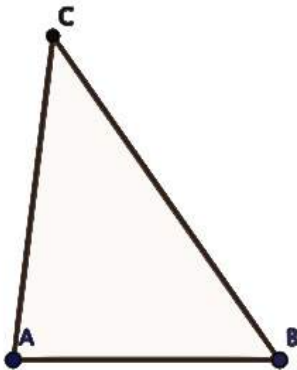
Task



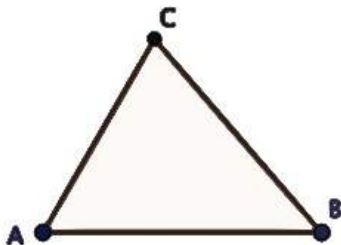
根据以下条件构造三角形 ABC:
 $AB=7, AC=4, BC=6$



根据以下条件构造三角形 ABC:
 $AB=6, \angle A=45^\circ, \angle B=75^\circ$



根据以下条件构造三角形 ABC:
 $AB=5, \angle C=42^\circ, \angle A=83^\circ$



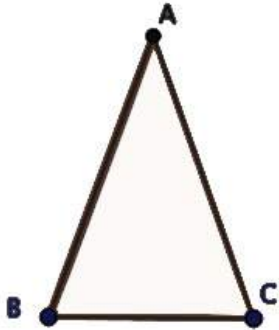
根据以下条件构造三角形 ABC:
 $AB=5, AC=4, \angle A=60^\circ$

Task in Mr. W's lesson

Task

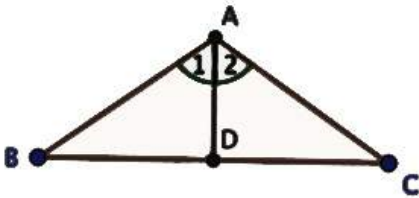
如图，已知 $AB=AC$ ， $\angle B=70^\circ$ ，求：

- (1) $\angle C$ 的度数
- (2) $\angle A$ 的度数



如图，已知 $AB=AC$ ， $\angle BAC=110^\circ$ ， AD 是 $\triangle ABC$ 的中线

- (1) 求 $\angle 1$ 和 $\angle 2$ 的度数
- (2) $AD \perp BC$ 吗？为什么？



Task in Mr. X's lesson

Task

如图，函数 $y=-x+7$ 与 $y=\frac{4}{3}x$ 交于点 A，交 x 轴于点 B

- (1) 求点 A 和点 B 的坐标
- (2) 如果 $AC \perp y$ 轴，垂足为 C，直线 l 平行于 y 轴，并且经过点 B，点 P 以每秒 1 单位的速度从 O 点，沿 O-C-A 的路线运动。同时，直线 l 从点 B 开始以同样的速度朝 O 点运动。直线 l 与 x 轴交于点 R，交线段 BA 或者线段 AO 于点 Q。设时间为 t，则

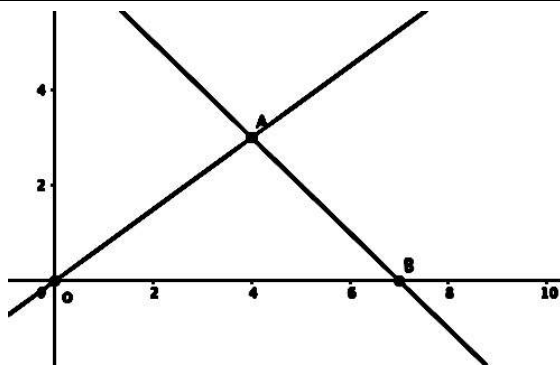


图 1

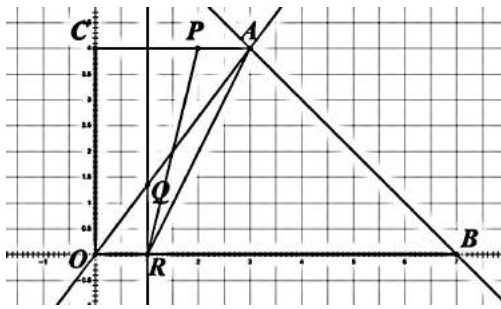


图 a

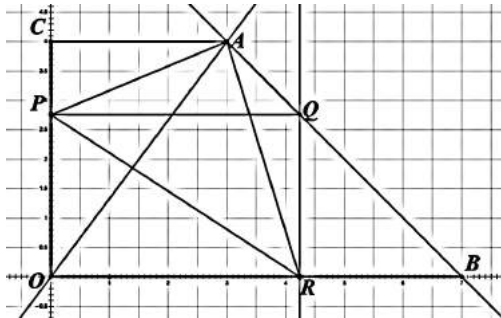


图 b

- a, 当三角形 APR 的面积为 8 时, 求 t 的值
 b, 三角形 APQ 是否能够成为等腰三角形, 如果是求出 t 的值, 如果不是, 说明理由

Task in Louisa's lesson

Nom:

Classe:

T.P. Géométrie dynamique

Consignes

On utilise le logiciel GeoGebra <http://www.geogebra.org> Pour chacun des programmes de construction suivants :

1. construis une figure à **main levée et au crayon à papier** dans la colonne centrale ;
2. construis une figure **aux instruments et au crayon à papier** dans la colonne de droite ;
3. construis une figure sur ordinateur à **l'aide du logiciel**, puis bouge les points. **Après chaque construction, appelle le professeur et enregistre ton travail.**

Cercles

Programme	Figure à main levée	Figure aux instruments
1. Place deux points A et B. Trace le segment [AB]. 2. Trace le cercle de centre A et qui passe par B. 3. Trace le cercle de centre B et qui passe		

par A.		
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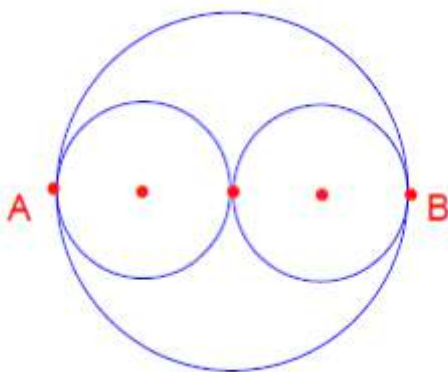
Programme	Figure à main levée	Figure aux instruments
1. Trace un cercle de centre A et de rayon 3cm. 2. Place deux points B et C sur ce cercle. 3. Trace le segment [BC].		



Programme	Figure à main levée	Figure aux instruments
1. Place deux points A et B. Trace le segment [AB]. 2. Trace le cercle de diamètre [AB].		



Bilan des menus utilisés : Écrire la fonction de chaque icône : :
:



1. Ecrire un programme de construction de la figure ci dessus en commençant par : Construire deux points A et B...
2. Construire la figure aux instruments et au crayon de papier.
3. Construire une figure sur ordinateur à l'aide du logiciel.