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Ismaël Rafai

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# THÈSE DE DOCTORAT

## Prise en Compte de l'Attention Limitée dans l'Analyse Economique.

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Groupe de Recherche en Droit, Economie et Gestion (GREDEG, CNRS)

**Présentée en vue de l'obtention du grade de  
Docteur en Sciences économiques  
d'Université Côte d'Azur**

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# Prise en Compte de l'Attention Limitée dans l'Analyse Économique

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# Integration of Limited Attention in Economic Theory

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*À la mémoire de Pierre.*





# Résumé: Prise en Compte de l'Attention Limitée dans l'Analyse Economique

Cette thèse contribue à la prise en compte de l'attention limitée dans l'analyse économique. Nous défendons l'idée que les processus d'allocation de l'attention peuvent être étudiés à travers un processus de production avec en input l'*attention allouée* (la quantité de ressources attentionnelles investies dans la décision), et en output l'*attention effective* (la quantité d'information contenue dans la décision). Afin d'améliorer la compréhension de ces processus, nous proposons trois essais empruntant des méthodes à la psychologie et aux sciences cognitives.

Dans le premier chapitre, nous manipulons l'ordre de présentation entre une information sur les incitations et un stimulus visuel, dans un paradigme de *choix forcé à deux alternatives*. L'*attention allouée* y est contrôlée, et nous mesurons l'*attention effective* à l'aide d'un modèle de *detection du signal*. Nous montrons que la dernière information présentée a un plus grand poids dans la décision et attribuons cet effet à une division de l'attention.

Le second chapitre propose une expérience dans laquelle les participants allouent une attention coûteuse, afin de réduire l'incertitude d'une *tâche de discrimination*. Ainsi, nous mesurons à la fois l'*attention allouée* (par le biais du temps de réponse) et l'*attention effective* (par le biais de la performance). Cette expérience nous permet d'étudier les *dilemmes sociaux attentionnels* (situations où l'attention est coûteuse pour l'individu mais bénéfique pour le groupe) et de mettre en lumière une divergence entre les préférences sociales – mesurées traditionnellement par des choix d'allocations monétaires – et les comportements observés dans notre dilemme social attentionnel.

Le dernier chapitre prouve qu'il est possible d'implémenter empiriquement et de tester la validité d'un modèle de *préférences révélées avec attention aléatoire*. Nous proposons une nouvelle caractérisation et un nouveau théorème des préférences révélées dans le cadre d'une version plus générale du modèle de Brady et Rehbeck (2016, *Econometrica*). Nous développons des procédures statistiques – que nous analysons à l'aide de simulations numériques – afin de tester les axiomes du modèle, de révéler les préférences, et d'obtenir une mesure de l'*attention effective*. Nous testons la validité du modèle à l'aide d'une tâche d'attention sélective dans laquelle les participants choisissent un gain monétaire parmi un ensemble de distracteurs. Les comportements observés dans cette expérience sont cohérents avec le modèle et les préférences induites expérimentalement.

**Mots-clés :** **Économie Comportementale ; Économie Expérimentale ; Rationalité Limitée ; Attention limitée ; Allocation de l'attention ; Ensembles de Considération ; (In)Attention Rationnelle ; Préférences révélées ; Préférences sociales**

# Abstract: Integration of Limited Attention in Economic Theory

This thesis contributes to the integration of limited attention within the economic theory. We argue that attentional allocation processes can be understood as a production process with the *allocated attention* (the quantity of attentional resources invested in a decision) as an input and the *effective attention* (the amount of information contained in that decision) as an output. Borrowing methods from psychology and cognitive sciences, we propose three essays to shedding light on these processes.

In the first chapter, we manipulate the presentation order between reward information and perceptual evidence in a *two-alternative forced-choice task*. The *allocated attention* is controlled and we measure the *effective attention* with a *Signal Detection* model. We found that the last information presented is more weighted in the decision. We attribute this effect to the division of attention.

The second chapter proposes an experiment where participants pay costly attention to reduce the uncertainty of a discrimination task. We measure both *allocated attention* (through the response time) and *effective attention* (through performance). This experiment allows the study of *attentional social dilemmas* (situations where attention is costly for individuals but benefits the group). We highlight a discrepancy between *monetary elicited social preferences* and the behaviors exhibited in our attentional social dilemma.

The last chapter proves that a model of *revealed preferences under stochastic attention* can be implemented and tested empirically. We provide new characterization and revealed preference theorems for a general version of Brady and Rehbeck's model (2016, *Econometrica*). We propose and analyze – with numerical simulations –

statistic procedures to test the axioms, to reveal preferences, and to measure *effective attention*. We test the internal validity of the model with a selective attention task, where participants choose an alternative among distractors and we find that most of the subjects behave in accordance with the model and reveal coherent preferences.

**Keywords:** Behavioral Economics; Experimental Economics; Bounded Rationality; Limited Attention; Attention Allocation; Consideration Set; Rational (In)Attention; Revealed Preferences; Social Preference

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# General Introduction

The number of different products in a typical US grocery store rose from less than 9,000 in the 70s to more than 40,000 nowadays. As a consequence, each class of products includes dozens of available distinctive specimens that differ in their price, brand, flavor, color, packaging and other characteristics.<sup>1</sup>

A belief shared for a long time by many economists was that thanks to the competitive forces, an increase in the amount of information and in the number of available products naturally enhances (or at least cannot decrease) the efficiency of the market and consumer welfare. Indeed, the higher the number of products available on the market, the higher the likelihood that consumers' needs can be satisfied. In addition more information helps the consumers to reduce uncertainty, and improves their decisions – *ceteris paribus*. A hypothesis behind this rationale is that individuals consider all the (relevant) alternatives and process all the information available for them when making a choice. However, empirical evidence from psychology and management science contradicts this hypothesis. Individuals can only process a finite amount of information and are thus limited in the number of brands, products and characteristics they consider when making a purchase (Miller, 1956; Howard and Sheth, 1969; Hauser and Wernerfelt, 1990; Roberts and Lattin, 1997).

A reason why individuals do not consider all the alternatives is because the cognitive resources needed to select, process and retain information are limited. Humans have limited attention.

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<sup>1</sup><https://www.consumerreports.org/cro/magazine/2014/03/too-many-product-choices-in-supermarkets/index.htm>, last consultation: September, 10<sup>th</sup>, 2019

## 0.1 Economic of Attention

“What information consumes is rather obvious: it consumes the attention of its recipients. Hence a wealth of information creates a poverty of attention and a need to allocate that attention efficiently among the overabundance of information sources that might consume it.” — Simon, 1971, p.40

### Attention as a scarce resource

Attention is a scarce resource and falls within the scope of economics as the science which studies *“human behaviour as a relationship between ends and scarce means which have alternative causes”*(Robbins, 1932, p.15). Considering attention as an economic resource is not a new concept. As early as in 1929, Hotelling already supported the idea of the taxation of advertisements based on the fact that unwanted messages cause negative externalities by dispossessing some part of the recipients’ limited attention.<sup>2</sup>

From a historical perspective, economists have considered it important to integrate uncertainty and information asymmetry into their models (Stiglitz, 2002). However, until recently, they have dedicated little effort towards understanding how economic agents (should) allocate their attention and how limited capacity for information processing impacts the market. Possible reasons are that economists believed that the issues related to limited attention confound those induced by a lack of information; or that the deviations from full rationality stemming from individual cognitive limitations can be treated as statistical errors that cancel out at the aggregated level or in the long run (Stigler, 1961). As a consequence, economists usually relegated the study of those phenomena to management and psychological sciences, alongside with the idea that from a market perspective, more information

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<sup>2</sup>“Another thing of limited quantity for which the demand exceeds the supply is the attention of people. Attention is desired for a variety of commercial, political, and other purposes, and is obtained with the help of billboards, newspaper, radio, and other advertising. [...] Taxation of advertising on this basis would be in addition to any taxation imposed for the purpose of diminishing its quantity with a view to restoring the property of attention to its rightful owners” (Hotelling 1938, p.257)

improve decisions' quality and social welfare.

The popularization of behavioral economics and the recent digital revolution are both challenging to these former beliefs. On one hand, behavioral economics, borrowing insights from psychology, have shown that many deviations from rationality are systematic, biased and are canceled out neither at the aggregated level nor in the long run. On the other hand, the digital revolution, characterized by the development of computers, the democratization of internet and worldwide connection, allowed the production, copying, storage, diffusion and access to information at negligible costs. It sparked an exponential increase in the overall available amount of information that elevates the scarcity of attention to an entirely new scale. For example, the digitalization of information allows firms like Amazon to reference more than 3 billion products in its catalogues. Such a number of products cannot be proposed in any single standard brick and mortar store, nor be printed in any paper catalogue.<sup>3</sup>

### **Attention as a means of exchange**

For a market to exist, it is required that the resource is exchangeable. While being intangible, attention is an exchangeable resource. This characteristic is at the core of advertising and one of the major sources of media revenue. Indeed, media companies (channel TV, newspaper, website) produce informational content that can be desirable for individuals (there is a demand for information). In addition to potential subscription fees, the consumers always need to pay attention in order to acquire and process this informational content. However, the media company is able to capture some part of consumers' attention and to transfer it to other firms through the presence of advertisements. As it will be discussed later, attentional processes are such that even if consumers are not interested in a message, if it is

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<sup>3</sup>A popular argument used to convince readers about the recent explosion of information is to quote Wurman (1989), who professed before the advent of internet that "a weekday edition of The New York Times contains more information than the average person was likely to come across in a lifetime in seventeenth-century England". Nowadays a simple search of "The New York Times" on "Google.com" returns 2.29 billion results. This is more than 38,000 times the number of published issues of the New York Times since its first publication and it would take more than 72 years to review all the results at one result per second, that is more or less the expected remaining life of an English child who just learned to read.

salient enough and placed into a strategic position (either spatially or temporally), the consumers cannot avoid paying (some) attention to it.

While advertisements are far from being new, the digital revolution inflated the phenomenons of attentional transfer and monetization to a broader magnitude, by allowing atomicity in the media market. With internet and social networks, anyone who owns a computer or a smartphone can enter the media market by publishing articles, images, or videos to capture the attention of its audience. This attention, measured by the website traffic, the number of views, or the numbers of “followers” can be converted into money either with advertisement or with product placement. Moreover, the different media may exchange and share part of the captured attention by referencing or collaborating between themselves.

There are potential adverse effects that can be associated with the increase in media competition. Indeed producing valuable informational content is not the only means to achieve the ultimate goal of capturing consumers’ attention. When competition is tough, proposing valuable informational content is not necessarily the most efficient strategy. Valuable information can be more complex and thus more costly in terms of attention if compared to simple messages that consume less attentional resources. Moreover, more attention remain and can be transferred and monetarized through advertisements if the message consume only few resources.<sup>4</sup>

Another sector where attention is traded is scientific production (Franck, 1999; 2002). According to Frank, attention is “a mode of payment, as well as the main input to scientific production”. Indeed, he argues that in academics, the main

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<sup>4</sup>In 2004, Patrick Lelay the CEO of the first French tv channel (TF1) depicted the business of TF1 as “selling to Coca-Cola available human brain time”, by “entertaining and relaxing” TV’s viewers to “prepare them in between two advertiser’s messages”. It followed an important controversy, since TF1’s shows were accused to be mindless not (only) for the purpose of increasing its audience but also to undermine viewers’ judgment (Molfese, 2004; Benoit-Browaays, 2005). To soften his declaration, Patrick Lelay referred directly to the market of attention: “*We sell to our clients a mass audience, a number of individuals that potentially watch an advertising spot. For the advertiser, the airtime is nothing more than consumers contact. It is **human attention**. In particular the one of the famous under-50 housewife, widely responsible of the purchase of food, house cleaning and cosmetic products.*” (in *Télérama*, n 2852, 2004, translated by the author)

It is worth noting that human attention, due to evolutionary reasons, is particularly prone to being captured by stimuli associated with the most basic and natural needs such as surviving, feeding, and reproducing. Therefore, contents related to food, security, violence, or sexuality are particularly good candidates for capturing attention, in particular when attentional competition is tough.

incentive for researchers is not money but the recognition they get for their work. This recognition can be understood as the attention that they receive from their peers. To attract attention, scientists need to publish innovative and “good quality” research. For this purpose, they need to allocate their limited attention to the “best” and most appropriate articles in order to process the information and to reuse it in their own works. As researchers have limited attention and as the number of publications is increasing, researchers cannot pay attention to every paper. Therefore they have to rely on signals about the quality of the paper, such as impact factor and previous reputation of the authors and the journal, to decide if it is worth it to read it. This can explain the increasing importance of evaluation based on bibliometrics for job promotion, as well as the market power of editors owning reputable and famous journals.

## Attention Competition

The value of attention being proportional to its scarcity, the share of the economy devoted to attracting consumer attention is increasing, especially on the internet. US digital advertising revenue has increased by 22% in 2016 to reach \$72.5 billion. Firms are developing digital strategies to maximize the likelihood of appearing at the top of the first page of web search engines.<sup>5</sup>

Since the beginning of the twenty-first century, economists have thus started to integrate limited attention to competitive models. There are two main approaches to model limited attention in economics: introducing constraints on information capacity, or constraints on the number of pieces of information considered.<sup>6</sup>

Constraints on information capacity were popularized by Sims (2003) and are referred to as “rational (in)attention theory”. In this framework, information is quantified through the reduction in uncertainty due to information acquisition, and limited attention is based on the fact that information channels can transfer information with a limited capacity, associated with an attentional cost. Basically

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<sup>5</sup><https://www.iab.com/news/internet-advertising-revenue-first-time-ever-total-digital-ad-spend-hits-landmark-72-5-billion-2016/>, last consultation: September, 10<sup>th</sup>, 2019

<sup>6</sup>see Hefti and Heinke (2015) for a review of competition models with limited attention; see Spiegler (2011) for a review of behavioral industrial organization models.



this implies two-stage decisions where the decision maker first chooses optimally how to process information (i.e. it is assumed that information is coded in its minimal form) and then chooses the action that maximizes her utility. In competition analysis, rational (in)attention can be expressed towards many dimensions of the economic decisions. Consumers may be rationally (in)attentive to prices (Matejka and McKay, 2015) or quality (Martin, 2017). In Armstrong and Chen (2009), some consumers are rationally (in)attentive to the quality of a monopoly product and buy uniquely on the basis of its price. In Li et al. (2016), a monopoly fixes the quantity of a cheap input with adverse effect on consumers (loss in utility) and decides either to disclose information or not about its presence and quantity towards consumers, some portion of them being unaware of the adverse effect.

The second approach has a long history in management sciences (Howard and Sheth, 1969; Hauser and Wernerfelt, 1990) and is based on the empirical observation that consumers can consider only a subset of the available alternatives. In this approach, referred as “consideration set approach”, consumers have a well-defined preference relation that they maximize over a subset of the available alternatives, the “consideration set”. While a consideration set may be an outcome of a rational (in)attention allocation process (Caplin et al., 2018), the literature considers many heuristics leading to the formation of a consideration set (Laroche et al., 2003; Hauser, 2014). The consideration set can be formed as a function of the products’ salience or producers’ marketing effort, as in Eliaz and Spiegler (2011) where firms compete to be part of the consumers’ consideration set and in Hefti (2018) where firms compete but the consumers can endogenously choose the threshold at which a producer is perceived and thus may increase the size of their consideration set. In De Clippel et al. (2014), consumers purchase products on several markets but are limited in the number of markets they can pay attention to. Consumers always know the price of the market’s leader but need to pay attention to the market to know about challenger offers.

One common result of both approaches is that limitation in attention is detrimental for competition: firms can take advantages of limited attention and could charge higher prices and/or lower their quality. These adverse effects are increasing with

the share of inattentive consumers and the attentional costs. Moreover, Hefti (2018) shows that endogenous attention leads to a socially sub-optimal level of attention. Indeed, as firms strategically respond to the global level of attention on the market, more attentive consumers create positive externalities for the other consumers. In Rafai (2014), we show that the more inattentive the market, the more sustainable the collusion. Therefore, policies trying to enhance attention through “*nudges*” (Thaler and Sunstein, 2009), are not only beneficial for the nudges’ recipients, but may also profit all the consumers.

## 0.2 Conceptual and Methodological Framework

“To formulate an allocation problem properly, ways must be found to measure the quantities of the scarce resource (...) A relatively straightforward way of measuring how much scarce resource a message consumes is by noting how much time the recipient spends on it.” — Simon, 1971, p.41

### Attention allocation as a Production Process

Integrating limited attention within economics represents an ongoing theoretical challenge. In addition of being a scarce resource that individuals need to allocate, attention is also a fundamental input used to make informed decisions: attention represents to information what labor is to capital. The attention allocation processes can be understood through production perspective by defining an input and an output. In this thesis the input dimension is referred to as “*allocated attention*” and represents the cognitive resources invested by individuals to process information. The output dimension is referred as “*effective attention*” and represents the amount of information that has been well processed and that is considered at the time of the decision.

While being related, these two notions are not confounded. They require different measures and are involved in different problems. As suggested by Simon (1971), the best measure of “*allocated attention*” is not an information-based measure (for

example the “bit”, since these measures depend on how information has been coded) but the time made by an individual to process the information.<sup>7</sup> For example, McDonald and Cranor (2008) estimate the annual opportunity cost of all US website users reading all the privacy policy rules they face to \$781 billion, based on an average reading speed. If all users were processing all the information contained in these policies rules, the only economic issue would be linked to the cost of this “*allocated attention*” that is supported by the economy. However, regardless of the amount of *allocated attention* paid to these rules, processing information can be too complex and may require specialized legal knowledge. It is thus possible that consumers cannot consider this information when making the decision to give their personal data. This is a problem of *effective attention*, since firms are able to extract consumers’ surplus as a result of their ignorance of the privacy rules.

Both rational (in)attention and consideration set approaches can be interpreted with those two notions. Rational (in)attention suggests explicitly the allocated dimension of attention through the existence of information acquisition costs. The *effective attention* dimension is represented by the posterior beliefs of the decision maker and the uncertainty reduction. Moreover, the rational (in)attention framework imposes an optimality condition on this process: a rational (in)attentive decision maker allocates his attention such that the marginal benefit of *effective attention* equals the marginal cost of *allocated attention*. In contrast, consideration sets are a measure of the *effective attention*: they represent the alternatives that are considered by the decision maker at the time of the choice.<sup>8</sup>

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<sup>7</sup> “Unfortunately, [bit] is not the right unit [to measure an information-processing system’s capacity for attention]. Roughly, the trouble is that the bit capacity of any device (or person) for receiving information depends entirely upon how the information is encoded. Bit capacity is not an invariant, hence is an unsuitable measure of the scarcity of attention. A relatively straightforward way of measuring how much scarce resource a message consumes is by noting how much time the recipient spends on it.” (Simon, 1971, p.41)

Eye-tracking data (e.g. in Devetag et al., 2016) can provide more detailed measure than the response time, as it measures the time spent on different pieces of information or on different components of the same piece of information.

<sup>8</sup>In the last chapter of this thesis we develop how it is possible to build such measure with a revealed (in)attention approach.

## Attention allocation as a Dual Process

To properly integrate attention into economics, we have a lot to learn from psychologists who have studied the concept of attention since the foundation of their discipline. One of the most important debates on attention was to understand to what extent its allocation was voluntary or not. Indeed, psychologists have identified two types of mechanisms that interplay in the allocation of attention: top-down attentional processes and bottom-up attentional processes (Theeuwes, 2010; Bugg and Crump, 2012).

The top-down attentional processes are goal-driven mechanisms involved in the allocation of attention. They consist in the ability to direct attention at will and to allocate more resources towards the stimuli that are the more appropriate for a particular goal. For example, a PhD student who needs to finish writing their dissertation thesis in a hurry will not allocate their attention out office windows to watch flying birds in the sky, regardless how beautifully they can be. These processes are well captured by the rational (in)attention framework that assumes fully goal-driven processes of attention allocation, as the information processed is the one maximizing the expected ex-ante utility of the individual. A rational (in)attentive consumer should not for example be impacted by a change in characteristics that are neither informative nor utility relevant.

In contrast, bottom-up attentional processes are stimuli-driven mechanisms. They correspond to the fact that attention is automatically attracted by “salient” stimuli. The salience of a stimulus represents its capacity to attract individual attention and is a function of its physical (or relational) properties (e.g. the color, brightness, size, location of a visual stimulus or the frequency, signification, of an auditory stimulus). For example, despite being entirely focused on their thesis dissertation, the attention of the PhD student cannot avoid being captured by a colleague who enters the office, calls their name and offers a drink.

The consideration set approach allows for the integration of bottom-up processes by integrating stimuli-driven components to the attentional allocation processes. Many different heuristics that do not require optimality, can be consciously or un-

consciously used in the formation of a consideration set. For example the individuals may consider the products associated with the higher marketing effort in the market (Eliaz and Spiegler, 2011), the three cheapest suppliers in the market (Dulleck et al., 2008), the products that appear on the first page of a web search (Hotchkiss et al., 2004) or the alternatives first available according to an exogenous order (Salant and Rubinstein, 2008). The consideration set approach may allow for goal-driven control as well, either in the choice of the search heuristic or in the number of alternatives considered.

Top-down and bottom-up processes are associated with different neural networks and are often studied separately with different experimental paradigms; it is their interaction that defines the attention allocation process (Corbetta and Shulman, 2002; Katsuki and Constantinidis, 2014). The existence and interaction between these two processes is one of the reasons why it is difficult to propose a general and unified framework that account for all the aspects of attention allocation.<sup>9</sup>

## Methodological Framework

This thesis presents three essays dedicated to improve the study of limited attention within economic theory. For this purpose, we suggest that economists should borrow more methodological and theoretical insights from psychology and cognitive sciences. We apply this approach here.

From a methodological perspective, in each chapter we used experimental tasks based on existing psychological paradigms: a *two-alternatives-forced-choice task* for the first chapter, a *discrimination task* for the second chapter and a *selective attention task* for the last chapter. All are *perceptual decision tasks* that are widely used in cognitive sciences to study attention. These tasks consisted in the presentation of a stimulus (two circles containing dots in the first chapter, a set of white and black balls in the second chapter, monetary rewards among distractors in the last chapter), followed by a decision impacting rewards (“choose the circle containing the most dots”, “report if the stimulus contains more black or more white balls”,

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<sup>9</sup>According to Awh et al. (2012), these two mechanisms fail to explain a growing number of cases in which neither current goals nor physical salience can account for strong selection biases, and other processes such as selection history may also play a role.

“choose a monetary reward”). While being incentivized, these tasks have practical and appealing characteristics: their simplicity makes them easy understandable by participants and their level of abstraction reduces biases (due for example to contextualization or to participants’ social background) and allows the researcher to better identify the underlying cognitive processes. Moreover, the short decision time permits gathering plentiful data in short experimental sessions, increasing statistical power.

All of these tasks allow for possible control and measure of *allocated* and *effective attention*. The *attention allocated* to the perceptual stimuli was experimentally controlled in the first and the third chapters, as the stimuli were disclosed during a brief period of time. In the second chapter, individuals may invest as much *allocated attention* as they want and we measured this quantity through the time needed to classify each stimulus. In the first chapter, using a computational model (Signal Detection Theory) that allows to disentangle decision strategy and perception, we can measure the *effective attention* towards the visual stimuli with a discriminability measure ( $d'$ ) which indicates the amount of evidence extracted by the individuals from the stimuli. Participants’ heterogeneity in  $d'$  can be reduced by using a staircase procedure to fix the difference in the number of dots between the two circles. In the second chapter, we simply measured the *effective attention* through the error rate, as it reflects the uncertainty reduction (from  $\frac{1}{2}$  to 0) and the efficiency of the attention allocation process can be controlled by task difficulty – measured by the difference between the number of black and white balls. In the last chapter, we use an axiomatic model to reveal from choices and measure the *effective attention* as the probability of considering each set of alternatives.

### 0.3 Organization and contributions of the Thesis

The main contribution of the thesis is to enrich existing research and open new avenues in the study of limited attention in economics. The thesis is organized as follows.

## Manipulating Payoff Processing Order in Perceptual Decision-Making

The first chapter is entitled “Manipulating Payoff Processing Order in Perceptual Decision-Making”. This chapter presents an experimental study where we manipulate the order between the presentation of a perceptual stimulus and information about incentives in an incentivized two-alternatives-forced choice task. The stimuli consisted in the short presentation of two circles with a different number of dots inside. Subjects were asymmetrically incentivized to report the circle that contains the most dots (i.e. in each trial, one of the circles was more rewarded in the case of a correct answer) and these incentives were communicated through a payoff matrix.

We analyze the results using Signal Detection Theory (Green and Swets, 1966; Wickens, 2001). Signal Detection Theory (SDT) models an observer who has to report the presence or the absence of a signal from a Gaussian noise, based on an amount of perceptual evidence  $x$ . Thus, there are two possible states of the world: “noise” or “signal + noise” and the error rate in each of these states revealed both the discriminability  $d'$  (the normalized difference between the average amount of evidence  $x$  in the two states) and the decision strategy (i.e. the evidence threshold to report or not the signal). The discriminability  $d'$  can be considered as a measure of *effective attention*, since it quantifies the amount of information that has been processed after the perception of the stimuli. Moreover, in presence of incentives, the SDT offers an “optimal classifier benchmark” as the optimal decision criterion  $\beta^*$  which maximizes the expected payoff does not depend on the perceptual ability of the individual. Therefore, SDT allows to measure the distance between the strategy used by an individual and the optimal one.

We found an order (recency) effect: decisions are closer to optimal when payoff information is presented last and the discriminability is higher when perceptual stimuli are presented last. Moreover, increasing payoff magnitude leads to more optimal decisions. This result can be interpreted as evidence in favor of a division of *allocated attention* between the two types of information, impacted by the order in which information is presented. We also confirmed the robust result that individuals

use “conservative criterion” instead of optimal decisions in this type of experiment: they rely too heavily on the perceptual stimuli rather than the payoff information. We argue that this division of attention is one of the explanations of why individuals fail to make optimal decisions.

This chapter was started and elaborated during a five-month research visiting at Queensland University of Technology (QUT) and is coauthored with Thomas Garcia and Sébastien Massoni. This chapter is also part of Thomas Garcia’s PhD thesis (Garcia, 2018, Chap.3). The current version of the chapter was submitted as a short note and is under revision.

### **Willingness to pay attention for others**

The second chapter is entitled “Willingness to pay attention for others: Do social preferences predict contribution in attentional social dilemmas?” This chapter presents a new experiment, the “dustbin experiment”, that is based on a discrimination task. The task allows measuring both the input (*allocated attention*, through response time) and the output (*effective attention*, through error rate) of the attentional allocation process as well as its interaction with other variables. More precisely, this experiment aims to study the interaction between incentives, social preferences and attention allocation. In the “dustbin experiment”, the participants allocate costly attention to classify stimuli based on the difference between the number of black and white balls they contain. Participants may allocate attention as much they want in the task. There is an opportunity cost since participants have the choice how to allocate a fixed amount of time between surfing the internet and processing information to perform the task.

We implement three types of incentives for performance in a between-subject design: absence of incentive, self-interested incentives and pro-social incentives. Since we showed that attention in the task responds to incentives, the behaviors in the pro-social treatment (where participant performance increases strangers’ payoff) should reveal individual social preferences. We elicited social preferences through a standard Social Value Orientation measure (Murphy et al., 2011) and we highlighted a discrepancy between the two elicitation procedure. This discrepancy suggests that



the expression of social preferences is either dependent on the resource used for their elicitation (money or attention) or prevented by the attention allocation processes.

This chapter was coauthored with Mira Toumi. A previous version of the chapter is part of her PhD Thesis (Toumi, 2018, Chap.4). The current version of the chapter is published in “Revue d’Économie Politique” (Rafai et al., 2018).

## Revealed Preferences under Stochastic Attention

The last chapter, entitled “Revealed Preferences under Stochastic Attention: Characterization, Statistical Tests and Experimental Implementation” proposes an experimental test of Brady and Rehbeck’s Random Conditional Choice Set Rule (RCCSR) (2016). RCCSR is an extension of Manzini and Mariotti’s model (2014) where a decision maker maximizes a complete and deterministic preference relation over a stochastic consideration set. The axiomatization proposed by Brady and Rehbeck (2016) allows an observer to uniquely disentangle, from choice probabilities, a preference relation and a probability measure over the set of possible consideration sets. This latter probability is a measure of “*effective attention*” revealed by choice probabilities.

In this chapter, we identify obstacles to the statistical test of the model’s axioms. We overcome those issues with a slight extension of RCCSR that allows degenerate measure of attention. We provide new characterization and revealed preferences theorems based on weaker axioms. Those axioms are more adapted to statistical testing and we develop specific statistical procedure to test the axioms, reveal preferences and measure the effective attention. We investigate the characteristics of those tests with numerical simulations (level and power).

Furthermore, we conduct an experiment, based on a selective attention task where participants have to choose alternatives among distractors, disclosed over a short period of time. In a within-subject design we vary the number of available alternatives in order to fully characterize and test the model. We find that most of the participants do not violate the model’s axioms. Moreover, as alternatives were monetary rewards, we found that the preferences revealed by our method were coherent with the induced preference relation.

This chapter is single authored but builds upon a project started with Pierre Garrouste and Dino Borie at the earlier stage of my thesis. A paper issued from this project has been published (in French) in “Revue Économique” under the title “Définition Théorique d’une mesure expérimentale de l’attention effective” (Theoretical Definition of an experimental measure of effective attention) (Garrouste et al., 2017).



# CHAPTER 1

## Manipulating Information Order in Perceptual Decision-Making

— *This chapter is coauthored with Thomas Garcia and Sébastien Massoni*

### 1.1 Introduction

Perceptual decision-making is based on two types of information: perceptual evidence and valuation of various options. Combining these two sources of information leads to what we define as the optimal behavior: maximizing expected reward. However rewards are sometimes neglected by individuals: they base their decision mainly on the amount of perceptual evidence favoring one or the other alternative. Behavioral divergences between these two strategies depend on the structure of rewards. If reward differentials between successes and errors are constant across the possible options, maximizing accuracy or maximizing expected reward leads to the same behavior. However, introducing asymmetric rewards implies divergences in behavior. Maximizing accuracy will be no more optimal in terms of rewards and the need to process both sources of information becomes salient.

Since attaining accuracy is not necessarily equivalent to being optimal, individuals achieve optimality at the expense of accuracy. Indeed if some decisions lead to higher additional rewards than others, individuals have to concede lower accuracy for the low rewarded decisions compared to the highly rewarded ones. This ultimately results in a loss of accuracy to the benefit of maximal payoffs. As an example

consider a physician facing a patient exhibiting ambiguous symptoms that may be related to either a serious disease or to a minor one. Correctly identifying that the patient suffers from the serious disease is associated with higher stakes than correctly classifying the disease as minor. To maximize her patient’s welfare, the physician should sometimes diagnose the major disease even if the minor one is more likely.

Facing such asymmetrical decisions, a robust finding is that individuals fail to adjust their behaviors. They use a decision strategy leading to over-accurate but sub-optimal answers (e.g. Pitz and Downing, 1967; Maddox and Bohil, 1998; Bohil and Maddox, 2001; Garcia and Massoni, 2017). This inability to shift their behaviors away from accuracy-maximizing is referred as a conservative criterion placement.<sup>1</sup> Observing such conservatism in decision criteria placement shows that information about rewards is not enough weighted compared to perceptual evidence. In this case, how and when, both sources of information are conveyed may matter.

Experimental procedures are designed such as participants almost always know the payoffs associated with the decisions before observing the stimuli. Within this sequence of information proceeding, we may assume an order effect between rewards information and perceptual evidence. If this is the case, conservative placement and order effect are confounded in the behavioral measurements. The purpose of this study is to investigate if and to what extent information order can explain conservative criterion placement. To tackle this question we use a two-alternative forced choice (2-AFC) visual task with asymmetrical payoffs and rely on a Signal Detection Theory (SDT) for the analytical framework. SDT (Green and Swets, 1966) allows us to understand perceptual decision-making by disentangling the impact of stimuli perception — the discriminability — from the impact of the decision strategy — the decision criterion. It also offers a normative benchmark to predict what will be an optimal behavior i.e. a perfect combination of perceptual evidence and rewards information. We apply this framework to a numerosity task in which participants have to identify the largest magnitude in two visual stimuli. Decisions are associated with different payoff matrices linking the possible actions to asymmetric rewards.

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<sup>1</sup>This chapter focuses on asymmetry induced by payoffs. Conservative criterion placement has also been highlighted for asymmetry in prior probabilities that each answer is correct (base-rate; Ell et al., 2009; Bohil and Wismer, 2015).

We vary payoff matrices and stimuli trial-by-trial while their presentation order is changed between blocks of trials. Note that even if presenting payoff information first seems a natural way to proceed, nothing, from theoretical point of view nor evidence-based from real-life decision, gives a strong support for this assumption. SDT assumes that individuals decide on the basis of the likelihood of correctness and the payoffs of each decision. But it is agnostic on how these two types of information are proceed and on which one comes first. External validity does not impose such a sequence neither. To refine the example previously discussed, the order in which information is delivered reflects two possibilities available to the physician. The consequences of each treatment are established by a cost-benefit analysis based on the patient's health history. The likelihood of each disease is obtained after having examined the patient. The physician will examine the patient and investigate the consequences of each treatment but he is free to choose the order in which she performs these two tasks.

The existence of a potential order effect between rewards and perceptual information may find its roots in two distinct observed experimental facts: an established order effect on discriminability of sequential stimuli and evidence of a dual-decision process facing multiattribute choice alternatives. For 2-AFC discrimination tasks involving comparative judgments between two sequential stimuli, SDT postulates that the decision is taken only with respect to the internal magnitude difference between stimuli representation. While temporal order of stimuli presentation should not affect the decision, there are robust evidence contradicting this postulate. Indeed, when participants are asked to compare the magnitude of a constant standard against the magnitude of a variable comparison, the discrimination sensitivity is higher when the standard precedes rather than follows the comparison (Dyjas and Ulrich, 2014). This so-called Type B effect goes against SDT neutrality of temporal information (Ulrich and Vorberg, 2009) and has been observed across multiple stimuli attributes and sensory modalities (Ellinghaus et al., 2018). Our study will investigate whether a similar order effect can be observed beyond pure perceptual comparisons in a situation where reward information interferes with perceptual evidence. Another approach to investigate a potential payoff order effect is to be

found in a dynamic multiattribute choice approach. Breaking with the time-order neutrality, some multiattribute models show that the sources of information are processed separately in a dynamic way. Applying this method to the integration of payoffs and perceptual information, Diederich and Oswald (2014) propose a sequential sampling model that produces different predictions depending on the order in which attributes are considered. In a design related to ours, Diederich (2016) shows that this modelization provides the best fit to observed behaviors. Even if this approach does not rely on SDT but on its dynamic extension, it provides experimental evidence in favor of a dynamic integration of payoffs and perceptual evidence in which the order of the processing affects the decision.

The current study assesses a payoff order effect on perceptual decision by inverting the order in which both types of information are given to participants. By comparing discriminability and decision criteria placement for the different information orders, it contributes to understand how payoffs and perceptual evidence are processed over time. Our results show that individuals shift their decision criterion in the direction of optimality when payoff information is given last and we observe a trend suggesting a better discriminability when stimuli are displayed last. Although these effects confirm the investigated order effect, their impacts are insufficient to move away participants from the conservative criterion placement. Overall our study highlights that the order of processing payoffs and perceptual information matters but human tendency to maximize accuracy over rewards is too strong for being vanished by such manipulation.

## 1.2 Method

### Stimuli

We used 2-AFC stimuli previously used in Massoni et al. (2014) and Fleming et al. (2016). This consists of a 700ms long display of dots in two circles on either sides of the screen, one with 50 dots and the other with  $c$  more dots (Figure 1.2.1-A). Participants evaluated which circle contained the most dots.  $c$  was calibrated at a participant level to control for heterogeneity in visual abilities and kept constant

during the experiment.<sup>2</sup>

### Task and Procedure

*Participants* — 58 students from Queensland University of Technology were enrolled in four 1-hour sessions (age:  $M = 24.89$ ,  $SD = 5.9$ ; 45% were females). They were paid on average AUD 22.8 ( $SD = 4.83$ ) based on the number of points earned during the experiment.

*Experiment phase* — The experiment was a within-subject design consisting of series of individual perceptual decisions organized in 4 blocks of 72 trials. To make a decision, participants received two different kinds of information: on stimuli and on payoffs. Payoffs were communicated by a payoff matrix providing a matching rule between the four possible decision outcomes and the number of points earned in each case. By combining both kinds of information, participants answered either “Left” or “Right” for the chosen circle. To help participants in their decision-making, we gave them feedback on accuracy and results for points after each trial as well as additional aggregated feedback on accuracy and points every 12 trials.<sup>3</sup> 28 randomly selected decisions were paid (10 *points* = AUD 1). In the “Stimuli-First” treatment, participants observed stimuli before the payoff matrix. In the “Payoff-First” treatment, the order was reversed (Figure 1.2.1-C&D). Treatments were varied between blocks (two blocks per treatment).<sup>4</sup>

*Payoff Matrices* — Payoff Matrices are  $2 \times 2$  matrices that indicate, for each trial, the monetary reward (in points) associated with correct and incorrect decisions. Stimuli need to be varied trial-by-trial, we thus vary also payoff matrices trial-by-trial to limit artificial differences between stimuli information and information on payoffs. To make this manipulation relevant, we used a set of  $3 \times 2$  payoff matrices that were invariant regarding the optimal decision criterion but varied regarding the payoff variance (difference between correct and incorrect payoffs).

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<sup>2</sup>Additional information about stimuli and their calibration are in appendix 1.A.

<sup>3</sup>Feedback on aggregated accuracy indicate the mean accuracy and feedback on aggregated points indicates how close participants were from perfect performance. Additional information are in appendix 1.A

<sup>4</sup>We randomly generated the same sequence of stimuli and payoff matrices for all the participants. The target circle’s frequency, payoff matrices and the number of paid decisions were controlled such that they were equal in each block. The order of treatments was randomly generated.



This consists of matrices with “Low” ( $M_{low}$ ), “Intermediate” ( $M_{int}$ ) and “High” ( $M_{high}$ ) payoff variances with more points earned for successes either for “Left” or “Right” (see Table 1.2.1 and Figure 1.2.1-B). Values have been chosen to respect these theoretical constraints while being visually as close as possible and rewarding similarly perfect performances.<sup>5</sup> We use negative and positive values as previous studies have obtained that pure-gain, pure-loss and mixed gain/loss frameworks do not affect decision criterion placement (Maddox et al., 2003; Garcia and Massoni, 2017).

Payoff Matrix	Success Left	Success Right	Error Left	Error Right
Low ( $M_{low}$ )	12	10	9	9
	10	12	9	9
Intermediate ( $M_{int}$ )	17	5	-1	-1
	5	17	-1	-1
High ( $M_{high}$ )	21	1	-9	-9
	1	21	-9	-9

Table 1.2.1: Set of payoff matrices.

### Data analysis

Based on our 2-AFC stimuli and the three different payoff variances, we specify our model as a Gaussian equal-variance SDT model with signal being “the highly rewarded circle contains more dots”. Reporting the “signal” thus consists in choosing the highly rewarded alternative.<sup>6</sup> The optimal decision criterion is  $\beta^* = \frac{1}{3}$  for all matrices.<sup>7</sup>

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<sup>5</sup> $M_{high}$  is obtained from  $M_{low}$  by reversing the two digits of the most rewarded success (12 to 21), suppressing the 0 of the less rewarded success (10 to 1) and adding minus to errors (9 to -9).  $M_{int}$  preserves a similar pattern with intermediate values. All matrices give the same expected payoff for perfect performance (11 points) but vary regarding the expected payoff for random guesses (1 point for  $M_{high}$ , 5 points for  $M_{int}$  and 10 points for  $M_{low}$ ).

<sup>6</sup>The equivalence between the 2-AFC specification induced by the perceptive task and our framework is detailed in Appendix 1.A

<sup>7</sup>The optimal decision criterion is given by the ratio between the additional payoff of a success for the “noise” alternative and the additional payoff of a success for the “signal” alternative (Wickens, 2001, p.36):

$$\beta_{high}^* = \frac{1 - (-9)}{21 - (-9)} = \beta_{int}^* = \frac{5 - (-1)}{17 - (-1)} = \beta_{low}^* = \frac{10 - 9}{12 - 9} = \beta^* = \frac{1}{3}$$

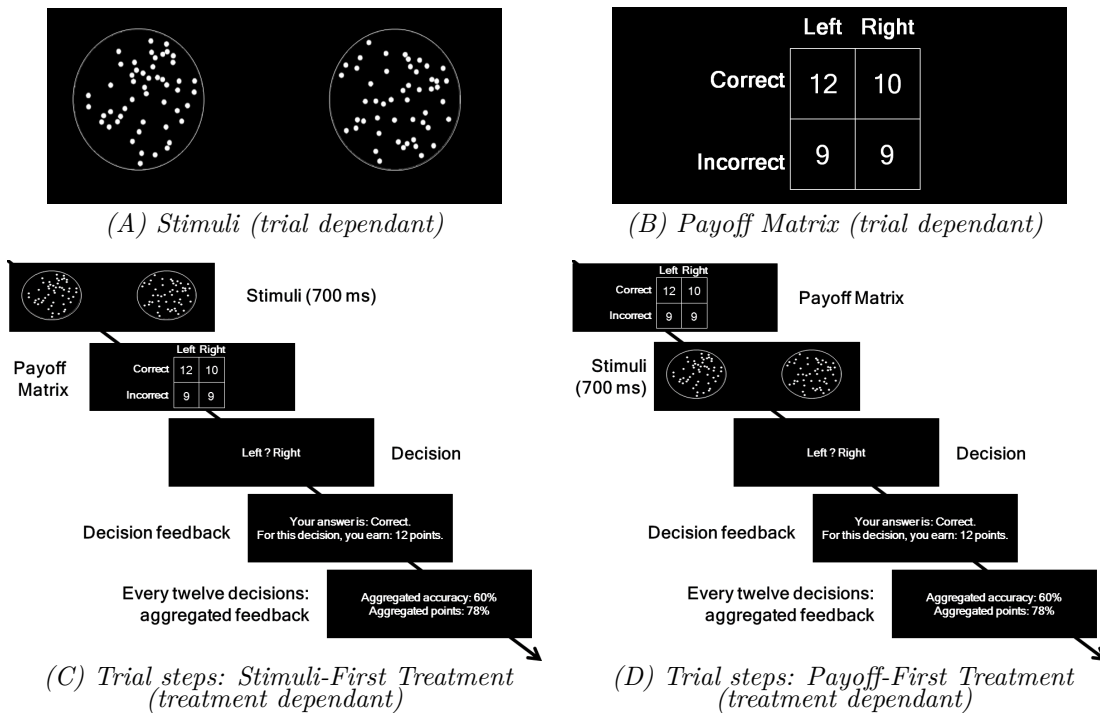


Figure 1.2.1: Experimental design.

**Note:** (A) Example of stimuli. Participants evaluated which circle contained the most dots. (B) Example of a Payoff Matrix. (C) Timeline of a trial for the Stimuli-First treatment. Facing a fixation cross, participants initiated the trial. Stimuli appeared for 700 ms before being replaced by the fixation cross. The payoff matrix was displayed with no time constraints. Participants made their decisions (right or left). Feedback on the accuracy of the answer and the number of points earned was given. Every twelve trials, aggregated feedback on accuracy and points over these trials was provided. (D) Timeline of a trial for the Payoff-First Treatment. The display order of the payoff matrix and stimuli is reversed compared to the previous timeline.

Gaussian equal-variance SDT provides a computational model to estimate both participant discriminability ( $d'$ ) and decision criterion ( $\beta$ ). To evaluate treatment effects, we estimate one couple ( $\hat{d}', \hat{\beta}$ ) for each order of information at a participant level (Estimation (1)). To assess the impact of each payoff matrix on the decision strategy, we estimated one decision criterion for each combination of matrices and information orders at a participant level. We estimated a single discriminability for each order of information. Estimation (2) thus consists in estimating a vector ( $\hat{d}', \hat{\beta}_{low}, \hat{\beta}_{int}, \hat{\beta}_{high}$ ) for each order of information at a participant level. We compute estimators using maximum likelihood because the analytic form of the estimators is unknown for Estimation (2) and equivalent to the analytic form for Estimation (1) (cf. Ackermann and Landy (2015) for a similar estimation procedure).<sup>8</sup>

Paired t-tests are used to compare two balanced samples. Matrix effects across our 6 samples (3 matrices  $\times$  2 treatments) are compared using repeated measures analysis of variance (rANOVA) followed by Tukey's range tests for multiple comparisons of means. We report Cohen's  $d_z$  and partial Eta-squared ( $\eta_p^2$ ) as effect size measures.

### 1.3 Results

Estimated discriminabilities and decision criteria for each treatment based on Estimation (1) are presented in Figure 1.3.1. Estimated decision criteria for each matrix in each treatment based on Estimation (2) are presented in Figure 1.3.2.

We observe a mean decision criterion of 0.93 ( $SD = 0.202$ ) for the Payoff-First treatment and 0.88 ( $SD = 0.158$ ) for the Stimuli-First treatment. Both decision criteria are lower than the accuracy maximizing decision criterion, ( $\beta^a = 1$ ), [ $t(57) = -2.535, p = 0.014$ ] for the Payoff-First treatment and [ $t(57) = -5.847, p < 0.001$ ] for the Stimuli-First treatment; and higher than the optimal decision criterion ( $\beta^* = \frac{1}{3}$ ), [ $t(57) = 22.58, p < 0.001$ ] for PF and [ $t(57) = 26.31, p < 0.001$ ] for Stimuli-First

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<sup>8</sup>Estimation methods, comparison between both estimations types and reasons for assuming constant discriminability across payoff contrasts are detailed in Appendices 1.B and 1.D. The equivalent analytic forms of Estimation (1) are (Wickens, 2001, p.24 & p.30):  $\hat{d}' = Z(HR) - Z(FAR)$  and  $\hat{\beta} = \exp(\frac{1}{2}[Z(FAR)^2 - Z(HR)^2])$  with  $Z$  the inverse Gaussian distribution,  $HR$  the hit rate and  $FAR$  the false alarm rate.

treatment. We conclude that participants use conservative decision criteria.

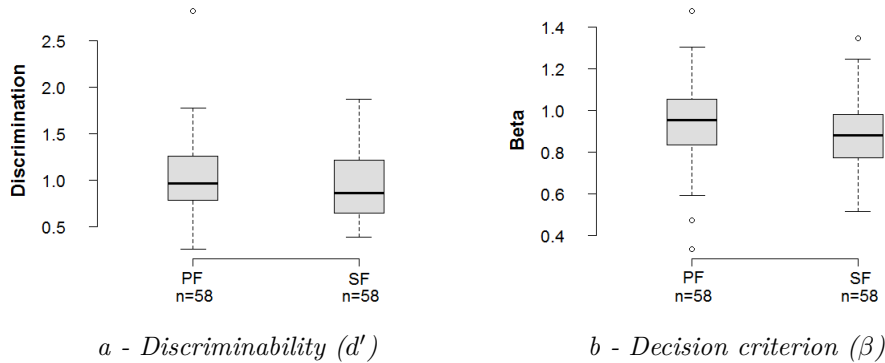


Figure 1.3.1: Tukey box-and-whisker plot - Discriminability and decision criteria for Payoff-First treatment (PF) and Stimuli-First treatment (SF).

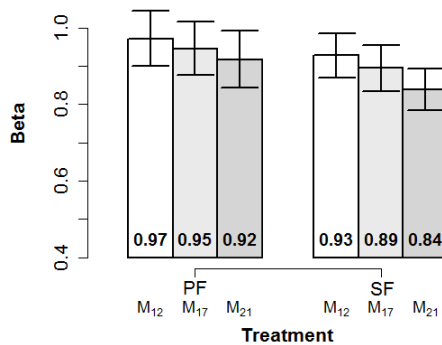


Figure 1.3.2: Estimated decision criteria for each matrix by treatment.

**Note:**  $M_{low}$ ,  $M_{int}$  and  $M_{high}$  respectively correspond to “Low”, “Intermediate” and “High” payoff variances. PF and SF respectively correspond to Payoff-First treatment and Stimuli-First treatment. Error bars correspond to 95% confidence intervals. Bold figures inside bars are mean values.

Participants’ decision criteria are lower in the Stimuli-First treatment ( $M = 0.88$ ,  $SD = 0.158$ ) than in the Payoff-First treatment ( $M = 0.93$ ,  $SD = 0.202$ ), [ $t(57) = 2.280$ ,  $p = 0.026$ ,  $d_z = 0.299$ ]. To assess the robustness of this finding, we conduct a 2 (information order: “Stimuli First” and “Payoff First”)  $\times$  3 (payoff variance: “Low”, “Intermediate” and “High”) repeated measures analysis of variance (rANOVA) on the decision criterion ( $\beta$ ). We find a statistically significant main effect of the information order [ $F(1, 57) = 5.725$ ,  $p = 0.020$ ,  $\eta_p^2 = 0.091$ ]. We thus conclude that participants’ decision strategies shift in the direction of optimality in the Stimuli-First treatment compared to the Payoff-First treatment. Descriptively,

the gap between optimal decision criterion and actual decision criterion is reduced by 9.0% in the Stimuli-First treatment.

Based on the rANOVA, we also find a statistically significant main effect of the payoff variance [ $F(2, 114) = 3.144, p = 0.047, \eta_p^2 = 0.052$ ]. The presence of the "High" payoff variance ( $M_{high}$ ) leads participants to adopt a lower decision criterion compared to the "Low" payoff variance ( $M_{low}$ ) resulting in a shift in the direction of optimality [ $q = -2.496, p = 0.034, d_z = 0.328$ ]. Descriptively, the gap between optimal decision criterion and actual decision criterion is reduced by 11.7% for  $M_{high}$  compared to  $M_{low}$ . The decision criterion associated with the intermediate payoff variance ( $M_{int}$ ) is included between decision criteria of the two other payoff variances. However, it is not statistically different from either of them [ $M_{low} : q = -1.044, p = 0.549, d_z = 0.137; M_{high} : q = -1.453, p = 0.314, d_z = 0.191$ ]. We conclude that participants' decision strategies shift in the direction of optimality in presence of higher payoff variances. Finally, we do not find a statistically significant effect of interaction between payoff variance and information order on strategies [ $F(2, 114) = 0.37, p = 0.691, \eta_p^2 = 0.006$ ].

Analyzing discriminabilities, we find a trend suggesting that participants exhibit greater discriminability in the Payoff-First treatment ( $M = 1.02, SD = 0.415$ ) compared to the Stimuli-First ( $M = 0.95, SD = 0.391$ ) treatment [ $t(57) = 1.854, p = 0.069, d_z = 0.243$ ].

## 1.4 Discussion

The purpose of this study was to investigate a potential confound between the conservative criteria placement and a payoff order effect. Our results show that this order effect exists: displaying payoffs last shifts decision criterion in the direction of optimality while receiving visual stimuli last involves a trend suggesting a greater discriminability. But the magnitudes of the changes in behaviors are not large enough to explain the conservative criterion placement. We thus confirm the robustness of this bias to a payoff order effect and stay in line with previous studies showing the difficulty to change behaviors toward optimality (see Rahnev and Denison, 2018, for

a review).

Even if we cannot shift the decisions in a large extent, we still observe an effect of processing order. Its existence informs on how payoffs and perceptual information are combined. In line with the evidence from the dynamic integration of multiattribute component of a choice, our results are in favor of a two-stage model in which both attributes (payoffs and perceptual evidence) are processed separately and sequentially over time. The differences we observed in terms of decision criteria (and to a lesser extent in discriminability) according to the order the information is displayed fit in the recent evidence supporting such dual-stage model to combine perceptual decision-making and payoffs (Diederich and Busemeyer, 2006; Diederich, 2008; 2016; Rorie et al., 2010; Gao et al., 2011). In particular, Diederich (2016) studies how providing information about payoffs before, after or at the same time than the stimuli affects choice probability and reaction time. While our design is almost identical to hers, both approaches are different. She performs sequential sampling models comparison and provides evidence for a dual-stage model while our main motivation is to disentangle the conservative criterion placement in static SDT from an information order effect. This divergence of analytical framework, allows us to conclude on different points of interest on the basis of a similar design.

Our results extend previous research on the Type B order effect to multiattribute choice. Type B order effect for comparison of consecutive stimuli is formalized based on the weighted difference model (Dyjas et al., 2014). According to this model, participants weight the sensory input from the first and second stimulus differently. Applying this model to experimental evidence (e.g. Nachmias, 2006; Ulrich, 2010; Bruno et al., 2012) typically leads to the conclusion that the weight associated with the second stimulus is larger than the weight associated with the first stimulus (Dyjas et al., 2014). In our framework, the decision is not based on the difference between two stimuli but on the difference between a likelihood ratio (extracted from the stimuli) and a decision criterion (extracted from the payoff matrix). Weighted difference model can be applied by associating a weight to both the likelihood ratio and the decision criterion. Our results could thus be explained similarly to Type B effects by a larger weight on the first of two decision components.

As detailed in Dyjas et al. (2014), this difference in weighting can be interpreted as an higher attention allocation to the second component of the decision. This attentional switch described by the dual-stage model of multiattribute choice would thus result in an unbalanced attention within attributes. This unbalanced attention may be explained by an optimal reaction to a memory decay. As suggested by Hellström and Rammsayer (2004), the quality of the information collected first may be altered due to forgetting. Adapting respective weights would thus correspond to giving more importance to the best quality information.

We observe that increasing the payoff variance leads to a shift of behaviors toward a less conservative criterion placement. This effect is in line with previous studies investigating how payoff matrix multiplication affects criterion learning (Bohil and Maddox, 2003; Maddox et al., 2003). These studies increase payoff variance by multiplying all payoffs by a constant factor and find that decision criterion learning is improved for high variance payoff matrices. The fact that we observe the same effect, while in our design decision criterion learning effects are less salient, highlights the strength of the variance effect.<sup>9</sup> Indeed, criteria adjustment is more difficult when the decision environment is changed trial-by-trial than block-by-block thus our experimental design gives a more conservative evaluation of the variance effect (Brown and Steyvers, 2005; Starns and Olchowski, 2015). Bohil and Maddox (2003); Maddox et al. (2003) explain this improved criterion learning by a change in the slope of the objective reward function. The reduction of the learning component suggests that allocation of attention mechanisms may also play a role in the process. A higher payoff variance may direct the attention toward the payoff matrix which would positively shift the decision criterion. While we assume a direct effect of attention on criterion placement, we cannot exclude the existence of an indirect effect on discriminability. The attention allocated toward payoffs may take place at the expense of the perceptual information processing. This depletion of attention will result in a lower discriminability for higher payoff variances. We do not address this point for statistical reasons in the present study (see appendix 1.D for details).

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<sup>9</sup>In our experiment participants made a smaller number of decision (288 against 1920 to 6480) and payoffs were varied at a decision level rather than at a block level (60 to 120 decisions).

The order in which payoffs are displayed in a perceptual task fails to explain the observed conservative criterion placement. Yet the sequence of information is not neutral on behaviors. We highlight a new order effect between perceptual information and payoffs. It opens questions on how payoff information and stimuli evidence are dynamically integrated to set the decision strategy. In particular, understanding how attention is allocated between both information sources might provide a deeper knowledge of the perceptual decision process.





# Appendices — Manipulating Information Order in Perceptual Decision-Making

## 1.A Procedure

This section details the stimuli characteristics and the feedback system. The study was programmed using MATLAB with the Psychophysics Toolbox version 3 (Brainard, 1997) and was run on 1920×1080 resolution computers. Participants were enrolled using ORSEE (Greiner, 2015).

### 1.A.1 Stimuli

*Calibration* — Stimuli difficulty was given by the difference in number of dots between the two circles (hereafter referred as  $c$ ). Calibration for task difficulty was done using a psychophysic staircase at the participant level in order to control for individual heterogeneity regarding vision acuity (Levitt, 1971). This one-up two-down staircase consists of removing one dot after two consecutive correct answers and adding one dot after one failure. The calibration ended when participants had achieved 35 reversals in the staircase (a reversal means a change in the  $c$ 's value). The number of dots difference  $c$  used in the experiment was computed as the mean number of dots across the last two staircase reversals. In this non-incentivized calibration period, participants received only the stimuli information and had to determine which circle contains the most dots. Feedback on decision accuracy was provided after each decision. The targeted accuracy rate was 71%. This level was chosen for participants to outperform random guesses (relevance of stimuli) and to avoid being too close to perfect discriminability (relevance of the payoff matrix).

*Stimuli generation* — To ensure the same level of calibration, we presented the exact same set of stimuli in each block to participants with the same difficulty level. Stimuli were randomly generated prior to the experiment under constraints concerning dot spacing. All dots were of the same size (diameter  $0.4^\circ$ ) and the average distance between dots was kept constant. They appeared in random positions inside two outlined circles (diameter  $5.1^\circ$ ). Before and after dot appearance, fixation crosses were displayed in the center of both circles at eccentricities of  $\pm 8.9^\circ$ .

### 1.A.2 Feedback

The feedback system was composed of trial feedback and aggregated feedback. We aimed not to influence participants to consider more stimuli or payoff matrices. Thus, we gave them feedback on stimuli — if their decisions were correct or incorrect — and feedback on the payoff matrix — the number of points earned for the decision. In addition to this trial-by-trial feedback, every 12 decisions participants received two types of aggregated feedback. First, we showed their accuracy rate for the last 12 decisions. Second, we provided an aggregated point feedback indicating how close they were from perfect performance. Formally the aggregated point feedback (apf) was given by the following formula:

$$apf = \frac{x - y}{z - y}$$

With  $x$  the actual number of points earned,  $y$  the number of points earned if all answers had been incorrect,  $z$  the number of points earned if all answers had been correct. We provide participants with this kind of aggregated feedback i) to induce a match between subjective beliefs and actual performances, ii) to allow non-decision periods within blocks.

### 1.A.3 Model specification: Gaussian equal-variance SDT

This section presents how we specify our model as a Gaussian equal-variance SDT. We start by modelling the perceptual part of the decision based on our 2AFC stimuli. Then, we use the payoffs provided by the payoff matrix to distinguish the four

different outcomes of SDT.

The perceptual task was a 2AFC task in which participants faced two circles with dots inside them and aimed to identify the circle with the most dots. The SDT theory specifies that evidence received from each circle follows a normal distribution. For a larger number of dots, the amount of evidence is in average larger. The variance parameters represent a participant's visual capacity to evaluate stimuli. The difference in dot number being relatively small, we make the usual assumption that both variance parameters are equal.

We note  $X_s$  the perception of the circle with the most dots and  $X_n$  the perception of the circle with the least dots:

$$X_n \sim \mathcal{N}(\mu_n, \sigma^2) \text{ and } X_s \sim \mathcal{N}(\mu_s, \sigma^2)$$

with  $\mu_n < \mu_s$ .

In addition to containing relatively greater or fewer dots, each answer is either associated with high or low payoffs when correct. To include this dimension, we note  $X_H$  the perception of the circle associated with higher rewards and  $X_L$  the perception of the circle associated with lower rewards. We note  $e$  the perceived difference between these two circles:

$$e = X_H - X_L = \begin{cases} X_s - X_n \sim \mathcal{N}(\mu_s - \mu_n, 2 \times \sigma^2) & \text{if the most reward. circ. has more dots} \\ X_n - X_s \sim \mathcal{N}(\mu_n - \mu_s, 2 \times \sigma^2) & \text{if the most reward. circ. has less dots} \end{cases}$$

The model can be specified as a Gaussian equal-variance Signal detection task by defining “signal” the event “the highly rewarded circle contains more dots” and “noise” the opposite event “the highly rewarded circle contains less dots”. In fact,  $e$  can be written as:

$$e \sim \begin{cases} \mathcal{N}(0, 1) & \text{if “noise”} \\ \mathcal{N}(d', 1) & \text{if “signal”} \end{cases}$$

$$\text{with } d' = \frac{2 \times (\mu_s - \mu_n)}{\sqrt{2} \times \sigma} \text{ and } \epsilon = \frac{Y + (\mu_s - \mu_n)}{\sqrt{2} \times \sigma}.$$

It is thus equivalent to the Gaussian equal variance SDT in its general specification. Under this framework, the four different outcomes are defined as:

- Hit: Reporting the highly rewarded circle when it contains more dots.
- Correct rejection: Reporting the lowly rewarded circle when it contains more dots.
- Miss: Reporting the lowly rewarded circle when it contains less dots.
- False alarm: Reporting the highly rewarded circle when it contains less dots.

Based on  $e$ , information extracted from stimuli is given by the likelihood function:

$$\mathcal{L}(e) = \frac{\mathbb{P}(e|S)}{\mathbb{P}(e|N)}$$

The decision strategy  $\beta$  is summarized by the minimal value of the likelihood function required to answer signal (reporting the circle associated with higher rewards). This value has to be interpreted as follows: an individual with a decision strategy  $\beta$  reports the circle associated with the higher rewards when it is (at least)  $\beta$  times more likely than the other circle to contain more dots.

The optimal decision strategy is defined by reporting the presence of the signal if and only if:

$$\mathcal{L}(e) > \frac{\Pi(\text{correct rejection}) - \Pi(\text{false alarm})}{\Pi(\text{hit}) - \Pi(\text{miss})} \times \frac{\mathbb{P}(N)}{\mathbb{P}(S)} \equiv \beta^*$$

Where  $\Pi$  refers to the payoff function. In our study participants are informed that we implement equal base rates i.e. signal and noise are equally likely ( $\mathbb{P}(S) = \mathbb{P}(N) = \frac{1}{2}$ ).

To conclude, we address the question of potential position bias. Position biases are biases to choose either “Left” or “Right”. Our modelization is orthogonal to spatial location as it takes as its reference the circle with the most dots and not its position. However, this does not affect our results as the experiment is designed

to cancel out these potential biases as target circles and highly rewarded circles are balanced on both sides.

## 1.B SDT estimation

This section details how we compute the descriptive statistics and how we estimate SDT parameters.

*Descriptive statistics* - The hit rate ( $HR$ ) is equal to the number of hits ( $n_H$ ) divided by the number of signal trials ( $n_s$ ). Likewise, false alarm rate ( $FAR$ ) is equal to the number of false alarms ( $n_{FA}$ ) divided by the number of noise trials ( $n_n$ )

$$HR = \frac{n_H}{n_s} \text{ and } FAR = \frac{n_{FA}}{n_n}$$

*Estimations* - To estimate a couple  $(\hat{d}', \hat{\beta})$  for each information order:

- (i) We express the probabilities of hit ( $P_H$ ) and false alarm ( $P_{FA}$ ) as functions of  $(d', \beta)$ :

$$P_H(d', \beta) = 1 - \phi\left(\frac{\log(\beta)}{d'} - d'/2\right) \quad (1.1)$$

$$P_{FA}(d', \beta) = 1 - \phi\left(\frac{\log(\beta)}{d'} + d'/2\right) \quad (1.2)$$

Where  $\phi$  refers to the cumulative distribution function of the standard normal distribution.

- (ii) We compute the probability that, for a couple  $(P_H, P_{FA})$ , we obtain the observed number of hits and false alarms using binomial distributions.

$$f(n_H; n_s; P_H) = \binom{n_s}{n_H} (P_H)^{n_H} (1 - P_H)^{n_s - n_H} \quad (1.3)$$

$$f(n_{FA}; n_n; P_{FA}) = \binom{n_n}{n_{FA}} (P_{FA})^{n_{FA}} (1 - P_{FA})^{n_n - n_{FA}} \quad (1.4)$$

- (iii) We compute the couple  $(d', \beta)$  that maximizes the joint probability of  $f(n_H; n_s; P_H)$

and  $f(n_{FA}; n_n; P_{FA})$  by maximum likelihood:

$$(\hat{d}', \hat{\beta}) = \operatorname{argmax} (\log(f(n_H; n_s; P_H(d', \beta))) + \log(f(n_{FA}; n_n; P_{FA}(d', \beta)))) \quad (1.5)$$

We use a similar approach to compute one discriminability and three matrix-specific decision criteria  $(\hat{d}', \hat{\beta}_{low}, \hat{\beta}_{int}, \hat{\beta}_{high})$  for each order of information:

(i) We compute three couples  $(P_H, P_{FA})$ :

$$P_H(d', \beta_{low}), P_{FA}(d', \beta_{low}), P_H(d', \beta_{int}), P_{FA}(d', \beta_{int}), P_H(d', \beta_{high}) \text{ and } P_{FA}(d', \beta_{high}).$$

(ii) We compute the associated binomial distributions based on the number of hits and false alarms per matrix.

(iii) We compute the vector  $(\hat{d}', \hat{\beta}_{low}, \hat{\beta}_{int}, \hat{\beta}_{high})$  that maximizes the joint probability of the binomial distributions by maximum likelihood.

## 1.C Robustness test of decision criterion estimation

In this chapter, we present an estimation of one couple  $(\hat{d}', \hat{\beta})$  for each information order at a participant level (Estimation (1)) and another estimation of one discriminability and three matrix-specific decision criteria  $(\hat{d}', \hat{\beta}_{low}, \hat{\beta}_{int}, \hat{\beta}_{high})$  for each information order at a participant level (Estimation (2)). Estimation (1) leads to slightly higher  $\hat{d}'$  ( $d'$  were in average 1.39% higher when estimated with (2) than with (1) [ $t(115) = -11.672, p < 0.0001$ ]. The maximum difference was 6.08%.) compared to Estimation (2). Another method consists in a two-step estimation. Estimate  $d'$  as in Estimation (1) then estimate the three matrix-specific decision criteria using  $d'$  previously estimated.

The choice could impact the existence of a payoff contrast effect. This section produces results obtained with this second methodology and concludes that this result is robust to the methodology change.

The repeated measures ANOVA rejects the hypothesis that matrix-specific mean decision strategies are equal [ $F(2, 114) = 3.168, p = 0.0458, \eta_p^2 = 0.05265$ ]. The Tukey's test reveals that the only significant difference is between matrix  $M_{high}$  and  $M_{low}$  [ $q = -2.505, p = 0.0329, d_z = 0.329$ ].

## 1.D Constant discriminability accross payoff variance

In our data treatment, we assume that the discriminability is independent from payoff variances while decision criteria are allowed to vary within payoff variances. This specification has been selected based on general considerations and data analysis.

As detailed in the main text of the chapter, the effect of payoff variances on decision criteria is analyzed through the scope of attention allocation: individuals pay more attention to higher payoff variances. There is thus a direct effect of attention on decision criteria. This increasing attention toward payoff matrices may take place at the expense of the processing of stimuli information. This indirect effect would lead to a decreased discriminability for higher payoff variances. Choosing not to address this indirect effect provides a better statistical power to test the direct effect. In fact, estimating six parameters at the participant level out of 288 trials (144 signal trials and 144 noise trials) would lead to unreliable estimations. For example, 5 different participants have a higher number of false alarms than their number of hits for one of the combinations of [information order  $\times$  payoff variances] which implies an unreasonable negative discriminability.

The hypothesis of independence is also supported by our data. We compute discriminabilities for each combination of information order (Stimuli First and Payoff First) and payoff variance (Low, Intermediate, High) by maximum likelihood. We conduct a 3 (payoff variances: "Low", "Int", "High") repeated measures analysis of variance (rANOVA) on the discriminability. We find no statistically significant main effect of the payoff variance [ $F(2, 114) = 0.454, p = 0.636, \eta_p^2 = 0.008$ ].

Finally, our specification follows existing studies on the influence of payoff variances on decisions based on SDT (Maddox et al., 2003; Bohil and Maddox, 2003).



Indeed, these studies also compute a discriminability for each task resulting in discriminabilities independent from payoff information.

To conclude, we believe that the estimation implemented is the best suited for our experiment and enhances comparability with previous studies. However new studies may explore further the impact of different payoff variances on SDT parameters as well as the underlying attentional process by allowing discriminability to vary within payoff variances.

## 1.E Descriptive Statistics

Table 1.E.1 describes participant hit rates and false alarm rates for each payoff matrix in each treatment. It also gives these rates at a treatment level (referred as “*All*”).

The order effect presented in the main text is driven mainly by the false alarm rate. In fact, the hit rate of both treatments is not different [ $t(57) = -0.8967$ ,  $p = 0.374$ ,  $d_z = 0.118$ ] while the false alarm rate is higher in the Stimuli-First treatment compared to the Payoff-First treatment [ $t(57) = -2.886$ ,  $p = < 0.0055$ ,  $d_z = 0.379$ ].

The matrix effect is salient for both hit rates [ $F(2, 114) = 4.758$ ,  $p = 0.0104$ ,  $\eta_p^2 = 0.077$ ] and false alarm rates [ $F(2, 114) = 5.694$ ,  $p = 0.0044$ ,  $\eta_p^2 = 0.091$ ].  $M_{high}$  is the matrix inducing this effect. With  $M_{high}$  participants achieve more hits than with  $M_{int}$  [ $q = 3.076$ ,  $p = 0.0059$ ,  $d_z = 0.404$ ] and more false alarms than with  $M_{low}$  [ $q = 3.277$ ,  $p = 0.0032$ ,  $d_z = 0.430$ ]. Additionally, we also observe slightly more false alarms with  $M_{int}$  than with  $M_{low}$  [ $q = 2.337$ ,  $p = 0.051$ ,  $d_z = 0.307$ ].

Statistic n=58	Treatment	Payoff matrix	$M$ (%)	$SD$	Min (%)	Max (%)
Hit Rate	$PF$	$M_{low}$	72.56	9.17	50.00	91.67
		$M_{int}$	71.05	14.32	37.50	100.00
		$M_{high}$	74.57	10.37	50.00	95.83
		$All$	72.73	8.49	52.78	94.44
	$SF$	$M_{low}$	74.21	9.32	54.17	95.83
		$M_{int}$	70.69	15.69	29.17	100.00
		$M_{high}$	76.08	9.12	54.17	95.83
		$All$	73.66	8.62	52.78	95.83
False Alarm Rate	$PF$	$M_{low}$	33.91	12.74	8.33	62.50
		$M_{int}$	35.56	15.06	4.17	79.17
		$M_{high}$	36.85	16.01	8.33	79.17
		$All$	35.44	13.00	11.11	70.83
	$SF$	$M_{low}$	35.78	14.24	4.17	87.50
		$M_{int}$	40.01	16.18	12.50	91.67
		$M_{high}$	41.09	15.97	8.33	91.67
		$All$	38.96	13.56	15.28	90.28

Table 1.E.1: Mean performances in terms of hit rate and false alarm rate.

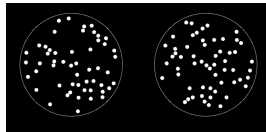
**Note:** Treatments are Payoff-First ( $PF$ ) and Stimuli-First ( $SF$ ).  $M_{low}$ ,  $M_{int}$  and  $M_{high}$  respectively correspond to “Low”, “Intermediate” and “High” payoff variances.

## 1.F Instruction

### Task

In this experiment, you will play a total of 4 blocks of 72 trials. In each trial, you will receive two pieces of information.

- Stimuli: two circles containing different numbers of dots. Each circle has a 50% chance of containing more dots than the other :



- Payoff Table: number of points earned according to the outcome of your decision. The Payoff Table will be randomly selected among one of the six following tables.

Decision	<i>Left</i>	<i>Right</i>	Decision	<i>Left</i>	<i>Right</i>	Decision	<i>Left</i>	<i>Right</i>
<i>Correct</i>	12	10	<i>Correct</i>	21	1	<i>Correct</i>	17	5
<i>Incorrect</i>	9	9	<i>Incorrect</i>	- 9	- 9	<i>Incorrect</i>	- 1	- 1
Decision	<i>Left</i>	<i>Right</i>	Decision	<i>Left</i>	<i>Right</i>	Decision	<i>Left</i>	<i>Right</i>
<i>Correct</i>	10	12	<i>Correct</i>	1	21	<i>Correct</i>	5	17
<i>Incorrect</i>	9	9	<i>Incorrect</i>	- 9	- 9	<i>Incorrect</i>	- 1	- 1

### Decision

In each trial, once you have received both kinds of information, you need to select one of the stimuli circles.

- If the selected circle is the one with the most dots, it is **correct**. Otherwise, it is **incorrect**.
- According to your decision and the Payoff Table, a different number of points is associated with correct and incorrect decisions.
- To select the left circle, press the key  on your keyboard.
- To select the right circle, press the key  on your keyboard.

## Blocks

- From one block to another, the information is not displayed in the same order:
  - In “Type A blocks”, you first receive the **Payoff Table** then the **Stimuli** before the decision.
  - In “Type 1 blocks”, you first receive the **Stimuli** then the **Payoff Table** before the decision.
- You will play a total of 4 blocks alternating between the two types.

## Feedback

- After each trial you will know if you were correct or not and the number of points earned.
- Every 12 trials, you will receive:
  - Aggregated accuracy feedback: the percentage of correct answers in the last 12 decisions.
  - Aggregated points feedback: how close you were from the highest number of points compared to the lowest number of points in the last 12 decisions.

## Earnings

Additionally to the AUD 5 show-up reward, your variable payment will be based on :

- 6 trials randomly selected in each of the 4 blocks.
- Your variable payment is the sum of all points accumulated during the 24 selected decisions (equal proportion of each Payoff Table).
- The conversion rate between points and AUD is:

$$10 \text{ points} = AUD1$$



## CHAPTER 2

# Willingness to Pay Attention for Others: Do Social Preferences Predict Contribution in Attentional Social Dilemmas?

— *This chapter is coauthored with Mira Toumi*

### 2.1 Introduction

An individual can choose to deposit waste in dustbin  $A$  (which means it cannot be recycled) or dustbin  $B$  (where it will be recycled). Disposing of the waste in dustbin  $A$  increases the waste treatment cost whereas recycling the waste in dustbin  $B$  is of value to the community. How can we model and predict which dustbin will be chosen by the individual? The most intuitive framework to investigate these issues is the Public Good Game (henceforth PGG) (Samuelson, 1954). In the PGG, players choose a level of contribution that benefits all the players. Since the contribution is costly for the individual, the Nash equilibrium predicts that a rational self-interested player will not contribute based on an optimal cost-benefit trade-off. However, the total welfare is maximized if the whole community contributes (Pareto optimum). This tension between the Nash equilibrium and the Pareto optimum describes a social dilemma.

Experimental results show that individuals do not behave as own payoff maximiz-

ers (Frey and Meier, 2004; Chaudhuri, 2011). To explain their larger contributions, economists have introduced the notion of “social preferences” (Rabin, 1993; Charness and Rabin, 2002; Fehr and Fischbacher, 2002; Bénabou and Tirole, 2006; Fischbacher and Gächter, 2010; Villeval, 2012; Graf et al., 2013). Social preferences are preferences that take others into account. These preferences can be represented by an individual’s utility as a function of the distribution of player outcomes (“distributive” preferences) or as a function of player beliefs about behaviors (“belief-dependent” preferences).<sup>1</sup> As a result, individuals with pro-social preferences are willing to reduce their own monetary reward to increase the payoff of strangers and the expression of such preferences shifts the contributions toward the Pareto optimum.

We could still expect from a rational individual with social preferences that their level of contribution would be the result of an optimal cost-benefit trade-off. In the experimental designs with “induced effort”, the experimenter controls the cost and the outcome of the contribution that are common knowledge to participants. Thus participants explicitly choose a level of contribution that should reveal their social preferences to some extent: e.g. the more altruistic an individual, the higher her contribution (Offerman et al., 1996; Festré et al., 2017).<sup>2</sup>

Induced effort may lack external validity with respect to real effort. Indeed, real effort may be impacted by several aspects, such as the knowledge of the cost and efficiency function, timing of decisions, the difference between planned actions and actual behaviors and the differences between types of efforts (e.g. cognitive or physical effort) (Charness et al., 2018). In the above waste-dustbin example – as in many pro-social behaviors – the contribution to public good is the amount of attention paid to reach a decision rather than an induced effort.

In Psychology, attentional processes refer to the cognitive processes in charge of the selection and processing of information (stimuli) to improve its evaluation

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<sup>1</sup>For more details about the theoretical and experimental distinction of these two types of preferences see Attanasi and Nagel (2008). It has been shown that individuals can express both types of social preferences and their expression can be confounded. For methodological convenience, we did not experimentally disentangle these two types of preference and have elicited only a standard measure of the expression of social preferences.

<sup>2</sup>As the experimenter controls all variables but social preferences, the latter is revealed assuming that the participants behave as utility maximizers. However these elicited preferences are contingent to the underlying model.

at the expense of other information. More precisely, information competes for limited perceptual and processing capacity (Pashler and Sutherland, 1998). Translated into economic terms, attention given to the decision regarding which dustbin to throw waste in is associated with an opportunity cost as cognitive resources could have been used for other goals. As a benefit, paying attention increases the differentiation between the two dustbins, and thus decreases the waste treatment costs. An economist would expect that the level of attention paid by a rational individual is the result of an optimal cost-benefit trade-off. The optimal attentional contribution should maximize the difference between an utility and a cost function shaped by the interaction between an individual's attentional processes and their social preferences.<sup>3</sup>

However, there exist several reasons to expect that contributions in term of attention are not necessary equivalent to other types of effort and may reveal different social preferences. These reasons are due to the particular nature of attention and its interaction with incentives. A first reason is that incentives can enhance the cognitive mechanisms involved in the allocation of attention (Pessoa, 2015). For example, Padmala and Pessoa (2011) show that the presence of rewards increase performance (decrease both response time and error rate) in a selective attention task where participants have to discriminate between house and building pictures with the presence of textual cue or distractors. These results suggest that incentives may have an effect on the efficiency of the attentional contribution.<sup>4</sup>

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<sup>3</sup>The study of attention as an economic concept has gained importance since the late 1990s (Festré and Garrouste, 2015a). To express their preferences, individuals need to pay attention to investigate any possible alternative outcomes. Paying attention reduces the uncertainty of the choice (Sims, 2003). However, since the available amount of attention is limited, an attention allocation problem appears: the individual may not consider all the available alternatives, and may not examine all the considered alternatives' characteristics (Caplin et al., 2011; Masatlioglu et al., 2012; Manzini and Mariotti, 2014; Sitzia et al., 2015; Li et al., 2016).

<sup>4</sup>Psychologists usually distinguish two types of mechanisms that interplay in the allocation of attention (Bugg and Crump, 2012). One is a stimuli-driven (or bottom-up) mechanism: attention is allocated toward stimuli in function of their salience, that is defined by their physical properties (e.g. color, brightness, size, position). The other is a goal-driven (or top-down) mechanism: it refers to the individual ability to exert attentional control to achieve goals by directing attention toward particular stimuli, locations, or characteristics. While these two mechanisms are often studied separately with different experimental paradigms, it is their interaction that defines the attention allocation process. However, according to Awh et al. (2012), these two mechanisms fail to explain a growing number of cases in which neither current goals nor physical salience can account for strong selection biases, and other processes such as selection history may play also a role. In economics, Hefti and Heinke's reviews (2015) of market models with limited attention are



A second reason is that the values assigned to stimuli at the time of choice depend on the amount of attention that they received during the decision making process (Krajbich et al., 2010; Armel and Rangel, 2008; Shimojo et al., 2003; Milosavljevic et al., 2012). This interplay between attentional and valuation processes is not only expressed in term of behaviors but seems to have deeper roots in the brain: manipulation of attention (through the use of cues or distractors) activates similar regions and neural networks in the brain that those activated by rewards manipulation (Pessoa and Engelmann, 2010). Furthermore, from a theoretical point of view, choosing an optimal amount of attention suggests an infinite regress problem. Indeed, an attention allocation problem would require itself cognitive resources to evaluate the cost and the benefit of each potential allocation of attention (Lipman, 1991).

Therefore, the interaction between social preferences and attention contribution is not straightforward: if the individual chooses the dustbin  $A$ , how can we distinguish between an expression of their preferences against “recycling” (as if guided by destructive goals), and an unintended insufficient level of attention? To what extent is the expression of social preferences affected by the particular cost and efficiency structure of the attentional contribution? This is an important issue when we consider that many pro-social behaviors require a contribution in the form of attention rather than a salient or a monetary effort with a known cost function. Moreover, understanding the interaction between social preferences and attention allocation is an important step in designing effective policies. Indeed, *nudges* can be understood as mechanisms that help desirable alternatives capture the attention of individuals (Thaler and Sunstein, 2009).

An useful way to investigate the effect of the peculiar cost and efficiency structure of an attentional contribution is in an experiment where participants invest real attention, in a setting similar to the dustbin example described above. We propose a new experiment where participants pay attention in order to reduce uncertainty in a discrimination task, by choosing where to sort an item according to its composition. Our design allows us to measure both sides of an attention allocation process: the

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organized around these notions of goal and stimuli-driven attention.

amount of *attentional resources* allocated in the decision measured by the Response Time (RT), and the amount of information properly processed (the uncertainty reduction) measured by the Error Rate (ER). We provide three treatments representing different incentive schemes. A baseline with no monetary incentives (T0), a treatment where the participant's accuracy increases her own payoff (T1), and a treatment where participant's accuracy increases the payoffs of others (T2). A social preferences measure (Social Value Orientation, SVO) is introduced prior to the task. According to economic theory, incentives should increase the attention allocation. In particular, in the pro-social treatment (T2), the pro-social individuals who revealed high social preferences should pay more attention than individualistic participants.

Our experimental results show that monetary incentives increase the amount of the attention paid by the participants without interaction with intrinsic motivation. These results prove that participants' attention responds to incentives in our task. Furthermore, contrary to standard PGG results (Balliet et al., 2009), we show that social preferences, elicited through standard monetary elicitation methods, do not explain the attention contribution of participants.

The chapter is organized as follows. The experimental design and the protocol are described in section 2.2. section 2.3 presents the results that are discussed in section 2.4.

## 2.2 Experimental design

We use a similar effort task as in Caplin and Dean (2014) and study attentional contribution and its correspondence with monetary elicited social preferences. Indeed, Caplin and Dean (2014) have shown that decisions made in this task are compatible with the class of rational inattention models.

A large amount of real effort task have been used in experimental economics to study effort provision in different contexts like contest, principal-agent and coordination. These tasks are chosen to be “fastidious and boring” and to “induce sufficient disutility” in order to simulate real – and potentially useful – working

effort (Charness et al., 2013). If some of these tasks require attention, several skills and cognitive processes may shape the cost function and the efficiency of the effort. Moreover, in all these tasks, the effort cannot be observed by the experimenter but only the resulting performance.<sup>5</sup>

We argue that the task we use provides two main advantages for studying attentional contribution: first it requires mainly – if not solely – participants’ attention, second it allows both for the measurement of the quantity of attention allocated in the task (through the response time) and the efficiency of the process (through the accuracy in replying). More precisely, this task follows a standard psychological paradigm widely used to study perceptual decision making and attention namely a *discrimination task*. In discrimination tasks, participants have to classify stimuli based on perceptual evidence (e.g. in Massoni et al. 2014; Fleming et al. 2016; Garcia et al. 2017 participants have to discriminate between two circles one of which contains more dots than the other). In our task, built on the introductory example (throwing waste into the correct or over the wrong dustbin); participants face a stimuli set composed of forty nine black and white balls and have to indicate the dominant color of the majority of the balls. The main difference when compared to the standard discrimination paradigm is that we allow the participants to allocate as much time as they want in order to gather perceptual evidence, instead of controlling the disclosing time.<sup>6</sup>

The response time (RT) is thus indicative of the quantity of attention allocated to the task. The error rate (ER) measures the quantity of information processed (50% when no information has been processed to 100% when all the information has been processed) which is basically the outcome of the attentional process. This is coherent with an economic interpretation of attention, as a production process in

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<sup>5</sup>For instance, in Fahr and Irlenbusch (2000), participants have to produce some mass of cracked walnuts with a nutcracker in a given period of time; in Gneezy and Rustichini (2000) participants go from house to house to collect money for charity; in Dutcher et al. (2015) participants enter data used for another research project; in Abeler et al. 2011, participants have to report the exact number of 0’s from a grid of 0 and 1. See Charness et al. 2018 for an extensive review.

<sup>6</sup>A reason why psychologists usually control the disclosing time is that it allows the interpretation of differences in performance with differences in the efficiency of the attentional process. In Caplin and Dean (2014) participants can allocate as much time/attention as they want in the task, however, in contrast with our design, there is no extrinsic value of time/attention and the authors did not measure individuals’ response time.

which individuals invest *attentional resources* – as an input – to achieve an *effective* level of attention – as an output (Garrouste et al., 2017).

Moreover, our design is a step forward towards more external validity when compared to how real effort tasks are usually employed in PGG. In these designs, the maximal amount participants can contribute (endowment) is not exogenously fixed by the experimenter but endogenously earned through a real effort exerted by the participants (Muehlbacher and Kirchler, 2009). However participants still explicitly decide how to divide this endowment within an induced method.<sup>7</sup> In our experiment, the contribution is real in the sense that participants have an endowment of attentional resources represented by an amount of time that they can invest either to perform the task or to enjoy an alternative free activity for themselves.

Paying attention to the task decreases the inherent uncertainty in the choice (thus increasing performance) but comes with an opportunity cost since the alternative activity may be more enjoyable. Therefore the experimenter has some control over the cost and benefit structure of the contribution. Indeed she could modify the efficiency of the attentional process by controlling the difficulty of the task through the number and the difference between the number of black and white balls or modify the opportunity cost of the attentional contribution by proposing more or less enjoyable alternative activities. The next section presents more details concerning the effort task.

### 2.2.1 The dustbin task

Before beginning the task, participants are matched randomly in anonymous groups of  $N=3$ .<sup>8</sup> Subjects have a time endowment of 45 minutes to complete a one-hundred trials task. They can allocate this time according to what they want, with no going back and knowing that once they complete the hundred trials, they are free to surf the internet until time is up. In each trial, participants have to place *waste* (called

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<sup>7</sup>At the moment of the contribution decision, the real effort exerted by the participants is a sunk cost and should not impact contribution. There is mixed evidence regarding the effect of the origin of the endowment on contribution (see Muehlbacher and Kirchler 2009 and Cherry et al. 2005 for discussion).

<sup>8</sup>We chose to group participants by three to keep a PGG structure with the lowest number of participants.

“item” in the instructions) in a white or a black *dustbin* (“box” in the instruction). The items are composed of 49 balls. Subjects are told that each ball has a prior probability of a half to be black or white. Subjects know that the color of the majority of the item balls is indicative of the color of the correct box: thus, an item with a majority of white (black) balls should be sorted into the white (black) box. Trials follow on each other with no waiting time or feedback. Once a participant has classified an item, the next one appears but remains masked on the screen until the participant clicks on it. When the item and the two boxes are revealed, the participant selects one box by clicking on it (see Figure 2.2.1).<sup>9</sup>

Subjects invest attention to partially or fully reduce the uncertainty (e.g. based on their intuition, by counting  $x$  times all the balls, by counting a subset of the balls, etc.). The time spent on the task has an opportunity cost for participants: once they complete the hundred trials, (and after answering a motivation questionnaire, see subsection 2.2.3) they are free to surf the internet.<sup>10</sup>

Each correct answer (properly recycled item) earns the participant  $X$  *Experimental Currency Unit* (ECU) and earns each of the other two group members  $Y$  ECU. Different values of  $X$  and  $Y$  are implemented depending on the treatment, as described in the following section.

## 2.2.2 The treatments

We propose three treatments with different monetary incentives for accuracy: a baseline with no incentive (T0); a self-interest incentive (T1), and a pro-social

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<sup>9</sup>The masking between Items limits the dependence among the trials in two ways: 1) it prevents retinal persistence; 2) it forces participants to relocate the mouse at the center of the screen which reduces participant bias for selecting the same box in consecutive trials. The average time taken to click on a new item (*Timeb*) controls for heterogeneity of participants’ natural speed in the experiment. The absence of feedback also reduces the dependency between trials. As the objective prior probability ( $\frac{1}{2}$ ) is given, we limit the effect of probability learning and the resulting distortions of decision from experience (de Palma et al., 2014).

<sup>10</sup>This is not the first experiment to use time rather than money as a reward: Noussair and Stoop (2015) propose a dictator, an ultimatum, and a trustgame experiment where participants earn the right to leave the laboratory earlier. Also, Corgnet et al. (2015) propose an alternative free activity that the agent can perform instead of spending time on the task allocated by the principal. We prefer the use of the alternative task rather than allowing participants to leave the laboratory for the following reasons: 1) the payoff can be computed (and participants paid) only if all the participants have completed the task, and 2) observing other participants leaving the laboratory may induce peer-effect (Rosaz et al., 2016).

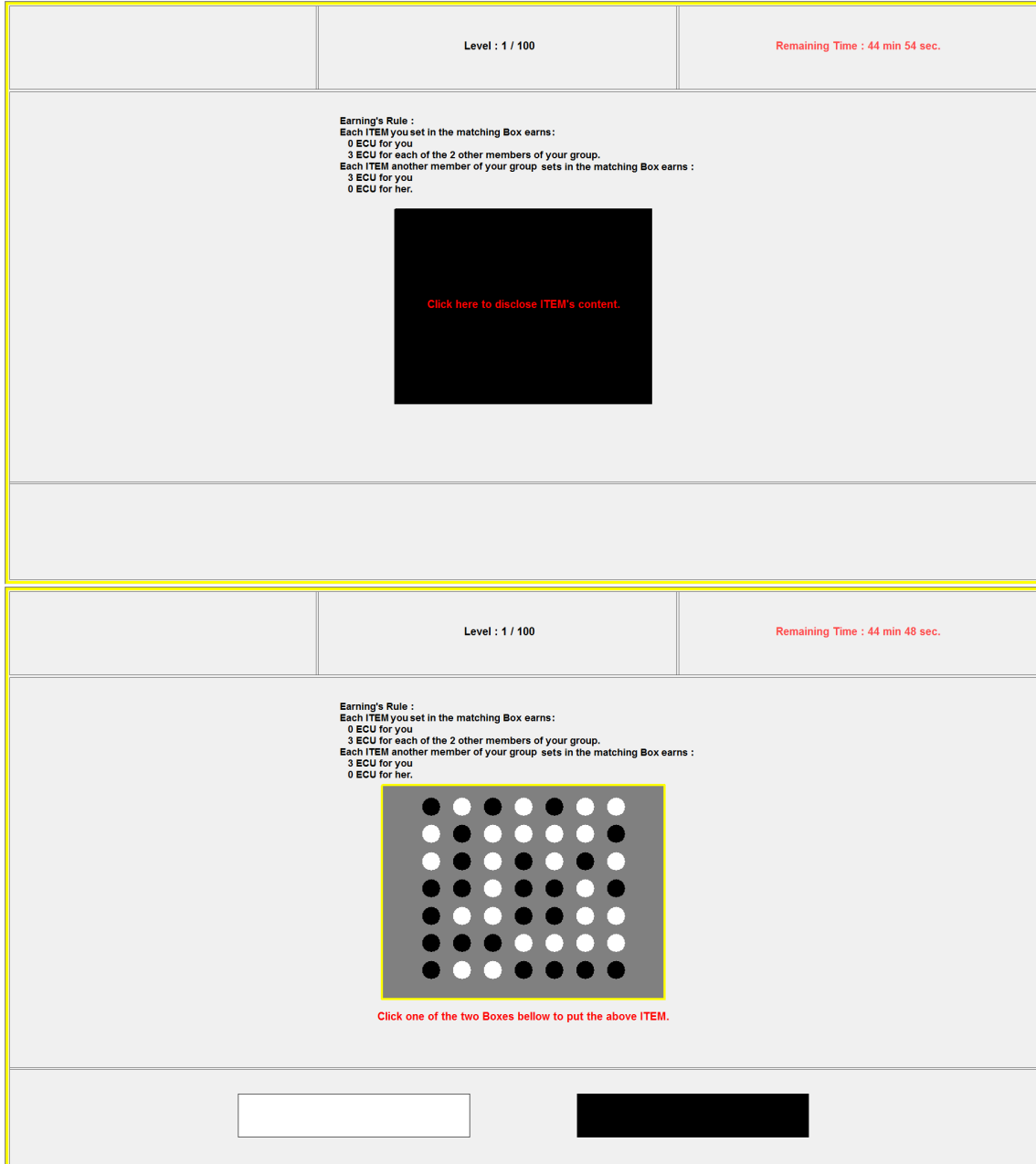


Figure 2.2.1: Screenshot of masked then unmasked Item.

**Note:** Translated by the authors, the original material in French.

incentive (T2). Table 2.2.1 reports the characteristics of each one expressed in ECU. Instructions are identical across treatments, only the values of  $X$  and  $Y$  differ.<sup>11</sup>

<b>Treatment</b>	<b>X for me</b>	<b>Y for eachother players</b>
Baseline No incentives (T0)	0	0
Monetary Self-interested Incentives (T1)	6	0
Monetary Pro-social Incentives (T2)	0	3 (x2)

Table 2.2.1: Treatments

The baseline treatment, with no monetary incentives (T0:  $X = 0$  and  $Y = 0$ ) allows us to know how participants allocate their attention between our task and the free-time, with no involvement of a monetary aspect. Thus, it reveals the intrinsic motivation to complete the task.

In the case of the self-interested monetary incentive (T1:  $X = 6$  and  $Y = 0$ ), we obtain the direct impact of the monetary incentive on the allocation of attention: compared to  $T0$ , we ascertain how much attention participants are willing to pay to increase their expected payoff.

The pro-social incentive has a PGG-like payment structure since a correct decision from a player benefits the other two members of the group (T2:  $X = 0$  and  $Y = 3$ ), and differs only by the nature of the contribution (in the form of attention rather than money or some other effort). The values of  $X$  and  $Y$  in  $T2$  are chosen in order : 1) to keep similar incentives between  $T2$  and  $T0$  for self-interested players ( $X = 0$  in both treatment), 2) to keep similar incentives between  $T2$  and  $T1$  for players indifferent between their own and their group's outcome as the expected social welfare is identical in both treatments ( $X = 6$  in  $T1$  and  $2 \times Y = 6$  in  $T2$ ). More precisely, the strategies that correspond to the Nash Equilibrium in  $T1$  are Pareto Optimal both in  $T1$  and  $T2$ . Therefore, even if we do not control contribution cost, the Nash Equilibrium in  $T2$ , if players are self-interested and rational, should be revealed by the behaviors in  $T0$  while the behaviors in  $T1$  should described the Pareto Optimum in  $T2$ . Therefore, social preferences should predict potential

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<sup>11</sup>Subjects know that the values of  $X$  and  $Y$  are the same for all the players in their group. This notion of group is only relevant for  $T2$ . However, we chose to frame all treatments with the same "group structure" in order to keep instructions as similar as possible.

difference in behavior between  $T2$  and the two other environments.

The way we measure social preference and other controls is described below.

### 2.2.3 Controls

#### Social preferences (SVO)

We use the zTree implementation of Murphy et al. (2011) Social Value Orientation Ring Measure proposed by Crosetto et al. (2012).<sup>12</sup> Each participant, as a “donor”, faces 15 successive choices among several potential outcomes distribution for them and an anonymous “receiver”. The outcome distribution goes from 50 to 100 points for the donor and 15 to 100 points for the receiver.<sup>13</sup> We implement an exchange rate of 100 points = 1 euro. Subjects are acknowledged that only one of their choices will be randomly chosen to be implemented. For a session with  $N$  participants, the “Ring matching” works as follows. Subjects are randomly assigned a number between 1 and  $N$ . Subjects number  $0 < n < N$  choose the outcome distributions for her and participants  $n + 1$ . Subject number  $N$  chooses the outcome distributions between their and participant 1. Therefore each participant is paid as a receiver and a donor for two different choices. This procedure generates for each participant a *svo\_angle* which represents the marginal rate of substitution between their own and an anonymous stranger’s earnings. This measure provides four *svo\_profiles* according to the closest extreme behavior: “*altruistic*” (maximizing others’ outcomes); “*pro-social*” (maximizing the shared outcome); “*individualistic*” (maximizing one’s own outcome); and “*competitor*” (maximizing the difference between one’s own and the outcome of others).<sup>14</sup>

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<sup>12</sup>Social Value Orientation has been introduced by Messick and McClintock (1968) and McClintock (1972).

<sup>13</sup>All the possible range of distributions are presented in the appendix

<sup>14</sup>Individuals with *svo\_angle*  $< 0$  are willing to reduce their own earnings to reduce the earnings of others; individuals with *svo\_angle* = 0 are not willing to reduce their own earnings for others, and individuals with *svo\_angle* = 45 are willing to reduce their own earnings by 1 in order to increase the earnings of others by 1. In our study we divided the population into two groups: *pro-social* and *individualistic*. Only one of our participants fitted the *competitive* profile and he was grouped with the *individualistic*.



**Intrinsic Motivation (IMI)**

We measure the participants' intrinsic motivation with Deci and Ryan's Intrinsic Motivation Inventory (IMI) (2003). IMI is based on Deci et al.'s Self Determination Theory (1982), a measure of intrinsic motivation and self-regulation that is popular in Cognitive Psychology and in Industrial and Organizational Psychology (Ryan et al., 1983). It is based on a self-reporting questionnaire about the feelings the player experienced during the task. It includes 31 Lickert-like items split across 5 dimensions: enjoyment/interest, perceived competence, effort, pressure/tension, relatedness to the group. We administrated the complete IMI questionnaire to the participants, but we refer only to the interest/enjoyment subscale in the data analysis since this is the only subscale that assesses intrinsic motivation.

**Risk aversion (HL)**

We control risk aversion using Holt and Laury's risk preferences elicitation method (2002) (HL). Subjects were confronted with 10 decisions among two lotteries that differ in their degree of risk. The most risky lottery earns either 385 or 10 ECU while the safest earns either 200 or 180 ECU. For both lotteries, the probability of earning the higher outcome goes from 1 to 0.1 across the 10 decisions. All the choices are presented simultaneously on a single screen, in a  $10 \times 2$  table where lines represent choices and columns represent lotteries. The most risky lottery was always presented on the same side (left or right) which was balanced between participants.

A rational participant is expected to choose the most risky lottery and switch to the less risky lottery as the probability of earning the higher outcome decreases. The switching point thus reveals a range for the curvature of the utility function. As recommended by the authors, we gave participants the possibility of switching twice – multiple switches are considered a violation of expected utility theory – and we count the number of times the risky lottery was chosen as a measure of risk aversion. At the end of the 10 choices, one choice is randomly selected to generate the earnings for each participant. However, the amount of earnings is communicated at the end of the experiment to avoid any interference with the other stages of the

experiment.

The next section provides more details on the implementation of the whole experiment.

### 2.2.4 Protocol

The average duration of the experimental sessions was one hour. The sessions were conducted as follows: 1) participants enter the room and are randomly assigned to a computer; 2) the experimenter reads the instructions for all the experimental step out loud in French; 3) SVO preferences test; 4) risk aversion test; 5) participants are informed about which treatment they are assigned to for the main task; 6) they play two practice rounds, with a trial example and comprehension questions related to the payoff structure, and provision of a help window to allow them to correct mistakes; 7) the 45 minutes countdown begins and participants start the 100 trials; 8) after completing the 100 trials, participants complete the IMI questionnaire with a submission time of a minimum of 3 minutes (to prevent random answers); 9) participants spend the remaining time surfing the internet; 10) participants are informed about the results of each step; 11) individuals are paid before completing a final questionnaire.

During the whole experiment, written instructions for all the experimental steps were available, and reminders were provided on the computer screen. We measured social and risk preferences before the main task so that the experiment would finish with the "free-time" since we had no control over this last step. In order not to interfere with the task, the results of each step were given only at the end of the experiment. We tried to avoid interference from the social and risk preferences elicitation by giving players' treatment information ( $X$  and  $Y$  value) only after these measures had been completed. Subjects were provided with headphones during the experiment to avoid communication among them, and to allow them to watch videos or listen to music during the free time. The instructions concluded by providing a non-exhaustive list of the most popular websites (including social networks, news, games, blogs, music, videos). Subjects had access to calculators (e.g. if they wanted to compute the expected payoff during HL).

The experiment included a total of 114 participants and 7 sessions (15 or 18 participants per session) and was held in the “Laboratoire d’Économie Expérimentale de Nice” (LEEN) at “Université Cote d’Azur” in April and May 2016. The participants were aged between 17 and 55 years ( $M = 22.53, SD = 5.99$ ).<sup>15</sup>

All participants were recruited using ORSEE (Greiner, 2015), and the experiment was implemented in zTree v3.5.1 (Fischbacher, 2007). Subjects were mainly undergraduate students from multiple disciplines (economics, management, science, literature). All participants received a show-up fee of 5 euros. The experimental currency units were converted into cash at the rate 100 ECU = 1 euro, and the average payment was 13.3 euros.

### 2.2.5 Behavioral Hypothesis

This section elaborates the behavioral hypotheses tested in our experiment. We first present the set of main hypotheses needed to answer our research question then we present secondary hypotheses that are more operational and due to the specific implementation of our task.

#### Main Hypothesis

First and foremost, we have to ensure that attentional effort responds to incentives in our game. The positive effect of incentives on effort provision and performance in an experimental task should not be taken as granted and have to be tested empirically. Indeed, Camerer and Hogarth (1999) review the effect of financial incentives in 74 experiments and show that “most studies do not show a clear improvement in mean performance” in the presence of incentives. One explanation for this inelasticity between incentives and effort (measured through performance in most effort task) is that the utility derived from the reward is negligible compared to the intrinsic motivation or the experimental demand. In-elasticity may also appear when

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<sup>15</sup>In France, undergraduate studies may start at the age of 17 so students can participate in the experiment before the age of majority. We excluded one participant from the analysis. She did not understand the instructions, took twice as long as the other participants at the training levels, failed many times in comprehension questions, and showed a significantly higher ER than if she were playing randomly (82%).

performance is not directly linked to an effort or does not increase sufficiently with (i.e. in-elasticity between effort and performance) (Camerer and Hogarth, 1999).<sup>16</sup> We design our experiment in order to have a large and positive effect of incentives on attentional contribution. Indeed, the larger the effect of incentives, the more likely we will be able to observe any potential effect of social preferences on attentional contribution. This leads to the test of our first hypothesis:

*H1: Monetary incentives increase allocated attention.*

In general, individuals prefer to receive a fixed reward for them rather than for others (e.g. Bekkers et al. 2007 found that only 5.7% of the participants choose to give in all-or-nothing dictator game). We thus expect a stronger effect of incentives within the Self-interested incentive treatment (T1) if compared to the Pro-social one (T2). However, it has also been shown that individuals may prefer to exert effort for rewarding others than for rewarding themselves when the incentives are low (Charness et al., 2016; Imas, 2014). An explanation of this “warm glow” effect is that some part of the utility that individuals derive from giving is fixed and independent of the given amount. In our experiment, we aim to provide sufficient incentives in order to ensure that the reward associated with performance variation dominates such a warm glow effect. Such dominance would be supported if the following hypothesis is verified:

*H1b: Self Interested monetary incentives increase allocated attention more than pro-social ones.*

An increase in performance in our task may be due either to an increase in the allocated attention or to an increase of the efficiency of the attentional process. As discussed in the introduction, interaction between incentives and attention processes has been documented and rewards may enhance the efficiency of the attentional process. Such effects usually lead to a decrease of both response time and error rate (Padmala and Pessoa, 2011). By measuring both the Response Time and the Error Rate, we are able to measure the efficiency of the attention allocation process (i.e. how much an increase in allocated attention increases the effective attention). We

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<sup>16</sup>Camerer and Hogarth (1999) also describe cases where incentives have a negative effect on effort and performance. Indeed, the presence of incentives may deteriorate intrinsic motivation or increase anxiety. These particular issues will be more detailed in our third hypothesis.

thus test the following hypothesis:

*H2: Monetary incentives enhance attention efficiency.*

In the pro-social treatment the incentives for paying attention are identical to the baseline treatment for individuals maximizing their own payoff and identical to the self interested treatment for individuals maximizing group payoff. Therefore, the strength of the pro-social incentives should be an increasing function of individual social preferences. If participants have been shown to respond to incentives (*H1* true), then we could expect that the pro-social individuals are willing to pay more attention than individualistic ones. The measure of social preferences we administer (SVO) is widely recognized as a laboratory elicitation tool of social preferences (Messick and McClintock, 1968; Van Lange et al., 2013) and has been shown to be a good predictor of behavior in social dilemmas (Balliet et al., 2009).

A rejection of the following hypothesis could be interpreted either as an instability of social preferences over time (due to the timing or interference of the elicitation and our task), over effort (money or attention) or by the particular nature of attention that could prevent the expression of social preferences.

*H3: Social value orientation predicts attentional contributions.*

### **Secondary hypothesis**

Moreover, our experiment allows us to test other hypotheses that are not the core of this chapter.

Intrinsically motivated individuals engage in a task only for the pleasure and satisfaction of doing or completing it. The attention paid by participants in the baseline treatment reveals their intrinsic motivation to complete the task compared to surfing the internet. However, intrinsic motivation and incentives may interact and generate crowding out effects on voluntary contributions (Bénabou and Tirole, 2006; Ariely et al., 2009; Gneezy et al., 2011; Frey and Oberholzer-Gee, 1997; Frey and Jegen, 2001).<sup>17</sup>

By measuring individuals' intrinsic motivation in our experiment, we aim to disentangle the direct and indirect (through intrinsic motivation) effects of incen-

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<sup>17</sup>See Festré and Garrouste (2015b) for a review of crowding out in economics and psychology.

tives on the attention allocation. According to Deci et al.'s meta-analysis (1999), the undermining effect of monetary incentives on individual's performance in a task (crowding out) is limited to conditions in which the incentive is completely independent from the performance in the task. As the incentives we proposed are directly linked to the performance, we expect not to find any crowding out effect. We test the following hypothesis.

*H4: Monetary incentives impact intrinsic motivation.*

In our experiment, allocating attention to the task can reduce its inherent uncertainty and results in an increase of the probability of being right. It mechanically increases the expected performance and decreases the variance in performance. Therefore, due to this latter effect, risk averse participants may be more willing to pay attention to our task. As we control for risk preferences, we can test the following hypothesis.

*H5: Risk aversion increases allocated attention.*

Our task is demanding in term of participants' attention which may vary during the task. Indeed, participants could get tired or gain experience in the processing of the information. We can investigate at a trial level if the level of allocated attention and its efficiency vary over trials.

*H6: Attention allocation varies over trials.*

By manipulating the difference in the number of black and white balls, we increase or decrease the difficulty of the task. When complexity increases, we conjecture that participants need to allocate more attention to achieve a similar level of effective attention. We test the following hypothesis.

*H7: Complexity increases allocated attention and decreases efficiency.*

The next session presents our statistical methodology and our results.

## 2.3 Results

Since participants can neither observe the actions nor communicate with the other participants until the end of the experiment, individuals decisions can be treated as independent between participants. Table 2.3.1 summarizes participants' aggregated

variables. We observe an average RT of 9.8 seconds and an ER of 8%. 49% of the participants are closer to pro-social than individualistic behavior in the SVO test.

Variable	Description	M	SD	Min	Max	Obs
RT	Response Time: number of second before the participant reach a decision.	9.83	8.62	.39	98.671	113
ER	Error Rate: number of items misplaced.	8.16	11.03	0	51	113
Prosocial	1 if the participant expressed high social preferences in SVO test, 0 otherwise	.49	.50	0	1	113
Interest	Result of IMI scale on the interest to the task	28.50	10.45	7	49	113
risk aversion	number of risky choice (out of 10) in the H&L test	6.44	1.94	0	10	113
svo_angle	SVO test result	19.62	15.97	-16.26	46.65	113
Male	1 for male, 0 otherwise	.372	.485	0	1	113
Age	Age of the participant in years	22.53	5.97	17	55	113
timeb	Time spent before disclosing a new item	0.802	0.266	0.420	1.746	113
difference	Difference between the number of black and white balls	5.55	4.22	1	23	700

Table 2.3.1: Descriptive statistics of the sample

We first analyze the data at the participant level ( $n_s = 113$ , aggregating for each participant the 100 trials). We assess treatment effects using one-way ANOVA followed by pairwise mean comparison using a two-sided t-test. We report  $R^2$  and Cohens'  $d$  as measures of effect size. According to Cohen's guidelines (1992) and our number of participants and conditions, the probability of detecting level medium effect sizes when they are present are equal to 0.651 for the ANOVA analysis and to 0.865 for the pairwise comparisons.

To increase statistical power by controlling for trial complexity – measured by the absolute difference between the numbers of black and white balls – and evolution across trials, we also regress the logarithm of the RT (OLS) and ER (logit) at a trial level ( $n_t = 11300$ ).<sup>18</sup> Table 2.3.2 presents regressions for all treatments to assess the effects of incentives, trials, complexity and individual characteristics on attention allocation, while Table 2.3.3 presents regressions to assess the effect of social preferences in Monetary Pro-social treatment (T2). In all regressions, standard errors are clustered at the participant level.

### Result 1: Monetary incentives increase allocated attention.

<sup>18</sup>Matejka and McKay (2015) show how limited attention induces the choice probability to follow a logit formula.

Figure 2.3.1 shows participants' RT (left) and ER (right) for each treatment. We find a significant effect of treatments on RT [ $F(2, 110) = 7.81, p < .001, R^2 = 0.1243$ ].

More precisely, RT are longer in Monetary Self-interested treatment (T1) ( $M = 11.98, SD = 4.005$ ) compared to Baseline treatment (T0) ( $M = 7.94, SD = 4.480$ ) [ $t(73) = -4.102, p < 0.001, d = 0.960$ ]. This observed effect size is considered as "large" according to Cohen's guidelines (1992).

**Result 1': Self-Interested incentives increase more allocated attention than Pro-social incentives.**

The effect of pro-social monetary incentives (T2) is less clear and pronounced. The RT in T2 ( $M = 9.72, SD = 4.734$ ) are longer only at the 10% significance level compared to absence of monetary incentive (T0) [ $t(75) = -1.721, p = 0.089, d = 0.397$ ] and shorter compared to Self-interested treatment (T1) [ $t(72) = 2.182, p = 0.032, d = 0.514$ ]. The regression analysis at the trial level confirmed that both type of incentives increased the attention allocated in the task (see Table 2.3.2, Model (5): [ $\beta = 0.601, t = 4.07, p < .001$ ] for T1, [ $\beta = 0.415, t = 2.46, p = 0.015$ ] for T2). We conclude that monetary incentives increase the attention allocated in the task and that this effect is stronger for T1 if compared to T2.



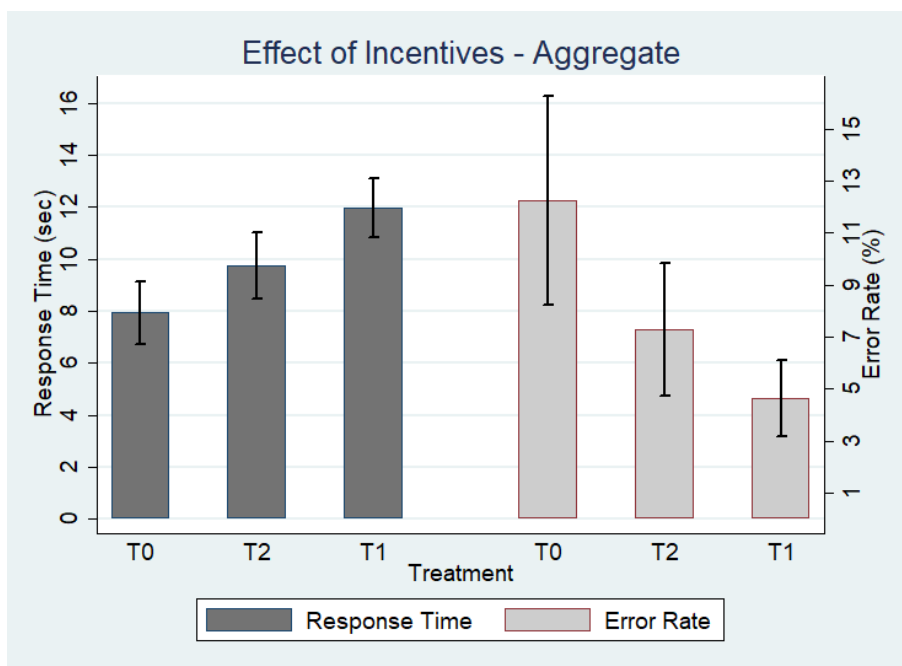


Figure 2.3.1: Incentives impact on Response Time (RT) and Error Rate (ER)

VARIABLES	(1) log(RT)	(2) ER	(3) log(RT)	(4) ER	(5) log(RT)	(6) ER
T1	0.605*** (0.154)		0.603*** (0.154)		.601*** (0.147)	
T2	0.304 · (0.166)		0.299 · (0.166)		0.414* (.168)	
RT		-0.082*** (0.009)		-0.127*** (9.51e-03)		-0.127*** (0.018)
RT×T1		-0.014 (0.010)		-3.84e-03 (0.011)		-1.73e-03 (0.024)
RT×T2		-0.012 (0.010)		-8.13e-03 (0.0108)		-0.011 (0.024)
trial			-0.021*** (0.002)	-7.24e-03 (5.28e-03)	-0.020*** (0.002)	-6.62e-03 (4.64e-03)
(trial) <sup>2</sup>			1.40e-04*** (1.77e-05)	5.09e-05 (4.98e-05)	1.40e-04*** (1.77e-05)	4.59e-05 (4.28e-05)
difference			-0.075*** (0.0043)	-0.253*** (0.040)	-0.076*** (4.27e-03)	-0.253*** (0.040)
prosocial					0.226 · (0.124)	-0.219 (0.214)
risk aversion					0.015 (0.033)	0.043 (0.058)
interest					-8.602e-03 (5.92e-03)	-7.88e-03 (0.0115)
age					-6.66e-03 (0.012)	3.30e-03 (0.019)
male					0.065 (0.132)	0.101 (0.233)
log(timeb)					0.644* (0.263)	
Constant	1.525*** (0.123)	-1.448*** (0.236)	2.508*** (0.135)	-0.0748 (0.147)	2.810*** (0.349)	-0.160 (0.455)
Observations	11,300	11,300	11,300	11,300	11,300	11,300
R <sup>2</sup>	0.036		0.166		0.185	

Robust standard errors clustered by participant in parentheses.

\*\*\* p<0.001, \*\* p<0.01, \* p<0.05, ·p < 0.1

Table 2.3.2: Attention allocation

**Result 2: Monetary incentives do not enhance attention efficiency.**

At the aggregate level, we found an higher ER in the Baseline treatment ( $M = 12.25\%$ ,  $SD = 0.149$ ) compared to Monetary Self-interested treatment ( $M = 4.64\%$ ,  $SD = 0.0516$ ) [ $t(73) = 2.945, p = 0.0049, d = 0.680$ ]. The ER in the Monetary Pro-social treatment ( $M = 9.72$ ,  $SD = 0.0930$ ) is between but is not statistically different to the two other treatments ([ $t(75) = 1.747, p = 0.0848, d = 0.403$ ] for T0 and [ $t(72) = -1.533, p = 0.1371, d = 0.354$ ] for T1).

The reduction of ER in T1 may be explained either by the higher amount of attention allocated to the task or by an increase in the efficiency of the attention allocation process. The regression analysis conducted at the trial level disentangles the two explanations and shows that the marginal effect of RT on ER is not statistically different between treatments (Table 2.3.2, Model (6): [ $\beta = -0.00173, z = -0.07, p = 0.942$ ] for Monetary Self-interested treatment and [ $\beta = -0.0110, z = -0.46, p = 0.649$ ] for Monetary Pro-social treatment).

We conclude that the treatment effect on effective attention can be mainly attributed to the increase in allocated attention rather than an increase in efficiency of the attentional process.

**Result 3: Social preferences do not predict attentional contributions.**

Figure 2.3.2 shows the RT and the ER in Monetary Pro-social treatment (T2) for pro-social ( $n = 16$ ) and individualistic ( $n = 22$ ) participants. The average behavior of the two groups is not statistically different [RT:  $t(36) = -0.340, p = 0.736, d = 0.112$ ; ER:  $t(36) = 0.759, p = 0.453, d = 0.249$ ].

These results are confirmed by a regression analysis at the trial level, restricted to T2 (Table 2.3.3: [Model (11):  $\beta = 0.110, t = 0.54, p = 0.591$ ] for RT and [Model (12):  $\beta = 0.0403, z = 1.27, p = 0.205$ ] for ER). SVO elicited social preferences do not explain the differences in attentional contribution in T2.

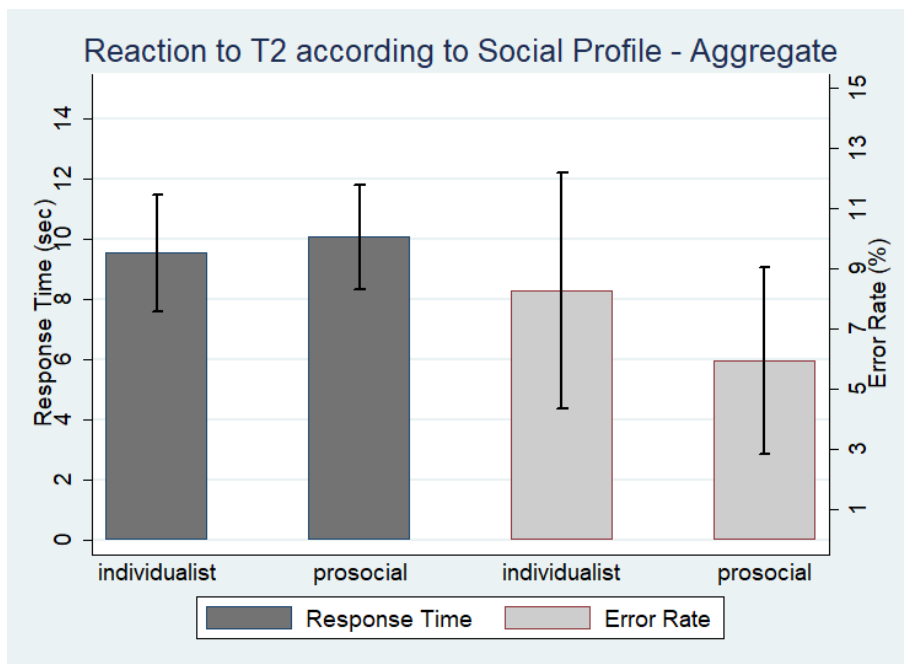


Figure 2.3.2: Impact of Social Preferences in Monetary Pro-social treatment

VARIABLES	(7) log(RT)	(8) ER	(9) log(RT)	(10) ER	(11) log(RT)	(12) ER	(13) log(RT)
prosocial	0.0991 (0.201)		0.111 (0.207)		0.110 (0.203)		
svo_angle							3.39e-03 (8.36e-03)
RT		-0.081*** (0.026)		-0.129*** (0.0210)		-0.132*** (0.022)	
RT×prosocial		0.016 (0.028)		0.022 (0.029)		0.040 (0.032)	
trial			-0.0155*** (2.59e-03)	-9.90e-03 (8.22e-03)	-0.0156*** (2.59e-03)	-8.60e-03 (8.33e-03)	-.0156*** (2.59e-03)
(trial) <sup>2</sup>			9.28e-05*** (2.29e-05)	8.56e-05 (7.42e-05)	9.32e-05*** (2.29e-05)	7.79e-05 (7.59e-05)	9.31e-05*** (2.30e-05)
difference			-0.0797*** (6.65e-03)	-0.295*** (0.072)	-0.080*** (6.65e-03)	-0.295*** (0.072)	-0.791*** (6.65e-03)
risk aversion					-0.0516 (0.048)	0.115 (0.066)	-0.050 (0.047)
interest					-4.30e-03 (9.23e-03)	-0.340* (0.016)	-4.40e-03 (8.89e-03)
age					-8.73e-03 (0.0163)	5.13e-03 (0.020)	-7.56e-03 (0.0159)
male					0.113 (0.221)	-0.232 (0.417)	0.109 (0.221)
log(timeb)					0.410 (0.411)		0.367 (0.415)
Constant	1.787*** (0.160)	-1.957*** (0.324)	2.692*** (0.177)	-0.123 (0.240)	3.466*** (0.537)	-0.124 (0.743)	3.402*** (0.549)
Observations	3,800	3,800	3,800	3,800	3,800	3,800	3,800
R <sup>2</sup>	0.0022	0.0309	0.144	0.132	0.167	0.159	0.167

Robust standard errors clustered by participant in parentheses.

Pseudo R<sup>2</sup> are computed for logistic regressions of ER.

\*\*\* p<0.001, \*\* p<0.01, \* p<0.05, ·p < 0.1

Table 2.3.3: Attention allocation in Monetary Pro-social treatment

This null result may be due to an absence or a trivial effect and/or to small statistical power. Indeed, we have only 38 participants to test this hypothesis. However, the large effect size found in *Result1* ( $d = 0.960$ ) suggests that attention in our task responds enough to incentives to be able to detect an effect of social preferences.<sup>19</sup> To disentangle between these two explanations, we compare the size of the effect of being pro-social on attentional contribution in our study with those usually found in comparable studies. In particular, the study of Balliet et al. (2009) reviewed 82 studies that measured both social preferences through SVO, and cooperation in social dilemma. In their meta-study, they found an average effect size of  $d_0 = 0.629$  of being pro-social on contributions. This effect is higher than the one we find here ( $\hat{d} = 0.112$ ) [ $Z = -1.574, p = 0.0621$ ].<sup>20</sup>

To increase statistical power, we also regress RT with the raw svo-angle as a measure of social preferences (see Table 2.3.3). This latter robustness check does not change the results and even increases the p-value [Model (13):  $\beta = 0.00334, t = 0.41, p = 0.687$ ]. We thus conclude that monetary elicited social preferences through SVO are not a good predictor of contribution in our attentional task.

**Secondary Result 4: Incentives do not impact intrinsic motivation in our task.** Figure 2.3.3 shows participants' self reported level of interest ( $M = 28.50, SD = 10.45$ ) according to the treatment. In our task, we find no effect of incentives on intrinsic motivation [ $F(2, 110) = 0.25, p = 0.774, R^2 = 0.0046$ ].

We conclude that we are allowed to interpret the effect of incentives on attention in our task as a direct effect, with no mediation of intrinsic motivation.

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<sup>19</sup>Even with 38 participants, the probability to detect such a large effect when it is present, with a type I error rate of  $\alpha = 0.05$ , is more than 0.98.

<sup>20</sup>We conducted the following statistical test.  $\mu_1$  ( $\bar{X}_1$ ) and  $\mu_2$  ( $\bar{X}_2$ ) are the expected (average) RT for pro-social and individualists in Monetary Pro-social treatment.  $\sigma$  is the standard deviation.

Under the Null ( $d = \frac{\mu_1 - \mu_2}{\sigma} = d_0$ ), the statistic  $Z = \frac{\hat{d} - d_0}{\hat{\sigma} \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{\bar{X}_1 - \bar{X}_2}{\hat{\sigma} \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} - \frac{d_0}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$  follows a Student's distribution with 36 degrees of freedom.

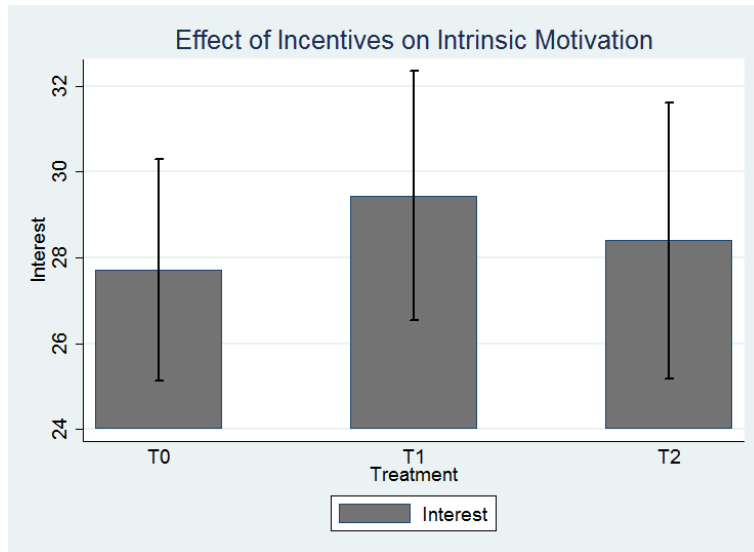


Figure 2.3.3: Incentives impact on Intrinsic Motivation

**Secondary Result 5: No effect of risk aversion neither on the quantity nor the efficiency of the allocated attention.**

We did not find any evidence in favor of an effect of risk preferences on RT nor ER (Table 2.3.2, [Model (5):  $\beta = 0.0149, t = 0.45, p = 0.656$ ] for RT and [Model (6):  $\beta = -0.043, z = 0.75, p = 0.455$ ] for ER). This absence of effect could be explained by the fact that all the trials are paid, reducing the variance in performance and so the expression of risk preferences.

**Secondary Result 6: The quantity of attention allocation follows a U-shape and efficiency of the attentional process does not vary across trials**

Figure 2.3.4 shows the average RT in each of the hundred trials for each treatment. The attention spent on the task decreases until the last third of trials (Table 2.3.2, [Model (5):  $\beta = -0.0204, t = -9.90, p < .001$ ] for the first order and [ $\beta = 1.40e^{-04}, t = 7.85, p < .001$ ] for the second order; marginal effect of the trial becomes positive for the 70<sup>st</sup> trial). We find no evidence of the influence of trial on the efficiency ([Model (6):  $\beta = -6.62e^{-03}, t = -1.43, p = 0.154$ ] for the first order, [ $\beta = 4.59e^{-05}, t = 1.07, p = 0.283$ ] for the second order).

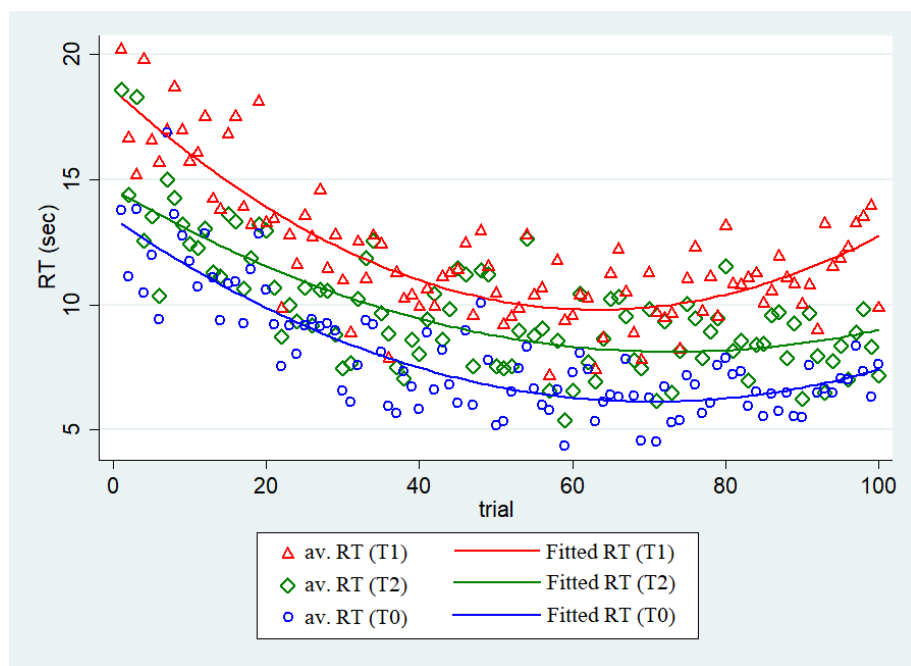


Figure 2.3.4: Average response time across trials by treatments

**Secondary Result 7: Complexity decreases the efficiency of the attentional process but increases the quantity of allocated attention.**

We find that both RT and ER increase with the difference between white and black balls (Table 2.3.2, [Model (5):  $\beta = -0.0756, t = -17.71, p < .001$ ] for RT and [Model (6):  $\beta = -0.253, t = -6.32, p < 0.001$ ] for ER). We conclude that complexity increases allocated attention but decreases the efficiency of the attentional process.

## 2.4 Discussion

In everyday experience, numerous pro-social behaviors imply investment in attention rather than salient effort provision with known cost and efficiency structures. In this chapter, we examine how standard PGG experimental results obtained with induced effort, could be applied to attentional social dilemma. To the best of our knowledge, there are no other studies dedicated to understand whether paying attention is equivalent to monetary contribution in a PGG.

To address this question, we proposed an experiment in which participants choose to allocate attention between reducing the uncertainty in a discrimination



task, and enjoying an alternative activity (surfing the internet) for 45 minutes. This design could be adapted to investigate a broader range of questions involving attention. It provides the following advantages. The task reproduces the cost and the efficiency structure of an attentional process, and enables measurement of the amount of attentional resources allocated (through RT) and the outcome of such investment (through ER) for each decision. These two measures reveal the input and the output of the individual attentional process (and permit to study the efficiency of such process) rather than impose a particular *ad hoc* structure. From a practical perspective, the design is easily understandable by participants, and allows the researcher to gather numerous decisions for each participant, based on an average RT of less than 10 seconds, increasing statistical power. In this framework, players were asked to complete one of three conditions which vary only by incentive manipulation: baseline without incentives (T0), self-interest incentives (T1), and pro-social incentives as in a PGG-like environment (T2). As predicted, monetary incentives increase both allocated and effective attention, and participants react more to self-interest incentives than to pro-social incentives. Furthermore, the absence of interference with intrinsic motivation avoids any crowding out effect and allows the researcher to interpret the responses to incentives as direct effects.

These results show the appropriateness of our design to investigate the expression of social preferences in an attentional public good game. According to the theory, T0 and T1 should respectively indicate the Nash-Equilibrium (if players are selfish) and the Pareto-Optimum in an environment (T2) where pro-social players should pay more attention than individualists. However, in our study participants qualified as pro-socials based on the SVO test, although being the most willing to reduce their monetary earnings in order to increase others' payoffs, they are not more willing to pay attention in order to benefit others. This is an interesting result.

It provides evidence that revealed social preferences depend on the nature of their elicitation, with no monotonous relation between attention and monetary contribution. Individuals may reveal pro-social preferences in terms of monetary or effort provision but not in terms of attention. This might be explained by the peculiar nature of attention compared to other resources: until individuals pay

enough attention, they ignore how their decisions impact the welfare of others. Indeed, Grossman and van der Weele (2016) and Dana et al. (2007) show that pro-social individuals who choose a fair option in a complete information game, may prefer willfully to remain ignorant in order to justify their selfish behavior. Thus, since remaining inattentive prevents the expression of social preferences, we cannot exclude the possibility that individuals may strategically remain ignorant if they prefer for their acts to be perceived — by others and/or their selves — as inattentive rather than selfish.

There are other potential explanations to the absence of correspondence between elicited social preferences through the SVO and the dustbin task. First it may be due to identical social preferences but different beliefs about the behavior of others according to the nature of the task. More precisely, individuals may have belief-dependent social preferences such that they are willing to contribute if they believe that others do. If such individuals believe that others will contribute in the SVO but not in the dustbin task, then they will contribute in the former but not the latter (and vice versa). Our experiment is not able to address this question but a simple extension with belief elicitation would.

A second explanation could be found in a “moral compensation” effect which suggests that individuals preferences are not stable over time. More precisely, it describes situations where people who initially behave “morally” express less moral behavior later (Blanken et al., 2015). In our experiment, moral compensation might occur for some individuals identified as pro-social who have already satisfied their individual demand for self image and therefore will not express pro-social preferences in the subsequent dustbin task. Our design could be easily modified to investigate the importance of this effect by inverting the order within the two tasks (playing the dustbin task before the SVO). If the “moral compensation” effect is important, individuals who participated to T1 should be more pro-social in the SVO than those who participated to T2. This effect might also be undermined by paying participants in only one of the two tasks or by administrating one of them days before the other.

A last explanation for our result could be linked to the work of Rand et al. (2012). Their main result is that cooperative strategies could take less time to be selected

and are more intuitive than individualistic ones. In our task, participants have to decide how much attention/time they want to dedicate to process information to increase the earnings of others. Our design does not allow us to directly disentangle between the time participants dedicate in choosing how long to process information and the time they dedicate to process the information. It thus could be possible that pro-social participants used less time to decide how long to process information but process information longer compared to individualistic participants. The overall response time could thus be similar among the two groups. However, we believe this explanation not to be very likely. Indeed, such explanation would suggest a difference in efficiency between the two profiles as pro-social participants would invest a higher share of their overall response time to reduce the error rate.

Despite the need of more research to disentangle between these explanations, our results highlight the need for more care on the part of researchers and policy makers when generalizing results obtained using induced effort games to environments where individuals' contributions mainly consume attention. Indeed, a deeper understanding of both the role of and interaction between social preferences and attentional processes is required and more research should be dedicated to investigate the role of attention in social dilemmas.

# Appendices — Willingness to pay attention for others: Do social preferences predict attentional contribution?

*The following text is an English translation of the instructions for the experiment*

## 2.A Instruction

By agreeing to participate in this experiment, you are signaling your complete agreement with the Laboratory Regulations which are available on the website or on request. You will participate in an experiment where your decisions will be anonymous, and will determine your final payment in part. So please read the following instructions carefully. In addition to the earnings collected in the experiment, and irrespective of your decisions, you will receive a fixed amount of 5 euros to cover your travel expenses. A variable amount will be added to this, depending on the decisions you make during the experiment. The total amount of your earnings will be distributed to you individually and confidentially at the end of the experiment after you have completed a final questionnaire. The currency used in this experiment is the experimental currency unit (ECU). However, at the end of the experiment you will be paid in euros at the exchange rate of  $100 \text{ ECU} = 1 \text{ euro}$ . A calculator will be available for the duration of the experiment. In order not to distort the results of the experiment, we ask you not to communicate or interfere with other participants. We would ask you also to turn off your mobile phones and refrain from using them for the duration of the experiment. In order to limit communication, we ask that you put on the headphones provided as soon as the instructions have

been read out. Non-compliance with these rules will result in the experiment being interrupted and your earnings canceled. If you encounter a technical problem, we would ask you to raise your hand and wait for the experimenter to come to you. All participants in the room have access to the same instructions and will participate in the same experiment.

**Experiment description:** The experiment in which you are participating today consists of 3 successive stages. A summary of the instructions will appear on your computer screen at the beginning of each step.

**Step1** The first stage consists of 15 consecutive rounds during which you will have to make anonymous decisions to allocate ECUs to yourself and another participant whose identity will be unknown to you. All participants in the experiment will be given the same problems. There are no good or bad answers; the choices you make will be based on personal preferences. In each round, you will be able to make only one choice from the 9 proposed. You will be asked to indicate the distribution of money (ECU) that you prefer by clicking on the point corresponding to the column of your choice.

Gain in step 1: At the end of the step, the computer will make a random selection of one from the 15 rounds, for each participant. The distribution chosen by the participant in the selected round will be awarded to him or her and another participant. Each participant will receive an amount that her or she has chosen plus an amount chosen by another participant. Thus, your earnings at this stage will depend on your decisions and those of another participant. To avoid influencing successive decisions, the information on the round selected and the earnings collected in this stage will be communicated to you only at the end of the experiment.

**Step2** The second stage of the experiment involves choices (numbered 1 to 10) between two lotteries A and B. Each lottery is associated with two possible earnings which are fixed during each of the choices, namely: Lottery A will always allow you to earn either 200 ECU or 160 ECU; Lot B will always allow you to win either 385 ECU or 10 ECU. However, the chances of receiving the higher amounts will be different for each of the 10 choices. The 10 choices will be presented in rows on the same screen, with each row corresponding to one of the choices. For each choice,

you will have to click on the description of the lottery in which you would like to participate.

For example, in choice number 1 corresponding to the first row of the table, you can choose between two lotteries:

- If you choose Lottery A, the computer will generate a random integer between 1 and 100.
  - If the number randomly drawn is between 1 and 100 (100% chances), then you will earn 200 ECU.
  - On the other hand, if the number is between 101 and 100 (0% chance), then you will earn 160 ECU.
- If you choose Lottery B, the computer will generate a random integer between 1 and 100.
  - If the number randomly drawn is between 1 and 100 (100% chances), then you will earn 385 ECU.
  - On the other hand, if the number is between 101 and 100 (0% chance), then you will earn 10 ECU.

Gain in step 2: At the end of the second step, a single row from the table will be drawn randomly by the computer to determine your winnings. The lottery chosen from this row will be activated. Therefore, your winnings at this stage will depend on your decisions and luck. In order not to influence your decisions in the next step, the number chosen and the amount gained during this step will be communicated to you only at the end of the experiment.

### **Step3**

Step 3 will begin and end at the same times for all participants. It will last for exactly 45 minutes. The time can be split among three periods, and each participant is free to decide on the allocation of time between the three periods:

- Period 1: where you have a task to perform;
- Period 2 : where you will be asked to complete a task evaluation questionnaire

- Period 3 : where you will be free to surf the internet.

Once a participant completes one of the periods, she or he will move automatically to the next period; there is no requirement to wait for the other participants.

**Period1: task description** Before starting the period, the computer will assign you randomly to a group with two other participants, whose identity you do not know. You will remain in the same group throughout the stage. The first period is 100 trials. In each trial, an ITEM will appear in the center of the screen. This ITEM will be hidden and requires you to click on it to reveal its contents. Clicking on the ITEM will reveal a content composed of 49 balls which may be white balls or black balls. Each ball is as likely to be black as to be white. Once the content of the ITEM is displayed, you must place it in one of the two boxes - the White box or the Black box - which will appear on your screen. The payment in this step will depend on the earnings rule and the number of items placed by you and the other members of your group in the corresponding box (see next page (Period 1 - Earnings during the period)).

Each ITEM corresponds to the box whose color is the same as that of the majority of the balls: That is, ITEMS composed of a larger number of white balls than black balls correspond to the white box; ITEMS composed of a larger number of black balls than white balls correspond to the black box.

To place the ITEM in one of the two boxes requires you to click on the box of your choice. In the example above, the ITEM contains 20 white balls and 29 black balls. Therefore, the box corresponding to this ITEM is the Black Box.

As soon as an ITEM has been placed in one of the boxes, you will pass automatically and immediately to the next turn; there is no possibility of going back.

The task will end when you have completed 100 trials, i.e. when you have placed all the ITEMS in one of the two boxes.

**Period 1: Earnings during task** The payment in this step will depend on the earnings rule and the number of ITEMS placed in the corresponding boxes by you and the other members of your group. At the beginning of the stage, the computer will randomly assign an earnings rule that will be applied to your group. This rule will inform you of two numbers which we will refer to here as X and Y, and will tell

you how many ECUs are associated with the ITEMS placed in the corresponding boxes. ITEMS placed in the wrong box will not earn any ECUs for any of the group members.

The values of X and Y will be communicated to you at the beginning of the step. They may be positive or equal to 0. These values may differ according to the group but will be the same for the members of the same group and will not change throughout the step.

In this step, the gain will depend on the number of ITEMS placed in the corresponding box by the members of the group, and the values of X and Y:

Whenever a participant places an ITEM in the correspondingly colored box it will earn him or her X ECU. Each time a participant places an ITEM in the correspondingly colored box it will earn Y ECU for the other members of his or her group (not to him). ITEMS placed in the incorrect (not corresponding) boxes will not earn any ECUs for any of the group members.

The values of X and Y will be repeated on the screen during each trial, and above the ITEM. Please pay attention to the instructions on the screen. Before the task begins, you will have two example rounds on screen.

**Two training trials:** Before you start the stage, you will be given two examples trials on the screen, so that you can familiarize yourself with the task described above and make sure that you understand the earning rule. The time spent during the two training trials will not be deducted from the 45 minutes and the answers will not be taken into account in the calculation of your earnings. On the right half of the screen the example will be shown, and on the left half a series of questions to ensure you understand the task and rules. The training trials will not be validated when you choose one of the two boxes, but when you correctly answer all the questions asked. In the case of incorrect answers, an information bubble will appear with an explanation, in order to help you to correct your answers.

The stage will begin and the 45 minutes count will begin as soon as all participants have completed the two training trials.

**Period 2: Questionnaire** Once you have placed each of the 100 ITEMS in one of the two boxes, you will be given a questionnaire about the task you performed in



the previous period.

The questionnaire consists of 30 statements. For each you must indicate how much you agree or disagree by changing the position of the cursor. All questionnaire items will be displayed on the same screen. To ensure that you take the questionnaire seriously and think about your answers, you will be unable to submit your responses until 3 minutes have elapsed (when the button will appear).

**Period 3: Internet browsing** Once the task is completed and the responses to the questionnaire have been submitted, an internet page will open and you will be able to use an time remaining from the 45 minutes allocated for the previous period to browse the internet. During this time, you can watch videos, listen to music, read articles, check your emails, play mini-games, consult social networks. You can also browse merchants' websites but we would discourage you from purchasing during the experiment, and we will not be responsible in the event of a problem. A directory of websites that potentially might interest you is provided in the appendix to these instructions. This directory is information only. You can visit other websites (in compliance with the charter of use of the Internet of the University of Nice). In order to guarantee anonymity and confidentiality of your navigation, the web page will open in "private navigation" mode thus, no navigation data can be recorded by the experimenter.

**End of Step3** As soon as the 45 minutes allocated to step 3 have passed, the web page will close automatically and you will be redirected to the experiment screen where your results will be displayed. We will tell you how many ITEMS you and the members of your group placed in the corresponding boxes, and the earnings resulting. We will also display the choices made and the earnings accrued in the first two steps. The total earnings will be displayed along with the euro value based on the rate  $100 \text{ ECU} = 1 \text{ euro}$  and rounded to 50 euro cents.

At that stage, the experiment has been completed. We ask you not to stand up and refer to the last page of the instructions describing the payment procedure.

All the instructions have now been provided and the experiment will start in a few moments. We ask you to focus on the computer screen and to put on your headphones. In case of a problem, do not hesitate to raise your hand and wait for

the experimenter to come to you. We remind you that communication with other participants in the room, and use of mobile phones are prohibited during the entire experiment. The following page of the instructions describes the end of experiment and the payment procedure. They will be read out to you when the experimenter informs you orally that the experiment has ended.

We thank you for your attention and hope you enjoy the experiment

## List of websites

<a href="http://www.google.com">www.google.com</a>	<a href="http://www.leconomiste.eu">www.leconomiste.eu</a>
<a href="http://www.pagesjaunes.fr">www.pagesjaunes.fr</a>	<a href="http://www.lesechos.fr">www.lesechos.fr</a>
<a href="http://www.yahoo.com">www.yahoo.com</a>	<a href="http://www.facebook.com">www.facebook.com</a>
<a href="http://www.Lemonde.fr">www.Lemonde.fr</a>	<a href="http://www.twitter.com">www.twitter.com</a>
<a href="http://www.lefigaro.fr">www.lefigaro.fr</a>	<a href="http://www.linkedin.com">www.linkedin.com</a>
<a href="http://www.lequipe.fr">www.lequipe.fr</a>	<a href="http://www.tumblr.com">www.tumblr.com</a>
<a href="http://www.allocine.fr">www.allocine.fr</a>	<a href="http://www.youtube.com">www.youtube.com</a>
<a href="http://www.msn.com">www.msn.com</a>	<a href="http://www.jeuxvideo.com">www.jeuxvideo.com</a>
<a href="http://www.wikipedia.org">www.wikipedia.org</a>	<a href="http://www.minijeux.com">www.minijeux.com</a>
<a href="http://www.commentcamarche.net">www.commentcamarche.net</a>	<a href="http://www.dailymotion.com/fr">www.dailymotion.com/fr</a>
<a href="http://www.nouvelobs.com">www.nouvelobs.com</a>	<a href="http://www.leboncoin.fr">www.leboncoin.fr</a>
<a href="http://www.meteofrance.com">www.meteofrance.com</a>	<a href="http://www.amazon.fr">www.amazon.fr</a>
<a href="http://www.journaldunet.com">www.journaldunet.com</a>	<a href="http://www.ebay.fr">www.ebay.fr</a>
<a href="http://www.worldpress.com">www.worldpress.com</a>	<a href="http://www.cdiscount.com">www.cdiscount.com</a>



## CHAPTER 3

# Revealed Preferences under Stochastic Attention: Characterization, Statistical Tests and Experimental Implementation

### 3.1 Introduction

Introduced by Samuelson (1938), revealed preference theory is one of the cornerstones of modern economic theory as it provides testable necessary and sufficient conditions for the observed choices to be consistent with complete and transitive preferences maximization. Choices reveal deterministic preferences if they respect the Weak Axiom of Revealed Preference (WARP), which is equivalent to the statement that in every set of alternatives  $A$ , the decision maker (DM) has a “preferred” alternative  $x_A^\succ$ , such that no alternatives contained in  $A$  but  $x_A^\succ$  could be chosen from any set  $B$  where  $x_A^\succ$  is available. However empirical studies have shown systematic violation of WARP (Sippel, 1997). These violations are often in conflict with any deterministic theory of choice: individuals do not always replicate choices when confronted to the same set of alternatives. In these conditions choices may still be consistent with the maximization of random preference relations, as long as the choice probabilities respect stochastic versions of WARP (a typical example is the Regularity Condition which requires that extending the choice set cannot increase former alternatives’ choice probabilities, see Dasgupta and Pattanaik 2007). Such random choice models are referred to the class of Random Utility Models (RUM,

hereafter) (Block and Marschak, 1960; Gul et al., 2014).

Instead of considering maximization of random preferences, this chapter focuses on randomness in DM’s attention to explain randomness in choices. In this chapter, we provide theoretical characterization of choices under stochastic attention and deterministic preferences, we derive statistical methods to test the axioms, measure attention and to reveal underlying preferences, and apply those methods to data collected from a laboratory experiment.

Considering limited attention in economic theory is an important theoretical and practical challenge that has receiving a growing interest until becoming a central topic in Behavioral Economics (Gabaix, 2019). When surrounded by a huge amount of information and facing a wide variety of alternatives, a rational individual with well defined preferences but limited attention may not being able or willing to consider and compare all the available alternatives. Individuals are thus likely not to select the preferred alternative. From a theoretical point of view, violations of standard rationality axioms occur. As a consequence, it exists no preference relation that can be directly revealed by the choice, and further standard utility-based analysis will be flawed. From a practical point of view, limited attention is linked with welfare adverse effects (Falkinger, 2008) that are aggravated with information abundance (as it enhances attention scarcity). Therefore, the production and diffusion of information at almost zero marginal costs caused by the digital revolution, requires a better integration of the concept of attention in economic analysis.<sup>1</sup>

Two main approaches have been developed so far to integrate the concept of attention into economic theory: the “*rational (in)attention*” approach and the “*consideration set*” approach. Both approaches share the same desire not to open the black-box of *how* attention is allocated and entrust the examination of those mecha-

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<sup>1</sup>One of the first identifications of attention as an economic concept and discussion about the adverse effects of its scarcity could be attributed to Hotelling: “*Another thing of limited quantity for which the demand exceeds the supply is the attention of people. [...] Taxation of advertising on this basis would be in addition to any taxation imposed for the purpose of diminishing its quantity with a view to restoring the property of attention to its rightful owners*” (Hotelling 1938, p.257). However, the popularization of the concept is due to Simon: “*a wealth of information creates a poverty of attention and a need to allocate that attention efficiently among the overabundance of information sources that might consume it.*” (Simon, 1971, p.40)

nisms to psychologists.<sup>2</sup> The rational (in)attention approach was introduced by Sims (2003) based on Shannon’s information theory (1948). Paying attention reduces the inherent uncertainty of the choice problem. Limited attention is modeled through limitations in the DM’s capacity to process information, represented by costly information acquisition (usually a function of the Shannon entropy). Information acquisition costs are integrated into the DM’s optimization program, who remains rational in the sense that she allocates optimally her attention, in order to maximize her expected utility, according to her *ex-ante* beliefs. Important equivalence results between rational (in)attention models and RUM have been established recently.<sup>3</sup>

The consideration set approach has a long history in marketing literature (Hauser and Wernerfelt, 1990) and has been reintroduced quite recently in economics. The DM is assumed to maximize a well-defined preference relation over only a subset of the available alternatives, called *consideration set*, ignoring the alternatives that do not belong to the consideration set. This framework can be used for market analysis (Eliaz and Spiegel 2011; see Hefti and Heinke 2015 for a review) or for revealed preference analysis. The chapter focuses on this latter application, introduced by Masatlioglu et al. (2012) and referred as “revealed (in)attention”. In “revealed (in)attention” theories, consideration sets are treated as primitives of the model at a similar level as preferences or beliefs. In principle, no assumption – not even an efficiency one – are made on how attention is allocated, how information is processed and how the consideration set is formed. Instead, the theory confines itself to derive necessary and sufficient conditions for an observable choice function to be consistent with the maximization of a preference relation over a consideration set that respects some desirable properties (Masatlioglu et al., 2012; Manzini and Mariotti, 2014; Brady and Rehbeck, 2016; Lleras et al., 2017). This approach presents numerous

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<sup>2</sup>Psychologists have widely documented how humans allocate their attention and the underlying processes (see for instance Kahneman (1973) or Pashler (2016)). Attention processes are divided into two classes of processes that respectively refer to the ability for the attention to be attracted by salient stimuli or characteristics (bottom-up processes) and to be directed toward stimuli or characteristics that interest the individual (top-down processes). These processes are often studied using clues, distractors and parallel tasks (the most famous paradigms have been proposed by Stroop 1935 and Posner et al. 1980).

<sup>3</sup>Matejka and McKay (2015) have shown that rational inattention modeled by Shannon Entropy based costly information acquisition implies the well-known multinomial logit-RUM. Fosgerau et al. (2016) generalize this result to any additive RUM using a more general form of entropy.

appeals. First it extends the scope of rationalizable behaviors outside the class of RUM. Second it provides behavioral and axiomatic foundations to numerous market models with inattentive consumers. Third, identifying the appropriate revealed (in)attention model for a particular data set may be informative about the properties of the underlying consideration sets. Fourth, it allows to disentangle the preferences and the alternatives considered by the DM and provide a measure of the DM effective attention, revealed by choices (Garrouste et al., 2017).<sup>4</sup>

Since *revealed (in)attention theories* impose precise axioms over the choice functions or the choice probabilities, the validity of these axioms should be tested empirically. Proposing a thorough empirical test of any model would require the following steps: (1) the existing axioms expressed in terms of choice functions or probabilities need to be translated into testable hypotheses, (2) adapted statistical procedures have to be developed to test those hypotheses and (3) empirical data have to be collected to apply those statistical procedures.<sup>5</sup>

This chapter contributes to each of these steps. We decide to concentrate on testing the Random Conditional Choice Set Rule (RCCSR) proposed by Brady and Rehbeck (2016) as it is an “extension” of two seminal articles in the *revealed (in)attention* literature: Masatlioglu et al. (2012) and Manzini and Mariotti (2014). In Section 3.2, we review those models and highlight statistical difficulties that hinder proper empirical test of RCCSR’s axioms. We thus propose a weaker form of the model (namely a weak Random Conditional Consideration Set Rule, wRCCSR) which allows degenerate measures of attention and present new characterization and revealed preference theorems in Section 3.3. In Section 3.4 we provide statistical procedures to test the axioms and to reveal the preferences. We study the asymptotic

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<sup>4</sup>Consideration set theories may account for many violations of the standard rational choice theory commonly observed in experiments (e.g. menu-dependent choice reversals and violations of the regularity condition). The different models aimed to capture different patterns induced by limited attention. For example, choices characterized by consideration sets *a la* Masatlioglu et al. (2012) would reveal unawareness to the unconsidered alternatives while choices characterized by consideration sets *a la* Lleras et al. (2017) would reveal an attentional competition between the alternatives.

<sup>5</sup>In a related spirit, and in the case of the axiom of preference transitivity, Regenwetter et al. (2010) review common issues related to each of these steps and propose theoretical and empirical solution to tackle those issues. Dean and Neligh (2019) experimentally test rational (in)attention models using the data from the experiment presented in Caplin and Dean (2014).

level and power of the tests using simulations. The results from a laboratory experiment, implemented in Section 3.5, suggest that most of the participants do not violate the axioms of the model and that the induced preferences can be recovered by our method.

## 3.2 Revealed Preferences and (in)Attention

### 3.2.1 Attention Filter and Consideration Set

The first axiomatization of *choice with limited attention* is proposed by Masatlioglu et al. (2012) (MNO hereafter). In their model, there is a finite set of feasible alternatives  $X$ . The decision maker (DM) faces non empty menus of alternatives  $A \subseteq X$  and must choose exactly one alternative inside  $A$ . The choice with limited attention function is characterized by a complete preference relation  $\succ$  over  $X$  and a consideration set mapping  $\Gamma$ .  $\Gamma$  is a mapping that associates, for each menu  $A$  proposed to the DM, a subset  $\Gamma(A) \subseteq A$  of alternatives that she considers. Moreover, MNO propose the following property for a consideration set mapping to be defined as an *Attention Filter* (AF).

**Definition 3.2.1.** *Attention Filter (MNO, 2012).*

*A consideration set mapping  $\Gamma : 2^X \rightarrow 2^X$  is an attention filter (AF) if and only if for any  $A \subseteq X$ ,  $A \neq \emptyset$ ,  $\Gamma(A) = \Gamma(A \setminus a)$  whenever  $a \notin \Gamma(A)$ .<sup>6</sup>*

This definition states that an Attention Filter is a Consideration Set that remains stable when the alternatives that are not considered are no longer available. The rationale behind this definition is that if a DM is not attentive to alternatives inside a menu, then she should not be able to discriminate among situations that change only with respect to the presence or the absence of those alternatives. Removing an alternative that does not attract attention should not be noticed by the DM and should have no impact on the set of alternatives she considers.<sup>7</sup>

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<sup>6</sup>Throughout the chapter, we abuse the notation by suppressing set delimiters, e.g., writing  $p(a, ab)$  instead of  $p(a, \{a, b\})$ ,  $\Gamma(ab)$  instead of  $\Gamma(\{a, b\})$ ,  $A \setminus a$  instead of  $A \setminus \{a\}$ .

<sup>7</sup>Masatlioglu et al.'s AF (2012) captures a conception of inattention linked to unawareness. It is possible to assume and characterize other properties for the consideration set. For example,



Based on this definition, a *choice with limited attention* (CLA) consists in choosing the  $\succ$ -preferred alternatives within an AF.

**Definition 3.2.2.** *Choice with limited attention (MNO, 2012).* A choice function  $c$  is a choice with limited attention (CLA) if there exists a complete and transitive preference  $\succ$  over  $X$  and an attention filter  $\Gamma$  such that for all non-empty  $A \subseteq X$ ,  $c(A)$  is the  $\succ$ -best element in  $\Gamma(A)$ .

MNO characterize a deterministic choice function and a deterministic AF. However, there are several arguments in favor of modeling stochastic consideration set. First, as discussed in the introduction, violations of deterministic theory of choice are frequent and can be rationalized by stochastic attention. Second, attention allocation could be viewed either as a fundamental random process or as a process influenced by unobserved random factors in the environment. Third, expressing consideration sets as probability measures over the possible alternatives could be interpreted as different levels in intensities in attention. Fourth, markets with inattentive consumers are often modeled through random consideration set (Hefti and Heinke, 2015), thus characterizing random consideration set would provide testable behavioral foundations to those models.

Manzini and Mariotti (2014) (MM hereafter) proposes a characterization of choice under stochastic consideration set. In their model, there is a nonempty finite set of alternatives  $X$  and a *domain*  $\mathcal{D}$  of subsets of  $X$ , that are the different menus  $A \subseteq X$  in which choices are observable.<sup>8</sup> Moreover, the DM is allowed “not to choose” any alternative from a menu. This outside option is modeled by the presence of a default alternative, noted  $o$ , that is available in any menu. Therefore, for any menu  $A$  proposed to a DM, we observe choices in the set  $A \cup o$ .<sup>9</sup>

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Lleras et al. (2017) define a “Competitive Consideration Set” to capture better the idea of choice overload. “Competitive Consideration Set” are consideration sets such that the consideration set of a menu  $B \subset A$  is a subset of the consideration set of the menu  $A$ . The two models are not disjoint, for example standard rational choice theory is a special case of both.

<sup>8</sup> $\mathcal{D}$  satisfies the following richness assumption:  $\{a, b, c\} \in \mathcal{D}$  for all distinct  $a, b$  and  $c \in X$  and  $A \in \mathcal{D}$  whenever  $B \in \mathcal{D}$  and  $A \subseteq B$ .

<sup>9</sup>Manzini and Mariotti (2014) and Brady and Rehbeck (2016) discuss the implication of removing  $o$ . They show that if the default alternative is removed from their framework, it is no longer possible to identify preferences over the two least preferred alternatives.

**Definition 3.2.3.** *Random choice rule (MM, 2014).*

A random choice rule is a map  $p : X \cup o \times \mathcal{D} \rightarrow [0, 1]$  such that for all  $A \in \mathcal{D}$ ,  $\sum_{a \in A \cup o} p(a, A) = 1$ ; for all  $a \notin A \cup o$ ,  $p(a, A) = 0$ ; and for all  $A \in \mathcal{D} \setminus \emptyset$ ,  $p(a, A) \in (0, 1)$  for all  $a \in A \cup o$ .

MM's model is based on non-degenerate random choice rules: the probability to choose an available alternative is always positive as well as the probability of not choosing any alternative inside a menu (choosing the default). In MM, the choice randomness is explained by the randomness in the alternatives considered by a DM with a deterministic, complete and transitive preference relation. Using MNO notations, each alternative has a probability  $\gamma(a) = \mathbb{P}(a \in \Gamma(A))$  of being considered and will be chosen if it is the  $\succ$ -maximal alternative considered.

**Definition 3.2.4.** *Random consideration set rule (MM, 2014).*

A random consideration set rule (RCSR) is a random choice rule  $p_{\succ, \gamma}$  for which there exists a pair  $(\succ, \gamma)$ , where  $\succ$  is a strict total order on  $X$  and  $\gamma$  is a map  $\gamma : X \rightarrow (0, 1)$  such that:

$$p_{\succ, \gamma}(a, A) = \gamma(a) \times \prod_{\substack{b \in A \\ b \succ a}} (1 - \gamma(b)) \text{ for all } A \in \mathcal{D}, \text{ for all } a \in A$$

The probability to choose an alternative  $a$  is thus the probability that  $a$  belongs to the consideration set and all the alternatives  $b$  preferred to  $a$  do not belong to the consideration set.

Several important consequences derive from this definition. First,  $\gamma$  is not a probability measure over  $X$  as the events  $a \in \Gamma(A)$  and  $b \in \Gamma(A)$  are not mutually exclusive. Second, the model assumes that attention is paid independently between the available alternatives and across the different menus. Indeed, the probability to pay attention to  $a$  does not depend on the presence nor on the absence of other alternatives in the consideration set : we have  $\gamma(a) = \mathbb{P}(a \in \Gamma(A) | b \in \Gamma(A)) = \mathbb{P}(a \in \Gamma(A) | b \notin \Gamma(A))$  for all  $A \in \mathcal{D}$ , and for all  $a$  and  $b \in A$ . Moreover, this probability is independent from the presence or from the absence of other alternatives in the menu:  $\gamma(a) = \mathbb{P}(a \in \Gamma(A)) = \mathbb{P}(a \in \Gamma(B))$  for all  $A$  and  $B$  such that  $a \in A \cap B$ .

This “attentional menu-independence” imposes strong restrictions on the behaviors that are rationalizable. Indeed one could easily imagine situations falsifying these conditions. For example, let consider an arbitrary large set of alternatives  $X$ , with the following preference order:  $(a_1 \succ \dots \succ a_N)$ . If  $p$  is a RCSR, then we must have  $p(a_1, X) = p(a_1, \{a_1\}) = \gamma(a_1)$ , that is the probability to consider and to choose the best available alternative does not depend on the number of other alternatives proposed.

### 3.2.2 Random Conditional Choice Set Rule

MM are aware about this limitation and open the way towards the characterization of menu-dependent consideration set in the last section of their article. They show that if menu-dependent consideration sets are allowed with no more assumptions, then the model is no longer falsifiable and fails to place any empirical restrictions on the random choice rules. That is any random choice rule could be represented by any preference relation with different menu-dependent random consideration sets.<sup>10</sup>

In a comment to MM’s article, Brady and Rehbeck (2016) (BR hereafter) propose some restrictions that accommodate for menu-dependence and have an empirical content. The general random choice rule considered here is identical to the one used in MM (see definition 3.2.3). Choices are observed over a domain of menu  $\mathcal{D}$  and are made by a DM who has always a positive probability to choose any alternative and a positive probability to choose the default alternative within any menu  $A \in \mathcal{D}$ . Moreover  $\mathcal{D}$  satisfies a slightly weaker richness assumption as in MM :  $\{a, b\} \in \mathcal{D}$  for all distinct  $a, b \in X$  and  $A \in \mathcal{D}$  whenever  $B \in \mathcal{D}$  and  $A \subseteq B$ .

BR model a DM with a complete preference order  $\succ$  and a stochastic consideration set. The probability that, when facing a menu  $A \in \mathcal{D}$ , the DM considers a particular subset of alternatives  $B$ , is given by a full support probability measure

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<sup>10</sup>“In some circumstances, it may be plausible to assume that the attention parameter of an alternative depends on which other alternatives are feasible. For example, a brightly colored object will stand out more in a menu whose other elements are all gray than in a menu that only contains brightly colored objects. [...] So, once we allow the attention parameters to be menu dependent, not only does the model fail to place any observable restriction on choice data, but the preference relation is also entirely unidentified. Strong assumptions on the function  $\gamma$  are needed to make the model with menu dependent attention useful, but we find it difficult to determine a priori what assumptions would be appropriate ” (Manzini and Mariotti, 2014, p. 1164-65).

over the domain  $\pi : \mathcal{D} \rightarrow \mathbb{R}_+^*$  as follows:

$$\forall A \in \mathcal{D}, \mathbb{P}(\Gamma(A) = B) = \frac{\pi(\{B\})}{\sum_{C \subseteq A} \pi(\{C\})} \text{ if } B \subseteq A \text{ and } 0 \text{ otherwise.}^{11}$$

To unify the notations of MNO, MM and BR, we have for all  $a \in A$  and for all  $A \in \mathcal{D}$ .

$$\mathbb{P}(a \in \Gamma(A)) = \gamma(a, A) = \frac{\sum_{\substack{B \subseteq A \\ a \in B}} \pi(B)}{\sum_{C \subseteq A} \pi(C)}$$

We use the same example as in BR (p.1211) to show how a RCCSR models menu-dependent attention.

**Example 3.2.1.** *Let consider the following probability measure:  $\pi(bc) = \frac{8}{20}$ ,  $\pi(abc) = \frac{4}{20}$ ,  $\pi(b) = \frac{3}{20}$  and  $\pi(ab) = \pi(ac) = \pi(a) = \pi(c) = \pi(\emptyset) = \frac{1}{20}$ .*

*The probability to pay attention to the alternative  $c$  decreases when we remove the alternative  $b$  from the menu  $\{a, b, c\}$  or conditional on  $b$  not being considered:*

$$\begin{aligned} \mathbb{P}(c \in \Gamma(abc)) &= \frac{14}{20} \\ \mathbb{P}(c \in \Gamma(abc) | b \notin \Gamma(abc)) &= \frac{1}{2} \\ \mathbb{P}(c \in \Gamma(ac)) &= \frac{1}{2} \end{aligned}$$

It is not surprising to have  $\mathbb{P}(c \in \Gamma(ac)) = \mathbb{P}(c \in \Gamma(abc) | b \notin \Gamma(abc))$  in the previous example. Indeed it can be easily shown (using Bayes Rules and simple algebra) that regardless of the values taken by  $\pi$ , we have:

$$\mathbb{P}(\Gamma(A) = B | a \notin \Gamma(A)) = \mathbb{P}(\Gamma(A \setminus a) = B) \text{ for all } A \in \mathcal{D}, B \subseteq A \text{ and } a \in X.$$

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<sup>11</sup>To avoid any confusion, we insist on the fact that  $\pi$  is not a probability measure over  $X$  but over  $\mathcal{D}$ , which is a collection of subsets of  $X$ . Therefore, all elements of  $\mathcal{D}$  are considered as mutually exclusive realisations of a random variable  $\Gamma(A)$  taking value in  $2^A$ . Throughout this chapter,  $\pi$  is only used to measure elementary events – as well as it is in BR’s article. Therefore we decide from now to unequivocally avoid the use of multiple brackets and we adopt identical notations as in BR:  $\pi(A)$  instead of  $\pi(\{A\})$  and  $\pi(ab)$  instead of  $\pi(\{\{a, b\}\})$ . Therefore, one should not expect to have  $B \subset A \Rightarrow \pi(B) \leq \pi(A)$  (as normally required by  $\sigma$ -additivity), and thus should not be surprised to read  $\pi(\emptyset) > 0$ .

This property states that, when facing a menu  $A$ , the conditional probability to pay attention to an alternative  $a$ , given not paying attention to the alternative  $b$ , is equal to the probability to pay attention to  $a$  when  $b$  is removed from the menu  $A$ . The rationale behind this property is similar as the one behind MNO's definition 3.2.1. Removing an alternative whenever it is not considered does not impact how DM's attention is allocated toward the other alternatives.

Based on this definition of stochastic consideration set, BR model the choice of a DM maximizing a complete preference order  $\succ$ . Therefore, as in MM and MNO, the probability to choose an alternative  $a$  when facing a menu  $A$  is simply the probability that  $a$  is the  $\succ$ -best considered alternative when facing a menu  $A$ . a RCCSR thus may be seen as a stochastic counterpart of MNO's CLA.

**Definition 3.2.5.** *Random Conditional Choice Set Rule (BR, 2016).*

A random conditional choice set rule (RCCSR) is a random choice rule  $p_{\succ, \pi}$  for which there exists a pair  $(\succ, \pi)$  where  $\succ$  is a strict preference ordering on  $X$  and  $\pi : \mathcal{D} \rightarrow (0, 1)$  a full support probability measure over  $\mathcal{D}$ , such that, for all  $A \in \mathcal{D} \setminus \{\emptyset\}$  and for all  $a \in A$ ,

$$p_{\succ, \pi}(a, A) = \mathbb{P}(\Gamma(A) \in A_{a \succ}) = \frac{\sum_{B \in A_{a \succ}} \pi(B)}{\sum_{C \subseteq A} \pi(C)},$$

with  $A_{a \succ} = \{B \subseteq A \text{ s.t. } a \in B \text{ and } \forall b \in B \setminus a, a \succ b\}$ , the set of subsets of  $A$  where  $a$  is the  $\succ$ -best alternative.

The key property on choices induced by BR's form of attentional menu-dependence is called Sequential Independence (SI). BR's characterization and revealed preferences theorems stand on this property. This property links two alternatives within a menu as follows.

**Definition 3.2.6.** *Sequential Independence (BR, 2016).*

An alternative  $b \in X$  is sequentially independent from an alternative  $a \in X$ , in a menu  $A \in 2^X$  (and we note  $bI_A a$ ), iff

$$A \in \mathcal{D}, a, b \in A \text{ and } a \neq b \Rightarrow p(b, A \setminus a) = p_{-a}(b, A),$$

with  $p_{-a}(b, A) = \frac{p(b, A)}{1 - p(a, A)}$  the conditional probability of choosing  $b$  in  $A$ , given  $a$  not being chosen.

Therefore, an alternative  $b$  is sequentially independent from a distinct alternative  $a$  in a menu  $A \in \mathcal{D}$  if the probability to choose  $b$  when  $a$  is removed from  $A$  is the same as the conditional probability of choosing  $b$  in  $A$ , given  $a$  not being chosen. To simplify further notations, we extend SI from alternatives to menus of alternatives as follows:

**Notation.** A menu  $A \subseteq X$  is sequentially independent from an alternative  $a \in A$  (and we note  $AIa$ ) iff all alternatives  $b$  are sequentially independent from  $a$  in  $A$ .

$$AIa \Leftrightarrow \forall b \in A \setminus a, bI_Aa.$$

The following axioms specify the collection of SI relations that uniquely characterizes a RCCSR.<sup>12</sup>

**Definition 3.2.7.** (BR, 2016)

*ASI — Asymmetric Sequential Independence:* For all distinct  $a, b \in X$ , exactly one of the following holds:

$$aI_{ab}b \text{ or } bI_{ab}a.$$

*TSI — Transitive Sequential Independence:* For all distinct  $a, b, c \in X$ ,

$$aI_{ab}b \text{ and } bI_{bc}c \Rightarrow aI_{ac}c.$$

*ESI — Extensive Sequential Independence:* For all  $a \in X$  and all menus  $A, B \in \mathcal{D}$  such that  $a \in A \cap B$ :

$$AIa \text{ and } BIa \Rightarrow A \cup BIa.$$

---

<sup>12</sup>To limit the number and the use of notations, we have modified BR's notations and presented an equivalent form of the IFO axiom that is used in BR's proof (BR state in their article the equivalence between the original definition of the IFO axiom and the version presented here).

*I*FO — *I*ncreasing *F*easible *O*dds: For any  $A \in \mathcal{D}$ ,  $|A| \geq 2$ ,

$$\sum_{B \subseteq A} (-1)^{|A \setminus B|} \frac{1}{p(o, B)} > 0.$$

ASI requires that exactly one SI relation holds in every binary menu, TSI requires that binary SI relations are transitive, and ESI requires that union of sets that are sequentially independent from a same alternative is also sequentially independent from this alternative. The IFO axiom is a kind of regularity condition on the outside option. It states roughly that the probability to choose the default option decreases at an increasing rate as we remove alternatives. Taken together, those axioms are equivalent to a RCCSR:

**Theorem 3.2.1.** *Characterization (BR, 2016).*

*A random choice rule  $p$  satisfies ASI, TSI, ESI and IFO if and only if it is a RCCSR  $p_{\succ, \pi}$ . Moreover, both  $\succ$  and  $\pi$  are unique.*

The intuition behind the characterization is the following: if  $p$  is a RCCSR, then in any menu, all the alternatives must be sequentially independent to the  $\succ$ -best alternative. Indeed, let  $a$  be the  $\succ$ -best alternative of a menu  $A$ , then choosing or paying attention to  $a$  in  $A$  corresponds to the same event. So for any  $b$  in  $A$ , the conditional probability of choosing  $b$ , given  $a$  not being chosen should equals the probability of choosing  $b$  when  $a$  has been removed from  $A$ .

Therefore, ASI requires that in each binary menu  $\{a, b\}$ , exactly one SI relation holds, so that each binary menu can have exactly one potential maximum. The SI relations within binary menus would now define a binary relation:  $a \succ b \Leftrightarrow bI_{ab}a$ . ASI imposes  $\succ$  to be complete and asymmetric, while TSI guarantees that this relation is transitive. ESI extends the SI to higher level menu to ensure that if  $a$  is the potential maximal alternative in two menus, then  $a$  should be also a potential maximal alternative in the union of these menus. ASI, TSI and ESI are sufficient for the existence of a complete preference order and a mapping  $\lambda$  that satisfies the property of a RCCSR. Technically IFO ensures the positivity of the  $\lambda$  and thus allows to built  $\pi$  as a probability measure over  $\mathcal{D}$ .

Knowing that  $p$  is a RCCSR, it is possible, from choice probabilities, to disentangle the preference relation  $\succ$  from the attentional measure  $\pi$ .

**Proposition 3.2.1.** (BR, 2016). *Let  $p_{\succ, \pi}$  a RCCSR. For all  $A \in \mathcal{D}$ , for all  $a, b \in A$ , we have the following properties:*

- i)  $a \succ b$  iff  $bI_{aba}$ ;
- ii)  $\mathbb{P}(a \in \Gamma(A)) = 1 - \frac{p(o, A)}{p(o, A \setminus a)}$ ;
- iii)  $\pi(A) = C \times \sum_{B \subseteq A} \left( (-1)^{|A \setminus B|} \times \frac{1}{p(o, B)} \right)$ , with  $C \in \mathbb{R}_+$  a scaling factor such that  $\sum_{A \in \mathcal{D}} \pi(A) = 1$ .

i) is directly obtained from the characterization theorem: the binary SI relations represent a preference order since they are complete, asymmetric (ASI) and transitive (TSI), ii) can be directly derived from the definition of the RCCSR, and iii) is obtained by construction of the RCCSR using Möbius inversion. If  $\mathcal{D} = 2^X$  then  $C = p(o, X)$ .

### 3.2.3 Issues related to statistical testing of RCCSR

When choices follow a RCCSR, it is theoretically possible to disentangle the preference relation and the probability to consider any subset of available alternatives. In this chapter, we are interested in testing RCCSR's axioms and to identify preferences using empirical data from a laboratory experiment. For this purpose, we first need to derive testable hypotheses from BR axioms. The fact that the underlying choice probabilities are not directly observable raises difficulties that we highlight in this section. As an illustration, we start this section by borrowing BR's example 1.

**Example 3.2.2** (Brady and Rehbeck). *Consider the following set of alternatives  $X = \{a, b, c\}$  and the random choice rule described in Table 3.2.1 (lines represent alternatives and columns menu).*

*From those probabilities, it is easy to check that ASI, ESI, TSI and IFO hold and that the random choice rule is a RCCSR with  $a \succ b \succ c$ ,  $\pi(\emptyset) = \frac{1}{20}$ ,  $\pi(a) = \frac{1}{20}$ ,  $\pi(b) = \frac{3}{20}$ ,  $\pi(c) = \frac{1}{20}$ ,  $\pi(ab) = \frac{1}{20}$ ,  $\pi(ac) = \frac{1}{20}$ ,  $\pi(bc) = \frac{8}{20}$  and  $\pi(abc) = \frac{4}{20}$ .*



	abc	ab	ac	bc	a	b	c
a	7/20	1/3	1/2	0	1/2	0	0
b	11/20	1/2	0	11/13	0	3/4	0
c	1/20	0	1/4	1/13	0	0	1/2
<i>o</i>	1/20	1/6	1/4	1/13	1/2	1/4	1/2

Table 3.2.1: Example of RCCSR (Brady and Rehbeck, 2016, p.1211).

However, in real situations, an observer (e.g. a researcher, a social planner, an economist, a firm, etc.) does not have access to these choice probabilities, but observe their realizations. Consider an observer who has access to 100 independent choices in all of the 7 possible menus as described in Table 3.2.2.

	abc	ab	ac	bc	a	b	c
a	41	34	41	0	47	0	0
b	50	56	0	85	0	80	0
c	7	0	33	9	0	0	60
<i>o</i>	2	10	26	6	53	20	40

Table 3.2.2: Example of observed choices

**Note:** Hundred choices for each menu have been simulated using Table 3.2.1 probabilities.

The observer wants i) to test if the underlying random choice rule  $p$  can be characterized by a RCCSR and ii) disentangle the preference relation  $\succ$  and the attentional measure  $\pi$ . Moreover a researcher could be interested to test the different axioms independently. We shall construct statistical tests to assess the likelihood of the observed data conditional on ASI, ESI, TSI and IFO.

A main requirement for any statistical test is to control its level, that is the probability of rejection of the tested “null” hypothesis, conditionally of the hypothesis being true. Applied to our context, this implies the construction of a statistical test that should be rejected with a probability lower than  $\alpha$  when observing choices drawn from a RCCSR, where  $\alpha$  is a nominal level accepted and fixed by the observer.

We will investigate in this section the existence of such tests for BR’s axioms. Throughout this section, we assume that we observe independent and identically distributed choices across all the possible menus, produced by a RCCSR. Moreover, without loss of generality, we assume  $a \succ b$ . For the sake of simplicity we also

assume that the number of observed decisions per menu  $N$  is equal across menus. This latter assumption could be easily relaxed.

Let decompose the different hypotheses that compose ASI:

$$H0^{ASI} \Leftrightarrow \bigcap_{\substack{a,b \in \mathcal{D} \\ a \neq b}} H0_{ab}^{ASI}$$

Where

$$H0_{ab}^{ASI} \Leftrightarrow H0_{a>b}^{ASI} \cup H0_{b>a}^{ASI},$$

$$H0_{a>b}^{ASI} \Leftrightarrow H0_{bIa}^{ab} \cap H1_{-aIb}^{ab},$$

$$H0_{bIa}^{ab} \Leftrightarrow p(b, b) = p_{-a}(b, ab) \quad \text{and} \quad H1_{-aIb}^{ab} \Leftrightarrow p(a, a) \neq p_{-b}(a, ab).$$

Since  $H0_{a>b}^{ASI}$  and  $H0_{b>a}^{ASI}$  are symmetric and mutually exclusive hypotheses, we will consider, without loss of generality, the case where  $H0_{a>b}^{ASI}$  is true and investigate if it is possible to construct a test where the level is controlled. Under  $H0_{a>b}^{ASI}$ ,  $H0_{bIa}^{ab}$  is true, and the probability to reject the hypothesis can be controlled - as long as  $N$  is large enough to guaranty reasonable asymptotic properties - independently from the model behind  $p$  (e.g.  $p$  is a RCCSR with attentional measure  $\pi$ ) and the number of observations  $N$ .<sup>13</sup>

**Proposition 3.2.2.** *Let  $A \in \mathcal{D}$ , Let  $a, b \in A$ , let  $n(a, A)$  the number of times an alternative  $a$  is chosen in a menu  $A$ . Under the null hypothesis that  $b$  is sequentially independent to  $a$  in the menu  $A$  ( $H0_{bIa}^A : p(b, A \setminus a) = p_{-a}(b, A)$ ), the statistics:*

$$Z_{bIa}^A = \sqrt{N} \times \frac{\hat{\mu}_{bIa}^A}{\hat{\sigma}_{bIa}^A} \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N}(0, 1),$$

*follows asymptotically a standard Gaussian distribution, with*

$$\hat{\mu}_{bIa}^A = \frac{n(b, A)}{N - n(a, A)} - \frac{n(b, A \setminus a)}{N},$$

---

<sup>13</sup>See Section 3.B for the detailed procedure.

and

$$\hat{\sigma}_{bIa}^A = \sqrt{\frac{n(b, A \setminus a) \times [N - n(b, A \setminus a)]}{N^2} + \frac{N \times n(b, A) \times [N - n(a, A) - n(b, A)]}{[N - n(a, A)]^3}}.$$

However, under  $H0_{a>b}^{ASI}$ ,  $H1_{-aIb}^{ab}$  is true and  $p(a, a) \neq p_{-b}(a, ab)$ . However, this difference can be arbitrarily close to 0, so the probability to reject  $H1_{-aIb}^{ab}$  and to reject  $H0_{aIb}^{ab}$  can be arbitrarily close. Only the probability of rejection of  $H0_{aIb}^{ab}$  under  $H0_{aIb}^{ab}$  can be controlled without placing further assumption on  $p$ . As a consequence the only way to control a priori the probability to reject  $H1_{-aIb}^{ab}$  when  $H1_{-aIb}^{ab}$  is true, is to construct a test with no power (i.e. its level will equal its power).

Therefore, to control the level of the test of  $H0_{ab}^{ASI}$ , it is required to test the weaker hypotheses  $H0_{ab}^{ASI'} \supset H0_{ab}^{ASI}$  :

$$H0_{ab}^{ASI'} \Leftrightarrow (H0_{aIb}^{ab} \cup H0_{bIa}^{ab}),$$

that is an observer should reject  $H0_{ab}^{ASI'}$  only if he rejects both  $H0_{aIb}^{ab}$  and  $H0_{bIa}^{ab}$ . However, further statistical and empirical issues arise when one of those hypotheses is not correctly rejected. Indeed, TSI and ESI are expressed in term of implication of equalities and their testability is thus impacted if false binary sequential independence relation have not been correctly rejected. For example, let consider a data generating process  $p$  that follow a RCCSR on  $\mathcal{D} = 2^{\{a,b,c\}}$ . Let assume that the observer has rejected none of the following hypotheses:  $H0_{aIb}^{ab}$ ,  $H0_{bIc}^{bc}$ ,  $H0_{bIa}^{ab}$  and  $H0_{cIb}^{bc}$ . TSI requires that the two former hypotheses imply  $H0_{aIc}^{ac}$  and the two latter hypotheses imply  $H0_{cIa}^{ac}$ , so any correct rejection of a binary SI in  $\{a, c\}$  required by the model, could be naively interpreted as a falsification of TSI. However, if  $p$  is a RCCSR we know that only one of those two hypotheses is true and we cannot control the rejection probability for the other one. Here again, TSI cannot be tested directly and a much weaker version of the hypothesis should be considered (for example, only cyclical rejections of SI relations, such that rejecting both  $H0_{aIb}^{ab}$ ,  $H0_{bIc}^{bc}$  and  $H0_{cIa}^{ac}$  should be considered as a violation of TSI). A similar argument stands for ESI: if the observer failed to reject any SI relations within the binary

menus, then any correct rejection of SI in a larger menu could be interpreted as a (false) violation of ESI. Moreover, in BR the preference relation  $\succ$  is defined as follows:  $a \succ b \Leftrightarrow bI_{ab}a$ . Therefore the inability to correctly reject SI also undermines the proper identification of the preference relation. To understand how serious these issues are, we address now the question of how likely it is to correctly reject false binary SI relations when  $p$  is a RCCSR.

Let assume  $p$  is a RCCSR with  $a \succ b$ , therefore  $H0_{aIb}^{ab}$  is false. Basically, the probability  $\beta_{a \succ b}$  to correctly reject  $H0_{aIb}^{ab}$  can be computed as a function of the number of observation  $N$  and the difference  $\mu(\pi)$  between  $p(a, a)$  and  $p_{-b}(a, ab)$ . More precisely,  $\beta_{a \succ b}$  is the power of a test of a null hypothesis ( $H0_{aIb}^{ab} : p(a, a) = p_{-b}(a, ab)$ ) under the alternative hypothesis:

$$\left( H1_{-aIb}^{ab} : p_{-b}(a, ab) - p(a, a) = \frac{\pi(a)}{\pi(a) + \pi(\emptyset)} - \frac{\pi(a) + \pi(ab)}{\pi(a) + \pi(\emptyset) + \pi(ab)} \neq 0 \right).$$

We can compute  $\beta_{a \succ b}$  as a function of the nominal level  $\alpha$  used to accept statistical significance, the number of observations per menu  $N$  and the attentional measures  $\pi(\emptyset), \pi(a), \pi(b)$  and  $\pi(ab)$ , using the following proposition.<sup>14</sup>

**Proposition 3.2.3.** *The probability to reject the null hypothesis ( $H0_{aIb}^{ab} : p_{-b}(a, ab) = p(a, a)$ ) with the  $\alpha$  level two-tailed test proposed in proposition 3.2.2, under the alternative hypothesis ( $H1_{-aIb}^{ab} : p_{-b}(a, ab) - p(a, a) = \mu_{a \succ b} \neq 0$ ) that the data generating process  $p$  follows a RCCSR with  $a \succ b$  is given by:*

$$\beta_{b \succ a} = \Phi \left( z_{\alpha/2} - \sqrt{N} \times \frac{\mu_{a \succ b}}{\sigma_{a \succ b}} \right) + \Phi \left( z_{\alpha/2} + \sqrt{N} \times \frac{\mu_{a \succ b}}{\sigma_{a \succ b}} \right),$$

with  $\Phi$  and  $z_{\alpha/2}$  the cumulative distribution function and the  $\frac{\alpha}{2}$  quantile of the standard normal distribution,

$$\mu_{a \succ b} = \frac{\pi(ab)\pi(\emptyset)}{[\pi(a) + \pi(ab) + \pi(\emptyset)][\pi(a) + \pi(\emptyset)]},$$

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<sup>14</sup>More details and demonstration about the proposition are provided in Section 3.B and a script function to compute  $\beta_{a \succ b}$  is provided in the electronic supplementary material.

$$\text{and } \sigma_{a \succ b} = \sqrt{\frac{\pi(a)\pi(\emptyset)}{[\pi(a) + \pi(\emptyset)]^2} + \frac{[\pi(a) + \pi(ab)][\pi(ab) + \pi(a) + \pi(b) + \pi(\emptyset)]\pi(\emptyset)}{[\pi(a) + \pi(ab) + \pi(\emptyset)]^3}}.$$

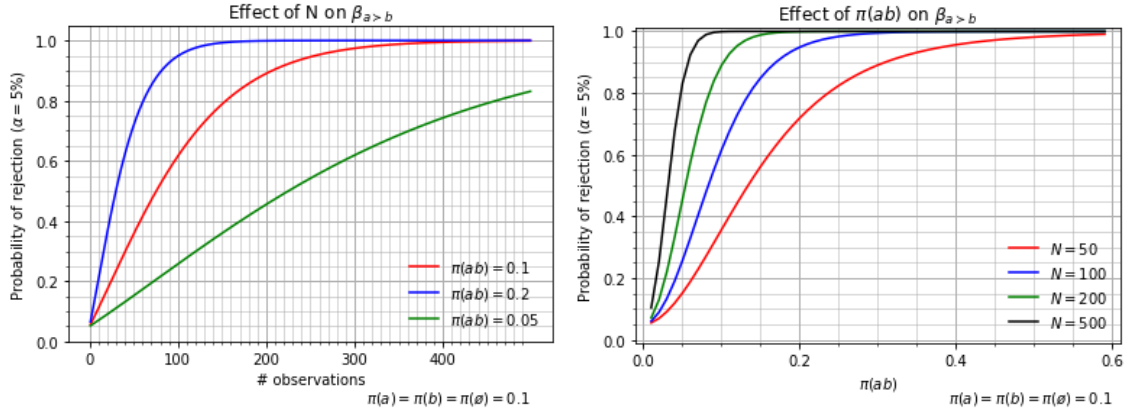
It follows immediately that this probability will increase and converge to 1 with the number of observations per menu  $N$ , and with the measure  $\pi(ab)$ .

**Proposition 3.2.4.**

Let  $p$  a RCCSR, for all  $a, b$  with  $a \succ b$  we have:

$$\begin{aligned} \lim_{\pi(ab) \rightarrow 0} \beta_{a \succ b} &= \alpha \\ \frac{d\beta_{a \succ b}}{dN} &> 0 \text{ and } \frac{d\beta_{a \succ b}}{d\pi(ab)} > 0 \\ \frac{d\beta_{a \succ b}}{d\pi(\emptyset)} &< 0 ; \frac{d\beta_{a \succ b}}{d\pi(a)} < 0 \text{ and } \frac{d\beta_{a \succ b}}{d\pi(b)} < 0 \end{aligned}$$

We now investigate numerically  $\beta_{a \succ b}$  for different values of  $\pi(\emptyset), \pi(a), \pi(b), \pi(ab)$  and  $N$ . If we take the RCCSR defined in Example 3.2.2, we have the same expected differences  $\mu_{a \succ b} = \mu_{a \succ c} = \mu_{b \succ c} = \frac{1}{6}$ . However, if we fix  $\alpha = .05$  and  $N = 100$  observations per menu, we have:  $\beta_{a \succ b} = 51.60\%$  chance to correctly reject  $aI_{abb}$ ,  $\beta_{a \succ c} = 61.60\%$  chance to correctly reject  $aI_{acc}$  and  $89.36\%$  chance to correctly reject  $bI_{bcc}$ . Let assume that under  $H0_{a \succ b}^{ASI}$ , an observer wants to correctly accept  $H0_{bIa}^{ab}$  and correctly reject  $H0_{aIb}^{ab}$  each in 95% of the cases, then if  $\pi(ab) = \pi(a) = \pi(b) = \pi(\emptyset)$ ,  $N = 256$  independent observations per menu are required. This number of observations increases to  $N = 762$  per menu if  $\pi(ab)$  is half as large as  $\pi(a) = \pi(b) = \pi(\emptyset)$  (that is the probability to consider  $a$  and  $b$  simultaneously is the same that the probability to consider only one of them). More generally, the lower  $\pi(ab)$ , the lower the effect of increasing  $N$  on  $\beta_{a \succ b}$ . Figure 3.2.1 shows the probability of correctly rejecting  $H0_{aIb}^{ab}$  under  $H0_{a \succ b}^{ASI}$  as a function of  $\pi(ab)$  and  $N$ .


 Figure 3.2.1:  $\beta_{a>b}$  as a function of  $N$  and  $\pi(ab)$ 

Moreover, the total number of observations to gather has to be multiplied by the number of menu which is increasing with the number of available alternatives. If the observer has to test too many SI relations separately, then the family-wise type-I error rate (FWER) will be controlled only if the observer use corrected p-values, that would even decreases statistical power and the probability to correctly reject the false SI relations.

The aim of this chapter, is to characterize and disentangle preferences from real choices made by a RCCSR. The range of RCCSR that can be properly tested and preferences revealed (i.e. the RCCSR with  $\pi(ab)$  large enough) is therefore highly restricted by the finite number of independent observations that we can gathered. If we test the minimal non-trivial version of the model (i.e. with 3 alternatives  $a \succ b \succ c$  and a default  $o$ ) and cannot propose more than 50 decisions for each of the 7 menus, we would have a probability  $\beta_{a>b}(N = 50, \pi(ab) = \pi(a) = \pi(b) = \pi(\emptyset), \alpha = .95) = 36\%$ . And  $\pi(ab)$  should be 2.5 times larger than  $\pi(a) = \pi(b) = \pi(\emptyset)$  for  $\beta_{a>b}$  to exceed 80%.

Based on the difficulty to properly test ASI, TSI and ESI axioms and to identify preferences using binary sequential independence relations, we argue that another characterization of RCCSR could be preferred to enable a proper empirical implementation and statistical testing of the model. The following section proposes such a characterization.

### 3.3 Weak Random Consideration Choice Set Rule

The difficulty to test BR's axioms and to properly characterize a RCCSR from choice data is due to the fact that  $a \succ b$  requires  $\neg aI_{ab}b$ . Therefore, a slight difference between  $p(a, a)$  and  $p_{\neg b}(a, ab)$  hinders proper tests of ASI, TSI and ESI as well as the identification of preferences. A characterization that does not explicitly require nested axioms and differences between choice probabilities will be more adapted for empirical implementation. If  $p$  is a RCCSR with  $a \succ b$ , then the difference between  $p(a, a)$  and  $p_{\neg b}(a, ab)$  is a direct consequence of  $\pi(ab) > 0$ , which is necessary as  $\pi$  is a full support probability measure. In this section, we relax this latter assumption and characterize a weak version of RCCSR allowing  $\pi(A) = 0$  for some  $A \in \mathcal{D}$ .

#### 3.3.1 Characterization

**Definition 3.3.1.** *Weak Consideration Choice Set Rule (wRCCSR)*

A random choice rule  $p$  is a wRCCSR iff there exists a pair  $(\succ, \pi)$  where  $\succ$  is a strict preference ordering on  $X$  and  $\pi : \mathcal{D} \rightarrow [0, 1]$  a probability distribution over  $\mathcal{D}$ , with  $\pi(\emptyset) > 0$ , such that,  $\forall A \in \mathcal{D} \setminus \{\emptyset\}$  and  $\forall a \in A$ :

$$p_{\succ, \pi}(a, A) = \frac{\sum_{B \in A_{a \succ}} \pi(B)}{\sum_{B \subseteq A} \pi(B)},$$

with  $A_{a \succ} = \{B \subseteq A \text{ s.t. } a \in B \text{ and } \forall b \in B \setminus a, a \succ b\}$ , is the set of subsets of  $A$  where  $a$  is the  $\succ$ -best alternative.

Our definition of wRCCSR is the same as the definition of a RCCSR, excepted that it allows some consideration sets to have a null probability of attracting attention. Empirically, the behaviors captured by a RCCSR and a wRCCSR are similar. Indeed, since a RCCSR accepts arbitrarily small attentional measures, any wRCCSR has arbitrarily precise RCCSR approximations. However, from a theoretical point of view, since ASI does not hold if  $\exists a, b \in X : \pi(ab) = 0$ , there exist 1) series of RCCSR

such that BR's axioms are falsified at the limit, and 2) series of RCCSR with different preference representation that are identical at the limit. That is, while the RCCSR framework assumes a theoretical uniqueness of the preference representation, it is possible to find arbitrarily close RCCSR that has different preference representations while being identical in the behaviors they capture. The wRCCSR extension aims to explicitly capture those situations, where the observed behavior can be rationalized by more than a single preference relation.<sup>15</sup>

On the contrary, the definition of wRCCSR only requires  $\pi(\emptyset) > 0$ . We consider this hypothesis as technical (it is sufficient to guaranty that  $p_{\succ, \pi}$  is well defined). Its behavioral interpretation is that there is always a positive probability that the DM is fully inattentive and chooses the default option  $o$ .<sup>16</sup> Another consequence of this refinement is to integrate the Luce's rule (1959) as a special case of the wRCCSR (the Luce Rule is not a RCCSR because it violates ASI).<sup>17</sup>

The characterization of a wRCCSR stands on the Sequential Independence (SI) relation that has been defined in Section 3.2. However, ASI, TSI and ESI axioms are replaced by the following single axiom.

**Definition 3.3.2.** *Weak Axiom of revealed preferences with stochastic limited attention (WARP-SLA).*

*A random choice rule satisfies WARP-SLA, if each non empty menu  $A \in 2^X$  contains an alternative  $x_A^\succ$  such that any subset of  $A$  is sequentially independent*

<sup>15</sup>For example let  $\mathcal{D} = 2^{\{abc\}} \setminus \{abc\}$  and the series of RCCSR  $(p(\succ_1, \pi_n))_{n \in \mathbb{N}}$  and  $(p(\succ_2, \pi_n))_{n \in \mathbb{N}}$ , such that  $a \succ_1 b$ ,  $b \succ_2 a$ ,  $\pi_n(ab) = \frac{1}{n+1}$  and  $\pi_n(A) = \frac{1}{6} - \frac{1}{6(n+1)}$  if  $A \in \mathcal{D} \setminus \{ab\}$ . We have  $\lim_{n \rightarrow \infty} p(\succ_1, \pi_n) = \lim_{n \rightarrow \infty} p(\succ_2, \pi_n) = p(\succ_1, \pi) = p(\succ_2, \pi)$ , where  $p(\succ_1, \pi)$  and  $p(\succ_2, \pi)$  are two different wRCCSR representations of the same random choice rule falsifying ASI.

<sup>16</sup>As  $p_{\succ, \pi}$  is a random choice rule, we have  $0 < p(a, a) < 1$  for all  $a \in X$ , therefore, we must have  $\pi(A) > 0$  for singleton menus. It could be possible to relax  $\pi(\emptyset) > 0$  and only keep  $\pi(A) > 0$  for all singleton menus. In this case,  $p_{\succ, \pi}$  is well defined but is no longer a random choice rule and we obtain a model with no default option. Such a model with no default option admits arbitrarily close wRCCSR approximation, but since our revealed preference theorem requires that  $\pi(\emptyset) > 0$ , preferences cannot be revealed with the method we proposed when  $\pi(\emptyset) \approx 0$ .

<sup>17</sup>Indeed let  $p$  a wRCCSR, if  $\pi(A) = 0$  for all  $|A| > 1$  it follows immediately and regardless of the preference relation  $\succ$ , that  $p$  is also a Luce Rule as we have for all  $A$  and all  $a \in A$   $p(a, A) = \frac{w(a)}{\sum_{b \in A \cup o} w(b)}$ , with  $w(a) = \pi(a)$ . The weight  $w$  of the Luce Rule are usually interpreted as a measure of preferences (such that for all  $A$  and for all  $a, b \in A$   $a \succ b \Leftrightarrow w(a) > w(b)$ ). On the other side, one may interpret the weight  $w$  in a wRCCSR framework, as a measure of attention paid to an alternative. In this latter condition, any preference relation is a valid representation since their expression is prevented by  $\pi(A) = 0$  for all  $|A| > 1$ .



to  $x_A^\succ$ .

$$\forall A \in 2^X \setminus \{\emptyset\}, \exists x_A^\succ \in A \text{ such that } \forall B \subseteq A, BIx_A^\succ.$$

Instead of requiring Asymmetry, Transitivity and Extensivity for the Sequential Independence, our characterization requires that each menu contains at least one alternative such that all subsets of the menu are sequentially independent to this alternative. Using a similar intuition as in BR, WARP-SLA guarantees the existence of a maximal alternative in any non empty menu  $A \in 2^X$  that is also maximal in any subset  $B \subseteq A$  whenever it is present. This condition is obviously sufficient for the existence of a preference relation over  $X$ . It is easy to show that ASI, TSI and ESI implies WARP-SLA. The main advantage to characterize a RCCSR using WARP-SLA instead of ASI, TSI and ESI is that WARP-SLA consists in a set of equalities, and thus can be statistically tested with a nominal level  $\alpha$  that is asymptotically controlled for any model specification (see Section 3.4).<sup>18</sup>

The following lemma states that WARP-SLA is sufficient to characterize a model with a preference relation and menu-dependent attention intensities  $\lambda$ .

**Lemma 3.3.1.** *Let  $p$  a random choice rule. If  $p$  satisfies WARP-SLA, then there exist a complete and transitive preference relation  $\succ$  over  $X$  and a mapping  $\lambda : \mathcal{D} \rightarrow \mathbb{R}$ , such that  $\forall A \in \mathcal{D} \setminus \{\emptyset\}, \forall a \in A$ :*

$$p(a, A) = \frac{\sum_{B \in A_{a^\succ}} \lambda(B)}{\sum_{B \subseteq A} \lambda(B)},$$

with  $A_{a^\succ} = \{B \subseteq A : a \in B \text{ and } \forall b \in B \setminus a, a \succ b\}$ , and  $\lambda(A) = \sum_{B \subseteq A} (-1)^{|A \setminus B|} \frac{1}{p(o, B)}$ .

Here, there is no restriction on the attentional intensities that could be null or negative (however by construction we have  $\sum_{B \subseteq A} \lambda(B) \neq 0$  for all  $A \in \mathcal{D}$ ).

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<sup>18</sup>One may note that while the random choice rules is defined only for the menu  $A \in \mathcal{D}$ , WARP-SLA requires that every possible menu  $A \in 2^X$  contains an alternative  $x_A^\succ$  such that any subset  $B \subseteq A$  is sequentially independent to  $x_A^\succ$ . Indeed, by definition, SI are assumed to hold for any alternatives in any set where choices are unobserved. If this assumption is rather technical and is made only to simplify the proof, it can be viewed as a conservative approach of an observer who choose not to reject any sequential independence relation in the menus where choices are not observed.

Therefore, we use a weaker version of BR's IFO axiom in order to guaranty weak positivity that allows some menus to have a zero probability to attract attention.

**Definition 3.3.3.** *weak Increasing Feasible Odds (wIFO)*

A random choice rule satisfies wIFO if for any  $A \in \mathcal{D}$ ,

$$\sum_{B \subseteq A} (-1)^{|A \setminus B|} \frac{1}{p(o, B)} \geq 0.$$

The interpretation of this axiom is the same as the interpretation of IFO.

**Theorem 3.3.1** (Characterization). *p satisfies WARP-SLA and wIFO iff  $p_{\succ, \pi}$  is a wRCCSR. Moreover,  $\pi$  is unique and*

$$\forall A \in \mathcal{D}, \pi(A) = C \times \sum_{B \subseteq A} \left( (-1)^{|A \setminus B|} \times \frac{1}{p(o, B)} \right),$$

with  $C \in \mathbb{R}_+$  a scaling factor such that  $\sum_{A \in \mathcal{D}} \pi(A) = 1$ .

The proof is detailed in appendix 3.A. The characterization follows immediately from Lemma 3.3.1 and wIFO. Although Theorem 3.3.1 guarantees the existence of a wRCCSR with a unique  $\pi$ , it does not guaranty the uniqueness of the preference relation. However, we show in the following example that it is possible to have a random choice rule with a unique wRCCSR representation but with no RCCSR representation.

**Example 3.3.1.**

Let consider the following random choice rule summarized in Table 3.3.1. Rows represent alternatives and columns represent menus.

	abc	ab	ac	bc	a	b	c
a	1/3	1/2	2/7	0	2/3	0	0
b	1/4	1/4	0	3/8	0	1/2	0
c	1/3	0	4/7	1/2	0	0	4/5
o	1/12	1/4	1/7	1/8	1/3	1/2	1/5

Table 3.3.1: Example of wRCCSR with unique preferences representation.

WARP-SLA holds since  $bI_{abc}a$ ,  $cI_{abc}a$ ,  $bI_{ab}a$ ,  $cI_{ac}a$  and  $cI_{bc}b$ . ASI is falsified since we also have:  $aI_{ab}b$  and  $aI_{ac}c$ . TSI holds but ESI is falsified since we have

$(\{bc\}Ib \cap \{ab\}Ib)$  but not  $(\{abc\}Ib)$ . *wIFO* is satisfied but not *IFO* since

$$\frac{1}{p(o, ab)} - \frac{1}{p(o, a)} - \frac{1}{p(o, b)} + 1 = 0 = \frac{1}{p(o, ac)} - \frac{1}{p(o, a)} - \frac{1}{p(o, c)} + 1$$

So  $p$  is a *wRCCSR* but not a *RCCSR*. Knowing that  $p$  is a *wRCCSR*, it is possible to check each of the potential preference relation and to verify that  $a \succ b \succ c$  is the only preference relation. Indeed  $c$  cannot be the maximum alternative in  $\{bc\}$  because we have not  $bI_{bc}c$  and  $b$  cannot be the maximum alternative in  $\{abc\}$ , since we have not  $aI_{abc}b$  so  $a$  must be the maximum alternative and preferences can be recovered. This illustrates that considering *wRCCSR* offers a more rich framework to identify preferences if compared to *RCCSR*. In *RCCSR* framework, preferences are defined based on asymmetric *SI* in binary menu. If we consider only binary *SI*, both  $a$  and  $c$  are potential maximum in  $\{ac\}$  and both  $a$  and  $b$  are potential maximum in  $\{ab\}$ .

We now present in table 3.3.2 another example where the random choice rule has several *wRCCSR* representations but where it is still possible to infer information about the underlying preferences.

**Example 3.3.2.**

Consider the following random choice rule  $p$ :

	abc	ab	ac	bc	a	b	c
a	1/3	1/2	2/7	0	2/3	0	0
b	1/4	3/8	0	3/8	0	3/4	0
c	1/3	0	4/7	1/2	0	0	4/5
o	1/12	1/8	1/7	1/8	1/3	1/4	1/5

Table 3.3.2: Example of *wRCCSR* with no unique preferences representations.

The choice probabilities respect three unions of sequential independence relations, each of them satisfying *WARP-SLA*:  $(bI_{abc}a, cI_{abc}a, bI_{ab}a, cI_{ac}a$  and  $cI_{bc}b)$ ;  $(bI_{abc}a, cI_{abc}a, bI_{ab}a, cI_{ac}a$  and  $bI_{bc}c)$  and  $(bI_{abc}c, aI_{abc}c, aI_{ac}c, bI_{bc}c$  and  $bI_{ab}a)$ .

So  $p$  has three possible *wRCCSR* representation  $p_{\succ_i, \pi}$ , with  $(\pi(abc) = \pi(ac) = \pi(bc) = 0, \pi(ab) = \pi(a) = 1/6, \pi(b) = 1/4, \pi(c) = 1/3$  and  $\pi(\emptyset) = 1/12)$  and  $(a \succ_1 b \succ_1 c; a \succ_2 c \succ_2 b$  and  $c \succ_3 a \succ_3 b)$ . The fact that an observer cannot infer the relative rank of  $c$  based on choices probability is due to the fact  $c$  is never

considered together with  $a$  and  $b$ . A real life example of such situation is an individual in the hurry who is shopping in a supermarket where  $a$  and  $b$  are products located in the same shelf but where  $c$  is located in a far side of the store.

### 3.3.2 Revealed preferences

Despite the fact that the preferences representation is not necessarily unique, choices may partially reveal preferences. Indeed, in Example 3.3.2, we have  $a \succ_i b$  for any of the potential preference representation  $(\succ_i)_{i=1,2,3}$  of  $p$ . So regardless the rank of  $c$ , the observer could infer that  $a$  is revealed to be preferred to  $b$ . More generally, as in Masatlioglu et al. (2012), we say that an alternative  $a$  is revealed to be preferred to another alternative  $b$  if  $a$  is preferred to  $b$  for any preference representations of  $p$ .

#### Definition 3.3.4. Revealed Preferences

Assume  $p$  is a wRCCSR and  $(\succ_i)_{i=1,\dots,N}$  are the valid preference representations of  $p$ .  $a$  is revealed to be preferred to  $b$  iff  $a \succ_i b$  for all  $i = 1, \dots, N$ .

However, testing all the preference relations to assess what are the possible representations of  $p$  may be delicate in the presence of many alternatives. Moreover, we could imagine situations where an observer has good reasons to assume that the underlying random choice rule is a wRCCSR (e.g. previous studies have shown that choices can be described by a wRCCSR in similar environments) and is merely interested in the relative preference between two specific alternatives. Our revealed preference theorem is based on the following binary relation:

**Notation.** Let  $p$  be a random choice rule and  $X$  the set of available alternatives. For all  $a$  and  $b$  distinct alternatives in  $X$ , we define the following binary relation  $P$ :

$$aPb \text{ if } \exists A \in \mathcal{D}, a, b \in A, \text{ such that } \frac{p(a, A)}{p(o, A)} \neq \frac{p(a, A \setminus b)}{p(o, A \setminus b)}$$

We note  $P_R$  the transitive closure of  $P$ .

If  $p$  is a wRCCSR, then we have an equivalence between  $P_R$  and revealed preferences:

**Theorem 3.3.2** (Revealed Preferences). *Let  $p$  be a wRCCSR,  $a$  is revealed to be preferred to  $b$  iff  $aP_R b$ .*

An hint of the proof provided in appendix 3.A.2 is that for any wRCCSR  $p_{\succ, \pi}$ , with  $b \succ a$  it is possible to show that  $\frac{p(a,A)}{p(o,A)} = \frac{p(a,A \setminus b)}{p(o,A \setminus b)}$ , for all  $A$  containing  $a$ .

This theorem is useful as an observer may infer preferences without observing choices in the full domain  $\mathcal{D}$ . Indeed, it is sufficient to observe a (significant) difference between  $\frac{p(a,A)}{p(o,A)}$  and  $\frac{p(a,A \setminus b)}{p(o,A \setminus b)}$  in at least one menu  $A$  to infer that  $a$  is preferred to  $b$  for all the possible wRCCSR representation of  $p$ . As an illustration, let assume the observer is a firm aiming to introduce a new product  $a$  in a market with numerous competitors. The firm already knows from previous studies that wRCCSR is an appropriate choice model for its market. The firm may not be interested to know the full preference order among the products, but would like to know which of the competitors' products  $b \in A$  are preferred to  $a$ . Thanks to Theorem 3.3.2 and just by comparing market shares in  $A \setminus a$  (before the introduction of the product) and in  $A \cup a$  (after the introduction of the product), the firm may partly identify those products.

## 3.4 Statistical methods to test wRCCSR

In the previous section, we provided theorems to characterize a wRCCSR and to disentangle preferences from limited attention when an observer knows the choice probabilities in a rich domain  $\mathcal{D}$ . However, this is never the case in the field, where only realisations of random variables are observed. In this section we propose statistical methods to test WARP-SLA and wIFO, to reveal preferences and to measure effective attention. We discuss the characteristics of those methods and investigate their asymptotic performance (level and power) using numerical simulations.

### 3.4.1 Test of WARP-SLA

From the proof of lemma 3.3.1, we know that WARP-SLA holds if and only if it exists at least one linear order  $\succ$  over  $X$  such that every non empty subset  $A \in 2^X$

contains a  $\succ$ -best element  $x_A^\succ$  such that  $A$  is sequentially independent to  $x_A^\succ$ . Since sequential independence holds trivially in menu outside  $\mathcal{D}$ , to test WARP-SLA, one can test the following hypothesis:

$$\begin{aligned} H0_{WARP\text{SLA}} &= \bigcup_{\succ_i \in \mathcal{L}} H0_{\succ_i}^{WARP\text{SLA}} \\ &= \bigcup_{\succ \in \mathcal{L}} \left( \bigcap_{\substack{A \in \mathcal{D} \\ |A| > 1}} AIx_A^\succ \right), \end{aligned}$$

with  $\mathcal{L}$  the set of linear order over  $X$ , and  $x_A^\succ$  the  $\succ$  –best alternative in  $A$ , such that  $\forall a \in A \setminus x_A^\succ, x_A^\succ \succ a$ .

For example, if  $X = \{a, b, c\}$  and  $\mathcal{D} = 2^X$ , there is 6 possible linear orders and each of them requires 5 SI relations to hold. So WARP-SLA is satisfied if and only if the 5 associated SI relations hold for at least one of these orders, as indicated in Table 3.4.1.

$H0_{a \succ b \succ c}^{WARP\text{SLA}}$	$H0_{a \succ c \succ b}^{WARP\text{SLA}}$	$H0_{b \succ a \succ c}^{WARP\text{SLA}}$	$H0_{b \succ c \succ a}^{WARP\text{SLA}}$	$H0_{c \succ a \succ b}^{WARP\text{SLA}}$	$H0_{c \succ b \succ a}^{WARP\text{SLA}}$
$bI_{abc}a$	$bI_{abc}a$	$aI_{abc}b$	$aI_{abc}b$	$bI_{abc}c$	$bI_{abc}c$
$cI_{abc}a$	$cI_{abc}a$	$cI_{abc}b$	$cI_{abc}b$	$aI_{abc}c$	$aI_{abc}c$
$bI_{ab}a$	$bI_{ab}a$	$aI_{ab}b$	$aI_{ab}b$	$bI_{bc}c$	$bI_{bc}c$
$cI_{ac}a$	$cI_{ac}a$	$cI_{bc}b$	$cI_{bc}b$	$aI_{ac}c$	$aI_{ac}c$
$cI_{bc}b$	$bI_{bc}c$	$cI_{ac}a$	$aI_{ac}c$	$bI_{ab}a$	$aI_{ab}b$

Table 3.4.1: SI relations necessary to satisfy WARP-SLA.

**Note:** Each column represents a possible preference relation  $\succ_i$  and the 5 rows represent the 5 Sequential Independence (SI) jointly required for the preference relation  $\succ_i$  to be a valid representation of  $p$ . WARP-SLA is rejected if and only if there is no column where the associated SI hold jointly.

#### a) Separate tests of sequential independence relation (SI).

An immediate solution is to test each SI relation with successive tests at nominal level  $\alpha$  followed by a proper correction for multiple testing. For each menu  $A \in \mathcal{D}$ , and each alternatives  $a, b \in A$ , the SI relation  $bI_Aa$  could be tested using a z-test as in the proposition 3.2.2 presented in section 3.2.3.

**Proposition 3.2.2.** *Let  $A \in \mathcal{D}$ , Let  $a, b \in A$ , let  $n(a, A)$  the number of times an alternative  $a$  is chosen in a menu  $A$ . Under the null hypothesis that  $b$  is sequentially independent to  $a$  in the menu  $A$  ( $H0_{bIa}^A : p(b, A \setminus a) = p_{-a}(b, A)$ ), the statistics:*

$$Z_{bIa}^A = \sqrt{N} \times \frac{\hat{\mu}_{bIa}^A}{\hat{\sigma}_{bIa}^A} \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N}(0, 1),$$

*follows asymptotically a standard Gaussian distribution, with*

$$\hat{\mu}_{bIa}^A = \frac{n(b, A)}{N - n(a, A)} - \frac{n(b, A \setminus a)}{N},$$

*and*

$$\hat{\sigma}_{bIa}^A = \sqrt{\frac{n(b, A \setminus a) \times [N - n(b, A \setminus a)]}{N^2} + \frac{N \times n(b, A) \times [N - n(a, A) - n(b, A)]}{[N - n(a, A)]^3}}.$$

Based on this proposition, we define p-values for a centred two-sided test as follows:

$$p_{bIa}^A = 2 \times \left(1 - \Phi(|Z_{bIa}^A|)\right),$$

with  $\Phi$  the cumulative distribution function (*cdf*) of the standard normal distribution. Under  $H0_{bIa}^A$  those p-values follow asymptotically a uniform distribution  $\mathcal{U}_{[0,1]}$ . A test of nominal level  $\alpha$  consists in rejecting  $H0_{bIa}^A$  whenever  $p_{bIa}^A < \alpha$ .

We must distinguish the *nominal level* of a test  $\alpha$  (i.e. a threshold used by the observer to reject the null hypothesis) with its *actual level*  $\hat{\alpha}$  (i.e. the probability to observe a p-value lower than  $\alpha$  under the null hypothesis). Since the test we propose is not exact,  $\alpha$  and  $\hat{\alpha}$  are not equal, but since the test is asymptotic,  $\hat{\alpha}$  is expected to converge to  $\alpha$  under  $H0_{bIa}^A$ . It is thus important to study 1) how fast this convergence occurs and 2) if the actual level of the test does not exceed “too much” the nominal level  $\alpha$  accepted by the observer. Ideally, one would prefer to have:  $\mathbb{P}\left(p_{bIa}^A \leq \alpha | H0_{bIa}^A\right) \leq \alpha$ . Such tests are called conservative since they ensure fewer type-I errors than accepted by the observer. Conservatism is far from an issue per se, however over-conservative tests are often associated with lack of power. A

last concern is how model parameters could influence the convergence speed. While proposition 3.2.2 ensures that  $p_{bIa}^A$  is asymptotically uniformly distributed whenever the data generating process is a wRCCSR (with  $a \succ b$ ), we do not provide theoretical results on how model's attention parameters  $\pi$  influence the speed of convergence. Those features are important to understand the number of decisions per menu an observer needs to gather to reasonably test the SI relations. For this purpose, we estimate the distribution of  $p_{bIa}^A$  when data are produced by several wRCCSR specifications. We use the following simulation method:

**Simulation 3.4.1.** *Choices are made among 3 different alternatives  $X = \{a, b, c\}$  (and a default option  $o$ ) in every menu of a domain that is the power set of  $X$ :  $\mathcal{D} = \{\{abc\}, \{ab\}, \{ac\}, \{bc\}, \{a\}, \{b\}, \{c\}, \emptyset\}$ . We generate 10.000 different wRCCSR  $(p_{\succ, \pi_i})_{i=1, \dots, 10.000}$ , all of them associated with the same preference relation  $a \succ b \succ c$  (without loss of generality), but different set of attention parameters  $\pi_i = \{\pi_i(abc), \pi_i(ab), \pi_i(ac), \pi_i(bc), \pi_i(a), \pi_i(b), \pi_i(c), \pi_i(\emptyset)\}$ . Each set of parameter  $\pi_i$  is generated as follows. We randomly draw  $(\pi_i(A))_{A \in \mathcal{D}}$ , identically and independently from a uniform distribution  $\mathcal{U}_{[0,1]}$  and we divide  $\pi_i(A)$  by  $\sum_{A \in \mathcal{D}} \pi_i(A)$  to guaranty that  $\pi_i$  is a probability measure. For each of the 10.000 different wRCCSR  $p_{\succ, \pi_i}$ , we vary the number of decisions ( $N = 20, 40, 100$  and 1000) made in each menu  $A \in \mathcal{D}$  and independently and identically simulate  $N$  decisions following  $p_{\succ, \pi_i}$  in each menu  $A \in \mathcal{D}$ . Each of the simulated agent corresponds to the realisation of a collection of independent multinomial distributions  $(\mathcal{M}_{|A|}(N, p_{\succ, \pi_i}(\cdot, A)))_{A \in \mathcal{D}}$  which indicates the number of times each alternative is chosen in each menu.*

We test each SI relation for each agent simulated according to Simulation 3.4.1. For each nominal level  $\alpha = 1\%$ ,  $5\%$ , and  $10\%$ , we report in the left columns of Table 3.4.2 the average frequency of rejection for the SI-tests associated with the preference representation  $a \succ b \succ c$ . This frequency is an estimator of the expected actual level of the test, when model parameters are uniformly distributed. The right columns in Table 3.4.2 reports the frequency of rejection for the other SI-tests. Under simulation assumption ( $a \succ b \succ c$ ), those SI relations are false and the frequency of rejection corresponds to the actual power to reject false SI relations (the probability  $\beta_{b \succ a}$  described in Section 3.2.3). Since our purpose is to evaluate



the quality of each SI-test, we apply no correction yet on the p-values presented in Table 3.4.2.

N	$p < \alpha$	SI-tests associated to $a \succ b \succ c$ .					SI-tests associated with other preference relations.						
		$cI_{abc}a$	$bI_{abc}a$	$bI_{ab}a$	$cI_{ac}a$	$cI_{bc}b$	$cI_{abc}b$	$aI_{abc}b$	$bI_{abc}c$	$aI_{abc}c$	$aI_{ab}b$	$aI_{ac}c$	$bI_{bc}c$
20	$\alpha = 0.1$	0.139	0.152	0.153	0.155	0.152	0.2	0.329	0.214	0.225	0.337	0.341	0.352
	$\alpha = 0.05$	0.08	0.096	0.096	0.098	0.095	0.13	0.242	0.133	0.16	0.261	0.263	0.271
	$\alpha = 0.01$	0.038	0.046	0.051	0.049	0.049	0.057	0.122	0.053	0.069	0.159	0.155	0.16
40	$\alpha = 0.1$	0.123	0.126	0.132	0.128	0.129	0.26	0.45	0.284	0.294	0.45	0.449	0.458
	$\alpha = 0.05$	0.07	0.071	0.078	0.074	0.075	0.179	0.356	0.205	0.214	0.366	0.369	0.375
	$\alpha = 0.01$	0.019	0.025	0.03	0.029	0.027	0.081	0.206	0.099	0.105	0.245	0.248	0.252
100	$\alpha = 0.1$	0.116	0.106	0.116	0.118	0.116	0.398	0.652	0.406	0.433	0.608	0.604	0.593
	$\alpha = 0.05$	0.061	0.057	0.064	0.066	0.063	0.309	0.57	0.325	0.348	0.535	0.532	0.523
	$\alpha = 0.01$	0.015	0.014	0.018	0.017	0.019	0.17	0.407	0.206	0.222	0.405	0.406	0.399
1000	$\alpha = 0.1$	0.099	0.098	0.103	0.102	0.097	0.852	0.967	0.75	0.776	0.885	0.877	0.878
	$\alpha = 0.05$	0.05	0.05	0.051	0.051	0.049	0.807	0.957	0.704	0.734	0.863	0.851	0.853
	$\alpha = 0.01$	0.009	0.011	0.011	0.011	0.01	0.714	0.928	0.617	0.66	0.815	0.805	0.802

Table 3.4.2: Probability to reject test of SI when choices follow a wRCCSR.

**Note:** This table indicates for each nominal type-I error  $\alpha$  and for each SI-relation, the proportion of observed p-values lower than  $\alpha$ , based on simulated choices following a wRCCSR with  $a \succ b \succ c$  (see simulation 3.4.1). The SI relations associated with the preference representation  $a \succ b \succ c$  hold under the null hypothesis, so the probability of rejection indicates the actual type-I error of the tests. The other SI relations are not required under  $H_0$ , so the probability of rejection indicates the power of the test to reject false sequential independence relation of a RCCSR.

Figure 3.4.1 and Figure 3.4.2 respectively show the p-values' empirical cumulative distribution function (*cdf*) of the tests of each SI associated to the preference relation  $a \succ b \succ c$  and  $c \succ b \succ a$  for  $N = 20, 40, 100$  and  $1000$  observation per menu. When  $N$ , is lower than 40, the actual levels of the tests are not controlled as in average more type I errors than expected by the observer occur. For  $N = 40$ , testing SI at a nominal level  $\alpha = 0.05$  implies the following expected actual levels:  $\hat{\alpha}(bI_{abc}a) = 0.071$ ,  $\hat{\alpha}(cI_{abc}a) = 0.07$ ,  $\hat{\alpha}(bI_{ab}a) = 0.078$ ,  $\hat{\alpha}(cI_{ac}a) = 0.074$ ,  $\hat{\alpha}(cI_{bc}b) = 0.075$ .

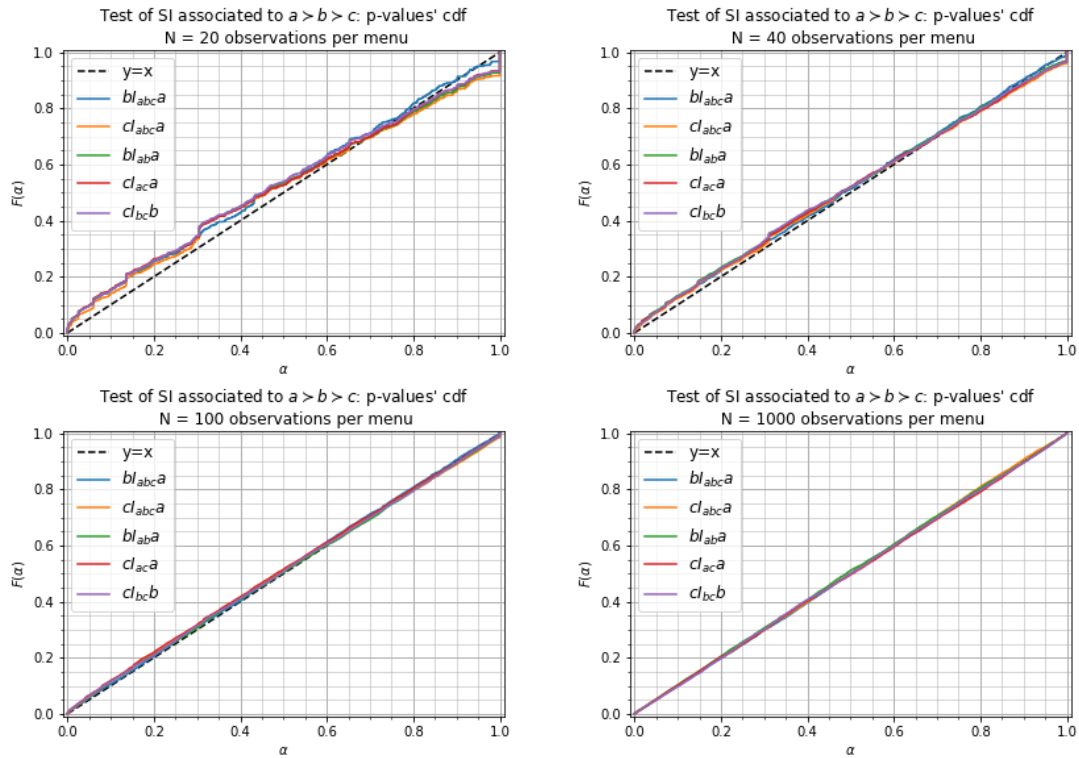


Figure 3.4.1: p-values' cdf for the tests of the (true) SI, when choices follow a wRCCSR.

**Note:** The figures present p-values' cdf for the test of each test of Sequential Independence (SI) associated with the true preference relation  $a \succ b \succ c$ , for wRCCSR simulated agent with  $a \succ b \succ c$  and taking  $N=20, 40, 100$  and  $1000$  decisions per menus (see simulation 3.4.1). Each cdf indicates the proportion of simulated agent ( $y$  axis) rejecting a given SI, for a given nominal level  $\alpha$  ( $x$ -axis). The null hypotheses  $H0_{bI_a}^A$  are true for all these figures. Therefore the cdf indicate the actual level of the test. The dash line represents the uniform distribution the p-values of any exact test must follow under the null hypothesis. A cdf below this line characterizes a conservative test.

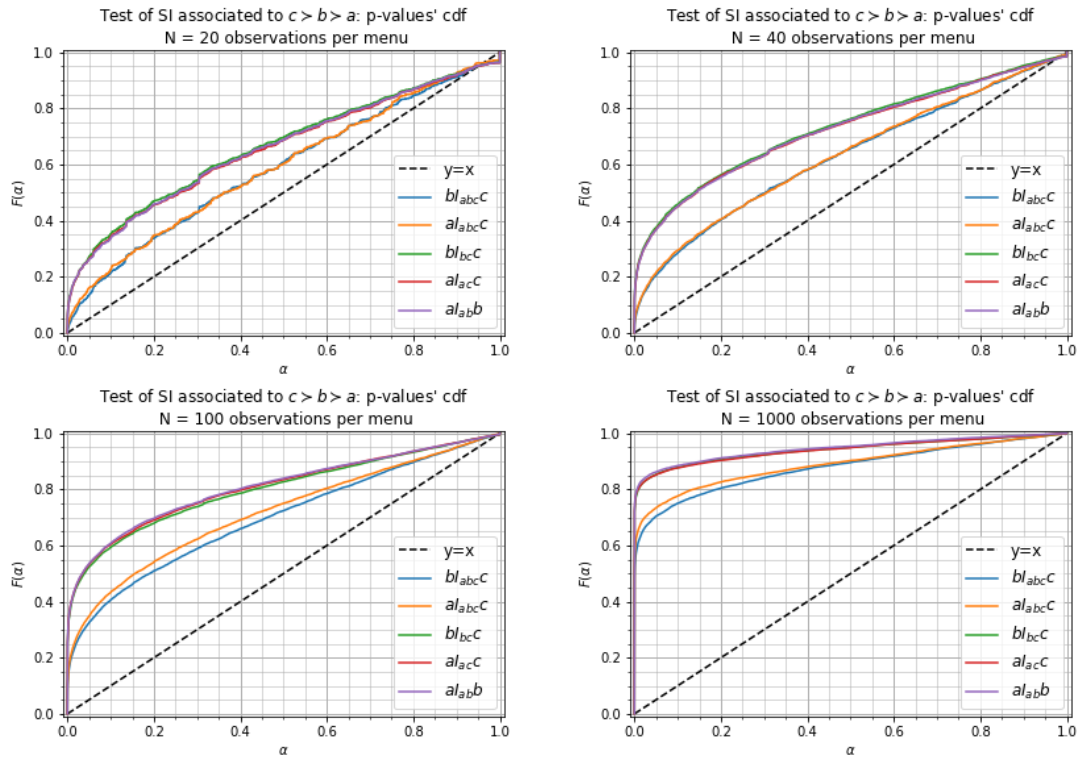


Figure 3.4.2: p-values' cdf for the tests of (false) SI, when choices follow a wRCCSR.

**Note:** The figures present p-values' cdf for the test of each test of Sequential Independence (SI) associated with the false preference relation  $c \succ b \succ a$ , for wRCCSR simulated agent with  $a \succ b \succ c$  and taking  $N=20, 40, 100$  and  $1000$  decisions per menus (see simulation 3.4.1). Each cdf indicates the proportion of simulated agent ( $y$  axis) rejecting a given SI, for a given nominal level  $\alpha$  ( $x$ -axis). The alternative hypotheses  $H1_{-bIa}^A$  are true for all these figures. Therefore the cdf indicate the expected power of the test to reject each incorrect SI relation. A cdf above the dash line characterizes a test with positive power.

To test WARP-SLA, each preference relation  $\succ$  can be tested through a disjunction of SI relations and a preference relation  $\succ$  should be rejected if at least one of the associated SI relation is rejected. Even if the actual level  $\hat{\alpha}$  for each SI was controlled to  $\alpha$ , the family-wise error rate (FWER) (i.e. the probability to make at least one false rejection) will be greater than  $\alpha$ . We must control the FWER when testing simultaneously the SI relation associated to a specific preference relation. This could be done by correcting the p-values obtained previously. We propose to use the correction from Simes (1986), which is less conservative than the usual Bonferroni correction, in particular when the p-values are not independent but positively correlated as in our case.<sup>19</sup>

For a hypothesis  $H0 = \bigcap_{j=1}^k H0_j$ , Simes' correction, works as follow. The p-values obtained from the  $k$  different tests are ranked from the smallest to the largest ( $p_{(1)} \leq \dots \leq p_{(k)}$ ) and then corrected:

$$p_{(i)}^c = \frac{k \times p_{(i)}}{i}.$$

The p-value of the disjoint null hypothesis  $H0_{\succ}^{WARP\text{SLA}}$  is defined as the smallest of the corrected p-values :  $p_{\succ}^{WARP\text{SLA}} = \min_{1 \leq i \leq k} p_{(i)}^c$ , where the  $p_{(i)}^c$  corresponds to the p-values associated to the test of SI relations associated to  $H0_{\succ}^{WARP\text{SLA}}$ , ranked and corrected. Thus,  $H0_{\succ}^{WARP\text{SLA}}$  is rejected at a nominal level  $\alpha$  if  $p_{\succ}^{WARP\text{SLA}} < \alpha$ .

$H0_{WARP\text{SLA}}$  holds if there is at least one preferences representation  $\succ$  for which  $H0_{\succ}^{WARP\text{SLA}}$  is not rejected. Therefore, we define the p-value of the test of  $H0_{WARP\text{SLA}}$  as the maximum p-value among those resulting from the test of each possible

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<sup>19</sup>The Bonferroni correction consists in multiplying all p-values by the number of hypotheses tested. It is probably the most used correction method because of its conservatism and simplicity of use. However superior methods exist. We refer to Blakesley et al. (2009) for a comparison of several multiple testing correction methods. Test of SI are correlated because some of the tests are based on decisions in the same menu. These correlations are positive under  $H0_{\succ}^{WARP\text{SLA}}$  (e.g. difference between  $\frac{n(b,bc)}{N}$  and  $p(b,bc)$ , due to sampling error impacts the probability to reject  $cI_{abc}a$  and  $bI_{abc}a$  in the same direction). For example, based on our simulations for  $N = 40$ , we have the

following correlations between tests' p-values: 
$$\begin{pmatrix} & cI_{abc}a & cI_{bc}b & cI_{ac}a \\ bI_{abc}a & 0.2632 & 0.0131 & 0.0007 \\ cI_{abc}a & & 0.0677 & 0.0030 \\ cI_{bc}b & & & 0.1180 \end{pmatrix}.$$

preference representation:

$$p_{WARPSLA} = \max_{\succ \in \mathcal{L}} \{p_{\succ}^{WARPSLA}\},$$

and WARP-SLA is rejected at a nominal level  $\alpha$  iff  $p_{WARPSLA} < \alpha$ .

Indeed, if the level is controlled for each test of potential preferences representation (i.e.  $\forall \succ \in \mathcal{L}, \forall \alpha \in [0, 1], \lim_{N \rightarrow \infty} \mathbb{P}(p_{\succ}^{WARPSLA} \leq \alpha) \leq \alpha$ ), we must have:

$$\lim_{N \rightarrow \infty} \mathbb{P}(p_{WARPSLA} \leq \alpha) \leq \alpha.$$

That is testing several preference orders cannot increase the probability to falsely reject  $H0_{WARPSLA}$  when WARP-SLA holds (Regenwetter et al., 2010). However, this leads to a conservative test. Indeed several hypothesis  $H0_{\succ}^{WARPSLA}$  can be simultaneously true under  $H0_{WARPSLA}$  and false  $H0_{\succ}^{WARPSLA}$  have a positive probability not to be correctly rejected.

Figure 3.4.3 and table 3.4.3 shows p-values' cdf for each  $H0_{\succ}^{WARPSLA}$  for  $N=20, 40, 100$  and  $1000$  decisions per-menu as well as the p-values' cumulative distributions for the test of  $H0_{WARPSLA}$  for  $N=20, 40, 100$  and  $1000$ . The lack of control for each test of SI, when  $N$  is not large enough is not corrected when we consider the disjunction hypothesis corresponding to the true preference distribution. Based on another set of simulations, we investigate the variation in the actual level due to variation in wRCCSR specification, and estimate p-values' confidence intervals (see appendix 3.B.7). For small number of observations ( $N < 100$ ), the actual level of the test of  $H0_{\succ}^{ARP-SLA}$  based on corrected p-values is not well controlled.

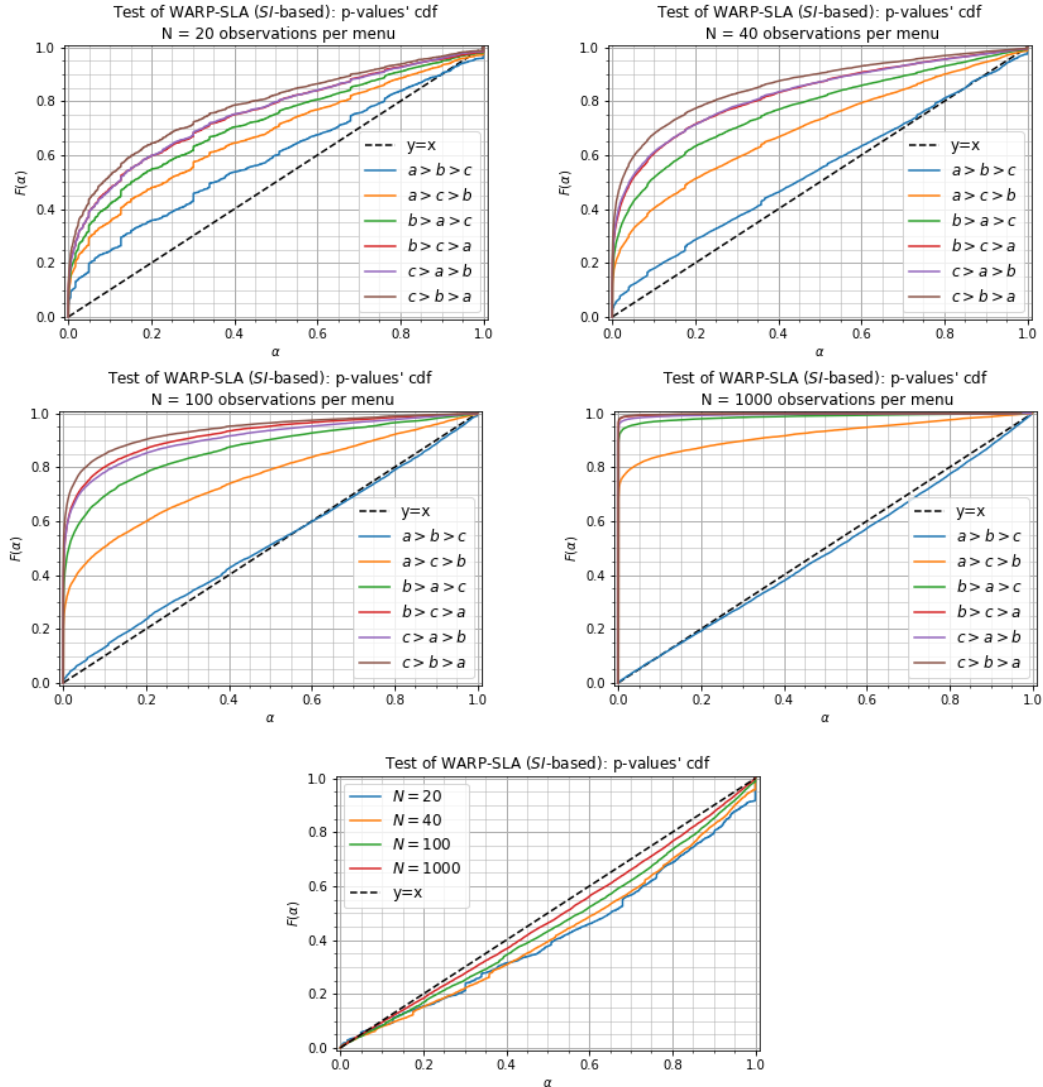


Figure 3.4.3: p-values' cdf for the (SI) test of  $H0_{WARPSLA}$  when choices follow a WRCCSR.

**Note:** Data from simulation 3.4.1. First two rows figures present p-values' cdf for each  $H0_{WARPSLA}$  based on SI corrected p-values when simulated agents take  $N=20, 40, 100,$  and  $1000$  decisions per menu. In these figures, only  $H0_{a>b>c}^{WARPSLA}$  is the true null hypothesis and its (blue) curve indicates the actual level of the test while other curves represent test's actual power to reject false preference representations. Last row figure represents the actual level of  $H0_{WARPSLA}$ , that is the frequency of not rejecting  $H0_{a>b>c}^{WARPSLA}$  for at least one preference order, for  $N = 20, 40, 100,$  and  $1000$  obs per menu.

N	$\alpha$	$p_{\succ}^{WARP\text{SLA}} < \alpha$						
		WARP-SLA	$a \succ b \succ c$	$a \succ c \succ b$	$b \succ a \succ c$	$b \succ c \succ a$	$c \succ a \succ b$	$c \succ b \succ a$
20	$\alpha = 0.1$	0.083	0.243	0.352	0.417	0.476	0.471	0.524
	$\alpha = 0.05$	0.057	0.199	0.294	0.34	0.391	0.389	0.437
	$\alpha = 0.01$	0.018	0.1	0.165	0.195	0.234	0.228	0.266
40	$\alpha = 0.1$	0.079	0.18	0.402	0.516	0.606	0.611	0.679
	$\alpha = 0.05$	0.044	0.12	0.324	0.426	0.514	0.523	0.593
	$\alpha = 0.01$	0.014	0.058	0.213	0.281	0.352	0.367	0.432
100	$\alpha = 0.1$	0.082	0.131	0.504	0.694	0.801	0.781	0.849
	$\alpha = 0.05$	0.043	0.079	0.436	0.616	0.733	0.721	0.795
	$\alpha = 0.01$	0.011	0.029	0.326	0.478	0.595	0.59	0.669
1000	$\alpha = 0.1$	0.094	0.098	0.841	0.971	0.994	0.988	0.996
	$\alpha = 0.05$	0.049	0.052	0.813	0.961	0.992	0.984	0.994
	$\alpha = 0.01$	0.012	0.013	0.762	0.939	0.985	0.972	0.987

Table 3.4.3: Probability to reject  $H0^{WARP\text{SLA}}$  and  $H0_{\succ}^{WARP\text{SLA}}$  based on corrected SI-tests. Simulated choices following a wRCCSR with  $a \succ b \succ c$ .

**b) Testing all the SI simultaneously for a given preferences representation.**

Applying multiple testing correction on family of test of SI may either lead to over-conservative tests (resulting in loss of statistical power) or to a loss of control when the tested hypotheses are correlated. Those issues increase with the number of tested hypotheses - which in our case explodes combinatorially with the number of alternatives. Therefore, it is preferable to conduct a single test which considers simultaneously all the SI relations required for a particular linear order to be a representation of  $p$ , satisfying WARP-SLA. This is what we are proposing now. The test statistics is obtained based on the ratio between the likelihoods of the observed data under the null hypothesis  $H0_{\succ}^{WARP\text{SLA}}$  ( $p$  satisfies WARP-SLA with  $\succ$  as a valid representation), and under the alternative hypothesis  $H1$  such that  $p$  can be any random choice rule.<sup>20</sup>

**Proposition 3.4.1.** *Let  $n^A = (n_a^A)_{a \in A \cup o}$  with  $A \in \mathcal{D}$  the realisation of independent multinomial variables with parameters  $p^A = (p_a^A)_{a \in A \cup o}$  and  $N$ , that indicate the number of times each alternative have been chosen in each menu  $A$ . Let  $\succ$  a linear order over  $X$ .  $\forall A \in \mathcal{D}$  and  $\forall a \in X$ , we note:  $x_A^{\succ}$  the  $\succ$ -best alternative in  $A$ ,  $\tilde{a} = \{b \in X : \neg(a \succ b)\}$  the set of alternatives that are  $\succ$  preferred to  $a$ ,  $n(\tilde{a}, A) =$*

<sup>20</sup>The proof is given in appendix 3.B.

$\sum_{b \in A \cap \check{a}} n_b^A$  the number of times an alternative  $\succ$ -preferred to  $a$  is chosen in the menu  $A$ , and  $\Psi_{\succ}^a(A) = \{B \in \mathcal{D} : B \setminus \check{a} = A \setminus \check{a}\}$  the set of menus where the alternatives  $\succ$ -dominated by  $a$  are the same as in the menu  $A$ .

Under the null hypothesis  $H0_{\succ}^{WARPSLA} = (\forall A \in \mathcal{D}, AIx_A^{\succ})$  that WARP-SLA holds for the specific preference relation  $\succ$ , the statistics:

$$\mathcal{J}_{\succ} = \sum_{A \in \mathcal{D}} \sum_{a \in A \cup \check{a}} n_a^A \log \left( \frac{n_a^A}{N \hat{p}_{\succ}(a, A)} \right) \xrightarrow[N \rightarrow +\infty]{d} \chi_k^2,$$

follows asymptotically a  $\chi^2$  distribution with  $k$  degrees of freedom, where  $k$  is the number of (non-trivial) SI relations required for the preference relation  $\succ$  to be a valid representation of  $p$ .

With

$$\hat{p}_{\succ}(a, A) = \begin{cases} \frac{\sum_{B \in \Psi_{\succ}^a(A)} n_a^B}{\sum_{B \in \Psi_{\succ}^a(A)} [N - n(\check{a}, B)]} & \text{if } a = x_A^{\succ} \\ \hat{p}_{\succ}(a, A \setminus \check{a}) \times \prod_{b \in \check{a}} [1 - \hat{p}_{\succ}(b, A \setminus \check{b})] & \text{otherwise} \end{cases},$$

the maximum-likelihood estimator of  $p_a^A$  under  $H0_{\succ}^{WARPSLA}$ .

We define the p-value for this  $\chi^2$ -based test of  $H0_{\succ}^{WARPSLA}$  as follows:

$$p_{\succ}^{WARPSLA2} = 1 - F_k(\mathcal{J}_{\succ}),$$

with  $F_k$  the cdf of the  $\chi^2$  distribution with  $k$  the degrees of freedom equals to the number of SI relations characterizing  $H0_{\succ}^{WARPSLA}$ .

Under  $H0_{\succ}^{WARPSLA}$ :  $p_{\succ}^{WARPSLA2}$  follows asymptotically a uniform distribution, and  $H0_{\succ}^{WARPSLA}$  is rejected at a nominal level  $\alpha$  whenever  $p_{\succ}^{WARPSLA2} < \alpha$ . As in the previous section, the p-value of the test of  $H0_{WARPSLA}$  is given by:

$$p_{WARPSLA2} = \max_{\succ \in \mathcal{L}} \{p_{\succ}^{WARPSLA2}\}.$$

We use the same simulated data produced by Simulation 3.4.1 and perform



the  $\chi^2$ -based test of  $H0_{\succ}^{WARPSLA}$  for all possible order as well as the conjunction test  $H0_{WARPSLA}$  for each simulated agent. Table 3.4.4 presents for each potential preference order and  $N=20, 40, 100,$  and  $1000$  decisions per menu, the probabilities that those p-values are lower than 1%, 5% and 10%. Figure 3.4.4 shows the p-values' cdf.

N	$\alpha$	$p_{\succ}^{WARPSLA2} < \alpha$						
		WARP-SLA2	$a \succ b \succ c$	$a \succ c \succ b$	$b \succ a \succ c$	$b \succ c \succ a$	$c \succ a \succ b$	$c \succ b \succ a$
20	0.1	0.045	0.13	0.316	0.345	0.483	0.468	0.58
	0.05	0.018	0.067	0.227	0.251	0.383	0.368	0.485
	0.01	0.002	0.014	0.112	0.124	0.231	0.212	0.32
40	0.1	0.056	0.114	0.416	0.481	0.658	0.633	0.759
	0.05	0.025	0.059	0.332	0.382	0.566	0.546	0.683
	0.01	0.004	0.011	0.206	0.242	0.408	0.391	0.54
100	0.1	0.077	0.113	0.558	0.685	0.854	0.812	0.91
	0.05	0.038	0.06	0.484	0.609	0.804	0.762	0.875
	0.01	0.006	0.012	0.358	0.469	0.689	0.651	0.798
1000	0.1	0.097	0.103	0.866	0.97	0.996	0.992	0.998
	0.05	0.047	0.051	0.841	0.96	0.995	0.987	0.997
	0.01	0.009	0.01	0.792	0.938	0.99	0.978	0.994

Table 3.4.4: Probability to reject  $H0^{WARPSLA}$  and  $H0_{\succ}^{WARPSLA}$  based on  $\chi^2$ -test. Simulated choices following a wRCCSR with  $a \succ b \succ c$ .

While the expected level of the test is not perfectly controlled for less than 40 observations per menu, it converges quicker to the nominal level if compared to the test based on the SI-tests' corrected pvalues. The  $\chi^2$ -based test is also more likely in average to reject false preference representation, which is indicative of an higher power. Moreover, we conduct a linear regression with the p-value of the test of  $H0_{a \succ b \succ c}^{WARPSLA}$  as a dependent variable and the attention parameters  $\pi()$  as independent variables. Indeed, a desirable quality of the test is the independence between type-I errors and model's parameters (such independence can be limited, since the actual level depends on the convergence of the multinomial to the normal distribution, which is shaped by model parameters). Using the simulated data, we find that  $\pi()$  explains respectively  $R^2 = 0.08\%, 0.07\%, 0.02\%,$  an  $0.04\%$  of the p-values' variance for  $N = 20, 40, 100,$  and  $1000$ . Therefore, model's parameters have a limited impact on the actual level of the tests. Those results are confirmed by another set of simulations, where we explicitly variation in actual level of different wRCCSR: the  $\chi^2$ -based test present also much less variation in the actual level if

compared to the test based on corrected p-values (see appendix 3.B.7).

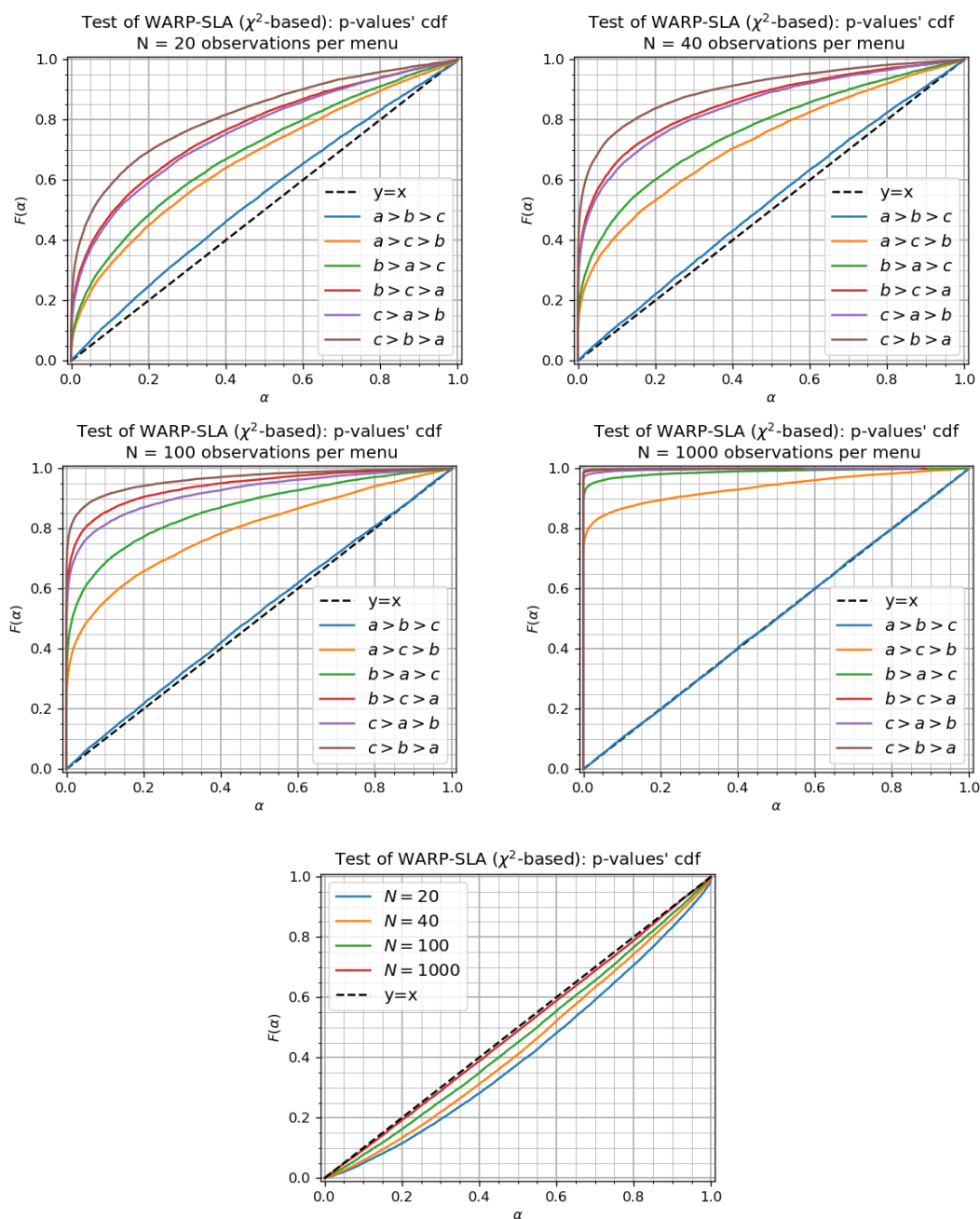


Figure 3.4.4: p-values's cdf for the ( $\chi^2$ ) test of WARP-SLA. Simulated choices following a wRCCSR.

**Note:** Data obtained from simulation 3.4.1. First two rows figures present present p-values' cdf for the test of each possible  $H0_{>}^{WARP\text{SLA}}$  based on the  $\chi^2$  test, when simulated agents take  $N=20, 40, 100,$  and  $1000$  decisions in each menu. In these figures, the only true null hypothesis is  $H0_{a>b>c}^{WARP\text{SLA}}$  (blue curve) and thus indicate the actual level of the test. Other curves represent the test's actual power to reject false preferences representations. Last row figures represent test of  $H0_{>}^{WARP\text{SLA}}$ , that is the frequency of not rejecting  $H0_{>}^{WARP\text{SLA}}$  for at least one preference order, for  $N = 20, 40, 100,$  and  $1000$  obs per menu).

### 3.4.2 Method to test wIFO and to reveal effective attention

To have a wRCCSR representation, a random choice rule  $p$  also needs to satisfy wIFO. We assume in this section that the observer already knows that WARP-SLA holds for at least a specific preference relation  $a \succ b \succ c$ . Therefore, we will construct a test of wIFO, conditional to  $H0_{\succ}^{WARP\text{SLA}}$ , with respect to this specific preference relation. Our test is constructed based on Lemma 3.3.1:

**Lemma 3.3.1.** *Let  $p$  a random choice rule. If  $p$  satisfies WARP-SLA, then there exist a complete and transitive preference relation  $\succ$  over  $X$  and a mapping  $\lambda : \mathcal{D} \rightarrow \mathbb{R}$ , such that  $\forall A \in \mathcal{D} \setminus \{\emptyset\}, \forall a \in A$ :*

$$p(a, A) = \frac{\sum_{B \in A_{a \succ}} \lambda(B)}{\sum_{B \subseteq A} \lambda(B)},$$

with  $A_{a \succ} = \{B \subseteq A : a \in B \text{ and } \forall b \in B \setminus a, a \succ b\}$ , and  $\lambda(A) = \sum_{B \subseteq A} (-1)^{|A \setminus B|} \frac{1}{p(o, B)}$ .

Given WARP-SLA, wIFO is thus equivalent to the positivity of  $\lambda$  that is a set of linear constraints, which can be jointly tested (Davis-Stober, 2009). More precisely, to test the positivity of  $\lambda$ , an observer should conduct a one-tailed test  $H0_{\succ}^{wIFO} : \forall A \in \mathcal{D}, \lambda(A) \geq 0$  versus  $H1 : \exists A \in \mathcal{D}, \lambda(A) < 0$ . Such test can be performed with a Likelihood Ratio Test:

$$LR = 2 \times (LL(\hat{\lambda}) - LL(\hat{\pi}))$$

Where LL is the log-likelihood function,  $\hat{\lambda}$  is the maximum likelihood estimator of  $\lambda$  over  $\mathbb{R}^{|\mathcal{D}|}$  and  $\hat{\pi}$  is the maximum likelihood estimator of  $\lambda$  over  $\mathbb{R}_+^{|\mathcal{D}|}$ . Contrary to the previous section, we do not provide closed form for the maximum likelihood estimators and estimate those values numerically.<sup>21</sup>

As the test is one-tailed, if  $\hat{\lambda} = \hat{\pi} > 0$ , we have  $LR = 0$  and since the data are

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<sup>21</sup>We apply the “*optimize.minimize*” function (Jones et al., 2001, from the Scipy library for Python) to  $-LL(\lambda)$  with and without constraints

less likely to be produced under  $H1$  than under  $H0_{\succ}^{wIFO}$ , we cannot reject  $H0_{\succ}^{wIFO}$ , at any statistical level. Therefore, the p-value of the test is equal to 1.

If  $\hat{\lambda} \neq \hat{\pi}$ , we have  $LR > 0$  and we must study the distribution of  $LR$  under  $H0_{\succ}^{wIFO}$ , to find the critical region and make a rejection decision. Under  $H0_{\succ}^{wIFO}$ , the LR statistic does not follow asymptotically a  $\chi^2$  distribution, since the number of constraints violated by  $\hat{\lambda}$  is itself a random variable, but a  $\bar{\chi}^2$  distribution which is a weighted sum of  $\chi^2$  (Shapiro, 1988; Davis-Stober, 2009):

$$\bar{\chi}^2 = \sum_{i=1}^k w_i \chi_i^2,$$

with  $w_i \geq 0$  for all  $i = (0, \dots, |\mathcal{D}|)$  and  $\sum w_i = 1$ .

The  $w_i$  represent the probability under the null, that  $\hat{\lambda}$  violates exactly  $i$  constraints fixed by  $H0_{\succ}^{wIFO}$ , that is the probability that there are  $i$  menus  $(A_j)_{j=1, \dots, i} \in \mathcal{D}$  such that  $\hat{\lambda}(A_j) < 0$ . General analytic forms for  $w_i$  are unknown, and depend on the true value of the parameter  $\lambda$  and the covariance matrix of the maximum likelihood estimator  $\hat{\lambda}$  under  $H0_{\succ}^{wIFO}$ . However, it is possible to use numerical methods to estimate these values. In particular, we use an algorithm proposed by Silvapulle (1996).<sup>22</sup>

Once the  $w_i$  are computed, the p-value of the test is computed as follows:

$$p_{\succ}^{wIFO} = \sum_i w_i [1 - F_i(LR)],$$

with  $F_k$  the cdf of the  $\chi^2$  distribution with  $k$  the degrees of freedom. We report in Figure 3.4.5 the p-values' cdf for the test of  $H0_{\succ}^{wIFO}$  for each preference relation and for each wCRCCSR agents, simulated according to Simulation 3.4.1.

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<sup>22</sup>See Section 3.B.4 for more details about the algorithm used to compute the  $w_i$ . This algorithm is also used in Davis-Stober (2009) and Regenwetter et al. (2010). See in particular Davis-Stober (2009) for an extensive review on the distribution of constrained likelihood ratio tests and its applications to axiomatic measurement theory.

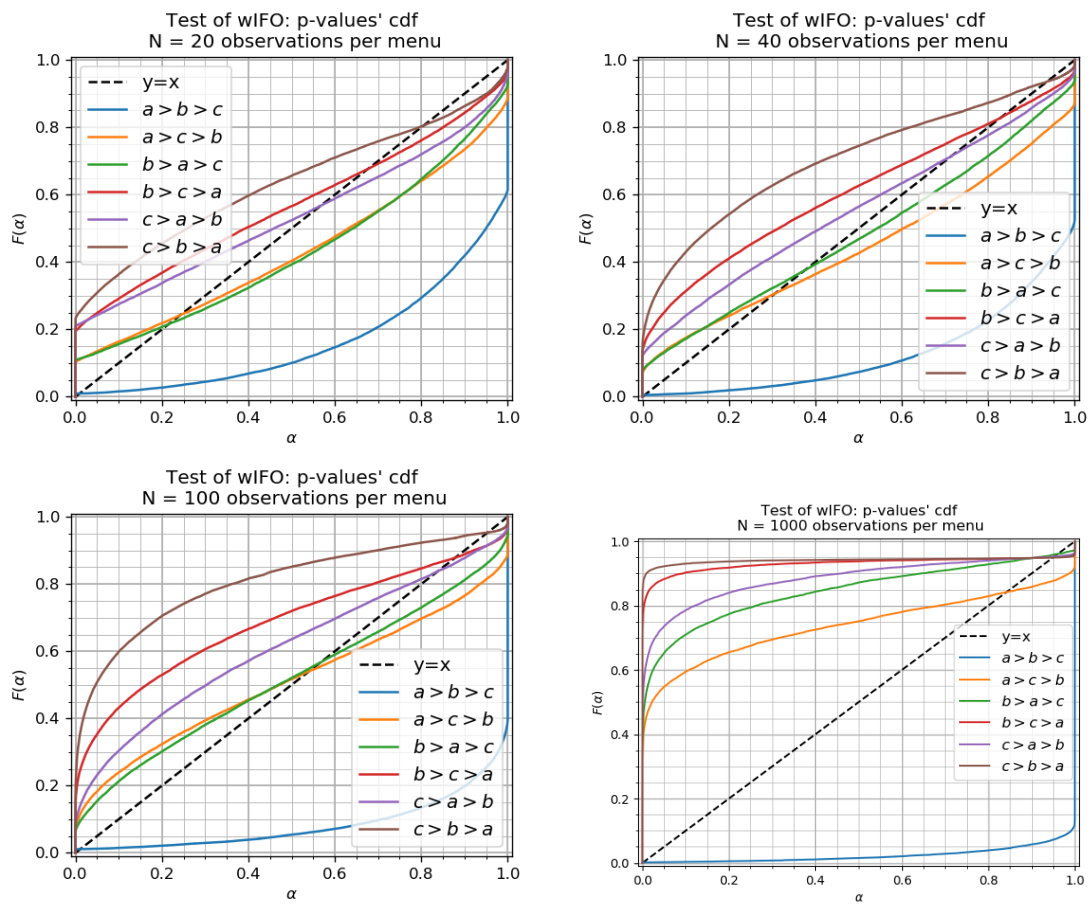


Figure 3.4.5: p-values' distribution for the tests of  $H_0^{wIFO}$

We observe that our method to test  $H0_{a \succ b \succ c}^{wIFO}$  is very conservative. The rejection probability goes to 0 as  $N$  increases. This is not surprising, since the test we used is one-tailed so to control the level of the test, the  $LR$  distribution is estimated in a least favorable case where  $\lambda$  is on its boundary. For  $\lambda$  in the interior of the permissive space, (as it is for the agents simulated according to simulation 3.4.1) the method is conservative by construction. Therefore, we confront this method to agents simulated on the boundary of  $H0_{\succ}^{wIFO}$ , where  $\pi(A) = 0$  for all  $|A| > 1$ . This case corresponds to the Luce Rule (1959). We perform the following simulation analysis.

**Simulation 3.4.2.** *We simulate agents making choices according to a wRCCSR with 3 alternatives  $a \succ b \succ c$  in  $\mathcal{D} = 2^{\{a,b,c\}}$ . For each agent  $i$ , we fix the attention parameters  $\pi_i(abc) = \pi_i(ab) = \pi_i(ac) = \pi_i(bc) = 0$  and randomly draw the others ( $\pi_i(a)$ ,  $\pi_i(b)$ ,  $\pi_i(c)$  and  $\pi_i(\emptyset)$ ) from independent uniform distributions  $\mathcal{U}_{[0,1]}$ . Then, we divide all the parameters by  $\pi_i(a) + \pi_i(b) + \pi_i(c) + \pi_i(\emptyset)$ , to guaranty that  $\pi$  is a probability measure. We generate the random choice rule  $p_{\pi_i, \succ}$  (which is independent of  $\succ$ ). Then for each agent, and for each possible menu  $A \in \mathcal{D}$ , we simulate a vector of choices  $n^A$  that is the realisation of the multinomial  $\mathcal{M}(N, p_{\pi_i, \succ}(\cdot, A))$ , where  $N$  is the number of choices each simulated agent makes per menu. We vary  $N$  and simulate  $K=10.000$  agents for each  $N=20, 40, 100$  and  $1000$ .*

We report in Figure 3.4.6 the actual p-values' cdf obtained when testing if the agents simulated in Simulation 3.4.2 respect wIFO associated to the preference relation  $a \succ b \succ c$ . We observe that when the attentional parameters lie on the boundary, the test is less conservative and asymptotically exact: wIFO is falsely rejected with the nominal level  $\alpha = 5\%$  for 2.7% of the simulated agents making  $N = 40$  decisions per menu.

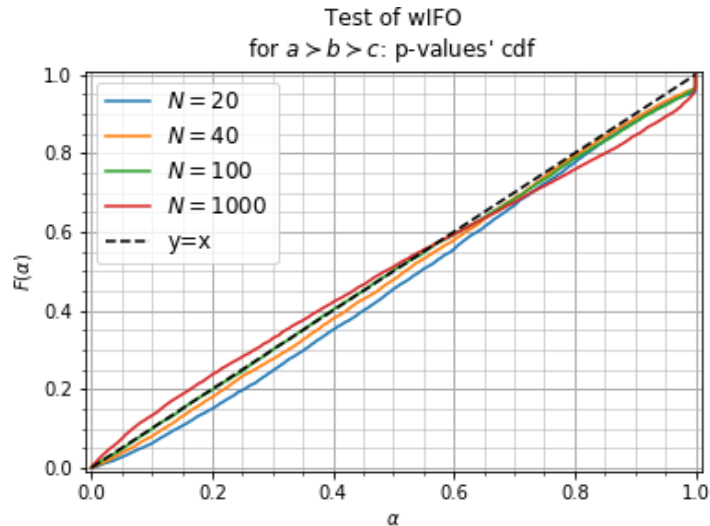


Figure 3.4.6: p-values' cdf for the test of  $H0_{a>b>c}^{wIFO}$  when p follows the Luce Rule.

The likelihood estimation required to test wIFO provides a measure  $\hat{\pi}$  of the attention parameters  $\pi$ . To assess how precise these estimators are, we compute for each menu  $A \in \mathcal{D}$  the square of the Pearson correlation coefficient  $R^2$  between the value  $\pi(A)$  used in Simulation 3.4.1 to generate the choices and the value  $\hat{\pi}(A)$  estimated by maximum likelihood. Figure 3.4.7 shows for each wRCCSR simulated agent the true and the estimated value of the attention parameters and reports the  $R^2$ .

We observe that all the estimators converge to their true values as the number of choices per menu increases. However, the estimators of the attentional measure associated with larger menu of alternatives are less precise. This difference in the estimation quality is explained by the fact that different number of observations are involved in their estimation: for example  $\pi(abc)$  is only involved in the choice probability in the menu  $\{abc\}$ , while  $\pi(\emptyset)$  is involved in the choice probabilities of all menus.

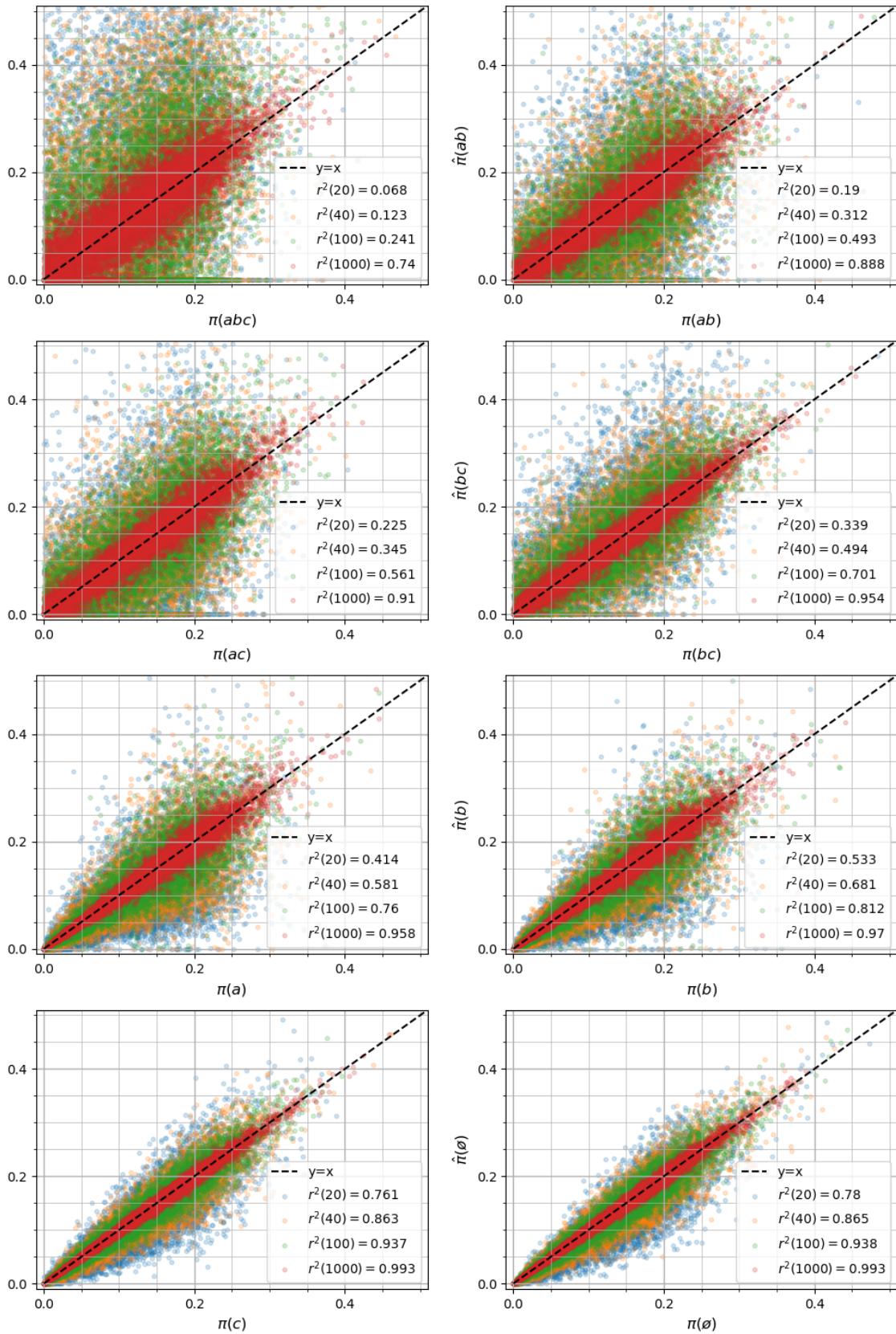


Figure 3.4.7: Real attention parameters and ML-estimated measure of attention.

**Note:** Estimated ( $y$ -axis) vs true ( $x$ -axis) attentional parameters for each simulated agents. Color represents the number of decisions  $N=20$ , (blue), 40 (orange), 100 (green) and 1000 (red) taken in each menu. The menus  $\pi(abc)$ ,  $\pi(ab)$ ,  $\pi(ac)$ ,  $\pi(bc)$ ,  $\pi(a)$ ,  $\pi(b)$ ,  $\pi(c)$  and  $\pi(\emptyset)$  are presented from left to right and top to bottom.  $R^2$  reports the square of the Pearson correlation coefficient between real and estimated parameters.



### 3.4.3 Join Test of WARP-SLA and wIFO

Using a similar approach, we propose a method to test simultaneously WARP-SLA and wIFO, with respect to a specific preference representation. For this purpose, we could rewrite any random choice rule to express WARP-SLA and wIFO as a set of linear inequality and equality constraints.<sup>23</sup>

**Proposition 3.4.2.** *Let  $p$  a random choice rule. We have for all  $a \in X$  and for all  $A \in \mathcal{D}$ :*

$$p(a, A) = \frac{\sum_{\substack{B \subseteq A \\ a \in B}} \mu(a, B)}{\sum_{B \subseteq A} \sum_{b \in B} \mu(b, B)}$$

With  $\mu : X \cup o \times \mathcal{D} \rightarrow \mathbb{R}$  such that for all  $a \in X$  and for all  $A \in \mathcal{D}$ :

$$\mu(a, A) = \kappa \times \sum_{B \subseteq A} (-1)^{|A \setminus B|} \frac{p(a, B)}{p(o, B)},$$

with  $\kappa$  a scaling factor such that  $\sum_{a \in X \cup o} \sum_{A \in \mathcal{D}} \mu(a, A) = 1$ .

Moreover, WARP-SLA and wIFO hold iff there exists a complete and transitive preference relation  $\succ$  over  $X$  such that  $\mu(a, A) \geq 0$  for all  $a$  and  $A$  and  $\mu(b, A) = 0$  whenever  $\exists a \in A$ , such that  $a \succ b$ .

Therefore, we can express the choice probability in terms of  $\mu$  and test for each linear order  $\succ$  the following hypothesis:

$$\left( H0_{\succ}^{\text{WARP-SLA} \cap \text{wIFO}} : \mu \geq 0, \text{ and } \forall A \in \mathcal{D} \forall b \neq x_A^{\succ}, \mu(b, A) = 0 \right),$$

against the alternative hypothesis  $H1$  that  $p$  is any random choice rule.

For example, if  $\mathcal{D} = 2^{abc}$ ,  $H0_{a \succ b \succ c}^{\text{WARP-SLA} \cap \text{wIFO}}$  holds iff:  $\mu(a, abc) \geq 0$ ,  $\mu(a, ab) \geq 0$ ,  $\mu(a, ac) \geq 0$ ,  $\mu(a, a) \geq 0$ ,  $\mu(b, b) \geq 0$ ,  $\mu(c, c) \geq 0$  and  $\mu(b, abc) = \mu(c, abc) = \mu(b, ab) = \mu(c, ac) = \mu(b, bc) = 0$ .

Therefore, WARP-SLA and wIFO can be tested using a maximum likelihood ratio test:

$$LR = 2 \times (LL(\hat{\mu}) - LL(\bar{\mu})) \xrightarrow[N \rightarrow +\infty]{d} \bar{\chi}^2,$$

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<sup>23</sup>See proof in appendix 3.B

### 3.4. STATISTICAL METHODS TO TEST WRCCSR

where LL is the log-likelihood function,  $\hat{\mu}$  is the unconstrained maximum likelihood estimator of  $\mu$  and  $\bar{\mu}$  is the constrained maximum likelihood estimator of  $\mu$ . Under  $H0_{\succ}^{WARPSLA \cap wIFO}$ ,  $LR$  follows a  $\bar{\chi}^2$  distribution and the rest of the method is similar as the one described in the previous section and in the appendix.

We confront this method to simulated data. We report in Figure 3.4.8 the p-values' cdf of the simultaneous test of wIFO and WARP-SLA for each preference order, for wRCCSR simulated agents. Table 3.4.5 reports the proportion of wRCCSR-simulated agents rejecting the simultaneous test at the 1%, 5% and 10% significance level for each potential preference relation and for N=20, 40, 100 and 1000 decisions per menu.

N	$\alpha$	$F(p_{\succ}^{WARPSLA \cap wIFO} < \alpha)$					
		$a \succ b \succ c$	$a \succ c \succ b$	$b \succ a \succ c$	$b \succ c \succ a$	$c \succ a \succ b$	$c \succ b \succ a$
20	0.1	0.044	0.236	0.275	0.461	0.425	0.604
	0.05	0.021	0.163	0.195	0.365	0.322	0.508
	0.01	0.006	0.08	0.093	0.217	0.179	0.335
40	0.1	0.036	0.344	0.408	0.649	0.602	0.792
	0.05	0.015	0.270	0.329	0.561	0.519	0.720
	0.01	0.004	0.175	0.203	0.406	0.366	0.575
100	0.1	0.031	0.503	0.634	0.855	0.801	0.935
	0.05	0.013	0.438	0.566	0.81	0.749	0.907
	0.01	0.002	0.336	0.431	0.701	0.637	0.837
1000	0.1	0.023	0.851	0.964	0.997	0.991	1
	0.05	0.011	0.826	0.954	0.996	0.988	1
	0.01	0.002	0.787	0.932	0.993	0.98	0.999

Table 3.4.5: Probability to reject  $H0_{\succ}^{WARPSLA \cap wIFO}$  for each preference order. Simulated choices following wRCCSR with  $a \succ b \succ c$ .

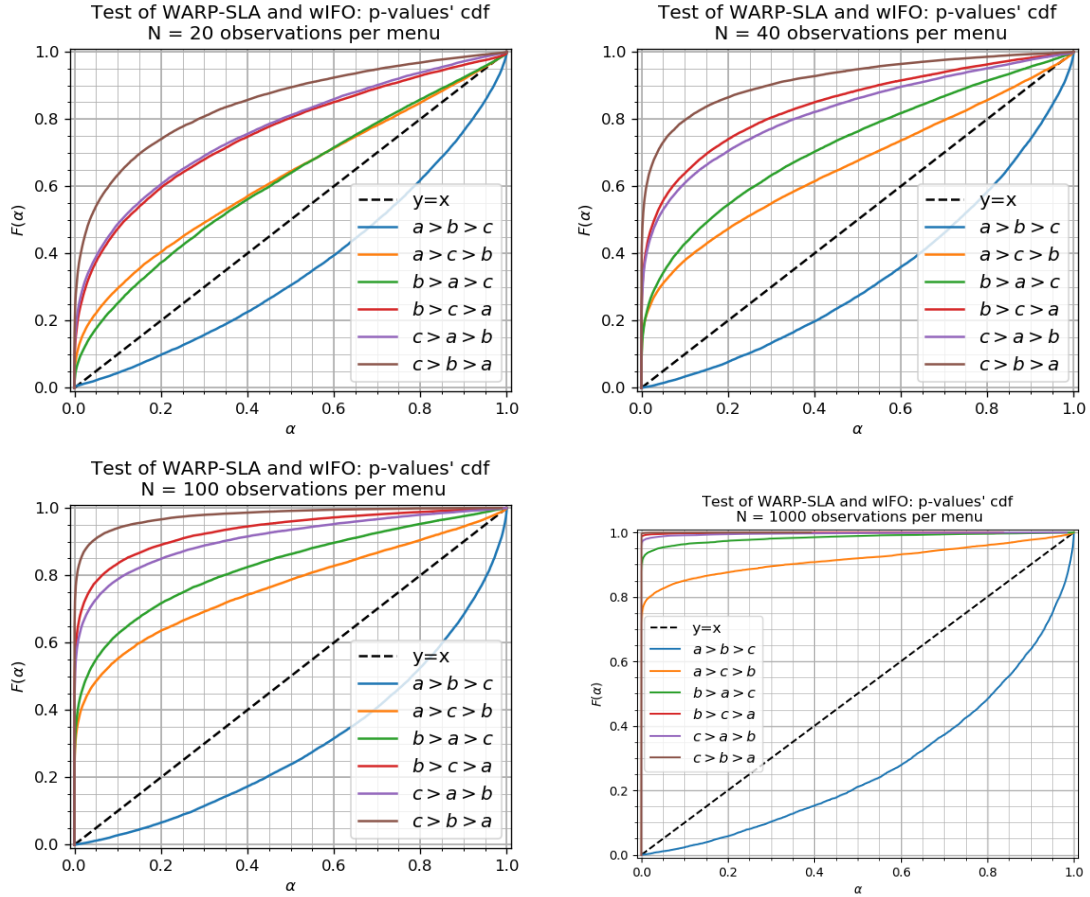


Figure 3.4.8: p-values' distribution for the simultaneous test of wIFO and WARP-SLA.

**Note:** The figures represent p-values' cdf for the test of  $H0_{\sim}^{WARP\text{SLA} \cap \text{wIFO}}$ . Each cdf indicates for each preference relation, the proportion of wRCCSR-simulated agent ( $y$  axis) rejecting  $H0_{\sim}^{WARP\text{SLA} \cap \text{wIFO}}$  at a given nominal level  $\alpha$  ( $x$ -axis). In all figures, the only true null hypothesis is  $H0_{a>b>c}^{WARP\text{SLA} \cap \text{wIFO}}$  and is represented by the blue curve, which thus indicates the actual expected level of the test. The other curves are thus indicative of the power of the test to reject the false preference representations when  $p$  is a wRCCSR.

For the same reason as the one discussed in the previous section, when the attentional measure does not lie on the boundary of  $H0_{\succ}^{WARP\text{SLA} \cap wIFO}$ , this test is very conservative. Indeed if we compared it to the tests of  $H0^{WARP\text{SLA}}$ , for  $N = 40$ , only 2.1% of the simulated wRCCSR agents reject the test of  $H0_{a>b>c}^{WARP\text{SLA} \cap wIFO}$  at the 5% level (vs 5.9% for the  $\chi^2$ -based test and 12% for the SI-based test). Besides being more conservative, using this test, an observer is more likely to reject false wRCCSR representations. For example the expected probability to reject the alternative hypothesis  $H1_{c>b>a}^{WARP\text{SLA} \cap wIFO}$  at the 5% level under  $H0_{a>b>c}^{WARP\text{SLA} \cap wIFO}$  is 0.72 of the simulated agents reject (vs respectively 0.64 and 0.59 for rejecting the alternative hypothesis  $H1_{c>b>a}^{WARP\text{SLA}}$  at the 5% level under  $H0_{a>b>c}^{WARP\text{SLA}}$  with the  $\chi^2$  and SI based tests). We investigate the expected level in the boundary conditions, that is when the data generating process follows the Luce Rule (see Simulation 3.4.2), we observe that the expected actual level of the test is very close to the nominal one, even with small sample ( $N = 20$ ) (see Figure 3.4.9).

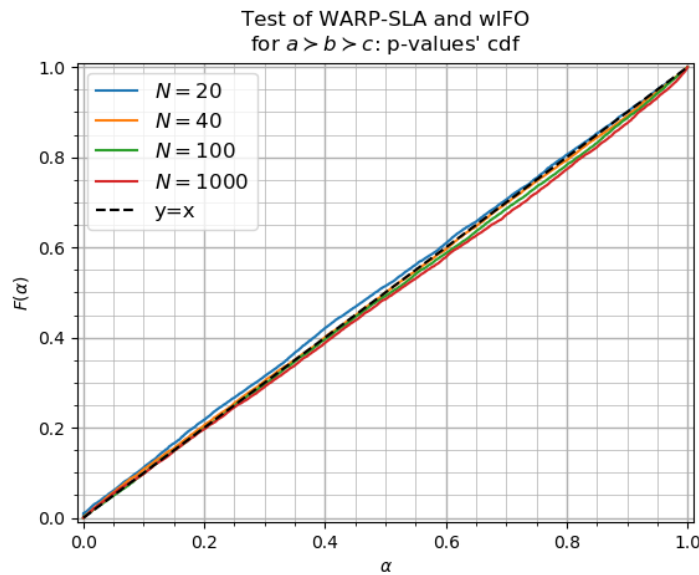


Figure 3.4.9: p-values' distributions for the test of  $H0^{WARP\text{SLA} \cap wIFO}$ , when  $p$  follows the Luce Rule.

For each preference representation  $\succ$  if we set  $\bar{\pi}(A) := \bar{\mu}(x_A^{\succ}, A)$ , we obtain exactly identical estimators of the attention parameters than those obtained in the previous version when testing  $H0_{\succ}^{wIFO}$ .

### 3.4.4 Power of the tests

Another important feature to investigate is the power of these tests, that is the probability to correctly reject  $H0_{WARPSLA}$  and  $H0_{WARPSLA \cap wIFO}$  when the choices were not produced by a wRCCSR. Basically, the power of the tests will depend both on the number of observations and on the distance between the true random choice rule underlying the observed decisions and the closest wRCCSR. To avoid the choice of any particular random choice rule, we decided to simulate uniformly several random choice rules among the set of possible random choice rule (i.e. the set of functions  $p : X \cup o \times \mathcal{D} \rightarrow [0, 1]$ , such that for all  $A \in \mathcal{D}$ ,  $\sum_{a \in A \cup o} p(a, A) = 1$ ,  $p(a, A) = 0$  if  $a \notin A \cup o$  and  $p(a, A) \in (0, 1)$  if  $a \in A$ ).

Therefore, the average power of the test calculated on the basis of these uniformly distributed random choice rules is an estimator of the proportion of random choice rules for which  $H0_{WARPSLA}$  and  $H0_{WARPSLA \cap wIFO}$  would be rejected at a level  $\alpha$ . More precisely, we conduct the following simulation analysis.<sup>24</sup>

**Simulation 3.4.3.** *Choices are made among 3 different alternatives  $X = \{a, b, c\}$  (and a default option  $o$ ) in every menu of a domain that is the power set of  $X$ :  $\mathcal{D} = \{\{abc\}, \{ab\}, \{ac\}, \{bc\}, \{a\}, \{b\}, \{c\}, \emptyset\}$ . We generate 10.000 different random choice rules ( $p_i$ ). Each random choice rule is generated as follows: for each menu  $A \in 2^X$  and each alternative  $a \in A \cup o$  we independently generate  $p_i(a, A) \sim \mathcal{U}_{[0,1]}$  and thus divide  $p_i(a, A)$  by  $\sum_{b \in A \cup o} p_i(b, A)$ . For each of the 10.000 different random choice rules  $p_i$ , we simulate independently and identically the realisation of  $(\mathcal{M}_{|A|}(N, p_i(\cdot, A)))_{A \in \mathcal{D}}$  which indicates the number of times each alternative is chosen in each menu. We repeat the simulation for a number of decisions per menu  $N$  in (20, 40, 100 and 1000).*

For each agent simulated according to Simulation 3.4.3 we test all the SI relations using Proposition 3.2.2 and use corrected pvalues to test  $H0_{\succsim}^{WARPSLA}$  for each preference representation and  $H0_{WARPSLA}$ . We also tested  $H0_{\succsim}^{WARPSLA}$  for each pref-

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<sup>24</sup>It could be interesting, in further research, to calculate the power under a more restricted set of random choice rules. A good candidate could be the set of RUM, as it is an important class of models, distinct from a wRCCSR. The average power would thus be a measure of how empirically distinct RUM and wRCCSR are.

erence representation and  $H0_{WARP\text{SLA}}$  using the  $\chi^2$ -based test described in Proposition 3.4.1 and  $H0_{\zeta}^{WARP\text{SLA}} \cap w\text{IFO}$  for each preference orders and  $H0_{WARP\text{SLA}} \cap w\text{IFO}$  with the method described in section 3.4.3. Figure 3.4.10 shows p-values' cdf for all of those tests.

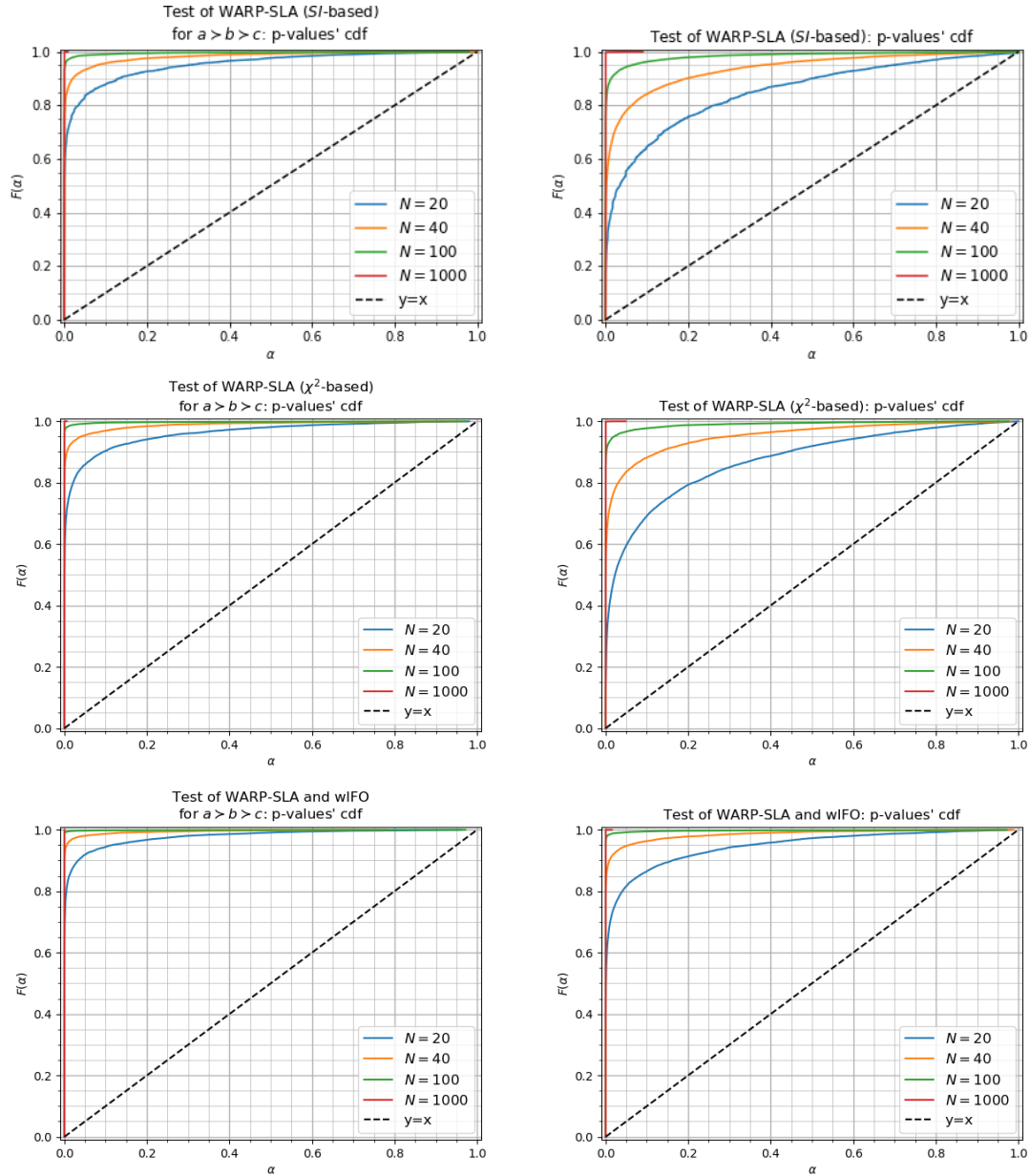


Figure 3.4.10: Power of separate  $SI$  and  $\chi^2$  based test of WARP-SLA and simultaneous test of wIFO and WARP-SLA.

**Note:** Choices simulated according to Simulation 3.4.3. Left (Right) figures present actual p-values' cdf for the tests of  $H0_{a>b>c}^{WARP\text{SLA}}$  ( $H0_{WARP\text{SLA}}$ ). Top, middle and bottom figures represent respectively separate  $SI$ ,  $\chi^2$  methods test of.

We conclude that the  $\chi^2$ -based test from proposition 3.4.1 is more powerful than

the SI-based test. And that despite being more conservative, our method to test simultaneously  $H0_{WARPSLA} \cap wIFO$  is the best to discriminate random choice rules. With only  $N = 40$  decisions per menu, it is rejected at the 5% level for 96.9% of the simulated random choice rules. With  $N = 100$  decisions per menu only 2 over the 10.000 simulated random choice rules return a  $p - value > 0.05$  and 4 return a  $p - value > 0.01$ .

### 3.4.5 Test of Revealed Preferences

Testing all possible preference representation allows to identify if some alternatives are revealed to be preferred to others. For example, if the observer accepts  $H0_{WARPSLA}$  but rejects  $H0_{c \succ a \succ b}^{WARPSLA}$ ,  $H0_{c \succ b \succ a}^{WARPSLA}$  and  $H0_{b \succ c \succ a}^{WARPSLA}$ , she can concludes that  $a$  is revealed to be preferred to  $c$  since there is no preferences representation ranking  $c$  above  $a$ . However, the number of linear order over a set of cardinality  $n$  is equal to  $n!$ , so testing all possible linear orders can be computationally difficult when the number of alternatives grows, if compare to computing all pairwise binary preferences (their number is equal to  $n \times (n - 1)$ ).

In this section, we assume that the observer knows that the data generating process  $p$  is a wRCCSR but has no further information about the attentional parameters nor the underlying preference relations. However, from theorem 3.3.2, we know that if  $p$  is a wRCCSR and accept at least one valid preference representation  $\succ$  such that  $b \succ a$ , then the following hypothesis is true :

$$H0^{-aPb} : \frac{p(a, A)}{p(o, A)} = \frac{p(a, A \setminus b)}{p(o, A \setminus b)} \text{ for all } A \in \mathcal{D}.$$

We test this null hypothesis against the alternative hypothesis:

$$H1^{aPb} : \exists A \in \mathcal{D}, \text{ such that } \frac{p(a, A)}{p(o, A)} \neq \frac{p(A, A \setminus b)}{p(o, A \setminus b)}.$$

If  $p$  is a wRCCSR and if we reject  $H0^{-aPb}$ , then we accept  $H1^{aPb}$  and reject the statement that it exists a valid preference representation of  $p$  such that  $b \succ a$ . Therefore we have  $a \succ_i b$ , for all  $\succ_i$ , that is  $a$  is revealed to be preferred to  $b$ .

$H0^{-aPb}$  should be rejected if the observer identifies a statistical difference between  $\frac{p(a, A)}{p(o, A)}$  and  $\frac{p(a, A \setminus b)}{p(o, A \setminus b)}$  for at least one menu  $A \in \mathcal{D}$ . Therefore, in a practical application of the test, preferences may be revealed without gathering data in all the menus of the domain. Indeed, we formulate the following statistical test which requires to observe decisions in an arbitrary number of pairs of menus, including the alternative  $a$  and only differing in the presence/absence of the alternative  $b$ .

**Proposition 3.4.3.** *Let  $p$  a wRCCSR. Let  $a$  and  $b \in X$ . Let  $\mathcal{S} \subseteq \mathcal{D}$ ,  $\mathcal{S} \neq \emptyset$ , such that  $A \in \mathcal{S} \Rightarrow (a \in A, A \setminus b \in \mathcal{S} \text{ and } A \cup b \in \mathcal{S})$ . Let  $(n(\cdot, A))_{A \in \mathcal{S}}$  the realisation of a collection of independent multinomial variables  $(\mathcal{M}(p(\cdot, A), N))_{A \in \mathcal{S}}$  that indicates the number of times each alternative is chosen in each menu  $A$ .*

*Under  $\left( H0^{-aPb} : \forall A \in \mathcal{D}, \frac{p(a, A)}{p(o, A)} = \frac{p(a, A \setminus b)}{p(o, A \setminus b)} \right)$ , the following statistics:*

$$\mathcal{P} = \sum_{\substack{A \in \mathcal{S} \\ b \in A}} \frac{1}{\hat{\sigma}_{-aPb, A}^2} \times \left( \log \left( \frac{n(a, A) \times n(o, A \setminus b)}{n(o, A) \times n(a, A \setminus b)} \right) \right)^2 \xrightarrow[N \rightarrow +\infty]{d} \chi_k^2,$$

*follows asymptotically a  $\chi^2$  distribution with  $k = |\mathcal{S}|/2$  degrees of freedom, with*

$$\hat{\sigma}_{-aPb, A}^2 = 2 \times \frac{n(a, A) + n(o, A) + n(o, A \setminus b) + n(a, A \setminus b)}{n(a, A)n(o, A \setminus b) + n(a, A \setminus b)n(o, A)},$$

*an estimator of the variance of  $\log \left( \frac{n(a, A) \times n(o, A \setminus b)}{n(o, A) \times n(a, A \setminus b)} \right)$  under  $H0^{-aPb}$ .*

Under the null hypothesis  $H0^{-aPb}$  that  $a$  is not revealed to be preferred to  $b$ , the above statistics  $\mathcal{P}$  follows asymptotically a  $\chi^2$  distribution. We thus define the p-value of the test as follow :

$$p^{a \succ b} = 1 - F_k(\mathcal{P}),$$

with  $F_k$  the cdf of the  $\chi^2$  distribution with  $k$  degrees of freedom. We rejected  $H0^{-aPb}$  and we accept  $H1^{aPb}$  at a nominal level  $\alpha$  whenever  $p^{a \succ b} < \alpha$ . In this case, we say that “ $a$  is statistically revealed to be preferred to  $b$  at a nominal level  $\alpha$ ” and we note  $a \hat{\succ} b$ . Since the test jointly investigates differences between  $\frac{p(a, A)}{p(o, A)}$  and  $\frac{p(a, A \setminus b)}{p(o, A \setminus b)}$  in all menus  $A \in \mathcal{S}$ , the test may reveal preferences that cannot be revealed by the



standard test of binary SI presented in Proposition 3.2.2.

We study the actual level and the power of the test through simulations. As Proposition 3.4.3 can be applied only under the assumption that  $p$  is a wRCCSR, we only confront this test to the wRCCSR-simulated agents (Simulation 3.4.1). Figure 3.4.11 reports p-values' cdf for the test of each "true" ( $a \succ b$ ,  $b \succ c$ ,  $a \succ c$ ) and "false" ( $b \succ a$ ,  $c \succ b$  and  $b \succ c$ ) binary preference, and for  $N=20$ , 40, 100, and 1000 decisions per menu. The cdf indicates for each test the frequency of p-values lower than  $\alpha$  and corresponds to the expected actual level for the tests of the false preferences and the expected power for the test of true preferences. Table 3.4.6 reports, for the test of each of those relations, the probabilities to reject the test at the 1%, 5% and 10% level.

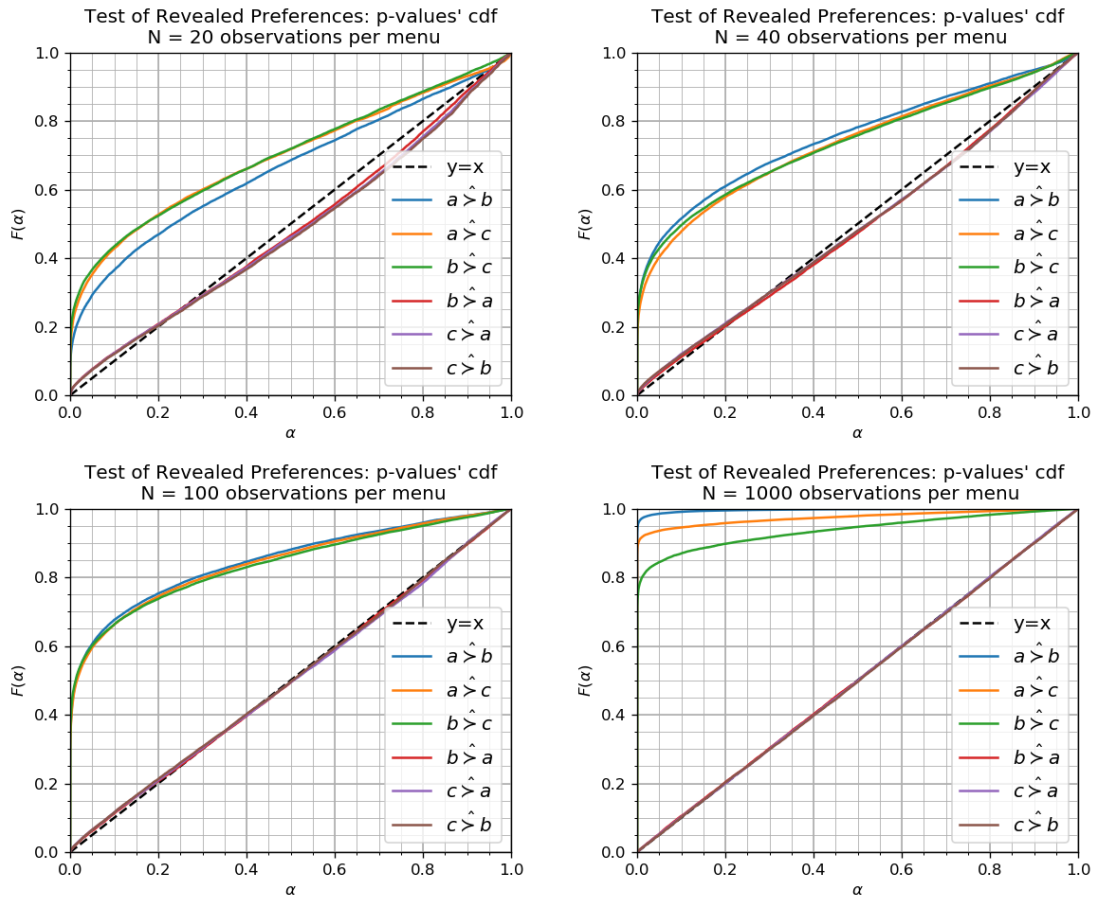


Figure 3.4.11: p-values' distributions for revealed preferences tests.

N	$\alpha$	Probability to reveal					
		true pref. (Power)			false pref. (level)		
		$a \hat{>} b$	$a \hat{>} c$	$b \hat{>} c$	$b \hat{>} a$	$c \hat{>} a$	$c \hat{>} b$
20	0.1	0.375	0.376	0.364	0.119	0.104	0.105
	0.05	0.306	0.306	0.301	0.072	0.06	0.064
	0.01	0.2	0.199	0.199	0.026	0.022	0.022
40	0.1	0.516	0.518	0.492	0.124	0.11	0.115
	0.05	0.441	0.441	0.421	0.073	0.064	0.065
	0.01	0.318	0.321	0.307	0.024	0.021	0.021
100	0.1	0.697	0.687	0.628	0.114	0.117	0.116
	0.05	0.629	0.623	0.564	0.063	0.068	0.068
	0.01	0.506	0.505	0.45	0.02	0.022	0.021
1000	0.1	0.953	0.949	0.885	0.102	0.107	0.099
	0.05	0.942	0.936	0.866	0.051	0.052	0.05
	0.01	0.913	0.904	0.825	0.012	0.013	0.011

Table 3.4.6: Probability to reveal “statistically significant” preferences.

We observe that our test is not conservative for small values of  $N$  but is asymptotically exact as  $N$  increases. The probability to reveal the true preferences depends heavily on the underlying attentional parameters. Since in our simulation, those parameters were uniformly drawn, the proportion of simulated agents revealing their underlying preferences can be interpreted as the expected probability to reveal statistically significant preferences at a given nominal level, under uninformative priors. Testing all revealed preferences at a nominal level  $\alpha = 5\%$  and with  $N = 40$  observations per menu,  $a$  is revealed to be preferred to  $b$  for 44.1%,  $a$  is revealed to be preferred to  $c$  for 44.1% and  $b$  is revealed to be preferred to  $c$  for 42.1% of the wRCCSR.

To investigate the influence of the model parameters, we conduct a multivariate linear regression with the 3 p-values associated to the test of the true preferences ( $p^{a \hat{>} b}$ ,  $p^{a \hat{>} c}$  and  $p^{b \hat{>} c}$ ) as dependent variables and the attention parameters  $\pi()$  as independent variables. We find that attentional parameters  $\pi()$  explain respectively for  $N=20, 40, 100$  and  $1000$  decisions per menu,  $R^2 = 24.28\%, 27.32\%, 29.23\%$  and  $16.78\%$  of p-values’ variance. The attentional parameters of the wRCSR are thus determinant to the probability to reveal the true preference relations.<sup>25</sup>

<sup>25</sup>We conduct the same regression with the 3 other false preference relations  $c \hat{>} b$ ,  $c \hat{>} a$  and  $b \hat{>} a$ . And find that the attentional parameters explain respectively 1.81%, 0.68%, 0.15% and 0.06% of

### 3.5 Experimental Implementation

In the previous section, we provide statistical methods to test wRCCSR axioms and to reveal preferences and analyze their level and power through simulation. It is thus possible to test empirically the model with real data using those methods. The aim of this section is two folds. First we want to see if it is possible to have environment where the decisions taken by individuals are coherent with a wRCCSR model. Second, we want to assess if it is possible to disentangle preferences from limited attention, in a real choice task, assuming that decisions follow a wRCCSR. For this purpose, we conducted a laboratory experiment where individuals take decisions in an environment where preferences are induced and attention limited.

It seems reasonable to expect that limited attention models are more adapted to capture situations with a high number of alternatives, where individuals are not able to be attentive to all of them. However, a theoretical requirement to test the axioms is to propose to participants to make their choices in at least all the possible binary and singletons menus that can be formed with a set of available alternatives  $X$ . This number grows quickly with the number of available alternatives (If  $|X| = M$ , we must propose at least  $|\mathcal{D}| = \frac{M \times (M+1)}{2}$  different menus). To ensure enough statistical power, another requirement is to observe a sufficient number of decisions  $N$  in each of these menus. Therefore, another contribution of this chapter is to propose an environment that satisfies the following conditions:

- i) Preferences are induced, so we can track their correct identification;
- ii) Participants are not able to pay attention to all the available alternatives;
- iii) Participants make a sufficiently high number of decisions in each menu;
- iv) The overall length of the experiment is not excessively long.

To meet these four constraints, we implemented a “Perceptual Decision” task, where participants face few alternatives represented by monetary gain (up to 3,

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the variance of the p-values for  $N=20, 40, 100$  and  $1000$ . So attention parameters have a limited impact on the probability to reveal false preferences and the actual level. A deeper analysis of the influence of model parameter based on another set of simulation is provided in the appendix 3.B.7

plus a default option), during a short period of time (less than one second) and among numerous distractors. Participants have to allocate their attention to avoid distractors and to perceive alternatives. The short period of time ensures that the participants do not have the time to perceive all the information and thus may not consider all the alternatives. In case a participant does not perceive any alternative, a default option is always available, as required by the model. We implemented a penalty when the individual chooses a distractor and we calibrated the values of the different alternatives in order to undermine incentives to guess randomly when an individual perceives no alternatives but the default.

Another advantage of our design is that participants' response time is short (less than 5 seconds in average). This allows us to propose a high number of trials within a single experimental session. We decided to propose all the menus in the powerset  $\mathcal{D} = 2^X$  with  $X = \{a, b, c\}$ . From the simulation analysis conducted in the previous section, we estimate that with  $N = 40$  choices per menu we have reasonable power and acceptable asymptotic properties. To have a balance in the experimental condition, we decided to repeat choices 42 times in each menu. Therefore each participant makes a total of  $42 \times 7 = 294$  decisions. As our experiment may appear repetitive and exhausting for the eyes, we decided to group decision into 7 blocks of 42 decisions and we imposed a minimum of one-minute break between each block. Participants are allowed and encouraged to rest themselves whenever they want, and we acknowledged them that this will not affect negatively their payoff.

### 3.5.1 Design

*Stimuli and choice.* — Our perceptual task consisted in the presentation of a stimuli composed of 1, 2 or 3 alternatives, plus a default option, among distractors, followed by a decision screen where participants have to choose one of the alternatives. A trial began with a reminder of the instructions that remains on screen until the participant initiate the trial by pressing the space bar.

The stimuli was disclosed 750ms after trial initiation and lasted 900ms. It consisted in the presentation of 14 symbols of  $0.7^\circ$  vision angle height, uniformly located on a centered circle, with a  $7^\circ$  vision angle radius. Depending on the

condition, one to three of the symbols are one-digit numbers that represent the alternatives. We have the following three possible alternatives:  $(a : 9 \text{ ECU}) > (b : 8 \text{ ECU}) > (c : 6 \text{ ECU})$ . The other symbols are a letter “P” and represent the distractors. At the center of the screen, another symbol of height =  $1.2^\circ$  represents the default alternative  $(o : 4 \text{ ECU})$  (see Figure 3.5.1, Top).<sup>26</sup>

Once the 900ms have expired, the decision screen appears. In the decision screen, all symbols but the default were replaced by an empty circle and a text invited the participant to choose one of the locations with the mouse. If participants clicked on the location of an alternative or of the default, they earned the associated amount in ECU. Otherwise, if they click on a distractor, they bore a cost of 15 ECU (see Figure 3.5.1, Bottom). Once the participant had made her choice, the next trial followed with a reminder of the rule and an invitation to initiate the trial. No feedback were provided between trials.

***Spatial location of alternatives, trials and menus.*** — Each participant played a sequence of 7 blocks  $\times$  42 trials. Trials are uniquely defined by their serial position within a block and the potential location of the three alternatives (alternatives are located from 1 to  $14 \times \frac{\pi}{7}$  on the unit circle, anticlockwise from the extreme right). For example, the first trial of a block always corresponds to the following potential alternatives’ location  $(x_a = 4, x_b = 5, x_c = 12)$ . The second trial corresponds to the following potential alternatives’ location:  $(x_a = 13, x_b = 5, x_c = 7)$ .

Blocks are defined by a sequence of conditions that indicates for each trial which alternatives are available or not. In each block, each trial is proposed in one of the 7 different conditions that correspond to all the possible non-empty menus:  $(\{abc\}, \{ab\}, \{ac\}, \{bc\}, \{a\}, \{b\}, \{c\})$ . For example, in Block 1, the first trial was proposed in the  $(\{b\})$  condition (only alternative  $b$  was available, and located at position  $x_b = 5$ ) and the second trial in the  $(\{abc\})$  condition (all alternatives were available at the positions  $x_a = 13, x_b = 5, x_c = 7$ ). While in Block 2, the former was proposed in the  $(\{abc\})$  condition and the latter in the  $(\{bc\})$  condition.

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<sup>26</sup>All the vision angles are based on the size, the resolution and the distance between the participant and the computer screen. Implemented angles were defined to ensure that several saccades were necessary to capture all the information.

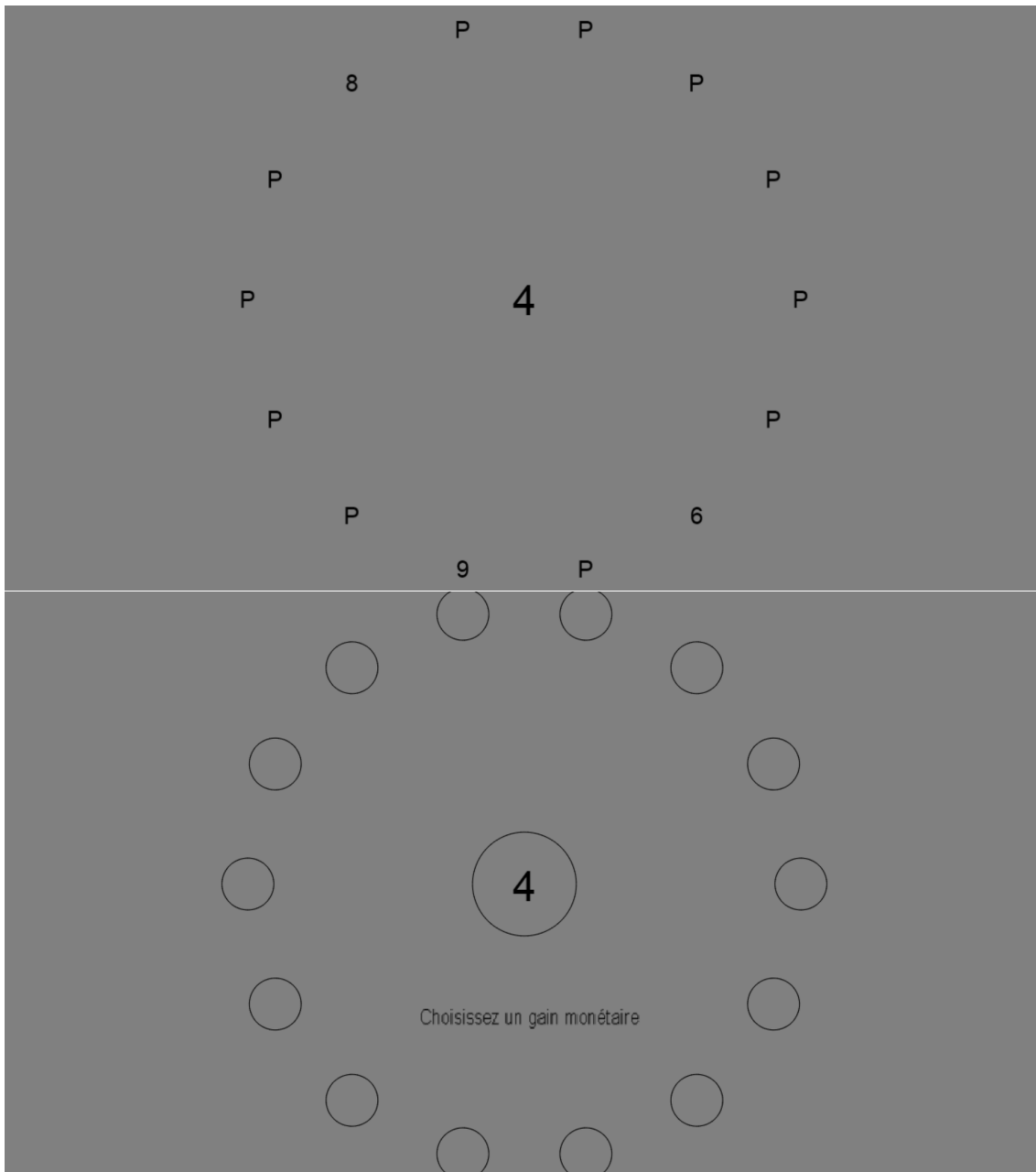


Figure 3.5.1: Screenshot of a trial

**Note:** Example of **Stimuli screen (Top)** for the menu  $a : 9 > b : 8 > c : 6$  and a default option  $o : 4$ . Each alternative earns its associated value in ECU if chosen. Alternatives' location defined as:  $x_a = 10$ ,  $x_b = 5$ ,  $x_c = 12$ . Other locations are occupied by "P" letters and represent costly penalties if chosen (-15 ECU). Stimuli screen last 900ms and is followed by the **Decision screen (Bottom)** where participants had to select one location by clicking on a circle to choose one of the previously disclosed alternatives (or distractors). No time limit to make the decision.

Position of the alternatives and the sequence of the conditions were randomly generated before the experiment and identical between the different sessions. We balanced the positions of the alternatives across trials (each alternative was located 3 times in each of the 14 locations). The conditions were balanced across blocks (each condition was played 6 times in each of the blocks). Moreover, we balanced 7 possible block orders between participants. Thus each trial was played in each of the 7 conditions (in the same order) by  $\frac{1}{7}$  of the participants.<sup>27</sup>

**General information and protocol** — The experiment was implemented on 13" (29.4cm) screen laptop with a 1280 × 800 resolution. Distance between participants and screen was approximately 70cm. We code the experiment with the PsychoPy 1.8 library for Python (Peirce, 2009).<sup>28</sup> Sessions took place at the Laboratoire d’Economie Experimentale de Nice (LEEN), France from 2018 April, 23<sup>rd</sup> to 28<sup>th</sup>. We recruited 231 participants (40% were male) aged between 17 and 58 years old (M=23.15, SD=6.47) with normal or corrected to normal vision, using the ORSEE platform (Greiner, 2015). Participants participated to only one of the 10 sessions that lasted approximately 45-minutes, including payment. The total payment ranged from 5.75 to 18.45 euros (M=15.80, SD=1.77). It was composed of three independent parts: a 5 euros show up fee, a payment for the main task and a payment for a risk-aversion elicitation task. The payment for the main task is based on the total of the earning minus the penalties realized on 30 randomly drawn trials or 0 ECU if the total is negative. Once the participants had finished the main task they were invited to complete a risk-elicitation task under gain and loss frames which consists in two successive choices among lotteries (Eckel and Grossman, 2008; Garcia and Massoni, 2017). Participants could earn between 1 and 35 ECU (14 ECU in average) in this task. At the end of the session, participants were asked their gender and their age (answers were not mandatory) and were paid individually

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<sup>27</sup>Each participant played each of the 42 trials in each of the 7 conditions, in the same order within block. Each participant participated to one of the 7 Block sequences. The 7 block sequences were balanced between participants and “quasi-balanced” across sessions (number of participants was not always a multiple of 7). Details of block and trial sequences are given in appendix 3.C.2

<sup>28</sup>PsychoPy is an Open-Source Python-based framework to program experiments in Cognitive Science. Compared to standard software used in economics, it provides the advantage of having a better precision in time measurement. The code of this experiment is available on the online supplementary material.

in a separated room. One-page instruction was provided and read aloud before beginning the experiment and a short reminder of the instruction was presented on the screen between each trial. Earnings in ECU were converted into euros at the exchange rate: 20 ECU = 1 euro.<sup>29</sup>

### 3.5.2 Confirmatory Results

For each participant we collect 42 decisions in all the 7 possible menus made by the 3 alternatives  $a > b > c$ . Table 3.5.1 shows the average number of choices for each alternative and for each menu aggregated by participants.

	<i>abc</i>	<i>ab</i>	<i>ac</i>	<i>bc</i>	<i>a</i>	<i>b</i>	<i>c</i>
<i>a</i>	22.04	24.22	25.13	0	29.03	0	0
<i>b</i>	12.73	13.41	0	24.22	0	29.63	0
<i>c</i>	5.00	0	11.13	12.37	0	0	26.67
<i>o</i>	2.22	4.37	5.74	5.41	12.97	12.37	15.33

Table 3.5.1: Average number of choices

We test the axioms needed to characterize a wRCCSR and to reveal participants' preferences and attention parameters at an individual level, following the procedures described in Section 3.4. In Figure 3.5.2 we report the cumulative distribution function of the p-values for all the tests developed in Section 3.4 when applied to our experimental dataset. More precisely, it corresponds to the tests of Sequential Independence required for the preference relation  $a \succ b \succ c$  to be a valid representation (top left), the test of WARP-SLA based on corrected p-values of SI tests for each preference representation (top right), WARP-SLA tested through  $\chi^2$  test for each preference representation (mid left), the test of wIFO for each preference representation (mid right), WARP-SLA and wIFO tested simultaneously for each preference representation (bottom left) and the test of revealed preferences for each binary preference relations (bottom right).

Table 3.5.2 shows the proportion of the participants that exhibit p-values lower than 0.01, 0.05 and 0.1 for the separated and the simultaneous tests of WARP-SLA and wIFO for each preference representation, as well as the proportion of the

<sup>29</sup>Instruction and description of the risk-elicitation task are provided in the appendices.



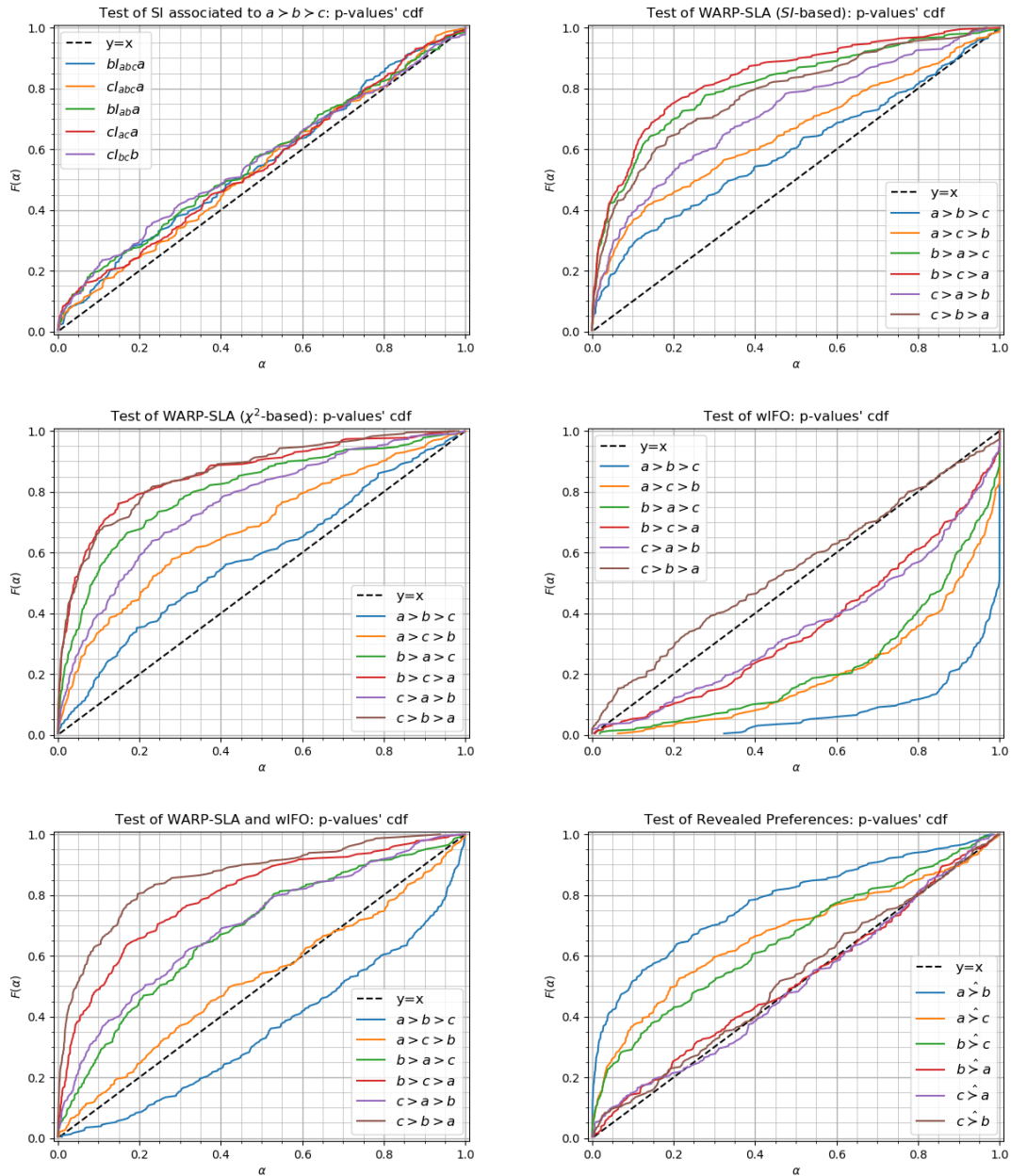


Figure 3.5.2: p-values' cdf for the tests of the axioms and revealed preferences applied to an experimental dataset.

*Note:* The figures represent p-values' cdf of the following test: The SI relations required for  $a > b > c$  to be a valid representation (top left); for each preference order, the tests of WARP-SLA based on the corrected p-values of SI test (top right), the tests of WARP-SLA based on  $\chi^2$  statistics (mid left), the tests of wIFO (mid-right), the simultaneous tests of WARP-SLA and wIFO (bottom left) and the tests of each binary revealed preference (bottom right)

participants that exhibit p-values lower than 0.01, 0.05 and 0.1 for the test of each binary revealed preference relation. We also report in this table the proportion of participants that reject a test of the Luce Rule at the 0.01, 0.05 and 0.1 significance level.<sup>30</sup>

		$p < 0.01$	$p < 0.05$	$p < 0.1$
$H0_{\succ}^{WARPSLA}$ ( $\chi^2$ based)	$a \succ b \succ c$	0.035	0.096	0.187
	$a \succ c \succ b$	0.061	0.213	0.335
	$b \succ a \succ c$	0.143	0.343	0.539
	$b \succ c \succ a$	0.278	0.526	0.674
	$c \succ a \succ b$	0.096	0.265	0.396
	$c \succ b \succ a$	0.278	0.522	0.661
$H0_{\succ}^{wIFO}$	$a \succ b \succ c$	0.000	0.000	0.000
	$a \succ c \succ b$	0.004	0.004	0.009
	$b \succ a \succ c$	0.004	0.017	0.017
	$b \succ c \succ a$	0.009	0.026	0.052
	$c \succ a \succ b$	0.026	0.043	0.052
	$c \succ b \succ a$	0.043	0.104	0.152
$H0_{\succ}^{WARPSLA \cap wIFO}$	$a \succ b \succ c$	0.000	0.017	0.035
	$a \succ c \succ b$	0.022	0.074	0.139
	$b \succ a \succ c$	0.052	0.152	0.270
	$b \succ c \succ a$	0.135	0.383	0.487
	$c \succ a \succ b$	0.057	0.191	0.309
	$c \succ b \succ a$	0.239	0.496	0.622
Rev. (True) Pref.	$a \hat{\succ} b$	0.265	0.422	0.517
	$a \hat{\succ} c$	0.104	0.261	0.365
	$b \hat{\succ} c$	0.104	0.239	0.296
Rev. (False) Pref.	$b \hat{\succ} a$	0.013	0.078	0.143
	$c \hat{\succ} a$	0.052	0.100	0.148
	$c \hat{\succ} b$	0.030	0.096	0.126
Luce Rule		0.209	0.474	0.604

Table 3.5.2: Proportion of the participants rejecting the axioms and revealing preferences.

The rejection rate of  $H0_{a \succ b \succ c}^{WARP-SLA}$ , based on the  $\chi^2$  procedure, is higher than the nominal level (0.096 > 0.05). However this value is close to the upper-bound of the 90% confidence interval of the actual level of the test corresponding to a nominal level of 5% (see appendix 3.B.7). Therefore, this value may be either due to a

<sup>30</sup>The test of the Luce Rule is the same test as the simultaneous test of wIFO and WARP-SLA described in Section 3.4.3, except that we add an additional constraint for the ML-estimation of  $\hat{\pi} : \hat{\pi}(x, S) = 0 \forall |S| > 2$ .

portion of participants that does not respect WARP-SLA or by lack of control due to specific attention parameters. If We test the model using the more conservative and powerful test of  $H0_{a>b}^{WARP\text{SLA} \cap wIFO}$ , only 1.70% of our participants reject this test at a 5% significance level. From these results, we conclude that most of the participants in this experiment make decision that can be considered coherent with a wRCCSR. For comparison, we found that 47.4% of the participants have a behavior that is statistically different from the Luce Rule at the 5% level.

Concerning the revealed preferences, it is worth noting that the probability to reveal the true preference relations depends heavily on the attentional parameters of the model. However, under the null hypothesis that participants made choices following a wRCCSR, the probability to reveal a false preference relation (the level of the test) is controlled. In this experiment, we found that most of the participants (61.30% ) have revealed at least one of the true binary preference relation at the 5% level (i.e. either  $a \hat{>} b$ ,  $a \hat{>} c$ , or  $b \hat{>} c$ ).  $a$  is revealed to be preferred to  $b$ ,  $a$  is revealed to be preferred to  $c$  and  $b$  to  $c$  respectively for 42.2%, 26.1% and 23.9% of the participants at the 5% level. Regardless of the chosen nominal level, the proportions of individuals who revealed incorrect preferences ( $b \hat{>} a$ ,  $c \hat{>} a$  or  $c \hat{>} b$ ) are close to the nominal level of the test and based on Kolmogorov-Smirnov tests, we cannot reject the hypothesis that those p-values follow a uniform distribution [ $D(230) = 0.060, p = 0.361$  for  $b \hat{>} a$  ;  $D(230) = 0.055, p = 0.476$  for  $c \hat{>} a$  ;  $D(230) = 0.057, p = 0.440$  for  $c \hat{>} b$ ]. This also suggest that participants behave in accordance with the model. We observed 20.8% of the participants who revealed at least one false preference at the 5% level.

However, since we are testing separately several preferences relation for each participants, we must control for multiple testing at the participant level. We use the Benjamini and Hochberg's procedure (1995). This procedure does not control the FWER (the probability to make at least one false inference) but the False Discovery Rate (FDR), that is equals to ratio between the number of false rejection (rejecting the null hypothesis when it is true) and the number of rejections (multiplied by the probability to make at least one rejection). In presence of multiple testing, controlling the FDR is often more adapted than controlling the FWER. The reason

is that FWER control is increasingly conservative with the number of hypothesis and an observer may find acceptable to make a few number of incorrect inference to keep high the number of correct inference. We apply the procedure as follows. For each participant, we rank the 6 p-values of the test of revealed preferences in ascending order  $p_1 < \dots < p_6$  and apply the following correction:  $p_i^c = \frac{6 \times p_i}{i}$ . Then we look for the highest  $i$  such that  $p_i^c < \alpha = 0.05$  and we reject all the tests corresponding to the p-values  $p_j$  such that  $j < i$ . Theoretically, this correction would control the FDR to 5%. Applying this correction, participants reveal in average 0.54 true preference relations and 0.13 false preference relations (instead of 0.92 and 0.27 without correction) and the estimated *FDR* is equal to 5.1% (instead of 11.4% without correction).

### 3.5.3 Exploratory Results

Our design allows us to measure the effective attention of the participants and their preferences at an individual level so it is possible to investigate the correlation of those measures with individual characteristics.

In our experiment, we collected the following individual characteristics about the participants: their age ( $M = 23.20$ ,  $SD = 6.47$ ) and their gender (40.74% are male), as well as their risk aversion under a gain and a loss frame (see Section 3.C.1). The risk elicitation tasks in the gain frame consists in the presentation of 6 lotteries that can be ranked by their associated level of risk. The risk elicitation task in the loss frame is identical than in the gain frame, excepted that we endowed 10 ECU to all participants just before they made their decision and subtract those 10 ECU from all the outcomes of the lotteries (if compared to the lottery in the gain frame). For each frame (gain and loss), we use the chosen lottery as a proxy for risk aversion:  $risk\_av = k$ , ( $k = 1, \dots, 6$ ) if the individual has chosen the  $k$ -*iest* less risky lottery.

Concerning revealed attention, we use the following measure for effective attention:

$$\mathbb{E}(|\Gamma|) = 3 \times \hat{\pi}(abc) + 2 \times (\hat{\pi}(ab) + \hat{\pi}(ac) + \hat{\pi}(bc)) + \hat{\pi}(a) + \hat{\pi}(b) + \hat{\pi}(c),$$

where  $\hat{\pi}()$  is the measure of attention estimated by maximum likelihood.  $\mathbb{E}(|\Gamma|)$  correspond to the estimator of the expected cardinal of the consideration set, which represents the average number of alternatives considered when making a choice. Using Ordinary Least Square, we regress this measure as a linear function of age, gender and risk aversion under both frames. We report the results in Table 3.5.3. To investigate the influence of individual characteristics on the probability to reveal true and false preferences, we construct the two following variables:  $TruePref = 1$  if the participant has revealed at least one of the true preference at a 0.05 significance level (i.e. one of the p-values associated to the test of  $a \succ b$ ,  $a \succ c$  or  $b \succ c$  is lower than 0.05) and 0 otherwise ;  $FalsePref = 1$  if the participant has revealed at least one of the false preference at a 0.05 significance level (i.e. one of the p-values associated to the test of  $b \succ a$ ,  $c \succ a$  or  $c \succ b$  is lower than 0.05) and 0 otherwise. We regress these two variables as a function of individuals' characteristics (age, gender and risk aversion) using a logit regression and we report the results in Table 3.5.3.

	$\mathbb{E}( \Gamma )$	$\mathbb{P}(TruePref)$	$\mathbb{P}(FalsePref)$
Intercept	1.79*** (0.07)	0.95 (0.57)	-2.46*** (0.69)
age	-0.01*** (0.00)	-0.04 (0.02)	0.01 (0.02)
male	0.01 (0.04)	0.48 (0.30)	0.56 (0.35)
risk_aversion (gain)	-0.01 (0.01)	0.01 (0.08)	0.03 (0.9)
risk_aversion (loss)	0.01 (0.01)	0.13 (0.08)	0.17 (0.10)
$R^2$	0.09		
N	230	230	230

Table 3.5.3: Probability to reveal (false and true) preferences and measure of attention as a function of individual characteristics

We observe that older participants are less effective in their attention than younger ones [ $z = -4.30$ ,  $p = 0.000$ ] and less likely to reveal true preferences [ $z = -2.00$ ,  $p = 0.045$ ]. The age of participants does not impact their probability to reveal false preferences [ $z = 0.539$ ,  $p = 0.590$ ]. We find no impact of the gender on the attentional measure [ $z = 0.388$ ,  $p = 0.699$ ], nor on the probability to

reveal true [ $z = 1.605$ ,  $p = 0.109$ ] or false preferences [ $z = 1.609$ ,  $p = 0.108$ ]. Risk preference does not impact effective attention nor the probability to reveal preferences. However, if we apply Benjamini and Hochberg's correction (1995) for multiple testing, the only robust result is that older participants are less effective in their attention, compared to younger ones.

## 3.6 Discussion

In this chapter, we tested Brady and Rehbeck's model (2016) of revealed preferences under stochastic attention. For this purpose we identified statistical obstacles to test their axioms in the limit of their model. We proposed a slight extension of their model and a new characterization to overcome those obstacles. After having proposed statistical tests and showed their appropriateness, we conducted a novel experiment to assess the internal validity of the theory.

Our experiment was designed to meet several requirements. Theoretically, a rich domain must be proposed to fully characterize and test the model. Statistically, a sufficient number of decisions per menu is needed to ensure reasonable asymptotic properties of our methods. Experimentally, the total length of the experiment should not be too long and the design should reproduce an environment where the participants allocate freely their attention but fail to be attentive to all the alternatives. To meet these constraints, we developed a specific Perceptual Decision task, where participants have to detect monetary rewards, briefly disclosed among numerous distractors. Based on this particular environment and the statistical procedure developed earlier, we were able to test the model.

The results of the experiment support the internal validity of the model, since the behaviors of most of the participants is in accordance with the model. Moreover, we show that it is possible to disentangle participants' preferences from their stochastic attention. Indeed, in our experiment we induced the preference relation over the alternatives (alternatives were monetary rewards) and we showed that at the individual level most of the participants revealed at least one true preference, while the probability for a participant to reveal a false preference is close to the nominal level.

At a more aggregated level, the induced preference relation underlying the choices is easily recoverable simply by looking participants p-values' cdf.

There are many applications that could be derived from the possibility to disentangle preferences and attention. For example, a firm aiming to increase its sales will not adopt the same strategy if its product is not bought because consumers prefer another product or because they are not considering it. In the former case, the firm should differentiate, decrease the price or increase the quality of its product, while in the latter case, the firm should advertise, change the packaging or negotiate with its distributors to highlight more the product. While demonstrating the internal validity of Brady and Rehbeck's model (2016) is encouraging, there is still more research needed to apply it to real situations. Indeed, our experiment was designed especially for facilitating the test of a minimal version of the model: the preference relation was trivial since alternatives were monetary rewards and we implemented a perceptual decision making where we controlled and limit the display time of the alternatives. Other environments could lead to different results and it would be worth considering to test the model with (more) ecological choice situations, involving real consumption decisions. However, such situations are often characterized by a high number of available alternatives and as we argued, testing the model with more alternatives is challenging.

Another application of this framework is to build a non-invasive measure of effective attention. Indeed, the model reveals the alternatives considered by the individual when making a choice, which corresponds to the output of the individual's attention allocation process. Combining this measure with standard measures of allocated attention (though response time and eye-tracking) could allow the researchers to study the efficiency of the attention allocation process and its interaction with other variables. More research is needed for this purpose. Indeed, we shall investigate in future work if the revealed measure of attention is coherent with modifications of the design (e.g. the probability to consider an alternative should increase with its disclosure duration and its salience), or correlated with standard individual measure of attentional ability, used in psychology (e.g. the performance in the Stroop's task (1935)), with measures of allocated attention.

This chapter represents a first step towards more thorough empirical tests of models of choice under limited attention. As we have shown in this paper, expressing mathematical axioms in terms of testable hypotheses is not always straightforward and we argue that economists should dedicate more effort to assess whether our models are implementable. Indeed, the “implementability” is an important characteristic of any model that aims to describe human behaviors. This characteristic is necessary to allow a two-way connection between the theory and the practice. Implementable models can be used to make quantitative predictions and testing the axioms of the models is necessary to identify and understand the conditions under which a model is an accurate simplification of the reality.

In this line, other models could be tested with our (or similar) datasets and compared with the wRCCSR. For example, we showed that the Luce’s Rule (1959) failed to explain the choices in our experiment. From this result, it is also possible to affirm that the extension proposed by Ahumada and Ülkü (2018), the Luce Rule with Limited Consideration Set (LRLC) cannot explain choices in our experiment. Indeed, the LRLC aims to accommodate for the decision maker limited attention and is characterized by a deterministic Consideration set over the set of available alternatives and a utility function. The probability to choose an alternative considered is given by a Luce Rule over the set of considered alternatives and there is a 0 probability to choose alternatives not considered. A direct consequence is that if all the available alternatives have a strict probability to be chosen — which is the case in our data —, then the LRLC and the Luce Rule are confounded. Of course, there must be numerous environment where a LRLC would explain choice better than a wRCCSR; it is therefore primordial to identify empirically the domain of application of each model.

Another model that could be tested following a similar procedure as in this chapter is the perception adjusted Luce Model (PALM) proposed by Echenique et al. (2018). In their model, the DM considers alternatives sequentially following a deterministic order and has a probability to choose an alternative when considered, defined by a Luce Rule. The class of PALM models is distinct from the class of wRCCSR but their intersection is not empty (for example, both classes admit the



Luce Rule as a special case). The class of PALM is also distinct from the class of RUM as a PALM may account for the violation of the regularity condition. Moreover, an interesting feature of the PALM is that when the axioms are satisfied, choices would reveal in which order the alternatives have been considered. Such order of consideration could then be compared with eye-tracking or other choice processing data (Caplin, 2016).

Another model related with wRCCSR is Aguiar’s Random Categorization Rule (2017) (RCG). Both wRCCSR and RCG are generalizations of the model of Manzini and Mariotti (2014) discussed in this chapter. However, the class of RCG is included in the class of RUM and cannot account for violation of the regularity condition. In an RCG, the DM chooses the best item of a random consideration set defined as the intersection between a random categorization rule (a random set of alternatives that is independent from the menu) and the menu of alternatives. The characterization of the RCG stands in particular on the acyclicity of the revealed preference relation defined as follows. An alternative  $x$  is revealed to be preferred to an alternative  $y$  if removing  $x$  impacts (positively or negatively) the choice of  $y$ . Therefore, it should be possible to develop appropriate statistical tests of their model and confront it with our experimental dataset. For example, participants highlighting statistical differences between both  $p(c, c) \neq p(c, bc)$  and  $p(b, b) \neq p(b, bc)$  do not satisfy the conditions of an RCG. While the wRCCSR integrates more the unawareness dimension of inattention (and could be view as a stochastic counterpart of Masatlioglu et al. 2012), the RCG integrates more attentional competition between alternatives (it is a stochastic counterpart of Lleras et al. 2017). Therefore, identifying situations where one model outperforms the other would be informative on which aspect of limited attention dominates in a particular environment.

# Appendices — Revealed Preferences under Stochastic Attention: Characteriza- tion, Statistical Tests and Experimental Implementation

## 3.A Proofs

In this section we present the proof of the main results.

### 3.A.1 Proof of Characterization Theorem

The proof of our characterization theorem is very similar from the one proposed by BR. The central idea is the same: first, one should define a preference relation  $\succ$  that respects a desired property such that each menu must be sequentially independent to its  $\succ$ -best alternative. In BR, the preference relation is uniquely defined thanks to ASI and TSI by the binary sequential independence relation and they show with their Lemma 2, that the former property is obtained when the random choice rule also respects ESI. In this chapter, we construct a preference relation over  $X$ , based on WARP-SLA that is not necessarily unique. However it is obvious that any preference relation that is constructed in our demonstration, respects the desired property. In both proofs, the appropriate definition of the RCCSR/wRCCSR is obtained by induction on menu size and the fact that the attentional measure  $\pi$  is a probability, directly follows from IFO/wIFO.

We now give the detail of the proof and first prove lemma 3.3.1.

**Lemma 3.3.1.** *Let  $p$  a random choice rule. If  $p$  satisfies WARP-SLA, then there exist a complete and transitive preference relation  $\succ$  over  $X$  and a mapping  $\lambda : \mathcal{D} \rightarrow \mathbb{R}$ , such that  $\forall A \in \mathcal{D} \setminus \{\emptyset\}, \forall a \in A$ :*

$$p(a, A) = \frac{\sum_{B \in A_{a \succ}} \lambda(B)}{\sum_{B \subseteq A} \lambda(B)},$$

with  $A_{a \succ} = \{B \subseteq A : a \in B \text{ and } \forall b \in B \setminus a, a \succ b\}$ , and  $\lambda(A) = \sum_{B \subseteq A} (-1)^{|A \setminus B|} \frac{1}{p(o, B)}$ .

*Proof.* Let  $X$  a finite set of available alternatives  $|X| = N$  and  $p$  a random choice rule satisfying WARP-SLA. We first define a transitive and complete preference relation based on menu sequential independence. We note  $S_1 = X$ . WARP-SLA implies,  $\exists x_1 \in S_1$  such that  $\forall A \subseteq S_1, AIx_1$ . By applying WARP-SLA sequentially, it is possible to construct a series of subsets  $(S_1, \dots, S_N)$  and a series of alternatives  $(x_1, \dots, x_N)$  such that  $\forall 1 < i \leq N, S_i = S_{i-1} \setminus x_{i-1}$  and  $x_i$  s.t.  $\forall A \subseteq S_i, AIx_i$ .

We thus define the following relation  $\succ : x_i \succ x_j \Leftrightarrow i < j$ . The relation  $\succ$  is thus strict, complete, transitive and respects the following property:  $AIa$  whenever  $a$  is the  $\succ$ -best alternative of  $A$ .

We now define a mapping  $\lambda : \mathcal{D} \rightarrow \mathbb{R}$  such that:

$$\lambda(A) = \sum_{B \subseteq A} (-1)^{|A \setminus B|} \frac{1}{p(o, B)}.$$

We apply Shafer's version of Möbius Inversion (1976) to obtain for all  $A \in \mathcal{D}$ :<sup>1</sup>

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**Theorem 3.A.1.** *Möbius Inversion (Shafer, 1976)*  
 If  $\Theta$  is a finite set with  $f$  and  $g$  functions on  $2^\Theta$ , then

$$f(A) = \sum_{B \subseteq A} g(B)$$

for all  $A \subseteq \Theta$  if and only if

$$g(a) = \sum_{B \subseteq A} (-1)^{|A \setminus B|} f(B)$$

for all  $A \subseteq \Theta$ .

$$\frac{1}{p(o, A)} = \sum_{B \subseteq A} \lambda(B). \quad (3.1)$$

Note that we have by construction  $\sum_{B \subseteq A} \lambda(B) > 0$  for all  $A \in \mathcal{D}$ , since  $p$  is a random choice rule. Moreover, we have  $\lambda(\emptyset) = 1$  since  $p(o, \emptyset) = 1$ .

We will show by induction on alternatives' rank and menus' cardinality, that in any non empty menu  $A \in \mathcal{D}$ , the following property holds for any alternative  $a \in A$ :

$$p(a, A) = \frac{\sum_{B \in A_{a \succ}} \lambda(B)}{\sum_{B \subseteq A} \lambda(B)}. \quad (3.2)$$

With  $A_{a \succ} = \{B \subseteq A \text{ s.t. } a \in B \text{ and } \forall b \in B \setminus a, a \succ b\}$  the set of subsets of  $A$  where  $a$  is the  $\succ$ -best alternative.

We first show that in any menu  $A \in \mathcal{D}$ ,  $|A| \geq 1$ , the Equation (3.2) holds for its 1<sup>st</sup>  $\succ$ -best alternative.

If  $A = \{a\}$  is a singleton, then using Equation (3.1):

$$p(a, A) = 1 - p(o, A) = 1 - \frac{1}{\lambda(a) + \lambda(\emptyset)} = \frac{\lambda(a)}{\lambda(a) + \lambda(\emptyset)}, \text{ since } \lambda(\emptyset) = 1,$$

so Equation (3.2) holds.

Let consider  $A \in \mathcal{D}$  with  $|A| > 1$ . Let  $x_k$  be the  $\succ$ -best alternative of  $A$ . By construction of  $\succ$ , we have  $AIx_k$ .

$$\begin{aligned} AIx_k &\Rightarrow \forall b \in A \setminus x_k, p(b, A \setminus x_k) = \frac{p(b, A)}{1 - p(x_k, A)} \\ &\Rightarrow \sum_{b \in A \setminus x_k} p(b, A \setminus x_k) = \frac{1}{1 - p(x_k, A)} \times \sum_{b \in A \setminus x_k} p(b, A) \\ &\Leftrightarrow 1 - p(o, A \setminus x_k) = \frac{1 - p(x_k, A) - p(o, A)}{1 - p(x_k, A)} \\ &\Leftrightarrow p(x_k, A) = 1 - \frac{p(o, A)}{p(o, A \setminus x_k)} \end{aligned}$$

Using Equation (3.1), we obtain:

$$p(x_k, A) = 1 - \frac{\sum_{B \subseteq A \setminus x_k} \lambda(B)}{\sum_{B \subseteq A} \lambda(B)} \quad (3.3)$$

$$= \frac{\sum_{B \subseteq A} \lambda(B) - \sum_{B \subseteq A \setminus x_k} \lambda(B)}{\sum_{B \subseteq A} \lambda(B)}. \quad (3.4)$$

Moreover, because  $x_k$  is the  $\succ$ -best alternative in  $A$ , we have:

$$\begin{aligned} \{B \subseteq A\} \setminus \{B \subseteq A \setminus x_k\} &= \{B \subseteq A, x_k \in B\} \\ &= \{B \subseteq A, x_k \in B \text{ and } \forall b \in B \setminus x_k, x_k \succ b\} \\ &= A_{x_k \succ}. \end{aligned}$$

Therefore,

$$p(x_k, A) = \frac{\sum_{B \subseteq A_{x_k \succ}} \lambda(B)}{\sum_{B \subseteq A} \lambda(B)}.$$

We have shown that in any menu  $A \in \mathcal{D}$  with  $|A| \geq 1$ , Equation (3.2) is true for the 1<sup>st</sup>  $\succ$ -best alternative.

We will now prove that if the property is true for rank  $K - 1 \geq 1$  then it is true for the rank  $K$ . Let  $K \in \mathbb{N}$ ,  $K > 1$  and assume that in any menu  $A \in \mathcal{D}$ , with  $|A| \geq K - 1$ , Equation (3.2) is true for its  $(K - 1)^{th}$   $\succ$ -best alternatives.

Let  $A \in \mathcal{D}$ , with  $|A| \geq K$ . Let  $x_k$  be the 1<sup>st</sup>  $\succ$ -best alternative in  $A$ . Using Equation (3.3):

$$1 - p(x_k, A) = \frac{\sum_{B \subseteq A \setminus x_k} \lambda(B)}{\sum_{B \subseteq A} \lambda(B)} \quad (3.5)$$

Let  $x_l$  the  $K^{th}$   $\succ$ -best alternative in  $A$ .  $x_k \in A$  and  $x_k \succ x_l$ , so  $x_l$  is the  $(K - 1)^{th}$   $\succ$ -best alternative in  $A \setminus x_k$ . Moreover  $A \setminus x_k \in \mathcal{D}$  by richness and  $|A \setminus x_k| \geq K - 1$ .

Therefore, by inductive assumption:

$$p(x_l, A \setminus x_k) = \frac{\sum_{B \in (A \setminus x_k)_{x_l \succ}} \lambda(B)}{\sum_{B \subseteq A \setminus x_k} \lambda(B)}. \quad (3.6)$$

By construction of  $\succ$  and since  $x_k$  is the  $\succ$ -best alternative in  $A$ , we have  $AIx_k$  and in particular  $x_l I_A x_k$ , i.e.

$$p(x_l, A) = p(x_l, A \setminus x_k) \times (1 - p(x_k, A)).$$

Using Equation (3.5) and Equation (3.6):

$$p(x_l, A) = \frac{\sum_{B \subseteq (A \setminus x_k)_{x_l \succ}} \lambda(B)}{\sum_{B \subseteq A} \lambda(B)}.$$

Moreover, since  $x_k \succ x_l$ , we have :

$$\begin{aligned} (A \setminus x_k)_{x_l \succ} &= \{B \subseteq A \setminus x_k, s.t. x_l \in B \text{ and } \forall b \in B \setminus x_l, x_l \succ b\} \\ &= \{B \subseteq A, s.t. x_l \in B \text{ and } \forall b \in B \setminus x_l, x_l \succ b\} \\ &= A_{x_l \succ}. \end{aligned}$$

Therefore,

$$p(x_l, A) = \frac{\sum_{B \subseteq A_{x_l \succ}} \lambda(B)}{\sum_{B \subseteq A} \lambda(B)}.$$

We have shown that Equation (3.2) is true for the 1<sup>st</sup>  $\succ$ -best alternative in any menu  $A \in \mathcal{D}$  with  $|A| \geq 1$ . Moreover, we have shown that if Equation (3.2) is true for the  $(K - 1)^{th}$   $\succ$ -best alternative in any menu  $A \in \mathcal{D}$  with  $|A| \geq (K - 1)$ , then Equation (3.2) must holds for the  $K^{th}$   $\succ$ -best alternative in any menu  $A \in \mathcal{D}$  with  $|A| \geq K$ . By induction, Equation (3.2) holds for any alternative in any menu  $|A| \in \mathcal{D}$ , that is:

$$\forall A \in \mathcal{D} \setminus \{\emptyset\}, \forall a \in A, p(a, A) = \frac{\sum_{B \in A_{a \succ}} \lambda(B)}{\sum_{B \subseteq A} \lambda(B)},$$

with  $A_{a \succ} = \{B \subseteq A \text{ s.t. } a \in B \text{ and } \forall b \in B \setminus a, a \succ b\}$ ;

$$\text{and } \lambda(A) = \sum_{B \subseteq A} (-1)^{|A \setminus B|} \frac{1}{p(o, B)}.$$

□

We now prove theorem 3.3.1:

**Theorem 3.3.1** (Characterization). *p satisfies WARP-SLA and wIFO iff  $p_{\succ, \pi}$  is a wRCCSR. Moreover,  $\pi$  is unique and*

$$\forall A \in \mathcal{D}, \pi(A) = C \times \sum_{B \subseteq A} \left( (-1)^{|A \setminus B|} \times \frac{1}{p(o, B)} \right),$$

with  $C \in \mathbb{R}_+$  a scaling factor such that  $\sum_{A \in \mathcal{D}} \pi(A) = 1$ .

*Proof. (Only if part)* Let  $p$  a random choice rule satisfying WARP-SLA and wIFO. Therefore, from lemma 3.3.1 we know that there exist  $\succ$  a complete preference ordering and  $\lambda : \mathcal{D} \rightarrow \mathbb{R}$  such that for any menu  $A \in \mathcal{D} \setminus \{\emptyset\}$  and any alternative  $a \in A$ :

$$p(a, A) = \frac{\sum_{B \subseteq A_{a \succ}} \lambda(B)}{\sum_{B \subseteq A} \lambda(B)},$$

$$\text{with } \lambda(A) = \sum_{B \subseteq A} (-1)^{|A \setminus B|} \frac{1}{p(o, B)}.$$

As  $p$  satisfies wIFO, we have  $\lambda(A) \geq 0$  for all  $A \in \mathcal{D}$ . Moreover,  $\lambda(\emptyset) = \frac{1}{p(o, \emptyset)} = 1 > 0$  since  $p$  is a random choice rule. However, we need to normalize  $\lambda$  to obtain a probability measure over  $\mathcal{D}$ .

We set

$$\pi(A) = \frac{\lambda(A)}{\sum_{B \in \mathcal{D}} \lambda(B)}.$$

Therefore, for any alternative  $a$  in any menu  $A \in \mathcal{D} \setminus \{\emptyset\}$ , we have:

$$p(a, A) = \frac{\sum_{B \in A_{x_A^\succ}} \pi(B)}{\sum_{B \subseteq A} \pi(B)}.$$

We have  $\pi \in [0, 1]$  and  $\sum_{A \in \mathcal{D}} \pi(A) = 1$ . So  $\pi$  is a probability measure over  $\mathcal{D}$ , with  $\pi(\emptyset) > 0$ .  $\square$

**(If part)** We first show that if  $p$  is a wRCCSR, it must respect WARP-SLA.

Let  $p_{\succ, \pi}$  a wRCCSR. Let  $A \in 2^X$ ,  $A \neq \emptyset$  and let  $x_A^\succ$  the  $\succ$ -best alternative in  $A$ . We will show that  $\forall B \subseteq A$ ,  $BIx_A^\succ$ .

Let  $B \subseteq A$ . If  $B \notin \mathcal{D}$  or  $x_A^\succ \notin B$ , then  $BIx_A^\succ$  trivially holds. If  $B \in \mathcal{D}$ , we have:  $\forall b \in B \setminus x_A^\succ$ ,

$$p(b, B) = \frac{\sum_{C \in B_{b^\succ}} \pi(C)}{\sum_{C \subseteq B} \pi(C)},$$

$$p(b, B \setminus x_A^\succ) = \frac{\sum_{C \in (B \setminus x_A^\succ)_{b^\succ}} \pi(C)}{\sum_{C \subseteq B \setminus x_A^\succ} \pi(C)} = \frac{\sum_{C \in B_{b^\succ}} \pi(C)}{\sum_{C \subseteq B \setminus x_A^\succ} \pi(C)},$$

and

$$\frac{1}{1 - p(x_A^\succ, B)} = \frac{\sum_{C \subseteq B} \pi(C)}{\sum_{C \subseteq B} \pi(C) - \sum_{C \in B_{x_A^\succ}^\succ} \pi(C)} = \frac{\sum_{C \subseteq B} \pi(C)}{\sum_{C \subseteq B \setminus x_A^\succ} \pi(C)}.$$

So  $p(b, B \setminus x_A^\succ) = \frac{p(b, B)}{1 - p(x_A^\succ, B)}$  for all  $b \in B \setminus x_A^\succ$ , so WARP-SLA is satisfied.

We now show that if  $p_{\succ, \pi}$  is a wRCCSR it should satisfy wIFO. Let  $p$  a wRCCSR.

We have for all menu  $A \in \mathcal{D}$ :

$$\frac{1}{p(o, A)} = \frac{\sum_{B \subseteq A} \pi(B)}{\pi(\emptyset)} = \sum_{B \subseteq A} \mu(A),$$

with  $\mu(A) = \frac{\pi(A)}{\pi(\emptyset)}$ .

Using Shafer's version of Mobius inversion:  $\mu(A) = \sum_{B \subseteq A} (-1)^{|A \setminus B|} \frac{1}{p(o, B)}$ .



Therefore, for all  $A \in \mathcal{D}$ ,  $\sum_{B \subseteq A} (-1)^{|A \setminus B|} \frac{1}{p(o, B)} = \frac{\pi(A)}{\pi(\emptyset)} \geq 0$  since  $\pi$  is a probability measure over  $\mathcal{D}$  with  $\pi(\emptyset) > 0$ . So wIFO holds.  $\square$

We now show that if  $p$  is a wRCCSR, then  $\pi$  is unique. Let  $p$  a wRCCSR. We have just shown that any representation  $(\succ, \pi)$  of  $p$  must respect:

$$\frac{\pi(A)}{\pi(\emptyset)} = \sum_{B \subseteq A} (-1)^{|A \setminus B|} \frac{1}{p(o, B)}, \text{ for all } A \in \mathcal{D}.$$

The right part of the equation does not depend on the chosen representation, therefore, if  $(\succ_1, \pi_1)$  and  $(\succ_2, \pi_2)$  are two possible representations of  $p$ , they must respect:

$$\begin{aligned} \forall A \in \mathcal{D}, \quad \frac{\pi_1(A)}{\pi_1(\emptyset)} &= \frac{\pi_2(A)}{\pi_2(\emptyset)} \\ \Rightarrow \sum_{A \in \mathcal{D}} \frac{\pi_1(A)}{\pi_1(\emptyset)} &= \sum_{A \in \mathcal{D}} \frac{\pi_2(A)}{\pi_2(\emptyset)}. \end{aligned}$$

Since  $\pi_1$  and  $\pi_2$  are probability measures over  $\mathcal{D}$ , we have  $\frac{1}{\pi_1(\emptyset)} = \frac{1}{\pi_2(\emptyset)}$ . Therefore  $\pi_1 = \pi_2$ .  $\square$

### 3.A.2 Proof of Revealed Preference Theorem

We first present and demonstrate some lemmas and then will prove theorem 3.3.2.

We remind the definition of the relation  $P$ .

$$aPb \Leftrightarrow \exists A \in \mathcal{D} \text{ s.t. } a, b \in A, a \neq b, \frac{p(a, A)}{p(o, A)} \neq \frac{p(a, A \setminus b)}{p(o, A \setminus b)}$$

**Lemma 3.A.1.** *Let  $p_{\succ, \pi}$  a wRCCSR.*

$$a \succ b \Rightarrow \neg(bPa)$$

*Proof.* From definition of wRCCSR, we have  $\forall A \in \mathcal{D}$  and  $\forall a$  and  $b \in A$ ,  $a \neq b$ :

$$\frac{p(b, A)}{p(o, A)} - \frac{p(b, A \setminus a)}{p(o, A \setminus a)} = \frac{\sum_{B \in A_{b \succ}} \pi(B) - \sum_{B \in (A \setminus a)_{b \succ}} \pi(B)}{\pi(\emptyset)},$$

with  $A_{a \succ} = \{B \subseteq A \text{ s.t. } a \in B \text{ and } \forall b \in B \setminus a, a \succ b\}$ .

$$\text{If } a \succ b \text{ then } A_{b \succ} = (A \setminus a)_{b \succ} \text{ then } \frac{p(b, A)}{p(o, A)} = \frac{p(b, A \setminus a)}{p(o, A \setminus a)}.$$

So  $a \succ b \Rightarrow \neg(bPa)$  □

**Lemma 3.A.2.**  $\forall A \in \mathcal{D}$ ,  $AIa$  iff,

$$\forall b \in A \setminus a, \frac{p(b, A)}{p(o, A)} = \frac{p(b, A \setminus a)}{p(o, A \setminus a)}.$$

*Proof.* If  $a \notin A$  the proposition holds trivially.

**(If part)** Let assume  $A \in \mathcal{D}$ ,  $a \in A$  and  $AIa$ . We have:

$$\begin{aligned} p(o, A) &= 1 - p(a, A) - \sum_{b \in A \setminus a} p(b, A) \\ &= 1 - p(a, A) - \sum_{b \in A \setminus a} p(b, A \setminus a) \times (1 - p(a, A)) \\ &= (1 - p(a, A)) \times \left(1 - \sum_{b \in A \setminus a} p(b, A \setminus a)\right) \\ &= (1 - p(a, A)) \times p(o, A \setminus a) \end{aligned}$$

So we have for all  $b \in A \setminus a$ ,  $\frac{p(b, A)}{p(b, A \setminus a)} = \frac{p(o, A)}{p(o, A \setminus a)} = 1 - p(a, A)$ . □

**(Only if part)** Let assume  $A \in \mathcal{D}$  and  $\frac{p(b, A)}{p(b, A \setminus a)} = \frac{p(o, A)}{p(o, A \setminus a)}$  for all  $b \in A \setminus a$ .

We have:

$$\begin{aligned} 1 - p(a, A) &= p(o, A) + \sum_{b \in A \setminus a} p(b, A) \\ &= p(o, A) + \frac{p(o, A)}{p(o, A \setminus a)} \times \sum_{b \in A \setminus a} p(b, A \setminus a) \\ &= p(o, A) + \frac{p(o, A)}{p(o, A \setminus a)} \times (1 - p(o, A \setminus a)) \\ &= \frac{p(o, A)}{p(o, A \setminus a)}. \end{aligned}$$

So we have: for all  $b \in A \setminus a$ ,  $\frac{p(b, A)}{p(b, A \setminus a)} = \frac{p(o, A)}{p(o, A \setminus a)} = 1 - p(a, A)$ , so  $AIa$ .  $\square$

**Lemma 3.A.3.**  *$P$  is acyclic if  $p$  satisfies WARP-SLA.*

*Proof.* We assume  $p$  satisfies WARP-SLA. Let  $\succ$  an order constructed following the proof of Lemma 3.3.1.  $\succ$  is acyclic by construction. Following the proof of Lemma 3.A.1, we have  $a \succ b \Rightarrow \neg bPa$ , so  $aPb \Rightarrow a \succ b$ . Therefore  $\succ$  contains  $P$ . So  $P$  has no cycle.  $\square$

We now prove the revealed preference theorem:

**Theorem 3.3.2** (Revealed Preferences). *Let  $p$  be a wRCCSR,  $a$  is revealed to be preferred to  $b$  iff  $aP_R b$ .*

With  $P_R$  defined as the transitive closure of the relation  $P$ .

*Proof.*

**(If part)** Let  $p$  be a wRCCSR and  $(\succ_i)$  its possible preference representations.

Since any transitive relation containing  $P$  contains also its transitive closure  $P_R$ , it is sufficient to show that  $P$  is contained in all  $(\succ_i)$ , that is  $aPb \Rightarrow a \succ_i b$  for all preference representation  $\succ_i$ .

From lemma 3.A.1, we have for all preference representations  $\succ_i$ :  $b \succ_i a \Rightarrow \neg(aPb)$ . Therefore, for all preference representation  $\succ_i$ ,  $aPb$  implies  $\neg(b \succ_i a)$ . So  $aPb$  implies  $a$  is revealed to be preferred to  $b$ .  $\square$

**(Only if part)** We show that if  $p$  is a wRCCSR, and  $\neg(aP_R b)$  then there exists  $\succ'$  such that  $b \succ' a$  and  $p_{\succ', \pi}$  is a wRCCSR.

As  $p$  is a wRCCSR it satisfies WARP-SLA, so from lemma 3.A.3,  $P_R$  is acyclic, so there is a linear order  $\succ'$  containing  $P_R$  such that  $b \succ' a$ . So we can rank all the available alternatives as follows:  $(x'_1 \succ' \dots \succ' x'_k \succ' \dots \succ' x'_l \succ' \dots \succ' x'_n)$ , with  $b = x'_k$  and  $a = x'_l$ .

As  $\succ'$  contains  $P_R$  (and  $P_R$  contains  $P$ ), we have:

$$\begin{aligned} x'_i \succ' x'_j &\Rightarrow \neg(x'_j P x'_i) \\ &\Rightarrow \forall A \in \mathcal{D}, \text{ and } x'_i, x'_j \in A, \frac{p(x'_j, A)}{p(o, A)} = \frac{p(x'_j, A \setminus x'_i)}{p(o, A \setminus x'_i)} \end{aligned}$$

Therefore, for any menu  $A \in \mathcal{D}$ , if  $x_A^\succ$  is the  $\succ'$ -best element in  $A$  then  $\frac{p(y, A)}{p(o, A)} = \frac{p(y, A \setminus x_A^\succ)}{p(o, A \setminus x_A^\succ)}$  for all  $y \in A \setminus x_A^\succ$ . Using lemma 3.A.2 we have that for any  $A$ , if  $x_A^\succ$  is the  $\succ'$ -best element of  $A$ , then  $AIx_A^\succ$ .

Therefore, we can construct a series  $(S'_n)$ , s.t.  $S_1 = X$  and  $S_i = S_{i-1} \setminus x'_{i-1}$ . Those series  $(S'_n)$  and  $(x'_n)$  have exactly the same properties than the series  $(x_n)$  and  $(S_n)$  constructed in the proof of lemma A.1. Moreover, as  $p$  satisfies wIFO, we can show that  $p_{\succ', \pi}$  is a wRCCSR in a similar way as in the proof of theorem 3.3.1.  $\square$

## 3.B Statistical Methods

This section details the construction of the different statistical tests of model axioms and revealed preferences. Throughout this section, we assume that we observe for each menu  $A \in \mathcal{D} \setminus \{\emptyset\}$  the realization of a random variable  $n(\cdot, A)$  following a multinomial distribution  $\mathcal{M}(N, p(\cdot, A))$ . We assume independence between menus. We assume  $\mathcal{D}$  satisfies the following richness assumption:  $\forall a \neq b \in X, \{a, b\} \in \mathcal{D}$  and  $(B \in \mathcal{D}) \cap (A \subseteq B) \Rightarrow A \in \mathcal{D}$ .

### 3.B.1 Statistical Tests of Sequential Independence

We show Proposition 3.2.2.

**Proposition 3.2.2.** *Let  $A \in \mathcal{D}$ , Let  $a, b \in A$ , let  $n(a, A)$  the number of times an alternative  $a$  is chosen in a menu  $A$ . Under the null hypothesis that  $b$  is sequentially independent to  $a$  in the menu  $A$  ( $H0_{b|a}^A : p(b, A \setminus a) = p_{-a}(b, A)$ ), the statistics:*

$$Z_{b|a}^A = \sqrt{N} \times \frac{\hat{\mu}_{b|a}^A}{\hat{\sigma}_{b|a}^A} \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N}(0, 1),$$

*follows asymptotically a standard Gaussian distribution, with*

$$\hat{\mu}_{b|a}^A = \frac{n(b, A)}{N - n(a, A)} - \frac{n(b, A \setminus a)}{N},$$

and

$$\hat{\sigma}_{b|a}^A = \sqrt{\frac{n(b, A \setminus a) \times [N - n(b, A \setminus a)]}{N^2} + \frac{N \times n(b, A) \times [N - n(a, A) - n(b, A)]}{[N - n(a, A)]^3}}.$$

*Proof.* We want to test the null hypothesis ( $H0_{b|a}^A$ ) such that  $b$  is sequentially independent to  $a$  in the menu  $A$ :

$$H0_{b|a}^A \Leftrightarrow p(b, A \setminus a) = p_{\neg}(b, A).$$

To simplify notation, we note for each alternative and for each menu:  $n_a^A = n(a, A)$  the number of times the alternative  $a$  is chosen in the menu  $A$  and  $p_a^A = p(a, A)$  the probability to choose  $a$  in  $A$ . For each  $A \in \mathcal{D}$ , we independently observe  $n^A = (n_a^A)_{a \in A \cup \emptyset}$  the realisation of the multinomial variable  $\mathcal{M}(p^A, N)$ , with  $p^A = (p_a^A)_{a \in A \cup \emptyset}$ .

Thanks to the multivariate central limit theorem (Anderson, 1962), we have for any  $A \in \mathcal{D}$ :

$$\sqrt{N} \times \left[ \frac{n^A}{N} - p^A \right] \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N}(0, \Sigma), \quad (3.7)$$

with

$$\Sigma = \begin{pmatrix} p_a^A(1 - p_a^A) & -p_a^A p_b^A & \dots \\ -p_a^A p_b^A & p_b^A(1 - p_b^A) & \dots \\ \dots & \dots & \dots \end{pmatrix}.$$

Therefore, we have

$$\frac{n_b^{A \setminus a}}{N} \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N}\left(p_b^{A \setminus a}, \frac{\sigma_1^2}{N}\right), \quad (3.8)$$

with  $\sigma_1^2 = p_b^{A \setminus a} \times (1 - p_b^{A \setminus a})$ .

We now construct, using the delta method, a random variable that converges in distribution to a Gaussian of mean  $p_{\neg a}(b, A) = \frac{p_b^A}{1 - p_a^A}$ .

Let  $k = |A| + 1$ , we define the following function  $g : (0, 1)^k \rightarrow \mathbb{R}$  such that:

$$g(x_1, x_2, \dots, x_k) = \frac{x_2}{1 - x_1}.$$

We have  $g\left(\frac{n^A}{N}\right) = \frac{n_b^A}{N - n_a^A}$  and  $g(p^A) = p_{-a}(b, A)$ . Moreover,  $g$  is differentiable on its domain and its Jacobian matrix is given by:

$$\nabla g(x_1, x_2) = \left( \frac{x_2}{(1 - x_1)^2}, \frac{1}{1 - x_1}, 0, \dots, 0 \right).$$

Using the multivariate version of the delta method (Oehlert, 1992), we obtain:

$$\begin{aligned} \sqrt{N} \times \left[ g\left(\frac{n^A}{N}\right) - g(p^A) \right] &\xrightarrow[N \rightarrow +\infty]{d} \mathcal{N}\left(0, \nabla g(p^A) \Sigma \nabla g(p^A)^T\right) \\ &\Leftrightarrow \frac{n_b^A}{N - n_a^A} \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N}\left(p_{-a}(b, A), \frac{\sigma_2^2}{N}\right), \end{aligned}$$

with

$$\begin{aligned} \sigma_2^2 &= \left( \frac{p_b^A}{(1 - p_a^A)^2} \times p_a^A(1 - p_a^A) + \frac{1}{1 - p_a^A} \times (-p_a^A p_b^A) \right) \times \frac{p_b^A}{(1 - p_a^A)^2} \\ &+ \left( \frac{p_b^A}{(1 - p_a^A)^2} \times (-p_a^A p_b^A) + \frac{1}{1 - p_a^A} \times p_b^A(1 - p_b^A) \right) \times \frac{1}{1 - p_a^A} \\ &= \frac{-p_a^A (p_b^A)^2 + p_b^A (1 - p_b^A) (1 - p_a^A)}{(1 - p_a^A)^3} \\ &= \frac{(1 - p_a^A - p_b^A) \times p_b^A}{(1 - p_a^A)^3}. \end{aligned}$$

By independence between menus, the empirical difference  $\hat{\mu}_{bIa}^A = \frac{n_b^A}{N - n_a^A} - \frac{n_b^{A \setminus a}}{N}$  asymptotically follows a normal distribution:

$$\hat{\mu}_{bIa}^A \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N}\left(\mu_{bIa}^A, \frac{\sigma_{bAa}^2}{N}\right), \quad (3.9)$$

with  $\mu_{bIa}^A = p_{-a}(b, A) - p(b, A \setminus a)$  and  $\sigma_{bAa}^2 = \sigma_1^2 + \sigma_2^2$ .

Moreover, we have the following convergence in probability:

$$\hat{\sigma}_{bAa} = \sqrt{\frac{n_b^{A \setminus a} \times (N - n_b^{A \setminus a})}{N^2} + \frac{N \times (N - n_a^A - n_b^A) \times n_b^A}{(N - n_a^A)^3}} \xrightarrow[N \rightarrow +\infty]{p} \sigma_{bAa}. \quad (3.10)$$

Therefore, it is possible to use Slutsky's theorem (Cramer, 1954, p.254-255) to eq. (3.9) and eq. (3.10), and obtain the following result:

$$Z_{bAa}^A = \sqrt{N} \times \frac{\hat{\mu}_{bAa}^A}{\hat{\sigma}_{bAa}} \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N}\left(\sqrt{N} \times \frac{\mu_{bAa}^A}{\sigma_{bAa}}, 1\right). \quad (3.11)$$

With  $\sqrt{N} \times \frac{\mu_{bAa}^A}{\sigma_{bAa}} = 0$  under the null hypothesis  $H0_{bAa}^A$ . □

### 3.B.2 Power Test of binary SI

In section 3.B.1 we have constructed a test of sequential independence relation in any menu. In this section, we are now interested to calculate the power of the test to reject false binary SI relations when the data generating process is a RCCSR as a function of the model parameters, the nominal level, and the number of observations. We demonstrate the following proposition:

**Proposition 3.2.3.** *The probability to reject the null hypothesis ( $H0_{aIb}^{ab} : p_{-b}(a, ab) = p(a, a)$ ) with the  $\alpha$  level two-tailed test proposed in proposition 3.2.2, under the alternative hypothesis ( $H1_{-aIb}^{ab} : p_{-b}(a, ab) - p(a, a) = \mu_{a>b} \neq 0$ ) that the data generating process  $p$  follows a RCCSR with  $a \succ b$  is given by:*

$$\beta_{b>a} = \Phi\left(z_{\alpha/2} - \sqrt{N} \times \frac{\mu_{a>b}}{\sigma_{a>b}}\right) + \Phi\left(z_{\alpha/2} + \sqrt{N} \times \frac{\mu_{a>b}}{\sigma_{a>b}}\right),$$

with  $\Phi$  and  $z_{\alpha/2}$  the cumulative distribution function and the  $\frac{\alpha}{2}$  quantile of the standard normal distribution,

$$\mu_{a>b} = \frac{\pi(ab)\pi(\emptyset)}{[\pi(a) + \pi(ab) + \pi(\emptyset)][\pi(a) + \pi(\emptyset)]},$$

$$\text{and } \sigma_{a \succ b} = \sqrt{\frac{\pi(a)\pi(\emptyset)}{[\pi(a) + \pi(\emptyset)]^2} + \frac{[\pi(a) + \pi(ab)][\pi(ab) + \pi(a) + \pi(b) + \pi(\emptyset)]\pi(\emptyset)}{[\pi(a) + \pi(ab) + \pi(\emptyset)]^3}}.$$

*Proof.* We focus here on sequential independence in the binary menu  $A = \{a, b\}$  and assume that  $p$  is a RCCSR with  $a \succ b$  (without loss of generality). The null hypothesis ( $H0_{aIb}^{ab} : p_{-b}(a, ab) = p(a, a)$ ) tested by the observer is therefore false, but the alternative hypothesis

$$H1_{-aIb}^{ab} : \mu_{aIb}^{ab} = p_{-b}(a, ab) - p(a, a) = \frac{\pi(a) + \pi(ab)}{\pi(a) + \pi(ab) + \pi(\emptyset)} - \frac{\pi(a)}{\pi(a) + \pi(\emptyset)} \neq 0,$$

is true.

We use the result shown in eq. (3.11) of the previous section, applied to the binary menu  $A = \{a, b\}$ . Indeed, this result holds under both the null hypothesis  $H0_{aIb}^{ab}$  tested by the observer and the alternative hypothesis  $H1_{-aIb}^{ab}$  we are now considering. Under the alternative hypothesis  $H1_{-aIb}^{ab}$ , the statistics  $Z_{aIb}^{ab}$  follows asymptotically a biased Gaussian distribution.

$$Z_{aIb}^{ab} \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N}\left(\sqrt{N} \frac{\mu_{a \succ b}}{\sigma_{a \succ b}}, 1\right), \quad (3.12)$$

$$\begin{aligned} \text{with } \mu_{a \succ b} &= \frac{p(a, ab)}{p(a, ab) + p(o, ab)} - p(a, a) \\ &= \frac{\pi(a) + \pi(ab)}{\pi(a) + \pi(ab) + \pi(\emptyset)} - \frac{\pi(a)}{\pi(a) + \pi(\emptyset)} \\ &= \frac{\pi(ab)\pi(\emptyset)}{[\pi(a) + \pi(ab) + \pi(\emptyset)][\pi(a) + \pi(\emptyset)]}, \end{aligned}$$

$$\begin{aligned} \text{and } \sigma_{a \succ b} &= p(a, a)[1 - p(a, a)] + \frac{p(a, ab)[1 - p(b, ab) - p(a, ab)]}{[1 - p(b, ab)]^3} \\ &= \sqrt{\frac{\pi(a)\pi(\emptyset)}{[\pi(a) + \pi(\emptyset)]^2} + \frac{[\pi(a) + \pi(ab)][\pi(ab) + \pi(a) + \pi(b) + \pi(\emptyset)]\pi(\emptyset)}{[\pi(a) + \pi(ab) + \pi(\emptyset)]^3}}. \end{aligned}$$

The observer will (correctly) reject the null hypothesis  $H0_{aIb}^{ab}$  whenever the statistic  $Z_{aIb}^{ab}$  lies in the critical region of the test. This region depends on the



nominal type I error  $\alpha$  and the type of test (e.g. one or two-sided test). We will here assume that the observer use a two-side test of significance level  $\alpha$ .<sup>2</sup> Therefore, the probability  $\beta_{a>b}$  of a correct rejection is given by:

$$\begin{aligned}\beta_{a>b} &= \mathbb{P}\left(|Z_{aIb}^{ab}| > z_{1-\alpha/2}\right) \\ &= 1 - \mathbb{P}\left(z_{\alpha/2} < Z_{aIb}^{ab} < -z_{\alpha/2}\right) \\ &= 1 - \mathbb{P}\left(z_{\alpha/2} - \sqrt{N} \times \frac{\mu_{a>b}}{\sigma_{a>b}} < Z_{aIb}^{ab} - \sqrt{N} \times \frac{\mu_{a>b}}{\sigma_{a>b}} < -z_{\alpha/2} - \sqrt{N} \times \frac{\mu_{a>b}}{\sigma_{a>b}}\right),\end{aligned}$$

with  $z_{\alpha/2}$  the  $\frac{\alpha}{2}$  quantile of the standard normal distribution. Under  $H1_{-aIb}^{ab}$ , we have:

$$Z_{aIb}^{ab} - \sqrt{N} \times \frac{\mu_{a>b}}{\sigma_{a>b}} \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N}(0, 1).$$

Therefore,

$$\begin{aligned}\beta_{a>b} &= 1 + \Phi\left(z_{\alpha/2} - \sqrt{N} \times \frac{\mu_{a>b}}{\sigma_{a>b}}\right) - \Phi\left(-z_{\alpha/2} - \sqrt{N} \times \frac{\mu_{a>b}}{\sigma_{a>b}}\right) \\ &= \Phi\left(z_{\alpha/2} - \sqrt{N} \times \frac{\mu_{a>b}}{\sigma_{a>b}}\right) + \Phi\left(z_{\alpha/2} + \sqrt{N} \times \frac{\mu_{a>b}}{\sigma_{a>b}}\right),\end{aligned}$$

with  $\Phi$  the cumulative distribution function of the standard normal distribution. □

### 3.B.3 Statistical Test of WARP-SLA

In this section we construct a statistic a test statistics of WARP-SLA with respect to a specific linear order. From the proof of lemma 3.3.1, we know that testing WARP-SLA is equivalent to testing the following hypothesis:

$$H0_{WARP-SLA} = \bigcup_{\succ \in \mathcal{L}} H0_{\succ}^{WARP-SLA},$$

---

<sup>2</sup>Since under the alternative hypothesis that  $p$  is a RCCSR with  $a > b$  we must have  $\mu_{a>b} < 0$ , it could also make sense to consider one-side test.

where  $\mathcal{L}$  is the set of linear order over  $X$ , and

$$H0_{\succ}^{\text{WARPSLA}} = \bigcap_{A \in \mathcal{D}} (AIx_A^{\succ}),$$

with  $x_A^{\succ}$  the  $\succ$ -best alternative in  $A$ .

We demonstrate the following proposition to test  $H0_{\succ}^{\text{WARPSLA}}$ :

**Proposition 3.4.1.** *Let  $n^A = (n_a^A)_{a \in A \cup o}$  with  $A \in \mathcal{D}$  the realisation of independent multinomial variables with parameters  $p^A = (p_a^A)_{a \in A \cup o}$  and  $N$ , that indicate the number of times each alternative have been chosen in each menu  $A$ . Let  $\succ$  a linear order over  $X$ .  $\forall A \in \mathcal{D}$  and  $\forall a \in X$ , we note:  $x_A^{\succ}$  the  $\succ$ -best alternative in  $A$ ,  $\tilde{a} = \{b \in X : \neg(a \succ b)\}$  the set of alternatives that are  $\succ$  preferred to  $a$ ,  $n(\tilde{a}, A) = \sum_{b \in A \cap \tilde{a}} n_b^A$  the number of times an alternative  $\succ$ -preferred to  $a$  is chosen in the menu  $A$ , and  $\Psi_{\succ}^a(A) = \{B \in \mathcal{D} : B \setminus \tilde{a} = A \setminus \tilde{a}\}$  the set of menus where the alternatives  $\succ$ -dominated by  $a$  are the same as in the menu  $A$ .*

*Under the null hypothesis  $H0_{\succ}^{\text{WARPSLA}} = (\forall A \in \mathcal{D}, AIx_A^{\succ})$  that WARP-SLA holds for the specific preference relation  $\succ$ , the statistics:*

$$\mathcal{J}_{\succ} = \sum_{A \in \mathcal{D}} \sum_{a \in A \cup o} n_a^A \log \left( \frac{n_a^A}{N \hat{p}_{\succ}(a, A)} \right) \xrightarrow[N \rightarrow +\infty]{d} \chi_k^2,$$

*follows asymptotically a  $\chi^2$  distribution with  $k$  degrees of freedom, where  $k$  is the number of (non-trivial) SI relations required for the preference relation  $\succ$  to be a valid representation of  $p$ .*

*With*

$$\hat{p}_{\succ}(a, A) = \begin{cases} \frac{\sum_{B \in \Psi_{\succ}^a(A)} n_a^B}{\sum_{B \in \Psi_{\succ}^a(A)} [N - n(\tilde{a}, B)]} & \text{if } a = x_A^{\succ} \\ \hat{p}_{\succ}(a, A \setminus \tilde{a}) \times \prod_{b \in \tilde{a}} [1 - \hat{p}_{\succ}(b, A \setminus \tilde{b})] & \text{otherwise} \end{cases},$$

*the maximum-likelihood estimator of  $p_a^A$  under  $H0_{\succ}^{\text{WARPSLA}}$ .*

*Proof.* Let  $X$  the set of alternatives and  $\mathcal{D}$  the domain where choices are observed.

We assume  $\mathcal{D}$  satisfies the following richness assumption:

$$\forall a \neq b \in X, \{a, b\} \in \mathcal{D} \text{ and } (B \in \mathcal{D}) \cap (A \subseteq B) \Rightarrow A \in \mathcal{D}.$$

Let  $\succ$  a linear order over  $X$ . For all  $A \in \mathcal{D}, |A| = k > 0$ , we note  $N^A = (N_a^A)_{a \in A \cup o}$  the random variable that indicates the number of times each alternative (including the default) is chosen in the menu  $A$  and  $n^A = (n_a^A)_{a \in A \cup o}$  its realisation.  $N^A$  follows a multinomial of parameters  $(N, p^A = (p_a^A)_{a \in A \cup o})$ . We assume that for any menus  $A$  and  $B \in \mathcal{D}$  distinct,  $N^A$  and  $N^B$  are independent.

For any strict linear order  $\succ$  we will construct a test statistics of WARP-SLA based on a likelihood ratio test.

For any menu  $A$ , the probability mass function of  $N_A$  is given by:

$$f(n^A, p^A) = \frac{N!}{\prod_{a \in A \cup o} n_a^A!} \times \prod_{a \in A \cup o} (p_a^A)^{n_a^A}. \quad (3.13)$$

Since  $(N^A)_{A \in \mathcal{D}}$  are independent, their joint probability mass function is given by:

$$g(n^A, n^B, \dots, p^A, p^B, \dots) = \prod_{A \in \mathcal{D}} \left( \frac{N!}{\prod_{a \in A \cup o} n_a^A!} \times \prod_{a \in A \cup o} (p_a^A)^{n_a^A} \right). \quad (3.14)$$

We note  $p = (p^A)_{A \in \mathcal{D}}$  and  $n = (n^A)_{A \in \mathcal{D}}$ . And the log-likelihood is given by:

$$\log(g(n, p)) = LL(p, n) + C(n),$$

with

$$LL(p, n) = \sum_{A \in \mathcal{D}} \left[ \sum_{a \in A \cup o} n_a^A \log(p_a^A) \right], \quad (3.15)$$

and

$$C(n) = \sum_{A \in \mathcal{D}} \left( \log(N!) - \sum_{a \in A \cup o} \log(n_a^A!) \right),$$

a constant that does not depend on the distribution parameters  $p$ .

The test consists in testing the null hypothesis  $(H0_{\succ}^{WARP\text{SLA}} : p \in \theta_0)$  against

the alternative hypothesis ( $H_1 : p \in \theta_1$ ), where  $\theta_0 \subset \theta_1$  indicates the set of linear constraints imposed on  $p$  by  $H0_{\succ}^{WARPSLA}$  and  $\theta_1$  indicates the full set of permissive parameters (i.e. any  $p \geq 0$  such that  $\sum_{a \in A \cup o} p_a^A = 1$  for all  $A \in \mathcal{D}$ ).

The Wilks's theorem (1938), ensures that under the null hypothesis  $H0_{\succ}^{WARPSLA}$ , the likelihood ratio defined by

$$LR = 2[LL(\hat{p}_1, n) - LL(\hat{p}_0, n)] \xrightarrow[N \rightarrow +\infty]{d} \chi_k^2,$$

with  $\hat{p}_1 = \arg \max_{p \in \theta_1} LL(p, n)$  and  $\hat{p}_0 = \arg \max_{p \in \theta_0} LL(p, n)$ , is asymptotically distributed as a  $\chi^2$  with  $k = \dim(\theta_1) - \dim(\theta_0)$  degrees of freedom. Moreover, it is obvious that  $k$  is equal to the number of (non-trivial) SI relations required for WARP-SLA to hold with respect to  $\succ$ .

We now provide closed form expressions for  $LL(\hat{p}_1, n)$  and  $LL(\hat{p}_0, n)$ . Under  $H_1$ , it is immediate that  $\hat{p}_1 = \left( \frac{n^A}{N} \right)_{A \in \mathcal{D}}$ , therefore:

$$LL(\hat{p}_1, n) = \sum_{A \in \mathcal{D}} \left[ \sum_{a \in A \cup o} n_a^A \log \left( \frac{n_a^A}{N} \right) \right]. \quad (3.16)$$

To complete the proof, we show that under  $H0_{\succ}^{WARPSLA}$ , we have:

$$LL(\hat{p}_0, n) = \sum_{A \in \mathcal{D}} \left[ \sum_{a \in A \cup o} n_a^A \log (\hat{p}_{\succ}(a, A)) \right], \quad (3.17)$$

with

$$\hat{p}_{\succ}(a, A) = \hat{p}_{\succ}(a, A \setminus \check{a}) \times \prod_{b \in \check{a}} \left[ 1 - \hat{p}_{\succ}(b, A \setminus \check{b}) \right], \quad (3.18)$$

and

$$\hat{p}_{\succ}(a, A \setminus \check{a}) = \frac{\sum_{B \in \psi_{\succ}^a(A)} n_a^B}{\sum_{B \in \psi_{\succ}^a(A)} [N - n(\check{a}, B)]}, \quad (3.19)$$

and with  $\check{a} = \{b \in X : \neg a \succ b\}$  the set of alternatives  $\succ$ -preferred to  $a$  (and we have  $\check{o} = X$ ) and  $n(\check{a}, A) = \sum_{b \in A \cap \check{a}} n_b^A$  the number of times those alternatives have been chosen in  $A$ .

We first show, that if  $p$  satisfies  $H0_{\succ}^{WARPSLA}$ , then we have:

$$\forall A \in \mathcal{D}, \forall a \in A, p(a, A) = p(a, A \setminus \check{a}) \times \prod_{b \in \check{a}} \left(1 - p(b, A \setminus \check{b})\right).$$

This is immediate for any menu  $A$  and for any alternative  $a \in A$ , by applying recursively the SI from the  $\succ$ -best alternatives to  $a$ . Indeed, let reorganize  $X$  according to  $\succ$  as follows:  $X = \{x_1 \succ x_2 \succ \dots\}$ , we have for all  $A \in \mathcal{D}$  and for all  $x_k \in A$ :

$$\begin{aligned} p(x_k, A) &= p(x_k, A \setminus x_1) [1 - p(x_1, A)] \\ &= p(x_k, A \setminus \{x_1, x_2\}) [1 - p(x_2, A \setminus x_1)] [1 - p(x_1, A)] \\ &= p(x_k, A \setminus \{x_1, x_2, x_3\}) [1 - p(x_3, A \setminus \{x_1, x_2\})] [1 - p(x_2, A \setminus x_1)] [1 - p(x_1, A)] \\ &= p(x_k, A \setminus \{x_1, \dots, x_{k-1}\}) [1 - p(x_{k-1}, A \setminus \check{x}_{k-1})] \times \dots \times [1 - p(x_1, A \setminus \check{x}_1)] \\ &= p(x_k, A \setminus \check{x}_k) \prod_{i=1}^{k-1} \left(1 - p(x_i, A \setminus \check{x}_i)\right). \end{aligned}$$

We can thus substitute the latter expression in eq. (3.17):

$$\begin{aligned} LL(p, n)_{|p \in \theta_0} &= \sum_{A \in \mathcal{D}} \left[ \sum_{a \in AU_0} n_a^A \log \left( p(a, A \setminus \check{a}) \times \prod_{b \in \check{a}} \left(1 - p(b, A \setminus \check{b})\right) \right) \right] \\ &= \sum_{A \in \mathcal{D}} \sum_{a \in AU_0} n_a^A \log \left( p(a, A \setminus \check{a}) \right) \\ &\quad + \sum_{A \in \mathcal{D}} \sum_{a \in AU_0} n_a^A \sum_{b \in A: b \succ a} \log \left( 1 - p(b, A \setminus \check{b}) \right) \\ &= \sum_{A \in \mathcal{D}} \sum_{a \in AU_0} n_a^A \log \left( p(a, A \setminus \check{a}) \right) \\ &\quad + \sum_{A \in \mathcal{D}} \sum_{b \in AU_0} \left( \log \left( 1 - p(b, A \setminus \check{b}) \right) \sum_{a \in A: b \succ a} n_a^A \right) \\ &= \sum_{A \in \mathcal{D}} \sum_{a \in AU_0} n_a^A \log \left( p(a, A \setminus \check{a}) \right) \\ &\quad + \sum_{A \in \mathcal{D}} \sum_{b \in AU_0} \log \left( 1 - p(b, A \setminus \check{b}) \right) [N - n(\check{b}, A) - n_b^A]. \end{aligned}$$

Since  $n_a^A = 0$  whenever  $a \notin A \cup o$ , we have:

$$LL(p, n)|_{p \in \theta_0} = \sum_{a \in X \cup o} \sum_{A \in \mathcal{D}} \left[ n_a^A \log \left( p(a, A \setminus \check{a}) \right) + [N - n(\check{a}, A) - n_a^A] \log \left( 1 - p(a, A \setminus \check{a}) \right) \right].$$

From the latter equation, we see that under  $H0_{\succ}^{WARPSLA}$ , only the choice probabilities of the  $\succ$ -preferred alternatives in any menu need to be estimated and  $LL(p, n)|_{p \in \theta_0}$  can be factorized by those choice probabilities. For each alternative  $a$ , we define  $\Psi(a) = \{A \in \mathcal{D} : x_A^{\succ} = a\}$ , the set of distinct menus where  $a$  is the  $\succ$ -preferred alternative, and  $\psi_a^{\succ}(A) = \{B \in \mathcal{D} : a \in A \cap B, B \setminus \check{a} = A \setminus \check{a}\}$ , the set of menus containing the same alternatives  $\succ$ -dominated by  $a$  as in  $A$ . For all  $A \in \Psi(a)$  and for all  $B \in \psi_a^{\succ}(A)$ , we have  $p(a, B \setminus \check{a}) = p_a^A$  and we rewrite  $LL(p, n)|_{p \in \theta_0}$  as a function of  $\left( (p_a^A)_{A \in \Psi(a)} \right)_{a \in X \cap o}$ :

$$LL(p, n)|_{p \in \theta_0} = \sum_{a \in X \cup o} \sum_{A \in \Psi(a)} \left( \log(p_a^A) \sum_{B \in \psi_a^{\succ}(A)} n_a^B + \log(1 - p_a^A) \sum_{B \in \psi_a^{\succ}(A)} [N - n(\check{a}, B) - n_a^B] \right).$$

Taking the derive of  $LL(p, n)|_{p \in \theta_0}$  with respect to  $p$ , we obtain, for all  $a \in X$  and for all  $A \in \Psi(a)$ :

$$\begin{aligned} \frac{\partial LL(p, n)|_{p \in \theta_0}}{\partial p_a^A} = 0 &\Leftrightarrow \frac{\sum_{B \in \psi_a^{\succ}(A)} n_a^B}{p_a^A} = \frac{\sum_{B \in \psi_a^{\succ}(A)} [N - n(\check{a}, B) - n_a^B]}{1 - p_a^A} \\ &\Leftrightarrow \frac{1 - p_a^A}{p_a^A} = \frac{\sum_{B \in \psi_a^{\succ}(A)} [N - n(\check{a}, B) - n_a^B]}{\sum_{B \in \psi_a^{\succ}(A)} n_a^B} \\ &\Leftrightarrow \frac{1}{p_a^A} = \frac{\sum_{B \in \psi_a^{\succ}(A)} [N - n(\check{a}, B)]}{\sum_{B \in \psi_a^{\succ}(A)} n_a^B} \\ &\Leftrightarrow \hat{p}_{\succ}(a, A \setminus \check{a}) = \frac{\sum_{B \in \psi_a^{\succ}(A)} n_a^B}{\sum_{B \in \psi_a^{\succ}(A)} [N - n(\check{a}, B)]}. \end{aligned}$$

Therefore, the likelihood ratio:

$$LR = 2 \times \sum_{A \in \mathcal{D}} \left[ \sum_{La \in A \cap o} n_a^A \log \left( \frac{n_a^A}{N \hat{p}_{\succ}(a, A)} \right) \right],$$

follows a  $\chi^2$  with  $k$  degrees of freedom, where  $k$  is the number of non trivial SI relations. With

$$\hat{p}_{\succ}(a, A) = \hat{p}_{\succ}(a, A \setminus \check{a}) \times \prod_{b \in \check{a}} \left[ 1 - \hat{p}_{\succ}(b, A \setminus \check{b}) \right],$$

the MLL estimator of  $p(a, A)$  under  $H_0$ , and with

$$\hat{p}_{\succ}(a, A \setminus \check{a}) = \frac{\sum_{B \in \Psi_{\check{a}}^a(A)} n_a^B}{\sum_{B \in \Psi_{\check{a}}^a(A)} [N - n(\check{a}, B)]}.$$

□

### 3.B.4 Method to test wIFO

Let  $\succ$  a linear order over the set of alternative  $X$ . In this section, we described the method used to estimate the weights  $w_i$  of the  $\bar{\chi}^2$  distribution, of the likelihood ratio statistics, under  $H0_{\succ}^{wIFO}$ :

$$LR = 2 \times [LL_{\succ}(\hat{\lambda}_1, n) - LL(\hat{\pi}, n)] \xrightarrow[N \rightarrow +\infty]{d} \bar{\chi}^2 = \sum_i w_i \chi_i^2, \quad (3.20)$$

with

$$LL_{\succ}(\lambda, n) = \sum_{A \in \mathcal{D}} \sum_{a \in A \cup o} n_a^A \log \left( \sum_{B \in A_{a \succ}} \lambda_B \right) - N \times \sum_{A \in \mathcal{D}} \log \left( \sum_{B \subseteq A} \lambda_B \right),$$

where  $\hat{\lambda}_1 = \arg \max_{\lambda \in \mathbb{R}^{|\mathcal{D}|}} LL_{\succ}(\lambda, n)$  and  $\hat{\pi} = \arg \max_{\lambda \in \mathbb{R}_+^{|\mathcal{D}|}} LL_{\succ}(\lambda, n)$ .

We estimate  $w_i = \mathbb{P} \left( \sum_{A \in \mathcal{D}} 1 \left( \hat{\lambda}_1^{\succ}(A) < 0 \right) = i \right)$  that is the probability that exactly  $i$  constraints contained in  $H0_{\succ}^{wIFO}$  are violated by  $\hat{\lambda}_1$  (Shapiro, 1988).

The maximum likelihood estimator is asymptotically normal:

$$\hat{\lambda}_1 \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N} \left( \lambda, I(\lambda)^{-1} \right),$$

with  $I(\lambda)$  the Fisher information matrix. However, the true value of  $\lambda$  is un-

known. In a similar case, Davis-Stober (2009) computed the  $w_i$  using Silvapulle's algorithm (1996) on the point  $\lambda = 0$ , since it represent the least favorable case. However, we cannot do the same in our case, since the choice function is not defined if  $\lambda = 0$  for all menus. However, we use  $\hat{\pi}$  as an approximation of the true  $\lambda$  and estimate the  $w_i$  with Monte-Carlo method. More precisely:

1. Compute the Fisher Information matrix at the point  $\hat{\pi}$  and take its inverse  $I(\hat{\pi})^{-1}$ ;
2. Generate  $Z \sim \mathcal{N}_{\mathcal{D}}(\hat{\pi}, I(\hat{\pi})^{-1})$  ;
3. Compute  $\Pi(Z)$ , the projection of  $Z$  on the cone  $C$  that represents our linear constraints ( $C = \mathbb{R}_+^{\mathcal{D}}$ ) with respect to the inner product associated to  $I(\hat{\pi})^{-1}$ :  

$$\Pi(Z) = \min_{x \in C} \{Z^T I(\hat{\pi})^{-1} x\}$$
 ;
4. Compute  $\phi$ , the number of dimensions such that  $\Pi(Z) = 0$ .
5. Repeat steps 2 to 4 a large number of times ( $>500$ ) to generate a robust estimator of  $w_i = \mathbb{P}(\phi = i)$ .

### 3.B.5 Joint Test of WARP-SLA and wIFO.

In this section we propose a method to test jointly WARP-SLA and wIFO based on a likelihood ratio test approach. The method is similar as the one described in the previous section. The only one difference remains on the form of the Likelihood function and the Fisher Information Matrix. The Likelihood function is obtained by expressing any random choice rule using the following proposition:

**Proposition 3.4.2.** *Let  $p$  a random choice rule. We have for all  $a \in X$  and for all  $A \in \mathcal{D}$ :*

$$p(a, A) = \frac{\sum_{\substack{B \subseteq A \\ a \in B}} \mu(a, B)}{\sum_{B \subseteq A} \sum_{b \in B} \mu(b, B)}$$

With  $\mu : X \cup o \times \mathcal{D} \rightarrow \mathbb{R}$  such that for all  $a \in X$  and for all  $A \in \mathcal{D}$ :

$$\mu(a, A) = \kappa \times \sum_{B \subseteq A} (-1)^{|A \setminus B|} \frac{p(a, B)}{p(o, B)},$$



with  $\kappa$  a scaling factor such that  $\sum_{a \in X \cup o} \sum_{A \in \mathcal{D}} \mu(a, A) = 1$ .

Moreover, *WARP-SLA* and *wIFO* hold iff there exists a complete and transitive preference relation  $\succ$  over  $X$  such that  $\mu(a, A) \geq 0$  for all  $a$  and  $A$  and  $\mu(b, A) = 0$  whenever  $\exists a \in A$ , such that  $a \succ b$ .

*Proof.* **(Existence of the function)** Let  $p$  a random choice rule. We define, for all  $a \in X \cup o$  and for all  $A \in \mathcal{D}$ ,

$$\mu(a, A) = \kappa \times \sum_{B \subseteq A} (-1)^{|A \setminus B|} \frac{p(a, B)}{p(o, B)},$$

with  $\kappa \in \mathbb{R}_+$  a scaling factor such that  $\sum_{A \in \mathcal{D}} \sum_{a \in A \cup o} \mu(a, A) = 1$ . Using Mobius inversion, we have for all  $A \in \mathcal{D}$  and for all  $a \in A \cup o$ :

$$p(a, A) = \frac{p(o, A)}{\kappa} \times \sum_{B \subseteq A} \mu(a, B)$$

Since  $p$  is a random choice rule, we have

$$p(a, A) = \frac{p(a, A)}{\sum_{b \in A \cup o} p(b, A)} = \frac{\sum_{B \subseteq A} \mu(a, B)}{\sum_{b \in A \cup o} \sum_{B \subseteq A} \mu(b, B)}.$$

Moreover, since  $p$  is a random choice rule,  $\mu$  must respect the following constraints:  $\mu(a, A) = 0$  if  $a \notin A \cup o$ ,  $\mu(o, \emptyset) = \kappa$ ,  $\mu(o, A) = 0$  if  $|A| > 1$  and  $\sum_{B \subseteq A} \mu(b, B) > 0$  for all  $A \in \mathcal{D}$  and for all  $b \in A$ . □

**(WARP-SLA and wIFO as linear constraints)** We show that if  $p$  is a random choice rule, *WARP-SLA* and *wIFO* are equivalent to the following constraints on  $\mu$ :  $\exists \succ \in \mathcal{L}$  a linear order over  $X$ , such that  $\mu(b, A) = 0$  whenever  $\exists a \in A$  such that  $a \succ b$ , and  $\mu(a, A) \geq 0$  for all  $a$  and for all  $A$ .

**If part.** If *WARP-SLA* and *wIFO* hold, then  $p_{\succ, \pi}$  is a *wRCCSR*. Let assume

$a \succ b$  and let  $A \in \mathcal{D}$  such that  $a, b \in A$ . We have:

$$\begin{aligned} \mu(b, A) &= \kappa \times \sum_{B \subseteq A} (-1)^{|A \setminus B|} \frac{p(b, B)}{p(o, B)} \\ &= \kappa \times \sum_{\substack{B \subseteq A \\ a \in B}} (-1)^{|A \setminus B|} \left( \frac{p(b, B)}{p(o, B)} - \frac{p(b, B \setminus a)}{p(o, B \setminus a)} \right). \end{aligned}$$

Using lemma 3.A.1, we have  $\frac{p(b, B)}{p(o, B)} - \frac{p(b, B \setminus a)}{p(o, B \setminus a)} = 0$  for all  $B \in \mathcal{D}$ , so  $\mu(b, A) = 0$ , whenever  $b$  is not the  $\succ$ -best alternative in  $A$ .

By construction of  $\pi$  (see the proof of theorem 3.3.1), and by definition of  $\mu$ , we have  $\pi(A) = \sum_{b \in A \cup o} \mu(b, A)$  for all  $A \in \mathcal{D}$ . Since  $\mu(b, A) = 0$  if  $b$  is not the  $\succ$ -best alternative in  $A$ , we have  $\mu(a, A) = \pi(A) \geq 0$ .

(Only if part.) Let  $p$  is a random choice rule and there exists a linear order  $\succ$  such that  $\mu \geq 0$  and  $\mu(a, A) = 0$  whenever  $a$  is not the  $\succ$ -best alternative in  $A$ . If we set  $\pi(A) = \mu(a, A)$ , with  $a$  the  $\succ$ -best alternative, then  $p_{\succ, \pi}$  is a wRCCSR, so WARP-SLA and wIFO hold.  $\square$

The rest of the method is similar as in the previous section.

### 3.B.6 Test of revealed preferences

We show Proposition 3.4.3.

**Proposition 3.4.3.** *Let  $p$  a wRCCSR. Let  $a$  and  $b \in X$ . Let  $\mathcal{S} \subseteq \mathcal{D}$ ,  $\mathcal{S} \neq \emptyset$ , such that  $A \in \mathcal{S} \Rightarrow (a \in A, A \setminus b \in \mathcal{S} \text{ and } A \cup b \in \mathcal{S})$ . Let  $(n(\cdot, A))_{A \in \mathcal{S}}$  the realisation of a collection of independent multinomial variables  $(\mathcal{M}(p(\cdot, A), N))_{A \in \mathcal{S}}$  that indicates the number of times each alternative is chosen in each menu  $A$ .*

Under  $\left( H_0^{-aPb} : \forall A \in \mathcal{D}, \frac{p(a, A)}{p(o, A)} = \frac{p(a, A \setminus b)}{p(o, A \setminus b)} \right)$ , the following statistics:

$$\mathcal{P} = \sum_{\substack{A \in \mathcal{S} \\ b \in A}} \frac{1}{\hat{\sigma}_{-aPb, A}^2} \times \left( \log \left( \frac{n(a, A) \times n(o, A \setminus b)}{n(o, A) \times n(a, A \setminus b)} \right) \right)^2 \xrightarrow[N \rightarrow +\infty]{d} \chi_k^2,$$

follows asymptotically a  $\chi^2$  distribution with  $k = |\mathcal{S}|/2$  degrees of freedom, with

$$\hat{\sigma}_{-aPb,A}^2 = 2 \times \frac{n(a, A) + n(o, A) + n(o, A \setminus b) + n(a, A \setminus b)}{n(a, A)n(o, A \setminus b) + n(a, A \setminus b)n(o, A)},$$

an estimator of the variance of  $\log \left( \frac{n(a, A) \times n(o, A \setminus b)}{n(o, A) \times n(a, A \setminus b)} \right)$  under  $H0^{-aPb}$ .

*Proof.* We want to test the hypothesis

$$H0^{-aPb} : \forall A \in \mathcal{D}, \frac{p(a, A)}{p(o, A)} = \frac{p(a, A \setminus b)}{p(o, A \setminus b)}.$$

Let  $\mathcal{S} \subset \mathcal{D}$  a set of menus where the observer observe the impact of the presence of the alternative  $b$  on the choice frequency of  $a$ . Formally,  $\forall A \in \mathcal{S}, a \in A, A \setminus b \in \mathcal{S}$  and  $A \cup b \in \mathcal{S}$ . The null hypothesis  $H0^{-aPb}$  implies that for all  $A \in \mathcal{S}$ , such that  $a \in A$  the following hypothesis is true :  $H0_A^{-aPb} : \frac{p(a, A)}{p(o, A)} = \frac{p(a, A \setminus b)}{p(o, A \setminus b)}$ .

Let  $A \in \mathcal{S}, a \in A$ . Let  $(n(a, A))_{a \in A \cup o}$  the realisation of the multinomial  $N(a, A)_{a \in A \cup o}$  of parameter  $(p(a, A))_{a \in A \cup o}$  and  $N$  which indicates the number of times each alternative is chosen in the menu  $A$ . Such multinomial is also the sum of  $N$  independent discrete random variables  $(C_i^A)_{i=1, \dots, N}$  which take values  $c_i^A \in A \cup o$ , with probabilities  $(p(a, A))_{a \in A \cup o}$  and indicate, for each trial  $i$ , which alternative has been chosen in  $A$ . For each  $i = (1, \dots, N)$ , we define  $X_i^A = \begin{pmatrix} \mathbb{1}_{c_i^A=a} \\ \mathbb{1}_{c_i^A=o} \end{pmatrix}$  the random variable that indicates if the alternative  $a$  or  $o$  have been chosen in the trial  $i$ . By construction, the  $X_i^A$  are iid, with expectation  $\mu := \mathbb{E}(X_i^A) = \begin{pmatrix} p(a, A) \\ p(o, A) \end{pmatrix}$  and

covariance matrix  $\Sigma = \begin{pmatrix} [1 - p(a, A)]p(a, A) & -p(a, A)p(o, A) \\ -p(a, A)p(o, A) & [1 - p(o, A)]p(o, A) \end{pmatrix}$ .

Let  $\bar{X}_n^A = \frac{1}{N} \sum_{i=1}^N X_i^A = \frac{1}{N} \begin{pmatrix} n(a, A) \\ n(o, A) \end{pmatrix}$ , the empirical mean of  $X_i^A$ . Using the Central Limit Theorem, we have:

$$\bar{X}_n^A \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N}_2 \left( \mu, \frac{\Sigma}{N} \right).$$

The test statistics is constructed using the *delta method* (Oehlert, 1992). Let

define the following function:  $g : ]0, 1[^2 \rightarrow \mathbb{R}$  such that  $g \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \log \left( \frac{x_1}{x_2} \right)$ . Its

Jacobian matrix is given by :  $\nabla g \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \left( \frac{1}{x_1}, -\frac{1}{x_2} \right)$ . We have:

$$\begin{aligned} & g(\bar{X}_n(A)) \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N} \left( g(\mu), \nabla g(\mu) \frac{\Sigma}{N} \nabla g(\mu)^T \right) \\ \Rightarrow & \log \left( \frac{n(a, A)}{n(o, A)} \right) \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N} \left( \log \left( \frac{p(a, A)}{p(o, A)} \right), \frac{1}{N} \left[ \frac{1}{p(a, A)} + \frac{1}{p(o, A)} \right] \right). \end{aligned}$$

Similarly, we have:

$$\log \left( \frac{n(a, A \setminus b)}{n(o, A \setminus b)} \right) \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N} \left( \log \left( \frac{p(a, A \setminus b)}{p(o, A \setminus b)} \right), \frac{1}{N} \left[ \frac{1}{p(a, A \setminus b)} + \frac{1}{p(o, A \setminus b)} \right] \right).$$

Under  $H0_A^{-aPb}$  :  $\frac{p(a, A)}{p(o, A)} = \frac{p(a, A \setminus b)}{p(o, A \setminus b)}$ , we have:

$$\log \left( \frac{n(a, A) \times n(o, A \setminus b)}{n(o, A) \times n(a, A \setminus b)} \right) \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N} \left( 0, \sigma_{-aPb, A}^2 \right),$$

with

$$\begin{aligned} \sigma_{-aPb, A}^2 &= \frac{1}{N} \left[ \frac{1}{p(a, A)} + \frac{1}{p(o, A)} + \frac{1}{p(a, A \setminus b)} + \frac{1}{p(o, A \setminus b)} \right] \\ &= \frac{1}{N} \left[ \frac{p(a, A) + p(o, A \setminus b)}{p(a, A)p(o, A \setminus b)} + \frac{p(o, A) + p(a, A \setminus b)}{p(o, A)p(a, A \setminus b)} \right] \\ &= \frac{1}{N} \left[ \frac{p(a, A) + p(o, A) + p(a, A \setminus b) + p(o, A \setminus b)}{p(a, A)p(o, A \setminus b)} \right]. \end{aligned}$$

Under  $H0_A^{-aPb}$ :

$$\hat{\sigma}_{-aPb, A} = \left( 2 \times \frac{n(a, A) + n(o, A) + n(a, A \setminus b) + n(o, A \setminus b)}{n(a, A)n(o, A \setminus b) + n(o, A)n(a, A \setminus b)} \right)^{1/2} \xrightarrow[N \rightarrow +\infty]{p} \sigma_{-aPb, A}.$$

Therefore, using Slutsky's theorem:

$$T_A^{-aPb} = \frac{1}{\hat{\sigma}_{-aPb, A}} \times \log \left( \frac{n(a, A) \times n(o, A \setminus b)}{n(o, A) \times n(a, A \setminus b)} \right) \xrightarrow[N \rightarrow +\infty]{d} \mathcal{N}(0, 1).$$

$\forall A, B \in \mathcal{S}$ , s.t.  $A \neq B$  and  $b \in A \cap B$ , if  $H0^{-aPb}$ , then  $H0_A^{-aPb}$  and  $H0_B^{-aPb}$ . Therefore,  $(T_A^{-aPb})^2$  and  $(T_B^{-aPb})^2$  are independently and identically asymptotically distributed as  $\chi_1^2$ , so their sum is asymptotically distributed as a  $\chi_2^2$ . Therefore,

$$\mathcal{P} = \sum_{\substack{A \in \mathcal{S} \\ b \in A}} (T_A^{-aPb})^2 \xrightarrow[N \rightarrow +\infty]{d} \chi_k^2,$$

with  $k = \frac{|\mathcal{S}|}{2}$  degrees of freedom. □

### 3.B.7 Additional Simulations: variation in tests' level and power

In the main text, we investigated the expected actual level and the expected power of the tests through simulations. Indeed, each agents was simulated using a different random choice rule or a different wRCCSR specification. Those simulations, are adapted to provide an estimator of the expected power and actual level for random choice rules and wRCCSR uniformly draw. However, they are not well adapted to asses the variance in levels and power among different model specifications. Indeed, to address this question, it is necessary to sample more than one agent to have the p-values' distribution for different specifications. Therefore, we conducted two additional set of simulations. In the first one, we generate 60 different wRCCSR  $(p_{\pi_i, \succ})_{i=1, \dots, 60}$  and simulate 400 agents for each wRCCSR and for each  $N=20, 40, 100$  and 1000 decisions per menu. In the other set of simulations, we generate 60 different random choice rule  $(p_i)_{i=1, \dots, 60}$  and again we simulate 400 agents for each wRCCSR and for each  $N=20, 40, 100$  and 1000 decisions per menu.

**Simulation 3.B.1.** *Choices are made among 3 different alternatives  $X = \{a, b, c\}$  (and a default option  $o$ ) in every menu of a domain that is the power set of  $X$ :  $\mathcal{D} = \{\{abc\}, \{ab\}, \{ac\}, \{bc\}, \{a\}, \{b\}, \{c\}, \emptyset\}$ . We generate 60 different wRCCSR  $(p_{\succ, \pi_i})_{i=1, \dots, 60}$ , all of them associated with the same preference relation  $a \succ b \succ c$  (without loss of generality), but different set of attention parameters  $\pi_i = \{\pi_i(abc), (\pi_i(ab), \pi_i(ac), \pi_i(bc), \pi_i(a), \pi_i(b), \pi_i(c), \pi_i(\emptyset))\}$ . Each set of parameter  $\pi_i$  is generated as follows. We randomly draw  $(\pi_i(A))_{A \in \mathcal{D}}$ , identically and independently from a*

uniform distribution  $\mathcal{U}_{[0,100]}$  and we divide  $\pi_i(A)$  by  $\sum_{A \in \mathcal{D}} \pi_i(A)$ . For each of the 60 different wRCCSR  $p_{\succ, \pi_i}$ , we vary the number of decisions ( $N = 20, 40, 100$  and  $1000$ ) made in each menu  $A \in \mathcal{D}$  and independently and identically simulate 400 agents for each  $N$ . Each of the  $60 \times 4 \times 400 = 96.000$  simulated agents is the realisation of a collection of independent multinomial distributions  $(\mathcal{M}_{|A|}(N, p_{\succ, \pi_i}(\cdot, A)))_{A \in \mathcal{D}}$  which indicates the number of times each alternative is chosen in each menu.

**Simulation 3.B.2.** Choices are made among 3 different alternatives  $X = \{a, b, c\}$  (and a default option  $o$ ) in every menu of a domain that is the power set of  $X$ :  $\mathcal{D} = \{\{abc\}, \{ab\}, \{ac\}, \{bc\}, \{a\}, \{b\}, \{c\}, \emptyset\}$ . We generate 60 different random choice rules  $(p_i)_{i=1, \dots, 60}$ . Each random set rules is uniformly draw as follows: for each menu  $A \in 2^X$  and each alternative  $a \in A \cup o$  we independently generate  $p_i(a, A) \sim \mathcal{U}(0, 100)$  and thus divide  $p_i(a, A)$  by  $\sum_{b \in A \cup o} p_i(b, A)$ . For each of the 60 different random choice rules  $p_i$ , we vary the number of decisions ( $N = 20, 40, 100$  and  $1000$ ) made in each menu  $A \in \mathcal{D}$  and independently and identically simulate 400 agents for each  $N$ . Each of the  $60 \times 4 \times 400 = 96.000$  simulated agents is the realisation of a collection of independent multinomial distributions  $(\mathcal{M}_{|A|}(N, p_i(\cdot, A)))_{A \in \mathcal{D}}$  which indicates the number of times each alternative is chosen in each menu.

Based on those simulation, we calculate confidence intervals for each test as follows. First, for each generated wRCCSR or random choice rule specification  $(p_i)_{i=1, \dots, 60}$ , and for each hypothesis tested, we estimate the actual rejection probability  $\hat{\alpha}_i(\alpha) = \mathbb{P}(\text{p-value}_i < \alpha)$  that is the proportion of agents simulated from a wRCCSR/random choice rule  $p_i$  rejecting a test at the nominal level  $\alpha$ . Then, we take the 5%-lower bound of the obtained rejection frequency, which correspond to the 3<sup>rd</sup> (among 60) lowest value of  $\hat{\alpha}_i(\alpha)$  and 5% upper bound, which corresponds to the 3<sup>rd</sup> (among 60) highest value of  $\hat{\alpha}_i(\alpha)$ .

We report in Table 3.B.1 the 90% Confidence Intervals of the rejection probabilities at the 5% nominal level for the  $\chi^2$  and SI-based tests of  $H0_{a>b>c}^{WARP\text{SLA}}$  as well as for the test of  $H0_{a>b>c}^{WARP\text{SLA}} \cap wIFO$  and for the tests of each revealed preferences, for wRCCSR agents simulated according to Simulation 3.B.1. We plot in Figure 3.B.1 symmetric 90%-confidence intervals for the p-values distributions of the tests of

N	20	40	100	1000
(1)	[ 0.065, 0.598 ]	[ 0.03, 0.483 ]	[ 0.035, 0.193 ]	[ 0.035, 0.070 ]
(2)	[ 0.020, 0.100 ]	[ 0.02, 0.09 ]	[ 0.032, 0.075 ]	[ 0.035, 0.068 ]
(3)	[ 0.002, 0.070 ]	[ 0.002, 0.038 ]	[ 0.002, 0.028 ]	[ 0, 0.035 ]
$a \succ b$	[ 0, 0.823 ]	[ 0.015, 0.995 ]	[ 0.105, 1 ]	[ 0.873, 1 ]
$a \succ c$	[ 0.002, 0.938 ]	[ 0.022, 1 ]	[ 0.050, 1 ]	[ 0.235, 1 ]
$b \succ c$	[ 0.020, 0.999 ]	[ 0.0175, 1 ]	[ 0.070, 1 ]	[ 0.08, 1 ]
$b \succ a$	[ 0.030, 0.110 ]	[ 0.025, 0.085 ]	[ 0.040, 0.093 ]	[ 0.032, 0.073 ]
$c \succ a$	[ 0.012, 0.125 ]	[ 0.015, 0.093 ]	[ 0.040, 0.093 ]	[ 0.032, 0.070 ]
$c \succ b$	[ 0.007, 0.113 ]	[ 0.010, 0.105 ]	[ 0.040, 0.088 ]	[ 0.032, 0.070 ]

Table 3.B.1: 90% Confidence Intervals for the rejection probabilities at the 5% nominal level when choices follow a wRCCSR.

*Note: (1) Test of  $H0_{a>b>c}^{WARPSLA}$  based on corrected p-values of SI tests ; (2) Test of  $H0_{a>b>c}^{WARPSLA}$  based on  $\chi^2$  statistics ; (3) Test of  $H0_{a>b>c}^{WARPSLA \cap wIFO}$*

$H0_{WARPSLA}$  and  $H0_{\succ}^{WARPSLA}$  (based on  $\chi^2$  and SI-corrected p-values) as well as the test of  $H0_{WARPSLA \cap wIFO}$  and  $H0_{\succ}^{WARPSLA \cap wIFO}$ , for wRCCSR agents simulated according to Simulation 3.B.1. We present in Figure 3.B.2 the symmetric 90%-confidence intervals for the power of those tests. That is the probability to reject the above null hypotheses, when the choices follow a random choice rules and are simulated according to Simulation 3.B.2. Finally, we present in Figure 3.B.3 the 90%-confidence intervals for the level (the probability to statistically reveal a false preference) and the power (the probability to statistically reveal the underlying preference) for the reveal preference tests, when choices follow a wRCCSR (Simulation 3.B.1).

The actual level of the  $\chi^2$  based test of  $H0_{\succ}^{WARPSLA}$  has less variance and converge quicker to the nominal value if compared to the test based on the corrected p-values of the SI-test. Indeed, with  $N = 40$  observation per menu, the  $\chi^2$  based test has an actual level (associated to the nominal level 0.05)  $\hat{\alpha}(\alpha) < 0.09\%$  for 95%, while the test based on corrected SI is not controlled even with a high number of observations. The  $\chi^2$  based test has also more power and less variance in that power. The method to test simultaneously WARP-SLA and wIFO is both more powerful and more conservative than the tests of  $H0^{WARPSLA}$ : the test is conservative for all the tested wRCCSR.

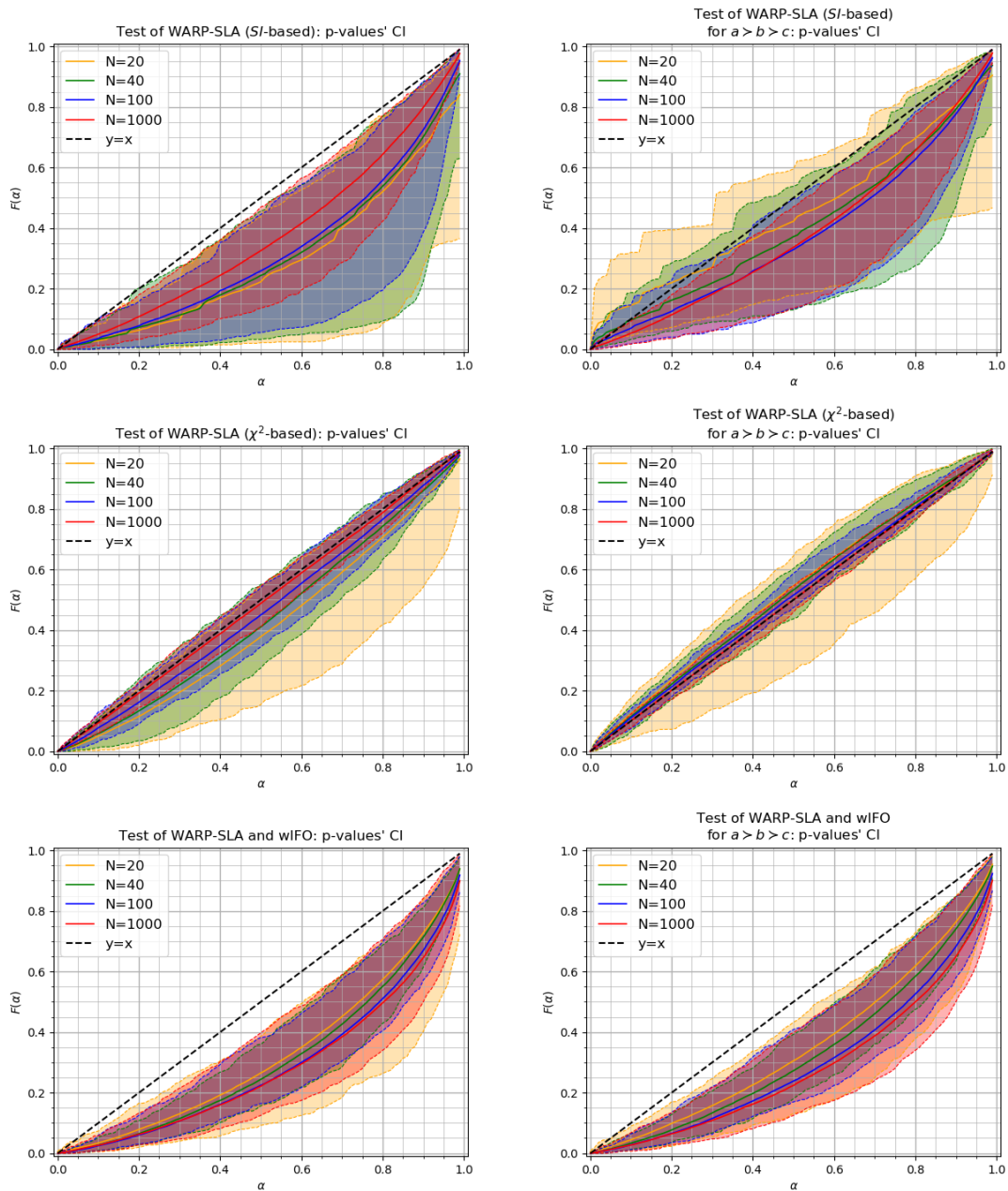


Figure 3.B.1: Confidence Intervals for the actual level of the tests of WARP-SLA and wIFO.

**Note:** Figures present 90% confidence intervals for p-values'cdf of the SI-corrected (top) and  $\chi^2$  (mid) based tests of WARP-SLA, as well as the simultaneous test of WARP-SLA and wIFO, when choices follow a wRCCSR. Colors represent the number of decisions  $N=20$  (yellow), 40 (green), 100 (blue) and 1000 (red) decisions per menu. The upper and lower limits of the CI shows for each nominal level  $\alpha$  the 3<sup>rd</sup> higher/lower rejection rate among the 60 generated wRCCSR.



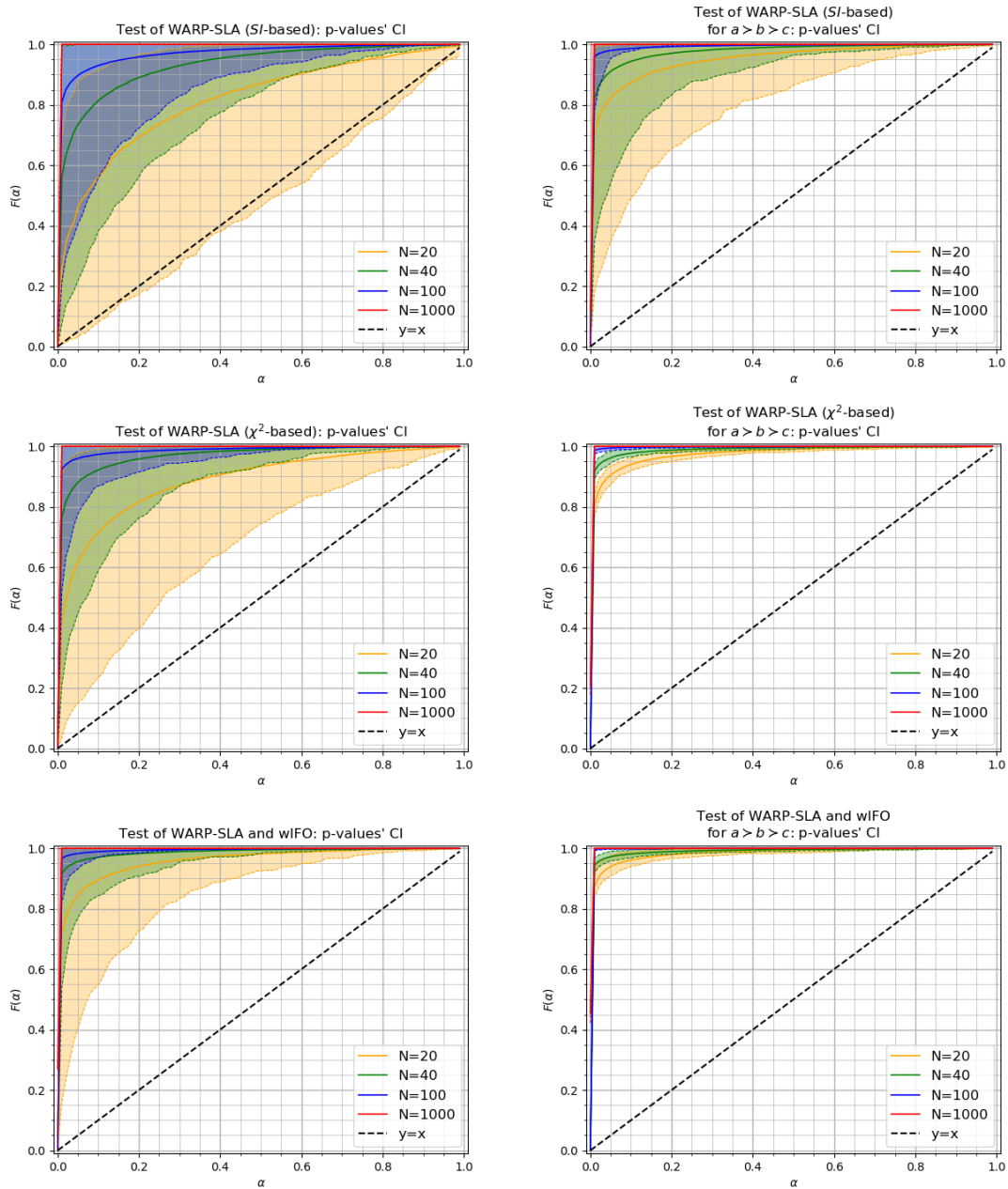


Figure 3.B.2: Confidence Intervals for the power of the tests of WARP-SLA and wIFO

**Note:** Figures present 90% confidence intervals for p-values' cdf of the SI-corrected (top) and  $\chi^2$  (mid) based tests of WARP-SLA, as well as the simultaneous test of WARP-SLA and wIFO, when choices follow a random choice rule. Colors represent the number of decisions  $N=20$  (yellow), 40 (green), 100 (blue) and 1000 (red) decisions per menu. The upper and lower limits of the CI shows for each nominal level  $\alpha$  the 3<sup>rd</sup> higher/lower rejection rate among the 60 generated random choice rules.

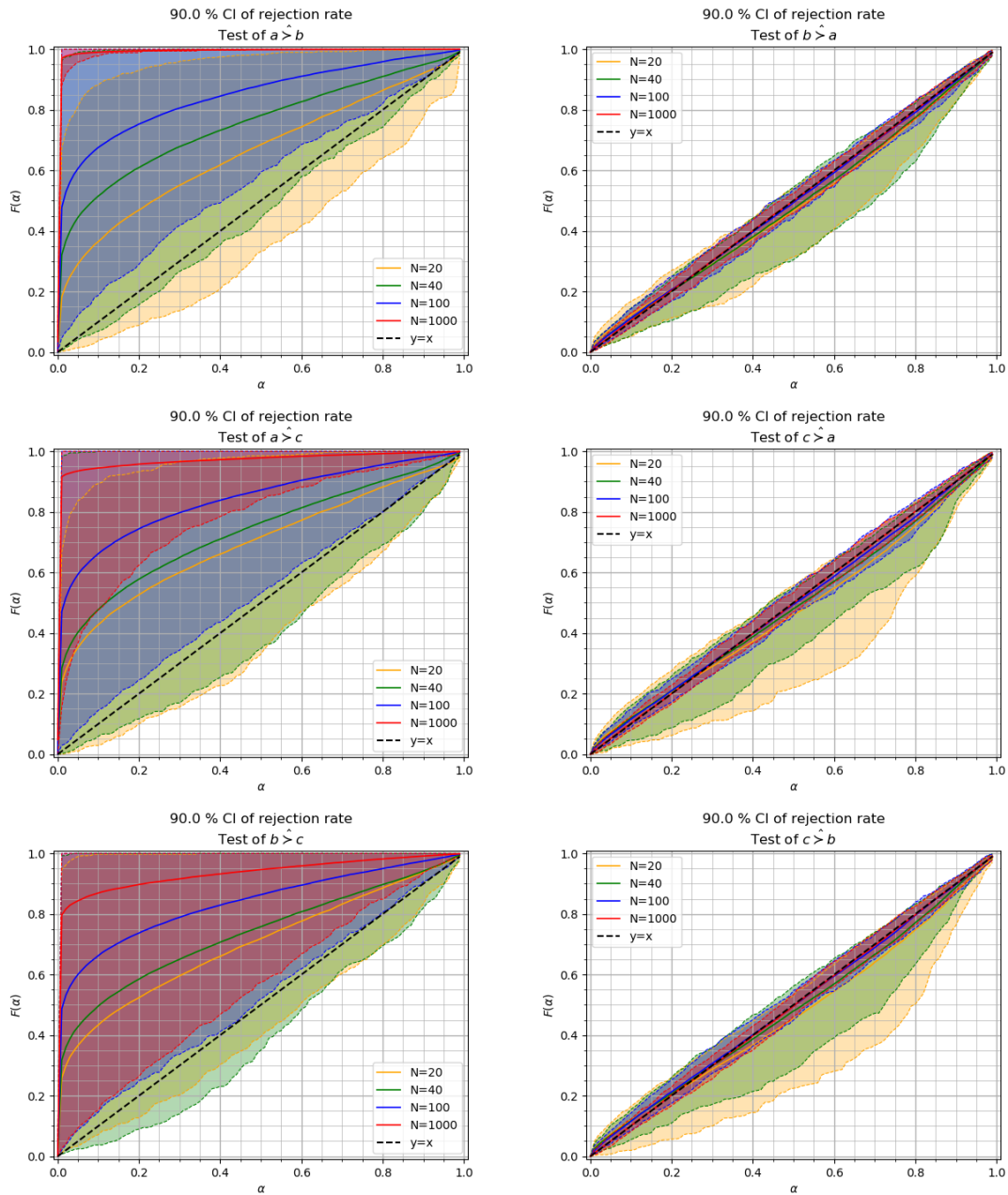


Figure 3.B.3: Confidence Intervals for revealed preferences tests.

**Note:** Figures present 90% confidence intervals for  $p$ -values'cdf of the tests of revealed preferences, when choices follow a  $wRCCSR$ . Colors represent the number of decisions  $N=20$  (yellow), 40 (green), 100 (blue) and 1000 (red) decisions per menu.  $H_0^{-aPb}$  ( $H_1^{aPb}$ ) is true for the left (right) figures which represent confidence interval for the actual level (power) of the tests. The upper and lower limits of the CI shows for each nominal level  $\alpha$  the 3<sup>rd</sup> higher/lower rejection rate among the 60 generated  $wRCCSR$ .

## 3.C Experimental Material

### 3.C.1 Elicitation of risk aversion under gain and loss frames

*Design* - The elicitation task was composed of two successive lotteries decisions. Subjects had to select a 50/50 lottery out of six possibilities. For the decision 1, all possible earnings are gains. Decision 2's earnings are obtained by subtracting 10 ECU from decision 1's outcomes such that some earnings are presented as losses. These potential losses are compensated by a 10 ECU endowment at the beginning of decision 2. Therefore both decisions involve identical stakes for the participants. Outcome earnings were constructed as follows: moving clockwise from lottery (B) the expected payoff and its variance increase, exception made of the last lottery (A) for which only the variance of the payoff is higher compared to the previous one (F).

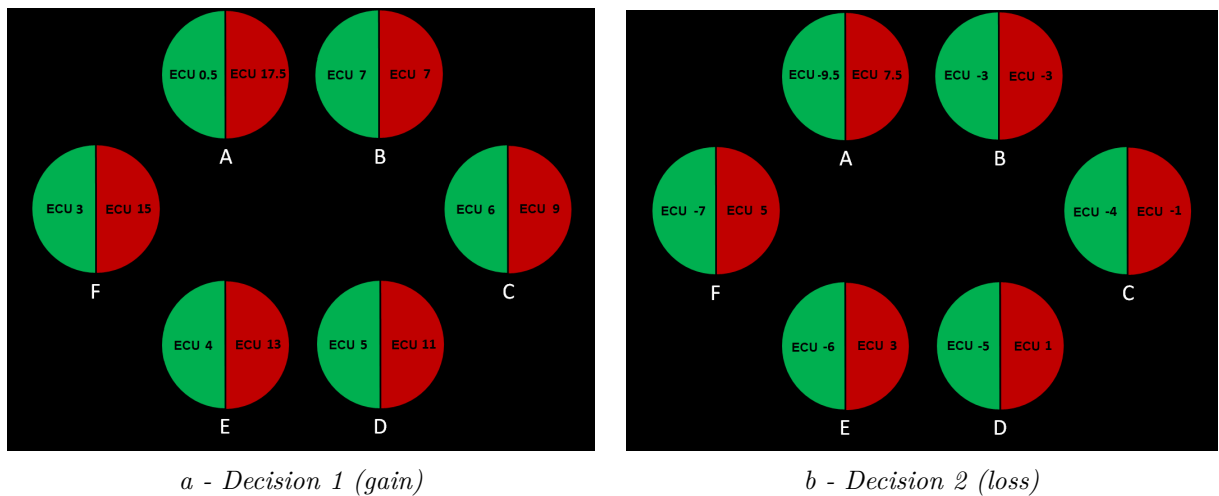


Figure 3.C.1: Lottery-based decisions used to measure risk aversion in the domains of gains and losses.

Based on this elicitation task, we used the lotteries chosen by the subjects as a measure of their risk aversion under the gain and the loss frames. We rank the lotteries from 0 (the safest lottery B) to 5 (the riskiest lottery A) according to their level of risk. An individual who chooses the lottery  $k$  ( $j$ ) in the gain (loss) frame is attributed a measure of risk aversion of  $6 - k$  ( $6 - j$ ) in the gain (loss) frame.

### 3.C.2 Trials and Block Sequences

Each subject participates to one of the seven following block sequences reported in table 3.C.1. The trial sequence for each block is reported in table 3.C.2

Treatment	1 <sup>st</sup> Block	2 <sup>nd</sup> Block	3 <sup>rd</sup> Block	4 <sup>th</sup> Block	5 <sup>th</sup> Block	6 <sup>th</sup> Block	7 <sup>th</sup> Block
1	Block1	Block2	Block3	Block4	Block5	Block6	Block7
2	Block2	Block3	Block4	Block5	Block6	Block7	Block1
3	Block3	Block4	Block5	Block6	Block7	Block1	Block2
4	Block4	Block5	Block6	Block7	Block1	Block2	Block3
5	Block5	Block6	Block7	Block1	Block2	Block3	Block4
6	Block6	Block7	Block1	Block2	Block3	Block4	Block5
7	Block7	Block1	Block2	Block3	Block4	Block5	Block6

Table 3.C.1: Block Sequences

trial	$pos_a$	$pos_b$	$pos_c$	Block1	Block2	Block3	Block4	Block5	Block6	Block7
1	4	5	12	b	abc	bc	c	ac	a	ab
2	13	5	7	abc	bc	ab	ac	a	b	c
3	8	5	6	ab	b	abc	a	c	ac	bc
4	11	12	4	bc	c	a	b	ab	abc	ac
5	9	6	3	a	ab	ac	abc	bc	c	b
6	11	6	13	c	ac	b	ab	abc	bc	a
7	3	11	2	ac	a	c	bc	b	ab	abc
8	14	7	6	ab	a	bc	abc	c	b	ac
9	2	9	10	ac	bc	ab	c	a	abc	b
10	12	10	2	bc	b	a	ac	abc	c	ab
11	10	13	6	c	abc	b	ab	ac	bc	a
12	5	11	1	b	c	ac	bc	ab	a	abc
13	6	10	3	abc	ac	c	a	b	ab	bc
14	6	9	2	a	ab	abc	b	bc	ac	c
15	5	13	7	ab	a	c	ac	bc	b	abc
16	2	7	8	c	abc	ac	b	a	bc	ab
17	3	4	7	a	c	b	bc	ab	abc	ac
18	5	6	11	bc	ac	a	ab	abc	c	b
19	7	9	1	b	bc	abc	c	ac	ab	a
20	1	3	14	ac	ab	bc	abc	b	a	c
21	6	4	9	abc	b	ab	a	c	ac	bc
22	4	10	13	c	ab	abc	ac	a	bc	b
23	13	1	12	b	ac	c	a	abc	ab	bc
24	10	1	9	ac	abc	ab	bc	c	b	a
25	8	7	11	bc	c	a	ab	b	ac	abc
26	14	11	12	abc	bc	b	c	ab	a	ac
27	13	2	3	ab	a	ac	b	bc	abc	c
28	7	1	11	a	b	bc	abc	ac	c	ab
29	7	4	8	abc	ac	b	ab	c	a	bc
30	14	3	8	ac	ab	bc	a	abc	c	b
31	12	3	10	ab	a	c	abc	bc	b	ac
32	9	8	10	a	bc	abc	c	b	ac	ab
33	12	8	9	b	abc	ac	bc	a	ab	c
34	10	14	13	bc	c	ab	b	ac	abc	a
35	9	12	14	c	b	a	ac	ab	bc	abc
36	2	12	1	bc	c	a	b	abc	ab	ac
37	1	14	5	c	ac	bc	ab	a	b	abc
38	8	13	5	ab	bc	abc	ac	b	a	c
39	11	2	14	ac	abc	ab	a	bc	c	b
40	3	14	4	a	b	ac	c	ab	abc	bc
41	4	2	5	abc	a	b	bc	c	ac	ab
42	1	8	4	b	ab	c	abc	ac	bc	a

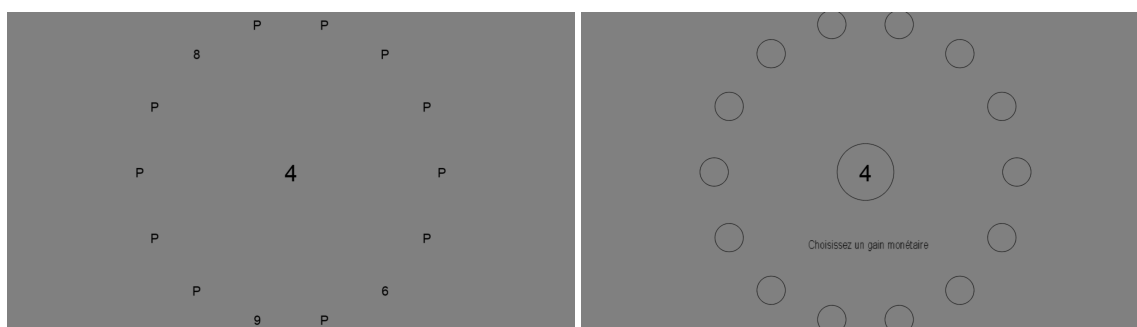
Table 3.C.2: Table of trial sequence across blocks

**Note:** Rows represent trial numbers within blocks. Column  $pos_x$  indicates the circle where the alternative  $x$  is located when available in the menu. Column Block  $Y$ , indicates the set of available alternatives in the Block  $Y$ .

### 3.C.3 Original Instructions (French)

#### DESCRIPTION DE L'EXPERIENCE

Cette expérience est constituée de 7 périodes comportant chacune 42 essais. Lors de chacun des essais, nous vous présenterons brièvement plusieurs gains monétaires à l'écran. Les valeurs des gains monétaires seront exprimées en ECU et pourront être égales à : 4, 6, 8 et 9 ECU. Un gain monétaire égal à 4 ECU sera toujours disposé au centre de l'écran. Les autres gains monétaires apparaitront autour, à des positions qui varieront à chaque essai. Chaque gain aura les mêmes chances d'apparaître à chacune des positions. Les positions non occupées par les gains monétaires représenteront des pénalités.



Pour choisir un gain monétaire, vous devrez ensuite cliquer sur le cercle qui apparaitra à la même position que le gain en question.

**ATTENTION :** En plus des gains monétaires, apparaitront des signes “ P ”, qui représentent une PENALITE. A chaque fois que vous vous trompez et cliquez à la position de l'un de ces signes, vous PERDREZ 15 ECU !

A la fin de tous les essais, **nous tirerons au hasard 30 essais** et vous recevrez la somme des gains et des pénalités que vous aurez choisis lors des essais sélectionnés.

Vos gains seront alors convertis en EURO selon le taux de change suivant : 20 ECU = 1 EURO.

Si vous faites un nombre trop élevé d'erreurs, le total de vos gains pourra alors être négatif. Il sera alors ramené à 0€. Cependant, même si vous n'arrivez pas à percevoir les gains monétaires, il est toujours possible de gagner un montant minimum d'argent égal à 6€ dans cette expérience. Le montant maximum réalisable est de 13€50.

#### Rappels Importants :

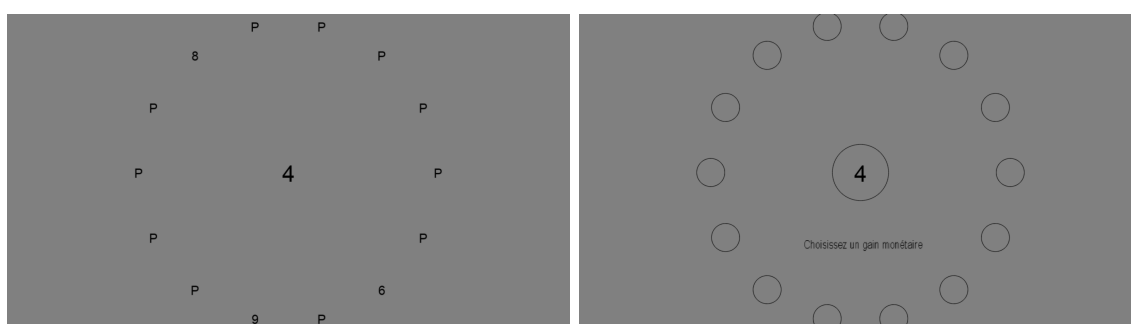
- 7 périodes de 42 essais.

- Le gain de 4 ECU est toujours au centre.
- Les gains de 6, 8 et 9 ECU ont les mêmes chances d'apparaître à chacune des autres positions.
- Cliquer sur une pénalité vous fait perdre 15 ECU.
- Paiement basé sur 30 essais sélectionnés au hasard.

### 3.C.4 Translated Instructions (English)

#### EXPERIMENT DESCRIPTION

This experiment is composed of 7 periods. Each period is composed of 42 trials. During each trial, several monetary rewards will briefly appear at the screen. Their values will be expressed in ECU and may be equal to: 4, 6, 8 and 9 ECU. A 4 ECU reward will be always displayed at the center of the screen. The other rewards will appear around, and their locations will vary across trials. All rewards have the same chances to appear in any of the locations. The location with no reward will represent penalties.



To choose a reward, click on the circle that will appear at the same location.

**WARNING:** In addition to the rewards, “P” signs will appear. They represent PENALTIES. Each time you made a mistake and click on the location of one of those signs, you WILL LOSE 15 ECU!

At the end of all the trials, we will randomly select 30 trials and you will receive the sum of the rewards and penalties you have chosen during the selected trials.

Your earning thus will be converted into EURO according to the following exchange rate: 20 ECU = 1 EURO.

If you make too many mistakes, your total earnings can be negative. In this case the latter will be set at 0 €. However, even if you failed to perceive any reward it is still possible to earn a minimum of 6 € in this experiment. The maximum possible earnings are 13 €50.

#### Important reminders:

- 7 periods of 42 trials.
- A 4 ECU earnings always displayed at the center.
- 6, 8 and 9 ECU earnings have the same chances to appear in all the other locations.



- Click on a penalty make you lose 15 ECU.
- Final payment based on 30 randomly selected trials.

# General Conclusion

Due to the digital revolution and the explosion of information, and because attention is one of the roots of decision making, integrating limited attention in economics is both an empirical and theoretical challenge. This thesis attempted to contribute to this challenge borrowing insights and methods from psychology and cognitive sciences.

## Main contributions

In the first chapter, we found an order (recency) effect: decisions are closer to optimality when payoff information is presented last and the discriminability is higher when perceptual stimuli are presented last. Moreover, increasing payoff magnitude leads to more optimal decisions. Our results also confirm the robust result that individuals use “conservative criterion” instead of optimal decisions in this type of experiment: they rely too heavily on the perceptual stimuli rather than the payoff information.

The second chapter, by using an experiment based on a discrimination task allows to measure both the input (allocated attention, through response time) and the output (effective attention, through error rate) of the attentional allocation process as well as interaction with other variables. We applied this experiment to study the interaction between incentives, social preferences and attention allocation and identify a discrepancy in the expression of social preferences between a monetary elicitation method and an “attentional social dilemma”.

The last chapter of this thesis tested the internal validity of Brady and Rehbeck’s Random Consideration Choice Set Rule (RCCSR) (2016). In this chapter we showed that a slight generalization of their model was necessary and sufficient to allow the

construction of statistical tests of the axioms, to reveal both preference relation and to measure effective attention. In a laboratory experiment, we found that most of the participants behave in accordance with the model. Therefore, revealed preference theories under limited attention could be applied to various empirical situations. Moreover, the applications of this framework go beyond the scope of economic analysis and may also be useful for researchers in psychology. Indeed, the axiomatic approach allows the consideration sets and their underlying properties to be revealed from the observation of choices. Therefore the identification of the proper model is informative as regards the characteristics of the underlying attentional process.

### **Possible Extension**

The work presented in this thesis is only a starting point and a call for more research. Indeed, it is possible to consider several extensions and research avenues based on the different chapters.

Concerning the first chapter. Based on the observation that individuals use conservative decision strategies and rely too heavily on perceptual information rather than the payoff information, it could be possible to imagine a refinement of the signal detection optimal classifier benchmark, that integrates limited attention. The idea behind such a model is that individuals improve their decision strategies at the expense of their perceptual ability, due to limited attention. From this assumption, the ratio between the discriminability and the decision criterion of an optimal classifier with limited attention would reveal the ratio between the marginal costs of allocating more attention towards perceptual evidence and allocating more attention towards the payoff information.

Introduction of Signal Detection Theory or related frameworks into competition analysis could also be considered. One example would be to introduce a Drift Diffusion Model (DDM) into a competition model. Drift Diffusion Models (DDMs), introduced by Ratcliff (1978) are dynamic extensions of Signal Detection Theory, where individuals continuously gather noisy perceptual evidence and make a decision as soon as the accumulated perceptual evidence in favor of an alternative exceeds

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a threshold. Krajbich et al. (2014) advocates for the introduction of DDMs in Economic Theory. Indeed, DDMs present several qualities: they offer a good fit of both error rate and response time for perceptual and purchase decisions (Krajbich et al., 2012), they are neurally founded (Liu and Pleskac, 2011), they are mathematically equivalent to optimal speed-accuracy trade-offs (Bogacz et al., 2006) and thus are coherent with rationality hypothesis. DDMs offer the opportunity to integrate the time needed by the consumers to reach their purchase decisions into economic models. This time is costly both for the consumers (opportunity cost) and for the firms (because it requires more sale forces), regardless of the final purchase decision. Thus, a policy maker should be concerned by the time needed to make decisions. We could imagine a model where two firms compete (price and spatial location) in a linear Hotelling's spatial competition framework (1929) and face consumers making decisions through a DDM. Some intuitive conjectures about the market dynamic of the model can be drawn. On one hand, the linear transportation cost creates an incentive for firms not to differentiate their product as in Hotelling (1929). On the other hand, there is an incentive to differentiate, since consumers' response time is costly for the firms and decreases with product differentiation. Therefore, at the equilibrium, the degree of differentiation would be the result of a trade-off between these two forces.

Drift Diffusion Model can also be used to analyze the data obtained from the second chapter's experiment. The advantage of a DDM-based data analysis is the estimation of the different model parameters that can be interpreted: the drift rate (speed of the evidence accumulation, i.e. the efficiency of the attentional allocation process in our case); the threshold (the minimal perceived difference to make a decision); the noise of the process and the decision time (the time needed to implement the choice). This latter measure of decision time is important since in our current analysis, we cannot disentangle the time dedicated to process information and the time dedicated to implement the decision. Using such a framework could thus bring new insights and confirm the discrepancy we highlighted between the "monetary elicited" and the "attention elicited" social preferences. Since we believe in the importance of this discrepancy, we recognize that it needs to be confirmed.

Indeed, as discussed in the second chapter, there are several explanations for this discrepancy that our actual design cannot rule out. Built upon the work presented in the second chapter, a series of additional experiments should be conducted to disentangle these potential explanations.

It is possible that the origin of the discrepancy between monetary and attentional social preferences comes from a discrepancy in the beliefs about others' behaviors in situations where attention or money is invested to increase others' welfare. In this case, identical social preferences in the two environments can lead to different behaviors in the SVO and the dustbin task. This hypothesis would be addressed through a simple incentive compatible belief elicitation procedure. A moral compensation phenomenon could also explain our null result as the demand for self-image (and thus pro-social behavior) in the dustbin task may diminishes more for the participants who acted pro-socially in the SVO test (performed before the task) if compared to those who acted individualistically. A between-participant manipulation of the order between the SVO and the dustbin task could be implemented to test this particular hypothesis: the expression of moral compensation should result in larger contributions in the SVO (or in the attentional dilemma), when played first.

If as we hypothesize, the discrepancy remains and the expression of social preferences depends on the nature of the invested resource, the “dustbin task”, proposed in the second chapter, or more precisely a slightly modified within-participant version, could be used as a measure of *attentional social preferences*. Such a measure may be more adapted if compared to standard elicitation of social preferences through monetary allocation, when the researcher wants a predictor of pro-social behaviors in a context where the effort is linked to attention (for example in a field experiment on energy consumption or on waste recycling, where the pro-social behavior consumes mainly individual attention).

Concerning the last chapter, we only tested the internal validity of the model in a particular environment, designed for this particular purpose. External validity is another required quality to qualify a model as good. To assess external validity, it is possible either to directly test the model on more ecological situations or to stress the experimental conditions to understand the model's scope of application.

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An increase in the number of available alternatives responds to these two possibilities. For experimental convenience, we restricted the model and tested a version with  $N = 3$  (plus a default) alternatives. However, models of limited attention should be able to describe ecological situations with many alternatives. Similarly, we only tested the characteristics of our statistical test with  $N = 3$ . Increasing the number of available alternatives makes the full characterization of the model very difficult, since the number of possible menus explodes exponentially. We shall therefore work to develop alternative statistical tests that does not require observing choices in all the menus.

Another characteristic that we controlled was the attention participants may invest in our task. External validity would suggest that a consumer can somehow choose the quantity of attention to allocate to a decision. In our task, this was not possible. Moreover, the fast disclosure of stimuli implies that the attentional processes involved are mostly stimuli-driven. Relaxing the time constraint would allow us to provide a more realistic environment and to investigate the limits in the applicability of the model. For this purpose, it is possible to slightly adapt our experimental design in order to elicit choice process data as in (Caplin et al., 2011, exp.2). In such a design, the alternatives would be displayed longer on screen and participants would be allowed to change the selected alternative whenever they want. The decision implemented for the payoff computation being the selected alternative at a randomly drawn point of time. Caplin et al.'s procedure (2011) thus reveals in an incentive compatible way the choice function at any point of time and we could assess model validity as a function of search time.

Moreover, the RCCSR should be compared with other classes of models. In particular the class of random utility models, or the class of rational (in)attention models. Other models, based on consideration set and related to the RCCSR could also be compared (Aguiar, 2017; Echenique et al., 2018). Indeed Aguiar's model (2017) also reveals consideration set but with a different property than the RCCSR and it would be interesting to investigate its correspondence with the attentional measure revealed by a RCCSR. Echenique et al.'s model (2018) reveals the order in which alternatives were considered. This comparison can be done in two-ways.

First, we can investigate, with simulations, the “empirical similitude” between the RCCSR and other classes of models. For this purpose we can generate choices following the tested models and use the statistical procedure we developed to test the proportion of the tested models that pass the test of RCCSR. The converse would require developing an appropriate test for each of the tested model. Second, we should develop a procedure similar to the one described in the last chapter of this thesis for other models.

This thesis highlighted the importance of limited attention in economics. This importance is already well-incorporated in the strategy of the firms competing for the increasingly scarce attention of consumers. While economists are expressing a great interest for these questions, integrating limited attention in economic theory represents an important challenge and is still at an early stage. Indeed, the existence of conscious goal-driven and unconscious stimuli-driven attentional processes hinders the emergence of an unifying and consensual theory. However, such a theory could be a suitable candidate to unify behavioral economics in a near future. Indeed, limited attention may account for many phenomenon described by separated theories (e.g. prospect theory, hyperbolic discounting) (Gabaix, 2019). We are supportive of this view and believe that attention could and should be treated by economists as a notion that is as important as preferences or beliefs. In the meanwhile, the integration of more methodological and theoretical insights from cognitive sciences and psychology allows us to measure the input and output of the attention allocation process. This is a necessary step to understand attention allocation and to identify the properties required for proper limited attention models.

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# À la recherche des remerciements parfaits: Le grand jeu concours des remerciements de thèse

## Résumé

Ce chapitre bonus propose d'étudier et d'apporter une réponse au problème des remerciements de thèse parfaits, ou optimaux. Après une brève définition des remerciements de thèse et une étude des pratiques standards, nous proposons un peu n'importe quoi.

## Introduction: note sur les remerciements optimaux.

Il est communément admis que les remerciements de thèse (RDT ci-après) constituent généralement la section la plus lue, la mieux appréciée et comprise de toute thèse de doctorat. C'est bien souvent la seule, appliquons-nous donc.

La rédaction de RDT consiste à choisir un texte (i.e. une "suite de signes linguistiques constituant un écrit ou une oeuvre"<sup>1</sup>) potentiellement véhiculeur de sens mais non relié au contenu scientifique de la thèse de doctorat (quelques clins d'oeil ont cependant déjà été observés), dans le but de satisfaire le lecteur que ce soit par l'usage de la flatterie ou de la commémoration d'événements et d'anecdotes heureuses. Dans la majorité des cas, le texte est choisi librement, sous contraintes de place, de temps, d'imagination, et quelquefois de courage. En dépit de l'immensité de l'ensemble des textes rédigeables, nous défendons dans ce chapitre la thèse selon laquelle les RDT ne diffèrent qu'en réalité assez peu, ni dans le fond, ni dans la forme.

Comme très bien enseigné dans toute faculté d'économie qui se respecte (et également à Nice), toute action est entreprise dans le but de maximiser l'*utilité* individuelle. Ainsi, le doctorant cherchera à minimiser l'impact négatif que pourra avoir un potentiel lecteur de ses RDT sur sa vie future. Cependant, écrire des RDT optimaux implique un problème de dimensionnalité et de complexité élevé, qui résiste encore à la communauté scientifique. Aucune solution analytique n'a

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<sup>1</sup>Centre National de Ressource Textuelles et Lexicales, définition de texte. <https://www.cnrtl.fr/lexicographie/texte>. Dernière consultation: 4 octobre 2019



pu être apportée à ce jour et la formulation d’une réponse explicite à ce problème reste l’une des questions mobilisant le plus les doctorants en fin de thèse.<sup>2</sup>

Comme il a possiblement pu en être vaguement question dans cette thèse dont vous n’allez certainement lire que les remerciements, les individus disposent d’une attention limitée qui les empêche de proposer des RDT optimaux. Néanmoins, il est assez facile de comprendre et de prédire, par le biais de la *statique comparative* du modèle de RDT “standard”, certaines régularités. Plus précisément, dans un but de se rapprocher de l’optimalité, les RDT standards remercient le plus tôt possible dans le texte les personnes qui ont à la fois le plus de chances de tomber un jour sur la thèse et qui ont la plus grande amplitude dans leur capacité de nuisance ou de bienfaisance vis à vis du futur docteur (certains lecteurs possédant même le pouvoir d’annuler des soutenances).<sup>3</sup>

Ces importants résultats théoriques ont également été confirmés empiriquement. En effet, d’après les résultats d’une méta-étude sur les RDT standards réalisés à la MSHS de Nice entre 2014 et 2019, il est d’usage de remercier dans l’ordre : 1) les rapporteurs pour avoir accepté de rapporter la thèse du doctorant, 2) les examinateurs pour avoir accepté d’examiner la thèse du doctorant, 3) les directeurs de thèse pour avoir accepté de diriger la thèse du doctorant, 4) le laboratoire d’accueil pour avoir accepté d’accueillir le doctorant. Assez régulièrement, le doctorant se permet quelques folies et une pointe d’originalité lorsque vient le tour de la famille qui a accepté de bien élever le doctorant ou d’un certain nombre de personnes qui ont accepté d’aller boire des coups avec le doctorant, mais pas trop quand même.<sup>4</sup>

En effet, certaines mythologies populaires ont introduit l’idée que des RCT optimaux devaient également être originaux, voire burlesques.<sup>5</sup> Dans tous les cas, l’idée sous-jacente est que davantage que l’ordre et l’identité des personnes remerciées, le faire brillamment augmenterait la probabilité

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<sup>2</sup> Formellement, le doctorant rationnel choisit parmi l’ensemble  $\mathcal{L}$  des textes possibles (intégrant les contraintes susmentionnées), un texte  $L$  (l’unicité n’ayant pas été prouvée) afin de maximiser la fonction suivante:

$$\max_{L \in \mathcal{L}} \int_{t=0}^{\infty} \left( \sum_{x \in X} f(x, t) \sum_{l \in L} g(x, l, t) \times A(x, l, t) \right) dt, \quad (3.21)$$

où  $X$  est l’ensemble des personnes qui existent ou existeront,  $f(x, t)$  la probabilité qu’une personne  $x \in X$  tombe sur la thèse à la date  $t$ ,  $g(x, l, t)$  la probabilité que le lecteur  $x$  lise la ligne  $l$  des remerciements à la date  $t$  et  $A(x, l, t)$  la valeur actualisée de l’utilité que retirera le doctorant si la personne  $x$  lit la ligne  $l$  à la date  $t$ , c’est à dire:

$$A(x, l, t_0) = \int_{t_0}^{\infty} \exp(-\rho t_1) \times a(x, l, t_0, t_1) dt_1, \quad (3.22)$$

avec  $a(x, l, t_0, t_1)$  l’impact (positif ou négatif) de l’action qu’effectuera envers le doctorant la personne  $x$  à la date  $t_1$ , après avoir lu la ligne  $l$  à la date  $t_0$ .

<sup>3</sup> Le résultat est immédiat en partant du principe que (1) une personne lisant des RDT aura une action d’autant plus négative sur le doctorant qu’elle est citée tardivement (car cela l’oblige de lire beaucoup de lignes et d’avoir l’impression d’être moins importante que les personnes précédemment remerciées), l’action minimale étant généralement atteinte en cas d’oubli (i.e.  $\min\{A(x, l, t)\} \Leftrightarrow x \notin l$  et  $\frac{dA(x, l, t)}{dl} < 0$  si  $x \in l$ ); (2) la probabilité de lecture de tout texte est une fonction décroissante de la ligne à laquelle il apparaît (i.e.  $\frac{dg(x, l, t)}{dl} < 0$ ).

<sup>4</sup>Un habile exemple de RDT standards a été discrètement glissé au début du manuscrit, sauras-tu le retrouver ?

<sup>5</sup>Et à défaut de l’être, l’auteur peut également s’essayer à un style pompeux sur deux bonnes pages et demi afin d’impressionner – s’il en est – des lecteurs moins lettrés que lui. Il s’en excuse pour les autres.

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de lecture et la positivité des actions futures entreprises envers le doctorant. Cependant, si nous n'évacuons pas l'hypothèse que d'autres doctorants aient pu oser sortir du carcan des RDT standards, il n'existe malheureusement que peu de traces de ces exercices de style et nous n'avons connaissance d'aucune recherche ni de méta-étude sérieuse sur le sujet. Plus précisément, l'effet (positif ou négatif) de la publication de RDT originaux sur la carrière future d'un candidat n'a jamais été étudiée.<sup>6</sup>

Les présents RDT apportent, humblement, une réponse à l'ensemble des problèmes soulevés. Premièrement, ils proposent de sonner le glas du manque d'originalité des RDT standards. Deuxièmement, ils apportent une solution au problème des RDT optimaux en ambitionnant d'être les meilleurs RDT jamais rédigés jusqu'alors. Troisièmement, ils ouvrent un vaste programme de recherche qui permettra de combler le manque de connaissance sur les RDT originaux et leur influence sur les carrières de leurs auteurs. Les présent RDT concluent ainsi cette thèse et constituent la pièce maîtresse de ce manuscrit.

## **Méthode: Le grand jeu concours des remerciements de thèse.**

Afin de réaliser les délires de son auteur, les présents RDT proposent simplement d'inaugurer "Le concours des meilleurs remerciements de thèse de Université Côte d'Azur". Ce nouveau concours référencera et élira chaque année les remerciements les plus originaux, drôles, émouvants, courageux et/ou bien écrits. Il y aura peut-être même des trucs cool à gagner. Bien évidemment, les présents RDT sont destinés à être les gagnants de la première édition de ce nouveau concours. Pour ce faire, nous décidons d'introduire dès la prochaine page un nouveau genre de remerciement participatif et extensif, supérieur en tout point aux RDT standards : "le grand jeu concours des remerciements de thèse" (GJCDRDT, ci après).<sup>7</sup>

Ainsi, en lieu et place de RDT standard, l'auteur propose (1) d'organiser un grand jeu concours auquel pourront participer ses lecteurs (le GJCDRDT) et (2) de créer (et d'en être le premier vainqueur) un méta-jeu concours, chargé d'élire chaque année les meilleurs remerciements. Balaise le mec. Je sais pas vous, mais perso, je trouve ça impressionnant. Bref, le règlement du GJCDRDT

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<sup>6</sup>Par exemple, certaine rumeurs mentionnent l'existence de "non-remerciement de thèse", consistant à explicitement "ne pas remercier" les "personnes qui ne méritent pas d'être remerciées" (ironiquement, ou non). Malgré tous nos efforts, nous n'avons pu nous les procurer.

<sup>7</sup>Un premier avantage du GJCDRDT par rapport aux RDT standards provient de son utilisation explicite de la méthode dite de "randomisation de l'ordre de remerciement" qui permet aux remerciés de ne jamais se vexer de l'ordre d'apparition et permet de créer de l'attente et de la surprise chez le lecteur. (Une technique similaire est celle du "remerciement par ordre alphabétique explicite". Le remerciement par ordre alphabétique explicite est cependant moins efficace, car il fatigue de manière injustement systématique les personnes dont le nom est à la fin de l'alphabet qui développent une certaine... on pourrait ainsi préférer la technique du "remerciement par ordre alphabétique inversé explicite", mais si tout le monde s'y met, on inverse juste le problème). Un second avantage est que le GJCDRDT est un RDT "participatif" (également appelé RDT inversé), qui maximise ainsi l'attention du lecteur en le transformant en réel acteur des remerciements. Enfin, un dernier avantage est que le GJCDRDT est une technique de RDT dite "extensive", qui permet au lecteur de se rajouter en cas d'oubli de la part de l'auteur, ou de se remercier à plusieurs reprises et un peu partout.

est donné ci-après. Le règlement de l'autre restera à définir, en tout cas, il récompensera chaque année les meilleurs RDT écrits avant le 22 novembre. Merci pour l'hommage.

## Règlement du GJCDRDT (jeu gratuit sans obligation de lire de la thèse)

### Principe du GJCDRDT.

Un long texte, remerciant plein de gens, de manière assez personnelle et ridicule parfois, a été écrit, mais les noms des personnalités initialement remerciées ont été remplacés par des *pointillés*. C'est-à-dire des points successifs, légèrement espacés, ressemblant assez à un truc comme ça :

“ . . . . . ”

Le jeu consiste simplement à remplacer, dans votre bulletin de participation, les *pointillés* par le nom de la personnalité qui – selon vous – correspondrait le mieux, sachant que les réponses correctes vous rapporteront des points.

Afin de vous faciliter la tâche, la *liste des personnalités initialement remerciées* est communiquée en entête du bulletin de participation. Une même personnalité peut correspondre à plusieurs emplacements. Afin de vous compliquer la tâche, et pour que le jeu ait un quelconque intérêt, l'ordre selon lequel les personnalités sont remerciées et l'ordre selon lequel les personnalités figurent dans la *liste des personnalités initialement remerciées* ont tout deux été mélangés, sans le contrôle d'un huissier.

Afin d'éviter tout favoritisme dans l'évaluation de vos réponses, les bulletins de participations seront anonymisés grâce à un identifiant unique de participant (IUP) qui sera indiqué sur chacune des feuilles de votre bulletin. Afin de favoriser le favoritisme dans l'évaluation de vos réponses, je demeure l'unique juge de la validité de vos réponses et je saurais reconnaître vos écritures.

### Décompte des points.

Les points sont comptabilisés ainsi :

- Chaque fois qu'une personnalité sera correctement placée à un endroit où “je suis d'accord, c'est la personnalité qui correspond le mieux”, **vous gagnerez dix points.**
- Chaque fois qu'une personnalité sera mal placée à un endroit où “j'avoue, cette personnalité correspond pas mal aussi”, **vous gagnerez cinq points.**
- Chaque fois qu'une personnalité sera mal placée à un endroit où “j'avoue, c'est encore plus drôle de placer cette personnalité ici!”, **vous gagnerez quinze points.**

Vous pouvez également décider **de changer la liste des personnalités à remercier**, si vous pensez qu'une personnalité mériterait plus d'y figurer qu'une autre. Attention, cette stratégie est toutefois risquée (mais peut vous rapporter gros !).

- 
- Chaque fois que vous retirerez une personnalité de la liste des personnalités à remercier, mais que “j’avoue j’étais pas obligé de la remercier !”, **vous gagnerez cinq points.**
  - Chaque fois que vous ajouterez une personnalité qui n’est pas présente dans la liste et que “j’avoue comment ai-je pu oublier de la remercier !?”, **vous gagnerez vingt points.**
  - Chaque fois que vous ajouterez une personnalité qui n’est pas présente dans la liste mais que “j’avais justement fait ce jeu pour ne pas avoir à la remercier !”, **vous perdrez trente points.**
  - Chaque fois que vous retirerez une personnalité de la liste des personnalités à remercier, mais que “sans cette personnalité, ce doctorat n’aurait pas été le même !”, **vous perdrez trente points.**
  - Chaque fois que vous retirerez une personnalité de la liste des personnalités à remercier, mais que “je trouve que cette personnalité a plus sa place dans cette liste que vous !”, **vous perdrez cent points.**

#### **Points bonus:**

Une zone de création libre est également mise à votre disposition.

- En fonction de la manière dont vous utiliserez cette zone, **vous pourrez gagner jusqu’à cinq cent points** sur décision souveraine du grand jury du GJCDRDT (moi).

J’ai “volontairement” glissé des fautes d’orthographe, de grammaire ou de conjugaison tout au long de ces remerciements (environs 187 fautes au total).

- Chaque fois que vous identifierez, reporterez et corrigerez correctement une faute, **vous gagnerez un dixième de point.**
- Chaque fois que vous reporterez abusivement une faute d’orthographe qui “en réalité, n’en est pas une”, **vous perdrez cinq points.**

Vous l’aurez compris, il est donc théoriquement possible pour vous d’écrire de meilleurs remerciements que la version originale de mes propres remerciements (la solution). Dans le cas où l’un des participants proposerait de meilleurs remerciements que moi, je m’engage à les remplacer lors du dépôt final de la thèse (trois mois après la soutenance).

#### **Modalités de participation, lots à gagner et désignation des vainqueurs**

Pour participer au jeu, rien de plus simple (à part peut-être ne pas participer au jeu) ! Il suffit de remplir le bulletin de participation et de retourner votre version complétée des remerciements avant la fin de la soirée de soutenance, le vendredi 22 novembre 2019 à 23h59, directement sur place pour mes vrais amis qui sont venus me soutenir (une urne ou un truc du genre sera mis à disposition) ou alors par mail pour ceux qui croient vraiment avoir une chance de gagner alors qu’ils ne sont même pas venus me soutenir : [grandconcoursdesremerciements@gmail.com](mailto:grandconcoursdesremerciements@gmail.com)

## Supplementary Acknowledgment

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Les participants seront classés en fonction du nombre de points obtenus et les gagnants seront les participants ayant obtenu le plus de point. En cas d'égalité entre un ou plusieurs participants, un tournoi de "shifumi" (pierre-papier-ciseaux) en 3 manches gagnantes sera organisé afin de les départager. Tout usage du puits entraînera une disqualification immédiate. Le classement et les gagnants du grand jeu concours des remerciements de thèse seront rendu publiques au plus tard une semaine après la soutenance. Les lots à gagner sont les suivants:

- Gros lot: Un week-end (2j/1n) "all inclusive" avec moi dans un pays d'Europe (transport, logement, repas et boissons pris en charge) ;
- Moyen lot: Un dîner avec moi, dans un bon restaurant ;
- Petit lot (de participation) : Une photo (de charme) de moi dédiée par moi.

Bonne chance, et attention, il y a beaucoup de pièges !

## Annexe: Bulletin de Participation au Grand Jeu Concours des Remerciements de Thèse

Nom du participant / de la participante: . . . . .

Courriel du participant / de la participante: . . . . .

Identifiant unique de participation (IUP) (ne pas remplir) . . . . .

### Liste des personnalités à remercier:

Martine — Patrice — Cathy C. — La Bouhleb — Cath L. — Sara — Boutzero — Francesco — Grand Alex — Anthony — Nabila — Alain — Maxime — Natalia — Mehdi — Marie — Nicolitto — Mon papa — Cheikh — Luffy — André Gaufrier — Pompidou — Agnès M. — Agnès F. — ma conseillère pôle emploi — Ma maman — Joe — Beer district — Adil Mitroglou — Cécile Pajova — Souleiman — Megane — Camille — Alexander — Psychanopé — Rizlaine — Benjamin A. — Sihem — Elise R. — Dino — Thomas B. — Muriel — Morgane — Romane — Ankinée — Flo N — Shadi — Noémie — Guillaume C. — Micka — Paul ? — Sami — Driss — Delia — Megalol — Svetlana — Victor — La Gioff — Guilhem — Alexandre F. — Dams — Le canapé du LAPCOS — Paolo — Le lac de Côme — Max — Waël — Vincent L. — Jean-Charles — Duy Vu — Zack — La #teamMSHS — Flora — Benjamin M. — Aissam — Aurore — La vie — Candice — Joslem — Tom — Ali D. — Nahid — Nicolas B. — Robinson — Mustapha — Gautier — Nino — Eric N. — Didier — Dav — Zizi — Mahmoud — Henri — Laurene — Vivien — Paul B. — Un poulet — Claire L. — Dodo — Selim — Le Grooving — Jazz — Abdias — Hanane Ramzouille — Elise ? — Laura L. — Dominique T. — L'école Doctorale DESPEG — Mira — Eric G. — Elise M. — Stéphanie — Sébastien D. — Petit Alex — Christophe C. — Aurélien — Cyrielle — Savère — George — David fdp — Blaise — Sonia — Céline — Sophie — Loubna M. — Sébastien M. — Clara — Brice — Roza — Margot — Cyril C. — Loubna E. — Sabine — Ginette — Raphaëlle — Maëlle — Jeremie B. — Patrice R. — "Tata" Laurence — Vincent — Lise — Gabi — Nathalie L. — Petit chat des bois — B06 — Giuseppe — Le LEEN — Ismael Jr — Cyprien — Mejdj — Vincent C. — Alexandra — Michela — Aymeric — Cyril A. — Nathalie P. — Thomas G. — Emma — Laura C. — Anaïs — Stéphane — Françoise — Tania — Alexia — Manuel — Claire A. — Guillaume D. — Jerem — Lou — Ibuanga — Séverine — Miriam — Olivier — Nobi — Paolo Z. — Lisa — Nathalie A. — Fabien M. — Karine —

### Personnalités oubliées:

. . . . .  
. . . . .  
. . . . .  
. . . . .  
. . . . .

**Remerciements**

- Merci monsieur le directeur . . . . . pour les blagues, les bouteilles cachées dans ton bureau et ton formidable jeu d'acteur.
- Merci . . . . . d'avoir fait revivre le groupe facebook jusqu'à sa mort. J'ai honte, mais j'avoue : je suis forcé de rire face à quelqu'un qui fait caca dans le métro !
- Merci . . . . . pour le soutien et la bienveillance tout au long de la thèse. Désolé de t'avoir bousculé le temps d'une mauvaise impro !
- Merci à mes frères et soeurs de sang . . . . ., . . . . . et . . . . .  
. . . . ., pour tout votre amour. Désolé pour les fois où je vous ai embetté quand nous étions petits!
- Merci . . . . . pour ton soutien et tes conseils dans ce moment difficile et pour avoir préféré, au fond de toi, te réveiller dans une autre ville, faisant naître mon amour pour . . . . .
- Merci à . . . . . pour cette très ancienne amitié et ta situation actuelle qui boost un peu ma street crédibilité.
- Merci . . . . . pour ton attitude "toujours digne" en soirée, tes pâtes et spiritueux délicieux. Merci de m'avoir fait chevalier de la carbo, c'est certainement mieux que d'être docteur ! Bref, quel honneur pour moi d'être ton premier "doctifié"! Quel malheur pour toi de commencer par le meilleur et d'être condamner à ne plus participer qu'à des jurys sans saveur !
- Merci à tout le personnel administratif de l'ISEM, en particulier : . . . . ., . . . . ., . . . . ., . . . . ., . . . . ., . . . . ., . . . . .
- Merci . . . . . pour les bons délires avec les "copains" ! Pour le grain de folie que tu sais mettre quand tu fais la bringue et bravo d'avoir élevé une âme aussi pure que celle de . . . . ., c'est lui le plus beau !
- Merci à . . . . ., d'avoir transformé un bâtiment de recherche en une vraie maison, d'incarner l'interdisciplinirité, le partage et le mélange, jusqu'à devenir un famille ch'ti à l'accent du sud !
- Merci . . . . . pour l'inspiration et la douceur de vivre.
- Merci . . . . ., pour tes répliques supérieures à celle d'Audiard (j'espère notamment aller en enfer...), merci pour ton sens de l'honneur, tes conneries. Merci car sans toi je ne serais peut être jamais venu à Nice et n'aurai pas pu écrire les meilleurs





- Merci à . . . . . pour ta sympathie et la soirée aux cabinets !
- Merci . . . . . pour ton nindo et ta conception de l'amitié, pour avoir suivi mes aventures sentimentales et écouté mes états ou vagues à l'âme quand j'en avais, pour m'avoir suivi dans mes folies nocturnes, pour les soirées à l'appart, les afters, les fioles, les 32, les mcdo, la table en noix de cajou, les soirées foot, les nouveaux ans chinois, et tes exs. Et merci pour le soutien aussi et les restes de pâtes qui m'ont évité de mourir quand je travaillais tellement que je ne me faisais plus à manger ! Merci pour les bons plats. Bref t'es le meilleur mec !
- Merci aux Lapcouz de m'avoir laissé squatter votre salle, en particulier : . . . . .  
. . . , . . . , . . . , . . . , . . .  
. . . , . . . , . . .
- Merci à la colloc d'Arcole (et mon co-vacataire !) pour m'avoir fait prendre gout à la vie en communauté : . . . . . , . . . . . , . . . . . , et . . .  
. . . . .
- Merci . . . . . pour cette "amitié secrète" !
- Merci à l'équipe du . . . . . : . . . . . , . . . . . , . . . . . , . . . . . , . . . . . , . . . . . , . . . . . , . . . . . , . . . . . , . . . . .  
. . . . . pour les flechettes, les jeux de sociétés et les réductions ! Et même merci à . . . . . , pour m'avoir fait retrouver le plaisir d'avoir un ennemi et de pouvoir lui souhaiter du mal !
- Merci à . . . . . d'être l'homme le plus lent, merci à . . . . .  
. . . d'être le plus beau, à . . . . . d'être le plus dynamique, à . . . . .  
. . . . . d'être le plus gentil, à . . . . . d'être le plus long et à . . . . .  
. . . . . d'être le plus rêveur. Merci à vous tous pour cette amitié inchangée malgré ces quatre cinq dernières années sans trop donner de nouvelles ! Merci pour toutes nos aventures, nos voyages un peu partout (mais souvent barcelone !), pour toutes ces fois où on a quand même un peu risqué nos vies... C'est grace à vous que je suis devenu une machine de guerre !
- Merci . . . . . pour toute la nourriture que tu nous as donné et de penser toujours aux doctorants ! Merci pour cette folle soirée qui aura fait naître tant de rumeurs. Ne bois pas trop aujourd'hui !
- Merci à . . . . . et . . . . . d'avoir supporté ma phobie administrative, pour m'avoir quand même aidé et permis de toujours m'inscrire en thèse, ou rembourser mes missions, même hors délais.

- Chantilly, Cacahuète ! Chantilly, Cacahuète ! Merci . . . . . d'avoir décidé de ne jamais décider et de rire de ton absence d'humour. Merci pour ton amitié précieuse, tous les messages et les chorégraphies super motivantes ! Pour les moments passés en pause, en soirée, ou en afters à l'époque et les questions parfois gênantes (ou peut être juste ennuyantes ? :p ). Merci de t'intéresser à la vie de tes amis. Et merci d'avoir mis le "game" dans une autre dimension !
- Merci au "Groupe du Groupe" : . . . . . , . . . . . , . . . . .  
. . . . . , . . . . . , . . . . . , . . . . . et . . . . .  
. . . . . Merci de m'avoir prouvé que passé 18 ans, c'était encore possible de s'amuser sans alcool !
- Merci . . . . . pour la compagnie à la colloc, les bières, les pétards, les découvertes musicales et aussi les punaises de lit !
- Merci aux anciens du . . . . . ( . . . . . , . . . . . ,  
et . . . . . ) pour m'avoir tout de suite accepté dans le meilleur des bureaux et m'avoir fait bénéficier de votre expérience : je n'ai eu qu'à suivre votre voix !
- Merci monsieur l'éminent professeur . . . . . , pour m'avoir fait l'honneur de votre compagnie lors des dernières semaines de ma première dernière année de thèse. Merci pour la soirée CocoLab, merci pour la voiture et désolé pour les amendes, j'aurais tout fait pour que la police vienne te chercher !
- Merci . . . . . de toujours tout dégommer ! Merci pour les disputes à cause des retards et annulations au foot: il n'y a pas de vraie équipe sans embrouille de vestiaires !
- Merci . . . . . pour les soirées en début de thèse avec la version 1 !! Pour tes retours incessants et la folie qui t'accompagnait !
- Merci à . . . . . d'être venu me voir et de m'avoir fait me rendre compte, sandwiches aidant, que rap et recherche, c'était le même milieu !
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Espace libre de remerciation créative:

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## Résumé : Prise en Compte de l'Attention Limitée dans l'Analyse Économique

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Cette thèse contribue à la prise en compte de l'attention limitée dans l'analyse économique. Nous défendons l'idée que les processus d'allocation de l'attention peuvent être étudiés à travers un processus de production avec en input l'*attention allouée* (la quantité de ressources attentionnelles investies dans la décision), et en output l'*attention effective* (la quantité d'information contenue dans la décision). Afin d'améliorer la compréhension de ces processus, nous proposons trois essais empruntant des méthodes à la psychologie et aux sciences cognitives. Dans le premier chapitre, nous manipulons l'ordre de présentation entre une information sur les incitations et un stimulus visuel, dans un paradigme de *choix forcé à deux alternatives*. L'*attention allouée* y est contrôlée, et nous mesurons l'*attention effective* à l'aide d'un modèle de *détection du signal*. Nous montrons que la dernière information présentée a un plus grand poids dans la décision et attribuons cet effet à une division de l'attention. Le second chapitre propose une expérience dans laquelle les participants allouent une attention coûteuse, afin de réduire l'incertitude d'une *tâche de discrimination*. Ainsi, nous mesurons à la fois l'*attention allouée* (par le biais du temps de réponse) et l'*attention effective* (par le biais de la performance). Cette expérience nous permet d'étudier les *dilemmes sociaux attentionnels* (situations où l'attention est coûteuse pour l'individu mais bénéfique pour le groupe) et de mettre en lumière une divergence entre les préférences sociales – mesurées traditionnellement par des choix d'allocations monétaires – et les comportements observés dans notre dilemme social attentionnel. Le dernier chapitre prouve qu'il est possible d'implémenter empiriquement et de tester la validité d'un modèle de *préférences révélées avec attention aléatoire*. Nous proposons une nouvelle caractérisation et un nouveau théorème des préférences révélées dans le cadre d'une version plus générale du modèle de Brady et Rehbeck (2016, *Econometrica*). Nous développons des procédures statistiques – que nous analysons à l'aide de simulations numériques – afin de tester les axiomes du modèle, de révéler les préférences, et d'obtenir une mesure de l'*attention effective*. Nous testons la validité du modèle à l'aide d'une tâche d'*attention sélective* dans laquelle les participants choisissent un gain monétaire parmi un ensemble de distracteurs. Les comportements observés dans cette expérience sont cohérents avec le modèle et les préférences induites expérimentalement.

**Mots-clés : Économie Comportementale ; Économie Expérimentale ; Rationalité Limitée ; Attention limité ; Allocation de l'attention ; (In)Attention Rationnelle ; Ensembles de Considération ; Préférences révélées ; Préférences sociales ;**

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## Abstract: Integration of Limited Attention in Economic Theory

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This thesis contributes to the integration of limited attention within the economic theory. We argue that attentional allocation processes can be understood as a production process with the *allocated attention* (the quantity of attentional resources invested in a decision) as an input and the *effective attention* (the amount of information contained in that decision) as an output. Borrowing methods from psychology and cognitive sciences, we propose three essays to shedding light on these processes. In the first chapter, we manipulate the presentation order between reward information and perceptual evidence in a *two-alternative forced-choice task*. The *allocated attention* is controlled, and we measure *effective attention* with a Signal Detection model. We found that the last information presented is more weighted in the decision. We attribute this effect to the division of attention. The second chapter proposes an experiment where participants pay costly attention to reduce the uncertainty of a *discrimination task*. We measure both *allocated attention* (through the response time) and *effective attention* (through performance). This experiment allows the study of *attentional social dilemmas* (situations where attention is costly for individuals but beneficial for the group). We highlight a discrepancy between *monetary elicited social preferences* and the behaviors exhibited in our attentional social dilemma. The last chapter proves that a model of *revealed preferences under stochastic attention* can be implemented and tested empirically. We provide new characterization and revealed preference theorems for a general version of Brady and Rehbeck's model (2016, *Econometrica*). We propose and analyze – with numerical simulations – statistic procedures to test the axioms, to reveal preferences, and to measure *effective attention*. We test the internal validity of the model with a *selective attention task*, where participants choose an alternative among distractors and we find that most of the participants behave in accordance with the model and reveal coherent preferences.

**Keywords: Behavioral Economics; Experimental Economics; Bounded Rationality; Limited Attention; Attention Allocation; Rational (In)Attention; Consideration Set; Revealed Preferences; Social Preference**