

Metamaterials with extreme properties for the control of acoustic waves

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THÈSE DE DOCTORAT DE

LE MANS UNIVERSITÉ

ÉCOLE DOCTORALE N° 602 Sciences pour l'Ingénieur Spécialité : Acoustique

Par Matthieu MALLÉJAC

Metamaterials with extreme properties for the control of acoustic waves

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Title: Metamaterials with extreme properties for the control of acoustic waves

Abstract:

Zero-index metamaterials, for which at least one of the effective parameters (density or dynamic compressibility for acoustics) vanishes, have received considerable attention in recent years. These materials have the particularity of inducing a considerable increase in the effective wavelength, thus offering numerous application possibilities, including, among others, propagation without phase change, acoustic hiding of diffusers, directivity control, etc. This PhD work focuses particularly on the near-zero effective density regime in acoustic metamaterials made of thin plates in air.

Through an in-depth study of a periodic arrangement of thin elastic plates embedded in a waveguide, we have been able to explore analytically, numerically and experimentally some of the above effects.

Particular attention is paid to the losses inherent to this type of system and their consequences on the expected behavior.

We begin by studying numerically and experimentally observing a phase-change-free propagation through the metamaterial at a frequency in a stopband of the finite system. We then transpose the concept of photonic doping to acoustics. The addition of an impurity, here a well-chosen Helmholtz resonator, to the system allows to transform the regime of zero density into one where density and compressibility are simultaneously near zero. Thus, propagation without phase change is accompanied by a unitary transmission, due to the impedance matching of the system with the surrounding air. Finally, we study the possibility of performing acoustic hiding or masking of an object using the acoustic wavelength stretching offered by the zero density.

Keywords:

Plate-type Acoustic Metamaterials, Zero-phase propagation, Doping, Hiding, Density Near Zero, Zero-Index Medium

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Titre : Métamatériaux avec propriétés extrêmes pour le contrôle des ondes acoustiques

Résumé :

Les métamatériaux à indice nul, pour lesquels au moins un des paramètres effectifs s'annule (densité ou compressibilité dynamique pour l'acoustique), ont fait l'objet d'une attention considérable au cours de ces dernières années. Ces matériaux ont la particularité d'induire une augmentation remarquable de la longueur d'onde effective, offrant ainsi de nombreuses possibilités d'application, incluant entre autres la propagation sans changement de phase, la dissimulation acoustique de diffuseurs, le contrôle de la directivité, etc. Ce travail de doctorat se concentre particulièrement sur le régime de densité effective quasi-nulle dans des métamatériaux acoustiques constitués de plaques fines dans l'air.

Grâce à une étude approfondie d'un arrangement périodique de fines plaques élastiques encastrées dans un guide d'onde, nous avons pu explorer analytiquement, numériquement et expérimentalement certains des effets ci-dessus. Une attention particulière est portée sur les pertes inhérentes à ce type de système et à leurs conséquences sur les comportements attendus.

Nous débutons par l'étude numérique et l'observation expérimentale d'une propagation sans changement de phase à travers le métamatériau, à une fréquence située dans une bande interdite du système fini. Nous transposons ensuite le concept de dopage photonique à l'acoustique. L'ajout dans le système d'une impureté, ici un résonateur de Helmholtz bien choisi, permet de transformer le régime de densité nulle en un régime où la densité et la compressibilité sont simultanément quasi-nulles. Ainsi, la propagation sans changement de phase est accompagnée d'une transmission unitaire, due à l'accord d'impédance du système avec l'air environnant. Nous étudions enfin la possibilité de réaliser une dissimulation ou un masquage acoustique d'un objet en utilisant l'extension de la longueur d'onde acoustique, offerte par la densité nulle.

Mots-Clés :

Arrangement périodique de plaques, Propagation sans changement de phase, Dopage, Dissimulation acoustique, Densité nulle, Matériau à index de réfraction nul

Thèse préparée au Laboratoire d'Acoustique de l'Université du Mans (LAUM UMR CNRS 6613), Avenue Olivier Messiaen, 72085 Le Mans Cedex 9, FRANCE.

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Nomenclature

List of Ab	breviations	e [.]	Exponential
1-2-3D	One-two-three dimensional	i	Imaginiry unit
CNZ	Compressibility Near Zero	arg	Argument
DCNS	Density and Compressibility Near	atan	Arc tangent
	Zero	Ι	Modified Bessel function of the
DM	Decorated Membranes		first kind
DNZ	Density Near Zero	Im	Imaginary part
EMNZ	Epsilon and Mu Near Zero	J	Bessel function of the first kind
ENZ	Epsilon Near Zero	К	Modified Bessel function of the
FEM	Finite Element Method		second kind
MNZ	Mu Near Zero	ln	Napierian logarithm
PAM	Plate-type Acoustic Metamate-	log	Logarithm to the base 10
	rial	Re	Real part
TMM	Transfer Matrix Method	Y	Bessel function of the second kind
ZIM	Zero Index Metamaterial	List of Syn	nbols
List of ma	thematical functions	α	Absorption
cos	Cosinus	$ar{f}_d$	Averaged force density
С	Cost function	β	Loss factor
\mathfrak{F}	Function of	ΔL	Length correction
\sin	Sinus	ΔP	Pressure difference
tan	Tangent	$\delta_{ u}$	Viscous penetration depth

ϵ	Permittivity	N	Number of plates/unit cells
γ	Adiabatic constant	n	Refraction index/mode number
κ	Bulk modulus	p	Pressure
Λ	Eigenvalues	Pr	Prandtl number
λ	Wavelength	Q	Quality factor
\mathbf{E}	Electric field	q	Bloch wavenumber
н	Magnetic field	R	${\rm Radius}/{\rm Reflection}\ {\rm coefficient}$
Т	Transfer matrix	r	Radius variable
V	Eigenvectors matrix	S	Cross-section
v	Particle velocity	T	Transmission coefficient
μ	Permeability	U	Flux
μ_0	Dynamic viscosity of air	w	Transverse displacement
ν	Poisson's ratio	x	Displacement
ω	Circular frequency	Ζ	Acoustic impedance
ϕ	Phase of the transmission coeffi-	List of sul	oscripts and notations
	cient	$.(\omega)$	Effective properties
ρ	Density	•0	Surrounding medium (air)
θ	Angle variable	$\cdot a$	Waveguide
A,B,C,D	Input and output waves	$\cdot d$	Dopant/diaphragm
C	Compressibility	$\cdot r$	Resonance
С	Celerity	$\cdot asym$	Asymmetric
D	Flexural rigidity	$\cdot Biot$	Viscothermal transition
E	Young modulus	$\cdot B$	Bragg
F	Harmonic excitation force	$\cdot ch$	Channel
f	Frequency	·c	Cut-off/cavity
K	Spring constant	$\cdot diss$	Disspative
k	Wavenumber	$\cdot FP$	Fabry-Perot
L	Length	$\cdot gap$	Air gap/band gap
M	Dynamic effective mass	$\cdot g$	Group
m	Mass/integer	\cdot holder	Plate holder

$\cdot HR$	Helmholtz resonator	·0	Outer
$\cdot h$	Inner boundary of the annular	$\cdot p$	Plate
	plate (hole)	$\cdot RB$	Rigidly-backed
·i	Inner	$\cdot SB$	Side branch element
$\cdot leak$	Leakage	$\cdot sym$	Symmetric
$\cdot lg$	Large guide	$\cdot S$	Series element
•meas	Measurement	.+	Total
$\cdot m$	Matching		Unit cell
$\cdot n$	Optimal	- <i>unii</i>	Cross soctional average
$\cdot opt$	Opiniai	•	Cross-sectional average

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Chapter

General paradigm

Overview

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Acoustic and elastic metamaterials have attracted considerable attention in the last few decades, certainly fueled by the possibilities of tailoring their wave-dispersion properties and making previously unexpected behaviors real. Actually, a plethora of applications have been motivated from these new ways of controlling always further the sound. These include perfect absorbers, topological systems, acoustic diodes, wavefront shaping devices, or cloaking systems, among others.

This PhD work focuses on the acoustic wave propagation in media with almost zero refraction index, i.e., zero-index media, in which at least one of the effective constitutive parameters (homogenized density and/or compressibility) vanishes. In particular, plate-type acoustic metamaterials, i.e., metamaterials consisting in a periodic arrangement of thin elastic plates, are deeply studied and specific attention is paid to the impact of viscothermal and viscoelastic losses on the resulting applications.

In this first Chapter, we present a non-exhaustive historical overview of the concept of metamaterials and remind some elements of the analogy between acoustics and electromagnetism. Then, we review the main applications associated with the different operating regimes of these artificial materials. Finally, we highlight the objectives and organization of this PhD thesis.

I.1 Definition and historical background of metamaterials

"Metamaterials"...

This new type of architectured medium has allowed a breakthrough in the field of wave physics in recent decades. Despite the appealing number of publications, its definition is still not universal, remains evasive, and sometimes controversial. Most of the time, the definition is contrasted with conventional materials, the constitutive parameters of which solely depend on the atomic composition of the material. Conversely, metamaterials are generally defined as artificial, i.e., man-made structures designed to make a hitherto unprecedented behavior real by overcoming the physical constraints of conventional materials.

These unusual properties are produced by the careful design of the meta-atoms, i.e., the elementary subwavelength building blocks constituting the locally resonant structure. If the size of the meta-atoms and the distance between them are both smaller than the incident wavelength, a homogenization scheme can be applied. The metamaterials can then be characterized by using effective constitutive dynamic parameters, depending on both the properties of the constituent materials and the resonance of the meta-atoms. If non-resonant atoms (conventional material) are considered, both constitutive parameters are positive. On the contrary, the constitutive dynamic parameters of a metamaterial are frequency-dependent and can take negative values. The resonant aspect of the building blocks is therefore essential. A more refined definition of metamaterials could therefore be "Heterogeneous artificial devices that present new responses that could not occur in the resonant constituent elements alone due to physical constraints" [1, 2].

Metamaterials are grounded on electromagnetic periodic media (referred to as photonic crystals hereafter), i.e., a periodic lattice of scatterers embedded in a host medium. In the diffraction regime, i.e., when the lattice constant is of the order of magnitude of the wavelength, destructive interferences open electromagnetic bandgaps, in which waves cannot propagate, and produce a strong dispersion allowing to tailor the refraction [3, 4]. The first theorizing of metamaterials dates back to 1968 [5]. A Russian physicist, Victor Veselago, considered in a pioneering theoretical work that an electromagnetic medium could simultaneously have a negative permittivity ϵ , and permeability μ , thus having a negative refractive index $n = \sqrt{\epsilon\mu} = -\sqrt{|\epsilon||\mu|}$. In these materials, named double negative (or lefthanded) metamaterials, the phase velocity of a wave is antiparallel to its group velocity. The resulting unconventional propagation had to wait nearly 30 years to be successfully demonstrated using the prototype proposed by Sir John Pendry's research group: meta-atoms made of split metal wires and rings. This simple design provided experimental evidence of the existence of double negative metamaterials [6, 7], setting a turning point for the community and attracting much attention and research work for the following decades [8, 9].

I.2 Analogy Acoustics/Electromagnetism

The craze generated by these major discoveries has not been limited to electromagnetism and has also taken hold of the acoustics and elasticity community [1, 2] fueled by the mathematical similarities between the governing equations.

In particular, a strong analogy [10] can be drawn between Maxwell's lossless equations linking the electric **E** and magnetic **H** fields in electromagnetism through the permeability ϵ and the permittivity μ

$$-\epsilon \frac{\partial \mathbf{E}}{\partial t} + \nabla \times \mathbf{H} = 0, \tag{I.1}$$

$$\mu \frac{\partial \mathbf{H}}{\partial t} + \nabla \times \mathbf{E} = 0, \tag{I.2}$$

and Euler's equations and mass conservation equations linking pressure p and velocity \mathbf{v} in acoustics through density ρ and compressibility C,

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \nabla p = 0, \tag{I.3}$$

$$C\frac{\partial p}{\partial t} + \nabla \mathbf{.v} = 0. \tag{I.4}$$

Both systems can be cast in a wave equation displaying the celerity of the respective wave in terms of the constitutive parameters, i.e., $c = (\epsilon \mu)^{-1/2}$ and $c = (\rho C)^{-1/2}$.

Despite the different types of solutions involved - longitudinal scalar for acoustic waves in fluids and transverse vector with two polarizations for electromagnetic waves - it is therefore not surprising that researchers in each community are looking for similar exotic properties, by acting on ρ (resp. ϵ) and/or C (resp. μ).

As a first step, the concept of photonic crystals was translated into elastic and acoustic media through the work of Kushwaha *et al.* [11] and Economou *et al.* [12, 13] in 1993 thus implying the appearance of the term phononic crystals. In 2000, the Professor Ping Sheng's research group added inner local resonances in phononic crystals [14], enabling to open bandgaps at frequencies much lower than that of the Bragg scattering, thus allowing negative dynamic acoustic properties, and paving the way for the development of acoustic metamaterials [15–17].

I.3 Extreme constitutive parameters: different operating regimes

Acoustic metamaterials can be classified depending on the operating regime of the constitutive parameters, that is positive, negative or near-zero, as it can be summarized in the diagram of Fig. I.1. It is worth noting that in the longwavelength limit, i.e., when both the periodicity and the dimensions of the meta-atoms are small compared to the acoustic wavelength, the meta-structure can be considered as an effective medium characterized by the effective constitutive dynamic parameters $\rho(\omega)$ and $C(\omega)$ determined by a homogenization process.



Figure I.1: Diagram of the operating regimes of acoustic metamaterials: examples of metamaterials with extreme constitutive parameters (adapted from [17]). Left upper quadrant: single-negative metamaterials with a negative effective dynamic mass density. (a) 3D deeply subwavelength lattice made of metallic spheres coated with a soft layer of sillicon rubber [14], (b) normal velocity field of a membrane-type metamaterial [18]. Left lower quadrant: double negative metamaterials with simultaneously negative compressibility and mass density. (c) Acoustic waveguide loaded with both side holes (responsible for the negative compressibility) and resonant membranes (responsible for the negative dynamic density) [19]. Right lower quadrant: single negative metamaterials with a negative compressibility. (d) 2D lattice of borehole resonators [20], 1D waveguide loaded with a periodic array of Helmholtz resonators [21]. Right upper quadrant: conventional materials or structured materials with gradient properties with positive constitutive parameters. (e) 3D printed graded porous material for broadband perfect absorption of sound [22], (g) axisymmetric gradient index lens made of rigid toroidal scatterers embedded in air [23]. (h) Zero index media with an almost zero mass density, made of a periodic array of anisotropic scatterers (structured cylinders) embedded in a 2D waveguide [24] (i) Zero index media with an almost zero compressibility composed of a Helmholtz resonator loaded in parallel to a waveguide [25]. (j) Double zero metamaterials with simultaneous zero compressibility and density made of an array of cylindrical bling holes [26].

I.3.1 Double positivity: conventional materials

The upper right quadrant of the diagram represents the class of conventional materials with positive mass density and compressibility. As previously mentioned, these materials, grouped under the term

"conventional", include natural media (glass, metals,...), but also man-made structured media that do not have inner local resonances but overcome the constraints encountered with natural materials. We can cite for example the realization of 3D printed porous materials illustrated in Fig. I.1(f), in which the microstructure is made up of an optimized gradient of properties enabling broadband absorption [22]. Besides, wave focusing leading to sound amplification can be achieved by using an axisymmetric gradient index lens consisting of rigid toroidal scatterers embedded in air [23]. Improved media can therefore be obtained using non resonant structured systems (gradient of properties or periodicity).

I.3.2 Single negativity

The introduction of local resonances in periodic media and the use of the produced extreme constitutive dynamic parameters is another strategy that will be detailed in the following.

The constitutive parameters of a medium have a direct impact on the propagation of an acoustic wave, the effective wavenumber of which reads as $k(\omega)^2 = \omega^2 \rho(\omega) C(\omega)$. If only one constitutive parameter is negative, the effective wavevector becomes purely imaginary. Consequently, the acoustic waves become evanescent and bandgaps, in which waves cannot propagate, are opened. Such condition can be achieved acting on either the compressibility or the mass density.

I.3.2.1 Negative dynamic mass density

To understand what negative mass density means, we should start again with the locally resonant sonic crystals designed by Liu *et al.* in 2000 [14]. In that cubic lattice illustrated in Fig. I.1(a), the soft coated metal spheres are acting as local mass-spring resonators. Under harmonic excitation F, the heavy metal sphere, acting as mass m_2 , can have a displacement x_2 around its steady position within the cavity formed by the soft silicone rubber shell. The coating, playing the role of a spring with a constant K and a mass m_1 , also moves with a displacement x_1 . By solving Newton's laws, the dynamics of the system can be modeled as

$$F = \left(m_1 + \frac{K}{\omega_r^2 - \omega^2}\right) \ddot{x}_1,\tag{I.5}$$

where the inner resonance pulsation $\omega_r^2 = K/m_2$ is clearly apparent. From an external point of view, i.e., when the inner structure cannot be seen, the system appears to react to a frequency dependent dynamic effective mass of the form $M(\omega) = m_1 + K/(\omega_r^2 - \omega^2)$. At the metamaterial level, one can define an effective dynamic mass density accounting for the relative motion of each meta-atom

$$\rho(\omega) = \frac{\overline{f}_d}{\ddot{x}},\tag{I.6}$$

linking the average (over the unit cell surface area) force density \overline{f}_d with the second order time derivative of the meta-atom displacement x. Further details on dynamic effective mass and effective mass density can be found in the work of Milton and Willis [27], Mei *et. al* [28], and in Yao *et al.* for an experimental verification on a 1D mass-spring system [29].

Examining equation (I.6), the negative dynamic mass density implies that the directions of force and acceleration are antiparallel. In the case of the considered resonant sonic crystals, negative density occurs above the inner resonance frequency. The out-of-phase motion of the masses produces Fano interferences, responsible for the opening of a low frequency stopband. In contrast to the Bragg bandgaps produced by the periodicity-dependent destructive interferences in the diffraction regime, Fano interferences are only due to the inner resonances. The independence on the spatial periodicity further allows the design of deep subwavelength structures.

Similar behaviors can also be obtained with membrane-type metamaterials in air, as demonstrated by the pioneering work of Yang *et al.* in 2008 [18]. A more thorough exploration of membrane- and plate-type metamaterials is presented in Chapter II.

I.3.2.2 Negative dynamic compressibility

Negative dynamic compressibility can be interpreted in opposite, as an unintuitive behavior: the expansion of a medium when compressed. Acoustic resonators such as quarter-wavelength or Helmholtz resonators are very good candidates since the underlying physics involves compression and expansion of air.

Fang *et al.* evidenced for the first time a dispersive negative compressibility with an underwater metamaterial made of a periodic set of coupled Helmholtz resonators loaded to a waveguide (Fig. I.1(e)) and working in the kHz frequency range [21]. Another example with a 2D periodic arrangement of bored hole resonators can be found in [20]. In both cases, the collective resonance produces a strong dispersion making the dynamic compressibility negative and leading to evanescent waves in the opened bandgap.

It should be noted here that the dispersion of systems composed of periodically loaded resonators was already previously studied in the pioneering work of Bradley [30] and Sugimoto [31], although they introduced neither the homogenization procedure nor the effective dynamic properties.

Symmetry of the resonances: To summarize, single negativity can be achieved by introducing local resonances in periodic structures. In doing so, the strong dispersion induced by the resonances gives rise to Fano destructive interferences, generating bandgaps in which the waves are evanescent.

Resonance-induced effective abnormal constitutive parameters can be classified into two categories according to the symmetry of resonance. Li *et al.* have indeed theoretically proved in 2004 that monopolar symmetry modes have mainly an impact on compressibility, since it leads to a compression-dilatation response, while dipolar symmetry resonances produce a dispersive effective dynamic mass density, with a localized contribution at the center of mass [32].

The development of single negative metamaterial has led to a plethora of applications including but not limited to deep-subwavelength perfect absorbers and high efficiency isolation devices involving either monopolar inclusions [33–36] or dipolar ones [18, 37–42].

I.3.3 Double negativity

The last quadrant (lower left) of Fig. I.1, represents certainly the most unexpected regime for acoustic metamaterials, double negativity. This regime, in which both constitutive parameters are negative, is analogous to negative-index in electromagnatism, leading to negative phase velocity and abnormal refraction. Interestingly and counter-intuitively, despite the fact that a single negativity prevents the waves from propagating, the simultaneous combination of the two negative constitutive parameters switches the medium from opaque to transparent, allowing the waves to propagate.

Double negativity requires coupling monopolar and dipolar resonances in the same frequency range. Two strategies can be employed. First, a combination of different types of resonators can be used to overlap the frequency response related to each symmetry [43–45]. Lee *et al.* experimentally realized the first double negative acoustic metamaterial [19] with an air filled waveguide periodically segmented with clamped membranes and loaded by periodic side holes (see Fig I.1(c)), generating respectively frequency dependent effective density (dipolar response) [37] and effective compressibility (monopolar response) [46]. The second strategy is to design resonators exhibiting eigenmodes with distinct symmetries. Careful tuning of these eigenmodes can then enable to control dispersion so that the effective density and compressibility are simultaneously negative, as demonstrated with coupled membranes with attached masses [47] or with soft 3D metamaterials consisting of sub-wavelength macroporous silicone rubber microbeads [48], exploiting the Mie resonance phenomenon [32].

How does such a medium behave physically speaking? Conciliating negative effective density and compressibility requires that the metamaterial expands upon compression as it moves in the opposite direction to that of the excitation for certain frequency ranges. This peculiar response has drawn considerable attention and inspired many applications such as reverse Doppler effect [49, 50], cloaking [51], super-focusing [52, 53], and subwavelength imaging [54, 55]. However, these structures also face the constant challenge of managing viscothermal losses. Cutanda *et al.* [56] demonstrated numerically, for example, the total disappearance of the double negativity features in metamaterials consisting of structured rigid cylinders [24], solely due to losses.

I.3.4 Zero-Index

Another interesting class of metamaterials is that of Zero-Index Metamaterials (ZIMs), in which one or both of the constitutive dynamic parameters are almost zero. We have seen in the aforementioned works that resonances induce a strong dispersion in the system, resulting in frequency-dependent effective parameters that may vary from positive to negative, thus passing through zero.

First discussed in plasmonics and electromagnetism [57, 58], theorized by Ziolkowski in 2004 [59], and extensively developed thanks to the pioneering work of Professor Engheta's research group [60], ZIMs have shown their ability to considerably extend the possibilities of wave manipulation. For instance, epsilon near zero metamaterials have been used to squeeze electromagnetic energy through narrow waveguide channels [61–65], to reduce scattering induced by sharp bends [66], to achieve

transparency and cloaking [67, 68], or to tailor the radiation phase pattern [69]. Acoustics ZIMs can also be designed seeking out similar properties.

The waves propagate in ZIMs with an extremely high phase velocity (almost infinite in the lossless case), $c(\omega) = [\rho(\omega)C(\omega)]^{-1/2}$. As a result, the effective wavelength is stretched and gives rise to quasi-static field distribution, providing a wide scope for wave manipulation. It allows, among others, radiation patterning and phase front manipulation, extraordinary propagation through corners and perfect power divider [24], supercoupling and tunneling effects [70, 71], or cloaking [72].

Single near-zero metamaterials, i.e., in which only one of the constitutive parameters is almost zero, can be obtained on the one hand by using anisotropic scatterers [24] (see Fig. I.1(h)) or membranetype metamaterials [70] for near-zero density. On the other hand, the use of Helmholtz resonators allows the compressibility to vanish [25] as depicted in Fig. I.1(i). Nonetheless, every application of a single near-zero acoustic metamaterial is limited because of a strong impedance mismatch with the surrounding fluid; the effective impedance $Z(\omega) = \sqrt{\rho(\omega)/C(\omega)}$ being either nearly zero ($\rho(\omega) \approx 0$) or nearly infinite ($C(\omega) \approx 0$).

Double near-zero metamaterials provide a solution to this impedance mismatch. These media simultaneously have a density and a compressibility close to zero, which makes it possible to maintain a constant finite effective impedance that can match that of the surrounding medium. In this way, perfect transmission can be achieved while maintaining the above-mentioned properties. Figure I.1(j) shows an experimental realization of double zero acoustic metamaterials made of a 2D square lattice of symmetric blind holes, evidencing the total transmission and revealing a Dirac-like cone [26].

Other kinds of metamaterials

The previous review on metamaterials focuses only on passive acoustic metamaterials with extreme parameters. It is worth noting here, that many other types exist, including, but not limited to, Willis, topological, or active metamaterials, which are out of the scope of this manuscript. Thorough reviews can be found in Refs. [15, 17].

I.4 Objective of the thesis

As we have seen, a substantial work has already been done on the developments of acoustic metamaterials using extreme parameters, however, much remains to be done.

The objective of this PhD thesis is to study analytically, numerically, and experimentally the specific case of a realistic Plate-type Acoustic Metamaterial (PAM), i.e., a periodic arrangement of thin clamped elastic plates, paying careful attention to the role of losses inherent to such systems.

Although the plates are governed by a 4th order differential equation, unlike a 2nd order differential equation for a membrane, PAMs are also exhibiting near zero effective dynamic mass density around the resonance frequency of the plates. Through an in-depth examination of the behavior of these metamaterials, we will pay particular attention to three main phenomena: zero-phase propagation, acoustic doping, and hiding/cloaking.

Due to the strong dispersion around the resonance frequency, a near-zero density leading to a stretching of the effective wavelength is expected. As a result, the metamaterial displays a quasi-static field distribution making zero-phase propagation possible. Nevertheless, to what extent can this zero-phase propagation be effectively observed and measured?

Two main limitations will be encountered: first, the impedance mismatch related to the single nearzero condition. Second, losses, not avoidable in real operating systems, are a constant and crucial challenge in the design of acoustic devices and perhaps even more so in the case of metamaterials, in that they have a considerable impact on dispersion. As a result, the expected unusual phenomenon can be radically altered and even annihilated. The origin and influence of losses in Plate-type Acoustic Metamaterials must then be thoroughly investigated and characterized, so that the design of the realistic sample accounts for this as a key factor.

The design and realization of a sample allowing experimental demonstration of the relevant properties will then pave the way for the conceptualization of potential applications using the unique zero-phase propagation property. Special attention will be devoted to phase patterning and wavefront shaping with the design of a subwavelength acoustic dipole. Moreover, other useful applications could arise from these extreme media. One can easily imagine that due to acoustic wavelength stretching, the embedment of scatterers could have a limited impact on the behavior of the metamaterial if the longwavelength constraint is met, i.e., if the dimensions of the embedded scatterers are very small relative to the wavelength. Such promising conditions could set the stage for the design of hiding and/or cloaking devices.

Double zero metamaterials have the particularity of conciliating both the exotic properties of DNZ media and impedance matching, thus alleviating the constraints for realistic applications. It is therefore of great interest to find a way to transform our single near-zero medium, the PAM, into a Density and Compressibility Near Zero (DCNZ) medium. By examining what is being developed in electromagnetism, we will propose an acoustic analogous of photonic doping which consists in adding a single impurity to a single near-zero medium to convert it to double near-zero. In doing so, we expect to meet the requirements for achieving super-coupling, i.e., total transmission, zero density, and zero-phase propagation simultaneously.

I.5 Organization of the manuscript

This manuscript is divided into 7 Chapters including the introduction, the conclusion, and a sidework highlight.

This first Chapter sets out the general paradigm of this work and reviews the historical background of metamaterials in general, focusing on the extreme values of the constitutive parameters achievable in acoustics. In Chapter II, we remind the features related to periodic and locally resonant periodic structures with the prism of a Plate-type Acoustic Metamaterial.

Chapter III is dedicated to the classification of the Density Near Zero regime of the PAM and of the experimental observation of a zero-phase propagation.

In Chapter IV, we transpose the concept of photonic doping to acoustics so as to transform the Density Near Zero regime into Density and Compressibility Near Zero and enable to impedance match the PAM to the surrounding medium at the zero-phase frequency.

We explore analytically and numerically in Chapter V the feasibility to cloak an obstacle in the PAM making use of doping. Accounting for the strong limitation induced by the losses, we evidence experimentally a hiding zone.

Finally, Chapter VI gathers a general conclusion, a discussion and some prospects to the presented work as well as a presentation of the PhD work realized in parallel to the Near-zero regime in Plate-type Acoustic Metamaterials, with a focus on sound absorption devices made of annular plates in Chapter VII.

Bibliography

- R. V. Craster and S. Guenneau, eds. Acoustic Metamaterials: Negative Refraction, Imaging, Lensing and Cloaking. Vol. 166. Springer Series in Materials Science. Dordrecht: Springer Netherlands, 2013. DOI: 10.1007/978-94-007-4813-2.
- [2] V. Romero-García and A.-Ch. Hladky-Hennion, eds. Fundamentals and Applications of Acoustic Metamaterials: From Seismic to Radio Frequency. 1st ed. Wiley, 2019. DOI: 10.1002/ 9781119649182.
- [3] N. Engheta. "Invited Metamaterials with Negative Permittivity and Permeability: Background, Salient Features, and New Trends." *IEEE* 03 (2003), pp. 187–190.
- [4] A. Boardman. "Pioneers in metamaterials: John Pendry and Victor Veselago." Journal of Optics 13 (2011), p. 020401. DOI: 10.1088/2040-8978/13/2/020401.
- [5] V. G. Veselago. "The electrodynamic of substances with simultaneously negative values of ϵ and μ ." Soviet Physics Uspekhi 10 (1968), pp. 509–514. DOI: 10.1070/pu1968v010n04abeh003699.
- [6] D. R. Smith et al. "Composite Medium with Simultaneously Negative Permeability and Permittivity." *Physical Review Letters* 84 (2000), pp. 4184–4187. DOI: 10.1103/PhysRevLett.84.4184.
- [7] R. A. Shelby. "Experimental Verification of a Negative Index of Refraction." Science 292 (2001), pp. 77–79. DOI: 10.1126/science.1058847.
- [8] J. B. Pendry. "Negative refraction." Contemporary Physics 45 (2004), pp. 191–202. DOI: 10. 1080/00107510410001667434.
- S. A. Ramakrishna. "Physics of negative refractive index materials." Reports on Progress in Physics 68 (2005), pp. 449–521. DOI: 10.1088/0034-4885/68/2/R06.
- [10] J. M. Carcione and F. Cavallini. "On the acoustic-electromagnetic analogy." Wave Motion 21 (1995), pp. 149–162.
- M. S. Kushwaha et al. "Acoustic band structure of periodic elastic composites." *Physical Review Letters* 71 (1993), pp. 2022–2025. DOI: 10.1103/PhysRevLett.71.2022.

- [12] M. M. Sigalas and E. N. Economou. "Elastic and acoustic wave band structure." Journal of Sound and Vibration 158 (1992), pp. 377–382. DOI: 10.1016/0022-460X(92)90059-7.
- M. Sigalas and E. N. Economou. "Band structure of elastic waves in two dimensional systems." Solid State Communications 86 (1993), pp. 141–143. DOI: 10.1016/0038-1098(93)90888-T.
- P. Sheng et al. "Locally resonant sonic materials." *Physica B: Condensed Matter* 338 (2003), pp. 201–205. DOI: 10.1016/S0921-4526(03)00487-3.
- [15] S. A. Cummer, J. Christensen, and A. Alù. "Controlling sound with acoustic metamaterials." *Nature Reviews Materials* 1 (2016). DOI: 10.1038/natrevmats.2016.1.
- [16] G. Ma and P. Sheng. "Acoustic metamaterials: From local resonances to broad horizons." Science Advances 2 (2016), e1501595. DOI: 10.1126/sciadv.1501595.
- [17] F. Zangeneh-Nejad and R. Fleury. "Active times for acoustic metamaterials." *Reviews in Physics* 4 (2019), p. 100031. DOI: 10.1016/j.revip.2019.100031.
- Z. Yang et al. "Membrane-Type Acoustic Metamaterial with Negative Dynamic Mass." Phys. Rev. Lett. 101 (2008), p. 204301. DOI: 10.1103/PhysRevLett.101.204301.
- [19] S. H. Lee et al. "Composite Acoustic Medium with Simultaneously Negative Density and Modulus." *Physical Review Letters* 104 (2010). DOI: 10.1103/PhysRevLett.104.054301.
- [20] L. Schwan et al. "Complex dispersion relation of surface acoustic waves at a lossy metasurface." Applied Physics Letters 110 (2017), p. 051902. DOI: 10.1063/1.4975120.
- [21] N. Fang et al. "Ultrasonic metamaterials with negative modulus." Nature Materials 5 (2006), pp. 452-456. DOI: 10.1038/nmat1644.
- J. Boulvert et al. "Optimally graded porous material for broadband perfect absorption of sound." Journal of Applied Physics 126 (2019), p. 175101. DOI: 10.1063/1.5119715.
- [23] V. Romero-García et al. "Wave focusing using symmetry matching in axisymmetric acoustic gradient index lenses." Applied Physics Letters 103 (2013), p. 264106. DOI: 10.1063/1.4860535.
- [24] R. Graciá-Salgado et al. "Negative mass density and rho-near-zero quasi-two-dimensional metamaterials: Design and applications." *Physical Review B* 88 (2013). DOI: 10.1103/PhysRevB.88. 224305.
- [25] N. Cselyuszka, M. Sečujski, and V. Crnojević Bengin. "Compressibility-near-zero acoustic metamaterial." *Physics Letters A* 378 (2014), pp. 1153–1156. DOI: 10.1016/j.physleta.2014.02.
 022.
- [26] M. Dubois et al. "Observation of acoustic Dirac-like cone and double zero refractive index." *Nature Communications* 8 (2017). DOI: 10.1038/ncomms14871.
- [27] G. W. Milton and J. R. Willis. "On modifications of Newton's second law and linear continuum elastodynamics." *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 463 (2007), pp. 855–880. DOI: 10.1098/rspa.2006.1795.

- [28] J. Mei et al. "Dynamic Mass Density and Acoustic Metamaterials." In: Acoustic Metamaterials and Phononic Crystals. Ed. by Pierre A. Deymier. Vol. 173. Berlin, Heidelberg: Springer Berlin Heidelberg, 2013, pp. 159–199. DOI: 10.1007/978-3-642-31232-8_5.
- [29] S. Yao, X. Zhou, and G. Hu. "Experimental study on negative effective mass in a 1D mass-spring system." New Journal of Physics 10 (2008), p. 043020. DOI: 10.1088/1367-2630/10/4/043020.
- [30] C. E. Bradley. "Linear and Nonlinear Acoustic Bloch Wave Propagation in Periodic Waveguides." PhD thesis. The University of Texas at Austin, 1993.
- [31] N. Sugimoto and T. Horioka. "Dispersion characteristics of sound waves in a tunnel with an array of Helmholtz resonators." The Journal of the Acoustical Society of America 97 (1995), pp. 1446–1459. DOI: 10.1121/1.412085.
- [32] J. Li and C. T. Chan. "Double-negative acoustic metamaterial." *Physical Review E* 70 (2004).
 DOI: 10.1103/PhysRevE.70.055602.
- [33] S.-H. Kim and S.-H. Lee. "Air transparent soundproof window." AIP Advances 4 (2014), p. 117123.
 DOI: 10.1063/1.4902155.
- [34] N. Jiménez et al. "Ultra-thin metamaterial for perfect and quasi-omnidirectional sound absorption." Applied Physics Letters 109 (2016), p. 121902. DOI: 10.1063/1.4962328.
- [35] N. Jiménez et al. "Quasiperfect absorption by subwavelength acoustic panels in transmission using accumulation of resonances due to slow sound." *Physical Review B* 95 (2017). DOI: 10. 1103/PhysRevB.95.014205.
- [36] J. Li et al. "A sound absorbing metasurface with coupled resonators." Applied Physics Letters 109 (2016), p. 091908. DOI: 10.1063/1.4961671.
- [37] S. H. Lee et al. "Acoustic metamaterial with negative density." *Physics Letters A* 373 (2009), pp. 4464-4469. DOI: 10.1016/j.physleta.2009.10.013.
- [38] C. J. Naify et al. "Transmission loss and dynamic response of membrane-type locally resonant acoustic metamaterials." *Journal of Applied Physics* 108 (2010), p. 114905. DOI: 10.1063/1. 3514082.
- [39] C. J. Naify et al. "Transmission loss of membrane-type acoustic metamaterials with coaxial ring masses." Journal of Applied Physics 110 (2011), p. 124903. DOI: 10.1063/1.3665213.
- [40] C. J. Naify et al. "Membrane-type metamaterials: Transmission loss of multi-celled arrays." Journal of Applied Physics 109 (2011), p. 104902. DOI: 10.1063/1.3583656.
- [41] G. Ma et al. "Acoustic metasurface with hybrid resonances." Nature Materials 13 (2014), pp. 873– 878. DOI: 10.1038/nmat3994.
- [42] M. Yang et al. "Subwavelength total acoustic absorption with degenerate resonators." Applied Physics Letters 107 (2015), p. 104104. DOI: 10.1063/1.4930944.

- [43] Y. Ding et al. "Metamaterial with Simultaneously Negative Bulk Modulus and Mass Density." *Physical Review Letters* 99 (2007). DOI: 10.1103/PhysRevLett.99.093904.
- [44] L. Fok and X. Zhang. "Negative acoustic index metamaterial." *Physical Review B* 83 (2011).
 DOI: 10.1103/PhysRevB.83.214304.
- [45] J. Christensen, Z. Liang, and M. Willatzen. "Metadevices for the confinement of sound and broadband double-negativity behavior." *Physical Review B* 88 (2013). DOI: 10.1103/PhysRevB. 88.100301.
- [46] S. H. Lee et al. "Acoustic metamaterial with negative modulus." Journal of Physics: Condensed Matter 21 (2009), p. 175704. DOI: 10.1088/0953-8984/21/17/175704.
- [47] M. Yang et al. "Coupled Membranes with Doubly Negative Mass Density and Bulk Modulus." *Physical Review Letters* 110 (2013). DOI: 10.1103/PhysRevLett.110.134301.
- [48] T. Brunet et al. "Soft 3D acoustic metamaterial with negative index." Nature Materials 14 (2015), pp. 384–388. DOI: 10.1038/nmat4164.
- [49] S. H. Lee et al. "Reversed Doppler effect in double negative metamaterials." *Physical Review B* 81 (2010). DOI: 10.1103/PhysRevB.81.241102.
- [50] S. L. Zhai et al. "Inverse Doppler Effects in Broadband Acoustic Metamaterials." Scientific Reports 6 (2016). DOI: 10.1038/srep32388.
- [51] D. Torrent and J. Sánchez-Dehesa. "Acoustic cloaking in two dimensions: a feasible approach." New Journal of Physics 10 (2008), p. 063015. DOI: 10.1088/1367-2630/10/6/063015.
- [52] S. Zhang, L. Yin, and N. Fang. "Focusing Ultrasound with an Acoustic Metamaterial Network." *Physical Review Letters* 102 (2009). DOI: 10.1103/PhysRevLett.102.194301.
- [53] M. Chen et al. "Design of an acoustic superlens using single-phase metamaterials with a starshaped lattice structure." *Scientific Reports* 8 (2018). DOI: 10.1038/s41598-018-19374-2.
- [54] K. Deng et al. "Theoretical study of subwavelength imaging by acoustic metamaterial slabs." Journal of Applied Physics 105 (2009), p. 124909. DOI: 10.1063/1.3153976.
- [55] H. Jia et al. "Subwavelength imaging by a simple planar acoustic superlens." Applied Physics Letters 97 (2010), p. 173507. DOI: 10.1063/1.3507893.
- [56] V. C. Henríquez, V. M. García-Chocano, and J. Sánchez-Dehesa. "Viscothermal Losses in Double Negative Acoustic Metamaterials." *Phys. Rev. Applied* 8 (1 2017), p. 014029. DOI: 10.1103/ PhysRevApplied.8.014029.
- [57] S. Enoch et al. "A Metamaterial for Directive Emission." *Phys. Rev. Lett.* 89 (2002), p. 213902.
 DOI: 10.1103/PhysRevLett.89.213902.
- [58] N. Garcia, E. V. Ponizovskaya, and John Q. Xiao. "Zero permittivity materials: Band gaps at the visible." Appl. Phys. Lett. 80 (2002), pp. 1120–1122. DOI: 10.1063/1.1449529.

- [59] R. W. Ziolkowski. "Propagation in and scattering from a matched metamaterial having a zero index of refraction." *Phys. Rev. E* 70 (2004), p. 046608. DOI: 10.1103/PhysRevE.70.046608.
- [60] N. Engheta. "Pursuing Near-Zero Response." Science 340 (2013), pp. 286–287. DOI: 10.1126/ science.1235589.
- [61] M. Silveirinha and N. Engheta. "Tunneling of Electromagnetic Energy through Subwavelength Channels and Bends using Epsilon-Near-Zero Materials." *Phys. Rev. Lett.* 97 (2006), p. 157403.
 DOI: 10.1103/PhysRevLett.97.157403.
- [62] M. G. Silveirinha and N. Engheta. "Theory of supercoupling, squeezing wave energy, and field confinement in narrow channels and tight bends using Epsion-near-zero metamaterials." *Phys. Rev. B* 76 (2007), p. 245109. DOI: 10.1103/PhysRevB.76.245109.
- [63] R. Liu et al. "Experimental Demonstration of Electromagnetic Tunneling Through an Epsilon-Near-Zero Metamaterial at Microwave Frequencies." *Phys. Rev. Lett.* 100 (2008), p. 023903. DOI: 10.1103/PhysRevLett.100.023903.
- [64] A. Alù, M. G. Silveirinha, and N. Engheta. "Transmission-line analysis of Epsilon-near-zero-filled narrow channels." *Phys. Rev. E* 78 (2008), p. 016604. DOI: 10.1103/PhysRevE.78.016604.
- [65] B. Edwards et al. "Experimental Verification of Epsilon-Near-Zero Metamaterial Coupling and Energy Squeezing Using a Microwave Waveguide." *Phys. Rev. Lett.* 100 (2008), p. 033903. DOI: 10.1103/PhysRevLett.100.033903.
- [66] B. Edwards et al. "Reflectionless sharp bends and corners in waveguides using epsilon-near-zero effects." Journal of Applied Physics 105 (2009), p. 044905. DOI: 10.1063/1.3074506.
- [67] A. Alù and N. Engheta. "Achieving transparency with plasmonic and metamaterial coatings." *Phys. Rev. E* 72 (2005), p. 016623. DOI: 10.1103/PhysRevE.72.016623.
- [68] W. Hao J.and Yan and M. Qiu. "Super-reflection and cloaking based on zero index metamaterial." Appl. Phys. Lett. 96 (2010), p. 101109. DOI: 10.1063/1.3359428.
- [69] A. Alù et al. "Epsilon-near-zero metamaterials and electromagnetic sources: Tailoring the radiation phase pattern." *Physical Review B* 75 (2007). DOI: 10.1103/PhysRevB.75.155410.
- [70] R. Fleury and A. Alù. "Extraordinary Sound Transmission through Density-Near-Zero Ultranarrow Channels." *Physical Review Letters* 111 (2013). DOI: 10.1103/PhysRevLett.111.055501.
- [71] H. Esfahlani et al. "Acoustic Supercoupling in a Zero-Compressibility Waveguide." Research 2019 (2019), pp. 1–10. DOI: 10.34133/2019/2457870.
- [72] Y. Wu and J. Li. "Total reflection and cloaking by zero index metamaterials loaded with rectangular dielectric defects." *Applied Physics Letters* 102 (2013), p. 183105. DOI: 10.1063/1. 4804201.

Chapter

Plate-type Acoustic Metamaterials

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II.1 Introduction

In the development of locally resonant acoustic metamaterials, membranes and plates have played a central role, certainly because of their high tunability and their small size and weight, allowing deep subwavelengths and lightweight designs that meet the requirements of very demanding applications such as in the aeronautics or aerospace industry.

Two main types of membrane-type acoustic metamaterials have been developed. The first type, based on decorated membranes (DM), composed of a clamped stretched membrane with additional mass platelets, are widely used for high insulation and sound absorption. The noise attenuation efficiency of weighted membranes was demonstrated, in building acoustics, in 1996 by Hashimoto *et al.* [1, 2] on a large-scale membrane with attached additional masses. The real development of DM had to wait until 2008 with the pioneering work of Yang [3], in which a simple membrane with a small attached mass acts counter-intuitively as an efficient sound reflector at frequencies between the first two eigenmodes of the structure. The coupling of the membrane and the added mass platelets gives rise to two simple vibration modes. The first eigenmode, the frequency of which depends on the weight of the mass, corresponds to an in-phase vibration of the mass and the membrane, while the mass platelet is motionless in the second. The out-of-phase superposition of the two eigenmodes amplitude at the antiresonance results in an averaged almost zero in-plane displacement, i.e., almost zero transmission. Detailed studies on the influence of the different design parameters (position, shape, number of additional masses, number of cells in the plane or stacking units,...) on the attenuation efficiency and frequency range followed [4–8].

In parallel, absorbing metamaterials have also been designed using DM. The optimization of the absorption of a structure is based on the concept of critical coupling. Total losses in resonators can be characterized by a quality factor Q, related to the half bandwidth of the resonance peak, and broken down into a dissipative part Q_{diss} and a leakage part Q_{leak} . The maximum absorption of a structure is reached when the inherent dissipation balances the energy leakage rate Q_{diss} = Q_{leak} , i.e., when the critical coupling condition is met [9–13]. In the problem of pure reflection, the critical coupling condition means perfect absorption regardless of the type of resonators (monopolar or dipolar). However, in the transmission problem with point resonators, the problem is more complicated as the critical coupling condition does not mean perfect absorption. If the resonators are monopolar (resp. dipolar), i.e., symmetric (resp. antisymmetric), the critical coupling condition implies that only 50% of the energy can be absorbed since only half of the problem is critically coupled. Therefore, in the case of a membrane or plate resonator, critical coupling means |R| = |T| = 0.5. This can be achieved by using membranes with semi-circular mass platelets as theoretically developed in Refs. [14, 15] and experimentally demonstrated in Ref. [16]. Fortunately, different strategies can be used to overcome this upper limit of absorption. On the one hand, one can use the anti-symmetric (resp. symmetric) coherent perfect absorption [13, 16, 17] by eliminating the monopolar (resp. dipolar) component of the incident wave. On the other hand, dipolar and monopolar motions being decoupled, the 50% absorption threshold can be exceeded until perfect absorption is reached, by combining these two types of resonance, i.e., by designing degenerated resonators [18, 19] or by using hybrid resonances [20]. All these studies have proven that membrane-type metamaterials have high absorption and insulation properties despite their subwavelength dimension, thus breaking the mass density law, stating that acoustic transmission through a layer of material is inversely proportional to the product of the layer thickness, mass density and sound frequency.

Besides decorated membranes, a second main type of membrane/plate-based metamaterial is a structure consisting of a periodic series stacking of membranes, which revealed exciting behaviors such as extraordinary transmission [21], tunneling effect [22], negative and almost zero density [23, 24] which will be further developed in Chapter III. In this work, the term "Plate-type Acoustic Metamaterial" (PAM) will refer to a periodic arrangement of thin clamped elastic plates, that is a one dimensional (1D) resonant sonic crystal. This Chapter aims to show the general acoustic behavior of a PAM and to explain how such a system can be modeled analytically, following the theory of sonic crystals and periodic media. It is organized as follows. First, we will focus on the simple case of a 1D non-resonant sonic crystal so as to introduce some generalities on periodic media, such as the derivation of the dispersion relation, the significance of the bandgaps, the transfer matrix method (TMM), ... Then, we will develop precisely the particular case of PAM modeling and the general behavior of such systems. Finally, we will present the experimental method for the acoustic characterization of the plates and the experimental set-up used.

II.2 "Non-resonant" 1D sonic crystals: General approach

"Crystals are solid materials whose constituents, such as atoms, molecules or ions, are arranged in a highly ordered microscopic structure, forming a lattice that extends in all directions". Periodic media can therefore be classified under this general definition [25].

At the end of the 19th century, Lord Rayleigh studied wave propagation in one-dimensional (1D) periodic stacks made of multilayer dielectric media and showed that over certain frequency ranges, named band-gaps, the incident waves undergo strong reflection [26, 27]. Due to the periodic spacing, destructive interference occurs with the multiple reflection at each layer, preventing propagation. This research laid the groundwork for the development of photonic crystals about one hundred years before the birth of the official denomination, with the simultaneous work of Yablonovitch [28] and John [29] in 1987. The periodic variation of the dielectric constant in photonic crystals produces similar effects to those of the periodic potential in semiconductor crystals, i.e., it allows or prevents the propagation of light (resp. electrons) in the passband or bandgap (resp. in the allowed or forbidden electronic energy bands).

Simultaneously with these works on electromagnetic waves [30–32], similar crystals have been developed with mechanical waves [33, 34]. The year 1993 marked the first design of a phononic crystal, i.e., a periodic distribution of solid scatterer embedded in a solid host medium [35, 36],
followed two years later by sonic crystals, i.e., particular cases of phononic crystals in which the host medium is a fluid [37–39]. Sonic crystals can be designed in one or several dimensions of the space (1D, 2D or 3D), depending on the dimensions in which the periodic variation of the density ρ and/or the compressibility C occurs as sketches by figure II.1.



Figure II.1: Examples of 1D, 2D and 3D crystals: the density or compressibility of which varies periodically along one, two or three spatial axes respectively. Each color represents a material with different constitutive parameters (ρ and C). The figure is taken from Ref. [30].

In this work, we will only focus on a 1D sonic crystal. Let us first consider a cylindrical waveguide of section $S = \pi R_a^2$, filled with one or two different materials as shown in Fig. II.2(b). We will use this simple example to introduce the Transfer Matrix Method (TMM) and the dispersion relation calculation.

II.2.1 Propagation in an air-filled waveguide

If the waveguide is first only filled with air (density ρ_0 and compressibility C_0), the lossless propagation over a length L_{unit} of a plane wave, the frequency of which is lower than the waveguide cut-off frequency, is governed by the constitutive equations

$$\frac{\partial p}{\partial x} = -i\omega \frac{\rho_0}{S} U,\tag{II.1}$$

$$\frac{\partial U}{\partial x} = -i\omega \frac{S}{\kappa_0} p,\tag{II.2}$$

written under the time convention $e^{i\omega t}$, where ω is the circular frequency, $\kappa_0 = 1/C_0$ is the air bulk modulus, p is the sound pressure, and U = Sv is the flux, with v being the particle velocity.

These governing equations can be rearranged in the following matrix form

$$\frac{\partial}{\partial x} \begin{cases} p \\ U \end{cases} = \begin{bmatrix} 0 & -i\omega\frac{\rho_0}{S} \\ -i\omega\frac{S}{\kappa_0} & 0 \end{bmatrix} \begin{cases} p \\ U \end{cases} = \mathbf{M} \begin{cases} p \\ U \end{cases},$$
(II.3)

the solution of which being

$$\begin{cases} p \\ U \end{cases}_{L_{unit}} = \exp(\mathbf{M}L_{unit}) \begin{cases} p \\ U \end{cases}_{0} = \mathbf{V} \cdot \begin{bmatrix} e^{\Lambda_{1}L_{unit}} & 0 \\ 0 & e^{\Lambda_{2}L_{unit}} \end{bmatrix} \cdot \mathbf{V}^{-1} \begin{cases} p \\ U \end{pmatrix}_{0} = \mathbf{T} \begin{cases} p \\ U \end{pmatrix}_{0}, \quad (\text{II.4})$$

with Λ_j the eigenvalues of **M**, and **V** the corresponding eigenvector matrix.

The transfer matrix \mathbf{T}_{unit} linking pressure and flux at one side and at the other of the L_{unit} -long slab reads as follows (the detailed calculation can be found in Appendix A)

$$\mathbf{T_{unit}} = \begin{bmatrix} \cos\left(k_0 L_{unit}\right) & -iZ_0 \sin\left(k_0 L_{unit}\right) \\ -iZ_0^{-1} \sin\left(k_0 L_{unit}\right) & \cos\left(k_0 L_{unit}\right) \end{bmatrix},\tag{II.5}$$

with $k_0 = \omega/c_0 = \omega\sqrt{\rho_0/\kappa_0}$ the wavenumber and $Z_0 = \sqrt{\rho_0\kappa_0}/S$ the acoustic characteristic impedance.

The dispersion relation of an infinite periodic system can then be deduced from the total matrix of a single unit cell \mathbf{T}_{unit} by applying the Bloch-Floquet theorem

$$\cos\left(qL_{unit}\right) = \frac{\operatorname{Tr}(\mathbf{T}_{unit})}{2},\tag{II.6}$$

with q the Bloch wavenumber (details can be found in Appendix A).

In the present case of an air-filled waveguide with an "artificial" periodicity, this operation leads to the straightforward dispersion relation

$$\omega = cq = c\left(k_0 + 2m\pi/L_{unit}\right),\tag{II.7}$$

with m an integer.

As demonstrated in Fig. II.2(c), thanks to the symmetry of the system, all relevant information on the dispersion relation is located within the $[-\pi/L_{unit} \pi/L_{unit}]$ k-space band, named the 1st Brillouin zone and bounded by the red dashed vertical lines. Figure II.2(h) is another representation of the dispersion relation with normalized axes, i.e., $\omega/\omega_B = \mathcal{F}(\text{Re}(qL_{unit}/\pi))$. By doing so, we can clearly see that the degeneracy point (where the scattering curve is folded on the edges of the Brillouin zone, i.e., $\text{Re}(qL_{unit}/\pi) = 1$) is located at the Bragg frequency $\omega_B = \pi c_0/L_{unit}$. The acoustic wavelength λ at the degeneracy is therefore

$$\lambda = 2L_{unit}.\tag{II.8}$$

It is worth noting here that no dispersion is encountered, since lossless propagation in the air is considered. Therefore, the phase velocity $c = \omega/\text{Re}(q)$ and the group velocity $c_g = \partial \omega/\partial \text{Re}(q)$ are equal and constant (Fig. II.2(d)).



Figure II.2: **1D** non resonant sonic crystals: "artificial" periodic medium made of a single material (a,c,d,g,h,i) and bi-layer periodic medium (b,e,f,j,k,l). Sketch of the considered media (a,b). Real part (c,e,h,k), and imaginary part (i,l) of the dispersion relation $\omega/\omega_B = \mathcal{F} (\text{Re}(qL_{unit}/\pi))$, phase (black solid line) and group (red dashed line) velocity (d,f). Red vertical dashed lines in Figs. (c, e) delimits the 1st Brillouin zone. Red color surfaces in Figs. (e,f,j,k,l) represent the location of the Bragg bandgaps.

II.2.2 Propagation in a bi-layer periodic medium

We now consider a real periodic medium made of alternating $L_{unit}/2$ -long layers of two different materials (ρ_0 , κ_0 and $\rho_1 = 2\rho_0$, $\kappa = 18\kappa_0$) arranged with a periodicity $L_{unit} = 1$ cm as sketched in Fig. II.2(b).

The total transfer matrix of the unit-cell \mathbf{T}_{unit} is now the matrix product of the elementary transfer matrices in the air \mathbf{T}_0 and in the material \mathbf{T}_1

$$\begin{cases} P \\ U \end{cases}_{L_{unit}} = \mathbf{T_0} \cdot \mathbf{T_1} \begin{cases} P \\ U \end{cases}_0 = \mathbf{T_{unit}} \begin{cases} P \\ U \end{cases}_0,$$
(II.9)

$$\mathbf{T}_{\mathbf{unit}} = \begin{bmatrix} \cos\left(k_0 \frac{L_{unit}}{2}\right) & -iZ_0 \sin\left(k_0 \frac{L_{unit}}{2}\right) \\ -iZ_0^{-1} \sin\left(k_0 \frac{L_{unit}}{2}\right) & \cos\left(k_0 \frac{L_{unit}}{2}\right) \end{bmatrix} \cdot \begin{bmatrix} \cos\left(k_1 \frac{L_{unit}}{2}\right) & -iZ_1 \sin\left(k_1 \frac{L_{unit}}{2}\right) \\ -iZ_1^{-1} \sin\left(k_1 \frac{L_{unit}}{2}\right) & \cos\left(k_1 \frac{L_{unit}}{2}\right) \end{bmatrix} . \quad (\text{II.10})$$

The dispersion relation shown in Fig. II.2 (e,j,k,l) is derived by applying Eq. (II.6). Contrary to the previous case, $\cos(qL_{unit})$ now can take values larger or smaller than 1 or -1 respectively over some frequency bands (see Fig. II.2 (j)), giving a purely imaginary wavevector q (see Fig. II.2 (l)). As a result, the waves become evanescent and a Bragg bandgap (red-mapped surface) is opened around the Bragg frequency, that is $f_B = 17.15$ kHz. The formation of a bandgap can be understood as a consequence of the multiple reflections induced by the periodicity. At each layer interface, part of the incident energy is reflected, generating multiple reflected waves interacting with each other. Constructive interference of the reflection while destructive interference allows total transmission through the crystal. The first condition results in the generation of a bandgap, while the second leads to a propagating band. It should be noted here that higher order Bragg bandgaps are also opened around odd multiples of f_B , i.e., for integer multiples of the wavelength.

Changing the periodicity of the system thus results in a shift of the bandgap, the center frequency of which is inversely proportional to L_{unit} . In the opposite, the width of the bandgap is related to the impedance contrast between each layer: the greater the contrast, the wider the bandgap. In addition to the opening of the forbidden bands, where the waves cannot propagate, a strong dispersion occurs in the vicinity of the degeneration point. Phase and group velocities are no longer equal and vary with frequency as shown in Fig. II.2(f). These outstanding features and the high tunability initiate a wide range of applications, including isolation devices and sound barriers.

II.3 Resonant sonic crystal: Modeling of a PAM

As demonstrated in the previous Section, non-resonant sonic crystals have opened up a new field of research for tailoring the propagation of sound waves. However, these applications are severely limited in the low-frequency regime, since the diffraction regime, in which Bragg bands can be generated, requires the crystal periodicity to be of the same order of magnitude as the wavelength. The pioneering works of Bradley [40], Sugimoto [41], Liu [42] and the large number of publications that have followed, have shown that introducing local resonances in a sonic crystal allows the opening of similar forbidden frequency bands in which waves cannot propagate, at frequencies much lower than that of Bragg. One of the main advantage of such resonant sonic crystals is that the stopband location can be shifted independently of the periodicity, thus allowing deep subwavelength designs.

In this work, we will use a thin elastic plate as the resonant element of our one dimensional sonic crystal. The TMM-based analytical modeling of the system will be first presented, followed by the thin plate theory derivation, and the viscoelastic and viscothermal models to account for the losses in the system.

II.3.1 Analytical model

A periodic arrangement of N thin elastic plates clamped in a circular waveguide of cross-sectional section $S = \pi R_a^2$, similar to that sketched in Fig. II.4 is considered. The metamaterial unit cell is symmetric and consists in a thin plate of thickness h_p surrounded by two air layers of length $L_{gap}/2$, giving a periodicity $L_{unit} = L_{gap} + h_p$ to the system. The metamaterial is modeled with the TMM. We thus only consider plane wave propagation and neglect the mutual coupling between the elements, which is a good approximation for the objectives of this work.



Figure II.3: Side branch and series elements: schematic drawing of the side branch (parallel) resonator (a) and the series resonator (b)

Two types of elements can be considered: parallel or series elements, both of which are considered punctual, i.e., of dimension $\Delta x = 2\delta x$ much smaller than the acoustic wavelength as illustrated in Fig. II.3. Parallel or side branch resonators of acoustic impedance Z_{SB} are characterized by a constant pressure $p(x + \delta x) = p(x - \delta x)$ and a flux balance $U(x + \delta x) = U_{SB}(x) + U(x - \delta x)$, with $U_{SB}(x) = p(x)/Z_{SB}$ the flux entering the resonator. In contrast, series resonator of acoustic impedance Z_S are characterized by a constant flux $U(x + \delta x) = U(x - \delta x)$ and a discontinuity of pressure $p(x + \delta x) = p(x - \delta x) + Z_S U_S$, with $U_S = p(x)/Z_S$. As a result, the elementary transfer matrix of both elements reads as

Side branch element:
$$\mathbf{T}_{\mathbf{SB}} = \begin{bmatrix} 1 & 0 \\ 1/Z_{SB} & 1 \end{bmatrix}$$
, Series element: $\mathbf{T}_{\mathbf{S}} = \begin{bmatrix} 1 & Z_S \\ 0 & 1 \end{bmatrix}$. (II.11)

A plate laterally clamped into a 1D waveguide causes pressure discontinuity and acoustic velocity continuity [24], i.e., a element in series, the transfer matrix of which is

$$\mathbf{T}_{\mathbf{p}} = \begin{bmatrix} 1 & Z_p \\ 0 & 1 \end{bmatrix},\tag{II.12}$$

involving the acoustic impedance Z_p of the circular plate considered as punctual $(h_p \ll \lambda_0)$ and defined in Subsection II.3.2.

The air gaps are defined by the elementary transfer matrix of a circular waveguide of length $L_{gap}/2$,

$$\mathbf{T_{cav}} = \begin{bmatrix} \cos\frac{k_0 L_{gap}}{2} & -iZ_0 \sin\frac{k_0 L_{gap}}{2} \\ -iZ_0^{-1} \sin\frac{k_0 L_{gap}}{2} & \cos\frac{k_0 L_{gap}}{2} \end{bmatrix},$$
(II.13)

with Z_0 and k_0 the acoustic impedance and wavenumber of the fluid respectively.

The matrix representing the unit cell is then obtained by multiplying the above-mentioned elementary matrices

$$\mathbf{T}_{\mathbf{unit}} = \mathbf{T}_{\mathbf{cav}} \cdot \mathbf{T}_{\mathbf{p}} \cdot \mathbf{T}_{\mathbf{cav}},\tag{II.14}$$

while that of the L-long metamaterial composed of N unit cells read as

$$\mathbf{T} = \mathbf{T}_{\mathbf{unit}}{}^{N} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}, \qquad (II.15)$$

and connects pressure and flux to the input (x = 0) and output (x = L) of the finite system.



Figure II.4: **3D** sketch of the **1D** PAM: whole system (a), close-up on the unit cell with a compacted (b) or exploded (c) view.

II.3.2 Thin plate impedance

The transverse displacement w of a thin circular plate of section $S = \pi R_a^2$, thickness h_p , Young modulus E_p , Poisson's ratio ν_p , and density ρ_p satisfies the flexural wave equation

$$D\nabla^4 w(r,\theta,t) + \rho_p h_p \frac{\partial}{\partial t} w(r,\theta,t) = \Delta P(t), \qquad (\text{II.16})$$

with $\nabla^4 = (\nabla^2)^2 = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)^2$ the bilaplacian in polar coordinates, $\Delta P = p(x+h_p) - p(x)$ the pressure difference applied on the plate, where p(x) and $p(x+\delta x)$ are the acoustic pressure on the upstream and downstream faces respectively, and D the flexural rigidity $D = \frac{E_p h_p^3}{12(1-\nu_p^2)}$.

The full derivation of the solution can be found in Appendix B. We only remind here the Kirchhoff-Love plate acoustic impedance Z_p resulting from a separation of variables and the application of the clamped boundary condition $w(r = R_a, \theta) = \frac{\partial}{\partial r}w(r = R_a, \theta) = 0$

$$Z_p = \frac{\int_S \Delta P(r) dS}{i\omega \overline{w} S^2} = -\frac{i\omega m}{S^2} \frac{I_1(k_p R_a) J_0(k_p R_a) + J_1(k_p R_a) I_0(k_p R_a)}{I_1(k_p R_a) J_2(k_p R_a) - J_1(k_p R_a) I_2(k_p R_a)},$$
(II.17)

where \overline{w} is the cross-sectional average of the displacement and $k_p = (\omega^2 \rho_p h_p / D)^{1/4}$ is the wavenumber of the flexural waves in the plate, with $m = \rho_p S h_p$ the mass of the plate, and J and I respectively the Bessel and modified Bessel functions of the 1st kind. A uniform pressure distribution over the plate (only small displacements and plane wave excitation), and only axi-symmetric modes are assumed in this expression. Please note that although the plate introduces a discontinuity of pressure ΔP , the normal velocity and the mean flow $U = i\omega \overline{w}S$ are continuous across the plate.

II.3.3 Losses modeling

As most acoustic systems, PAM presents unavoidable losses. In this case, two main loss sources have to be accounted for.

First, viscothermal losses should be considered near the guide walls, within the viscous penetration thickness $\delta_{\nu} = \sqrt{2\mu_0/\rho_0\omega}$ (where μ_0 is dynamic viscosity, ρ_0 is the density of air). Propagation in the waveguide is thus modeled by complex wavenumbers k_0 and impedance Z_0 , as defined by Zwikker and Kosten [43, 44]

$$k_0 = \frac{\omega}{c_0} \left(1 + \frac{(1-i)}{\sqrt{2R_a/\delta_\nu}} \frac{(1+(\gamma-1))}{\sqrt{Pr}} \right),$$
 (II.18)

$$Z_0 = \frac{\rho_0 c_0}{\pi R_a^2} \left(1 + \frac{(1-i)}{\sqrt{2R_a/\delta_\nu}} \frac{(1-(\gamma-1))}{\sqrt{Pr}} \right),\tag{II.19}$$

with Pr the Prandtl number, γ the ratio of the thermal capacity of the air and c_0 the sound speed in the air. It should be noted here that the losses in the bulk are neglected in front of those in the penetration thickness.

Second, the plates also introduce dissipation. The viscoelastic losses inherent to the plates are modeled assuming a loss factor β , i.e., by adding an imaginary part to the Young's modulus

$$E_p = E_{p_0}(1+i\beta).$$
 (II.20)

The loss factor β is defined as frequency-independent since the study focuses only on a narrow frequency range in the vicinity of resonance.

II.3.4 Scattering coefficients and effective properties

The total transfer matrix can be related to the scattering coefficients, i.e., the transmission T and reflection R coefficients

$$T = \frac{2}{T_{11} - T_{12}/Z_0 - Z_0 T_{21} + T_{22}},$$
(II.21)

$$R = \frac{T_{11} - T_{12}/Z_0 + Z_0 T_{21} - T_{22}}{T_{11} - T_{12}/Z_0 - Z_0 T_{21} + T_{22}},$$
(II.22)

for symmetric $T_{11} = T_{22}$ and reciprocal $T_{11}T_{22} - T_{12}T_{21} = 1$ systems [45, 46]. These two coefficients are then used to define the reflectance $|R|^2$, the transmittance $|T|^2$ and the absorbance $\alpha = 1 - |R|^2 - |T|^2$ of the metamaterial.

Homogenization

Moreover, in the long wavelength approximation, i.e., when the condition $L_{unit} \ll \lambda$ is fulfilled, a sonic crystals can be homogenized, the result of which is an equivalent medium with effective properties [47–53]. Our system may also be considered as an equivalent, homogeneous medium of length L

$$\begin{cases} p \\ U \end{cases}_{L} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{cases} p \\ U \end{cases}_{0} = \begin{bmatrix} \cos k(\omega)L & -iZ(\omega)\sin k(\omega)L \\ -iZ(\omega)^{-1}\sin k(\omega)L & \cos k(\omega)L \end{bmatrix} \begin{cases} p \\ U \end{cases}_{0}.$$
(II.23)

Two approaches can be followed to retrieve the effective properties of the homogenized medium. First, one can consider an infinite system and then extract directly the effective impedance $Z(\omega)$ and effective wavenumber $k(\omega)$ from the unit cell transfer matrix \mathbf{T}_{unit} [46]

$$k(\omega)L_{unit} = \cos^{-1}\left(\frac{\operatorname{Tr}[\mathbf{T}_{unit}]}{2}\right),$$
 (II.24)

$$Z(\omega) = \pm \sqrt{\frac{T_{unit_{12}}}{T_{unit_{21}}}}.$$
(II.25)

The sign of $Z(\omega)$ has to be chosen to fulfill the passivity condition, that is $\operatorname{Re}(Z(\omega)) \ge 0$.

These effective parameters do not account for the effect of the finite number of unit cells. Another option is to determine the effective parameters using the transmission and reflection coefficients of the finite system [47, 48, 54, 55].

The derivation of the effective impedance is rather direct,

$$Z(\omega) = \pm Z_0 \sqrt{\frac{(1+R)^2 - T^2}{(1-R)^2 - T^2}},$$
(II.26)

while the effective wavenumber

$$k(\omega) = -\frac{\ln(|e^{ik(\omega)L}|) + i\arg(e^{ik(\omega)L})}{iL} + \frac{2\pi m}{L},$$
(II.27)

results from the inversion of

$$e^{ik(\omega)L} = \frac{T\left[1 - Z(\omega)/Z_0\right]}{R\left[1 + Z(\omega)/Z_0\right] - Z(\omega)/Z_0 + 1},$$
(II.28)

with m a natural integer to correct the phase after inversion, the exponential being 2π periodic.

Knowing the effective impedance and the wavenumber, by either method, the effective density $\rho(\omega)$ and bulk modulus $\kappa(\omega)$ can then be determined

$$\rho(\omega) = S \frac{Z(\omega)}{c(\omega)} = S Z(\omega) \frac{k(\omega)}{\omega}, \qquad (\text{II.29})$$

$$\kappa(\omega) = \rho(\omega)c(\omega)^2 = SZ(\omega)\frac{\omega}{k(\omega)}.$$
(II.30)

II.4 Resonant sonic crystal: acoustic behavior of a PAM

The comprehensive analytical model developed in the previous Section now allows for an in-depth study of the behavior of a PAM. We will first focus on an infinite system, with the study of both its dispersion relation and its homogenized equivalent medium. Then, the influence of the design parameters of a finite PAM will be studied.

II.4.1 Dispersion relation and effective parameters

Following a procedure similar to that used for non-resonant sonic crystals (Section II.2), we will first derive the dispersion relation of an infinite number of plates arranged with a periodicity $L_{unit} = 1$ cm, using Eq. (II.24). Note that for an infinite system, the effective and Bloch wavenumbers are equal, i.e., $qL_{unit} = k(\omega)L_{unit}$.

The introduction of local resonances in a periodic structure induces multiple modifications in the dispersion relation. Some of these can be seen with a comparison of the solid and dashed lines in subplots (a-b) of Fig. II.5, that are respectively the analytical dispersion relations of an infinite PAM (considering only the axisymmetric modes of the plate) and of a geometrically similar two-layer infinite periodic medium with similar mechanical properties (except for the resonance behavior of the plate). First, the plate resonance opens a forbidden band [42, 56–58], named hybridization stopband, in frequency regions that are significantly lower than the Bragg bandgap, e.g., $[0.04, 0.05]f_B$ in this case. The hybridization stopbands are due to resonance-induced scattering of local inclusions contrary to the Bragg bandgaps which are due to Bragg scattering.

Each type of bandgaps can either exist independently or be coupled to each other. To prevent any confusion, the term bandgap will be kept for the Bragg type interferences while stopband will refer to



Figure II.5: **Dispersion relation and effective parameters of an infinite PAM**. Normalized by the Bragg frequency: real (a) and imaginary (b) part of the normalized dispersion relation $\omega/\omega_B = \mathcal{F}(qL_{unit}/\pi)$ of an infinite PAM. The black dashed lines represent the dispersion for an equivalent two-layers system with similar dimensions and mechanical properties as the PAM, but without the resonance behavior.

Close-up around the first pass band, normalization by the plate resonance frequency: real part (c) and imaginary part (d) of the dispersion relation, real part of the effective density (e), phase (black) and group (red) velocity (g), real part of the effective bulk modulus (f). The red-colored areas represent the stopbands. The frequency range of the close-up is represented by the dotted orange frame in (a-b).

Single plate: Imaginary part of the plate acoustic impedance (f), and transmission (black), reflection (red) magnitude (i).

the hybridization type. The strong dispersion induced by the resonances affects the phase velocity [59, 60] as we can see in Fig. II.5 (f). As a result, the Bragg bandgaps can be shifted to lower frequencies approaching those of hybridization. Efficient coupling can lead to specific features such as a widening of the forbidden bands, which can be in the opposite very narrow and sharp due to the resonance mechanism involved [59, 61–64]. Another interesting outcome of the coupling, highlighted with the current system, is the generation of local resonance-induced acoustic transparency bands within the Bragg bandgaps [60, 65–67], analogously to the electromagnetic induced transparency phenomenon [68, 69]. It can indeed be observed that narrow passbands can be opened in the Bragg bands (nonzero dotted line in Fig. II.5 (b)), where in the absence of local resonance a total reflection would occur. In addition, the strong dispersion enables the control of the attenuation [62, 70]. The sharp and asymmetric attenuation (proportional to the imaginary part of the dispersion relation) generated around the resonance in the hybridization gaps contrasts with the smoothly-varying dissipation due to the scattering gaps (dotted line). Well designed coupling can lead to highly dissipative devices that can be used for broadband absorbing structures [59, 71, 72].

Focus on the first passband

Although the unique properties resulting from this coupling may allow the design of promising applications, we will only focus on the first PAM passband in the following, i.e., within the low-frequency range delimited by the dotted orange box in Fig. II.5 (a-b).

The dispersion being closely related to the resonance of the plate, the study of the infinite system effective properties is performed with respect to the plate resonance frequency f_p , which corresponds to a cancellation of the imaginary part of the acoustic impedance as shown in Fig. II.5 (f). At this particular frequency, see Fig. II.5 (i), the plate is totally transparent with a zero reflection $|R_p| = 0$, and a total transmission $|T_p| = 1$. The strong dispersion generated by the resonance opens two low frequency stopbands in the investigated frequency range. These forbidden bands end around the resonance frequency of the plates and are characterized by a negative effective mass density (Fig. II.5 (e)). The beginning of the passbands is therefore accompanied by a frequency region, in the vicinity of resonance, where the effective density is almost zero. Note that the effect of series inclusions is small on the compressibility of the system. The effective bulk modulus, represented in Fig. II.5 (h), indeed remains positive with values close to that of the air medium, and is only slightly affected at the end of the passband and in the second stopband. We will therefore focus on the near-zero density frequency range of the first mode of the plate.

II.4.2 Finite size system: influence of the number of plates, the periodicity and the losses

We now consider a finite thickness PAM composed of N plates. In this Section, we will study the behavior of our system as a function of the number of unit cells, the periodicity L_{unit} , and the thickness of the plate h_p .



Figure II.6: Scattering properties of a finite PAM: (a-c) Transmission (black) and reflection (red) magnitude of a 4-unit long PAM, without losses (a), only with the viscothermal losses, and with both the viscothermal and viscoelastic losses ($\beta = 0.1$). (d) Imaginary part (red dashed line) and real part (black solid line) of the effective density of the full-lossy problem.

Considering a finite system makes it possible to study the scattering properties of a real PAM using Eq. (II.22). Figure II.6 shows the scattering parameters, i.e., the reflection |R| and transmission |T| magnitude of a 4 unit-long PAM. As expected, almost all of the energy is reflected in the negative density frequency ranges (red colored areas) corresponding to the hybridization bands. The plates are opaque, i.e., they behave as a rigid wall, below their resonance frequency. The plates begin to vibrate for frequencies close to their resonance and allow the wave to propagate through. The

transmission magnitude therefore increases in the vicinity of resonance until it reaches unity in the lossless case at the resonance frequency. In the passband, i.e., in the positive effective density regime, high transmission and low reflection are observed. Transmission maxima and reflection minima are obtained at Fabry-Perot frequencies that are related to the number of unit cells in the system, e.g., four of each in the present case.

The distinct role of the two types of losses is also examined in Figs. II.6 (b-c). The second subplot (b) presents the scattering parameters when only viscothermal losses are accounted for (Eq. (II.19)) while the subplot (c) shows the full lossy configuration, i.e., when both viscothermal and viscoelastic losses (Eq. (II.20), with $\beta = 0.1$) are accounted for. We can clearly see that for a circular waveguide of radius $R_a = 15$ mm, viscoelastic losses are the predominant source of losses and cause a drastic drop in the transmission amplitude.



Figure II.7: Effective properties of a finite PAM. Effect of the design parameters: viscoelastic losses β (a), number of unit cells N, periodicity L_{unit} , and plate thickness h_p . Real (1) and imaginary (2) part of the normalized dispersion relation $f/f_p = \mathcal{F}(k(\omega)L)$, transmission magnitude |T| (3), and real part of the effective density normalized to that of the air (4). Solid, dashed, dotted lines represent respectively the cases $\beta = 0.01$, $\beta = 0.1$, and $\beta = 0.5$ for (a), N = 2, N = 5, and N = 10 for (b), $L'_{unit} = 0.5L_{unit}$, $L'_{unit} = 5L_{unit}$, and $L'_{unit} = 10L_{unit}$ for (c), and finally $h'_p = 0.5h_p$, $h'_p = h_p$, and $h'_p = 2h_p$. In each case, only one parameter varies. The variation of the periodicity L'_{unit} and the plate thickness h'_p is defined according to the reference periodicity L_{unit} and thickness h_p used for all other cases. An indication of the width and location of the hybridization bands is given by the thick solid, dashed and dotted lines between the subplots and corresponding to each value of the variable.

The subplots (a1-4) in Fig. II.7 show that controlling viscoelastic losses is a key element in the design of the PAM, as it can lead to high dissipation in the system and thus to low transmission

accompanied by a broadening of the Fabry-Perot peaks (lower quality factor). Depending on the intended applications, these inherent losses can be an advantage or a disadvantage. In this work, we will focus on the properties resulting from transmission in the density-near-zero region. We should therefore use plates with low viscoelastic losses in order to be able to measure a sufficiently high transmission. Nevertheless, viscoelastic losses only have a small effect on the first stopband, and smoothen the dispersion relation at the end of the stopband as we can observe in Figs. II.7 (a1-2). In parallel, the number of unit cells, i.e., the number of plates composing the system have a drastic effect on the transmission amplitude. As expected, Figs. II.7 (b1-4) show that the increase of the number of plates leads to an increase of the viscoelastic losses sources, and thus to a decrease in transmission. In contrast to the number of plates which have only a limited impact on the location and width of the stopbands (represented by the thick solid, dashed and dotted line between the subplots), the increase of the periodicity constant greatly reduces the stopbands and passbands width. Consequently, an increase of the periodicity constant shifts down the zero density frequency while the frequency and amplitude of the first Fabry-Perot mode remain unchanged, as evidenced in Figs. II.7 (c1-4). Finally, the plate thickness is also an important element of the design, see Figs. II.7 (d1-4), as it directly influences the resonance frequency and losses amount. Changing the plate thickness then results in a change in the location and width of the stopbands, a change in the zero density frequency, and in the amplitude and frequency of the transmission maxima.

To summarize, the structure must be carefully designed for the intended application. In order to be able to measure a maximum of transmission in the vicinity of the DNZ region, plates with low viscoelastic losses and a resonance frequency in the target frequency range should be preferred. This can be achieved by varying both the mechanical properties of the material as well as the thickness of the plates, the fundamental resonance frequency being approximated by [24, 73]

$$f_p = 0.4694 \frac{h_p}{R_a^2} \sqrt{\frac{E_p}{\rho_p (1 - \nu_p^2)}}.$$
 (II.31)

Second, it appears from the previous study that increasing the periodicity constant is a better option for covering a given length with the PAM than multiplying the number of unit cells. Nevertheless, a compromise has to be found between these two options since the frequency of the zero density is also shifted according to the periodicity constant of the system.

II.5 Plate characterization

The choice of the plate is the key element of our design to be able to measure properly the peculiar phenomena related to the Density Near Zero (DNZ) regime. A prior characterization of the plate is thus required to estimate its mechanical properties and particularly its loss factor. In this Section, we will present an acoustic characterization procedure, as well as the experimental set-up used.

II.5.1 Experimental set-up

The plate characterizations and the measurements of the scattering parameters are performed in a 4 microphones impedance tube of inner radius $R_a = 15$ mm and ended by an anechoic termination as shown in Fig. II.8(a-b).

Plates or membranes can be clamped either by permanently gluing them to a ring support or by applying pressure between two surrounding rings. In order to obtain a non-destructive characterization, the second solution is preferred. The clamping condition is hard to achieve, although essential for good reproducibility of the measurements.

A sample holder shown in Fig. II.8 (d) is placed between the two microphone pairs of the impedance tube. One end of the sample holder is fixed. The plates are clamped between annular rings placed in this sample holder. The second end is screwed on so that the rings are compressed together. The sample holder and the rings have been dimensioned to ensure a continuity of section within the impedance tube.

The main difficulty of these measurements is to achieve reproducibility. A lot of attention has been devoted to the design of the sample holder so as to improve it. A brief history of the evolution of the set-up is reminded in the following. In the first holder shown by Fig. II.8(c), the rings used were made of plastic and were not perfectly flat. In view of the difficulties in obtaining satisfactory reproducibility of the resonance frequency, a new brass sample holder with aluminum rings was manufactured. Despite the better surface state of the aluminum rings, the reproducibility of the measurements was still unsatisfactory. One of the reasons for this was that the upper ring was entrained during the screwing operation, thus applying a torsional movement to the plate. Notches were therefore machined to ensure that the rings are firmly connected to each other. In addition, two annular Teflon rings of thickness 500 μ m are placed on either side of the plates to improve the clamping surface condition. The new unit cell is therefore composed of a plate symmetrically surrounded by Teflon ring and an aluminum ring as shown in Figs. II.4(b-c) and II.8(b). These aluminum rings have been machined to the exact outer and inner diameters of the Teflon pieces by 900 μ m thick, so that they perfectly host the plates surrounded by the two layers of Teflon. The plate clamped condition is ensured by applying a uniform pressure on the metallic rings by screwing on one extremity of the holder, the other one remaining fixed.

II.5.1.1 Measurement principle

A relative calibration is carried out between the microphones, considering the first one as a reference, in order to prevent from differences in absolute calibration and phase relationships. The frequency band under study, $f \in [0 - 2000]$ Hz, is low relative to the first cut-off frequency of the tube ($f_c = 1.841c_0/2\pi R_a = 6700$ Hz). A plane wave propagation is therefore considered. The use of 4 microphones makes it possible to account for the imperfection of the anechoic termination.

The pressure fields P_1 , P_2 (microphones M_1 and M_2) and P_3 , P_4 (M_3 and M_4), obtained from the



Figure II.8: **Experimental set-up:** Sketch (a) and photography (b) of the impedance tube of inner radius $R_a = 15$ mm, hold sample holder (c), exploded view of the actual plate holder (d), and different plates tested (e).

measurements of frequency responses and relative sensitivities between microphones M_2 , M_3 , M_4 and the reference microphone M_1 can be decomposed as a superposition of forward and backward waves upstream (A and B) and downstream (C and D) of the metamaterial [46, 55]

$$P_1 = (Ae^{-ikx_1} + Be^{ikx_1}), \tag{II.32}$$

$$P_2 = (Ae^{-ikx_2} + Be^{ikx_2}), \tag{II.33}$$

$$P_3 = (Ce^{-ikx_3} + De^{ikx_3}), \tag{II.34}$$

$$P_4 = (Ce^{-ikx_4} + De^{ikx_4}). \tag{II.35}$$

The scattering matrix connects, through the transmission and reflection coefficients, the incoming A, D and outgoing waves B, C from the metamaterial

$$\begin{cases} B \\ C \end{cases} = \begin{bmatrix} R & T \\ T & R \end{bmatrix} \begin{cases} A \\ D \end{cases}.$$
 (II.36)

 ${\cal R}$ and ${\cal T}$ therefore read as

$$R = \frac{AB - CD}{A^2 - D^2},$$
 (II.37)

$$T = \frac{AC - BD}{A^2 - D^2},\tag{II.38}$$

and enable to recover the effective properties of the sample by using equations (II.30). More details on the measurement principle can be found in Appendix C.

II.5.1.2 Characterization principle

Before any further measurement, the plates are characterized experimentally to estimate their mechanical properties [74]. Each plate of radius R = 1.615 cm used in the PAM is clamped between the two Téflon/aluminum rings with an inner radius of $R_i = 1.5$ cm and an outer radius of $R_o = 1.615$ cm.

Each of the unit cell is placed one after the other into the impedance tube. The characterization is then performed using a multi-objective optimization based on a least mean square procedure, the cost function of which is given by

$$\mathbb{C}(\rho_p, E_p, \beta, \nu_p) = \min\left(||R_{meas} - R_{TMM}||^2 + ||T_{meas} - T_{TMM}||^2\right),$$
(II.39)

between the measured R_{meas} , T_{meas} (Eq. (C.11)) and the analytical R_{TMM} , T_{TMM} (Eq. (II.22)) reflection and transmission coefficients.

Measurements are conducted in a temperature-controlled room so as to limit the impact of weather fluctuation on the measurements. In parallel, temperature, relative humidity and atmospheric pressure are collected during measurements to evaluate the air parameters (density, dynamic viscosity, thermal conductivity, velocity, ...) [75].

II.5.2 Choice of the plates

Two materials, illustrated in Fig. II.8(e), are tested: transparent plastic films, of thickness $h_p = 100 \,\mu\text{m}$, and a plastic material, known as plastic shim, which is available in different thicknesses (each corresponding to a different color), from 24 μm to 254 μm , thus allowing to control the resonance frequency and the intrinsic viscoelastic losses. Figure II.9 shows several measurements of the scattering parameters of transparent plates of thickness $h_p = 100 \,\mu\text{m}$ (a), and on plastic shims of thicknesses $h_p = 76 \,\mu\text{m}$ (b), $h_p = 102 \,\mu\text{m}$ (c), and $h_p = 254 \,\mu\text{m}$ (d). Each color corresponds to a realization, i.e., a different plate is clamped into the system.

The thinner the plates, the more difficult to get reproducible measurements. Indeed, a plastic deformation of the plate modifying the resonant behavior, appears as soon as the pressure applied to the rings is too important for the thinner plates. In the opposite, the thicker the plates, the greater the losses. A compromise must therefore be found between good reproducibility enabling to have a metamaterial made of identical unit cells and reasonable losses. The best option found with the tested samples are plastic shim of thickness $h_p = 102 \,\mu\text{m}$ (yellow plates), the mechanical properties of which have been estimated acoustically and read as $\rho_p = 1400 \,\text{kg.m}^{-3}$, $\nu_p = 0.41$, $E_p = 4.6(1 + 0.13i)$ GPa as evidenced in Fig. II.10



Figure II.9: Measurements of the scattering parameters of single clamped plates of different thicknesses: transmission (solid lines) and reflection (dashed lines) magnitudes for a transparent plate of thickness $h_p = 100 \ \mu\text{m}$, and for plastic shims of thicknesses $h_p = 76 \ \mu\text{m}$ (b), $h_p = 102 \ \mu\text{m}$ (c), and $h_p = 254 \ \mu\text{m}$ (d). Each color corresponds to a different measurement.



Figure II.10: Characterization of a 102 µm thick plastic shim: transmission (black) and reflection (red) magnitude. Circle symbols and solid lines correspond to the experimental and the TMM optimized analytical data.

II.6 Conclusion

In this Chapter, the general TMM is presented through the prism of sonic crystals. A general derivation of the dispersion relation of an infinite bilayer acoustic medium is used to introduce the Bragg bandgaps generated in the sonic crystals at frequencies within the diffraction regime, i.e., when the acoustic wavelength is of the order of the lattice constant. We then introduced the analytical models of a PAM and showed that the introduction of local resonances in a periodic medium results in the opening of the hybridization stopbands at lower frequencies. Derivation of the effective properties of an infinite PAM showed that the series arrangement of the plates has a predominant effect on the effective density as compared to the effective bulk modulus. The hybridization stopband corresponds to negative effective density regime, while a positive density is found in the passband. A density near zero regime is therefore observed at the transition between the stopband and the passband, occuring in the vicinity of the plate resonance. This almost zero density is accompanied by a high phase velocity, which can give rise to zero-phase propagation which will be studied in the next Chapter III.

The study of a finite thickness PAM revealed that transmission maxima are observed in the

passbands at frequencies corresponding to Fabry-Perot interferences, the number of which is related to the number of unit cells composing the system. The predominant losses are due to the inherent viscoelasticity of the plates and can significantly alter the magnitude of the transmission. It is therefore a key element of our device. A parametric analysis of the design parameters, such as the plate thickness, the periodicity, the number of unit cells and the losses, allows us to define the requirements of an optimal PAM to measure the particular properties in DNZ regimes. In the following, we will focus on exciting applications related to the stretching of the acoustic wavelength and the high phase velocity, e.g., zero-phase propagation, supercoupling effect or hiding capabilities, using a periodic arrangement made of the thin elastic plates characterized acoustically in this Chapter.

Bibliography

- N. Hashimoto et al. "Sound insulation of a rectangular thin membrane with additional weights." Applied Acoustics 33 (1991), pp. 21–43. DOI: 10.1016/0003-682X(91)90063-K.
- [2] N. Hashimoto et al. "Experimental study on sound insulation of membranes with small weights for application to membrane structures." *Applied Acoustics* 48 (1996), pp. 71–84. DOI: 10.1016/0003-682X(95)00062-E.
- [3] Z. Yang et al. "Membrane-Type Acoustic Metamaterial with Negative Dynamic Mass." *Physical Review Letters* 101 (2008). DOI: 10.1103/PhysRevLett.101.204301.
- [4] Z. Yang et al. "Acoustic metamaterial panels for sound attenuation in the 50–1000 Hz regime." Appl. Phys. Lett. 96 (2010), p. 041906. DOI: 10.1063/1.3299007.
- C. J. Naify et al. "Transmission loss and dynamic response of membrane-type locally resonant acoustic metamaterials." *Journal of Applied Physics* 108 (2010), p. 114905. DOI: 10.1063/1. 3514082.
- [6] C. J. Naify et al. "Transmission loss of membrane-type acoustic metamaterials with coaxial ring masses." *Journal of Applied Physics* 110 (2011), p. 124903. DOI: 10.1063/1.3665213.
- [7] C. J. Naify et al. "Membrane-type metamaterials: Transmission loss of multi-celled arrays." Journal of Applied Physics 109 (2011), p. 104902. DOI: 10.1063/1.3583656.
- [8] C. J. Naify et al. "Scaling of membrane-type locally resonant acoustic metamaterial arrays." *The Journal of the Acoustical Society of America* 132 (2012), pp. 2784–2792. DOI: 10.1121/1. 4744941.
- K. Y. Bliokh et al. "Colloquium : Unusual resonators: Plasmonics, metamaterials, and random media." Rev. Mod. Phys. 80 (2008), pp. 1201–1213. DOI: 10.1103/RevModPhys.80.1201.
- T. S. Luk et al. "Directional perfect absorption using deep subwavelength low-permittivity films." *Phys. Rev. B* 90 (8 2014), p. 085411. DOI: 10.1103/PhysRevB.90.085411.

- J.-P. Groby, R. Pommier, and Y. Aurégan. "Use of slow sound to design perfect and broadband passive sound absorbing materials." *The Journal of the Acoustical Society of America* 139 (2016), pp. 1660–1671. DOI: 10.1121/1.4945101.
- [12] V. Romero-García et al. "Use of complex frequency plane to design broadband and sub-wavelength absorbers." The Journal of the Acoustical Society of America 139 (2016), pp. 3395–3403. DOI: 10.1121/1.4950708.
- [13] V. Romero-García et al. "Perfect and broadband acoustic absorption by critically coupled subwavelength resonators." Sci Rep 6 (2016), p. 19519. DOI: 10.1038/srep19519.
- Y. Chen et al. "Analytical coupled vibroacoustic modeling of membrane-type acoustic metamaterials: Plate model." The Journal of the Acoustical Society of America 136 (2014), pp. 2926– 2934. DOI: 10.1121/1.4901706.
- Y. Chen et al. "Analytical coupled vibroacoustic modeling of membrane-type acoustic metamaterials: Membrane model." *The Journal of the Acoustical Society of America* 136 (2014), pp. 969–979. DOI: 10.1121/1.4892870.
- [16] M. Yang et al. "Sound absorption by subwavelength membrane structures: A geometric perspective." Comptes Rendus Mécanique 343 (2015), pp. 635–644. DOI: 10.1016/j.crme.2015.06.008.
- P. Wei et al. "Symmetrical and anti-symmetrical coherent perfect absorption for acoustic waves." Appl. Phys. Lett. 104 (2014), p. 121902. DOI: 10.1063/1.4869462.
- [18] M. Yang et al. "Subwavelength total acoustic absorption with degenerate resonators." Applied Physics Letters 107 (2015), p. 104104. DOI: 10.1063/1.4930944.
- [19] C. Meng et al. "Acoustic Coherent Perfect Absorbers as Sensitive Null Detectors." Sci Rep 7 (2017), p. 43574. DOI: 10.1038/srep43574.
- [20] G. Ma et al. "Acoustic metasurface with hybrid resonances." Nature Materials 13 (2014), pp. 873– 878. DOI: 10.1038/nmat3994.
- [21] J. J. Park et al. "Giant Acoustic Concentration by Extraordinary Transmission in Zero-Mass Metamaterials." *Phys. Rev. Lett.* 110 (2013), p. 244302. DOI: 10.1103/PhysRevLett.110. 244302.
- [22] R. Fleury and A. Alù. "Extraordinary Sound Transmission through Density-Near-Zero Ultranarrow Channels." *Physical Review Letters* 111 (2013). DOI: 10.1103/PhysRevLett.111.055501.
- [23] S. H. Lee et al. "Acoustic metamaterial with negative density." *Physics Letters A* 373 (2009), pp. 4464-4469. DOI: 10.1016/j.physleta.2009.10.013.
- [24] F. Bongard, H. Lissek, and J. R. Mosig. "Acoustic transmission line metamaterial with negative/zero/positive refractive index." *Phys. Rev. B* 82 (2010), p. 094306. DOI: 10.1103/PhysRevB. 82.094306.

- [25] L. Brillouin. Wave Propagation in Periodic Structures, Electric Filters and Crystal Lattices. First Edition. New-York: Mcgraw-hil Book Company, Inc., 1946. 272 pp.
- [26] Lord Rayleigh. "On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure." *Philosophy Magazine* 24 (1887), pp. 145–159.
- [27] Lord Rayleigh. "On the influence of obstacles arranged in rectangular order upon the properties of a medium." *Philosophy Magazine* 34 (1892), pp. 481–502.
- [28] E. Yablonovitch. "Inhibited Spontaneous Emission in Solid-State Physics and Electronics." Phys. Rev. Lett. 58 (1987), pp. 2059–2062. DOI: 10.1103/PhysRevLett.58.2059.
- [29] S. John. "Strong localization of photons in certain disordered dielectric superlattices." *Phys. Rev. Lett.* 58 (1987), pp. 2486–2489. DOI: 10.1103/PhysRevLett.58.2486.
- [30] J. D. Joannopoulos, ed. Photonic crystals: molding the flow of light. 2nd ed. Princeton: Princeton University Press, 2008. 286 pp.
- [31] M. Skorobogatiy and J. Yang. Fundamentals of Photonic Crystal Guiding. Cambridge: Cambridge University Press, 2008. DOI: 10.1017/CB09780511575228.
- [32] P. Markoš and C. M. Soukoulis. Wave propagation: from electrons to photonic crystals and left-handed materials. Princeton; Oxford: Princeton University Press, 2008. 352 pp.
- [33] P. A. Deymier, ed. Acoustic Metamaterials and Phononic Crystals. Vol. 173. Springer Series in Solid-State Sciences. Berlin, Heidelberg: Springer Berlin Heidelberg, 2013. DOI: 10.1007/978-3-642-31232-8.
- [34] A. Khelif and A. Adibi, eds. *Phononic Crystals*. New York, NY: Springer New York, 2016. DOI: 10.1007/978-1-4614-9393-8.
- [35] M. M. Sigalas and E. N. Economou. "Elastic and acoustic wave band structure." Journal of Sound and Vibration 158 (1992), pp. 377–382. DOI: 10.1016/0022-460X(92)90059-7.
- [36] M. Sigalas and E. N. Economou. "Band structure of elastic waves in two dimensional systems." Solid State Communications 86 (1993), pp. 141–143. DOI: 10.1016/0038-1098(93)90888-T.
- [37] R. Martínez-Sala et al. "Sound attenuation by sculpture." Nature 378 (1995), pp. 241–241. DOI: 10.1038/378241a0.
- [38] M. S. Kushwaha. "Stop-bands for periodic metallic rods: Sculptures that can filter the noise." Appl. Phys. Lett. 70 (1997), pp. 3218–3220. DOI: 10.1063/1.119130.
- [39] J. V. Sánchez-Pérez et al. "Sound Attenuation by a Two-Dimensional Array of Rigid Cylinders." *Phys. Rev. Lett.* 80 (1998), pp. 5325–5328. DOI: 10.1103/PhysRevLett.80.5325.
- [40] C. E. Bradley. "Linear and Nonlinear Acoustic Bloch Wave Propagation in Periodic Waveguides." PhD thesis. The University of Texas at Austin, 1993.

- [41] N. Sugimoto and T. Horioka. "Dispersion characteristics of sound waves in a tunnel with an array of Helmholtz resonators." *The Journal of the Acoustical Society of America* 97 (1995), pp. 1446–1459. DOI: 10.1121/1.412085.
- [42] P. Sheng et al. "Locally resonant sonic materials." *Physica B: Condensed Matter* 338 (2003), pp. 201–205. DOI: 10.1016/S0921-4526(03)00487-3.
- [43] C. Zwikker and C. W. Kosten. Sound absorbing materials. Amsterdam: Elsevier Publishing Company, 1949.
- [44] M. R. Stinson. "The propagation of plane sound waves in narrow and wide circular tubes, and generalization to uniform tubes of arbitrary cross-sectional shape." The Journal of the Acoustical Society of America 89 (1991), pp. 550–558. DOI: 10.1121/1.400379.
- [45] A. D. Pierce. Acoustics: an introduction to its physical principles and applications. 1989 Edition. Cham, Switzerland: Springer, 1990.
- [46] B. H. Song and J. S. Bolton. "A transfer-matrix approach for estimating the characteristic impedance and wave numbers of limp and rigid porous materials." *The Journal of the Acoustical Society of America* 107 (2000), pp. 1131–1152. DOI: 10.1121/1.428404.
- [47] A. M. Nicolson and G. F. Ross. "Measurement of the Intrinsic Properties of Materials by Time-Domain Techniques." *IEEE Transactions on Instrumentation and Measurement* 19 (1970), pp. 377–382.
- [48] W. B. Weir. "Automatic measurement of complex dielectric constant and permeability at microwave frequencies." *Proceedings of the IEEE* 62 (1974), pp. 33–36.
- [49] J. G. Berryman. "Long-wavelength propagation in composite elastic media II. Ellipsoidal inclusions." The Journal of the Acoustical Society of America 68 (1980), pp. 1820–1831. DOI: 10.1121/1.385172.
- [50] J. G. Berryman. "Long-wavelength propagation in composite elastic media I. Spherical inclusions." The Journal of the Acoustical Society of America 68 (1980), pp. 1809–1819. DOI: 10.1121/1.385171.
- [51] D. Torrent et al. "Homogenization of Two-Dimensional Clusters of Rigid Rods in Air." Phys. Rev. Lett. 96 (2006), p. 204302. DOI: 10.1103/PhysRevLett.96.204302.
- [52] D. Torrent and J. Sánchez-Dehesa. "Effective parameters of clusters of cylinders embedded in a nonviscous fluid or gas." *Phys. Rev. B* 74 (2006), p. 224305. DOI: 10.1103/PhysRevB.74.224305.
- [53] J. Mei et al. "Effective Mass Density of Fluid-Solid Composites." Phys. Rev. Lett. 96 (2006),
 p. 024301. DOI: 10.1103/PhysRevLett.96.024301.
- [54] D. R. Smith et al. "Determination of effective permittivity and permeability of metamaterials from reflection and transmission coefficients." *Phys. Rev. B* 65 (2002), p. 195104. DOI: 10.1103/ PhysRevB.65.195104.

- [55] M. Niskanen et al. "Deterministic and statistical characterization of rigid frame porous materials from impedance tube measurements." *The Journal of the Acoustical Society of America* 142 (2017), pp. 2407–2418. DOI: 10.1121/1.5008742.
- [56] R. Sainidou et al. "Locally resonant phononic crystals made of hollow spheres or cylinders." *Phys. Rev. B* 73 (2006), p. 024302. DOI: 10.1103/PhysRevB.73.024302.
- [57] J.-C. Hsu. "Local resonances-induced low-frequency band gaps in two-dimensional phononic crystal slabs with periodic stepped resonators." J. Phys. D: Appl. Phys. 44 (2011), p. 055401.
 DOI: 10.1088/0022-3727/44/5/055401.
- [58] M. Oudich et al. "Experimental evidence of locally resonant sonic band gap in two-dimensional phononic stubbed plates." *Phys. Rev. B* 84 (2011), p. 165136. DOI: 10.1103/PhysRevB.84. 165136.
- [59] B. Yuan et al. "On the coupling of resonance and Bragg scattering effects in three-dimensional locally resonant sonic materials." *Ultrasonics* 53 (2013), pp. 1332–1343. DOI: 10.1016/j.ultras. 2013.03.019.
- [60] A. Santillán and S. I. Bozhevolnyi. "Acoustic transparency and slow sound using detuned acoustic resonators." *Phys. Rev. B* 84 (2011), p. 064304. DOI: 10.1103/PhysRevB.84.064304.
- [61] R. Sainidou, N. Stefanou, and A. Modinos. "Widening of Phononic Transmission Gaps via Anderson Localization." *Phys. Rev. Lett.* 94 (2005), p. 205503. DOI: 10.1103/PhysRevLett.94. 205503.
- Y. Xiao et al. "Formation and coupling of band gaps in a locally resonant elastic system comprising a string with attached resonators." *Physics Letters A* 375 (2011), pp. 1485–1491. DOI: 10.1016/j.physleta.2011.02.044.
- [63] Y. Xiao, J. Wen, and X. Wen. "Broadband locally resonant beams containing multiple periodic arrays of attached resonators." *Physics Letters A* 376 (2012), pp. 1384–1390. DOI: 10.1016/j. physleta.2012.02.059.
- Y. Xiao, J. Wen, and X. Wen. "Flexural wave band gaps in locally resonant thin plates with periodically attached spring-mass resonators." J. Phys. D: Appl. Phys. 45 (2012), p. 195401.
 DOI: 10.1088/0022-3727/45/19/195401.
- [65] F. Liu et al. "Acoustic analog of electromagnetically induced transparency in periodic arrays of square rods." *Phys. Rev. E* 82 (2010), p. 026601. DOI: 10.1103/PhysRevE.82.026601.
- [66] J. Sánchez-Dehesa, D. Torrent, and L.-W. Cai. "Acoustic transparency in two-dimensional sonic crystals." New J. Phys. 11 (2009), p. 013039. DOI: 10.1088/1367-2630/11/1/013039.
- [67] G. Yu and X. Wang. "Acoustical "transparency" induced by local resonance in Bragg bandgaps." Journal of Applied Physics 115 (2014), p. 044913. DOI: 10.1063/1.4863400.

- [68] S. E. Harris. "Electromagnetically Induced Transparency." *Physics Today* 50 (1997), pp. 36–42.
 DOI: 10.1063/1.881806.
- [69] M. D. Lukin and A. Imamoğlu. "Controlling photons using electromagnetically induced transparency." Nature 413 (2001), pp. 273–276. DOI: 10.1038/35095000.
- [70] R. Zhu et al. "Experimental and numerical study of guided wave propagation in a thin metamaterial plate." *Physics Letters A* 375 (2011), pp. 2863–2867. DOI: 10.1016/j.physleta.2011. 06.006.
- [71] C. Croenne et al. "Band gaps in phononic crystals: Generation mechanisms and interaction effects." AIP Advances 1 (2011), p. 041401. DOI: 10.1063/1.3675797.
- [72] A. Kaina, M. Fink, and G. Lerosey. "Composite media mixing Bragg and local resonances for highly attenuating and broad bandgaps." *Scientific Reports* 3 (2013), p. 3240.
- [73] P. M. Morse and H. Feshbach. Methods of theoretical physics. Minneapolis, MN: McGraw-Hill, 1981.
- [74] A. Geslain et al. "Acoustic characterization of silica aerogel clamped plates for perfect absorption." Journal of Non-Crystalline Solids 499 (2018), pp. 283–288. DOI: 10.1016/j.jnoncrysol. 2018.07.021.
- [75] P. T. Tsilingiris. "Thermophysical and transport properties of humid air at temperature range between 0 and 100 degree Celsius." *Energy Conversion and Management* 49 (2008), pp. 1098– 1110. DOI: 10.1016/j.enconman.2007.09.015.

Chapter III

Zero-phase propagation

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III.1 Introduction

As pointed out in the previous Chapter, the strong dispersion generated in a Plate-type Acoustic Metamaterial allows the effective dynamic mass density to be controlled so that zero values can be achieved at specific frequencies.

In the density near zero regime, that is $\rho(\omega) \approx 0$, the effective acoustic wavelength $\lambda(\omega)$ and therefore the phase velocity $c(\omega)$ tend towards infinity

$$\lambda(\omega) = \frac{c(\omega)}{f} = \sqrt{\frac{\kappa(\omega)}{\rho(\omega)f^2}} \to \infty.$$
(III.1)

An almost zero density (resp. compressibility) also implies a decoupling of the spatial and temporal field variations (see Eqs. (I.3)-(I.4)) and results in a static-like spatial distribution and an almost constant phase distribution in the steady-state regime [1–4]. It is worth noting here that although the spatial distribution of the field has a static character, the monochromatic field still dynamically oscillates in time. Moreover, it is important to bear in mind that causality compliance is always ensured by the transient time needed to reach the steady-state regime necessary for the establishment of such behaviors.

In light of these features, various extraordinary properties have either been experimentally observed, like the measurement of a giant acoustic transmission through rigid diaphragms covered with membranes [5], or numerically predicted considering lossless metamaterials, such as unity transmission through sharp bends and perfect power dividers [6], or extraordinary sound transmission through ultranarrow channels, referred as the supersqueezing effect [3]. This latter has drawn lots of attention and will be briefly developed now.

III.1.1 Supersqueezing effect

The supercoupling or supersqueezing phenomenon, demonstrated in electromagnetism with Epsilon Near Zero metamaterials (ENZ) [7–10] and adapted to the acoustic field with DNZ metamaterials by Fleury and Alù [3] in 2013, consists in counterbalancing a strong impedance mismatch making use of the extreme value of one of the effective constitutive parameters.

The demonstration of this phenomenon, although not limited to that configuration, is made on two large waveguides of section S_{lg} connected by a very narrow duct of section S_{ch} . The reflection and coefficient at the entrance and exit of the narrow duct are given by

$$R = \frac{(Z_{ch}^2 - Z_{lg}^2)\tan(k_{ch}L_{ch})}{(Z_{ch}^2 + Z_{lg}^2)\tan(k_{ch}L_{ch}) - 2iZ_{ch}Z_{lg}},$$
(III.2)

$$T = \frac{-2iZ_{ch}Z_{lg}\cos^{-1}(k_{ch}L_{ch})}{(Z_{ch}^2 + Z_{lg}^2)\tan(k_{ch}L_{ch}) - 2iZ_{ch}Z_{lg}},$$
(III.3)

with $Z = \sqrt{\rho \kappa}/S$ being the acoustic impedance and k being the wavenumbers of the narrow $._{ch}$ and

wide $._{lg}$ waveguides.

When the section of the wide guide is very large ahead of the channel section, $S_{lg} \gg S_{ch}$, the section discontinuity results in a large impedance mismatch $Z_{lg} \ll Z_{ch}$. Almost all of the incident wave is therefore reflected. However, two solutions can be considered according to Eq. III.3 to obtain a total transmission. The first solution is to use the Fabry-Perot resonances of the duct $(\tan (k_{ch}L_{ch}) = 0)$, the resonance frequencies of which are determined by $k_{ch} = m\pi/L_{ch}$, where m is an integer. The other way to cancel the reflection coefficient and thus to have perfect transmission is to fulfill the impedance matching condition between the two guides, namely

$$Z_{lg} = Z_{ch}.$$
 (III.4)

If the large waveguide is filled with a conventional material, e.g., air, the cross-sectional discontinuity can be counterbalanced by filling the narrow channel with a very low impedance metamaterial, i.e.,

$$\sqrt{\rho_{ch}\kappa_{ch}} \ll \sqrt{\rho_{lg}\kappa_{lg}},\tag{III.5}$$

since $S_{ch} \ll S_{lg}$.

Two options are possible if only one of the two parameters is used to lower the effective impedance: a metamaterial with almost zero bulk modulus $\kappa(\omega) \to 0$ [11–13] or a metamaterial with almost zero density $\rho(\omega) \to 0$ [14]. However, effective parameters can only be defined within the long wavelength limit compared to the system periodicity L_{unit} . When the bulk modulus of a metamaterial tends towards extremely low values, its acoustic wavenumber increases and therefore its wavelength decreases. Above a certain limit, the wavenumber becomes of the same order of magnitude as the periodicity constant of the structure. Homogenization of the metamaterial can no longer be applied and thus its effective properties cannot be defined anymore. Therefore, the cross-sectional area ratio cannot exceed a limit value in order to observe the phenomenon of supersqueezing and tunneling with a metamaterial with zero bulk modulus.

Zero-density metamaterials do not have this limitation. The zero density regime corresponds to an infinite wavelength leading to a quasi-static pressure field, thus respecting the conditions necessary for homogenization. In their work, Fleury and Alù used a membrane-type metamaterial to achieve the supercoupling condition. With their configuration (geometries and mechanical properties of the membranes), two distinct total transmission peaks were identified. One of them depends on the number of unit cells (on the total length L of the channel) and thus corresponds to a Fabry-Perot frequency. The other transmission peak is independent of the unit cells number and corresponds to the impedance matching condition related to the DNZ regime. It is worth noting here that the mass density should not be zero, since it would imply an infinite impedance mismatch, but its value needs to be in the near zero region in order to compensate the cross-sectional ratio. In the absence of huge cross-sectional change, the impedance matching should be achieved when the bulk modulus is in addition nearly infinite [15]. As a result of the quasi-static pressure field distribution when the narrow channel is filled with a lossless DNZ medium, the impedance matching and the wave tunneling depend on neither the channel length nor the presence of bends, twists, and even absorbing sections, along the channel. Supercoupling is thus ideal for long-distant waveguide coupling with a high transmittance and no phase delay [3, 16], light concentration and harvesting [7, 8, 17], sensing [18], filtering [1] and nonlinear applications [19, 20].

III.1.2 Zero-phase propagation

Both the numerical work on supercoupling with membranes discussed in the previous paragraph, and the work of Gracia-Salgado *et al.*. [6] on zero density metamaterials obtained from a periodic distribution of structured cylindrical scatterers, have evidenced that zero-phase propagation was made possible with the stretching of the effective wavelength in the DNZ medium.

This Chapter aims at evaluating the feasibility of an experimental observation of this peculiar transmission making use of the PAM developed in the previous Chapter. In the considered case, the absence of the cross-sectional variation will induce impedance mismatch.

We will thus propose a design enabling the measurement of a zero-phase propagation within the PAM, through an in-depth comprehension of the underlying physics, and parametric studies from the lossless case to the full lossy problem, paying considerable attention on both the magnitude and the phase of the transmission coefficient of a L-long PAM surrounded by air

$$|T| = \left(\cos^2(k(\omega)L) + \frac{1}{4} \left[\frac{Z(\omega)}{Z_0} + \frac{Z_0}{Z(\omega)}\right]^2 \sin^2(k(\omega)L)\right)^{-1/2},$$
 (III.6)

$$\phi = -\operatorname{atan}\left(\frac{1}{2}\left[\frac{Z(\omega)}{Z_0} + \frac{Z_0}{Z(\omega)}\right] \operatorname{tan}\left(k(\omega)L\right)\right).$$
(III.7)

The different operating regimes of the PAM will be analyzed in a first Section. The study of the dependence of three characteristic frequencies on the number of unit cells and the viscoelastic loss factor will lead to an optimized configuration for the intended application, i.e., the measurement of a zero-phase propagation in a realistic PAM. In the second Section, a twofold procedure including full-wave numerical simulations and experiments will then be used to validate the analytical predictions. Finally we will show that the optimized PAM enables to experimentally evidence a zero-phase propagation, with a sufficiently high transmission to be used in realistic applications. As an example, a sub-wavelength acoustic dipole is designed numerically using that peculiar transmission property.

III.2 Classification of the PAM operating regimes

In order to optimize our design for the experimental observation of a zero-phase transmission with a PAM, the different operating regimes of the metamaterial under consideration are studied analytically,

numerically, and experimentally. The PAM is composed of the 102 µm thick plates characterized in the previous Chapter ($\rho_p = 1400 \text{ kg.m}^{-3}$, $\nu_p = 0.41$, $E_p = 4.6(1 + 0.13i)$ GPa), the resonance frequency of which is measured at $f_r = 438$ Hz and analytically estimated at $f_r = 423$ Hz in the lossless case.

III.2.1 Lossless PAM behavior

We first consider an arrangement of N = 1, 3, 6 and 9 plates omitting the viscoelastic and viscothermal losses. The periodicity constant of the system is $L_{unit} = 1$ cm.

Figure III.1(a) shows the effective mass density of the system $\rho(\omega) = SZ(\omega)k(\omega)/\omega$, derived from the effective wavenumber $k(\omega)$ and effective impedance $Z(\omega)$ calculated over a single unit cell, i.e., a fully periodic structure. We parallel-up the evolution of the effective density with the scattering amplitudes (Fig. III.1(b)) and the phase of the transmission coefficient (Fig. III.1(c)) of a finite PAM. In the frequency range of interest, three frequencies relevant for the analysis of the effective density can be identified: the impedance matching frequency f_m , the exact zero-density frequency $f_{\rho=0}$ and the zero-phase frequency $f_{\phi=0}$.



Figure III.1: Effective mass density and scattering properties of the PAM in the lossless case: (a) effective dynamic mass density, (b) and (c) represent the amplitude of the transmission [left vertical axis of (b)] and reflection [right vertical axis of (b)] coefficients as well as the normalized phase of the transmission coefficient for a finite PAM made of N = 1, 3, 6 and 9 plates respectively. Vertical blue, gray and green lines in (a), (b) and (c) represent the frequencies for the zero-phase, $f_{\phi=0}$, zero mass density, $f_{\rho=0}$, and impedance matching, f_m , respectively. The grey mapped area delimits the zero-frequency stopband of an infinite system.

First, the impedance matching frequency is reached at the resonance frequency of a single plate, $f_m = 423$ Hz in the lossless case. At this frequency, the effective density of the metamaterial is equal to that of the surrounding fluid, in this case the air medium, which leads to perfect transmission and zero reflection (III.1(b)). The unit transmission is accompanied by a phase shift related to the number of unit cells and therefore to the length $L = NL_{unit}$ of the PAM. In other words, the phase of the transmission coefficient equals the one produced in an air-filled cavity of same length at f_m .

The second important frequency is that at which the effective dynamic mass density cancels out, $f_{\rho=0}$. At this particular frequency, zero-phase delay propagation and a constant wave field in the metamaterial are expected, yet the PAM is not impedance matched to the surrounding environment. Therefore, the amplitude of the transmission coefficient is not unitary. Moreover, although a weaker phase shift is found at $f_{\rho=0}$ than at the impedance matching frequency f_m , propagation without phase change is still not supported by the system. More importantly, both the modulus and the phase of the transmission coefficient depend on the number of unit cells forming the PAM.

The last frequency of interest is the zero-phase frequency $f_{\phi=0}$, that is the frequency at which the phase of the transmission coefficient is exactly zero. At this frequency, the effective density is negative and equal to $\rho(f_{\phi=0}) = -\rho_0 \kappa_0 / \kappa(f_{\phi=0})$. This can indeed also be seen by examining Eq. (III.7). The phase of the transmission coefficient ϕ vanishes if either $k(2\pi f_{\phi=0})L = n\pi$, $n \in \mathbb{Z}$ or $Z(\omega)^2 = -Z_0^2$. The later condition provides

$$\rho(\omega) = -\frac{\rho_0 \kappa_0}{\kappa(\omega)}.$$
(III.8)

The effective bulk modulus being always positive in the case of a PAM (see Chapter II), Eq. (III.8) implies a negative value of $\rho(\omega)$. The negative effective density regime (grey mapped areas in Fig. III.1) corresponds to a stopband for an infinite system. Although $f_{\phi=0}$ lies in the negative effective density range, the transmission remains considerable due to the small size of the considered PAM. The modulus of the transmission (resp. reflection) coefficient is highly dependent on the number of unit cells, contrary to its phase, which remains constant and equal to 0. When no losses are considered, there is therefore a frequency, close to the zero-density frequency, for which a wave can propagate in the PAM without phase change and regardless of its length.

In the lossless case, a system made of 6 unit cells allows to transmit 90% of the incident wave through the metamaterial at the zero-phase frequency. This number of unit cell is a good compromise between finite size of the system and sufficiently high transmission magnitude. This target configuration is therefore the one considered in the following with the study of the influence of losses.

III.2.2 Effect of the losses

We now analyze the limits of zero-phase transmission in the presence of losses when N = 6. Both viscothermal losses in the waveguide and viscoelasticity of the plastic shims are accounted for. We first start by investigating the dependence of the zero-phase frequency, $f_{\phi=0}$, on the viscoelastic loss factor β only, the viscoelasticity being the predominant loss source in usual PAM. Figure III.2(c) shows that the zero-phase frequency slightly decreases with the increase of the loss factor, thus entering more and more in the PAM stopband. However, the frequency variation remains very limited and lower than 10% compared to the lossless case. Figures III.2(a-b) respectively depict the amplitude and the phase

the zero-phase frequency for the plastic shims characterized ($\beta = 0.13$).



of the transmission coefficient of the N = 6 PAM for different values of the loss factor.

Figure III.2: Analytical analysis of the zero-phase frequency and the scattering properties of the **PAM considering the viscoelastic and viscothermal losses:** (a) and (b) represent the amplitude of the transmission coefficient as well as its normalized phase for a finite PAM made of N = 6 plates depending on the losses. (c) Dependence of the zero-phase frequency on the viscoelastic loss factor. Analytical analysis of the zero-phase frequency depending on the number of plates: normalized phase for a finite PAM made of N = 2, 3, 6 and 9 plates (d). Horizontal and vertical blue lines in (a-d) represent

The transmitted amplitude is reduced because of the losses but remains reasonable to allow experimental observation of this phenomenon. The red curves show the particular case of the selected plates ($\beta = 0.13$) for which the zero-phase frequency is obtained at 390 Hz (marked by the blue horizontal and vertical lines). At this frequency, the amplitude of the transmission coefficient is sufficient for the application of zero-phase propagation in realistic situations.

Moreover, we represent in Fig. III.2(d) the dependence of the phase of the transmission coefficient on the number of unit cells, i.e., plates, considered in the finite length PAM.

In contrast to the lossless case, the zero-phase frequency now varies with the number of unit cells. There is no longer a frequency for which, whatever the number of unit cells considered, the phase of the coefficient maintains a constant value. However, it should be noted that the variation remains lower than 8% for metamaterials composed of up to 6 plates, with the considered losses.

III.3 Experimental observation of the zero-phase propagation

Given the previous observations, a 6-unit long PAM is being studied experimentally and numerically in order to validate the analytical results (twofold validation).

III.3.1 Numerical modeling of the PAM

First, we perform a full-wave numerical simulation in a 2D-axisymmetric configuration using the finite element method. The plates are modeled as elastic solids of thickness h_p , filled with a viscoelastic material of aforementioned properties, clamped at the outer boundary of the guide, and interacting with a fluid domain on both sides. The vibroacoustic coupling of the plates is thus fully accounted for. Frequency-dependent density and compressibility (derived from Eq. (II.19)) are assigned to the fluid domains to account for the viscothermal losses in the vicinity of the circular duct walls. The system is numerically meshed with 4961 structured quadrangles and is excited by a plane wave incident from the left end of the structure. As the maximum frequency of the study is lower than the cut-off frequency of the duct, only the plane mode is excited. A plane wave radiation condition is therefore applied at the other end of the guide in order to achieve an anechoic termination and thus to prevent from spurious reflection. The scattering parameters, and the effective properties are then obtained from four pressure measurements, two upstream and two downstream of the metamaterial, as measured experimentally.

III.3.2 Experimental demonstration of the acoustic wave propagation without phase change

The metamaterial is mounted in the impedance tube presented in Section II.5.1 so as to analyze its acoustic behavior. Particular attention is paid to the scattering parameters, the transmission phase, and the effective density (Fig. III.3).



Figure III.3: Experimental analysis of the scattering parameters and effective density of a 6-unit long PAM made of 102 µm thick plastic shims: (a) Transmission (black, left axis) and reflection (red, right axis) magnitude. (b) Phase of the transmission coefficient (black, left axis) and effective density (red, right axis). The symbols represent the experimental data, the solid lines represent the analytical results (TMM) and the dashed lines represent the numerical simulations (FEM).

Dashed lines and symbols in Fig. III.3 show respectively the full numerical and experimental validation of the analytical predictions for the 6-unit long PAM. Figure III.3(a) depicts the amplitude of the transmission and reflection coefficients of the finite system, while Fig. III.3(b) shows the transmission phase and the real part of the effective density. A very good agreement is observed between the analytical and numerical results which follow well the trend of the measured modulus and phase of the transmission coefficient (Fig. III.3(a,b), black symbols). Differences are however visible on the reflection coefficient, and consequently on the effective density (which is estimated from the two scattering parameters). A slight remaining variability between the different unit cells, due either to clamping defects or to the intrinsic variability of the mechanical properties of the material, is responsible for these discrepancies.

A maximum of the transmission is found at $f_m = 439$ Hz due to the quasi impedance matching condition around the plate resonance. A zero-phase propagation is measured at $f_{\phi=0} = 389$ Hz, in agreement with the predictions, thus experimentally confirming the feasibility of propagation without phase change within PAMs.

Note on the imaginary part of the effective dynamic mass density

Figure III.4 depicts the real and imaginary parts of the effective constitutive parameters characterizing the PAM.



Figure III.4: Analysis of the lossy effective dynamic mass density and effective bulk modulus: real part (black color, left axis) and imaginary part (red color, right axis) of the effective dynamic mass density (a) and effective bulk modulus (b). Continuous lines, dashed lines and symbols represent respectively the analytical, numerical and experimental results.

In the propagative regime, the real part of the effective dynamic mass density is positive while its imaginary counterpart is negative, thus fulfilling the causality principle. In contrast, both real and imaginary parts of the density are negative in the forbidden stopband. Moreover, the real and imaginary parts of the density are of the same order of magnitude over the frequency range of interest, contrary to the bulk modulus, the imaginary part of which is much smaller than its real part (b). Thus, most of the losses can be attributed to the complex effective density.

III.3.3 Applications of zero-phase propagation: control of the directivity

Since propagation without phase change is supported within a PAM, a potential use of this kind of metamaterials to control the outgoing wavefront is also possible. For example, imagine an "augmented" metamaterial consisting of a lateral juxtaposition of several PAMs, each composed of a different number of unit cells. A plane wave incident on that "super-metamaterial" can theoretically emerge distorted since the zero-phase transmission will not occur on the same length in each PAM.

Alternatively, an acoustic dipole can also be designed using a propagation without phase change. A dipole source can be approximated by two out-of-phase monopoles of equal flow rates, resulting into two symmetric lobes in the polar directivity pattern. One possibility to reach this condition is to use two waveguides, excited by the same incident plane wave and opening out into a semi-infinite space. One of the waveguide is filled with the PAM, the other being a coiled-up Fabry-Perot resonator (FPR). By designing the length of this FPR such as its first resonance coincides with the zero-phase frequency of the PAM, a phase shift of π will be encountered by the wave propagating in the FPR, while there will be no phase change in the PAM. As a result, the acoustic fields at both waveguide boundaries are out-of-phase at $f_{\phi=0}$. Then, the output flow rate of each guide must be adjusted, i.e., the attenuation and the cross-section of the Fabry-Perot resonator must be corrected, so that the absolute value of the two output flows is equal. The full-wave simulation displayed in Fig. III.5 attests the feasibility of this application with a 6-unit long PAM (placed in the right waveguide).

The acoustic behavior of the dipole is simulated with a 2D finite element model. A 6 plastic shims arrangement (of aforementioned mechanical properties) with a periodicity $L_{unit} = 1$ cm is placed in the right guide of length L = 6 cm and width $w_g = 2$ cm. The zero-phase frequency of the square section metamaterial is determined at $f_{\phi=0} = 489$ Hz. At this frequency, the wave propagates through the guide with no phase shift.

The left waveguide plays the role of the FPR. To resonate at $f_{\phi=0}$, the resonator length must be $L_{FP} = c/2f_{\phi=0} \approx 35$ cm. The Fabry-Perot resonator is coiled-up to reduce the total volume of the system (total width 19.7 cm and height 6 cm). The geometry is meshed with 53621 structured quadrangles. Both waveguides are excited by a plane wave pressure field at the top boundary and radiate in a semi infinite domain. The cross-section of the Fabry-Perot resonator is chosen equal



Figure III.5: Subwavelength dipole: 2D full-wave simulation of a subwavelength dipole device built of a coiled-up Fabry-Perot waveguide of length $L_{FP} \approx 35$ cm and a 6 plates PAM. The y component of the velocity field is shown as well as the normalized directivity polar plot of the dipole (purple line).

to that of the PAM to ensure an equal flow at the output of both guides. To reproduce the sound attenuation of the PAM, the Fabry-Perot waveguide is filled with a porous medium (porosity 0.96, flow resistivity 2847 Pa.s.m⁻², viscous length 273 µm, thermal length 672 µm, and tortuosity 1.07). This porous material has a viscoinertial transition frequency (transition between the diffusive and propagative regimes) of $f_{Biot} = 334$ Hz [21–23] and is therefore efficient at the dipole operating frequency $f_{\phi=0} = 489$ Hz.

An out-of-phase equal mean flow rate is found at each outer boundary, leading to a two lobes directivity pattern (purple line in Fig. III.5) and thus evidencing the efficiency of the device as a subwavelength dipole ($L_x \approx \lambda/4$ for the width and $L_y \approx \lambda/12$ for the height).

III.4 Conclusion

This Chapter has evidenced the feasibility of achieving zero-phase propagation within a PAM. A careful investigation of the effects of the design parameters has been carried out with a particular attention paid on the impact of the viscothermal and viscoelastic losses on the zero-phase propagation regime of a PAM. We have shown that the zero-phase propagation appears at frequencies in the negative mass density regime. In this regime, corresponding to the stopband, the amplitude of the transmission coefficient depends on the number of the unit cells in the system. In contrast, the phase of the transmission coefficient remains constant in the lossless case. Therefore a compromise between the number of unit cells and the variation of the amplitude should be reached. In this case, we consider systems made of $N \leq 6$, leading to a lossless transmission amplitude of $|T| \geq 0.9$. Once the losses are introduced in the system, a weak dependence of both the zero-phase frequency, $f_{\phi=0}$, and the phase of the transmission coefficient, ϕ , on the number of unit cells and the amount of losses is observed. In the two cases, the variation is less than 10%. These analytical results have been numerically reproduced by full-wave simulations and experimentally validated by measuring the scattering parameters of a PAM made of N = 6 plates as well as the effective mass density. The agreement between the analytical predictions, the numerical simulations and the experimental results are found very good. The results of this Chapter pave the way to design devices based on PAM with zero-phase propagation. As an example, a subwavelength acoustic dipole has been designed through numerical simulations. The content developed in this Chapter was published in Applied Physics Letter: "Zero-phase propagation in realistic Plate-type Acoustic Metamaterials" [24].
Bibliography

- A. Alù, M. G. Silveirinha, and N. Engheta. "Transmission-line analysis of Epsilon-near-zero-filled narrow channels." *Phys. Rev. E* 78 (2008), p. 016604. DOI: 10.1103/PhysRevE.78.016604.
- B. Edwards et al. "Experimental Verification of Epsilon-Near-Zero Metamaterial Coupling and Energy Squeezing Using a Microwave Waveguide." *Phys. Rev. Lett.* 100 (2008), p. 033903. DOI: 10.1103/PhysRevLett.100.033903.
- [3] R. Fleury and A. Alù. "Extraordinary Sound Transmission through Density-Near-Zero Ultranarrow Channels." *Physical Review Letters* 111 (2013). DOI: 10.1103/PhysRevLett.111.055501.
- [4] I. Liberal and N. Engheta. "Near-zero refractive index photonics." Nature Photon 11 (2017), pp. 149–158. DOI: 10.1038/nphoton.2017.13.
- J. J. Park et al. "Giant Acoustic Concentration by Extraordinary Transmission in Zero-Mass Metamaterials." *Phys. Rev. Lett.* 110 (2013), p. 244302. DOI: 10.1103/PhysRevLett.110. 244302.
- [6] R. Graciá-Salgado et al. "Negative mass density and rho-near-zero quasi-two-dimensional metamaterials: Design and applications." *Physical Review B* 88 (2013). DOI: 10.1103/PhysRevB.88. 224305.
- M. Silveirinha and N. Engheta. "Tunneling of Electromagnetic Energy through Subwavelength Channels and Bends using Epsilon-Near-Zero Materials." *Phys. Rev. Lett.* 97 (2006), p. 157403.
 DOI: 10.1103/PhysRevLett.97.157403.
- [8] M. G. Silveirinha and N. Engheta. "Theory of supercoupling, squeezing wave energy, and field confinement in narrow channels and tight bends using Epsion-near-zero metamaterials." *Phys. Rev. B* 76 (2007), p. 245109. DOI: 10.1103/PhysRevB.76.245109.
- R. Liu et al. "Experimental Demonstration of Electromagnetic Tunneling Through an Epsilon-Near-Zero Metamaterial at Microwave Frequencies." *Phys. Rev. Lett.* 100 (2008), p. 023903. DOI: 10.1103/PhysRevLett.100.023903.
- [10] B. Edwards et al. "Reflectionless sharp bends and corners in waveguides using epsilon-near-zero effects." *Journal of Applied Physics* 105 (2009), p. 044905. DOI: 10.1063/1.3074506.

- [11] S. H. Lee et al. "Composite Acoustic Medium with Simultaneously Negative Density and Modulus." *Physical Review Letters* 104 (2010). DOI: 10.1103/PhysRevLett.104.054301.
- [12] K. J. B. Lee, M. K. Jung, and S. H. Lee. "Highly tunable acoustic metamaterials based on a resonant tubular array." *Phys. Rev. B* 86 (2012), p. 184302. DOI: 10.1103/PhysRevB.86.184302.
- [13] Y. M. Seo et al. "Acoustic metamaterial exhibiting four different sign combinations of density and modulus." *Journal of Applied Physics* 111 (2012), p. 023504. DOI: 10.1063/1.3676262. arXiv: 1102.1767.
- F. Bongard, H. Lissek, and J. R. Mosig. "Acoustic transmission line metamaterial with negative/zero/positive refractive index." *Phys. Rev. B* 82 (2010), p. 094306. DOI: 10.1103/PhysRevB. 82.094306.
- [15] R. W. Ziolkowski. "Propagation in and scattering from a matched metamaterial having a zero index of refraction." *Phys. Rev. E* 70 (2004), p. 046608. DOI: 10.1103/PhysRevE.70.046608.
- [16] A. Alu and N. Engheta. "Coaxial-to-Waveguide Matching With Epsilon-Near-Zero Ultranarrow Channels and Bends." *IEEE Trans. Antennas Propagat.* 58 (2010), pp. 328–339. DOI: 10.1109/ TAP.2009.2037714.
- [17] A. Alù and N. Engheta. "Light squeezing through arbitrarily shaped plasmonic channels and sharp bends." *Phys. Rev. B* 78 (2008), p. 035440. DOI: 10.1103/PhysRevB.78.035440.
- [18] A. Alù and N. Engheta. "Dielectric sensing in Epsilon-near-zero narrow waveguide channels." *Phys. Rev. B* 78 (2008), p. 045102. DOI: 10.1103/PhysRevB.78.045102.
- [19] D. A. Powell et al. "Nonlinear control of tunneling through an epsilon-near-zero channel." Phys. Rev. B 79 (2009), p. 245135. DOI: 10.1103/PhysRevB.79.245135.
- [20] C. Argyropoulos et al. "Boosting optical nonlinearities in Epsilon-near-zero plasmonic channels." *Phys. Rev. B* 85 (2012), p. 045129. DOI: 10.1103/PhysRevB.85.045129.
- [21] J.-Ph. Groby, W. Lauriks, and T. E. Vigran. "Total absorption peak by use of a rigid frame porous layer backed by a rigid multi-irregularities grating." *The Journal of the Acoustical Society* of America 127 (2010), pp. 2865–2874. DOI: 10.1121/1.3337235.
- [22] J.-F. Allard and N. Atalla. Propagation of sound in porous media: modelling sound absorbing materials. 2nd ed. Hoboken, N.J: Wiley, 2009. 358 pp.
- [23] L. De Ryck et al. "Acoustic wave propagation in a macroscopically inhomogeneous porous medium saturated by a fluid." Appl. Phys. Lett. 90 (2007), p. 181901. DOI: 10.1063/1.2431570.
- [24] M. Malléjac et al. "Zero-phase propagation in realistic plate-type acoustic metamaterials." Applied Physics Letters 115 (2019), p. 134101. DOI: 10.1063/1.5121295.

Chapter IV

Acoustic doping of a PAM

Overview

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IV.1 Introduction

The detailed study of the density near zero regime of a plate-type acoustic metamaterials, carried out in the previous Chapters, has highlighted the possibility of experimentally demonstrating zerophase propagation due to the stretching of the wavelength and the high phase velocity generated in such metamaterials. Nevertheless, the frequency at which this specific transmission occurs lies in the negative density regime, i.e., in the PAM stopband. In addition, the metamaterial is not impedance matched to the surrounding environment, which prevents unitary transmission. Consequently, the requirement of a supercoupling effect, that is perfect transmission without phase change, cannot be met. The expressions of the amplitude and phase of the transmission coefficient, rewritten to make the effective parameters appear explicitly,

$$|T| = \left(\cos^2\left(\omega\sqrt{\rho C}L\right) + \frac{1}{4}\left[\sqrt{\frac{\rho C_0}{C\rho_0}}\frac{S_0}{S} + \sqrt{\frac{\rho_0 C}{C_0\rho}}\frac{S}{S_0}\right]^2 \sin^2\left(\omega\sqrt{\rho C}L\right)\right)^{-1/2},\tag{IV.1}$$

$$\phi = -\operatorname{atan}\left(\frac{1}{2}\left[\sqrt{\frac{\rho C_0}{C\rho_0}}\frac{S_0}{S} + \sqrt{\frac{\rho_0 C}{C_0\rho}}\frac{S}{S_0}\right] \operatorname{tan}\left(\omega\sqrt{\rho C}L\right)\right),\tag{IV.2}$$

give only two solutions to meet the supercoupling condition.

The first one consists in compensating the extreme value of the density by an important crosssectional ratio, $S/S_0 \rightarrow 0$, between the surrounding medium of section S_0 and the PAM of section S, as done in the work of Fleury and Alù [1] that was discussed in the previous Chapter. Please note that if a metamaterial with zero compressibility is considered instead of DNZ, the solution to obtain a supercoupling will still be to have a strong cross-sectional difference, but with an inverse ratio $S_0/S \rightarrow 0$, i.e., narrow surrounding waveguides connected to a large waveguide filled with the Compressibility Near Zero medium [2].

The second alternative, which does not require a change in cross-section $(S = S_0)$, is to transform the DNZ medium into a medium with an effective density and compressibility close to zero. Similarly to their electromagnetic counterpart, the Epsilon and Mu Near Zero (EMNZ) metamaterials [3–6], DCNZ media have the particularity of being able to satisfy both the zero phase propagation, associated with the static-like DNZ field distribution, and the impedance matching condition to free space, given for a normal incidence by $Z(\omega) = \sqrt{\rho(\omega)/C(\omega)}/S = Z_0$. This can be achieved by combining two types of resonators, that is resonators giving rise either to almost zero compressibility [2, 7] (resp. an almost zero permeability [8–10], $\mu \approx 0$) or to an almost zero density (resp. to an almost zero permittivity [11–16], $\epsilon \approx 0$).

Another interesting way to design EMNZ structures is to use doping, i.e., a control of the material macroscopic parameters by embedding locally appropriate inclusions/impurities, in an ENZ metamaterial [17–20]. Liberal *et al.* [21] showed that the inclusion of a single well-designed impurity can transform the effective properties of an ENZ body into those of an EMNZ, thus leading to full transmission without phase delay, regardless of the host geometry and of the location of the doping impurity. It is important to note here that homogenization remains possible even with a small number of potentially large impurities thanks to the DNZ effective stretching of the wavelength [21]. Moreover, although there is almost no spatial variation, the field still oscillates in time. The wavelength enlargement is accompanied by a decoupling of the spatial and temporal field variations.

The objective of this Chapter is to transpose the doping phenomenon to acoustics and implement it to transform the PAM, i.e., a DNZ body, into a DCNZ metamaterial ($\rho(\omega) \approx 0, C(\omega) \approx 0$), thereby enabling supercoupling.

A first Section will be devoted to a numerical study of the feasibility of an acoustic analogue of the photonic doping effect on a two-dimensional DNZ system. Then, we will show that our onedimensional lossless PAM can be efficiently doped using a single doping impurity: a tuned Helmholtz resonator. Finally, the effect of both the dopant location and losses will be discussed in a final Section, accompanied by experiments.

IV.2 Evidence of acoustic doping

The strong analogy between the constitutive equations of acoustics and of the transverse magnetic mode in electromagnetism, enables in many cases to adapt phenomena observed with light to sound waves. This Section aims at evidencing numerically an acoustic equivalent to that of photonic doping on a random DNZ medium.

We start by a full-wave simulation of the doping phenomenon on a two-port random 2D medium. The input and output ports (light blue areas in Fig. IV.1 (a)) are air-filled waveguides, plugged into the "L" and "M" shapes respectively. A monochromatic wave of frequency lower than the cut-off frequency of the ports (above which other modes than the plane wave one can propagate) impinges the structure from the left ensuring plane wave propagation. The "LAUM"-shaped medium is filled with air (b), a DNZ medium (c, e), or a DCNZ medium (f). The dimension of the "LAUM" structure (width $L_x = 3$ m and height $L_y = 1.15$ m) is chosen much larger than the acoustic wavelength in the air ($\lambda_0 = 27$ cm).

Figure IV.1(b) shows a non-uniform pressure field distribution inside the 2D medium with high order modes and a weak transmission (7%). The pressure field is strongly dependent on the geometry of the host. If the host medium is replaced by either a DNZ or a DCNZ medium (Fig. IV.1(c-f)), the pressure field becomes uniform within the medium due to the stretching of the effective acoustic wavelength, allowing a wave propagation without phase delay. In both cases, the pressure field is geometry independent. A tunneling effect is also achieved when the host medium is a DCNZ (Fig. IV.1(f)) in contrast to the case of a DNZ (subplot (c)). The host medium is impedance matched to the ports in the DCNZ case, which leads to a zero-phase total transmission.

We propose to reproduce the DCNZ behavior by doping the DNZ host using only a single impurity, i.e., by attributing different medium properties only to a small part of the geometry. We choose here



Figure IV.1: Full-wave simulation of a 2D acoustic doping with a single dopant: sketch of structure (a), pressure field when the medium is filled with air (b), with a DNZ medium (c), with a doped DNZ medium (dopant: transverse bar of the "A" letter, filled with a medium of bulk modulus κ_d and of width H) (e) or with a DCNZ medium (f). Inset (d) shows the effective bulk modulus of the entire host "LAUM" depending on the geometry of the dopant H. The doping condition to turn the host DNZ medium into an effective DCNZ medium occurs when $C \approx 0$, thus for $\kappa \to \infty$, at H = 10.1 cm (arrow in (d)).

to use the transverse bar of the "A" letter as a dopant. Doping can be achieved by tuning either the bulk modulus κ_d or the geometry of the dopant. Figure IV.1(d) shows the dependence of the effective bulk modulus of the whole system on the transverse bar geometry (the width H) for a fixed value $\kappa_d = 1.11 \cdot 10^4$ Pa. A resonant behavior is observed. The optimal geometry to dope the DNZ host is H = 10.1 cm, that is the width for which the system effective bulk modulus is the largest, i.e., an almost zero effective compressibility, while the zero effective density remains unchanged. It is worth noting here, that the sensitivity of doping to the dopant geometry directly depends on the variation of the bulk modulus. Sharp variation requires a high degree of design precision to achieve a maximum value of $\kappa(\omega)$. Figure IV.1(e) depicts the pressure field obtained for the doped DNZ host. The system "DNZ & dopant" exhibits a similar response to that of a medium integrally filled with a DCNZ material, thus evidencing acoustic doping. Videos of the harmonic evolution of the pressure field in each cases can be found in Appendix D (the use of Acrobat Reader is required to see the videos).

IV.3 Doping of a lossless PAM

Now that we have proven the existence of the doping analogue in acoustics, we will apply this phenomenon to our plate-type acoustic metamaterial, first in the lossless case. We consider a periodic arrangement made up of 20 units of the plastic shims used previously and spaced with a periodicity constant of 1 cm.

Figure IV.2 depicts the scattering properties and the effective density of the considered PAM.



Figure IV.2: Scattering parameters of a lossless PAM composed of 20 plastic shims with a periodicity constant $L_{unit} = 1$ cm: Transmission (black color) and reflection coefficients (red color) (a), phase of the transmission coefficient (b), and real part of the effective density (c).

Impedance matching, zero-density, and zero-phase propagation occur at three different frequencies, respectively $f_m = 422$ Hz, $f_{\rho=0} = 414$ Hz, and $f_{\phi=0} = 405$ Hz. As expected, an impedance mismatch at the zero-phase propagation frequency prevents such a PAM from total transmission.

IV.3.1 Design of the doping element

Doping the PAM should result in shifting both the zero-phase and the impedance-matching frequencies towards the zero-density one. Therefore, doping should allow to achieve supercoupling effect with a PAM even without a large section change. As evidenced in Section IV.2, doping a DNZ medium requires to produce a zero compressibility, that is an infinite bulk modulus, simultaneously to the zero density.

Since the effective bulk modulus is mainly sensitive to parallel elements (quarter wavelength resonators, side holes, Helmholtz resonators) [7, 22–24], a doping inclusion mounted in parallel to the waveguide is considered in this system. The chosen dopant is a Helmholtz resonator, which is easily tunable in practice and is of subwavelength dimensions in contrast to side holes or quarter wavelength resonators that require large volumes in this frequency range.

The elementary transfer matrix of this element is given by

$$\mathbf{T}_{\mathbf{HR}} = \begin{bmatrix} 1 & 0\\ 1/Z_{HR} & 1 \end{bmatrix},\tag{IV.3}$$

with

$$Z_{HR} = \frac{1}{i} \frac{\cos(k_n L_n) \cos(k_c L_c) - Z_n k_n \Delta L \cos(k_n L_n) \sin(k_c L_c) / Z_c - Z_n \sin(k_n L_n) \sin(k_c L_c) / Z_c}{\sin(k_n L_n) \cos(k_c L_c) / Z_n - k_n \Delta L \sin(k_n L_n) \sin(k_c L_c) / Z_c + \cos(k_n L_n) \sin(k_c L_c) / Z_c},$$
(IV.4)

the impedance of the Helmholtz resonator, where ΔL stands for the correction length of the neck accounting for the radiation at both ends

$$\Delta L = 0.82 \left[1 - 1.35 R_n / R_c + 0.31 \left(R_n / R_c \right)^3 \right] R_n$$
 (IV.5)

+0.82
$$\left[1 - 0.235R_n/R_a - 1.32(R_n/R_a)^2 + 1.54(R_n/R_a)^3 - 0.86(R_n/R_a)^4\right]R_a.$$
 (IV.6)

The total transfer matrix \mathbf{T} of the N unit cells metamaterial, with a dopant element placed in the middle, as illustrated by Fig. IV.3(a), then takes the form

$$\begin{pmatrix} p \\ U \end{pmatrix}_{L} = \mathbf{T} \begin{pmatrix} p \\ U \end{pmatrix}_{0}$$
 (IV.7)

$$= \left(\mathbf{T_{cav}} \cdot \mathbf{T_{p}} \cdot \mathbf{T_{cav}}\right)^{N/2} \cdot \mathbf{T_{HR}} \cdot \left(\mathbf{T_{cav}} \cdot \mathbf{T_{p}} \cdot \mathbf{T_{cav}}\right)^{N/2} \begin{cases} p \\ U \end{cases}_{0}.$$
(IV.8)

To find the optimal dimensions for achieving doping, we vary one parameter of the geometry, here the cavity length. The neck length $L_n = 20$ mm and the radii of the neck $R_n = 2$ mm and of the cavity $R_c = 10$ mm are fixed, while the cavity length L_c is adjustable with a piston as shown in Fig. IV.3(a).

We first analyze the lossless case depicted in Fig. IV.3, where the Helmholtz resonator is mounted between the 10-th and 11-th plates of the PAM. In order to find the configuration where doping occurs, we apply a similar procedure to that in Fig. IV.1(d), i.e., we look for the configuration that produces a maximum value of effective bulk modulus. This optimal configuration corresponds to a length $L_c = 32.06$ mm and requires to be dimensioned with an extreme precision. Figures IV.3(b,c,e,f) show respectively the amplitude of the scattering parameters, the effective dynamic mass density, the phase of the transmission coefficient and the effective bulk modulus for the configuration mentioned above. The analytical results are validated against those from a 3D full-wave simulation shown by the square symbols in Fig. IV.3. A zero-density accompanied by a maximum of bulk modulus is found at f = 414 Hz. At this particular frequency, the total system behaves as a DCNZ metamaterial as evidenced by the scattering parameters. The zero value of the transmission phase occurs with a zero reflection and an unitary transmission, i.e., non-delayed propagation and impedance matching are combined. We thus confirm the possibility to realize supercoupling with a PAM using doping.

Figures IV.3 (g,h) show the total pressure field, respectively without and with the doping impurity, to illustrate the impact of the dopant at the zero-phase frequency. In both cases, we observe a quasi static field distribution, giving rise to a non-delayed propagation either with an impedance mismatch or with a full transmission. In the latter case, the pressure field is perfectly symmetric with respect



Figure IV.3: Doping of a 20-unit long lossless PAM by adding a Helmholtz resonator: 3D sketches of the unit cell, of the 20-unit long PAM doped in its middle by a Helmholtz resonator, and of a close-up on the resonator (a). Transmission (black) and reflection (red) magnitudes (b), real part of the system effective density (c), phase of the transmission coefficient (e), and real part of the system effective bulk modulus versus frequency (f). Continuous lines and square symbols represent the analytical and numerical results respectively. Inset (d) shows the dependence of the effective bulk modulus on the Helmholtz cavity length L_c . Figures (g) and (h) depict the total pressure field of a 20-unit long PAM without and with a dopant at the zero-phase propagation frequency f = 405 Hz and f = 414 Hz respectively (full-wave simulation).

to the PAM, which is characteristic of the supercoupling effect. In contrast, although zero-phase propagation is satisfied when doping is not reached, the pressure field is not symmetric (see Fig. 2(g)) due to the non-unitary transmission (impedance mismatch).

IV.3.2 Independence of the dopant location

Another property of interest of DCNZ doping is its independence from the dopant location. Whatever the Helmholtz resonator location in the PAM, doping should occur [21]. To illustrate this property, Fig. IV.4 shows two examples with different locations of the Helmholtz resonator. The dopant is mounted between the 2nd and 3rd plates in the first case, Figs. IV.4(a-b), while it is placed between the 15-th and the 16-th in the second, Figs. IV.4(c-d).

It is worth noting here that the system remains reciprocal despite the breaking of the symmetry. The reflection coefficient of a wave incoming the system from the left R^+ is no longer equal to that of a wave incoming from the opposite side R^- . As a result, the scattering parameters now read as

$$T = \frac{2}{T_{11} - T_{12}/Z_0 - Z_0 T_{21} + T_{22}},$$
 (IV.9)

$$R^{+} = \frac{T_{11} - T_{12}/Z_0 + Z_0 T_{21} - T_{22}}{T_{11} - T_{12}/Z_0 - Z_0 T_{21} + T_{22}},$$
(IV.10)

$$R^{-} = \frac{-T_{11} - T_{12}/Z_0 + Z_0 T_{21} + T_{22}}{T_{11} - T_{12}/Z_0 - Z_0 T_{21} + T_{22}},$$
 (IV.11)

and the effective parameters as [25]

$$Z(\omega)^{\pm} = \pm Z_0 \frac{\sqrt{(1 - R^+ R^- + T^2)^2 - 4T^2}}{(1 - R^+)(1 - R^-) - T^2} (\pm 1 - iW), \qquad (IV.12)$$

$$e^{ik(\omega)L} = \frac{1 - R^+R^- + T^2 \pm \sqrt{(1 - R^+R^- + T^2)^2 - 4T^2}}{2T},$$
 (IV.13)

with $W = \pm \sqrt{R^+ - R^-} / \sqrt{(1 - R^+ R^- + T^2)^2 - 4^2}$ a parameter accounting for the asymmetry in the system.



Figure IV.4: **Influence of the dopant position:** (a,c) transmission magnitude (black) and phase of the transmission coefficient (red) (b,d) effective density (black) and effective bulk modulus (red), and pressure field along the metamaterial. Tuned Helmholtz resonator placed between the 2nd and 3rd plates (a-b) or between the 15th and 16th plates (c-d). Continuous lines and square symbols represent the analytical and numerical results respectively. Dashed vertical lines represent the doping frequency (zero-density, infinite bulk modulus, unit transmission, and zero-phase).

The choice of sign for $Z(\omega)^{\pm}$ and $k(\omega)$ are dictated by the fact that for a passive system the real part of $Z(\omega)$ must be positive. The sign of W must be positive within the quasi-static limit and may fluctuate at higher frequencies to ensure a continuous function of the frequency.

Zero-phase total transmission is observed in both configurations at f = 414 Hz. The location of the dopant does not affect the supercoupling condition. The two systems respond with a similar acoustic signature in terms of scattering and effective properties as evidenced in Fig. IV.4. This independence on the location is due to the large wavelength in the PAM and allows a high freedom in the design of the system.

IV.4 Doping of a lossy PAM

We now analyze the robustness of doping on the presence of losses. The full lossy problem is solved with both the viscothermal losses in the main waveguide as well as in the Helmholtz resonator and the viscoelastic losses in the plates.

The number of plates is reduced to 6, so as to control the overall viscoelastic losses. The realistic PAM, characterized in the previous Chapter, is again considered. As a reminder, Figs. IV.6(b-c) remind the scattering parameters magnitude and the phase of the transmission coefficient of the 6-unit long PAM. The amplitude of the transmission at the zero-phase frequency $f_{\phi=0} = 390$ Hz is $|T_{f_{\phi=0}}| = 0.47$ (0.42 measured in Chapter III). A large frequency offset separates this zero-phase frequency from the zero-density and impedance matched frequencies, respectively occurring at $f_{\rho=0} = 414$ Hz and $f_m = 439$ Hz.

A Helmholtz resonator is mounted between the 3rd and 4th plates of the system. The inset in Fig. IV.6(f) shows the evolution of the effective lossy bulk modulus with respect to the length of the Helmholtz resonator cavity. A maximum of real part of the bulk modulus is obtained for the optimal length $L_c = 36.75$ mm. It is worth noting here that the value of this maximum is much lower (by a factor 10^3) than in the previously presented lossless case.



Figure IV.5: Photographs of the experimental set-up with the modified plate-holder including a adjustable Helmholtz resonator.

To confirm the analytical and numerical simulations, the scattering of the real doped system is also measured using the set-up previously presented with a modified plate holder including the adjustable Helmholtz resonator as shown in Fig. IV.5.

In the following, solid lines, dashed lines and circle symbols represent the analytical, numerical, and experimental results respectively, the agreement of which is found to be very good.

Figures IV.6(d-f) give the response of the lossy system using this optimal configuration. The zerophase frequency is up shifted to $f_{\phi=0} = 412$ Hz and gets closer to the zero-density frequency of the system, i.e., $f_{\rho=0} = 414$ Hz.



Figure IV.6: **Doping in presence of losses:** (a,d) pressure field (full-wave simulation), (b,e) reflection (red) and transmission (black) magnitude, and (c,f) phase of the transmission coefficient of a 6-unit long lossy PAM (a-c) and a 6-unit long lossy PAM doped with a Helmholtz resonator ($L_c = 36.75 \text{ mm}$, $R_c = 10 \text{ mm}$, $L_n = 20 \text{ mm}$, and $R_n = 2 \text{ mm}$) mounted between plates 3 and 4 (d-f). Inset in (f) depicts the geometry optimization on the cavity length of the dopant. Continuous lines and dashed lines represent the analytical and numerical results. The experimental data are presented in a statistical form, with the symbols being the average of the 23 observations (system disassembled then reassembled) and the colored area the statistical standard error, i.e., the std around the mean value.

The careful design of the dopant thus allows to strongly reduce the frequency offset between zerophase, maximum of transmission, and zero-density frequencies. As a result the zero-phase propagation frequency $f_{\phi=0}$ gets closer to f_m . In contrast to the lossless case, the losses prevent from a perfect coincidence of maximal transmission and zero-phase propagation frequencies. As such, losses that are inherently present in any acoustic system can clearly limit the effectiveness of doping. Nonetheless, it is worth noting here that the doping condition allows to have a non-delayed propagation with a 13% higher transmission (according to the analytical and numerical, and 40% according to the measurements), the measured (resp. analytical and numerical) magnitude of which goes from 0.42 (resp. 0.47) without dopant to 0.59 (resp. 0.53) in the doped configuration. Reducing the losses, i.e., finding plates with lower viscoelastic losses, would lead to a better efficiency of the process.

IV.5 Conclusion

This Chapter demonstrates theoretically and numerically and shows experimentally the evidence for acoustic doping, i.e., the modification of the effective parameters of a complex medium by incorporating a single well-designed impurity. This equivalent phenomenon is based on previous works on the electromagnetic equivalent: the photonic doping of Epsilon Near Zero media. Based on the analogy between the constitutive equations of electromagnetism and acoustics, we have adapted the concept to acoustics and revealed the specific methodology to optimize the effect with the doping of a DNZ metamaterial, highlighting the limitation due to losses.

By combining analytical and numerical tools, we have unveiled the possibility of doping an entire DNZ medium, here a PAM, through the careful design of a single embedded element, a Helmholtz resonator. The dopant can be designed on the basis of its effective bulk modulus value or its geometry. As a result, the overall effective compressibility of the system is changed, and the behavior of the PAM switches from DNZ to Density and Compressibility Near Zero (DCNZ) at the exact zero density frequency of the non-doped system, thus combining acoustic wavelength stretching, zero-phase propagation and impedance matching. Moreover, the DCNZ condition is achieved independently of both the host geometry and the dopant location. Doping is therefore a good alternative to the large cross-sectional change required to observe supercoupling with a PAM, since it requires only one element to be added to the system. The realistic design and manufacture of this device is therefore greatly simplified.

In addition, the effect of losses on doping is being studied. We have found that losses, mainly those of plate viscoelasticity for a PAM, prevent from achieving exact zero compressibility together with zero density. Although the doping efficiency of the system is limited by the losses, a noticeable enhancement in the zero-phase transmission amplitude is experimentally evidenced. In the considered case of a 6-unit long PAM, doping significantly reduces the frequency offset between the zero-phase, zero-density, and maximum of transmission frequencies. As a result, the transmission amplitude of the zero-phase wave is increased by 13% compared to the non-doped system. The choice of plates with lower losses would increase the effectiveness of doping. Although complex geometries with sharp angles may restrict the use of PAMs, the doping phenomenon can be applied to any other DNZ systems, thus enabling a high freedom in the design as well as the filling of more complex geometries such as the 2D "LAUM" shape.

This work paves the way for further engineering of acoustic metamaterials and has the potential

to lead to several new applications such as acoustic tunneling devices, which were previously limited by the transmission drop induced by the impedance mismatch at the DNZ interface, or other acoustic elements aiming at controlling always further the sound. Another interesting application that could result from doping is the phenomenon of hiding or cloaking, which will be discussed in the next Chapter. This work has been published in Physical Review B: Rapid Communication, "Doping of a plate-type acoustic metamaterial" [26].

Bibliography

- R. Fleury and A. Alù. "Extraordinary Sound Transmission through Density-Near-Zero Ultranarrow Channels." *Physical Review Letters* 111 (2013). DOI: 10.1103/PhysRevLett.111.055501.
- H. Esfahlani et al. "Acoustic Supercoupling in a Zero-Compressibility Waveguide." Research 2019 (2019), pp. 1–10. DOI: 10.34133/2019/2457870.
- [3] A. M. Mahmoud and N. Engheta. "Wave-matter interactions in epsilon-and-mu-near-zero structures." Nat Commun 5 (2014), p. 5638. DOI: 10.1038/ncomms6638.
- [4] J. J. Yang et al. "Mu and epsilon near zero metamaterials for perfect coherence and new antenna designs." Opt. Express 22 (2014), p. 9107. DOI: 10.1364/0E.22.009107.
- [5] A. M. A. Mahmoud. "Wave Interaction With Epsilon-znd-Mu-Near-Zero (emnz) Platforms and Nonreciprocal Metastructures." Degree of Doctor of Philosophy dissertation. Faculties of the University of Pennsylvania, 2016. 151 pp.
- [6] I. Liberal and N. Engheta. "Near-zero refractive index photonics." Nature Photon 11 (2017), pp. 149–158. DOI: 10.1038/nphoton.2017.13.
- [7] N. Fang et al. "Ultrasonic metamaterials with negative modulus." Nature Materials 5 (2006), pp. 452-456. DOI: 10.1038/nmat1644.
- [8] J. S. Marcos, M. G. Silveirinha, and N. Engheta. "Mu-near-zero supercoupling." *Phys. Rev. B* 91 (2015), p. 195112. DOI: 10.1103/PhysRevB.91.195112.
- H. Feng Ma et al. "Experimental verification of supercoupling and cloaking using mu-near-zero materials based on a waveguide." *Appl. Phys. Lett.* 103 (2013), p. 021908. DOI: 10.1063/1. 4813555.
- [10] G. Lipworth et al. "Quasi-Static Magnetic Field Shielding Using Longitudinal Mu-Near-Zero Metamaterials." Sci Rep 5 (2015), p. 12764. DOI: 10.1038/srep12764.
- M. G. Silveirinha et al. "Overview of Theory and Applications of Epsilon-Near-Zero Materials." in URSI General Assembly (2008), p. 4.

- B. Edwards et al. "Experimental Verification of Epsilon-Near-Zero Metamaterial Coupling and Energy Squeezing Using a Microwave Waveguide." *Phys. Rev. Lett.* 100 (2008), p. 033903. DOI: 10.1103/PhysRevLett.100.033903.
- [13] A. Alù and N. Engheta. "Dielectric sensing in Epsilon-near-zero narrow waveguide channels." *Phys. Rev. B* 78 (2008), p. 045102. DOI: 10.1103/PhysRevB.78.045102.
- [14] A. Alù, M. G. Silveirinha, and N. Engheta. "Transmission-line analysis of Epsilon-near-zero-filled narrow channels." *Phys. Rev. E* 78 (2008), p. 016604. DOI: 10.1103/PhysRevE.78.016604.
- [15] A. Alu and N. Engheta. "Coaxial-to-Waveguide Matching With Epsilon-Near-Zero Ultranarrow Channels and Bends." *IEEE Trans. Antennas Propagat.* 58 (2010), pp. 328–339. DOI: 10.1109/ TAP.2009.2037714.
- [16] M. H. Javani and M. I. Stockman. "Real and imaginary properties of epsilon-near-zero materials." *Physical Review Letter* (2016). DOI: 10.1103/PhysRevLett.117.107404.
- [17] M. G. Silveirinha and N. Engheta. "Design of matched zero-index metamaterials using nonmagnetic inclusions in epsilon-near-zero media." *Phys. Rev. B* 75 (2007), p. 075119. DOI: 10.1103/ PhysRevB.75.075119.
- [18] I. Liberal, Y. Li, and N. Engheta. "Reconfigurable epsilon-near-zero metasurfaces via photonic doping." Nanophotonics 7 (2018), pp. 1117–1127. DOI: 10.1515/nanoph-2018-0012.
- [19] E. Nahvi, I. Liberal, and N. Engheta. "Nonperturbative Effective Magnetic Nonlinearity in ENZ Media Doped with Kerr Dielectric Inclusions." ACS Photonics 6 (2019), pp. 2823–2831. DOI: 10.1021/acsphotonics.9b00913.
- [20] Z. Zhou et al. "Substrate-integrated photonic doping for near-zero-index devices." Nat Commun 10 (2019), p. 4132. DOI: 10.1038/s41467-019-12083-y.
- [21] I. Liberal et al. "Photonic doping of epsilon-near-zero media." Science 355 (2017), pp. 1058–1062. DOI: 10.1126/science.aal2672.
- [22] N. Cselyuszka, M. Sečujski, and V. Crnojević Bengin. "Compressibility-near-zero acoustic metamaterial." *Physics Letters A* 378 (2014), pp. 1153–1156. DOI: 10.1016/j.physleta.2014.02.
 022.
- [23] S. H. Lee et al. "Acoustic metamaterial with negative modulus." Journal of Physics: Condensed Matter 21 (2009), p. 175704. DOI: 10.1088/0953-8984/21/17/175704.
- [24] L. Schwan et al. "Complex dispersion relation of surface acoustic waves at a lossy metasurface." *Applied Physics Letters* 110 (2017), p. 051902. DOI: 10.1063/1.4975120.
- [25] M. B. Muhlestein et al. "Experimental evidence of Willis coupling in a one-dimensional effective material element." Nat Commun 8 (2017), p. 15625. DOI: 10.1038/ncomms15625.
- [26] M. Malléjac et al. "Doping of a plate-type acoustic metamaterial." *Phys. Rev. B* 102 (2020), 060302(R). DOI: 10.1103/PhysRevB.102.060302.

| Chapter

Cloaking/Hiding zone with a PAM

V _____

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V.1 Introduction

The extraordinary large phase velocity, generated in the near-zero density regime of the PAM, revealed interesting behaviors discussed in Chapters III and IV, leading to zero-phase propagation and a supercoupling effect. The stretching of the effective acoustic wavelength should also enable to dissimulate an obstacle inside the PAM, since the embedded object would be small compared to the wavelength, and thus to realize cloaking devices.

Cloaking is perhaps one of the most intriguing phenomenon achieved through the development of metamaterials. Several strategies [1, 2] have been considered to reach the quest of invisibility including both passive designs based on transformation acoustics, carpet cloaking, or extreme parameters and active ones [3] including parity-time symmetry designs [4, 5]. Transformation acoustics consists in using a coordinate transformation [6-10], as it is done in transformation optics [11, 12], to deeply control the propagation of an acoustic wave. The aim is to reroute the incident wave around the obstacle to avoid any disturbance of the external field, including scattering and shadowing. A metafluid, i.e., a fluid-like material, with a controllable anisotropic (directionally varying) density and a controllable inhomogeneity (spatially varying properties) is required to achieve that goal [13]. These particular features can be obtained with a careful design of metamaterials based on either a solid inclusion-type unit cell (inertia metafluids) [7, 14–17] or an interconnected network of solid bridges (pentamode metafluids) [18–21]. An alternative path to attain cloaking, is to use topological-optimization to control wave interference in order to cancel the acoustic scattering induced by the presence of an obstacle [22, 23]. Another cloaking strategy, known as carpet or ground cloaking, is to cover an object placed on a reflective surface with a designed shield to make it invisible [24-26]. The main limitation of this "carpet cloaking" is the large size of the device compared to the object to be masked. The development of thin metasurfaces overcomes this difficulty by using either Helmholtz resonators [27–29] or membranes [30, 31].

An additional approach is to use extreme parameters [32, 33] to cloak an obstacle from an incoming field. Since the acoustic wavelength is strongly stretched in zero-index metamaterials, the presence of an obstacle inside the DNZ medium becomes almost imperceptible to an external observer. Different designs can be considered. Zhao *et al.* proposed a device based on elastic copper pieces [33], which allows at one particular frequency to achieve extraordinary transmission while maintaining an unchanged wave front and phase in presence of scatterer. Gu *et al.* states that cloaking is achievable in a membrane-type metamaterial and reports numerically an enhanced transmission through obstacles embedded in a two dimensional square arrangement of lossless membranes [34]. Nevertheless, a trade-off has to be made between the transmission magnitude and the phase shift induced by the membranes array. Yet, a perfect cloak requires canceling the effect of the object to be hidden both in amplitude and phase. Conciliating the static-like field distribution and the impedance matching condition offered by acoustic doping (developed in the previous Chapter) should provide a way to exceed this limit and achieve cloaking with a periodic arrangement of plates. The purpose of this Chapter is to assess the feasibility of such an application with our 1D PAM. It will be divided into two main Sections, including numerical simulations and analytical predictions as well as experimental validation.

We investigate in a first Section the ability to perfectly cloak a diaphragm using a one dimensional lossless PAM. Two distinct strategies are envisaged. The first consists in using the diaphragm itself as an impurity to realize doping and thus cloaking, while a configuration with a Helmholtz resonator, similar to that investigated in Chapter V, is used to dope the system, leading to a full cloak of the diaphragm in the second case. As in most acoustic systems, viscothermal and viscoelastic losses cannot be neglected and are moreover known to potentially lead to the drastic annihilation of the expected phenomenon [35]. In the case of doping, even if the efficiency is also limited by the losses, an enhanced transmission at the zero-phase frequency has been reported. The impact of losses on the effectiveness of cloaking for the two strategies considered is studied here.

Giving the difficulty of achieving cloaking with a realistic PAM, emphasis is placed on the possibility of hiding a diaphragm into a PAM in the second Section. The strategy is different from cloaking. Instead of trying to shield the diaphragm in the PAM to turn the whole system undetectable, we use the peculiar DNZ property to maintain the same acoustic behavior of the PAM (scattering and phase) with and without the embedded diaphragm. In this case, perfect transmission is no longer required, and a realistic (lossy) application can be considered. Finally a hiding zone is reported experimentally.

V.2 Cloaking

We first study the cloaking capability of our 1D PAM. We start by presenting the design of the obstacle to be masked, its analytical modeling as well as its acoustic behavior. Then, we study the cloaking efficiency from the lossless to the full lossy system and for the two doping configurations considered, that is with only the diaphragm impedance control or with the addition of a Helmholtz resonator.

V.2.1 Design of the object to cloak

Among the many possibilities of obstacles to conceal, the choice made in this study is a rigid diaphragm of orifice radius R_d (sketched in Fig. V.1 (a)) because of the ease of controlling the scattering properties and of manufacturing. Depending on its aperture, the diaphragm switches from transparent (transmission close to 1 for large R_d) to opaque (reflection close to 1 for small R_d), thus adjusting its influence on scattering.

The rigid diaphragm is modeled as a short waveguide of radius R_d and length $L_d = 2$ mm. An end correction $\Delta L_d = \frac{8}{3\pi} 2R_d$ accounting for the radiation at both ends is considered and leads to an elementary transfer matrix of the form

$$\mathbf{T}_{\mathbf{d}} = \begin{bmatrix} 1 & iZ_d k_d L_d + i\omega \rho_0 \Delta L_d \\ \frac{i}{Z_d} k_d L_d + i\omega \left[\kappa_0\right]^{-1} \Delta L_d & 1 \end{bmatrix},$$
(V.1)

in the $k_d L_d \ll 1$ approximation, with Z_d and k_d the characteristic impedance and wavenumber of the orifice.



Figure V.1: Acoustic signature of a single rigid diaphragm: (a) sketch of the diaphragm, (b) 3D plot of the transmission magnitude versus frequency and the orifice radius R_d of the diaphragm, (c) scattering parameters of a diaphragm of radius $R_d = 4$ mm, (d) phase of the transmission coefficient (black, left axis) and normalized effective density (red, right axis). Analytical, full-wave simulation, and experimental results are given respectively by solid lines, dashed-lines, and symbols.

Figure V.1(b) shows the dependence of the transmission magnitude through the diaphragm on both the frequency and R_d . The smaller the orifice, the greater the reflection, thus the lower the transmission. In addition, the transmission amplitude also decreases with frequency for a same diaphragm aperture, due to the frequency dependent viscothermal losses in the aperture [36].

The shaded surface in Fig. V.1 highlights the particular case of a diaphragm with a $R_d = 4$ mm orifice, the scattering parameters of which are shown in Fig. V.1(c). The transmission remains high over the considered frequency range, while the magnitude of the reflection varies from 0.2 to 0.5, thus confirming the strong scattering power of the obstacle. Furthermore, the presence of the diaphragm in a duct induces an increasing phase delay with frequency as shown by the phase of the transmission coefficient in Fig. V.1(d). The designed device will then have to act on both the amplitude and phase of the scattering coefficients to enable cloaking.

V.2.2 Lossless system

In a first step, we analyze the lossless case, i.e., when neither the viscoelastic losses of the plates nor the viscothermal losses in both the diaphragm orifice and the waveguide are accounted for. The diaphragm to be concealed is embedded in the middle of a 6-unit long PAM, i.e., between the 3rd and 4th plates. The two cloaking configurations under examination are optimized to find the ideal geometry to cloak the diaphragm. The efficiency of these lossless optimized systems is then tested.

V.2.2.1 Cloaking configurations

The first approach, depicted in Fig. V.2(a) is to use the diaphragm itself as a dopant to match the total system impedance to the surrounding waveguide impedance. This impedance matching condition would lead to full transmission, zero reflection, and zero-phase propagation, as if the diaphragm was not present in the waveguide.

The total transfer matrix characterizing the whole system reads as

$$\begin{cases} p \\ U \end{cases}_{L} = (\mathbf{T_{cav}} \cdot \mathbf{T_{p}} \cdot \mathbf{T_{cav}})^{3} \cdot \mathbf{T_{d}} \cdot (\mathbf{T_{cav}} \cdot \mathbf{T_{p}} \cdot \mathbf{T_{cav}})^{3} \begin{cases} p \\ U \end{cases}_{0}.$$
(V.2)

 R_d/R_a

Figure V.2: Analytical geometry optimization for cloaking a diaphragm using a 6-unit long lossless PAM: (a) sketch of the configuration, (b) influence of the diaphragm radius on the system's scattering parameters

The variation of the diaphragm geometry provides the optimal design for achieving cloaking. The lossless scattering magnitudes of the total system, that is the PAM with the embedded diaphragm, are calculated analytically for different orifice radii R_d as shown in Fig. V.2(b). A perfect transmission with zero reflection is found for the optimal radius $R_d = 0.15$ mm.



Figure V.3: Analytical geometry optimization for cloaking a diaphragm using a doped 6-unit long lossless PAM: (a) sketch of the configuration, (b) optimal geometry for cloaking by doping a 6-unit long lossless PAM with a Helmholtz resonator. Evolution of the effective bulk modulus with the cavity length of the Helmholtz resonator.

In the second configuration, an additional dopant consisting in a cylindrical Helmholtz resonator is used to dope the whole system, so as to enable cloaking. The doping condition is reached when the effective density and the effective compressibility are simultaneously equal to zero. A Helmholtz resonator (cavity length L_c , cavity radius $R_c = 1$ cm, neck length $L_n = 2$ cm, and neck radius $R_n = 2$ mm) is mounted on the main waveguide, between the first two plates as sketched in Fig. V.3(a).

The total transfer matrix can then be defined as follows

$$\begin{cases} p \\ U \end{cases}_{L} = (\mathbf{T_{cav}} \cdot \mathbf{T_{p}} \cdot \mathbf{T_{cav}}) \cdot \mathbf{T_{HR}} \cdot (\mathbf{T_{cav}} \cdot \mathbf{T_{p}} \cdot \mathbf{T_{cav}})^{2} \cdot \mathbf{T_{d}} \cdot (\mathbf{T_{cav}} \cdot \mathbf{T_{p}} \cdot \mathbf{T_{cav}})^{3} \begin{cases} p \\ U \end{cases}_{0}.$$
(V.3)

The cloaking condition is found in Fig. V.3(b) by varying the length of the Helmholtz resonator cavity and looking for a maximal bulk modulus, i.e., almost zero compressibility. A maximum is found for a length $L_c = 4.1$ cm. With this particular geometry, an impedance matching condition coupled with zero-phase propagation could be achieved.

V.2.2.2 Cloaking efficiency

The two cloaking configurations being now designed, an analytical (solid line) and a three dimensional FEM numerical study (dashed line) of the scattering parameters (magnitude and phase) and of the effective density are performed and presented in Fig. V.4.



Figure V.4: Lossless cloaking of a diaphragm: Pressure field from the full-wave simulation, transmission (black) and reflection (red) magnitudes (a,c), and phase of the transmission coefficient (black, left axis) and normalized effective density (red, right axis) of both systems (b,d). Figures (a,b) show the cloaking of a $R_d = 0.5$ mm diaphragm in a conventional 6-unit long PAM while Figs. (c,d) show the cloaking of a $R_d = 4$ mm diaphragm in a 6-unit long PAM doped with a Helmholtz resonator ($R_n = 2$ mm, $L_n = 2$ cm, $R_c = 1$ cm, and $L_c = 40.95$ mm). Solid line represents the analytical results and dashed line the full-wave simulation ones.

The two lossless configurations - self-doped diaphragm (Fig. V.4(a-b)) and doped with a Helmholtz resonator (Fig. V.4(c-d)) - lead to cloaking. In the first configuration, the full transmission (black color) and zero reflection (red color) observed over a very narrow frequency range in Fig. V.4(a) result from the high impedance of the diaphragm (due to its narrow aperture). The resulting transparency

is also accompanied by zero effective density (red color) and zero-phase propagation (Fig. V.4(b)) at $f_{\phi_t=0} = 88.3$ Hz.

Doping occurs as expected in the second configuration, when the designed Helmholtz resonator is mounted on the system. As a result, impedance matching $(|T| \approx 1, |R| \approx 0)$ (Fig. V.4(c)) and zero-phase propagation (Fig. V.4(d)) are also achieved at $f_{\phi_t=0} = 403$ Hz.

The pressure field is constant (quasi-static distribution) along the system in the "Helmholtz doped configuration". In contrast, the pressure varies within the metamaterial in the "self-doped configuration" (when the diaphragm itself enables cloaking). That change in the pressure field is however imperceptible to an outside observer.

Although the presence of the diaphragm becomes imperceptible in terms of scattering magnitude, both configurations do not reproduce the correct apparent phase produced by wave propagation. Indeed, measuring the phase advance of a wave propagating over a length L = 6 cm in an air-filled waveguide or through the proposed designs (PAM + diaphragm + potentially an additional dopant) lead to different results. The current configurations give a non-delayed propagation, that is $\Delta \phi \approx 0$, while the phase shift to be reproduced with the device should be $\Delta \phi = k_0 L = 2\pi f_{\phi_t=0}/c_0$ (with $f_{\phi_t=0} = 88.3$ Hz or $f_{\phi_t=0} = 403$ Hz depending on the configuration) to account for the propagation. The realization of a full cloak then requires the addition of another constraint. As the whole system (PAM & obstacle) must be undetectable both in terms of scattering and phase, the total length L of the PAM must be chosen so that propagation over a L-thick slab of filling material (here air) results in a phase advance of $\Delta \phi = m2\pi$, with m an integer. Consequently, the length of the PAM must be

$$L_{opt} = 2m\pi \frac{c_0}{\omega_{\phi_t=0}},\tag{V.4}$$

that is $L_{opt} = 3.88$ m for the first configuration, and $L_{opt} = 85.11$ cm for the second. As indicated in Chapter II, the increase in periodicity would not greatly affect the DNZ regime of the PAM and in particular the zero-phase propagation. The geometries described in Fig. V.4 can then be easily adapted to the optimal length of the device by changing the periodicity only. Nevertheless, the overall dimensions of the cloaking device are no longer subwavelength.

V.2.3 Lossy system

Whilst cloaking seems to occur in lossless PAMs, we now consider the case of a realistic system, with the impact of losses, the role of which has been crucial in the designs presented in the previous Chapters.

Figure V.5 shows the effect of total losses (viscoelasticity of the plates and viscothermal losses in both the aperture of the diaphragm and the duct) on the effectiveness of the two cloaking systems.

In the first configuration shown in Fig. V.5(a-b), viscothermal losses in the small aperture turn the diaphragm totally opaque. The incident wave is totally reflected, as if a rigid wall were encountered. In addition, the phase of the transmission coefficient no longer passes through zero. Considering a



Figure V.5: Lossy cloaking of a diaphragm: Pressure field from the full-wave simulation, transmission (black) and reflection (red) magnitudes (a,c), and phase of the transmission coefficient (black, left axis) and real part of the normalized effective density (red, right axis) of both systems (b,d). Figures (a,b) show the cloaking of a $R_d = 0.5$ mm diaphragm in a conventional 6-unit long PAM while Figs. (c,d) show the cloaking of a $R_d = 4$ mm diaphragm in a 6-unit long PAM doped with a Helmholtz resonator ($R_n = 2$ mm, $L_n = 2$ cm, $R_c = 1$ cm, and $L_c = 40.95$ mm). Solid line represents the analytical results and dashed line the full-wave simulation ones.

realistic diaphragm with such orifice then totally annihilate the cloaking effect.

In the second configuration, the viscoelasticy of the plates limits the doping efficiency as already observed in Chapter V. As a result, the presence of the system in the waveguide induces a scattering (non zero reflection). Losses thus also prevent cloaking in the second configuration.

V.2.4 Feasibility of cloaking

To summarize on the feasibility of cloaking, despite claims in the literature on the subject, this work shows that realistic applications of cloaking with membranes or plate-type metamaterials are in fact very limited. First of all, the presence of losses in the system prevents total transparency, i.e., perfect transmission. Moreover, although the stretching of the effective wavelength produces a static-like field distribution making the obstacle imperceptible in terms of apparent phase to an outside observer, full cloaking requires, on the contrary, reproducing the phase of an air-filled waveguide of the same length, as if the object to be cloaked and the occultation device were both non-existent. Full cloaking therefore requires specific lengths of the PAM, which are for low frequencies very large, i.e., no longer sub-wavelength.

V.3 Hiding

Although PAM can lead to full cloaking in the lossless case, losses in both conventional and doped PAM have been shown to prevent the peculiar propagation intended. Another application of PAMs is now studied: the phenomenon of hiding. The philosophy of hiding differs from that of cloaking in the sense it does not aim to render an external field unchanged by the presence of both the obstacle and the masking device. The goal here is to use the stretch of the effective wavelength in the metamaterial to hide the diaphragm (or any other obstacle) inside, without disturbing the acoustic scattering of the PAM itself, i.e., by maintaining a constant magnitude of reflection and transmission at the zero-phase frequency $f_{\phi=0}$.

V.3.1 Centered diaphragm

The phase shift induced by the presence of the diaphragm in a waveguide (observable in Fig. V.1(d)) can be inhibited by the presence of a PAM in the DNZ regime. Figure V.6 shows a comparison of the acoustic response (scattering parameters and effective density) of a conventional 6-unit long PAM (Figs. V.6(a-c)) and a 6-unit long PAM with an embedded diaphragm (Fig. V.6(d)), the orifice of which is either $R_d = 4$ mm (Fig. V.6(e-f)) or $R_d = 1$ mm (Fig. V.6(h-i)). In both cases, the presence of a PAM provides both zero-phase and zero-density to the whole system as evidenced in Fig. V.6(f,i). A twofold procedure, including 2D axisymmetric full-wave simulations (dashed line) and experimental measurements (circle symbols), is followed to valid the TMM-based analytical predictions. A very good agreement between the numerical results, the analytical calculation, and the measurements is found. The weak discrepancies on the reflection coefficient are attributed to a remaining variability on the unit cells, due to either the clamping condition or the intrinsic properties of the plates as previously observed.

The presence of the DNZ metamaterial makes it possible to lower and even to cancel the scattering effect induced by the diaphragm. The total system (metamaterial and diaphragm) indeed achieves the non-delayed propagation property. The zero-phase frequency of the whole system $f_{\phi_t=0}$ depends directly on the impedance of the obstacle, related here to the open area ratio R_d/R_a . As shown in Fig. V.6(g), the smaller the orifice radius, and therefore the larger the impedance of the diaphragm, the lower the zero-phase frequency of the system. The zero-phase frequency goes from $f_{\phi=0} = 390$ Hz for the metamaterial alone to $f_{\phi_t=0} = 373$ Hz for the system with a diaphragm of radius $R_d = 4$ mm (green marker), and to $f_{\phi_t=0} = 303$ Hz for the diaphragm of $R_d = 1$ mm (orange marker).

At the zero-phase frequency of the system with a $R_d = 4$ mm diaphragm, $f_{\phi_t=0} = 373$ Hz, the amplitude of the measured (respectively analytical and numerical) transmission and reflection coefficients are 0.45 (resp. 0.47) and 0.75 (resp. 0.57), and are equal (approximately for experimental data) to that of the metamaterial alone at its zero-phase frequency $f_{\phi=0}$, respectively 0.42 (resp. 0.47) and 0.79 (resp. 0.57). For the 1 mm diaphragm, the scattering parameters of the total system and of the metamaterial alone at their respective zero-phase frequencies $f_{\phi_t=0}$ and $f_{\phi=0}$ are not exactly the



Figure V.6: Hiding of a centered embedded diaphragm: (Upper part) 6-unit long PAM behavior ($h_p = 102 \ \mu m$, $L_{unit} = 1 \ cm$) a) sketch, b) modulus of the transmission (black, left label) and reflection (red, right label) coefficients from the measurements (circle markers), the full-wave simulations (dashed line) and the TMM (solid line), and c) phase of the transmission coefficient (black, left label) and real part of the effective dynamic mass density (red, right label).

(Lower part) 6-unit long PAM behavior with embedded $L_d = 2 \text{ mm}$ thick annular diaphragm with an aperture of radius R_d : d) sketch, g) relative frequency shift (ratio of the frequency of the zero of the transmission coefficient for the total system (diaphragm embedded into the metamaterial) to the one of the metamaterial alone) versus the diaphragm to the waveguide radii ratio (TMM), e) and h) modulus of the transmission (black, left label) and reflection (red, right label) coefficients of the total system, metamaterial with an embedded diaphragm of aperture $R_d = 4 \text{ mm}$ and $R_d = 1 \text{ mm}$ respectively, and f) and i) phase of the transmission coefficient (black color) and real part of the effective dynamic mass density (red color) of the total system for $R_d = 4 \text{ mm}$ and $R_d = 1 \text{ mm}$ respectively. Black dashed curves in Fig. e) and h) represent the transmission of the $R_d = 4 \text{ mm}$ and $R_d = 1 \text{ mm}$ diaphragms alone.

same with a transmission of 0.39 (resp. 0.38) and a reflection of 0.59. (resp. 0.62). The efficiency of the hiding phenomenon is therefore restricted when the impedance of the obstacle is excessively large. A trade-off need to be found between the diaphragm scattering power related to its acoustic impedance, i.e., its opening area ratio, and the efficiency of the PAM to hide the scatterer, as investigated in Fig. V.7.

Figure V.7(b) shows that the amplitude of the scattering parameters at $f_{\phi_t=0}$ remains constant and equal to those of the metamaterial alone at $f_{\phi=0}$ as long as the ratio orifice/total cross-section remains larger than 12% (grey mapped surface). Due to the slight change of the zero-phase frequency in that range, a small phase shift is noticeable for the PAM alone at the zero-phase frequency of the whole system $f_{\phi_t=0}$ in Fig. V.7(c). Beyond that 12% ratio, the impedance of the diaphragm becomes too large. The zero-phase frequency of the whole system is consequently significantly down shifted. The 12% cross-sectional area ratio corresponds to a variation of $f_{\phi_t=0}$ of 10% with respect to the zero-phase frequency of the metamaterial alone $f_{\phi=0}$. Due to this frequency shift, the DNZ power of the PAM is no longer sufficient to allow a proper hiding phenomenon. The system behaves like two distinct DNZ media separated by an obstacle of a given impedance. The scattering amplitude then varies drastically with the variation of $f_{\phi_t=0}$.



Figure V.7: Influence of the diaphragm aperture ratio on its scattering parameters (analytically calculated): Variation of the zero-phase frequency (a), evolution of the amplitude of the scattering parameters: transmission (black, left axis) and reflection (red, right axis) at the zero-phase frequency of the total system $f_{\phi_t=0}$ (b) and phase of the transmission coefficient of the metamaterial alone at the zero-phase frequency of the global system $f_{\phi_t=0}$ for the different diaphragm radii (c). The shaded area shows the aperture range for which the amplitude of the scattering parameters is independent of the orifice radius.

However, it is interesting to note that the hiding of a diaphragm (or any other obstacle) is possible with a lossy DNZ metamaterial, as soon as the impedance of the object to be hidden is well controlled. In this periodic arrangement of plates, despite the small variation of the zero-phase frequency (10% in the operating range) which results in a small phase change, the effect of the diaphragm is very limited. It is then possible to hide the diaphragm in the metamaterial in this frequency range even in the presence of losses.

V.3.2 Hiding zone

Moreover, the stretch of the effective wavelength in the PAM should allow to expand the hiding phenomenon to different locations of the scatterer. Figure V.8 shows the evolution of the scattering amplitude and of the zero-phase frequency for different locations of the diaphragm along the metamaterial and in its vicinity. The two subplots Fig. V.8 (b,e) represent, in the lossy and lossless cases respectively, the transmission magnitude (in black) and the reflection magnitude (in red) of the whole system at the zero-phase frequency $f_{\phi_t=0}$ for each location x_d of the diaphragm.

The presence of a non centered obstacle causes an asymmetry in the system. It is then necessary to differentiate the reflection coefficients of the waves incident from each side of the device. The reflection coefficient R^+ refers to the incidence from the left, while R^- refers to the incidence from the right. Since only one obstacle is considered here, the reflection R^+ generated by an obstacle placed between the 5th and 6th plates for a wave incident from the left is equivalent to the reflection $R^$ generated by an obstacle placed between the 1st and 2nd plates for a wave incident from the right. In the following only the coefficient R^+ will thus be presented. In addition, it is worth noting here that the system preserves its reciprocity condition, but not the symmetry.



Figure V.8: Hiding zone of a 4 mm radius diaphragm in the PAM: with losses (a-c) and no losses (d-f). Sketches of the considered system (a,d), magnitude of the scattering parameters (transmission, black, left axis and reflection, red right axis) at the zero-phase frequency of the system (b,e) and zero-phase frequency of the system (c,f). The circles, dots symbols and dotted lines represent respectively the experimental, analytical data for the total system and the experimental amplitude of the scattering parameters of the metamaterial alone at its zero-phase frequency.

In the lossless case, no matter where the 4 mm radius diaphragm is along the metamaterial, the amplitudes of transmission and reflection remain constant and equal to those of the metamaterial alone. The hiding phenomenon covers the entire structure which acts as a homogeneous and almost symmetric material $(R^+ \approx R^- \approx R)$. As soon as the viscothermal and viscoelastic losses are accounted for, the asymmetry of the system becomes much more visible, $R^+ \neq R^-$, which results in a slight change of the reflection with the location of the diaphragm as we can see in Fig. V.8(b). The agreement of the experimental results, represented by the circle symbols, and the analytical predictions, represented by the solid points, is very good on the transmission and the zero-phase frequency. More significant differences are however noticeable on the reflection, already noted in the case of the metamaterial alone. However, we find that regardless the location of the diaphragm, the measured reflection and transmission of the whole system equal those of the PAM without diaphragm, the measured values of which are reminded by the dotted horizontal lines in Fig. **??**(b).

According to these results it is therefore possible to define a hiding zone corresponding to the whole metamaterial. The diaphragm can be dissimulated when placed between any plates, as long as the losses and impedance of the object to hide are controlled.

V.4 Conclusions

In this Chapter, both the cloaking and hiding efficiency of a plate-type metamaterial device have been investigated, using the DNZ regime associated with such systems. The study has been restricted to the case of a thin rigid diaphragm to be hidden due to the ease of manufacturing, but can nevertheless be extended to any other obstacle.

We have shown that acoustic doping can be used to attain full cloaking in the lossless case, i.e., canceling any scattering from the diaphragm (total transmission, zero reflection, and zero-phase), by using either the impedance of the element to be concealed or an external impurity such as a designed Helmholtz resonator. We have reported that both configurations have transformed the DNZ regime into DCNZ, thus fulfilling the requirement for cloaking. However, the effectiveness is significantly altered when the full viscothermal and viscoelastic losses are turned on. In addition, the length of the DNZ medium required, for the apparent phase to be equal with and without the cloaking device, results in a system size that is no longer subwavelength. In the self-doped configuration, the diaphragm aperture necessary to have a strong enough impedance to dope the system is so small that the viscothermal losses in the orifice make the diaphragm opaque. When doping is accomplished using a Helmholtz resonator impurity, the viscoelastic losses have been shown to avoid impedance matching, thus preventing full transmission and cloaking.

Accounting for the difficulties of obtaining cloaking condition using a device based on a 1D realistic PAM, we have shown that instead of using the PAM to suppress any scattering of the external sound field, the effective wavelength stretch produced by the DNZ regime of the PAM can be used to hide an obstacle inside. In doing so, the acoustic behavior of a medium filled with DNZ material (here the PAM) remains unchanged regardless of whether an obstacle is present inside or not, as soon as its impedance is controlled. Furthermore, we have shown that the long effective wavelength allows to change the position of the obstacle inside the metamaterials without affecting the effectiveness of the hiding at all in the lossless case, and with only a slight impact on the reflection in the lossy case. We were thus able to report analytically, numerically and, experimentally the presence of a hiding zone along the entire PAM. These promising results could help in the design of new acoustic metamaterials and pave the way for improved hiding strategies using DNZ metamaterials involving plates or membranes.

Bibliography

- [1] A. N. Norris. "Acoustic Cloaking." Acoustics Today 11 (2015): Winter 2015,1, pp. 38–46.
- R. Fleury, F. Monticone, and A. Alù. "Invisibility and Cloaking: Origins, Present, and Future Perspectives." *Phys. Rev. Applied* 4 (2015), p. 037001. DOI: 10.1103/PhysRevApplied.4. 037001.
- [3] F. Zangeneh-Nejad and R. Fleury. "Active times for acoustic metamaterials." *Reviews in Physics* 4 (2019), p. 100031. DOI: 10.1016/j.revip.2019.100031.
- [4] F. Yang and Z. Lei Mei. "Parity-time symmetric cloak with isotropic modulation." J. Phys. D: Appl. Phys. 49 (2016), 21LT01. DOI: 10.1088/0022-3727/49/21/21LT01.
- [5] H.-X. Li et al. "Ultrathin Acoustic Parity-Time Symmetric Metasurface Cloak." Research 2019 (2019), pp. 1–7. DOI: 10.34133/2019/8345683.
- [6] H. Chen and C. T. Chan. "Acoustic cloaking in three dimensions using acoustic metamaterials." Appl. Phys. Lett. 91 (2007), p. 183518. DOI: 10.1063/1.2803315.
- J. B. Pendry and J. Li. "An acoustic metafluid: realizing a broadband acoustic cloak." New J. Phys. 10 (2008), p. 115032. DOI: 10.1088/1367-2630/10/11/115032.
- [8] A. N. Norris. "Acoustic cloaking theory." Proc. R. Soc. A 464 (2008), pp. 2411–2434. DOI: 10.1098/rspa.2008.0076.
- S. A Cummer and D. Schurig. "One path to acoustic cloaking." New J. Phys. 9 (2007), pp. 45–45.
 DOI: 10.1088/1367-2630/9/3/045.
- [10] H. Chen and C. T. Chan. "Acoustic cloaking and transformation acoustics." J. Phys. D: Appl. Phys. 43 (2010), p. 113001. DOI: 10.1088/0022-3727/43/11/113001.
- [11] J. B. Pendry. "Controlling Electromagnetic Fields." Science 312 (2006), pp. 1780–1782. DOI: 10.1126/science.1125907.
- [12] D. Schurig, J. B. Pendry, and D. R. Smith. "Calculation of material properties and ray tracing in transformation media." Opt. Express 14 (2006), p. 9794. DOI: 10.1364/0E.14.009794.

- [13] S. A. Cummer, J. Christensen, and A. Alù. "Controlling sound with acoustic metamaterials." *Nature Reviews Materials* 1 (2016). DOI: 10.1038/natrevmats.2016.1.
- S. A. Cummer et al. "Scattering Theory Derivation of a 3D Acoustic Cloaking Shell." *Phys. Rev. Lett.* 100 (2008), p. 024301. DOI: 10.1103/PhysRevLett.100.024301.
- [15] D. Torrent and J. Sánchez-Dehesa. "Acoustic cloaking in two dimensions: a feasible approach." New Journal of Physics 10 (2008), p. 063015. DOI: 10.1088/1367-2630/10/6/063015.
- [16] B.-I. Popa and S. A. Cummer. "Design and characterization of broadband acoustic composite metamaterials." *Phys. Rev. B* 80 (2009), p. 174303. DOI: 10.1103/PhysRevB.80.174303.
- [17] L. Zigoneanu et al. "Design and measurements of a broadband two-dimensional acoustic metamaterial with anisotropic effective mass density." *Journal of Applied Physics* 109 (2011), p. 054906.
 DOI: 10.1063/1.3552990.
- [18] G. W. Milton and A. V. Cherkaev. "Which Elasticity Tensors are Realizable?" Journal of Engineering Materials and Technology 117 (1995), pp. 483–493. DOI: 10.1115/1.2804743.
- [19] C. L. Scandrett, J. E. Boisvert, and T. R. Howarth. "Acoustic cloaking using layered pentamode materials." *The Journal of the Acoustical Society of America* 127 (2010), pp. 2856–2864. DOI: 10.1121/1.3365248.
- [20] N. H. Gokhale, J. L. Cipolla, and A. N. Norris. "Special transformations for pentamode acoustic cloaking." The Journal of the Acoustical Society of America 132 (2012), pp. 2932–2941. DOI: 10.1121/1.4744938.
- [21] Y. Chen, X. Liu, and G. Hu. "Latticed pentamode acoustic cloak." Sci Rep 5 (2015), p. 15745.
 DOI: 10.1038/srep15745.
- [22] V. M. García-Chocano et al. "Acoustic cloak for airborne sound by inverse design." Appl. Phys. Lett. 99 (2011), p. 074102. DOI: 10.1063/1.3623761.
- [23] L. Sanchis et al. "Three-Dimensional Axisymmetric Cloak Based on the Cancellation of Acoustic Scattering from a Sphere." *Phys. Rev. Lett.* 110 (2013), p. 124301. DOI: 10.1103/PhysRevLett. 110.124301.
- [24] B.-I. Popa, L. Zigoneanu, and S. A. Cummer. "Experimental Acoustic Ground Cloak in Air." Phys. Rev. Lett. 106 (2011), p. 253901. DOI: 10.1103/PhysRevLett.106.253901.
- [25] L. Zigoneanu, B.-I. Popa, and S. A. Cummer. "Three-dimensional broadband omnidirectional acoustic ground cloak." *Nature Mater* 13 (2014), pp. 352–355. DOI: 10.1038/nmat3901.
- [26] Y. Jin et al. "Engineered Diffraction Gratings for Acoustic Cloaking." Phys. Rev. Applied 11 (2019), p. 011004. DOI: 10.1103/PhysRevApplied.11.011004.
- [27] Y. Yang et al. "A metasurface carpet cloak for electromagnetic, acoustic and water waves." Sci Rep 6 (2016), p. 20219. DOI: 10.1038/srep20219.

- [28] C. Faure et al. "Experiments on metasurface carpet cloaking for audible acoustics." Appl. Phys. Lett. 108 (2016), p. 064103. DOI: 10.1063/1.4941810.
- [29] M. Dubois et al. "A thin and conformal metasurface for illusion acoustics of rapidly changing profiles." Appl. Phys. Lett. 110 (2017), p. 151902. DOI: 10.1063/1.4979978.
- [30] S. Zhai et al. "Ultrathin skin cloaks with metasurfaces for audible sound." J. Phys. D: Appl. Phys. 49 (2016), p. 225302. DOI: 10.1088/0022-3727/49/22/225302.
- [31] H. Esfahlani et al. "Acoustic carpet cloak based on an ultrathin metasurface." Phys. Rev. B 94 (2016), p. 014302. DOI: 10.1103/PhysRevB.94.014302.
- [32] L.-Y. Zheng et al. "Acoustic cloaking by a near-zero-index phononic crystal." Appl. Phys. Lett. 104 (2014), p. 161904. DOI: 10.1063/1.4873354.
- [33] J. Zhao et al. "Acoustic cloaking by extraordinary sound transmission." Journal of Applied Physics 117 (2015), p. 214507. DOI: 10.1063/1.4922120.
- [34] Y. Gu et al. "Controlling sound transmission with density-near-zero acoustic membrane network." Journal of Applied Physics 118 (2015), p. 024505. DOI: 10.1063/1.4922669.
- [35] V. C. Henríquez, V. M. García-Chocano, and J. Sánchez-Dehesa. "Viscothermal Losses in Double-Negative Acoustic Metamaterials." *Phys. Rev. Applied* 8 (2017), p. 014029. DOI: 10. 1103/PhysRevApplied.8.014029.
- [36] M. R. Stinson. "The propagation of plane sound waves in narrow and wide circular tubes, and generalization to uniform tubes of arbitrary cross-sectional shape." The Journal of the Acoustical Society of America 89 (1991), pp. 550–558. DOI: 10.1121/1.400379.

Chapter

General conclusion

In conclusion, this work is devoted to the near-zero index regime in a 1D Plate-type Acoustic Metamaterial and the underlying applications offered by the stretching of the effective wavelength characteristic of these media.

By the mean of analytical calculation, numerical simulation and experimentation, we first designed a PAM evidencing some of the specific features related to this stretching. In particular we studied and measured three main phenomena: (i) the zero-phase propagation (ii) the acoustic doping of a PAM allowing to meet the requirements of supercoupling, and (iii) the effectiveness of a PAM in cloaking or hiding an obstacle. Throughout this work, we have paid great attention to the study and demonstration of the limitation induced by the inherent losses, which are too often neglected in many studies even though they can have drastic effects if they are not properly accounted for as a key element of the design.

This sixth chapter gathers a general conclusion, a discussion and some prospects to the presented work. We also quickly discuss and present some side works that have been done during this PhD.

As each Chapter ends with its own conclusion, we do not recall here the details of the results obtained, but give a general overview of the work.

Overview

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VI.1 Conclusion

VI.1.1 Summary of the main results

In this PhD thesis, we studied various effects of the wavelength stretching in acoustics near-zero index media, with a simple device: a one dimensional Plate-type Acoustic Metamaterials. The strong dispersion produced in the periodic arrangement of clamped elastic plates around the resonance, mostly affects its effective dynamic mass density which varies with frequencies from negative to positive values. An hybridization stopband, corresponding to a negative effective density regime, is opened below the resonance, while a positive density is found in the passband. A Density Near Zero regime is therefore observed at the transition between the stopband and the passband, occuring near the plate resonance. This almost zero density is accompanied by a high phase velocity. As a result, the effective wavelength is stretched and becomes large compared to the system. The acoustic pressure field can therefore present a quasi-static distribution allowing zero-phase propagation. This latter property can be used to tailor the directivity of a source, as evidenced numerically in Chapter III with the design of an acoustic dipole with a 1D PAM. The large stretching of the effective wavelength also enables dissimulation applications, the dimensions of the object to conceal being small relatively to the acoustic wavelength.

The main motivation for this work was to analyze the effective applicability and to experimentally observe some of the consequences of such behaviors (in particular the zero phase propagation and the phenomena of doping, cloaking and hiding) with realistic designs that inevitably involve losses. Losses are however known to significantly affect the acoustic dispersion and may lead to the annihilation of the expected phenomena. We analyzed the contribution of the different sources of losses in these media, i.e., viscothermal and viscoelastic losses. We found that viscoelastic losses are the predominant loss source in PAMs and are responsible for a drastic drop in transmission, thus revealing the importance of considering it as a key design element. We developed an experimental set-up to acoustically characterize the mechanical properties and loss factor of the plates. Using a parametric analysis and a characterization of different plates (different materials and thicknesses), we designed a sample enabling experimental observation of the Density Near Zero regime specificities.

This particular regime can be described using three distinct frequencies: zero-phase, zero-density, and impedance matching frequencies. We highlighted that the zero-phase propagation does not occur at the exact zero-density frequency, but in the negative dynamic mass density regime, that is in the PAM stopband. As a result, even when losses are not accounted for, the PAM is not impedance matched to the surrounding medium. We were nonetheless able to measure a zero-phase propagation with a realistic PAM, with a trade-off on the number of plates composing the system to maintain a sufficient transmission in the system despite the viscoelastic losses and the impedance mismatch.

To overcome this mismatch at the zero-phase frequency, $f_{\phi=0}$, we proposed in Chapter IV an analogue of photonic doping in acoustics. We presented a simple way and explained the methodology to transform a single near-zero medium, the PAM in the present case, into a Density and Compressibility Near Zero (DCNZ) medium by adding a single well designed impurity. In contrast to DNZ (single zero) metamaterials, double zero ones (with both zero density and zero compressibility simultaneously achieved) have the particularity of conciliating both the interesting properties of DNZ media (wavelength stretching, zero-phase propagation) and impedance matching, thus relaxing the constraints for realistic applications. A Helmholtz resonator was used as an impurity to dope the PAM and to meet the requirements for supercoupling, i.e., total transmission, zero density, and zero-phase propagation simultaneously. We showed that for specific geometries of the Helmholtz resonators mounted in parallel to the PAM, both the dynamic compressibility (corresponding to a very large bulk modulus) and dynamic density of the overall system vanished simultaneously. Consequently, the zero-phase frequency was up-shifted towards the impedance matching frequency and a total transmission accompanied by a zero phase propagation was reached when losses were not accounted for. In contrast, we evidenced that the viscoelastic losses limited the maximum value of bulk modulus achievable and therefore restricted the doping phenomenon. Nonetheless, for the geometry producing the maximum of bulk modulus, we were able to measure a substantial increase of the transmission amplitude. Moreover we showed that the acoustic doping is independent of the location of the impurity in the system, thus giving a high freedom in the design. We have also numerically proven that acoustic doping can be applied to DNZ systems of greater dimensions, with an example on a 2D complex shape medium.

Gathering the peculiar properties highlighted in the previous chapters, we finally evaluated in Chapter V the possibility of dissimulating obstacles using the large wavelength stretching offered by the DNZ regime and the impedance matching offered by the doping of the PAM. Two strategies were investigated. First, we used the own impedance of the diaphragm to be cloaked to achieve doping and second we added another impurity in the system, a Helmholtz resonator. We evidenced analytically and numerically that in the absence of losses the diaphragm can be cloaked with both strategies (either with or without additional dopant). The diaphragm was then acoustically imperceptible to an outside observer both in term of scattering amplitude and phase. We showed that the limitation of the doping phenomenon induced by the viscoelastic losses drastically annihilates the cloaking efficiency of the device, the transmission magnitude being no longer unitary. Accounting for these limitations to observe cloaking, we studied the hiding efficiency of a PAM instead. The strategy of hiding differs from that of cloaking in the sense that the objective is no longer to make invisible the entire system, i.e., the PAM and the diaphragm, but to dissimulate an obstacle within the PAM. Impedance matching is therefore no longer required. We demonstrated that at the zero-phase frequency of the system, constant amplitudes of the scattering parameters were observed, with or without the obstacle inside, and even in presence of losses. Thus, the diaphragm was hidden in the PAM. We showed analytically, numerically and experimentally that the efficiency of the hiding phenomenon is dependent on the impedance of the obstacle to hide (here, the ratio orifice/diaphragm cross-section). We also observed experimentally that the PAM allows to hide the obstacle wherever its location within the system thanks to the stretching of the acoustic wavelength.

Although most of the phenomena related to the DNZ regime in a PAM can be strongly limited

by losses in real operating systems, we were able to experimentally observe some of these interesting effects for the first time with a carefully designed realistic device.

VI.1.2 Prospects

The specific wave propagations made possible by the near-zero density and/or compressibility regimes of a PAM and highlighted in this PhD work, open up a wide range of applications, such as acoustic tunneling devices, which were previously limited by the transmission drop induced by the impedance mismatch at the DNZ interface, wavefront and directivity tailoring devices, and hiding devices.

The main limitation of this system being the viscoelasticity of the plates, a major improvement would be to find a way to ensure better reproducibility of the clamping condition even for much thinner plates (the current plastic shims are distributed with thickness up to 10 μ m). During this PhD, we thought about a system consisting of a plate holder with small screws all around its outer edges in order to be able to adjust and control carefully the pressure applied on both sides of the plate. Unfortunately, this would have meant modifying the whole set-up, as it would have required a larger diameter of the clamping rings. However, the required manufacturing time was not compatible with the constraints of the three-year doctoral period.

The other solution that can be considered to reduce overall losses would be to choose another material for the plates, with lower viscoelastic losses. In doing so, it should be possible to observe all of the phenomena demonstrated here with greater amplitude and in particular to increase the doping efficiency, which could potentially make cloaking more feasible.

Moreover, both the particular properties (zero-phase propagation, doping, hiding,...) and the inherent limitations (impedance mismatch, losses,...) as well as the acoustic doping methodology presented here can be observed in and adapted to any other DNZ system, thus allowing great design freedom.

Although only a one-dimensional system is studied in this work, the evidenced peculiar behaviors can also be observed with systems of greater dimensions (as we have evidenced for the doping of a 2D DNZ system in Chapter IV). A redesign of the set-up to extend the system to two or three dimensions, as technically challenging as this may be, would pave the way for further engineering of acoustic metamaterials and metasurfaces and facilitate its applicability.

An interesting idea would also be to find a way to design a surface that could respond with a similar DNZ regime and that could support the same specific propagation.

VI.2 Side works

Besides the work on Plate-type Acoustic Metamaterials presented in this manuscript, we have also investigated other interesting behaviors obtained with acoustic metamaterials working with systems different from plates and at frequencies outside the near-zero index regime. Some of these subjects are still under study. Although these studies do not fall within the scope of this manuscript (near-zero index Plate-type Acoustic Metamaterials), we will briefly explain the respective projects in order to draw an overall picture in this section.

VI.2.1 Reconfigurable metamaterials

Reconfigurable metamaterials have great potential as they allow to switch from one application to another by simply changing the geometry of the systems. We are particularly interested here in room acoustics applications using Helmholtz resonators.

VI.2.1.1 Accordion like Helmholtz resonators

In the framework of the ANR/RGC METARoom project (ANR-18-CE08-0021), we are developing metasurfaces consisting of periodic arrangements of Helmholtz resonators for the control of room acoustics. This project involves the University of Science and Technology of Hong Kong (research group of Professor Ping Sheng), the Acoustics Laboratory of Le Mans University (LAUM UMR CNRS 6613), and two companies/start-ups: Acoustic Metamaterials Group (AMG) in Hong Kong and Metacoustic in Le Mans.

Metamaterials or metasurfaces based on Helmholtz resonators have been shown to produce a strong dispersion accompanied by a deep slow sound below the resonance frequency. Consequently, such structures give rise either to deep subwavelength perfect absorption devices [1–4] or to metadiffusers [5].

In this project, we intend to extend existing designs, which are generally static and intended only for a single functionality: absorption or diffusion, to reconfigurable designs, which can then operate on both. The principle used is fairly simple. A change in the geometry of the system changes the resonance frequency of the Helmholtz resonators and thus the dispersion produced. In doing so, the behavior of the medium can be passively changed from perfect absorption to controllable diffusion, i.e., from an anechoic audio sensation to that of a larger room.

Perfect absorption is achieved by critically coupling successive Helmholtz resonators of different geometries using the high dissipation and the slow sound around the resonance frequency. Careful adjustments of the resonators geometry, leakage and losses prevent reflection and result in perfect absorption over a broadband frequency range.

Conversely, the audio feel of a larger room requires the ability to control with the metasurface the reflection at the interface (wall of the room) to mimic that of a farther wall. In other words, we control the reflection of the metasurface to time-delay a pulse, as if it had propagated over a given distance. The design is a slightly tougher challenge since it requires to achieve a deep slow sound while limiting dispersion and absorption over a frequency band corresponding to the spectral content of the pulse. In doing so we are able to reproduce a large and almost constant group delay and prevent excessive distortion of the pulse.

This project sets a new frontier in room acoustics and would remove the significant dimensional constraints of traditional materials, which are effective at high frequencies but result in large and heavy structures at low frequencies. We are currently considering the fabrication of an accordion-like system, which would allow the desired diffusion or perfect absorption to be obtained by simply pushing on the wall.

VI.2.1.2 Multi-stable origami Helmholtz resonators

This other project, carried out in collaboration with two members of the Katia Bertholdi research group at Harvard, focuses on the design of multi-stable Helmholtz resonators using origami-like structures and tessellations. Many results are expected, some of which could be coupled to the METARoom project for an application in room acoustics. We also plan to use these bi-stable origami structures as actuators or switches. If we manage to transform the geometry from one to the other stable position, the resonance will be shifted and the amplification phenomenon will stop.

VI.2.2 Asymmetric reflection: Willis coupling

Willis coupling, analogous to bianisotropy in electromagnetic metamaterials, has recently received considerable attention. In contrast to symmetrical passive reciprocal media, where both transmission and reflection are independent of the side from which the wave is irradiated, Willis media exhibit asymmetry (lack of mirror symmetry) due to a coupling between potential and kinetic energy in the system. The Willis coupling term of a system is therefore defined as the cross-coupling parameter between stress and velocity in the acoustic constitutive equations. In this side work, we derive analytically and measure experimentally these Willis parameters for various lossy systems consisting of clamped plates, Helmholtz resonators or quarter-wavelength resonators.

VI.2.3 Broadband absorbing Plate-type Acoustic Metamaterials

Finally, we have also investigated the possibility of designing perfect broadband absorption devices with a series arrangement of annular plates.

The use of annular plates instead of circular plates allows to control both the resonance frequency and the losses, necessary to reach a critical coupling condition. In addition, annular plates are thin and are therefore perfect candidates for the design of a deep subwavelength structure for broadband sound absorption. As the system used here is similar to that presented in this manuscript, we will develop further the work done on this subject in the following.

Bibliography

- A. Merkel et al. "Control of acoustic absorption in one-dimensional scattering by resonant scatterers." *Applied Physics Letters* 107 (2015), p. 244102. DOI: 10.1063/1.4938121.
- [2] N. Jiménez et al. "Ultra-thin metamaterial for perfect and quasi-omnidirectional sound absorption." Applied Physics Letters 109 (2016), p. 121902. DOI: 10.1063/1.4962328.
- [3] N. Jiménez et al. "Rainbow-trapping absorbers: Broadband, perfect and asymmetric sound absorption by subwavelength panels for transmission problems." Scientific Reports 7 (2017), p. 13595. DOI: 10.1038/s41598-017-13706-4.
- [4] N. Jiménez et al. "Quasiperfect absorption by subwavelength acoustic panels in transmission using accumulation of resonances due to slow sound." *Physical Review B* 95 (2017). DOI: 10. 1103/PhysRevB.95.014205.
- [5] N. Jiménez et al. "Metadiffusers: Deep-subwavelength sound diffusers." Scientific Reports 7 (2017), p. 5389.

Chapter VII

Side work highlight: Annular plates for the control of absorption

Overview

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Decorated membranes or plates, i.e., membranes or plates with additional mass platelets, have been widely used for their high tuning, insulation and absorption capacity [1-11]. A brief review is also provided in the introduction to Chapter II.

Helmholtz resonators [12] are another type of resonators widely studied for sound absorption. A periodic arrangement of Helmholtz resonators produces a strong dispersion in the waveguide on which they are mounted [13], accompanied by a slow sound [14–16] and an increase in the density of state below resonance. Due to the dispersion in the periodic structure, the group velocity depends on the frequency and vanishes (or reaches a very low value if losses are accounted for) at the edges of the stopband, which considerably reduces the resonance frequency of the system and facilitates the design of deep subwavelength devices. In addition, the dissipation in the vicinity of the Helmholtz resonance is very high. Careful adjustment of the resonator intrinsic losses (viscothermal losses in the neck and cavity) and the energy leakage rate (coupling of the resonator with the main waveguide) can lead to perfect absorption, when one is in perfect balance with the other, i.e., when the critical coupling condition is fulfilled. This results in an impedance matching that prevents reflections and ensures energy trapping around the resonators, either in a reflection configuration (rigid backing) [17] or in a transmission configuration [18]. Moreover, broadband absorption can be achieved by successively critically coupling a set of graded Helmholtz resonators, i.e., a rainbow trapping metamaterials [19].

Here we propose an alternative strategy, which consists in arranging several annular plates in series in a 1D waveguide, instead of Helmholtz resonators, to mimic the rainbow trapping-like behavior. An annular plate is a good candidate for critical coupling since its overall losses can be adjusted by the hole diameter. In a first Section, we will briefly develop the analytical modeling of an annular plate and then analyze its acoustic signature. Then the study will be separated in two different configurations: reflection and transmission.

VII .1 Acoustic signature of an annular plate

To begin, we characterize the acoustic propagation through a thin clamped annular plate of outer radius R_a and inner radius (radius of the hole) R_h .

VII .1.1 Analytical modeling

The analytical modeling, based on a modal representation of the displacement w, is fully developed in Appendix B. We present only the main results in this Section.

The displacement of the plate is defined as a superposition of the displacement of an infinite number of modes (modal analysis form) and reads as follows

$$w(r,\theta) = \sum_{n=0}^{\infty} [A_n \mathcal{J}_n(\lambda r) + B_n \mathcal{Y}_n(\lambda r) + C_n \mathcal{I}_n(\lambda r) + D_n \mathcal{K}_n(\lambda r)] \cos(n\theta) = \sum_{n=0}^{\infty} w_n(r,\theta), \quad (\text{VII .1})$$

with n the mode number, J and I, and Y and K the Bessel and modified Bessel functions of the first

and second kind respectively.

The annular plate is clamped at its outer edge (in $r = R_a$), that is

$$w(r = a, \theta) = \frac{\partial}{\partial r}w(r = a, \theta) = 0,$$
 (VII .2)

and free at its inner edge. At $r = R_h$, the bending moment M_r and the shear forces V_r should therefore vanish, i.e.,

$$\begin{cases} M_r = -D \left[\frac{\partial^2}{\partial r^2} w(r,\theta) + \nu \left(\frac{1}{r} \frac{\partial}{\partial r} w(r,\theta) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right]_{r=R_h} = 0, \\ V_r = -D \left[\frac{\partial}{\partial r} \nabla^2 w(r,\theta) + \frac{1-\nu}{r^2} \frac{\partial^2}{\partial \theta^2} \left(\frac{\partial}{\partial r} w(r,\theta) - \frac{w(r,\theta)}{r} \right) \right]_{r=R_h} = 0. \end{cases}$$
(VII .3)

After multiple analytical manipulations of the resulting system (integrations, symbolic resolutions,...), the use of the orthogonality of the mode, and the application of the initial conditions, the general solution to the forced vibration problem of a clamped-free annular plate in vacuum then takes the form

$$w(r,\theta) = \sum_{n=0}^{\infty} \frac{\int_{S} \frac{\Delta P(r)}{D} w_n(r,\theta) dS}{\frac{\rho_p h_p}{D} (\omega_n^2 - \omega^2) \int_{S} \sum_{n=0}^{\infty} [w_n(r,\theta)]^2 dS} w_n(r,\theta).$$
(VII .4)

The acoustic impedance of the plate in vacuum can then be derived from the surface averaged transverse displacement \overline{w}

$$\overline{w} = \frac{1}{S} \int_0^{2\pi} \int_{R_h}^{R_a} w(r,\theta) r dr d\theta, \qquad (\text{VII .5})$$

leading to

$$Z_p = \frac{Z_m}{S^2} = \frac{\int_S \Delta P(r) dS}{i\omega \overline{w}}.$$
 (VII .6)

Since the plate is embedded in air, i.e., in a light fluid, the vibrations of the plate in the air-filled waveguide are considered to be those of plates in vacuum. However, the air flow through the internal orifice must be accounted for.

The orifice of section $S_h = \pi R_h^2$ is treated as a h_p -thick tube, for which an approximation of the acoustic impedance is given by

$$Z_h = i\omega\rho_{air}h_p \frac{1}{1 - \frac{2}{\sqrt{-i}k_h} \frac{J_1(\sqrt{-i}k_h)}{J_0(\sqrt{-i}k_h)}} \frac{1}{S_h} + \frac{2i\omega\rho_{air}\Delta L}{S_h}, \qquad (\text{VII .7})$$

where k_h is the ratio of the radius of the hole to the viscous penetration thickness and considering length corrections of the form $\Delta L = \frac{4}{3\pi} 2R_h$ at each end.

The total impedance of an annular plate is then given in the first order by

$$Z_{tot} = \frac{1}{\frac{1}{Z_{plate}} \frac{R_a^2}{R_a^2 - R_h^2} + \frac{1}{Z_h} \frac{R_a^2}{R_h^2}}.$$
 (VII .8)

This expression of the impedance depends on the open area ratio (ratio between the cross-section of the hole and that of the plate).

It is worth noting at this point that the derivation of the annular plate impedance is time consuming as it requires multiple numerical integrations and system resolutions. Moreover to be accurate, a calculation considering multiple modes is necessary (i.e., a wide frequency range required).

VII .1.2 Acoustic scattering

We first analyze the influence of the hole radius R_h and the plate thickness h_p on the scattering amplitude of an annular plate in Fig. VII .1. Plates with outer radius $R_a = 15$ mm and made of plastic shims, similar to those used in the previous Chapters, are considered here.

VII .1.2.1 Eigenfrequency and scattering coefficients

Figure VII .1(a) shows the evolution of the first eigenfrequency as a function of the orifice/plate cross-section ratio for plates of thicknesses $h_p = 102, 127$ and 254 µm respectively represented by the orange, green and cyan solid lines. We represent in the inset of Fig VII .1(a) the non-dimensional parameter $k_p R_a$ instead. We see that, regardless of the thickness, the greater the ratio R_h/R_a , the higher the resonance frequency. One can therefore tune the plate by dimensioning the orifice. As expected, the greater the thickness, the higher the resonance frequency. For very narrow apertures, the acoustic signature of an annular plates tends toward that of an ordinary circular plate, both in terms of resonance frequency and scattering amplitude, as evidenced in Figures VII .1(c1,d1). We find a behavior very similar to that of the 102 µm -thick circular plate shown in Fig. II.10 of Chapter II.

Figures VII .1(c1-4) and Figs. VII .1(d1-4) respectively represent the lossless and lossy magnitude of the scattering coefficients (transmission in black, reflection in red, and absorption in yellow) for different orifice radii and plate thicknesses indicated by the grey dots in Fig. VII .1(a). In contrast to the ordinary circular and very narrow orifice annular plates, the reflection (resp. transmission) magnitude of which being unitary (resp. zero) in the very low frequency range, the annular plates with larger apertures are transparent at low frequency with an almost unitary transmission.

The analytical behavior of the annular plates, computed considering only three modes, is validated against 2D axisymmetric full-wave simulations. The agreement between the analytical and the numerical results is judged to be very good. Small discrepancies are found out of the resonance and are attributed to the low number of modes considered for the computation. As this modeling will be used for the optimization process, we had to compromise here between the number of modes accounted for, i.e., the computation time, and the accuracy out of the resonance. Moreover, Figs. VII .1(c3,d3), representing the case of a R_h/R_a ratio of 1/3, shows that the analytical modeling is losing some accuracy for large aperture as evidenced by the stronger deviation compared to the numerical simulation. However, it is worth noting here that the analytical model is fairly well accurate and can be used for annular plates of aperture radius of up to 5 mm.



Figure VII .1: Scattering by an annular plate: (a) Evolution of the 1st eigenfrequency as a function of the orifice to plate cross-section area ratio R_h/R_a for plates of thickness 102 µm (orange line), 127 µm (green line), and 254 µm (cyan line). Inset of Fig. (a) shows the evolution of the dimensionless parameter k_pR_p relative to R_h/R_a . (b1-2) Dispersion relation for an infinite periodic arrangement of clamped annular plates with a periodicity $L_{unit} = 1$ cm, (b3-5) transmission, reflection magnitudes and absorption for a 102 µm-thick annular plate. Each colored line refers to a different hole radius R_h and the colored areas to the corresponding stopbands. (c1-c4) Lossless and (d1-d4) lossy scattering coefficients magnitude for single annular plate of different radii and thickness represented by the grey dots in (a): $R_h = 0.005$ mm and h = 102 µm in (1), $R_h = 1$ mm and $h_p = 102$ µm in (2), $R_h = 5$ mm and $h_p = 102$ µm in (3), and $R_h = 1$ mm and $h_p = 254$ µm in (4). The solid lines represent the analytical and the square symbols the full-wave results.

VII .1.2.2 Dispersion relation

Figures VII .1(b1-2) shows the real and imaginary part of the dispersion relation calculated on a unit cell composed of two layers of air in the middle of which the annular plate is clamped. The width of the unit-cell, i.e., the periodicity constant is $L_{unit} = 1$ cm. The dispersion relation is studied in parallel to the transmission, reflection magnitude, and absorption shown in Figs. VII .2(b2-3). A periodic arrangement of annular plates presents a successive passband and stopband below the resonance frequency. Another passband is reopened from the resonance frequency. Thus, propagation is almost unitary at low frequency, then passes through an anti-resonance characterized by a minimum of transmission and a maximum of reflection below resonance, and finally reaches again a maximum of transmission (unitary when losses are not accounted for) at the resonance frequency. The colored areas in Figs. VII .1(b1-5) represent the stopband of the infinite system, each color corresponding to a different radius orifice R_h . The narrower the orifice, the wider the stopband, as also evidenced by comparing Figs. VII .1(c2,d2) and Figs. VII .1(c3,d3) corresponding respectively to an annular plate with an orifice $R_h = 1$ mm and $R_h = 5$ mm. It can be observed that the antiresonance is sharper in the latter case.

VII .1.2.3 Role of the plate and orifice

So as to understand the reason for the minimum of transmission below the resonance, we plot in Fig. VII .2(a) the numerically computed mean flux generated in the extreme vicinity of either the plate (cyan color) or the orifice (green color). We consider here a lossless annular plate of orifice radius $R_h = 2.7$ mm and of thickness $h_p = 76 \,\mu\text{m}$. We parallel-up these results with the transmission (black color) and reflection (red color) magnitudes as well as the phase of the transmission coefficient in Figs. VII .2(b,c). The red, black and grey vertical lines represent respectively the zero reflection, unitary transmission, and zero transmission frequencies.



Figure VII .2: Fluxes and pole and zero of the scattering matrix eigenvalue of a single annular plate ($R_h = 2.7 \text{ mm}$ and $h_p = 76 \mu \text{m}$): (a) Averaged fluxes generated by the plate (cyan symbols) and by the hole (green symbols) from full-wave simulations when neither the viscoelastic nor the viscothermal losses are accounted for, (b) magnitude of transmission (black symbols) and reflection (red symbols), and (c) phase of the transmission coefficient. The red, black, and grey vertical dashed lines highlight the minimum reflection, maximum transmission, and minimum transmission respectively. The cyan and green areas respectively represent the frequency ranges with a positive flux for the plate and a negative flux for the hole and vice versa. Analytical lossless (d) and lossy (f) transmittance (black), reflectance (red), and absorption (yellow) coupled respectively to the complex plane representation of the eigenvalue |T - R| of the scattering matrix in (e) and (f).

The mean fluxes U_p and U_h , respectively generated by the plate of section $S_p = \pi (R_a^2 - R_h^2)$ and the hole of section $S_h = \pi R_h^2$, are calculated from the mean acoustic velocity averaged over the respective cross-sectional area, that is $U_p = \int_{S_p} v_z dS_p / S_p^2$ and $U_h = \int_{S_h} v_z dS_h / S_h^2$. The signs of the fluxes can be classified in three categories. The frequency ranges in which both fluxes are in the same direction (same sign) correspond to high transmission bands, i.e., the uncolored areas in Figs. VII .2(a-b). In contrast, the stopband corresponds to a frequency range in which the fluxes U_p and U_h are antiparallel (opposite signs), preventing large transmission. The green colored area refers to the frequency range in which the flux generated by the hole is positive and that of the plate is negative, and the cyan colored area to the opposite case. The transmission minimum and reflection maximum, i.e., the antiresonance of the annular plate, are found at the frequency at which both fluxes vanish (grey dashed line between the cyan and green colored areas). The zero reflection (red dashed vertical line) is located in the negative U_h range, at a frequency slightly different to that of the maximum of transmission (resonance frequency of the plate) for which the flux from the plate is positive while that of the orifice is zero. The minimum of reflection is moreover accompanied by a zero value of the transmission coefficient phase.

VII .1.2.4 Maximal absorption of an annular plate

We now investigate the maximal absorption achievable with a single annular plate.

The total losses of a resonant system can be characterized by its quality factor composed of a dissipative part Q_{diss} and a leakage part Q_{leak} . The maximal absorption of a structure is reached when the inherent dissipation balances the energy leakage rate $Q_{diss} = Q_{leak}$, i.e., when the critical coupling condition is met [20–24].

One way to visualize the optimal balance between energy leakage rate and inherent losses is to plot the eigenvalues of the scattering matrix in a complex plane, i.e., in the plane $\text{Im}(f) = \mathfrak{F}(\text{Re}(f))$, with $f = f_r + if_i$ the complex frequency.

The scattering matrix S connects through the transmission and reflection coefficients, the incoming complex amplitudes A, D and outgoing complex amplitudes B, C from the annular plate

$$\begin{bmatrix} B \\ C \end{bmatrix} = \begin{bmatrix} R^+ & T \\ T & R^- \end{bmatrix} \begin{bmatrix} A \\ D \end{bmatrix}.$$
 (VII .9)

The two eigenvalues of the **S**-matrix are therefore $\Lambda_{1,2} = T \pm \sqrt{R^+ R^-}$.

For mirror-symmetric configurations, i.e, when $R^+ = R^- = R$, the whole problem can be decoupled into two subconfigurations in reflection [18, 25]. The two eigenvalues of the **S**-matrix can indeed be regarded as the reflection coefficient of the symmetric, $R_{sym} = T - R$, and antisymmetric, $R_{asym} = T + R$, subproblems in reflection, that is when a Neumann boundary condition (rigid wall, $\partial p / \partial n = 0$) or a Dirichlet condition (soft boundary, p = 0) are respectively applied to the symmetry plane of the system. The absorption of the whole problem is then recovered from that of each subproblem $\alpha_{asym,sym} = 1 - |R_{asym,sym}|^2$, i.e., $\alpha = (\alpha_{asym} + \alpha_{sym})/2$.

In the present case, the transmission and reflection of a single punctual annular plate are given by

$$T = \frac{2}{2 + Z_p/Z_0},$$
 (VII .10)

$$R = \frac{Z_p/Z_0}{2 + Z_p/Z_0}.$$
 (VII .11)

The only non unitary eigenvalue thus reads as

$$T - R = \frac{2 - Z_p/Z_0}{2 + Z_p/Z_0},$$
 (VII .12)

the pole and zero of which are found at $Z_p/Z_0 = \pm 2$. The position of the poles and zeros in the complex frequency plane can help to characterize the scattering of a given problem.

Figures VII .2(e-g) show the non-constant eigenvalue $\log(|T - R|)$ of the **S**-matrix represented in the complex frequency plane. The resonance of a given system can be identified by the presence of pairs of poles and zeros that are complex conjugates of each other in the lossless case, i.e., that are symmetrical with respect to the real frequency axis (see Fig. VII .2(e)). The energy leakage rate of the system is given by the imaginary part of the poles complex frequency, while the resonance frequency is given by the real part. With the time convention considered here, $e^{i\omega t}$, the poles are located in the upper half of the complex plane, i.e., the positive imaginary plane, while the zeros are located in the negative one. When losses (viscothermal and viscoelastic) are introduced , the poles and zeros are translated in the complex plane. Eventually, the zeros move towards the real frequency axis when the energy leakage rate is perfectly balanced by the resonator intrinsic losses, giving a maximum of absorption, $\alpha = 0.5$. Perfect absorption would have implied having the zeros of the two eigenvalues on the real frequency axis at the same frequency, that is to critically couple the symmetric and antisymmetric problem, which is not achievable with a punctual symmetric resonator (only one non constant eigenvalue).

Different strategies can be used to overcome this upper absorption limit. In what follows, we will discuss two of them, a rigidly backed configuration and an asymmetrical configuration, coupling different resonators.

VII .2 Absorption in a rigid backing configuration

In a reflection configuration, i.e., when the system is rigidly backed, perfect absorption can be achieved by critically coupling the resonances of a single subwavelength resonator and of the cavity of length L_{cav} separating the resonator from the rigid backing [24].

VII .2.1 Perfect absorption with a single plate

We start by optimizing the geometry of both an annular plate (thickness h_p and orifice radius R_h) and the backing cavity (length L_{cav}) to balance the leakage with the inherent losses.

The reflection coefficient R_{RB} of the system can be written as follow,

$$R_{RB} = R + \frac{T^2 R_{term} e^{-i2k_0 L_{cav}}}{1 - R R_{term} e^{-i2k_0 L_{cav}}},$$
(VII .13)

with R and T the reflection and transmission coefficient of the annular plate respectively, R_{term} the

reflection coefficient of the termination ($R_{term} = 1$ in the case of a rigid backing), and k_0 the acoustic wavenumber of the air-filled backing cavity.

We perform a multiparametric optimization to obtain a perfect absorption at f = 300 Hz, using a cost function of the form

$$\mathbb{C}(R_h, h_p, L_{cav}) = \min\left(|R_{RB}|^2\right).$$
(VII .14)



Figure VII .3: Perfect absorption with an annular plate in a rigidly-backed configuration: Left: Optimized configuration: (a) Analytical transmittance (black) and reflectance (red) of the optimized annular plate ($R_h = 0.90 \text{ mm}$, $h_p = 127 \text{ µm}$). (b) Absorption magnitude (yellow) and reflectance (red) of the optimized whole system (plate, cavity of length $L_{cav} = 36 \text{ mm}$, and rigid backing). (c) Complex frequency plane representation of the reflection coefficient of the whole system log $|R_{RB}|$.

Right: Parametric analysis of the influence of R_h , h_p , L_{cav} on the maximum of absorption: (d) Evolution of the zero of R_{RB} with R_h , h_p , L_{cav} in the complex frequency plane. (e) Absorption magnitude at f = 300 Hz depending on R_h and L_{cav} . (f) Absorption magnitude for $L_{cav} = 36$ mm depending on R_h and the frequency.

The reflectance and transmittance of the optimized annular plate are given in Fig. VII .3(a). The transmission amplitude is high while the reflection is almost minimal at the target frequency f = 300 Hz. Figure VII .3(b-c) shows respectively the reflectance and the corresponding complex plane for the whole system (plate and backing cavity of length $L_{cav} = 3.6$ cm). Perfect absorption (zero reflectance) is obtained as expected at 300 Hz. The zero of the reflection coefficient lies on the real frequency axis in the complex plane, which proves the critical coupling for this deep-subwavelength structure of dimension almost 32 times thinner than the acoustic wavelength in air at 300 Hz.

The dependence of the critical coupling condition on the geometrical parameters of the system is now studied. Figure VII .3(d) shows a zoom on the first zero of the reflection coefficient in the complex plane. The orange, purple, and cyan trajectories illustrate the evolution of the location of the zero of the reflection coefficient with respect to the values of R_h , L_{cav} , and h_p respectively. Perfect absorption and critical coupling are reached when at least one of the trajectories crosses the real frequency axis. The only solution meeting this requirement is, in this case, the one where all the paths cross at the same point, corresponding to the optimized configuration.

Another representation of this dependency is presented in Figs. VII .3(e-f). The first represents the value of the absorption coefficient at 300 Hz for different pairs of values (L_{cav}, R_h) . The absorption magnitude is given by the color, the yellow being $\alpha \approx 1$ and the blue $\alpha \approx 0$. We can see that quasi perfect absorption ($\alpha > 0.95$) can be achieved for several pairs of orifice radii and cavity lengths around the optimum, ranging in $L_{cav} = [2.3 - 4.1]$ cm and $R_h = [0.78 - 1]$ mm. Figure VII .3(f) gives the absorption of the structure as a function of frequency for different R_h and a constant $L_{cav} = 3.6$ cm. It can be observed that an almost continuous perfect absorption can be obtained over the frequency range of interest just by varying the orifice radius and keeping the cavity length constant.

VII .2.2 Broadband absorption in a rigidly-backed configuration

We then complexify our system to achieve broadband absorption. We now consider a system composed of N = 8 plates, stacked in front of a rigid backing as depicted in the inset of Fig. VII .4(a).



Figure VII .4: Broadband absorption in a rigidly-backed configuration: (a) Optimized absorption. The solid line and the square symbols represent the analytical and the numerical results respectively. The optimized device consists in 8 annular plates of thicknesses comprised between 96 µm and 185 µm, and has a total length of 25 cm. (b) Complex frequency plane representation of the reflection coefficient of the optimized configuration. (c) Reflectance (red), absorption (yellow) and (d) complex plane at the 2nd step of the optimization (2 plates). (e) Reflectance (red), absorption (yellow), and (f) complex plane at the 6th step of the optimization (6 plates). Parameters of the final optimized geometry: $h_p = [183.9; 186.5; 155.9; 168.4; 139.8; 117.1; 70.9; 93.1]$ m, $L_{cav} = [26.3; 36; 24; 24; 28.6; 24; 31; 56]$ mm, and $R_h = [0.34; 0.68; 3; 2.7; 6.8; 3.8; 6.5; 7]$ mm.

We use a cascade optimization (step-by-step optimization) to critically couple each plate to the

previous ones. The spirit of the optimization is to create a cascade of stopbands, so that part of the frequency content of the incident waves is trapped at each step of the device. The dispersion relation in Figs. VII .1(b1-2) gives us an idea on how to initialize the radius of the hole for each plate. Since we want to decrease the frequency of the lower edge of the system's stopband as we approach the rigid backing, we initialize the optimization with decreasing orifice radii (from the free to the rigidly backed end of the device).

The target frequency range for the optimization is 200-1100 Hz. We use a total of 8 plates (plastic shims previously characterized) of different available thicknesses ranging from 25 μ m to 254 μ m. At each step l of the optimization, we tune the radius orifice R_{b_l} , the plate thickness h_{p_l} , and the cavity length L_{cav_l} between plate l and l-1. We first optimize, in a similar way to the previous Section, the plate geometry and the length of the cavity of the first resonator placed in front of the rigid backing, to have $\alpha(f_{target_1}) = 1$ at $f_{target_1} = 200$ Hz. Once optimized, another plate is added and the optimization is repeated for a frequency higher than the previous one. The result of the second optimization is presented in Figs. VII .4(c-d). As expected, two unitary absorption peaks (black line) are found at 200 and 335 Hz. If we look at the corresponding complex plane, we can observe two zeros lying on the real frequency axis at these two frequencies. The first two plates are then perfectly critically coupled to each other and to the rigid backing. The sixth optimization step is also depicted in Figs. VII .4(e-f). We can observe 6 zeros in the complex plane as expected. However, unlike the second optimization step, the zeros are no longer lying exactly on the real frequency axis. One of the reasons for this more chaotic behavior is the coupling between each of the plates. Each time we add a new plate, the overall acoustic behavior of the system is influenced. Since we only optimize at one single frequency at each step of the process, we do not search for new optimal geometries for the already optimized plates. However, after the 8th optimization, we carry out a final optimization using the initial values of the cascade optimization. We are then able to adjust the geometry of each plate and the length of each cavity to maximize absorption over the target frequency range, i.e.,

$$\mathbb{C}(R_{b_l}, h_{p_l}, L_{cav_l}) = \min || \int_{200}^{1100} |R_{RB}|^2(f) df ||.$$
(VII .15)

The resulting optimized absorption is shown in Fig. VII .4(a) together with the corresponding representation of the optimized reflection coefficient in the complex plane in Fig. VII .4(c). The analytical absorption is validated against a full-wave simulation on the optimized geometry. A fairly good agreement is found. The small visible discrepancies can be attributed to the off-resonance deviations of the annular plate modeling already observed in Figs. VII .1(c,d), due to the truncation on the modal displacement summation. Only two modes are accounted for in the analytical modeling because of a computational time constraint. Moreover the TMM does not account for the mutual coupling between resonators in contrast to the Finite Element Method.

The optimized configuration gives an almost perfect absorption (for analytical, solid line) and a very strong absorption (more than 0.9 on average, for numerical, square symbols) over the frequency range we are interested in. The complex plane is composed of a succession of 12 zeros on (or in the vicinity of) the real frequency axis. The greater number of pairs of poles and zeros relatively to the number of plates is due to the second modes of the annular plates which are also coupled. It is worth noting here that the location of the zeros follows a logarithmic-type distribution as observed in Refs. [19, 26].

Although the total thickness $L_{tot} = 25$ cm is subwavelength, i.e., a panel almost 7 times thinner than the wavelength in air at 200 Hz, the overall dimensions remains quite large for realistic applications. The reduction in thickness results in a deterioration of the absorption capacities, as predicted by the causality constraint inequality [26]

$$L_{tot} \ge \frac{1}{4\pi^2} \frac{\kappa(\omega)}{\kappa_0} \int_0^\infty \ln|1 - \alpha(\lambda)d\lambda| = L_{min}, \qquad (\text{VII .16})$$

with $\kappa(\omega)$ and κ_0 the effective bulk modulus of the structure in the static limit and that of the air respectively, α the absorption, λ the sound wavelength of air, and L_{min} the minimal thickness possible.

The optimal limit of inequality (VII .16) involves trade-offs between the thickness of the device, the target frequency range, and the maximum value of absorption obtained in that frequency band.

VII .3 Absorption in a transmission configuration

A similar optimization can be performed with a transmission configuration, i.e., not terminated by a rigid backing. In such configurations, perfect absorption requires that reflection and transmission vanish simultaneously. The design of degenerated resonators, e.g., decorated membranes [9], makes it possible to achieve this condition. The critical coupling of the symmetrical and antisymmetrical problems then results from the use of both the monopolar and dipolar behaviors of the degenerate resonator [27].

Quasi perfect absorption in transmission configuration can also be obtained with a periodic array of identical monopolar source, e.g., Helmholtz resonators, [18]. In this case, the accumulation of the cavity resonances (Fabry-Perot), due to slow sound propagation and the strong dispersion below the Helmholtz resonance frequency, allows to almost critically couple both the symmetric and antisymmetric subproblems. As a result, a quasi perfect absorption can be reached. Another strategy consists in breaking the symmetry of the system by coupling different resonators interacting with each other. Such an asymmetrical system has been experimentally demonstrated in acoustics with Helmholtz resonators [25]. At resonance, a Helmholtz resonator has almost zero transmission, i.e., it behaves like a rigid wall. It is then possible to tune the second Helmholtz resonator to critically couple the reflection problem and achieve perfect absorption. A cascade optimization with various different Helmholtz resonators can then be used to design a rainbow-trapping absorber, i.e., broadband [19].

We will here use this latter idea to achieve perfect absorption in transmission with annular plates instead of Helmholtz resonators.

VII .3.1 Optimization scheme

In contrast to the Helmholtz resonator, the transmission of an annular plate does not vanish at its resonance frequency, but at its antiresonance. However a similar optimization to that presented with the Helmholtz resonator can be applied.

First we optimize the radius and thickness of the latter plate of the system, tagged with a subscript 1, so as to minimize its transmission (zero in the lossless case) at a given frequency. We then successively critically couple each of the following plates.

As an example, we optimize here a system made of 2 annular plates in transmission, as sketched in the insets of Fig. VII .5(c-4), to achieve perfect absorption at a targeted frequency $f_{target} = 250$ Hz. The dotted circle in Fig. VII .5(b-1) shows the scattering amplitude of the latter plate, after optimization ($R_{b_1} = 0.5$ mm and $h_{p_1} = 110 \ \mu$ m), with a minimum of transmission at f_{target} .



Figure VII .5: Perfect absorption optimization for a system composed of 2 annular plates in transmission: (a1-2) Scattering amplitudes (transmittance in black, reflectance in red, and absorbance in yellow) and corresponding complex frequency plane of the first eigenvalue $\log |T-R|$ of the scattering matrix of the system's first plate (left plate, $R_{b2} = 1.1 \text{ mm}$, $h_{p2} = 110 \text{ µm}$). (b1-2) Scattering amplitudes (transmittance in black, reflectance in red, and absorbance in black, reflectance in red, and absorbance in yellow) and corresponding complex frequency plane of the first eigenvalue $\log |T-R|$ of the scattering matrix of the system's second plate (right plate, $R_{b1} = 0.5 \text{ mm}$, $h_{p1} = 110 \text{ µm}$). (c,d1-2) Scattering amplitudes (transmittance in black, reflectance in red, and absorbance in yellow) for a left or right incidence, and (c,d3-4) complex frequency plane of the first ($\log |T - \sqrt{R^+R^-}|$) and the second ($\log |T + \sqrt{R^+R^-}|$) eigenvalues of the scattering value of the whole system with a cavity length separating the two plates of $L_{cav} = 6 \text{ cm}$ (c) or $L_{cav} = 17 \text{ cm}$ (d).

We then add a second plate in front and optimize its geometry ($R_{b_2} = 1.1 \text{ mm}, h_{p_2} = 110 \text{ }\mu\text{m}$) as well as the cavity length L_{cav_1} to attain a maximum of absorption, i.e., critical coupling at f_{target} . Figures VII .5(a1-2) show the amplitude of the scattering parameters and the complex plane of the non constant eigenvalue $\log |T - R|$ of the second plate.

Figures VII .5(c1-4, d1-4) show respectively two optimizations with different upper limits for the length of the whole system, 6 cm or 17 cm. We can see that in the latter, an asymmetric perfect absorption can be achieved at 250 Hz (see Fig. VII .5(d-1)) when the wave impinges from the left, while a maximum of reflection is found for the opposite incidence (see Fig. VII .5(d-2)). If we look at the pole and zero of the eigenvalues of the scattering matrix, we can observe that, in this case, the zero of both eigenvalues (symmetric problem in Fig. VII .5(d-3) and antisymmetric problem in Fig. VII .5(d-4)) reach the real frequency axis at f_{target} , thus proving that the critical coupling condition is fulfilled.

In contrast, when the cavity length is decreased to 6 cm (see Figs. VII .5(c1-4)), the zero of the eigenvalues are effectively on the real frequency axis, but at frequency slightly different. Consequently, only a quasi perfect absorption is achieved with the system at 250 Hz. The maximum value of the absorbance is 0.97, as evidenced in Fig. VII .5(c-1). A compromise has to be found between the total thickness of the device and the maximum of absorption achievable.

One way to extend this quasi perfect absorption is to reiterate the optimization process adding plates successively and optimizing at slightly higher frequency at each step. The second step of the optimization can for example be done around 320 Hz, since the whole system present a nearly zero transmission at this frequency, as highlighted by the purple dotted circle in Fig. VII .5(c-1).

VII .3.2 Broadband asymmetric absorber

Applying the optimization scheme presented in the previous Section, we design a broadband asymmetric absorber consisting of 8 annular plates (plastic shims) with a target frequency range of $f \in [250 - 950]$ Hz. We constrain the optimization to a maximal thickness of L = 18 cm for the device.

After the cascade optimization, a global optimization over the whole frequency range is performed, using as cost function $\mathbb{C}(R_{b_l}, h_{p_l}, L_{cav_l}) = \min || \int_{250}^{950} |R^+|^2 + |T|^2 df ||.$

The magnitude of the scattering coefficients is shown in Fig VII .6. Although perfect broadband absorption is not achieved, the device nevertheless exhibits a high absorption capacity as shown by an absorption above 0.9 over the target frequency range. A succession of 10 zeros lies in the immediate vicinity of the real frequency axis on the complex frequency plane of each of the eigenvalues of the scattering matrix. However, they are located at slightly different real frequencies. One of the reasons for the absence of critical coupling is the constraint imposed on the total thickness of the device, as discussed in the previous Section with a system consisting of two annular plates.



Figure VII .6: Broadband absorption in a transmission configuration: (a) Optimized absorption (yellow), transmittance (black) and reflectance for a left incident wave. Inset of (a) shows the configuration under study. The solid line and the square symbols represent the analytical and the numerical results respectively. (b-c) Complex frequency plane representation of the first (b) and second (b) eigenvalues of the scattering matrix of the optimized configuration. The optimized device consists in 8 annular plates of thicknesses comprised between 90 µm and 230 µm, and has a total length of 17 cm: $h_p = [230; 145; 124; 171; 90; 150; 90; 190]$ µm, $L_{cav} = [6.3; 17.2; 18.5; 17.3; 39.7; 11; 62.2]$ mm, and $R_h = [0.53; 0.541.29; 0.9; 1.5; 2.6, 3.4; 4.8]$ mm

VII .4 Conclusion

In this side work, we investigated the possibility of using one-dimensional arrangements of annular plates as effective sound absorption devices. After deriving the analytical acoustic impedance on an annular plate and studying the acoustic signature of various samples, we focused on two particular configurations. First, we examined a rigid backing configuration, i.e., when the plates are arranged in front of a rigid wall, and second, a transmission configuration. Both optimizations were based on critical coupling. We found that perfect absorption can be achieved at a single low frequency in both cases. However, unlike the rigidly-backed configuration, in which the critical coupling condition can be met with a deep subwavelength device $(L \approx \lambda_0/32)$, the transmission configuration requires large dimensions (L = 17 cm). The latter configuration requires using the antiresonant frequency drop of an annular plate (behaving as a rigid wall at this particular frequency) to critically couple a second plate. To extend broadband absorption, cascade optimization was used to sequentially critically couple each of the plates. Although perfect broadband absorption was not obtained for a constrained total thickness of the device (upper limit allowed: L < 25 cm), high absorption ($\alpha > 0.9$) was observed both analytically and numerically in reflection and transmission configurations for frequencies ranging from [200-1100] Hz and [250-950] Hz, respectively. The relatively large total thickness of the devices required to achieve such absorption may, however, limit their applicability in realistic situations where space is a major problem.

Bibliography

- Z. Yang et al. "Membrane-Type Acoustic Metamaterial with Negative Dynamic Mass." *Physical Review Letters* 101 (2008). DOI: 10.1103/PhysRevLett.101.204301.
- Z. Yang et al. "Acoustic metamaterial panels for sound attenuation in the 50–1000 Hz regime." Appl. Phys. Lett. 96 (2010), p. 041906. DOI: 10.1063/1.3299007.
- C. J. Naify et al. "Transmission loss and dynamic response of membrane-type locally resonant acoustic metamaterials." *Journal of Applied Physics* 108 (2010), p. 114905. DOI: 10.1063/1.3514082.
- C. J. Naify et al. "Transmission loss of membrane-type acoustic metamaterials with coaxial ring masses." *Journal of Applied Physics* 110 (2011), p. 124903. DOI: 10.1063/1.3665213.
- C. J. Naify et al. "Membrane-type metamaterials: Transmission loss of multi-celled arrays." Journal of Applied Physics 109 (2011), p. 104902. DOI: 10.1063/1.3583656.
- [6] C. J. Naify et al. "Scaling of membrane-type locally resonant acoustic metamaterial arrays." *The Journal of the Acoustical Society of America* 132 (2012), pp. 2784–2792. DOI: 10.1121/1. 4744941.
- Y. Chen et al. "Analytical coupled vibroacoustic modeling of membrane-type acoustic metamaterials: Plate model." *The Journal of the Acoustical Society of America* 136 (2014), pp. 2926– 2934. DOI: 10.1121/1.4901706.
- [8] M. Yang et al. "Sound absorption by subwavelength membrane structures: A geometric perspective." Comptes Rendus Mécanique 343 (2015), pp. 635–644. DOI: 10.1016/j.crme.2015.06.008.
- M. Yang et al. "Subwavelength total acoustic absorption with degenerate resonators." Applied Physics Letters 107 (2015), p. 104104. DOI: 10.1063/1.4930944.
- G. Ma et al. "Acoustic metasurface with hybrid resonances." Nature Materials 13 (2014), pp. 873– 878. DOI: 10.1038/nmat3994.
- P. Wei et al. "Symmetrical and anti-symmetrical coherent perfect absorption for acoustic waves." Appl. Phys. Lett. 104 (2014), p. 121902. DOI: 10.1063/1.4869462.

- [12] V. Romero-García, N. Jiménez, and J.-P. Groby. "Slow Sound and Critical Coupling to Design Deep Subwavelength Acoustic Metamaterials for Perfect Absorption and Efficient Diffusion." In: *Fundamentals and Applications of Acoustic Metamaterials*. John Wiley & Sons, Ltd, 2019. Chap. 3, pp. 47–72. DOI: 10.1002/9781119649182.ch3.
- [13] N. Sugimoto and T. Horioka. "Dispersion characteristics of sound waves in a tunnel with an array of Helmholtz resonators." The Journal of the Acoustical Society of America 97 (1995), pp. 1446–1459. DOI: 10.1121/1.412085.
- [14] A. Santillán and S. I. Bozhevolnyi. "Acoustic transparency and slow sound using detuned acoustic resonators." *Phys. Rev. B* 84 (2011), p. 064304. DOI: 10.1103/PhysRevB.84.064304.
- [15] A. Santillán and S. I. Bozhevolnyi. "Demonstration of slow sound propagation and acoustic transparency with a series of detuned resonators." *Phys. Rev. B* 89 (18 2014), p. 184301. DOI: 10.1103/PhysRevB.89.184301.
- [16] G. Theocharis et al. "Limits of slow sound propagation and transparency in lossy, locally resonant periodic structures." New Journal of Physics 16 (2014), p. 093017. DOI: 10.1088/1367-2630/ 16/9/093017.
- [17] N. Jiménez et al. "Ultra-thin metamaterial for perfect and quasi-omnidirectional sound absorption." Applied Physics Letters 109 (2016), p. 121902. DOI: 10.1063/1.4962328.
- [18] N. Jiménez et al. "Quasiperfect absorption by subwavelength acoustic panels in transmission using accumulation of resonances due to slow sound." *Physical Review B* 95 (2017). DOI: 10. 1103/PhysRevB.95.014205.
- [19] N. Jiménez et al. "Rainbow-trapping absorbers: Broadband, perfect and asymmetric sound absorption by subwavelength panels for transmission problems." *Scientific Reports* 7 (2017), p. 13595. DOI: 10.1038/s41598-017-13706-4.
- [20] K. Y. Bliokh et al. "Colloquium : Unusual resonators: Plasmonics, metamaterials, and random media." Rev. Mod. Phys. 80 (2008), pp. 1201–1213. DOI: 10.1103/RevModPhys.80.1201.
- T. S. Luk et al. "Directional perfect absorption using deep subwavelength low-permittivity films." *Phys. Rev. B* 90 (8 2014), p. 085411. DOI: 10.1103/PhysRevB.90.085411.
- J.-P. Groby, R. Pommier, and Y. Aurégan. "Use of slow sound to design perfect and broadband passive sound absorbing materials." *The Journal of the Acoustical Society of America* 139 (2016), pp. 1660–1671. DOI: 10.1121/1.4945101.
- [23] V. Romero-García et al. "Use of complex frequency plane to design broadband and sub-wavelength absorbers." The Journal of the Acoustical Society of America 139 (2016), pp. 3395–3403. DOI: 10.1121/1.4950708.
- [24] V. Romero-García et al. "Perfect and broadband acoustic absorption by critically coupled subwavelength resonators." Sci Rep 6 (2016), p. 19519. DOI: 10.1038/srep19519.

- [25] A. Merkel et al. "Control of acoustic absorption in one-dimensional scattering by resonant scatterers." Applied Physics Letters 107 (2015), p. 244102. DOI: 10.1063/1.4938121.
- [26] Y. Min et al. "Optimal sound-absorbing structures." *Mater. Horiz.* 4 (4 2017), pp. 673–680. DOI: 10.1039/C7MH00129K.
- [27] J. R. Piper, V. Liu, and S. Fan. "Total absorption by degenerate critical coupling." Applied Physics Letters 104 (2014), p. 251110. DOI: 10.1063/1.4885517.

Appendices

Appendix

Supplement on the Transfer Matrix Method

A.1 Transfer matrix of a layer of material

The propagation of a plane wave over a length L_{unit} in a waveguide of cross-section S filled with air (density ρ_0 and compressibility C_0) is governed by the constitutive equations

$$\frac{\partial p}{\partial x} = -i\omega \frac{\rho_0}{S} U,\tag{A.1}$$

$$\frac{\partial U}{\partial x} = -i\omega \frac{S}{\kappa_0} p,\tag{A.2}$$

written with the time convention $e^{i\omega t}$, where ω is the circular frequency, $\kappa_0 = 1/C_0$ is the air bulk modulus, p is the sound pressure, and U = Sv is the flux, with v being the particle velocity.

These governing equations can be rearranged in the following matrix form

$$\frac{\partial}{\partial x} \begin{cases} P \\ U \end{cases} = \begin{bmatrix} 0 & -i\omega\frac{\rho_0}{S} \\ -i\omega\frac{S}{\kappa_0} & 0 \end{bmatrix} \begin{cases} p \\ U \end{cases} = \mathbf{M} \begin{cases} p \\ U \end{cases},$$
(A.3)

the solution of which being

$$\begin{cases} P \\ U \end{cases}_{L_{unit}} = \exp(\mathbf{M}L_{unit}) \begin{cases} p \\ U \end{cases}_{0} = \mathbf{V} \cdot \begin{bmatrix} e^{\Lambda_{1}L_{unit}} & 0 \\ 0 & e^{\Lambda_{2}L_{unit}} \end{bmatrix} \cdot \mathbf{V}^{-1} \begin{cases} p \\ U \end{pmatrix}_{0} = \mathbf{T} \begin{cases} p \\ U \end{pmatrix}_{0}.$$
(A.4)

 $\Lambda_{1,2}$ refers to the eigenvalues of **M**, that is

$$\det \left(\mathbf{M} - \Lambda \mathbf{Id}\right) = \Lambda^2 + \omega^2 \frac{\rho_0}{\kappa_0} = 0, \tag{A.5}$$

with det the determinant function, giving

$$\Lambda_{1,2} = \pm i\omega \sqrt{\frac{\rho_0}{\kappa_0}} = ik_0. \tag{A.6}$$

 ${f V}$ corresponds to the eigenvector matrix composed of the two eigenvectors of ${f M}$ and reads as

$$\mathbf{V} = \begin{bmatrix} V_{11} & V_{21} \\ V_{12} & V_{22} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{\rho_0 \kappa_0}}{S} & -\frac{\sqrt{\rho_0 \kappa_0}}{S} \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} Z_0 & -Z_0 \\ 1 & 1 \end{bmatrix}.$$
 (A.7)

The transfer matrix therefore becomes

$$\mathbf{T} = \frac{1}{2} \begin{bmatrix} Z_0 & -Z_0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} e^{-ik_0 L_{unit}} & 0 \\ 0 & e^{ik_0 L_{unit}} \end{bmatrix} \cdot \begin{bmatrix} Z_0^{-1} & 1 \\ -Z_0^{-1} & 1 \end{bmatrix}$$
(A.8)

$$= \frac{1}{2} \begin{bmatrix} e^{-ik_0 L_{unit}} + e^{ik_0 L_{unit}} & Z_0 \left(e^{-ik_0 L_{unit}} - e^{ik_0 L_{unit}} \right) \\ Z_0^{-1} \left(e^{-ik_0 L_{unit}} - e^{ik_0 L_{unit}} \right) & e^{-ik_0 L_{unit}} + e^{ik_0 L_{unit}} \end{bmatrix}$$
(A.9)

$$= \begin{bmatrix} \cos(k_0 L_{unit}) & -iZ_0 \sin(k_0 L_{unit}) \\ -iZ_0^{-1} \sin(k_0 L_{unit}) & \cos(k_0 L_{unit}) \end{bmatrix}.$$
 (A.10)

A.2 Dispersion relation of a periodic medium

A periodic medium of periodicity L_{unit} is now considered. Periodicity implies that the state vector components, i.e., pressure and flux, are equal to those at a L_{unit} further position, that is

$$\begin{cases} p(x) = p(x + L_{unit}), \\ U(x) = U(x + L_{unit}). \end{cases}$$
(A.11)

In other words, the state vector fulfils, in a periodic medium, the Bloch-Floquet theorem and can be analyzed only in one of the unit cells of the system.

The transfer matrix of a unit cell can therefore reads as follows

$$\begin{cases} P \\ U \end{cases}_{L_{unit}} = \mathbf{T} \begin{cases} P \\ U \end{cases}_{0} = \begin{bmatrix} T_{11} & T_{21} \\ T_{12} & T_{22} \end{bmatrix} \begin{cases} P \\ U \end{cases}_{0}$$
 (A.12)

$$= \begin{bmatrix} T_{11} & T_{21} \\ T_{12} & T_{22} \end{bmatrix} e^{iqL_{unit}} \begin{cases} P \\ U \end{cases}_{L_{unit}},$$
(A.13)

where q is the Bloch Floquet wavenumber.

This equality can be rewritten in the following form

$$\begin{pmatrix} \begin{bmatrix} T_{11} & T_{21} \\ T_{12} & T_{22} \end{bmatrix} - \begin{bmatrix} e^{-iqL_{unit}} & 0 \\ 0 & e^{-iqL_{unit}} \end{bmatrix} \end{pmatrix} \begin{pmatrix} P \\ U \end{pmatrix}_0 = 0,$$
 (A.14)

the solution of which only exists if

$$\det \begin{pmatrix} T_{11} - e^{-iqL_{unit}} & T_{21} \\ T_{12} & T_{22} - e^{-iqL_{unit}} \end{pmatrix} = 0.$$
(A.15)

This results in

$$T_{11}T_{22} - T_{12}T_{21} - e^{-iqL_{unit}}(T_{11} + T_{22}) + e^{-2iqL_{unit}} = 0$$
(A.16)

$$\Rightarrow T_{11}T_{22} - T_{12}T_{21} + e^{-2iqL_{unit}} = e^{-iqL_{unit}}(T_{11} + T_{22}).$$
(A.17)

Under the reciprocity condition, i.e., $T_{11}T_{22} - T_{12}T_{21} = 1$, Eq. (A.17) leads to the dispersion relation

$$\cos(qL_{unit}) = \frac{T_{11} + T_{22}}{2} = \frac{\text{Tr}(\mathbf{T})}{2}.$$
(A.18)

Appendix B

Analytical modeling of the plate impedance

A plate is a specific type of shell with zero curvature [1–3]. We only consider here the particular case of thin plates, that is plates fulfilling the Kirchhoff-Love approximation. In this approximation, the transverse vibration of a h_p -thick plate is represented by the vibration of its mid-plane, neglecting the effects of rotary inertia, shear deformations, in-plane forces, thickness variation, non-homogeneity. Only small deflections are also considered.

We simply remind in a first section the plate impedance of a thick plate accounting for the shear deformations through the thickness of the plate.

B.1 Analytical modeling of the acoustic impedance of a thick plate

The following thick plate acoustic impedance is based on the Mindlin and Deresiewicz [4] plate theory. Linear variation is assumed. The thickness variation during vibration is not accounted for.

The circular plate with radius R_a , Young's modulus E_p , Poisson's ratio ν_p , density ρ_p , section S and thickness h_p is modeled by its averaged acoustic impedance,

$$Z_{plate_{thick}} = \frac{i\omega M_{plate}/S^2}{1 - 2(\frac{1}{\delta_1} + \frac{1}{\delta_2})\frac{J_1(\delta_1)I_1(\delta_2)}{J_0(\delta_1)I_1(\delta_2) + J_1(\delta_1)I_0(\delta_2)}},$$
(B.1)

where J_n and I_n denote the Bessel and modified Bessel functions of the first kind.

The mass of the plate is $M_{plate} = \rho_p h_p S$. The variables $\delta_{1,2}$ read as [5–7]

$$\begin{cases} \delta_1^2 = \frac{1}{2} \zeta^4 \left((R+F) + \sqrt{(R-F)^2 + 4\zeta^{-4}} \right), \\ \delta_2^2 = \frac{1}{2} \zeta^4 \left(\sqrt{(R-F)^2 + 4\zeta^{-4}} - (R+F) \right), \end{cases}$$
(B.2)

where the parameters R and F as well as the non-dimensional frequency parameter ζ , the flexural

rigidity D, and the shear correction factor α are defined as follows

$$\begin{cases}
R = \frac{1}{12} \left(\frac{h_p}{R_a} \right)^2, \\
F = \frac{1}{6(1 - \nu_p)\alpha^2} \left(\frac{h_p}{R_a} \right)^2, \\
\zeta^4 = \frac{\rho_p h_p R_a^4 \omega^2}{D}, \\
D = \frac{E_p h_p^3}{12(1 - \nu_p^2)}, \\
\alpha = \frac{\pi}{12}.
\end{cases}$$
(B.3)

B.2 Analytical modeling of thin circular plates

Using the Kirchhoff-Love approximation, the transverse vibrations in thin circular plates of radius R_a and cross-section $S = \pi R_a^2$ follow the equation of plate motion in polar coordinates

$$D\nabla^4 \tilde{w}(r,\theta,t) + \rho_p h_p \frac{\partial^2}{\partial t^2} \tilde{w}(r,\theta,t) = \Delta \tilde{P}(t), \tag{B.4}$$

with $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$ the Laplacian in polar coordinates, and $\Delta P = p(x + h_p) - p(x)$ the difference of pressure applied to each side of the plate, where p(x) and $p(x + \delta x)$ are the acoustic pressure on the upstream and downstream faces respectively. The $\tilde{.}$ symbol is used to refer to time dependent displacement.

The use of variable separation provides a way to solve the homogeneous equation, by assuming $\tilde{w}(r,\theta,t) = R(r)\Theta(\theta)e^{i\omega t}$. The equation of motion then becomes

$$(\nabla^4 - k_p^4) R(r) \Theta(\theta) = (\nabla^4 - k_p^4) w(r, \theta) = 0,$$
(B.5)

with $k_p = \left(\omega^2 \rho_p h_p / D\right)^{1/4}$.

This results in

$$\frac{d}{dr^2}R(r) + \frac{1}{r}\frac{d}{dr}R(r) + (k_p^4 - \frac{n^2}{r^2}R(r)) = 0,$$
(B.6)

and

$$\frac{d}{dr^2}R(r) + \frac{1}{r}\frac{d}{dr}R(r) - (k_p^4 + \frac{n^2}{r^2}R(r)) = 0,$$
(B.7)

which leads to the solution for an axi-symmetric problem of the form

$$w(r,\theta) = R(r)\Theta(\theta) = [A_1 J_n(k_p r) + A_2 Y_n(k_p r) + A_3 I_n(k_p r) + A_4 K_n(k_p r)]\cos(n\theta),$$
(B.8)

where \mathbf{Y}_m and \mathbf{K}_m are the Bessel and modified Bessel functions of the second kind.

To get back to the plate impedance, it is then necessary to average the transverse displacement \overline{w}

over the plate surface.

$$\overline{w} = \frac{1}{S} \int_0^{2\pi} \int_0^{R_a} w(r,\theta) r dr d\theta.$$
(B.9)

The acoustic impedance of the plate is then given by

$$Z_p = \frac{Z_m}{S^2} = \frac{\int_S \Delta P(r) dS}{i\omega \overline{w}}.$$
(B.10)

B.2.1 Solution for a clamped circular plate

The clamping boundary conditions are

$$w(r = R_a, \theta) = \frac{\partial}{\partial r} w(r = R_a, \theta) = 0.$$
(B.11)

In addition, the displacement of the plate in its center (r = 0) must remain finite $w(r = 0, \theta) < \infty$. Since the Bessel and modified Bessel functions of the second kind Y_n and K_n diverge when the arguments are zero, the coefficients B and D must disappear.

If only the first mode is considered, n = 0, the solution of the homogeneous problem becomes

$$w(r) = A_1 J_0(k_p r) + A_3 I_0(k_p r).$$
(B.12)

Applying the boundary conditions in Eq. (B.11) then leads to

$$\begin{cases} A_1 J_0(k_p R_a) + A_3 I_0(k_p R_a) = 0, \\ -k_p A_1 J_1(k_p R_a) + k_p A_3 I_1(k_p R_a) = 0, \end{cases}$$
(B.13)

and then provides the condition

$$\frac{A_3}{A_1} = \frac{J_1(k_p R_a)}{I_1(k_p R_a)}.$$
(B.14)

The homogeneous solution is then written

$$w_h(r) = A_1 \frac{J_0(k_p r) I_1(k_p R_a) + J_1(k_p R_a) I_0(k_p r)}{I_1(k_p R_a)},$$
(B.15)

while a particular solution to the forced problem is

$$w_p(r) = \frac{-\Delta P}{Dk_p^4}.\tag{B.16}$$

Combining the homogeneous and particular equations and applying the initial conditions ($\tilde{w}(r, t = 0) = \hat{w}(r, t = 0) = 0$) on the resulting general solution [8], yields to

$$\begin{cases} A_{1} = \frac{\Delta P}{Dk_{p}^{4}} \frac{I_{1}(k_{p}R_{a})}{J_{O}(k_{p}R_{a})I_{1}(k_{p}R_{a})+J_{1}(k_{p}R_{a})I_{0}(k_{p}R_{a})}, \\ A_{3} = \frac{J_{1}(k_{p}R_{a})}{I_{1}(k_{p}R_{a})}A_{1} = \frac{\Delta P}{Dk_{p}^{4}} \frac{J_{1}(k_{p}R_{a})}{J_{O}(k_{p}R_{a})I_{1}(k_{p}R_{a})+J_{1}(k_{p}R_{a})I_{0}(k_{p}R_{a})}. \end{cases}$$
(B.17)
The displacement of a circular plate when it is subjected to harmonic excitation is therefore written as follows

$$w(r) = \left[\frac{-\Delta P}{Dk_p^4} + A_1 J_0(k_p r) + A_3 I_0(k_p r)\right].$$
 (B.18)

By inserting the expression of w(r) in equations (B.9) and (B.10), one can derive the acoustic impedance of a thin plate, that is

$$Z_p = \frac{\int_S \Delta P(r) dS}{i\omega \overline{w} S^2} = -\frac{i\omega m}{S^2} \frac{I_1(k_p R_a) J_0(k_p R_a) + J_1(k_p R_a) I_0(k_p R_a)}{I_1(k_p R_a) J_2(k_p R_a) - J_1(k_p R_a) I_2(k_p R_a)},$$
(B.19)

where $k_p = (\omega^2 \rho_p h_p / D)^{1/4}$ is the wavenumber of the flexural waves excited in the plate, with $m = \rho_p S h_p$ the mass of the plate. This expression assumes a uniform pressure distribution over the plate (only small displacements and plane wave excitation), and considers only axi-symmetric modes.

B.2.2 Solution for an annular plate

An annular plate clamped at its outer edge (in $r = R_a$), that is

$$w(r = a, \theta) = \frac{\partial}{\partial r}w(r = a, \theta) = 0,$$
 (B.20)

and free at its inner edge (in $r = R_h$) is now considered. This last boundary condition results in the cancellation of the bending moment M_r and of the shear forces V_r

$$\begin{cases} M_r = -D \left[\frac{\partial^2}{\partial r^2} w(r,\theta) + \nu \left(\frac{1}{r} \frac{\partial}{\partial r} w(r,\theta) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right]_{r=R_h} = 0, \\ V_r = -D \left[\frac{\partial}{\partial r} \nabla^2 w(r,\theta) + \frac{1-\nu}{r^2} \frac{\partial^2}{\partial \theta^2} \left(\frac{\partial}{\partial r} w(r,\theta) - \frac{w(r,\theta)}{r} \right) \right]_{r=R_h} = 0. \end{cases}$$
(B.21)

The general solution for an annular plate reads as follows

$$w(r,\theta) = \sum_{n=0}^{\infty} [A_n \mathcal{J}_n(k_{p_n}r) + B_n \mathcal{Y}_n(k_{p_n}r) + C_n \mathcal{I}_n(k_{p_n}r) + D_n \mathcal{K}(k_{p_n}(r)]\cos(n\theta) = \sum_{n=0}^{\infty} w_n(r,\theta).$$
(B.22)

One can insert Eq. (B.22) within the boundary conditions Eqs. (B.11)-(B.21), leading to the following system of linear and homogeneous equations [9]

$$A_{n}J_{n}(k_{p_{n}}R_{a}) + B_{n}Y_{n}(k_{p_{n}}R_{a}) + C_{n}I_{n}(k_{p_{n}}R_{a}) + D_{n}K_{n}(k_{p_{n}}R_{a}) = 0,$$

$$A_{n}\frac{\partial}{\partial r}J_{n}(k_{p_{n}}r)_{|R_{a}} + B_{n}\frac{\partial}{\partial r}Y_{n}(k_{p_{n}}r)_{|R_{a}} + C_{n}\frac{\partial}{\partial r}I_{n}(k_{p_{n}}r)_{|R_{a}} + D_{n}\frac{\partial}{\partial r}K_{n}(k_{p_{n}}r)_{|R_{a}} = 0,$$

$$A_{n}F_{1}(\nu, n, k_{p_{n}}R_{h}) + B_{n}F_{2}(\nu, n, k_{p_{n}}R_{h}) - C_{n}F_{3}(\nu, n, k_{p_{n}}R_{h}) - D_{n}F_{4}(\nu, n, k_{p_{n}}R_{h}) = 0,$$

$$A_{n}\Phi_{1}(\nu, n, k_{p_{n}}R_{h}) + B_{n}\Phi_{2}(\nu, n, k_{p_{n}}R_{h}) - C_{n}\Phi_{3}(\nu, n, k_{p_{n}}R_{h}) - D_{n}\Phi_{4}(\nu, n, k_{p_{n}}R_{h}) = 0,$$

$$(B.23)$$

with

$$\begin{cases} \frac{\partial}{\partial r} J_n(k_{p_n}r)|_{R_a} = \frac{n}{k_{p_n}R_a} J_n(k_{p_n}R_a) - J_{n+1}(k_{p_n}R_a), \\ \frac{\partial}{\partial r} Y_n(k_{p_n}r)|_{R_a} = \frac{n}{k_{p_n}R_a} Y_n(k_{p_n}R_a) - Y_{n+1}(k_{p_n}R_a), \\ \frac{\partial}{\partial r} I_n(k_{p_n}r)|_{R_a} = \frac{n}{k_{p_n}R_a} I_n(k_{p_n}R_a) + I_{n+1}(k_{p_n}R_a), \\ \frac{\partial}{\partial r} K_n(k_{p_n}r)|_{R_a} = \frac{n}{k_{p_n}R_a} K_n(k_{p_n}R_a) - K_{n+1}(k_{p_n}R_a), \end{cases}$$
(B.24)

$$F_{1}(\nu, n, k_{p_{n}}R_{h}) = \left(J_{n}(k_{p_{n}}R_{h}) - (1-\nu) \begin{bmatrix} \frac{n(n-1)}{(k_{p_{n}}R_{h})^{2}} J_{n}(k_{p_{n}}R_{h}) + \frac{1}{k_{p_{n}}R_{h}} J_{n+1}(k_{p_{n}}R_{h}) \end{bmatrix}\right),$$

$$F_{2}(\nu, n, k_{p_{n}}R_{h}) = \left(Y_{n}(k_{p_{n}}R_{h}) - (1-\nu) \begin{bmatrix} \frac{n(n-1)}{(k_{p_{n}}R_{h})^{2}} Y_{n}(k_{p_{n}}R_{h}) + \frac{1}{k_{p_{n}}R_{h}} Y_{n+1}(k_{p_{n}}R_{h}) \end{bmatrix}\right),$$

$$F_{3}(\nu, n, k_{p_{n}}R_{h}) = \left(I_{n}(k_{p_{n}}R_{h}) + (1-\nu) \begin{bmatrix} \frac{n(n-1)}{(k_{p_{n}}R_{h})^{2}} I_{n}(k_{p_{n}}R_{h}) - \frac{1}{k_{p_{n}}R_{h}} I_{n+1}(k_{p_{n}}R_{h}) \end{bmatrix}\right),$$

$$F_{4}(\nu, n, k_{p_{n}}R_{h}) = \left(K_{n}(k_{p_{n}}R_{h}) + (1-\nu) \begin{bmatrix} \frac{n(n-1)}{(k_{p_{n}}R_{h})^{2}} K_{n}(k_{p_{n}}R_{h}) + \frac{1}{k_{p_{n}}R_{h}} J_{n+1}(k_{p_{n}}R_{h}) \end{bmatrix}\right),$$
(B.25)

and

$$\Phi_{1}(\nu, n, k_{p_{n}}R_{h}) = nJ_{n}(k_{p_{n}}R_{h}) - (k_{p_{n}}R_{h})J_{n+1}(k_{p_{n}}R_{h}) + \frac{n^{2}(1-\nu)}{(k_{p_{n}}R_{h})^{2}}\left[(n-1)J_{n}(k_{p_{n}}R_{h}) - (k_{p_{n}}R_{h})J_{n+1}(k_{p_{n}}R_{h})\right], \\ \Phi_{2}(\nu, n, k_{p_{n}}R_{h}) = nY_{n}(k_{p_{n}}R_{h}) - (k_{p_{n}}R_{h})Y_{n+1}(k_{p_{n}}R_{h}) + \frac{n^{2}(1-\nu)}{(k_{p_{n}}R_{h})^{2}}\left[(n-1)Y_{n}(k_{p_{n}}R_{h}) - (k_{p_{n}}R_{h})Y_{n+1}(k_{p_{n}}R_{h})\right], \\ \Phi_{3}(\nu, n, k_{p_{n}}R_{h}) = nI_{n}(k_{p_{n}}R_{h}) + (k_{p_{n}}R_{h})I_{n+1}(k_{p_{n}}R_{h}) - \frac{n^{2}(1-\nu)}{(k_{p_{n}}R_{h})^{2}}\left[(n-1)I_{n}(k_{p_{n}}R_{h}) + (k_{p_{n}}R_{h})Y_{n+1}(k_{p_{n}}R_{h})\right], \\ \Phi_{4}(\nu, n, k_{p_{n}}R_{h}) = nK_{n}(k_{p_{n}}R_{h}) - (k_{p_{n}}R_{h})K_{n+1}(k_{p_{n}}R_{h}) - \frac{n^{2}(1-\nu)}{(k_{p_{n}}R_{h})^{2}}\left[(n-1)K_{n}(k_{p_{n}}R_{h}) - (k_{p_{n}}R_{h})K_{n+1}(k_{p_{n}}R_{h})\right]. \end{aligned}$$

The term $k_{p_n} = (\omega_n^2 \rho_p h_p / D)^{1/4}$ can be computed numerically from Eq.(B.23) by canceling the determinant of the resulting matrix

$$\begin{vmatrix} J_{n}(k_{p_{n}}R_{a}) & Y_{n}(k_{p_{n}}R_{a}) & I_{n}(k_{p_{n}}R_{a}) & K_{n}(k_{p_{n}}R_{a}) \\ \frac{n}{k_{p_{n}}R_{a}}J_{n}(k_{p_{n}}R_{a}) - J_{n+1}(k_{p_{n}}R_{a}) & \frac{n}{k_{p_{n}}R_{a}}Y_{n}(k_{p_{n}}R_{a}) - Y_{n+1}(k_{p_{n}}R_{a}) & \frac{n}{k_{p_{n}}R_{a}}I_{n}(k_{p_{n}}R_{a}) + I_{n+1}(k_{p_{n}}R_{a}) & \frac{n}{k_{p_{n}}R_{a}}K_{n}(k_{p_{n}}R_{a}) - K_{n+1}(k_{p_{n}}R_{a}) \\ F_{1}(\nu, n, k_{p_{n}}R_{h}) & F_{2}(\nu, n, k_{p_{n}}R_{h}) & -F_{3}(\nu, n, k_{p_{n}}R_{h}) & -F_{4}(\nu, n, k_{p_{n}}R_{h}) \\ \Phi_{1}(\nu, n, k_{p_{n}}R_{h}) & \Phi_{2}(\nu, n, k_{p_{n}}R_{h}) & -\Phi_{3}(\nu, n, k_{p_{n}}R_{h}) & -\Phi_{4}(\nu, n, k_{p_{n}}R_{h}) \end{vmatrix} = 0. \quad (B.27)$$

and using a Muller's algorithm with as initial hypothesis 3 values around the plain circular plate resonance frequency.

Knowing the eigenvalues, it is then possible to trace back to the non-trivial eigenvectors (coefficients A_n , B_n , C_n and D_n), and thus to obtain the eigenfunctions.

Returning to the forced problem (equation (B.4)) and projecting onto the modal basis

$$\tilde{w}(r,\theta,t) = \sum_{n=0}^{\infty} w_n(r,\theta) \tilde{q}_n(t), \qquad (B.28)$$

we get

$$\int_{S} \sum_{n=0}^{\infty} [\nabla^4 w_n(r,\theta) \tilde{q}_n(t) + \frac{\rho_p h_p}{D} w_n(r,\theta) \ddot{\tilde{q}}_n(t)] w_m(r,\theta) dS = \int_{S} \frac{\Delta P(r)}{D} e^{i\omega t} w_m(r,\theta) dS$$
(B.29)

$$= \int_{S} \sum_{n=0}^{\infty} \left[\frac{\rho h}{D}\ddot{\tilde{q}}_{n}(t) + k_{p}^{4}\tilde{q}_{n}(t)\right] w_{n}(r,\theta) w_{m}(r,\theta) dS.$$
(B.30)

The orthogonality of the modal basis leads, if a harmonic excitation is considered, to

$$\frac{\rho_p h_p}{D} \ddot{\tilde{q}}_n(t) + k_p^4 \tilde{q}_n(t) = \frac{\rho_p h_p}{D} (\omega_n^2 - \omega^2) \tilde{q}(t) = \frac{\int_S \frac{\Delta P(r)}{D} e^{i\omega t} w_n(r,\theta) dS}{\int_S \sum_{n=0}^\infty [w_n(r,\theta)]^2 dS}.$$
(B.31)

The general solution to the forced vibration problem of a clamped-free annular plate in vacuum then takes the form

$$w(r,\theta) = \sum_{n=0}^{\infty} \frac{\int_S \frac{\Delta P(r)}{D} w_n(r,\theta) dS}{\frac{\rho_p h_p}{D} (\omega_n^2 - \omega^2) \int_S \sum_{n=0}^{\infty} [w_n(r,\theta)]^2 dS} w_n(r,\theta).$$
(B.32)

N.B.: This method can also be applied to solve the problem of a circular plate.

The average transverse displacement \overline{w} over the plate surface

$$\overline{w} = \frac{1}{S} \int_0^{2\pi} \int_{R_h}^{R_a} w(r,\theta) r dr d\theta,$$
(B.33)

can then lead to the acoustic impedance of the annular plate.

B.3 Vibroacoustic coupling

Since a light fluid is considered (air), a complete vibroacoustic coupling between the plate and the air is not necessary. The vibrations of the plate in the air-filled waveguide are considered to be those of plates in vacuum. Nevertheless, in the case of an annular plate, the air flow through the internal orifice should still be accounted for.

To do this, the orifice of section $S_h = \pi R_h^2$ is treated as a h_p -thick tube, for which an approximation of the acoustic impedance is given by

$$Z_{hole} = i\omega\rho_{air}h_p \frac{1}{1 - \frac{2}{\sqrt{-i}k_h} \frac{J_1(\sqrt{-i}k_h)}{J_0(\sqrt{-i}k_h)}} \frac{1}{S_h} + \frac{2i\omega\rho_{air}\Delta l}{S_h},\tag{B.34}$$

$$\approx i\rho_{air}c_{air}\tan\left(\frac{\omega}{c_{air}}(h_p+2\Delta l)\right)\frac{1}{S_h}.$$
 (B.35)

considering length corrections of the form $\Delta l = \frac{4}{3\pi} 2R_h$ at each end [10–12].

An often-used approximation for micro-perforated plates is

$$Z_{hole} = \frac{32\mu h_p}{4R_h^2} \left[\sqrt{1 + \frac{k_h^2}{32}} + \frac{\sqrt{2}}{32} k_h \frac{2R_h}{h_h} \right] + i\omega \rho_{air} h_p \left[1 + \frac{1}{\sqrt{9 + \frac{k_h^2}{2}}} + \frac{8}{3\pi} \frac{2R_h}{h_p} \right] \frac{1}{S_h}, \tag{B.36}$$

where k_h is the ratio of the radius of the hole to the viscous penetration thickness [10, 13].

The total impedance of an annular plate is then given in the 1st order by

$$Z_{tot} = \frac{1}{\frac{1}{Z_{plate}} \frac{R_t^2}{R_a^2 - R_h^2} + \frac{1}{Z_{hole}} \frac{R_t^2}{R_h^2}}.$$
(B.37)

This expression of the impedance depends on the open area ratio (orifice to plate cross-sectional ratio).

Bibliography

- A. W. Leissa. Vibration of plates. Ed. by NASA SP-160. NASA Scientific And Technical Publications. NASA SP-160 vols. 1970. 353 pp.
- [2] S. Timoshenko and S. Wornowky Krieger. Theory of Plates and Shells. Second edition. McGraw-Hill Classic Textbook Reissue Series. New-York: McGraw-Hill Book Company, 1959. 591 pp.
- [3] L. L. Faulkner, ed. Vibrations of Shells and Plates. Third edition. MECHANICAL ENGINEER-ING: A Series of Textbooks and Reference Books. New-York: Marcel Dekker, Inc., 2004. 586 pp.
- [4] R. D. Mindlin and H. Deresiewicz. "ThicknessShear and Flexural Vibrations of a Circular Disk" (1954), p. 5.
- S. M. Vogel and D. W. Skinner. "Natural Frequencies of Transversely Vibrating Uniform Annular Plates." *Journal of Applied Mechanics* 32 (1965), pp. 926–931. DOI: 10.1115/1.3627337.
- [6] H. Lee and R. Singh. "Acoustic radiation from out-of-plane modes of an annular disk using thin and thick plate theories." *Journal of Sound and Vibration* 282 (2005), pp. 313–339. DOI: 10.1016/j.jsv.2004.02.059.
- M. D. Guild et al. "Aerogel as a Soft Acoustic Metamaterial for Airborne Sound." *Phys. Rev.* Applied 5 (2016), p. 034012. DOI: 10.1103/PhysRevApplied.5.034012.
- [8] Z. Skvor. Vibrating Systems and thei Equivalent Circuits. II. Studies in electrical and electronic engineering. Amsterdam: Elsevier, 1991. 243 pp.
- S. Vishwakarma et al. "Contribution of Acoustic Losses in the Quality Factor of a Micromechanical Resonator." 20th International Congress on Sound and Vibration 2013, ICSV 2013 1 (2012), pp. 1–11.
- [10] D.-Y. Maa. "Potential of microperforated panel absorber." The Journal of the Acoustical Society of America 104 (1998), p. 2861.
- [11] I. B. Crandall. Theory of vibrating systems and sound. New York: D. Van Nostrand company, 1926, p. 229.
- [12] M. Morse and K. U. Ingard. *Theoretical Acoustics*. Princeton University Press, 1968, pp. 460–63.

[13] T. Bravo, C. Maury, and C. Pinhède. "Enhancing sound absorption and transmission through flexible multi-layer micro-perforated structures." *The Journal of the Acoustical Society of America* 134 (2013), pp. 3663–3673. DOI: 10.1121/1.4821215.

Appendix

Supplement on the Measurement Principle

A relative calibration is carried out between the microphones, taking the first one as a reference, in order to prevent from differences in absolute calibration and phase relationships. As the frequency band under study, $f \in [0 - 2000]$ Hz, is low relative to the first cut-off frequency of the tube, that is $f_c = 1.841c_0/2\pi R_a = 6700$ Hz, plane wave propagation is considered. The use of 4 microphones makes it possible to take into account the imperfection of the anechoic termination and therefore that part of the transmitted wave is reflected at this termination.

The pressure fields P_1 , P_2 (microphones M_1 and M_2) and P_3 , P_4 (M_3 and M_4), obtained from measurements of frequency responses and relative sensitivities between microphones M_2 , M_3 , M_4 and the reference microphone M_1 can be decomposed as a superposition of forward and backward waves upstream (A and B) and downstream (C and D) of the metamaterial

$$P_1 = (Ae^{-ikx_1} + Be^{ikx_1}), \tag{C.1}$$

$$P_2 = (Ae^{-ikx_2} + Be^{ikx_2}), \tag{C.2}$$

$$P_3 = (Ce^{-ikx_3} + De^{ikx_3}), \tag{C.3}$$

$$P_4 = (Ce^{-ikx_4} + De^{ikx_4}), \tag{C.4}$$

as illustrated in Fig. C.1.



Figure C.1: Experimental set-up: Sketch (a) and photography (b) of the impedance tube of inner radius $R_a = 15$ mm.

The previous system of equations can be rewritten in terms of the amplitudes

$$A = \frac{i\left(P_1 e^{ikx_2} - P_2 e^{ikx_1}\right)}{2\sin k(x_1 - x_2)} = \frac{P_1 e^{-ikx_1} - P_2 e^{-ikx_2}}{e^{-2ikx_1} - e^{-2ikx_2}},$$
(C.5)

$$B = \frac{i\left(P_2 e^{ikx_1} - P_1 e^{ikx_2}\right)}{2\sin k(x_1 - x_2)} = \frac{P_1 e^{ikx_1} - P_2 e^{ikx_2}}{e^{2ikx_1} - e^{2ikx_2}},$$
(C.6)

$$C = \frac{i\left(P_3 e^{ikx_4} - P_1 e^{ikx_3}\right)}{2\sin k(x_3 - x_4)} = \frac{P_3 e^{-ikx_3} - P_2 e^{-ikx_4}}{e^{-2ikx_3} - e^{-2ikx_4}},$$
(C.7)

$$D = \frac{i\left(P_4 e^{ikx_3} - P_3 e^{ikx_4}\right)}{2\sin k(x_3 - x_4)} = \frac{P_3 e^{ikx_3} - P_2 e^{ikx_4}}{e^{2ikx_3} - e^{2ikx_4}}.$$
 (C.8)

C.1 Scattering matrix

The scattering matrix connects, through the transmission and reflection coefficients, the incoming A, D and outgoing waves B, C from the metamaterial

$$\begin{bmatrix} B \\ C \end{bmatrix} = \begin{bmatrix} R & T \\ T & R \end{bmatrix} \begin{bmatrix} A \\ D \end{bmatrix}.$$
 (C.9)

 ${\cal R}$ and ${\cal T}$ therefore reads as

$$R = \frac{AB - CD}{A^2 - D^2},$$
 (C.10)

$$T = \frac{AC - BD}{A^2 - D^2},\tag{C.11}$$

and enables to recover the effective properties using equations (II.30).

C.2 Transfer matrix

On the other hand, both the transmission and the effective parameters can be recovered using a transfer matrix approach.

The metamaterial transfer matrix reads as

$$\begin{bmatrix} p \\ V \end{bmatrix}_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p \\ V \end{bmatrix}_{x=L},$$
 (C.12)

the elements of which can be determined using the 4 complex pressure measurements

$$T_{11} = \frac{P_{|x=L}V_{|x=L} + P_{|x=0}V_{|x=0}}{P_{|x=0}V_{|x=L} + P_{|x=L}V_{|x=0}},$$
(C.13)

$$T_{12} = \frac{P_{|x=0}^2 - P_{|x=d}^2}{P_{|x=0}V_{|x=L} + P_{|x=L}V_{|x=0}},$$
(C.14)

$$T_{21} = \frac{V_{|x=0}^2 - V_{|x=d}^2}{P_{|x=0}V_{|x=L} + P_{|x=L}V_{|x=0}},$$
(C.15)

$$T_{22} = \frac{P_{|x=L}V_{|x=L} + P_{|x=0}V_{|x=0}}{P_{|x=0}V_{|x=L} + P_{|x=L}V_{|x=0}}.$$
(C.16)

The state vector \mathbf{w} , formed by the pressure P and the normal velocity V, can be written as follows

$$P_{|x=0} = A + B = 1 + R, \tag{C.17}$$

$$V_{|x=0} = \frac{A-B}{Z_0} = \frac{1-R}{Z_0},$$
(C.18)

$$P_{|x=L} = Ce^{-ikL} + De^{ikL} \approx Te^{-ikL}, \tag{C.19}$$

$$V_{|x=L} = \frac{Ce^{-ikL} - De^{ikL}}{Z_0} \approx \frac{Te^{-ikL}}{Z_0},$$
 (C.20)

with R = B/A the reflection coefficient at x = 0 and T = C/A the transmission coefficient at x = L, if, due to the presence of the anechoic termination, D is neglected.

Appendix D

Doping movies

This appendix shows movies of a 2D acoustic doping from the full-wave simulation.

A two-port random 2D medium is considered. The input and output ports are air-filled waveguides, plugged into the "L" and "M" shapes respectively. A monochromatic wave of frequency lower than the cut-off frequency of the ports impinges the structure from the left ensuring plane wave propagation.

The "LAUM"-shaped medium is filled with air (upper left quadrant), with a DNZ medium (upper right quadrant), with a doped DNZ medium (lower left quadrant) or with a DCNZ medium (lower right quadrant). In the lower left quadrant, the thickness of the "A" tranverse bar is chosen so that doping occurs. The host DNZ medium switches to an effective DCNZ medium when $C \approx 0$, thus for $\kappa \to \infty$ when the "A" bar have a thickness H = 10.1 cm.

Appendix

Extended abstract (in French)

Introduction

Les métamatériaux ont suscité beaucoup d'intérêt au cours des dernières décennies, dans les communautés de l'électromagnétisme, des milieux élastiques et de l'acoustique en raison de la possibilité qu'ils offrent de contrôler la dispersion des ondes. Ainsi certains comportements jusqu'alors inattendus ont été rendus réalisables. Une multitude d'applications ont été motivées par ces nouvelles façons de contrôler toujours plus le son. Citons entre autres les absorbeurs parfaits, les systèmes topologiques, les diodes acoustiques, les dispositifs de contrôle des fronts d'onde ou les systèmes d'occultation.

Ce travail de doctorat se concentre sur la propagation des ondes acoustiques dans des milieux à indice de réfraction nul, dans lesquels au moins un des paramètres constitutifs effectifs s'annule. En particulier, les métamatériaux constitués d'un arrangement périodique de fines plaques élastiques encastrées sont étudiés et une attention particulière est portée sur l'impact des pertes sur les applications qui en résultent.

Modélisation du métamatériau

On considère un arrangement périodique de N fines plaques élastiques encastrées dans un guide d'onde circulaire de section transversale $S = \pi R_a^2$, similaire à celui esquissé à la Figure E.1. La cellule unitaire du métamatériau est définie symétrique et est composée d'une fine plaque d'épaisseur h_p entourée de deux couches d'air de longueur $L_{qap/2}$, ce qui donne une périodicité $L_{unit} = L_{gap} + h_p$ au système.

Le métamatériau est modélisé par la méthode des matrices de transferts, liant pression et débit de part et d'autre de chaque élément du système. Nous ne considérons donc que la propagation des ondes planes et négligeons le couplage mutuel entre les éléments.

Une étude paramétrique sur l'influence des différents paramètres du métamatériau (géométrie, périodicidité, ...) et la caractérisation acoustique des propriétés mécaniques de différentes plaques (épaisseurs, matériaux) nous a permis de dimensionner et fabriquer une structure périodique perme-



Figure E.1: **Vue schématique du métamatériau :** système complet (a), zoom sur la cellule unitaire avec vue compacte (b) ou éclatée (c).

ttant l'étude expérimentale de la dispersion et des propriétés liées aux régimes d'index de réfraction nul.

Comme énoncé précédemment, une attention particulière est portée sur l'influence des pertes trop souvent négligées et pouvant pourtant complètement détruire les comportements intéressants liés à la dispersion dans les métamatériaux. Deux sources principales de dissipation ont été identifiées. D'une part les pertes viscothermiques liés aux frottements à proximité des parois du guide, et d'autre part les pertes viscoélastiques inhérentes aux plaques. L'influence de ce dernier type de pertes est prédominant et engendre une forte diminution de la transmission. C'est donc un des paramètres essentiels du dimensionnement de la structure. Le choix de plaques très fines, $h_p = 102 \ \mu m$, et d'un nombre limité de cellule unitaire ($N \leq 6$) permet de contrôler cette chute de la transmission et d'observer expérimentalement les phénomènes prédits.

Dispersion dans un arrangement périodique de plaques encastrées

La forte dispersion produite autour de la résonance dans ce métamatériau (Figs. E.2(a-b)), affecte principalement sa densité dynamique effective $\rho(\omega)$ qui varie de valeurs négatives à positives en fonction de la fréquence (Fig. E.2(c)). Une bande interdite d'hybridation, correspondant à un régime de densité effective négative (aplat rouge), est ouverte en dessous de la résonance, tandis que la densité devient positive dans la bande passante. Un régime de densité quasi-nulle est donc observé à la transition entre la bande interdite et la bande passante, c'est-à-dire à une fréquence proche de la résonance de la plaque.

Ce régime de densité quasi-nulle s'accompagne d'une vitesse de phase élevée, tendant vers l'infini dans le cas sans pertes (Fig. E.2(f)). En conséquence, la longueur d'onde effective est étirée et devient très grande par rapport au système. Le champ de pression acoustique peut donc présenter une distribution quasi-statique permettant une propagation sans changement de phase. L'allongement important de la longueur d'onde effective permet également des applications de dissimulation ou masquage acoustique (c'est-à-dire rendant imperceptible des obstacles placés dans le métamatériau), les dimensions de l'objet à dissimuler devenant faibles par rapport à la longueur d'onde acoustique.

Nous avons donc cherché à comprendre, étudier analytiquement et numériquement et à observer



Figure E.2: **Plaque seule (a) :** amplitude de la transmission (noir) et de la réflexion (rouge) **Relation de dispersion et paramètres effectifs d'un arrangement infini de plaque (b-f) :** partie réelle (b) et partie imaginaire (c) de la relation de dispersion, partie réelle de la densité effective (d) et du bulk modulus effectif (e), vitesse de phase (noire) et de groupe (rouge) (f). Les zones de couleur rouge représentent les bandes d'arrêt.

expérimentalement ces comportements.

Propagation sans changement de phase

Trois fréquences pertinentes peuvent être identifiées pour l'analyse du régime de densité quasi-nulle : la fréquence d'adaptation d'impédance f_m , la fréquence à laquelle la densité s'annule exactement $f_{\rho=0}$ et la fréquence de propagation sans changement de phase $f_{\phi=0}$.

La Figure E.3(a) montre la densité dynamique effective du système $\rho(\omega) = SZ(\omega)k(\omega)/\omega$, obtenue à partir du nombre d'onde effectif $k(\omega)$ et de l'impédance effective $Z(\omega)$ calculés sur une seule cellule unitaire, c'est-à-dire une structure infiniment périodique. L'évolution de la densité est étudiée parallèlement avec les amplitudes de scattering (Fig. E.3(b)) et la phase du coefficient de transmission (Fig. E.3(c)) d'un métamatériau fini.

Premièrement, l'accord d'impédance est atteint à la fréquence de résonance d'une plaque seule, $f_m = 423$ Hz dans le cas sans pertes. À cette fréquence, la densité effective du métamatériau est égale à celle du fluide environnant, ici l'air, ce qui conduit à une transmission parfaite et à une réflexion nulle (Fig. E.3(b)). La transmission unitaire s'accompagne d'un déphasage lié au nombre de cellules unitaires et donc à la longueur $L = NL_{unit}$ du métamatériau. En d'autres termes, la phase du coefficient de transmission est égale à celle produite dans une cavité remplie d'air de même longueur à f_m .

La deuxième fréquence importante pour l'étude de ce régime est celle à laquelle la densité effective dynamique s'annule, $f_{\rho=0}$. À cette fréquence, on s'attend à une propagation sans changement de phase et à un champ de pression quasi-statique dans le métamatériau. L'impédance effective du metamatériau n'est cependant pas accordée à celle du milieu environnant. Par conséquent, l'amplitude du coefficient de transmission n'est pas unitaire. De plus, bien qu'un déphasage plus faible soit trouvé à $f_{\rho=0}$ qu'à la fréquence d'adaptation d'impédance f_m , la propagation sans changement de phase n'est toujours pas atteinte dans le système. Plus important encore, tant le module que la phase du coefficient de transmission dépendent du nombre de cellules unitaires constituant le métamatériau.

La dernière fréquence qui nous intéresse est la fréquence de propagation sans changement de phase



Figure E.3: Densité effective et paramètres de scattering dans le cas sans perte : (a) partie réelle de la densité dynamique effective, (b) et (c) représentent l'amplitude des coefficients de transmission [noir, axe vertical gauche de (b)] et de réflexion [rouge, axe vertical droit de (b)] ainsi que la phase normalisée du coefficient de transmission pour un système fini composé de N = 1, 3, 6 et 9 plaques respectivement. Les lignes verticales bleues, grises et vertes sur les figures (a), (b) et (c) représentent les fréquences de propagation sans changement de phase, $f_{\phi=0}$, de densité effective nulle, $f_{\rho=0}$, et d'adaptation d'impédance, f_m , respectivement. La zone grisée délimite la bande d'arrêt d'un système infini.

Densité effective et paramètres de scattering mesurés (avec perte) : (a) amplitude des coefficients de transmission [axe vertical gauche] et de réflexion [axe vertical droit], (b) phase normalisée du coefficient de transmission [noir, axe vertical gauche] et partie réelle de la densité effective [rouge, axe vertical droit]. Les symboles représentent les données expérimentales, les lignes représentent les résultats analytiques (TMM) et les lignes pointillées les simulations numériques (FEM).

 $f_{\phi=0}$, c'est-à-dire la fréquence à laquelle la phase du coefficient de transmission s'annule. À cette fréquence, la densité effective est négative et égale à $\rho(f_{\phi=0}) = -\rho_0 \kappa_0 / \kappa(f_{\phi=0})$. Le régime de densité effective négatif (zones grisées dans la Fig. E.3) correspond à une bande d'arrêt pour un système infini. Bien que $f_{\phi=0}$ se situe dans la plage de densité négative, la transmission reste considérable en raison du nombre restreint de cellules unitaires du métamatériau considéré. Le module du coefficient de transmission (resp. de réflexion) dépend fortement du nombre de cellules unitaires, contrairement à sa phase, qui reste constante et égale à 0. Lorsque les pertes ne sont pas considérées, il existe donc une fréquence, proche de la fréquence de densité nulle, pour laquelle une onde peut se propager sans changement de phase et quelle que soit la longueur du système.

Le nombre de plaque influant fortement l'amplitude de la transmission à la fréquence de propagation sans changement de phase, un arrangement de 6 plaques est utilisé pour la validation expérimentale. La Figure E.3(d) représente l'amplitude mesurée des coefficients de transmission et de réflexion du système fini et la Figure E.3(e) montre la phase du coefficient de transmission ainsi que la partie réelle de la densité effective. Une très bonne concordance est observée entre les résultats analytiques et numériques qui suivent bien la tendance du module et de la phase du coefficient de transmission mesuré (E.3(e,f), symboles noirs). Des différences sont cependant visibles sur le coefficient de réflexion, et par conséquent sur la densité effective (qui est estimée à partir des deux coefficients de scattering). Une légère variabilité résiduelle entre les différentes cellules unitaires, due soit à des défauts de serrage, soit à la variabilité intrinsèque des propriétés mécaniques du matériau, est responsable de ces écarts.

Une fois les pertes considérées, on observe une faible dépendance de la fréquence de phase nulle, $f_{\phi=0}$, et de la phase du coefficient de transmission ϕ , en fonction du nombre de cellules unitaires et de la quantité de pertes. Dans les deux cas, la variation est inférieure à 10%.

Un maximum de transmission est observé à $f_m = 439$ Hz du fait de la condition de quasi-adaptation d'impédance autour de la résonance de la plaque. Une propagation sans changement de phase est mesurée à $f_{\phi=0} = 389$ Hz, en accord avec les prédictions, confirmant ainsi expérimentalement la faisabilité d'une propagation sans changement de phase dans ce type de métamatériau. Cette propagation sans changement de phase peut être utilisée pour adapter la directivité d'une source et contrôler le front d'onde émergeant d'une structure. En particulier, nous montrons numériquement que notre metamatériau placé dans un guide d'onde 1D, peut-être utilisé pour créer une source acoustique dipolaire en utilisant la propagation sans changement de phase.

Dopage acoustique

Un arrangement périodique de plaque rend donc possible la propagation sans changement de phase. Cependant, comme nous l'avons vu précédemment, cette propriété aussi intéressante soit elle, peut fortement être limitée par un désaccord d'impédance entraînant une chute de la transmission même lorsqu'aucune source de dissipation n'est considérée.

Les métamatériaux double zéro peuvent permettre de surmonter cette difficulté en cela qu'ils ont la particularité de concilier à la fois les propriétés exotiques des milieux à densité quasi-nulle et l'adaptation d'impédance, ce qui permet d'alléger les contraintes pour des applications réalistes. Il est donc très intéressant de trouver un moyen de transformer notre métamatariau pour lequel seul la densité est quasi-nulle, en un milieu où à la fois la densité et la compressibilité sont proches de zéro simultanément.

En examinant ce qui est développé en électromagnétisme, nous proposons un analogue acoustique du dopage photonique qui consiste en l'ajout d'une seule impureté à un milieu où seul l'un des paramètres dynamiques constitutifs est proche de zéro pour le convertir en un milieu double zéro.

Dans le cas présent, nous proposons d'ajouter un résonateur de Helmholtz comme élément dopant afin de modifier la compressibilité effective du milieu et la faire tendre vers zéro à la fréquence de densité nulle. Le résonateur de Helmholtz est équipé d'un piston permettant de régler la longueur de cavité L_c , les autres dimensions restant constantes.

Nous analysons d'abord le cas d'un métamatériau constitué de 20 cellules unitaires sans pertes illustré à la Figure E.4, sur lequel un résonateur de Helmholtz est monté entre les 10ème et 11ème

plaques. Afin de trouver la configuration dans laquelle le dopage se produit, nous recherchons la géométrie optimale du résonateur de Helmholtz qui produit une valeur maximale de bulk modulus effectif. Cette configuration optimale correspond à une longueur $L_c = 32,06$ mm et nécessite d'être dimensionnée avec une extrême précision. Les Figures E.4(b,c,e,f) montrent respectivement l'amplitude des coefficients de transmission et de réflexion, la densité dynamique effective, la phase du coefficient de transmission et le bulk modulus effectif pour la configuration mentionnée ci-dessus. Les résultats analytiques sont validés par une simulation éléments-finis 3D illustrée par les symboles carrés de la Figure E.4.



Figure E.4: Dopage d'un arrangement de 20 plaques sans perte avec dopant : un résonateur de Helmholtz: Croquis 3D de la cellule unitaire, du métamatériau composé des 20 plaques et dopé en son milieu par un résonateur de Helmholtz, et zoom sur le résonateur (a). Amplitudes de transmission (noir) et de réflexion (rouge) (b), partie réelle de la densité effective dynamique (c), phase du coefficient de transmission (e), et partie réelle du bulk modulus effectif du système en fonction de la fréquence (f). Les lignes continues et les symboles carrés représentent respectivement les résultats analytiques et numériques. L'encadré (d) montre la dépendance du bulk modulus effectif par rapport à la longueur de la cavité du résonateur de Helmholtz L_c . Les Figures (g) et (h) représentent le champ de pression total d'un métamatériau composé de 20 cellules unitaires sans et avec dopant à la fréquence de propagation sans changement de phase f = 405 Hz et f = 414 Hz respectivement (simulation éléments-finis).

Une densité nulle accompagnée d'un module effectif maximum est trouvée à f = 414 Hz. À cette fréquence particulière, l'ensemble du système se comporte comme un métamatériau double zero, comme le montre l'amplitude des coefficients de transmission et réflexion. L'annulation de la phase du coefficient de transmission est accompagnée d'une absence de réflexion et d'une transmission totale, c'est-à-dire que la propagation sans changement de phase et l'adaptation d'impédance sont combinées.

Les Figures. E.4(g,h) montrent le champ de pression total, respectivement sans et avec l'élément dopant, pour illustrer l'effet de ce dernier sur le champ à la fréquence de propagation sans changement de phase. Dans les deux cas, on observe une distribution quasi-statique du champ, donnant lieu à une propagation sans changement de phase, soit avec un désaccord d'impédance, soit avec une transmission totale. Dans ce dernier cas, le champ de pression est parfaitement symétrique par rapport au metamateriau, ce qui est caractéristique de l'effet de supercouplage, c'est-à-dire une transmission totale, une densité nulle, et une propagation sans changement de phase simultanées.

L'extension de la longueur d'onde effective engendrée permet aussi de déplacer l'élément dopant le long du métamatériau sans influence sur l'efficacité du dopage, offrant ainsi une grande liberté dans le design du métamatériau.

Le nombre de plaques est réduit à 6 pour l'étude du cas réaliste (avec pertes) afin de maîtriser les pertes viscoélastiques. Un résonateur de Helmholtz (avec une longueur de cavité optimale $L_c = 36,75$ mm) est monté entre les 3ème et 4ème plaques du système. Ce faisant, la fréquence de phase nulle est décalée à $f_{\phi=0} = 412$ Hz et se rapproche de la fréquence de densité nulle du système, c'est-à-dire $f_{\rho=0} = 414$ Hz.

La conception minutieuse du dopant permet donc de réduire fortement le décalage de fréquence entre les fréquences de phase nulle, de maximum de transmission et de densité nule. En conséquence, la fréquence de propagation sans changement de phase $f_{\phi=0}$ se rapproche de f_m . Contrairement au cas sans pertes, la dissipation empêche une coïncidence parfaite des fréquences de transmission maximale et de propagation sans changement de phase. Ainsi, les pertes qui sont intrinsèquement présentes dans tout système acoustique peuvent clairement limiter l'efficacité du dopage. Néanmoins, il convient de noter ici que la condition de dopage permet d'avoir une propagation sans changement de phase avec une transmission supérieure de 13%, avec une amplitude mesurée (resp. analytique et numérique) passant de 0,42 (resp. 0,47) sans dopant à 0,59 (resp. 0,53) dans la configuration dopée. La réduction des pertes (plaques avec des pertes viscoélastiques plus faibles) permettrait d'améliorer l'efficacité du processus.

Masquage et dissimulation acoustique

À la lumière des propriétés mises en évidence précédemment, nous évaluons finalement la possibilité de dissimuler des obstacles (ici un diaphragme rigide) en utilisant l'étirement de la longueur d'onde offert par le régime de densité quasi-nulle et l'adaptation d'impédance offerte par le dopage du métamatériau. Deux stratégies sont étudiées.

Premièrement, nous utilisons la propre impédance du diaphragme à dissimuler pour réaliser le dopage (Figs. E.5(a-b,e-f)) et deuxièmement nous ajoutons une autre impureté dans le système, un résonateur de Helmholtz (Figs. E.5(c-d,g-h)). Nous démontrons analytiquement et numériquement qu'en l'absence de pertes, le diaphragme peut être dissimulé avec les deux stratégies (avec ou sans dopant supplémentaire). Le diaphragme est alors acoustiquement imperceptible pour un observateur



extérieur, tant en termes d'amplitude que de phase du coefficient de transmission.

Figure E.5: Dissimulation acoustique d'un diaphragme sans pertes (a,c) et avec pertes (b,d) : Champ de pression (FEM), amplitudes de transmission (noir) et de réflexion (rouge) (a1,b1,c1,d1) et phase du coefficient de transmission (noir, axe gauche) et partie réelle de la densité effective normalisée (rouge, axe droit) des deux systèmes (a2,b2,c2,d2). Les figures (a,b) montrent l'occultation d'un diaphragme $R_d = 0,5$ mm dans un métamatériau de 6 unités tandis que les figures (c,d) montrent l'occultation d'un diaphragme $R_d = 4$ mm dans un métamatériau composé de plaques et dopé avec un résonateur de Helmholtz ($R_n = 2$ mm, $L_n = 2$ cm, $R_c = 1$ cm, et $L_c = 40,95$ cm). La ligne continue représente les résultats analytiques et la ligne en pointillés ceux des simulations éléments-finis.

Cependant nous montrons aussi que la limitation du phénomène de dopage induite par les pertes viscoélastiques impacte radicalement l'efficacité d'occultation du dispositif, la transmission n'étant plus unitaire, et ce quel que soit la stratégie d'occultation envisagée.

Bien que le métamatériau puisse conduire à une occultation totale dans le cas sans pertes, nous avons prouvé que les pertes du métamatériau avec et sans dopage empêchent la propagation unitaire requise. Compte tenu de cette forte limitation liée intrinsèquement au métamatériau étudié et de l'impossibilité d'occulter (au sens de cloaking) l'obstacle, une autre application est maintenant étudiée : le phénomène de dissimulation ou de masquage. La philosophie du masquage diffère de celle de l'occultation en ce sens qu'elle ne vise pas à rendre un champ extérieur inchangé par la présence à la fois de l'obstacle et du dispositif d'occultation. Le but est ici d'utiliser l'étirement de la longueur d'onde effective dans le métamatériau pour cacher le diaphragme (ou tout autre obstacle) à l'intérieur, sans perturber la signature acoustique du métamatériau lui-même, c'est-à-dire en maintenant constante l'amplitude de réflexion et de transmission à la fréquence de phase nulle.

La présence du métamatériau permet de diminuer et même d'annuler l'effet de diffraction induit par le diaphragme, rendant ainsi la propagation sans changement de phase atteignable par le système complet (métamatériau et diaphragme). La fréquence de phase nulle du système total $f_{\phi_t=0}$ dépend directement de l'impédance de l'obstacle, liée ici au ratio de surface ouverte R_d/R_a . Comme le montre la Figure E.6(a), plus le rayon de l'orifice est petit, et donc plus l'impédance du diaphragme est grande, plus la fréquence de phase nulle du système est faible. La fréquence de phase nulle varie de $f_{\phi=0} = 390$ Hz pour le métamatériau seul à $f_{\phi_t=0} = 373$ Hz pour le système avec un diaphragme de rayon $R_d = 4$ mm et à $f_{\phi_t=0} = 303$ Hz pour un diaphragme de rayon $R_d = 1$ mm.



Figure E.6: Influence du rayon de l'orifice du diaphragme sur ses paramètres de scattering (calculés analytiquement) : Variation de la fréquence de phase nulle (a), évolution de l'amplitude des paramètres de scattering : transmission (noir, axe gauche) et réflexion (rouge, axe droit) à la fréquence de phase nulle du système global $f_{\phi_t=0}$ (b) et phase du coefficient de transmission du métamatériau seul à la fréquence de phase nulle du système global $f_{\phi_t=0}$ pour différents rayons du diaphragme (c). Les zones grisées montrent la plage de rayons d'orifice pour lesquels l'amplitude des paramètres de scattering est indépendante du rayon de l'orifice.

La Figure E.6(b) montre que l'amplitude des coefficients de transmission et reflexion à $f_{\phi_t=0}$ reste constante et égale à celle du métamatériau seul à $f_{\phi=0}$ tant que le rapport orifice/section totale reste supérieur à 12% (surface grisée). En raison de la légère modification de la fréquence de phase nulle dans cette plage de fréquence, un léger déphasage est perceptible pour le métamatériau seul à la fréquence de propagation sans changement de phase de l'ensemble du système $f_{\phi_t=0}$ (Fig. E.6(c)). Au-delà de ce ratio de 12%, l'impédance du diaphragme devient trop importante. La fréquence de phase nulle de l'ensemble du système est donc décalée vers les basses fréquences de manière significative. Le rapport de section de 12% correspond à une variation de $f_{\phi_t=0}$ de 10% par rapport à la fréquence de phase nulle du métamatériau seul $f_{\phi=0}$. En raison de ce décalage de fréquence, la puissance du régime de densité quasi-nulle du métamatériau n'est plus suffisante pour permettre le phénomène de masquage souhaité. L'amplitude des paramètres de scattering varie alors considérablement avec la variation de $f_{\phi_t=0}$.

Cependant, il est intéressant de noter que le masquage d'un diaphragme (ou de tout autre obstacle) est possible avec un métamatériau à densité quasi-nulle même en présence de pertes, dès que l'impédance de l'objet à dissimuler est bien contrôlée. Avec cet arrangement périodique de plaques, malgré la faible variation de la fréquence de la phase nulle (10% dans la plage de fonctionnement) qui entraîne un faible changement de phase, l'effet du diaphragme est très limité. Il est alors possible de cacher la diaphragme dans le métamatériau dans cette gamme de fréquence même en présence de pertes.

De plus, l'extension de la longueur d'onde effective dans le métamatériau permet d'étendre le phénomène de masquage à l'ensemble du métamatériau. Le masquage a lieu indépendamment de

la localisation de l'obstacle. La Figure E.7 montre l'évolution de l'amplitude des coefficients de transmission et réflexion et de la fréquence de phase nulle pour différents emplacements du diaphragme le long du métamatériau et à son voisinage.



Figure E.7: **Zone de masquage d'un diaphragme de 4 mm de rayon dans le PAM:** avec (a-c) et sans pertes (d-f). Croquis du système considéré (a,d), amplitude des paramètres de scattering (transmission, noir, axe gauche et réflexion, axe droit rouge) à la fréquence de phase nulle du système (b,e) et valeur de la fréquence de phase nulle du système (c,f). Les cercles, les points et les lignes pointillées horizontales représentent respectivement les données expérimentales et analytiques pour le système total et l'amplitude expérimentale des paramètres de scattering du métamatériau seul à sa fréquence de phase nulle.

Dans le cas sans pertes (Figs. E.7(d-f)), quel que soit l'emplacement du diaphragme de 4 mm de rayon le long du métamatériau, les amplitudes de transmission et de réflexion restent constantes et égales à celles du métamatériau seul. Le phénomène de dissimulation couvre l'ensemble de la structure qui agit comme un matériau homogène et quasiment symétrique ($R^+ \approx R^- \approx R$). Dès que l'on considère les pertes viscothermiques et viscoélastiques (Fig. E.7(a-c)), l'asymétrie du système devient beaucoup plus visible, $R^+ \neq R^-$, ce qui entraîne une légère modification de la réflexion avec l'emplacement du diaphragme. L'accord entre les résultats expérimentaux, représentés par les cercles, et les prédictions analytiques, représentées par les points solides, est très bonne sur la transmission et la fréquence de propagation sans changement de phase. Des différences plus significatives sont cependant perceptibles sur la réflexion, déjà notée dans le cas du métamatériau seul. Cependant, nous constatons que quelle que soit la position du diaphragme, les données expérimentales des amplitudes de réflexion et transmission de l'ensemble du système sont égales à celles du métamatériau sans diaphragme, dont les valeurs mesurées sont rappelées par les lignes horizontales en pointillés de la Figure E.6(a).

D'après ces résultats il est donc possible de définir une zone de masquage correspondant à l'ensemble du métamatériau. Le diaphragme peut être dissimulé peu importe sa position, à condition de contrôler les pertes et l'impédance de la zone de dissimulation.

Conclusions

En conclusion, ce travail est consacré au régime d'indice de réfraction quasi-nul dans un métamatériau constitué d'un arrangement périodique de plaques fines encastrées dans un guide d'onde 1D, et aux applications sous-jacentes offertes par l'étirement de la longueur d'onde effective caractéristique de ces milieux.

Par le biais de calculs analytiques, de simulations numériques et d'expérimentations, nous avons d'abord conçu un métamatériau mettant en évidence certaines des caractéristiques spécifiques liées à cet étirement. En particulier, nous avons étudié et mesuré trois phénomènes principaux : (i) la propagation sans changement de phase, (ii) le dopage acoustique d'un métamatériau permettant de répondre aux exigences du supercouplage, et (iii) l'efficacité d'un arrangement de plaques à masquer ou à cacher un obstacle. Tout au long de ce travail, nous avons accordé une grande attention à l'étude et à la démonstration de la limitation induite par les pertes inhérentes au système, trop souvent négligées dans de nombreuses études alors qu'elles peuvent avoir des effets drastiques si elles ne sont pas correctement prises en compte comme un élément clé de la conception.

En parallèle de ce travail sur le régime de densité nulle, nous avons aussi exploré différents sujets tels que l'absorption large bande à partir de plaques annulaires, le couplage de Willis dans différents systèmes asymétriques composés de plaques ou de résonateurs de Helmholtz, le contrôle de la diffusion et de l'absorption avec des métamatériaux reconfigurables,... Ces derniers ont un grand potentiel car ils permettent de passer d'une application à une autre en modifiant simplement la géométrie des systèmes. Nous nous sommes particulièrement intéressés ici à une application pour le traitement acoustique des salles en utilisant des résonateurs de Helmholtz.

DOCTORAT BRETAGNE SCIENCES LOIRE POUR L'INGENIEUR



Titre : Métamatériaux avec propriétés extraordinaires pour le contrôle des ondes acoustiques

Mots clés : Arrangement périodique de plaques, Matériau à index de réfraction nul, Propagation sans changement de phase, Densité nulle, Dopage, Dissimulation acoustique

Résumé : Les métamatériaux à indice nul, pour lesquels au moins un des paramètres effectifs s'annule (densité ou compressibilité dynamique pour l'acoustique), ont fait l'objet d'une attention considérable au cours de ces dernières années. Ces matériaux ont la particularité d'induire une augmentation remarquable de la longueur d'onde effective, offrant ainsi de nombreuses possibilités d'application, incluant entre autres la propagation sans changement de phase, la dissimulation acoustique de diffuseurs, le contrôle de la directivité, etc. Ce travail de doctorat se concentre particulièrement sur le régime de effective densité quasi-nulle dans des métamatériaux acoustiques constitués de plaques fines dans l'air.

Grâce à une étude approfondie d'un arrangement périodique de fines plaques élastiques encastrées dans un guide d'onde, nous avons pu explorer certains des effets cidessus analytiquement, numériquement et expérimentalement. Une attention particulière est portée sur les pertes inhérentes à ce type de système et à leurs conséquences sur les comportements attendus.

Nous débutons par l'étude numérique et l'observation expérimentale d'une propagation sans changement de phase à travers le métamatériau, à une fréquence située dans une bande interdite du système fini. Nous transposons ensuite le concept de dopage photonique à l'acoustique. L'ajout dans le système d'une impureté, ici un résonateur de Helmholtz bien choisi, permet de transformer le régime de densité nulle en un régime où la densité et la compressibilité sont simultanément quasi-nulles. Ainsi, la propagation sans changement de phase est accompagnée d'une transmission unitaire, due à l'accord d'impédance du système avec l'air environnant. Nous étudions enfin la possibilité de réaliser une dissimulation ou un masquage acoustique d'un objet en utilisant l'extension de la longueur d'onde acoustique, offerte par la densité nulle.

Title: Metamaterials with extraordinary properties for the control of acoustic waves

Keywords: Plate-type Acoustic Metamaterials, Zero-index medium, Zero-phase propagation, Density Near Zero, Doping, Hiding

Abstract: Zero-index metamaterials, for which at least one of the effective parameters (density or dynamic compressibility for acoustics) vanishes, have received considerable attention in recent years. These materials have the particularity of inducing a considerable increase in the effective wavelength, thus offering numerous application possibilities, including, among others, propagation without phase change, acoustic hiding of diffusers, directivity control, etc. This PhD work focuses particularly on the near-zero effective density regime in acoustic metamaterials made of thin plates in air.

Through an in-depth study of a periodic arrangement of thin elastic plates embedded in a waveguide, we have been able to explore analytically, numerically and experimentally some of the above effects.

Particular attention is paid to the losses inherent to this type of system and their consequences on the expected behavior.

We begin by studying numerically and experimentally observing a phase-change-free propagation through the metamaterial at a frequency in a stopband of the finite system. We then transpose the concept of photonic doping to acoustics. The addition of an impurity, here a wellchosen Helmholtz resonator, to the system allows to transform the regime of zero density into one compressibility where density and are simultaneously near zero. Thus, propagation without phase change is accompanied by a unitary transmission, due to the impedance matching of the system with the surrounding air.

Finally, we study the possibility of performing acoustic hiding or masking of an object using the acoustic wavelength stretching offered by the zero density.