



Numerical methodologies for topology optimization of electromagnetic devices

Yilun Li

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Sorbonne Université
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Ecole Doctorale Science Mécanique Acoustique Electronique et Robotique

Laboratoire d'Electronique et Electromagnétism (L2E)

**Numerical Methodologies for Topology Optimization of
Electromagnetic Devices**

Par Yilun LI

Thèse de doctorat de Electronique

Dirigée par Zhuoxiang REN, Shiyu YANG

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Dédicace

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Résumé

L'optimisation de la topologie est la conception conceptuelle d'un produit. En comparaison avec les approches de conception conventionnelles, il peut créer une nouvelle topologie, qui ne pouvait être imaginée à l'avance, en particulier pour la conception d'un produit sans expérience préalable ni connaissance. En effet, la technique de la topologie consistant à rechercher des topologies efficaces à partir de brouillon devient un sérieux atout pour les concepteurs. Bien qu'elle provienne de l'optimisation de la structure, l'optimisation de la topologie en champ électromagnétique a prospéré au cours des deux dernières décennies. De nos jours, l'optimisation de la topologie est devenue le paradigme des techniques d'ingénierie prédominantes pour fournir une méthode de conception quantitative pour la conception technique moderne.

Cependant, en raison de sa nature complexe, le développement de méthodes et de stratégies applicables pour l'optimisation de la topologie est toujours en cours. Pour traiter les problèmes et défis typiques rencontrés dans le processus d'optimisation de l'ingénierie, en considérant les méthodes existantes dans la littérature, cette thèse se concentre sur les méthodes d'optimisation de la topologie basées sur des algorithmes déterministes et stochastiques. Les travaux principaux et la réalisation peuvent être résumés comme suit:

Premièrement, pour résoudre la convergence prématurée vers un point optimal local de la méthode ON/OFF existante, un Tabu-ON/OFF, un Quantum-inspiré Evolutif Algorithme (QEA) amélioré et une Génétique Algorithme (GA) amélioré sont proposés successivement. Les caractéristiques de chaque algorithme sont élaborées et ses performances sont comparées de manière exhaustive.

Deuxièmement, pour résoudre le problème de densité intermédiaire rencontré dans les méthodes basées sur la densité et le problème que la topologie optimisée est peu utilisée directement pour la production réelle, deux méthodes d'optimisation de la topologie, à savoir Matériau Isotrope solide avec pénalisation (SIMP)-Fonction de Base Radiale (RBF) et Méthode du Level Set (LSM)-Fonction de Base Radiale (RBF). Les deux méthodes calculent les informations de sensibilité de la fonction objectif et utilisent des optimiseurs déterministes pour guider le processus d'optimisation. Pour le problème posé par un grand nombre de variables de conception, le coût de calcul des méthodes proposées est

considérablement réduit par rapport à celui des méthodes de comptabilisation sur des algorithmes stochastiques. Dans le même temps, en raison de l'introduction de la technique de lissage par interpolation de données RBF, la topologie optimisée est plus adaptée aux productions réelles.

Troisièmement, afin de réduire les coûts informatiques excessifs lorsqu'un algorithme de recherche stochastique est utilisé dans l'optimisation de la topologie, une stratégie de redistribution des variables de conception est proposée. Dans la stratégie proposée, l'ensemble du processus de recherche d'une optimisation de la topologie est divisé en structures en couches. La solution de la couche précédente est définie comme topologie initiale pour la couche d'optimisation suivante, et seuls les éléments adjacents à la limite sont choisis comme variables de conception. Par conséquent, le nombre de variables de conception est réduit dans une certaine mesure; le temps de calcul du processus est ainsi raccourci.

Enfin, une méthodologie d'optimisation de topologie multi-objectif basée sur l'algorithme d'optimisation hybride multi-objectif combinant l'Algorithme Génétique de Tri Non dominé II (NSGAI) et l'algorithme d'Evolution Différentielle (DE) est proposée. Les résultats de la comparaison des fonctions de test indiquent que la performance de l'algorithme hybride proposé est supérieure à celle des algorithmes traditionnels NSGAI et Strength Pareto Evolutionary 2 (SPEA2), qui garantissent la bonne capacité globale optimale de la méthodologie proposée et permettent au concepteur de gérer les conditions de contrainte de manière directe.

Pour valider les méthodologies d'optimisation de topologie proposées, deux cas d'étude sont optimisés et analysés. L'application du problème d'optimisation de la topologie d'un actionneur électromagnétique montre que les performances des méthodes proposées sont supérieures à celles des méthodes existantes ; En adoptant la méthode d'optimisation de la topologie basée sur l'algorithme hybride proposé, il est possible d'obtenir de nombreuses topologies de conception nouvelles, capables de réduire autant que possible la consommation de matériau tout en garantissant que l'armature est soumise à une force électromagnétique relativement importante. Cela pourrait fournir une base de référence et une base théorique importante pour le travail d'un designer. L'application de la simulation sur le récupérateur d'énergie piézoélectrique montre qu'il est possible d'obtenir davantage de topologies optimales réalisables en adoptant la méthode proposée.

Abstract

Topology optimization is the conceptual design of a product. Comparing with conventional design approaches, it can create a novel topology, which could not be imagined beforehand, especially for the design of a product without prior-experiences or knowledge. Indeed, the topology optimization technique with the ability of finding efficient topologies starting from scratch has become a serious asset for the designers. Although originated from structure optimization, topology optimization in electromagnetic field has flourished in the past two decades. Nowadays, topology optimization has become the paradigm of the predominant engineering techniques to provide a quantitative design method for modern engineering design.

However, due to its inherent complex nature, the development of applicable methods and strategies for topology optimization is still in progress. To address the typical problems and challenges encountered in an engineering optimization process, considering the existing methods in the literature, this thesis focuses on topology optimization methods based on deterministic and stochastic algorithms. The main work and achievement can be summarized as:

Firstly, to solve the premature convergence to a local optimal point of existing ON/OFF method, a Tabu-ON/OFF, an improved Quantum-inspired Evolutionary Algorithm (QEA) and an improved Genetic Algorithm (GA) are proposed successively. The characteristics of each algorithm are elaborated, and its performance is compared comprehensively.

Secondly, to solve the intermediate density problem encountered in density-based methods and the engineering infeasibility of the finally optimized topology, two topology optimization methods, namely Solid Isotropic Material with Penalization-Radial Basis Function (SIMP-RBF) and Level Set Method-Radial Basis Function (LSM-RBF) are proposed. Both methods calculate the sensitivity information of the objective function, and use deterministic optimizers to guide the optimizing process. For the problem with a large number of design variables, the computational cost of the proposed methods is greatly reduced compared with those of the methods accounting on stochastic algorithms. At the

same time, due to the introduction of RBF data interpolation smoothing technique, the optimized topology is more conducive in actual productions.

Thirdly, to reduce the excessive computing costs when a stochastic searching algorithm is used in topology optimization, a design variable redistribution strategy is proposed. In the proposed strategy, the whole searching process of a topology optimization is divided into layered structures. The solution of the previous layer is set as the initial topology for the next optimization layer, and only elements adjacent to the boundary are chosen as design variables. Consequently, the number of design variables is reduced to some extent; and the computation time is thereby shortened.

Finally, a multi-objective topology optimization methodology based on the hybrid multi-objective optimization algorithm combining Non-dominated Sorting Genetic Algorithm II (NSGAI) and Differential Evolution (DE) algorithm is proposed. The comparison results on test functions indicate that the performance of the proposed hybrid algorithm is better than those of the traditional NSGAI and Strength Pareto Evolutionary Algorithm 2 (SPEA2), which guarantee the good global optimal ability of the proposed methodology, and enables a designer to handle constraint conditions in a direct way.

To validate the proposed topology optimization methodologies, two study cases are optimized and analyzed. The simulation application on the electromagnetic actuator topology optimization problem demonstrates that the performance of the proposed methods is superior to those of existing methods; by adopting the topology optimization method based on the proposed hybrid algorithm, many new design topologies can be obtained, which are able to reduce the material consumption as much as possible while ensuring that the armature is subjected to a comparatively large electromagnetic force. This could provide an important reference and theory basis for a designer's work. The simulation application on the piezoelectric energy harvester shows that optimized topology with better manufacturing feasibility can be gained by using the proposed methods.

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Introduction

Background

Today, because of the rapid depletion of energy resources, scarcity of economic and material resources, strong technological competition and increasing environmental awareness, engineers are under immense pressure to produce optimal designs in order to survive. On the other hand, the advent of new technology and materials, as well as the imminent introduction of many mandatory international regulations on electrical products, are making it increasingly difficult to obtain an optimal design of an electromagnetic device or system using traditional analytical and synthetic approaches. In this regard, numerical methodology based on multi-physics field computations becomes a topical area in design optimizations and inverse problems in computational electromagnetics in the last three decades [1].

The structure optimization design can be divided into three levels according to the type of design variables [2][3]: size optimization, shape optimization, and topology optimization. Topology optimization makes it possible that the design object can achieve some performance indexes under certain constraints by seeking the optimal topology layout of the structure. Compared to the first two category optimization techniques, topology optimization can change the topology of a structure to produce novel ones. Consequently, topology optimization is the highest level of structure designs and belongs to conceptual design.

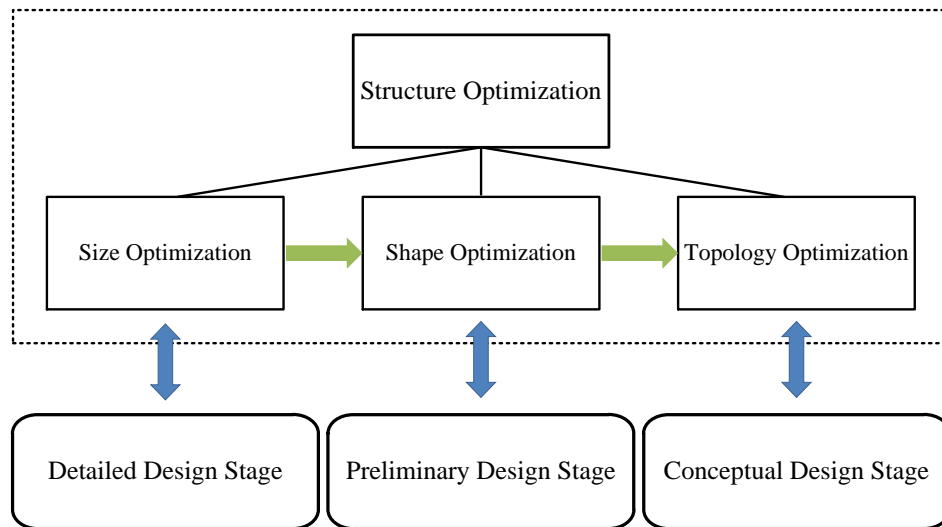


Figure 1 : Structural optimization design and corresponding design stage.

In designing an electromagnetic device, one starts from the definition of an initial geometry. In order to assess the device performances, a model is then developed based on this geometry, from which a parametric optimization is carried out to determine the optimal device dimensions. The main disadvantage of this approach lies in the choice of the initial geometry. Indeed, the initial geometry either comes from the literature or from the knowledge of the designer, who tends to rely on designs that have already proved to be efficient. Even if this approach is generally justified, it does not support any creation. A designer may indeed be faced with a new design problem about which no previous knowledge exists or with a problem where an unexpected design may outperform the conventional one. In this case, topology optimization (TO) exhibits its advantages. In fact, TO has now become the paradigm of the predominant engineering techniques to provide a quantitative design method for modern engineering design. The practical scope of topology optimization has covered many areas and disciplines including combinations of structures, heat transfer, acoustics, fluid flow, aeroelasticity, materials design, and other multiphysics problems.

Topology optimization

Defination

The goal of topology optimization is finding the optimal topology of a given design problem, i.e., determining the material that should be placed in a region in order to optimize the objective function while satisfying some specifications. In the case of finite element discretization, the design space can be represented as elements and the optimization goal is formulated as finding the optimal distribution of materials inside these elements. Figure 2 conceptually presents a 2D topology optimization problem with material library including four materials.

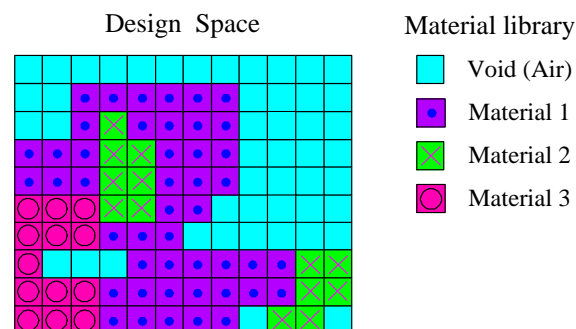


Figure 2 : 2D topology optimization problem.

Mathematical description

The theoretical description of topology optimization originates from the mechanical domain, which can be traced back to the design of the Michell truss in 1904 [4]. As the development of numerical analysis methods and optimization techniques, especially with the introduction of versatile finite element method, topology optimization has been made great progress in the last few decades. Literatures have reported a lot of work in the area of topological description, analysis means and optimization methods, among which, the description of topological problems is the basis of the analysis and optimization. The current main description methods include homogenization method, density based method, boundary based method, hard kill method, ON/OFF method and so on.

Optimization methods for solving topology optimization problems can be roughly divided into two categories: methods based on deterministic algorithms and those based on stochastic algorithms. Optimality criteria method, mathematical programming method and method of moving asymptotes are within the scope of deterministic methods. These methods use the gradient information of the objective function, and have a quick convergence speed. Such methods are suitable for topology optimization problems with a large number of design variables. However, their global searching ability is poor, and they are prone to fall into local optimal solutions. Moreover, such methods are difficult in dealing with multi-objective optimization problems directly.

Stochastic methods are optimization methods that use random mechanisms. The injected random principle may enable the method to escape from a local optimum; therefore, these methods have generally a stronger global searching ability than the deterministic algorithms, it is more suitable for the global optimal solution for non-convex mathematical programming problems with quite many local extremum. However, the main disadvantage of this kind of methods is that the computational cost is high and the convergence speed is slow. Currently, the global optimization methods commonly applied in the electromagnetic field are genetic algorithm, simulated annealing algorithm, tabu search algorithm, particle swarm optimization algorithm, quantum evolution algorithm and so on [5]. The basic idea in these algorithms is to eliminate the inferior solution and keep the superior solution through the iterative process, and finally find the optimal solution of the problem by introducing biological evolutionary ideas (such as Darwin's survival theory of the fittest).

Key issues

Topology optimization was at first applied to structure optimization problems. Although much research has already been conducted and different methods are proposed, the application of TO in electromagnetic field is comparatively much less studied and nowadays still faces many challenges. The main issues to be solved can be classified into the following four aspects.

1. Local optima

In the existing methods, quite a few use the gradient-based methods as optimizers, which utilize the sensitivity information to drive the optimization procedure as the typical deterministic optimization methods. Generally speaking, such methods have a good convergence performance. However, the gradient-based methods have their disadvantage namely, the stagnation on local optima. When facing with complex non-convex problems (most of practical optimization problems belong to such problems), the gradient-based methods may converge to one of the optima, and then stuck in this local minima, the optimization stops. On the contrary, heuristic algorithms are well designed to exploit the global solutions of an optimization problem. Unfortunately, their computation burden is comparatively heavier than that of the gradient-based methods.

2. Intermediate density and manufacturing infeasibility

- (1) The density-based method often encounters the problem of the so-called grayscales, regions of intermediate density that are allowed to exist in the optimal configurations. Although the penalization scheme will eliminate grayscales in a friction of engineering topology design problems, such filtering schemes crucially depend on artificial parameters that lack rational guidelines for determining appropriate priori parameter values;
- (2) In the level-set method, the obtained boundaries are still represented by a discretized, likely unsmooth, mesh in the analysis domain unless alternative techniques are applied.

3. Heavy computational expense

Although a wealth of efforts have been devoted to structure TO field, such as beam, truss or other supporting structure, which are relatively simple, for more complex electromagnetic TO problems, the excessive computation time using

finite element analysis (FEA) has been highlighted as a tough issue. The situation is much worse when the mesh density in discretization is large. Moreover, the sensitivity information needed in gradient-based methods or numerous iteration times demanded in heuristic methods require extra computations. All these factors accumulate heavy requirement on computation resources to make it an overwhelming computational task for topology optimization no matter what optimizer (deterministic or heuristic) is used.

4. Constraint handling

In the current existing methods, the way to handle constraint conditions is generally to combine them with the original objective to form a new optimization objective, which in fact changes the original problem.

Approaches

Aimed at the key issues mentioned above, different approaches and strategies are proposed in this thesis to address the problems.

1. To solve the premature convergence issue in the existing ON/OFF method, stochastic algorithms are introduced as the optimizers of topology optimization methods. A Tabu-ON/OFF, an improved Quantum-inspired Evolutionary Algorithm (QEA) and an improved Genetic Algorithm (GA) are proposed successively.
2. To solve the intermediate density and manufacture infeasibility problem, two topology optimization methods, namely Solid Isotropic Material with Penalization-Radial Basis Function (SIMP-RBF) and Level Set Method-Radial Basis Function (LSM-RBF). By introduction of RBF data interpolation technique, the optimized topology is more manufacturing friendly.
3. To reduce the excessive computation burden when applying a stochastic algorithm as the optimizer, a design variable redistribution mechanism is proposed. In this mechanism, the whole searching process of a topology optimization is divided into layered structures. In the each layer, only elements adjacent to the boundary are chosen as design variables. In this way, the number of design variables is reduced and the computation time is consequently shortened.
4. To consider the constraint conditions in the actual topology optimization problems, a multi-objective topology optimization method is proposed, which relies on the

proposed hybrid multi-objective optimization algorithm. The proposed TO methodology enables designers to handle different constraints in a rational and direct way.

Thesis organization

Originated from structure optimization, topology optimization has a wide scope of topics. However, in this thesis, the application is limited to the electromagnetic relevant devices. The thesis is organized as follows.

A systematic review of the state of the art of topology optimization is presented in Chapter 1. Theoretical characteristics and applicative optimizers of homogenization method, density based method, ON/OFF method, boundary based method and discrete method are summarized and analyzed respectively. The general mathematical representation of a topology optimization problem using discrete based method is given. In the end, engineering applications in the electromagnetic domain are listed.

Chapter 2 elaborates the different topology optimization methods proposed in this thesis, including the ON/OFF method, the combined tabu-ON/OFF method, an improved Quantum-inspired Evolutionary Algorithm (QEA) and an improved Genetic Algorithm (GA) based methods of the discrete methods; the combined Solid Isotropic Material with Penalization (SIMP)-Radial Basis Function (RBF) method of density based ones, and the Level Set Method (LSM)-Radial Basis Function (RBF) topology optimization method of boundary based ones.

In Chapter 3, the classification and development of multi-objective algorithms is firstly introduced and summarized. The related concepts of the Pareto optimal, the strong and weak dominance relation and the common performance indicators of multi-objective optimization algorithms are enumerated. Then a multi-objective topology optimization methodology based on the hybrid algorithm (JNSGA-DE) which combines Non-dominated Sorting Genetic Algorithm (NSGA) and Differential Evolution (DE) algorithms is proposed. The performance of the hybrid algorithm is assessed by using 9 typical test functions with different Pareto front features. The test results are compared with the NSGAI and Strength Pareto Evolutionary Algorithm 2 (SPEA2).

In Chapter 4, numerical examples of topology optimizations are elaborated. The numerical results of the proposed topology optimization methods are compared and analyzed. The ON/OFF-finite difference method, Tabu-ON/OFF method, improved genetic algorithm and improved quantum evolution algorithm are applied to a prototype of electromagnetic actuators for a single-objective topology optimization. In the piezoelectric energy harvester problem, the topology under static and harmonic conditions is optimized respectively by using SIMP-RBF method and LSM-RBF method. Finally, the proposed hybrid optimization algorithm is used to optimize the multi-objective topology of the electromagnetic actuator. A design variable redistribution strategy is introduced to alleviate the computation burden. And a set of typical Pareto non-dominated solutions and their corresponding optimized topologies are obtained.

And in the end, the thesis is concluded. Possible focus and direction of future researches is explored.

1. State of the art

The first established topology optimization application can be traced back to the beginning of the twentieth century when Michell [4] derived the optimality criteria for the least weight layout of trusses to an analytical optimization. Then, Rozvany and Prager [6]~[8] extended the principles to derive the topology optimization theory. Although the first paper on topology optimization was published over a century ago by the versatile Australian inventor Michell, it is only after the landmark paper of Bendsoe and Kikuchi [9] in the late 1980s, in which the homogenization method was proposed, which is based on the homogenization theory. Since then homogenization method has received lots of attention by the researchers who are notably represented by Kikuchi [10] and Allaire [11], and numerical methods for topology optimization have been proposed and investigated [8], topology optimization techniques been applied to solve a wide scope of problems.

Almost at the same time, one of the typical density based methods, namely the Solid Isotropic Microstructure with Penalization (SIMP) method, was proposed by Bendsoe [12]. Later Rozvany and Zhou [13] studied and developed the SIMP method. Compared to the homogenization approach, it was easier to be implemented, however the method had its own defects, such as mesh dependencies, checkerboard patterns and local minima [14].

As the development of numerical computation methods, especially after the introduction of finite element (FE) method in the field of topology optimization, the topology optimization approaches are booming prosperously to have the ability to solve various types of structure topologies involving possibly several materials in different engineering disciplines, such as in the design of piezoelectric transducers [15]~[20], fluid flow and heat [21]~[24], ultrasonic wave transducers [25][26], acoustic devices [27][28], photonic crystals [29]~[32] or aerodynamic designs [33]~[35].

Topology optimization was firstly applied to electromagnetic (EM) field by Dyck and Lowther, who proposed the so called optimized material distribution (OMD) method [36]. Besides of the OMD method, they analyzed the problems that would be encountered when topology optimization techniques applied in electromagnetic devices and defined some certain rules. After a few years tepid development, Yoo et al. [37] applied the homogenization method to the H-shaped electromagnet to obtain the optimal topology, in which the change of

inner hole size and rotational angle of unit cell determined the material distribution. However, after SIMP method gained its popularity in structure optimizations, literature of topology optimization in EM domain based on density-based methods have sprouted in electromagnetics [38]~[42]. Among which, the ON/OFF method proposed by Takahashi et al. [43] and the reluctivity-based method by Choi and Yoo [44] are two typical methods applied to electromagnetic devices. Later, when the level-set method grasped the public notice, many applications based on this boundary-based methods have emerged [45]~[49]. It should be noticed that, although TO methods based on evolutionary algorithms came up not much later than the density-based methods [50], the range of applications [51]~[53] were much narrower compared with the above mentioned methods.

Following the similar traces of fundamental research and development as other topics and directions in computational electromagnetics, it can be seen that the study of topology optimization of electromagnetic devices is also promoted by and stimulated from the corresponding studies in other related engineering disciplines, especially in the structure optimization of computational mechanics.

Also, its development and prosperity is synchronized with the evolutionary progress of computer software and hardware, as well as the continuously progressing and maturing in computational theory and numerical method. In this regard, the state of the art about the topology optimization researches and engineering applications for design optimizations of electromagnetic devices will be reviewed mainly based on the developments and progresses in the fields of topology optimization of the fellow engineering disciplines.

1.1 Homogenization method

In homogenization methods, the optimal shape of a structure is transformed into the optimal material distribution. The design domain is divided into a finite number of finite elements. Each finite element can be decomposed into infinite number of unit cells that have rectangular perforation whose size is updated in each iteration. Figure 1.1 shows the design domain Ω that is composed of a composite material with perforated microstructure [54]. Each microstructure is described by three design variables: θ_e , x_e and y_e , e is the index number of the microstructure. The values of homogenized permeability are defined by homogenization theory and are assigned at each iteration process in accordance with the design variable updates during the design process. By defining a mapping relationship between permeability

and design variables, in homogenization method, the macroscopic property of the design domain is derived from a large number of microstructure layouts using FE method.

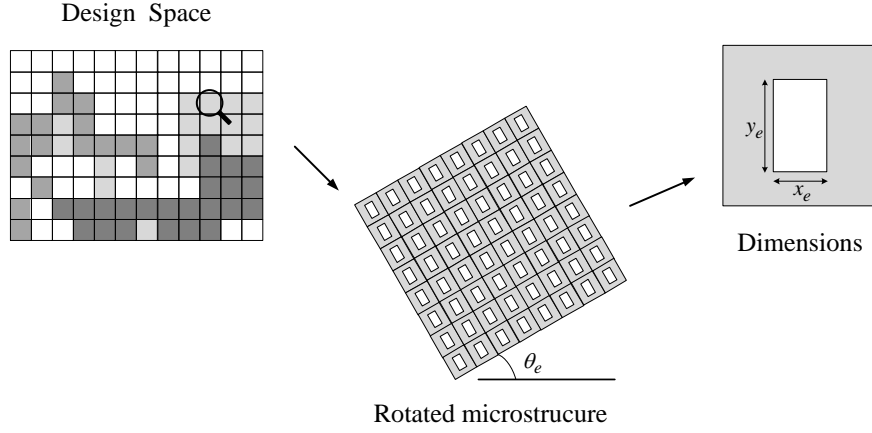


Figure 1.1 : Microstructure and design variables in homogenization method.

The resulting homogenization problem can be solved by using different algorithms. Moving asymptotes (MMA) method originally proposed by Svanberg [55] based on its simplicity and good feasibility was widely adopted by Yoo et al. and several scholars including Bruyneel et al. [56]. In other topology optimizations, the sequential linear programming method was also adopted (in the structure optimization field [57]; in electromagnetic domain [58][59]).

1.2 Density based method

In structural topology optimization, the most popular methodologies are the density-based methods. Given a fixed domain of finite elements, density based methods optimize the objective function by determining whether each element should consist of solid material or void. Thus it poses an extremely large-scale combination optimization problem. By adopting interpolation functions where the material properties are explicitly interpreted as the continuous design variables (usually the density of materials), the discrete variables are transferred to continuous variables and some optimizer is used to iteratively steer the solution towards a discrete solid/void topology. Usually, penalty methods are utilized to impel the solution to a crisp “0/1” or “solid/void” topologies. Besides, regularization and filter techniques are adopted to alleviate the checkerboard problem and mesh-dependency issue.

In the following, the SIMP method, a typical density-based method is simply introduced. As an example in the mechanical field, SIMP method can be described using a stiffness tensor of intermediate materials,

$$E_{ijkl}(x) = \rho(x)^p E_{ijkl}^0 \quad (1.1)$$

where $\rho(x)$ is the material proportion in position x , p is a penalty factor and E_{ijkl}^0 is the stiffness matrix of the solid material. Apparently, when p is large than 1, the cell stiffness matrix is exponential scale to the material proportion. In this way, intermediate materials are penalized, which impels the intermediate density to disappear in the final optimized topology. Literatures normally recommend the value of p from 2 to 4.

To utilize a gradient-based optimization method, the proportion of material should be limited by a lower bound to avoid the singularity, namely

$$\rho(x) > 10^{-3} \quad (1.2)$$

And the gradient of the stiffness matrix is then given as

$$\frac{\partial E_{ijkl}(x)}{\partial \rho} = p \rho^{p-1} E_{ijkl}^0 \quad (1.3)$$

It should be noted that the gradient-based algorithm would be stuck when $\rho(x) = 0$ and $p \neq 1$ since the gradient is equal to zero.

In the TO designs of electromagnetic devices, the method is usually implemented to model the following relation between the iron proportion and the permeability [37][39]:

$$\mu(x) = \mu_0(1 + (\mu_r - 1)\rho(x)^p) \quad (1.4)$$

with μ_0 is the air permeability and μ_r is the relative iron permeability.

Various optimization techniques can be implemented as optimizers to solve the density problem such as the classical optimality criteria method (OCM) [60]~[64]; sequential

linear programming (SLP) ([65]~[67]) or the method of moving asymptotes (MMA) [68]. Besides, the steepest-descent method was used as well by Okamoto and Takahashi [69], Labbé and Dehez [70].

Though the density-based method takes the dominated position in the topology optimization based on its versatility, effectiveness and easiness to be implemented, it also has a few distinct disadvantages. In the mechanical domain, Sigmund and Petersson [14] discussed the difficulties of the SIMP method such as local optimal, mesh-dependent structures and checkerboard patterns. Byun [40] and Okamoto [69] observed that the optimized results depend on the initial conditions. Besides, one typical problem is the intermediate density. Despite that many literatures advocate to choose the value of penalty factor between 2 and 4 to help to converge to a solid design without intermediate density, it is found that the final optimized topology is usually composed of quite a few elements with intermediate densities.

1.3 ON/OFF method

In the ON/OFF method, the design domain is subdivided into different elements, as shown in Figure 1.2. Each element has only one state, void or solid: a gray cell is identified as a solid (the state is called “ON”), and a white cell is a void (the state is called “OFF”) from which the method obtained its name. The material attribute of an element is updated iteratively in order to find a promising topology in terms of the objective function. The gradient information of the objective function is usually needed to determine the material state of each element whether solid or void [42][43][71].

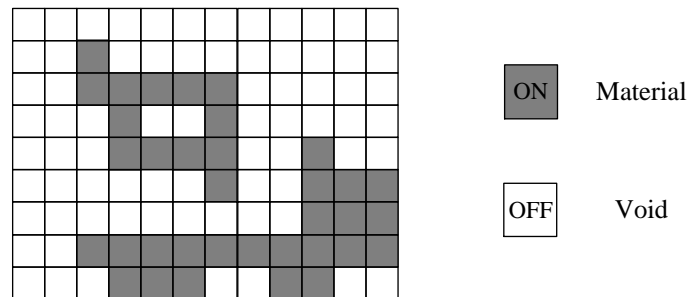


Figure 1.2 : Schematic of ON/OFF method.

For an air-magnetic material optimization problem, the iterative procedure is described as.

- 1) Decision of an initial topology.
- 2) Evaluation of the objective function using FE method.
- 3) Calculation of sensitivity information using adjoint variable method.
- 4) Modification of topology.
 - If the value of sensitivity is negative, the relevant material property in an element should be decreased.
 - If the value of sensitivity is positive, the relevant material property should be increased.
- 5) Topology smoothing.
 - a) Cut step: a magnetic material element surrounded by air elements (or just one magnetic material element) is changed to air.
 - b) Attached step: an air element surrounded by 4 or 5 magnetic material elements is transformed to magnetic material.
- 6) Evaluation of the objective function corresponding to the updated topology using FE method.
- 7) Annealing procedure.
 - If the objective function is improved, repeat steps 3 to 7.
 - Otherwise, reduce the number of changeable elements- N by a factor $0 < \gamma < 1$ and repeat steps 3 to 7.
- 8) The optimization is terminated when $N < 1$.

The advantage of the ON/OFF method is its convenience to be implemented on different topology optimization problems; however, the initial topology has a serious influence on the final optimized result of the ON/OFF method. Besides, according to the mechanism of the optimization process of the ON/OFF method, the sensitivity information is used to guide the optimization, which cannot avoid the method to be trapped into the local optima. It should be noted that, Choi and Yoo [52] proposed a combined genetic algorithm and ON/OFF method to optimize the topology of the magnetic actuators. The genetic algorithm is used at the first stage to acquire an initial optimized solution, and ON/OFF sensitivity is applied only to the surfaces to further optimize the optimal topology. In this way, the initial conditions are avoided to be given and the computational cost is not relatively heavy.

1.4 Boundary based methods

Apart from density-based methods introduced, boundary based methods are another most widely spread topology optimization methods, especially in the recent decades. The root of boundary variation methods lays in shape optimization techniques. Different from the density-based methods in which design domain is parameterized in an explicit function, the boundary based methods are based on an implicit function that defines the structural boundary [72]. Here, boundary based methods are mainly elaborated using level-set method and phase-field method as two typical ones. Figure 1.3 illustrates the difference between an explicit parameterization of variables and an implicit representation, which in Figure 1.3(a), the variable of design domain is explicitly parameterized between 0 and 1; while in Figure 1.3(b), the structural boundary is implicitly specified as a contour line of the field Φ , which is a function of x .

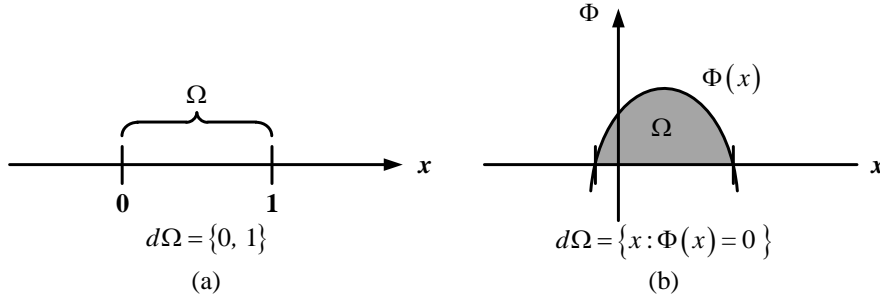


Figure 1.3 : (a) Explicit representation of design domain (b) implicit representation of boundaries [72].

Another difference between density based methods and boundary based methods lies in the product of optimization, in density based methods, the optimized topology usually contain a lot of intermediate density elements where post-processing procedure for interpreting topology is needed; however, the latter methods could generate the optimized topology with crisp and smooth edges. However, it is still needed to be noted that, despite that some literatures advocate that boundary based methods can improve the accuracy of mechanical response in the vicinity of boundaries and avoid the intermediate density in the optimized topology, many boundary based methods operate on the discretized finite elements, boundaries are still represented in a discretized way, which generally result in a unsmooth, toothed optimize topology.

1.4.1 Level-set method

In the level set method, the boundary is represented as the zero level curve using a scalar function Φ , and material distribution of the design domain is determined according to the values of level set functions in the certain region. The topology optimization is achieved through the evolvement of the design boundary including motion, merge and introduction of new holes. A 2D example of topology optimization using the level set method is shown in Figure 1.4.

$$\rho = \begin{cases} 0: \forall x \in \Omega: \Phi(x) < 0 \\ 1: \forall x \in \Omega: \Phi(x) \geq 0 \end{cases} \quad (1.5)$$

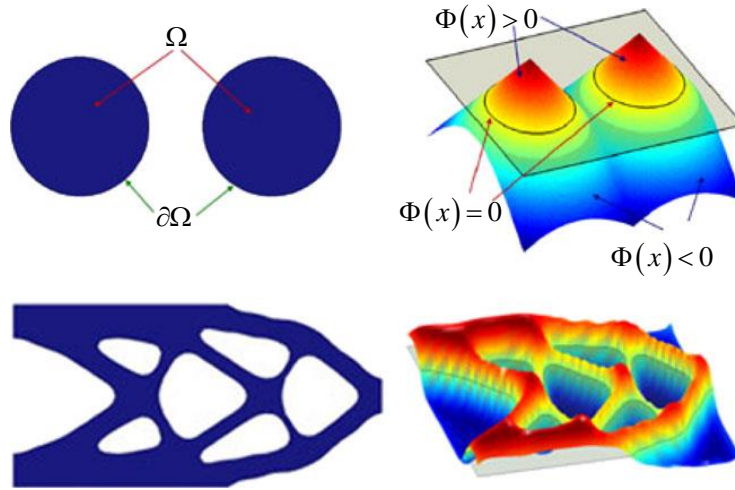


Figure 1.4 : Representations of level-set method: 2D topology example [73].

To be more general, a level set model which describes a surface in an implicit form using an iso-surface scalar function of a 3D structure can be given [74]

$$S = \{x : \Phi(x) = k\} \quad (1.6)$$

where k is an arbitrary iso-value, and x is a point in space on the iso-surface. Considering the process of structural optimization is dynamically changed in time, we can have (1.7).

$$S(t) = \{x(t) : \Phi(x(t), t) = k\} \quad (1.7)$$

Differentiate both sides of (1.7) and apply the chain rule, the following “Hamilton-Jacobi-type” equation can be obtained

$$\frac{\partial \Phi(x, t)}{\partial t} + \nabla \Phi(x, t) \frac{dx}{dt} = 0, \quad \Phi(x, 0) = \Phi_0(x) \quad (1.8)$$

which defines an initial value problem for the time dependent function Φ . Let dx/dt be the movement of a point on the surface driven by the objective function so that it can be expressed in terms of the position x and the geometry of the surface at that point. Then, optimal structural boundary can be expressed as the solution of a partial differential equation (1.9)

$$\frac{\partial \Phi(x)}{\partial t} = -\nabla \Phi(x) \frac{dx}{dt} \equiv -\nabla \Phi(x) V(x, \Phi), \quad \Phi(x, 0) = \Phi_0(x) \quad (1.9)$$

where $V(x, \Phi)$ is the “speed vector” and depends on the objective function, which is usually determined by sensitivity analysis respect to the objective function. To maintain a uniform spatial gradient value in Hamilton-Jacobi equation, the level set function needs to be re-initialized after several iterations of update. And to consider the constraint condition, augmented Lagrange multiplier formulation is typically used.

Since the conventional level set method mentioned is unable to create new holes and the resulting solutions are heavily dependent on the initial state of the design domain, the original Hamilton-Jacobi equation is transformed into the following form

$$\frac{\partial \Phi}{\partial t} + \nabla \Phi V - \mathcal{D}(\Phi) - \mathcal{R}(\Phi) = 0 \quad (1.10)$$

where $\mathcal{D}(\Phi)$ is the diffusive operator which smoothies the level set field typically using an isotropic or anisotropic, linear or nonlinear diffusion model [75]. These diffusion models are similar to the models adopted in the phase field method; and $\mathcal{R}(\Phi)$ is the reactive term which enables the nucleation of new holes, typically as the topological derivatives [76].

In topology optimization on electromagnetic devices, Zhou et al. presented a level-set framework for the design of a typical dipole antenna [45] and design of the negative

permeability metamaterials [46]; Choi et al. proposed a method based on reaction-diffusion equation and applied it to maximize the force of magnetic actuators [47]; Otomori et al. used a level-set based method to find the optimized configuration of the ferrite material in an electromagnetic cloak [49].

1.4.2 Phase-field method

The phase-field method proposed in topology optimization is based on the theories originally developed from phase-transition field [77][78]. In these theories, a phase field function ϕ is defined over the design domain Ω , which is classified into two phases, A and B, distinguished by values α and β of ϕ , and ε is the interfacial thickness of transition region between two boundary phases.

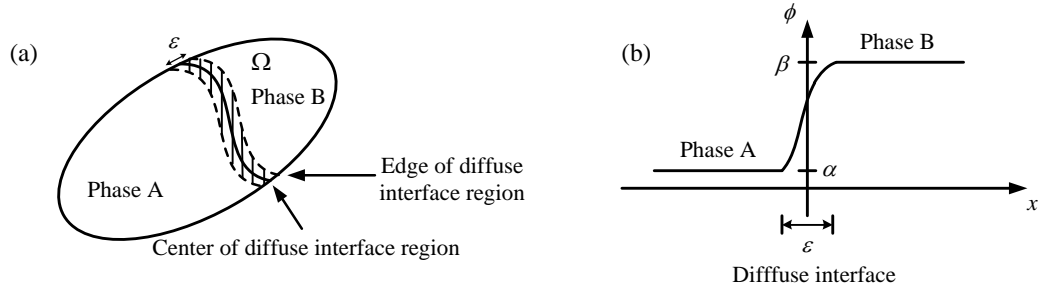


Figure 1.5 : (a) A 2D domain in phase field function (b) 1D phase field function [79].

In the phase field method, density variables in the design domain are directly handled, and the following function is minimized,

$$J(\mathbf{u}(\rho), \rho) = \int_{\Omega} f(\mathbf{u}(\rho)) dV + \int_{\Omega} \left(\varepsilon \|\nabla \rho\|^2 + \frac{1}{\varepsilon} w(\rho) \right) dV \quad (1.11)$$

where ρ is the density variable, $f(\mathbf{u})$ is the original objective function in the topology optimization problem, \mathbf{u} is the specific field which satisfies the linear or nonlinear state equation, i.e. displacement field in structural topology optimization, $w(\rho)$ is a double well potential function that takes value 0 when $\rho = 0$ or 1; And the parameter is the interfacial thickness of transition phases between solid and void.

Taking the derivative with respect to ρ on the both sides of (1.11),

$$J'_{\rho} = f'_{\rho} - \varepsilon \Delta \rho + \frac{1}{\varepsilon} w'_{\rho} \quad (1.12)$$

where $'_{\rho}$ represents the differentiation with respect to ρ . By introducing the time-dependent evolutionary Cahn-Hilliard equation, the above function is minimized,

$$\frac{\partial \rho}{\partial t} = -M \nabla \cdot (\nabla J'_{\rho}) \quad (1.13)$$

where M is a diffusion coefficient. For simple implementation, the above equation is often solved by splitting into two coupled equations.

$$\begin{aligned} \frac{\partial \rho}{\partial t} &= -M \nabla \cdot (\nabla \mu) \\ \mu &= f'_{\rho} - \varepsilon \Delta \rho + \frac{1}{\varepsilon} w'_{\rho} \end{aligned} \quad (1.14)$$

After the phase-field method is firstly proposed by Bourdin and Chambolle [80], a lot of researchers applied this method to the structure optimization application adopting different double well potential model and regularization techniques to alleviate the ill-posed problem. Among which, Wang and Zhou (2004a, b) used van der Waals-Cahn-Hilliard phase transition theory and the Γ -convergence theory to solve the bi-material phases and multi-phases optimization problems [81][82]. Wallin et al. solved the minimizing problem using volume preserving Cahn-Hilliard model in conjunction with an adaptive finite element method to lower the computational cost [83]. Although literatures claimed that phase field methods could be able to control the perimeter of the optimal topology, avoid the re-initialization procedure in level set methods, and be implementation easier, no fully and detailed comparison with other methods has been done. Besides, according to Sigmund's review, a number of the phase field methods minimize the functional directly without using the auxiliary field μ in, which makes these methods similar to density approaches [75].

In electromagnetic devices optimization, the phase-field approaches have not been received much attention compared to the level-set methods.

1.5 Hard-kill methods

1.5.1 Evolutionary structure optimization

Firstly proposed by Xie and Steven [84][85], Evolutionary Structural Optimization (ESO) has gained its attention and by now recognized as the most well-known hard-kill method of topology optimization. Different from density-based methods, which relax the combination optimization problem with discrete variables to the one with continuous variables by introducing interpolation functions, and by identifying a means to iteratively steer the solution towards a discrete solid/void solution, hard-kill methods directly handle the discrete variables optimization problem by gradually removing (or adding) the material in the design domain, and the decision of removing or adding material is based on heuristic criteria, the sensitivity information is often used as well.

The most distinct advantage of hard-kill methods like ESO is its simplicity of implementation; it can be easily incorporated with commercial finite element softwares; another advantage is that topology optimization results are without intermediate or gray material since the material property of the finite elements are defined only as solid or void. The common minimum compliance optimization problem using ESO method is given as:

$$\begin{aligned}
 \min : \quad & c = \mathbf{U}^T \mathbf{K} \mathbf{U} \\
 \text{subject to : } & V \leq V_0 \\
 & \mathbf{K} \mathbf{U} = \mathbf{F} \\
 & x = [0,1]
 \end{aligned} \tag{1.15}$$

where c is the objective function, \mathbf{K} is the global stiffness matrix, \mathbf{U} is the displacement vector, x is the vector of element design variables and $V < V_0$ is the constraint of material usage. It is obvious that different from density-based method where continuous variables are utilized, in the basic formulation of a hard-kill method, the design variables are taken as the existence ($x_e = 1$) or absence ($x_e = 0$) of finite elements.

Although the original proposed ESO method [86] allows for only the removal of the elements with material, soon a bidirectional ESO (BESO) that could both add and remove the elements is proposed by Querin et al. [87]. Huang et al. [88], Huang and Xie [89] then

proposed a modified BESO that uses nodal sensitivity numbers to solve the mesh dependence and checkerboard issue for the compliance minimization problem.

1.5.2 Heuristic searching algorithms

With the development of all variations of ESO methods, heuristic searching algorithms has gained its popularity based on its characteristics that convenient to be adopted in hard-kill methods and strong global searching ability. As one of the typical heuristic algorithms, genetic algorithm (GA) [90] is firstly tuned and applied in the topology optimization; particle swarm optimization (PSO) method for TO is then proposed [91]; recently, quantum-inspired evolutionary algorithm [92] was applied to the TO of modular cabled-trusses.

Topology optimization using the bit-array representation [93][94] is the most common and straightforward method to model the problem, which describes the solid or void status for elements in the design domain using binary digits; similar to the previous method Wang et al. [90], Guest and Genut [95], Bureerat and Limtragool [96] utilize bit array representations of the design domain.

On the contrary, Liu et al. proposed a different mechanism for representing the genes of individuals in the population [97]. This idea is achieved by giving every individual a n bits length binary string originally made of number '1'. After sensitivity numbers of each element are calculated, genetic operations such as selection, crossover and mutation will be employed to the chromosomes of each element. Only when all gene values in a chromosome are '0', will the relevant element be permanently removed. This method is termed genetic evolutionary structural optimization (GESO). The search of GESO is based on Darwin's survival-of-the-fittest principle. The fittest elements have higher probability to be kept in the population without apprehension of being deleted in early generations in ESO. Comparing with the conventional ESO method, the introduction of GA as the optimizer to lead the optimization helps to avoid the premature of ESO falling into local optima.

Zuo et al. developed a genetic BESO method that utilizes the similar bit array representation formulation as that in GESO [98]. In each iteration, a finite element analysis is firstly used to evaluate the sensitivities of every element, then GA operators of crossover and mutation are performed over the chromosomes of all individuals according to the sensitivity information.

It is worth to note that, in the aforementioned genetic BESO or GESO methods, the sensitivity information of elements are calculated to guide the searching direction in the evolution of adopted GA. Whereas, in some proposed methods [90][99], the evolution of the optimization algorithm is mainly dependent on the fitness value in GA searching process, especially in the cases where the derivative of objective function versus design variables is hard to be formulated into analytical form, the element sensitivity is then uneasy to be obtained.

The drawbacks to heuristic algorithm based TO methods are quite obvious. Since the heuristic algorithms are used, a large number of iterations are needed for the algorithm to find the promising optimal topology. This situation even get worse when element sensitivity information cannot be easily obtained from an analytical form rather than from computing the objective function using finite element analysis (FEA) directly.

Another issue is handling of constraints. Compared to density-based topology optimization approaches where additional constraints can simply be added to the optimization problems, in heuristic algorithms based TO methods, constraints can either be combined with the original objective function to form a fitness function or be treated as other objective to form a multi-objective optimization problem.

Generally, an optimization problem with constraints can be defined as finding a vector $\mathbf{x}^* = [x_1^*, x_2^*, \dots, x_n^*]$ to minimize the objective function and subjects to the m unequal constraints and p equal ones at the same time.

$$\begin{cases} \min & f(\mathbf{x}) \\ \text{s. t.} & g_i(\mathbf{x}) \leq 0, \quad i = 1, 2, \dots, m \\ & h_j(\mathbf{x}) = 0, \quad j = 1, 2, \dots, p \end{cases} \quad (1.16)$$

For the first solving approach to handle constraints mentioned before, penalty function which penalizes the unfeasible solution according to its violation of constraints is often used to combine the constraints with the original objective function. Various penalty functions have been proposed, such as, static penalty function [100][101], dynamic penalty function [102], and adaptive penalty function [103]. Su et al. adopted a traditional static penalty function for its simplicity and efficiency [104].

$$f_p(\mathbf{x}) = \sum_{i=1}^m c_i g_i^*(\mathbf{x})^2 + \sum_{j=1}^p r_j |h_j(\mathbf{x})|^2 \quad (1.17)$$

$$g_i^*(\mathbf{x}) = \max\{g_i(\mathbf{x}), 0\}$$

where c_i and r_j are constant penalty factors. The fitness function including the objective and the penalty term can be formulated as,

$$f_{fit}(\mathbf{x}) = Q - [f(\mathbf{x}) + f_p(\mathbf{x})] \quad (1.18)$$

where Q is a positive number large enough to ensure a positive fitness function.

The other approach to handle constraints is to form a multi-objective optimization problem. Kunnakote and Bureerat [105] proposed to set structural mass constraint as the second objective to be optimized. Garcia-Lopez et al. [106] treat the volume of the structure as an additional objective to avoid the penalty parameter tuning; to consider robust performance, the normal structure compliance to be minimized is replaced by the expected value and a variation response subject to different sources of uncertainty ω , thus the optimization problem can be stated as

$$\begin{aligned} \min \quad & \begin{cases} f_1(\mathbf{x}, \boldsymbol{\omega}) = E(c(\mathbf{x}, \boldsymbol{\omega})) \\ f_2(\mathbf{x}, \boldsymbol{\omega}) = Var(c(\mathbf{x}, \boldsymbol{\omega})) \\ f_3(\mathbf{x}, \boldsymbol{\omega}) = V(\mathbf{x}) \end{cases} \\ \text{subject to} \quad & \mathbf{KU} = \mathbf{F} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned} \quad (1.19)$$

While heuristic algorithms (especially GAs) have been applied to many topology optimization problems, their comparatively separate characteristic of optimization algorithms allows them easy to be implemented accompany with FEA softwares, and good global ability capacitates to find the global optima compared with deterministic methods. However, heuristic searching methods are far less popular than density-based or boundary-based methods because of their own limitations. One of the most adverse factors is that the computational expense of heuristic methods is much heavier than that of deterministic methods (for example, gradient methods). Another problem is how to guarantee the

connectivity of structure in the stochastic optimization process. Besides, the selection of rational and efficient convergence criterion needs to be paid extra attention.

1.6 Multi-objective topology optimization methods

Topology optimization (TO) problem as an optimal material distribution problem, with various interested parameters considered to be improved; the material consumption is certainly expected to be as little as possible. In this sense, its nature is a multi-objective optimization (MOO) problem. Moreover, to handle constraint conditions, as mentioned above, one way is to add the constraints to the original optimization problem to form a new optimization problem; the another way to is to treat them as other objectives to form a multi-objective optimization problem.

Despite that multi-objective topology optimization (MOTO) has not been received much attention by the scholars, however, there are still lots of literatures that proposed different MOTO methodologies, which were used to solve kinds of MOTO problems. According to the applied methods to solve MOO problem, these TO methodologies can be roughly divided into two categories: one is based on the classical MOO methods, such as weighted sum method (Chen et al. [107]), fuzzy logical method (Chen and Shieh [108]), compromise programming method (Luo et al. [109]), physical programming method (Lin et al. [110]) or so on; the other one is based on evolutionary MOO methods, such as GA (Madeira et al. [111]), immune algorithm (Sato et al. [112]), or other evolutionary algorithm. Commonly, in those MOTO methodologies that used classical MOO methods, homogenization method (Min et al. [113]) or density based method (like SIMP (Marck et al. [114])) was used in combination to optimize the structure topology; while in those MOTO methodologies that used evolutionary MOO methods, hard-kill methods were often applied and the formulated MOTO problem was solved directly by the evolutionary algorithm as an optimizer.

Besides the above mentioned MOTO methods, Tai and Prasad [115] used weighted sum approach to handle MOO and chose GA as an optimizer; Olympio and Gandhi [116] proposed a multi-objective GA coupled with a local search optimizer using density information to optimize the morphing aircraft structures; and Isakari et al. [117] used weighted sum method to formulate the original objectives as a new functional, applied level set method to solve the MOTO problem.

Most of the multi-objective TO methods are concentrated on the structure optimization field, and many of them were applied to the TO of a cantilever or beam structure, applications on electromagnetic devices are far more less.

1.7 Applications

Topology optimization has already been applied to a wide range of disciplines, and the followings are parts of the applications in electromagnetic devices designs:

- jumping rings: produce a sufficient vertical force on a copper ring to levitate it while minimizing the power dissipation (Dyck and Lowther, 1996[36]),
- linear motors: minimize the power required to generate a given force (Dyck and Lowther, 1997[118]),
- induction motors: maximize the magnetic energy under the constraint of volume (Wang et al. 2004a[66]); minimize the cogging torque (Im et al. 2003[50]; Choi et al. 2011[119]),
- reluctant motor: optimize the topology of stator to maximize the average torque (Labbé and Dehez, 2010[120]),
- inverse problems: find the material distribution (Byun et al., 2000[121]; Dorn et al. [122]; De Lima et al. 2007[59]),
- magnetic resonance imaging (MRI) systems: achieve a uniform magnetic flux density in a given region (Byun et al., 2004[40]; Lee and Yoo, 2010[123]),
- microspeaker: maximize the Lorentz force by designing the magnetic circuit (Kim and Kim, 2008[124]),
- magnetic recording heads: maximize the recording field (Okamoto et al., 2006[42]; Park et al., 2009a,b[125][126]; Takahashi et al., 2008[127]),
- H-magnets (Yoo and Hong, 2004[128]; Yoo and Kikuchi, 2001[54]),
- C-core actuators (Choi and Yoo, 2008, 2009[44][129]; Kang et al., 2004[130]; Wang et al., 2004b[131]; Yoo, 2004[132]; Yoo and Hong, 2004[128]; Yoo and Soh, 2005[38]),
- piezoelectric actuators: maximize output displacement in a given direction (Kögl and Silva, 2005[133]),
- electromagnetic couplers: maximize the actuating force in a given direction while limiting the power consumption (Yoo et al., 2008[64]),

- magnetostrictive sensors: increase the sensor output voltage for a given stress (Choi and Yoo, 2009[129]),
- superconducting coils (Elhaut et al., 2011[134]; Kim et al., 2005a, b[68][135]; Park et al., 2003[136]),
- negative permeability metamaterials: (Zhou et al., 2011[46]; Otomori et al. 2012[48]),
- magnetic shielding: minimize the maximum flux density in the target region(Sato et al., 2014[112]; Okamoto et al., 2014[137]),
- dipole antenna: (Zhou et al., 2010[45]),
- electromagnetic waves (Isakari et al., 2016[117]; Deng and Jan G., 2018[138]),
- synchronous motor (Watanabe et al., 2018[139])

1.8 Chapter summary

This chapter provides a systematic review of the main topology optimization methods and lists engineering application examples of topology optimization in electromagnetic devices. Considering the theoretical principles and characteristics of existing different methods such as homogenization method, density based method, ON/OFF method, boundary based method and discrete method are given, and it is found that deficiency still exists in the aforementioned methods:

1. In homogenization method, it uses three design variables to describe the microstructure of each cell in the design domain, which resulting in a large number of variables, facing large systems. Besides, it adopts a non-smooth estimate of the topology boundary;
2. In density based method, the appropriate material model is firstly needed to be considered. Then regularization and filter techniques need to be chosen carefully; in addition, the intermediate density issue encountered most in density based method will give rise to the optimized topology hard for the direct manufacturing;
3. Boundary based method like level-set method also generates unsmooth and toothed optimized topology and sometimes resulting solutions are heavily dependent on the initial state;

4. ON/OFF method, despite of the simplicity, the optimized results are dependent on the initial condition. And for the methods mentioned above, they are prone to fall into local optima when facing complex non-convex optimization problem;
5. As for heuristic algorithm based methods, although they are well designed to exploit the global solutions, however, the computational cost is frequently high. Constraint handling and convergence criterion selection are the other issues to be taken care of.

Considering the above deficiency and issues to be addressed, to equip the TO method a stronger global optimization ability, an improved ON/OFF method, a Tabu-ON/OFF method, a revised QEA method and a revised GA method are proposed respectively. Among which, a redistribute mechanism of design variables is put forward to reduce the computation cost of the heuristic algorithm based methods; to solve the intermediate density issue in SIMP method and unsmooth toothed optimized topology in level set method, the SIMP-RBF method and the LSM-RBF method is proposed to solve large systems for better manufacturing feasibility; and to solve multi-objective topology optimization problems and handle constraint conditions, a multi-objective topology optimization methodology is proposed.

All the proposed topology optimization methodologies will be elaborated in Chapter 2 and Chapter 3.

2. Methodologies of single-objective topology optimization on electromagnetic devices

The main existing methodologies for topology optimization (TO) have been reviewed in Chapter 1, the pros and cons of each method are summarized, among which, SIMP and LSM are suitable for TO problems with a large number of variables; ON/OFF method was simple in principle and easy to be implement; and hard-kill methods which uses evolutionary algorithms like GA have the potential ability to overcome the local optima issue. Therefore, based on the characteristics of the above mentioned TO methods, different original improved methodologies are proposed and elaborated in sequence in this chapter, which includes improved ON/OFF method, Tabu-ON/OFF method, revised QEA method, revised GA method, SIMP and RBF combined method and LSM and RBF combined method.

The first few methods enhance the global optimization ability by introducing stochastic algorithms, thereby address the local optima trapping issue; the SIMP-RBF and LSM-RBF method utilizes the data interpolation technique to solve the unsmooth optimized topology issue. In addition, to alleviate the computation burden, a design variable redistribution mechanism is proposed and applied in revised QEA method. The principle of each methodology is elaborated, key points of each method are explained and each methodology is described in details by giving a flowchart. The verification of numerical examples and comparative analysis between different proposed methodologies will be presented in Chapter 4.

2.1 ON/OFF and finite-difference method

2.1.1 ON/OFF method

In the conventional density method for topology optimizations, the material density is set as design variable, which changes continuously from zero to one. As a result, some gray scale elements occur in the final optimal topology obtained by using this type of methods, while in ON/OFF method, design variable only has two states, the design domain is subdivided into different elements and the material attribute of an element is updated iteratively in order to find a promising topology in terms of a performance parameter Q . As shown in Figure 2.1, each element has only one condition, void or solid. A gray cell is

identified as a solid (the state is called “ON”), and a white cell is a void (the state is called “OFF”) [71].

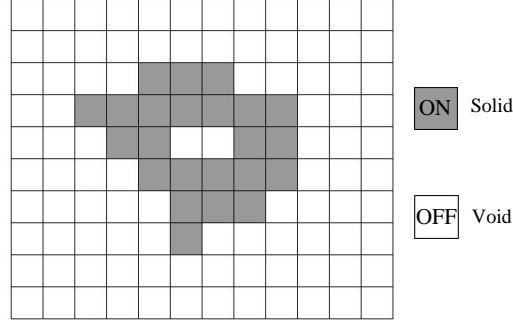


Figure 2.1 : Schematic of the ON/OFF method.

To determine the material attribute (ON/OFF state) of an element, the sensitivity, dQ / dp (p is the design variable, and usually the material density, permeability or other electromagnetic parameters of element; Q is the performance parameter) is used. If the sensitivity is negative, the cell is a void (air). If the sensitivity is positive, the cell is solid (material). The sensitivity of the i^{th} element is computed using a finite-difference approach as

$$\frac{dQ}{d\mu_i} = \frac{|Q(\mu_1, \mu_2, \dots, \mu'_i, \dots, \mu_N)| - |Q(\mu_1, \mu_2, \dots, \mu_i, \dots, \mu_N)|}{\Delta\mu} \quad (2.1)$$

where μ_i is the permeability of the i^{th} element, $\Delta\mu$ is the perturbation of μ , N is the number of total elements in the design domain.

2.1.2 An improved ON/OFF method

In order to strengthen global search ability and improve the convergence performance of the traditional ON/OFF method, an improved ON/OFF method is proposed. To facilitate the description of the proposed topology optimization methodology, its flow chart is given in Figure 2.2. The methodology starts from an initial phase. In this phase, the initial topology, the mesh and algorithm parameters are defined. After the initialization, the algorithm is transformed to compute the performance parameter using FEM and calculate sensitivity of each element using (2.1). According to these sensitivities, the attribute of every element is updated using the following rules: If the sensitivity of an element i is negative, the permeability in the element i will be decreased, and the material will be set as air; Otherwise,

the permeability in the element should be increased, and the element material is set as a magnetic material in next iteration. After the updating of element materials, the performance parameter corresponding to the new topology is computed.

In the proposed methodology, an annealing mechanism is proposed for refinement and efficient topology optimizations. For this purpose, one introduces a successful updating as: if the performance parameter of the new topology is better than the current one, this updating of the new topology is called a successful updating. The algorithm will start an annealing process once an updating is not successful. After the annealing process, the algorithm will check if the stop criterion is satisfied.

A promising byproduct of the annealing process is that a simple stop criterion is designed as: if the changeable number N_m of the elements in the design domain is less than 1, the optimal procedure will be terminated.

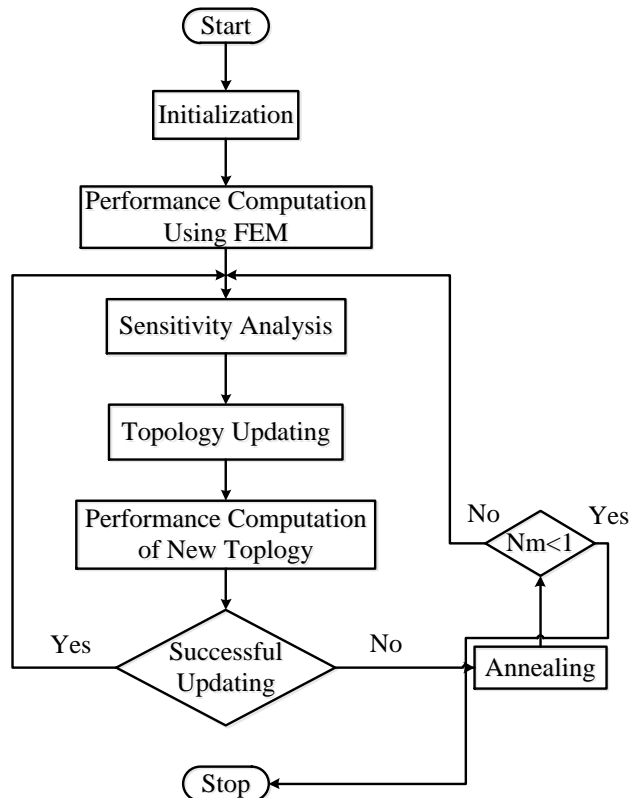


Figure 2.2 : Flowchart of the ON/OFF finite-difference methodology.

1. Virtual material

In this study, each cell in the design space possesses two different material attributes, air and magnetic materials, related to different permeabilities, for example. With the

advancement of the optimization procedures, the permeability of each cell is determined. However, to determine the attribute of an element in the next step, the sensitivity information is needed and will be computed using a finite difference approach as given in (2.1). Therefore, some perturbation technique is required to define the perturbation of μ , $\Delta\mu$. For this purpose, two virtual materials are introduced.

In the topology optimization of a magnetic actuator, the relative permeability of the air is 1, and that of the magnetic material is μ_m . Accordingly, two virtual materials with $0.5\mu_m$ and $1.5\mu_m$ permeability values of the magnetic material are introduced. In computing the sensitivity of an element, the virtual material with a relative permeability of $0.5\mu_m$ will be used if the current material in this element is air; and the virtual material with a relative permeability of $1.5\mu_m$ will be used if the current material in this element is magnetic material. By using this approach, the $\Delta\mu$ in (2.1) can be guaranteed to be positive. As shown in Figure 2.3, the material library of the proposed methodology includes air, magnetic material and two kinds of virtual materials.

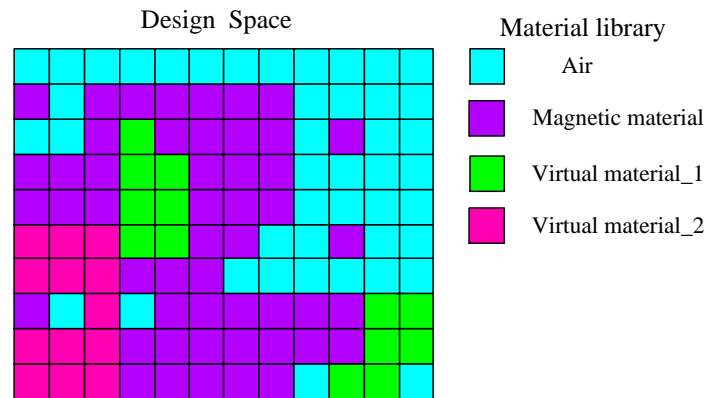


Figure 2.3 : Illustration of introduced virtual material on a 2D design space.

2. Annealing mechanism

In the proposed methodology, two different annealing mechanisms are proposed and used in order to provide different alternatives for engineering applications. The first mechanism is based on an annealing parameter and the second one is based on an index to counter un-updated number of the materials in an element.

- **Mechanism A**

In this mechanism, the algorithm will start an annealing process if the newest updating is not successful. In the annealing process, the number of changeable elements - N_m is decreased using the following equation

$$N_m^{k+1} = \gamma \cdot N_m^k \quad (2.2)$$

where N_m^k is the changeable number of elements in k^{th} iteration; γ is an annealing parameter, which is set as 0.85.

If the changeable number N_m of the elements in the design domain is less than 1, the iterative procedure will be terminated.

Based on this mechanism, four strategies are proposed to transfer changeable elements into unchangeable ones. These four strategies are illustrated in Figure 2.4. Since the design domain is regularly meshed, the whole search space is divided into a number of rows and several columns. *Strategy 1* and *Strategy 2* are implemented by changing the attributes of row elements. *Strategy 3* and *Strategy 4* are realized by changing the attributes of column elements. Comparing *Strategy 1* with *Strategy 2*, the elements that will become unchangeable are conducted in a way from down to up in *Strategy 1*, while the direction for material changing in *Strategy 2* is just opposite, from up to down. As for *Strategy 3* and *Strategy 4*, the elements become unchangeable from left to right in *Strategy 3*, and from right to left in *Strategy 4*.

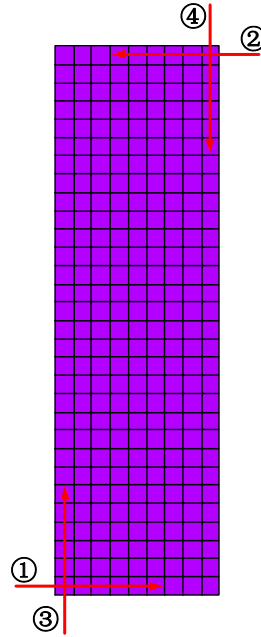


Figure 2.4 : Four different strategies for annealing mechanism A.

- **Mechanism B**

In this annealing mechanism, instead of the annealing equation (2.2), an index to counter the material updating information of all the cells in the design space is introduced and used. If the material attribute of the same element in

the design domain is not changed for consecutive α times, the material attribute of this element is fixed, namely this element will keep unchangeable for the residual iterations. Here, α is a parameter predefined by the user. In this study case, α is set to be 7.

Similar to the termination criterion based on **Mechanism A**, if the changeable number of the elements in the design domain is less than 1, the iterative procedure will be stopped.

2.2 A combined Tabu-ON/OFF methodology

Due to its easiness in implementation and ability to deal with large number of variables, ON/OFF method has been widely applied to the design of topology optimization. However, the ON/OFF method has its own limitation and deficiency. To be specific, the finite difference method for approximation of the gradient information will inevitably lead to inaccuracy; the other disadvantage is that the method is prone to be trapped into a local optimal.

On the contrary, as a stochastic algorithm Tabu search algorithm has a good global searching ability, but a slow convergence speed. For the topology optimization problem with a good number of design variables, these stochastic algorithms are hardly to be adopted directly since its slow convergence and costly computation burden.

To fully take advantages of ON/OFF method and Tabu algorithm, a Tabu-ON/OFF combined methodology is proposed.

2.2.1 Tabu searching algorithm

Tabu search algorithm is a stochastic heuristic searching algorithm. It is first proposed by *F. Glover* [140] to solve the combinational optimization problems and now been widely applied to various kind of combinational optimization problems [141]~[144]. For a minimization problem $\min f(\mathbf{x})$ ($\mathcal{R}^n \rightarrow \mathcal{R}$), the procedures of a basic Tabu algorithm can be simplified as:

1. Define the objective function $f(\mathbf{x})$, and the domain of definition \mathcal{D} ;

2. Define the neighborhood of point \mathbf{x} , namely $N(\mathbf{x})$. Randomly generate a series of new points $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_p$;
3. Calculate the objective function $f(\mathbf{x}_i)$ of new points which satisfies the domain of definition D and choose the “best” point \mathbf{x}^* (for minimization problem, $f(\mathbf{x}) = \min[f(\mathbf{x}_1), f(\mathbf{x}_2), \dots, f(\mathbf{x}_p)]$);
4. Set $\mathbf{x} = \mathbf{x}^*$, repeat 2 to 4 until a stop criterion is satisfied.

As a heuristic searching algorithm, in Tabu searching algorithm, each “movement” operation only chooses the best solution \mathbf{x}^* in a series of trial solution $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_p$ generated in the neighborhood of the current solution \mathbf{x} as the next initial point, and it does not require that new point \mathbf{x}^* must be “better” than \mathbf{x} (for minimization problem, $f(\mathbf{x}^*) < f(\mathbf{x})$), equipping a Tabu searching algorithm with the global searching ability [141].

2.2.2 The proposed topology optimization methodology

The flowchart of the proposed methodology is shown in Figure 2.5. The methodology starts from an initial topology, and then objective function of initial topology is calculated using FEM; Next, a starting searching point is generated randomly, the neighborhood of this point is determined. The objective function values corresponding to the change of material properties of each element in the neighborhood are calculated respectively. The element in the neighborhood which owns the “best” solution is chosen as the next starting searching point; when the stop criterion is satisfied, the methodology is terminated.

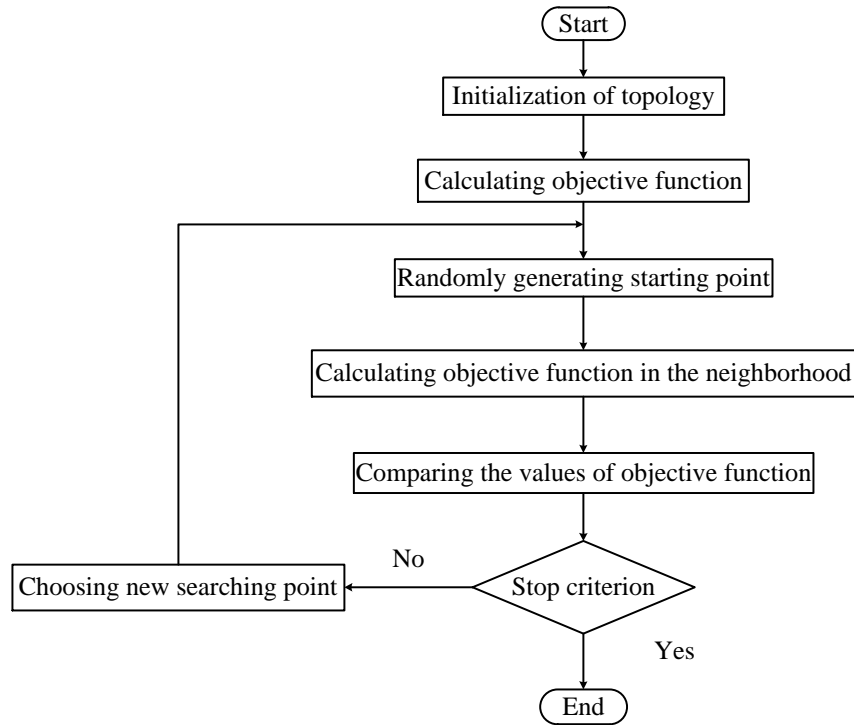


Figure 2.5 : Flowchart of the combined ON/OFF-Tabu methodology.

1. Combined with ON/OFF method

According to the concept of ON/OFF method, the material properties of elements in the design domain are set as design variables. Each element in the design domain possesses two material properties, which is expressed in binary value 0 and 1. More specifically, considering the neighborhood concept and searching mechanism in Tabu algorithm, in the proposed methodology, the elements in the design domain constitute the searching space, and the neighbor elements in geometry space of each element is the neighborhood solution space in the basic Tabu searching algorithm. When calculating the objective function values of the neighbor elements, the material property of each neighbor element is changed: that is, if the current material is air, it is changed to magnetic material and vice versa. At each iteration, the objective function value of each neighbor element is calculated respectively, according to the above mentioned material changing mechanism. Choosing the element corresponding to the “best” objective function value as the starting point for the next search, and the material property of the relevant element is updated at the same time.

2. The design of element neighborhood

To combine the ON/OFF method and discrete Tabu searching algorithm, and consider the region limits of the design domain, the element neighborhood is introduced and de-fined. As shown in Figure 2.6, the neighborhood of elements depending on the location of the

current element (red color) can be classified into three kinds, respectively, 3 neighborhoods, 5 neighborhoods, and 8 neighborhoods (yellow color). In the searching procedure, the type of neighborhood of the current element is firstly determined, and then the objective function values corresponding to the neighbor elements are calculated; finally, the relevant element having the “best” objective function value is selected as the new starting point for the next iteration.

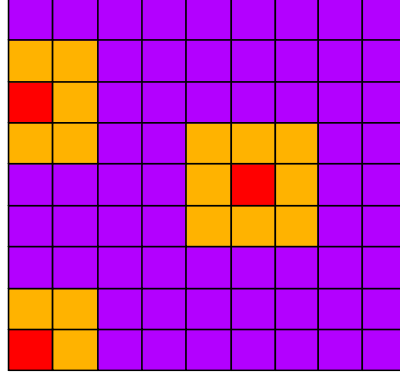


Figure 2.6 : Schematic diagram of neighborhood elements.

3. About the Tabu list

Generally, in Tabu searching algorithm, Tabu list is designed and adopted to prevent the algorithm from repeatedly searching the state space which has been searched. That is, the state points to which the current point is prohibited to go are recorded in the Tabu list, so as to avoid the algorithm being trapped into a local optimal solution.

Nevertheless, in this proposed combined ON/OFF-Tabu methodology, taking into account that the application of the Tabu list may limit the number of effective neighborhoods, and then reduce the number of new states generated from the current state, which worsens the global searching ability of the algorithm [142][145], the Tabu list is removed.

On the other hand, the scale of topology optimization problem is greatly reduced by introducing the ON/OFF method and set the elements in the design domain as searching space. In this way, the computational efficiency of the algorithm is improved. Meanwhile, a reset mechanism is set in the proposed methodology to avoid the searching process being stagnated into the dead loop in the movement operation. Specifically, after some consecutive iterations, a new staring searching position is randomly generated.

2.3 A revised quantum-inspired evolutionary algorithm

Quantum-inspired evolutionary algorithm (QEA) is a probabilistic evolutionary algorithm based on the principle of quantum computing. It is proposed by Han and Kim [146]. It introduces quantum bits and describes the different states of quantum bits with probability. The quantum chromosomes of the population are mutated by using some mutation method of Q-bit, and gradually evolved until the algorithm converges. Although a small size of population is adopted in QEA, it is still able to find the global optimal solution. It has a strong global searching ability. In order to improve the convergence speed of QEA algorithm and achieve a balance between global search (exploration) and local search (exploitation), this section proposes an improved QEA, which realizes the adaptive change of the angle in quantum rotation gate.

2.3.1 Quantum-inspired evolutionary algorithm

Quantum bit coding is employed in QEA and a quantum bit can be expressed as [147],

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (2.3)$$

where, $|0\rangle$ and $|1\rangle$ represent two different quantum states, α and β are both complex numbers. A q-bit may be in the states of $|0\rangle$ or $|1\rangle$, or any superposition of the two states, with the probability of q-bit respectively indicating that q-bit is at $|0\rangle$ or at $|1\rangle$. In a QEA, encoding is based on q-bit; specifically, a quantum bit is defined by a pair of complex numbers. A quantum chromosome of length l can be expressed as

$$\mathbf{q} = \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_l \\ \beta_1 & \beta_2 & \cdots & \beta_l \end{bmatrix} \quad (2.4)$$

where, α_i and β_i satisfy the normalization condition, namely, $|\alpha_i|^2 + |\beta_i|^2 = 1, i = 1, 2, \dots, l$. Using a quantum bit encoding, a chromosome of length l can simultaneously represent the superposition of 2^l candidate solutions. On the contrary, a traditional binary coding can only indicate a certain candidate solution, and therefore QEA using quantum coding could still guarantee the diversity of the population even when the population size is small compared to other evolutionary algorithms.

By using the random observation of a quantum chromosome, a binary state \mathbf{x} is obtained, that is, $\mathbf{x} = [x_1, x_2, \dots, x_l]$ (where x_i is 0 or 1). In the actual process of implementation, a random number - rnd between 0 and 1 is generated for each q-bit, $[\alpha_i, \beta_i]^T$. If $rnd < |\beta_i|^2$, then $x_i = 1$, otherwise, $x_i = 0$.

In a QEA, the quantum chromosomes are evolved and updated by using the rotation gate. The i^{th} bit of \mathbf{q} is updated by the following equation,

$$\begin{bmatrix} \alpha_i' \\ \beta_i' \end{bmatrix} = \begin{bmatrix} \cos \Delta\theta_i & -\sin \Delta\theta_i \\ \sin \Delta\theta_i & \cos \Delta\theta_i \end{bmatrix} \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} \quad (2.5)$$

where, $[\alpha_i', \beta_i']^T$ is the i^{th} bit q-bit after updated; $\Delta\theta_i$ is the rotation angle, and its value is determined by looking up a table [147]~[149].

Table 2.1 : Looking up table of rotation angle - $\Delta\theta_i$

x_i	b_i	$f(\mathbf{x}) \geq f(\mathbf{b})$	$\Delta\theta_i$	
			$\alpha_i\beta_i > 0$	$\alpha_i\beta_i < 0$
0	0	false	0	0
0	0	true	0	0
0	1	false	θ	$-\theta$
0	1	true	0	0
1	0	false	θ	$-\theta$
1	0	true	0	0
1	1	false	0	0
1	1	true	0	0

In the table, b_i is the i^{th} bit of the current optimal solution \mathbf{b} , $f(\cdot)$ is the fitness function, and θ is the magnitude of $\Delta\theta_i$, whose value is generally between $0.001\pi \sim 0.05\pi$.

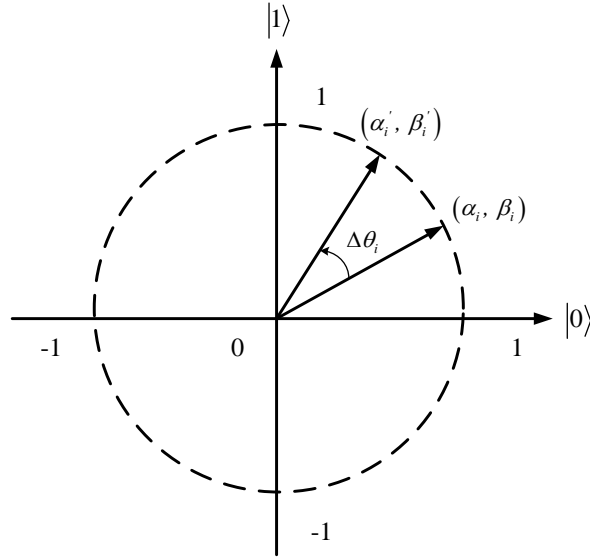


Figure 2.7 : Schematic diagram of rotation gate.

2.3.2 Improvements

1. Adaptive rotation angle strategy

In a traditional QEA algorithm, the rotation angle is determined by using a looking up table. As the quantum chromosome evolves continuously, the algorithm constantly updates the searched global optimal solution. However, since the rotation angle is set as a fixed value: a smaller rotation angle helps to search the solution space meticulously, but it is adverse to the convergence of the algorithm; conversely, although a larger rotation angle is conducive to speed up the convergence of the algorithm, it becomes easier to fall into the local extreme. Based on this observation, an adaptive rotation angle update strategy is proposed with reference to the concept of entropy in information theory [150].

Let the population size be n and the chromosome length be l , the population of the t^{th} generation can be expressed as $B(t) = (B_1^t, \dots, B_j^t, \dots, B_n^t)$, where $B_j^t = (b_1^t b_2^t \dots b_l^t)$ represents the j^{th} individual in binary encoding, and the proposed adaptive rotation angle update strategy can be described as follows,

$$\Delta\theta^{t+1} = e^{\left(1 - \frac{H(B^t)}{H(B^1)}\right)} \cdot \Delta\theta$$

$$H(B_j^t) = \sum_{i=1}^l -q_i^{\alpha^2} \ln q_i^{\alpha^2} - (1 - q_i^{\alpha^2}) \ln (1 - q_i^{\alpha^2}) \quad (2.6)$$

$$H(B^t) = \max(H(B_j^t))$$

In this equation, $\Delta\theta^{t+1}$ is the rotation angle of the $(t + 1)^{\text{th}}$ iteration, $\Delta\theta_{\min}$ is the minimum rotation angle predefined, and $q_i^{\alpha^2}$ represents the probability that the i^{th} q-bit of the chromosome is 0.

Considering the quantum population as an information system, according to (2.6), it is shown that in the initial stage of evolution, the whole system is unstable, the information entropy is large, and hence small rotation angle is adopted. The algorithm can fully exploit the searching space; with the continuous evolution of quantum populations, the system gradually becomes stable, the information entropy becomes smaller, and at this time the rotation angle gradually increases, which helps the algorithm to converge quickly.

2. Variable redistribution mechanism

For the optimization of a design problem, if the number of design variables is too large, the computation cost would be too high and the convergence speed be slow. Here, a design variable redistribution mechanism is proposed, which reorganizes the design variables and the optimization procedure is executed stepwisely to solve this issue. In this way, the computational burden is reduced and the whole optimization process is speeded up. The schematic diagram of the proposed redistribution mechanism is shown in Figure 2.8. The discrete elements are reconstructed after the finite element discretization of design domain. For example, in Step 1, the adjacent design elements (variables) in the design domain are combined with each other to form different super-cells (indicated by different colors), each super cell is regarded as a new design variable, and the proposed algorithm is applied to optimize the topology; in Step 2, the boundary of the optimized topology obtained after Step 1 is detected. The elements adjacent to the boundary are chosen as the design variable to be optimized in the next step; according to this rule, the design area is searched in a continuously refined way, and finally the optimized topology of the entire design area is obtained.

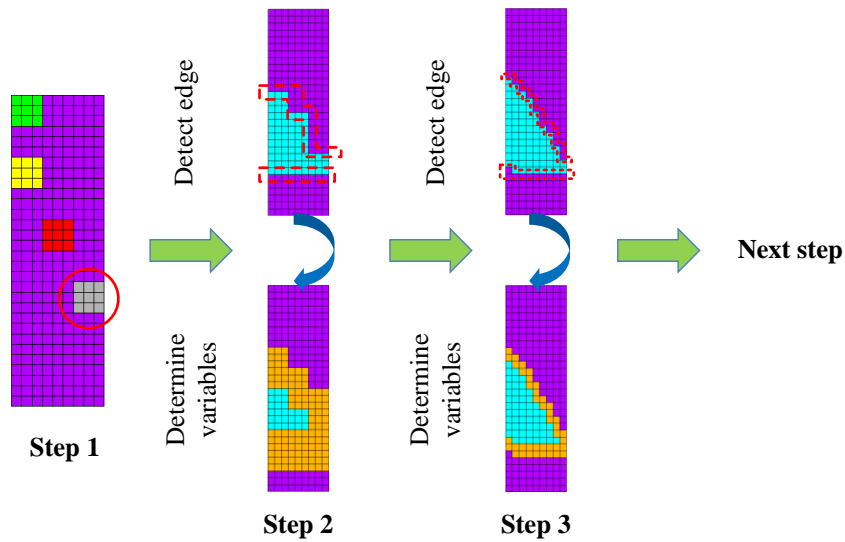


Figure 2.8 : The proposed design variables redistribution mechanism.

2.3.3 Algorithm flowchart

The flowchart of the proposed topology optimization methodology based on the improved QEA algorithm is shown in Figure 2.9. The specific process of quantum observation, objective function calculation, quantum chromosome update and optimal solution update is shown in the right of the figure. Each individual in the quantum population possesses an optimal solution found in its own individual search history, and the entire population has a continuously updated global optimal solution. In the quantum population evolution, neighboring individuals promote local evolution through local learning, which helps to fully search the variable space. The global optimal solution guides all individuals, which helps to speed up the convergence. In the numerical implementation, the global learning is set to be performed once every 20 generations, and the local learning is set to be performed once every 10 generations.

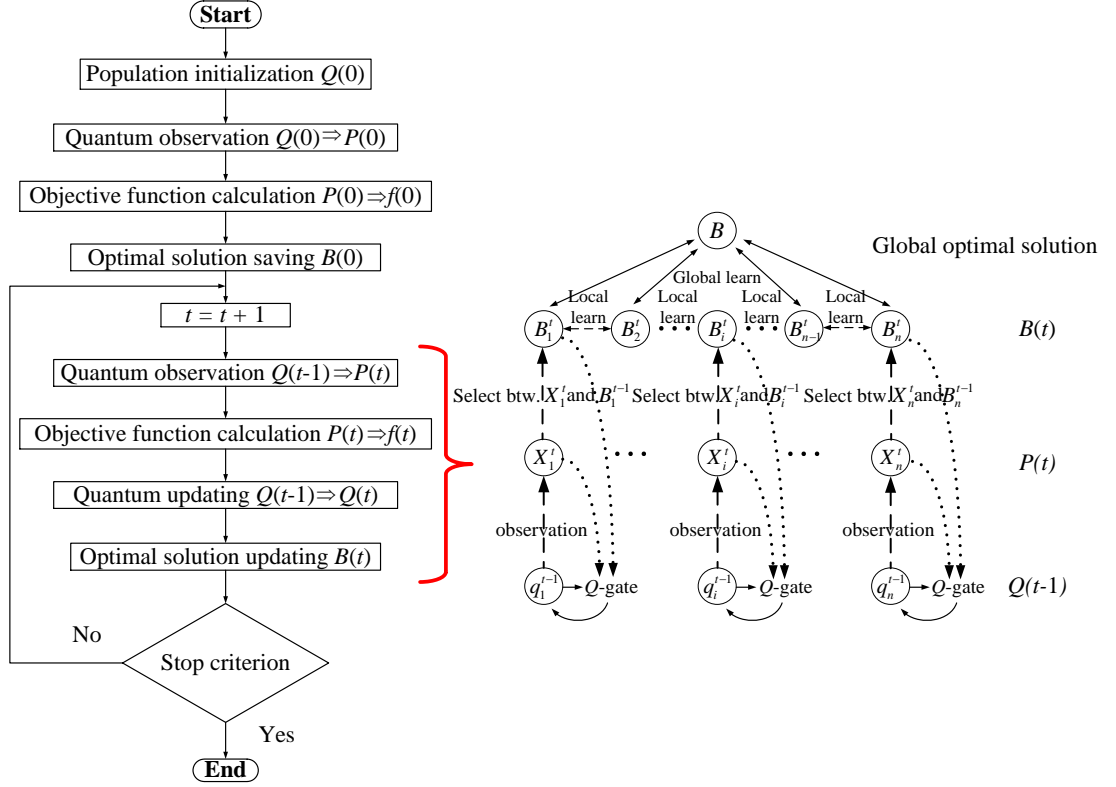


Figure 2.9 : Flowchart of the proposed QEA.

2.4 A revised genetic algorithm

2.4.1 Revised GA

As explained previously, the ON/OFF method is powerful in solving optimization problems with a large number of variables. However, it may converge to a local optimal solution. On the other hand, the stochastic algorithms, such as genetic algorithms (GA), have been recognized as the standards of global optimizers in solving electromagnetic design problems. Nevertheless, such algorithms are sometime inefficient in dealing with topology optimizations with a large number of design variables, since the computational cost is generally extremely high. To fully utilize the advantages of both the ON/OFF method of the fast convergence speed and the GA of the strong global searching ability, a modified GA is combined with the ON/OFF method to develop an efficient global topology optimizer for electromagnetic device designs.

It should be noted that there are two directions in combining an evolutionary algorithm with a local search one in literature. The first direction is to start from a local search method to give a relative better (initial) solution and then to transfer to an evolutionary algorithm to

improve the solution and find finally the global one. And the goal of such efforts is thus to enhance the convergence speed of the hybridized algorithm without worsening the global search ability. The second direction is to use an evolutionary algorithm to roughly locate the attract regions of the global optimal solution and then refine the searches using a local search method. Consequently, the proposed methodology belongs to the first category. Moreover, a methodology in the first category is more efficient for problems where the objective is globally convex, but the minimum is noised by metastabilities, as the case study of magnetic actuator; while an optimal technique in the second category is more dedicated to one rather flat global minimum with few well splitted metastabilities.

Genetic algorithm (GA) is a stochastic searching algorithm, which imitates the Darwin's biological evolution process including genetic selection, natural elimination and other procedures. It is a computation method, which simulates the mechanism of life evolution on the computer. Proposed by Goldberg and Holland [151][152], GA has been applied to nearly all kinds of optimizations of practical problems, and has been recognized as the standard of global optimizers in solving electromagnetic design problems.

A GA starts from the initialization of N randomly generated populations; through reproduction, crossover, mutation and other operators, the populations evolves to be better, until a stop criterion is satisfied. The basic procedures of a GA can be simplified as:

- 1) Initialization: randomly generate the initial population $P(t)$, calculates the objective function values and fitness values;
- 2) Operators: executes reproduction, crossover and mutation procedures to generate new population $P(t + 1)$;
- 3) Termination condition: determine whether the stop criterion is satisfied; if satisfied, the algorithm is terminated; otherwise, set $P(t) = P(t + 1)$, repeat 2,3.

In the proposed methodology, a binary encoded GA is employed to enhance the global searching ability of the methodology. Since the design variable is a discrete one indicating the element material state in ON/OFF method, it is easy to combine the two optimal techniques. In the traditional GA [151][153], the genetic operators are normally composed of selection, crossover and mutation. However, the new population is generated by using a different mechanism in the proposed improved GA.

1. Generation of new population

Inspired from Particle Swarm Optimization (PSO) algorithms [154]~[157], two best solutions, p_{best} and g_{best} are introduced to utilize the individual cognition and social experiences to guide the search process simultaneously. The first best, p_{best} , is the best solution that an individual has ever searched; and the second one, g_{best} , is the best solution that all individuals have so far found. The genes of an individual are updated according to the following rules as implemented using the MATLAB language as:

```

if (rand > max(c1,c2)/(c1+c2))
    newpop(i,j)=gbest(ItNum).bestindividual(j);
elseif (rand > min(c1,c2)/(c1+c2))
    newpop(i,j)=pbest(i).bestindividual(j);
else
    end
    
```

where, $rand$ is a random number between 0 and 1, c_1 and c_2 are two acceleration factors that change nonlinearly as the iteration increases, i represents the i^{th} individual in the population, j indicates the j^{th} gene in the chromosome, $ItNum$ is the current number of iterations. Moreover, c_1 and c_2 vary nonlinearly based on the following equations [158]:

$$\begin{aligned}
 c_1 &= c_{1_min} + (c_{1_max} - c_{1_min}) \times \left(1 - \cos^{-1} \left(\frac{-2 \times (CurrItNum - 1)}{MaxItNum} + 1 \right) / \pi \right) \\
 c_2 &= c_{2_max} - (c_{2_max} - c_{2_min}) \times \left(1 - \cos^{-1} \left(\frac{-2 \times (CurrItNum - 1)}{MaxItNum} + 1 \right) / \pi \right)
 \end{aligned} \tag{2.7}$$

where, c_{1_min} , c_{1_max} , c_{2_min} , c_{2_max} are the minimum value of c_1 , the maximum value of c_1 , the minimum value of c_2 and the maximum value of c_2 , respectively; $CurrItNum$ represents the current iteration number; $MaxItNum$ is the maximum iteration number predefined by the user.

In the iterative procedures, c_1 decreases nonlinearly, while c_2 increases nonlinearly. By adopting this strategy, it is hoped that the algorithm explores the whole space with enough diversity in the initial stage. In this stage, each individual searches mainly based on its own cognition (p_{best}), less affected by social experience (g_{best}). As the iteration advances, the

social experience plays more and more important role, and the algorithm learns more from the group experiences to enhance the convergence speed. Moreover, the purpose for using nonlinear changing of c_1 and c_2 is to guarantee that the generation procedure operates more smoothly, not sharply, to avoid premature convergence.

2. Adaptive mutation probability

Mutation operator plays a main role in keeping the diversity of population in randomly searching stage to avoid premature convergences. While in the stage of local searching, the mutation is hoped to accelerate the convergence speed for finding the global optimum solution. In this regard, an adaptive mutation probability is used in the improved genetic algorithm [159]:

$$p_m = p_{m_0} \left(1.0 - \frac{H}{L}\right)^3 + \alpha \quad (2.8)$$

where, p_{m_0} is the initial mutation probability, H is the hamming distance of the chromosomes between the best individual (with best fitness value) and the worst individual (with worst fitness value) in some iteration, L is the length of the chromosome, α is a constant: In the randomly searching stage, α is set to be 0.005; and in the local optimization stage, α is set to be 0.002.

In this study, p_{m_0} is set as the reciprocal of the length of chromosome. To identify the state of the current searching stage, the following criteria are used, and the corresponding mutation probabilities are updated, as implemented using a MATLAB language form:

```
if fitvalue_ave>threshold_1&&fitvalue_ave<threshold_2
    pm=pm0*(1.0-H/L)^3+0.005;
elseif fitvalue_ave>=threshold_2
    pm=pm0*(1.0-H/L)^3+0.002;
else
    pm=pm0;
end
```

where, threshold_1 and threshold_2 are two constants set beforehand to judge the searching stage, $\text{fitvalue}_{\text{avg}}$ is the average fitness value.

2.4.2 The proposed topology optimization methodology

The whole optimization procedure of the proposed methodology is shown in Figure 2.10. The methodology starts from ON/OFF method to obtain a relatively promising topology, and activates the annealing procedure. After the annealing procedure, the methodology is transferred to the initialization of GA. In this initialization phase, the related parameters are defined and the initial population is generated. It is worthy to be noted that the optimized topology by ON/OFF method is set as one initial individual of GA. Once the initialization phase is finished, the methodology will activate GA to find improved topologies with better performances until a stop criterion is satisfied. After the stop criterion is satisfied, the methodology will run a topology shape check process to modify the optimized geometry shape in order to guarantee its geometry feasibility.

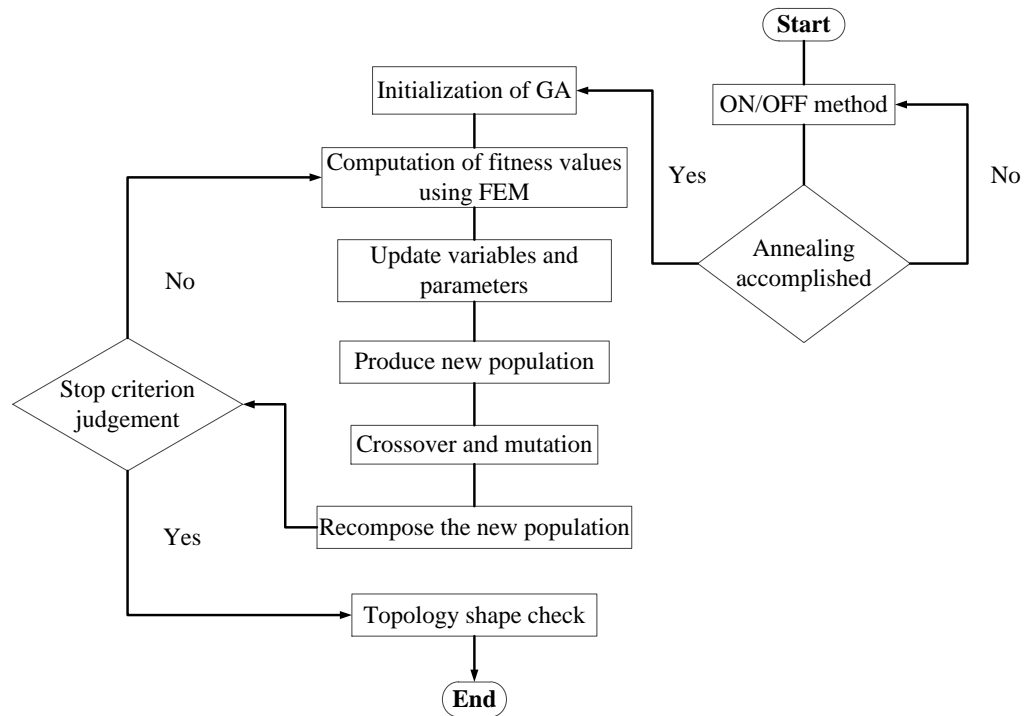


Figure 2.10 : Flowchart of the hybrid method combining ON/OFF method and GA.

2.5 A combined SIMP-RBF method

The solid isotropic material with penalization (SIMP) method has been widely employed in topology optimizations for its effectiveness and low computational cost. However, the finally optimized topology that contains a quite lot of gray elements is often blurry, which is not feasible and cannot be embraced by a designer. In this section a topology optimization methodology based on SIMP and radial basis function (RBF) is proposed. The SIMP is executed to obtain a preliminary optimized topology, and a post-processor based on RBF is then used to smooth the topology of the blurry area of the preliminary optimized topology. To validate the proposed method, a 3D finite element model of piezoelectric energy harvester is developed and the proposed methodology is applied to the topology optimization of the piezoelectric energy harvester.

2.5.1 SIMP model and RBF post-processor

1. SIMP model

Proposed by Bendsøe, Rozvany, Zhou and other collaborators, SIMP has gained its reputation in the field of structural topology optimization and widely applied in the TO of structures, heat transfer, aeroelasticity and multiphysics area [75]. In a SIMP method, a power-law interpolation function is introduced to relax a large-scale discrete variable problem to a continuous one, and then identified the optimization means, iteratively leading the continuous design towards a solid/void solution. To apply the SIMP method to a piezoelectric material, the following interpolation model [20] is adopted.

$$\begin{aligned}\mathbf{C} &= \gamma^{p_1} \mathbf{C}_0 \\ \mathbf{e} &= \gamma^{p_2} \mathbf{e}_0 \\ \boldsymbol{\varepsilon} &= \gamma^{p_3} \boldsymbol{\varepsilon}_0 \\ \rho &= \gamma \rho_0\end{aligned}\tag{2.9}$$

where γ ($0 < \gamma \leq 1$) is the pseudo-density as a design variable which differs in each finite element; \mathbf{C}_0 , \mathbf{e}_0 , and $\boldsymbol{\varepsilon}_0$ are the elasticity, piezoelectric, and permittivity matrices of the piezoelectric material with penalization factors p_1 , p_2 , and p_3 , respectively; ρ is the density of the piezoelectric material.

The appropriate selection of the penalization factor- p is a challenge issue: a too small penalization parameter may cause the optimization process to stagnate; on the other hand, too large penalization factor may results the algorithm to be trapped in a local optimum. In structure topology optimization studies, it is often suggested to slowly increase the value of penalization factor in the optimization procedure. In this study, a smooth decreasing strategy of p is proposed as (2.10). In (2.10), k is the iteration number; p_k is the value of p in k^{th} iteration, p_{max} and p_{min} are the maximum constant value and the minimum constant value, which are given beforehand. The effect of combinations of penalization factor on optimized topology will be analyzed in the next section.

$$p_{k+1} = \begin{cases} p_{max} & k \leq 20 \\ \max(p_{min}, 0.98p_k) & else \end{cases} \quad (2.10)$$

2. RBF post-processor

To smooth the edge of the obtained topology and make it more feasible for engineering production, a post-processor based on RBF is proposed. Given a series of sampling points and the corresponding function values (x_j, f_j) and the radial basis function H , which is a monotonic function of a n -dimensional Euclidean space $\|\cdot\|$, the interpolation function can be expressed as [160]:

$$f(x) = \sum_{j=1}^N c_j H(\|x - x_j\|) \quad (2.11)$$

where $\|x - x_j\|$ is the Euclidean norm between point x and center x_j . c_j is the constant coefficient. And in the proposed RBF post-processor, the multiquadrics function is used,

$$H(\|x - x_i\|) = ((\|x - x_i\|)^2 + h)^\beta \quad (2.12)$$

where h and β are two control parameters. After a series of sampling points are input as the trained samples, the coefficients $c_j (j = 1, 2, \dots, N)$ can be obtained. For a given point x , the reconstructed $f(x)$ can be calculated from (2.11).

The procedure of RBF post-processing is explained as: After the initial optimized topology that contains a lot of intermediate density is acquired by using SIMP methodology, a binarization step is executed to obtain a solid-void topology. In this step, a threshold value is set to filtrate the intermediate density; the boundary of the topology is thus detected and the coordinates of the boundary nodes are calculated to be used as the training samples; The RBF model is then constructed by inputting the training samples, and the boundary is reconstructed by recalculating the boundary nodes coordinates from the just constructed RBF model.

2.5.2 The proposed topology optimization methodology

In the proposed methodology, the SIMP method is firstly employed to obtain an initial optimal topology. 3D finite element analysis and sensitivity analysis are developed, and the optimality criteria (OC) method is employed in the SIMP. The RBF post-processor is executed to smooth the blurry topology. The iterative procedures of the proposed methodology are shown in Figure 2.11.

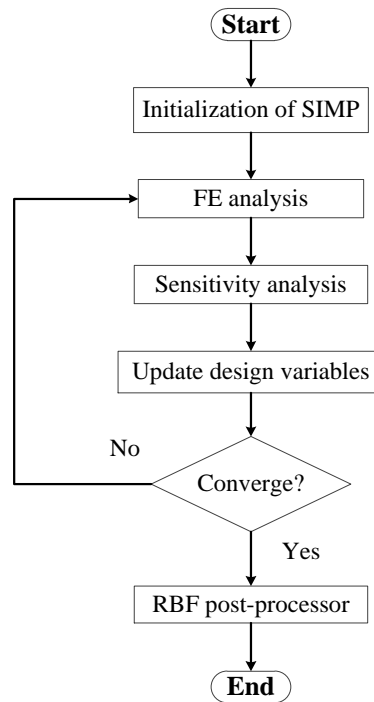


Figure 2.11 : Flowchart of the proposed SIMP-RBF methodology.

2.6 A combined LSM-RBF method

As a boundary based topology optimization method, the level-set method was firstly applied in the field of boundary evolution and image segmentation of multiphase flow [161][162], and has then been widely used in topology optimization of mechanical structures. Compared to the density based methods such as SIMP, one advantage of level set method is that the topology optimization results obtained by the density methods usually contain many intermediate density elements, which require post-processing procedure to analyze the obtained topological structure; while level set method and other boundary based topology optimization methods usually can obtain a topology with a clear boundary. In this section, a topology optimization methodology based on level set method and radial basis function (RBF) is proposed.

2.6.1 Level set method

Level set method is a numerical technique for tracking moving interfaces. It optimizes the topology by gradually moving and merging the initially defined shape. For a given region Ω , a level set function can be described as

$$\psi(\mathbf{p}) \begin{cases} < 0 & \text{if } \mathbf{p} \in \Omega \\ = 0 & \text{if } \mathbf{p} \in \partial\Omega \\ > 0 & \text{if } \mathbf{p} \notin \Omega \end{cases} \quad (2.13)$$

where \mathbf{p} is any point in the design area and $\partial\Omega$ is the boundary of the region Ω . The level set function is solved and updated using the following equation,

$$\frac{\partial\psi}{\partial t} + v|\nabla\psi| = 0 \quad (2.14)$$

where, t is the time variable and v is the scalar velocity field that determines the geometrical motion of the boundary, which is usually determined by the shape sensitivity of the objective function. In the numerical implementation, ψ is initialized to a signed distance function, and the well-known upwind difference format is often applied to solve the level set function.

The topology optimization method proposed in this section is based on a classical level set method [74][163][164]. The level set function ψ is first discretized in accordance with the grid points at the center of the element, satisfying the following,

$$\psi(p_e) \begin{cases} < 0 & \text{if } x_e = 1 \\ > 0 & \text{if } x_e = 0 \end{cases} \quad (2.15)$$

where, p_e is the center of element e in the region Ω . In order to consider the volume constraint, the augmented *Lagrangian* multiplier method [164] is used here.

$$L = \eta(\mathbf{x}) + \lambda_k (V(\mathbf{x}) - V_{req}) + \frac{1}{2\Lambda_k} [V(\mathbf{x}) - V_{req}]^2 \quad (2.16)$$

where, k is the iteration number, $\eta(\mathbf{x})$ is the objective function, λ_k and Λ_k are two parameters, which are updated using the following equation:

$$\begin{aligned} \lambda_{k+1} &= \lambda_k + \frac{1}{\Lambda_k} (V(\mathbf{x}) - V_{req}) \\ \Lambda_{k+1} &= \alpha \Lambda_k \end{aligned} \quad (2.17)$$

where α is a fixed constant between 0 and 1.

The velocity item v is determined according to the gradient direction of the shape sensitivity of the *Lagrangian* term L ,

$$v|_e = -\frac{\delta L}{\delta \Omega} \Big|_e = \frac{\delta \eta}{\delta \Omega} - \lambda_k - \frac{1}{\Lambda_k} (V(\mathbf{x}) - V_{req}) \quad (2.18)$$

2.6.2 Material interpolation and RBF post-processor

As described in the previous section of SIMP-based topology optimization methodology, the same material interpolation model is adopted to calculate the shape sensitivity of the level set function. The general form of the model is given as below,

$$\begin{aligned} \mathbf{C} &= x_e^{p_1} \mathbf{C}_0 \\ \mathbf{e} &= x_e^{p_2} \mathbf{e}_0 \\ \boldsymbol{\varepsilon} &= x_e^{p_3} \boldsymbol{\varepsilon}_0 \\ \rho &= x_e \rho_0 \end{aligned} \quad (2.19)$$

where x_e is the pseudo-density of the element; and definitions of other terms are the same as in the previous section. It is worth noting that, different from the density variable method before, where the design variable is a continuous value between 0 and 1, x_e here takes 1 or a small threshold value (to avoid generating singular matrices during optimization).

In order to obtain the optimized topology with a smooth boundary and make it more suitable for industrial production, the topology optimization method based on the level set method proposed in this section also adopts RBF-based post-processing procedure. Compared with numerical interpolation techniques such as *Lagrangian* polynomial interpolation, sequential linear interpolation and spline interpolation, RBF interpolation can still possess a well fitted curve even with few sampling points; in addition, RBF interpolation is easy to numerically implemented, thus helpful to facilitate a relatively automated post-processing.

The detail of RBF post-processing process is described as follows:

1. Since design variables in the proposed level set method are discrete values, numerical instability or checkerboard phenomenon may occur in the obtained optimized topology results. As shown in Figure 2.12(a), the initial optimized topology obtained by the level set method contains many burrs on the edges. Therefore, after obtaining the preliminary optimized topology, it is necessary to first perform a post-processing to remove the scatter and fill the cavity to obtain a smooth discrete topology containing only material or air. Here a direct mechanism for generating a smooth discrete topology is adopted: assuming that the material property of current element is air (or material), if at least three of its four adjacent elements are material (or air), the material property of the current element is changed to solid (void); Besides this condition, the material property of the element would be not changed. At last, the discrete optimization topology is obtained by extracting the largest connected domain, as shown in Figure 2.12(b);
2. After obtaining the discrete optimization topology, its boundary is detected, the coordinates of the adjacent boundary nodes are calculated, and these coordinates are regarded as a set of training samples for the radial basis function. In order to increase the number of sample points and improve the interpolation effect of RBF, here, the midpoint of the boundary is also counted in the training sample in addition to the boundary endpoint, marked by the cross as shown in Figure 2.13(a);

3. The RBF approximation of the boundary is obtained by using the training sample as an input data, and finally the smoothing boundary is reconstructed from the RBF model just built, as shown in Figure 2.13(b).

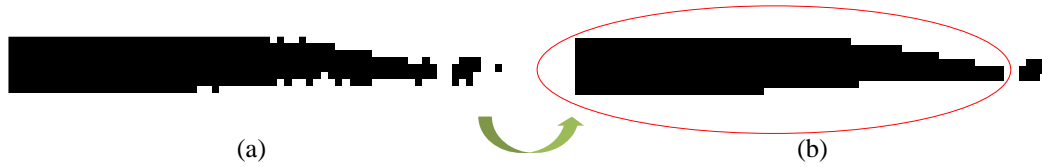


Figure 2.12 : Post-processing procedure of obtaining discrete topology.

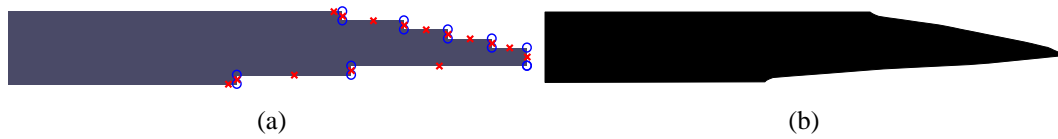


Figure 2.13 : Post-processing procedure of RBF interpolation.

2.7 Chapter summary

This chapter proposes different topology optimization methods. More specially, the discrete topology optimization methods include the ON/OFF method, the combined Tabu-ON/OFF method, the improved QEA based method, and the improved GA based method; a topology optimization methodology based on SIMP and RBF, and a methodology based on LSM and RBF are also introduced.

In the ON/OFF method, virtual materials are introduced and different annealing mechanisms are proposed to enhance the global searching ability and improve the convergence performance. In the combined Tabu-ON/OFF method, ON/OFF method is combined with Tabu search algorithm. The concept of element neighborhood is introduced to adjust to the topology optimization problem and Tabu list is removed. In the improved QEA topology optimization method, the adaptive rotation angle is used to improve the convergence performance. In order to reduce the computational burden, a redistribution mechanism of design variables is designed. To further improve the global optimization ability and avoid falling into the local optima, in the proposed improved GA method, the related concepts of PSO algorithm are introduced and the adaptive mutation probability is adopted.

And in the SIMP-RBF based method, the continuous variation penal factor is used in the corresponding SIMP model and the principle of RBF-post processor is explained in detail.

As for the LSM-RBF based method, discrete formulation of material interpolation model is used, and RBF post-processing procedure is elaborated.

3. Methodology of multi-objective topology optimization

As stated in Chapter 1, the nature of a topology optimization problem can be regarded as a multi-objective optimization problem; and to handle constraint conditions, in existing TO methods like level-set method and density based method, augmented *Lagrangian* multiplier is often used. By this means, the constraints and the original objective function are added together to form a new optimization problem, which inevitably changes the characteristic of the original problem. Besides, parameter tuning is another issue to be considered. Given all this, a MOTO methodology based on a new hybrid MOO algorithm which integrates Non-dominated Sorting Genetic Algorithm II (NSGAI) and Differential Evolutionary (DE) is proposed in this chapter, the framework of the proposed algorithm is elaborated, and the performance of the algorithm is evaluated by typical academic test functions, at last the flowchart of the proposed methodology of MOTO is given. To be noted that, the numerical application will be exhibited in Chapter 4.

3.1 Multi-objective optimization method

Multi-objective optimization, also called multicriteria optimization or Pareto optimization is of an area of multiple criteria decision making, which concerns optimizing more than one objective simultaneously using certain mathematical optimization techniques. In most cases, these multi-objectives are conflicting with each other, no single solution exists that can optimize all the objectives simultaneously; the optimized result is in fact a set of trade-off and compromised solutions between different objectives. In practical life, multi-objective optimization methods have been applied in many fields of disciplines such as finance, economics, civil engineering, chemical process, industrial manufacturing and so on, aiming at making best decision between two or more conflicting goals.

In order to obtain trade-off solutions between multiple conflicting objectives, the early methods mainly construct a new objective that can reflect the influence of multiple objectives simultaneously by certain ways, thus converts the MOO problem into a single-objective one, and then adopt single optimization methods to solve the problem; later, as the development of the intelligent optimization algorithms, a variety of MOO methods based on evolutionary algorithms have emerged, which can optimize multiple objectives at the same time.

3.1.1 Classical MOO method

As a priori method, a classical objective optimization method generally determines the importance degree of each objective (linear weighted sum method) or the bound value of each objective (constraint method) based on experience, and integrates multiple objective functions of an optimization problem into one objective function to convert the original problem into a single-objective optimization (SOO) one.

1. Linear weighted sum method

In solving a MOO problem using a linear weighted sum method, all the objective functions are firstly assigned a weight by some prior knowledge or analysis of the optimization goal, and the weighted sum of all the objectives is then to be optimized. The weighted sum optimal method can be expressed as follows [165]:

$$\min F(\mathbf{x}) = \sum_{i=1}^m w_i f_i(\mathbf{x}) = \mathbf{w} \cdot \mathbf{f}(\mathbf{x}) \quad (3.1)$$

where, $\mathbf{w} = [w_1, w_2, \dots, w_m]$ is the vector of weights, which satisfies,

$$\sum_{i=1}^m w_i = 1 \quad (3.2)$$

After a set of weight vectors is determined, the optimal solution of the original MOO problem under as a certain linear weighted meaning is gained by solving the converted SOO problem. By changing value of the weight vector, different Pareto solutions of the original problem can be obtained.

2. The ε -constraint method

In the ε -constraint method, one “most important” objective function of the m ones is chosen to form a SOO problem according to the needs of the actual application or the characteristics of the problem, and the other $(m - 1)$ objective functions are transformed into

constraint conditions by setting an upper bound (or lower bound), choosing the r^{th} objective function to be optimized [165],

$$\begin{aligned} \min \quad & f_r(\mathbf{x}) \\ \text{s.t.} \quad & f_i(\mathbf{x}) \leq \varepsilon_i, \quad 1 \leq i \leq m, \quad i \neq r \end{aligned} \quad (3.3)$$

where, ε_i is the upper bound constraint of the i^{th} objective function $f_i(\mathbf{x})$. In order to determine the bound value of each objective function, it is usually necessary to optimize each single objective function using some mathematical programming method. In addition, to give decision makers more information, it is also necessary to select different values of r and repeat the above optimization process.

Apart from the above two typical methods, the classical MOO algorithm also include methods such as goal programming method, goal attainment method and others. These methods are straightforward and easy to understand. But the drawbacks are also obvious: first, the weighted class method needs to set different weights to convert the MOO problem into a SOO problem. The determination of reasonable weight values is the main issue. Second, when setting the bound value of an objective, it is often necessary to optimize that objective separately, which increases the computational cost. In summary, for the simple convex optimization problems with prior experience, the classical MOO algorithm is simple and effective. However, for complex non-convex MOO problems, with no prior experience given, the classical MOO algorithm is hard to obtain satisfactory optimization results.

3.1.2 Evolutionary MOO method

In order to overcome the shortcomings and deficiencies of the above classical MOO methods, a variety of MOO algorithms based on evolutionary algorithms are proposed. Compared with the classical ones, evolutionary MOO algorithms optimize objective functions by means of evolutionary intelligent algorithms like genetic algorithm simulating natural biological evolution process, which greatly reduce the dependence on prior experience; besides, evolutionary MOO algorithms are able to optimize multiple objective functions simultaneously and obtain the Pareto solution set of the original MOO problem directly, which is convenient for decision makers to collect information. Since Schaffer [166] proposed the Vector Evaluated Genetic Algorithm (VEGA) in 1985, scholars such as Fonseca, Deb, Horn, Zilter and Knowles have proposed fruitful different evolutionary algorithms. According

to the development process and characteristics of MOO evolutionary algorithms, it can be roughly divided into three generations.

(1) First generation

The first generation began when Goldberg [167] proposed the use of non-dominated sorting methods and niche techniques to solve MOO problems in 1989. The non-dominated sorting method compares the advantages and disadvantages of different individuals of the evolutionary algorithm and preserves the non-dominated individuals to enter the next generation. This kind of survival of the fittest evolutionary thought is conducive to maintain superior individuals, thus ensuring the global optimization ability of the algorithm; the niche technique is mainly used to guarantee the diversity of the population, to avoid the Pareto solution set is clustered round by a certain point. Based on these two ideas, later scholars proposed Multi-objective Genetic Algorithm (MOGA) [168], Non-dominated Sorting Genetic Algorithm (NSGA) [169] and Niche Pareto Genetic Algorithm (NPGA) [170].

(2) Second generation

The second generation of multi-objective evolutionary algorithms is marked by the proposition of the Strength Pareto Evolutionary Algorithm (SPEA [171], SPEA2 [172]) using the elitism strategy by Zitzler. The algorithm added an external population besides the evolutionary population to preserve the non-dominated individuals (the elite solutions) in the evolutionary process. This idea led to the evolutionary multi-objective algorithms using different elitism strategies. Among which, there were Pareto Envelop based Selection Algorithm (PESA [173], PESAI [174]) with an external population like in SPEA; and without external populations, instead comparing and choosing the non-dominated individuals between the offspring individuals and father ones like in Pareto Archived Evolution Strategy (PAES) [175], and NSGAI [176].

Another feature of the second generation of multi-objective evolutionary algorithm is that it no longer used the niche techniques of fitness sharing to ensure the diversity of the algorithm, but utilized the neighbor information (NSGAI, SPEA2), introduced spatial grid (PAES), or used region concept (PESAI) to guarantee the diversity of the population.

(3) Third generation

Following the prestigious NSGAII and SPEA2, which are still widely used in the field of MOO, researchers have proposed a variety of MOO algorithms with diverse characteristics and specialties for different problems. The third generation of multi-objective evolutionary algorithm can roughly be divided into evolutionary pattern, dominating mechanism, high-dimensional problem and uncertainty consideration four facets: for evolutionary pattern, Coello et al. [177] proposed the Multi-objective Particle Swarm Optimization (MOPSO), which used an adaptive grid mechanism to preserve the external population to make the Pareto front more uniform, and adaptive mutation method to improve the convergence performance. Freschi [178], Tan [179] and Gong et al. [180] applied the principle and model of artificial immune system in MOO, respectively proposed Vector Immune System (VIS) algorithm, Evolutionary Multi-objective Immune Algorithm (EMOIA) and Non-dominated Neighbor Immune Algorithm (NNIA); In improving the dominance mechanism, Laumanns et al. [181] proposed the ε -dominance mechanism that is different from the traditional Pareto definition. Brockhoff et al. [182] proposed a partial dominance concept. Thiele [183], Zilter et al. [184] proposed multi-objective algorithms based on preference, in the iterative process, decision makers gave preference information to guide the evolution; To solve high dimensional MOO problems, Wagner et al. [185] applied the aggregate function method proposed by Hughes [186] to handle the multiple objectives in parallel. While Deb and Saxena [187][188] used principal component analysis to extract the main objectives, abandon redundant ones to reduce the order of the original high-dimensional problem; Aiming at problem with uncertainties, Zhang et al. [189] used a multi-objective PSO algorithm to plan the robot path under an uncertain environment. Mlakar et al. [190] introduced the Gaussian process model and combined DE algorithm to consider the influence of uncertainty factors.

3.2 Basic concepts of the multi-objective optimization

Without loss of generality, a minimization MOO problem with n decision variables and m objective functions can be defined as [191]

$$\begin{cases} \min & y = F(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x})]^T \\ \text{s. t.} & g_i(\mathbf{x}) \leq 0, \quad i = 1, 2, \dots, p \\ & h_i(\mathbf{x}) = 0, \quad i = 1, 2, \dots, q \end{cases} \quad (3.4)$$

where, $\mathbf{x} = (x_1, x_2, \dots, x_n) \in \mathbf{X} \subset R^n$ is the decision vector of a n -dimension Euclidean space; $\mathbf{y} = (y_1, y_2, \dots, y_m) \in \mathbf{Y} \subset R^m$ is the vector of the objective function in a m dimensional Euclidean space; $\mathbf{F}(\mathbf{x})$ is a mapping of the n dimensional decision variable space \mathbf{X} to the m dimensional objective space \mathbf{Y} , i.e. $\mathbf{F}: R^n \rightarrow R^m$, $g_i(\mathbf{x}) \leq 0$ and $h(\mathbf{x}) = 0$ is respectively the inequality constraint and the equality constraint.

3.2.1 Feasible solution and feasible solution set

The decision variable $\mathbf{x} \in \mathbf{X} \subset R^n$ for minimization problem that satisfies the constraint conditions is called a feasible solution, and its mathematical definition is as follows.

Definition 3-1 feasible solution \mathbf{x} and feasible solution set \mathbf{X}_f :

1. \mathbf{x} is a feasible solution to problem (3.4) if and only if \mathbf{x} satisfies all constraints of the problem, i.e., $\{\mathbf{x} \in R^n | (g_i(\mathbf{x}) \leq 0, i = 1, 2, \dots, p) \wedge (h_i(\mathbf{x}) = 0, i = 1, 2, \dots, q)\}$;
2. Let \mathbf{X}_f be a feasible solution set of problem (3.4) if and only if all $\mathbf{x} \in \mathbf{X}_f \subset R^n$ are feasible solutions, i.e., $\{\forall \mathbf{x} \in \mathbf{X}_f \subset R^n | (g_i(\mathbf{x}) \leq 0, i = 1, 2, \dots, p) \wedge (h_i(\mathbf{x}) = 0, i = 1, 2, \dots, q)\}$.

3.2.2 Dominance relation and Pareto frontier

Definition 3-2 strong dominance and Pareto frontier:

Consider a system with a function $\mathbf{F}: R^n \rightarrow R^m$, let \mathbf{X} be the compact set of feasible solution spaces in the n dimensional metric space, \mathbf{Y} is the set of vectors in the m dimensional space, then $\mathbf{Y} = \{\mathbf{y} \in R^m: \mathbf{y} = \mathbf{f}(\mathbf{x}) \in R^n, \mathbf{x} \in \mathbf{X}\}$. Let $\mathbf{y}_1 = \mathbf{f}(\mathbf{x}_1)$ and $\mathbf{y}_2 = \mathbf{f}(\mathbf{x}_2)$, among which, $\mathbf{y}_1 = (y_{11}, y_{12}, \dots, y_{1m})$, $\mathbf{y}_2 = (y_{21}, y_{22}, \dots, y_{2m})$. Let $\mathbf{M} = \{1, 2, \dots, m\}$, we define that \mathbf{y}_1 strictly dominates \mathbf{y}_2 ($\mathbf{y}_1 < \mathbf{y}_2$) if and only for all $i = 1, 2, \dots, m$, $y_{1i} < y_{2i}$ holds, namely,

$$\mathbf{y}_1 < \mathbf{y}_2 \Leftrightarrow \{y_{1i} < y_{2i}, \forall i \in \mathbf{M}\} \quad (3.5)$$

It could also be written as $\mathbf{x}_1 < \mathbf{x}_2$. Then the corresponding non-strict dominance concept is defined as

$$\neg(\mathbf{y}_1 < \mathbf{y}_2) \Leftrightarrow \{y_{1i} \geq y_{2i}, \exists i \in \mathbf{M}\} \quad (3.6)$$

The definition of Pareto frontier is then,

$$P(\mathbf{Y}) = \{\mathbf{y}_1 \in \mathbf{Y} : \{\mathbf{y}_2 \in \mathbf{Y} : \mathbf{y}_2 \succ \mathbf{y}_1, \mathbf{y}_2 \neq \mathbf{y}_1\} = \emptyset\} \quad (3.7)$$

Definition 3-3 weak dominance and weak Pareto frontier:

Given a similar system as above, let $\mathbf{y}_1 = \mathbf{f}(\mathbf{x}_1)$ and $\mathbf{y}_2 = \mathbf{f}(\mathbf{x}_2)$, among which, $\mathbf{y}_1 = (y_{11}, y_{12}, \dots, y_{1m})$, $\mathbf{y}_2 = (y_{21}, y_{22}, \dots, y_{2m})$. Let $\mathbf{M} = \{1, 2, \dots, m\}$, we define that \mathbf{y}_1 weakly dominates \mathbf{y}_2 ($\mathbf{y}_1 \preceq \mathbf{y}_2$) if and only for all $i = 1, 2, \dots, m$, $y_{1i} \leq y_{2i}$ holds, and at least one $i = 1, 2, \dots, m$ exists satisfying $y_{1i} < y_{2i}$, namely,

$$\mathbf{y}_1 \preceq \mathbf{y}_2 \Leftrightarrow \{y_{1i} \leq y_{2i}, \forall i \in \mathbf{M}\} \cap \{y_{1j} < y_{2j}, \exists j \in \mathbf{M}\} \quad (3.8)$$

Also known as $\mathbf{x}_1 \preceq \mathbf{x}_2$. Then the non-dominance concept is defined as

$$\neg(\mathbf{y}_1 \preceq \mathbf{y}_2) \Leftrightarrow \{y_{1i} \geq y_{2i}, \exists i \in \mathbf{M}\} \cup \{y_{1j} \geq y_{2j}, \forall j \in \mathbf{M}\} \quad (3.9)$$

The definition of (weak) Pareto frontier is then,

$$P^*(\mathbf{Y}) = \{\mathbf{y}_1 \in \mathbf{Y} : \{\mathbf{y}_2 \in \mathbf{Y} : \mathbf{y}_2 \succ \mathbf{y}_1, \mathbf{y}_2 \neq \mathbf{y}_1\} = \emptyset\} \quad (3.10)$$

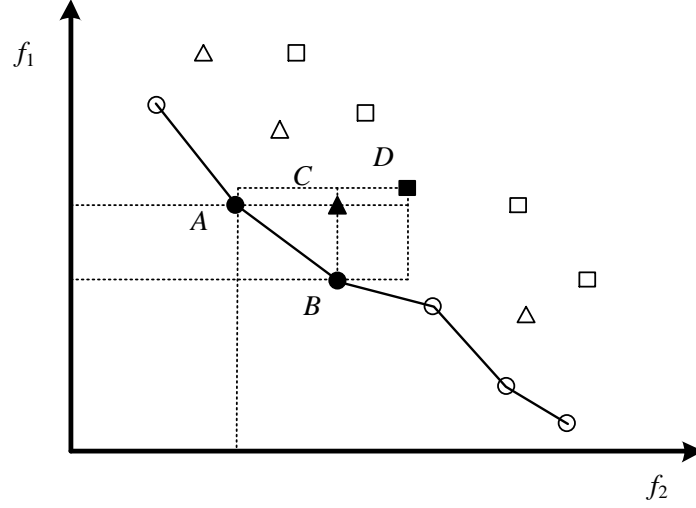


Figure 3.1 : Schematic diagram of Pareto dominance.

A diagram of Pareto dominance is as shown above, in which, $f_1(D) > f_1(A) = f_1(C) > f_1(B)$, $f_2(D) > f_2(C) = f_2(B) > f_2(A)$. From Definition 3-2, Definition 3-3, it can be seen that point D is strictly dominated by points A , B and C , point A weakly dominates point C , point B weakly dominates point C , and A and B are non-dominated by each other.

3.2.3 Performance metrics for multi-objective algorithms

For the MOO, unlike the single-objective optimization, the optimized result is not an optimal solution but a set of solutions; meanwhile, only a finite number of Pareto solutions on the real frontier can be obtained, and the solution sets obtained by different MOO algorithms are generally different. Therefore, it is necessary to determine reasonable metrics to evaluate the performance of multi-objective algorithms. The performance of an MOO algorithm is commonly considered by both the quality of Pareto solution set and the computational cost. The quality of a Pareto solution set is generally evaluated by the degree of closeness between obtained Pareto frontier to true frontier and the uniformity of the distribution of the frontier; and the computational cost is usually measured by the time required by the algorithm. The performance metrics currently used in literature include:

(1) Number of Pareto solutions (NPS)

Give the other circumstances the same, the more number of Pareto optimal solutions is, the stronger the searching ability of the algorithm is, and the more information can be provided for decision makers.

(2) Ratio of Non-dominated Individuals (RNI)

Ratio of non-dominated individuals can be used as a metric to evaluate the dominance of solutions between different solution sets [192], and an example of comparison between two solution sets is given in the following. Suppose that set \mathbf{A} and \mathbf{B} are two different Pareto solution sets of the problem, and $P(\mathbf{S})$ represents the set composed of all non-dominated solutions in the union set \mathbf{S} of \mathbf{A} and \mathbf{B} , that is,

$$P(\mathbf{S}) = \{\mathbf{x}' \in \mathbf{S} : \{\mathbf{x}'' \in \mathbf{S} : \mathbf{x}'' \succ \mathbf{x}', \mathbf{x}'' \neq \mathbf{x}'\} = \emptyset\} \quad (3.11)$$

then the ratio of non-dominated individuals of set \mathbf{A} and set \mathbf{B} can be calculated as,

$$\begin{aligned} ratio(\mathbf{A}) &= \frac{\sum_{\mathbf{x} \in \mathbf{A}} |\mathbf{x} \in P(\mathbf{S})|}{|P(\mathbf{S})|} \\ ratio(\mathbf{B}) &= \frac{\sum_{\mathbf{x} \in \mathbf{B}} |\mathbf{x} \in P(\mathbf{S})|}{|P(\mathbf{S})|} \end{aligned} \quad (3.12)$$

in the above, $\mathbf{x}'' \succ \mathbf{x}'$ indicates that solution \mathbf{x}'' is governed by \mathbf{x}' , and the higher the ratio of the non-dominated individual $ratio(\cdot)$ is, the higher the quality of the solution is. Since NRI indicator does not have closeness under sum operation, generally $ratio(\mathbf{A}) + ratio(\mathbf{B}) \neq 1$. When comparing the quality of multiple solution sets, NRI metric is very intuitive to evaluate and the quality of each solution set.

(3) Convergence metric - Υ

The convergence metric [176] reflects the proximity of the obtained non-dominated frontier Q and the real Pareto frontier P^* . The smaller the value of Υ is, the closer the non-dominated frontier of the algorithm is to the real frontier, namely the better the convergence of the algorithm is.

$$\Upsilon = \frac{\sum_{i=1}^{N_Q} d_i}{N_Q} \quad (3.13)$$

where d_i is the closest distance between the i^{th} non-dominated solution and the real Pareto frontier. N_Q stands for the number of non-dominated solutions in Pareto frontier obtained by the algorithm.

(4) Diversity metric - Δ

Diversity metric [176] reflects the distance between neighboring points in a non-dominated solution set Q , and describes the degree of uniformity of the

distribution of the non-dominated solution set. The smaller the value of Δ is, the more uniform the distribution of Pareto frontier is.

$$\Delta = \frac{d_f + d_l + \sum_{j=1}^{N_Q-1} |d_j - \bar{d}|}{d_f + d_l + (N_Q - 1)\bar{d}} \quad (3.14)$$

where, d_j is the Euclidean distance (objective space) between two neighboring points in the optimized non-dominated solution set, \bar{d} is the average value of the distance between adjacent points; d_f and d_l represent the distance between end point of obtained Pareto frontier Q and that of real Pareto frontier P^* .

(5) Hypervolume indicator - V_H

This is an indicator used to measure the volume covered by the Pareto solution set in the objective domain [193]. Its value is the area of the rectangle surrounded by all the objective function values of non-dominated solutions in the non-dominated solution set and a reference point $f(x^*) = (f_p, f_q)$, as shown in the Figure 3.2. The closer the obtained Pareto solution set is to the real frontier, the larger the corresponding value of hypervolume indicator is. Meanwhile, the more solutions the optimal solution set has or the better the diversity is, the larger the value of hypervolume is.

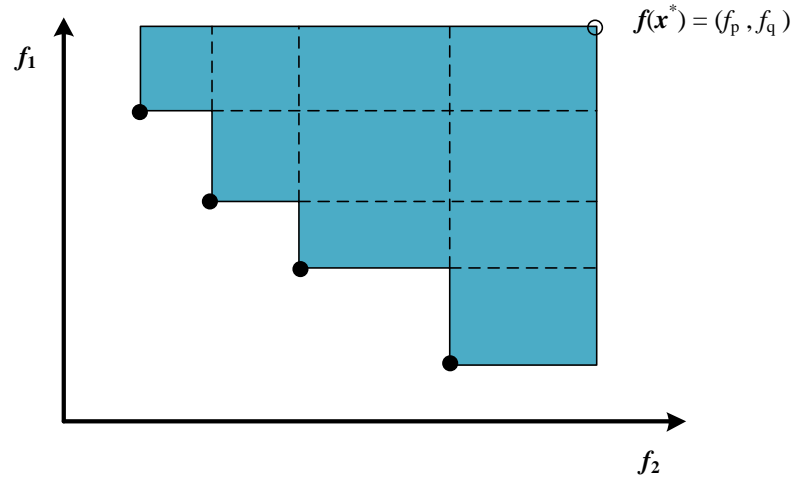


Figure 3.2 : Schematic diagram of hypervolume indicator.

3.3 A new hybrid multi-objective optimization algorithm

Although the NSGAII proposed by Deb [176] and the SPEA2 proposed by Zitzler [172] enjoy a reputation in the field of multi-objective optimization, numerical experiments have demonstrated that when binary coded NSGAII and SPEA2 are used to find the Pareto frontier of some typical multi-objective test functions, their performance is not ideal. Considering that the essence of the topology optimization problem is the problem of determining material distribution of the optimization region, and after the discretization using finite element method, the design domain can naturally be regarded as a binary encoded chromosome; therefore the evolutionary search algorithm is very natural and convenient to be applied. Based on this observation, we propose a multi-objective topology optimization methodology based on a hybrid algorithm which integrates binary encoded NSGAII and DE.

First of all, a large number of literatures have shown that the NSGAII is simple in structure, easy to be implemented, and has strong global search ability. Therefore, the NSGAII is chosen as the main algorithm in the proposed hybrid one and a new mutation operator is introduced to update the population. In this way, the main component being the NSGAII can guarantee that the proposed hybrid algorithm has good global search ability.

Secondly, to further enhance the global optimization ability of the binary encoded NSGAII so as to avoid the algorithm to fall into local optima, a mutation operator is introduced in the hybrid algorithm.

In addition, theoretical studies and numerical experiments show that despite that binary NSGAII has strong global optimization ability, nevertheless, when dealing with complex multi-objective optimization problems, the number of non-dominated solutions in the Pareto frontier is sometimes insufficient. The distribution of the final solutions may also be uneven. In order to find more non-dominated solutions and make the Pareto frontier distribution more uniform, the binary DE is integrated. The DE algorithm was proposed and improved by R. Storn and K. Price [194]. The algorithm is simple, fast, and robust. The main idea of the algorithm is to generate new individuals by performing differential operations on different individuals in the population. Based on this, the hybrid algorithm applies the differential evolution strategy to the elite archive, which enhances the local refinement ability of the algorithm, and thus improves the uniformity of the non-dominated frontier obtained by the algorithm.

Finally, NSGAII itself does not have the elite archive, the hybrid algorithm sets the elite archive solution set with reference to the SPEA2, and updates the so far found non-dominated solutions by continuously maintaining the elite archive; at the same time, uses differential evolution strategy in the elite archive to generate new solutions.

3.3.1 Improved NSGA

The iterative steps of the NSGAII are briefly explained as:

- 1) Initialize the population size $popsiz$, the maximum number of iterations $MaxIt$ and related parameters;
- 2) Randomly generating an initial population P_0 and calculating the objective function value of each individual;
- 3) For the individuals in the initial population P_0 , a population is Q_0 generated by tournament selection, crossover and mutation, objective function values of the individuals in population Q_0 is calculated, and an iteration counter $t = 0$ is set;
- 4) Let $R_t = P_t \cup Q_t$, all individuals in R_t are non-dominated sorted according to the objective function values of individuals, and the crowding distance of each individual is calculated. All the individuals are then divided into different non-dominated front ends $F = (F_1, F_2, \dots, F_i, \dots)$ according to the dominance between them. Find the value i of front-end which satisfies the expression $|F_1| + \dots + |F_{i-1}| < popsiz \leq |F_1| + \dots + |F_i|$, and sort all the individuals according to the crowding distance;
- 5) All the individuals in F_1, F_2, \dots, F_{i-1} are copied into the population P_{t+1} , and the first $popsiz - \sum_{m=1}^{i-1} |F_m|$ individuals in F_i are copied to the population P_{t+1} , as shown in Figure 3.3;
- 6) For individuals in population P_{t+1} , a new population Q_{t+1} is generated by tournament selection, crossover and mutation;
- 7) If $t > MaxIt$, the algorithm terminates, and all non-dominated solutions in P_t are output as optimization results; otherwise, let $t = t + 1$, go to step 4).

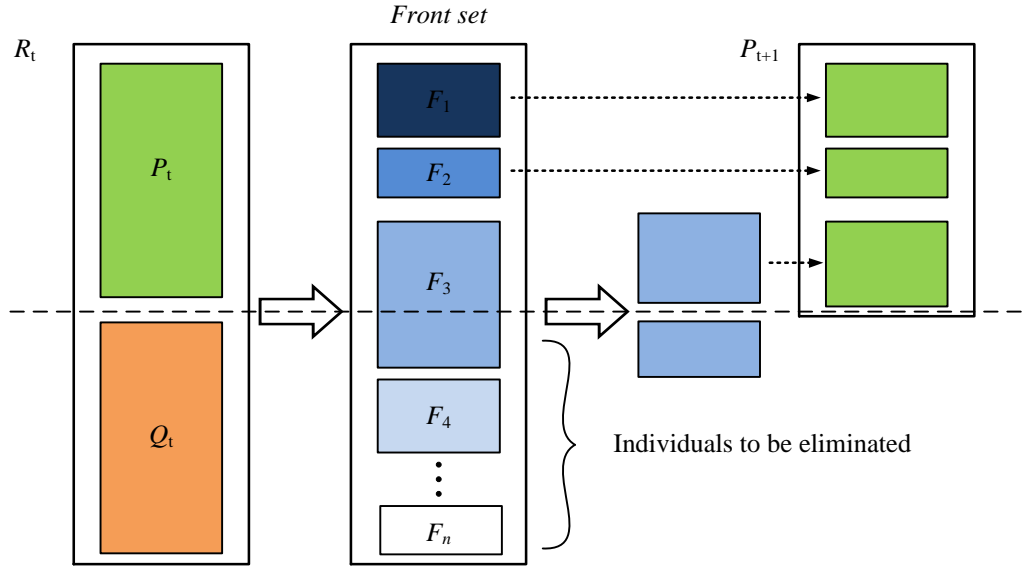


Figure 3.3 : Population updating process in NSGAIL.

1. Non-dominated sorting and crowding distance calculation

Non-dominated sorting is the core of the NSGAIL. It selects the superior individuals in the population to enter the next generation of population. The detailed procedures are described as follows:

- 1) The number of being dominated n_p of all the individuals p in population P is initialized to 0, and the dominating set S_p is initialized to an empty set. Among which, the number n_p is used to record the number of individuals in population P that dominate p , and the set S_p is used to store the individuals in population P that are dominated by p ;
- 2) Compare any two individuals in the population P , update the corresponding number of being dominated and dominating set of each individual, move the individuals with the dominated number $n_p = 0$ into the non-dominated front end F_i , and set the non-dominated front end counter $i = 1$;
- 3) Accessing each solution q in dominating set S_p of the non-dominated front end F_i , and for solution q , set the number of being dominated $n_q = n_q - 1$. Set a new population Q , if $n_q = 0$, then $Q = Q \cup q$;
- 4) If population Q is not empty, then let $i = i + 1$, copy the individuals in the population Q to the non-dominated front end F_i , and go to step 3). Repeat the above process until all the non-dominated fronts are determined.

Non-dominated sorting divides the individuals in a population into different front ends. For individuals at the same front end, it is necessary to sort by calculating and comparing the crowding distance. The crowding distance measures the intensity around the individual in the population. After the crowding distance is sorted, NSGAI algorithm preferentially selects the individuals with fewer surrounding individuals, namely, individuals with large crowding distance is preferred, so that the optimized Pareto front can have a uniform distribution.

The crowding distance $Distance(p)$ of individual p is defined as follows:

$$Distance(p) = \sum_{i=1}^m |f_i(p+1) - f_i(p-1)| / (f_i^{max} - f_i^{min}) \quad (3.15)$$

where, m is the number of objective functions for optimization, $f_i(p+1)$ and $f_i(p-1)$ are respectively the values of objective function of two neighboring individuals corresponding to the individual p . For an individual on the boundary, namely, one of its objective function values is the largest or smallest among all the individuals, which as shown in the Figure 3.4, the crowding distance of individual B at the boundary is set to positive infinity.

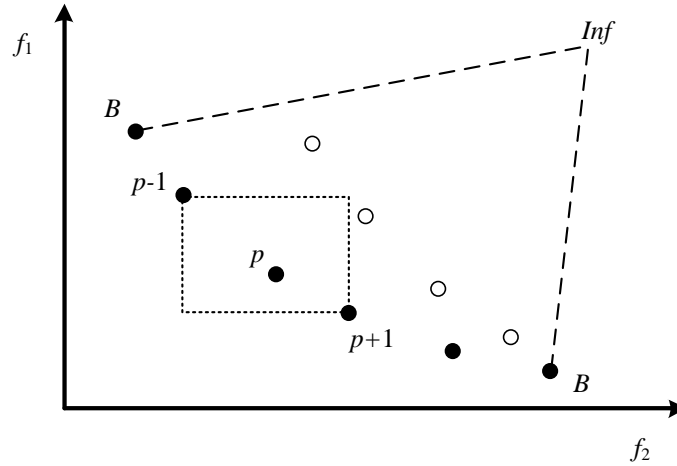


Figure 3.4 : Calculation of crowding distance.

2. Selection, crossover and mutation procedures

NSGAI algorithm primarily generates new population from superior individuals through selection, crossover, and mutation operations. In the improved NSGAI, a tournament selection method is used. First, an individual s is randomly selected from the population and a

random integer $tour$ is generated. Repeat $tour$ times to select another individual s' from the population P_t . If s' dominates s , then let $s' = s$. This process is repeated m times to obtain a population R_t containing m individuals. For all individuals in the population R_t , each two pairs are cross-operated according to the crossover probability to obtain a new population U_t ; then all individuals in the population U_t are mutated according to the mutation probability to obtain a new population Q_t .

It should be noted that the crossover operation adopted here is crossed based on the dimension of the variable, that is, the binary bit string under the same variable dimension of the parent individual is cross-operated; and the mutation operation is to uniformly mutate the chromosomes under all variable dimensions according to the mutation probability.

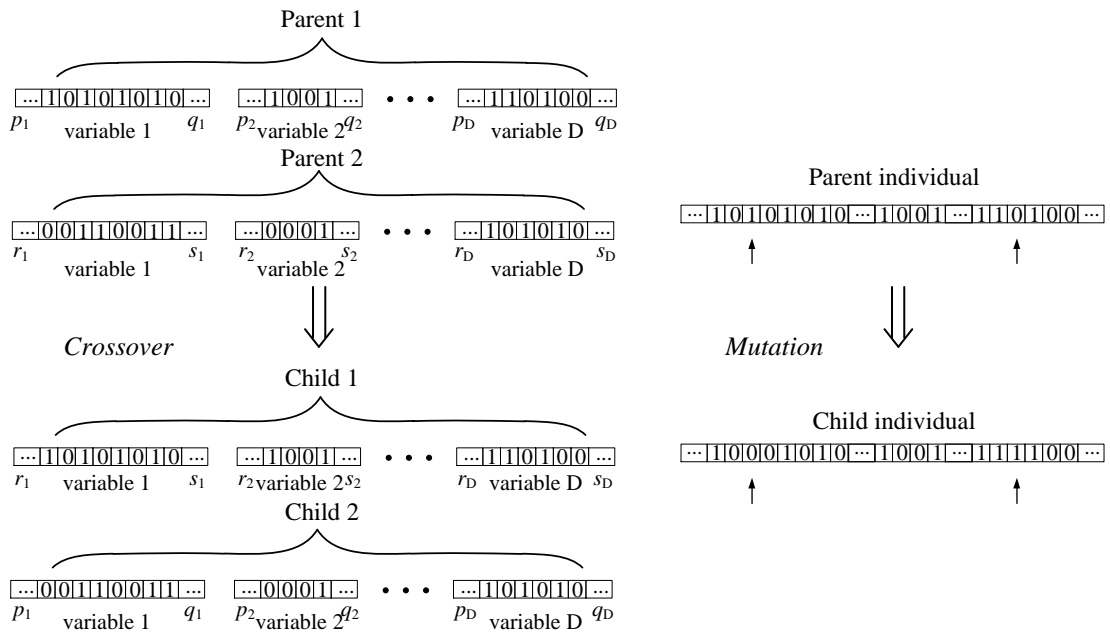


Figure 3.5 : Crossover and mutation operation in improved NSGAII.

In the field of biology, DNA was initially thought to be stable and unchanged. Until the late 1960s, scientists succeeded in extracting jumping genes from bacteria and calling them transposons. Later, the role of transposons in the resistance of bacteria to antibiotics has gradually become known. Almost simultaneously, studies have found that transposons can also produce genetic variation (diversity) in natural populations. These extra chromosomal transposons are not essential for normal life, but they can confer properties such as resistance and toxicity, providing survival advantages under the certain conditions. In fact, nearly 20% of an organism's genome may contain transposons [195].

The proposed mutation operator mimics the way in which transposon variation occurs in natural genetics, adopting an insertion and substitution method. Define the mutation probability of the jump gene operation is P_{jump} , and the individuals in the population are sequentially mutated by the jump gene operator according to the probability P_{jump} . As shown in Figure 3.6, two positions p and q on the original binary chromosomal bit strings are randomly generated, and the chromosomes in the middle of the two positions are replaced with random binary bit strings of the same length.

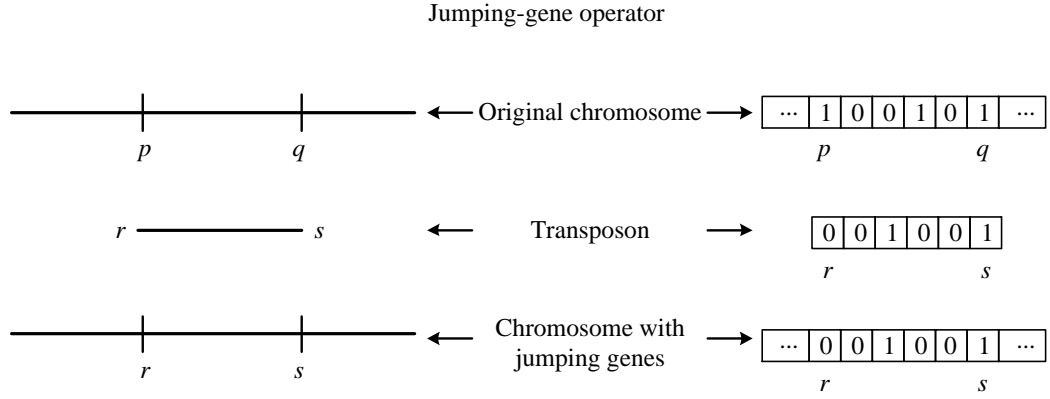


Figure 3.6 : Jumping gene mutation operator.

The improved NSGAI algorithm (hereafter referred to as JNSGA) preserves the non-dominated sorting of NSGAI, the calculation of crowding distance, and the genetic operations of selection, crossover and mutation. However, the elite retention strategy of NSGAI reduces the diversity of population to some extent. Inspired by jumping genes (or transposons) in biology, this hybrid algorithm introduces jumping gene mutation operator to increase the population diversity: thereby improving the overall global search ability of the algorithm.

3.3.2 Binary DE algorithm

In order to deal with the issue of binary coding in the topology optimization problem, it is necessary to handle the continuous variables in the original DE algorithm. The commonly used methods for the discretization of continuous variable in binary DE algorithms are rounding and mapping transformation. The rounding method limits the variables to 0~1, and rounds off to get 0 or 1 discrete variable; the mapping transformation method, with reference to binary particle swarm optimization (BPSO) algorithm uses a secondary search space, using a surjection to convert the variable to the discrete field $\{0,1\}^n$. The first method obtains

discrete variables by rounding, which has low coding efficiency and affects the search performance of the algorithm; the second mapping method needs to introduce other functions to convert continuous variables into discrete ones, and the numerical experiments show that this method is undesirable. In this regard, a parameter-free binary mutation operator [196] is used to form a binary DE algorithm. The mutation operator can directly generate the mutation vector without the variation parameter, and the details are as follows:

$$v_{i,j}^t = x_{r_1,j}^t + (-1)^{x_{r_1,j}^t} \cdot |x_{r_2,j}^t - x_{r_3,j}^t| \quad (3.16)$$

$$j = 1, 2, \dots, D$$

where i represents the i^{th} individual of the mutated population, j represents the dimension of variables, t is the number of the current iteration; $x_{r_1}^t$, $x_{r_2}^t$ and $x_{r_3}^t$ are three individuals different from each other in the population.

Since the value of the variable in the topology optimization is only 0 or 1, the absolute value of the vector difference is also only 0 or 1. It can be seen from the (3.16) that the variable obtained after the mutation is still a 0-1 variable, thus the operation satisfies the closeness. In the variation procedure, whether a certain dimensional variable is mutated depends on the value of the difference vector. When the absolute value of the j^{th} component in the difference vector ($x_{r_2}^t - x_{r_3}^t$) is 1, namely $|x_{r_2,j}^t - x_{r_3,j}^t| = 1$, the corresponding base vector component is mutated (from 0 to 1, or from 1 to 0), to increase the diversity of the population, improve the global search ability of the algorithm, prevent falling into the local optima; when the absolute value $|x_{r_2,j}^t - x_{r_3,j}^t| = 0$, $x_{r_1,j}^t$ remains unchanged, which is good for retaining the information of superior individuals and speeding up the convergence of the algorithm.

The basic flowchart of the BDE algorithm is as follows:

- 1) Randomly generate popsize D-dimensional vectors as the initial population,
 $\mathbf{x}_i^t = \{x_{i,1}^t, x_{i,2}^t, \dots, x_{i,D}^t\}$, $i = 1, 2, \dots, \text{popsize}$;
- 2) Binary parameter-free mutation operator. The variation is carried out bitwise by randomly selecting three different individuals in the population;

$$\mathbf{v}_i^t = \mathbf{x}_{r_1}^t + (-1)^{x_{r_1}^t} \cdot |\mathbf{x}_{r_2}^t - \mathbf{x}_{r_3}^t| \quad (3.17)$$

- 3) Crossover operation. As a supplement to the mutation operator, DE algorithm retains part of the information of the individual \mathbf{x}_i^t , and generates the individuals of child $\mathbf{u}_i^t = u_{i,1}^t, u_{i,2}^t, \dots, u_{i,D}^t$, the specific steps are:

$$u_{i,j}^t = \begin{cases} v_{i,j}^t, & \text{if } rand < CR \text{ or } j = j_{rand} \\ x_{i,j}^t, & \text{otherwise} \end{cases} \quad (3.18)$$

where CR is the crossover probability; j_{rand} is a random integer from 1 to D .

- 4) Selection operation. Use the greedy principle to compare the parent and the offspring, the individuals with the best objective function values constitute the next generation of the population;
- 5) Terminating condition judgment. If the number of the current iteration t exceeds the maximum evolution iteration number t_{max} , the search is stopped and the optimization result is output; otherwise, returning to step 2), the loop is continued.

The proposed binary DE algorithm adopts a parameter-free binary mutation operator, which can directly perform binary mutation according to the difference between individuals. The structure is simple and easy to be implemented. The advantages of DE algorithm relying on the population differences for evolution are well preserved at the same time. By analyzing the evolutionary strategies and characteristics of the real-coded DE algorithm, it can be concluded that the differential operation in the proposed binary DE evolutionary algorithm finds better individuals to evolve by refining the search in the space around the elite solutions. The crossover operation partially retains the information of the original individual and the mutated individual enhances the diversity of the population. This local refinement capability is just needed to be enhanced by the original NSGA.

3.3.3 Flowchart of the proposed algorithm

Based on the improved NSGAII and BDE algorithm, a new hybrid multi-objective optimization algorithm, JNSGA-DE, is proposed. To introduce the binary DE algorithm, the hybrid algorithm sets up the elite solution set *Archive* and continuously updates the

individuals in the *Archive* in the iterations. The search strategy of the hybrid algorithm is: JNSGA explores the whole search space to ensure that the algorithm has a good global search ability; the DE algorithm acts on the current elite solution set *Archive*, searches in the vicinity of the currently found optimal solutions, to enhance the local refinement search ability of the hybrid algorithm. The flowchart of the algorithm is shown in Figure 3.7.

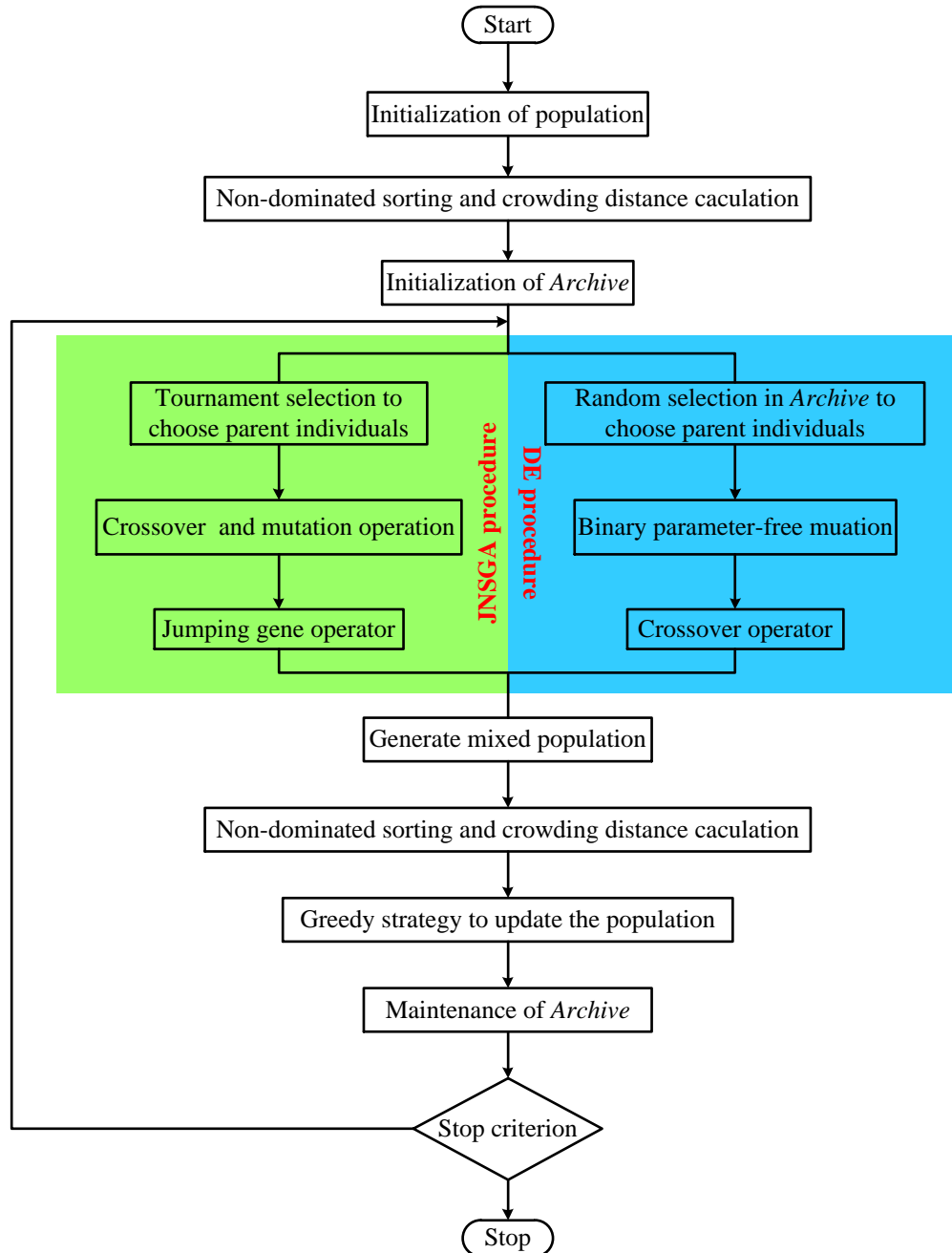


Figure 3.7 : Flowchart of the proposed hybrid algorithm JNSGA-DE.

1. Greedy strategy

The hybrid algorithm uses a greedy strategy to update the population, that is, the individuals in the mixed population of JNSGA and DE are ranked by using non-dominated sorting and crowding distance. The lower the value of non-dominated front end (in Section 3.3.1) is and the larger crowding distance is, the better the individual's rank is. Select the top ranked *popsiz*e individuals to form the next generation of populations. By using the greedy strategy, the genetic information closer to the real frontier is preserved, helping to speed up the convergence of the algorithm.

2. Maintenance of archive

Archive is used as an evolutionary population of the DE algorithm on the one hand and storage of non-dominated solutions to output the final optimization result on the other hand. In the hybrid algorithm, the size of the Archive changes dynamically; as shown in Figure 3.7, in each iteration, after the new population is generated, it is compared with the original elite solutions, and the non-dominated solutions are selected to become the Archive of the next generation. The upper limit of the number of solutions in Archive is set to the size of the population.

3.4 Algorithm performance analysis and validation

3.4.1 Test functions

In order to test the performance of the proposed JNSGA-DE hybrid algorithm and compare it with the existing renowned multi-objective optimization algorithms, it is used to solve the standard test functions as given in [176] and [197]. The characteristics of these test functions and their Pareto Frontiers (PF) are tabulated in Table 3.1.

Table 3.1 : Test functions

$f(\cdot)$	n	Domain	Objective function	Solution	PF
<i>SCH</i>	1	$[-10^3, 10^3]$	$\begin{cases} f_1(x) = x^2 \\ f_2(x) = (x - 2)^2 \end{cases}$	$x \in [0, 2]$	convex
<i>FON</i>	3	$[-4, 4]$	$\begin{cases} f_1(x) = 1 - \exp \left[- \sum_{i=1}^3 \left(x_i - \frac{1}{\sqrt{3}} \right)^2 \right] \\ f_2(x) = 1 - \exp \left[- \sum_{i=1}^3 \left(x_i + \frac{1}{\sqrt{3}} \right)^2 \right] \end{cases}$	$x_1 = x_2 = x_3$ $\left[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right]$	non-convex
<i>POL</i>	2	$[-\pi, \pi]$	$\begin{cases} f_1(x) = 1 + (A_1 - B_1)^2 + (A_2 - B_2)^2 \\ f_2(x) = (x_1 + 3)^2 + (x_2 + 1)^2 \end{cases}$	<i>ref</i>	non-convex discont inuuous

<i>KUR</i>	3	$[-5, 5]$	$\begin{cases} f_1(x) = \sum_{i=1}^{n-1} \left[-10 \exp \left(-0.2 \sqrt{x_i^2 + x_{i+1}^2} \right) \right] \\ f_2(x) = \sum_{i=1}^n (x ^{0.8} + 5 \sin x_i^3) \end{cases}$	<i>ref</i>	non-convex discontinuous
<i>ZDT1</i>	30	$[0, 1]$	$\begin{cases} f_1(x) = x_1 \\ f_2(x) = g(x) \left[1 - \sqrt{\frac{x_1}{g(x)}} \right] \\ g(x) = 1 + 9 \left(\sum_{i=2}^n x_i \right) / (n-1) \end{cases}$	$x_1 \in [0, 1]$ $x_{i \neq 1} = 0$ $i = 2, \dots, n$	convex
<i>ZDT2</i>	30	$[0, 1]$	$\begin{cases} f_1(x) = x_1 \\ f_2(x) = g(x) \left[1 - \left(\frac{x_1}{g(x)} \right)^2 \right] \\ g(x) = 1 + 9 \left(\sum_{i=2}^n x_i \right) / (n-1) \end{cases}$	$x_1 \in [0, 1]$ $x_{i \neq 1} = 0$ $i = 2, \dots, n$	non-convex
<i>ZDT3</i>	30	$[0, 1]$	$\begin{cases} f_1(x) = x_1 \\ f_2(x) = g(x) \left[1 - \sqrt{\frac{x_1}{g(x)}} - \frac{x_1}{g(x)} \cdot \sin(10\pi x_1) \right] \\ g(x) = 1 + 9 \left(\sum_{i=2}^n x_i \right) / (n-1) \end{cases}$	$x_1 \in [0, 1]$ $x_{i \neq 1} = 0$ $i = 2, \dots, n$	convex discontinuous
<i>ZDT4</i>	10	$x_1 \in [0, 1]$ $x_{i \neq 1} \in [-5, 5]$ $i = 2, \dots, n$	$\begin{cases} f_1(x) = x_1 \\ f_2(x) = g(x) \left[1 - \sqrt{\frac{x_1}{g(x)}} \right] \\ g(x) = 1 + 10(n-1) + \sum_{i=2}^n [x_i^2 - 10 \cos(4\pi x_i)] \end{cases}$	$x_1 \in [0, 1]$ $x_{i \neq 1} = 0$ $i = 2, \dots, n$	non-convex
<i>ZDT6</i>	10	$[0, 1]$	$\begin{cases} f_1(x) = 1 - \exp(-4x_1) \sin^6(6\pi x_1) \\ f_2(x) = g(x) \left[1 - \left(\frac{f_1(x)}{g(x)} \right)^2 \right] \\ g(x) = 1 + 9 \left(\sum_{i=2}^n x_i / (n-1) \right)^{0.25} \end{cases}$	$x_1 \in [0, 1]$ $x_{i \neq 1} = 0$ $i = 2, \dots, n$	non-convex uneven distribution

Note: in the table, n represents the dimension of the optimization problem, and in the objective function of *POL* optimization problem,

$$\begin{aligned} A_1 &= 0.5 \sin 1 - 2 \cos 1 + \sin 2 - 1.5 \cos 2 \\ A_2 &= 1.5 \sin 1 - \cos 1 + 2 \sin 2 - 0.5 \cos 2 \\ B_1 &= 0.5 \sin x_1 - 2 \cos x_1 + \sin x_2 - 1.5 \cos x_2 \\ B_2 &= 1.5 \sin x_1 - \cos x_1 + 2 \sin x_2 - 0.5 \cos x_2 \end{aligned}$$

3.4.2 Algorithm verification

1. Test counterparts

In order to verify the performance of the hybrid algorithm, the well-known multi-objective evolutionary algorithms, NSGAII and SPEA2, are used as comparison counterparts.

NSGAII and SPEA2 are programmed according to the relevant literatures. MATLAB platform is used for programming implementation and relevant parameters are set according to the literatures.

Table 3.2 : Algorithm parameters in JNSGA-DE, NSGAII and SPEA2

@ 8 G RAM, 3.4GHz	Hybrid	NSGAII	SPEA2
<i>popsiz</i> e	100	100	100
<i>MaxIt</i>	250	250	250
p_c	0.9	0.9	0.8
p_m	$1/n$	$1/n$	$1/n$
p_{jump}	0.7	—	—
<i>CR</i>	0.5	—	—
<i>Size of Archive</i>	100	—	100

Note: in the table, n is the length of the chromosome. For instance, if the number of design variables is 10, and 30 binary bits are used for each variable, then the length of chromosome is 300.

2. Performance indicators

To evaluate the performance of an algorithm, each algorithm is performed 30 independent runs on each test function for a statistical comparison. The performance indicators of multi-objective algorithms have been described previously, and two statistical analysis indicators are adopted in this section. The first one is the convergence metric Υ , which is used to characterize the distance between the PF frontier obtained by the algorithm and the true PF frontier of the test function; and the second one is the diversity metric Δ , which indicates the uniformity of the distribution of the PF frontier solution obtained by the optimization algorithm, namely, the diversity. The calculation method of the above performance indicators is detailed in section 3.2.3.

The smaller the convergence metric Υ is, the closer is the final solution of an algorithm to the true PF frontier, that means the better the convergence performance of the algorithm is; the diversity metric Δ reflects the space between adjacent solutions. The smaller the metric is, the more uniform the distribution is. For the test functions listed in the table, there are infinite points on the true frontier. For the convenience of calculation, 500 pairs of uniform sampling points on the true frontier are used to represent the true PF .

The two indicators obtained by 30 independent runs of each algorithm are statistically analyzed to evaluate the statistical average performance of the algorithm. Mean values $E(Y)$, $E(\Delta)$, and standard deviations $\sigma(Y)$, $\sigma(\Delta)$ of the indicator Y , Δ are respectively used. The above mean value and standard deviation reflect the value and distribution stability in the statistical sense, respectively. Therefore, they can be used to evaluate the overall performance of the algorithm.

3. Optimization indicator analysis

The optimization results of the three algorithms JNSGA-DE, NSGAI and SPEA2 for different test functions are shown in Table 3.3 and Table 3.4. Table 3.3 compares the convergence indicator of the three algorithms, and Table 3.4 shows the comparison of the diversity indicator. For each algorithm, the mean value and standard deviation of the corresponding indicators when optimizing different test functions are analyzed statistically.

(1) Convergence indicator analysis

a) Comparing JNSGA-DE algorithm with the NSGAI, only in the SCH test function, the convergence value of the later is slightly smaller than the former, and only the difference of 4 digits after the decimal point; on the other test functions, the convergence indicator of the JNSGA-DE algorithm are better than that of the NSGAI, especially for the *ZDT1*, *ZDT2*, *ZDT3*, *ZDT4*, and *ZDT6* test functions, the optimization result of JNSGA-DE is much better than the NSGAI. Not only the statistical mean value of the convergence indicator is smaller, but also the standard deviation is smaller, that means the stability of the former algorithm is better.

b) Compared with the SPEA2, the JNSGA-DE only has a larger mean value of the convergence indicator in *ZDT2*. On the other test functions, the mean and standard deviation of the convergence indicator of the JNSGA-DE algorithm are better, especially for the *ZDT4* function (this function has 21^9 local optima in the whole decision space, and therefore has higher requirement for the global optimization ability of the algorithm), the convergence indicator of the former is much better than that of the latter.

c) It can be seen from the optimization results of the convergence indicator that the overall convergence of the JNSGA-DE is better than the NSGAI and SPEA2

algorithms: that is, the optimized Pareto frontier is closer to the true frontier. At the same time, the JNSGA-DE algorithm is more stable and more reliable.

Table 3.3 : Comparison of statistical results of the convergence indicator

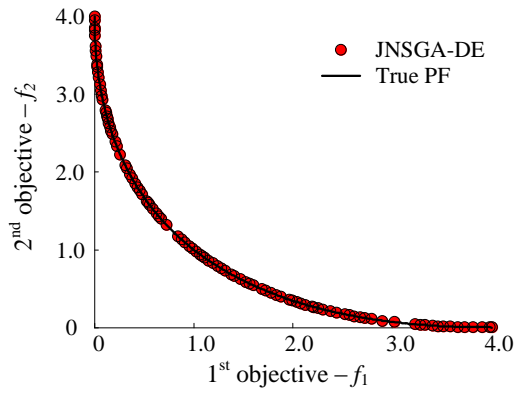
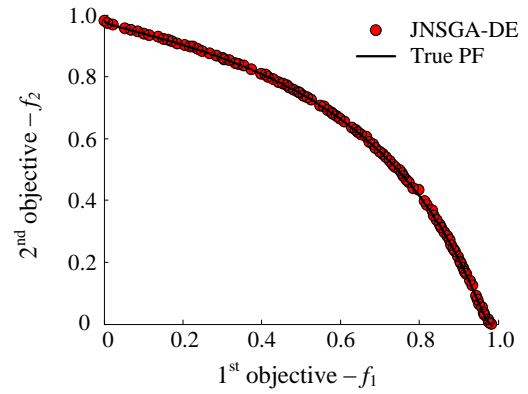
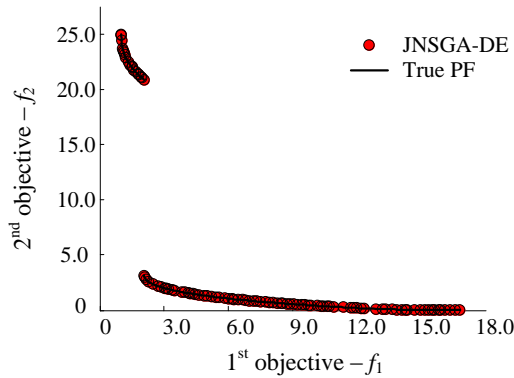
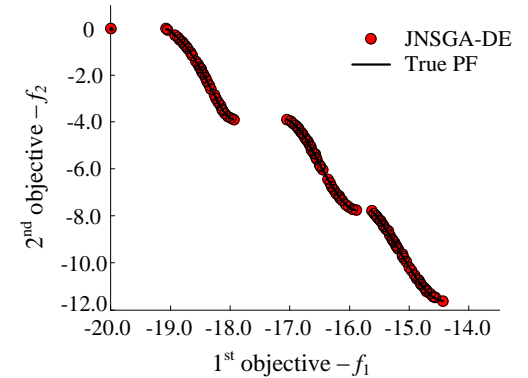
$f(\cdot)$	JNSGA-DE		NSGAI		SPEA2	
	$E(Y)$	$\sigma(Y)$	$E(Y)$	$\sigma(Y)$	$E(Y)$	$\sigma(Y)$
<i>SCH</i>	0.0033	5.94×10^{-5}	0.0032	1.72×10^{-4}	0.0033	2.65×10^{-4}
<i>FON</i>	0.0021	0.0018	0.0074	0.0058	0.0052	0.0035
<i>POL</i>	0.0168	0.0012	0.0347	0.0346	0.0255	0.0210
<i>KUR</i>	0.0125	8.88×10^{-4}	0.0749	0.1438	0.0505	0.1361
<i>ZDT1</i>	0.0190	0.0049	0.1786	0.0711	0.0382	0.0171
<i>ZDT2</i>	0.0761	0.0231	0.5118	0.1776	0.0644	0.0316
<i>ZDT3</i>	0.0095	0.0019	0.0710	0.0346	0.0120	0.0046
<i>ZDT4</i>	0.3902	0.2236	5.6711	2.1459	8.4953	4.8732
<i>ZDT6</i>	0.1901	0.1239	1.4942	0.7083	0.3088	0.1309

(2) Diversity indicator analysis

- Compared with the NSGAI, the JNSGA-DE algorithm has better statistical performance of the diversity indicator on all test functions, especially for the ZDT series problems, the advantage is obvious; while on ZDT2, ZDT4 and ZDT6 problem, NSGAI has smaller standard deviation of the diversity indicator.
- Compared with the SPEA2, for the statistical mean value of the diversity indicator, JNSGA-DE is better than SPEA2 on all test functions; SPEA2 has smaller standard deviation on the ZDT2, ZDT4 and ZDT6 problems. On the other test functions, hybrid algorithm has better performance.
- According to the optimization results of the diversity indicator, although the stability index of the hybrid algorithm is slightly worse (the standard deviation is slightly larger) on the ZDT2, ZDT4 and ZDT6, the uniformity of the Pareto solution obtained by the JNSGA-DE algorithm is better overall, the optimized Pareto frontier is more uniform.

Table 3.4 : Comparison of statistical results of the diversity indicator

$f(\cdot)$	JNSGA-DE		NSGAII		SPEA2	
	$E(\Delta)$	$\sigma(\Delta)$	$E(\Delta)$	$\sigma(\Delta)$	$E(\Delta)$	$\sigma(\Delta)$
<i>SCH</i>	0.3612	0.0084	0.4430	0.0350	0.8311	0.0891
<i>FON</i>	0.2906	0.0300	0.4867	0.0700	0.4336	0.0910
<i>POL</i>	0.9475	0.0175	0.9654	0.0245	1.0285	0.0415
<i>KUR</i>	0.4017	0.0248	0.5538	0.0570	0.6154	0.0598
<i>ZDT1</i>	0.3454	0.0912	0.7495	0.1833	0.6389	0.1068
<i>ZDT2</i>	0.6948	0.2564	1.0171	0.1472	0.7013	0.1820
<i>ZDT3</i>	0.6631	0.0776	0.8913	0.1361	0.9039	0.0832
<i>ZDT4</i>	0.6465	0.1313	0.9219	0.0440	0.9693	0.0247
<i>ZDT6</i>	0.6301	0.1240	1.0606	0.0760	0.9720	0.0955

SCH test function*FON* test function*POL* test function*KUR* test function*ZDT1* test function*ZDT2* test function

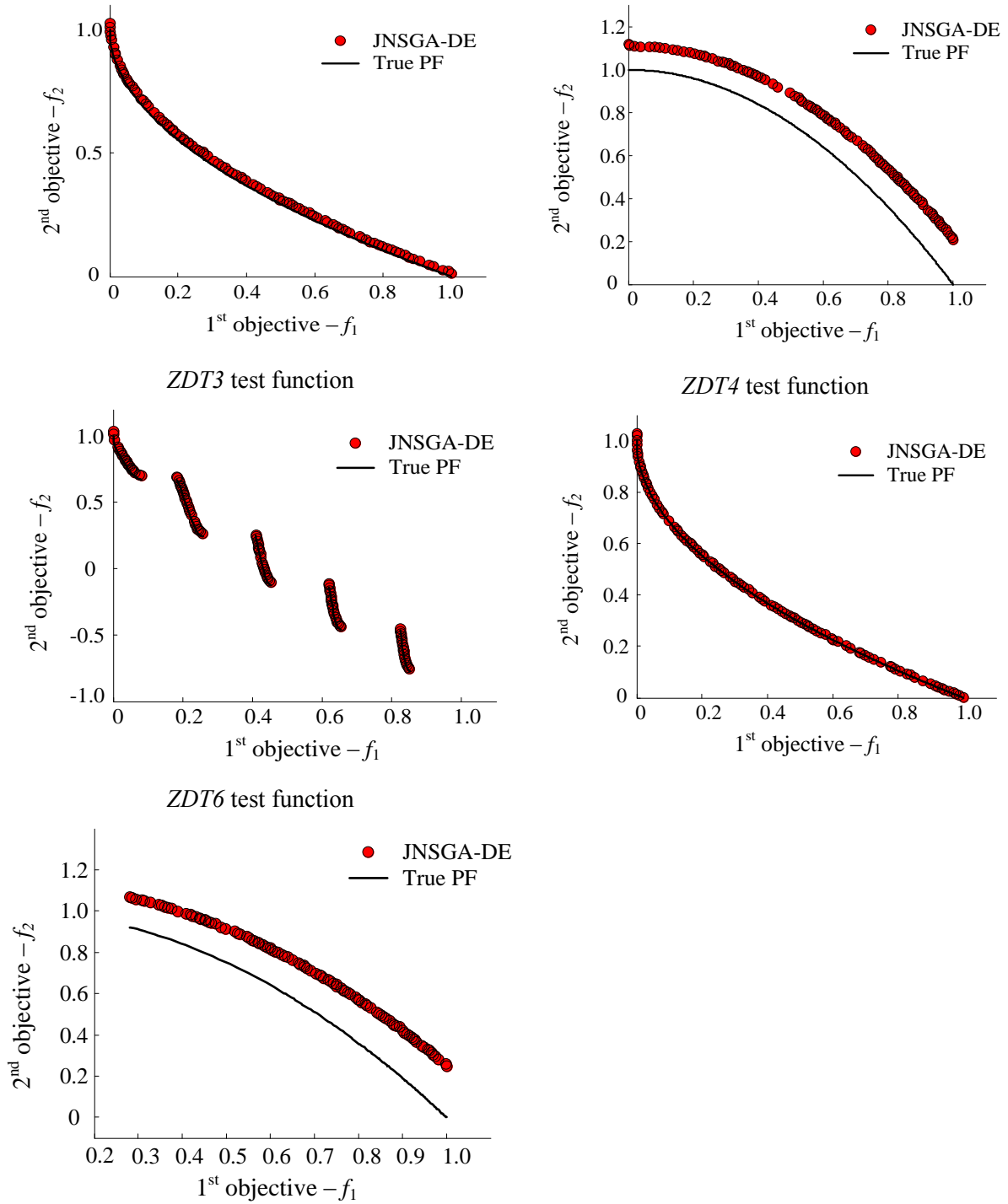


Figure 3.8 : Optimization results of different test functions by JNSGA-DE.

4. Conclusion

The optimization results of the test functions demonstrate that the proposed JNSGA-DE hybrid algorithm effectively combines the advantages of NSGAII and DE. By introducing the jump gene mutation operator, the global search ability of NSGAII is further enhanced. At the same time, the introduction of the DE algorithm effectively improves the distribution

uniformity of the obtained non-dominated solutions. For the 9 test functions with different Pareto frontier features, the hybrid algorithm has strong global and local search ability, especially for the *ZDT4* test function with a large number of local optima, the performance of the hybrid algorithm is much better than those of the NSGAI and SPEA2.

3.5 A multi-objective topology optimization methodology

Based on the proposed hybrid MOO algorithm, a MOTO methodology is constructed. The flowchart of the TO method is presented in Figure 3.9. The whole optimization process can be explained as below,

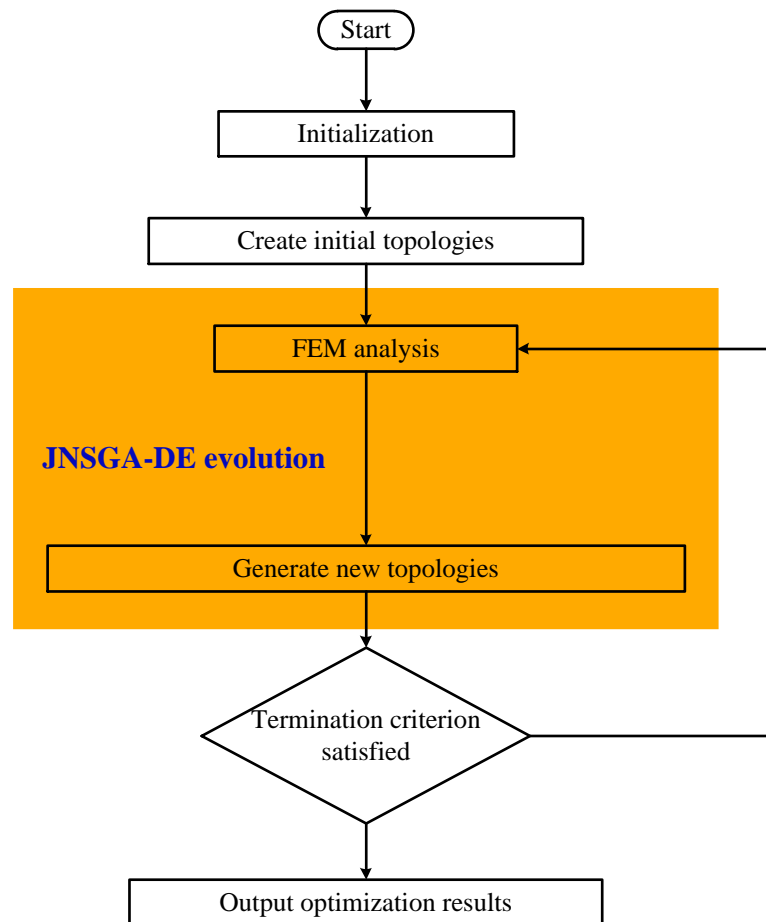


Figure 3.9 : Flowchart of the proposed MOTO methodology

1. Variables are declared, storage memory is prepared, and relevant parameters are set;
2. The population is randomly sampled to generate initial topologies;

3. Finite element analysis is carried out to calculate the values of multiple objectives corresponding to each topology;
4. New topologies are generated by executing the evolution process proposed in JNGSA-DE algorithm;
5. Check if termination criterion is satisfied;
6. The optimization is terminated and meanwhile the MOTO results are output.

3.6 Chapter summary

This chapter first briefly reviews multi-objective optimal algorithms; introduces the related terminologies, the fitness assignment, and enumerates several common performance indicators for evaluating multi-objective algorithms. A hybrid algorithm–JNSGA-DE by integrating NSGA and DE algorithms is then proposed. The jumping gene mutation and DE evolution strategy in the hybrid algorithm are explained in detail. The performance of the hybrid algorithm is tested on 9 test functions with different Pareto frontier distributions. And in the end, a multi-objective topology optimization methodology is proposed.

The numerical results of academic test functions indicate that the proposed hybrid algorithm has good global and local searching ability. Especially in the cases of non-convex and uneven distributed Pareto frontiers (such as *ZDT4* and *ZDT6*), the performance of the proposed algorithm is better than those of NSGAI and SPEA2. Considering that the actual electromagnetic topology optimization design problem is mostly non-convex, the introduction of the constraint may make the Pareto frontier of the objective function space unevenly distributed. Therefore, the proposed multi-objective topology optimization methodology is quite suitable for the topology optimization design of electromagnetic devices.

4. Numerical applications

4.1 Case study 1 : single-objective topology optimization

Electromagnetic actuators convert electrical and mechanical energy one into another. The energy conversion takes place in the so-called air gap separating the stationary part (stator or fixed contact) and moving part (rotor or moving contact) of the actuator. It is widely used in the mechanical manufacturing industry from precise control using small actuators to the quite large powerful units using electrical drives. Therefore, it is quite meaningful to optimize the topological shape of the electromagnetic actuator to improve its performance by using a topology optimization technology. In this chapter, the proposed ON/OFF method, the combined Tabu-ON/OFF method, the improved QEA method and the improved GA topology optimization method are applied to the topology optimization of a prototype electromagnetic actuator. The optimization goal of this study is to maximize the electromagnetic force of the armature in a specified direction by optimizing the topology under a fixed input power.

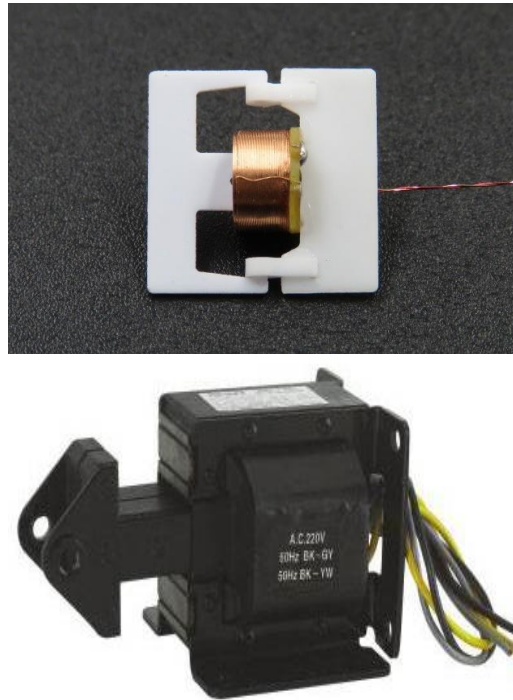


Figure 4.1 : An electromagnetic actuator [198].

4.1.1 Solid model

The initial geometry of an electromagnetic actuator commonly used in engineering applications is shown in Figure 4.2, which utilizes an iron core surrounded by electrified coils to attract the armature or hold a mechanical workpiece in a fixed position. The armature and the core are ferromagnetic material and the relative magnetic permeability of the armature is 2000 and that of the core is 1000. The core is wound by a coil of 1A current with 420 turns. The air gap between the armature and the right end of the core is fixed at 2 mm. The design domain of the electromagnetic actuator is shown in Figure 4.3.

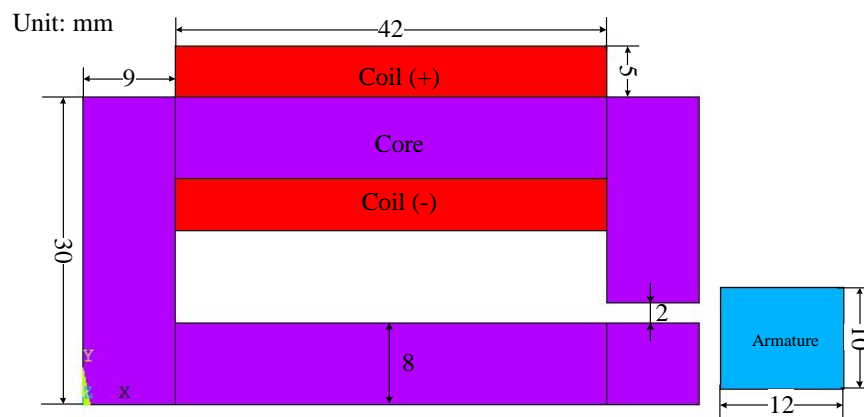


Figure 4.2 : Initial geometry of the electromagnetic actuator.



Figure 4.3 : Design domain of the electromagnetic actuator.

4.1.2 Mathematical formulation

The actuating force is determined using a virtual magnetic energy method from the finite element solution. Therefore, the mathematical model for the magnetic actuator topology optimization can be formulated as [64]:

$$\begin{aligned}
 &\text{Maximize } F = -\frac{\partial W_{\text{magnetic}}}{\partial x} \\
 &\text{Subject to } \sum_{e=1}^N V_e < V_{\text{initial}}
 \end{aligned} \tag{4.1}$$

where V_e is the volume of each element, N is the number of the total elements of magnetic material in the design domain. V_{initial} is the volume of the design domain in the initial model.

4.1.3 Numerical results

1. ON/OFF method

For the sensitivity analysis, adjoint variable methods can generally be used [42][199]. However, the forward finite difference approximation is used in this paper for versatility:

$$\frac{dF}{dp} = \lim_{\Delta p \rightarrow 0} \frac{\Delta F}{\Delta p} \approx \frac{F(p + \Delta p) - F(p)}{\Delta p} \tag{4.2}$$

where p is the design variable (here is the permeability of an element) and Δp represents the perturbation of p . The performance parameter F is computed by using finite element analysis.

Under the previous conditions, a prototype magnetic actuator is optimized using the proposed methodology. To demonstrate the performances of different annealing mechanisms, the proposed methodologies using different annealing mechanisms and strategies, Traditional ON/OFF method (*Traditional*), Strategy 1 under the annealing mechanism A (*Strategy 1_A*), Strategy 2 under the annealing mechanism A (*Strategy 2_A*), Strategy 3 under the annealing mechanism A (*Strategy 3_A*), Strategy 4 under the annealing mechanism A (*Strategy 4_A*), and the annealing *mechanism B*, other conditions being equal, are experimented on this case study. The optimized topology is shown in Figure 4.4 and Figure 4.5 depicts the

corresponding magnetic flux lines. The optimized results and performances of the methodology in different annealing strategies and mechanisms are given in Table 4.1. The trajectory of the absolute value of the objective function is shown in Figure 4.6.

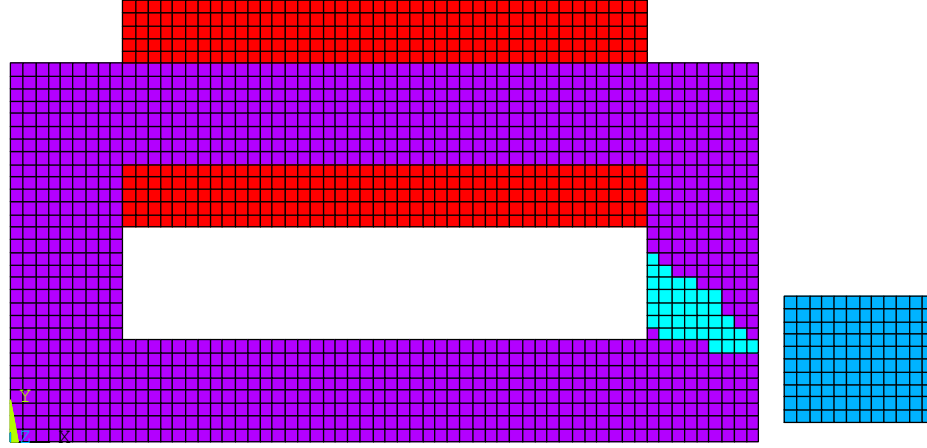


Figure 4.4 : Optimized topology by ON/OFF-finite difference method.

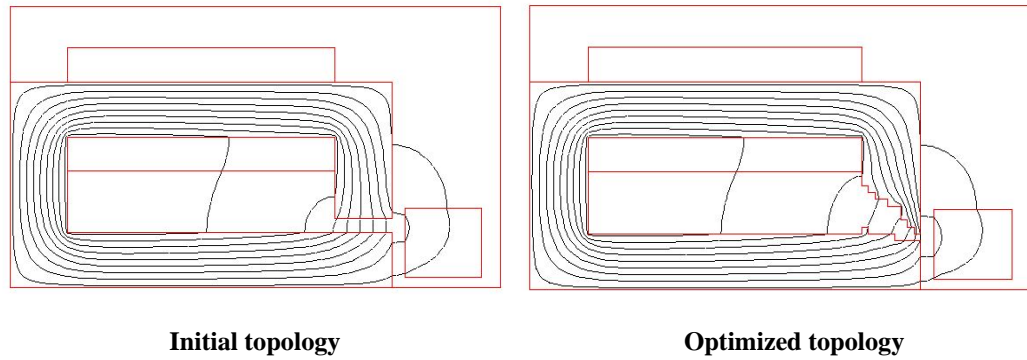


Figure 4.5 : Magnetic flux lines of initial and optimized topology.

Table 4.1 : Comparison of objective function using different annealing mechanisms

Annealing mechanism	Objective function (N/m)	CPU time (s)
Traditional	-49.22	9385.783
Strategy 1_A	-52.90	10153.509
Strategy 2_A	-52.90	10138.097
Strategy 3_A	-52.90	10178.531
Strategy 4_A	-52.90	9637.734
Mechanism B	-52.90	5842.414

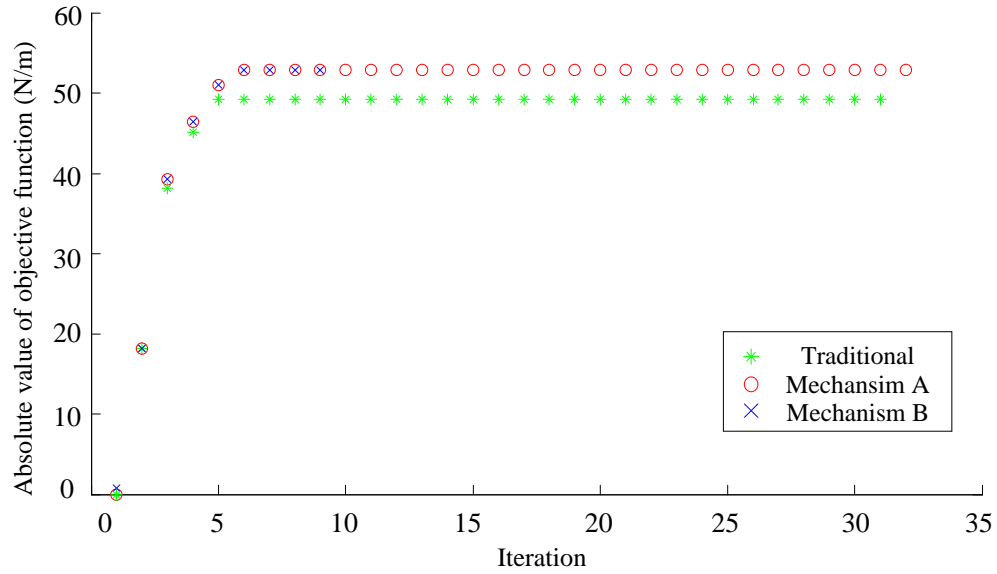


Figure 4.6 : Trajectory of objective function.

From the numerical results, it can be observed that,

- a) The actuating forces for the initial and optimized topologies are, respectively, 42.67 and 52.90 (N/m). In other words, the magnetic force is increased about 24% compared to that of the initial design, while the magnetic material consumption is small.
- b) The finally optimized objective function using different strategies are almost the same, but are all better than the results using traditional ON/OFF method. And the CPU time used under the annealing mechanism B is tremendously reduced, about 60%, compared to those using other annealing strategies. To intuitively explain these results, Figure 4.6 shows the convergence trajectories of the objective function using different annealing mechanisms. It is observed that the finally optimized objective functions using mechanism A and mechanism B are the same and keep unchanged after some number of generations (10 for this case study). However, the iterative procedure using mechanism A will continue because the number of changeable elements is greater than 1. On the contrary, the iterative procedures under the annealing mechanism method B will be immediately stopped since the material attribute is not changed for consecutive 7 iterations. And to some extent, the setting of parameter α is reasonable.

2. The combined Tabu-ON/OFF method

The proposed combined Tabu-ON/OFF topology optimization method is applied to this case study, and the related materials and parameter settings are the same as the previous section. The optimized topology is shown in Figure 4.7, Figure 4.8 depicts the distribution of magnetic lines corresponding to the original design and the optimized topology. The trajectory of the objective function value in the optimization process is shown in Figure 4.9.

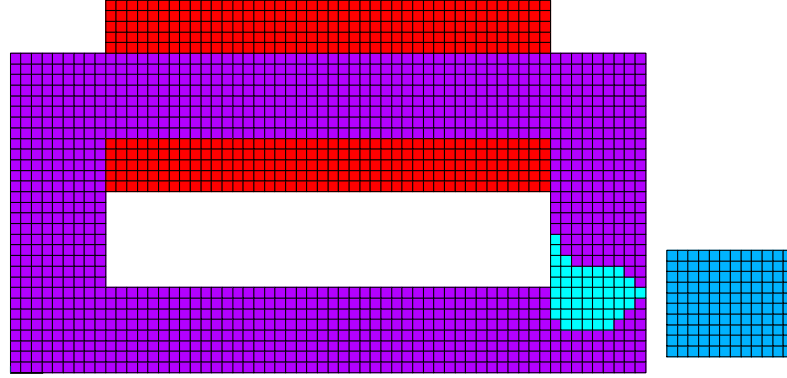


Figure 4.7 : Optimized topology by Tabu-ON/OFF method.

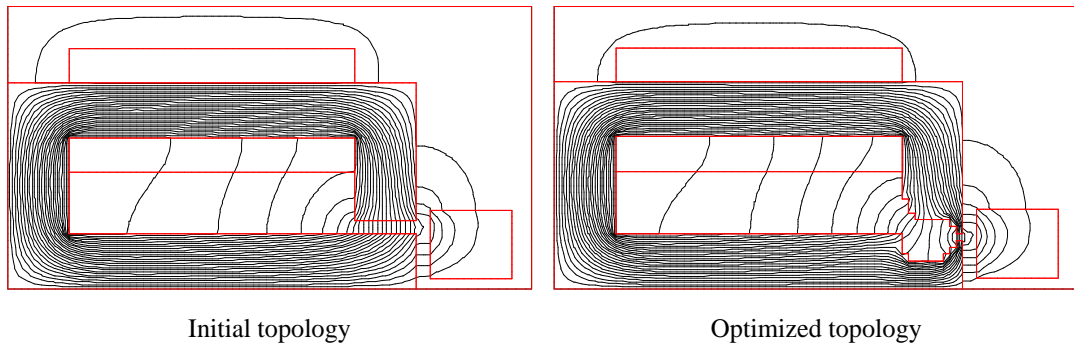


Figure 4.8 : Magnetic flux lines of initial and optimized topology.

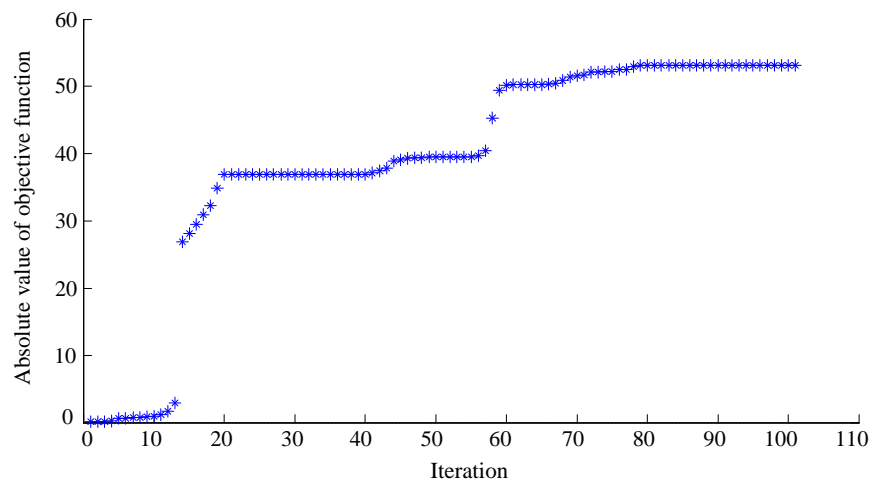


Figure 4.9 : Trajectory of objective function.

From these numerical results, it can be seen that, after the topology optimization, the electromagnetic force of the armature is significantly increased despite of the reduced use of magnetic material in core; Comparing with the initial design, the electromagnetic force is increased about 25% from 42.67 to 53.16 N/m . As shown in Figure 4.8, the optimized magnetic field lines are more concentrated in the vicinity of the armature under optimized topology design, which is why the electromagnetic force of the armature is increased. Figure 4.9 reveals the good convergence of the proposed algorithm, that is, as the optimization process goes on, better values of the objective function are continuously found. However, it can be seen from the convergence curve that the algorithm is also easy to stagnate near local optimal points, which proves that the adoption of the reset mechanism in the algorithm, using a counter to determine whether the method is stagnant into local optimal, is reasonable and reliable.

3. The improved QEA

The prototype of the electromagnetic actuator is also optimized by using the proposed topology optimization method based on improved quantum evolution algorithm (QEA). In order to evaluate the performance of the improved QEA and verify the proposed design variable redistribution mechanism (RM), in this section four different methods, namely, the original quantum evolution algorithm (noted as OQ), the improved quantum evolution algorithm (IQ), and the original QEA with variable redistribution mechanism (OQ with RM) and the improved QEA with the variable redistribution mechanism (IQ with RM) are employed to optimize the topology of the electromagnetic actuator, and the obtained numerical results are compared. Table 4.2 shows the objective function, the number of iterations, and the time consumed by different methods. In the cases of the variable redistribution mechanism, the corresponding objective function value of each optimization step is listed. The optimized topology obtained by the original QEA and the improved QEA methods are shown in Figure 4.10 and Figure 4.11 respectively, and Figure 4.12 describes the trajectories of the objective function. For the cases of the design variable redistribution mechanism, Figure 4.13 shows the optimized topology result corresponding to each step using the original QEA with the variable redistribution mechanism; and Figure 4.14 exhibits the optimized topology of each step by using the improved QEA algorithm with the variable distribution mechanism. Figure 4.15 depicts stepwise optimization trajectory of the objective function of the original QEA method and the improved QEA method using the variable

redistribution mechanism. In these results, OQ step1 stands for the first step of the original QEA algorithm; similarly, IQ step3 represents the third step of the improved QEA.

To facilitate the illustration of the selection of variables when adopting the design variable redistribution mechanism, Figure 4.16 shows the selection of design variables in the design domain for each step using improved QEA method. In Figure 4.16, the yellow color on the left side of each optimization step is the design variable, and the right side is the optimized topology corresponding to this step.

Table 4.2 : Optimization results comparison of original QEA and revised QEA

Method	Objective function (N/m)			Iteration number	CPU time(s)
OQ	-54.74			3000	110078.37
IQ	-54.91			2900	107843.47
OQ with RM	-51.54	-54.72	-54.95	1800	61679.10
IQ with RM	-51.54	-54.89	-54.95	1600	58682.19

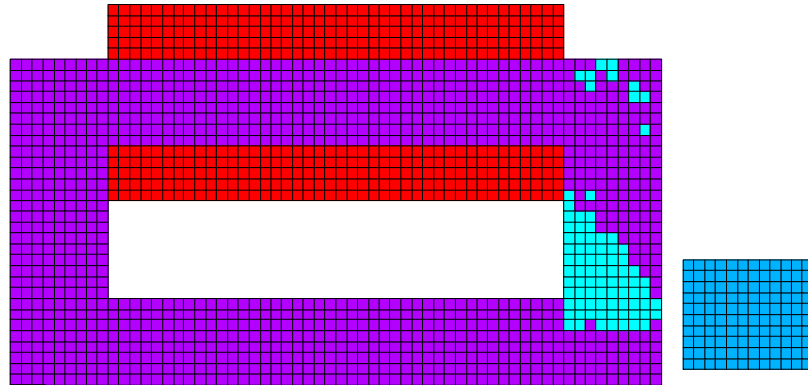


Figure 4.10 : Optimized topology of the original QEA.

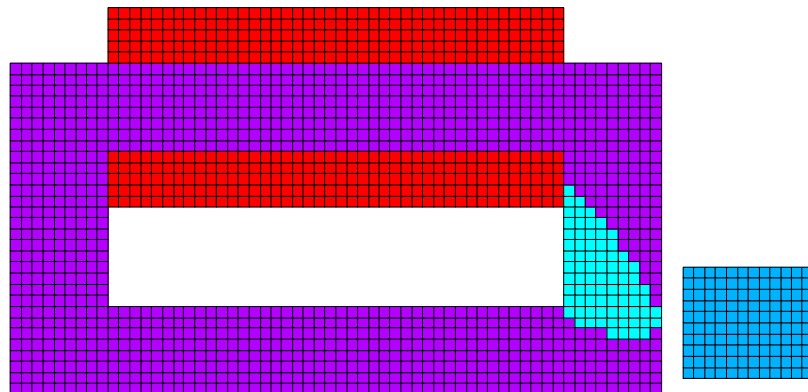


Figure 4.11 : Optimized topology of the revised QEA.

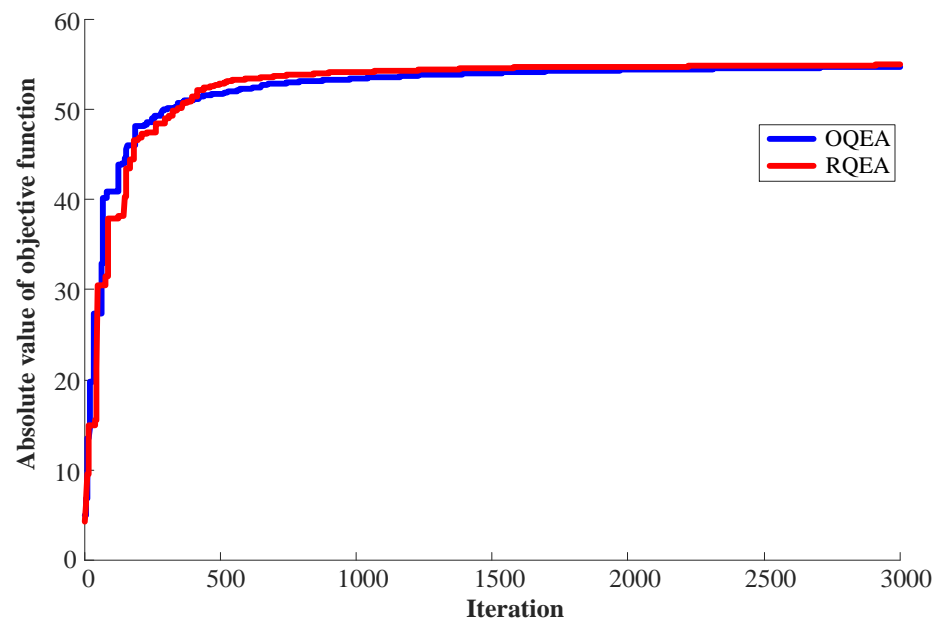


Figure 4.12 : Objective function trajcetories of the original and revised QEA.

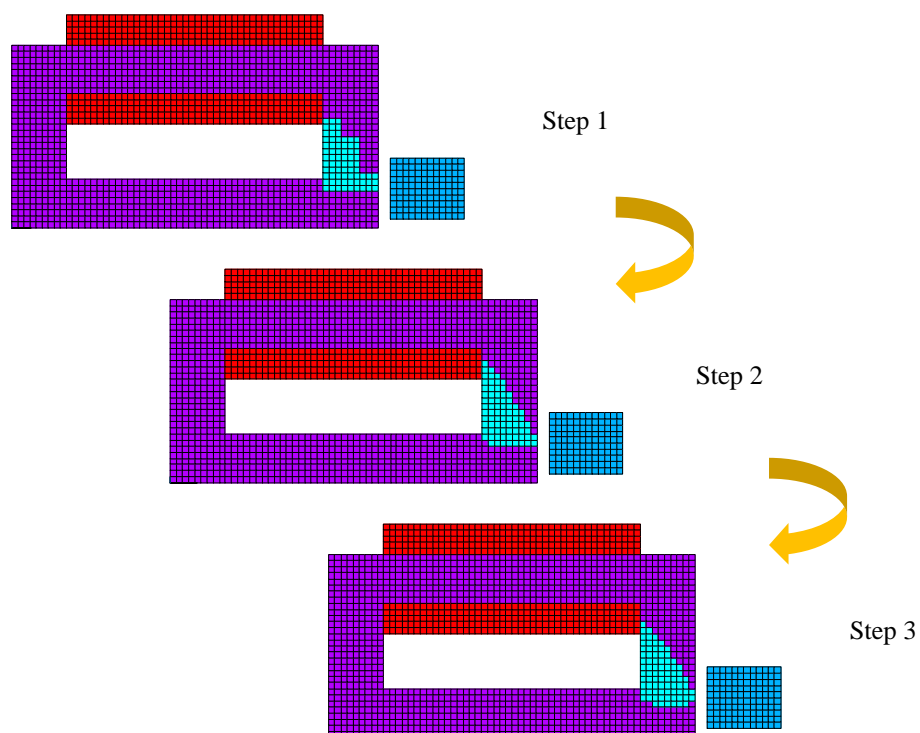


Figure 4.13 : Optimized topology of each step in the original QEA with RM.

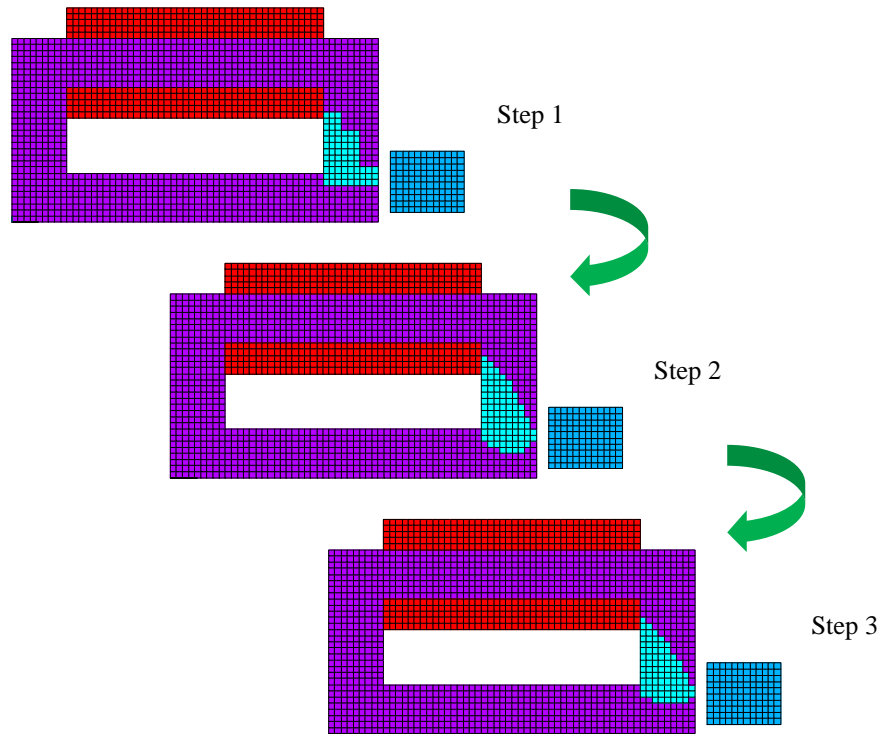


Figure 4.14 : Optimized topology of each step in the revised QEA with RM.

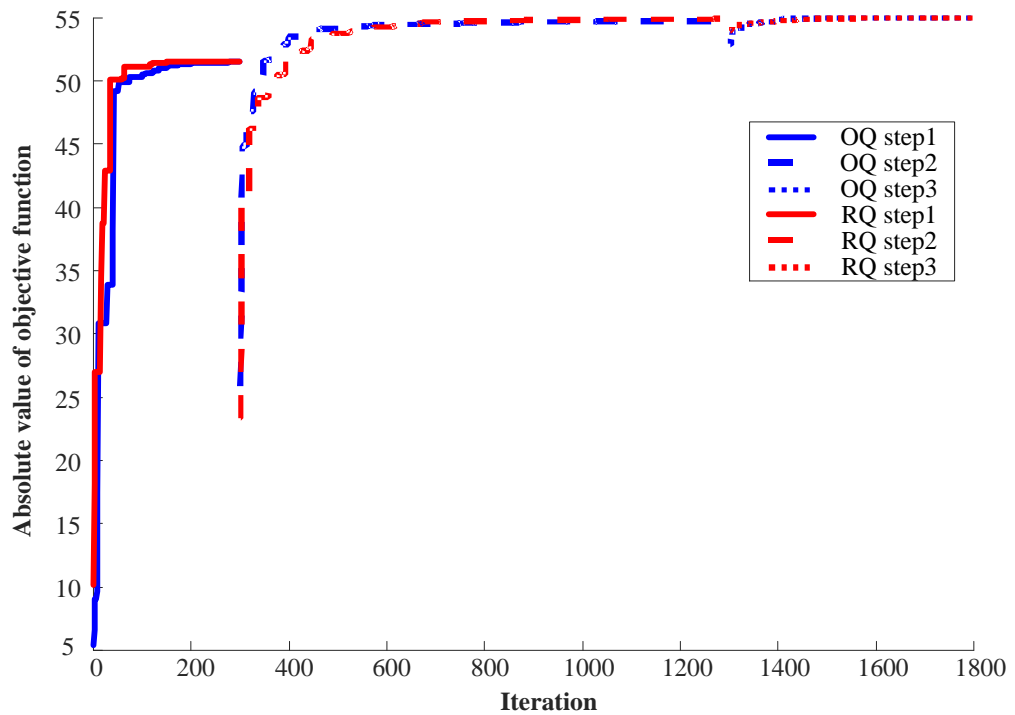


Figure 4.15 : Objective function trajectory of each step by adopting the RM.

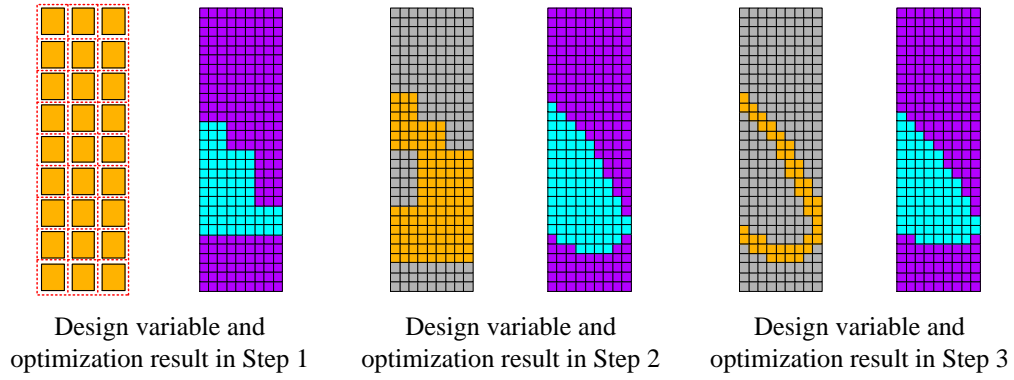


Figure 4.16 : Selection of design variables in each step and corresponding optimized topology.

From these numerical results, it can be seen that

- a) The objective function value corresponding to the optimized topology by the traditional QEA algorithm is 54.74 (N/m), and the objective function value corresponding to the improved QEA algorithm is 54.91. Using the design variable redistribution mechanism, the objective function values in the first, the second and third step of the original QEA, are 51.54, 54.72 and 54.95 respectively; while the objective function value in the first, the second and the third step of the revised QEA are 51.54, 54.89 and 54.95 respectively. Considering that the objective function under the original design is 42.67 (N/m), the electromagnetic force is eventually increased about 28.8% by using the improved QEA with RM, and at the same time, the material cost is reduced.
- b) The objective function value of the improved QEA is larger than that of the traditional QEA method. And meanwhile, it can be seen from the optimized topology in Figure 4.10 and Figure 4.11 that there are cavities in the optimized topology obtained by the original QEA algorithm, and the optimized topology obtained by the improved QEA algorithm has no this checkerboard issue. Therefore, it can be concluded that the former's global search ability is better than the latter.
- c) The number of iterations in Table 4.2 refers to the number of iterations when the algorithm finds the optimal topology (the maximum value of the objective function). It can be observed from the table that the improved QEA algorithm converges slightly faster than the original QEA algorithm; the variable redistribution mechanism will reduce greatly the number of design variables for each step, so that the number of iterations required per step is greatly reduced. The

computation time consumed is approximately half of the time required for the case when redistribution mechanism is unemployd.

- d) As can be seen from the objective function trajectory of Figure 4.12 and Figure 4.15, the objective function value of the improved QEA is slightly smaller than the original QEA algorithm at the beginning of the search process, but as search continues, the value of the improved QEA is gradually larger than that of the original QEA, which also demonstrates that the global search ability of the improved QEA algorithm is stronger, and it is less likely to be trapped into the local optimal solutions as compared to the original QEA.

4. The improved GA

The proposed improved GA in Chapter 2.4 is finally applied to the topology optimization of the electromagnetic actuator. In the numerical implementation, the topology of a prototype magnetic actuator is optimized using different methods for performance comparison. The methods include the traditional ON/OFF method ($T_{ON/OFF}$), the merely improved ON/OFF method ($I_{ON/OFF}$), the merely improved GA (I_{GA}), and the proposed hybrid method (ON/OFF_{GA}). Figure 4.17, Figure 4.18, Figure 4.19 and Figure 4.20 depict, respectively, the optimized topology of the traditional ON/OFF method, the improved ON/OFF method, the improved GA and the hybrid method. The corresponding performances are compared in Table 4.3.

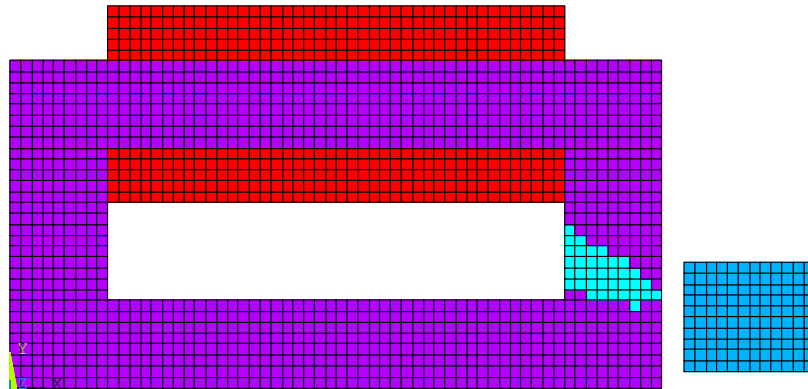


Figure 4.17 : Optimized topology by the traditional ON/OFF method.

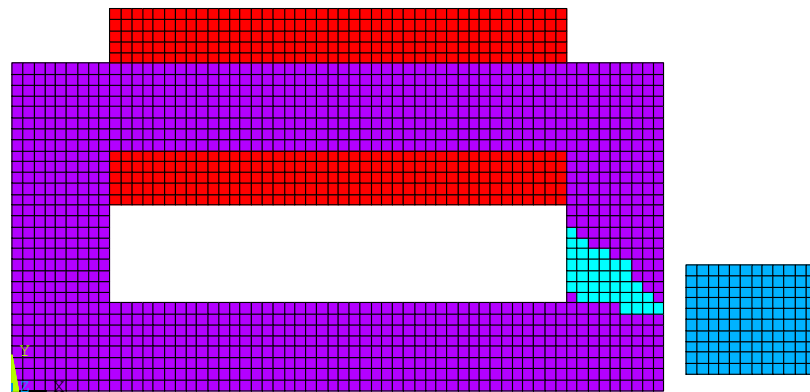


Figure 4.18 : Optimized topology by the improved ON/OFF method.

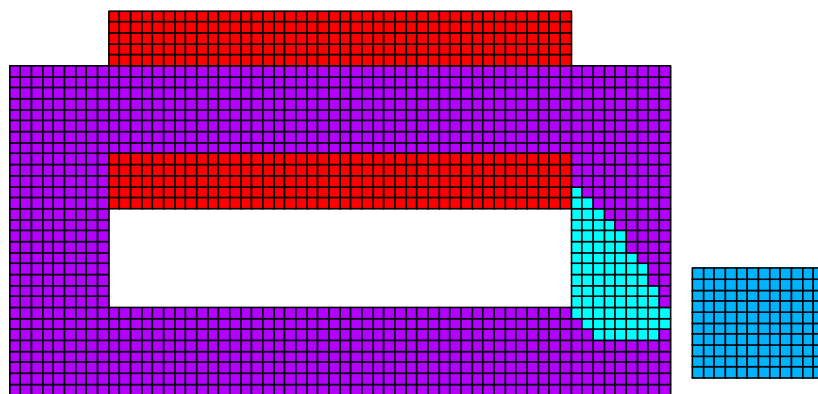


Figure 4.19 : Optimized topology by the improved GA method.

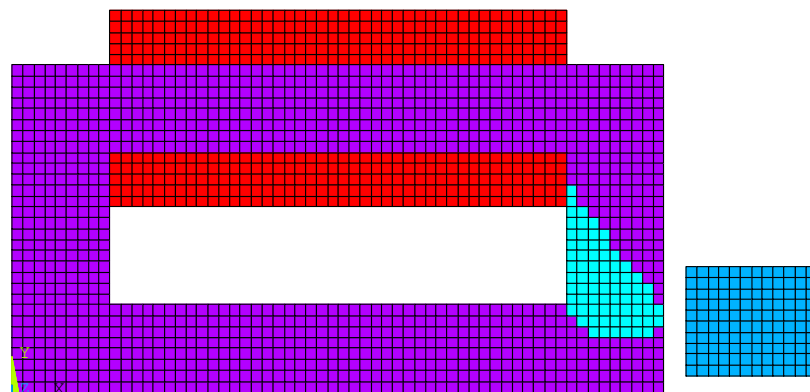


Figure 4.20 : Optimized topology by the ON/OFF-GA hybrid method.

Table 4.3 : Optimization results comparison of different methods

Optimization method	Objective function(N/m)	CPU time (s)
T_ON/OFF	-49.22	9385.783
I_ON/OFF	-52.90	5842.414
I_GA	-54.94	36699.565

ON/OFF_GA

-54.91

31870.162

From these numerical results, it is observed that:

- a) The objective functions are 49.22 (N/m) under the optimized topology of ON/OFF method, 52.90 (N/m) under the optimized topology of improved ON/OFF method, 54.94 under the optimized topology of I_GA , and 54.91 under the optimized topology of *ON/OFF_GA* method. Considering the actuating force of the initial topology of 42.67 (N/m), the magnetic force can be increased about 29% and the magnetic material consumption is nearly 74% under the optimized topologies of I_GA method and *GA_ON/OFF* topology optimization methodology.
- b) The proposed *I_ON/OFF* method outperforms T_ON/OFF . The absolute value of the objective function of the former is larger than that of the latter, which is mainly contributed by the introducing of virtual material. The convergence is faster since new a new annealing strategy is proposed.
- c) The optimized results of I_GA and *ON/OFF_GA* are better than those of T_ON/OFF and *I_ON/OFF*, validating that the global searching ability of methodologies is increased by adopting improved GA.
- d) The final solution of I_GA and *ON/OFF_GA* are merely the same while the CPU time used by the former is about 1.15 times of the latter.

4.2 Case study 2 : single-objective topology optimization

Micro-electromechanical systems (MEMS) have a wide range of applications in remote and embedded systems due to their miniaturization and good adaptability. Piezoelectric energy recovery (PEH) device, as one of the MEMSs that converts mechanical vibrational energy to electrical energy, shows great potential in remote sensors and embedded devices due to its compact size [16][18][20]. When a piezoelectric energy harvester device is subjected to external vibration, the upper and lower surfaces of the piezoelectric material will generate a potential difference. In this section, the topology optimization methodologies based on the SIMP method the level set method are applied to optimize the topology of a piezoelectric energy harvester (PEH), and then be verified.

4.2.1 Solid model

Figure 4.21 is a schematic diagram of a cantilever piezoelectric energy harvester model. The whole system consists of substrate, piezoelectric layer, electrodes and load. The left side surface of the base and the piezoelectric layer and are applied fixed constraints. The upper and lower surface of the piezoelectric layer is respectively set as equipotential surface condition, and electrodes are added on the two surfaces (the influence of the introduction of electrodes on the model is ignored here), and the external load is connected between the two electrodes. Under external mechanical vibration, voltage will be generated on the piezoelectric material layer of the PEH, and two electrodes respectively connected to the upper and lower surfaces of the piezoelectric layer will generate a potential difference.

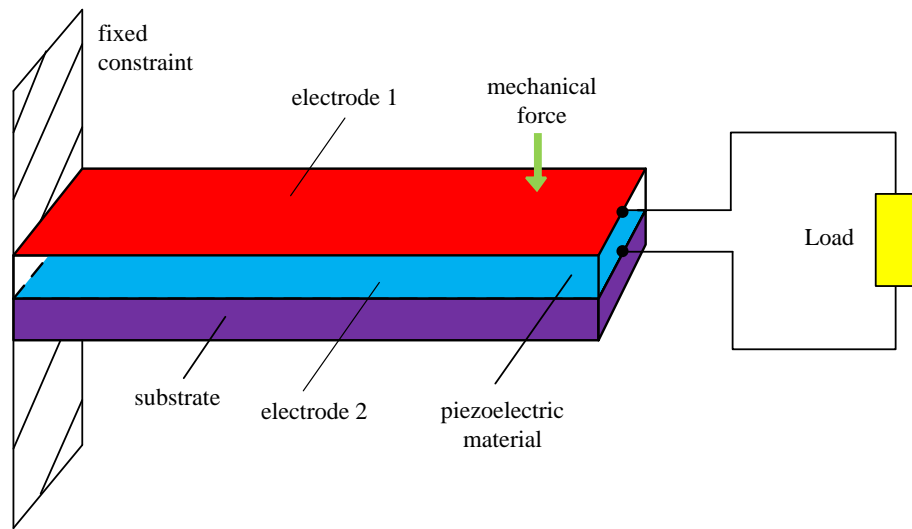


Figure 4.21 : Schematic of piezoelectric energy harvester model.

1. Objective function

To evaluate the performance of the piezoelectric energy harvesting devices, an energy conversion factor is introduced. The energy generated by an external force F is stored in the strain energy Π_S and the electric energy Π_E , which are defined as:

$$\begin{aligned}\Pi_S &= \frac{1}{2} \mathbf{u}^T \mathbf{K}_{uu} \mathbf{u} \\ \Pi_E &= \frac{1}{2} \boldsymbol{\varphi}^T \mathbf{K}_{\varphi\varphi} \boldsymbol{\varphi}\end{aligned}\tag{4.3}$$

The topology optimization of the piezoelectric energy harvester is thus formulated as

$$\begin{aligned}
 & \text{Maximize} && \eta = \frac{\Pi_E}{\Pi_E + \Pi_S} \\
 & \text{s.t.} && \sum_{e=1}^N \gamma_e v_e < V_0 \\
 & && 0 < \gamma_e \leq 1, \quad e \in [1, N]
 \end{aligned} \tag{4.4}$$

where v_i denotes the volume of the i^{th} element, N is the number of the finite elements in the design domain. V_0 is a given upper bound volume constraint and is set 60% of the design domain.

2. Finite element formulation

In this study, a 3D finite element model of piezoelectric energy harvester is developed to analyze the performance. Electrodes are deposited on the piezoelectric part and imposed equipotential electrical conditions. The influence of the electrodes in mechanic is neglected. In the static case, the external is an open circuit and the bottom surface of the piezoelectric material layer is equipotential grounding. The coupled mechanical and electric finite element formulation under the static case is thus given as

$$\begin{aligned}
 & \mathbf{K}_{uu} \mathbf{U} + \mathbf{K}_{u\varphi} \mathbf{\Phi} = \mathbf{F} \\
 & \mathbf{K}_{\varphi u} \mathbf{U} - \mathbf{K}_{\varphi\varphi} \mathbf{\Phi} = \mathbf{Q}
 \end{aligned} \tag{4.5}$$

where $\mathbf{K}_{u\varphi}$ represents the piezoelectric coupling matrix; \mathbf{K}_{uu} and $\mathbf{K}_{\varphi\varphi}$ denote the structural stiffness and dielectric conductivity matrices; \mathbf{U} and $\mathbf{\Phi}$ denote displacement and electrical potential vectors; \mathbf{F} and \mathbf{Q} are the applied force and electric charge vectors, respectively.

For the harmonic case, the above equation can be formulated as

$$\begin{aligned}
 & \mathbf{M}_{uu} \ddot{\mathbf{U}} + \mathbf{K}_{uu} \mathbf{U} + \mathbf{K}_{u\varphi} \mathbf{\Phi} = \mathbf{F} \\
 & \mathbf{K}_{\varphi u} \mathbf{U} - \mathbf{K}_{\varphi\varphi} \mathbf{\Phi} = \mathbf{Q}
 \end{aligned} \tag{4.6}$$

where \mathbf{M}_{uu} is the structural mass matrix; $\ddot{\mathbf{U}}$ denotes the displacement acceleration vectors. Considering the structural damping, Rayleigh damping is added to the above matrix. Electric circuits are described by using the equivalent capacitance matrix method [200] and the resultant matrices are compromised into the dielectric matrix. Equation (4.6) can thus be formulated as

$$\begin{aligned}\mathbf{M}_{uu}\ddot{\mathbf{U}} + \mathbf{C}_{uu}\dot{\mathbf{U}} + \mathbf{K}_{uu}\mathbf{U} + \mathbf{K}_{u\varphi}\boldsymbol{\Phi} &= \mathbf{F} \\ \mathbf{K}_{\varphi u}\mathbf{U} - (\mathbf{K}_{\varphi\varphi} + \hat{\mathbf{K}}_{\varphi\varphi})\boldsymbol{\Phi} &= \mathbf{Q}\end{aligned}\quad (4.7)$$

where \mathbf{C}_{uu} is the damping matrix; $\hat{\mathbf{K}}_{\varphi\varphi}$ is the effective capacitance matrix. In both the static and harmonic case, as the applied electric charge is zero, \mathbf{Q} is always zero.

3. Sensitivity analysis

Sensitivity information of the objective function with respect to the design variables is computed to bias iterative process. From (4.4), the first order derivative of the objective function is formulated as

$$\frac{\partial \eta}{\partial \gamma_e} = \frac{\frac{\partial \Pi_E}{\partial \gamma_e}(\Pi_E + \Pi_S) - \Pi_E \left(\frac{\partial \Pi_E}{\partial \gamma_e} + \frac{\partial \Pi_S}{\partial \gamma_e} \right)}{(\Pi_E + \Pi_S)^2} \quad (4.8)$$

According to (4.3), $\frac{\partial \Pi_E}{\partial \gamma_e}$ and $\frac{\partial \Pi_S}{\partial \gamma_e}$ can be written as

$$\begin{aligned}\frac{\partial \Pi_E}{\partial \gamma_e} &= \frac{1}{2} \frac{\partial \boldsymbol{\varphi}^T}{\partial \gamma_e} \mathbf{K}_{\varphi\varphi} \boldsymbol{\varphi} + \frac{1}{2} \boldsymbol{\varphi}^T \frac{\partial \mathbf{K}_{\varphi\varphi}}{\partial \gamma_e} \boldsymbol{\varphi} + \frac{1}{2} \boldsymbol{\varphi}^T \mathbf{K}_{\varphi\varphi} \frac{\partial \boldsymbol{\varphi}}{\partial \gamma_e} \\ \frac{\partial \Pi_S}{\partial \gamma_e} &= \frac{1}{2} \frac{\partial \mathbf{u}^T}{\partial \gamma_e} \mathbf{K}_{uu} \mathbf{u} + \frac{1}{2} \mathbf{u}^T \frac{\partial \mathbf{K}_{uu}}{\partial \gamma_e} \mathbf{u} + \frac{1}{2} \mathbf{u}^T \mathbf{K}_{uu} \frac{\partial \mathbf{u}}{\partial \gamma_e}\end{aligned}\quad (4.9)$$

By the convenience of introducing the material interpolation model, $\frac{\partial \mathbf{K}_{uu}}{\partial \gamma_e}$, and $\frac{\partial \mathbf{K}_{\varphi\varphi}}{\partial \gamma_e}$ can be easily obtained. As for partial derivative $\frac{\partial \mathbf{u}}{\partial \gamma_e}$ and $\frac{\partial \boldsymbol{\varphi}}{\partial \gamma_e}$, the adjoint matrix method is employed by solving the following equation:

$$\begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\varphi} \\ \mathbf{K}_{\varphi u} & -\mathbf{K}_{\varphi\varphi} \end{bmatrix} \begin{bmatrix} \frac{\partial \mathbf{u}}{\partial \gamma_e} \\ \frac{\partial \boldsymbol{\varphi}}{\partial \gamma_e} \end{bmatrix} = - \begin{bmatrix} \frac{\partial \mathbf{K}_{uu}}{\partial \gamma_e} & \frac{\partial \mathbf{K}_{u\varphi}}{\partial \gamma_e} \\ \frac{\partial \mathbf{K}_{\varphi u}}{\partial \gamma_e} & -\frac{\partial \mathbf{K}_{\varphi\varphi}}{\partial \gamma_e} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\varphi} \end{bmatrix} \quad (4.10)$$

It should be noted that, the above equations are valid only for the sensitivity analysis under static case. For the harmonic case, equation (4.3) is adjusted to

$$\begin{aligned} \Pi_S^* &= \frac{1}{2} \bar{\mathbf{u}}^T \mathbf{K}_{uu} \mathbf{u} \\ \Pi_E^* &= \frac{1}{2} \bar{\boldsymbol{\varphi}}^T \mathbf{K}_{\varphi\varphi} \boldsymbol{\varphi} \end{aligned} \quad (4.11)$$

where $\bar{\mathbf{u}}$ is the conjugate vector of displacement vector \mathbf{u} , and $\bar{\boldsymbol{\varphi}}$ is the conjugate vector of electrical potential vector of $\boldsymbol{\varphi}$. Moreover, according to (4.7), the coupled mechanical and electric finite element equation under the harmonic case can be given as

$$\begin{bmatrix} \mathbf{K}_{eff} & \mathbf{K}_{u\varphi} \\ \mathbf{K}_{\varphi u} & -\mathbf{K}_{\varphi\varphi}^* \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \boldsymbol{\Phi} \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{Q} \end{bmatrix} \quad (4.12)$$

where \mathbf{K}_{eff} and $\mathbf{K}_{\varphi\varphi}^*$ is respectively the following equation,

$$\mathbf{K}_{eff} = -\omega^2 \mathbf{M}_{uu} + j\omega \mathbf{C}_{uu} + \mathbf{K}_{uu} \quad (4.13)$$

$$\mathbf{C}_{uu} = \alpha \mathbf{M}_{uu} + \beta \mathbf{K}_{uu}$$

$$\mathbf{K}_{\varphi\varphi}^* = \mathbf{K}_{\varphi\varphi} + \hat{\mathbf{K}}_{\varphi\varphi} \quad (4.14)$$

And then, the adjoint equation for sensitivity analysis can formulated be

$$\begin{bmatrix} \mathbf{K}_{eff} & \mathbf{K}_{u\varphi} \\ \mathbf{K}_{\varphi u} & -\mathbf{K}_{\varphi\varphi}^* \end{bmatrix} \begin{bmatrix} \frac{\mathbf{u}}{\partial\gamma_e} \\ \frac{\partial\boldsymbol{\varphi}}{\partial\gamma_e} \end{bmatrix} = - \begin{bmatrix} \frac{\partial\mathbf{K}_{eff}}{\partial\gamma_e} & \frac{\partial\mathbf{K}_{u\varphi}}{\partial\gamma_e} \\ \frac{\partial\mathbf{K}_{\varphi u}}{\partial\gamma_e} & -\frac{\partial\mathbf{K}_{\varphi\varphi}^*}{\partial\gamma_e} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\varphi} \end{bmatrix} \quad (4.15)$$

As for the derivative of the objective function, the equation can be composed accordingly based on (4.8) and (4.9).

4.2.2 Mathematical formulation

1. Static case

In the static case, the optimal piezoelectric material distribution of a unimorph energy harvester under static point load is considered, as shown in Figure 4.22. The whole structure is composed of substrate, piezoelectric material and mass proof. An external force is applied to the middle point of the mass proof. The arrows on the piezoelectric plate show the polarization direction of the piezoelectric materials. Two electrodes placed on the top and bottom surfaces of the piezoelectric material. The entire piezoelectric material layers are set as the design domain. The design variables of top, middle and bottom layers are treated as the same along the z -direction in one hand to prevent the complex or impractical porous topology. The top and bottom layers of the piezoelectric material are equipotential boundary condition and the bottom layer is grounded. Finite element model and sensitivity analysis are in Chapter 4.2.1; the objective is to maximize the conversion factor subject to a volume constraint where the usage of piezoelectric material is limited to up to 60%.

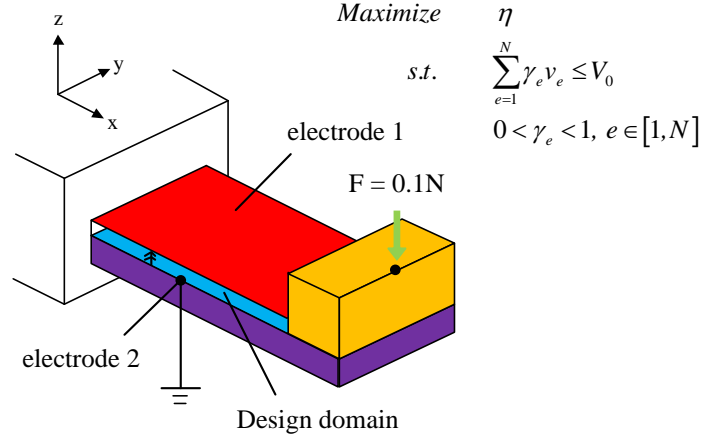


Figure 4.22 : Problem defination of static case.

2. Harmonic case

In this case, an optimization problem under a harmonic body force load is considered. An external body force loading is applied to the whole structure as shown in Figure 4.23. The FE discretization is the same as that in the static case. The piezoelectric layer is set as the design domain. The upper bound of the volume constraint is set to be 60%. The first-order eigen frequency of the whole system with an initial design, is calculated from a numerical analysis, is 202.89 Hz . Consequently, the excitation frequency is set to be 200 Hz and value of the resistor of external circuit is $500 \text{ k}\Omega$.

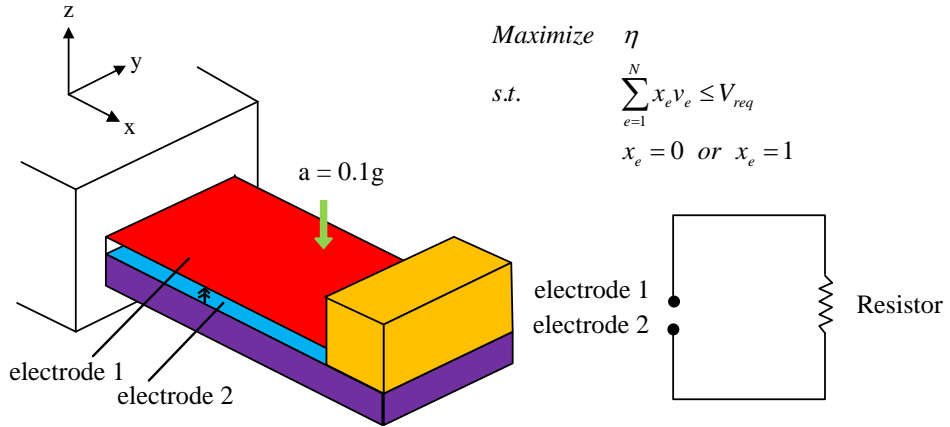


Figure 4.23 : Problem defination of harmonic case.

4.2.3 Numerical results

1. The SIMP method

Based on the finite element formulation, the solid model of piezoelectric energy harvester is analyzed. In this study case, the topology of design domain is optimized respectively under the static condition and the harmonic one. And the optimized topology under different working condition is obtained. The numerical results are validated by using FEA software COMSOL.

(1) Static case

First, three combinations of the penalization factors- $[p_1 \ p_2 \ p_3]$ are compared. For the first two combinations, the penalization factors are fixed constants; for the third one, p_1 is set to be continuous varied according to (2.10). For easiness of demonstration of the optimized topology, a vertical view of the initial optimized topology by using SIMP is given in Figure 4.24. The objective function trajectories using different penalization factors are plotted in Figure 4.25. The solid line represents the objective function value and the dotted line denotes the maximum density change between two neighboring iterations. From Figure 4.24 and Figure 4.25, it can be observed that different combinations of the penalization factors will produce different topologies even though the objective function values are close. However, the density variation in the continuous penalization factor case is the smallest. The updating process of the element density is steady to proceed. From this perspective, the convergence performance of the optimization is improved by adopting the proposed continuously varied penalization factor mechanism.

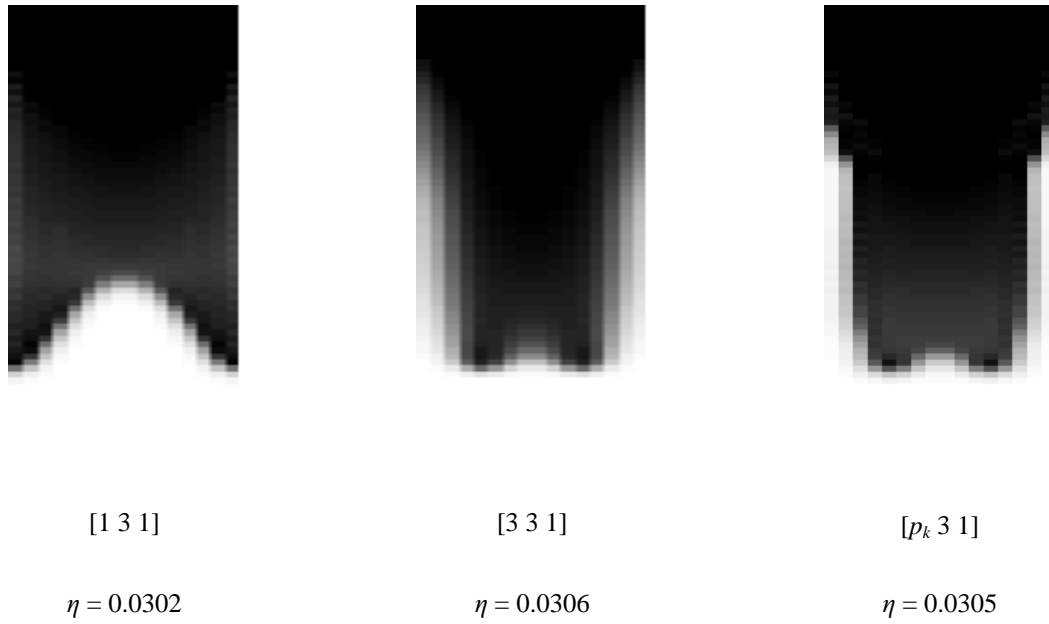


Figure 4.24 : Optimized topology with different combinations of penalization factors.

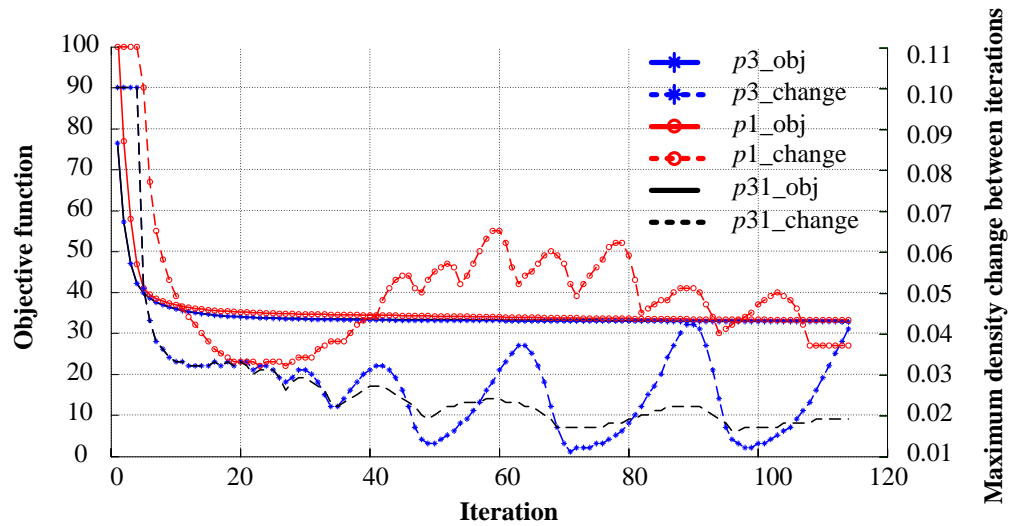


Figure 4.25 : Objective function trajectories of different combinations of penalization factors.

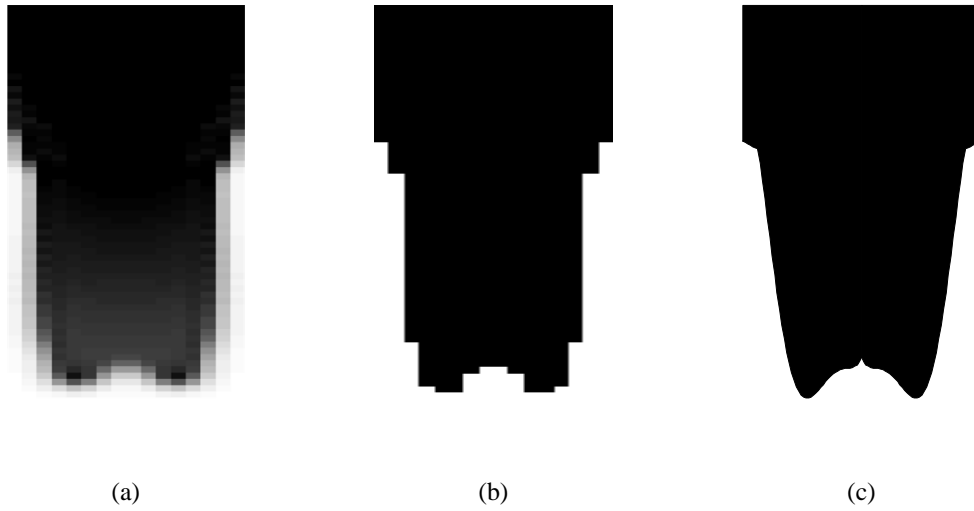


Figure 4.26 : Optimization results of static case.

Figure 4.26 also presents the blurry topology (a) obtained by the SIMP method, the discrete topology (b) after binarization and the smoothed topology (c) after adopting the RBF post-processor.

(2) Harmonic case

The optimized topology under the harmonic case is given in Figure 4.27.

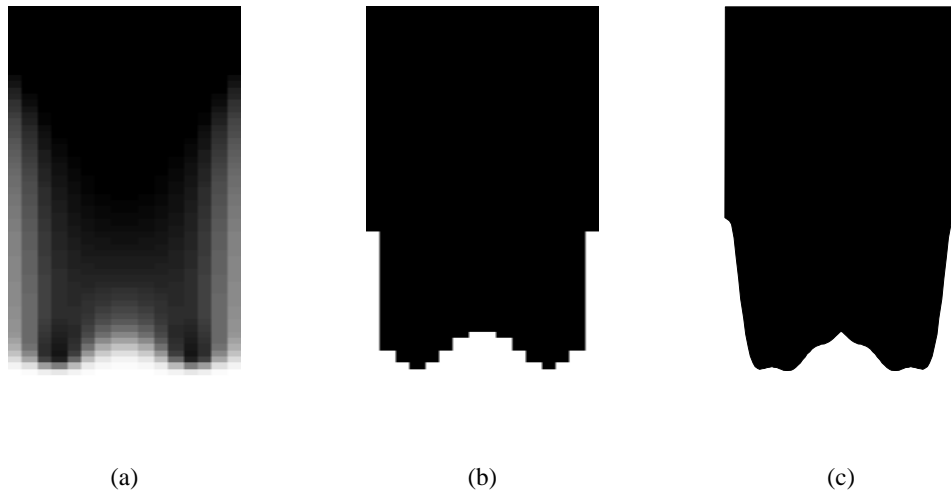


Figure 4.27 : Optimization results of harmonic case.

To validate the performance of the proposed methodology, both the static and the harmonic optimization results are validated using the commercial FEA software COMSOL. The elastic energy, electric energy and energy conversion factor in the initial design, the obtained discrete topology and the smoothed topology are given in Table 4.4 and Table 4.5.

From the numerical results, it can be seen that compared to the initial design, both the discrete topology and the smoothed topology can increase the energy conversion factor. Moreover, the smoothed topology has better performance.

Table 4.4 : Verification results of static case using SIMP-RBF method

Topology	E_s	E_e	η
<i>Initial topology</i>	1.4225×10^{-4}	4.2782×10^{-6}	0.0292
<i>Discrete topology</i>	1.5489×10^{-4}	5.4275×10^{-6}	0.0339
<i>Smoothed topology</i>	1.5498×10^{-4}	5.6078×10^{-6}	0.0353

Table 4.5 : Verification results of harmonic case using SIMP-RBF method

Topology	E_s^*	E_e^*	η
<i>Initial topology</i>	4.3447×10^{-11}	1.1925×10^{-12}	0.0267
<i>Discrete topology</i>	5.5424×10^{-11}	1.8846×10^{-12}	0.0329
<i>Smoothed topology</i>	4.6764×10^{-11}	1.6568×10^{-12}	0.0342

It should be noted that, in Table 4.4, E_s is the elastic energy, E_e is the electric energy and η is the energy conversion factor, namely the objective function; And in Table 4.5, E_s^* and E_e^* are the average energy of one period.

2. The level-set method

(1) Static case

The proposed topology optimization methodology based on the level-set method is applied to optimize the topology of the PEH model. In the level-set method, one also compares the effect of different combinations of penalty factors $[p_1 \ p_2 \ p_3]$ on the optimization results under static case. In order to facilitate the demonstration of the optimized topology, a vertical view of the initial optimized topology is shown in Figure 4.28. The objective function trajectories corresponding to different penalty factor combinations are shown in Figure 4.29. The solid line here indicates the objective function value, and the dotted line represents the material volume change during the iterative process. Obviously, different optimization topologies are obtained by using different penalty factor combinations.

The optimized topology obtained by using the continuously varying penalty factor is the most regular; the objective function values obtained by different penalty factor combinations are very close. Compared with the initial design, the value of objective function is increased by about 9%. In addition, it can be observed from the trajectories of the objective function that the updating process by using continuous variation strategy of penal factors is smoother and faster to achieve the volume constraint, which shows that the convergence performance of the algorithm after adopting the proposed continuous variation penalty factor is improved.

The initial optimized topology is post-processed by using the RBF post-processor, and finally the optimized topology under static conditions is shown in Figure 4.30, in which, (a) is the discrete topology obtained after initial post-processing, and (b) shows the optimized topology after RBF smoothing.

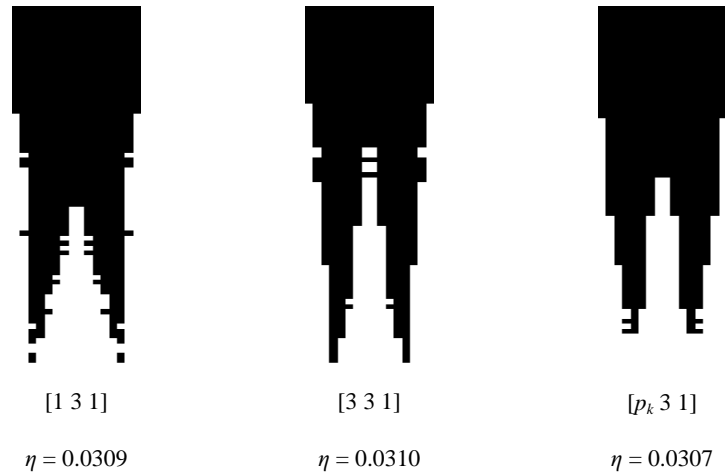


Figure 4.28 : Optimized topology with different combinations of penalization factors.

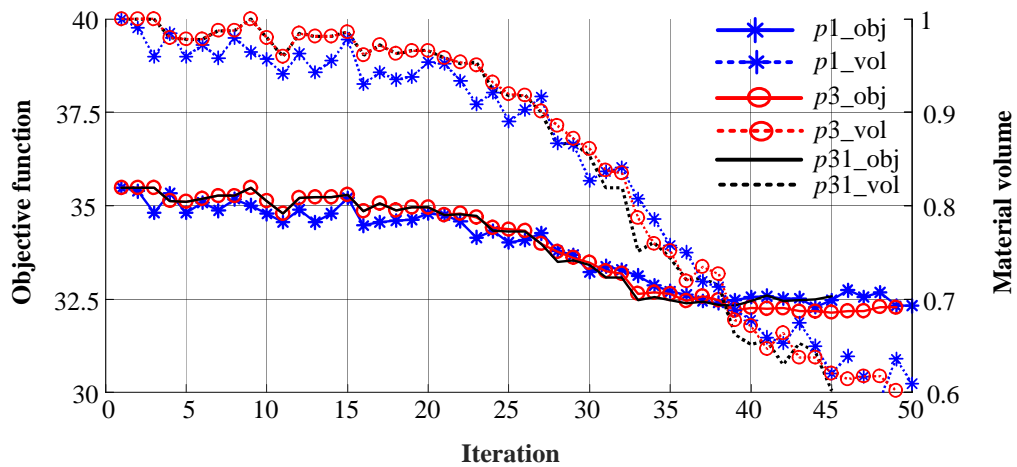


Figure 4.29 : Objective function trajectories of different combinations of penalization factors.

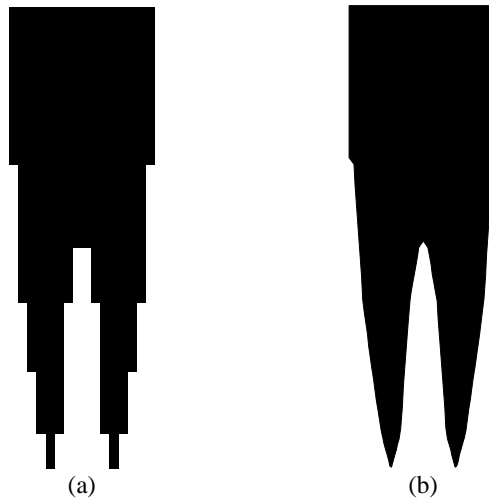


Figure 4.30 : Optimization results of static case. (a) Discrete topology; (b) Smooth topology

(2) Harmonic case

The optimized topology of the harmonic case is shown in Figure 4.31.

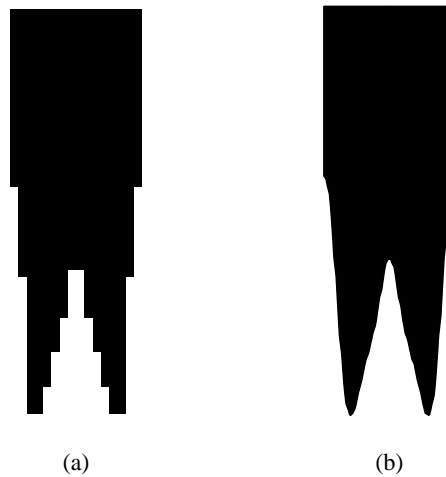


Figure 4.31 : Optimization results of harmonic case. (a) Discrete topology; (b) Smooth topology

In order to verify the optimization results of the method and evaluate the effect of the RBF post-processing process, the commercial FEA software COMSOL is used to verify the optimization results of the static and harmonious cases. The relevant performance indexes under the discrete topology and the smoothed topology are calculated, which is shown in Table 4.6 and Table 4.7. It can be seen from the numerical calculation results that the discrete topology and the smooth topology can improve the energy conversion factor compared with the original design. For static case, the smooth topology has better performance; while for harmonic case, the discrete topology corresponds to a larger energy conversion factor. So far,

the effectiveness of the proposed topology optimization method based on level set method is verified. At the same time, the RBF post-processor is introduced to obtain a smoother and manufacturing-easier topology structure, but the smooth processing may deteriorate the performance.

Table 4.6 : Verification results of static case using LSM-RBF method

Topology	Π_s	Π_e	η
<i>Initial topology</i>	1.4225×10^{-4}	4.2782×10^{-6}	0.0292
<i>Discrete topology</i>	1.5890×10^{-4}	6.0527×10^{-6}	0.0367
<i>Smoothed topology</i>	1.5942×10^{-4}	6.1209×10^{-6}	0.0370

Table 4.7 : Verification results of harmonic case using LSM-RBF method

Topology	Π_s^*	Π_e^*	η
<i>Initial topology</i>	4.3447×10^{-11}	1.1925×10^{-12}	0.0267
<i>Discrete topology</i>	4.1637×10^{-11}	1.4872×10^{-12}	0.0345
<i>Smoothed topology</i>	4.6161×10^{-11}	1.5833×10^{-12}	0.0332

In Table 4.6, Π_s is the elastic energy, Π_e is the electric energy and η is the energy conversion factor; and in Table 4.7, Π_s^* and Π_e^* are the average energy of one period.

4.3 Case study 3 : multi-objective topology optimization

The hybrid multi-objective optimization algorithm (JNSGA-DE) proposed in Chapter 3 is applied to optimize the topology of the electromagnetic actuator prototype proposed in section 4.1 by converting the single-objective optimization problem with fixed input power to a dual-objective optimization problem with simultaneously maximizing electromagnetic force and minimizing the material consumption.

4.3.1 Mathematical formulation

The two-objective optimization problem is expressed as,

$$\begin{aligned}
 & \max \quad f_1(x, u) \\
 & \min \quad f_2(x) \\
 & \text{subject to} \quad K(x)u = F(x) \\
 & \quad \quad \quad x = [0, 1]
 \end{aligned} \tag{4.16}$$

where $f_1(x, u)$ represents the electromagnetic force and $f_2(x)$ is the magnetic material consumption. x is the design variable of the optimization problem. In the optimization process, the objective function f_1 and f_2 are transformed into a minimization problem by inverting the objective function f_1 . The two-objective optimization problem is reformulated as,

$$\begin{aligned}
 & \min \quad \begin{cases} \text{Threshold} - f_1(x, u) \\ f_2(x) \end{cases} \\
 & \text{subject to} \quad K(x)u = F(x) \\
 & \quad \quad \quad x = [0, 1]
 \end{aligned} \tag{4.17}$$

4.3.2 Numerical results

The proposed hybrid multi-objective algorithm is applied to solve this case study. In order to reduce the computational burden and speed up the convergence, the design variable redistribution mechanism proposed in Section 2.3.2 is used. In the optimization process, the number of design variables gradually changes from small to large ones, and the design domain is gradually refined. The whole process is divided into three steps. In the first step, in order to compare the performance of different multi-objective optimization methods, the electromagnetic actuator is optimized by using the NSGAI, SPEA2 and the proposed hybrid algorithms respectively. The non-dominated ratios of Pareto solutions obtained by different algorithms are shown in Table 4.8. The obtained Pareto front is shown in Figure 4.32. In the figure, the Pareto frontier of different algorithms are denoted by using different symbols; the ordinate represents the first objective, and the abscissa is the second objective. From the figure, it can be observed that the hybrid algorithm not only finds more non-dominated solutions than the other two algorithms, but also finds the better non-dominated solutions that are closer to the true frontier. Table 4.9 represents the optimized topology corresponding to each solution on the Pareto front obtained by the hybrid algorithm. Based on the above results, it can be seen that when there is no magnetic material in the design area, the material consumption is the smallest, namely the function value corresponding to the second optimization objective is the smallest, and the corresponding first objective function is the

largest, that is, the electromagnetic force is the smallest; as the usage of ferromagnetic material in the design domain gradually increases, the function value of the second objective gradually increases, and the function value of the first objective gradually decreases, that is, the electromagnetic force becomes larger.

It is more meaningful that, by observing the change of the topology in the change process of the above two objective functions, we can understand the distribution of ferromagnetic materials in which design area will contribute to the first objective, that is, the increase of electromagnetic force. This optimization results provide a reference for designers to design an electromagnetic actuator to better achieve the balance between increasing the electromagnetic force and reducing material consumption.

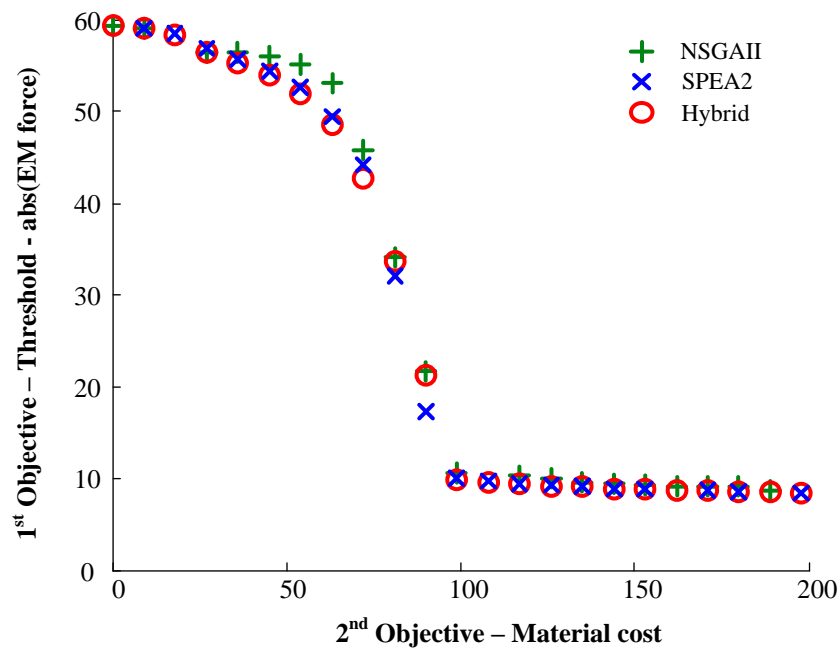


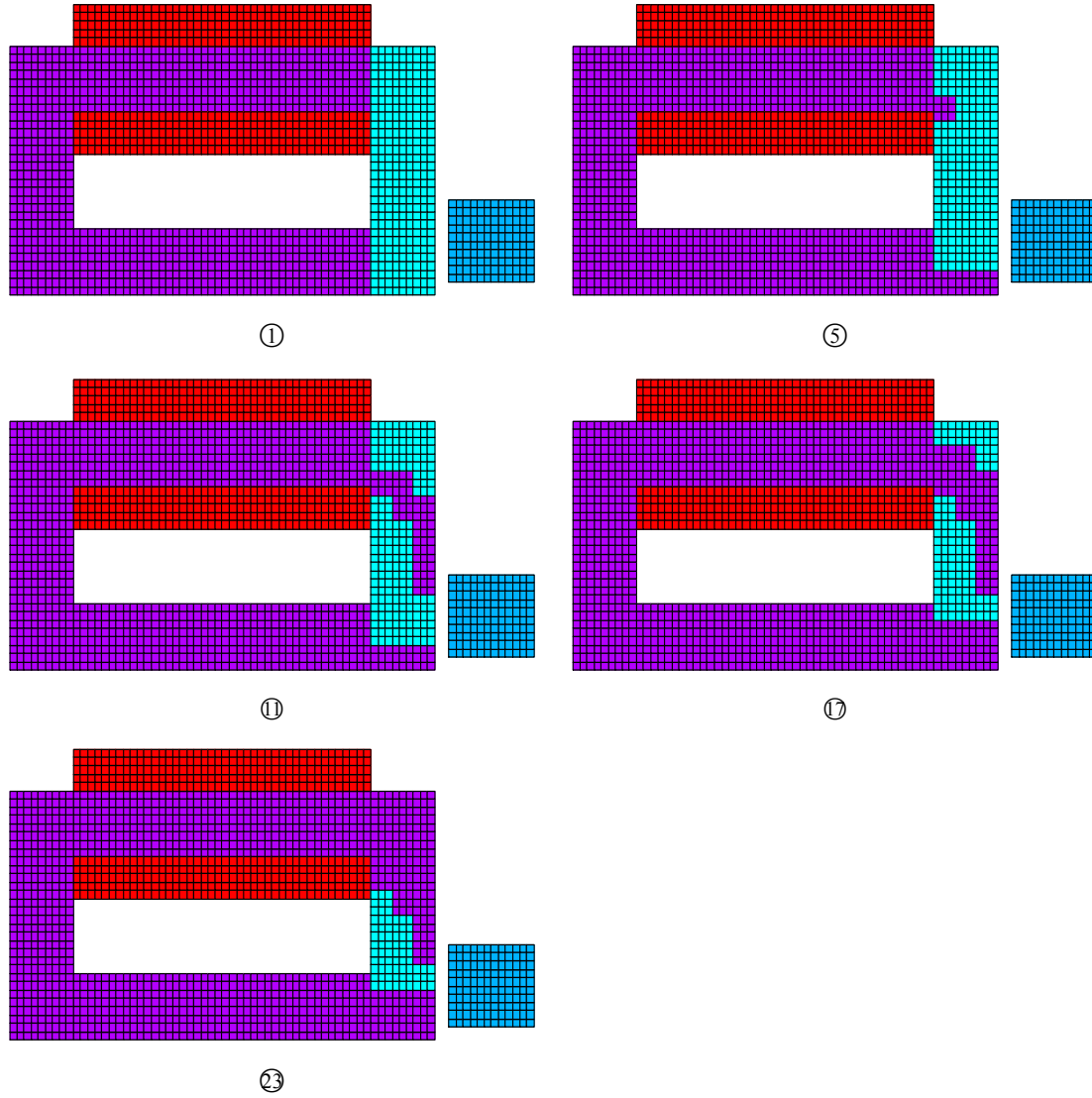
Figure 4.32 : Obtained Pareto frontier of different multi-objective optimization methods (Step 1).

Table 4.8 : Non-dominated ratio of Pareto solutions by different algorithms

	Hybrid		NGSAII		SPEA2	
<i>NDR</i>	21/23	91.3%	2/23	8.7%	7/23	30.4%

Note: The calculation of *NDR* takes the union of non-dominated solution set obtained by different optimization algorithms, and then performs non-dominated sorting to obtain the true non-dominated solutions. The ratio of the non-dominated solutions of different optimization algorithms with the true non-dominated solutions is *NDR*.

Table 4.9 : The corresponding topologies of typical Pareto solutions (Step 1)



In the variable redistribution mechanism, the boundary detection is performed on the current optimized topology, and the elements adjacent to the boundary are taken as the design variables of the next step. The corresponding topology of the 20th Pareto frontier solution obtained in the first step is chosen as the initial topology for the second step of the optimization. The hybrid algorithm is applied according to the above procedure, and the Pareto frontier of the second step is shown in Figure 4.33. Some typical solutions (marked as solid circle in the figure) are selected, and the corresponding topologies are presented in Table 4.10. The numbers in the table represent the number of Pareto solution which corresponds to the topology. The topology of the non-dominated solution 46th is selected as the initial structure for the third step optimization. It should be noted that, in the optimization process of the third step, the selection rule of the design variables is different from the previous step.

Except for the boundary elements detected, the elements whose material property is air in the upper right corner are all selected as design variables, as shown in Figure 4.34.

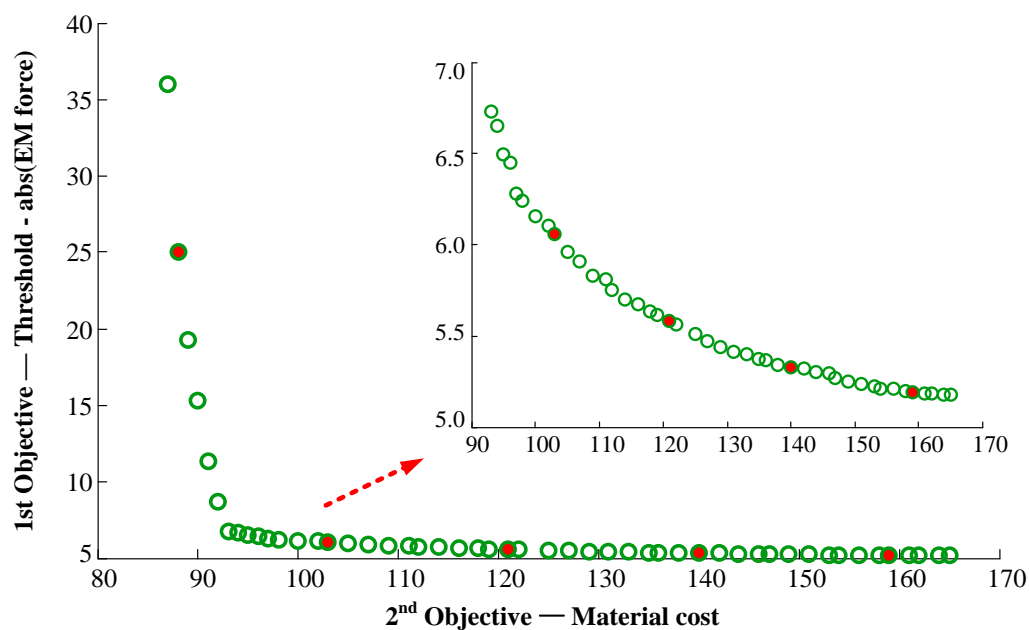
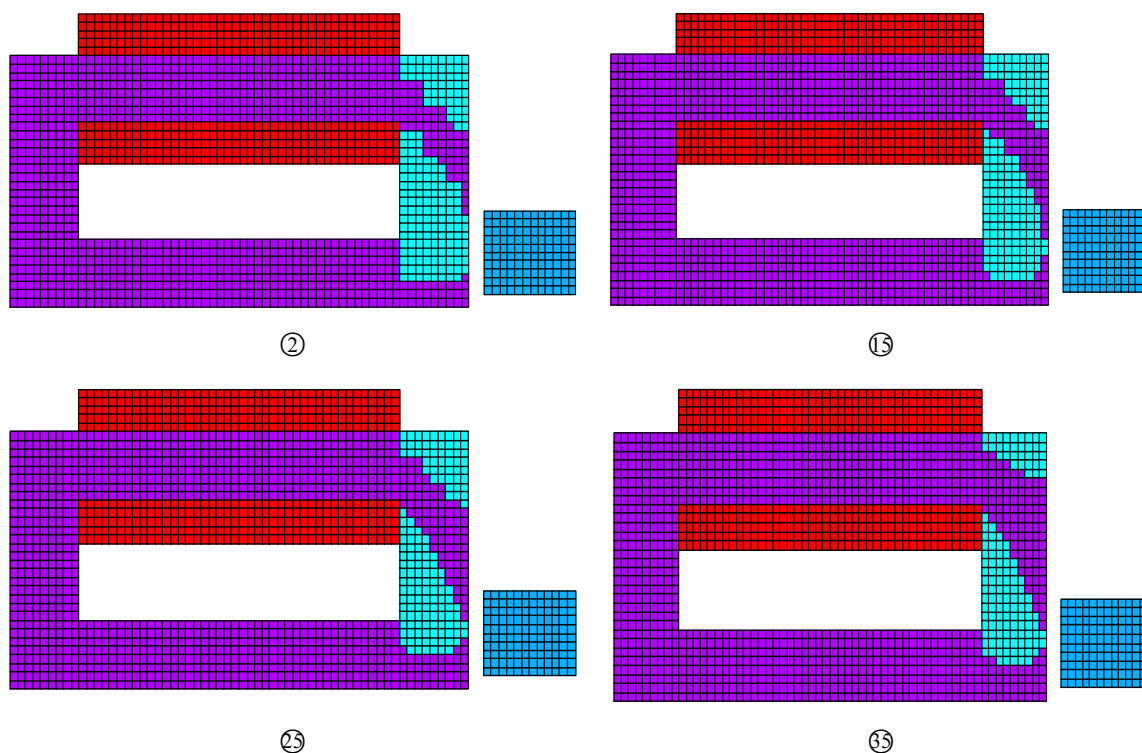


Figure 4.33 : Obtained Pareto frontier by hybrid algorithm (Step 2).

Table 4.10 : The corresponding topologies of typical Pareto solutions (Step 2)



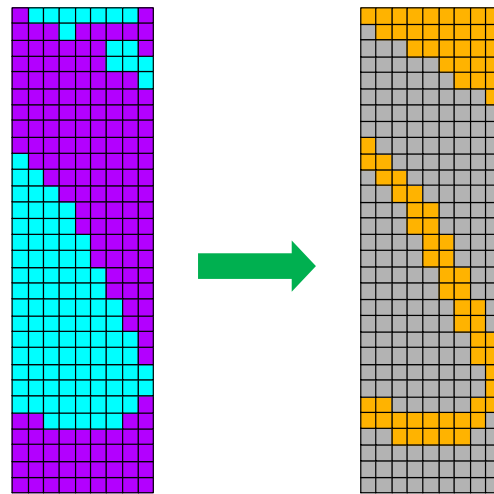
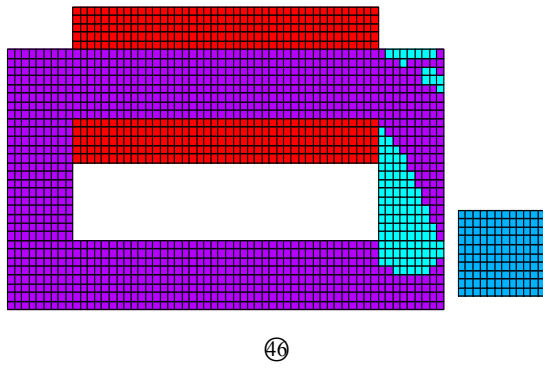


Figure 4.34 : Selection of design variables in Step 3.

According to the above selection rule of design variables, the searched Pareto front of the third step is shown in Figure 4.35. Table 4.11 presents the corresponding topological structure of the selected typical solutions (marked as solid circle). The numbers in the table represent the number of Pareto solutions corresponding to the topology.

Based on the topology optimization results of three steps, typical multi-objective topology optimization results for the electromagnetic actuator can be obtained as shown in Table 4.12. In the table, the material consumption is represented by the number of elements in the design domain; the up arrow indicates increase, and the down arrow stands for decrease. Compared with the original design, under the optimized topology the electromagnetic force can be maximally increased about 29%, and the material consumption is maximally reduced by 59%. After multi-objective topology optimization, it can be seen that the electromagnetic force of the armature is significantly increased, and the material consumption in the design area is also remarkably decreased.

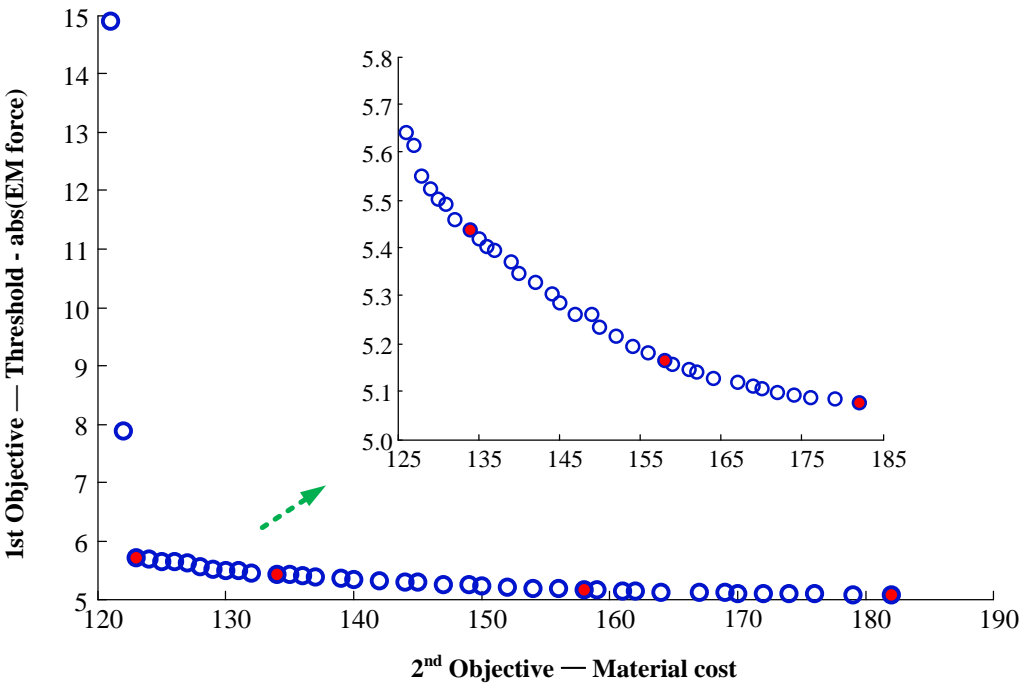


Figure 4.35 : Optimized Pareto frontier (Step 3).

Table 4.11 : The corresponding topologies of typical Pareto solutions (Step 3)

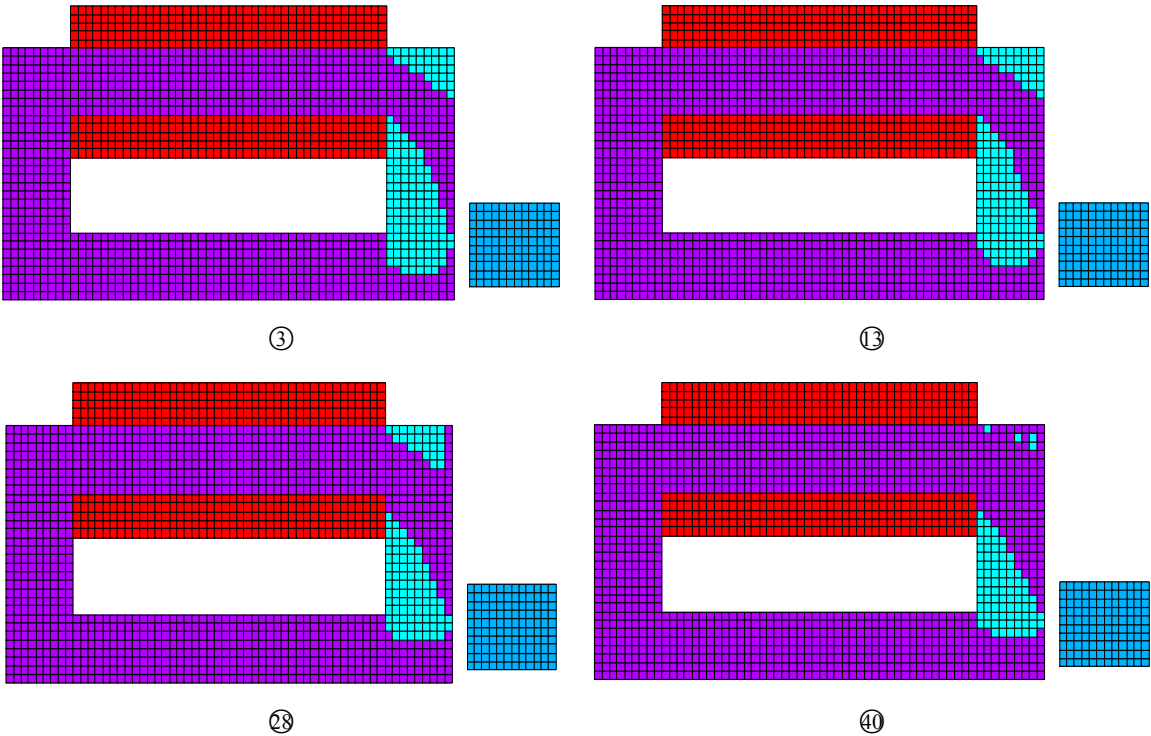
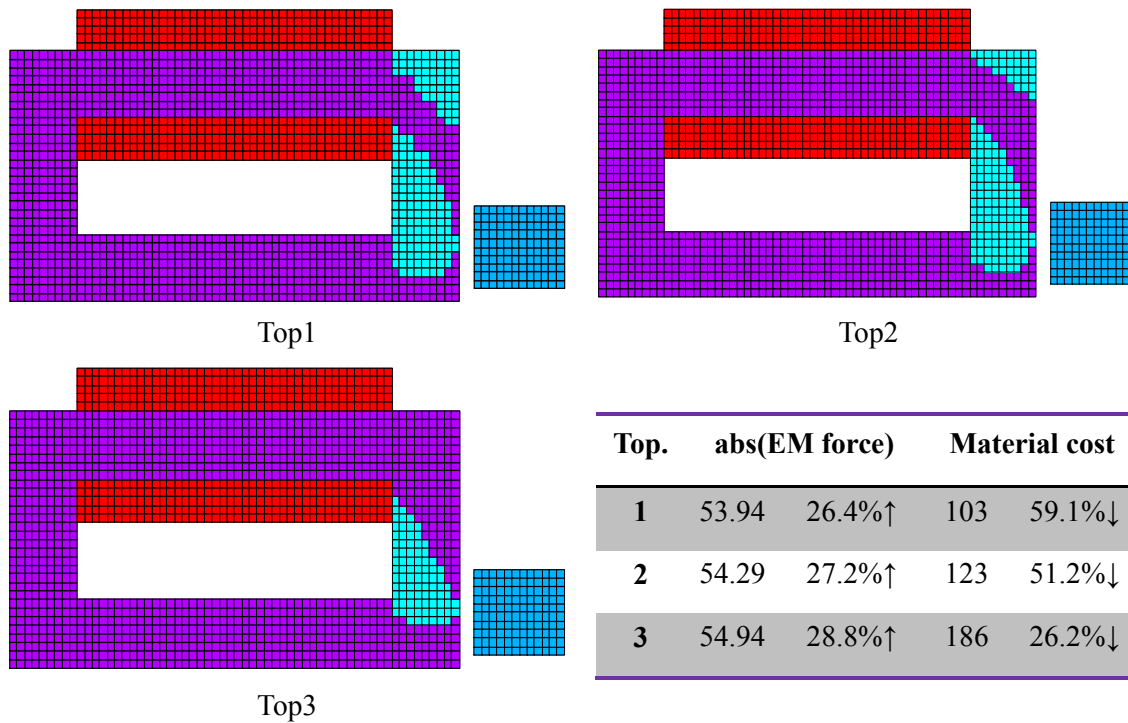


Table 4.12 : The optimized topologies of multi-objective topology optimization



4.4 Comparatively remarks

4.4.1 Single-objective topology optimization method

For the single-objective topology optimization of the electromagnetic actuator, this chapter presents numerical results using on the ON/OFF method, the combined Tabu-ON/OFF method, the improved GA method, and the improved QEA method. The ON/OFF method uses the sensitivity of the objective function to guide the update of the topology of the design domain in the optimization process; while in the Tabu-ON/OFF method; the improved GA method and the improved QEA method; the three Tabu, GA, and QEA random search algorithms are used to guide the topology optimization process.

Table 4.13 compares the optimization results of the aforementioned methodologies. Obviously, GA and QEA based methods can find the topology corresponding to the better objective function value, nevertheless more optimization time is needed. The Tabu-ON/OFF method is superior to the ON/OFF method in both the objective function and the calculation time, however, in the actual optimization process, the selection of the initial point of the Tabu-ON/OFF method has a great influence on the final result, by contrast the ON/OFF method is more robust. The final optimized topologies obtained by QEA based method and

GA based method are nearly the same, but the value of objective function of QEA is a little larger at the expense of more optimization time. Thus it can be seen that for the topology optimization problem with a large number of design variables, ON/OFF method and Tabu-ON/OFF method are more adjustable since the computation cost is less than that of QEA method and GA method. However, if a designer takes more care of the global optimal design, QEA method and GA method are suitable methods to be applied.

Table 4.13 : Comparison of different single-objective topology optimization methods (1)

Method	Objective function (N/m)	CPU time (s)
<i>ON/OFF</i>	-52.90	5842.41
<i>ON/OFF-Tabu</i>	-53.16	2616.67
<i>Improved QEA</i>	-54.95	58682.19
<i>Improved GA</i>	-54.94	36699.57

For the single-objective topology optimization of the PEH, considering the finite element discretization of the design domain, design variables are numerous, and the computation cost based on the random search algorithm is too high. At the same time, due to the introduction of the interpolation model of piezoelectric material, the original discrete variable optimization problem can be transformed into a continuous variable optimization one. Using the sensitivity information, the gradient-based solving methods such as optimization criterion method and moving asymptotes method can be used to solve the optimization problem. Therefore, for the topology optimization of piezoelectric layer of a PEH, this thesis uses SIMP and the level-set method based methodologies. The optimization results of the two methods are shown in Table 4.14.

Table 4.14 : Comparison of different single-objective topology optimization methods (2)

Method	Π_s	Π_e	η	Working case
<i>SIMP-RBF</i>	1.5498×10^{-4}	5.6078×10^{-6}	0.0353	static
<i>LSM-RBF</i>	1.5942×10^{-4}	6.1209×10^{-6}	0.0370	static
<i>SIMP-RBF</i>	4.6764×10^{-11}	1.6568×10^{-12}	0.0342	harmonic
<i>LSM-RBF</i>	4.6161×10^{-11}	1.5833×10^{-12}	0.0332	harmonic

It can be seen from this table that the topology optimization method based on the level set method has better optimization results in static cases, and the topology optimization

method based on SIMP works better under harmonic cases. Since the level set method adopts a binaryzation of design variables with 0 or 1 value, which avoids the intermediate density element generated during the optimization as in the SIMP method, but the numerical instability issue (such as the checkerboard problem) is more serious. By introducing RBF post-processing, smoother and manufacturing easier topology is obtained. Finally, the verification results of COMSOL also confirmed the effectiveness of the two optimization methods.

4.4.2 Multi-objective topology optimization method

For the multi-objective topology optimization of the electromagnetic actuator, the numerical results demonstrate that the proposed multi-objective hybrid algorithm is superior to the traditional NSGAI and SPEA2. Moreover, from the comparison of multi-objective and single topology optimization results of Table 4.15, it can be seen that the multi-objective topology optimization algorithm can obtain the topologies with less material consumption and at the same time a relatively large electromagnetic force can be guaranteed. However, in terms of computation times, the multi-objective topology optimization method takes much longer times than the single-objective optimization method.

Table 4.15 : Comparison of different single- and multi-objective topology optimization methods

Method		EM force (N/m)	Material cost	Computation time (s)
Single-	<i>I-QEA</i>	-54.95	190	58682.19
	<i>I-GA</i>	-54.94	190	36699.57
	①	-53.94	103	
Multi-	②	-54.29	123	763021.89
	③	-54.94	186	

4.5 Chapter summary

This chapter applies the previously proposed topology optimization technique to solve different case studies, compares and analyzes the numerical optimization results of various topology optimization methods.

For single-objective topology optimization of the electromagnetic actuator, the ON/OFF method, the Tabu-ON/OFF method, the improved GA method, and the improved

QEA method are applied respectively. The results reflect and demonstrate the characteristics and advantages of various methodologies. Among them, due to its strong global search ability, the QEA based method obtains the topology with the largest objective function value (electromagnetic force), but the computation cost required for the optimization is the highest.

For the topology optimization of the piezoelectric energy harvester, this chapter applies the method based on SIMP and the discrete level set method, analyzes the static and harmonic cases. From the optimization results, the optimized topology obtained by the two methods is different. And for the objective function value, the level set method is slightly larger than the SIMP method. From the numerical optimization results, it is obvious that after the implementation of the RBF post-processing procedure, the blurry topology with intermediate density (SIMP method) and the zigzag discrete topology (level set method) are smoothed to obtain a smooth topology structure that is easier to actually manufacture.

Finally, the proposed hybrid algorithm is applied to the multi-objective topology optimization of the electromagnetic actuator model. Using the design variable redistribution mechanism, the variables of the design domain are stepwise optimized. Finally, a set of typical Pareto non-dominated solutions are obtained. From the numerical results, and observing the characteristics of the Pareto frontier, it can be found that as the material consumption increases, the electromagnetic force increases as well. It is found that the region where the material is gradually filled is just a region having positive sensitivity to the increase of electromagnetic force. This conclusion can be used to guide the designer to minimize the material consumption while ensuring that the main design performance parameters (such as the electromagnetic force in this example) are met.

Conclusions and perspectives

Given a design problem, topology optimization techniques find its optimal topology to optimize the objective function, and meet the requirements. Compared to traditional design methods, topology optimization can obtain a novel topology that is previously unimagined, especially for product designs that lack prior experience and knowledge. Therefore, topology optimization is of great significance for the optimal design of products. It has been deemed as an important tool for designers.

This thesis aims to develop and implement topology optimization methodologies for electromagnetic device design. In this regard, the thesis proposes different topology optimization methodologies based on deterministic methods, such as ON/OFF method, SIMP-RBF method, LSM-RBF method; and topology optimization method based on random search algorithms, such as the Tabu-ON/OFF method, the improved GA method and the improved QEA method. At the same time, considering the various constraints of practical problems, a topology optimization methodology based on a multi-objective hybrid optimization algorithm is proposed. To summary, the main innovations and contributions are as follows:

- (1) To address the prone convergence to local optima in the optimization process, the topology optimization methods based on ON/OFF, Tabu-ON/OFF, improved QEA and improved GA are proposed. The characteristics of each algorithm are elaborated, and the performance of each algorithm is compared comprehensively. As the global search ability of the algorithm increases, the optimization time inevitably increases.
- (2) To solve the intermediate density as encountered in density-based methods and infeasibility topology in engineering manufacturing, the topology optimization methodologies based on SIMP-RBF and LSM-RBF are proposed. The methods use the sensitivity information in a deterministic optimizer to guide the optimizing process. For problems with numerous design variables, the computation cost is greatly reduced compared with using the methods based on random search algorithms. Moreover, the introduction of the RBF interpolation technology, the obtained optimized topology is more conducive to engineering applications.
- (3) To reduce excessive computation cost of stochastic search algorithms for topology optimization, a design variable redistribution mechanism is proposed, which

determine and select the design variables by boundary detection, and takes the optimization result of the previous step as the initial topology of the next step optimization, divides the whole optimization process in several steps. The number of design variables is thus reduced, and the computation time is shortened.

- (4) To provide more freedom for a decision maker, a hybrid multi-objective optimization algorithm based on NSGAI and DE algorithm is proposed. The comparison results of test functions demonstrate that the performance of the proposed hybrid algorithm is better than that of the traditional NSGAI and SPEA2. The application of the electromagnetic actuator topology optimization problem shows that many novel topologies can be obtained by using this topology optimization methodology, which reduces the material consumption as much as possible while ensuring that the armature is subjected to a large electromagnetic force. This provides an important reference and basis for the designer's work.

Although the numerical results of the applications of a prototype electromagnetic actuator and a PEH demonstrate that the proposed optimization methods can find a topology with better performance than the original design, there still exist many issues to be addressed in the future, especially in the following aspects:

- (1) First of all, for the topology optimization of complex problems, the excessive computation cost issue has always been the problem needed to be addressed. Although this thesis proposes a design variable redistribution mechanism to carry out the topology optimization process stepwisely, using the optimization result of the previous step as the initial solution for the next step, the number of design variables is reduced to some extent, thereby shortening the optimization time. However, for the multi-physics complex topology optimization problems with a large number of variables, it still takes a lot of computation time despite of adopting this method. Therefore, reducing computation cost is of great significance for topology optimization research. At present, a promising method seems to be adopting model reduction techniques in numerical computation field to decompose the original large-scale computation problem into a combination of multiple small-scale problems, and to reduce the complexity of the problem while ensuring the necessary solution accuracy. In addition, the adaptive parameters (such as stopping the criterion parameter) in the algorithm optimization process

will improve the convergence speed and reduce the number of iterations of the optimization.

- (2) Secondly, the topology optimization methodologies proposed in this thesis have good optimization performance on two comparatively simple numerical examples. However, the performance of the proposed methods on other topological optimization problems, especially on complex optimization problems with multi-physics coupling, needs to be testified. In addition, when applying topology optimization methods to deal with three-dimensional practical problems, how to ensure the continuity of the spatial structure, how to solve the checkerboard problem generated in the optimization process, and so on, are all key research directions.
- (3) Finally, the theoretical results obtained by using the proposed topology optimization methods can be further compared with the experimental model adopting the optimized topology design. In the theoretical analysis phase, the models and materials in the practical production should be used, and various parameters of the actual model need to be considered. According to the optimized topology obtained by theoretical analysis, the actual prototype can be manufactured, thereby topology optimization result of the simulation analysis is verified by experiments.

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Résumé :

L'optimisation de la topologie est la conception conceptuelle d'un produit. En comparaison avec les approches de conception conventionnelles, il peut créer une nouvelle topologie, qui ne pouvait être imaginée à l'avance, en particulier pour la conception d'un produit sans expérience préalable ni connaissance. En effet, la technique de la topologie consistant à rechercher des topologies efficaces à partir de brouillon devient un sérieux atout pour les concepteurs. Bien qu'elle provienne de l'optimisation de la structure, l'optimisation de la topologie en champ électromagnétique a prospéré au cours des deux dernières décennies. De nos jours, l'optimisation de la topologie est devenue le paradigme des techniques d'ingénierie prédominantes pour fournir une méthode de conception quantitative pour la conception technique moderne.

Cependant, en raison de sa nature complexe, le développement de méthodes et de stratégies applicables pour l'optimisation de la topologie est toujours en cours. Pour traiter les problèmes et défis typiques rencontrés dans le processus d'optimisation de l'ingénierie, en considérant les méthodes existantes dans la littérature, cette thèse se concentre sur les méthodes d'optimisation de la topologie basées sur des algorithmes déterministes et stochastiques. Les travaux principaux et la réalisation peuvent être résumés comme suit :

Premièrement, pour résoudre la convergence prématurée vers un point optimal local de la méthode ON/OFF existante, un Tabu-ON/OFF, un Quantum-inspiré Evolutif Algorithme (QEA) amélioré et une Génétique Algorithme (GA) amélioré sont proposés successivement. Les caractéristiques de chaque algorithme sont élaborées et ses performances sont comparées de manière exhaustive.

Deuxièmement, pour résoudre le problème de densité intermédiaire rencontré dans les méthodes basées sur la densité et le problème que la topologie optimisée est peu utilisée directement pour la production réelle, deux méthodes d'optimisation de la topologie, à savoir Matériau Isotrope solide avec pénalisation (SIMP)-Fonction de Base Radiale (RBF) et Méthode du Level Set (LSM)-Fonction de Base Radiale (RBF). Les deux méthodes calculent les informations de sensibilité de la fonction objectif et utilisent des optimiseurs déterministes pour guider le processus d'optimisation. Pour le problème posé par un grand nombre de variables de conception, le coût de calcul des méthodes proposées est considérablement réduit par rapport à celui des méthodes de comptabilisation sur des algorithmes stochastiques. Dans le même temps, en raison de l'introduction de la technique de lissage par interpolation de données RBF, la topologie optimisée est plus adaptée aux productions réelles.

Troisièmement, afin de réduire les coûts informatiques excessifs lorsqu'un algorithme de recherche stochastique est utilisé dans l'optimisation de la topologie, une stratégie de redistribution des variables de conception est proposée. Dans la stratégie proposée, l'ensemble du processus de recherche d'une optimisation de la topologie est divisé en structures en couches. La solution de la couche précédente est définie comme topologie initiale pour la couche d'optimisation suivante, et seuls les éléments adjacents à la limite sont choisis comme variables de conception. Par conséquent, le nombre de variables de conception est réduit dans une certaine mesure; le temps de calcul du processus est ainsi raccourci.

Enfin, une méthodologie d'optimisation de topologie multi-objectif basée sur l'algorithme d'optimisation hybride multi-objectif combinant l'Algorithme Génétique de Tri Non dominé II (NSGAI) et l'algorithme d'Evolution Différentielle (DE) est proposée. Les résultats de la comparaison des fonctions de test indiquent que la performance de l'algorithme hybride proposé est supérieure à celle des algorithmes traditionnels NSGAI et Strength Pareto Evolutionary 2 (SPEA2), qui garantissent la bonne capacité globale optimale de la méthodologie proposée et permettent au concepteur de gérer les conditions de contrainte de manière directe.

Pour valider les méthodologies d'optimisation de topologie proposées, deux cas d'étude sont optimisés et analysés. L'application du problème d'optimisation de la topologie d'un actionneur électromagnétique montre que les performances des méthodes proposées sont supérieures à celles des méthodes existantes ; En adoptant la méthode d'optimisation de la topologie basée sur l'algorithme hybride proposé, il est possible d'obtenir de nombreuses topologies de conception nouvelles, capables de réduire autant que possible la consommation de matériau tout en garantissant que l'armature est soumise à une force électromagnétique relativement importante. Cela pourrait fournir une base de référence et une base théorique importante pour le travail d'un designer. L'application de la simulation sur le récupérateur d'énergie piézoélectrique montre qu'il est possible d'obtenir davantage de topologies optimales réalisables en adoptant la méthode proposée.

Mots clés : Optimisation de la topologie, methode des elements finis, algorithm genetique, microstructure isotrope solide avec pénalisation, methode des surfaces de niveau, la fonction radiale, optimisation multi-objectifs

Numerical Methodologies for Topology Optimization of Electromagnetic Devices

Abstract :

Topology optimization is the conceptual design of a product. Comparing with conventional design approaches, it can create a novel topology, which could not be imagined beforehand, especially for the design of a product without prior-experiences or knowledge. Indeed, the topology optimization technique with the ability of finding efficient topologies starting from scratch has become a serious asset for the designers. Although originated from structure optimization, topology optimization in electromagnetic field has flourished in the past two decades. Nowadays, topology optimization has become the paradigm of the predominant engineering techniques to provide a quantitative design method for modern engineering design.

However, due to its inherent complex nature, the development of applicable methods and strategies for topology optimization is still in progress. To address the typical problems and challenges encountered in an engineering optimization process, considering the existing methods in the literature, this thesis focuses on topology optimization methods based on deterministic and stochastic algorithms. The main work and achievement can be summarized as:

Firstly, to solve the premature convergence to a local optimal point of existing ON/OFF method, a Tabu-ON/OFF, an improved Quantum-inspired Evolutionary Algorithm (QEA) and an improved Genetic Algorithm (GA) are proposed successively. The performance is compared comprehensively.

Secondly, to solve the intermediate density problem encountered in density-based methods and the engineering infeasibility of the finally optimized topology, two topology optimization methods, Solid Isotropic Material with Penalization-Radial Basis Function (SIMP-RBF) and Level Set Method-Radial Basis Function (LSM-RBF) are proposed. Both methods calculate the sensitivity information of the objective function, and use deterministic optimizers to guide the optimizing process. For the problem with a large number of design variables, the computational cost of the proposed methods is greatly reduced compared with those of the methods accounting on stochastic algorithms. At the same time, due to the introduction of RBF data interpolation smoothing technique, the optimized topology is more conducive in actual productions.

Thirdly, to reduce the excessive computing costs when a stochastic searching algorithm is used in topology optimization, a design variable redistribution strategy is proposed. In the proposed strategy, the whole searching process of a topology optimization is divided into layered structures. The solution of the previous layer is set as the initial topology for the next optimization layer, and only elements adjacent to the boundary are chosen as design variables. Consequently, the number of design variables is reduced to some extent; and the computation time is thereby shortened.

Finally, a multi-objective topology optimization methodology based on the hybrid multi-objective optimization algorithm combining Non-dominated Sorting Genetic Algorithm II (NSGAI) and Differential Evolution (DE) algorithm is proposed. The comparison results on test functions indicate that the performance of the proposed hybrid algorithm is better than those of the traditional NSGAI and Strength Pareto Evolutionary Algorithm 2 (SPEA2), which guarantee the good global optimal ability of the proposed methodology.

To validate the proposed topology optimization methodologies, two study cases are optimized and analyzed. The simulation application on the electromagnetic actuator topology optimization problem demonstrates that the performance of the proposed methods is superior to those of existing methods; by adopting the topology optimization method based on the proposed hybrid algorithm, many new design topologies can be obtained, which are able to reduce the material consumption as much as possible while ensuring that the armature is subjected to a comparatively large electromagnetic force. This could provide an important reference and theory basis for a designer's work. The simulation application on the piezoelectric energy harvester shows that optimized topology with better manufacturing feasibility can be gained by using the proposed methods.

Keywords : Topology optimization, FEA, GA, SIMP, LSM, RBF, multi-objective optimization