

Micromorphic Continua: Advanced Multiphysic Modelling and Numerical Simulation of Metal Forming

Evangelia Diamantopoulou

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Thèse de doctorat de l'UTT

Evangelia DIAMANTOPOULOU

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Evangelia DIAMANTOPOULOU

le 13 février 2018

Micromorphic Continua: Advanced Multiphysic Modelling and Numerical Simulation of Metal Forming

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Abstract

The main goal of this work is to show the efficiency of the modified fully coupled constitutive equations in the framework of the micromorphic continua in getting really mesh independent solutions even with high values of ductile damage. The generalized framework of the micromorphic continua is presented in order to extract three balance equations from the generalized principle of virtual power after introducing new micromorphic degrees of freedom (dofs) and their first gradients: the classical equilibrium equation and the micromorphic balance equations concerning the damage and the isotropic hardening. In the framework of the thermodynamics of irreversible processes, the state relations and the evolution equations are derived from the state and dissipation potentials. These potentials are enriched by introducing the microcracks closure phenomenon as well as the damage effect on the isotropic hardening.

The strong forms defining the IBVP are used to derive their associated weak forms. The latter are discretized in time through an Euler scheme with a global dynamic explicit solver and an implicit iterative local integration scheme and in a space domain using finite elements. The discretized mass matrices and the internal forces are given for the dynamic explicit resolution scheme as well as the local resolution scheme for the integration of the overall constitutive equations on each Gauss point.

The associated numerical aspects are treated in the framework of ABAQUS[®]/Explicit thanks to the appropriate user's subroutine VUEL for the implementation of the new micromorphic 2D quadrilateral assumed strain and 2D axisymmetric finite elements.

The proposed numerical methodology is validated after performing a detailed parametric study of the complete micromorphic model in order to analyze the role and investigate the effect of each micromorphic material parameter.

A methodology for identifying and properly choosing the value of the micromorphic internal length related to the micromorphic damage by associating it to the width of the shear bands that appear during the localized necking stage is also introduced thanks to the experimental ESPI strain rate field measurements.

Finally, applications are made to simulate tensile tests, bending and blanking operations on metallic components of 430 stainless steel, DP1000 and DP600 dual phase steels respectively in order to validate the proposed nonlocal micromorphic formulation and to show its efficiency in giving mesh independent solutions compared to the purely local model.

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General Introduction

Nowadays, the fully local constitutive equations have been well established to model the induced material softening behavior due to thermal, damage and other microstructure-dependent phenomena. However, the solutions of the evolution problem based on these fully local constitutive equations are highly sensitive to the space and time discretization. The natural way to overcome this drawback is to account for an appropriate neighborhood of each material point by introducing some characteristic lengths, representative of the materials' microstructures, into constitutive equations.

The mechanics of generalized continua makes possible the straightforward introduction of these characteristic lengths, related to the microstructure, into the constitutive equations of the materials. The most recent and comprehensive reviews of these generalized continua and their use to solve various problems in mechanics of solids and fluids is found in the recent books by (Eringen, 1999, 2002) and older works (Mindlin and Tiersten,1962; Toupin, 1962, 1964; Mindlin,1964, 1965; Eringen and Suhubi,1964; Green and Rivlin 1964a, 1964b, 1965; Green,1965; Truesdell and Noll, 1965; Eringen, 1965a, 1965b; Bringen, 1970). The application of this micromorphic framework to model the materials' behavior in plasticity with damage can be found in (Forest, 2006; Saanouni, 2012; Saanouni and Hamed, 2013; Hamed 2012).

Various generalized continuum theories have been developed during the last decades in order to account for some effects of the characteristic lengths related to the material's microstructure and leading to a wide range of models (Sidoroff, 1975; Maugin, 1979; Aifantis, 1987; Forest et al., 2002; Aifantis, 2003; Forest and Sievert, 2003, 2006; Liebe et al., 2003; Forest, 2006; Forest, 2008; Forest, 2009; Hirschberger and Steinmann, 2009; Forest and Aifantis, 2010 among many others). As summarized by Forest (Forest, 2006, 2009), all these generalized continuum theories, based on the assumption of local action (Truesdell and Noll, 2004), can be classified into two classes: (i) the higher grade continua and (ii) the higher order continua. Higher grade continua are those based on higher order spatial derivatives of the displacement field as originally proposed by (Mindlin, 1965; Mindlin and Eshel, 1968). Higher order continua are based on the introduction of additional degrees of freedom as pioneered by the Cosserat brothers (Cosserat, 1896, 1909, 2009), and extensively developed by Eringen in 1960s (Eringen and Suhubi, 1964; Eringen, 1965a, 1965b, 1999). A third class of generalized continuum theories is the so called strictly nonlocal continuum field theories, summarized in the recent book (Eringen, 2002) where a unified foundation of the basic field equations is presented and various main contributing works in the field are referenced. The class of nonlocal theories, which are not based on the principle of the local action, are "concerned with the physics of material bodies whose behavior at a material point is influenced by the state of all points of the body" as stated by Eringen in his introduction (Eringen, 2002).

Note that, the micromorphic theory initially proposed by (Eringen and Suhubi, 1964; Mindlin, 1964), introduces a general non compatible full field of micro-strains as an extra degree of freedom additional to the classical displacement field. It can be applied to any macroscopic quantity in order to introduce a characteristic length scale in the original classical continuum model in a systematic way, as presented by (Forest, 2009). From the comparison between nonlocal and micromorphic theories presented in (Forest and Aifantis, 2010) it can be concluded that when the micromorphic variable remains as close as possible to the plastic strain, the micromorphic model reduces to the largely used strain gradient theory.

Additionally to the generalized continuum theories (higher grade, higher order and nonlocal), other approximate regularization methods have been widely used. For the mechanics of materials exhibiting damage induced softening, a great number of papers have been published in this simplified framework with the main goal to regularize the associated IBVPs (Initial and Boundary Value Problems) by proposing the introduction of some "localization limiters" in the classical local constitutive equations, using either strain-gradients, damage-gradients or equivalently some averaging using specific integral equations (Pijaudier-Cabot and Bazant, 1987; Bazant and Pijaudier-Cabot, 1988; Saanouni et al., 1989; Lesne and Saanouni, 1990; Tvergaard and Needleman, 1995; Frémond and Nedjar, 1996; Li and Cescotto, 1997; Svedberg and Runesson, 1997; Borino et al., 1999; De Borst et al., 1999; Ganghoffer et al., 1999; Kuhl and Ramm, 1999; Svedberg and Runesson, 2000; Nedjar, 2001; Peerlings et al., 1996, 2001, 2002; Bazant and Jirasek, 2002; Areias, 2003; Engelen et al., 2003; Geers et al., 2003; Lorentz and Andrieux, 2003; Jirasek and Rolshoven, 2003; Geers, 2004; Yuan and Chen, 2004; Santaoja, 2002; Cesar de Sa et al., 2006; Sornin and Saanouni, 2011). It has been shown in recent works that the major part of the approximated models are in fact a particular case of the straightforward generalized continuum theories (Hirschberger and Steinmann, 2009; Forest, 2009; Forest and Aifantis, 2010; Saanouni, 2012).

On the other hand, to account for the damage activation (under tension) and deactivation (under compression) various approaches have been proposed (Marigo, 1985; Ju, 1989a, 1989b; Chaboche, 1992, 1993; Lemaitre, 1992; Ladveze, 1993; Chaboche et al., 1995; Desmorat, 2000; Lemaitre and Desmorat, 2005; Desmorat and Cantournet, 2007; Lemaitre et al., 2009; Voyiadjis et al., 2009; Ganczarski and Cegielski, 2010, ...). In order to introduce this aspect in the present model, the simplest idea consists of introducing the damage effect (through coupling) differently into the positive and negative parts of each tensorial state variable under concern. Such a kind of modelling poses the problem of convexity loss due to the discontinuity of the potential depending on positive and negative parts (Ladeveze, 1993; Lemaitre and Desmorat, 2005; Lemaitre et al, 2009; Ganczarski and Cegielski, 2010; Issa et al, 2012; Saanouni, 2012). To avoid these difficulties, in this work we will follow the same approach given in (Issa et al., 2012; Badreddine et al., 2015). According to this approach, the decomposition into positive and negative parts is accounted only for the damage energy density release rate Y in order to have higher values in tension than in compression.

All these simplified regularization methods allow us to achieve a mesh independent solution. In the meanwhile, the macroscopic crack, defined as the location of the completely damaged elements, seems to be always depending on the mesh size limited in a single row of elements (Sornin, 2007; Sornin and Saanouni, 2011). In fact, the Helmholtz equation that comes to regularize the damage is simply postulated in which the nonlocal damage simply replaces the local damage in the constitutive equations to reassure the coupling effects.

The present work is part of the ANR project "Micromorfing" which brings together three academic partners: the UTT (PI), the UTC and the Ecole des Mines de Paris (Mines ParisTech). The main objective of this project is to develop an 'advanced' modeling of multiphysic thermomechanical coupling in the framework of the generalized continuum mechanics (micromorphic theory) to introduce the concept of internal lengths that are representative of the materials microstructures while accounting for the various initial and induced anisotropies under large deformations. The targeted applications concern mainly the simulation of metal forming processes in order to improve them with respect to the ductile damage occurrence either to avoid the damage occurrence or in contrary to enhance the damage occurrence. In fact, optimizing various manufacturing processes in

order to obtain metallic parts with controlled defects while reducing the overall cost in terms of raw materials, energy required for their manufacturing or their use in service and environmental impact, is now a vital necessity. The fracture of metallic components during their manufacture or during their industrial use is a consequence of the strong localization of thermomechanical fields (as strains, temperature or damage) inside more or less narrow zones. The strong interactions (or coupling) between the intensive thermomechanical fields (stress, strain, hardening, heat, damage ...) inside these localization zones, result in some induced softening due to the damage effect and/or to the temperature increase. For quasi-static problems, the mechanics of materially simple (or Cauchy) continua has been shown to be no longer sufficient to model these highly localized phenomena in presence of the damage-induced softening. In the dynamic case including damage effect, the numerical solution of these problems leads to the localization bands whose width is very small and not related to the underlying microstructure.

Roughly speaking, the project "Micromorfing" is composed by four main scientific tasks. The first one concerns the theoretical aspects related to both (i) the derivation of the additional (micromorphic) balance equations and (ii) the derivation of the fully coupled constitutive equations. The second task is of experimental nature and aims to propose a "fine" full field based experimental method to measure the localization zones in order to identify the additional micromorphic material parameters directly related to the internal length scales linked to each targeted micromorphic phenomenon. The task three is related to the computational aspects in the framework of the FE method using fully adaptive numerical techniques (adaptive remeshing, adaptive time incrementation) together with the hyper-reduction of the model. Finally, the fourth task aims to apply the methodology to the numerical simulation of various metal forming processes. Note that the hyper-reduction of the models is investigated by CdM/MineParistech (Post doc), the adaptive remeshing is performed by the team of LRM at UTC (PhD and Post doc) while, the experimental method (a part of PhD work at LNIO/UTT) and the theoretical part as well as its implementation into ABAQUS®/Explicit FE software (the present PhD work at LASMIS/UTT) was performed by the two research teams at UTT.

In the present PhD thesis we present our works in the framework of the generalized continua in order to reformulate the IBVP for elastoplastic solids undergoing induced material softening due to the presence of the ductile damage. For this reason and in order to develop our model, we have chosen a formulation based on the micromorphic theory which belongs to the category of the higher order continua.

In the first chapter we focus on the presentation of a macroscopic formulation in time independent plasticity with a strong coupling with the ductile damage in the mechanical framework of the generalized continua but under isothermal conditions. The general framework of the generalized continua is then presented in the framework of the micromorphic theory. In this framework we deduce the micromorphic balance equations from the generalized principle of virtual power with additional micromorphic degrees of freedom (dofs). Then, the use of the thermodynamics of irreversible processes with an enhanced space of state variables, allows us to formulate the nonlocal constitutive equations with both micromorphic isotropic ductile damage and micromorphic isotropic hardening. The elastoplastic damage model in the framework of the materially simple continua is given as a special case derived from the nonlocal model by deactivating the micromorphic effects. We also dedicate a paragraph for the notion of the micro-cracks closure since some properties and the behavior of a representative volume element are affected by the

presence of induced softening. We conclude with a paragraph concerning the extension of the model to finite plastic strains and a discussion on the objectivity requirement.

The second chapter is dedicated to the numerical aspects related to the solution of the IBVP. The weak forms associated to the problem are formulated starting from the complete set of the balance equations. A nonlinear algebraic system is then obtained starting from the displacement-based finite element space discretization of the three weak forms. The resolution of this system by a dynamic explicit scheme and the local integration of the constitutive equations at each Gauss point are extensively discussed. A paragraph concerning the stability conditions for the micromorphic damage problem is added, as well as the formulation of two micromorphic 2D elements. We close this chapter with a paragraph containing a brief representation of the 2D adaptive remeshing methodology to be applied on the last chapter of the thesis.

In the third chapter a relatively exhaustive parametric study of the proposed micromorphic model is conducted. We present an extended and relatively complete parametric study in order to investigate the effect and the influence of each micromorphic parameter as well as their role to the IBVP solution. The study is made for a model containing a) only micromorphic damage, b) only micromorphic isotropic hardening and c) a complete model containing both. We close this chapter by clarifying the conditions that have to be respected before choosing the appropriate values of these micromorphic parameters in order to make sure that the solution is physically acceptable and independent from the mesh size. This parametric study of the micromorphic model was performed and the results were carefully analyzed to well understand the predictive possibilities of the proposed micromorphic fully coupled constitutive equations.

In the fourth chapter we will only focus on the parametric study concerning the elasticity, plasticity and local damage parameters and we will propose a methodology for identifying and properly choosing the values of the micromorphic internal length related to the micromorphic damage for a current material by associating it with the width of the shear bands that appear during the localized necking stage of a simple uniaxial tensile test.

Finally, the fifth chapter is dedicated to some applications including a bending and a blanking test, showing the efficiency and the predictive capabilities of a complete micromorphic model.

The manuscript is afterwards complete by general conclusions as well as the main perspectives of our work.

Chapter 1

Formulation of an elastoplastic damaged model

in the framework of the generalized continua.

The micromorphic theory.

1.1 Introduction

This chapter is dedicated to the presentation of the formulation of an elastoplastic model with damage in the framework of the generalized continua based on the micromorphic theory. We start by presenting the general framework of the mechanics of generalized continua by classifying them into 'higher grade' and 'higher order'. The generalized framework of the micromorphic continua is briefly presented in order to extract additional balance equations: the classical equilibrium equation and the micromorphic balance equations related to the additional micromorphic phenomena under concern. In the framework of the thermodynamics of irreversible processes with state variables, the state relations and the evolution relations are derived from the state and dissipation potentials. These potentials are enriched by introducing the micro-cracks closure phenomenon as well as different damage effect on the isotropic hardening. We close this chapter by one paragraph concerning the extension to finite strains under the formulation of a rotated frame.

1.2 Mechanics of the generalized continua: the micromorphic theory

The Mechanics of Materially Simple Continua (MMSC) suppose that the mechanical state at any material point of area \wp is completely determined by the history of state variables in an arbitrarily small neighborhood surrounding this point (Truesdell and Toupin, 1960). In this context, the knowledge of the first transformation gradient <u>F</u> (or first displacement gradient) is sufficient in order to determine the mechanical state (kinematic, behavior) of this point and the continuum is called materially simple or local (Truesdell and Noll, 1965).

However, in several situations, the displacement vector and its first gradient are not sufficient to define the mechanical state in a material point. In such cases, it is necessary to add other kinematic variables or dofs (degrees of freedom) to enrich the kinematical description of the continuum, as well as their gradients of first, second or higher order, in the principle of virtual power and as new arguments in the state and dissipation potentials. We call this the mechanics of generalized continua (MGC) or materially non-simple continua (Truesdell and Noll, 2004). These theories all seek to define the mechanical state at a material point in terms of a more or less vast domain surrounding the point, or even of the whole domain. Finally, this introduces a kind of scale effect in terms of the morphology of the spatial distribution of the different material phases inside the representative volume element (RVE) and of the size of the various constituent elements, or the effects of the gradients of physical fields. Different variations of these theories can be found in the literature and reference books such as those of the Cosserat brothers, whose book was first published in 1909 (Cosserat, 1909) and republished recently in 2009 (Cosserat, 2009). Also among these, (Kröner, 1967; Stojanovic, 1970; Brulin and Hsieh, 1982; Mülhaus, 1995; Eringen, 1999; Eringen, 2001; Eringen, 2002; Truesdell and Noll, 2004 and Forest, 2006).

It is often proposed to classify MMCG theories into three distinct theories (Forest and Sievert, 2003; Saanouni, 2012; Saanouni and Hamed, 2012; Labergère, 2015) as summarized in Fig. 1.1:

Higher grade continua, based on the introduction of higher order of the spatial derivatives of the displacement vector \vec{u} (in addition to the first displacement vector, which defines the strain tensor) in the principle of virtual power, as initially proposed by (Toupin, 1962; Toupin, 1964; Mindlin, 1968).

Higher order continua, consisting of the use of additional degrees of freedom (dof) introduced in the principle of virtual power, leading to additional balance equations with their appropriate boundary conditions to be added to the local classical equilibrium equations. This assumption was initially proposed by the Cosserat brothers (Cosserat, 1909) and subsequently applied by (Eringen, 1964), (Eringen, 1970). The additional dofs are new kinematic variables along with their higher order gradients. The simplest of these theories is limited to the first gradients of these new kinematic variables, as with MMSC, which utilizes the displacement and its first gradient. Moreover, by using the thermodynamics of irreversible processes in a way identical to the one used for MMSC, we obtain generalized constitutive equations as functions of the internal lengths, characteristic of the continuum's microstructure. With regard to higher order generalized continua, Forest (Forest, 2006) shows that the main equations suggested in the literature, including the Cosserat brothers' theory, can be obtained as special cases of a micromorphic theory first introduced by Eringen (Eringen, 1999), (Eringen, 2002).



Figure 1.1: Classification of the nonlocal theories, found in the recent work of Eringen (Eringen, 2002)

All of the MGC theories presented above are based on the principle of local action. So by excluding the strictly nonlocal continua based on integral formulations, in which the localization of the integral equations to a given material point causes the occurrence of "residual" localization terms with appropriate (complex) discontinuity conditions (Eringen, 2002), then the MGC theories are

formally grouped into the first two relatively distinct families from a conceptual perspective (Forest, 2006).

Regarding the theoretical framework which we have chosen to build our nonlocal formulation, we will start by giving the principal and the major steps. We have developed a formulation of an elastoplastic damaged model in the context of generalized continua in the framework of the micromorphic theory. The kinematics of a micromorphic continuum rely on the displacement \vec{u} and its first gradient $\nabla \vec{u}$ as well as a certain number of mciromorphic variables noted as $\underline{\breve{z}}_n$ and their first gradients $\nabla \underline{\breve{z}}_n$. These additional dofs are chosen in order to represent the micromorphic phenomena involved in our model.

We consider a materially simple continuum whose behavior is described by local state variables. By following the approach recommended by Samuel Forest (Forest, 2009), the development of a micromorphic model is described by the following steps:

- Rewrite the principle of virtual power by taking into consideration the micromorphic contributions.
- By applying the generalized principle of virtual power, we obtain the differential balance equations along with their boundary conditions. Thus, one additional balance equation corresponds to each micromorphic phenomenon.
- Expand the space of the state variables by introducing the micromorphic variables and their first gradients.
- Extend and apply the principle of energy conservation (1st principle of thermodynamics) as well as the entropy inequality (2nd principle of thermodynamics) by using the internal state variables, including the contribution of the micromorphic variables $\underline{\underline{z}}$ and $\overline{\nabla} \underline{\underline{z}}$.
- Use of the generalized Clausius-Duhem inequality in order to obtain:
 - o generalized state relations from a state potential, and
 - o generalized evolution equations.

The choice of the state and dissipation potentials is made in a way similar to the one of the classical local theory in order to extract all the constitutive equations of the dissipative phenomena.

This approach is particularly suited to describe the behavior of nonlinear solids with various dissipative phenomena, including the role of state variable gradients, as we will see in the following. In this section, we will use this approach to extend the thermodynamic framework in order to formulate the overall equations (conservation laws and constitutive equations) in the framework of a micromorphic continuum.

1.3 Theoretical modeling of the nonlocal micromorphic damage model

In metal forming through large inelastic strains, inelastic flow localization phenomena in specific areas, known as intense shear bands, are very often observed. These shear bands are generally the result of softening behavior provoked by microscopic physical phenomena. Among these phenomena, which are responsible for the formation of intense shear bands in metals there is softening, whether thermal or caused by any other physical phenomenon, in particular ductile damage. Generally, in these intense (visco)plastic shear bands, ductile fracture develops following from the three stages of nucleation, growth, and coalescence of microvoids at the source of the formation of macroscopic cracks.



Figure 1.2: Schematic representation of the damage effect on the force-elongation curve (Saanouni, 2012)

Thus, constitutive equations which take account of thermal and damage effects (or coupling) are characterized by a softening or negative hardening behavior, even if the material initially exhibits positive hardening, as schematized in Fig. 1.2. This is so for all time-independent models with hardening and ductile damage which are characterized by a relationship between stress and strain rates involving the continuous tangent operators. However, the introduction of viscosity (viscoplasticity models) does not remove all the "locks" resulting from the study of the mathematical properties and the local nature of these tangent operators especially if the softening is mainly due to the disappearance of all or of a part of the components of the tangent stiffness matrix of the structure as a result of the coupling with damage.

These theoretical difficulties are obstacles to the validity of the local models formulated in the framework of the mechanics of materially simple continua (MMSC). In order to introduce the induced softening phenomena, it is compulsory to reformulate the constitutive equations in the framework of the mechanics of generalized continua (MGC). In fact, the MGC enable the natural introduction of various internal lengths characteristic of the material's microstructure.

Despite the fact that the micromorphic effects can be linked to various phenomena (Saanouni 2012, Saanouni and Hamed 2013), in this thesis we limit ourselves to the micromorphic effects linked to the isotropic hardening and the isotropic ductile damage under isothermal conditions. For this, we place ourselves within the context of the micromorphic theory presented in Paragraph 1.2. We will reconsider the elastoplastic behavior with isotropic ductile damage by limiting ourselves to purely mechanical aspects of an isothermal elastoplastic damageable solid totally isotropic without considering the quasi-unilateral effect as well as initial and induced anisotropies. However, the consideration of these phenomena as an aspect of a micromorphic formulation does not pose any obstacles (Liu, 2017).

In addition to the four classical pairs of state variables defined as $(\underline{\varepsilon}^e, \underline{\sigma})$ for the plastic flow, (d, Y) for the ductile damage, (r, R) for the isotropic and $(\underline{\alpha}, \underline{X})$ for the kinematic hardening, we introduce four different pairs of micromorphic state variables noted as $(\overline{r}, \overline{R})$ and $(\overline{d}, \overline{Y})$ accounting for the micromorphic isotropic hardening and the micromorphic isotropic ductile damage as well as the pairs of their respective first gradients: $(\overline{\nabla}\overline{r}, \overline{R})$ and $(\overline{\nabla}\overline{d}, \overline{Y})$.

1.3.1 The Generalized principle of virtual power

The virtual power of internal forces δP_{int} of the classical local continuum is extended to the generalized micromorphic continuum level by using the two additional micromorphic degrees of freedom \breve{r} and \breve{d} associated to the micromorphic isotropic hardening and the micromorphic isotropic damage respectively. Starting from the generalized formulation given in (Saanouni, 2012; Saanouni and Hamed, 2013) and keeping the damage and isotropic hardening as the targeted micromorphic phenomena, we conclude to the following expression:

$$\delta P_{\rm int} = -\int_{\Omega} \left(\underline{\sigma} : \vec{\nabla} \vec{u}^* + \vec{Y} \vec{d}^* + \vec{R} \vec{\dot{r}}^* + \vec{Y} \cdot \vec{\nabla} \vec{d}^* + \vec{R} \cdot \vec{\nabla} \vec{r}^* \right) dV$$
(1.1)

where, $\underline{\sigma}$ indicates the Cauchy stress tensor, \underline{Y} and \underline{R} are the stress-like variables associated to \underline{d} and \underline{r} , while \underline{Y} and \underline{R} are the stress-like variables linked to their first gradients.

Similarly, the virtual power of external forces (body forces and contact forces via the boundaries of the solid) δP_{ext} are enriched by:

$$\delta P_{ext} = \rho \int_{\Omega} \left[\left(\vec{f}^{u} \cdot \dot{\vec{u}}^{*} \right) + \left(f^{\vec{d}} \cdot \dot{\vec{d}}^{*} + \vec{f}^{g\vec{d}} \cdot \vec{\nabla} \dot{\vec{d}}^{*} \right) + \left(f^{\vec{r}} \cdot \dot{\vec{r}}^{*} + \vec{f}^{g\vec{r}} \cdot \vec{\nabla} \dot{\vec{r}}^{*} \right) \right] dV + \int_{\Gamma} \left[\left(\vec{F}^{u} \cdot \dot{\vec{u}}^{*} \right) + \left(F^{\vec{d}} \cdot \dot{\vec{d}}^{*} \right) + \left(F^{\vec{r}} \cdot \dot{\vec{r}}^{*} \right) \right] dS$$

$$(1.2)$$

where \vec{f}^{u} is the classical body force while $f^{\bar{d}}$, $\vec{f}^{g\bar{d}}$, $f^{\bar{r}}$ and $\vec{f}^{g\bar{r}}$ are the micromorphic body forces associated to the micromorphic variables and their respective first gradients. Also \vec{F}^{u} is the classical external force while $F^{\bar{d}}$ and $F^{\bar{r}}$ are the micromorphic external forces applied on the boundary Γ . Finally, the virtual power of inertia forces can be expressed as:

$$\delta P_a = \rho \int_V \left(\ddot{\vec{u}} \cdot \dot{\vec{u}}^* + \zeta_{\vec{a}} \ddot{\vec{d}} \cdot \dot{\vec{d}}^* + \zeta_{\vec{r}} \ddot{\vec{r}} \cdot \dot{\vec{r}}^* \right) dV$$
(1.3)

where $\ddot{\vec{u}}$ is the acceleration, \vec{d} and $\ddot{\vec{r}}$ denote the acceleration associated to the micromorphic damage and the micromorphic isotropic hardening respectively. Finally, $\zeta_{\vec{a}}$ and $\zeta_{\vec{r}}$ are the scale factors which map the local material density to the micromorphic level (Nedjar, 2001). If these two parameters are zero, then the micromorphic acceleration quantities can be ignored.

The generalized virtual power enhanced with the micromorphic damage takes the following form for any given kinematically admissible fields:

$$\delta P_{\text{int}} + \delta P_{\text{ext}} = \delta P_a \quad \forall \dot{\vec{u}}^*, \dot{\vec{d}}^*, \dot{\vec{r}}^* K.A.$$
(1.4)

In the meanwhile, it is also worth presenting how this model can 'restore' its classical local behavior if we neglect all the micromorphic effects. We will reconsider the theoretical formulations in order to obtain the original classical local damage model as well as to compare and distinguish it from the above extended micromorphic (nonlocal) model.

By neglecting the additional micromorphic dofs related to the micromorphic damage d and the micromorphic isotropic hardening \breve{r} , the virtual power of the internal forces writes under the classical form:

$$\delta P_{\rm int} = -\int_{\Omega} \left(\underline{\sigma} : \underline{\dot{\varepsilon}}^*\right) dV \tag{1.5}$$

Accordingly, the virtual power of the external forces will retrieve its classical form, by dropping the generalized body and external forces associated with micromorphic fields:

$$\delta P_{ext} = \rho \int_{\Omega} \left(\vec{f}^{u} \cdot \vec{u}^{*} \right) dV + \int_{\Gamma} \left(\vec{F}^{u} \cdot \vec{u}^{*} \right) dS$$
(1.6)

Finally, the virtual power of inertia reduces to:

$$\delta P_a = \rho \int_V \left(\vec{\vec{u}} \cdot \vec{\vec{u}}^* \right) dV \tag{1.7}$$

1.3.2 Balance equations

The application of the virtual power thus leads to three partial differential equations, with their associated boundary conditions. By substituting Eq.(1.1) to Eq.(1.3) into the generalized form of virtual power Eq.(1.4) and using of the divergence theorem to transform the volume integration, leads to the local (well known) balance of momentum while the other two express the micromorphic forces equilibrium equations:

$$\begin{cases} \vec{\nabla} \cdot \underline{\sigma} + \rho \vec{f}^{u} = \rho \vec{\ddot{u}} & \text{in } \Omega \\ \underline{\sigma} \cdot \vec{n} = \vec{F}^{u} & \text{on } \Gamma \end{cases}$$
(a)

$$\begin{cases} \left(\vec{\nabla} \cdot \vec{\vec{R}} - \vec{R}\right) + \rho \left(f^{\vec{r}} - \vec{\nabla} \cdot \vec{f}^{g\bar{r}}\right) = \rho \zeta_{\bar{r}} \vec{\vec{r}} & in \Omega \\ \left(\vec{\vec{R}} - \rho \vec{f}^{g\bar{r}}\right) \cdot \vec{n} = F^{\bar{r}} & on \Gamma \end{cases}$$
(b) (1.8)

$$\begin{cases} \left(\vec{\nabla} \cdot \vec{\vec{Y}} - \vec{Y}\right) + \rho \left(f^{\vec{a}} - \vec{\nabla} \cdot \vec{f}^{g\vec{a}}\right) = \rho \zeta_{\vec{a}} \overset{:}{\vec{d}} & in \Omega \\ \left(\left(\vec{\vec{Y}} - \rho \vec{f}^{g\vec{a}}\right) \cdot \vec{n} = F^{\vec{d}} & on \Gamma \end{cases}$$
(c)

We will return to these two partial differential equations later in order to express them in the strain space. Note that each micromorphic phenomenon introduced into the principle of virtual power gives rise to an additional differential problem of the type given by Eq.(1.8b) or Eq.(1.8c). The same approach can therefore be followed to obtain other differential equations governing other micromorphic phenomena, such as, kinematic hardening and plastic strain. Let us note here that if all the micromorphic variables are canceled, the equations Eq.(1.8b) and Eq.(1.8c) disappear and we retrieve the local case characterized by the equilibrium of a materially simple continuum Eq.(18.a).

By following exactly the same procedure as for the micromorphic model, we obtain the classical equilibrium equation associated to the displacement, as the sole degree of freedom for the classical local case:

$$\begin{cases} \vec{\nabla} \cdot \underline{\sigma} + \rho \vec{f}^{u} = \rho \vec{u} & \text{in } \Omega \\ \underline{\sigma} \cdot \vec{n} = \vec{F}^{u} & \text{on } \Gamma \end{cases}$$
(1.9)

1.4 Formulation of the micromorphic damage model

In the terms of the local state in its current configuration and under isothermal conditions, a deformable solid is defined by:

- The displacement vector \vec{u} as the sole degree of freedom;
- Several pairs of local state variables $(\underline{\varepsilon}^{e}, \underline{\sigma})$, (d, Y), (r, R) and $(\underline{\alpha}, \underline{X})$, where $\underline{\varepsilon} = \int_{t} \underline{D} dt$ is the Eulerian total strain measure;
- A state potential expressed in the strain space by the Helmholtz free energy $\psi(\underline{\varepsilon}^{e}, \underline{\alpha}, r, d)$ where $\underline{\varepsilon}^{e} = \int_{r} D^{e} dt$ and $\underline{D} = \underline{D}^{e} + \underline{D}^{ir}$;
- A dual dissipation potential in the stress space φ^{*}(<u>σ</u>, Y, R, <u>X</u>) if the irreversible deformation process is time-dependent; or a yield function f(<u>σ</u>, <u>X</u>, R, d) and a dissipation potential F(<u>σ</u>, <u>X</u>, R, d) if the irreversible deformation process is time-independent (Saanouni 2012).

As a matter of fact, the extension of this materially simple continuum to a micromorphic one (or a generalized continuum) depends on selecting some additional pairs of state variables that will

determine the targeted micromorphic effects. Let us suppose that a pair of scalar variables (z,Z) is responsible for a specific micromorphic effect. Consequently, we have introduced one pair of micromorphic (scalar) variables called (\tilde{z}, \tilde{Z}) and the micromorphic continuum will be defined by:

- The following dofs: \vec{u} and \vec{z} to be introduced in the virtual power;
- The following pairs of state variables: $(\underline{\varepsilon}^e, \underline{\sigma}), (d, Y), (r, R), (\underline{\alpha}, \underline{X}), (\overline{z}, \overline{Z}) \text{ and } (\nabla \overline{z}, \overline{Z})$.
- We need a state potential expressed in the strain space by the Helmholtz free energy: $\psi(\varepsilon^e, d, r, \alpha, \tilde{z}, \vec{\nabla}\tilde{z})$;
- A dual dissipation potential in the stress space: $\varphi^*(\underline{\sigma}, Y, R, \underline{X}, \overline{Z}, \overline{Z})$.

For this type of micromorphic continuum, we can rewrite the energy balance as well as the entropy inequality and deduce from these the Clausius–Duhem inequality for micromorphic continuum in a manner similar to local theory. Assuming, as a first approximation, that the "entropy production" vector remains defined for micromorphic continuum, as in the MMSC theory, by the ratio of the heat-flow vector to absolute zero, the only difference comes from the internal forces power. The differentiation of the state potential with respect to all local and micromorphic state variables with the additional decomposition of the Eulerian strain rate leads to rewriting the inequality and deducing the expressions of the local stress-like variables as well as the micromorphic ones. The remaining residual term of the inequality defines the classical local volume dissipation. It is interesting to note that this dissipation is perfectly local and has the same form as the one resulting for the materially simple continua theory. This comes out from the hypothesis that the micromorphic variables do not dissipate due to the lack of any relative experimental information on the evolution of these variables. However, an additional decomposition of the micromorphic rate variables in reversible and irreversible parts allows overcoming this drawback under the current situation.

This procedure of the theory of generalized continuum mechanics will be used throughout this section in order to extend the constitutive equations to account for the damage induced softening of the micromorphic continua.

1.4.1 Choice of the state variables

Based on the local state method (Germain, 1973) we use state variables for which the values at each instant *t* and in each material point determine the material response. This approach is quite suitable for the formulation of the constitutive equations for a deformable solid with several dissipative phenomena. Thus, each dissipative phenomenon has its own state variable the evolution of which is governed by its own evolution equation. In this thesis, we will limit ourselves to the formulation of constitutive equations for deformable solids, using exclusively the local state method, which is the method best adapted to the modeling of deformable solids in large plasticity accounting for many dissipative phenomena.

By restricting ourselves to exclusively isothermal conditions and with the provision of virtual metal forming applications, for an elastoplastic deformable solid, we assume the following hypotheses:

-H1: The reversible (elastic) strains remain small compared to the irreversible (plastic or viscoplastic) inelastic strains. In this case, we can consider rigorously the additive decomposition of the total deformation rates.

-H2: Volume variations generated by inelastic (or irreversible) strains are negligible. This hypothesis is acceptable as long as the microvoids remain small until the final rupture of the RVE.

-H3: The updated Lagrangian formulation is used for the development of the behavior model. We consider that the two successive configurations are very close and we make the hypothesis of the total incompressibility despite the presence of damage. These simplifying hypotheses allow us to consider that the density remains constant ($\rho = \rho_0$) and as a result, the Kirchhoff and the Cauchy stress tensors are almost identical. Note that taking into account the compressibility induced by the damage does not pose any particular difficulties (Chaboche et al, 2006; Saanouni, 2012).

-H4: Two types of non-linear hardenings are taken into account:

- the isotropic hardening that controls the variation in the radius of the elastic domain,
- the kinematic hardening that governs the translation of the elastic domain in the loading space.

-H5: The damage is of ductile nature. To simplify the coupling with damage, we consider that the ductile damage is developed under the form of small isotropic spherical cavities. The damage effect at the level of an RVE will then be introduced by means of a scalar variable $0 \le d \le d_c \approx 1$ such that if d = 0 the RVE will be considered as safe or undamaged while if d reaches a critical limit d_c , the RVE is supposed fully damaged.

Under these hypotheses, we have associated a pair of state variables to each physical phenomenon. Each pair consists of a strain-like variable together with its dual force or stress-like variable as given in **Table 1.1**.

Phenomenon	Internal Variable	Dual Variable
Observable state variables		
Mechanical variables	<u>E</u>	σ
	Eulerian total strain tensor	Cauchy stress tensor
Non observable state variables		
Plastic flow	$\underline{\mathcal{E}}^{e}$	σ
Kinematic hardening	<u> </u>	<u>X</u>
Isotropic hardening	r	R
Isotropic ductile damage	d	Y
	$\int \breve{r}$	$\int \breve{R}$
Micromorphic isotropic hardening	$\left(\vec{\nabla} \vec{r} \right)$	$\left\{ec{ec{R}} ight.$
Micromorphic isotronic damage	$\int \breve{d}$	$\int \breve{Y}$
Micromorphic isotropic damage	$\Big(\vec{ abla} \vec{d}$	$\left(ec{ec{Y}} ight)$

Table 1.1: State variables associated with phenomena field

These local and micromorphic (or nonlocal) state variables will be used inside the state potential in order to define the state relations as well as in the dissipation potential for obtaining the evolution equations of each physical local or micromorphic phenomenon.

1.4.2 Definition of the total energy equivalence hypothesis and effective state variables

In order to formulate a behavior model in the context of the thermodynamics of irreversible processes by following the local state method, we should ensure the continuity property of the concerned media. However, the presence of the damage, which can be represented by a more or less random distribution of microvoids, introduces discontinuities in the RVE. Extensive discussions in the literature propose to define an equivalent homogeneous "safe" RVE, without damage by a homogeneous transformation adapted of the damaged RVE. This fictive transformation leads to the definition of effective variables as a function of the actual state variables. Throughout all this work, the hypothesis of total energy equivalence was chosen to construct the set of effective state variables (Saanouni, 1994, 2012).

More precisely, at any time (t), an RVE in its real deformed and damaged configuration, and where the thermomechanical state, at this time, is defined by the set of pairs of state variables from Table 1.1; we associate an equivalent undamaged fictive configuration, the state of which is described by the effective state variables listed in Table 1.2, in such a manner that the total energy defined over both real and fictive configurations is the same. This hypothesis is indeed a generalization of the elastic energy equivalence hypothesis initially proposed by (Cordebois and Sidoroff, 1982).

We consider a damaged RVE whose mechanical state is described by the couples of the state variables: $(\underline{\varepsilon}^{e}, \underline{\sigma}), (\underline{\alpha}, \underline{X}), (r, R)$ and (d, Y) in the rotated configuration. We constract an equivalent homogeneous undamaged RVE the mechanical state of which is described by the effective variables: $(\underline{\tilde{\varepsilon}}^{e}, \underline{\tilde{\sigma}}), (\underline{\tilde{\alpha}}, \underline{\tilde{X}}), (\tilde{r}, \tilde{R})$ so that the total energy defined in the two configurations is the same.

In terms of the Helmhold's free energy, we assume that the total energy W is decomposed in the elastic energy W_e and the inelastic energies W_r and W_{α} associated to the isotropic and kinematic hardenings respectively.

$\left[W_{ela} = \frac{1}{2}\underline{\sigma}:\underline{\varepsilon}^e = \frac{1}{2}\underline{\tilde{\sigma}}:\underline{\tilde{\varepsilon}}^e$	
$\begin{cases} W_{kin} = \frac{1}{2} \underline{X} : \underline{\alpha} = \frac{1}{2} \underline{\tilde{X}} : \underline{\tilde{\alpha}} \end{cases}$	(1.10)
$W_{iso} = \frac{1}{2}Rr = \frac{1}{2}\tilde{R}\tilde{r}$	
$\begin{cases} W_{miso} = \frac{1}{2} \vec{R} \vec{r} = \frac{1}{2} \vec{\tilde{R}} \vec{\tilde{r}} \\ W_{miso}^{g} = \frac{1}{2} \vec{\tilde{R}} \cdot \vec{\nabla} \vec{r} = \frac{1}{2} \vec{\tilde{R}} \cdot \vec{\nabla} \vec{\tilde{r}} \end{cases}$	(1.11)



Table 1.2: Definition of the total energy equivalence hypothesis in strain space

In the isotropic damage case, the general relations between the state and the effective variables can be written under the following form:

$$\begin{cases} \tilde{\underline{\sigma}} = \frac{\underline{\sigma}}{g_{ela}(d)}, & \tilde{\underline{\varepsilon}}^{e} = (g_{ela}(d))\underline{\varepsilon}^{e} \\ \\ \tilde{\underline{X}} = \frac{\underline{X}}{g_{kin}(d)}, & \tilde{\underline{\alpha}} = (g_{kin}(d))\underline{\alpha} \\ \\ \tilde{R} = \frac{R}{g_{iso}(d)}, & \tilde{r} = (g_{iso}(d))r \end{cases}$$
(1.12)

$$\begin{cases} \tilde{\vec{R}} = \frac{\vec{R}}{g_{miso}(d)}, & \tilde{\vec{r}} = g_{miso}(d)\vec{r} \\ \\ \tilde{\vec{R}} = \frac{\vec{\tilde{R}}}{g_{miso}(d)} & \nabla \tilde{\vec{r}} = g_{miso}(d)\nabla \vec{r} \end{cases}$$
(1.13)

There are many possible options for the choice of the functions $g_{el}(d)$, $g_{cin}(d)$ and $g_{iso}(d)$. These functions are positive and decreasing with respect to the scalar isotropic damage variable d; they tend to unity when d = 0 and to zero if d approaches its critical value d_c , generally close to 1.

In order to justify our choice, we consider the following hypotheses:

- The damage effect on the elastic behavior and the kinematic hardening are $g_{el}(d) = g_{kin}(d) = \sqrt{1-d}$ in order to simplify our numerical developments.
- The coupling function associated to the isotropic hardening is treated differently: $g_{iso}(d) \neq g_{el}(d)$
- The coupling function associated to the micromorphic isotropic hardening is treated with the approximately the same function of local isotropic hardening :

For defining the damage-isotropic hardening coupling, we choose: $g_{iso}(d) = \sqrt{1-d^{\gamma}}$. The parameter γ is the material parameter governing the effect of the damage on the isotropic hardening to be defined for each material type. The micromorphic isotropic hardening damage function is $g_{miso}(d) = \sqrt{1-\theta_r}d^{\gamma}$. The value of the parameter θ_r activates $(\theta_r = 1)$ or deactivates $(\theta_r = 0)$ the coupling with the local damage variable.

1.4.3 State potential and state relations

1.4.3.1 Thermodynamics of irreversible processes

By assuming the isothermal conditions, the local form of the Clausius-Duhem inequality, derived by combining the first and the second principles of thermodynamics, is enriched with the micromorphic variables:

$$\left(\underline{\sigma}:\underline{D}+\breve{Y}\dot{\vec{d}}+\breve{R}\dot{\vec{r}}+\breve{\breve{Y}}\cdot\vec{\nabla}\dot{\vec{d}}+\breve{\vec{R}}\cdot\vec{\nabla}\dot{\vec{r}}\right)-\rho\dot{\psi}\geq0$$
(1.14)

where, ψ is the specific Helmholtz free energy for a micromorphic solid to be chosen as the state potential. As in local continuum mechanics, the state potential is assumed to be a closed convex function of the overall classical local strain-like state variables (elastic strain $\underline{\varepsilon}^e$, kinematic hardening strain $\underline{\alpha}$, isotropic hardening strain r and the local damage d) as well as the micromorphic variables variables \overline{d} and \overline{r} along with their respective first gradients $\nabla \overline{d}$ and $\nabla \overline{r}$. Accordingly, the time derivative of the specific Helmholtz free energy $\rho \psi (\underline{\varepsilon}^e, d, r, \underline{\alpha}, \overline{d}, \overline{r}, \nabla \overline{d}, \nabla \overline{r})$ is given by:

$$\dot{\psi} = \frac{\partial \psi}{\partial \underline{\varepsilon}^{e}} : \underline{\dot{\varepsilon}}^{e} + \frac{\partial \psi}{\partial d} \dot{d} + \frac{\partial \psi}{\partial r} \dot{r} + \frac{\partial \psi}{\partial \underline{\alpha}} : \underline{\dot{\alpha}} + \frac{\partial \psi}{\partial \overline{d}} \dot{\vec{d}} + \frac{\partial \psi}{\partial \overline{r}} \dot{\vec{r}} + \frac{\partial \psi}{\partial \overline{\nabla} \overline{d}} \cdot \vec{\nabla} \dot{\vec{d}} + \frac{\partial \psi}{\partial \overline{\nabla} \overline{r}} \cdot \vec{\nabla} \dot{\vec{r}}$$
(1.15)

By substituting Eq.(1.15) into Eq.(1.14) and assuming that the total strain rate tensor decomposes according to the additive decomposition of total strain rate $\underline{D} = \underline{\dot{\varepsilon}}^{eJ} + \underline{D}^{p}$ ($\underline{\dot{\varepsilon}}^{eJ}$ being the Jaumann derivative of the small elastic strain tensor) and under the assumption that the micromorphic variables do not dissipate:

$$\left(\underline{\sigma} - \rho \frac{\partial \psi}{\partial \underline{\varepsilon}^{e}}\right) : \underline{\dot{\varepsilon}}^{e} + \left(\overline{Y} - \rho \frac{\partial \psi}{\partial \overline{d}}\right) \dot{\overline{d}} + \left(\overline{R} - \rho \frac{\partial \psi}{\partial \overline{r}}\right) \dot{\overline{r}}
+ \left(\overline{Y} - \rho \frac{\partial \psi}{\partial \overline{\nabla} \overline{d}}\right) \cdot \overline{\nabla} \dot{\overline{d}} + \left(\overline{R} - \rho \frac{\partial \psi}{\partial \overline{\nabla} \overline{r}}\right) \cdot \overline{\nabla} \dot{\overline{r}}
+ \underline{\sigma} : \underline{D}^{p} - \left(\rho \frac{\partial \psi}{\partial d} \dot{d} + \rho \frac{\partial \psi}{\partial r} \dot{r} + \rho \frac{\partial \psi}{\partial \underline{\alpha}} : \underline{\dot{\alpha}}\right) \ge 0$$
(1.16)

For the sake of simplicity, we assume that the five terms $\left(\underline{\sigma} - \rho \frac{\partial \psi}{\partial \underline{\varepsilon}^{e}}\right), \left(\overline{Y} - \rho \frac{\partial \psi}{\partial \overline{d}}\right),$

 $\left(\vec{R} - \rho \frac{\partial \psi}{\partial \vec{r}}\right), \left(\vec{Y} - \rho \frac{\partial \psi}{\partial \vec{\nabla} \vec{d}}\right)$ and $\left(\vec{R} - \rho \frac{\partial \psi}{\partial \vec{\nabla} \vec{r}}\right)$ do not depend on their rates respectively, as

well as that the micromorphic variables do not dissipate. Consequently, by following the standard arguments, we retrieve the following state relations together with the local residual dissipations:

The classical local state relations:

$$\underline{\sigma} = \rho \frac{\partial \psi}{\partial \underline{\varepsilon}^{e}}(a), \quad \underline{X} = \rho \frac{\partial \psi}{\partial \underline{\alpha}}(b), \quad R = \rho \frac{\partial \psi}{\partial r}(c), \quad Y = -\rho \frac{\partial \psi}{\partial d}(d)$$
(1.17)

The micromorphic state relations:

$$\vec{Y} = \rho \frac{\partial \psi}{\partial \vec{d}}, (a) \quad \vec{R} = \rho \frac{\partial \psi}{\partial \vec{r}} (b)$$

$$\vec{\tilde{Y}} = \rho \frac{\partial \psi}{\partial \vec{\nabla} \vec{d}} (c), \quad \vec{\tilde{R}} = \rho \frac{\partial \psi}{\partial \vec{\nabla} \vec{r}} (d)$$
(1.18)

The classical local intrinsic dissipation (note that the micromorphic damage is assumed with no intrinsic dissipation (Saanouni, 2012; Saanouni and Hamed, 2013)):

$$\Phi^{in} = \underline{\sigma} : \underline{D}^p - \underline{X} : \underline{\dot{\alpha}} - R\dot{r} + Yd \ge 0 \tag{1.19}$$

The stress-like variables $\underline{\sigma}$, Y, R, \underline{X} are given by the state relations (Eq.(1.17)). Their associated rates of strain-like variables: \underline{D}^p , \dot{d} , \dot{r} , $\underline{\dot{\alpha}}$ have to be deduced from an appropriate local yield function together with a local dissipation potential. These are defined in the effective rotated configuration as closed and convex scalar-valued functions of the associated stress-like variables in the effective stress space according to the classical thermodynamics of irreversible processes. We assume here that the plastic flow governs all the macroscopic dissipative phenomena, so a single surface formulation is used with a unified yield function $f(\underline{\sigma}, R, \underline{X}; d)$ and a plastic potential $F(\underline{\sigma}, R, \underline{X}, Y; d)$ from which all the local rates of the strain-like variables are deduced based on the generalized normality rule (see Saanouni, 2012 among others):

$$\underline{D}^{p} = \dot{\lambda} \frac{\partial F}{\partial \underline{\sigma}} = \dot{\lambda} \frac{\partial f}{\partial \underline{\sigma}} (a), \quad \dot{r} = -\dot{\lambda} \frac{\partial F}{\partial R} (b), \quad \underline{\dot{\alpha}} = -\dot{\lambda} \frac{\partial F}{\partial \underline{X}} (c), \quad \dot{d} = \dot{\lambda} \frac{\partial F}{\partial Y} (d)$$
(1.20)

where the plastic multiplier $\dot{\lambda}$ is a positive Lagrange multiplier derived from the consistency condition with respect to the yield function.

1.4.3.2 Choice of state potential and state relations

Similar to the local formulation. we choose Helmholtz the free energy $\rho\psi(\underline{\varepsilon}^{e}, d, r, \underline{\alpha}, \overline{d}, \overline{r}, \overline{\nabla}\overline{d}, \overline{\nabla}\overline{r})$ to be our state potential written in the fictive rotated configuration as a closed convex function of all local and micromorphic arguments defined in the effective strain space. On the other hand, the micromorphic state variables are assumed to contribute to the Helmholtz free energy in terms of the relative difference with respect to the local variables of the same nature. This introduces a coupling between macro and micromorphic variables (Forest, 2009).

Under these assumptions, and with the ability divide the generalized potential into two terms by an additive decomposition, the Helmholtz free energy can be written under the following form:

$$\rho \psi_{g} = \rho \psi \left(\underline{\tilde{\varepsilon}}^{e}, \underline{\tilde{\alpha}}, \overline{\tilde{r}} \right) + \rho \psi_{m} \left(\overline{d}, \overline{\tilde{r}}, \overline{\nabla} \overline{d}, \overline{\nabla} \overline{\tilde{r}}; d \right)$$
(1.21)

with,

$$\rho \psi = \frac{1}{2} \underline{\tilde{\varepsilon}}^{e} : \underline{\tilde{\Delta}} : \underline{\tilde{\varepsilon}}^{e} + \frac{1}{3} C \underline{\tilde{\alpha}} : \underline{\tilde{\alpha}} + \frac{1}{2} Q \tilde{r}^{2}
= (1-d) \frac{1}{2} \underline{\varepsilon}^{e} : \underline{\tilde{\Delta}} : \underline{\varepsilon}^{e} + \frac{1}{3} (1-d) C \underline{\alpha} : \underline{\alpha} + \frac{1}{2} (1-d^{\gamma}) Q r^{2}$$
(1.22)

and

$$\rho \psi_m = \frac{1}{2} \vec{Q} \left(\sqrt{1 - d^\gamma} r - \sqrt{1 - \theta_r d^\gamma} \vec{r} \right)^2 + \frac{1}{2} \left(1 - \theta_r d^\gamma \right) \vec{Q}^s \vec{\nabla} \vec{r} \cdot \vec{\nabla} \vec{r} + \frac{1}{2} \vec{H} \left(d - \vec{d} \right)^2 + \frac{1}{2} \vec{H}^s \vec{\nabla} \vec{d} \cdot \vec{\nabla} \vec{d}$$
(1.23)

where, $\underline{\Lambda} = \lambda_e 1 \otimes \underline{1} + 2\mu_e \underline{1}$ is the positive definite and symmetric forth-rank tensor of the elastic moduli, C and Q are the kinematic and isotropic hardening macro moduli, \overline{Q} and \overline{H} are the coupling moduli with respect to the isotropic hardening and the damage respectively while \overline{Q}^g and \overline{H}^g are the micromorphic moduli related to the first gradients of the isotropic hardening and damage respectively (all assumed to be positive or zero) while the parameter γ is the parameter defining the coupling between the damage and the isotropic hardening.

Using the local and micromorphic state relations in Eq.(1.17) and Eq.(1.18) with the state potential (1.21), we obtain the local the stress-like variables that are associated with all strain-like variables:

Cauchy stress tensor:

$$\underline{\sigma} = \rho \frac{\partial \psi_g}{\partial \underline{\varepsilon}^e} = (1 - d) \underline{\underline{\Lambda}} : \underline{\varepsilon}^e = (1 - d) \left(\lambda_e tr(\underline{\varepsilon}^e) \underline{1} + 2\mu_e \underline{\varepsilon}^e \right)$$
(1.24)

Kinematic hardening stress:

$$\underline{X} = \rho \frac{\partial \Psi_s}{\partial \underline{\alpha}} = \frac{2}{3} (1 - d) C \underline{\alpha}$$
(1.25)

- Isotropic hardening stress:

$$R = \rho \frac{\partial \psi_g}{\partial r} = (1 - d^{\gamma})Qr + \breve{Q} \left[(1 - d^{\gamma})r - \sqrt{1 - d^{\gamma}}\sqrt{1 - \theta_r d^{\gamma}} \breve{r} \right]$$
(1.26)

- Damage stress:

$$Y = -\rho \frac{\partial \psi_g}{\partial d} = Y_E + Y_A + Y_R + Y_d$$
(1.27)

with,

$$\begin{cases} Y_{E} = \frac{1}{2} \underline{\varepsilon}^{e} : \underline{\Lambda} : \underline{\varepsilon}^{e} & (a) \\ Y_{A} = \frac{1}{3} C \underline{\alpha} : \underline{\alpha} & (b) \\ Y_{R} = \frac{1}{2} \gamma d^{\gamma - 1} Q r^{2} + \frac{1}{2} \gamma d^{\gamma - 1} \overline{Q} \left(\sqrt{1 - d^{\gamma}} r - \sqrt{1 - \theta_{r}} d^{\gamma} \overline{r} \right) \left(\frac{r}{\sqrt{1 - d^{\gamma}}} - \theta_{r} \frac{\overline{r}}{\sqrt{1 - \theta_{r}} d^{\gamma}} \right) \\ + \frac{1}{2} \theta_{r} \gamma d^{\gamma - 1} \overline{Q}^{g} \overline{\nabla} \overline{r} \cdot \overline{\nabla} \overline{r} \quad (c) \\ Y_{d} = -\overline{H} \left(d - \overline{d} \right) & (d) \end{cases}$$

$$(1.28)$$

- Stress-like variables of micromorphic damage and its first gradient:

$$\vec{Y} = \rho \frac{\partial \psi}{\partial \vec{d}} = -\vec{H} \left(d - \vec{d} \right) \quad (a)$$

$$\vec{\vec{Y}} = \rho \frac{\partial \psi}{\partial \vec{\nabla} \vec{d}} = \vec{H}^{s} \vec{\nabla} \vec{d} \qquad (b)$$
(1.29)

- Stress-like variables of micromorphic isotropic hardening and its first gradient:

$$\vec{R} = \rho \frac{\partial \psi_g}{\partial \vec{r}} = -\vec{Q} \left(\sqrt{1 - d^\gamma} \sqrt{1 - \theta_r d^\gamma} r - (1 - \theta_r d^\gamma) \vec{r} \right) \quad (a)$$

$$\vec{R} = \rho \frac{\partial \psi_g}{\partial \vec{\nabla} \vec{r}} = (1 - \theta_r d^\gamma) \vec{Q}^g (\vec{\nabla} \vec{r}) \qquad (b)$$

At this point, it is interesting to note that the state relations Eq.(1.24)-Eq.(1.30) can be decomposed into classical local and the extended nonlocal parts with the help of the micromorphic state relations as given in Eq.(1.33) and Eq(1.34) below:

$$\underline{\sigma} = \underline{\sigma}_{loc} = (1 - d) \underline{\underline{\Lambda}} : \underline{\underline{\varepsilon}}^{e}$$
(1.31)

$$\underline{X} = \underline{X}_{loc} = \frac{2}{3} (1 - d) C \underline{\alpha}$$
(1.32)

$$\begin{cases} Y = Y_{loc} + Y_{nloc} \\ Y_{loc} = \frac{1}{2} \underline{\varepsilon}^{e} : \underline{\Lambda} : \underline{\varepsilon}^{e} + \frac{1}{3} C \underline{\alpha} : \underline{\alpha} + \frac{1}{2} Q r^{2} \\ Y_{nloc} = \overline{Y} + \frac{1}{2} \gamma d^{\gamma - 1} \overline{Q} \Big(\sqrt{1 - d^{\gamma}} r - \sqrt{1 - \theta_{r} d^{\gamma}} \overline{r} \Big) \Big(\frac{r}{\sqrt{1 - \theta_{r} d^{\gamma}}} - \theta_{r} \frac{\overline{r}}{\sqrt{1 - \theta_{r} d^{\gamma}}} \Big) + \frac{1}{2} \theta_{r} \gamma d^{\gamma - 1} \overline{Q}^{g} \overline{\nabla} \overline{r} \cdot \overline{\nabla} \overline{r} \end{cases}$$
(1.33)

$$\begin{cases} R = R_{loc} + R_{nloc} \\ R_{loc} = (1 - d^{\gamma})Qr \\ R_{nloc} = \vec{Q} \left((1 - d^{\gamma})r - \sqrt{1 - d^{\gamma}}\sqrt{1 - \theta_r d^{\gamma}}\vec{r} \right) \end{cases}$$
(1.34)

It is clear that the Cauchy stress Eq.(1.31) and the kinematic hardening stress tensor Eq.(1.32) remain under a purely local form while the other two thermodynamic forces associated to damage Eq.(1.33) and the isotropic hardening Eq.(1.34) are the sums of the classical local (indicated as *loc*) and and the nonlocal micromorphic contributions (indicated as *nloc*) resulting from the micromorphic state variables. The micromorphic isotropic hardening \vec{R} in Eq.(1.30a) affects the stress force R Eq.(1.26) and equally the thermodynamic damage force Y Eq. (1.27) as shown clearly in Eq.(1.28c).

1.4.3.3 Local state potential and state relations

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In the absence of any micromorphic fields, the differential form of the first law of thermodynamics transforms to the traditional one:

$$-\rho \dot{e} + (\underline{\sigma} : \underline{\dot{e}}) = 0 \tag{1.35}$$

Consequently, the Helmholtz free energy $\psi(\underline{\varepsilon}^e, d, r, \underline{\alpha})$ reduces to a closed convex function of the classical local state variables: $\underline{\varepsilon}^e, d, r$ and $\underline{\alpha}$.

Using the second law of thermodynamics and the above equations, the following classical state relations are obtained:

$$\underline{\sigma} = \rho \frac{\partial \psi}{\partial \underline{\varepsilon}^{e}} \quad (a)$$

$$Y = -\rho \frac{\partial \psi}{\partial d} \quad (b) \quad R = \rho \frac{\partial \psi}{\partial r} \quad (c) \quad \underline{X} = \rho \frac{\partial \psi}{\partial \underline{\alpha}} \quad (d)$$
(1.36)

along with the local dissipations:

 $\Phi = \Phi^{in} = \underline{\sigma} : \underline{D}^{p} + Y\dot{d} - R\dot{r} - \underline{X} : \dot{\underline{\alpha}} \ge 0$ (1.37)

By setting the micromorphic moduli equal to zero ($\vec{Q}=0, \vec{H}=0, \vec{Q}^s=0, and \vec{H}^s=0$), the generalized Helmholtz free energy that we had already chosen as the generalized state potential in Eq.(1.21), reduces to its local form:

$$\rho \psi_{g} = \rho \psi \left(\underline{\tilde{\varepsilon}}^{e}, \underline{\tilde{\alpha}}, \tilde{r} \right)$$
(1.38)

Using the state relations in Eq.(1.17), the classical local stress-like variables are obtained as follows:

- Cauchy stress tensor (always under a purely local form):

$$\underline{\sigma} = \rho \frac{\partial \psi}{\partial \underline{\varepsilon}^{e}} = (1 - d) \underline{\Lambda} : \underline{\varepsilon}^{e} = (1 - d) \left(\lambda_{e} tr(\underline{\varepsilon}^{e}) \underline{1} + 2\mu_{e} \underline{\varepsilon}^{e} \right)$$
(1.39)

Damage stress force:

$$Y = -\rho \frac{\partial \psi}{\partial d} = \frac{1}{2} \underline{\varepsilon}^{e} : \underline{\Lambda} : \underline{\varepsilon}^{e} + \frac{1}{3} \underline{\alpha} : \underline{C} : \underline{\alpha} + \frac{1}{2} \gamma d^{\gamma - 1} Q r^{2}$$
(1.40)

Isotropic hardening stress force:

$$R = \rho \frac{\partial \psi}{\partial r} = \tilde{Q}r = (1 - d^{\gamma})Qr$$
(1.41)

- Kinematic hardening stress force:

$$\underline{X} = \rho \frac{\partial \psi}{\partial \underline{\alpha}} = \underline{\tilde{C}} : \underline{\alpha} = \frac{2}{3} (1 - d) \underline{\underline{C}} : \underline{\alpha}$$
(1.42)

1.4.4 The effect of the micro-cracks closure

Another phenomenon that we have introduced is the notion of microcracks closure. When an RVE undergoes stress which leads to the formation of microdefects in tension, the physical properties and the behavior of this RVE are affected by the presence of induced softening. However, during the unloading and progressive transformation into the compressive phase, the micro-cracks close progressively until complete closure if the compression force is sufficient. Two main consequences arise during the compressive phase of the loading path:

- a) The damage rate will be much lower, if not zero, when the previous microcracks tend to close in compression,
- b) The physical properties of the damage material and mainly the moduli of elasticity and hardening, tend to return to their initial values before the damage occurrence when the micro-cracks are fully closed in compression.

The point (b) is not an easy task because it introduces a loss of continuity and/or convexity of the state and dissipation potentials. This aspect will not be considered in this work and the physical properties of the damaged material will be affected irreversibly by the damage even when all the

microcracks are closed in compression. However, the fact that the damage evolves more slowly (or not at all) in compression than in tension will be considered following the simple approach presented in (Ladeveze, 1984; Lemaitre, 1985; Desmorat, 2007; Pirondi, 2006; Saanouni, 2012). This simple approach, useful only with the isotropic damage, will be summarized here after. It preserves the continuity and the convexity of the state and dissipation potentials while giving a different damage rate under positive (tension) and negative (compression) loading paths.

For the isotropic damage case, we could deal with this effect by decomposing the effective state variables into positive and negative parts (Challamel, 2005, 2010; Murakami and Kamiya, 1997) with the help of the spectral decomposition of any symmetric second-rank tensor \underline{T} in the form:

$$\underline{T} = \langle \underline{T} \rangle_{+} + \langle \underline{T} \rangle_{-}$$

$$\langle \underline{T} \rangle_{+} = \sum_{i=1}^{3} \langle T_{i} \rangle \vec{e}_{i} \otimes \vec{e}_{i}$$
(1.43)

where, T_i and \vec{e}_i are the three eigenvalues and corresponding eigenvectors of the tensor \underline{T} and $\langle T_i \rangle$ is the positive part of T_i defined by: $\langle T_i \rangle = \max(0, T_i)$. These positive and negative parts of the second-rank tensor \underline{T} verify the following orthogonality and differentiability properties: $\langle \underline{T} \rangle_- : \langle \underline{T} \rangle_+ = \langle \underline{T} \rangle_+ : \langle \underline{T} \rangle_- = 0$ $d(\langle \underline{T} \rangle_+ : \langle \underline{T} \rangle_+) = 2 \langle \underline{T} \rangle_+ : d\underline{T}$ (1.44) $d(\langle \underline{T} \rangle_- : \langle \underline{T} \rangle_-) = 2 \langle \underline{T} \rangle_- : d\underline{T}$

Then the following derivatives of the positive and negative parts of the tensor \underline{T} can easily be obtained:

$$\frac{\partial \langle \underline{T} \rangle_{+}}{\partial \underline{T}} = \frac{\partial \langle T_{ij} \rangle_{+}}{\partial T_{kl}} = \begin{cases} \delta_{ik} \delta_{jl} & \text{if } T_{i} > 0 \\ 0 & \text{if } T_{i} < 0 \end{cases} = \langle \underline{1} \rangle_{+} \\
\frac{\partial \langle \underline{T} \rangle_{-}}{\partial \underline{T}} = \frac{\partial \langle T_{ij} \rangle_{-}}{\partial T_{kl}} = \begin{cases} \delta_{ik} \delta_{jl} & \text{if } T_{i} < 0 \\ 0 & \text{if } T_{i} > 0 \end{cases} = \langle \underline{1} \rangle_{-} \\
\frac{\partial \langle \underline{T} : \underline{1} \rangle_{+}}{\partial \underline{T}} = \frac{\partial \langle T_{ij} \delta_{ij} \rangle_{+}}{\partial T_{kl}} = \begin{cases} \left(\delta_{ik} \delta_{jl} \right) \delta_{ij} & \text{if } T_{i} > 0 \\ 0 & \text{if } T_{i} < 0 \end{cases} = \langle \underline{1} \rangle_{+} \\
0 & \text{if } T_{i} < 0 \end{cases} = \langle \underline{1} \rangle_{+} \\
\frac{\partial \langle \underline{T} : \underline{1} \rangle_{-}}{\partial \underline{T}} = \frac{\partial \langle T_{ij} \delta_{ij} \rangle_{-}}{\partial T_{kl}} = \begin{cases} \left(\delta_{ik} \delta_{jl} \right) \delta_{ij} & \text{if } T_{i} < 0 \\ 0 & \text{if } T_{i} < 0 \end{cases} = \langle \underline{1} \rangle_{-} \\
0 & \text{if } T_{i} < 0 \end{cases} = \langle \underline{1} \rangle_{-}$$
(1.45)

We wish to consider the microcracks closure effect only on the damage evolution. More complex formulations can be found in literature (Issa, 2010; Issa et al., 2012; Saanouni, 2012; Yue, 2014). To achieve this, we do not change the state relations but we introduce a new energy density release rate noted as:

$$Y^* = Y^{e^*} + \left[\left\langle Sign(tr(\underline{\varepsilon}^e)) \right\rangle + \left\langle -\hbar \ Sign(tr(\underline{\varepsilon}^e)) \right\rangle \right] (Y_A + Y_R + Y_d)$$
(1.46)

with respect to which the damage rate is derived and the energy density release rate Y^* is used instead of Y in the damage evolution. The new form of Y^{e^*} shall be derived from an appropriate form of the state potential accounting for the spectral decomposition of the effective small elastic strain tensor $\underline{\tilde{\varepsilon}}^e = \sqrt{1-d} \ \underline{\varepsilon}^e_+ + \sqrt{1-\hbar d} \ \underline{\varepsilon}^e$ for the elastic part of energy density release rate Y^{e^*} given in the following form:

$$Y^{e^*} = Y^{e^*}_+ + Y^{e^*}_-$$

= $\mu_e \underline{\varepsilon}^e_+ : \underline{\varepsilon}^e_+ + 3\lambda_e \left\langle tr(\underline{\varepsilon}^e) \right\rangle^2 + \hbar \left[\mu_e \underline{\varepsilon}^e_- : \underline{\varepsilon}^e_- + 3\lambda_e \left\langle -tr(\underline{\varepsilon}^e_-) \right\rangle^2 \right]$ (1.47)

where the parameter \hbar ($0 \le \hbar \le 1$) is related to the microcracks closure effect and allows reducing the damage rate under the negative parts of the applied load. The tensors $\underline{\varepsilon}_{+}^{e}$ and $\underline{\varepsilon}_{-}^{e}$ are the positive and negative parts of the small elastic strain tensor. Also, the new form of energy density release rate induces a sensibility to the micro-cracks closure effect of the hardening parts (Y_A, Y_R) as well as the micromorphic part (\breve{Y}) through the function $\left[\left\langle Sign(tr(\underline{\varepsilon}^{e}))\right\rangle + \left\langle -\hbar Sign(tr(\underline{\varepsilon}^{e}))\right\rangle\right]$ which takes the value 1 if the hydrostatic strain is positive $(tr(\varepsilon^e) > 0)$ and the value \hbar if the hydrostatic stress is negative $(tr(\varepsilon^e) < 0)$. The value of this parameter is usually taken equal to $\hbar = 0.2$ (Lemaitre, 1985; Krajcinovic, 1985), leading to a lower damage growth under the compressive phase of the loading path, with two extreme cases: (i) $\hbar = 1$ makes no difference between the damage growth under positive and negative loading paths and, (ii) $\hbar = 0$ for which the damage growth takes place only under the positive part of the applied loading path.

1.4.5 Intrinsic dissipation analysis for time independent plasticity

In section 1.4.3.2, we introduced the form of the state potential in the strain space, from which we have deduced the state relations, based on the Clausius–Duhem inequality. We now need to analyze the different dissipative phenomena, in order to define their evolution equations with the help of an appropriate yield function and a dissipation potential by following the local state method. We limit ourselves to the case where the micromorphic variables do not dissipate, which leads to the classical local dissipation defined in Eq.(1.19).

Since the force-like variables $(\underline{\sigma}, R, \underline{X}, Y)$ are given by the state relations Eq.(1.24)-Eq.(1.27) above, the associated flux variables $(\underline{D}^p, \dot{r}, \dot{\underline{\alpha}}, \dot{d})$ will result from the application of the generalized normality rule and the maximum volumic dissipation analysis. Of course, the micromorphic strain-like variables (\breve{r}, \breve{d}) are nothing but the micromorphic dofs, solutions of the two partial differential equations Eq.(1.8b) and Eq.(1.8c). By focusing our attention on the isothermal case for the isotropic elastoplastic and incompressible solids, we limit ourselves to the single yield surface for both plasticity and damage (choice of two yield surfaces can be found in Saanouni, 2012).

1.4.5.1 Plastic potential and evolution equations

To account for the damage effects, and limiting ourselves to the isotropic plastic flow for the sake of simplicity, the von Mises equivalent stress in both the yield function and the plastic potential is chosen. In this case, the micromorphic variables \tilde{R} and \tilde{Y} are included in the yield function and the plastic potential via the state variables R (Eq.(1.26)) and Y (Eq.(1.27)). The von Mises yield function and the plastic dissipation potential taken from (Saanouni et al., 1994; Saanouni, 2012) are then written under the following forms:

$$f_{p}\left(\underline{\sigma}, \underline{X}, R, d\right) = \frac{\left\|\underline{\sigma} - \underline{X}\right\|}{\sqrt{1-d}} - \frac{R}{\sqrt{1-d^{\gamma}}} - \sigma_{y}$$
(1.48)

$$F_{p}(\underline{\sigma}, \underline{X}, R, d) = f_{p} + \frac{3}{4} \frac{a(\underline{X}:\underline{X})}{C(1-d)} + \frac{1}{2} \frac{bR^{2}}{Q(1-d^{\gamma})} + \frac{S}{(s+1)(1-d)^{\beta}} \left\langle \frac{Y-Y_{0}}{S} \right\rangle^{s+1}$$
(1.49)

with the norm $\|\underline{\sigma} - \underline{X}\| = \sqrt{\frac{3}{2}}(\underline{\sigma} - \underline{X}):(\underline{\sigma} - \underline{X})$ defining the von Mises equivalent stress, while σ_y is the initial size of the plastic yield surface. The parameters a and b in Eq. (1.49) characterize the nonlinearity of the kinematic and isotropic respectively and finally, S, s, β and Y_0 characterize the nonlinear evolution of the ductile damage. It is worth noting that, despite the fact that Eq.(1.48) and Eq.(1.49) are identical to the classical local theory, the indirect contribution of the micromorphic state variables is taken into consideration since the state variables Y and R carry the micromorphic effects as shown by Eq.(1.33) and Eq.(1.34).

Applying the generalized normality rule to the local yield function (Saanouni and Chaboche, 2003; Saanouni, 2012; Forest 2016) and dissipation potential above, leads to the following evolution equations:

Plastic strain rate

$$\underline{D}^{p} = \dot{\lambda} \frac{\partial F_{p}}{\partial \underline{\sigma}} = \dot{\lambda} \frac{\underline{\tilde{n}}}{\sqrt{1-d}}, \quad with \qquad \underline{\tilde{n}} = \frac{3}{2} \frac{(\underline{\sigma}^{dev} - \underline{X})}{\|\underline{\sigma} - \underline{X}\|}$$
(1.50)

where, \tilde{n} is the outside normal to the yield surface and λ is the Lagrange plastic multiplier.

- Kinematic hardening strain rate

$$\dot{\underline{\alpha}} = -\dot{\lambda} \frac{\partial F_p}{\partial \underline{X}} = \underline{D}^p - a\dot{\underline{\lambda}}\underline{\alpha}$$
(1.51)

- Isotropic hardening strain rate

$$\dot{r} = -\dot{\lambda} \frac{\partial F_p}{\partial R} = \dot{\lambda} \left(\frac{1}{\sqrt{1 - d^{\lambda}}} - b \frac{R}{Q(1 - d^{\gamma})} \right) = \dot{\lambda} \left(\frac{1}{\sqrt{1 - d^{\gamma}}} - br - b \frac{\overline{Q}}{Q(1 - d^{\gamma})} \left((1 - d^{\gamma})r - \sqrt{1 - d^{\gamma}} \sqrt{1 - \theta_r d^{\gamma}} \overline{r} \right) \right)$$
(1.52)
Isotropic damage rate

$$\dot{d} = \dot{\lambda} \frac{\partial F_p}{\partial Y} = \frac{\dot{\lambda}}{(1-d)^{\beta}} \left\langle \frac{(Y_{loc} + Y_{nloc}) - Y_0}{S} \right\rangle^s$$
(1.53)

with the Macauley brackets $\langle x \rangle$ indicating the positive part of x.

We can note here that, if the scalar parameters a and b are zero, the kinematic and isotropic hardening evolution equations (Eq.(1.51) and Eq(1.52)) will retrieve their linear forms.

We recall that in Eq.(1.27) the thermodynamic force Y contains the micromorphic contributions as indicated by Eq.(1.33). We also note that Eq.(1.48) indicates that the evolution of isotropic hardening contains the classical local term together with a nonlocal contribution based on the micromorphic isotropic hardening variable.

As indicated in the paragraph about the micromorphic dissipation analysis, the assumption that the micromorphic variables do not dissipate leads to purely local intrinsic dissipation.

1.5 Transformation of the micromorphic balance equations to the strain space

The micromorphic balance equations (1.8b, 1.8c) are expressed in the stress space as functions of the micromorphic thermodynamic force variables. Going back to these equations, it is possible to transform them in the strain space as functions of the strain-like variables obtained in the previous paragraph. To achieve this, we substitute the micromorphic stress-like variables for the micromorphic ductile damage and the isotropic hardening as expressed in the equations Eq.(1.29) and Eq.(1.30) in the equations Eq.(1.8b) and Eq.(1.8c) respectively.

By noting Lap(X) being the Laplacian of X, we remark that as long as the micromorphic moduli \tilde{Q}^{g} and \tilde{H}^{g} remain constant, we obtain:

$$div(\vec{R}) = div(\vec{Q}^{g}\left(1 - \theta_{r}d^{\gamma}\right)\vec{\nabla}\vec{r}) = \vec{Q}^{g}\left(\left(1 - \theta_{r}d^{\gamma}\right)Lap(\vec{r}) - \theta_{r}\gamma d^{\gamma-1}\vec{\nabla}d.\vec{\nabla}\vec{r}\right)$$
(1.54)

$$div(\breve{Y}) = div(\breve{H}^{s} \vec{\nabla} \breve{d}) = \breve{H}^{s} Lap(\breve{d})$$
(1.55)

By substituting the micromorphic variables in Eq.(1.29) and Eq.(1.30) in the previously obtained balance equations, we get:

- The balance equation of the micromorphic isotropic hardening

$$\begin{cases} \bar{Q}^{g} \left(\left(1 - \theta_{r} d^{\gamma} \right) Lap(\bar{r}) - \theta_{r} \gamma d^{\gamma - 1} \vec{\nabla} d. \vec{\nabla} \vec{r} \right) + \bar{Q} \left(\sqrt{1 - d^{\gamma}} \sqrt{1 - \theta_{r} d^{\gamma}} r - \left(1 - \theta_{r} d^{\gamma} \right) \vec{r} \right) + \left(f^{\bar{r}} - \vec{\nabla} \cdot \vec{f}^{g\bar{r}} \right) = \rho \zeta_{\bar{r}} \ddot{\vec{r}} \quad in \Omega \\ \begin{bmatrix} \left(1 - \theta_{r} d^{\gamma} \right) \vec{Q}^{g} \vec{\nabla} \vec{r} - \rho \vec{f}^{g\bar{r}} \end{bmatrix} \cdot \vec{n} = F^{\bar{r}} & on \Gamma \end{cases}$$

$$(1.56)$$

- The balance equation of the micromorphic damage

$$\begin{cases} \vec{H}^{g} Lap(\vec{d}) + \vec{H}(d - \vec{d}) + (f^{\vec{d}} - \vec{\nabla} \cdot \vec{f}^{g\vec{d}}) = \rho \zeta_{\vec{d}} \vec{\vec{d}} & in \Omega \\ \left[\vec{H}^{g} (\vec{\nabla} \vec{d}) - \vec{f}^{g\vec{d}} \right] \cdot \vec{n} = F^{\vec{d}} & on \Gamma \end{cases}$$
(1.57)

For the sake of simplicity and without any experimental information, if the overall micromorphic $F^{\vec{d}}=F^{\vec{r}}=0$

body and contact forces are neglected: $f^{\breve{d}} = f^{\breve{r}}$

$$f^{a} = f^{r} = 0$$
(1.58)
$$\vec{f}^{g\bar{d}} = \vec{f}^{g\bar{r}} = \vec{0}$$

the above balance equations (Eq.(1.56) and Eq.(1.57)) can be rewritten in the strain space under the following form:

$$\begin{cases} \ell_{\vec{r}}^{2} \left(\left(1 - \theta_{r} d^{\gamma}\right) Lap(\vec{r}) - \theta_{r} \gamma d^{\gamma - 1} \vec{\nabla} d. \vec{\nabla} \vec{r} \right) + \left(\sqrt{1 - d^{\gamma}} \sqrt{1 - \theta_{r} d^{\gamma}} r - \left(1 - \theta_{r} d^{\gamma}\right) \vec{r} \right) = \rho \frac{\zeta_{\vec{r}} \vec{\vec{r}}}{\vec{Q}} \\ \vec{Q}^{g} \left(\vec{\nabla} \vec{r} \right) \cdot \vec{n} = 0 \end{cases}$$
(1.59)

$$\begin{cases} \ell_{\vec{a}}^{2} Lap(\vec{d}) + (d - \vec{d}) = \rho \frac{\zeta_{\vec{d}} \ddot{\vec{d}}}{\vec{H}} \\ \vec{H}^{g}(\vec{\nabla}\vec{d}) \cdot \vec{n} = 0 \end{cases}$$
(1.60)

where, $\ell_{\bar{r}}$ and $\ell_{\bar{d}}$ are the internal length scale parameters related to the micromorphic isotropic hardening and the micromorphic damage, defined by the ratio of the micromorphic moduli:

$$\ell_{\bar{r}}^{2} = \frac{\breve{Q}^{s}}{\breve{Q}} (a) \quad \ell_{\bar{a}}^{2} = \frac{\breve{H}^{s}}{\breve{H}} (b)$$
(1.61)

We prefer to use the Eq.(1.59) and Eq(1.60) as strong forms from which their associated weak forms will be deduced in order to solve the fully coupled IBVP as discussed in next chapter, instead of Eq.(1.8b) and Eq.(1.8c).

Let us mention that, if the micromorphic damage inertia is neglected (i.e., $\zeta_{\bar{d}} = 0$) from the micromorphic damage balance equation (Eq.(1.60)), we are leaded to the well-known Helmholtz equation proposed in the framework of the so called implicit nonlocal damage formulations as in Engelen et al. (2003), Geers et al. (2003), Geers (2004) and the framework of the so-called gradient-enhanced models (Engelen et al., 2003; Peerlings, 1999; Peerlings et al., 2001; Peerlings et al., 2004).

1.6 Extension to the finite plastic strains and the objectivity requirement

The constitutive equations presented above were formulated under the small strain assumption. We will now proceed to the extension of this model to the finite transformations in order to be able to simulate the metal forming applications under large strains. For this end, we will start by briefly mentioning the kinematics of finite transformations with an emphasis to the objectivity principle.

1.6.1 Kinematics of finite transformations

We consider a deformable solid S_1 of volume Ω_0 in its initial configuration C_0 at the time t_0 . At any instant t, the solid occupies a volume Ω_t in its actual configuration C_t . In an orthonormal Euclidian space, the initial position of a material point P_0 in C_0 of coordinates \vec{X} transforms into the point P_t of coordinates $\vec{x}(\vec{X},t)$ in C_t (see Fig. 1.3) by $\vec{x} = \vec{\phi}(\vec{X},t)$.



Figure 1.3: Kinematics of finite transformation (Saanouni, 2012)

To describe the deformation of the solid we introduce the vectorial field $\vec{\phi}(\vec{X})$ as:

$$\vec{\phi} : \vec{X} \Rightarrow \vec{\phi}(\vec{X}) = \vec{x}(\vec{X}, t) \tag{1.62}$$

The displacement and the velocity vectors of any material point in S_1 at time (*t*) are defined by: $\vec{u}(\vec{x},t) = \vec{x}(\vec{X},t) - \vec{X}$ and $\vec{v} = \frac{\partial \vec{u}}{\partial t}$ (1.63)

The gradient of the transformation $\vec{\phi}(\vec{X},t)$ is given by:

$$\underline{F} = \overrightarrow{Grad}(\vec{\phi}) = \frac{\partial \phi}{\partial \vec{X}} = \frac{\partial \vec{x}}{\partial \vec{X}} = \underline{1} + \frac{\partial \vec{u}}{\partial \vec{X}} = \underline{1} + \vec{\nabla}\vec{u}$$
(1.64)

According to the polar decomposition theorem, any homogeneous transformation can be seen as the product of a pure rotation and of a pure stretch (or elongation). This means that any non-singular

gradient of a homogeneous transformation \underline{F} can be multiplicatively decomposed, under the following form:

$$\underline{F} = \underline{R} \cdot \underline{U} = \underline{V} \cdot \underline{R} \tag{1.65}$$

where the symmetric and positive definite second-rank tensors \underline{U} and \underline{V} are called left and right pure stretch tensors, and \underline{R} is the rigid body orthogonal rotation tensor. \underline{U} is a Lagrangian tensor defined defined in C_0 while \underline{V} is purely Eulerian tensor, defined in C_t (Fig. 1.4).



Figure 1.4: Schematic representation of the polar decomposition (Saanouni, 2012)

To define the deformations of the solid, we introduce the Cauchy-Green tensors; the right one \underline{C} in C_0 and the left one \underline{B} in C_t . The scalar vector of two non-collinear transformed vectors in the configuration C_t is given by:

$$d\vec{x}^{T}.d\vec{x} = \left(d\vec{X}^{T}.\underline{F}^{T}\right).\left(\underline{F}.d\vec{X}\right) = d\vec{X}^{T}.\underline{C}.d\vec{X} \qquad \Rightarrow \underline{C} = \underline{F}^{T}.\underline{F}$$
(1.66)

$$d\vec{X}^{T}.d\vec{X} = \left(d\vec{x}^{T}.\underline{F}^{-T}\right).\underline{F}^{-1}.d\vec{x} = d\vec{x}^{T}.\underline{B}^{-1}.d\vec{x} \qquad \Rightarrow \underline{B} = \underline{F}.\underline{F}^{T}$$
(1.67)

Accordingly, the Lagrangian Green–Lagrange strain tensor \underline{E} defined in a point P_0 in C_0 and the Eulerian Euler–Almansi strain tensor \underline{A} defined in a point P_t in C_t are given by:

$$\underline{\mathbf{E}} = \frac{1}{2}(\underline{C} - \underline{I}) \qquad \underline{\mathbf{A}} = \frac{1}{2}(\underline{I} - \underline{B}^{-1}) \tag{1.68}$$

Moreover, the Eulerian velocity gradient in C_T is defined by:

$$\overrightarrow{grad}(\vec{v}) = \frac{\partial \vec{v}}{\partial \vec{x}} = \frac{\partial \vec{v}}{\partial \vec{X}} \cdot \frac{\partial \vec{X}}{\partial \vec{x}} = \underline{\dot{F}} \cdot \underline{F}^{-1} = \underline{L}$$
(1.69)

It is possible to decompose \underline{L} in a symmetric part noted as \underline{D} defining the local strain rates and an antisymmetric part noted as \underline{W} defining the local rotation rates:

$$\underline{D} = \frac{1}{2} (\underline{L} + \underline{L}^{T}) \qquad \underline{W} = \frac{1}{2} (\underline{L} - \underline{L}^{T})$$
(1.70)

The most widely used measure of the stress in a point of a continuum is the Cauchy stress, defined using the measure of the elementary internal force in a point P_t of the current configuration C_t . It is an Eulerian measurement defined as

$$\underline{\sigma}.d\vec{S} = d\vec{F} \tag{1.71}$$

However, we can also define the Kirchhoff stress tensor as

$$\underline{\tau} = J\underline{\sigma} \tag{1.72}$$

with $J = \det(\underline{F}) = \frac{\rho_0}{\rho} = \frac{V_t}{V_0}$ where ρ_0, ρ representing the material density in C_0 and C_t respectively.

Two other Lagrangien stress tensors can be defined as following:

The unsymmetric second-rank operator called the Boussinesq or Piola–Lagrange stress tensor, non-symmetric and neither purely Eulerian nor purely Lagrangian, defined by:

$$\underline{\pi}.dS_0 = d\vec{F} \tag{1.73}$$

> The second-rank tensor \underline{S} , which is symmetric due to the symmetry of $\underline{\sigma}$ and is purely Lagrangian, is called the Piola–Kirchhoff stress tensor, defined by:

$$\underline{S}.d\overline{S}_0 = d\overline{F}_0 \tag{1.74}$$

In all the above formulations, $d\vec{F}$ is the elementary force in C_t applied on an element of area dS while $d\vec{F}_0$ is obtained by the convective transport of $d\vec{F}$ from C_t to C_0 applied on an element of area dS_0 .

The density of massic power expressed in terms of the different stress and strain measures introduced above writes under the following form:

$$P = \frac{\sigma}{\rho} : \underline{D} = \tau : \underline{D} = \frac{\pi}{\rho_0} : \underline{\dot{F}} = \frac{\underline{S}}{\rho_0} : \underline{\dot{E}}$$
(1.75)

This is nothing but the so-called stress-strain conjugacy principle which defines stress-strain measure relation in order to define a constitutive equation.

1.6.2 Total strain rate decomposition

For metal forming processes it is vital that we consider the context of finite transformations for any behavior model that uses tensorial state variables. Accordingly, the formulation of nonlinear constitutive equations for inelastic solids under finite transformations, faces two basic problems:

- 1. How could we decompose the total strain tensors into reversible and irreversible parts?
- 2. Which formalism should we use in order to formulate the constitutive equations fulfilling the objectivity requirement?

These two aspects are widely discussed in the literature and an exhaustive analysis can be found in (Stolz, 1987; Doghri, 2000; Maugin, 1992; Rougee, 1997; Sidoroff, 2001; Nemat-Nasser, 2004; Stolz, 2009; Badreddine, 2006; Saanouni 2012)

In this work we consider the framework of elastoplastic finite strains based on the classical multiplicative decomposition of the total transformation gradient \underline{F} into elastic \underline{F}^{e} and plastic \underline{F}^{p} parts (Lee et al., 1983)., i.e.

$$\underline{F} = \underline{F}^{e} \cdot \underline{F}^{p} \tag{1.76}$$

Despite the fact that the choice of a purely Eulerian formalism leads to the best description of the finite transformations on the current deformed configuration, crucial problems of objectivity could appear. On the other hand, objectivity problems posed by a Lagrangian formalism, which consists of working on the initial configuration, lead to very complex constitutive equations.

In order to fulfil the objectivity requirement, the concept of the rotated frame formulation (RFF) is used. This supposes that all the constitutive equations will be written on the current configuration locally rotated by the orthogonal rotation tensor \underline{Q} (Mandel, 1971; Rice, 1971; Sidoroff, 1973, 1982; Simo and Ortiz, 1985; Sidoroff and Dogui, 2001, Badreddine et al, 2010; Saanouni 2012 among others), where the rotated configuration is Lagrangian by its orientation and Eulerian by the eigenvalues of the physical quantities.

As shown in Fig. 1.5, the configuration \overline{C}_{t}^{p} where all the tensorial variables will be transferred is called "isocline" and has exactly the same orientation with the initial configuration C_{0} and it is obtained through a multiplicative decomposition of the transformation gradient as well as the polar decomposition in the rotating frame:

$$\underline{F} = \underline{F}^{e} \cdot \underline{\overline{F}}^{p} = \underline{Q} \cdot \underline{\overline{F}} = \underline{V}^{e} \cdot \underline{Q} \cdot \underline{\overline{F}}^{p} = \underline{V}^{e} \cdot \underline{\hat{F}}^{p}$$
(1.77)

where $\underline{\overline{F}}^{p}$ is the rotated gradient of the plastic transformation \underline{F}^{p} , $\underline{\hat{F}}^{p}$ is the gradient of plastic transformation and \underline{V}^{e} is the left and purely elastic strain tensor.



Figure 1.5: Definition of two locally rotated objective configurations (Saanouni, 2012)

It is obvious that the configurations \overline{C}_t and \overline{C}_t^p have been rotated by \underline{Q} so that they have the same Lagrangian orientation as the initial configuration C_0 while the configuration C_t^e keeps the same orientation as the actual configuration C_t .

This rotation could be defined by two different ways. The first consists of using a privileged frame and calculating its rotation while it moves. This privileged frame is generally defined by the material microstructure as the crystallographic orientation for the monocrystalline materials. The second relies on postulating, a priori, a kinematic equation that governs the evolution of the orthogonal tensor Q:

$$\begin{cases} \underline{\dot{Q}} \cdot \underline{Q}^T = \underline{W}_Q \\ \underline{Q}(t_0) = \underline{1} \end{cases}$$
(1.78)

where \underline{W}_{o} characterizes the choice of the rotated frame.

For any second-rank tensor \underline{T} defined in the current configuration, its transfer to the locally rotated configuration (\overline{C}_t or \overline{C}_t^p in Fig. 1.5) by the rotation tensor \underline{Q} according to:

$$\underline{\overline{T}} = \underline{Q}^T \cdot \underline{T} \cdot \underline{Q} \tag{1.79}$$

By using the Eq.(1.77) and writing the velocity gradient with respect to \bar{C} we obtain:

$$\begin{bmatrix} \underline{V}^{e^{-1}} \cdot \underline{L} \cdot \underline{V}^{e} \end{bmatrix}^{s} = \begin{bmatrix} \underline{V}^{e^{-1}} \cdot \underline{V}^{e} \end{bmatrix}^{s} + \underline{D}^{p} \qquad (a)$$

$$\underline{W}_{Q} = \underline{\dot{Q}} \cdot \underline{Q}^{T} = \underline{\dot{W}}^{p} - \underline{Q} \cdot \underline{W}^{p} \cdot \underline{Q}^{T} \qquad (b)$$

$$\underline{W}_{Q} = \underline{\dot{Q}} \cdot \underline{Q}^{T} = \underline{W} - \underline{Q} \cdot \underline{W} \cdot \underline{Q}^{T} \qquad (c)$$

with $\underline{\vec{L}} = \underline{\dot{\vec{F}}} \cdot \underline{\vec{F}}^{-1}$ is the velocity gradient of the total strain with respect to \overline{C} . $\underline{\vec{D}}^{p} = \left[\underline{\dot{\vec{F}}}^{e} \cdot \underline{\vec{F}}^{e-1}\right]^{s}, \ \underline{\vec{W}}^{p} = \left[\underline{\dot{\vec{F}}}^{p} \cdot \underline{\vec{F}}^{p-1}\right]^{A}$ are the plastic strain and the plastic rotation rates with respect to \overline{C}_{t}^{ir} and $\underline{\hat{W}}^{p} = \left[\underline{\dot{\vec{F}}}^{p} \cdot \underline{\hat{\vec{F}}}^{p-1}\right]$ is the velocity of the plastic rotations.

The Eq.(1.80a) represents the decomposition of the strain rates which is equivalent to $\underline{\dot{\varepsilon}} = \underline{\dot{\varepsilon}}^e + \underline{\dot{\varepsilon}}^p$ under the small strains assumption. Considering the strains of the metallic materials, we assume that the reversible (elastic) strains are very small compared with irreversible ones. This assumption is translated as $\underline{V}^e = \underline{1} + \underline{\tilde{\varepsilon}}^e$ with $\|\underline{\tilde{\varepsilon}}^e\| \ll \underline{1}$ and the relation in Eq.(1.80a) transforms to:

$$\underline{\overline{D}} = \underline{\underline{\dot{\varepsilon}}}^e + 2\left[\underline{\overline{\varepsilon}}^e \underline{\overline{W}}_{ir}\right]^s + \underline{\overline{D}}^p = \underline{\underline{\dot{\varepsilon}}}^{ed_p} + \underline{\overline{D}}^p$$
(1.81)

with $\underline{\dot{\varepsilon}}^{e^{J_p}}$ being the Jaumann derivative with respect to \underline{W}^p . Between the quantities \underline{W}^p and \underline{W} we deduce the following relation:

$$\underline{\overline{W}} = \underline{\overline{W}}^{p} + 2 \left[\underline{\overline{\varepsilon}}^{e} \, \underline{\overline{D}}^{p} \right]^{A} \tag{1.82}$$

By neglecting the term $\left[\underline{\overline{e}}^{e}\underline{\overline{D}}^{p}\right]^{A}$, the Eq.(1.81) rewrites under the following form:

$$\underline{\overline{D}} = \underline{\underline{\dot{\varepsilon}}}^{e} + 2\left[\underline{\underline{\varepsilon}}^{e} \underline{\overline{W}}^{p}\right]^{s} + \underline{\overline{D}}^{p} = \underline{\underline{\dot{\varepsilon}}}^{ed_{T}} + \underline{\overline{D}}^{p}$$
(1.83)

with $\underline{\dot{\varepsilon}}^{e_T}$ being the Jaumann derivative with respect to \underline{W} . The equations (1.80b) and (1.80c) represent the rotation rates of the two rotated frames defined in Fig. 1.5.

1.6.3 Extension to finite transformations

Let us suppose that \underline{z} is a second-rank tensorial variable to which we associate its rotated objective variable $\overline{\underline{z}}_{\varrho}$ defined by:

$$\overline{\underline{z}}_{\underline{Q}} = \underline{\underline{Q}}^T \cdot \underline{\underline{z}} \cdot \underline{\underline{Q}}$$
(1.84)

The rotated frame defined by the rotation \underline{Q} is an orthonormal rotation depending on the material and its rotated derivatives with respect to this frame is given by:

$$\frac{D_{\varrho}\underline{z}}{Dt} = \underline{Q} \cdot \left[\frac{d(\underline{Q}^{T} \cdot \underline{z} \cdot \underline{Q})}{dt} \right] \cdot \underline{Q}^{T} = \underline{\dot{z}} + \underline{z} \cdot \underline{W}_{\varrho} - \underline{W}_{\varrho} \cdot \underline{z}$$

$$= \underline{\dot{z}} - \underline{z} \cdot \underline{W}_{\varrho}^{T} - \underline{W}_{\varrho} \cdot \underline{z}$$

$$= \underline{\dot{z}} + \underline{z} \cdot \underline{W}_{\varrho} + \underline{W}_{\varrho}^{T} \cdot \underline{z}$$
(1.85)

where $\underline{\dot{z}} = \frac{d\underline{z}}{dt}$ is the classical material derivative of the variable \underline{z} . We note that the choice of the rotated frame is made through the choice of the rotation tensor \underline{W}_Q . There are many possible choices, among them the two well-known defined by:

- > If $\underline{W}_{Q} = \underline{W}$, then the classical Jaumann derivative is obtained.
- → If $\underline{W}_{Q} = \underline{\dot{R}} \cdot \underline{R}^{T}$, then the Green-Naghdi rotation derivative is obtained, where <u>R</u> is described in Eq(1.65).

The material derivative of Eq. (1.84) gives:

$$\underline{\dot{z}}_{\underline{Q}} = \underline{\dot{Q}}^{T} \cdot \underline{z} \cdot \underline{Q} + \underline{Q}^{T} \cdot \underline{\dot{z}} \cdot \underline{Q} + \underline{Q}^{T} \cdot \underline{z} \cdot \underline{\dot{Q}}$$
(1.86)

By exploiting the equations Eq.(1.85) and Eq.(1.86), we obtain a relation between the material derivatives and the rotational ones of the variable \underline{z}_{ϱ}

$$\underline{\dot{z}}_{\varrho} = \underline{\varrho}^{T} \cdot \frac{D_{\varrho} \underline{z}}{Dt} \cdot \underline{\varrho} \text{ or } \frac{D_{\varrho} \underline{z}}{Dt} = \cdot \underline{\varrho} \cdot \underline{\dot{z}}_{\varrho} \cdot \underline{\varrho}^{T}$$
(1.87)

The choice of the rotational objective derivative depends on the chosen frame configuration. The objectivity of the stress and strain tensors by the rotating frame principle is the simplest to implement. Indeed, at each calculation increment, the reference configuration coincides with the current configuration during the calculation. Among these rotational derivatives, we find the derivative of Green-Naghdi in proper rotation (Dafalias, 1983). In fact, in the set of applications, we will limit ourselves to using the rotating reference frame proper or Green-Naghdi such that $\underline{Q} = \underline{R}$, where the rigid body tensor is the tensor resulting from the polar decomposition of the gradient of the transformation. The rotation increment is obtained from the total rotation tensor. Indeed, the derivative is as follows:

$$\underline{W}_{Q} = \underline{\dot{R}} \cdot \underline{R}^{T}$$
(1.88)

1.7 Conclusions

In this chapter, the theoretical formulation of the thermomechanical nonlocal micromorphic model is systematically presented: A complete set of generalized micromorphic constitutive equations is derived in the context of the thermodynamics of irreversible processes to capture the strongly nonlocal behavior of the materials in the localization regions. It contains the formulations of the balance equations, the state relations and the evolution equations. The micromorphic model coupled with elato-plasticity, ductile damage and mixed hardening as well as the nonlocal effects. We have resumed the principal nonlocal formulations used in order to regularize an IBVP and obtain one unique solution independent of the space discretization. As a result, we proposed a more robust nonlocal formulation in the framework of the generalized continua and more specifically in the framework of the micromorphic theory where we presented a complete formulation consisting of the overall constitutive equations for the damage and the isotropic hardening. We also showed that the equations of a simple local continuum can be easily retrieved and studied through the micromorphic theory. The complete set of equations will be discretized in space by the finite element method and in time by the finite difference method in order to resolve numerically the IBVP as presented in the following Chapter 2.

Chapter 2

Numerical Aspects.

2.1 Introduction

In Chapter 1, we formulated a complete set of constitutive equations of a micromorphic solid submitted to large irreversible strains accounting, among others, for the isotropic ductile damage evolution. Also partial differential equations (PDEs) describing the momentum balance were deduced from the generalized principle of virtual powers. Similarly, the constitutive equations governing the evolution of the dissipative phenomena were obtained as ordinary differential equations (ODEs). The complete set of these equations (PDEs and ODEs) define the so-called Initial and Boundary Value Problem (IBVP). The actual chapter is dedicated to the numerical aspects and the procedures related to the resolution of the micromorphic evolution problem in the framework of the Finite Element Method (FEM). To achieve this, we start by giving the strong forms of the IBVP and its associated variational or weak forms. Then, we describe the time discretization based on appropriate finite differences schemes and the space discretization based on the finite element method including the formulation of different finite elements with additional micromorphic degrees of freedom. Regarding the global resolution scheme, we examine the global resolution of the three balance equations (i.e. classical mechanical as well as micromorphic isotropic damage and micromorphic isotropic hardening equilibrium equations). We give a detailed description of the local integration scheme of the fully coupled constitutive equations in order to compute all the state variables at the end of each load increment on each integration or Gauss point for each element for the upcoming computation of the internal forces which appear in the weak forms associated to the balance equations. The local integration algorithm consists of discretizing all the constitutive equations over a typical time integral of size Δt defined by $t \in [t_n \ t_{n+1} = t_n + \Delta t]$ where t_n is the origin of this time interval. This leads to calculating the values of all the mechanical fields at the end of increment t_{n+1} as well as the loading increment $\Delta \underline{\varepsilon} = \Delta t_{n+1} \underline{D}_{n+\frac{1}{2}}$ and the rotation increment $\Delta \underline{Q}_{n+1} = Q_{n+1} \cdot \underline{Q}_n^T$ while the values of all the fields at the beginning of time increment t_n are known. This integration algorithm is based on the use of a purely implicit Euler scheme combined with an appropriate asymptotic scheme (Saanouni, 2012). For each integration point concerned by the plastic flow, the iterative method of elastic prediction-plastic correction is used to integrate numerically the complete set of constitutive equations thanks to a Newton-Raphson resolution procedure in order to converge towards a point lying on the current loading surface when the plastic admissibility condition is fulfilled.

Note that a huge number of academic books related to the numerical aspects connected to the time and space discretization of nonlinear solid mechanics are found in literature, e.g (Bathe, 1996; Zienkiewicz, 2005; Hughes, 1987; Crisfield, 1991; Bonnet 1997, Simo 1998, Belytcshko 2000...)

2.2 Initial and boundary value problem associated to virtual metal forming processes

In this section, we will pose the problem of nonlinear evolution for a micromorphic solid with ductile damage in finite strains, the constitutive equations of which were given in the first chapter. For the sake of brevity and simplicity, we consider only elastoplastic (rate-independent) solids with isotropic damage with a single yield surface, the state relations of which are developed in section 1.4.3 with the possible microcracks closure effect analyzed in section 1.4.4, and the evolution equations of the dissipative phenomena addressed in section 1.4.5 for time-independent plasticity.

2.2.1 Strong forms of the IBVP

Let us consider a deformable and damageable micromorphic solid made of an elastoplastic metallic material with ductile damage. At time t this solid occupies a volume Ω_t with a boundary Γ_t as shown in Fig. 2.1 and it contains a completely damaged area of volume Ω_t^d complementary to Ω_t with a boundary Γ_t^d , so that Ω_t^{nd} is the not totally damaged deformable volume.

At eat time $t \in [t_0, t_f]$, where t_0 and t_f are the beginning and the end of the applied loading path respectively, and the solid undergoes body forces noted as \vec{f}_t^u , boundary forces noted as \vec{F}_t^u on Γ_t^F , contact forces noted as \vec{F}_t^C on Γ_t^C , micromorphic body forces $\vec{f}^{g\bar{r}}, f^{\bar{r}}, \vec{f}^{g\bar{d}}, f^{\bar{d}}$ as well as micromorphic boundary forces noted as $F^{\bar{r}}$ on Γ_t^{F} for the micromorphic isotropic hardening and $F^{\bar{d}}$ on $\Gamma^{F_{\bar{d}}}$ for the micromorphic damage. Due to lack of information and for the sake of simplicity, we neglect all the micromorphic forces by taking $\vec{f}^{g\bar{r}} = \vec{0}$, $f^{\bar{r}} = 0$, $F^{\bar{r}} = 0$, $\vec{f}^{g\bar{d}} = \vec{0}$, $f^{\bar{d}} = 0$.



Figure 2.1: Schematization of the IBVP (Saanouni, 2012)

The problem posed is thus the following: compute all the IBVP unknowns at each time $t \in [t_0, t_f]$ at any material point $\vec{x}(\vec{X}, t)$ of the solid in the current configuration Ω_t by determining all the mechanical fields. The unknowns can be distinguished in the two following groups:

- > The kinematic unknown variables (or degrees of freedom) which in this case are: $\vec{u}(\vec{x},t), \vec{r}(\vec{x},t)$ and $\vec{d}(\vec{x},t)$.
- The pairs of state variables on the current deformed and damaged configuration: the local state variables $\left(\underline{\varepsilon}^{ir}(\vec{x},t), \underline{\sigma}(\vec{x},t)\right), \left(\underline{\alpha}(\vec{x},t), \underline{X}(\vec{x},t)\right), \left(r(\vec{x},t), R(\vec{x},t)\right), \left(d(\vec{x},t), Y(\vec{x},t)\right)$ and the micromorphic forces, $\breve{R}(\vec{x},t), \breve{Y}(\vec{x},t)$ associated to the two micromorphic dofs, as well as the two following pairs of micromorphic state variables $\left(\vec{\nabla}\breve{r}(\vec{x},t), \vec{R}(\vec{x},t)\right)$ and $\left(\vec{\nabla}\breve{d}(\vec{x},t), \vec{Y}(\vec{x},t)\right)$.

These kinematic and state variables should fulfill:

- 1. All of the following fields equations concerning together with their appropriate initial and boundary conditions:
 - The classical equilibrium in Eq.(1.8a)
 - > The additional micromorphic balance equations shown in Eq.(1.59) and Eq.(1.60).

All these three strong forms are summarized again in Table 2.1 where θ_r is chosen to be equal to zero in Eq.(1.59) as the particular case which will be implemented.

Classical local equilibrium	$\begin{cases} \vec{\nabla} \cdot \underline{\sigma} + \rho \vec{f}^{u} = \rho \vec{\vec{u}} & \text{in } \Omega \\ \underline{\sigma} \cdot \vec{n} = \vec{F}^{u} & \text{on } \Gamma \end{cases}$
Micromorphic damage equilibrium	$\begin{cases} \ell_{\tilde{a}}^{2} Lap(\breve{d}) + (d - \breve{d}) = \rho \frac{\zeta_{\breve{d}} \overset{\cdots}{\breve{d}}}{\breve{H}} \\ \breve{H}^{g}(\vec{\nabla}\breve{d}) \cdot \vec{n} = 0 \end{cases}$
Micromorphic isotropic hardening equilibrium	$\int \ell_{\bar{r}}^{2} \left(\left(1 - \theta_{r} d^{\gamma} \right) Lap(\bar{r}) - \theta_{r} \gamma d^{\gamma - 1} \vec{\nabla} d. \vec{\nabla} \vec{r} \right) + \left(\sqrt{1 - d^{\gamma}} \sqrt{1 - \theta_{r} d^{\gamma}} r - \left(1 - \theta_{r} d^{\gamma} \right) \vec{r} \right) = \rho \frac{\zeta_{\bar{r}} \ddot{\vec{r}}}{\vec{Q}}$
	$\bigcup \vec{Q}^{s} \left(\vec{\nabla} \vec{r} \right) \cdot \vec{n} = 0$

Table 2.1: The local and micromorphic balance equations

2. The kinematic equations:

$$\underline{F} = \underline{1} + \vec{\nabla}\vec{u}, \ \underline{L} = \underline{\dot{F}} \cdot \underline{F}^{-1}, \ \underline{D} = \frac{1}{2} \left(\underline{L} + \underline{L}^T \right), \ \underline{W} = \frac{1}{2} \left(\underline{L} - \underline{L}^T \right)$$
(2.1)

3. All the constitutive equations deduced in Chapter 1: the state relations (Eq.(1.24)-Eq. (1.30)) and their associated evolution equations (Eq.(1.50)-Eq.(1.53)) summarized in Table 2.2. For the rest of the thesis, we neglect the coupling between the local damage and the micromorphic isotropic hardening stress variable. The parameter θ_r is then set to 0 ($\theta_r = 0$).

Local State variables	Evolution equations	
$\frac{-Cauchy stress}{\underline{\sigma} = (1-d)\underline{\Lambda} : \underline{\varepsilon}^e = (1-d)(\lambda_e tr(\underline{\varepsilon}^e)\underline{1} + 2\mu_e \underline{\varepsilon}^e)$	$\underline{D}^{p} = \dot{\lambda} \frac{\underline{\tilde{n}}}{\sqrt{1-d}}$	
- Kinematic hardening	$\dot{\underline{\alpha}} = \underline{D}^p - a\dot{\lambda}\underline{\alpha}$	
$\underline{X} = \frac{2}{3}(1-d)C\underline{\alpha}$		
$\frac{-\text{Isotropic hardening}}{R = \rho \frac{\partial \psi_g}{\partial r} = (1 - d^{\gamma})Qr + \breve{Q} \Big[(1 - d^{\gamma})r - \sqrt{1 - d^{\gamma}}\breve{r} \Big]$	$\dot{r} = \left(\frac{1}{\sqrt{1-d^{\gamma}}} - br - b\frac{\breve{Q}}{Q(1-d^{\gamma})}\left(\left(1-d^{\gamma}\right)r - \sqrt{1-d^{\gamma}}\breve{r}\right)\right)$	
$\left(heta_r = 0 ight)$	$(\theta_r = 0)$	
$\frac{-\text{Isotropic ductile damage}}{Y = Y_E + Y_A + Y_R + Y_d}$ with		
$\left\{Y_E = \frac{1}{2}\underline{\varepsilon}^e : \underline{\Lambda} : \underline{\varepsilon}^e\right\}$	$\dot{d} = \frac{\dot{\lambda}}{(1-d)^{\beta}} \left\langle \frac{(Y_{loc} + Y_{nloc}) - Y_0}{S} \right\rangle^s$	
$Y_A = \frac{1}{3}C\underline{\alpha}:\underline{\alpha}$		
$ Y_R = \frac{1}{2} \gamma d^{\gamma - 1} Q r^2 + \frac{1}{2} \gamma d^{\gamma - 1} \widetilde{Q} \left(\sqrt{1 - d^{\gamma}} r - \widetilde{r} \right) \left(\frac{r}{\sqrt{1 - d^{\gamma}}} \right) $		
$\left Y_d = -\breve{H} \left(d - \breve{d} \right) \right $		
Micromorphic state variables		
- Micromorphic damage		
$egin{array}{l} egin{array}{l} egin{array}$		
$\vec{\breve{Y}} = \rho \frac{\partial \psi}{\partial \vec{\nabla} \vec{d}} = \vec{H}^{s} \vec{\nabla} \vec{d}$		
- Micromorphic isotropic hardening		
$\breve{R} = -\breve{Q} \Big(\sqrt{1 - d^{\gamma}} r - \breve{r} \Big)$		
$\vec{\breve{R}} = \breve{Q}^s (\vec{\nabla}\breve{r})$		

Table 2.2: The overall state relations and evolution equations of the behavior model

2.2.2 Weak forms of the IBVP

Following the FEM, the weak forms associated to the strong forms of the IBVP can be obtained using the weighted residuals method. According to the configuration on which these equations are defined, several weak formulations are possible:

- total Lagrangian formulation, in which all kinematic and state variables used are defined on the reference configuration at the initial time (undeformed configuration),
- total Eulerian formulation, in which all kinematic and state variables used are defined on the current deformed configuration,
- updated Lagrangian formulation, in which the reference configuration is a completely known deformed configuration;
- Arbitrary Lagrangien Eulerian (ALE) formulation in which the material points and the fixed grid (mesh) are governed by two different kinematics (or transformation gradients).

In the mechanics of nonlinear solids and more particularly for metal forming problems, the updated Lagrangian formulation is quite satisfactory, especially when it is associated with an efficient adaptive remeshing procedure. This makes it efficient enough to follow the history of each material point and to handle the large geometrical changes of the deforming solid and related boundary conditions.

In this thesis concerned by metal forming under large strains, we focus on the updated Lagrangian formulations in order to deduce the weak forms of the balance equations for the classical and the micromorphic problem. This allows us to solve the IBVP using the displacement-based finite element method.

2.2.2.1 Weak forms associated to the equilibrium equations

By using the updated Lagrangian formulation, the weak forms associated to the strong forms (Eq.(1.8)) can be easily obtained thanks to the weighted residuals method (Belytschko, 2001).

We start from the extension of the Hu-Washizu weak form proposed by Fish and Belytschko (Fish and Belytschko, 1990) to derive the following weak forms for:

The weak form associated to the classical local equilibrium:

$$\delta\pi\left(\vec{u},\underline{\dot{e}}^{a},\underline{\sigma}^{a}\right) = \int_{\Omega_{e}} \left(\delta\underline{\dot{e}}^{a}\right)^{T} : \underline{\sigma}^{a} d\Omega + \delta\int_{\Omega_{e}} \left(\underline{\sigma}^{a}\right)^{T} : \left(\vec{\nabla}_{s}\vec{u} - \underline{\dot{e}}^{a}\right) d\Omega + \int_{\Omega_{e}} \left[\rho\left(\vec{u} - \vec{f}^{u}\right) \cdot \delta\vec{u}\right] dV - \int_{\Gamma_{e}} \left(\vec{F}^{u} \cdot \delta\vec{u}\right) dS = 0$$
(2.2)

where, δ denotes a variation, $\underline{\dot{\varepsilon}}^a$ is the assumed strain rate, $\underline{\sigma}^a$ the assumed stress evaluated by the constitutive law and $\nabla_{\vec{u}} \vec{u}$ the symmetric part of the velocity gradient.

..

The weak form associated to the micromorphic damage:

$$J(\vec{d},\delta\vec{\vec{d}}) = -\int_{\Omega_t} \ell_{\vec{d}}^2 \vec{\nabla} \vec{d} \cdot \vec{\nabla} \delta \dot{\vec{d}} dV + \int_{\Omega_t} (d-\vec{d}) \delta \dot{\vec{d}} dV - \int_{\Omega_t} \rho \frac{\zeta_{\vec{d}} \vec{d}}{\vec{H}} \delta \dot{\vec{d}} dV = 0 \quad \forall \delta \dot{\vec{d}} K.A.$$
(2.3)

The weak form associated to the micromorphic isotropic hardening:

$$J(\breve{r},\delta\dot{\breve{r}}) = -\int_{\Omega_{t}} (1-d^{\gamma}) \ell_{\breve{r}}^{2} \vec{\nabla} \breve{r} \cdot \vec{\nabla} \delta\dot{\breve{r}} dV + \int_{\Omega_{t}} (1-d^{\gamma})(r-\breve{r}) \delta\dot{\breve{r}} dV - \int_{\Omega_{t}} \rho \frac{\zeta_{\breve{r}} \breve{r}}{\breve{Q}} \delta\dot{\breve{r}} dV = 0 \quad \forall \delta\dot{\breve{r}} K.A.$$

$$(2.4)$$

where $\delta \vec{u}$ is the kinematically admissible virtual velocity field and $\delta \dot{\vec{d}}$ and $\delta \dot{\vec{r}}$ are the kinematically admissible virtual velocity fields associated to the micromorphic damage and the micromorphic isotropic hardening respectively.

These three forms are strongly coupled as well as highly nonlinear so their analytical resolution is not possible. Accordingly, numerical methods based on the appropriate time and space discretization are required to solve the discretized nonlinear equations defining the IBVP. This leads to a nonlinear algebraic system that should be resolved using the appropriate Newton-type iterative scheme or direct non-iterative scheme.

2.3 Time and space discretization

In this paragraph we will discuss the time discretization of the weak forms of the IBVP introduced in section 2.2.2.1, over the interval $[t_0, t_f]$ as well as the spatial discretization over the domain Ω_t . In the context of FEM, the time interval $[t_0, t_f]$ is divided into subintervals of size Δt that are not necessarily constant while the domain Ω_t is decomposed into geometrically simple subdomains in order to express the IBVP in an incremental form of a nonlinear algebraic system to be solved over each time increment.

2.3.1 Time discretization of the IBVP

The typical time interval is divided into subintervals of the form $\begin{bmatrix} t_0, t_f \end{bmatrix} = \bigcup_{n=1}^{N_t} \begin{bmatrix} t_n, t_{n+1} = t_n + \Delta t_{n+1} \end{bmatrix}$, where $\Delta t_{n+1} = t_{n+1} - t_n$ is the incremental time step, not necessarily constant and N_t is the total number of the time increments. The purpose is solving a nonlinear problem in each one of these subintervals in order to compute all the unknowns of the IBVP at the time t_{n+1} since their values are supposed to be known at any time from the beginning of the time interval at t_0 until the time t_n . More precisely, for our micromorphic problem, the discrete nodal fields $\{u\}_n, \{\vec{d}\}_n$ and $\{\vec{r}\}_n$ are supposed to be known at the beginning of the increment t_n . The goal is to calculate $\{u\}_{n+1}, \{\vec{d}\}_{n+1}$ and $\{\vec{r}\}_{n+1}$ at the time t_{n+1} by using the discretized weak forms to be mentioned in the following paragraph.

2.3.2 Space discretization of the IBVP using finite elements

In the framework of the FEM, the domain Ω_t is discretized into a finite number N_{te} of subdomains or finite elements noted as Ω_e so that $\Omega_t = \bigcup_{e=1}^{N_{te}} \Omega_e$, $\Omega_i \cap \Omega_j = \emptyset$ where (e) indicates the element, $\bigcup_{e=1}^{N_{te}}$ is the operator indicating the addition over all the elements and $i, j \in [1, N_{te}]$.

In each FE Ω_e the main unknowns of the IBVP (here the displacement, the micromorphic damage and isotropic hardening fields) are approximated, based on a nodal approximation by subdomains, using the appropriate polynomial interpolation functions. For the sake of simplicity, the elements are considered as isoparametric, e.g., the geometry of each element is approached by the same interpolation functions with the unknown fields. Any reference element called Ω_r of the reference space expressed by the local coordinates $\vec{\xi}^e$ corresponding to Ω_e can be transformed into its real correspondent element by following the coordinate transformation relationship:

$$\vec{x}^{e}(\vec{\xi}^{e}) = \sum_{i=1}^{Nen} \overline{N}^{e}_{i}(\vec{\xi}) \vec{x}^{e}_{i} \quad or \quad \left\{ x^{e}(\vec{\xi}) \right\} = \left[\overline{N}^{e}(\vec{\xi}) \right] \left\{ x^{e}_{i} \right\}$$
(2.5)

where *Nen* are the nodes defining Ω_e , \vec{x}_i^e are the nodal coordinates of the specific element and $\overline{N}_i^e(\vec{\xi})$ are the interpolation polynomials shape functions in terms of local coordinates $\vec{\xi}$ in the reference space.

By applying the nodal approximation to the main real and virtual unknowns of the IBVP involved in the three weak forms (Eq.(2.2)-Eq.(2.4)), the real and virtual displacement vectors, the micromorphic damage and isotropic hardening are expressed on a typical reference element, in the following matrix forms:

$$\begin{cases} \left\{ u^{e}(\vec{\xi},t) \right\} = \left[N^{e}(\vec{\xi}) \right] \left\{ u^{e}_{i}(t) \right\} \\ \left\{ \delta u^{e}(\vec{\xi},t) \right\} = \left[N^{e}(\vec{\xi}) \right] \left\{ \delta u^{e}_{i}(t) \right\} \end{cases}$$
(a)
$$\begin{cases} \vec{d}^{e}(\vec{\xi},t) = \left\langle \vec{N}^{e}_{\vec{d}}(\vec{\xi}) \right\rangle \left\{ \vec{d}^{e}_{i}(t) \right\} \\ \delta \vec{d}^{e}(\vec{\xi},t) = \left\langle \vec{N}^{e}_{\vec{d}}(\vec{\xi}) \right\rangle \left\{ \delta \vec{d}^{e}_{i}(t) \right\} \end{cases}$$
(b)
$$(2.6)$$

$$\begin{cases} \vec{r}^{e}(\vec{\xi},t) = \left\langle \vec{N}_{\vec{r}}^{e}(\vec{\xi}) \right\rangle \left\{ \vec{r}_{i}^{e}(t) \right\} \\ \delta \vec{r}^{e}(\vec{\xi},t) = \left\langle \vec{N}_{\vec{r}}^{e}(\vec{\xi}) \right\rangle \left\{ \delta \vec{r}_{i}^{e}(t) \right\} \end{cases}$$
(c)

where $\left[N^{e}\right], \left[\overline{N}_{\overline{d}}^{e}\right], \left[\overline{N}_{\overline{r}}^{e}\right]$ are the matrices of the interpolation functions for each element (*e*) for the displacement, the micromorphic damage field and the micromorphic isotropic hardening field respectively. These functions depend only of the node coordinates and are independent of time. The terms $u_{i}^{e}, \overline{d}_{i}^{e}, \overline{r}_{i}^{e}$ correspond to the nodal displacement, micromorphic damage and isotropic hardening for each node *i* expressed in the global reference space.

Following the same procedure for the first time derivative and the acceleration fields, we get:

$$\begin{cases} \left\{ \dot{u}^{e}(\vec{\xi},t) \right\} = \left[N^{e}(\vec{\xi}) \right] \left\{ \dot{u}_{i}^{e}(t) \right\} \\ \left\{ \delta \dot{u}^{e}(\vec{\xi},t) \right\} = \left[N^{e}(\vec{\xi}) \right] \left\{ \delta \dot{u}_{i}^{e}(t) \right\} \end{cases}$$
(a)
$$\begin{cases} \left\{ \ddot{u}^{e}(\vec{\xi},t) \right\} = \left[N^{e}(\vec{\xi}) \right] \left\{ \delta \ddot{u}_{i}^{e}(t) \right\} \\ \left\{ \delta \ddot{u}^{e}(\vec{\xi},t) \right\} = \left[N^{e}(\vec{\xi}) \right] \left\{ \delta \ddot{u}_{i}^{e}(t) \right\} \end{cases}$$
(a)

$$\begin{cases} \dot{\vec{d}}^{e}(\vec{\xi},t) = \left\langle \vec{N}_{\vec{d}}^{e}(\vec{\xi}) \right\rangle \left\{ \dot{\vec{d}}_{i}^{e}(t) \right\} \\ \delta \dot{\vec{d}}^{e}(\vec{\xi},t) = \left\langle \vec{N}_{\vec{d}}^{e}(\vec{\xi}) \right\rangle \left\{ \delta \dot{\vec{d}}_{i}^{e}(t) \right\} \end{cases} \qquad (b) \text{ and } \begin{cases} \ddot{\vec{d}}^{e}(\vec{\xi},t) = \left\langle \vec{N}_{\vec{d}}^{e}(\vec{\xi}) \right\rangle \left\{ \ddot{\vec{d}}_{i}^{e}(t) \right\} \\ \delta \ddot{\vec{d}}^{e}(\vec{\xi},t) = \left\langle \vec{N}_{\vec{d}}^{e}(\vec{\xi}) \right\rangle \left\{ \delta \ddot{\vec{d}}_{i}^{e}(t) \right\} \end{cases} \qquad (b)$$

$$\begin{cases} \dot{\vec{r}}^{e}(\vec{\xi},t) = \left\langle \vec{N}_{\vec{r}}^{e}(\vec{\xi}) \right\rangle \left\{ \dot{\vec{r}}_{i}^{e}(t) \right\} \\ \delta \dot{\vec{r}}^{e}(\vec{\xi},t) = \left\langle \vec{N}_{\vec{r}}^{e}(\vec{\xi}) \right\rangle \left\{ \delta \dot{\vec{r}}_{i}^{e}(t) \right\} \end{cases} \qquad (c) \qquad \begin{cases} \ddot{\vec{r}}^{e}(\vec{\xi},t) = \left\langle \vec{N}_{\vec{r}}^{e}(\vec{\xi}) \right\rangle \left\{ \ddot{\vec{r}}_{i}^{e}(t) \right\} \\ \delta \ddot{\vec{r}}^{e}(\vec{\xi},t) = \left\langle \vec{N}_{\vec{r}}^{e}(\vec{\xi}) \right\rangle \left\{ \delta \ddot{\vec{r}}_{i}^{e}(t) \right\} \end{cases} \qquad (c) \qquad (2.7)$$

Moreover, the calculation of the first gradients of the real and virtual nodal variables involved in the three weak forms leads to:

$$\begin{cases} \vec{\nabla}(\dot{u}^{e}) = \left[\frac{\partial N^{e}}{\partial \vec{x}}\right] \left\{ \dot{u}_{i}^{e} \right\} = \left[\frac{\partial N^{e}}{\partial \vec{\xi}}\right] \left[\frac{\partial \vec{\xi}}{\partial \vec{x}}\right] \left\{ \dot{u}_{i}^{e} \right\} = \left[B^{e}\right] \left\{ \dot{u}_{i}^{e} \right\} \\ \vec{\nabla}(\delta \dot{u}^{e}) = \left[\frac{\partial N^{e}}{\partial \vec{x}}\right] \left\{ \delta \dot{u}_{i}^{e} \right\} = \left[\frac{\partial N^{e}}{\partial \vec{\xi}}\right] \left[\frac{\partial \vec{\xi}}{\partial \vec{x}}\right] \left\{ \delta \dot{u}_{i}^{e} \right\} = \left[B^{e}\right] \left\{ \delta \dot{u}_{i}^{e} \right\} \end{cases}$$
(2.8)

$$\begin{cases} \vec{\nabla}(\vec{d}^{e}) = \left[\frac{\partial \vec{N}_{\vec{d}}^{e}}{\partial \vec{x}}\right] \left\{\vec{d}_{i}^{e}\right\} = \left[\frac{\partial \vec{N}_{\vec{d}}^{e}}{\partial \vec{\xi}}\right] \left[\frac{\partial \vec{\xi}}{\partial \vec{x}}\right] \left\{\vec{d}_{i}^{e}\right\} = \left[\vec{B}_{\vec{d}}^{e}\right] \left\{\vec{d}_{i}^{e}\right\} \\ \vec{\nabla}(\delta \vec{d}^{e}) = \left[\frac{\partial \vec{N}_{\vec{d}}^{e}}{\partial \vec{x}}\right] \left\{\delta \vec{d}_{i}^{e}\right\} = \left[\frac{\partial \vec{N}_{\vec{d}}^{e}}{\partial \vec{\xi}}\right] \left[\frac{\partial \vec{\xi}}{\partial \vec{\xi}}\right] \left\{\delta \vec{d}_{i}^{e}\right\} = \left[\vec{B}_{\vec{d}}^{e}\right] \left\{\delta \vec{d}_{i}^{e}\right\}$$
(2.9)

$$\begin{cases} \vec{\nabla}(\vec{r}^{e}) = \left[\frac{\partial \vec{N}_{\vec{a}}^{e}}{\partial \vec{x}}\right] \left\{ \vec{r}_{i}^{e} \right\} = \left[\frac{\partial \vec{N}_{\vec{r}}^{e}}{\partial \vec{\xi}}\right] \left[\frac{\partial \vec{\xi}}{\partial \vec{x}}\right] \left\{ \vec{r}_{i}^{e} \right\} = \left[\vec{B}_{\vec{r}}^{e}\right] \left\{ \vec{r}_{i}^{e} \right\} \\ \vec{\nabla}(\delta\vec{r}^{e}) = \left[\frac{\partial \vec{N}_{\vec{r}}^{e}}{\partial \vec{x}}\right] \left\{ \delta\vec{r}_{i}^{e} \right\} = \left[\frac{\partial \vec{N}_{\vec{r}}^{e}}{\partial \vec{\xi}}\right] \left[\frac{\partial \vec{\xi}}{\partial \vec{x}}\right] \left\{ \delta\vec{r}_{i}^{e} \right\} = \left[\vec{B}_{\vec{r}}^{e}\right] \left\{ \delta\vec{r}_{i}^{e} \right\}$$
(2.10)

Substituting Eq.(2.6)-Eq.(2.10) into Eq.(2.2)-Eq.(2.4), the elementary discretized weak forms for a typical element (e) take the following form:

$$J_{u}^{e}\left(\delta\vec{v}^{e}\right) = \left\langle\delta\dot{u}_{i}^{e}\right\rangle\left(\left[M^{e}\right]\left\{\ddot{u}_{i}^{e}\right\} + \left\{F_{\text{int}}^{e}\right\} - \left\{F_{ext}^{e}\right\}\right) \qquad (a)$$

$$J_{\vec{d}}^{e}\left(\delta\vec{d}^{e}\right) = -\left\langle\delta\dot{\vec{d}}_{i}^{e}\right\rangle\left(\left[M_{\vec{d}}^{e}\right]\left\{\ddot{\vec{d}}_{i}^{e}\right\} + \left\{F_{\text{int}-\vec{d}}^{e}\right\} - \left\{F_{ext-\vec{d}}^{e}\right\}\right) \qquad (b)$$

$$J_{\vec{d}}^{e}\left(\delta\vec{r}^{e}\right) = -\left\langle\delta\dot{\vec{r}}_{i}^{e}\right\rangle\left(\left[M_{\vec{r}}^{e}\right]\left\{\ddot{\vec{r}}_{i}^{e}\right\} + \left\{F_{\text{int}-\vec{r}}^{e}\right\} - \left\{F_{ext-\vec{r}}^{e}\right\}\right) \qquad (c)$$

where, the consistent mass matrices, the internal and external forces for a typical reference element (e) are given in the reference element by:

- Displacement field:

$$\left[M^{e}\right] = \int_{\Omega_{r}^{e}} \rho \left[N^{e}\right]^{T} \left[N^{e}\right] J_{v}^{e} dV$$
(2.12)

$$\left\{F_{\text{int}}^{e}\right\} = \int_{\Omega_{r}^{e}} \left[B^{e}\right]^{T} \left\{\sigma^{e}\right\} J_{\nu}^{e} dV$$
(2.13)

$$\left\{F_{ext}^{e}\right\} = \int_{\Omega_{r}^{e}} \rho\left[N^{e}\right]^{T} \left\{f^{u}\right\} J_{v}^{e} dV + \int_{\Gamma_{t}^{u}} \left[N^{e}\right]^{T} \left\{F^{u}\right\} J_{s}^{e} dS$$
(2.14)

with $J_v^e = \det\left(\left[J^e\right]\right) = \det\left(\left[\partial \vec{x}/\partial \vec{\xi}\right]\right)$ is the determinant of the Jacobian matrix of the volume mapping transformation between the real and the reference elements Ω_r^e while J_s^e is the Jacobian of the boundaries (or surface) transformation.

- Micromorphic damage:

$$\left[\tilde{M}_{\bar{d}}^{e}\right] = \int_{\Omega_{\tau}^{e}} \rho \frac{\zeta_{\bar{d}}}{\tilde{H}} \left\langle \tilde{N}_{\bar{d}}^{e} \right\rangle \left\{ \tilde{N}_{\bar{d}}^{e} \right\} J_{\nu}^{e} dV$$
(2.15)

$$\left\{ \vec{F}_{\text{int}-\vec{d}}^{e} \right\} = \int_{\Omega_{r}^{e}} \left(\ell_{\vec{d}}^{2} \left[\vec{B}_{\vec{d}}^{e} \right]^{T} \left[\vec{B}_{\vec{d}}^{e} \right] + \left\{ \vec{N}_{\vec{d}}^{e} \right\} \left\langle \vec{N}_{\vec{d}}^{e} \right\rangle \right) \left\{ \vec{d}_{n}^{e} \right\} J_{\nu}^{e} dV$$

$$(2.16)$$

$$\left\{ \breve{F}_{ext-\breve{d}}^{e} \right\} = \int_{\Omega_{r}^{e}} d^{e} \left\{ \breve{N}_{\breve{d}}^{e} \right\} J_{v}^{e} dV$$
(2.17)

with d^{e} being the local ductile damage of the element e.

- Micromorphic isotropic hardening:

$$\left[\tilde{M}_{\bar{r}}^{e}\right] = \int_{\Omega_{\bar{r}}^{e}} \rho \frac{\zeta_{\bar{r}}}{\tilde{Q}} \left\{\tilde{N}_{\bar{r}}^{e}\right\} \left\langle\tilde{N}_{\bar{r}}^{e}\right\rangle J_{v}^{e} dV$$
(2.18)

$$\left\{ \breve{F}_{\text{int}-\breve{r}}^{e} \right\} = \int_{\Omega_{r}^{e}} \left(1 - \left(d^{e} \right)^{\gamma} \right) \left(\ell_{\breve{r}}^{2} \left[\breve{B}_{r}^{e} \right]^{T} \left[\breve{B}_{\breve{r}}^{e} \right] + \left\{ \breve{N}_{\breve{r}}^{e} \right\} \left\langle \breve{N}_{\breve{r}}^{e} \right\rangle \right) \left\{ \breve{r}_{n}^{e} \right\} J_{v}^{e} dV$$

$$(2.19)$$

$$\left\{\vec{F}_{ext-\bar{r}}^{e}\right\} = \int_{\Omega_{r}^{e}} \left(1 - \left(d^{e}\right)^{\gamma}\right) r^{e} \left\{\vec{N}_{\bar{r}}^{e}\right\} J_{\nu}^{e} dV$$
(2.20)

with r^{e} indicating the classical isotropic hardening of the element. The consistent mass matrices are often favorably replaced by their diagonal forms called the lumped mass matrices, obtained by concentrating the constant mass of the element in its different nodes. Several methods are used to

diagonalize the mass matrices, as can be found in literature (Jurgen, 1996; T J R, 1987; Zienkiewicz and Taylor, 2005).

The lumped mass matrix is used to diagonalize the global mass matrix. For each row, the diagonal lumped mass term is obtained by summing the different terms associated to each column:

$$M_{ii}^{lp} = \sum_{j=1}^{nddls} M_{ij}^{e} = \sum_{j=1}^{nddls} \int_{\Omega_{r}^{e}} \rho N_{i}^{e} N_{j}^{e} J_{\nu}^{e} dV = \int_{\Omega_{r}^{e}} \rho N_{i}^{e} \sum_{j=1}^{nddls} N_{j}^{e} J_{\nu}^{e} dV = \int_{\Omega_{r}^{e}} \rho N_{i}^{e} J_{\nu}^{e} dV = \int_{\Omega_{r}^{e}} \rho N_{i}^{e} J_{\nu}^{e} dV$$
(2.21)

2.4 Numerical computation of the large strain rate tensor

To integrate the behavior model, in the case of large deformations and rotations, it is important to choose a kinematic approximation over a time step to evaluate the strain rate tensor $\underline{D}_{n+\frac{1}{2}}$, according to an incrementally objective integration algorithm. The most usual approach is to proceed by the calculation of the material position and to sequence as follows,

$$\vec{x}_{n+1} = \vec{x}_n + \Delta \vec{u}_{n+1}$$
(2.22)

The transformation tensor at the time t_{n+1} is written in the following form,

$$\underline{F}_{n+1} = \frac{\overrightarrow{\partial x}_{n+1}}{\overrightarrow{\partial X}} = \frac{\overrightarrow{\partial x}_{n+1}}{\overrightarrow{\partial x}_{n}} \cdot \frac{\overrightarrow{\partial x}_{n}}{\overrightarrow{\partial X}} = \Delta \underline{F}_{n+1} \cdot \underline{F}_{n} = \left(\underline{I} + \frac{\overrightarrow{\partial \Delta u}_{n+1}}{\overrightarrow{\partial x}_{n}}\right) \cdot \underline{F}_{n}$$
(2.23)

Nevertheless, assuming the hypothesis of approximation of the real path by a linear scheme at constant velocity, the schema under consideration causes a variation of volume. Winget and Hughes (Winget and Hughes, 1980) then proposed as a linear relation the following expression,

$$\vec{x}_{n+\theta} = \vec{x}_n + \theta \Delta \vec{u}_{n+1}$$
(2.24)

and

$$\underline{F}_{n+\theta} = \Delta \underline{F}_{n+\theta} \cdot \underline{F}_n = \frac{\partial \vec{x}_{n+\theta}}{\partial \vec{x}_n} \cdot \underline{F}_n$$
(2.25)

We use a half-step implementation scheme assuming an implicit Euler schema in time with $\theta = \frac{1}{2}$ In order to calculate the symmetric strain rate tensor $\underline{D}_{n+\frac{1}{2}}$ or, half-step consider the development of the tensor gradient strain rate $\underline{L}_{n+\frac{1}{2}}$, such as,

$$\underline{L}_{n+\frac{1}{2}} = \underline{\dot{F}}_{n+\frac{1}{2}} \cdot \underline{F}_{n+\frac{1}{2}}^{-1},$$
(2.26)

a classic central difference scheme is used for the computation of the first term:

$$\underline{\dot{F}}_{n+\frac{1}{2}} = \frac{\underline{F}_{n+1} - \underline{F}_n}{\Delta t_{n+1}} = \frac{1}{\Delta t_{n+1}} \frac{\partial \Delta u_{n+1}}{\partial x_n} \cdot \underline{F}_n$$
(2.27)

The inversion of the increment of the transformation gradient gives:

$$\underline{F}_{n+\frac{1}{2}}^{-1} = \underline{F}_{n}^{-1} \cdot \Delta \underline{F}_{n+\frac{1}{2}}^{-1} = \underline{F}_{n}^{-1} \cdot \left(\frac{\partial \vec{x}_{n+\frac{1}{2}}}{\partial \vec{x}_{n}}\right)^{-1} = \underline{F}_{n}^{-1} \cdot \frac{\partial \vec{x}_{n}}{\partial \vec{x}_{n+\frac{1}{2}}}$$
(2.28)

The strain rate gradient tensor at the half-step is:

$$\underline{L}_{n+\frac{1}{2}} = \underline{\dot{F}}_{n+\frac{1}{2}} \cdot \underline{F}_{n+\frac{1}{2}}^{-1} = \frac{1}{\Delta t_{n+1}} \frac{\partial \Delta \vec{u}_{n+1}}{\partial \vec{x}_n} \cdot \underline{F}_n \cdot \underline{F}_n^{-1} \cdot \frac{\partial \vec{x}_n}{\partial \vec{x}_{n+\frac{1}{2}}} = \frac{1}{\Delta t_{n+1}} \frac{\partial \Delta \vec{u}_{n+1}}{\partial \vec{x}_{n+\frac{1}{2}}}$$
(2.29)

The gradient tensor of strain velocities can be decomposed in symmetric and antisymmetric parts. The symmetric part of the tensor is the total deformation rate, noted as $\underline{D}_{n+\frac{1}{2}}$, and described as:

$$\underline{D}_{n+\frac{1}{2}} = \frac{1}{2} \left(\underline{L}_{n+\frac{1}{2}} + \underline{L}_{n+\frac{1}{2}}^{T} \right)$$
(2.30)

In the same way, we express the rate of rotation, noted $\underline{\omega}_{n+\frac{1}{2}}$, by

$$\underline{\omega}_{n+\frac{1}{2}} = \frac{1}{2} \left(\underline{L}_{n+\frac{1}{2}} - \underline{L}_{n+\frac{1}{2}}^T \right)$$
(2.31)

2.5 Formulation of some finite elements

The introduction of the new micromorphic balance equations in the IBVP requires the construction of special elements based on appropriate mixed variational forms as the Hu-Washizu form, with a particular decomposition of the transformation gradients.

2.5.1 Formulation of the mixed variational form

As already presented, Fish and Belytschko (Fish and Belytschko, 1990) proposed an extension of Hu-Washizu's variational principle in the case of nonlinear solids mechanics. We start from there in order to derive the following form for our micromorphic IBVP:

$$\delta\pi\left(\vec{u},\underline{\dot{e}}^{a},\underline{\sigma}^{a}\right) = \int_{\Omega_{e}} \left(\delta\underline{\dot{e}}^{a}\right)^{T} : \underline{\sigma}^{a} d\Omega + \delta\int_{\Omega_{e}} \left(\underline{\sigma}^{a}\right)^{T} : \left(\vec{\nabla}_{s}\vec{u} - \underline{\dot{e}}^{a}\right) d\Omega + \int_{\Omega_{e}} \left[\rho\left(\vec{u} - \vec{f}^{u}\right) \cdot \delta\vec{u}\right] dV - \int_{\Gamma_{e}} \left(\vec{F}^{u} \cdot \delta\vec{u}\right) dS = 0$$
(2.32)

where, δ denotes a variation, $\underline{\dot{e}}^a$ is the assumed strain rate, $\underline{\sigma}^a$ the assumed stress evaluated by the constitutive law and $\nabla_s \vec{\dot{u}}$ the symmetric part of the velocity gradient.

2.5.1.1 A 4-node quadrilateral assumed strain element

We have impemented in Abaqus/Explicit[®] a quadrilateral 2D element initially proposed by Wang (Wang et. al, 2004). This element is called 'assumed strain' element and it is defined in such a way to avoid the occurrence of volumetric expansion locking and shear locking.

We discretize this variational form in an elementary domain Ω_e and we focus on a typical element (e) between the instants t_n and t_{n+1} by switching to matrix notations. Simo and Hughes (Simo and Hughes, 1998), suggested the projection of the discretized velocity gradient $\begin{bmatrix} B_{n+\frac{1}{2}} \end{bmatrix}$, so that $\{\nabla_s(\vec{u^e})\}_{n+1} = \begin{bmatrix} B_{n+\frac{1}{2}} \end{bmatrix}\{\dot{u}^e\}_{n+\frac{1}{2}}$. This new operator $\begin{bmatrix} B_{n+\frac{1}{2}}^a \end{bmatrix}$ is chosen in a way to be able to evaluate the assumed strains rate tensor $\{\dot{\varepsilon}^a\}_{n+1} = \begin{bmatrix} B_{n+\frac{1}{2}} \end{bmatrix}\{\dot{u}^e\}_{n+\frac{1}{2}}$ as well as generate an assumed stress tensor $\{\sigma_{n+1}^a\}$ orthogonal to the difference between the symmetric part of the velocity gradient and the assumed strain rate tensor. This choice allows us to simplify the Hu-Washizu variational principle and rewrite it under the following form:

$$\delta\pi(\underline{\dot{\varepsilon}}^{a}) = \int_{\Omega_{\epsilon}} \left(\delta\underline{\dot{\varepsilon}}^{a}\right)^{T} : \underline{\sigma}^{a} d\Omega + \int_{\Omega_{\epsilon}} \left[\rho(\overline{\ddot{u}} - \overline{f}^{u}) \cdot \delta\overline{\dot{u}}\right] dV - \int_{\Gamma_{\epsilon}} \left(\overline{F}^{u} \cdot \delta\overline{\dot{u}}\right) dS = 0$$
(2.33)



Figure 2.2: The quadrilateral element

The isoparametric shape functions of the element for the displacement field in the reference space are expressed as:

$$N_i(\xi,\eta) = \frac{1}{4}(1+\xi_i\xi)(1+\eta_i\eta)$$
(2.34)

and for the micromorphic fields as:

$$N_{\bar{d}_{i}}(\xi,\eta) = N_{\bar{r}_{i}}(\xi,\eta) = \frac{1}{4}(1+\xi_{i}\xi)(1+\eta_{i}\eta)$$
(2.35)

All the above shape functions can be also expanded in terms of a set of orthogonal base vectors as:

$$\{N(\xi,\eta)\} = \{N_{\bar{d}}(\xi,\eta)\} = \{N_{\bar{r}}(\xi,\eta)\} = \frac{1}{4}\{s\} + \frac{1}{4}\{\xi\}\xi + \frac{1}{4}\{\eta\}\eta + \frac{1}{4}\{h\}\xi\eta$$
(2.36)

where

$$\langle \boldsymbol{\xi} \rangle = \langle -1 \quad 1 \quad 1 \quad -1 \rangle \qquad \langle \boldsymbol{\eta} \rangle = \langle -1 \quad -1 \quad 1 \quad 1 \rangle \tag{2.37}$$

are the vectors of the nodal co-ordinates in the reference space and

$$\langle s \rangle = \langle 1 \quad 1 \quad 1 \quad 1 \rangle \quad \langle h \rangle = \langle 1 \quad -1 \quad 1 \quad -1 \rangle$$
(2.38)

being the translation and the hourglass vectors respectively.

To ensure the objectivity conditions of the model, we use a corotational system defined in the centroid of the element (see Fig. 2.3) built by the shape functions defined in the reference space (ξ, η) . The orientation of this corotational triad is governed by the orthogonal rotation tensor \underline{Q} which is expressed under the following form:

$$\begin{bmatrix} Q \end{bmatrix} = \begin{bmatrix} e_1^x & e_1^y \\ e_2^x & e_2^y \end{bmatrix}, \quad \vec{g}_1 = \frac{\partial x}{\partial \xi} \vec{x} + \frac{\partial x}{\partial \eta} \vec{y}, \quad \vec{e}_1 = \frac{\vec{g}_1}{\|\vec{g}_1\|}, \quad \vec{e}_2 \cdot \vec{e}_1 = 0, \quad \|\vec{e}_2\| = 1$$
(2.39)



Figure 2.3: Corotational definition of the Q4-URI element

All the operators in the form of a vector or matrix and the various mechanical fields (stress tensor, strain rate tensor, internal variable tensor) will be transported in the corotational coordinate frame in order to guarantee the objectivity of the tensorial increments. Only the nodal elementary forces (internal and external) will be turned at the end of the calculation in the global reference of the part.

The jacobian matrix [J] is written in this 2D case as:

$$\begin{bmatrix} J \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{4}(\langle \boldsymbol{\xi} \rangle + \eta \langle \boldsymbol{h} \rangle)\{\mathbf{X}\} & \frac{1}{4}(\langle \boldsymbol{\xi} \rangle + \eta \langle \boldsymbol{h} \rangle)\{\mathbf{Y}\} \\ \frac{1}{4}(\langle \eta \rangle + \boldsymbol{\xi} \langle \boldsymbol{h} \rangle)\{\mathbf{X}\} & \frac{1}{4}(\langle \eta \rangle + \boldsymbol{\xi} \langle \boldsymbol{h} \rangle)\{\mathbf{Y}\} \end{bmatrix}$$
(2.40)

with $\langle X \rangle = \langle x_1 \ x_2 \ x_3 \ x_4 \rangle$ and $\langle Y \rangle = \langle y_1 \ y_2 \ y_3 \ y_4 \rangle$ being the coordinates of the node in the corotational coordinate frame. The determinant of the jacobian can be written explicitly under the following form:

$$J_{\xi\eta} = \det\left(\left[J\right]\right) = J_0 + J_{\xi}\xi + J_{\eta}\eta$$
(2.41)

where

$$J_{0} = \frac{1}{16} \Big[\big(\langle \xi \rangle \{X\} \big) \big(\langle \eta \rangle \{Y\} \big) - \big(\langle \xi \rangle \{Y\} \big) \big(\langle \eta \rangle \{X\} \big) \Big] = \frac{1}{4} A$$

$$J_{\xi} = \frac{1}{16} \Big[\big(\langle h \rangle \{Y\} \big) \big(\langle \xi \rangle \{X\} \big) - \big(\langle h \rangle \{X\} \big) \big(\langle \xi \rangle \{Y\} \big) \Big]$$

$$J_{\eta} = \frac{1}{16} \Big[\big(\langle h \rangle \{X\} \big) \big(\langle \eta \rangle \{Y\} \big) - \big(\langle h \rangle \{Y\} \big) \big(\langle \eta \rangle \{X\} \big) \Big]$$
(2.42)

where A is the area of the element.

The gradient operator $\begin{bmatrix} B_{n+\frac{1}{2}}^{a} \end{bmatrix}$ is decomposed into two parts; the strain tensor $\begin{bmatrix} B_{n+\frac{1}{2}}^{0} \end{bmatrix}$ which gives the overall strain rate components $\{\dot{\varepsilon}^{a}\}_{n+1}$ in the center of the element ($\xi = \eta = 0$); and the tensor $\begin{bmatrix} B_{n+\frac{1}{2}}^{ha} \end{bmatrix} = \xi \begin{bmatrix} B_{n+\frac{1}{2}}^{\xi} \end{bmatrix} + \eta \begin{bmatrix} B_{n+\frac{1}{2}}^{\eta} \end{bmatrix}$ that completes the formulation of the strain rate tensor components for any other point of the element and can be expressed as:

$$\begin{bmatrix} B_{n+\frac{1}{2}}^{0} \end{bmatrix} = \begin{bmatrix} \langle b_{x} \rangle & \langle 0 \rangle \\ \langle 0 \rangle & \langle b_{y} \rangle \\ \langle b_{y} \rangle & \langle b_{x} \rangle \end{bmatrix}$$
(2.43)

with

$$\{b_{x}\} = \frac{1}{4A} \Big[(\langle \eta \rangle \{Y\}) \{\xi\} - (\langle \xi \rangle \{Y\}) \{\eta\} \Big] = \frac{\partial \{N\}}{\partial x} \Big|_{(\xi=\eta=0)}$$

$$\{b_{y}\} = \frac{1}{4A} \Big[- (\langle \eta \rangle \{X\}) \{\xi\} + (\langle \xi \rangle \{X\}) \{\eta\} \Big] = \frac{\partial \{N\}}{\partial y} \Big|_{(\xi=\eta=0)}$$

$$(2.44)$$

where (x, y) the co-ordinates of an arbitrary point in the corotational space frame.

Finally, the assumed strain rate field can be expressed under the following analytical form:

$$\begin{cases} \Delta \varepsilon_x^0 \\ \Delta \varepsilon_y^0 \\ 2\Delta \varepsilon_{xy}^0 \\ 2\Delta \varepsilon_{xy}^0 \end{cases} = \begin{bmatrix} B_{n+\frac{1}{2}}^0 \\ B_{n+\frac{1}{2}}^0 \end{bmatrix} \{ \Delta U_{n+1} \}$$
(2.45)

with

$$\langle \Delta U_{n+1} \rangle = \langle \Delta u_{x1} \quad \Delta u_{x2} \quad \Delta u_{x3} \quad \Delta u_{x4} \quad \Delta u_{y1} \quad \Delta u_{y2} \quad \Delta u_{y3} \quad \Delta u_{y4} \rangle_{n+1}$$

$$\langle \Delta U_{n+1} \rangle = \langle \langle \Delta U_x \rangle \quad \langle \Delta U_y \rangle \rangle$$

$$(2.46)$$

The form of the second part of the gradient operator is based on the expression developed by Wang (Wang et. al, 2001). To eliminate volumetric and shear locking phenomena, an assumed strain rate field is proposed to replace the classical strain rate fields obtained by the symmetric gradient operator. Hence, the volumetric energy and the shear energy corresponding to the hourglass terms tend to be zero. So, we use the following assumed strain rate field:

$$\left\{ \dot{\varepsilon}^{a} \right\}_{n+1} = \begin{cases} \Delta \varepsilon_{x}^{a} \\ \Delta \varepsilon_{y}^{a} \\ 2\Delta \varepsilon_{xy}^{a} \end{cases} = \begin{bmatrix} B_{n+\frac{1}{2}}^{a} \end{bmatrix} \left\{ \dot{u}^{e} \right\}_{n+\frac{1}{2}} = \begin{bmatrix} B_{n+\frac{1}{2}}^{0} \end{bmatrix} \left\{ \dot{u}^{e} \right\}_{n+\frac{1}{2}} + \left\{ \xi \begin{bmatrix} B_{n+\frac{1}{2}}^{\xi} \end{bmatrix} + \eta \begin{bmatrix} B_{n+\frac{1}{2}}^{\eta} \end{bmatrix} \right\} \left\{ \dot{u}^{e} \right\}_{n+\frac{1}{2}} =$$

$$\left\{ \Delta \varepsilon_{x}^{0} \\ \Delta \varepsilon_{y}^{0} \\ 2\Delta \varepsilon_{xy}^{0} \end{bmatrix} + \left\{ \Delta \varepsilon_{x}^{ha} \\ \Delta \varepsilon_{y}^{ha} \\ 2\Delta \varepsilon_{xy}^{ha} \end{bmatrix}$$

$$(2.47)$$

with

$$\begin{bmatrix} B_{n+\frac{1}{2}}^{ha} \end{bmatrix} = \xi \begin{bmatrix} B_{n+\frac{1}{2}}^{\xi} \end{bmatrix} + \eta \begin{bmatrix} B_{n+\frac{1}{2}}^{\eta} \end{bmatrix} = \xi \begin{bmatrix} e_1 \langle b_{x\xi} \rangle & e_2 \langle b_{y\xi} \rangle \\ e_2 \langle b_{x\xi} \rangle & e_1 \langle b_{y\xi} \rangle \\ e_3 \langle b_{y\xi} \rangle & e_3 \langle b_{x\xi} \rangle \end{bmatrix} + \eta \begin{bmatrix} e_1 \langle b_{x\eta} \rangle & e_2 \langle b_{y\eta} \rangle \\ e_2 \langle b_{x\eta} \rangle & e_1 \langle b_{y\eta} \rangle \\ e_3 \langle b_{y\xi} \rangle \end{bmatrix}$$
(2.48)

where

$$\{b_{x\xi}\} = -\frac{1}{4J_{\xi\eta}} (\langle \xi \rangle \{Y\}) \{\gamma\} \quad \{b_{x\eta}\} = \frac{1}{4J_{\xi\eta}} (\langle \eta \rangle \{Y\}) \{\gamma\}$$

$$\{b_{y\xi}\} = \frac{1}{4J_{\xi\eta}} (\langle \xi \rangle \{X\}) \{\gamma\} \quad \{b_{y\eta}\} = -\frac{1}{4J_{\xi\eta}} (\langle \eta \rangle \{X\}) \{\gamma\}$$

$$(2.49)$$

The vector $\{\gamma\}$ is calculated by the following expression:

$$\{\boldsymbol{\gamma}\} = \frac{1}{4} \Big[\{h\} - (\langle h \rangle \{X\}) \{b_x\} - (\langle h \rangle \{Y\}) \{b_y\} \Big]$$
(2.50)

and it is called hourglass stabilization vector, identical to the $\{\gamma\}$ -projection operator proposed by Belytschko (Belytschko, 2001).

Different values of the parameters e_1 , e_2 and e_3 can be chosen. In the case of $e_1=1$, $e_2=0$ and $e_3=1$, we find the expression of the classical strain rate fields obtained by a symmetric gradient operator:

$$\left\{\nabla_{s}\left(\vec{u}^{\vec{e}}\right)\right\}_{n+1} = \begin{cases}\Delta\varepsilon_{x}^{0}\\\Delta\varepsilon_{y}^{0}\\2\Delta\varepsilon_{xy}^{0}\end{cases} + \begin{cases}\Delta\varepsilon_{x}^{h}\\\Delta\varepsilon_{y}^{h}\\2\Delta\varepsilon_{xy}^{h}\end{cases} \text{ with } \Delta\varepsilon_{y}^{h} = \left(\xi\left\langle b_{x\xi}\right\rangle + \eta\left\langle b_{x\eta}\right\rangle\right)\left\{\Delta U_{x}\right\} \\\Delta\varepsilon_{y}^{h} = \left(\xi\left\langle b_{y\xi}\right\rangle + \eta\left\langle b_{y\eta}\right\rangle\right)\left\{\Delta U_{y}\right\} \\2\Delta\varepsilon_{xy}^{h} = \left(\xi\left\langle b_{y\xi}\right\rangle + \eta\left\langle b_{y\eta}\right\rangle\right)\left\{\Delta U_{x}\right\} + \left(\xi\left\langle b_{x\xi}\right\rangle + \eta\left\langle b_{x\eta}\right\rangle\right)\left\{\Delta U_{y}\right\} \end{cases}$$

$$\left\{2\Delta\varepsilon_{xy}^{h} = \left(\xi\left\langle b_{y\xi}\right\rangle + \eta\left\langle b_{y\eta}\right\rangle\right)\left\{\Delta U_{x}\right\} + \left(\xi\left\langle b_{x\xi}\right\rangle + \eta\left\langle b_{x\eta}\right\rangle\right)\left\{\Delta U_{y}\right\} \right\}$$

But, to eliminate volumetric locking phenomena, the volume dilatation rate corresponding to the hourglass terms can be expressed as:

$$\dot{\theta}_{v}^{ha} = \Delta \varepsilon_{x}^{ha} + \Delta \varepsilon_{y}^{ha} = (e_{1} + e_{2}) \left(\left(\xi \left\langle b_{x\xi} \right\rangle + \eta \left\langle b_{x\eta} \right\rangle \right) \left\{ \Delta U_{x} \right\} + \left(\xi \left\langle b_{y\xi} \right\rangle + \eta \left\langle b_{y\eta} \right\rangle \right) \left\{ \Delta U_{y} \right\} \right) \quad (a)$$

$$\dot{\theta}_{v}^{ha} = (e_{1} + e_{2}) \left(\Delta \varepsilon_{x}^{h} + \Delta \varepsilon_{y}^{h} \right) = (e_{1} + e_{2}) \dot{\theta}_{v}^{h} \qquad (b)$$

If we choose $e_1 = -e_2 = \frac{1}{2}$ the volumetric locking associated to the new assumed strain rate operator can be eliminated.

The shear strain rate corresponding to the hourglass terms can be expressed as: $2\Delta \varepsilon_{xy}^{ha} = e_3 2\Delta \varepsilon_{xy}^{h}$ If we let $e_3 = 0$, the shear locking can also be eliminated.

The new expression of the assumed strain rate operator as the following expression: $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix}$

$$\begin{bmatrix} B_{n+\frac{1}{2}}^{ha} \end{bmatrix} = \xi \begin{bmatrix} B_{n+\frac{1}{2}}^{\xi} \end{bmatrix} + \eta \begin{bmatrix} B_{n+\frac{1}{2}}^{\eta} \end{bmatrix} = \xi \begin{bmatrix} \frac{1}{2} \langle b_{x\xi} \rangle & -\frac{1}{2} \langle b_{y\xi} \rangle \\ -\frac{1}{2} \langle b_{x\xi} \rangle & \frac{1}{2} \langle b_{y\xi} \rangle \\ \langle 0 \rangle & \langle 0 \rangle \end{bmatrix} + \eta \begin{bmatrix} \frac{1}{2} \langle b_{x\eta} \rangle & -\frac{1}{2} \langle b_{y\eta} \rangle \\ -\frac{1}{2} \langle b_{y\eta} \rangle \\ \langle 0 \rangle & \langle 0 \rangle \end{bmatrix}$$
(2.53)

Finally, the assumed strain rate field can be expressed under this following form:

$$\forall \xi \in \begin{bmatrix} -1 & 1 \end{bmatrix} and \eta \in \begin{bmatrix} -1 & 1 \end{bmatrix} \quad \left\{ \Delta \underline{\varepsilon}_{n+1}^{a} \right\}_{\xi,\eta} = \left\{ \begin{array}{c} \Delta \varepsilon_{x}^{a} \\ \Delta \varepsilon_{y}^{a} \\ 2\Delta \varepsilon_{xy}^{a} \end{array} \right\} = \left(\begin{bmatrix} B_{n+\frac{1}{2}}^{0} \end{bmatrix} + \xi \begin{bmatrix} B_{n+\frac{1}{2}}^{\xi} \end{bmatrix} + \eta \begin{bmatrix} B_{n+\frac{1}{2}}^{\eta} \end{bmatrix} \right) \left\{ \Delta U_{n+1} \right\}$$

$$(2.54)$$

~

The assumed Cauchy stress tensor at the n+1 step is updated from calling the elastoplastic behavior model:

$$\left\{\underline{\sigma}_{n+1}^{a}\right\}_{\xi,\eta} = \begin{cases} \sigma_{x} \\ \sigma_{y} \\ \sigma_{xy} \end{cases} = \left\{\underline{\sigma}_{n}^{a}\right\}_{\xi,\eta} + \left[C^{ep}\right] \left\{\Delta\underline{\varepsilon}_{n+1}^{a}\right\}_{\xi,\eta} = \left\{\underline{\sigma}_{n+1}^{0}\right\}_{0,0} + \xi\left\{\underline{\sigma}_{n+1}^{\xi}\right\}_{\xi,\eta} + \eta\left\{\underline{\sigma}_{n+1}^{\eta}\right\}_{\xi,\eta}$$
(2.55)

where $\left\lceil C^{ep} \right\rceil$ is the material tangential modulus matrix of an elastolastic behavior model.

The vector of the nodal internal forces is then evaluated by mixed integration Gauss point as:

$$\left\{F_{\text{int}}^{e}\right\}_{n+1} = \left\{F_{\text{int}}^{0}\right\}_{n+1} + \left\{F_{\text{int}}^{a}\right\}_{n+1}$$
(2.56)

with

$$\left\{F_{\text{int}}^{0}\right\}_{n+1} = \int_{\Omega_{e}} \left[B_{n+\frac{1}{2}}^{0}\right]^{T} \left\{\underline{\sigma}_{n+1}^{a}\right\}_{0,0} dV = \int_{-1-1}^{+1+1} \left[B_{n+\frac{1}{2}}^{0}\right]^{T} \left\{\underline{\sigma}_{n+1}^{a}\right\}_{0,0} J_{0} d\xi d\eta = \left[B_{n+\frac{1}{2}}^{0}\right]^{T} \left\{\underline{\sigma}_{n+1}^{a}\right\}_{0,0} A$$
(2.57)

$$\left\{F_{\text{int}}^{a}\right\}_{n+1} = \underbrace{\int_{\Omega_{e}} \left(\xi \left[B_{n+\frac{1}{2}}^{\xi}\right]^{T} + \eta \left[B_{n+\frac{1}{2}}^{\eta}\right]^{T}\right) \left\{\sigma_{n+1}^{a}\right\}_{\xi,\eta} dV = \int_{-1-1}^{+1+1} \left(\xi \left[B_{n+\frac{1}{2}}^{\xi}\right]^{T} + \eta \left[B_{n+\frac{1}{2}}^{\eta}\right]^{T}\right) \left\{\sigma_{n+1}^{a}\right\}_{\xi,\eta} J_{\xi\eta} d\xi d\eta$$
(2.58)

$$\left\{F_{\text{int}}^{a}\right\}_{n+1} = \sum_{I=1}^{4} \omega_{I}^{PG} \left[\xi_{I}^{PG} \left[B_{n+\frac{1}{2}}^{\xi}\right]^{T} + \eta_{I}^{PG} \left[B_{n+\frac{1}{2}}^{\eta}\right]^{T}\right] \left\{\sigma_{n+1}^{a}\right\}_{\xi_{I}^{PG}, \eta_{I}^{PG}} J_{\xi_{I}^{PG} \eta_{I}^{PG}} J_{\xi_{I}^{PG} \eta_{I}^{PG}}$$
(2.59)

with

$$\omega_{1}^{PG} = \omega_{2}^{PG} = \omega_{3}^{PG} = \omega_{4}^{PG} = 1.$$

$$\xi_{1}^{PG} = -\frac{1}{\sqrt{3}}, \ \xi_{2}^{PG} = \frac{1}{\sqrt{3}}, \ \xi_{3}^{PG} = \frac{1}{\sqrt{3}}, \ \xi_{4}^{PG} = -\frac{1}{\sqrt{3}}$$

$$\eta_{1}^{PG} = -\frac{1}{\sqrt{3}}, \ \eta_{2}^{PG} = -\frac{1}{\sqrt{3}}, \ \eta_{3}^{PG} = \frac{1}{\sqrt{3}}, \ \eta_{4}^{PG} = \frac{1}{\sqrt{3}}$$
(2.60)

To add of the contribution of the micromorphic variables, we need to define a gradient operator of the scalars:

$$\vec{\nabla}(\vec{d}^{e}) = \begin{bmatrix} \vec{B}_{\vec{d}}^{e} \end{bmatrix} \{ \vec{d}_{i}^{e} \} \quad \text{with} \quad \left\langle \vec{d}_{i}^{e} \right\rangle = \left\langle \vec{d}_{1} \quad \vec{d}_{2} \quad \vec{d}_{3} \quad \vec{d}_{4} \right\rangle$$
$$\vec{\nabla}(\vec{r}^{e}) = \begin{bmatrix} \vec{B}_{\vec{r}}^{e} \end{bmatrix} \{ \vec{r}_{i}^{e} \} \quad \text{with} \quad \left\langle \vec{r}_{i}^{e} \right\rangle = \left\langle \vec{r}_{1} \quad \vec{r}_{2} \quad \vec{r}_{3} \quad \vec{r}_{4} \right\rangle$$

We use the same operator to build the gradient of the micromorphic damage and micromorphic isotropic hardening:

$$\begin{bmatrix} \breve{B}_{\bar{d}}^{e} \end{bmatrix} = \begin{bmatrix} \breve{B}_{\bar{r}}^{e} \end{bmatrix} = \begin{bmatrix} \langle b_{x} \rangle \\ \langle b_{y} \rangle \end{bmatrix} + \xi \begin{bmatrix} \langle b_{x\xi} \rangle \\ \langle b_{y\xi} \rangle \end{bmatrix} + \eta \begin{bmatrix} \langle b_{x\eta} \rangle \\ \langle b_{y\eta} \rangle \end{bmatrix} = \begin{bmatrix} B_{g}^{0} \end{bmatrix} + \xi \begin{bmatrix} B_{g}^{\xi} \end{bmatrix} + \eta \begin{bmatrix} B_{g}^{\eta} \end{bmatrix}$$
(2.61)

The internal micromorphic damage force vector is then computed:

$$\left\{ \vec{F}_{\text{int}-\vec{d}}^{e} \right\} = \int_{\Omega_{r}^{e}} \left(\ell_{\vec{d}}^{2} \left[\vec{B}_{\vec{d}}^{e} \right]^{T} \left[\vec{B}_{\vec{d}}^{e} \right] + \left\{ \vec{N}_{\vec{d}}^{e} \right\} \left\langle \vec{N}_{\vec{d}}^{e} \right\rangle \right) \left\{ \vec{d}_{i}^{e} \right\} J_{\nu}^{e} dV =$$

$$\int_{\underline{J}-1-1}^{+1} \left(\ell_{\vec{d}}^{2} \left[\vec{B}_{\vec{d}}^{e} \right]^{T} \left[\vec{B}_{\vec{d}}^{e} \right] + \left\{ \vec{N}_{\vec{d}}^{e} \right\} \left\langle \vec{N}_{\vec{d}}^{e} \right\rangle \right) J_{\xi\eta} d\xi d\eta \left\{ \vec{d}_{i}^{e} \right\}$$

$$(2.62)$$

4 Gauss integration points are used for the integration:

$$\begin{bmatrix} K_{\bar{d}} \end{bmatrix} = \sum_{I=1}^{4} \omega_{I}^{PG} \begin{bmatrix} \left(\begin{bmatrix} B_{g}^{0} \end{bmatrix}^{T} + \xi_{I}^{PG} \begin{bmatrix} B_{g}^{\xi} \end{bmatrix}^{T} + \eta_{I}^{PG} \begin{bmatrix} B_{g}^{\eta} \end{bmatrix}^{T} \right) \left(\begin{bmatrix} B_{g}^{0} \end{bmatrix}^{T} + \xi_{I}^{PG} \begin{bmatrix} B_{g}^{\xi} \end{bmatrix}^{T} + \eta_{I}^{PG} \begin{bmatrix} B_{g}^{\eta} \end{bmatrix}^{T} \right) + \left\{ \breve{N}_{\bar{d}}^{e} \right\}_{\xi_{I}^{PG}, \eta_{I}^{PG}}^{e} \left\langle \breve{N}_{\bar{d}}^{e} \right\rangle_{\xi_{I}^{PG}, \eta_{I}^{PG}}^{e} \end{bmatrix} J_{\xi_{I}^{PG}, \eta_{I}^{PG}}^{e}$$

$$(2.63)$$

The external micromorphic damage force vector is also integrated with 4 Gauss integration points:

$$\left\{ \breve{F}_{ext-\bar{d}}^{e} \right\} = \int_{\Omega_{r}^{e}} \left\{ \breve{N}_{\bar{d}}^{e} \right\} d^{e} J_{\nu}^{e} dV = \int_{-1-1}^{+1+1} \left\{ \breve{N}_{\bar{d}}^{e} \right\} d^{e} J_{\xi\eta} d\xi d\eta = \sum_{I=1}^{4} \omega_{I}^{PG} \left\{ \breve{N}_{\bar{d}}^{e} \right\}_{\xi_{I}^{PG} \eta_{I}^{PG}} d_{\xi_{I}^{PG} \eta_{I}^{PG}} J_{\xi_{I}^{PG} \eta_{I}^{PG}}$$
(2.64)

In the same way, we compute the internal and external force associated to the micromorphic isotropic hardening:

$$\left\{ \breve{F}_{\text{int}-\bar{r}}^{e} \right\} = \int_{\Omega_{r}^{e}} \left(1 - \left(d^{e} \right)^{\gamma} \right) \left(\ell_{\bar{r}}^{2} \left[\breve{B}_{\bar{r}}^{e} \right]^{T} \left[\breve{B}_{\bar{r}}^{e} \right] + \left\{ \breve{N}_{\bar{r}}^{e} \right\} \left\langle \breve{N}_{\bar{r}}^{e} \right\rangle \right) \left\{ \breve{r}_{n}^{e} \right\} J_{\nu}^{e} dV = \\ \underbrace{\int_{-1-1}^{+1} \left(1 - \left(d^{e} \right)^{\gamma} \right) \left(\ell_{\bar{r}}^{2} \left[\breve{B}_{\bar{r}}^{e} \right]^{T} \left[\breve{B}_{\bar{r}}^{e} \right] + \left\{ \breve{N}_{\bar{r}}^{e} \right\} \left\langle \breve{N}_{\bar{r}}^{e} \right\rangle \right) J_{\bar{\xi}\eta} d\xi d\eta \left\{ \breve{r}_{i}^{e} \right\} }_{\left[\breve{K}_{r}\right]}$$

$$(2.65)$$

$$\begin{bmatrix} K_{\bar{r}} \end{bmatrix} = \sum_{I=1}^{4} \omega_I^{PG} \begin{bmatrix} \left(\begin{bmatrix} B_g^0 \end{bmatrix}^T + \xi_I^{PG} \begin{bmatrix} B_g^{\xi} \end{bmatrix}^T + \eta_I^{PG} \begin{bmatrix} B_g^{\eta} \end{bmatrix}^T \right) \left(\begin{bmatrix} B_g^0 \end{bmatrix}^T + \xi_I^{PG} \begin{bmatrix} B_g^{\xi} \end{bmatrix}^T + \eta_I^{PG} \begin{bmatrix} B_g^{\eta} \end{bmatrix}^T \right) + \\ \begin{bmatrix} \left(\bar{N}_{\bar{r}}^e \right)_{\xi_I^{PG}, \eta_I^{PG}} & \left(\bar{N}_{\bar{r}}^e \right)_{\xi_I^{PG}, \eta_I^{PG}} \end{bmatrix} \end{bmatrix} \begin{bmatrix} 1 - \left(d_{\xi_I^{PG}, \eta_I^{PG}} \right)^{\gamma} \right) J_{\xi_I^{PG}, \eta_I^{PG}} \end{bmatrix}$$
(2.66)

$$\left\{ \vec{F}_{ext-\bar{r}}^{e} \right\} = \int_{\Omega_{r}^{e}} \left(1 - \left(d^{e} \right)^{\gamma} \right) r^{e} \left\{ \vec{N}_{\bar{r}}^{e} \right\} J_{\nu}^{e} dV = \int_{-1-1}^{+1+1} \left\{ \vec{N}_{\bar{r}}^{e} \right\} \left(1 - \left(d^{e} \right)^{\gamma} \right) r^{e} J_{\xi\eta} d\xi d\eta =$$

$$\sum_{I=1}^{4} \omega_{I}^{PG} \left\{ \vec{N}_{\bar{r}}^{e} \right\}_{\xi_{I}^{PG} \eta_{I}^{PG}} \left(1 - \left(d_{\xi_{I}^{PG} \eta_{I}^{PG}} \right)^{\gamma} \right) r_{\xi_{I}^{PG} \eta_{I}^{PG}} J_{\xi_{I}^{PG} \eta_{I}^{PG}} J_{\xi_{I}^{PG} \eta_{I}^{PG}}$$

$$(2.67)$$

2.5.1.2 A 4-node quadrilateral axisymmetric element

By following the assumed strain method introduced above, we develop a simple four-node axisymmetric element (Fredriksson and Ottosen, 2006). The strain-driven format obtained is well suited for materials with non-linear stress-strain relations. For this element, the orthogonal projection of the stress and strain fields through the B-bar approach by Simo and Hughes (Simo, 1998) and the $\{\gamma\}$ -projection operator suggested by Flanagan and Belytschko (Flanagan and Belytschko, 1981) are applied.

The shape functions remain the same and the mapping of the four-node quiadrilateral element is defined by:

$$r = \langle N(\xi, \eta) \rangle \{r\}, \quad z = \langle N(\xi, \eta) \rangle \{z\}$$
(2.68)

where r and z denote the polar co-ordinates and the nodal coordinates:

$$\langle r \rangle = \langle r_1 \ r_2 \ r_3 \ r_4 \rangle, \quad \langle z \rangle = \langle z_1 \ z_2 \ z_3 \ z_4 \rangle$$
 (2.69)

Similarly, the radial displacement u_r and the axial displacement u_z are approximated by:

$$\{u\} = \begin{cases} u_r \\ u_z \end{cases} = \begin{bmatrix} \langle N \rangle & \langle 0 \rangle \\ \langle 0 \rangle & \langle N \rangle \end{bmatrix} \begin{cases} \{U_r\} \\ \{U_z\} \end{cases}$$
(2.70)

where $\langle U_r \rangle = \langle u_{r1} u_{r2} u_{r3} u_{r3} \rangle$ and $\langle U_z \rangle = \langle u_{z1} u_{z2} u_{z3} u_{z4} \rangle$ contain the nodal displacements in the r and z direction respectively.

The strain rate components become:

.

$$\left\{\Delta\underline{\varepsilon}_{n+1}^{a}\right\}_{\xi,\eta} = \begin{cases}\Delta\varepsilon_{r}\\\Delta\varepsilon_{z}\\\Delta\varepsilon_{r}\\\Delta\varepsilon_{\theta}\end{cases} = \begin{cases}\Delta\varepsilon_{r}\\\Delta\varepsilon_{z}\\\Delta\varepsilon_{\theta}\end{cases} + \begin{cases}0\\0\\\Delta\varepsilon_{r}\\0\end{cases} + \begin{cases}0\\0\\\Delta\varepsilon_{\theta}\end{cases} = \begin{cases}\left\{\Delta\underline{\varepsilon}_{n+1}^{Plan}\right\}_{\xi,\eta}\\0\\\Delta\varepsilon_{\theta}\end{cases} + \begin{cases}0\}\\\Delta\varepsilon_{\theta}\end{cases}$$
(2.71)

The strain rate tensor $\left\{\Delta \underline{\mathcal{E}}_{n+1}^{Plan}\right\}_{\xi,\eta}$ is computed with the same equation used for the 2D plane strain element:

$$\left\{\Delta\underline{\varepsilon}_{n+1}^{plan}\right\}_{\xi,\eta} = \left[B_{n+\frac{1}{2}}^{plan}\right] \left\{\Delta U_{n+1}^{axi}\right\} = \left(\left[B_{n+\frac{1}{2}}^{0}\right] + \xi\left[B_{n+\frac{1}{2}}^{\xi}\right] + \eta\left[B_{n+\frac{1}{2}}^{\eta}\right]\right) \left\{\Delta U_{n+1}^{axi}\right\}$$
(2.72)

with

$$\left\{\Delta U_{n+1}^{axi}\right\} = \left\langle\Delta u_{r1} \quad \Delta u_{r2} \quad \Delta u_{r3} \quad \Delta u_{r4} \quad \Delta u_{z1} \quad \Delta u_{z2} \quad \Delta u_{z3} \quad \Delta u_{z4}\right\rangle_{n+1}$$
(2.73)

And the ortho-radial strain rate component is defined by

$$\Delta \varepsilon_{\theta} = \left\langle B_{n+\frac{1}{2}}^{\theta} \right\rangle \left\{ \Delta U_{n+1} \right\} = \frac{1}{r} \left\langle N_{1} \quad N_{2} \quad N_{3} \quad N_{4} \quad 0 \quad 0 \quad 0 \quad 0 \right\rangle \left\{ \Delta U_{n+1} \right\}$$
(2.74)

By following Flangan and Belytschko, we introduce the average operator:

$$\left[\hat{B}_{n+\frac{1}{2}}^{plan}\right] = \frac{1}{V} \int_{A} \left[B_{n+\frac{1}{2}}^{plan}\right] r \, dA \tag{2.75}$$

and

$$\left\langle \hat{B}_{n+\frac{1}{2}}^{\theta} \right\rangle = \frac{1}{V} \int_{A} \left\langle B_{n+\frac{1}{2}}^{\theta} \right\rangle r \, dA \tag{2.76}$$

where $V = \int_{A} r dA$ is the element volume per radian.

The orthoradial operator becomes:

$$\left\langle \hat{B}_{n+\frac{1}{2}}^{\theta} \right\rangle = \frac{1}{V} \left\langle \left\langle c \right\rangle \quad \left\langle 0 \right\rangle \right\rangle$$

$$\text{with } \left\{ c \right\} = \int_{A} \left\{ N \right\} dA \,.$$

$$(2.77)$$

So, the different gradient operators can be now expressed under this form:

$$\begin{bmatrix} \boldsymbol{B}_{n+\frac{1}{2}}^{plan} \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{B}}_{n+\frac{1}{2}}^{plan} \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_{n+\frac{1}{2}}^{plan} \end{bmatrix} - \begin{bmatrix} \hat{\boldsymbol{B}}_{n+\frac{1}{2}}^{plan} \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{B}}_{n+\frac{1}{2}}^{plan} \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{B}}_{n+\frac{1}{2}}^{pl} \end{bmatrix}$$
(2.78)

$$\left\langle B_{n+\frac{1}{2}}^{\theta} \right\rangle = \left\langle \hat{B}_{n+\frac{1}{2}}^{\theta} \right\rangle + \left\langle \underline{B_{n+\frac{1}{2}}^{\theta}} \right\rangle - \left\langle \hat{B}_{n+\frac{1}{2}}^{\theta} \right\rangle = \left\langle \hat{B}_{n+\frac{1}{2}}^{\theta} \right\rangle + \left\langle \tilde{B}_{n+\frac{1}{2}}^{\theta} \right\rangle$$

$$(2.79)$$

The assumed Cauchy stress tensor at the n+1 step is updated as we call the elastoplastic behavior model:

$$\left\{\underline{\sigma}_{n+1}^{a}\right\}_{\xi,\eta} = \begin{cases} \sigma_{r} \\ \sigma_{z} \\ \sigma_{rz} \\ \sigma_{\theta} \end{cases} = \left\{\underline{\sigma}_{n}^{a}\right\}_{\xi,\eta} + \left[C^{ep}\right] \left\{\Delta\underline{\varepsilon}_{n+1}^{a}\right\}_{\xi,\eta}$$
(2.80)

where $\left[C^{ep}\right]$ is the material tangential modulus matrix of an elastolastic behavior model.

The vector of the nodal internal forces is again evaluated by this new expression:

$$\left\{F_{\text{int}}^{e}\right\}_{n+1} = \left\{\hat{F}_{\text{int}}^{plan}\right\}_{n+1} + \left\{\tilde{F}_{\text{int}}^{plan}\right\}_{n+1} + \left\{\hat{F}_{\text{int}}^{\theta}\right\}_{n+1} + \left\{\tilde{F}_{\text{int}}^{\theta}\right\}_{n+1}$$
(2.81)

with

$$\left\{\hat{F}_{\text{int}}^{plan}\right\}_{n+1} = \left[\hat{B}_{n+\frac{1}{2}}^{plan}\right]^T \left\{\underline{\sigma}_{n+1}^{plan}\right\}_{0,0} V$$
(2.82)

$$\left\{\tilde{F}_{\text{int}}^{plan}\right\}_{n+1} = \sum_{I=1}^{4} \omega_{I}^{PG} \left[\tilde{B}_{n+\frac{1}{2}}^{plan}\right] \left\{\sigma_{n+1}^{plan}\right\}_{\xi_{I}^{PG}, \eta_{I}^{PG}} r_{\xi_{I}^{PG} \eta_{I}^{PG}} J_{\xi_{I}^{PG} \eta_{I}^{PG}} J_{\xi_{I}^{PG} \eta_{I}^{PG}}$$
(2.83)

$$\left\{\hat{F}_{\text{int}}^{\theta}\right\}_{n+1} = \left\{\hat{B}_{n+\frac{1}{2}}^{\theta}\right\} \left(\sigma_{\theta}\right)_{n+1} V \tag{2.84}$$

$$\left\{\tilde{F}_{\text{int}}^{\theta}\right\}_{n+1} = \sum_{I=1}^{4} \omega_{I}^{PG} \left\{\tilde{B}_{n+\frac{1}{2}}^{\theta}\right\} (\sigma_{\theta})_{n+1} r_{\xi_{I}^{PG} \eta_{I}^{PG}} J_{\xi_{I}^{PG} \eta_{I}^{PG}}$$
(2.85)

For the four-node axisymmetric element, we only add the micromorphic damage as the new degree of freedom. The new expressions of internal and external micromorphic force vectors are:

$$\begin{bmatrix} K_{\bar{d}} \end{bmatrix} = \sum_{I=1}^{4} \omega_{I}^{PG} \begin{bmatrix} \left(\begin{bmatrix} B_{g}^{0} \end{bmatrix}^{T} + \xi_{I}^{PG} \begin{bmatrix} B_{g}^{\xi} \end{bmatrix}^{T} + \eta_{I}^{PG} \begin{bmatrix} B_{g}^{\eta} \end{bmatrix}^{T} \right) \left(\begin{bmatrix} B_{g}^{0} \end{bmatrix}^{T} + \xi_{I}^{PG} \begin{bmatrix} B_{g}^{\xi} \end{bmatrix}^{T} + \eta_{I}^{PG} \begin{bmatrix} B_{g}^{\eta} \end{bmatrix}^{T} \right) + \begin{bmatrix} \left\{ \breve{N}_{\bar{d}}^{e} \right\}_{\xi_{I}^{PG}, \eta_{I}^{PG}} \left\langle \breve{N}_{\bar{d}}^{e} \right\rangle_{\xi_{I}^{PG}, \eta_{I}^{PG}} \left\langle \breve{N}_{\bar{d}}^{e} \right\rangle_{\xi_{I}^{PG}, \eta_{I}^{PG}} \left\langle \breve{N}_{\bar{d}}^{e} \right\rangle_{\xi_{I}^{PG}, \eta_{I}^{PG}} \right)$$
(2.86)

The external micromorphic damage force vector is integrated with also 4 Gauss integration point:

$$\left\{ \breve{F}_{ext-\breve{d}}^{e} \right\} = \sum_{I=1}^{4} \omega_{I}^{PG} \left\{ \breve{N}_{\breve{d}}^{e} \right\}_{\xi_{I}^{PG} \eta_{I}^{PG}} d_{\xi_{I}^{PG} \eta_{I}^{PG}} r_{\xi_{I}^{PG} \eta_{I}^{PG}} J_{\xi_{I}^{PG} \eta_{I}^{PG}} J_{\xi_{I}^{PG} \eta_{I}^{PG}}$$
(2.87)

2.6 Global resolution scheme

2.6.1 Dynamic explicit analysis

Considering the explicit dynamic analysis procedure in Abaqus[®]/Explicit the solutions of the IBVP with the three forms presented above are obtained simultaneously by an explicit coupling, based upon the implementation of an explicit integration rule together with the use of diagonal or lumped

mass matrices. Many dynamic explicit schemes that appear in the literature (e.g., Hughes 1987, Belytschko, 2001) can be used to treat the different coupled problems.

For the present IBVP that consists of many equations, a sequential resolution method is preferred, meaning that the three problems are successively resolved at the same time increment $[t_n, t_{n+1} = t_n + \Delta t]$. The objective is to obtain all the nodal unknowns and all the state variables at time t_{n+1} .

We start by the resolution of the mechanical problem in order to determine its solution at time t_{n+1} by keeping the values of the micromorphic damage and the micromorphic isotropic hardening constant at time t_n . The mechanical equation is then written under the following form:

$$\left[M\right]_{n}\left\{\ddot{U}\right\}_{n}+\left(\left\{F_{\text{int}}\right\}_{n}-\left\{F_{ext}\right\}_{n}\right)=0$$
(2.88)

Since the internal forces vector is already known at time t_n , the acceleration vector at the same time is expressed as:

$$\left\{ \ddot{U} \right\}_{n} = \left[M_{u} \right]_{n}^{-1} \left\{ \left\{ F_{ext} \right\}_{n} - \left\{ F_{int} \right\}_{n} \right\}$$
(2.89)

The equations of motion for the body are integrated using the explicit central difference integration scheme and the vectors of the global velocity and the global displacements are respectively:

$$\left\{ \dot{U} \right\}_{n+\frac{1}{2}} = \left\{ \dot{U} \right\}_{n-\frac{1}{2}} + \frac{\Delta t_{n+1} + \Delta t_n}{2} \left\{ \ddot{U} \right\}_n$$

$$\left\{ U \right\}_{n+1} = \left\{ U \right\}_n + \Delta t_{n+1} \left\{ \dot{U} \right\}_{n+\frac{1}{2}}$$

$$(2.90)$$

where the indices $\left(n+\frac{1}{2}\right)$, $\left(n-\frac{1}{2}\right)$ indicate the middle of two successive time increments.

The solutions of the micromorphic damage and the micromorphic isotropic hardening are also integrated in time using the above explicit central difference scheme as presented for the displacement fields.

At each time increment $[t_n, t_{n+1} = t_n + \Delta t]$, the overall state variables at t_n and the increments of all the displacement-like variables are provided to compute the state variables at t_{n+1} and the accelerations of the displacement-like variables by solving the weak forms of the IBVP. The main steps of the dynamic explicit resolution scheme are summarized as follows:

- 1) Resolution of the displacement equation (Eq.(2.2)) and the micromorphic equations (damage and isotropic hardening) in Eq.(2.3) and Eq.(2.4).
 - a) Compute the lumped mass matrix $\begin{bmatrix} M_u^e \end{bmatrix}$ for once, using the Eq.(2.21) and the lumped mass matrix of the micromorphic fields $\begin{bmatrix} \breve{M}_{\vec{d}}^e \end{bmatrix}$ for the damage (Eq.(2.15)) and $\begin{bmatrix} \breve{M}_{\vec{r}}^e \end{bmatrix}$ (Eq.(2.18)) for the isotropic hardening at the beginning of the analysis step;

- b) Estimate the new stable time increment $\Delta t_{n+1} = \min(\Delta t_u, \Delta t_{\bar{d}}, \Delta t_r)$ using the Eq.(2.93);
- c) Update the displacement field $\{U\}_{n+1} = \{U\}_n + \{\dot{U}\}_{n+\frac{1}{2}} \Delta t_{n+1}$ using the Eq.(2.90) and the micromorphic variables $\{\breve{d}\}_{n+1} = \{\breve{d}\}_n + \Delta t \{\dot{\breve{d}}\}_{n+\frac{1}{2}}$ and $\{\breve{r}\}_{n+1} = \{\breve{r}\}_n + \Delta t \{\dot{\breve{r}}\}_{n+\frac{1}{2}};$
- d) Call the material behavior model and update the state variables, e.g. $\underline{\sigma}_{n+1}$, $\underline{\alpha}_{n+1}$, r_{n+1} , d_{n+1} , by the local integration scheme (Paragraph 2.8);
- e) Compute the internal forces $\{F_{int}^e\}$ and external forces $\{F_{ext}^e\}$ using the Eq.(2.13) and Eq.(2.14);
- f) Compute the micromorphic internal $\left\{ \vec{F}_{int-\vec{d}}^{e} \right\}$ and $\left\{ \vec{F}_{int-\vec{r}}^{e} \right\}$ from Eq.(2.16) and Eq.(2.19), and external forces $\left\{ \vec{F}_{ext-\vec{d}}^{e} \right\}$ and $\left\{ \vec{F}_{ext-\vec{r}}^{e} \right\}$, using Eq.(2.17) and Eq.(2.20);

g) Compute the acceleration $\{\ddot{U}\}_{n+1}$ and the velocity $\{\dot{U}\}_{n+\frac{3}{2}}$ the accelerations $\{\ddot{\vec{d}}\}_{n+1}$ and $\{\ddot{\vec{r}}\}_{n+1}$, and the time rates of the micromorphic fields $\{\dot{\vec{d}}\}_{n+\frac{3}{2}}$ and $\{\dot{\vec{r}}\}_{n+\frac{3}{2}}$ according to the Eq.(2.89) and Eq.(2.90).

2) Go to the next time step.

The explicit dynamic procedure requires no iterations and no tangent stiffness matrix. However, the explicit procedure, which integrates through time by using many time increments, is conditionally stable. The stable time increment considering the damping effect is given by:

$$\Delta t_{u} \leq \frac{2}{\omega_{\max}\left(\sqrt{\left(1+\varsigma^{2}\right)}-\varsigma\right)}$$
(2.91)

In which ω_{\max} is the elementary highest eigenvalue of the mechanical system and $\zeta \leq 1$ is the damping parameter. A conservative estimate of the stable time increment is given by the minimum taken over all the elements. The above stability limit can be rewritten as:

$$\Delta t_u \le \min\left(\frac{L_e}{C_d}\right) \tag{2.92}$$

where, L_e is the characteristic element size and C_d is the current effective, dilatational wave speed of the material approximated by $C_d = \sqrt{(\lambda_e + 2\mu_e)/\rho}$.

Similarly, considering different values of the micromorphic density $\zeta_{\bar{d}}, \zeta_{\bar{r}}$, leads to the stability limits associated to each micromorphic field in order for the associated stable time increments $(\Delta t_{\bar{d}}, \Delta t_{\bar{r}})$ to be defined. Since each IBVP has one critical stable time increment, the global stability limit is governed by the minimum value of all the critical stable conditions:

$$\Delta t \le \min\left(\Delta t_u, \Delta t_{\bar{d}}, \Delta t_{\bar{r}}\right) \tag{2.93}$$

2.6.2 Stability condition for the micromorphic damage problem

For the explicit case (VUEL subroutine) we take into account the micromorphic inertia and we compute the diagonal lumped mass matrix (Eq.(2.21)) for both the mechanical and the micromorphic

damage problem by using
$$\left[M^e\right] = \int_{\Omega_r^e} \rho \left[N^e\right]^T \left[N^e\right] J_v^e dV$$
 and $\left[\breve{M}^e_{\bar{d}}\right] = \int_{\Omega_r^e} \rho \frac{\zeta_{\bar{d}}}{\breve{H}} \langle \breve{N}^e_{\bar{d}} \rangle \left\{\breve{N}^e_{\bar{d}}\right\} J_v^e dV$ as

introduced in section 2.3.2. The following equation (Eq.(2.94)) is then solved in 1D for one element

$$\det\left[\left[K^{e}\right] - \omega^{2}\left[M^{e}\right]\right] = 0 \tag{2.94}$$

with $\omega = \omega_{\max}$ being the elementary highest eigenvalue of the mechanical system and K^e the elementary stiffness matrix. Considering the stability condition provided by Abaqus[®]/Explicit, we define the stable time increment for the mechanical problem as: $\Delta t_u \leq \frac{2}{\omega_{\max}} \simeq \min\left(\frac{L_e}{C_d}\right)$.

We follow the same steps to define the stable time increment for the micromorphic damage problem by solving the same equation adapted properly as:

$$\det\left[\left[K_{\vec{d}}^{e}\right] - \omega_{\vec{d}}^{2}\left[\breve{M}_{\vec{d}}^{e}\right]\right] = 0$$
(2.95)

where

$$K_{\bar{d}}^{e} = \ell_{\bar{d}}^{2} \int_{\Omega_{e}} \left\{ B_{\bar{d}} \right\} \left\langle B_{\bar{d}} \right\rangle J d\Omega + \int_{\Omega_{e}} \left\{ N_{\bar{d}} \right\} \left\langle N_{\bar{d}} \right\rangle J d\Omega$$
(2.96)

Let us also note that in 1D the isoparametric shape functions of the element for the displacement and the micromorphic damage fields in the reference space are expressed as:

$$\langle N_{\tilde{d}} \rangle = \left\langle \frac{1-\xi}{2} \quad \frac{1+\xi}{2} \right\rangle$$
 (2.97)

For the jacobian : $J = \frac{L_e}{2}$ where l^e is the length of the 1D element. The gradient operator as the

following expression:

$$\left\langle B_{\bar{d}} \right\rangle = \frac{1}{L_e} \left\langle -1 \quad 1 \right\rangle \tag{2.98}$$

After integration, we get the following different matrices:

The micromorphic lumped mass matrix:

$$\left[\vec{M}_{\vec{d}}^{e}\right] = \rho \frac{\zeta_{\vec{d}}}{\vec{H}} \frac{A^{e}L_{e}}{2} \begin{bmatrix} 1 & 0\\ 0 & 1 \end{bmatrix}$$
(2.99)

where A^e is the section of the 1D element.

The micromorphic stiffness matrix:

$$\begin{bmatrix} \breve{K}_{\bar{d}}^{e} \end{bmatrix} = \int_{-1}^{+1} \left(\left(\frac{\ell_{\bar{d}}}{L_{e}} \right)^{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} (1-\xi)^{2} & 1-\xi^{2} \\ 1-\xi^{2} & (1+\xi)^{2} \end{bmatrix} \right) \frac{l^{e}}{2} A^{e} d\xi = \frac{\ell_{\bar{d}}^{2}}{L_{e}} A^{e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{L_{e}}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} A^{e} (2.100)$$
For the same reasons, $\Delta t_{\bar{d}} \leq \frac{2}{\omega_{\bar{d} \max}}$. The resolution of Eq.(2.95) gives that

$$\Delta t_{\vec{d}} \leq \frac{2}{\omega_{\vec{d}\max}} = \frac{2\sqrt{3\rho\zeta_{\vec{d}}}L_e}{\sqrt{\vec{H}\left(12\ell_{\vec{d}}^2 + (L_e)^2\right)}}$$
(2.101)

If we suppose the local damage d to be constant in the 1D element, we can also show with the same procedure that:

$$\Delta t_{\bar{r}} = \frac{2\sqrt{3\rho\zeta_{\bar{r}}}L_e}{\sqrt{\bar{\mathcal{Q}}\left(12\ell_{\bar{r}}^2 + (L_e)^2\right)}}\frac{1}{\sqrt{1-d^{\gamma}}}$$
(2.102)

2.7 Local integration scheme

For solving the algebraic system given later in Eq.(2.127), the computation of the internal and the external forces demands the evaluation of the local stress tensor $\underline{\sigma}_{n+1}$, the local damage d_{n+1} , the isotropic hardening r_{n+1} and the kinematic hardening $\underline{\alpha}_{n+1}$ at each integration point for each element at time step $(t_{n+1} = t_n + \Delta t)$, according to the already known state variables at time t_n , the state relations Eq.(1.24) to Eq.(1.27) and the evolution equations Eq.(1.50) to Eq.(1.53). To calculate these state variables at the end of each time step, we should numerically integrate the overall ordinary differential equations. Many integration methods are widely discussed within the literature (Wilkins, 1964; Dautray, 1984).

In presence of the nonlinear isotropic and kinematic hardenings, it has been shown in (Nesnas and Saanouni (2002), Saanouni and Chaboche (2003), Badreddine et al. (2010), Saanouni (2012b)) that combining the asymptotic scheme (Walker and Freed, 1991) with the return-mapping algorithm, leads to an efficient and robust unconditionally stable integration scheme in presence of the ductile damage. This method is followed here to integrate the above fully coupled constitutive equations with micromorphic damage.

The equations of our behavior model are first order ordinary differential equations and can formally be expressed under two forms, either as:

$$\begin{cases} \dot{y} = \varphi(y,t) \\ y(t) = y_n \quad \text{for } t = t_n \end{cases} \forall t \in [t_n, t_{n+1}]$$
(2.103)

or

$$\begin{cases} \dot{y} = \varphi(y,t) \left[\phi(y,t) - y \right] \\ y(t) = y_n \quad \text{for } t = t_n \end{cases} \quad \forall t \in [t_n, t_{n+1}]$$
(2.104)

By applying the classical θ -method, we obtain

$$y_{n+\theta} = y_n + \Delta t \left(\theta \dot{y}_{n+1} + (1-\theta) \dot{y}_n\right) \quad \text{for } 0 \le \theta \le 1$$
(2.105)

as the solution of Eq.(2.103), and

$$y_{n+\theta} = y_n \exp\left(-\theta\varphi(y_{n+\theta})\Delta_t\right) + \left[1 - \exp\left(-\theta\varphi(y_{n+\theta})\Delta t\right)\right]\phi(y_{n+\theta}) \text{ for } 0 \le \theta \le 1$$
(2.106)

as the solution of Eq.(2.104).

Applying the solutions Eq.(2.105) and Eq.(2.106) under the fully implicit assumption ($\theta = 1$), allows us to rewrite the state variables, at the end of the time step t_{n+1} for the time-independent plasticity, under the following form:

• The Cauchy stress and the plastic strain tensors:

$$\begin{cases} \underline{\varepsilon}_{n+1}^{p} = \underline{\varepsilon}_{n}^{p} + \Delta \lambda^{p} \frac{\underline{\tilde{n}}_{n+1}}{\sqrt{1 - d_{n+1}}}, & \underline{\tilde{n}}_{n+1} = \frac{3}{2} \frac{\underline{Z}_{n+1}}{\|\underline{Z}_{n+1}\|} \\ \underline{\sigma}_{n+1} = (1 - d_{n+1}) \left(\lambda_{e} tr(\underline{\varepsilon}_{n+1}^{e}) \underline{1} + 2\mu_{e} \underline{\varepsilon}_{n+1}^{e} \right) \end{cases}$$
(2.107)

with the elastic deformation

$$\underline{\varepsilon}_{n+1}^{e} = \underline{\varepsilon}_{n}^{e} + \underline{\Delta}\varepsilon - \underline{\Delta}\varepsilon^{p} = \underline{\varepsilon}_{n}^{e} + \underline{\Delta}\varepsilon - \Delta\lambda^{p} \frac{\underline{\tilde{n}}_{n+1}}{\sqrt{1 - d_{n+1}}}$$
(2.108)

• The ductile damage:

$$\begin{cases} d_{n+1} = d_n + \frac{\Delta\lambda^p}{(1 - d_{n+1})^\beta} \left\langle \frac{Y_{n+1} - Y_0}{S} \right\rangle^s \\ Y_{n+1} = \frac{1}{2} \underline{\varepsilon}_{n+1}^e : \underline{\Delta} : \underline{\varepsilon}_{n+1}^e + \frac{1}{3} C \underline{\alpha}_{n+1} : \underline{\alpha}_{n+1} + \frac{1}{2} \gamma d^{\gamma - 1} Q r_{n+1}^2 + \frac{1}{2} \gamma d^{\gamma - 1} \overline{Q} \left(r_{n+1} - \overline{r}_{n+1} \right)^2 - \overline{H} (d_{n+1} - \overline{d}_{n+1}) \end{cases}$$

$$(2.109)$$

• The isotropic hardening:

$$\begin{cases} r_{n+1} = r_n \exp\left(-b\Delta\lambda^p \left(\frac{Q+\bar{Q}}{Q}\right)\right) + \left(1 - \exp\left(-b\Delta\lambda^p \left(\frac{Q+\bar{Q}}{Q}\right)\right)\right) \left(\frac{Q+b\bar{Q}\bar{r}_{n+1}}{b\sqrt{1-d_{n+1}^{\gamma}}(Q+\bar{Q})}\right) - \bar{Q}\bar{r}_{n+1}\sqrt{1-d_{n+1}^{\gamma}} \\ R_{n+1} = (1-d_{n+1}^{\gamma})Qr_{n+1} + \bar{Q}\left[r_{n+1}(1-d_{n+1}^{\gamma}) - \sqrt{1-d_{n+1}^{\gamma}}\bar{r}_{n+1}\right] \\ (2.110)\end{cases}$$

• The kinematic hardening:

$$\begin{cases} \underline{\alpha}_{n+1} = \underline{\alpha}_n \exp(-a\Delta\lambda^p) + (1 - \exp(-a\Delta\lambda^p)) \left(\frac{\underline{\tilde{n}}_{n+1}}{a\sqrt{1 - d_{n+1}}}\right) \\ \underline{X}_{n+1} = \frac{2}{3}(1 - d_{n+1})C\underline{\alpha}_{n+1} \end{cases}$$
(2.111)

2.7.1 Local integration of time-independent plasticity

2.7.1.1 Elastic prediction

Let us suppose that the total incremental strain $\Delta \varepsilon$ over the current time step is completely elastic meaning the absence of any induced plastic flow, hardening or damage i.e., $\Delta \lambda^p = 0$. In this case, the elastic trial strain at time t_{n+1} is given by:

$$\underline{\varepsilon}_{n+1}^{trial} = \underline{\varepsilon}_{n}^{e} + \underline{\Delta}\varepsilon \tag{2.112}$$

which results to the trial stress deduced from Eq.(2.107):

$$\underline{\sigma}_{n+1}^{trial} = (1 - d_n) \underline{\underline{\Lambda}} : \underline{\underline{\varepsilon}}_{n+1}^{trial}$$
(2.113)

Since $\Delta \lambda^p = 0$, for this trial elastic loading increment, the von Mises yield criterion (Eq.(1.48)) corresponding to this trial stress rewrites as:

$$f_{n+1}^{trial} = \frac{\left\|\underline{\sigma}_{n+1}^{trial} - \underline{X}_{n}\right\|}{\sqrt{1 - d_{n}}} - \frac{R_{n}}{\sqrt{1 - d_{n}^{\gamma}}} - \sigma_{y}$$
(2.114)

If $f_{n+1}^{trial} < 0$, then the solution is effectively elastic, meaning that the trial stress state $\underline{\sigma}_{n+1}^{trial}$ is lying inside the yield surface (e.g. elastic unloading), and the state variables are updated as follows giving the elastic solution:

$$\underline{\varepsilon}_{n+1}^{e} = \underline{\varepsilon}_{n+1}^{trial,e}, \quad \underline{\varepsilon}_{n+1}^{p} = \underline{\varepsilon}_{n}^{p}, \quad \underline{\sigma}_{n+1} = \underline{\sigma}_{n+1}^{trial}$$

$$r_{n+1} = r_{n}, \quad R_{n+1} = R_{n}$$

$$\underline{\alpha}_{n+1} = \underline{\alpha}_{n+1}, \quad \underline{X}_{n+1} = \underline{X}_{n}$$

$$d_{n+1} = d_{n}, \quad Y_{n+1} = Y_{n}$$

$$(2.115)$$

2.7.1.2 Plastic correction

If $f_{n+1}^{trial} > 0$, it means that the trial stress state lies outside of the yield surface, something not admissible in time independent plasticity. The solution is then plastic and the trial solution should be corrected to determine the final values of the state variables $\underline{\sigma}, \underline{X}, R, Y$ and make sure that the yield condition $f_{n+1}(\underline{\sigma}_{n+1}, \underline{X}_{n+1}, R_{n+1}, Y_{n+1}) = 0$ is fulfilled. Under this case, the von Mises yield function takes the following form:

$$f_{n+1} = \frac{\left\|\underline{\sigma}_{n+1} - \underline{X}_{n+1}\right\|}{\sqrt{1 - d_{n+1}}} - \frac{R_{n+1}}{\sqrt{1 - d_{n+1}^{\gamma}}} - \sigma_{y}$$
(2.116)

To proceed with this viscoplastic correction, the discretized, nonlinear and fully coupled evolution equations must be solved using the Newton-Raphson iterative scheme to determine the stress-like variables and the admissibility conditions at t_{n+1} , the so-called return mapping algorithm.

The stress-like variables at the end of the time increment are expressed as:

• The Cauchy stress tensor:

$$\underline{\sigma}_{n+1} = (1 - d_{n+1}) \underline{\Delta} : \left(\underline{\varepsilon}_{n}^{e} + \underline{\Delta}\varepsilon - \Delta\lambda^{p} \frac{\underline{\tilde{n}}_{n+1}}{\sqrt{1 - d_{n+1}}} \right) = (1 - d_{n+1}) \underline{\Delta} : \left(\underline{\varepsilon}_{n+1}^{trial} - \Delta\lambda^{p} \frac{\underline{\tilde{n}}_{n+1}}{\sqrt{1 - d_{n+1}}} \right)$$
(2.117)

• The stress-like variable for the isotropic hardening:

$$R_{n+1} = \left(1 - d_{n+1}^{\gamma}\right) \left(Q + \bar{Q}\right) \left[r_n \exp\left(-b\left(\frac{Q + \bar{Q}}{Q}\right)\Delta\lambda^p\right) + \left(1 - \exp\left(-b\left(\frac{Q + \bar{Q}}{Q}\right)\Delta\lambda^p\right)\right) \frac{Q + b\bar{Q}\bar{r}_{n+1}}{b\left(Q + \bar{Q}\right)\sqrt{1 - d_{n+1}^{\gamma}}}\right] - \sqrt{1 - d_{n+1}^{\gamma}}\bar{Q}\bar{r}_{n+1}$$

$$(2.118)$$

• The stress-like variable of kinematic hardening:

$$\underline{X}_{n+1} = \frac{2}{3} (1 - d_{n+1}) C \left[\underline{\alpha}_n \exp(-a\Delta\lambda^p) + (1 - \exp(-a\Delta\lambda^p)) \left(\frac{\underline{\tilde{n}}_{n+1}}{a\sqrt{1 - d_{n+1}}} \right) \right]$$
(2.119)

• The deviatoric stress tensor:

$$\underline{Z}_{n+1} = \underline{\sigma}_{n+1}^{dev} - \underline{X}_{n+1}$$
(2.120)

The tensor \underline{Z}_{n+1} is written under the form:

$$\underline{Z}_{n+1} = \frac{1 - d_{n+1}}{1 - d_n} \underline{Z}_{n+1}^* - 2\sqrt{1 - d_{n+1}} \left[\mu_e \Delta \lambda^p + \frac{C}{3a} \left(1 - \exp\left(-a\left(\frac{C + \overline{C}}{C}\right) \Delta \lambda^p\right) \right) \right] \underline{\tilde{n}}_{n+1}$$
(2.121)

where

$$\underline{Z}_{n+1}^{*} = \underline{\sigma}_{n+1}^{*dev} - \underline{X}_{n} \left(\sqrt{1 - d_{n}} - \frac{1 - d_{n}}{\sqrt{1 - d_{n+1}}} \right) \frac{2}{3} \overline{C} \overline{\alpha}_{n+1} \exp\left(-a \left(\frac{C + \overline{C}}{C} \right) \Delta \lambda^{p} \right)$$
(2.122)

Since $\underline{\tilde{n}}_{n+1} = \frac{3}{2} \frac{\underline{Z}_{n+1}}{\|\underline{Z}_{n+1}\|}$, the tensor can be rewritten as:

$$\underline{Z}_{n+1} = \frac{2}{3} \|\underline{Z}_{n+1}\| \underline{\tilde{n}}_{n+1}$$
(2.123)

and by using the plasticity criterion obtain:

$$\underline{Z}_{n+1} = \frac{2}{3} \left[\sqrt{1 - d_{n+1}} \left(\frac{R_{n+1}}{\sqrt{1 - d_{n+1}^{\gamma}}} - \sigma_y \right) \right] \underline{\tilde{n}}_{n+1}$$
(2.124)

By substituting Eq.(2.118) to Eq.(2.124) we retrieve a second expression of the deviatoric tensor:

$$\underline{Z}_{n+1} = \frac{2}{3} \begin{bmatrix} \sqrt{1 - d_{n+1}} \left(\frac{\sqrt{1 - d_{n+1}^{\gamma}}}{1 - d_n^{\gamma}} R_n + \left(\frac{\sqrt{1 - d_{n+1}^{\gamma}} - \sqrt{1 - d_n^{\gamma}}}{\sqrt{1 - d_{n+1}^{\gamma}}} \right) \overline{Q} \overline{r}_{n+1} \right) \exp\left(-b \left(\frac{Q + \overline{Q}}{Q} \right) \Delta \lambda^p \right) + \int_{n+1} \underline{Q} \left(1 - \exp\left(-b \left(\frac{Q + \overline{Q}}{Q} \right) \Delta \lambda^p \right) \right) + \sqrt{1 - d_{n+1}} \sigma_y \end{bmatrix}$$

$$(2.125)$$

Finally, the equality of the two expressions (in Eq.(2.121) and Eq.(2.125)), gives:

.

$$\left\|\underline{Z}_{n+1}^{*}\right\| \frac{1-d_{n}}{\sqrt{1-d_{n+1}}} \left[\begin{aligned} 3\mu_{e}\Delta\lambda^{p} + \frac{C}{a} \left(1 - \exp\left(-a\left(\frac{C+\bar{C}}{C}\right)\Delta\lambda^{p}\right) \right) + \left(\frac{\sqrt{1-d_{n+1}^{\gamma}}}{1-d_{n}^{\gamma}}R_{n} + \left(\frac{\sqrt{1-d_{n+1}^{\gamma}} - \sqrt{1-d_{n}^{\gamma}}}{\sqrt{1-d_{n+1}^{\gamma}}}\right) \bar{Q}\bar{r}_{n+1} \right) \right] \\ \exp\left(-b\left(\frac{Q+\bar{Q}}{Q}\right)\Delta\lambda^{p}\right) + \frac{Q}{b}\left(1 - \exp\left(-b\left(\frac{Q+\bar{Q}}{Q}\right)\Delta\lambda^{p}\right)\right) + \sigma_{y} \right)$$

$$(2.126)$$

So, considering the fully isotropic plasticity, we obtain the following two highly nonlinear equations with two independent variables $\Delta \lambda^p$ and d:

$$\begin{cases} \left\{ f_{n+1} = \left\| \underline{Z}_{n+1}^{*} \right\| - \frac{1 - d_{n}}{\sqrt{1 - d_{n+1}}} \left\{ \begin{array}{l} 3\mu_{e}\Delta\lambda^{p} + \frac{C}{a} \left(1 - \exp\left(-a\left(\frac{C + \breve{C}}{C}\right)\Delta\lambda^{p}\right) \right) + \\ \left(\frac{\sqrt{1 - d_{n+1}^{\gamma}}}{1 - d_{n}^{\gamma}} R_{n} + \left(\frac{\sqrt{1 - d_{n+1}^{\gamma}} - \sqrt{1 - d_{n}^{\gamma}}}{\sqrt{1 - d_{n+1}^{\gamma}}} \right) \breve{Q}\breve{r}_{n+1} \right) \exp\left(-b\left(\frac{Q + \breve{Q}}{Q}\right)\Delta\lambda^{p}\right) + \\ \frac{Q}{b} \left(1 - \exp\left(-b\left(\frac{Q + \breve{Q}}{Q}\right)\Delta\lambda^{p}\right) \right) + \sigma_{y} \\ g_{n+1} = d_{n+1} - d_{n} - \frac{\Delta\lambda^{p}}{(1 - d_{n})^{\beta}} \left\langle \frac{Y_{n+1} - Y_{0}}{S} \right\rangle^{s} = 0 \end{cases}$$

$$(2.127)$$

This system of two equations with two unknowns is resolved by an iterative Newton-Raphson algorithm in order to linearize them under the following form:

$$\begin{cases} f \\ g \\ g \\ _{n+1} \end{cases}^{s} + \begin{bmatrix} \frac{\partial f}{\partial \Delta \lambda^{p}} & \frac{\partial f}{\partial d} \\ \frac{\partial g}{\partial \Delta \lambda^{p}} & \frac{\partial g}{\partial d} \end{bmatrix}_{n+1}^{s} \begin{cases} \delta \Delta \lambda^{p} \\ \partial d \\ _{n+1} \end{cases}^{s} + 0 \times (...) = \begin{cases} 0 \\ 0 \end{cases}$$
(2.128)

with s being the number of iterations and the corrections of $\delta\!\Delta\!\lambda^p\,$ and $\delta d_{\scriptscriptstyle n+1}$ given by:

$$\begin{cases} \delta \Delta \lambda^{p} = \left(\Delta \lambda^{p}\right)^{s+1} - \left(\Delta \lambda^{p}\right)^{s} \\ \delta d = \left(d_{n+1}\right)^{s+1} - \left(d_{n+1}\right)^{s} \end{cases}$$
(2.129)

After calculating the four terms $\frac{\partial f_{n+1}}{\partial \Delta \lambda^p}$, $\frac{\partial f_{n+1}}{\partial d_{n+1}}$, $\frac{\partial g_{n+1}}{\partial \Delta \lambda^p}$, $\frac{\partial g_{n+1}}{\partial d_{n+1}}$ of the jacobian matrix, we obtain:

$$\frac{\partial \overline{f}_{n+1}}{\partial \Delta \lambda^{p}} = \frac{\partial \left\| Z_{n+1}^{*} \right\|}{\partial \Delta \lambda^{p}} - \frac{1 - d_{n}}{\sqrt{1 - d_{n+1}}} \begin{cases} 3\mu_{e} + (C + \overline{C}) \exp\left(-a\left(\frac{C + \overline{C}}{C}\right)\Delta \lambda^{p}\right) + \\ (Q + \overline{Q}) \exp\left(-b\left(\frac{Q + \overline{Q}}{Q}\right)\Delta \lambda^{p}\right) \left[1 - \frac{b}{Q}\left(\frac{\sqrt{1 - d_{n+1}^{\gamma}}}{1 - d_{n}^{\gamma}}R_{n} + \left(\frac{\sqrt{1 - d_{n+1}^{\gamma}} - \sqrt{1 - d_{n}^{\gamma}}}{\sqrt{1 - d_{n+1}^{\gamma}}}\right)\overline{Q}\overline{r}_{n+1}\right) \right] \end{cases}$$

$$(2.130)$$

$$\frac{\partial \bar{f}_{n+1}}{\partial d_{n+1}} = \frac{\partial \left\| Z_{n+1}^* \right\|}{\partial d_{n+1}} - \frac{1}{2} \frac{(1-d_n)}{(1-d_{n+1})^{3/2}} \begin{bmatrix} 3\mu_e \Delta \lambda^p + \frac{C}{a} \left(1 - \exp\left(-a\left(\frac{C+\bar{C}}{C}\right)\Delta \lambda^p\right) \right) + \left(\frac{\sqrt{1-d_{n+1}^{\gamma}}}{1-d_n^{\gamma}}R_n + \left(\frac{\sqrt{1-d_{n+1}^{\gamma}}}{\sqrt{1-d_{n+1}^{\gamma}}}\right)\bar{Q}\bar{r}_{n+1} \right) \end{bmatrix} + \frac{1-d_n}{\sqrt{1-d_n}} \begin{bmatrix} \frac{\gamma d_{n+1}^{\gamma}}{2} \left(\frac{R_n}{(1-d_n^{\gamma})\sqrt{1-d_{n+1}^{\gamma}}} + \frac{\bar{Q}\bar{r}_{n+1}}{(1-d_n^{\gamma})^{3/2}} \right) \exp\left(-b\left(\frac{Q+\bar{Q}}{Q}\right)\Delta \lambda^p\right) \end{bmatrix}$$

$$(2.131)$$

$$\frac{\partial g_{n+1}}{\partial \Delta \lambda^{p}} = -\frac{1}{\left(1 - d_{n+1}\right)^{\beta}} \left\langle \frac{Y_{n+1}\left(d_{n+1}, \Delta \lambda^{p}\right) - Y_{0}}{S} \right\rangle - \frac{\Delta \lambda^{p}}{\left(1 - d_{n+1}\right)^{\beta}} \left\langle \frac{Y_{n+1}\left(d_{n+1}, \Delta \lambda^{p}\right) - Y_{0}}{S} \right\rangle \frac{\partial Y_{n+1}}{\partial \Delta \lambda^{p}}$$
(2.132)

$$\frac{\partial g_{n+1}}{\partial d_{n+1}} = 1 - \Delta \lambda^{p} \left[\frac{1}{(1 - d_{n+1})^{\beta}} \left\langle \frac{Y_{n+1}(d_{n+1}, \Delta \lambda^{p}) - Y_{0}}{S} \right\rangle^{s-1} \frac{\partial Y_{n+1}}{\partial d_{n+1}} + \frac{\beta}{(1 - d_{n+1})^{\beta+1}} \left\langle \frac{Y_{n+1}(d_{n+1}, \Delta \lambda^{p}) - Y_{0}}{S} \right\rangle^{s} \right]$$
(2.133)

From the system Eq.(2.128) and for each iteration, we get:

$$\begin{cases} \delta \Delta \lambda^{p} \\ \delta d_{n+1} \end{cases} = - \begin{bmatrix} \frac{\partial f_{n+1}}{\partial \Delta \lambda^{p}} & \frac{\partial f_{n+1}}{\partial d_{n+1}} \\ \frac{\partial g_{n+1}}{\partial \Delta \lambda^{p}} & \frac{\partial g_{n+1}}{\partial d_{n+1}} \end{bmatrix}^{s} \begin{cases} f_{n+1} \\ g_{n+1} \end{cases}^{s}$$

$$(2.134)$$

This allows us to obtain the values of $\delta\Delta\lambda^p$ and δd_{n+1} for each iteration which as a result, allows us to deduce the values of $(\Delta\lambda^p)^{s+1}$ and $(d_{n+1})^{s+1}$ until convergence. At convergence, we obtain the final values of $\Delta\lambda^p$ and d_{n+1} leading to the calculation of all the other variables at time t_{n+1} .

It is worth noting that we can simplify the system with two equations and two unknowns in one highly non-linear scalar equation with one unknown $\Delta \lambda^p$ under the form $f(\Delta \lambda^p) = 0$. It is enough to exit the equation g_{n+1} from the Newton-Raphson loop, supposing that the damage at time t_{n+1} is written under the following form:

$$g_{n+1} = d_{n+1} - d_n - \frac{\Delta \lambda^p}{(1 - d_n)^\beta} \left\langle \frac{Y_{n+1}(d_n, \Delta \lambda^p) - Y_0}{S} \right\rangle^s = 0$$
(2.135)

which is quite useful for accelerating the convergence and reducing the computation time.

2.7.2 Implementation in Abaqus[®]/Explicit

In order to solve the IBVP, we have implemented some finite elements with additional micromorphic degrees of freedom as well as a micromorphic behavior model combined with an integration algorithm to compute each state variable for each integration/Gauss point. In this paragraph we will give a short discussion with some details on the methodology followed for the implementation of the new elements based on a micromorphic behavior model and the finite elements in Abaqus/Explicit[®].

By using Abaqus/Explicit[®] we have the ability to implement our own element along with its numerical integration in order to compute all the state variables at each load increment. This is done by following the form of a proposed subroutine called inside the VUEL (Abaqus User Subroutines Reference Manual).

The current VUEL, built to implement the integration algorithm of the proposed micromorphic model, is written in Fortran90. Without going into many details and without giving any parts of the code, we present the major steps for a Gauss point of a typical element for a load increment in $[t_n, t_{n+1} = t_n + \Delta t]$:

- > At the beginning of a time increment: we read all the variables related to the current time increment and we restore the values of all the variables at the end of the previous step (t_n) .
- ▶ If $d_n = d_c = 1$ at a Gauss point, we exit the routine, by maintaining the state variables and keeping them unchanged at (t_n) .
- Else, (if for example $d_n < d_c$) the elastic prediction- plastic correction code is applied:
 - Calculation of the trial criterion by using the loading increment in $[t_n, t_{n+1} = t_n + \Delta t]$:
 - If $f_{n+1}^{trial} \leq 0$ we preserve all the state variables at (t_n) , keep $\underline{\sigma}_{n+1} = \underline{\sigma}_{n+1}^{trial}$ and we exit the routine
 - If $f_{n+1}^{trial} > 0$ we continue to the plastic correction by applying the Newton-Raphson iterative scheme until the convergence ($f_{n+1}^{trial} \le 0$).

- Update all the state variables at (t_{n+1}) .
- ▶ In case $d_{n+1} \ge d_c$ (total rupture test) in one Gauss point, we set $d_{n+1} = 1$, we neglect all the stress-like stress variables and preserve the latest values of the strain-like variables.

We highlight this case where a completely damaged element appears in the dynamic explicit resolution scheme. We note again that all the stress-like variables are equal to zero for an element the Gauss points of which are totally damaged. As a consequence, the vectors $\{F_{int}^e\}, \{F_{ext}^e\}$ are vanished. In our case, the completely damaged elements remain in the mesh but without any mechanical state and, apparently, the nodes connecting these elements do not have any contribution to the global internal force vector. At any case, this element should be removed from the whole structure with the use of an adaptive remeshing procedure (Borouchaki 2002, 2005; Labergère, 2009, 2011, 2014; Issa, 2011). Whenever a remeshing technique is not applied, these elements are 'hidden' during the post-processing of the results after the end of the simulation.

Since the post-processing step for elements with additional degrees of freedom is not available on the version 6.13-1 of Abaqus/Explicit[®], we had to overcome this difficulty by using an external software for this purpose (GID). By adapting and using this software we succeed in treating properly and quite fast the long and difficult post-processing operations resulting from the large quantities of information obtained after the end of a simulation.

2.8 Adaptive analysis in 2D: The adaptive remeshing methodology

During metal forming processes, the large plastic strains that occur in regions of the part undergoing deformation, lead to highly localized strain zones. Inside these necking zones, microscopic defects may appear leading frequently to the initiation of macroscopic cracks. The numerical simulation of these processes using Finite Element Methods based on updated Lagrangian formulation leads to large distortion of the initial mesh. For this reason, adaptive meshing procedures are necessary in order to perform the full analysis. If the constitutive equations take into account the description of the macroscopic cracks initiation, the simulation of crack growth becomes thereafter possible. A number of authors (Andrade Pires et al., 2004; Borouchaki et al., 2005; Bouchard et al., 2000; Bouchard et al., 2003; Boussetta et al., 2006; Ceretti et al., 1997; Labergere et al., 2007; Labergere et al., 2008; Labergere et al., 2010; Vaz and Owen, 2001) have proposed an adaptive meshing frame in which fully damaged elements are removed from the mesh leading to the determination of new boundaries inside the deformed part. Frequent remeshing must be thereafter performed in order to avoid large element distortion and to enhance the accuracy of the analysis in areas where gradients are highly localized. Accordingly, a robust mesh generation procedure is needed (Coupez et al., 2000). The objective to describe, with reasonable accuracy, macroscopic crack initiation and growth, can only be reached if the new moving boundaries are discretized with an optimal mesh size.

The previous works have been published regarding the mesh adaptation in elastoplasticity or viscoplasticity, while using appropriate local criteria to remove the cracked elements. A 2D adaptive meshing procedure for metal forming by large plastic deformation has been proposed by using linear triangular elements and the procedure has been tested on a number of examples (Labergere, 2014). However, some basic properties of the linear elements may lead to inaccuracies when high nonlinearities appear.

In this thesis, a 2D adaptive methodology using 2D bilinear assumed strain quadrangular elements in axisymmetric configuration is proposed and successfully applied to highly non-linear metal forming

problems with damage occurrence. The proposed methodology is based on the following tools and techniques:

- ABAQUS/EXPLICIT[®] software to solve the initial and boundary value problem. At each Gauss point, appropriate constitutive equations are used through the user defined material subroutine VUMAT.
- Appropriate error indicators used to compute the map of the mesh size including refinement and coarsening of the mesh.
- 2D mesh generator (DIAMESH2D)
- Fields transfer procedures between the old and the new mesh based on diffuse approximation (Brancherie and Villon, 2006; Labergere, 2014)
- Specific procedure to adapt the loading sequences that depend on the size and the number of the fully damaged elements.

The efficiency of this methodology is illustrated through the simulation of some blanking operations detailed in Chapter 5.

2.8.1 2D Adaptive numerical methodology

I. 2D adaptive meshing resolution scheme

The procedure adapts the load sequence and the mesh size using appropriate error indicators. The mesh generator DIAMESH2D is thereafter performed using linear or quadratic Quadrangular or Triangular elements (Rassineux, 1991), according to the following steps (see Fig.2.4):

- Step n°1: The initial part is coarsely meshed with respect to a maximum element size Δx_{max} and the local curvature of the tools;
- Step n°2: ABAQUS/EXPLICIT is used with a user defined VUEL user subroutine in order to solve the problem for the first load sequence and the final solution (displacement and state variables) at the end of the current loading sequence for the current mesh (M_i) is obtained;
- Step n°3: If the number of fully damaged elements does not exceed a given threshold and if all the fully damaged elements have the smallest mesh size Δx_{\min}^{dam} , these fully damaged elements are removed. New boundaries are then defined with respect to a new mesh size based on the error indicators. If the total number of the fully damaged elements exceeds the known threshold or if the size of any fully damaged element exceeds Δx_{\min}^{dam} , the analysis is cancelled for this loading sequence. A new load sequence is then carried out for a reduced loading amplitude.
- Step n°4: Error indicators based either on the local curvature of the tools at the contact boundaries and/or the Hessian of the plastic strain and damage rates are calculated and a new map of mesh sizes is derived from those error indicators;
- Step n°5: Knowing the mesh sizes map a new mesh (M_{i+1}) is generated using DIAMESH2D software.
- Step n°6: Data from mesh (M_i) are transferred to the newly created mesh (M_{i+1}) based on diffuse approximation procedure.
- Step n°7: A new input file for the analysis is prepared for a new loading sequence and the analysis is restarted from step 2.



Figure 2.4: 2D adaptive remeshing scheme (Labergere et. al, 2014)

II. Field transfer

As the domain geometry is time dependent, emphasis must be given to the transfer of the different fields during the process. In order to recover the different fields, a second order diffuse interpolation method is used. The main steps of the diffuse approximation method (Breitkopf et al., 2000 among others) are given here. The different state variables namely " S_{phys} " (plastic strain, stress, damage, isotropic hardening,...) are computed at each integration point of each element using the local integration procedure. Thereafter, a Monge patch surface of equation $\tilde{S}(x, y)$ is built:

$$\tilde{S} = \tilde{S}(x, y) = \langle 1, x, y, \frac{x^2}{2}, xy, \frac{y^2}{2} \rangle \{a\} = \langle P(\underline{X}) \rangle \{a\}$$
 (2.136)

with $\underline{X} = (x, y)$, where x and y are the global system of coordinates and $\{a\}$ is the approximation parameters vector calculated by minimizing a criterion based on the interpolation of the set of neighboring nodes, as:

$$J_{\underline{X}}(\{a\}) = \sum_{i=1}^{i=n_p} w(\underline{X}_i, \underline{X}) \times \left(\left\langle P(\underline{X}_i - \underline{X}) \right\rangle \{a\} - S_{\text{phys}}(\underline{X}_i) \right)^2$$
(2.137)

 \underline{X}_i denotes the vector coordinates at which the quantity $S_{\text{phys}}(\underline{X}_i)$ is given; n_p is the number of points at which $S_{\text{phys}}(\underline{X}_i)$ is provided. \underline{X} denotes the coordinate vector at which the approximation is computed. The contribution of each nodal value to the approximation is influenced by a weighting function $w(\underline{X}_i, \underline{X}) = w_i(\underline{X})$ such that $w(\underline{X}_i, ...) > 0$ inside the domain of influence of the node i and $w(\underline{X}_i, ...) = 0$ otherwise, providing a local character to the approximation.



Figure 2.5: Diffuse transfer procedure

Figure 2.5 represents the influence zone at each old Gauss point of the old mesh (M_i) and the evaluation of the mechanical fields for each new Gauss point of the new Mesh (M_{i+1}) For the nodal variables a polynomial interpolation based on the FE interpolation shape functions is used depending on the element type.

III. The error indicator

When dealing with strong material and kinematical non linearities, the mesh adaptation to the higher field gradients is a crucial issue. Many authors have proposed different error estimators based on different physical consideration (Babuska and Rheinboldt, 1978; Georges, 2001; Zienkiewicz et al., 1988). The mesh size, all over the domain, can be derived in a first step from geometrical considerations. An adaptation of the contour lines to the curvature in the first step of the analysis greatly improves the robustness of contact analysis especially whenever bilateral contact due to self-contact occurs. In this context, a masterslave surface method is used and many projections are performed. A coarse representation of the contact zone may impede the convergence of the process. For instance, the vicinity of the tools and more generally potential contact as well as auto contact between surfaces is taken into account even before a contact may occur. In a 2D context, the contact between bodies as well as the auto-contact problems are rather easy to detect and therefore to handle. Once the contact is detected an adequate element size is locally affected.

Mesh size map based on damage and equivalent plastic strain

The prediction of the mesh size is based on the values of some selected fields weighted by their time derivatives. Therefore, we also developed an empirical formula to predict the mesh size map based on the values of the main non linearity sources such as: plasticity and damage. On a number of predefined intervals, mesh size gradation has a predefined expression and therefore thresholds are defined. These intervals enable us to describe the main steps of the evolution of the mechanical behavior (elasticity, plasticity, damage). The whole meshing process is controlled by four element sizes $\Delta x_{max} > \Delta x_{max}^p > \Delta x_{min}^{dam}$

where:

 $\Delta x_{\rm max}$: maximum element size

 Δx_{\max}^p : maximum mesh size for plasticity

 Δx_{\min}^p : minimum mesh size for plasticity

 Δx_{\min}^{dam} : The smallest mesh size of fully damaged elements which are thereafter removed

Whenever the accumulated plastic strain p does not exceed a given threshold denoted by p_{\max} , the mesh size variation evolves linearly from Δx_{\max} to Δx_{\max}^p and indeed $\Delta x_{\max}^p < \Delta x_{\max}$. When $p > p_{\max}$, we consider that the size decreases exponentially to reach the minimum value Δx_{\min}^p .

In a second step, the mesh size under plasticity denoted as Δx^p is weighted by the damage variable D. The lowest value of mesh size denoted as Δx^d is Δx^{dam}_{min} . The different functions used to compute these mesh sizes are given in Table 2.3.

The initiation and propagation of macroscopic cracks are simulated here by deleting the fully damaged elements having the same smallest size Δx_{\min}^{dam} for a given material. Accordingly, there is no significant loss of mass if the removed elements have the smallest size. On the other hand, this ensures a kind of mesh independency, since the same material Δx_{\min}^{dam} has a constant same value inside the damaged zones.

	Areas of mechanical behavior	parameter	Evolution of the element size
Elastoplastic size indicator	Purely elastic area $\left(p = 0 ight)$	$\Delta x_{\rm max}$	$\Delta x^{p} = \Delta x_{\max}$
	Homogeneous plastic zone $\left(0$	p^* , $\Delta x^p_{ m max}$	$\Delta x^{p} = \Delta x_{\max} + \frac{\Delta x_{\max}^{p} - \Delta x_{\max}}{p^{*}} p$
	Highly localized plastic zone $(p > p^*)$	p^* , κ_1 , Δx^p_{\max} , Δx^p_{\min}	$\Delta x^{p} = \left(\Delta x_{\min}^{p} - \Delta x_{\max}^{p}\right) \left(1 - \exp\left(-\kappa_{1}\left(p - p^{*}\right)\right)\right) + \Delta x_{\max}^{p}$
icator	Plastic zone with low damage value $\left(d \leq d_{\min} ight)$	d_{\min}	$\Delta x^{dam} = \Delta x^p$
Damage size ind	Plastic zone with moderate damage value $\left(d_{\min} < d \le d_{\max}\right)$	d_{\min} , d_{\max} , Δx_{\min}^{dam}	$\Delta x^{dam} = \left(\Delta x^{dam}_{\min} - \Delta x^{p}\right) \frac{d - d_{\min}}{d_{\max} - d_{\min}} + \Delta x^{p}$
	Plastic zone with high damage value $\left(d > d_{\max}\right)$	d_{\min} , d_{\max} , Δx_{\min}^{dam}	$\Delta x^{dam} = \Delta x^{dam}_{\min}$

Table 2.3: Element size with respect to the cumulated plastic strain p and the damage variable d

2.9 Conclusions

In this chapter dedicated to the numerical aspects of the initial boundary value problem, we have presented the weak forms of the IBVP. We discussed hereafter the finite difference time discretization and the finite element space discretization as well as the numerical computation of the large strain rate tensor.

Two types of 2D elements, a 4-node quadrilateral assumed strain and a 4-node quadrilateral axisymmetric element, have been formulated to which we have added the new micromorphic degrees of freedom. We have also presented the numerical methods associated to this IBVP in terms of the global resolution scheme. Finally, we give attention to the local integration of the evolution equations and a brief representation of the 2D adaptive remeshing methodology.

This numerical methodology will be used in the following chapters in order to validate the proposed model through some examples concerning several metal forming applications.

Chapter 3

Parametric study of the micromorphic model.

3.1 Introduction

In the first chapter we formulated a micromorphic model with multiphysic coupling. To the local phenomena (hardenings, damage) we added the equivalent micromorphic variables for the isotropic hardening and the ductile damage as new degrees of freedom and additional state variables. In the second chapter, the following numerical aspects associated to this model were presented: (i) the variational forms of the IBVP, (ii) the global resolution scheme, (iii) the new finite elements with the additional micromorphic degrees of freedom and (iv) the local integration scheme of the complete set of the fully coupled constitutive equations.

This chapter is dedicated to the validation of the proposed model from both theoretical and numerical points of view. A detailed parametric study is performed on a simple uniaxial tensile test in order to explore the performance of the proposed nonlocal formulation and its ability to achieve acceptable solutions. In particular, the role of the micromorphic material parameters is studied for the two micromorphic phenomena concerning three models: the first only with micromorphic damage, the second only with micromorphic isotropic hardening and the third with the combination of them both.

3.2 Behavior model and specimen

3.2.1 Choice of the parameters

For all the validation examples and the parametric study, the model with mixed hardening strongly coupled with ductile damage is used (see Table 2.2 in Chapter 2). For this model, the local parameters for the 430(X8Cr17) steel were identified through an inverse method based on the appropriate experimental force-displacement curves and they are presented in Table 3.1.

	430(X8Cr17) steel
Elasticity parameters	E = 220000.0 MPa
	v = 0.29
Plasticity parameters	$\sigma_y = 300.0 MPa$
	Q = 1135.0 MPa
	b = 2.0
	C = 4300.0 MPa
	$\alpha = 40.0$
Damage parameters	<i>S</i> = 4.5
	$\beta = 1.0$
	<i>s</i> = 1.0
	$Y_0 = 0.0$

Table 3.1: Local material parameters for the 430 steel specimen

For the micromorphic (nonlocal) parameters, different values were taken into consideration for the pairs: (\breve{H},\breve{H}^g) for the micromorphic damage and (\breve{Q},\breve{Q}^g) for the micromorphic isotropic hardening. These different values are carefully selected in order to investigate their effects on the plastic flow and damage localization. More particularly, the impact of the values of these micromorphic parameters on the thickness of the macroscopic crack defined as the area of all the completely damaged elements, as well as their ability to provide a mesh independent solution at convergence, are examined.

3.2.2 Specimen

The thin metallic specimens tested were made of 430(X8Cr17) steel with effective length of 45.0 mm and thickness of 0.2 mm as shown in Fig. 3.1. For the sake of element validation, they were all modeled with Q4 quadrilateral plane stress elements and we have only treated the case of a tensile test. Figure 3.2 presents the geometry and the three different meshes using the micromorphic finite elements developed in Chapter 2.



Figure 3.1: Specimen geometry (Bao et al., 2015)



Figure 3.2: The specimen meshed with three different element sizes (h = 0.2 mm, h = 0.4 mm and h = 0.8 mm)

The specimen is subjected to an imposed displacement applied to the upper head with $\dot{u} = 0.015 \text{ mm/min}$. In the following we discuss, in some details, the distribution of the mechanical fields in the specimen as predicted by the fully coupled model. We recall that the elements were

implemented in Abaqus[®]/Explicit through the VUEL user's subroutine while the numerical integration of the overall constitutive equations are implemented in the same code through the VUMAT user's subroutine as detailed in Chapter 2.

3.3 Parametric study of the micromorphic model

In this section a relatively exhaustive parametric study of the micromorphic model is presented by focusing on the role of the four micromorphic parameters related to both micromorphic damage, e.g., (\breve{H},\breve{H}^g) and micromorphic isotropic hardening, e.g., (\breve{Q},\breve{Q}^g) . We examine the effect of the different values of each one of these parameters on the quality of the numerical solution in terms of the global force-displacement and stress-strain responses as well as the distribution of the mechanical fields inside the specimen.

Since the micromorphic moduli H and Q are the most important parameters which control the localization phenomena (as can be seen in the strong forms in Table 2.1 of Chapter 2), we will focus on these. For this, we start by examining the model with micromorphic damage only, followed by the model which carries the effect of the micromorphic hardening only. Finally, we conclude with the model including both micromorphic damage and isotropic hardening which is our complete micromorphic model. For all the cases, we use only the Q4 elements with the required micromorphic dofs and we compare the results with the solutions obtained by the local model.

3.3.1 Model only with micromorphic damage

In this case all the phenomena are supposed to remain of local nature and only the damage is taken as micromorphic, giving one additional degree of freedom and one additional micromorphic balance equation. We recall that in this case only one internal length is activated, the one related to the micromorphic damage given by $\ell_{\vec{x}} = \sqrt{\breve{H}^g / \breve{H}}$.

Before proceeding to the parametric study of the micromorphic damage model, let us retrieve and examine the purely local solution. We remind that the results for this tensile test for the local damage model as well, were obtained using the ABAQUS/EXPLICIT VUEL and VUMAT user's subroutines applied to the Q4 elements with the displacement being the sole degree of freedom by deactivating the micromorphic moduli \breve{H} and \breve{H}^{s} .

Figures 3.3, 3.4 and 3.5 show the distribution of d, p, σ_{eq} respectively, one step before the first crack appears, (e.g., d = 1.0 for the central element located at the intersection of the two shear bands) while Figure 3.6 illustrates the typical force-displacement curves obtained at the end of each simulation for the three different mesh sizes. In Figures 3.3a, 3.4a and 3.5a we have added the damage isovalues at the final fracture of the specimen showing the macroscopic crack path for each mesh size. From these results we observe that, as expected, the solutions are highly dependent on the mesh size mainly during the damage induced softening stage (Figure 3.6). As expected, the consequence of this mesh dependency is clearly shown by the non-unique force-displacement curves at the damage induced softening stage. The thickness of the totally damaged zone (macroscopic crack) is reduced to a single row of elements for all the cases but without following the same orientation as shown in Figures 3.3a, 3.4a and 3.5a. Indeed, the final crack appears to be horizontal for the coarse mesh size of 0.8 mm (Figure 3.3a), while for the two other finest meshes the macroscopic crack follows one of the shear bands with two different orientations. This difference

seems clearly governed by the numerical difference in local damage values. Let us also note that, for this local model, the equivalent plastic strain at fracture (ductility) is "nearly" independent from the mesh size with $p_{rupt} \simeq 56\%$ for h = 0.8 mm and h = 0.4 mm while $p_{rupt} = 60\%$ for h = 0.2 mm.



Figure 3.3: Local damage, equivalent plastic strain and von Mises stress eigenvalues before the final rupture of the first element for h = 0.8 mm giving $u_{rupt} = 11.73$ mm



Figure 3.4: Local damage, equivalent plastic strain and von Mises stress eigenvalues before the final rupture of the first element for h = 0.4 mm giving $u_{rupt} = 11.4$ mm



Figure 3.5: Local damage, equivalent plastic strain and von Mises stress eigenvalues before the final rupture of the first element for h = 0.2 mm giving $u_{rupt} = 11.1$ mm



Figure 3.6: Force-displacement responses of the local model for three mesh sizes

3.3.1.1 Effect of the internal length $\ell_{\tilde{d}}$ for a constant mesh

We examine the effect of the micromorphic damage modulus \breve{H} by taking three different values: a low value $\breve{H} = 6.4$, a medium value $\breve{H} = 60.0$ and a high value $\breve{H} = 1000.0$. For each of these three values, the values of the modulus \breve{H}^{g} are varied in order to obtain the following three values of the damage internal length $\ell_{\breve{d}}$ of 0.1, 1.0 and 5.0 mm.

$\breve{H}^{g}(N)$	$\ell_{ec{d}}$	U _{rupt}
0.064	0.1 mm	11.1 mm
6.4	1.0 mm	11.1 mm
160.0	5.0 mm	11.13 mm

(i) For $\tilde{H} = 6.4$: the values of \tilde{H}^s giving the three targeted values of the damage micromorphic internal lengths together with the associated fracture displacements are summarized in Table 3.2.

Table 3.2: Values of the micromorphic material parameters and fracture displacement for H = 6.4

As shown in Table 3.2, in Figure 3.7 by the force displacement curves and in Figure 3.8 by the stress-strain curves, due to the low value of the modulus H = 6.4, the regularization effect is small and the mechanical responses are close to the local solution with a fracture occurring for a displacement of 11.1 mm.

In terms of the distribution of the mechanical fields inside the specimen for this case with a low value of \vec{H} , when $\ell_{\vec{d}} = \sqrt{0.064/6.4} = 0.1 \text{ mm}$, the maximum value of local damage just before the final fracture of the specimen is $d_{\text{max}} = 50.0\%$ (Figure 3.9a.1), the maximum micromorphic damage is $\vec{d}_{\text{max}} = 19.0\%$ (Figure 3.9b), the maximum accumulated plastic strain is $p_{\text{max}} = 58.0\%$ (Figure 3.9c) and the maximum von Mises stress $\sigma_{eq.Max} = 659.0$ MPa (Figure 3.9d). For $\ell_{\vec{d}} = \sqrt{6.4/6.4} = 1.0 \text{ mm}$ we obtain $d_{\text{max}} = 37.0\%$ (Figure 3.10a.1), $\vec{d}_{\text{max}} = 40.0\%$ (Figure 3.10b), $p_{\text{max}} = 55.0\%$ (Figure 3.10c) and $\sigma_{eq}^{\text{max}} = 663.0 \text{ MPa}$ (Figure 3.10d). While for the highest value of micromorphic damage internal length $\ell_{\vec{d}} = \sqrt{160.0/6.4} = 5.0 \text{ mm}$, we obtain $d_{\text{max}} = 34.0\%$ (Figure 3.11a.1), $\vec{d}_{\text{max}} = 11.0\%$ (Figure 3.11b), $p_{\text{max}} = 56.0\%$ (Figure 3.11c) and $\sigma_{eq}^{\text{max}} = 664.0 \text{ MPa}$ (Figure 3.11d).

Figures 3.9a.2, 3.10a.2 and 3.11a.2 show the shape of the finale crack for the three cases of H = 6.4 along the band that concentrated the highest amount of plastic strain leading to the final rupture. Figures 3.8 and 3.12 show the comparisons of the $\sigma_{eq} - p$ responses of the local and the micromorphic model and the local damage evolution for the local and the micromorphic model respectively for a point lying in the middle of the specimen in the intersection of the two shear bands.

Note, that for this case, the symmetry of the localized shear bands is perfectly preserved.



Figure 3.7: Comparison of the global force-displacement responses of the local and the micromorphic model for a fixed $\breve{H} = 6.4$ and three different values of \breve{H}^s



Figure 3.8: Comparison of the $\sigma_{eq} - p$ responses of the local and the micromorphic model for a fixed $\breve{H} = 6.4$ and three different values of \breve{H}^{g}

It is obvious that the weak presence of nonlocality, mainly due to the low value of the modulus H (here H = 6.4), does not have a great influence on the evolution of the demonstrated mechanical fields and their maximum values stay close to the ones obtained by the local model as expected. The more the values of the micromorphic moduli tend to zero, the more the solution approaches the local one.

It's worth noting than even the nonlocal effect coming from the relatively high value $\vec{H}^{g} = 160.0$ is not obvious and this can be explained by the fact that it is the parameter \vec{H} that controls Y_{nloc} and that seems to have the biggest influence on the nonlocality effect.



Figure 3.9: Local and micromorphic damage, equivalent plastic strain and von Mises stress distribution for $\breve{H} = 6.4$, $\ell_{\breve{d}} = 0.1$ mm (i.e. $\breve{H}^s = 0.064$) at $u_{nupt} = 11.1$ mm



Figure 3.10: Local and micromorphic damage, equivalent plastic strain and von Mises stress distribution for $\breve{H} = 6.4$, $\ell_{\breve{d}} = 1.0$ mm at $u_{rupt} = 11.1$ mm (i.e. $\breve{H}^g = 6.4$)



Figure 3.11: Local and micromorphic damage, equivalent plastic strain and von Mises stress distributions for $\breve{H} = 6.4$, $\ell_{\bar{d}} = 5.0 \text{ mm}$ at $u_{rupt} = 11.13 \text{ mm}$ (i.e. $\breve{H}^g = 160.0$)



Figure 3.12: comparison of the local damage evolution for the local and the micromorphic model for $\breve{H} = 6.4$ at the end of the imposed displacement u = 15.0 mm

(ii) For H = 60.0: The values of H^{g} giving the three targeted values of the damage micromorphic internal lengths together with the associated fracture displacements (if any) are summarized in Table 3.3.

$\breve{H}^{g}(N)$	$\ell_{ar{d}}$	u_{rupt}
0.6	0.1 mm	11.34 mm
60.0	1.0 mm	11.9 mm
1500.0	5.0 mm	No crack

Table 3.3: Values of the micromorphic material parameters and fracture displacement (if any) for $\breve{H} = 60.0$



Figure 3.13: Comparison of the global force-displacement responses of the local and the micromorphic model for a fixed $\breve{H} = 60.0$

As shown in Table 3.3, by the force-displacement curves (Figure 3.13) and by the stress-strain curves (Figure 3.14), the regularization effect exists but it is less essential for an $\ell_{\tilde{a}}$ of 0.1 mm and 1.0 mm for which the mechanical responses are higher but still close to the local solution with a fracture occurring for displacement values of 11.34 mm and 11.9 mm respectively. However, no fracture is observed at the end of the applied load for the highest value of $\ell_{\tilde{a}} = 5.0$ mm.

In terms of the distribution of the mechanical fields inside the specimen, when $\ell_{\vec{d}} = \sqrt{0.06/60.0} = 0.1 \,\mathrm{mm}$ takes its lowest value, the maximum value of local damage just before the final fracture of the specimen is $d_{\max} = 41.0\%$ (Figure 3.15a.1), the maximum micromorphic damage is $\vec{d}_{\max} = 25.0\%$ (Figure 3.15b), the maximum accumulated plastic strain is $p_{\max} = 75.0\%$ (Figure 3.15c) and the maximum von Mises stress $\sigma_{eq,Max} = 698.0$ MPa (Figure 3.15d). For $\ell_{\vec{d}} = \sqrt{60.0/60.0} = 1.0 \,\mathrm{mm}$ we obtain $d_{\max} = 41.0\%$ (Figure 3.16a.1), $\vec{d}_{\max} = 21.0\%$ (Figure 3.16b),

 $p_{\rm max} = 89.0\%$ (Figure 3.16c) and $\sigma_{eq}^{\rm max} = 720.0$ MPa (Figure 3.16d). Finally, for the highest value of micromorphic damage internal length $\ell_{\vec{a}} = \sqrt{1500.0/60.0} = 5.0$ mm, we obtain $d_{\rm max} = 24.0\%$ (Figure 3.18a), $\vec{d}_{\rm max} = 12.0\%$ (Figure 3.18b), $p_{\rm max} = 290.0\%$ (Figure 3.18c) and $\sigma_{eq}^{\rm max} = 873.0$ MPa (Figure 3.18d) which are the values obtained at the end of the applied load with no fracture occurrence. Figures 3.15a.2 and 3.16a.2 show the shape of the finale crack for $\ell_{\vec{a}} = \sqrt{0.06/60.0} = 0.1$ mm and $\ell_{\vec{a}} = \sqrt{60.0/60.0} = 1.0$ mm, along the band that concentrated the highest amount of plastic strain leading to the final rupture. Figures 3.14 and 3.19 show the comparisons of the $\sigma_{eq} - p$ responses of the local and the micromorphic model and the local damage evolution for the local and the micromorphic model respectively for a point lying in the middle of the specimen in the intersection of the two shear bands. Figure 3.17 shows the evolution of d, \vec{d}, p and σ_{eq} the case of $\ell_{\vec{d}} = \sqrt{1500.0/60.0} = 5.0$ mm and for a relatively large displacement of 12.0 mm in the absence of final rupture. Once again for this case, the symmetry of the localized shear bands is perfectly preserved.



Figure 3.14: Comparison of the σ_{eq} – p responses of the local and the micromorphic model for a fixed \breve{H} = 60.0

For the case $\ell_{\vec{a}} = \sqrt{1500.0/60.0} = 5.0 \text{ mm}$ it is clearly shown that the micromorphic damage effect is excessively strong due to $\vec{H}^g = 1500.0$ and yields to no appearance of the final fracture up to the imposed displacement of 15.0 mm (Figure 3.18) despite the extremely high values of $p_{\max} = 290\%$ and $\sigma_{eq}^{\max} = 872.0 \text{ MPa}$. The effect is directly mapped to the very slow evolution and the low values of the local damage (Figure 3.19). We can note here that clearly the material ductility increases by increasing both \vec{H} and \vec{H}^g .



Figure 3.15: Local damage, micromorphic damage, equivalent plastic strain and von Mises stress distributions for $\breve{H} = 60.0$, $\ell_{\breve{d}} = 0.1$ mm at $u_{rupt} = 11.13$ mm (i.e. $\breve{H}^g = 0.6$)



Figure 3.16: Local and micromorphic damage, equivalent plastic strain and von Mises stress distributions for $\breve{H} = 60.0$, $\ell_{\breve{d}} = 1.0$ mm at $u_{rupt} = 11.5$ mm (i.e. $\breve{H}^g = 60.0$)



Figure 3.17: Local damage, micromorphic damage, equivalent plastic strain and von Mises stress distributions at u = 12.0 mm for $\breve{H} = 60.0$, $\ell_{\vec{d}} = 5.0 \text{ mm}$ (i.e. $\breve{H}^s = 1500.0$)



Figure 3.18: Local damage, micromorphic damage, equivalent plastic strain and von Mises stress distributions at the end of the applied load (u = 15 mm) for $\breve{H} = 60.0$, $\ell_{\breve{d}} = 5.0 \text{ mm}$ (i.e. $\breve{H}^g = 1500.0$)



Figure 3.19: comparison of the local damage evolution for the local and the micromorphic model for $\breve{H} = 60.0$ at the end of the imposed displacement u = 15.0 mm

(iii) For H = 1000.0: The values of H^s giving the three targeted values of the damage micromorphic internal lengths together with the associated fracture displacements (if any) are summarized in Table 3.4.

\breve{H}^{g} (N)	$\ell_{\breve{d}}$	U _{rupt}
10.0	0.1 mm	14.1 mm
1000.0	1.0 mm	no crack
25000.0	5.0 mm	no crack

Table 3.4: Values of the micromorphic material parameters and fracture displacement (if any) for $\breve{H} = 1000.0$

As shown in Table 3.4, by the force-displacement curves (Figure 3.20) and by the stress-strain curves (Figure 3.21), the final fracture is observed at 14.1 mm displacement only for $\ell_{\bar{d}} = 0.1$ mm far from the fracture displacement predicted by the local model. No fracture is observed at the end of the applied load for $\ell_{\bar{d}}$ of 1.0 mm and 5.0 mm.

When $\ell_{\bar{d}} = \sqrt{10.0/1000.0} = 0.1 \,\mathrm{mm}$ the maximum value of local damage just before the final fracture reaches $d_{\max} = 86.0\%$ (Figure 3.22a.1), the maximum micromorphic damage $\tilde{d}_{\max} = 80.0\%$ (Figure 3.22b), the maximum accumulated plastic strain $p_{\max} = 229.0\%$ (Figure 3.22c) and the maximum equivalent von Mises stress $\sigma_{eq} = 825.0 \,\mathrm{MPa}$ (Figure 3.22d). For $\ell_{\bar{d}} = \sqrt{1000.0/1000.0} = 1.0 \,\mathrm{mm}$ we obtain $d_{\max} = 14.6\%$ (Figure 3.24a) close to $\tilde{d}_{\max} = 14.3\%$ (Figure 3.24b), $p_{\max} = 287.0\%$ (Figure 3.24c) and $\sigma_{eq}^{\max} = 901.0 \,\mathrm{MPa}$ (Figure 3.24d). Finally, concerning the

highest value of the internal length $\ell_{\bar{d}} = \sqrt{25000.0/1000.0} = 5.0 \text{ mm}$ we obtained $d_{\text{max}} = 4.2\%$ (Figure 3.26a) very close to the micromorphic damage $\vec{d}_{\text{max}} = 3.9\%$ (Figure 3.26b), $p_{\text{max}} = 124.0\%$ (Figure 3.26c)and $\sigma_{eq}^{\text{max}} = 907.0 \text{ MPa}$ (Figure 3.26d).

Figure 3.22a.2 shows the shape of the finale crack for $\ell_{\vec{a}} = \sqrt{10.0/1000.0} = 0.1 \text{ mm}$ along the band that concentrated the highest amount of plastic strain leading to the final rupture. Figures 3.21 and 3.27 show the comparisons of the $\sigma_{eq} - p$ responses of the local and the micromorphic model and the local damage evolution for the local and the micromorphic model respectively for a point lying in the middle of the specimen in the intersection of the two shear bands. It is also interesting to take a look at the evolution of d, d, p and σ_{eq} in Figures 3.23 and 3.25 for $\ell_{\vec{a}} = \sqrt{1000.0/1000.0} = 1.0 \text{ mm}$ and $\ell_{\vec{a}} = \sqrt{25000.0/1000.0} = 5.0 \text{ mm}$ respectively for a relatively large displacement of 12.0 mm in the absence of final rupture.



Figure 3.20: Comparison of the global force-displacement responses of the local and the micromorphic model for a fixed high value of the modulus $\breve{H} = 1000.0$



Figure 3.21: Comparison of the $\sigma_{eq} - p$ responses of the local and the micromorphic model for a fixed high value of the modulus $\breve{H} = 1000.0$



Figure 3.22: Local and micromorphic damage, equivalent plastic strain and von Mises stress distributions for $\vec{H} = 1000.0$, $\ell_{\vec{d}} = 0.1$ mm at $u_{nupt} = 14.1$ mm (i.e. $\vec{H}^{g} = 10.0$)



Figure 3.23: Local and micromorphic damage, equivalent plastic strain and von Mises stress distributions at u = 12 mm for $\breve{H} = 1000.0$, $\ell_{\breve{d}} = 1.0 \text{ mm}$ (i.e. $\breve{H}^g = 1000.0$)



Figure 3.24: Local and micromorphic damage, equivalent plastic strain and von Mises stress at the end of the applied displacement (u = 15.0 mm) for $\breve{H} = 1000.0$, $\ell_{\breve{d}} = 1.0 \text{ mm}$ (i.e. $\breve{H}^g = 1000.0$)



Figure 3.25: Local and micromorphic damage, equivalent plastic strain and von Mises stress at u = 12.0 mm

for \breve{H} =1000.0, $\ell_{\vec{d}}$ = 5.0 mm (i.e. $\breve{H}^{\,g}$ = 25000.0)



Figure 3.26: Local and micromorphic damage, eq. plastic strain and von Mises stress distributions at the end of the applied displacement (u = 15.0 mm) for $\vec{H} = 1000.0$, $\ell_{\tilde{d}} = 5.0 \text{ mm}$ (i.e. $\vec{H}^g = 25000.0$)



Figure 3.27: comparison of the local damage evolution for the local and the micromorphic model for $\breve{H} = 1000.0$ at the end of the imposed displacement u = 15.0 mm

Similarly to the previous case ($\ell_{\vec{a}} = \sqrt{1500.0/60.0} = 5.0 \text{ mm}$), the same observations are made for $\ell_{\vec{d}} = \sqrt{1000.0/1000.0} = 1.0 \text{ mm}$ and $\ell_{\vec{d}} = \sqrt{25000.0/1000.0} = 5.0 \text{ mm}$. It is clearly shown that the micromorphic damage effect is excessively strong due to $\vec{H} = 1000.0 \text{ and } \vec{H}^s$ up to $\vec{H}^s = 25000.0$ and yields to no appearance of rupture although the displacement reaches up to 15.0 mm (Figure 3.20) and the $p_{\text{max}} > 200\%$. The effect is directly mapped to the very slow evolution the low values of the local damage as shown in Figure 3.27 while due to high diffusion, d and \vec{d} tend to evolve identically giving again a very small and negligible value to the term $(d - \vec{d})$ close to zero. As a matter of fact, these two solutions are yet not acceptable due to the lack of their physical substance.

By examining the F-u curve in Figure 3.20 and comparing the three micromorphic solutions to the local one, we see that for $\ell_{\bar{d}} = \sqrt{10.0/1000.0} = 0.1 \text{ mm}$, $u_{rupt} = 14.1 \text{ mm}$ and it is the largest observed for this study resulting from the extremely high value of $\breve{H} = 1000.0$.

<u>Summary</u>

We keep H constant and equal to 6.4, 60.0 and 1000.0 respectively by varying the value of the internal length $\ell_{\tilde{d}}$ between 0.1, 1.0 and 5.0 mm in order to observe the effect of the micromorphic damage moduli in terms of the global F - u and $\sigma_{eq} - p$ curves, distribution of mechanical fields, local damage evolution and regularization of the solution.

For all the tests, we examine the evolution of d, d, p and σ_{eq} as well as the evolution of the local damage, the $\sigma_{eq} - p$ and force-displacement curves for each case and we compare with the results obtained from the purely local model as shown above.

From all the force-displacement figures obtained we observe that:

- → \tilde{H} has a preponderant effect on the rupture displacement: the higher the value of \tilde{H} is, the more delayed the rupture is. In fact, for $\tilde{H} = 6.4$ the rupture displacement is $u_{rupt} \simeq 11.1$ mm, for $\tilde{H} = 60.0 u_{rupt} \simeq 12$. mm and for $\tilde{H} = 1000.0 u_{rupt} \simeq 14.0$ mm only for the case of $\ell_{\tilde{d}} = 5.0$ mm.
- ▶ If the value of \tilde{H} is low, the more the solution approaches the local one. This is explained by the fact that Y_{nloc} is directly proportional to \tilde{H} in a way that low values of \tilde{H} give a low value of Y_{nloc} so the micromorphic model tends to match the local one.
- For a constant value of \vec{H} , the more \vec{H}^{g} increases (larger $\ell_{\vec{d}}$), the more the rupture is delayed as well, for example, for $\vec{H} = 60.0$, $\vec{H}^{g} = 0.6 u_{rupt} = 11.0 \text{ mm}$, while for $\vec{H} = 60.0$, $\vec{H}^{g} = 60.0 u_{rupt} = 11.9 \text{ mm}$.
- We also compare now the distribution of (d, p, σ^{eq}) for the local and all the nonlocal cases; the isovalues of the damage, the equivalent plastic strain and the von Mises stress corresponding to the moment of the rupture. The more the values of \breve{H} and \breve{H}^{g} increase, the more the damage is homogenized around the macroscopic crack, which propagates along a single row of elements for all the cases. For the distribution of p, we can note the progressive growth of the ductility and consequently of the necking as the values of \breve{H} and \breve{H}^{g} increase.

In conclusion, it is the parameter \tilde{H} that controls Y_{nloc} and that seems to have the biggest influence on the nonlocality. In fact, the solution is almost identical to the local one for the lower values of \tilde{H} while the nonlocality effect becomes more important to the global response as the values of \tilde{H} increase, which is obvious from the F-u and the $\sigma_{eq} - p$ curves. All these figures represent the the responses of one Gauss point of an element in the center of the specimen in the intersection of the two bands.

3.3.1.2 Effect of the mesh size

In order to examine the influence of the mesh size on the micromorphic IBVP regularization ability for different values of the micromorphic moduli (\breve{H},\breve{H}^{g}) giving three values of the damage internal length of 0.6 mm, 0.8 mm and 1.0 mm (see table 3.5), the following three Q4 element sizes have been selected; h=0.2 mm, h=0.4 mm and h=0.8 mm. Clearly, h=0.2 mm is three times lower than the smallest value of damage internal length $\ell_{\breve{d}} = 0.6$ mm; h=0.4 mm is 1.5 times lower than the smallest value of the damage internal length, while h=0.8 mm is equal to the middle value of the damage internal length $\ell_{\breve{d}} = 0.8$ mm.

\breve{H} (MPa)	\breve{H}^{g} (N)	$\ell_{ec d}$
10.0	3.6	0.6 mm
100.0	36	0.6 mm
10.0	6.4	0.8 mm
100.0	64.0	0.8 mm
10.0	10.0	1.0 mm
100.0	100.0	1.0 mm

Table 3.5: Values of the material micromorphic parameters and the generated values of the damage internal length to be used with three different mesh sizes

Figures 3.28, 3.30, 3.32, 3.34 and 3.36 represent the global local and micromorphic F-u responses for all the six the cases for the three different meshes compared with the local case $(\breve{H} = \breve{H}^g = 0)$. Also the distribution of the local damage field d at the moment of the crack initiation are shown in Figures 3.29, 3.31, 3.33, 3.35, 3.37 and 3.39.

We present the results for all the cases and we make a discussion on them followed by a conclusion at the end of the paragraph.



▶ Case (I.a), $\ell_{d} = 0.6 \text{ mm}, H = 10.0, H^{g} = 3.6$

Figure 3.28: comparison of the global force-displacement responce of the local and the micromorphic model for three mesh sizes ($\ell_{\tilde{d}} = 0.6 \text{ mm}$, $\breve{H} = 10.0$, $\breve{H}^s = 3.6$)



Figure 3.29: local damage isovalues at the corresponding u_{rupt} for three mesh sizes

 $(\ell_{\bar{d}} = 0.6 \, mm, \, \breve{H} = 10.0, \, \breve{H}^{g} = 3.6)$

> Case (I.b), $\ell_{\tilde{d}} = 0.6 \text{ mm}, \breve{H} = 100.0, \breve{H}^{s} = 36.0$



Figure 3.30: comparison of the global force-displacement responce of the local and the micromorphic model for three mesh sizes ($\ell_{\vec{a}} = 0.6 \text{ mm}$, $\vec{H} = 100.0$, $\vec{H}^s = 36.0$)


Figure 3.31: local damage isovalues at the corresponding u_{rupt} for three mesh sizes ($\ell_{\tilde{d}} = 0.6 \text{ mm}, \tilde{H} = 100.0, \tilde{H}^g = 36.0$)

> Case (II.a), $\ell_{\tilde{d}} = 0.8 \text{ mm}, \tilde{H} = 10, \tilde{H}^{g} = 6.4$



Figure 3.32: comparison of the global force-displacement responce of the local and the micromorphic model for three mesh sizes ($\ell_{\tilde{d}} = 0.8 \text{ mm}, \tilde{H} = 10.0, \tilde{H}^s = 6.4$)



Figure 3.33: local damage isovalues at the corresponding u_{rupt} for three mesh sizes ($\ell_{\tilde{d}} = 0.8 \text{ mm}, \tilde{H} = 10.0, \tilde{H}^g = 6.4$)

Case (II.b), $\ell_{\vec{a}} = 0.8 \text{ mm}$, $\vec{H} = 100.0$, $\vec{H}^s = 64.0$



Figure 3.34: comparison of the global force-displacement responce of the local and the micromorphic model for three mesh sizes $\ell_{\bar{d}} = 0.8 \text{ mm}$, $\breve{H} = 100.0$, $\breve{H}^g = 64.0$



Figure 3.35: local damage isovalues at the corresponding u_{rupt} for three mesh sizes

($\ell_{\vec{d}} = 0.8 \text{ mm}, \vec{H} = 100.0, \vec{H}^{s} = 64.0$)

Case (III.a),
$$\ell_{i} = 1.0 \text{ mm}, H = 10.0, H^{g} = 10.0$$



Figure 3.36: comparison of the global force-displacement responce of the local and the micromorphic model for three mesh sizes ($\ell_{\bar{d}} = 1.0 \text{ mm}$, $\breve{H} = 10.0$, $\breve{H}^s = 10.0$)



Figure 3.37: local damage isovalues at the corresponding u_{rupt} for three mesh sizes ($\ell_{\tilde{d}} = 1.0 \text{ mm}, \breve{H} = 10.0, \breve{H}^g = 10.0$)



Figure 3.38: comparison of the global force-displacement responce of the local and the micromorphic model for three mesh sizes ($\ell_{\bar{d}} = 1.0 \text{ mm}$, $\breve{H} = 100.0$, $\breve{H}^g = 100.0$)



Figure 3.39: local damage isovalues at the corresponding u_{rupt} for three mesh sizes ($\ell_{\tilde{d}} = 1.0 \text{ mm}, \breve{H} = 100.0, \breve{H}^s = 100.0$)

Obviously, the fact that we use a micromorphic damage model and the activation of the micromorphic moduli are not enough to guarantee unconditional mesh independent solutions.

By observing the cases where $\ell_{\tilde{d}} = 0.6 \text{ mm}$ for $(\breve{H} = 10.0, \breve{H}^s = 3.6)$ (Figure 3.28), $\ell_{\tilde{d}} = 0.8 \text{ mm}$ for $(\breve{H} = 10.0, \breve{H}^s = 6.4)$ (Figure 3.32) and $\ell_{\tilde{d}} = 1.0 \text{ mm}$ for $(\breve{H} = 10.0, \breve{H}^s = 10.0)$ (Figure 3.36) we see that we obtain the exact local solutions due to the low values of \breve{H} and \breve{H}^s that are not sufficient to indicate the presence and the effect of the micromorphic contribution which seems negligible.

On the other hand, when $\ell_{\tilde{d}} = 0.6 \text{ mm}$ for $(\breve{H} = 100.0, \breve{H}^g = 36.0)$ (Figure 3.30), and $\ell_{\tilde{d}} = 0.8 \text{ mm}$ for $(\breve{H} = 100.0, \breve{H}^g = 64.0)$ (Figure 3.34), the solutions seem mesh independent only for h = 0.4 mm and h = 0.2 mm by giving rupture displacements at $u_{rupt} \approx 11.7 \text{ mm}$ and $u_{rupt} \approx 12 \text{ mm}$ respectively. Let us note here that the difference of 0.3 mm is due to the increase of \breve{H}^g from 36.0 N to 64.0 N since the value of \breve{H} is fixed to 10.0.

Finally, the best regularization seems to be achieved for $\ell_{\tilde{d}} = 1.0 \text{ mm}$ when $(\breve{H} = 100.0, \breve{H}^s = 100.0)$ (Figure 3.38). For this case, the internal length is bigger than the three

mesh sizes and the values of H and H^{g} are high enough in order to indicate the micromorphic effects by giving the same rupture displacement at $u_{rupt} \simeq 12.4$ mm yet the biggest among the 6 cases tested.

We conclude by saying that in order to achieve better regularization and as more mesh independent solutions as possible:

- 1) the size of the chosen internal length should be sufficiently bigger than the size of the elements $(\ell_{\vec{a}} > h)$, and
- 2) The most appropriate choice for the micromorphic moduli should depend on the fact that the values of \breve{H} and \breve{H}^s should be restricted in an "intermediate value" interval so that they are neither too low, in order to retrieve the local solutions, nor too high so that the micromorphic damage effect is so excessively strong that leads to no rupture appearance despite the large displacement as shown for example for the case of $\breve{H} = \breve{H}^s = 1000.0$.

3.3.2 Model with micromorphic isotropic hardening

The model only with micromorphic isotropic hardening gives one more partial differential equation supplementary to the local equilibrium equation, which is governed by the internal length $\ell_{\bar{r}} = \sqrt{\bar{Q}^{g}/\bar{Q}}$. As a result, the micromorphic scalar \bar{r} becomes a nodal unknown variable additionally to the displacements. This isotropic hardening model described by the pair of the parameters (\bar{Q}, \bar{Q}^{g}) is the objective of the following parametric study similar to the one shown before regarding the micromorphic damage moduli.

3.3.2.1 Uncoupled model only with micromorphic isotropic hardening

For this case, we activate only the micromorphic isotropic hardening in order to investigate any effects of coming from the several pairs of (\vec{Q}, \vec{Q}^s) but without indicating the coupling with the local ductile damage.

First, we fix the value of \bar{Q}^s and want to vary \bar{Q} by making the following choices of the representative internal length:

For $\breve{Q}^g = 2.5 (N)$

\breve{Q} (MPa)	$\ell_{ec{r}}$
250	0.1 mm
2.5	1.0 mm
0.1	5.0 mm

Table 3.6: Values of the micromorphic material parameters for $\overline{\check{Q}}^{g} = 2.5$



Figure 3.40: Force-displacement response of the uncoupled micromorphic model (\breve{Q}^s = 2.5)

We continue by fixing the value of \tilde{Q} and varying \tilde{Q} by making the following choices of the representative internal length:

For $\overline{Q} = 0.1 (MPa)$

$ar{Q}^{g}$ (N)	ℓ_r
0.001	0.1 mm
0.1	1.0 mm
2.5	5.0 mm

Table 3.7: Values of the micromorphic material parameters for $\, ec{Q} \, = 0.1 \,$



Figure 3.41: force-displacement response of the uncoupled micromorphic model ($\breve{Q}=0.1$)

As shown from the F-u curves in figures 3.40 and 3.41 there is no effect resulting from the presence of micromorphic for the current choices.

3.3.2.2 Model only with micromorphic isotropic hardening coupled with local damage-effect of $\ell_{\bar{r}}$ for a fixed mesh

We continue with a brief study on the effect coming from the micromorphic isotropic hardening moduli by fixing the value of \tilde{Q}^{g} and varying the value of $\ell_{\tilde{r}}$ between 0.1, 1.0 and 5.0 mm for a fixed mesh of h = 0.2 mm.

(i) For $\tilde{Q} = 0.1 (MPa)$: the values of \tilde{Q}^s giving the three targeted values of the micromorphic isotropic hardening internal lengths together with the associated fracture displacements are summarized in Table 3.8.

\breve{Q}^{g} (N)	$\ell_{\check{r}}$	u_{rupt}
0.001	0.1 mm	11.07 mm
0.1	1.0 mm	11.07 mm
2.5	5.0 mm	11.07 mm

Table 3.8: Values of the micromorphic material parameters and fracture displacement for



 $\overline{Q} = 0.1$

Figure 3.42: Local damage, equivalent plastic strain, von Mises stress, local and micromorphic isotropic hardening distributions at $u_{rupt} = 11.07$ mm for $\tilde{Q} = 0.1$, $\ell_{\bar{r}} = 0.1$, 1.0, 5.0 mm (i.e. $\tilde{Q}^s = 0.001$, 0.1, 2.5)



Figure 3.43 comparison of the global force-displacement responce of the local and the micromorphic model for $\,\,{ar Q}\,\,=0.1\,$

Figure 3.43 show the F-u curves obtained for the 3 cases. We note that only the parameter \tilde{Q} that defines R_{nloc} and consequantly Y_{nloc} could sensibly affect the global. No influence is observed from varying the parameter \tilde{Q}^{g} .

Moreover and for these explored values, the distribution of the demonstrated mechanical fields (Figures 3.42) remain unaffected and identical for all the internal lengths since for all the cases $p_{rupt} \simeq 52\%$, $d_{max} \simeq 40\%$, $\sigma_{eq} \simeq 660$, $r \simeq 0.32$ and $\breve{r} = 0.28$.

3.3.3 Model with micromorphic damage and micromorphic isotropic hardening

For this section we complete the micromorphic model by indicating the presence of both micromorphic damage and micromorphic isotropic hardening and we compare the F-u responses with the proportional obtained by the experimental tensile tests in order to validate the choice of all the micromorphic parameters.

3.3.3.1 Effect of $\ell_{\check{a}}$ for a fixed $\ell_{\check{r}}$ and a fixed mesh

We keep the mesh size fixed at h = 0.2 mm, $\ell_{\bar{r}} = \sqrt{\bar{Q}^s / \bar{Q}} = 5.0 \text{ mm}$ and $\bar{H} = 60.0$ and we vary the value of $\ell_{\bar{d}}$ in order to examine its effect on the F - u responses and the distribution of the mechanical fields for the mixed model.

 \vec{Q} =1000.0 (*MPa*), \vec{Q}^{s} =25000.0 (*N*), ℓ_{r} =5.0 mm

Й (MPa)	$\breve{H}^{g}(N)$	l _ă	U _{rupt}
60.0	0.6	0.1 mm	12.3 mm
60.0	60.0	1.0 mm	12.45 mm
60.0	1500.0	5.0 mm	13.9 mm

Table 3.9: Values of the micromorphic material parameters and fracture displacement for

 \vec{Q} =1000.0, \vec{Q}^{g} = 25000.0, ℓ_{r} = 5.0mm, \vec{H} = 60.0



Figure 3.44: Local damage, equivalent plastic strain, von Mises stress, local and micromorphic isotropic hardening distributions at $u_{rupr} = 12.3 \text{ mm}$ for $\tilde{Q} = 1000.0$, $\tilde{Q}^{g} = 25000.0$, $\tilde{H} = 60.0$,

 $\breve{H}^{\scriptscriptstyle g}$ = 0.6 (i.e. $\ell_{\scriptscriptstyle ec{r}}$ = 5.0mm, $\ell_{\scriptscriptstyle ec{d}}$ = 0.1mm)





isotropic hardening distributions at $u_{rupt} = 12.45 \text{ mm}$ for $\overline{Q} = 1000.0$, $\overline{Q}^{g} = 25000.0$, $\overline{H} = 60.0$,

 $\breve{H}^{g} = 60.0$ (i.e. $\ell_{r} = 5.0mm$, $\ell_{\breve{d}} = 1.0mm$)





Figure 3.46: Local damage, equivalent plastic strain, von Mises stress, local and micromorphic isotropic hardening distributions at $u_{rupt} = 13.9 \text{ mm}$ for $\breve{Q} = 1000.0$, $\breve{Q}^g = 25000.0$, $\breve{H} = 60.0$,

 \breve{H}^{g} =1500.0 (i.e. $\ell_{\breve{r}}$ =5.0mm, $\ell_{\breve{d}}$ =5.0mm)





Figure 3.47: Comparison of the global force-displacement responce of the experimental and the micromorphic model for $\ell_{\tilde{r}} = 5.0mm$, $\ell_{\tilde{d}} = 0.1mm$, $\ell_{\tilde{d}} = 1.0mm$, $\ell_{\tilde{d}} = 5.0mm$

For this case we till keep the mesh fixed at h = 0.2 mm and $l_{\tilde{r}} = 5.0 \text{ mm}$ and we investigate the effect of $\ell_{\tilde{d}}$ at the presence of micromorphic isotropic hardening. In order to enhance the micromorphic effect coming from the isotropic hardening, we increase the values of the parameters $(\tilde{Q} = 1000.0 \text{ and } \tilde{Q}^g = 25000.0)$ and to assure the final rupture of the specimen we choose an intermediate value for $\tilde{H} = 60.0$ and we vary the internal length among $\ell_{\tilde{d}} = 0.1, 1.0$ and 5.0 mm.

As expected, the more we increase the value of \vec{H}^{g} e.g, the value of $\ell_{\vec{d}}$, the more the rupture displacement is enlarged e.g, $u_{rupt} = 14.0 \text{ mm}$ for $\vec{H}^{g} = 1500.0 \text{ as shown}$ from the F - u curve in Figure 3.47. In terms of d, p_{rupt} , σ_{eq} and r their values tend to increase as we increase the value of $\ell_{\vec{d}}$ (Figures 3.44 to 3.46). On the other hand, the value of \vec{r} remains the same ($\vec{r} \approx 0.24$) for $\ell_{\vec{d}} = 0.1 \text{ mm}$ and $\ell_{\vec{d}} = 1.0 \text{ mm}$ but slightly increases up to $\vec{r} \approx 0.27$ when $\vec{H}^{g} = 1500.0$. Another observation is that the orientation of the crack is not respected and appears to be horizontal when $\ell_{\vec{d}} = 1.0 \text{ mm}$ but it always propagates along a single row of elements for all the cases. Unlike d, p_{rupt} , σ_{eq} and r, the values of the micromorphic damage \vec{d} seem to decrease by increasing the difference $(d - \vec{d})$ from case to case respectively.

Moreover, it is interesting to take a look at the results (Tables 3.10 to 3.12) for the same choice of internal length but by keeping the isotropic hardening parameters $\breve{Q} = \breve{Q}^g = 0$. It's worth noting that the presence of micromorphic isotropic hardening leads to larger rupture displacements, however a crack is achieved for the last test case ($\ell_{\bar{d}} = 5.0mm \ H = 60.0, \ H^g = 1500.0$) in contrast with keeping the isotropic hardening under a purely local form:

d	P _{rupt}	r	$\sigma_{_{eq}}$	u_{rupt}
0.41	0.75	0.38	698.0	11.3mm

Table 3.10: Local damage, equivalent plastic strain, von Mises stress, local isotropic hardening maximum values at $u_{rupt} = 11.3 \text{ mm}$ for $\ell_{\tilde{d}} = 0.1 \text{mm}$, $\breve{H} = 60$, $\breve{H}^g = 0.6$

d	P _{rupt}	r	$\sigma_{_{eq}}$	u_{rupt}
0.41	0.89	0.41	719.0	12.0 mm

Table 3.11: Local damage, equivalent plastic strain, von Mises stress, local isotropic hardening maximum values at $u_{rupt} = 12.46$ mm for $\ell_{\tilde{d}} = 1.0mm$, $\breve{H} = 60.0$, $\breve{H}^{g} = 60.0$

$d_{ m max}$	$P_{\rm max}$	r	$\sigma_{_{eq}}$	u_{rupt}
0.24	2.90	0.5	872	no rupture

Table 3.12: Local damage, equivalent plastic strain, von Mises stress, local isotropic hardening maximum values at u = 15.0 mm for $\ell_{\vec{d}} = 5.0 \text{ mm}$, $\vec{H} = 60.0$, $\vec{H}^g = 1500.0$

3.3.3.2 Effect of $\ell_{\tilde{r}}$ for a fixed $\ell_{\tilde{d}}$ and a fixed mesh

We continue the study by inverting the process and fixing the value of $\ell_{\tilde{d}} = 1.0$ mm and varying $\ell_{\tilde{r}}$ in order to examine its effect for a fixed mesh size of h = 0.2 mm. We compare again with the experimental results.

$\breve{H} = 100.0 \ (MPa), \ \breve{H}^{g} = 100.0 \ (N), \ \ell_{\breve{d}} = 1.0 mm$

\breve{Q} (MPa)	$ar{\mathcal{Q}}^{s}$ (N)	$\ell_{ec{r}}$	<i>U</i> _{rupt}
1000.0	25000.0	5.0 mm	12.66 mm
1000.0	100000.0	10.0 mm	13.6 mm
10.0	2250.0	15.0 mm	12.48 mm

Table 3.13: Values of the micromorphic material parameters and fracture displacement for \breve{H} =100.0, \breve{H}^{g} =100.0, $\ell_{\vec{d}}$ =1.0mm



Figure 3.48: comparison of the global force-displacement responce of the experimental and the micromorphic model for $\vec{H} = 100.0$, $\vec{H}^g = 100.0$, $\ell_{\vec{A}} = 1.0mm$, $\ell_{\vec{r}} = 5.0mm$, $\ell_{\vec{r}} = 10.0mm$, $\ell_{\vec{r}} = 15.0mm$

The mesh size remains fixed at h = 0.2 mm and $l_{\tilde{d}} = 1.0 \text{ mm}$ and we investigate the effect of $\ell_{\tilde{r}}$ at the presence of micromorphic damage. We choose $\ell_{\tilde{d}} = 1.0 \text{ mm}$ with $\breve{H} = \breve{H}^g = 100.0 \text{ and we vary}$ the internal length among $\ell_{\tilde{r}} = 5.0, 10.0 \text{ and } 10.0 \text{ mm}$ (Table 3.13).

Clearly the choice of \tilde{Q} and \tilde{Q}^{g} can affect the rupture displacement. As observed from the F-u curves in Figure 3.48, the lowest value of \tilde{Q} ($\tilde{Q} = 10.0$) gives $u_{nupt} = 12.48$ mm and similarly to the micromorphic damage model, the rupture displacement can be affected by the gradient modulus if \tilde{Q} remains fixed e.g, for $\tilde{Q} = 1000.0$ and $\tilde{Q}^{g} = 25000.0 u_{nupt} = 12.66$ mm, while for $\tilde{Q} = 1000.0$ and $\tilde{Q}^{g} = 100000.0 u_{nupt} = 13.6$ mm.

In terms of of d, p_{rupt} and σ_{eq} (Figures 3.49 to 3.51) their values tend to increase as we increase the value of $\ell_{\vec{r}}$. On the other hand, r and \breve{r} tend to behave differently this time. By keeping $\breve{Q} = 1000.0$ they both tend to decrease by increasing the internal length (increasing the value of \breve{Q}^g). As expected, \breve{r} takes its lowest value for $\breve{Q} = 10.0$ ($\breve{r} \approx 0.17$) but for all the three cases, the difference $(r - \breve{r})$ remains positive.

Finally, we also note that the combination of $\ell_{\tilde{r}} = 5.0mm$, $\ell_{\tilde{d}} = 1.0mm$ leads to a horizontal rather than a diagonal crack.





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\breve{H}^{s} =100.0 (i.e. \ell_{\bar{r}} =10.0mm, \ell_{\bar{d}} =1.0mm )
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3.3.3.3 Effect of the mesh size

Finally, we study the effect of the mesh size including both micromorphic damage and isotropic hardening. We choose three different mesh sizes h = 0.8mm, h = 0.4mm and h = 0.2mm; we give strong values to the isotropic hardening parameters ($\vec{Q} = 1000, \vec{Q}^s = 2500, \ell_r = 5.0 mm$) and acceptable intermediate values to the micromorphic damage moduli (H = 60, $H^s = 60$, $\ell_{\lambda} = 1.0$ mm). Figure 3.52 shows the global responses in terms of the F-u curves obtained for the three mesh sizes by using the Q4 element. The figure clearly shows the independence of the solution regarding the mesh size (for h = 0.8 mm, $u_{rupt} = 12.9 mm$, for h = 0.4 mm, $u_{rupt} = 12.7 mm$ and for msh = 0.2 mm, $u_{rupt} = 12.55mm$) while Figures 3.53 to 3.55 confirm the independence of the thickness of the crack even though it appears to be horizontal for all the mesh sizes but always on a single row of elements. In terms of the distribution of the studied mechanical fields, d, p_{ruot} and σ_{ea} , they seem to obtain their highest values for h = 0.2 mm ($d_{\max} = 67\%$, $p_{rupt} = 70\%$, $\sigma_{eq} = 712.0 \text{ MPa}$ as seen in Figure 3.55), while r and \breve{r} remain unaffected by the mesh size with $r \simeq 0.34$ and $\breve{r} \simeq 0.25$ for the three cases.



Figure 3.52: Comparison of the global force-displacement responce of the experimental and the micromorphic model for

 $(\breve{H} = \breve{H}^{g} = 60, \ell_{\breve{d}} = 1.0mm), (\breve{Q} = 1000.0, \breve{Q}^{g} = 2500.0, \ell_{\breve{r}} = 5.0mm), h = 0.8mm, h = 0.4mm, h = 0.2mm$



Figure 3.53: Local damage, equivalent plastic strain, von Mises stress, local and micromorphic isotropic hardening distributions at $u_{rupt} = 12.9$ mm for h = 0.8 mm







3.3.3.4 Effect of the mesh for a micromorphic isotropic hardening model size for a fixed ℓ_r

For a last test we want to study the effect of the mesh size for the Q4 element by taking the same mesh sizes used before (h=0.8mm, h=0.4mm, h=0.2mm). We deactivate the micromorphic damage moduli $(\breve{H}=\breve{H}^g=0)$ and we fix the value of $\ell_{\breve{r}}=\sqrt{\breve{Q}^g}/\breve{Q}=\sqrt{100000.0/1000.0}=10.0$ mm. Figure 3.56 shows the global F-u responses obtained for the three mesh sizes. The figure clearly shows the dependence of the solution on the size of the element, as seen for the purely local model and we conclude that the model with micromorphic isotropic hardening coupled with local damage fails to regularize the problem by giving an independent and unique solution.

As a result and after summarizing the results concerning the effect of the mesh size for a model with only micromorphic damage or micromorphic isotropic hardening, we confirm that a micromorphic damage model is necessary and enough in order to achieve the regularization of the IBVP solution undergoing induced softening due to the damage effect.



Figure 3.56: Global force-displacement response of the micromorphic isotropic hardening model for $\ell_{\tilde{a}} = 0.0$, $\ell_{\tilde{r}} = 10.0mm$, h = 0.8mm, h = 0.4mm, h = 0.2mm

3.4 Conclusions

In this chapter we have performed a relatively complete parametric study on three nonlocal micromorphic behavior models:

- One micromorphic damage model,
- One micromorphic isotropic hardening model and
- One model with the contributions of both micromorphic damage and isotropic hardening.

The metallic specimens of 430 stainless steel were meshed with the 2D quadrilateral assumed strain element and the numerical tensile tests were implemented in Abaqus[®]/Explicit.

The purpose of this study was the terms to regularize a micromorphic model and to examine the capacity of a micromorphic behavior model to give solutions independent from the mesh size. Finally, the numerical results concerning the model with both micromorphic damage and isotropic hardening were compared to the experimental F - u curve obtained by the tensile tests for specimens of the same geometry and material.

The principle conclusions of this study are:

- The micromorphic damage is necessary and enough to regularize the numerical solution of the IBVP;
- the size of the chosen micromorphic damage internal length should be sufficiently bigger than the size of the elements $(\ell_{\bar{a}} > h)$;
- for a constant mesh, the more the values of \breve{H} and \breve{H}^{g} increase, the more the rupture is retarded;
- the choice for the micromorphic moduli should depend on the fact that the values of H
 and H^g stay close to an "intermediate value", neither too low (so that the presence of
 nonlocality is negligible) nor too high (so that the micromorphic damage effect is so
 excessively strong that leads to no rupture appearance and non-physical solutions);
- the nonlocal model with both micromorphic damage and micromorphic isotropic hardening is highly capable to obtaining mesh independent solutions.

These conclusions allow us to make a proper choice of the values of the micromorphic material parameters with respect to the desirable mesh size and were carefully followed and applied for the metal forming applications presented in Chapter 5.

Chapter 4

Experimental aspects: local and micromorphic damage material parameters identification methodology.

4.1 Introduction

As presented in Chapter 2, the micromorphic constitutive equations derived from the framework of the irreversible thermodynamics and the generalized principle of virtual power are characterized by a group of specific material parameters which need to be determined from the experimental data. Before proceeding to the material parameter identification in this chapter, the parametric study of the micromorphic model was performed in chapter 4 and the results were carefully analyzed to well understand the predictive possibilities of the proposed micromorphic fully coupled constitutive equations. The detailed study of the effect of the material parameters entering the fully coupled constitutive equations is discussed in (Saanouni, 2012). Furthermore, here we will only focus on the parametric study concerning the elasticity, plasticity and local damage parameters and we will propose a methodology for identifying and properly choosing the value of the micromorphic moduli $(\tilde{H}, \tilde{H}^{g})$ related to the micromorphic isotropic ductile damage for this current material by associating it with the width of the shear bands that appear during the localized necking stage of a simple uniaxial tensile test.

4.2.1 Material parameters identification methodology

For this thesis, in order to identify the material parameters (for elasticity, local isotropic and kinematic hardening, local and nonlocal micromorphic damage parameters), we use the uniaxial tensile test for a specimen of 430 stainless steel with isotropic behavior the geometry of which is given in Fig. 4.2. Figure 4.1 shows the tensile response Force vs displacement u (F - u) obtained by the experiment:



Figure 4.1: Experimental F - u curve obtained from the tensile test for a specimen made of 430(X8Cr17) stainless steel

The specimens on which the tests were performed are made of stainless steel 430(X8Cr17) with effective length of $l_0 = 45.0$ mm, width of $b_0 = 15.0$ mm, and thickness of $e_0 = 0.2$ mm, and cut along

45° with respect to the rolling direction. The specimen was set in the tensile machine with the bottom clamp fixed, the top clamp lifted under controlled speed, which was kept constant at 0.5 mm/min in the early stage and decreased to 0.015 mm/min near and beyond to the maximum force load to observe precisely the state of diffuse and local necking (Bao et al., 2015). It should be pointed out that the tensile test evolution is monitored through the average total strain computed from the cross-beam displacement and the initial gauge length of the specimens as $l_n(l/l_0)$.



Figure 4.2: Geometry of the used specimen

We divide the curve (F - u curve in Fig. 4.1) into three stages according to the nature of the material parameters to be identified and we follow three steps respectively to complete the procedure:

- Stage I considering only the elastic behavior (I),
- Stage II considering a homogeneous elasto-plastic flow of the specimen. In this step, the effect of isotropic ductile damage on the global behavior can be neglected (II)
- Stage III considering a heterogeneous elasto-plastic flow with a significant effect of ductile damage.

When all mechanical fields are considered homogeneous in the useful zone of the specimen and the value of the damage is really small, the stress tensor is simplified in the framework of uniaxial stress:

$$\underline{\sigma} = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ with } \sigma_{xx} = \frac{F}{S_0} \left(1 + \frac{u}{l_0} \right)$$
(4.1)

where σ_{xx} is the uniaxial Cauchy stress component and $S_0 = b_0 e_0$ is the initial area of the specimen;

And, for the strain tensor:

$$\underline{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0\\ 0 & \varepsilon_{yy} & 0\\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \text{ with } \varepsilon_{xx} = \ln\left(1 + \frac{u}{l_0}\right), \tag{4.2}$$

The uniaxial loading path and Poisson effect linked to the different strain component:

$$\begin{cases} \varepsilon_{yy}^{e} = \varepsilon_{zz}^{e} = -v\varepsilon_{xx}^{e} & \text{for elastic behavior}(Stage I) \\ \varepsilon_{yy} = \varepsilon_{zz} = -\frac{1}{2}\varepsilon_{xx} & \text{for elastoplastic behavior}(Stages II + III) \end{cases}$$
(4.3)

where ν is the Poisson coefficient, ε_{xx} and ε_{xx}^{e} are respectively the total strain and elastic strain in the tensile direction *x*.

The Hooke law and the decomposition of the total strain for monotonic loading tensile path give the different relations:

$$\sigma_{xx} = E\varepsilon_{xx} \tag{4.4}$$

$$\mathcal{E}_{xx} = \mathcal{E}_{xx}^e + \mathcal{E}_{xx}^p \tag{4.5}$$

where E is the Young's modulus and \mathcal{E}_{xx}^{p} is the plastic strain in the tensile direction x.

<u>Step n°1 : Identification of the elastic material parameters (E, v and σ_{y})</u>

For the stage I, it is possible to translate the F - u curve into a $\sigma_{xx} - \varepsilon_{xx}^e$ curve in order to identify the elasticity material parameters regarding the Young's modulus E and the yield stress σ_y . After this study and for the specific material we conclude that: E = 220. GPa, v = 0,3 and $\sigma_y = 300$.MPa

Step n°2 : Identification of the isotropic (Q, b) and kinematic (C, a) hardening parameters

Since we are still before the value F_{max} of the maximum force and before the heterogeneity of the fringes followed by the diffuse necking, we can still work on the $\sigma - \varepsilon$ curve by taking into consideration this time the plastic part of the curve where the values of the local damage d are still relatively low.

The cumulative plastic strain norm is then defined by

$$p = \sqrt{\frac{2}{3}\underline{\varepsilon}^{p} : \underline{\varepsilon}^{p}} = \varepsilon_{xx}^{p} = \varepsilon_{xx} - \varepsilon_{xx}^{e} = \varepsilon_{xx} - \frac{\sigma_{xx}}{E}$$
(4.6)

We can now work on the proportional $\sigma - p$ curve as shown in Fig. 4.3.



Figure 4.3: Experimental vs numerical plastic part

The elastoplastic behavior model given in Chapter 1 can be simplified in an analytic form in the case of a homogeneous tensile loading path (see Saanouni, 2012 for more details):

$$\sigma_{xx}(p) = \sigma_{y} + \frac{C}{a} \left(1 - e^{-ap} \right) + \frac{Q}{b} \left(1 - e^{-bp} \right)$$
(4.7)

By using the formula above, the value of the yield stress σ_y obtained by the step 1 and respecting the following condition:

$$\frac{C}{a} < \frac{Q}{b} \tag{4.8}$$

To clarify the decomposition of the isotropic and kinematic hardening phenomena, a cycle loading path is generally used (a cyclic shear test for exemple). However, in the case of this stainless steel, only monotonic tensile tests have been realized. In the case of this steel, we assume that the kinematic hardening phenomena are activated and appear just after the elastic behavior. The parameters *C* and *a* associated to the kinematic hardening are calibrated to fit the first part of the plastic curve for (p < 0.05). So, we use the second part of the plastic curve to fit the parameters *Q* and *b* associated to the isotropic hardening. An optimization procedure based on BFGS quasi-Newton algorithm is used to find the most suitable values for the local isotropic and kinematic hardening parameters and fit the 'test' curve to the experimental one under the best agreement.

At the end of this step, we conclude that:

$$C = 3400.0MPa, a = 40.0$$

 $Q = 1135.0MPa, b = 2.0$ (4.9)

<u>Step n°3 : Identification of the local damage (S, s, β , γ , Y_0) and micromorphic damage (\breve{H}^s, \breve{H}) </u> parameters

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For the third and the last step of the identification process we use the stage (III) of the F-u curve. The damage parameters that have to be identified are both of local and nonlocal nature and thus we can divide this step in two:

- Step 3-i, for the identification of the local damage parameters, e.g., S, s, β, γ, Y_0 .
- Step3-ii, for the identification of the internal length $\ell_{\tilde{d}} = \sqrt{\frac{\breve{H}^s}{\breve{H}}}$ related to the micromorphic

damage.

For the overall third step, we have reached the value of F_{max} meaning that we pass from the diffuse to the highly localized necking and since all the mechanical fields have turned inhomogeneous, the use of the $\sigma - \varepsilon$ curve is no more possible. As a result, we switch to an inverse identification and with the introduction of the numerical simulation of the same tensile test, respecting the geometry, the velocity and the boundary conditions, we perform several tests for the identification of the material parameters related to the ductile damage by comparing the numerical and the experimental F - u curves.

Step 3-i: The sensitivity of each local damage parameter on the force/displacement response in the case of a tensile test has been studied in different references (Saanouni 2012; Labergere et. al, 2014 among others). The knowledge issue on these studies can be resumed in different points:

I. The parameter *s* governs the nonlinearity of the damage evolution: the higher value of *s*, the earlier the final fracture time, the smaller the plastic strain and the more brittle behavior of the material. Considering the formula of damage evolution in Eq.(1.53), when the parameter *s* approaches to zero, the damage rate will become approximately proportional to the

plastic multiplier (effective plastic strain rate) $\dot{d} = \frac{\dot{\lambda}}{(1-d)^{\beta}}$ if the effect of the parameter β

representing the nonlinearity of damage is negligible.

- II. When the local damage d > 0.1, the increase of parameter β accelerates the evolution of the local damage variable d.
- III. The parameter γ controls the influence of the local damage on the isotropic hardening. Considering the formula of Y_R in Eq.(1.28c), the higher values of γ provide a smaller value of Y_R for damage $d \ll 0.1$, but accelerate the evolution of Y_R when damage approaches the critical damage.
- IV. The parameter S is directly linked to the material ductility and governs the apparition of the fracture after the diffuse necking. Meanwhile, it is observed that its effect is to delay or accelerate the damage growth without modifying the shape of the damage evolution or the F-u curves.

No extended study or special treatment has been performed on the parameters s, β and γ in order to further identify their effect. Different metallic materials have been identified in different works (see Liu, 2017). Generally, a good agreement for their values are found between $1 \le s \le 2$ and $\frac{1}{2} \le \beta \le 4$ while some further investigation has been done for the parameter *S* due to its higher sensitivity. For the present material we use s = 1 and $\beta = 1$. We note that we also set the initial damage force $Y_0 = 0$ and $\hbar = 0$ for the effect of the microcracks closure.

<u>Step 3-ii</u>: The last group of material parameters left to be identified are the ones related to the micromorphic damage and more precisely the micromorphic internal length. We note that we deactivate the coupling with the micromorphic isotropic hardening as well as the respective micromorphic parameters \tilde{Q} and \tilde{Q}^{g} and for the sake of simplicity we intend to propose a methodology to identify $\ell_{\tilde{d}} = \sqrt{\tilde{H}^{g}/\tilde{H}}$.

As shown in the previous chapter, the numerical and the experimental F-u curves were used in order to investigate the effects of \breve{H} and \breve{H}^{g} and approach their most appropriate choice but without taking into account the notion of the micromorphic internal length $\ell_{\vec{d}}$. For this purpose and under the perceptible existence of nonlocality, the experimental strain field measurements coming from the Electronic Speckle Pattern Interferometry (ESPI) during the tensile test of the 430 stainless steel has been done. The strain rate component $\dot{\varepsilon}_{xx}$ measured by ESPI is used to determine the evolution of the band width during the localization. The idea is to define a link between $\ell_{\vec{d}}$ and the measured integral band width.

Necking as an unstable behavior in ductile solids has been studied for many years. However, less work has been done on the measurement of the width of the localization bands and its evolution during loading. It is also well known that the mesh size dependence of the post-bifurcation necking behavior is still an open problem in numerical analysis (Bažant et al., 1984). Indeed, with a purely local constitutive behavior, the solution for the strain distribution is no longer unique and can only be computed for a given mesh size (Labergere et al., 2014). Generalized continua schemes can overcome this difficulty (Jirásek and Rolshoven, 2009; Mazière and Forest, 2013) by introducing additional parameters that need special treatment for their identification. For this reason, the study of the strain localization behavior from an experimental perspective using photomechanical methods is introduced. Electronic Speckle Pattern Interferometry (ESPI) has been used for the deformation measurement, because of its specific advantages, such as high spatial and displacement resolutions.

The principles of ESPI are simple: a speckle field, e.g. a seemingly random intensity pattern, is produced by the interference of the wave fronts, when a coherent light illuminates a rough diffusing surface. A micro-displacement or deformation of the object surface leads to changes of the speckle field and thus the deformation information can be obtained from the correlated evolution of the initial speckle pattern (Labergere et al., 2014; Guelorget et al., 2006). The experimental setup is represented in Figures 4.4 and 4.5.



Figure 4.4: Experimental set-up: in-plane speckle interferometry



Figure 4.5: The laser beam path schematized as a red line

The fringe patterns shown in Fig. 4.6 represent the displacement field increment between the capture of two phases maps separated by the duration indicated besides each picture. Constant and homogenous strain appears as a uniform fringe pattern in which the fringes are straight and perpendicular to the tensile direction. So, in the elastic and homogenous plastic deformation range, there is no change in the fringe pattern appearance. As soon as the plastic deformation becomes inhomogeneous, the fringes start to curve, with a non-uniform spacing, and concentrate in the middle of the specimen forming an hourglass shape which can be interpreted as two crossing localization bands, as can be seen in Fig. 4.6.

This hourglass shape narrows progressively and, around 23% of average strain, turns into an "X" shape. The fringe spatial frequency is higher in the middle of the specimen than along the two edges of the specimen which means that the strain is higher in the middle. In the area outside the "X", where there is no fringe, the material stops deforming plastically and undergoes elastic unloading when the force decreases. At the beginning of the localization, the hourglass shape is symmetric, and then it loses the symmetry when it turns into an "X". Then the fringes (deformations) start to concentrate in one of the bands (or one of the branches of "X"), thus the other one tends to disappear gradually and finally the specimen breaks along the band left. The transition between the symmetric hourglass ")(" and the asymmetric "X" is interpreted as the onset of the localized necking.

The localized necking stage can be divided into two sub-stages, one where the disappearing band is still active and another where it disappears (Figure 4.6).



Figure 4.6: Evolutions of the fringe patterns (top row) obtained by ESPI and the corresponding strain rate maps (bottom row) during the reference test

For extracting global information and physically relevant parameters, the strain rate pattern is modeled assuming that the strain rate field is a system of two straight crossing bands based on the "X" shape of the fringe pattern. The strain rate distribution along each band is supposed to be constant. The whole strain rate distribution is assumed to be a simple superposition (sum) of the strain rate distribution of these two bands. This model is not a mechanical model but a purely analytical model used to describe the strain rate pattern and extract global parameters and completely independent of the constitutive behavior of the material. The strain rate distribution in each band is described by a linear combination of a Gaussian and a Lorentzian function (Bao et al., 2015):

$$\dot{\varepsilon}_{xx}(x, y) = \dot{\varepsilon}_{1}^{B} + \dot{\varepsilon}_{2}^{B}$$
with $\dot{\varepsilon}_{i}^{B} \left(\dot{\varepsilon}_{i}^{\max}, \eta_{i}, a_{i}, b_{i}, B_{i}, x, y \right) = \dot{\varepsilon}_{i}^{\max} \left(\eta_{i} G \left(a_{i}, b_{i}, B_{i}, x, y \right) + (1 - \eta_{i}) L \left(a_{i}, b_{i}, B_{i}, x, y \right) \right)$
(4.10)

where i = 1 or 2 indicates one of the two localized bands, and (x, y) are the coordinates of the points (x is parallel to the tensile direction), with:

- $\dot{\varepsilon}_i^{\max}$ is the maximum strain rate,
- G_i , Gaussian function,
- L_i , Lorentzian function,
- η_i and $1-\eta_i$ are the weights of the Gaussian and the Lorentzian functions respectively.

The Gaussian and Lorentzian functions are defined as following:

$$G(a_i, b_i, B_i, x, y) = \exp\left(-\pi \left(\frac{x - a_i y - b_i}{B_i}\right)^2\right)$$
(4.11)

$$L(a_i, b_i, B_i, x, y) = \frac{1}{1 + \left(\pi \frac{x - a_i y - b_i}{B_i}\right)^2}$$
(4.12)

with (Figure 4.7):

- a_i : represents the inclination of the band i with respect to the transverse direction,
- b_i : represents the location of the band i,
- B_i : represents the integral width of the band i.



Figure 4.7: Illustration of the parameters, the image on the left is a fringe pattern and that on the right is the corresponding strain rate map obtained

The inclination angles of the bands are defined as the angles between the band length and the tensile direction. The experimental parameters of the model are identified by a least squares fitting procedure from the velocity maps obtained by ESPI.

The obtained evolution of the strain rate map is shown on Fig. 4.6 for a test at 0.015 mm/min. The evolutions with the global average strain of the bands widths are shown. In all cases, the specimens failed in the same direction, along the band that was oriented almost perpendicular to the rolling direction or along the band 1 (B₁) as illustrated in Figure 4.7. Thereby for the sake of simplicity, this band will be called dominant band and the other one disappearing band, and the superscript "dom" and "dis" will be used to distinguish them. On the strain rate maps evolution, the observations made on the fringe patterns are clearer. In a first stage, the maps exhibit a symmetric hourglass shape. In a second stage, defined as the localized necking, one band starts to predominate and the shape turns into an asymmetric "X" shape. The strain rate in the dominant band increases progressively until fracture.

The maximum strain rate of the band^{40m} (dominant band B_1) increases exponentially with the cross-beam displacement while the maximum strain rate of band^{40s} (disappearing band B_2) increases slowly and even starts to decrease close to the end of the test. During the diffuse necking, the hourglass pattern formed by the two bands is symmetric and the maximum strain rates of the two bands are equal. Then, the strain rate of the two bands starts to diverge.

The bandwidths B^{dom} and B^{dis} of the two bands, in all cases, decrease with the total average strain or time. At the beginning, the two bandwidths decrease simultaneously and then they separate: B^{dis} tends to stabilize while B^{dom} continues decreasing.



Figure 4.8: Evolutions of the localization band characters vs total average strain (or time), (Bao et al., 2015)

The widths of the two bands decrease at a similar rate during the diffuse necking. Then, when the localized necking stage starts, the width of the dominant band continues decreasing until the rupture, while the width of the disappearing band continues to decrease for a while and then stabilizes. Based on this observation, which is consistent with the strain rate evolutions, two substages are distinguished in the localized necking regime. The transition of these two sub-stages is defined as the stabilization of the disappearing band width.

As already mentioned, generalized continua schemes introduce additional parameters that need special treatment for their identification. One of these is the internal length related to the micromorphic damage. Since ESPI allows the identification of the physical characteristics of the localization, such as bandwidths, band orientations and maximum strain rates of the bands at any moment of the necking, one first attempt is done under the idea of 'linking' the notion of the micromorphic internal length to the one of the bandwidths in order to give it some physical meaning and propose a numerical methodology to identify it.

In this spirit, and by numerically implementing the mathematical model given by Eq.(4.10)-Eq.(4.12) and calculating the numerical strain rate at a given time $\dot{\varepsilon}_{xx}^{num}(x, y, t)$ at each gauss point of an element located in the central zone we manage to extract the values of the numerical bandwidth B^{dom} during the diffuse and the localized necking and compare the numerical results with the experimental ones shown in Figure 4.8. The steps of the proposed methodology are given in the following Table 4.1.

We note $P \in \left[\varepsilon_1^{\max}, \eta_1, a_1, b_1, B_1, \varepsilon_2^{\max}, \eta_2, a_2, b_2, B_2\right]$ the best optimal parameters to fit the gradient evolution of the strain rate on the tensile direction.
Experimental (EPSI) data

Numerical data

Computation of $\dot{\varepsilon}_{\mathrm{xx}}$ in one direction at a defined experimental time t

$$\dot{\varepsilon}_{xx}^{exp}(\mathbf{x}, \mathbf{y}, t) \qquad \dot{\varepsilon}_{xx}^{num}(\mathbf{x}, \mathbf{y}, t)$$
$$\dot{\varepsilon}_{xx}^{fit}(P, x, y) = \dot{\varepsilon}_{1}^{B} \left(\dot{\varepsilon}_{1}^{\max}, \eta_{1}, a_{1}, b_{1}, B_{1}, x, y \right) + \dot{\varepsilon}_{2}^{B} \left(\dot{\varepsilon}_{2}^{\max}, \eta_{2}, a_{2}, b_{2}, B_{2}, x, y \right)$$

Least square optimization to minimize the space of the experimental parameters

$$\min_{P_{\exp}} \Re^{\exp}(P)$$
$$\Re^{\exp}(P) = \sum_{i=1}^{N_{\exp}} \left\| \dot{\varepsilon}_{xx}^{\exp}\left(x_i^{\exp}, y_i^{\exp}\right) - \dot{\varepsilon}_{xx}^{fit}\left(P, x_i^{\exp}, y_i^{\exp}\right) \right\|^2$$
where $P_{\exp} = \left[\dot{\varepsilon}_1^{\max}, a_1^{\exp}, b_1^{\exp}, B_1^{\exp}, \dot{\varepsilon}_2^{\max}, a_2^{\exp}, b_2^{\exp}, B_2^{\exp} \right]$

and N_{exp} is the number of measured experimental point. x_i^{exp}, y_i^{exp} are the coordinate of the experimental point

Least square optimization to minimize the space of the numerical parameters

$$\min_{P_{num}} \mathfrak{R}^{num}(P)$$
$$\mathfrak{R}^{num}(P) = \sum_{i=1}^{N_{ele}} \left\| \dot{\varepsilon}_{xx}^{num} \left(x_i^{PG}, y_i^{PG} \right) - \dot{\varepsilon}_{xx}^{fit} \left(P, x_i^{PG}, y_i^{PG} \right) \right\|^2$$

where

$$P_{num} = \left[\dot{\varepsilon}_1^{\max}, a_1^{num}, b_1^{num}, B_1^{num}, \dot{\varepsilon}_2^{\max}, a_2^{num}, b_2^{num}, B_2^{num} \right]$$

and $N_{\rm ele}$ is the number element located in the usefull area of the specimen. x_i^{PG}, y_i^{PG} are the coordinate of the Gauss point



Numerical B^{dom}



Table 4.1: experimental vs numerical identification methodology

4.3 Iterative procedure for the identification of the local damage parameter *S* and micromorphic moduli (\breve{H}^s, \breve{H})

Before presenting the results comparing B_{dom}^{exp} with B_{dom}^{num} we make a brief study of the effect of the parameter *S* and its sensitivity regarding the mesh size and the micromorphic regularization length.

First we fix the mesh size at h = 0.2 mm and the micromorphic regularization length at $\ell_{\tilde{d}} = \sqrt{\tilde{H}^g / \tilde{H}} = \sqrt{36.0/100.0} = 0.6 \text{ mm} > h$. We test the cases of S = 4.5, S = 5.0 and S = 6.0. The results on the effect on the numerical F - u curves are shown in Fig. 4.9.

For S = 4.5 we have that $u_{rupt} = 11.67$ mm, for S = 5.0 $u_{rupt} = 12.27$ mm and finally for S = 6.0 $u_{rupt} = 13.35$ mm.



Figure 4.9: Experimental vs numerical F - u curve, $\ell_{d} = 0.6mm$, h = 0.2mm

For a second test, we keep the same value of $\ell_{\tilde{d}} = \sqrt{\tilde{H}^s / \tilde{H}} = \sqrt{36.0/100.0} = 0.6 \text{ mm}$ and we fix the mesh size at h = 0.4 mm. The results on the effect on the numerical F - u curves are shown in Fig. 4.10.

For S = 4.5 we have that $u_{rupt} = 11.91$ mm, for S = 5.0 $u_{rupt} = 12.45$ mm and finally for S = 6.0 $u_{rupt} = 13.44$ mm.



Figure 4.10: Experimental vs numerical F - u curve, $\ell_{\tilde{d}} = 0.6mm$, h = 0.4mm

For the third test, we fix the mesh size at h = 0.2 mm and the internal length at $\ell_{\tilde{d}} = \sqrt{\tilde{H}^g / \tilde{H}} = \sqrt{64.0/100.0} = 0.8 \text{ mm}$. We test the cases of S = 4.5, S = 5.0 and S = 6.0. The results on the effect on the numerical F - u curves are shown in Fig. 4.11.

For S = 4.5 we get a $u_{rupt} = 11.94$ mm, for S = 5.0 $u_{rupt} = 12.58$ mm and finally for S = 6.0 $u_{rupt} = 13.3$ mm



Figure 4.11: Experimental vs numerical F-u curve, $\ell_{\vec{d}} = 0.8mm$, h = 0.2mm

Obviously as expected and as mentioned at the beginning of the chapter, the parameter S has an essential effect on the ductility of the material by retarding or accelerating the final fracture depending highly on the choice of the internal length and the mesh size. The same effect on the final rupture displacement can also result from the choices of (\vec{H}, \vec{H}^g) as discussed in the previous chapter. The goal is not to precise the value of the micromorphic regularization length according to the material and its microstructure, but to verify our strategy and the accuracy of connecting the width of the dominant band to the micromorphic regularization length scale related to the micromorphic damage in an effort to determine it physically and give some information about its measurement.

For these tests we varied the value of the parameter *S* between S = 4.5, S = 5.0 and S = 6.0 due to its impact on the numerical F - u curves. The mesh sizes which we chose are the finest ones with h = 0.2 mm and h = 0.4 mm and the micromorphic regularization length takes either the value of $\ell_{\bar{d}} = 0.6$ mm or $\ell_{\bar{d}} = 0.8$ mm with different choices of the respective micromorphic moduli $(\breve{H}, \breve{H}^{g})$.

4.3.1 Influence of the moduli (\breve{H}^s, \breve{H}) on the evolution of the integral bandwidth with S = 4.5

> Case 1.a: h = 0.2 mm, $\ell_{\vec{a}} = \sqrt{\breve{H}^{g} / \breve{H}} = \sqrt{3.6/10.0} = 0.6 \text{ mm}$ compared with h = 0.2 mm, $\ell_{\vec{a}} = \sqrt{\breve{H}^{g} / \breve{H}} = \sqrt{36.0/100.0} = 0.6 \text{ mm}$.



Figure 4.12: Evolution of the localization dominant band for $(\vec{H} = 100, \vec{H}^g = 36)$ and $(\vec{H} = 10, \vec{H}^g = 3, 6)$, numerical-experimental comparison for h = 0, 2mm

Case 1.b: h = 0.2 mm, $\ell_{\vec{a}} = \sqrt{\vec{H}^g / \vec{H}} = \sqrt{6.4/10.0} = 0.8 \text{ mm}$ compared with h = 0.2 mm, $\ell_{\vec{a}} = \sqrt{\vec{H}^g / \vec{H}} = \sqrt{64.0/100.0} = 0.8 \text{ mm}$.



Figure 4.13: Evolution of the localization dominant band for $(\breve{H} = 100.0, \breve{H}^g = 64.0)$ and $(\breve{H} = 10.0, \breve{H}^g = 6.4)$, numerical-experimental comparison for h = 0, 2mm

Case 1.c: h = 0.4 mm, $\ell_{\vec{d}} = \sqrt{\vec{H}^{g} / \vec{H}} = \sqrt{3.6/10.0} = 0.6 \text{ mm}$ compared with h = 0.4 mm, $\ell_{\vec{d}} = \sqrt{\vec{H}^{g} / \vec{H}} = \sqrt{36.0/100.0} = 0.6 \text{ mm}$.



Figure 4.14: Evolution of the localization dominant band for $(\breve{H} = 100.0, \breve{H}^g = 36.0)$ and $(\breve{H} = 10.0, \breve{H}^g = 3.6)$, numerical-experimental comparison for h = 0, 4mm

> Case 1.d: h = 0.4 mm, $\ell_{\vec{a}} = \sqrt{\vec{H}^{g} / \vec{H}} = \sqrt{6.4/10.0} = 0.8 \text{ mm}$ compared with h = 0.4 mm, $\ell_{\vec{a}} = \sqrt{\vec{H}^{g} / \vec{H}} = \sqrt{64.0/100.0} = 0.8 \text{ mm}$.



Figure 4.15: Evolution of the localization dominant band for $(\breve{H} = 100.0, \breve{H}^g = 64.0)$ and $(\breve{H} = 10.0, \breve{H}^g = 6.4)$, numerical-experimental comparison for h = 0, 4mm

After taking a look at the results of this first test case we can see that the variation of the mesh size does not affect the evolution of the band and the tendency to decrease is respected as observed in the experiment. While using h = 0.2 mm the bandwidth starts approximately at 20 mm for 20.6% of total average strain for both $\ell_{\vec{d}} = 0.6$ mm and $\ell_{\vec{d}} = 0.8$ mm (Figures 4.12 and 4.13) and it seems like the behavior of the numerical curves are not affected by the variation of (\vec{H}, \vec{H}^g) . On the other hand, for h = 0.4 mm (Figures 4.14 and 4.15) the width of the dominant band starts lower at around 10-15 mm except the case of $\ell_{\vec{d}} = \sqrt{\vec{H}^g / \vec{H}} = \sqrt{64.0/100.0} = 0.8$ mm where it starts at 20 mm for 20.6% of total average strain. For all the cases, the decreasing tendency is in agreement with the experimental results, however for S = 4.5 (case 1) the widths evolve at a much lower level which does not allow an appropriate fitting. We note here that from the numerical point of view, for h = 0.4 mm all the specimens failed at the opposite direction, so the experimental B^{dom} is represented by the numerical band B₂.

4.3.2 Influence of the moduli (\breve{H}^g, \breve{H}) on the evolution of the integral bandwidth with S = 5.0

> Case 2: h = 0.2 mm, $\ell_{\vec{d}} = \sqrt{\vec{H}^{g} / \vec{H}} = \sqrt{36.0 / 100.0} = 0.6 \text{ mm}$ compared with h = 0.4 mm, $\ell_{\vec{d}} = \sqrt{\vec{H}^{g} / \vec{H}} = \sqrt{36.0 / 100.0} = 0.6 \text{ mm}$.



Figure 4.16: Evolution of the localization dominant band for $(\breve{H} = 100.0, \breve{H}^s = 36.0)$, numericalexperimental comparison for two varying meshes h = 0, 2mm and h = 0, 4mm

We increase the value of S = 5.0 and we want to observe the behavior of the evolution of the numerically obtained bandwidths in the terms of a constant $\ell_{\bar{d}} = \sqrt{H^g / H} = \sqrt{36.0/100.0} = 0.6$ mm for two different mesh sizes. Similarly, to the case 1, we conclude that the mesh size does not affect the evolution of the band and the tendency is still decreasing but the numerical curves still fail to approach enough the experimental one. The size of the bandwidths is still lower than the experimental one, however by increasing the value of S we see that the starting point of B₂ is higher than 15 mm and higher than the previous case 1 where S = 4.5. The analysis of these previous results shows that if we continue increasing the value of S, the fitting between the numerical and experimental bandwidth becomes more accurate.

4.3.3 Influence of the moduli (\breve{H}^g, \breve{H}) in the evolution of the integral bandwidth with S = 6.0

The same parametric studies are analyzed.

> Case 3.a: h = 0.2 mm, $\ell_{\vec{a}} = \sqrt{\vec{H}^{g} / \vec{H}} = \sqrt{36.0/100.0} = 0.6 \text{ mm}$ compared with h = 0.2 mm, $\ell_{\vec{a}} = \sqrt{\vec{H}^{g} / \vec{H}} = \sqrt{21.6/60.0} = 0.6 \text{ mm}$.



Figure 4.17: Evolution of the localization dominant band for $(\breve{H} = 100.0, \breve{H}^s = 36.0)$ and $(\breve{H} = 60.0, \breve{H}^s = 21.6)$, numerical-experimental comparison for h = 0, 2mm

> Case 3.b: h = 0.4 mm, $\ell_{\vec{a}} = \sqrt{\vec{H}^g / \vec{H}} = \sqrt{36.0/100.0} = 0.6 \text{ mm}$ compared with h = 0.4 mm, $\ell_{\vec{a}} = \sqrt{\vec{H}^g / \vec{H}} = \sqrt{21.6/60.0} = 0.6 \text{ mm}$.



Figure 4.18: Evolution of the localization dominant band for $(\vec{H} = 100.0, \vec{H}^s = 36.0)$ and $(\vec{H} = 60.0, \vec{H}^s = 21.6)$, numerical-experimental comparison for h = 0, 4mm

For this last case, we fix S = 6.0 and $\ell_{\vec{d}} = 0.6 \text{ mm}$ with two different combinations ($\ell_{\vec{d}} = \sqrt{\vec{H}^{g} / \vec{H}} = \sqrt{36.0/100.0} = 0.6 \text{ mm}$ and $\ell_{\vec{d}} = \sqrt{\vec{H}^{g} / \vec{H}} = \sqrt{21.6/60} = 0.6 \text{ mm}$) for the two different mesh sizes.

The results seem quite interesting this time for h = 0.2 mm (Figure 4.17). For the choice of $\ell_{\tilde{d}} = \sqrt{\breve{H}^g / \breve{H}} = \sqrt{21.6/60.0} = 0.6 \text{ mm}$ the band width starts higher than all the previous cases (around 25 mm) followed by a good fitting between 21% and 23% of total average strain but with a big error at 25% of total average strain. For $\ell_{\tilde{d}} = \sqrt{\breve{H}^g / \breve{H}} = \sqrt{36.0/100.0} = 0.6 \text{ mm}$ we observe the most accurate behavior since the starting and ending values of B₁ are in absolute agreement with the experimental B_{dom} while the error remains relatively small and even negligible.

Approximately the same tendency is observed in the case of h = 0.4 mm (Figure 4.18). For $\ell_{\tilde{d}} = \sqrt{\tilde{H}^g / \tilde{H}} = \sqrt{21.6/60} = 0.6 \text{ mm}$ the dominant band starts evolving at around 24 mm, approaches sufficiently the experimental curve between 21% and 25% of total average strain but drops rapidly giving a big error until 25% of the total average strain. On the other hand, when $\ell_{\tilde{d}} = \sqrt{\tilde{H}^g / \tilde{H}} = \sqrt{36.0/100.0} = 0.6 \text{ mm}$, the band starts correctly at 20 mm, follows exactly the same behavior as the bandwidth obtained for $\tilde{H} = 60.0$ but does not achieve an exact fitting since it also decreases significantly at 24% of total average strain yet giving a smaller error than the previous case.

4.4 Material characterization based on local indentation test

Instrusmented Indentation Testing (IIT) is a form of mechanical testing that significantly expands on the capabilities of traditional hardness testing. IIT employs high-resolution instrumentation to continuously control and monitor the loads and displacements of an indenter as it is driven into and withdrawn from a material. Mechanical properties are derived from the indentation loaddisplacement data obtained in simple tests. The advantages of IIT are numerous, as indentation loaddisplacement data contain a wealth of information, and techniques have been developed for characterizing a variety of mechanical properties. The technique most frequently employed measures the hardness, but it also gives the elastic modulus (Young's modulus) from the same data. Mechanical properties are routinely measured from submicron indentations, and with careful technique, properties have even been determined from indentations only a few nanometers deep.

The most frequently used indenter in IIT testing is the Berkovich indenter, a three-sided pyramid with the same depth-to-area relation as the four-sided Vickers pyramid used commonly in microhardness work. The Berkovich geometry is preferred to the Vickers because a three-sided pyramid can be ground to a point, thus maintaining its self-similar geometry to very small scales. An example of small indentations located at specific points in an aluminium and a nickel alloy is shown in Fig. 4.19 (Hay and Phaar, 2001) and Fig.4.20 (Hay, 2009) respectively.



Figure 4.19: Berkovich indentation in aluminum

Figure 4.20: Residual impression in nickel, made by a Berkovich diamond indenter

As shown schematically in Fig. 4.21, equipment for performing instrumented indentation tests consists of three basic components: (a) an indenter of specific geometry usually mounted to a rigid column through which the force is transmitted, (b) an actuator for applying the force, and (c) a sensor for measuring the indenter displacements.



Figure 4.21: Schematic representation of the basic components of an instrumented indentation testing system

The two mechanical properties measured most frequently by IIT methods are hardness (H) and elastic modulus (Young's modulus) (E). A simple methodology has been developed to measure these two quantities (H and E) for isotropic materials exhibiting no time dependence in their deformation behavior, that is, no creep or viscoelasticity (Oliver and Pharr, 1992). A schematic of the indentation process for an axisymmetric indenter of arbitrary profile is shown in Fig. 4.22. As the indenter is driven into the material, both elastic and plastic deformation processes occur, producing a hardness impression that conforms to the shape of the indenter to some contact depth h_c . The radius of the circle of contact is a. As the indenter is withdrawn, only the elastic portion of the displacement is recovered, which effectively allows one to separate the elastic properties of the material from the plastic. A schematic representation of indentation load(P) versus displacement (h) data obtained during one full cycle of loading and unloading is presented in Fig. 4.23. The important quantities are the peak load (P_{max}) , the maximum depth (h_{max}) , the final or residual depth after unloading (h_f) , and the slope of the upper portion of the unloading curve $(\eta = \frac{dP}{dL})$. The parameter η has the dimensions of force per unit distance and is known as the elastic contact stiffness, or more simply, the contact stiffness. The hardness and elastic modulus are derived from these quantities. The fundamental relations from which H and E are determined are:

$$H = P / A \tag{4.13}$$

where P is the load and A is the projected contact area at that load, and:

$$E_r = \frac{\sqrt{\pi}}{2\chi} \frac{S}{\sqrt{A}}$$
(4.14)

where E_r is the reduced elastic modulus and χ is a constant shape factor that depends on the geometry of the indenter (Bulychev et. al, 1975; Oliver and Pharr, 1992). Eq.(4.13) is a working definition for the hardness as measured by instrumented indentation testing. The reduced modulus, E_r , is used in Eq.(4.14) to account for the fact that elastic displacements occur in both the indenter and the sample. The elastic modulus of the test material, E, is calculated from E_r using:

$$\frac{1}{E_r} = \frac{1 - v^2}{E} + \frac{1 - v_{ind}^2}{E_{ind}} \to E = \left(1 - v^2\right) \left(\frac{1}{E_r} - \frac{1 - v_{ind}^2}{E_{ind}}\right)^{-1}$$
(4.15)

where ν is the Poisson's ratio for the test material, and E_{ind} and ν_{ind} are the elastic modulus and Poisson's ratio, respectively, of the indenter. For diamond, the elastic constants $E_{ind} = 1141$ GPa and $\nu_{ind} = 0.07$ (Oliver and Pharr, 1992; Simmons and Wang, 1971).





Figure 4.22 axisymmetric indentation showing various quantities used in analysis

Figure 4.23 indentation load displacement data during one complete cycle of loading and unloading

In the framework of the micromorphic regularization length identification, a second thought was to realize a nanoindentation test. The idea here is to obtain some additional experimental information regarding the Young's modulus this time in the area of the localized necking, and link it directly to $\ell_{\tilde{d}}$; accordingly, the measurements of the hardness could be also linked to identify

$$\ell_{\tilde{r}} = \sqrt{Q^g} / Q \; .$$

For the indentation tests, we changed the material and the geometry of the specimen and we replaced the 430 stainless steel component by a low carbon steel, DC04 (containing 0.06% of carbon) with 0.8 mm thickness, the geometry of which is given in Fig. 4.24.



Figure 4.24: DC04 specimen geometry (Bao et al., 2015)

The specimen underwent a uniaxial tensile test before reaching the final rupture (Figure 4.25). In order to capture the necking area with the highest damage concentration, it was cut along the thickness in two lines of approximately 20.mm length in its central area (Figure 4.26a), embedded and then polished to remove possible microstructural changes and surface defects during the cutting (Figure 4.26b).



Figure 4.26: Cut, embedded and polished samples

We afterwards performed the nanoindentation tests of 60 Berkovich indents along the central line (Figure 4.27) in order to measure and observe the variation of the hardness and the Young's modulus (Figure 4.28) in the necking area.



Figure 4.27: Small Berkovich indentations located at specific points in the DC04 sample





Figure 4.28: Variations of the hardness (H) and the Young's modulus (E) in the localized necking area located in the central line of the sample

As can be seen from Fig. 4.28, there is an obvious variation and a drop in the middle of the axis for the Young's modulus at 125.0 GPa. The first thing that has to be further investigated is the origin of this drop at the localized necking, e.g, if it is a result of the damage flow or if the nanoindenter has

gone outside the plastic area and has performed some Berkovich indentations on the resin and not on the sample due to its small thickness.

With the help of the effective Young's Modulus $\tilde{E} = (1-d)E$, we can calculate the damage isovalues and plot it as well along the axis (Figure 4.29). For example, for the maximum value of the damage d_{max} at (0,0) we obtain: $d = 1.0 - \tilde{E}/E = 1.0 - 125.0/225.0 = 0.44$.

With indentation testing, we can capture better and extract useful information about the behavior of the material at the area of the highly localized necking stage before the final rupture while the ESPI measurements give the evolution of strain rates from the early stage of the diffused necking until the localization. According to these measurements, as already presented, the integral bandwidth of B^{dom} at 25% of the total average strain is around 0.2 mm (Figure 4.8). To describe mathematically the strain rate distributions, a versatile pseudo-Voigt function was chosen. This time, for a first approximation and for the sake of simplicity, we choose a Lorentzian function (Eq.(4.12)) of same width B = 0.2 mm and we plot it along the x-axis as seen in Figure 4.29.

By substituting $d(x) = \frac{d_{\max}}{1 + \left(\pi \frac{x}{B}\right)^2}$ into the micromorphic balance equation (Eq.(1.60)) $\ell_a^2 Lap(\vec{d}) + (d - \vec{d}) = \rho \frac{\zeta_{\vec{d}} \vec{d}}{\vec{H}}$ under its static form ($\rho \frac{\zeta_{\vec{d}} \vec{d}}{\vec{H}} = 0$) we get that: $\left(1 + \left(\pi \frac{x}{B}\right)^2\right) \left(\ell_a^2 Lap(\vec{d}) - \vec{d}\right) = -d_{\max}$ (4.16)

With the help of Fig. 4.29 and a possible analytical form of \tilde{d} coming from the Eq.(4.16), we might succeed in finding another link between the bandwidth B and ℓ_{3} .



Figure 4.29: Local damage vs Lorentzial function L(B)

4.5 Conclusions and perspectives

This chapter was dedicated to the material parameters identification for a 430 stainless steel specimen.

We presented the identification process given in three steps that analyze the techniques followed for each group of parameters according to associated phenomena. During the first and the second steps for the local parameters, due to the homogeneity of the mechanical fields we can transform the experimental F - u curve into:

- a) A proportional to the elastic part (I) $\sigma_{xx} \varepsilon_{xx}^e$ curve in order to identify the elasticity material parameters regarding the Young's modulus E, and the yield stress σ_y .
- b) A proportional to the plastic part (II) σp curve in order to identify the isotropic (*Q*,*b*) and kinematic (*C*,*a*) hardening parameters.

For the third step due to the induced softening caused by the damage effect and the inhomogeneity of the mechanical fields, we use the inverse identification methodology based on a virtual tensile FEM model. The identification of the local material parameters related to the ductile damage (S, s, β , γ , Y₀) has been done by several numerical simulation loops.

Regarding the last group of material parameters related to the micromorphic damage and more precisely the micromorphic regularization length $\ell_{\tilde{d}} = \sqrt{\breve{H}^g / \breve{H}}$, we made use of both the ESPI

experimental measurement and the numerical tensile test results to relate it to the width of the dominant band and propose a methodology to identify the couple of (\breve{H}, \breve{H}^s) .

Due to time constraints, the results of the indentation test were not further treated and we did not continue to any numerical implementations or analysis of the hardness curve which seems more complicated since its variations are less visible according to these first results. However, we presented the general idea and the test process and we leave it for further investigation in future works as a perspective and a possible second accurate methodology for the identification of the micromorphic internal lengths related to the damage and the isotropic hardening.

Chapter 5

2D metal forming applications.

5.1 Introduction

The Chapter 3 was dedicated to the parametric study of the micromorphic model and the Chapter 4 to the identification of the local and the micromorphic material parameters based the on experimental tensile tests. In this last chapter, the validation of our numerical methodology based on a micromorphic approach is investigated through two metal forming processes: the bending of a DP1000 sheet and the blanking operation of a DP600 sheet. According to the conclusions of Chapter 3, we will use the model only with micromorphic damage due to its capacity to provide us with unique solutions. Consequently, only one pair of micromorphic parameters (\breve{H},\breve{H}^g) will be activated and their choice is carefully made to assure physical solutions and acceptable crack propagation in the completely damaged zone.

5.2 Bending test Introduction

Cold bending is an increasingly used process to form various lightweight mechanical components with more and more advanced high-strength steels, mainly to meet the needs of aircraft and automotive industries. The air bending test is taken from the experimental work of (Soyarslan et al., 2012; Gharbi et.al, 2011) in which the 3D measurement of the strain field conducted via Aramis-GOM system (Figure 5.1) makes possible the measurement of the strain distribution over the specimen until the final fracture. The microstructure investigation shows that the cracks have an alternative behavior in the apex of the bended surface. The cracks in the cross section have a zigzag-shaped path. This behavior can be explained just by localization induced softening in the material.



Figure 5.1: Air bending setup enhanced with online measurement system GOM-Aramis

5.2.1 Identification of the DP1000 local material parameters

We need the simulation of a tensile test in order to determine the local material parameters of the behavior model. The DP1000 steel material parameters have been determined by the identification procedure described in Chapter 4 but only with a local damage behavior model. The tensile tests are conducted on Zwick Roell testing machine of type B0066550 (sensor Extensometer Zwick Roell, type B0066791). The geometry of the tensile specimen was followed regarding EN 10002-1:2001(D) with a thickness of 1.55 mm. All the experimental and bending operations have

been conducted by the international Partner the Institute of Forming Technology and Lightweight Construction, TU Dortmund.



Figure 5.2: Force-displacement response of a DP1000 tensile test

A good agreement is observed between the experimental and numerical force displacement response (Figure 5.2) and a critical damage threshold $d_c = 0.9$ is chosen to initiate the initiation of the microcracks. A 2D adaptive remeshing procedure was used to adapt the mesh size for the plasticity and damage evolution only for the tensile test model. The maximum mesh size $\Delta x_{\text{max}} = 8.0 \text{ mm}$, the minimum mesh size for plasticity $\Delta x_{\text{min}}^p = 0.2 \text{ mm}$ and the minimum mesh size for damage $\Delta x_{\text{min}}^{dam} = 0.02 \text{ mm}$.



Figure 5.3: Evolution of the mesh size and cumulated plastic strain for different displacements U

Starting from the initial coarse mesh, the mesh refinement follows the plastic strain and strain rate giving an isotropic mesh distribution of size Δx_{\min}^p inside the central area of the specimen where the plastic flow is highly active (Figure 5.3) and a clear formation of a diffuse necking is observed. For U = 7.45 mm a localized neck is, as expected, clearly observed inside one shear bands.

All the local material parameters are given in the Table 5.1:

	DP1000 dual phase steel		
Elasticity parameters	E = 220. GPa		
	v = 0.3		
Plasticity parameters	$\sigma_y = 810.0 MPa$		
	Q = 3800.0 MPa		
	b = 15.0		
	C = 69500.0 MPa		
	a = 240.0		
Damage parameters	S = 12.0 MPa		
	$\beta = 1.0$		
	<i>s</i> = 1.2		
	$Y_0 = 0.0$		
	$\gamma = 4.0$		

Table 5.1: Local materi	al parameters for the	DP1000 steel sheet
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5.2.2 Simulation of the bending procedure

This test was modelled in 2D plane strain without considering any initial symmetry in order to avoid enforcing any symmetric solution. The sheet made of DP1000 steel is discretized with the Q4 quadrangular plane strain element, proposed in Chapter 2, has been implemented in the software ABAQUS[®]/Explicit via the subroutine VUEL as discussed in the same chapter. Friction between the tools and the specimen is defined by the Coulomb model with a constant friction parameter $\eta = 0.15$. The punch velocity used is equal to 18.0 mm/s and the total time used depends on the test configuration. A following configuration is chosen for the bending operation (Figure 5.2):

- Die distance 24.0 mm;
- Die radius 1.0 mm;
- Punch radius 1.0 mm;
- The specimen geometry was fixed with 50×25 mm (length times width). The thickness is 1.55 mm.

Three small mesh sizes (M_s) have been used to mesh the sheet in the area where the failure is expected to appear:

- $M_s = 0.1 \text{ mm}$ (2340 elements),
- $M_s = 0.08 \text{ mm}$ (2602 elements),
- $M_s = 0.06 \text{ mm}$ (3500 elements).

5.2.2.1 Behavior model with local damage ($\breve{H} = 0, \breve{H}^s = 0$)

Figure 5.5 compares the experimental F-u curve to the numerically predicted ones illustrating the expected mesh dependence since the force-displacement curves are sensitive to the mesh size. As expected, the displacement at the final fracture is clearly mesh dependent: $U_{fr} = 10.98 \text{ mm}$ for $M_s = 0.1 \text{ mm}$, $U_{fr} = 10.26 \text{ mm}$ for $M_s = 0.08 \text{ mm}$ and $U_{fr} = 9.36 \text{ mm}$ for $M_s = 0.06 \text{ mm}$.



Figure 5.4: Local damage values of the failed bending specimen in the framework of the local approach for different mesh sizes

The simulation of the bending process gives the local damage distribution as shown in Fig.5.4 for the different mesh sizes at the values of punch displacement for which the crack has initiated. A phenomenon of network localization begins to appear; shear bands oriented around 45° to the axis

of the folding are initiated on the lower edge of the blank and seem to propagate in the thickness. For the three mesh size configurations, a macro-crack has clearly developed with a stair-step morphology. Clearly, the maximum damage values are highly concentrated inside one row of elements with a zig-zag evolution during the crack propagation.



Figure 5.5: Experimental and numerical force-displacement curves of DP1000 for air-bending for the local model

5.2.2.2 Behavior model with micromorphic damage ($\ell_{\vec{A}} = \sqrt{\vec{H}^s / \vec{H}}$)

The micromorphic formulation is now taken into consideration by activating the contribution of the micromorphic damage variable \tilde{d} . The micromorphic damage model the is applied to the air bending test in order to prove the agreement between the numerical solutions and the experimental results and reduce the dependence of the responses against the refinement of the mesh (Diamantopoulou et al., 2017). We set $\breve{H} = 80.0 \ N.mm^{-2}$, $\breve{H}^g = 3.6 \ N$ giving the internal length $\ell_{\tilde{d}} = \sqrt{\breve{H}^g / \breve{H}} \approx 0.21$ mm, two times higher than the highest mesh size and 3.5 times than higher than the lowest mesh size.



Figure 5.6: Experimental and numerical force-displacement curves of DP1000 for air-bending for the micromorphic model

Unlike the local model (see Figure 5.5), the F-u curves predicted by the micromorphic model give the same fracture displacement value as indicated in Fig. 5.6. Oscillations observed on the evolution of the cutting force are due to a modelling problem of the contact between the die and the sheet. The contact procedure based on a master surface approach (matrices) slave nodes (sheet) remains dependent on the conditions of the discretization the contact surfaces.



Figure 5.7 Local (left) and micromorphic (right) damage distributions at the end of the bending load as predicted by the micromorphic approach for different mesh sizes.



Figure 5.8: Local vs micromorphic damage evolution at the moment of U_{fr} for the three mesh sizes

The final distribution of the local damage as predicted by the micromorphic formulation is shown in Figure 5.7. We can see that the damage localization bandwidth becomes smoother than the local damage case (i.e. the damage gradient varies less than in the local case).

A horizontal line (dashed lines in Fig. 5.7) is used to plot the evolution of local damage (noted as "d_loc") and nonlocal damage (noted as "d_n.loc") when the macro fissure has spread through about half of the thickness of the sheet. Figure 5.8 shows the evolution of both local and micromorphic damage along these horizontal lines of the ruptured specimen at the moment of U_{fr} for $M_s = 0.1, M_s = 0.08$ and $M_s = 0.06$ mm. Clearly, the micromorphic damage is lower than the local damage ($\vec{d} < d$). At the beginning of the damage flow we have $\vec{d} = d$, but as the damage increases inside the localization zone the local damage increases more rapidly than the micromorphic damage. Accordingly, the difference between d and \vec{d} increases inside the damaged zone and at the final fracture $d = d_c = 1.0$ while $\vec{d} \approx 0.55$. Note that the fracture criterion uses the local damage d instead of the micromorphic damage \vec{d} which never reaches the critical value of the damage at fracture, i.e., $\vec{d} < 1.0$.

The evolution of local and nonlocal damage appears practically identical for the three mesh cases but the crack always propagates in final stage along a single row of elements.

5.3 Metal forming process: blanking (or cutting) operation

The adaptive analysis methodology described in Paragraph 2.8 of Chapter 2 is now applied to a blanking operation of a circular thin sheet. The axisymmetric sheet has an initial external radius R = 200.0 mm and a thickness e = 1.2 mm. The punch diameter is 12.0 mm and the clearance between the fixed die and the moving punch is 0.12 mm as shown in Figure 5.9 (Labergere et. al, 2010, 2014).



Figure 5.9: Schematic representation of the axisymmetric blanking process

The material is a DP600 steel and its parameters are given in Table 5.2. Only the micromorphic formulation is taken into consideration by activating the contribution of the micromorphic damage variable \vec{d} by setting $\vec{H} = 50.0 N.mm^{-2}$, $\vec{H}^{g} = 0.02 N$ giving the internal length $\ell_{\vec{d}} = \sqrt{\vec{H}^{g} / \vec{H}} = 0.02 \text{ mm}.$

	DP600 dual phase steel		
Elasticity parameters	$E = 220.0 \ GPa$		
	v = 0.29		
Plasticity parameters	$\sigma_y = 410.0 MPa$		
	Q = 1010.0 MPa		
	b = 2.0		
	C = 7000.0 MPa		
	$\alpha = 40.0$		
Damage parameters	S = 22.0 MPa		
	$\beta = 2.0$		
	<i>s</i> = 1.2		
	$Y_0 = 0.0$		
	$\gamma = 4.0$		

|--|

The sheet is discretized with quadrangular bilinear axisymmetric micromorphic elements presented in Chapter 3 and implemented in a VUEL ABAQUS[®]/Explicit subroutine. Friction between the tools and the sheet is modelled by the classical Coulomb model with a constant friction coefficient $\eta = 0.15$. The different element size parameters used by different error size estimator for the adaptive remeshing procedure are given in Table 5.3:

$\Delta x_{\rm max}$	Δx_{\max}^p	p^{*}	Δx_{\min}^p	ĸ	Δx_{\min}^{dam}	d_{\min}	d_{\max}
(mm)	(mm)		(mm)		(µm)		
1.0	0.4	0.22	0.1	5.0	3.0,6.0 or 9.0	0.005	0.05

Table 5.3: mesh size indicator

Three small mesh size configurations (indicated as M_i) $M_1 \rightarrow \Delta x_{\min}^{dam} = 3.0 \,\mu\text{m}$, $M_2 \rightarrow \Delta x_{\min}^{dam} = 6.0 \,\mu\text{m}$ and $M_3 \rightarrow \Delta x_{\min}^{dam} = 9.0 \,\mu\text{m}$ have been chosen to refine the area of the sheet where the crack is expected to appear. Figure 5.10 shows the first mesh adaptation performed using the geometrical error indicator based on the curvature of the tools and the cutting edges. The mesh is refined in the areas of the smallest curvature of the punch and the die. The size distribution of the initial mesh is important because it determines the accuracy of the contact algorithm between the tools and the sheet.



Figure 5.10: Initial mesh of the sheet

Figures 5.11 present the isovalues of the von Mises stress for the punch displacement U = 0.5 mm and for the three different mesh configurations M_i . The 2D mesh adaptation proved to be quite efficient as the mesh refinement follows the contact areas together with the zones with higher plasticity and damage. In addition, the mesh is clearly coarsened inside the inactive areas (no plastic flow). For the three configurations M_i , a crack initiates and appears near the contact between the sheet and the fillet of the die. The value of the von Mises stress near the crack is high and reaches an average value of 960.0 MPa. In the case of $\Delta x_{\min}^{dam} = 9.0 \,\mu\text{m}$ and $\Delta x_{\min}^{dam} = 6.0 \,\mu\text{m}$ the von Mises gradient is concentered along the principal shear band where the crack can propagate.



Figure 5.11: von Mises stress distribution inside the sheet thickness for punch displacement $U\,{=}\,0.5\,{
m mm}$

In order to complete the analysis of these results, we also plot the evolution of both local and micromorphic damage on the same plane for the three different mesh configurations M_i . The measurements were done on three lines of the deformed sheet as shown in Figure 5.12: a top line indicated as *LH* line, a middle line indicated as *LM* line and a bottom line indicated as *LB* (Figure 5.12) for a punch displacement equal to U = 0.5 mm, before the final rupture. Figures 5.13 to 5.15 show the evolution of local vs micromorphic damage along the three drawn lines for $\Delta x_{\min}^{dam} = 9.0 \,\mu\text{m}$, $\Delta x_{\min}^{dam} = 6.0 \,\mu\text{m}$ and $\Delta x_{\min}^{dam} = 3.0 \,\mu\text{m}$ respectively.

For $\Delta x_{\min}^{dam} = 9.0 \,\mu\text{m}$, we observe that the micromorphic damage is higher than the local one along all the three lines. When $\vec{d} > d$ the term of the damage force $Y_{\vec{d}} = -\vec{H}(d-\vec{d})$ is positive, the evolution of the local damage d is then accelerated to fill the difference with the nonlocal damage \vec{d} . For the line LB near the crack, the maximum nonlocal damage reaches the value of 0.12. For the two other lines LM and LH, the maximum nonlocal damage reaches up to 0.05.

In contrast, for the two other configurations $\Delta x_{\min}^{dam} = 6.0 \,\mu\text{m}$ and $\Delta x_{\min}^{dam} = 3.0 \,\mu\text{m}$, the difference $(d - \vec{d})$ remains always positive. In these two configurations, the term of damage force $Y_d = -\vec{H} \left(d - \vec{d} \right)$ is negative; the evolution of the local damage d is slower. The evolutions of the nonlocal damage for the three lines and for the two configurations M_1 and M_2 seem to be quasi identical. For the line LB, the value of the maximum nonlocal damage is equal to 0.1 for the two smallest mesh sizes $\Delta x_{\min}^{dam} = 6.0 \,\mu\text{m}$ and $\Delta x_{\min}^{dam} = 3.0 \,\mu\text{m}$.



Figure 5.12: Schematization of high (purple) middle (blue) and bottom (green) lines of the deformed sheet for the measurement of local and micromorphic damage along the axis at U = 0.5 mm



Figure 5.13: Local vs micromorphic damage evolution at $U=0.5\,{
m mm}$ for $\Delta x_{
m min}^{dam}$ = $9.0\,{
m \mu m}$



Figure 5.14: Local vs micromorphic damage evolution at $\,U=0.5\,{
m mm}$ for $\Delta x_{
m min}^{dam}=6.0\,{
m \mu m}$



Figure 5.15: Local vs micromorphic damage evolution at U = 0.5 mm for $\Delta x_{\min}^{dam} = 3.0 \,\mu m$

For the same punch displacement (U = 0.73 mm), we also present mesh adaptation in Fig. 5.16. The macroscopic crack lengths on the actual meshed configurations are shown and the adaptive character of the 2D mesh is clearly observed. The crack that initiated for U = 0.5 mm has now propagated along the shear band. We clearly see that the smallest mesh size is well applied to mesh the sheet along the shear band and the length of the cracks $\approx 0.25 \text{ mm}$ is approximatively the same for the three configurations.

For the same punch displacement (U = 0.73 mm), we also show the isovalues of the micromorphic damage in Fig. 5.17. For the two configurations M_2 and M_3 , the nonlocal damage is higher at the tip of the crack and reaches a maximum value of 0.2. The width of the localization band corresponding to the nonlocal damage remains also identical. For the configuration M_1 , the evolution of the gradient of the nonlocal damage is no more clear but the width of the localization band located in the sheet between the tip of the crack and the fillet of the punch seems to have the same shape with the two other configurations M_2 and M_3 .



Figure 5.16: Crack propagation and adaptive mesh for punch displacement U = 0.73 mm





Figure 5.17: Micromorphic damage isovalues for punch displacement U = 0.73 mm

The experimentally and the numerically predicted fracture shape are represented in Fig. 5.18 for the three minimum mesh sizes at $U \simeq 0.9$ mm. The obtained results are in good agreement concerning the convex zone (0.12 mm), sheared zone (0.33 mm), fracture zone (0.75 mm) and the bur (0.04 mm). For the three configurations, the shape of the crack is approximatively the same. We have also a relatively good agreement with the experimental crack geometry.


Figure 5.18: Experiental vs numerical cut for the three mesh sizes at the end of the applied load

Finally, the F-u curves predicted by the micromorphic model give almost the same value of fracture displacement as shown in Fig. 5.19 independent from the mesh size. However, we observe a slight difference between the three numerical and the experimental results in the softening phase.

This difference can be explained by the need to introduce a micromorphic variable associated to the isotropic hardening in order to enhance the regularization at this stage.



Figure 5.19: Experimental and numerical force-displacement curves of DP600 for blanking predicted by the micromorphic model.

5.4 Conclusions

With the simple uniaxial tensile test presented extensively in the previous chapter and these last two more realistic applications, we have confirmed the ability of our micromorphic model to obtain nearly unique solutions for different mesh sizes. In this Chapter 5, we have applied our micromorphic model for the prediction of ductile cracks initiation and propagation on an air-bending test and on an industrial blanking process. For the two metal forming processes the nonlocal model only with micromorphic damage was used. What is left, is the application of the mixed model with both micromorphic damage and isotropic hardening as presented in Chapter 3 by activating \tilde{Q} and \tilde{Q}^{g} and optimizing the numerical implementation, in order to validate its accuracy by obtaining mesh independent solutions.

General conclusions and perspectives

This work was focused on a nonlocal formulation of an elastoplastic behavior model with isotropic and kinematic hardening together with softening or negative hardening induced by the presence of isotropic ductile damage. In framework of the generalized continua, the micromorphic theory has been applied assuming that the isotropic ductile damage and the isotropic hardening are the two principal micromorphic phenomena. As a result, two additional micromorphic kinematic variables, \vec{d} and \vec{r} have enriched the space of degrees of freedom of the IBVP. These variables, as well as the body, contact and inertia forces are included in the generalized extended form of the principal of virtual power. This leads to obtaining, additionally to the classic local equilibrium equation, two new partial differential equations (PDEs) along with their Neumann-type boundary conditions, as the balance equations related to the micromorphic phenomena that are taken into consideration. By using the appropriate additional state relations (stress-like micromorphic variables), these two additional PDEs obtained in the stress space, can be expressed in the strain space.

Furthermore, the space of the state variables is also enriched by new pairs of micromorphic state variables (\vec{d}, \vec{Y}) , (\vec{r}, \vec{R}) as well as their first gradients $(\vec{\nabla}\vec{d}, \vec{Y})$ and $(\vec{\nabla}\vec{r}, \vec{R})$. This allows a micromorphic extension of the thermodynamics of irreversible processes through the construction of a state potential, a yield criterion and a plastic potential as appropriate convex functions of the overall strain-like state variables that allows us to extract the complete set of evolution equations for an elastoplastic micromorphic solid with damage and hardening. It is worth noting that in this isotropic and isothermal formulation, the dissipation of the micromorphic phenomena is neglected due to the absence of adequate experimental information. It is also remarkable, that the evolution equations of the targeted micromorphic phenomena can be written under additive terms, one related to the classical local evolution and a second one, nonlocal, coming from the evolution of the micromorphic variables.

In the second chapter the numerical aspects of the IBVP related to the motion of the micromorphic solid under a given loading path, are presented. The principal equations that define the IBVP are firstly given and we continued by obtaining the three associated highly nonlinear and strongly coupled weak forms. The time and space discretization (by finite differences and finite elements respectively) are also discussed and the dynamic explicit global resolution scheme is also described. We continued with the formulation of two 2D micromorphic finite elements and the stability condition for the micromorphic damage problem. An implicit algorithm that combines the θ -method with an asymptotic scheme for the reduction of the size of the material Jacobian matrix is discussed in detail. Finally, we gave a brief representation of the technical aspects regarding:

- the implementation of the finite elements and the behavior model into Abaqus/Explicit by using the associated user's subroutines;
- the 2D adaptive remeshing methodology.

In the third chapter an extended parametric study is performed on the model with micromorphic isotropic damage and micromorphic isotropic hardening. The objective is to explore the effect and the influence of the micromorphic model on the mesh independency of the solutions by using different mesh sizes and different values of the associated internal lengths. According to the results obtained from this parametic study we can conclude that:

The presence of micromorphic damage is necessary and enough to regularize the IBVP in contrast to the micromorphic isotropic hardening;

- The micromorphic damage model can give mesh independent solutions under the following conditions:
 - the size of the chosen internal length should be sufficiently bigger than the size of the elements $(\ell_{\bar{J}} > h)$, and
 - The most appropriate choice for the micromorphic moduli should depend on the fact that the values of \breve{H} and \breve{H}^s should be restricted in an "intermediate value" interval so that they are neither too low, in order to retrieve the local solutions, nor too high so that the micromorphic damage effect is so excessively strong that leads to no rupture appearance despite the large displacements.

In Chapter 4 we have given the major steps for the material parameters identification concerning the elasticity, plasticity and local damage. Afterwards, we proposed a numerical methodology for identifying the value of the micromorphic internal length related to the micromorphic damage by linking it to the width of the shear bands that appear during the localized necking and comparing with the experimental bandwidths obtained by the ESPI measurements in order to validate the accuracy of this first identification effort.

Finally, the fifth chapter was dedicated to two 2D applications, a bending and a blanking test, showing the efficiency and the predictive capabilities of a complete micromorphic damage model.

This work has opened a new field for further investigation on the formulation of nonlocal micromorphic behavior models. Concerning the "Micromorfing" ANR program, many open questions raise, the deeper investigation of which is essential.

- Bring together the numerical developments performed by CdM/MinesParisTech concerning the hyper-reduction of the models (HROM) and those performed by LRM/UTC concerning the 3D adaptive remeshing and their application to our formulation_using our elements extended to 3D.
- <u>From the theoretical point of view</u>: some extension could be added concerning:
 - The introduction of induced anisotropies due to the plastic flow, damage, distortion of the yield surface etc...;

> The introduction of the micromorphic dissipation including the micromorphic plastic strains as well as the dissipation relative to the micromorphic phenomena;

The extension to viscoplastisity and strong coupling with temperature;

> The exploration of the kinematic aspects related to the micromorphic variables in finite transformations and the impact on the objectivity of the model (choice of rotation frames);

A link between the micromorphic and the micro-macro formulations, in the framework of the appropriate homogenization methods in order to improve the calculation of media with microstructure (Forest, 2006);

The analytical study of some closed form solutions.

- From the numerical point of view:
 - The formulation of 3D micromorphic elements;
 - > Introduction of additional degrees of freedom (kinematic hardening, plastic strain...);
 - Adaptive remeshing for the micromorphic model in 3D;
 - Meshless methods adapted for gradient models;

- > Use of Implicit (quasi-static or dynamic) solver and coupling of different solvers;
- <u>Parametric study and applications</u>:
 - > The role and the influence of the micromorphic densities;
 - > The role and the influence of the micromorphic body and contact forces;
 - The improvement of the identification methodology based on 3D full-field measurement in order to determine as uniquely as possible the best values of the micromorphic internal lengths related to the micromorphic phenomena;
 - > More and more complex industrial metal forming applications.
- From the experimental point of view:
 - The numerical treatment and validation of the nanoindntation test for the introduction of a second micromorphic internal length identification methodology;
 - Adaptation and implementation of the 3D-DHI microscope for 3D displacement/velocity fields measurements with nanometer accuracy on loaded mechanical specimens;
 - Complementary characterizations for the measurement of the surface topography of the localization zones at several stages before fracture.

All these aspects are either under development, in the framework of the ANR program, or taken into high consideration as scientific priorities of the 'Formage virtuelle' team of LASMIS.

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Résumé en français

1. Contexte théorique.

Cette thèse est consacrée à la présentation de la formulation d'un modèle de comportement élastoplastique fortement couplé à l'endommagement ductile isotrope dans le cadre des milieux généralisés basés sur la théorie micromorphique. Les milieux continus matériellement simples supposent que l'état mécanique à n'importe quel point matériel de la zone \wp est complètement déterminé par l'histoire des variables d'état dans un voisinage arbitrairement petit entourant ce point (Truesdell and Toupin, 1960). Dans ce contexte, la connaissance du premier gradient de transformation F (ou premier gradient de déplacement) est suffisante pour déterminer l'état mécanique de ce point et le milieu est appelé matériellement simple ou local. (Truesdell and Noll, 1965). Cependant, dans plusieurs situations, le vecteur de déplacement et son premier gradient ne sont pas suffisants pour définir l'état mécanique d'un point matériel. Dans de tels cas, il est nécessaire d'ajouter d'autres variables cinématiques ou ddls (degrés de liberté) pour enrichir la description cinématique du milieu. Ainsi que leurs gradients de premier, deuxième ou plus haut ordre, dans le principe des puissances virtuelles et comme nouveau arguments dans l'état et les potentiels de dissipation. Cette approche est décrite comme la mécanique des milieux généralisés ou milieux matériellement non simple (Truesdell and Noll, 2004). Il est souvent proposé de classer les théories des milieux généralisés en trois théories distinctes (Forest et Sievert, 2003) et (Saanouni, 2012), présentées dans la Figure 1.1:

Milieux continus de degré supérieur, qui sont basés sur l'introduction des dérivées spatiales d'ordre supérieurs des déplacements dans le principe des puissances virtuelles comme initialement proposé par: (Toupin, 1962), (Toupin, 1964), (Mindlin, 1968).

Milieux continus d'ordre supérieur, qui s'appuient sur le rajout de degrés de liberté additionnelles dans le principe des puissances virtuelles comme l'ont proposé les frères Cosserat (Cosserat, 1986, 1909, 2009) et utilisé par (Eringen, 1964, 1970).

Milieux continus non locaux dont une synthèse récente se trouve dans (Eringen, 2002).



Figure 1.1. Schéma des différentes théories en mécanique des milieux continus

En ce qui concerne le cadre théorique que nous avons choisi pour construire notre formulation non locale, nous avons développé une formulation d'un modèle endommagé élastoplastique des milieux généralisés dans le cadre de la théorie micromorphique. La cinématique d'un milieu micromorphique repose sur un le déplacement \vec{u} et son premier gradient $\nabla \vec{u}$ ainsi que sur un certain nombre de variables micromorphiques notées \underline{z}_n et leurs premiers gradients $\nabla \underline{z}_n$.

La puissance virtuelle des forces internes du milieu local est étendue au niveau du milieu micromorphique généralisé en utilisant les deux degrés de liberté micromorphiques supplémentaires associés respectivement à l'écrouissage isotrope micromorphique \breve{r} et à l'endommagement isotrope micromorphique \breve{d} .

L'application de la puissance virtuelle conduit ainsi à trois équations aux dérivées partielles, avec leurs conditions aux limites associées ; l'équation de bilan locale tandis que les deux autres expriment des équations de bilan de forces micromorphiques:

$$\begin{cases} \vec{\nabla} \cdot \vec{\sigma} + \rho \vec{f}^{u} = \rho \vec{u} \quad dans \,\Omega \\ \sigma \cdot \vec{n} = \vec{F}^{u} \quad sur \,\Gamma \end{cases}$$
(a)
$$\begin{cases} \left(\vec{\nabla} \cdot \vec{\vec{R}} - \vec{R} \right) + \rho \left(f^{\vec{r}} - \vec{\nabla} \cdot \vec{f}^{g\vec{r}} \right) = \rho \zeta_{\vec{r}} \vec{\vec{r}} \quad dans \,\Omega \\ \left(\vec{\vec{R}} - \rho \vec{f}^{g\vec{r}} \right) \cdot \vec{n} = F^{\vec{r}} \quad sur \,\Gamma \end{cases}$$
(b)
$$\begin{cases} \left(\vec{\nabla} \cdot \vec{\vec{Y}} - \vec{Y} \right) + \rho \left(f^{\vec{d}} - \vec{\nabla} \cdot \vec{f}^{g\vec{d}} \right) = \rho \zeta_{\vec{d}} \vec{\vec{d}} \quad dans \,\Omega \\ \left(\vec{\vec{Y}} - \rho \vec{f}^{g\vec{d}} \right) \cdot \vec{n} = F^{\vec{d}} \quad sur \,\Gamma \end{cases}$$
(c)

En se basant sur la méthode de l'état local (Germain, 1973), nous utilisons des variables d'état pour lesquelles les valeurs à chaque instant *t* et à chaque point matériel, déterminent la réponse matérielle. Chaque phénomène dissipatif a sa propre variable d'état dont l'évolution est dirigée par sa propre équation d'évolution. Dans cette thèse, nous nous limitons à des conditions exclusivement isothermes et aux modélisations et simulations numériques d'opération de mise en forme par déformation plastique (emboutissage, forgeage, découpage, pliage, ...). Ainsi, nous avons utilisons un couple de variables internes d'état de déformation et duale en contrainte pour décrire chaque phénomène physique. Nous proposons les couples de variables regroupés dans le tableau (Tableau 1.1).

Phénomène	Variable interne	Variable duale	
Variables d'état observables			
Variables mécaniques	<u>£</u>	₫	
	mesure eulérienne	contrainte de	
	des déformations totales	Cauchy	
Variables d'état non observables			
Déformation élastique réversible	$\underline{\varepsilon}^{e}$	σ	
Ecrouissage cinématique	<u>a</u>	<u>X</u>	
Ecrouissage isotrope	r	R	
Endommagement ductile isotrope	d	Y	
Ecrouissage isotrope micromorphique	$\int \breve{r}$	$\int \breve{R}$	
	$\int \vec{\nabla} \vec{r}$	$\left\{ ec{ec{R}} ight.$	
Endommagement ductile isotrope micromorphique	$\int \vec{d}$	$\begin{cases} \breve{Y} \\ \vec{Y} \end{cases}$	
	$\left(ec abla ec d ight)$	$\left(\overset{\rightarrow}{ec{Y}} ight)$	

Table 1.3 Variables d'état associées aux phénomènes physiques à décrire

Nous choisissons l'énergie libre de Helmholtz comme potentiel d'état écrit sur la configuration fictive en fonction de toutes les variables internes locales et micromorphiques définis dans l'espace de déformation effective. En utilisant les relations d'état locales et micromorphiques couplé au potentiel d'état, nous obtenons les contraintes locales et les relations d'état micromorphiques:

- Contrainte de Cauchy:

$$\underline{\sigma} = \rho \frac{\partial \psi_g}{\partial \underline{\varepsilon}^e} = (1 - d) \underline{\Delta} : \underline{\varepsilon}^e = (1 - d) \left(\lambda_e tr(\underline{\varepsilon}^e) \underline{1} + 2\mu_e \underline{\varepsilon}^e \right)$$
(1.2)

- Ecrouissage cinématique:

$$\underline{X} = \rho \frac{\partial \psi_s}{\partial \underline{\alpha}} = \frac{2}{3} (1 - d) C \underline{\alpha}$$
(1.3)

- Ecrouissage isotrope:

$$R = \rho \frac{\partial \psi_g}{\partial r} = (1 - d^{\gamma})Qr + \vec{Q} \left[(1 - d^{\gamma})r - \sqrt{1 - d^{\gamma}}\sqrt{1 - \theta_r d^{\gamma}}\vec{r} \right]$$
(1.4)

- Endommagement ductile isotrope:

$$Y = -\rho \frac{\partial \psi_g}{\partial d} = Y_E + Y_A + Y_R + Y_d$$
(1.5)

avec,

$$\begin{cases} Y_E = \frac{1}{2} \underline{\varepsilon}^e : \underline{\Lambda} : \underline{\varepsilon}^e & (a) \\ Y_A = \frac{1}{3} C \underline{\alpha} : \underline{\alpha} & (b) \\ Y_R = \frac{1}{2} \gamma d^{\gamma - 1} Q r^2 + \frac{1}{2} \gamma d^{\gamma - 1} \overline{Q} \left(\sqrt{1 - d^{\gamma}} r - \sqrt{1 - \theta_r d^{\gamma}} \overline{r} \right) \left(\frac{r}{\sqrt{1 - d^{\gamma}}} - \theta_r \frac{\overline{r}}{\sqrt{1 - \theta_r d^{\gamma}}} \right) + \frac{1}{2} \theta_r \gamma d^{\gamma - 1} \overline{Q}^g \, \overline{\nabla} \overline{r} \cdot \overline{\nabla} \overline{r} \quad (c) \\ Y_d = -\overline{H} \left(d - \overline{d} \right) & (d) \end{cases}$$

$$(1.6)$$

- Endommagement ductile isotrope micomorphique et son premier gradient:

$$\begin{split} \vec{Y} &= \rho \frac{\partial \psi_s}{\partial \vec{d}} = -\vec{H} \left(d - \vec{d} \right) \quad (a) \\ \vec{Y} &= \rho \frac{\partial \psi_s}{\partial \vec{\nabla} \vec{d}} = \vec{H}^s \vec{\nabla} \vec{d} \qquad (b) \end{split}$$

- Ecrouissage isotrope micromorphique et son premier gradient:

$$\vec{R} = \rho \frac{\partial \psi_g}{\partial \vec{r}} = -\vec{Q} \left(\sqrt{1 - d^\gamma} \sqrt{1 - \theta_r d^\gamma} r - (1 - \theta_r d^\gamma) \vec{r} \right) \quad (a)$$

$$\vec{\tilde{R}} = \rho \frac{\partial \psi_g}{\partial \vec{\nabla} \vec{r}} = (1 - \theta_r d^\gamma) \vec{Q}^g (\vec{\nabla} \vec{r}) \qquad (b)$$

où, $\underline{\Lambda} = \lambda_e 1 \otimes \underline{1} + 2\mu_e \underline{1}$ est l'opérateur d'élasticité linéaire, C et Q sont les modules respectifs de l'écrouissage cinématique et isotrope, \underline{Q} , \underline{H} , \underline{Q}^g et \underline{H}^g (positifs ou nuls) sont les propriétés micromorphiques du solide et le paramètre γ défini le couplage entre l'endommagement local et l'écrouissage isotrope.

Puisque les variables duales $(\underline{\sigma}, R, \underline{X}, Y)$ sont données par les relations d'état, les variables de flux associées (\underline{D}^p , \dot{r} , $\underline{\dot{\alpha}}$, \dot{d}) découleront de l'application de la règle de normalité généralisée et de l'analyse de dissipation volumique maximale. Pour tenir compte des effets de l'endommagement, et en nous limitant au flux plastique isotrope pour des raisons de simplicité, nous avons choisis la contrainte équivalente de von Mises comme norme de contrainte pour définir la fonction de charge et un potentiel plastique. La fonction de charge de von Mises et le potentiel de dissipation plastique proposées sont référencées dans (Saanouni et al., 1994; Saanouni, 2012) et s'écrivent alors sous les formes suivantes:

$$f_p(\underline{\sigma}, \underline{X}, R, d) = \frac{\|\underline{\sigma} - \underline{X}\|}{\sqrt{1 - d}} - \frac{R}{\sqrt{1 - d^{\gamma}}} - \sigma_y$$
(1.9)

$$F_{p}\left(\underline{\sigma}, \underline{X}, R, d\right) = f_{p} + \frac{3}{4} \frac{a(\underline{X}:\underline{X})}{C(1-d)} + \frac{1}{2} \frac{bR^{2}}{Q(1-d^{\gamma})} + \frac{S}{(s+1)(1-d)^{\beta}} \left\langle \frac{Y-Y_{0}}{S} \right\rangle^{s+1}$$
(1.10)

avec la norme $\|\underline{\sigma} - \underline{X}\| = \sqrt{\frac{3}{2}(\underline{\sigma} - \underline{X}):(\underline{\sigma} - \underline{X})}$ définissant la contrainte équivalente von Mises, tandis que σ_y la limite d'élasticité du matériau vierge. Les paramètres a et b en Eq. (1.10) caractérisent respectivement la non-linéarité de l'écrouissage cinématique et isotrope et finalement, S, s, β et Y_0 caractérisent l'évolution non linéaire de l'endommagement ductile. L'application de la règle de normalité généralisée à la fonction de charge locale (Saanouni et Chaboche, 2003, Saanouni, 2012, Forest 2016) et au potentiel de dissipation ci-dessus conduit aux équations d'évolution suivantes:

Déformation plastique:

$$\underline{D}^{p} = \dot{\lambda} \frac{\partial F_{p}}{\partial \underline{\sigma}} = \dot{\lambda} \frac{\underline{\tilde{n}}}{\sqrt{1-d}}, \quad with \qquad \underline{\tilde{n}} = \frac{3}{2} \frac{(\underline{\sigma}^{dev} - \underline{X})}{\|\underline{\sigma} - \underline{X}\|}$$
(1.11)

où, $\underline{\tilde{n}}$ est la normale à l'extérieur de la surface de rendement et λ est le multiplicateur plastique de Lagrange.

- Ecrouissage cinématique:

$$\dot{\underline{\alpha}} = -\dot{\lambda} \frac{\partial F_p}{\partial \underline{X}} = \underline{D}^p - a\dot{\lambda}\underline{\alpha}$$
(1.12)

Ecrouissage isotrope:

$$\dot{r} = -\dot{\lambda}\frac{\partial F_p}{\partial R} = \dot{\lambda}\left(\frac{1}{\sqrt{1-d^{\lambda}}} - b\frac{R}{Q(1-d^{\gamma})}\right) = \dot{\lambda}\left(\frac{1}{\sqrt{1-d^{\gamma}}} - br - b\frac{\overline{Q}}{Q(1-d^{\gamma})}\left((1-d^{\gamma})r - \sqrt{1-d^{\gamma}}\sqrt{1-\theta_r}d^{\gamma}\overline{r}\right)\right)$$
(1.13)

Endommagement ductile isotrope :

$$\dot{d} = \dot{\lambda} \frac{\partial F_p}{\partial Y} = \frac{\dot{\lambda}}{(1-d)^{\beta}} \left\langle \frac{(Y_{loc} + Y_{nloc}) - Y_0}{S} \right\rangle^s$$
(1.14)

En utilisant les expressions des variables forces thermodynamiques ((1.2)-(1.8)) dans ((1.1) a, c), il est tout à fait possible d'exprimer les équations de bilan micromorphiques en fonction de variables déformations :

$$\begin{cases} \ell_{\bar{r}}^{2} \left(\left(1 - \theta_{r} d^{\gamma} \right) Lap(\bar{r}) - \theta_{r} \gamma d^{\gamma-1} \overline{\nabla} d. \overline{\nabla} \bar{r} \right) + \left(\sqrt{1 - d^{\gamma}} \sqrt{1 - \theta_{r} d^{\gamma}} r - \left(1 - \theta_{r} d^{\gamma} \right) \overline{r} \right) = \rho \frac{\zeta_{\bar{r}} \ddot{\bar{r}}}{\underline{\breve{Q}}} \\ Q^{g} \left(\overline{\nabla} \bar{r} \right) \cdot \vec{n} = 0 \end{cases}$$

(1.15)

$$\begin{cases} \ell^{2}_{\vec{a}} Lap(\vec{d}) + (d - \vec{d}) = \rho \frac{\zeta_{\vec{b}} \ddot{\vec{d}}}{\vec{H}} \\ \vec{H}^{g}(\vec{\nabla}\vec{d}) \cdot \vec{n} = 0 \end{cases}$$
(1.16)

où, $\ell_{\tilde{r}}$ et $\ell_{\tilde{d}}$ sont les longueurs internes relatives aux deux phénomènes micromorphiques comme étant la racine carrée des rapports entre les modules micromorphiques:

$$\ell_{\tilde{r}}^{2} = \frac{\tilde{Q}^{s}}{\tilde{Q}}(a) \quad \ell_{\tilde{d}}^{2} = \frac{\tilde{H}^{s}}{\tilde{H}}(b)$$
(1.17)

Nous préférons utiliser les équations (1.16)-(1.17) comme des formes fortes à partir desquelles les formes faibles associées seront déduites pour résoudre le PVIL. Nous notons que pour le reste de la thèse, nous négligerons le couplage entre l'éndommagement local et l'écrouissage isotrope micromorphique (le paramètre $\theta_r = 0$.)

2. Aspects numériques.

En utilisant la formulation lagrangienne actualisée, les formes faibles associées aux formes fortes (Eq.(1.1a), Eq.(1.15) et Eq.(1.16)) peuvent être facilement obtenues grâce à la méthode des résidus pondérés (Belytschko, 2001). Nous partons de l'extension de la forme faible de Hu-Washizu proposée par Fish et Belytschko (Fish et Belytschko, 1990) pour dériver les formes faibles suivantes:

Forme faible associée aux équations d'équilibre classiques:

$$\delta\pi\left(\vec{u},\underline{\dot{e}}^{a},\underline{\sigma}^{a}\right) = \int_{\Omega_{e}} \left(\delta\underline{\dot{e}}^{a}\right)^{T} : \underline{\sigma}^{a} d\Omega + \delta\int_{\Omega_{e}} \left(\underline{\sigma}^{a}\right)^{T} : \left(\vec{\nabla}_{s}\vec{u} - \underline{\dot{e}}^{a}\right) d\Omega + \int_{\Omega_{e}} \left[\rho\left(\vec{u} - \vec{f}^{u}\right) \cdot \delta\vec{u}\right] dV - \int_{\Gamma_{e}} \left(\vec{F}^{u} \cdot \delta\vec{u}\right) dS = 0$$

$$(2.1)$$

où, δ dénote une variation, $\dot{\underline{\varepsilon}}^a$ est le taux des déformations postulées, $\underline{\sigma}^a$ est le tenseur des contraintes postulées et $\vec{\nabla}_{,\vec{u}}$ représente la partie symétrique du gradient de vitesse.

> Forme faible associée aux équations de bilan de l'endommagement micromorphique:

$$J(\vec{d},\delta\dot{\vec{d}}) = -\int_{\Omega_{t}} \ell_{\vec{d}}^{2} \vec{\nabla}\vec{d} \cdot \vec{\nabla}\delta\dot{\vec{d}}dV + \int_{\Omega_{t}} \left(d - \vec{d}\right)\delta\dot{\vec{d}}dV - \int_{\Omega_{t}} \rho \frac{\zeta_{\vec{D}}\dot{\vec{d}}}{\vec{H}}\delta\dot{\vec{d}}dV = 0 \quad \forall \delta\dot{\vec{d}} \ K.A.$$
(2.2)

Forme faible associée aux équations de bilan de l'écrouissage isotrope micromorphique:

$$J_{\bar{r}}\left(\breve{r},\delta\dot{\breve{r}}\right) = -\int_{\Omega_{t}} \left(1-d^{\gamma}\right)\ell_{\bar{r}}^{2}\vec{\nabla}\breve{r}\cdot\vec{\nabla}\delta\dot{\breve{r}}dV + \int_{\Omega_{t}} \left(1-d^{\gamma}\right)\left(r-\breve{r}\right)\delta\dot{\breve{r}}dV - \int_{\Omega_{t}} \rho \frac{\zeta_{\bar{r}}\breve{r}}{\breve{Q}}\delta\dot{\breve{r}}dV = 0 \quad \forall\delta\dot{\breve{r}}\ K.A$$
(2.3)

où $\delta \vec{u}$ est le champ de vitesse virtuelle cinématiquement admissible et $\delta \vec{d}$ et $\delta \dot{\vec{r}}$ sont les champs de vitesse virtuels cinématiquement admissibles associés respectivement à l'endommagement micromorphique et à l'écrouissage isotrope micromorphique.

Ces trois formes sont fortement couplées et non linéaires de sorte que leurs résolutions analytiques sont impossible. En conséquence, des méthodes numériques basées sur la discrétisation temporelle et spatiale par des Eléments Finis appropriés, est nécessaire pour résoudre les équations non linéaires discrétisées définissant le PVIL. Cette approche conduit à un système algébrique non linéaire qui peut être résolu en utilisant le schéma itératif de type Newton.

L'introduction des nouvelles équations d'équilibre micromorphiques dans le PVIL nécessite la construction d'éléments spéciaux, basée sur des formes variationnelles appropriées comme celles de Hu-Washizu, avec une décomposition particulière des gradients de transformation. Nous avons implémenté dans Abaqus/Explicit[®] un élément 2D quadrilatéral initialement proposé par Wang et. al, (2004). Cet élément 2D bilinéaire quadrangle à déformation postulée est défini de manière à éviter l'apparition de verrouillage au cisaillement transverse et à l'incompressibilité. En suivant la méthode de déformation postulée, nous développons un simple élément 2D axisymétrique à quatre nœuds. Pour cet élément, la projection orthogonale des champs de contrainte et de déformation par

l'approche B-bar de Simo et Hughes (Simo, 1998) et l'opérateur de $\{\gamma\}$ -projection, suggérée par Flanagan et Belytschko (Flanagan et Belytschko, 1981) est appliquée.

Considérant la procédure d'analyse dynamique explicite dans Abaqus/Explicit les solutions du PVIL à trois formes présentées ci-dessus, sont obtenues simultanément par un couplage explicite, basé sur l'implémentation d'une règle d'intégration explicite avec l'utilisation de matrices de masse diagonales. Pour chaque rangée, le terme de masse localisée diagonale est obtenu en additionnant les différents termes associés à chaque colonne:

$$M_{ii}^{lp} = \sum_{j=1}^{nddls} M_{ij}^{e} = \sum_{j=1}^{nddls} \int_{\Omega_{r}^{e}} \rho N_{i}^{e} N_{j}^{e} J_{v}^{e} dV = \int_{\Omega_{r}^{e}} \rho N_{i}^{e} \sum_{j=1}^{nddls} N_{j}^{e} J_{v}^{e} dV = \int_{\Omega_{r}^{e}} \rho N_{i}^{e} J_{v}^{e} dV$$
(2.4)

où M_{ij}^{e} est la matrice de masse élémentaire, N_{i}^{e} représente les fonctions d'interpolation, ρ est la densité et J_{v}^{e} est le déterminant de la matrice jacobienne.

Pour résoudre le système algébrique donné dans l'équation (2.11), le calcul des forces internes et externes exige l'évaluation du tenseur des contraintes locales $\underline{\sigma}_{n+1}$, l'endommagement local d_{n+1} , l'écrouissage isotrope r_{n+1} et cinématique $\underline{\alpha}_{n+1}$, à chaque point d'intégration de chaque élément au

pas de temps $(t_{n+1} = t_n + \Delta t)$, en fonction des variables d'état déjà connues à l'instant t_n , les relations d'état Eq.(1.2) à Eq.(1.5) et les équations d'évolution Eq.(1.11) à Eq.(1.14). Afin de calculer ces variables d'état à la fin de chaque pas de temps, nous devons intégrer numériquement les équations différentielles ordinaires globales. En présence des écrouissages isotrope et cinématique non linéaires, il a été montré (Nesnas et Saanouni (2002), Saanouni et Chaboche (2003), Badreddine et al. (2010), Saanouni (2012b)) que la combinaison d'un schéma asymptotique (Walker et Freed, 1991) avec l'algorithme de prédiction élastique/correction plastique par retour radial, conduit à un schéma d'intégration efficace inconditionnellement stable en présence d'endommagement ductile. Cette méthode est suivie ici pour intégrer les équations constitutives complètement couplées à l'endommagement micromorphique.

Supposons que la déformation incrémentale totale $\Delta \underline{\varepsilon}$ à l'échelle actuelle de temps soit complètement élastique, ce qui signifie l'absence de tout écoulement plastique induit, écrouissage ou endommagement, c'est-à-dire $\Delta \lambda^p = 0$. Dans ce cas, la déformation élastique essai à l'instant t_{n+1} est donnée par:

$$\underline{\varepsilon}_{n+1}^{essat} = \underline{\varepsilon}_{n}^{e} + \underline{\Delta}\varepsilon$$
(2.5)

qui résulte de la contrainte d'essai:

.....

$$\underline{\sigma}_{n+1}^{essai} = (1 - d_n) \underline{\underline{\Delta}} : \underline{\varepsilon}_{n+1}^{essai}$$
(2.6)

Comme $\Delta \lambda^{p} = 0$, pour cet incrément de chargement élastique essai, le critère de charge de von Mises (Eq. (1.9)) correspondant à cette contrainte d'essai se réécrit comme:

$$f_{n+1}^{essai} = \frac{\left\|\underline{\sigma}_{n+1}^{essai} - \underline{X}_{n}\right\|}{\sqrt{1 - d_{n}}} - \frac{R_{n}}{\sqrt{1 - d_{n}^{\gamma}}} - \sigma_{y}$$

$$(2.7)$$

Si $f_{n+1}^{trial} < 0$, alors que la solution soit effectivement élastique, cela signifie que l'état de contrainte d'essai se trouve à l'intérieur de la surface d'écoulement, et les variables d'état sont mises à jour comme suit pour donner la solution élastique:

$$\underline{\varepsilon}_{n+1}^{e} = \underline{\varepsilon}_{n+1}^{essai,e}, \quad \underline{\varepsilon}_{n+1}^{p} = \underline{\varepsilon}_{n}^{p}, \quad \underline{\sigma}_{n+1} = \underline{\sigma}_{n+1}^{essai}$$

$$r_{n+1} = r_{n}, \quad R_{n+1} = R_{n}$$

$$\underline{\alpha}_{n+1} = \underline{\alpha}_{n+1}, \quad \underline{X}_{n+1} = \underline{X}_{n}$$

$$d_{n+1} = d_{n}, \quad Y_{n+1} = Y_{n}$$
(2.8)

Si $f_{n+1}^{trial} > 0$, cela signifie que l'état de contrainte d'essai se trouve en dehors de la surface de charge, ce qui n'est pas admissible dans la plasticité indépendante du temps. La solution est alors plastique et la solution d'essai doit être corrigée pour déterminer les valeurs finales des variables d'état $\underline{\sigma}, \underline{X}, R, Y$ et s'assurer que la condition de charge $f_{n+1}(\underline{\sigma}_{n+1}, \underline{X}_{n+1}, R, H_n) = 0$ est remplie. Dans ce cas, la fonction de charge de von Mises prend la forme suivante:

$$f_{n+1} = \frac{\left\|\underline{\sigma}_{n+1} - \underline{X}_{n+1}\right\|}{\sqrt{1 - d_{n+1}}} - \frac{R_{n+1}}{\sqrt{1 - d_{n+1}^{\gamma}}} - \sigma_{y}$$
(2.9)

Pour procéder à cette correction plastique, les équations d'évolution discrétisées, non linéaires et fortement couplées doivent être résolues en utilisant le schéma itératif de Newton-Raphson, pour déterminer les variables de contrainte et les conditions d'admissibilité à t_{n+1} . Ainsi, compte tenu de la plasticité isotrope, nous obtenons les deux équations fortement non linéaires suivantes avec deux variables indépendantes $\Delta \lambda^p$ et d_{n+1} :

$$\begin{cases} \int d\mu_{e}\Delta\lambda^{p} + \frac{C}{a}\left(1 - \exp\left(-a\left(\frac{C+\breve{C}}{C}\right)\Delta\lambda^{p}\right)\right) + \\ \int d\mu_{e}\Delta\lambda^{p} + \frac{C}{a}\left(1 - \exp\left(-a\left(\frac{C+\breve{C}}{C}\right)\Delta\lambda^{p}\right)\right) + \\ \int d\mu_{e}\Delta\lambda^{p} + \frac{C}{a}\left(1 - \exp\left(-a\left(\frac{C+\breve{C}}{C}\right)\Delta\lambda^{p}\right)\right) + \\ \int d\mu_{e}\Delta\lambda^{p} + \frac{C}{a}\left(1 - \exp\left(-\frac{\sqrt{1-d_{n+1}^{\gamma}}}{\sqrt{1-d_{n+1}^{\gamma}}}\right)\breve{Q}\breve{r}_{n+1}\right)\exp\left(-b\left(\frac{Q+\breve{Q}}{Q}\right)\Delta\lambda^{p}\right) + \\ \int d\mu_{e}\Delta\lambda^{p} + \frac{C}{b}\left(1 - \exp\left(-b\left(\frac{Q+\breve{Q}}{Q}\right)\Delta\lambda^{p}\right)\right) + \sigma_{y} + \\ \int d\mu_{e}\Delta\lambda^{p} + \frac{C}{b}\left(1 - \exp\left(-b\left(\frac{Q+\breve{Q}}{Q}\right)\Delta\lambda^{p}\right)\right) + \sigma_{y} + \\ \int d\mu_{e}\Delta\lambda^{p} + \frac{C}{b}\left(1 - \exp\left(-b\left(\frac{Q+\breve{Q}}{Q}\right)\Delta\lambda^{p}\right)\right) + \\ \int d\mu_{e}\Delta\lambda^{p} + \frac{C}{b}\left(1 - \exp\left(-b\left(\frac{Q+\breve{Q}\right)\Delta\lambda^{p}\right)\right) + \\ \int d\mu_{e}\Delta\lambda^{p} + \frac{C}{b}\left(1 - \exp\left(-b\left(\frac{Q+\breve{Q}\right)\Delta\lambda^{p}\right)\right) +$$

Ce système de deux équations à deux inconnues est résolu par un schéma itératif Newton-Raphson afin de les linéariser sous la forme suivante:

$$\begin{cases} f \\ g \end{cases}_{n+1}^{s} + \begin{bmatrix} \frac{\partial f}{\partial \Delta \lambda^{p}} & \frac{\partial f}{\partial d} \\ \frac{\partial g}{\partial \Delta \lambda^{p}} & \frac{\partial g}{\partial d} \end{bmatrix}_{n+1}^{s} \begin{cases} \delta \Delta \lambda^{p} \\ \partial d \end{cases}_{n+1}^{s} + 0 \times (...) = \begin{cases} 0 \\ 0 \end{cases}$$
(2.11)

où *s* est le nombre d'itérations et $\delta \Delta \lambda^p$ et δd_{n+1} les corrections données par:

$$\begin{cases} \delta \Delta \lambda^{p} = \left(\Delta \lambda^{p}\right)^{s+1} - \left(\Delta \lambda^{p}\right)^{s} \\ \delta d = \left(d_{n+1}\right)^{s+1} - \left(d_{n+1}\right)^{s} \end{cases}$$
(2.12)

Au cours des simulations numériques de divers procédés de mise en forme, les grandes déformations plastiques qui se produisent dans les zones de forte déformation plastique conduisent à des zones de déformation hautement localisées. À l'intérieur de ces zones de strictions, des défauts microscopiques peuvent apparaître fréquemment conduisant à l'apparition de fissures macroscopiques. La simulation numérique de ces processus, en utilisant la méthode des éléments finis basée sur la formulation lagrangienne actualisée, conduit à une grande distorsion d'élément au cours de la simulation. Pour cette raison, les procédures de maillage adaptatif sont nécessaires pour effectuer une analyse complète. Dans cette thèse, une méthodologie de remaillage adaptatif 2D est proposée et appliquée avec succès à des problèmes de mise en forme hautement non linéaires avec apparition de l'endommagement. La procédure adapte la séquence de chargement et la taille du maillage, en utilisant des indicateurs d'erreur appropriés. Le générateur de maillage DIAMESH2D est ensuite réalisé à l'aide d'éléments quadrangulaires ou triangulaires linéaires ou quadratiques (Rassineux, 1991), selon les étapes suivantes (Fig.2.1):



Figure 2.1: méthodologie de remaillage adaptatif 2D (Labergere et. al, 2014)

3. Etude paramétrique.

Dans cette section, une étude paramétrique relativement exhaustive du modèle micromorphique est présentée en mettant l'accent sur le rôle des deux paramètres micromorphiques (\breve{H},\breve{H}^g) liés à l'endommagement micromorphique. Nous examinons l'effet des différentes valeurs de chacun de ces paramètres sur la qualité de la solution numérique en termes de réponses force-déplacement et contrainte-déformation globales dans le cas d'essais de traction uniaxiale à partir d'éprouvette plate. Les éprouvettes plates métalliques testés sont en acier 430 X8Cr17) et ont tous été modélisés avec des éléments quadrangles à déformation postulée Q4. Nous rappelons que dans ce cas une seule longueur interne est activée, celle liée à l'endommagement micromorphique donnée par $\ell_{\vec{d}} = \sqrt{\vec{H}^s / \vec{H}}$.



Figure 3.1 Le spécimen maillé avec h = 0.2 mm

Nous examinons l'effet du module d'endommagement micromorphique \breve{H} en prenant une valeur moyenne de $\breve{H} = 60.0$. Les valeurs du module \breve{H}^{g} sont variées afin d'obtenir les trois valeurs suivantes de la longueur interne $\ell_{\breve{d}}$ de 0.1, 1.0 et 5.0 mm.

$\breve{H}^{g}(N)$	$\ell_{\vec{a}}$	<i>U</i> _{rupt}	
0.6	0.1 mm	11.34 mm	
60.0	1.0 mm	11.9 mm	
1500.0	5.0 mm	pas de rupture observée	

Table 3.1 Valeurs des paramètres de matériau micromorphiques et déplacement de la fracture pour

H = 60.0



Figure 3.2 Comparaison des réponses force-déplacement globales du modèle local et du modèle micromorphique pour un $\breve{H} = 60.0$



Figure 3.3 Courbes $\sigma_{\rm eq} - p$ du modèle local et du modèle micromorphique pour $\breve{H} = 60.0$

Comme le montrent le Tableau 3.1, les courbes force-déplacement (Figure 3.2) et les courbes contrainte-déformation (Figure 3.3), l'effet de régularisation existe mais il est moins essentiel pour un $\ell_{\bar{d}}$ de 0.1 mm et de 1.0 mm pour lesquels les réponses mécaniques sont plus élevées mais toujours proches de la solution locale avec une rupture se produisant pour des valeurs de déplacement de u_{rupt} =11.34 mm et u_{rupt} =11.9 mm respectivement. Cependant, aucune rupture n'est observée à la fin de la charge appliquée pour la valeur la plus élevée de $\ell_{\bar{d}}$ = 5.0 mm.

Pour le cas de $\ell_{\vec{d}} = \sqrt{1500.0/60.0} = 5.0 \text{ mm}$ il est clairement montré que l'effet de l'endommagement micromorphique est excessivement fort du fait que $\breve{H}^g = 1500.0$ et ne permet pas de prédire une rupture finale jusqu'à la fin de déplacement imposé de 15.0 mm (Figure 3.2) malgré les valeurs extrêmement élevées de $p_{max} = 290\%$ et $\sigma_{eq}^{max} = 872$ MPa observées. L'effet est directement mis en correspondance avec l'évolution très lente de l'endommagement local ainsi que ses faibles valeurs (Figure 3.4). Nous pouvons noter clairement ici que la ductilité du matériau augmente en augmentant les deux valeurs des modules \breve{H} et \breve{H}^g .



Figure 3.4 évolution de l'endommagement local du modèle local et micromorphique à la fin de déplacement $u = 15.0 \,\mathrm{mm}$



Figure 3.5 Endommagement local avant la fin de la charge appliquée (à u_{rupt}) et la forme de la fissure

finale pour les trois valeurs de $\ell_{\,\vec{d}}$ ($\breve{H}=60.0\,$)



Figure 3.6 Endommagement micromorphique avant la fin de la charge appliquée (à u_{rupt}) pour les trois valeurs de $\ell_{\vec{d}}$ ($\vec{H} = 60.0$)

En termes de distribution de l'endommagement à l'intérieur de l'échantillon, quand $\ell_{\bar{d}} = \sqrt{0.06/60.0} = 0.1 \,\mathrm{mm}$, la valeur maximale de l'endommagement local juste avant la rupture finale de l'échantillon est $d_{\max} = 41.0\%$ (Figure 3.5a.1) et l'endommagement micromorphique maximale est $\breve{d}_{\max} = 25.0\%$ (Figure 3.6a). Pour $\ell_{\bar{d}} = \sqrt{60.0/60.0} = 1.0 \,\mathrm{mm}$ nous obtenons $d_{\max} = 41.0\%$ (Figure 3.5a.2) et $\breve{d}_{\max} = 21.0\%$ (Figure 3.6a.2) tandis que pour $\ell_{\bar{d}} = \sqrt{1500.0/60.0} = 5.0 \,\mathrm{mm}$, nous obtenons $d_{\max} = 24.0\%$ (Figure 3.5a.5), $\breve{d}_{\max} = 12.0\%$ (Figure 3.6a.3).

En conclusion, c'est le paramètre H qui contrôle Y_{nloc} et qui semble avoir le plus d'effet sur la non-localité. En fait, la solution est presque identique à la solution locale pour les valeurs les plus faibles de H alors que l'effet de non-localité devient plus important pour la réponse globale comme les valeurs de H augmentent, ce qui est évident à partir des courbes F - u et $\sigma_{eq} - p$. Toutes ces figures représentent les réponses d'un point de Gauss d'un élément au centre de l'échantillon à l'intersection des deux bandes.

4. <u>Méthodologie d'identification de la longueur interne</u> $\ell_{\tilde{d}}$.

Comme indiqué dans la section précédente, les courbes F - u numériques ont été utilisées pour étudier les effets de H et H^{s} et choisir l'approche la plus appropriée, sans prendre en compte la notion de longueur micromorphique interne ℓ_{d} . A cause de cet effet et sous l'existence perceptible de non-localité, nous avons réalisé une mesure expérimentale des champs des taux de déformation à l'aide de l'Interférométrie Electronique Spectroscopique (ESPI) au cours d'un essai de traction de l'acier inoxydable 430. Le taux de déformation $\dot{\varepsilon}_{xx}$ mesuré par ESPI est utilisé pour déterminer l'évolution de la largeur de bande de localisation. L'idée est de définir un lien entre $\ell_{\bar{d}}$ et la largeur de bande intégrale mesurée. La distribution du taux de déformation totale est supposée être une simple superposition (somme) de la distribution du taux de déformation de ces deux bandes. Ce modèle n'est pas un modèle mécanique mais un modèle purement analytique utilisé pour décrire le l'évolution spatiale du taux de déformation et extraire les paramètres globaux et complètement indépendants du comportement constitutif du matériau. La distribution du taux de déformation dans chaque bande est décrite par une combinaison linéaire d'une fonction gaussienne et d'une fonction lorentzienne:

$$\dot{\varepsilon}_{xx}(x,y) = \dot{\varepsilon}_{1}^{B} + \dot{\varepsilon}_{2}^{B}$$

$$avec \ \dot{\varepsilon}_{i}^{B} \left(\dot{\varepsilon}_{i}^{\max}, \eta_{i}, a_{i}, b_{i}, B_{i}, x, y \right) = \dot{\varepsilon}_{i}^{\max} \left(\eta_{i} G \left(a_{i}, b_{i}, B_{i}, x, y \right) + (1 - \eta_{i}) L \left(a_{i}, b_{i}, B_{i}, x, y \right) \right)$$
(4.1)

où i = 1 ou 2 représente l'une des deux bandes localisées, et (x, y) sont les coordonnées des points (x est parallèle à la direction de traction), avec :

- *ἐ_i^{max}* le taux de déformation maximum,
- G_i , la fonction Gaussienne,
- *L_i*, la fonction Lorentzienne,
- η_i et $1-\eta_i$ les poids des fonctions gaussienne et lorentzienne respectivement.
- Les fonctions Gaussienne et Lorentzienne sont définies comme suit :

$$G(a_i, b_i, B_i, x, y) = \exp\left(-\pi \left(\frac{x - a_i y - b_i}{B_i}\right)^2\right)$$
(4.2)

$$L(a_i, b_i, B_i, x, y) = \frac{1}{1 + \left(\pi \frac{x - a_i y - b_i}{B_i}\right)^2}$$
(4.3)

a_i : représente l'inclinaison de la bande *i* par rapport à la direction transversale,

- b_i : représente l'emplacement de la bande i,
- B_i : représente la largeur intégrale de la bande *i* (Figure 4.1).



Figure 4.1. Illustration des paramètres, l'image de gauche est un diagramme de frange et celle de droite est la carte de taux de déformation correspondante obtenue.

Les largeurs B^{dom} et B^{dis} des deux bandes, dans tous les cas, diminuent avec la contrainte moyenne totale ou le temps. Au début, les deux largeurs de bande diminuent simultanément et ensuite elles se séparent: B^{dis} tend à se stabiliser tandis que B^{dom} continue de diminuer.



Figure 4.2. Evolution des caractères de la bande de localisation par rapport à la déformation moyenne totale (ou temps), (Bao, François, Le Joncour, 2015)

Dans cet esprit, et en implémentant numériquement le modèle mathématique donné par les équations (4.1) à (4.3) et en calculant le taux de déformation numérique à un instant donné $\dot{\epsilon}_{xx}^{num}(x, y, t)$ à chaque point gauss des éléments situés dans la zone centrale, on arrive à extraire les valeurs de la largeur de bande numérique B^{dom} pendant la striction diffuse et la striction localisée et comparer les résultats numériques avec ceux expérimentaux montrés dans la Figure 4.2.

Influence des modules (\breve{H}^s,\breve{H}) dans l'évolution de la bande intégrale avec

S = 6.0.

Nous notons que le paramètre S est directement lié à la ductilité et régit l'apparition de la fracture après la striction diffuse. En attendant, on observe que son effet est de retarder ou d'accélérer la croissance de l'endommagement sans modifier la forme de l'évolution de l'endommagement ou les courbes F-u.

Les études paramétriques suivantes sont analysées.

> 1.a.
$$h = 0.2 \text{ mm}, \quad \ell_{\bar{d}} = \sqrt{\frac{\bar{H}^s}{\bar{H}}} = \sqrt{\frac{36.0}{100.0}} = 0.6 \text{ mm} \quad \text{comparé} \quad a \quad h = 0.2 \text{ mm},$$

 $\ell_{\bar{d}} = \sqrt{\frac{\bar{H}^s}{\bar{H}}} = \sqrt{\frac{21.6}{60.0}} = 0.6 \text{ mm}.$



Figure 4.3. Evolution de la bande dominante de localisation pour $(\vec{H} = 100.0, \vec{H}^s = 36.0)$ et $(\vec{H} = 60.0, \vec{H}^s = 21.6)$, et comparaison entre les données numériques et expérimentales pour h = 0, 2mm



Figure 4.4. Evolution de la bande dominante de localisation pour $(\breve{H} = 100.0, \breve{H}^s = 36.0)$ et $(\breve{H} = 60.0, \breve{H}^s = 21.6)$, et comparaison entre les données numériques et expérimentales pour h = 0, 4 mm

Pour ce cas, nous réparons S = 6.0 et $\ell_{\bar{d}} = 0.6 \text{ mm}$ avec deux combinaisons différentes $\left(\ell_{\bar{d}} = \sqrt{\frac{H^s}{H}} = \sqrt{\frac{36.0}{100.0}} = 0.6 \text{ mm}$ et $\ell_{\bar{d}} = \sqrt{\frac{H^s}{H}} = \sqrt{\frac{21.6}{60.0}} = 0.6 \text{ mm}$) pour les deux tailles de maille différentes. Les résultats semblent être très intéressants pour h = 0.2 mm (Figure 4.3). Nous avons choisi $\ell_{\bar{d}} = \sqrt{\frac{H^s}{H}} = \sqrt{\frac{21.6}{60.0}} = 0.6 \text{ mm}$ puisque la largeur de bande commence plus haut que tous les cas précédents (environ 24 mm) suivi d'un bon ajustement entre 21% et 23% de la contrainte moyenne totale, mais avec une grosse erreur à 25% de la contrainte moyenne totale. Pour $\ell_{\bar{d}} = \sqrt{\frac{H^s}{H}} = \sqrt{\frac{36.0}{100.0}} = 0.6 \text{ mm}$ on observe le comportement le plus précis puisque les valeurs de début et de fin de B₁ sont en accord absolu avec le B_{dom} expérimental alors que l'erreur reste relativement faible et même négligeable. À peu près la même tendance est observée dans le cas de h = 0.4 mm (Figure 4.4). Pour

 $\ell_{\vec{d}} = \sqrt{\frac{\vec{H}^{g}}{\vec{H}}} = \sqrt{\frac{21.6}{60.0}} = 0.6 \text{ mm}$ la bande dominante commence à évoluer à 24 mm, et s'approche suffisamment de la courbe expérimentale entre 21% et 25% de la contrainte moyenne totale, mais chute rapidement donnant une grosse erreur jusqu'à 25% de la contrainte moyenne totale. D'un autre côté, quand $\ell_{\vec{d}} = \sqrt{\frac{\vec{H}^{g}}{\vec{H}}} = \sqrt{\frac{36.0}{100.0}} = 0.6 \text{ mm}$, la bande commence correctement à 20 mm, suit exactement le même comportement que la largeur de bande obtenue pour $\vec{H} = 60.0$ mais n'atteint pas un ajustement exact car elle diminue significativement à la déformation moyenne totale de 24%,

Concernant le groupe des paramètres de matériaux liés à l'endommagement micromorphique et plus précisément la longueur de régularisation micromorphique $\ell_{\bar{d}} = \sqrt{\frac{\breve{H}^s}{\breve{H}}}$, nous avons utilisé les mesures expérimentales de l'ESPI et les résultats des essais de traction numériques pour la relier à la largeur de la bande dominante et proposer une méthodologie pour identifier une paire de (\breve{H},\breve{H}^s) .

5. Applications à des opérations de pliage et de découpage

tout en donnant une erreur plus petite que dans le cas précédent.

Dans ce dernier chapitre, nous validons la prédiction de notre méthodologie numérique basée sur une approche micromorphique dans le cas de deux procédés de mise en forme, un essai de pliage d'une tôle de DP1000 et le découpage d'une tôle en DP600. Selon les conclusions du Chapitre 3, nous n'utiliserons le modèle qu'avec l'endommagement micromorphiques dus à sa capacité à donner des solutions uniques. Par conséquent, une seule paire de paramètres micromorphiques, (\breve{H},\breve{H}^g) sera activée et leurs choix seront soigneusement effectués pour assurer des solutions physiques et une propagation de fissure acceptable dans la zone complètement endommagée.

Simulation d'un essai de pliage

Ce test a été modélisé en déformation plane 2D sans tenir compte de la symétrie. La feuille en acier DP1000 est discrétisée avec l'élément de déformation plane quadrangulaire Q4 proposé au chapitre 2, a été implémenté dans le logiciel ABAQUS[®]/Explicit via la routine VUEL.

Trois petites tailles d'éléments (M_s) ont été utilisées pour mailler la tôle dans la zone où la rupture peut apparaître:

- ➤ M_s = 0.1 mm (2340 éléments),
- M_s = 0.08 mm (2602 éléments),
- $M_s = 0.06 \,\mathrm{mm}$ (3500 éléments).

La formulation micromorphique est maintenant prise en compte en activant la contribution de la variable d'endommagement micromorphique \vec{d} en fixant $\vec{H} = 80.0 \ N.mm^{-2}$, $\vec{H}^g = 3.6 \ N$ nous obtenons la valeur de la longueur interne $\ell_{\vec{d}} = \sqrt{\vec{H}^g / \vec{H}} \approx 0.21 \ mm$.



Figure 5.1. Courbes numériques et expérimentale de force-déplacement du DP1000 pour le pliage pour le modèle micromorphique


Figure 5.2. Évolution de l'endommagement local et micromorphique pour les trois tailles de maille

Contrairement au modèle local, la courbe prédite par le modèle micromorphique donne la même valeur de déplacement de la fracture comme indiquée à la Figure 5.1. La distribution finale de l'endommagement local, telle que prédit par la formulation micromorphique, est illustrée à la Figure 5.3. Nous pouvons voir que la bande de localisation de l'endommagement devient plus lisse que le cas de l'endommagement local (c'est-à-dire que le gradient l'endommagement varie moins que dans le cas local). La Figure 5.3 montre l'évolution de l'endommagement local et micromorphique le long des lignes horizontales de l'échantillon de rompue au moment de u_{rupt} pour $M_s = 0.1$, $M_s = 0.08$ et l'endommagement micromorphique est plus faible $M_{\rm s} = 0.06 \,\rm{mm}.$ Clairement, que l'endommagement local (d < d). Au début de l'écoulement de l'endommagement, nous avons d = d, mais comme l'endommagement augmente à l'intérieur de la zone de localisation, l'endommagement local augmentent plus rapidement que l'endommagement micromorphique. En conséquence, la différence entre d et d augmente à l'intérieur de la zone endommagée et au niveau de la fracture finale $d = d_c = 1.0$ et $d \approx 0.55$.





Figure 5.3: Isovaleurs de l'endommagement local et micromorphique de l'échantillon de pliage rompu dans le cadre de l'approche micromorphique pour les différents maillages à la fin du déplacement imposé

Simulation d'une opération de découpage

La méthodologie d'analyse adaptative décrite au paragraphe 2 est validée lors d'une opération de découpage d'une tôle mince circulaire. La tôle circulaire a une épaisseur de e=1.2 mm. Le diamètre du poinçon est de 12.0 mm et le jeu entre la matrice fixe et le poinçon mobile est de 0.12 mm, comme montré dans la figure 5.4 (Labergere et al., 2010, 2014).



Figure 5.4: Schéma du procédé de découpage axisymétrique

Le matériau est un acier DP600 et seule la formulation micromorphique en endommagement est prise en compte en activant la contribution des variable d'endommagement micromorphique $\breve{H} = 50.0 \ N.mm^{-2}$, $\breve{H}^g = 0.02 \ N$ donnant la longueur interne $\ell_{\vec{d}} = \sqrt{\breve{H}^g / \breve{H}} = 0.02 \ \text{mm}$. La tôle est discrétisée avec des éléments micromorphiques quadrangulaires bilinéaires axisymétriques implémentés ABAQUS[®]/Explicit via la routine VUEL et trois configurations de maillage (nommées M_i) $M_1 \rightarrow \Delta x_{\min}^{dam} = 3.0 \ \mu\text{m}$, $M_2 \rightarrow \Delta x_{\min}^{dam} = 6.0 \ \mu\text{m}$ et $M_3 \rightarrow \Delta x_{\min}^{dam} = 9.0 \ \mu\text{m}$ ont été choisis pour raffiner la zone de la tôle où les fissures apparaissent.

Pour le même déplacement de poinçon (U = 0.73 mm), nous montrons les isovaleurs de l'endommagement micromorphique de la Figure 5.5. Pour les deux configurations M₂ et M₃, l'endommagement non local est plus élevé à l'extrémité de la fissure et atteint une valeur maximale de 0.2. La largeur de la bande de localisation correspondant à l'endommagement non locale reste également identique. Pour la configuration M₁, l'évolution du gradient de l'endommagement non local n'est pas plus claire mais la largeur de la bande de localisation située dans la tôle entre l'extrémité de la fissure et le congé du poinçon semble avoir la même forme que les deux autres configurations M₂ et M₃.

Enfin, la courbe F-u prédite par le modèle micromorphique donne la même valeur de déplacement à rupture $U \approx 0.9 \text{ mm}$ comme indiquée sur la figure 5.6. Cependant, nous observons une légère différence entre les trois réponses numériques et les résultats expérimentaux dans la phase de l'adoucissement de la Figure 5.7. Cette différence peut être expliquée par la nécessité d'introduire une variable micromorphique associée à l'écrouissage isotrope afin d'améliorer la régularisation à ce stade.





Figure 5.5. Isovaleurs de l'endommagement micromorphique pour un déplacement du poinçon $U=0.73~{\rm mm}$



Figure 5.6. Réponses globales numériques et expérimentale de découpage pour les trois tailles de maillage

6. Conclusions générales

Ce travail a été consacré à une formulation non locale d'un modèle de comportement élastoplastique à l'écrouissage isotrope et cinématique, associés à un découlement induit par la présence de l'endommagement ductile isotrope. Dans le cadre des milieux généralisés, la théorie micromorphique a été appliquée en supposant que l'endommagement ductile isotrope et l'écrouissage isotrope sont les deux principaux phénomènes micromorphiques. En conséquence, deux variables cinématiques micromorphiques supplémentaires, \vec{d} et \vec{r} ont enrichi l'espace des degrés de liberté du PVIL.

Dans le deuxième chapitre, les aspects numériques du PVIL liés au mouvement du solide micromorphique, sous un chemin de chargement donné ont été présentés. Les équations principales qui définissent le PVIL ont d'abord été données et ensuite trois formes faiblement couplées et fortement non linéaires ont été obtenues. La discrétisation temporelle et spatiale (par différences et éléments finis respectivement) est également discutée suivie d'un schéma explicite de résolution globale dynamique. Finalement, une brève représentation des aspects techniques a été décrite, concernant:

- I'implémentation des éléments finis et du modèle de comportement dans Abaqus[®]/Explicit en utilisant la routine VUEL;
- Ia méthodologie de remaillage adaptatif 2D.

Dans le troisième chapitre, une étude paramétrique est réalisée sur le modèle pour l'endommagement isotrope micromorphique et pour l'écrouissage isotrope micromorphique. Les résultats obtenus par cette étude paramétrique ont montré que:

- La présence de l'endommagement micromorphique est nécessaire et suffisante pour régulariser l'IBVP, contrairement à l'écrouissage isotrope micromorphique;
- Le modèle à l'endommagement micromorphique peut donner des solutions indépendantes du maillage sous les conditions suivantes:
 - > la taille de la longueur interne choisie doit être considérablement plus grande que la taille des éléments ($\ell_{i} > h$), et
 - ▶ Le choix le plus approprié des modules micromorphiques va dépendre des valeurs de H et H^s qui doivent être restreintes dans un intervalle de "valeur intermédiaire", afin qu'elles ne soient ni trop faibles, pour récupérer les solutions locales, ni trop élevées sinon l'effet de l'endommagement micromorphique sera excessivement fort et va dissimuler l'apparition de rupture malgré des grands déplacements.

Dans le quatrième chapitre, les principales étapes pour l'identification des paramètres de matériaux ont été citées concernant l'élasticité, la plasticité et l'endommagement local. Par la suite, une méthodologie numérique a été proposée pour l'identification des valeurs des longueurs internes micromorphiques, liées à l'endommagement micromorphique dû à la largeur des bandes de cisaillement qui apparaissent lors de la striction localisée. Et par la suite une comparaison avec les largeurs expérimentales obtenues par les mesures ESPI, a permis de valider la précision de ce premier effort d'identification.

Finalement, le cinquième chapitre a été consacré à deux applications en 2D, un essai de pliage et une opération de découpage, montrant l'efficacité et les capacités prédictives d'un modèle à l'endommagement micromorphique. En conclusion, ce travail a ouvert un nouveau champ d'investigation sur la formulation des modèles de comportement micromorphiques non locaux. A propos du programme ANR "Micromorfing", de nombreuses questions ouvertes concernant les aspects théoriques, numériques et expérimentaux se posent, dont l'approfondissement est essentiel. Tous ces aspects sont soit en cours de développement dans le cadre du programme ANR, soit pris en compte en tant que priorités scientifiques de l'équipe «Formage Virtuel» de LASMIS.

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Milieux micromorphes : modélisation multiphysique et simulation numérique avancées de procédés de mise en forme

L'objectif de cette thèse est de démontrer l'efficacité des modèles de comportement élastoplastique fortement couplés à l'endommagement ductile isotrope dans le cadre des milieux micromorphes afin de s'affranchir de la dépendance au maillage lors de la prévision de l'amorçage et de la propagation de la rupture ductile. Cette approche repose sur (i) l'ajout de variables cinématiques micromorphes dans le principe des puissances virtuelles conduisant à de nouvelles équations de bilan : (ii) l'ajout de nouveaux couples de variables d'état conduisant à de nouvelles équations de comportement ; (iii) une discrétisation spatiale par éléments finis et temporelle par un schéma d'Euler avec un solveur global dynamique explicite et une intégration locale itérative implicite. Les aspects numériques associés sont implémentés dans ABAQUS®/Explicit. Deux éléments bilinéaires quadrangles à "déformation postulée" (2D)déformation plane et axisymétrique) ont été développés afin d'introduire les nouvelles formes variationnelles. Les modèles sont validés avec une étude paramétrique pour étudier l'effet de chaque paramètre micromorphe et une méthodologie d'identification de la longueur interne micromorphe liée à l'endommagement micromorphe est proposée. Des essais de traction uniaxiale d'éprouvettes en acier inoxydable 430, des opérations de pliage et de découpage de tôles métalliques en DP1000 et DP600, sont simulées afin de valider la formulation proposée et montrer son efficacité à donner des solutions indépendantes du maillage par rapport au modèle local.

Mots clés : endommagement, mécanique de l' (milieux continus) - élastoplasticité - éléments finis, méthode des - simulation par ordinateur.

Micromorphic Continua: Advanced Multiphysic Modelling and Numerical Simulation of Metal Forming

The objective of this work is to demonstrate the efficiency of advanced elastoplastic constitutive equations strongly coupled with isotropic ductile damage in the framework of the micromorphic continua in order to overcome the mesh dependency in the prediction of the ductile cracks initiation and propagation. This approach is based on (i) the introduction, in the principle of virtual power, of additional micromorphic kinematic variables leading to additional balance equations; (ii) addition of new micromorphic pairs of state variables leading to additional micromorphic constitutive equations; (iii) spatial discretization by finite elements and time discretization by finite difference scheme with an explicit dynamic global solver and an implicit iterative local integration scheme. The associated are numerical aspects implemented in ABAQUS®/Explicit. Two bilinear quadrilateral assumed strain elements (2D plane strain and axisymmetric) have been developed in order to introduce new weak forms. The models are validated with a parametric study in order to investigate the effect of each micromorphic parameter, and a methodology for the identification of the micromorphic internal length related to the micromorphic damage is proposed. Simple uniaxial tensile tests, sheet bending and blanking processes of metallic components in 430 stainless steel, DP1000 and DP600 dual phase steels respectively are simulated in order to validate the proposed formulation and to show its efficiency in giving mesh independent solutions compared to the purely local models.

Keywords: continuum damage mechanics - elastoplasticity - finite element method – computer simulation.

Thèse réalisée en partenariat entre :





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